Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.3-Tangent/218-4.3.3.1

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**855**]. This is test number [218].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (855)	0.00 (0)
Mathematica	95.79 (819)	4.21 (36)
Fricas	91.58 (783)	8.42 (72)
Maple	90.88 (777)	9.12 (78)
Mupad	61.87 (529)	38.13 (326)
Maxima	50.06 (428)	49.94 (427)
Giac	36.37 (311)	63.63 (544)
Sympy	24.44 (209)	75.56 (646)
Reduce	17.43 (149)	82.57 (706)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

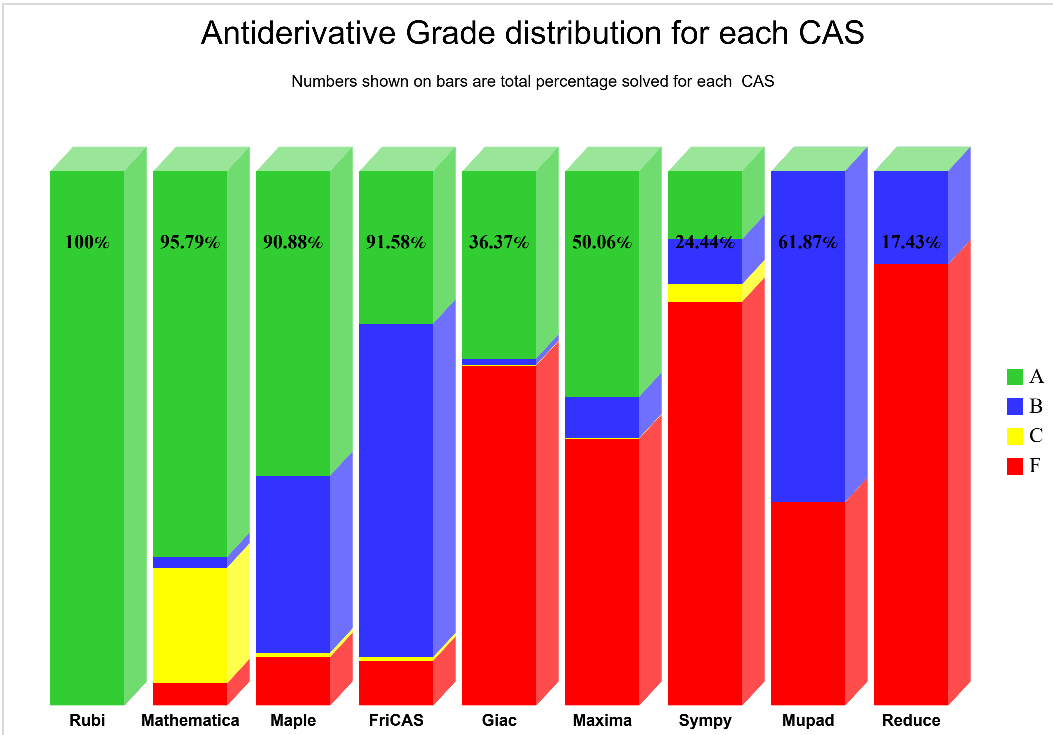
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

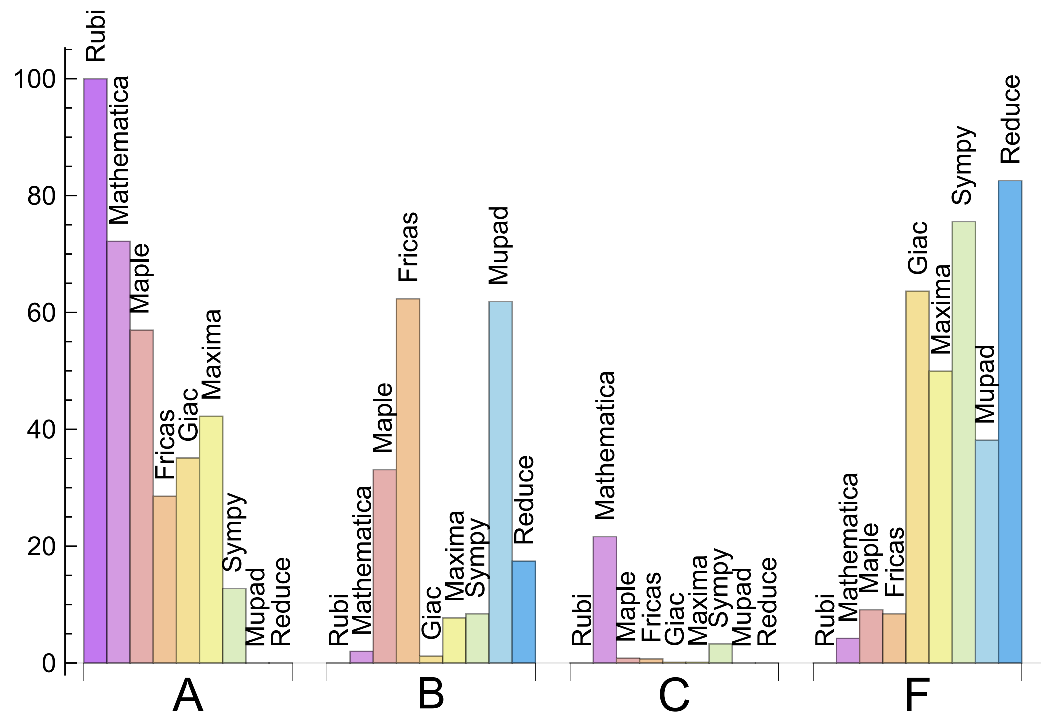
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	72.164	1.988	21.637	4.211
Maple	56.959	33.099	0.819	9.123
Maxima	42.222	7.719	0.117	49.942
Giac	35.088	1.170	0.117	63.626
Fricas	28.538	62.339	0.702	8.421
Sympy	12.749	8.421	3.275	75.556
Mupad	0.000	61.871	0.000	38.129
Reduce	0.000	17.427	0.000	82.573

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	36	100.00	0.00	0.00
Fricas	72	100.00	0.00	0.00
Maple	78	92.31	7.69	0.00
Mupad	326	0.00	100.00	0.00
Maxima	427	41.69	13.58	44.73
Giac	544	27.39	8.09	64.52
Sympy	646	69.04	27.55	3.41
Reduce	706	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Giac	0.44
Maple	0.60
Reduce	0.85
Sympy	0.86
Rubi	1.11
Mathematica	3.21
Fricas	3.59
Mupad	8.48

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Giac	146.86	1.06	117.00	0.92
Reduce	187.65	1.45	127.00	1.18
Rubi	199.12	1.02	189.00	1.03
Maxima	324.83	1.97	185.50	1.11
Mathematica	351.32	1.48	158.00	0.94
Sympy	509.15	3.99	258.00	2.06
Mupad	3191.36	14.78	221.00	1.39
Fricas	3233.19	12.94	488.00	2.75
Maple	218641.93	871.57	223.00	1.15

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

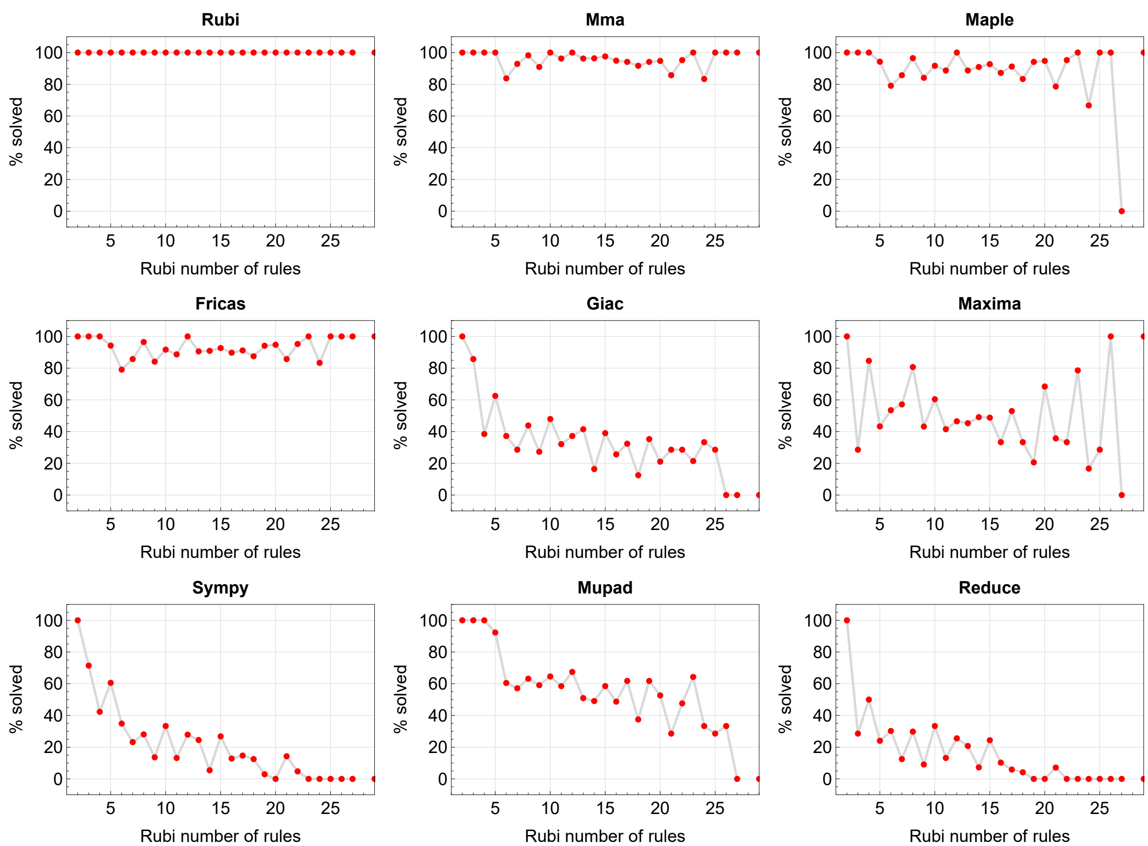


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

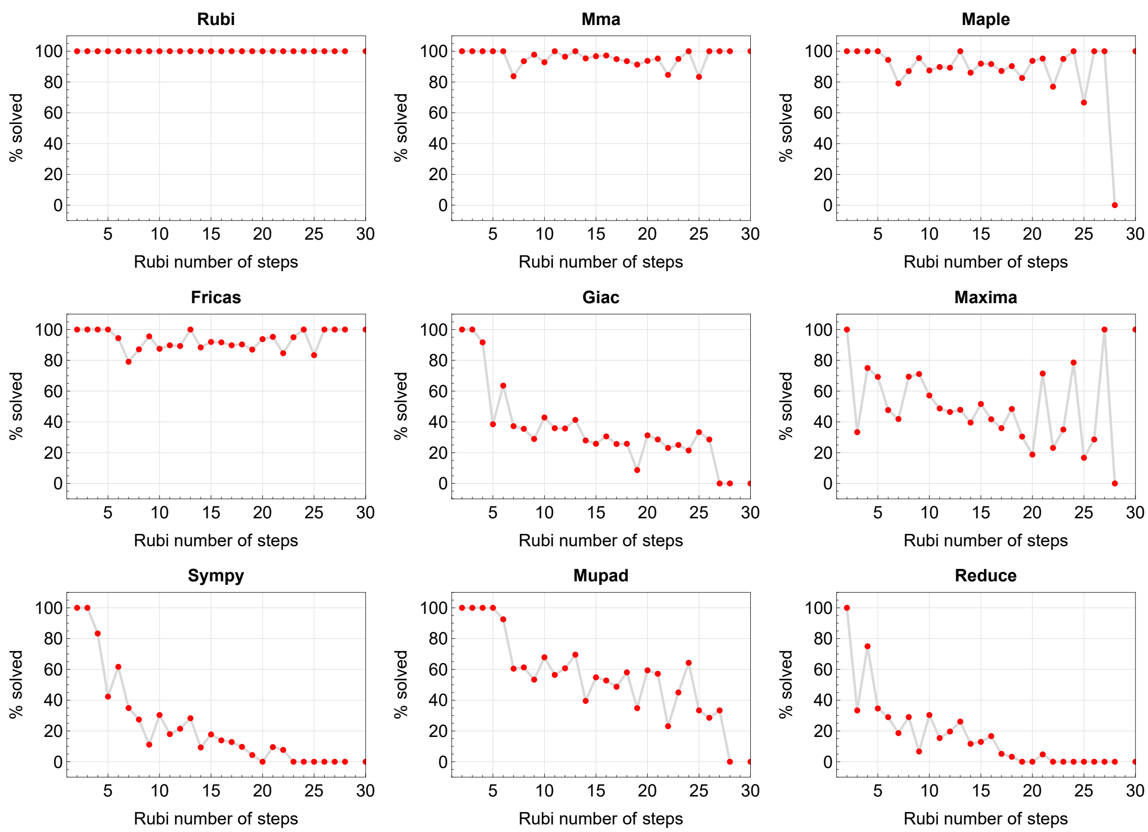


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

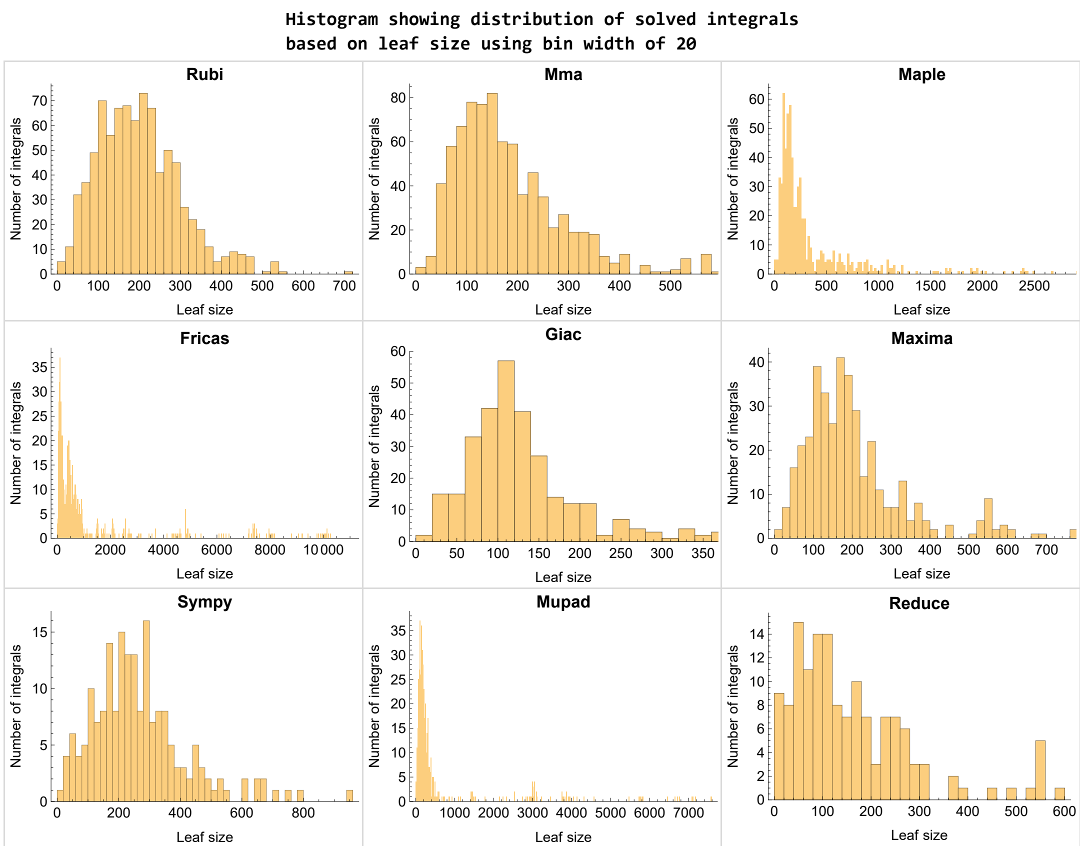


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

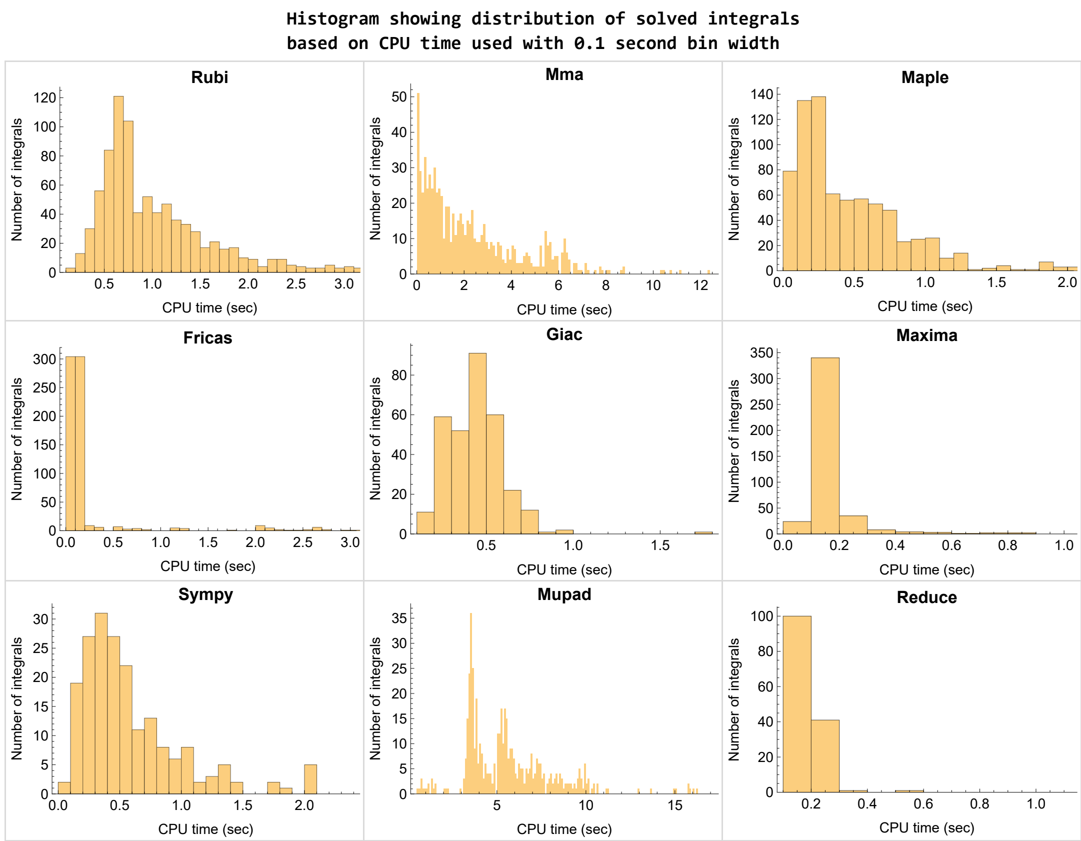


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

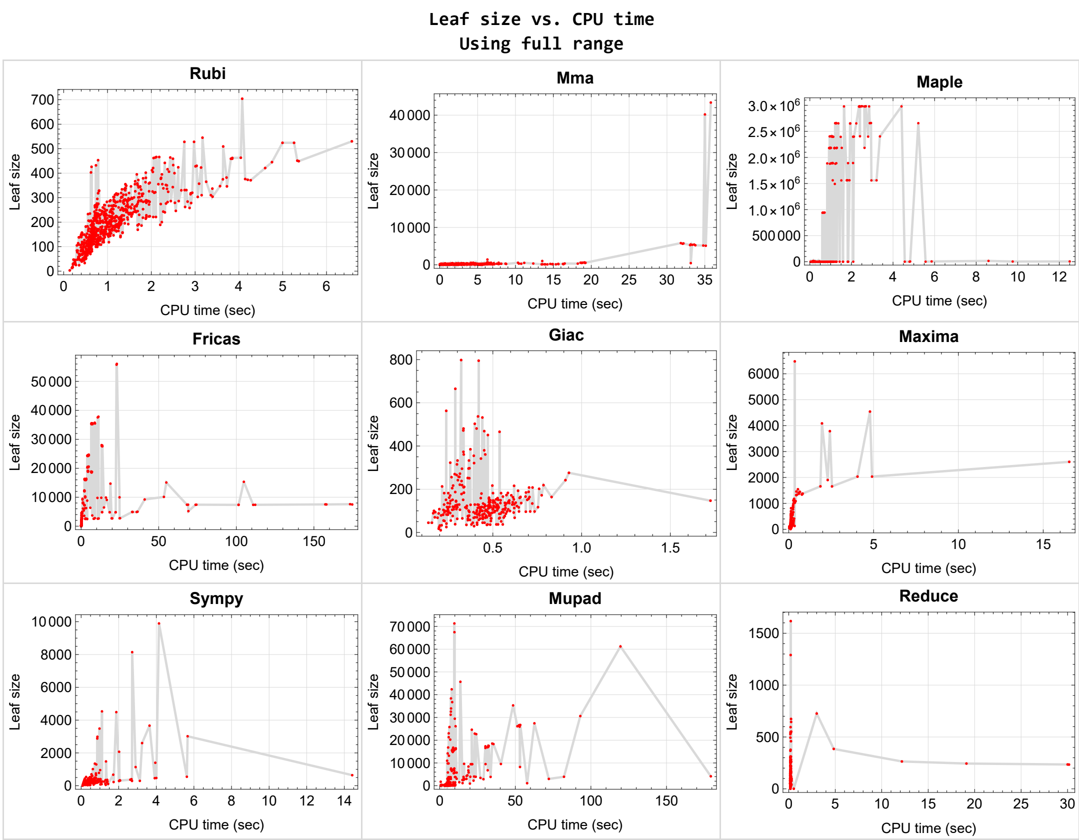


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {196, 197, 198, 199, 200, 201, 202, 203, 212, 213, 214, 215, 216, 217, 226, 227, 228, 229, 230, 231, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 473, 474, 475, 476, 477, 478, 487, 488, 489, 490, 491, 492, 502, 503, 505, 572, 573, 574, 575, 576, 577, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 661, 662, 663, 664}

Mathematica {140, 146, 147, 148, 149, 190, 338, 339, 410, 441, 448, 449, 450, 451, 534, 535, 538, 539, 570, 623, 624, 631, 632, 633, 634, 639, 805, 817, 818, 822, 823, 824}

Maple {1, 2, 3, 9, 10, 11, 17, 18, 19, 26, 27, 28, 240, 248, 257, 258, 299, 300, 301, 303, 305, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 668, 669, 670, 680, 681, 682, 693, 694, 695, 696}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

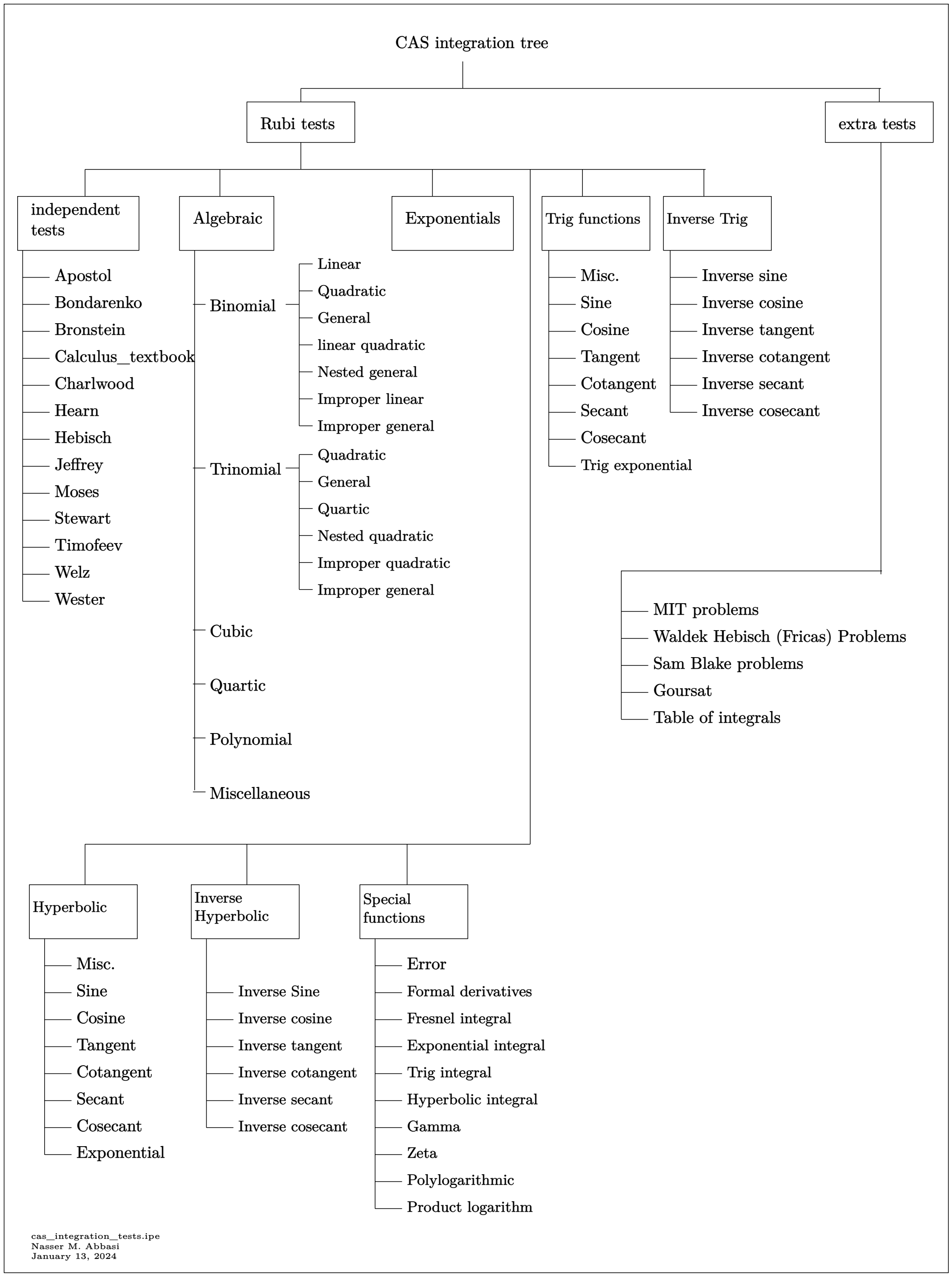
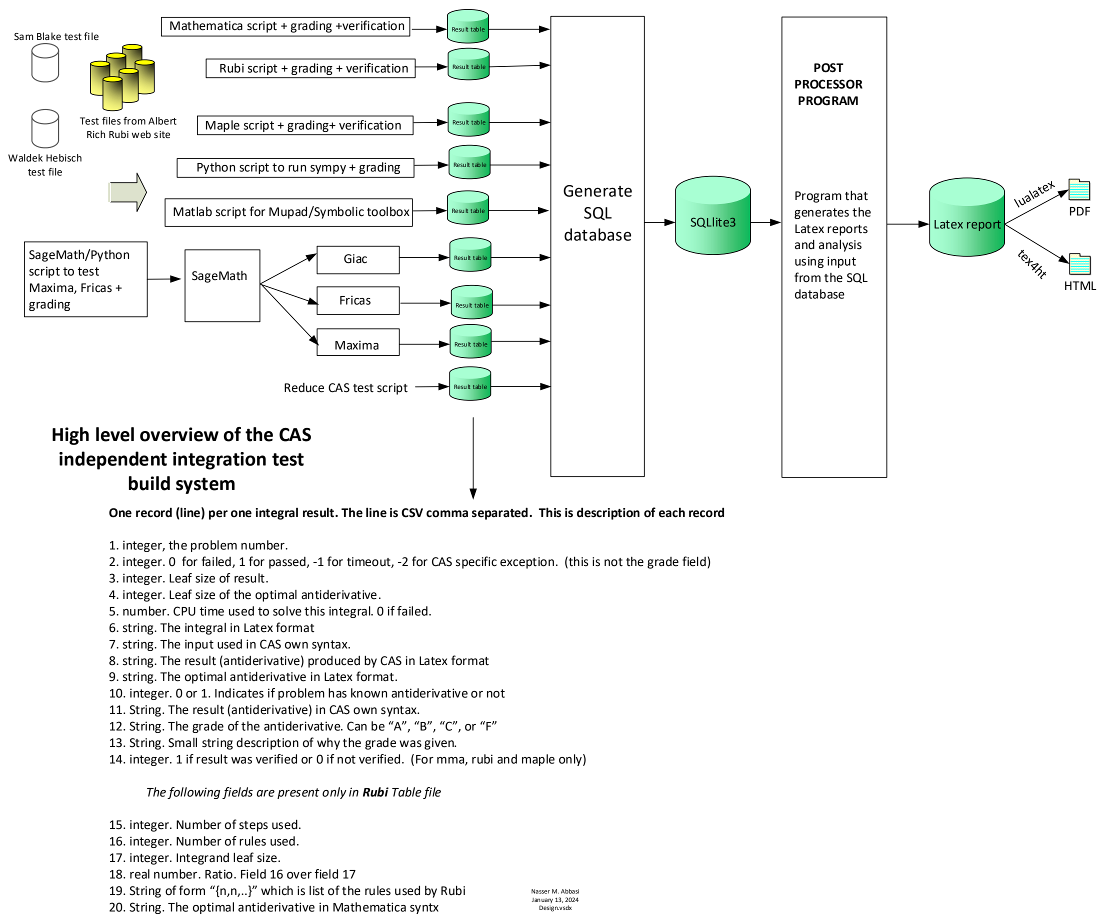


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462,

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786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804,
805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823,
824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842,
843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28,
29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 59, 60,
61, 62, 63, 64, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87,
88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 105, 106, 107, 109, 110, 111, 112,
113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133,
134, 135, 136, 137, 140, 141, 142, 143, 146, 147, 148, 149, 150, 151, 154, 155, 156, 157, 158,
159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 179, 180,

181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 219, 220, 221, 222, 223, 224, 225, 232, 233, 234, 235, 270, 298, 299, 300, 301, 302, 304, 315, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 340, 343, 344, 345, 346, 347, 348, 349, 352, 354, 355, 356, 360, 362, 363, 364, 367, 369, 370, 373, 374, 382, 383, 384, 400, 401, 416, 417, 418, 419, 420, 421, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 447, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 494, 495, 496, 497, 498, 499, 500, 501, 508, 509, 510, 511, 513, 514, 515, 516, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 625, 626, 627, 628, 629, 630, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 772, 773, 774, 775, 776, 781, 782, 783, 784, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 819, 820, 821, 822, 823, 825, 826, 830, 831, 832, 833, 834, 835, 838, 839, 840, 841, 842, 843, 844, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855 }

B grade { 164, 174, 178, 314, 324, 336, 337, 463, 486, 554, 677, 817, 818, 824, 827, 828, 829 }

C grade { 5, 6, 7, 8, 41, 42, 43, 49, 50, 57, 58, 65, 66, 100, 104, 108, 117, 118, 138, 139, 144, 145, 152, 153, 200, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 316, 338, 339, 341, 342, 350, 351, 353, 357, 358, 359, 361, 365, 366, 368, 371, 372, 375, 376, 377, 378, 379, 380, 381, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 422, 423, 424, 425, 426, 441, 446, 448, 449, 450, 506, 507, 512, 517, 518, 519, 520, 623, 624, 631, 632, 633, 634, 635, 769, 770, 771, 777, 778, 779, 780, 785, 786, 836, 837, 845 }

F normal fail { 212, 213, 214, 215, 216, 217, 218, 226, 227, 228, 229, 230, 231, 487, 488, 489, 490, 491, 492, 493, 502, 503, 504, 505, 571, 572, 573, 574, 575, 576, 577, 660, 661, 662, 663, 664 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 119, 126, 127, 128, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 196, 197, 198, 199, 200, 201, 202, 203, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 517, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 581, 582, 583, 584, 585, 586, 592, 596, 597, 598, 601, 602, 603, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 793, 794, 795, 796, 797, 798, 801, 802, 803, 804, 806, 807, 808, 812, 813, 814, 815, 817, 819, 820, 825, 826, 827, 828, 829, 833, 834, 835, 839, 840, 841, 842, 846, 847, 848, 849, 850, 854, 855 }

B grade { 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 129, 130, 131, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 518, 519, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 578, 579, 580, 587, 588, 589, 590, 591, 593, 594, 595, 599, 600, 604, 605, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 791, 799, 800, 805, 809, 810, 811, 816, 818, 821, 822, 823, 824, 830, 831, 832, 836, 837, 838, 843, 844, 845, 853 }

C grade { 449, 473, 474, 475, 476, 477, 478 }

F normal fail { 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 571, 572, 573, 574, 575, 576, 577, 660, 661, 662, 663, 664, 705, 715, 727, 851, 852 }

F(-1) timedout fail { 629, 630, 631, 632, 633, 634 }

F(-2) exception fail { }

Fricas

A grade { 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 278, 298, 299, 300, 301, 302, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 365, 375, 416, 417, 418, 419, 420, 421, 665, 667, 668, 670, 671, 672, 673, 674, 677, 678, 679, 681, 682, 683, 684, 685, 686, 687, 688, 690, 691, 692, 694, 696, 697, 698, 699, 700, 701, 702, 703, 704, 708, 709, 710, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 724, 725, 726, 728, 729, 730, 731, 732, 733, 734, 735, 737, 738, 739, 741, 742, 743, 744, 745, 746, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 785, 786, 792, 793, 794, 801, 802, 803, 804, 812, 813, 814, 815, 825, 826, 827, 828, 829, 833, 834, 835, 839, 840, 841, 842, 846, 847, 848, 849, 850, 853, 854 }

B grade { 1, 7, 8, 29, 30, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 303, 305, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 666, 675, 676, 689, 695, 706, 707, 740, 747, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 787, 788, 789, 790, 791, 795, 796, 797, 798, 799, 800, 805, 806, 807, 808, 809, 810, 811, 816, 817, 818, 819, 820, 821, 822, 823, 824, 830, 831, 832, 836, 837, 838, 843, 844, 845, 855 }

C grade { 669, 680, 693, 711, 723, 736 }

F normal fail { 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 571, 572, 573, 574, 575, 576, 577, 660, 661, 662, 663, 664, 705, 715, 727, 851, 852 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 119, 120, 121, 125, 126, 127, 128, 129, 130, 131, 132, 133, 196, 197, 198, 199, 200, 201, 202, 203, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 512, 516, 517, 518, 519, 520, 521, 522, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 666, 667, 668, 669, 670, 677, 678, 679, 680, 681, 682, 690, 691, 692, 693, 694, 695, 696, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 800, 801, 802, 803, 804, 811, 813, 814, 815, 824, 826, 827, 828, 829, 832, 833, 838, 841, 845, 846, 855 }

B grade { 112, 113, 114, 115, 116, 117, 118, 122, 123, 124, 292, 293, 294, 302, 373, 506, 507, 508, 509, 510, 511, 513, 514, 515, 540, 541, 542, 545, 546, 547, 551, 552, 553, 665, 676, 689, 787, 788, 789, 790, 795, 796, 797, 798, 799, 805, 806, 807, 808, 809, 810, 812, 816, 817, 818, 819, 820, 821, 822, 823, 825, 830, 831, 836, 844, 850 }

C grade { 301 }

F normal fail { 154, 155, 156, 161, 162, 163, 169, 170, 204, 205, 206, 207, 212, 213, 214, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 317, 318, 319, 321, 322, 323, 324, 325, 326, 328, 329, 330, 333, 335, 336, 343, 344, 345, 347, 348, 349, 352, 354, 362, 375, 376, 377, 427, 428, 429, 431, 433, 434, 435, 436, 438, 440, 442, 443, 444, 446, 448, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 463, 464, 465, 466, 469, 470, 471, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 543, 544, 549, 550, 557, 571, 572, 573, 574, 575, 576, 577, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 851, 852 }

F(-1) timedout fail { 157, 158, 159, 160, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 230, 331, 332, 337, 338, 339, 350, 351, 355, 356, 357, 358, 359, 360, 363, 364, 367, 369, 430, 432, 437, 439, 441, 445, 447, 449, 450, 461, 462, 467, 468, 472, 504, 505, 548, 554, 555, 556, 627, 636, 659 }

F(-2) exception fail { 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 208, 209, 210, 211, 215, 216, 217, 320, 327, 334, 340, 341, 342, 346, 353, 361, 365, 366, 368, 370, 371, 372, 374, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 671, 672, 673, 674, 675, 683, 684, 685, 686, 687, 688, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 791, 792, 793, 794, 834, 835, 837, 839, 840, 842, 843, 847, 848, 849, 853, 854 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 196, 197, 198, 199, 200, 201, 202, 203, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 475, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 667, 668, 669, 670, 671, 672, 673, 674, 675, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 854 }

B grade { 302, 373, 374, 375, 376, 477, 666, 677, 690, 691 }

C grade { 301 }

F normal fail { 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 350, 358, 377, 382, 383, 384, 390, 391, 395, 396, 397, 407, 409, 414, 415, 419, 420, 421, 427, 428, 434, 435, 442, 443, 452, 453, 458, 459, 462, 463, 464, 465, 467, 473, 474, 476, 478, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 599, 600, 601, 602, 604, 605, 606, 610, 611, 612, 613, 614, 642, 647, 660, 661, 662, 663, 664, 665, 676, 689, 705, 715, 727, 787, 788, 789, 790, 795, 796, 797, 798, 805, 806, 807, 808, 816, 817, 818, 819, 820, 833, 834, 835, 840, 841, 842, 848, 849, 850, 851, 852, 853 }

F(-1) timedout fail { 215, 216, 217, 346, 349, 351, 357, 365, 366, 370, 405, 406, 410, 411, 412, 413, 504, 592, 598, 603, 607, 608, 609, 615, 616, 617, 618, 619, 620, 621, 622, 640, 641, 643, 644, 645, 646, 648, 650, 651, 653, 655, 657, 659 }

F(-2) exception fail { 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 352, 353, 354, 355, 356, 359, 360, 361, 362, 363, 364, 367, 368, 369, 371, 372, 378, 379, 380, 381, 385, 386, 387, 388, 389, 392, 393, 394, 398, 399, 400, 401, 402, 403, 404, 408, 416, 417, 418, 422, 423, 424, 425, 426, 429, 430, 431, 432, 433, 436, 437, 438, 439, 440, 441, 444, 445, 446, 447, 448, 449, 450, 451, 454, 455, 456, 457, 460, 461, 466, 468, 469, 470, 471, 472, 479, 480, 481, 482, 483, 484, 485, 486, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 649, 652, 654, 656, 658, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 791, 792, 793, 794, 799, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 814, 815, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 836, 837, 838, 839, 843, 844, 845, 846, 847, 855 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 180, 181, 196, 197, 198, 199, 200, 201, 202, 203, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 429, 453, 454, 473, 474, 475, 476, 477, 478, 610, 611, 612, 613, 614, 615, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 790, 791, 792, 793, 794, 801, 802, 803, 804, 812, 813, 814, 815, 825, 826, 827, 828, 829, 832, 833, 834, 835, 839, 840, 841, 842, 846, 847, 848, 849, 850, 853, 854, 855 }

C grade { }

F normal fail { }

F(-1) timedout fail { 154, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 427, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 705, 715, 727, 787, 788, 789, 795, 796, 797, 798, 799, 800, 805, 806, 807, 808, 809, 810, 811, 816, 817, 818, 819, 820, 821, 822, 823, 824, 830, 831, 836, 837, 838, 843, 844, 845, 851, 852 }

F(-2) exception fail { }

Sympy

A grade { 2, 3, 5, 6, 9, 11, 12, 13, 14, 15, 16, 17, 19, 23, 24, 25, 26, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 232, 233, 240, 241, 242, 243, 250, 251, 252, 259, 260, 261, 262, 263, 265, 266, 301, 305, 670, 671, 682, 683, 684, 696, 697, 698, 699, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 721, 722, 723, 724, 725, 726, 728, 729, 730, 733, 734, 735, 736, 737, 738, 739, 854 }

B grade { 1, 4, 7, 8, 10, 18, 20, 21, 22, 27, 28, 29, 30, 32, 234, 235, 236, 237, 238, 239, 244, 245, 246, 247, 248, 249, 253, 254, 255, 256, 257, 258, 264, 298, 299, 300, 302, 303, 304, 314, 665, 666, 667, 668, 672, 673, 674, 675, 676, 677, 678, 679, 681, 685, 686, 687, 688, 689, 690, 691, 692, 694, 695, 700, 701, 702, 703, 704, 720, 731, 732, 853 }

C grade { 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 306, 307, 308, 309, 310, 311, 312, 313, 315, 316, 669, 680, 693 }

F normal fail { 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 140, 141, 142, 154, 155, 156, 157, 158, 162, 163, 164, 165, 172, 173, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 435, 436, 437, 438, 444, 445, 446, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 462, 463, 464, 465, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 504, 505, 508, 509, 510, 511, 514, 515, 516, 521, 522, 524, 525, 526, 527, 528, 529, 530, 531, 532, 534, 535, 536, 543, 544, 549, 550, 559, 560, 561, 563, 564, 567, 571, 574, 575, 576, 579, 580, 581, 584, 585, 586, 590, 591, 592, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 605, 606, 611, 612, 613, 614, 615, 620, 621, 622, 628, 629, 630, 642, 643, 644, 645, 647, 648, 649, 650, 653, 657, 658, 659, 662, 663, 664, 705, 715, 727, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 776, 777, 778, 780, 781, 782, 783, 784, 785, 786, 788, 789, 790, 791, 792, 793, 794, 797, 798, 799, 800, 801, 802, 808, 809, 810, 811, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 842, 845, 846, 847, 848, 849, 850, 851, 852, 855 }

F(-1) timedout fail { 81, 85, 86, 87, 88, 89, 119, 127, 139, 145, 146, 147, 148, 149, 150, 153, 159, 160, 161, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 184, 189, 193, 194, 195, 212, 226, 227, 335, 336, 337, 338, 339, 385, 392, 408, 409, 410, 411, 412, 413, 414, 415, 432, 433, 434, 439, 440, 441, 442, 443, 447, 448, 449, 450, 457, 466, 467, 468, 487, 502, 506, 507, 512, 513, 517, 518, 519, 520, 523, 533, 537, 538, 539, 540, 541, 542, 545, 546, 547, 548, 551, 552, 553, 554, 555, 556, 557, 558, 562, 565, 566, 568, 569, 570, 572, 573, 577, 578, 582, 583, 587, 588, 589, 593, 603, 607, 608, 609, 610, 616, 617, 618, 619, 623, 624, 625, 626, 627, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 646, 651, 652, 654, 655, 656, 660, 661, 771, 779, 787, 795, 796, 803, 804, 805, 806, 807, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 836, 843, 844 }

F(-2) exception fail { 137, 138, 143, 144, 151, 152, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 188, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 259, 260, 261, 262, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 665, 666, 667, 668, 669, 670, 676, 677, 678, 679, 680, 681, 682, 689, 690, 691, 692, 693, 694, 695, 696, 711, 723, 736, 740, 741, 742, 748, 749, 750, 756, 757, 758, 833, 841, 850 }

C grade { }

F normal fail { 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 255, 256, 263, 264, 265, 266, 289, 297, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 671, 672, 673, 674, 675, 683, 684, 685, 686, 687, 688, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 737, 738, 739, 743, 744, 745, 746, 747, 751, 752, 753, 754, 755, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 834, 835, 836, 837, 838, 839, 840, 842, 843, 844, 845, 846, 847, 848, 849, 851, 852, 853, 854, 855 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	97	95	82	170	167	100	101	82
N.S.	1	1.00	1.07	1.04	0.90	1.87	1.84	1.10	1.11	0.90
time (sec)	N/A	0.515	0.504	0.191	0.114	0.076	0.368	0.428	0.162	3.538

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	72	68	109	109	63	76	59
N.S.	1	1.00	0.96	1.04	0.99	1.58	1.58	0.91	1.10	0.86
time (sec)	N/A	0.352	0.187	0.086	0.135	0.095	0.289	0.359	0.165	3.625

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	66	50	50	64	53	35	55	38
N.S.	1	1.00	1.43	1.09	1.09	1.39	1.15	0.76	1.20	0.83
time (sec)	N/A	0.255	0.018	0.106	0.122	0.069	0.260	0.351	0.160	3.447

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	41	44	49	36	94	37	93	36
N.S.	1	1.05	1.02	1.10	1.22	0.90	2.35	0.92	2.32	0.90
time (sec)	N/A	0.408	0.026	0.174	0.140	0.090	1.091	0.379	0.196	3.673

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	46	68	58	64	62	53	58	129	39
N.S.	1	1.05	1.55	1.32	1.45	1.41	1.20	1.32	2.93	0.89
time (sec)	N/A	0.374	0.033	0.208	0.150	0.073	0.251	0.370	0.163	3.591

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	112	74	84	111	109	76	180	60
N.S.	1	1.03	1.65	1.09	1.24	1.63	1.60	1.12	2.65	0.88
time (sec)	N/A	0.503	0.040	0.315	0.162	0.074	0.326	0.360	0.170	3.510

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	91	135	88	103	166	168	95	222	80
N.S.	1	1.02	1.52	0.99	1.16	1.87	1.89	1.07	2.49	0.90
time (sec)	N/A	0.625	0.047	0.353	0.115	0.099	0.360	0.409	0.170	3.631

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	113	152	105	116	206	218	114	251	100
N.S.	1	1.02	1.37	0.95	1.05	1.86	1.96	1.03	2.26	0.90
time (sec)	N/A	0.786	0.043	0.372	0.151	0.099	0.921	0.427	0.169	3.838

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	141	147	101	121	112	236	236	148	125	153
N.S.	1	1.04	0.72	0.86	0.79	1.67	1.67	1.05	0.89	1.09
time (sec)	N/A	0.786	0.866	0.131	0.125	0.079	0.468	0.501	0.171	3.468

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	107	99	82	99	92	175	178	114	102	111
N.S.	1	0.93	0.77	0.93	0.86	1.64	1.66	1.07	0.95	1.04
time (sec)	N/A	0.491	0.527	0.110	0.160	0.089	0.360	0.415	0.172	3.531

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	80	70	58	76	71	121	122	74	79	76
N.S.	1	0.88	0.72	0.95	0.89	1.51	1.52	0.92	0.99	0.95
time (sec)	N/A	0.356	0.317	0.142	0.122	0.085	0.347	0.361	0.227	3.394

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	46	63	67	97	109	58	197	70
N.S.	1	1.03	0.61	0.84	0.89	1.29	1.45	0.77	2.63	0.93
time (sec)	N/A	0.611	0.320	0.194	0.119	0.093	1.286	0.421	0.238	3.548

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	81	54	76	74	102	109	70	173	87
N.S.	1	1.03	0.68	0.96	0.94	1.29	1.38	0.89	2.19	1.10
time (sec)	N/A	0.644	0.480	0.270	0.122	0.090	1.437	0.460	0.209	3.803

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	100	111	78	94	123	119	92	182	67
N.S.	1	1.06	1.18	0.83	1.00	1.31	1.27	0.98	1.94	0.71
time (sec)	N/A	0.630	0.331	0.301	0.116	0.081	0.386	0.499	0.171	3.686

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	126	143	93	111	181	182	115	224	93
N.S.	1	1.08	1.22	0.79	0.95	1.55	1.56	0.98	1.91	0.79
time (sec)	N/A	0.802	0.433	0.395	0.136	0.078	0.680	0.532	0.169	3.748

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	152	174	110	132	227	235	137	253	113
N.S.	1	1.09	1.25	0.79	0.95	1.63	1.69	0.99	1.82	0.81
time (sec)	N/A	0.971	0.748	0.441	0.123	0.079	0.564	0.568	0.186	3.949

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	182	192	123	146	132	291	292	182	149	230
N.S.	1	1.05	0.68	0.80	0.73	1.60	1.60	1.00	0.82	1.26
time (sec)	N/A	1.078	1.011	0.145	0.124	0.084	0.682	0.501	0.173	3.431

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	138	128	105	123	114	227	235	148	126	176
N.S.	1	0.93	0.76	0.89	0.83	1.64	1.70	1.07	0.91	1.28
time (sec)	N/A	0.605	0.644	0.119	0.118	0.088	0.462	0.479	0.180	3.389

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	99	73	100	96	175	184	115	103	125
N.S.	1	0.90	0.66	0.91	0.87	1.59	1.67	1.05	0.94	1.14
time (sec)	N/A	0.452	0.578	0.130	0.127	0.081	0.421	0.426	0.176	3.378

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	109	70	76	89	172	224	90	377	87
N.S.	1	1.02	0.65	0.71	0.83	1.61	2.09	0.84	3.52	0.81
time (sec)	N/A	0.816	0.558	0.245	0.116	0.088	1.727	0.477	0.196	3.471

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	118	94	84	84	141	219	86	301	76
N.S.	1	1.02	0.81	0.72	0.72	1.22	1.89	0.74	2.59	0.66
time (sec)	N/A	0.892	0.646	0.294	0.129	0.088	1.051	0.536	0.176	3.540

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	125	75	77	96	179	226	92	230	88
N.S.	1	1.02	0.61	0.63	0.78	1.46	1.84	0.75	1.87	0.72
time (sec)	N/A	0.908	0.728	0.612	0.145	0.097	1.084	0.617	0.174	3.491

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	145	89	91	115	181	182	116	224	93
N.S.	1	1.08	0.66	0.68	0.86	1.35	1.36	0.87	1.67	0.69
time (sec)	N/A	0.931	0.658	0.481	0.144	0.083	0.537	0.646	0.175	3.402

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	172	111	110	134	228	235	138	253	114
N.S.	1	1.10	0.71	0.70	0.85	1.45	1.50	0.88	1.61	0.73
time (sec)	N/A	1.109	0.924	0.674	0.124	0.077	1.387	0.629	0.182	3.575

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	195	207	127	151	287	296	161	282	140
N.S.	1	1.08	1.15	0.71	0.84	1.59	1.64	0.89	1.57	0.78
time (sec)	N/A	1.315	0.785	0.753	0.150	0.083	0.796	0.478	0.181	3.973

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	225	240	138	168	150	345	348	216	171	308
N.S.	1	1.07	0.61	0.75	0.67	1.53	1.55	0.96	0.76	1.37
time (sec)	N/A	1.460	1.146	0.375	0.118	0.091	0.878	0.620	0.210	3.175

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	168	157	119	145	132	279	291	182	148	240
N.S.	1	0.93	0.71	0.86	0.79	1.66	1.73	1.08	0.88	1.43
time (sec)	N/A	0.737	0.748	0.307	0.128	0.084	0.499	0.546	0.177	3.151

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	140	128	86	122	114	227	241	149	125	181
N.S.	1	0.91	0.61	0.87	0.81	1.62	1.72	1.06	0.89	1.29
time (sec)	N/A	0.565	0.702	0.357	0.122	0.103	0.458	0.486	0.169	3.129

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	144	89	108	107	246	289	130	528	133
N.S.	1	1.01	0.63	0.76	0.75	1.73	2.04	0.92	3.72	0.94
time (sec)	N/A	1.064	0.750	0.582	0.130	0.098	2.014	0.621	0.171	3.297

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	146	121	91	102	254	260	117	494	110
N.S.	1	1.01	0.84	0.63	0.71	1.76	1.81	0.81	3.43	0.76
time (sec)	N/A	1.138	0.858	0.586	0.119	0.096	1.402	0.754	0.173	3.585

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	162	122	97	108	255	252	106	367	102
N.S.	1	1.04	0.78	0.62	0.69	1.63	1.62	0.68	2.35	0.65
time (sec)	N/A	1.204	3.054	0.583	0.116	0.105	1.227	0.499	0.173	3.582

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	165	143	98	117	249	292	116	270	113
N.S.	1	1.01	0.88	0.60	0.72	1.53	1.79	0.71	1.66	0.69
time (sec)	N/A	1.198	0.719	0.717	0.130	0.103	2.707	0.521	0.171	3.683

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	193	91	114	136	228	235	138	253	114
N.S.	1	1.09	0.51	0.64	0.77	1.29	1.33	0.78	1.43	0.64
time (sec)	N/A	1.246	0.708	0.680	0.127	0.075	0.690	0.589	0.208	3.584

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	219	120	132	153	287	296	160	282	140
N.S.	1	1.10	0.60	0.66	0.76	1.44	1.48	0.80	1.41	0.70
time (sec)	N/A	1.454	1.070	0.857	0.134	0.085	3.124	0.651	0.241	4.013

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	245	150	150	172	332	347	183	311	162
N.S.	1	1.10	0.67	0.67	0.77	1.49	1.56	0.82	1.39	0.73
time (sec)	N/A	1.690	2.441	0.929	0.122	0.097	1.298	0.585	0.236	4.821

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	122	137	146	0	186	196	116	116	141
N.S.	1	0.95	1.06	1.13	0.00	1.44	1.52	0.90	0.90	1.09
time (sec)	N/A	0.606	1.289	0.312	0.000	0.120	0.489	0.451	0.163	3.693

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	99	103	125	0	127	151	87	95	95
N.S.	1	0.98	1.02	1.24	0.00	1.26	1.50	0.86	0.94	0.94
time (sec)	N/A	0.474	0.896	0.261	0.000	0.085	0.333	0.452	0.166	3.430

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	64	75	86	0	66	119	68	53	81
N.S.	1	0.96	1.12	1.28	0.00	0.99	1.78	1.01	0.79	1.21
time (sec)	N/A	0.377	0.327	0.287	0.000	0.088	0.222	0.392	0.169	3.315

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	56	54	0	42	87	70	44	45
N.S.	1	1.00	1.19	1.15	0.00	0.89	1.85	1.49	0.94	0.96
time (sec)	N/A	0.223	0.177	0.148	0.000	0.079	0.148	0.399	0.166	3.235

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	67	86	85	0	68	116	88	57	98
N.S.	1	1.08	1.39	1.37	0.00	1.10	1.87	1.42	0.92	1.58
time (sec)	N/A	0.429	0.524	0.341	0.000	0.090	0.262	0.400	0.173	3.328

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	98	140	0	129	158	111	76	126
N.S.	1	1.00	0.96	1.37	0.00	1.26	1.55	1.09	0.75	1.24
time (sec)	N/A	0.597	0.801	0.333	0.000	0.090	0.372	0.474	0.170	3.548

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	127	111	159	0	188	199	128	61	153
N.S.	1	0.97	0.85	1.21	0.00	1.44	1.52	0.98	0.47	1.17
time (sec)	N/A	0.748	1.163	0.412	0.000	0.101	0.467	0.478	0.209	3.695

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	150	114	185	0	249	253	146	91	174
N.S.	1	0.97	0.74	1.19	0.00	1.61	1.63	0.94	0.59	1.12
time (sec)	N/A	0.913	1.222	0.390	0.000	0.082	0.457	0.505	0.177	4.039

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	147	197	173	0	150	262	101	75	141
N.S.	1	1.04	1.39	1.22	0.00	1.06	1.85	0.71	0.53	0.99
time (sec)	N/A	0.736	0.949	0.333	0.000	0.088	0.485	0.435	0.160	3.853

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	106	158	122	0	84	223	86	75	114
N.S.	1	1.03	1.53	1.18	0.00	0.82	2.17	0.83	0.73	1.11
time (sec)	N/A	0.606	0.531	0.381	0.000	0.086	0.358	0.422	0.165	3.660

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	82	92	73	0	52	167	80	73	106
N.S.	1	1.08	1.21	0.96	0.00	0.68	2.20	1.05	0.96	1.39
time (sec)	N/A	0.389	0.429	0.207	0.000	0.078	0.205	0.441	0.166	3.222

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	79	89	73	0	54	162	83	64	70
N.S.	1	0.99	1.11	0.91	0.00	0.68	2.02	1.04	0.80	0.88
time (sec)	N/A	0.310	0.548	0.223	0.000	0.071	0.215	0.412	0.170	3.247

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	109	105	122	0	86	219	102	77	129
N.S.	1	1.15	1.11	1.28	0.00	0.91	2.31	1.07	0.81	1.36
time (sec)	N/A	0.665	1.133	0.444	0.000	0.125	0.309	0.410	0.195	3.413

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	147	130	177	0	152	267	129	81	164
N.S.	1	1.04	0.92	1.26	0.00	1.08	1.89	0.91	0.57	1.16
time (sec)	N/A	0.923	1.387	0.475	0.000	0.104	0.523	0.454	0.175	3.591

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	181	143	209	0	215	323	149	81	188
N.S.	1	1.06	0.84	1.23	0.00	1.26	1.90	0.88	0.48	1.11
time (sec)	N/A	1.130	1.773	0.529	0.000	0.084	0.540	0.544	0.166	3.689

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	203	270	214	0	174	337	121	315	184
N.S.	1	1.06	1.41	1.12	0.00	0.91	1.76	0.63	1.65	0.96
time (sec)	N/A	1.097	1.118	0.371	0.000	0.090	0.647	0.504	0.194	4.050

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	159	166	160	0	104	296	100	99	146
N.S.	1	1.07	1.12	1.08	0.00	0.70	2.00	0.68	0.67	0.99
time (sec)	N/A	0.873	1.370	0.394	0.000	0.084	0.633	0.441	0.189	3.773

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	135	147	128	0	78	258	102	99	111
N.S.	1	1.09	1.19	1.03	0.00	0.63	2.08	0.82	0.80	0.90
time (sec)	N/A	0.649	0.713	0.218	0.000	0.081	0.289	0.524	0.192	3.614

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	116	148	128	0	74	260	100	97	147
N.S.	1	1.05	1.35	1.16	0.00	0.67	2.36	0.91	0.88	1.34
time (sec)	N/A	0.486	0.970	0.231	0.000	0.093	0.306	0.457	0.185	3.608

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	111	109	128	0	76	258	100	88	111
N.S.	1	0.99	0.97	1.14	0.00	0.68	2.30	0.89	0.79	0.99
time (sec)	N/A	0.416	0.719	0.211	0.000	0.098	0.276	0.441	0.203	3.316

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	153	132	159	0	104	292	120	1404	164
N.S.	1	1.17	1.01	1.21	0.00	0.79	2.23	0.92	10.72	1.25
time (sec)	N/A	0.917	1.026	0.420	0.000	0.095	0.426	0.531	0.268	3.398

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	197	164	214	0	173	340	146	1393	197
N.S.	1	1.08	0.90	1.17	0.00	0.95	1.86	0.80	7.61	1.08
time (sec)	N/A	1.264	1.677	0.531	0.000	0.112	0.534	0.590	0.272	3.684

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	233	173	246	0	235	398	166	1507	221
N.S.	1	1.08	0.80	1.14	0.00	1.09	1.84	0.77	6.98	1.02
time (sec)	N/A	1.515	2.351	0.540	0.000	0.104	1.036	0.684	0.277	3.940

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	203	264	197	0	120	359	123	573	178
N.S.	1	1.10	1.43	1.06	0.00	0.65	1.94	0.66	3.10	0.96
time (sec)	N/A	1.180	0.846	0.505	0.000	0.101	1.388	0.486	0.207	3.673

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	181	158	147	0	88	301	116	351	178
N.S.	1	1.14	0.99	0.92	0.00	0.55	1.89	0.73	2.21	1.12
time (sec)	N/A	0.952	1.076	0.319	0.000	0.079	0.508	0.484	0.225	3.773

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	153	144	109	0	78	241	114	115	135
N.S.	1	1.06	0.99	0.75	0.00	0.54	1.66	0.79	0.79	0.93
time (sec)	N/A	0.737	1.030	0.336	0.000	0.112	0.546	0.477	0.230	3.786

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	148	141	109	0	78	246	111	113	172
N.S.	1	1.03	0.99	0.76	0.00	0.55	1.72	0.78	0.79	1.20
time (sec)	N/A	0.587	0.499	0.312	0.000	0.085	0.337	0.501	0.229	3.747

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	143	132	147	0	88	299	117	104	143
N.S.	1	0.99	0.91	1.01	0.00	0.61	2.06	0.81	0.72	0.99
time (sec)	N/A	0.509	0.852	0.292	0.000	0.077	0.368	0.433	0.174	3.546

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	191	152	196	0	122	359	136	0	196
N.S.	1	1.18	0.94	1.21	0.00	0.75	2.22	0.84	0.00	1.21
time (sec)	N/A	1.212	1.099	0.558	0.000	0.089	0.440	0.543	0.365	3.495

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	241	191	251	0	190	406	163	0	226
N.S.	1	1.10	0.87	1.14	0.00	0.86	1.85	0.74	0.00	1.03
time (sec)	N/A	1.650	2.320	0.614	0.000	0.122	1.008	0.613	0.369	3.979

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	279	204	283	0	253	466	180	0	251
N.S.	1	1.09	0.80	1.11	0.00	0.99	1.83	0.71	0.00	0.98
time (sec)	N/A	1.886	2.988	0.763	0.000	0.114	0.786	0.436	0.365	4.513

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	213	130	163	153	440	0	0	52	216
N.S.	1	1.10	0.67	0.84	0.79	2.27	0.00	0.00	0.27	1.11
time (sec)	N/A	1.094	1.342	0.452	0.146	0.117	0.000	0.000	0.169	1.343

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	150	111	124	130	383	0	0	52	168
N.S.	1	1.05	0.78	0.87	0.91	2.68	0.00	0.00	0.36	1.17
time (sec)	N/A	0.725	0.793	0.260	0.126	0.091	0.000	0.000	0.167	4.134

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	92	82	107	332	0	0	50	120
N.S.	1	1.02	0.88	0.78	1.02	3.16	0.00	0.00	0.48	1.14
time (sec)	N/A	0.453	0.450	0.230	0.139	0.110	0.000	0.000	0.165	4.122

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	75	63	87	272	0	0	69	96
N.S.	1	1.03	1.00	0.84	1.16	3.63	0.00	0.00	0.92	1.28
time (sec)	N/A	0.312	0.199	0.184	0.138	0.093	0.000	0.000	0.166	0.579

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	89	79	72	113	447	0	0	54	493
N.S.	1	1.03	0.92	0.84	1.31	5.20	0.00	0.00	0.63	5.73
time (sec)	N/A	0.568	0.707	0.210	0.142	0.090	0.000	0.000	0.184	3.863

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	134	118	114	145	649	0	0	58	168
N.S.	1	1.09	0.96	0.93	1.18	5.28	0.00	0.00	0.47	1.37
time (sec)	N/A	0.839	1.289	0.270	0.137	0.095	0.000	0.000	0.231	4.051

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	184	138	146	202	730	0	0	58	702
N.S.	1	1.09	0.82	0.86	1.20	4.32	0.00	0.00	0.34	4.15
time (sec)	N/A	1.134	1.866	0.272	0.137	0.115	0.000	0.000	0.234	4.110

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	236	157	171	249	823	0	0	58	735
N.S.	1	1.12	0.75	0.81	1.19	3.92	0.00	0.00	0.28	3.50
time (sec)	N/A	1.498	3.021	0.269	0.131	0.119	0.000	0.000	0.191	4.152

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	213	133	164	153	467	0	0	103	211
N.S.	1	1.08	0.68	0.83	0.78	2.37	0.00	0.00	0.52	1.07
time (sec)	N/A	1.104	1.244	0.240	0.131	0.094	0.000	0.000	0.180	4.422

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	114	123	130	415	0	0	101	163
N.S.	1	1.00	0.83	0.90	0.95	3.03	0.00	0.00	0.74	1.19
time (sec)	N/A	0.563	0.665	0.187	0.165	0.092	0.000	0.000	0.182	4.202

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	106	98	99	111	359	0	0	97	139
N.S.	1	0.99	0.92	0.93	1.04	3.36	0.00	0.00	0.91	1.30
time (sec)	N/A	0.395	0.362	0.170	0.155	0.106	0.000	0.000	0.179	3.872

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	115	108	87	130	514	0	0	115	553
N.S.	1	1.02	0.96	0.77	1.15	4.55	0.00	0.00	1.02	4.89
time (sec)	N/A	0.762	0.418	0.206	0.144	0.097	0.000	0.000	0.221	3.800

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	132	122	111	145	685	0	0	123	2338
N.S.	1	1.06	0.98	0.89	1.16	5.48	0.00	0.00	0.98	18.70
time (sec)	N/A	0.843	2.150	0.208	0.133	0.095	0.000	0.000	0.292	5.384

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	182	139	148	203	762	0	0	123	3027
N.S.	1	1.06	0.81	0.87	1.19	4.46	0.00	0.00	0.72	17.70
time (sec)	N/A	1.129	1.986	0.215	0.144	0.111	0.000	0.000	0.295	5.181

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	234	158	179	253	856	0	0	123	3084
N.S.	1	1.10	0.74	0.84	1.19	4.02	0.00	0.00	0.58	14.48
time (sec)	N/A	1.483	3.319	0.217	0.148	0.149	0.000	0.000	0.230	5.149

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	266	151	206	176	531	0	0	157	258
N.S.	1	1.08	0.61	0.84	0.72	2.16	0.00	0.00	0.64	1.05
time (sec)	N/A	1.462	2.242	0.314	0.130	0.118	0.000	0.000	0.226	5.066

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	168	132	165	153	477	0	0	155	212
N.S.	1	0.98	0.77	0.96	0.89	2.79	0.00	0.00	0.91	1.24
time (sec)	N/A	0.684	0.967	0.250	0.131	0.107	0.000	0.000	0.198	1.508

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	137	116	141	134	421	0	0	152	188
N.S.	1	0.97	0.82	1.00	0.95	2.99	0.00	0.00	1.08	1.33
time (sec)	N/A	0.519	0.557	0.187	0.117	0.090	0.000	0.000	0.200	4.459

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	149	126	125	154	610	0	0	181	597
N.S.	1	1.01	0.86	0.85	1.05	4.15	0.00	0.00	1.23	4.06
time (sec)	N/A	1.036	0.744	0.216	0.131	0.097	0.000	0.000	0.249	4.244

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	166	129	131	163	705	0	0	193	2947
N.S.	1	1.05	0.82	0.83	1.03	4.46	0.00	0.00	1.22	18.65
time (sec)	N/A	1.105	0.920	0.222	0.134	0.093	0.000	0.000	0.286	5.485

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	181	141	143	206	774	0	0	193	2991
N.S.	1	1.05	0.82	0.83	1.19	4.47	0.00	0.00	1.12	17.29
time (sec)	N/A	1.171	4.439	0.272	0.124	0.105	0.000	0.000	0.265	5.603

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	232	160	179	249	868	0	0	193	3048
N.S.	1	1.07	0.74	0.82	1.15	4.00	0.00	0.00	0.89	14.05
time (sec)	N/A	1.495	3.459	0.288	0.179	0.107	0.000	0.000	0.255	5.213

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	285	177	206	293	944	0	0	193	3094
N.S.	1	1.09	0.68	0.79	1.12	3.62	0.00	0.00	0.74	11.85
time (sec)	N/A	1.910	3.008	0.314	0.188	0.118	0.000	0.000	0.265	5.356

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	218	130	167	157	447	0	0	155	236
N.S.	1	1.06	0.63	0.81	0.77	2.18	0.00	0.00	0.76	1.15
time (sec)	N/A	1.131	1.551	0.292	0.134	0.103	0.000	0.000	0.200	4.720

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	165	111	128	134	391	0	0	155	188
N.S.	1	1.04	0.70	0.81	0.84	2.46	0.00	0.00	0.97	1.18
time (sec)	N/A	0.781	1.124	0.243	0.136	0.124	0.000	0.000	0.174	4.350

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	111	109	88	110	327	0	0	153	141
N.S.	1	1.02	1.00	0.81	1.01	3.00	0.00	0.00	1.40	1.29
time (sec)	N/A	0.470	0.769	0.191	0.137	0.092	0.000	0.000	0.179	0.997

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	84	82	71	91	328	0	0	283	117
N.S.	1	1.02	1.00	0.87	1.11	4.00	0.00	0.00	3.45	1.43
time (sec)	N/A	0.319	0.289	0.173	0.126	0.084	0.000	0.000	0.175	3.869

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	125	120	99	133	575	0	0	167	515
N.S.	1	1.10	1.05	0.87	1.17	5.04	0.00	0.00	1.46	4.52
time (sec)	N/A	0.791	1.178	0.233	0.123	0.119	0.000	0.000	0.190	3.818

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	183	131	139	184	742	0	0	175	2961
N.S.	1	1.10	0.78	0.83	1.10	4.44	0.00	0.00	1.05	17.73
time (sec)	N/A	1.142	2.086	0.245	0.149	0.114	0.000	0.000	0.188	5.275

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	237	150	171	232	835	0	0	175	3037
N.S.	1	1.08	0.68	0.78	1.06	3.81	0.00	0.00	0.80	13.87
time (sec)	N/A	1.514	2.649	0.226	0.130	0.138	0.000	0.000	0.224	5.292

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	219	149	154	160	440	0	0	650	233
N.S.	1	1.05	0.71	0.74	0.77	2.11	0.00	0.00	3.11	1.11
time (sec)	N/A	1.114	1.833	0.270	0.119	0.129	0.000	0.000	0.203	4.015

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	166	148	116	137	375	0	0	667	186
N.S.	1	0.99	0.89	0.69	0.82	2.25	0.00	0.00	3.99	1.11
time (sec)	N/A	0.774	1.067	0.226	0.126	0.118	0.000	0.000	0.226	3.740

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	126	118	96	116	369	0	0	520	163
N.S.	1	1.06	0.99	0.81	0.97	3.10	0.00	0.00	4.37	1.37
time (sec)	N/A	0.550	0.847	0.207	0.135	0.098	0.000	0.000	0.227	3.626

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	119	76	96	113	372	0	0	616	162
N.S.	1	0.98	0.63	0.79	0.93	3.07	0.00	0.00	5.09	1.34
time (sec)	N/A	0.420	0.421	0.165	0.150	0.086	0.000	0.000	0.236	3.479

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	167	156	126	161	624	0	0	103	563
N.S.	1	1.07	1.00	0.81	1.03	4.00	0.00	0.00	0.66	3.61
time (sec)	N/A	1.076	1.776	0.215	0.117	0.103	0.000	0.000	0.204	3.621

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	233	168	168	215	814	0	0	107	3051
N.S.	1	1.07	0.77	0.77	0.99	3.75	0.00	0.00	0.49	14.06
time (sec)	N/A	1.539	2.284	0.226	0.139	0.139	0.000	0.000	0.240	5.378

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	287	182	198	259	903	0	0	107	3106
N.S.	1	1.07	0.68	0.74	0.97	3.37	0.00	0.00	0.40	11.59
time (sec)	N/A	1.884	3.507	0.219	0.136	0.150	0.000	0.000	0.198	5.486

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	272	162	181	185	457	0	0	149	279
N.S.	1	1.07	0.64	0.71	0.73	1.79	0.00	0.00	0.58	1.09
time (sec)	N/A	1.564	1.996	0.285	0.126	0.123	0.000	0.000	0.161	3.851

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	219	204	142	162	392	0	0	149	230
N.S.	1	1.04	0.97	0.67	0.77	1.86	0.00	0.00	0.71	1.09
time (sec)	N/A	1.157	2.521	0.274	0.133	0.088	0.000	0.000	0.159	3.814

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	179	177	124	141	393	0	0	149	187
N.S.	1	1.07	1.06	0.74	0.84	2.35	0.00	0.00	0.89	1.12
time (sec)	N/A	0.841	1.425	0.247	0.137	0.089	0.000	0.000	0.159	3.639

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	158	147	121	136	391	0	0	147	186
N.S.	1	1.03	0.96	0.79	0.89	2.56	0.00	0.00	0.96	1.22
time (sec)	N/A	0.670	1.282	0.194	0.136	0.108	0.000	0.000	0.155	3.598

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	153	76	123	139	392	0	0	138	205
N.S.	1	0.99	0.49	0.79	0.90	2.53	0.00	0.00	0.89	1.32
time (sec)	N/A	0.533	0.545	0.172	0.149	0.107	0.000	0.000	0.185	3.598

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	211	217	153	186	644	0	0	151	528
N.S.	1	1.10	1.13	0.80	0.97	3.35	0.00	0.00	0.79	2.75
time (sec)	N/A	1.373	2.171	0.239	0.153	0.123	0.000	0.000	0.214	3.729

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	283	269	195	240	834	0	0	155	3002
N.S.	1	1.09	1.04	0.75	0.93	3.22	0.00	0.00	0.60	11.59
time (sec)	N/A	1.962	4.036	0.240	0.169	0.112	0.000	0.000	0.162	5.435

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	336	298	225	283	923	0	0	155	3048
N.S.	1	1.08	0.96	0.72	0.91	2.96	0.00	0.00	0.50	9.77
time (sec)	N/A	2.358	5.677	0.240	0.137	0.159	0.000	0.000	0.161	5.553

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	108	272	202	480	0	118	155	161
N.S.	1	1.00	0.83	2.09	1.55	3.69	0.00	0.91	1.19	1.24
time (sec)	N/A	0.698	0.952	0.184	0.151	0.129	0.000	0.438	0.177	7.341

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	251	186	432	0	94	137	130
N.S.	1	1.00	0.85	2.39	1.77	4.11	0.00	0.90	1.30	1.24
time (sec)	N/A	0.567	0.597	0.139	0.154	0.109	0.000	0.426	0.184	5.583

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	226	170	372	0	70	106	99
N.S.	1	1.00	0.89	2.82	2.12	4.65	0.00	0.88	1.32	1.24
time (sec)	N/A	0.439	0.340	0.082	0.152	0.103	0.000	0.411	0.189	4.860

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	201	151	314	0	45	86	68
N.S.	1	1.00	1.00	3.65	2.75	5.71	0.00	0.82	1.56	1.24
time (sec)	N/A	0.341	0.373	0.086	0.117	0.090	0.000	0.387	0.169	4.198

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	202	151	367	0	45	76	67
N.S.	1	1.00	1.02	3.81	2.85	6.92	0.00	0.85	1.43	1.26
time (sec)	N/A	0.337	0.336	0.129	0.139	0.085	0.000	0.411	0.224	4.104

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	220	171	427	0	68	85	99
N.S.	1	1.00	0.74	2.82	2.19	5.47	0.00	0.87	1.09	1.27
time (sec)	N/A	0.490	0.381	0.084	0.134	0.092	0.000	0.457	0.176	4.705

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	57	236	187	486	0	93	85	123
N.S.	1	1.00	0.55	2.29	1.82	4.72	0.00	0.90	0.83	1.19
time (sec)	N/A	0.629	0.526	0.087	0.170	0.100	0.000	0.475	0.172	5.768

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	186	130	298	230	555	0	164	197	326
N.S.	1	1.02	0.71	1.63	1.26	3.03	0.00	0.90	1.08	1.78
time (sec)	N/A	1.092	2.917	0.223	0.274	0.160	0.000	0.656	0.181	8.134

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	161	111	275	212	501	0	136	179	291
N.S.	1	1.03	0.71	1.76	1.36	3.21	0.00	0.87	1.15	1.87
time (sec)	N/A	0.901	1.342	0.161	0.129	0.123	0.000	0.660	0.183	6.350

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	134	93	252	194	441	0	108	151	256
N.S.	1	1.04	0.72	1.95	1.50	3.42	0.00	0.84	1.17	1.98
time (sec)	N/A	0.711	0.861	0.106	0.169	0.107	0.000	0.527	0.178	4.887

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	107	75	229	174	389	0	79	122	221
N.S.	1	1.03	0.72	2.20	1.67	3.74	0.00	0.76	1.17	2.12
time (sec)	N/A	0.590	1.239	0.154	0.154	0.087	0.000	0.425	0.181	4.012

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	74	217	170	386	0	63	111	203
N.S.	1	1.00	0.76	2.21	1.73	3.94	0.00	0.64	1.13	2.07
time (sec)	N/A	0.580	0.855	0.094	0.275	0.087	0.000	0.443	0.239	3.888

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	105	86	222	177	441	0	78	122	222
N.S.	1	1.03	0.84	2.18	1.74	4.32	0.00	0.76	1.20	2.18
time (sec)	N/A	0.596	1.042	0.098	0.150	0.094	0.000	0.485	0.201	4.319

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	135	102	240	195	502	0	107	131	258
N.S.	1	1.06	0.80	1.89	1.54	3.95	0.00	0.84	1.03	2.03
time (sec)	N/A	0.785	1.696	0.095	0.133	0.107	0.000	0.560	0.192	5.214

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	162	116	258	213	561	0	134	131	293
N.S.	1	1.05	0.75	1.68	1.38	3.64	0.00	0.87	0.85	1.90
time (sec)	N/A	0.937	2.323	0.107	0.143	0.131	0.000	0.576	0.180	6.725

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	206	129	299	234	561	0	164	221	327
N.S.	1	1.04	0.65	1.51	1.18	2.83	0.00	0.83	1.12	1.65
time (sec)	N/A	1.198	2.038	0.083	0.138	0.133	0.000	0.831	0.198	8.251

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	181	112	276	216	498	0	136	193	292
N.S.	1	1.06	0.65	1.61	1.26	2.91	0.00	0.80	1.13	1.71
time (sec)	N/A	0.994	1.304	0.075	0.141	0.106	0.000	0.593	0.206	6.370

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	154	92	253	196	447	0	108	166	257
N.S.	1	1.05	0.63	1.73	1.34	3.06	0.00	0.74	1.14	1.76
time (sec)	N/A	0.864	1.857	0.083	0.151	0.092	0.000	0.500	0.234	4.413

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	137	94	241	190	405	0	94	145	239
N.S.	1	1.02	0.70	1.80	1.42	3.02	0.00	0.70	1.08	1.78
time (sec)	N/A	0.804	1.113	0.079	0.167	0.102	0.000	0.550	0.257	4.118

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	140	94	234	191	438	0	93	156	240
N.S.	1	1.03	0.69	1.72	1.40	3.22	0.00	0.68	1.15	1.76
time (sec)	N/A	0.834	1.280	0.081	0.153	0.127	0.000	0.559	0.183	4.326

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	152	100	240	197	504	0	107	166	258
N.S.	1	1.06	0.69	1.67	1.37	3.50	0.00	0.74	1.15	1.79
time (sec)	N/A	0.846	1.791	0.080	0.145	0.089	0.000	0.587	0.182	4.577

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	182	118	258	215	561	0	134	175	293
N.S.	1	1.08	0.70	1.53	1.27	3.32	0.00	0.79	1.04	1.73
time (sec)	N/A	1.055	2.237	0.134	0.134	0.132	0.000	0.722	0.189	6.989

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	270	138	166	0	707	0	128	69	305
N.S.	1	1.07	0.55	0.66	0.00	2.81	0.00	0.51	0.27	1.21
time (sec)	N/A	0.927	1.578	0.159	0.000	0.111	0.000	0.482	0.164	8.387

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	237	114	143	0	621	0	105	67	270
N.S.	1	1.09	0.53	0.66	0.00	2.86	0.00	0.48	0.31	1.24
time (sec)	N/A	0.715	0.995	0.128	0.000	0.097	0.000	0.460	0.167	7.668

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	206	96	124	0	570	0	88	154	184
N.S.	1	1.11	0.52	0.67	0.00	3.06	0.00	0.47	0.83	0.99
time (sec)	N/A	0.566	0.708	0.198	0.000	0.092	0.000	0.478	0.191	5.870

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	204	97	121	0	571	0	89	152	184
N.S.	1	1.11	0.53	0.66	0.00	3.10	0.00	0.48	0.83	1.00
time (sec)	N/A	0.592	0.987	0.147	0.000	0.103	0.000	0.491	0.244	4.535

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	237	115	140	0	703	0	106	283	266
N.S.	1	1.11	0.54	0.66	0.00	3.30	0.00	0.50	1.33	1.25
time (sec)	N/A	0.747	0.841	0.120	0.000	0.155	0.000	0.556	0.196	4.814

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	260	118	160	0	795	0	125	303	303
N.S.	1	1.06	0.48	0.65	0.00	3.23	0.00	0.51	1.23	1.23
time (sec)	N/A	0.892	1.001	0.125	0.000	0.116	0.000	0.569	0.217	6.969

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	289	191	159	0	664	0	128	89	334
N.S.	1	1.10	0.73	0.61	0.00	2.53	0.00	0.49	0.34	1.27
time (sec)	N/A	1.069	2.239	0.200	0.000	0.106	0.000	0.526	0.173	7.050

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	244	178	147	0	664	0	116	87	318
N.S.	1	1.07	0.78	0.65	0.00	2.93	0.00	0.51	0.38	1.40
time (sec)	N/A	0.887	2.103	0.237	0.000	0.118	0.000	0.450	0.166	7.361

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	246	176	144	0	658	0	116	79	318
N.S.	1	1.07	0.77	0.63	0.00	2.87	0.00	0.51	0.34	1.39
time (sec)	N/A	0.867	1.870	0.178	0.000	0.141	0.000	0.422	0.157	7.168

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	243	178	144	0	664	0	117	110	318
N.S.	1	1.05	0.77	0.62	0.00	2.87	0.00	0.51	0.48	1.38
time (sec)	N/A	0.884	1.605	0.174	0.000	0.100	0.000	0.515	0.162	6.982

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	286	180	157	0	763	0	128	111	338
N.S.	1	1.08	0.68	0.59	0.00	2.89	0.00	0.48	0.42	1.28
time (sec)	N/A	1.050	1.314	0.167	0.000	0.122	0.000	0.521	0.208	7.178

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	315	180	176	0	861	0	148	121	373
N.S.	1	1.06	0.61	0.59	0.00	2.90	0.00	0.50	0.41	1.26
time (sec)	N/A	1.242	1.303	0.162	0.000	0.121	0.000	0.592	0.208	7.837

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	372	300	204	0	779	0	173	113	431
N.S.	1	1.10	0.88	0.60	0.00	2.30	0.00	0.51	0.33	1.27
time (sec)	N/A	1.704	7.522	0.181	0.000	0.162	0.000	0.577	0.173	7.617

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	339	286	181	0	685	0	151	113	395
N.S.	1	1.09	0.92	0.58	0.00	2.21	0.00	0.49	0.36	1.27
time (sec)	N/A	1.406	5.947	0.205	0.000	0.123	0.000	0.530	0.176	4.606

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	285	254	163	0	681	0	129	113	308
N.S.	1	1.10	0.98	0.63	0.00	2.63	0.00	0.50	0.44	1.19
time (sec)	N/A	1.150	4.906	0.219	0.000	0.119	0.000	0.526	0.172	3.247

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	287	183	152	0	635	0	123	111	239
N.S.	1	1.10	0.70	0.58	0.00	2.43	0.00	0.47	0.43	0.92
time (sec)	N/A	1.146	2.782	0.164	0.000	0.134	0.000	0.458	0.173	2.195

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	291	179	152	0	631	0	123	343	239
N.S.	1	1.09	0.67	0.57	0.00	2.36	0.00	0.46	1.28	0.90
time (sec)	N/A	1.141	3.161	0.159	0.000	0.112	0.000	0.474	0.201	2.098

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	288	168	157	0	682	0	129	340	308
N.S.	1	1.09	0.63	0.59	0.00	2.57	0.00	0.49	1.28	1.16
time (sec)	N/A	1.158	2.811	0.191	0.000	0.099	0.000	0.609	0.248	2.999

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	338	194	177	0	784	0	151	652	389
N.S.	1	1.09	0.63	0.57	0.00	2.53	0.00	0.49	2.10	1.25
time (sec)	N/A	1.414	1.946	0.151	0.000	0.105	0.000	0.694	0.285	3.847

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	367	194	196	0	875	0	172	676	425
N.S.	1	1.07	0.57	0.57	0.00	2.55	0.00	0.50	1.97	1.24
time (sec)	N/A	1.632	1.992	0.150	0.000	0.143	0.000	0.775	0.268	5.916

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	213	208	700	0	771	0	0	64	0
N.S.	1	1.06	1.04	3.50	0.00	3.86	0.00	0.00	0.32	0.00
time (sec)	N/A	1.268	5.594	0.426	0.000	0.118	0.000	0.000	0.187	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	158	189	577	0	673	0	0	124	2225
N.S.	1	1.04	1.24	3.80	0.00	4.43	0.00	0.00	0.82	14.64
time (sec)	N/A	0.905	3.437	0.266	0.000	0.116	0.000	0.000	0.183	22.335

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	141	469	0	539	0	0	126	372
N.S.	1	1.00	1.26	4.19	0.00	4.81	0.00	0.00	1.12	3.32
time (sec)	N/A	0.639	1.271	0.266	0.000	0.113	0.000	0.000	0.194	8.252

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	102	278	0	435	0	0	115	0
N.S.	1	1.00	1.13	3.09	0.00	4.83	0.00	0.00	1.28	0.00
time (sec)	N/A	0.438	0.603	0.204	0.000	0.100	0.000	0.000	0.181	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	142	131	354	0	483	0	0	148	0
N.S.	1	1.05	0.97	2.62	0.00	3.58	0.00	0.00	1.10	0.00
time (sec)	N/A	0.688	1.348	0.209	0.000	0.082	0.000	0.000	0.235	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	196	154	555	0	543	0	0	234	0
N.S.	1	1.10	0.87	3.12	0.00	3.05	0.00	0.00	1.31	0.00
time (sec)	N/A	0.993	4.063	0.208	0.000	0.089	0.000	0.000	0.284	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	250	173	630	0	593	0	0	293	0
N.S.	1	1.13	0.78	2.85	0.00	2.68	0.00	0.00	1.33	0.00
time (sec)	N/A	1.296	4.837	0.220	0.000	0.114	0.000	0.000	0.184	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	267	415	652	0	911	0	0	129	0
N.S.	1	1.08	1.67	2.63	0.00	3.67	0.00	0.00	0.52	0.00
time (sec)	N/A	1.658	6.726	0.286	0.000	0.108	0.000	0.000	0.224	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	212	283	565	0	829	0	0	158	0
N.S.	1	1.04	1.39	2.77	0.00	4.06	0.00	0.00	0.77	0.00
time (sec)	N/A	1.277	3.000	0.232	0.000	0.109	0.000	0.000	0.217	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	161	257	486	0	743	0	0	147	0
N.S.	1	1.03	1.65	3.12	0.00	4.76	0.00	0.00	0.94	0.00
time (sec)	N/A	0.931	2.147	0.227	0.000	0.103	0.000	0.000	0.201	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	301	521	0	748	0	0	138	0
N.S.	1	1.00	2.06	3.57	0.00	5.12	0.00	0.00	0.95	0.00
time (sec)	N/A	0.908	4.863	0.251	0.000	0.101	0.000	0.000	0.203	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	140	255	618	0	524	0	0	176	0
N.S.	1	1.02	1.86	4.51	0.00	3.82	0.00	0.00	1.28	0.00
time (sec)	N/A	0.704	5.436	0.218	0.000	0.093	0.000	0.000	0.233	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	194	323	705	0	593	0	0	235	0
N.S.	1	1.07	1.78	3.90	0.00	3.28	0.00	0.00	1.30	0.00
time (sec)	N/A	0.989	4.980	0.235	0.000	0.118	0.000	0.000	0.262	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	248	356	794	0	638	0	0	294	0
N.S.	1	1.10	1.58	3.53	0.00	2.84	0.00	0.00	1.31	0.00
time (sec)	N/A	1.356	4.679	0.288	0.000	0.117	0.000	0.000	0.217	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	302	242	885	0	701	0	0	353	0
N.S.	1	1.12	0.90	3.29	0.00	2.61	0.00	0.00	1.31	0.00
time (sec)	N/A	1.732	14.598	0.243	0.000	0.109	0.000	0.000	0.214	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	320	563	742	0	1017	0	0	197	0
N.S.	1	1.07	1.89	2.49	0.00	3.41	0.00	0.00	0.66	0.00
time (sec)	N/A	2.066	6.782	0.318	0.000	0.129	0.000	0.000	0.245	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	265	503	653	0	932	0	0	227	0
N.S.	1	1.05	2.00	2.59	0.00	3.70	0.00	0.00	0.90	0.00
time (sec)	N/A	1.664	6.604	0.275	0.000	0.138	0.000	0.000	0.241	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	212	304	564	0	849	0	0	215	0
N.S.	1	1.03	1.48	2.74	0.00	4.12	0.00	0.00	1.04	0.00
time (sec)	N/A	1.289	8.133	0.231	0.000	0.120	0.000	0.000	0.229	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	202	288	565	0	857	0	0	214	0
N.S.	1	1.03	1.47	2.88	0.00	4.37	0.00	0.00	1.09	0.00
time (sec)	N/A	1.273	5.280	0.212	0.000	0.100	0.000	0.000	0.245	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	310	618	0	846	0	0	248	0
N.S.	1	1.00	1.63	3.25	0.00	4.45	0.00	0.00	1.31	0.00
time (sec)	N/A	1.195	6.741	0.231	0.000	0.133	0.000	0.000	0.269	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	193	447	709	0	608	0	0	237	0
N.S.	1	1.04	2.42	3.83	0.00	3.29	0.00	0.00	1.28	0.00
time (sec)	N/A	1.033	7.134	0.222	0.000	0.131	0.000	0.000	0.212	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	247	363	798	0	657	0	0	296	0
N.S.	1	1.07	1.57	3.45	0.00	2.84	0.00	0.00	1.28	0.00
time (sec)	N/A	1.371	16.498	0.271	0.000	0.092	0.000	0.000	0.216	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	301	246	887	0	722	0	0	355	0
N.S.	1	1.09	0.89	3.20	0.00	2.61	0.00	0.00	1.28	0.00
time (sec)	N/A	1.703	16.268	0.251	0.000	0.116	0.000	0.000	0.229	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	355	328	976	0	771	0	0	414	0
N.S.	1	1.10	1.02	3.02	0.00	2.39	0.00	0.00	1.28	0.00
time (sec)	N/A	2.161	18.220	0.226	0.000	0.097	0.000	0.000	0.234	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	566	620	0	900	0	0	246	0
N.S.	1	1.00	2.98	3.26	0.00	4.74	0.00	0.00	1.29	0.00
time (sec)	N/A	1.298	10.334	0.339	0.000	0.101	0.000	0.000	0.225	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	214	178	1135	0	807	0	0	617	0
N.S.	1	1.04	0.87	5.54	0.00	3.94	0.00	0.00	3.01	0.00
time (sec)	N/A	1.285	1.719	0.359	0.000	0.203	0.000	0.000	0.246	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	160	157	900	0	718	0	0	434	4040
N.S.	1	1.03	1.01	5.77	0.00	4.60	0.00	0.00	2.78	25.90
time (sec)	N/A	0.908	1.393	0.234	0.000	0.203	0.000	0.000	0.258	22.719

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	111	639	0	416	0	0	264	426
N.S.	1	1.00	1.12	6.45	0.00	4.20	0.00	0.00	2.67	4.30
time (sec)	N/A	0.444	0.809	0.241	0.000	0.106	0.000	0.000	0.198	9.103

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	145	128	701	0	473	0	0	363	0
N.S.	1	1.01	0.90	4.90	0.00	3.31	0.00	0.00	2.54	0.00
time (sec)	N/A	0.715	1.038	0.257	0.000	0.166	0.000	0.000	0.200	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	199	151	746	0	534	0	0	348	0
N.S.	1	1.04	0.79	3.91	0.00	2.80	0.00	0.00	1.82	0.00
time (sec)	N/A	0.997	2.308	0.244	0.000	0.131	0.000	0.000	0.207	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	253	172	821	0	596	0	0	486	0
N.S.	1	1.07	0.73	3.46	0.00	2.51	0.00	0.00	2.05	0.00
time (sec)	N/A	1.373	2.454	0.248	0.000	0.118	0.000	0.000	0.208	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	211	179	1223	0	758	0	0	259	0
N.S.	1	1.04	0.88	6.02	0.00	3.73	0.00	0.00	1.28	0.00
time (sec)	N/A	1.280	2.977	0.286	0.000	0.170	0.000	0.000	0.237	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	154	148	868	0	443	0	0	254	0
N.S.	1	1.03	0.99	5.79	0.00	2.95	0.00	0.00	1.69	0.00
time (sec)	N/A	0.728	1.860	0.222	0.000	0.123	0.000	0.000	0.303	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	151	152	868	0	441	0	0	687	0
N.S.	1	1.02	1.03	5.86	0.00	2.98	0.00	0.00	4.64	0.00
time (sec)	N/A	0.722	1.245	0.237	0.000	0.132	0.000	0.000	0.514	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	199	166	931	0	511	0	0	1	0
N.S.	1	1.03	0.86	4.80	0.00	2.63	0.00	0.00	0.01	0.00
time (sec)	N/A	1.057	1.702	0.229	0.000	0.119	0.000	0.000	0.538	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	253	185	1012	0	569	0	0	819	0
N.S.	1	1.05	0.77	4.22	0.00	2.37	0.00	0.00	3.41	0.00
time (sec)	N/A	1.362	2.779	0.242	0.000	0.124	0.000	0.000	0.574	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	264	296	1530	0	777	0	0	163	0
N.S.	1	1.06	1.19	6.14	0.00	3.12	0.00	0.00	0.65	0.00
time (sec)	N/A	1.619	3.817	0.295	0.000	0.184	0.000	0.000	0.239	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	206	224	1096	0	459	0	0	161	0
N.S.	1	1.06	1.15	5.65	0.00	2.37	0.00	0.00	0.83	0.00
time (sec)	N/A	1.062	2.229	0.220	0.000	0.122	0.000	0.000	0.169	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	207	222	1096	0	461	0	0	153	0
N.S.	1	1.06	1.13	5.59	0.00	2.35	0.00	0.00	0.78	0.00
time (sec)	N/A	1.047	1.710	0.230	0.000	0.149	0.000	0.000	0.170	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	205	242	1096	0	460	0	0	180	0
N.S.	1	1.06	1.25	5.65	0.00	2.37	0.00	0.00	0.93	0.00
time (sec)	N/A	1.068	2.820	0.247	0.000	0.117	0.000	0.000	0.171	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	253	216	1158	0	529	0	0	181	0
N.S.	1	1.05	0.90	4.82	0.00	2.20	0.00	0.00	0.75	0.00
time (sec)	N/A	1.418	2.667	0.236	0.000	0.134	0.000	0.000	0.170	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	307	259	1239	0	588	0	0	191	0
N.S.	1	1.07	0.91	4.33	0.00	2.06	0.00	0.00	0.67	0.00
time (sec)	N/A	1.790	4.760	0.249	0.000	0.133	0.000	0.000	0.175	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	139	186	159	167	405	0	256	44	365
N.S.	1	0.69	0.93	0.79	0.83	2.01	0.00	1.27	0.22	1.82
time (sec)	N/A	0.402	0.520	0.162	0.118	0.086	0.000	0.267	0.169	1.190

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	216	196	223	210	660	0	333	55	436
N.S.	1	0.80	0.73	0.83	0.78	2.44	0.00	1.23	0.20	1.61
time (sec)	N/A	0.826	3.235	0.250	0.117	0.114	0.000	0.365	0.245	4.870

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	171	165	181	187	571	0	295	53	390
N.S.	1	0.74	0.71	0.78	0.81	2.46	0.00	1.27	0.23	1.68
time (sec)	N/A	0.564	0.773	0.176	0.119	0.100	0.000	0.326	0.168	4.566

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	141	138	159	168	489	0	266	44	367
N.S.	1	0.70	0.68	0.79	0.83	2.42	0.00	1.32	0.22	1.82
time (sec)	N/A	0.393	0.399	0.113	0.114	0.083	0.000	0.295	0.163	4.641

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	218	127	248	252	711	0	355	57	1761
N.S.	1	0.75	0.44	0.86	0.87	2.46	0.00	1.23	0.20	6.09
time (sec)	N/A	0.761	2.747	0.181	0.121	0.104	0.000	0.392	0.179	0.641

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	267	251	294	298	1096	0	469	61	5825
N.S.	1	0.78	0.73	0.86	0.87	3.20	0.00	1.37	0.18	17.03
time (sec)	N/A	1.031	2.171	0.217	0.130	0.154	0.000	0.451	0.177	5.361

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	150	150	166	172	547	0	270	44	383
N.S.	1	0.70	0.70	0.78	0.81	2.57	0.00	1.27	0.21	1.80
time (sec)	N/A	0.398	0.557	0.149	0.126	0.095	0.000	0.285	0.164	4.143

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	150	198	166	171	493	0	257	44	390
N.S.	1	0.70	0.93	0.78	0.80	2.31	0.00	1.21	0.21	1.83
time (sec)	N/A	0.414	0.474	0.143	0.118	0.105	0.000	0.311	0.160	3.939

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	315	173	0	0	0	0	0	737	0
N.S.	1	1.09	0.60	0.00	0.00	0.00	0.00	0.00	2.54	0.00
time (sec)	N/A	1.788	2.012	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	266	119	0	0	0	0	0	392	0
N.S.	1	1.30	0.58	0.00	0.00	0.00	0.00	0.00	1.91	0.00
time (sec)	N/A	1.187	1.617	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	164	81	0	0	0	0	0	319	0
N.S.	1	1.24	0.61	0.00	0.00	0.00	0.00	0.00	2.42	0.00
time (sec)	N/A	0.758	1.004	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	97	52	0	0	0	0	0	117	0
N.S.	1	1.39	0.74	0.00	0.00	0.00	0.00	0.00	1.67	0.00
time (sec)	N/A	0.392	0.330	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	164	137	0	0	0	0	0	166	0
N.S.	1	0.98	0.82	0.00	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.560	1.218	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	227	175	0	0	0	0	0	81	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.952	2.080	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	319	227	0	0	0	0	0	359	0
N.S.	1	1.04	0.74	0.00	0.00	0.00	0.00	0.00	1.17	0.00
time (sec)	N/A	1.492	2.845	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	386	405	277	0	0	0	0	0	121	0
N.S.	1	1.05	0.72	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	2.035	2.291	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	315	326	0	0	0	0	0	0	270	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	1.634	0.000	0.000	0.000	0.000	0.000	0.000	0.385	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	235	0	0	0	0	0	0	199	0
N.S.	1	1.05	0.00	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	1.129	0.000	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	159	0	0	0	0	0	0	163	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.700	0.000	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F(-2)	F	F	F(-1)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	222	0	0	0	0	0	0	537	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	2.49	0.00
time (sec)	N/A	1.050	0.000	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F(-2)	F	F	F(-1)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	284	296	0	0	0	0	0	0	0	0
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.566	0.000	0.000	0.000	0.000	0.000	0.000	147.353	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F(-2)	F	F	F(-1)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	381	0	0	0	0	0	0	155	0
N.S.	1	1.05	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	2.224	0.000	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	174	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.634	0.000	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	260	189	0	0	0	0	0	53	0
N.S.	1	1.06	0.77	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.254	2.788	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	172	137	0	0	0	0	0	53	0
N.S.	1	1.05	0.84	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.758	0.962	0.000	0.000	0.000	0.000	0.000	0.280	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	117	87	0	0	0	0	0	51	0
N.S.	1	1.05	0.78	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.455	0.506	0.000	0.000	0.000	0.000	0.000	0.329	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	84	60	0	0	0	0	0	78	0
N.S.	1	1.08	0.77	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.302	0.192	0.000	0.000	0.000	0.000	0.000	0.328	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	103	124	0	0	0	0	0	55	0
N.S.	1	1.06	1.28	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.517	0.799	0.000	0.000	0.000	0.000	0.000	0.316	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	144	156	0	0	0	0	0	59	0
N.S.	1	1.10	1.19	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.746	1.889	0.000	0.000	0.000	0.000	0.000	0.362	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	205	221	0	0	0	0	0	59	0
N.S.	1	1.11	1.19	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	1.153	2.041	0.000	0.000	0.000	0.000	0.000	0.357	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	383	421	0	0	0	0	0	0	67	0
N.S.	1	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	2.041	0.000	0.000	0.000	0.000	0.000	0.000	0.349	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	330	0	0	0	0	0	0	65	0
N.S.	1	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.404	0.000	0.000	0.000	0.000	0.000	0.000	0.322	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	247	0	0	0	0	0	0	170	0
N.S.	1	1.15	0.00	0.00	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.961	0.000	0.000	0.000	0.000	0.000	0.000	0.326	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	176	0	0	0	0	0	0	172	0
N.S.	1	1.11	0.00	0.00	0.00	0.00	0.00	0.00	1.09	0.00
time (sec)	N/A	0.652	0.000	0.000	0.000	0.000	0.000	0.000	0.262	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F(-1)	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	220	0	0	0	0	0	0	350	0
N.S.	1	1.13	0.00	0.00	0.00	0.00	0.00	0.00	1.80	0.00
time (sec)	N/A	0.943	0.000	0.000	0.000	0.000	0.000	0.000	0.263	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	280	0	0	0	0	0	0	218	0
N.S.	1	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	1.358	0.000	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	124	88	86	85	136	119	81	84
N.S.	1	1.00	1.43	1.01	0.99	0.98	1.56	1.37	0.93	0.97
time (sec)	N/A	0.515	0.394	0.079	0.114	0.095	0.124	0.275	0.215	3.528

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	69	66	66	104	82	65	63
N.S.	1	1.00	1.03	1.06	1.02	1.02	1.60	1.26	1.00	0.97
time (sec)	N/A	0.373	0.270	0.057	0.111	0.108	0.107	0.246	0.200	3.555

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	59	47	50	50	73	53	42	55
N.S.	1	1.00	1.40	1.12	1.19	1.19	1.74	1.26	1.00	1.31
time (sec)	N/A	0.261	0.026	0.044	0.115	0.088	0.089	0.227	0.288	3.429

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	39	36	47	52	59	78	55	96	69
N.S.	1	1.05	0.97	1.27	1.41	1.59	2.11	1.49	2.59	1.86
time (sec)	N/A	0.413	0.047	0.125	0.109	0.108	0.216	0.263	0.310	3.582

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	45	62	65	68	73	121	76	93	87
N.S.	1	1.05	1.44	1.51	1.58	1.70	2.81	1.77	2.16	2.02
time (sec)	N/A	0.386	0.029	0.129	0.107	0.103	0.313	0.274	0.229	3.565

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	107	89	86	95	148	95	161	108
N.S.	1	1.03	1.62	1.35	1.30	1.44	2.24	1.44	2.44	1.64
time (sec)	N/A	0.515	0.031	0.165	0.114	0.098	0.750	0.368	0.230	3.569

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	89	129	106	104	121	178	112	163	127
N.S.	1	1.02	1.48	1.22	1.20	1.39	2.05	1.29	1.87	1.46
time (sec)	N/A	0.642	0.047	0.191	0.104	0.086	0.846	0.352	0.238	3.576

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	110	146	124	122	138	209	129	219	145
N.S.	1	1.02	1.35	1.15	1.13	1.28	1.94	1.19	2.03	1.34
time (sec)	N/A	0.791	0.040	0.216	0.110	0.081	1.364	0.398	0.241	3.593

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	148	160	221	148	147	146	246	211	130	151
N.S.	1	1.08	1.49	1.00	0.99	0.99	1.66	1.43	0.88	1.02
time (sec)	N/A	0.838	6.147	0.089	0.109	0.086	0.157	0.339	0.218	3.415

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	172	120	120	119	192	161	104	121
N.S.	1	1.00	1.54	1.07	1.07	1.06	1.71	1.44	0.93	1.08
time (sec)	N/A	0.536	1.243	0.077	0.107	0.088	0.126	0.254	0.248	3.294

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	96	90	91	91	143	105	78	91
N.S.	1	1.00	1.10	1.03	1.05	1.05	1.64	1.21	0.90	1.05
time (sec)	N/A	0.406	0.310	0.062	0.104	0.077	0.115	0.271	0.297	3.320

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	72	93	80	85	92	129	90	169	90
N.S.	1	1.03	1.33	1.14	1.21	1.31	1.84	1.29	2.41	1.29
time (sec)	N/A	0.455	0.197	0.134	0.113	0.098	0.314	0.260	0.233	3.448

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	100	84	93	112	167	101	169	100
N.S.	1	1.03	1.39	1.17	1.29	1.56	2.32	1.40	2.35	1.39
time (sec)	N/A	0.468	0.194	0.167	0.108	0.114	0.593	0.363	0.217	3.503

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	91	123	107	120	122	214	129	182	127
N.S.	1	1.03	1.40	1.22	1.36	1.39	2.43	1.47	2.07	1.44
time (sec)	N/A	0.635	0.233	0.210	0.118	0.103	0.805	0.414	0.224	3.425

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	121	152	136	149	157	260	156	223	156
N.S.	1	1.03	1.29	1.15	1.26	1.33	2.20	1.32	1.89	1.32
time (sec)	N/A	0.790	0.817	0.247	0.114	0.080	1.117	0.478	0.198	3.369

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	152	180	162	175	191	313	182	263	182
N.S.	1	1.01	1.19	1.07	1.16	1.26	2.07	1.21	1.74	1.21
time (sec)	N/A	1.013	2.035	0.282	0.105	0.116	2.072	0.226	0.210	3.350

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	201	216	241	207	214	213	384	321	184	217
N.S.	1	1.07	1.20	1.03	1.06	1.06	1.91	1.60	0.92	1.08
time (sec)	N/A	1.093	1.513	0.126	0.106	0.082	0.192	0.394	0.193	3.350

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	209	180	179	178	311	251	153	181
N.S.	1	1.00	1.27	1.09	1.08	1.08	1.88	1.52	0.93	1.10
time (sec)	N/A	0.754	1.029	0.102	0.113	0.089	0.159	0.367	0.247	3.376

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	130	141	143	142	240	182	115	142
N.S.	1	1.00	0.93	1.01	1.02	1.01	1.71	1.30	0.82	1.01
time (sec)	N/A	0.605	0.671	0.080	0.104	0.081	0.137	0.249	0.263	3.367

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	119	115	121	124	133	204	140	459	118
N.S.	1	1.02	0.98	1.03	1.06	1.14	1.74	1.20	3.92	1.01
time (sec)	N/A	0.736	0.401	0.185	0.104	0.093	0.465	0.485	0.237	3.549

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	121	113	118	125	145	223	134	262	114
N.S.	1	1.02	0.95	0.99	1.05	1.22	1.87	1.13	2.20	0.96
time (sec)	N/A	0.763	0.386	0.218	0.130	0.120	0.791	0.445	0.268	3.547

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	129	126	136	142	162	262	151	264	135
N.S.	1	1.02	0.99	1.07	1.12	1.28	2.06	1.19	2.08	1.06
time (sec)	N/A	0.774	0.292	0.218	0.108	0.093	1.047	0.273	12.171	3.547

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	164	172	180	181	332	187	244	169
N.S.	1	1.05	1.06	1.12	1.17	1.18	2.16	1.21	1.58	1.10
time (sec)	N/A	1.053	0.811	0.243	0.105	0.085	2.061	0.288	19.119	3.551

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	204	199	209	215	225	400	221	33	204
N.S.	1	1.07	1.04	1.09	1.13	1.18	2.09	1.16	0.17	1.07
time (sec)	N/A	1.322	0.507	0.293	0.117	0.119	2.677	0.314	200.024	3.589

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	242	237	243	250	266	471	258	33	238
N.S.	1	1.04	1.02	1.04	1.07	1.14	2.02	1.11	0.14	1.02
time (sec)	N/A	1.604	0.779	0.334	0.112	0.129	4.001	0.307	200.024	3.665

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	263	275	290	274	290	289	536	451	249	300
N.S.	1	1.05	1.10	1.04	1.10	1.10	2.04	1.71	0.95	1.14
time (sec)	N/A	1.378	3.948	0.155	0.108	0.089	0.266	0.472	0.177	3.421

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	257	242	246	245	437	361	206	251
N.S.	1	1.00	1.14	1.07	1.09	1.08	1.93	1.60	0.91	1.11
time (sec)	N/A	1.036	2.323	0.132	0.112	0.118	0.199	0.457	0.186	3.416

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	240	201	202	201	347	272	165	205
N.S.	1	1.00	1.19	1.00	1.00	1.00	1.72	1.35	0.82	1.01
time (sec)	N/A	0.858	2.586	0.105	0.106	0.119	0.168	0.314	0.231	3.474

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	174	149	172	175	185	291	217	674	151
N.S.	1	1.01	0.87	1.00	1.02	1.08	1.69	1.26	3.92	0.88
time (sec)	N/A	1.097	1.023	0.222	0.105	0.133	0.746	0.497	0.243	3.588

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	177	134	159	164	193	289	184	595	142
N.S.	1	1.01	0.77	0.91	0.94	1.10	1.65	1.05	3.40	0.81
time (sec)	N/A	1.168	0.755	0.252	0.107	0.101	0.992	0.336	0.194	3.601

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	192	140	168	173	199	309	184	385	149
N.S.	1	1.03	0.75	0.90	0.93	1.07	1.66	0.99	2.07	0.80
time (sec)	N/A	1.246	0.469	0.260	0.108	0.098	2.043	0.337	4.838	3.572

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	189	167	195	202	222	369	209	33	177
N.S.	1	1.01	0.89	1.04	1.08	1.19	1.97	1.12	0.18	0.95
time (sec)	N/A	1.186	0.844	0.278	0.106	0.106	2.615	0.335	200.018	3.620

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	238	211	240	246	249	459	252	33	218
N.S.	1	1.06	0.94	1.07	1.09	1.11	2.04	1.12	0.15	0.97
time (sec)	N/A	1.571	0.661	0.322	0.113	0.113	3.915	0.358	200.022	3.627

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	288	257	284	289	300	546	297	33	263
N.S.	1	1.05	0.94	1.04	1.06	1.10	2.00	1.09	0.12	0.96
time (sec)	N/A	1.899	1.215	0.380	0.121	0.096	5.624	0.368	200.022	3.774

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	338	299	330	333	350	643	342	33	307
N.S.	1	1.05	0.93	1.02	1.03	1.08	1.99	1.06	0.10	0.95
time (sec)	N/A	2.276	0.916	0.471	0.115	0.085	14.420	0.433	200.021	4.071

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	144	138	127	130	190	1297	146	27	144
N.S.	1	1.13	1.09	1.00	1.02	1.50	10.21	1.15	0.21	1.13
time (sec)	N/A	0.925	1.068	0.148	0.110	0.126	0.807	0.261	0.196	3.882

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	108	118	101	109	149	1015	119	15	117
N.S.	1	1.07	1.17	1.00	1.08	1.48	10.05	1.18	0.15	1.16
time (sec)	N/A	0.625	0.459	0.110	0.109	0.104	0.625	0.256	0.197	3.552

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	75	98	87	94	110	700	100	16	100
N.S.	1	0.94	1.22	1.09	1.18	1.38	8.75	1.25	0.20	1.25
time (sec)	N/A	0.509	0.132	0.102	0.111	0.133	0.550	0.229	0.163	3.813

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	66	82	88	76	524	100	1	93
N.S.	1	1.00	1.14	1.41	1.52	1.31	9.03	1.72	0.02	1.60
time (sec)	N/A	0.335	0.084	0.068	0.109	0.080	0.459	0.168	0.188	3.805

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	82	113	95	107	118	952	118	31	115
N.S.	1	1.02	1.41	1.19	1.34	1.48	11.90	1.48	0.39	1.44
time (sec)	N/A	0.451	0.313	0.202	0.111	0.100	1.012	0.242	0.173	4.095

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	111	138	122	131	177	2071	146	17	140
N.S.	1	1.08	1.34	1.18	1.27	1.72	20.11	1.42	0.17	1.36
time (sec)	N/A	0.679	0.631	0.207	0.107	0.110	2.021	0.292	0.169	5.008

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	153	163	152	158	234	2599	178	67	175
N.S.	1	1.12	1.19	1.11	1.15	1.71	18.97	1.30	0.49	1.28
time (sec)	N/A	1.052	0.979	0.286	0.107	0.128	3.243	0.323	0.183	5.485

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	192	194	189	200	293	3016	219	28	208
N.S.	1	1.14	1.15	1.12	1.18	1.73	17.85	1.30	0.17	1.23
time (sec)	N/A	1.474	1.941	0.315	0.152	0.121	5.665	0.327	0.195	5.760

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	225	193	172	220	434	4534	242	81	210
N.S.	1	1.08	0.93	0.83	1.06	2.09	21.80	1.16	0.39	1.01
time (sec)	N/A	1.196	4.326	0.164	0.119	0.131	1.109	0.303	0.226	4.424

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	171	146	155	197	311	3485	217	55	165
N.S.	1	1.09	0.93	0.99	1.25	1.98	22.20	1.38	0.35	1.05
time (sec)	N/A	0.799	1.979	0.125	0.113	0.141	0.977	0.276	0.203	3.756

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	126	140	145	185	221	2987	213	47	163
N.S.	1	1.10	1.22	1.26	1.61	1.92	25.97	1.85	0.41	1.42
time (sec)	N/A	0.573	1.571	0.122	0.111	0.087	0.866	0.214	0.231	3.606

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	122	190	141	177	222	2878	205	48	153
N.S.	1	1.10	1.71	1.27	1.59	2.00	25.93	1.85	0.43	1.38
time (sec)	N/A	0.528	1.702	0.097	0.124	0.092	0.858	0.271	0.206	3.678

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	161	183	163	208	323	4488	238	105	180
N.S.	1	1.18	1.34	1.19	1.52	2.36	32.76	1.74	0.77	1.31
time (sec)	N/A	0.848	0.656	0.305	0.109	0.116	1.872	0.323	0.225	5.568

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	222	193	196	262	465	8145	296	169	230
N.S.	1	1.16	1.01	1.02	1.36	2.42	42.42	1.54	0.88	1.20
time (sec)	N/A	1.221	2.534	0.480	0.111	0.129	2.716	0.363	0.218	6.808

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	288	220	227	325	590	9896	380	234	284
N.S.	1	1.15	0.88	0.91	1.30	2.36	39.58	1.52	0.94	1.14
time (sec)	N/A	1.712	3.406	0.581	0.119	0.158	4.142	0.451	30.153	8.187

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	363	275	263	389	890	0	385	312	335
N.S.	1	1.10	0.83	0.79	1.18	2.69	0.00	1.16	0.94	1.01
time (sec)	N/A	1.868	3.522	0.254	0.116	0.171	0.000	0.373	0.174	5.013

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	285	222	242	366	666	0	361	277	307
N.S.	1	1.14	0.89	0.97	1.46	2.66	0.00	1.44	1.11	1.23
time (sec)	N/A	1.290	5.596	0.206	0.113	0.200	0.000	0.336	0.180	4.014

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	216	288	223	333	478	0	333	198	280
N.S.	1	1.14	1.52	1.18	1.76	2.53	0.00	1.76	1.05	1.48
time (sec)	N/A	0.994	3.866	0.166	0.126	0.121	0.000	0.301	0.193	3.865

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	201	188	213	330	488	0	323	259	282
N.S.	1	1.12	1.05	1.19	1.84	2.73	0.00	1.80	1.45	1.58
time (sec)	N/A	0.845	2.577	0.185	0.123	0.116	0.000	0.260	0.167	3.804

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	197	243	208	321	482	0	315	212	279
N.S.	1	1.13	1.39	1.19	1.83	2.75	0.00	1.80	1.21	1.59
time (sec)	N/A	0.832	2.795	0.145	0.113	0.112	0.000	0.298	0.181	3.688

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	257	254	243	372	683	0	373	541	315
N.S.	1	1.20	1.18	1.13	1.73	3.18	0.00	1.73	2.52	1.47
time (sec)	N/A	1.341	2.554	0.683	0.135	0.143	0.000	0.334	0.217	5.674

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	333	288	289	454	917	0	466	726	380
N.S.	1	1.16	1.00	1.01	1.58	3.20	0.00	1.62	2.53	1.32
time (sec)	N/A	1.766	6.279	1.029	0.120	0.242	0.000	0.538	3.006	8.689

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	406	320	320	541	1065	0	563	33	434
N.S.	1	1.15	0.91	0.91	1.54	3.03	0.00	1.60	0.09	1.23
time (sec)	N/A	2.404	6.328	1.165	0.121	0.254	0.000	0.237	200.027	10.334

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	402	307	343	583	1113	0	532	645	486
N.S.	1	1.15	0.87	0.98	1.66	3.17	0.00	1.52	1.84	1.38
time (sec)	N/A	1.874	2.338	0.343	0.137	0.198	0.000	0.441	0.253	4.607

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	346	465	318	550	813	0	503	552	470
N.S.	1	1.16	1.56	1.07	1.85	2.73	0.00	1.69	1.85	1.58
time (sec)	N/A	1.536	6.220	0.277	0.120	0.106	0.000	0.398	0.264	4.379

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	300	411	301	526	836	0	481	548	446
N.S.	1	1.15	1.57	1.15	2.02	3.20	0.00	1.84	2.10	1.71
time (sec)	N/A	1.309	6.215	0.280	0.125	0.125	0.000	0.335	0.196	4.336

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	283	248	287	523	838	0	481	546	447
N.S.	1	1.13	0.99	1.15	2.09	3.35	0.00	1.92	2.18	1.79
time (sec)	N/A	1.229	0.966	0.276	0.119	0.142	0.000	0.408	0.215	4.022

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	280	327	284	514	815	0	472	543	442
N.S.	1	1.13	1.32	1.15	2.08	3.30	0.00	1.91	2.20	1.79
time (sec)	N/A	1.184	6.206	0.240	0.128	0.150	0.000	0.334	0.193	3.922

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	358	308	335	580	1126	0	537	1290	484
N.S.	1	1.19	1.02	1.11	1.92	3.73	0.00	1.78	4.27	1.60
time (sec)	N/A	1.945	2.356	1.030	0.128	0.196	0.000	0.416	0.202	7.282

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	459	357	400	698	1510	0	665	1617	576
N.S.	1	1.15	0.89	1.00	1.75	3.78	0.00	1.67	4.05	1.44
time (sec)	N/A	2.494	4.358	2.087	0.131	0.341	0.000	0.289	0.212	8.619

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	545	417	429	815	1732	0	798	33	664
N.S.	1	1.14	0.87	0.90	1.71	3.63	0.00	1.67	0.07	1.39
time (sec)	N/A	3.168	6.484	1.432	0.135	0.312	0.000	0.321	200.027	9.976

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	30	30	31	53	32	28	28
N.S.	1	1.00	0.90	1.03	1.03	1.07	1.83	1.10	0.97	0.97
time (sec)	N/A	0.256	0.022	0.053	0.105	0.101	0.492	0.201	0.158	3.461

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	25	17	22	19	36	24	16	16
N.S.	1	1.00	1.56	1.06	1.38	1.19	2.25	1.50	1.00	1.00
time (sec)	N/A	0.206	0.010	0.036	0.110	0.093	0.363	0.197	0.162	3.508

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	17	19	37	17	17	17
N.S.	1	1.00	1.00	1.38	1.31	1.46	2.85	1.31	1.31	1.31
time (sec)	N/A	0.190	0.008	0.034	0.107	0.089	0.336	0.195	0.164	3.427

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	10	3	2	10	3	3
N.S.	1	1.00	1.00	1.33	3.33	1.00	0.67	3.33	1.00	1.00
time (sec)	N/A	0.138	0.000	0.036	0.104	0.069	0.076	0.206	0.157	3.297

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	14	12	18	29	20	49	33	32	27
N.S.	1	1.17	1.00	1.50	2.42	1.67	4.08	2.75	2.67	2.25
time (sec)	N/A	0.191	0.006	0.063	0.103	0.079	0.469	0.211	0.182	3.400

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	30	17	23	42	37	25	16	16
N.S.	1	1.00	1.76	1.00	1.35	2.47	2.18	1.47	0.94	0.94
time (sec)	N/A	0.201	0.016	0.066	0.105	0.073	0.473	0.238	0.163	3.378

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	32	30	30	40	53	76	55	68	37
N.S.	1	1.07	1.00	1.00	1.33	1.77	2.53	1.83	2.27	1.23
time (sec)	N/A	0.265	0.011	0.083	0.107	0.083	1.023	0.222	0.162	3.385

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	34	28	38	90	49	38	29	32
N.S.	1	0.94	1.10	0.90	1.23	2.90	1.58	1.23	0.94	1.03
time (sec)	N/A	0.259	0.014	0.099	0.111	0.088	1.311	0.287	0.164	3.428

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	115	108	93	104	144	782	114	109	114
N.S.	1	1.13	1.06	0.91	1.02	1.41	7.67	1.12	1.07	1.12
time (sec)	N/A	0.757	0.266	0.108	0.113	0.104	0.949	0.246	0.165	3.580

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	89	92	80	89	119	660	98	81	98
N.S.	1	1.07	1.11	0.96	1.07	1.43	7.95	1.18	0.98	1.18
time (sec)	N/A	0.582	0.264	0.096	0.108	0.092	0.790	0.240	0.154	3.548

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	77	79	59	75	95	442	82	52	81
N.S.	1	0.95	0.98	0.73	0.93	1.17	5.46	1.01	0.64	1.00
time (sec)	N/A	0.496	0.028	0.079	0.107	0.119	0.695	0.219	0.163	3.614

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	67	51	71	65	282	82	48	79
N.S.	1	1.00	1.40	1.06	1.48	1.35	5.88	1.71	1.00	1.65
time (sec)	N/A	0.334	0.078	0.061	0.111	0.106	0.597	0.220	0.158	3.533

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	77	51	72	64	272	83	49	76
N.S.	1	1.00	1.64	1.09	1.53	1.36	5.79	1.77	1.04	1.62
time (sec)	N/A	0.310	0.015	0.059	0.111	0.079	0.702	0.239	0.198	3.435

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	70	79	77	88	104	672	99	106	99
N.S.	1	1.01	1.14	1.12	1.28	1.51	9.74	1.43	1.54	1.43
time (sec)	N/A	0.414	0.100	0.142	0.107	0.090	1.708	0.249	0.170	3.477

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	92	97	95	105	147	1137	118	170	113
N.S.	1	1.08	1.14	1.12	1.24	1.73	13.38	1.39	2.00	1.33
time (sec)	N/A	0.608	0.275	0.174	0.114	0.121	2.898	0.285	0.192	3.548

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	125	107	115	130	192	1401	143	235	143
N.S.	1	1.12	0.96	1.03	1.16	1.71	12.51	1.28	2.10	1.28
time (sec)	N/A	0.836	0.441	0.206	0.116	0.103	3.946	0.293	30.004	3.646

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	62	33	35	44	39	41	32	49
N.S.	1	1.00	2.48	1.32	1.40	1.76	1.56	1.64	1.28	1.96
time (sec)	N/A	0.279	0.042	0.062	0.108	0.084	0.116	0.239	0.254	3.437

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	65	85	95	78	235	104	88	112
N.S.	1	1.00	1.12	1.47	1.64	1.34	4.05	1.79	1.52	1.93
time (sec)	N/A	0.341	0.083	0.099	0.112	0.097	0.465	0.221	0.202	3.834

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	112	187	125	161	191	1348	171	298	152
N.S.	1	1.11	1.85	1.24	1.59	1.89	13.35	1.69	2.95	1.50
time (sec)	N/A	0.528	1.802	0.114	0.107	0.089	0.730	0.259	0.169	3.804

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	260	212	937	0	1312	0	0	49	1093
N.S.	1	1.12	0.91	4.02	0.00	5.63	0.00	0.00	0.21	4.69
time (sec)	N/A	1.473	1.963	0.450	0.000	0.113	0.000	0.000	0.205	57.798

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	180	169	873	0	1265	0	0	49	938
N.S.	1	0.97	0.91	4.69	0.00	6.80	0.00	0.00	0.26	5.04
time (sec)	N/A	1.030	1.425	0.158	0.000	0.108	0.000	0.000	0.191	19.960

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	145	140	829	0	1228	0	0	47	864
N.S.	1	0.99	0.96	5.68	0.00	8.41	0.00	0.00	0.32	5.92
time (sec)	N/A	0.750	0.363	0.140	0.000	0.105	0.000	0.000	0.195	9.744

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	103	120	812	0	1199	0	0	38	845
N.S.	1	0.84	0.98	6.66	0.00	9.83	0.00	0.00	0.31	6.93
time (sec)	N/A	0.592	0.093	0.123	0.000	0.107	0.000	0.000	0.194	6.066

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	118	219	976	0	2449	0	0	51	9785
N.S.	1	0.90	1.67	7.45	0.00	18.69	0.00	0.00	0.39	74.69
time (sec)	N/A	1.032	0.445	0.145	0.000	0.356	0.000	0.000	0.353	5.837

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	156	235	1029	0	2576	0	0	33	10987
N.S.	1	0.93	1.41	6.16	0.00	15.43	0.00	0.00	0.20	65.79
time (sec)	N/A	1.343	1.741	0.164	0.000	0.730	0.000	0.000	200.034	4.878

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	218	271	1145	0	2688	0	0	33	14195
N.S.	1	1.00	1.24	5.23	0.00	12.27	0.00	0.00	0.15	64.82
time (sec)	N/A	1.810	3.211	0.166	0.000	2.928	0.000	0.000	200.014	5.248

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	286	566	1295	0	2766	0	0	33	16796
N.S.	1	1.03	2.03	4.64	0.00	9.91	0.00	0.00	0.12	60.20
time (sec)	N/A	2.445	6.298	0.165	0.000	8.867	0.000	0.000	200.016	5.556

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	209	252	1697	0	3148	0	0	79	2993
N.S.	1	0.98	1.18	7.93	0.00	14.71	0.00	0.00	0.37	13.99
time (sec)	N/A	1.340	1.809	0.225	0.000	0.319	0.000	0.000	0.193	72.162

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	160	192	1668	0	3102	0	0	77	2868
N.S.	1	0.91	1.10	9.53	0.00	17.73	0.00	0.00	0.44	16.39
time (sec)	N/A	0.947	0.916	0.151	0.000	0.370	0.000	0.000	0.195	28.913

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	131	140	1657	0	3058	0	0	68	2823
N.S.	1	0.87	0.93	11.05	0.00	20.39	0.00	0.00	0.45	18.82
time (sec)	N/A	0.766	0.361	0.135	0.000	0.291	0.000	0.000	0.212	16.246

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	139	144	1654	0	6162	0	0	87	20255
N.S.	1	0.91	0.95	10.88	0.00	40.54	0.00	0.00	0.57	133.26
time (sec)	N/A	1.296	0.248	0.139	0.000	2.409	0.000	0.000	0.361	6.953

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	158	282	1689	0	6245	0	0	33	21319
N.S.	1	0.93	1.67	9.99	0.00	36.95	0.00	0.00	0.20	126.15
time (sec)	N/A	1.343	0.407	0.150	0.000	3.202	0.000	0.000	200.031	5.918

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	218	195	1805	0	6354	0	0	33	23016
N.S.	1	1.00	0.89	8.24	0.00	29.01	0.00	0.00	0.15	105.10
time (sec)	N/A	1.882	1.752	0.164	0.000	5.913	0.000	0.000	200.021	5.937

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	282	241	1985	0	6449	0	0	33	25789
N.S.	1	1.01	0.87	7.14	0.00	23.20	0.00	0.00	0.12	92.77
time (sec)	N/A	2.373	3.851	0.152	0.000	14.716	0.000	0.000	200.022	6.427

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	246	296	2438	0	4963	0	0	109	4139
N.S.	1	0.98	1.17	9.67	0.00	19.69	0.00	0.00	0.43	16.42
time (sec)	N/A	1.636	3.255	0.234	0.000	0.713	0.000	0.000	0.204	179.323

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	216	258	2412	0	4916	0	0	107	3932
N.S.	1	1.01	1.21	11.32	0.00	23.08	0.00	0.00	0.50	18.46
time (sec)	N/A	1.165	1.129	0.158	0.000	0.686	0.000	0.000	0.208	82.180

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	169	233	2392	0	4855	0	0	98	3863
N.S.	1	0.90	1.24	12.72	0.00	25.82	0.00	0.00	0.52	20.55
time (sec)	N/A	1.040	0.765	0.133	0.000	0.705	0.000	0.000	0.202	33.629

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	169	177	2373	0	9754	0	0	123	29441
N.S.	1	0.93	0.97	13.04	0.00	53.59	0.00	0.00	0.68	161.76
time (sec)	N/A	1.691	0.810	0.148	0.000	10.509	0.000	0.000	4.820	9.957

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F(-1)	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	186	400	2390	0	9812	0	0	33	31186
N.S.	1	0.95	2.04	12.19	0.00	50.06	0.00	0.00	0.17	159.11
time (sec)	N/A	1.777	0.727	0.158	0.000	12.981	0.000	0.000	200.035	7.219

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	B	F(-1)	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	214	448	2488	0	9885	0	0	33	32561
N.S.	1	0.97	2.04	11.31	0.00	44.93	0.00	0.00	0.15	148.00
time (sec)	N/A	1.873	1.628	0.170	0.000	15.975	0.000	0.000	200.016	7.347

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	F(-2)	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	276	40192	2668	0	9962	0	0	33	33949
N.S.	1	1.00	145.10	9.63	0.00	35.96	0.00	0.00	0.12	122.56
time (sec)	N/A	2.337	35.014	0.164	0.000	24.744	0.000	0.000	200.020	7.475

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	F(-2)	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	357	43365	2914	0	10104	0	0	33	36736
N.S.	1	1.04	126.80	8.52	0.00	29.54	0.00	0.00	0.10	107.42
time (sec)	N/A	3.091	35.791	0.184	0.000	53.337	0.000	0.000	200.020	8.229

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	121	193	1375	0	1472	0	0	98	3441
N.S.	1	0.80	1.28	9.11	0.00	9.75	0.00	0.00	0.65	22.79
time (sec)	N/A	0.689	0.954	0.113	0.000	0.114	0.000	0.000	0.190	26.919

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	432	183	986	0	956	0	0	45	2529
N.S.	1	1.39	0.59	3.18	0.00	3.08	0.00	0.00	0.15	8.16
time (sec)	N/A	0.737	0.326	0.112	0.000	0.099	0.000	0.000	0.186	14.988

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	453	157	814	0	719	0	0	39	581
N.S.	1	1.41	0.49	2.54	0.00	2.24	0.00	0.00	0.12	1.81
time (sec)	N/A	0.785	0.164	0.106	0.000	0.103	0.000	0.000	0.188	5.765

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	221	170	2022	0	1767	0	0	22	3054
N.S.	1	1.04	0.80	9.49	0.00	8.30	0.00	0.00	0.10	14.34
time (sec)	N/A	1.175	2.755	0.251	0.000	0.136	0.000	0.000	0.182	11.190

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	157	139	1958	0	1725	0	0	22	2981
N.S.	1	0.95	0.84	11.80	0.00	10.39	0.00	0.00	0.13	17.96
time (sec)	N/A	0.849	1.112	0.168	0.000	0.126	0.000	0.000	0.192	7.338

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	109	118	1919	0	1706	0	0	20	2930
N.S.	1	0.88	0.95	15.48	0.00	13.76	0.00	0.00	0.16	23.63
time (sec)	N/A	0.573	0.367	0.133	0.000	0.118	0.000	0.000	0.191	6.329

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	83	101	1894	0	1669	0	0	13	2909
N.S.	1	0.81	0.99	18.57	0.00	16.36	0.00	0.00	0.13	28.52
time (sec)	N/A	0.416	0.080	0.488	0.000	0.117	0.000	0.000	0.176	5.514

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	118	170	4002	0	3455	0	0	20	7099
N.S.	1	0.90	1.30	30.55	0.00	26.37	0.00	0.00	0.15	54.19
time (sec)	N/A	0.915	0.501	0.147	0.000	0.776	0.000	0.000	0.225	7.435

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	164	201	4057	0	3579	0	0	22	9790
N.S.	1	0.97	1.19	24.01	0.00	21.18	0.00	0.00	0.13	57.93
time (sec)	N/A	1.250	2.015	0.167	0.000	2.001	0.000	0.000	0.663	5.288

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	230	360	4175	0	3685	0	0	22	13182
N.S.	1	1.03	1.61	18.64	0.00	16.45	0.00	0.00	0.10	58.85
time (sec)	N/A	1.723	6.197	0.161	0.000	7.174	0.000	0.000	0.349	6.071

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	273	300	3751	0	4421	0	0	34	5811
N.S.	1	1.03	1.14	14.21	0.00	16.75	0.00	0.00	0.13	22.01
time (sec)	N/A	1.457	2.434	0.244	0.000	0.582	0.000	0.000	0.193	19.110

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	186	248	3709	0	4379	0	0	34	5768
N.S.	1	1.11	1.49	22.21	0.00	26.22	0.00	0.00	0.20	34.54
time (sec)	N/A	0.995	0.911	0.162	0.000	0.656	0.000	0.000	0.193	11.200

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	151	229	3684	0	4329	0	0	32	5742
N.S.	1	1.07	1.62	26.13	0.00	30.70	0.00	0.00	0.23	40.72
time (sec)	N/A	0.680	0.954	0.141	0.000	0.561	0.000	0.000	0.176	10.003

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	144	113	3679	0	4318	0	0	59	5737
N.S.	1	1.04	0.82	26.66	0.00	31.29	0.00	0.00	0.43	41.57
time (sec)	N/A	0.625	0.178	0.132	0.000	0.551	0.000	0.000	0.184	9.765

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	195	186	7982	0	8817	0	0	32	26139
N.S.	1	1.14	1.09	46.68	0.00	51.56	0.00	0.00	0.19	152.86
time (sec)	N/A	1.395	0.842	0.148	0.000	3.946	0.000	0.000	0.183	10.468

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	253	208	8043	0	9072	0	0	34	38368
N.S.	1	1.16	0.95	36.73	0.00	41.42	0.00	0.00	0.16	175.20
time (sec)	N/A	1.812	2.703	0.170	0.000	16.560	0.000	0.000	0.554	7.425

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	324	409	8164	0	9216	0	0	33	42371
N.S.	1	1.14	1.44	28.65	0.00	32.34	0.00	0.00	0.12	148.67
time (sec)	N/A	2.385	6.176	0.166	0.000	40.935	0.000	0.000	200.038	7.911

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	395	450	4592	0	7358	0	0	50	9547
N.S.	1	1.06	1.21	12.38	0.00	19.83	0.00	0.00	0.13	25.73
time (sec)	N/A	2.242	6.201	0.272	0.000	2.075	0.000	0.000	0.270	40.449

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	298	309	4543	0	7284	0	0	50	9498
N.S.	1	1.14	1.18	17.41	0.00	27.91	0.00	0.00	0.19	36.39
time (sec)	N/A	1.555	2.361	0.187	0.000	2.095	0.000	0.000	0.179	30.291

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	225	260	4487	0	7218	0	0	50	9468
N.S.	1	1.14	1.31	22.66	0.00	36.45	0.00	0.00	0.25	47.82
time (sec)	N/A	1.101	0.726	0.156	0.000	2.090	0.000	0.000	0.175	21.525

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	213	325	4477	0	7209	0	0	48	9464
N.S.	1	1.13	1.73	23.81	0.00	38.35	0.00	0.00	0.26	50.34
time (sec)	N/A	0.942	2.415	0.151	0.000	2.287	0.000	0.000	0.180	20.533

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	206	115	4472	0	7181	0	0	150	9457
N.S.	1	1.11	0.62	24.17	0.00	38.82	0.00	0.00	0.81	51.12
time (sec)	N/A	0.889	0.145	0.145	0.000	2.105	0.000	0.000	0.176	21.148

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	272	242	12870	0	14697	0	0	48	45681
N.S.	1	1.21	1.08	57.46	0.00	65.61	0.00	0.00	0.21	203.93
time (sec)	N/A	1.948	3.506	0.148	0.000	18.893	0.000	0.000	0.166	13.658

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	343	306	12968	0	15079	0	0	50	67465
N.S.	1	1.19	1.06	44.87	0.00	52.18	0.00	0.00	0.17	233.44
time (sec)	N/A	2.490	3.486	0.176	0.000	54.902	0.000	0.000	78.479	9.735

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	423	373	13092	0	15322	0	0	33	71314
N.S.	1	1.16	1.02	35.97	0.00	42.09	0.00	0.00	0.09	195.92
time (sec)	N/A	3.136	4.769	0.181	0.000	104.840	0.000	0.000	200.022	9.682

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	403	88	662	0	399	0	0	15	3033
N.S.	1	1.49	0.32	2.44	0.00	1.47	0.00	0.00	0.06	11.19
time (sec)	N/A	0.623	0.053	0.156	0.000	0.086	0.000	0.000	0.157	5.396

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	426	88	1575	0	805	0	0	56	6453
N.S.	1	1.41	0.29	5.20	0.00	2.66	0.00	0.00	0.18	21.30
time (sec)	N/A	0.636	0.022	0.122	0.000	0.103	0.000	0.000	0.177	8.008

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	108	112	374	0	1671	0	0	34	2142
N.S.	1	0.91	0.94	3.14	0.00	14.04	0.00	0.00	0.29	18.00
time (sec)	N/A	0.851	0.055	0.186	0.000	0.125	0.000	0.000	0.175	5.878

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	125	106	1955	0	2174	0	0	147	9618
N.S.	1	1.02	0.86	15.89	0.00	17.67	0.00	0.00	1.20	78.20
time (sec)	N/A	0.576	0.097	0.093	0.000	0.118	0.000	0.000	0.170	16.092

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	176	166	1779	0	4575	0	0	50	7172
N.S.	1	1.14	1.08	11.55	0.00	29.71	0.00	0.00	0.32	46.57
time (sec)	N/A	1.254	0.792	0.185	0.000	0.191	0.000	0.000	0.185	9.067

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-1)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	85	109	1895	0	1115	0	0	97	2731
N.S.	1	0.83	1.07	18.58	0.00	10.93	0.00	0.00	0.95	26.77
time (sec)	N/A	0.420	0.125	0.102	0.000	0.098	0.000	0.000	0.179	5.550

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	129	154	2291	0	2320	0	0	265	5475
N.S.	1	0.98	1.17	17.36	0.00	17.58	0.00	0.00	2.01	41.48
time (sec)	N/A	0.621	0.277	0.110	0.000	0.131	0.000	0.000	0.177	9.235

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	182	156	3055	0	3695	0	0	514	8437
N.S.	1	1.05	0.90	17.56	0.00	21.24	0.00	0.00	2.95	48.49
time (sec)	N/A	0.879	0.212	0.112	0.000	0.179	0.000	0.000	0.195	18.989

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	35	45	729	6480	249	0	159	95	1410
N.S.	1	0.78	1.00	16.20	144.00	5.53	0.00	3.53	2.11	31.33
time (sec)	N/A	0.249	0.271	0.203	0.353	0.104	0.000	0.527	0.181	5.680

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	35	45	741	0	267	0	159	96	1410
N.S.	1	0.78	1.00	16.47	0.00	5.93	0.00	3.53	2.13	31.33
time (sec)	N/A	0.245	0.036	0.138	0.000	0.085	0.000	0.432	0.168	4.697

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	61	54	0	28	0	65	54	31
N.S.	1	1.00	2.03	1.80	0.00	0.93	0.00	2.17	1.80	1.03
time (sec)	N/A	0.213	0.487	0.260	0.000	0.103	0.000	0.328	0.176	0.709

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	61	52	0	47	0	57	56	35
N.S.	1	1.00	2.26	1.93	0.00	1.74	0.00	2.11	2.07	1.30
time (sec)	N/A	0.215	0.567	0.069	0.000	0.073	0.000	0.359	0.182	4.266

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	134	0	157	0	0	91	147
N.S.	1	1.00	0.88	1.58	0.00	1.85	0.00	0.00	1.07	1.73
time (sec)	N/A	0.395	0.068	0.274	0.000	0.076	0.000	0.000	0.177	4.377

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	255	151	275	227	937	0	0	107	1522
N.S.	1	1.13	0.67	1.22	1.00	4.15	0.00	0.00	0.47	6.73
time (sec)	N/A	0.929	1.112	0.095	0.119	0.096	0.000	0.000	0.179	12.939

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	231	134	253	210	921	0	0	121	1492
N.S.	1	1.14	0.66	1.25	1.04	4.56	0.00	0.00	0.60	7.39
time (sec)	N/A	0.762	0.711	0.068	0.119	0.103	0.000	0.000	0.186	8.950

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	206	114	229	192	901	0	0	76	1456
N.S.	1	1.16	0.64	1.29	1.08	5.06	0.00	0.00	0.43	8.18
time (sec)	N/A	0.616	0.293	0.061	0.127	0.107	0.000	0.000	0.167	6.931

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	182	94	203	174	886	0	0	78	1420
N.S.	1	1.20	0.62	1.34	1.14	5.83	0.00	0.00	0.51	9.34
time (sec)	N/A	0.508	0.112	0.062	0.131	0.099	0.000	0.000	0.173	5.964

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	182	158	203	174	916	0	0	58	1420
N.S.	1	1.19	1.03	1.33	1.14	5.99	0.00	0.00	0.38	9.28
time (sec)	N/A	0.503	0.419	0.059	0.133	0.096	0.000	0.000	0.168	5.567

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	206	178	222	192	944	0	0	67	1448
N.S.	1	1.16	1.01	1.25	1.08	5.33	0.00	0.00	0.38	8.18
time (sec)	N/A	0.633	0.523	0.063	0.138	0.103	0.000	0.000	0.173	7.166

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	231	198	240	211	961	0	0	67	1473
N.S.	1	1.14	0.98	1.18	1.04	4.73	0.00	0.00	0.33	7.26
time (sec)	N/A	0.816	0.814	0.064	0.127	0.112	0.000	0.000	0.169	10.186

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	349	205	374	329	1546	0	0	190	3914
N.S.	1	1.06	0.62	1.13	1.00	4.68	0.00	0.00	0.58	11.86
time (sec)	N/A	1.429	4.233	0.130	0.125	0.114	0.000	0.000	0.178	23.798

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	317	178	333	302	1527	0	0	150	3869
N.S.	1	1.07	0.60	1.13	1.02	5.18	0.00	0.00	0.51	13.12
time (sec)	N/A	1.201	1.510	0.061	0.126	0.132	0.000	0.000	0.184	15.722

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	283	151	292	275	1492	0	0	140	3825
N.S.	1	1.08	0.57	1.11	1.05	5.67	0.00	0.00	0.53	14.54
time (sec)	N/A	0.986	0.841	0.058	0.115	0.147	0.000	0.000	0.174	10.055

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	249	119	251	248	1472	0	0	105	3773
N.S.	1	1.09	0.52	1.10	1.09	6.46	0.00	0.00	0.46	16.55
time (sec)	N/A	0.818	0.365	0.068	0.125	0.116	0.000	0.000	0.173	6.616

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	228	211	238	240	1490	0	0	105	3749
N.S.	1	1.08	1.00	1.12	1.13	7.03	0.00	0.00	0.50	17.68
time (sec)	N/A	0.754	0.633	0.065	0.122	0.128	0.000	0.000	0.178	5.755

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	235	119	243	248	1514	0	0	84	3745
N.S.	1	1.08	0.55	1.11	1.14	6.94	0.00	0.00	0.39	17.18
time (sec)	N/A	0.782	0.493	0.062	0.122	0.113	0.000	0.000	0.173	7.273

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	269	120	270	276	1534	0	0	93	3782
N.S.	1	1.06	0.47	1.06	1.09	6.04	0.00	0.00	0.37	14.89
time (sec)	N/A	0.939	0.405	0.062	0.123	0.137	0.000	0.000	0.177	10.469

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	395	221	432	398	2124	0	0	233	6774
N.S.	1	1.02	0.57	1.11	1.03	5.47	0.00	0.00	0.60	17.46
time (sec)	N/A	1.669	2.366	0.067	0.134	0.146	0.000	0.000	0.185	31.721

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	355	197	372	363	2094	0	0	169	6716
N.S.	1	1.03	0.57	1.08	1.05	6.07	0.00	0.00	0.49	19.47
time (sec)	N/A	1.409	1.391	0.065	0.122	0.139	0.000	0.000	0.186	18.925

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	312	153	314	327	2051	0	0	169	6657
N.S.	1	1.03	0.50	1.03	1.08	6.75	0.00	0.00	0.56	21.90
time (sec)	N/A	1.171	0.995	0.074	0.133	0.150	0.000	0.000	0.193	9.597

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	303	264	286	310	2074	0	0	132	7108
N.S.	1	1.02	0.89	0.96	1.04	6.98	0.00	0.00	0.44	23.93
time (sec)	N/A	1.136	1.772	0.070	0.119	0.135	0.000	0.000	0.187	7.526

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	302	165	278	310	2077	0	0	132	7578
N.S.	1	1.02	0.56	0.94	1.04	6.99	0.00	0.00	0.44	25.52
time (sec)	N/A	1.151	0.856	0.071	0.128	0.136	0.000	0.000	0.187	7.474

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	312	166	290	327	2099	0	0	110	7591
N.S.	1	1.03	0.55	0.95	1.08	6.90	0.00	0.00	0.36	24.97
time (sec)	N/A	1.224	0.932	0.069	0.131	0.166	0.000	0.000	0.179	9.655

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	289	187	292	258	2674	0	0	18	16441
N.S.	1	1.09	0.71	1.11	0.98	10.13	0.00	0.00	0.07	62.28
time (sec)	N/A	1.440	0.776	0.122	0.132	20.059	0.000	0.000	0.167	10.635

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	254	165	265	234	2560	0	0	35	15701
N.S.	1	1.07	0.70	1.12	0.99	10.80	0.00	0.00	0.15	66.25
time (sec)	N/A	1.108	0.232	0.084	0.144	8.259	0.000	0.000	0.165	9.695

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	228	195	244	216	2508	0	0	9	15090
N.S.	1	1.06	0.90	1.13	1.00	11.61	0.00	0.00	0.04	69.86
time (sec)	N/A	0.812	0.229	0.085	0.117	2.723	0.000	0.000	0.157	8.305

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	228	194	244	217	2488	0	0	18	14816
N.S.	1	1.05	0.89	1.12	1.00	11.47	0.00	0.00	0.08	68.28
time (sec)	N/A	0.831	0.225	0.088	0.135	3.866	0.000	0.000	0.167	8.623

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	254	153	265	235	2648	0	0	18	15318
N.S.	1	1.08	0.65	1.12	1.00	11.22	0.00	0.00	0.08	64.91
time (sec)	N/A	1.096	0.410	0.088	0.120	11.306	0.000	0.000	0.157	9.129

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	289	174	286	256	2746	0	0	18	16111
N.S.	1	1.09	0.66	1.08	0.97	10.36	0.00	0.00	0.07	60.80
time (sec)	N/A	1.477	2.319	0.088	0.166	25.083	0.000	0.000	0.166	10.630

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	379	275	351	377	4897	0	0	30	18313
N.S.	1	1.04	0.76	0.96	1.04	13.45	0.00	0.00	0.08	50.31
time (sec)	N/A	1.812	1.604	0.121	0.126	33.238	0.000	0.000	0.158	35.533

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	335	230	336	354	4800	0	0	28	17579
N.S.	1	1.06	0.73	1.06	1.12	15.14	0.00	0.00	0.09	55.45
time (sec)	N/A	1.387	1.278	0.090	0.144	18.420	0.000	0.000	0.155	32.154

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	334	220	332	341	4816	0	0	118	17089
N.S.	1	1.05	0.69	1.04	1.07	15.14	0.00	0.00	0.37	53.74
time (sec)	N/A	1.331	0.965	0.094	0.117	13.563	0.000	0.000	0.180	32.110

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	335	204	336	356	4803	0	0	31	17494
N.S.	1	1.06	0.65	1.06	1.13	15.20	0.00	0.00	0.10	55.36
time (sec)	N/A	1.392	0.777	0.096	0.143	22.295	0.000	0.000	0.160	32.409

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	381	239	352	396	4991	0	0	33	22667
N.S.	1	1.04	0.65	0.96	1.08	13.60	0.00	0.00	0.09	61.76
time (sec)	N/A	1.876	1.605	0.095	0.118	36.358	0.000	0.000	0.169	24.144

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	433	287	373	449	5186	0	0	33	24620
N.S.	1	1.03	0.68	0.89	1.07	12.38	0.00	0.00	0.08	58.76
time (sec)	N/A	2.500	2.641	0.099	0.164	69.116	0.000	0.000	0.155	21.204

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	528	1034	466	571	7496	0	0	46	27429
N.S.	1	1.03	2.01	0.91	1.11	14.58	0.00	0.00	0.09	53.36
time (sec)	N/A	2.755	13.533	0.202	0.129	158.039	0.000	0.000	0.155	62.618

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	466	372	451	551	7355	0	0	46	26614
N.S.	1	1.04	0.83	1.01	1.23	16.42	0.00	0.00	0.10	59.41
time (sec)	N/A	2.116	4.676	0.187	0.166	101.415	0.000	0.000	0.161	52.526

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	462	333	450	538	7400	0	0	44	25944
N.S.	1	1.03	0.74	1.00	1.20	16.52	0.00	0.00	0.10	57.91
time (sec)	N/A	2.051	4.078	0.100	0.123	68.186	0.000	0.000	0.194	52.847

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	464	344	451	537	7394	0	0	38	26133
N.S.	1	1.05	0.77	1.02	1.21	16.65	0.00	0.00	0.09	58.86
time (sec)	N/A	2.092	4.300	0.104	0.124	73.748	0.000	0.000	0.200	51.356

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	466	288	452	551	7359	0	0	47	26707
N.S.	1	1.04	0.64	1.01	1.23	16.43	0.00	0.00	0.10	59.61
time (sec)	N/A	2.181	3.131	0.106	0.216	110.873	0.000	0.000	0.158	53.289

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	528	341	466	607	7591	0	0	49	35300
N.S.	1	1.03	0.66	0.90	1.18	14.74	0.00	0.00	0.10	68.54
time (sec)	N/A	2.976	5.936	0.103	0.130	173.301	0.000	0.000	0.168	48.525

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	157	89	102	123	114	0	0	20	16727
N.S.	1	1.35	0.77	0.88	1.06	0.98	0.00	0.00	0.17	144.20
time (sec)	N/A	0.416	0.105	0.048	0.142	0.081	0.000	0.000	0.159	8.856

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	155	139	102	123	113	0	0	36	16060
N.S.	1	1.35	1.21	0.89	1.07	0.98	0.00	0.00	0.31	139.65
time (sec)	N/A	0.420	0.096	0.044	0.115	0.101	0.000	0.000	0.165	8.552

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	138	60	90	109	102	0	0	11	15753
N.S.	1	1.39	0.61	0.91	1.10	1.03	0.00	0.00	0.11	159.12
time (sec)	N/A	0.345	0.041	0.075	0.134	0.076	0.000	0.000	0.164	8.459

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	138	110	90	112	102	0	0	20	15437
N.S.	1	1.41	1.12	0.92	1.14	1.04	0.00	0.00	0.20	157.52
time (sec)	N/A	0.353	0.051	0.067	0.123	0.079	0.000	0.000	0.155	8.370

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	155	78	102	122	145	0	0	20	15569
N.S.	1	1.36	0.68	0.89	1.07	1.27	0.00	0.00	0.18	136.57
time (sec)	N/A	0.415	0.089	0.045	0.113	0.082	0.000	0.000	0.164	8.066

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	157	82	102	124	154	0	0	20	16545
N.S.	1	1.34	0.70	0.87	1.06	1.32	0.00	0.00	0.17	141.41
time (sec)	N/A	0.419	0.091	0.046	0.114	0.085	0.000	0.000	0.157	8.951

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	228	156	242	188	1870	0	0	32	18514
N.S.	1	1.13	0.77	1.20	0.93	9.26	0.00	0.00	0.16	91.65
time (sec)	N/A	0.977	0.118	0.076	0.119	0.176	0.000	0.000	0.175	34.730

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	203	228	226	173	1784	0	0	30	16878
N.S.	1	1.09	1.23	1.22	0.93	9.59	0.00	0.00	0.16	90.74
time (sec)	N/A	0.753	0.129	0.072	0.121	0.143	0.000	0.000	0.164	31.606

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	203	205	225	172	1792	0	0	115	17323
N.S.	1	1.10	1.11	1.22	0.93	9.74	0.00	0.00	0.62	94.15
time (sec)	N/A	0.764	0.058	0.071	0.133	0.124	0.000	0.000	0.181	30.219

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	203	229	226	172	1780	0	0	33	16598
N.S.	1	1.10	1.24	1.22	0.93	9.62	0.00	0.00	0.18	89.72
time (sec)	N/A	0.770	0.077	0.072	0.167	0.144	0.000	0.000	0.162	30.226

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	228	132	242	188	1942	0	0	35	22906
N.S.	1	1.13	0.65	1.20	0.93	9.61	0.00	0.00	0.17	113.40
time (sec)	N/A	0.985	0.327	0.072	0.125	0.152	0.000	0.000	0.160	23.143

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	264	265	304	2183235	0	16118	0	0	61	0
N.S.	1	1.00	1.15	8269.83	0.00	61.05	0.00	0.00	0.23	0.00
time (sec)	N/A	1.421	2.390	2.629	0.000	3.334	0.000	0.000	0.197	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	201	199	239	2180690	0	15912	0	0	53	61200
N.S.	1	0.99	1.19	10849.20	0.00	79.16	0.00	0.00	0.26	304.48
time (sec)	N/A	0.948	1.912	0.924	0.000	3.021	0.000	0.000	0.179	119.553

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	169	179	204	2178123	0	15940	0	0	190	1141
N.S.	1	1.06	1.21	12888.30	0.00	94.32	0.00	0.00	1.12	6.75
time (sec)	N/A	1.177	0.587	0.954	0.000	2.369	0.000	0.000	0.194	15.096

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	154	170	169	2178365	0	7865	0	0	206	0
N.S.	1	1.10	1.10	14145.23	0.00	51.07	0.00	0.00	1.34	0.00
time (sec)	N/A	0.867	0.593	0.954	0.000	1.189	0.000	0.000	0.190	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	199	222	194	2181119	0	7946	0	0	231	0
N.S.	1	1.12	0.97	10960.40	0.00	39.93	0.00	0.00	1.16	0.00
time (sec)	N/A	1.204	1.041	0.955	0.000	1.221	0.000	0.000	0.213	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	250	289	226	2183172	0	7964	0	0	273	0
N.S.	1	1.16	0.90	8732.69	0.00	31.86	0.00	0.00	1.09	0.00
time (sec)	N/A	1.769	2.020	0.970	0.000	1.214	0.000	0.000	0.222	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	314	355	265	2185304	0	8039	0	0	370	0
N.S.	1	1.13	0.84	6959.57	0.00	25.60	0.00	0.00	1.18	0.00
time (sec)	N/A	2.251	2.656	1.000	0.000	1.220	0.000	0.000	0.229	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	323	326	347	2403184	0	24426	0	0	98	0
N.S.	1	1.01	1.07	7440.20	0.00	75.62	0.00	0.00	0.30	0.00
time (sec)	N/A	1.949	2.921	1.085	0.000	4.968	0.000	0.000	0.229	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	260	263	290	2400957	0	24274	0	0	90	0
N.S.	1	1.01	1.12	9234.45	0.00	93.36	0.00	0.00	0.35	0.00
time (sec)	N/A	1.521	1.702	1.058	0.000	4.474	0.000	0.000	0.203	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	204	200	243	2398415	0	24220	0	0	228	0
N.S.	1	0.98	1.19	11756.94	0.00	118.73	0.00	0.00	1.12	0.00
time (sec)	N/A	1.074	0.688	1.096	0.000	4.208	0.000	0.000	0.208	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	209	200	416	2394889	0	24297	0	0	229	0
N.S.	1	0.96	1.99	11458.80	0.00	116.25	0.00	0.00	1.10	0.00
time (sec)	N/A	1.023	2.823	1.482	0.000	3.945	0.000	0.000	0.189	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	196	225	238	2398858	0	12024	0	0	332	0
N.S.	1	1.15	1.21	12239.07	0.00	61.35	0.00	0.00	1.69	0.00
time (sec)	N/A	1.321	0.625	1.077	0.000	2.140	0.000	0.000	0.236	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	259	289	286	2400946	0	12093	0	0	326	0
N.S.	1	1.12	1.10	9270.06	0.00	46.69	0.00	0.00	1.26	0.00
time (sec)	N/A	1.880	2.124	1.050	0.000	2.147	0.000	0.000	0.246	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	311	352	346	2403086	0	12128	0	0	429	0
N.S.	1	1.13	1.11	7726.96	0.00	39.00	0.00	0.00	1.38	0.00
time (sec)	N/A	2.333	3.614	0.951	0.000	2.095	0.000	0.000	0.257	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	382	430	5445	2405433	0	12202	0	0	499	0
N.S.	1	1.13	14.25	6296.95	0.00	31.94	0.00	0.00	1.31	0.00
time (sec)	N/A	3.035	33.125	1.013	0.000	2.100	0.000	0.000	0.261	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	397	400	411	2656933	0	35458	0	0	135	0
N.S.	1	1.01	1.04	6692.53	0.00	89.31	0.00	0.00	0.34	0.00
time (sec)	N/A	2.529	3.083	5.217	0.000	8.943	0.000	0.000	0.235	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	316	325	345	2657119	0	35244	0	0	127	0
N.S.	1	1.03	1.09	8408.60	0.00	111.53	0.00	0.00	0.40	0.00
time (sec)	N/A	2.030	3.199	2.881	0.000	7.412	0.000	0.000	0.225	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	260	257	291	2652267	0	35212	0	0	266	0
N.S.	1	0.99	1.12	10201.03	0.00	135.43	0.00	0.00	1.02	0.00
time (sec)	N/A	1.541	1.816	2.233	0.000	6.711	0.000	0.000	0.209	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	241	237	358	2651144	0	35230	0	0	271	0
N.S.	1	0.98	1.49	11000.60	0.00	146.18	0.00	0.00	1.12	0.00
time (sec)	N/A	1.527	1.721	1.934	0.000	6.859	0.000	0.000	0.216	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	240	241	417	2651450	0	35349	0	0	275	0
N.S.	1	1.00	1.74	11047.71	0.00	147.29	0.00	0.00	1.15	0.00
time (sec)	N/A	1.429	2.869	1.944	0.000	6.532	0.000	0.000	0.237	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	247	283	321	5393	0	18802	0	0	429	0
N.S.	1	1.15	1.30	21.83	0.00	76.12	0.00	0.00	1.74	0.00
time (sec)	N/A	1.815	1.408	12.499	0.000	4.383	0.000	0.000	0.270	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	309	350	5641	5427	0	18856	0	0	482	0
N.S.	1	1.13	18.26	17.56	0.00	61.02	0.00	0.00	1.56	0.00
time (sec)	N/A	2.356	32.061	5.857	0.000	4.448	0.000	0.000	0.267	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	378	428	5725	15788	0	18922	0	0	558	0
N.S.	1	1.13	15.15	41.77	0.00	50.06	0.00	0.00	1.48	0.00
time (sec)	N/A	3.010	32.177	8.594	0.000	4.405	0.000	0.000	0.314	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	460	509	5809	2659382	0	18981	0	0	632	0
N.S.	1	1.11	12.63	5781.27	0.00	41.26	0.00	0.00	1.37	0.00
time (sec)	N/A	3.641	31.833	2.940	0.000	4.363	0.000	0.000	0.315	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	253	255	356	1491744	0	16341	0	0	479	0
N.S.	1	1.01	1.41	5896.22	0.00	64.59	0.00	0.00	1.89	0.00
time (sec)	N/A	1.612	3.047	1.200	0.000	2.219	0.000	0.000	0.219	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	206	207	245	1888526	0	20328	0	0	27	0
N.S.	1	1.00	1.19	9167.60	0.00	98.68	0.00	0.00	0.13	0.00
time (sec)	N/A	0.968	1.309	1.076	0.000	4.603	0.000	0.000	0.176	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	205	1887830	0	20234	0	0	21	30600
N.S.	1	1.00	1.22	11237.08	0.00	120.44	0.00	0.00	0.12	182.14
time (sec)	N/A	1.092	0.861	1.021	0.000	3.997	0.000	0.000	0.154	92.977

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	137	1878820	0	9913	0	0	159	8223
N.S.	1	1.00	1.11	15274.96	0.00	80.59	0.00	0.00	1.29	66.85
time (sec)	N/A	0.609	0.159	0.965	0.000	2.560	0.000	0.000	0.192	53.085

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	159	166	172	1888108	0	10036	0	0	29	0
N.S.	1	1.04	1.08	11874.89	0.00	63.12	0.00	0.00	0.18	0.00
time (sec)	N/A	0.849	0.348	1.102	0.000	2.614	0.000	0.000	0.159	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	203	219	195	1890767	0	10067	0	0	29	0
N.S.	1	1.08	0.96	9314.12	0.00	49.59	0.00	0.00	0.14	0.00
time (sec)	N/A	1.163	1.309	1.836	0.000	2.650	0.000	0.000	0.169	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	256	282	227	1892796	0	10125	0	0	29	0
N.S.	1	1.10	0.89	7393.73	0.00	39.55	0.00	0.00	0.11	0.00
time (sec)	N/A	1.699	3.734	2.076	0.000	2.618	0.000	0.000	0.184	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	219	253	341	1560668	0	37564	0	0	39	0
N.S.	1	1.16	1.56	7126.34	0.00	171.53	0.00	0.00	0.18	0.00
time (sec)	N/A	1.134	1.322	3.207	0.000	10.700	0.000	0.000	0.175	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	170	202	239	1559493	0	18532	0	0	163	0
N.S.	1	1.19	1.41	9173.49	0.00	109.01	0.00	0.00	0.96	0.00
time (sec)	N/A	0.958	1.089	2.960	0.000	6.807	0.000	0.000	0.202	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	175	203	202	1559531	0	18563	0	0	42	0
N.S.	1	1.16	1.15	8911.61	0.00	106.07	0.00	0.00	0.24	0.00
time (sec)	N/A	0.965	0.574	1.513	0.000	6.648	0.000	0.000	0.185	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	216	258	248	4054	0	18689	0	0	44	0
N.S.	1	1.19	1.15	18.77	0.00	86.52	0.00	0.00	0.20	0.00
time (sec)	N/A	1.326	1.481	9.757	0.000	6.698	0.000	0.000	0.182	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	276	316	299	4644	0	18754	0	0	121	0
N.S.	1	1.14	1.08	16.83	0.00	67.95	0.00	0.00	0.44	0.00
time (sec)	N/A	1.812	2.121	4.825	0.000	7.007	0.000	0.000	0.183	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	282	344	596	2978162	0	55768	0	0	57	0
N.S.	1	1.22	2.11	10560.86	0.00	197.76	0.00	0.00	0.20	0.00
time (sec)	N/A	1.747	6.254	4.408	0.000	22.811	0.000	0.000	0.183	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	244	293	308	2975178	0	27576	0	0	173	0
N.S.	1	1.20	1.26	12193.35	0.00	113.02	0.00	0.00	0.71	0.00
time (sec)	N/A	1.516	2.154	2.415	0.000	13.723	0.000	0.000	0.196	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	244	297	320	2975224	0	27593	0	0	49	0
N.S.	1	1.22	1.31	12193.54	0.00	113.09	0.00	0.00	0.20	0.00
time (sec)	N/A	1.479	2.388	2.360	0.000	13.579	0.000	0.000	0.180	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	247	299	273	2978185	0	27639	0	0	58	0
N.S.	1	1.21	1.11	12057.43	0.00	111.90	0.00	0.00	0.23	0.00
time (sec)	N/A	1.551	1.787	2.850	0.000	13.563	0.000	0.000	0.170	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	301	361	326	2979708	0	27749	0	0	201	0
N.S.	1	1.20	1.08	9899.36	0.00	92.19	0.00	0.00	0.67	0.00
time (sec)	N/A	1.965	2.590	2.481	0.000	13.365	0.000	0.000	0.193	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	359	424	383	2982515	0	27865	0	0	256	0
N.S.	1	1.18	1.07	8307.84	0.00	77.62	0.00	0.00	0.71	0.00
time (sec)	N/A	2.615	2.489	2.388	0.000	13.629	0.000	0.000	0.214	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	155	148	190	944366	0	9186	0	0	41	0
N.S.	1	0.95	1.23	6092.68	0.00	59.26	0.00	0.00	0.26	0.00
time (sec)	N/A	0.425	0.454	0.630	0.000	0.893	0.000	0.000	0.178	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	117	114	124	939563	0	4509	0	0	160	0
N.S.	1	0.97	1.06	8030.45	0.00	38.54	0.00	0.00	1.37	0.00
time (sec)	N/A	0.330	0.029	0.677	0.000	0.545	0.000	0.000	0.220	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	111	108	125	939324	0	4469	0	0	44	0
N.S.	1	0.97	1.13	8462.38	0.00	40.26	0.00	0.00	0.40	0.00
time (sec)	N/A	0.351	0.031	0.605	0.000	0.607	0.000	0.000	0.186	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	150	149	158	943933	0	4608	0	0	46	0
N.S.	1	0.99	1.05	6292.89	0.00	30.72	0.00	0.00	0.31	0.00
time (sec)	N/A	0.473	0.255	0.709	0.000	0.544	0.000	0.000	0.173	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	269	263	99	0	5405	0	0	40	3945
N.S.	1	0.71	0.69	0.26	0.00	14.26	0.00	0.00	0.11	10.41
time (sec)	N/A	0.717	0.677	0.780	0.000	1.724	0.000	0.000	0.184	20.263

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	267	347	97	0	2785	0	0	40	2537
N.S.	1	0.71	0.92	0.26	0.00	7.39	0.00	0.00	0.11	6.73
time (sec)	N/A	0.711	0.653	0.705	0.000	0.194	0.000	0.000	0.199	15.745

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	233	227	72	0	4121	0	147	40	3228
N.S.	1	0.65	0.64	0.20	0.00	11.54	0.00	0.41	0.11	9.04
time (sec)	N/A	0.555	0.297	0.725	0.000	0.286	0.000	1.726	0.196	15.900

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	233	305	69	0	6073	0	0	40	4562
N.S.	1	0.65	0.85	0.19	0.00	17.01	0.00	0.00	0.11	12.78
time (sec)	N/A	0.952	0.172	0.785	0.000	1.167	0.000	0.000	0.176	18.294

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	105	104	42	0	278	0	795	40	2982
N.S.	1	0.71	0.70	0.28	0.00	1.88	0.00	5.37	0.27	20.15
time (sec)	N/A	0.302	0.416	0.635	0.000	0.078	0.000	0.420	0.193	17.681

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	235	330	72	0	443	0	0	41	4308
N.S.	1	0.79	1.10	0.24	0.00	1.48	0.00	0.00	0.14	14.41
time (sec)	N/A	0.600	0.257	0.847	0.000	0.089	0.000	0.000	0.194	17.797

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	403	416	355	0	0	0	0	0	789	0
N.S.	1	1.03	0.88	0.00	0.00	0.00	0.00	0.00	1.96	0.00
time (sec)	N/A	2.371	3.113	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	283	232	0	0	0	0	0	354	0
N.S.	1	1.06	0.87	0.00	0.00	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	1.452	1.598	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	198	155	0	0	0	0	0	290	0
N.S.	1	1.02	0.80	0.00	0.00	0.00	0.00	0.00	1.49	0.00
time (sec)	N/A	0.900	0.497	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	108	0	0	0	0	0	86	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.472	0.330	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	178	144	0	0	0	0	0	10	0
N.S.	1	0.96	0.78	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.867	0.684	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	292	239	0	0	0	0	0	126	0
N.S.	1	1.04	0.85	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	1.497	2.192	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	438	464	530	0	0	0	0	0	75	0
N.S.	1	1.06	1.21	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	2.433	6.254	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	659	704	1425	0	0	0	0	0	108	0
N.S.	1	1.07	2.16	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	4.075	6.294	0.000	0.000	0.000	0.000	0.000	0.244	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	193	0	0	0	0	0	0	131	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.646	0.000	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	189	0	0	0	0	0	0	93	0
N.S.	1	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.661	0.000	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	187	0	0	0	0	0	0	55	0
N.S.	1	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.633	0.000	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	187	0	0	0	0	0	0	22	0
N.S.	1	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.618	0.000	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	193	0	0	0	0	0	0	482	0
N.S.	1	0.91	0.00	0.00	0.00	0.00	0.00	0.00	2.28	0.00
time (sec)	N/A	0.679	0.000	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	193	0	0	0	0	0	0	90	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.680	0.000	0.000	0.000	0.000	0.000	0.000	0.271	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	183	0	0	0	0	0	0	57	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.545	0.000	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	387	412	384	0	0	0	0	0	51	0
N.S.	1	1.06	0.99	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	2.015	4.079	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	319	281	0	0	0	0	0	51	0
N.S.	1	1.10	0.97	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	1.311	1.744	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	219	234	169	0	0	0	0	0	51	0
N.S.	1	1.07	0.77	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.865	1.024	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	174	125	0	0	0	0	0	49	0
N.S.	1	1.04	0.74	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.572	0.200	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	145	120	0	0	0	0	0	40	0
N.S.	1	1.01	0.84	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.394	0.144	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	196	169	0	0	0	0	0	31	0
N.S.	1	1.03	0.88	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.791	0.298	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	229	237	202	0	0	0	0	0	33	0
N.S.	1	1.03	0.88	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.106	0.312	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	317	230	0	0	0	0	0	33	0
N.S.	1	1.08	0.78	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.721	0.390	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	173	0	0	0	0	0	0	63	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.632	0.000	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	173	0	0	0	0	0	0	55	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.632	0.000	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F(-1)	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	167	0	0	0	0	0	0	540	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	3.20	0.00
time (sec)	N/A	0.629	0.000	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F(-1)	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	161	0	0	0	0	0	0	426	0
N.S.	1	0.94	0.00	0.00	0.00	0.00	0.00	0.00	2.49	0.00
time (sec)	N/A	0.612	0.000	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	55	551	190	434	0	93	145	0
N.S.	1	1.00	0.53	5.35	1.84	4.21	0.00	0.90	1.41	0.00
time (sec)	N/A	0.744	0.512	0.572	0.117	0.099	0.000	0.227	0.238	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	54	528	174	382	0	68	105	0
N.S.	1	1.00	0.69	6.77	2.23	4.90	0.00	0.87	1.35	0.00
time (sec)	N/A	0.580	0.354	0.575	0.117	0.090	0.000	0.188	0.227	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	64	508	155	316	0	45	117	0
N.S.	1	1.00	1.21	9.58	2.92	5.96	0.00	0.85	2.21	0.00
time (sec)	N/A	0.511	0.509	0.409	0.120	0.088	0.000	0.156	0.224	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	63	422	155	364	0	45	72	0
N.S.	1	1.00	1.15	7.67	2.82	6.62	0.00	0.82	1.31	0.00
time (sec)	N/A	0.497	0.454	0.440	0.121	0.092	0.000	0.137	0.219	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	86	220	177	429	0	70	105	0
N.S.	1	1.00	1.08	2.75	2.21	5.36	0.00	0.88	1.31	0.00
time (sec)	N/A	0.626	0.679	0.539	0.121	0.089	0.000	0.173	0.202	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	99	470	191	482	0	94	105	0
N.S.	1	1.00	0.94	4.48	1.82	4.59	0.00	0.90	1.00	0.00
time (sec)	N/A	0.742	1.273	0.377	0.131	0.114	0.000	0.176	0.207	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	133	74	253	198	448	0	107	207	0
N.S.	1	1.04	0.58	1.98	1.55	3.50	0.00	0.84	1.62	0.00
time (sec)	N/A	0.912	1.046	0.730	0.123	0.098	0.000	0.430	0.295	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	106	78	230	180	394	0	78	167	0
N.S.	1	1.03	0.76	2.23	1.75	3.83	0.00	0.76	1.62	0.00
time (sec)	N/A	0.747	1.964	0.759	0.123	0.108	0.000	0.366	0.273	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	69	218	174	385	0	63	177	0
N.S.	1	1.00	0.70	2.20	1.76	3.89	0.00	0.64	1.79	0.00
time (sec)	N/A	0.735	1.536	0.442	0.128	0.088	0.000	0.299	0.261	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	108	90	225	180	439	0	79	118	0
N.S.	1	1.03	0.86	2.14	1.71	4.18	0.00	0.75	1.12	0.00
time (sec)	N/A	0.769	2.864	0.546	0.116	0.091	0.000	0.208	0.212	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	138	97	242	199	505	0	108	167	0
N.S.	1	1.06	0.75	1.86	1.53	3.88	0.00	0.83	1.28	0.00
time (sec)	N/A	0.938	3.151	0.558	0.120	0.096	0.000	0.198	0.218	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	181	113	275	218	508	0	134	227	0
N.S.	1	1.06	0.66	1.61	1.27	2.97	0.00	0.78	1.33	0.00
time (sec)	N/A	1.215	1.244	1.263	0.124	0.114	0.000	0.258	0.365	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	154	113	252	200	449	0	107	269	0
N.S.	1	1.05	0.77	1.73	1.37	3.08	0.00	0.73	1.84	0.00
time (sec)	N/A	1.051	2.232	1.257	0.125	0.104	0.000	0.279	0.337	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	141	113	241	194	409	0	93	227	0
N.S.	1	1.02	0.82	1.75	1.41	2.96	0.00	0.67	1.64	0.00
time (sec)	N/A	1.022	1.846	1.161	0.120	0.094	0.000	0.249	0.341	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	146	113	236	197	442	0	94	235	0
N.S.	1	1.03	0.80	1.66	1.39	3.11	0.00	0.66	1.65	0.00
time (sec)	N/A	1.008	1.871	0.406	0.120	0.095	0.000	0.391	0.281	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	156	110	243	203	502	0	108	162	0
N.S.	1	1.05	0.74	1.64	1.37	3.39	0.00	0.73	1.09	0.00
time (sec)	N/A	1.070	4.041	0.471	0.129	0.102	0.000	0.295	0.204	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	186	132	261	221	563	0	136	227	0
N.S.	1	1.08	0.76	1.51	1.28	3.25	0.00	0.79	1.31	0.00
time (sec)	N/A	1.272	4.070	0.497	0.125	0.111	0.000	0.245	0.255	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	265	186	165	0	716	0	125	75	0
N.S.	1	1.07	0.75	0.67	0.00	2.90	0.00	0.51	0.30	0.00
time (sec)	N/A	1.125	2.189	0.558	0.000	0.124	0.000	0.289	0.188	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	234	159	144	0	623	0	106	166	0
N.S.	1	1.09	0.74	0.67	0.00	2.91	0.00	0.50	0.78	0.00
time (sec)	N/A	0.926	1.500	0.549	0.000	0.100	0.000	0.247	0.216	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	205	137	125	0	570	0	89	59	0
N.S.	1	1.11	0.74	0.68	0.00	3.08	0.00	0.48	0.32	0.00
time (sec)	N/A	0.742	2.033	0.653	0.000	0.095	0.000	0.164	0.185	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	207	136	127	0	571	0	88	81	0
N.S.	1	1.11	0.73	0.68	0.00	3.05	0.00	0.47	0.43	0.00
time (sec)	N/A	0.747	2.143	0.749	0.000	0.095	0.000	0.194	0.183	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	242	154	143	0	700	0	105	89	0
N.S.	1	1.09	0.69	0.64	0.00	3.15	0.00	0.47	0.40	0.00
time (sec)	N/A	0.926	2.556	0.581	0.000	0.157	0.000	0.315	0.182	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	267	178	161	0	794	0	128	89	0
N.S.	1	1.04	0.69	0.63	0.00	3.09	0.00	0.50	0.35	0.00
time (sec)	N/A	1.088	3.561	0.557	0.000	0.105	0.000	0.375	0.183	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	286	221	159	0	669	0	128	91	0
N.S.	1	1.09	0.84	0.60	0.00	2.54	0.00	0.49	0.35	0.00
time (sec)	N/A	1.295	3.410	0.676	0.000	0.106	0.000	0.299	0.169	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	248	198	146	0	666	0	117	79	0
N.S.	1	1.08	0.86	0.63	0.00	2.90	0.00	0.51	0.34	0.00
time (sec)	N/A	1.071	2.744	0.663	0.000	0.117	0.000	0.300	0.174	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	241	196	146	0	663	0	116	110	0
N.S.	1	1.08	0.88	0.65	0.00	2.96	0.00	0.52	0.49	0.00
time (sec)	N/A	1.054	2.075	0.675	0.000	0.100	0.000	0.249	0.179	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	246	197	144	0	666	0	116	146	0
N.S.	1	1.05	0.84	0.62	0.00	2.85	0.00	0.50	0.62	0.00
time (sec)	N/A	1.074	2.847	0.657	0.000	0.099	0.000	0.345	0.172	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	288	215	157	0	761	0	128	158	0
N.S.	1	1.09	0.81	0.59	0.00	2.87	0.00	0.48	0.60	0.00
time (sec)	N/A	1.286	3.322	0.648	0.000	0.107	0.000	0.420	0.174	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	342	248	180	0	688	0	151	115	0
N.S.	1	1.09	0.79	0.58	0.00	2.20	0.00	0.48	0.37	0.00
time (sec)	N/A	1.708	3.988	0.533	0.000	0.117	0.000	0.333	0.221	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	293	223	162	0	683	0	129	103	0
N.S.	1	1.09	0.83	0.60	0.00	2.55	0.00	0.48	0.38	0.00
time (sec)	N/A	1.402	2.971	0.566	0.000	0.103	0.000	0.310	0.218	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	291	196	155	0	635	0	123	153	0
N.S.	1	1.09	0.74	0.58	0.00	2.39	0.00	0.46	0.58	0.00
time (sec)	N/A	1.378	3.803	0.592	0.000	0.099	0.000	0.353	0.201	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	287	203	150	0	637	0	123	169	0
N.S.	1	1.10	0.78	0.58	0.00	2.45	0.00	0.47	0.65	0.00
time (sec)	N/A	1.362	2.890	0.641	0.000	0.092	0.000	0.472	0.221	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	288	217	158	0	685	0	129	169	0
N.S.	1	1.10	0.83	0.60	0.00	2.61	0.00	0.49	0.65	0.00
time (sec)	N/A	1.386	3.739	0.674	0.000	0.102	0.000	0.225	0.216	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	346	242	178	0	783	0	151	169	0
N.S.	1	1.11	0.77	0.57	0.00	2.50	0.00	0.48	0.54	0.00
time (sec)	N/A	1.748	4.663	0.601	0.000	0.145	0.000	0.236	0.234	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	217	154	634	1409	481	0	0	72	0
N.S.	1	1.10	0.78	3.20	7.12	2.43	0.00	0.00	0.36	0.00
time (sec)	N/A	1.312	3.861	1.112	0.579	0.100	0.000	0.000	0.225	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	163	122	557	1157	430	0	0	72	0
N.S.	1	1.05	0.79	3.59	7.46	2.77	0.00	0.00	0.46	0.00
time (sec)	N/A	0.917	2.651	0.927	0.312	0.108	0.000	0.000	0.255	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	111	102	438	558	370	0	0	68	0
N.S.	1	1.01	0.93	3.98	5.07	3.36	0.00	0.00	0.62	0.00
time (sec)	N/A	0.624	1.915	0.924	0.222	0.091	0.000	0.000	0.218	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	133	161	506	0	576	0	0	56	0
N.S.	1	0.88	1.06	3.33	0.00	3.79	0.00	0.00	0.37	0.00
time (sec)	N/A	0.863	2.209	1.032	0.000	0.097	0.000	0.000	0.199	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	179	233	315	0	793	0	0	72	0
N.S.	1	0.93	1.21	1.64	0.00	4.13	0.00	0.00	0.38	0.00
time (sec)	N/A	1.166	2.390	0.901	0.000	0.103	0.000	0.000	0.179	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	269	320	802	3787	568	0	0	151	0
N.S.	1	1.10	1.31	3.27	15.46	2.32	0.00	0.00	0.62	0.00
time (sec)	N/A	1.665	8.711	0.817	2.423	0.095	0.000	0.000	0.290	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	215	325	715	1457	510	0	0	151	0
N.S.	1	1.07	1.62	3.56	7.25	2.54	0.00	0.00	0.75	0.00
time (sec)	N/A	1.262	4.576	0.785	0.446	0.096	0.000	0.000	0.298	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	161	291	624	1113	460	0	0	151	0
N.S.	1	1.03	1.85	3.97	7.09	2.93	0.00	0.00	0.96	0.00
time (sec)	N/A	0.960	4.277	0.782	0.271	0.098	0.000	0.000	0.277	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	167	301	526	0	674	0	0	143	0
N.S.	1	0.90	1.62	2.83	0.00	3.62	0.00	0.00	0.77	0.00
time (sec)	N/A	1.163	5.830	0.896	0.000	0.096	0.000	0.000	0.251	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	182	280	492	0	827	0	0	119	0
N.S.	1	0.93	1.43	2.51	0.00	4.22	0.00	0.00	0.61	0.00
time (sec)	N/A	1.168	5.744	0.842	0.000	0.106	0.000	0.000	0.236	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	233	306	454	0	913	0	0	151	0
N.S.	1	0.95	1.25	1.86	0.00	3.74	0.00	0.00	0.62	0.00
time (sec)	N/A	1.548	8.075	0.912	0.000	0.104	0.000	0.000	0.186	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	322	354	893	4543	629	0	0	235	0
N.S.	1	1.08	1.19	3.01	15.30	2.12	0.00	0.00	0.79	0.00
time (sec)	N/A	2.091	12.350	1.829	4.788	0.101	0.000	0.000	0.353	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	268	332	806	4087	577	0	0	235	0
N.S.	1	1.07	1.32	3.21	16.28	2.30	0.00	0.00	0.94	0.00
time (sec)	N/A	1.665	10.444	1.819	1.957	0.094	0.000	0.000	0.360	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	214	306	715	1545	515	0	0	235	0
N.S.	1	1.04	1.49	3.49	7.54	2.51	0.00	0.00	1.15	0.00
time (sec)	N/A	1.303	8.676	1.842	0.539	0.097	0.000	0.000	0.362	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	211	575	626	0	780	0	0	235	0
N.S.	1	0.92	2.50	2.72	0.00	3.39	0.00	0.00	1.02	0.00
time (sec)	N/A	1.461	7.775	1.841	0.000	0.139	0.000	0.000	0.338	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	223	288	571	0	849	0	0	223	0
N.S.	1	0.94	1.22	2.42	0.00	3.60	0.00	0.00	0.94	0.00
time (sec)	N/A	1.546	6.432	0.793	0.000	0.105	0.000	0.000	0.326	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	233	304	572	0	923	0	0	187	0
N.S.	1	0.95	1.24	2.33	0.00	3.75	0.00	0.00	0.76	0.00
time (sec)	N/A	1.551	10.757	0.898	0.000	0.104	0.000	0.000	0.253	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	286	524	541	0	1013	0	0	235	0
N.S.	1	0.98	1.79	1.85	0.00	3.47	0.00	0.00	0.80	0.00
time (sec)	N/A	1.939	11.156	0.958	0.000	0.112	0.000	0.000	0.205	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	220	141	748	0	487	0	0	203	0
N.S.	1	1.04	0.67	3.55	0.00	2.31	0.00	0.00	0.96	0.00
time (sec)	N/A	1.287	3.485	1.036	0.000	0.131	0.000	0.000	0.241	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	166	121	709	0	426	0	0	195	0
N.S.	1	1.02	0.74	4.35	0.00	2.61	0.00	0.00	1.20	0.00
time (sec)	N/A	0.950	2.634	1.016	0.000	0.103	0.000	0.000	0.210	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	120	111	647	0	423	0	0	171	0
N.S.	1	1.01	0.93	5.44	0.00	3.55	0.00	0.00	1.44	0.00
time (sec)	N/A	0.647	1.816	1.014	0.000	0.088	0.000	0.000	0.197	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	181	191	603	0	736	0	0	219	0
N.S.	1	0.92	0.97	3.08	0.00	3.76	0.00	0.00	1.12	0.00
time (sec)	N/A	1.149	1.813	1.058	0.000	0.095	0.000	0.000	0.198	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	220	196	931	0	465	0	0	192	0
N.S.	1	1.03	0.92	4.35	0.00	2.17	0.00	0.00	0.90	0.00
time (sec)	N/A	1.312	4.619	0.954	0.000	0.093	0.000	0.000	0.234	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	172	146	868	0	463	0	0	1390	0
N.S.	1	1.02	0.87	5.17	0.00	2.76	0.00	0.00	8.27	0.00
time (sec)	N/A	0.973	2.989	1.001	0.000	0.098	0.000	0.000	0.369	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	175	148	657	0	461	0	0	339	0
N.S.	1	1.03	0.87	3.86	0.00	2.71	0.00	0.00	1.99	0.00
time (sec)	N/A	0.960	2.917	0.904	0.000	0.093	0.000	0.000	0.238	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	232	232	1225	0	782	0	0	371	0
N.S.	1	0.95	0.95	5.04	0.00	3.22	0.00	0.00	1.53	0.00
time (sec)	N/A	1.565	3.400	1.059	0.000	0.101	0.000	0.000	0.278	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	274	223	1156	0	482	0	0	165	0
N.S.	1	1.05	0.86	4.45	0.00	1.85	0.00	0.00	0.63	0.00
time (sec)	N/A	1.683	5.802	0.988	0.000	0.098	0.000	0.000	0.179	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	226	221	1094	0	481	0	0	153	0
N.S.	1	1.06	1.03	5.11	0.00	2.25	0.00	0.00	0.71	0.00
time (sec)	N/A	1.323	3.604	1.010	0.000	0.120	0.000	0.000	0.172	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	228	222	848	0	482	0	0	180	0
N.S.	1	1.06	1.03	3.93	0.00	2.23	0.00	0.00	0.83	0.00
time (sec)	N/A	1.333	3.337	0.955	0.000	0.094	0.000	0.000	0.174	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	227	224	1096	0	482	0	0	216	0
N.S.	1	1.06	1.05	5.12	0.00	2.25	0.00	0.00	1.01	0.00
time (sec)	N/A	1.373	3.984	0.950	0.000	0.089	0.000	0.000	0.199	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	285	320	1542	0	799	0	0	228	0
N.S.	1	0.99	1.11	5.34	0.00	2.76	0.00	0.00	0.79	0.00
time (sec)	N/A	1.921	7.136	0.990	0.000	0.108	0.000	0.000	0.177	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	200	0	0	0	0	0	0	59	0
N.S.	1	1.12	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.906	0.000	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	247	301	0	0	0	0	0	0	73	0
N.S.	1	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.606	0.000	0.000	0.000	0.000	0.000	0.000	0.229	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	241	0	0	0	0	0	0	69	0
N.S.	1	1.24	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	1.111	0.000	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	197	0	0	0	0	0	0	57	0
N.S.	1	1.25	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.821	0.000	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	268	0	0	0	0	0	0	73	0
N.S.	1	1.25	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.159	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	351	0	0	0	0	0	0	73	0
N.S.	1	1.21	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	1.689	0.000	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	383	442	0	0	0	0	0	0	73	0
N.S.	1	1.15	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.308	0.000	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	206	198	535	196	936	0	0	81	0
N.S.	1	1.16	1.12	3.02	1.11	5.29	0.00	0.00	0.46	0.00
time (sec)	N/A	0.770	0.633	0.474	0.116	0.096	0.000	0.000	0.224	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	182	179	512	178	884	0	0	97	0
N.S.	1	1.19	1.17	3.35	1.16	5.78	0.00	0.00	0.63	0.00
time (sec)	N/A	0.657	0.322	0.332	0.118	0.098	0.000	0.000	0.208	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	182	178	426	178	886	0	0	56	0
N.S.	1	1.20	1.17	2.80	1.17	5.83	0.00	0.00	0.37	0.00
time (sec)	N/A	0.657	0.139	0.364	0.116	0.103	0.000	0.000	0.186	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	206	198	223	198	910	0	0	81	0
N.S.	1	1.16	1.11	1.25	1.11	5.11	0.00	0.00	0.46	0.00
time (sec)	N/A	0.834	0.346	0.424	0.123	0.097	0.000	0.000	0.205	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	283	255	293	279	1534	0	0	158	0
N.S.	1	1.08	0.97	1.11	1.06	5.83	0.00	0.00	0.60	0.00
time (sec)	N/A	1.181	1.244	0.642	0.118	0.113	0.000	0.000	29.326	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	249	226	252	252	1506	0	0	115	0
N.S.	1	1.09	0.99	1.10	1.10	6.58	0.00	0.00	0.50	0.00
time (sec)	N/A	0.985	0.794	0.624	0.118	0.114	0.000	0.000	3.796	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	228	221	239	244	1458	0	0	130	0
N.S.	1	1.08	1.04	1.13	1.15	6.88	0.00	0.00	0.61	0.00
time (sec)	N/A	0.920	0.545	0.360	0.120	0.113	0.000	0.000	1.328	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	235	226	244	254	1482	0	0	82	0
N.S.	1	1.08	1.04	1.12	1.17	6.83	0.00	0.00	0.38	0.00
time (sec)	N/A	0.964	0.344	0.362	0.121	0.107	0.000	0.000	0.265	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	269	255	271	282	1502	0	0	115	0
N.S.	1	1.06	1.00	1.07	1.11	5.91	0.00	0.00	0.45	0.00
time (sec)	N/A	1.138	0.734	0.375	0.116	0.109	0.000	0.000	0.217	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	355	326	1192	366	2129	0	0	33	0
N.S.	1	1.03	0.94	3.46	1.06	6.17	0.00	0.00	0.10	0.00
time (sec)	N/A	1.627	2.428	1.097	0.113	0.175	0.000	0.000	200.016	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	312	286	1134	330	2099	0	0	33	0
N.S.	1	1.03	0.94	3.73	1.09	6.90	0.00	0.00	0.11	0.00
time (sec)	N/A	1.362	1.545	1.120	0.118	0.129	0.000	0.000	200.019	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	303	270	1104	314	2069	0	0	33	0
N.S.	1	1.01	0.90	3.69	1.05	6.92	0.00	0.00	0.11	0.00
time (sec)	N/A	1.320	1.337	1.125	0.124	0.130	0.000	0.000	200.033	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	302	270	1104	316	2042	0	0	33	0
N.S.	1	1.02	0.92	3.74	1.07	6.92	0.00	0.00	0.11	0.00
time (sec)	N/A	1.339	1.371	0.376	0.116	0.123	0.000	0.000	200.019	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	312	287	956	334	2062	0	0	108	0
N.S.	1	1.03	0.94	3.14	1.10	6.78	0.00	0.00	0.36	0.00
time (sec)	N/A	1.423	0.807	0.381	0.150	0.125	0.000	0.000	16.663	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	358	327	328	370	2104	0	0	149	0
N.S.	1	1.04	0.95	0.95	1.07	6.10	0.00	0.00	0.43	0.00
time (sec)	N/A	1.682	1.639	0.424	0.131	0.133	0.000	0.000	0.199	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	286	272	772	262	2726	0	0	18	0
N.S.	1	1.08	1.03	2.91	0.99	10.29	0.00	0.00	0.07	0.00
time (sec)	N/A	1.765	1.075	0.472	0.124	24.793	0.000	0.000	0.160	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	250	249	686	239	2572	0	0	35	0
N.S.	1	1.06	1.06	2.91	1.01	10.90	0.00	0.00	0.15	0.00
time (sec)	N/A	1.418	0.580	0.373	0.115	10.724	0.000	0.000	0.171	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	224	215	531	221	2488	0	0	9	0
N.S.	1	1.03	0.99	2.45	1.02	11.47	0.00	0.00	0.04	0.00
time (sec)	N/A	1.064	0.249	0.393	0.119	3.883	0.000	0.000	0.164	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	224	215	244	220	2508	0	0	18	0
N.S.	1	1.04	1.00	1.13	1.02	11.61	0.00	0.00	0.08	0.00
time (sec)	N/A	1.090	0.264	0.400	0.125	2.668	0.000	0.000	0.163	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	249	251	265	238	2560	0	0	18	0
N.S.	1	1.05	1.06	1.12	1.00	10.80	0.00	0.00	0.08	0.00
time (sec)	N/A	1.444	0.355	0.376	0.121	8.079	0.000	0.000	0.171	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	285	272	286	262	2690	0	0	18	0
N.S.	1	1.08	1.03	1.08	0.99	10.19	0.00	0.00	0.07	0.00
time (sec)	N/A	1.862	0.618	0.415	0.121	19.662	0.000	0.000	0.170	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	377	383	2217	383	4891	0	0	28	0
N.S.	1	1.03	1.05	6.06	1.05	13.36	0.00	0.00	0.08	0.00
time (sec)	N/A	2.217	3.597	0.448	0.123	35.800	0.000	0.000	0.173	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	331	341	1936	362	4803	0	0	22	0
N.S.	1	1.04	1.08	6.11	1.14	15.15	0.00	0.00	0.07	0.00
time (sec)	N/A	2.801	1.761	0.444	0.129	22.169	0.000	0.000	0.451	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	330	336	332	347	4816	0	0	35	0
N.S.	1	1.04	1.06	1.05	1.09	15.19	0.00	0.00	0.11	0.00
time (sec)	N/A	2.685	1.851	0.420	0.125	13.475	0.000	0.000	0.181	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	331	342	336	360	4800	0	0	39	0
N.S.	1	1.04	1.08	1.06	1.13	15.09	0.00	0.00	0.12	0.00
time (sec)	N/A	2.760	1.640	0.407	0.129	18.359	0.000	0.000	0.172	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	375	390	351	402	4917	0	0	39	0
N.S.	1	1.03	1.07	0.96	1.10	13.47	0.00	0.00	0.11	0.00
time (sec)	N/A	3.635	2.216	0.396	0.122	32.886	0.000	0.000	0.174	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	524	617	4581	577	7491	0	0	33	0
N.S.	1	1.02	1.20	8.90	1.12	14.55	0.00	0.00	0.06	0.00
time (sec)	N/A	4.995	6.345	0.461	0.131	174.664	0.000	0.000	200.022	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	462	533	4191	558	7379	0	0	38	0
N.S.	1	1.03	1.19	9.35	1.25	16.47	0.00	0.00	0.08	0.00
time (sec)	N/A	3.850	4.466	0.439	0.140	112.164	0.000	0.000	86.556	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	459	529	451	544	7415	0	0	57	0
N.S.	1	1.03	1.18	1.01	1.22	16.59	0.00	0.00	0.13	0.00
time (sec)	N/A	3.818	4.114	0.470	0.131	74.266	0.000	0.000	0.164	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	459	534	450	545	7418	0	0	63	0
N.S.	1	1.03	1.20	1.01	1.22	16.67	0.00	0.00	0.14	0.00
time (sec)	N/A	3.847	4.148	0.421	0.136	68.909	0.000	0.000	4.414	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	463	563	451	556	7375	0	0	63	0
N.S.	1	1.03	1.26	1.01	1.24	16.46	0.00	0.00	0.14	0.00
time (sec)	N/A	4.043	4.402	0.452	0.133	101.485	0.000	0.000	0.172	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	524	639	466	612	7516	0	0	63	0
N.S.	1	1.02	1.24	0.91	1.19	14.62	0.00	0.00	0.12	0.00
time (sec)	N/A	5.259	6.347	0.415	0.143	157.335	0.000	0.000	5.672	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	157	89	102	127	398	0	0	20	64
N.S.	1	1.34	0.76	0.87	1.09	3.40	0.00	0.00	0.17	0.55
time (sec)	N/A	0.703	0.154	0.276	0.125	0.094	0.000	0.000	0.167	6.415

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	155	140	102	127	338	0	0	36	65
N.S.	1	1.36	1.23	0.89	1.11	2.96	0.00	0.00	0.32	0.57
time (sec)	N/A	0.690	0.120	0.244	0.113	0.082	0.000	0.000	0.175	6.040

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	138	60	90	113	310	0	0	11	42
N.S.	1	1.41	0.61	0.92	1.15	3.16	0.00	0.00	0.11	0.43
time (sec)	N/A	0.575	0.049	0.305	0.117	0.083	0.000	0.000	0.191	5.548

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	138	110	90	116	310	0	0	20	47
N.S.	1	1.39	1.11	0.91	1.17	3.13	0.00	0.00	0.20	0.47
time (sec)	N/A	0.570	0.028	0.300	0.122	0.095	0.000	0.000	0.184	5.401

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	155	78	102	126	423	0	0	20	64
N.S.	1	1.35	0.68	0.89	1.10	3.68	0.00	0.00	0.17	0.56
time (sec)	N/A	0.684	0.089	0.256	0.120	0.086	0.000	0.000	0.179	5.571

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	157	82	102	128	429	0	0	20	65
N.S.	1	1.35	0.71	0.88	1.10	3.70	0.00	0.00	0.17	0.56
time (sec)	N/A	0.689	0.135	0.273	0.342	0.109	0.000	0.000	0.168	6.300

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	354	376	291	2185255	0	8179	0	0	33	0
N.S.	1	1.06	0.82	6173.04	0.00	23.10	0.00	0.00	0.09	0.00
time (sec)	N/A	4.143	2.850	1.298	0.000	1.195	0.000	0.000	200.023	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	290	310	252	2183123	0	8104	0	0	33	0
N.S.	1	1.07	0.87	7528.01	0.00	27.94	0.00	0.00	0.11	0.00
time (sec)	N/A	3.149	1.591	1.188	0.000	1.209	0.000	0.000	200.020	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	239	243	216	2178910	0	8070	0	0	33	0
N.S.	1	1.02	0.90	9116.78	0.00	33.77	0.00	0.00	0.14	0.00
time (sec)	N/A	2.273	1.303	1.176	0.000	1.180	0.000	0.000	200.015	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	194	191	189	2178316	0	7949	0	0	33	0
N.S.	1	0.98	0.97	11228.43	0.00	40.97	0.00	0.00	0.17	0.00
time (sec)	N/A	1.714	0.329	1.281	0.000	1.178	0.000	0.000	200.018	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	229	200	238	2178068	0	16183	0	0	33	0
N.S.	1	0.87	1.04	9511.21	0.00	70.67	0.00	0.00	0.14	0.00
time (sec)	N/A	2.240	0.538	1.184	0.000	2.159	0.000	0.000	200.029	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	261	220	293	2180708	0	16171	0	0	69	0
N.S.	1	0.84	1.12	8355.20	0.00	61.96	0.00	0.00	0.26	0.00
time (sec)	N/A	1.886	2.390	1.011	0.000	3.204	0.000	0.000	0.275	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	324	286	356	2183187	0	16383	0	0	69	0
N.S.	1	0.88	1.10	6738.23	0.00	50.56	0.00	0.00	0.21	0.00
time (sec)	N/A	2.626	3.240	1.534	0.000	3.306	0.000	0.000	0.280	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	422	451	5158	2405378	0	12342	0	0	33	0
N.S.	1	1.07	12.22	5699.95	0.00	29.25	0.00	0.00	0.08	0.00
time (sec)	N/A	5.333	34.793	2.410	0.000	2.099	0.000	0.000	200.019	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	351	373	5099	2403031	0	12268	0	0	33	0
N.S.	1	1.06	14.53	6846.24	0.00	34.95	0.00	0.00	0.09	0.00
time (sec)	N/A	4.201	35.126	3.371	0.000	2.078	0.000	0.000	200.022	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	299	310	286	2398647	0	12233	0	0	33	0
N.S.	1	1.04	0.96	8022.23	0.00	40.91	0.00	0.00	0.11	0.00
time (sec)	N/A	3.360	2.058	2.802	0.000	2.080	0.000	0.000	200.032	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	236	246	244	2398803	0	12148	0	0	33	0
N.S.	1	1.04	1.03	10164.42	0.00	51.47	0.00	0.00	0.14	0.00
time (sec)	N/A	2.560	0.697	2.118	0.000	2.050	0.000	0.000	200.016	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	269	221	266	2397204	0	24440	0	0	33	0
N.S.	1	0.82	0.99	8911.54	0.00	90.86	0.00	0.00	0.12	0.00
time (sec)	N/A	2.043	1.643	2.079	0.000	4.123	0.000	0.000	200.017	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	264	221	263	2396110	0	24471	0	0	33	0
N.S.	1	0.84	1.00	9076.17	0.00	92.69	0.00	0.00	0.12	0.00
time (sec)	N/A	2.105	0.850	1.128	0.000	4.250	0.000	0.000	200.019	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	320	284	310	0	0	24563	0	0	33	0
N.S.	1	0.89	0.97	0.00	0.00	76.76	0.00	0.00	0.10	0.00
time (sec)	N/A	2.829	2.276	180.000	0.000	4.470	0.000	0.000	200.025	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	383	347	367	0	0	24697	0	0	33	0
N.S.	1	0.91	0.96	0.00	0.00	64.48	0.00	0.00	0.09	0.00
time (sec)	N/A	3.569	4.060	180.000	0.000	4.936	0.000	0.000	200.026	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F(-1)	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	500	530	5419	0	0	19121	0	0	33	0
N.S.	1	1.06	10.84	0.00	0.00	38.24	0.00	0.00	0.07	0.00
time (sec)	N/A	6.579	33.592	180.000	0.000	4.382	0.000	0.000	200.022	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F(-1)	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	418	449	5348	0	0	19062	0	0	33	0
N.S.	1	1.07	12.79	0.00	0.00	45.60	0.00	0.00	0.08	0.00
time (sec)	N/A	5.366	33.324	180.000	0.000	4.388	0.000	0.000	200.014	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F(-1)	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	349	371	5276	0	0	18996	0	0	33	0
N.S.	1	1.06	15.12	0.00	0.00	54.43	0.00	0.00	0.09	0.00
time (sec)	N/A	4.270	33.051	180.000	0.000	4.335	0.000	0.000	200.025	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F(-1)	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	304	5229	0	0	18942	0	0	33	0
N.S.	1	1.06	18.22	0.00	0.00	66.00	0.00	0.00	0.11	0.00
time (sec)	N/A	3.398	33.748	180.000	0.000	4.360	0.000	0.000	200.017	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	300	262	417	2654016	0	35564	0	0	33	0
N.S.	1	0.87	1.39	8846.72	0.00	118.55	0.00	0.00	0.11	0.00
time (sec)	N/A	2.771	3.595	1.256	0.000	6.667	0.000	0.000	200.018	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	301	258	363	2653710	0	35373	0	0	33	0
N.S.	1	0.86	1.21	8816.31	0.00	117.52	0.00	0.00	0.11	0.00
time (sec)	N/A	2.874	2.127	1.370	0.000	7.040	0.000	0.000	200.022	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	320	278	311	2652345	0	35483	0	0	33	0
N.S.	1	0.87	0.97	8288.58	0.00	110.88	0.00	0.00	0.10	0.00
time (sec)	N/A	2.909	2.354	1.253	0.000	7.063	0.000	0.000	200.022	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	376	346	365	2657129	0	35533	0	0	33	0
N.S.	1	0.92	0.97	7066.83	0.00	94.50	0.00	0.00	0.09	0.00
time (sec)	N/A	3.726	4.163	1.215	0.000	7.282	0.000	0.000	200.028	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	457	421	456	2659501	0	35731	0	0	33	0
N.S.	1	0.92	1.00	5819.48	0.00	78.19	0.00	0.00	0.07	0.00
time (sec)	N/A	4.600	33.145	1.268	0.000	8.780	0.000	0.000	200.027	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	296	303	244	1892755	0	10265	0	0	33	0
N.S.	1	1.02	0.82	6394.44	0.00	34.68	0.00	0.00	0.11	0.00
time (sec)	N/A	3.060	4.241	1.099	0.000	2.583	0.000	0.000	200.020	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	243	240	213	1890726	0	10191	0	0	29	0
N.S.	1	0.99	0.88	7780.77	0.00	41.94	0.00	0.00	0.12	0.00
time (sec)	N/A	2.310	1.843	1.082	0.000	2.635	0.000	0.000	0.245	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	199	187	193	1888059	0	10120	0	0	27	0
N.S.	1	0.94	0.97	9487.73	0.00	50.85	0.00	0.00	0.14	0.00
time (sec)	N/A	1.685	1.063	1.080	0.000	2.634	0.000	0.000	10.200	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	163	144	157	1878877	0	10053	0	0	21	0
N.S.	1	0.88	0.96	11526.85	0.00	61.67	0.00	0.00	0.13	0.00
time (sec)	N/A	1.298	0.260	1.224	0.000	2.783	0.000	0.000	0.194	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	228	189	225	1885968	0	20493	0	0	29	0
N.S.	1	0.83	0.99	8271.79	0.00	89.88	0.00	0.00	0.13	0.00
time (sec)	N/A	2.204	1.024	0.834	0.000	4.160	0.000	0.000	0.234	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	266	228	354	1888591	0	20571	0	0	29	0
N.S.	1	0.86	1.33	7099.97	0.00	77.33	0.00	0.00	0.11	0.00
time (sec)	N/A	1.887	1.689	1.273	0.000	4.862	0.000	0.000	0.245	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	316	337	301	1562552	0	18876	0	0	33	0
N.S.	1	1.07	0.95	4944.78	0.00	59.73	0.00	0.00	0.10	0.00
time (sec)	N/A	3.394	2.855	1.811	0.000	6.496	0.000	0.000	200.021	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	256	279	255	1561945	0	18757	0	0	39	0
N.S.	1	1.09	1.00	6101.35	0.00	73.27	0.00	0.00	0.15	0.00
time (sec)	N/A	2.539	1.597	1.905	0.000	6.523	0.000	0.000	0.223	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	215	224	222	1561037	0	18703	0	0	33	0
N.S.	1	1.04	1.03	7260.64	0.00	86.99	0.00	0.00	0.15	0.00
time (sec)	N/A	1.967	0.842	1.773	0.000	6.467	0.000	0.000	0.196	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	210	223	259	1561051	0	18672	0	0	46	0
N.S.	1	1.06	1.23	7433.58	0.00	88.91	0.00	0.00	0.22	0.00
time (sec)	N/A	1.946	1.476	1.099	0.000	6.492	0.000	0.000	0.176	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	279	274	361	1562182	0	37815	0	0	50	0
N.S.	1	0.98	1.29	5599.22	0.00	135.54	0.00	0.00	0.18	0.00
time (sec)	N/A	2.216	1.551	1.802	0.000	11.162	0.000	0.000	0.210	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	399	445	385	2982444	0	27987	0	0	33	0
N.S.	1	1.12	0.96	7474.80	0.00	70.14	0.00	0.00	0.08	0.00
time (sec)	N/A	4.755	2.792	2.507	0.000	13.205	0.000	0.000	200.023	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	341	382	334	2976849	0	27817	0	0	33	0
N.S.	1	1.12	0.98	8729.76	0.00	81.57	0.00	0.00	0.10	0.00
time (sec)	N/A	3.717	2.860	2.618	0.000	13.419	0.000	0.000	200.020	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	287	320	293	2975320	0	27790	0	0	49	0
N.S.	1	1.11	1.02	10366.97	0.00	96.83	0.00	0.00	0.17	0.00
time (sec)	N/A	2.935	2.499	2.382	0.000	13.428	0.000	0.000	61.384	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	284	318	340	2978186	0	27743	0	0	68	0
N.S.	1	1.12	1.20	10486.57	0.00	97.69	0.00	0.00	0.24	0.00
time (sec)	N/A	2.900	3.130	1.641	0.000	13.361	0.000	0.000	0.190	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	284	314	328	2978061	0	27726	0	0	74	0
N.S.	1	1.11	1.15	10486.13	0.00	97.63	0.00	0.00	0.26	0.00
time (sec)	N/A	2.896	2.590	2.670	0.000	13.273	0.000	0.000	0.352	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	342	365	617	2981045	0	56055	0	0	74	0
N.S.	1	1.07	1.80	8716.51	0.00	163.90	0.00	0.00	0.22	0.00
time (sec)	N/A	3.253	6.272	2.692	0.000	23.003	0.000	0.000	0.174	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	151	128	145	892	0	4609	0	0	35	0
N.S.	1	0.85	0.96	5.91	0.00	30.52	0.00	0.00	0.23	0.00
time (sec)	N/A	0.779	0.077	5.566	0.000	0.556	0.000	0.000	0.181	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	157	134	144	1030	0	4649	0	0	48	0
N.S.	1	0.85	0.92	6.56	0.00	29.61	0.00	0.00	0.31	0.00
time (sec)	N/A	0.803	0.066	4.564	0.000	0.537	0.000	0.000	0.171	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	215	168	210	1026	0	9437	0	0	52	0
N.S.	1	0.78	0.98	4.77	0.00	43.89	0.00	0.00	0.24	0.00
time (sec)	N/A	0.924	0.665	4.794	0.000	0.868	0.000	0.000	0.248	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	216	0	0	0	0	0	0	33	0
N.S.	1	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.261	0.000	0.000	0.000	0.000	0.000	0.000	200.023	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	182	0	0	0	0	0	0	33	0
N.S.	1	1.06	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.260	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	188	0	0	0	0	0	0	33	0
N.S.	1	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.233	0.000	0.000	0.000	0.000	0.000	0.000	200.018	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	194	0	0	0	0	0	0	71	0
N.S.	1	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.272	0.000	0.000	0.000	0.000	0.000	0.000	11.449	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	194	0	0	0	0	0	0	71	0
N.S.	1	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.286	0.000	0.000	0.000	0.000	0.000	0.000	9.369	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	50	80	311	93	394	0	46	128
N.S.	1	1.02	0.79	1.27	4.94	1.48	6.25	0.00	0.73	2.03
time (sec)	N/A	0.495	0.616	1.592	0.209	0.096	0.582	0.000	0.154	1.131

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	53	77	56	95	100	155	124	96	100
N.S.	1	0.90	1.31	0.95	1.61	1.69	2.63	2.10	1.63	1.69
time (sec)	N/A	0.462	0.912	0.375	0.128	0.068	0.306	0.504	0.152	5.151

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	53	70	56	72	86	136	94	72	76
N.S.	1	0.90	1.19	0.95	1.22	1.46	2.31	1.59	1.22	1.29
time (sec)	N/A	0.466	1.941	0.336	0.134	0.070	0.262	0.471	0.152	5.188

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	66	53	54	49	55	74	117	64	50	50
N.S.	1	0.80	0.82	0.74	0.83	1.12	1.77	0.97	0.76	0.76
time (sec)	N/A	0.449	1.222	0.304	0.134	0.069	0.206	0.428	0.161	5.195

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	32	27	29	54	82	29	25	25
N.S.	1	1.00	1.33	1.12	1.21	2.25	3.42	1.21	1.04	1.04
time (sec)	N/A	0.351	0.025	0.112	0.120	0.070	0.153	0.394	0.160	5.130

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	66	50	50	64	53	35	55	38
N.S.	1	1.00	1.43	1.09	1.09	1.39	1.15	0.76	1.20	0.83
time (sec)	N/A	0.431	0.029	0.181	0.119	0.117	0.209	0.466	0.150	5.506

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	43	44	74	0	43	88	45	96	51
N.S.	1	0.80	0.81	1.37	0.00	0.80	1.63	0.83	1.78	0.94
time (sec)	N/A	0.458	1.422	0.235	0.000	0.094	0.186	0.453	0.159	5.330

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	46	0	46	153	36	132	51
N.S.	1	1.00	0.96	1.00	0.00	1.00	3.33	0.78	2.87	1.11
time (sec)	N/A	0.422	1.599	0.278	0.000	0.097	0.200	0.485	0.156	5.416

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	49	41	43	0	59	201	37	179	63
N.S.	1	0.89	0.75	0.78	0.00	1.07	3.65	0.67	3.25	1.15
time (sec)	N/A	0.450	0.311	0.289	0.000	0.070	0.236	0.520	0.163	5.332

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	51	41	44	0	81	304	37	221	73
N.S.	1	0.89	0.72	0.77	0.00	1.42	5.33	0.65	3.88	1.28
time (sec)	N/A	0.458	0.348	0.296	0.000	0.075	0.302	0.556	0.164	5.477

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	49	41	45	0	94	348	37	257	82
N.S.	1	0.89	0.75	0.82	0.00	1.71	6.33	0.67	4.67	1.49
time (sec)	N/A	0.458	1.057	0.348	0.000	0.074	0.384	0.529	0.170	5.575

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	102	89	167	660	206	1482	0	127	193
N.S.	1	0.94	0.82	1.53	6.06	1.89	13.60	0.00	1.17	1.77
time (sec)	N/A	0.594	1.375	1.595	0.265	0.085	1.323	0.000	0.149	7.912

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	82	270	83	151	152	243	208	144	158
N.S.	1	0.83	2.73	0.84	1.53	1.54	2.45	2.10	1.45	1.60
time (sec)	N/A	0.564	1.255	0.556	0.115	0.087	0.810	0.723	0.151	5.296

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	82	103	83	116	136	224	140	98	120
N.S.	1	0.83	1.04	0.84	1.17	1.37	2.26	1.41	0.99	1.21
time (sec)	N/A	0.546	1.453	0.484	0.117	0.071	0.521	0.586	0.154	5.099

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	82	95	83	101	124	206	140	98	108
N.S.	1	0.83	0.96	0.84	1.02	1.25	2.08	1.41	0.99	1.09
time (sec)	N/A	0.552	4.887	0.407	0.114	0.072	0.563	0.501	0.161	5.058

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	49	53	53	72	105	160	72	52	82
N.S.	1	0.79	0.85	0.85	1.16	1.69	2.58	1.16	0.84	1.32
time (sec)	N/A	0.475	0.116	0.187	0.107	0.084	0.295	0.460	0.154	5.104

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	51	54	51	53	98	158	64	50	50
N.S.	1	0.80	0.84	0.80	0.83	1.53	2.47	1.00	0.78	0.78
time (sec)	N/A	0.433	0.665	0.207	0.112	0.071	0.213	0.428	0.150	5.156

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	80	70	58	76	71	121	122	74	79	76
N.S.	1	0.88	0.72	0.95	0.89	1.51	1.52	0.92	0.99	0.95
time (sec)	N/A	0.610	0.240	0.351	0.109	0.080	0.306	0.443	0.150	5.488

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	65	84	146	0	111	134	78	95	105
N.S.	1	0.70	0.90	1.57	0.00	1.19	1.44	0.84	1.02	1.13
time (sec)	N/A	0.519	5.349	0.355	0.000	0.083	0.370	0.479	0.152	5.445

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	65	56	103	0	62	160	65	205	104
N.S.	1	0.71	0.62	1.13	0.00	0.68	1.76	0.71	2.25	1.14
time (sec)	N/A	0.542	2.818	0.304	0.000	0.112	0.295	0.535	0.159	5.604

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	76	62	69	0	49	167	68	277	87
N.S.	1	0.82	0.67	0.74	0.00	0.53	1.80	0.73	2.98	0.94
time (sec)	N/A	0.534	2.469	0.397	0.000	0.073	0.287	0.539	0.179	5.450

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	74	51	68	0	64	218	51	339	78
N.S.	1	0.81	0.56	0.75	0.00	0.70	2.40	0.56	3.73	0.86
time (sec)	N/A	0.547	5.197	0.351	0.000	0.078	0.327	0.547	0.186	5.295

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	78	62	69	0	89	332	69	393	108
N.S.	1	0.82	0.65	0.73	0.00	0.94	3.49	0.73	4.14	1.14
time (sec)	N/A	0.535	5.251	0.477	0.000	0.074	0.406	0.561	0.189	5.466

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	74	60	66	0	104	379	68	477	118
N.S.	1	0.81	0.66	0.73	0.00	1.14	4.16	0.75	5.24	1.30
time (sec)	N/A	0.548	5.269	0.424	0.000	0.081	0.513	0.623	0.204	5.430

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	138	154	271	1056	335	3665	0	270	323
N.S.	1	0.91	1.02	1.79	6.99	2.22	24.27	0.00	1.79	2.14
time (sec)	N/A	0.661	2.656	1.260	0.333	0.098	3.639	0.000	0.167	9.908

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	109	185	108	193	194	325	276	190	208
N.S.	1	0.81	1.37	0.80	1.43	1.44	2.41	2.04	1.41	1.54
time (sec)	N/A	0.625	5.624	0.798	0.114	0.074	0.978	0.929	0.177	5.264

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	109	98	108	166	182	306	242	166	174
N.S.	1	0.81	0.73	0.80	1.23	1.35	2.27	1.79	1.23	1.29
time (sec)	N/A	0.595	5.477	0.655	0.125	0.088	0.825	0.909	0.160	5.205

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	106	131	108	151	170	287	208	144	156
N.S.	1	0.80	0.99	0.82	1.14	1.29	2.17	1.58	1.09	1.18
time (sec)	N/A	0.598	1.397	0.543	0.119	0.090	0.661	0.704	0.162	5.094

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	84	61	65	75	106	147	224	106	74	120
N.S.	1	0.73	0.77	0.89	1.26	1.75	2.67	1.26	0.88	1.43
time (sec)	N/A	0.507	0.194	0.206	0.113	0.069	0.524	0.567	0.186	5.042

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	101	84	91	94	105	152	250	140	98	108
N.S.	1	0.83	0.90	0.93	1.04	1.50	2.48	1.39	0.97	1.07
time (sec)	N/A	0.540	3.652	0.289	0.117	0.069	0.413	0.477	0.160	5.404

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	61	55	70	63	72	132	218	94	72	72
N.S.	1	0.90	1.15	1.03	1.18	2.16	3.57	1.54	1.18	1.18
time (sec)	N/A	0.461	2.128	0.235	0.123	0.070	0.298	0.509	0.169	5.054

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	99	73	100	96	175	184	115	103	125
N.S.	1	0.90	0.66	0.91	0.87	1.59	1.67	1.05	0.94	1.14
time (sec)	N/A	0.767	0.599	0.234	0.138	0.077	0.370	0.430	0.165	5.190

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	81	120	178	0	164	206	113	114	139
N.S.	1	0.68	1.01	1.50	0.00	1.38	1.73	0.95	0.96	1.17
time (sec)	N/A	0.563	5.446	0.302	0.000	0.112	0.470	0.621	0.166	5.243

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	85	90	189	0	138	236	99	277	182
N.S.	1	0.69	0.73	1.54	0.00	1.12	1.92	0.80	2.25	1.48
time (sec)	N/A	0.577	3.883	0.384	0.000	0.087	0.490	0.597	0.171	5.809

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	101	74	124	0	77	212	87	373	141
N.S.	1	0.78	0.57	0.96	0.00	0.60	1.64	0.67	2.89	1.09
time (sec)	N/A	0.559	3.189	0.410	0.000	0.124	0.405	0.543	0.184	5.577

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	91	79	88	0	49	167	97	515	118
N.S.	1	0.92	0.80	0.89	0.00	0.49	1.69	0.98	5.20	1.19
time (sec)	N/A	0.508	2.112	0.384	0.000	0.073	0.369	0.580	0.198	5.644

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	96	78	87	0	64	218	97	541	128
N.S.	1	0.79	0.64	0.71	0.00	0.52	1.79	0.80	4.43	1.05
time (sec)	N/A	0.572	5.416	0.490	0.000	0.076	0.469	0.647	0.205	5.655

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	101	80	90	0	89	332	96	639	140
N.S.	1	0.80	0.63	0.71	0.00	0.70	2.61	0.76	5.03	1.10
time (sec)	N/A	0.567	5.429	0.398	0.000	0.071	0.539	0.686	0.216	5.538

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	99	80	89	0	104	379	97	711	151
N.S.	1	0.79	0.64	0.71	0.00	0.83	3.03	0.78	5.69	1.21
time (sec)	N/A	0.573	5.215	0.704	0.000	0.075	0.708	0.735	0.232	5.768

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	101	84	90	0	131	496	97	787	160
N.S.	1	0.80	0.66	0.71	0.00	1.03	3.91	0.76	6.20	1.26
time (sec)	N/A	0.574	4.717	0.457	0.000	0.081	0.721	0.710	0.243	6.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	119	98	0	0	0	0	0	149	0
N.S.	1	1.03	0.85	0.00	0.00	0.00	0.00	0.00	1.30	0.00
time (sec)	N/A	0.590	0.910	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	110	150	224	0	298	328	166	324	205
N.S.	1	0.70	0.96	1.43	0.00	1.90	2.09	1.06	2.06	1.31
time (sec)	N/A	0.606	0.969	0.452	0.000	0.087	0.526	0.557	0.178	5.435

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	85	124	191	0	219	265	113	118	136
N.S.	1	0.70	1.02	1.58	0.00	1.81	2.19	0.93	0.98	1.12
time (sec)	N/A	0.560	3.237	0.442	0.000	0.086	0.471	0.571	0.167	5.136

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	68	83	154	0	153	194	78	98	110
N.S.	1	0.71	0.86	1.60	0.00	1.59	2.02	0.81	1.02	1.15
time (sec)	N/A	0.540	2.794	0.401	0.000	0.103	0.414	0.461	0.185	5.519

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	48	47	83	0	67	112	45	97	54
N.S.	1	0.84	0.82	1.46	0.00	1.18	1.96	0.79	1.70	0.95
time (sec)	N/A	0.471	0.466	0.298	0.000	0.079	0.184	0.442	0.164	5.322

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	56	54	0	42	87	70	44	45
N.S.	1	1.00	1.19	1.15	0.00	0.89	1.85	1.49	0.94	0.96
time (sec)	N/A	0.370	0.172	0.225	0.000	0.078	0.139	0.466	0.161	5.380

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	43	54	0	58	165	53	58	40
N.S.	1	1.11	0.96	1.20	0.00	1.29	3.67	1.18	1.29	0.89
time (sec)	N/A	0.507	0.127	0.346	0.000	0.074	0.186	0.489	0.165	5.270

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	96	123	130	0	80	284	120	237	129
N.S.	1	0.85	1.09	1.15	0.00	0.71	2.51	1.06	2.10	1.14
time (sec)	N/A	0.598	4.191	0.449	0.000	0.078	0.258	0.528	0.171	5.583

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	123	149	195	0	92	328	138	89	161
N.S.	1	0.83	1.00	1.31	0.00	0.62	2.20	0.93	0.60	1.08
time (sec)	N/A	0.635	4.006	0.529	0.000	0.101	0.329	0.502	0.177	5.374

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	146	141	238	0	115	439	156	131	204
N.S.	1	0.81	0.78	1.31	0.00	0.64	2.43	0.86	0.72	1.13
time (sec)	N/A	0.660	2.385	0.493	0.000	0.074	0.371	0.701	0.180	5.914

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	119	152	0	0	0	0	0	93	0
N.S.	1	1.03	1.32	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.585	1.570	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	138	197	267	0	329	445	188	419	282
N.S.	1	0.71	1.02	1.38	0.00	1.70	2.29	0.97	2.16	1.45
time (sec)	N/A	0.662	4.156	0.650	0.000	0.095	0.683	0.702	0.184	5.256

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	113	129	239	0	249	377	140	349	207
N.S.	1	0.72	0.82	1.51	0.00	1.58	2.39	0.89	2.21	1.31
time (sec)	N/A	0.620	5.645	0.476	0.000	0.106	0.603	0.612	0.183	5.198

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	94	92	200	0	177	309	99	279	194
N.S.	1	0.73	0.72	1.56	0.00	1.38	2.41	0.77	2.18	1.52
time (sec)	N/A	0.589	5.409	0.495	0.000	0.091	0.510	0.511	0.178	5.767

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	73	58	114	0	88	206	65	205	104
N.S.	1	0.75	0.60	1.18	0.00	0.91	2.12	0.67	2.11	1.07
time (sec)	N/A	0.552	5.416	0.401	0.000	0.082	0.291	0.491	0.174	5.241

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	60	46	0	47	158	36	133	50
N.S.	1	1.00	1.25	0.96	0.00	0.98	3.29	0.75	2.77	1.04
time (sec)	N/A	0.432	0.780	0.378	0.000	0.080	0.206	0.411	0.173	5.021

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	79	89	73	0	54	162	83	64	70
N.S.	1	0.99	1.11	0.91	0.00	0.68	2.02	1.04	0.80	0.88
time (sec)	N/A	0.511	0.519	0.278	0.000	0.080	0.185	0.442	0.169	5.162

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	100	123	130	0	79	296	120	234	129
N.S.	1	0.85	1.05	1.11	0.00	0.68	2.53	1.03	2.00	1.10
time (sec)	N/A	0.605	5.444	0.548	0.000	0.072	0.261	0.482	0.173	5.285

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	53	96	0	90	360	65	107	53
N.S.	1	1.08	0.75	1.35	0.00	1.27	5.07	0.92	1.51	0.75
time (sec)	N/A	0.529	0.169	0.387	0.000	0.087	0.336	0.489	0.180	5.009

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	148	173	209	0	115	454	153	486	208
N.S.	1	0.81	0.95	1.14	0.00	0.63	2.48	0.84	2.66	1.14
time (sec)	N/A	0.663	5.631	0.481	0.000	0.072	0.413	0.618	0.171	5.799

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	177	213	281	0	127	498	172	147	247
N.S.	1	0.80	0.96	1.27	0.00	0.57	2.25	0.78	0.67	1.12
time (sec)	N/A	0.720	4.146	0.557	0.000	0.074	0.527	0.604	0.168	6.885

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	198	233	303	0	151	605	193	173	291
N.S.	1	0.79	0.93	1.21	0.00	0.60	2.41	0.77	0.69	1.16
time (sec)	N/A	0.754	5.429	0.658	0.000	0.093	0.571	0.642	0.178	6.961

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	119	210	0	0	0	0	0	340	0
N.S.	1	1.03	1.83	0.00	0.00	0.00	0.00	0.00	2.96	0.00
time (sec)	N/A	0.577	1.583	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	137	152	285	0	267	473	164	350	233
N.S.	1	0.72	0.80	1.49	0.00	1.40	2.48	0.86	1.83	1.22
time (sec)	N/A	0.655	3.102	0.683	0.000	0.096	0.782	0.707	0.185	5.425

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	118	107	248	0	197	403	122	318	266
N.S.	1	0.72	0.65	1.51	0.00	1.20	2.46	0.74	1.94	1.62
time (sec)	N/A	0.618	5.448	0.513	0.000	0.102	0.909	0.636	0.187	7.510

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	109	76	135	0	103	258	87	375	149
N.S.	1	0.81	0.56	1.00	0.00	0.76	1.91	0.64	2.78	1.10
time (sec)	N/A	0.577	5.633	0.496	0.000	0.091	0.400	0.574	0.199	5.444

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	82	62	69	0	49	172	68	277	87
N.S.	1	0.83	0.63	0.70	0.00	0.49	1.74	0.69	2.80	0.88
time (sec)	N/A	0.556	5.213	0.500	0.000	0.070	0.271	0.443	0.184	5.231

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	53	41	43	0	60	206	37	181	62
N.S.	1	0.90	0.69	0.73	0.00	1.02	3.49	0.63	3.07	1.05
time (sec)	N/A	0.469	5.092	0.396	0.000	0.077	0.263	0.470	0.178	5.013

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	111	109	128	0	76	258	100	88	111
N.S.	1	0.99	0.97	1.14	0.00	0.68	2.30	0.89	0.79	0.99
time (sec)	N/A	0.697	0.627	0.270	0.000	0.076	0.294	0.468	0.174	5.221

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	127	148	195	0	91	340	138	89	161
N.S.	1	0.83	0.97	1.27	0.00	0.59	2.22	0.90	0.58	1.05
time (sec)	N/A	0.638	5.547	0.592	0.000	0.078	0.339	0.512	0.166	5.410

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	150	172	209	0	115	452	153	504	208
N.S.	1	0.81	0.93	1.13	0.00	0.62	2.44	0.83	2.72	1.12
time (sec)	N/A	0.679	5.661	0.608	0.000	0.079	0.413	0.518	0.170	5.866

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	104	63	138	0	126	508	76	152	64
N.S.	1	1.05	0.64	1.39	0.00	1.27	5.13	0.77	1.54	0.65
time (sec)	N/A	0.547	0.187	0.401	0.000	0.074	0.529	0.567	0.160	5.049

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	198	221	261	0	151	605	185	830	286
N.S.	1	0.79	0.88	1.04	0.00	0.60	2.41	0.74	3.31	1.14
time (sec)	N/A	0.756	5.906	0.524	0.000	0.076	0.593	0.687	0.176	6.850

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	225	261	335	0	159	646	202	173	319
N.S.	1	0.78	0.91	1.17	0.00	0.55	2.25	0.70	0.60	1.11
time (sec)	N/A	0.800	6.129	0.681	0.000	0.074	0.791	0.761	0.170	7.286

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	248	278	392	0	185	753	219	173	352
N.S.	1	0.78	0.87	1.23	0.00	0.58	2.36	0.69	0.54	1.10
time (sec)	N/A	0.833	6.410	0.748	0.000	0.077	0.787	0.785	0.185	7.691

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	58	55	48	108	0	0	106	101
N.S.	1	1.02	0.94	0.89	0.77	1.74	0.00	0.00	1.71	1.63
time (sec)	N/A	0.518	2.274	0.935	0.038	0.166	0.000	0.000	0.172	9.554

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	60	55	48	96	0	0	84	101
N.S.	1	1.02	0.97	0.89	0.77	1.55	0.00	0.00	1.35	1.63
time (sec)	N/A	0.503	1.301	0.692	0.044	0.110	0.000	0.000	0.173	10.197

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	54	55	48	78	0	0	58	99
N.S.	1	1.02	0.87	0.89	0.77	1.26	0.00	0.00	0.94	1.60
time (sec)	N/A	0.498	0.938	0.780	0.040	0.092	0.000	0.000	0.174	7.962

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	45	66	48	66	0	0	75	102
N.S.	1	1.02	0.75	1.10	0.80	1.10	0.00	0.00	1.25	1.70
time (sec)	N/A	0.498	0.738	0.500	0.036	0.084	0.000	0.000	0.190	0.795

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	59	42	53	48	55	0	0	463	164
N.S.	1	1.02	0.72	0.91	0.83	0.95	0.00	0.00	7.98	2.83
time (sec)	N/A	0.480	0.981	0.536	0.040	0.078	0.000	0.000	0.208	5.681

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	57	53	45	75	0	0	645	170
N.S.	1	1.02	0.95	0.88	0.75	1.25	0.00	0.00	10.75	2.83
time (sec)	N/A	0.499	1.550	0.520	0.039	0.078	0.000	0.000	0.310	6.280

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	58	53	46	92	0	0	288	232
N.S.	1	1.02	0.94	0.85	0.74	1.48	0.00	0.00	4.65	3.74
time (sec)	N/A	0.497	2.294	0.440	0.039	0.087	0.000	0.000	0.165	6.746

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	58	53	46	113	0	0	0	157
N.S.	1	1.02	0.94	0.85	0.74	1.82	0.00	0.00	0.00	2.53
time (sec)	N/A	0.509	3.590	0.506	0.034	0.110	0.000	0.000	1.635	7.224

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	98	104	84	78	149	0	0	133	132
N.S.	1	0.93	0.99	0.80	0.74	1.42	0.00	0.00	1.27	1.26
time (sec)	N/A	0.621	5.561	0.989	0.037	0.237	0.000	0.000	0.171	9.002

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	98	104	84	78	133	0	0	109	132
N.S.	1	0.93	0.99	0.80	0.74	1.27	0.00	0.00	1.04	1.26
time (sec)	N/A	0.606	3.341	0.857	0.039	0.146	0.000	0.000	0.156	9.972

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	98	105	84	78	119	0	0	85	130
N.S.	1	0.93	1.00	0.80	0.74	1.13	0.00	0.00	0.81	1.24
time (sec)	N/A	0.614	2.616	0.837	0.040	0.109	0.000	0.000	0.159	8.744

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	96	76	84	78	104	0	0	157	241
N.S.	1	0.93	0.74	0.82	0.76	1.01	0.00	0.00	1.52	2.34
time (sec)	N/A	0.594	1.963	0.832	0.037	0.084	0.000	0.000	0.196	2.263

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	94	62	94	81	93	0	0	641	176
N.S.	1	0.93	0.61	0.93	0.80	0.92	0.00	0.00	6.35	1.74
time (sec)	N/A	0.607	2.516	0.563	0.038	0.092	0.000	0.000	0.221	6.137

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	94	81	79	79	81	0	0	807	158
N.S.	1	0.93	0.80	0.78	0.78	0.80	0.00	0.00	7.99	1.56
time (sec)	N/A	0.613	4.583	0.629	0.039	0.112	0.000	0.000	0.363	6.059

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	96	77	80	76	102	0	0	437	208
N.S.	1	0.93	0.75	0.78	0.74	0.99	0.00	0.00	4.24	2.02
time (sec)	N/A	0.621	5.397	0.500	0.041	0.083	0.000	0.000	0.166	6.610

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	98	79	80	76	123	0	0	0	167
N.S.	1	0.93	0.75	0.76	0.72	1.17	0.00	0.00	0.00	1.59
time (sec)	N/A	0.622	5.655	0.619	0.042	0.127	0.000	0.000	0.993	6.760

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	131	123	121	104	179	0	0	155	349
N.S.	1	0.91	0.85	0.84	0.72	1.24	0.00	0.00	1.08	2.42
time (sec)	N/A	0.673	5.563	1.285	0.039	0.349	0.000	0.000	0.160	9.633

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	131	128	121	104	173	0	0	133	349
N.S.	1	0.91	0.89	0.84	0.72	1.20	0.00	0.00	0.92	2.42
time (sec)	N/A	0.640	4.521	0.992	0.038	0.208	0.000	0.000	0.169	8.875

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	131	128	121	104	153	0	0	107	335
N.S.	1	0.91	0.89	0.84	0.72	1.06	0.00	0.00	0.74	2.33
time (sec)	N/A	0.646	2.823	0.927	0.038	0.122	0.000	0.000	0.170	8.404

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	129	93	121	104	137	0	0	214	313
N.S.	1	0.91	0.65	0.85	0.73	0.96	0.00	0.00	1.51	2.20
time (sec)	N/A	0.626	2.387	0.795	0.037	0.100	0.000	0.000	0.208	9.337

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	127	83	135	108	127	0	0	809	351
N.S.	1	0.91	0.59	0.96	0.77	0.91	0.00	0.00	5.78	2.51
time (sec)	N/A	0.623	3.213	0.603	0.038	0.083	0.000	0.000	0.240	8.526

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	127	95	118	105	117	0	0	1080	221
N.S.	1	0.91	0.68	0.84	0.75	0.84	0.00	0.00	7.71	1.58
time (sec)	N/A	0.641	5.473	0.684	0.038	0.084	0.000	0.000	0.405	6.817

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	127	100	105	107	102	0	0	585	208
N.S.	1	0.91	0.71	0.75	0.76	0.73	0.00	0.00	4.18	1.49
time (sec)	N/A	0.629	5.670	0.549	0.037	0.090	0.000	0.000	0.181	6.456

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	129	95	105	102	123	0	0	0	161
N.S.	1	0.91	0.67	0.74	0.72	0.87	0.00	0.00	0.00	1.13
time (sec)	N/A	0.634	5.851	0.672	0.038	0.091	0.000	0.000	1.148	6.983

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	188	148	192	190	466	0	0	299	298
N.S.	1	0.85	0.67	0.87	0.86	2.12	0.00	0.00	1.36	1.35
time (sec)	N/A	0.677	5.851	0.606	0.114	0.107	0.000	0.000	0.198	1.371

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	161	134	150	167	406	0	0	182	245
N.S.	1	0.89	0.74	0.83	0.93	2.26	0.00	0.00	1.01	1.36
time (sec)	N/A	0.670	3.765	0.537	0.117	0.112	0.000	0.000	0.191	6.255

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	132	108	111	138	330	0	0	179	189
N.S.	1	0.92	0.75	0.77	0.96	2.29	0.00	0.00	1.24	1.31
time (sec)	N/A	0.629	2.396	0.592	0.124	0.101	0.000	0.000	0.189	1.016

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	108	98	90	116	326	0	0	138	159
N.S.	1	0.99	0.90	0.83	1.06	2.99	0.00	0.00	1.27	1.46
time (sec)	N/A	0.591	1.832	0.551	0.113	0.080	0.000	0.000	0.179	5.916

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	137	113	121	143	355	0	0	635	212
N.S.	1	0.97	0.80	0.86	1.01	2.52	0.00	0.00	4.50	1.50
time (sec)	N/A	0.629	2.405	0.573	0.118	0.107	0.000	0.000	0.234	6.242

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	171	103	141	167	388	0	0	530	261
N.S.	1	0.93	0.56	0.77	0.91	2.11	0.00	0.00	2.88	1.42
time (sec)	N/A	0.673	2.906	0.530	0.112	0.091	0.000	0.000	0.180	6.895

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	205	127	168	195	418	0	0	858	308
N.S.	1	0.92	0.57	0.75	0.87	1.87	0.00	0.00	3.85	1.38
time (sec)	N/A	0.695	3.920	0.461	0.122	0.109	0.000	0.000	0.190	7.091

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	229	161	220	244	513	0	0	472	402
N.S.	1	0.83	0.59	0.80	0.89	1.87	0.00	0.00	1.72	1.46
time (sec)	N/A	0.714	6.355	0.643	0.117	0.107	0.000	0.000	0.181	6.917

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	202	182	179	221	458	0	0	378	349
N.S.	1	0.85	0.76	0.75	0.93	1.92	0.00	0.00	1.59	1.47
time (sec)	N/A	0.652	6.437	0.618	0.116	0.111	0.000	0.000	0.167	6.699

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	173	167	139	193	381	0	0	280	294
N.S.	1	0.87	0.84	0.70	0.97	1.91	0.00	0.00	1.41	1.48
time (sec)	N/A	0.642	5.758	0.565	0.119	0.086	0.000	0.000	0.174	6.223

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	146	144	118	166	372	0	0	184	267
N.S.	1	0.91	0.90	0.74	1.04	2.32	0.00	0.00	1.15	1.67
time (sec)	N/A	0.637	5.791	0.634	0.117	0.082	0.000	0.000	0.171	5.863

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	155	145	120	172	362	0	0	90	264
N.S.	1	0.97	0.91	0.75	1.08	2.28	0.00	0.00	0.57	1.66
time (sec)	N/A	0.638	3.435	0.625	0.117	0.087	0.000	0.000	0.163	6.110

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	182	153	149	195	390	0	0	144	305
N.S.	1	0.93	0.78	0.76	1.00	2.00	0.00	0.00	0.74	1.56
time (sec)	N/A	0.683	3.887	0.544	0.122	0.086	0.000	0.000	0.160	0.827

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	216	155	176	217	420	0	0	843	353
N.S.	1	0.96	0.69	0.78	0.96	1.86	0.00	0.00	3.73	1.56
time (sec)	N/A	0.715	4.762	0.568	0.124	0.093	0.000	0.000	0.195	6.792

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	250	142	196	245	446	0	0	829	399
N.S.	1	0.92	0.52	0.72	0.90	1.63	0.00	0.00	3.04	1.46
time (sec)	N/A	0.747	5.462	0.485	0.126	0.114	0.000	0.000	0.192	6.668

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	243	190	207	266	471	0	0	449	441
N.S.	1	0.84	0.65	0.71	0.91	1.62	0.00	0.00	1.54	1.52
time (sec)	N/A	0.701	6.815	0.550	0.133	0.107	0.000	0.000	0.306	5.841

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	216	161	168	240	402	0	0	508	386
N.S.	1	0.86	0.64	0.67	0.95	1.60	0.00	0.00	2.02	1.53
time (sec)	N/A	0.678	6.626	0.595	0.130	0.091	0.000	0.000	0.287	6.277

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	189	174	147	212	403	0	0	446	360
N.S.	1	0.89	0.82	0.69	1.00	1.89	0.00	0.00	2.09	1.69
time (sec)	N/A	0.675	6.121	0.570	0.123	0.092	0.000	0.000	0.268	6.476

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	196	174	139	212	395	0	0	384	360
N.S.	1	0.93	0.82	0.66	1.00	1.87	0.00	0.00	1.82	1.71
time (sec)	N/A	0.661	5.940	0.597	0.125	0.089	0.000	0.000	0.207	6.190

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	200	173	147	220	386	0	0	323	355
N.S.	1	0.96	0.83	0.70	1.05	1.85	0.00	0.00	1.55	1.70
time (sec)	N/A	0.653	3.608	0.563	0.122	0.091	0.000	0.000	0.218	5.873

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	227	211	178	241	408	0	0	0	394
N.S.	1	0.93	0.86	0.73	0.98	1.67	0.00	0.00	0.00	1.61
time (sec)	N/A	0.713	3.011	0.560	0.121	0.115	0.000	0.000	1.163	6.698

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	261	191	205	263	438	0	0	193	443
N.S.	1	0.95	0.70	0.75	0.96	1.60	0.00	0.00	0.70	1.62
time (sec)	N/A	0.758	4.502	0.596	0.128	0.101	0.000	0.000	0.172	7.314

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	295	187	232	291	464	0	0	0	490
N.S.	1	0.95	0.60	0.75	0.94	1.49	0.00	0.00	0.00	1.58
time (sec)	N/A	0.767	5.789	0.560	0.130	0.114	0.000	0.000	0.382	7.206

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	265	171	349	1343	604	0	0	280	0
N.S.	1	0.97	0.63	1.28	4.94	2.22	0.00	0.00	1.03	0.00
time (sec)	N/A	0.755	5.582	1.068	0.768	0.101	0.000	0.000	0.240	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	216	149	285	1085	547	0	0	207	0
N.S.	1	1.00	0.69	1.31	5.00	2.52	0.00	0.00	0.95	0.00
time (sec)	N/A	0.710	3.279	0.631	0.401	0.110	0.000	0.000	0.210	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	167	129	223	771	459	0	0	130	0
N.S.	1	1.02	0.79	1.36	4.70	2.80	0.00	0.00	0.79	0.00
time (sec)	N/A	0.691	2.286	0.787	0.243	0.107	0.000	0.000	0.196	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	110	104	121	447	293	0	0	63	133
N.S.	1	1.06	1.00	1.16	4.30	2.82	0.00	0.00	0.61	1.28
time (sec)	N/A	0.596	1.471	0.597	0.198	0.101	0.000	0.000	0.177	7.442

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	115	110	321	0	332	0	0	216	266
N.S.	1	1.06	1.01	2.94	0.00	3.05	0.00	0.00	1.98	2.44
time (sec)	N/A	0.629	2.212	0.660	0.000	0.112	0.000	0.000	0.173	9.609

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	111	81	84	0	96	0	0	364	145
N.S.	1	1.09	0.79	0.82	0.00	0.94	0.00	0.00	3.57	1.42
time (sec)	N/A	0.640	2.997	0.763	0.000	0.079	0.000	0.000	0.246	1.424

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	167	110	98	0	113	0	0	168	171
N.S.	1	1.08	0.71	0.63	0.00	0.73	0.00	0.00	1.08	1.10
time (sec)	N/A	0.673	4.626	0.740	0.000	0.080	0.000	0.000	0.177	7.145

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	221	132	135	0	131	0	0	361	246
N.S.	1	1.06	0.63	0.65	0.00	0.63	0.00	0.00	1.74	1.18
time (sec)	N/A	0.734	5.999	0.928	0.000	0.091	0.000	0.000	0.365	7.377

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	265	236	412	1655	682	0	0	316	0
N.S.	1	0.95	0.85	1.48	5.93	2.44	0.00	0.00	1.13	0.00
time (sec)	N/A	0.732	8.769	0.719	2.549	0.103	0.000	0.000	0.239	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	216	206	350	1375	614	0	0	280	0
N.S.	1	0.96	0.91	1.55	6.08	2.72	0.00	0.00	1.24	0.00
time (sec)	N/A	0.714	5.264	0.653	0.814	0.106	0.000	0.000	0.230	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	159	181	179	853	433	0	0	138	0
N.S.	1	1.01	1.15	1.14	5.43	2.76	0.00	0.00	0.88	0.00
time (sec)	N/A	0.672	3.488	0.704	0.262	0.111	0.000	0.000	0.206	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	167	182	223	771	459	0	0	128	0
N.S.	1	1.04	1.14	1.39	4.82	2.87	0.00	0.00	0.80	0.00
time (sec)	N/A	0.676	2.693	0.587	0.243	0.103	0.000	0.000	0.186	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	170	149	497	599	438	0	0	361	0
N.S.	1	1.01	0.88	2.94	3.54	2.59	0.00	0.00	2.14	0.00
time (sec)	N/A	0.691	3.453	0.632	0.222	0.104	0.000	0.000	0.206	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	168	218	406	170	380	0	0	808	0
N.S.	1	1.08	1.41	2.62	1.10	2.45	0.00	0.00	5.21	0.00
time (sec)	N/A	0.685	5.018	0.684	0.201	0.116	0.000	0.000	0.258	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	111	111	90	154	98	0	0	335	190
N.S.	1	1.09	1.09	0.88	1.51	0.96	0.00	0.00	3.28	1.86
time (sec)	N/A	0.637	5.679	0.767	0.211	0.079	0.000	0.000	0.170	6.538

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	167	131	113	186	117	0	0	1210	215
N.S.	1	1.08	0.85	0.73	1.20	0.75	0.00	0.00	7.81	1.39
time (sec)	N/A	0.688	13.936	0.892	0.214	0.077	0.000	0.000	0.567	8.056

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	219	148	136	260	136	0	0	537	290
N.S.	1	1.05	0.71	0.65	1.25	0.65	0.00	0.00	2.58	1.39
time (sec)	N/A	0.720	13.559	0.716	0.214	0.075	0.000	0.000	0.645	9.375

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	275	179	158	312	155	0	0	670	315
N.S.	1	1.05	0.69	0.61	1.20	0.59	0.00	0.00	2.57	1.21
time (sec)	N/A	0.787	14.879	0.830	0.218	0.076	0.000	0.000	0.832	9.575

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	267	568	478	2033	756	0	0	428	0
N.S.	1	0.93	1.97	1.66	7.06	2.62	0.00	0.00	1.49	0.00
time (sec)	N/A	0.754	18.776	0.878	4.052	0.113	0.000	0.000	0.284	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	210	148	221	1445	585	0	0	214	0
N.S.	1	0.99	0.69	1.04	6.78	2.75	0.00	0.00	1.00	0.00
time (sec)	N/A	0.700	5.861	0.634	0.655	0.106	0.000	0.000	0.205	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	216	222	350	1375	614	0	0	278	0
N.S.	1	0.97	1.00	1.58	6.19	2.77	0.00	0.00	1.25	0.00
time (sec)	N/A	0.714	5.579	0.707	0.813	0.111	0.000	0.000	0.240	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	216	198	285	1087	547	0	0	207	0
N.S.	1	1.00	0.91	1.31	5.01	2.52	0.00	0.00	0.95	0.00
time (sec)	N/A	0.703	3.562	0.588	0.384	0.120	0.000	0.000	0.205	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	221	170	565	993	530	0	0	508	0
N.S.	1	0.97	0.75	2.49	4.37	2.33	0.00	0.00	2.24	0.00
time (sec)	N/A	0.720	6.370	0.644	0.274	0.107	0.000	0.000	0.230	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	222	179	667	795	493	0	0	1109	0
N.S.	1	0.98	0.79	2.95	3.52	2.18	0.00	0.00	4.91	0.00
time (sec)	N/A	0.725	7.209	0.702	0.239	0.105	0.000	0.000	0.305	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	218	203	507	214	418	0	0	506	0
N.S.	1	1.07	1.00	2.50	1.05	2.06	0.00	0.00	2.49	0.00
time (sec)	N/A	0.721	15.675	0.617	0.215	0.102	0.000	0.000	0.188	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	111	121	106	166	104	0	0	851	192
N.S.	1	1.09	1.19	1.04	1.63	1.02	0.00	0.00	8.34	1.88
time (sec)	N/A	0.634	13.454	0.716	0.211	0.081	0.000	0.000	0.664	8.536

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	167	135	119	198	125	0	0	0	217
N.S.	1	1.08	0.87	0.77	1.28	0.81	0.00	0.00	0.00	1.40
time (sec)	N/A	0.687	13.846	0.608	0.222	0.080	0.000	0.000	0.825	8.057

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	221	156	156	276	146	0	0	884	292
N.S.	1	1.06	0.75	0.75	1.33	0.70	0.00	0.00	4.25	1.40
time (sec)	N/A	0.725	14.872	0.711	0.240	0.077	0.000	0.000	1.076	9.489

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	275	183	169	332	167	0	0	894	191
N.S.	1	1.05	0.70	0.65	1.27	0.64	0.00	0.00	3.43	0.73
time (sec)	N/A	0.763	16.338	0.627	0.250	0.079	0.000	0.000	1.313	9.966

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	318	213	604	2603	882	0	0	574	0
N.S.	1	0.91	0.61	1.73	7.44	2.52	0.00	0.00	1.64	0.00
time (sec)	N/A	0.785	7.728	0.831	16.537	0.116	0.000	0.000	0.341	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	261	535	269	1901	693	0	0	286	0
N.S.	1	0.98	2.00	1.01	7.12	2.60	0.00	0.00	1.07	0.00
time (sec)	N/A	0.714	18.839	0.829	2.295	0.109	0.000	0.000	0.228	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	267	572	478	2033	756	0	0	426	0
N.S.	1	0.94	2.01	1.68	7.16	2.66	0.00	0.00	1.50	0.00
time (sec)	N/A	0.738	18.586	0.751	4.909	0.110	0.000	0.000	0.266	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	265	237	412	1657	682	0	0	316	0
N.S.	1	0.95	0.85	1.48	5.94	2.44	0.00	0.00	1.13	0.00
time (sec)	N/A	0.747	7.835	0.710	1.866	0.106	0.000	0.000	0.238	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	265	221	349	1345	604	0	0	281	0
N.S.	1	0.97	0.81	1.28	4.94	2.22	0.00	0.00	1.03	0.00
time (sec)	N/A	0.728	5.130	0.606	0.784	0.109	0.000	0.000	0.226	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	270	183	627	1329	590	0	0	651	0
N.S.	1	0.95	0.65	2.22	4.70	2.08	0.00	0.00	2.30	0.00
time (sec)	N/A	0.760	6.108	0.645	0.475	0.097	0.000	0.000	0.241	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	273	517	731	1176	571	0	0	1409	0
N.S.	1	0.96	1.81	2.56	4.13	2.00	0.00	0.00	4.94	0.00
time (sec)	N/A	0.772	19.221	0.773	0.313	0.106	0.000	0.000	0.324	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	274	528	833	936	515	0	0	676	0
N.S.	1	0.97	1.87	2.94	3.31	1.82	0.00	0.00	2.39	0.00
time (sec)	N/A	0.774	19.157	0.697	0.262	0.122	0.000	0.000	0.183	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	268	570	598	246	434	0	0	0	0
N.S.	1	1.07	2.27	2.38	0.98	1.73	0.00	0.00	0.00	0.00
time (sec)	N/A	0.765	19.240	0.747	0.232	0.102	0.000	0.000	0.604	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	111	121	106	166	104	0	0	1019	192
N.S.	1	1.09	1.19	1.04	1.63	1.02	0.00	0.00	9.99	1.88
time (sec)	N/A	0.633	14.214	0.593	0.235	0.075	0.000	0.000	0.993	7.831

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	167	135	119	198	125	0	0	0	217
N.S.	1	1.08	0.87	0.77	1.28	0.81	0.00	0.00	0.00	1.40
time (sec)	N/A	0.668	15.337	0.763	0.250	0.091	0.000	0.000	1.434	8.375

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	221	495	156	276	146	0	0	1117	167
N.S.	1	1.06	2.38	0.75	1.33	0.70	0.00	0.00	5.37	0.80
time (sec)	N/A	0.735	18.702	0.706	0.277	0.088	0.000	0.000	1.695	9.874

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	275	577	169	332	167	0	0	1330	191
N.S.	1	1.05	2.21	0.65	1.27	0.64	0.00	0.00	5.10	0.73
time (sec)	N/A	0.788	18.858	0.767	0.294	0.076	0.000	0.000	2.184	9.941

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	329	655	206	412	188	0	0	1319	229
N.S.	1	1.05	2.09	0.66	1.31	0.60	0.00	0.00	4.20	0.73
time (sec)	N/A	0.808	19.097	0.621	0.314	0.107	0.000	0.000	2.697	10.176

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	222	162	566	1319	548	0	0	505	0
N.S.	1	0.97	0.71	2.48	5.79	2.40	0.00	0.00	2.21	0.00
time (sec)	N/A	0.737	4.590	0.835	0.437	0.099	0.000	0.000	0.218	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	171	136	499	898	456	0	0	361	0
N.S.	1	1.01	0.80	2.95	5.31	2.70	0.00	0.00	2.14	0.00
time (sec)	N/A	0.679	2.768	0.835	0.249	0.106	0.000	0.000	0.193	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	116	119	323	138	348	0	0	217	250
N.S.	1	1.05	1.08	2.94	1.25	3.16	0.00	0.00	1.97	2.27
time (sec)	N/A	0.610	1.809	0.736	0.185	0.095	0.000	0.000	0.167	8.507

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	101	50	92	124	114	0	0	61	143
N.S.	1	1.10	0.54	1.00	1.35	1.24	0.00	0.00	0.66	1.55
time (sec)	N/A	0.615	1.489	0.691	0.191	0.079	0.000	0.000	0.155	0.754

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	163	97	110	0	147	0	0	305	146
N.S.	1	1.04	0.62	0.70	0.00	0.94	0.00	0.00	1.94	0.93
time (sec)	N/A	0.662	2.885	0.882	0.000	0.080	0.000	0.000	0.190	5.789

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	217	116	148	0	161	0	0	214	186
N.S.	1	1.02	0.54	0.69	0.00	0.76	0.00	0.00	1.00	0.87
time (sec)	N/A	0.693	4.761	0.717	0.000	0.078	0.000	0.000	0.157	1.329

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	275	160	733	1373	589	0	0	1389	0
N.S.	1	0.96	0.56	2.55	4.78	2.05	0.00	0.00	4.84	0.00
time (sec)	N/A	0.789	7.151	0.867	0.600	0.145	0.000	0.000	0.324	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	224	141	669	0	511	0	0	1101	0
N.S.	1	0.98	0.62	2.92	0.00	2.23	0.00	0.00	4.81	0.00
time (sec)	N/A	0.735	4.635	0.796	0.000	0.104	0.000	0.000	0.293	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	170	137	408	168	400	0	0	811	0
N.S.	1	1.08	0.87	2.60	1.07	2.55	0.00	0.00	5.17	0.00
time (sec)	N/A	0.704	3.244	0.855	0.207	0.098	0.000	0.000	0.252	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	113	81	103	0	92	0	0	376	195
N.S.	1	1.09	0.78	0.99	0.00	0.88	0.00	0.00	3.62	1.88
time (sec)	N/A	0.599	1.911	0.705	0.000	0.075	0.000	0.000	0.220	6.576

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	162	97	152	0	145	0	0	320	170
N.S.	1	1.07	0.64	1.00	0.00	0.95	0.00	0.00	2.11	1.12
time (sec)	N/A	0.676	1.975	0.790	0.000	0.086	0.000	0.000	0.184	6.267

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	160	79	113	200	150	0	0	84	198
N.S.	1	1.05	0.52	0.74	1.32	0.99	0.00	0.00	0.55	1.30
time (sec)	N/A	0.698	4.144	0.731	0.220	0.080	0.000	0.000	0.154	6.363

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	273	152	199	0	181	0	0	673	196
N.S.	1	1.01	0.57	0.74	0.00	0.67	0.00	0.00	2.50	0.73
time (sec)	N/A	0.769	6.699	0.717	0.000	0.081	0.000	0.000	0.219	6.411

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	327	247	899	0	610	0	0	826	0
N.S.	1	0.95	0.72	2.62	0.00	1.78	0.00	0.00	2.41	0.00
time (sec)	N/A	0.805	15.641	0.767	0.000	0.110	0.000	0.000	0.180	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	276	205	835	1025	533	0	0	662	0
N.S.	1	0.97	0.72	2.94	3.61	1.88	0.00	0.00	2.33	0.00
time (sec)	N/A	0.757	14.914	0.814	0.357	0.109	0.000	0.000	0.175	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	220	167	509	216	436	0	0	494	0
N.S.	1	1.07	0.81	2.48	1.05	2.13	0.00	0.00	2.41	0.00
time (sec)	N/A	0.718	6.342	0.706	0.226	0.103	0.000	0.000	0.173	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	113	92	92	153	98	0	0	328	240
N.S.	1	1.09	0.88	0.88	1.47	0.94	0.00	0.00	3.15	2.31
time (sec)	N/A	0.633	3.171	0.905	0.212	0.077	0.000	0.000	0.182	7.554

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	169	99	127	0	112	0	0	164	246
N.S.	1	1.08	0.63	0.81	0.00	0.71	0.00	0.00	1.04	1.57
time (sec)	N/A	0.669	2.106	0.816	0.000	0.077	0.000	0.000	0.164	7.664

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	216	115	186	0	161	0	0	214	246
N.S.	1	1.02	0.54	0.88	0.00	0.76	0.00	0.00	1.01	1.16
time (sec)	N/A	0.738	2.485	0.796	0.000	0.114	0.000	0.000	0.172	7.444

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	225	152	199	0	181	0	0	661	249
N.S.	1	1.03	0.70	0.91	0.00	0.83	0.00	0.00	3.03	1.14
time (sec)	N/A	0.722	5.515	0.827	0.000	0.087	0.000	0.000	0.230	7.082

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	218	90	124	334	185	0	0	105	249
N.S.	1	1.06	0.44	0.60	1.62	0.90	0.00	0.00	0.51	1.21
time (sec)	N/A	0.702	6.959	0.662	0.225	0.082	0.000	0.000	0.164	7.217

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	165	122	0	0	0	0	0	206	0
N.S.	1	1.10	0.81	0.00	0.00	0.00	0.00	0.00	1.37	0.00
time (sec)	N/A	0.634	0.700	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	152	133	0	0	0	0	0	265	0
N.S.	1	1.01	0.88	0.00	0.00	0.00	0.00	0.00	1.75	0.00
time (sec)	N/A	0.670	5.265	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	92	0	54	66	0	176	90
N.S.	1	1.00	0.94	2.79	0.00	1.64	2.00	0.00	5.33	2.73
time (sec)	N/A	0.480	5.283	1.250	0.000	0.079	0.665	0.000	0.173	5.725

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	109	93	154	0	84	296	117	136	159
N.S.	1	1.05	0.89	1.48	0.00	0.81	2.85	1.12	1.31	1.53
time (sec)	N/A	0.797	1.269	0.421	0.000	0.093	0.324	0.489	0.165	5.886

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	154	141	131	142	619	0	0	1136	245
N.S.	1	1.05	0.96	0.89	0.97	4.21	0.00	0.00	7.73	1.67
time (sec)	N/A	1.082	1.597	0.740	0.112	0.118	0.000	0.000	0.309	1.561

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [604] had the largest ratio of [.87878800000000014]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	8	1.00	32	0.250
2	A	6	6	1.00	30	0.200
3	A	4	4	1.00	24	0.167
4	A	8	8	1.05	30	0.267
5	A	7	7	1.05	32	0.219
6	A	10	10	1.03	32	0.312
7	A	12	12	1.02	32	0.375
8	A	15	15	1.02	32	0.469
9	A	10	10	1.04	34	0.294
10	A	8	8	0.93	32	0.250
11	A	6	6	0.88	26	0.231
12	A	10	10	1.03	32	0.312
13	A	10	10	1.03	34	0.294
14	A	10	10	1.06	34	0.294
15	A	12	12	1.08	34	0.353
16	A	15	15	1.09	34	0.441
17	A	12	12	1.05	34	0.353
18	A	10	10	0.93	32	0.312
19	A	8	8	0.90	26	0.308
20	A	13	13	1.02	32	0.406
21	A	12	12	1.02	34	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	14	14	1.02	34	0.412
23	A	13	13	1.08	34	0.382
24	A	17	17	1.10	34	0.500
25	A	18	18	1.08	34	0.529
26	A	16	16	1.07	34	0.471
27	A	12	12	0.93	32	0.375
28	A	10	10	0.91	26	0.385
29	A	16	16	1.01	32	0.500
30	A	15	15	1.01	34	0.441
31	A	15	15	1.04	34	0.441
32	A	16	16	1.01	34	0.471
33	A	15	15	1.09	34	0.441
34	A	17	17	1.10	34	0.500
35	A	21	21	1.10	34	0.618
36	A	8	8	0.95	34	0.235
37	A	6	6	0.98	34	0.176
38	A	7	7	0.96	32	0.219
39	A	3	3	1.00	26	0.115
40	A	7	7	1.08	32	0.219
41	A	10	10	1.00	34	0.294
42	A	12	12	0.97	34	0.353
43	A	15	15	0.97	34	0.441
44	A	9	9	1.04	34	0.265
45	A	11	11	1.03	34	0.324
46	A	5	5	1.08	32	0.156
47	A	5	5	0.99	26	0.192
48	A	10	10	1.15	32	0.312
49	A	13	13	1.04	34	0.382
50	A	15	15	1.06	34	0.441
51	A	12	12	1.06	34	0.353
52	A	13	13	1.07	34	0.382
53	A	7	7	1.09	34	0.206

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	7	7	1.05	32	0.219
55	A	7	7	0.99	26	0.269
56	A	13	13	1.17	32	0.406
57	A	16	16	1.08	34	0.471
58	A	18	18	1.08	34	0.529
59	A	17	17	1.10	34	0.500
60	A	11	11	1.14	34	0.324
61	A	11	11	1.06	34	0.324
62	A	9	9	1.03	32	0.281
63	A	9	9	0.99	26	0.346
64	A	16	16	1.18	32	0.500
65	A	19	19	1.10	34	0.559
66	A	22	22	1.09	34	0.647
67	A	14	13	1.10	36	0.361
68	A	11	10	1.05	36	0.278
69	A	8	7	1.02	34	0.206
70	A	6	5	1.03	28	0.179
71	A	9	8	1.03	34	0.235
72	A	12	11	1.09	36	0.306
73	A	15	14	1.09	36	0.389
74	A	18	17	1.12	36	0.472
75	A	14	13	1.08	36	0.361
76	A	10	9	1.00	34	0.265
77	A	8	7	0.99	28	0.250
78	A	12	11	1.02	34	0.324
79	A	12	11	1.06	36	0.306
80	A	15	14	1.06	36	0.389
81	A	18	17	1.10	36	0.472
82	A	17	16	1.08	36	0.444
83	A	12	11	0.98	34	0.324
84	A	10	9	0.97	28	0.321
85	A	15	14	1.01	34	0.412

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	15	14	1.05	36	0.389
87	A	15	14	1.05	36	0.389
88	A	18	17	1.07	36	0.472
89	A	21	20	1.09	36	0.556
90	A	14	13	1.06	36	0.361
91	A	11	10	1.04	36	0.278
92	A	8	7	1.02	34	0.206
93	A	6	5	1.02	28	0.179
94	A	12	11	1.10	34	0.324
95	A	15	14	1.10	36	0.389
96	A	18	17	1.08	36	0.472
97	A	14	13	1.05	36	0.361
98	A	11	10	0.99	36	0.278
99	A	8	7	1.06	34	0.206
100	A	8	7	0.98	28	0.250
101	A	15	14	1.07	34	0.412
102	A	18	17	1.07	36	0.472
103	A	21	20	1.07	36	0.556
104	A	17	16	1.07	36	0.444
105	A	14	13	1.04	36	0.361
106	A	11	10	1.07	36	0.278
107	A	10	9	1.03	34	0.265
108	A	10	9	0.99	28	0.321
109	A	18	17	1.10	34	0.500
110	A	21	20	1.09	36	0.556
111	A	23	22	1.08	36	0.611
112	A	12	11	1.00	34	0.324
113	A	10	9	1.00	34	0.265
114	A	8	7	1.00	34	0.206
115	A	6	5	1.00	34	0.147
116	A	6	5	1.00	34	0.147
117	A	9	8	1.00	34	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	11	10	1.00	34	0.294
119	A	15	14	1.02	36	0.389
120	A	13	12	1.03	36	0.333
121	A	11	10	1.04	36	0.278
122	A	9	8	1.03	36	0.222
123	A	9	8	1.00	36	0.222
124	A	10	9	1.03	36	0.250
125	A	12	11	1.06	36	0.306
126	A	15	14	1.05	36	0.389
127	A	16	15	1.04	36	0.417
128	A	14	13	1.06	36	0.361
129	A	12	11	1.05	36	0.306
130	A	12	11	1.02	36	0.306
131	A	12	11	1.03	36	0.306
132	A	13	12	1.06	36	0.333
133	A	16	15	1.08	36	0.417
134	A	20	19	1.07	36	0.528
135	A	18	17	1.09	36	0.472
136	A	15	14	1.11	36	0.389
137	A	15	14	1.11	36	0.389
138	A	18	17	1.11	36	0.472
139	A	20	19	1.06	36	0.528
140	A	20	19	1.10	36	0.528
141	A	18	17	1.07	36	0.472
142	A	17	16	1.07	36	0.444
143	A	17	16	1.05	36	0.444
144	A	20	19	1.08	36	0.528
145	A	22	21	1.06	36	0.583
146	A	25	24	1.10	36	0.667
147	A	24	23	1.09	36	0.639
148	A	21	20	1.10	36	0.556
149	A	20	19	1.10	36	0.528

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	20	19	1.09	36	0.528
151	A	20	19	1.09	36	0.528
152	A	24	23	1.09	36	0.639
153	A	25	24	1.07	36	0.667
154	A	15	14	1.06	38	0.368
155	A	12	11	1.04	38	0.289
156	A	9	8	1.00	38	0.211
157	A	7	6	1.00	38	0.158
158	A	10	9	1.05	38	0.237
159	A	13	12	1.10	38	0.316
160	A	16	15	1.13	38	0.395
161	A	18	17	1.08	38	0.447
162	A	15	14	1.04	38	0.368
163	A	12	11	1.03	38	0.289
164	A	12	11	1.00	38	0.289
165	A	10	9	1.02	38	0.237
166	A	13	12	1.07	38	0.316
167	A	16	15	1.10	38	0.395
168	A	19	18	1.12	38	0.474
169	A	21	20	1.07	38	0.526
170	A	18	17	1.05	38	0.447
171	A	15	14	1.03	38	0.368
172	A	15	14	1.03	38	0.368
173	A	15	14	1.00	38	0.368
174	A	13	12	1.04	38	0.316
175	A	15	14	1.07	38	0.368
176	A	19	18	1.09	38	0.474
177	A	22	21	1.10	38	0.553
178	A	15	14	1.00	46	0.304
179	A	15	14	1.04	38	0.368
180	A	12	11	1.03	38	0.289
181	A	7	6	1.00	38	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	10	9	1.01	38	0.237
183	A	13	12	1.04	38	0.316
184	A	16	15	1.07	38	0.395
185	A	15	14	1.04	38	0.368
186	A	10	9	1.03	38	0.237
187	A	10	9	1.02	38	0.237
188	A	13	12	1.03	38	0.316
189	A	16	15	1.05	38	0.395
190	A	18	17	1.06	38	0.447
191	A	13	12	1.06	38	0.316
192	A	13	12	1.06	38	0.316
193	A	13	12	1.06	38	0.316
194	A	16	15	1.05	38	0.395
195	A	19	18	1.07	38	0.474
196	A	9	8	0.69	28	0.286
197	A	14	13	0.80	36	0.361
198	A	11	10	0.74	34	0.294
199	A	9	8	0.70	28	0.286
200	A	14	13	0.75	34	0.382
201	A	17	16	0.78	36	0.444
202	A	9	8	0.70	28	0.286
203	A	9	8	0.70	28	0.286
204	A	15	14	1.09	34	0.412
205	A	12	11	1.30	34	0.324
206	A	10	9	1.24	34	0.265
207	A	7	6	1.39	32	0.188
208	A	8	7	0.98	34	0.206
209	A	11	10	1.00	34	0.294
210	A	14	13	1.04	34	0.382
211	A	17	16	1.05	34	0.471
212	A	18	17	1.03	36	0.472
213	A	15	14	1.05	36	0.389

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	12	11	1.01	36	0.306
215	A	15	14	1.03	36	0.389
216	A	18	17	1.04	36	0.472
217	A	21	20	1.05	36	0.556
218	A	12	11	1.00	34	0.324
219	A	14	13	1.06	34	0.382
220	A	11	10	1.05	34	0.294
221	A	8	7	1.05	32	0.219
222	A	6	5	1.08	26	0.192
223	A	8	7	1.06	32	0.219
224	A	10	9	1.10	34	0.265
225	A	12	11	1.11	34	0.324
226	A	23	22	1.10	36	0.611
227	A	20	19	1.13	36	0.528
228	A	17	16	1.15	36	0.444
229	A	14	13	1.11	36	0.361
230	A	17	16	1.13	36	0.444
231	A	20	19	1.13	36	0.528
232	A	8	8	1.00	29	0.276
233	A	6	6	1.00	27	0.222
234	A	4	4	1.00	21	0.190
235	A	8	8	1.05	27	0.296
236	A	7	7	1.05	29	0.241
237	A	10	10	1.03	29	0.345
238	A	12	12	1.02	29	0.414
239	A	15	15	1.02	29	0.517
240	A	11	11	1.08	31	0.355
241	A	8	8	1.00	29	0.276
242	A	6	6	1.00	23	0.261
243	A	7	7	1.03	29	0.241
244	A	7	7	1.03	31	0.226
245	A	10	10	1.03	31	0.323

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
246	A	12	12	1.03	31	0.387
247	A	15	15	1.01	31	0.484
248	A	13	13	1.07	31	0.419
249	A	10	10	1.00	29	0.345
250	A	8	8	1.00	23	0.348
251	A	11	11	1.02	29	0.379
252	A	10	10	1.02	31	0.323
253	A	11	11	1.02	31	0.355
254	A	13	13	1.05	31	0.419
255	A	17	17	1.07	31	0.548
256	A	18	18	1.04	31	0.581
257	A	15	15	1.05	31	0.484
258	A	12	12	1.00	29	0.414
259	A	10	10	1.00	23	0.435
260	A	14	14	1.01	29	0.483
261	A	13	13	1.01	31	0.419
262	A	13	13	1.03	31	0.419
263	A	13	13	1.01	31	0.419
264	A	15	15	1.06	31	0.484
265	A	17	17	1.05	31	0.548
266	A	21	21	1.05	31	0.677
267	A	12	11	1.13	31	0.355
268	A	10	9	1.07	31	0.290
269	A	8	8	0.94	29	0.276
270	A	4	4	1.00	23	0.174
271	A	6	6	1.02	29	0.207
272	A	8	8	1.08	31	0.258
273	A	12	12	1.12	31	0.387
274	A	14	14	1.14	31	0.452
275	A	13	12	1.08	31	0.387
276	A	10	9	1.09	31	0.290
277	A	6	6	1.10	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	6	6	1.10	23	0.261
279	A	8	8	1.18	29	0.276
280	A	10	10	1.16	31	0.323
281	A	13	13	1.15	31	0.419
282	A	16	15	1.10	31	0.484
283	A	12	11	1.14	31	0.355
284	A	9	9	1.14	31	0.290
285	A	8	8	1.12	29	0.276
286	A	8	8	1.13	23	0.348
287	A	11	11	1.20	29	0.379
288	A	13	13	1.16	31	0.419
289	A	17	17	1.15	31	0.548
290	A	16	15	1.15	31	0.484
291	A	13	13	1.16	31	0.419
292	A	11	11	1.15	31	0.355
293	A	10	10	1.13	29	0.345
294	A	10	10	1.13	23	0.435
295	A	14	14	1.19	29	0.483
296	A	16	16	1.15	31	0.516
297	A	19	19	1.14	31	0.613
298	A	5	5	1.00	34	0.147
299	A	4	4	1.00	34	0.118
300	A	3	3	1.00	32	0.094
301	A	2	2	1.00	26	0.077
302	A	4	4	1.17	32	0.125
303	A	4	4	1.00	34	0.118
304	A	8	8	1.07	34	0.235
305	A	6	6	0.94	34	0.176
306	A	14	13	1.13	34	0.382
307	A	11	10	1.07	34	0.294
308	A	8	8	0.95	34	0.235
309	A	5	5	1.00	32	0.156

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	5	5	1.00	26	0.192
311	A	7	7	1.01	32	0.219
312	A	9	9	1.08	34	0.265
313	A	13	13	1.12	34	0.382
314	A	5	5	1.00	21	0.238
315	A	4	4	1.00	28	0.143
316	A	6	6	1.11	23	0.261
317	A	18	17	1.12	33	0.515
318	A	15	14	0.97	33	0.424
319	A	12	11	0.99	31	0.355
320	A	10	9	0.84	25	0.360
321	A	13	12	0.90	31	0.387
322	A	17	16	0.93	33	0.485
323	A	20	19	1.00	33	0.576
324	A	23	22	1.03	33	0.667
325	A	17	16	0.98	33	0.485
326	A	14	13	0.91	31	0.419
327	A	12	11	0.87	25	0.440
328	A	16	15	0.91	31	0.484
329	A	17	16	0.93	33	0.485
330	A	20	19	1.00	33	0.576
331	A	23	22	1.01	33	0.667
332	A	19	18	0.98	33	0.545
333	A	16	15	1.01	31	0.484
334	A	14	13	0.90	25	0.520
335	A	19	18	0.93	31	0.581
336	A	20	19	0.95	33	0.576
337	A	20	19	0.97	33	0.576
338	A	23	22	1.00	33	0.667
339	A	26	25	1.04	33	0.758
340	A	13	12	0.80	27	0.444
341	A	14	13	1.39	27	0.481

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	14	13	1.41	27	0.481
343	A	16	15	1.04	33	0.455
344	A	13	12	0.95	33	0.364
345	A	10	9	0.88	31	0.290
346	A	8	7	0.81	25	0.280
347	A	13	12	0.90	31	0.387
348	A	17	16	0.97	33	0.485
349	A	20	19	1.03	33	0.576
350	A	16	15	1.03	33	0.455
351	A	13	12	1.11	33	0.364
352	A	10	9	1.07	31	0.290
353	A	10	9	1.04	25	0.360
354	A	17	16	1.14	31	0.516
355	A	20	19	1.16	33	0.576
356	A	23	22	1.14	33	0.667
357	A	19	18	1.06	33	0.545
358	A	15	14	1.14	33	0.424
359	A	13	12	1.14	33	0.364
360	A	12	11	1.13	31	0.355
361	A	12	11	1.11	25	0.440
362	A	20	19	1.21	31	0.613
363	A	23	22	1.19	33	0.667
364	A	26	25	1.16	33	0.758
365	A	12	11	1.49	28	0.393
366	A	12	11	1.41	28	0.393
367	A	15	14	0.91	34	0.412
368	A	11	10	1.02	28	0.357
369	A	18	17	1.14	34	0.500
370	A	8	7	0.83	27	0.259
371	A	11	10	0.98	27	0.370
372	A	13	12	1.05	27	0.444
373	A	6	5	0.78	27	0.185

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	6	5	0.78	27	0.185
375	A	4	3	1.00	15	0.200
376	A	4	3	1.00	17	0.176
377	A	8	7	1.00	25	0.280
378	A	19	18	1.13	31	0.581
379	A	18	17	1.14	31	0.548
380	A	16	15	1.16	31	0.484
381	A	13	12	1.20	31	0.387
382	A	13	12	1.19	31	0.387
383	A	16	15	1.16	31	0.484
384	A	18	17	1.14	31	0.548
385	A	23	22	1.06	33	0.667
386	A	21	20	1.07	33	0.606
387	A	19	18	1.08	33	0.545
388	A	17	16	1.09	33	0.485
389	A	15	14	1.08	33	0.424
390	A	16	15	1.08	33	0.455
391	A	18	17	1.06	33	0.515
392	A	24	23	1.02	33	0.697
393	A	22	21	1.03	33	0.636
394	A	20	19	1.03	33	0.576
395	A	19	18	1.02	33	0.545
396	A	20	19	1.02	33	0.576
397	A	20	19	1.03	33	0.576
398	A	23	22	1.09	33	0.667
399	A	20	19	1.07	33	0.576
400	A	16	15	1.06	33	0.455
401	A	16	15	1.05	33	0.455
402	A	20	19	1.08	33	0.576
403	A	23	22	1.09	33	0.667
404	A	24	23	1.04	33	0.697
405	A	21	20	1.06	33	0.606

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
406	A	21	20	1.05	33	0.606
407	A	21	20	1.06	33	0.606
408	A	24	23	1.04	33	0.697
409	A	27	26	1.03	33	0.788
410	A	27	26	1.03	33	0.788
411	A	24	23	1.04	33	0.697
412	A	24	23	1.03	33	0.697
413	A	24	23	1.05	33	0.697
414	A	24	23	1.04	33	0.697
415	A	27	26	1.03	33	0.788
416	A	15	14	1.35	36	0.389
417	A	15	14	1.35	36	0.389
418	A	13	12	1.39	36	0.333
419	A	13	12	1.41	36	0.333
420	A	15	14	1.36	36	0.389
421	A	15	14	1.34	36	0.389
422	A	21	20	1.13	36	0.556
423	A	18	17	1.09	36	0.472
424	A	17	16	1.10	36	0.444
425	A	17	16	1.10	36	0.444
426	A	21	20	1.13	36	0.556
427	A	12	11	1.00	35	0.314
428	A	9	8	0.99	35	0.229
429	A	13	12	1.06	35	0.343
430	A	11	10	1.10	35	0.286
431	A	14	13	1.12	35	0.371
432	A	17	16	1.16	35	0.457
433	A	20	19	1.13	35	0.543
434	A	15	14	1.01	35	0.400
435	A	12	11	1.01	35	0.314
436	A	9	8	0.98	35	0.229
437	A	9	8	0.96	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	14	13	1.15	35	0.371
439	A	17	16	1.12	35	0.457
440	A	20	19	1.13	35	0.543
441	A	23	22	1.13	35	0.629
442	A	18	17	1.01	35	0.486
443	A	15	14	1.03	35	0.400
444	A	12	11	0.99	35	0.314
445	A	12	11	0.98	35	0.314
446	A	12	11	1.00	35	0.314
447	A	17	16	1.15	35	0.457
448	A	20	19	1.13	35	0.543
449	A	23	22	1.13	35	0.629
450	A	26	25	1.11	35	0.714
451	A	12	11	1.01	43	0.256
452	A	9	8	1.00	35	0.229
453	A	13	12	1.00	35	0.343
454	A	8	7	1.00	35	0.200
455	A	11	10	1.04	35	0.286
456	A	14	13	1.08	35	0.371
457	A	17	16	1.10	35	0.457
458	A	9	8	1.16	35	0.229
459	A	11	10	1.19	35	0.286
460	A	11	10	1.16	35	0.286
461	A	14	13	1.19	35	0.371
462	A	17	16	1.14	35	0.457
463	A	12	11	1.22	35	0.314
464	A	14	13	1.20	35	0.371
465	A	14	13	1.22	35	0.371
466	A	14	13	1.21	35	0.371
467	A	17	16	1.20	35	0.457
468	A	20	19	1.18	35	0.543
469	A	6	5	0.95	38	0.132

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
470	A	8	7	0.97	38	0.184
471	A	6	5	0.97	38	0.132
472	A	11	10	0.99	38	0.263
473	A	12	11	0.71	25	0.440
474	A	12	11	0.71	25	0.440
475	A	10	9	0.65	25	0.360
476	A	10	9	0.65	25	0.360
477	A	7	6	0.71	27	0.222
478	A	10	9	0.79	26	0.346
479	A	16	15	1.03	31	0.484
480	A	14	13	1.06	31	0.419
481	A	11	10	1.02	31	0.323
482	A	8	7	1.00	29	0.241
483	A	10	9	0.96	31	0.290
484	A	12	11	1.04	31	0.355
485	A	16	15	1.06	31	0.484
486	A	19	18	1.07	31	0.581
487	A	7	6	0.90	33	0.182
488	A	7	6	0.88	33	0.182
489	A	7	6	0.87	33	0.182
490	A	7	6	0.89	33	0.182
491	A	7	6	0.91	33	0.182
492	A	7	6	0.90	33	0.182
493	A	7	6	0.99	31	0.194
494	A	18	17	1.06	31	0.548
495	A	15	14	1.10	31	0.452
496	A	12	11	1.07	31	0.355
497	A	9	8	1.04	29	0.276
498	A	7	6	1.01	23	0.261
499	A	11	10	1.03	29	0.345
500	A	15	14	1.03	31	0.452
501	A	17	16	1.08	31	0.516

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	8	7	0.99	33	0.212
503	A	8	7	0.99	33	0.212
504	A	8	7	0.99	33	0.212
505	A	8	7	0.94	33	0.212
506	A	12	11	1.00	34	0.324
507	A	10	9	1.00	34	0.265
508	A	8	7	1.00	34	0.206
509	A	8	7	1.00	34	0.206
510	A	10	9	1.00	34	0.265
511	A	13	12	1.00	34	0.353
512	A	13	12	1.04	36	0.333
513	A	11	10	1.03	36	0.278
514	A	11	10	1.00	36	0.278
515	A	12	11	1.03	36	0.306
516	A	14	13	1.06	36	0.361
517	A	16	15	1.06	36	0.417
518	A	14	13	1.05	36	0.361
519	A	14	13	1.02	36	0.361
520	A	14	13	1.03	36	0.361
521	A	15	14	1.05	36	0.389
522	A	17	16	1.08	36	0.444
523	A	21	20	1.07	36	0.556
524	A	20	19	1.09	36	0.528
525	A	18	17	1.11	36	0.472
526	A	18	17	1.11	36	0.472
527	A	20	19	1.09	36	0.528
528	A	23	22	1.04	36	0.611
529	A	22	21	1.09	36	0.583
530	A	20	19	1.08	36	0.528
531	A	21	20	1.08	36	0.556
532	A	21	20	1.05	36	0.556
533	A	24	23	1.09	36	0.639

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	26	25	1.09	36	0.694
535	A	23	22	1.09	36	0.611
536	A	23	22	1.09	36	0.611
537	A	23	22	1.10	36	0.611
538	A	23	22	1.10	36	0.611
539	A	26	25	1.11	36	0.694
540	A	15	14	1.10	38	0.368
541	A	12	11	1.05	38	0.289
542	A	9	8	1.01	38	0.211
543	A	11	10	0.88	38	0.263
544	A	14	13	0.93	38	0.342
545	A	18	17	1.10	38	0.447
546	A	15	14	1.07	38	0.368
547	A	12	11	1.03	38	0.289
548	A	14	13	0.90	38	0.342
549	A	14	13	0.93	38	0.342
550	A	17	16	0.95	38	0.421
551	A	21	20	1.08	38	0.526
552	A	17	16	1.07	38	0.421
553	A	15	14	1.04	38	0.368
554	A	17	16	0.92	38	0.421
555	A	17	16	0.94	38	0.421
556	A	17	16	0.95	38	0.421
557	A	20	19	0.98	38	0.500
558	A	15	14	1.04	38	0.368
559	A	12	11	1.02	38	0.289
560	A	9	8	1.01	38	0.211
561	A	14	13	0.92	38	0.342
562	A	15	14	1.03	38	0.368
563	A	12	11	1.02	38	0.289
564	A	12	11	1.03	38	0.289
565	A	17	16	0.95	38	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	18	17	1.05	38	0.447
567	A	15	14	1.06	38	0.368
568	A	15	14	1.06	38	0.368
569	A	15	14	1.06	38	0.368
570	A	20	19	0.99	38	0.500
571	A	14	13	1.12	34	0.382
572	A	22	21	1.22	36	0.583
573	A	19	18	1.24	36	0.500
574	A	16	15	1.25	36	0.417
575	A	19	18	1.25	36	0.500
576	A	22	21	1.21	36	0.583
577	A	25	24	1.15	36	0.667
578	A	17	16	1.16	31	0.516
579	A	15	14	1.19	31	0.452
580	A	16	15	1.20	31	0.484
581	A	18	17	1.16	31	0.548
582	A	21	20	1.08	33	0.606
583	A	19	18	1.09	33	0.545
584	A	18	17	1.08	33	0.515
585	A	19	18	1.08	33	0.545
586	A	21	20	1.06	33	0.606
587	A	24	23	1.03	33	0.697
588	A	22	21	1.03	33	0.636
589	A	21	20	1.01	33	0.606
590	A	22	21	1.02	33	0.636
591	A	23	22	1.03	33	0.667
592	A	25	24	1.04	33	0.727
593	A	26	25	1.08	33	0.758
594	A	23	22	1.06	33	0.667
595	A	20	19	1.03	33	0.576
596	A	19	18	1.04	33	0.545
597	A	23	22	1.05	33	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
598	A	26	25	1.08	33	0.758
599	A	27	26	1.03	33	0.788
600	A	24	23	1.04	33	0.697
601	A	24	23	1.04	33	0.697
602	A	24	23	1.04	33	0.697
603	A	27	26	1.03	33	0.788
604	A	30	29	1.02	33	0.879
605	A	27	26	1.03	33	0.788
606	A	27	26	1.03	33	0.788
607	A	27	26	1.03	33	0.788
608	A	27	26	1.03	33	0.788
609	A	30	29	1.02	33	0.879
610	A	15	14	1.34	36	0.389
611	A	15	14	1.36	36	0.389
612	A	13	12	1.41	36	0.333
613	A	13	12	1.39	36	0.333
614	A	15	14	1.35	36	0.389
615	A	15	14	1.35	36	0.389
616	A	22	21	1.06	35	0.600
617	A	19	18	1.07	35	0.514
618	A	16	15	1.02	35	0.429
619	A	13	12	0.98	35	0.343
620	A	15	14	0.87	35	0.400
621	A	11	10	0.84	35	0.286
622	A	14	13	0.88	35	0.371
623	A	25	24	1.07	35	0.686
624	A	22	21	1.06	35	0.600
625	A	19	18	1.04	35	0.514
626	A	16	15	1.04	35	0.429
627	A	11	10	0.82	35	0.286
628	A	11	10	0.84	35	0.286
629	A	14	13	0.89	35	0.371

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
630	A	17	16	0.91	35	0.457
631	A	28	27	1.06	35	0.771
632	A	25	24	1.07	35	0.686
633	A	22	21	1.06	35	0.600
634	A	19	18	1.06	35	0.514
635	A	14	13	0.87	35	0.371
636	A	14	13	0.86	35	0.371
637	A	14	13	0.87	35	0.371
638	A	17	16	0.92	35	0.457
639	A	20	19	0.92	35	0.543
640	A	19	18	1.02	35	0.514
641	A	16	15	0.99	35	0.429
642	A	13	12	0.94	35	0.343
643	A	10	9	0.88	35	0.257
644	A	15	14	0.83	35	0.400
645	A	11	10	0.86	35	0.286
646	A	19	18	1.07	35	0.514
647	A	16	15	1.09	35	0.429
648	A	13	12	1.04	35	0.343
649	A	13	12	1.06	35	0.343
650	A	11	10	0.98	35	0.286
651	A	22	21	1.12	35	0.600
652	A	19	18	1.12	35	0.514
653	A	16	15	1.11	35	0.429
654	A	16	15	1.12	35	0.429
655	A	16	15	1.11	35	0.429
656	A	14	13	1.07	35	0.371
657	A	8	7	0.85	38	0.184
658	A	10	9	0.85	38	0.237
659	A	8	7	0.78	38	0.184
660	A	9	8	1.10	31	0.258
661	A	10	9	1.06	33	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
662	A	10	9	1.11	33	0.273
663	A	10	9	1.11	33	0.273
664	A	10	9	1.11	33	0.273
665	A	5	4	1.02	39	0.103
666	A	6	5	0.90	39	0.128
667	A	6	5	0.90	39	0.128
668	A	6	5	0.80	39	0.128
669	A	4	3	1.00	37	0.081
670	A	4	4	1.00	24	0.167
671	A	6	5	0.80	39	0.128
672	A	5	4	1.00	39	0.103
673	A	6	5	0.89	39	0.128
674	A	6	5	0.89	39	0.128
675	A	6	5	0.89	39	0.128
676	A	6	5	0.94	41	0.122
677	A	6	5	0.83	41	0.122
678	A	6	5	0.83	41	0.122
679	A	6	5	0.83	41	0.122
680	A	7	6	0.79	41	0.146
681	A	6	5	0.80	39	0.128
682	A	6	6	0.88	26	0.231
683	A	6	5	0.70	41	0.122
684	A	6	5	0.71	41	0.122
685	A	6	5	0.82	41	0.122
686	A	6	5	0.81	41	0.122
687	A	6	5	0.82	41	0.122
688	A	6	5	0.81	41	0.122
689	A	6	5	0.91	41	0.122
690	A	6	5	0.81	41	0.122
691	A	6	5	0.81	41	0.122
692	A	6	5	0.80	41	0.122
693	A	8	7	0.73	41	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
694	A	6	5	0.83	41	0.122
695	A	6	5	0.90	39	0.128
696	A	8	8	0.90	26	0.308
697	A	6	5	0.68	41	0.122
698	A	6	5	0.69	41	0.122
699	A	7	6	0.78	41	0.146
700	A	6	5	0.92	41	0.122
701	A	6	5	0.79	41	0.122
702	A	6	5	0.80	41	0.122
703	A	6	5	0.79	41	0.122
704	A	6	5	0.80	41	0.122
705	A	6	5	1.03	41	0.122
706	A	6	5	0.70	41	0.122
707	A	6	5	0.70	41	0.122
708	A	6	5	0.71	41	0.122
709	A	6	5	0.84	39	0.128
710	A	3	3	1.00	26	0.115
711	A	7	6	1.11	41	0.146
712	A	6	5	0.85	41	0.122
713	A	6	5	0.83	41	0.122
714	A	6	5	0.81	41	0.122
715	A	6	5	1.03	41	0.122
716	A	6	5	0.71	41	0.122
717	A	6	5	0.72	41	0.122
718	A	6	5	0.73	41	0.122
719	A	6	5	0.75	41	0.122
720	A	5	4	1.00	39	0.103
721	A	5	5	0.99	26	0.192
722	A	6	5	0.85	41	0.122
723	A	8	7	1.08	41	0.171
724	A	6	5	0.81	41	0.122
725	A	6	5	0.80	41	0.122

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	6	5	0.79	41	0.122
727	A	6	5	1.03	41	0.122
728	A	6	5	0.72	41	0.122
729	A	6	5	0.72	41	0.122
730	A	7	6	0.81	41	0.146
731	A	6	5	0.83	41	0.122
732	A	6	5	0.90	39	0.128
733	A	7	7	0.99	26	0.269
734	A	6	5	0.83	41	0.122
735	A	6	5	0.81	41	0.122
736	A	9	8	1.05	41	0.195
737	A	6	5	0.79	41	0.122
738	A	6	5	0.78	41	0.122
739	A	6	5	0.78	41	0.122
740	A	5	4	1.02	41	0.098
741	A	5	4	1.02	41	0.098
742	A	5	4	1.02	41	0.098
743	A	5	4	1.02	41	0.098
744	A	5	4	1.02	41	0.098
745	A	5	4	1.02	41	0.098
746	A	5	4	1.02	41	0.098
747	A	5	4	1.02	41	0.098
748	A	6	5	0.93	43	0.116
749	A	6	5	0.93	43	0.116
750	A	6	5	0.93	43	0.116
751	A	6	5	0.93	43	0.116
752	A	6	5	0.93	43	0.116
753	A	6	5	0.93	43	0.116
754	A	6	5	0.93	43	0.116
755	A	6	5	0.93	43	0.116
756	A	6	5	0.91	43	0.116
757	A	6	5	0.91	43	0.116

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
758	A	6	5	0.91	43	0.116
759	A	6	5	0.91	43	0.116
760	A	6	5	0.91	43	0.116
761	A	6	5	0.91	43	0.116
762	A	6	5	0.91	43	0.116
763	A	6	5	0.91	43	0.116
764	A	10	9	0.85	43	0.209
765	A	9	8	0.89	43	0.186
766	A	8	7	0.92	43	0.163
767	A	7	6	0.99	43	0.140
768	A	8	7	0.97	43	0.163
769	A	9	8	0.93	43	0.186
770	A	10	9	0.92	43	0.209
771	A	11	10	0.83	43	0.233
772	A	10	9	0.85	43	0.209
773	A	9	8	0.87	43	0.186
774	A	8	7	0.91	43	0.163
775	A	8	7	0.97	43	0.163
776	A	9	8	0.93	43	0.186
777	A	10	9	0.96	43	0.209
778	A	11	10	0.92	43	0.233
779	A	11	10	0.84	43	0.233
780	A	10	9	0.86	43	0.209
781	A	9	8	0.89	43	0.186
782	A	9	8	0.93	43	0.186
783	A	9	8	0.96	43	0.186
784	A	10	9	0.93	43	0.209
785	A	11	10	0.95	43	0.233
786	A	12	11	0.95	43	0.256
787	A	9	8	0.97	45	0.178
788	A	8	7	1.00	45	0.156
789	A	7	6	1.02	45	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
790	A	6	5	1.06	45	0.111
791	A	6	5	1.06	45	0.111
792	A	5	4	1.09	45	0.089
793	A	6	5	1.08	45	0.111
794	A	7	6	1.06	45	0.133
795	A	9	8	0.95	45	0.178
796	A	8	7	0.96	45	0.156
797	A	7	6	1.01	45	0.133
798	A	7	6	1.04	45	0.133
799	A	7	6	1.01	45	0.133
800	A	7	6	1.08	45	0.133
801	A	5	4	1.09	45	0.089
802	A	6	5	1.08	45	0.111
803	A	7	6	1.05	45	0.133
804	A	8	7	1.05	45	0.156
805	A	9	8	0.93	45	0.178
806	A	8	7	0.99	45	0.156
807	A	8	7	0.97	45	0.156
808	A	8	7	1.00	45	0.156
809	A	8	7	0.97	45	0.156
810	A	8	7	0.98	45	0.156
811	A	8	7	1.07	45	0.156
812	A	5	4	1.09	45	0.089
813	A	6	5	1.08	45	0.111
814	A	7	6	1.06	45	0.133
815	A	8	7	1.05	45	0.156
816	A	10	9	0.91	45	0.200
817	A	9	8	0.98	45	0.178
818	A	9	8	0.94	45	0.178
819	A	9	8	0.95	45	0.178
820	A	9	8	0.97	45	0.178
821	A	9	8	0.95	45	0.178

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
822	A	9	8	0.96	45	0.178
823	A	9	8	0.97	45	0.178
824	A	9	8	1.07	45	0.178
825	A	5	4	1.09	45	0.089
826	A	6	5	1.08	45	0.111
827	A	7	6	1.06	45	0.133
828	A	8	7	1.05	45	0.156
829	A	9	8	1.05	45	0.178
830	A	8	7	0.97	45	0.156
831	A	7	6	1.01	45	0.133
832	A	6	5	1.05	45	0.111
833	A	5	4	1.10	45	0.089
834	A	6	5	1.04	45	0.111
835	A	7	6	1.02	45	0.133
836	A	9	8	0.96	45	0.178
837	A	8	7	0.98	45	0.156
838	A	7	6	1.08	45	0.133
839	A	5	4	1.09	45	0.089
840	A	6	5	1.07	45	0.111
841	A	6	5	1.05	45	0.111
842	A	8	7	1.01	45	0.156
843	A	10	9	0.95	45	0.200
844	A	9	8	0.97	45	0.178
845	A	8	7	1.07	45	0.156
846	A	5	4	1.09	45	0.089
847	A	6	5	1.08	45	0.111
848	A	7	6	1.02	45	0.133
849	A	7	6	1.03	45	0.133
850	A	7	6	1.06	45	0.133
851	A	6	5	1.10	41	0.122
852	A	6	5	1.01	47	0.106
853	A	4	3	1.00	46	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
854	A	5	5	1.05	36	0.139
855	A	8	7	1.05	38	0.184

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \tan^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 91

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -a(A-iB)x+\frac{a(iA+B)\log(\cos(c+dx))}{d}+\frac{a(A-iB)\tan(c+dx)}{d} + \frac{a(iA+B)\tan^2(c+dx)}{2d}+\frac{iaB\tan^3(c+dx)}{3d}$$

output
```
-a*(A-I*B)*x+a*(I*A+B)*ln(cos(d*x+c))/d+a*(A-I*B)*tan(d*x+c)/d+1/2*a*(I*A+
B)*tan(d*x+c)^2/d+1/3*I*a*B*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{a(-6(A-iB)\arctan(\tan(c+dx))+6iA\log(\cos(c+dx))+6B\log(\cos(c+dx))+3(iA+B)\sec^2(c+}{6d}$$

input
```
Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(a*(-6*(A - I*B)*ArcTan[Tan[c + d*x]] + (6*I)*A*Log[Cos[c + d*x]] + 6*B*Lo
g[Cos[c + d*x]] + 3*(I*A + B)*Sec[c + d*x]^2 + 6*A*Tan[c + d*x] - (6*I)*B*
Tan[c + d*x] + (2*I)*B*Tan[c + d*x]^3))/(6*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4075, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int \tan^2(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx+\frac{iaB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a(A-iB)+a(iA+B)\tan(c+dx))dx+\frac{iaB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int \tan(c+dx)(a(A-iB)\tan(c+dx)-a(iA+B))dx+\frac{a(B+iA)\tan^2(c+dx)}{2d}+\frac{iaB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a(A-iB)\tan(c+dx)-a(iA+B))dx+\frac{a(B+iA)\tan^2(c+dx)}{2d}+\frac{iaB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4008}$$

$$-a(B+iA)\int \tan(c+dx)dx+\frac{a(B+iA)\tan^2(c+dx)}{2d}+\frac{a(A-iB)\tan(c+dx)}{d}-ax(A-iB)+\frac{iaB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-a(B+iA)\int \tan(c+dx)dx+\frac{a(B+iA)\tan^2(c+dx)}{2d}+\frac{a(A-iB)\tan(c+dx)}{d}-ax(A-iB)+\frac{iaB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a(B+iA)\tan^2(c+dx)}{2d}+\frac{a(A-iB)\tan(c+dx)}{d}+\frac{a(B+iA)\log(\cos(c+dx))}{d}-ax(A-iB)+\frac{iaB\tan^3(c+dx)}{3d}$$

input
```
Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-(a*(A - I*B)*x) + (a*(I*A + B)*Log[Cos[c + d*x]])/d + (a*(A - I*B)*Tan[c
+ d*x])/d + (a*(I*A + B)*Tan[c + d*x]^2)/(2*d) + ((I/3)*a*B*Tan[c + d*x]^3
)/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

method	result
parts	$\frac{(iaA+Ba)\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{aA(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{iaB\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+\right.}{d}$
norman	$(iaB-aA)\,x+\frac{(-iaB+aA)\tan(dx+c)}{d}+\frac{(iaA+Ba)\tan(dx+c)^2}{2d}+\frac{iaB\tan(dx+c)^3}{3d}-\frac{(iaA+Ba)\ln\left(1+\mathrm{t}\right.}{2d}$
derivativedivides	$\frac{a\left(\frac{iB\tan(dx+c)^3}{3}+\frac{iA\tan(dx+c)^2}{2}-iB\tan(dx+c)+\frac{B\tan(dx+c)^2}{2}+A\tan(dx+c)+\frac{(-iA-B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(iB-A)\,\mathrm{a}\right.}{d}$
default	$\frac{a\left(\frac{iB\tan(dx+c)^3}{3}+\frac{iA\tan(dx+c)^2}{2}-iB\tan(dx+c)+\frac{B\tan(dx+c)^2}{2}+A\tan(dx+c)+\frac{(-iA-B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(iB-A)\,\mathrm{a}\right.}{d}$
parallelrisch	$-\frac{-2iaB\tan(dx+c)^3-3iA\tan(dx+c)^2a-6iBxad+3iA\ln\left(1+\tan(dx+c)^2\right)a+6Axad+6iB\tan(dx+c)a-3B\tan(dx+}{6d}$
risch	$-\frac{2iaBc}{d}+\frac{2aAc}{d}+\frac{2a\left(6iA\,\mathrm{e}^{4i(dx+c)}+9B\,\mathrm{e}^{4i(dx+c)}+9iA\,\mathrm{e}^{2i(dx+c)}+9B\,\mathrm{e}^{2i(dx+c)}+3iA+4B\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}+\right.}{d}$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
(I*a*A+B*a)/d*(1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)^2))+a*A/d*(tan(d*x+c)-
arctan(tan(d*x+c)))+I*a*B/d*(1/3*tan(d*x+c)^3-tan(d*x+c)+arctan(tan(d*x+c)
))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(77) = 154$.

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.87

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{6\,(-2i\,A-3\,B)ae^{(4i\,dx+4i\,c)}+18\,(-i\,A-B)ae^{(2i\,dx+2i\,c)}+2\,(-3i\,A-4\,B)a+3\,\left((-i\,A-B)ae^{(6i\,dx+6}\right.}{3\,(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/3*(6*(-2*I*A - 3*B)*a*e^(4*I*d*x + 4*I*c) + 18*(-I*A - B)*a*e^(2*I*d*x
+ 2*I*c) + 2*(-3*I*A - 4*B)*a + 3*((-I*A - B)*a*e^(6*I*d*x + 6*I*c) + 3*(-
I*A - B)*a*e^(4*I*d*x + 4*I*c) + 3*(-I*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*
A - B)*a)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*
I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 0.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.84

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{ia(A-iB)\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{6iAa+8Ba+(18iAae^{2ic}+18Bae^{2ic})\,e^{2idx}+(12iAae^{4ic}+18Bae^{4ic})\,e^{4idx}}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}$$

input
```
integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
I*a*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (6*I*A*a + 8*B*a + (18*I
*A*a*exp(2*I*c) + 18*B*a*exp(2*I*c))*exp(2*I*d*x) + (12*I*A*a*exp(4*I*c) +
 18*B*a*exp(4*I*c))*exp(4*I*d*x))/(3*d*exp(6*I*c)*exp(6*I*d*x) + 9*d*exp(4
*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{-2i\,Ba\tan\left(dx+c\right)^3+3\left(-i\,A-B\right)a\tan\left(dx+c\right)^2+6\left(dx+c\right)(A-i\,B)a+3\left(i\,A+B\right)a\log\left(\tan\left(\right.\right.}{6\,d}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/6*(-2*I*B*a*tan(d*x + c)^3 + 3*(-I*A - B)*a*tan(d*x + c)^2 + 6*(d*x + c
)*(A - I*B)*a + 3*(I*A + B)*a*log(tan(d*x + c)^2 + 1) - 6*(A - I*B)*a*tan(
d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{\left(-i\,Aa-Ba\right)\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{-2i\,Bad^2\tan\left(dx+c\right)^3-3i\,Aad^2\tan\left(dx+c\right)^2-3\,Bad^2\tan\left(dx+c\right)^2-6\,Aad^2\tan\left(dx+c\right)+6i\,Ba}{6\,d^3}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
(-I*A*a - B*a)*log(tan(d*x + c) + I)/d - 1/6*(-2*I*B*a*d^2*tan(d*x + c)^3
- 3*I*A*a*d^2*tan(d*x + c)^2 - 3*B*a*d^2*tan(d*x + c)^2 - 6*A*a*d^2*tan(d*
x + c) + 6*I*B*a*d^2*tan(d*x + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{\tan(c+d\,x)^2\,\left(\frac{B\,a}{2}+\frac{A\,a\,1\mathrm{i}}{2}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B\,a+A\,a\,1\mathrm{i}\right)}{d}+\frac{\tan(c+d\,x)\,\left(A\,a-B\,a\,1\mathrm{i}\right)}{d}+\frac{B\,a\,\tan(c+d\,x)^3\,1\mathrm{i}}{3\,d}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
(tan(c + d*x)^2*((A*a*1i)/2 + (B*a)/2))/d - (log(tan(c + d*x) + 1i)*(A*a*1
i + B*a))/d + (tan(c + d*x)*(A*a - B*a*1i))/d + (B*a*tan(c + d*x)^3*1i)/(3
*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{a\left(-3\log\left(\tan\left(dx+c\right)^2+1\right)ai-3\log\left(\tan\left(dx+c\right)^2+1\right)b+2\tan\left(dx+c\right)^3bi+3\tan\left(dx+c\right)^2ai+3\right.}{6d}$$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*( - 3*log(tan(c + d*x)**2 + 1)*a*i - 3*log(tan(c + d*x)**2 + 1)*b + 2*t
an(c + d*x)**3*b*i + 3*tan(c + d*x)**2*a*i + 3*tan(c + d*x)**2*b + 6*tan(c
 + d*x)*a - 6*tan(c + d*x)*b*i - 6*a*d*x + 6*b*d*i*x))/(6*d)
```

3.2 $\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 69

$$\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -a(iA+B)x - \frac{a(A-iB)\log(\cos(c+dx))}{d} + \frac{a(iA+B)\tan(c+dx)}{d} + \frac{iaB\tan^2(c+dx)}{2d}$$

output
```
-a*(I*A+B)*x-a*(A-I*B)*ln(cos(d*x+c))/d+a*(I*A+B)*tan(d*x+c)/d+1/2*I*a*B*t
an(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{a((-2iA-2B)\arctan(\tan(c+dx)) + iB\sec^2(c+dx) - 2(A-iB)(\log(\cos(c+dx)) - i\tan(c+dx)))}{2d}$$

input
```
Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(a*(((-2*I)*A - 2*B)*ArcTan[Tan[c + d*x]] + I*B*Sec[c + d*x]^2 - 2*(A - I*
B)*(Log[Cos[c + d*x]] - I*Tan[c + d*x])))/(2*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4075, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int \tan(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx+\frac{iaB\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx+\frac{iaB\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4008}$$

$$a(A-iB)\int \tan(c+dx)dx+\frac{a(B+iA)\tan(c+dx)}{d}-ax(B+iA)+\frac{iaB\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$a(A-iB)\int \tan(c+dx)dx+\frac{a(B+iA)\tan(c+dx)}{d}-ax(B+iA)+\frac{iaB\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a(B+iA)\tan(c+dx)}{d} - \frac{a(A-iB)\log(\cos(c+dx))}{d} - ax(B+iA) + \frac{iaB\tan^2(c+dx)}{2d}$$

input
```
Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-(a*(I*A + B)*x) - (a*(A - I*B)*Log[Cos[c + d*x]])/d + (a*(I*A + B)*Tan[c
+ d*x])/d + ((I/2)*a*B*Tan[c + d*x]^2)/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a\left(\frac{iB\tan(dx+c)^2}{2}+iA\tan(dx+c)+B\tan(dx+c)+\frac{(-iB+A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-iA-B)\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{iB\tan(dx+c)^2}{2}+iA\tan(dx+c)+B\tan(dx+c)+\frac{(-iB+A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-iA-B)\arctan(\tan(dx+c))\right)}{d}$
norman	$(-iaA-Ba)\,x+\frac{(iaA+Ba)\tan(dx+c)}{d}+\frac{iaB\tan(dx+c)^2}{2d}+\frac{(-iaB+aA)\ln\left(1+\tan(dx+c)^2\right)}{2d}$
parts	$\frac{(iaA+Ba)(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{aA\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{iaB\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}$
parallelrisch	$-\frac{2iAxad-iaB\tan(dx+c)^2-2iA\tan(dx+c)a+iB\ln\left(1+\tan(dx+c)^2\right)a+2Bxad-A\ln\left(1+\tan(dx+c)^2\right)a-2B\tan(dx}{2d}$
risch	$\frac{2aBc}{d}+\frac{2iaAc}{d}+\frac{2ia\left(iA\,e^{2i(dx+c)}+2B\,e^{2i(dx+c)}+iA+B\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{ia\ln\left(e^{2i(dx+c)}+1\right)B}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)A}{d}$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/d*a*(1/2*I*B*tan(d*x+c)^2+I*A*tan(d*x+c)+B*tan(d*x+c)+1/2*(A-I*B)*ln(1+t
an(d*x+c)^2)+(-I*A-B)*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.58

$$\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\,(A-2i\,B)ae^{(2i\,dx+2i\,c)}+2\,(A-i\,B)a+\left((A-i\,B)ae^{(4i\,dx+4i\,c)}+2\,(A-i\,B)ae^{(2i\,dx+2i\,c)}+(A-i\,B)\right.}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-(2*(A - 2*I*B)*a*e^(2*I*d*x + 2*I*c) + 2*(A - I*B)*a + ((A - I*B)*a*e^(4*
I*d*x + 4*I*c) + 2*(A - I*B)*a*e^(2*I*d*x + 2*I*c) + (A - I*B)*a)*log(e^(2
*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d
)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.58

$$\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{a(A-iB)\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-2Aa+2iBa+\left(-2Aae^{2ic}+4iBae^{2ic}\right)e^{2idx}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
-a*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-2*A*a + 2*I*B*a + (-2*A
*a*exp(2*I*c) + 4*I*B*a*exp(2*I*c))*exp(2*I*d*x))/(d*exp(4*I*c)*exp(4*I*d*
x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{-i\,Ba\tan\left(dx+c\right)^2-2\left(dx+c\right)(-i\,A-B)a-(A-i\,B)a\log\left(\tan\left(dx+c\right)^2+1\right)+2\left(-i\,A-B\right)a\,t}{2\,d}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/2*(-I*B*a*tan(d*x + c)^2 - 2*(d*x + c)*(-I*A - B)*a - (A - I*B)*a*log(t
an(d*x + c)^2 + 1) + 2*(-I*A - B)*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{(Aa-iBa)\log(\tan(dx+c)+i)}{d}-\frac{-iBad\tan(dx+c)^2-2iAad\tan(dx+c)-2Bad\tan(dx+c)}{2d^2}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
(A*a - I*B*a)*log(tan(d*x + c) + I)/d - 1/2*(-I*B*a*d*tan(d*x + c)^2 - 2*I
*A*a*d*tan(d*x + c) - 2*B*a*d*tan(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(A\,a-B\,a\,1\mathrm{i})}{d}+\frac{\tan(c+d\,x)\,(B\,a+A\,a\,1\mathrm{i})}{d}+\frac{B\,a\tan(c+d\,x)^2\,1\mathrm{i}}{2\,d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
(log(tan(c + d*x) + 1i)*(A*a - B*a*1i))/d + (tan(c + d*x)*(A*a*1i + B*a))/
d + (B*a*tan(c + d*x)^2*1i)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \tan(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{a\left(\log\left(\tan\left(dx+c\right)^{2}+1\right)a-\log\left(\tan\left(dx+c\right)^{2}+1\right)bi+\tan\left(dx+c\right)^{2}bi+2\tan\left(dx+c\right)ai+2\tan\left(dx\right.\right.}{2d}$$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*(log(tan(c + d*x)**2 + 1)*a - log(tan(c + d*x)**2 + 1)*b*i + tan(c + d*
x)**2*b*i + 2*tan(c + d*x)*a*i + 2*tan(c + d*x)*b - 2*a*d*i*x - 2*b*d*x))/
(2*d)
```

3.3 $\int (a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int (a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = a(A-iB)x - \frac{a(iA+B)\log(\cos(c+dx))}{d} + \frac{iaB\tan(c+dx)}{d}$$

output
```
a*(A-I*B)*x-a*(I*A+B)*ln(cos(d*x+c))/d+I*a*B*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int (a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = aAx - \frac{iaB\arctan(\tan(c+dx))}{d} - \frac{iaA\log(\cos(c+dx))}{d} - \frac{aB\log(\cos(c+dx))}{d} + \frac{iaB\tan(c+dx)}{d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
a*A*x - (I*a*B*ArcTan[Tan[c + d*x]])/d - (I*a*A*Log[Cos[c + d*x]])/d - (a*
B*Log[Cos[c + d*x]])/d + (I*a*B*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4008}$$

$$a(B+iA)\int \tan(c+dx)dx+ax(A-iB)+\frac{iaB\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a(B+iA)\int \tan(c+dx)dx+ax(A-iB)+\frac{iaB\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{a(B+iA)\log(\cos(c+dx))}{d}+ax(A-iB)+\frac{iaB\tan(c+dx)}{d}$$

input
```
Int[(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
a*(A - I*B)*x - (a*(I*A + B)*Log[Cos[c + d*x]])/d + (I*a*B*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{a\left(iB\tan(dx+c)+\frac{(iA+B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-iB+A)\arctan(\tan(dx+c))\right)}{d}$	50
default	$\frac{a\left(iB\tan(dx+c)+\frac{(iA+B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-iB+A)\arctan(\tan(dx+c))\right)}{d}$	50
norman	$(-iaB+aA)\,x+\frac{iaB\tan(dx+c)}{d}+\frac{(iaA+Ba)\ln\left(1+\tan(dx+c)^2\right)}{2d}$	52
parts	$Aax+\frac{(iaA+Ba)\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{iaB(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$	55
parallelrisch	$\frac{-2iBxad+iA\ln\left(1+\tan(dx+c)^2\right)a+2Axad+2iB\tan(dx+c)a+B\ln\left(1+\tan(dx+c)^2\right)a}{2d}$	61
risch	$\frac{2iaBc}{d}-\frac{2aAc}{d}-\frac{2aB}{d\left(e^{2i(dx+c)}+1\right)}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)B}{d}-\frac{ia\ln\left(e^{2i(dx+c)}+1\right)A}{d}$	78

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*a*(I*B*tan(d*x+c)+1/2*(I*A+B)*ln(1+tan(d*x+c)^2)+(A-I*B)*arctan(tan(d*
x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int (a + ia\tan(c + dx))(A + B\tan(c + dx))\,dx = -\frac{2\,Ba - \left((-i\,A - B)ae^{(2i\,dx+2i\,c)} + (-i\,A - B)a\right)\log\left(e^{(2i\,dx+2i\,c)} + 1\right)}{de^{(2i\,dx+2i\,c)} + d}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-(2*B*a - ((-I*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*log(e^(2*I*d*x
 + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int (a + ia\tan(c + dx))(A + B\tan(c + dx))\,dx = -\frac{2Ba}{de^{2ic}e^{2idx} + d} - \frac{ia(A - iB)\log\left(e^{2idx} + e^{-2ic}\right)}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
-2*B*a/(d*exp(2*I*c)*exp(2*I*d*x) + d) - I*a*(A - I*B)*log(exp(2*I*d*x) +
exp(-2*I*c))/d
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int (a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{2\,(dx+c)(A-i\,B)a-(-i\,A-B)a\log\left(\tan\left(dx+c\right)^2+1\right)+2i\,Ba\tan\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*(d*x + c)*(A - I*B)*a - (-I*A - B)*a*log(tan(d*x + c)^2 + 1) + 2*I*
B*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int (a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{i\,Ba\tan\left(dx+c\right)}{d}+\frac{(i\,Aa+Ba)\log\left(\tan\left(dx+c\right)+i\right)}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
I*B*a*tan(d*x + c)/d + (I*A*a + B*a)*log(tan(d*x + c) + I)/d
```

Mupad [B] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int (a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B\,a+A\,a\,1\mathrm{i}\right)}{d}+\frac{B\,a\tan(c+d\,x)\,1\mathrm{i}}{d}$$

input
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
(log(tan(c + d*x) + 1i)*(A*a*1i + B*a))/d + (B*a*tan(c + d*x)*1i)/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int (a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{a\left(\log\left(\tan\left(dx+c\right)^{2}+1\right)ai+\log\left(\tan\left(dx+c\right)^{2}+1\right)b+2\tan\left(dx+c\right)bi+2adx-2bdix\right)}{2d}$$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*(log(tan(c + d*x)**2 + 1)*a*i + log(tan(c + d*x)**2 + 1)*b + 2*tan(c +
d*x)*b*i + 2*a*d*x - 2*b*d*i*x))/(2*d)
```

3.4 $\int \cot(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 40

$$\int \cot(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = a(iA+B)x-\frac{iaB\log(\cos(c+dx))}{d}+\frac{aA\log(\sin(c+dx))}{d}$$

output
```
a*(I*A+B)*x-I*a*B*ln(cos(d*x+c))/d+a*A*ln(sin(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \cot(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = iaAx+aBx-\frac{iaB\log(\cos(c+dx))}{d}+\frac{aA\log(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
I*a*A*x + a*B*x - (I*a*B*Log[Cos[c + d*x]])/d + (a*A*Log[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4072}$$

$$\int \cot(c+dx)(aA+a(iA+B)\tan(c+dx))dx+iaB\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aA+a(iA+B)\tan(c+dx)}{\tan(c+dx)}dx+iaB\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3956}$$

$$\int \frac{aA+a(iA+B)\tan(c+dx)}{\tan(c+dx)}dx-\frac{iaB\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{4014}$$

$$aA\int \cot(c+dx)dx+ax(B+iA)-\frac{iaB\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$aA\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+ax(B+iA)-\frac{iaB\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$-aA\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+ax(B+iA)-\frac{iaB\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3956}$$

$$ax(B+iA)+\frac{aA\log(-\sin(c+dx))}{d}-\frac{iaB\log(\cos(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
a*(I*A + B)*x - (I*a*B*Log[Cos[c + d*x]])/d + (a*A*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

method	result	size
parallelrisch	$\frac{a\left(\frac{(iB-A)\ln\left(\sec(dx+c)^2\right)}{2}+A\ln(\tan(dx+c))+(iA+B)xd\right)}{d}$	44
derivativedivides	$\frac{a\left(\frac{(iB-A)\ln\left(1+\tan(dx+c)^2\right)}{2}+\arctan(\tan(dx+c))(iA+B)+A\ln(\tan(dx+c))\right)}{d}$	51
default	$\frac{a\left(\frac{(iB-A)\ln\left(1+\tan(dx+c)^2\right)}{2}+\arctan(\tan(dx+c))(iA+B)+A\ln(\tan(dx+c))\right)}{d}$	51
norman	$(iaA+Ba)\,x+\frac{aA\ln(\tan(dx+c))}{d}-\frac{(-iaB+aA)\ln\left(1+\tan(dx+c)^2\right)}{2d}$	51
risch	$-\frac{2iaAc}{d}-\frac{2aBc}{d}+\frac{aA\ln\left(e^{2i(dx+c)}-1\right)}{d}-\frac{ia\ln\left(e^{2i(dx+c)}+1\right)B}{d}$	57

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
a/d*(1/2*(-A+I*B)*ln(sec(d*x+c)^2)+A*ln(tan(d*x+c))+(I*A+B)*x*d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int\cot(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{-i\,Ba\log\left(e^{(2i\,dx+2i\,c)}+1\right)+Aa\log\left(e^{(2i\,dx+2i\,c)}-1\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
(-I*B*a*log(e^(2*I*d*x + 2*I*c) + 1) + A*a*log(e^(2*I*d*x + 2*I*c) - 1))/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(36) = 72.

Time = 1.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.35

$$\int \cot(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{Aa\log\left(\frac{-Aa-iBa}{Aae^{2ic}+iBae^{2ic}}+e^{2idx}\right)}{d} - \frac{iBa\log\left(\frac{Aa+iBa}{Aae^{2ic}+iBae^{2ic}}+e^{2idx}\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
A*a*log((-A*a - I*B*a)/(A*a*exp(2*I*c) + I*B*a*exp(2*I*c)) + exp(2*I*d*x))
/d - I*B*a*log((A*a + I*B*a)/(A*a*exp(2*I*c) + I*B*a*exp(2*I*c)) + exp(2*I
*d*x))/d
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \cot(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,(dx+c)(i\,A+B)a-(A-i\,B)a\log\left(\tan\left(dx+c\right)^2+1\right)+2\,Aa\log\left(\tan\left(dx+c\right)\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/2*(2*(d*x + c)*(I*A + B)*a - (A - I*B)*a*log(tan(d*x + c)^2 + 1) + 2*A*a
*log(tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \cot(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{Aa\log(|\tan(dx+c)|)}{d} - \frac{(Aa-iBa)\log(\tan(dx+c)+i)}{d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
A*a*log(abs(tan(d*x + c)))/d - (A*a - I*B*a)*log(tan(d*x + c) + I)/d
```

Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \cot(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{A\,a\,\ln(\tan(c+d\,x))}{d} - \frac{a\,\ln(\tan(c+d\,x)+1\mathrm{i})\,(A-B\,1\mathrm{i})}{d}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
(A*a*log(tan(c + d*x)))/d - (a*log(tan(c + d*x) + 1i)*(A - B*1i))/d
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.32

$$\int \cot(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{a\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)a+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)bi-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)bi-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{d}$$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*( - log(tan((c + d*x)/2)**2 + 1)*a + log(tan((c + d*x)/2)**2 + 1)*b*i -
 log(tan((c + d*x)/2) - 1)*b*i - log(tan((c + d*x)/2) + 1)*b*i + log(tan((
c + d*x)/2))*a + a*d*i*x + b*d*x))/d
```

3.5 $\int \cot^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 44

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -a(A-iB)x - \frac{aA\cot(c+dx)}{d} + \frac{a(iA+B)\log(\sin(c+dx))}{d}$$

output
```
-a*(A-I*B)*x-a*A*cot(d*x+c)/d+a*(I*A+B)*ln(sin(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = iaBx - \frac{aA\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d} + \frac{iaA\log(\sin(c+dx))}{d} + \frac{aB\log(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
I*a*B*x - (a*A*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^
2])/d + (I*a*A*Log[Sin[c + d*x]])/d + (a*B*Log[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 4074, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4074}$$

$$-\frac{aA\cot(c+dx)}{d}+\int \cot(c+dx)(a(iA+B)-a(A-iB)\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{aA\cot(c+dx)}{d}+\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4014}$$

$$a(B+iA)\int \cot(c+dx)dx-ax(A-iB)-\frac{aA\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a(B+iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-ax(A-iB)-\frac{aA\cot(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-a(B+iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-ax(A-iB)-\frac{aA\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a(B+iA)\log(-\sin(c+dx))}{d}-ax(A-iB)-\frac{aA\cot(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-(a*(A - I*B)*x) - (a*A*Cot[c + d*x])/d + (a*(I*A + B)*Log[-Sin[c + d*x]])
/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

method	result	size
parallelrisch	$\frac{a\left(-\frac{(iA+B)\ln\left(\sec(dx+c)^2\right)}{2}+(iA+B)\ln(\tan(dx+c))-A\cot(dx+c)+(iB-A)xd\right)}{d}$	58
derivativedivides	$\frac{a\left(\frac{(-iA-B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(iB-A)\arctan(\tan(dx+c))-\frac{A}{\tan(dx+c)}+(iA+B)\ln(\tan(dx+c))\right)}{d}$	69
default	$\frac{a\left(\frac{(-iA-B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(iB-A)\arctan(\tan(dx+c))-\frac{A}{\tan(dx+c)}+(iA+B)\ln(\tan(dx+c))\right)}{d}$	69
risch	$-\frac{2iaBc}{d}+\frac{2aAc}{d}-\frac{2iaA}{d\left(e^{2i(dx+c)}-1\right)}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)B}{d}+\frac{ia\ln\left(e^{2i(dx+c)}-1\right)A}{d}$	78
norman	$\frac{(iaB-aA)x\tan(dx+c)-\frac{aA}{d}}{\tan(dx+c)}+\frac{(iaA+Ba)\ln(\tan(dx+c))}{d}-\frac{(iaA+Ba)\ln\left(1+\tan(dx+c)^2\right)}{2d}$	82

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
a/d*(-1/2*(I*A+B)*ln(sec(d*x+c)^2)+(I*A+B)*ln(tan(d*x+c))-A*cot(d*x+c)+(-A
+I*B)*x*d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.41

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{-2iAa + \left((iA+B)ae^{(2i\,dx+2i\,c)} + (-iA-B)a\right)\log\left(e^{(2i\,dx+2i\,c)}-1\right)}{de^{(2i\,dx+2i\,c)}-d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
(-2*I*A*a + ((I*A + B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*log(e^(2*I*d*
x + 2*I*c) - 1))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{2iAa}{de^{2ic}e^{2idx}-d} + \frac{ia(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d}$$

input
```
integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
-2*I*A*a/(d*exp(2*I*c)*exp(2*I*d*x) - d) + I*a*(A - I*B)*log(exp(2*I*d*x)
- exp(-2*I*c))/d
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{2\,(dx+c)(A-i\,B)a+(i\,A+B)a\log\left(\tan\left(dx+c\right)^2+1\right)-2\,(i\,A+B)a\log\left(\tan\left(dx+c\right)\right)+\frac{2\,Aa}{\tan(dx+c)}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/2*(2*(d*x + c)*(A - I*B)*a + (I*A + B)*a*log(tan(d*x + c)^2 + 1) - 2*(I
*A + B)*a*log(tan(d*x + c)) + 2*A*a/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{(-i\,Aa-Ba)\log\left(\tan\left(dx+c\right)+i\right)}{d} + \frac{(i\,Aa+Ba)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d} - \frac{Aa}{d\tan\left(dx+c\right)}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
(-I*A*a - B*a)*log(tan(d*x + c) + I)/d + (I*A*a + B*a)*log(abs(tan(d*x + c
)))/d - A*a/(d*tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=-\frac{A\,a\cot(c+d\,x)}{d}+\frac{a\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\,\left(B+A\,1\mathrm{i}\right)\,2\mathrm{i}}{d}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
(a*atan(2*tan(c + d*x) + 1i)*(A*1i + B)*2i)/d - (A*a*cot(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.93

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{a\left(-\cos\left(dx+c\right)a-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+c\right)ai-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+c\right)b+\mathrm{lo}\right.}{\sin\left(d\right.}$$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*( - cos(c + d*x)*a - log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a*i - lo
g(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*b + log(tan((c + d*x)/2))*sin(c +
d*x)*a*i + log(tan((c + d*x)/2))*sin(c + d*x)*b - sin(c + d*x)*a*d*x + sin
(c + d*x)*b*d*i*x))/(sin(c + d*x)*d)
```

3.6 $\int \cot^3(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 68

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -a(iA+B)x-\frac{a(iA+B)\cot(c+dx)}{d} - \frac{aA\cot^2(c+dx)}{2d}-\frac{a(A-iB)\log(\sin(c+dx))}{d}$$

output

```
-a*(I*A+B)*x-a*(I*A+B)*cot(d*x+c)/d-1/2*a*A*cot(d*x+c)^2/d-a*(A-I*B)*ln(si
n(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{aA\csc^2(c+dx)}{2d} - \frac{iaA\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}$$
$$-\frac{aB\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}$$
$$-\frac{aA\log(\sin(c+dx))}{d} + \frac{iaB\log(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(a*A*Csc[c + d*x]^2)/d - (I*a*A*Cot[c + d*x]*Hypergeometric2F1[-1/2,
1, 1/2, -Tan[c + d*x]^2])/d - (a*B*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1,
 1/2, -Tan[c + d*x]^2])/d - (a*A*Log[Sin[c + d*x]])/d + (I*a*B*Log[Sin[c +
 d*x]])/d
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4074, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$
$$\Big\downarrow \text{4074}$$
$$-\frac{aA\cot^2(c+dx)}{2d} + \int \cot^2(c+dx)(a(iA+B)-a(A-iB)\tan(c+dx))dx$$
$$\Big\downarrow \text{3042}$$

$$-\frac{aA\cot^2(c+dx)}{2d}+\int\frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{ 4012}$$

$$\int -\cot(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx-\frac{a(B+iA)\cot(c+dx)}{d}-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \cot(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx-\frac{a(B+iA)\cot(c+dx)}{d}-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\int\frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a(B+iA)\cot(c+dx)}{d}-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 4014}$$

$$-a(A-iB)\int\cot(c+dx)dx-\frac{a(B+iA)\cot(c+dx)}{d}-ax(B+iA)-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$-a(A-iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a(B+iA)\cot(c+dx)}{d}-ax(B+iA)-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 25}$$

$$a(A-iB)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a(B+iA)\cot(c+dx)}{d}-ax(B+iA)-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{a(B+iA)\cot(c+dx)}{d}-\frac{a(A-iB)\log(-\sin(c+dx))}{d}-ax(B+iA)-\frac{aA\cot^2(c+dx)}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-(a*(I*A + B)*x) - (a*(I*A + B)*Cot[c + d*x])/d - (a*A*Cot[c + d*x]^2)/(2*
d) - (a*(A - I*B)*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

method	result
parallelrisch	$-\frac{a\left(\frac{(iB-A)\ln\left(\sec(dx+c)^{2}\right)}{2}+(-iB+A)\ln(\tan(dx+c))+\frac{A\cot(dx+c)^{2}}{2}+\cot(dx+c)(iA+B)+(iA+B)xd\right)}{d}$
derivativedivides	$\frac{a\left(\frac{(-iB+A)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-iA-B)\arctan(\tan(dx+c))-\frac{A}{2\tan(dx+c)^{2}}+(iB-A)\ln(\tan(dx+c))-\frac{iA+B}{\tan(dx+c)}\right)}{d}$
default	$\frac{a\left(\frac{(-iB+A)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-iA-B)\arctan(\tan(dx+c))-\frac{A}{2\tan(dx+c)^{2}}+(iB-A)\ln(\tan(dx+c))-\frac{iA+B}{\tan(dx+c)}\right)}{d}$
norman	$\frac{(-iaA-Ba)x\tan(dx+c)^{2}-\frac{aA}{2d}-\frac{(iaA+Ba)\tan(dx+c)}{d}}{\tan(dx+c)^{2}}-\frac{(-iaB+aA)\ln(\tan(dx+c))}{d}+\frac{(-iaB+aA)\ln\left(1+\tan(dx+\right.}{2d}$
risch	$\frac{2aBc}{d}+\frac{2iaAc}{d}-\frac{2ia\left(2iA\,e^{2i(dx+c)}+B\,e^{2i(dx+c)}-iA-B\right)}{d\left(e^{2i(dx+c)}-1\right)^{2}}+\frac{ia\ln\left(e^{2i(dx+c)}-1\right)B}{d}-\frac{aA\ln\left(e^{2i(dx+c)}-1\right)}{d}$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-a/d*(1/2*(-A+I*B)*ln(sec(d*x+c)^2)+(A-I*B)*ln(tan(d*x+c))+1/2*A*cot(d*x+c
)^2+cot(d*x+c)*(I*A+B)+(I*A+B)*x*d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.63

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{2\,(2\,A-i\,B)ae^{(2i\,dx+2i\,c)}-2\,(A-i\,B)a-\left((A-i\,B)ae^{(4i\,dx+4i\,c)}-2\,(A-i\,B)ae^{(2i\,dx+2i\,c)}+(A-i\,B)a\right.}{de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
(2*(2*A - I*B)*a*e^(2*I*d*x + 2*I*c) - 2*(A - I*B)*a - ((A - I*B)*a*e^(4*I
*d*x + 4*I*c) - 2*(A - I*B)*a*e^(2*I*d*x + 2*I*c) + (A - I*B)*a)*log(e^(2*
I*d*x + 2*I*c) - 1))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.60

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{a(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d} + \frac{-2Aa+2iBa+\left(4Aae^{2ic}-2iBae^{2ic}\right)e^{2idx}}{de^{4ic}e^{4idx}-2de^{2ic}e^{2idx}+d}$$

input
```
integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
-a*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-2*A*a + 2*I*B*a + (4*A*
a*exp(2*I*c) - 2*I*B*a*exp(2*I*c))*exp(2*I*d*x))/(d*exp(4*I*c)*exp(4*I*d*x
) - 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.24

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,(dx+c)(-i\,A-B)a+(A-i\,B)a\log\left(\tan\left(dx+c\right)^2+1\right)-2\,(A-i\,B)a\log\left(\tan\left(dx+c\right)\right)+\frac{2\,(-i\,A-B}{\mathrm{ta}}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/2*(2*(d*x + c)*(-I*A - B)*a + (A - I*B)*a*log(tan(d*x + c)^2 + 1) - 2*(A
 - I*B)*a*log(tan(d*x + c)) + (2*(-I*A - B)*a*tan(d*x + c) - A*a)/tan(d*x
+ c)^2)/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{(Aa-iBa)\log(\tan(dx+c)+i)}{d}-\frac{(Aa-iBa)\log(|\tan(dx+c)|)}{d}-\frac{Aa+2(iAa+Ba)\tan(dx+c)}{2d\tan(dx+c)^2}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
(A*a - I*B*a)*log(tan(d*x + c) + I)/d - (A*a - I*B*a)*log(abs(tan(d*x + c)
))/d - 1/2*(A*a + 2*(I*A*a + B*a)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=-\frac{\frac{Aa}{2}+\tan(c+dx)\,(Ba+Aa\,1\mathrm{i})}{d\tan(c+dx)^2}-\frac{a\,\mathrm{atan}(2\tan(c+dx)+1\mathrm{i})\,(A-B\,1\mathrm{i})\,2\mathrm{i}}{d}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
- ((A*a)/2 + tan(c + d*x)*(A*a*1i + B*a))/(d*tan(c + d*x)^2) - (a*atan(2*t
an(c + d*x) + 1i)*(A - B*1i)*2i)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.65

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{a\left(-4\cos(dx+c)\sin(dx+c)\,ai - 4\cos(dx+c)\sin(dx+c)\,b + 4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)^2\right.}{}$$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*( - 4*cos(c + d*x)*sin(c + d*x)*a*i - 4*cos(c + d*x)*sin(c + d*x)*b + 4
*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a - 4*log(tan((c + d*x)/2)**
2 + 1)*sin(c + d*x)**2*b*i - 4*log(tan((c + d*x)/2))*sin(c + d*x)**2*a + 4
*log(tan((c + d*x)/2))*sin(c + d*x)**2*b*i - 4*sin(c + d*x)**2*a*d*i*x + s
in(c + d*x)**2*a - 4*sin(c + d*x)**2*b*d*x - 2*a))/(4*sin(c + d*x)**2*d)
```

3.7 $\int \cot^4(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 89

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = a(A-iB)x+\frac{a(A-iB)\cot(c+dx)}{d}-\frac{a(iA+B)\cot^2(c+dx)}{2d} - \frac{aA\cot^3(c+dx)}{3d}-\frac{a(iA+B)\log(\sin(c+dx))}{d}$$

output
```
a*(A-I*B)*x+a*(A-I*B)*cot(d*x+c)/d-1/2*a*(I*A+B)*cot(d*x+c)^2/d-1/3*a*A*co
t(d*x+c)^3/d-a*(I*A+B)*ln(sin(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.52

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{iaA\csc^2(c+dx)}{2d} - \frac{aB\csc^2(c+dx)}{2d} - \frac{aA\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d} - \frac{iaB\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d} - \frac{iaA\log(\sin(c+dx))}{d} - \frac{aB\log(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
((-1/2*I)*a*A*Csc[c + d*x]^2)/d - (a*B*Csc[c + d*x]^2)/(2*d) - (a*A*Cot[c
+ d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) - (I*a*B
*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d - (I*a*A
*Log[Sin[c + d*x]])/d - (a*B*Log[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4074, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4074}$$

$$-\frac{aA\cot^3(c+dx)}{3d}+\int \cot^3(c+dx)(a(iA+B)-a(A-iB)\tan(c+dx))dx$$

$\Big\downarrow$ 3042

$$-\frac{aA\cot^3(c+dx)}{3d}+\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\tan(c+dx)^3}dx$$

$\Big\downarrow$ 4012

$$\int -\cot^2(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx-\frac{a(B+iA)\cot^2(c+dx)}{2d}-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 25

$$-\int \cot^2(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx-\frac{a(B+iA)\cot^2(c+dx)}{2d}-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$-\int \frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{a(B+iA)\cot^2(c+dx)}{2d}-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 4012

$$-\int \cot(c+dx)(a(iA+B)-a(A-iB)\tan(c+dx))dx-\frac{a(B+iA)\cot^2(c+dx)}{2d}+\frac{a(A-iB)\cot(c+dx)}{d}-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$-\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a(B+iA)\cot^2(c+dx)}{2d}+\frac{a(A-iB)\cot(c+dx)}{d}-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 4014

$$-a(B+iA)\int \cot(c+dx)dx-\frac{a(B+iA)\cot^2(c+dx)}{2d}+\frac{a(A-iB)\cot(c+dx)}{d}+ax(A-iB)-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$-a(B+iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a(B+iA)\cot^2(c+dx)}{2d}+\frac{a(A-iB)\cot(c+dx)}{d}+ax(A-iB)-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 25

$$a(B+iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a(B+iA)\cot^2(c+dx)}{2d}+\frac{a(A-iB)\cot(c+dx)}{d}+ax(A-iB)-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 3956

$$-\frac{a(B+iA)\cot^2(c+dx)}{2d}+\frac{a(A-iB)\cot(c+dx)}{d}-\frac{a(B+iA)\log(-\sin(c+dx))}{d}+ax(A-iB)-\frac{aA\cot^3(c+dx)}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
a*(A - I*B)*x + (a*(A - I*B)*Cot[c + d*x])/d - (a*(I*A + B)*Cot[c + d*x]^2
)/(2*d) - (a*A*Cot[c + d*x]^3)/(3*d) - (a*(I*A + B)*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{\left(\frac{(iA+B)\ln\left(\sec(dx+c)^2\right)}{2}-(iA+B)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^3}{3}-\frac{\cot(dx+c)^2(iA+B)}{2}+(-iB+A)\cot(dx+c)+(-iB+A)\right.}{d}$
derivativedivides	$\frac{a\left(\frac{(iA+B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-iB+A)\arctan(\tan(dx+c))-\frac{A}{3\tan(dx+c)^3}-\frac{iB-A}{\tan(dx+c)}+(-iA-B)\ln(\tan(dx+c))-\frac{}{2\tan}\right.}{d}$
default	$\frac{a\left(\frac{(iA+B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-iB+A)\arctan(\tan(dx+c))-\frac{A}{3\tan(dx+c)^3}-\frac{iB-A}{\tan(dx+c)}+(-iA-B)\ln(\tan(dx+c))-\frac{}{2\tan}\right.}{d}$
norman	$\frac{\frac{(-iaB+aA)\tan(dx+c)^2}{d}+(-iaB+aA)x\tan(dx+c)^3-\frac{aA}{3d}-\frac{(iaA+Ba)\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{(iaA+Ba)\ln(\tan(dx+c))}{d}+\frac{(iaA-}{}$
risch	$\frac{2iaBc}{d}-\frac{2aAc}{d}+\frac{2a\left(9iA\,e^{4i(dx+c)}+6B\,e^{4i(dx+c)}-9iA\,e^{2i(dx+c)}-9B\,e^{2i(dx+c)}+4iA+3B\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}-\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
(1/2*(I*A+B)*ln(sec(d*x+c)^2)-(I*A+B)*ln(tan(d*x+c))-1/3*A*cot(d*x+c)^3-1/
2*cot(d*x+c)^2*(I*A+B)+(A-I*B)*cot(d*x+c)+(A-I*B)*x*d)*a/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(77) = 154$.

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.87

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{6\,(-3i\,A-2\,B)ae^{(4i\,dx+4i\,c)}+18\,(i\,A+B)ae^{(2i\,dx+2i\,c)}+2\,(-4i\,A-3\,B)a+3\,\left((i\,A+B)ae^{(6i\,dx+6i\,c)}\right.}{3\,\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+\right.}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/3*(6*(-3*I*A - 2*B)*a*e^(4*I*d*x + 4*I*c) + 18*(I*A + B)*a*e^(2*I*d*x +
 2*I*c) + 2*(-4*I*A - 3*B)*a + 3*((I*A + B)*a*e^(6*I*d*x + 6*I*c) + 3*(-I*
A - B)*a*e^(4*I*d*x + 4*I*c) + 3*(I*A + B)*a*e^(2*I*d*x + 2*I*c) + (-I*A -
 B)*a)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d
*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(73) = 146$.

Time = 0.36 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.89

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{ia(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d} + \frac{8iAa+6Ba+\left(-18iAae^{2ic}-18Bae^{2ic}\right)e^{2idx}+\left(18iAae^{4ic}+12Bae^{4ic}\right)e^{4idx}}{3de^{6ic}e^{6idx}-9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}-3d}$$

input
```
integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
-I*a*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (8*I*A*a + 6*B*a + (-18
*I*A*a*exp(2*I*c) - 18*B*a*exp(2*I*c))*exp(2*I*d*x) + (18*I*A*a*exp(4*I*c)
 + 12*B*a*exp(4*I*c))*exp(4*I*d*x))/(3*d*exp(6*I*c)*exp(6*I*d*x) - 9*d*exp
(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) - 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{6\,(dx+c)(A-i\,B)a-3\,(-i\,A-B)a\log\left(\tan\left(dx+c\right)^2+1\right)+6\,(-i\,A-B)a\log\left(\tan\left(dx+c\right)\right)+\frac{6\,(A-}{}}{6\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/6*(6*(d*x + c)*(A - I*B)*a - 3*(-I*A - B)*a*log(tan(d*x + c)^2 + 1) + 6*
(-I*A - B)*a*log(tan(d*x + c)) + (6*(A - I*B)*a*tan(d*x + c)^2 + 3*(-I*A -
 B)*a*tan(d*x + c) - 2*A*a)/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{(iAa+Ba)\log(\tan(dx+c)+i)}{d}+\frac{(-iAa-Ba)\log(|\tan(dx+c)|)}{d}$$
$$+\frac{6(Aa-iBa)\tan(dx+c)^2-2Aa-3(iAa+Ba)\tan(dx+c)}{6d\tan(dx+c)^3}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
(I*A*a + B*a)*log(tan(d*x + c) + I)/d + (-I*A*a - B*a)*log(abs(tan(d*x + c
)))/d + 1/6*(6*(A*a - I*B*a)*tan(d*x + c)^2 - 2*A*a - 3*(I*A*a + B*a)*tan(
d*x + c))/(d*tan(d*x + c)^3)
```

Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=-\frac{(-A\,a+B\,a\,1\mathrm{i})\,\tan(c+d\,x)^2+\left(\frac{B\,a}{2}+\frac{A\,a\,1\mathrm{i}}{2}\right)\,\tan(c+d\,x)+\frac{A\,a}{3}}{d\tan\left(c+d\,x\right)^3}$$
$$-\frac{a\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\,(B+A\,1\mathrm{i})\,2\mathrm{i}}{d}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
- ((A*a)/3 + tan(c + d*x)*((A*a*1i)/2 + (B*a)/2) - tan(c + d*x)^2*(A*a - B
*a*1i))/(d*tan(c + d*x)^3) - (a*atan(2*tan(c + d*x) + 1i)*(A*1i + B)*2i)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.49

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{a\left(16\cos\left(dx+c\right)\sin\left(dx+c\right)^2 a - 12\cos\left(dx+c\right)\sin\left(dx+c\right)^2 bi - 4\cos\left(dx+c\right)a + 12\log\left(\tan\left(\frac{dx}{2}+\right.\right.\right.}{}$$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*(16*cos(c + d*x)*sin(c + d*x)**2*a - 12*cos(c + d*x)*sin(c + d*x)**2*b*
i - 4*cos(c + d*x)*a + 12*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*a*i
 + 12*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*b - 12*log(tan((c + d*x
)/2))*sin(c + d*x)**3*a*i - 12*log(tan((c + d*x)/2))*sin(c + d*x)**3*b + 1
2*sin(c + d*x)**3*a*d*x + 3*sin(c + d*x)**3*a*i - 12*sin(c + d*x)**3*b*d*i
*x + 3*sin(c + d*x)**3*b - 6*sin(c + d*x)*a*i - 6*sin(c + d*x)*b))/(12*sin
(c + d*x)**3*d)
```

3.8 $\int \cot^5(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 111

$$\begin{aligned}\int \cot^5(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx \\ = a(iA+B)x+\frac{a(iA+B)\cot(c+dx)}{d}+\frac{a(A-iB)\cot^2(c+dx)}{2d} \\ -\frac{a(iA+B)\cot^3(c+dx)}{3d}-\frac{aA\cot^4(c+dx)}{4d}+\frac{a(A-iB)\log(\sin(c+dx))}{d}\end{aligned}$$

output
```
a*(I*A+B)*x+a*(I*A+B)*cot(d*x+c)/d+1/2*a*(A-I*B)*cot(d*x+c)^2/d-1/3*a*(I*A
+B)*cot(d*x+c)^3/d-1/4*a*A*cot(d*x+c)^4/d+a*(A-I*B)*ln(sin(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{aA\csc^2(c+dx)}{d} - \frac{iaB\csc^2(c+dx)}{2d} - \frac{aA\csc^4(c+dx)}{4d} - \frac{iaA\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d} - \frac{aB\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d} + \frac{aA\log(\sin(c+dx))}{d} - \frac{iaB\log(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(a*A*Csc[c + d*x]^2)/d - ((I/2)*a*B*Csc[c + d*x]^2)/d - (a*A*Csc[c + d*x]^
4)/(4*d) - ((I/3)*a*A*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan
[c + d*x]^2])/d - (a*B*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Ta
n[c + d*x]^2])/(3*d) + (a*A*Log[Sin[c + d*x]])/d - (I*a*B*Log[Sin[c + d*x]
])/d
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {3042, 4074, 3042, 4012, 25, 3042, 4012, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{4074}$$

$$-\frac{aA\cot^4(c+dx)}{4d}+\int \cot^4(c+dx)(a(iA+B)-a(A-iB)\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{aA\cot^4(c+dx)}{4d}+\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4012}$$

$$\int -\cot^3(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot^3(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\tan(c+dx)^3}dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{4012}$$

$$-\int \cot^2(c+dx)(a(iA+B)-a(A-iB)\tan(c+dx))dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}+\frac{a(A-iB)\cot^2(c+dx)}{2d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}+\frac{a(A-iB)\cot^2(c+dx)}{2d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{4012}$$

$$-\int -\cot(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}+\frac{a(A-iB)\cot^2(c+dx)}{2d}+\frac{a(B+iA)\cot(c+dx)}{d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{25}$$

$$\int \cot(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}+\frac{a(A-iB)\cot^2(c+dx)}{2d}+\frac{a(B+iA)\cot(c+dx)}{d}-\frac{aA\cot^4(c+dx)}{4d}$$

$\Big\downarrow$ 3042

$$\int \frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}+\frac{a(A-iB)\cot^2(c+dx)}{2d}+\frac{a(B+iA)\cot(c+dx)}{d}-\frac{aA\cot^4(c+dx)}{4d}$$

$\Big\downarrow$ 4014

$$a(A-iB)\int \cot(c+dx)dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}+\frac{a(A-iB)\cot^2(c+dx)}{2d}+\frac{a(B+iA)\cot(c+dx)}{d}+ax(B+iA)-\frac{aA\cot^4(c+dx)}{4d}$$

$\Big\downarrow$ 3042

$$a(A-iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}+\frac{a(A-iB)\cot^2(c+dx)}{2d}+\frac{a(B+iA)\cot(c+dx)}{d}+ax(B+iA)-\frac{aA\cot^4(c+dx)}{4d}$$

$\Big\downarrow$ 25

$$-a(A-iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a(B+iA)\cot^3(c+dx)}{3d}+\frac{a(A-iB)\cot^2(c+dx)}{2d}+\frac{a(B+iA)\cot(c+dx)}{d}+ax(B+iA)-\frac{aA\cot^4(c+dx)}{4d}$$

$\Big\downarrow$ 3956

$$-\frac{a(B+iA)\cot^3(c+dx)}{3d}+\frac{a(A-iB)\cot^2(c+dx)}{2d}+\frac{a(B+iA)\cot(c+dx)}{d}+\frac{a(A-iB)\log(-\sin(c+dx))}{d}+ax(B+iA)-\frac{aA\cot^4(c+dx)}{4d}$$

input
```
Int[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
a*(I*A + B)*x + (a*(I*A + B)*Cot[c + d*x])/d + (a*(A - I*B)*Cot[c + d*x]^2
)/(2*d) - (a*(I*A + B)*Cot[c + d*x]^3)/(3*d) - (a*A*Cot[c + d*x]^4)/(4*d)
 + (a*(A - I*B)*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{\left(\frac{(iB-A)\ln\left(\sec(dx+c)^2\right)}{2}+(-iB+A)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^4}{4}-\frac{\cot(dx+c)^3(iA+B)}{3}+\frac{(-iB+A)\cot(dx+c)^2}{2}+\cot(dx+\right.}{d}$
derivativedivides	$\frac{a\left(\frac{(iB-A)\ln\left(1+\tan(dx+c)^2\right)}{2}+\arctan(\tan(dx+c))(iA+B)-\frac{A}{4\tan(dx+c)^4}-\frac{iB-A}{2\tan(dx+c)^2}+(-iB+A)\ln(\tan(dx+c))-\right.}{d}$
default	$\frac{a\left(\frac{(iB-A)\ln\left(1+\tan(dx+c)^2\right)}{2}+\arctan(\tan(dx+c))(iA+B)-\frac{A}{4\tan(dx+c)^4}-\frac{iB-A}{2\tan(dx+c)^2}+(-iB+A)\ln(\tan(dx+c))-\right.}{d}$
norman	$\frac{\frac{(iaA+Ba)\tan(dx+c)^3}{d}+(iaA+Ba)x\tan(dx+c)^4-\frac{aA}{4d}+\frac{(-iaB+aA)\tan(dx+c)^2}{2d}-\frac{(iaA+Ba)\tan(dx+c)}{3d}}{\tan(dx+c)^4}+\frac{(-iaB+aA)\ln}{d}$
risch	$-\frac{2aBc}{d}-\frac{2iaAc}{d}+\frac{2ia\left(12iA\,e^{6i(dx+c)}+9B\,e^{6i(dx+c)}-18iA\,e^{4i(dx+c)}-18B\,e^{4i(dx+c)}+16iA\,e^{2i(dx+c)}+13B\,e^{2i(d}\right.}{3d\left(e^{2i(dx+c)}-1\right)^4}$

input
```
int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
(1/2*(-A+I*B)*ln(sec(d*x+c)^2)+(A-I*B)*ln(tan(d*x+c))-1/4*A*cot(d*x+c)^4-1
/3*cot(d*x+c)^3*(I*A+B)+1/2*(A-I*B)*cot(d*x+c)^2+cot(d*x+c)*(I*A+B)+(I*A+B
)*x*d)*a/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(95) = 190.

Time = 0.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.86

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{6\,(4\,A-3i\,B)ae^{(6i\,dx+6i\,c)}-36\,(A-i\,B)ae^{(4i\,dx+4i\,c)}+2\,(16\,A-13i\,B)ae^{(2i\,dx+2i\,c)}-8\,(A-i\,B)a-3}{3\,(de^{(8i\,dx+8i\,c)}-}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/3*(6*(4*A - 3*I*B)*a*e^(6*I*d*x + 6*I*c) - 36*(A - I*B)*a*e^(4*I*d*x +
4*I*c) + 2*(16*A - 13*I*B)*a*e^(2*I*d*x + 2*I*c) - 8*(A - I*B)*a - 3*((A -
 I*B)*a*e^(8*I*d*x + 8*I*c) - 4*(A - I*B)*a*e^(6*I*d*x + 6*I*c) + 6*(A - I
*B)*a*e^(4*I*d*x + 4*I*c) - 4*(A - I*B)*a*e^(2*I*d*x + 2*I*c) + (A - I*B)*
a)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x +
 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(92) = 184$.

Time = 0.92 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.96

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{a(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d} + \frac{8Aa-8iBa+\left(-32Aae^{2ic}+26iBae^{2ic}\right)e^{2idx}+\left(36Aae^{4ic}-36iBae^{4ic}\right)e^{4idx}+\left(-24Aae^{6ic}+18iBae^{6ic}\right)e^{6idx}}{3de^{8ic}e^{8idx}-12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}-12de^{2ic}e^{2idx}+3d}$$

input
```
integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
a*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (8*A*a - 8*I*B*a + (-32*A*
a*exp(2*I*c) + 26*I*B*a*exp(2*I*c))*exp(2*I*d*x) + (36*A*a*exp(4*I*c) - 36
*I*B*a*exp(4*I*c))*exp(4*I*d*x) + (-24*A*a*exp(6*I*c) + 18*I*B*a*exp(6*I*c
))*exp(6*I*d*x))/(3*d*exp(8*I*c)*exp(8*I*d*x) - 12*d*exp(6*I*c)*exp(6*I*d*
x) + 18*d*exp(4*I*c)*exp(4*I*d*x) - 12*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{12\,(dx+c)(i\,A+B)a-6\,(A-i\,B)a\log\left(\tan\left(dx+c\right)^2+1\right)+12\,(A-i\,B)a\log\left(\tan\left(dx+c\right)\right)-\frac{12\,(-i\,\ldots}{\ldots}}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/12*(12*(d*x + c)*(I*A + B)*a - 6*(A - I*B)*a*log(tan(d*x + c)^2 + 1) + 1
2*(A - I*B)*a*log(tan(d*x + c)) - (12*(-I*A - B)*a*tan(d*x + c)^3 - 6*(A -
 I*B)*a*tan(d*x + c)^2 + 4*(I*A + B)*a*tan(d*x + c) + 3*A*a)/tan(d*x + c)^
4)/d
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.03

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{(Aa-i\,Ba)\log(\tan(dx+c)+i)}{d}+\frac{(Aa-i\,Ba)\log(|\tan(dx+c)|)}{d} - \frac{12\,(-i\,Aa-Ba)\tan(dx+c)^3-6\,(Aa-i\,Ba)\tan(dx+c)^2+3\,Aa+4\,(i\,Aa+Ba)\tan(dx+c)}{12\,d\tan(dx+c)^4}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-(A*a - I*B*a)*log(tan(d*x + c) + I)/d + (A*a - I*B*a)*log(abs(tan(d*x + c
)))/d - 1/12*(12*(-I*A*a - B*a)*tan(d*x + c)^3 - 6*(A*a - I*B*a)*tan(d*x +
 c)^2 + 3*A*a + 4*(I*A*a + B*a)*tan(d*x + c))/(d*tan(d*x + c)^4)
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{(-B\,a-A\,a\,1\mathrm{i})\,\tan(c+d\,x)^3+\left(-\frac{A\,a}{2}+\frac{B\,a\,1\mathrm{i}}{2}\right)\,\tan(c+d\,x)^2+\left(\frac{B\,a}{3}+\frac{A\,a\,1\mathrm{i}}{3}\right)\,\tan(c+d\,x)+\frac{A\,a}{4}}{d\tan(c+d\,x)^4} + \frac{a\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\,(A-B\,1\mathrm{i})\,2\mathrm{i}}{d}$$

input
```
int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
(a*atan(2*tan(c + d*x) + 1i)*(A - B*1i)*2i)/d - ((A*a)/4 + tan(c + d*x)*((
A*a*1i)/3 + (B*a)/3) - tan(c + d*x)^3*(A*a*1i + B*a) - tan(c + d*x)^2*((A*
a)/2 - (B*a*1i)/2))/(d*tan(c + d*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.26

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{a\Big(128\cos(dx+c)\sin(dx+c)^3\,ai+128\cos(dx+c)\sin(dx+c)^3\,b-32\cos(dx+c)\sin(dx+c)\,ai-3}{}$$

input
```
int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*(128*cos(c + d*x)*sin(c + d*x)**3*a*i + 128*cos(c + d*x)*sin(c + d*x)**
3*b - 32*cos(c + d*x)*sin(c + d*x)*a*i - 32*cos(c + d*x)*sin(c + d*x)*b -
96*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**4*a + 96*log(tan((c + d*x)/2
)**2 + 1)*sin(c + d*x)**4*b*i + 96*log(tan((c + d*x)/2))*sin(c + d*x)**4*a
 - 96*log(tan((c + d*x)/2))*sin(c + d*x)**4*b*i + 96*sin(c + d*x)**4*a*d*i
*x - 39*sin(c + d*x)**4*a + 96*sin(c + d*x)**4*b*d*x + 24*sin(c + d*x)**4*
b*i + 96*sin(c + d*x)**2*a - 48*sin(c + d*x)**2*b*i - 24*a))/(96*sin(c + d
*x)**4*d)
```

3.9 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 141

$$\begin{aligned}\int \tan^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx &= -2a^2(A-iB)x+\frac{2a^2(iA+B)\log(\cos(c+dx))}{d} \\ &\quad +\frac{2a^2(A-iB)\tan(c+dx)}{d}+\frac{a^2(iA+B)\tan^2(c+dx)}{d} \\ &\quad -\frac{a^2(4A-5iB)\tan^3(c+dx)}{12d}+\frac{iB\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\end{aligned}$$

output
```
-2*a^2*(A-I*B)*x+2*a^2*(I*A+B)*ln(cos(d*x+c))/d+2*a^2*(A-I*B)*tan(d*x+c)/d
+a^2*(I*A+B)*tan(d*x+c)^2/d-1/12*a^2*(4*A-5*I*B)*tan(d*x+c)^3/d+1/4*I*B*ta
n(d*x+c)^3*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2(-4iA-5B-24i(A-iB)\log(i+\tan(c+dx))+24(A-iB)\tan(c+dx)+12(iA+B)\tan^2(c+d}{12d}$$

input
```
Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(a^2*((-4*I)*A - 5*B - (24*I)*(A - I*B)*Log[I + Tan[c + d*x]] + 24*(A - I*
B)*Tan[c + d*x] + 12*(I*A + B)*Tan[c + d*x]^2 - 4*(A - (2*I)*B)*Tan[c + d*
x]^3 - 3*B*Tan[c + d*x]^4))/(12*d)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4077, 3042, 4075, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \tan(c+dx)^2(a+ia\tan(c+dx))^2(A+B\tan(c+dx))dx$$
$$\Big\downarrow \; 4077$$
$$\frac{1}{4}\int \tan^2(c+dx)(i\tan(c+dx)a+a)(a(4A-3iB)+a(4iA+5B)\tan(c+dx))dx+\frac{iB\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$
$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int \tan(c+dx)^2(i\tan(c+dx)a+a)(a(4A-3iB)+a(4iA+5B)\tan(c+dx))dx+\frac{iB\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\big\downarrow$ 4075

$$\frac{1}{4}\left(\int \tan^2(c+dx)\left(8(A-iB)a^2+8(iA+B)\tan(c+dx)a^2\right)dx-\frac{a^2(4A-5iB)\tan^3(c+dx)}{3d}\right)+\frac{iB\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\int \tan(c+dx)^2\left(8(A-iB)a^2+8(iA+B)\tan(c+dx)a^2\right)dx-\frac{a^2(4A-5iB)\tan^3(c+dx)}{3d}\right)+\frac{iB\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\big\downarrow$ 4011

$$\frac{1}{4}\left(\int \tan(c+dx)\left(8a^2(A-iB)\tan(c+dx)-8a^2(iA+B)\right)dx-\frac{a^2(4A-5iB)\tan^3(c+dx)}{3d}+\frac{4a^2(B+iA)\text{ta}}{d}\right.$$
$$\frac{iB\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\int \tan(c+dx)\left(8a^2(A-iB)\tan(c+dx)-8a^2(iA+B)\right)dx-\frac{a^2(4A-5iB)\tan^3(c+dx)}{3d}+\frac{4a^2(B+iA)\text{ta}}{d}\right.$$
$$\frac{iB\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\big\downarrow$ 4008

$$\frac{1}{4}\left(-8a^2(B+iA)\int \tan(c+dx)dx-\frac{a^2(4A-5iB)\tan^3(c+dx)}{3d}+\frac{4a^2(B+iA)\tan^2(c+dx)}{d}+\frac{8a^2(A-iB)\text{t}}{d}\right.$$
$$\frac{iB\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(-8a^2(B+iA)\int \tan(c+dx)dx-\frac{a^2(4A-5iB)\tan^3(c+dx)}{3d}+\frac{4a^2(B+iA)\tan^2(c+dx)}{d}+\frac{8a^2(A-iB)\,\mathrm{t}}{d}\right.$$
$$\frac{iB\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\Big\downarrow$ 3956

$$\frac{1}{4}\left(-\frac{a^2(4A-5iB)\tan^3(c+dx)}{3d}+\frac{4a^2(B+iA)\tan^2(c+dx)}{d}+\frac{8a^2(A-iB)\tan(c+dx)}{d}+\frac{8a^2(B+iA)\log(\mathrm{co}}{d}\right.$$
$$\frac{iB\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

input
```
Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
((I/4)*B*Tan[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x]))/d + (-8*a^2*(A - I*B)*
x + (8*a^2*(I*A + B)*Log[Cos[c + d*x]])/d + (8*a^2*(A - I*B)*Tan[c + d*x])
/d + (4*a^2*(I*A + B)*Tan[c + d*x]^2)/d - (a^2*(4*A - (5*I)*B)*Tan[c + d*x
]^3)/(3*d))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a^2\left(\frac{2iB\tan(dx+c)^3}{3}-\frac{B\tan(dx+c)^4}{4}+iA\tan(dx+c)^2-\frac{A\tan(dx+c)^3}{3}-2iB\tan(dx+c)+B\tan(dx+c)^2+2A\tan(dx+c)+\frac{(}{}\right.}{d}$
default	$\frac{a^2\left(\frac{2iB\tan(dx+c)^3}{3}-\frac{B\tan(dx+c)^4}{4}+iA\tan(dx+c)^2-\frac{A\tan(dx+c)^3}{3}-2iB\tan(dx+c)+B\tan(dx+c)^2+2A\tan(dx+c)+\frac{(}{}\right.}{d}$
norman	$(2iB\,a^2-2A\,a^2)\,x+\frac{(iA\,a^2+B\,a^2)\tan(dx+c)^2}{d}-\frac{(-2iB\,a^2+A\,a^2)\tan(dx+c)^3}{3d}+\frac{2(-iB\,a^2+A\,a^2)\tan(c}{d}$
parts	$\frac{(2iA\,a^2+B\,a^2)\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{(2iB\,a^2-A\,a^2)\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}$
parallelrisch	$-\frac{-8iB\tan(dx+c)^3a^2+3B\tan(dx+c)^4a^2-12iA\tan(dx+c)^2a^2+4A\tan(dx+c)^3a^2-24iBx\,a^2d+12iA\ln\left(1+\tan(dx+\right.}{12d}$
risch	$-\frac{4ia^2Bc}{d}+\frac{4a^2Ac}{d}+\frac{2a^2\left(15iA\,e^{6i(dx+c)}+21B\,e^{6i(dx+c)}+33iA\,e^{4i(dx+c)}+36B\,e^{4i(dx+c)}+25iA\,e^{2i(dx+c)}+29B\,e^{}\right.}{3d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*a^2*(2/3*I*B*tan(d*x+c)^3-1/4*B*tan(d*x+c)^4+I*A*tan(d*x+c)^2-1/3*A*ta
n(d*x+c)^3-2*I*B*tan(d*x+c)+B*tan(d*x+c)^2+2*A*tan(d*x+c)+1/2*(-2*B-2*I*A)
*ln(1+tan(d*x+c)^2)+(2*I*B-2*A)*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.67

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(3\,(-5i\,A-7\,B)a^2e^{(6i\,dx+6i\,c)}+3\,(-11i\,A-12\,B)a^2e^{(4i\,dx+4i\,c)}+(-25i\,A-29\,B)a^2e^{(2i\,dx+2i\,c)}+(-\right.}{}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-2/3*(3*(-5*I*A - 7*B)*a^2*e^(6*I*d*x + 6*I*c) + 3*(-11*I*A - 12*B)*a^2*e^
(4*I*d*x + 4*I*c) + (-25*I*A - 29*B)*a^2*e^(2*I*d*x + 2*I*c) + (-7*I*A - 8
*B)*a^2 + 3*((-I*A - B)*a^2*e^(8*I*d*x + 8*I*c) + 4*(-I*A - B)*a^2*e^(6*I*
d*x + 6*I*c) + 6*(-I*A - B)*a^2*e^(4*I*d*x + 4*I*c) + 4*(-I*A - B)*a^2*e^(
2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I
*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^
(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.67

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{2ia^2(A-iB)\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$
$$+ \frac{14iAa^2+16Ba^2+\left(50iAa^2e^{2ic}+58Ba^2e^{2ic}\right)e^{2idx}+\left(66iAa^2e^{4ic}+72Ba^2e^{4ic}\right)e^{4idx}+\left(30iAa^2e^{6ic}+42\right.}{3de^{8ic}e^{8idx}+12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}+12de^{2ic}e^{2idx}+3d}$$

input
```
integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
2*I*a**2*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (14*I*A*a**2 + 16*B
*a**2 + (50*I*A*a**2*exp(2*I*c) + 58*B*a**2*exp(2*I*c))*exp(2*I*d*x) + (66
*I*A*a**2*exp(4*I*c) + 72*B*a**2*exp(4*I*c))*exp(4*I*d*x) + (30*I*A*a**2*e
xp(6*I*c) + 42*B*a**2*exp(6*I*c))*exp(6*I*d*x))/(3*d*exp(8*I*c)*exp(8*I*d*
x) + 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d*x) + 12*d*ex
p(2*I*c)*exp(2*I*d*x) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{3\,Ba^2\tan\left(dx+c\right)^4+4\left(A-2i\,B\right)a^2\tan\left(dx+c\right)^3+12\left(-i\,A-B\right)a^2\tan\left(dx+c\right)^2+24\left(dx+c\right)(A-}{12\,d}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/12*(3*B*a^2*tan(d*x + c)^4 + 4*(A - 2*I*B)*a^2*tan(d*x + c)^3 + 12*(-I*
A - B)*a^2*tan(d*x + c)^2 + 24*(d*x + c)*(A - I*B)*a^2 - 12*(-I*A - B)*a^2
*log(tan(d*x + c)^2 + 1) - 24*(A - I*B)*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{2\,(i\,Aa^2+Ba^2)\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{3\,Ba^2d^3\tan\left(dx+c\right)^4+4\,Aa^2d^3\tan\left(dx+c\right)^3-8i\,Ba^2d^3\tan\left(dx+c\right)^3-12i\,Aa^2d^3\tan\left(dx+c\right)^2-12}{12\,d^4}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-2*(I*A*a^2 + B*a^2)*log(tan(d*x + c) + I)/d - 1/12*(3*B*a^2*d^3*tan(d*x +
 c)^4 + 4*A*a^2*d^3*tan(d*x + c)^3 - 8*I*B*a^2*d^3*tan(d*x + c)^3 - 12*I*A
*a^2*d^3*tan(d*x + c)^2 - 12*B*a^2*d^3*tan(d*x + c)^2 - 24*A*a^2*d^3*tan(d
*x + c) + 24*I*B*a^2*d^3*tan(d*x + c))/d^4
```

Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)^3\left(\frac{a^2\,(B+A\,1\mathrm{i})\,1\mathrm{i}}{3}+\frac{B\,a^2\,1\mathrm{i}}{3}\right)}{d} - \frac{\tan(c+d\,x)\,\left(-A\,a^2+a^2\,(B+A\,1\mathrm{i})\,1\mathrm{i}+B\,a^2\,1\mathrm{i}\right)}{d} + \frac{\tan(c+d\,x)^2\left(\frac{a^2\,(B+A\,1\mathrm{i})}{2}+\frac{B\,a^2}{2}+\frac{A\,a^2\,1\mathrm{i}}{2}\right)}{d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(2\,B\,a^2+A\,a^2\,2\mathrm{i}\right)}{d} - \frac{B\,a^2\tan(c+d\,x)^4}{4\,d}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(tan(c + d*x)^3*((a^2*(A*1i + B)*1i)/3 + (B*a^2*1i)/3))/d - (tan(c + d*x)*
(a^2*(A*1i + B)*1i - A*a^2 + B*a^2*1i))/d + (tan(c + d*x)^2*((A*a^2*1i)/2
+ (a^2*(A*1i + B))/2 + (B*a^2)/2))/d - (log(tan(c + d*x) + 1i)*(A*a^2*2i +
 2*B*a^2))/d - (B*a^2*tan(c + d*x)^4)/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\left(-12\log\left(\tan\left(dx+c\right)^2+1\right)ai-12\log\left(\tan\left(dx+c\right)^2+1\right)b-3\tan\left(dx+c\right)^4b-4\tan\left(dx+c\right)^3a+\right.}{}$$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*( - 12*log(tan(c + d*x)**2 + 1)*a*i - 12*log(tan(c + d*x)**2 + 1)*b
- 3*tan(c + d*x)**4*b - 4*tan(c + d*x)**3*a + 8*tan(c + d*x)**3*b*i + 12*t
an(c + d*x)**2*a*i + 12*tan(c + d*x)**2*b + 24*tan(c + d*x)*a - 24*tan(c +
 d*x)*b*i - 24*a*d*x + 24*b*d*i*x))/(12*d)
```

3.10 $\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 107

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -2a^2(iA+B)x-\frac{2a^2(A-iB)\log(\cos(c+dx))}{d}+\frac{a^2(iA+B)\tan(c+dx)}{d}+\frac{A(a+ia\tan(c+dx))^2}{2d}-\frac{iB(a+ia\tan(c+dx))^3}{3ad}$$

output
```
-2*a^2*(I*A+B)*x-2*a^2*(A-I*B)*ln(cos(d*x+c))/d+a^2*(I*A+B)*tan(d*x+c)/d+1
/2*A*(a+I*a*tan(d*x+c))^2/d-1/3*I*B*(a+I*a*tan(d*x+c))^3/a/d
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{a^2(3A-2iB+12(A-iB)\log(i+\tan(c+dx))+12(iA+B)\tan(c+dx)-3(A-2iB)\tan^2(c+dx)}{6d}$$

input
```
Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(a^2*(3*A - (2*I)*B + 12*(A - I*B)*Log[I + Tan[c + d*x]] + 12*(I*A + B)*Ta
n[c + d*x] - 3*(A - (2*I)*B)*Tan[c + d*x]^2 - 2*B*Tan[c + d*x]^3))/(6*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4075, 3042, 4010, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int (i\tan(c+dx)a+a)^2(A\tan(c+dx)-B)dx-\frac{iB(a+ia\tan(c+dx))^3}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\int (i\tan(c+dx)a+a)^2(A\tan(c+dx)-B)dx-\frac{iB(a+ia\tan(c+dx))^3}{3ad}$$

$$\Big\downarrow \text{4010}$$

$$-(B+iA)\int (i\tan(c+dx)a+a)^2dx+\frac{A(a+ia\tan(c+dx))^2}{2d}-\frac{iB(a+ia\tan(c+dx))^3}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\int (i\tan(c+dx)a+a)^2dx+\frac{A(a+ia\tan(c+dx))^2}{2d}-\frac{iB(a+ia\tan(c+dx))^3}{3ad}$$

$$\Big\downarrow \text{3958}$$

$$-(B+iA)\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{A(a+ia\tan(c+dx))^2}{2d}-\frac{iB(a+ia\tan(c+dx))^3}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{A(a+ia\tan(c+dx))^2}{2d}-\frac{iB(a+ia\tan(c+dx))^3}{3ad}$$

$$\Big\downarrow \text{3956}$$

$$-(B+iA)\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{A(a+ia\tan(c+dx))^2}{2d}-\frac{iB(a+ia\tan(c+dx))^3}{3ad}$$

input
```
Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(A*(a + I*a*Tan[c + d*x])^2)/(2*d) - ((I/3)*B*(a + I*a*Tan[c + d*x])^3)/(a
*d) - (I*A + B)*(2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c +
d*x])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{a^2\left(iB\tan(dx+c)^2-\frac{B\tan(dx+c)^3}{3}+2iA\tan(dx+c)-\frac{A\tan(dx+c)^2}{2}+2B\tan(dx+c)+\frac{(-2iB+2A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-2\right.}{d}$
default	$\frac{a^2\left(iB\tan(dx+c)^2-\frac{B\tan(dx+c)^3}{3}+2iA\tan(dx+c)-\frac{A\tan(dx+c)^2}{2}+2B\tan(dx+c)+\frac{(-2iB+2A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-2\right.}{d}$
norman	$(-2iA\,a^2-2B\,a^2)\,x-\frac{(-2iB\,a^2+A\,a^2)\tan(dx+c)^2}{2d}+\frac{2(iA\,a^2+B\,a^2)\tan(dx+c)}{d}-\frac{B\,a^2\tan(dx+c)^3}{3d}+$
parallelrisch	$-\frac{12iAx\,a^2d-6iB\tan(dx+c)^2a^2+2B\tan(dx+c)^3a^2-12iA\tan(dx+c)a^2+3A\tan(dx+c)^2a^2+6iB\ln\left(1+\tan(dx+c)^2\right)}{6d}$
parts	$\frac{(2iA\,a^2+B\,a^2)(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{(2iB\,a^2-A\,a^2)\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{A\ln\left(1+\mathrm{ta}\right.}{}$
risch	$\frac{4a^2Bc}{d}+\frac{4ia^2Ac}{d}+\frac{2ia^2\left(9iA\,e^{4i(dx+c)}+15B\,e^{4i(dx+c)}+15iA\,e^{2i(dx+c)}+18B\,e^{2i(dx+c)}+6iA+7B\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{2ia^2\ln(e^{2i}}{}$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*a^2*(I*B*tan(d*x+c)^2-1/3*B*tan(d*x+c)^3+2*I*A*tan(d*x+c)-1/2*A*tan(d*
x+c)^2+2*B*tan(d*x+c)+1/2*(-2*I*B+2*A)*ln(1+tan(d*x+c)^2)+(-2*B-2*I*A)*arc
tan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.64

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(3\left(3A-5iB\right)a^2e^{(4i\,dx+4i\,c)}+3\left(5A-6iB\right)a^2e^{(2i\,dx+2i\,c)}+(6A-7iB)a^2+3\left((A-iB)a^2e^{(6i\,dx+6i}\right.\right.}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}\right.}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-2/3*(3*(3*A - 5*I*B)*a^2*e^(4*I*d*x + 4*I*c) + 3*(5*A - 6*I*B)*a^2*e^(2*I
*d*x + 2*I*c) + (6*A - 7*I*B)*a^2 + 3*((A - I*B)*a^2*e^(6*I*d*x + 6*I*c) +
 3*(A - I*B)*a^2*e^(4*I*d*x + 4*I*c) + 3*(A - I*B)*a^2*e^(2*I*d*x + 2*I*c)
 + (A - I*B)*a^2)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*I*c) + 3
*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(88) = 176.

Time = 0.36 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.66

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{2a^2(A-iB)\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$
$$+\frac{-12Aa^2+14iBa^2+\left(-30Aa^2e^{2ic}+36iBa^2e^{2ic}\right)e^{2idx}+\left(-18Aa^2e^{4ic}+30iBa^2e^{4ic}\right)e^{4idx}}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
-2*a**2*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-12*A*a**2 + 14*I*B
*a**2 + (-30*A*a**2*exp(2*I*c) + 36*I*B*a**2*exp(2*I*c))*exp(2*I*d*x) + (-
18*A*a**2*exp(4*I*c) + 30*I*B*a**2*exp(4*I*c))*exp(4*I*d*x))/(3*d*exp(6*I*
c)*exp(6*I*d*x) + 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x
) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{2\,Ba^2\tan\left(dx+c\right)^3+3\,(A-2i\,B)a^2\tan\left(dx+c\right)^2+12\,(dx+c)(i\,A+B)a^2-6\,(A-i\,B)a^2\log\left(\tan\right.}{6\,d}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/6*(2*B*a^2*tan(d*x + c)^3 + 3*(A - 2*I*B)*a^2*tan(d*x + c)^2 + 12*(d*x
+ c)*(I*A + B)*a^2 - 6*(A - I*B)*a^2*log(tan(d*x + c)^2 + 1) + 12*(-I*A -
B)*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{2\,(Aa^2-i\,Ba^2)\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{2\,Ba^2d^2\tan\left(dx+c\right)^3+3\,Aa^2d^2\tan\left(dx+c\right)^2-6i\,Ba^2d^2\tan\left(dx+c\right)^2-12i\,Aa^2d^2\tan\left(dx+c\right)-12}{6\,d^3}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
2*(A*a^2 - I*B*a^2)*log(tan(d*x + c) + I)/d - 1/6*(2*B*a^2*d^2*tan(d*x + c
)^3 + 3*A*a^2*d^2*tan(d*x + c)^2 - 6*I*B*a^2*d^2*tan(d*x + c)^2 - 12*I*A*a
^2*d^2*tan(d*x + c) - 12*B*a^2*d^2*tan(d*x + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)^2\,\left(\frac{a^2\,(B+A\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{B\,a^2\,1\mathrm{i}}{2}\right)}{d}+\frac{\tan(c+d\,x)\,\left(a^2\,(B+A\,1\mathrm{i})+B\,a^2+A\,a^2\,1\mathrm{i}\right)}{d}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(2\,A\,a^2-B\,a^2\,2\mathrm{i}\right)}{d}-\frac{B\,a^2\,\tan(c+d\,x)^3}{3\,d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(tan(c + d*x)^2*((a^2*(A*1i + B)*1i)/2 + (B*a^2*1i)/2))/d + (tan(c + d*x)*
(A*a^2*1i + a^2*(A*1i + B) + B*a^2))/d + (log(tan(c + d*x) + 1i)*(2*A*a^2
- B*a^2*2i))/d - (B*a^2*tan(c + d*x)^3)/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \tan(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\left(6\log\left(\tan\left(dx+c\right)^2+1\right)a-6\log\left(\tan\left(dx+c\right)^2+1\right)bi-2\tan\left(dx+c\right)^3b-3\tan\left(dx+c\right)^2a+6\tan\right.}{6d}$$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*(6*log(tan(c + d*x)**2 + 1)*a - 6*log(tan(c + d*x)**2 + 1)*b*i - 2*t
an(c + d*x)**3*b - 3*tan(c + d*x)**2*a + 6*tan(c + d*x)**2*b*i + 12*tan(c
+ d*x)*a*i + 12*tan(c + d*x)*b - 12*a*d*i*x - 12*b*d*x))/(6*d)
```

3.11 $\int (a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int (a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = 2a^2(A-iB)x-\frac{2a^2(iA+B)\log(\cos(c+dx))}{d} - \frac{a^2(A-iB)\tan(c+dx)}{d}+\frac{B(a+ia\tan(c+dx))^2}{2d}$$

output
```
2*a^2*(A-I*B)*x-2*a^2*(I*A+B)*ln(cos(d*x+c))/d-a^2*(A-I*B)*tan(d*x+c)/d+1/
2*B*(a+I*a*tan(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int (a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{a^2(B+4(iA+B)\log(i+\tan(c+dx))-2(A-2iB)\tan(c+dx)-B\tan^2(c+dx))}{2d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(a^2*(B + 4*(I*A + B)*Log[I + Tan[c + d*x]] - 2*(A - (2*I)*B)*Tan[c + d*x]
 - B*Tan[c + d*x]^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4010, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4010}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^2dx+\frac{B(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^2dx+\frac{B(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3958}$$

$$(A-iB)\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{B(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{B(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3956}$$

$$(A-iB)\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{B(a+ia\tan(c+dx))^2}{2d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(B*(a + I*a*Tan[c + d*x])^2)/(2*d) + (A - I*B)*(2*a^2*x - ((2*I)*a^2*Log[C
os[c + d*x]])/d - (a^2*Tan[c + d*x])/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{B\tan(dx+c)^{2}}{2}-A\tan(dx+c)+2iB\tan(dx+c)+\frac{(2iA+2B)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-2iB+2A)\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a^{2}\left(-\frac{B\tan(dx+c)^{2}}{2}-A\tan(dx+c)+2iB\tan(dx+c)+\frac{(2iA+2B)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-2iB+2A)\arctan(\tan(dx+c))\right)}{d}$
norman	$(-2iB\,a^{2}+2A\,a^{2})\,x-\frac{(-2iB\,a^{2}+A\,a^{2})\tan(dx+c)}{d}-\frac{B\,a^{2}\tan(dx+c)^{2}}{2d}+\frac{(iA\,a^{2}+B\,a^{2})\ln\left(1+\tan(dx+c\right.}{d}$
parallelrisch	$\frac{-4iBx\,a^{2}d+2iA\ln\left(1+\tan(dx+c)^{2}\right)a^{2}+4Ax\,a^{2}d+4iB\tan(dx+c)a^{2}-B\tan(dx+c)^{2}a^{2}-2A\tan(dx+c)a^{2}+2B\ln\left(1+\mathrm{t}\right.}{2d}$
parts	$A\,a^{2}x+\frac{(2iA\,a^{2}+B\,a^{2})\ln\left(1+\tan(dx+c)^{2}\right)}{2d}+\frac{(2iB\,a^{2}-A\,a^{2})(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}-\frac{B\,a^{2}\left(\frac{\tan(d}{2}\right.}{}$
risch	$\frac{4ia^{2}Bc}{d}-\frac{4a^{2}Ac}{d}-\frac{2a^{2}\left(iA\,\mathrm{e}^{2i(dx+c)}+3B\,\mathrm{e}^{2i(dx+c)}+iA+2B\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}-\frac{2a^{2}\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)B}{d}-\frac{2ia^{2}\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*a^2*(-1/2*B*tan(d*x+c)^2-A*tan(d*x+c)+2*I*B*tan(d*x+c)+1/2*(2*B+2*I*A)
*ln(1+tan(d*x+c)^2)+(-2*I*B+2*A)*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.51

$$\int (a+ia\tan(c+dx))^{2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(\left(i\,A+3\,B\right)a^{2}e^{(2i\,dx+2i\,c)}+\left(i\,A+2\,B\right)a^{2}+\left(\left(i\,A+B\right)a^{2}e^{(4i\,dx+4i\,c)}+2\left(i\,A+B\right)a^{2}e^{(2i\,dx+2i\,c)}+(i\,A\right.\right.}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2*((I*A + 3*B)*a^2*e^(2*I*d*x + 2*I*c) + (I*A + 2*B)*a^2 + ((I*A + B)*a^2
*e^(4*I*d*x + 4*I*c) + 2*(I*A + B)*a^2*e^(2*I*d*x + 2*I*c) + (I*A + B)*a^2
)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x +
2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.52

$$\int (a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{2ia^2(A-iB)\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-2iAa^2-4Ba^2+\left(-2iAa^2e^{2ic}-6Ba^2e^{2ic}\right)e^{2idx}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$

input
```
integrate((a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
-2*I*a**2*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-2*I*A*a**2 - 4*B
*a**2 + (-2*I*A*a**2*exp(2*I*c) - 6*B*a**2*exp(2*I*c))*exp(2*I*d*x))/(d*ex
p(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int (a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{Ba^2\tan\left(dx+c\right)^2-4\left(dx+c\right)(A-i\,B)a^2-2\left(i\,A+B\right)a^2\log\left(\tan\left(dx+c\right)^2+1\right)+2\left(A-2i\,B\right)a^2\,\text{t}}{2\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(B*a^2*tan(d*x + c)^2 - 4*(d*x + c)*(A - I*B)*a^2 - 2*(I*A + B)*a^2*l
og(tan(d*x + c)^2 + 1) + 2*(A - 2*I*B)*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int (a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{2\left(-i\,Aa^2 - Ba^2\right)\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{Ba^2d\tan\left(dx+c\right)^2 + 2\,Aa^2d\tan\left(dx+c\right) - 4i\,Ba^2d\tan\left(dx+c\right)}{2\,d^2}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
-2*(-I*A*a^2 - B*a^2)*log(tan(d*x + c) + I)/d - 1/2*(B*a^2*d*tan(d*x + c)^
2 + 2*A*a^2*d*tan(d*x + c) - 4*I*B*a^2*d*tan(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int (a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(2\,B\,a^2+A\,a^2\,2\mathrm{i}\right)}{d} + \frac{\tan(c+d\,x)\,\left(a^2\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}+B\,a^2\,1\mathrm{i}\right)}{d} - \frac{B\,a^2\tan(c+d\,x)^2}{2\,d}$$

input
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(log(tan(c + d*x) + 1i)*(A*a^2*2i + 2*B*a^2))/d + (tan(c + d*x)*(a^2*(A*1i
 + B)*1i + B*a^2*1i))/d - (B*a^2*tan(c + d*x)^2)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int (a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{a^2\left(2\log\left(\tan\left(dx+c\right)^2+1\right)ai+2\log\left(\tan\left(dx+c\right)^2+1\right)b-\tan\left(dx+c\right)^2b-2\tan\left(dx+c\right)a+4\tan\left(\right.\right.}{2d}$$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*(2*log(tan(c + d*x)**2 + 1)*a*i + 2*log(tan(c + d*x)**2 + 1)*b - tan
(c + d*x)**2*b - 2*tan(c + d*x)*a + 4*tan(c + d*x)*b*i + 4*a*d*x - 4*b*d*i
*x))/(2*d)
```

3.12 $\int \cot(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 75

$$\int \cot(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = 2a^2(iA+B)x+\frac{a^2(A-2iB)\log(\cos(c+dx))}{d} + \frac{a^2A\log(\sin(c+dx))}{d}+\frac{iB(a^2+ia^2\tan(c+dx))}{d}$$

output
```
2*a^2*(I*A+B)*x+a^2*(A-2*I*B)*ln(cos(d*x+c))/d+a^2*A*ln(sin(d*x+c))/d+I*B*
(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \cot(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{a^2(A\log(\tan(c+dx))-2(A-iB)\log(i+\tan(c+dx))-B\tan(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(a^2*(A*Log[Tan[c + d*x]] - 2*(A - I*B)*Log[I + Tan[c + d*x]] - B*Tan[c +
d*x]))/d
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4077, 3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4077}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)(aA+a(iA+2B)\tan(c+dx))dx+\frac{iB\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)(aA+a(iA+2B)\tan(c+dx))}{\tan(c+dx)}dx+\frac{iB\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4072}$$

$$-\left(a^2(A-2iB)\int \tan(c+dx)dx\right)+\int \cot(c+dx)\left(Aa^2+2(iA+B)\tan(c+dx)a^2\right)dx+\frac{iB\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\left(a^2(A-2iB)\int \tan(c+dx)dx\right)+\int \frac{Aa^2+2(iA+B)\tan(c+dx)a^2}{\tan(c+dx)}dx+\frac{iB\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3956}$$

$$\int \frac{Aa^2+2(iA+B)\tan(c+dx)a^2}{\tan(c+dx)}dx+\frac{a^2(A-2iB)\log(\cos(c+dx))}{d}+\frac{iB\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{4014}$$

$$a^2A\int \cot(c+dx)dx+\frac{a^2(A-2iB)\log(\cos(c+dx))}{d}+2a^2x(B+iA)+\frac{iB\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a^2A\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^2(A-2iB)\log(\cos(c+dx))}{d}+2a^2x(B+iA)+\frac{iB\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$-a^2A\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{a^2(A-2iB)\log(\cos(c+dx))}{d}+2a^2x(B+iA)+\frac{iB\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a^2(A-2iB)\log(\cos(c+dx))}{d}+2a^2x(B+iA)+\frac{a^2A\log(-\sin(c+dx))}{d}+\frac{iB\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

input
```
Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
2*a^2*(I*A + B)*x + (a^2*(A - (2*I)*B)*Log[Cos[c + d*x]])/d + (a^2*A*Log[-
Sin[c + d*x]])/d + (I*B*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

method	result
parallelrisch	$\frac{a^{2}\left(iB\ln\left(\sec(dx+c)^{2}\right)+A\left(\ln(\tan(dx+c))-\ln\left(\sec(dx+c)^{2}\right)\right)+2iAxd+2Bdx-B\tan(dx+c)\right)}{d}$
derivativedivides	$\frac{a^{2}\left(\frac{(2iB-2A)\ln\left(\cot(dx+c)^{2}+1\right)}{2}+(-2iA-2B)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)-\frac{B}{\cot(dx+c)}+(-2iB+A)\ln(\cot(dx+c))\right)}{d}$
default	$\frac{a^{2}\left(\frac{(2iB-2A)\ln\left(\cot(dx+c)^{2}+1\right)}{2}+(-2iA-2B)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)-\frac{B}{\cot(dx+c)}+(-2iB+A)\ln(\cot(dx+c))\right)}{d}$
norman	$(2iA\,a^{2}+2B\,a^{2})\,x-\frac{B\,a^{2}\tan(dx+c)}{d}+\frac{A\,a^{2}\ln(\tan(dx+c))}{d}-\frac{(-iB\,a^{2}+A\,a^{2})\ln\left(1+\tan(dx+c)^{2}\right)}{d}$
risch	$-\frac{4ia^{2}Ac}{d}-\frac{4a^{2}Bc}{d}-\frac{2iB\,a^{2}}{d\left(e^{2i(dx+c)}+1\right)}+\frac{A\,a^{2}\ln\left(e^{2i(dx+c)}-1\right)}{d}-\frac{2ia^{2}\ln\left(e^{2i(dx+c)}+1\right)B}{d}+\frac{a^{2}\ln\left(e^{2i(dx+c)}+\right.}{d}$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*a^2*(I*B*ln(sec(d*x+c)^2)+A*(ln(tan(d*x+c))-ln(sec(d*x+c)^2))+2*I*A*x*
d+2*B*d*x-B*tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int\cot(c+dx)(a+ia\tan(c+dx))^{2}(A+B\tan(c+dx))\,dx$$
$$=\frac{-2i\,Ba^{2}+\left((A-2i\,B)a^{2}e^{(2i\,dx+2i\,c)}+(A-2i\,B)a^{2}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)+\left(Aa^{2}e^{(2i\,dx+2i\,c)}+Aa^{2}\right)\log\left(e\right.}{de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
(-2*I*B*a^2 + ((A - 2*I*B)*a^2*e^(2*I*d*x + 2*I*c) + (A - 2*I*B)*a^2)*log(
e^(2*I*d*x + 2*I*c) + 1) + (A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2)*log(e^(2*I*
d*x + 2*I*c) - 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

$$\int \cot(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{Aa^2\log\left(e^{2idx}-e^{-2ic}\right)}{d} - \frac{2iBa^2}{de^{2ic}e^{2idx}+d} + \frac{a^2(A-2iB)\log\left(e^{2idx}+\frac{(-iAa^2-Ba^2+ia^2(A-2iB))e^{-2ic}}{Ba^2}\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
A*a**2*log(exp(2*I*d*x) - exp(-2*I*c))/d - 2*I*B*a**2/(d*exp(2*I*c)*exp(2*
I*d*x) + d) + a**2*(A - 2*I*B)*log(exp(2*I*d*x) + (-I*A*a**2 - B*a**2 + I*
a**2*(A - 2*I*B))*exp(-2*I*c)/(B*a**2))/d
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \cot(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{2\,(dx+c)(-i\,A-B)a^2+(A-i\,B)a^2\log\left(\tan\left(dx+c\right)^2+1\right)-Aa^2\log\left(\tan\left(dx+c\right)\right)+Ba^2\tan\left(dx\right.}{d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-(2*(d*x + c)*(-I*A - B)*a^2 + (A - I*B)*a^2*log(tan(d*x + c)^2 + 1) - A*a
^2*log(tan(d*x + c)) + B*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int \cot(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{Aa^2\log\left(|\tan\left(dx+c\right)|\right)}{d}-\frac{Ba^2\tan\left(dx+c\right)}{d}-\frac{2\left(Aa^2-i\,Ba^2\right)\log\left(\tan\left(dx+c\right)+i\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
A*a^2*log(abs(tan(d*x + c)))/d - B*a^2*tan(d*x + c)/d - 2*(A*a^2 - I*B*a^2
)*log(tan(d*x + c) + I)/d
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \cot(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{A\,a^2\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{2\,A\,a^2\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{d}$$
$$-\frac{B\,a^2\tan(c+d\,x)}{d}+\frac{B\,a^2\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,2\mathrm{i}}{d}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(A*a^2*log(tan(c + d*x)))/d - (2*A*a^2*log(tan(c + d*x) + 1i))/d + (B*a^2*
log(tan(c + d*x) + 1i)*2i)/d - (B*a^2*tan(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.63

$$\int \cot(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\left(-2\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)a+2\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)bi+\cos\left(dx+c\right)\mathrm{lo}\right.}{}$$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*( - 2*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*a + 2*cos(c + d*x)*l
og(tan((c + d*x)/2)**2 + 1)*b*i + cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a
 - 2*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*b*i + cos(c + d*x)*log(tan((c
+ d*x)/2) + 1)*a - 2*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*b*i + cos(c +
d*x)*log(tan((c + d*x)/2))*a + 2*cos(c + d*x)*a*d*i*x + 2*cos(c + d*x)*b*d
*x - sin(c + d*x)*b))/(cos(c + d*x)*d)
```

3.13 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 79

$$\begin{aligned}\int \cot^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx &= -2a^2(A-iB)x+\frac{a^2B\log(\cos(c+dx))}{d}\\ &\quad+\frac{a^2(2iA+B)\log(\sin(c+dx))}{d}-\frac{A\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\end{aligned}$$

output
```
-2*a^2*(A-I*B)*x+a^2*B*ln(cos(d*x+c))/d+a^2*(2*I*A+B)*ln(sin(d*x+c))/d-A*c
ot(d*x+c)*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{a^2(-A\cot(c+dx)+(2iA+B)\log(\tan(c+dx))-2i(A-iB)\log(i+\tan(c+dx)))}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(a^2*(-(A*Cot[c + d*x]) + ((2*I)*A + B)*Log[Tan[c + d*x]] - (2*I)*(A - I*B
)*Log[I + Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4076, 3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \ 4076$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)(a(2iA+B)+iaB\tan(c+dx))dx-\frac{A\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(i\tan(c+dx)a+a)(a(2iA+B)+iaB\tan(c+dx))}{\tan(c+dx)}dx-\frac{A\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \ 4072$$

$$\int \cot(c+dx)\left(a^2(2iA+B)-2a^2(A-iB)\tan(c+dx)\right)dx+a^2(-B)\int \tan(c+dx)dx-\frac{A\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a^2(2iA+B)-2a^2(A-iB)\tan(c+dx)}{\tan(c+dx)}dx+a^2(-B)\int \tan(c+dx)dx-\frac{A\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3956}$$

$$\int \frac{a^2(2iA+B)-2a^2(A-iB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{A\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}+\frac{a^2B\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{4014}$$

$$a^2(B+2iA)\int \cot(c+dx)dx-2a^2x(A-iB)-\frac{A\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}+\frac{a^2B\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$a^2(B+2iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-2a^2x(A-iB)-\frac{A\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}+\frac{a^2B\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$-a^2(B+2iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-2a^2x(A-iB)-\frac{A\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}+\frac{a^2B\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a^2(B+2iA)\log(-\sin(c+dx))}{d}-2a^2x(A-iB)-\frac{A\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}+\frac{a^2B\log(\cos(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
-2*a^2*(A - I*B)*x + (a^2*B*Log[Cos[c + d*x]])/d + (a^2*((2*I)*A + B)*Log[
-Sin[c + d*x]])/d - (A*Cot[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

method	result
parallelrisch	$-\frac{a^2\left(-iA\left(-\ln\left(\sec(dx+c)^2\right)+2\ln(\tan(dx+c))\right)-B\left(\ln(\tan(dx+c))-\ln\left(\sec(dx+c)^2\right)\right)-2iBdx+2Adx+A\cot(dx+c\right)}{d}$
derivativedivides	$\frac{-A\,a^2(dx+c)+B\,a^2\ln(\cos(dx+c))+2iA\,a^2\ln(\sin(dx+c))+2iB\,a^2(dx+c)+A\,a^2(-\cot(dx+c)-dx-c)+B\,a^2\ln(\sin(dx}{d}$
default	$\frac{-A\,a^2(dx+c)+B\,a^2\ln(\cos(dx+c))+2iA\,a^2\ln(\sin(dx+c))+2iB\,a^2(dx+c)+A\,a^2(-\cot(dx+c)-dx-c)+B\,a^2\ln(\sin(dx}{d}$
norman	$\frac{(2iB\,a^2-2A\,a^2)x\tan(dx+c)-\frac{A\,a^2}{d}}{\tan(dx+c)}+\frac{(2iA\,a^2+B\,a^2)\ln(\tan(dx+c))}{d}-\frac{(iA\,a^2+B\,a^2)\ln\left(1+\tan(dx+c)^2\right)}{d}$
risch	$-\frac{4ia^2Bc}{d}+\frac{4a^2Ac}{d}-\frac{2iA\,a^2}{d(\mathrm{e}^{2i(dx+c)}-1)}+\frac{a^2\ln(\mathrm{e}^{2i(dx+c)}-1)B}{d}+\frac{2ia^2\ln(\mathrm{e}^{2i(dx+c)}-1)A}{d}+\frac{a^2\ln(\mathrm{e}^{2i(dx+c)}+1}{d}$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-1/d*a^2*(-I*A*(-ln(sec(d*x+c)^2)+2*ln(tan(d*x+c)))-B*(ln(tan(d*x+c))-ln(s
ec(d*x+c)^2))-2*I*B*d*x+2*A*d*x+A*cot(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{-2i\,Aa^2+\left(Ba^2e^{(2i\,dx+2i\,c)}-Ba^2\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)+\left((2i\,A+B)a^2e^{(2i\,dx+2i\,c)}+(-2i\,A-B)a^2\right)\log}{de^{(2i\,dx+2i\,c)}-d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
(-2*I*A*a^2 + (B*a^2*e^(2*I*d*x + 2*I*c) - B*a^2)*log(e^(2*I*d*x + 2*I*c)
+ 1) + ((2*I*A + B)*a^2*e^(2*I*d*x + 2*I*c) + (-2*I*A - B)*a^2)*log(e^(2*I
*d*x + 2*I*c) - 1))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.38

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{2iAa^2}{de^{2ic}e^{2idx}-d}+\frac{Ba^2\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{ia^2\cdot(2A-iB)\log\left(e^{2idx}+\frac{(Aa^2-iBa^2-a^2\cdot(2A-iB))e^{-2ic}}{Aa^2}\right)}{d}$$

input
```
integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
-2*I*A*a**2/(d*exp(2*I*c)*exp(2*I*d*x) - d) + B*a**2*log(exp(2*I*d*x) + ex
p(-2*I*c))/d + I*a**2*(2*A - I*B)*log(exp(2*I*d*x) + (A*a**2 - I*B*a**2 -
a**2*(2*A - I*B))*exp(-2*I*c)/(A*a**2))/d
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{2\,(dx+c)(A-i\,B)a^2-(-i\,A-B)a^2\log\left(\tan\left(dx+c\right)^2+1\right)-(2i\,A+B)a^2\log\left(\tan\left(dx+c\right)\right)+\frac{}{\tan}}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-(2*(d*x + c)*(A - I*B)*a^2 - (-I*A - B)*a^2*log(tan(d*x + c)^2 + 1) - (2*
I*A + B)*a^2*log(tan(d*x + c)) + A*a^2/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{2\,(i\,Aa^2+Ba^2)\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{(-2i\,Aa^2-Ba^2)\log\left(|\tan\left(dx+c\right)|\right)}{d} - \frac{Aa^2}{d\tan\left(dx+c\right)}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-2*(I*A*a^2 + B*a^2)*log(tan(d*x + c) + I)/d - (-2*I*A*a^2 - B*a^2)*log(ab
s(tan(d*x + c)))/d - A*a^2/(d*tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{B\,a^2\,\ln\left(\tan(c+d\,x)\right)}{d} - \frac{2\,B\,a^2\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{d} - \frac{A\,a^2\cot(c+d\,x)}{d} + \frac{A\,a^2\,\ln\left(\tan(c+d\,x)\right)\,2\mathrm{i}}{d} - \frac{A\,a^2\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,2\mathrm{i}}{d}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(A*a^2*log(tan(c + d*x))*2i)/d + (B*a^2*log(tan(c + d*x)))/d - (A*a^2*log(
tan(c + d*x) + 1i)*2i)/d - (2*B*a^2*log(tan(c + d*x) + 1i))/d - (A*a^2*cot
(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.19

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\left(-\cos(dx+c)\,a-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)\,ai-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)\,b\right.}{}$$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*( - cos(c + d*x)*a - 2*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a*i
 - 2*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*b + log(tan((c + d*x)/2) -
1)*sin(c + d*x)*b + log(tan((c + d*x)/2) + 1)*sin(c + d*x)*b + 2*log(tan((
c + d*x)/2))*sin(c + d*x)*a*i + log(tan((c + d*x)/2))*sin(c + d*x)*b - 2*s
in(c + d*x)*a*d*x + 2*sin(c + d*x)*b*d*i*x))/(sin(c + d*x)*d)
```

3.14 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 94

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -2a^2(iA+B)x-\frac{a^2(3iA+2B)\cot(c+dx)}{2d} - \frac{2a^2(A-iB)\log(\sin(c+dx))}{d}-\frac{A\cot^2(c+dx)\,(a^2+ia^2\tan(c+dx))}{2d}$$

output
```
-2*a^2*(I*A+B)*x-1/2*a^2*(3*I*A+2*B)*cot(d*x+c)/d-2*a^2*(A-I*B)*ln(sin(d*x
+c))/d-1/2*A*cot(d*x+c)^2*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = a^2\left(-\frac{2iA\cot(c+dx)}{d}-\frac{B\cot(c+dx)}{d}-\frac{A\cot^2(c+dx)}{2d}-\frac{2A\log(\tan(c+dx))}{d} + \frac{2iB\log(\tan(c+dx))}{d}+\frac{2A\log(i+\tan(c+dx))}{d}-\frac{2iB\log(i+\tan(c+dx))}{d}\right)$$

input
```
Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
a^2*(((-2*I)*A*Cot[c + d*x])/d - (B*Cot[c + d*x])/d - (A*Cot[c + d*x]^2)/(
2*d) - (2*A*Log[Tan[c + d*x]])/d + ((2*I)*B*Log[Tan[c + d*x]])/d + (2*A*Lo
g[I + Tan[c + d*x]])/d - ((2*I)*B*Log[I + Tan[c + d*x]])/d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 4076, 3042, 4074, 27, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{2}\int \cot^2(c+dx)(i\tan(c+dx)a+a)(a(3iA+2B)-a(A-2iB)\tan(c+dx))dx-\frac{A\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)(a(3iA+2B)-a(A-2iB)\tan(c+dx))}{\tan(c+dx)^2}dx-\frac{A\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \text{4074}$$

$$\frac{1}{2}\left(\int -4\cot(c+dx)\left((A-iB)a^2+(iA+B)\tan(c+dx)a^2\right)dx-\frac{a^2(2B+3iA)\cot(c+dx)}{d}\right)-\frac{A\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-4\int \cot(c+dx)\left((A-iB)a^2+(iA+B)\tan(c+dx)a^2\right)dx-\frac{a^2(2B+3iA)\cot(c+dx)}{d}\right)-\frac{A\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-4\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\tan(c+dx)}dx-\frac{a^2(2B+3iA)\cot(c+dx)}{d}\right)-\frac{A\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \text{4014}$$

$$\frac{1}{2}\left(-4\left(a^2(A-iB)\int \cot(c+dx)dx+a^2x(B+iA)\right)-\frac{a^2(2B+3iA)\cot(c+dx)}{d}\right)-\frac{A\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-4\left(a^2(A-iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+a^2x(B+iA)\right)-\frac{a^2(2B+3iA)\cot(c+dx)}{d}\right)-\frac{A\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-4\left(a^2x(B+iA)-a^2(A-iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx\right)-\frac{a^2(2B+3iA)\cot(c+dx)}{d}\right)-\frac{A\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{2}\left(-\frac{a^2(2B+3iA)\cot(c+dx)}{d}-4\left(\frac{a^2(A-iB)\log(-\sin(c+dx))}{d}+a^2x(B+iA)\right)\right)-\frac{A\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(-((a^2*((3*I)*A + 2*B)*Cot[c + d*x])/d) - 4*(a^2*(I*A + B)*x + (a^2*(A -
I*B)*Log[-Sin[c + d*x]])/d))/2 - (A*Cot[c + d*x]^2*(a^2 + I*a^2*Tan[c + d*
x]))/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

method	result
parallelrisch	$-\frac{2a^2\left(\frac{(iB-A)\ln\left(\sec(dx+c)^2\right)}{2}+(-iB+A)\ln(\tan(dx+c))+\frac{A\cot(dx+c)^2}{4}+\cot(dx+c)\left(iA+\frac{B}{2}\right)+(iA+B)xd\right)}{d}$
derivativedivides	$\frac{-A\,a^2\ln(\sin(dx+c))-B\,a^2(dx+c)+2iA\,a^2(-\cot(dx+c)-dx-c)+2iB\,a^2\ln(\sin(dx+c))+A\,a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(\right.}{d}$
default	$\frac{-A\,a^2\ln(\sin(dx+c))-B\,a^2(dx+c)+2iA\,a^2(-\cot(dx+c)-dx-c)+2iB\,a^2\ln(\sin(dx+c))+A\,a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(\right.}{d}$
risch	$\frac{4a^2Bc}{d}+\frac{4ia^2Ac}{d}-\frac{2ia^2\left(3iA\,e^{2i(dx+c)}+B\,e^{2i(dx+c)}-2iA-B\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+\frac{2ia^2\ln\left(e^{2i(dx+c)}-1\right)B}{d}-\frac{2A\,a^2\ln\left(e^{2i(dx+c)}\right.}{d}$
norman	$\frac{(-2iA\,a^2-2B\,a^2)x\tan(dx+c)^2-\frac{A\,a^2}{2d}-\frac{\left(2iA\,a^2+B\,a^2\right)\tan(dx+c)}{d}}{\tan(dx+c)^2}+\frac{(-iB\,a^2+A\,a^2)\ln\left(1+\tan(dx+c)^2\right)}{d}-\frac{2(-iB}{}$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-2*a^2/d*(1/2*(-A+I*B)*ln(sec(d*x+c)^2)+(A-I*B)*ln(tan(d*x+c))+1/4*A*cot(d
*x+c)^2+cot(d*x+c)*(I*A+1/2*B)+(I*A+B)*x*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.31

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{2\left((3A-iB)a^2e^{(2i\,dx+2i\,c)}-(2A-iB)a^2-\left((A-iB)a^2e^{(4i\,dx+4i\,c)}-2(A-iB)a^2e^{(2i\,dx+2i\,c)}+(A-i\right.\right.}{de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
2*((3*A - I*B)*a^2*e^(2*I*d*x + 2*I*c) - (2*A - I*B)*a^2 - ((A - I*B)*a^2*
e^(4*I*d*x + 4*I*c) - 2*(A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + (A - I*B)*a^2)
*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2
*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{2a^2(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d}+\frac{-4Aa^2+2iBa^2+\left(6Aa^2e^{2ic}-2iBa^2e^{2ic}\right)e^{2idx}}{de^{4ic}e^{4idx}-2de^{2ic}e^{2idx}+d}$$

input
```
integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
-2*a**2*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-4*A*a**2 + 2*I*B*a
**2 + (6*A*a**2*exp(2*I*c) - 2*I*B*a**2*exp(2*I*c))*exp(2*I*d*x))/(d*exp(4
*I*c)*exp(4*I*d*x) - 2*d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{4\,(dx+c)(i\,A+B)a^2-2\,(A-i\,B)a^2\log\left(\tan\left(dx+c\right)^2+1\right)+4\,(A-i\,B)a^2\log\left(\tan\left(dx+c\right)\right)-\frac{2\,(\cdots}{\cdots}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/2*(4*(d*x + c)*(I*A + B)*a^2 - 2*(A - I*B)*a^2*log(tan(d*x + c)^2 + 1)
+ 4*(A - I*B)*a^2*log(tan(d*x + c)) - (2*(-2*I*A - B)*a^2*tan(d*x + c) - A
*a^2)/tan(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{2\,(Aa^2-i\,Ba^2)\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{2\,(Aa^2-i\,Ba^2)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$
$$-\frac{Aa^2+2i\,(2\,Aa^2-i\,Ba^2)\tan\left(dx+c\right)}{2\,d\tan\left(dx+c\right)^2}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
2*(A*a^2 - I*B*a^2)*log(tan(d*x + c) + I)/d - 2*(A*a^2 - I*B*a^2)*log(abs(
tan(d*x + c)))/d - 1/2*(A*a^2 + 2*I*(2*A*a^2 - I*B*a^2)*tan(d*x + c))/(d*t
an(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{\frac{A\,a^2}{2}+\tan(c+d\,x)\,\left(B\,a^2+A\,a^2\,2\mathrm{i}\right)}{d\tan\left(c+d\,x\right)^2} - \frac{4\,a^2\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\,\left(B+A\,1\mathrm{i}\right)}{d}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
- ((A*a^2)/2 + tan(c + d*x)*(A*a^2*2i + B*a^2))/(d*tan(c + d*x)^2) - (4*a^
2*atan(2*tan(c + d*x) + 1i)*(A*1i + B))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.94

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\left(-8\cos\left(dx+c\right)\sin\left(dx+c\right)ai - 4\cos\left(dx+c\right)\sin\left(dx+c\right)b + 8\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+c\right)\right.}{}$$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*( - 8*cos(c + d*x)*sin(c + d*x)*a*i - 4*cos(c + d*x)*sin(c + d*x)*b
+ 8*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a - 8*log(tan((c + d*x)/2
)**2 + 1)*sin(c + d*x)**2*b*i - 8*log(tan((c + d*x)/2))*sin(c + d*x)**2*a
+ 8*log(tan((c + d*x)/2))*sin(c + d*x)**2*b*i - 8*sin(c + d*x)**2*a*d*i*x
+ sin(c + d*x)**2*a - 8*sin(c + d*x)**2*b*d*x - 2*a))/(4*sin(c + d*x)**2*d
)
```

3.15 $\int \cot^4(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 117

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = 2a^2(A-iB)x+\frac{2a^2(A-iB)\cot(c+dx)}{d}-\frac{a^2(4iA+3B)\cot^2(c+dx)}{6d} - \frac{2a^2(iA+B)\log(\sin(c+dx))}{d}-\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

output
```
2*a^2*(A-I*B)*x+2*a^2*(A-I*B)*cot(d*x+c)/d-1/6*a^2*(4*I*A+3*B)*cot(d*x+c)^
2/d-2*a^2*(I*A+B)*ln(sin(d*x+c))/d-1/3*A*cot(d*x+c)^3*(a^2+I*a^2*tan(d*x+c
))/d
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = a^2\left(\frac{2A\cot(c+dx)}{d} - \frac{2iB\cot(c+dx)}{d} - \frac{iA\cot^2(c+dx)}{d} - \frac{B\cot^2(c+dx)}{2d} - \frac{A\cot^3(c+dx)}{3d} - \frac{2iA\log(\tan(c+dx))}{d} - \frac{2B\log(\tan(c+dx))}{d} + \frac{2iA\log(i+\tan(c+dx))}{d} + \frac{2B\log(i+\tan(c+dx))}{d}\right)$$

input
```
Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
a^2*((2*A*Cot[c + d*x])/d - ((2*I)*B*Cot[c + d*x])/d - (I*A*Cot[c + d*x]^2
)/d - (B*Cot[c + d*x]^2)/(2*d) - (A*Cot[c + d*x]^3)/(3*d) - ((2*I)*A*Log[T
an[c + d*x]])/d - (2*B*Log[Tan[c + d*x]])/d + ((2*I)*A*Log[I + Tan[c + d*x
]])/d + (2*B*Log[I + Tan[c + d*x]])/d)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4076, 3042, 4074, 27, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{3}\int \cot^3(c+dx)(i\tan(c+dx)a+a)(a(4iA+3B)-a(2A-3iB)\tan(c+dx))dx-\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$\downarrow$ 3042

$$\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{\tan(c+dx)^3}dx-\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$\downarrow$ 4074

$$\frac{1}{3}\left(\int -6\cot^2(c+dx)\left((A-iB)a^2+(iA+B)\tan(c+dx)a^2\right)dx-\frac{a^2(3B+4iA)\cot^2(c+dx)}{2d}\right)-\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$\downarrow$ 27

$$\frac{1}{3}\left(-6\int \cot^2(c+dx)\left((A-iB)a^2+(iA+B)\tan(c+dx)a^2\right)dx-\frac{a^2(3B+4iA)\cot^2(c+dx)}{2d}\right)-\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(-6\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\tan(c+dx)^2}dx-\frac{a^2(3B+4iA)\cot^2(c+dx)}{2d}\right)-\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$\downarrow$ 4012

$$\frac{1}{3}\left(-6\left(\int \cot(c+dx)\left(a^2(iA+B)-a^2(A-iB)\tan(c+dx)\right)dx-\frac{a^2(A-iB)\cot(c+dx)}{d}\right)-\frac{a^2(3B+4iA)\,c}{2d}\right.$$
$$-\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(-6\left(\int \frac{a^2(iA+B)-a^2(A-iB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a^2(A-iB)\cot(c+dx)}{d}\right)-\frac{a^2(3B+4iA)\cot^2(c+dx)}{2d}\right)-\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \text{4014}$$

$$\frac{1}{3}\left(-6\left(a^2(B+iA)\int \cot(c+dx)dx-\frac{a^2(A-iB)\cot(c+dx)}{d}-\left(a^2x(A-iB)\right)\right)-\frac{a^2(3B+4iA)\cot^2(c+dx)}{2d}\right.$$
$$\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-6\left(a^2(B+iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a^2(A-iB)\cot(c+dx)}{d}-\left(a^2x(A-iB)\right)\right)-\frac{a^2(3B+4iA)\cot}{2d}\right.$$
$$\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(-6\left(-a^2(B+iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a^2(A-iB)\cot(c+dx)}{d}-\left(a^2x(A-iB)\right)\right)-\frac{a^2(3B+4iA}{2}\right.$$
$$\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{3}\left(-\frac{a^2(3B+4iA)\cot^2(c+dx)}{2d}-6\left(-\frac{a^2(A-iB)\cot(c+dx)}{d}+\frac{a^2(B+iA)\log(-\sin(c+dx))}{d}-\left(a^2x(A-iB\right.\right.\right.$$
$$\frac{A\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(-1/2*(a^2*((4*I)*A + 3*B)*Cot[c + d*x]^2)/d - 6*(-(a^2*(A - I*B)*x) - (a^
2*(A - I*B)*Cot[c + d*x])/d + (a^2*(I*A + B)*Log[-Sin[c + d*x]])/d))/3 - (
A*Cot[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x]))/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{2\left(\frac{(iA+B)\ln\left(\sec(dx+c)^2\right)}{2}-(iA+B)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^3}{6}-\frac{\cot(dx+c)^2\left(iA+\frac{B}{2}\right)}{2}+(-iB+A)\cot(dx+c)+(-iB+\right.}{d}$
derivativedivides	$\frac{-A a^2(-\cot(dx+c)-dx-c)-B a^2\ln(\sin(dx+c))+2iA a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+2iB a^2(-\cot(dx+c)-dx-}{d}$
default	$\frac{-A a^2(-\cot(dx+c)-dx-c)-B a^2\ln(\sin(dx+c))+2iA a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+2iB a^2(-\cot(dx+c)-dx-}{d}$
risch	$\frac{4ia^2Bc}{d}-\frac{4a^2Ac}{d}+\frac{2a^2\left(15iA\,e^{4i(dx+c)}+9B\,e^{4i(dx+c)}-18iA\,e^{2i(dx+c)}-15B\,e^{2i(dx+c)}+7iA+6B\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}-\frac{2a^2\ln(e^{2i(d}{d}$
norman	$\frac{(-2iB a^2+2A a^2)x\tan(dx+c)^3-\frac{A a^2}{3d}+\frac{2\left(-iB a^2+A a^2\right)\tan(dx+c)^2}{d}-\frac{\left(2iA a^2+B a^2\right)\tan(dx+c)}{2d}}{\tan(dx+c)^3}+\frac{(iA a^2+B a^2)\ln\left(1\right.}{d}$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
2*(1/2*(I*A+B)*ln(sec(d*x+c)^2)-(I*A+B)*ln(tan(d*x+c))-1/6*A*cot(d*x+c)^3-
1/2*cot(d*x+c)^2*(I*A+1/2*B)+(A-I*B)*cot(d*x+c)+(A-I*B)*x*d)*a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.55

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(3\left(-5i\,A-3\,B\right)a^2e^{(4i\,dx+4i\,c)}+3\left(6i\,A+5\,B\right)a^2e^{(2i\,dx+2i\,c)}+\left(-7i\,A-6\,B\right)a^2+3\left(\left(i\,A+B\right)a^2e^{(6i\,d\cdots}\right.\right.}{3\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+\cdots}\right.}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-2/3*(3*(-5*I*A - 3*B)*a^2*e^(4*I*d*x + 4*I*c) + 3*(6*I*A + 5*B)*a^2*e^(2*
I*d*x + 2*I*c) + (-7*I*A - 6*B)*a^2 + 3*((I*A + B)*a^2*e^(6*I*d*x + 6*I*c)
 + 3*(-I*A - B)*a^2*e^(4*I*d*x + 4*I*c) + 3*(I*A + B)*a^2*e^(2*I*d*x + 2*I
*c) + (-I*A - B)*a^2)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(6*I*d*x + 6*I*c)
 - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.56

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$= -\frac{2ia^2(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d} + \frac{14iAa^2+12Ba^2+\left(-36iAa^2e^{2ic}-30Ba^2e^{2ic}\right)e^{2idx}+\left(30iAa^2e^{4ic}+18Ba^2e^{4ic}\right)e^{4idx}}{3de^{6ic}e^{6idx}-9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}-3d}$$

input
```
integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
-2*I*a**2*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (14*I*A*a**2 + 12*
B*a**2 + (-36*I*A*a**2*exp(2*I*c) - 30*B*a**2*exp(2*I*c))*exp(2*I*d*x) + (
30*I*A*a**2*exp(4*I*c) + 18*B*a**2*exp(4*I*c))*exp(4*I*d*x))/(3*d*exp(6*I*
c)*exp(6*I*d*x) - 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x
) - 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{12\,(dx+c)(A-i\,B)a^2+6\,(i\,A+B)a^2\log\left(\tan\left(dx+c\right)^2+1\right)-12\,(i\,A+B)a^2\log\left(\tan\left(dx+c\right)\right)+\frac{12\,(}{}}{6\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/6*(12*(d*x + c)*(A - I*B)*a^2 + 6*(I*A + B)*a^2*log(tan(d*x + c)^2 + 1)
- 12*(I*A + B)*a^2*log(tan(d*x + c)) + (12*(A - I*B)*a^2*tan(d*x + c)^2 +
3*(-2*I*A - B)*a^2*tan(d*x + c) - 2*A*a^2)/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{2\left(-i\,Aa^2-Ba^2\right)\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{2\left(i\,Aa^2+Ba^2\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$
$$-\frac{2\,Aa^2-12\left(Aa^2-i\,Ba^2\right)\tan\left(dx+c\right)^2+3\left(2i\,Aa^2+Ba^2\right)\tan\left(dx+c\right)}{6\,d\tan\left(dx+c\right)^3}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-2*(-I*A*a^2 - B*a^2)*log(tan(d*x + c) + I)/d - 2*(I*A*a^2 + B*a^2)*log(ab
s(tan(d*x + c)))/d - 1/6*(2*A*a^2 - 12*(A*a^2 - I*B*a^2)*tan(d*x + c)^2 +
3*(2*I*A*a^2 + B*a^2)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{\frac{A\,a^2}{3}-\tan(c+d\,x)^2\,\left(2\,A\,a^2-B\,a^2\,2\mathrm{i}\right)+\tan(c+d\,x)\,\left(\frac{B\,a^2}{2}+A\,a^2\,1\mathrm{i}\right)}{d\tan\left(c+d\,x\right)^3} - \frac{a^2\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\,\left(B+A\,1\mathrm{i}\right)\,4\mathrm{i}}{d}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
- ((A*a^2)/3 - tan(c + d*x)^2*(2*A*a^2 - B*a^2*2i) + tan(c + d*x)*(A*a^2*1
i + (B*a^2)/2))/(d*tan(c + d*x)^3) - (a^2*atan(2*tan(c + d*x) + 1i)*(A*1i
+ B)*4i)/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.91

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\left(28\cos\left(dx+c\right)\sin\left(dx+c\right)^2a-24\cos\left(dx+c\right)\sin\left(dx+c\right)^2bi-4\cos\left(dx+c\right)a+24\log\left(\tan\left(\frac{dx}{2}+\right.\right.\right.}{}$$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*(28*cos(c + d*x)*sin(c + d*x)**2*a - 24*cos(c + d*x)*sin(c + d*x)**2
*b*i - 4*cos(c + d*x)*a + 24*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*
a*i + 24*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*b - 24*log(tan((c +
d*x)/2))*sin(c + d*x)**3*a*i - 24*log(tan((c + d*x)/2))*sin(c + d*x)**3*b
+ 24*sin(c + d*x)**3*a*d*x + 6*sin(c + d*x)**3*a*i - 24*sin(c + d*x)**3*b*
d*i*x + 3*sin(c + d*x)**3*b - 12*sin(c + d*x)*a*i - 6*sin(c + d*x)*b))/(12
*sin(c + d*x)**3*d)
```

3.16 $\int \cot^5(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 139

$$\begin{aligned}&\int \cot^5(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx\\&=2a^2(iA+B)x+\frac{2a^2(iA+B)\cot(c+dx)}{d}\\&\quad+\frac{a^2(A-iB)\cot^2(c+dx)}{d}-\frac{a^2(5iA+4B)\cot^3(c+dx)}{12d}\\&\quad+\frac{2a^2(A-iB)\log(\sin(c+dx))}{d}-\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}\end{aligned}$$

output
```
2*a^2*(I*A+B)*x+2*a^2*(I*A+B)*cot(d*x+c)/d+a^2*(A-I*B)*cot(d*x+c)^2/d-1/12
*a^2*(5*I*A+4*B)*cot(d*x+c)^3/d+2*a^2*(A-I*B)*ln(sin(d*x+c))/d-1/4*A*cot(d
*x+c)^4*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.25

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= a^2\left(\frac{2iA\cot(c+dx)}{d}+\frac{2B\cot(c+dx)}{d}+\frac{A\cot^2(c+dx)}{d}-\frac{iB\cot^2(c+dx)}{d}\right.$$
$$-\frac{2iA\cot^3(c+dx)}{3d}-\frac{B\cot^3(c+dx)}{3d}-\frac{A\cot^4(c+dx)}{4d}+\frac{2A\log(\tan(c+dx))}{d}$$
$$\left.-\frac{2iB\log(\tan(c+dx))}{d}-\frac{2A\log(i+\tan(c+dx))}{d}+\frac{2iB\log(i+\tan(c+dx))}{d}\right)$$

input
```
Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
a^2*(((2*I)*A*Cot[c + d*x])/d + (2*B*Cot[c + d*x])/d + (A*Cot[c + d*x]^2)/
d - (I*B*Cot[c + d*x]^2)/d - (((2*I)/3)*A*Cot[c + d*x]^3)/d - (B*Cot[c + d
*x]^3)/(3*d) - (A*Cot[c + d*x]^4)/(4*d) + (2*A*Log[Tan[c + d*x]])/d - ((2*
I)*B*Log[Tan[c + d*x]])/d - (2*A*Log[I + Tan[c + d*x]])/d + ((2*I)*B*Log[I
 + Tan[c + d*x]])/d)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {3042, 4076, 3042, 4074, 27, 3042, 4012, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{ 4076}$$

$$\frac{1}{4}\int \cot^4(c+dx)(i\tan(c+dx)a+a)(a(5iA+4B)-a(3A-4iB)\tan(c+dx))dx-\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4}\int \frac{(i\tan(c+dx)a+a)(a(5iA+4B)-a(3A-4iB)\tan(c+dx))}{\tan(c+dx)^4}dx-\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{ 4074}$$

$$\frac{1}{4}\left(\int -8\cot^3(c+dx)\left((A-iB)a^2+(iA+B)\tan(c+dx)a^2\right)dx-\frac{a^2(4B+5iA)\cot^3(c+dx)}{3d}\right)-\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(-8\int \cot^3(c+dx)\left((A-iB)a^2+(iA+B)\tan(c+dx)a^2\right)dx-\frac{a^2(4B+5iA)\cot^3(c+dx)}{3d}\right)-\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4}\left(-8\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\tan(c+dx)^3}dx-\frac{a^2(4B+5iA)\cot^3(c+dx)}{3d}\right)-\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{ 4012}$$

$$\frac{1}{4}\left(-8\left(\int \cot^2(c+dx)\left(a^2(iA+B)-a^2(A-iB)\tan(c+dx)\right)dx-\frac{a^2(A-iB)\cot^2(c+dx)}{2d}\right)-\frac{a^2(4B+5iA}{3}\right.-\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4}\left(-8\left(\int \frac{a^2(iA+B)-a^2(A-iB)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{a^2(A-iB)\cot^2(c+dx)}{2d}\right)-\frac{a^2(4B+5iA)\cot^3(c+dx)}{3d}\right.$$
$$\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\Big\downarrow$ 4012

$$\frac{1}{4}\left(-8\left(\int -\cot(c+dx)\left((A-iB)a^2+(iA+B)\tan(c+dx)a^2\right)dx-\frac{a^2(A-iB)\cot^2(c+dx)}{2d}-\frac{a^2(B+iA)\text{co}}{d}\right.\right.$$
$$\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\Big\downarrow$ 25

$$\frac{1}{4}\left(-8\left(-\int \cot(c+dx)\left((A-iB)a^2+(iA+B)\tan(c+dx)a^2\right)dx-\frac{a^2(A-iB)\cot^2(c+dx)}{2d}-\frac{a^2(B+iA)\text{co}}{d}\right.\right.$$
$$\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(-8\left(-\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\tan(c+dx)}dx-\frac{a^2(A-iB)\cot^2(c+dx)}{2d}-\frac{a^2(B+iA)\cot(c+dx)}{d}\right)-\right.$$
$$\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\Big\downarrow$ 4014

$$\frac{1}{4}\left(-8\left(-a^2(A-iB)\int \cot(c+dx)dx-\frac{a^2(A-iB)\cot^2(c+dx)}{2d}-\frac{a^2(B+iA)\cot(c+dx)}{d}-a^2x(B+iA)\right)-\right.$$
$$\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(-8\left(-a^2(A-iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a^2(A-iB)\cot^2(c+dx)}{2d}-\frac{a^2(B+iA)\cot(c+dx)}{d}-a^2x(B\right.\right.$$
$$\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$\Big\downarrow$ 25

$$\frac{1}{4}\left(-8\left(a^2(A-iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a^2(A-iB)\cot^2(c+dx)}{2d}-\frac{a^2(B+iA)\cot(c+dx)}{d}-a^2x(B\right.\right.$$
$$\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{4}\left(-\frac{a^2(4B+5iA)\cot^3(c+dx)}{3d}-8\left(-\frac{a^2(A-iB)\cot^2(c+dx)}{2d}-\frac{a^2(B+iA)\cot(c+dx)}{d}-\frac{a^2(A-iB)\log(-}{d}\right.\right.$$
$$\frac{A\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

input
```
Int[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(-1/3*(a^2*((5*I)*A + 4*B)*Cot[c + d*x]^3)/d - 8*(-(a^2*(I*A + B)*x) - (a^
2*(I*A + B)*Cot[c + d*x])/d - (a^2*(A - I*B)*Cot[c + d*x]^2)/(2*d) - (a^2*
(A - I*B)*Log[-Sin[c + d*x]])/d))/4 - (A*Cot[c + d*x]^4*(a^2 + I*a^2*Tan[c
 + d*x]))/(4*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{2\left(\frac{(iB-A)\ln\left(\sec(dx+c)^2\right)}{2}+(-iB+A)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^4}{8}-\frac{\cot(dx+c)^3\left(iA+\frac{B}{2}\right)}{3}+\frac{(-iB+A)\cot(dx+c)^2}{2}+\cot(d\right.}{d}$
derivativedivides	$\frac{-A\,a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)-B\,a^2(-\cot(dx+c)-dx-c)+2iA\,a^2\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2iB\,a}{}$
default	$\frac{-A\,a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)-B\,a^2(-\cot(dx+c)-dx-c)+2iA\,a^2\left(-\frac{\cot(dx+c)^3}{3}+\cot(dx+c)+dx+c\right)+2iB\,a}{}$
risch	$-\frac{4a^2Bc}{d}-\frac{4ia^2Ac}{d}+\frac{2ia^2\left(21iA\,e^{6i(dx+c)}+15B\,e^{6i(dx+c)}-36iA\,e^{4i(dx+c)}-33B\,e^{4i(dx+c)}+29iA\,e^{2i(dx+c)}+25B\,e\right.}{3d\left(e^{2i(dx+c)}-1\right)^4}$
norman	$\frac{\frac{(-iB\,a^2+A\,a^2)\tan(dx+c)^2}{d}+(2iA\,a^2+2B\,a^2)x\tan(dx+c)^4-\frac{A\,a^2}{4d}+\frac{2(iA\,a^2+B\,a^2)\tan(dx+c)^3}{d}-\frac{(2iA\,a^2+B\,a^2)\tan(dx+c}{3d}}{\tan(dx+c)^4}$

input
```
int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
2*(1/2*(-A+I*B)*ln(sec(d*x+c)^2)+(A-I*B)*ln(tan(d*x+c))-1/8*A*cot(d*x+c)^4
-1/3*cot(d*x+c)^3*(I*A+1/2*B)+1/2*(A-I*B)*cot(d*x+c)^2+cot(d*x+c)*(I*A+B)+
(I*A+B)*x*d)*a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.63

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{2\left(3\left(7A-5iB\right)a^2e^{(6i\,dx+6i\,c)}-3\left(12A-11iB\right)a^2e^{(4i\,dx+4i\,c)}+\left(29A-25iB\right)a^2e^{(2i\,dx+2i\,c)}-\left(8A-7\right.\right.}{3\left(de^{(8i\,dx}\right.}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-2/3*(3*(7*A - 5*I*B)*a^2*e^(6*I*d*x + 6*I*c) - 3*(12*A - 11*I*B)*a^2*e^(4
*I*d*x + 4*I*c) + (29*A - 25*I*B)*a^2*e^(2*I*d*x + 2*I*c) - (8*A - 7*I*B)*
a^2 - 3*((A - I*B)*a^2*e^(8*I*d*x + 8*I*c) - 4*(A - I*B)*a^2*e^(6*I*d*x +
6*I*c) + 6*(A - I*B)*a^2*e^(4*I*d*x + 4*I*c) - 4*(A - I*B)*a^2*e^(2*I*d*x
+ 2*I*c) + (A - I*B)*a^2)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(8*I*d*x + 8*
I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x
+ 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.69

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{2a^2(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d} + \frac{16Aa^2 - 14iBa^2 + (-58Aa^2e^{2ic} + 50iBa^2e^{2ic})\,e^{2idx} + (72Aa^2e^{4ic} - 66iBa^2e^{4ic})\,e^{4idx} + (-42Aa^2e^{6ic} +}{3de^{8ic}e^{8idx} - 12de^{6ic}e^{6idx} + 18de^{4ic}e^{4idx} - 12de^{2ic}e^{2idx} + 3d}$$

input
```
integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
2*a**2*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (16*A*a**2 - 14*I*B*a
**2 + (-58*A*a**2*exp(2*I*c) + 50*I*B*a**2*exp(2*I*c))*exp(2*I*d*x) + (72*
A*a**2*exp(4*I*c) - 66*I*B*a**2*exp(4*I*c))*exp(4*I*d*x) + (-42*A*a**2*exp
(6*I*c) + 30*I*B*a**2*exp(6*I*c))*exp(6*I*d*x))/(3*d*exp(8*I*c)*exp(8*I*d*
x) - 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d*x) - 12*d*ex
p(2*I*c)*exp(2*I*d*x) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{24\,(dx+c)(-i\,A-B)a^2 + 12\,(A-i\,B)a^2\log\left(\tan\left(dx+c\right)^2+1\right) - 24\,(A-i\,B)a^2\log\left(\tan\left(dx+c\right)\right)}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/12*(24*(d*x + c)*(-I*A - B)*a^2 + 12*(A - I*B)*a^2*log(tan(d*x + c)^2 +
 1) - 24*(A - I*B)*a^2*log(tan(d*x + c)) - (24*(I*A + B)*a^2*tan(d*x + c)^
3 + 12*(A - I*B)*a^2*tan(d*x + c)^2 + 4*(-2*I*A - B)*a^2*tan(d*x + c) - 3*
A*a^2)/tan(d*x + c)^4)/d
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{2\left(Aa^2-i\,Ba^2\right)\log\left(\tan\left(dx+c\right)+i\right)}{d}+\frac{2\left(Aa^2-i\,Ba^2\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$
$$-\frac{24\left(-i\,Aa^2-Ba^2\right)\tan\left(dx+c\right)^3+3\,Aa^2-12\left(Aa^2-i\,Ba^2\right)\tan\left(dx+c\right)^2+4\left(2i\,Aa^2+Ba^2\right)\tan\left(dx\right.}{12\,d\tan\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-2*(A*a^2 - I*B*a^2)*log(tan(d*x + c) + I)/d + 2*(A*a^2 - I*B*a^2)*log(abs
(tan(d*x + c)))/d - 1/12*(24*(-I*A*a^2 - B*a^2)*tan(d*x + c)^3 + 3*A*a^2 -
 12*(A*a^2 - I*B*a^2)*tan(d*x + c)^2 + 4*(2*I*A*a^2 + B*a^2)*tan(d*x + c))
/(d*tan(d*x + c)^4)
```

Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)^2\,\left(A\,a^2-B\,a^2\,1{}\mathrm{i}\right)+\tan(c+d\,x)^3\,\left(2\,B\,a^2+A\,a^2\,2{}\mathrm{i}\right)-\frac{A\,a^2}{4}-\tan(c+d\,x)\,\left(\frac{B\,a^2}{3}+\frac{A\,a^2\,2{}\mathrm{i}}{3}\right)}{d\,\tan\left(c+d\,x\right)^4}$$
$$+\frac{4\,a^2\,\mathrm{atan}(2\,\tan(c+d\,x)+1{}\mathrm{i})\,\left(B+A\,1{}\mathrm{i}\right)}{d}$$

input
```
int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(tan(c + d*x)^2*(A*a^2 - B*a^2*1i) + tan(c + d*x)^3*(A*a^2*2i + 2*B*a^2) -
 (A*a^2)/4 - tan(c + d*x)*((A*a^2*2i)/3 + (B*a^2)/3))/(d*tan(c + d*x)^4) +
 (4*a^2*atan(2*tan(c + d*x) + 1i)*(A*1i + B))/d
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.82

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\Big(256\cos(dx+c)\sin(dx+c)^3 ai + 224\cos(dx+c)\sin(dx+c)^3 b - 64\cos(dx+c)\sin(dx+c)ai - 3}{}$$

input
```
int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*(256*cos(c + d*x)*sin(c + d*x)**3*a*i + 224*cos(c + d*x)*sin(c + d*x
)**3*b - 64*cos(c + d*x)*sin(c + d*x)*a*i - 32*cos(c + d*x)*sin(c + d*x)*b
 - 192*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**4*a + 192*log(tan((c + d
*x)/2)**2 + 1)*sin(c + d*x)**4*b*i + 192*log(tan((c + d*x)/2))*sin(c + d*x
)**4*a - 192*log(tan((c + d*x)/2))*sin(c + d*x)**4*b*i + 192*sin(c + d*x)*
*4*a*d*i*x - 63*sin(c + d*x)**4*a + 192*sin(c + d*x)**4*b*d*x + 48*sin(c +
 d*x)**4*b*i + 144*sin(c + d*x)**2*a - 96*sin(c + d*x)**2*b*i - 24*a))/(96
*sin(c + d*x)**4*d)
```

3.17 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 182

$$\begin{aligned}\int \tan^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= -4a^3(A-iB)x+\frac{4a^3(iA+B)\log(\cos(c+dx))}{d}\\ &\quad+\frac{4a^3(A-iB)\tan(c+dx)}{d}+\frac{2a^3(iA+B)\tan^2(c+dx)}{d}\\ &\quad-\frac{a^3(45A-47iB)\tan^3(c+dx)}{60d}+\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}\\ &\quad-\frac{(5A-7iB)\tan^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}\end{aligned}$$

output
```
-4*a^3*(A-I*B)*x+4*a^3*(I*A+B)*ln(cos(d*x+c))/d+4*a^3*(A-I*B)*tan(d*x+c)/d
+2*a^3*(I*A+B)*tan(d*x+c)^2/d-1/60*a^3*(45*A-47*I*B)*tan(d*x+c)^3/d+1/5*I*
a*B*tan(d*x+c)^3*(a+I*a*tan(d*x+c))^2/d-1/20*(5*A-7*I*B)*tan(d*x+c)^3*(a^3
+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.68

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{a^3(-15iA-17B-240i(A-iB)\log(i+\tan(c+dx))+240(A-iB)\tan(c+dx)+120(iA+B)\tan^2(}{60d}$$

input
```
Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(a^3*((-15*I)*A - 17*B - (240*I)*(A - I*B)*Log[I + Tan[c + d*x]] + 240*(A
- I*B)*Tan[c + d*x] + 120*(I*A + B)*Tan[c + d*x]^2 + (-60*A + (80*I)*B)*Ta
n[c + d*x]^3 - (15*I)*(A - (3*I)*B)*Tan[c + d*x]^4 - (12*I)*B*Tan[c + d*x]
^5))/(60*d)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4077, 3042, 4077, 3042, 4075, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^2(a+ia\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4077}$$

$$\frac{1}{5}\int \tan^2(c+dx)(i\tan(c+dx)a+a)^2(a(5A-3iB)+a(5iA+7B)\tan(c+dx))dx+\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \tan(c+dx)^2(i\tan(c+dx)a+a)^2(a(5A-3iB)+a(5iA+7B)\tan(c+dx))dx+\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\downarrow$ 4077

$$\frac{1}{5}\left(\frac{1}{4}\int \tan^2(c+dx)(i\tan(c+dx)a+a)\left((35A-33iB)a^2+(45iA+47B)\tan(c+dx)a^2\right)dx-\frac{(5A-7iB)\tan}{}\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\int \tan(c+dx)^2(i\tan(c+dx)a+a)\left((35A-33iB)a^2+(45iA+47B)\tan(c+dx)a^2\right)dx-\frac{(5A-7iB)\tan}{}\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\downarrow$ 4075

$$\frac{1}{5}\left(\frac{1}{4}\left(\int \tan^2(c+dx)\left(80(A-iB)a^3+80(iA+B)\tan(c+dx)a^3\right)dx-\frac{a^3(45A-47iB)\tan^3(c+dx)}{3d}\right)-\frac{(5A}{}\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(\int \tan(c+dx)^2\left(80(A-iB)a^3+80(iA+B)\tan(c+dx)a^3\right)dx-\frac{a^3(45A-47iB)\tan^3(c+dx)}{3d}\right)-\frac{(5A}{}\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\downarrow$ 4011

$$\frac{1}{5}\left(\frac{1}{4}\left(\int \tan(c+dx)\left(80a^3(A-iB)\tan(c+dx)-80a^3(iA+B)\right)dx-\frac{a^3(45A-47iB)\tan^3(c+dx)}{3d}+\frac{40a^3(B}{}\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(\int \tan(c+dx)\left(80a^3(A-iB)\tan(c+dx)-80a^3(iA+B)\right)dx-\frac{a^3(45A-47iB)\tan^3(c+dx)}{3d}+\frac{40a^3(B\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\Big\downarrow$ 4008

$$\frac{1}{5}\left(\frac{1}{4}\left(-80a^3(B+iA)\int \tan(c+dx)dx-\frac{a^3(45A-47iB)\tan^3(c+dx)}{3d}+\frac{40a^3(B+iA)\tan^2(c+dx)}{d}+\frac{80a^3(\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(-80a^3(B+iA)\int \tan(c+dx)dx-\frac{a^3(45A-47iB)\tan^3(c+dx)}{3d}+\frac{40a^3(B+iA)\tan^2(c+dx)}{d}+\frac{80a^3(\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\Big\downarrow$ 3956

$$\frac{1}{5}\left(\frac{1}{4}\left(-\frac{a^3(45A-47iB)\tan^3(c+dx)}{3d}+\frac{40a^3(B+iA)\tan^2(c+dx)}{d}+\frac{80a^3(A-iB)\tan(c+dx)}{d}+\frac{80a^3(B+i\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

input
```
Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
((I/5)*a*B*Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^2)/d + (-1/4*((5*A - (7*I
)*B)*Tan[c + d*x]^3*(a^3 + I*a^3*Tan[c + d*x]))/d + (-80*a^3*(A - I*B)*x +
 (80*a^3*(I*A + B)*Log[Cos[c + d*x]])/d + (80*a^3*(A - I*B)*Tan[c + d*x])/
d + (40*a^3*(I*A + B)*Tan[c + d*x]^2)/d - (a^3*(45*A - (47*I)*B)*Tan[c + d
*x]^3)/(3*d))/4)/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{a^3\left(-\frac{iB\tan(dx+c)^5}{5}-\frac{iA\tan(dx+c)^4}{4}+\frac{4iB\tan(dx+c)^3}{3}-\frac{3B\tan(dx+c)^4}{4}+2iA\tan(dx+c)^2-A\tan(dx+c)^3-4iB\tan(dx+c\right.}{d}$
default	$\frac{a^3\left(-\frac{iB\tan(dx+c)^5}{5}-\frac{iA\tan(dx+c)^4}{4}+\frac{4iB\tan(dx+c)^3}{3}-\frac{3B\tan(dx+c)^4}{4}+2iA\tan(dx+c)^2-A\tan(dx+c)^3-4iB\tan(dx+c\right.}{d}$
norman	$(4iB\,a^3-4A\,a^3)\,x-\frac{(iA\,a^3+3B\,a^3)\tan(dx+c)^4}{4d}-\frac{(-4iB\,a^3+3A\,a^3)\tan(dx+c)^3}{3d}+\frac{4(-iB\,a^3+A\,a^3)\tan}{d}$
parallelrisch	$-\frac{12iB\,a^3\tan(dx+c)^5+15iA\tan(dx+c)^4a^3-80iB\tan(dx+c)^3a^3+45B\tan(dx+c)^4a^3-120iA\tan(dx+c)^2a^3+60A\tan}{}$
risch	$-\frac{8ia^3Bc}{d}+\frac{8a^3Ac}{d}+\frac{2a^3\left(180iA\,e^{8i(dx+c)}+240B\,e^{8i(dx+c)}+525iA\,e^{6i(dx+c)}+585B\,e^{6i(dx+c)}+615iA\,e^{4i(dx+c)}+\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$
parts	$\frac{(-iA\,a^3-3B\,a^3)\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{(3iA\,a^3+B\,a^3)\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right.}{d}$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*a^3*(-1/5*I*B*tan(d*x+c)^5-1/4*I*A*tan(d*x+c)^4+4/3*I*B*tan(d*x+c)^3-3
/4*B*tan(d*x+c)^4+2*I*A*tan(d*x+c)^2-A*tan(d*x+c)^3-4*I*B*tan(d*x+c)+2*B*t
an(d*x+c)^2+4*A*tan(d*x+c)+1/2*(-4*I*A-4*B)*ln(1+tan(d*x+c)^2)+(4*I*B-4*A)
*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.60

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(60\left(-3i\,A-4\,B\right)a^3e^{(8i\,dx+8i\,c)}+15\left(-35i\,A-39\,B\right)a^3e^{(6i\,dx+6i\,c)}+5\left(-123i\,A-139\,B\right)a^3e^{(4i\,dx+4i\,c}\right.}{}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-2/15*(60*(-3*I*A - 4*B)*a^3*e^(8*I*d*x + 8*I*c) + 15*(-35*I*A - 39*B)*a^3
*e^(6*I*d*x + 6*I*c) + 5*(-123*I*A - 139*B)*a^3*e^(4*I*d*x + 4*I*c) + 5*(-
69*I*A - 77*B)*a^3*e^(2*I*d*x + 2*I*c) + (-75*I*A - 83*B)*a^3 + 30*((-I*A
- B)*a^3*e^(10*I*d*x + 10*I*c) + 5*(-I*A - B)*a^3*e^(8*I*d*x + 8*I*c) + 10
*(-I*A - B)*a^3*e^(6*I*d*x + 6*I*c) + 10*(-I*A - B)*a^3*e^(4*I*d*x + 4*I*c
) + 5*(-I*A - B)*a^3*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^3)*log(e^(2*I*d*x
+ 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e
^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) +
d)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.60

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{4ia^3(A-iB)\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$
$$+ \frac{150iAa^3+166Ba^3+\left(690iAa^3e^{2ic}+770Ba^3e^{2ic}\right)e^{2idx}+\left(1230iAa^3e^{4ic}+1390Ba^3e^{4ic}\right)e^{4idx}+(1050i}{15de^{10ic}e^{10idx}+75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}+150de^{4ic}e^{4idx}}$$

input
```
integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
4*I*a**3*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (150*I*A*a**3 + 166
*B*a**3 + (690*I*A*a**3*exp(2*I*c) + 770*B*a**3*exp(2*I*c))*exp(2*I*d*x) +
 (1230*I*A*a**3*exp(4*I*c) + 1390*B*a**3*exp(4*I*c))*exp(4*I*d*x) + (1050*
I*A*a**3*exp(6*I*c) + 1170*B*a**3*exp(6*I*c))*exp(6*I*d*x) + (360*I*A*a**3
*exp(8*I*c) + 480*B*a**3*exp(8*I*c))*exp(8*I*d*x))/(15*d*exp(10*I*c)*exp(1
0*I*d*x) + 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d*x) +
150*d*exp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) + 15*d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{12i\,Ba^3\tan{(dx+c)}^5+15\,(i\,A+3\,B)a^3\tan{(dx+c)}^4+20\,(3\,A-4i\,B)a^3\tan{(dx+c)}^3+120\,(-i\,A\ldots}{}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/60*(12*I*B*a^3*tan(d*x + c)^5 + 15*(I*A + 3*B)*a^3*tan(d*x + c)^4 + 20*
(3*A - 4*I*B)*a^3*tan(d*x + c)^3 + 120*(-I*A - B)*a^3*tan(d*x + c)^2 + 240
*(d*x + c)*(A - I*B)*a^3 + 120*(I*A + B)*a^3*log(tan(d*x + c)^2 + 1) - 240
*(A - I*B)*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= -\frac{4\,(i\,Aa^3+Ba^3)\log{(\tan{(dx+c)}+i)}}{d}$$

$$-\frac{12i\,Ba^3d^4\tan{(dx+c)}^5+15i\,Aa^3d^4\tan{(dx+c)}^4+45\,Ba^3d^4\tan{(dx+c)}^4+60\,Aa^3d^4\tan{(dx+c)}^3\ldots}{}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-4*(I*A*a^3 + B*a^3)*log(tan(d*x + c) + I)/d - 1/60*(12*I*B*a^3*d^4*tan(d*
x + c)^5 + 15*I*A*a^3*d^4*tan(d*x + c)^4 + 45*B*a^3*d^4*tan(d*x + c)^4 + 6
0*A*a^3*d^4*tan(d*x + c)^3 - 80*I*B*a^3*d^4*tan(d*x + c)^3 - 120*I*A*a^3*d
^4*tan(d*x + c)^2 - 120*B*a^3*d^4*tan(d*x + c)^2 - 240*A*a^3*d^4*tan(d*x +
 c) + 240*I*B*a^3*d^4*tan(d*x + c))/d^5
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.26

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{\tan(c+d\,x)^3\,\left(\frac{B\,a^3\,1\mathrm{i}}{3}-\frac{a^3\,(2\,A-B\,1\mathrm{i})}{3}+\frac{a^3\,(2\,B+A\,1\mathrm{i})\,1\mathrm{i}}{3}\right)}{d}+\frac{\tan(c+d\,x)\,\left(A\,a^3-B\,a^3\,1\mathrm{i}+a^3\,(2\,A-B\,1\mathrm{i})-a^3\,(2\,B+A\,1\mathrm{i})\,1\mathrm{i}\right)}{d}-\frac{\tan(c+d\,x)^4\,\left(\frac{B\,a^3}{4}+\frac{a^3\,(2\,B+A\,1\mathrm{i})}{4}\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(4\,B\,a^3+A\,a^3\,4\mathrm{i}\right)}{d}+\frac{\tan(c+d\,x)^2\,\left(\frac{A\,a^3\,1\mathrm{i}}{2}+\frac{B\,a^3}{2}+\frac{a^3\,(2\,A-B\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{a^3\,(2\,B+A\,1\mathrm{i})}{2}\right)}{d}-\frac{B\,a^3\,\tan(c+d\,x)^5\,1\mathrm{i}}{5\,d}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(tan(c + d*x)^3*((B*a^3*1i)/3 - (a^3*(2*A - B*1i))/3 + (a^3*(A*1i + 2*B)*1
i)/3))/d + (tan(c + d*x)*(A*a^3 - B*a^3*1i + a^3*(2*A - B*1i) - a^3*(A*1i
+ 2*B)*1i))/d - (tan(c + d*x)^4*((B*a^3)/4 + (a^3*(A*1i + 2*B))/4))/d - (l
og(tan(c + d*x) + 1i)*(A*a^3*4i + 4*B*a^3))/d + (tan(c + d*x)^2*((A*a^3*1i
)/2 + (B*a^3)/2 + (a^3*(2*A - B*1i)*1i)/2 + (a^3*(A*1i + 2*B))/2))/d - (B*
a^3*tan(c + d*x)^5*1i)/(5*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.82

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{a^3\left(-120\log\left(\tan\left(dx+c\right)^2+1\right)ai-120\log\left(\tan\left(dx+c\right)^2+1\right)b-12\tan\left(dx+c\right)^5bi-15\tan\left(dx+c\right)\right.}{}$$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*( - 120*log(tan(c + d*x)**2 + 1)*a*i - 120*log(tan(c + d*x)**2 + 1)*
b - 12*tan(c + d*x)**5*b*i - 15*tan(c + d*x)**4*a*i - 45*tan(c + d*x)**4*b
 - 60*tan(c + d*x)**3*a + 80*tan(c + d*x)**3*b*i + 120*tan(c + d*x)**2*a*i
 + 120*tan(c + d*x)**2*b + 240*tan(c + d*x)*a - 240*tan(c + d*x)*b*i - 240
*a*d*x + 240*b*d*i*x))/(60*d)
```

3.18 $\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 138

$$\begin{aligned}\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= -4a^3(iA+B)x-\frac{4a^3(A-iB)\log(\cos(c+dx))}{d}\\ &\quad+\frac{2a^3(iA+B)\tan(c+dx)}{d}+\frac{a(A-iB)(a+ia\tan(c+dx))^2}{2d}\\ &\quad+\frac{A(a+ia\tan(c+dx))^3}{3d}-\frac{iB(a+ia\tan(c+dx))^4}{4ad}\end{aligned}$$

output
```
-4*a^3*(I*A+B)*x-4*a^3*(A-I*B)*ln(cos(d*x+c))/d+2*a^3*(I*A+B)*tan(d*x+c)/d
+1/2*a*(A-I*B)*(a+I*a*tan(d*x+c))^2/d+1/3*A*(a+I*a*tan(d*x+c))^3/d-1/4*I*B
*(a+I*a*tan(d*x+c))^4/a/d
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3(4A-3iB+48(A-iB)\log(i+\tan(c+dx))+48(iA+B)\tan(c+dx)-6(3A-4iB)\tan^2(c+dx)}{12d}$$

input
```
Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(a^3*(4*A - (3*I)*B + 48*(A - I*B)*Log[I + Tan[c + d*x]] + 48*(I*A + B)*Ta
n[c + d*x] - 6*(3*A - (4*I)*B)*Tan[c + d*x]^2 - (4*I)*(A - (3*I)*B)*Tan[c
+ d*x]^3 - (3*I)*B*Tan[c + d*x]^4))/(12*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4075, 3042, 4010, 3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int (i\tan(c+dx)a+a)^3(A\tan(c+dx)-B)dx-\frac{iB(a+ia\tan(c+dx))^4}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$\int (i\tan(c+dx)a+a)^3(A\tan(c+dx)-B)dx-\frac{iB(a+ia\tan(c+dx))^4}{4ad}$$

$$\Big\downarrow \text{4010}$$

$$-(B+iA)\int (i\tan(c+dx)a+a)^3dx+\frac{A(a+ia\tan(c+dx))^3}{3d}-\frac{iB(a+ia\tan(c+dx))^4}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\int (i\tan(c+dx)a+a)^3dx+\frac{A(a+ia\tan(c+dx))^3}{3d}-\frac{iB(a+ia\tan(c+dx))^4}{4ad}$$

$$\Big\downarrow \text{3959}$$

$$-(B+iA)\left(2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{A(a+ia\tan(c+dx))^3}{3d}-\frac{iB(a+ia\tan(c+dx))^4}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\left(2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{A(a+ia\tan(c+dx))^3}{3d}-\frac{iB(a+ia\tan(c+dx))^4}{4ad}$$

$$\Big\downarrow \text{3958}$$

$$-(B+iA)\left(2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{A(a+ia\tan(c+dx))^3}{3d}-\frac{iB(a+ia\tan(c+dx))^4}{4ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\left(2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{A(a+ia\tan(c+dx))^3}{3d}-\frac{iB(a+ia\tan(c+dx))^4}{4ad}$$

$$\Big\downarrow \text{3956}$$

$$-(B+iA)\left(2a\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{A(a+ia\tan(c+dx))^3}{3d}-\frac{iB(a+ia\tan(c+dx))^4}{4ad}$$

input

```
Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(A*(a + I*a*Tan[c + d*x])^3)/(3*d) - ((I/4)*B*(a + I*a*Tan[c + d*x])^4)/(a
*d) - (I*A + B)*(((I/2)*a*(a + I*a*Tan[c + d*x])^2)/d + 2*a*(2*a^2*x - ((2
*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{a^3\left(-\frac{iB\tan(dx+c)^4}{4}-\frac{iA\tan(dx+c)^3}{3}+2iB\tan(dx+c)^2-B\tan(dx+c)^3+4iA\tan(dx+c)-\frac{3A\tan(dx+c)^2}{2}+4B\tan(dx+c\right.}{d}$
default	$\frac{a^3\left(-\frac{iB\tan(dx+c)^4}{4}-\frac{iA\tan(dx+c)^3}{3}+2iB\tan(dx+c)^2-B\tan(dx+c)^3+4iA\tan(dx+c)-\frac{3A\tan(dx+c)^2}{2}+4B\tan(dx+c\right.}{d}$
norman	$(-4iA\,a^3-4B\,a^3)\,x-\frac{(iA\,a^3+3B\,a^3)\tan(dx+c)^3}{3d}-\frac{(-4iB\,a^3+3A\,a^3)\tan(dx+c)^2}{2d}+\frac{4(iA\,a^3+B\,a^3)\,\mathrm{ta}}{d}$
parallelrisch	$\frac{-3iB\,a^3\tan(dx+c)^4-4iA\tan(dx+c)^3a^3-48iAx\,a^3d+24iB\tan(dx+c)^2a^3-12B\tan(dx+c)^3a^3+48iA\tan(dx+c)a^3-}{12d}$
risch	$\frac{8a^3Bc}{d}+\frac{8ia^3Ac}{d}+\frac{2ia^3\left(24iA\,e^{6i(dx+c)}+36B\,e^{6i(dx+c)}+57iA\,e^{4i(dx+c)}+69B\,e^{4i(dx+c)}+46iA\,e^{2i(dx+c)}+54B\,e^{2i(}\right.}{3d\left(e^{2i(dx+c)}+1\right)^4}$
parts	$\frac{(-iA\,a^3-3B\,a^3)\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{(3iA\,a^3+B\,a^3)(\tan(dx+c)-\arctan(\tan(dx+c))}{d}$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*a^3*(-1/4*I*B*tan(d*x+c)^4-1/3*I*A*tan(d*x+c)^3+2*I*B*tan(d*x+c)^2-B*t
an(d*x+c)^3+4*I*A*tan(d*x+c)-3/2*A*tan(d*x+c)^2+4*B*tan(d*x+c)+1/2*(-4*I*B
+4*A)*ln(1+tan(d*x+c)^2)+(-4*I*A-4*B)*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.64

$$\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(12\,(2\,A-3i\,B)a^3e^{(6i\,dx+6i\,c)}+3\,(19\,A-23i\,B)a^3e^{(4i\,dx+4i\,c)}+2\,(23\,A-27i\,B)a^3e^{(2i\,dx+2i\,c)}+(13\,A\right.}{3\,(de^{(8}}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-2/3*(12*(2*A - 3*I*B)*a^3*e^(6*I*d*x + 6*I*c) + 3*(19*A - 23*I*B)*a^3*e^(
4*I*d*x + 4*I*c) + 2*(23*A - 27*I*B)*a^3*e^(2*I*d*x + 2*I*c) + (13*A - 15*
I*B)*a^3 + 6*((A - I*B)*a^3*e^(8*I*d*x + 8*I*c) + 4*(A - I*B)*a^3*e^(6*I*d
*x + 6*I*c) + 6*(A - I*B)*a^3*e^(4*I*d*x + 4*I*c) + 4*(A - I*B)*a^3*e^(2*I
*d*x + 2*I*c) + (A - I*B)*a^3)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x
 + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I
*d*x + 2*I*c) + d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(114) = 228.

Time = 0.46 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.70

$$\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= -\frac{4a^3(A-iB)\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-26Aa^3+30iBa^3+\left(-92Aa^3e^{2ic}+108iBa^3e^{2ic}\right)e^{2idx}+\left(-114Aa^3e^{4ic}+138iBa^3e^{4ic}\right)e^{4idx}+(-48A}{3de^{8ic}e^{8idx}+12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}+12de^{2ic}e^{2idx}+3d}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
-4*a**3*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-26*A*a**3 + 30*I*B
*a**3 + (-92*A*a**3*exp(2*I*c) + 108*I*B*a**3*exp(2*I*c))*exp(2*I*d*x) + (
-114*A*a**3*exp(4*I*c) + 138*I*B*a**3*exp(4*I*c))*exp(4*I*d*x) + (-48*A*a*
*3*exp(6*I*c) + 72*I*B*a**3*exp(6*I*c))*exp(6*I*d*x))/(3*d*exp(8*I*c)*exp(
8*I*d*x) + 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d*x) + 1
2*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{3i\,Ba^3\tan\left(dx+c\right)^4+4\left(i\,A+3\,B\right)a^3\tan\left(dx+c\right)^3+6\left(3\,A-4i\,B\right)a^3\tan\left(dx+c\right)^2+48\left(dx+c\right)(i\,A}{12\,d}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/12*(3*I*B*a^3*tan(d*x + c)^4 + 4*(I*A + 3*B)*a^3*tan(d*x + c)^3 + 6*(3*
A - 4*I*B)*a^3*tan(d*x + c)^2 + 48*(d*x + c)*(I*A + B)*a^3 - 24*(A - I*B)*
a^3*log(tan(d*x + c)^2 + 1) + 48*(-I*A - B)*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

$$\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{4\left(Aa^3-i\,Ba^3\right)\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{3i\,Ba^3d^3\tan\left(dx+c\right)^4+4i\,Aa^3d^3\tan\left(dx+c\right)^3+12\,Ba^3d^3\tan\left(dx+c\right)^3+18\,Aa^3d^3\tan\left(dx+c\right)^2-}{12\,d^4}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
4*(A*a^3 - I*B*a^3)*log(tan(d*x + c) + I)/d - 1/12*(3*I*B*a^3*d^3*tan(d*x
+ c)^4 + 4*I*A*a^3*d^3*tan(d*x + c)^3 + 12*B*a^3*d^3*tan(d*x + c)^3 + 18*A
*a^3*d^3*tan(d*x + c)^2 - 24*I*B*a^3*d^3*tan(d*x + c)^2 - 48*I*A*a^3*d^3*t
an(d*x + c) - 48*B*a^3*d^3*tan(d*x + c))/d^4
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.28

$$\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{\tan(c+d\,x)^2\,\left(\frac{B\,a^3\,1\mathrm{i}}{2}-\frac{a^3\,\left(2\,A-B\,1\mathrm{i}\right)}{2}+\frac{a^3\,\left(2\,B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{2}\right)}{d}+\frac{\tan(c+d\,x)\,\left(A\,a^3\,1\mathrm{i}+B\,a^3+a^3\,\left(2\,A-B\,1\mathrm{i}\right)\,1\mathrm{i}+a^3\,\left(2\,B+A\,1\mathrm{i}\right)\right)}{d}-\frac{\tan(c+d\,x)^3\,\left(\frac{B\,a^3}{3}+\frac{a^3\,\left(2\,B+A\,1\mathrm{i}\right)}{3}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(4\,A\,a^3-B\,a^3\,4\mathrm{i}\right)}{d}-\frac{B\,a^3\,\tan(c+d\,x)^4\,1\mathrm{i}}{4\,d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(tan(c + d*x)^2*((B*a^3*1i)/2 - (a^3*(2*A - B*1i))/2 + (a^3*(A*1i + 2*B)*1
i)/2))/d + (tan(c + d*x)*(A*a^3*1i + B*a^3 + a^3*(2*A - B*1i)*1i + a^3*(A*
1i + 2*B)))/d - (tan(c + d*x)^3*((B*a^3)/3 + (a^3*(A*1i + 2*B))/3))/d + (l
og(tan(c + d*x) + 1i)*(4*A*a^3 - B*a^3*4i))/d - (B*a^3*tan(c + d*x)^4*1i)/
(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \tan(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{a^3\left(24\log\left(\tan\left(dx+c\right)^2+1\right)a-24\log\left(\tan\left(dx+c\right)^2+1\right)bi-3\tan\left(dx+c\right)^4bi-4\tan\left(dx+c\right)^3ai-\right.}{}$$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output

```
(a**3*(24*log(tan(c + d*x)**2 + 1)*a - 24*log(tan(c + d*x)**2 + 1)*b*i - 3
*tan(c + d*x)**4*b*i - 4*tan(c + d*x)**3*a*i - 12*tan(c + d*x)**3*b - 18*t
an(c + d*x)**2*a + 24*tan(c + d*x)**2*b*i + 48*tan(c + d*x)*a*i + 48*tan(c
 + d*x)*b - 48*a*d*i*x - 48*b*d*x))/(12*d)
```

3.19 $\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = 4a^3(A-iB)x-\frac{4a^3(iA+B)\log(\cos(c+dx))}{d}-\frac{2a^3(A-iB)\tan(c+dx)}{d}+\frac{a(iA+B)(a+ia\tan(c+dx))^2}{2d}+\frac{B(a+ia\tan(c+dx))^3}{3d}$$

output
```
4*a^3*(A-I*B)*x-4*a^3*(I*A+B)*ln(cos(d*x+c))/d-2*a^3*(A-I*B)*tan(d*x+c)/d+
1/2*a*(I*A+B)*(a+I*a*tan(d*x+c))^2/d+1/3*B*(a+I*a*tan(d*x+c))^3/d
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

$$\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{B(a+ia\tan(c+dx))^3+\frac{3}{2}a^3(iA+B)\left(8\log(i+\tan(c+dx))+6i\tan(c+dx)-\tan^2(c+dx)\right)}{3d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(B*(a + I*a*Tan[c + d*x])^3 + (3*a^3*(I*A + B)*(8*Log[I + Tan[c + d*x]] +
(6*I)*Tan[c + d*x] - Tan[c + d*x]^2))/2)/(3*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 4010, 3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4010}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^3dx+\frac{B(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^3dx+\frac{B(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3959}$$

$$(A-iB)\left(2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{B(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\left(2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{B(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3958}$$

$$(A-iB)\left(2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{B(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\left(2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{B(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3956}$$

$$(A-iB)\left(2a\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{B(a+ia\tan(c+dx))^3}{3d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(B*(a + I*a*Tan[c + d*x])^3)/(3*d) + (A - I*B)*(((I/2)*a*(a + I*a*Tan[c +
d*x])^2)/d + 2*a*(2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c +
 d*x])/d))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a^3\left(-\frac{iB\tan(dx+c)^3}{3}-\frac{iA\tan(dx+c)^2}{2}+4iB\tan(dx+c)-\frac{3B\tan(dx+c)^2}{2}-3A\tan(dx+c)+\frac{(4iA+4B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-\right.}{d}$
default	$\frac{a^3\left(-\frac{iB\tan(dx+c)^3}{3}-\frac{iA\tan(dx+c)^2}{2}+4iB\tan(dx+c)-\frac{3B\tan(dx+c)^2}{2}-3A\tan(dx+c)+\frac{(4iA+4B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-\right.}{d}$
norman	$(-4iB\,a^3+4A\,a^3)\,x-\frac{(iA\,a^3+3B\,a^3)\tan(dx+c)^2}{2d}-\frac{(-4iB\,a^3+3A\,a^3)\tan(dx+c)}{d}-\frac{iB\,a^3\tan(dx+c)^3}{3d}$
parallelrisch	$\frac{-2iB\tan(dx+c)^3a^3-3iA\tan(dx+c)^2a^3-24iBx\,a^3d+12iA\ln\left(1+\tan(dx+c)^2\right)a^3+24Ax\,a^3d+24iB\tan(dx+c)a^3-9}{6d}$
risch	$\frac{8ia^3Bc}{d}-\frac{8a^3Ac}{d}-\frac{2a^3\left(12iA\,\mathrm{e}^{4i(dx+c)}+24B\,\mathrm{e}^{4i(dx+c)}+21iA\,\mathrm{e}^{2i(dx+c)}+33B\,\mathrm{e}^{2i(dx+c)}+9iA+13B\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}-\frac{4a^3\ln(\mathrm{e}^{2}}{}$
parts	$A\,a^3x+\frac{(-iA\,a^3-3B\,a^3)\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{(3iA\,a^3+B\,a^3)\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{(3iB\,a^3-}{}$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*a^3*(-1/3*I*B*tan(d*x+c)^3-1/2*I*A*tan(d*x+c)^2+4*I*B*tan(d*x+c)-3/2*B
*tan(d*x+c)^2-3*A*tan(d*x+c)+1/2*(4*I*A+4*B)*ln(1+tan(d*x+c)^2)+(-4*I*B+4*
A)*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.59

$$\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{2\left(12\,(i\,A+2\,B)a^3e^{(4i\,dx+4i\,c)}+3\,(7i\,A+11\,B)a^3e^{(2i\,dx+2i\,c)}+(9i\,A+13\,B)a^3+6\left((i\,A+B)a^3e^{(6i\,dx+\ldots}\right.\right.}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}\ldots\right.}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-2/3*(12*(I*A + 2*B)*a^3*e^(4*I*d*x + 4*I*c) + 3*(7*I*A + 11*B)*a^3*e^(2*I
*d*x + 2*I*c) + (9*I*A + 13*B)*a^3 + 6*((I*A + B)*a^3*e^(6*I*d*x + 6*I*c)
+ 3*(I*A + B)*a^3*e^(4*I*d*x + 4*I*c) + 3*(I*A + B)*a^3*e^(2*I*d*x + 2*I*c
) + (I*A + B)*a^3)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*I*c) +
3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{4ia^3(A-iB)\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-18iAa^3-26Ba^3+\left(-42iAa^3e^{2ic}-66Ba^3e^{2ic}\right)e^{2idx}+\left(-24iAa^3e^{4ic}-48Ba^3e^{4ic}\right)e^{4idx}}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}$$

input
```
integrate((a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
-4*I*a**3*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-18*I*A*a**3 - 26
*B*a**3 + (-42*I*A*a**3*exp(2*I*c) - 66*B*a**3*exp(2*I*c))*exp(2*I*d*x) +
(-24*I*A*a**3*exp(4*I*c) - 48*B*a**3*exp(4*I*c))*exp(4*I*d*x))/(3*d*exp(6*
I*c)*exp(6*I*d*x) + 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d
*x) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.87

$$\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2i\,Ba^3\tan{(dx+c)}^3+3\,(i\,A+3\,B)a^3\tan{(dx+c)}^2-24\,(dx+c)(A-i\,B)a^3+12\,(-i\,A-B)a^3\log{(}}{6\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/6*(2*I*B*a^3*tan(d*x + c)^3 + 3*(I*A + 3*B)*a^3*tan(d*x + c)^2 - 24*(d*
x + c)*(A - I*B)*a^3 + 12*(-I*A - B)*a^3*log(tan(d*x + c)^2 + 1) + 6*(3*A
- 4*I*B)*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{4\,(-i\,Aa^3-Ba^3)\log{(\tan{(dx+c)}+i)}}{d}$$
$$-\frac{2i\,Ba^3d^2\tan{(dx+c)}^3+3i\,Aa^3d^2\tan{(dx+c)}^2+9\,Ba^3d^2\tan{(dx+c)}^2+18\,Aa^3d^2\tan{(dx+c)}-24}{6\,d^3}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
-4*(-I*A*a^3 - B*a^3)*log(tan(d*x + c) + I)/d - 1/6*(2*I*B*a^3*d^2*tan(d*x
 + c)^3 + 3*I*A*a^3*d^2*tan(d*x + c)^2 + 9*B*a^3*d^2*tan(d*x + c)^2 + 18*A
*a^3*d^2*tan(d*x + c) - 24*I*B*a^3*d^2*tan(d*x + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

$$\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= -\frac{\tan(c+d\,x)^2\,\left(\frac{B\,a^3}{2}+\frac{a^3\,(2\,B+A\,1\mathrm{i})}{2}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(4\,B\,a^3+A\,a^3\,4\mathrm{i}\right)}{d}$$
$$+\frac{\tan(c+d\,x)\,\left(B\,a^3\,1\mathrm{i}-a^3\,(2\,A-B\,1\mathrm{i})+a^3\,(2\,B+A\,1\mathrm{i})\,1\mathrm{i}\right)}{d}$$
$$-\frac{B\,a^3\,\tan(c+d\,x)^3\,1\mathrm{i}}{3\,d}$$

input
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(log(tan(c + d*x) + 1i)*(A*a^3*4i + 4*B*a^3))/d - (tan(c + d*x)^2*((B*a^3)
/2 + (a^3*(A*1i + 2*B))/2))/d + (tan(c + d*x)*(B*a^3*1i - a^3*(2*A - B*1i)
 + a^3*(A*1i + 2*B)*1i))/d - (B*a^3*tan(c + d*x)^3*1i)/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int (a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3\left(12\log\left(\tan(dx+c)^2+1\right)ai+12\log\left(\tan(dx+c)^2+1\right)b-2\tan(dx+c)^3bi-3\tan(dx+c)^2ai-9\right.}{6d}$$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*(12*log(tan(c + d*x)**2 + 1)*a*i + 12*log(tan(c + d*x)**2 + 1)*b - 2
*tan(c + d*x)**3*b*i - 3*tan(c + d*x)**2*a*i - 9*tan(c + d*x)**2*b - 18*ta
n(c + d*x)*a + 24*tan(c + d*x)*b*i + 24*a*d*x - 24*b*d*i*x))/(6*d)
```

3.20 $\int \cot(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 107

$$\int \cot(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= 4a^3(iA+B)x + \frac{a^3(3A-4iB)\log(\cos(c+dx))}{d} + \frac{a^3A\log(\sin(c+dx))}{d} + \frac{iaB(a+ia\tan(c+dx))^2}{2d} - \frac{(A-2iB)\left(a^3+ia^3\tan(c+dx)\right)}{d}$$

output
```
4*a^3*(I*A+B)*x+a^3*(3*A-4*I*B)*ln(cos(d*x+c))/d+a^3*A*ln(sin(d*x+c))/d+1/
2*I*a*B*(a+I*a*tan(d*x+c))^2/d-(A-2*I*B)*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

$$\int \cot(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3(2A\log(\tan(c+dx)) - 8(A-iB)\log(i+\tan(c+dx)) + (-2iA-6B)\tan(c+dx) - iB\tan^2(c+dx}{2d}$$

input
```
Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(a^3*(2*A*Log[Tan[c + d*x]] - 8*(A - I*B)*Log[I + Tan[c + d*x]] + ((-2*I)*
A - 6*B)*Tan[c + d*x] - I*B*Tan[c + d*x]^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {3042, 4077, 27, 3042, 4077, 3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{2}\int 2\cot(c+dx)(i\tan(c+dx)a+a)^2(aA+a(iA+2B)\tan(c+dx))dx+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)^2(aA+a(iA+2B)\tan(c+dx))dx+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^2(aA+a(iA+2B)\tan(c+dx))}{\tan(c+dx)}dx+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4077}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)\left(Aa^2+(3iA+4B)\tan(c+dx)a^2\right)dx-\frac{(A-2iB)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)\left(Aa^2+(3iA+4B)\tan(c+dx)a^2\right)}{\tan(c+dx)}dx-\frac{(A-2iB)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4072}$$

$$-\left(a^3(3A-4iB)\int\tan(c+dx)dx\right)+\int\cot(c+dx)\left(Aa^3+4(iA+B)\tan(c+dx)a^3\right)dx-\frac{(A-2iB)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\left(a^3(3A-4iB)\int\tan(c+dx)dx\right)+\int\frac{Aa^3+4(iA+B)\tan(c+dx)a^3}{\tan(c+dx)}dx-\frac{(A-2iB)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3956}$$

$$\int\frac{Aa^3+4(iA+B)\tan(c+dx)a^3}{\tan(c+dx)}dx-\frac{(A-2iB)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{a^3(3A-4iB)\log(\cos(c+dx))}{d}+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4014}$$

$$a^3A\int\cot(c+dx)dx-\frac{(A-2iB)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{a^3(3A-4iB)\log(\cos(c+dx))}{d}+4a^3x(B+iA)+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$a^3A\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{(A-2iB)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{a^3(3A-4iB)\log(\cos(c+dx))}{d}+4a^3x(B+iA)+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{25}$$

$$-a^3A\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{(A-2iB)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{a^3(3A-4iB)\log(\cos(c+dx))}{d}+4a^3x(B+iA)+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(A-2iB)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{a^3(3A-4iB)\log(\cos(c+dx))}{d}+4a^3x(B+iA)+\frac{a^3A\log(-\sin(c+dx))}{d}+\frac{iaB(a+ia\tan(c+dx))^2}{2d}$$

input
```
Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
4*a^3*(I*A + B)*x + (a^3*(3*A - (4*I)*B)*Log[Cos[c + d*x]])/d + (a^3*A*Log
[-Sin[c + d*x]])/d + ((I/2)*a*B*(a + I*a*Tan[c + d*x])^2)/d - ((A - (2*I)*
B)*(a^3 + I*a^3*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{4 a^{3} \left(\frac{\left(i B -A \right) \ln \left(\sec \left(d x +c \right)^{2}\right)}{2}+\frac{A \ln \left(\tan \left(d x +c \right)\right)}{4}-\frac{i B \tan \left(d x +c \right)^{2}}{8}-\frac{\left(i A +3 B \right) \tan \left(d x +c \right)}{4}+\left(i A +B \right) x d \right)}{d}$
derivativedivides	$\frac{a^{3} \left(\frac{\left(4 i B -4 A \right) \ln \left(\cot \left(d x +c \right)^{2}+1\right)}{2}+\left(-4 i A -4 B \right) \left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot \left(d x +c \right)\right)\right)+\left(-4 i B +3 A \right) \ln \left(\cot \left(d x +c \right)\right)-\frac{i A +3 B}{\cot \left(d x +c \right)}-\frac{\cdots}{2 \cot (\cdots}\right.}{d}$
default	$\frac{a^{3} \left(\frac{\left(4 i B -4 A \right) \ln \left(\cot \left(d x +c \right)^{2}+1\right)}{2}+\left(-4 i A -4 B \right) \left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot \left(d x +c \right)\right)\right)+\left(-4 i B +3 A \right) \ln \left(\cot \left(d x +c \right)\right)-\frac{i A +3 B}{\cot \left(d x +c \right)}-\frac{\cdots}{2 \cot (\cdots}\right.}{d}$
norman	$\left(4 i A \,a^{3}+4 B \,a^{3}\right) x -\frac{\left(i A \,a^{3}+3 B \,a^{3}\right) \tan \left(d x +c \right)}{d}-\frac{i B \,a^{3} \tan \left(d x +c \right)^{2}}{2 d}+\frac{A \,a^{3} \ln \left(\tan \left(d x +c \right)\right)}{d}-\frac{2 \left(-i B \,a^{3} \cdots\right.}{\cdots}$
risch	$-\frac{8 a^{3} B c}{d}-\frac{8 i a^{3} A c}{d}-\frac{2 i a^{3} \left(i A \,{\mathrm e}^{2 i \left(d x +c \right)}+4 B \,{\mathrm e}^{2 i \left(d x +c \right)}+i A +3 B \right)}{d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{2}}-\frac{4 i a^{3} \ln \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right) B}{d}+\frac{3 a^{3} \ln \left({\mathrm e}^{2 i \left(d x +c \right)} \cdots\right.}{d}$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
4*a^3/d*(1/2*(-A+I*B)*ln(sec(d*x+c)^2)+1/4*A*ln(tan(d*x+c))-1/8*I*B*tan(d*
x+c)^2-1/4*(I*A+3*B)*tan(d*x+c)+(I*A+B)*x*d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.61

$$\int \cot(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{2\,(A-4i\,B)a^3e^{(2i\,dx+2i\,c)}+2\,(A-3i\,B)a^3+\left((3\,A-4i\,B)a^3e^{(4i\,dx+4i\,c)}+2\,(3\,A-4i\,B)a^3e^{(2i\,dx+2i\,c)}+(\right.}{de^{(4i\,dx+4i\,c)}+2\,de}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
(2*(A - 4*I*B)*a^3*e^(2*I*d*x + 2*I*c) + 2*(A - 3*I*B)*a^3 + ((3*A - 4*I*B
)*a^3*e^(4*I*d*x + 4*I*c) + 2*(3*A - 4*I*B)*a^3*e^(2*I*d*x + 2*I*c) + (3*A
 - 4*I*B)*a^3)*log(e^(2*I*d*x + 2*I*c) + 1) + (A*a^3*e^(4*I*d*x + 4*I*c) +
 2*A*a^3*e^(2*I*d*x + 2*I*c) + A*a^3)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(
4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(94) = 188.

Time = 1.73 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.09

$$\int \cot(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{Aa^3\log\left(\frac{-Aa^3+2iBa^3}{Aa^3e^{2ic}-2iBa^3e^{2ic}}+e^{2idx}\right)}{d} + \frac{a^3\cdot(3A-4iB)\log\left(e^{2idx}+\frac{-2Aa^3+2iBa^3+a^3\cdot(3A-4iB)}{Aa^3e^{2ic}-2iBa^3e^{2ic}}\right)}{d} + \frac{2Aa^3-6iBa^3+\left(2Aa^3e^{2ic}-8iBa^3e^{2ic}\right)e^{2idx}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
A*a**3*log((-A*a**3 + 2*I*B*a**3)/(A*a**3*exp(2*I*c) - 2*I*B*a**3*exp(2*I*
c)) + exp(2*I*d*x))/d + a**3*(3*A - 4*I*B)*log(exp(2*I*d*x) + (-2*A*a**3 +
 2*I*B*a**3 + a**3*(3*A - 4*I*B))/(A*a**3*exp(2*I*c) - 2*I*B*a**3*exp(2*I*
c)))/d + (2*A*a**3 - 6*I*B*a**3 + (2*A*a**3*exp(2*I*c) - 8*I*B*a**3*exp(2*
I*c))*exp(2*I*d*x))/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*
x) + d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \cot(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{i\,Ba^3\tan\left(dx+c\right)^2+8\left(dx+c\right)(-i\,A-B)a^3+4\left(A-i\,B\right)a^3\log\left(\tan\left(dx+c\right)^2+1\right)-2\,Aa^3\log\left(\tan\right.}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/2*(I*B*a^3*tan(d*x + c)^2 + 8*(d*x + c)*(-I*A - B)*a^3 + 4*(A - I*B)*a^
3*log(tan(d*x + c)^2 + 1) - 2*A*a^3*log(tan(d*x + c)) + 2*(I*A + 3*B)*a^3*
tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84

$$\int \cot(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{Aa^3\log\left(|\tan\left(dx+c\right)|\right)}{d}-\frac{4\left(Aa^3-i\,Ba^3\right)\log\left(\tan\left(dx+c\right)+i\right)}{d} - \frac{i\,Ba^3d\tan\left(dx+c\right)^2+2i\,Aa^3d\tan\left(dx+c\right)+6\,Ba^3d\tan\left(dx+c\right)}{2\,d^2}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
A*a^3*log(abs(tan(d*x + c)))/d - 4*(A*a^3 - I*B*a^3)*log(tan(d*x + c) + I)
/d - 1/2*(I*B*a^3*d*tan(d*x + c)^2 + 2*I*A*a^3*d*tan(d*x + c) + 6*B*a^3*d*
tan(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \cot(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{A\,a^3\,\ln\left(\tan(c+d\,x)\right)}{d} - \frac{\tan(c+d\,x)\,\left(B\,a^3+a^3\,(2\,B+A\,1\mathrm{i})\right)}{d} - \frac{4\,a^3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,(A-B\,1\mathrm{i})}{d} - \frac{B\,a^3\tan(c+d\,x)^2\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(A*a^3*log(tan(c + d*x)))/d - (tan(c + d*x)*(B*a^3 + a^3*(A*1i + 2*B)))/d
- (4*a^3*log(tan(c + d*x) + 1i)*(A - B*1i))/d - (B*a^3*tan(c + d*x)^2*1i)/
(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.52

$$\int \cot(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3\left(2\cos\left(dx+c\right)\sin\left(dx+c\right)ai+6\cos\left(dx+c\right)\sin\left(dx+c\right)b-8\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+c\right)^2 c\right.}{}$$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*(2*cos(c + d*x)*sin(c + d*x)*a*i + 6*cos(c + d*x)*sin(c + d*x)*b - 8
*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a + 8*log(tan((c + d*x)/2)**
2 + 1)*sin(c + d*x)**2*b*i + 8*log(tan((c + d*x)/2)**2 + 1)*a - 8*log(tan(
(c + d*x)/2)**2 + 1)*b*i + 6*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a -
 8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b*i - 6*log(tan((c + d*x)/2)
- 1)*a + 8*log(tan((c + d*x)/2) - 1)*b*i + 6*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**2*a - 8*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b*i - 6*log(t
an((c + d*x)/2) + 1)*a + 8*log(tan((c + d*x)/2) + 1)*b*i + 2*log(tan((c +
d*x)/2))*sin(c + d*x)**2*a - 2*log(tan((c + d*x)/2))*a + 8*sin(c + d*x)**2
*a*d*i*x + 8*sin(c + d*x)**2*b*d*x + sin(c + d*x)**2*b*i - 8*a*d*i*x - 8*b
*d*x))/(2*d*(sin(c + d*x)**2 - 1))
```

3.21 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 116

$$\begin{aligned}\int \cot^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx\\ = -4a^3(A-iB)x+\frac{a^3(iA+3B)\log(\cos(c+dx))}{d}+\frac{a^3(3iA+B)\log(\sin(c+dx))}{d}\\ -\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}+\frac{(iA-B)\left(a^3+ia^3\tan(c+dx)\right)}{d}\end{aligned}$$

output

```
-4*a^3*(A-I*B)*x+a^3*(I*A+3*B)*ln(cos(d*x+c))/d+a^3*(3*I*A+B)*ln(sin(d*x+c
))/d-a*A*cot(d*x+c)*(a+I*a*tan(d*x+c))^2/d+(I*A-B)*(a^3+I*a^3*tan(d*x+c))/
d
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.81

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=a^3\left(-\frac{A\cot(c+dx)}{d}+\frac{3iA\log(\tan(c+dx))}{d}+\frac{B\log(\tan(c+dx))}{d}-\frac{4iA\log(i+\tan(c+dx))}{d}-\frac{4B\log(i+\tan(c+dx))}{d}-\frac{iB\tan(c+dx)}{d}\right)$$

input
```
Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
a^3*(-((A*Cot[c + d*x])/d) + ((3*I)*A*Log[Tan[c + d*x]])/d + (B*Log[Tan[c
+ d*x]])/d - ((4*I)*A*Log[I + Tan[c + d*x]])/d - (4*B*Log[I + Tan[c + d*x]
])/d - (I*B*Tan[c + d*x])/d)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 4076, 3042, 4077, 3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4076}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)^2(a(3iA+B)+a(A+iB)\tan(c+dx))dx-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^2(a(3iA+B)+a(A+iB)\tan(c+dx))}{\tan(c+dx)}dx-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{4077}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)\left(a^2(3iA+B)-a^2(A-3iB)\tan(c+dx)\right)dx+\frac{(-B+iA)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)\left(a^2(3iA+B)-a^2(A-3iB)\tan(c+dx)\right)}{\tan(c+dx)}dx+\frac{(-B+iA)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{4072}$$

$$-\left(a^3(3B+iA)\int \tan(c+dx)dx\right)+\int \cot(c+dx)\left(a^3(3iA+B)-4a^3(A-iB)\tan(c+dx)\right)dx+\frac{(-B+iA)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\left(a^3(3B+iA)\int \tan(c+dx)dx\right)+\int \frac{a^3(3iA+B)-4a^3(A-iB)\tan(c+dx)}{\tan(c+dx)}dx+\frac{(-B+iA)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3956}$$

$$\int \frac{a^3(3iA+B)-4a^3(A-iB)\tan(c+dx)}{\tan(c+dx)}dx+\frac{(-B+iA)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{a^3(3B+iA)\log(\cos(c+dx))}{d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{4014}$$

$$a^3(B+3iA)\int \cot(c+dx)dx+\frac{(-B+iA)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{a^3(3B+iA)\log(\cos(c+dx))}{d}-4a^3x(A-iB)-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$a^3(B+3iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{(-B+iA)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{a^3(3B+iA)\log(\cos(c+dx))}{d}-4a^3x(A-iB)-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{25}$$

$$-a^3(B+3iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{(-B+iA)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{a^3(3B+iA)\log(\cos(c+dx))}{d}-4a^3x(A-iB)-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(-B+iA)\left(a^3+ia^3\tan(c+dx)\right)}{d}+\frac{a^3(B+3iA)\log(-\sin(c+dx))}{d}+\frac{a^3(3B+iA)\log(\cos(c+dx))}{d}-4a^3x(A-iB)-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^2}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
-4*a^3*(A - I*B)*x + (a^3*(I*A + 3*B)*Log[Cos[c + d*x]])/d + (a^3*((3*I)*A
 + B)*Log[-Sin[c + d*x]])/d - (a*A*Cot[c + d*x]*(a + I*a*Tan[c + d*x])^2)/
d + ((I*A - B)*(a^3 + I*a^3*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{a^3\left(4iBdx+3iA\ln(\tan(dx+c))-2iA\ln\left(\sec(dx+c)^2\right)-4Adx-iB\tan(dx+c)+B\ln(\tan(dx+c))-2B\ln\left(\sec(dx+c)^2\right)-\right.}{d}$
derivativedivides	$\frac{a^3\left(-A\cot(dx+c)+\frac{(-4iA-4B)\ln\left(\cot(dx+c)^2+1\right)}{2}+(-4iB+4A)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)+(iA+3B)\ln(\cot(dx+c))-\right.}{d}$
default	$\frac{a^3\left(-A\cot(dx+c)+\frac{(-4iA-4B)\ln\left(\cot(dx+c)^2+1\right)}{2}+(-4iB+4A)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)+(iA+3B)\ln(\cot(dx+c))-\right.}{d}$
norman	$\frac{(4iB\,a^3-4A\,a^3)x\tan(dx+c)-\frac{A\,a^3}{d}-\frac{iB\,a^3\tan(dx+c)^2}{d}}{\tan(dx+c)}+\frac{(3iA\,a^3+B\,a^3)\ln(\tan(dx+c))}{d}-\frac{2(iA\,a^3+B\,a^3)\ln\left(1+\tan\right.}{d}$
risch	$-\frac{8ia^3Bc}{d}+\frac{8a^3Ac}{d}+\frac{2a^3\left(-iA\,e^{2i(dx+c)}+B\,e^{2i(dx+c)}-iA-B\right)}{d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)}+\frac{a^3\ln\left(e^{2i(dx+c)}-1\right)B}{d}+\frac{3ia^3\ln\left(e^{2i(dx+c)}-1\right.}{d}$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*a^3*(4*I*B*d*x+3*I*A*ln(tan(d*x+c))-2*I*A*ln(sec(d*x+c)^2)-4*A*d*x-I*B
*tan(d*x+c)+B*ln(tan(d*x+c))-2*B*ln(sec(d*x+c)^2)-A*cot(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.22

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\,(i\,A-B)a^3e^{(2i\,dx+2i\,c)}+2\,(i\,A+B)a^3-\left((i\,A+3\,B)a^3e^{(4i\,dx+4i\,c)}+(-i\,A-3\,B)a^3\right)\log\left(e^{(2i\,dx+2i\,c)}\right.}{de^{(4i\,dx+4i\,c)}-d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-(2*(I*A - B)*a^3*e^(2*I*d*x + 2*I*c) + 2*(I*A + B)*a^3 - ((I*A + 3*B)*a^3
*e^(4*I*d*x + 4*I*c) + (-I*A - 3*B)*a^3)*log(e^(2*I*d*x + 2*I*c) + 1) - ((
3*I*A + B)*a^3*e^(4*I*d*x + 4*I*c) + (-3*I*A - B)*a^3)*log(e^(2*I*d*x + 2*
I*c) - 1))/(d*e^(4*I*d*x + 4*I*c) - d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(99) = 198$.

Time = 1.05 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.89

$$\begin{aligned}
&\int \cot^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\
&= \frac{ia^3(A-3iB)\log\left(e^{2idx}+\frac{2Aa^3-2iBa^3-a^3(A-3iB)}{Aa^3e^{2ic}+iBa^3e^{2ic}}\right)}{d} \\
&\quad+\frac{ia^3\cdot(3A-iB)\log\left(e^{2idx}+\frac{2Aa^3-2iBa^3-a^3\cdot(3A-iB)}{Aa^3e^{2ic}+iBa^3e^{2ic}}\right)}{d} \\
&\quad+\frac{-2iAa^3-2Ba^3+(-2iAa^3e^{2ic}+2Ba^3e^{2ic})\,e^{2idx}}{de^{4ic}e^{4idx}-d}
\end{aligned}$$

input
```
integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
I*a**3*(A - 3*I*B)*log(exp(2*I*d*x) + (2*A*a**3 - 2*I*B*a**3 - a**3*(A - 3
*I*B))/(A*a**3*exp(2*I*c) + I*B*a**3*exp(2*I*c)))/d + I*a**3*(3*A - I*B)*l
og(exp(2*I*d*x) + (2*A*a**3 - 2*I*B*a**3 - a**3*(3*A - I*B))/(A*a**3*exp(2
*I*c) + I*B*a**3*exp(2*I*c)))/d + (-2*I*A*a**3 - 2*B*a**3 + (-2*I*A*a**3*e
xp(2*I*c) + 2*B*a**3*exp(2*I*c))*exp(2*I*d*x))/(d*exp(4*I*c)*exp(4*I*d*x)
- d)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{4\,(dx+c)(A-i\,B)a^3+2\,(i\,A+B)a^3\log\left(\tan\left(dx+c\right)^2+1\right)-(3i\,A+B)a^3\log\left(\tan\left(dx+c\right)\right)+i\,B}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-(4*(d*x + c)*(A - I*B)*a^3 + 2*(I*A + B)*a^3*log(tan(d*x + c)^2 + 1) - (3
*I*A + B)*a^3*log(tan(d*x + c)) + I*B*a^3*tan(d*x + c) + A*a^3/tan(d*x + c
))/d
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=-\frac{i\,Ba^3\tan\left(dx+c\right)}{d}-\frac{Aa^3}{d\tan\left(dx+c\right)}+\frac{4\,(-i\,Aa^3-Ba^3)\log\left(\tan\left(dx+c\right)+i\right)}{d}$$
$$-\frac{(-3i\,Aa^3-Ba^3)\log\left(|\tan\left(dx+c\right)|\right)}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-I*B*a^3*tan(d*x + c)/d - A*a^3/(d*tan(d*x + c)) + 4*(-I*A*a^3 - B*a^3)*lo
g(tan(d*x + c) + I)/d - (-3*I*A*a^3 - B*a^3)*log(abs(tan(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{a^3\,\ln\left(\tan(c+d\,x)\right)\,\left(B+A\,3\mathrm{i}\right)}{d}-\frac{4\,a^3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{d}-\frac{A\,a^3\cot(c+d\,x)}{d}-\frac{B\,a^3\tan(c+d\,x)\,1\mathrm{i}}{d}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(a^3*log(tan(c + d*x))*(A*3i + B))/d - (4*a^3*log(tan(c + d*x) + 1i)*(A*1i
 + B))/d - (A*a^3*cot(c + d*x))/d - (B*a^3*tan(c + d*x)*1i)/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.59

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{a^3\left(-4\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+c\right)ai-4\cos\left(dx+c\right)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(d\right.\right.}{}$$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*( - 4*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a*i - 4
*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*b + cos(c + d*x)*l
og(tan((c + d*x)/2) - 1)*sin(c + d*x)*a*i + 3*cos(c + d*x)*log(tan((c + d*
x)/2) - 1)*sin(c + d*x)*b + cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c +
 d*x)*a*i + 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*b + 3*co
s(c + d*x)*log(tan((c + d*x)/2))*sin(c + d*x)*a*i + cos(c + d*x)*log(tan((
c + d*x)/2))*sin(c + d*x)*b - 4*cos(c + d*x)*sin(c + d*x)*a*d*x + 4*cos(c
+ d*x)*sin(c + d*x)*b*d*i*x + sin(c + d*x)**2*a - sin(c + d*x)**2*b*i - a)
)/(cos(c + d*x)*sin(c + d*x)*d)
```

3.22 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 123

$$\begin{aligned}\int \cot^3(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= -4a^3(iA+B)x+\frac{ia^3B\log(\cos(c+dx))}{d}\\ &\quad-\frac{a^3(4A-3iB)\log(\sin(c+dx))}{d}-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}\\ &\quad-\frac{(2iA+B)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d}\end{aligned}$$

output
```
-4*a^3*(I*A+B)*x+I*a^3*B*ln(cos(d*x+c))/d-a^3*(4*A-3*I*B)*ln(sin(d*x+c))/d
-1/2*a*A*cot(d*x+c)^2*(a+I*a*tan(d*x+c))^2/d-(2*I*A+B)*cot(d*x+c)*(a^3+I*a
^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3((-6iA-2B)\cot(c+dx)-A\cot^2(c+dx)+(-8A+6iB)\log(\tan(c+dx))+8(A-iB)\log(i+\tan$$
$$2d$$

input
```
Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(a^3*(((-6*I)*A - 2*B)*Cot[c + d*x] - A*Cot[c + d*x]^2 + (-8*A + (6*I)*B)*
Log[Tan[c + d*x]] + 8*(A - I*B)*Log[I + Tan[c + d*x]]))/(2*d)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4076, 27, 3042, 4076, 25, 3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{ 4076}$$

$$\frac{1}{2}\int 2\cot^2(c+dx)(i\tan(c+dx)a+a)^2(a(2iA+B)+iaB\tan(c+dx))dx-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 27}$$

$$\int \cot^2(c+dx)(i\tan(c+dx)a+a)^2(a(2iA+B)+iaB\tan(c+dx))dx-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^2(a(2iA+B)+iaB\tan(c+dx))}{\tan(c+dx)^2}dx-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4076}$$

$$\int -\cot(c+dx)(i\tan(c+dx)a+a)\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)dx-\frac{(B+2iA)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(c+dx)(i\tan(c+dx)a+a)\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)dx-\frac{(B+2iA)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{(i\tan(c+dx)a+a)\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)}{\tan(c+dx)}dx-\frac{(B+2iA)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4072}$$

$$-\int \cot(c+dx)\left((4A-3iB)a^3+4(iA+B)\tan(c+dx)a^3\right)dx-ia^3B\int \tan(c+dx)dx-\frac{(B+2iA)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{(4A-3iB)a^3+4(iA+B)\tan(c+dx)a^3}{\tan(c+dx)}dx-ia^3B\int \tan(c+dx)dx-\frac{(B+2iA)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d}-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3956}$$

$$-\int \frac{(4A-3iB)a^3+4(iA+B)\tan(c+dx)a^3}{\tan(c+dx)}dx - \frac{(B+2iA)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d} + \frac{ia^3B\log(\cos(c+dx))}{d} - \frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 4014

$$-a^3(4A-3iB)\int \cot(c+dx)dx - \frac{(B+2iA)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d} - 4a^3x(B+iA) + \frac{ia^3B\log(\cos(c+dx))}{d} - \frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 3042

$$-a^3(4A-3iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx - \frac{(B+2iA)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d} - 4a^3x(B+iA) + \frac{ia^3B\log(\cos(c+dx))}{d} - \frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 25

$$a^3(4A-3iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx - \frac{(B+2iA)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d} - 4a^3x(B+iA) + \frac{ia^3B\log(\cos(c+dx))}{d} - \frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 3956

$$-\frac{a^3(4A-3iB)\log(-\sin(c+dx))}{d} - \frac{(B+2iA)\cot(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d} - 4a^3x(B+iA) + \frac{ia^3B\log(\cos(c+dx))}{d} - \frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^2}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
-4*a^3*(I*A + B)*x + (I*a^3*B*Log[Cos[c + d*x]])/d - (a^3*(4*A - (3*I)*B)*
Log[-Sin[c + d*x]])/d - (a*A*Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^2)/(2*d
) - (((2*I)*A + B)*Cot[c + d*x]*(a^3 + I*a^3*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.63

method	result
parallelrisch	$-\frac{4a^3\left(\frac{(iB-A)\ln\left(\sec(dx+c)^2\right)}{2}+\left(-\frac{3iB}{4}+A\right)\ln(\tan(dx+c))+\frac{A\cot(dx+c)^2}{8}+\frac{\cot(dx+c)(3iA+B)}{4}+(iA+B)xd\right)}{d}$
derivativedivides	$-\frac{a^3\left(\frac{(4iB-4A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(4iA+4B)\arctan(\tan(dx+c))+(-3iB+4A)\ln(\tan(dx+c))-\frac{-3iA-B}{\tan(dx+c)}+\frac{A}{2\tan(dx+}\right.}{d}$
default	$-\frac{a^3\left(\frac{(4iB-4A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(4iA+4B)\arctan(\tan(dx+c))+(-3iB+4A)\ln(\tan(dx+c))-\frac{-3iA-B}{\tan(dx+c)}+\frac{A}{2\tan(dx+}\right.}{d}$
norman	$\frac{(-4iA\,a^3-4B\,a^3)x\tan(dx+c)^2-\frac{A\,a^3}{2d}-\frac{(3iA\,a^3+B\,a^3)\tan(dx+c)}{d}}{\tan(dx+c)^2}-\frac{(-3iB\,a^3+4A\,a^3)\ln(\tan(dx+c))}{d}+\frac{2(-iB\,a^3}{}$
risch	$\frac{8a^3Bc}{d}+\frac{8ia^3Ac}{d}-\frac{2ia^3\left(4iA\,e^{2i(dx+c)}+B\,e^{2i(dx+c)}-3iA-B\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+\frac{3ia^3\ln\left(e^{2i(dx+c)}-1\right)B}{d}-\frac{4A\,a^3\ln\left(e^{2i(dx+c)}\right.}{d}$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-4*a^3/d*(1/2*(-A+I*B)*ln(sec(d*x+c)^2)+(-3/4*I*B+A)*ln(tan(d*x+c))+1/8*A*
cot(d*x+c)^2+1/4*cot(d*x+c)*(3*I*A+B)+(I*A+B)*x*d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.46

$$\int\cot^3(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{2\,(4\,A-i\,B)a^3e^{(2i\,dx+2i\,c)}-2\,(3\,A-i\,B)a^3+\left(i\,Ba^3e^{(4i\,dx+4i\,c)}-2i\,Ba^3e^{(2i\,dx+2i\,c)}+i\,Ba^3\right)\log\left(e^{(2i\,dx+2i}\right.}{de^{(4i\,dx+4i\,c)}-2}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
(2*(4*A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - 2*(3*A - I*B)*a^3 + (I*B*a^3*e^(4
*I*d*x + 4*I*c) - 2*I*B*a^3*e^(2*I*d*x + 2*I*c) + I*B*a^3)*log(e^(2*I*d*x
+ 2*I*c) + 1) - ((4*A - 3*I*B)*a^3*e^(4*I*d*x + 4*I*c) - 2*(4*A - 3*I*B)*a
^3*e^(2*I*d*x + 2*I*c) + (4*A - 3*I*B)*a^3)*log(e^(2*I*d*x + 2*I*c) - 1))/
(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(109) = 218$.

Time = 1.08 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.84

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{iBa^3\log\left(\frac{2Aa^3-iBa^3}{2Aa^3e^{2ic}-iBa^3e^{2ic}}+e^{2idx}\right)}{d} - \frac{a^3\cdot(4A-3iB)\log\left(e^{2idx}+\frac{2Aa^3-2iBa^3-a^3\cdot(4A-3iB)}{2Aa^3e^{2ic}-iBa^3e^{2ic}}\right)}{d} + \frac{-6Aa^3+2iBa^3+\left(8Aa^3e^{2ic}-2iBa^3e^{2ic}\right)e^{2idx}}{de^{4ic}e^{4idx}-2de^{2ic}e^{2idx}+d}$$

input
```
integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
I*B*a**3*log((2*A*a**3 - I*B*a**3)/(2*A*a**3*exp(2*I*c) - I*B*a**3*exp(2*I
*c)) + exp(2*I*d*x))/d - a**3*(4*A - 3*I*B)*log(exp(2*I*d*x) + (2*A*a**3 -
 2*I*B*a**3 - a**3*(4*A - 3*I*B))/(2*A*a**3*exp(2*I*c) - I*B*a**3*exp(2*I*
c)))/d + (-6*A*a**3 + 2*I*B*a**3 + (8*A*a**3*exp(2*I*c) - 2*I*B*a**3*exp(2
*I*c))*exp(2*I*d*x))/(d*exp(4*I*c)*exp(4*I*d*x) - 2*d*exp(2*I*c)*exp(2*I*d
*x) + d)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.78

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{8\,(dx+c)(i\,A+B)a^3-4\,(A-i\,B)a^3\log\left(\tan\left(dx+c\right)^2+1\right)+2\,(4\,A-3i\,B)a^3\log\left(\tan\left(dx+c\right)\right)-}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/2*(8*(d*x + c)*(I*A + B)*a^3 - 4*(A - I*B)*a^3*log(tan(d*x + c)^2 + 1)
+ 2*(4*A - 3*I*B)*a^3*log(tan(d*x + c)) - (2*(-3*I*A - B)*a^3*tan(d*x + c)
 - A*a^3)/tan(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{4\,(Aa^3-i\,Ba^3)\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{(4\,Aa^3-3i\,Ba^3)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$
$$-\frac{Aa^3+2\,(3i\,Aa^3+Ba^3)\tan\left(dx+c\right)}{2\,d\tan\left(dx+c\right)^2}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
4*(A*a^3 - I*B*a^3)*log(tan(d*x + c) + I)/d - (4*A*a^3 - 3*I*B*a^3)*log(ab
s(tan(d*x + c)))/d - 1/2*(A*a^3 + 2*(3*I*A*a^3 + B*a^3)*tan(d*x + c))/(d*t
an(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.72

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= -\frac{\frac{A\,a^3}{2}+\tan(c+d\,x)\,\left(B\,a^3+A\,a^3\,3\mathrm{i}\right)}{d\tan\left(c+d\,x\right)^2}-\frac{a^3\,\ln\left(\tan(c+d\,x)\right)\,\left(4\,A-B\,3\mathrm{i}\right)}{d}$$
$$+\frac{4\,a^3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{d}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(4*a^3*log(tan(c + d*x) + 1i)*(A - B*1i))/d - (a^3*log(tan(c + d*x))*(4*A
- B*3i))/d - ((A*a^3)/2 + tan(c + d*x)*(A*a^3*3i + B*a^3))/(d*tan(c + d*x)
^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.87

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3\left(-12\cos\left(dx+c\right)\sin\left(dx+c\right)ai-4\cos\left(dx+c\right)\sin\left(dx+c\right)b+16\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+\right.\right.}{}$$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*( - 12*cos(c + d*x)*sin(c + d*x)*a*i - 4*cos(c + d*x)*sin(c + d*x)*b
 + 16*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a - 16*log(tan((c + d*x
)/2)**2 + 1)*sin(c + d*x)**2*b*i + 4*log(tan((c + d*x)/2) - 1)*sin(c + d*x
)**2*b*i + 4*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b*i - 16*log(tan((c
 + d*x)/2))*sin(c + d*x)**2*a + 12*log(tan((c + d*x)/2))*sin(c + d*x)**2*b
*i - 16*sin(c + d*x)**2*a*d*i*x + sin(c + d*x)**2*a - 16*sin(c + d*x)**2*b
*d*x - 2*a))/(4*sin(c + d*x)**2*d)
```

3.23 $\int \cot^4(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 134

$$\begin{aligned}\int \cot^4(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= 4a^3(A-iB)x+\frac{a^3(17A-15iB)\cot(c+dx)}{6d} \\ &\quad -\frac{4a^3(iA+B)\log(\sin(c+dx))}{d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d} \\ &\quad -\frac{(5iA+3B)\cot^2(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d}\end{aligned}$$

output
```
4*a^3*(A-I*B)*x+1/6*a^3*(17*A-15*I*B)*cot(d*x+c)/d-4*a^3*(I*A+B)*ln(sin(d*
x+c))/d-1/3*a*A*cot(d*x+c)^3*(a+I*a*tan(d*x+c))^2/d-1/6*(5*I*A+3*B)*cot(d*
x+c)^2*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.66

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{a^3(6(4A-3iB)\cot(c+dx)+(-9iA-3B)\cot^2(c+dx)-2A\cot^3(c+dx)-24i(A-iB)(\log(\tan(c+}{6d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(a^3*(6*(4*A - (3*I)*B)*Cot[c + d*x] + ((-9*I)*A - 3*B)*Cot[c + d*x]^2 - 2
*A*Cot[c + d*x]^3 - (24*I)*(A - I*B)*(Log[Tan[c + d*x]] - Log[I + Tan[c +
d*x]])))/(6*d)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.382$, Rules used = {3042, 4076, 3042, 4076, 25, 3042, 4074, 27, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{3}\int \cot^3(c+dx)(i\tan(c+dx)a+a)^2(a(5iA+3B)-a(A-3iB)\tan(c+dx))dx-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)^2(a(5iA+3B)-a(A-3iB)\tan(c+dx))}{\tan(c+dx)^3}dx-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 4076

$$\frac{1}{3}\left(\frac{1}{2}\int -\cot^2(c+dx)(i\tan(c+dx)a+a)\left((17A-15iB)a^2+(7iA+9B)\tan(c+dx)a^2\right)dx-\frac{(3B+5iA)\cot^2}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 25

$$\frac{1}{3}\left(-\frac{1}{2}\int \cot^2(c+dx)(i\tan(c+dx)a+a)\left((17A-15iB)a^2+(7iA+9B)\tan(c+dx)a^2\right)dx-\frac{(3B+5iA)\cot^2}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)\left((17A-15iB)a^2+(7iA+9B)\tan(c+dx)a^2\right)}{\tan(c+dx)^2}dx-\frac{(3B+5iA)\cot^2(c+dx)\,(a^3}{2d}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 4074

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{a^3(17A-15iB)\cot(c+dx)}{d}-\int 24\cot(c+dx)\left(a^3(iA+B)-a^3(A-iB)\tan(c+dx)\right)dx\right)-\frac{(3B+5i}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{a^3(17A-15iB)\cot(c+dx)}{d}-24\int \cot(c+dx)\left(a^3(iA+B)-a^3(A-iB)\tan(c+dx)\right)dx\right)-\frac{(3B+5i}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{a^3(17A-15iB)\cot(c+dx)}{d}-24\int\frac{a^3(iA+B)-a^3(A-iB)\tan(c+dx)}{\tan(c+dx)}dx\right)-\frac{(3B+5iA)\cot^2(c+dx)}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 4014

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{a^3(17A-15iB)\cot(c+dx)}{d}-24\left(a^3(B+iA)\int\cot(c+dx)dx-a^3x(A-iB)\right)\right)-\frac{(3B+5iA)\cot^2(c}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{a^3(17A-15iB)\cot(c+dx)}{d}-24\left(a^3(B+iA)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx-a^3x(A-iB)\right)\right)-\frac{(3B+5iA}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 25

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{a^3(17A-15iB)\cot(c+dx)}{d}-24\left(-a^3(B+iA)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\left(a^3x(A-iB)\right)\right)\right)-\frac{(3B}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 3956

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{a^3(17A-15iB)\cot(c+dx)}{d}-24\left(\frac{a^3(B+iA)\log(-\sin(c+dx))}{d}-a^3x(A-iB)\right)\right)-\frac{(3B+5iA)\cot^2(}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
-1/3*(a*A*Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^2)/d + (((a^3*(17*A - (15*
I)*B)*Cot[c + d*x])/d - 24*(-(a^3*(A - I*B)*x) + (a^3*(I*A + B)*Log[-Sin[c
 + d*x]])/d))/2 - (((5*I)*A + 3*B)*Cot[c + d*x]^2*(a^3 + I*a^3*Tan[c + d*x
]))/(2*d))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{4\left(\frac{(iA+B)\ln\left(\sec(dx+c)^2\right)}{2}-(iA+B)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^3}{12}-\frac{\cot(dx+c)^2(3iA+B)}{8}+\left(-\frac{3iB}{4}+A\right)\cot(dx+c)+(-iB\right.}{d}$
derivativedivides	$\frac{a^3\left(-\frac{3iA\cot(dx+c)^2}{2}-\frac{A\cot(dx+c)^3}{3}-3iB\cot(dx+c)-\frac{B\cot(dx+c)^2}{2}+4A\cot(dx+c)+\frac{(4iA+4B)\ln\left(\cot(dx+c)^2+1\right)}{2}+(4iB\right.}{d}$
default	$\frac{a^3\left(-\frac{3iA\cot(dx+c)^2}{2}-\frac{A\cot(dx+c)^3}{3}-3iB\cot(dx+c)-\frac{B\cot(dx+c)^2}{2}+4A\cot(dx+c)+\frac{(4iA+4B)\ln\left(\cot(dx+c)^2+1\right)}{2}+(4iB\right.}{d}$
risch	$\frac{8ia^3Bc}{d}-\frac{8a^3Ac}{d}+\frac{2a^3\left(24iA\,e^{4i(dx+c)}+12B\,e^{4i(dx+c)}-33iA\,e^{2i(dx+c)}-21B\,e^{2i(dx+c)}+13iA+9B\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}-\frac{4a^3\ln(e^{2i}}{}$
norman	$\frac{\frac{\left(-3iB\,a^3+4A\,a^3\right)\tan(dx+c)^2}{d}+\left(-4iB\,a^3+4A\,a^3\right)x\tan(dx+c)^3-\frac{A\,a^3}{3d}-\frac{\left(3iA\,a^3+B\,a^3\right)\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{4\left(iA\,a^3+B\,a^3\right)\ln}{d}$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
4*(1/2*(I*A+B)*ln(sec(d*x+c)^2)-(I*A+B)*ln(tan(d*x+c))-1/12*A*cot(d*x+c)^3
-1/8*cot(d*x+c)^2*(3*I*A+B)+(-3/4*I*B+A)*cot(d*x+c)+(A-I*B)*x*d)*a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.35

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(12\left(-2i\,A-B\right)a^3e^{(4i\,dx+4i\,c)}+3\left(11i\,A+7\,B\right)a^3e^{(2i\,dx+2i\,c)}+\left(-13i\,A-9\,B\right)a^3+6\left(\left(i\,A+B\right)a^3e^{(6}\right.\right.}{3\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx-}\right.}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-2/3*(12*(-2*I*A - B)*a^3*e^(4*I*d*x + 4*I*c) + 3*(11*I*A + 7*B)*a^3*e^(2*
I*d*x + 2*I*c) + (-13*I*A - 9*B)*a^3 + 6*((I*A + B)*a^3*e^(6*I*d*x + 6*I*c
) + 3*(-I*A - B)*a^3*e^(4*I*d*x + 4*I*c) + 3*(I*A + B)*a^3*e^(2*I*d*x + 2*
I*c) + (-I*A - B)*a^3)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(6*I*d*x + 6*I*c
) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.36

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=-\frac{4ia^3(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d}$$
$$+\frac{26iAa^3+18Ba^3+\left(-66iAa^3e^{2ic}-42Ba^3e^{2ic}\right)e^{2idx}+\left(48iAa^3e^{4ic}+24Ba^3e^{4ic}\right)e^{4idx}}{3de^{6ic}e^{6idx}-9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}-3d}$$

input
```
integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
-4*I*a**3*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (26*I*A*a**3 + 18*
B*a**3 + (-66*I*A*a**3*exp(2*I*c) - 42*B*a**3*exp(2*I*c))*exp(2*I*d*x) + (
48*I*A*a**3*exp(4*I*c) + 24*B*a**3*exp(4*I*c))*exp(4*I*d*x))/(3*d*exp(6*I*
c)*exp(6*I*d*x) - 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x
) - 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{24\,(dx+c)(A-i\,B)a^3-12\,(-i\,A-B)a^3\log\left(\tan\left(dx+c\right)^2+1\right)-24\,(i\,A+B)a^3\log\left(\tan\left(dx+c\right)\right)+}{6\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/6*(24*(d*x + c)*(A - I*B)*a^3 - 12*(-I*A - B)*a^3*log(tan(d*x + c)^2 + 1
) - 24*(I*A + B)*a^3*log(tan(d*x + c)) + (6*(4*A - 3*I*B)*a^3*tan(d*x + c)
^2 + 3*(-3*I*A - B)*a^3*tan(d*x + c) - 2*A*a^3)/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=-\frac{4\,(-i\,Aa^3-Ba^3)\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{4\,(i\,Aa^3+Ba^3)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$
$$-\frac{2\,Aa^3-6\,(4\,Aa^3-3i\,Ba^3)\tan\left(dx+c\right)^2+3\,(3i\,Aa^3+Ba^3)\tan\left(dx+c\right)}{6\,d\tan\left(dx+c\right)^3}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-4*(-I*A*a^3 - B*a^3)*log(tan(d*x + c) + I)/d - 4*(I*A*a^3 + B*a^3)*log(ab
s(tan(d*x + c)))/d - 1/6*(2*A*a^3 - 6*(4*A*a^3 - 3*I*B*a^3)*tan(d*x + c)^2
 + 3*(3*I*A*a^3 + B*a^3)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= -\frac{\frac{A\,a^3}{3}-\tan(c+d\,x)^2\,(4\,A\,a^3-B\,a^3\,3\mathrm{i})+\tan(c+d\,x)\,\left(\frac{B\,a^3}{2}+\frac{A\,a^3\,3\mathrm{i}}{2}\right)}{d\tan\left(c+d\,x\right)^3} - \frac{a^3\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\,\left(B+A\,1\mathrm{i}\right)\,8\mathrm{i}}{d}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
- ((A*a^3)/3 - tan(c + d*x)^2*(4*A*a^3 - B*a^3*3i) + tan(c + d*x)*((A*a^3*
3i)/2 + (B*a^3)/2))/(d*tan(c + d*x)^3) - (a^3*atan(2*tan(c + d*x) + 1i)*(A
*1i + B)*8i)/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.67

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3\left(52\cos\left(dx+c\right)\sin\left(dx+c\right)^2a-36\cos\left(dx+c\right)\sin\left(dx+c\right)^2bi-4\cos\left(dx+c\right)a+48\log\left(\tan\left(\frac{dx}{2}+\right.\right.\right.}{}$$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*(52*cos(c + d*x)*sin(c + d*x)**2*a - 36*cos(c + d*x)*sin(c + d*x)**2
*b*i - 4*cos(c + d*x)*a + 48*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*
a*i + 48*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*b - 48*log(tan((c +
d*x)/2))*sin(c + d*x)**3*a*i - 48*log(tan((c + d*x)/2))*sin(c + d*x)**3*b
+ 48*sin(c + d*x)**3*a*d*x + 9*sin(c + d*x)**3*a*i - 48*sin(c + d*x)**3*b*
d*i*x + 3*sin(c + d*x)**3*b - 18*sin(c + d*x)*a*i - 6*sin(c + d*x)*b))/(12
*sin(c + d*x)**3*d)
```

3.24 $\int \cot^5(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 157

$$\begin{aligned}\int \cot^5(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= 4a^3(iA+B)x+\frac{4a^3(iA+B)\cot(c+dx)}{d}+\frac{a^3(15A-14iB)\cot^2(c+dx)}{12d}\\&\quad+\frac{4a^3(A-iB)\log(\sin(c+dx))}{d}-\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}\\&\quad-\frac{(3iA+2B)\cot^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d}\end{aligned}$$

output
```
4*a^3*(I*A+B)*x+4*a^3*(I*A+B)*cot(d*x+c)/d+1/12*a^3*(15*A-14*I*B)*cot(d*x+
c)^2/d+4*a^3*(A-I*B)*ln(sin(d*x+c))/d-1/4*a*A*cot(d*x+c)^4*(a+I*a*tan(d*x+
c))^2/d-1/6*(3*I*A+2*B)*cot(d*x+c)^3*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.71

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{ia^3(-((3A-4iB)(i+\cot(c+dx))^3)+3iA\cot(c+dx)(i+\cot(c+dx))^3-6i(A-iB)\left(6i\cot(c+dx)\right.}{12d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
((I/12)*a^3*(-((3*A - (4*I)*B)*(I + Cot[c + d*x])^3) + (3*I)*A*Cot[c + d*x
]*(I + Cot[c + d*x])^3 - (6*I)*(A - I*B)*((6*I)*Cot[c + d*x] + Cot[c + d*x
]^2 + 8*Log[Tan[c + d*x]] - 8*Log[I + Tan[c + d*x]])))/d
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4076, 27, 3042, 4076, 25, 3042, 4074, 27, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{4}\int 2\cot^4(c+dx)(i\tan(c+dx)a+a)^2(a(3iA+2B)-a(A-2iB)\tan(c+dx))dx-\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \cot^4(c+dx)(i\tan(c+dx)a+a)^2(a(3iA+2B)-a(A-2iB)\tan(c+dx))dx-\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)^2(a(3iA+2B)-a(A-2iB)\tan(c+dx))}{\tan(c+dx)^4}dx-\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 4076

$$\frac{1}{2}\left(\frac{1}{3}\int -\cot^3(c+dx)(i\tan(c+dx)a+a)\left((15A-14iB)a^2+(9iA+10B)\tan(c+dx)a^2\right)dx-\frac{(2B+3iA)\cot\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 25

$$\frac{1}{2}\left(-\frac{1}{3}\int \cot^3(c+dx)(i\tan(c+dx)a+a)\left((15A-14iB)a^2+(9iA+10B)\tan(c+dx)a^2\right)dx-\frac{(2B+3iA)\cot\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)\left((15A-14iB)a^2+(9iA+10B)\tan(c+dx)a^2\right)}{\tan(c+dx)^3}dx-\frac{(2B+3iA)\cot^3(c+dx)\,(a}{3d}\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 4074

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^3(15A-14iB)\cot^2(c+dx)}{2d}-\int 24\cot^2(c+dx)\left(a^3(iA+B)-a^3(A-iB)\tan(c+dx)\right)dx\right)-\frac{(2B+\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^3(15A-14iB)\cot^2(c+dx)}{2d}-24\int\cot^2(c+dx)\left(a^3(iA+B)-a^3(A-iB)\tan(c+dx)\right)dx\right)-\frac{(2B+\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^3(15A-14iB)\cot^2(c+dx)}{2d}-24\int\frac{a^3(iA+B)-a^3(A-iB)\tan(c+dx)}{\tan(c+dx)^2}dx\right)-\frac{(2B+3iA)\cot^3(c+\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 4012

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^3(15A-14iB)\cot^2(c+dx)}{2d}-24\left(\int-\cot(c+dx)\left((A-iB)a^3+(iA+B)\tan(c+dx)a^3\right)dx-\frac{a^3(B\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^3(15A-14iB)\cot^2(c+dx)}{2d}-24\left(-\int\cot(c+dx)\left((A-iB)a^3+(iA+B)\tan(c+dx)a^3\right)dx-\frac{a^3(B\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^3(15A-14iB)\cot^2(c+dx)}{2d}-24\left(-\int\frac{(A-iB)a^3+(iA+B)\tan(c+dx)a^3}{\tan(c+dx)}dx-\frac{a^3(B+iA)\cot(c+}{d}\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 4014

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^3(15A-14iB)\cot^2(c+dx)}{2d}-24\left(-a^3(A-iB)\int\cot(c+dx)dx-\frac{a^3(B+iA)\cot(c+dx)}{d}-(a^3x(B\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^3(15A-14iB)\cot^2(c+dx)}{2d}-24\left(-a^3(A-iB)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a^3(B+iA)\cot(c+dx)}{d}-\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 25

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^3(15A-14iB)\cot^2(c+dx)}{2d}-24\left(a^3(A-iB)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a^3(B+iA)\cot(c+dx)}{d}-\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

$\downarrow$ 3956

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^3(15A-14iB)\cot^2(c+dx)}{2d}-24\left(-\frac{a^3(B+iA)\cot(c+dx)}{d}-\frac{a^3(A-iB)\log(-\sin(c+dx))}{d}-(a^3x(B\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^2}{4d}$$

input
```
Int[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
-1/4*(a*A*Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^2)/d + (((a^3*(15*A - (14*
I)*B)*Cot[c + d*x]^2)/(2*d) - 24*(-(a^3*(I*A + B)*x) - (a^3*(I*A + B)*Cot[
c + d*x])/d - (a^3*(A - I*B)*Log[-Sin[c + d*x]])/d))/3 - (((3*I)*A + 2*B)*
Cot[c + d*x]^3*(a^3 + I*a^3*Tan[c + d*x]))/(3*d))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{4\left(\frac{(iB-A)\ln\left(\sec(dx+c)^2\right)}{2}+(-iB+A)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^4}{16}-\frac{\cot(dx+c)^3\left(iA+\frac{B}{3}\right)}{4}+\frac{\left(-\frac{3iB}{4}+A\right)\cot(dx+c)^2}{2}+\cot(\right.}{d}$
derivativedivides	$\frac{a^3\left(-iA\cot(dx+c)^3-\frac{A\cot(dx+c)^4}{4}-\frac{3iB\cot(dx+c)^2}{2}-\frac{B\cot(dx+c)^3}{3}+4iA\cot(dx+c)+2A\cot(dx+c)^2+4\cot(dx+c)B+\right.}{d}$
default	$\frac{a^3\left(-iA\cot(dx+c)^3-\frac{A\cot(dx+c)^4}{4}-\frac{3iB\cot(dx+c)^2}{2}-\frac{B\cot(dx+c)^3}{3}+4iA\cot(dx+c)+2A\cot(dx+c)^2+4\cot(dx+c)B+\right.}{d}$
risch	$-\frac{8a^3Bc}{d}-\frac{8ia^3Ac}{d}+\frac{2ia^3\left(36iA\,e^{6i(dx+c)}+24B\,e^{6i(dx+c)}-69iA\,e^{4i(dx+c)}-57B\,e^{4i(dx+c)}+54iA\,e^{2i(dx+c)}+46B\,e\right.}{3d\left(e^{2i(dx+c)}-1\right)^4}$
norman	$\frac{(4iA\,a^3+4B\,a^3)x\tan(dx+c)^4-\frac{A\,a^3}{4d}+\frac{\left(-3iB\,a^3+4A\,a^3\right)\tan(dx+c)^2}{2d}-\frac{\left(3iA\,a^3+B\,a^3\right)\tan(dx+c)}{3d}+\frac{4\left(iA\,a^3+B\,a^3\right)\tan(dx+}{d}}{\tan(dx+c)^4}$

input
```
int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
4*(1/2*(-A+I*B)*ln(sec(d*x+c)^2)+(A-I*B)*ln(tan(d*x+c))-1/16*A*cot(d*x+c)^
4-1/4*cot(d*x+c)^3*(I*A+1/3*B)+1/2*(-3/4*I*B+A)*cot(d*x+c)^2+cot(d*x+c)*(I
*A+B)+(I*A+B)*x*d)*a^3/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.45

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\,(12\,(3\,A-2i\,B)a^3e^{(6i\,dx+6i\,c)}-3\,(23\,A-19i\,B)a^3e^{(4i\,dx+4i\,c)}+2\,(27\,A-23i\,B)a^3e^{(2i\,dx+2i\,c)}-(15\,A}{3\,(de^{(8}}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-2/3*(12*(3*A - 2*I*B)*a^3*e^(6*I*d*x + 6*I*c) - 3*(23*A - 19*I*B)*a^3*e^(
4*I*d*x + 4*I*c) + 2*(27*A - 23*I*B)*a^3*e^(2*I*d*x + 2*I*c) - (15*A - 13*
I*B)*a^3 - 6*((A - I*B)*a^3*e^(8*I*d*x + 8*I*c) - 4*(A - I*B)*a^3*e^(6*I*d
*x + 6*I*c) + 6*(A - I*B)*a^3*e^(4*I*d*x + 4*I*c) - 4*(A - I*B)*a^3*e^(2*I
*d*x + 2*I*c) + (A - I*B)*a^3)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(8*I*d*x
 + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I
*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.50

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{4a^3(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d}$$
$$+ \frac{30Aa^3 - 26iBa^3 + \left(-108Aa^3e^{2ic} + 92iBa^3e^{2ic}\right)e^{2idx} + \left(138Aa^3e^{4ic} - 114iBa^3e^{4ic}\right)e^{4idx} + \left(-72Aa^3e^{6ic}\right.}{3de^{8ic}e^{8idx} - 12de^{6ic}e^{6idx} + 18de^{4ic}e^{4idx} - 12de^{2ic}e^{2idx} + 3d}$$

input
```
integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
4*a**3*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (30*A*a**3 - 26*I*B*a
**3 + (-108*A*a**3*exp(2*I*c) + 92*I*B*a**3*exp(2*I*c))*exp(2*I*d*x) + (13
8*A*a**3*exp(4*I*c) - 114*I*B*a**3*exp(4*I*c))*exp(4*I*d*x) + (-72*A*a**3*
exp(6*I*c) + 48*I*B*a**3*exp(6*I*c))*exp(6*I*d*x))/(3*d*exp(8*I*c)*exp(8*I
*d*x) - 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d*x) - 12*d
*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{48\,(dx+c)(-i\,A-B)a^3 + 24\,(A-i\,B)a^3\log\left(\tan\left(dx+c\right)^2+1\right) - 48\,(A-i\,B)a^3\log\left(\tan\left(dx+c\right)\right)}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/12*(48*(d*x + c)*(-I*A - B)*a^3 + 24*(A - I*B)*a^3*log(tan(d*x + c)^2 +
 1) - 48*(A - I*B)*a^3*log(tan(d*x + c)) - (48*(I*A + B)*a^3*tan(d*x + c)^
3 + 6*(4*A - 3*I*B)*a^3*tan(d*x + c)^2 + 4*(-3*I*A - B)*a^3*tan(d*x + c) -
 3*A*a^3)/tan(d*x + c)^4)/d
```

Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= -\frac{4\,(Aa^3 - i\,Ba^3)\log\left(\tan\left(dx+c\right)+i\right)}{d} + \frac{4\,(Aa^3 - i\,Ba^3)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$
$$- \frac{3\,Aa^3 + 48\,(-i\,Aa^3 - Ba^3)\tan\left(dx+c\right)^3 - 6\,(4\,Aa^3 - 3i\,Ba^3)\tan\left(dx+c\right)^2 + 4\,(3i\,Aa^3 + Ba^3)\tan\left(a\right. % text cut off at page edge
}{12\,d\tan\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-4*(A*a^3 - I*B*a^3)*log(tan(d*x + c) + I)/d + 4*(A*a^3 - I*B*a^3)*log(abs
(tan(d*x + c)))/d - 1/12*(3*A*a^3 + 48*(-I*A*a^3 - B*a^3)*tan(d*x + c)^3 -
 6*(4*A*a^3 - 3*I*B*a^3)*tan(d*x + c)^2 + 4*(3*I*A*a^3 + B*a^3)*tan(d*x +
c))/(d*tan(d*x + c)^4)
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.73

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)^2\,\left(2\,A\,a^3 - \frac{B\,a^3\,3\mathrm{i}}{2}\right) + \tan(c+d\,x)^3\,\left(4\,B\,a^3 + A\,a^3\,4\mathrm{i}\right) - \frac{A\,a^3}{4} - \tan(c+d\,x)\,\left(\frac{B\,a^3}{3} + A\,a^3\,1\mathrm{i}\right) % text cut off at page edge
}{d\tan\left(c+d\,x\right)^4}$$
$$+ \frac{8\,a^3\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\,\left(B+A\,1\mathrm{i}\right)}{d}$$

input
```
int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(tan(c + d*x)^2*(2*A*a^3 - (B*a^3*3i)/2) + tan(c + d*x)^3*(A*a^3*4i + 4*B*
a^3) - (A*a^3)/4 - tan(c + d*x)*(A*a^3*1i + (B*a^3)/3))/(d*tan(c + d*x)^4)
 + (8*a^3*atan(2*tan(c + d*x) + 1i)*(A*1i + B))/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.61

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3\left(480\cos(dx+c)\sin(dx+c)^3 ai + 416\cos(dx+c)\sin(dx+c)^3 b - 96\cos(dx+c)\sin(dx+c) ai - 3\right.}{}$$

input
```
int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*(480*cos(c + d*x)*sin(c + d*x)**3*a*i + 416*cos(c + d*x)*sin(c + d*x
)**3*b - 96*cos(c + d*x)*sin(c + d*x)*a*i - 32*cos(c + d*x)*sin(c + d*x)*b
 - 384*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**4*a + 384*log(tan((c + d
*x)/2)**2 + 1)*sin(c + d*x)**4*b*i + 384*log(tan((c + d*x)/2))*sin(c + d*x
)**4*a - 384*log(tan((c + d*x)/2))*sin(c + d*x)**4*b*i + 384*sin(c + d*x)*
*4*a*d*i*x - 111*sin(c + d*x)**4*a + 384*sin(c + d*x)**4*b*d*x + 72*sin(c
+ d*x)**4*b*i + 240*sin(c + d*x)**2*a - 144*sin(c + d*x)**2*b*i - 24*a))/(
96*sin(c + d*x)**4*d)
```

3.25 $\int \cot^6(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 180

$$\begin{aligned}&\int \cot^6(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx\\&=-4a^3(A-iB)x-\frac{4a^3(A-iB)\cot(c+dx)}{d}\\&\quad+\frac{2a^3(iA+B)\cot^2(c+dx)}{d}+\frac{a^3(47A-45iB)\cot^3(c+dx)}{60d}\\&\quad+\frac{4a^3(iA+B)\log(\sin(c+dx))}{d}-\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}\\&\quad-\frac{(7iA+5B)\cot^4(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}\end{aligned}$$

output
```
-4*a^3*(A-I*B)*x-4*a^3*(A-I*B)*cot(d*x+c)/d+2*a^3*(I*A+B)*cot(d*x+c)^2/d+1
/60*a^3*(47*A-45*I*B)*cot(d*x+c)^3/d+4*a^3*(I*A+B)*ln(sin(d*x+c))/d-1/5*a*
A*cot(d*x+c)^5*(a+I*a*tan(d*x+c))^2/d-1/20*(7*I*A+5*B)*cot(d*x+c)^4*(a^3+I
*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.15

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= a^3\left(-\frac{4A\cot(c+dx)}{d}+\frac{4iB\cot(c+dx)}{d}+\frac{2iA\cot^2(c+dx)}{d}+\frac{2B\cot^2(c+dx)}{d}+\frac{4A\cot^3(c+dx)}{3d}-\frac{iB\cot^3(c+dx)}{d}-\frac{3iA\cot^4(c+dx)}{4d}-\frac{B\cot^4(c+dx)}{4d}-\frac{A\cot^5(c+dx)}{5d}+\frac{4iA\log(\tan(c+dx))}{d}+\frac{4B\log(\tan(c+dx))}{d}-\frac{4iA\log(i+\tan(c+dx))}{d}-\frac{4B\log(i+\tan(c+dx))}{d}\right)$$

input
```
Integrate[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
a^3*((-4*A*Cot[c + d*x])/d + ((4*I)*B*Cot[c + d*x])/d + ((2*I)*A*Cot[c + d
*x]^2)/d + (2*B*Cot[c + d*x]^2)/d + (4*A*Cot[c + d*x]^3)/(3*d) - (I*B*Cot[
c + d*x]^3)/d - (((3*I)/4)*A*Cot[c + d*x]^4)/d - (B*Cot[c + d*x]^4)/(4*d)
- (A*Cot[c + d*x]^5)/(5*d) + ((4*I)*A*Log[Tan[c + d*x]])/d + (4*B*Log[Tan[
c + d*x]])/d - ((4*I)*A*Log[I + Tan[c + d*x]])/d - (4*B*Log[I + Tan[c + d*
x]])/d)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 4076, 3042, 4076, 25, 3042, 4074, 27, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^6}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{5}\int \cot^5(c+dx)(i\tan(c+dx)a+a)^2(a(7iA+5B)-a(3A-5iB)\tan(c+dx))dx-\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)^2(a(7iA+5B)-a(3A-5iB)\tan(c+dx))}{\tan(c+dx)^5}dx-\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{5}\left(\frac{1}{4}\int -\cot^4(c+dx)(i\tan(c+dx)a+a)\left((47A-45iB)a^2+(33iA+35B)\tan(c+dx)a^2\right)dx-\frac{(5B+7iA)\,\mathrm{c}}{}\right.$$
$$-\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(-\frac{1}{4}\int \cot^4(c+dx)(i\tan(c+dx)a+a)\left((47A-45iB)a^2+(33iA+35B)\tan(c+dx)a^2\right)dx-\frac{(5B+7iA)\,\mathrm{c}}{}\right.$$
$$-\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-\frac{1}{4}\int \frac{(i\tan(c+dx)a+a)\left((47A-45iB)a^2+(33iA+35B)\tan(c+dx)a^2\right)}{\tan(c+dx)^4}dx-\frac{(5B+7iA)\cot^4(c+dx)\,(}{4d}\right.$$
$$-\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{4074}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-\int 80\cot^3(c+dx)\left(a^3(iA+B)-a^3(A-iB)\tan(c+dx)\right)dx\right)-\frac{(5B+}{}\right.$$
$$-\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\int\cot^3(c+dx)\left(a^3(iA+B)-a^3(A-iB)\tan(c+dx)\right)dx\right)-\frac{(5B+\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\int\frac{a^3(iA+B)-a^3(A-iB)\tan(c+dx)}{\tan(c+dx)^3}dx\right)-\frac{(5B+7iA)\cot^4(c+\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 4012

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\left(\int-\cot^2(c+dx)\left((A-iB)a^3+(iA+B)\tan(c+dx)a^3\right)dx-\frac{a^3(B\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 25

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\left(-\int\cot^2(c+dx)\left((A-iB)a^3+(iA+B)\tan(c+dx)a^3\right)dx-\frac{a^3(B\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\left(-\int\frac{(A-iB)a^3+(iA+B)\tan(c+dx)a^3}{\tan(c+dx)^2}dx-\frac{a^3(B+iA)\cot^2(c+}{2d}\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 4012

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\left(-\int\cot(c+dx)\left(a^3(iA+B)-a^3(A-iB)\tan(c+dx)\right)dx-\frac{a^3(B\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\Bigg(-\int\frac{a^3(iA+B)-a^3(A-iB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a^3(B+iA)\cot^2(c+$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\Big\downarrow$ 4014

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\Bigg(-a^3(B+iA)\int\cot(c+dx)dx-\frac{a^3(B+iA)\cot^2(c+dx)}{2d}+\frac{a^3(A-$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\Bigg(-a^3(B+iA)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a^3(B+iA)\cot^2(c+dx)}{2d}+$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\Big\downarrow$ 25

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\Bigg(a^3(B+iA)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a^3(B+iA)\cot^2(c+dx)}{2d}$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

$\Big\downarrow$ 3956

$$\frac{1}{5}\Bigg(\frac{1}{4}\Bigg(\frac{a^3(47A-45iB)\cot^3(c+dx)}{3d}-80\Bigg(-\frac{a^3(B+iA)\cot^2(c+dx)}{2d}+\frac{a^3(A-iB)\cot(c+dx)}{d}-\frac{a^3(B+iA)\mathrm{l}}{}$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

input
```
Int[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
-1/5*(a*A*Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^2)/d + (((a^3*(47*A - (45*
I)*B)*Cot[c + d*x]^3)/(3*d) - 80*(a^3*(A - I*B)*x + (a^3*(A - I*B)*Cot[c +
 d*x])/d - (a^3*(I*A + B)*Cot[c + d*x]^2)/(2*d) - (a^3*(I*A + B)*Log[-Sin[
c + d*x]])/d))/4 - (((7*I)*A + 5*B)*Cot[c + d*x]^4*(a^3 + I*a^3*Tan[c + d*
x]))/(4*d))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{4a^3\left(-\frac{(iA+B)\ln\left(\sec(dx+c)^2\right)}{2}+(iA+B)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^5}{20}-\frac{\cot(dx+c)^4(3iA+B)}{16}+\cot(dx+c)^3\left(-\frac{iB}{4}+\frac{A}{3}\right)+\right.}{d}$
derivativedivides	$\frac{a^3\left(-\frac{3iA\cot(dx+c)^4}{4}-\frac{A\cot(dx+c)^5}{5}-iB\cot(dx+c)^3-\frac{B\cot(dx+c)^4}{4}+2iA\cot(dx+c)^2+\frac{4A\cot(dx+c)^3}{3}+4iB\cot(dx+c)+\right.}{d}$
default	$\frac{a^3\left(-\frac{3iA\cot(dx+c)^4}{4}-\frac{A\cot(dx+c)^5}{5}-iB\cot(dx+c)^3-\frac{B\cot(dx+c)^4}{4}+2iA\cot(dx+c)^2+\frac{4A\cot(dx+c)^3}{3}+4iB\cot(dx+c)+\right.}{d}$
risch	$-\frac{8ia^3Bc}{d}+\frac{8a^3Ac}{d}-\frac{2a^3\left(240iA\,e^{8i(dx+c)}+180B\,e^{8i(dx+c)}-585iA\,e^{6i(dx+c)}-525B\,e^{6i(dx+c)}+695iA\,e^{4i(dx+c)}+\right.}{15d\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{(4iB\,a^3-4A\,a^3)x\tan(dx+c)^5-\frac{A\,a^3}{5d}+\frac{\left(-3iB\,a^3+4A\,a^3\right)\tan(dx+c)^2}{3d}-\frac{\left(3iA\,a^3+B\,a^3\right)\tan(dx+c)}{4d}-\frac{4\left(-iB\,a^3+A\,a^3\right)\tan(dx}{d}}{\tan(dx+c)^5}$

input
```
int(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
4*a^3/d*(-1/2*(I*A+B)*ln(sec(d*x+c)^2)+(I*A+B)*ln(tan(d*x+c))-1/20*A*cot(d
*x+c)^5-1/16*cot(d*x+c)^4*(3*I*A+B)+cot(d*x+c)^3*(-1/4*I*B+1/3*A)+1/2*cot(
d*x+c)^2*(I*A+B)+cot(d*x+c)*(-A+I*B)+(-A+I*B)*x*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.59

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{2\left(60\,(4i\,A+3\,B)a^3e^{(8i\,dx+8i\,c)}+15\,(-39i\,A-35\,B)a^3e^{(6i\,dx+6i\,c)}+5\,(139i\,A+123\,B)a^3e^{(4i\,dx+4i\,c)}+\right.}{}$$

input
```
integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-2/15*(60*(4*I*A + 3*B)*a^3*e^(8*I*d*x + 8*I*c) + 15*(-39*I*A - 35*B)*a^3*
e^(6*I*d*x + 6*I*c) + 5*(139*I*A + 123*B)*a^3*e^(4*I*d*x + 4*I*c) + 5*(-77
*I*A - 69*B)*a^3*e^(2*I*d*x + 2*I*c) + (83*I*A + 75*B)*a^3 + 30*((-I*A - B
)*a^3*e^(10*I*d*x + 10*I*c) + 5*(I*A + B)*a^3*e^(8*I*d*x + 8*I*c) + 10*(-I
*A - B)*a^3*e^(6*I*d*x + 6*I*c) + 10*(I*A + B)*a^3*e^(4*I*d*x + 4*I*c) + 5
*(-I*A - B)*a^3*e^(2*I*d*x + 2*I*c) + (I*A + B)*a^3)*log(e^(2*I*d*x + 2*I*
c) - 1))/(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*
d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.64

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{4ia^3(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d} + \frac{-166iAa^3-150Ba^3+\left(770iAa^3e^{2ic}+690Ba^3e^{2ic}\right)e^{2idx}+\left(-1390iAa^3e^{4ic}-1230Ba^3e^{4ic}\right)e^{4idx}+(11}{15de^{10ic}e^{10idx}-75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}-150de^{4ic}e^{4}}$$

input
```
integrate(cot(d*x+c)**6*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
4*I*a**3*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-166*I*A*a**3 - 15
0*B*a**3 + (770*I*A*a**3*exp(2*I*c) + 690*B*a**3*exp(2*I*c))*exp(2*I*d*x)
+ (-1390*I*A*a**3*exp(4*I*c) - 1230*B*a**3*exp(4*I*c))*exp(4*I*d*x) + (117
0*I*A*a**3*exp(6*I*c) + 1050*B*a**3*exp(6*I*c))*exp(6*I*d*x) + (-480*I*A*a
**3*exp(8*I*c) - 360*B*a**3*exp(8*I*c))*exp(8*I*d*x))/(15*d*exp(10*I*c)*ex
p(10*I*d*x) - 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d*x)
 - 150*d*exp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) - 15*d)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.84

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{240\,(dx+c)(A-i\,B)a^3+120\,(i\,A+B)a^3\log\left(\tan\left(dx+c\right)^2+1\right)+240\,(-i\,A-B)a^3\log\left(\tan\left(dx+\right.\right.}{60\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/60*(240*(d*x + c)*(A - I*B)*a^3 + 120*(I*A + B)*a^3*log(tan(d*x + c)^2
+ 1) + 240*(-I*A - B)*a^3*log(tan(d*x + c)) + (240*(A - I*B)*a^3*tan(d*x +
 c)^4 - 120*(I*A + B)*a^3*tan(d*x + c)^3 - 20*(4*A - 3*I*B)*a^3*tan(d*x +
c)^2 - 15*(-3*I*A - B)*a^3*tan(d*x + c) + 12*A*a^3)/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.89

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= -\frac{4\,(i\,Aa^3+Ba^3)\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{4\,(-i\,Aa^3-Ba^3)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$

$$-\frac{240\,(Aa^3-i\,Ba^3)\tan\left(dx+c\right)^4+12\,Aa^3+120\,(-i\,Aa^3-Ba^3)\tan\left(dx+c\right)^3-20\,(4\,Aa^3-3i\,Ba^3)\,\mathrm{t}}{60\,d\tan\left(dx+c\right)^5}$$

input
```
integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-4*(I*A*a^3 + B*a^3)*log(tan(d*x + c) + I)/d - 4*(-I*A*a^3 - B*a^3)*log(ab
s(tan(d*x + c)))/d - 1/60*(240*(A*a^3 - I*B*a^3)*tan(d*x + c)^4 + 12*A*a^3
 + 120*(-I*A*a^3 - B*a^3)*tan(d*x + c)^3 - 20*(4*A*a^3 - 3*I*B*a^3)*tan(d*
x + c)^2 + 15*(3*I*A*a^3 + B*a^3)*tan(d*x + c))/(d*tan(d*x + c)^5)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.78

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{\frac{A\,a^3}{5}-\tan(c+d\,x)^2\left(\frac{4\,A\,a^3}{3}-B\,a^3\,1\mathrm{i}\right)+\tan(c+d\,x)^4\,(4\,A\,a^3-B\,a^3\,4\mathrm{i})-\tan(c+d\,x)^3\,(2\,B\,a^3+A\,a}{d\tan{(c+d\,x)}^5}$$
$$+\frac{a^3\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\,(B+A\,1\mathrm{i})\,8\mathrm{i}}{d}$$

input
```
int(cot(c + d*x)^6*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(a^3*atan(2*tan(c + d*x) + 1i)*(A*1i + B)*8i)/d - (tan(c + d*x)^4*(4*A*a^3
 - B*a^3*4i) - tan(c + d*x)^2*((4*A*a^3)/3 - B*a^3*1i) - tan(c + d*x)^3*(A
*a^3*2i + 2*B*a^3) + (A*a^3)/5 + tan(c + d*x)*((A*a^3*3i)/4 + (B*a^3)/4))/
(d*tan(c + d*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.57

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{a^3\left(-2656\cos(dx+c)\sin(dx+c)^4a+2400\cos(dx+c)\sin(dx+c)^4bi+832\cos(dx+c)\sin(dx+c)^2\right.}{}$$

input

```
int(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output

```
(a**3*( - 2656*cos(c + d*x)*sin(c + d*x)**4*a + 2400*cos(c + d*x)*sin(c +
d*x)**4*b*i + 832*cos(c + d*x)*sin(c + d*x)**2*a - 480*cos(c + d*x)*sin(c
+ d*x)**2*b*i - 96*cos(c + d*x)*a - 1920*log(tan((c + d*x)/2)**2 + 1)*sin(
c + d*x)**5*a*i - 1920*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**5*b + 19
20*log(tan((c + d*x)/2))*sin(c + d*x)**5*a*i + 1920*log(tan((c + d*x)/2))*
sin(c + d*x)**5*b - 1920*sin(c + d*x)**5*a*d*x - 705*sin(c + d*x)**5*a*i +
 1920*sin(c + d*x)**5*b*d*i*x - 555*sin(c + d*x)**5*b + 1680*sin(c + d*x)*
*3*a*i + 1200*sin(c + d*x)**3*b - 360*sin(c + d*x)*a*i - 120*sin(c + d*x)*
b))/(480*sin(c + d*x)**5*d)
```

3.26 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 225

$$\begin{aligned}&\int \tan^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx\\&=-8a^4(A-iB)x+\frac{8a^4(iA+B)\log(\cos(c+dx))}{d}\\&\quad+\frac{8a^4(A-iB)\tan(c+dx)}{d}+\frac{4a^4(iA+B)\tan^2(c+dx)}{d}\\&\quad-\frac{a^4(92A-93iB)\tan^3(c+dx)}{60d}+\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}\\&\quad-\frac{(2A-3iB)\tan^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d}\\&\quad-\frac{(12A-13iB)\tan^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{20d}\end{aligned}$$

output
```
-8*a^4*(A-I*B)*x+8*a^4*(I*A+B)*ln(cos(d*x+c))/d+8*a^4*(A-I*B)*tan(d*x+c)/d
+4*a^4*(I*A+B)*tan(d*x+c)^2/d-1/60*a^4*(92*A-93*I*B)*tan(d*x+c)^3/d+1/6*I*
a*B*tan(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d-1/10*(2*A-3*I*B)*tan(d*x+c)^3*(a^2
+I*a^2*tan(d*x+c))^2/d-1/20*(12*A-13*I*B)*tan(d*x+c)^3*(a^4+I*a^4*tan(d*x+
c))/d
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.61

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{a^4(-92iA-93B-480i(A-iB)\log(i+\tan(c+dx))+480(A-iB)\tan(c+dx)+240(iA+B)\tan^2(}{}$$

input
```
Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(a^4*((-92*I)*A - 93*B - (480*I)*(A - I*B)*Log[I + Tan[c + d*x]] + 480*(A
- I*B)*Tan[c + d*x] + 240*(I*A + B)*Tan[c + d*x]^2 - 20*(7*A - (8*I)*B)*Ta
n[c + d*x]^3 + ((-60*I)*A - 105*B)*Tan[c + d*x]^4 + 12*(A - (4*I)*B)*Tan[c
 + d*x]^5 + 10*B*Tan[c + d*x]^6))/(60*d)
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4077, 3042, 4075, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a+ia\tan(c+dx))^4(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{6}\int 3\tan^2(c+dx)(i\tan(c+dx)a+a)^3(a(2A-iB)+a(2iA+3B)\tan(c+dx))dx+\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \tan^2(c+dx)(i\tan(c+dx)a+a)^3(a(2A-iB)+a(2iA+3B)\tan(c+dx))dx+\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \tan(c+dx)^2(i\tan(c+dx)a+a)^3(a(2A-iB)+a(2iA+3B)\tan(c+dx))dx+\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{2}\left(\frac{1}{5}\int 2\tan^2(c+dx)(i\tan(c+dx)a+a)^2\left((8A-7iB)a^2+(12iA+13B)\tan(c+dx)a^2\right)dx-\frac{(2A-3iB)\tan}{}\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{2}{5}\int \tan^2(c+dx)(i\tan(c+dx)a+a)^2\left((8A-7iB)a^2+(12iA+13B)\tan(c+dx)a^2\right)dx-\frac{(2A-3iB)\tan^3}{}\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2}{5}\int \tan(c+dx)^2(i\tan(c+dx)a+a)^2\left((8A-7iB)a^2+(12iA+13B)\tan(c+dx)a^2\right)dx-\frac{(2A-3iB)\tan^3}{}\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{4}\int \tan^2(c+dx)(i\tan(c+dx)a+a)\left((68A-67iB)a^3+(92iA+93B)\tan(c+dx)a^3\right)dx-\frac{(12A-13i}{}\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{4}\int \tan(c+dx)^2(i\tan(c+dx)a+a)\left((68A-67iB)a^3+(92iA+93B)\tan(c+dx)a^3\right)dx-\frac{(12A-13i\right.\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{4075}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{4}\left(\int \tan^2(c+dx)\left(160(A-iB)a^4+160(iA+B)\tan(c+dx)a^4\right)dx-\frac{a^4(92A-93iB)\tan^3(c+dx)}{3d}\right)\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{4}\left(\int \tan(c+dx)^2\left(160(A-iB)a^4+160(iA+B)\tan(c+dx)a^4\right)dx-\frac{a^4(92A-93iB)\tan^3(c+dx)}{3d}\right)\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{4011}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{4}\left(\int \tan(c+dx)\left(160a^4(A-iB)\tan(c+dx)-160a^4(iA+B)\right)dx-\frac{a^4(92A-93iB)\tan^3(c+dx)}{3d}+\frac{8}{}\right.\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{4}\left(\int \tan(c+dx)\left(160a^4(A-iB)\tan(c+dx)-160a^4(iA+B)\right)dx-\frac{a^4(92A-93iB)\tan^3(c+dx)}{3d}+\frac{8}{}\right.\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{4008}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{4}\left(-160a^4(B+iA)\int \tan(c+dx)dx-\frac{a^4(92A-93iB)\tan^3(c+dx)}{3d}+\frac{80a^4(B+iA)\tan^2(c+dx)}{d}+\frac{1}{}\right.\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{4}\left(-160a^4(B+iA)\int \tan(c+dx)dx-\frac{a^4(92A-93iB)\tan^3(c+dx)}{3d}+\frac{80a^4(B+iA)\tan^2(c+dx)}{d}+\frac{1}{}\right.\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \; 3956$$

$$\frac{1}{2}\left(\frac{2}{5}\left(\frac{1}{4}\left(-\frac{a^4(92A-93iB)\tan^3(c+dx)}{3d}+\frac{80a^4(B+iA)\tan^2(c+dx)}{d}+\frac{160a^4(A-iB)\tan(c+dx)}{d}+\frac{160a^4}{}\right.\right.\right.$$
$$\frac{iaB\tan^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

input
```
Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
((I/6)*a*B*Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d + (-1/5*((2*A - (3*I
)*B)*Tan[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x])^2)/d + (2*(-1/4*((12*A - (1
3*I)*B)*Tan[c + d*x]^3*(a^4 + I*a^4*Tan[c + d*x]))/d + (-160*a^4*(A - I*B)
*x + (160*a^4*(I*A + B)*Log[Cos[c + d*x]])/d + (160*a^4*(A - I*B)*Tan[c +
d*x])/d + (80*a^4*(I*A + B)*Tan[c + d*x]^2)/d - (a^4*(92*A - (93*I)*B)*Tan
[c + d*x]^3)/(3*d))/4))/5)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{a^4\left(-\frac{4iB\tan(dx+c)^5}{5}+\frac{B\tan(dx+c)^6}{6}-iA\tan(dx+c)^4+\frac{A\tan(dx+c)^5}{5}+\frac{8iB\tan(dx+c)^3}{3}-\frac{7B\tan(dx+c)^4}{4}+4iA\tan(dx+c)^2\right.}{}$
default	$\frac{a^4\left(-\frac{4iB\tan(dx+c)^5}{5}+\frac{B\tan(dx+c)^6}{6}-iA\tan(dx+c)^4+\frac{A\tan(dx+c)^5}{5}+\frac{8iB\tan(dx+c)^3}{3}-\frac{7B\tan(dx+c)^4}{4}+4iA\tan(dx+c)^2\right.}{}$
norman	$(8iB\,a^4-8A\,a^4)\,x-\frac{(4iA\,a^4+7B\,a^4)\tan(dx+c)^4}{4d}-\frac{(-8iB\,a^4+7A\,a^4)\tan(dx+c)^3}{3d}+\frac{8(-iB\,a^4+A\,a^4)\,\mathrm{ta}}{d}$
parallelrisch	$-\frac{480iB\tan(dx+c)a^4-10B\tan(dx+c)^6a^4+48iB\tan(dx+c)^5a^4-12A\tan(dx+c)^5a^4-480iBx\,a^4d+105B\tan(dx+c)^4}{}$
risch	$-\frac{16ia^4Bc}{d}+\frac{16a^4Ac}{d}+\frac{4a^4\left(210iA\,\mathrm{e}^{10i(dx+c)}+270B\,\mathrm{e}^{10i(dx+c)}+765iA\,\mathrm{e}^{8i(dx+c)}+855B\,\mathrm{e}^{8i(dx+c)}+1210iA\,\mathrm{e}^{6i(dx}\right.}{1}$
parts	$\frac{(-4iA\,a^4-6B\,a^4)\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{(-4iB\,a^4+A\,a^4)\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\mathrm{ta}\right.}{d}$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*a^4*(-4/5*I*B*tan(d*x+c)^5+1/6*B*tan(d*x+c)^6-I*A*tan(d*x+c)^4+1/5*A*t
an(d*x+c)^5+8/3*I*B*tan(d*x+c)^3-7/4*B*tan(d*x+c)^4+4*I*A*tan(d*x+c)^2-7/3
*A*tan(d*x+c)^3-8*I*B*tan(d*x+c)+4*B*tan(d*x+c)^2+8*A*tan(d*x+c)+1/2*(-8*I
*A-8*B)*ln(1+tan(d*x+c)^2)+(-8*A+8*I*B)*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.53

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = -\frac{4\left(30\left(-7i\,A-9\,B\right)a^4e^{(10i\,dx+10i\,c)}+45\left(-17i\,A-19\,B\right)a^4e^{(8i\,dx+8i\,c)}+10\left(-121i\,A-135\,B\right)a^4e^{(6i\,dx+}\right.}{}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-4/15*(30*(-7*I*A - 9*B)*a^4*e^(10*I*d*x + 10*I*c) + 45*(-17*I*A - 19*B)*a
^4*e^(8*I*d*x + 8*I*c) + 10*(-121*I*A - 135*B)*a^4*e^(6*I*d*x + 6*I*c) + 1
5*(-68*I*A - 75*B)*a^4*e^(4*I*d*x + 4*I*c) + 6*(-74*I*A - 81*B)*a^4*e^(2*I
*d*x + 2*I*c) + (-79*I*A - 86*B)*a^4 + 30*((-I*A - B)*a^4*e^(12*I*d*x + 12
*I*c) + 6*(-I*A - B)*a^4*e^(10*I*d*x + 10*I*c) + 15*(-I*A - B)*a^4*e^(8*I*
d*x + 8*I*c) + 20*(-I*A - B)*a^4*e^(6*I*d*x + 6*I*c) + 15*(-I*A - B)*a^4*e
^(4*I*d*x + 4*I*c) + 6*(-I*A - B)*a^4*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^4
)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x
 + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^
(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.55

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{8ia^4(A-iB)\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$
$$+ \frac{316iAa^4+344Ba^4+\left(1776iAa^4e^{2ic}+1944Ba^4e^{2ic}\right)e^{2idx}+\left(4080iAa^4e^{4ic}+4500Ba^4e^{4ic}\right)e^{4idx}+(484}{15de^{12ic}e^{12idx}+90de^{10ic}e^{10idx}+225de^{8ic}e^{8idx}}$$

input
```
integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
8*I*a**4*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (316*I*A*a**4 + 344
*B*a**4 + (1776*I*A*a**4*exp(2*I*c) + 1944*B*a**4*exp(2*I*c))*exp(2*I*d*x)
 + (4080*I*A*a**4*exp(4*I*c) + 4500*B*a**4*exp(4*I*c))*exp(4*I*d*x) + (484
0*I*A*a**4*exp(6*I*c) + 5400*B*a**4*exp(6*I*c))*exp(6*I*d*x) + (3060*I*A*a
**4*exp(8*I*c) + 3420*B*a**4*exp(8*I*c))*exp(8*I*d*x) + (840*I*A*a**4*exp(
10*I*c) + 1080*B*a**4*exp(10*I*c))*exp(10*I*d*x))/(15*d*exp(12*I*c)*exp(12
*I*d*x) + 90*d*exp(10*I*c)*exp(10*I*d*x) + 225*d*exp(8*I*c)*exp(8*I*d*x) +
 300*d*exp(6*I*c)*exp(6*I*d*x) + 225*d*exp(4*I*c)*exp(4*I*d*x) + 90*d*exp(
2*I*c)*exp(2*I*d*x) + 15*d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{10\,Ba^4\tan(dx+c)^6 + 12\,(A-4i\,B)a^4\tan(dx+c)^5 - 15\,(4i\,A+7\,B)a^4\tan(dx+c)^4 - 20\,(7\,A-8i\,B$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/60*(10*B*a^4*tan(d*x + c)^6 + 12*(A - 4*I*B)*a^4*tan(d*x + c)^5 - 15*(4*
I*A + 7*B)*a^4*tan(d*x + c)^4 - 20*(7*A - 8*I*B)*a^4*tan(d*x + c)^3 - 240*
(-I*A - B)*a^4*tan(d*x + c)^2 - 480*(d*x + c)*(A - I*B)*a^4 - 240*(I*A + B
)*a^4*log(tan(d*x + c)^2 + 1) + 480*(A - I*B)*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{8\,(i\,Aa^4+Ba^4)\log(\tan(dx+c)+i)}{d}$$
$$+ \frac{10\,Ba^4d^5\tan(dx+c)^6 + 12\,Aa^4d^5\tan(dx+c)^5 - 48i\,Ba^4d^5\tan(dx+c)^5 - 60i\,Aa^4d^5\tan(dx+c)^4$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-8*(I*A*a^4 + B*a^4)*log(tan(d*x + c) + I)/d + 1/60*(10*B*a^4*d^5*tan(d*x
+ c)^6 + 12*A*a^4*d^5*tan(d*x + c)^5 - 48*I*B*a^4*d^5*tan(d*x + c)^5 - 60*
I*A*a^4*d^5*tan(d*x + c)^4 - 105*B*a^4*d^5*tan(d*x + c)^4 - 140*A*a^4*d^5*
tan(d*x + c)^3 + 160*I*B*a^4*d^5*tan(d*x + c)^3 + 240*I*A*a^4*d^5*tan(d*x
+ c)^2 + 240*B*a^4*d^5*tan(d*x + c)^2 + 480*A*a^4*d^5*tan(d*x + c) - 480*I
*B*a^4*d^5*tan(d*x + c))/d^6
```

Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.37

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)^3\left(-a^4\,(A-B\,1\mathrm{i})+\frac{a^4\,(B+A\,3\mathrm{i})\,1\mathrm{i}}{3}+\frac{B\,a^4\,1\mathrm{i}}{3}+\frac{a^4\,(3\,B+A\,1\mathrm{i})\,1\mathrm{i}}{3}\right)}{d} - \frac{\tan(c+d\,x)\,\left(-A\,a^4-3\,a^4\,(A-B\,1\mathrm{i})+a^4\,(B+A\,3\mathrm{i})\,1\mathrm{i}+B\,a^4\,1\mathrm{i}+a^4\,(3\,B+A\,1\mathrm{i})\,1\mathrm{i}\right)}{d} - \frac{\tan(c+d\,x)^5\left(\frac{B\,a^4\,1\mathrm{i}}{5}+\frac{a^4\,(3\,B+A\,1\mathrm{i})\,1\mathrm{i}}{5}\right)}{d} - \frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,\left(8\,B\,a^4+A\,a^4\,8\mathrm{i}\right)}{d} + \frac{\tan(c+d\,x)^2\left(\frac{A\,a^4\,1\mathrm{i}}{2}+\frac{a^4\,(A-B\,1\mathrm{i})\,3\mathrm{i}}{2}+\frac{a^4\,(B+A\,3\mathrm{i})}{2}+\frac{B\,a^4}{2}+\frac{a^4\,(3\,B+A\,1\mathrm{i})}{2}\right)}{d} - \frac{\tan(c+d\,x)^4\left(\frac{a^4\,(A-B\,1\mathrm{i})\,3\mathrm{i}}{4}+\frac{B\,a^4}{4}+\frac{a^4\,(3\,B+A\,1\mathrm{i})}{4}\right)}{d} + \frac{B\,a^4\tan(c+d\,x)^6}{6\,d}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(tan(c + d*x)^3*((a^4*(A*3i + B)*1i)/3 - a^4*(A - B*1i) + (B*a^4*1i)/3 + (
a^4*(A*1i + 3*B)*1i)/3))/d - (tan(c + d*x)*(a^4*(A*3i + B)*1i - 3*a^4*(A -
 B*1i) - A*a^4 + B*a^4*1i + a^4*(A*1i + 3*B)*1i))/d - (tan(c + d*x)^5*((B*
a^4*1i)/5 + (a^4*(A*1i + 3*B)*1i)/5))/d - (log(tan(c + d*x) + 1i)*(A*a^4*8
i + 8*B*a^4))/d + (tan(c + d*x)^2*((A*a^4*1i)/2 + (a^4*(A - B*1i)*3i)/2 +
(a^4*(A*3i + B))/2 + (B*a^4)/2 + (a^4*(A*1i + 3*B))/2))/d - (tan(c + d*x)^
4*((a^4*(A - B*1i)*3i)/4 + (B*a^4)/4 + (a^4*(A*1i + 3*B))/4))/d + (B*a^4*t
an(c + d*x)^6)/(6*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.76

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{a^4\left(-240\log\left(\tan(dx+c)^2+1\right)ai-240\log\left(\tan(dx+c)^2+1\right)b+10\tan(dx+c)^6\,b+12\tan(dx+c)^5\right.}{}$$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(a**4*( - 240*log(tan(c + d*x)**2 + 1)*a*i - 240*log(tan(c + d*x)**2 + 1)*
b + 10*tan(c + d*x)**6*b + 12*tan(c + d*x)**5*a - 48*tan(c + d*x)**5*b*i -
 60*tan(c + d*x)**4*a*i - 105*tan(c + d*x)**4*b - 140*tan(c + d*x)**3*a +
160*tan(c + d*x)**3*b*i + 240*tan(c + d*x)**2*a*i + 240*tan(c + d*x)**2*b
+ 480*tan(c + d*x)*a - 480*tan(c + d*x)*b*i - 480*a*d*x + 480*b*d*i*x))/(6
0*d)
```

3.27 $\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 168

$$\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = -8a^4(iA+B)x - \frac{8a^4(A-iB)\log(\cos(c+dx))}{d} + \frac{4a^4(iA+B)\tan(c+dx)}{d} + \frac{a(A-iB)(a+ia\tan(c+dx))^3}{3d} + \frac{A(a+ia\tan(c+dx))^4}{4d} - \frac{iB(a+ia\tan(c+dx))^5}{5ad} + \frac{(A-iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{d}$$

output
```
-8*a^4*(I*A+B)*x-8*a^4*(A-I*B)*ln(cos(d*x+c))/d+4*a^4*(I*A+B)*tan(d*x+c)/d
+1/3*a*(A-I*B)*(a+I*a*tan(d*x+c))^3/d+1/4*A*(a+I*a*tan(d*x+c))^4/d-1/5*I*B
*(a+I*a*tan(d*x+c))^5/a/d+(A-I*B)*(a^2+I*a^2*tan(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.71

$$\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{a^4(95A-92iB+480(A-iB)\log(i+\tan(c+dx))+480(iA+B)\tan(c+dx)-30(7A-8iB)\tan^2(c}{60d}$$

input
```
Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(a^4*(95*A - (92*I)*B + 480*(A - I*B)*Log[I + Tan[c + d*x]] + 480*(I*A + B
)*Tan[c + d*x] - 30*(7*A - (8*I)*B)*Tan[c + d*x]^2 + ((-80*I)*A - 140*B)*T
an[c + d*x]^3 + 15*(A - (4*I)*B)*Tan[c + d*x]^4 + 12*B*Tan[c + d*x]^5))/(6
0*d)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4075, 3042, 4010, 3042, 3959, 3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int (i\tan(c+dx)a+a)^4(A\tan(c+dx)-B)dx-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\int (i\tan(c+dx)a+a)^4(A\tan(c+dx)-B)dx-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

$$\Big\downarrow \text{4010}$$

$$-(B+iA)\int (i\tan(c+dx)a+a)^4dx+\frac{A(a+ia\tan(c+dx))^4}{4d}-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\int (i\tan(c+dx)a+a)^4dx+\frac{A(a+ia\tan(c+dx))^4}{4d}-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

$$\Big\downarrow \text{3959}$$

$$-(B+iA)\left(2a\int (i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{A(a+ia\tan(c+dx))^4}{4d}-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\left(2a\int (i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{A(a+ia\tan(c+dx))^4}{4d}-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

$$\Big\downarrow \text{3959}$$

$$-(B+iA)\left(2a\left(2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{A(a+ia\tan(c+dx))^4}{4d}-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\left(2a\left(2a\int (i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{A(a+ia\tan(c+dx))^4}{4d}-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

$$\Big\downarrow \text{3958}$$

$$-(B+iA)\left(2a\left(2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+}{3d}\right.$$
$$\frac{A(a+ia\tan(c+dx))^4}{4d}-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\left(2a\left(2a\left(2ia^2\int \tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{A(a+ia\tan(c+dx))^4}{4d}-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

$\downarrow$ 3956

$$-(B+iA)\left(2a\left(2a\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{A(a+ia\tan(c+dx))^4}{4d}-\frac{iB(a+ia\tan(c+dx))^5}{5ad}$$

input

```
Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output

```
(A*(a + I*a*Tan[c + d*x])^4)/(4*d) - ((I/5)*B*(a + I*a*Tan[c + d*x])^5)/(a
*d) - (I*A + B)*(((I/3)*a*(a + I*a*Tan[c + d*x])^3)/d + 2*a*(((I/2)*a*(a +
 I*a*Tan[c + d*x])^2)/d + 2*a*(2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d -
 (a^2*Tan[c + d*x])/d)))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a^4\left(-iB\tan(dx+c)^4+\frac{B\tan(dx+c)^5}{5}-\frac{4iA\tan(dx+c)^3}{3}+\frac{A\tan(dx+c)^4}{4}+4iB\tan(dx+c)^2-\frac{7B\tan(dx+c)^3}{3}+8iA\tan(dx+c)\right.}{d}$
default	$\frac{a^4\left(-iB\tan(dx+c)^4+\frac{B\tan(dx+c)^5}{5}-\frac{4iA\tan(dx+c)^3}{3}+\frac{A\tan(dx+c)^4}{4}+4iB\tan(dx+c)^2-\frac{7B\tan(dx+c)^3}{3}+8iA\tan(dx+c)\right.}{d}$
norman	$(-8iA\,a^4-8B\,a^4)\,x-\frac{(4iA\,a^4+7B\,a^4)\tan(dx+c)^3}{3d}-\frac{(-8iB\,a^4+7A\,a^4)\tan(dx+c)^2}{2d}+\frac{8(iA\,a^4+B\,a^4)\,\mathrm{t}}{d}$
parallelrisch	$-\frac{60iB\tan(dx+c)^4a^4-12B\tan(dx+c)^5a^4+80iA\tan(dx+c)^3a^4-15A\tan(dx+c)^4a^4+480iAx\,a^4d-240iB\tan(dx+c)}{}$
risch	$\frac{16a^4Bc}{d}+\frac{16ia^4Ac}{d}+\frac{4ia^4\left(150iA\,\mathrm{e}^{8i(dx+c)}+210B\,\mathrm{e}^{8i(dx+c)}+465iA\,\mathrm{e}^{6i(dx+c)}+555B\,\mathrm{e}^{6i(dx+c)}+565iA\,\mathrm{e}^{4i(dx+c)}+\right.}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$
parts	$\frac{(-4iA\,a^4-6B\,a^4)\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{(-4iB\,a^4+A\,a^4)\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln}{}\right.}{d}$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*a^4*(-I*B*tan(d*x+c)^4+1/5*B*tan(d*x+c)^5-4/3*I*A*tan(d*x+c)^3+1/4*A*t
an(d*x+c)^4+4*I*B*tan(d*x+c)^2-7/3*B*tan(d*x+c)^3+8*I*A*tan(d*x+c)-7/2*A*t
an(d*x+c)^2+8*B*tan(d*x+c)+1/2*(8*A-8*I*B)*ln(1+tan(d*x+c)^2)+(-8*I*A-8*B)
*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.66

$$\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = -\frac{4\left(30\,(5\,A-7i\,B)a^4e^{(8i\,dx+8i\,c)}+15\,(31\,A-37i\,B)a^4e^{(6i\,dx+6i\,c)}+5\,(113\,A-131i\,B)a^4e^{(4i\,dx+4i\,c)}+5\,(\right.}{}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-4/15*(30*(5*A - 7*I*B)*a^4*e^(8*I*d*x + 8*I*c) + 15*(31*A - 37*I*B)*a^4*e
^(6*I*d*x + 6*I*c) + 5*(113*A - 131*I*B)*a^4*e^(4*I*d*x + 4*I*c) + 5*(64*A
 - 73*I*B)*a^4*e^(2*I*d*x + 2*I*c) + (70*A - 79*I*B)*a^4 + 30*((A - I*B)*a
^4*e^(10*I*d*x + 10*I*c) + 5*(A - I*B)*a^4*e^(8*I*d*x + 8*I*c) + 10*(A - I
*B)*a^4*e^(6*I*d*x + 6*I*c) + 10*(A - I*B)*a^4*e^(4*I*d*x + 4*I*c) + 5*(A
- I*B)*a^4*e^(2*I*d*x + 2*I*c) + (A - I*B)*a^4)*log(e^(2*I*d*x + 2*I*c) +
1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x +
 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(138) = 276.

Time = 0.50 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.73

$$\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{8a^4(A-iB)\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-280Aa^4+316iBa^4+\left(-1280Aa^4e^{2ic}+1460iBa^4e^{2ic}\right)e^{2idx}+\left(-2260Aa^4e^{4ic}+2620iBa^4e^{4ic}\right)e^{4idx}+}{15de^{10ic}e^{10idx}+75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}+150de^{4}}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
-8*a**4*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-280*A*a**4 + 316*I
*B*a**4 + (-1280*A*a**4*exp(2*I*c) + 1460*I*B*a**4*exp(2*I*c))*exp(2*I*d*x
) + (-2260*A*a**4*exp(4*I*c) + 2620*I*B*a**4*exp(4*I*c))*exp(4*I*d*x) + (-
1860*A*a**4*exp(6*I*c) + 2220*I*B*a**4*exp(6*I*c))*exp(6*I*d*x) + (-600*A*
a**4*exp(8*I*c) + 840*I*B*a**4*exp(8*I*c))*exp(8*I*d*x))/(15*d*exp(10*I*c)
*exp(10*I*d*x) + 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d
*x) + 150*d*exp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) + 15*d)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

$$\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{12\,Ba^4\tan(dx+c)^5+15\,(A-4i\,B)a^4\tan(dx+c)^4-20\,(4i\,A+7\,B)a^4\tan(dx+c)^3-30\,(7\,A-8i\,B}{}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/60*(12*B*a^4*tan(d*x + c)^5 + 15*(A - 4*I*B)*a^4*tan(d*x + c)^4 - 20*(4*
I*A + 7*B)*a^4*tan(d*x + c)^3 - 30*(7*A - 8*I*B)*a^4*tan(d*x + c)^2 - 480*
(d*x + c)*(I*A + B)*a^4 + 240*(A - I*B)*a^4*log(tan(d*x + c)^2 + 1) - 480*
(-I*A - B)*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.08

$$\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{8\left(Aa^4 - i\,Ba^4\right)\log\left(\tan\left(dx+c\right)+i\right)}{d}$$
$$+ \frac{12\,Ba^4d^4\tan\left(dx+c\right)^5 + 15\,Aa^4d^4\tan\left(dx+c\right)^4 - 60i\,Ba^4d^4\tan\left(dx+c\right)^4 - 80i\,Aa^4d^4\tan\left(dx+c\right)^3}{}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
8*(A*a^4 - I*B*a^4)*log(tan(d*x + c) + I)/d + 1/60*(12*B*a^4*d^4*tan(d*x +
 c)^5 + 15*A*a^4*d^4*tan(d*x + c)^4 - 60*I*B*a^4*d^4*tan(d*x + c)^4 - 80*I
*A*a^4*d^4*tan(d*x + c)^3 - 140*B*a^4*d^4*tan(d*x + c)^3 - 210*A*a^4*d^4*t
an(d*x + c)^2 + 240*I*B*a^4*d^4*tan(d*x + c)^2 + 480*I*A*a^4*d^4*tan(d*x +
 c) + 480*B*a^4*d^4*tan(d*x + c))/d^5
```

Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.43

$$\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{\tan(c+d\,x)^2\left(-\frac{3\,a^4\,(A-B\,1\mathrm{i})}{2}+\frac{a^4\,(B+A\,3\mathrm{i})\,1\mathrm{i}}{2}+\frac{B\,a^4\,1\mathrm{i}}{2}+\frac{a^4\,(3\,B+A\,1\mathrm{i})\,1\mathrm{i}}{2}\right)}{d}$$
$$+\frac{\tan(c+d\,x)\,\left(A\,a^4\,1\mathrm{i}+a^4\,(A-B\,1\mathrm{i})\,3\mathrm{i}+a^4\,(B+A\,3\mathrm{i})+B\,a^4+a^4\,(3\,B+A\,1\mathrm{i})\right)}{d}$$
$$-\frac{\tan(c+d\,x)^4\left(\frac{B\,a^4\,1\mathrm{i}}{4}+\frac{a^4\,(3\,B+A\,1\mathrm{i})\,1\mathrm{i}}{4}\right)}{d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(8\,A\,a^4-B\,a^4\,8\mathrm{i}\right)}{d}$$
$$-\frac{\tan(c+d\,x)^3\left(a^4\,(A-B\,1\mathrm{i})\,1\mathrm{i}+\frac{B\,a^4}{3}+\frac{a^4\,(3\,B+A\,1\mathrm{i})}{3}\right)}{d}+\frac{B\,a^4\tan(c+d\,x)^5}{5\,d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(tan(c + d*x)^2*((a^4*(A*3i + B)*1i)/2 - (3*a^4*(A - B*1i))/2 + (B*a^4*1i)
/2 + (a^4*(A*1i + 3*B)*1i)/2))/d + (tan(c + d*x)*(A*a^4*1i + a^4*(A - B*1i
)*3i + a^4*(A*3i + B) + B*a^4 + a^4*(A*1i + 3*B)))/d - (tan(c + d*x)^4*((B
*a^4*1i)/4 + (a^4*(A*1i + 3*B)*1i)/4))/d + (log(tan(c + d*x) + 1i)*(8*A*a^
4 - B*a^4*8i))/d - (tan(c + d*x)^3*(a^4*(A - B*1i)*1i + (B*a^4)/3 + (a^4*(
A*1i + 3*B))/3))/d + (B*a^4*tan(c + d*x)^5)/(5*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

$$\int \tan(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{a^4\left(240\log\left(\tan\left(dx+c\right)^2+1\right)a-240\log\left(\tan\left(dx+c\right)^2+1\right)bi+12\tan\left(dx+c\right)^5b+15\tan\left(dx+c\right)^4a\right.}{}$$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(a**4*(240*log(tan(c + d*x)**2 + 1)*a - 240*log(tan(c + d*x)**2 + 1)*b*i +
 12*tan(c + d*x)**5*b + 15*tan(c + d*x)**4*a - 60*tan(c + d*x)**4*b*i - 80
*tan(c + d*x)**3*a*i - 140*tan(c + d*x)**3*b - 210*tan(c + d*x)**2*a + 240
*tan(c + d*x)**2*b*i + 480*tan(c + d*x)*a*i + 480*tan(c + d*x)*b - 480*a*d
*i*x - 480*b*d*x))/(60*d)
```

3.28 $\int (a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 140

$$\begin{aligned}
&\int (a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx \\
&= 8a^4(A-iB)x-\frac{8a^4(iA+B)\log(\cos(c+dx))}{d} \\
&\quad -\frac{4a^4(A-iB)\tan(c+dx)}{d}+\frac{a(iA+B)(a+ia\tan(c+dx))^3}{3d} \\
&\quad +\frac{B(a+ia\tan(c+dx))^4}{4d}+\frac{(iA+B)\left(a^2+ia^2\tan(c+dx)\right)^2}{d}
\end{aligned}$$

output
```
8*a^4*(A-I*B)*x-8*a^4*(I*A+B)*ln(cos(d*x+c))/d-4*a^4*(A-I*B)*tan(d*x+c)/d+
1/3*a*(I*A+B)*(a+I*a*tan(d*x+c))^3/d+1/4*B*(a+I*a*tan(d*x+c))^4/d+(I*A+B)*
(a^2+I*a^2*tan(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

$$\int (a + ia\tan(c + dx))^4 (A + B\tan(c + dx))\,dx$$
$$= \frac{B(a + ia\tan(c + dx))^4 + \frac{4}{3}a^4(iA + B)\left(4 + 24\log(i + \tan(c + dx)) + 21i\tan(c + dx) - 6\tan^2(c + dx)\right.}{4d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(B*(a + I*a*Tan[c + d*x])^4 + (4*a^4*(I*A + B)*(4 + 24*Log[I + Tan[c + d*x
]] + (21*I)*Tan[c + d*x] - 6*Tan[c + d*x]^2 - I*Tan[c + d*x]^3))/3)/(4*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 4010, 3042, 3959, 3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + ia\tan(c + dx))^4 (A + B\tan(c + dx))\,dx$$
$$\Big\downarrow \ 3042$$
$$\int (a + ia\tan(c + dx))^4 (A + B\tan(c + dx))dx$$
$$\Big\downarrow \ 4010$$
$$(A - iB)\int (i\tan(c + dx)a + a)^4 dx + \frac{B(a + ia\tan(c + dx))^4}{4d}$$
$$\Big\downarrow \ 3042$$
$$(A - iB)\int (i\tan(c + dx)a + a)^4 dx + \frac{B(a + ia\tan(c + dx))^4}{4d}$$
$$\Big\downarrow \ 3959$$

$$(A-iB)\left(2a\int(i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{B(a+ia\tan(c+dx))^4}{4d}$$

$\Big\downarrow$ 3042

$$(A-iB)\left(2a\int(i\tan(c+dx)a+a)^3dx+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{B(a+ia\tan(c+dx))^4}{4d}$$

$\Big\downarrow$ 3959

$$(A-iB)\left(2a\left(2a\int(i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{B(a+ia\tan(c+dx))^4}{4d}$$

$\Big\downarrow$ 3042

$$(A-iB)\left(2a\left(2a\int(i\tan(c+dx)a+a)^2dx+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+dx))^3}{3d}\right)+\frac{B(a+ia\tan(c+dx))^4}{4d}$$

$\Big\downarrow$ 3958

$$(A-iB)\left(2a\left(2a\left(2ia^2\int\tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+}{3d}\right.\frac{B(a+ia\tan(c+dx))^4}{4d}$$

$\Big\downarrow$ 3042

$$(A-iB)\left(2a\left(2a\left(2ia^2\int\tan(c+dx)dx-\frac{a^2\tan(c+dx)}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c+}{3d}\right.\frac{B(a+ia\tan(c+dx))^4}{4d}$$

$\Big\downarrow$ 3956

$$(A-iB)\left(2a\left(2a\left(-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x\right)+\frac{ia(a+ia\tan(c+dx))^2}{2d}\right)+\frac{ia(a+ia\tan(c-}{3d}\right.\frac{B(a+ia\tan(c+dx))^4}{4d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(B*(a + I*a*Tan[c + d*x])^4)/(4*d) + (A - I*B)*(((I/3)*a*(a + I*a*Tan[c +
d*x])^3)/d + 2*a*(((I/2)*a*(a + I*a*Tan[c + d*x])^2)/d + 2*a*(2*a^2*x - ((
2*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{a^4\left(-\frac{4iB\tan(dx+c)^3}{3}+\frac{B\tan(dx+c)^4}{4}-2iA\tan(dx+c)^2+\frac{A\tan(dx+c)^3}{3}+8iB\tan(dx+c)-\frac{7B\tan(dx+c)^2}{2}-7A\tan(dx+c)\right.}{d}$
default	$\frac{a^4\left(-\frac{4iB\tan(dx+c)^3}{3}+\frac{B\tan(dx+c)^4}{4}-2iA\tan(dx+c)^2+\frac{A\tan(dx+c)^3}{3}+8iB\tan(dx+c)-\frac{7B\tan(dx+c)^2}{2}-7A\tan(dx+c)\right.}{d}$
norman	$(-8iB\,a^4+8A\,a^4)\,x-\frac{(4iA\,a^4+7B\,a^4)\tan(dx+c)^2}{2d}-\frac{(-8iB\,a^4+7A\,a^4)\tan(dx+c)}{d}+\frac{(-4iB\,a^4+A\,a^4)}{3d}$
parallelrisch	$\frac{-16iB\tan(dx+c)^3a^4+3B\tan(dx+c)^4a^4-24iA\tan(dx+c)^2a^4+4A\tan(dx+c)^3a^4-96iBx\,a^4d+48iA\ln\left(1+\tan(dx+c\right.}{12d}$
risch	$\frac{16ia^4Bc}{d}-\frac{16a^4Ac}{d}-\frac{4a^4\left(18iA\,e^{6i(dx+c)}+30B\,e^{6i(dx+c)}+45iA\,e^{4i(dx+c)}+63B\,e^{4i(dx+c)}+38iA\,e^{2i(dx+c)}+50B\,e^{2}\right.}{3d\left(e^{2i(dx+c)}+1\right)^4}$
parts	$A\,a^4x+\frac{(-4iA\,a^4-6B\,a^4)\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{(-4iB\,a^4+A\,a^4)\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\mathrm{arct}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*a^4*(-4/3*I*B*tan(d*x+c)^3+1/4*B*tan(d*x+c)^4-2*I*A*tan(d*x+c)^2+1/3*A
*tan(d*x+c)^3+8*I*B*tan(d*x+c)-7/2*B*tan(d*x+c)^2-7*A*tan(d*x+c)+1/2*(8*B+
8*I*A)*ln(1+tan(d*x+c)^2)+(8*A-8*I*B)*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.62

$$\int (a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$
$$-\frac{4\left(6\,(3i\,A+5\,B)a^4e^{(6i\,dx+6i\,c)}+9\,(5i\,A+7\,B)a^4e^{(4i\,dx+4i\,c)}+2\,(19i\,A+25\,B)a^4e^{(2i\,dx+2i\,c)}+(11i\,A+\right.}{3\,(de^{(8i\,d}}$$

input
```
integrate((a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-4/3*(6*(3*I*A + 5*B)*a^4*e^(6*I*d*x + 6*I*c) + 9*(5*I*A + 7*B)*a^4*e^(4*I
*d*x + 4*I*c) + 2*(19*I*A + 25*B)*a^4*e^(2*I*d*x + 2*I*c) + (11*I*A + 14*B
)*a^4 + 6*((I*A + B)*a^4*e^(8*I*d*x + 8*I*c) + 4*(I*A + B)*a^4*e^(6*I*d*x
+ 6*I*c) + 6*(I*A + B)*a^4*e^(4*I*d*x + 4*I*c) + 4*(I*A + B)*a^4*e^(2*I*d*
x + 2*I*c) + (I*A + B)*a^4)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x +
8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(116) = 232.

Time = 0.46 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.72

$$\int (a + ia\tan(c + dx))^4 (A + B\tan(c + dx))\,dx = -\frac{8ia^4(A - iB)\log\left(e^{2idx} + e^{-2ic}\right)}{d} + \frac{-44iAa^4 - 56Ba^4 + (-152iAa^4e^{2ic} - 200Ba^4e^{2ic})\,e^{2idx} + (-180iAa^4e^{4ic} - 252Ba^4e^{4ic})\,e^{4idx} + (-72i}{3de^{8ic}e^{8idx} + 12de^{6ic}e^{6idx} + 18de^{4ic}e^{4idx} + 12de^{2ic}e^{2idx} + 3d}$$

input
```
integrate((a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
-8*I*a**4*(A - I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-44*I*A*a**4 - 56
*B*a**4 + (-152*I*A*a**4*exp(2*I*c) - 200*B*a**4*exp(2*I*c))*exp(2*I*d*x)
+ (-180*I*A*a**4*exp(4*I*c) - 252*B*a**4*exp(4*I*c))*exp(4*I*d*x) + (-72*I
*A*a**4*exp(6*I*c) - 120*B*a**4*exp(6*I*c))*exp(6*I*d*x))/(3*d*exp(8*I*c)*
exp(8*I*d*x) + 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d*x)
 + 12*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81

$$\int (a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{3\,Ba^4\tan{(dx+c)}^4+4\,(A-4i\,B)a^4\tan{(dx+c)}^3-6\,(4i\,A+7\,B)a^4\tan{(dx+c)}^2+96\,(dx+c)(A-i}{12\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(3*B*a^4*tan(d*x + c)^4 + 4*(A - 4*I*B)*a^4*tan(d*x + c)^3 - 6*(4*I*A
 + 7*B)*a^4*tan(d*x + c)^2 + 96*(d*x + c)*(A - I*B)*a^4 - 48*(-I*A - B)*a^
4*log(tan(d*x + c)^2 + 1) - 12*(7*A - 8*I*B)*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06

$$\int (a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = -\frac{8\,(-i\,Aa^4-Ba^4)\log{(\tan{(dx+c)}+i)}}{d}$$
$$+\frac{3\,Ba^4d^3\tan{(dx+c)}^4+4\,Aa^4d^3\tan{(dx+c)}^3-16i\,Ba^4d^3\tan{(dx+c)}^3-24i\,Aa^4d^3\tan{(dx+c)}^2-}{12\,d^4}$$

input
```
integrate((a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
-8*(-I*A*a^4 - B*a^4)*log(tan(d*x + c) + I)/d + 1/12*(3*B*a^4*d^3*tan(d*x
+ c)^4 + 4*A*a^4*d^3*tan(d*x + c)^3 - 16*I*B*a^4*d^3*tan(d*x + c)^3 - 24*I
*A*a^4*d^3*tan(d*x + c)^2 - 42*B*a^4*d^3*tan(d*x + c)^2 - 84*A*a^4*d^3*tan
(d*x + c) + 96*I*B*a^4*d^3*tan(d*x + c))/d^4
```

Mupad [B] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.29

$$\int (a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{B\,a^4\tan(c+d\,x)^4}{4\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(8\,B\,a^4+A\,a^4\,8\mathrm{i}\right)}{d}$$
$$-\frac{\tan(c+d\,x)^2\,\left(\frac{a^4\,(A-B\,1\mathrm{i})\,3\mathrm{i}}{2}+\frac{B\,a^4}{2}+\frac{a^4\,(3\,B+A\,1\mathrm{i})}{2}\right)}{d}$$
$$+\frac{\tan(c+d\,x)\,\left(-3\,a^4\,(A-B\,1\mathrm{i})+a^4\,(B+A\,3\mathrm{i})\,1\mathrm{i}+B\,a^4\,1\mathrm{i}+a^4\,(3\,B+A\,1\mathrm{i})\,1\mathrm{i}\right)}{d}$$
$$-\frac{\tan(c+d\,x)^3\,\left(\frac{B\,a^4\,1\mathrm{i}}{3}+\frac{a^4\,(3\,B+A\,1\mathrm{i})\,1\mathrm{i}}{3}\right)}{d}$$

input
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(log(tan(c + d*x) + 1i)*(A*a^4*8i + 8*B*a^4))/d - (tan(c + d*x)^3*((B*a^4*
1i)/3 + (a^4*(A*1i + 3*B)*1i)/3))/d - (tan(c + d*x)^2*((a^4*(A - B*1i)*3i)
/2 + (B*a^4)/2 + (a^4*(A*1i + 3*B))/2))/d + (tan(c + d*x)*(a^4*(A*3i + B)*
1i - 3*a^4*(A - B*1i) + B*a^4*1i + a^4*(A*1i + 3*B)*1i))/d + (B*a^4*tan(c
+ d*x)^4)/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int (a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{a^4\left(48\log\left(\tan\left(dx+c\right)^2+1\right)ai+48\log\left(\tan\left(dx+c\right)^2+1\right)b+3\tan\left(dx+c\right)^4b+4\tan\left(dx+c\right)^3a-16\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output

```
(a**4*(48*log(tan(c + d*x)**2 + 1)*a*i + 48*log(tan(c + d*x)**2 + 1)*b + 3
*tan(c + d*x)**4*b + 4*tan(c + d*x)**3*a - 16*tan(c + d*x)**3*b*i - 24*tan
(c + d*x)**2*a*i - 42*tan(c + d*x)**2*b - 84*tan(c + d*x)*a + 96*tan(c + d
*x)*b*i + 96*a*d*x - 96*b*d*i*x))/(12*d)
```

3.29 $\int \cot(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 142

$$\begin{aligned}\int \cot(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx &= 8a^4(iA+B)x+\frac{a^4(7A-8iB)\log(\cos(c+dx))}{d}\\&\quad+\frac{a^4A\log(\sin(c+dx))}{d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}\\&\quad-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{(3A-4iB)\left(a^4+ia^4\tan(c+dx)\right)}{d}\end{aligned}$$

output

```
8*a^4*(I*A+B)*x+a^4*(7*A-8*I*B)*ln(cos(d*x+c))/d+a^4*A*ln(sin(d*x+c))/d+1/
3*I*a*B*(a+I*a*tan(d*x+c))^3/d-1/2*(A-2*I*B)*(a^2+I*a^2*tan(d*x+c))^2/d-(3
*A-4*I*B)*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int \cot(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{a^4(6A\log(\tan(c+dx))-8(A-iB)(1+6\log(i+\tan(c+dx)))+(-24iA-42B)\tan(c+dx)+3(A-}{6d}$$

input
```
Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(a^4*(6*A*Log[Tan[c + d*x]] - 8*(A - I*B)*(1 + 6*Log[I + Tan[c + d*x]]) +
((-24*I)*A - 42*B)*Tan[c + d*x] + 3*(A - (4*I)*B)*Tan[c + d*x]^2 + 2*B*Tan
[c + d*x]^3))/(6*d)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4077, 3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{3}\int 3\cot(c+dx)(i\tan(c+dx)a+a)^3(aA+a(iA+2B)\tan(c+dx))dx+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)^3(aA+a(iA+2B)\tan(c+dx))dx+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^3(aA+a(iA+2B)\tan(c+dx))}{\tan(c+dx)}dx+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{2}\int 2\cot(c+dx)(i\tan(c+dx)a+a)^2\left(Aa^2+(3iA+4B)\tan(c+dx)a^2\right)dx-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)^2\left(Aa^2+(3iA+4B)\tan(c+dx)a^2\right)dx-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^2\left(Aa^2+(3iA+4B)\tan(c+dx)a^2\right)}{\tan(c+dx)}dx-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4077}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)\left(Aa^3+(7iA+8B)\tan(c+dx)a^3\right)dx-\frac{(3A-4iB)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)\left(Aa^3+(7iA+8B)\tan(c+dx)a^3\right)}{\tan(c+dx)}dx-\frac{(3A-4iB)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4072}$$

$$-\left(a^4(7A-8iB)\int \tan(c+dx)dx\right)+\int \cot(c+dx)\left(Aa^4+8(iA+B)\tan(c+dx)a^4\right)dx-\frac{(3A-4iB)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$\downarrow$ 3042

$$-\left(a^4(7A-8iB)\int \tan(c+dx)dx\right)+\int \frac{Aa^4+8(iA+B)\tan(c+dx)a^4}{\tan(c+dx)}dx-\frac{(3A-4iB)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$\downarrow$ 3956

$$\int \frac{Aa^4+8(iA+B)\tan(c+dx)a^4}{\tan(c+dx)}dx-\frac{(3A-4iB)\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{a^4(7A-8iB)\log(\cos(c+dx))}{d}-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$\downarrow$ 4014

$$a^4A\int \cot(c+dx)dx-\frac{(3A-4iB)\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{a^4(7A-8iB)\log(\cos(c+dx))}{d}+8a^4x(B+iA)-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$\downarrow$ 3042

$$a^4A\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{(3A-4iB)\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{a^4(7A-8iB)\log(\cos(c+dx))}{d}+8a^4x(B+iA)-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$\downarrow$ 25

$$-a^4A\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{(3A-4iB)\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{a^4(7A-8iB)\log(\cos(c+dx))}{d}+8a^4x(B+iA)-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(3A-4iB)\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{a^4(7A-8iB)\log(\cos(c+dx))}{d}+8a^4x(B+iA)+\frac{a^4A\log(-\sin(c+dx))}{d}-\frac{(A-2iB)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}+\frac{iaB(a+ia\tan(c+dx))^3}{3d}$$

input
```
Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
8*a^4*(I*A + B)*x + (a^4*(7*A - (8*I)*B)*Log[Cos[c + d*x]])/d + (a^4*A*Log
[-Sin[c + d*x]])/d + ((I/3)*a*B*(a + I*a*Tan[c + d*x])^3)/d - ((A - (2*I)*
B)*(a^2 + I*a^2*Tan[c + d*x])^2)/(2*d) - ((3*A - (4*I)*B)*(a^4 + I*a^4*Tan
[c + d*x]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a^4\left(-2iB\tan(dx+c)^2+\frac{B\tan(dx+c)^3}{3}-4iA\tan(dx+c)+\frac{A\tan(dx+c)^2}{2}-7B\tan(dx+c)+\frac{(8iB-8A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(8\right.}{d}$
default	$\frac{a^4\left(-2iB\tan(dx+c)^2+\frac{B\tan(dx+c)^3}{3}-4iA\tan(dx+c)+\frac{A\tan(dx+c)^2}{2}-7B\tan(dx+c)+\frac{(8iB-8A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(8\right.}{d}$
parallelrisch	$\frac{a^4\left(24iB\ln\left(\sec(dx+c)^2\right)+6A\left(\ln(\tan(dx+c))-4\ln\left(\sec(dx+c)^2\right)\right)+48iAxd-12iB\tan(dx+c)^2+2B\tan(dx+c)^3-24\right.}{6d}$
norman	$(8iA\,a^4+8B\,a^4)\,x-\frac{(4iA\,a^4+7B\,a^4)\tan(dx+c)}{d}+\frac{(-4iB\,a^4+A\,a^4)\tan(dx+c)^2}{2d}+\frac{B\,a^4\tan(dx+c)^3}{3d}+$
risch	$-\frac{16a^4Bc}{d}-\frac{16ia^4Ac}{d}-\frac{2ia^4\left(15iA\,e^{4i(dx+c)}+36B\,e^{4i(dx+c)}+27iA\,e^{2i(dx+c)}+54B\,e^{2i(dx+c)}+12iA+22B\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{8i}{}$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
a^4/d*(-2*I*B*tan(d*x+c)^2+1/3*B*tan(d*x+c)^3-4*I*A*tan(d*x+c)+1/2*A*tan(d
*x+c)^2-7*B*tan(d*x+c)+1/2*(-8*A+8*I*B)*ln(1+tan(d*x+c)^2)+(8*B+8*I*A)*arc
tan(tan(d*x+c))+A*ln(tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(122) = 244.

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.73

$$\int \cot(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{6\,(5\,A-12i\,B)a^4e^{(4i\,dx+4i\,c)}+54\,(A-2i\,B)a^4e^{(2i\,dx+2i\,c)}+4\,(6\,A-11i\,B)a^4+3\,((7\,A-8i\,B)a^4e^{(6i\,dx+6}}{}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/3*(6*(5*A - 12*I*B)*a^4*e^(4*I*d*x + 4*I*c) + 54*(A - 2*I*B)*a^4*e^(2*I*
d*x + 2*I*c) + 4*(6*A - 11*I*B)*a^4 + 3*((7*A - 8*I*B)*a^4*e^(6*I*d*x + 6*
I*c) + 3*(7*A - 8*I*B)*a^4*e^(4*I*d*x + 4*I*c) + 3*(7*A - 8*I*B)*a^4*e^(2*
I*d*x + 2*I*c) + (7*A - 8*I*B)*a^4)*log(e^(2*I*d*x + 2*I*c) + 1) + 3*(A*a^
4*e^(6*I*d*x + 6*I*c) + 3*A*a^4*e^(4*I*d*x + 4*I*c) + 3*A*a^4*e^(2*I*d*x +
 2*I*c) + A*a^4)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(6*I*d*x + 6*I*c) + 3*
d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(121) = 242.

Time = 2.01 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.04

$$\int \cot(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{Aa^4\log\left(\frac{-3Aa^4+4iBa^4}{3Aa^4e^{2ic}-4iBa^4e^{2ic}}+e^{2idx}\right)}{d} + \frac{a^4\cdot(7A-8iB)\log\left(e^{2idx}+\frac{-4Aa^4+4iBa^4+a^4\cdot(7A-8iB)}{3Aa^4e^{2ic}-4iBa^4e^{2ic}}\right)}{d} + \frac{24Aa^4-44iBa^4+(54Aa^4e^{2ic}-108iBa^4e^{2ic})e^{2idx}+(30Aa^4e^{4ic}-72iBa^4e^{4ic})e^{4idx}}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
A*a**4*log((-3*A*a**4 + 4*I*B*a**4)/(3*A*a**4*exp(2*I*c) - 4*I*B*a**4*exp(
2*I*c)) + exp(2*I*d*x))/d + a**4*(7*A - 8*I*B)*log(exp(2*I*d*x) + (-4*A*a*
*4 + 4*I*B*a**4 + a**4*(7*A - 8*I*B))/(3*A*a**4*exp(2*I*c) - 4*I*B*a**4*ex
p(2*I*c)))/d + (24*A*a**4 - 44*I*B*a**4 + (54*A*a**4*exp(2*I*c) - 108*I*B*
a**4*exp(2*I*c))*exp(2*I*d*x) + (30*A*a**4*exp(4*I*c) - 72*I*B*a**4*exp(4*
I*c))*exp(4*I*d*x))/(3*d*exp(6*I*c)*exp(6*I*d*x) + 9*d*exp(4*I*c)*exp(4*I*
d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \cot(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,Ba^4\tan(dx+c)^3+3\,(A-4i\,B)a^4\tan(dx+c)^2-48\,(dx+c)(-i\,A-B)a^4-24\,(A-i\,B)a^4\log(\tan}{6\,d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/6*(2*B*a^4*tan(d*x + c)^3 + 3*(A - 4*I*B)*a^4*tan(d*x + c)^2 - 48*(d*x +
 c)*(-I*A - B)*a^4 - 24*(A - I*B)*a^4*log(tan(d*x + c)^2 + 1) + 6*A*a^4*lo
g(tan(d*x + c)) - 6*(4*I*A + 7*B)*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

$$\int \cot(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{Aa^4\log(|\tan(dx+c)|)}{d} - \frac{8(Aa^4 - iBa^4)\log(\tan(dx+c)+i)}{d} + \frac{2Ba^4d^2\tan(dx+c)^3 + 3Aa^4d^2\tan(dx+c)^2 - 12iBa^4d^2\tan(dx+c)^2 - 24iAa^4d^2\tan(dx+c) - 4}{6d^3}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
A*a^4*log(abs(tan(d*x + c)))/d - 8*(A*a^4 - I*B*a^4)*log(tan(d*x + c) + I)
/d + 1/6*(2*B*a^4*d^2*tan(d*x + c)^3 + 3*A*a^4*d^2*tan(d*x + c)^2 - 12*I*B
*a^4*d^2*tan(d*x + c)^2 - 24*I*A*a^4*d^2*tan(d*x + c) - 42*B*a^4*d^2*tan(d
*x + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int \cot(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{A\,a^4\,\ln(\tan(c+d\,x))}{d} - \frac{\tan(c+d\,x)\,\left(a^4\,(A-B\,1\mathrm{i})\,3\mathrm{i} + B\,a^4 + a^4\,(3\,B + A\,1\mathrm{i})\right)}{d} - \frac{\tan(c+d\,x)^2\,\left(\frac{B\,a^4\,1\mathrm{i}}{2} + \frac{a^4\,(3\,B+A\,1\mathrm{i})\,1\mathrm{i}}{2}\right)}{d} - \frac{8\,a^4\,\ln(\tan(c+d\,x)+1\mathrm{i})\,(A-B\,1\mathrm{i})}{d} + \frac{B\,a^4\,\tan(c+d\,x)^3}{3\,d}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(A*a^4*log(tan(c + d*x)))/d - (tan(c + d*x)*(a^4*(A - B*1i)*3i + B*a^4 + a
^4*(A*1i + 3*B)))/d - (tan(c + d*x)^2*((B*a^4*1i)/2 + (a^4*(A*1i + 3*B)*1i
)/2))/d - (8*a^4*log(tan(c + d*x) + 1i)*(A - B*1i))/d + (B*a^4*tan(c + d*x
)^3)/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 528, normalized size of antiderivative = 3.72

$$\int \cot(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(a**4*( - 48*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a +
 48*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*b*i + 48*cos
(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*a - 48*cos(c + d*x)*log(tan((c + d*
x)/2)**2 + 1)*b*i + 42*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)
**2*a - 48*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b*i - 42
*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a + 48*cos(c + d*x)*log(tan((c + d
*x)/2) - 1)*b*i + 42*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**
2*a - 48*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*b*i - 42*c
os(c + d*x)*log(tan((c + d*x)/2) + 1)*a + 48*cos(c + d*x)*log(tan((c + d*x
)/2) + 1)*b*i + 6*cos(c + d*x)*log(tan((c + d*x)/2))*sin(c + d*x)**2*a - 6
*cos(c + d*x)*log(tan((c + d*x)/2))*a + 48*cos(c + d*x)*sin(c + d*x)**2*a*
d*i*x - 3*cos(c + d*x)*sin(c + d*x)**2*a + 48*cos(c + d*x)*sin(c + d*x)**2
*b*d*x + 12*cos(c + d*x)*sin(c + d*x)**2*b*i - 48*cos(c + d*x)*a*d*i*x - 4
8*cos(c + d*x)*b*d*x - 24*sin(c + d*x)**3*a*i - 44*sin(c + d*x)**3*b + 24*
sin(c + d*x)*a*i + 42*sin(c + d*x)*b))/(6*cos(c + d*x)*d*(sin(c + d*x)**2
 - 1))
```

3.30 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 144

$$\begin{aligned}
&\int \cot^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx \\
&= -8a^4(A-iB)x+\frac{a^4(4iA+7B)\log(\cos(c+dx))}{d} \\
&\quad+\frac{a^4(4iA+B)\log(\sin(c+dx))}{d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d} \\
&\quad+\frac{(2iA-B)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{3B(a^4+ia^4\tan(c+dx))}{d}
\end{aligned}$$

output
```
-8*a^4*(A-I*B)*x+a^4*(4*I*A+7*B)*ln(cos(d*x+c))/d+a^4*(4*I*A+B)*ln(sin(d*x
+c))/d-a*A*cot(d*x+c)*(a+I*a*tan(d*x+c))^3/d+1/2*(2*I*A-B)*(a^2+I*a^2*tan(
d*x+c))^2/d-3*B*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= a^4\left(-\frac{A\cot(c+dx)}{d}+\frac{4iA\log(\tan(c+dx))}{d}+\frac{B\log(\tan(c+dx))}{d}-\frac{8iA\log(i+\tan(c+dx))}{d}-\frac{8B\log(i+\tan(c+dx))}{d}+\frac{A\tan(c+dx)}{d}-\frac{4iB\tan(c+dx)}{d}+\frac{B\tan^2(c+dx)}{2d}\right)$$

input
```
Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
a^4*(-((A*Cot[c + d*x])/d) + ((4*I)*A*Log[Tan[c + d*x]])/d + (B*Log[Tan[c
+ d*x]])/d - ((8*I)*A*Log[I + Tan[c + d*x]])/d - (8*B*Log[I + Tan[c + d*x]
])/d + (A*Tan[c + d*x])/d - ((4*I)*B*Tan[c + d*x])/d + (B*Tan[c + d*x]^2)/
(2*d))
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.441, Rules used = {3042, 4076, 3042, 4077, 27, 3042, 4077, 3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4076}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)^3(a(4iA+B)+a(2A+iB)\tan(c+dx))dx-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^3(a(4iA+B)+a(2A+iB)\tan(c+dx))}{\tan(c+dx)}dx-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{2}\int 2\cot(c+dx)(i\tan(c+dx)a+a)^2\left((4iA+B)a^2+3iB\tan(c+dx)a^2\right)dx+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)^2\left((4iA+B)a^2+3iB\tan(c+dx)a^2\right)dx+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^2\left((4iA+B)a^2+3iB\tan(c+dx)a^2\right)}{\tan(c+dx)}dx+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{4077}$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)\left(a^3(4iA+B)-a^3(4A-7iB)\tan(c+dx)\right)dx-\frac{3B\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)\left(a^3(4iA+B)-a^3(4A-7iB)\tan(c+dx)\right)}{\tan(c+dx)}dx-\frac{3B\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{4072}$$

$$-\left(a^4(7B+4iA)\int \tan(c+dx)dx\right)+\int \cot(c+dx)\left(a^4(4iA+B)-8a^4(A-iB)\tan(c+dx)\right)dx-\frac{3B\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\left(a^4(7B+4iA)\int \tan(c+dx)dx\right)+\int \frac{a^4(4iA+B)-8a^4(A-iB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{3B\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3956}$$

$$\int \frac{a^4(4iA+B)-8a^4(A-iB)\tan(c+dx)}{\tan(c+dx)}dx+\frac{a^4(7B+4iA)\log(\cos(c+dx))}{d}-\frac{3B\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{4014}$$

$$a^4(B+4iA)\int \cot(c+dx)dx+\frac{a^4(7B+4iA)\log(\cos(c+dx))}{d}-8a^4x(A-iB)-\frac{3B\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$a^4(B+4iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^4(7B+4iA)\log(\cos(c+dx))}{d}-8a^4x(A-iB)-\frac{3B\left(a^4+ia^4\tan(c+dx)\right)}{d}+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{25}$$

$$-a^4(B+4iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{a^4(7B+4iA)\log(\cos(c+dx))}{d}-8a^4x(A-iB)-\frac{3B(a^4+ia^4\tan(c+dx))}{d}+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

$\Big\downarrow$ 3956

$$\frac{a^4(B+4iA)\log(-\sin(c+dx))}{d}+\frac{a^4(7B+4iA)\log(\cos(c+dx))}{d}-8a^4x(A-iB)-\frac{3B(a^4+ia^4\tan(c+dx))}{d}+\frac{(-B+2iA)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^3}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-8*a^4*(A - I*B)*x + (a^4*((4*I)*A + 7*B)*Log[Cos[c + d*x]])/d + (a^4*((4*
I)*A + B)*Log[-Sin[c + d*x]])/d - (a*A*Cot[c + d*x]*(a + I*a*Tan[c + d*x])
^3)/d + (((2*I)*A - B)*(a^2 + I*a^2*Tan[c + d*x])^2)/(2*d) - (3*B*(a^4 + I
*a^4*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{4a^4\left(\left(-iA-B\right)\ln\left(\sec(dx+c)^2\right)+\left(iA+\frac{B}{4}\right)\ln(\tan(dx+c))+\frac{B\tan(dx+c)^2}{8}+\left(-iB+\frac{A}{4}\right)\tan(dx+c)-\frac{A\cot(dx+c)}{4}+2(iB\right.}{d}$
derivativedivides	$\frac{a^4\left(\frac{B\tan(dx+c)^2}{2}+A\tan(dx+c)-4iB\tan(dx+c)+\frac{(-8iA-8B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(8iB-8A)\arctan(\tan(dx+c))-\frac{}{\tan(}\right.}{d}$
default	$\frac{a^4\left(\frac{B\tan(dx+c)^2}{2}+A\tan(dx+c)-4iB\tan(dx+c)+\frac{(-8iA-8B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(8iB-8A)\arctan(\tan(dx+c))-\frac{}{\tan(}\right.}{d}$
norman	$\frac{(8iB\,a^4-8A\,a^4)x\tan(dx+c)+\frac{\left(-4iB\,a^4+A\,a^4\right)\tan(dx+c)^2}{d}-\frac{A\,a^4}{d}+\frac{B\,a^4\tan(dx+c)^3}{2d}}{\tan(dx+c)}+\frac{(4iA\,a^4+B\,a^4)\ln(\tan(dx+c))}{d}$
risch	$-\frac{16ia^4Bc}{d}+\frac{16a^4Ac}{d}+\frac{2a^4\left(5B\,e^{4i(dx+c)}-2iA\,e^{2i(dx+c)}-B\,e^{2i(dx+c)}-2iA-4B\right)}{d\left(e^{2i(dx+c)}+1\right)^2\left(e^{2i(dx+c)}-1\right)}+\frac{7a^4\ln\left(e^{2i(dx+c)}+1\right)B}{d}+$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
4*a^4*((-I*A-B)*ln(sec(d*x+c)^2)+(I*A+1/4*B)*ln(tan(d*x+c))+1/8*B*tan(d*x+
c)^2+(-I*B+1/4*A)*tan(d*x+c)-1/4*A*cot(d*x+c)+2*(-A+I*B)*x*d)/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(126) = 252.

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.76

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{10\,Ba^4e^{(4i\,dx+4i\,c)}-2\,(2i\,A+B)a^4e^{(2i\,dx+2i\,c)}-4\,(i\,A+2\,B)a^4+\left((4i\,A+7\,B)a^4e^{(6i\,dx+6i\,c)}+(4i\,A+7\right.}{}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
(10*B*a^4*e^(4*I*d*x + 4*I*c) - 2*(2*I*A + B)*a^4*e^(2*I*d*x + 2*I*c) - 4*
(I*A + 2*B)*a^4 + ((4*I*A + 7*B)*a^4*e^(6*I*d*x + 6*I*c) + (4*I*A + 7*B)*a
^4*e^(4*I*d*x + 4*I*c) + (-4*I*A - 7*B)*a^4*e^(2*I*d*x + 2*I*c) + (-4*I*A
- 7*B)*a^4)*log(e^(2*I*d*x + 2*I*c) + 1) + ((4*I*A + B)*a^4*e^(6*I*d*x + 6
*I*c) + (4*I*A + B)*a^4*e^(4*I*d*x + 4*I*c) + (-4*I*A - B)*a^4*e^(2*I*d*x
+ 2*I*c) + (-4*I*A - B)*a^4)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(6*I*d*x +
 6*I*c) + d*e^(4*I*d*x + 4*I*c) - d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(124) = 248.

Time = 1.40 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.81

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{ia^4\cdot(4A-7iB)\log\left(e^{2idx}+\frac{(-4iAa^4-4Ba^4+ia^4\cdot(4A-7iB))e^{-2ic}}{3Ba^4}\right)}{d} + \frac{ia^4\cdot(4A-iB)\log\left(e^{2idx}+\frac{(-4iAa^4-4Ba^4+ia^4\cdot(4A-iB))e^{-2ic}}{3Ba^4}\right)}{d} + \frac{-4iAa^4+10Ba^4e^{4ic}e^{4idx}-8Ba^4+(-4iAa^4e^{2ic}-2Ba^4e^{2ic})\,e^{2idx}}{de^{6ic}e^{6idx}+de^{4ic}e^{4idx}-de^{2ic}e^{2idx}-d}$$

input
```
integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
I*a**4*(4*A - 7*I*B)*log(exp(2*I*d*x) + (-4*I*A*a**4 - 4*B*a**4 + I*a**4*(
4*A - 7*I*B))*exp(-2*I*c)/(3*B*a**4))/d + I*a**4*(4*A - I*B)*log(exp(2*I*d
*x) + (-4*I*A*a**4 - 4*B*a**4 + I*a**4*(4*A - I*B))*exp(-2*I*c)/(3*B*a**4)
)/d + (-4*I*A*a**4 + 10*B*a**4*exp(4*I*c)*exp(4*I*d*x) - 8*B*a**4 + (-4*I*
A*a**4*exp(2*I*c) - 2*B*a**4*exp(2*I*c))*exp(2*I*d*x))/(d*exp(6*I*c)*exp(6
*I*d*x) + d*exp(4*I*c)*exp(4*I*d*x) - d*exp(2*I*c)*exp(2*I*d*x) - d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{Ba^4\tan(dx+c)^2 - 16\,(dx+c)(A-i\,B)a^4 - 8\,(i\,A+B)a^4\log\left(\tan(dx+c)^2+1\right) + 2\,(4i\,A+B)a^4\log}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/2*(B*a^4*tan(d*x + c)^2 - 16*(d*x + c)*(A - I*B)*a^4 - 8*(I*A + B)*a^4*l
og(tan(d*x + c)^2 + 1) + 2*(4*I*A + B)*a^4*log(tan(d*x + c)) + 2*(A - 4*I*
B)*a^4*tan(d*x + c) - 2*A*a^4/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{Aa^4}{d\tan(dx+c)} + \frac{8\left(-i\,Aa^4 - Ba^4\right)\log\left(\tan(dx+c)+i\right)}{d} - \frac{\left(-4i\,Aa^4 - Ba^4\right)\log\left(|\tan(dx+c)|\right)}{d} + \frac{Ba^4d\tan(dx+c)^2 + 2\,Aa^4d\tan(dx+c) - 8i\,Ba^4d\tan(dx+c)}{2\,d^2}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-A*a^4/(d*tan(d*x + c)) + 8*(-I*A*a^4 - B*a^4)*log(tan(d*x + c) + I)/d - (
-4*I*A*a^4 - B*a^4)*log(abs(tan(d*x + c)))/d + 1/2*(B*a^4*d*tan(d*x + c)^2
 + 2*A*a^4*d*tan(d*x + c) - 8*I*B*a^4*d*tan(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \frac{B\,a^4\tan(c+d\,x)^2}{2\,d}+\frac{a^4\,\ln\left(\tan(c+d\,x)\right)\,\left(B+A\,4\mathrm{i}\right)}{d} - \frac{8\,a^4\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{d}-\frac{A\,a^4\cot(c+d\,x)}{d} - \frac{\tan(c+d\,x)\,\left(B\,a^4\,1\mathrm{i}+a^4\,\left(3\,B+A\,1\mathrm{i}\right)\,1\mathrm{i}\right)}{d}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(a^4*log(tan(c + d*x))*(A*4i + B))/d - (tan(c + d*x)*(B*a^4*1i + a^4*(A*1i
 + 3*B)*1i))/d - (8*a^4*log(tan(c + d*x) + 1i)*(A*1i + B))/d - (A*a^4*cot(
c + d*x))/d + (B*a^4*tan(c + d*x)^2)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 494, normalized size of antiderivative = 3.43

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(a**4*( - 4*cos(c + d*x)*sin(c + d*x)**2*a + 8*cos(c + d*x)*sin(c + d*x)**
2*b*i + 2*cos(c + d*x)*a - 16*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3
*a*i - 16*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*b + 16*log(tan((c +
 d*x)/2)**2 + 1)*sin(c + d*x)*a*i + 16*log(tan((c + d*x)/2)**2 + 1)*sin(c
+ d*x)*b + 8*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*a*i + 14*log(tan((c
 + d*x)/2) - 1)*sin(c + d*x)**3*b - 8*log(tan((c + d*x)/2) - 1)*sin(c + d*
x)*a*i - 14*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*b + 8*log(tan((c + d*x)
/2) + 1)*sin(c + d*x)**3*a*i + 14*log(tan((c + d*x)/2) + 1)*sin(c + d*x)**
3*b - 8*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*a*i - 14*log(tan((c + d*x)/
2) + 1)*sin(c + d*x)*b + 8*log(tan((c + d*x)/2))*sin(c + d*x)**3*a*i + 2*l
og(tan((c + d*x)/2))*sin(c + d*x)**3*b - 8*log(tan((c + d*x)/2))*sin(c + d
*x)*a*i - 2*log(tan((c + d*x)/2))*sin(c + d*x)*b - 16*sin(c + d*x)**3*a*d*
x + 16*sin(c + d*x)**3*b*d*i*x - sin(c + d*x)**3*b + 16*sin(c + d*x)*a*d*x
 - 16*sin(c + d*x)*b*d*i*x))/(2*sin(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.31 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 156

$$\begin{aligned}&\int \cot^3(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx\\&=-8a^4(iA+B)x-\frac{a^4(A-4iB)\log(\cos(c+dx))}{d}\\&\quad-\frac{a^4(7A-4iB)\log(\sin(c+dx))}{d}-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}\\&\quad-\frac{(5iA+2B)\cot(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{3A(a^4+ia^4\tan(c+dx))}{d}\end{aligned}$$

output
```
-8*a^4*(I*A+B)*x-a^4*(A-4*I*B)*ln(cos(d*x+c))/d-a^4*(7*A-4*I*B)*ln(sin(d*x
+c))/d-1/2*a*A*cot(d*x+c)^2*(a+I*a*tan(d*x+c))^3/d-1/2*(5*I*A+2*B)*cot(d*x
+c)*(a^2+I*a^2*tan(d*x+c))^2/d-3*A*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = a^4\left(-\frac{4iA\cot(c+dx)}{d}-\frac{B\cot(c+dx)}{d}-\frac{A\cot^2(c+dx)}{2d}-\frac{7A\log(\tan(c+dx))}{d}+\frac{4iB\log(\tan(c+dx))}{d}+\frac{8A\log(i+\tan(c+dx))}{d}-\frac{8iB\log(i+\tan(c+dx))}{d}+\frac{B\tan(c+dx)}{d}\right)$$

input
```
Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
a^4*(((-4*I)*A*Cot[c + d*x])/d - (B*Cot[c + d*x])/d - (A*Cot[c + d*x]^2)/(
2*d) - (7*A*Log[Tan[c + d*x]])/d + ((4*I)*B*Log[Tan[c + d*x]])/d + (8*A*Lo
g[I + Tan[c + d*x]])/d - ((8*I)*B*Log[I + Tan[c + d*x]])/d + (B*Tan[c + d*
x])/d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {3042, 4076, 3042, 4076, 27, 3042, 4077, 3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{2}\int \cot^2(c+dx)(i\tan(c+dx)a+a)^3(a(5iA+2B)+a(A+2iB)\tan(c+dx))dx-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)^3(a(5iA+2B)+a(A+2iB)\tan(c+dx))}{\tan(c+dx)^2}dx-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 4076

$$\frac{1}{2}\left(\int -2\cot(c+dx)(i\tan(c+dx)a+a)^2\left(a^2(7A-4iB)-3ia^2A\tan(c+dx)\right)dx-\frac{(2B+5iA)\cot(c+dx)\left(a^{2}\right.}{d}\right.$$
$$-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(-2\int \cot(c+dx)(i\tan(c+dx)a+a)^2\left(a^2(7A-4iB)-3ia^2A\tan(c+dx)\right)dx-\frac{(2B+5iA)\cot(c+dx)\left(a^{2}\right.}{d}\right.$$
$$-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-2\int \frac{(i\tan(c+dx)a+a)^2\left(a^2(7A-4iB)-3ia^2A\tan(c+dx)\right)}{\tan(c+dx)}dx-\frac{(2B+5iA)\cot(c+dx)\left(a^2+ia^2\tan(\right.}{d}\right.$$
$$-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 4077

$$\frac{1}{2}\left(-2\left(\int \cot(c+dx)(i\tan(c+dx)a+a)\left((7A-4iB)a^3+(iA+4B)\tan(c+dx)a^3\right)dx+\frac{3A(a^4+ia^4\tan(c-}{d}\right.\right.$$
$$-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-2\left(\int \frac{(i\tan(c+dx)a+a)\left((7A-4iB)a^3+(iA+4B)\tan(c+dx)a^3\right)}{\tan(c+dx)}dx+\frac{3A\left(a^4+ia^4\tan(c+dx)\right)}{d}\right)-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}\right.$$

$\downarrow$ 4072

$$\frac{1}{2}\left(-2\left(-\left(a^4(A-4iB)\int \tan(c+dx)dx\right)+\int \cot(c+dx)\left((7A-4iB)a^4+8(iA+B)\tan(c+dx)a^4\right)dx+\frac{3}{}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-2\left(-\left(a^4(A-4iB)\int \tan(c+dx)dx\right)+\int \frac{(7A-4iB)a^4+8(iA+B)\tan(c+dx)a^4}{\tan(c+dx)}dx+\frac{3A\left(a^4+ia^4\tan\right.}{d}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 3956

$$\frac{1}{2}\left(-2\left(\int \frac{(7A-4iB)a^4+8(iA+B)\tan(c+dx)a^4}{\tan(c+dx)}dx+\frac{a^4(A-4iB)\log(\cos(c+dx))}{d}+\frac{3A\left(a^4+ia^4\tan(c+\right.}{d}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 4014

$$\frac{1}{2}\left(-2\left(a^4(7A-4iB)\int \cot(c+dx)dx+\frac{a^4(A-4iB)\log(\cos(c+dx))}{d}+8a^4x(B+iA)+\frac{3A\left(a^4+ia^4\tan(c+\right.}{d}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-2\left(a^4(7A-4iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{a^4(A-4iB)\log(\cos(c+dx))}{d}+8a^4x(B+iA)+\frac{3A\left(a^4+ia\right.}{}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\downarrow$ 25

$$\frac{1}{2}\left(-2\left(-a^4(7A-4iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{a^4(A-4iB)\log(\cos(c+dx))}{d}+8a^4x(B+iA)+\frac{3A(a^4\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

$\Big\downarrow$ 3956

$$\frac{1}{2}\left(-2\left(\frac{a^4(7A-4iB)\log(-\sin(c+dx))}{d}+\frac{a^4(A-4iB)\log(\cos(c+dx))}{d}+8a^4x(B+iA)+\frac{3A(a^4+ia^4\tan(c}{d}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^3}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(a*A*Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^3)/d + (-((((5*I)*A + 2*B)
*Cot[c + d*x]*(a^2 + I*a^2*Tan[c + d*x])^2)/d) - 2*(8*a^4*(I*A + B)*x + (a
^4*(A - (4*I)*B)*Log[Cos[c + d*x]])/d + (a^4*(7*A - (4*I)*B)*Log[-Sin[c +
d*x]])/d + (3*A*(a^4 + I*a^4*Tan[c + d*x]))/d))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{a^4\left(B\tan(dx+c)+\frac{(-8iB+8A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-8iA-8B)\arctan(\tan(dx+c))-\frac{A}{2\tan(dx+c)^2}-\frac{4iA+B}{\tan(dx+c)}+(4iB-7\right.}{d}$
default	$\frac{a^4\left(B\tan(dx+c)+\frac{(-8iB+8A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-8iA-8B)\arctan(\tan(dx+c))-\frac{A}{2\tan(dx+c)^2}-\frac{4iA+B}{\tan(dx+c)}+(4iB-7\right.}{d}$
parallelrisch	$-\frac{a^4\left(16iAxd+16Bdx-8iB\left(\ln(\tan(dx+c))-\ln\left(\sec(dx+c)^2\right)\right)+2A\left(-4\ln\left(\sec(dx+c)^2\right)+7\ln(\tan(dx+c))\right)+A\cot(\right.}{2d}$
norman	$\frac{(-8iA\,a^4-8B\,a^4)x\tan(dx+c)^2+\frac{B\,a^4\tan(dx+c)^3}{d}-\frac{A\,a^4}{2d}-\frac{\left(4iA\,a^4+B\,a^4\right)\tan(dx+c)}{d}}{\tan(dx+c)^2}-\frac{(-4iB\,a^4+7A\,a^4)\ln(\tan(dx+}{d}$
risch	$\frac{16a^4Bc}{d}+\frac{16ia^4Ac}{d}-\frac{2ia^4\left(5iA\,e^{4i(dx+c)}+iA\,e^{2i(dx+c)}+2B\,e^{2i(dx+c)}-4iA-2B\right)}{d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)^2}+\frac{4ia^4\ln\left(e^{2i(dx+c)}+1\right)B}{d}-$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
a^4/d*(B*tan(d*x+c)+1/2*(8*A-8*I*B)*ln(1+tan(d*x+c)^2)+(-8*I*A-8*B)*arctan
(tan(d*x+c))-1/2*A/tan(d*x+c)^2-(4*I*A+B)/tan(d*x+c)+(4*I*B-7*A)*ln(tan(d*
x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.63

$$\int\cot^3(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{10\,Aa^4e^{(4i\,dx+4i\,c)}+2\,(A-2i\,B)a^4e^{(2i\,dx+2i\,c)}-4\,(2\,A-i\,B)a^4-\left((A-4i\,B)a^4e^{(6i\,dx+6i\,c)}-(A-4i\,B)a\right.}{}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
(10*A*a^4*e^(4*I*d*x + 4*I*c) + 2*(A - 2*I*B)*a^4*e^(2*I*d*x + 2*I*c) - 4*
(2*A - I*B)*a^4 - ((A - 4*I*B)*a^4*e^(6*I*d*x + 6*I*c) - (A - 4*I*B)*a^4*e
^(4*I*d*x + 4*I*c) - (A - 4*I*B)*a^4*e^(2*I*d*x + 2*I*c) + (A - 4*I*B)*a^4
)*log(e^(2*I*d*x + 2*I*c) + 1) - ((7*A - 4*I*B)*a^4*e^(6*I*d*x + 6*I*c) -
(7*A - 4*I*B)*a^4*e^(4*I*d*x + 4*I*c) - (7*A - 4*I*B)*a^4*e^(2*I*d*x + 2*I
*c) + (7*A - 4*I*B)*a^4)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(6*I*d*x + 6*I
*c) - d*e^(4*I*d*x + 4*I*c) - d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.62

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{a^4(A-4iB)\log\left(e^{2idx}+\frac{(4Aa^4-4iBa^4-a^4(A-4iB))e^{-2ic}}{3Aa^4}\right)}{d} - \frac{a^4\cdot(7A-4iB)\log\left(e^{2idx}+\frac{(4Aa^4-4iBa^4-a^4\cdot(7A-4iB))e^{-2ic}}{3Aa^4}\right)}{d} + \frac{10Aa^4e^{4ic}e^{4idx}-8Aa^4+4iBa^4+(2Aa^4e^{2ic}-4iBa^4e^{2ic})\,e^{2idx}}{de^{6ic}e^{6idx}-de^{4ic}e^{4idx}-de^{2ic}e^{2idx}+d}$$

input
```
integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
-a**4*(A - 4*I*B)*log(exp(2*I*d*x) + (4*A*a**4 - 4*I*B*a**4 - a**4*(A - 4*
I*B))*exp(-2*I*c)/(3*A*a**4))/d - a**4*(7*A - 4*I*B)*log(exp(2*I*d*x) + (4
*A*a**4 - 4*I*B*a**4 - a**4*(7*A - 4*I*B))*exp(-2*I*c)/(3*A*a**4))/d + (10
*A*a**4*exp(4*I*c)*exp(4*I*d*x) - 8*A*a**4 + 4*I*B*a**4 + (2*A*a**4*exp(2*
I*c) - 4*I*B*a**4*exp(2*I*c))*exp(2*I*d*x))/(d*exp(6*I*c)*exp(6*I*d*x) - d
*exp(4*I*c)*exp(4*I*d*x) - d*exp(2*I*c)*exp(2*I*d*x) + d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.69

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$

$$-\frac{16\,(dx+c)(i\,A+B)a^4-8\,(A-i\,B)a^4\log\left(\tan\left(dx+c\right)^2+1\right)+2\,(7\,A-4i\,B)a^4\log\left(\tan\left(dx+c\right)\right)-}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/2*(16*(d*x + c)*(I*A + B)*a^4 - 8*(A - I*B)*a^4*log(tan(d*x + c)^2 + 1)
 + 2*(7*A - 4*I*B)*a^4*log(tan(d*x + c)) - 2*B*a^4*tan(d*x + c) - (2*(-4*I
*A - B)*a^4*tan(d*x + c) - A*a^4)/tan(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{Ba^4\tan\left(dx+c\right)}{d}+\frac{8\,(Aa^4-i\,Ba^4)\log\left(\tan\left(dx+c\right)+i\right)}{d}$$
$$-\frac{(7\,Aa^4-4i\,Ba^4)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}-\frac{Aa^4+2\,(4i\,Aa^4+Ba^4)\tan\left(dx+c\right)}{2\,d\tan\left(dx+c\right)^2}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
B*a^4*tan(d*x + c)/d + 8*(A*a^4 - I*B*a^4)*log(tan(d*x + c) + I)/d - (7*A*
a^4 - 4*I*B*a^4)*log(abs(tan(d*x + c)))/d - 1/2*(A*a^4 + 2*(4*I*A*a^4 + B*
a^4)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.65

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{B\,a^4\tan(c+d\,x)}{d} - \frac{a^4\,\ln\left(\tan(c+d\,x)\right)\,\left(7\,A-B\,4\mathrm{i}\right)}{d} + \frac{8\,a^4\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{d} - \frac{\cot(c+d\,x)^2\,\left(\frac{A\,a^4}{2}+\tan(c+d\,x)\,\left(B\,a^4+A\,a^4\,4\mathrm{i}\right)\right)}{d}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(8*a^4*log(tan(c + d*x) + 1i)*(A - B*1i))/d - (a^4*log(tan(c + d*x))*(7*A
- B*4i))/d - (cot(c + d*x)^2*((A*a^4)/2 + tan(c + d*x)*(A*a^4*4i + B*a^4))
)/d + (B*a^4*tan(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.35

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{a^4\left(64\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)^2 a - 64\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(c\right.}{}$$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output

```
(a**4*(64*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a - 64
*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*b*i - 8*cos(c +
 d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a + 32*cos(c + d*x)*log(ta
n((c + d*x)/2) - 1)*sin(c + d*x)**2*b*i - 8*cos(c + d*x)*log(tan((c + d*x)
/2) + 1)*sin(c + d*x)**2*a + 32*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin
(c + d*x)**2*b*i - 56*cos(c + d*x)*log(tan((c + d*x)/2))*sin(c + d*x)**2*a
 + 32*cos(c + d*x)*log(tan((c + d*x)/2))*sin(c + d*x)**2*b*i - 64*cos(c +
d*x)*sin(c + d*x)**2*a*d*i*x + cos(c + d*x)*sin(c + d*x)**2*a - 64*cos(c +
 d*x)*sin(c + d*x)**2*b*d*x - 4*cos(c + d*x)*a + 32*sin(c + d*x)**3*a*i +
16*sin(c + d*x)**3*b - 32*sin(c + d*x)*a*i - 8*sin(c + d*x)*b))/(8*cos(c +
 d*x)*sin(c + d*x)**2*d)
```

3.32 $\int \cot^4(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 163

$$\begin{aligned}\int \cot^4(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx &= 8a^4(A-iB)x-\frac{a^4B\log(\cos(c+dx))}{d}-\frac{a^4(8iA+7B)\log(\sin(c+dx))}{d}\\ &\quad-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}\\ &\quad-\frac{(2iA+B)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}\\ &\quad+\frac{(4A-3iB)\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}\end{aligned}$$

output
```
8*a^4*(A-I*B)*x-a^4*B*ln(cos(d*x+c))/d-a^4*(8*I*A+7*B)*ln(sin(d*x+c))/d-1/
3*a*A*cot(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d-1/2*(2*I*A+B)*cot(d*x+c)^2*(a^2+
I*a^2*tan(d*x+c))^2/d+(4*A-3*I*B)*cot(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = a^4\left(\frac{7A\cot(c+dx)}{d}-\frac{4iB\cot(c+dx)}{d}-\frac{2iA\cot^2(c+dx)}{d}-\frac{B\cot^2(c+dx)}{2d}-\frac{A\cot^3(c+dx)}{3d}-\frac{8iA\log(\tan(c+dx))}{d}-\frac{7B\log(\tan(c+dx))}{d}+\frac{8iA\log(i+\tan(c+dx))}{d}+\frac{8B\log(i+\tan(c+dx))}{d}\right)$$

input
```
Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
a^4*((7*A*Cot[c + d*x])/d - ((4*I)*B*Cot[c + d*x])/d - ((2*I)*A*Cot[c + d*
x]^2)/d - (B*Cot[c + d*x]^2)/(2*d) - (A*Cot[c + d*x]^3)/(3*d) - ((8*I)*A*L
og[Tan[c + d*x]])/d - (7*B*Log[Tan[c + d*x]])/d + ((8*I)*A*Log[I + Tan[c +
 d*x]])/d + (8*B*Log[I + Tan[c + d*x]])/d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4076, 3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{3}\int 3\cot^3(c+dx)(i\tan(c+dx)a+a)^3(a(2iA+B)+iaB\tan(c+dx))dx-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot^3(c+dx)(i\tan(c+dx)a+a)^3(a(2iA+B)+iaB\tan(c+dx))dx-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^3(a(2iA+B)+iaB\tan(c+dx))}{\tan(c+dx)^3}dx-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{2}\int -2\cot^2(c+dx)(i\tan(c+dx)a+a)^2\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)dx-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{27}$$

$$-\int \cot^2(c+dx)(i\tan(c+dx)a+a)^2\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)dx-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{(i\tan(c+dx)a+a)^2\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)}{\tan(c+dx)^2}dx-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4076}$$

$$-\int \cot(c+dx)(i\tan(c+dx)a+a)\left((8iA+7B)a^3+iB\tan(c+dx)a^3\right)dx+\frac{(4A-3iB)\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{(i\tan(c+dx)a+a)\left((8iA+7B)a^3+iB\tan(c+dx)a^3\right)}{\tan(c+dx)}dx+\frac{(4A-3iB)\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4072}$$

$$-\int \cot(c+dx)\left(a^4(8iA+7B)-8a^4(A-iB)\tan(c+dx)\right)dx+a^4B\int \tan(c+dx)dx+\frac{(4A-3iB)\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{a^4(8iA+7B)-8a^4(A-iB)\tan(c+dx)}{\tan(c+dx)}dx+a^4B\int \tan(c+dx)dx+\frac{(4A-3iB)\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3956}$$

$$-\int \frac{a^4(8iA+7B)-8a^4(A-iB)\tan(c+dx)}{\tan(c+dx)}dx+\frac{(4A-3iB)\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{a^4B\log(\cos(c+dx))}{d}-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4014}$$

$$-a^4(7B+8iA)\int \cot(c+dx)dx+\frac{(4A-3iB)\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}+8a^4x(A-iB)-\frac{a^4B\log(\cos(c+dx))}{d}-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-a^4(7B+8iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{(4A-3iB)\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}+8a^4x(A-iB)-\frac{a^4B\log(\cos(c+dx))}{d}-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 25

$$a^4(7B+8iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{(4A-3iB)\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}+8a^4x(A-iB)-\frac{a^4B\log(\cos(c+dx))}{d}-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3956

$$-\frac{a^4(7B+8iA)\log(-\sin(c+dx))}{d}+\frac{(4A-3iB)\cot(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}+8a^4x(A-iB)-\frac{a^4B\log(\cos(c+dx))}{d}-\frac{(B+2iA)\cot^2(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{2d}-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
8*a^4*(A - I*B)*x - (a^4*B*Log[Cos[c + d*x]])/d - (a^4*((8*I)*A + 7*B)*Log
[-Sin[c + d*x]])/d - (a*A*Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/(3*d) -
 (((2*I)*A + B)*Cot[c + d*x]^2*(a^2 + I*a^2*Tan[c + d*x])^2)/(2*d) + ((4*A
 - (3*I)*B)*Cot[c + d*x]*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{8\left(\frac{(iA+B)\ln\left(\sec(dx+c)^2\right)}{2}-\left(iA+\frac{7B}{8}\right)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^3}{24}-\frac{\cot(dx+c)^2\left(iA+\frac{B}{4}\right)}{4}+\frac{\left(-iB+\frac{7A}{4}\right)\cot(dx+c)}{2}+(-i\right.}{d}$
derivativedivides	$\frac{a^4\left(\frac{(8iA+8B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-8iB+8A)\arctan(\tan(dx+c))-\frac{A}{3\tan(dx+c)^3}+(-8iA-7B)\ln(\tan(dx+c))-\frac{4iA-}{2\tan(d}\right.}{d}$
default	$\frac{a^4\left(\frac{(8iA+8B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-8iB+8A)\arctan(\tan(dx+c))-\frac{A}{3\tan(dx+c)^3}+(-8iA-7B)\ln(\tan(dx+c))-\frac{4iA-}{2\tan(d}\right.}{d}$
norman	$\frac{\frac{\left(-4iB\,a^4+7A\,a^4\right)\tan(dx+c)^2}{d}+\left(-8iB\,a^4+8A\,a^4\right)x\tan(dx+c)^3-\frac{A\,a^4}{3d}-\frac{\left(4iA\,a^4+B\,a^4\right)\tan(dx+c)}{2d}}{\tan(dx+c)^3}+\frac{4\left(iA\,a^4+B\,a^4\right)\ln}{a}$
risch	$\frac{16ia^4Bc}{d}-\frac{16a^4Ac}{d}+\frac{2a^4\left(36iA\,e^{4i(dx+c)}+15B\,e^{4i(dx+c)}-54iA\,e^{2i(dx+c)}-27B\,e^{2i(dx+c)}+22iA+12B\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}-\frac{7a^4\ln}{}$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
8*(1/2*(I*A+B)*ln(sec(d*x+c)^2)-(I*A+7/8*B)*ln(tan(d*x+c))-1/24*A*cot(d*x+
c)^3-1/4*cot(d*x+c)^2*(I*A+1/4*B)+1/2*(-I*B+7/4*A)*cot(d*x+c)+(A-I*B)*x*d)
*a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.53

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$
$$-\frac{6\,(-12i\,A-5\,B)a^4e^{(4i\,dx+4i\,c)}+54\,(2i\,A+B)a^4e^{(2i\,dx+2i\,c)}+4\,(-11i\,A-6\,B)a^4+3\,\left(Ba^4e^{(6i\,dx+6i\,c)}-\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-1/3*(6*(-12*I*A - 5*B)*a^4*e^(4*I*d*x + 4*I*c) + 54*(2*I*A + B)*a^4*e^(2*
I*d*x + 2*I*c) + 4*(-11*I*A - 6*B)*a^4 + 3*(B*a^4*e^(6*I*d*x + 6*I*c) - 3*
B*a^4*e^(4*I*d*x + 4*I*c) + 3*B*a^4*e^(2*I*d*x + 2*I*c) - B*a^4)*log(e^(2*
I*d*x + 2*I*c) + 1) + 3*((8*I*A + 7*B)*a^4*e^(6*I*d*x + 6*I*c) + 3*(-8*I*A
 - 7*B)*a^4*e^(4*I*d*x + 4*I*c) + 3*(8*I*A + 7*B)*a^4*e^(2*I*d*x + 2*I*c)
+ (-8*I*A - 7*B)*a^4)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(6*I*d*x + 6*I*c)
 - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(143) = 286$.

Time = 2.71 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.79

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{Ba^4\log\left(\frac{4Aa^4-3iBa^4}{4Aa^4e^{2ic}-3iBa^4e^{2ic}}+e^{2idx}\right)}{d} - \frac{ia^4\cdot(8A-7iB)\log\left(e^{2idx}+\frac{4Aa^4-4iBa^4-a^4\cdot(8A-7iB)}{4Aa^4e^{2ic}-3iBa^4e^{2ic}}\right)}{d} + \frac{44iAa^4+24Ba^4+(-108iAa^4e^{2ic}-54Ba^4e^{2ic})\,e^{2idx}+(72iAa^4e^{4ic}+30Ba^4e^{4ic})\,e^{4idx}}{3de^{6ic}e^{6idx}-9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}-3d}$$

input
```
integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
-B*a**4*log((4*A*a**4 - 3*I*B*a**4)/(4*A*a**4*exp(2*I*c) - 3*I*B*a**4*exp(
2*I*c)) + exp(2*I*d*x))/d - I*a**4*(8*A - 7*I*B)*log(exp(2*I*d*x) + (4*A*a
**4 - 4*I*B*a**4 - a**4*(8*A - 7*I*B))/(4*A*a**4*exp(2*I*c) - 3*I*B*a**4*e
xp(2*I*c)))/d + (44*I*A*a**4 + 24*B*a**4 + (-108*I*A*a**4*exp(2*I*c) - 54*
B*a**4*exp(2*I*c))*exp(2*I*d*x) + (72*I*A*a**4*exp(4*I*c) + 30*B*a**4*exp(
4*I*c))*exp(4*I*d*x))/(3*d*exp(6*I*c)*exp(6*I*d*x) - 9*d*exp(4*I*c)*exp(4*
I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) - 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.72

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{48\,(dx+c)(A-i\,B)a^4 - 24\,(-i\,A-B)a^4\log\left(\tan\left(dx+c\right)^2+1\right) + 6\,(-8i\,A-7\,B)a^4\log\left(\tan\left(dx+c\right)\right.}{6\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/6*(48*(d*x + c)*(A - I*B)*a^4 - 24*(-I*A - B)*a^4*log(tan(d*x + c)^2 + 1
) + 6*(-8*I*A - 7*B)*a^4*log(tan(d*x + c)) + (6*(7*A - 4*I*B)*a^4*tan(d*x
+ c)^2 + 3*(-4*I*A - B)*a^4*tan(d*x + c) - 2*A*a^4)/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{8\,(-i\,Aa^4-Ba^4)\log\left(\tan\left(dx+c\right)+i\right)}{d} + \frac{(-8i\,Aa^4-7\,Ba^4)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d} - \frac{2\,Aa^4 - 6\,(7\,Aa^4-4i\,Ba^4)\tan\left(dx+c\right)^2 + 3\,(4i\,Aa^4+Ba^4)\tan\left(dx+c\right)}{6\,d\tan\left(dx+c\right)^3}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-8*(-I*A*a^4 - B*a^4)*log(tan(d*x + c) + I)/d + (-8*I*A*a^4 - 7*B*a^4)*log
(abs(tan(d*x + c)))/d - 1/6*(2*A*a^4 - 6*(7*A*a^4 - 4*I*B*a^4)*tan(d*x + c
)^2 + 3*(4*I*A*a^4 + B*a^4)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.69

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{\frac{A\,a^4}{3} - \tan(c+d\,x)^2\,(7\,A\,a^4 - B\,a^4\,4\mathrm{i}) + \tan(c+d\,x)\,\left(\frac{B\,a^4}{2} + A\,a^4\,2\mathrm{i}\right)}{d\tan\left(c+d\,x\right)^3}$$
$$- \frac{a^4\,\ln\left(\tan(c+d\,x)\right)\,\left(7\,B + A\,8\mathrm{i}\right)}{d} + \frac{8\,a^4\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B + A\,1\mathrm{i}\right)}{d}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(8*a^4*log(tan(c + d*x) + 1i)*(A*1i + B))/d - (a^4*log(tan(c + d*x))*(A*8i
 + 7*B))/d - ((A*a^4)/3 - tan(c + d*x)^2*(7*A*a^4 - B*a^4*4i) + tan(c + d*
x)*(A*a^4*2i + (B*a^4)/2))/(d*tan(c + d*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.66

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{a^4\Big(88\cos\left(dx+c\right)\sin\left(dx+c\right)^2 a - 48\cos\left(dx+c\right)\sin\left(dx+c\right)^2 bi - 4\cos\left(dx+c\right)a + 96\log\Big(\tan\left(\frac{dx}{2}+\right.}{}$$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(a**4*(88*cos(c + d*x)*sin(c + d*x)**2*a - 48*cos(c + d*x)*sin(c + d*x)**2
*b*i - 4*cos(c + d*x)*a + 96*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*
a*i + 96*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*b - 12*log(tan((c +
d*x)/2) - 1)*sin(c + d*x)**3*b - 12*log(tan((c + d*x)/2) + 1)*sin(c + d*x)
**3*b - 96*log(tan((c + d*x)/2))*sin(c + d*x)**3*a*i - 84*log(tan((c + d*x
)/2))*sin(c + d*x)**3*b + 96*sin(c + d*x)**3*a*d*x + 12*sin(c + d*x)**3*a*
i - 96*sin(c + d*x)**3*b*d*i*x + 3*sin(c + d*x)**3*b - 24*sin(c + d*x)*a*i
 - 6*sin(c + d*x)*b))/(12*sin(c + d*x)**3*d)
```

3.33 $\int \cot^5(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 177

$$\begin{aligned}\int \cot^5(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx &= 8a^4(iA+B)x+\frac{a^4(67iA+64B)\cot(c+dx)}{12d}\\ &+\frac{8a^4(A-iB)\log(\sin(c+dx))}{d}-\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}\\ &-\frac{(7iA+4B)\cot^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d}\\ &+\frac{(19A-16iB)\cot^2(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{12d}\end{aligned}$$

output
```
8*a^4*(I*A+B)*x+1/12*a^4*(67*I*A+64*B)*cot(d*x+c)/d+8*a^4*(A-I*B)*ln(sin(d
*x+c))/d-1/4*a*A*cot(d*x+c)^4*(a+I*a*tan(d*x+c))^3/d-1/12*(7*I*A+4*B)*cot(
d*x+c)^3*(a^2+I*a^2*tan(d*x+c))^2/d+1/12*(19*A-16*I*B)*cot(d*x+c)^2*(a^4+I
*a^4*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.51

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{a^4(-3A(i+\cot(c+dx))^4+4(A-iB)\left(21i\cot(c+dx)+6\cot^2(c+dx)-i\cot^3(c+dx)+24(\log(\tan(\right.}{12d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(a^4*(-3*A*(I + Cot[c + d*x])^4 + 4*(A - I*B)*((21*I)*Cot[c + d*x] + 6*Cot
[c + d*x]^2 - I*Cot[c + d*x]^3 + 24*(Log[Tan[c + d*x]] - Log[I + Tan[c + d
*x]]))))/(12*d)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {3042, 4076, 3042, 4076, 27, 3042, 4076, 3042, 4074, 27, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{4}\int \cot^4(c+dx)(i\tan(c+dx)a+a)^3(a(7iA+4B)-a(A-4iB)\tan(c+dx))dx - \frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{(i\tan(c+dx)a+a)^3(a(7iA+4B)-a(A-4iB)\tan(c+dx))}{\tan(c+dx)^4}dx - \frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 4076

$$\frac{1}{4}\left(\frac{1}{3}\int -2\cot^3(c+dx)(i\tan(c+dx)a+a)^2\left((19A-16iB)a^2+(5iA+8B)\tan(c+dx)a^2\right)dx - \frac{(4B+7iA)\,c\right. \frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{2}{3}\int \cot^3(c+dx)(i\tan(c+dx)a+a)^2\left((19A-16iB)a^2+(5iA+8B)\tan(c+dx)a^2\right)dx - \frac{(4B+7iA)\cot\right. \frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(-\frac{2}{3}\int \frac{(i\tan(c+dx)a+a)^2\left((19A-16iB)a^2+(5iA+8B)\tan(c+dx)a^2\right)}{\tan(c+dx)^3}dx - \frac{(4B+7iA)\cot^3(c+dx)\,(a}{3d}\right. \frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 4076

$$\frac{1}{4}\left(-\frac{2}{3}\left(\frac{1}{2}\int \cot^2(c+dx)(i\tan(c+dx)a+a)\left(a^3(67iA+64B)-a^3(29A-32iB)\tan(c+dx)\right)dx - \frac{(19A-16}{}\right.\right. \frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(-\frac{2}{3}\left(\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)\left(a^3(67iA+64B)-a^3(29A-32iB)\tan(c+dx)\right)}{\tan(c+dx)^2}dx - \frac{(19A-16iB)\cot^2(c+}{}\right.\right. \frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 4074

$$\frac{1}{4}\left(-\frac{2}{3}\left(\frac{1}{2}\left(\int -96\cot(c+dx)\left((A-iB)a^4+(iA+B)\tan(c+dx)a^4\right)dx-\frac{a^4(64B+67iA)\cot(c+dx)}{d}\right)-\frac{(1}{}\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{2}{3}\left(\frac{1}{2}\left(-96\int \cot(c+dx)\left((A-iB)a^4+(iA+B)\tan(c+dx)a^4\right)dx-\frac{a^4(64B+67iA)\cot(c+dx)}{d}\right)-\frac{(1}{}\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(-\frac{2}{3}\left(\frac{1}{2}\left(-96\int \frac{(A-iB)a^4+(iA+B)\tan(c+dx)a^4}{\tan(c+dx)}dx-\frac{a^4(64B+67iA)\cot(c+dx)}{d}\right)-\frac{(19A-16iB)\,c}{}\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 4014

$$\frac{1}{4}\left(-\frac{2}{3}\left(\frac{1}{2}\left(-96\left(a^4(A-iB)\int \cot(c+dx)dx+a^4x(B+iA)\right)-\frac{a^4(64B+67iA)\cot(c+dx)}{d}\right)-\frac{(19A-16i}{}\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(-\frac{2}{3}\left(\frac{1}{2}\left(-96\left(a^4(A-iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+a^4x(B+iA)\right)-\frac{a^4(64B+67iA)\cot(c+dx)}{d}\right)-\frac{(19}{}\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 25

$$\frac{1}{4}\left(-\frac{2}{3}\left(\frac{1}{2}\left(-96\left(a^4x(B+iA)-a^4(A-iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx\right)-\frac{a^4(64B+67iA)\cot(c+dx)}{d}\right)-\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

$\downarrow$ 3956

$$\frac{1}{4}\left(-\frac{2}{3}\left(\frac{1}{2}\left(-\frac{a^4(64B+67iA)\cot(c+dx)}{d}-96\left(\frac{a^4(A-iB)\log(-\sin(c+dx))}{d}+a^4x(B+iA)\right)\right)-\frac{(19A-16\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

input
```
Int[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-1/4*(a*A*Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^3)/d + (-1/3*(((7*I)*A + 4
*B)*Cot[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x])^2)/d - (2*((-((a^4*((67*I)*A
 + 64*B)*Cot[c + d*x])/d) - 96*(a^4*(I*A + B)*x + (a^4*(A - I*B)*Log[-Sin[
c + d*x]])/d))/2 - ((19*A - (16*I)*B)*Cot[c + d*x]^2*(a^4 + I*a^4*Tan[c +
d*x]))/(2*d)))/3)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

method	result
parallelrisch	$\frac{8\left(\frac{(iB-A)\ln\left(\sec(dx+c)^2\right)}{2}+(-iB+A)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^4}{32}-\frac{\cot(dx+c)^3\left(iA+\frac{B}{4}\right)}{6}+\frac{\left(-iB+\frac{7A}{4}\right)\cot(dx+c)^2}{4}+\left(iA\right.\right.}{d}$
derivativedivides	$\frac{a^4\left(\frac{(8iB-8A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(8iA+8B)\arctan(\tan(dx+c))-\frac{A}{4\tan(dx+c)^4}+(-8iB+8A)\ln(\tan(dx+c))-\frac{-8iA-7B}{\tan(dx+c}\right.}{d}$
default	$\frac{a^4\left(\frac{(8iB-8A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(8iA+8B)\arctan(\tan(dx+c))-\frac{A}{4\tan(dx+c)^4}+(-8iB+8A)\ln(\tan(dx+c))-\frac{-8iA-7B}{\tan(dx+c}\right.}{d}$
risch	$-\frac{16a^4Bc}{d}-\frac{16ia^4Ac}{d}+\frac{4ia^4\left(30iA\,e^{6i(dx+c)}+18B\,e^{6i(dx+c)}-63iA\,e^{4i(dx+c)}-45B\,e^{4i(dx+c)}+50iA\,e^{2i(dx+c)}+38\right.}{3d\left(e^{2i(dx+c)}-1\right)^4}$
norman	$\frac{\frac{\left(8iA\,a^4+7B\,a^4\right)\tan(dx+c)^3}{d}+\left(8iA\,a^4+8B\,a^4\right)x\tan(dx+c)^4-\frac{A\,a^4}{4d}+\frac{\left(-4iB\,a^4+7A\,a^4\right)\tan(dx+c)^2}{2d}-\frac{\left(4iA\,a^4+B\,a^4\right)\tan(dx\right.}{3d}}{\tan(dx+c)^4}$

input
```
int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
8*(1/2*(-A+I*B)*ln(sec(d*x+c)^2)+(A-I*B)*ln(tan(d*x+c))-1/32*A*cot(d*x+c)^
4-1/6*cot(d*x+c)^3*(I*A+1/4*B)+1/4*(-I*B+7/4*A)*cot(d*x+c)^2+(I*A+7/8*B)*c
ot(d*x+c)+(I*A+B)*x*d)*a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.29

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = -\frac{4\left(6\left(5A-3iB\right)a^4e^{(6i\,dx+6i\,c)}-9\left(7A-5iB\right)a^4e^{(4i\,dx+4i\,c)}+2\left(25A-19iB\right)a^4e^{(2i\,dx+2i\,c)}-(14A-1}{3\left(de^{(8i\,c}}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-4/3*(6*(5*A - 3*I*B)*a^4*e^(6*I*d*x + 6*I*c) - 9*(7*A - 5*I*B)*a^4*e^(4*I
*d*x + 4*I*c) + 2*(25*A - 19*I*B)*a^4*e^(2*I*d*x + 2*I*c) - (14*A - 11*I*B
)*a^4 - 6*((A - I*B)*a^4*e^(8*I*d*x + 8*I*c) - 4*(A - I*B)*a^4*e^(6*I*d*x
+ 6*I*c) + 6*(A - I*B)*a^4*e^(4*I*d*x + 4*I*c) - 4*(A - I*B)*a^4*e^(2*I*d*
x + 2*I*c) + (A - I*B)*a^4)*log(e^(2*I*d*x + 2*I*c) - 1))/(d*e^(8*I*d*x +
8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.33

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$= \frac{8a^4(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d} + \frac{56Aa^4-44iBa^4+\left(-200Aa^4e^{2ic}+152iBa^4e^{2ic}\right)e^{2idx}+\left(252Aa^4e^{4ic}-180iBa^4e^{4ic}\right)e^{4idx}+(-120Aa}{3de^{8ic}e^{8idx}-12de^{6ic}e^{6idx}+18de^{4ic}e^{4idx}-12de^{2ic}e^{2idx}+3d}$$

input
```
integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
8*a**4*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (56*A*a**4 - 44*I*B*a
**4 + (-200*A*a**4*exp(2*I*c) + 152*I*B*a**4*exp(2*I*c))*exp(2*I*d*x) + (2
52*A*a**4*exp(4*I*c) - 180*I*B*a**4*exp(4*I*c))*exp(4*I*d*x) + (-120*A*a**
4*exp(6*I*c) + 72*I*B*a**4*exp(6*I*c))*exp(6*I*d*x))/(3*d*exp(8*I*c)*exp(8
*I*d*x) - 12*d*exp(6*I*c)*exp(6*I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d*x) - 12
*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = -\frac{96\,(dx+c)(-i\,A-B)a^4+48\,(A-i\,B)a^4\log\left(\tan\left(dx+c\right)^2+1\right)-96\,(A-i\,B)a^4\log\left(\tan\left(dx+c\right)\right)}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/12*(96*(d*x + c)*(-I*A - B)*a^4 + 48*(A - I*B)*a^4*log(tan(d*x + c)^2 +
 1) - 96*(A - I*B)*a^4*log(tan(d*x + c)) - (12*(8*I*A + 7*B)*a^4*tan(d*x +
 c)^3 + 6*(7*A - 4*I*B)*a^4*tan(d*x + c)^2 + 4*(-4*I*A - B)*a^4*tan(d*x +
c) - 3*A*a^4)/tan(d*x + c)^4)/d
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{8\,(Aa^4-i\,Ba^4)\log\left(\tan\left(dx+c\right)+i\right)}{d}+\frac{8\,(Aa^4-i\,Ba^4)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d} - \frac{3\,Aa^4+12\,(-8i\,Aa^4-7\,Ba^4)\tan\left(dx+c\right)^3-6\,(7\,Aa^4-4i\,Ba^4)\tan\left(dx+c\right)^2+4\,(4i\,Aa^4+Ba^4)\tan}{12\,d\tan\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-8*(A*a^4 - I*B*a^4)*log(tan(d*x + c) + I)/d + 8*(A*a^4 - I*B*a^4)*log(abs
(tan(d*x + c)))/d - 1/12*(3*A*a^4 + 12*(-8*I*A*a^4 - 7*B*a^4)*tan(d*x + c)
^3 - 6*(7*A*a^4 - 4*I*B*a^4)*tan(d*x + c)^2 + 4*(4*I*A*a^4 + B*a^4)*tan(d*
x + c))/(d*tan(d*x + c)^4)
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)^2\left(\frac{7\,A\,a^4}{2}-B\,a^4\,2\mathrm{i}\right)+\tan(c+d\,x)^3\left(7\,B\,a^4+A\,a^4\,8\mathrm{i}\right)-\frac{A\,a^4}{4}-\tan(c+d\,x)\,\left(\frac{B\,a^4}{3}+\frac{A\,a^4\,4\mathrm{i}}{3}\right)}{d\tan\left(c+d\,x\right)^4}$$
$$+\frac{16\,a^4\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\,\left(B+A\,1\mathrm{i}\right)}{d}$$

input
```
int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(tan(c + d*x)^2*((7*A*a^4)/2 - B*a^4*2i) + tan(c + d*x)^3*(A*a^4*8i + 7*B*
a^4) - (A*a^4)/4 - tan(c + d*x)*((A*a^4*4i)/3 + (B*a^4)/3))/(d*tan(c + d*x
)^4) + (16*a^4*atan(2*tan(c + d*x) + 1i)*(A*1i + B))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.43

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{a^4\left(896\cos(dx+c)\sin(dx+c)^3 ai+704\cos(dx+c)\sin(dx+c)^3 b-128\cos(dx+c)\sin(dx+c) ai-\right.}{}$$

input
```
int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(a**4*(896*cos(c + d*x)*sin(c + d*x)**3*a*i + 704*cos(c + d*x)*sin(c + d*x
)**3*b - 128*cos(c + d*x)*sin(c + d*x)*a*i - 32*cos(c + d*x)*sin(c + d*x)*
b - 768*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**4*a + 768*log(tan((c +
d*x)/2)**2 + 1)*sin(c + d*x)**4*b*i + 768*log(tan((c + d*x)/2))*sin(c + d*
x)**4*a - 768*log(tan((c + d*x)/2))*sin(c + d*x)**4*b*i + 768*sin(c + d*x)
**4*a*d*i*x - 183*sin(c + d*x)**4*a + 768*sin(c + d*x)**4*b*d*x + 96*sin(c
 + d*x)**4*b*i + 384*sin(c + d*x)**2*a - 192*sin(c + d*x)**2*b*i - 24*a))/
(96*sin(c + d*x)**4*d)
```

3.34 $\int \cot^6(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 200

$$\begin{aligned}
&\int \cot^6(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx \\
&= -8a^4(A-iB)x - \frac{8a^4(A-iB)\cot(c+dx)}{d} + \frac{a^4(148iA+145B)\cot^2(c+dx)}{60d} \\
&\quad + \frac{8a^4(iA+B)\log(\sin(c+dx))}{d} - \frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d} \\
&\quad - \frac{(8iA+5B)\cot^4(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{20d} \\
&\quad + \frac{(28A-25iB)\cot^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{30d}
\end{aligned}$$

output
```
-8*a^4*(A-I*B)*x-8*a^4*(A-I*B)*cot(d*x+c)/d+1/60*a^4*(148*I*A+145*B)*cot(d
*x+c)^2/d+8*a^4*(I*A+B)*ln(sin(d*x+c))/d-1/5*a*A*cot(d*x+c)^5*(a+I*a*tan(d
*x+c))^3/d-1/20*(8*I*A+5*B)*cot(d*x+c)^4*(a^2+I*a^2*tan(d*x+c))^2/d+1/30*(
28*A-25*I*B)*cot(d*x+c)^3*(a^4+I*a^4*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.60

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{a^4\left(-3i(4A-5iB)(i+\cot(c+dx))^4-12A\cot(c+dx)(i+\cot(c+dx))^4+20(A-iB)\left(-21\cot(c+d\right.\right.}{60d}$$

input
```
Integrate[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(a^4*((-3*I)*(4*A - (5*I)*B)*(I + Cot[c + d*x])^4 - 12*A*Cot[c + d*x]*(I +
 Cot[c + d*x])^4 + 20*(A - I*B)*(-21*Cot[c + d*x] + (6*I)*Cot[c + d*x]^2 +
 Cot[c + d*x]^3 + (24*I)*(Log[Tan[c + d*x]] - Log[I + Tan[c + d*x]]))))/(6
0*d)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4076, 3042, 4076, 27, 3042, 4076, 3042, 4074, 27, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^6}dx$$
$$\Big\downarrow \text{4076}$$
$$\frac{1}{5}\int \cot^5(c+dx)(i\tan(c+dx)a+a)^3(a(8iA+5B)-a(2A-5iB)\tan(c+dx))dx-\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)^3(a(8iA+5B)-a(2A-5iB)\tan(c+dx))}{\tan(c+dx)^5}dx-\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \; 4076$$

$$\frac{1}{5}\left(\frac{1}{4}\int -2\cot^4(c+dx)(i\tan(c+dx)a+a)^2\left((28A-25iB)a^2+3(4iA+5B)\tan(c+dx)a^2\right)dx-\frac{(5B+8iA)}{}\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(-\frac{1}{2}\int \cot^4(c+dx)(i\tan(c+dx)a+a)^2\left((28A-25iB)a^2+3(4iA+5B)\tan(c+dx)a^2\right)dx-\frac{(5B+8iA)\,\mathrm{co}}{}\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(-\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)^2\left((28A-25iB)a^2+3(4iA+5B)\tan(c+dx)a^2\right)}{\tan(c+dx)^4}dx-\frac{(5B+8iA)\cot^4(c+dx)\,(}{4d}\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \; 4076$$

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{(28A-25iB)\cot^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{3d}-\frac{1}{3}\int \cot^3(c+dx)(i\tan(c+dx)a+a)\left(a^3(148iA+1\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{(28A-25iB)\cot^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{3d}-\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)\left(a^3(148iA+145B)-a^3(9\right.}{\tan(c+dx)^3}\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \; 4074$$

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{a^4(145B+148iA)\cot^2(c+dx)}{2d}-\int -240\cot^2(c+dx)\left((A-iB)a^4+(iA+B)\tan(c+dx)a^4\right)dx\right)\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{1}{3}\left(240\int \cot^2(c+dx)\left((A-iB)a^4+(iA+B)\tan(c+dx)a^4\right)dx+\frac{a^4(145B+148iA)\cot^2(c+dx)}{2d}\right)+\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{1}{3}\left(240\int \frac{(A-iB)a^4+(iA+B)\tan(c+dx)a^4}{\tan(c+dx)^2}dx+\frac{a^4(145B+148iA)\cot^2(c+dx)}{2d}\right)+\frac{(28A-25iB)}{}\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 4012

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{1}{3}\left(240\left(\int \cot(c+dx)\left(a^4(iA+B)-a^4(A-iB)\tan(c+dx)\right)dx-\frac{a^4(A-iB)\cot(c+dx)}{d}\right)+\frac{a^4(145}{}\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{1}{3}\left(240\left(\int \frac{a^4(iA+B)-a^4(A-iB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a^4(A-iB)\cot(c+dx)}{d}\right)+\frac{a^4(145B+148iA)\,\mathrm{co}}{2d}\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 4014

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{1}{3}\left(240\left(a^4(B+iA)\int \cot(c+dx)dx-\frac{a^4(A-iB)\cot(c+dx)}{d}-\left(a^4x(A-iB)\right)\right)+\frac{a^4(145B+148iA)}{2d}\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{1}{3}\left(240\left(a^4(B+iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a^4(A-iB)\cot(c+dx)}{d}-\left(a^4x(A-iB)\right)\right)+\frac{a^4(145B\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$\downarrow$ 25

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{1}{3}\left(240\left(-a^4(B+iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a^4(A-iB)\cot(c+dx)}{d}-\left(a^4x(A-iB)\right)\right)+\frac{a^4(14\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

$\downarrow$ 3956

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{(28A-25iB)\cot^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{3d}+\frac{1}{3}\left(\frac{a^4(145B+148iA)\cot^2(c+dx)}{2d}+240\left(-\frac{a^4(A-\right.\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+ia\tan(c+dx))^3}{5d}$$

input
```
Int[Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-1/5*(a*A*Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^3)/d + (-1/4*(((8*I)*A + 5
*B)*Cot[c + d*x]^4*(a^2 + I*a^2*Tan[c + d*x])^2)/d + (((a^4*((148*I)*A + 1
45*B)*Cot[c + d*x]^2)/(2*d) + 240*(-(a^4*(A - I*B)*x) - (a^4*(A - I*B)*Cot
[c + d*x])/d + (a^4*(I*A + B)*Log[-Sin[c + d*x]])/d))/3 + ((28*A - (25*I)*
B)*Cot[c + d*x]^3*(a^4 + I*a^4*Tan[c + d*x]))/(3*d))/2)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{8a^4\left(-\frac{(iA+B)\ln\left(\sec(dx+c)^2\right)}{2}+(iA+B)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^5}{40}-\frac{\left(iA+\frac{B}{4}\right)\cot(dx+c)^4}{8}+\frac{\left(-iB+\frac{7A}{4}\right)\cot(dx+c)^3}{6}+\right.}{d}$
derivativedivides	$\frac{a^4\left(\frac{(-8iA-8B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(8iB-8A)\arctan(\tan(dx+c))-\frac{A}{5\tan(dx+c)^5}-\frac{-8iB+8A}{\tan(dx+c)}+(8iA+8B)\ln(\tan(dx+c)\right.}{d}$
default	$\frac{a^4\left(\frac{(-8iA-8B)\ln\left(1+\tan(dx+c)^2\right)}{2}+(8iB-8A)\arctan(\tan(dx+c))-\frac{A}{5\tan(dx+c)^5}-\frac{-8iB+8A}{\tan(dx+c)}+(8iA+8B)\ln(\tan(dx+c)\right.}{d}$
risch	$-\frac{16ia^4Bc}{d}+\frac{16a^4Ac}{d}-\frac{4a^4(210iA\,e^{8i(dx+c)}+150B\,e^{8i(dx+c)}-555iA\,e^{6i(dx+c)}-465B\,e^{6i(dx+c)}+655iA\,e^{4i(dx+c)}}{15d\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{(8iBa^4-8Aa^4)x\tan(dx+c)^5-\frac{Aa^4}{5d}+\frac{\left(-4iBa^4+7Aa^4\right)\tan(dx+c)^2}{3d}-\frac{\left(4iAa^4+Ba^4\right)\tan(dx+c)}{4d}-\frac{8\left(-iBa^4+Aa^4\right)\tan(dx}{d}}{\tan(dx+c)^5}$

input
```
int(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
8*a^4/d*(-1/2*(I*A+B)*ln(sec(d*x+c)^2)+(I*A+B)*ln(tan(d*x+c))-1/40*A*cot(d
*x+c)^5-1/8*(I*A+1/4*B)*cot(d*x+c)^4+1/6*(-I*B+7/4*A)*cot(d*x+c)^3+1/2*cot
(d*x+c)^2*(I*A+7/8*B)+cot(d*x+c)*(-A+I*B)+(-A+I*B)*x*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.44

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$
$$-\frac{4\left(30\,(7i\,A+5\,B)a^4e^{(8i\,dx+8i\,c)}+15\,(-37i\,A-31\,B)a^4e^{(6i\,dx+6i\,c)}+5\,(131i\,A+113\,B)a^4e^{(4i\,dx+4i\,c)}+\right.}{}$$

input
```
integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-4/15*(30*(7*I*A + 5*B)*a^4*e^(8*I*d*x + 8*I*c) + 15*(-37*I*A - 31*B)*a^4*
e^(6*I*d*x + 6*I*c) + 5*(131*I*A + 113*B)*a^4*e^(4*I*d*x + 4*I*c) + 5*(-73
*I*A - 64*B)*a^4*e^(2*I*d*x + 2*I*c) + (79*I*A + 70*B)*a^4 + 30*((-I*A - B
)*a^4*e^(10*I*d*x + 10*I*c) + 5*(I*A + B)*a^4*e^(8*I*d*x + 8*I*c) + 10*(-I
*A - B)*a^4*e^(6*I*d*x + 6*I*c) + 10*(I*A + B)*a^4*e^(4*I*d*x + 4*I*c) + 5
*(-I*A - B)*a^4*e^(2*I*d*x + 2*I*c) + (I*A + B)*a^4)*log(e^(2*I*d*x + 2*I*
c) - 1))/(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*
d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [A] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.48

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{8ia^4(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d} + \frac{-316iAa^4-280Ba^4+\left(1460iAa^4e^{2ic}+1280Ba^4e^{2ic}\right)e^{2idx}+\left(-2620iAa^4e^{4ic}-2260Ba^4e^{4ic}\right)e^{4idx}+(}{15de^{10ic}e^{10idx}-75de^{8ic}e^{8idx}+150de^{6ic}e^{6idx}-150de^{4ic}e}$$

input
```
integrate(cot(d*x+c)**6*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
8*I*a**4*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-316*I*A*a**4 - 28
0*B*a**4 + (1460*I*A*a**4*exp(2*I*c) + 1280*B*a**4*exp(2*I*c))*exp(2*I*d*x
) + (-2620*I*A*a**4*exp(4*I*c) - 2260*B*a**4*exp(4*I*c))*exp(4*I*d*x) + (2
220*I*A*a**4*exp(6*I*c) + 1860*B*a**4*exp(6*I*c))*exp(6*I*d*x) + (-840*I*A
*a**4*exp(8*I*c) - 600*B*a**4*exp(8*I*c))*exp(8*I*d*x))/(15*d*exp(10*I*c)*
exp(10*I*d*x) - 75*d*exp(8*I*c)*exp(8*I*d*x) + 150*d*exp(6*I*c)*exp(6*I*d*
x) - 150*d*exp(4*I*c)*exp(4*I*d*x) + 75*d*exp(2*I*c)*exp(2*I*d*x) - 15*d)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.76

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$

$$-\frac{480\,(dx+c)(A-i\,B)a^4 + 240\,(i\,A+B)a^4\log\left(\tan\left(dx+c\right)^2+1\right) + 480\,(-i\,A-B)a^4\log\left(\tan\left(dx+\right.\right.}{60\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/60*(480*(d*x + c)*(A - I*B)*a^4 + 240*(I*A + B)*a^4*log(tan(d*x + c)^2
+ 1) + 480*(-I*A - B)*a^4*log(tan(d*x + c)) + (480*(A - I*B)*a^4*tan(d*x +
 c)^4 - 30*(8*I*A + 7*B)*a^4*tan(d*x + c)^3 - 20*(7*A - 4*I*B)*a^4*tan(d*x
 + c)^2 - 15*(-4*I*A - B)*a^4*tan(d*x + c) + 12*A*a^4)/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$= -\frac{8\,(i\,Aa^4+Ba^4)\log\left(\tan\left(dx+c\right)+i\right)}{d} + \frac{8\,(i\,Aa^4+Ba^4)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$

$$-\frac{12\,Aa^4 + 480\,(Aa^4-i\,Ba^4)\tan\left(dx+c\right)^4 + 30\,(-8i\,Aa^4-7\,Ba^4)\tan\left(dx+c\right)^3 - 20\,(7\,Aa^4-4i\,Ba^4)}{60\,d\tan\left(dx+c\right)^5}$$

input
```
integrate(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-8*(I*A*a^4 + B*a^4)*log(tan(d*x + c) + I)/d + 8*(I*A*a^4 + B*a^4)*log(abs
(tan(d*x + c)))/d - 1/60*(12*A*a^4 + 480*(A*a^4 - I*B*a^4)*tan(d*x + c)^4
+ 30*(-8*I*A*a^4 - 7*B*a^4)*tan(d*x + c)^3 - 20*(7*A*a^4 - 4*I*B*a^4)*tan(
d*x + c)^2 + 15*(4*I*A*a^4 + B*a^4)*tan(d*x + c))/(d*tan(d*x + c)^5)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$
$$-\frac{\frac{A\,a^4}{5}-\tan(c+d\,x)^2\left(\frac{7\,A\,a^4}{3}-\frac{B\,a^4\,4i}{3}\right)+\tan(c+d\,x)^4\,(8\,A\,a^4-B\,a^4\,8i)-\tan(c+d\,x)^3\left(\frac{7\,B\,a^4}{2}+A\,a^4\right.}{d\tan(c+d\,x)^5}$$
$$+\frac{a^4\,\mathrm{atan}(2\tan(c+d\,x)+1i)\,(B+A\,1i)\,16i}{d}$$

input
```
int(cot(c + d*x)^6*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(a^4*atan(2*tan(c + d*x) + 1i)*(A*1i + B)*16i)/d - (tan(c + d*x)^4*(8*A*a^
4 - B*a^4*8i) - tan(c + d*x)^2*((7*A*a^4)/3 - (B*a^4*4i)/3) - tan(c + d*x)
^3*(A*a^4*4i + (7*B*a^4)/2) + (A*a^4)/5 + tan(c + d*x)*(A*a^4*1i + (B*a^4)
/4))/(d*tan(c + d*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.41

$$\int \cot^6(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{a^4\left(-5056\cos(dx+c)\sin(dx+c)^4a+4480\cos(dx+c)\sin(dx+c)^4bi+1312\cos(dx+c)\sin(dx+c\right.}{}$$

input
```
int(cot(d*x+c)^6*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(a**4*( - 5056*cos(c + d*x)*sin(c + d*x)**4*a + 4480*cos(c + d*x)*sin(c +
d*x)**4*b*i + 1312*cos(c + d*x)*sin(c + d*x)**2*a - 640*cos(c + d*x)*sin(c
 + d*x)**2*b*i - 96*cos(c + d*x)*a - 3840*log(tan((c + d*x)/2)**2 + 1)*sin
(c + d*x)**5*a*i - 3840*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**5*b + 3
840*log(tan((c + d*x)/2))*sin(c + d*x)**5*a*i + 3840*log(tan((c + d*x)/2))
*sin(c + d*x)**5*b - 3840*sin(c + d*x)**5*a*d*x - 1260*sin(c + d*x)**5*a*i
 + 3840*sin(c + d*x)**5*b*d*i*x - 915*sin(c + d*x)**5*b + 2880*sin(c + d*x
)**3*a*i + 1920*sin(c + d*x)**3*b - 480*sin(c + d*x)*a*i - 120*sin(c + d*x
)*b))/(480*sin(c + d*x)**5*d)
```

3.35 $\int \cot^7(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 223

$$\begin{aligned}
&\int \cot^7(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx \\
&= -8a^4(iA+B)x - \frac{8a^4(iA+B)\cot(c+dx)}{d} \\
&\quad - \frac{4a^4(A-iB)\cot^2(c+dx)}{d} + \frac{a^4(93iA+92B)\cot^3(c+dx)}{60d} \\
&\quad - \frac{8a^4(A-iB)\log(\sin(c+dx))}{d} - \frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d} \\
&\quad - \frac{(3iA+2B)\cot^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d} \\
&\quad + \frac{(13A-12iB)\cot^4(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{20d}
\end{aligned}$$

output
```
-8*a^4*(I*A+B)*x-8*a^4*(I*A+B)*cot(d*x+c)/d-4*a^4*(A-I*B)*cot(d*x+c)^2/d+1
/60*a^4*(93*I*A+92*B)*cot(d*x+c)^3/d-8*a^4*(A-I*B)*ln(sin(d*x+c))/d-1/6*a*
A*cot(d*x+c)^6*(a+I*a*tan(d*x+c))^3/d-1/10*(3*I*A+2*B)*cot(d*x+c)^5*(a^2+I
*a^2*tan(d*x+c))^2/d+1/20*(13*A-12*I*B)*cot(d*x+c)^4*(a^4+I*a^4*tan(d*x+c)
)/d
```

Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int \cot^7(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{a^4((13A-12iB)(i+\cot(c+dx))^4-4i(2A-3iB)\cot(c+dx)(i+\cot(c+dx))^4-10A\cot^2(c+dx)(i$$

input
```
Integrate[Cot[c + d*x]^7*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(a^4*((13*A - (12*I)*B)*(I + Cot[c + d*x])^4 - (4*I)*(2*A - (3*I)*B)*Cot[c
 + d*x]*(I + Cot[c + d*x])^4 - 10*A*Cot[c + d*x]^2*(I + Cot[c + d*x])^4 +
20*(I*A + B)*(-21*Cot[c + d*x] + (6*I)*Cot[c + d*x]^2 + Cot[c + d*x]^3 + (
24*I)*(Log[Tan[c + d*x]] - Log[I + Tan[c + d*x]]))))/(60*d)
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.618$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4076, 3042, 4074, 27, 3042, 4012, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^7(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^7}dx$$

$$\Big\downarrow \text{ 4076}$$

$$\frac{1}{6}\int 3\cot^6(c+dx)(i\tan(c+dx)a+a)^3(a(3iA+2B)-a(A-2iB)\tan(c+dx))dx-\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \cot^6(c+dx)(i\tan(c+dx)a+a)^3(a(3iA+2B)-a(A-2iB)\tan(c+dx))dx-\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)^3(a(3iA+2B)-a(A-2iB)\tan(c+dx))}{\tan(c+dx)^6}dx-\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{2}\left(\frac{1}{5}\int -2\cot^5(c+dx)(i\tan(c+dx)a+a)^2\left((13A-12iB)a^2+(7iA+8B)\tan(c+dx)a^2\right)dx-\frac{(2B+3iA)\,\mathrm{c}}{}\right.$$
$$-\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{2}{5}\int \cot^5(c+dx)(i\tan(c+dx)a+a)^2\left((13A-12iB)a^2+(7iA+8B)\tan(c+dx)a^2\right)dx-\frac{(2B+3iA)\cot}{}\right.$$
$$-\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2}{5}\int \frac{(i\tan(c+dx)a+a)^2\left((13A-12iB)a^2+(7iA+8B)\tan(c+dx)a^2\right)}{\tan(c+dx)^5}dx-\frac{(2B+3iA)\cot^5(c+dx)\,(a}{5d}\right.$$
$$-\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\int \cot^4(c+dx)(i\tan(c+dx)a+a)\left(a^3(93iA+92B)-a^3(67A-68iB)\tan(c+dx)\right)dx-\frac{(13A-12}{}\right.\right.$$
$$-\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\int\frac{(i\tan(c+dx)a+a)\left(a^3(93iA+92B)-a^3(67A-68iB)\tan(c+dx)\right)}{\tan(c+dx)^4}dx-\frac{(13A-12iB)\cot^4(c+\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4074}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(\int-160\cot^3(c+dx)\left((A-iB)a^4+(iA+B)\tan(c+dx)a^4\right)dx-\frac{a^4(92B+93iA)\cot^3(c+dx)}{3d}\right)\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-160\int\cot^3(c+dx)\left((A-iB)a^4+(iA+B)\tan(c+dx)a^4\right)dx-\frac{a^4(92B+93iA)\cot^3(c+dx)}{3d}\right)\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-160\int\frac{(A-iB)a^4+(iA+B)\tan(c+dx)a^4}{\tan(c+dx)^3}dx-\frac{a^4(92B+93iA)\cot^3(c+dx)}{3d}\right)-\frac{(13A-12iB\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4012}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-160\left(\int\cot^2(c+dx)\left(a^4(iA+B)-a^4(A-iB)\tan(c+dx)\right)dx-\frac{a^4(A-iB)\cot^2(c+dx)}{2d}\right)-\frac{a^4}{}\right.\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-160\left(\int\frac{a^4(iA+B)-a^4(A-iB)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{a^4(A-iB)\cot^2(c+dx)}{2d}\right)-\frac{a^4(92B+93iA)}{3d}\right.\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4012}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-160\left(\int -\cot(c+dx)\left((A-iB)a^4+(iA+B)\tan(c+dx)a^4\right)dx-\frac{a^4(A-iB)\cot^2(c+dx)}{2d}-\frac{a^4\right.\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-160\left(-\int \cot(c+dx)\left((A-iB)a^4+(iA+B)\tan(c+dx)a^4\right)dx-\frac{a^4(A-iB)\cot^2(c+dx)}{2d}-\frac{a^4\right.\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-160\left(-\int \frac{(A-iB)a^4+(iA+B)\tan(c+dx)a^4}{\tan(c+dx)}dx-\frac{a^4(A-iB)\cot^2(c+dx)}{2d}-\frac{a^4(B+iA)\cot(}{d}\right.\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{4014}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-160\left(-a^4(A-iB)\int \cot(c+dx)dx-\frac{a^4(A-iB)\cot^2(c+dx)}{2d}-\frac{a^4(B+iA)\cot(c+dx)}{d}-a^4x(\right.\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-160\left(-a^4(A-iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a^4(A-iB)\cot^2(c+dx)}{2d}-\frac{a^4(B+iA)\cot(c+dx}{d}\right.\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-160\left(a^4(A-iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a^4(A-iB)\cot^2(c+dx)}{2d}-\frac{a^4(B+iA)\cot(c+dx}{d}\right.\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$
$$\Big\downarrow \text{3956}$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(\frac{1}{4}\left(-\frac{a^4(92B+93iA)\cot^3(c+dx)}{3d}-160\left(-\frac{a^4(A-iB)\cot^2(c+dx)}{2d}-\frac{a^4(B+iA)\cot(c+dx)}{d}-\frac{a^4(\right.\right.\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

input
```
Int[Cot[c + d*x]^7*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-1/6*(a*A*Cot[c + d*x]^6*(a + I*a*Tan[c + d*x])^3)/d + (-1/5*(((3*I)*A + 2
*B)*Cot[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x])^2)/d - (2*((-1/3*(a^4*((93*I
)*A + 92*B)*Cot[c + d*x]^3)/d - 160*(-(a^4*(I*A + B)*x) - (a^4*(I*A + B)*C
ot[c + d*x])/d - (a^4*(A - I*B)*Cot[c + d*x]^2)/(2*d) - (a^4*(A - I*B)*Log
[-Sin[c + d*x]])/d))/4 - ((13*A - (12*I)*B)*Cot[c + d*x]^4*(a^4 + I*a^4*Ta
n[c + d*x]))/(4*d)))/5)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{8\left(\frac{(-iB+A)\ln\left(\sec(dx+c)^2\right)}{2}+(iB-A)\ln(\tan(dx+c))-\frac{A\cot(dx+c)^6}{48}-\frac{\cot(dx+c)^5\left(iA+\frac{B}{4}\right)}{10}+\frac{\left(-iB+\frac{7A}{4}\right)\cot(dx+c)^4}{8}+\frac{(iA}{}\right.}{d}$
derivativedivides	$\frac{a^4\left(\frac{(-8iB+8A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-8iA-8B)\arctan(\tan(dx+c))-\frac{A}{6\tan(dx+c)^6}+(8iB-8A)\ln(\tan(dx+c))-\frac{-8iB}{2\tan(da}\right.}{d}$
default	$\frac{a^4\left(\frac{(-8iB+8A)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-8iA-8B)\arctan(\tan(dx+c))-\frac{A}{6\tan(dx+c)^6}+(8iB-8A)\ln(\tan(dx+c))-\frac{-8iB}{2\tan(da}\right.}{d}$
risch	$\frac{16a^4Bc}{d}+\frac{16ia^4Ac}{d}-\frac{4ia^4\left(270iA\,e^{10i(dx+c)}+210B\,e^{10i(dx+c)}-855iA\,e^{8i(dx+c)}-765B\,e^{8i(dx+c)}+1350iA\,e^{6i(dx+}\right.}{15}$
norman	$\frac{(-8iA\,a^4-8B\,a^4)x\tan(dx+c)^6-\frac{A\,a^4}{6d}+\frac{\left(-4iB\,a^4+7A\,a^4\right)\tan(dx+c)^2}{4d}-\frac{\left(4iA\,a^4+B\,a^4\right)\tan(dx+c)}{5d}-\frac{4\left(-iB\,a^4+A\,a^4\right)\tan(}{d}}{\tan(dx+c)^6}$

input
```
int(cot(d*x+c)^7*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
8*(1/2*(A-I*B)*ln(sec(d*x+c)^2)+(-A+I*B)*ln(tan(d*x+c))-1/48*A*cot(d*x+c)^
6-1/10*cot(d*x+c)^5*(I*A+1/4*B)+1/8*(-I*B+7/4*A)*cot(d*x+c)^4+1/3*(I*A+7/8
*B)*cot(d*x+c)^3+1/2*cot(d*x+c)^2*(-A+I*B)-cot(d*x+c)*(I*A+B)-(I*A+B)*x*d)
*a^4/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.49

$$\int\cot^7(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{4\left(30\,(9\,A-7i\,B)a^4e^{(10i\,dx+10i\,c)}-45\,(19\,A-17i\,B)a^4e^{(8i\,dx+8i\,c)}+10\,(135\,A-121i\,B)a^4e^{(6i\,dx+6i\,c)}-1\right.}{}$$

input
```
integrate(cot(d*x+c)^7*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
4/15*(30*(9*A - 7*I*B)*a^4*e^(10*I*d*x + 10*I*c) - 45*(19*A - 17*I*B)*a^4*
e^(8*I*d*x + 8*I*c) + 10*(135*A - 121*I*B)*a^4*e^(6*I*d*x + 6*I*c) - 15*(7
5*A - 68*I*B)*a^4*e^(4*I*d*x + 4*I*c) + 6*(81*A - 74*I*B)*a^4*e^(2*I*d*x +
 2*I*c) - (86*A - 79*I*B)*a^4 - 30*((A - I*B)*a^4*e^(12*I*d*x + 12*I*c) -
6*(A - I*B)*a^4*e^(10*I*d*x + 10*I*c) + 15*(A - I*B)*a^4*e^(8*I*d*x + 8*I*
c) - 20*(A - I*B)*a^4*e^(6*I*d*x + 6*I*c) + 15*(A - I*B)*a^4*e^(4*I*d*x +
4*I*c) - 6*(A - I*B)*a^4*e^(2*I*d*x + 2*I*c) + (A - I*B)*a^4)*log(e^(2*I*d
*x + 2*I*c) - 1))/(d*e^(12*I*d*x + 12*I*c) - 6*d*e^(10*I*d*x + 10*I*c) + 1
5*d*e^(8*I*d*x + 8*I*c) - 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I
*c) - 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.56

$$\int \cot^7(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{8a^4(A-iB)\log\left(e^{2idx}-e^{-2ic}\right)}{d} + \frac{-344Aa^4+316iBa^4+\left(1944Aa^4e^{2ic}-1776iBa^4e^{2ic}\right)e^{2idx}+\left(-4500Aa^4e^{4ic}+4080iBa^4e^{4ic}\right)e^{4idx}+(\cdots}{15de^{12ic}e^{12idx}-90de^{10ic}e^{10idx}+225de^{8ic}e^{8i\cdots}}$$

input
```
integrate(cot(d*x+c)**7*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
-8*a**4*(A - I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/d + (-344*A*a**4 + 316*I
*B*a**4 + (1944*A*a**4*exp(2*I*c) - 1776*I*B*a**4*exp(2*I*c))*exp(2*I*d*x)
 + (-4500*A*a**4*exp(4*I*c) + 4080*I*B*a**4*exp(4*I*c))*exp(4*I*d*x) + (54
00*A*a**4*exp(6*I*c) - 4840*I*B*a**4*exp(6*I*c))*exp(6*I*d*x) + (-3420*A*a
**4*exp(8*I*c) + 3060*I*B*a**4*exp(8*I*c))*exp(8*I*d*x) + (1080*A*a**4*exp
(10*I*c) - 840*I*B*a**4*exp(10*I*c))*exp(10*I*d*x))/(15*d*exp(12*I*c)*exp(
12*I*d*x) - 90*d*exp(10*I*c)*exp(10*I*d*x) + 225*d*exp(8*I*c)*exp(8*I*d*x)
 - 300*d*exp(6*I*c)*exp(6*I*d*x) + 225*d*exp(4*I*c)*exp(4*I*d*x) - 90*d*ex
p(2*I*c)*exp(2*I*d*x) + 15*d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.77

$$\int \cot^7(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$

$$-\frac{480\,(dx+c)(i\,A+B)a^4-240\,(A-i\,B)a^4\log\left(\tan\left(dx+c\right)^2+1\right)+480\,(A-i\,B)a^4\log\left(\tan\left(dx+c\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^7*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/60*(480*(d*x + c)*(I*A + B)*a^4 - 240*(A - I*B)*a^4*log(tan(d*x + c)^2
+ 1) + 480*(A - I*B)*a^4*log(tan(d*x + c)) - (480*(-I*A - B)*a^4*tan(d*x +
 c)^5 - 240*(A - I*B)*a^4*tan(d*x + c)^4 + 20*(8*I*A + 7*B)*a^4*tan(d*x +
c)^3 + 15*(7*A - 4*I*B)*a^4*tan(d*x + c)^2 + 12*(-4*I*A - B)*a^4*tan(d*x +
 c) - 10*A*a^4)/tan(d*x + c)^6)/d
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.82

$$\int \cot^7(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$=\frac{8\,(Aa^4-i\,Ba^4)\log\left(\tan\left(dx+c\right)+i\right)}{d}-\frac{8\,(Aa^4-i\,Ba^4)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$

$$-\frac{480i\,(Aa^4-i\,Ba^4)\tan\left(dx+c\right)^5+10\,Aa^4-240i\,(i\,Aa^4+Ba^4)\tan\left(dx+c\right)^4-20i\,(8\,Aa^4-7i\,Ba^4)\,\mathrm{t}}{60\,d\tan\left(dx+c\right.}$$

input
```
integrate(cot(d*x+c)^7*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
8*(A*a^4 - I*B*a^4)*log(tan(d*x + c) + I)/d - 8*(A*a^4 - I*B*a^4)*log(abs(
tan(d*x + c)))/d - 1/60*(480*I*(A*a^4 - I*B*a^4)*tan(d*x + c)^5 + 10*A*a^4
 - 240*I*(I*A*a^4 + B*a^4)*tan(d*x + c)^4 - 20*I*(8*A*a^4 - 7*I*B*a^4)*tan
(d*x + c)^3 - 15*I*(-7*I*A*a^4 - 4*B*a^4)*tan(d*x + c)^2 + 12*I*(4*A*a^4 -
 I*B*a^4)*tan(d*x + c))/(d*tan(d*x + c)^6)
```

Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.73

$$\int \cot^7(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$
$$-\frac{\tan(c+d\,x)^4\,(4\,A\,a^4-B\,a^4\,4\mathrm{i})-\tan(c+d\,x)^2\,\left(\frac{7\,A\,a^4}{4}-B\,a^4\,1\mathrm{i}\right)+\tan(c+d\,x)^5\,(8\,B\,a^4+A\,a^4\,8\mathrm{i})-}{d\tan\left(c+d\,x\right)^6}$$
$$-\frac{16\,a^4\,\mathrm{atan}(2\tan(c+d\,x)+1\mathrm{i})\,\left(B+A\,1\mathrm{i}\right)}{d}$$

input
```
int(cot(c + d*x)^7*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
- (tan(c + d*x)^4*(4*A*a^4 - B*a^4*4i) - tan(c + d*x)^2*((7*A*a^4)/4 - B*a
^4*1i) + tan(c + d*x)^5*(A*a^4*8i + 8*B*a^4) - tan(c + d*x)^3*((A*a^4*8i)/
3 + (7*B*a^4)/3) + (A*a^4)/6 + tan(c + d*x)*((A*a^4*4i)/5 + (B*a^4)/5))/(d
*tan(c + d*x)^6) - (16*a^4*atan(2*tan(c + d*x) + 1i)*(A*1i + B))/d
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.39

$$\int \cot^7(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{a^4\left(-1376\cos\left(dx+c\right)\sin\left(dx+c\right)^5ai-1264\cos\left(dx+c\right)\sin\left(dx+c\right)^5b+512\cos\left(dx+c\right)\sin\left(dx+c\right)^3\right.}{}$$

input
```
int(cot(d*x+c)^7*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(a**4*( - 1376*cos(c + d*x)*sin(c + d*x)**5*a*i - 1264*cos(c + d*x)*sin(c
+ d*x)**5*b + 512*cos(c + d*x)*sin(c + d*x)**3*a*i + 328*cos(c + d*x)*sin(
c + d*x)**3*b - 96*cos(c + d*x)*sin(c + d*x)*a*i - 24*cos(c + d*x)*sin(c +
 d*x)*b + 960*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**6*a - 960*log(tan
((c + d*x)/2)**2 + 1)*sin(c + d*x)**6*b*i - 960*log(tan((c + d*x)/2))*sin(
c + d*x)**6*a + 960*log(tan((c + d*x)/2))*sin(c + d*x)**6*b*i - 960*sin(c
+ d*x)**6*a*d*i*x + 385*sin(c + d*x)**6*a - 960*sin(c + d*x)**6*b*d*x - 31
5*sin(c + d*x)**6*b*i - 960*sin(c + d*x)**4*a + 720*sin(c + d*x)**4*b*i +
270*sin(c + d*x)**2*a - 120*sin(c + d*x)**2*b*i - 20*a))/(120*sin(c + d*x)
**6*d)
```

3.36 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 129

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{3(iA-B)x}{2a} - \frac{(A+2iB)\log(\cos(c+dx))}{ad} - \frac{3(iA-B)\tan(c+dx)}{2ad} - \frac{(A+2iB)\tan^2(c+dx)}{2ad} + \frac{(iA-B)\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

output
```
3/2*(I*A-B)*x/a-(A+2*I*B)*ln(cos(d*x+c))/a/d-3/2*(I*A-B)*tan(d*x+c)/a/d-1/
2*(A+2*I*B)*tan(d*x+c)^2/a/d+1/2*(I*A-B)*tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.06

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$=\frac{\frac{2(2A+iB)\tan^2(c+dx)}{a+ia\tan(c+dx)}+\frac{2B\tan^3(c+dx)}{a+ia\tan(c+dx)}+\frac{(5A+7iB)\log(i-\tan(c+dx))-(A-iB)\log(i+\tan(c+dx))+\frac{6(-iA+B)}{-i+\tan(c+dx)}}{a}}{4d}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
((2*(2*A + I*B)*Tan[c + d*x]^2)/(a + I*a*Tan[c + d*x]) + (2*B*Tan[c + d*x]
^3)/(a + I*a*Tan[c + d*x]) + ((5*A + (7*I)*B)*Log[I - Tan[c + d*x]] - (A -
 I*B)*Log[I + Tan[c + d*x]] + (6*((-I)*A + B))/(-I + Tan[c + d*x]))/a)/(4*
d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 4078, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \tan^2(c+dx)(3a(iA-B)+2a(A+2iB)\tan(c+dx))dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \tan(c+dx)^2(3a(iA-B)+2a(A+2iB)\tan(c+dx))dx}{2a^2}$$

$$\Big\downarrow \text{4011}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \tan(c+dx)(3a(iA-B)\tan(c+dx)-2a(A+2iB))dx+\frac{a(A+2iB)\tan^2(c+dx)}{d}}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \tan(c+dx)(3a(iA-B)\tan(c+dx)-2a(A+2iB))dx+\frac{a(A+2iB)\tan^2(c+dx)}{d}}{2a^2}$$

$$\Big\downarrow \text{4008}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-2a(A+2iB)\int \tan(c+dx)dx+\frac{a(A+2iB)\tan^2(c+dx)}{d}+\frac{3a(-B+iA)\tan(c+dx)}{d}-3ax(-B+iA)}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-2a(A+2iB)\int \tan(c+dx)dx+\frac{a(A+2iB)\tan^2(c+dx)}{d}+\frac{3a(-B+iA)\tan(c+dx)}{d}-3ax(-B+iA)}{2a^2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\frac{a(A+2iB)\tan^2(c+dx)}{d}+\frac{3a(-B+iA)\tan(c+dx)}{d}+\frac{2a(A+2iB)\log(\cos(c+dx))}{d}-3ax(-B+iA)}{2a^2}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
((I*A - B)*Tan[c + d*x]^3)/(2*d*(a + I*a*Tan[c + d*x])) - (-3*a*(I*A - B)*
x + (2*a*(A + (2*I)*B)*Log[Cos[c + d*x]])/d + (3*a*(I*A - B)*Tan[c + d*x])
/d + (a*(A + (2*I)*B)*Tan[c + d*x]^2)/d)/(2*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.13

method	result
norman	$\frac{\frac{(-iA+B)\tan(dx+c)^{3}}{ad}-\frac{3(-iA+B)x}{2a}+\frac{2iB+A}{2ad}+\frac{3(-iA+B)\tan(dx+c)}{2ad}-\frac{3(-iA+B)x\tan(dx+c)^{2}}{2a}-\frac{iB\tan(dx+c)^{4}}{2ad}}{1+\tan(dx+c)^{2}}+\frac{(2iB+A}{}$
risch	$-\frac{7xB}{2a}+\frac{5ixA}{2a}+\frac{ie^{-2i(dx+c)}B}{4ad}+\frac{e^{-2i(dx+c)}A}{4ad}-\frac{4Bc}{ad}+\frac{2iAc}{ad}+\frac{2i\left(-iA\,e^{2i(dx+c)}-iA+B\right)}{da\left(e^{2i(dx+c)}+1\right)^{2}}-\frac{2i\ln(e^{2i(dx}{a}$
derivativedivides	$\frac{B\tan(dx+c)}{da}-\frac{iB\tan(dx+c)^{2}}{2da}-\frac{iA\tan(dx+c)}{da}+\frac{A\ln\left(1+\tan(dx+c)^{2}\right)}{2da}+\frac{3iA\arctan(\tan(dx+c))}{2da}+\frac{iB\ln\left(1\right.}{}$
default	$\frac{B\tan(dx+c)}{da}-\frac{iB\tan(dx+c)^{2}}{2da}-\frac{iA\tan(dx+c)}{da}+\frac{A\ln\left(1+\tan(dx+c)^{2}\right)}{2da}+\frac{3iA\arctan(\tan(dx+c))}{2da}+\frac{iB\ln\left(1\right.}{}$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
(1/a/d*(B-I*A)*tan(d*x+c)^3-3/2/a*(B-I*A)*x+1/2*(A+2*I*B)/a/d+3/2/a/d*(B-I
*A)*tan(d*x+c)-3/2/a*(B-I*A)*x*tan(d*x+c)^2-1/2*I*B/a/d*tan(d*x+c)^4)/(1+t
an(d*x+c)^2)+1/2*(A+2*I*B)/a/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.44

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{2\,(-5i\,A+7\,B)dxe^{(6i\,dx+6i\,c)}+(4\,(-5i\,A+7\,B)dx-9\,A-i\,B)e^{(4i\,dx+4i\,c)}+2\,((-5i\,A+7\,B)dx-5\,A\cdots}{4\,(ade^{(6i\,dx+\cdots}}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/4*(2*(-5*I*A + 7*B)*d*x*e^(6*I*d*x + 6*I*c) + (4*(-5*I*A + 7*B)*d*x - 9
*A - I*B)*e^(4*I*d*x + 4*I*c) + 2*((-5*I*A + 7*B)*d*x - 5*A - 5*I*B)*e^(2*
I*d*x + 2*I*c) + 4*((A + 2*I*B)*e^(6*I*d*x + 6*I*c) + 2*(A + 2*I*B)*e^(4*I
*d*x + 4*I*c) + (A + 2*I*B)*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) +
 1) - A - I*B)/(a*d*e^(6*I*d*x + 6*I*c) + 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*
e^(2*I*d*x + 2*I*c))
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.52

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{2Ae^{2ic}e^{2idx}+2A+2iB}{ade^{4ic}e^{4idx}+2ade^{2ic}e^{2idx}+ad} + \begin{cases} \frac{(A+iB)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic}\neq 0 \\ x\left(-\frac{5iA-7B}{2a}+\frac{(5iAe^{2ic}-iA-7Be^{2ic}+B)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(5iA-7B)}{2a} - \frac{(A+2iB)\log\left(e^{2idx}+e^{-2ic}\right)}{ad}$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(2*A*exp(2*I*c)*exp(2*I*d*x) + 2*A + 2*I*B)/(a*d*exp(4*I*c)*exp(4*I*d*x) +
 2*a*d*exp(2*I*c)*exp(2*I*d*x) + a*d) + Piecewise(((A + I*B)*exp(-2*I*c)*e
xp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*(-(5*I*A - 7*B)/(2*a) + (
5*I*A*exp(2*I*c) - I*A - 7*B*exp(2*I*c) + B)*exp(-2*I*c)/(2*a)), True)) +
x*(5*I*A - 7*B)/(2*a) - (A + 2*I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/(a*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx &= -\frac{(A-iB)\log(\tan(dx+c)+i)}{4\,ad} \\ &\quad+\frac{(5A+7iB)\log(\tan(dx+c)-i)}{4\,ad}-\frac{iA-B}{2\,ad(\tan(dx+c)-i)} \\ &\quad-\frac{iBad\tan(dx+c)^2+2iAad\tan(dx+c)-2Bad\tan(dx+c)}{2\,a^2d^2}\end{aligned}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-1/4*(A - I*B)*log(tan(d*x + c) + I)/(a*d) + 1/4*(5*A + 7*I*B)*log(tan(d*x
 + c) - I)/(a*d) - 1/2*(I*A - B)/(a*d*(tan(d*x + c) - I)) - 1/2*(I*B*a*d*t
an(d*x + c)^2 + 2*I*A*a*d*tan(d*x + c) - 2*B*a*d*tan(d*x + c))/(a^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.09

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\frac{A}{2\,a}-\frac{A+B\,1\mathrm{i}}{2\,a}+\frac{A+B\,2\mathrm{i}}{2\,a}}{d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)} - \frac{\tan(c+d\,x)\,\left(-\frac{B}{a}+\frac{A\,1\mathrm{i}}{a}\right)}{d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,a\,d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(5\,A+B\,7\mathrm{i}\right)}{4\,a\,d} - \frac{B\tan(c+d\,x)^2\,1\mathrm{i}}{2\,a\,d}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
(A/(2*a) - (A + B*1i)/(2*a) + (A + B*2i)/(2*a))/(d*(tan(c + d*x)*1i + 1))
- (tan(c + d*x)*((A*1i)/a - B/a))/d + (log(tan(c + d*x) + 1i)*(A*1i + B)*1
i)/(4*a*d) + (log(tan(c + d*x) - 1i)*(5*A + B*7i))/(4*a*d) - (B*tan(c + d*
x)^2*1i)/(2*a*d)
```

Reduce [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{-2\left(\int \frac{1}{\tan(dx+c)-i}dx\right)ad-2\left(\int \frac{1}{\tan(dx+c)-i}dx\right)bdi+\log\left(\tan\left(dx+c\right)^2+1\right)a+2\log\left(\tan\left(dx+c\right)^2+1\right)b}{2ad}$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
( - 2*int(1/(tan(c + d*x) - i),x)*a*d - 2*int(1/(tan(c + d*x) - i),x)*b*d*
i + log(tan(c + d*x)**2 + 1)*a + 2*log(tan(c + d*x)**2 + 1)*b*i - tan(c +
d*x)**2*b*i - 2*tan(c + d*x)*a*i + 2*tan(c + d*x)*b + 4*a*d*i*x - 4*b*d*x)
/(2*a*d)
```

3.37 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 101

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{(A+3iB)x}{2a}+\frac{(iA-B)\log(\cos(c+dx))}{ad} - \frac{(A+3iB)\tan(c+dx)}{2ad} + \frac{(iA-B)\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

output
```
1/2*(A+3*I*B)*x/a+(I*A-B)*ln(cos(d*x+c))/a/d-1/2*(A+3*I*B)*tan(d*x+c)/a/d+
1/2*(I*A-B)*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\frac{4B\tan^2(c+dx)}{a+ia\tan(c+dx)}+\frac{(-3iA+5B)\log(i-\tan(c+dx))-(iA+B)\log(i+\tan(c+dx))-\frac{2(A+3iB)}{-i+\tan(c+dx)}}{a}}{4d}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
((4*B*Tan[c + d*x]^2)/(a + I*a*Tan[c + d*x]) + (((-3*I)*A + 5*B)*Log[I - T
an[c + d*x]] - (I*A + B)*Log[I + Tan[c + d*x]] - (2*(A + (3*I)*B))/(-I + T
an[c + d*x]))/a)/(4*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4078, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 4078}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \tan(c+dx)(2a(iA-B)+a(A+3iB)\tan(c+dx))dx}{2a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \tan(c+dx)(2a(iA-B)+a(A+3iB)\tan(c+dx))dx}{2a^2}$$

$$\Big\downarrow \text{ 4008}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{2a(-B+iA)\int \tan(c+dx)dx+\frac{a(A+3iB)\tan(c+dx)}{d}-ax(A+3iB)}{2a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{2a(-B+iA)\int \tan(c+dx)dx+\frac{a(A+3iB)\tan(c+dx)}{d}-ax(A+3iB)}{2a^2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\frac{a(A+3iB)\tan(c+dx)}{d}-\frac{2a(-B+iA)\log(\cos(c+dx))}{d}-ax(A+3iB)}{2a^2}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
((I*A - B)*Tan[c + d*x]^2)/(2*d*(a + I*a*Tan[c + d*x])) - (-(a*(A + (3*I)*
B)*x) - (2*a*(I*A - B)*Log[Cos[c + d*x]])/d + (a*(A + (3*I)*B)*Tan[c + d*x
])/d)/(2*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

method	result
norman	$\frac{\frac{(3iB+A)x}{2a}+\frac{-iA+B}{2ad}-\frac{(3iB+A)\tan(dx+c)}{2ad}+\frac{(3iB+A)x\tan(dx+c)^2}{2a}-\frac{iB\tan(dx+c)^3}{ad}}{1+\tan(dx+c)^2}+\frac{(-iA+B)\ln\left(1+\tan(dx+c)^2\right)}{2ad}$
derivativedivides	$-\frac{iB\tan(dx+c)}{da}-\frac{A}{2da(-i+\tan(dx+c))}-\frac{iB}{2da(-i+\tan(dx+c))}-\frac{iA\ln\left(1+\tan(dx+c)^2\right)}{2da}+\frac{A\arctan(\tan(dx+}{2da}$
default	$-\frac{iB\tan(dx+c)}{da}-\frac{A}{2da(-i+\tan(dx+c))}-\frac{iB}{2da(-i+\tan(dx+c))}-\frac{iA\ln\left(1+\tan(dx+c)^2\right)}{2da}+\frac{A\arctan(\tan(dx+}{2da}$
risch	$\frac{5ixB}{2a}+\frac{3xA}{2a}+\frac{e^{-2i(dx+c)}B}{4ad}-\frac{ie^{-2i(dx+c)}A}{4ad}+\frac{2iBc}{ad}+\frac{2Ac}{ad}+\frac{2B}{da\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{2i(dx+c)}+1\right)B}{ad}+$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
(1/2*(A+3*I*B)*x/a+1/2/a/d*(B-I*A)-1/2*(A+3*I*B)*tan(d*x+c)/a/d+1/2*(A+3*I
*B)/a*x*tan(d*x+c)^2-I*B/a/d*tan(d*x+c)^3)/(1+tan(d*x+c)^2)+1/2/a/d*(B-I*A
)*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.26

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$=\frac{2\,(3\,A+5i\,B)dxe^{(4i\,dx+4i\,c)}+(2\,(3\,A+5i\,B)dx-i\,A+9\,B)e^{(2i\,dx+2i\,c)}-4\,\left((-i\,A+B)e^{(4i\,dx+4i\,c)}+(-i\right.}{4\,\left(ade^{(4i\,dx+4i\,c)}+ade^{(2i\,dx+2i\,c)}\right)}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/4*(2*(3*A + 5*I*B)*d*x*e^(4*I*d*x + 4*I*c) + (2*(3*A + 5*I*B)*d*x - I*A
+ 9*B)*e^(2*I*d*x + 2*I*c) - 4*((-I*A + B)*e^(4*I*d*x + 4*I*c) + (-I*A + B
)*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - I*A + B)/(a*d*e^(4*I
*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{2B}{ade^{2ic}e^{2idx}+ad} + \begin{cases} \frac{(-iA+B)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic}\neq 0 \\ x\left(-\frac{3A+5iB}{2a}+\frac{(3Ae^{2ic}-A+5iBe^{2ic}-iB)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases}$$
$$+ \frac{x(3A+5iB)}{2a} + \frac{i(A+iB)\log\left(e^{2idx}+e^{-2ic}\right)}{ad}$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
2*B/(a*d*exp(2*I*c)*exp(2*I*d*x) + a*d) + Piecewise(((-I*A + B)*exp(-2*I*c
)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*(-(3*A + 5*I*B)/(2*a)
+ (3*A*exp(2*I*c) - A + 5*I*B*exp(2*I*c) - I*B)*exp(-2*I*c)/(2*a)), True))
 + x*(3*A + 5*I*B)/(2*a) + I*(A + I*B)*log(exp(2*I*d*x) + exp(-2*I*c))/(a*
d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{(-iA-B)\log(\tan(dx+c)+i)}{4\,ad} + \frac{(-3iA+5B)\log(\tan(dx+c)-i)}{4\,ad} - \frac{iB\tan(dx+c)}{ad} - \frac{A+iB}{2\,ad(\tan(dx+c)-i)}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
1/4*(-I*A - B)*log(tan(d*x + c) + I)/(a*d) + 1/4*(-3*I*A + 5*B)*log(tan(d*
x + c) - I)/(a*d) - I*B*tan(d*x + c)/(a*d) - 1/2*(A + I*B)/(a*d*(tan(d*x +
 c) - I))
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(B+A\,1\mathrm{i})}{4\,a\,d} - \frac{B\tan(c+d\,x)\,1\mathrm{i}}{a\,d} - \frac{(A+B\,1\mathrm{i})\,1\mathrm{i}}{2\,a\,d\,(1+\tan(c+d\,x)\,1\mathrm{i})} - \frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(-5\,B+A\,3\mathrm{i})}{4\,a\,d}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
- (log(tan(c + d*x) + 1i)*(A*1i + B))/(4*a*d) - (B*tan(c + d*x)*1i)/(a*d)
- ((A + B*1i)*1i)/(2*a*d*(tan(c + d*x)*1i + 1)) - (log(tan(c + d*x) - 1i)*
(A*3i - 5*B))/(4*a*d)
```

Reduce [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{2\left(\int \frac{1}{\tan(dx+c)-i}dx\right)adi - 2\left(\int \frac{1}{\tan(dx+c)-i}dx\right)bd - \log\left(\tan\left(dx+c\right)^2+1\right)ai + \log\left(\tan\left(dx+c\right)^2+1\right)b -}{2ad}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(2*int(1/(tan(c + d*x) - i),x)*a*d*i - 2*int(1/(tan(c + d*x) - i),x)*b*d -
 log(tan(c + d*x)**2 + 1)*a*i + log(tan(c + d*x)**2 + 1)*b - 2*tan(c + d*x
)*b*i + 2*a*d*x + 4*b*d*i*x)/(2*a*d)
```

3.38 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 67

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{(iA-B)x}{2a}+\frac{iB\log(\cos(c+dx))}{ad} - \frac{A+iB}{2ad(1+i\tan(c+dx))}$$

output
```
-1/2*(I*A-B)*x/a+I*B*ln(cos(d*x+c))/a/d-1/2*(A+I*B)/a/d/(1+I*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{-((A+3iB)\log(i-\tan(c+dx)))+(A-iB)\log(i+\tan(c+dx))+\frac{2i(A+iB)}{-i+\tan(c+dx)}}{4ad}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
(-((A + (3*I)*B)*Log[I - Tan[c + d*x]]) + (A - I*B)*Log[I + Tan[c + d*x]]
+ ((2*I)*(A + I*B))/(-I + Tan[c + d*x]))/(4*a*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 4072, 27, 3042, 3956, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{4072}$$

$$-\frac{i\int \frac{(iA-B)\tan(c+dx)}{i\tan(c+dx)+1}dx}{a}-\frac{iB\int \tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{i(-B+iA)\int \frac{\tan(c+dx)}{i\tan(c+dx)+1}dx}{a}-\frac{iB\int \tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i(-B+iA)\int \frac{\tan(c+dx)}{i\tan(c+dx)+1}dx}{a}-\frac{iB\int \tan(c+dx)dx}{a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{iB\log(\cos(c+dx))}{ad}-\frac{i(-B+iA)\int \frac{\tan(c+dx)}{i\tan(c+dx)+1}dx}{a}$$

$$\Big\downarrow \text{4009}$$

$$\frac{iB\log(\cos(c+dx))}{ad}-\frac{i(-B+iA)\left(-\frac{i\int 1dx}{2}-\frac{1}{2d(1+i\tan(c+dx))}\right)}{a}$$

$$\Big\downarrow 24$$

$$\frac{iB\log(\cos(c+dx))}{ad}-\frac{i(-B+iA)\left(-\frac{1}{2d(1+i\tan(c+dx))}-\frac{ix}{2}\right)}{a}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
(I*B*Log[Cos[c + d*x]])/(a*d) - (I*(I*A - B)*((-1/2*I)*x - 1/(2*d*(1 + I*T
an[c + d*x]))))/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

method	result
risch	$\frac{3xB}{2a}-\frac{ixA}{2a}-\frac{i\mathrm{e}^{-2i(dx+c)}B}{4ad}-\frac{\mathrm{e}^{-2i(dx+c)}A}{4ad}+\frac{2Bc}{ad}+\frac{i\ln(\mathrm{e}^{2i(dx+c)}+1)B}{ad}$
derivativedivides	$\frac{iA}{2da(-i+\tan(dx+c))}-\frac{B}{2da(-i+\tan(dx+c))}-\frac{iA\arctan(\tan(dx+c))}{2da}-\frac{iB\ln\left(1+\tan(dx+c)^2\right)}{2da}+\frac{B\arctan(\tan}{2da}$
default	$\frac{iA}{2da(-i+\tan(dx+c))}-\frac{B}{2da(-i+\tan(dx+c))}-\frac{iA\arctan(\tan(dx+c))}{2da}-\frac{iB\ln\left(1+\tan(dx+c)^2\right)}{2da}+\frac{B\arctan(\tan}{2da}$
norman	$\frac{\frac{(-iA+B)x}{2a}-\frac{iB+A}{2ad}-\frac{(-iA+B)\tan(dx+c)}{2ad}+\frac{(-iA+B)x\tan(dx+c)^2}{2a}}{1+\tan(dx+c)^2}-\frac{iB\ln\left(1+\tan(dx+c)^2\right)}{2da}$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
3/2*x/a*B-1/2*I*x/a*A-1/4*I/a/d*exp(-2*I*(d*x+c))*B-1/4/a/d*exp(-2*I*(d*x+
c))*A+2/a/d*B*c+I*B/a/d*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{(2\,(i\,A-3\,B)dxe^{(2i\,dx+2i\,c)}-4i\,Be^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+A+i\,B)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/4*(2*(I*A - 3*B)*d*x*e^(2*I*d*x + 2*I*c) - 4*I*B*e^(2*I*d*x + 2*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) + A + I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{iB\log\left(e^{2idx}+e^{-2ic}\right)}{ad} + \begin{cases} \frac{(-A-iB)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic}\neq 0 \\ x\left(-\frac{-iA+3B}{2a}+\frac{(-iAe^{2ic}+iA+3Be^{2ic}-B)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(-iA+3B)}{2a}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
I*B*log(exp(2*I*d*x) + exp(-2*I*c))/(a*d) + Piecewise(((-A - I*B)*exp(-2*I
*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*(-(-I*A + 3*B)/(2*a)
 + (-I*A*exp(2*I*c) + I*A + 3*B*exp(2*I*c) - B)*exp(-2*I*c)/(2*a)), True))
 + x*(-I*A + 3*B)/(2*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{(A-iB)\log(\tan(dx+c)+i)}{4\,ad} - \frac{(A+3iB)\log(\tan(dx+c)-i)}{4\,ad} - \frac{-iA+B}{2\,ad(\tan(dx+c)-i)}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
1/4*(A - I*B)*log(tan(d*x + c) + I)/(a*d) - 1/4*(A + 3*I*B)*log(tan(d*x +
c) - I)/(a*d) - 1/2*(-I*A + B)/(a*d*(tan(d*x + c) - I))
```

Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{\frac{A}{2\,a}+\frac{B\,1\text{i}}{2\,a}}{d\,\left(1+\tan\left(c+d\,x\right)\,1\text{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\text{i}\right)\,\left(A-B\,1\text{i}\right)}{4\,a\,d} - \frac{\ln\left(\tan(c+d\,x)-\text{i}\right)\,\left(A+B\,3\text{i}\right)}{4\,a\,d}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
(log(tan(c + d*x) + 1i)*(A - B*1i))/(4*a*d) - (A/(2*a) + (B*1i)/(2*a))/(d*
(tan(c + d*x)*1i + 1)) - (log(tan(c + d*x) - 1i)*(A + B*3i))/(4*a*d)
```

Reduce [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\left(\int \frac{\tan(dx+c)^2}{\tan(dx+c)i+1}dx\right)b+\left(\int \frac{\tan(dx+c)}{\tan(dx+c)i+1}dx\right)a}{a}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(int(tan(c + d*x)**2/(tan(c + d*x)*i + 1),x)*b + int(tan(c + d*x)/(tan(c +
 d*x)*i + 1),x)*a)/a
```

3.39 $\int \frac{A+B\tan(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \frac{A+B\tan(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{(A-iB)x}{2a}+\frac{iA-B}{2d(a+ia\tan(c+dx))}$$

output
```
1/2*(A-I*B)*x/a+1/2*(I*A-B)/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \frac{A+B\tan(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{(A-iB)\arctan(\tan(c+dx))}{2ad}-\frac{A+iB}{2ad(i-\tan(c+dx))}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x]),x]
```

output
```
((A - I*B)*ArcTan[Tan[c + d*x]])/(2*a*d) - (A + I*B)/(2*a*d*(I - Tan[c + d
*x]))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{4009}$$

$$\frac{(A-iB)\int 1dx}{2a}+\frac{-B+iA}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{24}$$

$$\frac{-B+iA}{2d(a+ia\tan(c+dx))}+\frac{x(A-iB)}{2a}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x]),x]
```

output
```
((A - I*B)*x)/(2*a) + (I*A - B)/(2*d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

method	result	size
risch	$-\frac{ixB}{2a}+\frac{xA}{2a}-\frac{\mathrm{e}^{-2i(dx+c)}B}{4ad}+\frac{i\mathrm{e}^{-2i(dx+c)}A}{4ad}$	54
derivativedivides	$\frac{A\arctan(\tan(dx+c))}{2da}-\frac{iB\arctan(\tan(dx+c))}{2da}+\frac{A}{2da(-i+\tan(dx+c))}+\frac{iB}{2da(-i+\tan(dx+c))}$	76
default	$\frac{A\arctan(\tan(dx+c))}{2da}-\frac{iB\arctan(\tan(dx+c))}{2da}+\frac{A}{2da(-i+\tan(dx+c))}+\frac{iB}{2da(-i+\tan(dx+c))}$	76
norman	$\frac{\frac{(-iB+A)x}{2a}-\frac{-iA+B}{2ad}+\frac{(-iB+A)x\tan(dx+c)^2}{2a}+\frac{(iB+A)\tan(dx+c)}{2ad}}{1+\tan(dx+c)^2}$	81

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*x/a*B+1/2*x/a*A-1/4/a/d*exp(-2*I*(d*x+c))*B+1/4*I/a/d*exp(-2*I*(d*x
+c))*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{A+B\tan(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\left(2\,(A-i\,B)dxe^{(2i\,dx+2i\,c)}+i\,A-B\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*(A - I*B)*d*x*e^(2*I*d*x + 2*I*c) + I*A - B)*e^(-2*I*d*x - 2*I*c)/(
a*d)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

$$\int \frac{A + B\tan(c + dx)}{a + ia\tan(c + dx)}\,dx = \begin{cases} \frac{(iA-B)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(-\frac{A-iB}{2a} + \frac{(Ae^{2ic}+A-iBe^{2ic}+iB)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(A - iB)}{2a}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
Piecewise(((I*A - B)*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c),
 0)), (x*(-(A - I*B)/(2*a) + (A*exp(2*I*c) + A - I*B*exp(2*I*c) + I*B)*exp
(-2*I*c)/(2*a)), True)) + x*(A - I*B)/(2*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{a + ia\tan(c + dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{A+B\tan(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{(-iA-B)\log(\tan(dx+c)+i)}{4\,ad} - \frac{(iA+B)\log(\tan(dx+c)-i)}{4\,ad} + \frac{A+iB}{2\,ad(\tan(dx+c)-i)}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(-I*A - B)*log(tan(d*x + c) + I)/(a*d) - 1/4*(I*A + B)*log(tan(d*x +
c) - I)/(a*d) + 1/2*(A + I*B)/(a*d*(tan(d*x + c) - I))
```

Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{A+B\tan(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{-\frac{B}{2\,a}+\frac{A\,1\mathrm{i}}{2\,a}}{d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)} - \frac{x\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,a}$$

input
```
int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i),x)
```

output
```
((A*1i)/(2*a) - B/(2*a))/(d*(tan(c + d*x)*1i + 1)) - (x*(A*1i + B)*1i)/(2*
a)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(\int \frac{\tan(dx+c)}{\tan(dx+c)i+1}dx\right)b + \left(\int \frac{1}{\tan(dx+c)i+1}dx\right)a}{a}$$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output

```
(int(tan(c + d*x)/(tan(c + d*x)*i + 1),x)*b + int(1/(tan(c + d*x)*i + 1),x
)*a)/a
```

3.40 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 62

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{(iA-B)x}{2a} + \frac{A\log(\sin(c+dx))}{ad} + \frac{A+iB}{2d(a+ia\tan(c+dx))}$$

output
```
-1/2*(I*A-B)*x/a+A*ln(sin(d*x+c))/a/d+1/2*(A+I*B)/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.39

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{(3A+iB)\log(i-\tan(c+dx)) - 4A\log(\tan(c+dx)) + (A-iB)\log(i+\tan(c+dx)) + \frac{2i(A+iB)}{-i+\tan(c+dx)}}{4ad}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
-1/4*((3*A + I*B)*Log[I - Tan[c + d*x]] - 4*A*Log[Tan[c + d*x]] + (A - I*B
)*Log[I + Tan[c + d*x]] + ((2*I)*(A + I*B))/(-I + Tan[c + d*x]))/(a*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 4079, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \cot(c+dx)(2aA-a(iA-B)\tan(c+dx))dx}{2a^2}+\frac{A+iB}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2aA-a(iA-B)\tan(c+dx)}{\tan(c+dx)}dx}{2a^2}+\frac{A+iB}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4014}$$

$$\frac{2aA\int \cot(c+dx)dx-ax(-B+iA)}{2a^2}+\frac{A+iB}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2aA\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-ax(-B+iA)}{2a^2}+\frac{A+iB}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{-2aA\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-ax(-B+iA)}{2a^2}+\frac{A+iB}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{2aA\log(-\sin(c+dx))}{d} - ax(-B+iA)}{2a^2} + \frac{A+iB}{2d(a+ia\tan(c+dx))}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
(-(a*(I*A - B)*x) + (2*a*A*Log[-Sin[c + d*x]])/d)/(2*a^2) + (A + I*B)/(2*d
*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

method	result
risch	$\frac{xB}{2a}-\frac{3ixA}{2a}+\frac{i\mathrm{e}^{-2i(dx+c)}B}{4ad}+\frac{\mathrm{e}^{-2i(dx+c)}A}{4ad}-\frac{2iAc}{ad}+\frac{A\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{ad}$
derivativedivides	$-\frac{A\ln\left(1+\tan(dx+c)^2\right)}{2da}-\frac{iA\arctan(\tan(dx+c))}{2da}+\frac{B\arctan(\tan(dx+c))}{2da}-\frac{iA}{2da(-i+\tan(dx+c))}+\frac{I}{2da(-i+t}$
default	$-\frac{A\ln\left(1+\tan(dx+c)^2\right)}{2da}-\frac{iA\arctan(\tan(dx+c))}{2da}+\frac{B\arctan(\tan(dx+c))}{2da}-\frac{iA}{2da(-i+\tan(dx+c))}+\frac{I}{2da(-i+t}$
norman	$\frac{\frac{iB+A}{2ad}+\frac{(-iA+B)x}{2a}+\frac{(-iA+B)\tan(dx+c)}{2ad}+\frac{(-iA+B)x\tan(dx+c)^2}{2a}}{1+\tan(dx+c)^2}+\frac{A\ln(\tan(dx+c))}{da}-\frac{A\ln\left(1+\tan(dx+c)^2\right)}{2da}$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/2*x/a*B-3/2*I*x/a*A+1/4*I/a/d*exp(-2*I*(d*x+c))*B+1/4/a/d*exp(-2*I*(d*x+
c))*A-2*I*A/a/d*c+A/a/d*ln(exp(2*I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{\left(2\,(3i\,A-B)dxe^{(2i\,dx+2i\,c)}-4\,Ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}-1\right)-A-i\,B\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/4*(2*(3*I*A - B)*d*x*e^(2*I*d*x + 2*I*c) - 4*A*e^(2*I*d*x + 2*I*c)*log(
e^(2*I*d*x + 2*I*c) - 1) - A - I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.87

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{A\log\left(e^{2idx}-e^{-2ic}\right)}{ad} + \begin{cases} \frac{(A+iB)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(-\frac{-3iA+B}{2a} + \frac{(-3iAe^{2ic}-iA+Be^{2ic}+B)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(-3iA+B)}{2a}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
A*log(exp(2*I*d*x) - exp(-2*I*c))/(a*d) + Piecewise(((A + I*B)*exp(-2*I*c)
*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*(-(-3*I*A + B)/(2*a) +
(-3*I*A*exp(2*I*c) - I*A + B*exp(2*I*c) + B)*exp(-2*I*c)/(2*a)), True)) +
x*(-3*I*A + B)/(2*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.42

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{(A-iB)\log(\tan(dx+c)+i)}{4\,ad} - \frac{(3\,A+i\,B)\log(\tan(dx+c)-i)}{4\,ad} + \frac{A\log(|\tan(dx+c)|)}{ad} - \frac{i\,A-B}{2\,ad(\tan(dx+c)-i)}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
-1/4*(A - I*B)*log(tan(d*x + c) + I)/(a*d) - 1/4*(3*A + I*B)*log(tan(d*x +
 c) - I)/(a*d) + A*log(abs(tan(d*x + c)))/(a*d) - 1/2*(I*A - B)/(a*d*(tan(
d*x + c) - I))
```

Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\frac{A}{2\,a}+\frac{B\,1\mathrm{i}}{2\,a}}{d\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)} + \frac{A\,\ln\left(\tan(c+d\,x)\right)}{a\,d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,a\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(3\,A+B\,1\mathrm{i}\right)}{4\,a\,d}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
(A/(2*a) + (B*1i)/(2*a))/(d*(tan(c + d*x)*1i + 1)) + (A*log(tan(c + d*x)))
/(a*d) + (log(tan(c + d*x) + 1i)*(A*1i + B)*1i)/(4*a*d) - (log(tan(c + d*x
) - 1i)*(3*A + B*1i))/(4*a*d)
```

Reduce [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\left(\int \frac{\cot(dx+c)}{\tan(dx+c)i+1}dx\right)a+\left(\int \frac{\cot(dx+c)\tan(dx+c)}{\tan(dx+c)i+1}dx\right)b}{a}$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(int(cot(c + d*x)/(tan(c + d*x)*i + 1),x)*a + int((cot(c + d*x)*tan(c + d*
x))/(tan(c + d*x)*i + 1),x)*b)/a
```

3.41 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 102

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{(3A+iB)x}{2a} - \frac{(3A+iB)\cot(c+dx)}{2ad} - \frac{(iA-B)\log(\sin(c+dx))}{ad} + \frac{(A+iB)\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

output
```
-1/2*(3*A+I*B)*x/a-1/2*(3*A+I*B)*cot(d*x+c)/a/d-(I*A-B)*ln(sin(d*x+c))/a/d
+1/2*(A+I*B)*cot(d*x+c)/d/(a+I*a*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.80 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\frac{(A+iB)\cot^2(c+dx)}{i+\cot(c+dx)} - (3A+iB)\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right) + 2(-iA+B)(}{2ad}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
(((A + I*B)*Cot[c + d*x]^2)/(I + Cot[c + d*x]) - (3*A + I*B)*Cot[c + d*x]*
Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2] + 2*((-I)*A + B)*(Log[Cos
[c + d*x]] + Log[Tan[c + d*x]]))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4079, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \cot^2(c+dx)(a(3A+iB)-2a(iA-B)\tan(c+dx))dx}{2a^2}+\frac{(A+iB)\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(3A+iB)-2a(iA-B)\tan(c+dx)}{\tan(c+dx)^2}dx}{2a^2}+\frac{(A+iB)\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int -\cot(c+dx)(2a(iA-B)+a(3A+iB)\tan(c+dx))dx-\frac{a(3A+iB)\cot(c+dx)}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\int \cot(c+dx)(2a(iA-B)+a(3A+iB)\tan(c+dx))dx-\frac{a(3A+iB)\cot(c+dx)}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \frac{2a(iA-B)+a(3A+iB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a(3A+iB)\cot(c+dx)}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4014}$$

$$\frac{-2a(-B+iA)\int \cot(c+dx)dx-\frac{a(3A+iB)\cot(c+dx)}{d}-ax(3A+iB)}{2a^2}+\frac{(A+iB)\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-2a(-B+iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a(3A+iB)\cot(c+dx)}{d}-ax(3A+iB)}{2a^2}+\frac{(A+iB)\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{2a(-B+iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a(3A+iB)\cot(c+dx)}{d}-ax(3A+iB)}{2a^2}+\frac{(A+iB)\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3956}$$

$$\frac{-\frac{a(3A+iB)\cot(c+dx)}{d}-\frac{2a(-B+iA)\log(-\sin(c+dx))}{d}-ax(3A+iB)}{2a^2}+\frac{(A+iB)\cot(c+dx)}{2d(a+ia\tan(c+dx))}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
(-(a*(3*A + I*B)*x) - (a*(3*A + I*B)*Cot[c + d*x])/d - (2*a*(I*A - B)*Log[
-Sin[c + d*x]])/d)/(2*a^2) + ((A + I*B)*Cot[c + d*x])/(2*d*(a + I*a*Tan[c
+ d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{3ixB}{2a}-\frac{5xA}{2a}+\frac{\mathrm{e}^{-2i(dx+c)}B}{4ad}-\frac{i\mathrm{e}^{-2i(dx+c)}A}{4ad}-\frac{2iBc}{ad}-\frac{2Ac}{ad}-\frac{2iA}{ad\left(\mathrm{e}^{2i(dx+c)}-1\right)}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)B}{ad}$
norman	$\frac{-\frac{A}{ad}-\frac{(iB+3A)\tan(dx+c)^2}{2ad}-\frac{(iB+3A)x\tan(dx+c)}{2a}-\frac{(iB+3A)x\tan(dx+c)^3}{2a}+\frac{(-iA+B)\tan(dx+c)}{2ad}}{\tan(dx+c)\left(1+\tan(dx+c)^2\right)}+\frac{(-iA+B)\ln(\tan(dx+}{ad}$
derivativedivides	$\frac{iA\ln\left(1+\tan(dx+c)^2\right)}{2da}-\frac{3A\arctan(\tan(dx+c))}{2da}-\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2da}-\frac{iB\arctan(\tan(dx+c))}{2da}-\frac{}{2da(-i+}$
default	$\frac{iA\ln\left(1+\tan(dx+c)^2\right)}{2da}-\frac{3A\arctan(\tan(dx+c))}{2da}-\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2da}-\frac{iB\arctan(\tan(dx+c))}{2da}-\frac{}{2da(-i+}$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-3/2*I*x/a*B-5/2*x/a*A+1/4/a/d*exp(-2*I*(d*x+c))*B-1/4*I/a/d*exp(-2*I*(d*x
+c))*A-2*I/a/d*B*c-2/a/d*A*c-2*I*A/a/d/(exp(2*I*(d*x+c))-1)+1/a/d*ln(exp(2
*I*(d*x+c))-1)*B-I/a/d*ln(exp(2*I*(d*x+c))-1)*A
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.26

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{2\,(5\,A+3i\,B)dxe^{(4i\,dx+4i\,c)}-(2\,(5\,A+3i\,B)dx-9i\,A+B)e^{(2i\,dx+2i\,c)}+4\,\left((i\,A-B)e^{(4i\,dx+4i\,c)}+(-i\right.}{4\,\left(ade^{(4i\,dx+4i\,c)}-ade^{(2i\,dx+2i\,c)}\right)}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/4*(2*(5*A + 3*I*B)*d*x*e^(4*I*d*x + 4*I*c) - (2*(5*A + 3*I*B)*d*x - 9*I
*A + B)*e^(2*I*d*x + 2*I*c) + 4*((I*A - B)*e^(4*I*d*x + 4*I*c) + (-I*A + B
)*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) - 1) - I*A + B)/(a*d*e^(4*I
*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.55

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= -\frac{2iA}{ade^{2ic}e^{2idx}-ad} + \begin{cases} \frac{(-iA+B)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic}\neq 0 \\ x\left(-\frac{-5A-3iB}{2a}+\frac{(-5Ae^{2ic}-A-3iBe^{2ic}-iB)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases}$$
$$+\frac{x(-5A-3iB)}{2a} - \frac{i(A+iB)\log\left(e^{2idx}-e^{-2ic}\right)}{ad}$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
-2*I*A/(a*d*exp(2*I*c)*exp(2*I*d*x) - a*d) + Piecewise(((-I*A + B)*exp(-2*
I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*(-(-5*A - 3*I*B)/(2
*a) + (-5*A*exp(2*I*c) - A - 3*I*B*exp(2*I*c) - I*B)*exp(-2*I*c)/(2*a)), T
rue)) + x*(-5*A - 3*I*B)/(2*a) - I*(A + I*B)*log(exp(2*I*d*x) - exp(-2*I*c
))/(a*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{(iA+B)\log(\tan(dx+c)+i)}{4ad} - \frac{(-5iA+3B)\log(\tan(dx+c)-i)}{4ad} + \frac{(-iA+B)\log(|\tan(dx+c)|)}{ad} - \frac{(3A+iB)\tan(dx+c)-2iA}{2ad(\tan(dx+c)-i)\tan(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-1/4*(I*A + B)*log(tan(d*x + c) + I)/(a*d) - 1/4*(-5*I*A + 3*B)*log(tan(d*
x + c) - I)/(a*d) + (-I*A + B)*log(abs(tan(d*x + c)))/(a*d) - 1/2*((3*A +
I*B)*tan(d*x + c) - 2*I*A)/(a*d*(tan(d*x + c) - I)*tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.24

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{\frac{A}{a}+\tan(c+d\,x)\,\left(-\frac{B}{2\,a}+\frac{A\,3\mathrm{i}}{2\,a}\right)}{d\,\left({\tan\left(c+d\,x\right)}^2\,1\mathrm{i}+\tan\left(c+d\,x\right)\right)} - \frac{\ln\left(\tan(c+d\,x)\right)\,\left(-B+A\,1\mathrm{i}\right)}{a\,d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{4\,a\,d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-3\,B+A\,5\mathrm{i}\right)}{4\,a\,d}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
(log(tan(c + d*x) - 1i)*(A*5i - 3*B))/(4*a*d) - (log(tan(c + d*x))*(A*1i -
 B))/(a*d) - (log(tan(c + d*x) + 1i)*(A*1i + B))/(4*a*d) - (A/a + tan(c +
d*x)*((A*3i)/(2*a) - B/(2*a)))/(d*(tan(c + d*x) + tan(c + d*x)^2*1i))
```

Reduce [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$=\frac{\cot(dx+c)\,bi-\left(\int \frac{\cot(dx+c)^2}{\tan(dx+c)-i}dx\right)adi+\left(\int \frac{\cot(dx+c)^2}{\tan(dx+c)-i}dx\right)bd+bdix}{ad}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(cot(c + d*x)*b*i - int(cot(c + d*x)**2/(tan(c + d*x) - i),x)*a*d*i + int(
cot(c + d*x)**2/(tan(c + d*x) - i),x)*b*d + b*d*i*x)/(a*d)
```

3.42 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 131

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{3(iA-B)x}{2a}+\frac{3(iA-B)\cot(c+dx)}{2ad}-\frac{(2A+iB)\cot^2(c+dx)}{2ad}-\frac{(2A+iB)\log(\sin(c+dx))}{ad}+\frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

output
```
3/2*(I*A-B)*x/a+3/2*(I*A-B)*cot(d*x+c)/a/d-1/2*(2*A+I*B)*cot(d*x+c)^2/a/d-
(2*A+I*B)*ln(sin(d*x+c))/a/d+1/2*(A+I*B)*cot(d*x+c)^2/d/(a+I*a*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\frac{(A+iB)\cot^3(c+dx)}{i+\cot(c+dx)} + 3i(A+iB)\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right) - (2A+iB)\,(c}{2ad}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
(((A + I*B)*Cot[c + d*x]^3)/(I + Cot[c + d*x]) + (3*I)*(A + I*B)*Cot[c + d
*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2] - (2*A + I*B)*(Cot[c
+ d*x]^2 + 2*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4079, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\int \cot^3(c+dx)(2a(2A+iB)-3a(iA-B)\tan(c+dx))dx}{2a^2} + \frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{2a(2A+iB)-3a(iA-B)\tan(c+dx)}{\tan(c+dx)^3}dx}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int -\cot^2(c+dx)(3a(iA-B)+2a(2A+iB)\tan(c+dx))dx-\frac{a(2A+iB)\cot^2(c+dx)}{d}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\int \cot^2(c+dx)(3a(iA-B)+2a(2A+iB)\tan(c+dx))dx-\frac{a(2A+iB)\cot^2(c+dx)}{d}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \frac{3a(iA-B)+2a(2A+iB)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{a(2A+iB)\cot^2(c+dx)}{d}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4012}$$

$$\frac{-\int \cot(c+dx)(2a(2A+iB)-3a(iA-B)\tan(c+dx))dx-\frac{a(2A+iB)\cot^2(c+dx)}{d}+\frac{3a(-B+iA)\cot(c+dx)}{d}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \frac{2a(2A+iB)-3a(iA-B)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a(2A+iB)\cot^2(c+dx)}{d}+\frac{3a(-B+iA)\cot(c+dx)}{d}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4014}$$

$$\frac{-2a(2A+iB)\int \cot(c+dx)dx-\frac{a(2A+iB)\cot^2(c+dx)}{d}+\frac{3a(-B+iA)\cot(c+dx)}{d}+3ax(-B+iA)}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-2a(2A+iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx - \frac{a(2A+iB)\cot^2(c+dx)}{d} + \frac{3a(-B+iA)\cot(c+dx)}{d} + 3ax(-B+iA)}{2a^2} + \frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

$\Big\downarrow$ 25

$$\frac{2a(2A+iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx - \frac{a(2A+iB)\cot^2(c+dx)}{d} + \frac{3a(-B+iA)\cot(c+dx)}{d} + 3ax(-B+iA)}{2a^2} + \frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

$\Big\downarrow$ 3956

$$\frac{-\frac{a(2A+iB)\cot^2(c+dx)}{d} + \frac{3a(-B+iA)\cot(c+dx)}{d} - \frac{2a(2A+iB)\log(-\sin(c+dx))}{d} + 3ax(-B+iA)}{2a^2} + \frac{(A+iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
(3*a*(I*A - B)*x + (3*a*(I*A - B)*Cot[c + d*x])/d - (a*(2*A + I*B)*Cot[c +
 d*x]^2)/d - (2*a*(2*A + I*B)*Log[-Sin[c + d*x]])/d)/(2*a^2) + ((A + I*B)*
Cot[c + d*x]^2)/(2*d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{5xB}{2a}+\frac{7ixA}{2a}-\frac{i\mathrm{e}^{-2i(dx+c)}B}{4ad}-\frac{\mathrm{e}^{-2i(dx+c)}A}{4ad}-\frac{2Bc}{ad}+\frac{4iAc}{ad}-\frac{2i\left(B\,\mathrm{e}^{2i(dx+c)}+iA-B\right)}{ad\left(\mathrm{e}^{2i(dx+c)}-1\right)^{2}}-\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}\right.}{ad}$
norman	$\frac{-\frac{A}{2ad}-\frac{(-iA+B)\tan(dx+c)}{ad}-\frac{3(-iA+B)\tan(dx+c)^{3}}{2ad}-\frac{3(-iA+B)x\tan(dx+c)^{2}}{2a}-\frac{3(-iA+B)x\tan(dx+c)^{4}}{2a}-\frac{(iB+2A)\tan(dx+c}{2ad}}{\tan(dx+c)^{2}\left(1+\tan(dx+c)^{2}\right)}$
derivativedivides	$\frac{A\ln\left(1+\tan(dx+c)^{2}\right)}{da}+\frac{3iA\arctan(\tan(dx+c))}{2da}+\frac{iB\ln\left(1+\tan(dx+c)^{2}\right)}{2da}-\frac{3B\arctan(\tan(dx+c))}{2da}+\frac{}{2da(-i+}$
default	$\frac{A\ln\left(1+\tan(dx+c)^{2}\right)}{da}+\frac{3iA\arctan(\tan(dx+c))}{2da}+\frac{iB\ln\left(1+\tan(dx+c)^{2}\right)}{2da}-\frac{3B\arctan(\tan(dx+c))}{2da}+\frac{}{2da(-i+}$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-5/2*x/a*B+7/2*I*x/a*A-1/4*I/a/d*exp(-2*I*(d*x+c))*B-1/4/a/d*exp(-2*I*(d*x
+c))*A-2/a/d*B*c+4*I/a/d*A*c-2*I*(B*exp(2*I*(d*x+c))+I*A-B)/a/d/(exp(2*I*(
d*x+c))-1)^2-I/a/d*ln(exp(2*I*(d*x+c))-1)*B-2*A/a/d*ln(exp(2*I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.44

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{2\,(-7i\,A+5\,B)dxe^{(6i\,dx+6i\,c)}+(4\,(7i\,A-5\,B)dx+A+9i\,B)e^{(4i\,dx+4i\,c)}+2\,((-7i\,A+5\,B)dx-5\,A}{4\,(ade^{(6i\,dx+6}}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/4*(2*(-7*I*A + 5*B)*d*x*e^(6*I*d*x + 6*I*c) + (4*(7*I*A - 5*B)*d*x + A
+ 9*I*B)*e^(4*I*d*x + 4*I*c) + 2*((-7*I*A + 5*B)*d*x - 5*A - 5*I*B)*e^(2*I
*d*x + 2*I*c) + 4*((2*A + I*B)*e^(6*I*d*x + 6*I*c) - 2*(2*A + I*B)*e^(4*I*
d*x + 4*I*c) + (2*A + I*B)*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) -
1) + A + I*B)/(a*d*e^(6*I*d*x + 6*I*c) - 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*e
^(2*I*d*x + 2*I*c))
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.52

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{2A-2iBe^{2ic}e^{2idx}+2iB}{ade^{4ic}e^{4idx}-2ade^{2ic}e^{2idx}+ad}$$
$$+\begin{cases}\frac{(-A-iB)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic}\neq 0\\ x\left(-\frac{7iA-5B}{2a}+\frac{(7iAe^{2ic}+iA-5Be^{2ic}-B)e^{-2ic}}{2a}\right) & \text{otherwise}\end{cases}$$
$$+\frac{x(7iA-5B)}{2a}-\frac{(2A+iB)\log\left(e^{2idx}-e^{-2ic}\right)}{ad}$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(2*A - 2*I*B*exp(2*I*c)*exp(2*I*d*x) + 2*I*B)/(a*d*exp(4*I*c)*exp(4*I*d*x)
 - 2*a*d*exp(2*I*c)*exp(2*I*d*x) + a*d) + Piecewise(((-A - I*B)*exp(-2*I*c
)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*(-(7*I*A - 5*B)/(2*a)
+ (7*I*A*exp(2*I*c) + I*A - 5*B*exp(2*I*c) - B)*exp(-2*I*c)/(2*a)), True))
 + x*(7*I*A - 5*B)/(2*a) - (2*A + I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/(a*
d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{(A-iB)\log(\tan(dx+c)+i)}{4\,ad} + \frac{(7\,A+5i\,B)\log(\tan(dx+c)-i)}{4\,ad}$$
$$- \frac{(2\,A+i\,B)\log(|\tan(dx+c)|)}{ad}$$
$$- \frac{3\,(-i\,A+B)\tan(dx+c)^2-(A+2i\,B)\tan(dx+c)-i\,A}{2\,ad(\tan(dx+c)-i)\tan(dx+c)^2}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
1/4*(A - I*B)*log(tan(d*x + c) + I)/(a*d) + 1/4*(7*A + 5*I*B)*log(tan(d*x
+ c) - I)/(a*d) - (2*A + I*B)*log(abs(tan(d*x + c)))/(a*d) - 1/2*(3*(-I*A
+ B)*tan(d*x + c)^2 - (A + 2*I*B)*tan(d*x + c) - I*A)/(a*d*(tan(d*x + c) -
 I)*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= -\frac{\tan(c+d\,x)^2\,\left(\frac{3\,A}{2\,a}+\frac{B\,3\mathrm{i}}{2\,a}\right)+\frac{A}{2\,a}-\tan(c+d\,x)\,\left(-\frac{B}{a}+\frac{A\,1\mathrm{i}}{2\,a}\right)}{d\,\left(\tan(c+d\,x)^3\,1\mathrm{i}+\tan(c+d\,x)^2\right)}$$
$$- \frac{\ln(\tan(c+d\,x))\,(2\,A+B\,1\mathrm{i})}{a\,d} + \frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(A-B\,1\mathrm{i})}{4\,a\,d}$$
$$+ \frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(7\,A+B\,5\mathrm{i})}{4\,a\,d}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
(log(tan(c + d*x) + 1i)*(A - B*1i))/(4*a*d) - (log(tan(c + d*x))*(2*A + B*
1i))/(a*d) - (tan(c + d*x)^2*((3*A)/(2*a) + (B*3i)/(2*a)) + A/(2*a) - tan(
c + d*x)*((A*1i)/(2*a) - B/a))/(d*(tan(c + d*x)^2 + tan(c + d*x)^3*1i)) +
(log(tan(c + d*x) - 1i)*(7*A + B*5i))/(4*a*d)
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\left(\int \frac{\cot(dx+c)^3}{\tan(dx+c)i+1}dx\right)a+\left(\int \frac{\cot(dx+c)^3\tan(dx+c)}{\tan(dx+c)i+1}dx\right)b}{a}$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(int(cot(c + d*x)**3/(tan(c + d*x)*i + 1),x)*a + int((cot(c + d*x)**3*tan(
c + d*x))/(tan(c + d*x)*i + 1),x)*b)/a
```

3.43 $\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 155

$$\begin{aligned}\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx &= \frac{(5A+3iB)x}{2a}+\frac{(5A+3iB)\cot(c+dx)}{2ad}\\ &+\frac{(iA-B)\cot^2(c+dx)}{ad}\\ &-\frac{(5A+3iB)\cot^3(c+dx)}{6ad}\\ &+\frac{2(iA-B)\log(\sin(c+dx))}{ad}\\ &+\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}\end{aligned}$$

output
```
1/2*(5*A+3*I*B)*x/a+1/2*(5*A+3*I*B)*cot(d*x+c)/a/d+(I*A-B)*cot(d*x+c)^2/a/
d-1/6*(5*A+3*I*B)*cot(d*x+c)^3/a/d+2*(I*A-B)*ln(sin(d*x+c))/a/d+1/2*(A+I*B
)*cot(d*x+c)^3/d/(a+I*a*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.74

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\frac{3(A+iB)\cot^4(c+dx)}{i+\cot(c+dx)} - (5A+3iB)\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right) + 6i(A+i}{6ad}$$

input
```
Integrate[(Cot[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
((3*(A + I*B)*Cot[c + d*x]^4)/(I + Cot[c + d*x]) - (5*A + (3*I)*B)*Cot[c +
 d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2] + (6*I)*(A + I*B
)*(Cot[c + d*x]^2 + 2*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/(6*a*d)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {3042, 4079, 3042, 4012, 25, 3042, 4012, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^4(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \cot^4(c+dx)(a(5A+3iB)-4a(iA-B)\tan(c+dx))dx}{2a^2} + \frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(5A+3iB)-4a(iA-B)\tan(c+dx)}{\tan(c+dx)^4}dx}{2a^2}+\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int -\cot^3(c+dx)(4a(iA-B)+a(5A+3iB)\tan(c+dx))dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}}{2a^2}+\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\int \cot^3(c+dx)(4a(iA-B)+a(5A+3iB)\tan(c+dx))dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}}{2a^2}+\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \frac{4a(iA-B)+a(5A+3iB)\tan(c+dx)}{\tan(c+dx)^3}dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}}{2a^2}+\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4012}$$

$$\frac{-\int \cot^2(c+dx)(a(5A+3iB)-4a(iA-B)\tan(c+dx))dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}+\frac{2a(-B+iA)\cot^2(c+dx)}{d}}{2a^2}+\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \frac{a(5A+3iB)-4a(iA-B)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}+\frac{2a(-B+iA)\cot^2(c+dx)}{d}}{2a^2}+\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4012}$$

$$\frac{-\int -\cot(c+dx)(4a(iA-B)+a(5A+3iB)\tan(c+dx))dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}+\frac{2a(-B+iA)\cot^2(c+dx)}{d}+\frac{a(5A}{}}{2a^2}+\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cot(c+dx)(4a(iA-B)+a(5A+3iB)\tan(c+dx))dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}+\frac{2a(-B+iA)\cot^2(c+dx)}{d}+\frac{a(5A+3iB}{}}{2a^2}$$
$$\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{4a(iA-B)+a(5A+3iB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}+\frac{2a(-B+iA)\cot^2(c+dx)}{d}+\frac{a(5A+3iB)\cot(c+dx)}{d}}{2a^2}+$$
$$\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 4014$$

$$\frac{4a(-B+iA)\int \cot(c+dx)dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}+\frac{2a(-B+iA)\cot^2(c+dx)}{d}+\frac{a(5A+3iB)\cot(c+dx)}{d}+ax(5A+3iB)}{2a^2}$$
$$\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{4a(-B+iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}+\frac{2a(-B+iA)\cot^2(c+dx)}{d}+\frac{a(5A+3iB)\cot(c+dx)}{d}+ax(5A}{2a^2}$$
$$\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 25$$

$$\frac{-4a(-B+iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}+\frac{2a(-B+iA)\cot^2(c+dx)}{d}+\frac{a(5A+3iB)\cot(c+dx)}{d}+ax}{2a^2}$$
$$\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 3956$$

$$\frac{-\frac{a(5A+3iB)\cot^3(c+dx)}{3d}+\frac{2a(-B+iA)\cot^2(c+dx)}{d}+\frac{a(5A+3iB)\cot(c+dx)}{d}+\frac{4a(-B+iA)\log(-\sin(c+dx))}{d}+ax(5A+3iB)}{2a^2}+$$
$$\frac{(A+iB)\cot^3(c+dx)}{2d(a+ia\tan(c+dx))}$$

input
```
Int[(Cot[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
(a*(5*A + (3*I)*B)*x + (a*(5*A + (3*I)*B)*Cot[c + d*x])/d + (2*a*(I*A - B)
*Cot[c + d*x]^2)/d - (a*(5*A + (3*I)*B)*Cot[c + d*x]^3)/(3*d) + (4*a*(I*A
- B)*Log[-Sin[c + d*x]])/d)/(2*a^2) + ((A + I*B)*Cot[c + d*x]^3)/(2*d*(a +
 I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

method	result
risch	$\frac{7ixB}{2a}+\frac{9xA}{2a}-\frac{\mathrm{e}^{-2i(dx+c)}B}{4ad}+\frac{i\mathrm{e}^{-2i(dx+c)}A}{4ad}+\frac{4iBc}{ad}+\frac{4Ac}{ad}+\frac{4iA\,\mathrm{e}^{4i(dx+c)}-6iA\,\mathrm{e}^{2i(dx+c)}+2B\,\mathrm{e}^{2i(dx+c)}+}{ad\left(\mathrm{e}^{2i(dx+c)}-1\right)^{3}}$
norman	$\frac{-\frac{A}{3ad}-\frac{(-iA+B)\tan(dx+c)}{2ad}+\frac{(3iB+5A)\tan(dx+c)^{2}}{3ad}+\frac{(3iB+5A)\tan(dx+c)^{4}}{2ad}+\frac{(3iB+5A)x\tan(dx+c)^{3}}{2a}+\frac{(3iB+5A)x\tan(dx+c)}{2a}}{\tan(dx+c)^{3}\left(1+\tan(dx+c)^{2}\right)}$
derivativedivides	$\frac{iA}{2ad\tan(dx+c)^{2}}+\frac{5A\arctan(\tan(dx+c))}{2da}+\frac{B\ln\left(1+\tan(dx+c)^{2}\right)}{da}-\frac{iA\ln\left(1+\tan(dx+c)^{2}\right)}{da}+\frac{A}{2da(-i+\tan(dx}$
default	$\frac{iA}{2ad\tan(dx+c)^{2}}+\frac{5A\arctan(\tan(dx+c))}{2da}+\frac{B\ln\left(1+\tan(dx+c)^{2}\right)}{da}-\frac{iA\ln\left(1+\tan(dx+c)^{2}\right)}{da}+\frac{A}{2da(-i+\tan(dx}$

input
```
int(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
7/2*I*x/a*B+9/2*x/a*A-1/4/a/d*exp(-2*I*(d*x+c))*B+1/4*I/a/d*exp(-2*I*(d*x+
c))*A+4*I/a/d*B*c+4/a/d*A*c+2/3*(6*I*A*exp(4*I*(d*x+c))-9*I*A*exp(2*I*(d*x
+c))+3*B*exp(2*I*(d*x+c))+7*I*A-3*B)/a/d/(exp(2*I*(d*x+c))-1)^3-2/a/d*ln(e
xp(2*I*(d*x+c))-1)*B+2*I/a/d*ln(exp(2*I*(d*x+c))-1)*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.61

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{6\,(9\,A+7i\,B)dxe^{(8i\,dx+8i\,c)} - 3\,(6\,(9\,A+7i\,B)dx - 17i\,A+B)e^{(6i\,dx+6i\,c)} + 3\,(6\,(9\,A+7i\,B)dx - 27i\,A}{}$$

input
```
integrate(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/12*(6*(9*A + 7*I*B)*d*x*e^(8*I*d*x + 8*I*c) - 3*(6*(9*A + 7*I*B)*d*x - 1
7*I*A + B)*e^(6*I*d*x + 6*I*c) + 3*(6*(9*A + 7*I*B)*d*x - 27*I*A + 11*B)*e
^(4*I*d*x + 4*I*c) - (6*(9*A + 7*I*B)*d*x - 65*I*A + 33*B)*e^(2*I*d*x + 2*
I*c) - 24*((-I*A + B)*e^(8*I*d*x + 8*I*c) + 3*(I*A - B)*e^(6*I*d*x + 6*I*c
) + 3*(-I*A + B)*e^(4*I*d*x + 4*I*c) + (I*A - B)*e^(2*I*d*x + 2*I*c))*log(
e^(2*I*d*x + 2*I*c) - 1) - 3*I*A + 3*B)/(a*d*e^(8*I*d*x + 8*I*c) - 3*a*d*e
^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.63

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{12iAe^{4ic}e^{4idx} + 14iA - 6B + (-18iAe^{2ic} + 6Be^{2ic})\,e^{2idx}}{3ade^{6ic}e^{6idx} - 9ade^{4ic}e^{4idx} + 9ade^{2ic}e^{2idx} - 3ad}$$
$$+ \begin{cases} \frac{(iA-B)e^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(-\frac{9A+7iB}{2a} + \frac{(9Ae^{2ic}+A+7iBe^{2ic}+iB)e^{-2ic}}{2a}\right) & \text{otherwise} \end{cases}$$
$$+ \frac{x(9A+7iB)}{2a} + \frac{2i(A+iB)\log\left(e^{2idx} - e^{-2ic}\right)}{ad}$$

input
```
integrate(cot(d*x+c)**4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(12*I*A*exp(4*I*c)*exp(4*I*d*x) + 14*I*A - 6*B + (-18*I*A*exp(2*I*c) + 6*B
*exp(2*I*c))*exp(2*I*d*x))/(3*a*d*exp(6*I*c)*exp(6*I*d*x) - 9*a*d*exp(4*I*
c)*exp(4*I*d*x) + 9*a*d*exp(2*I*c)*exp(2*I*d*x) - 3*a*d) + Piecewise(((I*A
 - B)*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*(-(9*A
 + 7*I*B)/(2*a) + (9*A*exp(2*I*c) + A + 7*I*B*exp(2*I*c) + I*B)*exp(-2*I*c
)/(2*a)), True)) + x*(9*A + 7*I*B)/(2*a) + 2*I*(A + I*B)*log(exp(2*I*d*x)
- exp(-2*I*c))/(a*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{(-iA-B)\log(\tan(dx+c)+i)}{4\,ad} - \frac{(9iA-7B)\log(\tan(dx+c)-i)}{4\,ad} - \frac{2(-iA+B)\log(|\tan(dx+c)|)}{ad} + \frac{3(5A+3iB)\tan(dx+c)^3 - 3(3iA-B)\tan(dx+c)^2 + (A+3iB)\tan(dx+c) + 2iA}{6\,ad(\tan(dx+c)-i)\tan(dx+c)^3}$$

input
```
integrate(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-1/4*(-I*A - B)*log(tan(d*x + c) + I)/(a*d) - 1/4*(9*I*A - 7*B)*log(tan(d*
x + c) - I)/(a*d) - 2*(-I*A + B)*log(abs(tan(d*x + c)))/(a*d) + 1/6*(3*(5*
A + 3*I*B)*tan(d*x + c)^3 - 3*(3*I*A - B)*tan(d*x + c)^2 + (A + 3*I*B)*tan
(d*x + c) + 2*I*A)/(a*d*(tan(d*x + c) - I)*tan(d*x + c)^3)
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.12

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\tan(c+d\,x)^2\,\left(\frac{3\,A}{2\,a}+\frac{B\,1\mathrm{i}}{2\,a}\right)+\tan(c+d\,x)^3\,\left(-\frac{3\,B}{2\,a}+\frac{A\,5\mathrm{i}}{2\,a}\right)-\frac{A}{3\,a}+\tan(c+d\,x)\,\left(-\frac{B}{2\,a}+\frac{A\,1\mathrm{i}}{6\,a}\right)}{d\,\left(\tan\left(c+d\,x\right)^4\,1\mathrm{i}+\tan\left(c+d\,x\right)^3\right)} + \frac{2\,\ln\left(\tan(c+d\,x)\right)\,\left(-B+A\,1\mathrm{i}\right)}{a\,d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{4\,a\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-7\,B+A\,9\mathrm{i}\right)}{4\,a\,d}$$

input
```
int((cot(c + d*x)^4*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
(tan(c + d*x)^2*((3*A)/(2*a) + (B*1i)/(2*a)) + tan(c + d*x)^3*((A*5i)/(2*a
) - (3*B)/(2*a)) - A/(3*a) + tan(c + d*x)*((A*1i)/(6*a) - B/(2*a)))/(d*(ta
n(c + d*x)^3 + tan(c + d*x)^4*1i)) + (2*log(tan(c + d*x))*(A*1i - B))/(a*d
) + (log(tan(c + d*x) + 1i)*(A*1i + B))/(4*a*d) - (log(tan(c + d*x) - 1i)*
(A*9i - 7*B))/(4*a*d)
```

Reduce [F]

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\cot\left(dx+c\right)^3 bi-3\cot\left(dx+c\right)bi-3\left(\int\frac{\cot(dx+c)^4}{\tan(dx+c)-i}dx\right)adi+3\left(\int\frac{\cot(dx+c)^4}{\tan(dx+c)-i}dx\right)bd-3bdix}{3ad}$$

input
```
int(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(cot(c + d*x)**3*b*i - 3*cot(c + d*x)*b*i - 3*int(cot(c + d*x)**4/(tan(c +
 d*x) - i),x)*a*d*i + 3*int(cot(c + d*x)**4/(tan(c + d*x) - i),x)*b*d - 3*
b*d*i*x)/(3*a*d)
```

3.44 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 142

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{3(iA-3B)x}{4a^2}+\frac{(A+2iB)\log(\cos(c+dx))}{a^2d}+\frac{3(iA-3B)\tan(c+dx)}{4a^2d}+\frac{(A+2iB)\tan^2(c+dx)}{2a^2d(1+i\tan(c+dx))}+\frac{(iA-B)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

output
```
-3/4*(I*A-3*B)*x/a^2+(A+2*I*B)*ln(cos(d*x+c))/a^2/d+3/4*(I*A-3*B)*tan(d*x+
c)/a^2/d+1/2*(A+2*I*B)*tan(d*x+c)^2/a^2/d/(1+I*tan(d*x+c))+1/4*(I*A-B)*tan
(d*x+c)^3/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.39

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{(7A+17iB)\log(i-\tan(c+dx))+(A-iB)\log(i+\tan(c+dx))+2(-3iA+9B+(7iA-17B)\log(i$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x
]
```

output
```
((7*A + (17*I)*B)*Log[I - Tan[c + d*x]] + (A - I*B)*Log[I + Tan[c + d*x]]
+ 2*((-3*I)*A + 9*B + ((7*I)*A - 17*B)*Log[I - Tan[c + d*x]] + (I*A + B)*L
og[I + Tan[c + d*x]])*Tan[c + d*x] + (8*A + (28*I)*B - (7*A + (17*I)*B)*Lo
g[I - Tan[c + d*x]] - (A - I*B)*Log[I + Tan[c + d*x]])*Tan[c + d*x]^2 - 8*
B*Tan[c + d*x]^3)/(8*a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4078, 3042, 4078, 27, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{4078}$$
$$\frac{(-B+iA)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int \frac{\tan^2(c+dx)(3a(iA-B)+a(A+5iB)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int\frac{\tan(c+dx)^2(3a(iA-B)+a(A+5iB)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{4a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{-\frac{\int-2\tan(c+dx)(4a^2(A+2iB)-3a^2(iA-3B)\tan(c+dx))dx}{2a^2}-\frac{2(A+2iB)\tan^2(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\tan(c+dx)(4a^2(A+2iB)-3a^2(iA-3B)\tan(c+dx))dx}{a^2}-\frac{2(A+2iB)\tan^2(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\tan(c+dx)(4a^2(A+2iB)-3a^2(iA-3B)\tan(c+dx))dx}{a^2}-\frac{2(A+2iB)\tan^2(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}$$

$$\Big\downarrow \text{4008}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{4a^2(A+2iB)\int\tan(c+dx)dx-\frac{3a^2(-3B+iA)\tan(c+dx)}{d}+3a^2x(-3B+iA)}{a^2}-\frac{2(A+2iB)\tan^2(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{4a^2(A+2iB)\int\tan(c+dx)dx-\frac{3a^2(-3B+iA)\tan(c+dx)}{d}+3a^2x(-3B+iA)}{a^2}-\frac{2(A+2iB)\tan^2(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{-\frac{3a^2(-3B+iA)\tan(c+dx)}{d}-\frac{4a^2(A+2iB)\log(\cos(c+dx))}{d}+3a^2x(-3B+iA)}{a^2}-\frac{2(A+2iB)\tan^2(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I*A - B)*Tan[c + d*x]^3)/(4*d*(a + I*a*Tan[c + d*x])^2) - ((-2*(A + (2*I
)*B)*Tan[c + d*x]^2)/(d*(1 + I*Tan[c + d*x])) + (3*a^2*(I*A - 3*B)*x - (4*
a^2*(A + (2*I)*B)*Log[Cos[c + d*x]])/d - (3*a^2*(I*A - 3*B)*Tan[c + d*x])/
d)/a^2)/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.22

method	result
derivativedivides	$-\frac{B\tan(dx+c)}{d\,a^2}-\frac{A\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^2}-\frac{3iA\arctan(\tan(dx+c))}{4d\,a^2}-\frac{iB\ln\left(1+\tan(dx+c)^2\right)}{d\,a^2}+\frac{9B\arctan(\tan(}{4d\,a^2}$
default	$-\frac{B\tan(dx+c)}{d\,a^2}-\frac{A\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^2}-\frac{3iA\arctan(\tan(dx+c))}{4d\,a^2}-\frac{iB\ln\left(1+\tan(dx+c)^2\right)}{d\,a^2}+\frac{9B\arctan(\tan(}{4d\,a^2}$
risch	$\frac{17xB}{4a^2}-\frac{7ixA}{4a^2}-\frac{3i\mathrm{e}^{-2i(dx+c)}B}{4a^2d}-\frac{\mathrm{e}^{-2i(dx+c)}A}{2a^2d}+\frac{i\mathrm{e}^{-4i(dx+c)}B}{16a^2d}+\frac{\mathrm{e}^{-4i(dx+c)}A}{16a^2d}+\frac{4Bc}{a^2d}-\frac{2iAc}{a^2d}-\frac{}{d\,a^2(\mathrm{e}}$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVER
BOSE)
```

output
```
-1/d/a^2*B*tan(d*x+c)-1/2/d/a^2*A*ln(1+tan(d*x+c)^2)-3/4*I/d/a^2*A*arctan(
tan(d*x+c))-I/d/a^2*B*ln(1+tan(d*x+c)^2)+9/4/d/a^2*B*arctan(tan(d*x+c))+5/
4*I/d/a^2/(-I+tan(d*x+c))*A-7/4/d/a^2/(-I+tan(d*x+c))*B-1/4/d/a^2/(-I+tan(
d*x+c))^2*A-1/4*I/d/a^2/(-I+tan(d*x+c))^2*B
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int\frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx=$$
$$-\frac{4\,(7i\,A-17\,B)dxe^{(6i\,dx+6i\,c)}+4\,((7i\,A-17\,B)dx+2\,A+11i\,B)e^{(4i\,dx+4i\,c)}+(7\,A+11i\,B)e^{(2i\,dx+2i\,c)}}{16\,(a^2de^{(6i\,dx+6i\,c)}+a^2de^{(4}}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"fricas")
```

output
```
-1/16*(4*(7*I*A - 17*B)*d*x*e^(6*I*d*x + 6*I*c) + 4*((7*I*A - 17*B)*d*x +
2*A + 11*I*B)*e^(4*I*d*x + 4*I*c) + (7*A + 11*I*B)*e^(2*I*d*x + 2*I*c) - 1
6*((A + 2*I*B)*e^(6*I*d*x + 6*I*c) + (A + 2*I*B)*e^(4*I*d*x + 4*I*c))*log(
e^(2*I*d*x + 2*I*c) + 1) - A - I*B)/(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(
4*I*d*x + 4*I*c))
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.85

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2iB}{a^2de^{2ic}e^{2idx}+a^2d} + \begin{cases} \frac{\left(\left(4Aa^2de^{2ic}+4iBa^2de^{2ic}\right)e^{-4idx}+\left(-32Aa^2de^{4ic}-48iBa^2de^{4ic}\right)e^{-2idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(-\frac{-7iA+17B}{4a^2}+\frac{(-7iAe^{4ic}+4iAe^{2ic}-iA+17Be^{4ic}-6Be^{2ic}+B)e^{-4ic}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(-7iA+17B)}{4a^2} + \frac{(A+2iB)\log\left(e^{2idx}+e^{-2ic}\right)}{a^2d}$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-2*I*B/(a**2*d*exp(2*I*c)*exp(2*I*d*x) + a**2*d) + Piecewise((((4*A*a**2*d
*exp(2*I*c) + 4*I*B*a**2*d*exp(2*I*c))*exp(-4*I*d*x) + (-32*A*a**2*d*exp(4
*I*c) - 48*I*B*a**2*d*exp(4*I*c))*exp(-2*I*d*x))*exp(-6*I*c)/(64*a**4*d**2
), Ne(a**4*d**2*exp(6*I*c), 0)), (x*(-(-7*I*A + 17*B)/(4*a**2) + (-7*I*A*e
xp(4*I*c) + 4*I*A*exp(2*I*c) - I*A + 17*B*exp(4*I*c) - 6*B*exp(2*I*c) + B)
*exp(-4*I*c)/(4*a**2)), True)) + x*(-7*I*A + 17*B)/(4*a**2) + (A + 2*I*B)*
log(exp(2*I*d*x) + exp(-2*I*c))/(a**2*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.71

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(A-iB)\log(\tan(dx+c)+i)}{8a^2d} - \frac{(7A+17iB)\log(\tan(dx+c)-i)}{8a^2d} - \frac{B\tan(dx+c)}{a^2d} - \frac{(-5iA+7B)\tan(dx+c)-4A-6iB}{4a^2d(\tan(dx+c)-i)^2}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"giac")
```

output
```
-1/8*(A - I*B)*log(tan(d*x + c) + I)/(a^2*d) - 1/8*(7*A + 17*I*B)*log(tan(
d*x + c) - I)/(a^2*d) - B*tan(d*x + c)/(a^2*d) - 1/4*((-5*I*A + 7*B)*tan(d
*x + c) - 4*A - 6*I*B)/(a^2*d*(tan(d*x + c) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{(A+B\,2\mathrm{i})\,1\mathrm{i}}{a^2}+\frac{B}{2\,a^2}-\tan(c+d\,x)\,\left(\frac{5\,(A+B\,2\mathrm{i})}{4\,a^2}-\frac{B\,3\mathrm{i}}{4\,a^2}\right)}{d\,\left({\tan\left(c+d\,x\right)}^2\,1\mathrm{i}+2\,\tan\left(c+d\,x\right)-\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,a^2\,d} - \frac{B\tan(c+d\,x)}{a^2\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(7\,A+B\,17\mathrm{i}\right)}{8\,a^2\,d}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(((A + B*2i)*1i)/a^2 + B/(2*a^2) - tan(c + d*x)*((5*(A + B*2i))/(4*a^2) -
(B*3i)/(4*a^2)))/(d*(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i)) + (log(tan(
c + d*x) + 1i)*(A*1i + B)*1i)/(8*a^2*d) - (B*tan(c + d*x))/(a^2*d) - (log(
tan(c + d*x) - 1i)*(7*A + B*17i))/(8*a^2*d)
```

Reduce [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{-\left(\int \frac{\tan(dx+c)^4}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b - \left(\int \frac{\tan(dx+c)^3}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a}{a^2}$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(tan(c + d*x)**4/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*b + i
nt(tan(c + d*x)**3/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a))/a**2
```

3.45 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 103

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(A+3iB)x}{4a^2}+\frac{B\log(\cos(c+dx))}{a^2d} + \frac{iA-3B}{4a^2d(1+i\tan(c+dx))} + \frac{(iA-B)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

output
```
-1/4*(A+3*I*B)*x/a^2+B*ln(cos(d*x+c))/a^2/d+1/4*(I*A-3*B)/a^2/d/(1+I*tan(d
*x+c))+1/4*(I*A-B)*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.53

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{\sec^2(c+dx)(-2iA+4B+\cos(2(c+dx))(2iA-4B+(-iA+7B)\log(i-\tan(c+dx))+(iA+B)\log}{8a^2d(}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x
]
```

output
```
(Sec[c + d*x]^2*((-2*I)*A + 4*B + Cos[2*(c + d*x)]*((2*I)*A - 4*B + ((-I)*
A + 7*B)*Log[I - Tan[c + d*x]] + (I*A + B)*Log[I + Tan[c + d*x]]) + (-A -
(3*I)*B + (A + (7*I)*B)*Log[I - Tan[c + d*x]] - (A - I*B)*Log[I + Tan[c +
d*x]])*Sin[2*(c + d*x)]))/(8*a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4078, 27, 3042, 4072, 25, 27, 3042, 3956, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int \frac{2\tan(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{i\tan(c+dx)a+a}dx}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{i\tan(c+dx)a+a}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{i\tan(c+dx)a+a}dx}{2a^2}$$

$$\Big\downarrow \text{4072}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{2B\int\tan(c+dx)dx-\frac{i\int-\frac{a^2(A+3iB)\tan(c+dx)}{i\tan(c+dx)a+a}dx}{a}}{2a^2}$$

$$\Big\downarrow \ 25$$

$$\frac{(-B+iA)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{2B\int\tan(c+dx)dx+\frac{i\int\frac{a^2(A+3iB)\tan(c+dx)}{i\tan(c+dx)a+a}dx}{a}}{2a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(-B+iA)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{2B\int\tan(c+dx)dx+ia(A+3iB)\int\frac{\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(-B+iA)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{2B\int\tan(c+dx)dx+ia(A+3iB)\int\frac{\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}$$

$$\Big\downarrow \ 3956$$

$$\frac{(-B+iA)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{-\frac{2B\log(\cos(c+dx))}{d}+ia(A+3iB)\int\frac{\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}$$

$$\Big\downarrow \ 4009$$

$$\frac{(-B+iA)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{-\frac{2B\log(\cos(c+dx))}{d}+ia(A+3iB)\left(-\frac{i\int 1dx}{2a}-\frac{1}{2d(a+ia\tan(c+dx))}\right)}{2a^2}$$

$$\Big\downarrow \ 24$$

$$\frac{(-B+iA)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{-\frac{2B\log(\cos(c+dx))}{d}+ia(A+3iB)\left(-\frac{1}{2d(a+ia\tan(c+dx))}-\frac{ix}{2a}\right)}{2a^2}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I*A - B)*Tan[c + d*x]^2)/(4*d*(a + I*a*Tan[c + d*x])^2) - ((-2*B*Log[Cos
[c + d*x]])/d + I*a*(A + (3*I)*B)*(((-1/2*I)*x)/a - 1/(2*d*(a + I*a*Tan[c
+ d*x]))))/(2*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4078

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{7ixB}{4a^2}-\frac{xA}{4a^2}-\frac{e^{-2i(dx+c)}B}{2a^2d}+\frac{ie^{-2i(dx+c)}A}{4a^2d}+\frac{e^{-4i(dx+c)}B}{16a^2d}-\frac{ie^{-4i(dx+c)}A}{16a^2d}-\frac{2iBc}{a^2d}+\frac{B\ln(e^{2i(dx+c)}...}{a^2d}$
derivativedivides	$-\frac{A\arctan(\tan(dx+c))}{4d\,a^2}-\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^2}-\frac{3iB\arctan(\tan(dx+c))}{4d\,a^2}+\frac{5iB}{4d\,a^2(-i+\tan(dx+c))}+\frac{}{4d\,a^2(-...}$
default	$-\frac{A\arctan(\tan(dx+c))}{4d\,a^2}-\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^2}-\frac{3iB\arctan(\tan(dx+c))}{4d\,a^2}+\frac{5iB}{4d\,a^2(-i+\tan(dx+c))}+\frac{}{4d\,a^2(-...}$

input

```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVER
BOSE)
```

output

```
-7/4*I*x/a^2*B-1/4*x/a^2*A-1/2/a^2/d*exp(-2*I*(d*x+c))*B+1/4*I/a^2/d*exp(-
2*I*(d*x+c))*A+1/16/a^2/d*exp(-4*I*(d*x+c))*B-1/16*I/a^2/d*exp(-4*I*(d*x+c
))*A-2*I*B/a^2/d*c+B/a^2/d*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(4\,(A+7i\,B)dxe^{(4i\,dx+4i\,c)}-16\,Be^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+4\,(-i\,A+2\,B)e^{(2i\,dx+2i\,c)}+i\,A-B...}{16\,a^2d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"fricas")
```

output
```
-1/16*(4*(A + 7*I*B)*d*x*e^(4*I*d*x + 4*I*c) - 16*B*e^(4*I*d*x + 4*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) + 4*(-I*A + 2*B)*e^(2*I*d*x + 2*I*c) + I*A - B)
*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.17

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{B\log\left(e^{2idx}+e^{-2ic}\right)}{a^2d} + \begin{cases} \frac{\left(\left(-4iAa^2de^{2ic}+4Ba^2de^{2ic}\right)e^{-4idx}+\left(16iAa^2de^{4ic}-32Ba^2de^{4ic}\right)e^{-2idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(-\frac{-A-7iB}{4a^2}+\frac{\left(-Ae^{4ic}+2Ae^{2ic}-A-7iBe^{4ic}+4iBe^{2ic}-iB\right)e^{-4ic}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(-A-7iB)}{4a^2}$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
B*log(exp(2*I*d*x) + exp(-2*I*c))/(a**2*d) + Piecewise((((-4*I*A*a**2*d*ex
p(2*I*c) + 4*B*a**2*d*exp(2*I*c))*exp(-4*I*d*x) + (16*I*A*a**2*d*exp(4*I*c
) - 32*B*a**2*d*exp(4*I*c))*exp(-2*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(
a**4*d**2*exp(6*I*c), 0)), (x*(-(-A - 7*I*B)/(4*a**2) + (-A*exp(4*I*c) + 2
*A*exp(2*I*c) - A - 7*I*B*exp(4*I*c) + 4*I*B*exp(2*I*c) - I*B)*exp(-4*I*c)
/(4*a**2)), True)) + x*(-A - 7*I*B)/(4*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(i\,A+B)\log\left(\tan\left(dx+c\right)+i\right)}{8\,a^2d} - \frac{(-i\,A+7\,B)\log\left(\tan\left(dx+c\right)-i\right)}{8\,a^2d} + \frac{(3\,A+5i\,B)\tan\left(dx+c\right)-2i\,A+4\,B}{4\,a^2d(\tan\left(dx+c\right)-i)^2}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"giac")
```

output
```
-1/8*(I*A + B)*log(tan(d*x + c) + I)/(a^2*d) - 1/8*(-I*A + 7*B)*log(tan(d*
x + c) - I)/(a^2*d) + 1/4*((3*A + 5*I*B)*tan(d*x + c) - 2*I*A + 4*B)/(a^2*
d*(tan(d*x + c) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{A}{2\,a^2}+\frac{B\,1\mathrm{i}}{a^2}+\tan(c+d\,x)\,\left(-\frac{5\,B}{4\,a^2}+\frac{A\,3\mathrm{i}}{4\,a^2}\right)}{d\,\left(\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{8\,a^2\,d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-7\,B+A\,1\mathrm{i}\right)}{8\,a^2\,d}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(A/(2*a^2) + (B*1i)/a^2 + tan(c + d*x)*((A*3i)/(4*a^2) - (5*B)/(4*a^2)))/(
d*(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i)) - (log(tan(c + d*x) + 1i)*(A*
1i + B))/(8*a^2*d) + (log(tan(c + d*x) - 1i)*(A*1i - 7*B))/(8*a^2*d)
```

Reduce [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\left(\int \frac{\tan(dx+c)^3}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\tan(dx+c)^2}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a}{a^2}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(tan(c + d*x)**3/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*b + i
nt(tan(c + d*x)**2/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a))/a**2
```

3.46 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 76

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(iA+B)x}{4a^2}+\frac{A+3iB}{4a^2d(1+i\tan(c+dx))} - \frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

output
```
-1/4*(I*A+B)*x/a^2+1/4*(A+3*I*B)/a^2/d/(1+I*tan(d*x+c))-1/4*(A+I*B)/d/(a+I
*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{\sec^2(c+dx)(-4iB+(A+4iAdx+B(i+4dx))\cos(2(c+dx))+(-iA+B-4Adx+4iBdx)\sin(2(c}{16a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(Sec[c + d*x]^2*((-4*I)*B + (A + (4*I)*A*d*x + B*(I + 4*d*x))*Cos[2*(c + d
*x)] + ((-I)*A + B - 4*A*d*x + (4*I)*B*d*x)*Sin[2*(c + d*x)]))/(16*a^2*d*(
-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3042, 4073, 3042, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \ 4073$$

$$-\frac{i\int \frac{a(A+iB)+2aB\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{i\int \frac{a(A+iB)+2aB\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \ 4009$$

$$-\frac{i\left(\frac{1}{2}(A-iB)\int 1dx+\frac{-3B+iA}{2d(1+i\tan(c+dx))}\right)}{2a^2}-\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \ 24$$

$$-\frac{i\left(\frac{-3B+iA}{2d(1+i\tan(c+dx))}+\frac{1}{2}x(A-iB)\right)}{2a^2}-\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-1/2*I)*(((A - I*B)*x)/2 + (I*A - 3*B)/(2*d*(1 + I*Tan[c + d*x]))))/a^2
- (A + I*B)/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{xB}{4a^2}-\frac{ixA}{4a^2}+\frac{i e^{-2i(dx+c)}B}{4a^2d}-\frac{i e^{-4i(dx+c)}B}{16a^2d}-\frac{e^{-4i(dx+c)}A}{16a^2d}$
derivativedivides	$-\frac{iA\arctan(\tan(dx+c))}{4d\,a^2}-\frac{B\arctan(\tan(dx+c))}{4d\,a^2}+\frac{A}{4d\,a^2(-i+\tan(dx+c))^2}+\frac{iB}{4d\,a^2(-i+\tan(dx+c))^2}-\frac{}{4d\,a^2(}$
default	$-\frac{iA\arctan(\tan(dx+c))}{4d\,a^2}-\frac{B\arctan(\tan(dx+c))}{4d\,a^2}+\frac{A}{4d\,a^2(-i+\tan(dx+c))^2}+\frac{iB}{4d\,a^2(-i+\tan(dx+c))^2}-\frac{}{4d\,a^2(}$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
-1/4*x/a^2*B-1/4*I*x/a^2*A+1/4*I*B/a^2/d*exp(-2*I*(d*x+c))-1/16*I/a^2/d*ex
p(-4*I*(d*x+c))*B-1/16/a^2/d*exp(-4*I*(d*x+c))*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\left(4\,(i\,A+B)dxe^{(4i\,dx+4i\,c)}-4i\,Be^{(2i\,dx+2i\,c)}+A+i\,B\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
-1/16*(4*(I*A + B)*d*x*e^(4*I*d*x + 4*I*c) - 4*I*B*e^(2*I*d*x + 2*I*c) + A
 + I*B)*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.20

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{\left(16iBa^2de^{4ic}e^{-2idx}+\left(-4Aa^2de^{2ic}-4iBa^2de^{2ic}\right)e^{-4idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(-\frac{-iA-B}{4a^2}+\frac{\left(-iAe^{4ic}+iA-Be^{4ic}+2Be^{2ic}-B\right)e^{-4ic}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(-iA-B)}{4a^2}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((16*I*B*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + (-4*A*a**2*d*exp(2*I*
c) - 4*I*B*a**2*d*exp(2*I*c))*exp(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), N
e(a**4*d**2*exp(6*I*c), 0)), (x*(-(-I*A - B)/(4*a**2) + (-I*A*exp(4*I*c) +
 I*A - B*exp(4*I*c) + 2*B*exp(2*I*c) - B)*exp(-4*I*c)/(4*a**2)), True)) +
x*(-I*A - B)/(4*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{(A-i\,B)\log\left(\tan\left(dx+c\right)+i\right)}{8\,a^2d} - \frac{(A-i\,B)\log\left(\tan\left(dx+c\right)-i\right)}{8\,a^2d} - \frac{i\,(A+3i\,B)\tan\left(dx+c\right)+2i\,B}{4\,a^2d(\tan\left(dx+c\right)-i)^2}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="g
iac")
```

output
```
1/8*(A - I*B)*log(tan(d*x + c) + I)/(a^2*d) - 1/8*(A - I*B)*log(tan(d*x +
c) - I)/(a^2*d) - 1/4*(I*(A + 3*I*B)*tan(d*x + c) + 2*I*B)/(a^2*d*(tan(d*x
 + c) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.39

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{B}{2\,a^2}+\tan(c+d\,x)\,\left(\frac{A}{4\,a^2}+\frac{B\,3\mathrm{i}}{4\,a^2}\right)}{d\,\left({\tan\left(c+d\,x\right)}^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,a^2\,d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{8\,a^2\,d}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(B/(2*a^2) + tan(c + d*x)*(A/(4*a^2) + (B*3i)/(4*a^2)))/(d*(2*tan(c + d*x)
 + tan(c + d*x)^2*1i - 1i)) + (log(tan(c + d*x) - 1i)*(A*1i + B)*1i)/(8*a^
2*d) + (log(tan(c + d*x) + 1i)*(A - B*1i))/(8*a^2*d)
```

Reduce [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\left(\int \frac{\tan(dx+c)^2}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\tan(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a}{a^2}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(tan(c + d*x)**2/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*b + i
nt(tan(c + d*x)/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a))/a**2
```

3.47 $\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{(A-iB)x}{4a^2} + \frac{iA-B}{4d(a+ia\tan(c+dx))^2} + \frac{iA+B}{4d\,(a^2+ia^2\tan(c+dx))}$$

output
```
1/4*(A-I*B)*x/a^2+1/4*(I*A-B)/d/(a+I*a*tan(d*x+c))^2+1/4*(I*A+B)/d/(a^2+I*
a^2*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{(A-iB)\arctan(\tan(c+dx))}{4a^2d} - \frac{iA-B}{4a^2d(i-\tan(c+dx))^2} - \frac{A-iB}{4a^2d(i-\tan(c+dx))}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((A - I*B)*ArcTan[Tan[c + d*x]])/(4*a^2*d) - (I*A - B)/(4*a^2*d*(I - Tan[c
 + d*x])^2) - (A - I*B)/(4*a^2*d*(I - Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3042, 4009, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4009}$$

$$\frac{(A-iB)\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{-B+iA}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{-B+iA}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3960}$$

$$\frac{(A-iB)\left(\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}\right)}{2a}+\frac{-B+iA}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{-B+iA}{4d(a+ia\tan(c+dx))^2}+\frac{(A-iB)\left(\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}\right)}{2a}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
(I*A - B)/(4*d*(a + I*a*Tan[c + d*x])^2) + ((A - I*B)*(x/(2*a) + (I/2)/(d*
(a + I*a*Tan[c + d*x]))))/(2*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{ixB}{4a^2}+\frac{xA}{4a^2}+\frac{i\,\mathrm{e}^{-2i(dx+c)}A}{4a^2d}-\frac{\mathrm{e}^{-4i(dx+c)}B}{16a^2d}+\frac{i\,\mathrm{e}^{-4i(dx+c)}A}{16a^2d}$
derivativedivides	$\frac{A\arctan(\tan(dx+c))}{4d\,a^2}-\frac{iB\arctan(\tan(dx+c))}{4d\,a^2}+\frac{A}{4d\,a^2(-i+\tan(dx+c))}-\frac{iB}{4d\,a^2(-i+\tan(dx+c))}-\frac{}{4d\,a^2(-i+}$
default	$\frac{A\arctan(\tan(dx+c))}{4d\,a^2}-\frac{iB\arctan(\tan(dx+c))}{4d\,a^2}+\frac{A}{4d\,a^2(-i+\tan(dx+c))}-\frac{iB}{4d\,a^2(-i+\tan(dx+c))}-\frac{}{4d\,a^2(-i+}$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*x/a^2*B+1/4*x/a^2*A+1/4*I/a^2/d*exp(-2*I*(d*x+c))*A-1/16/a^2/d*exp(
-4*I*(d*x+c))*B+1/16*I/a^2/d*exp(-4*I*(d*x+c))*A
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(4\,(A-i\,B)dxe^{(4i\,dx+4i\,c)}+4i\,Ae^{(2i\,dx+2i\,c)}+i\,A-B\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/16*(4*(A - I*B)*d*x*e^(4*I*d*x + 4*I*c) + 4*I*A*e^(2*I*d*x + 2*I*c) + I*
A - B)*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.02

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(16iAa^2de^{4ic}e^{-2idx}+(4iAa^2de^{2ic}-4Ba^2de^{2ic})e^{-4idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(-\frac{A-iB}{4a^2}+\frac{(Ae^{4ic}+2Ae^{2ic}+A-iBe^{4ic}+iB)e^{-4ic}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(A-iB)}{4a^2}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Piecewise(((16*I*A*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + (4*I*A*a**2*d*exp(2*I
*c) - 4*B*a**2*d*exp(2*I*c))*exp(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne
(a**4*d**2*exp(6*I*c), 0)), (x*(-(A - I*B)/(4*a**2) + (A*exp(4*I*c) + 2*A*
exp(2*I*c) + A - I*B*exp(4*I*c) + I*B)*exp(-4*I*c)/(4*a**2)), True)) + x*(
A - I*B)/(4*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(-iA-B)\log(\tan(dx+c)+i)}{8a^2d} + \frac{(-iA-B)\log(\tan(dx+c)-i)}{8a^2d} + \frac{(A-iB)\tan(dx+c)-2iA}{4a^2d(\tan(dx+c)-i)^2}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/8*(-I*A - B)*log(tan(d*x + c) + I)/(a^2*d) + 1/8*(-I*A - B)*log(tan(d*x
 + c) - I)/(a^2*d) + 1/4*((A - I*B)*tan(d*x + c) - 2*I*A)/(a^2*d*(tan(d*x
+ c) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{A}{2\,a^2}+\tan(c+d\,x)\,\left(\frac{B}{4\,a^2}+\frac{A\,1\mathrm{i}}{4\,a^2}\right)}{d\,\left(\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)}-\frac{x\,(B+A\,1\mathrm{i})\,1\mathrm{i}}{4\,a^2}$$

input
```
int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(A/(2*a^2) + tan(c + d*x)*((A*1i)/(4*a^2) + B/(4*a^2)))/(d*(2*tan(c + d*x)
 + tan(c + d*x)^2*1i - 1i)) - (x*(A*1i + B)*1i)/(4*a^2)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\left(\int \frac{\tan(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b-\left(\int \frac{1}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a}{a^2}$$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(tan(c + d*x)/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*b + int(
1/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a))/a**2
```

3.48 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 95

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(3iA-B)x}{4a^2}+\frac{A\log(\sin(c+dx))}{a^2d}+\frac{3A+iB}{4a^2d(1+i\tan(c+dx))}+\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

output
```
-1/4*(3*I*A-B)*x/a^2+A*ln(sin(d*x+c))/a^2/d+1/4*(3*A+I*B)/a^2/d/(1+I*tan(d
*x+c))+1/4*(A+I*B)/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(7A+iB)\log(i-\tan(c+dx))-8A\log(\tan(c+dx))+(A-iB)\log(i+\tan(c+dx))+\frac{2(A+iB)}{(-i+\tan(c+dx}}{8a^2d}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/8*((7*A + I*B)*Log[I - Tan[c + d*x]] - 8*A*Log[Tan[c + d*x]] + (A - I*B
)*Log[I + Tan[c + d*x]] + (2*(A + I*B))/(-I + Tan[c + d*x])^2 - (2*((-3*I)
*A + B))/(-I + Tan[c + d*x]))/(a^2*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3042, 4079, 27, 3042, 4079, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{2\cot(c+dx)(2aA-a(iA-B)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cot(c+dx)(2aA-a(iA-B)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{2a^2}+\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2aA-a(iA-B)\tan(c+dx)}{\tan(c+dx)(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int \cot(c+dx)\left(4a^2A-a^2(3iA-B)\tan(c+dx)\right)dx}{2a^2}+\frac{3A+iB}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{4a^2A-a^2(3iA-B)\tan(c+dx)}{\tan(c+dx)}dx}{2a^2}+\frac{3A+iB}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\frac{4a^2A\int \cot(c+dx)dx-a^2x(-B+3iA)}{2a^2}+\frac{3A+iB}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4a^2A\int -\tan(c+dx+\frac{\pi}{2})dx-a^2x(-B+3iA)}{2a^2}+\frac{3A+iB}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{-4a^2A\int \tan(\frac{1}{2}(2c+\pi)+dx)dx-(a^2x(-B+3iA))}{2a^2}+\frac{3A+iB}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{\frac{4a^2A\log(-\sin(c+dx))}{d}-a^2x(-B+3iA)}{2a^2}+\frac{3A+iB}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{A+iB}{4d(a+ia\tan(c+dx))^2}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-(a^2*((3*I)*A - B)*x) + (4*a^2*A*Log[-Sin[c + d*x]])/d)/(2*a^2) + (3*A
+ I*B)/(2*d*(1 + I*Tan[c + d*x])))/(2*a^2) + (A + I*B)/(4*d*(a + I*a*Tan[c
 + d*x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

method	result
risch	$\frac{xB}{4a^2}-\frac{7ixA}{4a^2}+\frac{i\mathrm{e}^{-2i(dx+c)}B}{4a^2d}+\frac{\mathrm{e}^{-2i(dx+c)}A}{2a^2d}+\frac{i\mathrm{e}^{-4i(dx+c)}B}{16a^2d}+\frac{\mathrm{e}^{-4i(dx+c)}A}{16a^2d}-\frac{2iAc}{a^2d}+\frac{A\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{a^2d}$
derivativedivides	$-\frac{A\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^2}-\frac{3iA\arctan(\tan(dx+c))}{4d\,a^2}+\frac{B\arctan(\tan(dx+c))}{4d\,a^2}-\frac{A}{4d\,a^2(-i+\tan(dx+c))^2}-\frac{}{4d\,a^2(}$
default	$-\frac{A\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^2}-\frac{3iA\arctan(\tan(dx+c))}{4d\,a^2}+\frac{B\arctan(\tan(dx+c))}{4d\,a^2}-\frac{A}{4d\,a^2(-i+\tan(dx+c))^2}-\frac{}{4d\,a^2(}$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/4*x/a^2*B-7/4*I*x/a^2*A+1/4*I*B/a^2/d*exp(-2*I*(d*x+c))+1/2/a^2/d*exp(-2
*I*(d*x+c))*A+1/16*I/a^2/d*exp(-4*I*(d*x+c))*B+1/16/a^2/d*exp(-4*I*(d*x+c)
)*A-2*I/a^2/d*A*c+1/a^2*A/d*ln(exp(2*I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\left(4\,(7i\,A-B)dxe^{(4i\,dx+4i\,c)}-16\,Ae^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}-1\right)-4\,(2\,A+i\,B)e^{(2i\,dx+2i\,c)}-A-i\,B\right)}{16\,a^2d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
-1/16*(4*(7*I*A - B)*d*x*e^(4*I*d*x + 4*I*c) - 16*A*e^(4*I*d*x + 4*I*c)*lo
g(e^(2*I*d*x + 2*I*c) - 1) - 4*(2*A + I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*
e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.31

$$\begin{aligned}\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx &= \frac{A\log\left(e^{2idx}-e^{-2ic}\right)}{a^2d} \\ &+ \begin{cases} \frac{\left(\left(4Aa^2de^{2ic}+4iBa^2de^{2ic}\right)e^{-4idx}+\left(32Aa^2de^{4ic}+16iBa^2de^{4ic}\right)e^{-2idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(-\frac{-7iA+B}{4a^2}+\frac{\left(-7iAe^{4ic}-4iAe^{2ic}-iA+Be^{4ic}+2Be^{2ic}+B\right)e^{-4ic}}{4a^2}\right) & \text{otherwise} \end{cases} \\ &+ \frac{x(-7iA+B)}{4a^2}\end{aligned}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
A*log(exp(2*I*d*x) - exp(-2*I*c))/(a**2*d) + Piecewise((((4*A*a**2*d*exp(2
*I*c) + 4*I*B*a**2*d*exp(2*I*c))*exp(-4*I*d*x) + (32*A*a**2*d*exp(4*I*c) +
 16*I*B*a**2*d*exp(4*I*c))*exp(-2*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a
**4*d**2*exp(6*I*c), 0)), (x*(-(-7*I*A + B)/(4*a**2) + (-7*I*A*exp(4*I*c)
- 4*I*A*exp(2*I*c) - I*A + B*exp(4*I*c) + 2*B*exp(2*I*c) + B)*exp(-4*I*c)/
(4*a**2)), True)) + x*(-7*I*A + B)/(4*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(A-iB)\log(\tan(dx+c)+i)}{8a^2d} - \frac{(7A+iB)\log(\tan(dx+c)-i)}{8a^2d} + \frac{A\log(|\tan(dx+c)|)}{a^2d} - \frac{(3iA-B)\tan(dx+c)+4A+2iB}{4a^2d(\tan(dx+c)-i)^2}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm="g
iac")
```

output
```
-1/8*(A - I*B)*log(tan(d*x + c) + I)/(a^2*d) - 1/8*(7*A + I*B)*log(tan(d*x
 + c) - I)/(a^2*d) + A*log(abs(tan(d*x + c)))/(a^2*d) - 1/4*((3*I*A - B)*t
an(d*x + c) + 4*A + 2*I*B)/(a^2*d*(tan(d*x + c) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{B}{2a^2}-\frac{A\,1\mathrm{i}}{a^2}+\tan(c+d\,x)\,\left(\frac{3\,A}{4\,a^2}+\frac{B\,1\mathrm{i}}{4\,a^2}\right)}{d\,\left({\tan\left(c+d\,x\right)}^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)} + \frac{A\,\ln\left(\tan(c+d\,x)\right)}{a^2\,d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{8\,a^2\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(7\,A+B\,1\mathrm{i}\right)}{8\,a^2\,d}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(B/(2*a^2) - (A*1i)/a^2 + tan(c + d*x)*((3*A)/(4*a^2) + (B*1i)/(4*a^2)))/(
d*(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i)) + (A*log(tan(c + d*x)))/(a^2*
d) - (log(tan(c + d*x) + 1i)*(A - B*1i))/(8*a^2*d) - (log(tan(c + d*x) - 1
i)*(7*A + B*1i))/(8*a^2*d)
```

Reduce [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\left(\int \frac{\cot(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a-\left(\int \frac{\cot(dx+c)\tan(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b}{a^2}$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(cot(c + d*x)/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a + int(
(cot(c + d*x)*tan(c + d*x))/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*b)
)/a**2
```

3.49 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 141

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{3(3A+iB)x}{4a^2} - \frac{3(3A+iB)\cot(c+dx)}{4a^2d} - \frac{(2iA-B)\log(\sin(c+dx))}{a^2d} + \frac{(2A+iB)\cot(c+dx)}{2a^2d(1+i\tan(c+dx))} + \frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

output
```
-3/4*(3*A+I*B)*x/a^2-3/4*(3*A+I*B)*cot(d*x+c)/a^2/d-(2*I*A-B)*ln(sin(d*x+c
))/a^2/d+1/2*(2*A+I*B)*cot(d*x+c)/a^2/d/(1+I*tan(d*x+c))+1/4*(A+I*B)*cot(d
*x+c)/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.39 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{2(A+iB)\cot^3(c+dx)}{(i+\cot(c+dx))^2}+\frac{4(2A+iB)\cot^2(c+dx)}{i+\cot(c+dx)}-6(3A+iB)\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+\right.}{8a^2d}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x
]
```

output
```
((2*(A + I*B)*Cot[c + d*x]^3)/(I + Cot[c + d*x])^2 + (4*(2*A + I*B)*Cot[c
+ d*x]^2)/(I + Cot[c + d*x]) - 6*(3*A + I*B)*Cot[c + d*x]*Hypergeometric2F
1[-1/2, 1, 1/2, -Tan[c + d*x]^2] + 8*((-2*I)*A + B)*(Log[Cos[c + d*x]] + L
og[Tan[c + d*x]]))/(8*a^2*d)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {3042, 4079, 3042, 4079, 27, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{\cot^2(c+dx)(a(5A+iB)-3a(iA-B)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(5A+iB)-3a(iA-B)\tan(c+dx)}{\tan(c+dx)^2(i\tan(c+dx)a+a)}dx}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int 2\cot^2(c+dx)\left(3a^2(3A+iB)-4a^2(2iA-B)\tan(c+dx)\right)dx}{2a^2}+\frac{2(2A+iB)\cot(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \cot^2(c+dx)\left(3a^2(3A+iB)-4a^2(2iA-B)\tan(c+dx)\right)dx}{a^2}+\frac{2(2A+iB)\cot(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3a^2(3A+iB)-4a^2(2iA-B)\tan(c+dx)}{\tan(c+dx)^2}dx}{a^2}+\frac{2(2A+iB)\cot(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\int -\cot(c+dx)\left(4(2iA-B)a^2+3(3A+iB)\tan(c+dx)a^2\right)dx-\frac{3a^2(3A+iB)\cot(c+dx)}{d}}{a^2}+\frac{2(2A+iB)\cot(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{-\int \cot(c+dx)\left(4(2iA-B)a^2+3(3A+iB)\tan(c+dx)a^2\right)dx-\frac{3a^2(3A+iB)\cot(c+dx)}{d}}{a^2}+\frac{2(2A+iB)\cot(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\int \frac{4(2iA-B)a^2+3(3A+iB)\tan(c+dx)a^2}{\tan(c+dx)}dx-\frac{3a^2(3A+iB)\cot(c+dx)}{d}}{a^2}+\frac{2(2A+iB)\cot(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\frac{-4a^2(-B+2iA)\int \cot(c+dx)dx-\frac{3a^2(3A+iB)\cot(c+dx)}{d}-3a^2x(3A+iB)}{a^2}+\frac{2(2A+iB)\cot(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{-4a^2(-B+2iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{3a^2(3A+iB)\cot(c+dx)}{d}-3a^2x(3A+iB)}{a^2}+\frac{2(2A+iB)\cot(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{4a^2(-B+2iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{3a^2(3A+iB)\cot(c+dx)}{d}-3a^2x(3A+iB)}{a^2}+\frac{2(2A+iB)\cot(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\frac{-\frac{3a^2(3A+iB)\cot(c+dx)}{d}-\frac{4a^2(-B+2iA)\log(-\sin(c+dx))}{d}-3a^2x(3A+iB)}{a^2}+\frac{2(2A+iB)\cot(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((-3*a^2*(3*A + I*B)*x - (3*a^2*(3*A + I*B)*Cot[c + d*x])/d - (4*a^2*((2*I
)*A - B)*Log[-Sin[c + d*x]])/d)/a^2 + (2*(2*A + I*B)*Cot[c + d*x])/(d*(1 +
 I*Tan[c + d*x])))/(4*a^2) + ((A + I*B)*Cot[c + d*x])/(4*d*(a + I*a*Tan[c
+ d*x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{7ixB}{4a^2}-\frac{17xA}{4a^2}+\frac{e^{-2i(dx+c)}B}{2a^2d}-\frac{3ie^{-2i(dx+c)}A}{4a^2d}+\frac{e^{-4i(dx+c)}B}{16a^2d}-\frac{ie^{-4i(dx+c)}A}{16a^2d}-\frac{2iBc}{a^2d}-\frac{4Ac}{a^2d}-$
derivativedivides	$-\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^2}-\frac{3iB\arctan(\tan(dx+c))}{4d\,a^2}+\frac{iA\ln\left(1+\tan(dx+c)^2\right)}{a^2d}-\frac{9A\arctan(\tan(dx+c))}{4d\,a^2}-$
default	$-\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^2}-\frac{3iB\arctan(\tan(dx+c))}{4d\,a^2}+\frac{iA\ln\left(1+\tan(dx+c)^2\right)}{a^2d}-\frac{9A\arctan(\tan(dx+c))}{4d\,a^2}-$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVER
BOSE)
```

output
```
-7/4*I*x/a^2*B-17/4*x/a^2*A+1/2/a^2/d*exp(-2*I*(d*x+c))*B-3/4*I/a^2/d*exp(
-2*I*(d*x+c))*A+1/16/a^2/d*exp(-4*I*(d*x+c))*B-1/16*I/a^2/d*exp(-4*I*(d*x+
c))*A-2*I*B/a^2/d*c-4/a^2/d*A*c-2*I*A/a^2/d/(exp(2*I*(d*x+c))-1)+1/a^2/d*l
n(exp(2*I*(d*x+c))-1)*B-2*I/a^2/d*ln(exp(2*I*(d*x+c))-1)*A
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.08

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{4\,(17\,A+7i\,B)dxe^{(6i\,dx+6i\,c)}-4\,((17\,A+7i\,B)dx-11i\,A+2\,B)e^{(4i\,dx+4i\,c)}-(11i\,A-7\,B)e^{(2i\,dx+2i\,c)}}{16\,(a^2de^{(6i\,dx+6i\,c)}-a^2de^{(}}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"fricas")
```

output
```
-1/16*(4*(17*A + 7*I*B)*d*x*e^(6*I*d*x + 6*I*c) - 4*((17*A + 7*I*B)*d*x -
11*I*A + 2*B)*e^(4*I*d*x + 4*I*c) - (11*I*A - 7*B)*e^(2*I*d*x + 2*I*c) + 1
6*((2*I*A - B)*e^(6*I*d*x + 6*I*c) + (-2*I*A + B)*e^(4*I*d*x + 4*I*c))*log
(e^(2*I*d*x + 2*I*c) - 1) - I*A + B)/(a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^
(4*I*d*x + 4*I*c))
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.89

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= -\frac{2iA}{a^2de^{2ic}e^{2idx}-a^2d}$$
$$+\begin{cases} \frac{((-4iAa^2de^{2ic}+4Ba^2de^{2ic})e^{-4idx}+(-48iAa^2de^{4ic}+32Ba^2de^{4ic})e^{-2idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(-\frac{-17A-7iB}{4a^2}+\frac{(-17Ae^{4ic}-6Ae^{2ic}-A-7iBe^{4ic}-4iBe^{2ic}-iB)e^{-4ic}}{4a^2}\right) & \text{otherwise} \end{cases}$$
$$+\frac{x(-17A-7iB)}{4a^2}-\frac{i(2A+iB)\log\left(e^{2idx}-e^{-2ic}\right)}{a^2d}$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-2*I*A/(a**2*d*exp(2*I*c)*exp(2*I*d*x) - a**2*d) + Piecewise((((-4*I*A*a**
2*d*exp(2*I*c) + 4*B*a**2*d*exp(2*I*c))*exp(-4*I*d*x) + (-48*I*A*a**2*d*ex
p(4*I*c) + 32*B*a**2*d*exp(4*I*c))*exp(-2*I*d*x))*exp(-6*I*c)/(64*a**4*d**
2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*(-(-17*A - 7*I*B)/(4*a**2) + (-17*A*e
xp(4*I*c) - 6*A*exp(2*I*c) - A - 7*I*B*exp(4*I*c) - 4*I*B*exp(2*I*c) - I*B
)*exp(-4*I*c)/(4*a**2)), True)) + x*(-17*A - 7*I*B)/(4*a**2) - I*(2*A + I*
B)*log(exp(2*I*d*x) - exp(-2*I*c))/(a**2*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(iA+B)\log(\tan(dx+c)+i)}{8a^2d} - \frac{(-17iA+7B)\log(\tan(dx+c)-i)}{8a^2d} + \frac{(-2iA+B)\log(|\tan(dx+c)|)}{a^2d} - \frac{3(3A+iB)\tan(dx+c)^2+2(-7iA+2B)\tan(dx+c)-4A}{4a^2d(\tan(dx+c)-i)^2\tan(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"giac")
```

output
```
-1/8*(I*A + B)*log(tan(d*x + c) + I)/(a^2*d) - 1/8*(-17*I*A + 7*B)*log(tan
(d*x + c) - I)/(a^2*d) + (-2*I*A + B)*log(abs(tan(d*x + c)))/(a^2*d) - 1/4
*(3*(3*A + I*B)*tan(d*x + c)^2 + 2*(-7*I*A + 2*B)*tan(d*x + c) - 4*A)/(a^2
*d*(tan(d*x + c) - I)^2*tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.16

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(c+d\,x)^2\left(-\frac{3\,B}{4\,a^2}+\frac{A\,9\mathrm{i}}{4\,a^2}\right)-\frac{A\,1\mathrm{i}}{a^2}+\tan(c+d\,x)\,\left(\frac{7\,A}{2\,a^2}+\frac{B\,1\mathrm{i}}{a^2}\right)}{d\,\left(\tan\left(c+d\,x\right)^3\,1\mathrm{i}+2\tan\left(c+d\,x\right)^2-\tan\left(c+d\,x\right)\,1\mathrm{i}\right)} - \frac{\ln\left(\tan(c+d\,x)\right)\,\left(-B+A\,2\mathrm{i}\right)}{a^2\,d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{8\,a^2\,d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-7\,B+A\,17\mathrm{i}\right)}{8\,a^2\,d}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(log(tan(c + d*x) - 1i)*(A*17i - 7*B))/(8*a^2*d) - (log(tan(c + d*x))*(A*2
i - B))/(a^2*d) - (log(tan(c + d*x) + 1i)*(A*1i + B))/(8*a^2*d) - (tan(c +
 d*x)^2*((A*9i)/(4*a^2) - (3*B)/(4*a^2)) - (A*1i)/a^2 + tan(c + d*x)*((7*A
)/(2*a^2) + (B*1i)/a^2))/(d*(2*tan(c + d*x)^2 - tan(c + d*x)*1i + tan(c +
d*x)^3*1i))
```

Reduce [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\left(\int \frac{\cot(dx+c)^2}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a-\left(\int \frac{\cot(dx+c)^2\tan(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b}{a^2}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(cot(c + d*x)**2/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a + i
nt((cot(c + d*x)**2*tan(c + d*x))/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1)
,x)*b))/a**2
```

3.50 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 170

$$\begin{aligned}\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx &= \frac{3(5iA-3B)x}{4a^2}+\frac{3(5iA-3B)\cot(c+dx)}{4a^2d} \\ &\quad -\frac{(2A+iB)\cot^2(c+dx)}{a^2d} \\ &\quad -\frac{2(2A+iB)\log(\sin(c+dx))}{a^2d} \\ &\quad +\frac{(5A+3iB)\cot^2(c+dx)}{4a^2d(1+i\tan(c+dx))} \\ &\quad +\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}\end{aligned}$$

output
```
3/4*(5*I*A-3*B)*x/a^2+3/4*(5*I*A-3*B)*cot(d*x+c)/a^2/d-(2*A+I*B)*cot(d*x+c
)^2/a^2/d-2*(2*A+I*B)*ln(sin(d*x+c))/a^2/d+1/4*(5*A+3*I*B)*cot(d*x+c)^2/a^
2/d/(1+I*tan(d*x+c))+1/4*(A+I*B)*cot(d*x+c)^2/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.77 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{2(A+iB)\cot^4(c+dx)}{(i+\cot(c+dx))^2} + \frac{2(5A+3iB)\cot^3(c+dx)}{i+\cot(c+dx)} + 6(5iA-3B)\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c\right.}{8a^2d}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x
]
```

output
```
((2*(A + I*B)*Cot[c + d*x]^4)/(I + Cot[c + d*x])^2 + (2*(5*A + (3*I)*B)*Co
t[c + d*x]^3)/(I + Cot[c + d*x]) + 6*((5*I)*A - 3*B)*Cot[c + d*x]*Hypergeo
metric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2] - 8*(2*A + I*B)*(Cot[c + d*x]^2 +
 2*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]])))/(8*a^2*d)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$, Rules used = {3042, 4079, 27, 3042, 4079, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{2\cot^3(c+dx)(a(3A+iB)-2a(iA-B)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\cot^3(c+dx)(a(3A+iB)-2a(iA-B)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(3A+iB)-2a(iA-B)\tan(c+dx)}{\tan(c+dx)^3(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\int \cot^3(c+dx)(8a^2(2A+iB)-3a^2(5iA-3B)\tan(c+dx))dx}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{8a^2(2A+iB)-3a^2(5iA-3B)\tan(c+dx)}{\tan(c+dx)^3}dx}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\int -\cot^2(c+dx)(3(5iA-3B)a^2+8(2A+iB)\tan(c+dx)a^2)dx-\frac{4a^2(2A+iB)\cot^2(c+dx)}{d}}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{-\int \cot^2(c+dx)(3(5iA-3B)a^2+8(2A+iB)\tan(c+dx)a^2)dx-\frac{4a^2(2A+iB)\cot^2(c+dx)}{d}}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{-\int \frac{3(5iA-3B)a^2+8(2A+iB)\tan(c+dx)a^2}{\tan(c+dx)^2}dx-\frac{4a^2(2A+iB)\cot^2(c+dx)}{d}}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 4012

$$\frac{\frac{-\int \cot(c+dx)\left(8a^2(2A+iB)-3a^2(5iA-3B)\tan(c+dx)\right)dx-\frac{4a^2(2A+iB)\cot^2(c+dx)}{d}+\frac{3a^2(-3B+5iA)\cot(c+dx)}{d}}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\int \frac{8a^2(2A+iB)-3a^2(5iA-3B)\tan(c+dx)}{\tan(c+dx)}dx-\frac{4a^2(2A+iB)\cot^2(c+dx)}{d}+\frac{3a^2(-3B+5iA)\cot(c+dx)}{d}}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\frac{-8a^2(2A+iB)\int \cot(c+dx)dx-\frac{4a^2(2A+iB)\cot^2(c+dx)}{d}+\frac{3a^2(-3B+5iA)\cot(c+dx)}{d}+3a^2x(-3B+5iA)}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-8a^2(2A+iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4a^2(2A+iB)\cot^2(c+dx)}{d}+\frac{3a^2(-3B+5iA)\cot(c+dx)}{d}+3a^2x(-3B+5iA)}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{8a^2(2A+iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{4a^2(2A+iB)\cot^2(c+dx)}{d}+\frac{3a^2(-3B+5iA)\cot(c+dx)}{d}+3a^2x(-3B+5iA)}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{-\frac{4a^2(2A+iB)\cot^2(c+dx)}{d}+\frac{3a^2(-3B+5iA)\cot(c+dx)}{d}-\frac{8a^2(2A+iB)\log(-\sin(c+dx))}{d}+3a^2x(-3B+5iA)}{2a^2}+\frac{(5A+3iB)\cot^2(c+dx)}{2d(1+i\tan(c+dx))}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((3*a^2*((5*I)*A - 3*B)*x + (3*a^2*((5*I)*A - 3*B)*Cot[c + d*x])/d - (4*a^
2*(2*A + I*B)*Cot[c + d*x]^2)/d - (8*a^2*(2*A + I*B)*Log[-Sin[c + d*x]])/d
)/(2*a^2) + ((5*A + (3*I)*B)*Cot[c + d*x]^2)/(2*d*(1 + I*Tan[c + d*x])))/(
2*a^2) + ((A + I*B)*Cot[c + d*x]^2)/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{17xB}{4a^2}+\frac{31ixA}{4a^2}-\frac{3i\mathrm{e}^{-2i(dx+c)}B}{4a^2d}-\frac{\mathrm{e}^{-2i(dx+c)}A}{a^2d}-\frac{i\mathrm{e}^{-4i(dx+c)}B}{16a^2d}-\frac{\mathrm{e}^{-4i(dx+c)}A}{16a^2d}-\frac{4Bc}{a^2d}+\frac{8iAc}{a^2d}-\frac{2i(}{}$
derivativedivides	$-\frac{5B}{4d\,a^2(-i+\tan(dx+c))}+\frac{7iA}{4d\,a^2(-i+\tan(dx+c))}+\frac{A}{4d\,a^2(-i+\tan(dx+c))^2}+\frac{15iA\arctan(\tan(dx+c))}{4d\,a^2}+\frac{}{4d\,a}$
default	$-\frac{5B}{4d\,a^2(-i+\tan(dx+c))}+\frac{7iA}{4d\,a^2(-i+\tan(dx+c))}+\frac{A}{4d\,a^2(-i+\tan(dx+c))^2}+\frac{15iA\arctan(\tan(dx+c))}{4d\,a^2}+\frac{}{4d\,a}$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVER
BOSE)
```

output
```
-17/4*x/a^2*B+31/4*I*x/a^2*A-3/4*I/a^2/d*exp(-2*I*(d*x+c))*B-1/a^2/d*exp(-
2*I*(d*x+c))*A-1/16*I/a^2/d*exp(-4*I*(d*x+c))*B-1/16/a^2/d*exp(-4*I*(d*x+c
))*A-4/a^2/d*B*c+8*I/a^2/d*A*c-2*I*(-I*A*exp(2*I*(d*x+c))+B*exp(2*I*(d*x+c
))+2*I*A-B)/a^2/d/(exp(2*I*(d*x+c))-1)^2-2*I/a^2/d*ln(exp(2*I*(d*x+c))-1)*
B-4/a^2*A/d*ln(exp(2*I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.26

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{4\,(-31i\,A+17\,B)dxe^{(8i\,dx+8i\,c)}+4\,(2\,(31i\,A-17\,B)dx+12\,A+11i\,B)e^{(6i\,dx+6i\,c)}+(4\,(-31i\,A+17\,\ldots}{}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"fricas")
```

output
```
-1/16*(4*(-31*I*A + 17*B)*d*x*e^(8*I*d*x + 8*I*c) + 4*(2*(31*I*A - 17*B)*d
*x + 12*A + 11*I*B)*e^(6*I*d*x + 6*I*c) + (4*(-31*I*A + 17*B)*d*x - 95*A -
 55*I*B)*e^(4*I*d*x + 4*I*c) + 2*(7*A + 5*I*B)*e^(2*I*d*x + 2*I*c) + 32*((
2*A + I*B)*e^(8*I*d*x + 8*I*c) - 2*(2*A + I*B)*e^(6*I*d*x + 6*I*c) + (2*A
+ I*B)*e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c) - 1) + A + I*B)/(a^2*d
*e^(8*I*d*x + 8*I*c) - 2*a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*
I*c))
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.90

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{4A+2iB+\left(-2Ae^{2ic}-2iBe^{2ic}\right)e^{2idx}}{a^2de^{4ic}e^{4idx}-2a^2de^{2ic}e^{2idx}+a^2d} + \begin{cases} \frac{\left(\left(-4Aa^2de^{2ic}-4iBa^2de^{2ic}\right)e^{-4idx}+\left(-64Aa^2de^{4ic}-48iBa^2de^{4ic}\right)e^{-2idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(-\frac{31iA-17B}{4a^2}+\frac{\left(31iAe^{4ic}+8iAe^{2ic}+iA-17Be^{4ic}-6Be^{2ic}-B\right)e^{-4ic}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(31iA-17B)}{4a^2} - \frac{2\cdot(2A+iB)\log\left(e^{2idx}-e^{-2ic}\right)}{a^2d}$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
(4*A + 2*I*B + (-2*A*exp(2*I*c) - 2*I*B*exp(2*I*c))*exp(2*I*d*x))/(a**2*d*
exp(4*I*c)*exp(4*I*d*x) - 2*a**2*d*exp(2*I*c)*exp(2*I*d*x) + a**2*d) + Pie
cewise((((-4*A*a**2*d*exp(2*I*c) - 4*I*B*a**2*d*exp(2*I*c))*exp(-4*I*d*x)
+ (-64*A*a**2*d*exp(4*I*c) - 48*I*B*a**2*d*exp(4*I*c))*exp(-2*I*d*x))*exp(
-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*(-(31*I*A - 17*B)
/(4*a**2) + (31*I*A*exp(4*I*c) + 8*I*A*exp(2*I*c) + I*A - 17*B*exp(4*I*c)
- 6*B*exp(2*I*c) - B)*exp(-4*I*c)/(4*a**2)), True)) + x*(31*I*A - 17*B)/(4
*a**2) - 2*(2*A + I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/(a**2*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.88

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{(A-iB)\log(\tan(dx+c)+i)}{8a^2d} + \frac{(31A+17iB)\log(\tan(dx+c)-i)}{8a^2d} - \frac{2(2A+iB)\log(|\tan(dx+c)|)}{a^2d} - \frac{3(-5iA+3B)\tan(dx+c)^3-2(11A+7iB)\tan(dx+c)^2+4(iA-B)\tan(dx+c)-2A}{4a^2d(\tan(dx+c)-i)^2\tan(dx+c)^2}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"giac")
```

output
```
1/8*(A - I*B)*log(tan(d*x + c) + I)/(a^2*d) + 1/8*(31*A + 17*I*B)*log(tan(
d*x + c) - I)/(a^2*d) - 2*(2*A + I*B)*log(abs(tan(d*x + c)))/(a^2*d) - 1/4
*(3*(-5*I*A + 3*B)*tan(d*x + c)^3 - 2*(11*A + 7*I*B)*tan(d*x + c)^2 + 4*(I
*A - B)*tan(d*x + c) - 2*A)/(a^2*d*(tan(d*x + c) - I)^2*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{\tan(c+d\,x)^2\,\left(-\frac{7\,B}{2\,a^2}+\frac{A\,11\mathrm{i}}{2\,a^2}\right)-\tan(c+d\,x)^3\,\left(\frac{15\,A}{4\,a^2}+\frac{B\,9\mathrm{i}}{4\,a^2}\right)+\frac{A\,1\mathrm{i}}{2\,a^2}+\tan(c+d\,x)\,\left(\frac{A}{a^2}+\frac{B\,1\mathrm{i}}{a^2}\right)}{d\,\left(\tan\left(c+d\,x\right)^4\,1\mathrm{i}+2\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,1\mathrm{i}\right)}$$
$$-\frac{2\,\ln\left(\tan(c+d\,x)\right)\,\left(2\,A+B\,1\mathrm{i}\right)}{a^2\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{8\,a^2\,d}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(31\,A+B\,17\mathrm{i}\right)}{8\,a^2\,d}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(tan(c + d*x)^2*((A*11i)/(2*a^2) - (7*B)/(2*a^2)) - tan(c + d*x)^3*((15*A)
/(4*a^2) + (B*9i)/(4*a^2)) + (A*1i)/(2*a^2) + tan(c + d*x)*(A/a^2 + (B*1i)
/a^2))/(d*(2*tan(c + d*x)^3 - tan(c + d*x)^2*1i + tan(c + d*x)^4*1i)) - (2
*log(tan(c + d*x))*(2*A + B*1i))/(a^2*d) + (log(tan(c + d*x) + 1i)*(A - B*
1i))/(8*a^2*d) + (log(tan(c + d*x) - 1i)*(31*A + B*17i))/(8*a^2*d)
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{-\left(\int \frac{\cot(dx+c)^3}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a-\left(\int \frac{\cot(dx+c)^3\tan(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b}{a^2}$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(cot(c + d*x)**3/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a + i
nt((cot(c + d*x)**3*tan(c + d*x))/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1)
,x)*b))/a**2
```

3.51 $\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 191

$$\begin{aligned}\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{(7A+25iB)x}{8a^3}-\frac{(iA-3B)\log(\cos(c+dx))}{a^3d}+\frac{(7A+25iB)\tan(c+dx)}{8a^3d}\\&\quad+\frac{(iA-B)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{(5A+11iB)\tan^3(c+dx)}{24ad(a+ia\tan(c+dx))^2}-\frac{(iA-3B)\tan^2(c+dx)}{2d\,(a^3+ia^3\tan(c+dx))}\end{aligned}$$

output

```
-1/8*(7*A+25*I*B)*x/a^3-(I*A-3*B)*ln(cos(d*x+c))/a^3/d+1/8*(7*A+25*I*B)*ta
n(d*x+c)/a^3/d+1/6*(I*A-B)*tan(d*x+c)^4/d/(a+I*a*tan(d*x+c))^3+1/24*(5*A+1
1*I*B)*tan(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^2-1/2*(I*A-3*B)*tan(d*x+c)^2/d/
(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.41

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-3((15A+49iB)\log(i-\tan(c+dx))+(A-iB)\log(i+\tan(c+dx)))+3(14iA-50B+3(-15iA+\cdots}{}$$

input
```
Integrate[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x
]
```

output
```
(-3*((15*A + (49*I)*B)*Log[I - Tan[c + d*x]] + (A - I*B)*Log[I + Tan[c + d
*x]]) + 3*((14*I)*A - 50*B + 3*((-15*I)*A + 49*B)*Log[I - Tan[c + d*x]] +
((-3*I)*A - 3*B)*Log[I + Tan[c + d*x]])*Tan[c + d*x] + 3*(-2*(17*A + (63*I
)*B) + 3*(15*A + (49*I)*B)*Log[I - Tan[c + d*x]] + 3*(A - I*B)*Log[I + Tan
[c + d*x]])*Tan[c + d*x]^2 + ((-68*I)*A + 284*B + ((45*I)*A - 147*B)*Log[I
 - Tan[c + d*x]] + 3*(I*A + B)*Log[I + Tan[c + d*x]])*Tan[c + d*x]^3 + (48
*I)*B*Tan[c + d*x]^4)/(48*a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4078, 3042, 4078, 27, 3042, 4078, 27, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^4(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int\frac{\tan^3(c+dx)(4a(iA-B)+a(A+7iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{6a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int\frac{\tan(c+dx)^3(4a(iA-B)+a(A+7iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{6a^2}$$

$\Big\downarrow$ 4078

$$\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{-\frac{\int-\frac{3\tan^2(c+dx)\left(a^2(5A+11iB)-a^2(3iA-13B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}-\frac{a(5A+11iB)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\int\frac{\tan^2(c+dx)\left(a^2(5A+11iB)-a^2(3iA-13B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}-\frac{a(5A+11iB)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\int\frac{\tan(c+dx)^2\left(a^2(5A+11iB)-a^2(3iA-13B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}-\frac{a(5A+11iB)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$\Big\downarrow$ 4078

$$\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{4a^2(-3B+iA)\tan^2(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\int2\tan(c+dx)\left(8(iA-3B)a^3+(7A+25iB)\tan(c+dx)a^3\right)dx}{2a^2}\right)}{4a^2}-\frac{a(5A+11iB)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{4a^2(-3B+iA)\tan^2(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\int\tan(c+dx)\left(8(iA-3B)a^3+(7A+25iB)\tan(c+dx)a^3\right)dx}{a^2}\right)}{4a^2}-\frac{a(5A+11iB)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-}{\frac{3\left(\frac{4a^2(-3B+iA)\tan^2(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\int\tan(c+dx)\left(8(iA-3B)a^3+(7A+25iB)\tan(c+dx)a^3\right)dx}{a^2}\right)}{4a^2}-\frac{a(5A+11iB)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$$\Big\downarrow \textbf{4008}$$

$$\frac{\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-}{\frac{3\left(\frac{4a^2(-3B+iA)\tan^2(c+dx)}{d(a+ia\tan(c+dx))}-\frac{8a^3(-3B+iA)\int\tan(c+dx)dx+\frac{a^3(7A+25iB)\tan(c+dx)}{d}-\left(a^3x(7A+25iB)\right)}{a^2}\right)}{4a^2}-\frac{a(5A+11iB)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-}{\frac{3\left(\frac{4a^2(-3B+iA)\tan^2(c+dx)}{d(a+ia\tan(c+dx))}-\frac{8a^3(-3B+iA)\int\tan(c+dx)dx+\frac{a^3(7A+25iB)\tan(c+dx)}{d}-\left(a^3x(7A+25iB)\right)}{a^2}\right)}{4a^2}-\frac{a(5A+11iB)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$$\Big\downarrow \textbf{3956}$$

$$\frac{\frac{(-B+iA)\tan^4(c+dx)}{6d(a+ia\tan(c+dx))^3}-}{\frac{3\left(\frac{4a^2(-3B+iA)\tan^2(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\frac{a^3(7A+25iB)\tan(c+dx)}{d}-\frac{8a^3(-3B+iA)\log(\cos(c+dx))}{d}-\left(a^3x(7A+25iB)\right)}{a^2}\right)}{4a^2}-\frac{a(5A+11iB)\tan^3(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

input
```
Int[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I*A - B)*Tan[c + d*x]^4)/(6*d*(a + I*a*Tan[c + d*x])^3) - (-1/4*(a*(5*A
+ (11*I)*B)*Tan[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^2) + (3*((4*a^2*(I*A
 - 3*B)*Tan[c + d*x]^2)/(d*(a + I*a*Tan[c + d*x])) - (-(a^3*(7*A + (25*I)*
B)*x) - (8*a^3*(I*A - 3*B)*Log[Cos[c + d*x]])/d + (a^3*(7*A + (25*I)*B)*Ta
n[c + d*x])/d)/a^2))/(4*a^2))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{49ixB}{8a^3}-\frac{15xA}{8a^3}-\frac{23\,\mathrm{e}^{-2i(dx+c)}B}{16a^3d}+\frac{11i\mathrm{e}^{-2i(dx+c)}A}{16a^3d}+\frac{7\,\mathrm{e}^{-4i(dx+c)}B}{32a^3d}-\frac{5i\mathrm{e}^{-4i(dx+c)}A}{32a^3d}-\frac{\mathrm{e}^{-6i(dx+c)}B}{48a^3d}$
derivativedivides	$\frac{iB\tan(dx+c)}{d\,a^3}-\frac{A}{6d\,a^3(-i+\tan(dx+c))^3}-\frac{iB}{6d\,a^3(-i+\tan(dx+c))^3}-\frac{3B\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^3}-\frac{25iB\arctan(\tan}{8d\,a^3}$
default	$\frac{iB\tan(dx+c)}{d\,a^3}-\frac{A}{6d\,a^3(-i+\tan(dx+c))^3}-\frac{iB}{6d\,a^3(-i+\tan(dx+c))^3}-\frac{3B\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^3}-\frac{25iB\arctan(\tan}{8d\,a^3}$

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVER
BOSE)
```

output
```
-49/8*I*x/a^3*B-15/8*x/a^3*A-23/16/a^3/d*exp(-2*I*(d*x+c))*B+11/16*I/a^3/d
*exp(-2*I*(d*x+c))*A+7/32/a^3/d*exp(-4*I*(d*x+c))*B-5/32*I/a^3/d*exp(-4*I*
(d*x+c))*A-1/48/a^3/d*exp(-6*I*(d*x+c))*B+1/48*I/a^3/d*exp(-6*I*(d*x+c))*A
-6*I/a^3/d*B*c-2/a^3/d*A*c-2/d/a^3*B/(exp(2*I*(d*x+c))+1)+3/a^3/d*ln(exp(2
*I*(d*x+c))+1)*B-I/a^3/d*ln(exp(2*I*(d*x+c))+1)*A
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{12\,(15\,A+49i\,B)dxe^{(8i\,dx+8i\,c)}+6\,(2\,(15\,A+49i\,B)dx-11i\,A+55\,B)e^{(6i\,dx+6i\,c)}+3\,(-17i\,A+39\,B}{96\,(}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"fricas")
```

output
```
-1/96*(12*(15*A + 49*I*B)*d*x*e^(8*I*d*x + 8*I*c) + 6*(2*(15*A + 49*I*B)*d
*x - 11*I*A + 55*B)*e^(6*I*d*x + 6*I*c) + 3*(-17*I*A + 39*B)*e^(4*I*d*x +
4*I*c) - (-13*I*A + 19*B)*e^(2*I*d*x + 2*I*c) + 96*((I*A - 3*B)*e^(8*I*d*x
 + 8*I*c) + (I*A - 3*B)*e^(6*I*d*x + 6*I*c))*log(e^(2*I*d*x + 2*I*c) + 1)
- 2*I*A + 2*B)/(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*c))
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.76

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2B}{a^3de^{2ic}e^{2idx}+a^3d} + \begin{cases} \frac{((512iAa^6d^2e^{6ic}-512Ba^6d^2e^{6ic})e^{-6idx}+(-3840iAa^6d^2e^{8ic}+5376Ba^6d^2e^{8ic})e^{-4idx}+(16896iAa^6d^2e^{10ic}-35328Ba^6d^2e^{10ic})e^{-2idx})e}{24576a^9d^3} \\ x\left(-\frac{-15A-49iB}{8a^3}+\frac{(-15Ae^{6ic}+11Ae^{4ic}-5Ae^{2ic}+A-49iBe^{6ic}+23iBe^{4ic}-7iBe^{2ic}+iB)e^{-6ic}}{8a^3}\right) \end{cases} + \frac{x(-15A-49iB)}{8a^3} - \frac{i(A+3iB)\log\left(e^{2idx}+e^{-2ic}\right)}{a^3d}$$

input
```
integrate(tan(d*x+c)**4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
-2*B/(a**3*d*exp(2*I*c)*exp(2*I*d*x) + a**3*d) + Piecewise((((512*I*A*a**6
*d**2*exp(6*I*c) - 512*B*a**6*d**2*exp(6*I*c))*exp(-6*I*d*x) + (-3840*I*A*
a**6*d**2*exp(8*I*c) + 5376*B*a**6*d**2*exp(8*I*c))*exp(-4*I*d*x) + (16896
*I*A*a**6*d**2*exp(10*I*c) - 35328*B*a**6*d**2*exp(10*I*c))*exp(-2*I*d*x))
*exp(-12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*(-(-15*
A - 49*I*B)/(8*a**3) + (-15*A*exp(6*I*c) + 11*A*exp(4*I*c) - 5*A*exp(2*I*c
) + A - 49*I*B*exp(6*I*c) + 23*I*B*exp(4*I*c) - 7*I*B*exp(2*I*c) + I*B)*ex
p(-6*I*c)/(8*a**3)), True)) + x*(-15*A - 49*I*B)/(8*a**3) - I*(A + 3*I*B)*
log(exp(2*I*d*x) + exp(-2*I*c))/(a**3*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.63

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= -\frac{(-iA-B)\log(\tan(dx+c)+i)}{16a^3d} - \frac{(-15iA+49B)\log(\tan(dx+c)-i)}{16a^3d} + \frac{iB\tan(dx+c)}{a^3d} + \frac{3(17A+31iB)\tan(dx+c)^2 - 3(27iA-53B)\tan(dx+c) - 34A - 70iB}{24a^3d(\tan(dx+c)-i)^3}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"giac")
```

output
```
-1/16*(-I*A - B)*log(tan(d*x + c) + I)/(a^3*d) - 1/16*(-15*I*A + 49*B)*log
(tan(d*x + c) - I)/(a^3*d) + I*B*tan(d*x + c)/(a^3*d) + 1/24*(3*(17*A + 31
*I*B)*tan(d*x + c)^2 - 3*(27*I*A - 53*B)*tan(d*x + c) - 34*A - 70*I*B)/(a^
3*d*(tan(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\tan(c+dx)\left(\frac{B\,7\mathrm{i}}{2a^3} + \frac{(-3B+A\,1\mathrm{i})\,27\mathrm{i}}{8a^3}\right) + \frac{4B}{3a^3} - \tan(c+dx)^2\left(\frac{5B}{2a^3} + \frac{17(-3B+A\,1\mathrm{i})}{8a^3}\right) + \frac{17(-3B+A\,1\mathrm{i})}{12a^3}}{d\left(-\tan(c+dx)^3\,1\mathrm{i} - 3\tan(c+dx)^2 + \tan(c+dx)\,3\mathrm{i} + 1\right)} + \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(B+A\,1\mathrm{i})}{16a^3d} + \frac{B\tan(c+dx)\,1\mathrm{i}}{a^3d} + \frac{\ln(\tan(c+dx)-\mathrm{i})\,(-49B+A\,15\mathrm{i})}{16a^3d}$$

input
```
int((tan(c + d*x)^4*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(tan(c + d*x)*((B*7i)/(2*a^3) + ((A*1i - 3*B)*27i)/(8*a^3)) + (4*B)/(3*a^3
) - tan(c + d*x)^2*((5*B)/(2*a^3) + (17*(A*1i - 3*B))/(8*a^3)) + (17*(A*1i
 - 3*B))/(12*a^3))/(d*(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3
*1i + 1)) + (log(tan(c + d*x) + 1i)*(A*1i + B))/(16*a^3*d) + (B*tan(c + d*
x)*1i)/(a^3*d) + (log(tan(c + d*x) - 1i)*(A*15i - 49*B))/(16*a^3*d)
```

Reduce [F]

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-12\left(\int \frac{\tan(dx+c)^2}{\tan(dx+c)^3-3\tan(dx+c)^2i-3\tan(dx+c)+i}dx\right)adi+20\left(\int \frac{\tan(dx+c)^2}{\tan(dx+c)^3-3\tan(dx+c)^2i-3\tan(dx+c)+i}dx\right)bd-16\left(\int\right.}{}$$

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 12*int(tan(c + d*x)**2/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(
c + d*x) + i),x)*a*d*i + 20*int(tan(c + d*x)**2/(tan(c + d*x)**3 - 3*tan(c
 + d*x)**2*i - 3*tan(c + d*x) + i),x)*b*d - 16*int(tan(c + d*x)/(tan(c + d
*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x)*a*d - 30*int(tan(c +
 d*x)/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x)*b*d*
i + 6*int(1/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x
)*a*d*i - 12*int(1/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x)
 + i),x)*b*d + log(tan(c + d*x)**2 + 1)*a*i - 3*log(tan(c + d*x)**2 + 1)*b
 + 2*tan(c + d*x)*b*i - 6*a*d*x - 14*b*d*i*x)/(2*a**3*d)
```

3.52 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 148

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{(iA-7B)x}{8a^3} - \frac{iB\log(\cos(c+dx))}{a^3d} + \frac{(iA-B)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{(A+3iB)\tan^2(c+dx)}{8ad(a+ia\tan(c+dx))^2} + \frac{A+7iB}{8d\,(a^3+ia^3\tan(c+dx))}$$

output
```
1/8*(I*A-7*B)*x/a^3-I*B*ln(cos(d*x+c))/a^3/d+1/6*(I*A-B)*tan(d*x+c)^3/d/(a
+I*a*tan(d*x+c))^3+1/8*(A+3*I*B)*tan(d*x+c)^2/a/d/(a+I*a*tan(d*x+c))^2+1/8
*(A+7*I*B)/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{i\Big(3i((A+15iB)\log(i-\tan(c+dx))-(A-iB)\log(i+\tan(c+dx)))+6(A+7iB)\tan(c+dx)+}{4}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x
]
```

output
```
((-1/48*I)*((3*I)*((A + (15*I)*B)*Log[I - Tan[c + d*x]] - (A - I*B)*Log[I
+ Tan[c + d*x]]) + 6*(A + (7*I)*B)*Tan[c + d*x] + 24*B*Tan[c + d*x]^2 - 2*
(A + (7*I)*B)*Tan[c + d*x]^3 + (2*Tan[c + d*x]^4*(-6*(A + I*B) + ((-3*I)*A
 + 9*B)*Tan[c + d*x] + (A + (7*I)*B)*Tan[c + d*x]^2))/(-I + Tan[c + d*x])^
3))/(a^3*d)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4072, 27, 3042, 3956, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{4078}$$
$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{3\tan^2(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{6a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int\frac{\tan^2(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int\frac{\tan(c+dx)^2(a(iA-B)+2iaB\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{2a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{-\frac{\int-\frac{2\tan(c+dx)\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{i\tan(c+dx)a+a}dx}{4a^2}-\frac{a(A+3iB)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{2a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\int\frac{\tan(c+dx)\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(A+3iB)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\int\frac{\tan(c+dx)\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(A+3iB)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{2a^2}$$

$$\Big\downarrow \text{4072}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{-\frac{i\int\frac{a^3(iA-7B)\tan(c+dx)}{i\tan(c+dx)a+a}dx}{a}-4iaB\int\tan(c+dx)dx}{2a^2}-\frac{a(A+3iB)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{2a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{-ia^2(-7B+iA)\int\frac{\tan(c+dx)}{i\tan(c+dx)a+a}dx-4iaB\int\tan(c+dx)dx}{2a^2}-\frac{a(A+3iB)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{-ia^2(-7B+iA)\int\frac{\tan(c+dx)}{i\tan(c+dx)a+a}dx-4iaB\int\tan(c+dx)dx}{2a^2}-\frac{a(A+3iB)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{2a^2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{4iaB\log(\cos(c+dx))}{d}-ia^2(-7B+iA)\int\frac{\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(A+3iB)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{2a^2}$$

$$\Big\downarrow \text{4009}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{4iaB\log(\cos(c+dx))}{d}-ia^2(-7B+iA)\left(-\frac{i\int 1dx}{2a}-\frac{1}{2d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(A+3iB)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{2a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{4iaB\log(\cos(c+dx))}{d}-ia^2(-7B+iA)\left(-\frac{1}{2d(a+ia\tan(c+dx))}-\frac{ix}{2a}\right)}{2a^2}-\frac{a(A+3iB)\tan^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{2a^2}$$

input

```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output

```
((I*A - B)*Tan[c + d*x]^3)/(6*d*(a + I*a*Tan[c + d*x])^3) - (-1/4*(a*(A +
(3*I)*B)*Tan[c + d*x]^2)/(d*(a + I*a*Tan[c + d*x])^2) + (((4*I)*a*B*Log[Co
s[c + d*x]])/d - I*a^2*(I*A - 7*B)*(((-1/2*I)*x)/a - 1/(2*d*(a + I*a*Tan[c
 + d*x]))))/(2*a^2))/(2*a^2)
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{15xB}{8a^3}+\frac{ixA}{8a^3}+\frac{11i\,\mathrm{e}^{-2i(dx+c)}B}{16a^3d}+\frac{3\,\mathrm{e}^{-2i(dx+c)}A}{16a^3d}-\frac{5i\,\mathrm{e}^{-4i(dx+c)}B}{32a^3d}-\frac{3\,\mathrm{e}^{-4i(dx+c)}A}{32a^3d}+\frac{i\,\mathrm{e}^{-6i(dx+c)}B}{48a^3d}+$
derivativedivides	$\frac{iA\arctan(\tan(dx+c))}{8d\,a^3}+\frac{iB\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^3}-\frac{7B\arctan(\tan(dx+c))}{8d\,a^3}+\frac{17B}{8d\,a^3(-i+\tan(dx+c))}-$
default	$\frac{iA\arctan(\tan(dx+c))}{8d\,a^3}+\frac{iB\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^3}-\frac{7B\arctan(\tan(dx+c))}{8d\,a^3}+\frac{17B}{8d\,a^3(-i+\tan(dx+c))}-$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVER
BOSE)
```

output
```
-15/8*x/a^3*B+1/8*I*x/a^3*A+11/16*I/a^3/d*exp(-2*I*(d*x+c))*B+3/16/a^3/d*e
xp(-2*I*(d*x+c))*A-5/32*I/a^3/d*exp(-4*I*(d*x+c))*B-3/32/a^3/d*exp(-4*I*(d
*x+c))*A+1/48*I/a^3/d*exp(-6*I*(d*x+c))*B+1/48/a^3/d*exp(-6*I*(d*x+c))*A-2
*B/a^3/d*c-I*B/a^3/d*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.70

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(12\,(-i\,A+15\,B)dxe^{(6i\,dx+6i\,c)}+96i\,Be^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-6\,(3\,A+11i\,B)e^{(4i\,dx+4i\,c)}+3}{96\,a^3d}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"fricas")
```

output
```
-1/96*(12*(-I*A + 15*B)*d*x*e^(6*I*d*x + 6*I*c) + 96*I*B*e^(6*I*d*x + 6*I*
c)*log(e^(2*I*d*x + 2*I*c) + 1) - 6*(3*A + 11*I*B)*e^(4*I*d*x + 4*I*c) + 3
*(3*A + 5*I*B)*e^(2*I*d*x + 2*I*c) - 2*A - 2*I*B)*e^(-6*I*d*x - 6*I*c)/(a^
3*d)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.00

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{iB\log\left(e^{2idx}+e^{-2ic}\right)}{a^3d} + \begin{cases} \frac{((512Aa^6d^2e^{6ic}+512iBa^6d^2e^{6ic})e^{-6idx}+(-2304Aa^6d^2e^{8ic}-3840iBa^6d^2e^{8ic})e^{-4idx}+(4608Aa^6d^2e^{10ic}+16896iBa^6d^2e^{10ic})e^{-2idx})e^{-}}{24576a^9d^3} \\ x\left(-\frac{iA-15B}{8a^3}+\frac{(iAe^{6ic}-3iAe^{4ic}+3iAe^{2ic}-iA-15Be^{6ic}+11Be^{4ic}-5Be^{2ic}+B)e^{-6ic}}{8a^3}\right) \end{cases} + \frac{x(iA-15B)}{8a^3}$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
-I*B*log(exp(2*I*d*x) + exp(-2*I*c))/(a**3*d) + Piecewise((((512*A*a**6*d*
*2*exp(6*I*c) + 512*I*B*a**6*d**2*exp(6*I*c))*exp(-6*I*d*x) + (-2304*A*a**
6*d**2*exp(8*I*c) - 3840*I*B*a**6*d**2*exp(8*I*c))*exp(-4*I*d*x) + (4608*A
*a**6*d**2*exp(10*I*c) + 16896*I*B*a**6*d**2*exp(10*I*c))*exp(-2*I*d*x))*e
xp(-12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*(-(I*A -
15*B)/(8*a**3) + (I*A*exp(6*I*c) - 3*I*A*exp(4*I*c) + 3*I*A*exp(2*I*c) - I
*A - 15*B*exp(6*I*c) + 11*B*exp(4*I*c) - 5*B*exp(2*I*c) + B)*exp(-6*I*c)/(
8*a**3)), True)) + x*(I*A - 15*B)/(8*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.68

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(A-iB)\log(\tan(dx+c)+i)}{16a^3d} + \frac{(A+15iB)\log(\tan(dx+c)-i)}{16a^3d} - \frac{3(7iA-17B)\tan(dx+c)^2+27(A+3iB)\tan(dx+c)-10iA+34B}{24a^3d(\tan(dx+c)-i)^3}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"giac")
```

output
```
-1/16*(A - I*B)*log(tan(d*x + c) + I)/(a^3*d) + 1/16*(A + 15*I*B)*log(tan(
d*x + c) - I)/(a^3*d) - 1/24*(3*(7*I*A - 17*B)*tan(d*x + c)^2 + 27*(A + 3*
I*B)*tan(d*x + c) - 10*I*A + 34*B)/(a^3*d*(tan(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{5A}{12a^3} - \tan(c+dx)^2\left(\frac{7A}{8a^3} + \frac{B\,17\mathrm{i}}{8a^3}\right) + \frac{B\,17\mathrm{i}}{12a^3} + \tan(c+dx)\left(-\frac{27B}{8a^3} + \frac{A\,9\mathrm{i}}{8a^3}\right)}{d\left(-\tan(c+dx)^3\,1\mathrm{i} - 3\tan(c+dx)^2 + \tan(c+dx)\,3\mathrm{i} + 1\right)} - \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(A-B\,1\mathrm{i})}{16a^3d} + \frac{\ln(\tan(c+dx)-\mathrm{i})\,(A+B\,15\mathrm{i})}{16a^3d}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
((5*A)/(12*a^3) - tan(c + d*x)^2*((7*A)/(8*a^3) + (B*17i)/(8*a^3)) + (B*17
i)/(12*a^3) + tan(c + d*x)*((A*9i)/(8*a^3) - (27*B)/(8*a^3)))/(d*(tan(c +
d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*1i + 1)) - (log(tan(c + d*x) +
 1i)*(A - B*1i))/(16*a^3*d) + (log(tan(c + d*x) - 1i)*(A + B*15i))/(16*a^3
*d)
```

Reduce [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\tan(dx+c)^4}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c) i-1}dx\right) b - \left(\int \frac{\tan(dx+c)^3}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c) i-1}dx\right) a}{a^3}$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int(tan(c + d*x)**4/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c
+ d*x)*i - 1),x)*b + int(tan(c + d*x)**3/(tan(c + d*x)**3*i + 3*tan(c + d*
x)**2 - 3*tan(c + d*x)*i - 1),x)*a))/a**3
```

3.53 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 124

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(A-iB)x}{8a^3}+\frac{(iA-B)\tan^2(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{iA-7B}{24ad(a+ia\tan(c+dx))^2} + \frac{iA+17B}{24d\,(a^3+ia^3\tan(c+dx))}$$

output
```
-1/8*(A-I*B)*x/a^3+1/6*(I*A-B)*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))^3+1/24*(I
*A-7*B)/a/d/(a+I*a*tan(d*x+c))^2+1/24*(I*A+17*B)/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^3(c+dx)(-9(A-iB)\cos(c+dx)+2(A+iB-6iAdx-6Bdx)\cos(3(c+dx))-3iA\sin(c+dx)-}{96a^3d(-i}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x
]
```

output
```
(Sec[c + d*x]^3*(-9*(A - I*B)*Cos[c + d*x] + 2*(A + I*B - (6*I)*A*d*x - 6*
B*d*x)*Cos[3*(c + d*x)] - (3*I)*A*Sin[c + d*x] - 27*B*Sin[c + d*x] - (2*I)
*A*Sin[3*(c + d*x)] + 2*B*Sin[3*(c + d*x)] + 12*A*d*x*Sin[3*(c + d*x)] - (
12*I)*B*d*x*Sin[3*(c + d*x)]))/(96*a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4078, 3042, 4073, 3042, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan(c+dx)(2a(iA-B)-a(A-5iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{6a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan(c+dx)(2a(iA-B)-a(A-5iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{6a^2}$$

$$\Big\downarrow \text{4073}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{-\frac{i\int \frac{a^2(iA-7B)-2a^2(A-5iB)\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(-7B+iA)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{-\frac{i\int\frac{a^2(iA-7B)-2a^2(A-5iB)\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(-7B+iA)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$$\Big\downarrow \; 4009$$

$$\frac{(-B+iA)\tan^2(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{-\frac{i\left(\frac{3}{2}a(B+iA)\int 1dx+\frac{a^2(A-17iB)}{2d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(-7B+iA)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

$$\Big\downarrow \; 24$$

$$\frac{(-B+iA)\tan^2(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{-\frac{i\left(\frac{a^2(A-17iB)}{2d(a+ia\tan(c+dx))}+\frac{3}{2}ax(B+iA)\right)}{2a^2}-\frac{a(-7B+iA)}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I*A - B)*Tan[c + d*x]^2)/(6*d*(a + I*a*Tan[c + d*x])^3) - (-1/4*(a*(I*A
- 7*B))/(d*(a + I*a*Tan[c + d*x])^2) - ((I/2)*((3*a*(I*A + B)*x)/2 + (a^2*
(A - (17*I)*B))/(2*d*(a + I*a*Tan[c + d*x]))))/a^2)/(6*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03

method	result
risch	$\frac{ixB}{8a^3} - \frac{xA}{8a^3} + \frac{3\,\mathrm{e}^{-2i(dx+c)}B}{16a^3d} + \frac{i\mathrm{e}^{-2i(dx+c)}A}{16a^3d} - \frac{3\,\mathrm{e}^{-4i(dx+c)}B}{32a^3d} + \frac{i\mathrm{e}^{-4i(dx+c)}A}{32a^3d} + \frac{\mathrm{e}^{-6i(dx+c)}B}{48a^3d} - \frac{i\mathrm{e}^{-6i(dx}}{48a^3}$
derivativedivides	$-\frac{A}{8d\,a^3(-i+\tan(dx+c))} - \frac{7iB}{8d\,a^3(-i+\tan(dx+c))} + \frac{A}{6d\,a^3(-i+\tan(dx+c))^3} + \frac{iB}{6d\,a^3(-i+\tan(dx+c))^3} - \frac{A\,\mathrm{arc}}{}$
default	$-\frac{A}{8d\,a^3(-i+\tan(dx+c))} - \frac{7iB}{8d\,a^3(-i+\tan(dx+c))} + \frac{A}{6d\,a^3(-i+\tan(dx+c))^3} + \frac{iB}{6d\,a^3(-i+\tan(dx+c))^3} - \frac{A\,\mathrm{arc}}{}$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVER
BOSE)
```

output
```
1/8*I*x/a^3*B-1/8*x/a^3*A+3/16/a^3/d*exp(-2*I*(d*x+c))*B+1/16*I/a^3/d*exp(
-2*I*(d*x+c))*A-3/32/a^3/d*exp(-4*I*(d*x+c))*B+1/32*I/a^3/d*exp(-4*I*(d*x+
c))*A+1/48/a^3/d*exp(-6*I*(d*x+c))*B-1/48*I/a^3/d*exp(-6*I*(d*x+c))*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\left(12\,(A-i\,B)dxe^{(6i\,dx+6i\,c)}+6\,(-i\,A-3\,B)e^{(4i\,dx+4i\,c)}+3\,(-i\,A+3\,B)e^{(2i\,dx+2i\,c)}+2i\,A-2\,B\right)e^{(-6i\,d}}{96\,a^3d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"fricas")
```

output
```
-1/96*(12*(A - I*B)*d*x*e^(6*I*d*x + 6*I*c) + 6*(-I*A - 3*B)*e^(4*I*d*x +
4*I*c) + 3*(-I*A + 3*B)*e^(2*I*d*x + 2*I*c) + 2*I*A - 2*B)*e^(-6*I*d*x - 6
*I*c)/(a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.08

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{((-512iAa^6d^2e^{6ic}+512Ba^6d^2e^{6ic})e^{-6idx}+(768iAa^6d^2e^{8ic}-2304Ba^6d^2e^{8ic})e^{-4idx}+(1536iAa^6d^2e^{10ic}+4608Ba^6d^2e^{10ic})e^{-2idx})e^{-12ic}}{24576a^9d^3} \\ x\left(-\frac{-A+iB}{8a^3}+\frac{(-Ae^{6ic}+Ae^{4ic}+Ae^{2ic}-A+iBe^{6ic}-3iBe^{4ic}+3iBe^{2ic}-iB)e^{-6ic}}{8a^3}\right) \end{cases} + \frac{x(-A+iB)}{8a^3}$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((((-512*I*A*a**6*d**2*exp(6*I*c) + 512*B*a**6*d**2*exp(6*I*c))*e
xp(-6*I*d*x) + (768*I*A*a**6*d**2*exp(8*I*c) - 2304*B*a**6*d**2*exp(8*I*c)
)*exp(-4*I*d*x) + (1536*I*A*a**6*d**2*exp(10*I*c) + 4608*B*a**6*d**2*exp(1
0*I*c))*exp(-2*I*d*x))*exp(-12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12
*I*c), 0)), (x*(-(-A + I*B)/(8*a**3) + (-A*exp(6*I*c) + A*exp(4*I*c) + A*e
xp(2*I*c) - A + I*B*exp(6*I*c) - 3*I*B*exp(4*I*c) + 3*I*B*exp(2*I*c) - I*B
)*exp(-6*I*c)/(8*a**3)), True)) + x*(-A + I*B)/(8*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(iA+B)\log(\tan(dx+c)+i)}{16\,a^3d} - \frac{(-iA-B)\log(\tan(dx+c)-i)}{16\,a^3d} - \frac{3\,(A+7i\,B)\tan(dx+c)^2+3\,(i\,A+9\,B)\tan(dx+c)+2\,A-10i\,B}{24\,a^3d(\tan(dx+c)-i)^3}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"giac")
```

output
```
-1/16*(I*A + B)*log(tan(d*x + c) + I)/(a^3*d) - 1/16*(-I*A - B)*log(tan(d*
x + c) - I)/(a^3*d) - 1/24*(3*(A + 7*I*B)*tan(d*x + c)^2 + 3*(I*A + 9*B)*t
an(d*x + c) + 2*A - 10*I*B)/(a^3*d*(tan(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\tan(c+d\,x)^2\,\left(-\frac{7\,B}{8\,a^3}+\frac{A\,1\mathrm{i}}{8\,a^3}\right)+\frac{A\,1\mathrm{i}}{12\,a^3}+\frac{5\,B}{12\,a^3}-\tan(c+d\,x)\,\left(\frac{A}{8\,a^3}-\frac{B\,9\mathrm{i}}{8\,a^3}\right)}{d\,\left(-{\tan\left(c+d\,x\right)}^3\,1\mathrm{i}-3\,{\tan\left(c+d\,x\right)}^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1\right)}$$
$$+\frac{x\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{8\,a^3}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(tan(c + d*x)^2*((A*1i)/(8*a^3) - (7*B)/(8*a^3)) + (A*1i)/(12*a^3) + (5*B)
/(12*a^3) - tan(c + d*x)*(A/(8*a^3) - (B*9i)/(8*a^3)))/(d*(tan(c + d*x)*3i
 - 3*tan(c + d*x)^2 - tan(c + d*x)^3*1i + 1)) + (x*(A*1i + B)*1i)/(8*a^3)
```

Reduce [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\tan(dx+c)^3}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\tan(dx+c)^2}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)a}{a^3}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int(tan(c + d*x)**3/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c
+ d*x)*i - 1),x)*b + int(tan(c + d*x)**2/(tan(c + d*x)**3*i + 3*tan(c + d*
x)**2 - 3*tan(c + d*x)*i - 1),x)*a))/a**3
```

3.54 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 110

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(iA+B)x}{8a^3} - \frac{A+iB}{6d(a+ia\tan(c+dx))^3} + \frac{A+3iB}{8ad(a+ia\tan(c+dx))^2} + \frac{A-iB}{8d(a^3+ia^3\tan(c+dx))}$$

output
```
-1/8*(I*A+B)*x/a^3-1/6*(A+I*B)/d/(a+I*a*tan(d*x+c))^3+1/8*(A+3*I*B)/a/d/(a
+I*a*tan(d*x+c))^2+1/8*(A-I*B)/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{(\cos(3(c+dx))-i\sin(3(c+dx)))(3(A+3iB)\cos(c+dx)-2(A+6iAdx+B(i+6dx))\cos(3(c+dx)}{}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)])*(3*(A + (3*I)*B)*Cos[c + d*x] - 2
*(A + (6*I)*A*d*x + B*(I + 6*d*x))*Cos[3*(c + d*x)] + (9*I)*A*Sin[c + d*x]
 - 3*B*Sin[c + d*x] + (2*I)*A*Sin[3*(c + d*x)] - 2*B*Sin[3*(c + d*x)] + 12
*A*d*x*Sin[3*(c + d*x)] - (12*I)*B*d*x*Sin[3*(c + d*x)]))/(96*a^3*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3042, 4073, 3042, 4009, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4073}$$

$$-\frac{i\int \frac{a(A+iB)+2aB\tan(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{2a^2}-\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \frac{a(A+iB)+2aB\tan(c+dx)}{(i\tan(c+dx)a+a)^2}dx}{2a^2}-\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4009}$$

$$-\frac{i\left(\frac{1}{2}(A-iB)\int \frac{1}{i\tan(c+dx)a+a}dx+\frac{a(-3B+iA)}{4d(a+ia\tan(c+dx))^2}\right)}{2a^2}-\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}(A-iB)\int \frac{1}{i\tan(c+dx)a+a}dx+\frac{a(-3B+iA)}{4d(a+ia\tan(c+dx))^2}\right)}{2a^2}-\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3960}$$

$$-\frac{i\left(\frac{1}{2}(A-iB)\left(\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}\right)+\frac{a(-3B+iA)}{4d(a+ia\tan(c+dx))^2}\right)}{2a^2}-\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 24$$

$$-\frac{i\left(\frac{a(-3B+iA)}{4d(a+ia\tan(c+dx))^2}+\frac{1}{2}(A-iB)\left(\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}\right)\right)}{2a^2}-\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
-1/6*(A + I*B)/(d*(a + I*a*Tan[c + d*x])^3) - ((I/2)*((a*(I*A - 3*B))/(4*d
*(a + I*a*Tan[c + d*x])^2) + ((A - I*B)*(x/(2*a) + (I/2)/(d*(a + I*a*Tan[c
 + d*x]))))/2))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{xB}{8a^3}-\frac{ixA}{8a^3}+\frac{i\mathrm{e}^{-2i(dx+c)}B}{16a^3d}+\frac{\mathrm{e}^{-2i(dx+c)}A}{16a^3d}+\frac{i\mathrm{e}^{-4i(dx+c)}B}{32a^3d}-\frac{\mathrm{e}^{-4i(dx+c)}A}{32a^3d}-\frac{i\mathrm{e}^{-6i(dx+c)}B}{48a^3d}-\frac{\mathrm{e}^{-6i(dx-}}{48a^3}$
derivativedivides	$-\frac{iA\arctan(\tan(dx+c))}{8d\,a^3}-\frac{B\arctan(\tan(dx+c))}{8d\,a^3}-\frac{iA}{8d\,a^3(-i+\tan(dx+c))}-\frac{B}{8d\,a^3(-i+\tan(dx+c))}-\frac{}{6d\,a^3(-}$
default	$-\frac{iA\arctan(\tan(dx+c))}{8d\,a^3}-\frac{B\arctan(\tan(dx+c))}{8d\,a^3}-\frac{iA}{8d\,a^3(-i+\tan(dx+c))}-\frac{B}{8d\,a^3(-i+\tan(dx+c))}-\frac{}{6d\,a^3(-}$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
-1/8*x/a^3*B-1/8*I*x/a^3*A+1/16*I/a^3/d*exp(-2*I*(d*x+c))*B+1/16/a^3/d*exp
(-2*I*(d*x+c))*A+1/32*I/a^3/d*exp(-4*I*(d*x+c))*B-1/32/a^3/d*exp(-4*I*(d*x
+c))*A-1/48*I/a^3/d*exp(-6*I*(d*x+c))*B-1/48/a^3/d*exp(-6*I*(d*x+c))*A
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\left(12\,(i\,A+B)dxe^{(6i\,dx+6i\,c)}-6\,(A+i\,B)e^{(4i\,dx+4i\,c)}+3\,(A-i\,B)e^{(2i\,dx+2i\,c)}+2\,A+2i\,B\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/96*(12*(I*A + B)*d*x*e^(6*I*d*x + 6*I*c) - 6*(A + I*B)*e^(4*I*d*x + 4*I
*c) + 3*(A - I*B)*e^(2*I*d*x + 2*I*c) + 2*A + 2*I*B)*e^(-6*I*d*x - 6*I*c)/
(a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.36

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{((-512Aa^6d^2e^{6ic}-512iBa^6d^2e^{6ic})e^{-6idx}+(-768Aa^6d^2e^{8ic}+768iBa^6d^2e^{8ic})e^{-4idx}+(1536Aa^6d^2e^{10ic}+1536iBa^6d^2e^{10ic})e^{-2idx})e^{-12ic}}{24576a^9d^3} \\ x\left(-\frac{-iA-B}{8a^3}+\frac{(-iAe^{6ic}-iAe^{4ic}+iAe^{2ic}+iA-Be^{6ic}+Be^{4ic}+Be^{2ic}-B)e^{-6ic}}{8a^3}\right) \end{cases}$$
$$+\frac{x(-iA-B)}{8a^3}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((((-512*A*a**6*d**2*exp(6*I*c) - 512*I*B*a**6*d**2*exp(6*I*c))*e
xp(-6*I*d*x) + (-768*A*a**6*d**2*exp(8*I*c) + 768*I*B*a**6*d**2*exp(8*I*c)
)*exp(-4*I*d*x) + (1536*A*a**6*d**2*exp(10*I*c) + 1536*I*B*a**6*d**2*exp(1
0*I*c))*exp(-2*I*d*x))*exp(-12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12
*I*c), 0)), (x*(-(-I*A - B)/(8*a**3) + (-I*A*exp(6*I*c) - I*A*exp(4*I*c) +
 I*A*exp(2*I*c) + I*A - B*exp(6*I*c) + B*exp(4*I*c) + B*exp(2*I*c) - B)*ex
p(-6*I*c)/(8*a**3)), True)) + x*(-I*A - B)/(8*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{(A-iB)\log(\tan(dx+c)+i)}{16\,a^3d} - \frac{(A-iB)\log(\tan(dx+c)-i)}{16\,a^3d} - \frac{3\,(i\,A+B)\tan(dx+c)^2+3\,(3\,A+i\,B)\tan(dx+c)-2i\,A+2\,B}{24\,a^3d(\tan(dx+c)-i)^3}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="g
iac")
```

output
```
1/16*(A - I*B)*log(tan(d*x + c) + I)/(a^3*d) - 1/16*(A - I*B)*log(tan(d*x
+ c) - I)/(a^3*d) - 1/24*(3*(I*A + B)*tan(d*x + c)^2 + 3*(3*A + I*B)*tan(d
*x + c) - 2*I*A + 2*B)/(a^3*d*(tan(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.34

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{A}{12\,a^3}-\tan(c+d\,x)^2\,\left(\frac{A}{8\,a^3}-\frac{B\,1\mathrm{i}}{8\,a^3}\right)+\frac{B\,1\mathrm{i}}{12\,a^3}+\tan(c+d\,x)\,\left(-\frac{B}{8\,a^3}+\frac{A\,3\mathrm{i}}{8\,a^3}\right)}{d\,\left(-\tan\left(c+d\,x\right)^3\,1\mathrm{i}-3\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,a^3\,d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{16\,a^3\,d}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(A/(12*a^3) - tan(c + d*x)^2*(A/(8*a^3) - (B*1i)/(8*a^3)) + (B*1i)/(12*a^3
) + tan(c + d*x)*((A*3i)/(8*a^3) - B/(8*a^3)))/(d*(tan(c + d*x)*3i - 3*tan
(c + d*x)^2 - tan(c + d*x)^3*1i + 1)) + (log(tan(c + d*x) - 1i)*(A*1i + B)
*1i)/(16*a^3*d) + (log(tan(c + d*x) + 1i)*(A - B*1i))/(16*a^3*d)
```

Reduce [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\tan(dx+c)^2}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\tan(dx+c)}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)a}{a^3}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int(tan(c + d*x)**2/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c
+ d*x)*i - 1),x)*b + int(tan(c + d*x)/(tan(c + d*x)**3*i + 3*tan(c + d*x)*
*2 - 3*tan(c + d*x)*i - 1),x)*a))/a**3
```

3.55 $\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 112

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{(A-iB)x}{8a^3}+\frac{iA-B}{6d(a+ia\tan(c+dx))^3}+\frac{iA+B}{8ad(a+ia\tan(c+dx))^2}+\frac{iA+B}{8d\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
1/8*(A-I*B)*x/a^3+1/6*(I*A-B)/d/(a+I*a*tan(d*x+c))^3+1/8*(I*A+B)/a/d/(a+I*
a*tan(d*x+c))^2+1/8*(I*A+B)/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{-10A+2iB+3(iA+B)\arctan(\tan(c+dx))\sec^3(c+dx)(\cos(3(c+dx))+i\sin(3(c+dx)))+(-9iA}{24a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(-10*A + (2*I)*B + 3*(I*A + B)*ArcTan[Tan[c + d*x]]*Sec[c + d*x]^3*(Cos[3*
(c + d*x)] + I*Sin[3*(c + d*x)]) + ((-9*I)*A - 9*B)*Tan[c + d*x] + 3*(A -
I*B)*Tan[c + d*x]^2)/(24*a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4009, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4009}$$

$$\frac{(A-iB)\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{-B+iA}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{-B+iA}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3960}$$

$$\frac{(A-iB)\left(\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}\right)}{2a}+\frac{-B+iA}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\left(\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}\right)}{2a}+\frac{-B+iA}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3960}$$

$$\frac{(A-iB)\left(\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}\right)}{2a}+\frac{-B+iA}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{-B+iA}{6d(a+ia\tan(c+dx))^3}+\frac{(A-iB)\left(\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}\right)}{2a}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(I*A - B)/(6*d*(a + I*a*Tan[c + d*x])^3) + ((A - I*B)*((I/4)/(d*(a + I*a*T
an[c + d*x])^2) + (x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x])))/(2*a)))/(2*
a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{ixB}{8a^3}+\frac{xA}{8a^3}+\frac{e^{-2i(dx+c)}B}{16a^3d}+\frac{3ie^{-2i(dx+c)}A}{16a^3d}-\frac{e^{-4i(dx+c)}B}{32a^3d}+\frac{3ie^{-4i(dx+c)}A}{32a^3d}-\frac{e^{-6i(dx+c)}B}{48a^3d}+\frac{ie^{-6i(}}{48}$
derivativedivides	$\frac{A\arctan(\tan(dx+c))}{8d\,a^3}-\frac{iB\arctan(\tan(dx+c))}{8d\,a^3}+\frac{A}{8d\,a^3(-i+\tan(dx+c))}-\frac{iB}{8d\,a^3(-i+\tan(dx+c))}-\frac{}{6d\,a^3(-i+}$
default	$\frac{A\arctan(\tan(dx+c))}{8d\,a^3}-\frac{iB\arctan(\tan(dx+c))}{8d\,a^3}+\frac{A}{8d\,a^3(-i+\tan(dx+c))}-\frac{iB}{8d\,a^3(-i+\tan(dx+c))}-\frac{}{6d\,a^3(-i+}$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*x/a^3*B+1/8*x/a^3*A+1/16/a^3/d*exp(-2*I*(d*x+c))*B+3/16*I/a^3/d*exp
(-2*I*(d*x+c))*A-1/32/a^3/d*exp(-4*I*(d*x+c))*B+3/32*I/a^3/d*exp(-4*I*(d*x
+c))*A-1/48/a^3/d*exp(-6*I*(d*x+c))*B+1/48*I/a^3/d*exp(-6*I*(d*x+c))*A
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$=\frac{(12\,(A-i\,B)dxe^{(6i\,dx+6i\,c)}-6\,(-3i\,A-B)e^{(4i\,dx+4i\,c)}-3\,(-3i\,A+B)e^{(2i\,dx+2i\,c)}+2i\,A-2\,B)e^{(-6i\,dx-6}}{96\,a^3d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/96*(12*(A - I*B)*d*x*e^(6*I*d*x + 6*I*c) - 6*(-3*I*A - B)*e^(4*I*d*x + 4
*I*c) - 3*(-3*I*A + B)*e^(2*I*d*x + 2*I*c) + 2*I*A - 2*B)*e^(-6*I*d*x - 6*
I*c)/(a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.30

$$\int \frac{A + B\tan(c + dx)}{(a + ia\tan(c + dx))^3}\,dx$$
$$= \begin{cases} \frac{((512iAa^6d^2e^{6ic} - 512Ba^6d^2e^{6ic})e^{-6idx} + (2304iAa^6d^2e^{8ic} - 768Ba^6d^2e^{8ic})e^{-4idx} + (4608iAa^6d^2e^{10ic} + 1536Ba^6d^2e^{10ic})e^{-2idx})e^{-12ic}}{24576a^9d^3} \\ x\left(-\frac{A-iB}{8a^3} + \frac{(Ae^{6ic}+3Ae^{4ic}+3Ae^{2ic}+A-iBe^{6ic}-iBe^{4ic}+iBe^{2ic}+iB)e^{-6ic}}{8a^3}\right) \end{cases}$$
$$+ \frac{x(A - iB)}{8a^3}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Piecewise((((512*I*A*a**6*d**2*exp(6*I*c) - 512*B*a**6*d**2*exp(6*I*c))*ex
p(-6*I*d*x) + (2304*I*A*a**6*d**2*exp(8*I*c) - 768*B*a**6*d**2*exp(8*I*c))
*exp(-4*I*d*x) + (4608*I*A*a**6*d**2*exp(10*I*c) + 1536*B*a**6*d**2*exp(10
*I*c))*exp(-2*I*d*x))*exp(-12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*
I*c), 0)), (x*(-(A - I*B)/(8*a**3) + (A*exp(6*I*c) + 3*A*exp(4*I*c) + 3*A*
exp(2*I*c) + A - I*B*exp(6*I*c) - I*B*exp(4*I*c) + I*B*exp(2*I*c) + I*B)*e
xp(-6*I*c)/(8*a**3)), True)) + x*(A - I*B)/(8*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{(a + ia\tan(c + dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$=-\frac{(-iA-B)\log(\tan(dx+c)+i)}{16\,a^3d}-\frac{(iA+B)\log(\tan(dx+c)-i)}{16\,a^3d}$$
$$+\frac{3(A-iB)\tan(dx+c)^2-9(iA+B)\tan(dx+c)-10A+2iB}{24\,a^3d(\tan(dx+c)-i)^3}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/16*(-I*A - B)*log(tan(d*x + c) + I)/(a^3*d) - 1/16*(I*A + B)*log(tan(d*
x + c) - I)/(a^3*d) + 1/24*(3*(A - I*B)*tan(d*x + c)^2 - 9*(I*A + B)*tan(d
*x + c) - 10*A + 2*I*B)/(a^3*d*(tan(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$=-\frac{\tan(c+d\,x)^2\left(\frac{B}{8\,a^3}+\frac{A\,1\mathrm{i}}{8\,a^3}\right)-\frac{A\,5\mathrm{i}}{12\,a^3}-\frac{B}{12\,a^3}+\tan(c+d\,x)\left(\frac{3\,A}{8\,a^3}-\frac{B\,3\mathrm{i}}{8\,a^3}\right)}{d\left(-\tan(c+d\,x)^3\,1\mathrm{i}-3\tan(c+d\,x)^2+\tan(c+d\,x)\,3\mathrm{i}+1\right)}$$
$$-\frac{x\,(B+A\,1\mathrm{i})\,1\mathrm{i}}{8\,a^3}$$

input
```
int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
- (tan(c + d*x)^2*((A*1i)/(8*a^3) + B/(8*a^3)) - (A*5i)/(12*a^3) - B/(12*a
^3) + tan(c + d*x)*((3*A)/(8*a^3) - (B*3i)/(8*a^3)))/(d*(tan(c + d*x)*3i -
 3*tan(c + d*x)^2 - tan(c + d*x)^3*1i + 1)) - (x*(A*1i + B)*1i)/(8*a^3)
```

Reduce [F]

$$\int \frac{A + B\tan(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\tan(dx+c)}{\tan(dx+c)^3 i + 3\tan(dx+c)^2 - 3\tan(dx+c)i - 1}dx\right)b - \left(\int \frac{1}{\tan(dx+c)^3 i + 3\tan(dx+c)^2 - 3\tan(dx+c)i - 1}dx\right)a}{a^3}$$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int(tan(c + d*x)/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d
*x)*i - 1),x)*b + int(1/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c +
 d*x)*i - 1),x)*a))/a**3
```

3.56 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 131

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(7iA-B)x}{8a^3}+\frac{A\log(\sin(c+dx))}{a^3d}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}+\frac{3A+iB}{8ad(a+ia\tan(c+dx))^2}+\frac{7A+iB}{8d\,(a^3+ia^3\tan(c+dx))}$$

output
```
-1/8*(7*I*A-B)*x/a^3+A*ln(sin(d*x+c))/a^3/d+1/6*(A+I*B)/d/(a+I*a*tan(d*x+c
))^3+1/8*(3*A+I*B)/a/d/(a+I*a*tan(d*x+c))^2+1/8*(7*A+I*B)/d/(a^3+I*a^3*tan
(d*x+c))
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-3(15A+iB)\log(i-\tan(c+dx))+48A\log(\tan(c+dx))-3(A-iB)\log(i+\tan(c+dx))+\frac{8i(A-}{(-i+\tan(}}{48a^3d}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(-3*(15*A + I*B)*Log[I - Tan[c + d*x]] + 48*A*Log[Tan[c + d*x]] - 3*(A - I
*B)*Log[I + Tan[c + d*x]] + ((8*I)*(A + I*B))/(-I + Tan[c + d*x])^3 - (6*(
3*A + I*B))/(-I + Tan[c + d*x])^2 + (6*((-7*I)*A + B))/(-I + Tan[c + d*x])
)/(48*a^3*d)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{3\cot(c+dx)(2aA-a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cot(c+dx)(2aA-a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{2aA-a(iA-B)\tan(c+dx)}{\tan(c+dx)(i\tan(c+dx)a+a)^2}dx}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\int \frac{2\cot(c+dx)\left(4a^2A-a^2(3iA-B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{a(3A+iB)}{4d(a+ia\tan(c+dx))^2}}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\cot(c+dx)\left(4a^2A-a^2(3iA-B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{2a^2}+\frac{a(3A+iB)}{4d(a+ia\tan(c+dx))^2}}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{4a^2A-a^2(3iA-B)\tan(c+dx)}{\tan(c+dx)(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{a(3A+iB)}{4d(a+ia\tan(c+dx))^2}}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\frac{\int \cot(c+dx)\left(8a^3A-a^3(7iA-B)\tan(c+dx)\right)dx}{2a^2}+\frac{a^2(7A+iB)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(3A+iB)}{4d(a+ia\tan(c+dx))^2}}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{8a^3A-a^3(7iA-B)\tan(c+dx)}{\tan(c+dx)}dx}{2a^2}+\frac{a^2(7A+iB)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(3A+iB)}{4d(a+ia\tan(c+dx))^2}}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 4014

$$\frac{\frac{\frac{8a^3A\int \cot(c+dx)dx-a^3x(-B+7iA)}{2a^2}+\frac{a^2(7A+iB)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(3A+iB)}{4d(a+ia\tan(c+dx))^2}}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{8a^3A\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx-a^3x(-B+7iA)}{2a^2}+\frac{a^2(7A+iB)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(3A+iB)}{4d(a+ia\tan(c+dx))^2}}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{-8a^3A\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\left(a^3x(-B+7iA)\right)}{2a^2}+\frac{a^2(7A+iB)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(3A+iB)}{4d(a+ia\tan(c+dx))^2}}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{\frac{a^2(7A+iB)}{2d(a+ia\tan(c+dx))}+\frac{\frac{8a^3A\log(-\sin(c+dx))}{d}-a^3x(-B+7iA)}{2a^2}}{2a^2}+\frac{a(3A+iB)}{4d(a+ia\tan(c+dx))^2}}{2a^2}+\frac{A+iB}{6d(a+ia\tan(c+dx))^3}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
(A + I*B)/(6*d*(a + I*a*Tan[c + d*x])^3) + ((a*(3*A + I*B))/(4*d*(a + I*a*
Tan[c + d*x])^2) + ((-(a^3*((7*I)*A - B)*x) + (8*a^3*A*Log[-Sin[c + d*x]])
/d)/(2*a^2) + (a^2*(7*A + I*B))/(2*d*(a + I*a*Tan[c + d*x])))/(2*a^2))/(2*
a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.21

method	result
risch	$\frac{xB}{8a^3} - \frac{15ixA}{8a^3} + \frac{3i\mathrm{e}^{-2i(dx+c)}B}{16a^3d} + \frac{11\,\mathrm{e}^{-2i(dx+c)}A}{16a^3d} + \frac{3i\mathrm{e}^{-4i(dx+c)}B}{32a^3d} + \frac{5\,\mathrm{e}^{-4i(dx+c)}A}{32a^3d} + \frac{i\mathrm{e}^{-6i(dx+c)}B}{48a^3d} + \mathrm{e}^{-}$
derivativedivides	$\frac{B\arctan(\tan(dx+c))}{8d\,a^3} - \frac{A\ln\left(1+\tan(dx+c)^2\right)}{2a^3d} - \frac{7iA\arctan(\tan(dx+c))}{8d\,a^3} + \frac{iA}{6d\,a^3(-i+\tan(dx+c))^3} - \frac{}{6d\,a^3(-i}$
default	$\frac{B\arctan(\tan(dx+c))}{8d\,a^3} - \frac{A\ln\left(1+\tan(dx+c)^2\right)}{2a^3d} - \frac{7iA\arctan(\tan(dx+c))}{8d\,a^3} + \frac{iA}{6d\,a^3(-i+\tan(dx+c))^3} - \frac{}{6d\,a^3(-i}$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/8*x/a^3*B-15/8*I*x/a^3*A+3/16*I/a^3/d*exp(-2*I*(d*x+c))*B+11/16/a^3/d*ex
p(-2*I*(d*x+c))*A+3/32*I/a^3/d*exp(-4*I*(d*x+c))*B+5/32/a^3/d*exp(-4*I*(d*
x+c))*A+1/48*I/a^3/d*exp(-6*I*(d*x+c))*B+1/48/a^3/d*exp(-6*I*(d*x+c))*A-2*
I*A/a^3/d*c+A/a^3/d*ln(exp(2*I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(12\,(15i\,A-B)dxe^{(6i\,dx+6i\,c)}-96\,Ae^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}-1\right)-6\,(11\,A+3i\,B)e^{(4i\,dx+4i\,c)}-3\,(5\ldots}{96\,a^3d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/96*(12*(15*I*A - B)*d*x*e^(6*I*d*x + 6*I*c) - 96*A*e^(6*I*d*x + 6*I*c)*
log(e^(2*I*d*x + 2*I*c) - 1) - 6*(11*A + 3*I*B)*e^(4*I*d*x + 4*I*c) - 3*(5
*A + 3*I*B)*e^(2*I*d*x + 2*I*c) - 2*A - 2*I*B)*e^(-6*I*d*x - 6*I*c)/(a^3*d
)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.23

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{A\log\left(e^{2idx}-e^{-2ic}\right)}{a^3d} + \begin{cases} \frac{((512Aa^6d^2e^{6ic}+512iBa^6d^2e^{6ic})e^{-6idx}+(3840Aa^6d^2e^{8ic}+2304iBa^6d^2e^{8ic})e^{-4idx}+(16896Aa^6d^2e^{10ic}+4608iBa^6d^2e^{10ic})e^{-2idx})e^{-1\ldots}}{24576a^9d^3} \\ x\left(-\frac{-15iA+B}{8a^3}+\frac{(-15iAe^{6ic}-11iAe^{4ic}-5iAe^{2ic}-iA+Be^{6ic}+3Be^{4ic}+3Be^{2ic}+B)e^{-6ic}}{8a^3}\right) \end{cases} + \frac{x(-15iA+B)}{8a^3}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
A*log(exp(2*I*d*x) - exp(-2*I*c))/(a**3*d) + Piecewise((((512*A*a**6*d**2*
exp(6*I*c) + 512*I*B*a**6*d**2*exp(6*I*c))*exp(-6*I*d*x) + (3840*A*a**6*d*
*2*exp(8*I*c) + 2304*I*B*a**6*d**2*exp(8*I*c))*exp(-4*I*d*x) + (16896*A*a*
*6*d**2*exp(10*I*c) + 4608*I*B*a**6*d**2*exp(10*I*c))*exp(-2*I*d*x))*exp(-
12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*(-(-15*I*A +
B)/(8*a**3) + (-15*I*A*exp(6*I*c) - 11*I*A*exp(4*I*c) - 5*I*A*exp(2*I*c) -
 I*A + B*exp(6*I*c) + 3*B*exp(4*I*c) + 3*B*exp(2*I*c) + B)*exp(-6*I*c)/(8*
a**3)), True)) + x*(-15*I*A + B)/(8*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx \\
&= -\frac{(A-iB)\log(\tan(dx+c)+i)}{16\,a^3d} \\
&\quad -\frac{(15\,A+i\,B)\log(\tan(dx+c)-i)}{16\,a^3d}+\frac{A\log(|\tan(dx+c)|)}{a^3d} \\
&\quad -\frac{3\,(7i\,A-B)\tan(dx+c)^2+3\,(17\,A+3i\,B)\tan(dx+c)-34i\,A+10\,B}{24\,a^3d(\tan(dx+c)-i)^3}
\end{aligned}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/16*(A - I*B)*log(tan(d*x + c) + I)/(a^3*d) - 1/16*(15*A + I*B)*log(tan(
d*x + c) - I)/(a^3*d) + A*log(abs(tan(d*x + c)))/(a^3*d) - 1/24*(3*(7*I*A
- B)*tan(d*x + c)^2 + 3*(17*A + 3*I*B)*tan(d*x + c) - 34*I*A + 10*B)/(a^3*
d*(tan(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.25

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{17A}{12a^3} - \tan(c+dx)^2\left(\frac{7A}{8a^3} + \frac{B\,1\mathrm{i}}{8a^3}\right) + \frac{B\,5\mathrm{i}}{12a^3} + \tan(c+dx)\left(-\frac{3B}{8a^3} + \frac{A\,17\mathrm{i}}{8a^3}\right)}{d\left(-\tan\left(c+dx\right)^3 1\mathrm{i} - 3\tan\left(c+dx\right)^2 + \tan\left(c+dx\right)3\mathrm{i} + 1\right)}$$
$$+ \frac{A\ln\left(\tan(c+dx)\right)}{a^3 d} + \frac{\ln\left(\tan(c+dx)+1\mathrm{i}\right)\left(B+A\,1\mathrm{i}\right)1\mathrm{i}}{16a^3 d}$$
$$- \frac{\ln\left(\tan(c+dx)-\mathrm{i}\right)\left(15A+B\,1\mathrm{i}\right)}{16a^3 d}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
((17*A)/(12*a^3) - tan(c + d*x)^2*((7*A)/(8*a^3) + (B*1i)/(8*a^3)) + (B*5i
)/(12*a^3) + tan(c + d*x)*((A*17i)/(8*a^3) - (3*B)/(8*a^3)))/(d*(tan(c + d
*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*1i + 1)) + (A*log(tan(c + d*x))
)/(a^3*d) + (log(tan(c + d*x) + 1i)*(A*1i + B)*1i)/(16*a^3*d) - (log(tan(c
 + d*x) - 1i)*(15*A + B*1i))/(16*a^3*d)
```

Reduce [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output

```
(120*int(tan((c + d*x)/2)**4/(tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*
i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2
)**2 - 6*tan((c + d*x)/2)*i - 1),x)*a*d*i - 72*int(tan((c + d*x)/2)**4/(ta
n((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*
tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1)
,x)*b*d + 396*int(tan((c + d*x)/2)**3/(tan((c + d*x)/2)**6 - 6*tan((c + d*
x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c
 + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1),x)*a*d + 210*int(tan((c + d*x)/2
)**3/(tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)*
*4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2
)*i - 1),x)*b*d*i - 624*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**6 - 6*t
an((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i +
 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1),x)*a*d*i + 324*int(tan
((c + d*x)/2)**2/(tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((
c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan
((c + d*x)/2)*i - 1),x)*b*d - 504*int(tan((c + d*x)/2)/(tan((c + d*x)/2)**
6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)/2)
**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1),x)*a*d - 252*in
t(tan((c + d*x)/2)/(tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan
((c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - ...
```

3.57 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 183

$$\begin{aligned}\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{(25A+7iB)x}{8a^3}-\frac{(25A+7iB)\cot(c+dx)}{8a^3d}\\&\quad-\frac{(3iA-B)\log(\sin(c+dx))}{a^3d}\\&\quad+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}\\&\quad+\frac{(11A+5iB)\cot(c+dx)}{24ad(a+ia\tan(c+dx))^2}\\&\quad+\frac{(3A+iB)\cot(c+dx)}{2d\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

output
```
-1/8*(25*A+7*I*B)*x/a^3-1/8*(25*A+7*I*B)*cot(d*x+c)/a^3/d-(3*I*A-B)*ln(sin
(d*x+c))/a^3/d+1/6*(A+I*B)*cot(d*x+c)/d/(a+I*a*tan(d*x+c))^3+1/24*(11*A+5*
I*B)*cot(d*x+c)/a/d/(a+I*a*tan(d*x+c))^2+1/2*(3*A+I*B)*cot(d*x+c)/d/(a^3+I
*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.68 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.90

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$=\frac{\frac{4(A+iB)\cot^4(c+dx)}{(i+\cot(c+dx))^3}+\frac{(11A+5iB)\cot^3(c+dx)}{(i+\cot(c+dx))^2}+\frac{12(3A+iB)\cot^2(c+dx)}{i+\cot(c+dx)}-3\left((25A+7iB)\cot(c+dx)\,\text{Hypergeometric}\right.}{24a^3d}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x
]
```

output
```
((4*(A + I*B)*Cot[c + d*x]^4)/(I + Cot[c + d*x])^3 + ((11*A + (5*I)*B)*Cot
[c + d*x]^3)/(I + Cot[c + d*x])^2 + (12*(3*A + I*B)*Cot[c + d*x]^2)/(I + C
ot[c + d*x]) - 3*((25*A + (7*I)*B)*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1,
 1/2, -Tan[c + d*x]^2] + 8*((3*I)*A - B)*(Log[Cos[c + d*x]] + Log[Tan[c +
d*x]])))/(24*a^3*d)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4079, 3042, 4079, 27, 3042, 4079, 27, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{\cot^2(c+dx)(a(7A+iB)-4a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a(7A+iB)-4a(iA-B)\tan(c+dx)}{\tan(c+dx)^2(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 4079$$

$$\frac{\frac{\int \frac{3\cot^2(c+dx)\left(a^2(13A+3iB)-a^2(11iA-5B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{\cot^2(c+dx)\left(a^2(13A+3iB)-a^2(11iA-5B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\int \frac{a^2(13A+3iB)-a^2(11iA-5B)\tan(c+dx)}{\tan(c+dx)^2(i\tan(c+dx)a+a)}dx}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 4079$$

$$\frac{\frac{3\left(\frac{\int 2\cot^2(c+dx)\left(a^3(25A+7iB)-8a^3(3iA-B)\tan(c+dx)\right)dx}{2a^2}+\frac{4a^2(3A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(\frac{\int \cot^2(c+dx)\left(a^3(25A+7iB)-8a^3(3iA-B)\tan(c+dx)\right)dx}{a^2}+\frac{4a^2(3A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\left(\frac{\int \frac{a^3(25A+7iB)-8a^3(3iA-B)\tan(c+dx)}{\tan(c+dx)^2}dx}{a^2}+\frac{4a^2(3A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

↓ 4012

$$\frac{\frac{3\left(\frac{\int -\cot(c+dx)\left(8(3iA-B)a^3+(25A+7iB)\tan(c+dx)a^3\right)dx-\frac{a^3(25A+7iB)\cot(c+dx)}{d}}{a^2}+\frac{4a^2(3A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

↓ 25

$$\frac{\frac{3\left(\frac{-\int \cot(c+dx)\left(8(3iA-B)a^3+(25A+7iB)\tan(c+dx)a^3\right)dx-\frac{a^3(25A+7iB)\cot(c+dx)}{d}}{a^2}+\frac{4a^2(3A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

↓ 3042

$$\frac{\frac{3\left(\frac{-\int \frac{8(3iA-B)a^3+(25A+7iB)\tan(c+dx)a^3}{\tan(c+dx)}dx-\frac{a^3(25A+7iB)\cot(c+dx)}{d}}{a^2}+\frac{4a^2(3A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

↓ 4014

$$\frac{\frac{3\left(\frac{-8a^3(-B+3iA)\int \cot(c+dx)dx-\frac{a^3(25A+7iB)\cot(c+dx)}{d}-\left(a^3x(25A+7iB)\right)}{a^2}+\frac{4a^2(3A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

↓ 3042

$$\frac{\frac{3\left(\frac{\frac{-8a^3(-B+3iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{a^3(25A+7iB)\cot(c+dx)}{d}-\left(a^3x(25A+7iB)\right)}{a^2}+\frac{4a^2(3A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3\left(\frac{\frac{8a^3(-B+3iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{a^3(25A+7iB)\cot(c+dx)}{d}-\left(a^3x(25A+7iB)\right)}{a^2}+\frac{4a^2(3A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{3\left(\frac{4a^2(3A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}+\frac{-\frac{a^3(25A+7iB)\cot(c+dx)}{d}-\frac{8a^3(-B+3iA)\log(-\sin(c+dx))}{d}-\left(a^3x(25A+7iB)\right)}{a^2}\right)}{4a^2}+\frac{a(11A+5iB)\cot(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((A + I*B)*Cot[c + d*x])/(6*d*(a + I*a*Tan[c + d*x])^3) + ((a*(11*A + (5*I
)*B)*Cot[c + d*x])/(4*d*(a + I*a*Tan[c + d*x])^2) + (3*((-(a^3*(25*A + (7*
I)*B)*x) - (a^3*(25*A + (7*I)*B)*Cot[c + d*x])/d - (8*a^3*((3*I)*A - B)*Lo
g[-Sin[c + d*x]])/d)/a^2 + (4*a^2*(3*A + I*B)*Cot[c + d*x])/(d*(a + I*a*Ta
n[c + d*x]))))/(4*a^2))/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{15ixB}{8a^3}-\frac{49xA}{8a^3}+\frac{11\,\mathrm{e}^{-2i(dx+c)}B}{16a^3d}-\frac{23i\mathrm{e}^{-2i(dx+c)}A}{16a^3d}+\frac{5\,\mathrm{e}^{-4i(dx+c)}B}{32a^3d}-\frac{7i\mathrm{e}^{-4i(dx+c)}A}{32a^3d}+\frac{\mathrm{e}^{-6i(dx+c)}B}{48a^3d}$
derivativedivides	$-\frac{A\cot(dx+c)}{a^3d}+\frac{A}{6a^3d(i+\cot(dx+c))^3}+\frac{iB}{6a^3d(i+\cot(dx+c))^3}+\frac{9iA}{8a^3d(i+\cot(dx+c))^2}-\frac{7B}{8a^3d(i+\cot(dx+c))^2}$
default	$-\frac{A\cot(dx+c)}{a^3d}+\frac{A}{6a^3d(i+\cot(dx+c))^3}+\frac{iB}{6a^3d(i+\cot(dx+c))^3}+\frac{9iA}{8a^3d(i+\cot(dx+c))^2}-\frac{7B}{8a^3d(i+\cot(dx+c))^2}$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVER
BOSE)
```

output
```
-15/8*I*x/a^3*B-49/8*x/a^3*A+11/16/a^3/d*exp(-2*I*(d*x+c))*B-23/16*I/a^3/d
*exp(-2*I*(d*x+c))*A+5/32/a^3/d*exp(-4*I*(d*x+c))*B-7/32*I/a^3/d*exp(-4*I*
(d*x+c))*A+1/48/a^3/d*exp(-6*I*(d*x+c))*B-1/48*I/a^3/d*exp(-6*I*(d*x+c))*A
-2*I/a^3/d*B*c-6/a^3/d*A*c-2*I*A/a^3/d/(exp(2*I*(d*x+c))-1)+1/a^3/d*ln(exp
(2*I*(d*x+c))-1)*B-3*I/a^3/d*ln(exp(2*I*(d*x+c))-1)*A
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.95

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{12\,(49\,A+15i\,B)dxe^{(8i\,dx+8i\,c)}-6\,(2\,(49\,A+15i\,B)dx-55i\,A+11\,B)e^{(6i\,dx+6i\,c)}+3\,(-39i\,A+17\,B}{96\,(a}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"fricas")
```

output
```
-1/96*(12*(49*A + 15*I*B)*d*x*e^(8*I*d*x + 8*I*c) - 6*(2*(49*A + 15*I*B)*d
*x - 55*I*A + 11*B)*e^(6*I*d*x + 6*I*c) + 3*(-39*I*A + 17*B)*e^(4*I*d*x +
4*I*c) - (19*I*A - 13*B)*e^(2*I*d*x + 2*I*c) + 96*((3*I*A - B)*e^(8*I*d*x
+ 8*I*c) + (-3*I*A + B)*e^(6*I*d*x + 6*I*c))*log(e^(2*I*d*x + 2*I*c) - 1)
- 2*I*A + 2*B)/(a^3*d*e^(8*I*d*x + 8*I*c) - a^3*d*e^(6*I*d*x + 6*I*c))
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.86

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2iA}{a^3de^{2ic}e^{2idx}-a^3d} + \begin{cases} \frac{((-512iAa^6d^2e^{6ic}+512Ba^6d^2e^{6ic})e^{-6idx}+(-5376iAa^6d^2e^{8ic}+3840Ba^6d^2e^{8ic})e^{-4idx}+(-35328iAa^6d^2e^{10ic}+16896Ba^6d^2e^{10ic})e^{-2ic}}{24576a^9d^3} \\ x\left(-\frac{-49A-15iB}{8a^3}+\frac{(-49Ae^{6ic}-23Ae^{4ic}-7Ae^{2ic}-A-15iBe^{6ic}-11iBe^{4ic}-5iBe^{2ic}-iB)e^{-6ic}}{8a^3}\right) \end{cases} + \frac{x(-49A-15iB)}{8a^3} - \frac{i(3A+iB)\log\left(e^{2idx}-e^{-2ic}\right)}{a^3d}$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
-2*I*A/(a**3*d*exp(2*I*c)*exp(2*I*d*x) - a**3*d) + Piecewise((((-512*I*A*a
**6*d**2*exp(6*I*c) + 512*B*a**6*d**2*exp(6*I*c))*exp(-6*I*d*x) + (-5376*I
*A*a**6*d**2*exp(8*I*c) + 3840*B*a**6*d**2*exp(8*I*c))*exp(-4*I*d*x) + (-3
5328*I*A*a**6*d**2*exp(10*I*c) + 16896*B*a**6*d**2*exp(10*I*c))*exp(-2*I*d
*x))*exp(-12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*(-(
-49*A - 15*I*B)/(8*a**3) + (-49*A*exp(6*I*c) - 23*A*exp(4*I*c) - 7*A*exp(2
*I*c) - A - 15*I*B*exp(6*I*c) - 11*I*B*exp(4*I*c) - 5*I*B*exp(2*I*c) - I*B
)*exp(-6*I*c)/(8*a**3)), True)) + x*(-49*A - 15*I*B)/(8*a**3) - I*(3*A + I
*B)*log(exp(2*I*d*x) - exp(-2*I*c))/(a**3*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.80

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(iA+B)\log(\tan(dx+c)+i)}{16\,a^3d} - \frac{(-49i\,A+15\,B)\log(\tan(dx+c)-i)}{16\,a^3d} + \frac{(-3i\,A+B)\log(|\tan(dx+c)|)}{a^3d} - \frac{3\,(25\,A+7i\,B)\tan(dx+c)^3+3\,(-63i\,A+17\,B)\tan(dx+c)^2-2\,(71\,A+17i\,B)\tan(dx+c)+24i}{24\,a^3d(\tan(dx+c)-i)^3\tan(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"giac")
```

output
```
-1/16*(I*A + B)*log(tan(d*x + c) + I)/(a^3*d) - 1/16*(-49*I*A + 15*B)*log(
tan(d*x + c) - I)/(a^3*d) + (-3*I*A + B)*log(abs(tan(d*x + c)))/(a^3*d) -
1/24*(3*(25*A + 7*I*B)*tan(d*x + c)^3 + 3*(-63*I*A + 17*B)*tan(d*x + c)^2
- 2*(71*A + 17*I*B)*tan(d*x + c) + 24*I*A)/(a^3*d*(tan(d*x + c) - I)^3*tan
(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.08

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\tan(c+d\,x)^3\,\left(\frac{25\,A}{8\,a^3}+\frac{B\,7\mathrm{i}}{8\,a^3}\right)-\tan(c+d\,x)^2\,\left(-\frac{17\,B}{8\,a^3}+\frac{A\,63\mathrm{i}}{8\,a^3}\right)+\frac{A\,1\mathrm{i}}{a^3}-\tan(c+d\,x)\,\left(\frac{71\,A}{12\,a^3}+\frac{B\,17\mathrm{i}}{12\,a^3}\right)}{d\,\left({\tan\left(c+d\,x\right)}^4-{\tan\left(c+d\,x\right)}^3\,3\mathrm{i}-3\,{\tan\left(c+d\,x\right)}^2+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)} - \frac{\ln\left(\tan(c+d\,x)\right)\,\left(-B+A\,3\mathrm{i}\right)}{a^3\,d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{16\,a^3\,d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-15\,B+A\,49\mathrm{i}\right)}{16\,a^3\,d}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(log(tan(c + d*x) - 1i)*(A*49i - 15*B))/(16*a^3*d) - (log(tan(c + d*x))*(A
*3i - B))/(a^3*d) - (log(tan(c + d*x) + 1i)*(A*1i + B))/(16*a^3*d) - (tan(
c + d*x)^3*((25*A)/(8*a^3) + (B*7i)/(8*a^3)) - tan(c + d*x)^2*((A*63i)/(8*
a^3) - (17*B)/(8*a^3)) + (A*1i)/a^3 - tan(c + d*x)*((71*A)/(12*a^3) + (B*1
7i)/(12*a^3)))/(d*(tan(c + d*x)*1i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*3i
+ tan(c + d*x)^4))
```

Reduce [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 121*cos(c + d*x)*a - 80*cos(c + d*x)*b*i + 148*int(tan((c + d*x)/2)**3
/(tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 +
 20*tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i
- 1),x)*sin(c + d*x)*a*d*i - 108*int(tan((c + d*x)/2)**3/(tan((c + d*x)/2)
**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)/
2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1),x)*sin(c + d*
x)*b*d + 560*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**6 - 6*tan((c + d*x
)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c
+ d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1),x)*sin(c + d*x)*a*d + 392*int(tan
((c + d*x)/2)**2/(tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((
c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan
((c + d*x)/2)*i - 1),x)*sin(c + d*x)*b*d*i - 920*int(tan((c + d*x)/2)/(tan
((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*t
an((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1),
x)*sin(c + d*x)*a*d*i + 632*int(tan((c + d*x)/2)/(tan((c + d*x)/2)**6 - 6*
tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i
+ 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1),x)*sin(c + d*x)*b*d +
 60*int(1/(tan((c + d*x)/2)**8 - 6*tan((c + d*x)/2)**7*i - 15*tan((c + d*x
)/2)**6 + 20*tan((c + d*x)/2)**5*i + 15*tan((c + d*x)/2)**4 - 6*tan((c + d
*x)/2)**3*i - tan((c + d*x)/2)**2),x)*sin(c + d*x)*a*d + 40*int(1/(tan(...
```

3.58 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 216

$$\begin{aligned}
&\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx \\
&= \frac{5(11iA-5B)x}{8a^3}+\frac{5(11iA-5B)\cot(c+dx)}{8a^3d}-\frac{(7A+3iB)\cot^2(c+dx)}{2a^3d} \\
&\quad -\frac{(7A+3iB)\log(\sin(c+dx))}{a^3d}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3} \\
&\quad +\frac{(13A+7iB)\cot^2(c+dx)}{24ad(a+ia\tan(c+dx))^2}+\frac{5(11A+5iB)\cot^2(c+dx)}{24d\left(a^3+ia^3\tan(c+dx)\right)}
\end{aligned}$$

output
```
5/8*(11*I*A-5*B)*x/a^3+5/8*(11*I*A-5*B)*cot(d*x+c)/a^3/d-1/2*(7*A+3*I*B)*c
ot(d*x+c)^2/a^3/d-(7*A+3*I*B)*ln(sin(d*x+c))/a^3/d+1/6*(A+I*B)*cot(d*x+c)^
2/d/(a+I*a*tan(d*x+c))^3+1/24*(13*A+7*I*B)*cot(d*x+c)^2/a/d/(a+I*a*tan(d*x
+c))^2+5/24*(11*A+5*I*B)*cot(d*x+c)^2/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.35 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.80

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{4(A+iB)\cot^5(c+dx)}{(i+\cot(c+dx))^3}+\frac{(13A+7iB)\cot^4(c+dx)}{(i+\cot(c+dx))^2}+\frac{5(11A+5iB)\cot^3(c+dx)}{i+\cot(c+dx)}+15(11iA-5B)\cot(c+dx)\,\text{Hypergeometric}}{24}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x
]
```

output
```
((4*(A + I*B)*Cot[c + d*x]^5)/(I + Cot[c + d*x])^3 + ((13*A + (7*I)*B)*Cot
[c + d*x]^4)/(I + Cot[c + d*x])^2 + (5*(11*A + (5*I)*B)*Cot[c + d*x]^3)/(I
 + Cot[c + d*x]) + 15*((11*I)*A - 5*B)*Cot[c + d*x]*Hypergeometric2F1[-1/2
, 1, 1/2, -Tan[c + d*x]^2] - 12*(7*A + (3*I)*B)*(Cot[c + d*x]^2 + 2*(Log[C
os[c + d*x]] + Log[Tan[c + d*x]])))/(24*a^3*d)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 4079, 3042, 4079, 27, 3042, 4079, 27, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{\cot^3(c+dx)(2a(4A+iB)-5a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a(4A+iB)-5a(iA-B)\tan(c+dx)}{\tan(c+dx)^3(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int \frac{2\cot^3(c+dx)\left(a^2(29A+11iB)-2a^2(13iA-7B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\cot^3(c+dx)\left(a^2(29A+11iB)-2a^2(13iA-7B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{2a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(29A+11iB)-2a^2(13iA-7B)\tan(c+dx)}{\tan(c+dx)^3(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\frac{\int 3\cot^3(c+dx)\left(8a^3(7A+3iB)-5a^3(11iA-5B)\tan(c+dx)\right)dx}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \cot^3(c+dx)\left(8a^3(7A+3iB)-5a^3(11iA-5B)\tan(c+dx)\right)dx}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\int \frac{8a^3(7A+3iB)-5a^3(11iA-5B)\tan(c+dx)}{\tan(c+dx)^3}dx}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\frac{3\left(\int -\cot^2(c+dx)\left(5(11iA-5B)a^3+8(7A+3iB)\tan(c+dx)a^3\right)dx-\frac{4a^3(7A+3iB)\cot^2(c+dx)}{d}\right)}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{3\left(-\int \cot^2(c+dx)\left(5(11iA-5B)a^3+8(7A+3iB)\tan(c+dx)a^3\right)dx-\frac{4a^3(7A+3iB)\cot^2(c+dx)}{d}\right)}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(-\int \frac{5(11iA-5B)a^3+8(7A+3iB)\tan(c+dx)a^3}{\tan(c+dx)^2}dx-\frac{4a^3(7A+3iB)\cot^2(c+dx)}{d}\right)}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\frac{3\left(-\int \cot(c+dx)\left(8a^3(7A+3iB)-5a^3(11iA-5B)\tan(c+dx)\right)dx-\frac{4a^3(7A+3iB)\cot^2(c+dx)}{d}+\frac{5a^3(-5B+11iA)\cot(c+dx)}{d}\right)}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3\left(-\int \frac{8a^3(7A+3iB)-5a^3(11iA-5B)\tan(c+dx)}{\tan(c+dx)}dx-\frac{4a^3(7A+3iB)\cot^2(c+dx)}{d}+\frac{5a^3(-5B+11iA)\cot(c+dx)}{d}\right)}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A+7iB)\cot^2(c+dx)}{4d(a+ia\tan(c+dx))^2}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 4014

$$\frac{\frac{\frac{3\left(-8a^3(7A+3iB)\int \cot(c+dx)dx-\frac{4a^3(7A+3iB)\cot^2(c+dx)}{d}+\frac{5a^3(-5B+11iA)\cot(c+dx)}{d}+5a^3x(-5B+11iA)\right)}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A+7iB}{4d(a+ia\,\mathrm{ta}}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\left(-8a^3(7A+3iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{4a^3(7A+3iB)\cot^2(c+dx)}{d}+\frac{5a^3(-5B+11iA)\cot(c+dx)}{d}+5a^3x(-5B+11iA)\right)}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A}{4d(}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\downarrow$ 25

$$\frac{\frac{\frac{3\left(8a^3(7A+3iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{4a^3(7A+3iB)\cot^2(c+dx)}{d}+\frac{5a^3(-5B+11iA)\cot(c+dx)}{d}+5a^3x(-5B+11iA)\right)}{2a^2}+\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{a(13A}{4d(}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\downarrow$ 3956

$$\frac{\frac{\frac{5a^2(11A+5iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{3\left(-\frac{4a^3(7A+3iB)\cot^2(c+dx)}{d}+\frac{5a^3(-5B+11iA)\cot(c+dx)}{d}-\frac{8a^3(7A+3iB)\log(-\sin(c+dx))}{d}+5a^3x(-5B+11iA)\right)}{2a^2}}{2a^2}+\frac{a(13A+7}{4d(a+i}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((A + I*B)*Cot[c + d*x]^2)/(6*d*(a + I*a*Tan[c + d*x])^3) + ((a*(13*A + (7
*I)*B)*Cot[c + d*x]^2)/(4*d*(a + I*a*Tan[c + d*x])^2) + ((3*(5*a^3*((11*I)
*A - 5*B)*x + (5*a^3*((11*I)*A - 5*B)*Cot[c + d*x])/d - (4*a^3*(7*A + (3*I
)*B)*Cot[c + d*x]^2)/d - (8*a^3*(7*A + (3*I)*B)*Log[-Sin[c + d*x]])/d))/(2
*a^2) + (5*a^2*(11*A + (5*I)*B)*Cot[c + d*x]^2)/(2*d*(a + I*a*Tan[c + d*x]
)))/(2*a^2))/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{49xB}{8a^3}+\frac{111ixA}{8a^3}-\frac{23i\,\mathrm{e}^{-2i(dx+c)}B}{16a^3d}-\frac{39\,\mathrm{e}^{-2i(dx+c)}A}{16a^3d}-\frac{7i\mathrm{e}^{-4i(dx+c)}B}{32a^3d}-\frac{9\,\mathrm{e}^{-4i(dx+c)}A}{32a^3d}-\frac{i\mathrm{e}^{-6i(dx+c)}B}{48a^3d}$
derivativedivides	$-\frac{A\cot(dx+c)^2}{2a^3d}+\frac{3iA\cot(dx+c)}{a^3d}-\frac{B\cot(dx+c)}{a^3d}+\frac{11A}{8a^3d(i+\cot(dx+c))^2}+\frac{9iB}{8a^3d(i+\cot(dx+c))^2}+\frac{3iB\ln\left(c\right.}{}$
default	$-\frac{A\cot(dx+c)^2}{2a^3d}+\frac{3iA\cot(dx+c)}{a^3d}-\frac{B\cot(dx+c)}{a^3d}+\frac{11A}{8a^3d(i+\cot(dx+c))^2}+\frac{9iB}{8a^3d(i+\cot(dx+c))^2}+\frac{3iB\ln\left(c\right.}{}$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVER
BOSE)
```

output
```
-49/8*x/a^3*B+111/8*I*x/a^3*A-23/16*I/a^3/d*exp(-2*I*(d*x+c))*B-39/16/a^3/
d*exp(-2*I*(d*x+c))*A-7/32*I/a^3/d*exp(-4*I*(d*x+c))*B-9/32/a^3/d*exp(-4*I
*(d*x+c))*A-1/48*I/a^3/d*exp(-6*I*(d*x+c))*B-1/48/a^3/d*exp(-6*I*(d*x+c))*
A-6*B/a^3/d*c+14*I/a^3/d*A*c-2*I*(-2*I*A*exp(2*I*(d*x+c))+B*exp(2*I*(d*x+c
))+3*I*A-B)/a^3/d/(exp(2*I*(d*x+c))-1)^2-3*I/a^3/d*ln(exp(2*I*(d*x+c))-1)*
B-7*A/a^3/d*ln(exp(2*I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.09

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{12\,(-111i\,A+49\,B)dxe^{(10i\,dx+10i\,c)}+6\,(4\,(111i\,A-49\,B)dx+103\,A+55i\,B)e^{(8i\,dx+8i\,c)}+3\,(4\,(-111\cdots}{}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"fricas")
```

output
```
-1/96*(12*(-111*I*A + 49*B)*d*x*e^(10*I*d*x + 10*I*c) + 6*(4*(111*I*A - 49
*B)*d*x + 103*A + 55*I*B)*e^(8*I*d*x + 8*I*c) + 3*(4*(-111*I*A + 49*B)*d*x
 - 339*A - 149*I*B)*e^(6*I*d*x + 6*I*c) + 14*(13*A + 7*I*B)*e^(4*I*d*x + 4
*I*c) + (23*A + 17*I*B)*e^(2*I*d*x + 2*I*c) + 96*((7*A + 3*I*B)*e^(10*I*d*
x + 10*I*c) - 2*(7*A + 3*I*B)*e^(8*I*d*x + 8*I*c) + (7*A + 3*I*B)*e^(6*I*d
*x + 6*I*c))*log(e^(2*I*d*x + 2*I*c) - 1) + 2*A + 2*I*B)/(a^3*d*e^(10*I*d*
x + 10*I*c) - 2*a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*c))
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.84

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{6A+2iB+\left(-4Ae^{2ic}-2iBe^{2ic}\right)e^{2idx}}{a^3de^{4ic}e^{4idx}-2a^3de^{2ic}e^{2idx}+a^3d}$$
$$+\begin{cases} \frac{((-512Aa^6d^2e^{6ic}-512iBa^6d^2e^{6ic})e^{-6idx}+(-6912Aa^6d^2e^{8ic}-5376iBa^6d^2e^{8ic})e^{-4idx}+(-59904Aa^6d^2e^{10ic}-35328iBa^6d^2e^{10ic})e^{-2ic}}{24576a^9d^3} \\ x\left(-\frac{111iA-49B}{8a^3}+\frac{(111iAe^{6ic}+39iAe^{4ic}+9iAe^{2ic}+iA-49Be^{6ic}-23Be^{4ic}-7Be^{2ic}-B)e^{-6ic}}{8a^3}\right) \end{cases}$$
$$+\frac{x(111iA-49B)}{8a^3}-\frac{(7A+3iB)\log\left(e^{2idx}-e^{-2ic}\right)}{a^3d}$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
(6*A + 2*I*B + (-4*A*exp(2*I*c) - 2*I*B*exp(2*I*c))*exp(2*I*d*x))/(a**3*d*
exp(4*I*c)*exp(4*I*d*x) - 2*a**3*d*exp(2*I*c)*exp(2*I*d*x) + a**3*d) + Pie
cewise((((-512*A*a**6*d**2*exp(6*I*c) - 512*I*B*a**6*d**2*exp(6*I*c))*exp(
-6*I*d*x) + (-6912*A*a**6*d**2*exp(8*I*c) - 5376*I*B*a**6*d**2*exp(8*I*c))
*exp(-4*I*d*x) + (-59904*A*a**6*d**2*exp(10*I*c) - 35328*I*B*a**6*d**2*exp
(10*I*c))*exp(-2*I*d*x))*exp(-12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(
12*I*c), 0)), (x*(-(111*I*A - 49*B)/(8*a**3) + (111*I*A*exp(6*I*c) + 39*I*
A*exp(4*I*c) + 9*I*A*exp(2*I*c) + I*A - 49*B*exp(6*I*c) - 23*B*exp(4*I*c)
- 7*B*exp(2*I*c) - B)*exp(-6*I*c)/(8*a**3)), True)) + x*(111*I*A - 49*B)/(
8*a**3) - (7*A + 3*I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/(a**3*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.77

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{(A-iB)\log(\tan(dx+c)+i)}{16\,a^3d} + \frac{(111\,A+49i\,B)\log(\tan(dx+c)-i)}{16\,a^3d} - \frac{(7\,A+3i\,B)\log(|\tan(dx+c)|)}{a^3d} - \frac{15\,(-11i\,A+5\,B)\tan(dx+c)^4 - 3\,(137\,A+63i\,B)\tan(dx+c)^3 + 2\,(149i\,A-71\,B)\tan(dx+c)^2 +}{24\,a^3d(\tan(dx+c)-i)^3\tan(dx+c)^2}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"giac")
```

output
```
1/16*(A - I*B)*log(tan(d*x + c) + I)/(a^3*d) + 1/16*(111*A + 49*I*B)*log(t
an(d*x + c) - I)/(a^3*d) - (7*A + 3*I*B)*log(abs(tan(d*x + c)))/(a^3*d) -
1/24*(15*(-11*I*A + 5*B)*tan(d*x + c)^4 - 3*(137*A + 63*I*B)*tan(d*x + c)^
3 + 2*(149*I*A - 71*B)*tan(d*x + c)^2 + 12*(3*A + 2*I*B)*tan(d*x + c) + 12
*I*A)/(a^3*d*(tan(d*x + c) - I)^3*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\frac{A}{2a^3}+\tan(c+dx)^3\left(-\frac{63B}{8a^3}+\frac{A\,137\mathrm{i}}{8a^3}\right)+\tan(c+dx)^2\left(\frac{149A}{12a^3}+\frac{B\,71\mathrm{i}}{12a^3}\right)-\tan(c+dx)^4\left(\frac{55A}{8a^3}+\frac{B\,25\mathrm{i}}{8a^3}\right)-\mathrm{ta}}{d\left(-\tan(c+dx)^5\,1\mathrm{i}-3\tan(c+dx)^4+\tan(c+dx)^3\,3\mathrm{i}+\tan(c+dx)^2\right)} - \frac{\ln(\tan(c+dx))\,(7A+B\,3\mathrm{i})}{a^3 d}+\frac{\ln(\tan(c+dx)+1\mathrm{i})\,(A-B\,1\mathrm{i})}{16a^3d} + \frac{\ln(\tan(c+dx)-\mathrm{i})\,(111A+B\,49\mathrm{i})}{16a^3d}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(log(tan(c + d*x) + 1i)*(A - B*1i))/(16*a^3*d) - (log(tan(c + d*x))*(7*A +
 B*3i))/(a^3*d) - (tan(c + d*x)^3*((A*137i)/(8*a^3) - (63*B)/(8*a^3)) - ta
n(c + d*x)^4*((55*A)/(8*a^3) + (B*25i)/(8*a^3)) + tan(c + d*x)^2*((149*A)/
(12*a^3) + (B*71i)/(12*a^3)) + A/(2*a^3) - tan(c + d*x)*((A*3i)/(2*a^3) -
B/a^3))/(d*(tan(c + d*x)^2 + tan(c + d*x)^3*3i - 3*tan(c + d*x)^4 - tan(c
+ d*x)^5*1i)) + (log(tan(c + d*x) - 1i)*(111*A + B*49i))/(16*a^3*d)
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
(684*cos(c + d*x)*sin(c + d*x)*a*i - 484*cos(c + d*x)*sin(c + d*x)*b - 155
2*cos(c + d*x)*a - 1184*cos(c + d*x)*b*i + 1632*int(tan((c + d*x)/2)**2/(t
an((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20
*tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1
),x)*sin(c + d*x)**2*a*d*i - 1312*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2
)**6 - 6*tan((c + d*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)
/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1),x)*sin(c + d
*x)**2*b*d + 6600*int(tan((c + d*x)/2)/(tan((c + d*x)/2)**6 - 6*tan((c + d
*x)/2)**5*i - 15*tan((c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i + 15*tan((
c + d*x)/2)**2 - 6*tan((c + d*x)/2)*i - 1),x)*sin(c + d*x)**2*a*d + 5200*i
nt(tan((c + d*x)/2)/(tan((c + d*x)/2)**6 - 6*tan((c + d*x)/2)**5*i - 15*ta
n((c + d*x)/2)**4 + 20*tan((c + d*x)/2)**3*i + 15*tan((c + d*x)/2)**2 - 6*
tan((c + d*x)/2)*i - 1),x)*sin(c + d*x)**2*b*d*i + 776*int(1/(tan((c + d*x
)/2)**9 - 6*tan((c + d*x)/2)**8*i - 15*tan((c + d*x)/2)**7 + 20*tan((c + d
*x)/2)**6*i + 15*tan((c + d*x)/2)**5 - 6*tan((c + d*x)/2)**4*i - tan((c +
d*x)/2)**3),x)*sin(c + d*x)**2*a*d + 592*int(1/(tan((c + d*x)/2)**9 - 6*ta
n((c + d*x)/2)**8*i - 15*tan((c + d*x)/2)**7 + 20*tan((c + d*x)/2)**6*i +
15*tan((c + d*x)/2)**5 - 6*tan((c + d*x)/2)**4*i - tan((c + d*x)/2)**3),x)
*sin(c + d*x)**2*b*d*i + 4320*int(1/(tan((c + d*x)/2)**8 - 6*tan((c + d*x)
/2)**7*i - 15*tan((c + d*x)/2)**6 + 20*tan((c + d*x)/2)**5*i + 15*tan((...
```

3.59 $\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 185

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \frac{(A+15iB)x}{16a^4} - \frac{B\log(\cos(c+dx))}{a^4d} - \frac{iA-15B}{16a^4d(1+i\tan(c+dx))} - \frac{(iA-7B)\tan^2(c+dx)}{16a^4d(1+i\tan(c+dx))^2} + \frac{(iA-B)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{(A+3iB)\tan^3(c+dx)}{12ad(a+ia\tan(c+dx))^3}$$

output
```
1/16*(A+15*I*B)*x/a^4-B*ln(cos(d*x+c))/a^4/d-1/16*(I*A-15*B)/a^4/d/(1+I*ta
n(d*x+c))-1/16*(I*A-7*B)*tan(d*x+c)^2/a^4/d/(1+I*tan(d*x+c))^2+1/8*(I*A-B)
*tan(d*x+c)^4/d/(a+I*a*tan(d*x+c))^4+1/12*(A+3*I*B)*tan(d*x+c)^3/a/d/(a+I*
a*tan(d*x+c))^3
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.43

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^4(c+dx)(18iA-48B+8(-4iA+21B)\cos(2(c+dx))+2\cos(4(c+dx))(7iA-60B+(-3iA+9}{}$$

input
```
Integrate[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x
]
```

output
```
(Sec[c + d*x]^4*((18*I)*A - 48*B + 8*((-4*I)*A + 21*B)*Cos[2*(c + d*x)] +
2*Cos[4*(c + d*x)]*((7*I)*A - 60*B + ((-3*I)*A + 93*B)*Log[I - Tan[c + d*x
]] + 3*(I*A + B)*Log[I + Tan[c + d*x]]) + 16*A*Sin[2*(c + d*x)] + (144*I)*
B*Sin[2*(c + d*x)] - 11*A*Sin[4*(c + d*x)] - (117*I)*B*Sin[4*(c + d*x)] +
6*A*Log[I - Tan[c + d*x]]*Sin[4*(c + d*x)] + (186*I)*B*Log[I - Tan[c + d*x
]]*Sin[4*(c + d*x)] - 6*A*Log[I + Tan[c + d*x]]*Sin[4*(c + d*x)] + (6*I)*B
*Log[I + Tan[c + d*x]]*Sin[4*(c + d*x)]))/(192*a^4*d*(-I + Tan[c + d*x])^4
)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4078, 27, 3042, 4072, 25, 27, 3042, 3956, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^4(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\int \frac{4\tan^3(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\int \frac{\tan^3(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\int \frac{\tan(c+dx)^3(a(iA-B)+2iaB\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{2a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{-\frac{\int -\frac{3\tan^2(c+dx)\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{(i\tan(c+dx)a+a)^2}dx}{6a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\int \frac{\tan^2(c+dx)\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{(i\tan(c+dx)a+a)^2}dx}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\int \frac{\tan(c+dx)^2\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{(i\tan(c+dx)a+a)^2}dx}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\frac{(-7B+iA)\tan^2(c+dx)}{4d(1+i\tan(c+dx))^2}-\frac{\int \frac{2\tan(c+dx)\left((iA-7B)a^3+8iB\tan(c+dx)a^3\right)}{i\tan(c+dx)a+a}dx}{4a^2}}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\frac{(-7B+iA)\tan^2(c+dx)}{4d(1+i\tan(c+dx))^2}-\frac{\int\frac{\tan(c+dx)\left((iA-7B)a^3+8iB\tan(c+dx)a^3\right)}{i\tan(c+dx)a+a}dx}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}}{}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\frac{(-7B+iA)\tan^2(c+dx)}{4d(1+i\tan(c+dx))^2}-\frac{\int\frac{\tan(c+dx)\left((iA-7B)a^3+8iB\tan(c+dx)a^3\right)}{i\tan(c+dx)a+a}dx}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}}{}$$

$$\Big\downarrow \ 4072$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\frac{(-7B+iA)\tan^2(c+dx)}{4d(1+i\tan(c+dx))^2}-\frac{8a^2B\int\tan(c+dx)dx-\frac{i\int-\frac{a^4(A+15iB)\tan(c+dx)}{i\tan(c+dx)a+a}dx}{a}}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

$$\Big\downarrow \ 25$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\frac{(-7B+iA)\tan^2(c+dx)}{4d(1+i\tan(c+dx))^2}-\frac{8a^2B\int\tan(c+dx)dx+\frac{i\int\frac{a^4(A+15iB)\tan(c+dx)}{i\tan(c+dx)a+a}dx}{a}}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\frac{(-7B+iA)\tan^2(c+dx)}{4d(1+i\tan(c+dx))^2}-\frac{8a^2B\int\tan(c+dx)dx+ia^3(A+15iB)\int\frac{\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\frac{(-7B+iA)\tan^2(c+dx)}{4d(1+i\tan(c+dx))^2}-\frac{8a^2B\int\tan(c+dx)dx+ia^3(A+15iB)\int\frac{\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

$$\Big\downarrow \; 3956$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\frac{(-7B+iA)\tan^2(c+dx)}{4d(1+i\tan(c+dx))^2}-\frac{-\frac{8a^2B\log(\cos(c+dx))}{d}+ia^3(A+15iB)\int\frac{\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

$$\Big\downarrow \; 4009$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\frac{(-7B+iA)\tan^2(c+dx)}{4d(1+i\tan(c+dx))^2}-\frac{-\frac{8a^2B\log(\cos(c+dx))}{d}+ia^3(A+15iB)\left(-\frac{i\int 1dx}{2a}-\frac{1}{2d(a+ia\tan(c+dx))}\right)}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

$$\Big\downarrow \; 24$$

$$\frac{(-B+iA)\tan^4(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{\frac{(-7B+iA)\tan^2(c+dx)}{4d(1+i\tan(c+dx))^2}-\frac{-\frac{8a^2B\log(\cos(c+dx))}{d}+ia^3(A+15iB)\left(-\frac{1}{2d(a+ia\tan(c+dx))}-\frac{ix}{2a}\right)}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^3(c+dx)}{6d(a+ia\tan(c+dx))^3}}{2a^2}$$

input
```
Int[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I*A - B)*Tan[c + d*x]^4)/(8*d*(a + I*a*Tan[c + d*x])^4) - (-1/6*(a*(A +
(3*I)*B)*Tan[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) + (((I*A - 7*B)*Tan[
c + d*x]^2)/(4*d*(1 + I*Tan[c + d*x])^2) - ((-8*a^2*B*Log[Cos[c + d*x]])/d
 + I*a^3*(A + (15*I)*B)*(((-1/2*I)*x)/a - 1/(2*d*(a + I*a*Tan[c + d*x]))))
/(2*a^2))/(2*a^2))/(2*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06

method	result
risch	$\frac{31ixB}{16a^4}+\frac{xA}{16a^4}+\frac{13\,\mathrm{e}^{-2i(dx+c)}B}{16d\,a^4}-\frac{i\mathrm{e}^{-2i(dx+c)}A}{8d\,a^4}-\frac{\mathrm{e}^{-4i(dx+c)}B}{4d\,a^4}+\frac{3i\mathrm{e}^{-4i(dx+c)}A}{32d\,a^4}+\frac{\mathrm{e}^{-6i(dx+c)}B}{16d\,a^4}-\frac{i\mathrm{e}^{-6}}{}$
derivativedivides	$\frac{15iB\arctan(\tan(dx+c))}{16d\,a^4}+\frac{A\arctan(\tan(dx+c))}{16d\,a^4}+\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^4}-\frac{49iB}{16d\,a^4(-i+\tan(dx+c))}-\frac{}{16d\,a^4(}$
default	$\frac{15iB\arctan(\tan(dx+c))}{16d\,a^4}+\frac{A\arctan(\tan(dx+c))}{16d\,a^4}+\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^4}-\frac{49iB}{16d\,a^4(-i+\tan(dx+c))}-\frac{}{16d\,a^4(}$

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVER
BOSE)
```

output
```
31/16*I*x/a^4*B+1/16*x/a^4*A+13/16/d/a^4*exp(-2*I*(d*x+c))*B-1/8*I/d/a^4*e
xp(-2*I*(d*x+c))*A-1/4/d/a^4*exp(-4*I*(d*x+c))*B+3/32*I/d/a^4*exp(-4*I*(d*
x+c))*A+1/16/d/a^4*exp(-6*I*(d*x+c))*B-1/24*I/d/a^4*exp(-6*I*(d*x+c))*A-1/
128/d/a^4*exp(-8*I*(d*x+c))*B+1/128*I/d/a^4*exp(-8*I*(d*x+c))*A+2*I*B/d/a^
4*c-B/d/a^4*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.65

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$=\frac{(24\,(A+31i\,B)dxe^{(8i\,dx+8i\,c)}-384\,Be^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-24\,(2i\,A-13\,B)e^{(6i\,dx+6i\,c)}-12\,(-}{384\,a^4d}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"fricas")
```

output
```
1/384*(24*(A + 31*I*B)*d*x*e^(8*I*d*x + 8*I*c) - 384*B*e^(8*I*d*x + 8*I*c)
*log(e^(2*I*d*x + 2*I*c) + 1) - 24*(2*I*A - 13*B)*e^(6*I*d*x + 6*I*c) - 12
*(-3*I*A + 8*B)*e^(4*I*d*x + 4*I*c) - 8*(2*I*A - 3*B)*e^(2*I*d*x + 2*I*c)
+ 3*I*A - 3*B)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.94

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{B\log\left(e^{2idx}+e^{-2ic}\right)}{a^4d} + \begin{cases} \frac{((24576iAa^{12}d^3e^{12ic}-24576Ba^{12}d^3e^{12ic})e^{-8idx}+(-131072iAa^{12}d^3e^{14ic}+196608Ba^{12}d^3e^{14ic})e^{-6idx}+(294912iAa^{12}d^3e^{16ic}-786432}{3145728a^{16}d^4} \\ x\left(-\frac{A+31iB}{16a^4}+\frac{(Ae^{8ic}-4Ae^{6ic}+6Ae^{4ic}-4Ae^{2ic}+A+31iBe^{8ic}-26iBe^{6ic}+16iBe^{4ic}-6iBe^{2ic}+iB)e^{-8ic}}{16a^4}\right) \end{cases} + \frac{x(A+31iB)}{16a^4}$$

input
```
integrate(tan(d*x+c)**4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)
```

output
```
-B*log(exp(2*I*d*x) + exp(-2*I*c))/(a**4*d) + Piecewise((((24576*I*A*a**12
*d**3*exp(12*I*c) - 24576*B*a**12*d**3*exp(12*I*c))*exp(-8*I*d*x) + (-1310
72*I*A*a**12*d**3*exp(14*I*c) + 196608*B*a**12*d**3*exp(14*I*c))*exp(-6*I*
d*x) + (294912*I*A*a**12*d**3*exp(16*I*c) - 786432*B*a**12*d**3*exp(16*I*c
))*exp(-4*I*d*x) + (-393216*I*A*a**12*d**3*exp(18*I*c) + 2555904*B*a**12*d
**3*exp(18*I*c))*exp(-2*I*d*x))*exp(-20*I*c)/(3145728*a**16*d**4), Ne(a**1
6*d**4*exp(20*I*c), 0)), (x*(-(A + 31*I*B)/(16*a**4) + (A*exp(8*I*c) - 4*A
*exp(6*I*c) + 6*A*exp(4*I*c) - 4*A*exp(2*I*c) + A + 31*I*B*exp(8*I*c) - 26
*I*B*exp(6*I*c) + 16*I*B*exp(4*I*c) - 6*I*B*exp(2*I*c) + I*B)*exp(-8*I*c)/
(16*a**4)), True)) + x*(A + 31*I*B)/(16*a**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.66

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= -\frac{(-iA-B)\log(\tan(dx+c)+i)}{32a^4d} + \frac{(-iA+31B)\log(\tan(dx+c)-i)}{32a^4d} - \frac{3(15A+49iB)\tan(dx+c)^3 + 12(-7iA+29B)\tan(dx+c)^2 - (61A+291iB)\tan(dx+c) + 16}{48a^4d(\tan(dx+c)-i)^4}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"giac")
```

output
```
-1/32*(-I*A - B)*log(tan(d*x + c) + I)/(a^4*d) + 1/32*(-I*A + 31*B)*log(ta
n(d*x + c) - I)/(a^4*d) - 1/48*(3*(15*A + 49*I*B)*tan(d*x + c)^3 + 12*(-7*
I*A + 29*B)*tan(d*x + c)^2 - (61*A + 291*I*B)*tan(d*x + c) + 16*I*A - 84*B
)/(a^4*d*(tan(d*x + c) - I)^4)
```

Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\tan(c+dx)^2\left(-\frac{29B}{4a^4}+\frac{A\,7\mathrm{i}}{4a^4}\right) - \tan(c+dx)^3\left(\frac{15A}{16a^4}+\frac{B\,49\mathrm{i}}{16a^4}\right) - \frac{A\,1\mathrm{i}}{3a^4} + \frac{7B}{4a^4} + \tan(c+dx)\left(\frac{61A}{48a^4}+\frac{B\,97\mathrm{i}}{16a^4}\right)}{d\left(\tan(c+dx)^4 - \tan(c+dx)^3\,4\mathrm{i} - 6\tan(c+dx)^2 + \tan(c+dx)\,4\mathrm{i} + 1\right)} + \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(B+A\,1\mathrm{i})}{32a^4d} - \frac{\ln(\tan(c+dx)-\mathrm{i})\,(A+B\,31\mathrm{i})\,1\mathrm{i}}{32a^4d}$$

input
```
int((tan(c + d*x)^4*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(tan(c + d*x)^2*((A*7i)/(4*a^4) - (29*B)/(4*a^4)) - tan(c + d*x)^3*((15*A)
/(16*a^4) + (B*49i)/(16*a^4)) - (A*1i)/(3*a^4) + (7*B)/(4*a^4) + tan(c + d
*x)*((61*A)/(48*a^4) + (B*97i)/(16*a^4)))/(d*(tan(c + d*x)*4i - 6*tan(c +
d*x)^2 - tan(c + d*x)^3*4i + tan(c + d*x)^4 + 1)) + (log(tan(c + d*x) + 1i
)*(A*1i + B))/(32*a^4*d) - (log(tan(c + d*x) - 1i)*(A + B*31i)*1i)/(32*a^4
*d)
```

Reduce [F]

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{-16\left(\int \frac{\tan(dx+c)^3}{\tan(dx+c)^4i+4\tan(dx+c)^3-6\tan(dx+c)^2i-4\tan(dx+c)+i}dx\right)ad-40\left(\int \frac{\tan(dx+c)^3}{\tan(dx+c)^4i+4\tan(dx+c)^3-6\tan(dx+c)^2i-4\,\mathrm{ta}}\right.}{}$$

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

output
```
( - 16*int(tan(c + d*x)**3/(tan(c + d*x)**4*i + 4*tan(c + d*x)**3 - 6*tan(
c + d*x)**2*i - 4*tan(c + d*x) + i),x)*a*d - 40*int(tan(c + d*x)**3/(tan(c
 + d*x)**4*i + 4*tan(c + d*x)**3 - 6*tan(c + d*x)**2*i - 4*tan(c + d*x) +
i),x)*b*d*i + 24*int(tan(c + d*x)**2/(tan(c + d*x)**4*i + 4*tan(c + d*x)**
3 - 6*tan(c + d*x)**2*i - 4*tan(c + d*x) + i),x)*a*d*i - 80*int(tan(c + d*
x)**2/(tan(c + d*x)**4*i + 4*tan(c + d*x)**3 - 6*tan(c + d*x)**2*i - 4*tan
(c + d*x) + i),x)*b*d + 16*int(tan(c + d*x)/(tan(c + d*x)**4*i + 4*tan(c +
 d*x)**3 - 6*tan(c + d*x)**2*i - 4*tan(c + d*x) + i),x)*a*d + 60*int(tan(c
 + d*x)/(tan(c + d*x)**4*i + 4*tan(c + d*x)**3 - 6*tan(c + d*x)**2*i - 4*t
an(c + d*x) + i),x)*b*d*i - 4*int(1/(tan(c + d*x)**4*i + 4*tan(c + d*x)**3
 - 6*tan(c + d*x)**2*i - 4*tan(c + d*x) + i),x)*a*d*i + 16*int(1/(tan(c +
d*x)**4*i + 4*tan(c + d*x)**3 - 6*tan(c + d*x)**2*i - 4*tan(c + d*x) + i),
x)*b*d - log(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4
*tan(c + d*x)*i + 1)*a*i + 4*log(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6
*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1)*b + 2*log(tan(c + d*x)**2 + 1)*a*
i - 6*log(tan(c + d*x)**2 + 1)*b)/(4*a**4*d)
```

3.60 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 159

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \frac{(iA+B)x}{16a^4} - \frac{A-13iB}{48a^4d(1+i\tan(c+dx))^2} + \frac{5A-29iB}{48a^4d(1+i\tan(c+dx))} + \frac{(iA-B)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4} + \frac{(A+5iB)\tan^2(c+dx)}{24ad(a+ia\tan(c+dx))^3}$$

output
```
1/16*(I*A+B)*x/a^4-1/48*(A-13*I*B)/a^4/d/(1+I*tan(d*x+c))^2+1/48*(5*A-29*I
*B)/a^4/d/(1+I*tan(d*x+c))+1/8*(I*A-B)*tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))^4
+1/24*(A+5*I*B)*tan(d*x+c)^2/a/d/(a+I*a*tan(d*x+c))^3
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^4(c+dx)(36iB+16(A-4iB)\cos(2(c+dx))+3(A+iB+8iAdx+8Bdx)\cos(4(c+dx))+32iA\,\mathrm{s}}{3}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x
]
```

output
```
(Sec[c + d*x]^4*((36*I)*B + 16*(A - (4*I)*B)*Cos[2*(c + d*x)] + 3*(A + I*B
 + (8*I)*A*d*x + 8*B*d*x)*Cos[4*(c + d*x)] + (32*I)*A*Sin[2*(c + d*x)] + 3
2*B*Sin[2*(c + d*x)] - (3*I)*A*Sin[4*(c + d*x)] + 3*B*Sin[4*(c + d*x)] - 2
4*A*d*x*Sin[4*(c + d*x)] + (24*I)*B*d*x*Sin[4*(c + d*x)]))/(384*a^4*d*(-I
+ Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4078, 3042, 4078, 27, 3042, 4073, 25, 3042, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{4078}$$
$$\frac{(-B+iA)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\int \frac{\tan^2(c+dx)(3a(iA-B)-a(A-7iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\int\frac{\tan(c+dx)^2(3a(iA-B)-a(A-7iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{8a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{-\frac{\int-\frac{4\tan(c+dx)\left((A+5iB)a^2+2(iA+4B)\tan(c+dx)a^2\right)}{(i\tan(c+dx)a+a)^2}dx}{6a^2}-\frac{a(A+5iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{2\int\frac{\tan(c+dx)\left((A+5iB)a^2+2(iA+4B)\tan(c+dx)a^2\right)}{(i\tan(c+dx)a+a)^2}dx}{3a^2}-\frac{a(A+5iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{2\int\frac{\tan(c+dx)\left((A+5iB)a^2+2(iA+4B)\tan(c+dx)a^2\right)}{(i\tan(c+dx)a+a)^2}dx}{3a^2}-\frac{a(A+5iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}$$

$$\Big\downarrow \text{4073}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{2\left(\frac{A-13iB}{4d(1+i\tan(c+dx))^2}-\frac{i\int-\frac{a^3(A-13iB)-4a^3(iA+4B)\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}\right)}{3a^2}-\frac{a(A+5iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{2\left(\frac{i\int\frac{a^3(A-13iB)-4a^3(iA+4B)\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}+\frac{A-13iB}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}-\frac{a(A+5iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{2\left(\frac{i\int\frac{a^3(A-13iB)-4a^3(iA+4B)\tan(c+dx)}{i\tan(c+dx)a+a}dx}{2a^2}+\frac{A-13iB}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}-\frac{a(A+5iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}$$

$$\Big\downarrow \ 4009$$

$$\frac{\frac{(-B+iA)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{2\left(\frac{i\left(\frac{a^3(29B+5iA)}{2d(a+ia\tan(c+dx))}-\frac{3}{2}a^2(A-iB)\int 1dx\right)}{2a^2}+\frac{A-13iB}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}-\frac{a(A+5iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}$$

$$\Big\downarrow \ 24$$

$$\frac{\frac{(-B+iA)\tan^3(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{2\left(\frac{i\left(\frac{a^3(29B+5iA)}{2d(a+ia\tan(c+dx))}-\frac{3}{2}a^2x(A-iB)\right)}{2a^2}+\frac{A-13iB}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}-\frac{a(A+5iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I*A - B)*Tan[c + d*x]^3)/(8*d*(a + I*a*Tan[c + d*x])^4) - (-1/3*(a*(A +
(5*I)*B)*Tan[c + d*x]^2)/(d*(a + I*a*Tan[c + d*x])^3) + (2*((A - (13*I)*B)
/(4*d*(1 + I*Tan[c + d*x])^2) + ((I/2)*((-3*a^2*(A - I*B)*x)/2 + (a^3*((5*
I)*A + 29*B))/(2*d*(a + I*a*Tan[c + d*x]))))/a^2))/(3*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

method	result
risch	$\frac{xB}{16a^4}+\frac{ixA}{16a^4}-\frac{i e^{-2i(dx+c)}B}{8d\,a^4}+\frac{e^{-2i(dx+c)}A}{16d\,a^4}+\frac{3iB\,e^{-4i(dx+c)}}{32d\,a^4}-\frac{i e^{-6i(dx+c)}B}{24d\,a^4}-\frac{e^{-6i(dx+c)}A}{48d\,a^4}+\frac{i e^{-8i(d}}{128d}$
derivativedivides	$\frac{iA}{16d\,a^4(-i+\tan(dx+c))}+\frac{iA\arctan(\tan(dx+c))}{16d\,a^4}+\frac{B\arctan(\tan(dx+c))}{16d\,a^4}+\frac{A}{8d\,a^4(-i+\tan(dx+c))^4}-\frac{}{16d\,a^4(-}$
default	$\frac{iA}{16d\,a^4(-i+\tan(dx+c))}+\frac{iA\arctan(\tan(dx+c))}{16d\,a^4}+\frac{B\arctan(\tan(dx+c))}{16d\,a^4}+\frac{A}{8d\,a^4(-i+\tan(dx+c))^4}-\frac{}{16d\,a^4(-}$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVER
BOSE)
```

output
```
1/16*x/a^4*B+1/16*I*x/a^4*A-1/8*I/d/a^4*exp(-2*I*(d*x+c))*B+1/16/d/a^4*exp
(-2*I*(d*x+c))*A+3/32*I*B/d/a^4*exp(-4*I*(d*x+c))-1/24*I/d/a^4*exp(-6*I*(d
*x+c))*B-1/48/d/a^4*exp(-6*I*(d*x+c))*A+1/128*I/d/a^4*exp(-8*I*(d*x+c))*B+
1/128/d/a^4*exp(-8*I*(d*x+c))*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.55

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\left(24\,(-i\,A-B)dxe^{(8i\,dx+8i\,c)}-24\,(A-2i\,B)e^{(6i\,dx+6i\,c)}-36i\,Be^{(4i\,dx+4i\,c)}+8\,(A+2i\,B)e^{(2i\,dx+2i\,c)}-3\right.}{384\,a^4d}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"fricas")
```

output
```
-1/384*(24*(-I*A - B)*d*x*e^(8*I*d*x + 8*I*c) - 24*(A - 2*I*B)*e^(6*I*d*x
+ 6*I*c) - 36*I*B*e^(4*I*d*x + 4*I*c) + 8*(A + 2*I*B)*e^(2*I*d*x + 2*I*c)
- 3*A - 3*I*B)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.89

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(294912iBa^{12}d^3e^{16ic}e^{-4idx}+(24576Aa^{12}d^3e^{12ic}+24576iBa^{12}d^3e^{12ic})e^{-8idx}+(-65536Aa^{12}d^3e^{14ic}-131072iBa^{12}d^3e^{14ic})e^{-6idx}+(19660}{3145728a^{16}d^4} \\ x\left(-\frac{iA+B}{16a^4}+\frac{(iAe^{8ic}-2iAe^{6ic}+2iAe^{2ic}-iA+Be^{8ic}-4Be^{6ic}+6Be^{4ic}-4Be^{2ic}+B)e^{-8ic}}{16a^4}\right) \end{cases} + \frac{x(iA+B)}{16a^4}$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((294912*I*B*a**12*d**3*exp(16*I*c)*exp(-4*I*d*x) + (24576*A*a**
12*d**3*exp(12*I*c) + 24576*I*B*a**12*d**3*exp(12*I*c))*exp(-8*I*d*x) + (-
65536*A*a**12*d**3*exp(14*I*c) - 131072*I*B*a**12*d**3*exp(14*I*c))*exp(-6
*I*d*x) + (196608*A*a**12*d**3*exp(18*I*c) - 393216*I*B*a**12*d**3*exp(18*
I*c))*exp(-2*I*d*x))*exp(-20*I*c)/(3145728*a**16*d**4), Ne(a**16*d**4*exp(
20*I*c), 0)), (x*(-(I*A + B)/(16*a**4) + (I*A*exp(8*I*c) - 2*I*A*exp(6*I*c
) + 2*I*A*exp(2*I*c) - I*A + B*exp(8*I*c) - 4*B*exp(6*I*c) + 6*B*exp(4*I*c
) - 4*B*exp(2*I*c) + B)*exp(-8*I*c)/(16*a**4)), True)) + x*(I*A + B)/(16*a
**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$\begin{aligned}&\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx\\&= -\frac{(A-iB)\log(\tan(dx+c)+i)}{32a^4d} + \frac{(A-iB)\log(\tan(dx+c)-i)}{32a^4d}\\&\quad - \frac{3(-iA+15B)\tan(dx+c)^3 + 12(A-7iB)\tan(dx+c)^2 + (-13iA-61B)\tan(dx+c) - 4A + \text{I}}{48a^4d(\tan(dx+c)-i)^4}\end{aligned}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"giac")
```

output
```
-1/32*(A - I*B)*log(tan(d*x + c) + I)/(a^4*d) + 1/32*(A - I*B)*log(tan(d*x
 + c) - I)/(a^4*d) - 1/48*(3*(-I*A + 15*B)*tan(d*x + c)^3 + 12*(A - 7*I*B)
*tan(d*x + c)^2 + (-13*I*A - 61*B)*tan(d*x + c) - 4*A + 16*I*B)/(a^4*d*(ta
n(d*x + c) - I)^4)
```

Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{A}{12\,a^4}+\tan(c+d\,x)^3\,\left(-\frac{15\,B}{16\,a^4}+\frac{A\,1\mathrm{i}}{16\,a^4}\right)-\tan(c+d\,x)^2\,\left(\frac{A}{4\,a^4}-\frac{B\,7\mathrm{i}}{4\,a^4}\right)-\frac{B\,1\mathrm{i}}{3\,a^4}+\tan(c+d\,x)\,\left(\frac{61\,B}{48\,a^4}+\frac{A\,13\mathrm{i}}{48\,a^4}\right)}{d\,\left(\tan\left(c+d\,x\right)^4-\tan\left(c+d\,x\right)^3\,4\mathrm{i}-6\,\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,4\mathrm{i}+1\right)}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{32\,a^4\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{32\,a^4\,d}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(tan(c + d*x)^3*((A*1i)/(16*a^4) - (15*B)/(16*a^4)) - tan(c + d*x)^2*(A/(4
*a^4) - (B*7i)/(4*a^4)) + A/(12*a^4) - (B*1i)/(3*a^4) + tan(c + d*x)*((A*1
3i)/(48*a^4) + (61*B)/(48*a^4)))/(d*(tan(c + d*x)*4i - 6*tan(c + d*x)^2 -
tan(c + d*x)^3*4i + tan(c + d*x)^4 + 1)) + (log(tan(c + d*x) - 1i)*(A - B*
1i))/(32*a^4*d) + (log(tan(c + d*x) + 1i)*(A*1i + B)*1i)/(32*a^4*d)
```

Reduce [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{4\left(\int \frac{\tan(dx+c)^3}{\tan(dx+c)^4 i+4\tan(dx+c)^3-6\tan(dx+c)^2 i-4\tan(dx+c)+i}dx\right)adi-16\left(\int \frac{\tan(dx+c)^3}{\tan(dx+c)^4 i+4\tan(dx+c)^3-6\tan(dx+c)^2 i-4\tan(d}\right.}{}$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(4*int(tan(c + d*x)**3/(tan(c + d*x)**4*i + 4*tan(c + d*x)**3 - 6*tan(c +
d*x)**2*i - 4*tan(c + d*x) + i),x)*a*d*i - 16*int(tan(c + d*x)**3/(tan(c +
 d*x)**4*i + 4*tan(c + d*x)**3 - 6*tan(c + d*x)**2*i - 4*tan(c + d*x) + i)
,x)*b*d + 24*int(tan(c + d*x)**2/(tan(c + d*x)**4*i + 4*tan(c + d*x)**3 -
6*tan(c + d*x)**2*i - 4*tan(c + d*x) + i),x)*b*d*i + 16*int(tan(c + d*x)/(
tan(c + d*x)**4*i + 4*tan(c + d*x)**3 - 6*tan(c + d*x)**2*i - 4*tan(c + d*
x) + i),x)*b*d - 4*int(1/(tan(c + d*x)**4*i + 4*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2*i - 4*tan(c + d*x) + i),x)*b*d*i - log(tan(c + d*x)**4 - 4*tan(c
 + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1)*b*i + 2*log(tan(c
 + d*x)**2 + 1)*b*i)/(4*a**4*d)
```

3.61 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 145

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(A-iB)x}{16a^4}+\frac{iA+5B}{16a^4d(1+i\tan(c+dx))^2} - \frac{iA+B}{16a^4d(1+i\tan(c+dx))} + \frac{(iA-B)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4} - \frac{B}{6ad(a+ia\tan(c+dx))^3}$$

output
```
-1/16*(A-I*B)*x/a^4+1/16*(I*A+5*B)/a^4/d/(1+I*tan(d*x+c))^2-1/16*(I*A+B)/a
^4/d/(1+I*tan(d*x+c))+1/8*(I*A-B)*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))^4-1/6*
B/a/d/(a+I*a*tan(d*x+c))^3
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(\cos(4(c+dx))-i\sin(4(c+dx)))(-12iA-16B\cos(2(c+dx))+3(iA-B+8Adx-8iBdx)\cos(4$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x
]
```

output
```
-1/384*((Cos[4*(c + d*x)] - I*Sin[4*(c + d*x)])*((-12*I)*A - 16*B*Cos[2*(c
 + d*x)] + 3*(I*A - B + 8*A*d*x - (8*I)*B*d*x)*Cos[4*(c + d*x)] - (32*I)*B
*Sin[2*(c + d*x)] + 3*A*Sin[4*(c + d*x)] + (3*I)*B*Sin[4*(c + d*x)] + (24*
I)*A*d*x*Sin[4*(c + d*x)] + 24*B*d*x*Sin[4*(c + d*x)]))/(a^4*d)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4078, 27, 3042, 4073, 27, 3042, 4009, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\int \frac{2\tan(c+dx)(a(iA-B)-a(A-3iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\int \frac{\tan(c+dx)(a(iA-B)-a(A-3iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\int \frac{\tan(c+dx)(a(iA-B)-a(A-3iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{4a^2}$$

$\Big\downarrow$ 4073

$$\frac{(-B+iA)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{2aB}{3d(a+ia\tan(c+dx))^3}-\frac{i\int -\frac{2\left(2Ba^2+(A-3iB)\tan(c+dx)a^2\right)}{(i\tan(c+dx)a+a)^2}dx}{2a^2}}{4a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{i\int \frac{2Ba^2+(A-3iB)\tan(c+dx)a^2}{(i\tan(c+dx)a+a)^2}dx}{a^2}+\frac{2aB}{3d(a+ia\tan(c+dx))^3}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{i\int \frac{2Ba^2+(A-3iB)\tan(c+dx)a^2}{(i\tan(c+dx)a+a)^2}dx}{a^2}+\frac{2aB}{3d(a+ia\tan(c+dx))^3}}{4a^2}$$

$\Big\downarrow$ 4009

$$\frac{(-B+iA)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{i\left(-\frac{1}{2}a(B+iA)\int \frac{1}{i\tan(c+dx)a+a}dx-\frac{A-5iB}{4d(1+i\tan(c+dx))^2}\right)}{a^2}+\frac{2aB}{3d(a+ia\tan(c+dx))^3}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{i\left(-\frac{1}{2}a(B+iA)\int \frac{1}{i\tan(c+dx)a+a}dx-\frac{A-5iB}{4d(1+i\tan(c+dx))^2}\right)}{a^2}+\frac{2aB}{3d(a+ia\tan(c+dx))^3}}{4a^2}$$

$\Big\downarrow$ 3960

$$\frac{(-B+iA)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{i\left(-\frac{1}{2}a(B+iA)\left(\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}\right)-\frac{A-5iB}{4d(1+i\tan(c+dx))^2}\right)}{a^2}+\frac{2aB}{3d(a+ia\tan(c+dx))^3}}{4a^2}$$

$\Big\downarrow$ 24

$$\frac{(-B+iA)\tan^2(c+dx)}{8d(a+ia\tan(c+dx))^4}-\frac{\frac{i\left(-\frac{1}{2}a(B+iA)\left(\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}\right)-\frac{A-5iB}{4d(1+i\tan(c+dx))^2}\right)}{a^2}+\frac{2aB}{3d(a+ia\tan(c+dx))^3}}{4a^2}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((I*A - B)*Tan[c + d*x]^2)/(8*d*(a + I*a*Tan[c + d*x])^4) - ((2*a*B)/(3*d*
(a + I*a*Tan[c + d*x])^3) + (I*(-1/4*(A - (5*I)*B)/(d*(1 + I*Tan[c + d*x])
^2) - (a*(I*A + B)*(x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x]))))/2))/a^2)/
(4*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

method	result
risch	$\frac{ixB}{16a^4} - \frac{xA}{16a^4} + \frac{e^{-2i(dx+c)}B}{16d\,a^4} + \frac{ie^{-4i(dx+c)}A}{32d\,a^4} - \frac{e^{-6i(dx+c)}B}{48d\,a^4} + \frac{e^{-8i(dx+c)}B}{128d\,a^4} - \frac{ie^{-8i(dx+c)}A}{128d\,a^4}$
derivativedivides	$\frac{iB\arctan(\tan(dx+c))}{16d\,a^4} - \frac{A\arctan(\tan(dx+c))}{16d\,a^4} - \frac{iA}{8d\,a^4(-i+\tan(dx+c))^4} - \frac{A}{16d\,a^4(-i+\tan(dx+c))} + \frac{}{16d\,a^4(-}$
default	$\frac{iB\arctan(\tan(dx+c))}{16d\,a^4} - \frac{A\arctan(\tan(dx+c))}{16d\,a^4} - \frac{iA}{8d\,a^4(-i+\tan(dx+c))^4} - \frac{A}{16d\,a^4(-i+\tan(dx+c))} + \frac{}{16d\,a^4(-}$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVER
BOSE)
```

output
```
1/16*I*x/a^4*B-1/16*x/a^4*A+1/16/d/a^4*exp(-2*I*(d*x+c))*B+1/32*I*A/d/a^4*
exp(-4*I*(d*x+c))-1/48/d/a^4*exp(-6*I*(d*x+c))*B+1/128/d/a^4*exp(-8*I*(d*x
+c))*B-1/128*I/d/a^4*exp(-8*I*(d*x+c))*A
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\left(24\,(A-i\,B)dxe^{(8i\,dx+8i\,c)} - 24\,Be^{(6i\,dx+6i\,c)} - 12i\,Ae^{(4i\,dx+4i\,c)} + 8\,Be^{(2i\,dx+2i\,c)} + 3i\,A - 3\,B\right)e^{(-8i\,dx-8}}{384\,a^4d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"fricas")
```

output
```
-1/384*(24*(A - I*B)*d*x*e^(8*I*d*x + 8*I*c) - 24*B*e^(6*I*d*x + 6*I*c) -
12*I*A*e^(4*I*d*x + 4*I*c) + 8*B*e^(2*I*d*x + 2*I*c) + 3*I*A - 3*B)*e^(-8*
I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.66

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(98304iAa^{12}d^3e^{16ic}e^{-4idx}+196608Ba^{12}d^3e^{18ic}e^{-2idx}-65536Ba^{12}d^3e^{14ic}e^{-6idx}+(-24576iAa^{12}d^3e^{12ic}+24576Ba^{12}d^3e^{12ic})e^{-8idx})e^{-20}}{3145728a^{16}d^4} \\ x\left(-\frac{-A+iB}{16a^4}+\frac{(-Ae^{8ic}+2Ae^{4ic}-A+iBe^{8ic}-2iBe^{6ic}+2iBe^{2ic}-iB)e^{-8ic}}{16a^4}\right) \end{cases} + \frac{x(-A+iB)}{16a^4}$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((98304*I*A*a**12*d**3*exp(16*I*c)*exp(-4*I*d*x) + 196608*B*a**1
2*d**3*exp(18*I*c)*exp(-2*I*d*x) - 65536*B*a**12*d**3*exp(14*I*c)*exp(-6*I
*d*x) + (-24576*I*A*a**12*d**3*exp(12*I*c) + 24576*B*a**12*d**3*exp(12*I*c
))*exp(-8*I*d*x))*exp(-20*I*c)/(3145728*a**16*d**4), Ne(a**16*d**4*exp(20*
I*c), 0)), (x*(-(-A + I*B)/(16*a**4) + (-A*exp(8*I*c) + 2*A*exp(4*I*c) - A
 + I*B*exp(8*I*c) - 2*I*B*exp(6*I*c) + 2*I*B*exp(2*I*c) - I*B)*exp(-8*I*c)
/(16*a**4)), True)) + x*(-A + I*B)/(16*a**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= -\frac{(iA+B)\log(\tan(dx+c)+i)}{32a^4d} - \frac{(-iA-B)\log(\tan(dx+c)-i)}{32a^4d}$$
$$- \frac{3(A-iB)\tan(dx+c)^3 + 12(-iA+B)\tan(dx+c)^2 - (3A+13iB)\tan(dx+c) - 4B}{48a^4d(\tan(dx+c)-i)^4}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"giac")
```

output
```
-1/32*(I*A + B)*log(tan(d*x + c) + I)/(a^4*d) - 1/32*(-I*A - B)*log(tan(d*
x + c) - I)/(a^4*d) - 1/48*(3*(A - I*B)*tan(d*x + c)^3 + 12*(-I*A + B)*tan
(d*x + c)^2 - (3*A + 13*I*B)*tan(d*x + c) - 4*B)/(a^4*d*(tan(d*x + c) - I)
^4)
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\tan(c+d\,x)^2\,\left(-\frac{B}{4\,a^4}+\frac{A\,1\mathrm{i}}{4\,a^4}\right)-\tan(c+d\,x)^3\,\left(\frac{A}{16\,a^4}-\frac{B\,1\mathrm{i}}{16\,a^4}\right)+\frac{B}{12\,a^4}+\tan(c+d\,x)\,\left(\frac{A}{16\,a^4}+\frac{B\,13\mathrm{i}}{48\,a^4}\right)}{d\,\left(\tan\left(c+d\,x\right)^4-\tan\left(c+d\,x\right)^3\,4\mathrm{i}-6\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,4\mathrm{i}+1\right)}$$
$$+\frac{x\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,a^4}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(tan(c + d*x)^2*((A*1i)/(4*a^4) - B/(4*a^4)) - tan(c + d*x)^3*(A/(16*a^4)
- (B*1i)/(16*a^4)) + B/(12*a^4) + tan(c + d*x)*(A/(16*a^4) + (B*13i)/(48*a
^4)))/(d*(tan(c + d*x)*4i - 6*tan(c + d*x)^2 - tan(c + d*x)^3*4i + tan(c +
 d*x)^4 + 1)) + (x*(A*1i + B)*1i)/(16*a^4)
```

Reduce [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\left(\int \frac{\tan(dx+c)^3}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)b+\left(\int \frac{\tan(dx+c)^2}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}\right.}{a^4}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(int(tan(c + d*x)**3/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*
x)**2 + 4*tan(c + d*x)*i + 1),x)*b + int(tan(c + d*x)**2/(tan(c + d*x)**4
- 4*tan(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x)*a)/a*
*4
```

3.62 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 143

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(iA+B)x}{16a^4} - \frac{A+iB}{8d(a+ia\tan(c+dx))^4} + \frac{A+3iB}{12ad(a+ia\tan(c+dx))^3} + \frac{A-iB}{16d\left(a^2+ia^2\tan(c+dx)\right)^2} + \frac{A-iB}{16d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
-1/16*(I*A+B)*x/a^4-1/8*(A+I*B)/d/(a+I*a*tan(d*x+c))^4+1/12*(A+3*I*B)/a/d/
(a+I*a*tan(d*x+c))^3+1/16*(A-I*B)/d/(a^2+I*a^2*tan(d*x+c))^2+1/16*(A-I*B)/
d/(a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^4(c+dx)(12iB+16A\cos(2(c+dx))-3(A+8iAdx+B(i+8dx))\cos(4(c+dx))+32iA\sin(2(c+}{384a^4d(-i+\tan}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^4*((12*I)*B + 16*A*Cos[2*(c + d*x)] - 3*(A + (8*I)*A*d*x + B
*(I + 8*d*x))*Cos[4*(c + d*x)] + (32*I)*A*Sin[2*(c + d*x)] + (3*I)*A*Sin[4
*(c + d*x)] - 3*B*Sin[4*(c + d*x)] + 24*A*d*x*Sin[4*(c + d*x)] - (24*I)*B*
d*x*Sin[4*(c + d*x)]))/(384*a^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {3042, 4073, 3042, 4009, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{ 4073}$$

$$-\frac{i\int \frac{a(A+iB)+2aB\tan(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{2a^2}-\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{i\int \frac{a(A+iB)+2aB\tan(c+dx)}{(i\tan(c+dx)a+a)^3}dx}{2a^2}-\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 4009

$$-\frac{i\left(\frac{1}{2}(A-iB)\int \frac{1}{(i\tan(c+dx)a+a)^2}dx+\frac{a(-3B+iA)}{6d(a+ia\tan(c+dx))^3}\right)}{2a^2}-\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$-\frac{i\left(\frac{1}{2}(A-iB)\int \frac{1}{(i\tan(c+dx)a+a)^2}dx+\frac{a(-3B+iA)}{6d(a+ia\tan(c+dx))^3}\right)}{2a^2}-\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3960

$$-\frac{i\left(\frac{1}{2}(A-iB)\left(\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}\right)+\frac{a(-3B+iA)}{6d(a+ia\tan(c+dx))^3}\right)}{2a^2}-\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$-\frac{i\left(\frac{1}{2}(A-iB)\left(\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}\right)+\frac{a(-3B+iA)}{6d(a+ia\tan(c+dx))^3}\right)}{2a^2}-\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3960

$$-\frac{i\left(\frac{1}{2}(A-iB)\left(\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}\right)+\frac{a(-3B+iA)}{6d(a+ia\tan(c+dx))^3}\right)}{2a^2}-\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 24

$$-\frac{i\left(\frac{a(-3B+iA)}{6d(a+ia\tan(c+dx))^3}+\frac{1}{2}(A-iB)\left(\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}\right)\right)}{2a^2}-\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
-1/8*(A + I*B)/(d*(a + I*a*Tan[c + d*x])^4) - ((I/2)*((a*(I*A - 3*B))/(6*d
*(a + I*a*Tan[c + d*x])^3) + ((A - I*B)*((I/4)/(d*(a + I*a*Tan[c + d*x])^2
) + (x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x])))/(2*a)))/2))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{xB}{16a^4}-\frac{ixA}{16a^4}+\frac{e^{-2i(dx+c)}A}{16d\,a^4}+\frac{iB\,e^{-4i(dx+c)}}{32d\,a^4}-\frac{e^{-6i(dx+c)}A}{48d\,a^4}-\frac{ie^{-8i(dx+c)}B}{128d\,a^4}-\frac{e^{-8i(dx+c)}A}{128d\,a^4}$
derivativedivides	$-\frac{iB}{8d\,a^4(-i+\tan(dx+c))^4}-\frac{iA\arctan(\tan(dx+c))}{16d\,a^4}-\frac{B\arctan(\tan(dx+c))}{16d\,a^4}+\frac{iA}{12d\,a^4(-i+\tan(dx+c))^3}-\frac{}{16d}$
default	$-\frac{iB}{8d\,a^4(-i+\tan(dx+c))^4}-\frac{iA\arctan(\tan(dx+c))}{16d\,a^4}-\frac{B\arctan(\tan(dx+c))}{16d\,a^4}+\frac{iA}{12d\,a^4(-i+\tan(dx+c))^3}-\frac{}{16d}$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
-1/16*x/a^4*B-1/16*I*x/a^4*A+1/16/d/a^4*exp(-2*I*(d*x+c))*A+1/32*I*B/d/a^4
*exp(-4*I*(d*x+c))-1/48/d/a^4*exp(-6*I*(d*x+c))*A-1/128*I/d/a^4*exp(-8*I*(
d*x+c))*B-1/128/d/a^4*exp(-8*I*(d*x+c))*A
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.55

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\left(24\,(i\,A+B)dxe^{(8i\,dx+8i\,c)}-24\,Ae^{(6i\,dx+6i\,c)}-12i\,Be^{(4i\,dx+4i\,c)}+8\,Ae^{(2i\,dx+2i\,c)}+3\,A+3i\,B\right)e^{(-8i\,dx-8}}{384\,a^4d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
-1/384*(24*(I*A + B)*d*x*e^(8*I*d*x + 8*I*c) - 24*A*e^(6*I*d*x + 6*I*c) -
12*I*B*e^(4*I*d*x + 4*I*c) + 8*A*e^(2*I*d*x + 2*I*c) + 3*A + 3*I*B)*e^(-8*
I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.72

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{(196608Aa^{12}d^3e^{18ic}e^{-2idx}-65536Aa^{12}d^3e^{14ic}e^{-6idx}+98304iBa^{12}d^3e^{16ic}e^{-4idx}+(-24576Aa^{12}d^3e^{12ic}-24576iBa^{12}d^3e^{12ic})e^{-8idx})e^{-20}}{3145728a^{16}d^4} \\ x\left(-\frac{-iA-B}{16a^4}+\frac{(-iAe^{8ic}-2iAe^{6ic}+2iAe^{2ic}+iA-Be^{8ic}+2Be^{4ic}-B)e^{-8ic}}{16a^4}\right) \end{cases}$$
$$+\frac{x(-iA-B)}{16a^4}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((196608*A*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) - 65536*A*a**12*
d**3*exp(14*I*c)*exp(-6*I*d*x) + 98304*I*B*a**12*d**3*exp(16*I*c)*exp(-4*I
*d*x) + (-24576*A*a**12*d**3*exp(12*I*c) - 24576*I*B*a**12*d**3*exp(12*I*c
))*exp(-8*I*d*x))*exp(-20*I*c)/(3145728*a**16*d**4), Ne(a**16*d**4*exp(20*
I*c), 0)), (x*(-(-I*A - B)/(16*a**4) + (-I*A*exp(8*I*c) - 2*I*A*exp(6*I*c)
 + 2*I*A*exp(2*I*c) + I*A - B*exp(8*I*c) + 2*B*exp(4*I*c) - B)*exp(-8*I*c)
/(16*a**4)), True)) + x*(-I*A - B)/(16*a**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{(A-iB)\log(\tan(dx+c)+i)}{32a^4d} - \frac{(A-iB)\log(\tan(dx+c)-i)}{32a^4d} - \frac{3(iA+B)\tan(dx+c)^3+12(A-iB)\tan(dx+c)^2+(-19iA-3B)\tan(dx+c)-4A}{48a^4d(\tan(dx+c)-i)^4}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="g
iac")
```

output
```
1/32*(A - I*B)*log(tan(d*x + c) + I)/(a^4*d) - 1/32*(A - I*B)*log(tan(d*x
+ c) - I)/(a^4*d) - 1/48*(3*(I*A + B)*tan(d*x + c)^3 + 12*(A - I*B)*tan(d*
x + c)^2 + (-19*I*A - 3*B)*tan(d*x + c) - 4*A)/(a^4*d*(tan(d*x + c) - I)^4
)
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.20

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{\tan(c+dx)^2\left(\frac{A}{4a^4}-\frac{B\,1i}{4a^4}\right)+\tan(c+dx)^3\left(\frac{B}{16a^4}+\frac{A\,1i}{16a^4}\right)-\frac{A}{12a^4}-\tan(c+dx)\left(\frac{B}{16a^4}+\frac{A\,19i}{48a^4}\right)}{d\left(\tan(c+dx)^4-\tan(c+dx)^3\,4i-6\tan(c+dx)^2+\tan(c+dx)\,4i+1\right)} + \frac{\ln(\tan(c+dx)-i)\,(B+A\,1i)\,1i}{32a^4d} + \frac{\ln(\tan(c+dx)+1i)\,(A-B\,1i)}{32a^4d}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(log(tan(c + d*x) - 1i)*(A*1i + B)*1i)/(32*a^4*d) - (tan(c + d*x)^2*(A/(4*
a^4) - (B*1i)/(4*a^4)) + tan(c + d*x)^3*((A*1i)/(16*a^4) + B/(16*a^4)) - A
/(12*a^4) - tan(c + d*x)*((A*19i)/(48*a^4) + B/(16*a^4)))/(d*(tan(c + d*x)
*4i - 6*tan(c + d*x)^2 - tan(c + d*x)^3*4i + tan(c + d*x)^4 + 1)) + (log(t
an(c + d*x) + 1i)*(A - B*1i))/(32*a^4*d)
```

Reduce [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\left(\int \frac{\tan(dx+c)^2}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)b+\left(\int \frac{\tan(dx+c)}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)a}{a^4}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(int(tan(c + d*x)**2/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*
x)**2 + 4*tan(c + d*x)*i + 1),x)*b + int(tan(c + d*x)/(tan(c + d*x)**4 - 4
*tan(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x)*a)/a**4
```

3.63 $\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 145

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=\frac{(A-iB)x}{16a^4}+\frac{iA-B}{8d(a+ia\tan(c+dx))^4}+\frac{iA+B}{12ad(a+ia\tan(c+dx))^3}+\frac{iA+B}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{iA+B}{16d\left(a^4+ia^4\tan(c+dx)\right)}$$

output
```
1/16*(A-I*B)*x/a^4+1/8*(I*A-B)/d/(a+I*a*tan(d*x+c))^4+1/12*(I*A+B)/a/d/(a+
I*a*tan(d*x+c))^3+1/16*(I*A+B)/d/(a^2+I*a^2*tan(d*x+c))^2+1/16*(I*A+B)/d/(
a^4+I*a^4*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^3(c+dx)(18iA\cos(c+dx)+2(7iA+4B)\cos(3(c+dx))-(A-iB)(5\sin(c+dx)+11\sin(3(c+d}{96a^4d(-i+\tan(c+d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]^3*((18*I)*A*Cos[c + d*x] + 2*((7*I)*A + 4*B)*Cos[3*(c + d*x)
] - (A - I*B)*(5*Sin[c + d*x] + 11*Sin[3*(c + d*x)]) + 6*(A - I*B)*ArcTan[
Tan[c + d*x]]*Sec[c + d*x]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)])))/(96*a
^4*d*(-I + Tan[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 4009, 3042, 3960, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{4009}$$
$$\frac{(A-iB)\int \frac{1}{(i\tan(c+dx)a+a)^3}dx}{2a}+\frac{-B+iA}{8d(a+ia\tan(c+dx))^4}$$
$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int \frac{1}{(i\tan(c+dx)a+a)^3}dx}{2a}+\frac{-B+iA}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3960

$$\frac{(A-iB)\left(\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}\right)}{2a}+\frac{-B+iA}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$\frac{(A-iB)\left(\frac{\int \frac{1}{(i\tan(c+dx)a+a)^2}dx}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}\right)}{2a}+\frac{-B+iA}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3960

$$\frac{(A-iB)\left(\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}\right)}{2a}+\frac{-B+iA}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$\frac{(A-iB)\left(\frac{\frac{\int \frac{1}{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}\right)}{2a}+\frac{-B+iA}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3960

$$\frac{(A-iB)\left(\frac{\frac{\frac{\int 1dx}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}\right)}{2a}+\frac{-B+iA}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 24

$$\frac{-B+iA}{8d(a+ia\tan(c+dx))^4}+\frac{(A-iB)\left(\frac{\frac{\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}}{2a}+\frac{i}{4d(a+ia\tan(c+dx))^2}}{2a}+\frac{i}{6d(a+ia\tan(c+dx))^3}\right)}{2a}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(I*A - B)/(8*d*(a + I*a*Tan[c + d*x])^4) + ((A - I*B)*((I/6)/(d*(a + I*a*T
an[c + d*x])^3) + ((I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (x/(2*a) + (I/2)/(
d*(a + I*a*Tan[c + d*x])))/(2*a))/(2*a)))/(2*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{ixB}{16a^4}+\frac{xA}{16a^4}+\frac{\mathrm{e}^{-2i(dx+c)}B}{16d\,a^4}+\frac{i\mathrm{e}^{-2i(dx+c)}A}{8d\,a^4}+\frac{3i\mathrm{e}^{-4i(dx+c)}A}{32d\,a^4}-\frac{\mathrm{e}^{-6i(dx+c)}B}{48d\,a^4}+\frac{i\mathrm{e}^{-6i(dx+c)}A}{24d\,a^4}-$
derivativedivides	$-\frac{iA}{16d\,a^4(-i+\tan(dx+c))^2}+\frac{A\arctan(\tan(dx+c))}{16d\,a^4}-\frac{iB\arctan(\tan(dx+c))}{16d\,a^4}+\frac{A}{16d\,a^4(-i+\tan(dx+c))}+$
default	$-\frac{iA}{16d\,a^4(-i+\tan(dx+c))^2}+\frac{A\arctan(\tan(dx+c))}{16d\,a^4}-\frac{iB\arctan(\tan(dx+c))}{16d\,a^4}+\frac{A}{16d\,a^4(-i+\tan(dx+c))}+$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/16*I*x/a^4*B+1/16*x/a^4*A+1/16/d/a^4*exp(-2*I*(d*x+c))*B+1/8*I/d/a^4*ex
p(-2*I*(d*x+c))*A+3/32*I/d/a^4*exp(-4*I*(d*x+c))*A-1/48/d/a^4*exp(-6*I*(d*
x+c))*B+1/24*I/d/a^4*exp(-6*I*(d*x+c))*A-1/128/d/a^4*exp(-8*I*(d*x+c))*B+1
/128*I/d/a^4*exp(-8*I*(d*x+c))*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{(24\,(A-i\,B)dxe^{(8i\,dx+8i\,c)}-24\,(-2i\,A-B)e^{(6i\,dx+6i\,c)}+36i\,Ae^{(4i\,dx+4i\,c)}-8\,(-2i\,A+B)e^{(2i\,dx+2i\,c)}+3}{384\,a^4d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/384*(24*(A - I*B)*d*x*e^(8*I*d*x + 8*I*c) - 24*(-2*I*A - B)*e^(6*I*d*x +
 6*I*c) + 36*I*A*e^(4*I*d*x + 4*I*c) - 8*(-2*I*A + B)*e^(2*I*d*x + 2*I*c)
+ 3*I*A - 3*B)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.06

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(294912iAa^{12}d^3e^{16ic}e^{-4idx}+(24576iAa^{12}d^3e^{12ic}-24576Ba^{12}d^3e^{12ic})e^{-8idx}+(131072iAa^{12}d^3e^{14ic}-65536Ba^{12}d^3e^{14ic})e^{-6idx}+(393216}{3145728a^{16}d^4} \\ x\left(-\frac{A-iB}{16a^4}+\frac{(Ae^{8ic}+4Ae^{6ic}+6Ae^{4ic}+4Ae^{2ic}+A-iBe^{8ic}-2iBe^{6ic}+2iBe^{2ic}+iB)e^{-8ic}}{16a^4}\right) \end{cases} + \frac{x(A-iB)}{16a^4}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)
```

output
```
Piecewise(((294912*I*A*a**12*d**3*exp(16*I*c)*exp(-4*I*d*x) + (24576*I*A*a
**12*d**3*exp(12*I*c) - 24576*B*a**12*d**3*exp(12*I*c))*exp(-8*I*d*x) + (1
31072*I*A*a**12*d**3*exp(14*I*c) - 65536*B*a**12*d**3*exp(14*I*c))*exp(-6*
I*d*x) + (393216*I*A*a**12*d**3*exp(18*I*c) + 196608*B*a**12*d**3*exp(18*I
*c))*exp(-2*I*d*x))*exp(-20*I*c)/(3145728*a**16*d**4), Ne(a**16*d**4*exp(2
0*I*c), 0)), (x*(-(A - I*B)/(16*a**4) + (A*exp(8*I*c) + 4*A*exp(6*I*c) + 6
*A*exp(4*I*c) + 4*A*exp(2*I*c) + A - I*B*exp(8*I*c) - 2*I*B*exp(6*I*c) + 2
*I*B*exp(2*I*c) + I*B)*exp(-8*I*c)/(16*a**4)), True)) + x*(A - I*B)/(16*a*
*4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{(a + ia\tan(c + dx))^4} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

$$\begin{aligned}
&\int \frac{A + B\tan(c + dx)}{(a + ia\tan(c + dx))^4} \, dx \\
&= -\frac{(-i\,A - B)\log\left(\tan\left(dx + c\right) + i\right)}{32\,a^4 d} + \frac{(-i\,A - B)\log\left(\tan\left(dx + c\right) - i\right)}{32\,a^4 d} \\
&\quad + \frac{3\,(A - i\,B)\tan\left(dx + c\right)^3 - 12\,(i\,A + B)\tan\left(dx + c\right)^2 - 19\,(A - i\,B)\tan\left(dx + c\right) + 16i\,A + 4\,B}{48\,a^4 d(\tan\left(dx + c\right) - i)^4}
\end{aligned}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/32*(-I*A - B)*log(tan(d*x + c) + I)/(a^4*d) + 1/32*(-I*A - B)*log(tan(d
*x + c) - I)/(a^4*d) + 1/48*(3*(A - I*B)*tan(d*x + c)^3 - 12*(I*A + B)*tan
(d*x + c)^2 - 19*(A - I*B)*tan(d*x + c) + 16*I*A + 4*B)/(a^4*d*(tan(d*x +
c) - I)^4)
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.99

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{B}{12\,a^4}+\tan(c+d\,x)^3\,\left(\frac{A}{16\,a^4}-\frac{B\,1\mathrm{i}}{16\,a^4}\right)+\frac{A\,1\mathrm{i}}{3\,a^4}-\tan(c+d\,x)^2\,\left(\frac{B}{4\,a^4}+\frac{A\,1\mathrm{i}}{4\,a^4}\right)-\tan(c+d\,x)\,\left(\frac{19\,A}{48\,a^4}-\frac{B\,19\mathrm{i}}{48\,a^4}\right)}{d\,\left({\mathrm{tan}\left(c+d\,x\right)}^4-{\mathrm{tan}\left(c+d\,x\right)}^3\,4\mathrm{i}-6\,{\mathrm{tan}\left(c+d\,x\right)}^2+\mathrm{tan}\left(c+d\,x\right)\,4\mathrm{i}+1\right)}$$
$$-\frac{x\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{16\,a^4}$$

input
```
int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(tan(c + d*x)^3*(A/(16*a^4) - (B*1i)/(16*a^4)) - tan(c + d*x)^2*((A*1i)/(4
*a^4) + B/(4*a^4)) + (A*1i)/(3*a^4) + B/(12*a^4) - tan(c + d*x)*((19*A)/(4
8*a^4) - (B*19i)/(48*a^4)))/(d*(tan(c + d*x)*4i - 6*tan(c + d*x)^2 - tan(c
 + d*x)^3*4i + tan(c + d*x)^4 + 1)) - (x*(A*1i + B)*1i)/(16*a^4)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\left(\int \frac{\tan(dx+c)}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}dx\right)b+\left(\int \frac{1}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c)i+1}\right.}{a^4}$$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

output

```
(int(tan(c + d*x)/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*x)*
*2 + 4*tan(c + d*x)*i + 1),x)*b + int(1/(tan(c + d*x)**4 - 4*tan(c + d*x)*
*3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*i + 1),x)*a)/a**4
```

3.64 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 162

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(15iA-B)x}{16a^4}+\frac{A\log(\sin(c+dx))}{a^4d} + \frac{7A+iB}{16a^4d(1+i\tan(c+dx))^2} + \frac{15A+iB}{16a^4d(1+i\tan(c+dx))} + \frac{A+iB}{8d(a+ia\tan(c+dx))^4} + \frac{3A+iB}{12ad(a+ia\tan(c+dx))^3}$$

output
```
-1/16*(15*I*A-B)*x/a^4+A*ln(sin(d*x+c))/a^4/d+1/16*(7*A+I*B)/a^4/d/(1+I*ta
n(d*x+c))^2+1/16*(15*A+I*B)/a^4/d/(1+I*tan(d*x+c))+1/8*(A+I*B)/d/(a+I*a*ta
n(d*x+c))^4+1/12*(3*A+I*B)/a/d/(a+I*a*tan(d*x+c))^3
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{-3(31A+iB)\log(i-\tan(c+dx))+96A\log(\tan(c+dx))-3(A-iB)\log(i+\tan(c+dx))+\frac{12(A\cdots}{(-i+\tan(\cdots}}{96a^4d}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(-3*(31*A + I*B)*Log[I - Tan[c + d*x]] + 96*A*Log[Tan[c + d*x]] - 3*(A - I
*B)*Log[I + Tan[c + d*x]] + (12*(A + I*B))/(-I + Tan[c + d*x])^4 + ((24*I)
*A - 8*B)/(-I + Tan[c + d*x])^3 - (6*(7*A + I*B))/(-I + Tan[c + d*x])^2 +
(6*((-15*I)*A + B))/(-I + Tan[c + d*x]))/(96*a^4*d)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+ia\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{ 4079}$$
$$\frac{\int \frac{4\cot(c+dx)(2aA-a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{8a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\cot(c+dx)(2aA-a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{2aA-a(iA-B)\tan(c+dx)}{\tan(c+dx)(i\tan(c+dx)a+a)^3}dx}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\int \frac{3\cot(c+dx)\left(4a^2A-a^2(3iA-B)\tan(c+dx)\right)}{(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\cot(c+dx)\left(4a^2A-a^2(3iA-B)\tan(c+dx)\right)}{(i\tan(c+dx)a+a)^2}dx}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{4a^2A-a^2(3iA-B)\tan(c+dx)}{\tan(c+dx)(i\tan(c+dx)a+a)^2}dx}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\frac{\int \frac{2\cot(c+dx)\left(8a^3A-a^3(7iA-B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{7A+iB}{4d(1+i\tan(c+dx))^2}}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(8a^3A-a^3(7iA-B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{2a^2}+\frac{7A+iB}{4d(1+i\tan(c+dx))^2}}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{8a^3A-a^3(7iA-B)\tan(c+dx)}{\tan(c+dx)(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{7A+iB}{4d(1+i\tan(c+dx))^2}}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 4079$$

$$\frac{\frac{\frac{\frac{\int \cot(c+dx)\left(16a^4A-a^4(15iA-B)\tan(c+dx)\right)dx}{2a^2}+\frac{a^3(15A+iB)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{7A+iB}{4d(1+i\tan(c+dx))^2}}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\frac{\int \frac{16a^4A-a^4(15iA-B)\tan(c+dx)}{\tan(c+dx)}dx}{2a^2}+\frac{a^3(15A+iB)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{7A+iB}{4d(1+i\tan(c+dx))^2}}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 4014$$

$$\frac{\frac{\frac{\frac{16a^4A\int \cot(c+dx)dx-a^4x(-B+15iA)}{2a^2}+\frac{a^3(15A+iB)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{7A+iB}{4d(1+i\tan(c+dx))^2}}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\frac{16a^4A\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-a^4x(-B+15iA)}{2a^2}+\frac{a^3(15A+iB)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{7A+iB}{4d(1+i\tan(c+dx))^2}}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\frac{-16a^4A\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\left(a^4x(-B+15iA)\right)}{2a^2}+\frac{a^3(15A+iB)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{7A+iB}{4d(1+i\tan(c+dx))^2}}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \ 3956$$

$$\frac{\frac{\frac{\frac{a^3(15A+iB)}{2d(a+ia\tan(c+dx))}+\frac{\frac{16a^4A\log(-\sin(c+dx))}{d}-a^4x(-B+15iA)}{2a^2}}{2a^2}+\frac{7A+iB}{4d(1+i\tan(c+dx))^2}}{2a^2}+\frac{a(3A+iB)}{6d(a+ia\tan(c+dx))^3}}{2a^2}+\frac{A+iB}{8d(a+ia\tan(c+dx))^4}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
(A + I*B)/(8*d*(a + I*a*Tan[c + d*x])^4) + ((a*(3*A + I*B))/(6*d*(a + I*a*
Tan[c + d*x])^3) + ((7*A + I*B)/(4*d*(1 + I*Tan[c + d*x])^2) + ((-(a^4*((1
5*I)*A - B)*x) + (16*a^4*A*Log[-Sin[c + d*x]])/d)/(2*a^2) + (a^3*(15*A + I
*B))/(2*d*(a + I*a*Tan[c + d*x])))/(2*a^2))/(2*a^2))/(2*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.21

method	result
risch	$\frac{xB}{16a^4} - \frac{31ixA}{16a^4} + \frac{i\mathrm{e}^{-2i(dx+c)}B}{8d\,a^4} + \frac{13\,\mathrm{e}^{-2i(dx+c)}A}{16d\,a^4} + \frac{3iB\,\mathrm{e}^{-4i(dx+c)}}{32d\,a^4} + \frac{\mathrm{e}^{-4i(dx+c)}A}{4d\,a^4} + \frac{i\mathrm{e}^{-6i(dx+c)}B}{24d\,a^4} + \frac{\mathrm{e}^{-6}}{}$
derivativedivides	$-\frac{iB}{16d\,a^4(-i+\tan(dx+c))^2} + \frac{B\arctan(\tan(dx+c))}{16d\,a^4} - \frac{A\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^4} - \frac{15iA}{16d\,a^4(-i+\tan(dx+c))} + \frac{}{4d\,a^4}$
default	$-\frac{iB}{16d\,a^4(-i+\tan(dx+c))^2} + \frac{B\arctan(\tan(dx+c))}{16d\,a^4} - \frac{A\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^4} - \frac{15iA}{16d\,a^4(-i+\tan(dx+c))} + \frac{}{4d\,a^4}$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/16*x/a^4*B-31/16*I*x/a^4*A+1/8*I/d/a^4*exp(-2*I*(d*x+c))*B+13/16/d/a^4*e
xp(-2*I*(d*x+c))*A+3/32*I*B/d/a^4*exp(-4*I*(d*x+c))+1/4/d/a^4*exp(-4*I*(d*
x+c))*A+1/24*I/d/a^4*exp(-6*I*(d*x+c))*B+1/16/d/a^4*exp(-6*I*(d*x+c))*A+1/
128*I/d/a^4*exp(-8*I*(d*x+c))*B+1/128/d/a^4*exp(-8*I*(d*x+c))*A-2*I/a^4*A/
d*c+1/a^4*A/d*ln(exp(2*I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(24\,(31i\,A-B)dxe^{(8i\,dx+8i\,c)}-384\,Ae^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}-1\right)-24\,(13\,A+2i\,B)e^{(6i\,dx+6i\,c)}-12}{384\,a^4d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
-1/384*(24*(31*I*A - B)*d*x*e^(8*I*d*x + 8*I*c) - 384*A*e^(8*I*d*x + 8*I*c
)*log(e^(2*I*d*x + 2*I*c) - 1) - 24*(13*A + 2*I*B)*e^(6*I*d*x + 6*I*c) - 1
2*(8*A + 3*I*B)*e^(4*I*d*x + 4*I*c) - 8*(3*A + 2*I*B)*e^(2*I*d*x + 2*I*c)
- 3*A - 3*I*B)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.22

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \frac{A\log\left(e^{2idx}-e^{-2ic}\right)}{a^4d} + \begin{cases} \frac{((24576Aa^{12}d^3e^{12ic}+24576iBa^{12}d^3e^{12ic})e^{-8idx}+(196608Aa^{12}d^3e^{14ic}+131072iBa^{12}d^3e^{14ic})e^{-6idx}+(786432Aa^{12}d^3e^{16ic}+294912iBa}{3145728a^{16}d^4} \\ x\left(-\frac{-31iA+B}{16a^4}+\frac{(-31iAe^{8ic}-26iAe^{6ic}-16iAe^{4ic}-6iAe^{2ic}-iA+Be^{8ic}+4Be^{6ic}+6Be^{4ic}+4Be^{2ic}+B)e^{-8ic}}{16a^4}\right) \end{cases} + \frac{x(-31iA+B)}{16a^4}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)
```

output
```
A*log(exp(2*I*d*x) - exp(-2*I*c))/(a**4*d) + Piecewise((((24576*A*a**12*d*
*3*exp(12*I*c) + 24576*I*B*a**12*d**3*exp(12*I*c))*exp(-8*I*d*x) + (196608
*A*a**12*d**3*exp(14*I*c) + 131072*I*B*a**12*d**3*exp(14*I*c))*exp(-6*I*d*
x) + (786432*A*a**12*d**3*exp(16*I*c) + 294912*I*B*a**12*d**3*exp(16*I*c))
*exp(-4*I*d*x) + (2555904*A*a**12*d**3*exp(18*I*c) + 393216*I*B*a**12*d**3
*exp(18*I*c))*exp(-2*I*d*x))*exp(-20*I*c)/(3145728*a**16*d**4), Ne(a**16*d
**4*exp(20*I*c), 0)), (x*(-(-31*I*A + B)/(16*a**4) + (-31*I*A*exp(8*I*c) -
 26*I*A*exp(6*I*c) - 16*I*A*exp(4*I*c) - 6*I*A*exp(2*I*c) - I*A + B*exp(8*
I*c) + 4*B*exp(6*I*c) + 6*B*exp(4*I*c) + 4*B*exp(2*I*c) + B)*exp(-8*I*c)/(
16*a**4)), True)) + x*(-31*I*A + B)/(16*a**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(A-iB)\log(\tan(dx+c)+i)}{32\,a^4d} - \frac{(31\,A+i\,B)\log(\tan(dx+c)-i)}{32\,a^4d} + \frac{A\log(|\tan(dx+c)|)}{a^4d} - \frac{3\,(15i\,A-B)\tan(dx+c)^3+12\,(13\,A+i\,B)\tan(dx+c)^2+(-189i\,A+19\,B)\tan(dx+c)-84\,A-}{48\,a^4d(\tan(dx+c)-i)^4}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm="g
iac")
```

output
```
-1/32*(A - I*B)*log(tan(d*x + c) + I)/(a^4*d) - 1/32*(31*A + I*B)*log(tan(
d*x + c) - I)/(a^4*d) + A*log(abs(tan(d*x + c)))/(a^4*d) - 1/48*(3*(15*I*A
 - B)*tan(d*x + c)^3 + 12*(13*A + I*B)*tan(d*x + c)^2 + (-189*I*A + 19*B)*
tan(d*x + c) - 84*A - 16*I*B)/(a^4*d*(tan(d*x + c) - I)^4)
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.21

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{7A}{4a^4} - \tan(c+dx)^3\left(-\frac{B}{16a^4}+\frac{A\,15\mathrm{i}}{16a^4}\right) - \tan(c+dx)^2\left(\frac{13A}{4a^4}+\frac{B\,1\mathrm{i}}{4a^4}\right) + \frac{B\,1\mathrm{i}}{3a^4} + \tan(c+dx)\left(-\frac{19B}{48a^4}+\frac{A\,63\mathrm{i}}{16a^4}\right)}{d\left(\tan(c+dx)^4 - \tan(c+dx)^3\,4\mathrm{i} - 6\tan(c+dx)^2 + \tan(c+dx)\,4\mathrm{i} + 1\right)}$$
$$+ \frac{A\ln(\tan(c+dx))}{a^4 d} + \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(B+A\,1\mathrm{i})\,1\mathrm{i}}{32a^4 d}$$
$$- \frac{\ln(\tan(c+dx)-\mathrm{i})\,(31A+B\,1\mathrm{i})}{32a^4 d}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
((7*A)/(4*a^4) - tan(c + d*x)^3*((A*15i)/(16*a^4) - B/(16*a^4)) - tan(c +
d*x)^2*((13*A)/(4*a^4) + (B*1i)/(4*a^4)) + (B*1i)/(3*a^4) + tan(c + d*x)*(
(A*63i)/(16*a^4) - (19*B)/(48*a^4)))/(d*(tan(c + d*x)*4i - 6*tan(c + d*x)^
2 - tan(c + d*x)^3*4i + tan(c + d*x)^4 + 1)) + (A*log(tan(c + d*x)))/(a^4*
d) + (log(tan(c + d*x) + 1i)*(A*1i + B)*1i)/(32*a^4*d) - (log(tan(c + d*x)
 - 1i)*(31*A + B*1i))/(32*a^4*d)
```

Reduce [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{too large to display}$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

output
```
( - 320*int(tan((c + d*x)/2)**6/(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2
)**7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x
)/2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c +
 d*x)/2) + i),x)*a*d - 160*int(tan((c + d*x)/2)**6/(tan((c + d*x)/2)**8*i
+ 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**
5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan((c + d*x)/2
)**2*i - 8*tan((c + d*x)/2) + i),x)*b*d*i + 1520*int(tan((c + d*x)/2)**5/(
tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2)**6*i -
 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d*x)/2)**
3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i),x)*a*d*i - 672*int(
tan((c + d*x)/2)**5/(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*ta
n((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i +
56*tan((c + d*x)/2)**3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i
),x)*b*d + 3584*int(tan((c + d*x)/2)**4/(tan((c + d*x)/2)**8*i + 8*tan((c
+ d*x)/2)**7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan(
(c + d*x)/2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan((c + d*x)/2)**2*i - 8*
tan((c + d*x)/2) + i),x)*a*d + 1536*int(tan((c + d*x)/2)**4/(tan((c + d*x)
/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c +
d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan((c
 + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i),x)*b*d*i - 4880*int(tan((c + ...
```

3.65 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4} dx$

Optimal result

Integrand size = 34, antiderivative size = 220

$$\begin{aligned}\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{5(13A+3iB)x}{16a^4}-\frac{5(13A+3iB)\cot(c+dx)}{16a^4d}-\frac{(4iA-B)\log(\sin(c+dx))}{a^4d}\\ &\quad+\frac{(31A+9iB)\cot(c+dx)}{48a^4d(1+i\tan(c+dx))^2}+\frac{(4A+iB)\cot(c+dx)}{2a^4d(1+i\tan(c+dx))}\\ &\quad+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{(7A+3iB)\cot(c+dx)}{24ad(a+ia\tan(c+dx))^3}\end{aligned}$$

output
```
-5/16*(13*A+3*I*B)*x/a^4-5/16*(13*A+3*I*B)*cot(d*x+c)/a^4/d-(4*I*A-B)*ln(s
in(d*x+c))/a^4/d+1/48*(31*A+9*I*B)*cot(d*x+c)/a^4/d/(1+I*tan(d*x+c))^2+1/2
*(4*A+I*B)*cot(d*x+c)/a^4/d/(1+I*tan(d*x+c))+1/8*(A+I*B)*cot(d*x+c)/d/(a+I
*a*tan(d*x+c))^4+1/24*(7*A+3*I*B)*cot(d*x+c)/a/d/(a+I*a*tan(d*x+c))^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.87

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{6(A+iB)\cot^5(c+dx)}{(i+\cot(c+dx))^4}+\frac{2(7A+3iB)\cot^4(c+dx)}{(i+\cot(c+dx))^3}+\frac{(31A+9iB)\cot^3(c+dx)}{(i+\cot(c+dx))^2}+\frac{24(4A+iB)\cot^2(c+dx)}{i+\cot(c+dx)}-15(13A+3iB)\cot(c+}{48}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x
]
```

output
```
((6*(A + I*B)*Cot[c + d*x]^5)/(I + Cot[c + d*x])^4 + (2*(7*A + (3*I)*B)*Co
t[c + d*x]^4)/(I + Cot[c + d*x])^3 + ((31*A + (9*I)*B)*Cot[c + d*x]^3)/(I
+ Cot[c + d*x])^2 + (24*(4*A + I*B)*Cot[c + d*x]^2)/(I + Cot[c + d*x]) - 1
5*(13*A + (3*I)*B)*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d
*x]^2] + 48*((-4*I)*A + B)*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/(48*a^
4*d)
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.10, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.559$, Rules used = {3042, 4079, 3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \; 4079$$

$$\frac{\int \frac{\cot^2(c+dx)(a(9A+iB)-5a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a(9A+iB)-5a(iA-B)\tan(c+dx)}{\tan(c+dx)^2(i\tan(c+dx)a+a)^3}dx}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 4079$$

$$\frac{\frac{\int \frac{4\cot^2(c+dx)\left(a^2(17A+3iB)-2a^2(7iA-3B)\tan(c+dx)\right)}{(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{a(7A+3iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{\cot^2(c+dx)\left(a^2(17A+3iB)-2a^2(7iA-3B)\tan(c+dx)\right)}{(i\tan(c+dx)a+a)^2}dx}{3a^2}+\frac{a(7A+3iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\int \frac{a^2(17A+3iB)-2a^2(7iA-3B)\tan(c+dx)}{\tan(c+dx)^2(i\tan(c+dx)a+a)^2}dx}{3a^2}+\frac{a(7A+3iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 4079$$

$$\frac{\frac{2\left(\frac{\int \frac{3\cot^2(c+dx)\left(a^3(33A+7iB)-a^3(31iA-9B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+3iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\left(\frac{3\int \frac{\cot^2(c+dx)\left(a^3(33A+7iB)-a^3(31iA-9B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+3iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\left(\frac{3\int \frac{a^3(33A+7iB)-a^3(31iA-9B)\tan(c+dx)}{\tan(c+dx)^2(i\tan(c+dx)a+a)}dx}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+3iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 4079

$$\frac{\frac{2\left(\frac{3\left(\frac{\int 2\cot^2(c+dx)\left(5a^4(13A+3iB)-16a^4(4iA-B)\tan(c+dx)\right)dx}{2a^2}+\frac{8a^3(4A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+3iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\left(\frac{3\left(\frac{\int \cot^2(c+dx)\left(5a^4(13A+3iB)-16a^4(4iA-B)\tan(c+dx)\right)dx}{a^2}+\frac{8a^3(4A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+3iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\left(\frac{3\left(\frac{\int \frac{5a^4(13A+3iB)-16a^4(4iA-B)\tan(c+dx)}{\tan(c+dx)^2}dx}{a^2}+\frac{8a^3(4A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+3iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}+\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 4012

$$\frac{2\left(\frac{3\left(\frac{\int -\cot(c+dx)\left(16(4iA-B)a^4+5(13A+3iB)\tan(c+dx)a^4\right)dx-\frac{5a^4(13A+3iB)\cot(c+dx)}{d}}{a^2}+\frac{8a^3(4A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A}{3d(a}}{8a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 25

$$\frac{2\left(\frac{3\left(\frac{-\int \cot(c+dx)\left(16(4iA-B)a^4+5(13A+3iB)\tan(c+dx)a^4\right)dx-\frac{5a^4(13A+3iB)\cot(c+dx)}{d}}{a^2}+\frac{8a^3(4A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A}{3d(a}}{8a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 3042

$$\frac{2\left(\frac{3\left(\frac{-\int \frac{16(4iA-B)a^4+5(13A+3iB)\tan(c+dx)a^4}{\tan(c+dx)}dx-\frac{5a^4(13A+3iB)\cot(c+dx)}{d}}{a^2}+\frac{8a^3(4A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+3iB)\cot}{3d(a+ia\tan(c}}{8a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 4014

$$\frac{2\left(\frac{3\left(\frac{-16a^4(-B+4iA)\int \cot(c+dx)dx-\frac{5a^4(13A+3iB)\cot(c+dx)}{d}-5a^4x(13A+3iB)}{a^2}+\frac{8a^3(4A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+3iB)c}{3d(a+ia\tan(}}{8a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 3042

$$\frac{\frac{2\left(\frac{3\left(\frac{-16a^4(-B+4iA)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{5a^4(13A+3iB)\cot(c+dx)}{d}-5a^4x(13A+3iB)}{a^2}+\frac{8a^3(4A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+}{3d(a+}}{8a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 25

$$\frac{\frac{2\left(\frac{3\left(\frac{16a^4(-B+4iA)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{5a^4(13A+3iB)\cot(c+dx)}{d}-5a^4x(13A+3iB)}{a^2}+\frac{8a^3(4A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}\right)}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+}{3d(a+}}{8a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 3956

$$\frac{\frac{2\left(\frac{3\left(\frac{8a^3(4A+iB)\cot(c+dx)}{d(a+ia\tan(c+dx))}+\frac{-\frac{5a^4(13A+3iB)\cot(c+dx)}{d}-\frac{16a^4(-B+4iA)\log(-\sin(c+dx))}{d}-5a^4x(13A+3iB)}{a^2}\right)}{4a^2}+\frac{(31A+9iB)\cot(c+dx)}{4d(1+i\tan(c+dx))^2}\right)}{3a^2}+\frac{a(7A+3iB}{3d(a+ia\,ta}}{8a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

input

```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
```

output

```
((A + I*B)*Cot[c + d*x])/(8*d*(a + I*a*Tan[c + d*x])^4) + ((a*(7*A + (3*I)
*B)*Cot[c + d*x])/(3*d*(a + I*a*Tan[c + d*x])^3) + (2*(((31*A + (9*I)*B)*C
ot[c + d*x])/(4*d*(1 + I*Tan[c + d*x])^2) + (3*((-5*a^4*(13*A + (3*I)*B)*x
 - (5*a^4*(13*A + (3*I)*B)*Cot[c + d*x])/d - (16*a^4*((4*I)*A - B)*Log[-Si
n[c + d*x]])/d)/a^2 + (8*a^3*(4*A + I*B)*Cot[c + d*x])/(d*(a + I*a*Tan[c +
 d*x]))))/(4*a^2)))/(3*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{i e^{-8i(dx+c)} A}{128d\,a^4}-\frac{129xA}{16a^4}+\frac{13\,e^{-2i(dx+c)} B}{16d\,a^4}-\frac{9i e^{-2i(dx+c)} A}{4d\,a^4}+\frac{e^{-4i(dx+c)} B}{4d\,a^4}-\frac{15i e^{-4i(dx+c)} A}{32d\,a^4}+\frac{e^{-6i(dx}{16d}$
derivativedivides	$-\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^4}-\frac{iA}{8d\,a^4(-i+\tan(dx+c))^4}+\frac{2iA\ln\left(1+\tan(dx+c)^2\right)}{d\,a^4}-\frac{65A\arctan(\tan(dx+c))}{16d\,a^4}+\frac{}{16a}$
default	$-\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d\,a^4}-\frac{iA}{8d\,a^4(-i+\tan(dx+c))^4}+\frac{2iA\ln\left(1+\tan(dx+c)^2\right)}{d\,a^4}-\frac{65A\arctan(\tan(dx+c))}{16d\,a^4}+\frac{}{16a}$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVER
BOSE)
```

output
```
-1/128*I/d/a^4*exp(-8*I*(d*x+c))*A-129/16*x/a^4*A+13/16/d/a^4*exp(-2*I*(d*
x+c))*B-9/4*I/d/a^4*exp(-2*I*(d*x+c))*A+1/4/d/a^4*exp(-4*I*(d*x+c))*B-15/3
2*I/d/a^4*exp(-4*I*(d*x+c))*A+1/16/d/a^4*exp(-6*I*(d*x+c))*B-1/12*I/d/a^4*
exp(-6*I*(d*x+c))*A+1/128/d/a^4*exp(-8*I*(d*x+c))*B-2*I/a^4/d*B*c-2*I*A/a^
4/d/(exp(2*I*(d*x+c))-1)-31/16*I*x/a^4*B-8/a^4/d*A*c-4*I/a^4/d*ln(exp(2*I*
(d*x+c))-1)*A+1/a^4/d*ln(exp(2*I*(d*x+c))-1)*B
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.86

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{24\,(129\,A+31i\,B)dxe^{(10i\,dx+10i\,c)}-24\,((129\,A+31i\,B)dx-68i\,A+13\,B)e^{(8i\,dx+8i\,c)}+36\,(-19i\,A+}{}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"fricas")
```

output
```
-1/384*(24*(129*A + 31*I*B)*d*x*e^(10*I*d*x + 10*I*c) - 24*((129*A + 31*I*
B)*d*x - 68*I*A + 13*B)*e^(8*I*d*x + 8*I*c) + 36*(-19*I*A + 6*B)*e^(6*I*d*
x + 6*I*c) + 4*(-37*I*A + 18*B)*e^(4*I*d*x + 4*I*c) - (29*I*A - 21*B)*e^(2
*I*d*x + 2*I*c) + 384*((4*I*A - B)*e^(10*I*d*x + 10*I*c) + (-4*I*A + B)*e^
(8*I*d*x + 8*I*c))*log(e^(2*I*d*x + 2*I*c) - 1) - 3*I*A + 3*B)/(a^4*d*e^(1
0*I*d*x + 10*I*c) - a^4*d*e^(8*I*d*x + 8*I*c))
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.85

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2iA}{a^4de^{2ic}e^{2idx}-a^4d} + \begin{cases} \frac{((-24576iAa^{12}d^3e^{12ic}+24576Ba^{12}d^3e^{12ic})e^{-8idx}+(-262144iAa^{12}d^3e^{14ic}+196608Ba^{12}d^3e^{14ic})e^{-6idx}+(-1474560iAa^{12}d^3e^{16ic}+78}{3145728a^{16}d^4} \\ x\left(-\frac{-129A-31iB}{16a^4}+\frac{(-129Ae^{8ic}-72Ae^{6ic}-30Ae^{4ic}-8Ae^{2ic}-A-31iBe^{8ic}-26iBe^{6ic}-16iBe^{4ic}-6iBe^{2ic}-iB)e^{-8ic}}{16a^4}\right) \end{cases} + \frac{x(-129A-31iB)}{16a^4} - \frac{i(4A+iB)\log\left(e^{2idx}-e^{-2ic}\right)}{a^4d}$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)
```

output
```
-2*I*A/(a**4*d*exp(2*I*c)*exp(2*I*d*x) - a**4*d) + Piecewise((((-24576*I*A
*a**12*d**3*exp(12*I*c) + 24576*B*a**12*d**3*exp(12*I*c))*exp(-8*I*d*x) +
(-262144*I*A*a**12*d**3*exp(14*I*c) + 196608*B*a**12*d**3*exp(14*I*c))*exp
(-6*I*d*x) + (-1474560*I*A*a**12*d**3*exp(16*I*c) + 786432*B*a**12*d**3*ex
p(16*I*c))*exp(-4*I*d*x) + (-7077888*I*A*a**12*d**3*exp(18*I*c) + 2555904*
B*a**12*d**3*exp(18*I*c))*exp(-2*I*d*x))*exp(-20*I*c)/(3145728*a**16*d**4)
, Ne(a**16*d**4*exp(20*I*c), 0)), (x*(-(-129*A - 31*I*B)/(16*a**4) + (-129
*A*exp(8*I*c) - 72*A*exp(6*I*c) - 30*A*exp(4*I*c) - 8*A*exp(2*I*c) - A - 3
1*I*B*exp(8*I*c) - 26*I*B*exp(6*I*c) - 16*I*B*exp(4*I*c) - 6*I*B*exp(2*I*c
) - I*B)*exp(-8*I*c)/(16*a**4)), True)) + x*(-129*A - 31*I*B)/(16*a**4) -
I*(4*A + I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/(a**4*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.74

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(iA+B)\log(\tan(dx+c)+i)}{32\,a^4d} - \frac{(-129iA+31B)\log(\tan(dx+c)-i)}{32\,a^4d} + \frac{(-4iA+B)\log(|\tan(dx+c)|)}{a^4d} - \frac{15(13A+3iB)\tan(dx+c)^4 + 12(-57iA+13B)\tan(dx+c)^3 - (851A+189iB)\tan(dx+c)^2 +}{48\,a^4d(\tan(dx+c)-i)^4\tan(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"giac")
```

output
```
-1/32*(I*A + B)*log(tan(d*x + c) + I)/(a^4*d) - 1/32*(-129*I*A + 31*B)*log
(tan(d*x + c) - I)/(a^4*d) + (-4*I*A + B)*log(abs(tan(d*x + c)))/(a^4*d) -
 1/48*(15*(13*A + 3*I*B)*tan(d*x + c)^4 + 12*(-57*I*A + 13*B)*tan(d*x + c)
^3 - (851*A + 189*I*B)*tan(d*x + c)^2 + 4*(104*I*A - 21*B)*tan(d*x + c) +
48*A)/(a^4*d*(tan(d*x + c) - I)^4*tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.03

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{\frac{A}{a^4}+\tan(c+d\,x)^4\left(\frac{65\,A}{16\,a^4}+\frac{B\,15\mathrm{i}}{16\,a^4}\right)-\tan(c+d\,x)^2\left(\frac{851\,A}{48\,a^4}+\frac{B\,63\mathrm{i}}{16\,a^4}\right)-\tan(c+d\,x)^3\left(-\frac{13\,B}{4\,a^4}+\frac{A\,57\mathrm{i}}{4\,a^4}\right)+\tan(}{d\,\left(\tan\left(c+d\,x\right)^5-\tan\left(c+d\,x\right)^4\,4\mathrm{i}-6\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,4\mathrm{i}+\tan\left(c+\right.\right.}$$
$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(-B+A\,4\mathrm{i}\right)}{a^4\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{32\,a^4\,d}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-31\,B+A\,129\mathrm{i}\right)}{32\,a^4\,d}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(log(tan(c + d*x) - 1i)*(A*129i - 31*B))/(32*a^4*d) - (log(tan(c + d*x))*(
A*4i - B))/(a^4*d) - (log(tan(c + d*x) + 1i)*(A*1i + B))/(32*a^4*d) - (tan
(c + d*x)^4*((65*A)/(16*a^4) + (B*15i)/(16*a^4)) - tan(c + d*x)^3*((A*57i)
/(4*a^4) - (13*B)/(4*a^4)) - tan(c + d*x)^2*((851*A)/(48*a^4) + (B*63i)/(1
6*a^4)) + A/a^4 + tan(c + d*x)*((A*26i)/(3*a^4) - (7*B)/(4*a^4)))/(d*(tan(
c + d*x) + tan(c + d*x)^2*4i - 6*tan(c + d*x)^3 - tan(c + d*x)^4*4i + tan(
c + d*x)^5))
```

Reduce [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{too large to display}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

output
```
( - 281*cos(c + d*x)*a - 160*cos(c + d*x)*b*i - 408*int(tan((c + d*x)/2)**
5/(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2)**6*
i - 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d*x)/2
)**3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i),x)*sin(c + d*x)*
a*d - 260*int(tan((c + d*x)/2)**5/(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)
/2)**7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan((c + d
*x)/2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c
 + d*x)/2) + i),x)*sin(c + d*x)*b*d*i + 2200*int(tan((c + d*x)/2)**4/(tan(
(c + d*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2)**6*i - 56*
tan((c + d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d*x)/2)**3 -
28*tan((c + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i),x)*sin(c + d*x)*a*d*i -
 1344*int(tan((c + d*x)/2)**4/(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2)*
*7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x)/
2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c + d
*x)/2) + i),x)*sin(c + d*x)*b*d + 5448*int(tan((c + d*x)/2)**3/(tan((c + d
*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c
 + d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan
((c + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i),x)*sin(c + d*x)*a*d + 3260*in
t(tan((c + d*x)/2)**3/(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*
tan((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x)/2)**4...
```

3.66 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 255

$$\begin{aligned}\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx &= \frac{5(35iA-13B)x}{16a^4}+\frac{5(35iA-13B)\cot(c+dx)}{16a^4d}-\frac{(11A+4iB)\cot^2(c+dx)}{2a^4d}\\ &-\frac{(11A+4iB)\log(\sin(c+dx))}{a^4d}+\frac{(43A+17iB)\cot^2(c+dx)}{48a^4d(1+i\tan(c+dx))^2}\\ &+\frac{5(35A+13iB)\cot^2(c+dx)}{48a^4d(1+i\tan(c+dx))}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{(2A+iB)\cot^2(c+dx)}{6ad(a+ia\tan(c+dx))^3}\end{aligned}$$

output
```
5/16*(35*I*A-13*B)*x/a^4+5/16*(35*I*A-13*B)*cot(d*x+c)/a^4/d-1/2*(11*A+4*I
*B)*cot(d*x+c)^2/a^4/d-(11*A+4*I*B)*ln(sin(d*x+c))/a^4/d+1/48*(43*A+17*I*B
)*cot(d*x+c)^2/a^4/d/(1+I*tan(d*x+c))^2+5/48*(35*A+13*I*B)*cot(d*x+c)^2/a^
4/d/(1+I*tan(d*x+c))+1/8*(A+I*B)*cot(d*x+c)^2/d/(a+I*a*tan(d*x+c))^4+1/6*(
2*A+I*B)*cot(d*x+c)^2/a/d/(a+I*a*tan(d*x+c))^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.99 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.80

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{6(A+iB)\cot^6(c+dx)}{(i+\cot(c+dx))^4}+\frac{8(2A+iB)\cot^5(c+dx)}{(i+\cot(c+dx))^3}+\frac{(43A+17iB)\cot^4(c+dx)}{(i+\cot(c+dx))^2}+\frac{5(35A+13iB)\cot^3(c+dx)}{i+\cot(c+dx)}+15(35iA-13B)\cot(c$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x
]
```

output
```
((6*(A + I*B)*Cot[c + d*x]^6)/(I + Cot[c + d*x])^4 + (8*(2*A + I*B)*Cot[c
+ d*x]^5)/(I + Cot[c + d*x])^3 + ((43*A + (17*I)*B)*Cot[c + d*x]^4)/(I + C
ot[c + d*x])^2 + (5*(35*A + (13*I)*B)*Cot[c + d*x]^3)/(I + Cot[c + d*x]) +
 15*((35*I)*A - 13*B)*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
+ d*x]^2] - 24*(11*A + (4*I)*B)*(Cot[c + d*x]^2 + 2*(Log[Cos[c + d*x]] + L
og[Tan[c + d*x]])))/(48*a^4*d)
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.09, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{2\cot^3(c+dx)(a(5A+iB)-3a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{8a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cot^3(c+dx)(a(5A+iB)-3a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(5A+iB)-3a(iA-B)\tan(c+dx)}{\tan(c+dx)^3(i\tan(c+dx)a+a)^3}dx}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int \frac{2\cot^3(c+dx)\left(a^2(23A+7iB)-10a^2(2iA-B)\tan(c+dx)\right)}{(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{2a(2A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\cot^3(c+dx)\left(a^2(23A+7iB)-10a^2(2iA-B)\tan(c+dx)\right)}{(i\tan(c+dx)a+a)^2}dx}{3a^2}+\frac{2a(2A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(23A+7iB)-10a^2(2iA-B)\tan(c+dx)}{\tan(c+dx)^3(i\tan(c+dx)a+a)^2}dx}{3a^2}+\frac{2a(2A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\frac{\int \frac{2\cot^3(c+dx)\left(a^3(89A+31iB)-2a^3(43iA-17B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{(43A+17iB)\cot^2(c+dx)}{4d(1+i\tan(c+dx))^2}}{3a^2}+\frac{2a(2A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{\cot^3(c+dx)\left(a^3(89A+31iB)-2a^3(43iA-17B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{2a^2}+\frac{(43A+17iB)\cot^2(c+dx)}{4d(1+i\tan(c+dx))^2}}{3a^2}+\frac{2a(2A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{a^3(89A+31iB)-2a^3(43iA-17B)\tan(c+dx)}{\tan(c+dx)^3(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{(43A+17iB)\cot^2(c+dx)}{4d(1+i\tan(c+dx))^2}}{3a^2}+\frac{2a(2A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\frac{\frac{\int 3\cot^3(c+dx)\left(16a^4(11A+4iB)-5a^4(35iA-13B)\tan(c+dx)\right)dx}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{(43A+17iB)\cot^2(c+dx)}{4d(1+i\tan(c+dx))^2}}{3a^2}+\frac{2a(2A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{3\int \cot^3(c+dx)\left(16a^4(11A+4iB)-5a^4(35iA-13B)\tan(c+dx)\right)dx}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{(43A+17iB)\cot^2(c+dx)}{4d(1+i\tan(c+dx))^2}}{3a^2}+\frac{2a(2A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{3\int \frac{16a^4(11A+4iB)-5a^4(35iA-13B)\tan(c+dx)}{\tan(c+dx)^3}dx}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{(43A+17iB)\cot^2(c+dx)}{4d(1+i\tan(c+dx))^2}}{3a^2}+\frac{2a(2A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^3}}{4a^2}+\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\frac{\frac{3\left(\int -\cot^2(c+dx)\left(5(35iA-13B)a^4+16(11A+4iB)\tan(c+dx)a^4\right)dx-\frac{8a^4(11A+4iB)\cot^2(c+dx)}{d}\right)}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{(43A+17iB)\cot^2(c+dx)}{4d(1+i\tan(c+dx))^2}}{3a^2}+}{4a^2}$$
$$\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\frac{3\left(-\int \cot^2(c+dx)\left(5(35iA-13B)a^4+16(11A+4iB)\tan(c+dx)a^4\right)dx-\frac{8a^4(11A+4iB)\cot^2(c+dx)}{d}\right)}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{(43A+17iB)\cot^2(c+dx)}{4d(1+i\tan(c+dx))^2}}{3a^2}+\cdots}{4a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{3\left(-\int \frac{5(35iA-13B)a^4+16(11A+4iB)\tan(c+dx)a^4}{\tan(c+dx)^2}dx-\frac{8a^4(11A+4iB)\cot^2(c+dx)}{d}\right)}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{(43A+17iB)\cot^2(c+dx)}{4d(1+i\tan(c+dx))^2}}{3a^2}+\frac{2a(2A+iB\cdots}{3d(a+ia\cdots}}{4a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\big\downarrow$ 4012

$$\frac{\frac{\frac{\frac{3\left(-\int \cot(c+dx)\left(16a^4(11A+4iB)-5a^4(35iA-13B)\tan(c+dx)\right)dx-\frac{8a^4(11A+4iB)\cot^2(c+dx)}{d}+\frac{5a^4(-13B+35iA)\cot(c+dx)}{d}\right)}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}\cdots}{2a^2}\cdots}{3a^2}\cdots}{4a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{3\left(-\int \frac{16a^4(11A+4iB)-5a^4(35iA-13B)\tan(c+dx)}{\tan(c+dx)}dx-\frac{8a^4(11A+4iB)\cot^2(c+dx)}{d}+\frac{5a^4(-13B+35iA)\cot(c+dx)}{d}\right)}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{(43A+17i\cdots}{4d(1+i\cdots}}{3a^2}\cdots}{4a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\big\downarrow$ 4014

$$\frac{\frac{\frac{\frac{3\left(-16a^4(11A+4iB)\int \cot(c+dx)dx-\frac{8a^4(11A+4iB)\cot^2(c+dx)}{d}+\frac{5a^4(-13B+35iA)\cot(c+dx)}{d}+5a^4x(-13B+35iA)\right)}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{(43A+1\cdots}{4d(1+\cdots}}{3a^2}\cdots}{4a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{3\left(-16a^4(11A+4iB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{8a^4(11A+4iB)\cot^2(c+dx)}{d}+\frac{5a^4(-13B+35iA)\cot(c+dx)}{d}+5a^4x(-13B+35iA)\right)}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{(\cdots}{\cdots}}{3a^2}}{4a^2}$$
$$\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\frac{3\left(16a^4(11A+4iB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{8a^4(11A+4iB)\cot^2(c+dx)}{d}+\frac{5a^4(-13B+35iA)\cot(c+dx)}{d}+5a^4x(-13B+35iA)\right)}{2a^2}+\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}}{2a^2}+\frac{(\cdots}{\cdots}}{3a^2}}{4a^2}$$
$$\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 3956

$$\frac{\frac{\frac{\frac{5a^3(35A+13iB)\cot^2(c+dx)}{2d(a+ia\tan(c+dx))}+\frac{3\left(-\frac{8a^4(11A+4iB)\cot^2(c+dx)}{d}+\frac{5a^4(-13B+35iA)\cot(c+dx)}{d}-\frac{16a^4(11A+4iB)\log(-\sin(c+dx))}{d}+5a^4x(-13B+35iA)\right)}{2a^2}}{2a^2}+\frac{(43A\cdots}{4d(\cdots}}{3a^2}}{4a^2}$$
$$\frac{(A+iB)\cot^2(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((A + I*B)*Cot[c + d*x]^2)/(8*d*(a + I*a*Tan[c + d*x])^4) + ((2*a*(2*A + I
*B)*Cot[c + d*x]^2)/(3*d*(a + I*a*Tan[c + d*x])^3) + (((43*A + (17*I)*B)*C
ot[c + d*x]^2)/(4*d*(1 + I*Tan[c + d*x])^2) + ((3*(5*a^4*((35*I)*A - 13*B)
*x + (5*a^4*((35*I)*A - 13*B)*Cot[c + d*x])/d - (8*a^4*(11*A + (4*I)*B)*Co
t[c + d*x]^2)/d - (16*a^4*(11*A + (4*I)*B)*Log[-Sin[c + d*x]])/d))/(2*a^2)
 + (5*a^3*(35*A + (13*I)*B)*Cot[c + d*x]^2)/(2*d*(a + I*a*Tan[c + d*x])))/
(2*a^2))/(3*a^2))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{129xB}{16a^4}-\frac{2i\left(-3iA\,\mathrm{e}^{2i(dx+c)}+B\,\mathrm{e}^{2i(dx+c)}+4iA-B\right)}{a^4d\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}-\frac{4i\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)B}{a^4d}-\frac{75\,\mathrm{e}^{-2i(dx+c)}A}{16d\,a^4}+\frac{351ixA}{16a^4}-$
derivativedivides	$-\frac{A}{8d\,a^4(-i+\tan(dx+c))^4}-\frac{7iA}{12d\,a^4(-i+\tan(dx+c))^3}-\frac{49B}{16d\,a^4(-i+\tan(dx+c))}+\frac{175iA\arctan(\tan(dx+c))}{16d\,a^4}+$
default	$-\frac{A}{8d\,a^4(-i+\tan(dx+c))^4}-\frac{7iA}{12d\,a^4(-i+\tan(dx+c))^3}-\frac{49B}{16d\,a^4(-i+\tan(dx+c))}+\frac{175iA\arctan(\tan(dx+c))}{16d\,a^4}+$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVER
BOSE)
```

output
```
-129/16*x/a^4*B-2*I*(-3*I*A*exp(2*I*(d*x+c))+B*exp(2*I*(d*x+c))+4*I*A-B)/a
^4/d/(exp(2*I*(d*x+c))-1)^2-4*I/a^4/d*ln(exp(2*I*(d*x+c))-1)*B-75/16/d/a^4
*exp(-2*I*(d*x+c))*A+351/16*I*x/a^4*A-3/4/d/a^4*exp(-4*I*(d*x+c))*A-15/32*
I/d/a^4*exp(-4*I*(d*x+c))*B-5/48/d/a^4*exp(-6*I*(d*x+c))*A-1/12*I/d/a^4*ex
p(-6*I*(d*x+c))*B-1/128/d/a^4*exp(-8*I*(d*x+c))*A-9/4*I/d/a^4*exp(-2*I*(d*
x+c))*B-8/a^4/d*B*c+22*I/a^4/d*A*c-1/128*I/d/a^4*exp(-8*I*(d*x+c))*B-11/a^
4*A/d*ln(exp(2*I*(d*x+c))-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.99

$$\int\frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx=-\frac{72\,(-117i\,A+43\,B)dxe^{(12i\,dx+12i\,c)}+24\,(6\,(117i\,A-43\,B)dx+171\,A+68i\,B)e^{(10i\,dx+10i\,c)}+12\,(6\,(-$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"fricas")
```

output
```
-1/384*(72*(-117*I*A + 43*B)*d*x*e^(12*I*d*x + 12*I*c) + 24*(6*(117*I*A -
43*B)*d*x + 171*A + 68*I*B)*e^(10*I*d*x + 10*I*c) + 12*(6*(-117*I*A + 43*B
)*d*x - 532*A - 193*I*B)*e^(8*I*d*x + 8*I*c) + 8*(158*A + 67*I*B)*e^(6*I*d
*x + 6*I*c) + (211*A + 119*I*B)*e^(4*I*d*x + 4*I*c) + 2*(17*A + 13*I*B)*e^
(2*I*d*x + 2*I*c) + 384*((11*A + 4*I*B)*e^(12*I*d*x + 12*I*c) - 2*(11*A +
4*I*B)*e^(10*I*d*x + 10*I*c) + (11*A + 4*I*B)*e^(8*I*d*x + 8*I*c))*log(e^(
2*I*d*x + 2*I*c) - 1) + 3*A + 3*I*B)/(a^4*d*e^(12*I*d*x + 12*I*c) - 2*a^4*
d*e^(10*I*d*x + 10*I*c) + a^4*d*e^(8*I*d*x + 8*I*c))
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.83

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \frac{8A+2iB+\left(-6Ae^{2ic}-2iBe^{2ic}\right)e^{2idx}}{a^4de^{4ic}e^{4idx}-2a^4de^{2ic}e^{2idx}+a^4d} + \begin{cases} \frac{\left(\left(-24576Aa^{12}d^3e^{12ic}-24576iBa^{12}d^3e^{12ic}\right)e^{-8idx}+\left(-327680Aa^{12}d^3e^{14ic}-262144iBa^{12}d^3e^{14ic}\right)e^{-6idx}+\left(-2359296Aa^{12}d^3e^{16ic}-147\right.}{3145728a^{16}d^4} \\ x\left(-\frac{351iA-129B}{16a^4}+\frac{\left(351iAe^{8ic}+150iAe^{6ic}+48iAe^{4ic}+10iAe^{2ic}+iA-129Be^{8ic}-72Be^{6ic}-30Be^{4ic}-8Be^{2ic}-B\right)e^{-8ic}}{16a^4}\right) \end{cases} + \frac{x(351iA-129B)}{16a^4} - \frac{(11A+4iB)\log\left(e^{2idx}-e^{-2ic}\right)}{a^4d}$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)
```

output
```
(8*A + 2*I*B + (-6*A*exp(2*I*c) - 2*I*B*exp(2*I*c))*exp(2*I*d*x))/(a**4*d*
exp(4*I*c)*exp(4*I*d*x) - 2*a**4*d*exp(2*I*c)*exp(2*I*d*x) + a**4*d) + Pie
cewise((((-24576*A*a**12*d**3*exp(12*I*c) - 24576*I*B*a**12*d**3*exp(12*I*
c))*exp(-8*I*d*x) + (-327680*A*a**12*d**3*exp(14*I*c) - 262144*I*B*a**12*d
**3*exp(14*I*c))*exp(-6*I*d*x) + (-2359296*A*a**12*d**3*exp(16*I*c) - 1474
560*I*B*a**12*d**3*exp(16*I*c))*exp(-4*I*d*x) + (-14745600*A*a**12*d**3*ex
p(18*I*c) - 7077888*I*B*a**12*d**3*exp(18*I*c))*exp(-2*I*d*x))*exp(-20*I*c
)/(3145728*a**16*d**4), Ne(a**16*d**4*exp(20*I*c), 0)), (x*(-(351*I*A - 12
9*B)/(16*a**4) + (351*I*A*exp(8*I*c) + 150*I*A*exp(6*I*c) + 48*I*A*exp(4*I
*c) + 10*I*A*exp(2*I*c) + I*A - 129*B*exp(8*I*c) - 72*B*exp(6*I*c) - 30*B*
exp(4*I*c) - 8*B*exp(2*I*c) - B)*exp(-8*I*c)/(16*a**4)), True)) + x*(351*I
*A - 129*B)/(16*a**4) - (11*A + 4*I*B)*log(exp(2*I*d*x) - exp(-2*I*c))/(a*
*4*d)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.71

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \frac{(A-iB)\log(\tan(dx+c)+i)}{32\,a^4d} + \frac{3\,(117\,A+43i\,B)\log(\tan(dx+c)-i)}{32\,a^4d} - \frac{(11\,A+4i\,B)\log(|\tan(dx+c)|)}{a^4d} - \frac{15\,(-35i\,A+13\,B)\tan(dx+c)^5 - 36\,(51\,A+19i\,B)\tan(dx+c)^4 + (2269i\,A-851\,B)\tan(dx+c)^3}{48\,a^4d(\tan(dx+c)-i)^4\tan(dx}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"giac")
```

output
```
1/32*(A - I*B)*log(tan(d*x + c) + I)/(a^4*d) + 3/32*(117*A + 43*I*B)*log(t
an(d*x + c) - I)/(a^4*d) - (11*A + 4*I*B)*log(abs(tan(d*x + c)))/(a^4*d) -
 1/48*(15*(-35*I*A + 13*B)*tan(d*x + c)^5 - 36*(51*A + 19*I*B)*tan(d*x + c
)^4 + (2269*I*A - 851*B)*tan(d*x + c)^3 + 4*(271*A + 104*I*B)*tan(d*x + c)
^2 + 48*(-2*I*A + B)*tan(d*x + c) + 24*A)/(a^4*d*(tan(d*x + c) - I)^4*tan(
d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.98

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\tan(c+d\,x)^4\left(\frac{153\,A}{4\,a^4}+\frac{B\,57\mathrm{i}}{4\,a^4}\right)+\tan(c+d\,x)^5\left(-\frac{65\,B}{16\,a^4}+\frac{A\,175\mathrm{i}}{16\,a^4}\right)-\tan(c+d\,x)^2\left(\frac{271\,A}{12\,a^4}+\frac{B\,26\mathrm{i}}{3\,a^4}\right)-\tan(c+d}{d\,\left(\tan\left(c+d\,x\right)^6-\tan\left(c+d\,x\right)^5\,4\mathrm{i}-6\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)}$$
$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(11\,A+B\,4\mathrm{i}\right)}{a^4\,d}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{32\,a^4\,d}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(351\,A+B\,129\mathrm{i}\right)}{32\,a^4\,d}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
(tan(c + d*x)^4*((153*A)/(4*a^4) + (B*57i)/(4*a^4)) + tan(c + d*x)^5*((A*1
75i)/(16*a^4) - (65*B)/(16*a^4)) - tan(c + d*x)^2*((271*A)/(12*a^4) + (B*2
6i)/(3*a^4)) - tan(c + d*x)^3*((A*2269i)/(48*a^4) - (851*B)/(48*a^4)) - A/
(2*a^4) + tan(c + d*x)*((A*2i)/a^4 - B/a^4))/(d*(tan(c + d*x)^2 + tan(c +
d*x)^3*4i - 6*tan(c + d*x)^4 - tan(c + d*x)^5*4i + tan(c + d*x)^6)) - (log
(tan(c + d*x))*(11*A + B*4i))/(a^4*d) + (log(tan(c + d*x) + 1i)*(A - B*1i)
)/(32*a^4*d) + (log(tan(c + d*x) - 1i)*(351*A + B*129i))/(32*a^4*d)
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{too large to display}$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(1808*cos(c + d*x)*sin(c + d*x)*a*i - 1124*cos(c + d*x)*sin(c + d*x)*b - 4
816*cos(c + d*x)*a - 3264*cos(c + d*x)*b*i - 5888*int(tan((c + d*x)/2)**4/
(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2)**6*i
- 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d*x)/2)*
*3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i),x)*sin(c + d*x)**2
*a*d - 4256*int(tan((c + d*x)/2)**4/(tan((c + d*x)/2)**8*i + 8*tan((c + d*
x)/2)**7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan((c +
 d*x)/2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan((c + d*x)/2)**2*i - 8*tan(
(c + d*x)/2) + i),x)*sin(c + d*x)**2*b*d*i + 33768*int(tan((c + d*x)/2)**3
/(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2)**6*i
 - 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d*x)/2)
**3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i),x)*sin(c + d*x)**
2*a*d*i - 23904*int(tan((c + d*x)/2)**3/(tan((c + d*x)/2)**8*i + 8*tan((c
+ d*x)/2)**7 - 28*tan((c + d*x)/2)**6*i - 56*tan((c + d*x)/2)**5 + 70*tan(
(c + d*x)/2)**4*i + 56*tan((c + d*x)/2)**3 - 28*tan((c + d*x)/2)**2*i - 8*
tan((c + d*x)/2) + i),x)*sin(c + d*x)**2*b*d + 86464*int(tan((c + d*x)/2)*
*2/(tan((c + d*x)/2)**8*i + 8*tan((c + d*x)/2)**7 - 28*tan((c + d*x)/2)**6
*i - 56*tan((c + d*x)/2)**5 + 70*tan((c + d*x)/2)**4*i + 56*tan((c + d*x)/
2)**3 - 28*tan((c + d*x)/2)**2*i - 8*tan((c + d*x)/2) + i),x)*sin(c + d*x)
**2*a*d + 60432*int(tan((c + d*x)/2)**2/(tan((c + d*x)/2)**8*i + 8*tan(...
```

3.67 $\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 194

$$\begin{aligned}
&\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx \\
&= \frac{\sqrt{2}\sqrt{a}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{8(7A-iB)\sqrt{a+ia\tan(c+dx)}}{35d} \\
&\quad + \frac{2(7A-iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} \\
&\quad + \frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d} - \frac{2(7A-31iB)(a+ia\tan(c+dx))^{3/2}}{105ad}
\end{aligned}$$

output

```
2^(1/2)*a^(1/2)*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))/d-8/35*(7*A-I*B)*(a+I*a*tan(d*x+c))^(1/2)/d+2/35*(7*A-I*B)*tan(d*x+c)^
2*(a+I*a*tan(d*x+c))^(1/2)/d+2/7*B*tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/d
-2/105*(7*A-31*I*B)*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.67

$$\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{\sqrt{2}\sqrt{a}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$
$$+\frac{2\sqrt{a+ia\tan(c+dx)}(-91A+43iB+(-7iA-31B)\tan(c+dx)+3(7A-iB)\tan^2(c+dx)+15B}{105d}$$

input
```
Integrate[Tan[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x
]
```

output
```
(Sqrt[2]*Sqrt[a]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqr
t[a])])/d + (2*Sqrt[a + I*a*Tan[c + d*x]]*(-91*A + (43*I)*B + ((-7*I)*A -
31*B)*Tan[c + d*x] + 3*(7*A - I*B)*Tan[c + d*x]^2 + 15*B*Tan[c + d*x]^3))/
(105*d)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.361, Rules used = {3042, 4080, 27, 3042, 4080, 27, 3042, 4075, 3042, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)^3\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4080$$

$$\frac{2\int -\frac{1}{2}\tan^2(c+dx)\sqrt{i\tan(c+dx)a+a}(6aB-a(7A-iB)\tan(c+dx))dx}{7a}+\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\int \tan^2(c+dx)\sqrt{i\tan(c+dx)a+a}(6aB-a(7A-iB)\tan(c+dx))dx}{7a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\int \tan(c+dx)^2\sqrt{i\tan(c+dx)a+a}(6aB-a(7A-iB)\tan(c+dx))dx}{7a}$$

$$\Big\downarrow \; 4080$$

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\frac{2\int \frac{1}{2}\tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4(7A-iB)a^2+(7iA+31B)\tan(c+dx)a^2)dx}{5a}-\frac{2a(7A-iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}}{7a}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4(7A-iB)a^2+(7iA+31B)\tan(c+dx)a^2)dx}{5a}-\frac{2a(7A-iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}}{7a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4(7A-iB)a^2+(7iA+31B)\tan(c+dx)a^2)dx}{5a}-\frac{2a(7A-iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}}{7a}$$

$$\Big\downarrow \; 4075$$

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\frac{\int \sqrt{i\tan(c+dx)a+a}(4a^2(7A-iB)\tan(c+dx)-a^2(7iA+31B))dx+\frac{2a(7A-31iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{5a}-\frac{2a(7A-iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}}{7a}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\frac{\int\sqrt{i\tan(c+dx)a+a}\left(4a^2(7A-iB)\tan(c+dx)-a^2(7iA+31B)\right)dx+\frac{2a(7A-31iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{5a}-\frac{2a(7A-iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx}}{5d}}{7a}$$

$\downarrow$ 4010

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\frac{-35a^2(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx+\frac{8a^2(7A-iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2a(7A-31iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{5a}-\frac{2a(7A-iB)\tan^2(c+dx)\sqrt{a+ia\tan(c-}}{5d}}{7a}$$

$\downarrow$ 3042

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\frac{-35a^2(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx+\frac{8a^2(7A-iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2a(7A-31iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{5a}-\frac{2a(7A-iB)\tan^2(c+dx)\sqrt{a+ia\tan(c-}}{5d}}{7a}$$

$\downarrow$ 3961

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\frac{\frac{70ia^3(B+iA)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+\frac{8a^2(7A-iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2a(7A-31iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{5a}-\frac{2a(7A-iB)\tan^2(c+dx)\sqrt{a+}}{5d}}{7a}$$

$\downarrow$ 219

$$\frac{2B\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}-\frac{\frac{\frac{35i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{8a^2(7A-iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2a(7A-31iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{5a}-\frac{2a(7A-iB)\tan^2(c+dx)\sqrt{a+}}{5d}}{7a}$$

input
```
Int[Tan[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*Tan[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(7*d) - ((-2*a*(7*A - I*B)
*Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(5*d) + (((35*I)*Sqrt[2]*a^(5/
2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d + (8
*a^2*(7*A - I*B)*Sqrt[a + I*a*Tan[c + d*x]])/d + (2*a*(7*A - (31*I)*B)*(a
+ I*a*Tan[c + d*x])^(3/2))/(3*d))/(5*a))/(7*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{2iB(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{4iBa(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2Aa(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{4iB\,a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+\frac{2A\,a^{2}(a+ia\tan(dx+c}{3}}{a^{3}d}$
default	$\frac{\frac{2iB(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{4iBa(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2Aa(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{4iB\,a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+\frac{2A\,a^{2}(a+ia\tan(dx+c}{3}}{a^{3}d}$
parts	$\frac{2A\left(-\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-a^{2}\sqrt{a+ia\tan(dx+c)}+\frac{a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2}\right)}{d\,a^{2}}+\frac{2iB}{}$

input
```
int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2/d/a^3*(1/7*I*B*(a+I*a*tan(d*x+c))^(7/2)-2/5*I*B*a*(a+I*a*tan(d*x+c))^(5/
2)-1/5*A*a*(a+I*a*tan(d*x+c))^(5/2)+2/3*I*B*a^2*(a+I*a*tan(d*x+c))^(3/2)+1
/3*A*a^2*(a+I*a*tan(d*x+c))^(3/2)-A*a^3*(a+I*a*tan(d*x+c))^(1/2)+1/2*a^(7/
2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(151) = 302.

Time = 0.12 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.27

$$\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{105\sqrt{2}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{(A^2-2i\,AB-B^2)a}{d^2}}\log\left(-\frac{4\left((-i\,A-B)ae^{(i\,dx+i\,c)}-(i\,de^{(}\right.}{}\right.}{}$$

input
```
integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/210*(105*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*
e^(2*I*d*x + 2*I*c) + d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A -
 B)*a*e^(I*d*x + I*c) - (I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((A^2 - 2*I*A*
B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A +
 B)) - 105*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*
e^(2*I*d*x + 2*I*c) + d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A -
 B)*a*e^(I*d*x + I*c) - (-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((A^2 - 2*I*A
*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A
+ B)) - 4*sqrt(2)*((119*A - 92*I*B)*e^(7*I*d*x + 7*I*c) + 7*(37*A - 16*I*B
)*e^(5*I*d*x + 5*I*c) + 35*(7*A - 4*I*B)*e^(3*I*d*x + 3*I*c) + 105*A*e^(I*
d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(6*I*d*x + 6*I*c) + 3*
d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\left(A+B\tan\left(c+dx\right)\right)\tan^3\left(c+dx\right)dx$$

input
```
integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*tan(c + d*x)**3
, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.79

$$\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$
$$-\frac{105\sqrt{2}(A-iB)a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-60i\left(ia\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}Ba+84\left(ia\tan\left(dx+c\right)\right.}{210\,a^4 d}$$

input
```
integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/210*(105*sqrt(2)*(A - I*B)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan
(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 60*I*(I*
a*tan(d*x + c) + a)^(7/2)*B*a + 84*(I*a*tan(d*x + c) + a)^(5/2)*(A + 2*I*B
)*a^2 - 140*(I*a*tan(d*x + c) + a)^(3/2)*(A + 2*I*B)*a^3 + 420*sqrt(I*a*ta
n(d*x + c) + a)*A*a^4)/(a^4*d)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.11

$$\begin{aligned}&\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx\\&=-\frac{2A\sqrt{a+a\tan(c+dx)\ 1\mathrm{i}}}{d}+\frac{2A\,(a+a\tan(c+dx)\ 1\mathrm{i})^{3/2}}{3\,a\,d}\\&\quad-\frac{2A\,(a+a\tan(c+dx)\ 1\mathrm{i})^{5/2}}{5\,a^2\,d}+\frac{B\,(a+a\tan(c+dx)\ 1\mathrm{i})^{3/2}\,4\mathrm{i}}{3\,a\,d}\\&\quad-\frac{B\,(a+a\tan(c+dx)\ 1\mathrm{i})^{5/2}\,4\mathrm{i}}{5\,a^2\,d}+\frac{B\,(a+a\tan(c+dx)\ 1\mathrm{i})^{7/2}\,2\mathrm{i}}{7\,a^3\,d}\\&\quad-\frac{\sqrt{2}\,B\,\sqrt{-a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{d}-\frac{\sqrt{2}\,A\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}\,1\mathrm{i}}{2\,\sqrt{a}}\right)\,1\mathrm{i}}{d}\end{aligned}$$

input
```
int(tan(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(2*A*(a + a*tan(c + d*x)*1i)^(3/2))/(3*a*d) - (2*A*(a + a*tan(c + d*x)*1i)
^(1/2))/d - (2*A*(a + a*tan(c + d*x)*1i)^(5/2))/(5*a^2*d) + (B*(a + a*tan(
c + d*x)*1i)^(3/2)*4i)/(3*a*d) - (B*(a + a*tan(c + d*x)*1i)^(5/2)*4i)/(5*a
^2*d) + (B*(a + a*tan(c + d*x)*1i)^(7/2)*2i)/(7*a^3*d) - (2^(1/2)*B*(-a)^(
1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/d -
(2^(1/2)*A*a^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1
/2)))*1i)/d
```

Reduce [F]

$$\int \tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\sqrt{a}\left(\left(\int\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^4\,dx\right)b+\left(\int\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^3\,dx\right)a\right)$$

input
```
int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4,x)*b + int(sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x)**3,x)*a)
```

3.68 $\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 143

$$\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{\sqrt{2}\sqrt{a}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{8B\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{2B\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d} - \frac{2(5iA+B)(a+ia\tan(c+dx))^{3/2}}{15ad}$$

output
```
2^(1/2)*a^(1/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/
2))/d-8/5*B*(a+I*a*tan(d*x+c))^(1/2)/d+2/5*B*tan(d*x+c)^2*(a+I*a*tan(d*x+c
))^(1/2)/d-2/15*(5*I*A+B)*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{15\sqrt{2}\sqrt{a}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+2\sqrt{a+ia\tan(c+dx)}(-5iA-13B+(5A-iB)\tan(c+}{15d}$$

input
```
Integrate[Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x
]
```

output
```
(15*Sqrt[2]*Sqrt[a]*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*
Sqrt[a])] + 2*Sqrt[a + I*a*Tan[c + d*x]]*((-5*I)*A - 13*B + (5*A - I*B)*Ta
n[c + d*x] + 3*B*Tan[c + d*x]^2))/(15*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4080, 27, 3042, 4075, 3042, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4080}$$

$$\frac{2\int -\frac{1}{2}\tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4aB-a(5A-iB)\tan(c+dx))dx}{5a}+\frac{2B\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4aB-a(5A-iB)\tan(c+dx))dx}{5a}$$

$\downarrow$ 3042

$$\frac{2B\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4aB-a(5A-iB)\tan(c+dx))dx}{5a}$$

$\downarrow$ 4075

$$\frac{2B\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{\int \sqrt{i\tan(c+dx)a+a}(a(5A-iB)+4aB\tan(c+dx))dx+\frac{2(B+5iA)(a+ia\tan(c+dx))^{3/2}}{3d}}{5a}$$

$\downarrow$ 3042

$$\frac{2B\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{\int \sqrt{i\tan(c+dx)a+a}(a(5A-iB)+4aB\tan(c+dx))dx+\frac{2(B+5iA)(a+ia\tan(c+dx))^{3/2}}{3d}}{5a}$$

$\downarrow$ 4010

$$\frac{2B\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{5a(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2(B+5iA)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{8aB\sqrt{a+ia\tan(c+dx)}}{d}}{5a}$$

$\downarrow$ 3042

$$\frac{2B\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{5a(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2(B+5iA)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{8aB\sqrt{a+ia\tan(c+dx)}}{d}}{5a}$$

$\downarrow$ 3961

$$\frac{2B\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{-\frac{10ia^2(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+\frac{2(B+5iA)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{8aB\sqrt{a+ia\tan(c+dx)}}{d}}{5a}$$

$\downarrow$ 219

$$\frac{2B\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{-\frac{5i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2(B+5iA)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{8aB\sqrt{a+ia\tan(c+dx)}}{d}}{5a}$$

input
```
Int[Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(5*d) - (((-5*I)*Sqrt[2]*a
^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
+ (8*a*B*Sqrt[a + I*a*Tan[c + d*x]])/d + (2*((5*I)*A + B)*(a + I*a*Tan[c +
 d*x])^(3/2))/(3*d))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{2i\left(\frac{iB(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{iBa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+ia^2B\sqrt{a+ia\tan(dx+c)}+\frac{a^{\frac{5}{2}}(-iB+A)\sqrt{2}\,\mathrm{arct}}{}\right.}{d\,a^2}$
default	$\frac{2i\left(\frac{iB(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{iBa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+ia^2B\sqrt{a+ia\tan(dx+c)}+\frac{a^{\frac{5}{2}}(-iB+A)\sqrt{2}\,\mathrm{arct}}{}\right.}{d\,a^2}$
parts	$\frac{2iA\left(-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+\frac{a^{\frac{3}{2}}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2}\right)}{da}+\frac{2B\left(-\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right.}{}$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d/a^2*(1/5*I*B*(a+I*a*tan(d*x+c))^(5/2)-1/3*I*B*a*(a+I*a*tan(d*x+c))^(
3/2)-1/3*A*a*(a+I*a*tan(d*x+c))^(3/2)+I*a^2*B*(a+I*a*tan(d*x+c))^(1/2)+1/2
*a^(5/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1
/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(112) = 224$.

Time = 0.09 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.68

$$\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$

$$-\frac{15\sqrt{2}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{(A^2-2i\,AB-B^2)a}{d^2}}\log\left(-\frac{4\left((-i\,A-B)ae^{(i\,dx+i\,c)}+(de^{(2i\,dx+2i\,c)}+d)\sqrt{-\cdots}\right.}{i\,A\cdots}\right.}{\cdots}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/30*(15*sqrt(2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*sq
rt(-(A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) + (d
*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(2)*(d*e^(4*I*d
*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(A^2 - 2*I*A*B - B^2)*a/d
^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (d*e^(2*I*d*x + 2*I*c) + d)*sqr
t(-(A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d
*x - I*c)/(I*A + B)) + 4*sqrt(2)*((10*I*A + 17*B)*e^(5*I*d*x + 5*I*c) + 10
*(I*A + 2*B)*e^(3*I*d*x + 3*I*c) + 15*B*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\left(A+B\tan\left(c+dx\right)\right)\tan^2\left(c+dx\right)dx$$

input
```
integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*tan(c + d*x)**2
, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.91

$$\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{i\left(15\sqrt{2}(A-iB)a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-12i\,(ia\tan(dx+c)+a)^{\frac{5}{2}}Ba+20\,(ia\tan(dx+\right.}{30\,a^3d}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/30*I*(15*sqrt(2)*(A - I*B)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan
(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 12*I*(I*
a*tan(d*x + c) + a)^(5/2)*B*a + 20*(I*a*tan(d*x + c) + a)^(3/2)*(A + I*B)*
a^2 - 60*I*sqrt(I*a*tan(d*x + c) + a)*B*a^3)/(a^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17

$$\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= -\frac{2\,B\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{d} - \frac{A\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,2\mathrm{i}}{3\,a\,d} + \frac{2\,B\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}{3\,a\,d} - \frac{2\,B\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}{5\,a^2\,d} + \frac{\sqrt{2}\,A\,\sqrt{-a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{d} - \frac{\sqrt{2}\,B\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,1\mathrm{i}}{2\,\sqrt{a}}\right)\,1\mathrm{i}}{d}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(2*B*(a + a*tan(c + d*x)*1i)^(3/2))/(3*a*d) - (A*(a + a*tan(c + d*x)*1i)^(
3/2)*2i)/(3*a*d) - (2*B*(a + a*tan(c + d*x)*1i)^(1/2))/d - (2*B*(a + a*tan
(c + d*x)*1i)^(5/2))/(5*a^2*d) + (2^(1/2)*A*(-a)^(1/2)*atan((2^(1/2)*(a +
a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/d - (2^(1/2)*B*a^(1/2)*atan(
(2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*1i)/d
```

Reduce [F]

$$\int \tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \sqrt{a}\left(\left(\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)^3dx\right)b + \left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)^2dx\right)a\right)\right.$$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3,x)*b + int(sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x)**2,x)*a)
```

3.69 $\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 105

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{\sqrt{2}\sqrt{a}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2A\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2iB(a+ia\tan(c+dx))^{3/2}}{3ad}$$

output
```
-2^(1/2)*a^(1/2)*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1
/2))/d+2*A*(a+I*a*tan(d*x+c))^(1/2)/d-2/3*I*B*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{-3\sqrt{2}\sqrt{a}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right) + 2\sqrt{a+ia\tan(c+dx)}(3A-iB+B\tan(c+dx))}{3d}$$

input
```
Integrate[Tan[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(-3*Sqrt[2]*Sqrt[a]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*
Sqrt[a])] + 2*Sqrt[a + I*a*Tan[c + d*x]]*(3*A - I*B + B*Tan[c + d*x]))/(3*
d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4075, 3042, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int \sqrt{i\tan(c+dx)a+a}(A\tan(c+dx)-B)dx-\frac{2iB(a+ia\tan(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{i\tan(c+dx)a+a}(A\tan(c+dx)-B)dx-\frac{2iB(a+ia\tan(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{4010}$$

$$-(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2A\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2iB(a+ia\tan(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2A\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2iB(a+ia\tan(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \text{3961}$$

$$\frac{2ia(B+iA)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+\frac{2A\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2iB(a+ia\tan(c+dx))^{3/2}}{3ad}$$

$$\Big\downarrow \; 219$$

$$\frac{i\sqrt{2}\sqrt{a}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2A\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2iB(a+ia\tan(c+dx))^{3/2}}{3ad}$$

input

```
Int[Tan[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output

```
(I*Sqrt[2]*Sqrt[a]*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*S
qrt[a])])/d + (2*A*Sqrt[a + I*a*Tan[c + d*x]])/d - (((2*I)/3)*B*(a + I*a*T
an[c + d*x])^(3/2))/(a*d)
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\frac{2iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2aA\sqrt{a+ia\tan(dx+c)}-a^{\frac{3}{2}}(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{ad}$
default	$\frac{-\frac{2iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2aA\sqrt{a+ia\tan(dx+c)}-a^{\frac{3}{2}}(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{ad}$
parts	$\frac{A\left(2\sqrt{a+ia\tan(dx+c)}-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}+\frac{2iB\left(-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+\frac{a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+}}{}\right.}{2}\right.}{da}$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
2/d/a*(-1/3*I*B*(a+I*a*tan(d*x+c))^(3/2)+a*A*(a+I*a*tan(d*x+c))^(1/2)-1/2*
a^(3/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/
2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(80) = 160$.

Time = 0.11 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.16

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$

$$-\frac{3\sqrt{2}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{(A^2-2i\,AB-B^2)a}{d^2}}\log\left(-\frac{4\left((-i\,A-B)ae^{(i\,dx+i\,c)}-\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{(A^2-2i\,AB-B^2)a}{d^2}}\sqrt{e^{(2i\,\cdots}}\right.}{i\,A+B}\right.}{\cdots}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/6*(3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d
^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (I*d*e^(2*I*d*x + 2*I*c) + I*d)
*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-
I*d*x - I*c)/(I*A + B)) - 3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((A^2
- 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (-I*d*e^(2*
I*d*x + 2*I*c) - I*d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 4*sqrt(2)*((3*A - 2*I*B)*e^
(3*I*d*x + 3*I*c) + 3*A*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\left(A+B\tan\left(c+dx\right)\right)\tan\left(c+dx\right)\,dx$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*tan(c + d*x), x
)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{3\sqrt{2}(A-iB)a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-4i\left(ia\tan(dx+c)+a\right)^{\frac{3}{2}}Ba+12\sqrt{ia\tan(dx+c)+a}}{6a^2d}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/6*(3*sqrt(2)*(A - I*B)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
+ c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 4*I*(I*a*tan(
d*x + c) + a)^(3/2)*B*a + 12*sqrt(I*a*tan(d*x + c) + a)*A*a^2)/(a^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{2\,A\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{d}-\frac{B\,{\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)}^{3/2}\,2\mathrm{i}}{3\,a\,d}$$
$$+\frac{\sqrt{2}\,B\,\sqrt{-a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{d}-\frac{\sqrt{2}\,A\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(2*A*(a + a*tan(c + d*x)*1i)^(1/2))/d - (B*(a + a*tan(c + d*x)*1i)^(3/2)*2
i)/(3*a*d) + (2^(1/2)*B*(-a)^(1/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(
1/2))/(2*(-a)^(1/2)))*1i)/d - (2^(1/2)*A*a^(1/2)*atanh((2^(1/2)*(a + a*tan
(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/d
```

Reduce [F]

$$\int \tan(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\sqrt{a}\left(\left(\int\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)^{2}dx\right)b+\left(\int\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)dx\right)a\right)$$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*b + int(sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x),x)*a)
```

3.70 $\int \sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 75

$$\int \sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{\sqrt{2}\sqrt{a}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2B\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
-2^(1/2)*a^(1/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1
/2))/d+2*B*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{-i\sqrt{2}\sqrt{a}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+2B\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Integrate[Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
((-I)*Sqrt[2]*Sqrt[a]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2
]*Sqrt[a])] + 2*B*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3042, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4010}$$

$$(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2B\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2B\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3961}$$

$$\frac{2B\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2ia(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{2B\sqrt{a+ia\tan(c+dx)}}{d}-\frac{i\sqrt{2}\sqrt{a}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

input
```
Int[Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
((-I)*Sqrt[2]*Sqrt[a]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2
]*Sqrt[a])])/d + (2*B*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{2i\left(-iB\sqrt{a+ia\tan(dx+c)}-\frac{\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2}\right)}{d}$
default	$\frac{2i\left(-iB\sqrt{a+ia\tan(dx+c)}-\frac{\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2}\right)}{d}$
parts	$-\frac{iA\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{d}+\frac{B\left(2\sqrt{a+ia\tan(dx+c)}-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/d*(-I*B*(a+I*a*tan(d*x+c))^(1/2)-1/2*a^(1/2)*(A-I*B)*2^(1/2)*arctanh(1
/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(58) = 116$.

Time = 0.09 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.63

$$\int \sqrt{a + ia\tan(c+dx)}(A + B\tan(c+dx))\,dx$$

$$= \frac{4\sqrt{2}B\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)} + \sqrt{2}d\sqrt{-\frac{(A^2-2i\,AB-B^2)a}{d^2}}\log\left(-\frac{4\left((-i\,A-B)ae^{(i\,dx+i\,c)}+(de^{(2i\,dx+2i\,c)}+d)\sqrt{-\frac{(A^2-2i}{}}\right.}{i\,A+B}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(4*sqrt(2)*B*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) + sqrt(
2)*d*sqrt(-(A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*
c) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - sqrt(2)*d*sqrt(
-(A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (d*e^
(2*I*d*x + 2*I*c) + d)*sqrt(-(A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)))/d
```

Sympy [F]

$$\int \sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\left(A+B\tan\left(c+dx\right)\right)\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int \sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{i\left(\sqrt{2}(A-i\,B)a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)-4i\,\sqrt{i\,a\tan\left(dx+c\right)+a}Ba\right)}{2\,ad}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*I*(sqrt(2)*(A - I*B)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
+ c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 4*I*sqrt(I*a*
tan(d*x + c) + a)*B*a)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int \sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{2\,B\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{d}-\frac{\sqrt{2}\,A\,\sqrt{-a}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1{}\mathrm{i}}{d} - \frac{\sqrt{2}\,B\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{a}}\right)}{d}$$

input
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(2*B*(a + a*tan(c + d*x)*1i)^(1/2))/d - (2^(1/2)*A*(-a)^(1/2)*atan((2^(1/2
)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/d - (2^(1/2)*B*a^(1/2
)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/d
```

Reduce [F]

$$\int \sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a}\left(-2\sqrt{\tan\left(dx+c\right)i+1}\,ai+\left(\int\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)dx\right)adi+\left(\int\sqrt{\tan\left(dx+c\right)i+1}\right.}{d}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*a*i + int(sqrt(tan(c + d*x)*i + 1)
*tan(c + d*x),x)*a*d*i + int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*b*d)
)/d
```

3.71 $\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 86

$$\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{2\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{2}\sqrt{a}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

output
```
-2*a^(1/2)*A*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+2^(1/2)*a^(1/2)*(
A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a}\left(-2A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)\right)}{d}$$

input
```
Integrate[Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(Sqrt[a]*(-2*A*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]] + Sqrt[2]*(A -
I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]))/d
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4083}$$

$$(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{A\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3961}$$

$$\frac{A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{a}-\frac{2ia(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{a}-\frac{i\sqrt{2}\sqrt{a}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

$$\Big\downarrow \text{4082}$$

$$\frac{aA\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{i\sqrt{2}\sqrt{a}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{2iA\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{i\sqrt{2}\sqrt{a}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{i\sqrt{2}\sqrt{a}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}$$

input
```
Int[Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*Sqrt[a]*A*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - (I*Sqrt[2]*
Sqrt[a]*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{2a\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}-\frac{A\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{d}$	72
default	$\frac{2a\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}-\frac{A\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{d}$	72

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
2/d*a*(-1/2*(-A+I*B)*2^(1/2)/a^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*
2^(1/2)/a^(1/2))-A/a^(1/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(65) = 130$.

Time = 0.09 (sec) , antiderivative size = 447, normalized size of antiderivative = 5.20

$$\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{1}{2}\sqrt{2}\sqrt{\frac{(A^2-2iAB-B^2)a}{d^2}}\log\left(-\frac{4\left((-iA-B)ae^{(idx+ic)}-\left(ide^{(2idx+2ic)}+id\right)\sqrt{\frac{(A^2-2iAB-B^2)a}{d^2}}\sqrt{\frac{\ }{e}}\right.}{iA+B}\right.$$

$$-\frac{1}{2}\sqrt{2}\sqrt{\frac{(A^2-2iAB-B^2)a}{d^2}}\log\left(-\frac{4\left((-iA-B)ae^{(idx+ic)}-\left(-ide^{(2idx+2ic)}-id\right)\sqrt{\frac{(A^2-2iAB-B^2)a}{d^2}}\right.}{iA+B}\right.$$

$$-\frac{1}{2}\sqrt{\frac{A^2a}{d^2}}\log\left(\frac{16\left(3Aa^2e^{(2idx+2ic)}+Aa^2+2\sqrt{2}\left(ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{\frac{A^2a}{d^2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)e^{(-}}{A}\right.$$

$$+\frac{1}{2}\sqrt{\frac{A^2a}{d^2}}\log\left(\frac{16\left(3Aa^2e^{(2idx+2ic)}+Aa^2-2\sqrt{2}\left(ade^{(3idx+3ic)}+ade^{(idx+ic)}\right)\sqrt{\frac{A^2a}{d^2}}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\right)e^{(-}}{A}\right.$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/2*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*a*e^(I*d*
x + I*c) - (I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^
2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 1/2*sq
rt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*
c) - (-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 1/2*sqrt(A^
2*a/d^2)*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2 + 2*sqrt(2)*(a*d*e^(3
*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(A^2*a/d^2)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/A) + 1/2*sqrt(A^2*a/d^2)*log(16*(3*A*
a^2*e^(2*I*d*x + 2*I*c) + A*a^2 - 2*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) + a*d
*e^(I*d*x + I*c))*sqrt(A^2*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2
*I*d*x - 2*I*c)/A)
```

Sympy [F]

$$\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}(A+B\tan\left(c+dx\right))\cot\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*cot(c + d*x), x
)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.31

$$\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= -\frac{\sqrt{2}(A-i\,B)\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)-2\,A\sqrt{a}\log\left(\frac{\sqrt{i\,a\tan(dx+c)+a}-\sqrt{a}}{\sqrt{i\,a\tan(dx+c)+a}+\sqrt{a}}\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/2*(sqrt(2)*(A - I*B)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x +
 c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 2*A*sqrt(a)*lo
g((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqr
t(a))))/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 493, normalized size of antiderivative = 5.73

$$\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\,A\,\sqrt{a}\,\mathrm{atanh}\left(\frac{16\,A^3\,a^{9/2}\,d\,\sqrt{a+a\,\tan(c+d\,x)\,1\mathrm{i}}}{16\,d\,A^3\,a^5+32\mathrm{i}\,d\,A^2\,B\,a^5+16\,d\,A\,B^2\,a^5}+\frac{16\,A\,B^2\,a^{9/2}\,d\,\sqrt{a+a\,\tan(c+d\,x)\,1\mathrm{i}}}{16\,d\,A^3\,a^5+32\mathrm{i}\,d\,A^2\,B\,a^5+16\,d\,A\,B^2\,a^5}+\frac{A^2\,B\,a^{9/2}\,d\,\sqrt{a+a\,\tan(c+d\,x)\cdots}}{16\,d\,A^3\,a^5+32\mathrm{i}\,d\,A^2\,B\,a^5+16\,d\cdots}\cdots\right)}{d}$$

$$+\frac{\sqrt{2}\,\sqrt{-a}\,\mathrm{atan}\left(\frac{4\,\sqrt{2}\,A^3\,{(-a)}^{9/2}\,d\,\sqrt{a+a\,\tan(c+d\,x)\,1\mathrm{i}}}{8\,d\,A^3\,a^5+8\mathrm{i}\,d\,A^2\,B\,a^5+24\,d\,A\,B^2\,a^5-8\mathrm{i}\,d\,B^3\,a^5}-\frac{\sqrt{2}\,B^3\,{(-a)}^{9/2}\,d\,\sqrt{a+a\,\tan(c+d\,x)\,1\mathrm{i}}\,4\mathrm{i}}{8\,d\,A^3\,a^5+8\mathrm{i}\,d\,A^2\,B\,a^5+24\,d\,A\,B^2\,a^5-8\mathrm{i}\,d\,B^3\,a^5}+\frac{12\,\sqrt{2}\,A\,B\cdots}{8\,d\,A^3\,a^5+8\mathrm{i}\cdots}\cdots\right)}{d}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(2^(1/2)*(-a)^(1/2)*atan((4*2^(1/2)*A^3*(-a)^(9/2)*d*(a + a*tan(c + d*x)*1
i)^(1/2))/(8*A^3*a^5*d - B^3*a^5*d*8i + 24*A*B^2*a^5*d + A^2*B*a^5*d*8i) -
 (2^(1/2)*B^3*(-a)^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2)*4i)/(8*A^3*a^5*d
- B^3*a^5*d*8i + 24*A*B^2*a^5*d + A^2*B*a^5*d*8i) + (12*2^(1/2)*A*B^2*(-a)
^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2))/(8*A^3*a^5*d - B^3*a^5*d*8i + 24*A
*B^2*a^5*d + A^2*B*a^5*d*8i) + (2^(1/2)*A^2*B*(-a)^(9/2)*d*(a + a*tan(c +
d*x)*1i)^(1/2)*4i)/(8*A^3*a^5*d - B^3*a^5*d*8i + 24*A*B^2*a^5*d + A^2*B*a^
5*d*8i))*(A*1i + B)*1i)/d - (2*A*a^(1/2)*atanh((16*A^3*a^(9/2)*d*(a + a*ta
n(c + d*x)*1i)^(1/2))/(16*A^3*a^5*d + 16*A*B^2*a^5*d + A^2*B*a^5*d*32i) +
(16*A*B^2*a^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2))/(16*A^3*a^5*d + 16*A*B^
2*a^5*d + A^2*B*a^5*d*32i) + (A^2*B*a^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2
)*32i)/(16*A^3*a^5*d + 16*A*B^2*a^5*d + A^2*B*a^5*d*32i)))/d
```

Reduce [F]

$$\int \cot(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \sqrt{a}\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\,dx\right)a\right)$$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)*tan(c + d*x),x)*b + int
(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x),x)*a)
```

3.72 $\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 123

$$\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{\sqrt{a}(iA+2B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{\sqrt{2}\sqrt{a}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
-a^(1/2)*(I*A+2*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+2^(1/2)*a^(
1/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d-A*cot
(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

$$\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{-i\sqrt{a}(A-2iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt{2}\sqrt{a}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)-A\cot(c+dx)\sqrt{}}{d}$$

input
```
Integrate[Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x
]
```

output
```
((-I)*Sqrt[a]*(A - (2*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]] +
Sqrt[2]*Sqrt[a]*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt
[a])] - A*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.306, Rules used = {3042, 4081, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4081}$$

$$\frac{\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}(a(iA+2B)-aA\tan(c+dx))dx}{a}-\frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}(a(iA+2B)-aA\tan(c+dx))dx}{2a}-\frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+2B)-aA\tan(c+dx))}{\tan(c+dx)}dx}{2a}-\frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4083}$$

$$\frac{(2B+iA)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-2a(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2B+iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-2a(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3961}$$

$$\frac{\frac{4ia^2(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+(2B+iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a}-\frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2B+iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx+\frac{2i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4082}$$

$$\frac{\frac{a^2(2B+iA)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{2i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2ia(2B+iA)\int\frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a}-\frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^{3/2}(2B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}}{2a}-\frac{A\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Int[Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
((-2*a^(3/2)*(I*A + 2*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d +
((2*I)*Sqrt[2]*a^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[
2]*Sqrt[a])])/d)/(2*a) - (A*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

method	result	si
derivativedivides	$\frac{2ia^2\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{3}{2}}}+\frac{\frac{iA\sqrt{a+ia\tan(dx+c)}}{2a\tan(dx+c)}-\frac{(-2iB+A)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a}\right)}{d}$	11
default	$\frac{2ia^2\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{3}{2}}}+\frac{\frac{iA\sqrt{a+ia\tan(dx+c)}}{2a\tan(dx+c)}-\frac{(-2iB+A)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a}\right)}{d}$	11

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d*a^2*(-1/2*(-A+I*B)/a^(3/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1
/2)*2^(1/2)/a^(1/2))+1/a*(1/2*I*A*(a+I*a*tan(d*x+c))^(1/2)/a/tan(d*x+c)-1/
2*(A-2*I*B)/a^(1/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(96) = 192.

Time = 0.10 (sec) , antiderivative size = 649, normalized size of antiderivative = 5.28

$$\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/4*(2*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(A^2 - 2*I*A*B - B^2)*a/
d^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) + (d*e^(2*I*d*x + 2*I*c) + d)*sq
rt(-(A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*
d*x - I*c)/(I*A + B)) - 2*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(A^2 -
 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (d*e^(2*I*d*
x + 2*I*c) + d)*sqrt(-(A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - (d*e^(2*I*d*x + 2*I*c) - d)*sqrt
(-(A^2 - 4*I*A*B - 4*B^2)*a/d^2)*log(-16*(3*(-I*A - 2*B)*a^2*e^(2*I*d*x +
2*I*c) + (-I*A - 2*B)*a^2 + 2*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*
d*x + I*c))*sqrt(-(A^2 - 4*I*A*B - 4*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1)))*e^(-2*I*d*x - 2*I*c)/(I*A + 2*B)) + (d*e^(2*I*d*x + 2*I*c) - d)*
sqrt(-(A^2 - 4*I*A*B - 4*B^2)*a/d^2)*log(-16*(3*(-I*A - 2*B)*a^2*e^(2*I*d*
x + 2*I*c) + (-I*A - 2*B)*a^2 - 2*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) + a*d*e
^(I*d*x + I*c))*sqrt(-(A^2 - 4*I*A*B - 4*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/(I*A + 2*B)) + 4*sqrt(2)*(I*A*e^(3*I*d*
x + 3*I*c) + I*A*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^
(2*I*d*x + 2*I*c) - d)
```

Sympy [F]

$$\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}(A+B\tan\left(c+dx\right))\cot^2\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*cot(c + d*x)**2
, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.18

$$\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$
$$-\frac{i\left(\frac{\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{\sqrt{a}}-\frac{(A-2iB)\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2i\sqrt{ia\tan(dx+c)+a}A}{a\tan(dx+c)}\right)a}{2d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/2*I*(sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) - (A - 2*I*B)*
log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + s
qrt(a)))/sqrt(a) - 2*I*sqrt(I*a*tan(d*x + c) + a)*A/(a*tan(d*x + c)))*a/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.37

$$\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$

$$-\frac{\cot(c+d\,x)\,\left(A\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}+A\,\sqrt{a}\tan(c+d\,x)\,\mathrm{atanh}\left(\frac{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{\sqrt{a}}\right)\,1\mathrm{i}+2\,B\,\sqrt{a}\,\mathrm{tan}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
-(cot(c + d*x)*(A*(a + a*tan(c + d*x)*1i)^(1/2) + A*a^(1/2)*tan(c + d*x)*a
tanh((a + a*tan(c + d*x)*1i)^(1/2)/a^(1/2))*1i + 2*B*a^(1/2)*tan(c + d*x)*
atanh((a + a*tan(c + d*x)*1i)^(1/2)/a^(1/2)) - 2^(1/2)*A*a^(1/2)*atanh((2^
(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2)))*tan(c + d*x)*1i - 2^(1/2
)*B*a^(1/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2)))*tan
(c + d*x)))/d
```

Reduce [F]

$$\int \cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\sqrt{a}\left(\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)b+\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2*tan(c + d*x),x)*b +
int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2,x)*a)
```

3.73 $\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 169

$$\begin{aligned}\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx &= \frac{\sqrt{a}(7A-4iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d} - \frac{\sqrt{2}\sqrt{a}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} \\ &\quad - \frac{(iA+4B)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}\end{aligned}$$

output

```
1/4*a^(1/2)*(7*A-4*I*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d-2^(1/2
)*a^(1/2)*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d-
1/4*(I*A+4*B)*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/2*A*cot(d*x+c)^2*(a+
I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.82

$$\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \frac{\sqrt{a}(7A-4iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)-4\sqrt{2}\sqrt{a}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)-\cot(c+dx)(iA+}{4d}$$

input
```
Integrate[Cot[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x
]
```

output
```
(Sqrt[a]*(7*A - (4*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]] - 4*S
qrt[2]*Sqrt[a]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[
a])] - Cot[c + d*x]*(I*A + 4*B + 2*A*Cot[c + d*x])*Sqrt[a + I*a*Tan[c + d*
x]])/(4*d)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4081, 27, 3042, 4081, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4081}$$

$$\frac{\int \frac{1}{2}\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}(a(iA+4B)-3aA\tan(c+dx))dx}{2a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\,(a(iA+4B)-3aA\tan(c+dx))dx}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+4B)-3aA\tan(c+dx))}{\tan(c+dx)^2}dx}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \; 4081$$

$$\frac{\frac{\int -\frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left((7A-4iB)a^2+(iA+4B)\tan(c+dx)a^2\right)dx}{a}-\frac{a(4B+iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left((7A-4iB)a^2+(iA+4B)\tan(c+dx)a^2\right)dx}{2a}-\frac{a(4B+iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((7A-4iB)a^2+(iA+4B)\tan(c+dx)a^2\right)}{\tan(c+dx)}dx}{2a}-\frac{a(4B+iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \; 4083$$

$$\frac{-\frac{8a^2(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+a(7A-4iB)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{a(4B+iA)\cot(c+dx)\sqrt{a+ia\tan(c+}}{d}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{8a^2(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx+a(7A-4iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a}-\frac{a(4B+iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\downarrow$ 3961

$$\frac{-\frac{a(7A-4iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{16ia^3(B+iA)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a}-\frac{a(4B+iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\downarrow$ 219

$$\frac{-\frac{a(7A-4iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{8i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a(4B+iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\downarrow$ 4082

$$\frac{-\frac{\frac{a^3(7A-4iB)\int\frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{8i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a(4B+iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\downarrow$ 73

$$\frac{-\frac{-\frac{2ia^2(7A-4iB)\int\frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{8i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a(4B+iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\downarrow$ 221

$$\frac{-\frac{-\frac{2a^{5/2}(7A-4iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{8i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a(4B+iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

input
```
Int[Cot[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(A*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/d + (-1/2*((-2*a^(5/2)*
(7*A - (4*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - ((8*I)*Sq
rt[2]*a^(5/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a
])])/d)/a - (a*(I*A + 4*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d)/(4*
a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{2a^3\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{5}{2}}}+\frac{-\frac{\left(-\frac{iB}{2}+\frac{A}{8}\right)(a+ia\tan(dx+c))^{\frac{3}{2}}+\left(\frac{1}{2}iaB+\frac{1}{8}aA\right)\sqrt{a+ia\tan(dx+c)}}{a^2\tan(dx+c)^2}+\frac{(-}{}}{a^2}\right.}{d}$
default	$\frac{2a^3\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{5}{2}}}+\frac{-\frac{\left(-\frac{iB}{2}+\frac{A}{8}\right)(a+ia\tan(dx+c))^{\frac{3}{2}}+\left(\frac{1}{2}iaB+\frac{1}{8}aA\right)\sqrt{a+ia\tan(dx+c)}}{a^2\tan(dx+c)^2}+\frac{(-}{}}{a^2}\right.}{d}$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2/d*a^3*(-1/2*(A-I*B)/a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))+1/a^2*(-((-1/2*I*B+1/8*A)*(a+I*a*tan(d*x+c))^(3/2)+(1/2*
I*a*B+1/8*a*A)*(a+I*a*tan(d*x+c))^(1/2))/a^2/tan(d*x+c)^2+1/8*(7*A-4*I*B)/
a^(1/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(130) = 260$.

Time = 0.11 (sec) , antiderivative size = 730, normalized size of antiderivative = 4.32

$$\int \cot^3(c + dx)\sqrt{a + ia\tan(c + dx)}(A + B\tan(c + dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/16*(8*sqrt(2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqr
t((A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (I*d
*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 8*sqrt(2)*(d*e^(4*I*d
*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt((A^2 - 2*I*A*B - B^2)*a/d^
2)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (-I*d*e^(2*I*d*x + 2*I*c) - I*d)
*sqrt((A^2 - 2*I*A*B - B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-
I*d*x - I*c)/(I*A + B)) + (d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c)
 + d)*sqrt((49*A^2 - 56*I*A*B - 16*B^2)*a/d^2)*log(-16*(3*(-7*I*A - 4*B)*a
^2*e^(2*I*d*x + 2*I*c) + (-7*I*A - 4*B)*a^2 + 2*sqrt(2)*(I*a*d*e^(3*I*d*x
+ 3*I*c) + I*a*d*e^(I*d*x + I*c))*sqrt((49*A^2 - 56*I*A*B - 16*B^2)*a/d^2)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/(7*I*A + 4*B)) -
(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt((49*A^2 - 56*I*
A*B - 16*B^2)*a/d^2)*log(-16*(3*(-7*I*A - 4*B)*a^2*e^(2*I*d*x + 2*I*c) + (
-7*I*A - 4*B)*a^2 + 2*sqrt(2)*(-I*a*d*e^(3*I*d*x + 3*I*c) - I*a*d*e^(I*d*x
 + I*c))*sqrt((49*A^2 - 56*I*A*B - 16*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1)))*e^(-2*I*d*x - 2*I*c)/(7*I*A + 4*B)) - 4*sqrt(2)*((3*A - 4*I*B)*
e^(5*I*d*x + 5*I*c) + 4*A*e^(3*I*d*x + 3*I*c) + (A + 4*I*B)*e^(I*d*x + I*c
))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*
d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}(A+B\tan\left(c+dx\right))\cot^3\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*cot(c + d*x)**3
, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.20

$$\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\left(\frac{4\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}} - \frac{(7A-4iB)\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\left((ia\tan(dx+c)+a)^{\frac{3}{2}}(A-4iB)+\sqrt{ia\,\mathrm{ta}}\right.}{(ia\tan(dx+c)+a)^2a-2\,(ia\tan(d}\right.}{8d}$$

input

```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output

```
1/8*a^2*(4*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2) - (7*A - 4*
I*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a
) + sqrt(a)))/a^(3/2) + 2*((I*a*tan(d*x + c) + a)^(3/2)*(A - 4*I*B) + sqrt
(I*a*tan(d*x + c) + a)*(A + 4*I*B)*a)/((I*a*tan(d*x + c) + a)^2*a - 2*(I*a
*tan(d*x + c) + a)*a^2 + a^3))/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 702, normalized size of antiderivative = 4.15

$$\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(((A*a^2 + B*a^2*4i)*(a + a*tan(c + d*x)*1i)^(1/2))/(4*d) + ((A*a - B*a*4i
)*(a + a*tan(c + d*x)*1i)^(3/2))/(4*d))/((a + a*tan(c + d*x)*1i)^2 - 2*a*(
a + a*tan(c + d*x)*1i) + a^2) - (atan((17*A^3*a^4*d*(-a/2)^(1/2)*(a + a*ta
n(c + d*x)*1i)^(1/2))/(17*A^3*a^5*d - B^3*a^5*d*16i + 24*A*B^2*a^5*d - A^2
*B*a^5*d*9i) - (B^3*a^4*d*(-a/2)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*16i)/
(17*A^3*a^5*d - B^3*a^5*d*16i + 24*A*B^2*a^5*d - A^2*B*a^5*d*9i) + (24*A*B
^2*a^4*d*(-a/2)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(17*A^3*a^5*d - B^3*a
^5*d*16i + 24*A*B^2*a^5*d - A^2*B*a^5*d*9i) - (A^2*B*a^4*d*(-a/2)^(1/2)*(a
 + a*tan(c + d*x)*1i)^(1/2)*9i)/(17*A^3*a^5*d - B^3*a^5*d*16i + 24*A*B^2*a
^5*d - A^2*B*a^5*d*9i))*(A*1i + B)*(-a/2)^(1/2)*2i)/d + ((-a)^(1/2)*atan((
119*A^3*(-a)^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2))/(4*((119*A^3*a^5*d)/4
- B^3*a^5*d*16i + 36*A*B^2*a^5*d - A^2*B*a^5*d*3i)) - (B^3*(-a)^(9/2)*d*(a
 + a*tan(c + d*x)*1i)^(1/2)*16i)/((119*A^3*a^5*d)/4 - B^3*a^5*d*16i + 36*A
*B^2*a^5*d - A^2*B*a^5*d*3i) + (36*A*B^2*(-a)^(9/2)*d*(a + a*tan(c + d*x)*
1i)^(1/2))/((119*A^3*a^5*d)/4 - B^3*a^5*d*16i + 36*A*B^2*a^5*d - A^2*B*a^5
*d*3i) - (A^2*B*(-a)^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2)*3i)/((119*A^3*a
^5*d)/4 - B^3*a^5*d*16i + 36*A*B^2*a^5*d - A^2*B*a^5*d*3i))*(A*7i + 4*B)*1
i)/(4*d)
```

Reduce [F]

$$\int \cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \sqrt{a}\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3*tan(c + d*x),x)*b +
int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3,x)*a)
```

3.74 $\int \cot^4(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 210

$$\int \cot^4(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a}(9iA+14B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{8d} - \frac{\sqrt{2}\sqrt{a}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{(iA+6B)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d} - \frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

output
```
1/8*a^(1/2)*(9*I*A+14*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d-2^(1/
2)*a^(1/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d
+1/8*(7*A-2*I*B)*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/12*(I*A+6*B)*cot(
d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*A*cot(d*x+c)^3*(a+I*a*tan(d*x+c))^
(1/2)/d
```

Mathematica [A] (verified)

Time = 3.02 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.75

$$\int \cot^4(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{-3\sqrt{a}(9iA+14B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)+24\sqrt{2}\sqrt{a}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+\cot(c+a}{24d}$$

input
```
Integrate[Cot[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x
]
```

output
```
-1/24*(-3*Sqrt[a]*((9*I)*A + 14*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt
[a]] + 24*Sqrt[2]*Sqrt[a]*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sq
rt[2]*Sqrt[a])] + Cot[c + d*x]*(-21*A + (6*I)*B + 2*(I*A + 6*B)*Cot[c + d*
x] + 8*A*Cot[c + d*x]^2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4081, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4081}$$

$$\frac{\int \frac{1}{2}\cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}(a(iA+6B)-5aA\tan(c+dx))dx}{3a}-\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\Big\downarrow$ 27

$$\frac{\int \cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}(a(iA+6B)-5aA\tan(c+dx))dx}{6a}-\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+6B)-5aA\tan(c+dx))}{\tan(c+dx)^3}dx}{6a}-\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\Big\downarrow$ 4081

$$\frac{\frac{\int -\frac{3}{2}\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left((7A-2iB)a^2+(iA+6B)\tan(c+dx)a^2\right)dx}{2a}-\frac{a(6B+iA)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}}{6a}-\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\Big\downarrow$ 27

$$\frac{-\frac{3\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left((7A-2iB)a^2+(iA+6B)\tan(c+dx)a^2\right)dx}{4a}-\frac{a(6B+iA)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}}{6a}-\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}\left((7A-2iB)a^2+(iA+6B)\tan(c+dx)a^2\right)}{\tan(c+dx)^2}dx}{4a}-\frac{a(6B+iA)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}}{6a}-\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\Big\downarrow$ 4081

$$\frac{-\frac{3\left(\frac{\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(9iA+14B)-a^3(7A-2iB)\tan(c+dx)\right)dx}{a}-\frac{a^2(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}-\frac{a(6B+iA)\cot^2(c+dx)\sqrt{}}{2d}}{6a}-\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{-\frac{3\left(\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(9iA+14B)-a^3(7A-2iB)\tan(c+dx)\right)dx}{2a}-\frac{a^2(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}-\frac{a(6B+iA)\cot^2(c+dx)\sqrt{a}}{2d}}{6a}$$

$$\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{-\frac{3\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(9iA+14B)-a^3(7A-2iB)\tan(c+dx)\right)}{\tan(c+dx)}dx}{2a}-\frac{a^2(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}-\frac{a(6B+iA)\cot^2(c+dx)\sqrt{a+ia\tan}}{2d}}{6a}$$

$$\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 4083

$$\frac{-\frac{3\left(\frac{a^2(14B+9iA)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-16a^3(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{a^2(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}-a(}{6a}$$

$$\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{-\frac{3\left(\frac{a^2(14B+9iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-16a^3(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{a^2(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}-a(6B+iA}{6a}$$

$$\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 3961

$$\frac{-\frac{3\left(\frac{\frac{32ia^4(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+a^2(14B+9iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a}-\frac{a^2(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right.}{4a}}{6a}$$

$$\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 219

$$-\frac{3\left(\frac{\frac{a^2(14B+9iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx+\frac{16i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a^2(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right.}{4a}}{6a}$$

$$\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 4082

$$-\frac{3\left(\frac{\frac{a^4(14B+9iA)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{16i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a^2(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}-\frac{a}{6a}$$

$$\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 73

$$-\frac{3\left(\frac{\frac{16i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2ia^3(14B+9iA)\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a}-\frac{a^2(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+}}{d}\right.}{4a}}{6a}$$

$$\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 221

$$-\frac{3\left(\frac{\frac{16i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^{7/2}(14B+9iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}}{2a}-\frac{a^2(7A-2iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}}{6a}$$

$$\frac{A\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Int[Cot[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
-1/3*(A*Cot[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (-1/2*(a*(I*A + 6*B
)*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/d - (3*(((-2*a^(7/2)*((9*I)*A
 + 14*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + ((16*I)*Sqrt[2]*
a^(7/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
)/(2*a) - (a^2*(7*A - (2*I)*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d)
)/(4*a))/(6*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{2ia^4\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{7}{2}}}-\frac{i\left(\left(-\frac{iB}{8}+\frac{7A}{16}\right)(a+ia\tan(dx+c))^{\frac{5}{2}}-\frac{5Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{6}+\left(\frac{9}{16}A\right.\right.}{a^3\tan(dx+c)^3}\right.}{d}$
default	$\frac{2ia^4\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{7}{2}}}-\frac{i\left(\left(-\frac{iB}{8}+\frac{7A}{16}\right)(a+ia\tan(dx+c))^{\frac{5}{2}}-\frac{5Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{6}+\left(\frac{9}{16}A\right.\right.}{a^3\tan(dx+c)^3}\right.}{d}$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d*a^4*(-1/2*(A-I*B)/a^(7/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/
2)*2^(1/2)/a^(1/2))-1/a^3*(-I*((-1/8*I*B+7/16*A)*(a+I*a*tan(d*x+c))^(5/2)-
5/6*A*a*(a+I*a*tan(d*x+c))^(3/2)+(9/16*A*a^2+1/8*I*B*a^2)*(a+I*a*tan(d*x+c
))^(1/2))/a^3/tan(d*x+c)^3-1/16*(-14*I*B+9*A)/a^(1/2)*arctanh((a+I*a*tan(d
*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(163) = 326$.

Time = 0.12 (sec) , antiderivative size = 823, normalized size of antiderivative = 3.92

$$\int \cot^4(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/96*(48*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^
(2*I*d*x + 2*I*c) - d)*sqrt(-(A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A -
B)*a*e^(I*d*x + I*c) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(A^2 - 2*I*A*B -
B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B))
 - 48*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*
I*d*x + 2*I*c) - d)*sqrt(-(A^2 - 2*I*A*B - B^2)*a/d^2)*log(-4*((-I*A - B)*
a*e^(I*d*x + I*c) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(A^2 - 2*I*A*B - B^2
)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) -
3*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*
c) - d)*sqrt(-(81*A^2 - 252*I*A*B - 196*B^2)*a/d^2)*log(-16*(3*(-9*I*A - 1
4*B)*a^2*e^(2*I*d*x + 2*I*c) + (-9*I*A - 14*B)*a^2 + 2*sqrt(2)*(a*d*e^(3*I
*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(-(81*A^2 - 252*I*A*B - 196*B^2)*
a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/(9*I*A + 14
*B)) + 3*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x
 + 2*I*c) - d)*sqrt(-(81*A^2 - 252*I*A*B - 196*B^2)*a/d^2)*log(-16*(3*(-9*
I*A - 14*B)*a^2*e^(2*I*d*x + 2*I*c) + (-9*I*A - 14*B)*a^2 - 2*sqrt(2)*(a*d
*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(-(81*A^2 - 252*I*A*B - 19
6*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/(9*I
*A + 14*B)) + 4*sqrt(2)*((31*I*A + 18*B)*e^(7*I*d*x + 7*I*c) + (5*I*A + 6*
B)*e^(5*I*d*x + 5*I*c) + (I*A - 18*B)*e^(3*I*d*x + 3*I*c) - 3*(-9*I*A +...
```

Sympy [F]

$$\int \cot^4(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}(A+B\tan\left(c+dx\right))\cot^4\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*cot(c + d*x)**4
, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.19

$$\int \cot^4(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \frac{i\,a^3\left(\frac{2\left(3\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}(7\,A-2i\,B)-40\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}Aa+3\,\sqrt{i\,a\tan(dx+c)+a}(9\,A+2i\,B)a^2\right)}{(i\,a\tan(dx+c)+a)^3a^2-3\,(i\,a\tan(dx+c)+a)^2a^3+3\,(i\,a\tan(dx+c)+a)a^4-a^5}+\frac{24\,\sqrt{2}(A-i\,B)\log\left(-\frac{\sqrt{2}\sqrt{a}-}{\sqrt{2}\sqrt{a}+}\right.}{a^{\frac{5}{2}}}\right.}{48\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/48*I*a^3*(2*(3*(I*a*tan(d*x + c) + a)^(5/2)*(7*A - 2*I*B) - 40*(I*a*tan(
d*x + c) + a)^(3/2)*A*a + 3*sqrt(I*a*tan(d*x + c) + a)*(9*A + 2*I*B)*a^2)/
((I*a*tan(d*x + c) + a)^3*a^2 - 3*(I*a*tan(d*x + c) + a)^2*a^3 + 3*(I*a*ta
n(d*x + c) + a)*a^4 - a^5) + 24*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) -
sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))
)/a^(5/2) - 3*(9*A - 14*I*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(s
qrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot^4(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.50

$$\int \cot^4(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(a^(1/2)*atan((423*A^3*a^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2))/(8*((A^3*a
^5*d*423i)/8 + 119*B^3*a^5*d + (A*B^2*a^5*d*139i)/2 + (347*A^2*B*a^5*d)/4)
) - (B^3*a^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2)*119i)/((A^3*a^5*d*423i)/8
 + 119*B^3*a^5*d + (A*B^2*a^5*d*139i)/2 + (347*A^2*B*a^5*d)/4) + (139*A*B^
2*a^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2))/(2*((A^3*a^5*d*423i)/8 + 119*B^
3*a^5*d + (A*B^2*a^5*d*139i)/2 + (347*A^2*B*a^5*d)/4)) - (A^2*B*a^(9/2)*d*
(a + a*tan(c + d*x)*1i)^(1/2)*347i)/(4*((A^3*a^5*d*423i)/8 + 119*B^3*a^5*d
 + (A*B^2*a^5*d*139i)/2 + (347*A^2*B*a^5*d)/4)))*(A*9i + 14*B)*1i)/(8*d) -
 (atan((47*32^(1/2)*A^3*a^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2))/(8*(A^3*a
^5*d*47i + 68*B^3*a^5*d + A*B^2*a^5*d*64i + 51*A^2*B*a^5*d)) - (32^(1/2)*B
^3*a^(9/2)*d*(a + a*tan(c + d*x)*1i)^(1/2)*17i)/(2*(A^3*a^5*d*47i + 68*B^3
*a^5*d + A*B^2*a^5*d*64i + 51*A^2*B*a^5*d)) + (8*32^(1/2)*A*B^2*a^(9/2)*d*
(a + a*tan(c + d*x)*1i)^(1/2))/(A^3*a^5*d*47i + 68*B^3*a^5*d + A*B^2*a^5*d
*64i + 51*A^2*B*a^5*d) - (32^(1/2)*A^2*B*a^(9/2)*d*(a + a*tan(c + d*x)*1i)
^(1/2)*51i)/(8*(A^3*a^5*d*47i + 68*B^3*a^5*d + A*B^2*a^5*d*64i + 51*A^2*B*
a^5*d)))*(A*1i + B)*(a/32)^(1/2)*8i)/d - (((9*A*a^3 + B*a^3*2i)*(a + a*tan
(c + d*x)*1i)^(1/2)*1i)/(8*d) + ((7*A*a - B*a*2i)*(a + a*tan(c + d*x)*1i)^
(5/2)*1i)/(8*d) - (A*a^2*(a + a*tan(c + d*x)*1i)^(3/2)*5i)/(3*d))/(3*a*(a
+ a*tan(c + d*x)*1i)^2 - 3*a^2*(a + a*tan(c + d*x)*1i) - (a + a*tan(c + d*
x)*1i)^3 + a^3)
```

Reduce [F]

$$\int \cot^4(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \sqrt{a}\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\cot(dx+c)^4\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\cot(dx+c)^4\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**4*tan(c + d*x),x)*b +
int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**4,x)*a)
```

3.75 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 197

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{2\sqrt{2}a^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{8a(7iA+8B)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{2a(7iA+8B)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d} - \frac{4(21iA+19B)(a+ia\tan(c+dx))^{3/2}}{105d}$$

output
```
2*2^(1/2)*a^(3/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(
1/2))/d-8/35*a*(7*I*A+8*B)*(a+I*a*tan(d*x+c))^(1/2)/d+2/35*a*(7*I*A+8*B)*t
an(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d+2/7*I*a*B*tan(d*x+c)^3*(a+I*a*tan(d
*x+c))^(1/2)/d-4/105*(21*I*A+19*B)*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.68

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\frac{210\sqrt{2}a^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+2a\sqrt{a+ia\tan(c+dx)}(-126iA-13}{105}$$

input
```
Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
(210*Sqrt[2]*a^(3/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]
*Sqrt[a])] + 2*a*Sqrt[a + I*a*Tan[c + d*x]]*((-126*I)*A - 134*B + (42*A -
(38*I)*B)*Tan[c + d*x] + 3*((7*I)*A + 8*B)*Tan[c + d*x]^2 + (15*I)*B*Tan[c
 + d*x]^3))/(105*d)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.361, Rules used = {3042, 4077, 27, 3042, 4080, 25, 3042, 4075, 3042, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^2(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4077}$$

$$\frac{2}{7}\int \frac{1}{2}\tan^2(c+dx)\sqrt{i\tan(c+dx)a+a}(a(7A-6iB)+a(7iA+8B)\tan(c+dx))dx+\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\int \tan^2(c+dx)\sqrt{i\tan(c+dx)a+a}(a(7A-6iB)+a(7iA+8B)\tan(c+dx))dx+\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\int \tan(c+dx)^2\sqrt{i\tan(c+dx)a+a}(a(7A-6iB)+a(7iA+8B)\tan(c+dx))dx+\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \; 4080$$

$$\frac{1}{7}\left(\frac{2\int -\tan(c+dx)\sqrt{i\tan(c+dx)a+a}\left(2a^2(7iA+8B)-a^2(21A-19iB)\tan(c+dx)\right)dx}{5a}+\frac{2a(8B+7iA)\,\mathrm{ta}\right.$$
$$\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{7}\left(\frac{2a(8B+7iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{2\int\tan(c+dx)\sqrt{i\tan(c+dx)a+a}\left(2a^2(7iA+8B)-a^2\right.}{5a}\right.$$
$$\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{2a(8B+7iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{2\int\tan(c+dx)\sqrt{i\tan(c+dx)a+a}\left(2a^2(7iA+8B)-a^2\right.}{5a}\right.$$
$$\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \; 4075$$

$$\frac{1}{7}\left(\frac{2a(8B+7iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{2\left(\int\sqrt{i\tan(c+dx)a+a}\left((21A-19iB)a^2+2(7iA+8B\right.\right.}{5a}\right.$$
$$\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{2a(8B+7iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{2\left(\int\sqrt{i\tan(c+dx)a+a}\left((21A-19iB)a^2+2(7iA+8B\right.\right.}{5a}\right.$$

$$\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$\Big\downarrow$ 4010

$$\frac{1}{7}\left(\frac{2a(8B+7iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{2\left(35a^2(A-iB)\int\sqrt{i\tan(c+dx)a+a}dx+\frac{4a^2(8B+7iA)\sqrt{}}{}\right.}{5a}\right.$$

$$\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{2a(8B+7iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{2\left(35a^2(A-iB)\int\sqrt{i\tan(c+dx)a+a}dx+\frac{4a^2(8B+7iA)\sqrt{}}{}\right.}{5a}\right.$$

$$\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$\Big\downarrow$ 3961

$$\frac{1}{7}\left(\frac{2a(8B+7iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{2\left(-\frac{70ia^3(A-iB)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+\frac{4a^2(8B+7i}{}\right.}{5a}\right.$$

$$\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

$\Big\downarrow$ 219

$$\frac{1}{7}\left(\frac{2a(8B+7iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{2\left(-\frac{35i\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{4a^2(8B+7iA}{}\right.}{5a}\right.$$

$$\frac{2iaB\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

input
```
Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(((2*I)/7)*a*B*Tan[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + ((2*a*((7*I)
*A + 8*B)*Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(5*d) - (2*(((-35*I)*
Sqrt[2]*a^(5/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt
[a])])/d + (4*a^2*((7*I)*A + 8*B)*Sqrt[a + I*a*Tan[c + d*x]])/d + (2*a*((2
1*I)*A + 19*B)*(a + I*a*Tan[c + d*x])^(3/2))/(3*d)))/(5*a))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2i\left(\frac{iB(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{iBa(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{Aa(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{iB\,a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+ia^{3}B\sqrt{a+ia\tan(d\cdots}\right.}{d\,a^{2}}$
default	$\frac{2i\left(\frac{iB(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{iBa(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{Aa(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{iB\,a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+ia^{3}B\sqrt{a+ia\tan(d\cdots}\right.}{d\,a^{2}}$
parts	$\frac{2iA\left(-\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-a^{2}\sqrt{a+ia\tan(dx+c)}+a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{da}+\frac{2B\left(-\frac{(a+ia\tan(dx+c)\cdots}{7}\right.}{\cdots}$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d/a^2*(1/7*I*B*(a+I*a*tan(d*x+c))^(7/2)-1/5*I*B*a*(a+I*a*tan(d*x+c))^(
5/2)-1/5*A*a*(a+I*a*tan(d*x+c))^(5/2)+1/3*I*a^2*B*(a+I*a*tan(d*x+c))^(3/2)
+I*a^3*B*(a+I*a*tan(d*x+c))^(1/2)-A*a^3*(a+I*a*tan(d*x+c))^(1/2)+a^(7/2)*(
A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(152) = 304$.

Time = 0.09 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.37

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{105\sqrt{2}\sqrt{-\frac{(A^2-2i\,AB-B^2)a^3}{d^2}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left((-i\,A-B)a^2e^{(i\,dx+i\,c)}+\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/105*(105*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(6*I*d*x + 6
*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*
A - B)*a^2*e^(I*d*x + I*c) + sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(2*
I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((
-I*A - B)*a)) - 105*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(6*I
*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log
(4*((-I*A - B)*a^2*e^(I*d*x + I*c) - sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*
(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x -
 I*c)/((-I*A - B)*a)) + 2*sqrt(2)*((189*I*A + 211*B)*a*e^(7*I*d*x + 7*I*c)
 + 7*(57*I*A + 53*B)*a*e^(5*I*d*x + 5*I*c) + 35*(9*I*A + 11*B)*a*e^(3*I*d*
x + 3*I*c) + 105*(I*A + B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1)))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x +
 2*I*c) + d)
```

Sympy [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}(A+B\tan(c+dx))\tan^2(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*tan(c + d*x)
**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{i\left(105\sqrt{2}(A-iB)a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)-30i\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}Ba+42\,(i\,a\tan(dx+c)\right.}{105\,a^3d}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/105*I*(105*sqrt(2)*(A - I*B)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*t
an(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 30*I*(
I*a*tan(d*x + c) + a)^(7/2)*B*a + 42*(I*a*tan(d*x + c) + a)^(5/2)*(A + I*B
)*a^2 - 70*I*(I*a*tan(d*x + c) + a)^(3/2)*B*a^3 + 210*sqrt(I*a*tan(d*x + c
) + a)*(A - I*B)*a^4)/(a^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{2\,B\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}{3\,d}-\frac{A\,a\,\sqrt{a+a\tan\left(c+d\,x\right)\ 1\mathrm{i}}\,2\mathrm{i}}{d}-\frac{2\,B\,a\,\sqrt{a+a\tan\left(c+d\,x\right)\ 1\mathrm{i}}}{d}-\frac{A\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}\,2\mathrm{i}}{5\,a\,d}+\frac{2\,B\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}{5\,a\,d}-\frac{2\,B\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{7/2}}{7\,a^2\,d}-\frac{\sqrt{2}\,A\,(-a)^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,2\mathrm{i}}{d}-\frac{\sqrt{2}\,B\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,1\mathrm{i}}{2\,\sqrt{a}}\right)\,2\mathrm{i}}{d}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
(2*B*(a + a*tan(c + d*x)*1i)^(5/2))/(5*a*d) - (A*a*(a + a*tan(c + d*x)*1i)
^(1/2)*2i)/d - (2*B*a*(a + a*tan(c + d*x)*1i)^(1/2))/d - (A*(a + a*tan(c +
 d*x)*1i)^(5/2)*2i)/(5*a*d) - (2*B*(a + a*tan(c + d*x)*1i)^(3/2))/(3*d) -
(2*B*(a + a*tan(c + d*x)*1i)^(7/2))/(7*a^2*d) - (2^(1/2)*A*(-a)^(3/2)*atan
((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*2i)/d - (2^(1/2)*
B*a^(3/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*2i)
/d
```

Reduce [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^4\,dx\right)bi + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^3\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^3\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^2\,dx\right)a\right)$$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4,x)*b*i + int(sqrt(
tan(c + d*x)*i + 1)*tan(c + d*x)**3,x)*a*i + int(sqrt(tan(c + d*x)*i + 1)*
tan(c + d*x)**3,x)*b + int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*a)
```

3.76 $\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 137

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{2\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2a(A-iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2A(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2iB(a+ia\tan(c+dx))^{5/2}}{5ad}$$

output
```
-2*2^(1/2)*a^(3/2)*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^
(1/2))/d+2*a*(A-I*B)*(a+I*a*tan(d*x+c))^(1/2)/d+2/3*A*(a+I*a*tan(d*x+c))^(
3/2)/d-2/5*I*B*(a+I*a*tan(d*x+c))^(5/2)/a/d
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{-30\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+2a\sqrt{a+ia\tan(c+dx)}(20A-18iB}{15d}$$

input
```
Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
(-30*Sqrt[2]*a^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]
*Sqrt[a])] + 2*a*Sqrt[a + I*a*Tan[c + d*x]]*(20*A - (18*I)*B + ((5*I)*A +
6*B)*Tan[c + d*x] + (3*I)*B*Tan[c + d*x]^2))/(15*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {3042, 4075, 3042, 4010, 3042, 3959, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int (i\tan(c+dx)a+a)^{3/2}(A\tan(c+dx)-B)dx-\frac{2iB(a+ia\tan(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$\int (i\tan(c+dx)a+a)^{3/2}(A\tan(c+dx)-B)dx-\frac{2iB(a+ia\tan(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{4010}$$

$$-(B+iA)\int (i\tan(c+dx)a+a)^{3/2}dx+\frac{2A(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2iB(a+ia\tan(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\int (i\tan(c+dx)a+a)^{3/2}dx+\frac{2A(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2iB(a+ia\tan(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3959}$$

$$-(B+iA)\left(2a\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{2A(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2iB(a+ia\tan(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\left(2a\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{2A(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2iB(a+ia\tan(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{3961}$$

$$-(B+iA)\left(\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}-\frac{4ia^2\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}\right)+\frac{2A(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2iB(a+ia\tan(c+dx))^{5/2}}{5ad}$$

$$\Big\downarrow \text{219}$$

$$-(B+iA)\left(\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2i\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\right)+\frac{2A(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2iB(a+ia\tan(c+dx))^{5/2}}{5ad}$$

input
```
Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(2*A*(a + I*a*Tan[c + d*x])^(3/2))/(3*d) - (((2*I)/5)*B*(a + I*a*Tan[c + d
*x])^(5/2))/(a*d) - (I*A + B)*(((-2*I)*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + I*
a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d + ((2*I)*a*Sqrt[a + I*a*Tan[c + d*x]
])/d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{2iB(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2ia^2B\sqrt{a+ia\tan(dx+c)}+2A\,a^2\sqrt{a+ia\tan(dx+c)}-2a^{\frac{5}{2}}(-iB+A)\sqrt{}}{ad}$
default	$\frac{-\frac{2iB(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2ia^2B\sqrt{a+ia\tan(dx+c)}+2A\,a^2\sqrt{a+ia\tan(dx+c)}-2a^{\frac{5}{2}}(-iB+A)\sqrt{}}{ad}$
parts	$\frac{A\left(\frac{2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+ia\tan(dx+c)}-2a^{\frac{3}{2}}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}+\frac{2iB\left(-\frac{(a+ia\tan(dx+c))}{5}\right.}{}$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
2/d/a*(-1/5*I*B*(a+I*a*tan(d*x+c))^(5/2)+1/3*A*a*(a+I*a*tan(d*x+c))^(3/2)-
I*B*a^2*(a+I*a*tan(d*x+c))^(1/2)+A*a^2*(a+I*a*tan(d*x+c))^(1/2)-a^(5/2)*(A
-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(104) = 208.

Time = 0.09 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.03

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{15\sqrt{2}\sqrt{\frac{(A^2-2i\,AB-B^2)a^3}{d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left((-i\,A-B)a^2e^{(i\,dx+i\,c)}-\sqrt{\frac{(A^2-2i\,AB-B^2)a^3}{d^2}}(i\,de^{(2i\,}\right.}{(-i\,A-B)a}\right.}{}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/15*(15*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*
c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^2*e^(I*d*x + I*c) -
sqrt((A^2 - 2*I*A*B - B^2)*a^3/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - 15*sqrt(2)
*sqrt((A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d
*x + 2*I*c) + d)*log(4*((-I*A - B)*a^2*e^(I*d*x + I*c) - sqrt((A^2 - 2*I*A
*B - B^2)*a^3/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - 2*sqrt(2)*((25*A - 27*I*B)
*a*e^(5*I*d*x + 5*I*c) + 10*(4*A - 3*I*B)*a*e^(3*I*d*x + 3*I*c) + 15*(A -
I*B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x +
 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}(A+B\tan(c+dx))\tan(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*tan(c + d*x)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.95

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{15\sqrt{2}(A-iB)a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-6i\,(ia\tan(dx+c)+a)^{\frac{5}{2}}Ba+10\,(\cdots}{15\,a^2d}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/15*(15*sqrt(2)*(A - I*B)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*
x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 6*I*(I*a*ta
n(d*x + c) + a)^(5/2)*B*a + 10*(I*a*tan(d*x + c) + a)^(3/2)*A*a^2 + 30*sqr
t(I*a*tan(d*x + c) + a)*(A - I*B)*a^3)/(a^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{2\,A\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{3\,d} + \frac{2\,A\,a\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{d} - \frac{B\,a\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,2{}\mathrm{i}}{d} - \frac{B\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,2{}\mathrm{i}}{5\,a\,d} - \frac{\sqrt{2}\,B\,{\left(-a\right)}^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-a}}\right)\,2{}\mathrm{i}}{d} - \frac{2\,\sqrt{2}\,A\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{a}}\right)}{d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
(2*A*(a + a*tan(c + d*x)*1i)^(3/2))/(3*d) + (2*A*a*(a + a*tan(c + d*x)*1i)
^(1/2))/d - (B*a*(a + a*tan(c + d*x)*1i)^(1/2)*2i)/d - (B*(a + a*tan(c + d
*x)*1i)^(5/2)*2i)/(5*a*d) - (2^(1/2)*B*(-a)^(3/2)*atan((2^(1/2)*(a + a*tan
(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*2i)/d - (2*2^(1/2)*A*a^(3/2)*atanh((2
^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/d
```

Reduce [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a\Bigg(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^3\,dx\right)bi + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^2\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^2\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)\,dx\right)a\Bigg)$$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3,x)*b*i + int(sqrt(
tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*a*i + int(sqrt(tan(c + d*x)*i + 1)*
tan(c + d*x)**2,x)*b + int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a)
```

3.77 $\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{2\sqrt{2}a^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2a(iA+B)\sqrt{a+ia\tan(c+dx)}}{d} + \frac{2B(a+ia\tan(c+dx))^{3/2}}{3d}$$

output
```
-2*2^(1/2)*a^(3/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^
(1/2))/d+2*a*(I*A+B)*(a+I*a*tan(d*x+c))^(1/2)/d+2/3*B*(a+I*a*tan(d*x+c))^(
3/2)/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{-6i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right) + 2a\sqrt{a+ia\tan(c+dx)}(3iA+4B+}{3d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((-6*I)*Sqrt[2]*a^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt
[2]*Sqrt[a])] + 2*a*Sqrt[a + I*a*Tan[c + d*x]]*((3*I)*A + 4*B + I*B*Tan[c
+ d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4010, 3042, 3959, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4010}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^{3/2}dx+\frac{2B(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^{3/2}dx+\frac{2B(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{ 3959}$$

$$(A-iB)\left(2a\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{2B(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$(A-iB)\left(2a\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{2B(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3961

$$(A-iB)\left(\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}-\frac{4ia^2\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}\right)+\frac{2B(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 219

$$(A-iB)\left(\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2i\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\right)+\frac{2B(a+ia\tan(c+dx))^{3/2}}{3d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*(a + I*a*Tan[c + d*x])^(3/2))/(3*d) + (A - I*B)*(((-2*I)*Sqrt[2]*a^(3
/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d + ((2*I)*a*Sq
rt[a + I*a*Tan[c + d*x]])/d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2i\left(-\frac{iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-iaB\sqrt{a+ia\tan(dx+c)}+aA\sqrt{a+ia\tan(dx+c)}-a^{\frac{3}{2}}(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+}}{2\sqrt{a}}\right.\right.}{d}$
default	$\frac{2i\left(-\frac{iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-iaB\sqrt{a+ia\tan(dx+c)}+aA\sqrt{a+ia\tan(dx+c)}-a^{\frac{3}{2}}(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+}}{2\sqrt{a}}\right.\right.}{d}$
parts	$\frac{2iAa\left(\sqrt{a+ia\tan(dx+c)}-\sqrt{a}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}+\frac{B\left(\frac{2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+ia\tan(dx+c}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/d*(-1/3*I*B*(a+I*a*tan(d*x+c))^(3/2)-I*a*B*(a+I*a*tan(d*x+c))^(1/2)+a*
A*(a+I*a*tan(d*x+c))^(1/2)-a^(3/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(
d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(82) = 164$.

Time = 0.11 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.36

$$\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{3\sqrt{2}\sqrt{-\frac{(A^2-2i\,AB-B^2)a^3}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left((-i\,A-B)a^2e^{(i\,dx+i\,c)}+\sqrt{-\frac{(A^2-2i\,AB-B^2)a^3}{d^2}}\cdots\right.}{(-i\,A-\cdots}\right.\cdots}{\cdots}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(3*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c)
 + d)*log(4*((-I*A - B)*a^2*e^(I*d*x + I*c) + sqrt(-(A^2 - 2*I*A*B - B^2)*
a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^
(-I*d*x - I*c)/((-I*A - B)*a)) - 3*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^3
/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^2*e^(I*d*x + I*c) -
sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - 2*sqrt(2)*((-
3*I*A - 5*B)*a*e^(3*I*d*x + 3*I*c) + 3*(-I*A - B)*a*e^(I*d*x + I*c))*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \left(ia(\tan(c+dx)-i)\right)^{\frac{3}{2}}\left(A+B\tan(c+dx)\right)\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

$$\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\frac{i\left(3\sqrt{2}(A-iB)a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-2i\,(ia\tan(dx+c)+a)^{\frac{3}{2}}Ba+6\cdots}{3\,ad}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*I*(3*sqrt(2)*(A - I*B)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*
x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 2*I*(I*a*ta
n(d*x + c) + a)^(3/2)*B*a + 6*sqrt(I*a*tan(d*x + c) + a)*(A - I*B)*a^2)/(a
*d)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.30

$$\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{2\,B\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}{3\,d} + \frac{A\,a\,\sqrt{a+a\tan\left(c+d\,x\right)\ 1\mathrm{i}}\,2\mathrm{i}}{d} + \frac{2\,B\,a\,\sqrt{a+a\tan\left(c+d\,x\right)\ 1\mathrm{i}}}{d} + \frac{\sqrt{2}\,A\,(-a)^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,2\mathrm{i}}{d} - \frac{2\,\sqrt{2}\,B\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{d}$$

input
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
(2*B*(a + a*tan(c + d*x)*1i)^(3/2))/(3*d) + (A*a*(a + a*tan(c + d*x)*1i)^(
1/2)*2i)/d + (2*B*a*(a + a*tan(c + d*x)*1i)^(1/2))/d + (2^(1/2)*A*(-a)^(3/
2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*2i)/d - (2
*2^(1/2)*B*a^(3/2)*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2
))))/d
```

Reduce [F]

$$\int (a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a}\,a\left(-2\sqrt{\tan\left(dx+c\right)i+1}\,ai+\left(\int\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)^{2}dx\right)bdi+2\left(\right.$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
(sqrt(a)*a*( - 2*sqrt(tan(c + d*x)*i + 1)*a*i + int(sqrt(tan(c + d*x)*i +
1)*tan(c + d*x)**2,x)*b*d*i + 2*int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),
x)*a*d*i + int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*b*d))/d
```

3.78 $\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 113

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{2a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{2}a^{3/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

output

```
-2*a^(3/2)*A*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+2*2^(1/2)*a^(3/2)
*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d+2*I*a*B*(
a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{-2a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)+2\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+2ia}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
(-2*a^(3/2)*A*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]] + 2*Sqrt[2]*a^(3
/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])] + (2*I
)*a*B*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4077, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 4077}$$

$$2\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}(aA+a(iA+2B)\tan(c+dx))dx+\frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}(aA+a(iA+2B)\tan(c+dx))dx+\frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{i\tan(c+dx)a+a}(aA+a(iA+2B)\tan(c+dx))}{\tan(c+dx)}dx+\frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4083}$$

$$2a(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+A\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx+\frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$2a(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx+\frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3961}$$

$$-\frac{4ia^2(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx+\frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{219}$$

$$A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2i\sqrt{2}a^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4082}$$

$$\frac{a^2A\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{2i\sqrt{2}a^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2iaA\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{2i\sqrt{2}a^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{2i\sqrt{2}a^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2iaB\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a^(3/2)*A*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - ((2*I)*Sqrt
[2]*a^(3/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/d + ((2*I)*a*B*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{2a\left(iB\sqrt{a+ia\tan(dx+c)}+\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-\sqrt{a}\,A\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{\sqrt{a}}\right)\right)}{d}$
default	$\frac{2a\left(iB\sqrt{a+ia\tan(dx+c)}+\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-\sqrt{a}\,A\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{\sqrt{a}}\right)\right)}{d}$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
2/d*a*(I*B*(a+I*a*tan(d*x+c))^(1/2)+a^(1/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a
+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-a^(1/2)*A*arctanh((a+I*a*tan(d*x+c
))^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(86) = 172.

Time = 0.10 (sec) , antiderivative size = 514, normalized size of antiderivative = 4.55

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{-4i\sqrt{2}Ba\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}-2\sqrt{2}\sqrt{\frac{(A^2-2i\,AB-B^2)a^3}{d^2}}d\log\left(\frac{4\left((-i\,A-B)a^2e^{(i\,dx+i\,c)}-\sqrt{\frac{(A^2-2i\,AB-B^2)a^3}{d^2}}(i\,de\right.}{(-i\,A-B}\right.}{}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/2*(-4*I*sqrt(2)*B*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) -
 2*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a^3/d^2)*d*log(4*((-I*A - B)*a^2*e^(
I*d*x + I*c) - sqrt((A^2 - 2*I*A*B - B^2)*a^3/d^2)*(I*d*e^(2*I*d*x + 2*I*c
) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a
)) + 2*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a^3/d^2)*d*log(4*((-I*A - B)*a^2
*e^(I*d*x + I*c) - sqrt((A^2 - 2*I*A*B - B^2)*a^3/d^2)*(-I*d*e^(2*I*d*x +
2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A -
 B)*a)) + sqrt(A^2*a^3/d^2)*d*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2
+ 2*sqrt(2)*sqrt(A^2*a^3/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/A) - sqrt(A^2*a^3/
d^2)*d*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2 - 2*sqrt(2)*sqrt(A^2*a^
3/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/A))/d
```

Sympy [F]

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}(A+B\tan(c+dx))\cot(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*cot(c + d*x)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{\sqrt{2}(A-iB)a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right) - Aa^{\frac{3}{2}}\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right) - 2i\sqrt{ia\tan(dx+c)+a}\,aB}{d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-(sqrt(2)*(A - I*B)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - A*a^(3/2)*log((sqr
t(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))
 - 2*I*sqrt(I*a*tan(d*x + c) + a)*B*a)/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot(c+dx)(a + ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 553, normalized size of antiderivative = 4.89

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{B\,a\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,2\mathrm{i}}{d} - \frac{2\,A\,\mathrm{atanh}\left(-\frac{32\,A^3\,a^6\,d\,\sqrt{a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{-32\,d\,A^3\,a^8+128\mathrm{i}\,d\,A^2\,B\,a^8+64\,d\,A\,B^2\,a^8}+\frac{64\,A\,B^2\,a^6\,d\,\sqrt{a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{-32\,d\,A^3\,a^8+128\mathrm{i}\,d\,A^2\,B\,a^8+64\,d\,A\,B^2\,a^8}+\frac{A^2\,B\,a^6\,d\,\sqrt{a^3}\,\sqrt{a+a\tan(c+d}{-32\,d\,A^3\,a^8+128\mathrm{i}\,d\,A^2\,B\,a^8+64}\right.}{d} + \frac{2\,\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,A^3\,a^6\,d\,\sqrt{-a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,16\mathrm{i}}{32\,d\,A^3\,a^8-160\mathrm{i}\,d\,A^2\,B\,a^8-192\,d\,A\,B^2\,a^8+64\mathrm{i}\,d\,B^3\,a^8}-\frac{32\,\sqrt{2}\,B^3\,a^6\,d\,\sqrt{-a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{32\,d\,A^3\,a^8-160\mathrm{i}\,d\,A^2\,B\,a^8-192\,d\,A\,B^2\,a^8+64\mathrm{i}\,d\,B^3\,a^8}-\frac{\sqrt{2}}{32\,d\,A^3\,a}\right.}{d}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
(B*a*(a + a*tan(c + d*x)*1i)^(1/2)*2i)/d - (2*A*atanh((64*A*B^2*a^6*d*(a^3
)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(64*A*B^2*a^8*d - 32*A^3*a^8*d + A^
2*B*a^8*d*128i) - (32*A^3*a^6*d*(a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))
/(64*A*B^2*a^8*d - 32*A^3*a^8*d + A^2*B*a^8*d*128i) + (A^2*B*a^6*d*(a^3)^(
1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*128i)/(64*A*B^2*a^8*d - 32*A^3*a^8*d +
A^2*B*a^8*d*128i))*(a^3)^(1/2))/d + (2*2^(1/2)*atanh((2^(1/2)*A^3*a^6*d*(-
a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*16i)/(32*A^3*a^8*d + B^3*a^8*d*64
i - 192*A*B^2*a^8*d - A^2*B*a^8*d*160i) - (32*2^(1/2)*B^3*a^6*d*(-a^3)^(1/
2)*(a + a*tan(c + d*x)*1i)^(1/2))/(32*A^3*a^8*d + B^3*a^8*d*64i - 192*A*B^
2*a^8*d - A^2*B*a^8*d*160i) - (2^(1/2)*A*B^2*a^6*d*(-a^3)^(1/2)*(a + a*tan
(c + d*x)*1i)^(1/2)*96i)/(32*A^3*a^8*d + B^3*a^8*d*64i - 192*A*B^2*a^8*d -
 A^2*B*a^8*d*160i) + (80*2^(1/2)*A^2*B*a^6*d*(-a^3)^(1/2)*(a + a*tan(c + d
*x)*1i)^(1/2))/(32*A^3*a^8*d + B^3*a^8*d*64i - 192*A*B^2*a^8*d - A^2*B*a^8
*d*160i))*(A*1i + B)*(-a^3)^(1/2))/d
```

Reduce [F]

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\tan(dx+c)^2\,dx\right)bi + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\,dx\right)a\right)$$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)*tan(c + d*x)**2,x)*b*
i + int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)*tan(c + d*x),x)*a*i + int(sq
rt(tan(c + d*x)*i + 1)*cot(c + d*x)*tan(c + d*x),x)*b + int(sqrt(tan(c + d
*x)*i + 1)*cot(c + d*x),x)*a)
```

3.79 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 125

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{a^{3/2}(3iA+2B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{2}a^{3/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
-a^(3/2)*(3*I*A+2*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+2*2^(1/2)
*a^(3/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d-a
*A*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{-ia^{3/2}(3A-2iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)+2\sqrt{2}a^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+d}}{\sqrt{2}\sqrt{a}}\right.}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
((-I)*a^(3/2)*(3*A - (2*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]]
+ 2*Sqrt[2]*a^(3/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*
Sqrt[a])] - a*A*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 4076, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4076}$$

$$\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}(a(3iA+2B)-a(A-2iB)\tan(c+dx))dx - \frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}(a(3iA+2B)-a(A-2iB)\tan(c+dx))dx-\frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(3iA+2B)-a(A-2iB)\tan(c+dx))}{\tan(c+dx)}dx-\frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 4083$$

$$\frac{1}{2}\left((2B+3iA)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-4a(A-iB)\int \sqrt{i\tan(c+dx)a+a}d\right.$$
$$\frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\left((2B+3iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-4a(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx\right)-\frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 3961$$

$$\frac{1}{2}\left(\frac{8ia^2(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+(2B+3iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+}}{\tan(c+dx)}\right.$$
$$\frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left((2B+3iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx+\frac{4i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\right)-$$
$$\frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 4082$$

$$\frac{1}{2}\left(\frac{a^2(2B+3iA)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{4i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\right)-\frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}\left(\frac{4i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2ia(2B+3iA)\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}\right)-\frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{2}\left(\frac{4i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^{3/2}(2B+3iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}\right)-\frac{aA\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((-2*a^(3/2)*((3*I)*A + 2*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/
d + ((4*I)*Sqrt[2]*a^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(S
qrt[2]*Sqrt[a])])/d)/2 - (a*A*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2ia^2\left(-\frac{(2iB-2A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}+\frac{iA\sqrt{a+ia\tan(dx+c)}}{2a\tan(dx+c)}-\frac{(-2iB+3A)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{d}$
default	$\frac{2ia^2\left(-\frac{(2iB-2A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}+\frac{iA\sqrt{a+ia\tan(dx+c)}}{2a\tan(dx+c)}-\frac{(-2iB+3A)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{d}$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d*a^2*(-1/2*(2*I*B-2*A)*2^(1/2)/a^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))
^(1/2)*2^(1/2)/a^(1/2))+1/2*I*A*(a+I*a*tan(d*x+c))^(1/2)/a/tan(d*x+c)-1/2*
(-2*I*B+3*A)/a^(1/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(98) = 196$.

Time = 0.09 (sec) , antiderivative size = 685, normalized size of antiderivative = 5.48

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/4*(4*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c
) - d)*log(4*((-I*A - B)*a^2*e^(I*d*x + I*c) + sqrt(-(A^2 - 2*I*A*B - B^2)
*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e
^(-I*d*x - I*c)/((-I*A - B)*a)) - 4*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^
3/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(4*((-I*A - B)*a^2*e^(I*d*x + I*c) -
 sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - sqrt(-(9*A^2
 - 12*I*A*B - 4*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(-16*(3*(-3*I
*A - 2*B)*a^2*e^(2*I*d*x + 2*I*c) + (-3*I*A - 2*B)*a^2 + 2*sqrt(2)*sqrt(-(
9*A^2 - 12*I*A*B - 4*B^2)*a^3/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I
*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/(3*I*A + 2*B)
) + sqrt(-(9*A^2 - 12*I*A*B - 4*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*
log(-16*(3*(-3*I*A - 2*B)*a^2*e^(2*I*d*x + 2*I*c) + (-3*I*A - 2*B)*a^2 - 2
*sqrt(2)*sqrt(-(9*A^2 - 12*I*A*B - 4*B^2)*a^3/d^2)*(d*e^(3*I*d*x + 3*I*c)
+ d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*
c)/(3*I*A + 2*B)) + 4*sqrt(2)*(I*A*a*e^(3*I*d*x + 3*I*c) + I*A*a*e^(I*d*x
+ I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F]

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}(A+B\tan(c+dx))\cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*cot(c + d*x)
**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx =$$
$$-\frac{i\left(2\sqrt{2}(A-iB)\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-(3A-2iB)\sqrt{a}\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)-\frac{2i\sqrt{ia\tan(}{\tan(d}}{2d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/2*I*(2*sqrt(2)*(A - I*B)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d
*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - (3*A - 2*I
*B)*sqrt(a)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x +
 c) + a) + sqrt(a))) - 2*I*sqrt(I*a*tan(d*x + c) + a)*A/tan(d*x + c))*a/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 2338, normalized size of antiderivative = 18.70

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
- 2*atanh((6*d^4*(a + a*tan(c + d*x)*1i)^(1/2)*((3*B^2*a^3)/(2*d^2) - (17*
A^2*a^3)/(8*d^2) - ((A^4*a^18)/d^4 + (16*B^4*a^18)/d^4 - (8*A^2*B^2*a^18)/
d^4 + (A*B^3*a^18*32i)/d^4 + (A^3*B*a^18*8i)/d^4)^(1/2)/(8*a^6) + (A*B*a^3
*7i)/(2*d^2))^(1/2)*((A^4*a^18)/d^4 + (16*B^4*a^18)/d^4 - (8*A^2*B^2*a^18)
/d^4 + (A*B^3*a^18*32i)/d^4 + (A^3*B*a^18*8i)/d^4)^(1/2))/(A^3*a^11*d*10i
+ 32*B^3*a^11*d + A*B^2*a^11*d*72i - 32*A^2*B*a^11*d + A*a^2*d^3*((A^4*a^1
8)/d^4 + (16*B^4*a^18)/d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*32i)/d^4 +
 (A^3*B*a^18*8i)/d^4)^(1/2)*2i) + (2*A^2*a^6*d^2*(a + a*tan(c + d*x)*1i)^(
1/2)*((3*B^2*a^3)/(2*d^2) - (17*A^2*a^3)/(8*d^2) - ((A^4*a^18)/d^4 + (16*B
^4*a^18)/d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*32i)/d^4 + (A^3*B*a^18*8
i)/d^4)^(1/2)/(8*a^6) + (A*B*a^3*7i)/(2*d^2))^(1/2))/(A^3*a^8*d*10i + 32*B
^3*a^8*d + A*B^2*a^8*d*72i - 32*A^2*B*a^8*d + (A*d^3*((A^4*a^18)/d^4 + (16
*B^4*a^18)/d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*32i)/d^4 + (A^3*B*a^18
*8i)/d^4)^(1/2)*2i)/a) + (8*B^2*a^6*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((3*
B^2*a^3)/(2*d^2) - (17*A^2*a^3)/(8*d^2) - ((A^4*a^18)/d^4 + (16*B^4*a^18)/
d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*32i)/d^4 + (A^3*B*a^18*8i)/d^4)^(
1/2)/(8*a^6) + (A*B*a^3*7i)/(2*d^2))^(1/2))/(A^3*a^8*d*10i + 32*B^3*a^8*d
+ A*B^2*a^8*d*72i - 32*A^2*B*a^8*d + (A*d^3*((A^4*a^18)/d^4 + (16*B^4*a^18
)/d^4 - (8*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*32i)/d^4 + (A^3*B*a^18*8i)/d^4)
^(1/2)*2i)/a) + (A*B*a^6*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((3*B^2*a^3)...
```

Reduce [F]

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a\left(\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)bi+\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)ai+\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)b+\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2*tan(c + d*x)**2,x)
*b*i + int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2*tan(c + d*x),x)*a*i +
int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2*tan(c + d*x),x)*b + int(sqrt(
tan(c + d*x)*i + 1)*cot(c + d*x)**2,x)*a)
```

3.80 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 171

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\frac{a^{3/2}(11A-12iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d}-\frac{2\sqrt{2}a^{3/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{a(5iA+4B)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}-\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

output
```
1/4*a^(3/2)*(11*A-12*I*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d-2*2^
(1/2)*a^(3/2)*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)/d-1/4*a*(5*I*A+4*B)*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/2*a*A*cot(d*
x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.81

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{a^{3/2}(11A-12iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)-8\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right. \cdots}{4d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
(a^(3/2)*(11*A - (12*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]] - 8
*Sqrt[2]*a^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqr
t[a])] - a*Cot[c + d*x]*((5*I)*A + 4*B + 2*A*Cot[c + d*x])*Sqrt[a + I*a*Ta
n[c + d*x]])/(4*d)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4076, 27, 3042, 4081, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{2}\int \frac{1}{2}\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}(a(5iA+4B)-a(3A-4iB)\tan(c+dx))dx - \frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}(a(5iA+4B)-a(3A-4iB)\tan(c+dx))dx-\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(5iA+4B)-a(3A-4iB)\tan(c+dx))}{\tan(c+dx)^2}dx-\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{4081}$$

$$\frac{1}{4}\left(\frac{\int -\frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left((11A-12iB)a^2+(5iA+4B)\tan(c+dx)a^2\right)dx}{a}-\frac{a(4B+5iA)\cot(\right.$$
$$\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left((11A-12iB)a^2+(5iA+4B)\tan(c+dx)a^2\right)dx}{2a}-\frac{a(4B+5iA)\cot(c\right.$$
$$\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((11A-12iB)a^2+(5iA+4B)\tan(c+dx)a^2\right)}{\tan(c+dx)}dx}{2a}-\frac{a(4B+5iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)-$$
$$\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{4083}$$

$$\frac{1}{4}\left(-\frac{16a^2(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+a(11A-12iB)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)}}{2a}\right.$$
$$\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(-\frac{16a^2(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx + a(11A-12iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a} - \frac{a(4B+5iA\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\big\downarrow$ 3961

$$\frac{1}{4}\left(-\frac{a(11A-12iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx - \frac{32ia^3(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a} - \frac{a(4B+5iA\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\big\downarrow$ 219

$$\frac{1}{4}\left(-\frac{a(11A-12iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx - \frac{16i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a} - \frac{a(4B+5iA)\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\big\downarrow$ 4082

$$\frac{1}{4}\left(-\frac{\frac{a^3(11A-12iB)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d} - \frac{16i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a} - \frac{a(4B+5iA)\cot(c+dx)}{d}\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\big\downarrow$ 73

$$\frac{1}{4}\left(-\frac{-\frac{2ia^2(11A-12iB)\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d} - \frac{16i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a} - \frac{a(4B+5iA)\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\big\downarrow$ 221

$$\frac{1}{4}\left(-\frac{-\frac{2a^{5/2}(11A-12iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{16i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a(4B+5iA)\cot(c+\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(a*A*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/d + (-1/2*((-2*a^(5/2
)*(11*A - (12*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - ((16*
I)*Sqrt[2]*a^(5/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*S
qrt[a])])/d)/a - (a*((5*I)*A + 4*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]
])/d)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{2a^{3}\left(-\frac{(-2iB+2A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{3}{2}}}+\frac{-\frac{\left(-\frac{iB}{2}+\frac{5A}{8}\right)(a+ia\tan(dx+c))^{\frac{3}{2}}+\left(\frac{1}{2}iaB-\frac{3}{8}aA\right)\sqrt{a+ia\tan(dx+c)}}{a^{2}\tan(dx+c)^{2}}}{a}\right.}{d}$
default	$\frac{2a^{3}\left(-\frac{(-2iB+2A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{3}{2}}}+\frac{-\frac{\left(-\frac{iB}{2}+\frac{5A}{8}\right)(a+ia\tan(dx+c))^{\frac{3}{2}}+\left(\frac{1}{2}iaB-\frac{3}{8}aA\right)\sqrt{a+ia\tan(dx+c)}}{a^{2}\tan(dx+c)^{2}}}{a}\right.}{d}$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2/d*a^3*(-1/2*(-2*I*B+2*A)/a^(3/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^
(1/2)*2^(1/2)/a^(1/2))+1/a*(-((-1/2*I*B+5/8*A)*(a+I*a*tan(d*x+c))^(3/2)+(1
/2*I*a*B-3/8*a*A)*(a+I*a*tan(d*x+c))^(1/2))/a^2/tan(d*x+c)^2+1/8*(11*A-12*
I*B)/a^(1/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(132) = 264$.

Time = 0.11 (sec) , antiderivative size = 762, normalized size of antiderivative = 4.46

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/16*(16*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*
c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^2*e^(I*d*x + I*c) -
sqrt((A^2 - 2*I*A*B - B^2)*a^3/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - 16*sqrt(2)
*sqrt((A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d
*x + 2*I*c) + d)*log(4*((-I*A - B)*a^2*e^(I*d*x + I*c) - sqrt((A^2 - 2*I*A
*B - B^2)*a^3/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) + sqrt((121*A^2 - 264*I*A*B
- 144*B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*
log(-16*(3*(-11*I*A - 12*B)*a^2*e^(2*I*d*x + 2*I*c) + (-11*I*A - 12*B)*a^2
 + 2*sqrt(2)*sqrt((121*A^2 - 264*I*A*B - 144*B^2)*a^3/d^2)*(I*d*e^(3*I*d*x
 + 3*I*c) + I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*
I*d*x - 2*I*c)/(11*I*A + 12*B)) - sqrt((121*A^2 - 264*I*A*B - 144*B^2)*a^3
/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-16*(3*(-1
1*I*A - 12*B)*a^2*e^(2*I*d*x + 2*I*c) + (-11*I*A - 12*B)*a^2 + 2*sqrt(2)*s
qrt((121*A^2 - 264*I*A*B - 144*B^2)*a^3/d^2)*(-I*d*e^(3*I*d*x + 3*I*c) - I
*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c
)/(11*I*A + 12*B)) - 4*sqrt(2)*((7*A - 4*I*B)*a*e^(5*I*d*x + 5*I*c) + 4*A*
a*e^(3*I*d*x + 3*I*c) - (3*A - 4*I*B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\,(A+B\tan{(c+dx)})\cot^3{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*cot(c + d*x)
**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.19

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\frac{\left(\frac{8\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{\sqrt{a}}-\frac{(11A-12iB)\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\left((ia\tan(dx+c\right.}{(ia\tan}\right.}{8d}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/8*(8*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) - (11*A - 12*I*
B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a)
+ sqrt(a)))/sqrt(a) + 2*((I*a*tan(d*x + c) + a)^(3/2)*(5*A - 4*I*B) - sqrt
(I*a*tan(d*x + c) + a)*(3*A - 4*I*B)*a)/((I*a*tan(d*x + c) + a)^2 - 2*(I*a
*tan(d*x + c) + a)*a + a^2))*a^2/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 3027, normalized size of antiderivative = 17.70

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
2*atanh((3*d^4*(a + a*tan(c + d*x)*1i)^(1/2)*((249*A^2*a^3)/(128*d^2) - ((
49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*
a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^(1/2)/(64*a^6) - (17*B^2*a^3)/(8*d^2
) - (A*B*a^3*65i)/(16*d^2))^(1/2)*((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d
^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^
(1/2))/(2*((133*A^3*a^11*d)/16 - B^3*a^11*d*20i + 29*A*B^2*a^11*d + (A^2*B
*a^11*d*3i)/4 + (3*A*a^2*d^3*((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 +
(40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^(1/2)
)/8 - (B*a^2*d^3*((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + (40*A^2*B^2*
a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^(1/2)*1i)/2)) + (
7*A^2*a^6*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((249*A^2*a^3)/(128*d^2) - ((4
9*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a
^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4)^(1/2)/(64*a^6) - (17*B^2*a^3)/(8*d^2)
 - (A*B*a^3*65i)/(16*d^2))^(1/2))/(4*((133*A^3*a^8*d)/16 - B^3*a^8*d*20i +
 29*A*B^2*a^8*d + (A^2*B*a^8*d*3i)/4 + (3*A*d^3*((49*A^4*a^18)/(4*d^4) + (
64*B^4*a^18)/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a
^18*28i)/d^4)^(1/2))/(8*a) - (B*d^3*((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)
/d^4 + (40*A^2*B^2*a^18)/d^4 + (A*B^3*a^18*64i)/d^4 + (A^3*B*a^18*28i)/d^4
)^(1/2)*1i)/(2*a))) + (4*B^2*a^6*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((249*A
^2*a^3)/(128*d^2) - ((49*A^4*a^18)/(4*d^4) + (64*B^4*a^18)/d^4 + (40*A^...
```

Reduce [F]

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\tan(dx+c)^2\,dx\right)bi + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output

```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3*tan(c + d*x)**2,x)
*b*i + int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3*tan(c + d*x),x)*a*i +
int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3*tan(c + d*x),x)*b + int(sqrt(
tan(c + d*x)*i + 1)*cot(c + d*x)**3,x)*a)
```

3.81 $\int \cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 213

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{a^{3/2}(23iA+22B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{8d} - \frac{2\sqrt{2}a^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{a(9A-10iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{a(7iA+6B)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d} - \frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

output
```
1/8*a^(3/2)*(23*I*A+22*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d-2*2^
(1/2)*a^(3/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)/d+1/8*a*(9*A-10*I*B)*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/12*a*(7*I*A
+6*B)*cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*a*A*cot(d*x+c)^3*(a+I*a*
tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 3.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.74

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx =$$
$$-\frac{-3a^{3/2}(23iA+22B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)+48\sqrt{2}a^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+a\cot(c+}{24d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
-1/24*(-3*a^(3/2)*((23*I)*A + 22*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqr
t[a]] + 48*Sqrt[2]*a^(3/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(S
qrt[2]*Sqrt[a])] + a*Cot[c + d*x]*(-27*A + (30*I)*B + 2*((7*I)*A + 6*B)*Co
t[c + d*x] + 8*A*Cot[c + d*x]^2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4076, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{3}\int \frac{1}{2}\cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}(a(7iA+6B)-a(5A-6iB)\tan(c+dx))dx-\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\int \cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}(a(7iA+6B)-a(5A-6iB)\tan(c+dx))dx-\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(7iA+6B)-a(5A-6iB)\tan(c+dx))}{\tan(c+dx)^3}dx-\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 4081

$$\frac{1}{6}\left(\frac{\int -\frac{3}{2}\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left((9A-10iB)a^2+(7iA+6B)\tan(c+dx)a^2\right)dx}{2a}-\frac{a(6B+7iA)\cot^2\right.$$
$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\left(-\frac{3\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left((9A-10iB)a^2+(7iA+6B)\tan(c+dx)a^2\right)dx}{4a}-\frac{a(6B+7iA)\cot^2\right.$$
$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(-\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}\left((9A-10iB)a^2+(7iA+6B)\tan(c+dx)a^2\right)}{\tan(c+dx)^2}dx}{4a}-\frac{a(6B+7iA)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}\right)-$$
$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 4081

$$\frac{1}{6}\left(-\frac{3\left(\frac{\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(23iA+22B)-a^3(9A-10iB)\tan(c+dx)\right)dx}{a}-\frac{a^2(9A-10iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}-\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(-\frac{3\left(\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(23iA+22B)-a^3(9A-10iB)\tan(c+dx)\right)dx}{2a}-\frac{a^2(9A-10iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}-\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(-\frac{3\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(23iA+22B)-a^3(9A-10iB)\tan(c+dx)\right)}{\tan(c+dx)}dx}{2a}-\frac{a^2(9A-10iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{4a}-\frac{a(6B+7iA)}{}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{4083}$$

$$\frac{1}{6}\left(-\frac{3\left(\frac{a^2(22B+23iA)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-32a^3(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{a^2(9A-10iB)\cot(c+dx}{d}\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(-\frac{3\left(\frac{a^2(22B+23iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-32a^3(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{a^2(9A-10iB)\cot(c+dx)\sqrt{a+ia\tan(c}}{d}\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 3961

$$\frac{1}{6}\left(-\frac{3\left(\frac{\frac{64ia^4(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+a^2(22B+23iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a}-\frac{a^2(9A-10iB)\cot(c+dx)\sqrt{}}{d}\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 219

$$\frac{1}{6}\left(-\frac{3\left(\frac{a^2(22B+23iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx+\frac{32i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a^2(9A-10iB)\cot(c+dx)\sqrt{}}{d}\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 4082

$$\frac{1}{6}\left(-\frac{3\left(\frac{\frac{a^4(22B+23iA)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{32i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a^2(9A-10iB)\cot(c+dx)\sqrt{a+ia\tan}}{d}\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 73

$$\frac{1}{6}\left(-\frac{3\left(\frac{\frac{32i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2ia^3(22B+23iA)\int\frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a}-\frac{a^2(9A-10iB)\cot(c+dx}{d}}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{6}\left(-\frac{3\left(\frac{\frac{32i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^{7/2}(22B+23iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}}{2a}-\frac{a^2(9A-10iB)\cot(c+dx)\sqrt{a+ia}}{d}}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-1/3*(a*A*Cot[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (-1/2*(a*((7*I)*A
 + 6*B)*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/d - (3*(((-2*a^(7/2)*((
23*I)*A + 22*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + ((32*I)*S
qrt[2]*a^(7/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[
a])])/d)/(2*a) - (a^2*(9*A - (10*I)*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d
*x]])/d))/(4*a))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{2ia^4\left(-\frac{(-2iB+2A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{5}{2}}}+\frac{-\frac{i\left(\left(\frac{5iB}{8}-\frac{9A}{16}\right)(a+ia\tan(dx+c))^{\frac{5}{2}}+\left(-iaB+\frac{5}{6}aA\right)(a+ia\tan(dx+c)\right.}{a^3\tan(dx+c)^3}}{}\right.}{d}$
default	$\frac{2ia^4\left(-\frac{(-2iB+2A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{5}{2}}}+\frac{-\frac{i\left(\left(\frac{5iB}{8}-\frac{9A}{16}\right)(a+ia\tan(dx+c))^{\frac{5}{2}}+\left(-iaB+\frac{5}{6}aA\right)(a+ia\tan(dx+c)\right.}{a^3\tan(dx+c)^3}}{}\right.}{d}$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d*a^4*(-1/2*(-2*I*B+2*A)/a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c)
)^(1/2)*2^(1/2)/a^(1/2))+1/a^2*(-I*((5/8*I*B-9/16*A)*(a+I*a*tan(d*x+c))^(5
/2)+(-I*a*B+5/6*a*A)*(a+I*a*tan(d*x+c))^(3/2)+(3/8*I*B*a^2-7/16*A*a^2)*(a+
I*a*tan(d*x+c))^(1/2))/a^3/tan(d*x+c)^3+1/16*(-22*I*B+23*A)/a^(1/2)*arctan
h((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(166) = 332$.

Time = 0.15 (sec) , antiderivative size = 856, normalized size of antiderivative = 4.02

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/96*(96*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(6*I*d*x + 6*I*
c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*((-I*A -
 B)*a^2*e^(I*d*x + I*c) + sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(2*I*d
*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*
A - B)*a)) - 96*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e^(6*I*d*x
 + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*(
(-I*A - B)*a^2*e^(I*d*x + I*c) - sqrt(-(A^2 - 2*I*A*B - B^2)*a^3/d^2)*(d*e
^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c
)/((-I*A - B)*a)) - 3*sqrt(-(529*A^2 - 1012*I*A*B - 484*B^2)*a^3/d^2)*(d*e
^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d
)*log(-16*(3*(-23*I*A - 22*B)*a^2*e^(2*I*d*x + 2*I*c) + (-23*I*A - 22*B)*a
^2 + 2*sqrt(2)*sqrt(-(529*A^2 - 1012*I*A*B - 484*B^2)*a^3/d^2)*(d*e^(3*I*d
*x + 3*I*c) + d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*
I*d*x - 2*I*c)/(23*I*A + 22*B)) + 3*sqrt(-(529*A^2 - 1012*I*A*B - 484*B^2)
*a^3/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*
x + 2*I*c) - d)*log(-16*(3*(-23*I*A - 22*B)*a^2*e^(2*I*d*x + 2*I*c) + (-23
*I*A - 22*B)*a^2 - 2*sqrt(2)*sqrt(-(529*A^2 - 1012*I*A*B - 484*B^2)*a^3/d^
2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1)))*e^(-2*I*d*x - 2*I*c)/(23*I*A + 22*B)) - 4*sqrt(2)*(7*(-7*I*A - 6*B
)*a*e^(7*I*d*x + 7*I*c) - (11*I*A - 18*B)*a*e^(5*I*d*x + 5*I*c) - (-17*...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.19

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{i a^3 \left(\frac{48\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}} - \frac{3(23A-22iB)\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2\left(3(ia\tan\right.}{}\right.}{48d}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/48*I*a^3*(48*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
+ c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2) - 3*(23
*A - 22*I*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x
+ c) + a) + sqrt(a)))/a^(3/2) + 2*(3*(I*a*tan(d*x + c) + a)^(5/2)*(9*A - 1
0*I*B) - 8*(I*a*tan(d*x + c) + a)^(3/2)*(5*A - 6*I*B)*a + 3*sqrt(I*a*tan(d
*x + c) + a)*(7*A - 6*I*B)*a^2)/((I*a*tan(d*x + c) + a)^3*a - 3*(I*a*tan(d
*x + c) + a)^2*a^2 + 3*(I*a*tan(d*x + c) + a)*a^3 - a^4))/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 3084, normalized size of antiderivative = 14.48

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
2*atanh((6*d^4*(a + a*tan(c + d*x)*1i)^(1/2)*((249*B^2*a^3)/(128*d^2) - (1
041*A^2*a^3)/(512*d^2) - ((289*A^4*a^18)/(64*d^4) + (49*B^4*a^18)/(4*d^4)
+ (101*A^2*B^2*a^18)/(8*d^4) + (A*B^3*a^18*21i)/(2*d^4) + (A^3*B*a^18*51i)
/(8*d^4))^(1/2)/(64*a^6) + (A*B*a^3*509i)/(128*d^2))^(1/2)*((289*A^4*a^18)
/(64*d^4) + (49*B^4*a^18)/(4*d^4) + (101*A^2*B^2*a^18)/(8*d^4) + (A*B^3*a^
18*21i)/(2*d^4) + (A^3*B*a^18*51i)/(8*d^4))^(1/2))/((A^3*a^11*d*663i)/32 +
 (133*B^3*a^11*d)/4 + (A*B^2*a^11*d*387i)/8 + (89*A^2*B*a^11*d)/16 + (A*a^
2*d^3*((289*A^4*a^18)/(64*d^4) + (49*B^4*a^18)/(4*d^4) + (101*A^2*B^2*a^18
)/(8*d^4) + (A*B^3*a^18*21i)/(2*d^4) + (A^3*B*a^18*51i)/(8*d^4))^(1/2)*7i)
/4 + (3*B*a^2*d^3*((289*A^4*a^18)/(64*d^4) + (49*B^4*a^18)/(4*d^4) + (101*
A^2*B^2*a^18)/(8*d^4) + (A*B^3*a^18*21i)/(2*d^4) + (A^3*B*a^18*51i)/(8*d^4
))^(1/2))/2) + (17*A^2*a^6*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((249*B^2*a^3
)/(128*d^2) - (1041*A^2*a^3)/(512*d^2) - ((289*A^4*a^18)/(64*d^4) + (49*B^
4*a^18)/(4*d^4) + (101*A^2*B^2*a^18)/(8*d^4) + (A*B^3*a^18*21i)/(2*d^4) +
(A^3*B*a^18*51i)/(8*d^4))^(1/2)/(64*a^6) + (A*B*a^3*509i)/(128*d^2))^(1/2)
)/(4*((A^3*a^8*d*663i)/32 + (133*B^3*a^8*d)/4 + (A*B^2*a^8*d*387i)/8 + (89
*A^2*B*a^8*d)/16 + (A*d^3*((289*A^4*a^18)/(64*d^4) + (49*B^4*a^18)/(4*d^4)
 + (101*A^2*B^2*a^18)/(8*d^4) + (A*B^3*a^18*21i)/(2*d^4) + (A^3*B*a^18*51i
)/(8*d^4))^(1/2)*7i)/(4*a) + (3*B*d^3*((289*A^4*a^18)/(64*d^4) + (49*B^4*a
^18)/(4*d^4) + (101*A^2*B^2*a^18)/(8*d^4) + (A*B^3*a^18*21i)/(2*d^4) + ...
```

Reduce [F]

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^4\tan(dx+c)^2\,dx\right)bi + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^4\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^4\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^4\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**4*tan(c + d*x)**2,x)
*b*i + int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**4*tan(c + d*x),x)*a*i +
int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**4*tan(c + d*x),x)*b + int(sqrt(
tan(c + d*x)*i + 1)*cot(c + d*x)**4,x)*a)
```

3.82 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 246

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{4\sqrt{2}a^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{8a^2(45iA+46B)\sqrt{a+ia\tan(c+dx)}}{105d} + \frac{2a^2(45iA+46B)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{105d} - \frac{2a^2(3A-4iB)\tan^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d} - \frac{8a(60iA+59B)(a+ia\tan(c+dx))^{3/2}}{315d} + \frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

output
```
4*2^(1/2)*a^(5/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(
1/2))/d-8/105*a^2*(45*I*A+46*B)*(a+I*a*tan(d*x+c))^(1/2)/d+2/105*a^2*(45*I
*A+46*B)*tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d-2/21*a^2*(3*A-4*I*B)*tan(
d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/d-8/315*a*(60*I*A+59*B)*(a+I*a*tan(d*x+c
))^(3/2)/d+2/9*I*a*B*tan(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.61

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{2a^2\left(630\sqrt{2}\sqrt{a}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+\sqrt{a+ia\tan(c+dx)}(-780iA-\cdots$$

input
```
Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
(2*a^2*(630*Sqrt[2]*Sqrt[a]*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(
Sqrt[2]*Sqrt[a])] + Sqrt[a + I*a*Tan[c + d*x]]*((-780*I)*A - 788*B + 4*(60
*A - (59*I)*B)*Tan[c + d*x] + 3*((45*I)*A + 46*B)*Tan[c + d*x]^2 + (-45*A
+ (95*I)*B)*Tan[c + d*x]^3 - 35*B*Tan[c + d*x]^4)))/(315*d)
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4080, 27, 3042, 4075, 3042, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{2}{9}\int \frac{3}{2}\tan^2(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(3A-2iB)+a(3iA+4B)\tan(c+dx))dx+\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \tan^2(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(3A-2iB)+a(3iA+4B)\tan(c+dx))dx+\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \tan(c+dx)^2(i\tan(c+dx)a+a)^{3/2}(a(3A-2iB)+a(3iA+4B)\tan(c+dx))dx+\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{3}\left(\frac{2}{7}\int \frac{1}{2}\tan^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left((39A-38iB)a^2+(45iA+46B)\tan(c+dx)a^2\right)dx-\frac{2a^2(3A-4i\right.$$
$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{7}\int \tan^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left((39A-38iB)a^2+(45iA+46B)\tan(c+dx)a^2\right)dx-\frac{2a^2(3A-4iB)\right.$$
$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{1}{7}\int \tan(c+dx)^2\sqrt{i\tan(c+dx)a+a}\left((39A-38iB)a^2+(45iA+46B)\tan(c+dx)a^2\right)dx-\frac{2a^2(3A-4iB)\right.$$
$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 4080$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\int -2\tan(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(45iA+46B)-a^3(60A-59iB)\tan(c+dx)\right)dx}{5a}+\frac{2a^2(46B\ldots\right.\right.$$

$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46B+45iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{4\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(45iA+46\ldots\right.}{5a}\right.\right.$$

$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46B+45iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{4\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(45iA+46\ldots\right.}{5a}\right.\right.$$

$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 4075$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46B+45iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{4\left(\int \sqrt{i\tan(c+dx)a+a}\left((60A-59iB)a^3+(45i\ldots\right.\right.}{}\right.\right.$$

$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46B+45iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{4\left(\int \sqrt{i\tan(c+dx)a+a}\left((60A-59iB)a^3+(45i\ldots\right.\right.}{}\right.\right.$$

$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$$\Big\downarrow \; 4010$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46B+45iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{4\left(105a^3(A-iB)\int\sqrt{i\tan(c+dx)a+a}dx+\frac{2a^3(\cdots}{}\right.}{}\right.\right.$$

$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46B+45iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{4\left(105a^3(A-iB)\int\sqrt{i\tan(c+dx)a+a}dx+\frac{2a^3(\cdots}{}\right.}{}\right.\right.$$

$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$\big\downarrow$ 3961

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46B+45iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{4\left(-\frac{210ia^4(A-iB)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+\frac{2a\cdots}{}\right.}{}\right.\right.$$

$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

$\big\downarrow$ 219

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46B+45iA)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{4\left(-\frac{105i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2a\cdots}{}\right.}{}\right.\right.$$

$$\frac{2iaB\tan^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

input
```
Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(((2*I)/9)*a*B*Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d + ((-2*a^2*(
3*A - (4*I)*B)*Tan[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(7*d) + ((2*a^2*
((45*I)*A + 46*B)*Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(5*d) - (4*((
(-105*I)*Sqrt[2]*a^(7/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqr
t[2]*Sqrt[a])])/d + (2*a^3*((45*I)*A + 46*B)*Sqrt[a + I*a*Tan[c + d*x]])/d
 + (2*a^2*((60*I)*A + 59*B)*(a + I*a*Tan[c + d*x])^(3/2))/(3*d)))/(5*a))/7
)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{2i\left(\frac{iB(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{iBa(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{Aa(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{ia^2B(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{iB\,a^3(a+ia\tan(dx+c}{3}\right.}{}$
default	$\frac{2i\left(\frac{iB(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{iBa(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{Aa(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{ia^2B(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{iB\,a^3(a+ia\tan(dx+c}{3}\right.}{}$
parts	$\frac{2iA\left(-\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}a^2}{3}-2a^3\sqrt{a+ia\tan(dx+c)}+2a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{da}-$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d/a^2*(1/9*I*B*(a+I*a*tan(d*x+c))^(9/2)-1/7*I*B*a*(a+I*a*tan(d*x+c))^(
7/2)-1/7*A*a*(a+I*a*tan(d*x+c))^(7/2)+1/5*I*a^2*B*(a+I*a*tan(d*x+c))^(5/2)
+1/3*I*B*a^3*(a+I*a*tan(d*x+c))^(3/2)-1/3*A*a^3*(a+I*a*tan(d*x+c))^(3/2)+2
*I*B*a^4*(a+I*a*tan(d*x+c))^(1/2)-2*A*a^4*(a+I*a*tan(d*x+c))^(1/2)+2*a^(9/
2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(193) = 386.

Time = 0.12 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.16

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-2/315*(315*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(8*I*d*x + 8
*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x
 + 2*I*c) + d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) + sqrt(-(A^2 - 2*I*A*
B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 315*sqrt(2)*sqrt(-(A^2 - 2*I*A
*B - B^2)*a^5/d^2)*(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*
e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^3*e
^(I*d*x + I*c) - sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*
c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^
2)) + 2*sqrt(2)*(2*(300*I*A + 323*B)*a^2*e^(9*I*d*x + 9*I*c) + 27*(65*I*A
+ 61*B)*a^2*e^(7*I*d*x + 7*I*c) + 63*(35*I*A + 37*B)*a^2*e^(5*I*d*x + 5*I*
c) + 1365*(I*A + B)*a^2*e^(3*I*d*x + 3*I*c) + 315*(I*A + B)*a^2*e^(I*d*x +
 I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(
6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{5}{2}}(A+B\tan(c+dx))\tan^2(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(5/2)*(A + B*tan(c + d*x))*tan(c + d*x)
**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.72

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{2i\left(315\sqrt{2}(A-iB)a^{\frac{11}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-35i\,(ia\tan(dx+c)+a)^{\frac{9}{2}}Ba+45\,(ia\tan(dx+\right.}{}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-2/315*I*(315*sqrt(2)*(A - I*B)*a^(11/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*
tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 35*I*
(I*a*tan(d*x + c) + a)^(9/2)*B*a + 45*(I*a*tan(d*x + c) + a)^(7/2)*(A + I*
B)*a^2 - 63*I*(I*a*tan(d*x + c) + a)^(5/2)*B*a^3 + 105*(I*a*tan(d*x + c) +
 a)^(3/2)*(A - I*B)*a^4 + 630*sqrt(I*a*tan(d*x + c) + a)*(A - I*B)*a^5)/(a
^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \\ -\frac{2\,B\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{5\,d}-\frac{A\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,2{}\mathrm{i}}{3\,d} \\ -\frac{2\,B\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{3\,d}-\frac{A\,a^2\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,4{}\mathrm{i}}{d} \\ -\frac{A\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}\,2{}\mathrm{i}}{7\,a\,d}-\frac{4\,B\,a^2\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{d} \\ +\frac{2\,B\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}{7\,a\,d}-\frac{2\,B\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{9/2}}{9\,a^2\,d} \\ +\frac{\sqrt{2}\,A\,{\left(-a\right)}^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-a}}\right)\,4{}\mathrm{i}}{d} \\ -\frac{\sqrt{2}\,B\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,\sqrt{a}}\right)\,4{}\mathrm{i}}{d}\end{aligned}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
(2*B*(a + a*tan(c + d*x)*1i)^(7/2))/(7*a*d) - (A*a*(a + a*tan(c + d*x)*1i)
^(3/2)*2i)/(3*d) - (2*B*a*(a + a*tan(c + d*x)*1i)^(3/2))/(3*d) - (A*a^2*(a
 + a*tan(c + d*x)*1i)^(1/2)*4i)/d - (A*(a + a*tan(c + d*x)*1i)^(7/2)*2i)/(
7*a*d) - (4*B*a^2*(a + a*tan(c + d*x)*1i)^(1/2))/d - (2*B*(a + a*tan(c + d
*x)*1i)^(5/2))/(5*d) - (2*B*(a + a*tan(c + d*x)*1i)^(9/2))/(9*a^2*d) + (2^
(1/2)*A*(-a)^(5/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1
/2)))*4i)/d - (2^(1/2)*B*a^(5/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/
2)*1i)/(2*a^(1/2)))*4i)/d
```

Reduce [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a^2\Bigg(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^5\,dx\right)b - \left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^4\,dx\right)a + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^4\,dx\right)bi + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^3\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^3\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^2\,dx\right)a\Bigg)$$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**5,x)*b - int(s
qrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4,x)*a + 2*int(sqrt(tan(c + d*x)*i +
 1)*tan(c + d*x)**4,x)*b*i + 2*int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**
3,x)*a*i + int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3,x)*b + int(sqrt(ta
n(c + d*x)*i + 1)*tan(c + d*x)**2,x)*a)
```

3.83 $\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 171

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{4\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{4a^2(A-iB)\sqrt{a+ia\tan(c+dx)}}{d} + \frac{2a(A-iB)(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2A(a+ia\tan(c+dx))^{5/2}}{5d} - \frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

output
```
-4*2^(1/2)*a^(5/2)*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^
(1/2))/d+4*a^2*(A-I*B)*(a+I*a*tan(d*x+c))^(1/2)/d+2/3*a*(A-I*B)*(a+I*a*tan
(d*x+c))^(3/2)/d+2/5*A*(a+I*a*tan(d*x+c))^(5/2)/d-2/7*I*B*(a+I*a*tan(d*x+c
))^(7/2)/a/d
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.77

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{-420\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+2a^2\sqrt{a+ia\tan(c+dx)}(266A-26$$

input
```
Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
(-420*Sqrt[2]*a^(5/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2
]*Sqrt[a])] + 2*a^2*Sqrt[a + I*a*Tan[c + d*x]]*(266*A - (260*I)*B + ((77*I
)*A + 80*B)*Tan[c + d*x] + (-21*A + (45*I)*B)*Tan[c + d*x]^2 - 15*B*Tan[c
+ d*x]^3))/(105*d)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4075, 3042, 4010, 3042, 3959, 3042, 3959, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4075}$$

$$\int (i\tan(c+dx)a+a)^{5/2}(A\tan(c+dx)-B)dx-\frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

$$\Big\downarrow \text{ 3042}$$

$$\int (i\tan(c+dx)a+a)^{5/2}(A\tan(c+dx)-B)dx-\frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

$\downarrow$ 4010

$$-(B+iA)\int (i\tan(c+dx)a+a)^{5/2}dx+\frac{2A(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

$\downarrow$ 3042

$$-(B+iA)\int (i\tan(c+dx)a+a)^{5/2}dx+\frac{2A(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

$\downarrow$ 3959

$$-(B+iA)\left(2a\int (i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+\frac{2A(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

$\downarrow$ 3042

$$-(B+iA)\left(2a\int (i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+\frac{2A(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

$\downarrow$ 3959

$$-(B+iA)\left(2a\left(2a\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+\frac{2A(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

$\downarrow$ 3042

$$-(B+iA)\left(2a\left(2a\int \sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+\frac{2A(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

$\downarrow$ 3961

$$-(B+iA)\left(2a\left(\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}-\frac{4ia^2\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}\right)+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)$$
$$\frac{2A(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

$\downarrow$ 219

$$-(B+iA)\left(2a\left(\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2i\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\right)+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+$$
$$\frac{2A(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{2iB(a+ia\tan(c+dx))^{7/2}}{7ad}$$

input
```
Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(2*A*(a + I*a*Tan[c + d*x])^(5/2))/(5*d) - (((2*I)/7)*B*(a + I*a*Tan[c + d
*x])^(7/2))/(a*d) - (I*A + B)*((((2*I)/3)*a*(a + I*a*Tan[c + d*x])^(3/2))/
d + 2*a*(((-2*I)*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[
2]*Sqrt[a])])/d + ((2*I)*a*Sqrt[a + I*a*Tan[c + d*x]])/d))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{2iB(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{2Aa(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2iB\,a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+\frac{2A\,a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4ia^{3}B\sqrt{a+ia\tan}}{ad}$
default	$\frac{-\frac{2iB(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{2Aa(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2iB\,a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+\frac{2A\,a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4ia^{3}B\sqrt{a+ia\tan}}{ad}$
parts	$\frac{A\left(\frac{2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a^{2}\sqrt{a+ia\tan(dx+c)}-4a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}+$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
2/d/a*(-1/7*I*B*(a+I*a*tan(d*x+c))^(7/2)+1/5*A*a*(a+I*a*tan(d*x+c))^(5/2)-
1/3*I*B*a^2*(a+I*a*tan(d*x+c))^(3/2)+1/3*A*a^2*(a+I*a*tan(d*x+c))^(3/2)-2*
I*B*a^3*(a+I*a*tan(d*x+c))^(1/2)+2*A*a^3*(a+I*a*tan(d*x+c))^(1/2)-2*a^(7/2
)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(130) = 260$.

Time = 0.11 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.79

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(105\sqrt{2}\sqrt{\frac{(A^2-2i\,AB-B^2)a^5}{d^2}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left((-i\,A-B)a^3e^{(i\,dx+i\,c)}-\right.}{}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-2/105*(105*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(6*I*d*x + 6*
I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A
 - B)*a^3*e^(I*d*x + I*c) - sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2)*(I*d*e^(2*
I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/
((-I*A - B)*a^2)) - 105*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(
6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*
log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) - sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2
)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-
I*d*x - I*c)/((-I*A - B)*a^2)) - 2*sqrt(2)*(2*(91*A - 100*I*B)*a^2*e^(7*I*
d*x + 7*I*c) + 7*(61*A - 55*I*B)*a^2*e^(5*I*d*x + 5*I*c) + 350*(A - I*B)*a
^2*e^(3*I*d*x + 3*I*c) + 105*(A - I*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1)))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*
d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{5}{2}}(A+B\tan(c+dx))\tan(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(5/2)*(A + B*tan(c + d*x))*tan(c + d*x)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{2\left(105\sqrt{2}(A-iB)a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-15i\left(ia\tan(dx+c)+a\right)^{\frac{7}{2}}Ba+\cdots}{\cdots}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
2/105*(105*sqrt(2)*(A - I*B)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(
d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 15*I*(I*a
*tan(d*x + c) + a)^(7/2)*B*a + 21*(I*a*tan(d*x + c) + a)^(5/2)*A*a^2 + 35*
(I*a*tan(d*x + c) + a)^(3/2)*(A - I*B)*a^3 + 210*sqrt(I*a*tan(d*x + c) + a
)*(A - I*B)*a^4)/(a^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.24

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{2\,A\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{5\,d} + \frac{2\,A\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{3\,d} - \frac{B\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,2{}\mathrm{i}}{3\,d} + \frac{4\,A\,a^2\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{d} - \frac{B\,a^2\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,4{}\mathrm{i}}{d} - \frac{B\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}\,2{}\mathrm{i}}{7\,a\,d} + \frac{\sqrt{2}\,B\,{\left(-a\right)}^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-a}}\right)\,4{}\mathrm{i}}{d} + \frac{\sqrt{2}\,A\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,\sqrt{a}}\right)\,4{}\mathrm{i}}{d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
(2*A*(a + a*tan(c + d*x)*1i)^(5/2))/(5*d) + (2*A*a*(a + a*tan(c + d*x)*1i)
^(3/2))/(3*d) - (B*a*(a + a*tan(c + d*x)*1i)^(3/2)*2i)/(3*d) + (4*A*a^2*(a
 + a*tan(c + d*x)*1i)^(1/2))/d - (B*a^2*(a + a*tan(c + d*x)*1i)^(1/2)*4i)/
d - (B*(a + a*tan(c + d*x)*1i)^(7/2)*2i)/(7*a*d) + (2^(1/2)*B*(-a)^(5/2)*a
tan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*4i)/d + (2^(1/
2)*A*a^(5/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*
4i)/d
```

Reduce [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a^2\Bigg(-\left(\int \sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^4\,dx\right)b - \left(\int \sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^3\,dx\right)a + 2\left(\int \sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^3\,dx\right)bi + 2\left(\int \sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2\,dx\right)b + \left(\int \sqrt{\tan(dx+c)i+1}\,\tan(dx+c)\,dx\right)a\Bigg)$$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4,x)*b - int(s
qrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3,x)*a + 2*int(sqrt(tan(c + d*x)*i +
 1)*tan(c + d*x)**3,x)*b*i + 2*int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**
2,x)*a*i + int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*b + int(sqrt(ta
n(c + d*x)*i + 1)*tan(c + d*x),x)*a)
```

3.84 $\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 141

$$\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{4\sqrt{2}a^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{4a^2(iA+B)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2a(iA+B)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+ia\tan(c+dx))^{5/2}}{5d}$$

output
```
-4*2^(1/2)*a^(5/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^
(1/2))/d+4*a^2*(I*A+B)*(a+I*a*tan(d*x+c))^(1/2)/d+2/3*a*(I*A+B)*(a+I*a*tan
(d*x+c))^(3/2)/d+2/5*B*(a+I*a*tan(d*x+c))^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{-60i\sqrt{2}a^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+2a^2\sqrt{a+ia\tan(c+dx)}(35iA+38\ldots}{15d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((-60*I)*Sqrt[2]*a^(5/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqr
t[2]*Sqrt[a])] + 2*a^2*Sqrt[a + I*a*Tan[c + d*x]]*((35*I)*A + 38*B + (-5*A
 + (11*I)*B)*Tan[c + d*x] - 3*B*Tan[c + d*x]^2))/(15*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3042, 4010, 3042, 3959, 3042, 3959, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4010}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^{5/2}dx+\frac{2B(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^{5/2}dx+\frac{2B(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{ 3959}$$

$$(A-iB)\left(2a\int (i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+\frac{2B(a+ia\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$(A-iB)\left(2a\int(i\tan(c+dx)a+a)^{3/2}dx+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+\frac{2B(a+ia\tan(c+dx))^{5/2}}{5d}$$

$\big\downarrow$ 3959

$$(A-iB)\left(2a\left(2a\int\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+\frac{2B(a+ia\tan(c+dx))^{5/2}}{5d}$$

$\big\downarrow$ 3042

$$(A-iB)\left(2a\left(2a\int\sqrt{i\tan(c+dx)a+a}dx+\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+\frac{2B(a+ia\tan(c+dx))^{5/2}}{5d}$$

$\big\downarrow$ 3961

$$(A-iB)\left(2a\left(\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}-\frac{4ia^2\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}\right)+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+\frac{2B(a+ia\tan(c+dx))^{5/2}}{5d}$$

$\big\downarrow$ 219

$$(A-iB)\left(2a\left(\frac{2ia\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2i\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\right)+\frac{2ia(a+ia\tan(c+dx))^{3/2}}{3d}\right)+\frac{2B(a+ia\tan(c+dx))^{5/2}}{5d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*(a + I*a*Tan[c + d*x])^(5/2))/(5*d) + (A - I*B)*((((2*I)/3)*a*(a + I*
a*Tan[c + d*x])^(3/2))/d + 2*a*(((-2*I)*Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + I
*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d + ((2*I)*a*Sqrt[a + I*a*Tan[c + d*x
]])/d))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2i\left(-\frac{iB(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{iBa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+\frac{Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2ia^2B\sqrt{a+ia\tan(dx+c)}+2A\,a^2\sqrt{a+ia\tan}\right.}{d}$
default	$\frac{2i\left(-\frac{iB(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{iBa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+\frac{Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2ia^2B\sqrt{a+ia\tan(dx+c)}+2A\,a^2\sqrt{a+ia\tan}\right.}{d}$
parts	$\frac{2iAa\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2a\sqrt{a+ia\tan(dx+c)}-2a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{d}+\frac{B\left(\frac{2(a+ia\tan(dx+c))}{5}\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/d*(-1/5*I*B*(a+I*a*tan(d*x+c))^(5/2)-1/3*I*B*a*(a+I*a*tan(d*x+c))^(3/2
)+1/3*A*a*(a+I*a*tan(d*x+c))^(3/2)-2*I*B*a^2*(a+I*a*tan(d*x+c))^(1/2)+2*A*
a^2*(a+I*a*tan(d*x+c))^(1/2)-2*a^(5/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*
tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(108) = 216.

Time = 0.09 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.99

$$\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{2\left(15\sqrt{2}\sqrt{-\frac{(A^2-2i\,AB-B^2)a^5}{d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left((-i\,A-B)a^3e^{(i\,dx+}\right.}{}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
2/15*(15*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*
c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) +
sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 15*sqrt(2)*
sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d
*x + 2*I*c) + d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) - sqrt(-(A^2 - 2*I*
A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 2*sqrt(2)*(2*(-10*I*A - 13*B
)*a^2*e^(5*I*d*x + 5*I*c) + 35*(-I*A - B)*a^2*e^(3*I*d*x + 3*I*c) + 15*(-I
*A - B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*
d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \left(ia(\tan{(c+dx)}-i)\right)^{\frac{5}{2}}\left(A+B\tan{(c+dx)}\right)\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(5/2)*(A + B*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95

$$\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{2i\left(15\sqrt{2}(A-iB)a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-3i\left(ia\tan{(dx+c)}+a\right)^{\frac{5}{2}}Ba+\cdots\right.}{15\,ad}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
2/15*I*(15*sqrt(2)*(A - I*B)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(
d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 3*I*(I*a*
tan(d*x + c) + a)^(5/2)*B*a + 5*(I*a*tan(d*x + c) + a)^(3/2)*(A - I*B)*a^2
 + 30*sqrt(I*a*tan(d*x + c) + a)*(A - I*B)*a^3)/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 4.46 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.33

$$\begin{aligned}\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx &= \frac{2\,B\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}{5\,d}\\ &+\frac{A\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,2{}\mathrm{i}}{3\,d}+\frac{2\,B\,a\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{3\,d}\\ &+\frac{A\,a^2\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,4{}\mathrm{i}}{d}+\frac{4\,B\,a^2\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{d}\\ &-\frac{\sqrt{2}\,A\,{\left(-a\right)}^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-a}}\right)\,4{}\mathrm{i}}{d}\\ &+\frac{\sqrt{2}\,B\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,\sqrt{a}}\right)\,4{}\mathrm{i}}{d}\end{aligned}$$

input
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
(2*B*(a + a*tan(c + d*x)*1i)^(5/2))/(5*d) + (A*a*(a + a*tan(c + d*x)*1i)^(
3/2)*2i)/(3*d) + (2*B*a*(a + a*tan(c + d*x)*1i)^(3/2))/(3*d) + (A*a^2*(a +
 a*tan(c + d*x)*1i)^(1/2)*4i)/d + (4*B*a^2*(a + a*tan(c + d*x)*1i)^(1/2))/
d - (2^(1/2)*A*(-a)^(5/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*
(-a)^(1/2)))*4i)/d + (2^(1/2)*B*a^(5/2)*atan((2^(1/2)*(a + a*tan(c + d*x)*
1i)^(1/2)*1i)/(2*a^(1/2)))*4i)/d
```

Reduce [F]

$$\int (a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a}\,a^2\left(-2\sqrt{\tan(dx+c)\,i+1}\,ai-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^3\,dx\right)bd-\left(\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
(sqrt(a)*a**2*( - 2*sqrt(tan(c + d*x)*i + 1)*a*i - int(sqrt(tan(c + d*x)*i
 + 1)*tan(c + d*x)**3,x)*b*d - int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**
2,x)*a*d + 2*int(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*b*d*i + 3*int
(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i + int(sqrt(tan(c + d*x)*i
+ 1)*tan(c + d*x),x)*b*d))/d
```

3.85 $\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 147

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{2a^{5/2}A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{4\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

output
```
-2*a^(5/2)*A*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+4*2^(1/2)*a^(5/2)
*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d-2*a^2*(A-
2*I*B)*(a+I*a*tan(d*x+c))^(1/2)/d+2/3*I*a*B*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =$$
$$-\frac{2a^2\left(3\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)-6\sqrt{2}\sqrt{a}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+\sqrt{a+ia\tan(c+dx)}\right.}{3d}$$

input
```
Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
(-2*a^2*(3*Sqrt[a]*A*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]] - 6*Sqrt[
2]*Sqrt[a]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
 + Sqrt[a + I*a*Tan[c + d*x]]*(3*A - (7*I)*B + B*Tan[c + d*x])))/(3*d)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{2}{3}\int \frac{3}{2}\cot(c+dx)(i\tan(c+dx)a+a)^{3/2}(aA+a(iA+2B)\tan(c+dx))dx+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 27$$

$$\int \cot(c+dx)(i\tan(c+dx)a+a)^{3/2}(aA+a(iA+2B)\tan(c+dx))dx+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(i\tan(c+dx)a+a)^{3/2}(aA+a(iA+2B)\tan(c+dx))}{\tan(c+dx)}dx+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 4077$$

$$2\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(Aa^2+(3iA+4B)\tan(c+dx)a^2\right)dx-\frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 27$$

$$\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(Aa^2+(3iA+4B)\tan(c+dx)a^2\right)dx-\frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{i\tan(c+dx)a+a}\left(Aa^2+(3iA+4B)\tan(c+dx)a^2\right)}{\tan(c+dx)}dx-\frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 4083$$

$$4a^2(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+aA\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-\frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 3042$$

$$4a^2(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+aA\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 3961$$

$$-\frac{8ia^3(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+ aA\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+ \frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 219

$$aA\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx- \frac{4i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+ \frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 4082

$$\frac{a^3A\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{4i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}- \frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 73

$$-\frac{2ia^2A\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}- \frac{4i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+ \frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\big\downarrow$ 221

$$-\frac{4i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^{5/2}A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}- \frac{2a^2(A-2iB)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2iaB(a+ia\tan(c+dx))^{3/2}}{3d}$$

input
```
Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a^(5/2)*A*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - ((4*I)*Sqrt
[2]*a^(5/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/d - (2*a^2*(A - (2*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/d + (((2*I)/3)*a*B
*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2a\left(\frac{iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2iaB\sqrt{a+ia\tan(dx+c)}-aA\sqrt{a+ia\tan(dx+c)}+2a^{\frac{3}{2}}(-iB+A)\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx-}}{2\sqrt{a}}\right.\right.}{d}$
default	$\frac{2a\left(\frac{iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+2iaB\sqrt{a+ia\tan(dx+c)}-aA\sqrt{a+ia\tan(dx+c)}+2a^{\frac{3}{2}}(-iB+A)\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx-}}{2\sqrt{a}}\right.\right.}{d}$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
2/d*a*(1/3*I*B*(a+I*a*tan(d*x+c))^(3/2)+2*I*B*a*(a+I*a*tan(d*x+c))^(1/2)-a
*A*(a+I*a*tan(d*x+c))^(1/2)+2*a^(3/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*t
an(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-A*a^(3/2)*arctanh((a+I*a*tan(d*x+c))^(1/
2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(112) = 224$.

Time = 0.10 (sec) , antiderivative size = 610, normalized size of antiderivative = 4.15

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/6*(12*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c)
 + d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) - sqrt((A^2 - 2*I*A*B - B^2)*a
^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))
*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 12*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2
)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*
c) - sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 3*
sqrt(A^2*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(16*(3*A*a^3*e^(2*I*d*x +
 2*I*c) + A*a^3 + 2*sqrt(2)*sqrt(A^2*a^5/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e
^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/(A
*a)) + 3*sqrt(A^2*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(16*(3*A*a^3*e^(
2*I*d*x + 2*I*c) + A*a^3 - 2*sqrt(2)*sqrt(A^2*a^5/d^2)*(d*e^(3*I*d*x + 3*I
*c) + d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x -
2*I*c)/(A*a)) - 4*sqrt(2)*((3*A - 8*I*B)*a^2*e^(3*I*d*x + 3*I*c) + 3*(A -
2*I*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(2*I*d
*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =$$
$$-\frac{6\sqrt{2}(A-iB)a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-3Aa^{\frac{5}{2}}\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)-2i\left(ia\tan\left(dx+c\right)+a\right.}{3d}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/3*(6*sqrt(2)*(A - I*B)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
 + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 3*A*a^(5/2)*
log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + s
qrt(a))) - 2*I*(I*a*tan(d*x + c) + a)^(3/2)*B*a + 6*sqrt(I*a*tan(d*x + c)
+ a)*(A - 2*I*B)*a^2)/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 597, normalized size of antiderivative = 4.06

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\left(\frac{2\,a^2\,(A+B\,1\mathrm{i})}{d}-\frac{B\,a^2\,6\mathrm{i}}{d}\right)\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}+\frac{B\,a\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}\,2\mathrm{i}}{3\,d} - \frac{A\,\mathrm{atan}\left(\frac{A^3\,a^8\,d\,\sqrt{a^5}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,224\mathrm{i}}{-224\,d\,A^3\,a^{11}+512\mathrm{i}\,d\,A^2\,B\,a^{11}+256\,d\,A\,B^2\,a^{11}}-\frac{A\,B^2\,a^8\,d\,\sqrt{a^5}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,256\mathrm{i}}{-224\,d\,A^3\,a^{11}+512\mathrm{i}\,d\,A^2\,B\,a^{11}+256\,d\,A\,B^2\,a^{11}}+\frac{512\,A^2\,B\,a^8\,d\,\sqrt{a^5}\,\sqrt{a-}}{-224\,d\,A^3\,a^{11}+512\mathrm{i}\,d\,A^2\,B}\right.}{d} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{224\,\sqrt{2}\,A^3\,a^8\,d\,\sqrt{-a^5}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{448\,d\,A^3\,a^{11}-1472\mathrm{i}\,d\,A^2\,B\,a^{11}-1536\,d\,A\,B^2\,a^{11}+512\mathrm{i}\,d\,B^3\,a^{11}}+\frac{\sqrt{2}\,B^3\,a^8\,d\,\sqrt{-a^5}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,256\mathrm{i}}{448\,d\,A^3\,a^{11}-1472\mathrm{i}\,d\,A^2\,B\,a^{11}-1536\,d\,A\,B^2\,a^{11}+512\mathrm{i}\,d\,B^3\,a^{11}}\right.}{}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
(B*a*(a + a*tan(c + d*x)*1i)^(3/2)*2i)/(3*d) - ((2*a^2*(A + B*1i))/d - (B*
a^2*6i)/d)*(a + a*tan(c + d*x)*1i)^(1/2) - (A*atan((A^3*a^8*d*(a^5)^(1/2)*
(a + a*tan(c + d*x)*1i)^(1/2)*224i)/(256*A*B^2*a^11*d - 224*A^3*a^11*d + A
^2*B*a^11*d*512i) - (A*B^2*a^8*d*(a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)
*256i)/(256*A*B^2*a^11*d - 224*A^3*a^11*d + A^2*B*a^11*d*512i) + (512*A^2*
B*a^8*d*(a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(256*A*B^2*a^11*d - 224
*A^3*a^11*d + A^2*B*a^11*d*512i))*(a^5)^(1/2)*2i)/d + (2^(1/2)*atan((224*2
^(1/2)*A^3*a^8*d*(-a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(448*A^3*a^11
*d + B^3*a^11*d*512i - 1536*A*B^2*a^11*d - A^2*B*a^11*d*1472i) + (2^(1/2)*
B^3*a^8*d*(-a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*256i)/(448*A^3*a^11*d
 + B^3*a^11*d*512i - 1536*A*B^2*a^11*d - A^2*B*a^11*d*1472i) - (768*2^(1/2
)*A*B^2*a^8*d*(-a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(448*A^3*a^11*d
+ B^3*a^11*d*512i - 1536*A*B^2*a^11*d - A^2*B*a^11*d*1472i) - (2^(1/2)*A^2
*B*a^8*d*(-a^5)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*736i)/(448*A^3*a^11*d
+ B^3*a^11*d*512i - 1536*A*B^2*a^11*d - A^2*B*a^11*d*1472i))*(A*1i + B)*(-
a^5)^(1/2)*4i)/d
```

Reduce [F]

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\tan(dx+c)^3\,dx\right)b - \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\tan(dx+c)^2\,dx\right)a + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\tan(dx+c)^2\,dx\right)bi + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)\,dx\right)a\right)$$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output

```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)*tan(c + d*x)**3
,x)*b - int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)*tan(c + d*x)**2,x)*a + 2
*int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)*tan(c + d*x)**2,x)*b*i + 2*int(
sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)*tan(c + d*x),x)*a*i + int(sqrt(tan(c
 + d*x)*i + 1)*cot(c + d*x)*tan(c + d*x),x)*b + int(sqrt(tan(c + d*x)*i +
1)*cot(c + d*x),x)*a)
```

3.86 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 158

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{a^{5/2}(5iA+2B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{4\sqrt{2}a^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{a^2(iA-2B)\sqrt{a+ia\tan(c+dx)}}{d} - \frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

output

```
-a^(5/2)*(5*I*A+2*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+4*2^(1/2)
*a^(5/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d+a
^2*(I*A-2*B)*(a+I*a*tan(d*x+c))^(1/2)/d-a*A*cot(d*x+c)*(a+I*a*tan(d*x+c))^
(3/2)/d
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{-ia^{5/2}(5A-2iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)+4\sqrt{2}a^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+d}}{\sqrt{2}\sqrt{a}}\right.}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
((-I)*a^(5/2)*(5*A - (2*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]]
+ 4*Sqrt[2]*a^(5/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*
Sqrt[a])] + a^2*(-2*B - A*Cot[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4076, 27, 3042, 4077, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4076}$$

$$\int \frac{1}{2}\cot(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(5iA+2B)+a(A+2iB)\tan(c+dx))dx - \frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \cot(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(5iA+2B)+a(A+2iB)\tan(c+dx))dx-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(5iA+2B)+a(A+2iB)\tan(c+dx))}{\tan(c+dx)}dx-\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{2}\left(2\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^2(5iA+2B)-3a^2(A-2iB)\tan(c+dx)\right)dx+\frac{2a^2(-2B+iA)\sqrt{a}}{d}\right.$$
$$\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^2(5iA+2B)-3a^2(A-2iB)\tan(c+dx)\right)dx+\frac{2a^2(-2B+iA)\sqrt{a+i}}{d}\right.$$
$$\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(5iA+2B)-3a^2(A-2iB)\tan(c+dx)\right)}{\tan(c+dx)}dx+\frac{2a^2(-2B+iA)\sqrt{a+ia\tan(c+d}}{d}\right.$$
$$\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{4083}$$

$$\frac{1}{2}\left(-8a^2(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+a(2B+5iA)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a}\right.$$
$$\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-8a^2(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+a(2B+5iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx+\frac{2a}{}\right.$$
$$\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$\Big\downarrow$ 3961

$$\frac{1}{2}\left(\frac{16ia^3(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+a(2B+5iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a}}{\tan(c+dx)}\right.$$
$$\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(a(2B+5iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx+\frac{8i\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\right.$$
$$\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$\Big\downarrow$ 4082

$$\frac{1}{2}\left(\frac{a^3(2B+5iA)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{8i\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2a^2(-2B+i}{}\right.$$
$$\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$\Big\downarrow$ 73

$$\frac{1}{2}\left(-\frac{2ia^2(2B+5iA)\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}+\frac{8i\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\right.$$
$$\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

$\Big\downarrow$ 221

$$\frac{1}{2}\left(-\frac{2a^{5/2}(2B+5iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{8i\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{2a^2(-2B+i\right.$$

$$\frac{aA\cot(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-((a*A*Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d) + ((-2*a^(5/2)*((5*I)
*A + 2*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + ((8*I)*Sqrt[2]*
a^(5/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
 + (2*a^2*(I*A - 2*B)*Sqrt[a + I*a*Tan[c + d*x]])/d)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2ia^2\left(iB\sqrt{a+ia\tan(dx+c)}+2\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-a\left(-\frac{iA\sqrt{a+ia\tan(dx+c)}}{2a\tan(dx+c)}+\frac{(-2iB+5A}{}\right.\right.}{d}$
default	$\frac{2ia^2\left(iB\sqrt{a+ia\tan(dx+c)}+2\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)-a\left(-\frac{iA\sqrt{a+ia\tan(dx+c)}}{2a\tan(dx+c)}+\frac{(-2iB+5A}{}\right.\right.}{d}$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d*a^2*(I*B*(a+I*a*tan(d*x+c))^(1/2)+2*a^(1/2)*(A-I*B)*2^(1/2)*arctanh(
1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-a*(-1/2*I*A*(a+I*a*tan(d*x+c
))^(1/2)/a/tan(d*x+c)+1/2*(5*A-2*I*B)/a^(1/2)*arctanh((a+I*a*tan(d*x+c))^(
1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 705 vs. 2(125) = 250.

Time = 0.09 (sec) , antiderivative size = 705, normalized size of antiderivative = 4.46

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/4*(8*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c
) - d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) + sqrt(-(A^2 - 2*I*A*B - B^2)
*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e
^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 8*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*
a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c)
 - sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - sqrt(-(2
5*A^2 - 20*I*A*B - 4*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(-16*(3*
(-5*I*A - 2*B)*a^3*e^(2*I*d*x + 2*I*c) + (-5*I*A - 2*B)*a^3 + 2*sqrt(2)*sq
rt(-(25*A^2 - 20*I*A*B - 4*B^2)*a^5/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d
*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((5*I*A
 + 2*B)*a)) + sqrt(-(25*A^2 - 20*I*A*B - 4*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2
*I*c) - d)*log(-16*(3*(-5*I*A - 2*B)*a^3*e^(2*I*d*x + 2*I*c) + (-5*I*A - 2
*B)*a^3 - 2*sqrt(2)*sqrt(-(25*A^2 - 20*I*A*B - 4*B^2)*a^5/d^2)*(d*e^(3*I*d
*x + 3*I*c) + d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*
I*d*x - 2*I*c)/((5*I*A + 2*B)*a)) + 4*sqrt(2)*((I*A + 2*B)*a^2*e^(3*I*d*x
+ 3*I*c) + (I*A - 2*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{i\left(4\sqrt{2}(A-iB)a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-(5A-2iB)a^{\frac{3}{2}}\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)-4i\sqrt{ia\tan}\right.}{2d}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/2*I*(4*sqrt(2)*(A - I*B)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d
*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - (5*A - 2*I
*B)*a^(3/2)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x +
 c) + a) + sqrt(a))) - 4*I*sqrt(I*a*tan(d*x + c) + a)*B*a - 2*I*sqrt(I*a*t
an(d*x + c) + a)*A*a/tan(d*x + c))*a/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 2947, normalized size of antiderivative = 18.65

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
atan((d^4*(a + a*tan(c + d*x)*1i)^(1/2)*(((49*A^4*a^22)/d^4 + (784*B^4*a^2
2)/d^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*61
6i)/d^4)^(1/2)/(8*a^6) - (57*A^2*a^5)/(8*d^2) + (9*B^2*a^5)/(2*d^2) + (A*B
*a^5*21i)/(2*d^2))^(1/2)*((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A
^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^(1/2)*6
i)/(A^3*a^14*d*126i - 336*B^3*a^14*d - A*B^2*a^14*d*1032i + 876*A^2*B*a^14
*d + A*a^3*d^3*((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^2
2)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^(1/2)*2i - 4*B*a^
3*d^3*((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22)/d^4 +
(A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^(1/2)) + (A^2*a^8*d^2*(a +
 a*tan(c + d*x)*1i)^(1/2)*(((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 - (2328
*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^4)^(1/2)
/(8*a^6) - (57*A^2*a^5)/(8*d^2) + (9*B^2*a^5)/(2*d^2) + (A*B*a^5*21i)/(2*d
^2))^(1/2)*14i)/(A^3*a^11*d*126i - 336*B^3*a^11*d + A*d^3*((49*A^4*a^22)/d
^4 + (784*B^4*a^22)/d^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4
 - (A^3*B*a^22*616i)/d^4)^(1/2)*2i - 4*B*d^3*((49*A^4*a^22)/d^4 + (784*B^4
*a^22)/d^4 - (2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^2
2*616i)/d^4)^(1/2) - A*B^2*a^11*d*1032i + 876*A^2*B*a^11*d) - (B^2*a^8*d^2
*(a + a*tan(c + d*x)*1i)^(1/2)*(((49*A^4*a^22)/d^4 + (784*B^4*a^22)/d^4 -
(2328*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*2464i)/d^4 - (A^3*B*a^22*616i)/d^...
```

Reduce [F]

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\tan(dx+c)^3\,dx\right)b - \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)a + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)bi + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^2\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2*tan(c + d*x)
**3,x)*b - int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2*tan(c + d*x)**2,x)
*a + 2*int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2*tan(c + d*x)**2,x)*b*i
 + 2*int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2*tan(c + d*x),x)*a*i + in
t(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2*tan(c + d*x),x)*b + int(sqrt(ta
n(c + d*x)*i + 1)*cot(c + d*x)**2,x)*a)
```

3.87 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 173

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{a^{5/2}(23A-20iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4d} - \frac{4\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{a^2(7iA+4B)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

output
```
1/4*a^(5/2)*(23*A-20*I*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d-4*2^
(1/2)*a^(5/2)*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)/d-1/4*a^2*(7*I*A+4*B)*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/2*a*A*cot(
d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 4.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.82

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{a^{5/2}(23A-20iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)-16\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right) \ldots}{4d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
(a^(5/2)*(23*A - (20*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]] - 1
6*Sqrt[2]*a^(5/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sq
rt[a])] - a^2*Cot[c + d*x]*((9*I)*A + 4*B + 2*A*Cot[c + d*x])*Sqrt[a + I*a
*Tan[c + d*x]])/(4*d)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{2}\int \frac{1}{2}\cot^2(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(7iA+4B)-a(A-4iB)\tan(c+dx))dx - \frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \cot^2(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(7iA+4B)-a(A-4iB)\tan(c+dx))dx-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(7iA+4B)-a(A-4iB)\tan(c+dx))}{\tan(c+dx)^2}dx-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{4}\left(\int -\frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left((23A-20iB)a^2+3(3iA+4B)\tan(c+dx)a^2\right)dx-\frac{a^2(4B+7iA)\,\mathrm{c}\cdots}{\cdots}\right.$$
$$-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left((23A-20iB)a^2+3(3iA+4B)\tan(c+dx)a^2\right)dx-\frac{a^2(4B+7iA)\,\mathrm{c}\cdots}{\cdots}\right.$$
$$-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((23A-20iB)a^2+3(3iA+4B)\tan(c+dx)a^2\right)}{\tan(c+dx)}dx-\frac{a^2(4B+7iA)\cot(c+dx)\sqrt{\cdots}}{d}\right.$$
$$-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{4083}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-32a^2(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx-a(23A-20iB)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+\cdots}\right.\right.$$
$$-\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(-32a^2(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx-a(23A-20iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}d\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$\big\downarrow$ 3961

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{64ia^3(B+iA)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}-a(23A-20iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+}}{\tan(c+dx)}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$\big\downarrow$ 219

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{32i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-a(23A-20iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a}}{\tan(c+dx)}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$\big\downarrow$ 4082

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{32i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{a^3(23A-20iB)\int\frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}\right)-\frac{a^2(4}{}\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$\big\downarrow$ 73

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{2ia^2(23A-20iB)\int\frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}+\frac{32i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx}}{\sqrt{2}\sqrt{a}}\right.}{d}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

$\big\downarrow$ 221

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{2a^{5/2}(23A-20iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{32i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\right)-\frac{a^2(4B$$

$$\frac{aA\cot^2(c+dx)(a+ia\tan(c+dx))^{3/2}}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(a*A*Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2))/d + (((2*a^(5/2)*(2
3*A - (20*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + ((32*I)*S
qrt[2]*a^(5/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[
a])])/d)/2 - (a^2*((7*I)*A + 4*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2a^3\left(-\frac{(-4iB+4A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\left(-\frac{iB}{2}+\frac{9A}{8}\right)(a+ia\tan(dx+c))^{\frac{3}{2}}+\left(\frac{1}{2}iaB-\frac{7}{8}aA\right)\sqrt{a+ia\tan(dx+c)}}{a^2\tan(dx+c)^2}+\cdots\right.}{d}$
default	$\frac{2a^3\left(-\frac{(-4iB+4A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\left(-\frac{iB}{2}+\frac{9A}{8}\right)(a+ia\tan(dx+c))^{\frac{3}{2}}+\left(\frac{1}{2}iaB-\frac{7}{8}aA\right)\sqrt{a+ia\tan(dx+c)}}{a^2\tan(dx+c)^2}+\cdots\right.}{d}$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2/d*a^3*(-1/2*(-4*I*B+4*A)*2^(1/2)/a^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^
(1/2)*2^(1/2)/a^(1/2))-((-1/2*I*B+9/8*A)*(a+I*a*tan(d*x+c))^(3/2)+(1/2*I*a
*B-7/8*a*A)*(a+I*a*tan(d*x+c))^(1/2))/a^2/tan(d*x+c)^2+1/8*(23*A-20*I*B)/a
^(1/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(134) = 268.

Time = 0.11 (sec) , antiderivative size = 774, normalized size of antiderivative = 4.47

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/16*(32*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*
c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) -
sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 32*sqrt(
2)*sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I
*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) - sqrt((A^2 - 2*I
*A*B - B^2)*a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) + sqrt((529*A^2 - 920*I*
A*B - 400*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) +
 d)*log(-16*(3*(-23*I*A - 20*B)*a^3*e^(2*I*d*x + 2*I*c) + (-23*I*A - 20*B)
*a^3 + 2*sqrt(2)*sqrt((529*A^2 - 920*I*A*B - 400*B^2)*a^5/d^2)*(I*d*e^(3*I
*d*x + 3*I*c) + I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^
(-2*I*d*x - 2*I*c)/((23*I*A + 20*B)*a)) - sqrt((529*A^2 - 920*I*A*B - 400*
B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-1
6*(3*(-23*I*A - 20*B)*a^3*e^(2*I*d*x + 2*I*c) + (-23*I*A - 20*B)*a^3 + 2*s
qrt(2)*sqrt((529*A^2 - 920*I*A*B - 400*B^2)*a^5/d^2)*(-I*d*e^(3*I*d*x + 3*
I*c) - I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x
 - 2*I*c)/((23*I*A + 20*B)*a)) - 4*sqrt(2)*((11*A - 4*I*B)*a^2*e^(5*I*d*x
+ 5*I*c) + 4*A*a^2*e^(3*I*d*x + 3*I*c) - (7*A - 4*I*B)*a^2*e^(I*d*x + I*c)
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.19

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\left(16\sqrt{2}(A-iB)\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-(23A-20iB)\sqrt{a}\log\left(\frac{\sqrt{ia\tan(dx}}{\sqrt{ia\tan(dx}}\right.\right.}{8d}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/8*(16*sqrt(2)*(A - I*B)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
 + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - (23*A - 20*I
*B)*sqrt(a)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x +
 c) + a) + sqrt(a))) + 2*((I*a*tan(d*x + c) + a)^(3/2)*(9*A - 4*I*B)*a - s
qrt(I*a*tan(d*x + c) + a)*(7*A - 4*I*B)*a^2)/((I*a*tan(d*x + c) + a)^2 - 2
*(I*a*tan(d*x + c) + a)*a + a^2))*a^2/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 2991, normalized size of antiderivative = 17.29

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
2*atanh((17*A^2*a^8*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((1041*A^2*a^5)/(128
*d^2) - ((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (1752*A^2*B^2*a^22
)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^(1/2)/(64*a^6) - (
57*B^2*a^5)/(8*d^2) - (A*B*a^5*243i)/(16*d^2))^(1/2))/(4*((663*A^3*a^11*d)
/16 - B^3*a^11*d*252i - (7*A*d^3*((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)
/d^4 - (1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i
)/d^4)^(1/2))/8 + (B*d^3*((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (
1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^(
1/2)*1i)/2 + 507*A*B^2*a^11*d + (A^2*B*a^11*d*861i)/4)) - (3*d^4*(a + a*ta
n(c + d*x)*1i)^(1/2)*((1041*A^2*a^5)/(128*d^2) - ((289*A^4*a^22)/(4*d^4) +
 (3136*B^4*a^22)/d^4 - (1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 +
(A^3*B*a^22*884i)/d^4)^(1/2)/(64*a^6) - (57*B^2*a^5)/(8*d^2) - (A*B*a^5*24
3i)/(16*d^2))^(1/2)*((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (1752*
A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^(1/2))
/(2*((663*A^3*a^14*d)/16 - B^3*a^14*d*252i + 507*A*B^2*a^14*d + (A^2*B*a^1
4*d*861i)/4 - (7*A*a^3*d^3*((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 -
 (1752*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)
^(1/2))/8 + (B*a^3*d^3*((289*A^4*a^22)/(4*d^4) + (3136*B^4*a^22)/d^4 - (17
52*A^2*B^2*a^22)/d^4 + (A*B^3*a^22*5824i)/d^4 + (A^3*B*a^22*884i)/d^4)^(1/
2)*1i)/2)) + (28*B^2*a^8*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((1041*A^2*a...
```

Reduce [F]

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a^2\left(-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\tan(dx+c)^3\,dx\right)b-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\tan(dx+c)^2\,dx\right)a+2\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\tan(dx+c)^2\,dx\right)bi+2\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\tan(dx+c)\,dx\right)ai+\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\tan(dx+c)\,dx\right)b+\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^3\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3*tan(c + d*x)
**3,x)*b - int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3*tan(c + d*x)**2,x)
*a + 2*int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3*tan(c + d*x)**2,x)*b*i
 + 2*int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3*tan(c + d*x),x)*a*i + in
t(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3*tan(c + d*x),x)*b + int(sqrt(ta
n(c + d*x)*i + 1)*cot(c + d*x)**3,x)*a)
```

3.88 $\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 217

$$\begin{aligned}\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}(A&+B\tan(c+dx))\,dx = \frac{a^{5/2}(45iA+46B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{8d}\\
&-\frac{4\sqrt{2}a^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\\
&+\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}\\
&-\frac{a^2(3iA+2B)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}\\
&-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}\end{aligned}$$

output
```
1/8*a^(5/2)*(45*I*A+46*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d-4*2^
(1/2)*a^(5/2)*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2)
)/d+1/8*a^2*(19*A-18*I*B)*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/4*a^2*(3
*I*A+2*B)*cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*a*A*cot(d*x+c)^3*(a+
I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 3.46 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.74

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =$$
$$-\frac{-3a^{5/2}(45iA+46B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)+96\sqrt{2}a^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)+a^2\cot(c+}{24d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
-1/24*(-3*a^(5/2)*((45*I)*A + 46*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqr
t[a]] + 96*Sqrt[2]*a^(5/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(S
qrt[2]*Sqrt[a])] + a^2*Cot[c + d*x]*(-57*A + (54*I)*B + 2*((13*I)*A + 6*B)
*Cot[c + d*x] + 8*A*Cot[c + d*x]^2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4081, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{3}\int \frac{3}{2}\cot^3(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(3iA+2B)-a(A-2iB)\tan(c+dx))dx-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\frac{1}{2}\int \cot^3(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(3iA+2B)-a(A-2iB)\tan(c+dx))dx-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(3iA+2B)-a(A-2iB)\tan(c+dx))}{\tan(c+dx)^3}dx-\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4076

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{1}{2}\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left((19A-18iB)a^2+(13iA+14B)\tan(c+dx)a^2\right)dx-\frac{a^2(2B+3i\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(-\frac{1}{4}\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left((19A-18iB)a^2+(13iA+14B)\tan(c+dx)a^2\right)dx-\frac{a^2(2B+3iA)\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((19A-18iB)a^2+(13iA+14B)\tan(c+dx)a^2\right)}{\tan(c+dx)^2}dx-\frac{a^2(2B+3iA)\cot^2(c+dx}{2d}\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4081

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int\frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(45iA+46B)\right.}{a}\right.\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(45iA+46B)-\right.}{2a}\right.\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}(a^3(45iA+46B)-a^3(19A-18iB)\tan(c+dx))}{\tan(c+dx)}}{2a}\right.\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 4083

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{a^2(46B+45iA)\int\cot(c+dx)(a-ia\tan(c+dx))\sqrt{}}{}\right.\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{a^2(46B+45iA)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-}{2a}\right.\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 3961

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{128ia^4(A-iB)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+a^2(46B-}{2a}\right.\right.$$
$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{a^2(46B+45iA)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx+\right.\right.$$

$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 4082$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{a^4(46B+45iA)\int\frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{64i\sqrt{2}a^{7/2}(A-i}{}}{2a}\right.\right.$$

$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{64i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2ia^3(46B+45}{}}{2a}\right.\right.$$

$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19A-18iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{64i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{2a^{7/2}(46B+4}{}}{2a}\right.\right.$$

$$\frac{aA\cot^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-1/3*(a*A*Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d + (-1/2*(a^2*((3*
I)*A + 2*B)*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/d + (-1/2*((-2*a^(7
/2)*((45*I)*A + 46*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d + ((6
4*I)*Sqrt[2]*a^(7/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]
*Sqrt[a])])/d)/a + (a^2*(19*A - (18*I)*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c
+ d*x]])/d)/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{2ia^4\left(-\frac{(-4iB+4A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{3}{2}}}+\frac{-\frac{i\left(\left(\frac{9iB}{8}-\frac{19A}{16}\right)(a+ia\tan(dx+c))^{\frac{5}{2}}+\left(-2iaB+\frac{11}{6}aA\right)(a+ia\tan(dx}{a^3\tan(dx+c)^3}}{}\right.}{d}$
default	$\frac{2ia^4\left(-\frac{(-4iB+4A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{3}{2}}}+\frac{-\frac{i\left(\left(\frac{9iB}{8}-\frac{19A}{16}\right)(a+ia\tan(dx+c))^{\frac{5}{2}}+\left(-2iaB+\frac{11}{6}aA\right)(a+ia\tan(dx}{a^3\tan(dx+c)^3}}{}\right.}{d}$

input
```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d*a^4*(-1/2*(-4*I*B+4*A)/a^(3/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c)
)^(1/2)*2^(1/2)/a^(1/2))+1/a*(-I*((9/8*I*B-19/16*A)*(a+I*a*tan(d*x+c))^(5/
2)+(-2*I*a*B+11/6*a*A)*(a+I*a*tan(d*x+c))^(3/2)+(7/8*I*B*a^2-13/16*A*a^2)*
(a+I*a*tan(d*x+c))^(1/2))/a^3/tan(d*x+c)^3+1/16*(-46*I*B+45*A)/a^(1/2)*arc
tanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 868 vs. 2(170) = 340.

Time = 0.11 (sec) , antiderivative size = 868, normalized size of antiderivative = 4.00

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/96*(192*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(6*I*d*x + 6*I
*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*((-I*A
- B)*a^3*e^(I*d*x + I*c) + sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(2*I*
d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I
*A - B)*a^2)) - 192*sqrt(2)*sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(6*I
*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log
(4*((-I*A - B)*a^3*e^(I*d*x + I*c) - sqrt(-(A^2 - 2*I*A*B - B^2)*a^5/d^2)*
(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x -
 I*c)/((-I*A - B)*a^2)) - 3*sqrt(-(2025*A^2 - 4140*I*A*B - 2116*B^2)*a^5/d
^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*
I*c) - d)*log(-16*(3*(-45*I*A - 46*B)*a^3*e^(2*I*d*x + 2*I*c) + (-45*I*A -
 46*B)*a^3 + 2*sqrt(2)*sqrt(-(2025*A^2 - 4140*I*A*B - 2116*B^2)*a^5/d^2)*(
d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
)))*e^(-2*I*d*x - 2*I*c)/((45*I*A + 46*B)*a)) + 3*sqrt(-(2025*A^2 - 4140*I
*A*B - 2116*B^2)*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c)
 + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(-16*(3*(-45*I*A - 46*B)*a^3*e^(2*I*d*x
 + 2*I*c) + (-45*I*A - 46*B)*a^3 - 2*sqrt(2)*sqrt(-(2025*A^2 - 4140*I*A*B
- 2116*B^2)*a^5/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((45*I*A + 46*B)*a)) + 4*sq
rt(2)*((91*I*A + 66*B)*a^2*e^(7*I*d*x + 7*I*c) - 7*(I*A + 6*B)*a^2*e^(5...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.15

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{i\left(\frac{96\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{\sqrt{a}} - \frac{3(45A-46iB)\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{\sqrt{a}} + \frac{2(3(ia\tan\dots}{}\right)}{48d}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/48*I*(96*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) - 3*(45*A -
 46*I*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c)
 + a) + sqrt(a)))/sqrt(a) + 2*(3*(I*a*tan(d*x + c) + a)^(5/2)*(19*A - 18*I
*B) - 8*(I*a*tan(d*x + c) + a)^(3/2)*(11*A - 12*I*B)*a + 3*sqrt(I*a*tan(d*
x + c) + a)*(13*A - 14*I*B)*a^2)/((I*a*tan(d*x + c) + a)^3 - 3*(I*a*tan(d*
x + c) + a)^2*a + 3*(I*a*tan(d*x + c) + a)*a^2 - a^3))*a^3/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 3048, normalized size of antiderivative = 14.05

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
2*atanh((23*A^2*a^8*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((1041*B^2*a^5)/(128
*d^2) - (4073*A^2*a^5)/(512*d^2) - ((529*A^4*a^22)/(64*d^4) + (289*B^4*a^2
2)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3
*B*a^22*253i)/(8*d^4))^(1/2)/(64*a^6) + (A*B*a^5*2059i)/(128*d^2))^(1/2))/
(4*((A^3*a^11*d*1771i)/32 + (663*B^3*a^11*d)/4 - (A*d^3*((529*A^4*a^22)/(6
4*d^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22
*187i)/(2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^(1/2)*13i)/4 - (7*B*d^3*((529*
A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) +
 (A*B^3*a^22*187i)/(2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^(1/2))/2 + (A*B^2*
a^11*d*2167i)/8 - (797*A^2*B*a^11*d)/16)) - (6*d^4*(a + a*tan(c + d*x)*1i)
^(1/2)*((1041*B^2*a^5)/(128*d^2) - (4073*A^2*a^5)/(512*d^2) - ((529*A^4*a^
22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^
3*a^22*187i)/(2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^(1/2)/(64*a^6) + (A*B*a^
5*2059i)/(128*d^2))^(1/2)*((529*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4
) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*187i)/(2*d^4) + (A^3*B*a^22*2
53i)/(8*d^4))^(1/2))/((A^3*a^14*d*1771i)/32 + (663*B^3*a^14*d)/4 + (A*B^2*
a^14*d*2167i)/8 - (797*A^2*B*a^14*d)/16 - (A*a^3*d^3*((529*A^4*a^22)/(64*d
^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^4) + (A*B^3*a^22*18
7i)/(2*d^4) + (A^3*B*a^22*253i)/(8*d^4))^(1/2)*13i)/4 - (7*B*a^3*d^3*((529
*A^4*a^22)/(64*d^4) + (289*B^4*a^22)/(4*d^4) + (149*A^2*B^2*a^22)/(8*d^...
```

Reduce [F]

$$\int \cot^4(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^4\tan(dx+c)^3\,dx\right)b - \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^4\tan(dx+c)^2\,dx\right)a + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^4\tan(dx+c)^2\,dx\right)bi + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^4\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^4\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^4\,dx\right)a\right)$$

input

```
int(cot(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output

```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**4*tan(c + d*x)
**3,x)*b - int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**4*tan(c + d*x)**2,x)
*a + 2*int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**4*tan(c + d*x)**2,x)*b*i
 + 2*int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**4*tan(c + d*x),x)*a*i + in
t(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**4*tan(c + d*x),x)*b + int(sqrt(ta
n(c + d*x)*i + 1)*cot(c + d*x)**4,x)*a)
```

3.89 $\int \cot^5(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 261

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{3a^{5/2}(121A-120iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{64d} + \frac{4\sqrt{2}a^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{a^2(149iA+152B)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d} + \frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{96d} - \frac{a^2(11iA+8B)\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24d} - \frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

output
```
-3/64*a^(5/2)*(121*A-120*I*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/d+
4*2^(1/2)*a^(5/2)*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(
1/2))/d+1/64*a^2*(149*I*A+152*B)*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d+1/9
6*a^2*(107*A-104*I*B)*cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d-1/24*a^2*(11
*I*A+8*B)*cot(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/d-1/4*a*A*cot(d*x+c)^4*(a+
I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.68

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{-9a^{5/2}(121A-120iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)+768\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+i}}{}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
(-9*a^(5/2)*(121*A - (120*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]
] + 768*Sqrt[2]*a^(5/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt
[2]*Sqrt[a])] + a^2*Cot[c + d*x]*((447*I)*A + 456*B + (214*A - (208*I)*B)*
Cot[c + d*x] + ((-136*I)*A - 64*B)*Cot[c + d*x]^2 - 48*A*Cot[c + d*x]^3)*S
qrt[a + I*a*Tan[c + d*x]])/(192*d)
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.09, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{4}\int \frac{1}{2}\cot^4(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(11iA+8B)-a(5A-8iB)\tan(c+dx))dx-\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \cot^4(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(11iA+8B)-a(5A-8iB)\tan(c+dx))dx-\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(11iA+8B)-a(5A-8iB)\tan(c+dx))}{\tan(c+dx)^4}dx-\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{8}\left(\frac{1}{3}\int -\frac{1}{2}\cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}\left((107A-104iB)a^2+(85iA+88B)\tan(c+dx)a^2\right)dx-\frac{a^2(8B+\right.$$
$$-\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(-\frac{1}{6}\int \cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}\left((107A-104iB)a^2+(85iA+88B)\tan(c+dx)a^2\right)dx-\frac{a^2(8B+11\right.$$
$$-\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(-\frac{1}{6}\int\frac{\sqrt{i\tan(c+dx)a+a}\left((107A-104iB)a^2+(85iA+88B)\tan(c+dx)a^2\right)}{\tan(c+dx)^3}dx-\frac{a^2(8B+11iA)\cot^3(c+\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 4081

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\int\frac{3}{2}\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(149iA+\right.}{2a}\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\int\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(149iA+\right.}{4a}\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\int\frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(149iA+152B)-a^3(107A-104iB)\tan\right.}{\tan(c+dx)^2}}{4a}\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 4081

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\int-\frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(3(121A-120iB)a^4+(14\right.}{a}\right.}{}\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(-\frac{\int\cot(c+dx)\sqrt{i\tan(c+dx)a+a}(3(121A-120iB)a^4+(149\right.}{2a}}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left(3(121A-120iB)a^4+(149iA+152B)\tan(c\right.}{\tan(c+dx)}}{2a}\right.}{4}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 4083

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(-\frac{512a^4(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx+3a^3(121A-120iB\right.}{2a}}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(-\frac{512a^4(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx+3a^3(121A-120iB\right.}{2a}}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3961

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(-\frac{3a^3(121A-120iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 219

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(-\frac{3a^3(121A-120iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 4082

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(-\frac{\frac{3a^5(121A-120iB)\int\frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{512i\sqrt{2}a}{}}{2a}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 73

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(-\frac{-\frac{6ia^4(121A-120iB)\int\frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a}}{d}}{2a}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow 221$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107A-104iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(-\frac{-\frac{6a^{9/2}(121A-120iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{512}{}}{2a}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

input
```
Int[Cot[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-1/4*(a*A*Cot[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2))/d + (-1/3*(a^2*((11
*I)*A + 8*B)*Cot[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + ((a^2*(107*A -
 (104*I)*B)*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(2*d) - (3*(-1/2*((
-6*a^(9/2)*(121*A - (120*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]]
)/d - ((512*I)*Sqrt[2]*a^(9/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]
]/(Sqrt[2]*Sqrt[a])])/d)/a - (a^3*((149*I)*A + 152*B)*Cot[c + d*x]*Sqrt[a
+ I*a*Tan[c + d*x]])/d))/(4*a))/6)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.79

method	result
derivativedivides	$2a^5\left(-\frac{(4iB-4A)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{5}{2}}}-\frac{\left(-\frac{19iB}{16}+\frac{149A}{128}\right)(a+ia\tan(dx+c))^{\frac{7}{2}}+\left(\frac{145}{48}iaB-\frac{1127}{384}aA\right)(a+ia\tan(d\ldots}{}\right.$
default	$2a^5\left(-\frac{(4iB-4A)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2a^{\frac{5}{2}}}-\frac{\left(-\frac{19iB}{16}+\frac{149A}{128}\right)(a+ia\tan(dx+c))^{\frac{7}{2}}+\left(\frac{145}{48}iaB-\frac{1127}{384}aA\right)(a+ia\tan(d\ldots}{}\right.$

input
```
int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
2/d*a^5*(-1/2*(4*I*B-4*A)/a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(
1/2)*2^(1/2)/a^(1/2))-1/a^2*(((-19/16*I*B+149/128*A)*(a+I*a*tan(d*x+c))^(7
/2)+(145/48*I*a*B-1127/384*a*A)*(a+I*a*tan(d*x+c))^(5/2)+(-127/48*I*B*a^2+
1049/384*A*a^2)*(a+I*a*tan(d*x+c))^(3/2)+(13/16*I*B*a^3-107/128*A*a^3)*(a+
I*a*tan(d*x+c))^(1/2))/a^4/tan(d*x+c)^4+3/128*(121*A-120*I*B)/a^(1/2)*arct
anh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(206) = 412$.

Time = 0.12 (sec) , antiderivative size = 944, normalized size of antiderivative = 3.62

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/768*(1536*sqrt(2)*sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2)*(d*e^(8*I*d*x + 8*
I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x
+ 2*I*c) + d)*log(4*((-I*A - B)*a^3*e^(I*d*x + I*c) - sqrt((A^2 - 2*I*A*B
- B^2)*a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 1536*sqrt(2)*sqrt((A^2 - 2*I
*A*B - B^2)*a^5/d^2)*(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*
d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((-I*A - B)*a^3
*e^(I*d*x + I*c) - sqrt((A^2 - 2*I*A*B - B^2)*a^5/d^2)*(-I*d*e^(2*I*d*x +
2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A -
 B)*a^2)) + 9*sqrt((14641*A^2 - 29040*I*A*B - 14400*B^2)*a^5/d^2)*(d*e^(8*
I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e
^(2*I*d*x + 2*I*c) + d)*log(16*(3*(-121*I*A - 120*B)*a^3*e^(2*I*d*x + 2*I*
c) + (-121*I*A - 120*B)*a^3 + 2*sqrt(2)*sqrt((14641*A^2 - 29040*I*A*B - 14
400*B^2)*a^5/d^2)*(I*d*e^(3*I*d*x + 3*I*c) + I*d*e^(I*d*x + I*c))*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-121*I*A - 120*B)*a)) -
9*sqrt((14641*A^2 - 29040*I*A*B - 14400*B^2)*a^5/d^2)*(d*e^(8*I*d*x + 8*I*
c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x +
2*I*c) + d)*log(16*(3*(-121*I*A - 120*B)*a^3*e^(2*I*d*x + 2*I*c) + (-121*I
*A - 120*B)*a^3 + 2*sqrt(2)*sqrt((14641*A^2 - 29040*I*A*B - 14400*B^2)*a^5
/d^2)*(-I*d*e^(3*I*d*x + 3*I*c) - I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**5*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.12

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{a^4\left(\frac{768\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}}-\frac{9(121A-120iB)\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(3(ia\tan(dx+c)+a)^{\frac{7}{2}}(149A-\cdots\right.}{\cdots}\right.}{384d\cdots}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/384*a^4*(768*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
 + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2) - 9*(1
21*A - 120*I*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d
*x + c) + a) + sqrt(a)))/a^(3/2) + 2*(3*(I*a*tan(d*x + c) + a)^(7/2)*(149*
A - 152*I*B) - (I*a*tan(d*x + c) + a)^(5/2)*(1127*A - 1160*I*B)*a + (I*a*t
an(d*x + c) + a)^(3/2)*(1049*A - 1016*I*B)*a^2 - 3*sqrt(I*a*tan(d*x + c) +
 a)*(107*A - 104*I*B)*a^3)/((I*a*tan(d*x + c) + a)^4*a - 4*(I*a*tan(d*x +
c) + a)^3*a^2 + 6*(I*a*tan(d*x + c) + a)^2*a^3 - 4*(I*a*tan(d*x + c) + a)*
a^4 + a^5))/d
```

Giac [F(-2)]

Exception generated.

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 3094, normalized size of antiderivative = 11.85

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
(((107*A*a^6 - B*a^6*104i)*(a + a*tan(c + d*x)*1i)^(1/2))/(64*d) - ((149*A
*a^3 - B*a^3*152i)*(a + a*tan(c + d*x)*1i)^(7/2))/(64*d) - ((1049*A*a^5 -
B*a^5*1016i)*(a + a*tan(c + d*x)*1i)^(3/2))/(192*d) + ((1127*A*a^4 - B*a^4
*1160i)*(a + a*tan(c + d*x)*1i)^(5/2))/(192*d))/((a + a*tan(c + d*x)*1i)^4
 - 4*a^3*(a + a*tan(c + d*x)*1i) - 4*a*(a + a*tan(c + d*x)*1i)^3 + 6*a^2*(
a + a*tan(c + d*x)*1i)^2 + a^4) - 2*atanh((384*d^4*(a + a*tan(c + d*x)*1i)
^(1/2)*(((485809*A^4*a^22)/(262144*d^4) + (529*B^4*a^22)/(64*d^4) + (11229
*A^2*B^2*a^22)/(2048*d^4) + (A*B^3*a^22*1127i)/(128*d^4) + (A^3*B*a^22*341
53i)/(8192*d^4))^(1/2)/(64*a^6) + (262841*A^2*a^5)/(32768*d^2) - (4073*B^2
*a^5)/(512*d^2) - (A*B*a^5*32719i)/(2048*d^2))^(1/2)*((485809*A^4*a^22)/(2
62144*d^4) + (529*B^4*a^22)/(64*d^4) + (11229*A^2*B^2*a^22)/(2048*d^4) + (
A*B^3*a^22*1127i)/(128*d^4) + (A^3*B*a^22*34153i)/(8192*d^4))^(1/2))/((431
443*A^3*a^14*d)/256 - B^3*a^14*d*3542i + (21783*A*B^2*a^14*d)/4 + (A^2*B*a
^14*d*6993i)/32 + 214*A*a^3*d^3*((485809*A^4*a^22)/(262144*d^4) + (529*B^4
*a^22)/(64*d^4) + (11229*A^2*B^2*a^22)/(2048*d^4) + (A*B^3*a^22*1127i)/(12
8*d^4) + (A^3*B*a^22*34153i)/(8192*d^4))^(1/2) - B*a^3*d^3*((485809*A^4*a^
22)/(262144*d^4) + (529*B^4*a^22)/(64*d^4) + (11229*A^2*B^2*a^22)/(2048*d^
4) + (A*B^3*a^22*1127i)/(128*d^4) + (A^3*B*a^22*34153i)/(8192*d^4))^(1/2)*
208i) + (697*A^2*a^8*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*(((485809*A^4*a^22)
/(262144*d^4) + (529*B^4*a^22)/(64*d^4) + (11229*A^2*B^2*a^22)/(2048*d^...
```

Reduce [F]

$$\int \cot^5(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a^2\Bigg(-\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^5\tan(dx+c)^3\,dx\right)b - \left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^5\tan(dx+c)^2\,dx\right)a + 2\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^5\tan(dx+c)^2\,dx\right)bi + 2\left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^5\tan(dx+c)\,dx\right)ai + \left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^5\tan(dx+c)\,dx\right)b + \left(\int\sqrt{\tan(dx+c)\,i+1}\,\cot(dx+c)^5\,dx\right)a\Bigg)$$

input
```
int(cot(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**5*tan(c + d*x)
**3,x)*b - int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**5*tan(c + d*x)**2,x)
*a + 2*int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**5*tan(c + d*x)**2,x)*b*i
 + 2*int(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**5*tan(c + d*x),x)*a*i + in
t(sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**5*tan(c + d*x),x)*b + int(sqrt(ta
n(c + d*x)*i + 1)*cot(c + d*x)**5,x)*a)
```

3.90 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 205

$$\begin{aligned}\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{(iA-B)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} \\ &\quad + \frac{4(5A+7iB)\sqrt{a+ia\tan(c+dx)}}{5ad} - \frac{(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5ad} \\ &\quad - \frac{(25A+23iB)(a+ia\tan(c+dx))^{3/2}}{15a^2d}\end{aligned}$$

output
```
1/2*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
a^(1/2)/d+(I*A-B)*tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+4/5*(5*A+7*I*B)*
(a+I*a*tan(d*x+c))^(1/2)/a/d-1/5*(5*A+7*I*B)*tan(d*x+c)^2*(a+I*a*tan(d*x+c
))^(1/2)/a/d-1/15*(25*A+23*I*B)*(a+I*a*tan(d*x+c))^(3/2)/a^2/d
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{35A+61iB+(10iA-38B)\tan(c+dx)+2(5A+iB)\tan^2(c+dx)+6B\tan^3(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]]
,x]
```

output
```
((A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]
*Sqrt[a]*d) + (35*A + (61*I)*B + ((10*I)*A - 38*B)*Tan[c + d*x] + 2*(5*A +
 I*B)*Tan[c + d*x]^2 + 6*B*Tan[c + d*x]^3)/(15*d*Sqrt[a + I*a*Tan[c + d*x]
])
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.361, Rules used = {3042, 4078, 27, 3042, 4080, 27, 3042, 4075, 3042, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4078}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \frac{1}{2}\tan^2(c+dx)\sqrt{i\tan(c+dx)a+a}(6a(iA-B)+a(5A+7iB)\tan(c+dx))dx}{a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \tan^2(c+dx)\sqrt{i\tan(c+dx)a+a}(6a(iA-B)+a(5A+7iB)\tan(c+dx))dx}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \tan(c+dx)^2\sqrt{i\tan(c+dx)a+a}(6a(iA-B)+a(5A+7iB)\tan(c+dx))dx}{2a^2}$$

$\Big\downarrow$ 4080

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2\int -\frac{1}{2}\tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4a^2(5A+7iB)-a^2(25iA-23B)\tan(c+dx))dx}{5a}+\frac{2a(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}}{2a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4a^2(5A+7iB)-a^2(25iA-23B)\tan(c+dx))dx}{5a}}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4a^2(5A+7iB)-a^2(25iA-23B)\tan(c+dx))dx}{5a}}{2a^2}$$

$\Big\downarrow$ 4075

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{\int \sqrt{i\tan(c+dx)a+a}((25iA-23B)a^2+4(5A+7iB)\tan(c+dx)a^2)dx-\frac{2a(25A+23iB)(a+ia\tan(c+d}{3d}}{5a}}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{\int\sqrt{i\tan(c+dx)a+a}\left((25iA-23B)a^2+4(5A+7iB)\tan(c+dx)a^2\right)dx-\frac{2a(25A+23iB)(a+ia\tan(c+d\cdots}{3d}}{5a}}{2a^2}$$

$\Big\downarrow$ 4010

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{5a^2(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx+\frac{8a^2(5A+7iB)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(25A+23iB)(a+ia\tan(c+dx\cdots}{3d}}{5a}}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{5a^2(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx+\frac{8a^2(5A+7iB)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(25A+23iB)(a+ia\tan(c+dx\cdots}{3d}}{5a}}{2a^2}$$

$\Big\downarrow$ 3961

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{-\frac{10ia^3(B+iA)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+\frac{8a^2(5A+7iB)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(25A+23iB)(\cdots}{}}{5a}}{2a^2}$$

$\Big\downarrow$ 219

$$\frac{(-B+iA)\tan^3(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(5A+7iB)\tan^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{-\frac{5i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{8a^2(5A+7iB)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(25A+23iB)(a\cdots}{}}{5a}}{2a^2}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I*A - B)*Tan[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((2*a*(5*A + (
7*I)*B)*Tan[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(5*d) - (((-5*I)*Sqrt[2
]*a^(5/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])
/d + (8*a^2*(5*A + (7*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/d - (2*a*(25*A + (
23*I)*B)*(a + I*a*Tan[c + d*x])^(3/2))/(3*d))/(5*a))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{2iB(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{4iBa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{2Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4ia^2B\sqrt{a+ia\tan(dx+c)}+2Aa^2\sqrt{a+ia\tan(}}{a^3d}$
default	$\frac{\frac{2iB(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{4iBa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{2Aa(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4ia^2B\sqrt{a+ia\tan(dx+c)}+2Aa^2\sqrt{a+ia\tan(}}{a^3d}$
parts	$\frac{2A\left(-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+a\sqrt{a+ia\tan(dx+c)}+\frac{a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4}+\frac{a^2}{2\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^2}+\frac{2iB\left(\right.}{}$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
2/d/a^3*(1/5*I*B*(a+I*a*tan(d*x+c))^(5/2)-2/3*I*B*a*(a+I*a*tan(d*x+c))^(3/
2)-1/3*A*a*(a+I*a*tan(d*x+c))^(3/2)+2*I*B*a^2*(a+I*a*tan(d*x+c))^(1/2)+A*a
^2*(a+I*a*tan(d*x+c))^(1/2)+1/4*a^(5/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a
*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+1/2*a^3*(A+I*B)/(a+I*a*tan(d*x+c))^(1/
2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(163) = 326.

Time = 0.10 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.18

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$

$$-\frac{15\sqrt{2}\left(ade^{(5i\,dx+5i\,c)}+2\,ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)\sqrt{\frac{A^2-2i\,AB-B^2}{ad^2}}\log\left(-\frac{4\left((-i\,A-B)ae^{(i\,dx+i\,c)}+\left(i\,ade^{(2i\,dx+}\right.\right.}{}\right.}{}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
-1/60*(15*sqrt(2)*(a*d*e^(5*I*d*x + 5*I*c) + 2*a*d*e^(3*I*d*x + 3*I*c) + a
*d*e^(I*d*x + I*c))*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2))*log(-4*((-I*A - B)
*a*e^(I*d*x + I*c) + (I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2)))*e^(-I*d*x - I*c)/(I*
A + B)) - 15*sqrt(2)*(a*d*e^(5*I*d*x + 5*I*c) + 2*a*d*e^(3*I*d*x + 3*I*c)
+ a*d*e^(I*d*x + I*c))*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2))*log(-4*((-I*A -
 B)*a*e^(I*d*x + I*c) + (-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2)))*e^(-I*d*x - I*c)
/(I*A + B)) - 2*sqrt(2)*((35*A + 103*I*B)*e^(6*I*d*x + 6*I*c) + 5*(25*A +
41*I*B)*e^(4*I*d*x + 4*I*c) + 15*(7*A + 11*I*B)*e^(2*I*d*x + 2*I*c) + 15*A
 + 15*I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(a*d*e^(5*I*d*x + 5*I*c) + 2
*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^3(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**3/sqrt(I*a*(tan(c + d*x) - I))
, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.77

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{15\sqrt{2}(A-iB)a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-24i(ia\tan(dx+c)+a)^{\frac{5}{2}}Ba+40(ia\tan(dx+c)}{60a^4d}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
-1/60*(15*sqrt(2)*(A - I*B)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d
*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 24*I*(I*a*
tan(d*x + c) + a)^(5/2)*B*a + 40*(I*a*tan(d*x + c) + a)^(3/2)*(A + 2*I*B)*
a^2 - 120*sqrt(I*a*tan(d*x + c) + a)*(A + 2*I*B)*a^3 - 60*(A + I*B)*a^4/sq
rt(I*a*tan(d*x + c) + a))/(a^4*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{A}{d\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}} \\ &+ \frac{B\,1\mathrm{i}}{d\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}} \\ &+ \frac{2A\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}}{a\,d} \\ &- \frac{2A\,(a+a\tan(c+dx)\,1\mathrm{i})^{3/2}}{3a^2\,d} \\ &+ \frac{B\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}\,4\mathrm{i}}{a\,d} \\ &- \frac{B\,(a+a\tan(c+dx)\,1\mathrm{i})^{3/2}\,4\mathrm{i}}{3a^2\,d} \\ &+ \frac{B\,(a+a\tan(c+dx)\,1\mathrm{i})^{5/2}\,2\mathrm{i}}{5a^3\,d} \\ &+ \frac{\sqrt{2}\,B\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}}{2\sqrt{-a}}\right)\,1\mathrm{i}}{2\sqrt{-a}\,d} \\ &- \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}\,1\mathrm{i}}{2\sqrt{a}}\right)\,1\mathrm{i}}{2\sqrt{a}\,d}\end{aligned}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
A/(d*(a + a*tan(c + d*x)*1i)^(1/2)) + (B*1i)/(d*(a + a*tan(c + d*x)*1i)^(1
/2)) + (2*A*(a + a*tan(c + d*x)*1i)^(1/2))/(a*d) - (2*A*(a + a*tan(c + d*x
)*1i)^(3/2))/(3*a^2*d) + (B*(a + a*tan(c + d*x)*1i)^(1/2)*4i)/(a*d) - (B*(
a + a*tan(c + d*x)*1i)^(3/2)*4i)/(3*a^2*d) + (B*(a + a*tan(c + d*x)*1i)^(5
/2)*2i)/(5*a^3*d) + (2^(1/2)*B*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)
)/(2*(-a)^(1/2)))*1i)/(2*(-a)^(1/2)*d) - (2^(1/2)*A*atan((2^(1/2)*(a + a*t
an(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*1i)/(2*a^(1/2)*d)
```

Reduce [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^5}{\tan(dx+c)^2+1}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^4}{\tan(dx+c)^2+1}dx\right)ai+\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^4}{\tan(dx+c)^2+1}dx\right.}{a}$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**5)/(tan(c + d*x)*
*2 + 1),x)*b*i - int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4)/(tan(c + d
*x)**2 + 1),x)*a*i + int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4)/(tan(c
 + d*x)**2 + 1),x)*b + int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3)/(tan
(c + d*x)**2 + 1),x)*a))/a
```

3.91 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 159

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{(iA-B)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{4(iA-B)\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{(3iA-5B)(a+ia\tan(c+dx))^{3/2}}{3a^2d}$$

output
```
1/2*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
a^(1/2)/d+(I*A-B)*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-4*(I*A-B)*(a+I*a
*tan(d*x+c))^(1/2)/a/d+1/3*(3*I*A-5*B)*(a+I*a*tan(d*x+c))^(3/2)/a^2/d
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.70

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}+\frac{-9iA+7B+(6A+2iB)\tan(c+dx)+2B\tan^2(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]]
,x]
```

output
```
((I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]
*Sqrt[a]*d) + ((-9*I)*A + 7*B + (6*A + (2*I)*B)*Tan[c + d*x] + 2*B*Tan[c +
 d*x]^2)/(3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4078, 27, 3042, 4075, 3042, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \frac{1}{2}\tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4a(iA-B)+a(3A+5iB)\tan(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4a(iA-B)+a(3A+5iB)\tan(c+dx))dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4a(iA-B)+a(3A+5iB)\tan(c+dx))dx}{2a^2}$$

$$\Big\downarrow \text{4075}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \sqrt{i\tan(c+dx)a+a}(4a(iA-B)\tan(c+dx)-a(3A+5iB))dx-\frac{2(-5B+3iA)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \sqrt{i\tan(c+dx)a+a}(4a(iA-B)\tan(c+dx)-a(3A+5iB))dx-\frac{2(-5B+3iA)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}$$

$$\Big\downarrow \text{4010}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx-\frac{2(-5B+3iA)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{8a(-B+iA)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx-\frac{2(-5B+3iA)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{8a(-B+iA)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}$$

$$\Big\downarrow \text{3961}$$

$$\frac{\frac{(-B+iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-}{\frac{-\frac{2ia^2(A-iB)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+\frac{8a(-B+iA)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2(-5B+3iA)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{(-B+iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-}{\frac{-\frac{i\sqrt{2}a^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{8a(-B+iA)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2(-5B+3iA)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((I*A - B)*Tan[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((-I)*Sqrt[2]
*a^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/
d + (8*a*(I*A - B)*Sqrt[a + I*a*Tan[c + d*x]])/d - (2*((3*I)*A - 5*B)*(a +
 I*a*Tan[c + d*x])^(3/2))/(3*d))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{2i\left(\frac{iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-iaB\sqrt{a+ia\tan(dx+c)}-aA\sqrt{a+ia\tan(dx+c)}+\frac{a^{\frac{3}{2}}(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right.}{4}\right.}{d\,a^{2}}$
default	$\frac{2i\left(\frac{iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-iaB\sqrt{a+ia\tan(dx+c)}-aA\sqrt{a+ia\tan(dx+c)}+\frac{a^{\frac{3}{2}}(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right.}{4}\right.}{d\,a^{2}}$
parts	$\frac{2iA\left(-\sqrt{a+ia\tan(dx+c)}+\frac{\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4}-\frac{a}{2\sqrt{a+ia\tan(dx+c)}}\right)}{da}+\frac{2B\left(-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right.}{}$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d/a^2*(1/3*I*B*(a+I*a*tan(d*x+c))^(3/2)-I*a*B*(a+I*a*tan(d*x+c))^(1/2)
-a*A*(a+I*a*tan(d*x+c))^(1/2)+1/4*a^(3/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I
*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-1/2*a^2*(A+I*B)/(a+I*a*tan(d*x+c))^(
1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(127) = 254.

Time = 0.12 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.46

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$

$$-\frac{3\sqrt{2}\left(ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)\sqrt{-\frac{A^2-2i\,AB-B^2}{ad^2}}\log\left(-\frac{4\left((-i\,A-B)ae^{(i\,dx+i\,c)}+\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}}}\right.}{i\,A+B}\right.}{}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
-1/12*(3*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(-(A^
2 - 2*I*A*B - B^2)/(a*d^2))*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) + (a*d*e^
(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*
I*A*B - B^2)/(a*d^2)))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(2)*(a*d*e^(3*I
*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a*d^2))*
log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a*d^2)))*e^(-I
*d*x - I*c)/(I*A + B)) + 2*sqrt(2)*((15*I*A - 7*B)*e^(4*I*d*x + 4*I*c) + 1
8*(I*A - B)*e^(2*I*d*x + 2*I*c) + 3*I*A - 3*B)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1)))/(a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^2(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I))
, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\left(3\sqrt{2}(A-iB)a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-8i\left(ia\tan(dx+c)+a\right)^{\frac{3}{2}}Ba+24\sqrt{ia\tan(dx+c)}\right.}{12a^3d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
-1/12*I*(3*sqrt(2)*(A - I*B)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(
d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 8*I*(I*a*
tan(d*x + c) + a)^(3/2)*B*a + 24*sqrt(I*a*tan(d*x + c) + a)*(A + I*B)*a^2
+ 12*(A + I*B)*a^3/sqrt(I*a*tan(d*x + c) + a))/(a^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{A\,1\mathrm{i}}{d\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}} + \frac{B}{d\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}} - \frac{A\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,2\mathrm{i}}{a\,d} + \frac{2\,B\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{a\,d} - \frac{2\,B\,{\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)}^{3/2}}{3\,a^2\,d} - \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{2\,\sqrt{-a}\,d} - \frac{\sqrt{2}\,B\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,1\mathrm{i}}{2\,\sqrt{a}}\right)\,1\mathrm{i}}{2\,\sqrt{a}\,d}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
B/(d*(a + a*tan(c + d*x)*1i)^(1/2)) - (A*1i)/(d*(a + a*tan(c + d*x)*1i)^(1
/2)) - (A*(a + a*tan(c + d*x)*1i)^(1/2)*2i)/(a*d) + (2*B*(a + a*tan(c + d*
x)*1i)^(1/2))/(a*d) - (2*B*(a + a*tan(c + d*x)*1i)^(3/2))/(3*a^2*d) - (2^(
1/2)*A*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/(2
*(-a)^(1/2)*d) - (2^(1/2)*B*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i
)/(2*a^(1/2)))*1i)/(2*a^(1/2)*d)
```

Reduce [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^4}{\tan(dx+c)^2+1}dx\right)bi - \left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^3}{\tan(dx+c)^2+1}dx\right)ai + \left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^3}{\tan(dx+c)^2+1}dx\right.}{a}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4)/(tan(c + d*x)*
*2 + 1),x)*b*i - int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3)/(tan(c + d
*x)**2 + 1),x)*a*i + int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3)/(tan(c
 + d*x)**2 + 1),x)*b + int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan
(c + d*x)**2 + 1),x)*a))/a
```

3.92 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 109

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad}$$

output
```
-1/2*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)
/a^(1/2)/d-(A+I*B)/d/(a+I*a*tan(d*x+c))^(1/2)-2*I*B*(a+I*a*tan(d*x+c))^(1/
2)/a/d
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}} - \frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x
]
```

output
```
-(((A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[
2]*Sqrt[a]*d)) - (A + I*B)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((2*I)*B*Sqrt[
a + I*a*Tan[c + d*x]])/(a*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4075, 3042, 4009, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4075}$$

$$\int \frac{A\tan(c+dx)-B}{\sqrt{i\tan(c+dx)a+a}}dx - \frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\tan(c+dx)-B}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{4009}$$

$$-\frac{(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{3961}$$

$$\frac{i(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{219}$$

$$\frac{i(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}-\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2iB\sqrt{a+ia\tan(c+dx)}}{ad}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
(I*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[
2]*Sqrt[a]*d) - (A + I*B)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((2*I)*B*Sqrt[a
 + I*a*Tan[c + d*x]])/(a*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-2iB\sqrt{a+ia\tan(dx+c)}-\frac{\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2}-\frac{a(iB+A)}{\sqrt{a+ia\tan(dx+c)}}}{ad}$
default	$\frac{-2iB\sqrt{a+ia\tan(dx+c)}-\frac{\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2}-\frac{a(iB+A)}{\sqrt{a+ia\tan(dx+c)}}}{ad}$
parts	$\frac{A\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}-\frac{1}{\sqrt{a+ia\tan(dx+c)}}\right)}{d}+\frac{2iB\left(-\sqrt{a+ia\tan(dx+c)}+\frac{\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan}}{2\sqrt{}}\right.}{4}\right.}{da}$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/d/a*(-I*B*(a+I*a*tan(d*x+c))^(1/2)-1/4*a^(1/2)*(A-I*B)*2^(1/2)*arctanh(1
/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-1/2*a*(A+I*B)/(a+I*a*tan(d*x+
c))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.00

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(\sqrt{2}ad\sqrt{\frac{A^2-2i\,AB-B^2}{ad^2}}e^{(i\,dx+i\,c)}\log\left(-\frac{4\left((-i\,A-B)ae^{(i\,dx+i\,c)}+\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{A^2-2i\,AB-B^2}{ad^2}}\right)e^{(-i\,c}}{i\,A+B}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
1/4*(sqrt(2)*a*d*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2))*e^(I*d*x + I*c)*log(-
4*((-I*A - B)*a*e^(I*d*x + I*c) + (I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2)))*e^(-I*d
*x - I*c)/(I*A + B)) - sqrt(2)*a*d*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2))*e^(
I*d*x + I*c)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) + (-I*a*d*e^(2*I*d*x + 2
*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2
)/(a*d^2)))*e^(-I*d*x - I*c)/(I*A + B)) - 2*sqrt(2)*((A + 5*I*B)*e^(2*I*d*
x + 2*I*c) + A + I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/
(a*d)
```

Sympy [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan{(c+dx)}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}(A-i\,B)a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)-8i\,\sqrt{i\,a\tan{(dx+c)}+a}Ba-\frac{4\,(A+i\,B)a^2}{\sqrt{i\,a\tan(dx+c)+a}}}{4\,a^2d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
1/4*(sqrt(2)*(A - I*B)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x +
c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 8*I*sqrt(I*a*ta
n(d*x + c) + a)*B*a - 4*(A + I*B)*a^2/sqrt(I*a*tan(d*x + c) + a))/(a^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.29

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{A}{d\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}} - \frac{B\,1\mathrm{i}}{d\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}} - \frac{B\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,2\mathrm{i}}{a\,d} - \frac{\sqrt{2}\,B\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\sqrt{-a}}\right)\,1\mathrm{i}}{2\sqrt{-a}\,d} - \frac{\sqrt{2}\,A\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
- A/(d*(a + a*tan(c + d*x)*1i)^(1/2)) - (B*1i)/(d*(a + a*tan(c + d*x)*1i)^
(1/2)) - (B*(a + a*tan(c + d*x)*1i)^(1/2)*2i)/(a*d) - (2^(1/2)*B*atan((2^(
1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/(2*(-a)^(1/2)*d) -
 (2^(1/2)*A*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/(2
*a^(1/2)*d)
```

Reduce [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^3}{\tan(dx+c)^2+1}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2}{\tan(dx+c)^2+1}dx\right)ai+\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2}{\tan(dx+c)^2+1}dx\right.}{a}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3)/(tan(c + d*x)*
*2 + 1),x)*b*i - int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c + d
*x)**2 + 1),x)*a*i + int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c
 + d*x)**2 + 1),x)*b + int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c
+ d*x)**2 + 1),x)*a))/a
```

3.93 $\int \frac{A+B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 82

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}+\frac{iA-B}{d\sqrt{a+ia\tan(c+dx)}}$$

output
```
-1/2*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)
/a^(1/2)/d+(I*A-B)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}+\frac{iA-B}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(A + B*Tan[c + d*x])/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
-(((I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[
2]*Sqrt[a]*d)) + (I*A - B)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3042, 4009, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4009}$$

$$\frac{(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx}{2a}+\frac{-B+iA}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx}{2a}+\frac{-B+iA}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3961}$$

$$\frac{-B+iA}{d\sqrt{a+ia\tan(c+dx)}}-\frac{i(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{-B+iA}{d\sqrt{a+ia\tan(c+dx)}}-\frac{i(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}$$

input
```
Int[(A + B*Tan[c + d*x])/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-I)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*Sqrt[a]*d) + (I*A - B)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{2i\left(-\frac{\left(-\frac{iB}{2}+\frac{A}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}-\frac{-\frac{A}{2}-\frac{iB}{2}}{\sqrt{a+ia\tan(dx+c)}}\right)}{d}$
default	$\frac{2i\left(-\frac{\left(-\frac{iB}{2}+\frac{A}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}-\frac{-\frac{A}{2}-\frac{iB}{2}}{\sqrt{a+ia\tan(dx+c)}}\right)}{d}$
parts	$\frac{2iAa\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{3}{2}}}+\frac{1}{2a\sqrt{a+ia\tan(dx+c)}}\right)}{d}+\frac{B\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}-\frac{}{\sqrt{a+ia}}\right.}{d}$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d*(-1/2*(-1/2*I*B+1/2*A)*2^(1/2)/a^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c)
)^(1/2)*2^(1/2)/a^(1/2))-(-1/2*A-1/2*I*B)/(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(63) = 126$.

Time = 0.08 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.00

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{\left(\sqrt{2}ad\sqrt{-\frac{A^2-2i\,AB-B^2}{ad^2}}e^{(i\,dx+i\,c)}\log\left(-\frac{4\left((-i\,A-B)ae^{(i\,dx+i\,c)}+\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{A^2-2i\,AB-B^2}{ad^2}}\right)e^{(-}}{i\,A+B}\right.\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*a*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a*d^2))*e^(I*d*x + I*c)*log(
-4*((-I*A - B)*a*e^(I*d*x + I*c) + (a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a*d^2)))*e^(-I*d*x
 - I*c)/(I*A + B)) - sqrt(2)*a*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a*d^2))*e^(I
*d*x + I*c)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) - (a*d*e^(2*I*d*x + 2*I*c
) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a*
d^2)))*e^(-I*d*x - I*c)/(I*A + B)) - 2*sqrt(2)*((-I*A + B)*e^(2*I*d*x + 2*
I*c) - I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{A+B\tan{(c+dx)}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\left(\sqrt{2}(A-i\,B)\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\,(A+i\,B)a}{\sqrt{i\,a\tan(dx+c)+a}}\right)}{4\,ad}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*I*(sqrt(2)*(A - I*B)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
+ c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(A + I*B)*a
/sqrt(I*a*tan(d*x + c) + a))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.43

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{A\,1\mathrm{i}}{d\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}-\frac{B}{d\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}} + \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{2\,\sqrt{-a}\,d} - \frac{\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{2\,\sqrt{a}\,d}$$

input
```
int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(A*1i)/(d*(a + a*tan(c + d*x)*1i)^(1/2)) - B/(d*(a + a*tan(c + d*x)*1i)^(1
/2)) + (2^(1/2)*A*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/
2)))*1i)/(2*(-a)^(1/2)*d) - (2^(1/2)*B*atanh((2^(1/2)*(a + a*tan(c + d*x)*
1i)^(1/2))/(2*a^(1/2))))/(2*a^(1/2)*d)
```

Reduce [F]

$$\int \frac{A + B\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(-2\sqrt{\tan\left(dx+c\right)i+1}\,ai - \left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2}{\tan(dx+c)^2+1}dx\right)\tan\left(dx+c\right)^2 bdi - \left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\tan(dx}{\tan(dx+c)^2+1}\right.}{}$$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*a*i - int((sqrt(tan(c + d*x)*i + 1
)*tan(c + d*x)**2)/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*b*d*i - int((s
qrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c + d*x)**2 + 1),x)*b*d*i -
4*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*tan
(c + d*x)**2*a*d*i + int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c +
d*x)**2 + 1),x)*tan(c + d*x)**2*b*d - 4*int((sqrt(tan(c + d*x)*i + 1)*tan(
c + d*x))/(tan(c + d*x)**2 + 1),x)*a*d*i + int((sqrt(tan(c + d*x)*i + 1)*t
an(c + d*x))/(tan(c + d*x)**2 + 1),x)*b*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.94 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 114

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

output

```
-2*A*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d+1/2*(A-I*B)*arcta
nh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(1/2)/d+(A+I*B)
/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\left(\frac{4iA\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{2(iA-B)}{\sqrt{a+ia\tan(c+dx)}}\right)}{2d}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x
]
```

output
```
((I/2)*(((4*I)*A*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] - (S
qrt[2]*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sq
rt[a] - (2*(I*A - B))/Sqrt[a + I*a*Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4079, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \ 4079$$

$$\frac{\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}(2aA-a(iA-B)\tan(c+dx))dx}{a^2}+\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}(2aA-a(iA-B)\tan(c+dx))dx}{2a^2}+\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(2aA-a(iA-B)\tan(c+dx))}{\tan(c+dx)}dx}{2a^2}+\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 4083$$

$$\frac{a(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+2A\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{2a^2}+\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{a(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+2A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a^2}+\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 3961$$

$$\frac{2A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2ia^2(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a^2}+\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 219$$

$$\frac{2A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{i\sqrt{2}a^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a^2}+\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 4082$$

$$\frac{\frac{2a^2A\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{i\sqrt{2}a^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a^2}+\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \ 73$$

$$\frac{-\frac{4iaA\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{i\sqrt{2}a^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a^2}+\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{i\sqrt{2}a^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{4a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}}{2a^2}+\frac{A+iB}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((-4*a^(3/2)*A*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - (I*Sqrt[2]
*a^(3/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/
d)/(2*a^2) + (A + I*B)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{2a\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{3}{2}}}-\frac{-iB-A}{2a\sqrt{a+ia\tan(dx+c)}}-\frac{A\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)}{d}$	99
default	$\frac{2a\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{3}{2}}}-\frac{-iB-A}{2a\sqrt{a+ia\tan(dx+c)}}-\frac{A\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)}{d}$	99

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/d*a*(-1/4*(-A+I*B)/a^(3/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*
2^(1/2)/a^(1/2))-1/2*(-I*B-A)/a/(a+I*a*tan(d*x+c))^(1/2)-1/a^(3/2)*A*arcta
nh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(88) = 176.

Time = 0.12 (sec) , antiderivative size = 575, normalized size of antiderivative = 5.04

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$

$$-\frac{\left(\sqrt{2}ad\sqrt{\frac{A^2-2i\,AB-B^2}{ad^2}}e^{(i\,dx+i\,c)}\log\left(-\frac{4\left((-i\,A-B)ae^{(i\,dx+i\,c)}+(i\,ade^{(2i\,dx+2i\,c)}+i\,ad)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{A^2-2i\,AB-B^2}{ad^2}}\right)e^{(}}{i\,A+B}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-1/4*(sqrt(2)*a*d*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2))*e^(I*d*x + I*c)*log(
-4*((-I*A - B)*a*e^(I*d*x + I*c) + (I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2)))*e^(-I*
d*x - I*c)/(I*A + B)) - sqrt(2)*a*d*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2))*e^
(I*d*x + I*c)*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) + (-I*a*d*e^(2*I*d*x +
2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^
2)/(a*d^2)))*e^(-I*d*x - I*c)/(I*A + B)) + 2*a*d*sqrt(A^2/(a*d^2))*e^(I*d*
x + I*c)*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2 + 2*sqrt(2)*(a^2*d*e^
(3*I*d*x + 3*I*c) + a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(A^2/(a*d^2)))*e^(-2*I*d*x - 2*I*c)/A) - 2*a*d*sqrt(A^2/(a*d^2))*e^
(I*d*x + I*c)*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2 - 2*sqrt(2)*(a^2
*d*e^(3*I*d*x + 3*I*c) + a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(A^2/(a*d^2)))*e^(-2*I*d*x - 2*I*c)/A) - 2*sqrt(2)*((A + I*B)*
e^(2*I*d*x + 2*I*c) + A + I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*
x - I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(A+B\tan{(c+dx)})\cot{(c+dx)}}{\sqrt{ia(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\frac{\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{\sqrt{a}}-\frac{4\,A\log\left(\frac{\sqrt{i\,a\tan(dx+c)+a}-\sqrt{a}}{\sqrt{i\,a\tan(dx+c)+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{4\,(A+i\,B)}{\sqrt{i\,a\tan(dx+c)+a}}}{4\,d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
-1/4*(sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) - 4*A*log((sqrt(
I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/s
qrt(a) - 4*(A + I*B)/sqrt(I*a*tan(d*x + c) + a))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 515, normalized size of antiderivative = 4.52

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{A+B\,1i}{d\,\sqrt{a+a\tan\left(c+d\,x\right)\,1i}} - \frac{2\,A\,\mathrm{atanh}\left(\frac{28\,A^3\,a^{3/2}\,d\,\sqrt{a+a\tan(c+d\,x)\,1i}}{28\,d\,A^3\,a^2+8i\,d\,A^2\,B\,a^2+4\,d\,A\,B^2\,a^2}+\frac{4\,A\,B^2\,a^{3/2}\,d\,\sqrt{a+a\tan(c+d\,x)\,1i}}{28\,d\,A^3\,a^2+8i\,d\,A^2\,B\,a^2+4\,d\,A\,B^2\,a^2}+\frac{A^2\,B\,a^{3/2}\,d\,\sqrt{a+a\tan(c+d\,x)\,1i}\,8i}{28\,d\,A^3\,a^2+8i\,d\,A^2\,B\,a^2+4\,d\,A\,B^2\,a^2}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,A^3\,(-a)^{3/2}\,d\,\sqrt{a+a\tan(c+d\,x)\,1i}\,7i}{2\,(7\,d\,A^3\,a^2-5i\,d\,A^2\,B\,a^2+3\,d\,A\,B^2\,a^2-1i\,d\,B^3\,a^2)}+\frac{\sqrt{2}\,B^3\,(-a)^{3/2}\,d\,\sqrt{a+a\tan(c+d\,x)\,1i}}{2\,(7\,d\,A^3\,a^2-5i\,d\,A^2\,B\,a^2+3\,d\,A\,B^2\,a^2-1i\,d\,B^3\,a^2)}+\frac{\sqrt{2}\,A\,B^2\,(\ldots}{2\,(7\,d\,A^3\,a^2-5\ldots}\right.}{2\,\sqrt{-a}\,d}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
(A + B*1i)/(d*(a + a*tan(c + d*x)*1i)^(1/2)) - (2*A*atanh((28*A^3*a^(3/2)*
d*(a + a*tan(c + d*x)*1i)^(1/2))/(28*A^3*a^2*d + 4*A*B^2*a^2*d + A^2*B*a^2
*d*8i) + (4*A*B^2*a^(3/2)*d*(a + a*tan(c + d*x)*1i)^(1/2))/(28*A^3*a^2*d +
 4*A*B^2*a^2*d + A^2*B*a^2*d*8i) + (A^2*B*a^(3/2)*d*(a + a*tan(c + d*x)*1i
)^(1/2)*8i)/(28*A^3*a^2*d + 4*A*B^2*a^2*d + A^2*B*a^2*d*8i)))/(a^(1/2)*d)
+ (2^(1/2)*atanh((2^(1/2)*A^3*(-a)^(3/2)*d*(a + a*tan(c + d*x)*1i)^(1/2)*7
i)/(2*(7*A^3*a^2*d - B^3*a^2*d*1i + 3*A*B^2*a^2*d - A^2*B*a^2*d*5i)) + (2^
(1/2)*B^3*(-a)^(3/2)*d*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(7*A^3*a^2*d - B^
3*a^2*d*1i + 3*A*B^2*a^2*d - A^2*B*a^2*d*5i)) + (2^(1/2)*A*B^2*(-a)^(3/2)*
d*(a + a*tan(c + d*x)*1i)^(1/2)*3i)/(2*(7*A^3*a^2*d - B^3*a^2*d*1i + 3*A*B
^2*a^2*d - A^2*B*a^2*d*5i)) + (5*2^(1/2)*A^2*B*(-a)^(3/2)*d*(a + a*tan(c +
 d*x)*1i)^(1/2))/(2*(7*A^3*a^2*d - B^3*a^2*d*1i + 3*A*B^2*a^2*d - A^2*B*a^
2*d*5i)))*(A*1i + B))/(2*(-a)^(1/2)*d)
```

Reduce [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cot(dx+c)\tan(dx+c)^2}{\tan(dx+c)^2+1}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cot(dx+c)\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)ai+\left(\int \frac{\sqrt{\tan(dx+c)}}{t}\right.\right.}{a}$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)*tan(c + d*x)**2)/(
tan(c + d*x)**2 + 1),x)*b*i - int((sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)*t
an(c + d*x))/(tan(c + d*x)**2 + 1),x)*a*i + int((sqrt(tan(c + d*x)*i + 1)*
cot(c + d*x)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*b + int((sqrt(tan(c +
d*x)*i + 1)*cot(c + d*x))/(tan(c + d*x)**2 + 1),x)*a))/a
```

3.95 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 167

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(iA-2B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad}$$

output
```
(I*A-2*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d+1/2*(I*A+B)*
arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(1/2)/d+(A
+I*B)*cot(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-(2*A+I*B)*cot(d*x+c)*(a+I*a*ta
n(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i(A+2iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{-2iA+B-A\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]]
,x]
```

output
```
(I*(A + (2*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
+ ((I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[
2]*Sqrt[a]*d) + ((-2*I)*A + B - A*Cot[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*
x]])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4079, 27, 3042, 4081, 25, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{1}{2}\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}(2a(2A+iB)-3a(iA-B)\tan(c+dx))dx}{a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}(2a(2A+iB)-3a(iA-B)\tan(c+dx))dx}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(2a(2A+iB)-3a(iA-B)\tan(c+dx))}{\tan(c+dx)^2}dx}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 4081

$$\frac{\frac{\int -\cot(c+dx)\sqrt{i\tan(c+dx)a+a}((iA-2B)a^2+(2A+iB)\tan(c+dx)a^2)dx}{a}-\frac{2a(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}((iA-2B)a^2+(2A+iB)\tan(c+dx)a^2)dx}{a}-\frac{2a(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((iA-2B)a^2+(2A+iB)\tan(c+dx)a^2\right)}{\tan(c+dx)}dx}{a}-\frac{2a(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 4083

$$\frac{-\frac{a^2(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+a(-2B+iA)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}-\frac{2a(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+}}{d}}{2a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{a^2(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+a(-2B+iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{a}-\frac{2a(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\downarrow$ 3961

$$-\frac{\frac{a(-2B+iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2ia^3(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{a}-\frac{2a(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\downarrow$ 219

$$-\frac{\frac{a(-2B+iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{i\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{2a(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\downarrow$ 4082

$$-\frac{\frac{\frac{a^3(-2B+iA)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{i\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{2a(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\downarrow$ 73

$$-\frac{\frac{-\frac{2ia^2(-2B+iA)\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{i\sqrt{2}a^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{2a(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\downarrow$ 221

$$-\frac{-\frac{-\frac{2a^{5/2}(-2B+iA)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{i\sqrt{2}a^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{2a(2A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((A + I*B)*Cot[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (-(((-2*a^(5/2)*
(I*A - 2*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - (I*Sqrt[2]*a^
(5/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d)/
a) - (2*a*(2*A + I*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d)/(2*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2ia^2\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{5}{2}}}-\frac{iB+A}{2a^2\sqrt{a+ia\tan(dx+c)}}+\frac{\frac{iA\sqrt{a+ia\tan(dx+c)}}{2a\tan(dx+c)}+\frac{(2iB+A)\operatorname{arctanh}\left(\frac{\sqrt{a+ia}}{2\sqrt{a}}\right.}{}}{a^2}\right.}{d}$
default	$\frac{2ia^2\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{5}{2}}}-\frac{iB+A}{2a^2\sqrt{a+ia\tan(dx+c)}}+\frac{\frac{iA\sqrt{a+ia\tan(dx+c)}}{2a\tan(dx+c)}+\frac{(2iB+A)\operatorname{arctanh}\left(\frac{\sqrt{a+ia}}{2\sqrt{a}}\right.}{}}{a^2}\right.}{d}$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d*a^2*(-1/4*(-A+I*B)/a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1
/2)*2^(1/2)/a^(1/2))-1/2/a^2*(A+I*B)/(a+I*a*tan(d*x+c))^(1/2)+1/a^2*(1/2*I
*A*(a+I*a*tan(d*x+c))^(1/2)/a/tan(d*x+c)+1/2*(A+2*I*B)/a^(1/2)*arctanh((a+
I*a*tan(d*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 742 vs. 2(135) = 270.

Time = 0.11 (sec) , antiderivative size = 742, normalized size of antiderivative = 4.44

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
-1/4*(sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(-(A^2 -
 2*I*A*B - B^2)/(a*d^2))*log(-4*((-I*A - B)*a*e^(I*d*x + I*c) + (a*d*e^(2*
I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A
*B - B^2)/(a*d^2)))*e^(-I*d*x - I*c)/(I*A + B)) - sqrt(2)*(a*d*e^(3*I*d*x
+ 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a*d^2))*log(-
4*((-I*A - B)*a*e^(I*d*x + I*c) - (a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a*d^2)))*e^(-I*d*x
- I*c)/(I*A + B)) - (a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(-
(A^2 + 4*I*A*B - 4*B^2)/(a*d^2))*log(-16*(3*(I*A - 2*B)*a^2*e^(2*I*d*x + 2
*I*c) + (I*A - 2*B)*a^2 + 2*sqrt(2)*(a^2*d*e^(3*I*d*x + 3*I*c) + a^2*d*e^(
I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 + 4*I*A*B - 4*B
^2)/(a*d^2)))*e^(-2*I*d*x - 2*I*c)/(-I*A + 2*B)) + (a*d*e^(3*I*d*x + 3*I*c
) - a*d*e^(I*d*x + I*c))*sqrt(-(A^2 + 4*I*A*B - 4*B^2)/(a*d^2))*log(-16*(3
*(I*A - 2*B)*a^2*e^(2*I*d*x + 2*I*c) + (I*A - 2*B)*a^2 - 2*sqrt(2)*(a^2*d*
e^(3*I*d*x + 3*I*c) + a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(-(A^2 + 4*I*A*B - 4*B^2)/(a*d^2)))*e^(-2*I*d*x - 2*I*c)/(-I*A +
2*B)) + 2*sqrt(2)*((3*I*A - B)*e^(4*I*d*x + 4*I*c) + 2*I*A*e^(2*I*d*x + 2*
I*c) - I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(a*d*e^(3*I*d*x + 3*I*c
) - a*d*e^(I*d*x + I*c))
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\cot^2(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I))
, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ia\left(\frac{\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}}+\frac{2(A+2iB)\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{4((ia\tan(dx+c)+a)(2A+iB)-(A+iB)a)}{(ia\tan(dx+c)+a)^{\frac{3}{2}}a-\sqrt{ia\tan(dx+c)+a}\,a^2}\right)}{4d}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
-1/4*I*a*(sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2) + 2*(A + 2*I
*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a)
 + sqrt(a)))/a^(3/2) + 4*((I*a*tan(d*x + c) + a)*(2*A + I*B) - (A + I*B)*a
)/((I*a*tan(d*x + c) + a)^(3/2)*a - sqrt(I*a*tan(d*x + c) + a)*a^2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 2961, normalized size of antiderivative = 17.73

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
2*atanh((3*d^4*(a + a*tan(c + d*x)*1i)^(1/2)*((9*B^2)/(16*a*d^2) - (3*A^2)
/(16*a*d^2) - ((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4
 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2)/(16*a^6) - (A*B*3i)
/(8*a*d^2))^(1/2)*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)
/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2))/((A^3*a^5*d*1i
)/2 + (35*B^3*a^5*d)/2 + (A*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114
*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2)*3
i)/2 - (3*B*d^3*((A^4*a^10)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d
^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/d^4)^(1/2))/2 - (A*B^2*a^5*d
*57i)/2 - (15*A^2*B*a^5*d)/2) + (A^2*a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/2)
*((9*B^2)/(16*a*d^2) - (3*A^2)/(16*a*d^2) - ((A^4*a^10)/d^4 + (49*B^4*a^10
)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10*20i)/
d^4)^(1/2)/(16*a^6) - (A*B*3i)/(8*a*d^2))^(1/2))/((A^3*a^2*d*1i)/2 + (35*B
^3*a^2*d)/2 - (A*B^2*a^2*d*57i)/2 - (15*A^2*B*a^2*d)/2 + (A*d^3*((A^4*a^10
)/d^4 + (49*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4
 + (A^3*B*a^10*20i)/d^4)^(1/2)*3i)/(2*a^3) - (3*B*d^3*((A^4*a^10)/d^4 + (4
9*B^4*a^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*
a^10*20i)/d^4)^(1/2))/(2*a^3)) - (7*B^2*a^2*d^2*(a + a*tan(c + d*x)*1i)^(1
/2)*((9*B^2)/(16*a*d^2) - (3*A^2)/(16*a*d^2) - ((A^4*a^10)/d^4 + (49*B^4*a
^10)/d^4 - (114*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*140i)/d^4 + (A^3*B*a^10...
```

Reduce [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cot(dx+c)^2\tan(dx+c)^2}{\tan(dx+c)^2+1}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cot(dx+c)^2\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)ai+\left(\int \frac{\sqrt{\tan(dx+}\right.\right.}{a}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**2*tan(c + d*x)**2
)/(tan(c + d*x)**2 + 1),x)*b*i - int((sqrt(tan(c + d*x)*i + 1)*cot(c + d*x
)**2*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*a*i + int((sqrt(tan(c + d*x)*i
 + 1)*cot(c + d*x)**2*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*b + int((sqrt
(tan(c + d*x)*i + 1)*cot(c + d*x)**2)/(tan(c + d*x)**2 + 1),x)*a))/a
```

3.96 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 219

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(11A+4iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}d} - \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{(7iA-8B)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad} - \frac{(3A+2iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad}$$

output
```
1/4*(11*A+4*I*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d-1/2*(
A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(1/
2)/d+(A+I*B)*cot(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)+1/4*(7*I*A-8*B)*cot(d
*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a/d-1/2*(3*A+2*I*B)*cot(d*x+c)^2*(a+I*a*tan
(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.68

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\frac{(11A+4iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}+\frac{-7A-8iB+i(A+4iB)\cot(c+dx)-2A\cot^2(c+d}{\sqrt{a+ia\tan(c+dx)}}}{4d}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]]
,x]
```

output
```
(((11*A + (4*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] -
(2*Sqrt[2]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/Sqrt[a] + (-7*A - (8*I)*B + I*(A + (4*I)*B)*Cot[c + d*x] - 2*A*Cot[c + d
*x]^2)/Sqrt[a + I*a*Tan[c + d*x]])/(4*d)
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4079, 27, 3042, 4081, 25, 3042, 4081, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\int \frac{1}{2}\cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}(2a(3A+2iB)-5a(iA-B)\tan(c+dx))dx}{a^2}+\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{\int \cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}(2a(3A+2iB)-5a(iA-B)\tan(c+dx))dx}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(2a(3A+2iB)-5a(iA-B)\tan(c+dx))}{\tan(c+dx)^3}dx}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 4081

$$\frac{\frac{\int -\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}((7iA-8B)a^2+3(3A+2iB)\tan(c+dx)a^2)dx}{2a}-\frac{a(3A+2iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}((7iA-8B)a^2+3(3A+2iB)\tan(c+dx)a^2)dx}{2a}-\frac{a(3A+2iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((7iA-8B)a^2+3(3A+2iB)\tan(c+dx)a^2\right)}{\tan(c+dx)^2}dx}{2a}-\frac{a(3A+2iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 4081

$$-\frac{-\frac{\frac{\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(11A+4iB)-a^3(7iA-8B)\tan(c+dx)\right)dx}{a}-\frac{a^2(-8B+7iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a}-\frac{a(3A+2iB)\cot^2(c+dx)\sqrt{a}}{d}}{2a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\big\downarrow$ 27

$$-\frac{-\frac{\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^3(11A+4iB)-a^3(7iA-8B)\tan(c+dx)\right)dx}{2a}-\frac{a^2(-8B+7iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a}-\frac{a(3A+2iB)\cot^2(c+dx)\sqrt{a+}}{d}}{2a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\big\downarrow$ 3042

$$-\frac{-\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(11A+4iB)-a^3(7iA-8B)\tan(c+dx)\right)}{\tan(c+dx)}dx}{2a}-\frac{a^2(-8B+7iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a}-\frac{a(3A+2iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+}}{d}}{2a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\big\downarrow$ 4083

$$-\frac{-\frac{\frac{4a^3(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+a^2(11A+4iB)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{a^2(-8B+7iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a}-a(3A+}{2a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\big\downarrow$ 3042

$$-\frac{-\frac{\frac{4a^3(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+a^2(11A+4iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a}-\frac{a^2(-8B+7iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a}-a(3A+2iB)\,\mathrm{co}}{2a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\big\downarrow$ 3961

$$-\frac{\frac{\frac{a^2(11A+4iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{8ia^4(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a}-\frac{a^2(-8B+7iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a}-\cdots}{2a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 219

$$-\frac{\frac{\frac{a^2(11A+4iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{4i\sqrt{2}a^{7/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a^2(-8B+7iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a}-\cdots}{2a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 4082

$$-\frac{\frac{\frac{\frac{a^4(11A+4iB)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{4i\sqrt{2}a^{7/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a^2(-8B+7iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a}-\frac{a(3A\cdots}{\cdots}}{2a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 73

$$-\frac{\frac{\frac{-\frac{2ia^3(11A+4iB)\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{4i\sqrt{2}a^{7/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a^2(-8B+7iA)\cot(c+dx)\sqrt{a+ia\tan(c+\cdots}}{d}}{2a}\cdots}{2a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 221

$$-\frac{\frac{\frac{-\frac{2a^{7/2}(11A+4iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{4i\sqrt{2}a^{7/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{a^2(-8B+7iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a}-\cdots}{2a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((A + I*B)*Cot[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (-((a*(3*A + (
2*I)*B)*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/d) - (((-2*a^(7/2)*(11*
A + (4*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - ((4*I)*Sqrt[
2]*a^(7/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/d)/(2*a) - (a^2*((7*I)*A - 8*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d)/(2*a))/(2*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{2a^3\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{7}{2}}}-\frac{iB+A}{2a^3\sqrt{a+ia\tan(dx+c)}}+\frac{-\frac{\left(-\frac{iB}{2}-\frac{3A}{8}\right)(a+ia\tan(dx+c))^{\frac{3}{2}}+\left(\frac{1}{2}iaB+\frac{5}{8}a\right.}{a^2\tan(dx+c)^2}}{}\right.}{d}$
default	$\frac{2a^3\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{7}{2}}}-\frac{iB+A}{2a^3\sqrt{a+ia\tan(dx+c)}}+\frac{-\frac{\left(-\frac{iB}{2}-\frac{3A}{8}\right)(a+ia\tan(dx+c))^{\frac{3}{2}}+\left(\frac{1}{2}iaB+\frac{5}{8}a\right.}{a^2\tan(dx+c)^2}}{}\right.}{d}$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
2/d*a^3*(-1/4*(A-I*B)/a^(7/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))-1/2/a^3*(A+I*B)/(a+I*a*tan(d*x+c))^(1/2)+1/a^3*(-((-1/2*
I*B-3/8*A)*(a+I*a*tan(d*x+c))^(3/2)+(1/2*I*a*B+5/8*a*A)*(a+I*a*tan(d*x+c))
^(1/2))/a^2/tan(d*x+c)^2+1/8*(11*A+4*I*B)/a^(1/2)*arctanh((a+I*a*tan(d*x+c
))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(172) = 344.

Time = 0.14 (sec) , antiderivative size = 835, normalized size of antiderivative = 3.81

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
1/16*(4*sqrt(2)*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d
*e^(I*d*x + I*c))*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2))*log(-4*((-I*A - B)*a
*e^(I*d*x + I*c) + (I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2)))*e^(-I*d*x - I*c)/(I*A
+ B)) - 4*sqrt(2)*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a
*d*e^(I*d*x + I*c))*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2))*log(-4*((-I*A - B)
*a*e^(I*d*x + I*c) + (-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a*d^2)))*e^(-I*d*x - I*c)/(I
*A + B)) + (a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I
*d*x + I*c))*sqrt((121*A^2 + 88*I*A*B - 16*B^2)/(a*d^2))*log(-16*(3*(11*I*
A - 4*B)*a^2*e^(2*I*d*x + 2*I*c) + (11*I*A - 4*B)*a^2 + 2*sqrt(2)*(I*a^2*d
*e^(3*I*d*x + 3*I*c) + I*a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((121*A^2 + 88*I*A*B - 16*B^2)/(a*d^2)))*e^(-2*I*d*x - 2*I*c)/
(-11*I*A + 4*B)) - (a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) +
a*d*e^(I*d*x + I*c))*sqrt((121*A^2 + 88*I*A*B - 16*B^2)/(a*d^2))*log(-16*(
3*(11*I*A - 4*B)*a^2*e^(2*I*d*x + 2*I*c) + (11*I*A - 4*B)*a^2 + 2*sqrt(2)*
(-I*a^2*d*e^(3*I*d*x + 3*I*c) - I*a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt((121*A^2 + 88*I*A*B - 16*B^2)/(a*d^2)))*e^(-2*I*d*x
- 2*I*c)/(-11*I*A + 4*B)) - 4*sqrt(2)*(3*(A + 2*I*B)*e^(6*I*d*x + 6*I*c) -
 2*(3*A + I*B)*e^(4*I*d*x + 4*I*c) - (7*A + 6*I*B)*e^(2*I*d*x + 2*I*c) ...
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{(A+B\tan(c+dx))\cot^3(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**3/sqrt(I*a*(tan(c + d*x) - I))
, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.06

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{a^2\left(\frac{2\left((i\,a\tan(dx+c)+a)^2(7\,A+8i\,B)-(i\,a\tan(dx+c)+a)(13\,A+12i\,B)a+4\,(A+i\,B)a^2\right)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^2-2\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^3+\sqrt{i\,a\tan(dx+c)+a}\,a^4}-\frac{2\sqrt{2}(A-i\,B)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right.}{a^{\frac{5}{2}}}}{8\,d}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
-1/8*a^2*(2*((I*a*tan(d*x + c) + a)^2*(7*A + 8*I*B) - (I*a*tan(d*x + c) +
a)*(13*A + 12*I*B)*a + 4*(A + I*B)*a^2)/((I*a*tan(d*x + c) + a)^(5/2)*a^2
- 2*(I*a*tan(d*x + c) + a)^(3/2)*a^3 + sqrt(I*a*tan(d*x + c) + a)*a^4) - 2
*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sq
rt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2) + (11*A + 4*I*B)*log(
(sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(
a)))/a^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 3037, normalized size of antiderivative = 13.87

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
2*atanh((12*d^4*(a + a*tan(c + d*x)*1i)^(1/2)*((129*A^2)/(128*a*d^2) - ((1
2769*A^4*a^10)/(4*d^4) + (16*B^4*a^10)/d^4 - (3156*A^2*B^2*a^10)/d^4 - (A*
B^3*a^10*416i)/d^4 + (A^3*B*a^10*5876i)/d^4)^(1/2)/(64*a^6) - (3*B^2)/(16*
a*d^2) + (A*B*9i)/(16*a*d^2))^(1/2)*((12769*A^4*a^10)/(4*d^4) + (16*B^4*a^
10)/d^4 - (3156*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*416i)/d^4 + (A^3*B*a^10*58
76i)/d^4)^(1/2))/(B^3*a^5*d*8i - (1469*A^3*a^5*d)/2 + 9*A*d^3*((12769*A^4*
a^10)/(4*d^4) + (16*B^4*a^10)/d^4 - (3156*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*
416i)/d^4 + (A^3*B*a^10*5876i)/d^4)^(1/2) + B*d^3*((12769*A^4*a^10)/(4*d^4
) + (16*B^4*a^10)/d^4 - (3156*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*416i)/d^4 +
(A^3*B*a^10*5876i)/d^4)^(1/2)*6i + 156*A*B^2*a^5*d - A^2*B*a^5*d*789i) - (
226*A^2*a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((129*A^2)/(128*a*d^2) - ((1
2769*A^4*a^10)/(4*d^4) + (16*B^4*a^10)/d^4 - (3156*A^2*B^2*a^10)/d^4 - (A*
B^3*a^10*416i)/d^4 + (A^3*B*a^10*5876i)/d^4)^(1/2)/(64*a^6) - (3*B^2)/(16*
a*d^2) + (A*B*9i)/(16*a*d^2))^(1/2))/(B^3*a^2*d*8i - (1469*A^3*a^2*d)/2 +
156*A*B^2*a^2*d - A^2*B*a^2*d*789i + (9*A*d^3*((12769*A^4*a^10)/(4*d^4) +
(16*B^4*a^10)/d^4 - (3156*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*416i)/d^4 + (A^3
*B*a^10*5876i)/d^4)^(1/2))/a^3 + (B*d^3*((12769*A^4*a^10)/(4*d^4) + (16*B^
4*a^10)/d^4 - (3156*A^2*B^2*a^10)/d^4 - (A*B^3*a^10*416i)/d^4 + (A^3*B*a^1
0*5876i)/d^4)^(1/2)*6i)/a^3) + (16*B^2*a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/
2)*((129*A^2)/(128*a*d^2) - ((12769*A^4*a^10)/(4*d^4) + (16*B^4*a^10)/d...
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cot(dx+c)^3\tan(dx+c)^2}{\tan(dx+c)^2+1}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\cot(dx+c)^3\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)ai+\left(\int \frac{\sqrt{\tan(dx+}\right.\right.}{a}$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*cot(c + d*x)**3*tan(c + d*x)**2
)/(tan(c + d*x)**2 + 1),x)*b*i - int((sqrt(tan(c + d*x)*i + 1)*cot(c + d*x
)**3*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*a*i + int((sqrt(tan(c + d*x)*i
 + 1)*cot(c + d*x)**3*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*b + int((sqrt
(tan(c + d*x)*i + 1)*cot(c + d*x)**3)/(tan(c + d*x)**2 + 1),x)*a))/a
```

3.97 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 209

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(iA-B)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{(3A+5iB)\tan^2(c+dx)}{2ad\sqrt{a+ia\tan(c+dx)}} - \frac{2(3A+5iB)\sqrt{a+ia\tan(c+dx)}}{a^2d} + \frac{(11A+21iB)(a+ia\tan(c+dx))^{3/2}}{6a^3d}$$

output
```
1/4*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
a^(3/2)/d+1/3*(I*A-B)*tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)+1/2*(3*A+5*I
*B)*tan(d*x+c)^2/a/d/(a+I*a*tan(d*x+c))^(1/2)-2*(3*A+5*I*B)*(a+I*a*tan(d*x
+c))^(1/2)/a^2/d+1/6*(11*A+21*I*B)*(a+I*a*tan(d*x+c))^(3/2)/a^3/d
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{25iA-39B-3(13A+19iB)\tan(c+dx)+12(-iA+B)\tan^2(c+dx)-4iB\tan^3(c+dx)}{6ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/
2),x]
```

output
```
((A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[
2]*a^(3/2)*d) + ((25*I)*A - 39*B - 3*(13*A + (19*I)*B)*Tan[c + d*x] + 12*(
(-I)*A + B)*Tan[c + d*x]^2 - (4*I)*B*Tan[c + d*x]^3)/(6*a*d*(-I + Tan[c +
d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4075, 3042, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{3\tan^2(c+dx)(2a(iA-B)+a(A+3iB)\tan(c+dx))}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{\tan^2(c+dx)(2a(iA-B)+a(A+3iB)\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{\tan(c+dx)^2(2a(iA-B)+a(A+3iB)\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}$$

$$\Big\downarrow \; 4078$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{-\frac{\int -\frac{1}{2}\tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4a^2(3A+5iB)-a^2(11iA-21B)\tan(c+dx))dx}{a^2}-\frac{a(3A+5iB)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4a^2(3A+5iB)-a^2(11iA-21B)\tan(c+dx))dx}{2a^2}-\frac{a(3A+5iB)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}(4a^2(3A+5iB)-a^2(11iA-21B)\tan(c+dx))dx}{2a^2}-\frac{a(3A+5iB)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \; 4075$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int \sqrt{i\tan(c+dx)a+a}((11iA-21B)a^2+4(3A+5iB)\tan(c+dx)a^2)dx-\frac{2a(11A+21iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}-\frac{a(3A+5iB)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int \sqrt{i\tan(c+dx)a+a}((11iA-21B)a^2+4(3A+5iB)\tan(c+dx)a^2)dx-\frac{2a(11A+21iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}-\frac{a(3A+5iB)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \text{4010}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{-\left(a^2(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx\right)+\frac{8a^2(3A+5iB)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(11A+21iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}-\frac{a(3A+5iB)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{-\left(a^2(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx\right)+\frac{8a^2(3A+5iB)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(11A+21iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}-\frac{a(3A+5iB)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \text{3961}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{2ia^3(B+iA)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}+\frac{8a^2(3A+5iB)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(11A+21iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}-\frac{a(3A+5iB)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}+\frac{8a^2(3A+5iB)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(11A+21iB)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}-\frac{a(3A+5iB)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I*A - B)*Tan[c + d*x]^3)/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) - (-((a*(3*A
 + (5*I)*B)*Tan[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])) + ((I*Sqrt[2]*
a^(5/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d
 + (8*a^2*(3*A + (5*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/d - (2*a*(11*A + (21
*I)*B)*(a + I*a*Tan[c + d*x])^(3/2))/(3*d))/(2*a^2))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{\frac{2iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4iaB\sqrt{a+ia\tan(dx+c)}-2aA\sqrt{a+ia\tan(dx+c)}+\frac{a^{\frac{3}{2}}(-iB+A)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right.}{4}}{a^3d}$
default	$\frac{\frac{2iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4iaB\sqrt{a+ia\tan(dx+c)}-2aA\sqrt{a+ia\tan(dx+c)}+\frac{a^{\frac{3}{2}}(-iB+A)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right.}{4}}{a^3d}$
parts	$\frac{2A\left(-\sqrt{a+ia\tan(dx+c)}+\frac{\sqrt{a}\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8}-\frac{5a}{4\sqrt{a+ia\tan(dx+c)}}+\frac{a^2}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}\right)}{d\,a^2}+\frac{2iB\left(\right.}{}$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
2/d/a^3*(1/3*I*B*(a+I*a*tan(d*x+c))^(3/2)-2*I*a*B*(a+I*a*tan(d*x+c))^(1/2)
-a*A*(a+I*a*tan(d*x+c))^(1/2)+1/8*a^(3/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I
*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-1/4*a^2*(5*A+7*I*B)/(a+I*a*tan(d*x+c
))^(1/2)+1/6*a^3*(A+I*B)/(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(162) = 324.

Time = 0.13 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.11

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{3\sqrt{\frac{1}{2}}\left(a^2de^{(5i\,dx+5i\,c)}+a^2de^{(3i\,dx+3i\,c)}\right)\sqrt{\frac{A^2-2i\,AB-B^2}{a^3d^2}}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{A^2-2i}{a}}\right.}{i\,A+B}\right.}{}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
-1/12*(3*sqrt(1/2)*(a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))
*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^2*d*
e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2
 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)/(I*A + B)) - 3*sqrt(1/2)*(a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x
 + 3*I*c))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)
*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^
(-I*d*x - I*c)/(I*A + B)) + sqrt(2)*(2*(19*A + 26*I*B)*e^(6*I*d*x + 6*I*c)
 + 3*(17*A + 29*I*B)*e^(4*I*d*x + 4*I*c) + 6*(2*A + 3*I*B)*e^(2*I*d*x + 2*
I*c) - A - I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(a^2*d*e^(5*I*d*x + 5*I
*c) + a^2*d*e^(3*I*d*x + 3*I*c))
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3
/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.77

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{3\sqrt{2}(A-iB)a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-16i(ia\tan(dx+c)+a)^{\frac{3}{2}}Ba+48\sqrt{ia\tan(dx+c)+\cdots}}{24a^4d}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
-1/24*(3*sqrt(2)*(A - I*B)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*
x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 16*I*(I*a*t
an(d*x + c) + a)^(3/2)*B*a + 48*sqrt(I*a*tan(d*x + c) + a)*(A + 2*I*B)*a^2
 + 4*(3*(I*a*tan(d*x + c) + a)*(5*A + 7*I*B)*a^3 - 2*(A + I*B)*a^4)/(I*a*t
an(d*x + c) + a)^(3/2))/(a^4*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{\frac{B\,1\mathrm{i}}{3\,d}-\frac{B\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,7\mathrm{i}}{2\,a\,d}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}} \\ &\quad+\frac{\frac{A\,a}{3}-\frac{5\,A\,(a+a\tan(c+d\,x)\,1\mathrm{i})}{2}}{a\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}}-\frac{2\,A\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{a^2\,d} \\ &\quad-\frac{B\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,4\mathrm{i}}{a^2\,d}+\frac{B\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}\,2\mathrm{i}}{3\,a^3\,d} \\ &\quad-\frac{\sqrt{2}\,B\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{4\,(-a)^{3/2}\,d}+\frac{\sqrt{2}\,A\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{4\,a^{3/2}\,d}\end{aligned}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
((B*1i)/(3*d) - (B*(a + a*tan(c + d*x)*1i)*7i)/(2*a*d))/(a + a*tan(c + d*x
)*1i)^(3/2) + ((A*a)/3 - (5*A*(a + a*tan(c + d*x)*1i))/2)/(a*d*(a + a*tan(
c + d*x)*1i)^(3/2)) - (2*A*(a + a*tan(c + d*x)*1i)^(1/2))/(a^2*d) - (B*(a
+ a*tan(c + d*x)*1i)^(1/2)*4i)/(a^2*d) + (B*(a + a*tan(c + d*x)*1i)^(3/2)*
2i)/(3*a^3*d) - (2^(1/2)*B*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2
*(-a)^(1/2)))*1i)/(4*(-a)^(3/2)*d) + (2^(1/2)*A*atanh((2^(1/2)*(a + a*tan(
c + d*x)*1i)^(1/2))/(2*a^(1/2))))/(4*a^(3/2)*d)
```

Reduce [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 4*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 2*sqrt(tan(c +
d*x)*i + 1)*a + 5*int(sqrt(tan(c + d*x)*i + 1)/(tan(c + d*x)**3*i + tan(c
+ d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*a*d*i + 5*int(sqrt(tan(
c + d*x)*i + 1)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1)
,x)*a*d*i - 2*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**5)/(tan(c + d*x)
**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*b*d*i - 2
*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**5)/(tan(c + d*x)**3*i + tan(c
 + d*x)**2 + tan(c + d*x)*i + 1),x)*b*d*i - 2*int((sqrt(tan(c + d*x)*i + 1
)*tan(c + d*x)**4)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i +
 1),x)*tan(c + d*x)**2*a*d*i + 2*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x
)**4)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c
+ d*x)**2*b*d - 2*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4)/(tan(c +
d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*a*d*i + 2*int((sqrt(t
an(c + d*x)*i + 1)*tan(c + d*x)**4)/(tan(c + d*x)**3*i + tan(c + d*x)**2 +
 tan(c + d*x)*i + 1),x)*b*d + 3*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)
**2)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c +
 d*x)**2*a*d*i + 3*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c +
 d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*a*d*i))/(2*a**2*d*(t
an(c + d*x)**2 + 1))
```

3.98 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 167

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(iA-B)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{5iA-11B}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{(iA-7B)\sqrt{a+ia\tan(c+dx)}}{3a^2d}$$

output
```
1/4*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
a^(3/2)/d+1/3*(I*A-B)*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(3/2)+1/6*(5*I*A-1
1*B)/a/d/(a+I*a*tan(d*x+c))^(1/2)+1/3*(I*A-7*B)*(a+I*a*tan(d*x+c))^(1/2)/a
^2/d
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2B\tan^2(c+dx)}{d(a+ia\tan(c+dx))^{3/2}} + \frac{i\left(\frac{3\sqrt{2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}} - \frac{4a(A+7iB)}{(a+ia\tan(c+dx))^{3/2}} + \frac{6(3A+13iB)}{\sqrt{a+ia\tan(c+dx)}}\right)}{12ad}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/
2),x]
```

output
```
(2*B*Tan[c + d*x]^2)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((I/12)*((3*Sqrt[2
]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a]
 - (4*a*(A + (7*I)*B))/(a + I*a*Tan[c + d*x])^(3/2) + (6*(3*A + (13*I)*B))
/Sqrt[a + I*a*Tan[c + d*x]]))/(a*d)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4078, 27, 3042, 4075, 3042, 4009, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\tan(c+dx)(4a(iA-B)+a(A+7iB)\tan(c+dx))}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(-B+iA)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{\tan(c+dx)(4a(iA-B)+a(A+7iB)\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{\tan(c+dx)(4a(iA-B)+a(A+7iB)\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}$$

$$\Big\downarrow \; 4075$$

$$\frac{(-B+iA)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{4a(iA-B)\tan(c+dx)-a(A+7iB)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{2(-7B+iA)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{4a(iA-B)\tan(c+dx)-a(A+7iB)}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{2(-7B+iA)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}$$

$$\Big\downarrow \; 4009$$

$$\frac{(-B+iA)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{3}{2}(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx-\frac{a(-11B+5iA)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2(-7B+iA)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{3}{2}(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx-\frac{a(-11B+5iA)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2(-7B+iA)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}$$

$$\Big\downarrow \; 3961$$

$$\frac{(-B+iA)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{-\frac{3ia(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{a(-11B+5iA)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2(-7B+iA)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(-B+iA)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-}{\frac{-\frac{3i\sqrt{a}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}-\frac{a(-11B+5iA)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2(-7B+iA)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((I*A - B)*Tan[c + d*x]^2)/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) - (((-3*I)*S
qrt[a]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(S
qrt[2]*d) - (a*((5*I)*A - 11*B))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (2*(I*A
- 7*B)*Sqrt[a + I*a*Tan[c + d*x]])/d)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2i\left(iB\sqrt{a+ia\tan(dx+c)}+\frac{\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8}+\frac{a(5iB+3A)}{4\sqrt{a+ia\tan(dx+c)}}-\frac{a^2(iB+A)}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}\right)}{d\,a^2}$
default	$\frac{2i\left(iB\sqrt{a+ia\tan(dx+c)}+\frac{\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8}+\frac{a(5iB+3A)}{4\sqrt{a+ia\tan(dx+c)}}-\frac{a^2(iB+A)}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}\right)}{d\,a^2}$
parts	$\frac{2iA\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8\sqrt{a}}+\frac{3}{4\sqrt{a+ia\tan(dx+c)}}-\frac{a}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}\right)}{da}+\frac{2B\left(-\sqrt{a+ia\tan(dx+c)}+\right.}{}$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d/a^2*(I*B*(a+I*a*tan(d*x+c))^(1/2)+1/8*a^(1/2)*(A-I*B)*2^(1/2)*arctan
h(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+1/4*a*(3*A+5*I*B)/(a+I*a*t
an(d*x+c))^(1/2)-1/6*a^2*(A+I*B)/(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(128) = 256$.

Time = 0.12 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.25

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$
$$-\frac{\left(3\sqrt{\frac{1}{2}}a^2d\sqrt{-\frac{A^2-2i\,AB-B^2}{a^3d^2}}e^{(3i\,dx+3i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{A^2-2i\,AB-B^2}{a^3d^2}}+(-i\,A-B)\right.}{i\,A+B}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
-1/12*(3*sqrt(1/2)*a^2*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x
 + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (
-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*a^2
*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(
2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)) - (-I*A - B)*a*e^(I*d*x +
I*c))*e^(-I*d*x - I*c)/(I*A + B)) + sqrt(2)*(2*(-4*I*A + 19*B)*e^(4*I*d*x
+ 4*I*c) - (7*I*A - 13*B)*e^(2*I*d*x + 2*I*c) + I*A - B)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1)))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3
/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.82

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{i\left(3\sqrt{2}(A-iB)a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-48i\sqrt{ia\tan(dx+c)+a}Ba-\frac{4(3(ia\tan(dx+c)+a)(3A+\cdots}{(ia\tan(dx+c\cdots}\right.}{24a^3d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
-1/24*I*(3*sqrt(2)*(A - I*B)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(
d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 48*I*sqrt
(I*a*tan(d*x + c) + a)*B*a - 4*(3*(I*a*tan(d*x + c) + a)*(3*A + 5*I*B)*a^2
 - 2*(A + I*B)*a^3)/(I*a*tan(d*x + c) + a)^(3/2))/(a^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\frac{A\,1\mathrm{i}}{3\,d}-\frac{A\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,3\mathrm{i}}{2\,a\,d}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}} + \frac{\frac{B\,a}{3}-\frac{5\,B\,(a+a\tan(c+d\,x)\,1\mathrm{i})}{2}}{a\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}} - \frac{2\,B\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{a^2\,d} + \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{4\,(-a)^{3/2}\,d} + \frac{\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{4\,a^{3/2}\,d}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
((B*a)/3 - (5*B*(a + a*tan(c + d*x)*1i))/2)/(a*d*(a + a*tan(c + d*x)*1i)^(
3/2)) - ((A*1i)/(3*d) - (A*(a + a*tan(c + d*x)*1i)*3i)/(2*a*d))/(a + a*tan
(c + d*x)*1i)^(3/2) - (2*B*(a + a*tan(c + d*x)*1i)^(1/2))/(a^2*d) + (2^(1/
2)*A*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/(4*(
-a)^(3/2)*d) + (2^(1/2)*B*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2
*a^(1/2))))/(4*a^(3/2)*d)
```

Reduce [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 4*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a - 4*sqrt(tan(c + d*
x)*i + 1)*tan(c + d*x)*b*i + 2*sqrt(tan(c + d*x)*i + 1)*a*i - 2*sqrt(tan(c
 + d*x)*i + 1)*b + 5*int(sqrt(tan(c + d*x)*i + 1)/(tan(c + d*x)**3*i + tan
(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*a*d + 5*int(sqrt(tan
(c + d*x)*i + 1)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1
),x)*tan(c + d*x)**2*b*d*i + 5*int(sqrt(tan(c + d*x)*i + 1)/(tan(c + d*x)*
*3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*a*d + 5*int(sqrt(tan(c + d
*x)*i + 1)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*b
*d*i - 2*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4)/(tan(c + d*x)**3*i
 + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*b*d*i - 2*int(
(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4)/(tan(c + d*x)**3*i + tan(c + d*
x)**2 + tan(c + d*x)*i + 1),x)*b*d*i + 5*int((sqrt(tan(c + d*x)*i + 1)*tan
(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x
)*tan(c + d*x)**2*a*d + 3*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(
tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)*
*2*b*d*i + 5*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c + d*x)*
*3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*a*d + 3*int((sqrt(tan(c +
d*x)*i + 1)*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c
+ d*x)*i + 1),x)*b*d*i))/(2*a**2*d*(tan(c + d*x)**2 + 1))
```

3.99 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 119

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{A+3iB}{2ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
-1/4*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)
/a^(3/2)/d-1/3*(A+I*B)/d/(a+I*a*tan(d*x+c))^(3/2)+1/2*(A+3*I*B)/a/d/(a+I*a
*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{3\sqrt{2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}} + \frac{4a(iA-B)}{(a+ia\tan(c+dx))^{3/2}} - \frac{6(iA-3}{\sqrt{a+ia\tan}}\right.}{12ad}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2)
,x]
```

output
```
((I/12)*((3*Sqrt[2]*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*
Sqrt[a])])/Sqrt[a] + (4*a*(I*A - B))/(a + I*a*Tan[c + d*x])^(3/2) - (6*(I*
A - 3*B))/Sqrt[a + I*a*Tan[c + d*x]]))/(a*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4073, 3042, 4009, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4073}$$

$$-\frac{i\int \frac{a(A+iB)+2aB\tan(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}-\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \frac{a(A+iB)+2aB\tan(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}-\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4009}$$

$$-\frac{i\left(\frac{1}{2}(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{a(-3B+iA)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{2a^2}-\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{a(-3B+iA)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{2a^2}-\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3961}$$

$$-\frac{i\left(\frac{a(-3B+iA)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{ia(A-iB)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}\right)}{2a^2}-\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{i\left(\frac{a(-3B+iA)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{a}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}\right)}{2a^2}-\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
-1/3*(A + I*B)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/2)*(((-I)*Sqrt[a]*(A
 - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*d)
 + (a*(I*A - 3*B))/(d*Sqrt[a + I*a*Tan[c + d*x]])))/a^2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{\left(\frac{A}{4}-\frac{iB}{4}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\left(-\frac{A}{4}-\frac{3iB}{4}\right)}{\sqrt{a+ia\tan(dx+c)}}-\frac{a(iB+A)}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}}{ad}$
default	$\frac{-\frac{\left(\frac{A}{4}-\frac{iB}{4}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\left(-\frac{A}{4}-\frac{3iB}{4}\right)}{\sqrt{a+ia\tan(dx+c)}}-\frac{a(iB+A)}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}}{ad}$
parts	$\frac{A\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{4a^{\frac{3}{2}}}-\frac{1}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{1}{2a\sqrt{a+ia\tan(dx+c)}}\right)}{d}+\frac{2iB\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(a}}{2\sqrt{a}}\right.}{8\sqrt{a}}\right.}{}$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/d/a*(-1/2*(1/4*A-1/4*I*B)*2^(1/2)/a^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))
^(1/2)*2^(1/2)/a^(1/2))-(-1/4*A-3/4*I*B)/(a+I*a*tan(d*x+c))^(1/2)-1/6*a*(A
+I*B)/(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(88) = 176.

Time = 0.10 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.10

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(3\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{A^2-2iAB-B^2}{a^3d^2}}e^{(3idx+3ic)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}(ia^2de^{(2idx+2ic)}+\right.}{}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
1/12*(3*sqrt(1/2)*a^2*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x +
 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2)) +
(-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*a^
2*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt
(2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*e^(I*d
*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) + sqrt(2)*(2*(A + 4*I*B)*e^(4*I*d*x
 + 4*I*c) + (A + 7*I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1)))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{3\sqrt{2}(A-iB)\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\,(3\,(ia\tan(dx+c)+}{(ia\tan}}{24\,a^2d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
1/24*(3*sqrt(2)*(A - I*B)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
 + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3*(I*a*ta
n(d*x + c) + a)*(A + 3*I*B)*a - 2*(A + I*B)*a^2)/(I*a*tan(d*x + c) + a)^(3
/2))/(a^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.37

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\frac{B\,1i}{3\,d}-\frac{B\,(a+a\tan(c+d\,x)\,1i)\,3i}{2\,a\,d}}{(a+a\tan(c+d\,x)\,1i)^{3/2}}-\frac{\frac{A}{3}-\frac{A\,(a+a\tan(c+d\,x)\,1i)}{2\,a}}{d\,(a+a\tan(c+d\,x)\,1i)^{3/2}} + \frac{\sqrt{2}\,B\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1i}}{2\,\sqrt{-a}}\right)\,1i}{4\,(-a)^{3/2}\,d}-\frac{\sqrt{2}\,A\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1i}}{2\,\sqrt{a}}\right)}{4\,a^{3/2}\,d}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
(2^(1/2)*B*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*1i
)/(4*(-a)^(3/2)*d) - (A/3 - (A*(a + a*tan(c + d*x)*1i))/(2*a))/(d*(a + a*t
an(c + d*x)*1i)^(3/2)) - ((B*1i)/(3*d) - (B*(a + a*tan(c + d*x)*1i)*3i)/(2
*a*d))/(a + a*tan(c + d*x)*1i)^(3/2) - (2^(1/2)*A*atanh((2^(1/2)*(a + a*ta
n(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/(4*a^(3/2)*d)
```

Reduce [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-4\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)\,b-2\sqrt{\tan(dx+c)\,i+\right.}{}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 4*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 2*sqrt(tan(c + d*
x)*i + 1)*a + 2*sqrt(tan(c + d*x)*i + 1)*b*i + int(sqrt(tan(c + d*x)*i + 1
)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*
x)**2*a*d*i + 5*int(sqrt(tan(c + d*x)*i + 1)/(tan(c + d*x)**3*i + tan(c +
d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*b*d + int(sqrt(tan(c + d*
x)*i + 1)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*a*
d*i + 5*int(sqrt(tan(c + d*x)*i + 1)/(tan(c + d*x)**3*i + tan(c + d*x)**2
+ tan(c + d*x)*i + 1),x)*b*d + int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*
*2)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c +
d*x)**2*a*d*i + 5*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c +
d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*b*d +
 int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c
 + d*x)**2 + tan(c + d*x)*i + 1),x)*a*d*i + 5*int((sqrt(tan(c + d*x)*i + 1
)*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i +
 1),x)*b*d))/(2*a**2*d*(tan(c + d*x)**2 + 1))
```

3.100 $\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 121

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{(iA+B)\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{iA-B}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{iA+B}{2ad\sqrt{a+ia\tan(c+dx)}}$$

output

```
-1/4*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)
/a^(3/2)/d+1/3*(I*A-B)/d/(a+I*a*tan(d*x+c))^(3/2)+1/2*(I*A+B)/a/d/(a+I*a*t
an(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.63

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2iA-2B-3(A-iB)\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right.}{6d(a+ia\tan(c+dx))^{3/2}}$$

input

```
Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((2*I)*A - 2*B - 3*(A - I*B)*Hypergeometric2F1[-1/2, 1, 1/2, (1 + I*Tan[c
+ d*x])/2]*(-I + Tan[c + d*x]))/(6*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4009, 3042, 3960, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4009}$$

$$\frac{(A-iB)\int \frac{1}{\sqrt{i\tan(c+dx)a+a}}dx}{2a}+\frac{-B+iA}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int \frac{1}{\sqrt{i\tan(c+dx)a+a}}dx}{2a}+\frac{-B+iA}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3960}$$

$$\frac{(A-iB)\left(\frac{\int \sqrt{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{d\sqrt{a+ia\tan(c+dx)}}\right)}{2a}+\frac{-B+iA}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\left(\frac{\int \sqrt{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{d\sqrt{a+ia\tan(c+dx)}}\right)}{2a}+\frac{-B+iA}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3961}$$

$$\frac{(A-iB)\left(\frac{i}{d\sqrt{a+ia\tan(c+dx)}}-\frac{i\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}\right)}{2a}+\frac{-B+iA}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(A-iB)\left(\frac{i}{d\sqrt{a+ia\tan(c+dx)}}-\frac{i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}\right)}{2a}+\frac{-B+iA}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(I*A - B)/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((A - I*B)*(((-I)*ArcTanh[S
qrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*Sqrt[a]*d) + I/(d*S
qrt[a + I*a*Tan[c + d*x]])))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{2i\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}}-\frac{-\frac{A}{2}-\frac{iB}{2}}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{iB-A}{4a\sqrt{a+ia\tan(dx+c)}}\right)}{d}$
default	$\frac{2i\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}}-\frac{-\frac{A}{2}-\frac{iB}{2}}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{iB-A}{4a\sqrt{a+ia\tan(dx+c)}}\right)}{d}$
parts	$\frac{2iAa\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{5}{2}}}+\frac{1}{4a^{2}\sqrt{a+ia\tan(dx+c)}}+\frac{1}{6a(a+ia\tan(dx+c))^{\frac{3}{2}}}\right)}{d}+\frac{B\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ic}}{}\right.}{4a^{\frac{3}{}}}\right.}{}$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d*(-1/8*(A-I*B)/a^(3/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2
^(1/2)/a^(1/2))-1/3*(-1/2*A-1/2*I*B)/(a+I*a*tan(d*x+c))^(3/2)-1/4/a*(-A+I*
B)/(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(90) = 180.

Time = 0.09 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.07

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{\left(3\sqrt{\frac{1}{2}}a^{2}d\sqrt{-\frac{A^{2}-2i\,AB-B^{2}}{a^{3}d^{2}}}e^{(3i\,dx+3i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)\sqrt{\frac{}{e^{(2i\,c}}}\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(1/2)*a^2*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x
+ 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-
I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*a^2*
d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2
)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)) - (-I*A - B)*a*e^(I*d*x + I
*c))*e^(-I*d*x - I*c)/(I*A + B)) - sqrt(2)*(2*(-2*I*A - B)*e^(4*I*d*x + 4*
I*c) - (5*I*A + B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1)))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{(a + ia\tan(c + dx))^{3/2}}\,dx = \int \frac{A + B\tan{(c + dx)}}{(ia(\tan{(c + dx)} - i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \frac{A + B\tan(c + dx)}{(a + ia\tan(c + dx))^{3/2}}\,dx = \frac{i\left(\frac{3\sqrt{2}(A - iB)\log\left(-\frac{\sqrt{2}\sqrt{a} - \sqrt{ia\tan(dx + c) + a}}{\sqrt{2}\sqrt{a} + \sqrt{ia\tan(dx + c) + a}}\right)}{\sqrt{a}} + \frac{4(3(ia\tan(dx + c) + a)(A - iB) + 2(A + iB)a)}{(ia\tan(dx + c) + a)^{\frac{3}{2}}}\right)}{24ad}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/24*I*(3*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(3*(I*a*
tan(d*x + c) + a)*(A - I*B) + 2*(A + I*B)*a)/(I*a*tan(d*x + c) + a)^(3/2))
/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\frac{A\,1\mathrm{i}}{3\,d}+\frac{A\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,1\mathrm{i}}{2\,a\,d}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{3/2}} - \frac{\frac{B}{3}-\frac{B\,(a+a\tan(c+d\,x)\,1\mathrm{i})}{2\,a}}{d\,(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{3/2}} - \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{4\,{(-a)}^{3/2}\,d} - \frac{\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{4\,a^{3/2}\,d}$$

input
```
int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
((A*1i)/(3*d) + (A*(a + a*tan(c + d*x)*1i)*1i)/(2*a*d))/(a + a*tan(c + d*x
)*1i)^(3/2) - (B/3 - (B*(a + a*tan(c + d*x)*1i))/(2*a))/(d*(a + a*tan(c +
d*x)*1i)^(3/2)) - (2^(1/2)*A*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/
(2*(-a)^(1/2)))*1i)/(4*(-a)^(3/2)*d) - (2^(1/2)*B*atanh((2^(1/2)*(a + a*ta
n(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/(4*a^(3/2)*d)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*a*i - 2*sqrt(tan(c + d*x)*i + 1)*b
 + int(sqrt(tan(c + d*x)*i + 1)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan
(c + d*x)*i + 1),x)*tan(c + d*x)**2*a*d + int(sqrt(tan(c + d*x)*i + 1)/(ta
n(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*tan(c + d*x)**2
*b*d*i + int(sqrt(tan(c + d*x)*i + 1)/(tan(c + d*x)**3*i + tan(c + d*x)**2
 + tan(c + d*x)*i + 1),x)*a*d + int(sqrt(tan(c + d*x)*i + 1)/(tan(c + d*x)
**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*b*d*i + 9*int((sqrt(tan(c
 + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan
(c + d*x)*i + 1),x)*tan(c + d*x)**2*a*d + int((sqrt(tan(c + d*x)*i + 1)*ta
n(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),
x)*tan(c + d*x)**2*b*d*i + 9*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2
)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*a*d + int(
(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c + d*
x)**2 + tan(c + d*x)*i + 1),x)*b*d*i - 8*int((sqrt(tan(c + d*x)*i + 1)*tan
(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*t
an(c + d*x)**2*a*d*i - 8*int((sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(
c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*a*d*i))/(2*a**2*d
*(tan(c + d*x)**2 + 1))
```

3.101 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 156

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}+\frac{3A+iB}{2ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
-2*A*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/4*(A-I*B)*arcta
nh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(3/2)/d+1/3*(A+
I*B)/d/(a+I*a*tan(d*x+c))^(3/2)+1/2*(3*A+I*B)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{-6a^3A\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right) + \frac{3a^3(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}}}{3a^{9/2}d}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2)
,x]
```

output
```
(-6*a^3*A*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]] + (3*a^3*(A - I*B)*A
rcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(2*Sqrt[2]) + (a^(9/
2)*(A + I*B))/(a + I*a*Tan[c + d*x])^(3/2) + (3*a^(7/2)*(3*A + I*B))/(2*Sq
rt[a + I*a*Tan[c + d*x]]))/(3*a^(9/2)*d)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{3\cot(c+dx)(2aA-a(iA-B)\tan(c+dx))}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cot(c+dx)(2aA-a(iA-B)\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{2aA-a(iA-B)\tan(c+dx)}{\tan(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}(4a^2A-a^2(3iA-B)\tan(c+dx))dx}{a^2}+\frac{a(3A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}(4a^2A-a^2(3iA-B)\tan(c+dx))dx}{2a^2}+\frac{a(3A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(4a^2A-a^2(3iA-B)\tan(c+dx)\right)}{\tan(c+dx)}dx}{2a^2}+\frac{a(3A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 4083

$$\frac{\frac{a^2(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+4aA\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{2a^2}+\frac{a(3A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a^2(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+4aA\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a^2}+\frac{a(3A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 3961

$$\frac{\frac{4aA\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2ia^3(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a^2}+\frac{a(3A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{4aA\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a^2}+\frac{a(3A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4082}$$

$$\frac{\frac{\frac{4a^3A\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a^2}+\frac{a(3A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{-\frac{8ia^2A\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a^2}+\frac{a(3A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{-\frac{i\sqrt{2}a^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{8a^{5/2}A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}}{2a^2}+\frac{a(3A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
(A + I*B)/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((-8*a^(5/2)*A*ArcTanh[Sqr
t[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - (I*Sqrt[2]*a^(5/2)*(I*A + B)*ArcTanh
[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/d)/(2*a^2) + (a*(3*A + I*B
))/(d*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{2a\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{5}{2}}}-\frac{-iB-3A}{4a^{2}\sqrt{a+ia\tan(dx+c)}}-\frac{-iB-A}{6a(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{A\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx}}{\sqrt{a}}\right.}{a^{\frac{5}{2}}}\right.}{d}$
default	$\frac{2a\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{5}{2}}}-\frac{-iB-3A}{4a^{2}\sqrt{a+ia\tan(dx+c)}}-\frac{-iB-A}{6a(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{A\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx}}{\sqrt{a}}\right.}{a^{\frac{5}{2}}}\right.}{d}$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/d*a*(-1/8*(-A+I*B)/a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*
2^(1/2)/a^(1/2))-1/4/a^2*(-I*B-3*A)/(a+I*a*tan(d*x+c))^(1/2)-1/6*(-I*B-A)/
a/(a+I*a*tan(d*x+c))^(3/2)-1/a^(5/2)*A*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^
(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(117) = 234$.

Time = 0.10 (sec) , antiderivative size = 624, normalized size of antiderivative = 4.00

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
-1/12*(3*sqrt(1/2)*a^2*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x
+ 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2)) +
 (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*a
^2*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqr
t(2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*e^(I*
d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) + 6*a^2*d*sqrt(A^2/(a^3*d^2))*e^(3
*I*d*x + 3*I*c)*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2 + 2*sqrt(2)*(a
^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(A^2/(a^3*d^2)))*e^(-2*I*d*x - 2*I*c)/A) - 6*a^2*d*sqrt(A^2/
(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2
 - 2*sqrt(2)*(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt(A^2/(a^3*d^2)))*e^(-2*I*d*x - 2*I*c)/A) - sq
rt(2)*(2*(5*A + 2*I*B)*e^(4*I*d*x + 4*I*c) + (11*A + 5*I*B)*e^(2*I*d*x + 2
*I*c) + A + I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-3*I*d*x - 3*I*c)/(
a^2*d)
```

Sympy [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan(c+dx))\cot(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\frac{3\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}} - \frac{24A\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{4(3(ia\tan(dx+c)+a)(3A+iB)+2(A+iB)a)}{(ia\tan(dx+c)+a)^{\frac{3}{2}}a}}{24d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
-1/24*(3*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
 a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2) - 24*A*log((s
qrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)
))/a^(3/2) - 4*(3*(I*a*tan(d*x + c) + a)*(3*A + I*B) + 2*(A + I*B)*a)/((I*
a*tan(d*x + c) + a)^(3/2)*a))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.61

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\frac{A+B\,1\mathrm{i}}{3\,d}+\frac{(3\,A+B\,1\mathrm{i})\,(a+a\tan(c+d\,x)\,1\mathrm{i})}{2\,a\,d}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}} - \frac{2\,A\,\mathrm{atanh}\left(\frac{31\,A^3\,d\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{\sqrt{a^3}\,\left(\frac{31\,A^3\,d}{a}+\frac{A\,B^2\,d}{a}+\frac{A^2\,B\,d\,2\mathrm{i}}{a}\right)}+\frac{A\,B^2\,d\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{\sqrt{a^3}\,\left(\frac{31\,A^3\,d}{a}+\frac{A\,B^2\,d}{a}+\frac{A^2\,B\,d\,2\mathrm{i}}{a}\right)}+\frac{A^2\,B\,d\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,2\mathrm{i}}{\sqrt{a^3}\,\left(\frac{31\,A^3\,d}{a}+\frac{A\,B^2\,d}{a}+\frac{A^2\,B\,d\,2\mathrm{i}}{a}\right)}\right)}{d\,\sqrt{a^3}} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,A^3\,d\,\sqrt{-a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,31\mathrm{i}}{16\,\left(\frac{31\,d\,A^3\,a^2}{8}-\frac{29\mathrm{i}\,d\,A^2\,B\,a^2}{8}+\frac{3\,d\,A\,B^2\,a^2}{8}-\frac{1\mathrm{i}\,d\,B^3\,a^2}{8}\right)}+\frac{\sqrt{2}\,B^3\,d\,\sqrt{-a^3}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{16\,\left(\frac{31\,d\,A^3\,a^2}{8}-\frac{29\mathrm{i}\,d\,A^2\,B\,a^2}{8}+\frac{3\,d\,A\,B^2\,a^2}{8}-\frac{1\mathrm{i}\,d\,B^3\,a^2}{8}\right)}+\cdots\right)}{4\,a^3\,d}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
((A + B*1i)/(3*d) + ((3*A + B*1i)*(a + a*tan(c + d*x)*1i))/(2*a*d))/(a + a
*tan(c + d*x)*1i)^(3/2) - (2*A*atanh((31*A^3*d*(a + a*tan(c + d*x)*1i)^(1/
2))/((a^3)^(1/2)*((31*A^3*d)/a + (A*B^2*d)/a + (A^2*B*d*2i)/a)) + (A*B^2*d
*(a + a*tan(c + d*x)*1i)^(1/2))/((a^3)^(1/2)*((31*A^3*d)/a + (A*B^2*d)/a +
 (A^2*B*d*2i)/a)) + (A^2*B*d*(a + a*tan(c + d*x)*1i)^(1/2)*2i)/((a^3)^(1/2
)*((31*A^3*d)/a + (A*B^2*d)/a + (A^2*B*d*2i)/a))))/(d*(a^3)^(1/2)) + (2^(1
/2)*atanh((2^(1/2)*A^3*d*(-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*31i)/(
16*((31*A^3*a^2*d)/8 - (B^3*a^2*d*1i)/8 + (3*A*B^2*a^2*d)/8 - (A^2*B*a^2*d
*29i)/8)) + (2^(1/2)*B^3*d*(-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(16
*((31*A^3*a^2*d)/8 - (B^3*a^2*d*1i)/8 + (3*A*B^2*a^2*d)/8 - (A^2*B*a^2*d*2
9i)/8)) + (2^(1/2)*A*B^2*d*(-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*3i)/
(16*((31*A^3*a^2*d)/8 - (B^3*a^2*d*1i)/8 + (3*A*B^2*a^2*d)/8 - (A^2*B*a^2*
d*29i)/8)) + (29*2^(1/2)*A^2*B*d*(-a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2
))/(16*((31*A^3*a^2*d)/8 - (B^3*a^2*d*1i)/8 + (3*A*B^2*a^2*d)/8 - (A^2*B*a
^2*d*29i)/8)))*(A*1i + B)*(-a^3)^(1/2))/(4*a^3*d)
```

Reduce [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\int \frac{\cot(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx\right)a+\left(\int \frac{\cot(dx}{\sqrt{\tan(dx+c)i+1}\,\mathrm{ta}}\right.}{\sqrt{a}\,a}$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(int(cot(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c +
d*x)*i + 1)),x)*a + int((cot(c + d*x)*tan(c + d*x))/(sqrt(tan(c + d*x)*i +
 1)*tan(c + d*x)*i + sqrt(tan(c + d*x)*i + 1)),x)*b)/(sqrt(a)*a)
```

3.102 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 217

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(3iA-2B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{(13A+7iB)\cot(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{2a^2d}$$

output
```
(3*I*A-2*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/4*(I*A+B
)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(3/2)/d+
1/3*(A+I*B)*cot(d*x+c)/d/(a+I*a*tan(d*x+c))^(3/2)+1/6*(13*A+7*I*B)*cot(d*x
+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*(7*A+3*I*B)*cot(d*x+c)*(a+I*a*tan(d*x
+c))^(1/2)/a^2/d
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{12i(3A+2iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)+3\sqrt{2}(iA+B)\text{arctanh}}{12a}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/
2),x]
```

output
```
((12*I)*(3*A + (2*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]] + 3*Sq
rt[2]*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])] - (2
*Sqrt[a]*(-3*(7*A + (3*I)*B) + ((29*I)*A - 11*B)*Cot[c + d*x] + 6*A*Cot[c
+ d*x]^2))/((I + Cot[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]))/(12*a^(3/2)*d)
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4081, 25, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\int \frac{\cot^2(c+dx)(2a(4A+iB)-5a(iA-B)\tan(c+dx))}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\cot^2(c+dx)(2a(4A+iB)-5a(iA-B)\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{2a(4A+iB)-5a(iA-B)\tan(c+dx)}{\tan(c+dx)^2\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\frac{\int \frac{3}{2}\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}(2a^2(7A+3iB)-a^2(13iA-7B)\tan(c+dx))dx}{a^2}+\frac{a(13A+7iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}(2a^2(7A+3iB)-a^2(13iA-7B)\tan(c+dx))dx}{2a^2}+\frac{a(13A+7iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}\left(2a^2(7A+3iB)-a^2(13iA-7B)\tan(c+dx)\right)}{\tan(c+dx)^2}dx}{2a^2}+\frac{a(13A+7iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 4081}$$

$$\frac{\frac{3\left(\frac{\int -\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(2(3iA-2B)a^3+(7A+3iB)\tan(c+dx)a^3\right)dx}{a}-\frac{2a^2(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a(13A+7iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{3\left(-\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(2(3iA-2B)a^3+(7A+3iB)\tan(c+dx)a^3\right)dx}{a}-\frac{2a^2(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a(13A+7iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+$$
$$\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(2(3iA-2B)a^3+(7A+3iB)\tan(c+dx)a^3\right)}{\tan(c+dx)}dx}{a}-\frac{2a^2(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a(13A+7iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+$$
$$\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 4083

$$\frac{\frac{3\left(-\frac{a^3(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+2a^2(-2B+3iA)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}-\frac{2a^2(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+a(}{6a^2}$$
$$\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{3\left(-\frac{a^3(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+2a^2(-2B+3iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{a}-\frac{2a^2(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a(13A+7}{d\sqrt{a+i}}}{6a^2}$$
$$\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3961

$$\frac{\frac{3\left(-\frac{2a^2(-2B+3iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2ia^4(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{a}-\frac{2a^2(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right.}{2a^2}}{6a^2}$$
$$\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 219

$$\frac{3\left(-\frac{\frac{2a^2(-2B+3iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{2a^2(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 4082

$$\frac{3\left(-\frac{\frac{\frac{2a^4(-2B+3iA)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{2a^2(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}\right)}{6a^2}+$$

$$\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 73

$$\frac{3\left(-\frac{-\frac{4ia^3(-2B+3iA)\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{2a^2(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 221

$$\frac{3\left(-\frac{-\frac{4a^{7/2}(-2B+3iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{2a^2(7A+3iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}\right)}{6a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((A + I*B)*Cot[c + d*x])/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((a*(13*A +
(7*I)*B)*Cot[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*(-(((-4*a^(7/2)
*((3*I)*A - 2*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - (I*Sqrt[
2]*a^(7/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/d)/a) - (2*a^2*(7*A + (3*I)*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/
d))/(2*a^2))/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{2ia^2\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{7}{2}}}-\frac{3iB+5A}{4a^3\sqrt{a+ia\tan(dx+c)}}-\frac{iB+A}{6a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{\frac{iA\sqrt{a+ia\tan(dx+c)}}{2a\tan(dx+c)}+}{}\right.}{d}$
default	$\frac{2ia^2\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{7}{2}}}-\frac{3iB+5A}{4a^3\sqrt{a+ia\tan(dx+c)}}-\frac{iB+A}{6a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{\frac{iA\sqrt{a+ia\tan(dx+c)}}{2a\tan(dx+c)}+}{}\right.}{d}$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d*a^2*(-1/8*(-A+I*B)/a^(7/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1
/2)*2^(1/2)/a^(1/2))-1/4/a^3*(5*A+3*I*B)/(a+I*a*tan(d*x+c))^(1/2)-1/6/a^2*
(A+I*B)/(a+I*a*tan(d*x+c))^(3/2)+1/a^3*(1/2*I*A*(a+I*a*tan(d*x+c))^(1/2)/a
/tan(d*x+c)+1/2*(3*A+2*I*B)/a^(1/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/
2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 814 vs. 2(170) = 340.

Time = 0.14 (sec) , antiderivative size = 814, normalized size of antiderivative = 3.75

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
-1/12*(3*sqrt(1/2)*(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c))
*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*e
^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 -
 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c
)/(I*A + B)) - 3*sqrt(1/2)*(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x +
 3*I*c))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^3*d^2))*log(4*(sqrt(2)*sqrt(1/2)*(
a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
-(A^2 - 2*I*A*B - B^2)/(a^3*d^2)) - (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*
x - I*c)/(I*A + B)) - 3*(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*
I*c))*sqrt(-(9*A^2 + 12*I*A*B - 4*B^2)/(a^3*d^2))*log(-16*(3*(3*I*A - 2*B)
*a^2*e^(2*I*d*x + 2*I*c) + (3*I*A - 2*B)*a^2 + 2*sqrt(2)*(a^3*d*e^(3*I*d*x
 + 3*I*c) + a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
-(9*A^2 + 12*I*A*B - 4*B^2)/(a^3*d^2)))*e^(-2*I*d*x - 2*I*c)/(-3*I*A + 2*B
)) + 3*(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c))*sqrt(-(9*A^
2 + 12*I*A*B - 4*B^2)/(a^3*d^2))*log(-16*(3*(3*I*A - 2*B)*a^2*e^(2*I*d*x +
 2*I*c) + (3*I*A - 2*B)*a^2 - 2*sqrt(2)*(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d
*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(9*A^2 + 12*I*A*
B - 4*B^2)/(a^3*d^2)))*e^(-2*I*d*x - 2*I*c)/(-3*I*A + 2*B)) + sqrt(2)*(2*(
14*I*A - 5*B)*e^(6*I*d*x + 6*I*c) - (-13*I*A + B)*e^(4*I*d*x + 4*I*c) + 2*
(-8*I*A + 5*B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*...
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan(c+dx))\cot^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3
/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.99

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{ia\left(\frac{4\left(3\,(ia\tan(dx+c)+a)^2(7\,A+3i\,B)-(ia\tan(dx+c)+a)(13\,A+7i\,B)a-2\,(A+i\,B)a^2\right)}{(ia\tan(dx+c)+a)^{\frac{5}{2}}a^2-(ia\tan(dx+c)+a)^{\frac{3}{2}}a^3}+\frac{3\sqrt{2}(A-i\,B)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{a^{\frac{5}{2}}}}{24\,d}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
-1/24*I*a*(4*(3*(I*a*tan(d*x + c) + a)^2*(7*A + 3*I*B) - (I*a*tan(d*x + c)
 + a)*(13*A + 7*I*B)*a - 2*(A + I*B)*a^2)/((I*a*tan(d*x + c) + a)^(5/2)*a^
2 - (I*a*tan(d*x + c) + a)^(3/2)*a^3) + 3*sqrt(2)*(A - I*B)*log(-(sqrt(2)*
sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x
+ c) + a)))/a^(5/2) + 12*(3*A + 2*I*B)*log((sqrt(I*a*tan(d*x + c) + a) - s
qrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(5/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 3051, normalized size of antiderivative = 14.06

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
2*atanh((3*d^4*(a + a*tan(c + d*x)*1i)^(1/2)*((33*B^2)/(64*a^3*d^2) - (73*
A^2)/(64*a^3*d^2) - ((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B
^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^(1/2)/(64*a^
6) - (A*B*47i)/(32*a^3*d^2))^(1/2)*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4
 - (14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^
4)^(1/2))/((A^3*a^2*d*781i)/4 + (279*B^3*a^2*d)/4 - (A*B^2*a^2*d*1223i)/4
- (1717*A^2*B*a^2*d)/4 + (A*d^3*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 -
(14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^
(1/2)*13i)/(4*a) - (7*B*d^3*((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (140
06*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^(1/2
))/(4*a)) + (71*A^2*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((33*B^2)/(64*a^3*d^
2) - (73*A^2)/(64*a^3*d^2) - ((5041*A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14
006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076i)/d^4 + (A^3*B*a^6*13916i)/d^4)^(1/
2)/(64*a^6) - (A*B*47i)/(32*a^3*d^2))^(1/2))/((A^3*d*781i)/(4*a) + (279*B^
3*d)/(4*a) - (A*B^2*d*1223i)/(4*a) - (1717*A^2*B*d)/(4*a) + (A*d^3*((5041*
A^4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*60
76i)/d^4 + (A^3*B*a^6*13916i)/d^4)^(1/2)*13i)/(4*a^4) - (7*B*d^3*((5041*A^
4*a^6)/d^4 + (961*B^4*a^6)/d^4 - (14006*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*6076
i)/d^4 + (A^3*B*a^6*13916i)/d^4)^(1/2))/(4*a^4)) - (31*B^2*d^2*(a + a*tan(
c + d*x)*1i)^(1/2)*((33*B^2)/(64*a^3*d^2) - (73*A^2)/(64*a^3*d^2) - ((5...
```

Reduce [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\int \frac{\cot(dx+c)^2}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx\right)a+\left(\int \frac{\cot(d}{\sqrt{\tan(dx+c)i+1}\,t}\right.}{\sqrt{a}\,a}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(int(cot(c + d*x)**2/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c
 + d*x)*i + 1)),x)*a + int((cot(c + d*x)**2*tan(c + d*x))/(sqrt(tan(c + d*
x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c + d*x)*i + 1)),x)*b)/(sqrt(a)*a)
```

3.103 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 268

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(23A+12iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} - \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{(17A+11iB)\cot^2(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}} + \frac{7(3iA-2B)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4a^2d} - \frac{(22A+13iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{6a^2d}$$

output

```
1/4*(23*A+12*I*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/4*
(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(3
/2)/d+1/3*(A+I*B)*cot(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(3/2)+1/6*(17*A+11*I*B
)*cot(d*x+c)^2/a/d/(a+I*a*tan(d*x+c))^(1/2)+7/4*(3*I*A-2*B)*cot(d*x+c)*(a+
I*a*tan(d*x+c))^(1/2)/a^2/d-1/6*(22*A+13*I*B)*cot(d*x+c)^2*(a+I*a*tan(d*x+
c))^(1/2)/a^2/d
```

Mathematica [A] (verified)

Time = 3.51 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.68

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{3(23A+12iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)-3\sqrt{2}(A-iB)\text{arctanh}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/
2),x]
```

output
```
(3*(23*A + (12*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]] - 3*Sqrt[
2]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])] - (Sqrt
[a]*((63*I)*A - 42*B + (82*A + (58*I)*B)*Cot[c + d*x] + 3*((-3*I)*A + 4*B)
*Cot[c + d*x]^2 + 6*A*Cot[c + d*x]^3))/((I + Cot[c + d*x])*Sqrt[a + I*a*Ta
n[c + d*x]]))/(12*a^(3/2)*d)
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.07, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{\cot^3(c+dx)(2a(5A+2iB)-7a(iA-B)\tan(c+dx))}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\cot^3(c+dx)(2a(5A+2iB)-7a(iA-B)\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2a(5A+2iB)-7a(iA-B)\tan(c+dx)}{\tan(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4079$$

$$\frac{\frac{\int \frac{1}{2}\cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}\left(4a^2(22A+13iB)-5a^2(17iA-11B)\tan(c+dx)\right)dx}{a^2}+\frac{a(17A+11iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}\left(4a^2(22A+13iB)-5a^2(17iA-11B)\tan(c+dx)\right)dx}{2a^2}+\frac{a(17A+11iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(4a^2(22A+13iB)-5a^2(17iA-11B)\tan(c+dx)\right)}{\tan(c+dx)^3}dx}{2a^2}+\frac{a(17A+11iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4081$$

$$\frac{\frac{\frac{\int -6\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left(7(3iA-2B)a^3+(22A+13iB)\tan(c+dx)a^3\right)dx}{2a}-\frac{2a^2(22A+13iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{a(17A+11iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\frac{3\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left(7(3iA-2B)a^3+(22A+13iB)\tan(c+dx)a^3\right)dx}{a}-\frac{2a^2(22A+13iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{a(17A+11iB)\cot^2(c+\ldots}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}$$
$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}\left(7(3iA-2B)a^3+(22A+13iB)\tan(c+dx)a^3\right)}{\tan(c+dx)^2}dx}{a}-\frac{2a^2(22A+13iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}+\frac{a(17A+11iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+$$
$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 4081

$$\frac{-\frac{\frac{3\left(\frac{\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^4(23A+12iB)-7a^4(3iA-2B)\tan(c+dx)\right)dx}{a}-\frac{7a^3(-2B+3iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}-\frac{2a^2(22A+13iB)\cot^2(c+\ldots}{d}}{2a^2}}{6a^2}$$
$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{-\frac{\frac{3\left(\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^4(23A+12iB)-7a^4(3iA-2B)\tan(c+dx)\right)dx}{2a}-\frac{7a^3(-2B+3iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}-\frac{2a^2(22A+13iB)\cot^2(c+dx\ldots}{d}}{2a^2}}{6a^2}$$
$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{3\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^4(23A+12iB)-7a^4(3iA-2B)\tan(c+dx)\right)}{\tan(c+dx)}dx}{2a}-\frac{7a^3(-2B+3iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}-\frac{2a^2(22A+13iB)\cot^2(c+dx)\sqrt{a+ia\,t\ldots}}{d}}{2a^2}}{6a^2}$$
$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 4083

$$-\frac{\frac{3\left(\frac{2a^4(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+a^3(23A+12iB)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{2a}-\frac{7a^3(-2B+3iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}-\frac{2a}{}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{3\left(\frac{2a^4(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+a^3(23A+12iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a}-\frac{7a^3(-2B+3iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}-\frac{2a^2(22A+1}{}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3961

$$-\frac{\frac{3\left(\frac{a^3(23A+12iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{4ia^5(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a}-\frac{7a^3(-2B+3iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right.}{a}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 219

$$-\frac{\frac{3\left(\frac{a^3(23A+12iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2i\sqrt{2}a^{9/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a}-\frac{7a^3(-2B+3iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx}}{d}\right.}{a}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 4082

$$\frac{-\frac{3\left(\frac{\frac{a^5(23A+12iB)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}}{2a}-\frac{2i\sqrt{2}a^{9/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{7a^3(-2B+3iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}-2a}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{-\frac{3\left(-\frac{\frac{2ia^4(23A+12iB)\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a}-\frac{2i\sqrt{2}a^{9/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{7a^3(-2B+3iA)\cot(c+dx)\sqrt{a+ia\tan}}{d}\right.}{a}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{-\frac{2a^2(22A+13iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{3\left(-\frac{\frac{2a^{9/2}(23A+12iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}}{2a}-\frac{2i\sqrt{2}a^{9/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}\right.}{a}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

input

```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output

```
((A + I*B)*Cot[c + d*x]^2)/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((a*(17*A
+ (11*I)*B)*Cot[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((-2*a^2*(22*
A + (13*I)*B)*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/d - (3*(((-2*a^(9
/2)*(23*A + (12*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - ((2
*I)*Sqrt[2]*a^(9/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*
Sqrt[a])])/d)/(2*a) - (7*a^3*((3*I)*A - 2*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan
[c + d*x]])/d))/a)/(2*a^2))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{2a^3\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{9}{2}}}-\frac{5iB+7A}{4a^4\sqrt{a+ia\tan(dx+c)}}-\frac{iB+A}{6a^3(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{-\left(-\frac{iB}{2}-\frac{7A}{8}\right)(a+ia\,\mathrm{t}}{}\right.}{d}$
default	$\frac{2a^3\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{9}{2}}}-\frac{5iB+7A}{4a^4\sqrt{a+ia\tan(dx+c)}}-\frac{iB+A}{6a^3(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{-\left(-\frac{iB}{2}-\frac{7A}{8}\right)(a+ia\,\mathrm{t}}{}\right.}{d}$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
2/d*a^3*(-1/8/a^(9/2)*(A-I*B)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))-1/4/a^4*(7*A+5*I*B)/(a+I*a*tan(d*x+c))^(1/2)-1/6/a^3*(A+
I*B)/(a+I*a*tan(d*x+c))^(3/2)+1/a^4*(-((-1/2*I*B-7/8*A)*(a+I*a*tan(d*x+c))
^(3/2)+(1/2*I*a*B+9/8*a*A)*(a+I*a*tan(d*x+c))^(1/2))/a^2/tan(d*x+c)^2+1/8*
(23*A+12*I*B)/a^(1/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(209) = 418$.

Time = 0.15 (sec) , antiderivative size = 903, normalized size of antiderivative = 3.37

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
1/48*(12*sqrt(1/2)*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c
) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*log(-
4*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*
e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 12*sqrt(1/2)*(a^2*d*e^(7*I*
d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sq
rt((A^2 - 2*I*A*B - B^2)/(a^3*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(-I*a^2*d*e^
(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 -
 2*I*A*B - B^2)/(a^3*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c
)/(I*A + B)) + 3*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c)
+ a^2*d*e^(3*I*d*x + 3*I*c))*sqrt((529*A^2 + 552*I*A*B - 144*B^2)/(a^3*d^2
))*log(-16*(3*(23*I*A - 12*B)*a^2*e^(2*I*d*x + 2*I*c) + (23*I*A - 12*B)*a^
2 + 2*sqrt(2)*(I*a^3*d*e^(3*I*d*x + 3*I*c) + I*a^3*d*e^(I*d*x + I*c))*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((529*A^2 + 552*I*A*B - 144*B^2)/(a^3*d^
2)))*e^(-2*I*d*x - 2*I*c)/(-23*I*A + 12*B)) - 3*(a^2*d*e^(7*I*d*x + 7*I*c)
 - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt((529*A^2
+ 552*I*A*B - 144*B^2)/(a^3*d^2))*log(-16*(3*(23*I*A - 12*B)*a^2*e^(2*I*d*
x + 2*I*c) + (23*I*A - 12*B)*a^2 + 2*sqrt(2)*(-I*a^3*d*e^(3*I*d*x + 3*I*c)
 - I*a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((529*A^
2 + 552*I*A*B - 144*B^2)/(a^3*d^2)))*e^(-2*I*d*x - 2*I*c)/(-23*I*A + 12...
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{(A+B\tan{(c+dx)})\cot^3{(c+dx)}}{(ia\,(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3
/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.97

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{a^2\left(\frac{2\left(21\,(i\,a\tan(dx+c)+a)^3(3\,A+2i\,B)-(i\,a\tan(dx+c)+a)^2(107\,A+68i\,B)a+2\,(i\,a\tan(dx+c)+a)(17\,A+11i\,B)a^2+4\,(A+i\,B)a^3\right)}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^3-2\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^4+(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^5}-\frac{3\sqrt{2}(\cdots}{\cdots}\right.}{24\,d}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
-1/24*a^2*(2*(21*(I*a*tan(d*x + c) + a)^3*(3*A + 2*I*B) - (I*a*tan(d*x + c
) + a)^2*(107*A + 68*I*B)*a + 2*(I*a*tan(d*x + c) + a)*(17*A + 11*I*B)*a^2
 + 4*(A + I*B)*a^3)/((I*a*tan(d*x + c) + a)^(7/2)*a^3 - 2*(I*a*tan(d*x + c
) + a)^(5/2)*a^4 + (I*a*tan(d*x + c) + a)^(3/2)*a^5) - 3*sqrt(2)*(A - I*B)
*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sq
rt(I*a*tan(d*x + c) + a)))/a^(7/2) + 3*(23*A + 12*I*B)*log((sqrt(I*a*tan(d
*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(7/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 3106, normalized size of antiderivative = 11.59

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
2*atanh((48*d^4*(a + a*tan(c + d*x)*1i)^(1/2)*((531*A^2)/(128*a^3*d^2) - (
(277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 -
 (A*B^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/d^4)^(1/2)/(64*a^6) - (73*B^
2)/(64*a^3*d^2) + (A*B*137i)/(32*a^3*d^2))^(1/2)*((277729*A^4*a^6)/(4*d^4)
 + (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*39476i)/d^4
+ (A^3*B*a^6*146506i)/d^4)^(1/2))/(B^3*a^2*d*3124i - 25296*A^3*a^2*d + 190
48*A*B^2*a^2*d - A^2*B*a^2*d*38282i + (88*A*d^3*((277729*A^4*a^6)/(4*d^4)
+ (5041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*39476i)/d^4 +
 (A^3*B*a^6*146506i)/d^4)^(1/2))/a + (B*d^3*((277729*A^4*a^6)/(4*d^4) + (5
041*B^4*a^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*39476i)/d^4 + (A^
3*B*a^6*146506i)/d^4)^(1/2)*52i)/a) - (4216*A^2*d^2*(a + a*tan(c + d*x)*1i
)^(1/2)*((531*A^2)/(128*a^3*d^2) - ((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a
^6)/d^4 - (114701*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*39476i)/d^4 + (A^3*B*a^6*1
46506i)/d^4)^(1/2)/(64*a^6) - (73*B^2)/(64*a^3*d^2) + (A*B*137i)/(32*a^3*d
^2))^(1/2))/((B^3*d*3124i)/a - (25296*A^3*d)/a + (19048*A*B^2*d)/a - (A^2*
B*d*38282i)/a + (88*A*d^3*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 -
 (114701*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/d
^4)^(1/2))/a^4 + (B*d^3*((277729*A^4*a^6)/(4*d^4) + (5041*B^4*a^6)/d^4 - (
114701*A^2*B^2*a^6)/d^4 - (A*B^3*a^6*39476i)/d^4 + (A^3*B*a^6*146506i)/d^4
)^(1/2)*52i)/a^4) + (1136*B^2*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((531*A...
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\int \frac{\cot(dx+c)^3}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx\right)a+\left(\int \frac{\cot(d}{\sqrt{\tan(dx+c)i+1}\,t}\right.}{\sqrt{a}\,a}$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(int(cot(c + d*x)**3/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c
 + d*x)*i + 1)),x)*a + int((cot(c + d*x)**3*tan(c + d*x))/(sqrt(tan(c + d*
x)*i + 1)*tan(c + d*x)*i + sqrt(tan(c + d*x)*i + 1)),x)*b)/(sqrt(a)*a)
```

3.104 $\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 255

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}+\frac{(iA-B)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} +\frac{(11A+21iB)\tan^3(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}}-\frac{(39iA-89B)\tan^2(c+dx)}{20a^2d\sqrt{a+ia\tan(c+dx)}} +\frac{(39iA-89B)\sqrt{a+ia\tan(c+dx)}}{5a^3d}-\frac{(151iA-361B)(a+ia\tan(c+dx))^{3/2}}{60a^4d}$$

output
```
-1/8*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)
/a^(5/2)/d+1/5*(I*A-B)*tan(d*x+c)^4/d/(a+I*a*tan(d*x+c))^(5/2)+1/30*(11*A+
21*I*B)*tan(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(3/2)-1/20*(39*I*A-89*B)*tan(d
*x+c)^2/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+1/5*(39*I*A-89*B)*(a+I*a*tan(d*x+c)
)^(1/2)/a^3/d-1/60*(151*I*A-361*B)*(a+I*a*tan(d*x+c))^(3/2)/a^4/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.00 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.64

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{15(iA+B)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{}$$

input
```
Integrate[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/
2),x]
```

output
```
(15*(I*A + B)*Hypergeometric2F1[-1/2, 1, 1/2, (1 + I*Tan[c + d*x])/2]*(-I
+ Tan[c + d*x])^2 - 2*((151*I)*A - 361*B - 5*(77*A + (179*I)*B)*Tan[c + d*
x] + 60*((-5*I)*A + 11*B)*Tan[c + d*x]^2 + 20*(3*A + (5*I)*B)*Tan[c + d*x]
^3 + 20*B*Tan[c + d*x]^4))/(60*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Ta
n[c + d*x]])
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4078, 27, 3042, 4075, 3042, 4010, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^4(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan^3(c+dx)(8a(iA-B)+a(3A+13iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan^3(c+dx)(8a(iA-B)+a(3A+13iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan(c+dx)^3(8a(iA-B)+a(3A+13iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{\int -\frac{3\tan^2(c+dx)\left(2a^2(11A+21iB)-a^2(17iA-47B)\tan(c+dx)\right)}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}-\frac{a(11A+21iB)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{\tan^2(c+dx)\left(2a^2(11A+21iB)-a^2(17iA-47B)\tan(c+dx)\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}-\frac{a(11A+21iB)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{\tan(c+dx)^2\left(2a^2(11A+21iB)-a^2(17iA-47B)\tan(c+dx)\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}-\frac{a(11A+21iB)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-89B+39iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \frac{1}{2}\tan(c+dx)\sqrt{i\tan(c+dx)a+a}\left(4(39iA-89B)a^3+(151A+361iB)\tan(c+dx)a^3\right)dx}{a^2}}{2a^2}-\frac{a(11A+21iB)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-89B+39iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}\left(4(39iA-89B)a^3+(151A+361iB)\tan(c+dx)a^3\right)dx}{2a^2}}{2a^2}-\frac{a(11A+21iB)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$\downarrow$ 3042

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-89B+39iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \tan(c+dx)\sqrt{i\tan(c+dx)a+a}\left(4(39iA-89B)a^3+(151A+361iB)\tan(c+dx)a^3\right)dx}{2a^2}}{2a^2}-\frac{a(11A+21iB)\tan^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$\downarrow$ 4075

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-89B+39iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \sqrt{i\tan(c+dx)a+a}\left(4a^3(39iA-89B)\tan(c+dx)-a^3(151A+361iB)\right)dx-\frac{2a^2(-361B+151iA)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}}{2a^2}-\frac{a(11A+}{3d(a+}}{10a^2}$$

$\downarrow$ 3042

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-89B+39iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \sqrt{i\tan(c+dx)a+a}\left(4a^3(39iA-89B)\tan(c+dx)-a^3(151A+361iB)\right)dx-\frac{2a^2(-361B+151iA)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}}{2a^2}-\frac{a(11A+}{3d(a+}}{10a^2}$$

$\downarrow$ 4010

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-89B+39iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{5a^3(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{8a^3(-89B+39iA)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a^2(-361B+151iA)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}}{2a^2}-\frac{a(11A}{3d(a}}{10a^2}$$

$\downarrow$ 3042

$$\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-89B+39iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{5a^3(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+\frac{8a^3(-89B+39iA)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a^2(-361B+151iA)(a+ia\tan(c+dx))^{3/2}}{3d}}{2a^2}}{2a^2}-\frac{a(11A}{3d(a}}{10a^2}$$

$\downarrow$ 3961

$$\frac{\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\frac{a^2(-89B+39iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{-\frac{10ia^4(A-iB)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d} + \frac{8a^3(-89B+39iA)\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2a^2(-361B+151iA)(a+ia\tan(c+d}{3d}}{2a^2}}{2a^2}}{10a^2}$$

$\downarrow$ 219

$$\frac{\frac{(-B+iA)\tan^4(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\frac{a^2(-89B+39iA)\tan^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{-\frac{5i\sqrt{2}a^{7/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} + \frac{8a^3(-89B+39iA)\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2a^2(-361B+151iA)(a+ia\tan(c+d}{3d}}{2a^2}}{2a^2}}{10a^2}$$

input
```
Int[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I*A - B)*Tan[c + d*x]^4)/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) - (-1/3*(a*(
11*A + (21*I)*B)*Tan[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((a^2*
((39*I)*A - 89*B)*Tan[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((-5*I
)*Sqrt[2]*a^(7/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sq
rt[a])])/d + (8*a^3*((39*I)*A - 89*B)*Sqrt[a + I*a*Tan[c + d*x]])/d - (2*a
^2*((151*I)*A - 361*B)*(a + I*a*Tan[c + d*x])^(3/2))/(3*d))/(2*a^2))/(2*a^
2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{2i\left(-\frac{iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+3iaB\sqrt{a+ia\tan(dx+c)}+aA\sqrt{a+ia\tan(dx+c)}-\frac{a^{\frac{3}{2}}(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{2\sqrt{a}}\right.}{16}\right.}{d\,a^4}$
default	$\frac{2i\left(-\frac{iB(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+3iaB\sqrt{a+ia\tan(dx+c)}+aA\sqrt{a+ia\tan(dx+c)}-\frac{a^{\frac{3}{2}}(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}}{2\sqrt{a}}\right.}{16}\right.}{d\,a^4}$
parts	$\frac{2iA\left(\sqrt{a+ia\tan(dx+c)}-\frac{\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16}+\frac{17a}{8\sqrt{a+ia\tan(dx+c)}}-\frac{7a^2}{12(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{a^3}{10(a+ia\tan\right.}{d\,a^3}$

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d/a^4*(-1/3*I*B*(a+I*a*tan(d*x+c))^(3/2)+3*I*a*B*(a+I*a*tan(d*x+c))^(1
/2)+a*A*(a+I*a*tan(d*x+c))^(1/2)-1/16*a^(3/2)*(A-I*B)*2^(1/2)*arctanh(1/2*
(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+1/8*a^2*(31*I*B+17*A)/(a+I*a*tan
(d*x+c))^(1/2)-1/12*a^3*(9*I*B+7*A)/(a+I*a*tan(d*x+c))^(3/2)+1/10*a^4*(A+I
*B)/(a+I*a*tan(d*x+c))^(5/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(198) = 396.

Time = 0.12 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.79

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{15\sqrt{\frac{1}{2}}\left(a^3de^{(7i\,dx+7i\,c)}+a^3de^{(5i\,dx+5i\,c)}\right)\sqrt{-\frac{A^2-2i\,AB-B^2}{a^5d^2}}\log\left(-\frac{4}{\right.}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
1/120*(15*sqrt(1/2)*(a^3*d*e^(7*I*d*x + 7*I*c) + a^3*d*e^(5*I*d*x + 5*I*c)
)*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^3*d*
e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2
- 2*I*A*B - B^2)/(a^5*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*
c)/(I*A + B)) - 15*sqrt(1/2)*(a^3*d*e^(7*I*d*x + 7*I*c) + a^3*d*e^(5*I*d*x
 + 5*I*c))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*log(4*(sqrt(2)*sqrt(1/2)
*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)) - (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*
d*x - I*c)/(I*A + B)) + sqrt(2)*((463*I*A - 983*B)*e^(8*I*d*x + 8*I*c) - 3
*(-219*I*A + 509*B)*e^(6*I*d*x + 6*I*c) - 12*(-14*I*A + 29*B)*e^(4*I*d*x +
 4*I*c) + (-23*I*A + 33*B)*e^(2*I*d*x + 2*I*c) + 3*I*A - 3*B)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1)))/(a^3*d*e^(7*I*d*x + 7*I*c) + a^3*d*e^(5*I*d*x + 5*I
*c))
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(5
/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.73

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\left(15\sqrt{2}(A-iB)a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-160i\,(i\,a\,\mathrm{ta}\right.}{}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
1/240*I*(15*sqrt(2)*(A - I*B)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan
(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 160*I*(I
*a*tan(d*x + c) + a)^(3/2)*B*a + 480*sqrt(I*a*tan(d*x + c) + a)*(A + 3*I*B
)*a^2 + 4*(15*(I*a*tan(d*x + c) + a)^2*(17*A + 31*I*B)*a^3 - 10*(I*a*tan(d
*x + c) + a)*(7*A + 9*I*B)*a^4 + 12*(A + I*B)*a^5)/(I*a*tan(d*x + c) + a)^
(5/2))/(a^5*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.09

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\frac{A\,1\mathrm{i}}{5\,d}-\frac{A\,\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)\,7\mathrm{i}}{6\,a\,d}+\frac{A\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^2\,17\mathrm{i}}{4\,a^2\,d}}{{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{5/2}} + \frac{A\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}}\,2\mathrm{i}}{a^3\,d} - \frac{6\,B\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}}}{a^3\,d} + \frac{2\,B\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{3/2}}{3\,a^4\,d} - \frac{\frac{B\,a^2}{5}+\frac{31\,B\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^2}{4}-\frac{3\,B\,a\,\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}{2}}{a^2\,d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{5/2}} + \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{8\,{\left(-a\right)}^{5/2}\,d} + \frac{\sqrt{2}\,B\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}}\,1\mathrm{i}}{2\,\sqrt{a}}\right)\,1\mathrm{i}}{8\,a^{5/2}\,d}$$

input
```
int((tan(c + d*x)^4*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
((A*1i)/(5*d) - (A*(a + a*tan(c + d*x)*1i)*7i)/(6*a*d) + (A*(a + a*tan(c +
 d*x)*1i)^2*17i)/(4*a^2*d))/(a + a*tan(c + d*x)*1i)^(5/2) + (A*(a + a*tan(
c + d*x)*1i)^(1/2)*2i)/(a^3*d) - (6*B*(a + a*tan(c + d*x)*1i)^(1/2))/(a^3*
d) + (2*B*(a + a*tan(c + d*x)*1i)^(3/2))/(3*a^4*d) - ((B*a^2)/5 + (31*B*(a
 + a*tan(c + d*x)*1i)^2)/4 - (3*B*a*(a + a*tan(c + d*x)*1i))/2)/(a^2*d*(a
+ a*tan(c + d*x)*1i)^(5/2)) + (2^(1/2)*A*atan((2^(1/2)*(a + a*tan(c + d*x)
*1i)^(1/2))/(2*(-a)^(1/2)))*1i)/(8*(-a)^(5/2)*d) + (2^(1/2)*B*atan((2^(1/2
)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)/(2*a^(1/2)))*1i)/(8*a^(5/2)*d)
```

Reduce [F]

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\tan(dx+c)^5}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}\right.}{}$$

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(tan(c + d*x)**5/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*b + in
t(tan(c + d*x)**4/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c
 + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a))/(sqrt(a)*
a**2)
```

3.105 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 211

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{(iA-B)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(7A+17iB)\tan^2(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{41A+151iB}{60a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{(13A+83iB)\sqrt{a+ia\tan(c+dx)}}{30a^3d}$$

output
```
1/8*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
a^(5/2)/d+1/5*(I*A-B)*tan(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(5/2)+1/30*(7*A+17
*I*B)*tan(d*x+c)^2/a/d/(a+I*a*tan(d*x+c))^(3/2)+1/60*(41*A+151*I*B)/a^2/d/
(a+I*a*tan(d*x+c))^(1/2)+1/30*(13*A+83*I*B)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d
```

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-240a^{5/2}(A+5iB)\tan^2(c+dx)+240a^{5/2}B\tan^3(c+dx)+i\Big(}{}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/
2),x]
```

output
```
(-240*a^(5/2)*(A + (5*I)*B)*Tan[c + d*x]^2 + 240*a^(5/2)*B*Tan[c + d*x]^3
+ I*(72*a^(5/2)*((3*I)*A - 13*B) + 20*a^(5/2)*(19*A + (77*I)*B)*(-I + Tan[
c + d*x]) + 30*Sqrt[a]*(I*A + B)*(a + I*a*Tan[c + d*x])^2 - 15*Sqrt[2]*(I*
A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]*(a + I*a*Tan[
c + d*x])^(5/2)))/(120*a^(5/2)*d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4075, 3042, 4009, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan^2(c+dx)(6a(iA-B)+a(A+11iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan^2(c+dx)(6a(iA-B)+a(A+11iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan(c+dx)^2(6a(iA-B)+a(A+11iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{\int -\frac{\tan(c+dx)\left(4a^2(7A+17iB)-a^2(13iA-83B)\tan(c+dx)\right)}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}-\frac{a(7A+17iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{\tan(c+dx)\left(4a^2(7A+17iB)-a^2(13iA-83B)\tan(c+dx)\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}-\frac{a(7A+17iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{\tan(c+dx)\left(4a^2(7A+17iB)-a^2(13iA-83B)\tan(c+dx)\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}-\frac{a(7A+17iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{4075}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{(13iA-83B)a^2+4(7A+17iB)\tan(c+dx)a^2}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{2a(13A+83iB)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}-\frac{a(7A+17iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{(13iA-83B)a^2+4(7A+17iB)\tan(c+dx)a^2}{\sqrt{i\tan(c+dx)a+a}}dx-\frac{2a(13A+83iB)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}-\frac{a(7A+17iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{4009}$$

$$\frac{\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-}{\frac{-\frac{15}{2}a(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx-\frac{a^2(41A+151iB)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2a(13A+83iB)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}-\frac{a(7A+17iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-}{\frac{-\frac{15}{2}a(B+iA)\int\sqrt{i\tan(c+dx)a+a}dx-\frac{a^2(41A+151iB)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2a(13A+83iB)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}-\frac{a(7A+17iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$\Big\downarrow$ 3961

$$\frac{\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-}{\frac{\frac{15ia^2(B+iA)\int\frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{a^2(41A+151iB)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2a(13A+83iB)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}-\frac{a(7A+17iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{(-B+iA)\tan^3(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-}{\frac{\frac{15ia^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}-\frac{a^2(41A+151iB)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{2a(13A+83iB)\sqrt{a+ia\tan(c+dx)}}{d}}{6a^2}-\frac{a(7A+17iB)\tan^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I*A - B)*Tan[c + d*x]^3)/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) - (-1/3*(a*(
7*A + (17*I)*B)*Tan[c + d*x]^2)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((15*I
)*a^(3/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])
/(Sqrt[2]*d) - (a^2*(41*A + (151*I)*B))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (
2*a*(13*A + (83*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/d)/(6*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{2iB\sqrt{a+ia\tan(dx+c)}+\frac{\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8}+\frac{a(17iB+7A)}{4\sqrt{a+ia\tan(dx+c)}}-\frac{a^2(7iB+5A)}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{}{5(a+}}{a^3d}$
default	$\frac{2iB\sqrt{a+ia\tan(dx+c)}+\frac{\sqrt{a}\,(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8}+\frac{a(17iB+7A)}{4\sqrt{a+ia\tan(dx+c)}}-\frac{a^2(7iB+5A)}{6(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{}{5(a+}}{a^3d}$
parts	$\frac{2A\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16\sqrt{a}}+\frac{7}{8\sqrt{a+ia\tan(dx+c)}}-\frac{5a}{12(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{a^2}{10(a+ia\tan(dx+c))^{\frac{5}{2}}}\right)}{d\,a^2}+\frac{2iB\left(\sqrt{\right.}}{}$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
2/d/a^3*(I*B*(a+I*a*tan(d*x+c))^(1/2)+1/16*a^(1/2)*(A-I*B)*2^(1/2)*arctanh
(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+1/8*a*(7*A+17*I*B)/(a+I*a*t
an(d*x+c))^(1/2)-1/12*a^2*(5*A+7*I*B)/(a+I*a*tan(d*x+c))^(3/2)+1/10*a^3*(A
+I*B)/(a+I*a*tan(d*x+c))^(5/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(162) = 324.

Time = 0.09 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.86

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{A^2-2iAB-B^2}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{A^2-2i\,AB-B^2}{a^5d^2}}+(-i\,A-B)a\right.}{i\,A+B}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
-1/120*(15*sqrt(1/2)*a^3*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*
x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*
d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))
 + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2
)*a^3*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(
sqrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2)) + (-I*A - B)*a*e^
(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - sqrt(2)*((83*A + 463*I*B)*e^(
6*I*d*x + 6*I*c) + 2*(32*A + 97*I*B)*e^(4*I*d*x + 4*I*c) - 2*(8*A + 13*I*B
)*e^(2*I*d*x + 2*I*c) + 3*A + 3*I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^
(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5
/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.77

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{15\sqrt{2}(A-iB)a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)-480i\sqrt{ia\tan(dx+c)+a}Ba-\frac{4\left(15\,(ia\tan(dx+c)+a)^2(7A\right.}{}}{240\,a^4d}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
-1/240*(15*sqrt(2)*(A - I*B)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(
d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 480*I*sqr
t(I*a*tan(d*x + c) + a)*B*a - 4*(15*(I*a*tan(d*x + c) + a)^2*(7*A + 17*I*B
)*a^2 - 10*(I*a*tan(d*x + c) + a)*(5*A + 7*I*B)*a^3 + 12*(A + I*B)*a^4)/(I
*a*tan(d*x + c) + a)^(5/2))/(a^4*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.09

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\frac{B\,1\mathrm{i}}{5\,d}-\frac{B\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,7\mathrm{i}}{6\,a\,d}+\frac{B\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2\,17\mathrm{i}}{4\,a^2\,d}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}} + \frac{B\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,2\mathrm{i}}{a^3\,d} + \frac{\frac{A\,a^2}{5}+\frac{7\,A\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2}{4}-\frac{5\,A\,a\,(a+a\tan(c+d\,x)\,1\mathrm{i})}{6}}{a^2\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}} + \frac{\sqrt{2}\,B\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{8\,(-a)^{5/2}\,d} + \frac{\sqrt{2}\,A\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{8\,a^{5/2}\,d}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
((B*1i)/(5*d) - (B*(a + a*tan(c + d*x)*1i)*7i)/(6*a*d) + (B*(a + a*tan(c +
 d*x)*1i)^2*17i)/(4*a^2*d))/(a + a*tan(c + d*x)*1i)^(5/2) + (B*(a + a*tan(
c + d*x)*1i)^(1/2)*2i)/(a^3*d) + ((A*a^2)/5 + (7*A*(a + a*tan(c + d*x)*1i)
^2)/4 - (5*A*a*(a + a*tan(c + d*x)*1i))/6)/(a^2*d*(a + a*tan(c + d*x)*1i)^
(5/2)) + (2^(1/2)*B*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(
1/2)))*1i)/(8*(-a)^(5/2)*d) + (2^(1/2)*A*atanh((2^(1/2)*(a + a*tan(c + d*x
)*1i)^(1/2))/(2*a^(1/2))))/(8*a^(5/2)*d)
```

Reduce [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\tan(dx+c)^4}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}\right.}{}$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(tan(c + d*x)**4/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*b + in
t(tan(c + d*x)**3/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c
 + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a))/(sqrt(a)*
a**2)
```

3.106 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 167

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{(iA-B)\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{3iA-13B}{30ad(a+ia\tan(c+dx))^{3/2}} - \frac{iA-31B}{20a^2d\sqrt{a+ia\tan(c+dx)}}$$

output

```
1/8*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/
a^(5/2)/d+1/5*(I*A-B)*tan(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(5/2)+1/30*(3*I*A-
13*B)/a/d/(a+I*a*tan(d*x+c))^(3/2)-1/20*(I*A-31*B)/a^2/d/(a+I*a*tan(d*x+c)
)^(1/2)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.06

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2B\tan^2(c+dx)}{d(a+ia\tan(c+dx))^{5/2}} + \frac{i\left(\frac{15\sqrt{2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}} - \frac{24a(A-9iB)}{(a+ia\tan(c+dx))^{5/2}} + \frac{20(3A-19iB)}{(a+ia\tan(c+dx))^{3/2}} - \frac{30(A-iB)}{a\sqrt{a+ia\tan(c+dx)}}\right)}{120ad}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/
2),x]
```

output
```
(-2*B*Tan[c + d*x]^2)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + ((I/120)*((15*Sqr
t[2]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/a^(3
/2) - (24*a*(A - (9*I)*B))/(a + I*a*Tan[c + d*x])^(5/2) + (20*(3*A - (19*I
)*B))/(a + I*a*Tan[c + d*x])^(3/2) - (30*(A - I*B))/(a*Sqrt[a + I*a*Tan[c
+ d*x]])))/(a*d)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4078, 27, 3042, 4073, 3042, 4009, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan(c+dx)(4a(iA-B)-a(A-9iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(-B+iA)\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan(c+dx)(4a(iA-B)-a(A-9iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan(c+dx)(4a(iA-B)-a(A-9iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}$$

$$\Big\downarrow \; 4073$$

$$\frac{(-B+iA)\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{i\int \frac{a^2(3iA-13B)-2a^2(A-9iB)\tan(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}-\frac{a(-13B+3iA)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{i\int \frac{a^2(3iA-13B)-2a^2(A-9iB)\tan(c+dx)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}-\frac{a(-13B+3iA)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \; 4009$$

$$\frac{(-B+iA)\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{i\left(\frac{5}{2}a(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx-\frac{a^2(A+31iB)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{2a^2}-\frac{a(-13B+3iA)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{i\left(\frac{5}{2}a(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx-\frac{a^2(A+31iB)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{2a^2}-\frac{a(-13B+3iA)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \; 3961$$

$$\frac{(-B+iA)\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{i\left(-\frac{5ia^2(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{a^2(A+31iB)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{2a^2}-\frac{a(-13B+3iA)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{(-B+iA)\tan^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{i\left(-\frac{5ia^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}-\frac{a^2(A+31iB)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{2a^2}-\frac{a(-13B+3iA)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((I*A - B)*Tan[c + d*x]^2)/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) - (-1/3*(a*(
(3*I)*A - 13*B))/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/2)*(((-5*I)*a^(3/2
)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2
]*d) - (a^2*(A + (31*I)*B))/(d*Sqrt[a + I*a*Tan[c + d*x]])))/a^2)/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{2i\left(-\frac{\left(-\frac{A}{8}+\frac{iB}{8}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\frac{A}{8}+\frac{7iB}{8}}{\sqrt{a+ia\tan(dx+c)}}+\frac{a(5iB+3A)}{12(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{a^{2}(iB+A)}{10(a+ia\tan(dx+c))^{\frac{5}{2}}}\right)}{d\,a^{2}}$
default	$\frac{2i\left(-\frac{\left(-\frac{A}{8}+\frac{iB}{8}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\frac{A}{8}+\frac{7iB}{8}}{\sqrt{a+ia\tan(dx+c)}}+\frac{a(5iB+3A)}{12(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{a^{2}(iB+A)}{10(a+ia\tan(dx+c))^{\frac{5}{2}}}\right)}{d\,a^{2}}$
parts	$\frac{2iA\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{3}{2}}}+\frac{1}{4(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{1}{8a\sqrt{a+ia\tan(dx+c)}}-\frac{a}{10(a+ia\tan(dx+c))^{\frac{5}{2}}}\right)}{da}+\frac{2B\left(\cdots\right.}{}$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d/a^2*(-1/2*(-1/8*A+1/8*I*B)*2^(1/2)/a^(1/2)*arctanh(1/2*(a+I*a*tan(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))-(1/8*A+7/8*I*B)/(a+I*a*tan(d*x+c))^(1/2)+1/12
*a*(3*A+5*I*B)/(a+I*a*tan(d*x+c))^(3/2)-1/10*a^2*(A+I*B)/(a+I*a*tan(d*x+c)
)^(5/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(128) = 256.

Time = 0.09 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.35

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{-\frac{A^2-2i\,AB-B^2}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{A^2-2i\,AB-B^2}{a^5d^2}}+(-i\,A-B\right.}{i\,A+B}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
-1/120*(15*sqrt(1/2)*a^3*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d
*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)) +
 (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*
a^3*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sq
rt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)) - (-I*A - B)*a*e^(I*d*x
 + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - sqrt(2)*((-3*I*A + 83*B)*e^(6*I*d*x
 + 6*I*c) - 2*(-3*I*A - 32*B)*e^(4*I*d*x + 4*I*c) - 2*(-3*I*A + 8*B)*e^(2*
I*d*x + 2*I*c) - 3*I*A + 3*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-5*I*d
*x - 5*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5
/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.84

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{i\left(15\sqrt{2}(A-iB)\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(15(ia\tan(dx+c)+a)^2(A+7iB)a-10(ia\tan(dx+c)+a)(3A+5iB\right.}{(ia\tan(dx+c)+a)^{\frac{5}{2}}}}{240a^3d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
-1/240*I*(15*sqrt(2)*(A - I*B)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*ta
n(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(
I*a*tan(d*x + c) + a)^2*(A + 7*I*B)*a - 10*(I*a*tan(d*x + c) + a)*(3*A + 5
*I*B)*a^2 + 12*(A + I*B)*a^3)/(I*a*tan(d*x + c) + a)^(5/2))/(a^3*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{A\,1i}{20\,d\,(a+a\tan(c+d\,x)\,1i)^{5/2}} + \frac{A\tan(c+d\,x)^2\,1i}{4\,d\,(a+a\tan(c+d\,x)\,1i)^{5/2}} + \frac{\frac{B\,a^2}{5}+\frac{7\,B\,(a+a\tan(c+d\,x)\,1i)^2}{4}-\frac{5\,B\,a\,(a+a\tan(c+d\,x)\,1i)}{6}}{a^2\,d\,(a+a\tan(c+d\,x)\,1i)^{5/2}} - \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1i}}{2\,\sqrt{-a}}\right)\,1i}{8\,(-a)^{5/2}\,d} + \frac{\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1i}}{2\,\sqrt{a}}\right)}{8\,a^{5/2}\,d}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
(A*1i)/(20*d*(a + a*tan(c + d*x)*1i)^(5/2)) + (A*tan(c + d*x)^2*1i)/(4*d*(
a + a*tan(c + d*x)*1i)^(5/2)) + ((B*a^2)/5 + (7*B*(a + a*tan(c + d*x)*1i)^
2)/4 - (5*B*a*(a + a*tan(c + d*x)*1i))/6)/(a^2*d*(a + a*tan(c + d*x)*1i)^(
5/2)) - (2^(1/2)*A*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1
/2)))*1i)/(8*(-a)^(5/2)*d) + (2^(1/2)*B*atanh((2^(1/2)*(a + a*tan(c + d*x)
*1i)^(1/2))/(2*a^(1/2))))/(8*a^(5/2)*d)
```

Reduce [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\tan(dx+c)^3}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}\right.}{}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(tan(c + d*x)**3/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*b + in
t(tan(c + d*x)**2/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c
 + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a))/(sqrt(a)*
a**2)
```

3.107 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 153

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} - \frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{A+3iB}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{A-iB}{4a^2d\sqrt{a+ia\tan(c+dx)}}$$

output
```
-1/8*(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)
/a^(5/2)/d-1/5*(A+I*B)/d/(a+I*a*tan(d*x+c))^(5/2)+1/6*(A+3*I*B)/a/d/(a+I*a
*tan(d*x+c))^(3/2)+1/4*(A-I*B)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\left(\frac{15\sqrt{2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{a^{3/2}} + \frac{24a(iA-B)}{(a+ia\tan(c+dx))^{5/2}} - \frac{20(i\ldots}{(a+ia\tan\ldots}\right.}{120ad}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2)
,x]
```

output
```
((I/120)*((15*Sqrt[2]*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2
]*Sqrt[a])])/a^(3/2) + (24*a*(I*A - B))/(a + I*a*Tan[c + d*x])^(5/2) - (20
*(I*A - 3*B))/(a + I*a*Tan[c + d*x])^(3/2) - (30*(I*A + B))/(a*Sqrt[a + I*
a*Tan[c + d*x]])))/(a*d)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {3042, 4073, 3042, 4009, 3042, 3960, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4073}$$

$$-\frac{i\int \frac{a(A+iB)+2aB\tan(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{2a^2}-\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \frac{a(A+iB)+2aB\tan(c+dx)}{(i\tan(c+dx)a+a)^{3/2}}dx}{2a^2}-\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4009}$$

$$-\frac{i\left(\frac{1}{2}(A-iB)\int \frac{1}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{a(-3B+iA)}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a^2}-\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\left(\frac{1}{2}(A-iB)\int \frac{1}{\sqrt{i\tan(c+dx)a+a}}dx+\frac{a(-3B+iA)}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a^2}-\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 3960

$$-\frac{i\left(\frac{1}{2}(A-iB)\left(\frac{\int \sqrt{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{a(-3B+iA)}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a^2}-\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{i\left(\frac{1}{2}(A-iB)\left(\frac{\int \sqrt{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{d\sqrt{a+ia\tan(c+dx)}}\right)+\frac{a(-3B+iA)}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a^2}-\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 3961

$$-\frac{i\left(\frac{1}{2}(A-iB)\left(\frac{i}{d\sqrt{a+ia\tan(c+dx)}}-\frac{i\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}\right)+\frac{a(-3B+iA)}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a^2}-\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 219

$$-\frac{i\left(\frac{1}{2}(A-iB)\left(\frac{i}{d\sqrt{a+ia\tan(c+dx)}}-\frac{i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}\right)+\frac{a(-3B+iA)}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a^2}-\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
-1/5*(A + I*B)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((I/2)*((a*(I*A - 3*B))/
(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((A - I*B)*(((-I)*ArcTanh[Sqrt[a + I*
a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*Sqrt[a]*d) + I/(d*Sqrt[a + I*
a*Tan[c + d*x]])))/2))/a^2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}}-\frac{2\left(-\frac{A}{4}-\frac{3iB}{4}\right)}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{a(iB+A)}{5(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{iB-A}{4a\sqrt{a+ia\tan(dx+c)}}}{ad}$
default	$\frac{-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{3}{2}}}-\frac{2\left(-\frac{A}{4}-\frac{3iB}{4}\right)}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{a(iB+A)}{5(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{iB-A}{4a\sqrt{a+ia\tan(dx+c)}}}{ad}$
parts	$\frac{A\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{5}{2}}}-\frac{1}{5(a+ia\tan(dx+c))^{\frac{5}{2}}}+\frac{1}{4a^{2}\sqrt{a+ia\tan(dx+c)}}+\frac{1}{6a(a+ia\tan(dx+c))^{\frac{3}{2}}}\right)}{d}+\frac{2iB\Big(}{}$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/d/a*(-1/16*(A-I*B)/a^(3/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*
2^(1/2)/a^(1/2))-1/3*(-1/4*A-3/4*I*B)/(a+I*a*tan(d*x+c))^(3/2)-1/10*a*(A+I
*B)/(a+I*a*tan(d*x+c))^(5/2)-1/8/a*(-A+I*B)/(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(114) = 228.

Time = 0.11 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.56

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{A^2-2i\,AB-B^2}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}(i\,a^3de^{(2i\,dx+2i\,c)}\ldots\right.}{}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
1/120*(15*sqrt(1/2)*a^3*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*x
 + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))
+ (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)
*a^3*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(s
qrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2)) + (-I*A - B)*a*e^(
I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) + sqrt(2)*((17*A - 3*I*B)*e^(6*I
*d*x + 6*I*c) + 2*(8*A + 3*I*B)*e^(4*I*d*x + 4*I*c) - 2*(2*A - 3*I*B)*e^(2
*I*d*x + 2*I*c) - 3*A - 3*I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-5*I*
d*x - 5*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\frac{15\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{\sqrt{a}} + \frac{4\left(15(ia\tan(dx+c)+a)^2(A-iB)+1\cdots\right.}{(iat\cdots}}{240a^2d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
1/240*(15*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(15*(I*a
*tan(d*x + c) + a)^2*(A - I*B) + 10*(I*a*tan(d*x + c) + a)*(A + 3*I*B)*a -
 12*(A + I*B)*a^2)/(I*a*tan(d*x + c) + a)^(5/2))/(a^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\frac{A}{5}+\frac{A\,(a+a\tan(c+d\,x)\,1\mathrm{i})}{6\,a}+\frac{A\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2}{4\,a^2}}{d\,(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{5/2}} + \frac{B\,1\mathrm{i}}{20\,d\,(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{5/2}} + \frac{B\tan(c+d\,x)^2\,1\mathrm{i}}{4\,d\,(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{5/2}} - \frac{\sqrt{2}\,B\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{8\,(-a)^{5/2}\,d} - \frac{\sqrt{2}\,A\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{8\,a^{5/2}\,d}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
((A*(a + a*tan(c + d*x)*1i))/(6*a) - A/5 + (A*(a + a*tan(c + d*x)*1i)^2)/(
4*a^2))/(d*(a + a*tan(c + d*x)*1i)^(5/2)) + (B*1i)/(20*d*(a + a*tan(c + d*
x)*1i)^(5/2)) + (B*tan(c + d*x)^2*1i)/(4*d*(a + a*tan(c + d*x)*1i)^(5/2))
- (2^(1/2)*B*atan((2^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*(-a)^(1/2)))*
1i)/(8*(-a)^(5/2)*d) - (2^(1/2)*A*atanh((2^(1/2)*(a + a*tan(c + d*x)*1i)^(
1/2))/(2*a^(1/2))))/(8*a^(5/2)*d)
```

Reduce [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\tan(dx+c)^2}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}\,d\right.}{}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(tan(c + d*x)**2/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*b + in
t(tan(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c +
d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a))/(sqrt(a)*a**
2)
```

3.108 $\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 155

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{iA-B}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{iA+B}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{iA+B}{4a^2d\sqrt{a+ia\tan(c+dx)}}$$

output
```
-1/8*(I*A+B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)
/a^(5/2)/d+1/5*(I*A-B)/d/(a+I*a*tan(d*x+c))^(5/2)+1/6*(I*A+B)/a/d/(a+I*a*t
an(d*x+c))^(3/2)+1/4*(I*A+B)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.49

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{6iA-6B-5(A-iB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx\right.}{30d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((6*I)*A - 6*B - 5*(A - I*B)*Hypergeometric2F1[-3/2, 1, -1/2, (1 + I*Tan[c
 + d*x])/2]*(-I + Tan[c + d*x]))/(30*d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3042, 4009, 3042, 3960, 3042, 3960, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4009}$$

$$\frac{(A-iB)\int \frac{1}{(i\tan(c+dx)a+a)^{3/2}}dx}{2a}+\frac{-B+iA}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int \frac{1}{(i\tan(c+dx)a+a)^{3/2}}dx}{2a}+\frac{-B+iA}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3960}$$

$$\frac{(A-iB)\left(\frac{\int \frac{1}{\sqrt{i\tan(c+dx)a+a}}dx}{2a}+\frac{i}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a}+\frac{-B+iA}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\left(\frac{\int \frac{1}{\sqrt{i\tan(c+dx)a+a}}dx}{2a}+\frac{i}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a}+\frac{-B+iA}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3960}$$

$$\frac{(A-iB)\left(\frac{\frac{\int \sqrt{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{d\sqrt{a+ia\tan(c+dx)}}}{2a}+\frac{i}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a}+\frac{-B+iA}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\left(\frac{\frac{\int \sqrt{i\tan(c+dx)a+a}dx}{2a}+\frac{i}{d\sqrt{a+ia\tan(c+dx)}}}{2a}+\frac{i}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a}+\frac{-B+iA}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3961}$$

$$\frac{(A-iB)\left(\frac{\frac{i}{d\sqrt{a+ia\tan(c+dx)}}-\frac{i\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a}+\frac{i}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a}+\frac{-B+iA}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(A-iB)\left(\frac{\frac{i}{d\sqrt{a+ia\tan(c+dx)}}-\frac{i\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}\sqrt{a}d}}{2a}+\frac{i}{3d(a+ia\tan(c+dx))^{3/2}}\right)}{2a}+\frac{-B+iA}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
(I*A - B)/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((A - I*B)*((I/3)/(d*(a + I
*a*Tan[c + d*x])^(3/2)) + (((-I)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[
2]*Sqrt[a])])/(Sqrt[2]*Sqrt[a]*d) + I/(d*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a
)))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{2i\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{5}{2}}}-\frac{-\frac{A}{2}-\frac{iB}{2}}{5(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{iB-A}{12a(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{iB-A}{8a^{2}\sqrt{a+ia\tan(dx+c)}}\right)}{d}$
default	$\frac{2i\left(-\frac{(-iB+A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{5}{2}}}-\frac{-\frac{A}{2}-\frac{iB}{2}}{5(a+ia\tan(dx+c))^{\frac{5}{2}}}-\frac{iB-A}{12a(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{iB-A}{8a^{2}\sqrt{a+ia\tan(dx+c)}}\right)}{d}$
parts	$\frac{2iAa\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{7}{2}}}+\frac{1}{8a^{3}\sqrt{a+ia\tan(dx+c)}}+\frac{1}{12a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}+\frac{1}{10a(a+ia\tan(dx+c))^{\frac{5}{2}}}\right)}{d}+$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/d*(-1/16*(A-I*B)/a^(5/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*
2^(1/2)/a^(1/2))-1/5*(-1/2*A-1/2*I*B)/(a+I*a*tan(d*x+c))^(5/2)-1/12/a*(-A+
I*B)/(a+I*a*tan(d*x+c))^(3/2)-1/8/a^2*(-A+I*B)/(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(118) = 236.

Time = 0.11 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.53

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(15\sqrt{\frac{1}{2}}a^{3}d\sqrt{-\frac{A^{2}-2i\,AB-B^{2}}{a^{5}d^{2}}}e^{(5i\,dx+5i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^{3}de^{(2i\,dx+2i\,c)}+a^{3}d\right)\sqrt{\frac{}{e^{(2}}}\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/120*(15*sqrt(1/2)*a^3*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*
x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)) +
(-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*a
^3*d*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sqr
t(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)) - (-I*A - B)*a*e^(I*d*x
+ I*c))*e^(-I*d*x - I*c)/(I*A + B)) + sqrt(2)*((23*I*A + 17*B)*e^(6*I*d*x
+ 6*I*c) - 2*(-17*I*A - 8*B)*e^(4*I*d*x + 4*I*c) - 2*(-7*I*A + 2*B)*e^(2*I
*d*x + 2*I*c) + 3*I*A - 3*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-5*I*d*
x - 5*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{(a + ia\tan(c + dx))^{5/2}}\,dx = \int \frac{A + B\tan(c + dx)}{(ia(\tan(c + dx) - i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{A + B\tan(c + dx)}{(a + ia\tan(c + dx))^{5/2}}\,dx = \frac{i\left(\frac{15\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{a^{\frac{3}{2}}} + \frac{4\left(15\,(ia\tan(dx+c)+a)^2(A-iB)+10\,(ia\tan\right.}{(ia\tan(dx+c)}\right.}{240\,ad}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/240*I*(15*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c
) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2) + 4*(15*(I
*a*tan(d*x + c) + a)^2*(A - I*B) + 10*(I*a*tan(d*x + c) + a)*(A - I*B)*a +
 12*(A + I*B)*a^2)/((I*a*tan(d*x + c) + a)^(5/2)*a))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.32

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\frac{A\,1\mathrm{i}}{5\,d}+\frac{A\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,1\mathrm{i}}{6\,a\,d}+\frac{A\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2\,1\mathrm{i}}{4\,a^2\,d}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{5/2}} + \frac{-\frac{B}{5}+\frac{B\,(a+a\tan(c+d\,x)\,1\mathrm{i})}{6\,a}+\frac{B\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2}{4\,a^2}}{d\,(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{5/2}} + \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}}\right)\,1\mathrm{i}}{8\,(-a)^{5/2}\,d} - \frac{\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{2\,\sqrt{a}}\right)}{8\,a^{5/2}\,d}$$

input
```
int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
((A*1i)/(5*d) + (A*(a + a*tan(c + d*x)*1i)*1i)/(6*a*d) + (A*(a + a*tan(c +
 d*x)*1i)^2*1i)/(4*a^2*d))/(a + a*tan(c + d*x)*1i)^(5/2) + ((B*(a + a*tan(
c + d*x)*1i))/(6*a) - B/5 + (B*(a + a*tan(c + d*x)*1i)^2)/(4*a^2))/(d*(a +
 a*tan(c + d*x)*1i)^(5/2)) + (2^(1/2)*A*atan((2^(1/2)*(a + a*tan(c + d*x)*
1i)^(1/2))/(2*(-a)^(1/2)))*1i)/(8*(-a)^(5/2)*d) - (2^(1/2)*B*atanh((2^(1/2
)*(a + a*tan(c + d*x)*1i)^(1/2))/(2*a^(1/2))))/(8*a^(5/2)*d)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\tan(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}dx\right)b-\left(\ldots\right.}{\sqrt{a}\,a^2}$$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(tan(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(t
an(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*b + int(1
/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*ta
n(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a))/(sqrt(a)*a**2)
```

3.109 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 192

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{3A+iB}{6ad(a+ia\tan(c+dx))^{3/2}} + \frac{7A+iB}{4a^2d\sqrt{a+ia\tan(c+dx)}}$$

output
```
-2*A*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+1/8*(A-I*B)*arcta
nh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(5/2)/d+1/5*(A+
I*B)/d/(a+I*a*tan(d*x+c))^(5/2)+1/6*(3*A+I*B)/a/d/(a+I*a*tan(d*x+c))^(3/2)
+1/4*(7*A+I*B)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.13

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{3a^{13/2}(A+iB)+(a+ia\tan(c+dx))\left(\frac{5}{2}a^{11/2}(3A+iB)+\frac{15}{8}a^4(\right.}{}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2)
,x]
```

output
```
(3*a^(13/2)*(A + I*B) + (a + I*a*Tan[c + d*x])*((5*a^(11/2)*(3*A + I*B))/2
 + (15*a^4*(a + I*a*Tan[c + d*x])*(2*Sqrt[a]*(7*A + I*B) - 16*A*ArcTanh[Sq
rt[a + I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[a + I*a*Tan[c + d*x]] + Sqrt[2]*(A
- I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[a + I*a*
Tan[c + d*x]]))/8))/(15*a^(13/2)*d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{5\cot(c+dx)(2aA-a(iA-B)\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\cot(c+dx)(2aA-a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{2aA-a(iA-B)\tan(c+dx)}{\tan(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 4079$$

$$\frac{\frac{\int \frac{3\cot(c+dx)\left(4a^2A-a^2(3iA-B)\tan(c+dx)\right)}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\cot(c+dx)\left(4a^2A-a^2(3iA-B)\tan(c+dx)\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{4a^2A-a^2(3iA-B)\tan(c+dx)}{\tan(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 4079$$

$$\frac{\frac{\frac{\int \frac{1}{2}\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(8a^3A-a^3(7iA-B)\tan(c+dx)\right)dx}{a^2}+\frac{a^2(7A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(8a^3A-a^3(7iA-B)\tan(c+dx)\right)dx}{2a^2}+\frac{a^2(7A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(8a^3A-a^3(7iA-B)\tan(c+dx)\right)}{\tan(c+dx)}dx}{2a^2}+\frac{a^2(7A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 4083

$$\frac{\frac{\frac{a^3(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+8a^2A\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{2a^2}+\frac{a^2(7A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{a^3(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+8a^2A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{2a^2}+\frac{a^2(7A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 3961

$$\frac{\frac{\frac{8a^2A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2ia^4(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{2a^2}+\frac{a^2(7A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 219

$$\frac{\frac{\frac{8a^2A\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{i\sqrt{2}a^{7/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a^2}+\frac{a^2(7A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 4082

$$\frac{\frac{\frac{\frac{8a^4A\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{i\sqrt{2}a^{7/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a^2}+\frac{a^2(7A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{\frac{-\frac{16ia^3A\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{i\sqrt{2}a^{7/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{2a^2}+\frac{a^2(7A+iB)}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{a^2(7A+iB)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{-\frac{i\sqrt{2}a^{7/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{16a^{7/2}A\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}}{2a^2}}{2a^2}+\frac{a(3A+iB)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}+\frac{A+iB}{5d(a+ia\tan(c+dx))^{5/2}}$$

input

```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output

```
(A + I*B)/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((a*(3*A + I*B))/(3*d*(a +
I*a*Tan[c + d*x])^(3/2)) + (((-16*a^(7/2)*A*ArcTanh[Sqrt[a + I*a*Tan[c + d
*x]]/Sqrt[a]])/d - (I*Sqrt[2]*a^(7/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c
 + d*x]]/(Sqrt[2]*Sqrt[a])])/d)/(2*a^2) + (a^2*(7*A + I*B))/(d*Sqrt[a + I*
a*Tan[c + d*x]]))/(2*a^2))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{2a\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{7}{2}}}-\frac{-iB-7A}{8a^{3}\sqrt{a+ia\tan(dx+c)}}-\frac{-iB-3A}{12a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{-iB-A}{10a(a+ia\tan(dx+c))^{\frac{5}{2}}}-\right.}{d}$
default	$\frac{2a\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{7}{2}}}-\frac{-iB-7A}{8a^{3}\sqrt{a+ia\tan(dx+c)}}-\frac{-iB-3A}{12a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{-iB-A}{10a(a+ia\tan(dx+c))^{\frac{5}{2}}}-\right.}{d}$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/d*a*(-1/16*(-A+I*B)/a^(7/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)
*2^(1/2)/a^(1/2))-1/8/a^3*(-I*B-7*A)/(a+I*a*tan(d*x+c))^(1/2)-1/12*(-I*B-3
*A)/a^2/(a+I*a*tan(d*x+c))^(3/2)-1/10*(-I*B-A)/a/(a+I*a*tan(d*x+c))^(5/2)-
1/a^(7/2)*A*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(145) = 290$.

Time = 0.12 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.35

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
-1/120*(15*sqrt(1/2)*a^3*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*
x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*
d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))
 + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2
)*a^3*d*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(
sqrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2)) + (-I*A - B)*a*e^
(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) + 60*a^3*d*sqrt(A^2/(a^5*d^2))*
e^(5*I*d*x + 5*I*c)*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) + A*a^2 + 2*sqrt(2
)*(a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(A^2/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/A) - 60*a^3*d*sqrt
(A^2/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(16*(3*A*a^2*e^(2*I*d*x + 2*I*c) +
A*a^2 - 2*sqrt(2)*(a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(A^2/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/A)
 - sqrt(2)*((123*A + 23*I*B)*e^(6*I*d*x + 6*I*c) + 2*(72*A + 17*I*B)*e^(4*
I*d*x + 4*I*c) + 2*(12*A + 7*I*B)*e^(2*I*d*x + 2*I*c) + 3*A + 3*I*B)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\cot(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.97

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{\frac{15\sqrt{2}(A-iB)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)}{a^{\frac{5}{2}}}-\frac{240A\log\left(\frac{\sqrt{ia\tan(dx+c)+a}-\sqrt{a}}{\sqrt{ia\tan(dx+c)+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{4\left(15\,(ia\tan(dx+c)+a)^{2}(7A+iB)+10\,(ia\tan(dx+\right.}{(ia\tan(dx+c)+a)^{\frac{5}{2}}}}{240\,d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
-1/240*(15*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2) - 240*A*log
((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt
(a)))/a^(5/2) - 4*(15*(I*a*tan(d*x + c) + a)^2*(7*A + I*B) + 10*(I*a*tan(d
*x + c) + a)*(3*A + I*B)*a + 12*(A + I*B)*a^2)/((I*a*tan(d*x + c) + a)^(5/
2)*a^2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.75

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
((A + B*1i)/(5*d) + ((3*A + B*1i)*(a + a*tan(c + d*x)*1i))/(6*a*d) + ((7*A
 + B*1i)*(a + a*tan(c + d*x)*1i)^2)/(4*a^2*d))/(a + a*tan(c + d*x)*1i)^(5/
2) - (2*A*atanh((127*A^3*a*d*(1/a^3)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2))/
(127*A^3*d + A*B^2*d + A^2*B*d*2i) + (A*B^2*a*d*(1/a^3)^(1/2)*(a + a*tan(c
 + d*x)*1i)^(1/2))/(127*A^3*d + A*B^2*d + A^2*B*d*2i) + (A^2*B*a*d*(1/a^3)
^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*2i)/(127*A^3*d + A*B^2*d + A^2*B*d*2i
))*(1/a^3)^(1/2))/(a*d) + (2^(1/2)*atanh((2^(1/2)*A^3*a*d*(-1/a^3)^(1/2)*(
a + a*tan(c + d*x)*1i)^(1/2)*127i)/(32*((127*A^3*d)/16 - (B^3*d*1i)/16 + (
3*A*B^2*d)/16 - (A^2*B*d*125i)/16)) + (2^(1/2)*B^3*a*d*(-1/a^3)^(1/2)*(a +
 a*tan(c + d*x)*1i)^(1/2))/(32*((127*A^3*d)/16 - (B^3*d*1i)/16 + (3*A*B^2*
d)/16 - (A^2*B*d*125i)/16)) + (2^(1/2)*A*B^2*a*d*(-1/a^3)^(1/2)*(a + a*tan
(c + d*x)*1i)^(1/2)*3i)/(32*((127*A^3*d)/16 - (B^3*d*1i)/16 + (3*A*B^2*d)/
16 - (A^2*B*d*125i)/16)) + (125*2^(1/2)*A^2*B*a*d*(-1/a^3)^(1/2)*(a + a*ta
n(c + d*x)*1i)^(1/2))/(32*((127*A^3*d)/16 - (B^3*d*1i)/16 + (3*A*B^2*d)/16
 - (A^2*B*d*125i)/16)))*(A*1i + B)*(-1/a^3)^(1/2))/(8*a*d)
```

Reduce [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\cot(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}\,d$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(cot(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(t
an(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a + int((
cot(c + d*x)*tan(c + d*x))/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*s
qrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*b))/
(sqrt(a)*a**2)
```

3.110 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 259

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{(5iA-2B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d} + \frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(19A+9iB)\cot(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{(41A+15iB)\cot(c+dx)}{12a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{7(3A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4a^3d}$$

output

```
(5*I*A-2*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+1/8*(I*A+B
)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(5/2)/d+
1/5*(A+I*B)*cot(d*x+c)/d/(a+I*a*tan(d*x+c))^(5/2)+1/30*(19*A+9*I*B)*cot(d*
x+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)+1/12*(41*A+15*I*B)*cot(d*x+c)/a^2/d/(a+I
*a*tan(d*x+c))^(1/2)-7/4*(3*A+I*B)*cot(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^3
/d
```

Mathematica [A] (verified)

Time = 4.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.04

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{12a^{15/2}(A+iB)\cot(c+dx)-\frac{1}{2}a^7(i+\cot(c+dx))\tan^2(c+dx}{}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/
2),x]
```

output
```
(12*a^(15/2)*(A + I*B)*Cot[c + d*x] - (a^7*(I + Cot[c + d*x])*Tan[c + d*x]
^2*(2*Sqrt[a]*(-105*(3*A + I*B) + (5*I)*(85*A + (27*I)*B)*Cot[c + d*x] + 1
2*(6*A + I*B)*Cot[c + d*x]^2) + 120*(5*A + (2*I)*B)*ArcTanh[Sqrt[a + I*a*T
an[c + d*x]]/Sqrt[a]]*(1 - I*Cot[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]] + 15
*Sqrt[2]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]*(
1 - I*Cot[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]))/2)/(60*a^(15/2)*d*(a + I*
a*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.09, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4081, 25, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\int \frac{\cot^2(c+dx)(2a(6A+iB)-7a(iA-B)\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}+\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\cot^2(c+dx)(2a(6A+iB)-7a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{2a(6A+iB)-7a(iA-B)\tan(c+dx)}{\tan(c+dx)^2(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\int \frac{5\cot^2(c+dx)\left(2a^2(11A+3iB)-a^2(19iA-9B)\tan(c+dx)\right)}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{a(19A+9iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{5\int \frac{\cot^2(c+dx)\left(2a^2(11A+3iB)-a^2(19iA-9B)\tan(c+dx)\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(19A+9iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{5\int \frac{2a^2(11A+3iB)-a^2(19iA-9B)\tan(c+dx)}{\tan(c+dx)^2\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(19A+9iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 4079

$$\frac{\frac{5\left(\frac{\int \frac{3}{2}\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left(14a^3(3A+iB)-a^3(41iA-15B)\tan(c+dx)\right)dx}{a^2}+\frac{a^2(41A+15iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{6a^2}+\frac{a(19A+9iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{5\left(\frac{3\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left(14a^3(3A+iB)-a^3(41iA-15B)\tan(c+dx)\right)dx}{2a^2}+\frac{a^2(41A+15iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{6a^2}+\frac{a(19A+9iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{5\left(\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}\left(14a^3(3A+iB)-a^3(41iA-15B)\tan(c+dx)\right)}{\tan(c+dx)^2}dx}{2a^2}+\frac{a^2(41A+15iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{6a^2}+\frac{a(19A+9iB)\cot(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\big\downarrow$ 4081

$$\frac{\frac{5\left(\frac{3\left(\frac{\int -\cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(4(5iA-2B)a^4+7(3A+iB)\tan(c+dx)a^4\right)dx}{a}-\frac{14a^3(3A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a^2(41A+15iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{6a^2}}{10a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\big\downarrow$ 25

$$\frac{\frac{5\left(\frac{3\left(-\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(4(5iA-2B)a^4+7(3A+iB)\tan(c+dx)a^4\right)dx}{a}-\frac{14a^3(3A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a^2(41A+15iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{6a^2}}{10a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{5\left(\frac{3\left(-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(4(5iA-2B)a^4+7(3A+iB)\tan(c+dx)a^4\right)}{\tan(c+dx)}dx}{a}-\frac{14a^3(3A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a^2(41A+15iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}\right)}{6a^2}+\frac{a(19}{3d(}}{10a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 4083

$$\frac{\frac{5\left(\frac{3\left(-\frac{a^4(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+4a^3(-2B+5iA)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}-\frac{14a^3(3A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a}{}}{6a^2}}{10a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{5\left(\frac{3\left(-\frac{a^4(A-iB)\int \sqrt{i\tan(c+dx)a+a}dx+4a^3(-2B+5iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{a}-\frac{14a^3(3A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a^2(41A+1}{d\sqrt{a+i}}}{6a^2}}{10a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 3961

$$\frac{\frac{5\left(\frac{3\left(-\frac{4a^3(-2B+5iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2ia^5(A-iB)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{a}-\frac{14a^3(3A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+}}{d}}{2a^2}\right)}{6a^2}}{10a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 219

$$\frac{5\left(\frac{3\left(-\frac{\frac{4a^3(-2B+5iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{a}-\frac{i\sqrt{2}a^{9/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{}-\frac{14a^3(3A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+d\cdots}}{d}\cdots\right)}{2a^2}\cdots\right)}{6a^2}\cdots}{10a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4082}$$

$$\frac{5\left(\frac{3\left(-\frac{\frac{4a^5(-2B+5iA)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{i\sqrt{2}a^{9/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{14a^3(3A+iB)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\cdots\right)}{6a^2}\cdots}{10a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{5\left(\frac{3\left(-\frac{-\frac{8ia^4(-2B+5iA)\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}-\frac{i\sqrt{2}a^{9/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{14a^3(3A+iB)\cot(c+dx)\sqrt{a+ia\,\text{ta}\cdots}}{d}\cdots\right)}{2a^2}\cdots\right)}{6a^2}\cdots}{10a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{5\left(\frac{a^2(41A+15iB)\cot(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{3\left(-\frac{-\frac{8a^{9/2}(-2B+5iA)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{i\sqrt{2}a^{9/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{14a^3(3A+iB)\,c\cdots}{}\right)}{2a^2}\right)}{6a^2}}{10a^2}$$

$$\frac{(A+iB)\cot(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((A + I*B)*Cot[c + d*x])/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((a*(19*A +
(9*I)*B)*Cot[c + d*x])/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (5*((a^2*(41*A
 + (15*I)*B)*Cot[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (3*(-(((-8*a^(
9/2)*((5*I)*A - 2*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]])/d - (I*S
qrt[2]*a^(9/2)*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[
a])])/d)/a) - (14*a^3*(3*A + I*B)*Cot[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d))/(2*a^2)))/(6*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{2ia^2\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{9}{2}}}-\frac{7iB+17A}{8a^4\sqrt{a+ia\tan(dx+c)}}-\frac{3iB+5A}{12a^3(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{iB+A}{10a^2(a+ia\tan(dx+c))}\right.}{d}$
default	$\frac{2ia^2\left(-\frac{(iB-A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{9}{2}}}-\frac{7iB+17A}{8a^4\sqrt{a+ia\tan(dx+c)}}-\frac{3iB+5A}{12a^3(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{iB+A}{10a^2(a+ia\tan(dx+c))}\right.}{d}$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*I/d*a^2*(-1/16*(-A+I*B)/a^(9/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(
1/2)*2^(1/2)/a^(1/2))-1/8/a^4*(17*A+7*I*B)/(a+I*a*tan(d*x+c))^(1/2)-1/12/a
^3*(5*A+3*I*B)/(a+I*a*tan(d*x+c))^(3/2)-1/10/a^2*(A+I*B)/(a+I*a*tan(d*x+c)
)^(5/2)+1/a^4*(1/2*I*A*(a+I*a*tan(d*x+c))^(1/2)/a/tan(d*x+c)+1/2*(5*A+2*I*
B)/a^(1/2)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 834 vs. 2(204) = 408.

Time = 0.11 (sec) , antiderivative size = 834, normalized size of antiderivative = 3.22

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
-1/120*(15*sqrt(1/2)*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c
))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(a^3*d
*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(A^2
 - 2*I*A*B - B^2)/(a^5*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)/(I*A + B)) - 15*sqrt(1/2)*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*
x + 5*I*c))*sqrt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2))*log(4*(sqrt(2)*sqrt(1/2
)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt(-(A^2 - 2*I*A*B - B^2)/(a^5*d^2)) - (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I
*d*x - I*c)/(I*A + B)) - 30*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x
+ 5*I*c))*sqrt(-(25*A^2 + 20*I*A*B - 4*B^2)/(a^5*d^2))*log(-16*(3*(5*I*A -
 2*B)*a^2*e^(2*I*d*x + 2*I*c) + (5*I*A - 2*B)*a^2 + 2*sqrt(2)*(a^4*d*e^(3*
I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(-(25*A^2 + 20*I*A*B - 4*B^2)/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/(-5*I*A
 + 2*B)) + 30*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt
(-(25*A^2 + 20*I*A*B - 4*B^2)/(a^5*d^2))*log(-16*(3*(5*I*A - 2*B)*a^2*e^(2
*I*d*x + 2*I*c) + (5*I*A - 2*B)*a^2 - 2*sqrt(2)*(a^4*d*e^(3*I*d*x + 3*I*c)
 + a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-(25*A^2
+ 20*I*A*B - 4*B^2)/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/(-5*I*A + 2*B)) - sqr
t(2)*((-403*I*A + 123*B)*e^(8*I*d*x + 8*I*c) + (-151*I*A + 21*B)*e^(6*I*d*
x + 6*I*c) - 40*(-7*I*A + 3*B)*e^(4*I*d*x + 4*I*c) + (31*I*A - 21*B)*e^...
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{(A+B\tan(c+dx))\cot^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5
/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.93

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx=$$
$$-\frac{ia\left(\frac{4\left(105(ia\tan(dx+c)+a)^3(3A+iB)-5(ia\tan(dx+c)+a)^2(41A+15iB)a-2(ia\tan(dx+c)+a)(19A+9iB)a^2-12(A+iB)a^3\right)}{(ia\tan(dx+c)+a)^{\frac{7}{2}}a^3-(ia\tan(dx+c)+a)^{\frac{5}{2}}a^4}+\frac{15\sqrt{}}{}\right.}{240\,d}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
-1/240*I*a*(4*(105*(I*a*tan(d*x + c) + a)^3*(3*A + I*B) - 5*(I*a*tan(d*x +
 c) + a)^2*(41*A + 15*I*B)*a - 2*(I*a*tan(d*x + c) + a)*(19*A + 9*I*B)*a^2
 - 12*(A + I*B)*a^3)/((I*a*tan(d*x + c) + a)^(7/2)*a^3 - (I*a*tan(d*x + c)
 + a)^(5/2)*a^4) + 15*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*t
an(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(7/2)
+ 120*(5*A + 2*I*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt(a))/(sqrt(I*a*t
an(d*x + c) + a) + sqrt(a)))/a^(7/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 3002, normalized size of antiderivative = 11.59

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
2*atanh((12*a*d^4*(a + a*tan(c + d*x)*1i)^(1/2)*((129*B^2)/(256*a^5*d^2) -
 (801*A^2)/(256*a^5*d^2) - ((638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(1
6*d^4) - (307555*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*
B*a^2*256479i)/(4*d^4))^(1/2)/(64*a^6) - (A*B*319i)/(128*a^5*d^2))^(1/2)*(
(638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2
)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^(1/2
))/((A^3*d*31161i)/8 + (2159*B^3*d)/8 - (A*B^2*d*15867i)/8 - (38621*A^2*B*
d)/8 + (A*d^3*((638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307
555*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i
)/(4*d^4))^(1/2)*41i)/(2*a) - (15*B*d^3*((638401*A^4*a^2)/(16*d^4) + (1612
9*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4
*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^(1/2))/(2*a)) + (799*A^2*a^2*d^2*(a +
 a*tan(c + d*x)*1i)^(1/2)*((129*B^2)/(256*a^5*d^2) - (801*A^2)/(256*a^5*d^
2) - ((638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B
^2*a^2)/(8*d^4) - (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4)
)^(1/2)/(64*a^6) - (A*B*319i)/(128*a^5*d^2))^(1/2))/((A^3*d*31161i)/8 + (2
159*B^3*d)/8 - (A*B^2*d*15867i)/8 - (38621*A^2*B*d)/8 + (A*d^3*((638401*A^
4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - (307555*A^2*B^2*a^2)/(8*d^4)
- (A*B^3*a^2*40767i)/(4*d^4) + (A^3*B*a^2*256479i)/(4*d^4))^(1/2)*41i)/(2*
a) - (15*B*d^3*((638401*A^4*a^2)/(16*d^4) + (16129*B^4*a^2)/(16*d^4) - ...
```

Reduce [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\cot(dx+c)^2}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}\right.}{}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(cot(c + d*x)**2/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a + in
t((cot(c + d*x)**2*tan(c + d*x))/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2
 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x
)*b))/(sqrt(a)*a**2)
```

3.111 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 312

$$\begin{aligned}\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{(43A+20iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} \\ &- \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}a^{5/2}d}+\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{(23A+13iB)\cot^2(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}}+\frac{(337A+167iB)\cot^2(c+dx)}{60a^2d\sqrt{a+ia\tan(c+dx)}} \\ &+ \frac{21(2iA-B)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{4a^3d} \\ &- \frac{(85A+41iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12a^3d}\end{aligned}$$

output
```
1/4*(43*A+20*I*B)*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d-1/8*
(A-I*B)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/a^(5
/2)/d+1/5*(A+I*B)*cot(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(5/2)+1/30*(23*A+13*I*
B)*cot(d*x+c)^2/a/d/(a+I*a*tan(d*x+c))^(3/2)+1/60*(337*A+167*I*B)*cot(d*x+
c)^2/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+21/4*(2*I*A-B)*cot(d*x+c)*(a+I*a*tan(d
*x+c))^(1/2)/a^3/d-1/12*(85*A+41*I*B)*cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2
)/a^3/d
```

Mathematica [A] (verified)

Time = 5.68 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.96

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{12a^{17/2}(A+iB)\cot^2(c+dx)-\frac{1}{2}a^8(i+\cot(c+dx))\tan^2(c+d$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/
2),x]
```

output
```
(12*a^(17/2)*(A + I*B)*Cot[c + d*x]^2 - (a^8*(I + Cot[c + d*x])*Tan[c + d*
x]^2*(-1/2*(Sqrt[a]*Csc[c + d*x]^3*(-((961*A + (461*I)*B)*Cos[c + d*x]) +
(793*A + (413*I)*B)*Cos[3*(c + d*x)] + 18*((-57*I)*A + 27*B + ((83*I)*A -
43*B)*Cos[2*(c + d*x)])*Sin[c + d*x])) - 30*(43*A + (20*I)*B)*ArcTanh[Sqrt
[a + I*a*Tan[c + d*x]]/Sqrt[a]]*(I + Cot[c + d*x])*Sqrt[a + I*a*Tan[c + d*
x]] + 15*Sqrt[2]*(A - I*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqr
t[a])]*(I + Cot[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]))/2)/(60*a^(17/2)*d*(
a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.08, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4081, 27, 3042, 4081, 3042, 4083, 3042, 3961, 219, 4082, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{\cot^3(c+dx)(2a(7A+2iB)-9a(iA-B)\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}+\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cot^3(c+dx)(2a(7A+2iB)-9a(iA-B)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a(7A+2iB)-9a(iA-B)\tan(c+dx)}{\tan(c+dx)^3(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int \frac{\cot^3(c+dx)\left(4a^2(44A+19iB)-7a^2(23iA-13B)\tan(c+dx)\right)}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{a(23A+13iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\cot^3(c+dx)\left(4a^2(44A+19iB)-7a^2(23iA-13B)\tan(c+dx)\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(23A+13iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{4a^2(44A+19iB)-7a^2(23iA-13B)\tan(c+dx)}{\tan(c+dx)^3\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(23A+13iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\frac{\int \frac{5}{2}\cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}\left(4a^3(85A+41iB)-a^3(337iA-167B)\tan(c+dx)\right)dx}{a^2}+\frac{a^2(337A+167iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(23A+13iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{5\int \cot^3(c+dx)\sqrt{i\tan(c+dx)a+a}\left(4a^3(85A+41iB)-a^3(337iA-167B)\tan(c+dx)\right)dx}{2a^2}+\frac{a^2(337A+167iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(23A+13iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{5\int \frac{\sqrt{i\tan(c+dx)a+a}\left(4a^3(85A+41iB)-a^3(337iA-167B)\tan(c+dx)\right)}{\tan(c+dx)^3}dx}{2a^2}+\frac{a^2(337A+167iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(23A+13iB)\cot^2(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 4081

$$\frac{\frac{5\left(\frac{\int -6\cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left(42(2iA-B)a^4+(85A+41iB)\tan(c+dx)a^4\right)dx}{2a}-\frac{2a^3(85A+41iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a^2(337A+167iB)\cot^2(}{d\sqrt{a+ia\tan(c+dx}}}{6a^2}}{10a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{5\left(-\frac{3\int \cot^2(c+dx)\sqrt{i\tan(c+dx)a+a}\left(42(2iA-B)a^4+(85A+41iB)\tan(c+dx)a^4\right)dx}{a}-\frac{2a^3(85A+41iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a^2(337A+167iB)\cot^2(}{d\sqrt{a+ia\tan(c+dx}}}{6a^2}}{10a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{5\left(-\frac{3\int \frac{\sqrt{i\tan(c+dx)a+a}\left(42(2iA-B)a^4+(85A+41iB)\tan(c+dx)a^4\right)}{\tan(c+dx)^2}dx}{a}-\frac{2a^3(85A+41iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{2a^2}+\frac{a^2(337A+167iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+}{10a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 4081

$$\frac{\frac{5\left(-\frac{3\left(\frac{\int \cot(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^5(43A+20iB)-21a^5(2iA-B)\tan(c+dx)\right)dx}{a}-\frac{42a^4(-B+2iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}-\frac{2a^3(85A+41iB)\cot^2(c+\cdots}{d}\cdots\right)}{2a^2}\cdots}{6a^2}\cdots}{10a^2}\cdots$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{5\left(-\frac{3\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^5(43A+20iB)-21a^5(2iA-B)\tan(c+dx)\right)}{\tan(c+dx)}dx}{a}-\frac{42a^4(-B+2iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}-\frac{2a^3(85A+41iB)\cot^2(c+dx)\sqrt{a+i\cdots}}{d}\cdots\right)}{2a^2}\cdots}{6a^2}\cdots}{10a^2}\cdots$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 4083

$$\frac{\frac{5\left(-\frac{3\left(\frac{a^5(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+a^4(43A+20iB)\int \cot(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}-\frac{42a^4(-B+2iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}-\cdots\right)}{2a^2}\cdots}{6a^2}\cdots}{10a^2}\cdots$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{5\left(-\frac{3\left(\frac{a^5(B+iA)\int \sqrt{i\tan(c+dx)a+a}dx+a^4(43A+20iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx}{a}-\frac{42a^4(-B+2iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}-\frac{2a^3(85A+\cdots}{\cdots}\cdots\right)}{2a^2}\cdots}{6a^2}\cdots}{10a^2}\cdots$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 3961

$$\frac{5\left(-\frac{3\left(\frac{\frac{a^4(43A+20iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{2ia^6(B+iA)\int \frac{1}{a-ia\tan(c+dx)}d\sqrt{i\tan(c+dx)a+a}}{d}}{a}-\frac{42a^4(-B+2iA)\cot(c+dx)\sqrt{a+ia\tan(c+}}{d}\right.}{a}\right.}{2a^2}$$

$$6a^2$$

$$10a^2$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{5\left(-\frac{3\left(\frac{\frac{a^4(43A+20iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\tan(c+dx)}dx-\frac{i\sqrt{2}a^{11/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{42a^4(-B+2iA)\cot(c+dx)\sqrt{a+ia\tan(c+}}{d}\right.}{a}\right.}{2a^2}$$

$$6a^2$$

$$10a^2$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4082$$

$$\frac{5\left(-\frac{3\left(\frac{\frac{a^6(43A+20iB)\int \frac{\cot(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{i\sqrt{2}a^{11/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}}{a}-\frac{42a^4(-B+2iA)\cot(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}\right)}{a}\right.}{2a^2}$$

$$6a^2$$

$$10a^2$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{5\left(-\frac{3\left(-\frac{\frac{2ia^5(43A+20iB)\int \frac{1}{i-\frac{i(i\tan(c+dx)a+a)}{a}}d\sqrt{i\tan(c+dx)a+a}}{d}}{a}-\frac{i\sqrt{2}a^{11/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}-\frac{42a^4(-B+2iA)\cot(c+dx)\sqrt{a+ia}}{d}\right.}{a}\right.}{2a^2}}{6a^2}}{10a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 221

$$\frac{\frac{\frac{a^2(337A+167iB)\cot^2(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{5\left(-\frac{2a^3(85A+41iB)\cot^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}-\frac{3\left(\frac{-\frac{2a^{11/2}(43A+20iB)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}}\right)}{d}}{a}-\frac{i\sqrt{2}a^{11/2}(B+}{}\right.}{}\right.}{2a^2}}{6a^2}}{10a^2}$$

$$\frac{(A+iB)\cot^2(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((A + I*B)*Cot[c + d*x]^2)/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((a*(23*A
 + (13*I)*B)*Cot[c + d*x]^2)/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((a^2*(33
7*A + (167*I)*B)*Cot[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (5*((-2*
a^3*(85*A + (41*I)*B)*Cot[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/d - (3*((
(-2*a^(11/2)*(43*A + (20*I)*B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[a]]
)/d - (I*Sqrt[2]*a^(11/2)*(I*A + B)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sq
rt[2]*Sqrt[a])])/d)/a - (42*a^4*((2*I)*A - B)*Cot[c + d*x]*Sqrt[a + I*a*Ta
n[c + d*x]])/d))/a))/(2*a^2))/(6*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{2a^3\left(-\frac{(-iB+A)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{11}{2}}}-\frac{17iB+31A}{8a^5\sqrt{a+ia\tan(dx+c)}}-\frac{5iB+7A}{12a^4(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{iB+A}{10a^3(a+ia\tan(dx+c)}\right.}{d}$
default	$\frac{2a^3\left(-\frac{(-iB+A)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(dx+c)}\,\sqrt{2}}{2\sqrt{a}}\right)}{16a^{\frac{11}{2}}}-\frac{17iB+31A}{8a^5\sqrt{a+ia\tan(dx+c)}}-\frac{5iB+7A}{12a^4(a+ia\tan(dx+c))^{\frac{3}{2}}}-\frac{iB+A}{10a^3(a+ia\tan(dx+c)}\right.}{d}$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
2/d*a^3*(-1/16*(A-I*B)/a^(11/2)*2^(1/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/
2)*2^(1/2)/a^(1/2))-1/8/a^5*(17*I*B+31*A)/(a+I*a*tan(d*x+c))^(1/2)-1/12/a^
4*(7*A+5*I*B)/(a+I*a*tan(d*x+c))^(3/2)-1/10/a^3*(A+I*B)/(a+I*a*tan(d*x+c))
^(5/2)+1/a^5*(-((-1/2*I*B-11/8*A)*(a+I*a*tan(d*x+c))^(3/2)+(1/2*I*a*B+13/8
*a*A)*(a+I*a*tan(d*x+c))^(1/2))/a^2/tan(d*x+c)^2+1/8*(43*A+20*I*B)/a^(1/2)
*arctanh((a+I*a*tan(d*x+c))^(1/2)/a^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(243) = 486$.

Time $= 0.16$ (sec) , antiderivative size $= 923$, normalized size of antiderivative $= 2.96$

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
1/240*(30*sqrt(1/2)*(a^3*d*e^(9*I*d*x + 9*I*c) - 2*a^3*d*e^(7*I*d*x + 7*I*
c) + a^3*d*e^(5*I*d*x + 5*I*c))*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))*log(
-4*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2)) + (-I*A - B)*a
*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 30*sqrt(1/2)*(a^3*d*e^(9*I
*d*x + 9*I*c) - 2*a^3*d*e^(7*I*d*x + 7*I*c) + a^3*d*e^(5*I*d*x + 5*I*c))*s
qrt((A^2 - 2*I*A*B - B^2)/(a^5*d^2))*log(-4*(sqrt(2)*sqrt(1/2)*(-I*a^3*d*e
^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((A^2
- 2*I*A*B - B^2)/(a^5*d^2)) + (-I*A - B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*
c)/(I*A + B)) + 15*(a^3*d*e^(9*I*d*x + 9*I*c) - 2*a^3*d*e^(7*I*d*x + 7*I*c
) + a^3*d*e^(5*I*d*x + 5*I*c))*sqrt((1849*A^2 + 1720*I*A*B - 400*B^2)/(a^5
*d^2))*log(-16*(3*(43*I*A - 20*B)*a^2*e^(2*I*d*x + 2*I*c) + (43*I*A - 20*B
)*a^2 + 2*sqrt(2)*(I*a^4*d*e^(3*I*d*x + 3*I*c) + I*a^4*d*e^(I*d*x + I*c))*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((1849*A^2 + 1720*I*A*B - 400*B^2)/(
a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/(-43*I*A + 20*B)) - 15*(a^3*d*e^(9*I*d*x +
 9*I*c) - 2*a^3*d*e^(7*I*d*x + 7*I*c) + a^3*d*e^(5*I*d*x + 5*I*c))*sqrt((1
849*A^2 + 1720*I*A*B - 400*B^2)/(a^5*d^2))*log(-16*(3*(43*I*A - 20*B)*a^2*
e^(2*I*d*x + 2*I*c) + (43*I*A - 20*B)*a^2 + 2*sqrt(2)*(-I*a^4*d*e^(3*I*d*x
 + 3*I*c) - I*a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t((1849*A^2 + 1720*I*A*B - 400*B^2)/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/(-...
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\cot^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5
/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.91

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{a^2\left(\frac{4\left(315\,(i\,a\tan(dx+c)+a)^4(2\,A+i\,B)-5\,(i\,a\tan(dx+c)+a)^3(211\,A+104i\,B)a+(i\,a\tan(dx+c)+a)^2(337\,A+167i\,B)a^2+2\,(i\,a\tan(dx+c)+a}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a^4-2\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^5+(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^6}\right.}{240\,d}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
-1/240*a^2*(4*(315*(I*a*tan(d*x + c) + a)^4*(2*A + I*B) - 5*(I*a*tan(d*x +
 c) + a)^3*(211*A + 104*I*B)*a + (I*a*tan(d*x + c) + a)^2*(337*A + 167*I*B
)*a^2 + 2*(I*a*tan(d*x + c) + a)*(23*A + 13*I*B)*a^3 + 12*(A + I*B)*a^4)/(
(I*a*tan(d*x + c) + a)^(9/2)*a^4 - 2*(I*a*tan(d*x + c) + a)^(7/2)*a^5 + (I
*a*tan(d*x + c) + a)^(5/2)*a^6) - 15*sqrt(2)*(A - I*B)*log(-(sqrt(2)*sqrt(
a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c)
+ a)))/a^(9/2) + 30*(43*A + 20*I*B)*log((sqrt(I*a*tan(d*x + c) + a) - sqrt
(a))/(sqrt(I*a*tan(d*x + c) + a) + sqrt(a)))/a^(9/2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 3048, normalized size of antiderivative = 9.77

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
2*atanh((192*a*d^4*(a + a*tan(c + d*x)*1i)^(1/2)*((3699*A^2)/(256*a^5*d^2)
 - ((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2
*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4
*d^4))^(1/2)/(64*a^6) - (801*B^2)/(256*a^5*d^2) + (A*B*1719i)/(128*a^5*d^2
))^(1/2)*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (88775
85*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*636253
7i)/(4*d^4))^(1/2))/(B^3*d*62322i - 643278*A^3*d + 407502*A*B^2*d - A^2*B*
d*887274i + (680*A*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16
*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^
3*B*a^2*6362537i)/(4*d^4))^(1/2))/a + (B*d^3*((13667809*A^4*a^2)/(16*d^4)
+ (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1
375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(1/2)*328i)/a) - (59152*A
^2*a^2*d^2*(a + a*tan(c + d*x)*1i)^(1/2)*((3699*A^2)/(256*a^5*d^2) - ((136
67809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A^2*B^2*a^2
)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/(4*d^4))^(
1/2)/(64*a^6) - (801*B^2)/(256*a^5*d^2) + (A*B*1719i)/(128*a^5*d^2))^(1/2)
)/(B^3*d*62322i - 643278*A^3*d + 407502*A*B^2*d - A^2*B*d*887274i + (680*A
*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a^2)/(16*d^4) - (8877585*A
^2*B^2*a^2)/(8*d^4) - (A*B^3*a^2*1375079i)/(4*d^4) + (A^3*B*a^2*6362537i)/
(4*d^4))^(1/2))/a + (B*d^3*((13667809*A^4*a^2)/(16*d^4) + (638401*B^4*a...
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\cot(dx+c)^3}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}\right.}{}$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output

```
( - (int(cot(c + d*x)**3/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a + in
t((cot(c + d*x)**3*tan(c + d*x))/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2
 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x
)*b))/(sqrt(a)*a**2)
```

3.112 $\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 130

$$\begin{aligned}\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx\\ = -\frac{2\sqrt[4]{-1}a(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a(iA+B)\sqrt{\tan(c+dx)}}{d}\\ +\frac{2a(A-iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(iA+B)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}\end{aligned}$$

output
```
-2*(-1)^(1/4)*a*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-2*a*(I*A+B)*
tan(d*x+c)^(1/2)/d+2/3*a*(A-I*B)*tan(d*x+c)^(3/2)/d+2/5*a*(I*A+B)*tan(d*x+
c)^(5/2)/d+2/7*I*a*B*tan(d*x+c)^(7/2)/d
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{2a\left(-105\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}(-105i(A-iB)+35(A-iB)\right.}{105d}$$

input
```
Integrate[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*a*(-105*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + Sq
rt[Tan[c + d*x]]*((-105*I)*(A - I*B) + 35*(A - I*B)*Tan[c + d*x] + 21*(I*A
 + B)*Tan[c + d*x]^2 + (15*I)*B*Tan[c + d*x]^3)))/(105*d)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4075, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)^{5/2}(a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4075$$

$$\int \tan^{\frac{5}{2}}(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)^{5/2}(a(A-iB)+a(iA+B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 4011

$$\int \tan^{\frac{3}{2}}(c+dx)(a(A-iB)\tan(c+dx)-a(iA+B))dx+\frac{2a(B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 3042

$$\int \tan(c+dx)^{3/2}(a(A-iB)\tan(c+dx)-a(iA+B))dx+\frac{2a(B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 4011

$$\int \sqrt{\tan(c+dx)}(-a(A-iB)-a(iA+B)\tan(c+dx))dx+\frac{2a(B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a(A-iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 3042

$$\int \sqrt{\tan(c+dx)}(-a(A-iB)-a(iA+B)\tan(c+dx))dx+\frac{2a(B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a(A-iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 4011

$$\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2a(B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a(A-iB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2a(B+iA)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 3042

$$\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2a(B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a(A-iB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2a(B+iA)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 4016

$$\frac{2a^2(B+iA)^2 \int \frac{1}{a(iA+B)+a(A-iB)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{2a(B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a(A-iB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2a(B+iA)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{2\sqrt[4]{-1}a(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}+\frac{2a(B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a(A-iB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2a(B+iA)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

input
```
Int[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*(-1)^(1/4)*a*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d - (2*a
*(I*A + B)*Sqrt[Tan[c + d*x]])/d + (2*a*(A - I*B)*Tan[c + d*x]^(3/2))/(3*d
) + (2*a*(I*A + B)*Tan[c + d*x]^(5/2))/(5*d) + (((2*I)/7)*a*B*Tan[c + d*x]
^(7/2))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(105) = 210$.

Time = 0.18 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.09

method	result
derivativedivides	$\frac{a\left(\frac{2iB\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{2iA\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2B\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{2iB\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{2A\tan(dx+c)^{\frac{3}{2}}}{3}-2iA\sqrt{\tan(dx+c)}-2B\sqrt{\tan(dx+c)}\right.}{}$
default	$\frac{a\left(\frac{2iB\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{2iA\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2B\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{2iB\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{2A\tan(dx+c)^{\frac{3}{2}}}{3}-2iA\sqrt{\tan(dx+c)}-2B\sqrt{\tan(dx+c)}\right.}{}$
parts	$\frac{(iaA+Ba)\left(\frac{2\tan(dx+c)^{\frac{5}{2}}}{5}-2\sqrt{\tan(dx+c)}+\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\right.}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*a*(2/7*I*B*tan(d*x+c)^(7/2)+2/5*I*A*tan(d*x+c)^(5/2)+2/5*B*tan(d*x+c)^
(5/2)-2/3*I*B*tan(d*x+c)^(3/2)+2/3*A*tan(d*x+c)^(3/2)-2*I*A*tan(d*x+c)^(1/
2)-2*B*tan(d*x+c)^(1/2)+1/4*(I*A+B)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*
x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*
tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-A+I*B)*2^(1
/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*
x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*ta
n(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(100) = 200.

Time = 0.13 (sec) , antiderivative size = 480, normalized size of antiderivative = 3.69

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$= \frac{105\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a^2}{d^2}}\log\left(\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}+(de^{(2}\right.}{}\right.}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/210*(105*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d
*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(2*((A - I*B)*
a*e^(2*I*d*x + 2*I*c) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B
+ I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) - 105*(d*e^(6*I*d*x + 6*I*c) + 3
*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*
B + I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) - (d*e^(2*I*d*x
 + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)
*a)) - 4*((161*I*A + 176*B)*a*e^(6*I*d*x + 6*I*c) + (329*I*A + 284*B)*a*e^
(4*I*d*x + 4*I*c) + (259*I*A + 304*B)*a*e^(2*I*d*x + 2*I*c) + (91*I*A + 76
*B)*a)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^
(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = ia\left(\int A\tan^{\frac{7}{2}}(c+dx)\,dx + \int B\tan^{\frac{9}{2}}(c+dx)\,dx + \int \left(-iA\tan^{\frac{5}{2}}(c+dx)\right)\,dx + \int \left(-iB\tan^{\frac{7}{2}}(c+dx)\right)\,dx\right)$$

input
```
integrate(tan(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
I*a*(Integral(A*tan(c + d*x)**(7/2), x) + Integral(B*tan(c + d*x)**(9/2),
x) + Integral(-I*A*tan(c + d*x)**(5/2), x) + Integral(-I*B*tan(c + d*x)**(
7/2), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(100) = 200$.

Time = 0.15 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.55

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{-120i\,Ba\tan{(dx+c)}^{\frac{7}{2}}+168\,(-i\,A-B)a\tan{(dx+c)}^{\frac{5}{2}}-280\,(A-i\,B)a\tan{(dx+c)}^{\frac{3}{2}}+840\,(i\,A+$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/420*(-120*I*B*a*tan(d*x + c)^(7/2) + 168*(-I*A - B)*a*tan(d*x + c)^(5/2
) - 280*(A - I*B)*a*tan(d*x + c)^(3/2) + 840*(I*A + B)*a*sqrt(tan(d*x + c)
) - 105*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2
*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt
(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)*l
og(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(-(I + 1)*A +
(I - 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a)/d
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{-30i\,Ba\tan{(dx+c)}^{\frac{7}{2}}-42i\,Aa\tan{(dx+c)}^{\frac{5}{2}}-42\,Ba\tan{(dx+c)}^{\frac{5}{2}}-70\,Aa\tan{(dx+c)}^{\frac{3}{2}}+70i\,Ba$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/105*(-30*I*B*a*tan(d*x + c)^(7/2) - 42*I*A*a*tan(d*x + c)^(5/2) - 42*B*
a*tan(d*x + c)^(5/2) - 70*A*a*tan(d*x + c)^(3/2) + 70*I*B*a*tan(d*x + c)^(
3/2) + 105*sqrt(2)*(-(I - 1)*A*a - (I + 1)*B*a)*arctan(-(1/2*I - 1/2)*sqrt
(2)*sqrt(tan(d*x + c))) + 210*I*A*a*sqrt(tan(d*x + c)) + 210*B*a*sqrt(tan(
d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 7.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.24

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{2\,A\,a\,\tan(c+d\,x)^{3/2}}{3\,d} - \frac{A\,a\,\sqrt{\tan\left(c+d\,x\right)}\,2\mathrm{i}}{d} + \frac{A\,a\,\tan(c+d\,x)^{5/2}\,2\mathrm{i}}{5\,d} - \frac{2\,B\,a\,\sqrt{\tan\left(c+d\,x\right)}}{d} - \frac{B\,a\,\tan(c+d\,x)^{3/2}\,2\mathrm{i}}{3\,d} + \frac{2\,B\,a\,\tan(c+d\,x)^{5/2}}{5\,d} + \frac{B\,a\,\tan(c+d\,x)^{7/2}\,2\mathrm{i}}{7\,d} - \frac{(-1)^{1/4}\,A\,a\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\,1\mathrm{i}\right)\,2\mathrm{i}}{d} + \frac{\sqrt{2}\,B\,a\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,(1+1\mathrm{i})}{d}$$

input
```
int(tan(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
(2*A*a*tan(c + d*x)^(3/2))/(3*d) - (A*a*tan(c + d*x)^(1/2)*2i)/d + (A*a*ta
n(c + d*x)^(5/2)*2i)/(5*d) - (2*B*a*tan(c + d*x)^(1/2))/d - (B*a*tan(c + d
*x)^(3/2)*2i)/(3*d) + (2*B*a*tan(c + d*x)^(5/2))/(5*d) + (B*a*tan(c + d*x)
^(7/2)*2i)/(7*d) - ((-1)^(1/4)*A*a*atan((-1)^(1/4)*tan(c + d*x)^(1/2)*1i)*
2i)/d + (2^(1/2)*B*a*atan(2^(1/2)*tan(c + d*x)^(1/2)*(1/2 - 1i/2))*(1 + 1i
))/d
```

Reduce [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{a\left(2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 ai + 2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 b - 10\sqrt{\tan(dx+c)}\,ai - 10\sqrt{\tan(\right.}{}$$

input
```
int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*(2*sqrt(tan(c + d*x))*tan(c + d*x)**2*a*i + 2*sqrt(tan(c + d*x))*tan(c
+ d*x)**2*b - 10*sqrt(tan(c + d*x))*a*i - 10*sqrt(tan(c + d*x))*b + 5*int(
sqrt(tan(c + d*x))/tan(c + d*x),x)*a*d*i + 5*int(sqrt(tan(c + d*x))/tan(c
+ d*x),x)*b*d + 5*int(sqrt(tan(c + d*x))*tan(c + d*x)**4,x)*b*d*i + 5*int(
sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*a*d))/(5*d)
```

3.113 $\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 105

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{2\sqrt[4]{-1}a(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}+\frac{2a(A-iB)\sqrt{\tan(c+dx)}}{d}+\frac{2a(iA+B)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

output
```
2*(-1)^(1/4)*a*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d+2*a*(A-I*B)*t
an(d*x+c)^(1/2)/d+2/3*a*(I*A+B)*tan(d*x+c)^(3/2)/d+2/5*I*a*B*tan(d*x+c)^(5
/2)/d
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{2a\left(15\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}(15(A-iB)+5(iA+B)\tan(c+\right.}{15d}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*a*(15*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + Sqrt
[Tan[c + d*x]]*(15*(A - I*B) + 5*(I*A + B)*Tan[c + d*x] + (3*I)*B*Tan[c +
d*x]^2)))/(15*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {3042, 4075, 3042, 4011, 3042, 4011, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^{3/2}(a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4075}$$

$$\int \tan^{\frac{3}{2}}(c+dx)(a(A-iB)+a(iA+B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^{3/2}(a(A-iB)+a(iA+B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\Big\downarrow$ 4011

$$\int \sqrt{\tan(c+dx)}(a(A-iB)\tan(c+dx)-a(iA+B))dx+\frac{2a(B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\Big\downarrow$ 3042

$$\int \sqrt{\tan(c+dx)}(a(A-iB)\tan(c+dx)-a(iA+B))dx+\frac{2a(B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\Big\downarrow$ 4011

$$\int \frac{-a(A-iB)-a(iA+B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2a(B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(A-iB)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\Big\downarrow$ 3042

$$\int \frac{-a(A-iB)-a(iA+B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2a(B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(A-iB)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\Big\downarrow$ 4016

$$\frac{2a^2(A-iB)^2\int \frac{1}{a(iA+B)\tan(c+dx)-a(A-iB)}d\sqrt{\tan(c+dx)}}{d}+\frac{2a(B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(A-iB)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\Big\downarrow$ 218

$$\frac{2\sqrt[4]{-1}a(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}+\frac{2a(B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(A-iB)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

input
```
Int[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(2*(-1)^(1/4)*a*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d + (2*a*
(A - I*B)*Sqrt[Tan[c + d*x]])/d + (2*a*(I*A + B)*Tan[c + d*x]^(3/2))/(3*d)
 + (((2*I)/5)*a*B*Tan[c + d*x]^(5/2))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(85) = 170$.

Time = 0.14 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.39

method	result
derivativedivides	$\frac{a\left(\frac{2iB\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2iA\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{2B\tan(dx+c)^{\frac{3}{2}}}{3}-2iB\sqrt{\tan(dx+c)}+2A\sqrt{\tan(dx+c)}+\frac{(iB-A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+}{\tan(dx+c)-}\right.\right.}{}\right.}{}$
default	$\frac{a\left(\frac{2iB\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2iA\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{2B\tan(dx+c)^{\frac{3}{2}}}{3}-2iB\sqrt{\tan(dx+c)}+2A\sqrt{\tan(dx+c)}+\frac{(iB-A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+}{\tan(dx+c)-}\right.\right.}{}\right.}{}$
parts	$\frac{(iaA+Ba)\left(\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx\right.\right.}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*a*(2/5*I*B*tan(d*x+c)^(5/2)+2/3*I*A*tan(d*x+c)^(3/2)+2/3*B*tan(d*x+c)^
(3/2)-2*I*B*tan(d*x+c)^(1/2)+2*A*tan(d*x+c)^(1/2)+1/4*(-A+I*B)*2^(1/2)*(ln
((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1
/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c
)^(1/2)))+1/4*(-I*A-B)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)
/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1
/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(81) = 162.

Time = 0.11 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.11

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$

$$-\frac{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{(i\,A^2+2\,AB-i\,B^2)a^2}{d^2}}\log\left(\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}+\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{(\cdots}{\cdots}}\cdots\right.}{(i\,A+\cdots}\right.}{\cdots}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/30*(15*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I*A
^2 + 2*A*B - I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) + (I*d
*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*sqrt((-
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c
)/((I*A + B)*a)) - 15*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d
)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e^(2*I*d*x + 2
*I*c) + (-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2
/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*
I*d*x - 2*I*c)/((I*A + B)*a)) - 4*((20*A - 23*I*B)*a*e^(4*I*d*x + 4*I*c) +
 6*(5*A - 4*I*B)*a*e^(2*I*d*x + 2*I*c) + (10*A - 13*I*B)*a)*sqrt((-I*e^(2*
I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) + 2
*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= ia\left(\int A\tan^{\frac{5}{2}}(c+dx)\,dx+\int B\tan^{\frac{7}{2}}(c+dx)\,dx+\int\left(-iA\tan^{\frac{3}{2}}(c+dx)\right)dx\right.$$
$$\left.+\int\left(-iB\tan^{\frac{5}{2}}(c+dx)\right)dx\right)$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
I*a*(Integral(A*tan(c + d*x)**(5/2), x) + Integral(B*tan(c + d*x)**(7/2),
x) + Integral(-I*A*tan(c + d*x)**(3/2), x) + Integral(-I*B*tan(c + d*x)**(
5/2), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(81) = 162$.

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.77

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{-24i\,Ba\tan{(dx+c)}^{\frac{5}{2}}+40\,(-i\,A-B)a\tan{(dx+c)}^{\frac{3}{2}}-120\,(A-i\,B)a\sqrt{\tan{(dx+c)}}-15\left(2\sqrt{2}(-\right.}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/60*(-24*I*B*a*tan(d*x + c)^(5/2) + 40*(-I*A - B)*a*tan(d*x + c)^(3/2) -
 120*(A - I*B)*a*sqrt(tan(d*x + c)) - 15*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*
B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I +
 1)*A + (I - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) +
 sqrt(2)*((I - 1)*A + (I + 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x
+ c) + 1) - sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)
) + tan(d*x + c) + 1))*a)/d
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{-6i\,Ba\tan\left(dx+c\right)^{\frac{5}{2}} - 10i\,Aa\tan\left(dx+c\right)^{\frac{3}{2}} - 10\,Ba\tan\left(dx+c\right)^{\frac{3}{2}} + 15\,\sqrt{2}((i+1)\,Aa-(i-1)\,Ba}{15\,d}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/15*(-6*I*B*a*tan(d*x + c)^(5/2) - 10*I*A*a*tan(d*x + c)^(3/2) - 10*B*a*
tan(d*x + c)^(3/2) + 15*sqrt(2)*((I + 1)*A*a - (I - 1)*B*a)*arctan(-(1/2*I
 - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - 30*A*a*sqrt(tan(d*x + c)) + 30*I*B*a
*sqrt(tan(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.24

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,A\,a\,\sqrt{\tan\left(c+d\,x\right)}}{d} + \frac{A\,a\,{\tan\left(c+d\,x\right)}^{3/2}\,2{}\mathrm{i}}{3\,d} - \frac{B\,a\,\sqrt{\tan\left(c+d\,x\right)}\,2{}\mathrm{i}}{d}$$
$$+ \frac{2\,B\,a\,{\tan\left(c+d\,x\right)}^{3/2}}{3\,d} + \frac{B\,a\,{\tan\left(c+d\,x\right)}^{5/2}\,2{}\mathrm{i}}{5\,d}$$
$$+ \frac{\sqrt{2}\,A\,a\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-1-\mathrm{i}\right)}{d}$$
$$- \frac{{\left(-1\right)}^{1/4}\,B\,a\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{d}$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
(2*A*a*tan(c + d*x)^(1/2))/d + (A*a*tan(c + d*x)^(3/2)*2i)/(3*d) - (B*a*ta
n(c + d*x)^(1/2)*2i)/d + (2*B*a*tan(c + d*x)^(3/2))/(3*d) + (B*a*tan(c + d
*x)^(5/2)*2i)/(5*d) - (2^(1/2)*A*a*atan(2^(1/2)*tan(c + d*x)^(1/2)*(1/2 -
1i/2))*(1 + 1i))/d - ((-1)^(1/4)*B*a*atan((-1)^(1/4)*tan(c + d*x)^(1/2)*1i
)*2i)/d
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{a\left(2\sqrt{\tan\left(dx+c\right)}\,\tan\left(dx+c\right)^{2}bi+10\sqrt{\tan\left(dx+c\right)}\,a-10\sqrt{\tan\left(dx+c\right)}\,bi-5\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)ad\right.}{}$$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*(2*sqrt(tan(c + d*x))*tan(c + d*x)**2*b*i + 10*sqrt(tan(c + d*x))*a - 1
0*sqrt(tan(c + d*x))*b*i - 5*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*d +
5*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b*d*i + 5*int(sqrt(tan(c + d*x))*
tan(c + d*x)**2,x)*a*d*i + 5*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*b*d
))/(5*d)
```

3.114 $\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 80

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{2\sqrt[4]{-1}a(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} + \frac{2a(iA+B)\sqrt{\tan(c+dx)}}{d} + \frac{2iaB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

output
```
2*(-1)^(1/4)*a*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d+2*a*(I*A+B)*t
an(d*x+c)^(1/2)/d+2/3*I*a*B*tan(d*x+c)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{2a\left(3\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}(3iA+3B+iB\tan(c+dx))\right)}{3d}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*a*(3*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + Sqrt[
Tan[c + d*x]]*((3*I)*A + 3*B + I*B*Tan[c + d*x])))/(3*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4075, 3042, 4011, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int \sqrt{\tan(c+dx)}(a(A-iB)+a(iA+B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a(A-iB)+a(iA+B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{a(A-iB)\tan(c+dx)-a(iA+B)}{\sqrt{\tan(c+dx)}}dx+\frac{2a(B+iA)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a(A-iB)\tan(c+dx)-a(iA+B)}{\sqrt{\tan(c+dx)}}dx+\frac{2a(B+iA)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{4016}$$

$$\frac{2a^2(B+iA)^2\int \frac{1}{-a(iA+B)-a(A-iB)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{2a(B+iA)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt[4]{-1}a(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}+\frac{2a(B+iA)\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(2*(-1)^(1/4)*a*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d + (2*a*
(I*A + B)*Sqrt[Tan[c + d*x]])/d + (((2*I)/3)*a*B*Tan[c + d*x]^(3/2))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(65) = 130$.

Time = 0.08 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.82

method	result
derivativedivides	$a\left(\frac{2iB\tan(dx+c)^{\frac{3}{2}}}{3}+2iA\sqrt{\tan(dx+c)}+2B\sqrt{\tan(dx+c)}+\frac{(-iA-B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{\right.\right.}{4}\right.$
default	$a\left(\frac{2iB\tan(dx+c)^{\frac{3}{2}}}{3}+2iA\sqrt{\tan(dx+c)}+2B\sqrt{\tan(dx+c)}+\frac{(-iA-B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{\right.\right.}{4}\right.$
parts	$\frac{(iaA+Ba)\left(2\sqrt{\tan(dx+c)}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(\right.\right.}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*a*(2/3*I*B*tan(d*x+c)^(3/2)+2*I*A*tan(d*x+c)^(1/2)+2*B*tan(d*x+c)^(1/2
)+1/4*(-I*A-B)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*
x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*a
rctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(A-I*B)*2^(1/2)*(ln((tan(d*x+c)-2^
(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arcta
n(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(62) = 124$.

Time = 0.10 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.65

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$

$$-\frac{3\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a^2}{d^2}}\log\left(\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}+\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a^2}{d^2}}\sqrt{\cdots}\right.}{(i\,A+B)a}\right.}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/6*(3*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2
)*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) + (d*e^(2*I*d*x + 2*I*c) + d)*sqr
t(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) - 3*(d*e^(2*I
*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(2*((A - I*B
)*a*e^(2*I*d*x + 2*I*c) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*
B + I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) + 4*((-3*I*A - 4*B)*a*e^(2*I*d
*x + 2*I*c) + (-3*I*A - 2*B)*a)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) + 1)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= ia\left(\int A\tan^{\frac{3}{2}}(c+dx)\,dx + \int B\tan^{\frac{5}{2}}(c+dx)\,dx + \int\left(-iA\sqrt{\tan(c+dx)}\right)dx + \int\left(-iB\tan^{\frac{3}{2}}(c+dx)\right)dx\right)$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
I*a*(Integral(A*tan(c + d*x)**(3/2), x) + Integral(B*tan(c + d*x)**(5/2),
x) + Integral(-I*A*sqrt(tan(c + d*x)), x) + Integral(-I*B*tan(c + d*x)**(3
/2), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(62) = 124.

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.12

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{-8i\,Ba\tan(dx+c)^{\frac{3}{2}} + 24(-i\,A-B)a\sqrt{\tan(dx+c)} + 3\left(2\sqrt{2}((i-1)\,A+(i+1)\,B)\arctan\left(\frac{1}{2}\sqrt{\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/12*(-8*I*B*a*tan(d*x + c)^(3/2) + 24*(-I*A - B)*a*sqrt(tan(d*x + c)) +
3*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(
tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt(2)*(s
qrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(sqr
t(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(-(I + 1)*A + (I - 1
)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a)/d
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{-2i\,Ba\tan\left(dx+c\right)^{\frac{3}{2}}+3\sqrt{2}((i-1)\ Aa+(i+1)\ Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)-6i\,A}{3\,d}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/3*(-2*I*B*a*tan(d*x + c)^(3/2) + 3*sqrt(2)*((I - 1)*A*a + (I + 1)*B*a)*
arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - 6*I*A*a*sqrt(tan(d*x +
 c)) - 6*B*a*sqrt(tan(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{A\,a\,\sqrt{\tan\left(c+d\,x\right)}\,2\mathrm{i}}{d}+\frac{2\,B\,a\,\sqrt{\tan\left(c+d\,x\right)}}{d}+\frac{B\,a\,{\tan\left(c+d\,x\right)}^{3/2}\,2\mathrm{i}}{3\,d}$$
$$-\frac{2\,{\left(-1\right)}^{1/4}\,A\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$
$$+\frac{\sqrt{2}\,B\,a\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-1-\mathrm{i}\right)}{d}$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
(A*a*tan(c + d*x)^(1/2)*2i)/d + (2*B*a*tan(c + d*x)^(1/2))/d + (B*a*tan(c
+ d*x)^(3/2)*2i)/(3*d) - (2*(-1)^(1/4)*A*a*atanh((-1)^(1/4)*tan(c + d*x)^(
1/2)))/d - (2^(1/2)*B*a*atan(2^(1/2)*tan(c + d*x)^(1/2)*(1/2 - 1i/2))*(1 +
 1i))/d
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{a\left(2\sqrt{\tan(dx+c)}\,ai+2\sqrt{\tan(dx+c)}\,b-\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)adi-\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)bd+\left(\int\sqrt{\tan(dx\right.\right.}{d}$$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*(2*sqrt(tan(c + d*x))*a*i + 2*sqrt(tan(c + d*x))*b - int(sqrt(tan(c + d
*x))/tan(c + d*x),x)*a*d*i - int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b*d +
int(sqrt(tan(c + d*x)),x)*a*d + int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*
b*d*i))/d
```

3.115 $\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 55

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{2\sqrt[4]{-1}a(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}+\frac{2iaB\sqrt{\tan(c+dx)}}{d}$$

output
```
-2*(-1)^(1/4)*a*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d+2*I*a*B*tan(
d*x+c)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{2\sqrt[4]{-1}a(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}+\frac{2iaB\sqrt{\tan(c+dx)}}{d}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]]
,x]
```

output
```
(-2*(-1)^(1/4)*a*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d + ((2*
I)*a*B*Sqrt[Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3042, 4075, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4075}$$

$$\int \frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2iaB\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2iaB\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 4016}$$

$$\frac{2a^2(A-iB)^2\int \frac{1}{a(A-iB)-a(iA+B)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{2iaB\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2iaB\sqrt{\tan(c+dx)}}{d}-\frac{2\sqrt[4]{-1}a(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}$$

input
```
Int[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x]
```

output
```
(-2*(-1)^(1/4)*a*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d + ((2*
I)*a*B*Sqrt[Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(45) = 90$.

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.65

method	result
derivativedivides	$\frac{a\left(2iB\sqrt{\tan(dx+c)}+\frac{(-iB+A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan}\right.\right.}{4}\right.}{d}$
default	$\frac{a\left(2iB\sqrt{\tan(dx+c)}+\frac{(-iB+A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan}\right.\right.}{4}\right.}{d}$
parts	$\frac{(iaA+Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4d}$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*a*(2*I*B*tan(d*x+c)^(1/2)+1/4*(A-I*B)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*
tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^
(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(I*A+B)
*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*t
an(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/
2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 5.71

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx =$$

$$-\frac{-4i\,Ba\sqrt{\frac{-i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}+1}}-\sqrt{-\frac{(i\,A^2+2\,AB-i\,B^2)a^2}{d^2}}\,d\log\left(\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}+\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{(i\,A^2+2\,AB-i}{d^2}}\right.}{(i\,A+B)a}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-1/2*(-4*I*B*a*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)
) - sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*d*log(2*((A - I*B)*a*e^(2*I*d*x
 + 2*I*c) + (I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*
a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(
-2*I*d*x - 2*I*c)/((I*A + B)*a)) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*
d*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) + (-I*d*e^(2*I*d*x + 2*I*c) - I*d
)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)))/d
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = ia\left(\int A\sqrt{\tan{(c+dx)}}\,dx+\int B\tan^{\frac{3}{2}}{(c+dx)}\,dx+\int\left(-\frac{iA}{\sqrt{\tan{(c+dx)}}}\right)dx + \int\left(-iB\sqrt{\tan{(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
I*a*(Integral(A*sqrt(tan(c + d*x)), x) + Integral(B*tan(c + d*x)**(3/2), x
) + Integral(-I*A/sqrt(tan(c + d*x)), x) + Integral(-I*B*sqrt(tan(c + d*x)
), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(43) = 86$.

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.75

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{-8i\,Ba\sqrt{\tan{(dx+c)}}+\left(2\sqrt{2}(-(i+1)\,A+(i-1)\,B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan{(dx+c)}}\right)\right)+2\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
-1/4*(-8*I*B*a*sqrt(tan(d*x + c)) + (2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*ar
ctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A
 + (I - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt
(2)*((I - 1)*A + (I + 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c)
+ 1) - sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + t
an(d*x + c) + 1))*a)/d
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{\sqrt{2}(-(i+1)\ Aa+(i-1)\ Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)-2i\,Ba\sqrt{\tan\left(dx+c\right)}}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
-(sqrt(2)*(-(I + 1)*A*a + (I - 1)*B*a)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(
tan(d*x + c))) - 2*I*B*a*sqrt(tan(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{2\,{\left(-1\right)}^{1/4}\,B\,a\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}+\frac{B\,a\,\sqrt{\tan\left(c+d\,x\right)}\,2{}\mathrm{i}}{d}+\frac{\sqrt{2}\,A\,a\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(1+1{}\mathrm{i}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i))/tan(c + d*x)^(1/2),x)
```

output
```
(B*a*tan(c + d*x)^(1/2)*2i)/d + (2^(1/2)*A*a*atan(2^(1/2)*tan(c + d*x)^(1/
2)*(1/2 - 1i/2))*(1 + 1i))/d - (2*(-1)^(1/4)*B*a*atanh((-1)^(1/4)*tan(c +
d*x)^(1/2)))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{a\left(2\sqrt{\tan\left(dx+c\right)}\,bi+\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)ad-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)bdi+\left(\int \sqrt{\tan\left(dx+c\right)}dx\right)adi+\left(\int \sqrt{}\right.}{d}$$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(a*(2*sqrt(tan(c + d*x))*b*i + int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*d
- int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b*d*i + int(sqrt(tan(c + d*x)),x)
*a*d*i + int(sqrt(tan(c + d*x)),x)*b*d))/d
```

3.116 $\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 53

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\sqrt[4]{-1}a(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2aA}{d\sqrt{\tan(c+dx)}}$$

output
```
-2*(-1)^(1/4)*a*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-2*a*A/d/tan(
d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2ia\left(\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) - \frac{iA}{\sqrt{\tan(c+dx)}}\right)}{d}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2)
,x]
```

output
```
((-2*I)*a*((-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - (I
*A)/Sqrt[Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3042, 4074, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 4074}$$

$$-\frac{2aA}{d\sqrt{\tan(c+dx)}}+\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2aA}{d\sqrt{\tan(c+dx)}}+\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4016}$$

$$-\frac{2aA}{d\sqrt{\tan(c+dx)}}+\frac{2a^2(B+iA)^2\int \frac{1}{a(iA+B)+a(A-iB)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{2aA}{d\sqrt{\tan(c+dx)}}-\frac{2\sqrt[4]{-1}a(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}$$

input
```
Int[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x]
```

output
```
(-2*(-1)^(1/4)*a*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d - (2*a
*A)/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(44) = 88$.

Time = 0.13 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.81

method	result
derivativedivides	$\frac{a\left(-\frac{2A}{\sqrt{\tan(dx+c)}}+\frac{(iA+B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+\right.\right.}{4}}{d}$
default	$\frac{a\left(-\frac{2A}{\sqrt{\tan(dx+c)}}+\frac{(iA+B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+\right.\right.}{4}}{d}$
parts	$\frac{(iaA+Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4d}$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*a*(-2*A/tan(d*x+c)^(1/2)+1/4*(I*A+B)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*t
an(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(
1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-A+I*B)
*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*t
an(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/
2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 367, normalized size of antiderivative = 6.92

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a^2}{d^2}}\log\left(\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}+\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{\left(-i\,A^2-2\,AB+i\,B^2\right)a^2}{d^2}}\sqrt{\frac{-i\,e^{(2i}}{e^{(2i\,d}}}\right.}{(i\,A+B)a}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
1/2*((d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*l
og(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-
(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) - (d*e^(2*I*d*x
+ 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e
^(2*I*d*x + 2*I*c) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I
*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)
))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) - 4*(I*A*a*e^(2*I*d*x + 2*I*c) + I*
A*a)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(2
*I*d*x + 2*I*c) - d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = ia\left(\int \frac{A}{\sqrt{\tan(c+dx)}}\,dx+\int B\sqrt{\tan(c+dx)}\,dx+\int\left(-\frac{iA}{\tan^{\frac{3}{2}}(c+dx)}\right)dx+\int\left(-\frac{iB}{\sqrt{\tan(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
I*a*(Integral(A/sqrt(tan(c + d*x)), x) + Integral(B*sqrt(tan(c + d*x)), x)
 + Integral(-I*A/tan(c + d*x)**(3/2), x) + Integral(-I*B/sqrt(tan(c + d*x)
), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(43) = 86$.

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.85

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\left(2\sqrt{2}((i-1)\ A+(i+1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((i-1)\ A+(i+1)\ B)\,\mathrm{ar}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
1/4*((2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sq
rt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt(2)
*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(
sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(-(I + 1)*A + (I
- 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a - 8*A*a/sqr
t(tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{\sqrt{2}(-(i-1)\ Aa-(i+1)\ Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)+\frac{2\,Aa}{\sqrt{\tan(dx+c)}}}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
-(sqrt(2)*(-(I - 1)*A*a - (I + 1)*B*a)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(
tan(d*x + c))) + 2*A*a/sqrt(tan(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,(-1)^{1/4}\,A\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d} - \frac{2\,A\,a}{d\,\sqrt{\tan\left(c+d\,x\right)}} + \frac{\sqrt{2}\,B\,a\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(1+1\mathrm{i}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i))/tan(c + d*x)^(3/2),x)
```

output
```
(2*(-1)^(1/4)*A*a*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/d - (2*A*a)/(d*tan
(c + d*x)^(1/2)) + (2^(1/2)*B*a*atan(2^(1/2)*tan(c + d*x)^(1/2)*(1/2 - 1i/
2))*(1 + 1i))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = a\left(\left(\int \frac{\sqrt{\tan\left(dx+c\right)}}{\tan\left(dx+c\right)^{2}}dx\right)a+\left(\int \frac{\sqrt{\tan\left(dx+c\right)}}{\tan\left(dx+c\right)}dx\right)ai + \left(\int \frac{\sqrt{\tan\left(dx+c\right)}}{\tan\left(dx+c\right)}dx\right)b+\left(\int \sqrt{\tan\left(dx+c\right)}dx\right)bi\right)$$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
a*(int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a + int(sqrt(tan(c + d*x))/ta
n(c + d*x),x)*a*i + int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b + int(sqrt(ta
n(c + d*x)),x)*b*i)
```

3.117 $\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 78

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt[4]{-1}a(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2a(iA+B)}{d\sqrt{\tan(c+dx)}}$$

output
```
2*(-1)^(1/4)*a*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-2/3*a*A/d/tan
(d*x+c)^(3/2)-2*a*(I*A+B)/d/tan(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{-2aA-6ia(A-iB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},i\tan(c+dx)\right)\tan(c+dx)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2)
,x]
```

output
```
(-2*a*A - (6*I)*a*(A - I*B)*Hypergeometric2F1[-1/2, 1, 1/2, I*Tan[c + d*x]
]*Tan[c + d*x])/(3*d*Tan[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 4074, 3042, 4012, 25, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 4074}$$

$$-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}+\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}+\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 4012}$$

$$\int -\frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a(B+iA)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a(B+iA)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a(B+iA)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4016}$$

$$-\frac{2a^2(A-iB)^2\int \frac{1}{a(A-iB)-a(iA+B)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}-\frac{2a(B+iA)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt[4]{-1}a(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a(B+iA)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x]
```

output
```
(2*(-1)^(1/4)*a*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d - (2*a*
A)/(3*d*Tan[c + d*x]^(3/2)) - (2*a*(I*A + B))/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(64) = 128$.

Time = 0.08 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.82

method	result
derivativedivides	$a\left(\frac{(iB-A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-iA-B)}{}\right.$
default	$a\left(\frac{(iB-A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-iA-B)}{}\right.$
parts	$\frac{(iaA+Ba)\left(-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}-\frac{}{\sqrt{\tan}}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*a*(1/4*(-A+I*B)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(t
an(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-I*A-B)*2^(1/2)*(ln((tan(d*x
+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2
*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))
-2/3*A/tan(d*x+c)^(3/2)-2*(I*A+B)/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(62) = 124$.

Time = 0.09 (sec) , antiderivative size = 427, normalized size of antiderivative = 5.47

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{3\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{(i\,A^2+2\,AB-i\,B^2)a^2}{d^2}}\log\left(\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}+\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{(i\,A\cdots}{\cdots}}\cdots\right.}{(i\,A+\cdots}\cdots\right.}{\cdots}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
-1/6*(3*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I*A^2
 + 2*A*B - I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) + (I*d*e
^(2*I*d*x + 2*I*c) + I*d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*sqrt((-I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/
((I*A + B)*a)) - 3*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*s
qrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*
c) + (-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^
2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d
*x - 2*I*c)/((I*A + B)*a)) - 4*((4*A - 3*I*B)*a*e^(4*I*d*x + 4*I*c) + 2*A*
a*e^(2*I*d*x + 2*I*c) - (2*A - 3*I*B)*a)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I
*c) + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = ia\left(\int \frac{A}{\tan^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{B}{\sqrt{\tan(c+dx)}}\,dx + \int \left(-\frac{iA}{\tan^{\frac{5}{2}}(c+dx)}\right)dx + \int \left(-\frac{iB}{\tan^{\frac{3}{2}}(c+dx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
I*a*(Integral(A/tan(c + d*x)**(3/2), x) + Integral(B/sqrt(tan(c + d*x)), x
) + Integral(-I*A/tan(c + d*x)**(5/2), x) + Integral(-I*B/tan(c + d*x)**(3
/2), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(62) = 124.

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.19

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{3\left(2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}(-(i+1)\ A+(i-1)\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
1/12*(3*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) +
2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1/2*sq
rt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((I - 1)*A + (I + 1)*B)*
log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((I - 1)*A +
(I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a + 8*(3*(
-I*A - B)*a*tan(d*x + c) - A*a)/tan(d*x + c)^(3/2))/d
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{3\sqrt{2}(-(i+1)\ Aa+(i-1)\ Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)-\frac{2\,(3i\,Aa\tan(dx+c)+3\,Ba\tan(dx+c)+}{\tan(dx+c)^{\frac{3}{2}}}}{3\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
1/3*(3*sqrt(2)*(-(I + 1)*A*a + (I - 1)*B*a)*arctan(-(1/2*I - 1/2)*sqrt(2)*
sqrt(tan(d*x + c))) - 2*(3*I*A*a*tan(d*x + c) + 3*B*a*tan(d*x + c) + A*a)/
tan(d*x + c)^(3/2))/d
```

Mupad [B] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2\,(-1)^{1/4}\,B\,a\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d} - \frac{2\,B\,a}{d\,\sqrt{\tan\left(c+d\,x\right)}} - \frac{\frac{2\,A\,a}{3\,d}+\frac{A\,a\,\tan(c+d\,x)\,2\mathrm{i}}{d}}{\tan\left(c+d\,x\right)^{3/2}} + \frac{\sqrt{2}\,A\,a\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-1-\mathrm{i}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i))/tan(c + d*x)^(5/2),x)
```

output
```
(2*(-1)^(1/4)*B*a*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/d - (2*B*a)/(d*tan
(c + d*x)^(1/2)) - (2^(1/2)*A*a*atan(2^(1/2)*tan(c + d*x)^(1/2)*(1/2 - 1i/
2))*(1 + 1i))/d - ((2*A*a)/(3*d) + (A*a*tan(c + d*x)*2i)/d)/tan(c + d*x)^(
3/2)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= a\left(\left(\int\frac{\sqrt{\tan\left(dx+c\right)}}{\tan\left(dx+c\right)^{3}}dx\right)a+\left(\int\frac{\sqrt{\tan\left(dx+c\right)}}{\tan\left(dx+c\right)^{2}}dx\right)ai+\left(\int\frac{\sqrt{\tan\left(dx+c\right)}}{\tan\left(dx+c\right)^{2}}dx\right)b+\left(\int\frac{\sqrt{\tan\left(dx+c\right)}}{\tan\left(dx+c\right)}dx\right)bi\right)$$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
a*(int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*a + int(sqrt(tan(c + d*x))/ta
n(c + d*x)**2,x)*a*i + int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*b + int(s
qrt(tan(c + d*x))/tan(c + d*x),x)*b*i)
```

3.118 $\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 103

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt[4]{-1}a(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2a(iA+B)}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2a(A-iB)}{d\sqrt{\tan(c+dx)}}$$

output
```
2*(-1)^(1/4)*a*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-2/5*a*A/d/tan
(d*x+c)^(5/2)-2/3*a*(I*A+B)/d/tan(d*x+c)^(3/2)+2*a*(A-I*B)/d/tan(d*x+c)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\left(-3A-5i(A-iB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},i\tan(c+dx)\right)\tan(c+dx)\right)}{15d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2)
,x]
```

output
```
(2*a*(-3*A - (5*I)*(A - I*B)*Hypergeometric2F1[-3/2, 1, -1/2, I*Tan[c + d*
x]]*Tan[c + d*x]))/(15*d*Tan[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4074, 3042, 4012, 25, 3042, 4012, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \; 4074$$

$$-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}+\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}+\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\int -\frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a(B+iA)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a(B+iA)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{a(A-iB)+a(iA+B)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx-\frac{2a(B+iA)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4012}$$

$$-\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a(B+iA)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2a(A-iB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{a(iA+B)-a(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a(B+iA)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2a(A-iB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4016}$$

$$-\frac{2a^2(B+iA)^2\int \frac{1}{a(iA+B)+a(A-iB)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}-\frac{2a(B+iA)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2a(A-iB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt[4]{-1}a(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a(B+iA)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2a(A-iB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x]
```

output
```
(2*(-1)^(1/4)*a*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d - (2*a*
A)/(5*d*Tan[c + d*x]^(5/2)) - (2*a*(I*A + B))/(3*d*Tan[c + d*x]^(3/2)) + (
2*a*(A - I*B))/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.29

method	result
derivativedivides	$\frac{a\left(\frac{(-iA-B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-iB+}{}\right.}{}$
default	$\frac{a\left(\frac{(-iA-B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-iB+}{}\right.}{}$
parts	$\frac{(iaA+Ba)\left(-\frac{2}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(\right.\right.}{4}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*a*(1/4*(-I*A-B)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(t
an(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(A-I*B)*2^(1/2)*(ln((tan(d*x+
c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*
arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-
2/5*A/tan(d*x+c)^(5/2)-2*(-A+I*B)/tan(d*x+c)^(1/2)-2/3*(I*A+B)/tan(d*x+c)^
(3/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(81) = 162$.

Time = 0.10 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.72

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{15\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a^2}{d^2}}\log\left(\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}+(de\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
-1/30*(15*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*
x + 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(2*((A - I*B)*a
*e^(2*I*d*x + 2*I*c) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B +
 I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) - 15*(d*e^(6*I*d*x + 6*I*c) - 3*d
*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B
+ I*B^2)*a^2/d^2)*log(2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) - (d*e^(2*I*d*x +
 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt((-I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a
)) + 4*((-23*I*A - 20*B)*a*e^(6*I*d*x + 6*I*c) + (I*A + 10*B)*a*e^(4*I*d*x
 + 4*I*c) + (11*I*A + 20*B)*a*e^(2*I*d*x + 2*I*c) + (-13*I*A - 10*B)*a)*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(6*I*d*x
+ 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = ia\left(\int \frac{A}{\tan^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{B}{\tan^{\frac{3}{2}}(c+dx)}\,dx + \int \left(-\frac{iA}{\tan^{\frac{7}{2}}(c+dx)}\right)dx + \int \left(-\frac{iB}{\tan^{\frac{5}{2}}(c+dx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
I*a*(Integral(A/tan(c + d*x)**(5/2), x) + Integral(B/tan(c + d*x)**(3/2),
x) + Integral(-I*A/tan(c + d*x)**(7/2), x) + Integral(-I*B/tan(c + d*x)**(
5/2), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(81) = 162$.

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.82

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{15\left(2\sqrt{2}((i-1)\,A+(i+1)\,B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((i-1)\,A+(i+1)\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
-1/60*(15*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sq
rt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)
*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(-(I + 1)*A
+ (I - 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a - 8*(1
5*(A - I*B)*a*tan(d*x + c)^2 + 5*(-I*A - B)*a*tan(d*x + c) - 3*A*a)/tan(d*
x + c)^(5/2))/d
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{15\sqrt{2}((i-1)\ Aa+(i+1)\ Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)-\frac{2\left(15\,Aa\tan(dx+c)^{2}-15i\,Ba\tan(dx\right.}{}}{15\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
-1/15*(15*sqrt(2)*((I - 1)*A*a + (I + 1)*B*a)*arctan(-(1/2*I - 1/2)*sqrt(2
)*sqrt(tan(d*x + c))) - 2*(15*A*a*tan(d*x + c)^2 - 15*I*B*a*tan(d*x + c)^2
 - 5*I*A*a*tan(d*x + c) - 5*B*a*tan(d*x + c) - 3*A*a)/tan(d*x + c)^(5/2))/
d
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.19

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$= -\frac{\frac{2\,B\,a}{3\,d}+\frac{B\,a\tan(c+d\,x)\,2\mathrm{i}}{d}}{\tan\left(c+d\,x\right)^{3/2}}+\frac{\sqrt{2}\,B\,a\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{\tan\left(c+d\,x\right)}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-1-\mathrm{i}\right)}{d}$$
$$-\frac{2\,A\,a\,\left(15\,{\left(-1\right)}^{1/4}\,{\tan\left(c+d\,x\right)}^{5/2}\,\mathrm{atanh}\left({\left(-1\right)}^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)-15\,{\tan\left(c+d\,x\right)}^{2}+3+\tan(c+d\,x\right.}{15\,d\,{\tan\left(c+d\,x\right)}^{5/2}}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i))/tan(c + d*x)^(7/2),x)
```

output
```
- ((2*B*a)/(3*d) + (B*a*tan(c + d*x)*2i)/d)/tan(c + d*x)^(3/2) - (2^(1/2)*
B*a*atan(2^(1/2)*tan(c + d*x)^(1/2)*(1/2 - 1i/2))*(1 + 1i))/d - (2*A*a*(ta
n(c + d*x)*5i - 15*tan(c + d*x)^2 + 15*(-1)^(1/4)*tan(c + d*x)^(5/2)*atanh
((-1)^(1/4)*tan(c + d*x)^(1/2)) + 3))/(15*d*tan(c + d*x)^(5/2))
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = a\left(\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)a+\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)ai + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)b+\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)bi\right)$$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
a*(int(sqrt(tan(c + d*x))/tan(c + d*x)**4,x)*a + int(sqrt(tan(c + d*x))/ta
n(c + d*x)**3,x)*a*i + int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*b + int(s
qrt(tan(c + d*x))/tan(c + d*x)**2,x)*b*i)
```

3.119 $\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 183

$$\begin{aligned}\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx &= -\frac{4\sqrt[4]{-1}a^2(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{4a^2(iA+B)\sqrt{\tan(c+dx)}}{d} \\ &\quad + \frac{4a^2(A-iB)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{4a^2(iA+B)\tan^{\frac{5}{2}}(c+dx)}{5d} \\ &\quad - \frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{63d} + \frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}\end{aligned}$$

output
```
-4*(-1)^(1/4)*a^2*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-4*a^2*(I*A
+B)*tan(d*x+c)^(1/2)/d+4/3*a^2*(A-I*B)*tan(d*x+c)^(3/2)/d+4/5*a^2*(I*A+B)*
tan(d*x+c)^(5/2)/d-2/63*a^2*(9*A-11*I*B)*tan(d*x+c)^(7/2)/d+2/9*I*B*tan(d*
x+c)^(7/2)*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.71

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{2a^2\left((315+315i)\sqrt{2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{\tan(c+dx)}}{\sqrt{2}}\right)+\sqrt{\tan(c+dx)}(-630i(A-iB)+210(A-iB)\right.}{315d}$$

input
```
Integrate[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x])
,x]
```

output
```
(2*a^2*((315 + 315*I)*Sqrt[2]*(A - I*B)*ArcTanh[((1 + I)*Sqrt[Tan[c + d*x]
])/Sqrt[2]] + Sqrt[Tan[c + d*x]]*((-630*I)*(A - I*B) + 210*(A - I*B)*Tan[c
 + d*x] + 126*(I*A + B)*Tan[c + d*x]^2 - 45*(A - (2*I)*B)*Tan[c + d*x]^3 -
 35*B*Tan[c + d*x]^4)))/(315*d)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 4077, 27, 3042, 4075, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{5/2}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{2}{9}\int \frac{1}{2}\tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)(a(9A-7iB)+a(9iA+11B)\tan(c+dx))dx+\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\int \tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)(a(9A-7iB)+a(9iA+11B)\tan(c+dx))dx+\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\int \tan(c+dx)^{5/2}(i\tan(c+dx)a+a)(a(9A-7iB)+a(9iA+11B)\tan(c+dx))dx+\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \; 4075$$

$$\frac{1}{9}\left(\int \tan^{\frac{5}{2}}(c+dx)\left(18(A-iB)a^2+18(iA+B)\tan(c+dx)a^2\right)dx-\frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{7d}\right)+\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\int \tan(c+dx)^{5/2}\left(18(A-iB)a^2+18(iA+B)\tan(c+dx)a^2\right)dx-\frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{7d}\right)+\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \; 4011$$

$$\frac{1}{9}\left(\int \tan^{\frac{3}{2}}(c+dx)\left(18a^2(A-iB)\tan(c+dx)-18a^2(iA+B)\right)dx-\frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{7d}+\frac{36a^2(B\right.$$
$$\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\int \tan(c+dx)^{3/2}\left(18a^2(A-iB)\tan(c+dx)-18a^2(iA+B)\right)dx-\frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{7d}+\frac{36a^2(B\right.$$
$$\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$\downarrow$ 4011

$$\frac{1}{9}\left(\int \sqrt{\tan(c+dx)}\left(-18(A-iB)a^2-18(iA+B)\tan(c+dx)a^2\right)dx-\frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{7d}+\frac{36a^2(\right.$$
$$\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\int \sqrt{\tan(c+dx)}\left(-18(A-iB)a^2-18(iA+B)\tan(c+dx)a^2\right)dx-\frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{7d}+\frac{36a^2(\right.$$
$$\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$\downarrow$ 4011

$$\frac{1}{9}\left(\int \frac{18a^2(iA+B)-18a^2(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{7d}+\frac{36a^2(B+iA)\tan^{\frac{5}{2}}(c+\right.$$
$$\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\int \frac{18a^2(iA+B)-18a^2(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{7d}+\frac{36a^2(B+iA)\tan^{\frac{5}{2}}(c+\right.$$
$$\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$\downarrow$ 4016

$$\frac{1}{9}\left(\frac{648a^4(B+iA)^2\int \frac{1}{18(iA+B)a^2+18(A-iB)\tan(c+dx)a^2}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{7d}+\frac{36a^2(B\right.$$
$$\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

$\downarrow$ 218

$$\frac{1}{9}\left(-\frac{36\sqrt[4]{-1}a^2(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a^2(9A-11iB)\tan^{\frac{7}{2}}(c+dx)}{7d}+\frac{36a^2(B+iA)\tan^{\frac{5}{2}}}{5d}\right.$$

$$\frac{2iB\tan^{\frac{7}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

input
```
Int[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(((2*I)/9)*B*Tan[c + d*x]^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))/d + ((-36*(-1)
^(1/4)*a^2*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d - (36*a^2*(I
*A + B)*Sqrt[Tan[c + d*x]])/d + (12*a^2*(A - I*B)*Tan[c + d*x]^(3/2))/d +
(36*a^2*(I*A + B)*Tan[c + d*x]^(5/2))/(5*d) - (2*a^2*(9*A - (11*I)*B)*Tan[
c + d*x]^(7/2))/(7*d))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{a^2\left(-\frac{2B\tan(dx+c)^{\frac{9}{2}}}{9}+\frac{4iB\tan(dx+c)^{\frac{7}{2}}}{7}-\frac{2A\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{4iA\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{4B\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{4iB\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{4A\tan(dx}{3}\right.}{}$
default	$\frac{a^2\left(-\frac{2B\tan(dx+c)^{\frac{9}{2}}}{9}+\frac{4iB\tan(dx+c)^{\frac{7}{2}}}{7}-\frac{2A\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{4iA\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{4B\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{4iB\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{4A\tan(dx}{3}\right.}{}$
parts	$\frac{(2iA\,a^2+B\,a^2)\left(\frac{2\tan(dx+c)^{\frac{5}{2}}}{5}-2\sqrt{\tan(dx+c)}+\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\,a}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^2*(-2/9*B*tan(d*x+c)^(9/2)+4/7*I*B*tan(d*x+c)^(7/2)-2/7*A*tan(d*x+c)
^(7/2)+4/5*I*A*tan(d*x+c)^(5/2)+4/5*B*tan(d*x+c)^(5/2)-4/3*I*B*tan(d*x+c)^
(3/2)+4/3*A*tan(d*x+c)^(3/2)-4*I*A*tan(d*x+c)^(1/2)-4*B*tan(d*x+c)^(1/2)+1
/4*(2*B+2*I*A)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*
x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*a
rctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(2*I*B-2*A)*2^(1/2)*(ln((tan(d*x+c
)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*a
rctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(145) = 290$.

Time = 0.16 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.03

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/315*(315*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(8*I*d*x + 8*I*c)
+ 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I
*c) + d)*log(-2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + sqrt(-(-I*A^2 - 2*A*B
 + I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2))
- 315*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(8*I*d*x + 8*I*c) + 4*d
*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) +
 d)*log(-2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*
B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 2*(
(1011*I*A + 1091*B)*a^2*e^(8*I*d*x + 8*I*c) + 10*(285*I*A + 262*B)*a^2*e^(
6*I*d*x + 6*I*c) + 42*(84*I*A + 89*B)*a^2*e^(4*I*d*x + 4*I*c) + 10*(219*I*
A + 214*B)*a^2*e^(2*I*d*x + 2*I*c) + (501*I*A + 491*B)*a^2)*sqrt((-I*e^(2*
I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(8*I*d*x + 8*I*c) + 4
*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.26

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{140\,Ba^2\tan\left(dx+c\right)^{\frac{9}{2}}+180\,(A-2i\,B)a^2\tan\left(dx+c\right)^{\frac{7}{2}}+504\,(-i\,A-B)a^2\tan\left(dx+c\right)^{\frac{5}{2}}-840\,(A-$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/630*(140*B*a^2*tan(d*x + c)^(9/2) + 180*(A - 2*I*B)*a^2*tan(d*x + c)^(7
/2) + 504*(-I*A - B)*a^2*tan(d*x + c)^(5/2) - 840*(A - I*B)*a^2*tan(d*x +
c)^(3/2) + 2520*(I*A + B)*a^2*sqrt(tan(d*x + c)) - 315*(2*sqrt(2)*((I - 1)
*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*s
qrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d
*x + c)))) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c
)) + tan(d*x + c) + 1) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(-sqrt(2)*sqr
t(tan(d*x + c)) + tan(d*x + c) + 1))*a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.90

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(35\,Ba^2\tan\left(dx+c\right)^{\frac{9}{2}}+45\,Aa^2\tan\left(dx+c\right)^{\frac{7}{2}}-90i\,Ba^2\tan\left(dx+c\right)^{\frac{7}{2}}-126i\,Aa^2\tan\left(dx+c\right)^{\frac{5}{2}}-1\right.}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/315*(35*B*a^2*tan(d*x + c)^(9/2) + 45*A*a^2*tan(d*x + c)^(7/2) - 90*I*B
*a^2*tan(d*x + c)^(7/2) - 126*I*A*a^2*tan(d*x + c)^(5/2) - 126*B*a^2*tan(d
*x + c)^(5/2) - 210*A*a^2*tan(d*x + c)^(3/2) + 210*I*B*a^2*tan(d*x + c)^(3
/2) + 630*I*A*a^2*sqrt(tan(d*x + c)) + 630*B*a^2*sqrt(tan(d*x + c)) + 315*
sqrt(2)*(-(I - 1)*A*a^2 - (I + 1)*B*a^2)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqr
t(tan(d*x + c))))/d
```

Mupad [B] (verification not implemented)

Time = 8.13 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.78

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{4\,A\,a^2\tan(c+d\,x)^{3/2}}{3\,d} - \frac{A\,a^2\,\sqrt{\tan\left(c+d\,x\right)}\,4\mathrm{i}}{d} + \frac{A\,a^2\tan(c+d\,x)^{5/2}\,4\mathrm{i}}{5\,d} - \frac{2\,A\,a^2\tan(c+d\,x)^{7/2}}{7\,d} - \frac{4\,B\,a^2\,\sqrt{\tan\left(c+d\,x\right)}}{d} - \frac{B\,a^2\tan(c+d\,x)^{3/2}\,4\mathrm{i}}{3\,d} + \frac{4\,B\,a^2\tan(c+d\,x)^{5/2}}{5\,d} + \frac{B\,a^2\tan(c+d\,x)^{7/2}\,4\mathrm{i}}{7\,d} - \frac{2\,B\,a^2\tan(c+d\,x)^{9/2}}{9\,d} + \frac{\sqrt{2}\,A\,a^2\,\ln\left(-4\,A\,a^2\,d+\sqrt{2}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-2-2\mathrm{i})\right)\,(1+1\mathrm{i})}{d} - \frac{\sqrt{4\mathrm{i}}\,A\,a^2\,\ln\left(-4\,A\,a^2\,d+2\,\sqrt{4\mathrm{i}}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d} + \frac{\sqrt{2}\,B\,a^2\,\ln\left(B\,a^2\,d\,4\mathrm{i}+\sqrt{2}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-2+2\mathrm{i})\right)\,(1-\mathrm{i})}{d} - \frac{\sqrt{-4\mathrm{i}}\,B\,a^2\,\ln\left(B\,a^2\,d\,4\mathrm{i}+2\,\sqrt{-4\mathrm{i}}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(tan(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(4*A*a^2*tan(c + d*x)^(3/2))/(3*d) - (A*a^2*tan(c + d*x)^(1/2)*4i)/d + (A*
a^2*tan(c + d*x)^(5/2)*4i)/(5*d) - (2*A*a^2*tan(c + d*x)^(7/2))/(7*d) - (4
*B*a^2*tan(c + d*x)^(1/2))/d - (B*a^2*tan(c + d*x)^(3/2)*4i)/(3*d) + (4*B*
a^2*tan(c + d*x)^(5/2))/(5*d) + (B*a^2*tan(c + d*x)^(7/2)*4i)/(7*d) - (2*B
*a^2*tan(c + d*x)^(9/2))/(9*d) + (2^(1/2)*A*a^2*log(- 4*A*a^2*d - 2^(1/2)*
A*a^2*d*tan(c + d*x)^(1/2)*(2 + 2i))*(1 + 1i))/d - (4i^(1/2)*A*a^2*log(2*4
i^(1/2)*A*a^2*d*tan(c + d*x)^(1/2) - 4*A*a^2*d))/d + (2^(1/2)*B*a^2*log(B*
a^2*d*4i - 2^(1/2)*B*a^2*d*tan(c + d*x)^(1/2)*(2 - 2i))*(1 - 1i))/d - ((-4
i)^(1/2)*B*a^2*log(B*a^2*d*4i + 2*(-4i)^(1/2)*B*a^2*d*tan(c + d*x)^(1/2)))
/d
```

Reduce [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\left(-10\sqrt{\tan(dx+c)}\tan(dx+c)^4b+36\sqrt{\tan(dx+c)}\tan(dx+c)^2ai+36\sqrt{\tan(dx+c)}\tan(dx+\right.}{}$$

input
```
int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*( - 10*sqrt(tan(c + d*x))*tan(c + d*x)**4*b + 36*sqrt(tan(c + d*x))*
tan(c + d*x)**2*a*i + 36*sqrt(tan(c + d*x))*tan(c + d*x)**2*b - 180*sqrt(t
an(c + d*x))*a*i - 180*sqrt(tan(c + d*x))*b + 90*int(sqrt(tan(c + d*x))/ta
n(c + d*x),x)*a*d*i + 90*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b*d - 45*i
nt(sqrt(tan(c + d*x))*tan(c + d*x)**4,x)*a*d + 90*int(sqrt(tan(c + d*x))*t
an(c + d*x)**4,x)*b*d*i + 45*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*a*d
))/(45*d)
```

3.120 $\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 156

$$\begin{aligned}\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx &= \frac{4\sqrt[4]{-1}a^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} \\ &\quad+\frac{4a^2(A-iB)\sqrt{\tan(c+dx)}}{d}+\frac{4a^2(iA+B)\tan^{\frac{3}{2}}(c+dx)}{3d} \\ &\quad-\frac{2a^2(7A-9iB)\tan^{\frac{5}{2}}(c+dx)}{35d}+\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}\end{aligned}$$

output
```
4*(-1)^(1/4)*a^2*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d+4*a^2*(A-I*
B)*tan(d*x+c)^(1/2)/d+4/3*a^2*(I*A+B)*tan(d*x+c)^(3/2)/d-2/35*a^2*(7*A-9*I
*B)*tan(d*x+c)^(5/2)/d+2/7*I*B*tan(d*x+c)^(5/2)*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.71

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{2a^2\left((105+105i)\sqrt{2}(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{\tan(c+dx)}}{\sqrt{2}}\right)+\sqrt{\tan(c+dx)}(210(A-iB)+70(iA+B)\tan\right.}{105d}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x])
,x]
```

output
```
(2*a^2*((105 + 105*I)*Sqrt[2]*(I*A + B)*ArcTanh[((1 + I)*Sqrt[Tan[c + d*x]
])/Sqrt[2]] + Sqrt[Tan[c + d*x]]*(210*(A - I*B) + 70*(I*A + B)*Tan[c + d*x
] - 21*(A - (2*I)*B)*Tan[c + d*x]^2 - 15*B*Tan[c + d*x]^3)))/(105*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4077, 27, 3042, 4075, 3042, 4011, 3042, 4011, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{2}{7}\int \frac{1}{2}\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)(a(7A-5iB)+a(7iA+9B)\tan(c+dx))dx+\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)(a(7A-5iB)+a(7iA+9B)\tan(c+dx))dx+\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\int \tan(c+dx)^{3/2}(i\tan(c+dx)a+a)(a(7A-5iB)+a(7iA+9B)\tan(c+dx))dx+\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$\big\downarrow$ 4075

$$\frac{1}{7}\left(\int \tan^{\frac{3}{2}}(c+dx)\left(14(A-iB)a^2+14(iA+B)\tan(c+dx)a^2\right)dx-\frac{2a^2(7A-9iB)\tan^{\frac{5}{2}}(c+dx)}{5d}\right)+\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\int \tan(c+dx)^{3/2}\left(14(A-iB)a^2+14(iA+B)\tan(c+dx)a^2\right)dx-\frac{2a^2(7A-9iB)\tan^{\frac{5}{2}}(c+dx)}{5d}\right)+\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$\big\downarrow$ 4011

$$\frac{1}{7}\left(\int \sqrt{\tan(c+dx)}\left(14a^2(A-iB)\tan(c+dx)-14a^2(iA+B)\right)dx-\frac{2a^2(7A-9iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{28a^2(B+\right.$$
$$\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\int \sqrt{\tan(c+dx)}\left(14a^2(A-iB)\tan(c+dx)-14a^2(iA+B)\right)dx-\frac{2a^2(7A-9iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{28a^2(B+\right.$$
$$\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$\big\downarrow$ 4011

$$\frac{1}{7}\left(\int \frac{-14(A-iB)a^2-14(iA+B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(7A-9iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{28a^2(B+iA)\tan^{\frac{3}{2}}(c+\right.$$
$$\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\int \frac{-14(A-iB)a^2-14(iA+B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(7A-9iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{28a^2(B+iA)\tan^{\frac{3}{2}}(c+\right.$$
$$\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$$\Big\downarrow \text{4016}$$

$$\frac{1}{7}\left(\frac{392a^4(A-iB)^2\int \frac{1}{14a^2(iA+B)\tan(c+dx)-14a^2(A-iB)}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^2(7A-9iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{28a^2(B+\right.$$
$$\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{7}\left(\frac{28\sqrt[4]{-1}a^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a^2(7A-9iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{28a^2(B+iA)\tan^{\frac{3}{2}}(c\right.$$
$$\frac{2iB\tan^{\frac{5}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

input
```
Int[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(((2*I)/7)*B*Tan[c + d*x]^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))/d + ((28*(-1)^
(1/4)*a^2*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d + (28*a^2*(A
- I*B)*Sqrt[Tan[c + d*x]])/d + (28*a^2*(I*A + B)*Tan[c + d*x]^(3/2))/(3*d)
 - (2*a^2*(7*A - (9*I)*B)*Tan[c + d*x]^(5/2))/(5*d))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(130) = 260.

Time = 0.16 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.76

method	result
derivativedivides	$\frac{a^2\left(-\frac{2B\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{4iB\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{2A\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{4iA\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{4B\tan(dx+c)^{\frac{3}{2}}}{3}-4iB\sqrt{\tan(dx+c)}+4A\sqrt{\tan}\right.}{}$
default	$\frac{a^2\left(-\frac{2B\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{4iB\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{2A\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{4iA\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{4B\tan(dx+c)^{\frac{3}{2}}}{3}-4iB\sqrt{\tan(dx+c)}+4A\sqrt{\tan}\right.}{}$
parts	$\frac{(2iA\,a^2+B\,a^2)\left(\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{t}\right.\right.}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^2*(-2/7*B*tan(d*x+c)^(7/2)+4/5*I*B*tan(d*x+c)^(5/2)-2/5*A*tan(d*x+c)
^(5/2)+4/3*I*A*tan(d*x+c)^(3/2)+4/3*B*tan(d*x+c)^(3/2)-4*I*B*tan(d*x+c)^(1
/2)+4*A*tan(d*x+c)^(1/2)+1/4*(2*I*B-2*A)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*t
an(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(
1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-2*B-2*
I*A)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/
2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2
^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(124) = 248$.

Time = 0.12 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.21

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{105\sqrt{-\frac{(iA^2+2AB-iB^2)a^4}{d^2}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(-\frac{2\left((A-i\,B)a^2e^{(2i\,dx+2i\,c)}+\right.}{}\right.}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output

```
-1/105*(105*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(6*I*d*x + 6*I*c)
+ 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)
*a^2*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(I*d*e^(
2*I*d*x + 2*I*c) + I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*
I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 105*sqrt(-(I*A^2 + 2*
A*B - I*B^2)*a^4/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3
*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + sq
rt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt
((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*
I*c)/((-I*A - B)*a^2)) - 2*((301*A - 337*I*B)*a^2*e^(6*I*d*x + 6*I*c) + (6
79*A - 613*I*B)*a^2*e^(4*I*d*x + 4*I*c) + (539*A - 563*I*B)*a^2*e^(2*I*d*x
 + 2*I*c) + (161*A - 167*I*B)*a^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) + 1)))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) +
3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -a^2\left(\int\left(-A\tan^{\frac{3}{2}}(c+dx)\right)dx+\int A\tan^{\frac{7}{2}}(c+dx)\,dx + \int\left(-B\tan^{\frac{5}{2}}(c+dx)\right)dx+\int B\tan^{\frac{9}{2}}(c+dx)\,dx + \int\left(-2iA\tan^{\frac{5}{2}}(c+dx)\right)dx+\int\left(-2iB\tan^{\frac{7}{2}}(c+dx)\right)dx\right)$$

input

```
integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output

```
-a**2*(Integral(-A*tan(c + d*x)**(3/2), x) + Integral(A*tan(c + d*x)**(7/2
), x) + Integral(-B*tan(c + d*x)**(5/2), x) + Integral(B*tan(c + d*x)**(9/
2), x) + Integral(-2*I*A*tan(c + d*x)**(5/2), x) + Integral(-2*I*B*tan(c +
 d*x)**(7/2), x))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.36

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{60\,Ba^2\tan(dx+c)^{\frac{7}{2}}+84\,(A-2i\,B)a^2\tan(dx+c)^{\frac{5}{2}}+280\,(-i\,A-B)a^2\tan(dx+c)^{\frac{3}{2}}-840\,(A-i}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/210*(60*B*a^2*tan(d*x + c)^(7/2) + 84*(A - 2*I*B)*a^2*tan(d*x + c)^(5/2
) + 280*(-I*A - B)*a^2*tan(d*x + c)^(3/2) - 840*(A - I*B)*a^2*sqrt(tan(d*x
 + c)) - 105*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(
2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((I - 1)*A + (I + 1
)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((I - 1)
*A + (I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^2)/
d
```

Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{2\left(15\,Ba^2\tan(dx+c)^{\frac{7}{2}}+21\,Aa^2\tan(dx+c)^{\frac{5}{2}}-42i\,Ba^2\tan(dx+c)^{\frac{5}{2}}-70i\,Aa^2\tan(dx+c)^{\frac{3}{2}}-70\right.}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/105*(15*B*a^2*tan(d*x + c)^(7/2) + 21*A*a^2*tan(d*x + c)^(5/2) - 42*I*B
*a^2*tan(d*x + c)^(5/2) - 70*I*A*a^2*tan(d*x + c)^(3/2) - 70*B*a^2*tan(d*x
 + c)^(3/2) - 210*A*a^2*sqrt(tan(d*x + c)) + 210*I*B*a^2*sqrt(tan(d*x + c)
) + 105*sqrt(2)*((I + 1)*A*a^2 - (I - 1)*B*a^2)*arctan(-(1/2*I - 1/2)*sqrt
(2)*sqrt(tan(d*x + c))))/d
```

Mupad [B] (verification not implemented)

Time = 6.35 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.87

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{4\,A\,a^2\,\sqrt{\tan\left(c+d\,x\right)}}{d} + \frac{A\,a^2\,{\tan(c+d\,x)}^{3/2}\,4{}\mathrm{i}}{3\,d} - \frac{2\,A\,a^2\,{\tan(c+d\,x)}^{5/2}}{5\,d}$$
$$- \frac{B\,a^2\,\sqrt{\tan\left(c+d\,x\right)}\,4{}\mathrm{i}}{d} + \frac{4\,B\,a^2\,{\tan(c+d\,x)}^{3/2}}{3\,d}$$
$$+ \frac{B\,a^2\,{\tan(c+d\,x)}^{5/2}\,4{}\mathrm{i}}{5\,d} - \frac{2\,B\,a^2\,{\tan(c+d\,x)}^{7/2}}{7\,d}$$
$$+ \frac{\sqrt{2}\,A\,a^2\,\ln\left(-A\,a^2\,d\,4{}\mathrm{i}+\sqrt{2}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-2+2{}\mathrm{i}\right)\right)\,\left(1-\mathrm{i}\right)}{d}$$
$$- \frac{\sqrt{-4{}\mathrm{i}}\,A\,a^2\,\ln\left(-A\,a^2\,d\,4{}\mathrm{i}+2\,\sqrt{-4{}\mathrm{i}}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$
$$+ \frac{\sqrt{2}\,B\,a^2\,\ln\left(-4\,B\,a^2\,d+\sqrt{2}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-2-2{}\mathrm{i}\right)\right)\,\left(1+1{}\mathrm{i}\right)}{d}$$
$$- \frac{\sqrt{4{}\mathrm{i}}\,B\,a^2\,\ln\left(-4\,B\,a^2\,d+2\,\sqrt{4{}\mathrm{i}}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(4*A*a^2*tan(c + d*x)^(1/2))/d + (A*a^2*tan(c + d*x)^(3/2)*4i)/(3*d) - (2*
A*a^2*tan(c + d*x)^(5/2))/(5*d) - (B*a^2*tan(c + d*x)^(1/2)*4i)/d + (4*B*a
^2*tan(c + d*x)^(3/2))/(3*d) + (B*a^2*tan(c + d*x)^(5/2)*4i)/(5*d) - (2*B*
a^2*tan(c + d*x)^(7/2))/(7*d) + (2^(1/2)*A*a^2*log(- A*a^2*d*4i - 2^(1/2)*
A*a^2*d*tan(c + d*x)^(1/2)*(2 - 2i))*(1 - 1i))/d - ((-4i)^(1/2)*A*a^2*log(
2*(-4i)^(1/2)*A*a^2*d*tan(c + d*x)^(1/2) - A*a^2*d*4i))/d + (2^(1/2)*B*a^2
*log(- 4*B*a^2*d - 2^(1/2)*B*a^2*d*tan(c + d*x)^(1/2)*(2 + 2i))*(1 + 1i))/
d - (4i^(1/2)*B*a^2*log(2*4i^(1/2)*B*a^2*d*tan(c + d*x)^(1/2) - 4*B*a^2*d)
)/d
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{a^2\Big(-2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2\,a+4\sqrt{\tan(dx+c)}\,\tan(dx+c)^2\,bi+20\sqrt{\tan(dx+c)}\,a-20\sqrt{\tan}$$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*( - 2*sqrt(tan(c + d*x))*tan(c + d*x)**2*a + 4*sqrt(tan(c + d*x))*ta
n(c + d*x)**2*b*i + 20*sqrt(tan(c + d*x))*a - 20*sqrt(tan(c + d*x))*b*i -
10*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*d + 10*int(sqrt(tan(c + d*x))/
tan(c + d*x),x)*b*d*i - 5*int(sqrt(tan(c + d*x))*tan(c + d*x)**4,x)*b*d +
10*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*a*d*i + 5*int(sqrt(tan(c + d*
x))*tan(c + d*x)**2,x)*b*d))/(5*d)
```

3.121 $\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 129

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{4\sqrt[4]{-1}a^2(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}+\frac{4a^2(iA+B)\sqrt{\tan(c+dx)}}{d} - \frac{2a^2(5A-7iB)\tan^{\frac{3}{2}}(c+dx)}{15d}+\frac{2iB\tan^{\frac{3}{2}}(c+dx)\,(a^2+ia^2\tan(c+dx))}{5d}$$

output
```
4*(-1)^(1/4)*a^2*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d+4*a^2*(I*A+
B)*tan(d*x+c)^(1/2)/d-2/15*a^2*(5*A-7*I*B)*tan(d*x+c)^(3/2)/d+2/5*I*B*tan(
d*x+c)^(3/2)*(a^2+I*a^2*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2a^2\left((15+15i)\sqrt{2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{\tan(c+dx)}}{\sqrt{2}}\right)+\sqrt{\tan(c+dx)}(-30iA-30B+5(A-2iB)\tan(c+dx)+3B\tan^2(c+dx))\right)}{15d}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x])
,x]
```

output
```
(-2*a^2*((15 + 15*I)*Sqrt[2]*(A - I*B)*ArcTanh[((1 + I)*Sqrt[Tan[c + d*x]]
)/Sqrt[2]] + Sqrt[Tan[c + d*x]]*((-30*I)*A - 30*B + 5*(A - (2*I)*B)*Tan[c
+ d*x] + 3*B*Tan[c + d*x]^2)))/(15*d)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4077, 27, 3042, 4075, 3042, 4011, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)(a(5A-3iB)+a(5iA+7B)\tan(c+dx))dx+\frac{2iB\tan^{\frac{3}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)(a(5A-3iB)+a(5iA+7B)\tan(c+dx))dx+\frac{2iB\tan^{\frac{3}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)(a(5A-3iB)+a(5iA+7B)\tan(c+dx))dx+\frac{2iB\tan^{\frac{3}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$\downarrow$ 4075

$$\frac{1}{5}\left(\int \sqrt{\tan(c+dx)}\left(10(A-iB)a^2+10(iA+B)\tan(c+dx)a^2\right)dx-\frac{2a^2(5A-7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2iB\tan^{\frac{3}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\int \sqrt{\tan(c+dx)}\left(10(A-iB)a^2+10(iA+B)\tan(c+dx)a^2\right)dx-\frac{2a^2(5A-7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}\right)+\frac{2iB\tan^{\frac{3}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$\downarrow$ 4011

$$\frac{1}{5}\left(\int \frac{10a^2(A-iB)\tan(c+dx)-10a^2(iA+B)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(5A-7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{20a^2(B+iA)\sqrt{\tan(c+d}}{d}\right.$$
$$\frac{2iB\tan^{\frac{3}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\int \frac{10a^2(A-iB)\tan(c+dx)-10a^2(iA+B)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(5A-7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{20a^2(B+iA)\sqrt{\tan(c+d}}{d}\right.$$
$$\frac{2iB\tan^{\frac{3}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$\downarrow$ 4016

$$\frac{1}{5}\left(\frac{200a^4(B+iA)^2\int \frac{1}{-10(iA+B)a^2-10(A-iB)\tan(c+dx)a^2}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^2(5A-7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{20a^2(B\right.$$

$$\frac{2iB\tan^{\frac{3}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{5}\left(\frac{20\sqrt[4]{-1}a^2(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a^2(5A-7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{20a^2(B+iA)\sqrt{\tan(c}}{d}\right.$$

$$\frac{2iB\tan^{\frac{3}{2}}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(((2*I)/5)*B*Tan[c + d*x]^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))/d + ((20*(-1)^
(1/4)*a^2*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d + (20*a^2*(I*
A + B)*Sqrt[Tan[c + d*x]])/d - (2*a^2*(5*A - (7*I)*B)*Tan[c + d*x]^(3/2))/
(3*d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(108) = 216$.

Time = 0.11 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.95

method	result
derivativedivides	$\frac{a^2\left(-\frac{2B\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{4iB\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{2A\tan(dx+c)^{\frac{3}{2}}}{3}+4iA\sqrt{\tan(dx+c)}+4B\sqrt{\tan(dx+c)}+\frac{(-2iA-2B)\sqrt{2}\left(\ln\left(\frac{\tan(d}{\tan(d}\right.\right.}{}\right.}{}$
default	$\frac{a^2\left(-\frac{2B\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{4iB\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{2A\tan(dx+c)^{\frac{3}{2}}}{3}+4iA\sqrt{\tan(dx+c)}+4B\sqrt{\tan(dx+c)}+\frac{(-2iA-2B)\sqrt{2}\left(\ln\left(\frac{\tan(d}{\tan(d}\right.\right.}{}\right.}{}$
parts	$\frac{(2iA\,a^2+B\,a^2)\left(2\sqrt{\tan(dx+c)}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{}\right.\right.}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^2*(-2/5*B*tan(d*x+c)^(5/2)+4/3*I*B*tan(d*x+c)^(3/2)-2/3*A*tan(d*x+c)
^(3/2)+4*I*A*tan(d*x+c)^(1/2)+4*B*tan(d*x+c)^(1/2)+1/4*(-2*B-2*I*A)*2^(1/2
)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+
c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2)))+1/4*(-2*I*B+2*A)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)
^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(
d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(103) = 206$.

Time = 0.11 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.42

$$\int\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{15\sqrt{-\frac{(-iA^2-2AB+iB^2)a^4}{d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(-\frac{2\left((A-iB)a^2e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{(-iA^2-2AB+iB}{d^2}}\right.}{(-}\right.}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/15*(15*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(4*I*d*x + 4*I*c) +
 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) +
sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((
-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*
c)/((-I*A - B)*a^2)) - 15*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(4*
I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^2*e^(2*I
*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*
I*c) + d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^
(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) + 2*((-35*I*A - 43*B)*a^2*e^(4*I*d*x
+ 4*I*c) + 6*(-10*I*A - 9*B)*a^2*e^(2*I*d*x + 2*I*c) + (-25*I*A - 23*B)*a^
2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I
*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -a^2\left(\int \left(-A\sqrt{\tan{(c+dx)}}\right)dx + \int A\tan^{\frac{5}{2}}{(c+dx)}\,dx + \int \left(-B\tan^{\frac{3}{2}}{(c+dx)}\right)dx + \int B\tan^{\frac{7}{2}}{(c+dx)}\,dx + \int \left(-2iA\tan^{\frac{3}{2}}{(c+dx)}\right)dx + \int \left(-2iB\tan^{\frac{5}{2}}{(c+dx)}\right)dx\right)$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
-a**2*(Integral(-A*sqrt(tan(c + d*x)), x) + Integral(A*tan(c + d*x)**(5/2)
, x) + Integral(-B*tan(c + d*x)**(3/2), x) + Integral(B*tan(c + d*x)**(7/2
), x) + Integral(-2*I*A*tan(c + d*x)**(3/2), x) + Integral(-2*I*B*tan(c +
d*x)**(5/2), x))
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.50

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{12\,Ba^2\tan(dx+c)^{\frac{5}{2}}+20\,(A-2i\,B)a^2\tan(dx+c)^{\frac{3}{2}}+120\,(-i\,A-B)a^2\sqrt{\tan(dx+c)}+15\,\left(2\,\sqrt{2}(\right.}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/30*(12*B*a^2*tan(d*x + c)^(5/2) + 20*(A - 2*I*B)*a^2*tan(d*x + c)^(3/2)
 + 120*(-I*A - B)*a^2*sqrt(tan(d*x + c)) + 15*(2*sqrt(2)*((I - 1)*A + (I +
 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((
I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))
) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(
d*x + c) + 1) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(-sqrt(2)*sqrt(tan(d*x
 + c)) + tan(d*x + c) + 1))*a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(3\,Ba^2\tan(dx+c)^{\frac{5}{2}}+5\,Aa^2\tan(dx+c)^{\frac{3}{2}}-10i\,Ba^2\tan(dx+c)^{\frac{3}{2}}-30i\,Aa^2\sqrt{\tan(dx+c)}-30\,B\right.}{15\,d}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/15*(3*B*a^2*tan(d*x + c)^(5/2) + 5*A*a^2*tan(d*x + c)^(3/2) - 10*I*B*a^
2*tan(d*x + c)^(3/2) - 30*I*A*a^2*sqrt(tan(d*x + c)) - 30*B*a^2*sqrt(tan(d
*x + c)) + 15*sqrt(2)*((I - 1)*A*a^2 + (I + 1)*B*a^2)*arctan(-(1/2*I - 1/2
)*sqrt(2)*sqrt(tan(d*x + c))))/d
```

Mupad [B] (verification not implemented)

Time = 4.89 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.98

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{A\,a^2\,\sqrt{\tan\left(c+d\,x\right)}\,4\mathrm{i}}{d} - \frac{2\,A\,a^2\,{\tan(c+d\,x)}^{3/2}}{3\,d} + \frac{4\,B\,a^2\,\sqrt{\tan\left(c+d\,x\right)}}{d} + \frac{B\,a^2\,{\tan(c+d\,x)}^{3/2}\,4\mathrm{i}}{3\,d} - \frac{2\,B\,a^2\,{\tan(c+d\,x)}^{5/2}}{5\,d} + \frac{\sqrt{2}\,A\,a^2\,\ln\left(4\,A\,a^2\,d+\sqrt{2}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-2-2\mathrm{i})\right)\,(1+1\mathrm{i})}{d} - \frac{\sqrt{4\mathrm{i}}\,A\,a^2\,\ln\left(4\,A\,a^2\,d+2\,\sqrt{4\mathrm{i}}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d} + \frac{\sqrt{2}\,B\,a^2\,\ln\left(-B\,a^2\,d\,4\mathrm{i}+\sqrt{2}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-2+2\mathrm{i})\right)\,(1-\mathrm{i})}{d} - \frac{\sqrt{-4\mathrm{i}}\,B\,a^2\,\ln\left(-B\,a^2\,d\,4\mathrm{i}+2\,\sqrt{-4\mathrm{i}}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
(A*a^2*tan(c + d*x)^(1/2)*4i)/d - (2*A*a^2*tan(c + d*x)^(3/2))/(3*d) + (4*
B*a^2*tan(c + d*x)^(1/2))/d + (B*a^2*tan(c + d*x)^(3/2)*4i)/(3*d) - (2*B*a
^2*tan(c + d*x)^(5/2))/(5*d) + (2^(1/2)*A*a^2*log(4*A*a^2*d - 2^(1/2)*A*a^
2*d*tan(c + d*x)^(1/2)*(2 + 2i))*(1 + 1i))/d - (4i^(1/2)*A*a^2*log(4*A*a^2
*d + 2*4i^(1/2)*A*a^2*d*tan(c + d*x)^(1/2)))/d + (2^(1/2)*B*a^2*log(- B*a^
2*d*4i - 2^(1/2)*B*a^2*d*tan(c + d*x)^(1/2)*(2 - 2i))*(1 - 1i))/d - ((-4i)
^(1/2)*B*a^2*log(2*(-4i)^(1/2)*B*a^2*d*tan(c + d*x)^(1/2) - B*a^2*d*4i))/d
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\left(-2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 b+20\sqrt{\tan(dx+c)}\,ai+20\sqrt{\tan(dx+c)}\,b-10\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)\right.}{}$$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*( - 2*sqrt(tan(c + d*x))*tan(c + d*x)**2*b + 20*sqrt(tan(c + d*x))*a
*i + 20*sqrt(tan(c + d*x))*b - 10*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a
*d*i - 10*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b*d + 5*int(sqrt(tan(c +
d*x)),x)*a*d - 5*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*a*d + 10*int(sq
rt(tan(c + d*x))*tan(c + d*x)**2,x)*b*d*i))/(5*d)
```

3.122 $\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 104

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{4\sqrt[4]{-1}a^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2(3A-5iB)\sqrt{\tan(c+dx)}}{3d} + \frac{2iB\sqrt{\tan(c+dx)}(a^2+ia^2\tan(c+dx))}{3d}$$

output
```
-4*(-1)^(1/4)*a^2*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-2/3*a^2*(3
*A-5*I*B)*tan(d*x+c)^(1/2)/d+2/3*I*B*tan(d*x+c)^(1/2)*(a^2+I*a^2*tan(d*x+c
))/d
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.72

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{2a^2\left((3+3i)\sqrt{2}(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{\tan(c+dx)}}{\sqrt{2}}\right)+\sqrt{\tan(c+dx)}(3A-6iB+B\tan(c+dx))\right)}{3d}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x
]],x]
```

output
```
(-2*a^2*((3 + 3*I)*Sqrt[2]*(I*A + B)*ArcTanh[((1 + I)*Sqrt[Tan[c + d*x]])/
Sqrt[2]] + Sqrt[Tan[c + d*x]]*(3*A - (6*I)*B + B*Tan[c + d*x])))/(3*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4077, 27, 3042, 4075, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{2}{3}\int \frac{(i\tan(c+dx)a+a)(a(3A-iB)+a(3iA+5B)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx+\frac{2iB\sqrt{\tan(c+dx)}(a^2+ia^2\tan(c+dx))}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)(a(3A-iB)+a(3iA+5B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{2iB\sqrt{\tan(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)(a(3A-iB)+a(3iA+5B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{2iB\sqrt{\tan(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \; 4075$$

$$\frac{1}{3}\left(\int \frac{6(A-iB)a^2+6(iA+B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(3A-5iB)\sqrt{\tan(c+dx)}}{d}\right)+\frac{2iB\sqrt{\tan(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\int \frac{6(A-iB)a^2+6(iA+B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(3A-5iB)\sqrt{\tan(c+dx)}}{d}\right)+\frac{2iB\sqrt{\tan(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \; 4016$$

$$\frac{1}{3}\left(\frac{72a^4(A-iB)^2\int \frac{1}{6a^2(A-iB)-6a^2(iA+B)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^2(3A-5iB)\sqrt{\tan(c+dx)}}{d}\right)+\frac{2iB\sqrt{\tan(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

$$\Big\downarrow \; 218$$

$$\frac{1}{3}\left(-\frac{12\sqrt[4]{-1}a^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a^2(3A-5iB)\sqrt{\tan(c+dx)}}{d}\right)+\frac{2iB\sqrt{\tan(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

input
```
Int[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x]
```

output
```
((-12*(-1)^(1/4)*a^2*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d -
(2*a^2*(3*A - (5*I)*B)*Sqrt[Tan[c + d*x]])/d)/3 + (((2*I)/3)*B*Sqrt[Tan[c
+ d*x]]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(86) = 172.

Time = 0.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.20

method	result
derivativedivides	$\frac{a^2\left(-\frac{2B\tan(dx+c)^{\frac{3}{2}}}{3}-2A\sqrt{\tan(dx+c)}+4iB\sqrt{\tan(dx+c)}+\frac{(-2iB+2A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan(\right.}{4}}{}$
default	$\frac{a^2\left(-\frac{2B\tan(dx+c)^{\frac{3}{2}}}{3}-2A\sqrt{\tan(dx+c)}+4iB\sqrt{\tan(dx+c)}+\frac{(-2iB+2A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan(\right.}{4}}{}$
parts	$\frac{(2iA\,a^2+B\,a^2)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+\right.\right.}{4d}$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^2*(-2/3*B*tan(d*x+c)^(3/2)-2*A*tan(d*x+c)^(1/2)+4*I*B*tan(d*x+c)^(1/
2)+1/4*(-2*I*B+2*A)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(t
an(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(2*B+2*I*A)*2^(1/2)*(ln((tan(
d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)
)+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2
))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(82) = 164$.

Time = 0.09 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.74

$$\int \frac{(a + ia\tan(c + dx))^2(A + B\tan(c + dx))}{\sqrt{\tan(c + dx)}}\,dx$$

$$= \frac{3\sqrt{-\frac{(iA^2+2AB-iB^2)a^4}{d^2}}\left(de^{(2i\,dx+2i\,c)} + d\right)\log\left(-\frac{2\left((A-iB)a^2e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{(iA^2+2AB-iB^2)a^4}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\cdots}\right.}{(-iA-B)a^2}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algori
thm="fricas")
```

output
```
1/3*(3*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*
log(-2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*
a^4/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 3*sq
rt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((
A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*
(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 2*((3*A - 7*
I*B)*a^2*e^(2*I*d*x + 2*I*c) + (3*A - 5*I*B)*a^2)*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -a^2\left(\int\left(-\frac{A}{\sqrt{\tan(c+dx)}}\right)dx+\int A\tan^{\frac{3}{2}}(c+dx)\,dx + \int\left(-B\sqrt{\tan(c+dx)}\right)dx+\int B\tan^{\frac{5}{2}}(c+dx)\,dx + \int\left(-2iA\sqrt{\tan(c+dx)}\right)dx+\int\left(-2iB\tan^{\frac{3}{2}}(c+dx)\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
-a**2*(Integral(-A/sqrt(tan(c + d*x)), x) + Integral(A*tan(c + d*x)**(3/2)
, x) + Integral(-B*sqrt(tan(c + d*x)), x) + Integral(B*tan(c + d*x)**(5/2)
, x) + Integral(-2*I*A*sqrt(tan(c + d*x)), x) + Integral(-2*I*B*tan(c + d*
x)**(3/2), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(82) = 164.

Time = 0.15 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.67

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{4\,Ba^2\tan(dx+c)^{\frac{3}{2}}+12\,(A-2i\,B)a^2\sqrt{\tan(dx+c)}+3\left(2\sqrt{2}(-(i+1)\,A+(i-1)\,B)\arctan\left(\frac{1}{2}\sqrt{\right.\right.}}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algori
thm="maxima")
```

output
```
-1/6*(4*B*a^2*tan(d*x + c)^(3/2) + 12*(A - 2*I*B)*a^2*sqrt(tan(d*x + c)) +
 3*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqr
t(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1/2*sqrt(2)
*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((I - 1)*A + (I + 1)*B)*log(s
qrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((I - 1)*A + (I +
1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx =$$
$$-\frac{2\left(Ba^2\tan\left(dx+c\right)^{\frac{3}{2}}+3\,Aa^2\sqrt{\tan\left(dx+c\right)}-6i\,Ba^2\sqrt{\tan\left(dx+c\right)}+3\,\sqrt{2}(-(i+1)\;Aa^2+(i-1)\;B\right.}{3\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algori
thm="giac")
```

output
```
-2/3*(B*a^2*tan(d*x + c)^(3/2) + 3*A*a^2*sqrt(tan(d*x + c)) - 6*I*B*a^2*sq
rt(tan(d*x + c)) + 3*sqrt(2)*(-(I + 1)*A*a^2 + (I - 1)*B*a^2)*arctan(-(1/2
*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))))/d
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.12

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= -\frac{2\,A\,a^2\,\sqrt{\tan\left(c+d\,x\right)}}{d}+\frac{B\,a^2\,\sqrt{\tan\left(c+d\,x\right)}\,4\mathrm{i}}{d}-\frac{2\,B\,a^2\,{\tan\left(c+d\,x\right)}^{3/2}}{3\,d}$$
$$+\frac{\sqrt{2}\,A\,a^2\,\ln\left(A\,a^2\,d\,4\mathrm{i}+\sqrt{2}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-2+2\mathrm{i}\right)\right)\,\left(1-\mathrm{i}\right)}{d}$$
$$-\frac{\sqrt{-4\mathrm{i}}\,A\,a^2\,\ln\left(A\,a^2\,d\,4\mathrm{i}+2\,\sqrt{-4\mathrm{i}}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$
$$+\frac{\sqrt{2}\,B\,a^2\,\ln\left(4\,B\,a^2\,d+\sqrt{2}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-2-2\mathrm{i}\right)\right)\,\left(1+1\mathrm{i}\right)}{d}$$
$$-\frac{\sqrt{4\mathrm{i}}\,B\,a^2\,\ln\left(4\,B\,a^2\,d+2\,\sqrt{4\mathrm{i}}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2)/tan(c + d*x)^(1/2),x)
```

output
```
(B*a^2*tan(c + d*x)^(1/2)*4i)/d - (2*A*a^2*tan(c + d*x)^(1/2))/d - (2*B*a^
2*tan(c + d*x)^(3/2))/(3*d) + (2^(1/2)*A*a^2*log(A*a^2*d*4i - 2^(1/2)*A*a^
2*d*tan(c + d*x)^(1/2)*(2 - 2i))*(1 - 1i))/d - ((-4i)^(1/2)*A*a^2*log(A*a^
2*d*4i + 2*(-4i)^(1/2)*A*a^2*d*tan(c + d*x)^(1/2)))/d + (2^(1/2)*B*a^2*log
(4*B*a^2*d - 2^(1/2)*B*a^2*d*tan(c + d*x)^(1/2)*(2 + 2i))*(1 + 1i))/d - (4
i^(1/2)*B*a^2*log(4*B*a^2*d + 2*4i^(1/2)*B*a^2*d*tan(c + d*x)^(1/2)))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{a^2\left(-2\sqrt{\tan\left(dx+c\right)}\,a+4\sqrt{\tan\left(dx+c\right)}\,bi+2\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)ad-2\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)bdi+2\left(\int\sqrt{\mathrm{t}\cdots}\right.}{d}$$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(a**2*( - 2*sqrt(tan(c + d*x))*a + 4*sqrt(tan(c + d*x))*b*i + 2*int(sqrt(t
an(c + d*x))/tan(c + d*x),x)*a*d - 2*int(sqrt(tan(c + d*x))/tan(c + d*x),x
)*b*d*i + 2*int(sqrt(tan(c + d*x)),x)*a*d*i + int(sqrt(tan(c + d*x)),x)*b*
d - int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*b*d))/d
```

3.123 $\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 98

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{4\sqrt[4]{-1}a^2(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} + \frac{2a^2(iA-B)\sqrt{\tan(c+dx)}}{d} - \frac{2A(a^2+ia^2\tan(c+dx))}{d\sqrt{\tan(c+dx)}}$$

output
```
-4*(-1)^(1/4)*a^2*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d+2*a^2*(I*A
-B)*tan(d*x+c)^(1/2)/d-2*A*(a^2+I*a^2*tan(d*x+c))/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2a^2\left(A-(1+i)\sqrt{2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{\tan(c+dx)}}{\sqrt{2}}\right)\sqrt{\tan(c+dx)}+B\tan(c+dx)\right)}{d\sqrt{\tan(c+dx)}}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/
2),x]
```

output
```
(-2*a^2*(A - (1 + I)*Sqrt[2]*(A - I*B)*ArcTanh[((1 + I)*Sqrt[Tan[c + d*x]]
)/Sqrt[2]]*Sqrt[Tan[c + d*x]] + B*Tan[c + d*x]))/(d*Sqrt[Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4076, 27, 3042, 4075, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$2\int \frac{(i\tan(c+dx)a+a)(a(3iA+B)+a(A+iB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx-\frac{2A(a^2+ia^2\tan(c+dx))}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(i\tan(c+dx)a+a)(a(3iA+B)+a(A+iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2A(a^2+ia^2\tan(c+dx))}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)(a(3iA+B)+a(A+iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2A(a^2+ia^2\tan(c+dx))}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4075}$$

$$\int \frac{2a^2(iA+B)-2a^2(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2a^2(-B+iA)\sqrt{\tan(c+dx)}}{d}-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{2a^2(iA+B)-2a^2(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2a^2(-B+iA)\sqrt{\tan(c+dx)}}{d}-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{ 4016}$$

$$\frac{8a^4(B+iA)^2\int \frac{1}{2(iA+B)a^2+2(A-iB)\tan(c+dx)a^2}d\sqrt{\tan(c+dx)}}{d}+\frac{2a^2(-B+iA)\sqrt{\tan(c+dx)}}{d}-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{4\sqrt[4]{-1}a^2(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}+\frac{2a^2(-B+iA)\sqrt{\tan(c+dx)}}{d}-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x]
```

output
```
(-4*(-1)^(1/4)*a^2*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d + (2
*a^2*(I*A - B)*Sqrt[Tan[c + d*x]])/d - (2*A*(a^2 + I*a^2*Tan[c + d*x]))/(d
*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(85) = 170$.

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.21

method	result
derivativedivides	$\frac{a^2\left(-2B\sqrt{\tan(dx+c)}-\frac{2A}{\sqrt{\tan(dx+c)}}+\frac{(2iA+2B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\,\mathrm{ar}}{4}\right.}{}$
default	$\frac{a^2\left(-2B\sqrt{\tan(dx+c)}-\frac{2A}{\sqrt{\tan(dx+c)}}+\frac{(2iA+2B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\,\mathrm{ar}}{4}\right.}{}$
parts	$\frac{(2iA\,a^2+B\,a^2)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+\right.\right.}{4d}$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^2*(-2*B*tan(d*x+c)^(1/2)-2*A/tan(d*x+c)^(1/2)+1/4*(2*B+2*I*A)*2^(1/2
)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+
c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2)))+1/4*(2*I*B-2*A)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^
(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(82) = 164$.

Time = 0.09 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.94

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a^4}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(-\frac{2\left((A-i\,B)a^2e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{\left(-i\,A^2-2\,AB+i\,B^2\right)a^4}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{-i}{e}}\right.}{(-i\,A-B)a^2}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algori
thm="fricas")
```

output
```
(sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(-
2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/
d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - sqrt(-(-I*A^
2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(-2*((A - I*B)*
a^2*e^(2*I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(2*
I*d*x + 2*I*c) + d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 2*((I*A + B)*a^2*e^(2*I*d
*x + 2*I*c) + (I*A - B)*a^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) + 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -a^2\left(\int \left(-\frac{A}{\tan^{\frac{3}{2}}(c+dx)}\right)dx + \int A\sqrt{\tan(c+dx)}\,dx + \int \left(-\frac{B}{\sqrt{\tan(c+dx)}}\right)dx + \int B\tan^{\frac{3}{2}}(c+dx)\,dx + \int \left(-\frac{2iA}{\sqrt{\tan(c+dx)}}\right)dx + \int \left(-2iB\sqrt{\tan(c+dx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
-a**2*(Integral(-A/tan(c + d*x)**(3/2), x) + Integral(A*sqrt(tan(c + d*x))
, x) + Integral(-B/sqrt(tan(c + d*x)), x) + Integral(B*tan(c + d*x)**(3/2)
, x) + Integral(-2*I*A/sqrt(tan(c + d*x)), x) + Integral(-2*I*B*sqrt(tan(c
 + d*x)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(82) = 164$.

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.73

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{4\,Ba^2\sqrt{\tan(dx+c)} - \left(2\sqrt{2}((i-1)\,A+(i+1)\,B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right) + 2\sqrt{2}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algori
thm="maxima")
```

output
```
-1/2*(4*B*a^2*sqrt(tan(d*x + c)) - (2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arct
an(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A +
(I + 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)
*(-(I + 1)*A + (I - 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) +
1) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + ta
n(d*x + c) + 1))*a^2 + 4*A*a^2/sqrt(tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.64

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(Ba^2\sqrt{\tan\left(dx+c\right)}+\sqrt{2}(-(i-1)\ Aa^2-(i+1)\ Ba^2)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+\cdots\right.}{d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algori
thm="giac")
```

output
```
-2*(B*a^2*sqrt(tan(d*x + c)) + sqrt(2)*(-(I - 1)*A*a^2 - (I + 1)*B*a^2)*ar
ctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) + A*a^2/sqrt(tan(d*x + c))
)/d
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.07

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= -\frac{2\,A\,a^2}{d\,\sqrt{\tan\left(c+d\,x\right)}} - \frac{2\,B\,a^2\,\sqrt{\tan\left(c+d\,x\right)}}{d}$$
$$+ \frac{\sqrt{2}\,A\,a^2\,\ln\left(-4\,A\,a^2\,d+\sqrt{2}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-2-2\mathrm{i})\right)\,(1+1\mathrm{i})}{d}$$
$$- \frac{\sqrt{4\mathrm{i}}\,A\,a^2\,\ln\left(-4\,A\,a^2\,d+2\,\sqrt{4\mathrm{i}}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$
$$+ \frac{\sqrt{2}\,B\,a^2\,\ln\left(B\,a^2\,d\,4\mathrm{i}+\sqrt{2}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-2+2\mathrm{i})\right)\,(1-\mathrm{i})}{d}$$
$$- \frac{\sqrt{-4\mathrm{i}}\,B\,a^2\,\ln\left(B\,a^2\,d\,4\mathrm{i}+2\,\sqrt{-4\mathrm{i}}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2)/tan(c + d*x)^(3/2),x)
```

output
```
(2^(1/2)*A*a^2*log(- 4*A*a^2*d - 2^(1/2)*A*a^2*d*tan(c + d*x)^(1/2)*(2 + 2
i))*(1 + 1i))/d - (2*B*a^2*tan(c + d*x)^(1/2))/d - (2*A*a^2)/(d*tan(c + d*
x)^(1/2)) - (4i^(1/2)*A*a^2*log(2*4i^(1/2)*A*a^2*d*tan(c + d*x)^(1/2) - 4*
A*a^2*d))/d + (2^(1/2)*B*a^2*log(B*a^2*d*4i - 2^(1/2)*B*a^2*d*tan(c + d*x)
^(1/2)*(2 - 2i))*(1 - 1i))/d - ((-4i)^(1/2)*B*a^2*log(B*a^2*d*4i + 2*(-4i)
^(1/2)*B*a^2*d*tan(c + d*x)^(1/2)))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{a^2\left(-2\sqrt{\tan\left(dx+c\right)}\,b+\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)ad+2\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)adi+2\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)bd-\left(\int\sqrt{\right.}\right.}{d}$$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
(a**2*( - 2*sqrt(tan(c + d*x))*b + int(sqrt(tan(c + d*x))/tan(c + d*x)**2,
x)*a*d + 2*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*d*i + 2*int(sqrt(tan(c
 + d*x))/tan(c + d*x),x)*b*d - int(sqrt(tan(c + d*x)),x)*a*d + 2*int(sqrt(
tan(c + d*x)),x)*b*d*i))/d
```

3.124 $\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 102

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\sqrt[4]{-1}a^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2(5iA+3B)}{3d\sqrt{\tan(c+dx)}} - \frac{2A(a^2+ia^2\tan(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}$$

output
```
4*(-1)^(1/4)*a^2*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-2/3*a^2*(5*
I*A+3*B)/d/tan(d*x+c)^(1/2)-2/3*A*(a^2+I*a^2*tan(d*x+c))/d/tan(d*x+c)^(3/2
)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.84

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2\left(-A+(-6iA-3B)\tan(c+dx)+(3+3i)\sqrt{2}(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{\tan(c+dx)}}{\sqrt{2}}\right)\tan^{\frac{3}{2}}(c+dx)\right)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/
2),x]
```

output
```
(2*a^2*(-A + ((-6*I)*A - 3*B)*Tan[c + d*x] + (3 + 3*I)*Sqrt[2]*(I*A + B)*A
rcTanh[((1 + I)*Sqrt[Tan[c + d*x]])/Sqrt[2]]*Tan[c + d*x]^(3/2)))/(3*d*Tan
[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4076, 27, 3042, 4074, 27, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{3}\int \frac{(i\tan(c+dx)a+a)(a(5iA+3B)-a(A-3iB)\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2A(a^2+ia^2\tan(c+dx))}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)(a(5iA+3B)-a(A-3iB)\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)(a(5iA+3B)-a(A-3iB)\tan(c+dx))}{\tan(c+dx)^{3/2}}dx-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4074}$$

$$\frac{1}{3}\left(\int -\frac{6\left((A-iB)a^2+(iA+B)\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(3B+5iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-6\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(3B+5iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-6\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(3B+5iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4016}$$

$$\frac{1}{3}\left(-\frac{12a^4(A-iB)^2\int \frac{1}{a^2(A-iB)-a^2(iA+B)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^2(3B+5iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(\frac{12\sqrt[4]{-1}a^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a^2(3B+5iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x]
```

output
```
((12*(-1)^(1/4)*a^2*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d - (
2*a^2*((5*I)*A + 3*B))/(d*Sqrt[Tan[c + d*x]]))/3 - (2*A*(a^2 + I*a^2*Tan[c
 + d*x]))/(3*d*Tan[c + d*x]^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(85) = 170$.

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.18

method	result
derivativedivides	$a^2\left(\frac{(2iB-2A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-2iA \ldots$
default	$a^2\left(\frac{(2iB-2A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-2iA \ldots$
parts	$\frac{(2iA\,a^2+B\,a^2)\left(-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4} - \ldots\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^2*(1/4*(2*I*B-2*A)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+
1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^
(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-2*B-2*I*A)*2^(1/2)*(ln
((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1
/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c
)^(1/2)))-2/3*A/tan(d*x+c)^(3/2)-2*(2*I*A+B)/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(82) = 164$.

Time = 0.09 (sec) , antiderivative size = 441, normalized size of antiderivative = 4.32

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{3\sqrt{-\frac{(iA^2+2AB-iB^2)a^4}{d^2}}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(-\frac{2\left((A-i\,B)a^2e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{(i\,A^2+2\,AB-i\,B^2)a^4}{d^2}}\cdots\right.}{(-i\,\cdots}\right.\cdots}{\cdots}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algori
thm="fricas")
```

output
```
-1/3*(3*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*
d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + sqr
t(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((
-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*
c)/((-I*A - B)*a^2)) - 3*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(4*I*
d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^2*e^(2*I*d
*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(-I*d*e^(2*I*d*x + 2*
I*c) - I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*
e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 2*((7*A - 3*I*B)*a^2*e^(4*I*d*x +
 4*I*c) + 2*A*a^2*e^(2*I*d*x + 2*I*c) - (5*A - 3*I*B)*a^2)*sqrt((-I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*
d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -a^2\left(\int\left(-\frac{A}{\tan^{\frac{5}{2}}(c+dx)}\right)dx+\int\frac{A}{\sqrt{\tan(c+dx)}}\,dx+\int\left(-\frac{B}{\tan^{\frac{3}{2}}(c+dx)}\right)dx + \int B\sqrt{\tan(c+dx)}\,dx+\int\left(-\frac{2iA}{\tan^{\frac{3}{2}}(c+dx)}\right)dx+\int\left(-\frac{2iB}{\sqrt{\tan(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
-a**2*(Integral(-A/tan(c + d*x)**(5/2), x) + Integral(A/sqrt(tan(c + d*x))
, x) + Integral(-B/tan(c + d*x)**(3/2), x) + Integral(B*sqrt(tan(c + d*x))
, x) + Integral(-2*I*A/tan(c + d*x)**(3/2), x) + Integral(-2*I*B/sqrt(tan(
c + d*x)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(82) = 164.

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.74

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{3\left(2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(-(i+1)\ A+(i-1)\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algori
thm="maxima")
```

output
```
1/6*(3*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2
*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1/2*sqr
t(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((I - 1)*A + (I + 1)*B)*l
og(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((I - 1)*A + (
I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^2 + 4*(3*
(-2*I*A - B)*a^2*tan(d*x + c) - A*a^2)/tan(d*x + c)^(3/2))/d
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(3\sqrt{2}(-(i+1)\,Aa^2+(i-1)\,Ba^2)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)-\frac{6i\,Aa^2\tan(dx+c)+3\,Ba^2\tan(dx}{\tan(dx+c)^{\frac{3}{2}}}\right.}{3\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algori
thm="giac")
```

output
```
2/3*(3*sqrt(2)*(-(I + 1)*A*a^2 + (I - 1)*B*a^2)*arctan(-(1/2*I - 1/2)*sqrt
(2)*sqrt(tan(d*x + c))) - (6*I*A*a^2*tan(d*x + c) + 3*B*a^2*tan(d*x + c) +
 A*a^2)/tan(d*x + c)^(3/2))/d
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.18

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= -\frac{\frac{2\,A\,a^2}{3\,d}+\frac{A\,a^2\tan(c+d\,x)\,4\mathrm{i}}{d}}{\tan\left(c+d\,x\right)^{3/2}} - \frac{2\,B\,a^2}{d\,\sqrt{\tan\left(c+d\,x\right)}}$$
$$+ \frac{\sqrt{2}\,A\,a^2\,\ln\left(-A\,a^2\,d\,4\mathrm{i}+\sqrt{2}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-2+2\mathrm{i}\right)\right)\,\left(1-\mathrm{i}\right)}{d}$$
$$- \frac{\sqrt{-4\mathrm{i}}\,A\,a^2\,\ln\left(-A\,a^2\,d\,4\mathrm{i}+2\,\sqrt{-4\mathrm{i}}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$
$$+ \frac{\sqrt{2}\,B\,a^2\,\ln\left(-4\,B\,a^2\,d+\sqrt{2}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-2-2\mathrm{i}\right)\right)\,\left(1+1\mathrm{i}\right)}{d}$$
$$- \frac{\sqrt{4\mathrm{i}}\,B\,a^2\,\ln\left(-4\,B\,a^2\,d+2\,\sqrt{4\mathrm{i}}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2)/tan(c + d*x)^(5/2),x)
```

output
```
(2^(1/2)*A*a^2*log(- A*a^2*d*4i - 2^(1/2)*A*a^2*d*tan(c + d*x)^(1/2)*(2 -
2i))*(1 - 1i))/d - (2*B*a^2)/(d*tan(c + d*x)^(1/2)) - ((2*A*a^2)/(3*d) + (
A*a^2*tan(c + d*x)*4i)/d)/tan(c + d*x)^(3/2) - ((-4i)^(1/2)*A*a^2*log(2*(-
4i)^(1/2)*A*a^2*d*tan(c + d*x)^(1/2) - A*a^2*d*4i))/d + (2^(1/2)*B*a^2*log
(- 4*B*a^2*d - 2^(1/2)*B*a^2*d*tan(c + d*x)^(1/2)*(2 + 2i))*(1 + 1i))/d -
(4i^(1/2)*B*a^2*log(2*4i^(1/2)*B*a^2*d*tan(c + d*x)^(1/2) - 4*B*a^2*d))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= a^2\left(\left(\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)a+2\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)ai + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)b-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a + 2\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)bi-\left(\int \sqrt{\tan(dx+c)}dx\right)b\right)$$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
a**2*(int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*a + 2*int(sqrt(tan(c + d*x
))/tan(c + d*x)**2,x)*a*i + int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*b -
int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a + 2*int(sqrt(tan(c + d*x))/tan(c
+ d*x),x)*b*i - int(sqrt(tan(c + d*x)),x)*b)
```

3.125 $\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 127

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{4\sqrt[4]{-1}a^2(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2(7iA+5B)}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{4a^2(A-iB)}{d\sqrt{\tan(c+dx)}} - \frac{2A(a^2+ia^2\tan(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}$$

output
```
4*(-1)^(1/4)*a^2*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-2/15*a^2*(7
*I*A+5*B)/d/tan(d*x+c)^(3/2)+4*a^2*(A-I*B)/d/tan(d*x+c)^(1/2)-2/5*A*(a^2+I
*a^2*tan(d*x+c))/d/tan(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{2a^2\left(3A+5(2iA+B)\tan(c+dx)-30(A-iB)\tan^2(c+dx)+(15+15i)\sqrt{2}(A-iB)\text{arctanh}\left(\frac{(1+i)}{}\right.\right.}{15d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/
2),x]
```

output
```
(-2*a^2*(3*A + 5*((2*I)*A + B)*Tan[c + d*x] - 30*(A - I*B)*Tan[c + d*x]^2
+ (15 + 15*I)*Sqrt[2]*(A - I*B)*ArcTanh[((1 + I)*Sqrt[Tan[c + d*x]])/Sqrt[
2]]*Tan[c + d*x]^(5/2)))/(15*d*Tan[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 4076, 27, 3042, 4074, 27, 3042, 4012, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 4076}$$

$$\frac{2}{5}\int \frac{(i\tan(c+dx)a+a)(a(7iA+5B)-a(3A-5iB)\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2A(a^2+ia^2\tan(c+dx))}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)(a(7iA+5B)-a(3A-5iB)\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)(a(7iA+5B)-a(3A-5iB)\tan(c+dx))}{\tan(c+dx)^{5/2}}dx-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4074}$$

$$\frac{1}{5}\left(\int -\frac{10\left((A-iB)a^2+(iA+B)\tan(c+dx)a^2\right)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a^2(5B+7iA)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(-10\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a^2(5B+7iA)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-10\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\tan(c+dx)^{3/2}}dx-\frac{2a^2(5B+7iA)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4012}$$

$$\frac{1}{5}\left(-10\left(\int \frac{a^2(iA+B)-a^2(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(A-iB)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^2(5B+7iA)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-10\left(\int \frac{a^2(iA+B)-a^2(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(A-iB)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^2(5B+7iA)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4016

$$\frac{1}{5}\left(-10\left(\frac{2a^4(B+iA)^2\int \frac{1}{(iA+B)a^2+(A-iB)\tan(c+dx)a^2}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^2(A-iB)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^2(5B+7iA)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 218

$$\frac{1}{5}\left(-10\left(-\frac{2\sqrt[4]{-1}a^2(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a^2(A-iB)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^2(5B+7iA)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{5d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x]
```

output
```
(-10*((-2*(-1)^(1/4)*a^2*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/
d - (2*a^2*(A - I*B))/(d*Sqrt[Tan[c + d*x]])) - (2*a^2*((7*I)*A + 5*B))/(3
*d*Tan[c + d*x]^(3/2)))/5 - (2*A*(a^2 + I*a^2*Tan[c + d*x]))/(5*d*Tan[c +
d*x]^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(107) = 214$.

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.89

method	result
derivativedivides	$a^2\left(\frac{(-2iA-2B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-2\cdots}{\cdots}\right.$
default	$a^2\left(\frac{(-2iA-2B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-2\cdots}{\cdots}\right.$
parts	$\frac{(2iA\,a^2+B\,a^2)\left(-\frac{2}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\cdots\right.\right)}{4}\cdots}{d}$

input

```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RETUR
NVERBOSE)
```

output

```
1/d*a^2*(1/4*(-2*B-2*I*A)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)
+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)
^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-2*I*B+2*A)*2^(1/2)*(l
n((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(
1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+
c)^(1/2)))-2/5*A/tan(d*x+c)^(5/2)-2/3*(2*I*A+B)/tan(d*x+c)^(3/2)-2*(2*I*B-
2*A)/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(103) = 206$.

Time = 0.11 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.95

$$\int \frac{(a + ia\tan(c+dx))^2(A + B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{15\sqrt{-\frac{(-iA^2-2AB+iB^2)a^4}{d^2}}\left(de^{(6i\,dx+6i\,c)} - 3\,de^{(4i\,dx+4i\,c)} + 3\,de^{(2i\,dx+2i\,c)} - d\right)\log\left(-\frac{2\left((A-i\,B)a^2e^{(2i\,dx+2i\,c)} + \cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algori
thm="fricas")
```

output
```
-1/15*(15*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(6*I*d*x + 6*I*c) -
 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(-2*((A - I*B)*
a^2*e^(2*I*d*x + 2*I*c) + sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(2*
I*d*x + 2*I*c) + d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 15*sqrt(-(-I*A^2 - 2*A*B
+ I*B^2)*a^4/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e
^(2*I*d*x + 2*I*c) - d)*log(-2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) - sqrt(-
(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-
I*A - B)*a^2)) + 2*((-43*I*A - 35*B)*a^2*e^(6*I*d*x + 6*I*c) + (11*I*A + 2
5*B)*a^2*e^(4*I*d*x + 4*I*c) + (31*I*A + 35*B)*a^2*e^(2*I*d*x + 2*I*c) + (
-23*I*A - 25*B)*a^2)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x
 + 2*I*c) - d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$= -a^2\left(\int \left(-\frac{A}{\tan^{\frac{7}{2}}(c+dx)}\right)dx + \int \frac{A}{\tan^{\frac{3}{2}}(c+dx)}\,dx + \int \left(-\frac{B}{\tan^{\frac{5}{2}}(c+dx)}\right)dx + \int \frac{B}{\sqrt{\tan(c+dx)}}\,dx + \int \left(-\frac{2iA}{\tan^{\frac{5}{2}}(c+dx)}\right)dx + \int \left(-\frac{2iB}{\tan^{\frac{3}{2}}(c+dx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
-a**2*(Integral(-A/tan(c + d*x)**(7/2), x) + Integral(A/tan(c + d*x)**(3/2
), x) + Integral(-B/tan(c + d*x)**(5/2), x) + Integral(B/sqrt(tan(c + d*x)
), x) + Integral(-2*I*A/tan(c + d*x)**(5/2), x) + Integral(-2*I*B/tan(c +
d*x)**(3/2), x))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.54

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{15\left(2\sqrt{2}((i-1)\,A+(i+1)\,B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((i-1)\,A+(i+1)\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algori
thm="maxima")
```

output
```
-1/30*(15*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sq
rt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)
*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(-(I + 1)*A
+ (I - 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^2 - 4*
(30*(A - I*B)*a^2*tan(d*x + c)^2 + 5*(-2*I*A - B)*a^2*tan(d*x + c) - 3*A*a
^2)/tan(d*x + c)^(5/2))/d
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(15\sqrt{2}((i-1)\ Aa^2+(i+1)\ Ba^2)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)-\frac{30\,Aa^2\tan(dx+c)^2-30i\,Ba^2}{}\right.}{15\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algori
thm="giac")
```

output
```
-2/15*(15*sqrt(2)*((I - 1)*A*a^2 + (I + 1)*B*a^2)*arctan(-(1/2*I - 1/2)*sq
rt(2)*sqrt(tan(d*x + c))) - (30*A*a^2*tan(d*x + c)^2 - 30*I*B*a^2*tan(d*x
+ c)^2 - 10*I*A*a^2*tan(d*x + c) - 5*B*a^2*tan(d*x + c) - 3*A*a^2)/tan(d*x
 + c)^(5/2))/d
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.03

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$= -\frac{\frac{2Aa^2}{5d}-\frac{4Aa^2\tan(c+dx)^2}{d}+\frac{Aa^2\tan(c+dx)\,4\mathrm{i}}{3d}}{\tan(c+dx)^{5/2}}-\frac{\frac{2Ba^2}{3d}+\frac{Ba^2\tan(c+dx)\,4\mathrm{i}}{d}}{\tan(c+dx)^{3/2}}$$
$$+\frac{\sqrt{2}\,Aa^2\,\ln\left(4Aa^2d+\sqrt{2}\,Aa^2d\sqrt{\tan(c+dx)}\,(-2-2\mathrm{i})\right)\,(1+1\mathrm{i})}{d}$$
$$-\frac{\sqrt{4\mathrm{i}}\,Aa^2\,\ln\left(4Aa^2d+2\sqrt{4\mathrm{i}}\,Aa^2d\sqrt{\tan(c+dx)}\right)}{d}$$
$$+\frac{\sqrt{2}\,Ba^2\,\ln\left(-Ba^2d\,4\mathrm{i}+\sqrt{2}\,Ba^2d\sqrt{\tan(c+dx)}\,(-2+2\mathrm{i})\right)\,(1-\mathrm{i})}{d}$$
$$-\frac{\sqrt{-4\mathrm{i}}\,Ba^2\,\ln\left(-Ba^2d\,4\mathrm{i}+2\sqrt{-4\mathrm{i}}\,Ba^2d\sqrt{\tan(c+dx)}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2)/tan(c + d*x)^(7/2),x)
```

output
```
(2^(1/2)*A*a^2*log(4*A*a^2*d - 2^(1/2)*A*a^2*d*tan(c + d*x)^(1/2)*(2 + 2i)
)*(1 + 1i))/d - ((2*B*a^2)/(3*d) + (B*a^2*tan(c + d*x)*4i)/d)/tan(c + d*x)
^(3/2) - ((2*A*a^2)/(5*d) + (A*a^2*tan(c + d*x)*4i)/(3*d) - (4*A*a^2*tan(c
 + d*x)^2)/d)/tan(c + d*x)^(5/2) - (4i^(1/2)*A*a^2*log(4*A*a^2*d + 2*4i^(1
/2)*A*a^2*d*tan(c + d*x)^(1/2)))/d + (2^(1/2)*B*a^2*log(- B*a^2*d*4i - 2^(
1/2)*B*a^2*d*tan(c + d*x)^(1/2)*(2 - 2i))*(1 - 1i))/d - ((-4i)^(1/2)*B*a^2
*log(2*(-4i)^(1/2)*B*a^2*d*tan(c + d*x)^(1/2) - B*a^2*d*4i))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = a^2\left(\left(\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)a+2\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)ai + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)b-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)a + 2\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)b\right)$$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
a**2*(int(sqrt(tan(c + d*x))/tan(c + d*x)**4,x)*a + 2*int(sqrt(tan(c + d*x
))/tan(c + d*x)**3,x)*a*i + int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*b -
int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a + 2*int(sqrt(tan(c + d*x))/tan
(c + d*x)**2,x)*b*i - int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b)
```

3.126 $\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 154

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -\frac{4\sqrt[4]{-1}a^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{2a^2(9iA+7B)}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{4a^2(A-iB)}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{4a^2(iA+B)}{d\sqrt{\tan(c+dx)}} - \frac{2A(a^2+ia^2\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)}$$

output
```
-4*(-1)^(1/4)*a^2*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-2/35*a^2*(
9*I*A+7*B)/d/tan(d*x+c)^(5/2)+4/3*a^2*(A-I*B)/d/tan(d*x+c)^(3/2)+4*a^2*(I*
A+B)/d/tan(d*x+c)^(1/2)-2/7*A*(a^2+I*a^2*tan(d*x+c))/d/tan(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx =$$
$$-\frac{2a^2\left(15A+21(2iA+B)\tan(c+dx)-70(A-iB)\tan^2(c+dx)-210i(A-iB)\tan^3(c+dx)+210\right.}{105d\tan^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/
2),x]
```

output
```
(-2*a^2*(15*A + 21*((2*I)*A + B)*Tan[c + d*x] - 70*(A - I*B)*Tan[c + d*x]^
2 - (210*I)*(A - I*B)*Tan[c + d*x]^3 + 210*(-1)^(1/4)*(A - I*B)*ArcTan[(-1
)^(3/4)*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(7/2)))/(105*d*Tan[c + d*x]^(7/2)
)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4076, 27, 3042, 4074, 27, 3042, 4012, 3042, 4012, 25, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$
$$\Big\downarrow \text{ 4076}$$

$$\frac{2}{7}\int \frac{(i\tan(c+dx)a+a)(a(9iA+7B)-a(5A-7iB)\tan(c+dx))}{2\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2A(a^2+ia^2\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{(i\tan(c+dx)a+a)(a(9iA+7B)-a(5A-7iB)\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2A(a^2+ia^2\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{(i\tan(c+dx)a+a)(a(9iA+7B)-a(5A-7iB)\tan(c+dx))}{\tan(c+dx)^{7/2}}dx-\frac{2A(a^2+ia^2\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4074}$$

$$\frac{1}{7}\left(\int -\frac{14((A-iB)a^2+(iA+B)\tan(c+dx)a^2)}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2a^2(7B+9iA)}{5d\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2A(a^2+ia^2\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(-14\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2a^2(7B+9iA)}{5d\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2A(a^2+ia^2\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(-14\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\tan(c+dx)^{5/2}}dx-\frac{2a^2(7B+9iA)}{5d\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2A(a^2+ia^2\tan(c+dx))}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4012}$$

$$\frac{1}{7}\left(-14\left(\int \frac{a^2(iA+B)-a^2(A-iB)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a^2(A-iB)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2a^2(7B+9iA)}{5d\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(-14\left(\int \frac{a^2(iA+B)-a^2(A-iB)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx-\frac{2a^2(A-iB)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2a^2(7B+9iA)}{5d\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4012$$

$$\frac{1}{7}\left(-14\left(\int -\frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(A-iB)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a^2(B+iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^2(7B+9iA}{5d\tan^{\frac{5}{2}}(c+dx}\right.$$
$$-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{7}\left(-14\left(-\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(A-iB)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a^2(B+iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^2(7B+9i}{5d\tan^{\frac{5}{2}}(c+d}\right.$$
$$-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(-14\left(-\int \frac{(A-iB)a^2+(iA+B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(A-iB)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a^2(B+iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^2(7B+9i}{5d\tan^{\frac{5}{2}}(c+d}\right.$$
$$-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4016$$

$$\frac{1}{7}\left(-14\left(-\frac{2a^4(A-iB)^2\int \frac{1}{a^2(A-iB)-a^2(iA+B)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^2(A-iB)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a^2(B+iA)}{d\sqrt{\tan(c+dx)}}\right)-\right.$$
$$-\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow 218$$

$$\frac{1}{7}\left(-14\left(\frac{2\sqrt[4]{-1}a^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a^2(A-iB)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a^2(B+iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^2(7B}{5d\tan^{\frac{5}{2}}(}\right.$$
$$\frac{2A\left(a^2+ia^2\tan(c+dx)\right)}{7d\tan^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/2),x]
```

output
```
(-14*((2*(-1)^(1/4)*a^2*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d
 - (2*a^2*(A - I*B))/(3*d*Tan[c + d*x]^(3/2)) - (2*a^2*(I*A + B))/(d*Sqrt[
Tan[c + d*x]])) - (2*a^2*((9*I)*A + 7*B))/(5*d*Tan[c + d*x]^(5/2)))/7 - (2
*A*(a^2 + I*a^2*Tan[c + d*x]))/(7*d*Tan[c + d*x]^(7/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.68

method	result
derivativedivides	$\frac{a^2\left(\frac{(-2iB+2A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(2iA}{}\right.}{}$
default	$\frac{a^2\left(\frac{(-2iB+2A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(2iA}{}\right.}{}$
parts	$\frac{(2iA\,a^2+B\,a^2)\left(\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}-\frac{}{5}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^2*(1/4*(-2*I*B+2*A)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)
+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)
^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(2*B+2*I*A)*2^(1/2)*(ln
((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1
/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c
)^(1/2)))-2/7*A/tan(d*x+c)^(7/2)-2*(-2*B-2*I*A)/tan(d*x+c)^(1/2)-2/5*(2*I*
A+B)/tan(d*x+c)^(5/2)-2/3*(2*I*B-2*A)/tan(d*x+c)^(3/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(124) = 248.

Time = 0.13 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.64

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algori
thm="fricas")
```

output
```
1/105*(105*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(8*I*d*x + 8*I*c) -
 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*
c) + d)*log(-2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B -
 I*B^2)*a^4/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)
) - 105*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(8*I*d*x + 8*I*c) - 4*
d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c)
+ d)*log(-2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*
B^2)*a^4/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2))
- 2*((337*A - 301*I*B)*a^2*e^(8*I*d*x + 8*I*c) - 6*(46*A - 63*I*B)*a^2*e^(
6*I*d*x + 6*I*c) - 10*(5*A - 14*I*B)*a^2*e^(4*I*d*x + 4*I*c) + 18*(22*A -
21*I*B)*a^2*e^(2*I*d*x + 2*I*c) - (167*A - 161*I*B)*a^2)*sqrt((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(8*I*d*x + 8*I*c) - 4*d*
e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) +
d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -a^2\left(\int\left(-\frac{A}{\tan^{\frac{9}{2}}(c+dx)}\right)dx+\int\frac{A}{\tan^{\frac{5}{2}}(c+dx)}\,dx+\int\left(-\frac{B}{\tan^{\frac{7}{2}}(c+dx)}\right)dx+\int\frac{B}{\tan^{\frac{3}{2}}(c+dx)}\,dx+\int\left(-\frac{2iA}{\tan^{\frac{7}{2}}(c+dx)}\right)dx+\int\left(-\frac{2iB}{\tan^{\frac{5}{2}}(c+dx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)
```

output
```
-a**2*(Integral(-A/tan(c + d*x)**(9/2), x) + Integral(A/tan(c + d*x)**(5/2
), x) + Integral(-B/tan(c + d*x)**(7/2), x) + Integral(B/tan(c + d*x)**(3/
2), x) + Integral(-2*I*A/tan(c + d*x)**(7/2), x) + Integral(-2*I*B/tan(c +
 d*x)**(5/2), x))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.38

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{105\left(2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}(-(i+1)\ A+(i\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algori
thm="maxima")
```

output
```
-1/210*(105*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2
) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((I - 1)*A + (I + 1)
*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((I - 1)*
A + (I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^2 -
4*(210*(I*A + B)*a^2*tan(d*x + c)^3 + 70*(A - I*B)*a^2*tan(d*x + c)^2 + 21
*(-2*I*A - B)*a^2*tan(d*x + c) - 15*A*a^2)/tan(d*x + c)^(7/2))/d
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.87

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(105\sqrt{2}(-(i+1)\ Aa^2+(i-1)\ Ba^2)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+\frac{-210i\,Aa^2\tan(dx+c)^3-2}{}\right.}{105\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algori
thm="giac")
```

output
```
-2/105*(105*sqrt(2)*(-(I + 1)*A*a^2 + (I - 1)*B*a^2)*arctan(-(1/2*I - 1/2)
*sqrt(2)*sqrt(tan(d*x + c))) + (-210*I*A*a^2*tan(d*x + c)^3 - 210*B*a^2*ta
n(d*x + c)^3 - 70*A*a^2*tan(d*x + c)^2 + 70*I*B*a^2*tan(d*x + c)^2 + 42*I*
A*a^2*tan(d*x + c) + 21*B*a^2*tan(d*x + c) + 15*A*a^2)/tan(d*x + c)^(7/2))
/d
```

Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.90

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$
$$= -\frac{\frac{2\,A\,a^2}{7\,d}+\frac{A\,a^2\,\tan(c+d\,x)\,4\mathrm{i}}{5\,d}-\frac{4\,A\,a^2\,\tan(c+d\,x)^2}{3\,d}-\frac{A\,a^2\,\tan(c+d\,x)^3\,4\mathrm{i}}{d}}{\tan\left(c+d\,x\right)^{7/2}}$$
$$-\frac{\frac{2\,B\,a^2}{5\,d}-\frac{4\,B\,a^2\,\tan(c+d\,x)^2}{d}+\frac{B\,a^2\,\tan(c+d\,x)\,4\mathrm{i}}{3\,d}}{\tan\left(c+d\,x\right)^{5/2}}$$
$$+\frac{\sqrt{2}\,A\,a^2\,\ln\left(A\,a^2\,d\,4\mathrm{i}+\sqrt{2}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-2+2\mathrm{i}\right)\right)\,\left(1-\mathrm{i}\right)}{d}$$
$$-\frac{\sqrt{-4\mathrm{i}}\,A\,a^2\,\ln\left(A\,a^2\,d\,4\mathrm{i}+2\,\sqrt{-4\mathrm{i}}\,A\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$
$$+\frac{\sqrt{2}\,B\,a^2\,\ln\left(4\,B\,a^2\,d+\sqrt{2}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-2-2\mathrm{i}\right)\right)\,\left(1+1\mathrm{i}\right)}{d}$$
$$-\frac{\sqrt{4\mathrm{i}}\,B\,a^2\,\ln\left(4\,B\,a^2\,d+2\,\sqrt{4\mathrm{i}}\,B\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2)/tan(c + d*x)^(9/2),x)
```

output
```
(2^(1/2)*A*a^2*log(A*a^2*d*4i - 2^(1/2)*A*a^2*d*tan(c + d*x)^(1/2)*(2 - 2i
))*(1 - 1i))/d - ((2*B*a^2)/(5*d) + (B*a^2*tan(c + d*x)*4i)/(3*d) - (4*B*a
^2*tan(c + d*x)^2)/d)/tan(c + d*x)^(5/2) - ((2*A*a^2)/(7*d) + (A*a^2*tan(c
 + d*x)*4i)/(5*d) - (4*A*a^2*tan(c + d*x)^2)/(3*d) - (A*a^2*tan(c + d*x)^3
*4i)/d)/tan(c + d*x)^(7/2) - ((-4i)^(1/2)*A*a^2*log(A*a^2*d*4i + 2*(-4i)^(
1/2)*A*a^2*d*tan(c + d*x)^(1/2)))/d + (2^(1/2)*B*a^2*log(4*B*a^2*d - 2^(1/
2)*B*a^2*d*tan(c + d*x)^(1/2)*(2 + 2i))*(1 + 1i))/d - (4i^(1/2)*B*a^2*log(
4*B*a^2*d + 2*4i^(1/2)*B*a^2*d*tan(c + d*x)^(1/2)))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = a^2\left(\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^5}dx\right)a+2\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)ai + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)b-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)a + 2\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)b\right)$$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)
```

output
```
a**2*(int(sqrt(tan(c + d*x))/tan(c + d*x)**5,x)*a + 2*int(sqrt(tan(c + d*x
))/tan(c + d*x)**4,x)*a*i + int(sqrt(tan(c + d*x))/tan(c + d*x)**4,x)*b -
int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*a + 2*int(sqrt(tan(c + d*x))/tan
(c + d*x)**3,x)*b*i - int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*b)
```

3.127 $\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 198

$$\begin{aligned}
&\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\
&= \frac{8\sqrt[4]{-1}a^3(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} \\
&\quad+\frac{8a^3(A-iB)\sqrt{\tan(c+dx)}}{d}+\frac{8a^3(iA+B)\tan^{\frac{3}{2}}(c+dx)}{3d} \\
&\quad-\frac{16a^3(18A-19iB)\tan^{\frac{5}{2}}(c+dx)}{315d}+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d} \\
&\quad-\frac{2(9A-13iB)\tan^{\frac{5}{2}}(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{63d}
\end{aligned}$$

output
```
8*(-1)^(1/4)*a^3*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d+8*a^3*(A-I*
B)*tan(d*x+c)^(1/2)/d+8/3*a^3*(I*A+B)*tan(d*x+c)^(3/2)/d-16/315*a^3*(18*A-
19*I*B)*tan(d*x+c)^(5/2)/d+2/9*I*a*B*tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2
/d-2/63*(9*A-13*I*B)*tan(d*x+c)^(5/2)*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.65

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2a^3\left(1260\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}(1260(A-iB)+420(iA+B)}{315d}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x])
,x]
```

output
```
(2*a^3*(1260*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] +
Sqrt[Tan[c + d*x]]*(1260*(A - I*B) + 420*(I*A + B)*Tan[c + d*x] - 63*(3*A
- (4*I)*B)*Tan[c + d*x]^2 - (45*I)*(A - (3*I)*B)*Tan[c + d*x]^3 - (35*I)*B
*Tan[c + d*x]^4)))/(315*d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4075, 3042, 4011, 3042, 4011, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)^{3/2}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4077$$

$$\frac{2}{9}\int \frac{1}{2}\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^2(a(9A-5iB)+a(9iA+13B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{9}\int \tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^2(a(9A-5iB)+a(9iA+13B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\int \tan(c+dx)^{3/2}(i\tan(c+dx)a+a)^2(a(9A-5iB)+a(9iA+13B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{ 4077}$$

$$\frac{1}{9}\left(\frac{2}{7}\int 2\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)\left((27A-25iB)a^2+2(18iA+19B)\tan(c+dx)a^2\right)dx-\frac{2(9A-13iI}{}\right.$$
$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{9}\left(\frac{4}{7}\int \tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)\left((27A-25iB)a^2+2(18iA+19B)\tan(c+dx)a^2\right)dx-\frac{2(9A-13iB)}{}\right.$$
$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}\left(\frac{4}{7}\int \tan(c+dx)^{3/2}(i\tan(c+dx)a+a)\left((27A-25iB)a^2+2(18iA+19B)\tan(c+dx)a^2\right)dx-\frac{2(9A-13iI}{}\right.$$
$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{ 4075}$$

$$\frac{1}{9}\left(\frac{4}{7}\left(\int \tan^{\frac{3}{2}}(c+dx)\left(63(A-iB)a^3+63(iA+B)\tan(c+dx)a^3\right)dx-\frac{4a^3(18A-19iB)\tan^{\frac{5}{2}}(c+dx)}{5d}\right)-\frac{2(\right.$$
$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{4}{7}\left(\int \tan(c+dx)^{3/2}\left(63(A-iB)a^3+63(iA+B)\tan(c+dx)a^3\right)dx-\frac{4a^3(18A-19iB)\tan^{\frac{5}{2}}(c+dx)}{5d}\right)-\frac{2}{}\right.$$
$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$\downarrow$ 4011

$$\frac{1}{9}\left(\frac{4}{7}\left(\int \sqrt{\tan(c+dx)}\left(63a^3(A-iB)\tan(c+dx)-63a^3(iA+B)\right)dx-\frac{4a^3(18A-19iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{42a}{}\right.\right.$$
$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{4}{7}\left(\int \sqrt{\tan(c+dx)}\left(63a^3(A-iB)\tan(c+dx)-63a^3(iA+B)\right)dx-\frac{4a^3(18A-19iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{42a}{}\right.\right.$$
$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$\downarrow$ 4011

$$\frac{1}{9}\left(\frac{4}{7}\left(\int \frac{-63(A-iB)a^3-63(iA+B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx-\frac{4a^3(18A-19iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{42a^3(B+iA)\tan}{d}\right.\right.$$
$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\frac{4}{7}\left(\int \frac{-63(A-iB)a^3-63(iA+B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx-\frac{4a^3(18A-19iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{42a^3(B+iA)\tan}{d}\right.\right.$$
$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$\downarrow$ 4016

$$\frac{1}{9}\left(\frac{4}{7}\left(\frac{7938a^6(A-iB)^2\int\frac{1}{63a^3(iA+B)\tan(c+dx)-63a^3(A-iB)}d\sqrt{\tan(c+dx)}}{d}-\frac{4a^3(18A-19iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{42}{}\right.\right.$$

$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{9}\left(\frac{4}{7}\left(\frac{126\sqrt[4]{-1}a^3(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{4a^3(18A-19iB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{42a^3(B+iA)t}{d}\right.\right.$$

$$\frac{2iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

input
```
Int[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(((2*I)/9)*a*B*Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2)/d + ((-2*(9*A
 - (13*I)*B)*Tan[c + d*x]^(5/2)*(a^3 + I*a^3*Tan[c + d*x]))/(7*d) + (4*((12
6*(-1)^(1/4)*a^3*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d + (126
*a^3*(A - I*B)*Sqrt[Tan[c + d*x]])/d + (42*a^3*(I*A + B)*Tan[c + d*x]^(3/2
))/d - (4*a^3*(18*A - (19*I)*B)*Tan[c + d*x]^(5/2))/(5*d)))/7)/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{a^3\left(-\frac{2iB\tan(dx+c)^{\frac{9}{2}}}{9}-\frac{2iA\tan(dx+c)^{\frac{7}{2}}}{7}-\frac{6B\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{8iB\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{6A\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{8iA\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{8B\tan(d\cdots}{3}\cdots\right.}{\cdots}$
default	$\frac{a^3\left(-\frac{2iB\tan(dx+c)^{\frac{9}{2}}}{9}-\frac{2iA\tan(dx+c)^{\frac{7}{2}}}{7}-\frac{6B\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{8iB\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{6A\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{8iA\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{8B\tan(d\cdots}{3}\cdots\right.}{\cdots}$
parts	$\frac{(-iA\,a^3-3B\,a^3)\left(\frac{2\tan(dx+c)^{\frac{7}{2}}}{7}-\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\cdots\right.}{4}\cdots\right.}{d}$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^3*(-2/9*I*B*tan(d*x+c)^(9/2)-2/7*I*A*tan(d*x+c)^(7/2)-6/7*B*tan(d*x+
c)^(7/2)+8/5*I*B*tan(d*x+c)^(5/2)-6/5*A*tan(d*x+c)^(5/2)+8/3*I*A*tan(d*x+c
)^(3/2)+8/3*B*tan(d*x+c)^(3/2)-8*I*B*tan(d*x+c)^(1/2)+8*A*tan(d*x+c)^(1/2)
+1/4*(4*I*B-4*A)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(
d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2
*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-4*I*A-4*B)*2^(1/2)*(ln((tan(d*
x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+
2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(158) = 316$.

Time = 0.13 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.83

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-2/315*(315*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(8*I*d*x + 8*I*c)
+ 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I
*c) + d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B
- I*B^2)*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3
)) - 315*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(8*I*d*x + 8*I*c) + 4
*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
 + d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I
*B^2)*a^6/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((-I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3))
 - 2*((957*A - 1051*I*B)*a^3*e^(8*I*d*x + 8*I*c) + 5*(579*A - 547*I*B)*a^3
*e^(6*I*d*x + 6*I*c) + 21*(171*A - 173*I*B)*a^3*e^(4*I*d*x + 4*I*c) + 5*(4
29*A - 433*I*B)*a^3*e^(2*I*d*x + 2*I*c) + 4*(123*A - 124*I*B)*a^3)*sqrt((-
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(8*I*d*x + 8*I
*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x +
 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.18

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{70i\,Ba^3\tan\left(dx+c\right)^{\frac{9}{2}}+90\,(i\,A+3\,B)a^3\tan\left(dx+c\right)^{\frac{7}{2}}+126\,(3\,A-4i\,B)a^3\tan\left(dx+c\right)^{\frac{5}{2}}+840\,(-i\,\ldots}{\ldots}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/315*(70*I*B*a^3*tan(d*x + c)^(9/2) + 90*(I*A + 3*B)*a^3*tan(d*x + c)^(7
/2) + 126*(3*A - 4*I*B)*a^3*tan(d*x + c)^(5/2) + 840*(-I*A - B)*a^3*tan(d*
x + c)^(3/2) - 2520*(A - I*B)*a^3*sqrt(tan(d*x + c)) + 315*(2*sqrt(2)*((I
+ 1)*A - (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) +
 2*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(t
an(d*x + c)))) - sqrt(2)*((I - 1)*A + (I + 1)*B)*log(sqrt(2)*sqrt(tan(d*x
+ c)) + tan(d*x + c) + 1) + sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)*s
qrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.83

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(35i\,Ba^3\tan\left(dx+c\right)^{\frac{9}{2}}+45i\,Aa^3\tan\left(dx+c\right)^{\frac{7}{2}}+135\,Ba^3\tan\left(dx+c\right)^{\frac{7}{2}}+189\,Aa^3\tan\left(dx+c\right)^{\frac{5}{2}}-\right.}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/315*(35*I*B*a^3*tan(d*x + c)^(9/2) + 45*I*A*a^3*tan(d*x + c)^(7/2) + 13
5*B*a^3*tan(d*x + c)^(7/2) + 189*A*a^3*tan(d*x + c)^(5/2) - 252*I*B*a^3*ta
n(d*x + c)^(5/2) - 420*I*A*a^3*tan(d*x + c)^(3/2) - 420*B*a^3*tan(d*x + c)
^(3/2) - 1260*A*a^3*sqrt(tan(d*x + c)) + 1260*I*B*a^3*sqrt(tan(d*x + c)) +
 630*sqrt(2)*((I + 1)*A*a^3 - (I - 1)*B*a^3)*arctan(-(1/2*I - 1/2)*sqrt(2)
*sqrt(tan(d*x + c))))/d
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.65

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{8\,A\,a^3\,\sqrt{\tan\left(c+d\,x\right)}}{d}+\frac{A\,a^3\,{\tan\left(c+d\,x\right)}^{3/2}\,8{}\mathrm{i}}{3\,d}-\frac{6\,A\,a^3\,{\tan\left(c+d\,x\right)}^{5/2}}{5\,d}-\frac{A\,a^3\,{\tan\left(c+d\,x\right)}^{7/2}\,2{}\mathrm{i}}{7\,d}-\frac{B\,a^3\,\sqrt{\tan\left(c+d\,x\right)}\,8{}\mathrm{i}}{d}+\frac{8\,B\,a^3\,{\tan\left(c+d\,x\right)}^{3/2}}{3\,d}+\frac{B\,a^3\,{\tan\left(c+d\,x\right)}^{5/2}\,8{}\mathrm{i}}{5\,d}-\frac{6\,B\,a^3\,{\tan\left(c+d\,x\right)}^{7/2}}{7\,d}-\frac{B\,a^3\,{\tan\left(c+d\,x\right)}^{9/2}\,2{}\mathrm{i}}{9\,d}+\frac{\sqrt{2}\,A\,a^3\,\ln\left(-A\,a^3\,d\,8{}\mathrm{i}+\sqrt{2}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-4+4{}\mathrm{i}\right)\right)\,\left(2-2{}\mathrm{i}\right)}{d}-\frac{\sqrt{-16{}\mathrm{i}}\,A\,a^3\,\ln\left(-A\,a^3\,d\,8{}\mathrm{i}+2\,\sqrt{-16{}\mathrm{i}}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}+\frac{\sqrt{2}\,B\,a^3\,\ln\left(-8\,B\,a^3\,d+\sqrt{2}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-4-4{}\mathrm{i}\right)\right)\,\left(2+2{}\mathrm{i}\right)}{d}-\frac{\sqrt{16{}\mathrm{i}}\,B\,a^3\,\ln\left(-8\,B\,a^3\,d+2\,\sqrt{16{}\mathrm{i}}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(8*A*a^3*tan(c + d*x)^(1/2))/d + (A*a^3*tan(c + d*x)^(3/2)*8i)/(3*d) - (6*
A*a^3*tan(c + d*x)^(5/2))/(5*d) - (A*a^3*tan(c + d*x)^(7/2)*2i)/(7*d) - (B
*a^3*tan(c + d*x)^(1/2)*8i)/d + (8*B*a^3*tan(c + d*x)^(3/2))/(3*d) + (B*a^
3*tan(c + d*x)^(5/2)*8i)/(5*d) - (6*B*a^3*tan(c + d*x)^(7/2))/(7*d) - (B*a
^3*tan(c + d*x)^(9/2)*2i)/(9*d) + (2^(1/2)*A*a^3*log(- A*a^3*d*8i - 2^(1/2
)*A*a^3*d*tan(c + d*x)^(1/2)*(4 - 4i))*(2 - 2i))/d - ((-16i)^(1/2)*A*a^3*l
og(2*(-16i)^(1/2)*A*a^3*d*tan(c + d*x)^(1/2) - A*a^3*d*8i))/d + (2^(1/2)*B
*a^3*log(- 8*B*a^3*d - 2^(1/2)*B*a^3*d*tan(c + d*x)^(1/2)*(4 + 4i))*(2 + 2
i))/d - (16i^(1/2)*B*a^3*log(2*16i^(1/2)*B*a^3*d*tan(c + d*x)^(1/2) - 8*B*
a^3*d))/d
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3\left(-10\sqrt{\tan(dx+c)}\,\tan(dx+c)^4 bi - 54\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 a + 72\sqrt{\tan(dx+c)}\,\tan(dx+\right.}{}$$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*( - 10*sqrt(tan(c + d*x))*tan(c + d*x)**4*b*i - 54*sqrt(tan(c + d*x)
)*tan(c + d*x)**2*a + 72*sqrt(tan(c + d*x))*tan(c + d*x)**2*b*i + 360*sqrt
(tan(c + d*x))*a - 360*sqrt(tan(c + d*x))*b*i - 180*int(sqrt(tan(c + d*x))
/tan(c + d*x),x)*a*d + 180*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b*d*i -
45*int(sqrt(tan(c + d*x))*tan(c + d*x)**4,x)*a*d*i - 135*int(sqrt(tan(c +
d*x))*tan(c + d*x)**4,x)*b*d + 135*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,
x)*a*d*i + 45*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*b*d))/(45*d)
```

3.128 $\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 171

$$\begin{aligned}
&\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\
&= \frac{8\sqrt[4]{-1}a^3(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}+\frac{8a^3(iA+B)\sqrt{\tan(c+dx)}}{d} \\
&\quad -\frac{8a^3(21A-23iB)\tan^{\frac{3}{2}}(c+dx)}{105d}+\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d} \\
&\quad -\frac{2(7A-11iB)\tan^{\frac{3}{2}}(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{35d}
\end{aligned}$$

output

```
8*(-1)^(1/4)*a^3*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d+8*a^3*(I*A+
B)*tan(d*x+c)^(1/2)/d-8/105*a^3*(21*A-23*I*B)*tan(d*x+c)^(3/2)/d+2/7*I*a*B
*tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2/d-2/35*(7*A-11*I*B)*tan(d*x+c)^(3/2
)*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.65

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2a^3\left(420\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}(420(iA+B)-35(3A-4iB)\text{t}\right.}{105d}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x])
,x]
```

output
```
(2*a^3*(420*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + S
qrt[Tan[c + d*x]]*(420*(I*A + B) - 35*(3*A - (4*I)*B)*Tan[c + d*x] - (21*I
)*(A - (3*I)*B)*Tan[c + d*x]^2 - (15*I)*B*Tan[c + d*x]^3)))/(105*d)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4075, 3042, 4011, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{2}{7}\int \frac{1}{2}\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^2(a(7A-3iB)+a(7iA+11B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^2(a(7A-3iB)+a(7iA+11B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^2(a(7A-3iB)+a(7iA+11B)\tan(c+dx))dx+\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$\Big\downarrow$ 4077

$$\frac{1}{7}\left(\frac{2}{5}\int 2\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)\left(2(7A-6iB)a^2+(21iA+23B)\tan(c+dx)a^2\right)dx-\frac{2(7A-11iB)}{}\right.$$
$$\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(\frac{4}{5}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)\left(2(7A-6iB)a^2+(21iA+23B)\tan(c+dx)a^2\right)dx-\frac{2(7A-11iB)\,t}{}\right.$$
$$\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{4}{5}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)\left(2(7A-6iB)a^2+(21iA+23B)\tan(c+dx)a^2\right)dx-\frac{2(7A-11iB)\,t}{}\right.$$
$$\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$\Big\downarrow$ 4075

$$\frac{1}{7}\left(\frac{4}{5}\left(\int \sqrt{\tan(c+dx)}\left(35(A-iB)a^3+35(iA+B)\tan(c+dx)a^3\right)dx-\frac{2a^3(21A-23iB)\tan^{\frac{3}{2}}(c+dx)}{3d}\right)-\frac{2}{}\right.$$
$$\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{4}{5}\left(\left(\int \sqrt{\tan(c+dx)}\left(35(A-iB)a^3+35(iA+B)\tan(c+dx)a^3\right)dx-\frac{2a^3(21A-23iB)\tan^{\frac{3}{2}}(c+dx)}{3d}\right)-\frac{2}{}\right.\right.$$
$$\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$\big\downarrow$ 4011

$$\frac{1}{7}\left(\frac{4}{5}\left(\left(\int \frac{35a^3(A-iB)\tan(c+dx)-35a^3(iA+B)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(21A-23iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{70a^3(B+iA)\sqrt{\tan}}{d}\right.\right.\right.$$
$$\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{4}{5}\left(\left(\int \frac{35a^3(A-iB)\tan(c+dx)-35a^3(iA+B)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(21A-23iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{70a^3(B+iA)\sqrt{\tan}}{d}\right.\right.\right.$$
$$\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$\big\downarrow$ 4016

$$\frac{1}{7}\left(\frac{4}{5}\left(\frac{2450a^6(B+iA)^2\int \frac{1}{-35(iA+B)a^3-35(A-iB)\tan(c+dx)a^3}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^3(21A-23iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\right.\right.$$
$$\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

$\big\downarrow$ 218

$$\frac{1}{7}\left(\frac{4}{5}\left(\frac{70\sqrt[4]{-1}a^3(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a^3(21A-23iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{70a^3(B+iA)\sqrt{}}{d}\right.\right.$$
$$\frac{2iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(((2*I)/7)*a*B*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2)/d + ((-2*(7*A
- (11*I)*B)*Tan[c + d*x]^(3/2)*(a^3 + I*a^3*Tan[c + d*x]))/(5*d) + (4*((70
*(-1)^(1/4)*a^3*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d + (70*a
^3*(I*A + B)*Sqrt[Tan[c + d*x]])/d - (2*a^3*(21*A - (23*I)*B)*Tan[c + d*x]
^(3/2))/(3*d)))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.61

method	result
derivativedivides	$a^3\left(-\frac{2iB\tan(dx+c)^{\frac{7}{2}}}{7}-\frac{2iA\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{6B\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{8iB\tan(dx+c)^{\frac{3}{2}}}{3}-2A\tan(dx+c)^{\frac{3}{2}}+8iA\sqrt{\tan(dx+c)}+8B\sqrt{\mathrm{t}\right.$
default	$a^3\left(-\frac{2iB\tan(dx+c)^{\frac{7}{2}}}{7}-\frac{2iA\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{6B\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{8iB\tan(dx+c)^{\frac{3}{2}}}{3}-2A\tan(dx+c)^{\frac{3}{2}}+8iA\sqrt{\tan(dx+c)}+8B\sqrt{\mathrm{t}\right.$
parts	$\frac{(-iA\,a^3-3B\,a^3)\left(\frac{2\tan(dx+c)^{\frac{5}{2}}}{5}-2\sqrt{\tan(dx+c)}+\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+\right.}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^3*(-2/7*I*B*tan(d*x+c)^(7/2)-2/5*I*A*tan(d*x+c)^(5/2)-6/5*B*tan(d*x+
c)^(5/2)+8/3*I*B*tan(d*x+c)^(3/2)-2*A*tan(d*x+c)^(3/2)+8*I*A*tan(d*x+c)^(1
/2)+8*B*tan(d*x+c)^(1/2)+1/4*(-4*I*A-4*B)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*
tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^
(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-4*I*B
+4*A)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1
/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+
2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(137) = 274$.

Time = 0.11 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.91

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(105\sqrt{-\frac{(-iA^2-2AB+iB^2)a^6}{d^2}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(-\frac{2\left((A-iB)a^3e^{(2i\,dx}\right.}{}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-2/105*(105*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(6*I*d*x + 6*I*c)
 + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B
)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(
2*I*d*x + 2*I*c) + d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 105*sqrt(-(-I*A^2 - 2*A
*B + I*B^2)*a^6/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*
d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqr
t(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/
((-I*A - B)*a^3)) + 2*((-273*I*A - 319*B)*a^3*e^(6*I*d*x + 6*I*c) + 2*(-33
6*I*A - 323*B)*a^3*e^(4*I*d*x + 4*I*c) + (-567*I*A - 551*B)*a^3*e^(2*I*d*x
 + 2*I*c) + 4*(-42*I*A - 41*B)*a^3)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) + 1)))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) +
 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= -ia^3\left(\int\left(-3A\tan^{\frac{3}{2}}(c+dx)\right)dx + \int A\tan^{\frac{7}{2}}(c+dx)\,dx + \int\left(-3B\tan^{\frac{5}{2}}(c+dx)\right)dx + \int B\tan^{\frac{9}{2}}(c+dx)\,dx + \int iA\sqrt{\tan(c+dx)}\,dx + \int\left(-3iA\tan^{\frac{5}{2}}(c+dx)\right)dx + \int iB\tan^{\frac{3}{2}}(c+dx)\,dx + \int\left(-3iB\tan^{\frac{7}{2}}(c+dx)\right)dx\right)$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
-I*a**3*(Integral(-3*A*tan(c + d*x)**(3/2), x) + Integral(A*tan(c + d*x)**
(7/2), x) + Integral(-3*B*tan(c + d*x)**(5/2), x) + Integral(B*tan(c + d*x
)**(9/2), x) + Integral(I*A*sqrt(tan(c + d*x)), x) + Integral(-3*I*A*tan(c
 + d*x)**(5/2), x) + Integral(I*B*tan(c + d*x)**(3/2), x) + Integral(-3*I*
B*tan(c + d*x)**(7/2), x))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.26

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{30i\,Ba^3\tan(dx+c)^{\frac{7}{2}} + 42\,(i\,A+3\,B)a^3\tan(dx+c)^{\frac{5}{2}} + 70\,(3\,A-4i\,B)a^3\tan(dx+c)^{\frac{3}{2}} + 840\,(-i\,A}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/105*(30*I*B*a^3*tan(d*x + c)^(7/2) + 42*(I*A + 3*B)*a^3*tan(d*x + c)^(5
/2) + 70*(3*A - 4*I*B)*a^3*tan(d*x + c)^(3/2) + 840*(-I*A - B)*a^3*sqrt(ta
n(d*x + c)) + 105*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(s
qrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(-(I + 1)*A + (I
 - 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(-(I
 + 1)*A + (I - 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*
a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(15i\,Ba^3\tan\left(dx+c\right)^{\frac{7}{2}}+21i\,Aa^3\tan\left(dx+c\right)^{\frac{5}{2}}+63\,Ba^3\tan\left(dx+c\right)^{\frac{5}{2}}+105\,Aa^3\tan\left(dx+c\right)^{\frac{3}{2}}-1\right.}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/105*(15*I*B*a^3*tan(d*x + c)^(7/2) + 21*I*A*a^3*tan(d*x + c)^(5/2) + 63
*B*a^3*tan(d*x + c)^(5/2) + 105*A*a^3*tan(d*x + c)^(3/2) - 140*I*B*a^3*tan
(d*x + c)^(3/2) - 420*I*A*a^3*sqrt(tan(d*x + c)) - 420*B*a^3*sqrt(tan(d*x
+ c)) + 210*sqrt(2)*((I - 1)*A*a^3 + (I + 1)*B*a^3)*arctan(-(1/2*I - 1/2)*
sqrt(2)*sqrt(tan(d*x + c))))/d
```

Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.71

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{A\,a^3\,\sqrt{\tan\left(c+d\,x\right)}\,8\mathrm{i}}{d} - \frac{2\,A\,a^3\,{\tan(c+d\,x)}^{3/2}}{d} - \frac{A\,a^3\,{\tan(c+d\,x)}^{5/2}\,2\mathrm{i}}{5\,d}$$
$$+ \frac{8\,B\,a^3\,\sqrt{\tan\left(c+d\,x\right)}}{d} + \frac{B\,a^3\,{\tan(c+d\,x)}^{3/2}\,8\mathrm{i}}{3\,d}$$
$$- \frac{6\,B\,a^3\,{\tan(c+d\,x)}^{5/2}}{5\,d} - \frac{B\,a^3\,{\tan(c+d\,x)}^{7/2}\,2\mathrm{i}}{7\,d}$$
$$+ \frac{\sqrt{2}\,A\,a^3\,\ln\left(8\,A\,a^3\,d+\sqrt{2}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-4-4\mathrm{i})\right)\,(2+2\mathrm{i})}{d}$$
$$- \frac{\sqrt{16\mathrm{i}}\,A\,a^3\,\ln\left(8\,A\,a^3\,d+2\,\sqrt{16\mathrm{i}}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$
$$+ \frac{\sqrt{2}\,B\,a^3\,\ln\left(-B\,a^3\,d\,8\mathrm{i}+\sqrt{2}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-4+4\mathrm{i})\right)\,(2-2\mathrm{i})}{d}$$
$$- \frac{\sqrt{-16\mathrm{i}}\,B\,a^3\,\ln\left(-B\,a^3\,d\,8\mathrm{i}+2\,\sqrt{-16\mathrm{i}}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
(A*a^3*tan(c + d*x)^(1/2)*8i)/d - (2*A*a^3*tan(c + d*x)^(3/2))/d - (A*a^3*
tan(c + d*x)^(5/2)*2i)/(5*d) + (8*B*a^3*tan(c + d*x)^(1/2))/d + (B*a^3*tan
(c + d*x)^(3/2)*8i)/(3*d) - (6*B*a^3*tan(c + d*x)^(5/2))/(5*d) - (B*a^3*ta
n(c + d*x)^(7/2)*2i)/(7*d) + (2^(1/2)*A*a^3*log(8*A*a^3*d - 2^(1/2)*A*a^3*
d*tan(c + d*x)^(1/2)*(4 + 4i))*(2 + 2i))/d - (16i^(1/2)*A*a^3*log(8*A*a^3*
d + 2*16i^(1/2)*A*a^3*d*tan(c + d*x)^(1/2)))/d + (2^(1/2)*B*a^3*log(- B*a^
3*d*8i - 2^(1/2)*B*a^3*d*tan(c + d*x)^(1/2)*(4 - 4i))*(2 - 2i))/d - ((-16i
)^(1/2)*B*a^3*log(2*(-16i)^(1/2)*B*a^3*d*tan(c + d*x)^(1/2) - B*a^3*d*8i))
/d
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3\left(-2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2\,ai-6\sqrt{\tan(dx+c)}\,\tan(dx+c)^2\,b+40\sqrt{\tan(dx+c)}\,ai+40\sqrt{\mathrm{ta}}\right.}{}$$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*( - 2*sqrt(tan(c + d*x))*tan(c + d*x)**2*a*i - 6*sqrt(tan(c + d*x))*
tan(c + d*x)**2*b + 40*sqrt(tan(c + d*x))*a*i + 40*sqrt(tan(c + d*x))*b -
20*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*d*i - 20*int(sqrt(tan(c + d*x)
)/tan(c + d*x),x)*b*d + 5*int(sqrt(tan(c + d*x)),x)*a*d - 5*int(sqrt(tan(c
 + d*x))*tan(c + d*x)**4,x)*b*d*i - 15*int(sqrt(tan(c + d*x))*tan(c + d*x)
**2,x)*a*d + 15*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*b*d*i))/(5*d)
```

3.129 $\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 146

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{8\sqrt[4]{-1}a^3(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{16a^3(5A-6iB)\sqrt{\tan(c+dx)}}{15d} + \frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d} - \frac{2(5A-9iB)\sqrt{\tan(c+dx)}(a^3+ia^3\tan(c+dx))}{15d}$$

output

```
-8*(-1)^(1/4)*a^3*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-16/15*a^3*
(5*A-6*I*B)*tan(d*x+c)^(1/2)/d+2/5*I*a*B*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c
))^2/d-2/15*(5*A-9*I*B)*tan(d*x+c)^(1/2)*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{2a^3\left(60\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}(45A-60iB+5(iA+3B)\tan(c+dx)+3iB\tan^2(c+dx))\right)}{15d}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x
]],x]
```

output
```
(-2*a^3*(60*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + S
qrt[Tan[c + d*x]]*(45*A - (60*I)*B + 5*(I*A + 3*B)*Tan[c + d*x] + (3*I)*B*
Tan[c + d*x]^2)))/(15*d)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4075, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{2}{5}\int \frac{(i\tan(c+dx)a+a)^2(a(5A-iB)+a(5iA+9B)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx+\frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d}$$

$$\downarrow 27$$

$$\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)^2(a(5A-iB)+a(5iA+9B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d}$$

$$\downarrow 3042$$

$$\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)^2(a(5A-iB)+a(5iA+9B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d}$$

$$\downarrow 4077$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{2(i\tan(c+dx)a+a)\left((5A-3iB)a^2+2(5iA+6B)\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2(5A-9iB)\sqrt{\tan(c+dx)}(a^3}{3d}\right.$$
$$\frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d}$$

$$\downarrow 27$$

$$\frac{1}{5}\left(\frac{4}{3}\int \frac{(i\tan(c+dx)a+a)\left((5A-3iB)a^2+2(5iA+6B)\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2(5A-9iB)\sqrt{\tan(c+dx)}(a^3}{3d}\right.$$
$$\frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d}$$

$$\downarrow 3042$$

$$\frac{1}{5}\left(\frac{4}{3}\int \frac{(i\tan(c+dx)a+a)\left((5A-3iB)a^2+2(5iA+6B)\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2(5A-9iB)\sqrt{\tan(c+dx)}(a^3}{3d}\right.$$
$$\frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d}$$

$$\downarrow 4075$$

$$\frac{1}{5}\left(\frac{4}{3}\left(\int \frac{15(A-iB)a^3+15(iA+B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx-\frac{4a^3(5A-6iB)\sqrt{\tan(c+dx)}}{d}\right)-\frac{2(5A-9iB)\sqrt{\tan(}}{}\right.$$
$$\frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d}$$

$$\downarrow 3042$$

$$\frac{1}{5}\left(\frac{4}{3}\left(\int \frac{15(A-iB)a^3+15(iA+B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx-\frac{4a^3(5A-6iB)\sqrt{\tan(c+dx)}}{d}\right)-\frac{2(5A-9iB)\sqrt{\tan(}}{}\right.$$
$$\frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \; 4016$$

$$\frac{1}{5}\left(\frac{4}{3}\left(\frac{450a^6(A-iB)^2\int \frac{1}{15a^3(A-iB)-15a^3(iA+B)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}-\frac{4a^3(5A-6iB)\sqrt{\tan(c+dx)}}{d}\right)-\frac{2(5}{}\right.$$
$$\frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \; 218$$

$$\frac{1}{5}\left(\frac{4}{3}\left(-\frac{30\sqrt[4]{-1}a^3(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{4a^3(5A-6iB)\sqrt{\tan(c+dx)}}{d}\right)-\frac{2(5A-9iB)\sqrt{}}{}\right.$$
$$\frac{2iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}{5d}$$

input
```
Int[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x]
```

output
```
(((2*I)/5)*a*B*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^2)/d + ((4*((-30*
(-1)^(1/4)*a^3*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d - (4*a^3
*(5*A - (6*I)*B)*Sqrt[Tan[c + d*x]])/d))/3 - (2*(5*A - (9*I)*B)*Sqrt[Tan[c
 + d*x]]*(a^3 + I*a^3*Tan[c + d*x]))/(3*d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(122) = 244$.

Time = 0.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.73

method	result
derivativedivides	$\frac{a^3\left(-\frac{2iB\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{2iA\tan(dx+c)^{\frac{3}{2}}}{3}-2B\tan(dx+c)^{\frac{3}{2}}+8iB\sqrt{\tan(dx+c)}-6A\sqrt{\tan(dx+c)}+\frac{(-4iB+4A)\sqrt{2}\left(\ln\left(\frac{\tan}{\tan}\right.\right.}{}\right.}{}$
default	$\frac{a^3\left(-\frac{2iB\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{2iA\tan(dx+c)^{\frac{3}{2}}}{3}-2B\tan(dx+c)^{\frac{3}{2}}+8iB\sqrt{\tan(dx+c)}-6A\sqrt{\tan(dx+c)}+\frac{(-4iB+4A)\sqrt{2}\left(\ln\left(\frac{\tan}{\tan}\right.\right.}{}\right.}{}$
parts	$\frac{(-iA\,a^3-3B\,a^3)\left(\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\right.\right.}{4}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^3*(-2/5*I*B*tan(d*x+c)^(5/2)-2/3*I*A*tan(d*x+c)^(3/2)-2*B*tan(d*x+c)
^(3/2)+8*I*B*tan(d*x+c)^(1/2)-6*A*tan(d*x+c)^(1/2)+1/4*(-4*I*B+4*A)*2^(1/2
)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+
c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2)))+1/4*(4*I*A+4*B)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^
(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(116) = 232$.

Time = 0.09 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.06

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{2\left(15\sqrt{-\frac{(iA^2+2AB-iB^2)a^6}{d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(-\frac{2\left((A-iB)a^3e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{(iA^2+2AB-iB^2}{d^2}}\right.}{(}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algori
thm="fricas")
```

output
```
2/15*(15*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2
*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sq
rt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I
*c)/((-I*A - B)*a^3)) - 15*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(4*
I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^3*e^(2*I
*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(-I*d*e^(2*I*d*x +
2*I*c) - I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 2*((25*A - 39*I*B)*a^3*e^(4*I*d
*x + 4*I*c) + 3*(15*A - 19*I*B)*a^3*e^(2*I*d*x + 2*I*c) + 4*(5*A - 6*I*B)*
a^3)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4
*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -ia^3\left(\int\left(-3A\sqrt{\tan\left(c+dx\right)}\right)dx+\int A\tan^{\frac{5}{2}}\left(c+dx\right)dx + \int\left(-3B\tan^{\frac{3}{2}}\left(c+dx\right)\right)dx+\int B\tan^{\frac{7}{2}}\left(c+dx\right)dx+\int\frac{iA}{\sqrt{\tan\left(c+dx\right)}}\,dx + \int\left(-3iA\tan^{\frac{3}{2}}\left(c+dx\right)\right)dx+\int iB\sqrt{\tan\left(c+dx\right)}\,dx + \int\left(-3iB\tan^{\frac{5}{2}}\left(c+dx\right)\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
-I*a**3*(Integral(-3*A*sqrt(tan(c + d*x)), x) + Integral(A*tan(c + d*x)**(
5/2), x) + Integral(-3*B*tan(c + d*x)**(3/2), x) + Integral(B*tan(c + d*x)
**(7/2), x) + Integral(I*A/sqrt(tan(c + d*x)), x) + Integral(-3*I*A*tan(c
+ d*x)**(3/2), x) + Integral(I*B*sqrt(tan(c + d*x)), x) + Integral(-3*I*B*
tan(c + d*x)**(5/2), x))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.34

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx =$$
$$-\frac{6i\,Ba^3\tan\left(dx+c\right)^{\frac{5}{2}}+10\,(i\,A+3\,B)a^3\tan\left(dx+c\right)^{\frac{3}{2}}+30\,(3\,A-4i\,B)a^3\sqrt{\tan\left(dx+c\right)}-15\left(2\sqrt{2}(\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algori
thm="maxima")
```

output
```
-1/15*(6*I*B*a^3*tan(d*x + c)^(5/2) + 10*(I*A + 3*B)*a^3*tan(d*x + c)^(3/2
) + 30*(3*A - 4*I*B)*a^3*sqrt(tan(d*x + c)) - 15*(2*sqrt(2)*((I + 1)*A - (
I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)
*((I + 1)*A - (I - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c
)))) - sqrt(2)*((I - 1)*A + (I + 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + ta
n(d*x + c) + 1) + sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*
x + c)) + tan(d*x + c) + 1))*a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx =$$
$$-\frac{2\left(3i\,Ba^3\tan\left(dx+c\right)^{\frac{5}{2}}+5i\,Aa^3\tan\left(dx+c\right)^{\frac{3}{2}}+15\,Ba^3\tan\left(dx+c\right)^{\frac{3}{2}}+45\,Aa^3\sqrt{\tan\left(dx+c\right)}-60i\right.}{15\,a}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algori
thm="giac")
```

output
```
-2/15*(3*I*B*a^3*tan(d*x + c)^(5/2) + 5*I*A*a^3*tan(d*x + c)^(3/2) + 15*B*
a^3*tan(d*x + c)^(3/2) + 45*A*a^3*sqrt(tan(d*x + c)) - 60*I*B*a^3*sqrt(tan
(d*x + c)) + 30*sqrt(2)*(-(I + 1)*A*a^3 + (I - 1)*B*a^3)*arctan(-(1/2*I -
1/2)*sqrt(2)*sqrt(tan(d*x + c))))/d
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.76

$$\begin{aligned}
&\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx \\
&= -\frac{6\,A\,a^3\,\sqrt{\tan\left(c+d\,x\right)}}{d}-\frac{A\,a^3\,{\tan\left(c+d\,x\right)}^{3/2}\,2{}\mathrm{i}}{3\,d}+\frac{B\,a^3\,\sqrt{\tan\left(c+d\,x\right)}\,8{}\mathrm{i}}{d} \\
&\quad -\frac{2\,B\,a^3\,{\tan\left(c+d\,x\right)}^{3/2}}{d}-\frac{B\,a^3\,{\tan\left(c+d\,x\right)}^{5/2}\,2{}\mathrm{i}}{5\,d} \\
&\quad +\frac{\sqrt{2}\,A\,a^3\,\ln\left(A\,a^3\,d\,8{}\mathrm{i}+\sqrt{2}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-4+4{}\mathrm{i}\right)\right)\,\left(2-2{}\mathrm{i}\right)}{d} \\
&\quad -\frac{\sqrt{-16{}\mathrm{i}}\,A\,a^3\,\ln\left(A\,a^3\,d\,8{}\mathrm{i}+2\,\sqrt{-16{}\mathrm{i}}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d} \\
&\quad +\frac{\sqrt{2}\,B\,a^3\,\ln\left(8\,B\,a^3\,d+\sqrt{2}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-4-4{}\mathrm{i}\right)\right)\,\left(2+2{}\mathrm{i}\right)}{d} \\
&\quad -\frac{\sqrt{16{}\mathrm{i}}\,B\,a^3\,\ln\left(8\,B\,a^3\,d+2\,\sqrt{16{}\mathrm{i}}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}
\end{aligned}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3)/tan(c + d*x)^(1/2),x)
```

output
```
(B*a^3*tan(c + d*x)^(1/2)*8i)/d - (A*a^3*tan(c + d*x)^(3/2)*2i)/(3*d) - (6
*A*a^3*tan(c + d*x)^(1/2))/d - (2*B*a^3*tan(c + d*x)^(3/2))/d - (B*a^3*tan
(c + d*x)^(5/2)*2i)/(5*d) + (2^(1/2)*A*a^3*log(A*a^3*d*8i - 2^(1/2)*A*a^3*
d*tan(c + d*x)^(1/2)*(4 - 4i))*(2 - 2i))/d - ((-16i)^(1/2)*A*a^3*log(A*a^3
*d*8i + 2*(-16i)^(1/2)*A*a^3*d*tan(c + d*x)^(1/2)))/d + (2^(1/2)*B*a^3*log
(8*B*a^3*d - 2^(1/2)*B*a^3*d*tan(c + d*x)^(1/2)*(4 + 4i))*(2 + 2i))/d - (1
6i^(1/2)*B*a^3*log(8*B*a^3*d + 2*16i^(1/2)*B*a^3*d*tan(c + d*x)^(1/2)))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{a^3\left(-2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 bi - 30\sqrt{\tan(dx+c)}\,a + 40\sqrt{\tan(dx+c)}\,bi + 20\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(a**3*( - 2*sqrt(tan(c + d*x))*tan(c + d*x)**2*b*i - 30*sqrt(tan(c + d*x))
*a + 40*sqrt(tan(c + d*x))*b*i + 20*int(sqrt(tan(c + d*x))/tan(c + d*x),x)
*a*d - 20*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b*d*i + 15*int(sqrt(tan(c
 + d*x)),x)*a*d*i + 5*int(sqrt(tan(c + d*x)),x)*b*d - 5*int(sqrt(tan(c + d
*x))*tan(c + d*x)**2,x)*a*d*i - 15*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,
x)*b*d))/(5*d)
```

3.130 $\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 134

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{8\sqrt[4]{-1}a^3(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{16a^3B\sqrt{\tan(c+dx)}}{3d} - \frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}} + \frac{2(3iA-B)\sqrt{\tan(c+dx)}(a^3+ia^3\tan(c+dx))}{3d}$$

output
```
-8*(-1)^(1/4)*a^3*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-16/3*a^3*B
*tan(d*x+c)^(1/2)/d-2*a*A*(a+I*a*tan(d*x+c))^2/d/tan(d*x+c)^(1/2)+2/3*(3*I
*A-B)*tan(d*x+c)^(1/2)*(a^3+I*a^3*tan(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.70

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{2ia^3\left(-3iA+12\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}+3(A-3iB)\tan(c+\right.}{3d\sqrt{\tan(c+dx)}}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/
2),x]
```

output
```
(((-2*I)/3)*a^3*((-3*I)*A + 12*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt
[Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] + 3*(A - (3*I)*B)*Tan[c + d*x] + B*Tan[
c + d*x]^2))/(d*Sqrt[Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 4076, 27, 3042, 4077, 27, 3042, 4075, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$2\int \frac{(i\tan(c+dx)a+a)^2(a(5iA+B)+a(3A+iB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(i\tan(c+dx)a+a)^2(a(5iA+B)+a(3A+iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^2(a(5iA+B)+a(3A+iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4077}$$

$$\frac{2}{3}\int \frac{2(i\tan(c+dx)a+a)\left((3iA+B)a^2+2iB\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx+\frac{2(-B+3iA)\sqrt{\tan(c+dx)}(a^3+ia^3\tan(c+dx))}{3d}-\frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{3}\int \frac{(i\tan(c+dx)a+a)\left((3iA+B)a^2+2iB\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx+\frac{2(-B+3iA)\sqrt{\tan(c+dx)}(a^3+ia^3\tan(c+dx))}{3d}-\frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}\int \frac{(i\tan(c+dx)a+a)\left((3iA+B)a^2+2iB\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx+\frac{2(-B+3iA)\sqrt{\tan(c+dx)}(a^3+ia^3\tan(c+dx))}{3d}-\frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4075}$$

$$\frac{4}{3}\left(-\frac{4a^3B\sqrt{\tan(c+dx)}}{d}+\int \frac{3a^3(iA+B)-3a^3(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2(-B+3iA)\sqrt{\tan(c+dx)}(a^3+ia^3\tan(c+dx))}{3d}-\frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}\left(-\frac{4a^3B\sqrt{\tan(c+dx)}}{d}+\int \frac{3a^3(iA+B)-3a^3(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx\right)+\frac{2(-B+3iA)\sqrt{\tan(c+dx)}(a^3+ia^3\tan(c+dx))}{3d}-\frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4016}$$

$$\frac{4}{3}\left(-\frac{4a^3B\sqrt{\tan(c+dx)}}{d}+\frac{18a^6(B+iA)^2\int\frac{1}{3(iA+B)a^3+3(A-iB)\tan(c+dx)a^3}d\sqrt{\tan(c+dx)}}{d}\right)+\frac{2(-B+3iA)\sqrt{\tan(c+dx)}\left(a^3+ia^3\tan(c+dx)\right)}{3d}-\frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{218}$$

$$\frac{4}{3}\left(-\frac{4a^3B\sqrt{\tan(c+dx)}}{d}-\frac{6\sqrt[4]{-1}a^3(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}\right)+\frac{2(-B+3iA)\sqrt{\tan(c+dx)}\left(a^3+ia^3\tan(c+dx)\right)}{3d}-\frac{2aA(a+ia\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x]
```

output
```
(4*((-6*(-1)^(1/4)*a^3*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d
- (4*a^3*B*Sqrt[Tan[c + d*x]])/d))/3 - (2*a*A*(a + I*a*Tan[c + d*x])^2)/(d
*Sqrt[Tan[c + d*x]]) + (2*((3*I)*A - B)*Sqrt[Tan[c + d*x]]*(a^3 + I*a^3*Ta
n[c + d*x]))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(114) = 228$.

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{a^3\left(-\frac{2iB\tan(dx+c)^{\frac{3}{2}}}{3}-2iA\sqrt{\tan(dx+c)}-6B\sqrt{\tan(dx+c)}-\frac{2A}{\sqrt{\tan(dx+c)}}+\frac{(4iA+4B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+}\right.\right.}{}\right.}{}$
default	$\frac{a^3\left(-\frac{2iB\tan(dx+c)^{\frac{3}{2}}}{3}-2iA\sqrt{\tan(dx+c)}-6B\sqrt{\tan(dx+c)}-\frac{2A}{\sqrt{\tan(dx+c)}}+\frac{(4iA+4B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+}\right.\right.}{}\right.}{}$
parts	$\frac{(-iA\,a^3-3B\,a^3)\left(2\sqrt{\tan(dx+c)}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\right.\right.}{4}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^3*(-2/3*I*B*tan(d*x+c)^(3/2)-2*I*A*tan(d*x+c)^(1/2)-6*B*tan(d*x+c)^(
1/2)-2*A/tan(d*x+c)^(1/2)+1/4*(4*I*A+4*B)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*
tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^
(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(4*I*B-
4*A)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/
2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2
^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(108) = 216.

Time = 0.10 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.02

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2\left(3\sqrt{-\frac{(-iA^2-2AB+iB^2)a^6}{d^2}}\left(de^{(4i\,dx+4i\,c)}-d\right)\log\left(-\frac{2\left((A-iB)a^3e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{(-iA^2-2AB+iB^2)a^6}{d^2}}\left(de^{(2i\,dx+2i\,c)}+\right.\right.}{(-iA-B)a^3}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algori
thm="fricas")
```

output
```
2/3*(3*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) - d)
*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(-I*A^2 - 2*A*B + I*B^2
)*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(
e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 3*sqrt
(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) - d)*log(-2*((A
 - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*
(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 2*((3*I*A + 5*B)*
a^3*e^(4*I*d*x + 4*I*c) + (3*I*A - B)*a^3*e^(2*I*d*x + 2*I*c) - 4*B*a^3)*s
qrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x
 + 4*I*c) - d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -ia^3\left(\int\left(-\frac{3A}{\sqrt{\tan{(c+dx)}}}\right)dx+\int A\tan^{\frac{3}{2}}{(c+dx)}\,dx+\int\left(-3B\sqrt{\tan{(c+dx)}}\right)dx+\int B\tan^{\frac{5}{2}}{(c+dx)}\,dx+\int\frac{iA}{\tan^{\frac{3}{2}}{(c+dx)}}\,dx+\int\left(-3iA\sqrt{\tan{(c+dx)}}\right)dx+\int\frac{iB}{\sqrt{\tan{(c+dx)}}}\,dx+\int\left(-3iB\tan^{\frac{3}{2}}{(c+dx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
-I*a**3*(Integral(-3*A/sqrt(tan(c + d*x)), x) + Integral(A*tan(c + d*x)**(
3/2), x) + Integral(-3*B*sqrt(tan(c + d*x)), x) + Integral(B*tan(c + d*x)*
*(5/2), x) + Integral(I*A/tan(c + d*x)**(3/2), x) + Integral(-3*I*A*sqrt(t
an(c + d*x)), x) + Integral(I*B/sqrt(tan(c + d*x)), x) + Integral(-3*I*B*t
an(c + d*x)**(3/2), x))
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.42

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2i\,Ba^3\tan\left(dx+c\right)^{\frac{3}{2}}+6\left(i\,A+3\,B\right)a^3\sqrt{\tan\left(dx+c\right)}-3\left(2\,\sqrt{2}((i-1)\,A+(i+1)\,B)\arctan\left(\frac{1}{2}\,\sqrt{2}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algori
thm="maxima")
```

output
```
-1/3*(2*I*B*a^3*tan(d*x + c)^(3/2) + 6*(I*A + 3*B)*a^3*sqrt(tan(d*x + c))
- 3*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqr
t(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt(2)*
(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(s
qrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(-(I + 1)*A + (I -
 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^3 + 6*A*a^3/
sqrt(tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.70

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\left(i\,Ba^3\tan\left(dx+c\right)^{\frac{3}{2}}+3i\,Aa^3\sqrt{\tan\left(dx+c\right)}+9\,Ba^3\sqrt{\tan\left(dx+c\right)}+\frac{3\,Aa^3}{\sqrt{\tan(dx+c)}}+6\,\sqrt{2}(-(i-1)\,A\right.}{3\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algori
thm="giac")
```

output
```
-2/3*(I*B*a^3*tan(d*x + c)^(3/2) + 3*I*A*a^3*sqrt(tan(d*x + c)) + 9*B*a^3*
sqrt(tan(d*x + c)) + 3*A*a^3/sqrt(tan(d*x + c)) + 6*sqrt(2)*(-(I - 1)*A*a^
3 - (I + 1)*B*a^3)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))))/d
```

Mupad [B] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.78

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= -\frac{2\,A\,a^3}{d\,\sqrt{\mathrm{tan}\left(c+d\,x\right)}}-\frac{A\,a^3\,\sqrt{\mathrm{tan}\left(c+d\,x\right)}\,2{}\mathrm{i}}{d}-\frac{6\,B\,a^3\,\sqrt{\mathrm{tan}\left(c+d\,x\right)}}{d}-\frac{B\,a^3\,{\mathrm{tan}\left(c+d\,x\right)}^{3/2}\,2{}\mathrm{i}}{3\,d}+\frac{\sqrt{2}\,A\,a^3\,\ln\left(-8\,A\,a^3\,d+\sqrt{2}\,A\,a^3\,d\,\sqrt{\mathrm{tan}\left(c+d\,x\right)}\,\left(-4-4{}\mathrm{i}\right)\right)\,\left(2+2{}\mathrm{i}\right)}{d}-\frac{\sqrt{16{}\mathrm{i}}\,A\,a^3\,\ln\left(-8\,A\,a^3\,d+2\,\sqrt{16{}\mathrm{i}}\,A\,a^3\,d\,\sqrt{\mathrm{tan}\left(c+d\,x\right)}\right)}{d}+\frac{\sqrt{2}\,B\,a^3\,\ln\left(B\,a^3\,d\,8{}\mathrm{i}+\sqrt{2}\,B\,a^3\,d\,\sqrt{\mathrm{tan}\left(c+d\,x\right)}\,\left(-4+4{}\mathrm{i}\right)\right)\,\left(2-2{}\mathrm{i}\right)}{d}-\frac{\sqrt{-16{}\mathrm{i}}\,B\,a^3\,\ln\left(B\,a^3\,d\,8{}\mathrm{i}+2\,\sqrt{-16{}\mathrm{i}}\,B\,a^3\,d\,\sqrt{\mathrm{tan}\left(c+d\,x\right)}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3)/tan(c + d*x)^(3/2),x)
```

output
```
(2^(1/2)*A*a^3*log(- 8*A*a^3*d - 2^(1/2)*A*a^3*d*tan(c + d*x)^(1/2)*(4 + 4
i))*(2 + 2i))/d - (A*a^3*tan(c + d*x)^(1/2)*2i)/d - (6*B*a^3*tan(c + d*x)^
(1/2))/d - (B*a^3*tan(c + d*x)^(3/2)*2i)/(3*d) - (2*A*a^3)/(d*tan(c + d*x)
^(1/2)) - (16i^(1/2)*A*a^3*log(2*16i^(1/2)*A*a^3*d*tan(c + d*x)^(1/2) - 8*
A*a^3*d))/d + (2^(1/2)*B*a^3*log(B*a^3*d*8i - 2^(1/2)*B*a^3*d*tan(c + d*x)
^(1/2)*(4 - 4i))*(2 - 2i))/d - ((-16i)^(1/2)*B*a^3*log(B*a^3*d*8i + 2*(-16
i)^(1/2)*B*a^3*d*tan(c + d*x)^(1/2)))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{a^3\left(-2\sqrt{\tan\left(dx+c\right)}\,ai-6\sqrt{\tan\left(dx+c\right)}\,b+\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)ad+4\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)adi+4\left(\int\frac{\sqrt{\tan}}{\tan(}\right.\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
(a**3*( - 2*sqrt(tan(c + d*x))*a*i - 6*sqrt(tan(c + d*x))*b + int(sqrt(tan
(c + d*x))/tan(c + d*x)**2,x)*a*d + 4*int(sqrt(tan(c + d*x))/tan(c + d*x),
x)*a*d*i + 4*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b*d - 3*int(sqrt(tan(c
 + d*x)),x)*a*d + 3*int(sqrt(tan(c + d*x)),x)*b*d*i - int(sqrt(tan(c + d*x
))*tan(c + d*x)**2,x)*b*d*i))/d
```

3.131 $\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 136

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{8\sqrt[4]{-1}a^3(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} - \frac{16a^3A\sqrt{\tan(c+dx)}}{3d} - \frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(7iA+3B)\left(a^3+ia^3\tan(c+dx)\right)}{3d\sqrt{\tan(c+dx)}}$$

output
```
8*(-1)^(1/4)*a^3*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d-16/3*a^3*A*
tan(d*x+c)^(1/2)/d-2/3*a*A*(a+I*a*tan(d*x+c))^2/d/tan(d*x+c)^(3/2)-2/3*(7*
I*A+3*B)*(a^3+I*a^3*tan(d*x+c))/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a^3\left(-A+(-9iA-3B)\tan(c+dx)+12\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)-\right.}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/
2),x]
```

output
```
(2*a^3*(-A + ((-9*I)*A - 3*B)*Tan[c + d*x] + 12*(-1)^(1/4)*(A - I*B)*ArcTa
n[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2) - (3*I)*B*Tan[c + d*x]
^2))/(3*d*Tan[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4075, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{3}\int \frac{(i\tan(c+dx)a+a)^2(a(7iA+3B)+a(A+3iB)\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)^2(a(7iA+3B)+a(A+3iB)\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)^2(a(7iA+3B)+a(A+3iB)\tan(c+dx))}{\tan(c+dx)^{3/2}}dx-\frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4076$$

$$\frac{1}{3}\left(2\int -\frac{2(i\tan(c+dx)a+a)\left(a^2(5A-3iB)-2ia^2A\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2(3B+7iA)\left(a^3+ia^3\tan(c+dx)\right)}{d\sqrt{\tan(c+dx)}}\right) \frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(-4\int \frac{(i\tan(c+dx)a+a)\left(a^2(5A-3iB)-2ia^2A\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2(3B+7iA)\left(a^3+ia^3\tan(c+dx)\right)}{d\sqrt{\tan(c+dx)}}\right)- \frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(-4\int \frac{(i\tan(c+dx)a+a)\left(a^2(5A-3iB)-2ia^2A\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2(3B+7iA)\left(a^3+ia^3\tan(c+dx)\right)}{d\sqrt{\tan(c+dx)}}\right)- \frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4075$$

$$\frac{1}{3}\left(-4\left(\frac{4a^3A\sqrt{\tan(c+dx)}}{d}+\int \frac{3(A-iB)a^3+3(iA+B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(3B+7iA)\left(a^3+ia^3\tan(c+\right.}{d\sqrt{\tan(c+dx)}} \frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(-4\left(\frac{4a^3A\sqrt{\tan(c+dx)}}{d}+\int\frac{3(A-iB)a^3+3(iA+B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2(3B+7iA)\left(a^3+ia^3\tan(c+\right.}{d\sqrt{\tan(c+dx)}}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4016

$$\frac{1}{3}\left(-4\left(\frac{4a^3A\sqrt{\tan(c+dx)}}{d}+\frac{18a^6(A-iB)^2\int\frac{1}{3a^3(A-iB)-3a^3(iA+B)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}\right)-\frac{2(3B+7iA)\left(\right.}{d\sqrt{t}}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 218

$$\frac{1}{3}\left(-4\left(\frac{4a^3A\sqrt{\tan(c+dx)}}{d}-\frac{6\sqrt[4]{-1}a^3(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}\right)-\frac{2(3B+7iA)\left(a^3+ia^3\tan\right.}{d\sqrt{\tan(c+dx)}}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x]
```

output
```
(-2*a*A*(a + I*a*Tan[c + d*x])^2)/(3*d*Tan[c + d*x]^(3/2)) + (-4*((-6*(-1)
^(1/4)*a^3*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d + (4*a^3*A*S
qrt[Tan[c + d*x]])/d) - (2*((7*I)*A + 3*B)*(a^3 + I*a^3*Tan[c + d*x]))/(d*
Sqrt[Tan[c + d*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(114) = 228.

Time = 0.08 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{a^3\left(-2iB\sqrt{\tan(dx+c)}+\frac{(4iB-4A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\right.\right.}{4}\right.}{}$
default	$\frac{a^3\left(-2iB\sqrt{\tan(dx+c)}+\frac{(4iB-4A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\right.\right.}{4}\right.}{}$
parts	$\frac{(-iA\,a^3-3B\,a^3)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx\right.\right.}{4d}$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^3*(-2*I*B*tan(d*x+c)^(1/2)+1/4*(4*I*B-4*A)*2^(1/2)*(ln((tan(d*x+c)+2
^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arct
an(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*
(-4*I*A-4*B)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+
c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arc
tan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-2/3*A/tan(d*x+c)^(3/2)-2*(3*I*A+B)/tan(d
*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(110) = 220.

Time = 0.13 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.22

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2\left(3\sqrt{-\frac{(iA^2+2AB-iB^2)a^6}{d^2}}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(-\frac{2\left((A-iB)a^3e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{(iA^2+2AB-iB}{d^2}}\right.}{}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algori
thm="fricas")
```

output
```
-2/3*(3*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*
d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqr
t(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((
-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*
c)/((-I*A - B)*a^3)) - 3*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(4*I*
d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-2*((A - I*B)*a^3*e^(2*I*d
*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(-I*d*e^(2*I*d*x + 2*
I*c) - I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*
e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 2*((5*A - 3*I*B)*a^3*e^(4*I*d*x +
 4*I*c) + (A + 3*I*B)*a^3*e^(2*I*d*x + 2*I*c) - 4*A*a^3)*sqrt((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*
e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -ia^3\left(\int\left(-\frac{3A}{\tan^{\frac{3}{2}}(c+dx)}\right)dx+\int A\sqrt{\tan(c+dx)}\,dx + \int\left(-\frac{3B}{\sqrt{\tan(c+dx)}}\right)dx+\int B\tan^{\frac{3}{2}}(c+dx)\,dx+\int\frac{iA}{\tan^{\frac{5}{2}}(c+dx)}\,dx + \int\left(-\frac{3iA}{\sqrt{\tan(c+dx)}}\right)dx+\int\frac{iB}{\tan^{\frac{3}{2}}(c+dx)}\,dx+\int\left(-3iB\sqrt{\tan(c+dx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
-I*a**3*(Integral(-3*A/tan(c + d*x)**(3/2), x) + Integral(A*sqrt(tan(c + d
*x)), x) + Integral(-3*B/sqrt(tan(c + d*x)), x) + Integral(B*tan(c + d*x)*
*(3/2), x) + Integral(I*A/tan(c + d*x)**(5/2), x) + Integral(-3*I*A/sqrt(t
an(c + d*x)), x) + Integral(I*B/tan(c + d*x)**(3/2), x) + Integral(-3*I*B*
sqrt(tan(c + d*x)), x))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.40

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{6i\,Ba^3\sqrt{\tan(dx+c)}+3\left(2\sqrt{2}((i+1)\,A-(i-1)\,B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algori
thm="maxima")
```

output
```
-1/3*(6*I*B*a^3*sqrt(tan(d*x + c)) + 3*(2*sqrt(2)*((I + 1)*A - (I - 1)*B)*
arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I + 1)*
A - (I - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqr
t(2)*((I - 1)*A + (I + 1)*B)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c)
 + 1) + sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) +
tan(d*x + c) + 1))*a^3 - 2*(3*(-3*I*A - B)*a^3*tan(d*x + c) - A*a^3)/tan(d
*x + c)^(3/2))/d
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(3i\,Ba^3\sqrt{\tan\left(dx+c\right)}-6\sqrt{2}(-(i+1)\;Aa^3+(i-1)\;Ba^3)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\;\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)\right.}{3\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algori
thm="giac")
```

output
```
-2/3*(3*I*B*a^3*sqrt(tan(d*x + c)) - 6*sqrt(2)*(-(I + 1)*A*a^3 + (I - 1)*B
*a^3)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - (-9*I*A*a^3*tan(
d*x + c) - 3*B*a^3*tan(d*x + c) - A*a^3)/tan(d*x + c)^(3/2))/d
```

Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.76

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= -\frac{\frac{2\,A\,a^3}{3\,d}+\frac{A\,a^3\,\tan(c+d\,x)\,6\mathrm{i}}{d}}{\tan\left(c+d\,x\right)^{3/2}}-\frac{2\,B\,a^3}{d\,\sqrt{\tan\left(c+d\,x\right)}}-\frac{B\,a^3\,\sqrt{\tan\left(c+d\,x\right)}\,2\mathrm{i}}{d}$$
$$+\frac{\sqrt{2}\,A\,a^3\,\ln\left(-A\,a^3\,d\,8\mathrm{i}+\sqrt{2}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-4+4\mathrm{i})\right)\,(2-2\mathrm{i})}{d}$$
$$-\frac{\sqrt{-16\mathrm{i}}\,A\,a^3\,\ln\left(-A\,a^3\,d\,8\mathrm{i}+2\,\sqrt{-16\mathrm{i}}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$
$$+\frac{\sqrt{2}\,B\,a^3\,\ln\left(-8\,B\,a^3\,d+\sqrt{2}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-4-4\mathrm{i})\right)\,(2+2\mathrm{i})}{d}$$
$$-\frac{\sqrt{16\mathrm{i}}\,B\,a^3\,\ln\left(-8\,B\,a^3\,d+2\,\sqrt{16\mathrm{i}}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3)/tan(c + d*x)^(5/2),x)
```

output
```
(2^(1/2)*A*a^3*log(- A*a^3*d*8i - 2^(1/2)*A*a^3*d*tan(c + d*x)^(1/2)*(4 -
4i))*(2 - 2i))/d - (2*B*a^3)/(d*tan(c + d*x)^(1/2)) - (B*a^3*tan(c + d*x)^
(1/2)*2i)/d - ((2*A*a^3)/(3*d) + (A*a^3*tan(c + d*x)*6i)/d)/tan(c + d*x)^(
3/2) - ((-16i)^(1/2)*A*a^3*log(2*(-16i)^(1/2)*A*a^3*d*tan(c + d*x)^(1/2) -
 A*a^3*d*8i))/d + (2^(1/2)*B*a^3*log(- 8*B*a^3*d - 2^(1/2)*B*a^3*d*tan(c +
 d*x)^(1/2)*(4 + 4i))*(2 + 2i))/d - (16i^(1/2)*B*a^3*log(2*16i^(1/2)*B*a^3
*d*tan(c + d*x)^(1/2) - 8*B*a^3*d))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{a^3\left(-2\sqrt{\tan(dx+c)}\,bi+\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)ad+3\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)adi+\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)bd-3\left(\int \frac{\sqrt{}}{}\right.}{d}$$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
(a**3*( - 2*sqrt(tan(c + d*x))*b*i + int(sqrt(tan(c + d*x))/tan(c + d*x)**
3,x)*a*d + 3*int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a*d*i + int(sqrt(ta
n(c + d*x))/tan(c + d*x)**2,x)*b*d - 3*int(sqrt(tan(c + d*x))/tan(c + d*x)
,x)*a*d + 4*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b*d*i - int(sqrt(tan(c
+ d*x)),x)*a*d*i - 3*int(sqrt(tan(c + d*x)),x)*b*d))/d
```

3.132 $\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 144

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{8\sqrt[4]{-1}a^3(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} + \frac{16a^3(6A-5iB)}{15d\sqrt{\tan(c+dx)}} - \frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2(9iA+5B)\left(a^3+ia^3\tan(c+dx)\right)}{15d\tan^{\frac{3}{2}}(c+dx)}$$

output

```
8*(-1)^(1/4)*a^3*(I*A+B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d+16/15*a^3*(
6*A-5*I*B)/d/tan(d*x+c)^(1/2)-2/5*a*A*(a+I*a*tan(d*x+c))^2/d/tan(d*x+c)^(5
/2)-2/15*(9*I*A+5*B)*(a^3+I*a^3*tan(d*x+c))/d/tan(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^3\left(-3A+(-15iA-5B)\tan(c+dx)+15(4A-3iB)\tan^2(c+dx)+60\sqrt[4]{-1}(iA+B)\arctan\left((-1)^3\right.\right.}{15d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/
2),x]
```

output
```
(2*a^3*(-3*A + ((-15*I)*A - 5*B)*Tan[c + d*x] + 15*(4*A - (3*I)*B)*Tan[c +
 d*x]^2 + 60*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Ta
n[c + d*x]^(5/2)))/(15*d*Tan[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4074, 27, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{5}\int \frac{(i\tan(c+dx)a+a)^2(a(9iA+5B)-a(A-5iB)\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)^2(a(9iA+5B)-a(A-5iB)\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)^2(a(9iA+5B)-a(A-5iB)\tan(c+dx))}{\tan(c+dx)^{5/2}}dx-\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{5}\left(\frac{2}{3}\int -\frac{2(i\tan(c+dx)a+a)\left(2(6A-5iB)a^2+(3iA+5B)\tan(c+dx)a^2\right)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2(5B+9iA)\left(a^3+ia^3\tan(c\right.}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(-\frac{4}{3}\int \frac{(i\tan(c+dx)a+a)\left(2(6A-5iB)a^2+(3iA+5B)\tan(c+dx)a^2\right)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2(5B+9iA)\left(a^3+ia^3\tan(c\right.}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-\frac{4}{3}\int \frac{(i\tan(c+dx)a+a)\left(2(6A-5iB)a^2+(3iA+5B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{3/2}}dx-\frac{2(5B+9iA)\left(a^3+ia^3\tan(c\right.}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4074}$$

$$\frac{1}{5}\left(-\frac{4}{3}\left(\int \frac{15\left(a^3(iA+B)-a^3(A-iB)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx-\frac{4a^3(6A-5iB)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2(5B+9iA)\left(a^3+ia^3\tan(c+\right.}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}\left(-\frac{4}{3}\left(15\int \frac{a^3(iA+B)-a^3(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{4a^3(6A-5iB)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2(5B+9iA)\left(a^3+ia^3\tan(c+dx\right.}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\left(-\frac{4}{3}\left(15\int \frac{a^3(iA+B)-a^3(A-iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{4a^3(6A-5iB)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2(5B+9iA)\left(a^3+ia^3\tan(c+dx\right.}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 4016$$

$$\frac{1}{5}\left(-\frac{4}{3}\left(\frac{30a^6(B+iA)^2\int \frac{1}{(iA+B)a^3+(A-iB)\tan(c+dx)a^3}d\sqrt{\tan(c+dx)}}{d}-\frac{4a^3(6A-5iB)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2(5B+9iA)\left(a^3+\right.}{3d\tan^{\frac{3}{2}}(c\right.}$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 218$$

$$\frac{1}{5}\left(-\frac{4}{3}\left(-\frac{30\sqrt[4]{-1}a^3(B+iA)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{4a^3(6A-5iB)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2(5B+9iA)\left(a^3+ia^3\tan\right.}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x]
```

output
```
(-2*a*A*(a + I*a*Tan[c + d*x])^2)/(5*d*Tan[c + d*x]^(5/2)) + ((-4*((-30*(-
1)^(1/4)*a^3*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d - (4*a^3*(
6*A - (5*I)*B))/(d*Sqrt[Tan[c + d*x]])))/3 - (2*((9*I)*A + 5*B)*(a^3 + I*a
^3*Tan[c + d*x]))/(3*d*Tan[c + d*x]^(3/2)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{a^3\left(\frac{(-4iA-4B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-4}{}\right.}{}$
default	$\frac{a^3\left(\frac{(-4iA-4B)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-4}{}\right.}{}$
parts	$\frac{(-iA\,a^3-3B\,a^3)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx\right.\right.}{4d}$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^3*(1/4*(-4*I*A-4*B)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)
+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)
^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-4*I*B+4*A)*2^(1/2)*(l
n((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(
1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+
c)^(1/2)))-2/5*A/tan(d*x+c)^(5/2)-2/3*(3*I*A+B)/tan(d*x+c)^(3/2)-2*(3*I*B-
4*A)/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(116) = 232.

Time = 0.09 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.50

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(15\sqrt{-\frac{(-iA^2-2AB+iB^2)a^6}{d^2}}\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\log\left(-\frac{2\left((A-iB)a^3e^{(2i\,dx+}\right.}{}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algori
thm="fricas")
```

output
```
-2/15*(15*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(6*I*d*x + 6*I*c) -
 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(-2*((A - I*B)*
a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(2*
I*d*x + 2*I*c) + d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 15*sqrt(-(-I*A^2 - 2*A*B
+ I*B^2)*a^6/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e
^(2*I*d*x + 2*I*c) - d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt(-
(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-
I*A - B)*a^3)) + 2*((-39*I*A - 25*B)*a^3*e^(6*I*d*x + 6*I*c) + 2*(9*I*A +
10*B)*a^3*e^(4*I*d*x + 4*I*c) + (33*I*A + 25*B)*a^3*e^(2*I*d*x + 2*I*c) +
4*(-6*I*A - 5*B)*a^3)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*
x + 2*I*c) - d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -ia^3\left(\int\left(-\frac{3A}{\tan^{\frac{5}{2}}(c+dx)}\right)dx+\int\frac{A}{\sqrt{\tan(c+dx)}}\,dx+\int\left(-\frac{3B}{\tan^{\frac{3}{2}}(c+dx)}\right)dx+\int B\sqrt{\tan(c+dx)}\,dx+\int\frac{iA}{\tan^{\frac{7}{2}}(c+dx)}\,dx+\int\left(-\frac{3iA}{\tan^{\frac{3}{2}}(c+dx)}\right)dx+\int\frac{iB}{\tan^{\frac{5}{2}}(c+dx)}\,dx+\int\left(-\frac{3iB}{\sqrt{\tan(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
-I*a**3*(Integral(-3*A/tan(c + d*x)**(5/2), x) + Integral(A/sqrt(tan(c + d
*x)), x) + Integral(-3*B/tan(c + d*x)**(3/2), x) + Integral(B*sqrt(tan(c +
 d*x)), x) + Integral(I*A/tan(c + d*x)**(7/2), x) + Integral(-3*I*A/tan(c
+ d*x)**(3/2), x) + Integral(I*B/tan(c + d*x)**(5/2), x) + Integral(-3*I*B
/sqrt(tan(c + d*x)), x))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.37

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{15\left(2\sqrt{2}((i-1)\ A+(i+1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}((i-1)\ A+(i+1)\ \cdots}{\cdots}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algori
thm="maxima")
```

output
```
-1/15*(15*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sq
rt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)
*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*(-(I + 1)*A
+ (I - 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^3 - 2*
(15*(4*A - 3*I*B)*a^3*tan(d*x + c)^2 + 5*(-3*I*A - B)*a^3*tan(d*x + c) - 3
*A*a^3)/tan(d*x + c)^(5/2))/d
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.74

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2\left(30\sqrt{2}((i-1)\ Aa^3+(i+1)\ Ba^3)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)-\frac{60\,Aa^3\tan(dx+c)^2-45i\,Ba^3\cdots}{\cdots}\right.}{15\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algori
thm="giac")
```

output
```
-2/15*(30*sqrt(2)*((I - 1)*A*a^3 + (I + 1)*B*a^3)*arctan(-(1/2*I - 1/2)*sq
rt(2)*sqrt(tan(d*x + c))) - (60*A*a^3*tan(d*x + c)^2 - 45*I*B*a^3*tan(d*x
+ c)^2 - 15*I*A*a^3*tan(d*x + c) - 5*B*a^3*tan(d*x + c) - 3*A*a^3)/tan(d*x
 + c)^(5/2))/d
```

Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.79

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$= -\frac{\frac{2\,A\,a^3}{5\,d}-\frac{8\,A\,a^3\tan(c+d\,x)^2}{d}+\frac{A\,a^3\tan(c+d\,x)\,2\mathrm{i}}{d}}{\tan\left(c+d\,x\right)^{5/2}}-\frac{\frac{2\,B\,a^3}{3\,d}+\frac{B\,a^3\tan(c+d\,x)\,6\mathrm{i}}{d}}{\tan\left(c+d\,x\right)^{3/2}}$$
$$+\frac{\sqrt{2}\,A\,a^3\,\ln\left(8\,A\,a^3\,d+\sqrt{2}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-4-4\mathrm{i})\right)\,(2+2\mathrm{i})}{d}$$
$$-\frac{\sqrt{16\mathrm{i}}\,A\,a^3\,\ln\left(8\,A\,a^3\,d+2\,\sqrt{16\mathrm{i}}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$
$$+\frac{\sqrt{2}\,B\,a^3\,\ln\left(-B\,a^3\,d\,8\mathrm{i}+\sqrt{2}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,(-4+4\mathrm{i})\right)\,(2-2\mathrm{i})}{d}$$
$$-\frac{\sqrt{-16\mathrm{i}}\,B\,a^3\,\ln\left(-B\,a^3\,d\,8\mathrm{i}+2\,\sqrt{-16\mathrm{i}}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3)/tan(c + d*x)^(7/2),x)
```

output
```
(2^(1/2)*A*a^3*log(8*A*a^3*d - 2^(1/2)*A*a^3*d*tan(c + d*x)^(1/2)*(4 + 4i)
)*(2 + 2i))/d - ((2*B*a^3)/(3*d) + (B*a^3*tan(c + d*x)*6i)/d)/tan(c + d*x)
^(3/2) - ((2*A*a^3)/(5*d) + (A*a^3*tan(c + d*x)*2i)/d - (8*A*a^3*tan(c + d
*x)^2)/d)/tan(c + d*x)^(5/2) - (16i^(1/2)*A*a^3*log(8*A*a^3*d + 2*16i^(1/2
)*A*a^3*d*tan(c + d*x)^(1/2)))/d + (2^(1/2)*B*a^3*log(- B*a^3*d*8i - 2^(1/
2)*B*a^3*d*tan(c + d*x)^(1/2)*(4 - 4i))*(2 - 2i))/d - ((-16i)^(1/2)*B*a^3*
log(2*(-16i)^(1/2)*B*a^3*d*tan(c + d*x)^(1/2) - B*a^3*d*8i))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$= a^3\left(\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)a+3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)ai + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)b-3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)a + 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)ai - 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)b-\left(\int \sqrt{\tan(dx+c)}dx\right)bi\right)$$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
a**3*(int(sqrt(tan(c + d*x))/tan(c + d*x)**4,x)*a + 3*int(sqrt(tan(c + d*x
))/tan(c + d*x)**3,x)*a*i + int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*b -
3*int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a + 3*int(sqrt(tan(c + d*x))/t
an(c + d*x)**2,x)*b*i - int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*i - 3*int
(sqrt(tan(c + d*x))/tan(c + d*x),x)*b - int(sqrt(tan(c + d*x)),x)*b*i)
```

3.133 $\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 169

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -\frac{8\sqrt[4]{-1}a^3(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d} + \frac{8a^3(23A-21iB)}{105d\tan^{\frac{3}{2}}(c+dx)} + \frac{8a^3(iA+B)}{d\sqrt{\tan(c+dx)}} - \frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{2(11iA+7B)\left(a^3+ia^3\tan(c+dx)\right)}{35d\tan^{\frac{5}{2}}(c+dx)}$$

output
```
-8*(-1)^(1/4)*a^3*(A-I*B)*arctan((-1)^(3/4)*tan(d*x+c)^(1/2))/d+8/105*a^3*
(23*A-21*I*B)/d/tan(d*x+c)^(3/2)+8*a^3*(I*A+B)/d/tan(d*x+c)^(1/2)-2/7*a*A*
(a+I*a*tan(d*x+c))^2/d/tan(d*x+c)^(7/2)-2/35*(11*I*A+7*B)*(a^3+I*a^3*tan(d
*x+c))/d/tan(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.70

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{2a^3\left(15A+21(3iA+B)\tan(c+dx)-35(4A-3iB)\tan^2(c+dx)-420i(A-iB)\tan^3(c+dx)+42\right.}{105d\tan^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/
2),x]
```

output
```
(-2*a^3*(15*A + 21*((3*I)*A + B)*Tan[c + d*x] - 35*(4*A - (3*I)*B)*Tan[c +
 d*x]^2 - (420*I)*(A - I*B)*Tan[c + d*x]^3 + 420*(-1)^(1/4)*(A - I*B)*ArcT
an[(-1)^(3/4)*Sqrt[Tan[c + d*x]]*Tan[c + d*x]^(7/2)))/(105*d*Tan[c + d*x]
^(7/2))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4074, 27, 3042, 4012, 25, 3042, 4016, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \ 4076$$

$$\frac{2}{7}\int \frac{(i\tan(c+dx)a+a)^2(a(11iA+7B)-a(3A-7iB)\tan(c+dx))}{2\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{(i\tan(c+dx)a+a)^2(a(11iA+7B)-a(3A-7iB)\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{(i\tan(c+dx)a+a)^2(a(11iA+7B)-a(3A-7iB)\tan(c+dx))}{\tan(c+dx)^{7/2}}dx-\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{7}\left(\frac{2}{5}\int -\frac{2(i\tan(c+dx)a+a)\left((23A-21iB)a^2+2(6iA+7B)\tan(c+dx)a^2\right)}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2(7B+11iA)\left(a^3+ia^3\tan\right.}{5d\tan^{\frac{5}{2}}(c+dx)}\right.$$
$$-\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(-\frac{4}{5}\int \frac{(i\tan(c+dx)a+a)\left((23A-21iB)a^2+2(6iA+7B)\tan(c+dx)a^2\right)}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2(7B+11iA)\left(a^3+ia^3\tan\right.}{5d\tan^{\frac{5}{2}}(c+dx)}\right.$$
$$-\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(-\frac{4}{5}\int \frac{(i\tan(c+dx)a+a)\left((23A-21iB)a^2+2(6iA+7B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{5/2}}dx-\frac{2(7B+11iA)\left(a^3+ia^3\tan\right.}{5d\tan^{\frac{5}{2}}(c+dx)}\right.$$
$$-\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4074}$$

$$\frac{1}{7}\left(-\frac{4}{5}\left(\int \frac{35\left(a^3(iA+B)-a^3(A-iB)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a^3(23A-21iB)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7B+11iA)\left(a^3+ia^3\tan(c\right.}{5d\tan^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(-\frac{4}{5}\left(35\int \frac{a^3(iA+B)-a^3(A-iB)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a^3(23A-21iB)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7B+11iA)\left(a^3+ia^3\tan(c+\right.}{5d\tan^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(-\frac{4}{5}\left(35\int \frac{a^3(iA+B)-a^3(A-iB)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx-\frac{2a^3(23A-21iB)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7B+11iA)\left(a^3+ia^3\tan(c+\right.}{5d\tan^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4012

$$\frac{1}{7}\left(-\frac{4}{5}\left(35\left(\int -\frac{(A-iB)a^3+(iA+B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(B+iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^3(23A-21iB)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7B+}{}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 25

$$\frac{1}{7}\left(-\frac{4}{5}\left(35\left(-\int \frac{(A-iB)a^3+(iA+B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(B+iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^3(23A-21iB)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7B+}{}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(-\frac{4}{5}\left(35\left(-\int \frac{(A-iB)a^3+(iA+B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(B+iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^3(23A-21iB)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7B+}{}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4016}$$

$$\frac{1}{7}\left(-\frac{4}{5}\left(35\left(-\frac{2a^6(A-iB)^2\int\frac{1}{a^3(A-iB)-a^3(iA+B)\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^3(B+iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^3(23A-21i\ldots}{3d\tan^{\frac{3}{2}}(c+d\ldots}\right.\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{7}\left(-\frac{4}{5}\left(35\left(\frac{2\sqrt[4]{-1}a^3(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a^3(B+iA)}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^3(23A-21iB)}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2(\ldots}{\ldots}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^2}{7d\tan^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/2),x]
```

output
```
(-2*a*A*(a + I*a*Tan[c + d*x])^2)/(7*d*Tan[c + d*x]^(7/2)) + ((-4*(35*((2*
(-1)^(1/4)*a^3*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/d - (2*a^3
*(I*A + B))/(d*Sqrt[Tan[c + d*x]])) - (2*a^3*(23*A - (21*I)*B))/(3*d*Tan[c
 + d*x]^(3/2))))/5 - (2*((11*I)*A + 7*B)*(a^3 + I*a^3*Tan[c + d*x]))/(5*d*
Tan[c + d*x]^(5/2)))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.53

method	result
derivativedivides	$a^3 \left(\frac{(-4iB+4A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4} + \frac{(4iA$
default	$a^3 \left(\frac{(-4iB+4A)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4} + \frac{(4iA$
parts	$\frac{(-iA\,a^3-3B\,a^3)\left(-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*a^3*(1/4*(-4*I*B+4*A)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)
+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)
^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(4*I*A+4*B)*2^(1/2)*(ln
((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1
/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c
)^(1/2)))-2/7*A/tan(d*x+c)^(7/2)-2*(-4*I*A-4*B)/tan(d*x+c)^(1/2)-2/5*(3*I*
A+B)/tan(d*x+c)^(5/2)-2/3*(3*I*B-4*A)/tan(d*x+c)^(3/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(137) = 274.

Time = 0.13 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.32

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algori
thm="fricas")
```

output
```
2/105*(105*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(8*I*d*x + 8*I*c) -
 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*
c) + d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B -
 I*B^2)*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)
) - 105*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(8*I*d*x + 8*I*c) - 4*
d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c)
+ d)*log(-2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*
B^2)*a^6/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((-I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3))
- 2*((319*A - 273*I*B)*a^3*e^(8*I*d*x + 8*I*c) - 3*(109*A - 133*I*B)*a^3*e
^(6*I*d*x + 6*I*c) - 5*(19*A - 21*I*B)*a^3*e^(4*I*d*x + 4*I*c) + 3*(129*A
- 133*I*B)*a^3*e^(2*I*d*x + 2*I*c) - 4*(41*A - 42*I*B)*a^3)*sqrt((-I*e^(2*
I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(8*I*d*x + 8*I*c) - 4
*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -ia^3\left(\int\left(-\frac{3A}{\tan^{\frac{7}{2}}(c+dx)}\right)dx+\int\frac{A}{\tan^{\frac{3}{2}}(c+dx)}\,dx+\int\left(-\frac{3B}{\tan^{\frac{5}{2}}(c+dx)}\right)dx + \int\frac{B}{\sqrt{\tan(c+dx)}}\,dx+\int\frac{iA}{\tan^{\frac{9}{2}}(c+dx)}\,dx+\int\left(-\frac{3iA}{\tan^{\frac{5}{2}}(c+dx)}\right)dx + \int\frac{iB}{\tan^{\frac{7}{2}}(c+dx)}\,dx+\int\left(-\frac{3iB}{\tan^{\frac{3}{2}}(c+dx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)
```

output
```
-I*a**3*(Integral(-3*A/tan(c + d*x)**(7/2), x) + Integral(A/tan(c + d*x)**
(3/2), x) + Integral(-3*B/tan(c + d*x)**(5/2), x) + Integral(B/sqrt(tan(c
+ d*x)), x) + Integral(I*A/tan(c + d*x)**(9/2), x) + Integral(-3*I*A/tan(c
 + d*x)**(5/2), x) + Integral(I*B/tan(c + d*x)**(7/2), x) + Integral(-3*I*
B/tan(c + d*x)**(3/2), x))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.27

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{105\left(2\sqrt{2}((i+1)\ A-(i-1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((i+1)\ A-(i-1)\ B\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algori
thm="maxima")
```

output
```
1/105*(105*(2*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2)
+ 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-1/2*s
qrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((I - 1)*A + (I + 1)*B)
*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((I - 1)*A +
 (I + 1)*B)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*a^3 + 2*(
420*(I*A + B)*a^3*tan(d*x + c)^3 + 35*(4*A - 3*I*B)*a^3*tan(d*x + c)^2 + 2
1*(-3*I*A - B)*a^3*tan(d*x + c) - 15*A*a^3)/tan(d*x + c)^(7/2))/d
```

Giac [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx =$$

$$-\frac{2\left(210\sqrt{2}(-(i+1)\ Aa^3+(i-1)\ Ba^3)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan(dx+c)}\right)+\frac{-420i\,Aa^3\tan(dx+c)^3-}{}\right.}{105\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algori
thm="giac")
```

output
```
-2/105*(210*sqrt(2)*(-(I + 1)*A*a^3 + (I - 1)*B*a^3)*arctan(-(1/2*I - 1/2)
*sqrt(2)*sqrt(tan(d*x + c))) + (-420*I*A*a^3*tan(d*x + c)^3 - 420*B*a^3*ta
n(d*x + c)^3 - 140*A*a^3*tan(d*x + c)^2 + 105*I*B*a^3*tan(d*x + c)^2 + 63*
I*A*a^3*tan(d*x + c) + 21*B*a^3*tan(d*x + c) + 15*A*a^3)/tan(d*x + c)^(7/2
))/d
```

Mupad [B] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.73

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$
$$= -\frac{\frac{2\,A\,a^3}{7\,d}+\frac{A\,a^3\,\tan(c+d\,x)\,6{}\mathrm{i}}{5\,d}-\frac{8\,A\,a^3\,\tan(c+d\,x)^2}{3\,d}-\frac{A\,a^3\,\tan(c+d\,x)^3\,8{}\mathrm{i}}{d}}{\tan\left(c+d\,x\right)^{7/2}}$$
$$-\frac{\frac{2\,B\,a^3}{5\,d}-\frac{8\,B\,a^3\,\tan(c+d\,x)^2}{d}+\frac{B\,a^3\,\tan(c+d\,x)\,2{}\mathrm{i}}{d}}{\tan\left(c+d\,x\right)^{5/2}}$$
$$+\frac{\sqrt{2}\,A\,a^3\,\ln\left(A\,a^3\,d\,8{}\mathrm{i}+\sqrt{2}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-4+4{}\mathrm{i}\right)\right)\,\left(2-2{}\mathrm{i}\right)}{d}$$
$$-\frac{\sqrt{-16{}\mathrm{i}}\,A\,a^3\,\ln\left(A\,a^3\,d\,8{}\mathrm{i}+2\,\sqrt{-16{}\mathrm{i}}\,A\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$
$$+\frac{\sqrt{2}\,B\,a^3\,\ln\left(8\,B\,a^3\,d+\sqrt{2}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\left(-4-4{}\mathrm{i}\right)\right)\,\left(2+2{}\mathrm{i}\right)}{d}$$
$$-\frac{\sqrt{16{}\mathrm{i}}\,B\,a^3\,\ln\left(8\,B\,a^3\,d+2\,\sqrt{16{}\mathrm{i}}\,B\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3)/tan(c + d*x)^(9/2),x)
```

output
```
(2^(1/2)*A*a^3*log(A*a^3*d*8i - 2^(1/2)*A*a^3*d*tan(c + d*x)^(1/2)*(4 - 4i
))*(2 - 2i))/d - ((2*B*a^3)/(5*d) + (B*a^3*tan(c + d*x)*2i)/d - (8*B*a^3*t
an(c + d*x)^2)/d)/tan(c + d*x)^(5/2) - ((2*A*a^3)/(7*d) + (A*a^3*tan(c + d
*x)*6i)/(5*d) - (8*A*a^3*tan(c + d*x)^2)/(3*d) - (A*a^3*tan(c + d*x)^3*8i)
/d)/tan(c + d*x)^(7/2) - ((-16i)^(1/2)*A*a^3*log(A*a^3*d*8i + 2*(-16i)^(1/
2)*A*a^3*d*tan(c + d*x)^(1/2)))/d + (2^(1/2)*B*a^3*log(8*B*a^3*d - 2^(1/2)
*B*a^3*d*tan(c + d*x)^(1/2)*(4 + 4i))*(2 + 2i))/d - (16i^(1/2)*B*a^3*log(8
*B*a^3*d + 2*16i^(1/2)*B*a^3*d*tan(c + d*x)^(1/2)))/d
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$
$$= a^3\left(\left(\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^5}dx\right)a+3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)ai + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)b-3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)a + 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)ai - 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)b-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)bi\right)$$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)
```

output
```
a**3*(int(sqrt(tan(c + d*x))/tan(c + d*x)**5,x)*a + 3*int(sqrt(tan(c + d*x
))/tan(c + d*x)**4,x)*a*i + int(sqrt(tan(c + d*x))/tan(c + d*x)**4,x)*b -
3*int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*a + 3*int(sqrt(tan(c + d*x))/t
an(c + d*x)**3,x)*b*i - int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a*i - 3*
int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*b - int(sqrt(tan(c + d*x))/tan(c
 + d*x),x)*b*i)
```

3.134 $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 252

$$\begin{aligned}\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx &= -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)((4+i)A+(1+6i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad} \\ &\quad+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)((4+i)A+(1+6i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad} \\ &\quad-\frac{((3-5i)A+(5+7i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{4\sqrt{2}ad}-\frac{5(iA-B)\sqrt{\tan(c+dx)}}{2ad} \\ &\quad-\frac{(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{6ad}+\frac{(iA-B)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}\end{aligned}$$

output

```
(1/8+1/8*I)*((4+I)*A+(1+6*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2
)/a/d+(1/8+1/8*I)*((4+I)*A+(1+6*I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2
^(1/2)/a/d-1/8*((3-5*I)*A+(5+7*I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+t
an(d*x+c)))*2^(1/2)/a/d-5/2*(I*A-B)*tan(d*x+c)^(1/2)/a/d-1/6*(3*A+7*I*B)*t
an(d*x+c)^(3/2)/a/d+1/2*(I*A-B)*tan(d*x+c)^(5/2)/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.55

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{-3(-1)^{3/4}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-6(-1)^{3/4}(2A+3iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\tan(c+a\right.}}{6ad}$$

input
```
Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
,x]
```

output
```
(-3*(-1)^(3/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - 6*(-1)^(3
/4)*(2*A + (3*I)*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (Sqrt[Tan[c +
 d*x]]*(-15*(A + I*B) + 4*((-3*I)*A + 2*B)*Tan[c + d*x] - (4*I)*B*Tan[c +
d*x]^2))/(-I + Tan[c + d*x]))/(6*a*d)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.07, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4078, 27, 3042, 4011, 3042, 4011, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(c+dx)^{5/2}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \; 4078$$
$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \frac{1}{2}\tan^{\frac{3}{2}}(c+dx)(5a(iA-B)+a(3A+7iB)\tan(c+dx))dx}{2a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \tan^{\frac{3}{2}}(c+dx)(5a(iA-B)+a(3A+7iB)\tan(c+dx))dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \tan(c+dx)^{3/2}(5a(iA-B)+a(3A+7iB)\tan(c+dx))dx}{4a^2}$$

$$\Big\downarrow \text{4011}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \sqrt{\tan(c+dx)}(5a(iA-B)\tan(c+dx)-a(3A+7iB))dx+\frac{2a(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \sqrt{\tan(c+dx)}(5a(iA-B)\tan(c+dx)-a(3A+7iB))dx+\frac{2a(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}}{4a^2}$$

$$\Big\downarrow \text{4011}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \frac{-5a(iA-B)-a(3A+7iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2a(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{10a(-B+iA)\sqrt{\tan(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \frac{-5a(iA-B)-a(3A+7iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2a(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{10a(-B+iA)\sqrt{\tan(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\frac{2\int -\frac{a(5(iA-B)+(3A+7iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2a(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{10a(-B+iA)\sqrt{\tan(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-\frac{2\int\frac{a(5(iA-B)+(3A+7iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2a(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{10a(-B+iA)\sqrt{\tan(c+dx)}}{d}}{4a^2}$$

$\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-\frac{2a\int\frac{5(iA-B)+(3A+7iB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2a(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{10a(-B+iA)\sqrt{\tan(c+dx)}}{d}}{4a^2}$$

$\downarrow$ 1482

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+4i)A-(6+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B)\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{2a(3A+7iB)\text{ta}}{3d}}{4a^2}$$

$\downarrow$ 1476

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+4i)A-(6+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\right.}{d}}{4a^2}$$

$\downarrow$ 1082

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+4i)A-(6+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+}}{}\right.}{d}}{4a^2}$$

$\downarrow$ 217

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+4i)A-(6+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}}\right.}{d}}{4a^2}$$

$\downarrow$ 1479

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+4i)A-(6+i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((4+\cdots}{d}}{4a^2}$$

$\downarrow$ 25

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+4i)A-(6+i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+\cdots}{d}}{4a^2}$$

$\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+4i)A-(6+i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)\cdots}{d}}{4a^2}$$

$\downarrow$ 1103

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((1+4i)A-(6+i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c\cdots}{2\sqrt{2}}\right.}{d}}{4a^2}$$

input
```
Int[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
((I*A - B)*Tan[c + d*x]^(5/2))/(2*d*(a + I*a*Tan[c + d*x])) - ((-2*a*((1/2
 + I/2)*((4 + I)*A + (1 + 6*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + (1/2 + I/2)
*((1 + 4*I)*A - (6 + I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*
Sqrt[2]))))/d + (10*a*(I*A - B)*Sqrt[Tan[c + d*x]])/d + (2*a*(3*A + (7*I)*
B)*Tan[c + d*x]^(3/2))/(3*d))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-\frac{2iB\tan(dx+c)^{\frac{3}{2}}}{3}+2B\sqrt{\tan(dx+c)}-2iA\sqrt{\tan(dx+c)}-\frac{i\left(-\frac{i(iB+A)\sqrt{\tan(dx+c)}}{-i+\tan(dx+c)}+\frac{4(2iA-3B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2}+}{da}$
default	$\frac{-\frac{2iB\tan(dx+c)^{\frac{3}{2}}}{3}+2B\sqrt{\tan(dx+c)}-2iA\sqrt{\tan(dx+c)}-\frac{i\left(-\frac{i(iB+A)\sqrt{\tan(dx+c)}}{-i+\tan(dx+c)}+\frac{4(2iA-3B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2}+}{da}$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d/a*(-2/3*I*B*tan(d*x+c)^(3/2)+2*B*tan(d*x+c)^(1/2)-2*I*A*tan(d*x+c)^(1/
2)-1/2*I*(-I*(A+I*B)*tan(d*x+c)^(1/2)/(-I+tan(d*x+c))+4*(2*I*A-3*B)/(2^(1/
2)-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))+4*(-1/4*A+1/
4*I*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(183) = 366.

Time = 0.11 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.81

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/24*(3*(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((I*A^2 +
2*A*B - I*B^2)/(a^2*d^2))*log(2*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B -
I*B^2)/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I
*A + B)) - 3*(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((I*A
^2 + 2*A*B - I*B^2)/(a^2*d^2))*log(-2*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqr
t((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*
A*B - I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I
*c)/(I*A + B)) - 6*(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqr
t((-4*I*A^2 + 12*A*B + 9*I*B^2)/(a^2*d^2))*log(-((a*d*e^(2*I*d*x + 2*I*c)
+ a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((
-4*I*A^2 + 12*A*B + 9*I*B^2)/(a^2*d^2)) + 2*A + 3*I*B)*e^(-2*I*d*x - 2*I*c
)/(a*d)) + 6*(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((-4*
I*A^2 + 12*A*B + 9*I*B^2)/(a^2*d^2))*log(((a*d*e^(2*I*d*x + 2*I*c) + a*d)*
sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-4*I*A^
2 + 12*A*B + 9*I*B^2)/(a^2*d^2)) - 2*A - 3*I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)
) + 2*((-27*I*A + 19*B)*e^(4*I*d*x + 4*I*c) - 2*(15*I*A - 19*B)*e^(2*I*d*x
 + 2*I*c) - 3*I*A + 3*B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1)))/(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{i\left(\int \frac{A\tan^{\frac{5}{2}}(c+dx)}{\tan(c+dx)-i}\,dx + \int \frac{B\tan^{\frac{7}{2}}(c+dx)}{\tan(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate(tan(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*(Integral(A*tan(c + d*x)**(5/2)/(tan(c + d*x) - I), x) + Integral(B*tan
(c + d*x)**(7/2)/(tan(c + d*x) - I), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.51

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{6\sqrt{2}((2i+2)\,A+(3i-3)\,B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}((i-1)\,A+(i+1)\,B)\operatorname{arct}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
1/12*(6*sqrt(2)*((2*I + 2)*A + (3*I - 3)*B)*arctan((1/2*I + 1/2)*sqrt(2)*s
qrt(tan(d*x + c))) + 3*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-(1/2*I - 1/
2)*sqrt(2)*sqrt(tan(d*x + c))) - 8*I*B*tan(d*x + c)^(3/2) - 24*I*A*sqrt(ta
n(d*x + c)) + 24*B*sqrt(tan(d*x + c)) - 6*(A*sqrt(tan(d*x + c)) + I*B*sqrt
(tan(d*x + c)))/(tan(d*x + c) - I))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.21

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \operatorname{atan}\left(\frac{a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,1\mathrm{i}}{a^2\,d^2}}\,1\mathrm{i}}{A}\right)\,\sqrt{-\frac{A^2\,1\mathrm{i}}{a^2\,d^2}}\,2\mathrm{i}$$
$$-\operatorname{atan}\left(\frac{a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,1\mathrm{i}}{16\,a^2\,d^2}}\,4\mathrm{i}}{A}\right)\,\sqrt{\frac{A^2\,1\mathrm{i}}{16\,a^2\,d^2}}\,2\mathrm{i}$$
$$+\operatorname{atan}\left(\frac{2\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,9\mathrm{i}}{4\,a^2\,d^2}}}{3\,B}\right)\,\sqrt{\frac{B^2\,9\mathrm{i}}{4\,a^2\,d^2}}\,2\mathrm{i}$$
$$+\operatorname{atan}\left(\frac{4\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1\mathrm{i}}{16\,a^2\,d^2}}}{B}\right)\,\sqrt{-\frac{B^2\,1\mathrm{i}}{16\,a^2\,d^2}}\,2\mathrm{i}$$
$$-\frac{A\,\sqrt{\tan\left(c+d\,x\right)}\,2\mathrm{i}}{a\,d}+\frac{2\,B\,\sqrt{\tan\left(c+d\,x\right)}}{a\,d}-\frac{B\,{\tan\left(c+d\,x\right)}^{3/2}\,2\mathrm{i}}{3\,a\,d}$$
$$-\frac{A\,\sqrt{\tan\left(c+d\,x\right)}\,1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}+\frac{B\,\sqrt{\tan\left(c+d\,x\right)}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
atan((a*d*tan(c + d*x)^(1/2)*(-(A^2*1i)/(a^2*d^2))^(1/2)*1i)/A)*(-(A^2*1i)
/(a^2*d^2))^(1/2)*2i - atan((a*d*tan(c + d*x)^(1/2)*((A^2*1i)/(16*a^2*d^2)
)^(1/2)*4i)/A)*((A^2*1i)/(16*a^2*d^2))^(1/2)*2i + atan((2*a*d*tan(c + d*x)
^(1/2)*((B^2*9i)/(4*a^2*d^2))^(1/2))/(3*B))*((B^2*9i)/(4*a^2*d^2))^(1/2)*2
i + atan((4*a*d*tan(c + d*x)^(1/2)*(-(B^2*1i)/(16*a^2*d^2))^(1/2))/B)*(-(B
^2*1i)/(16*a^2*d^2))^(1/2)*2i - (A*tan(c + d*x)^(1/2)*2i)/(a*d) + (2*B*tan
(c + d*x)^(1/2))/(a*d) - (B*tan(c + d*x)^(3/2)*2i)/(3*a*d) - (A*tan(c + d*
x)^(1/2)*1i)/(2*a*d*(tan(c + d*x)*1i + 1)) + (B*tan(c + d*x)^(1/2))/(2*a*d
*(tan(c + d*x)*1i + 1))
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^3}{\tan(dx+c)i+1}dx\right)b+\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{\tan(dx+c)i+1}dx\right)a}{a}$$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(int((sqrt(tan(c + d*x))*tan(c + d*x)**3)/(tan(c + d*x)*i + 1),x)*b + int(
(sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)*i + 1),x)*a)/a
```

3.135 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 217

$$\begin{aligned}\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx &= -\frac{((1-3i)A+(3+5i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{4\sqrt{2}ad} \\ &\quad+\frac{((1-3i)A+(3+5i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{4\sqrt{2}ad} \\ &\quad+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)((2+i)A+(1+4i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}ad} \\ &\quad-\frac{(A+5iB)\sqrt{\tan(c+dx)}}{2ad}+\frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}\end{aligned}$$

output
```
1/8*((1-3*I)*A+(3+5*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a/d+
1/8*((1-3*I)*A+(3+5*I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a/d+(
1/8+1/8*I)*((2+I)*A+(1+4*I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x
+c)))*2^(1/2)/a/d-1/2*(A+5*I*B)*tan(d*x+c)^(1/2)/a/d+1/2*(I*A-B)*tan(d*x+c
)^(3/2)/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.53

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-2\sqrt[4]{-1}(A+2iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\frac{i\sqrt{\cdots}}{\cdots}}{2ad}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
,x]
```

output
```
((-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - 2*(-1)^(1/4)
*(A + (2*I)*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (I*Sqrt[Tan[c + d*
x]]*(A + (5*I)*B - 4*B*Tan[c + d*x]))/(-I + Tan[c + d*x]))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4078, 27, 3042, 4011, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \ 4078$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int \frac{1}{2}\sqrt{\tan(c+dx)}(3a(iA-B)+a(A+5iB)\tan(c+dx))dx}{2a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int\sqrt{\tan(c+dx)}(3a(iA-B)+a(A+5iB)\tan(c+dx))dx}{4a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int\sqrt{\tan(c+dx)}(3a(iA-B)+a(A+5iB)\tan(c+dx))dx}{4a^2}$$

$$\Big\downarrow \ 4011$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int\frac{3a(iA-B)\tan(c+dx)-a(A+5iB)}{\sqrt{\tan(c+dx)}}dx+\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\int\frac{3a(iA-B)\tan(c+dx)-a(A+5iB)}{\sqrt{\tan(c+dx)}}dx+\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \ 4017$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\frac{2\int-\frac{a(A+5iB-3(iA-B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \ 25$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}-\frac{2\int\frac{a(A+5iB-3(iA-B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a\int\frac{A+5iB-3(iA-B)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \ 1482$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1-3i)A+(3+5i)B)\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}}{4a^2}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1-3i)A+(3+5i)B)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d}\right.\right.}{d}}{4a^2}$$

$\downarrow$ 1082

$$\frac{\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1-3i)A+(3+5i)B)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{t\right.}}{\sqrt{2}}\right.\right.}{d}}{4a^2}$$

$\downarrow$ 217

$$\frac{\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1-3i)A+(3+5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{d}}{4a^2}$$

$\downarrow$ 1479

$$\frac{\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\right.\right.}{d}}{4a^2}$$

$\downarrow$ 25

$$\frac{\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(\right.\right.}{d}}{4a^2}$$

$\downarrow$ 27

$$\frac{\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right.}{d}}{4a^2}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-$$

$\downarrow$ 1103

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{2d(a+ia\tan(c+dx))}-$$

$$\frac{\frac{2a(A+5iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a\left(\frac{1}{2}((1-3i)A+(3+5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\left(\right.\right.}{d}}{4a^2}$$

input
```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
-1/4*((-2*a*((((1 - 3*I)*A + (3 + 5*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c
 + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 +
(1/2 + I/2)*((2 + I)*A + (1 + 4*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]/(2*Sqrt[2]))))/d + (2*a*(A + (5*I)*B)*Sqrt[Tan[c + d*x]])/d)/a^2 +
 ((I*A - B)*Tan[c + d*x]^(3/2))/(2*d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-2iB\sqrt{\tan(dx+c)}+\frac{i\left(-\frac{i(iA-B)\sqrt{\tan(dx+c)}}{-i+\tan(dx+c)}-\frac{4(2iB+A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2}+\frac{4\left(-\frac{iA}{4}-\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{da}$
default	$\frac{-2iB\sqrt{\tan(dx+c)}+\frac{i\left(-\frac{i(iA-B)\sqrt{\tan(dx+c)}}{-i+\tan(dx+c)}-\frac{4(2iB+A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2}+\frac{4\left(-\frac{iA}{4}-\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{da}$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d/a*(-2*I*B*tan(d*x+c)^(1/2)+1/2*I*(-I*(I*A-B)*tan(d*x+c)^(1/2)/(-I+tan(
d*x+c))-4*(A+2*I*B)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)
-I*2^(1/2))))+4*(-1/4*I*A-1/4*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(
1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(160) = 320$.

Time = 0.10 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.86

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/8*(a*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(
-2*((I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) - (A -
 I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a*d*sqrt((-I*
A^2 - 2*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*((-I*a*d*e^(2*I
*d*x + 2*I*c) - I*a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x
+ 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + 2*a*d*sqrt((I*A^2 - 4*A*B - 4*
I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(((a*d*e^(2*I*d*x + 2*I*c) + a*d)
*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2
- 4*A*B - 4*I*B^2)/(a^2*d^2)) + I*A - 2*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) - 2
*a*d*sqrt((I*A^2 - 4*A*B - 4*I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-((
a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((I*A^2 - 4*A*B - 4*I*B^2)/(a^2*d^2)) - I*A + 2*B)*e
^(-2*I*d*x - 2*I*c)/(a*d)) - 2*((A + 9*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)
*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x
 - 2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{i\left(\int \frac{A\tan^{\frac{3}{2}}(c+dx)}{\tan(c+dx)-i}\,dx + \int \frac{B\tan^{\frac{5}{2}}(c+dx)}{\tan(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*(Integral(A*tan(c + d*x)**(3/2)/(tan(c + d*x) - I), x) + Integral(B*tan
(c + d*x)**(5/2)/(tan(c + d*x) - I), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.48

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$

$$= \frac{2\sqrt{2}(-(i-1)\ A+(2i+2)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan(dx+c)}\right)+\sqrt{2}(-(i+1)\ A+(i-1)\ B)\,\mathrm{arc}}{4\,ad}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
1/4*(2*sqrt(2)*(-(I - 1)*A + (2*I + 2)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sqr
t(tan(d*x + c))) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-(1/2*I - 1/2)*
sqrt(2)*sqrt(tan(d*x + c))) - 8*I*B*sqrt(tan(d*x + c)) - 2*(-I*A*sqrt(tan(
d*x + c)) + B*sqrt(tan(d*x + c)))/(tan(d*x + c) - I))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.24

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\mathrm{atan}\left(\frac{2\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,1{}\mathrm{i}}{4\,a^2\,d^2}}}{A}\right)\,\sqrt{\frac{A^2\,1{}\mathrm{i}}{4\,a^2\,d^2}}\,2{}\mathrm{i} - \mathrm{atan}\left(\frac{4\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,1{}\mathrm{i}}{16\,a^2\,d^2}}}{A}\right)\,\sqrt{-\frac{A^2\,1{}\mathrm{i}}{16\,a^2\,d^2}}\,2{}\mathrm{i} + \mathrm{atan}\left(\frac{a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1{}\mathrm{i}}{a^2\,d^2}}\,1{}\mathrm{i}}{B}\right)\,\sqrt{-\frac{B^2\,1{}\mathrm{i}}{a^2\,d^2}}\,2{}\mathrm{i} - \mathrm{atan}\left(\frac{a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,1{}\mathrm{i}}{16\,a^2\,d^2}}\,4{}\mathrm{i}}{B}\right)\,\sqrt{\frac{B^2\,1{}\mathrm{i}}{16\,a^2\,d^2}}\,2{}\mathrm{i} - \frac{B\,\sqrt{\tan\left(c+d\,x\right)}\,2{}\mathrm{i}}{a\,d} - \frac{A\,\sqrt{\tan\left(c+d\,x\right)}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)} - \frac{B\,\sqrt{\tan\left(c+d\,x\right)}\,1{}\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
atan((a*d*tan(c + d*x)^(1/2)*(-(B^2*1i)/(a^2*d^2))^(1/2)*1i)/B)*(-(B^2*1i)
/(a^2*d^2))^(1/2)*2i - atan((4*a*d*tan(c + d*x)^(1/2)*(-(A^2*1i)/(16*a^2*d
^2))^(1/2))/A)*(-(A^2*1i)/(16*a^2*d^2))^(1/2)*2i - atan((2*a*d*tan(c + d*x
)^(1/2)*((A^2*1i)/(4*a^2*d^2))^(1/2))/A)*((A^2*1i)/(4*a^2*d^2))^(1/2)*2i -
 atan((a*d*tan(c + d*x)^(1/2)*((B^2*1i)/(16*a^2*d^2))^(1/2)*4i)/B)*((B^2*1
i)/(16*a^2*d^2))^(1/2)*2i - (B*tan(c + d*x)^(1/2)*2i)/(a*d) - (A*tan(c + d
*x)^(1/2))/(2*a*d*(tan(c + d*x)*1i + 1)) - (B*tan(c + d*x)^(1/2)*1i)/(2*a*
d*(tan(c + d*x)*1i + 1))
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{\tan(dx+c)i+1}dx\right)b+\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)}{\tan(dx+c)i+1}dx\right)a}{a}$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)*i + 1),x)*b + int(
(sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)*i + 1),x)*a)/a
```

3.136 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 186

$$\begin{aligned}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx &= -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(A+(2-i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad} \\ &\quad+\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(A+(2-i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad} \\ &\quad-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(A-(2+i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}ad}+\frac{(iA-B)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}\end{aligned}$$

output

```
(1/8-1/8*I)*(A+(2-I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a/d+(1
/8-1/8*I)*(A+(2-I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a/d-(1/8+
1/8*I)*(A-(2+I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2
)/a/d+1/2*(I*A-B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-2\sqrt[4]{-1}B\text{arctanh}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\frac{(A+iB)\sqrt{\tan(c+dx)}}{-i+\tan(c+dx)}}{2ad}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
,x]
```

output
```
((-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - 2*(-1)^(1/4)
*B*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + ((A + I*B)*Sqrt[Tan[c + d*x]])
/(-I + Tan[c + d*x]))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4078, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \ 4078$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{\int \frac{a(iA-B)-a(A-3iB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}}dx}{2a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{\int\frac{a(iA-B)-a(A-3iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{\int\frac{a(iA-B)-a(A-3iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{4a^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{\int\frac{a(iA-B-(A-3iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{2a^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{\int\frac{iA-B-(A-3iB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{2ad}$$

$$\Big\downarrow \text{1482}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-$$
$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)(A+(2-i)B)\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{2ad}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-$$
$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)(A+(2-i)B)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\right.}{2ad}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-$$
$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)(A+(2-i)B)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right.}{2ad}$$

$$\Big\downarrow \text{217}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-$$
$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)(A+(2-i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\text{arctan}}{}\right.}{2ad}$$

$$\Big\downarrow \text{1479}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\left(\frac{1}{2}-\right.}{2ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)(}{2ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-}{2ad}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)(A+(2-i}{2ad}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
-1/2*((-1/2 + I/2)*(A + (2 - I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]
]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + (1/2 + I/2
)*(A - (2 + I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]))
)/(a*d) + ((I*A - B)*Sqrt[Tan[c + d*x]])/(2*d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{\frac{i\left(-\frac{i(iB+A)\sqrt{\tan(dx+c)}}{-i+\tan(dx+c)}-\frac{4B\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2}+\frac{4\left(\frac{A}{4}-\frac{iB}{4}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{da}$	124
default	$\frac{\frac{i\left(-\frac{i(iB+A)\sqrt{\tan(dx+c)}}{-i+\tan(dx+c)}-\frac{4B\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2}+\frac{4\left(\frac{A}{4}-\frac{iB}{4}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{da}$	124

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d/a*(1/2*I*(-I*(A+I*B)*tan(d*x+c)^(1/2)/(-I+tan(d*x+c))-4*B/(2^(1/2)-I*2
^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))+4*(1/4*A-1/4*I*B)/
(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(139) = 278$.

Time = 0.09 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.06

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/8*(a*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(
2*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) + (A - I*B)*
e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a*d*sqrt((I*A^2 + 2
*A*B - I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*((a*d*e^(2*I*d*x + 2*I
*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(
-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a*d*sqrt(I*B^2/(a^2*d^2))*e^(2*I*d*x + 2*
I*c)*log(((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(I*B^2/(a^2*d^2)) + I*B)*e^(-2*I*d*x - 2*
I*c)/(a*d)) + 2*a*d*sqrt(I*B^2/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-((a*d*e
^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(I*B^2/(a^2*d^2)) - I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) - 2*
((I*A - B)*e^(2*I*d*x + 2*I*c) + I*A - B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{i\left(\int \frac{A\sqrt{\tan{(c+dx)}}}{\tan{(c+dx)}-i}\,dx + \int \frac{B\tan^{\frac{3}{2}}{(c+dx)}}{\tan{(c+dx)}-i}\,dx\right)}{a}$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*(Integral(A*sqrt(tan(c + d*x))/(tan(c + d*x) - I), x) + Integral(B*tan(
c + d*x)**(3/2)/(tan(c + d*x) - I), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{(2i-2)\ \sqrt{2}B\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan{(dx+c)}}\right)+\sqrt{2}((i-1)\ A+(i+1)\ B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\right.}{4\,ad}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/4*((2*I - 2)*sqrt(2)*B*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(tan(d*x + c)))
 + sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(
d*x + c))) - 2*(A*sqrt(tan(d*x + c)) + I*B*sqrt(tan(d*x + c)))/(tan(d*x +
c) - I))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= -\mathrm{atan}\left(\frac{2\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,1\mathrm{i}}{4\,a^2\,d^2}}}{B}\right)\,\sqrt{\frac{B^2\,1\mathrm{i}}{4\,a^2\,d^2}}\,2\mathrm{i}$$
$$-\mathrm{atan}\left(\frac{4\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1\mathrm{i}}{16\,a^2\,d^2}}}{B}\right)\,\sqrt{-\frac{B^2\,1\mathrm{i}}{16\,a^2\,d^2}}\,2\mathrm{i}$$
$$-\frac{2\,\sqrt{\frac{1}{16}\mathrm{i}}\,A\,\mathrm{atanh}\left(4\,\sqrt{\frac{1}{16}\mathrm{i}}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{a\,d}$$
$$+\frac{A\,\sqrt{\tan\left(c+d\,x\right)}\,1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}-\frac{B\,\sqrt{\tan\left(c+d\,x\right)}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
(A*tan(c + d*x)^(1/2)*1i)/(2*a*d*(tan(c + d*x)*1i + 1)) - atan((4*a*d*tan(
c + d*x)^(1/2)*(-(B^2*1i)/(16*a^2*d^2))^(1/2))/B)*(-(B^2*1i)/(16*a^2*d^2))
^(1/2)*2i - (2*(1i/16)^(1/2)*A*atanh(4*(1i/16)^(1/2)*tan(c + d*x)^(1/2)))/
(a*d) - atan((2*a*d*tan(c + d*x)^(1/2)*((B^2*1i)/(4*a^2*d^2))^(1/2))/B)*((
B^2*1i)/(4*a^2*d^2))^(1/2)*2i - (B*tan(c + d*x)^(1/2))/(2*a*d*(tan(c + d*x
)*1i + 1))
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{-2\sqrt{\tan(dx+c)}\,ai-\left(\int -\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2-\tan(dx+c)i}dx\right)ad-\left(\int -\frac{\sqrt{\tan(dx+c)}\tan(dx+c)^2}{\tan(dx+c)-i}dx\right)adi-\left(\int -\frac{\sqrt{\tan(dx}}{\tan}\right.}{ad}$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
( - 2*sqrt(tan(c + d*x))*a*i - int(( - sqrt(tan(c + d*x)))/(tan(c + d*x)**
2 - tan(c + d*x)*i),x)*a*d - int(( - sqrt(tan(c + d*x))*tan(c + d*x)**2)/(
tan(c + d*x) - i),x)*a*d*i - int(( - sqrt(tan(c + d*x))*tan(c + d*x))/(tan
(c + d*x) - i),x)*a*d + int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x
)*i + 1),x)*b*d)/(a*d)
```

3.137 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 184

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}\,dx = -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)((2+i)A+B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad} + \frac{\left(\frac{1}{4}-\frac{i}{4}\right)((2+i)A+B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad} + \frac{((3+i)A-(1+i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{4\sqrt{2}ad} + \frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

output
```
(1/8-1/8*I)*((2+I)*A+B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a/d+(1
/8-1/8*I)*((2+I)*A+B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a/d+1/8*(
(3+I)*A-(1+I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/
a/d+1/2*(A+I*B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.53

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}\,dx$$
$$= \frac{-\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-2\sqrt[4]{-1}A\operatorname{arctanh}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\frac{(-iA+B)\sqrt{}}{-i+\tan}}{2ad}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x]))
,x]
```

output
```
(-((-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]) - 2*(-1)^(1
/4)*A*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (((-I)*A + B)*Sqrt[Tan[c +
d*x]])/(-I + Tan[c + d*x]))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4079, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\int \frac{a(3A-iB)-a(iA-B)\tan(c+dx)}{2\sqrt{\tan(c+dx)}}dx}{2a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a(3A-iB)-a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{4a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(3A-iB)-a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{4a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\Big\downarrow$ 4017

$$\frac{\int \frac{a(3A-iB-(iA-B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{2a^2d}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{3A-iB-(iA-B)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{2ad}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\Big\downarrow$ 1482

$$\frac{\frac{1}{2}((3+i)A-(1+i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)(B+(2+i)A)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{2ad}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\Big\downarrow$ 1476

$$\frac{\frac{1}{2}((3+i)A-(1+i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)(B+(2+i)A)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\right.}{2ad}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\Big\downarrow$ 1082

$$\frac{\frac{1}{2}((3+i)A-(1+i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)(B+(2+i)A)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right.}{2ad}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\Big\downarrow$ 217

$$\frac{\frac{\frac{1}{2}((3+i)A-(1+i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)(B+(2+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan}{}\right.}{2ad}}{}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\downarrow$ 1479

$$\frac{\frac{1}{2}((3+i)A-(1+i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}-\right.}{2ad}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}((3+i)A-(1+i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)(}{2ad}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}((3+i)A-(1+i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+}{2ad}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

$\downarrow$ 1103

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(B+(2+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((3+i)A-(1+i)B)\left(\frac{\log\left(\tan(c\right.}{}\right.}{2ad}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{2d(a+ia\tan(c+dx))}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

output
```
((1/2 - I/2)*((2 + I)*A + B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqr
t[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + (((3 + I)*A - (1
 + I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2]
+ Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2)/(2*a
*d) + ((A + I*B)*Sqrt[Tan[c + d*x]])/(2*d*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{-\frac{i(iB+A)\sqrt{\tan(dx+c)}}{2(-i+\tan(dx+c))}-\frac{2iA\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{4\left(\frac{iA}{4}+\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{da}$	121
default	$\frac{-\frac{i(iB+A)\sqrt{\tan(dx+c)}}{2(-i+\tan(dx+c))}-\frac{2iA\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{4\left(\frac{iA}{4}+\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{da}$	121

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d/a*(-1/2*I*(A+I*B)*tan(d*x+c)^(1/2)/(-I+tan(d*x+c))-2*I*A/(2^(1/2)-I*2^
(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))+4*(1/4*I*A+1/4*B)/(2
^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(139) = 278.

Time = 0.10 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.10

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/8*(a*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log
(-2*((I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) - (A
- I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a*d*sqrt((-I
*A^2 - 2*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*((-I*a*d*e^(2*
I*d*x + 2*I*c) - I*a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x
 + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a*d*sqrt(I*A^2/(a^2*d^2))*e
^(2*I*d*x + 2*I*c)*log(((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(I*A^2/(a^2*d^2)) + I*A)*e^
(-2*I*d*x - 2*I*c)/(a*d)) + 2*a*d*sqrt(I*A^2/(a^2*d^2))*e^(2*I*d*x + 2*I*c
)*log(-((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(I*A^2/(a^2*d^2)) - I*A)*e^(-2*I*d*x - 2*I*
c)/(a*d)) - 2*((A + I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*sqrt((-I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + ia\tan(c + dx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + ia\tan(c + dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}\,dx =$$

$$-\frac{(2i-2)\,\sqrt{2}A\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+\sqrt{2}(-(i+1)\,A+(i-1)\,B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right.\right.}{4\,ad}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/4*((2*I - 2)*sqrt(2)*A*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(tan(d*x + c)))
 + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan
(d*x + c))) + 2*(I*A*sqrt(tan(d*x + c)) - B*sqrt(tan(d*x + c)))/(tan(d*x +
 c) - I))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}\,dx &= -\mathrm{atan}\left(\frac{2\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,1\mathrm{i}}{4\,a^2\,d^2}}}{A}\right)\,\sqrt{\frac{A^2\,1\mathrm{i}}{4\,a^2\,d^2}}\,2\mathrm{i}\\ &\quad+\mathrm{atan}\left(\frac{4\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,1\mathrm{i}}{16\,a^2\,d^2}}}{A}\right)\,\sqrt{-\frac{A^2\,1\mathrm{i}}{16\,a^2\,d^2}}\,2\mathrm{i}\\ &\quad-\frac{2\,\sqrt{\frac{1}{16}\mathrm{i}}\,B\,\mathrm{atanh}\left(4\,\sqrt{\frac{1}{16}\mathrm{i}}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{a\,d}\\ &\quad+\frac{A\,\sqrt{\tan\left(c+d\,x\right)}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}+\frac{B\,\sqrt{\tan\left(c+d\,x\right)}\,1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}\end{aligned}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
atan((4*a*d*tan(c + d*x)^(1/2)*(-(A^2*1i)/(16*a^2*d^2))^(1/2))/A)*(-(A^2*1
i)/(16*a^2*d^2))^(1/2)*2i - atan((2*a*d*tan(c + d*x)^(1/2)*((A^2*1i)/(4*a^
2*d^2))^(1/2))/A)*((A^2*1i)/(4*a^2*d^2))^(1/2)*2i - (2*(1i/16)^(1/2)*B*ata
nh(4*(1i/16)^(1/2)*tan(c + d*x)^(1/2)))/(a*d) + (A*tan(c + d*x)^(1/2))/(2*
a*d*(tan(c + d*x)*1i + 1)) + (B*tan(c + d*x)^(1/2)*1i)/(2*a*d*(tan(c + d*x
)*1i + 1))
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\,(a+ia\tan(c+dx))}\,dx = \frac{-2\sqrt{\tan(dx+c)}\,bi-\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2-\tan(dx+c)i}dx\right)adi+\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2-\tan(dx+c)i}dx\right)bd+\left(\int\frac{\sqrt{\tan(dx+c)}\tan(\cdots}{\tan(dx+c)-\cdots}\right.}{ad}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
( - 2*sqrt(tan(c + d*x))*b*i - int(sqrt(tan(c + d*x))/(tan(c + d*x)**2 - t
an(c + d*x)*i),x)*a*d*i + int(sqrt(tan(c + d*x))/(tan(c + d*x)**2 - tan(c
+ d*x)*i),x)*b*d + int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)
- i),x)*b*d*i + int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x) - i),x
)*b*d)/(a*d)
```

3.138 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 213

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx &= \frac{((5+3i)A-(3-i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{4\sqrt{2}ad} \\ &+ \frac{((-5-3i)A+(3-i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{4\sqrt{2}ad} \\ &+ \frac{((5-3i)A+(3+i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{4\sqrt{2}ad} \\ &- \frac{5A+iB}{2ad\sqrt{\tan(c+dx)}}+\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}\end{aligned}$$

output
```
-1/8*((5+3*I)*A+(-3+I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a/d+
1/8*((-5-3*I)*A+(3-I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a/d+1/
8*((5-3*I)*A+(3+I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(
1/2)/a/d-1/2*(5*A+I*B)/a/d/tan(d*x+c)^(1/2)+1/2*(A+I*B)/d/tan(d*x+c)^(1/2)
/(a+I*a*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.84 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.54

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \frac{-iA+B-2(2A+iB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-i\tan(c+dx)\right)(-i+\tan(c+dx))-(A-iB)}{2ad\sqrt{\tan(c+dx)}(-i+\tan(c+dx))}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x]))
,x]
```

output
```
((-I)*A + B - 2*(2*A + I*B)*Hypergeometric2F1[-1/2, 1, 1/2, (-I)*Tan[c + d
*x]]*(-I + Tan[c + d*x]) - (A - I*B)*Hypergeometric2F1[-1/2, 1, 1/2, I*Tan
[c + d*x]]*(-I + Tan[c + d*x]))/(2*a*d*Sqrt[Tan[c + d*x]]*(-I + Tan[c + d*
x]))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.472, Rules used = {3042, 4079, 27, 3042, 4012, 25, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(5A+iB)-3a(iA-B)\tan(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 27

$$\frac{\int \frac{a(5A+iB)-3a(iA-B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx}{4a^2}+\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 3042

$$\frac{\int \frac{a(5A+iB)-3a(iA-B)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx}{4a^2}+\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 4012

$$\frac{\int -\frac{3a(iA-B)+a(5A+iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a(5A+iB)}{d\sqrt{\tan(c+dx)}}}{4a^2}+\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 25

$$\frac{-\int \frac{3a(iA-B)+a(5A+iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a(5A+iB)}{d\sqrt{\tan(c+dx)}}}{4a^2}+\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 3042

$$\frac{-\int \frac{3a(iA-B)+a(5A+iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a(5A+iB)}{d\sqrt{\tan(c+dx)}}}{4a^2}+\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 4017

$$\frac{-\frac{2\int \frac{a(3(iA-B)+(5A+iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2a(5A+iB)}{d\sqrt{\tan(c+dx)}}}{4a^2}+\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 27

$$\frac{-\frac{2a\int \frac{3(iA-B)+(5A+iB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2a(5A+iB)}{d\sqrt{\tan(c+dx)}}}{4a^2}+\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 1482

$$\frac{-\frac{2a\left(\frac{1}{2}((5+3i)A-(3-i)B)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)((4+i)A+(1+2i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a(5A+iB)}{d\sqrt{\tan(c+dx)}}}{4a^2}+\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 1476

$$\frac{-\frac{2a\left(\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((4+i)A+\cdots}{d}}{4a^2}\ \frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 1082

$$\frac{-\frac{2a\left(\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((4+i)A+(1+2i)B)\int \frac{1-\tan\cdots}{\tan^2(\cdots}}{d}}{4a^2}\ \frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 217

$$\frac{-\frac{2a\left(\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((4+i)A+(1+2i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}}{4a^2}\ \frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 1479

$$\frac{-\frac{2a\left(\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((4+i)A+(1+2i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d\cdots}}}{2\sqrt{2}}\cdots\right.}{d}}{4a^2}\ \frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 25

$$-\frac{\frac{2a\left(\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((4+i)A+(1+2i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}}{2\sqrt{2}}\right.\right.}{d}}{4a^2}$$

$$\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 27

$$-\frac{\frac{2a\left(\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((4+i)A+(1+2i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}}{2\sqrt{2}}\right.\right.}{d}}{4a^2}$$

$$\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

$\downarrow$ 1103

$$-\frac{\frac{2a\left(\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((4+i)A+(1+2i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}\right.}{2\sqrt{2}}\right.\right.}{d}}{4a^2}$$

$$\frac{A+iB}{2d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
((-2*a*((((5 + 3*I)*A - (3 - I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]
]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 - (1/2 -
I/2)*((4 + I)*A + (1 + 2*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] +
Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/(2*Sqrt[2]))))/d - (2*a*(5*A + I*B))/(d*Sqrt[Tan[c + d*x]]))/(4*a^2) + (A
 + I*B)/(2*d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{\frac{i(iA-B)\sqrt{\tan(dx+c)}}{-2i+2\tan(dx+c)}-\frac{2(iB+2A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{4\left(-\frac{A}{4}+\frac{iB}{4}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}-\frac{2A}{\sqrt{\tan(dx+c)}}}{da}$	140
default	$\frac{\frac{i(iA-B)\sqrt{\tan(dx+c)}}{-2i+2\tan(dx+c)}-\frac{2(iB+2A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{4\left(-\frac{A}{4}+\frac{iB}{4}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}-\frac{2A}{\sqrt{\tan(dx+c)}}}{da}$	140

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d/a*(1/2*I*(I*A-B)*tan(d*x+c)^(1/2)/(-I+tan(d*x+c))-2*(2*A+I*B)/(2^(1/2)
-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))+4*(-1/4*A+1/4*I
*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))-2*A
/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(162) = 324.

Time = 0.15 (sec) , antiderivative size = 703, normalized size of antiderivative = 3.30

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/8*((a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((I*A^2 + 2*A
*B - I*B^2)/(a^2*d^2))*log(2*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B
^2)/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A
+ B)) - (a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((I*A^2 +
2*A*B - I*B^2)/(a^2*d^2))*log(-2*((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B -
 I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(
I*A + B)) + 2*(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((-4
*I*A^2 + 4*A*B + I*B^2)/(a^2*d^2))*log(((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-4*I*A^2
+ 4*A*B + I*B^2)/(a^2*d^2)) + 2*A + I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) - 2*(
a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((-4*I*A^2 + 4*A*B
+ I*B^2)/(a^2*d^2))*log(-((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-4*I*A^2 + 4*A*B + I*B^
2)/(a^2*d^2)) - 2*A - I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) + 2*((-9*I*A + B)*e
^(4*I*d*x + 4*I*c) - 8*I*A*e^(2*I*d*x + 2*I*c) + I*A - B)*sqrt((-I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(a*d*e^(4*I*d*x + 4*I*c) - a
*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia\tan(c + dx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.50

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$= \frac{2\sqrt{2}(-(2i+2)\,A-(i-1)\,B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)-\sqrt{2}(-(i-1)\,A-(i+1)\,B)\,\mathrm{arc}}{4\,ad}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
1/4*(2*sqrt(2)*(-(2*I + 2)*A - (I - 1)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sqr
t(tan(d*x + c))) - sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-(1/2*I - 1/2)*
sqrt(2)*sqrt(tan(d*x + c))) - 2*(5*A*tan(d*x + c) + I*B*tan(d*x + c) - 4*I
*A)/(tan(d*x + c)^(3/2) - I*sqrt(tan(d*x + c))))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.25

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$= 2\,\mathrm{atanh}\left(\frac{a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,1\mathrm{i}}{a^2\,d^2}}}{A}\right)\,\sqrt{-\frac{A^2\,1\mathrm{i}}{a^2\,d^2}}$$
$$+ 2\,\mathrm{atanh}\left(\frac{4\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,1\mathrm{i}}{16\,a^2\,d^2}}}{A}\right)\,\sqrt{\frac{A^2\,1\mathrm{i}}{16\,a^2\,d^2}}$$
$$- \mathrm{atan}\left(\frac{2\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,1\mathrm{i}}{4\,a^2\,d^2}}}{B}\right)\,\sqrt{\frac{B^2\,1\mathrm{i}}{4\,a^2\,d^2}}\,2\mathrm{i}$$
$$+ \mathrm{atan}\left(\frac{4\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1\mathrm{i}}{16\,a^2\,d^2}}}{B}\right)\,\sqrt{-\frac{B^2\,1\mathrm{i}}{16\,a^2\,d^2}}\,2\mathrm{i}$$
$$- \frac{\frac{2\,A}{a\,d}+\frac{A\tan(c+d\,x)\,5\mathrm{i}}{2\,a\,d}}{\sqrt{\tan\left(c+d\,x\right)}+{\tan\left(c+d\,x\right)}^{3/2}\,1\mathrm{i}} + \frac{B\,\sqrt{\tan\left(c+d\,x\right)}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
2*atanh((a*d*tan(c + d*x)^(1/2)*(-(A^2*1i)/(a^2*d^2))^(1/2))/A)*(-(A^2*1i)
/(a^2*d^2))^(1/2) + 2*atanh((4*a*d*tan(c + d*x)^(1/2)*((A^2*1i)/(16*a^2*d^
2))^(1/2))/A)*((A^2*1i)/(16*a^2*d^2))^(1/2) - atan((2*a*d*tan(c + d*x)^(1/
2)*((B^2*1i)/(4*a^2*d^2))^(1/2))/B)*((B^2*1i)/(4*a^2*d^2))^(1/2)*2i + atan
((4*a*d*tan(c + d*x)^(1/2)*(-(B^2*1i)/(16*a^2*d^2))^(1/2))/B)*(-(B^2*1i)/(
16*a^2*d^2))^(1/2)*2i - ((2*A)/(a*d) + (A*tan(c + d*x)*5i)/(2*a*d))/(tan(c
 + d*x)^(1/2) + tan(c + d*x)^(3/2)*1i) + (B*tan(c + d*x)^(1/2))/(2*a*d*(ta
n(c + d*x)*1i + 1))
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$= \frac{2\sqrt{\tan(dx+c)}\tan(dx+c)\,b+2\sqrt{\tan(dx+c)}\,bi-\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3-\tan(dx+c)^2i}dx\right)\tan(dx+c)\,adi+\left(\int\right.}{}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(2*sqrt(tan(c + d*x))*tan(c + d*x)*b + 2*sqrt(tan(c + d*x))*b*i - int(sqrt
(tan(c + d*x))/(tan(c + d*x)**3 - tan(c + d*x)**2*i),x)*tan(c + d*x)*a*d*i
 + int(sqrt(tan(c + d*x))/(tan(c + d*x)**3 - tan(c + d*x)**2*i),x)*tan(c +
 d*x)*b*d - int(sqrt(tan(c + d*x))/(tan(c + d*x)**2*i + tan(c + d*x)),x)*t
an(c + d*x)*b*d - int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)*i
 + 1),x)*tan(c + d*x)*b*d*i + int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c
 + d*x) - i),x)*tan(c + d*x)*b*d*i - int((sqrt(tan(c + d*x))*tan(c + d*x))
/(tan(c + d*x)*i + 1),x)*tan(c + d*x)*b*d)/(tan(c + d*x)*a*d)
```

3.139 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 246

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx &= \frac{((7-5i)A+(5+3i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{4\sqrt{2}ad} \\ &- \frac{\left(\frac{1}{4}-\frac{i}{4}\right)((6+i)A+(1+4i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}ad} \\ &+ \frac{((-7-5i)A+(5-3i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{4\sqrt{2}ad} - \frac{7A+3iB}{6ad\tan^{\frac{3}{2}}(c+dx)} \\ &+ \frac{5(iA-B)}{2ad\sqrt{\tan(c+dx)}} + \frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\end{aligned}$$

output
```
-1/8*((7-5*I)*A+(5+3*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a/d
+(-1/8+1/8*I)*((6+I)*A+(1+4*I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/
2)/a/d+1/8*((-7-5*I)*A+(5-3*I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(
d*x+c)))*2^(1/2)/a/d-1/6*(7*A+3*I*B)/a/d/tan(d*x+c)^(3/2)+5/2*(I*A-B)/a/d/
tan(d*x+c)^(1/2)+1/2*(A+I*B)/d/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.00 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.48

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$= \frac{3(-iA+B)-2(3A+2iB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-i\tan(c+dx)\right)(-i+\tan(c+dx))-(A-}{6ad\tan^{\frac{3}{2}}(c+dx)(-i+\tan(c+dx}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x]))
,x]
```

output
```
(3*((-I)*A + B) - 2*(3*A + (2*I)*B)*Hypergeometric2F1[-3/2, 1, -1/2, (-I)*
Tan[c + d*x]]*(-I + Tan[c + d*x]) - (A - I*B)*Hypergeometric2F1[-3/2, 1, -
1/2, I*Tan[c + d*x]]*(-I + Tan[c + d*x]))/(6*a*d*Tan[c + d*x]^(3/2)*(-I +
Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.06, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4079, 27, 3042, 4012, 25, 3042, 4012, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(7A+3iB)-5a(iA-B)\tan(c+dx)}{2\tan^{\frac{5}{2}}(c+dx)}dx}{2a^2}+\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(7A+3iB)-5a(iA-B)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}dx}{4a^2}+\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(7A+3iB)-5a(iA-B)\tan(c+dx)}{\tan(c+dx)^{5/2}}dx}{4a^2}+\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int -\frac{5a(iA-B)+a(7A+3iB)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}}{4a^2}+\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\int \frac{5a(iA-B)+a(7A+3iB)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}}{4a^2}+\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \frac{5a(iA-B)+a(7A+3iB)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx-\frac{2a(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}}{4a^2}+\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4012}$$

$$\frac{-\int \frac{a(7A+3iB)-5a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{10a(-B+iA)}{d\sqrt{\tan(c+dx)}}}{4a^2}+\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \frac{a(7A+3iB)-5a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{10a(-B+iA)}{d\sqrt{\tan(c+dx)}}}{4a^2}+\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4017}$$

$$\frac{-\frac{2\int \frac{a(7A+3iB-5(iA-B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2a(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{10a(-B+iA)}{d\sqrt{\tan(c+dx)}}}{4a^2}+\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{2a\int \frac{7A+3iB-5(iA-B)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2a(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{10a(-B+iA)}{d\sqrt{\tan(c+dx)}}}{4a^2}+\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 1482$$

$$\frac{-\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((7-5i)A+(5+3i)B)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2a(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{10a(\cdots}{d\sqrt{ta\cdots}}}{4a^2}$$

$$\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 1476$$

$$\frac{-\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((7-5i)A+(5+3i)B)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{\cdots}{\tan(c+\cdots}\right.\right.}{d}}{4a^2}$$

$$\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 1082$$

$$\frac{-\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((7-5i)A+(5+3i)B)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d(\sqrt{\cdots}}{\sqrt{2}\cdots}\right.\right.}{d}}{4a^2}$$

$$\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((7-5i)A+(5+3i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}-}{4a^2}$$

$$\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$\downarrow$ 1479

$$-\frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((7-5i)A+(5+}{d}}{4a^2}$$

$$\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$\downarrow$ 25

$$-\frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((7-5i)A+(5+3i)B}{d}}{4a^2}$$

$$\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$\downarrow$ 27

$$-\frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}((7-5i)A+(5+3}{d}}{4a^2}$$

$$\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

$\downarrow$ 1103

$$-\frac{\frac{2a\left(\frac{1}{2}((7-5i)A+(5+3i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((7+5i)A-(5-3i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right.}{d}}{4a^2}$$

$$\frac{A+iB}{2d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
((-2*a*((((7 - 5*I)*A + (5 + 3*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + (((7
+ 5*I)*A - (5 - 3*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqr
t[2])))/2))/d - (2*a*(7*A + (3*I)*B))/(3*d*Tan[c + d*x]^(3/2)) + (10*a*(I*
A - B))/(d*Sqrt[Tan[c + d*x]]))/(4*a^2) + (A + I*B)/(2*d*Tan[c + d*x]^(3/2
)*(a + I*a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{-\frac{i\left(\frac{i(iA-B)\sqrt{\tan(dx+c)}}{-i+\tan(dx+c)}-\frac{4(2iB+3A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2}+\frac{4\left(-\frac{iA}{4}-\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}-\frac{2A}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{2(-}{\sqrt{ta}}}{da}$
default	$\frac{-\frac{i\left(\frac{i(iA-B)\sqrt{\tan(dx+c)}}{-i+\tan(dx+c)}-\frac{4(2iB+3A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{2}+\frac{4\left(-\frac{iA}{4}-\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}-\frac{2A}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{2(-}{\sqrt{ta}}}{da}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d/a*(-1/2*I*(I*(I*A-B)*tan(d*x+c)^(1/2)/(-I+tan(d*x+c))-4*(3*A+2*I*B)/(2
^(1/2)-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))+4*(-1/4*
I*A-1/4*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2
)))-2/3*A/tan(d*x+c)^(3/2)-2*(B-I*A)/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(183) = 366$.

Time = 0.12 (sec) , antiderivative size = 795, normalized size of antiderivative = 3.23

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia\tan(c + dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/24*(3*(a*d*e^(6*I*d*x + 6*I*c) - 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*
d*x + 2*I*c))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2))*log(-2*((I*a*d*e^(2
*I*d*x + 2*I*c) + I*a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*
x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*(a*d*e^(6*I*d*x + 6*I*c) -
 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((-I*A^2 - 2*A*B
 + I*B^2)/(a^2*d^2))*log(-2*((-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt((-I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B
+ I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/
(I*A + B)) - 6*(a*d*e^(6*I*d*x + 6*I*c) - 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*
e^(2*I*d*x + 2*I*c))*sqrt((9*I*A^2 - 12*A*B - 4*I*B^2)/(a^2*d^2))*log(-((a
*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt((9*I*A^2 - 12*A*B - 4*I*B^2)/(a^2*d^2)) + 3*I*A - 2*
B)*e^(-2*I*d*x - 2*I*c)/(a*d)) + 6*(a*d*e^(6*I*d*x + 6*I*c) - 2*a*d*e^(4*I
*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((9*I*A^2 - 12*A*B - 4*I*B^2)
/(a^2*d^2))*log(((a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((9*I*A^2 - 12*A*B - 4*I*B^2)/(a^2
*d^2)) - 3*I*A + 2*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) - 2*((19*A + 27*I*B)*e^(
6*I*d*x + 6*I*c) - (19*A + 3*I*B)*e^(4*I*d*x + 4*I*c) - (35*A + 27*I*B)*e^
(2*I*d*x + 2*I*c) + 3*A + 3*I*B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.51

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx =$$

$$-\frac{6\sqrt{2}(-(3i-3)\ A+(2i+2)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}((i+1)\ A-(i-1)\ B)}{12}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/12*(6*sqrt(2)*(-(3*I - 3)*A + (2*I + 2)*B)*arctan((1/2*I + 1/2)*sqrt(2)
*sqrt(tan(d*x + c))) + 3*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-(1/2*I -
1/2)*sqrt(2)*sqrt(tan(d*x + c))) + 6*(-I*A*sqrt(tan(d*x + c)) + B*sqrt(tan
(d*x + c)))/(tan(d*x + c) - I) + 8*(-3*I*A*tan(d*x + c) + 3*B*tan(d*x + c)
 + A)/tan(d*x + c)^(3/2))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.23

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$=\operatorname{atan}\left(\frac{2\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,9{}\mathrm{i}}{4\,a^2\,d^2}}}{3\,A}\right)\,\sqrt{\frac{A^2\,9{}\mathrm{i}}{4\,a^2\,d^2}}\,2{}\mathrm{i}$$
$$-\operatorname{atan}\left(\frac{4\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,1{}\mathrm{i}}{16\,a^2\,d^2}}}{A}\right)\,\sqrt{-\frac{A^2\,1{}\mathrm{i}}{16\,a^2\,d^2}}\,2{}\mathrm{i}$$
$$+2\,\operatorname{atanh}\left(\frac{a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1{}\mathrm{i}}{a^2\,d^2}}}{B}\right)\,\sqrt{-\frac{B^2\,1{}\mathrm{i}}{a^2\,d^2}}$$
$$+2\,\operatorname{atanh}\left(\frac{4\,a\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,1{}\mathrm{i}}{16\,a^2\,d^2}}}{B}\right)\,\sqrt{\frac{B^2\,1{}\mathrm{i}}{16\,a^2\,d^2}}$$
$$-\frac{\frac{2\,A}{3\,a\,d}+\frac{5\,A\,{\tan\left(c+d\,x\right)}^2}{2\,a\,d}-\frac{A\,\tan\left(c+d\,x\right)\,4{}\mathrm{i}}{3\,a\,d}}{{\tan\left(c+d\,x\right)}^{3/2}+{\tan\left(c+d\,x\right)}^{5/2}\,1{}\mathrm{i}}-\frac{\frac{2\,B}{a\,d}+\frac{B\,\tan\left(c+d\,x\right)\,5{}\mathrm{i}}{2\,a\,d}}{\sqrt{\tan\left(c+d\,x\right)}+{\tan\left(c+d\,x\right)}^{3/2}\,1{}\mathrm{i}}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
atan((2*a*d*tan(c + d*x)^(1/2)*((A^2*9i)/(4*a^2*d^2))^(1/2))/(3*A))*((A^2*
9i)/(4*a^2*d^2))^(1/2)*2i - atan((4*a*d*tan(c + d*x)^(1/2)*(-(A^2*1i)/(16*
a^2*d^2))^(1/2))/A)*(-(A^2*1i)/(16*a^2*d^2))^(1/2)*2i + 2*atanh((a*d*tan(c
 + d*x)^(1/2)*(-(B^2*1i)/(a^2*d^2))^(1/2))/B)*(-(B^2*1i)/(a^2*d^2))^(1/2)
+ 2*atanh((4*a*d*tan(c + d*x)^(1/2)*((B^2*1i)/(16*a^2*d^2))^(1/2))/B)*((B^
2*1i)/(16*a^2*d^2))^(1/2) - ((2*A)/(3*a*d) - (A*tan(c + d*x)*4i)/(3*a*d) +
 (5*A*tan(c + d*x)^2)/(2*a*d))/(tan(c + d*x)^(3/2) + tan(c + d*x)^(5/2)*1i
) - ((2*B)/(a*d) + (B*tan(c + d*x)*5i)/(2*a*d))/(tan(c + d*x)^(1/2) + tan(
c + d*x)^(3/2)*1i)
```

Reduce [F]

$$\int \frac{A + B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$= \frac{6\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 bi + 2\sqrt{\tan(dx+c)}\,bi - 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4-\tan(dx+c)^3 i}dx\right)\tan(dx+c)^2 adi +}{}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(6*sqrt(tan(c + d*x))*tan(c + d*x)**2*b*i + 2*sqrt(tan(c + d*x))*b*i - 3*i
nt(sqrt(tan(c + d*x))/(tan(c + d*x)**4 - tan(c + d*x)**3*i),x)*tan(c + d*x
)**2*a*d*i + 3*int(sqrt(tan(c + d*x))/(tan(c + d*x)**4 - tan(c + d*x)**3*i
),x)*tan(c + d*x)**2*b*d + 3*int(sqrt(tan(c + d*x))/(tan(c + d*x)**2 - tan
(c + d*x)*i),x)*tan(c + d*x)**2*b*d - 3*int(sqrt(tan(c + d*x))/(tan(c + d*
x)**2*i + tan(c + d*x)),x)*tan(c + d*x)**2*b*d*i + 3*int((sqrt(tan(c + d*x
))*tan(c + d*x)**2)/(tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*b*d - 3*int((s
qrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)*i + 1),x)*tan(c + d*x)**2*b*
d*i)/(3*tan(c + d*x)**2*a*d)
```

3.140 $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 262

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{((9+5i)A-(25-21i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} - \frac{((9+5i)A-(25-21i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} + \frac{\left(\frac{1}{16}-\frac{i}{16}\right)((7+2i)A+(2+23i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}a^2d} + \frac{5(iA-5B)\sqrt{\tan(c+dx)}}{8a^2d} + \frac{(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{8a^2d(1+i\tan(c+dx))} + \frac{(iA-B)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

output

```
-1/32*((9+5*I)*A+(-25+21*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)
/a^2/d-1/32*((9+5*I)*A+(-25+21*I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^
(1/2)/a^2/d+(1/32-1/32*I)*((7+2*I)*A+(2+23*I)*B)*arctanh(2^(1/2)*tan(d*x+c
)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/a^2/d+5/8*(I*A-5*B)*tan(d*x+c)^(1/2)/a^2/d
+1/8*(3*A+7*I*B)*tan(d*x+c)^(3/2)/a^2/d/(1+I*tan(d*x+c))+1/4*(I*A-B)*tan(d
*x+c)^(5/2)/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (warning: unable to verify)

Time = 2.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{2\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))+\sqrt[4]{-1}(}{}$$

input
```
Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^2,x]
```

output
```
(2*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]
^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + (-1)^(1/4)*((-7*I)*A + 23*B)*
ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^2*(Cos[2*(c + d*x)] +
I*Sin[2*(c + d*x)]) + Sqrt[Tan[c + d*x]]*((-5*I)*A + 25*B + (7*A + (43*I)*
B)*Tan[c + d*x] - 16*B*Tan[c + d*x]^2))/(8*a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.10, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4078, 27, 3042, 4078, 25, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^{5/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \; 4078$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int\frac{\tan^{\frac{3}{2}}(c+dx)(5a(iA-B)+a(A+9iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)}dx}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int\frac{\tan^{\frac{3}{2}}(c+dx)(5a(iA-B)+a(A+9iB)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int\frac{\tan(c+dx)^{3/2}(5a(iA-B)+a(A+9iB)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{8a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{-\frac{\int-\sqrt{\tan(c+dx)}\left(3a^2(3A+7iB)-5a^2(iA-5B)\tan(c+dx)\right)dx}{2a^2}-\frac{(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{d(1+i\tan(c+dx))}}{8a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\sqrt{\tan(c+dx)}\left(3a^2(3A+7iB)-5a^2(iA-5B)\tan(c+dx)\right)dx}{2a^2}-\frac{(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{d(1+i\tan(c+dx))}}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\sqrt{\tan(c+dx)}\left(3a^2(3A+7iB)-5a^2(iA-5B)\tan(c+dx)\right)dx}{2a^2}-\frac{(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{d(1+i\tan(c+dx))}}{8a^2}$$

$$\Big\downarrow \text{4011}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\frac{5(iA-5B)a^2+3(3A+7iB)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{10a^2(-5B+iA)\sqrt{\tan(c+dx)}}{d}}{2a^2}-\frac{(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{d(1+i\tan(c+dx))}}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int \frac{5(iA-5B)a^2+3(3A+7iB)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{10a^2(-5B+iA)\sqrt{\tan(c+dx)}}{d}}{2a^2}-\frac{(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{d(1+i\tan(c+dx))}}{8a^2}$$

$\downarrow$ 4017

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{2\int \frac{a^2(5(iA-5B)+3(3A+7iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{10a^2(-5B+iA)\sqrt{\tan(c+dx)}}{d}}{2a^2}-\frac{(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{d(1+i\tan(c+dx))}}{8a^2}$$

$\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{2a^2\int \frac{5(iA-5B)+3(3A+7iB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{10a^2(-5B+iA)\sqrt{\tan(c+dx)}}{d}}{2a^2}-\frac{(3A+7iB)\tan^{\frac{3}{2}}(c+dx)}{d(1+i\tan(c+dx))}}{8a^2}$$

$\downarrow$ 1482

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{2a^2\left(\frac{1}{2}((9+5i)A-(25-21i)B)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{10a^2(-5B+iA)\sqrt{\tan(c+dx)}}{d}}{2a^2}-}{8a^2}$$

$\downarrow$ 1476

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{2a^2\left(\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i\right.}{d}}{2a^2}}{8a^2}$$

$\downarrow$ 1082

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{2a^2\left(\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\int \frac{1-\tan(c\cdots}{\tan^2(c+d\cdots}\right.}{d}}{2a^2}}{8a^2}$$

$\downarrow$ 217

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{2a^2\left(\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\cdots}{2a^2}}{8a^2}$$

$\downarrow$ 1479

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{2a^2\left(\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+\cdots}}{2\sqrt{2}}\cdots\right)\right)}{d}\cdots}{2a^2}}{8a^2}$$

$\downarrow$ 25

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{2a^2\left(\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\cdots}}{2\sqrt{2}}\cdots\right)\right)}{d}\cdots}{2a^2}}{8a^2}$$

$\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{2a^2\left(\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\cdots}}{2\sqrt{2}}\cdots\right)\right)}{d}\cdots}{2a^2}}{8a^2}$$

$\downarrow$ 1103

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{2a^2\left(\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\cdots\right)\right)}{d}\cdots}{2a^2}}{8a^2}$$

input
```
Int[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((I*A - B)*Tan[c + d*x]^(5/2))/(4*d*(a + I*a*Tan[c + d*x])^2) - (((2*a^2*(
(((9 + 5*I)*A - (25 - 21*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/S
qrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 - (1/2 - I/2)
*((7 + 2*I)*A + (2 + 23*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/
(2*Sqrt[2]))))/d - (10*a^2*(I*A - 5*B)*Sqrt[Tan[c + d*x]])/d)/(2*a^2) - ((
3*A + (7*I)*B)*Tan[c + d*x]^(3/2))/(d*(1 + I*Tan[c + d*x])))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{-2B\sqrt{\tan(dx+c)}+\frac{i\left(\frac{\left(-\frac{7iA}{2}+\frac{11B}{2}\right)\tan(dx+c)^{\frac{3}{2}}+\left(-\frac{5A}{2}-\frac{9iB}{2}\right)\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^2}+\frac{(7iA-23B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{4}+\frac{i(iA+}{}}{d\,a^2}$
default	$\frac{-2B\sqrt{\tan(dx+c)}+\frac{i\left(\frac{\left(-\frac{7iA}{2}+\frac{11B}{2}\right)\tan(dx+c)^{\frac{3}{2}}+\left(-\frac{5A}{2}-\frac{9iB}{2}\right)\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^2}+\frac{(7iA-23B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{4}+\frac{i(iA+}{}}{d\,a^2}$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^2*(-2*B*tan(d*x+c)^(1/2)+1/4*I*(((-7/2*I*A+11/2*B)*tan(d*x+c)^(3/2)+
(-5/2*A-9/2*I*B)*tan(d*x+c)^(1/2))/(-I+tan(d*x+c))^2+(7*I*A-23*B)/(2^(1/2)
-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))+1/2*I*(I*A+B)/
(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 664 vs. 2(197) = 394.

Time = 0.11 (sec) , antiderivative size = 664, normalized size of antiderivative = 2.53

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
1/32*(2*a^2*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*
log(2*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) + (A
 - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a^2*d*sqr
t((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*((a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) - (A - I*B)*e^(2*I*d
*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + a^2*d*sqrt((-49*I*A^2 + 322
*A*B + 529*I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(1/8*((a^2*d*e^(2*I*d*
x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt((-49*I*A^2 + 322*A*B + 529*I*B^2)/(a^4*d^2)) + 7*A + 23*I*B)*e
^(-2*I*d*x - 2*I*c)/(a^2*d)) - a^2*d*sqrt((-49*I*A^2 + 322*A*B + 529*I*B^2
)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I*c) + a^
2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-4
9*I*A^2 + 322*A*B + 529*I*B^2)/(a^4*d^2)) - 7*A - 23*I*B)*e^(-2*I*d*x - 2*
I*c)/(a^2*d)) - 2*(6*(-I*A + 7*B)*e^(4*I*d*x + 4*I*c) - (5*I*A - 9*B)*e^(2
*I*d*x + 2*I*c) + I*A - B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) + 1)))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= -\frac{\int \frac{A\tan^{\frac{5}{2}}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx + \int \frac{B\tan^{\frac{7}{2}}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-(Integral(A*tan(c + d*x)**(5/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1),
 x) + Integral(B*tan(c + d*x)**(7/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) -
 1), x))/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.49

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{2}((7i+7)\ A+(23i-23)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+2\sqrt{2}(-(i-1)\ A-(i+1)\ B}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output
```
-1/16*(sqrt(2)*((7*I + 7)*A + (23*I - 23)*B)*arctan((1/2*I + 1/2)*sqrt(2)*
sqrt(tan(d*x + c))) + 2*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-(1/2*I -
1/2)*sqrt(2)*sqrt(tan(d*x + c))) + 32*B*sqrt(tan(d*x + c)) - 2*(7*A*tan(d*
x + c)^(3/2) + 11*I*B*tan(d*x + c)^(3/2) - 5*I*A*sqrt(tan(d*x + c)) + 9*B*
sqrt(tan(d*x + c)))/(tan(d*x + c) - I)^2)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.27

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{5\,A\,\sqrt{\tan(c+d\,x)}}{8\,a^2\,d}+\frac{A\,\tan(c+d\,x)^{3/2}\,7\mathrm{i}}{8\,a^2\,d}}{\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}}+\frac{-\frac{11\,B\,\tan(c+d\,x)^{3/2}}{8\,a^2\,d}+\frac{B\,\sqrt{\tan(c+d\,x)}\,9\mathrm{i}}{8\,a^2\,d}}{\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}}$$
$$+2\,\mathrm{atanh}\left(\frac{8\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,1\mathrm{i}}{64\,a^4\,d^2}}}{A}\right)\,\sqrt{\frac{A^2\,1\mathrm{i}}{64\,a^4\,d^2}}$$
$$+2\,\mathrm{atanh}\left(\frac{16\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,49\mathrm{i}}{256\,a^4\,d^2}}}{7\,A}\right)\,\sqrt{-\frac{A^2\,49\mathrm{i}}{256\,a^4\,d^2}}$$
$$-\frac{2\,B\,\sqrt{\tan\left(c+d\,x\right)}}{a^2\,d}+\mathrm{atan}\left(\frac{8\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1\mathrm{i}}{64\,a^4\,d^2}}}{B}\right)\,\sqrt{-\frac{B^2\,1\mathrm{i}}{64\,a^4\,d^2}}\,2\mathrm{i}$$
$$-\mathrm{atan}\left(\frac{16\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,529\mathrm{i}}{256\,a^4\,d^2}}}{23\,B}\right)\,\sqrt{\frac{B^2\,529\mathrm{i}}{256\,a^4\,d^2}}\,2\mathrm{i}$$

input
```
int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
((5*A*tan(c + d*x)^(1/2))/(8*a^2*d) + (A*tan(c + d*x)^(3/2)*7i)/(8*a^2*d))
/(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i) + ((B*tan(c + d*x)^(1/2)*9i)/(8
*a^2*d) - (11*B*tan(c + d*x)^(3/2))/(8*a^2*d))/(2*tan(c + d*x) + tan(c + d
*x)^2*1i - 1i) + 2*atanh((8*a^2*d*tan(c + d*x)^(1/2)*((A^2*1i)/(64*a^4*d^2
))^(1/2))/A)*((A^2*1i)/(64*a^4*d^2))^(1/2) + 2*atanh((16*a^2*d*tan(c + d*x
)^(1/2)*(-(A^2*49i)/(256*a^4*d^2))^(1/2))/(7*A))*(-(A^2*49i)/(256*a^4*d^2)
)^(1/2) + atan((8*a^2*d*tan(c + d*x)^(1/2)*(-(B^2*1i)/(64*a^4*d^2))^(1/2))
/B)*(-(B^2*1i)/(64*a^4*d^2))^(1/2)*2i - atan((16*a^2*d*tan(c + d*x)^(1/2)*
((B^2*529i)/(256*a^4*d^2))^(1/2))/(23*B))*((B^2*529i)/(256*a^4*d^2))^(1/2)
*2i - (2*B*tan(c + d*x)^(1/2))/(a^2*d)
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^3}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a}{a^2}$$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int((sqrt(tan(c + d*x))*tan(c + d*x)**3)/(tan(c + d*x)**2 - 2*tan(c +
 d*x)*i - 1),x)*b + int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)
**2 - 2*tan(c + d*x)*i - 1),x)*a))/a**2
```

3.141 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 227

$$
\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx
&= \frac{((1+3i)A+(9+5i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} \\
&\quad - \frac{((1+3i)A+(9+5i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} \\
&\quad - \frac{((1-3i)A-(9-5i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{16\sqrt{2}a^2d} \\
&\quad + \frac{(A+5iB)\sqrt{\tan(c+dx)}}{8a^2d(1+i\tan(c+dx))} + \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}
\end{aligned}
$$

output
```
-1/32*((1+3*I)*A+(9+5*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a^
2/d-1/32*((1+3*I)*A+(9+5*I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/
a^2/d-1/32*((1-3*I)*A+(-9+5*I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(
d*x+c)))*2^(1/2)/a^2/d+1/8*(A+5*I*B)*tan(d*x+c)^(1/2)/a^2/d/(1+I*tan(d*x+c
))+1/4*(I*A-B)*tan(d*x+c)^(3/2)/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{-2\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))+\sqrt[4]{-}}{}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^2,x]
```

output
```
(-2*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x
]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + (-1)^(1/4)*(A - (7*I)*B)*Arc
Tanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*S
in[2*(c + d*x)]) + Sqrt[Tan[c + d*x]]*(-A - (5*I)*B + ((-3*I)*A + 7*B)*Tan
[c + d*x]))/(8*a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4078, 27, 3042, 4078, 25, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int\frac{\sqrt{\tan(c+dx)}(3a(iA-B)-a(A-7iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)}dx}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int\frac{\sqrt{\tan(c+dx)}(3a(iA-B)-a(A-7iB)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int\frac{\sqrt{\tan(c+dx)}(3a(iA-B)-a(A-7iB)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{8a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{-\frac{\int-\frac{(A+5iB)a^2+3(iA+3B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{(A+5iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\frac{(A+5iB)a^2+3(iA+3B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{(A+5iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\frac{(A+5iB)a^2+3(iA+3B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{(A+5iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\frac{a^2(A+5iB+3(iA+3B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{a^2d}-\frac{(A+5iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\frac{A+5iB+3(iA+3B)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{(A+5iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$$\Big\downarrow \text{1482}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1-3i)A-(9-5i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1+3i)A+(9+5i)B)\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{(A+5iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1-3i)A-(9-5i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1+3i)A+(9+5i)B)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{}{\tan(c+dx)+\sqrt{}}\right.}{d}}{8a^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1-3i)A-(9-5i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1+3i)A+(9+5i)B)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}}\right.}{d}}{8a^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1-3i)A-(9-5i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1+3i)A+(9+5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)}{d}-\frac{(A+5i}{d(1+}}{8a^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1-3i)A-(9-5i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((1+3i)A+(9+5i)B)\Big(}{d}}{8a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1-3i)A-(9-5i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((1+3i)A+(9+5i)B)\Big(\frac{\text{arcta}}{}}{d}}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{1}{2}((1-3i)A-(9-5i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}((1+3i)A+(9+5i)B)\Big(}{d}}{8a^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{1}{2}((1+3i)A+(9+5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((1-3i)A-(9-5i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log(t}{}\right.}{d}}{8a^2}$$

input
```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/8*(((((1 + 3*I)*A + (9 + 5*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + (((1 -
 3*I)*A - (9 - 5*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt
[2])))/2)/d - ((A + (5*I)*B)*Sqrt[Tan[c + d*x]])/(d*(1 + I*Tan[c + d*x])))
/a^2 + ((I*A - B)*Tan[c + d*x]^(3/2))/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{\frac{i\left(\frac{\left(-\frac{7iB}{2}-\frac{3A}{2}\right)\tan(dx+c)^{\frac{3}{2}}+\left(-\frac{5B}{2}+\frac{iA}{2}\right)\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^2}-\frac{(-7iB+A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{4}-\frac{i(-iB+A)\arctan\left(\frac{2\sqrt{\tan(dx+}}{\sqrt{2}+i\sqrt{2}}\right.}{2\left(\sqrt{2}+i\sqrt{2}\right)}}{d\,a^2}$
default	$\frac{\frac{i\left(\frac{\left(-\frac{7iB}{2}-\frac{3A}{2}\right)\tan(dx+c)^{\frac{3}{2}}+\left(-\frac{5B}{2}+\frac{iA}{2}\right)\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^2}-\frac{(-7iB+A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{4}-\frac{i(-iB+A)\arctan\left(\frac{2\sqrt{\tan(dx+}}{\sqrt{2}+i\sqrt{2}}\right.}{2\left(\sqrt{2}+i\sqrt{2}\right)}}{d\,a^2}$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^2*(1/4*I*(((-7/2*I*B-3/2*A)*tan(d*x+c)^(3/2)+(-5/2*B+1/2*I*A)*tan(d*
x+c)^(1/2))/(-I+tan(d*x+c))^2-(-7*I*B+A)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(
d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))-1/2*I*(A-I*B)/(2^(1/2)+I*2^(1/2))*arcta
n(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 664 vs. 2(172) = 344.

Time = 0.12 (sec) , antiderivative size = 664, normalized size of antiderivative = 2.93

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
1/32*(2*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)
*log(-2*((I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2
)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a^
2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*((
-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)) - (A -
 I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + a^2*d*sqrt((I
*A^2 + 14*A*B - 49*I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(1/8*((a^2*d*e
^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((I*A^2 + 14*A*B - 49*I*B^2)/(a^4*d^2)) + I*A + 7*B)*e^
(-2*I*d*x - 2*I*c)/(a^2*d)) - a^2*d*sqrt((I*A^2 + 14*A*B - 49*I*B^2)/(a^4*
d^2))*e^(4*I*d*x + 4*I*c)*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 1
4*A*B - 49*I*B^2)/(a^4*d^2)) - I*A - 7*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) +
2*(2*(A + 3*I*B)*e^(4*I*d*x + 4*I*c) + (A + 5*I*B)*e^(2*I*d*x + 2*I*c) - A
 - I*B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-
4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{A\tan^{\frac{3}{2}}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx+\int \frac{B\tan^{\frac{5}{2}}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-(Integral(A*tan(c + d*x)**(3/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1),
 x) + Integral(B*tan(c + d*x)**(5/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) -
 1), x))/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.51

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{2}((i-1)\ A+(7i+7)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan{(dx+c)}}\right)+2\sqrt{2}((i+1)\ A-(i-1)\ B)\arctan}{16\,a^2d}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output
```
-1/16*(sqrt(2)*((I - 1)*A + (7*I + 7)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sqrt
(tan(d*x + c))) + 2*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-(1/2*I - 1/2)*
sqrt(2)*sqrt(tan(d*x + c))) + 2*(3*I*A*tan(d*x + c)^(3/2) - 7*B*tan(d*x +
c)^(3/2) + A*sqrt(tan(d*x + c)) + 5*I*B*sqrt(tan(d*x + c)))/(tan(d*x + c)
- I)^2)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 7.36 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.40

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{-\frac{3A\tan(c+dx)^{3/2}}{8a^2d}+\frac{A\sqrt{\tan(c+dx)}\,1\mathrm{i}}{8a^2d}}{\tan(c+dx)^2\,1\mathrm{i}+2\tan(c+dx)-\mathrm{i}}+\frac{\frac{5B\sqrt{\tan(c+dx)}}{8a^2d}+\frac{B\tan(c+dx)^{3/2}\,7\mathrm{i}}{8a^2d}}{\tan(c+dx)^2\,1\mathrm{i}+2\tan(c+dx)-\mathrm{i}} - \mathrm{atan}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{-\frac{A^2\,1\mathrm{i}}{64a^4d^2}}}{A}\right)\sqrt{-\frac{A^2\,1\mathrm{i}}{64a^4d^2}}\,2\mathrm{i} - \mathrm{atan}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2\,1\mathrm{i}}{256a^4d^2}}}{A}\right)\sqrt{\frac{A^2\,1\mathrm{i}}{256a^4d^2}}\,2\mathrm{i} + 2\,\mathrm{atanh}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2\,1\mathrm{i}}{64a^4d^2}}}{B}\right)\sqrt{\frac{B^2\,1\mathrm{i}}{64a^4d^2}} + 2\,\mathrm{atanh}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{-\frac{B^2\,49\mathrm{i}}{256a^4d^2}}}{7B}\right)\sqrt{-\frac{B^2\,49\mathrm{i}}{256a^4d^2}}$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
((5*B*tan(c + d*x)^(1/2))/(8*a^2*d) + (B*tan(c + d*x)^(3/2)*7i)/(8*a^2*d))
/(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i) - ((A*tan(c + d*x)^(1/2)*1i)/(8
*a^2*d) - (3*A*tan(c + d*x)^(3/2))/(8*a^2*d))/(2*tan(c + d*x) + tan(c + d*
x)^2*1i - 1i) - atan((8*a^2*d*tan(c + d*x)^(1/2)*(-(A^2*1i)/(64*a^4*d^2))^
(1/2))/A)*(-(A^2*1i)/(64*a^4*d^2))^(1/2)*2i - atan((16*a^2*d*tan(c + d*x)^
(1/2)*((A^2*1i)/(256*a^4*d^2))^(1/2))/A)*((A^2*1i)/(256*a^4*d^2))^(1/2)*2i
 + 2*atanh((8*a^2*d*tan(c + d*x)^(1/2)*((B^2*1i)/(64*a^4*d^2))^(1/2))/B)*(
(B^2*1i)/(64*a^4*d^2))^(1/2) + 2*atanh((16*a^2*d*tan(c + d*x)^(1/2)*(-(B^2
*49i)/(256*a^4*d^2))^(1/2))/(7*B))*(-(B^2*49i)/(256*a^4*d^2))^(1/2)
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a}{a^2}$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)**2 - 2*tan(c +
 d*x)*i - 1),x)*b + int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)**2
 - 2*tan(c + d*x)*i - 1),x)*a))/a**2
```

3.142 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 229

$$\begin{aligned}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx &= \frac{((-1+3i)A+(1+3i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} \\ &\quad - \frac{((-1+3i)A+(1+3i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} \\ &\quad - \frac{((1+3i)A+(1-3i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{16\sqrt{2}a^2d} \\ &\quad + \frac{(iA+3B)\sqrt{\tan(c+dx)}}{8a^2d(1+i\tan(c+dx))} + \frac{(iA-B)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}\end{aligned}$$

output

```
-1/32*((-1+3*I)*A+(1+3*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a
^2/d-1/32*((-1+3*I)*A+(1+3*I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2
)/a^2/d-1/32*((1+3*I)*A+(1-3*I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan
(d*x+c)))*2^(1/2)/a^2/d+1/8*(I*A+3*B)*tan(d*x+c)^(1/2)/a^2/d/(1+I*tan(d*x+
c))+1/4*(I*A-B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{-2\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))+\sqrt[4]{-}}{}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^2,x]
```

output
```
(-2*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x
]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + (-1)^(1/4)*((-I)*A + B)*ArcT
anh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Si
n[2*(c + d*x)]) + Sqrt[Tan[c + d*x]]*((-3*I)*A - B + (A - (3*I)*B)*Tan[c +
 d*x]))/(8*a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4078, 27, 3042, 4079, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\int\frac{a(iA-B)-a(3A-5iB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{4a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\int\frac{a(iA-B)-a(3A-5iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{8a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\int\frac{a(iA-B)-a(3A-5iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{8a^2}$$

$\Big\downarrow$ 4079

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\frac{a^2(3iA+B)-a^2(A-3iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{(3B+iA)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\frac{a^2(3iA+B)-a^2(A-3iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{(3B+iA)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$\Big\downarrow$ 4017

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\frac{a^2(3iA+B-(A-3iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{a^2d}-\frac{(3B+iA)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int\frac{3iA+B-(A-3iB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{(3B+iA)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$\Big\downarrow$ 1482

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1+3i)B-(1-3i)A)\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{(3B+iA)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}$$

$\Big\downarrow$ 1476

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1+3i)B-(1-3i)A)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{}{\tan(c+dx)+\sqrt{}}\right.}{d}}{8a^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1+3i)B-(1-3i)A)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}}\right.}{d}}{8a^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((1+3i)B-(1-3i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)}{d}-\frac{(3B+i}{d(1+}}{8a^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((1+3i)B-(1-3i)A)\Big(}{d}}{8a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((1+3i)B-(1-3i)A)\left(\frac{\text{arcta}}{}\right.}{d}}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}((1+3i)B-(1-3i)A)\left(\cdots\right.}{d}}{8a^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{1}{2}((1+3i)B-(1-3i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((1+3i)A+(1-3i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log(\cdots}{}\right.}{d}}{8a^2}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
-1/8*(((((-1 + 3*I)*A + (1 + 3*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + (((1
+ 3*I)*A + (1 - 3*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqr
t[2])))/2)/d - ((I*A + 3*B)*Sqrt[Tan[c + d*x]])/(d*(1 + I*Tan[c + d*x])))/
a^2 + ((I*A - B)*Sqrt[Tan[c + d*x]])/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.63

method	result	si
derivativedivides	$\frac{\frac{\left(-\frac{3iB}{2}+\frac{A}{2}\right)\tan(dx+c)^{\frac{3}{2}}+\left(-\frac{B}{2}-\frac{3iA}{2}\right)\sqrt{\tan(dx+c)}}{4(-i+\tan(dx+c))^2}-\frac{(iB+A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}-\frac{i(iA+B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{2\left(\sqrt{2}+i\sqrt{2}\right)}}{d\,a^2}$	1
default	$\frac{\frac{\left(-\frac{3iB}{2}+\frac{A}{2}\right)\tan(dx+c)^{\frac{3}{2}}+\left(-\frac{B}{2}-\frac{3iA}{2}\right)\sqrt{\tan(dx+c)}}{4(-i+\tan(dx+c))^2}-\frac{(iB+A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}-\frac{i(iA+B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{2\left(\sqrt{2}+i\sqrt{2}\right)}}{d\,a^2}$	1

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^2*(1/4*((-3/2*I*B+1/2*A)*tan(d*x+c)^(3/2)+(-1/2*B-3/2*I*A)*tan(d*x+c
)^(1/2))/(-I+tan(d*x+c))^2-1/4*(A+I*B)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d*
x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))-1/2*I*(I*A+B)/(2^(1/2)+I*2^(1/2))*arctan(2
*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(174) = 348$.

Time = 0.14 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.87

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
-1/32*(2*a^2*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)
*log(2*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) + (
A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a^2*d*sq
rt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*((a^2*d*e
^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) - (A - I*B)*e^(2*I*
d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a^2*d*sqrt((-I*A^2 + 2*A*B
 + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(1/8*((a^2*d*e^(2*I*d*x + 2*I*
c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt((-I*A^2 + 2*A*B + I*B^2)/(a^4*d^2)) + A + I*B)*e^(-2*I*d*x - 2*I*c)/(a
^2*d)) + a^2*d*sqrt((-I*A^2 + 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c
)*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 + 2*A*B + I*B^2)/(a^4*d^2)
) - A - I*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) + 2*(2*(-I*A - B)*e^(4*I*d*x +
4*I*c) - (3*I*A + B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt((-I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{A\sqrt{\tan(c+dx)}}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx + \int \frac{B\tan^{\frac{3}{2}}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-(Integral(A*sqrt(tan(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1),
x) + Integral(B*tan(c + d*x)**(3/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) -
1), x))/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{2}((i+1)\ A+(i-1)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+2\sqrt{2}((i-1)\ A+(i+1)\ B)\arcta}{16\,a^2d}$$

input

```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output

```
-1/16*(sqrt(2)*((I + 1)*A + (I - 1)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(t
an(d*x + c))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-(1/2*I - 1/2)*sq
rt(2)*sqrt(tan(d*x + c))) - 2*(A*tan(d*x + c)^(3/2) - 3*I*B*tan(d*x + c)^(
3/2) - 3*I*A*sqrt(tan(d*x + c)) - B*sqrt(tan(d*x + c)))/(tan(d*x + c) - I)
^2)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{3A\sqrt{\tan(c+dx)}}{8a^2d}+\frac{A\tan(c+dx)^{3/2}\,1\mathrm{i}}{8a^2d}}{\tan(c+dx)^2\,1\mathrm{i}+2\tan(c+dx)-\mathrm{i}}-\frac{-\frac{3B\tan(c+dx)^{3/2}}{8a^2d}+\frac{B\sqrt{\tan(c+dx)}\,1\mathrm{i}}{8a^2d}}{\tan(c+dx)^2\,1\mathrm{i}+2\tan(c+dx)-\mathrm{i}}$$
$$-2\,\mathrm{atanh}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{A^2\,1\mathrm{i}}{64a^4d^2}}}{A}\right)\sqrt{\frac{A^2\,1\mathrm{i}}{64a^4d^2}}$$
$$+2\,\mathrm{atanh}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{-\frac{A^2\,1\mathrm{i}}{256a^4d^2}}}{A}\right)\sqrt{-\frac{A^2\,1\mathrm{i}}{256a^4d^2}}$$
$$-\mathrm{atan}\left(\frac{8a^2d\sqrt{\tan(c+dx)}\sqrt{-\frac{B^2\,1\mathrm{i}}{64a^4d^2}}}{B}\right)\sqrt{-\frac{B^2\,1\mathrm{i}}{64a^4d^2}}\,2\mathrm{i}$$
$$-\mathrm{atan}\left(\frac{16a^2d\sqrt{\tan(c+dx)}\sqrt{\frac{B^2\,1\mathrm{i}}{256a^4d^2}}}{B}\right)\sqrt{\frac{B^2\,1\mathrm{i}}{256a^4d^2}}\,2\mathrm{i}$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
((3*A*tan(c + d*x)^(1/2))/(8*a^2*d) + (A*tan(c + d*x)^(3/2)*1i)/(8*a^2*d))
/(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i) - ((B*tan(c + d*x)^(1/2)*1i)/(8
*a^2*d) - (3*B*tan(c + d*x)^(3/2))/(8*a^2*d))/(2*tan(c + d*x) + tan(c + d*
x)^2*1i - 1i) - 2*atanh((8*a^2*d*tan(c + d*x)^(1/2)*((A^2*1i)/(64*a^4*d^2)
)^(1/2))/A)*((A^2*1i)/(64*a^4*d^2))^(1/2) + 2*atanh((16*a^2*d*tan(c + d*x)
^(1/2)*(-(A^2*1i)/(256*a^4*d^2))^(1/2))/A)*(-(A^2*1i)/(256*a^4*d^2))^(1/2)
 - atan((8*a^2*d*tan(c + d*x)^(1/2)*(-(B^2*1i)/(64*a^4*d^2))^(1/2))/B)*(-(
B^2*1i)/(64*a^4*d^2))^(1/2)*2i - atan((16*a^2*d*tan(c + d*x)^(1/2)*((B^2*1
i)/(256*a^4*d^2))^(1/2))/B)*((B^2*1i)/(256*a^4*d^2))^(1/2)*2i
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b}{a^2}$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(sqrt(tan(c + d*x))/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a
+ int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)**2 - 2*tan(c + d*x)*
i - 1),x)*b))/a**2
```

3.143 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 231

$$\begin{aligned}
\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}\,dx &= \frac{\left(\frac{1}{16}+\frac{i}{16}\right)((-2+7i)A+(1+2i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d} \\
&+ \frac{((9-5i)A+(1-3i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} \\
&+ \frac{((9+5i)A-(1+3i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{16\sqrt{2}a^2d} \\
&+ \frac{(5A+iB)\sqrt{\tan(c+dx)}}{8a^2d(1+i\tan(c+dx))} + \frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}
\end{aligned}$$

output
```
(-1/32-1/32*I)*((-2+7*I)*A+(1+2*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*
2^(1/2)/a^2/d+1/32*((9-5*I)*A+(1-3*I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)*2^(1/2)/a^2/d+1/32*((9+5*I)*A-(1+3*I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2
)/(1+tan(d*x+c)))*2^(1/2)/a^2/d+1/8*(5*A+I*B)*tan(d*x+c)^(1/2)/a^2/d/(1+I*
tan(d*x+c))+1/4*(A+I*B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.77

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{2\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))+\sqrt[4]{-1}(}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^
2),x]
```

output
```
(2*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]
^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + (-1)^(1/4)*(7*A - I*B)*ArcTan
h[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[
2*(c + d*x)]) + Sqrt[Tan[c + d*x]]*(-7*A - (3*I)*B + ((-5*I)*A + B)*Tan[c
+ d*x]))/(8*a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4079, 27, 3042, 4079, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(7A-iB)-3a(iA-B)\tan(c+dx)}{2\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{4a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a(7A-iB)-3a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{8a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(7A-iB)-3a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{8a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\int \frac{3a^2(3A-iB)-a^2(5iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{2a^2}+\frac{(5A+iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{3a^2(3A-iB)-a^2(5iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{2a^2}+\frac{(5A+iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 4017

$$\frac{\frac{\int \frac{a^2(3(3A-iB)-(5iA-B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{a^2d}+\frac{(5A+iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{3(3A-iB)-(5iA-B)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{(5A+iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 1482

$$\frac{\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((9-5i)A+(1-3i)B)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{(5A+iB)\sqrt{\tan(c+dx)}}{d(1+i\tan(c+dx))}}{8a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 1476

$$\frac{\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((9-5i)A+(1-3i)B)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{}{\tan(c+dx)+\sqrt{}}\right.}{d}}{8a^2}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\downarrow$ 1082

$$\frac{\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((9-5i)A+(1-3i)B)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}}\right.}{d}}{8a^2}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\downarrow$ 217

$$\frac{\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((9-5i)A+(1-3i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)}{d}+\frac{(5A+i}{d(1+}}{8a^2}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\downarrow$ 1479

$$\frac{\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((9-5i)A+(1-3i)B)\left(\right.}{d}}{8a^2}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\downarrow$ 25

$$\frac{\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((9-5i)A+(1-3i)B)\left(\frac{\text{arcta}}{}\right.}{d}}{8a^2}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}((9-5i)A+(1-3i)B)\Big(}{d}}{8a^2}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\frac{1}{2}((9-5i)A+(1-3i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((9+5i)A-(1+3i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log(t}{}\right.}{d}}{8a^2}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{4d(a+ia\tan(c+dx))^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(((((9 - 5*I)*A + (1 - 3*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/S
qrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + (((9 + 5*I)
*A - (1 + 3*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]))
)/2)/d + ((5*A + I*B)*Sqrt[Tan[c + d*x]])/(d*(1 + I*Tan[c + d*x])))/(8*a^2
) + ((A + I*B)*Sqrt[Tan[c + d*x]])/(4*d*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{-\frac{\left(\frac{5iA}{2}-\frac{B}{2}\right)\tan(dx+c)^{\frac{3}{2}}+\left(\frac{7A}{2}+\frac{3iB}{2}\right)\sqrt{\tan(dx+c)}}{4(-i+\tan(dx+c))^{2}}-\frac{(7iA+B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{i(-iB+A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{2\sqrt{2}+2i\sqrt{2}}}{d\,a^{2}}$
default	$\frac{-\frac{\left(\frac{5iA}{2}-\frac{B}{2}\right)\tan(dx+c)^{\frac{3}{2}}+\left(\frac{7A}{2}+\frac{3iB}{2}\right)\sqrt{\tan(dx+c)}}{4(-i+\tan(dx+c))^{2}}-\frac{(7iA+B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{i(-iB+A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{2\sqrt{2}+2i\sqrt{2}}}{d\,a^{2}}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^2*(-1/4*((5/2*I*A-1/2*B)*tan(d*x+c)^(3/2)+(7/2*A+3/2*I*B)*tan(d*x+c)
^(1/2))/(-I+tan(d*x+c))^2-1/4*(7*I*A+B)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d
*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))+1/2*I*(A-I*B)/(2^(1/2)+I*2^(1/2))*arctan(
2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 664 vs. 2(174) = 348.

Time = 0.10 (sec) , antiderivative size = 664, normalized size of antiderivative = 2.87

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
-1/32*(2*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c
)*log(-2*((I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^
2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a
^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*(
(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)) - (A
- I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a^2*d*sqrt((
49*I*A^2 + 14*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(1/8*((a^2*d*
e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt((49*I*A^2 + 14*A*B - I*B^2)/(a^4*d^2)) + 7*I*A + B)*e
^(-2*I*d*x - 2*I*c)/(a^2*d)) + a^2*d*sqrt((49*I*A^2 + 14*A*B - I*B^2)/(a^4
*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*s
qrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((49*I*A^2
 + 14*A*B - I*B^2)/(a^4*d^2)) - 7*I*A - B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) -
 2*(2*(3*A + I*B)*e^(4*I*d*x + 4*I*c) + (7*A + 3*I*B)*e^(2*I*d*x + 2*I*c)
+ A + I*B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e
^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.51

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{2}((7i-7)\ A+(i+1)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\,\mathrm{ar}}{16\,a^2d}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output
```
-1/16*(sqrt(2)*((7*I - 7)*A + (I + 1)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sqrt
(tan(d*x + c))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-(1/2*I - 1/2)
*sqrt(2)*sqrt(tan(d*x + c))) + 2*(5*I*A*tan(d*x + c)^(3/2) - B*tan(d*x + c
)^(3/2) + 7*A*sqrt(tan(d*x + c)) + 3*I*B*sqrt(tan(d*x + c)))/(tan(d*x + c)
 - I)^2)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.38

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= -\frac{-\frac{5\,A\tan(c+d\,x)^{3/2}}{8\,a^2\,d}+\frac{A\,\sqrt{\tan(c+d\,x)}\,7\mathrm{i}}{8\,a^2\,d}}{\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}}+\frac{\frac{3\,B\,\sqrt{\tan(c+d\,x)}}{8\,a^2\,d}+\frac{B\tan(c+d\,x)^{3/2}\,1\mathrm{i}}{8\,a^2\,d}}{\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}}$$
$$+\operatorname{atan}\left(\frac{8\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,1\mathrm{i}}{64\,a^4\,d^2}}}{A}\right)\sqrt{-\frac{A^2\,1\mathrm{i}}{64\,a^4\,d^2}}\,2\mathrm{i}$$
$$-\operatorname{atan}\left(\frac{16\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,49\mathrm{i}}{256\,a^4\,d^2}}}{7\,A}\right)\sqrt{\frac{A^2\,49\mathrm{i}}{256\,a^4\,d^2}}\,2\mathrm{i}$$
$$-2\operatorname{atanh}\left(\frac{8\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,1\mathrm{i}}{64\,a^4\,d^2}}}{B}\right)\sqrt{\frac{B^2\,1\mathrm{i}}{64\,a^4\,d^2}}$$
$$+2\operatorname{atanh}\left(\frac{16\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1\mathrm{i}}{256\,a^4\,d^2}}}{B}\right)\sqrt{-\frac{B^2\,1\mathrm{i}}{256\,a^4\,d^2}}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
((3*B*tan(c + d*x)^(1/2))/(8*a^2*d) + (B*tan(c + d*x)^(3/2)*1i)/(8*a^2*d))
/(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i) - ((A*tan(c + d*x)^(1/2)*7i)/(8
*a^2*d) - (5*A*tan(c + d*x)^(3/2))/(8*a^2*d))/(2*tan(c + d*x) + tan(c + d*
x)^2*1i - 1i) + atan((8*a^2*d*tan(c + d*x)^(1/2)*(-(A^2*1i)/(64*a^4*d^2))^
(1/2))/A)*(-(A^2*1i)/(64*a^4*d^2))^(1/2)*2i - atan((16*a^2*d*tan(c + d*x)^
(1/2)*((A^2*49i)/(256*a^4*d^2))^(1/2))/(7*A))*((A^2*49i)/(256*a^4*d^2))^(1
/2)*2i - 2*atanh((8*a^2*d*tan(c + d*x)^(1/2)*((B^2*1i)/(64*a^4*d^2))^(1/2)
)/B)*((B^2*1i)/(64*a^4*d^2))^(1/2) + 2*atanh((16*a^2*d*tan(c + d*x)^(1/2)*
(-(B^2*1i)/(256*a^4*d^2))^(1/2))/B)*(-(B^2*1i)/(256*a^4*d^2))^(1/2)
```

Reduce [F]

$$\int \frac{A + B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{-\left(\int \frac{\tan(dx+c)}{\sqrt{\tan(dx+c)}\tan(dx+c)^2 - 2\sqrt{\tan(dx+c)}\tan(dx+c)i - \sqrt{\tan(dx+c)}}dx\right)b - \left(\int \frac{1}{\sqrt{\tan(dx+c)}\tan(dx+c)^2 - 2\sqrt{\tan(dx+c)}\tan(dx+c)i - \sqrt{\tan(dx+c)}}dx\right)a}{a^2}$$

input

```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x)
```

output

```
( - (int(tan(c + d*x)/(sqrt(tan(c + d*x))*tan(c + d*x)**2 - 2*sqrt(tan(c +
 d*x))*tan(c + d*x)*i - sqrt(tan(c + d*x))),x)*b + int(1/(sqrt(tan(c + d*x
))*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x))*tan(c + d*x)*i - sqrt(tan(c + d*
x))),x)*a))/a**2
```

3.144 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 264

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx \\
&= \frac{((25+21i)A-(9-5i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^2d} \\
&\quad - \frac{\left(\frac{1}{16}-\frac{i}{16}\right)((2+23i)A-(7+2i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d} \\
&\quad + \frac{\left(\frac{1}{16}-\frac{i}{16}\right)((23+2i)A+(2+7i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}a^2d} - \frac{5(5A+iB)}{8a^2d\sqrt{\tan(c+dx)}} \\
&\quad + \frac{7A+3iB}{8a^2d(1+i\tan(c+dx))\sqrt{\tan(c+dx)}} + \frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}
\end{aligned}$$

output

```
-1/32*((25+21*I)*A+(-9+5*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)
/a^2/d+(-1/32+1/32*I)*((2+23*I)*A-(7+2*I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(
1/2))*2^(1/2)/a^2/d+(1/32-1/32*I)*((23+2*I)*A+(2+7*I)*B)*arctanh(2^(1/2)*t
an(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/a^2/d-5/8*(5*A+I*B)/a^2/d/tan(d*x+
c)^(1/2)+1/8*(7*A+3*I*B)/a^2/d/(1+I*tan(d*x+c))/tan(d*x+c)^(1/2)+1/4*(A+I*
B)/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.68

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\sec^2(c+dx)\left(-2\cos(c+dx)((9A+5iB)\cos(c+dx)+(7iA-3B)\sin(c+dx))+2(23A+7iB)\,\text{Hyper}\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^
2),x]
```

output
```
(Sec[c + d*x]^2*(-2*Cos[c + d*x]*((9*A + (5*I)*B)*Cos[c + d*x] + ((7*I)*A
- 3*B)*Sin[c + d*x]) + 2*(23*A + (7*I)*B)*Hypergeometric2F1[-1/2, 1, 1/2,
(-I)*Tan[c + d*x]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + 4*(A - I*B)*H
ypergeometric2F1[-1/2, 1, 1/2, I*Tan[c + d*x]]*(Cos[2*(c + d*x)] + I*Sin[2
*(c + d*x)])))/(16*a^2*d*Sqrt[Tan[c + d*x]]*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.08, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4079, 27, 3042, 4079, 3042, 4012, 25, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(9A+iB)-5a(iA-B)\tan(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)}dx}{4a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a(9A+iB)-5a(iA-B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)}dx}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(9A+iB)-5a(iA-B)\tan(c+dx)}{\tan(c+dx)^{3/2}(i\tan(c+dx)a+a)}dx}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\int \frac{5a^2(5A+iB)-3a^2(7iA-3B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{7A+3iB}{d(1+i\tan(c+dx))\sqrt{\tan(c+dx)}}}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{5a^2(5A+iB)-3a^2(7iA-3B)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx}{2a^2}+\frac{7A+3iB}{d(1+i\tan(c+dx))\sqrt{\tan(c+dx)}}}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\int -\frac{3(7iA-3B)a^2+5(5A+iB)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{10a^2(5A+iB)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{7A+3iB}{d(1+i\tan(c+dx))\sqrt{\tan(c+dx)}}}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{-\int \frac{3(7iA-3B)a^2+5(5A+iB)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{10a^2(5A+iB)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{7A+3iB}{d(1+i\tan(c+dx))\sqrt{\tan(c+dx)}}}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{-\int \frac{3(7iA-3B)a^2+5(5A+iB)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx-\frac{10a^2(5A+iB)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{7A+3iB}{d(1+i\tan(c+dx))\sqrt{\tan(c+dx)}}}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{-\frac{2\int \frac{a^2(3(7iA-3B)+5(5A+iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{10a^2(5A+iB)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{7A+3iB}{d(1+i\tan(c+dx))\sqrt{\tan(c+dx)}}}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{2a^2\int \frac{3(7iA-3B)+5(5A+iB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{10a^2(5A+iB)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{7A+3iB}{d(1+i\tan(c+dx))\sqrt{\tan(c+dx)}}}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{-\frac{2a^2\left(\frac{1}{2}((25+21i)A-(9-5i)B)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{10a^2(5A+iB)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{\cdots}{d(1+i\tan(\cdots}}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{-\frac{2a^2\left(\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+\cdots}{d}}{2a^2}}{8a^2}+\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{\frac{2a^2\left(\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\int \frac{1-\tan}{\tan^2(c}}{d}}{2a^2}}{8a^2}$$

$$\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\downarrow$ 217

$$-\frac{\frac{2a^2\left(\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-}{2a^2}}{8a^2}$$

$$\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\downarrow$ 1479

$$-\frac{\frac{2a^2\left(\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}}}{2\sqrt{2}}\right.}{d}}{2a^2}}{8a^2}$$

$$\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\downarrow$ 25

$$-\frac{\frac{2a^2\left(\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}}{2\sqrt{2}}\right.}{d}}{2a^2}}{8a^2}$$

$$\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}}{2\sqrt{2}}\right.}{d}}{2a^2}}{8a^2}$$

$$\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+\right.}{2\sqrt{2}}\right.}{d}}{2a^2}}{8a^2}$$

$$\frac{A+iB}{4d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}$$

input

```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output

```
(((-2*a^2*((((25 + 21*I)*A - (9 - 5*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c
 + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 -
(1/2 - I/2)*((23 + 2*I)*A + (2 + 7*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]]/(2*Sqrt[2]))))/d - (10*a^2*(5*A + I*B))/(d*Sqrt[Tan[c + d*x]]))
/(2*a^2) + (7*A + (3*I)*B)/(d*(1 + I*Tan[c + d*x])*Sqrt[Tan[c + d*x]]))/(8
*a^2) + (A + I*B)/(4*d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\frac{\left(-\frac{9A}{2}-\frac{5iB}{2}\right)\tan\left(dx+c\right)^{\frac{3}{2}}+\left(\frac{11iA}{2}-\frac{7B}{2}\right)\sqrt{\tan\left(dx+c\right)}}{4\left(-i+\tan\left(dx+c\right)\right)^{2}}-\frac{\left(7iB+23A\right)\arctan\left(\frac{2\sqrt{\tan\left(dx+c\right)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{i\left(iA+B\right)\arctan\left(\frac{2\sqrt{\tan\left(dx+c\right)}}{\sqrt{2}+i\sqrt{2}}\right)}{2\sqrt{2}+2i\sqrt{2}}-}{d\,a^{2}}$
default	$\frac{\frac{\left(-\frac{9A}{2}-\frac{5iB}{2}\right)\tan\left(dx+c\right)^{\frac{3}{2}}+\left(\frac{11iA}{2}-\frac{7B}{2}\right)\sqrt{\tan\left(dx+c\right)}}{4\left(-i+\tan\left(dx+c\right)\right)^{2}}-\frac{\left(7iB+23A\right)\arctan\left(\frac{2\sqrt{\tan\left(dx+c\right)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{i\left(iA+B\right)\arctan\left(\frac{2\sqrt{\tan\left(dx+c\right)}}{\sqrt{2}+i\sqrt{2}}\right)}{2\sqrt{2}+2i\sqrt{2}}-}{d\,a^{2}}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^2*(1/4*((-9/2*A-5/2*I*B)*tan(d*x+c)^(3/2)+(11/2*I*A-7/2*B)*tan(d*x+c
)^(1/2))/(-I+tan(d*x+c))^2-1/4*(23*A+7*I*B)/(2^(1/2)-I*2^(1/2))*arctan(2*t
an(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))+1/2*I*(I*A+B)/(2^(1/2)+I*2^(1/2))*arc
tan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))-2*A/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(197) = 394$.

Time = 0.12 (sec) , antiderivative size = 763, normalized size of antiderivative = 2.89

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia\tan(c + dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
1/32*(2*(a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))*sqrt((I*A^
2 + 2*A*B - I*B^2)/(a^4*d^2))*log(2*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*s
qrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 +
2*A*B - I*B^2)/(a^4*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2
*I*c)/(I*A + B)) - 2*(a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x + 4*I*c
))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*log(-2*((a^2*d*e^(2*I*d*x + 2*I
*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e
^(-2*I*d*x - 2*I*c)/(I*A + B)) + (a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I
*d*x + 4*I*c))*sqrt((-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2))*log(1/8*(
(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt((-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2)) +
 23*A + 7*I*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) - (a^2*d*e^(6*I*d*x + 6*I*c)
- a^2*d*e^(4*I*d*x + 4*I*c))*sqrt((-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d
^2))*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-529*I*A^2 + 322*A*B + 49*I*B^
2)/(a^4*d^2)) - 23*A - 7*I*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) - 2*(6*(7*I*A
- B)*e^(6*I*d*x + 6*I*c) - (-33*I*A + B)*e^(4*I*d*x + 4*I*c) + 2*(-5*I*A +
 3*B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) + 1)))/(a^2*d*e^(6*I*d*x + 6*I*c) - a^2*d*e^(4*I*d*x ...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.48

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{2}((23i+23)\ A+(7i-7)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+2\sqrt{2}(-(i-1)\ A-(i+1)\ B}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output
```
-1/16*(sqrt(2)*((23*I + 23)*A + (7*I - 7)*B)*arctan((1/2*I + 1/2)*sqrt(2)*
sqrt(tan(d*x + c))) + 2*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-(1/2*I -
1/2)*sqrt(2)*sqrt(tan(d*x + c))) + 32*A/sqrt(tan(d*x + c)) + 2*(9*A*tan(d*
x + c)^(3/2) + 5*I*B*tan(d*x + c)^(3/2) - 11*I*A*sqrt(tan(d*x + c)) + 7*B*
sqrt(tan(d*x + c)))/(tan(d*x + c) - I)^2)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 7.18 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.28

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = -\frac{-\frac{5\,B\,{\tan\left(c+d\,x\right)}^{3/2}}{8\,a^2\,d}+\frac{B\,\sqrt{\tan\left(c+d\,x\right)}\,7{}\mathrm{i}}{8\,a^2\,d}}{{\tan\left(c+d\,x\right)}^2\,1{}\mathrm{i}+2\,\tan\left(c+d\,x\right)-\mathrm{i}} + 2\,\mathrm{atanh}\left(\frac{8\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,1{}\mathrm{i}}{64\,a^4\,d^2}}}{A}\right)\,\sqrt{\frac{A^2\,1{}\mathrm{i}}{64\,a^4\,d^2}} + 2\,\mathrm{atanh}\left(\frac{16\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,529{}\mathrm{i}}{256\,a^4\,d^2}}}{23\,A}\right)\,\sqrt{-\frac{A^2\,529{}\mathrm{i}}{256\,a^4\,d^2}} + \mathrm{atan}\left(\frac{8\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1{}\mathrm{i}}{64\,a^4\,d^2}}}{B}\right)\,\sqrt{-\frac{B^2\,1{}\mathrm{i}}{64\,a^4\,d^2}}\,2{}\mathrm{i} - \mathrm{atan}\left(\frac{16\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,49{}\mathrm{i}}{256\,a^4\,d^2}}}{7\,B}\right)\,\sqrt{\frac{B^2\,49{}\mathrm{i}}{256\,a^4\,d^2}}\,2{}\mathrm{i} - \frac{\frac{43\,A\,\tan\left(c+d\,x\right)}{8\,a^2\,d}-\frac{A\,2{}\mathrm{i}}{a^2\,d}+\frac{A\,{\tan\left(c+d\,x\right)}^2\,25{}\mathrm{i}}{8\,a^2\,d}}{2\,{\tan\left(c+d\,x\right)}^{3/2}-\sqrt{\tan\left(c+d\,x\right)}\,1{}\mathrm{i}+{\tan\left(c+d\,x\right)}^{5/2}\,1{}\mathrm{i}}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
2*atanh((8*a^2*d*tan(c + d*x)^(1/2)*((A^2*1i)/(64*a^4*d^2))^(1/2))/A)*((A^
2*1i)/(64*a^4*d^2))^(1/2) - ((B*tan(c + d*x)^(1/2)*7i)/(8*a^2*d) - (5*B*ta
n(c + d*x)^(3/2))/(8*a^2*d))/(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i) + 2
*atanh((16*a^2*d*tan(c + d*x)^(1/2)*(-(A^2*529i)/(256*a^4*d^2))^(1/2))/(23
*A))*(-(A^2*529i)/(256*a^4*d^2))^(1/2) + atan((8*a^2*d*tan(c + d*x)^(1/2)*
(-(B^2*1i)/(64*a^4*d^2))^(1/2))/B)*(-(B^2*1i)/(64*a^4*d^2))^(1/2)*2i - ata
n((16*a^2*d*tan(c + d*x)^(1/2)*((B^2*49i)/(256*a^4*d^2))^(1/2))/(7*B))*((B
^2*49i)/(256*a^4*d^2))^(1/2)*2i - ((43*A*tan(c + d*x))/(8*a^2*d) - (A*2i)/
(a^2*d) + (A*tan(c + d*x)^2*25i)/(8*a^2*d))/(2*tan(c + d*x)^(3/2) - tan(c
+ d*x)^(1/2)*1i + tan(c + d*x)^(5/2)*1i)
```

Reduce [F]

$$\int \frac{A + B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{-\left(\int \frac{1}{\sqrt{\tan(dx+c)}\,\tan(dx+c)^3-2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 i-\sqrt{\tan(dx+c)}\,\tan(dx+c)}dx\right)a-\left(\int \frac{1}{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2-2\sqrt{\tan(d}\right.}{a^2}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(1/(sqrt(tan(c + d*x))*tan(c + d*x)**3 - 2*sqrt(tan(c + d*x))*tan(
c + d*x)**2*i - sqrt(tan(c + d*x))*tan(c + d*x)),x)*a + int(1/(sqrt(tan(c
+ d*x))*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x))*tan(c + d*x)*i - sqrt(tan(c
 + d*x))),x)*b))/a**2
```

3.145 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 297

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx &= \frac{\left(\frac{1}{16}-\frac{i}{16}\right)((47+2i)A+(2+23i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d} \\ &\quad - \frac{\left(\frac{1}{16}-\frac{i}{16}\right)((47+2i)A+(2+23i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^2d} \\ &\quad - \frac{\left(\frac{1}{16}-\frac{i}{16}\right)((2+47i)A-(23+2i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}a^2d} \\ &\quad - \frac{7(7A+3iB)}{24a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{9A+5iB}{8a^2d(1+i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)} \\ &\quad + \frac{5(9iA-5B)}{8a^2d\sqrt{\tan(c+dx)}} + \frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\end{aligned}$$

output
```
(-1/32+1/32*I)*((47+2*I)*A+(2+23*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
*2^(1/2)/a^2/d+(-1/32+1/32*I)*((47+2*I)*A+(2+23*I)*B)*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))*2^(1/2)/a^2/d+(-1/32+1/32*I)*((2+47*I)*A-(23+2*I)*B)*arctan
h(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/a^2/d-7/24*(7*A+3*I*B)/
a^2/d/tan(d*x+c)^(3/2)+1/8*(9*A+5*I*B)/a^2/d/(1+I*tan(d*x+c))/tan(d*x+c)^(
3/2)+5/8*(9*I*A-5*B)/a^2/d/tan(d*x+c)^(1/2)+1/4*(A+I*B)/d/tan(d*x+c)^(3/2)
/(a+I*a*tan(d*x+c))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.61

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\sec^2(c+dx)\left(-6\cos(c+dx)((11A+7iB)\cos(c+dx)+(9iA-5B)\sin(c+dx))+2(47A+23iB)\,\mathrm{Hyp}\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^
2),x]
```

output
```
(Sec[c + d*x]^2*(-6*Cos[c + d*x]*((11*A + (7*I)*B)*Cos[c + d*x] + ((9*I)*A
 - 5*B)*Sin[c + d*x]) + 2*(47*A + (23*I)*B)*Hypergeometric2F1[-3/2, 1, -1/
2, (-I)*Tan[c + d*x]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + 4*(A - I*B
)*Hypergeometric2F1[-3/2, 1, -1/2, I*Tan[c + d*x]]*(Cos[2*(c + d*x)] + I*S
in[2*(c + d*x)])))/(48*a^2*d*Tan[c + d*x]^(3/2)*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.06, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4079, 27, 3042, 4079, 3042, 4012, 25, 3042, 4012, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}dx$$

$\Big\downarrow$ 3042

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+ia\tan(c+dx))^2}dx$$

$\Big\downarrow$ 4079

$$\frac{\int \frac{a(11A+3iB)-7a(iA-B)\tan(c+dx)}{2\tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)}dx}{4a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a(11A+3iB)-7a(iA-B)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)}dx}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(11A+3iB)-7a(iA-B)\tan(c+dx)}{\tan(c+dx)^{5/2}(i\tan(c+dx)a+a)}dx}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\int \frac{7a^2(7A+3iB)-5a^2(9iA-5B)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}dx}{2a^2}+\frac{9A+5iB}{d(1+i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{7a^2(7A+3iB)-5a^2(9iA-5B)\tan(c+dx)}{\tan(c+dx)^{5/2}}dx}{2a^2}+\frac{9A+5iB}{d(1+i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\int -\frac{5(9iA-5B)a^2+7(7A+3iB)\tan(c+dx)a^2}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{14a^2(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{9A+5iB}{d(1+i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{-\int \frac{5(9iA-5B)a^2+7(7A+3iB)\tan(c+dx)a^2}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{14a^2(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{9A+5iB}{d(1+i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\int \frac{5(9iA-5B)a^2+7(7A+3iB)\tan(c+dx)a^2}{\tan(c+dx)^{3/2}}dx-\frac{14a^2(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{9A+5iB}{d(1+i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{-\int \frac{7a^2(7A+3iB)-5a^2(9iA-5B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{14a^2(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{10a^2(-5B+9iA)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{9A+5iB}{d(1+i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\int \frac{7a^2(7A+3iB)-5a^2(9iA-5B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{14a^2(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{10a^2(-5B+9iA)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{9A+5iB}{d(1+i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{-\frac{2\int \frac{a^2(7(7A+3iB)-5(9iA-5B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{14a^2(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{10a^2(-5B+9iA)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{9A+5iB}{d(1+i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{2a^2\int\frac{7(7A+3iB)-5(9iA-5B)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{14a^2(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{10a^2(-5B+9iA)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{9A+5iB}{d(1+i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}}{8a^2}+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 1482

$$\frac{\frac{-\frac{2a^2\left(\frac{1}{2}((49+45i)A-(25-21i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)((47+2i)A+(2+23i)B)\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{14a^2(7A+3iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{10a^2(-5\cdots}{d\sqrt{\tan(\cdots}}\cdots}{2a^2}\cdots}{8a^2}\cdots+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 1476

$$\frac{\frac{-\frac{2a^2\left(\frac{1}{2}((49+45i)A-(25-21i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)((47+2i)A+(2+23i)B)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{\cdots}{\tan(c+\cdots}\cdots\right.}{d}\cdots}{2a^2}\cdots}{8a^2}\cdots+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 1082

$$\frac{\frac{-\frac{2a^2\left(\frac{1}{2}((49+45i)A-(25-21i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)((47+2i)A+(2+23i)B)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}\cdots}{\cdots}\cdots\right.}{d}\cdots}{2a^2}\cdots}{8a^2}\cdots+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{-\frac{2a^2\left(\frac{1}{2}((49+45i)A-(25-21i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)((47+2i)A+(2+23i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\cdots}{d}\cdots}{2a^2}\cdots}{8a^2}\cdots+\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \; 1479$$

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}((49+45i)A-(25-21i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)((47+2i)A\cdots}{d}}{2a^2}}{8a^2}$$

$$\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}((49+45i)A-(25-21i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)((47+2i)A+(2+\cdots}{d}}{2a^2}}{8a^2}$$

$$\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}((49+45i)A-(25-21i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)((47+2i)A+(2\cdots}{d}}{2a^2}}{8a^2}$$

$$\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \; 1103$$

$$-\frac{\frac{\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((47+2i)A+(2+23i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((49+45i)A-(25-21i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}\cdots\right.}{2\sqrt{2}}\cdots\right.}{d}}{2a^2}}{8a^2}$$

$$\frac{A+iB}{4d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}$$

input

```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(((-2*a^2*((1/2 - I/2)*((47 + 2*I)*A + (2 + 23*I)*B)*(-(ArcTan[1 - Sqrt[2]
*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqr
t[2]) + (((49 + 45*I)*A - (25 - 21*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]]/(2*Sqrt[2])))/2))/d - (14*a^2*(7*A + (3*I)*B))/(3*d*Tan[c + d*x
]^(3/2)) + (10*a^2*((9*I)*A - 5*B))/(d*Sqrt[Tan[c + d*x]]))/(2*a^2) + (9*A
 + (5*I)*B)/(d*(1 + I*Tan[c + d*x])*Tan[c + d*x]^(3/2)))/(8*a^2) + (A + I*
B)/(4*d*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{-\frac{i\left(\frac{\left(-\frac{13A}{2}-\frac{9iB}{2}\right)\tan(dx+c)^{\frac{3}{2}}+\left(\frac{15iA}{2}-\frac{11B}{2}\right)\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^2}-\frac{(23iB+47A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{4}-\frac{i(-iB+A)\arctan\left(\frac{2\sqrt{\tan}}{\sqrt{}}\right.}{2\left(\sqrt{2}+i\sqrt{2}\right)}}{d\,a^2}$
default	$\frac{-\frac{i\left(\frac{\left(-\frac{13A}{2}-\frac{9iB}{2}\right)\tan(dx+c)^{\frac{3}{2}}+\left(\frac{15iA}{2}-\frac{11B}{2}\right)\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^2}-\frac{(23iB+47A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{4}-\frac{i(-iB+A)\arctan\left(\frac{2\sqrt{\tan}}{\sqrt{}}\right.}{2\left(\sqrt{2}+i\sqrt{2}\right)}}{d\,a^2}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^2*(-1/4*I*(((-13/2*A-9/2*I*B)*tan(d*x+c)^(3/2)+(15/2*I*A-11/2*B)*tan
(d*x+c)^(1/2))/(-I+tan(d*x+c))^2-(47*A+23*I*B)/(2^(1/2)-I*2^(1/2))*arctan(
2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))-1/2*I*(A-I*B)/(2^(1/2)+I*2^(1/2))
*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))-2/3*A/tan(d*x+c)^(3/2)-2*(
-2*I*A+B)/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(220) = 440.

Time = 0.12 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.90

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
1/96*(6*(a^2*d*e^(8*I*d*x + 8*I*c) - 2*a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e
^(4*I*d*x + 4*I*c))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*log(-2*((I*a^
2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)) - (A - I*B)
*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 6*(a^2*d*e^(8*I*d*
x + 8*I*c) - 2*a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))*sqrt
((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*log(-2*((-I*a^2*d*e^(2*I*d*x + 2*I*c)
 - I*a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e
^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*(a^2*d*e^(8*I*d*x + 8*I*c) - 2*a^2*d*e^
(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))*sqrt((2209*I*A^2 - 2162*A*B
 - 529*I*B^2)/(a^4*d^2))*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqr
t((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((2209*I*A^2
 - 2162*A*B - 529*I*B^2)/(a^4*d^2)) + 47*I*A - 23*B)*e^(-2*I*d*x - 2*I*c)/
(a^2*d)) + 3*(a^2*d*e^(8*I*d*x + 8*I*c) - 2*a^2*d*e^(6*I*d*x + 6*I*c) + a^
2*d*e^(4*I*d*x + 4*I*c))*sqrt((2209*I*A^2 - 2162*A*B - 529*I*B^2)/(a^4*d^2
))*log(1/8*((a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt((-I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((2209*I*A^2 - 2162*A*B - 529*I*B^2
)/(a^4*d^2)) - 47*I*A + 23*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) - 2*(2*(101*A
+ 63*I*B)*e^(8*I*d*x + 8*I*c) - (103*A + 27*I*B)*e^(6*I*d*x + 6*I*c) - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia\tan(c + dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.50

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{3\sqrt{2}(-(47i-47)\ A+(23i+23)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+6\sqrt{2}((i+1)\ A-(i-1)$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output
```
-1/48*(3*sqrt(2)*(-(47*I - 47)*A + (23*I + 23)*B)*arctan((1/2*I + 1/2)*sqr
t(2)*sqrt(tan(d*x + c))) + 6*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-(1/2*
I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) + 32*(-6*I*A*tan(d*x + c) + 3*B*tan(d
*x + c) + A)/tan(d*x + c)^(3/2) + 6*(-13*I*A*tan(d*x + c)^(3/2) + 9*B*tan(
d*x + c)^(3/2) - 15*A*sqrt(tan(d*x + c)) - 11*I*B*sqrt(tan(d*x + c)))/(tan
(d*x + c) - I)^2)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 7.84 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.26

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$
$$= -\mathrm{atan}\left(\frac{8\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,1\mathrm{i}}{64\,a^4\,d^2}}}{A}\right)\,\sqrt{-\frac{A^2\,1\mathrm{i}}{64\,a^4\,d^2}}\,2\mathrm{i}$$
$$+\mathrm{atan}\left(\frac{16\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,2209\mathrm{i}}{256\,a^4\,d^2}}}{47\,A}\right)\,\sqrt{\frac{A^2\,2209\mathrm{i}}{256\,a^4\,d^2}}\,2\mathrm{i}$$
$$+2\,\mathrm{atanh}\left(\frac{8\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,1\mathrm{i}}{64\,a^4\,d^2}}}{B}\right)\,\sqrt{\frac{B^2\,1\mathrm{i}}{64\,a^4\,d^2}}$$
$$+2\,\mathrm{atanh}\left(\frac{16\,a^2\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,529\mathrm{i}}{256\,a^4\,d^2}}}{23\,B}\right)\,\sqrt{-\frac{B^2\,529\mathrm{i}}{256\,a^4\,d^2}}$$
$$+\frac{\frac{8\,A\tan(c+d\,x)}{3\,a^2\,d}-\frac{45\,A\tan(c+d\,x)^3}{8\,a^2\,d}+\frac{A\,2\mathrm{i}}{3\,a^2\,d}+\frac{A\tan(c+d\,x)^2\,221\mathrm{i}}{24\,a^2\,d}}{2\tan\left(c+d\,x\right)^{5/2}-\tan\left(c+d\,x\right)^{3/2}\,1\mathrm{i}+\tan\left(c+d\,x\right)^{7/2}\,1\mathrm{i}}$$
$$-\frac{\frac{43\,B\tan(c+d\,x)}{8\,a^2\,d}-\frac{B\,2\mathrm{i}}{a^2\,d}+\frac{B\tan(c+d\,x)^2\,25\mathrm{i}}{8\,a^2\,d}}{2\tan\left(c+d\,x\right)^{3/2}-\sqrt{\tan\left(c+d\,x\right)}\,1\mathrm{i}+\tan\left(c+d\,x\right)^{5/2}\,1\mathrm{i}}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
atan((16*a^2*d*tan(c + d*x)^(1/2)*((A^2*2209i)/(256*a^4*d^2))^(1/2))/(47*A
))*((A^2*2209i)/(256*a^4*d^2))^(1/2)*2i - atan((8*a^2*d*tan(c + d*x)^(1/2)
*(-(A^2*1i)/(64*a^4*d^2))^(1/2))/A)*(-(A^2*1i)/(64*a^4*d^2))^(1/2)*2i + 2*
atanh((8*a^2*d*tan(c + d*x)^(1/2)*((B^2*1i)/(64*a^4*d^2))^(1/2))/B)*((B^2*
1i)/(64*a^4*d^2))^(1/2) + 2*atanh((16*a^2*d*tan(c + d*x)^(1/2)*(-(B^2*529i
)/(256*a^4*d^2))^(1/2))/(23*B))*(-(B^2*529i)/(256*a^4*d^2))^(1/2) + ((A*2i
)/(3*a^2*d) + (8*A*tan(c + d*x))/(3*a^2*d) + (A*tan(c + d*x)^2*221i)/(24*a
^2*d) - (45*A*tan(c + d*x)^3)/(8*a^2*d))/(2*tan(c + d*x)^(5/2) - tan(c + d
*x)^(3/2)*1i + tan(c + d*x)^(7/2)*1i) - ((43*B*tan(c + d*x))/(8*a^2*d) - (
B*2i)/(a^2*d) + (B*tan(c + d*x)^2*25i)/(8*a^2*d))/(2*tan(c + d*x)^(3/2) -
tan(c + d*x)^(1/2)*1i + tan(c + d*x)^(5/2)*1i)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{-\left(\int \frac{1}{\sqrt{\tan(dx+c)}\,\tan(dx+c)^4-2\sqrt{\tan(dx+c)}\,\tan(dx+c)^3 i-\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}dx\right)a-\left(\int \frac{}{\sqrt{\tan(dx+c)}\,\tan(dx+c)^3-2\sqrt{\tan(\ldots}}\right.}{a^2}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(1/(sqrt(tan(c + d*x))*tan(c + d*x)**4 - 2*sqrt(tan(c + d*x))*tan(
c + d*x)**3*i - sqrt(tan(c + d*x))*tan(c + d*x)**2),x)*a + int(1/(sqrt(tan
(c + d*x))*tan(c + d*x)**3 - 2*sqrt(tan(c + d*x))*tan(c + d*x)**2*i - sqrt
(tan(c + d*x))*tan(c + d*x)),x)*b))/a**2
```

3.146 $\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 339

$$\begin{aligned}
&\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx \\
&= \frac{\left(\frac{1}{16}+\frac{i}{16}\right)((29+i)A+(1+76i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^3d} \\
&\quad - \frac{\left(\frac{1}{16}+\frac{i}{16}\right)((29+i)A+(1+76i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^3d} \\
&\quad + \frac{((28-30i)A+(75+77i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{16\sqrt{2}a^3d} \\
&\quad + \frac{15(2iA-5B)\sqrt{\tan(c+dx)}}{8a^3d} + \frac{7(4A+11iB)\tan^{\frac{3}{2}}(c+dx)}{24a^3d} \\
&\quad + \frac{(iA-B)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3} + \frac{(A+2iB)\tan^{\frac{7}{2}}(c+dx)}{4ad(a+ia\tan(c+dx))^2} - \frac{3(2iA-5B)\tan^{\frac{5}{2}}(c+dx)}{8d\left(a^3+ia^3\tan(c+dx)\right)}
\end{aligned}$$

output
```
(-1/32-1/32*I)*((29+I)*A+(1+76*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2
^(1/2)/a^3/d-(1/32+1/32*I)*((29+I)*A+(1+76*I)*B)*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))*2^(1/2)/a^3/d+1/32*((28-30*I)*A+(75+77*I)*B)*arctanh(2^(1/2)*tan
(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/a^3/d+15/8*(2*I*A-5*B)*tan(d*x+c)^(1
/2)/a^3/d+7/24*(4*A+11*I*B)*tan(d*x+c)^(3/2)/a^3/d+1/6*(I*A-B)*tan(d*x+c)^
(9/2)/d/(a+I*a*tan(d*x+c))^3+1/4*(A+2*I*B)*tan(d*x+c)^(7/2)/a/d/(a+I*a*tan
(d*x+c))^2-3/8*(2*I*A-5*B)*tan(d*x+c)^(5/2)/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 7.52 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.88

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sec^3(c+dx)(\cos(dx)+i\sin(dx))^3(A+B\tan(c+dx))\left(3\left(((30-28i)A+(77+75i)B)\arcsin(\cos(c+\right.\right.}{}$$

input
```
Integrate[(Tan[c + d*x]^(9/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^3,x]
```

output
```
(Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x])*(3*(((30 -
28*I)*A + (77 + 75*I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] + (1 + I)*((-
29 + I)*A + (1 - 76*I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c
+ d*x)]]])*(I*Cos[3*c] - Sin[3*c])*Sqrt[Sin[2*(c + d*x)]] + (I*Cos[3*d*x]
+ Sin[3*d*x])*(33*A + (69*I)*B + 2*(90*A + (241*I)*B)*Cos[2*(c + d*x)] + (
147*A + (349*I)*B)*Cos[4*(c + d*x)] + (194*I)*A*Sin[2*(c + d*x)] - 502*B*S
in[2*(c + d*x)] + (145*I)*A*Sin[4*(c + d*x)] - 347*B*Sin[4*(c + d*x)])*Tan
[c + d*x]))/(96*d*(A*Cos[c + d*x] + B*Sin[c + d*x])*Sqrt[Tan[c + d*x]]*(a
+ I*a*Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.10, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4078, 3042, 4011, 3042, 4011, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{9/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{3\tan^{\frac{7}{2}}(c+dx)(3a(iA-B)+a(A+5iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)^2}dx}{6a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan^{\frac{7}{2}}(c+dx)(3a(iA-B)+a(A+5iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan(c+dx)^{7/2}(3a(iA-B)+a(A+5iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{4a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{-\frac{\int -\frac{2\tan^{\frac{5}{2}}(c+dx)\left(7a^2(A+2iB)-a^2(5iA-16B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}-\frac{a(A+2iB)\tan^{\frac{7}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\int\frac{\tan^{\frac{5}{2}}(c+dx)\left(7a^2(A+2iB)-a^2(5iA-16B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(A+2iB)\tan^{\frac{7}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\int\frac{\tan(c+dx)^{5/2}\left(7a^2(A+2iB)-a^2(5iA-16B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(A+2iB)\tan^{\frac{7}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 4078

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\int\tan^{\frac{3}{2}}(c+dx)\left(15(2iA-5B)a^3+7(4A+11iB)\tan(c+dx)a^3\right)dx}{2a^2}}{2a^2}-\frac{a(A+2iB)\tan^{\frac{7}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\int\tan(c+dx)^{3/2}\left(15(2iA-5B)a^3+7(4A+11iB)\tan(c+dx)a^3\right)dx}{2a^2}}{2a^2}-\frac{a(A+2iB)\tan^{\frac{7}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 4011

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\int\sqrt{\tan(c+dx)}\left(15a^3(2iA-5B)\tan(c+dx)-7a^3(4A+11iB)\right)dx+\frac{14a^3(4A+11iB)\tan^{\frac{3}{2}}(c+dx)}{3d}}{2a^2}}{2a^2}-\frac{a(A+2iB)\tan^{\frac{7}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\int\sqrt{\tan(c+dx)}\left(15a^3(2iA-5B)\tan(c+dx)-7a^3(4A+11iB)\right)dx+\frac{14a^3(4A+11iB)\tan^{\frac{3}{2}}(c+dx)}{3d}}{2a^2}}{2a^2}-\frac{a(A+2iB)\tan^{\frac{7}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 4011

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\frac{\int\frac{-15(2iA-5B)a^3-7(4A+11iB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx+\frac{14a^3(4A+11iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{30a^3(-5B+2iA)\sqrt{\tan(c+dx)}}{d}}{2a^2}}{2a^2}-\frac{a(A+2iB)}{d(a+ia\,\mathrm{ta}}}{4a^2}$$

$\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\frac{\int\frac{-15(2iA-5B)a^3-7(4A+11iB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx+\frac{14a^3(4A+11iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{30a^3(-5B+2iA)\sqrt{\tan(c+dx)}}{d}}{2a^2}}{2a^2}-\frac{a(A+2iB)}{d(a+ia\,\mathrm{ta}}}{4a^2}$$

$\downarrow$ 4017

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\frac{\frac{2\int-\frac{a^3(15(2iA-5B)+7(4A+11iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{14a^3(4A+11iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{30a^3(-5B+2iA)\sqrt{\tan(c+dx)}}{d}}{2a^2}}{2a^2}-}{4a^2}$$

$\downarrow$ 25

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{-\frac{2\int\frac{a^3(15(2iA-5B)+7(4A+11iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{14a^3(4A+11iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{30a^3(-5B+2iA)\sqrt{\tan(c+dx)}}{d}}{2a^2}}{2a^2}-}{4a^2}$$

$\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{-\frac{2a^3\int\frac{15(2iA-5B)+7(4A+11iB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{14a^3(4A+11iB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{30a^3(-5B+2iA)\sqrt{\tan(c+dx)}}{d}}{2a^2}}{2a^2}-}{4a^2}$$

$\downarrow$ 1482

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{-\frac{-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+29i)A-(76+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((29+i)A+(1+76i)B)\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}}{2a^2}}{2a^2}}{4a^2}$$

$\Big\downarrow$ 1476

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+29i)A-(76+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((29+i)A+(1+76i)B)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+}}\right.}{d}}{2a^2}}{}$$

$\Big\downarrow$ 1082

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{-\frac{-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+29i)A-(76+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((29+i)A+(1+76i)B)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\right.}{\sqrt{2}}\right.\right.}{d}}{2a^2}}{2a^2}}{4a^2}$$

$\Big\downarrow$ 217

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{-\frac{-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+29i)A-(76+i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((29+i)A+(1+76i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}}{2a^2}}{2a^2}}{4a^2}$$

$\Big\downarrow$ 1479

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+29i)A-(76+i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{}}{2\sqrt{2}}\right.\right.}{d}}{}}{}$$

$\Big\downarrow$ 25

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-$$

$$\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+29i)A-(76+i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\cdots}{d}\cdots}{\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-$$

$$\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+29i)A-(76+i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\cdots}{d}\cdots}{\cdots}$$

$$\Big\downarrow \; 1103$$

$$\frac{(-B+iA)\tan^{\frac{9}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-$$

$$\frac{3a^2(-5B+2iA)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((29+i)A+(1+76i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((1+29i)A-(76+i)B\cdots}{d}\cdots}{2a^2\cdots}$$

input
```
Int[(Tan[c + d*x]^(9/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I*A - B)*Tan[c + d*x]^(9/2))/(6*d*(a + I*a*Tan[c + d*x])^3) - (-((a*(A +
 (2*I)*B)*Tan[c + d*x]^(7/2))/(d*(a + I*a*Tan[c + d*x])^2)) + ((3*a^2*((2*
I)*A - 5*B)*Tan[c + d*x]^(5/2))/(d*(a + I*a*Tan[c + d*x])) - ((-2*a^3*((1/
2 + I/2)*((29 + I)*A + (1 + 76*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + (1/2 + I
/2)*((1 + 29*I)*A - (76 + I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] +
 Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
]/(2*Sqrt[2]))))/d + (30*a^3*((2*I)*A - 5*B)*Sqrt[Tan[c + d*x]])/d + (14*a
^3*(4*A + (11*I)*B)*Tan[c + d*x]^(3/2))/(3*d))/(2*a^2))/(2*a^2))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.60

method	result
derivativedivides	$\frac{\frac{2iB\tan(dx+c)^{\frac{3}{2}}}{3}-6B\sqrt{\tan(dx+c)}+2iA\sqrt{\tan(dx+c)}+\frac{i\left(\frac{-5i(7iB+4A)\tan(dx+c)^{\frac{5}{2}}+\left(-\frac{182iB}{3}-\frac{98A}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(14iA-}{(-i+\tan(dx+c))^3}\right.}{8}}{d\,a^3}$
default	$\frac{\frac{2iB\tan(dx+c)^{\frac{3}{2}}}{3}-6B\sqrt{\tan(dx+c)}+2iA\sqrt{\tan(dx+c)}+\frac{i\left(\frac{-5i(7iB+4A)\tan(dx+c)^{\frac{5}{2}}+\left(-\frac{182iB}{3}-\frac{98A}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(14iA-}{(-i+\tan(dx+c))^3}\right.}{8}}{d\,a^3}$

input
```
int(tan(d*x+c)^(9/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^3*(2/3*I*B*tan(d*x+c)^(3/2)-6*B*tan(d*x+c)^(1/2)+2*I*A*tan(d*x+c)^(1
/2)+1/8*I*((-5*I*(7*I*B+4*A)*tan(d*x+c)^(5/2)+(-182/3*I*B-98/3*A)*tan(d*x+
c)^(3/2)+(-27*B+14*I*A)*tan(d*x+c)^(1/2))/(-I+tan(d*x+c))^3+2*(-76*B+29*I*
A)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))+4*(
1/16*A-1/16*I*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*
2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 779 vs. 2(256) = 512.

Time = 0.16 (sec) , antiderivative size = 779, normalized size of antiderivative = 2.30

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(9/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
-1/96*(3*(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((I*A
^2 + 2*A*B - I*B^2)/(a^6*d^2))*log(2*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*
sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 +
 2*A*B - I*B^2)/(a^6*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x -
2*I*c)/(I*A + B)) - 3*(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*
c))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*log(-2*((a^3*d*e^(2*I*d*x + 2*
I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*
e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(
6*I*d*x + 6*I*c))*sqrt((-841*I*A^2 + 4408*A*B + 5776*I*B^2)/(a^6*d^2))*log
(1/8*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-841*I*A^2 + 4408*A*B + 5776*I*B^2)/(a^
6*d^2)) + 29*A + 76*I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) + 3*(a^3*d*e^(8*I*d
*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((-841*I*A^2 + 4408*A*B + 577
6*I*B^2)/(a^6*d^2))*log(-1/8*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-841*I*A^2 + 44
08*A*B + 5776*I*B^2)/(a^6*d^2)) - 29*A - 76*I*B)*e^(-2*I*d*x - 2*I*c)/(a^3
*d)) + 2*(2*(-73*I*A + 174*B)*e^(8*I*d*x + 8*I*c) - (187*I*A - 492*B)*e^(6
*I*d*x + 6*I*c) + 3*(-11*I*A + 23*B)*e^(4*I*d*x + 4*I*c) - (-7*I*A + 10*B)
*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(9/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(9/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.51

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{3\sqrt{2}((29i+29)\ A+(76i-76)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}((i-1)\ A+(i+1)}{}$$

input
```
integrate(tan(d*x+c)^(9/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*(3*sqrt(2)*((29*I + 29)*A + (76*I - 76)*B)*arctan((1/2*I + 1/2)*sqrt
(2)*sqrt(tan(d*x + c))) + 3*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-(1/2*I
 - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - 32*I*B*tan(d*x + c)^(3/2) - 96*I*A*s
qrt(tan(d*x + c)) + 288*B*sqrt(tan(d*x + c)) - 2*(60*A*tan(d*x + c)^(5/2)
+ 105*I*B*tan(d*x + c)^(5/2) - 98*I*A*tan(d*x + c)^(3/2) + 182*B*tan(d*x +
 c)^(3/2) - 42*A*sqrt(tan(d*x + c)) - 81*I*B*sqrt(tan(d*x + c)))/(tan(d*x
+ c) - I)^3)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.27

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \operatorname{atan}\left(\frac{a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,1{}\mathrm{i}}{256\,a^6\,d^2}}\,16{}\mathrm{i}}{A}\right)\,\sqrt{\frac{A^2\,1{}\mathrm{i}}{256\,a^6\,d^2}}\,2{}\mathrm{i}$$
$$-\operatorname{atan}\left(\frac{a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,841{}\mathrm{i}}{256\,a^6\,d^2}}\,16{}\mathrm{i}}{29\,A}\right)\,\sqrt{-\frac{A^2\,841{}\mathrm{i}}{256\,a^6\,d^2}}\,2{}\mathrm{i}$$
$$-\operatorname{atan}\left(\frac{16\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1{}\mathrm{i}}{256\,a^6\,d^2}}}{B}\right)\,\sqrt{-\frac{B^2\,1{}\mathrm{i}}{256\,a^6\,d^2}}\,2{}\mathrm{i}$$
$$-\operatorname{atan}\left(\frac{4\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,361{}\mathrm{i}}{16\,a^6\,d^2}}}{19\,B}\right)\,\sqrt{\frac{B^2\,361{}\mathrm{i}}{16\,a^6\,d^2}}\,2{}\mathrm{i}$$
$$-\frac{\frac{49\,A\,{\tan\left(c+d\,x\right)}^{3/2}}{12\,a^3\,d}-\frac{A\,\sqrt{\tan\left(c+d\,x\right)}\,7{}\mathrm{i}}{4\,a^3\,d}+\frac{A\,{\tan\left(c+d\,x\right)}^{5/2}\,5{}\mathrm{i}}{2\,a^3\,d}}{-{\tan\left(c+d\,x\right)}^3\,1{}\mathrm{i}-3\,{\tan\left(c+d\,x\right)}^2+\tan\left(c+d\,x\right)\,3{}\mathrm{i}+1}$$
$$-\frac{\frac{27\,B\,\sqrt{\tan\left(c+d\,x\right)}}{8\,a^3\,d}-\frac{35\,B\,{\tan\left(c+d\,x\right)}^{5/2}}{8\,a^3\,d}+\frac{B\,{\tan\left(c+d\,x\right)}^{3/2}\,91{}\mathrm{i}}{12\,a^3\,d}}{-{\tan\left(c+d\,x\right)}^3\,1{}\mathrm{i}-3\,{\tan\left(c+d\,x\right)}^2+\tan\left(c+d\,x\right)\,3{}\mathrm{i}+1}$$
$$+\frac{A\,\sqrt{\tan\left(c+d\,x\right)}\,2{}\mathrm{i}}{a^3\,d}-\frac{6\,B\,\sqrt{\tan\left(c+d\,x\right)}}{a^3\,d}+\frac{B\,{\tan\left(c+d\,x\right)}^{3/2}\,2{}\mathrm{i}}{3\,a^3\,d}$$

input
```
int((tan(c + d*x)^(9/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
atan((a^3*d*tan(c + d*x)^(1/2)*((A^2*1i)/(256*a^6*d^2))^(1/2)*16i)/A)*((A^
2*1i)/(256*a^6*d^2))^(1/2)*2i - atan((a^3*d*tan(c + d*x)^(1/2)*(-(A^2*841i
)/(256*a^6*d^2))^(1/2)*16i)/(29*A))*(-(A^2*841i)/(256*a^6*d^2))^(1/2)*2i -
 atan((16*a^3*d*tan(c + d*x)^(1/2)*(-(B^2*1i)/(256*a^6*d^2))^(1/2))/B)*(-(
B^2*1i)/(256*a^6*d^2))^(1/2)*2i - atan((4*a^3*d*tan(c + d*x)^(1/2)*((B^2*3
61i)/(16*a^6*d^2))^(1/2))/(19*B))*((B^2*361i)/(16*a^6*d^2))^(1/2)*2i - ((4
9*A*tan(c + d*x)^(3/2))/(12*a^3*d) - (A*tan(c + d*x)^(1/2)*7i)/(4*a^3*d) +
 (A*tan(c + d*x)^(5/2)*5i)/(2*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2
- tan(c + d*x)^3*1i + 1) - ((27*B*tan(c + d*x)^(1/2))/(8*a^3*d) + (B*tan(c
 + d*x)^(3/2)*91i)/(12*a^3*d) - (35*B*tan(c + d*x)^(5/2))/(8*a^3*d))/(tan(
c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*1i + 1) + (A*tan(c + d*x)^
(1/2)*2i)/(a^3*d) - (6*B*tan(c + d*x)^(1/2))/(a^3*d) + (B*tan(c + d*x)^(3/
2)*2i)/(3*a^3*d)
```

Reduce [F]

$$\int \frac{\tan^{\frac{9}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^5}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^4}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)a}{a^3}$$

input
```
int(tan(d*x+c)^(9/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int((sqrt(tan(c + d*x))*tan(c + d*x)**5)/(tan(c + d*x)**3*i + 3*tan(c
 + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*b + int((sqrt(tan(c + d*x))*tan(c +
d*x)**4)/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)
*a))/a**3
```

3.147 $\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 310

$$\begin{aligned}
\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx
&= \frac{((5-7i)A+(28+30i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} \\
&+ \frac{\left(\frac{1}{16}+\frac{i}{16}\right)((1+6i)A-(29+i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^3d} \\
&- \frac{\left(\frac{1}{16}+\frac{i}{16}\right)((6+i)A+(1+29i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}a^3d} \\
&+ \frac{5(A+6iB)\sqrt{\tan(c+dx)}}{8a^3d} + \frac{(iA-B)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3} \\
&+ \frac{(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{12ad(a+ia\tan(c+dx))^2} - \frac{7(iA-4B)\tan^{\frac{3}{2}}(c+dx)}{24d\left(a^3+ia^3\tan(c+dx)\right)}
\end{aligned}$$

output
```
-1/32*((5-7*I)*A+(28+30*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/
a^3/d+(1/32+1/32*I)*((1+6*I)*A-(29+I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)*2^(1/2)/a^3/d-(1/32+1/32*I)*((6+I)*A+(1+29*I)*B)*arctanh(2^(1/2)*tan(d*x
+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/a^3/d+5/8*(A+6*I*B)*tan(d*x+c)^(1/2)/a^3
/d+1/6*(I*A-B)*tan(d*x+c)^(7/2)/d/(a+I*a*tan(d*x+c))^3+1/12*(2*A+5*I*B)*ta
n(d*x+c)^(5/2)/a/d/(a+I*a*tan(d*x+c))^2-7/24*(I*A-4*B)*tan(d*x+c)^(3/2)/d/
(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 5.95 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.92

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sec^2(c+dx)(\cos(dx)+i\sin(dx))^3(A+B\tan(c+dx))\left(-i\left(((7+5i)A-(30-28i)B)\arcsin(\cos(c+a\right.\right.}{}$$

input
```
Integrate[(Tan[c + d*x]^(7/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^3,x]
```

output
```
(Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^3*(A + B*Tan[c + d*x])*((-I)*(((7
+ 5*I)*A - (30 - 28*I)*B)*ArcSin[Cos[c + d*x] - Sin[c + d*x]] + (1 - I)*((
6 + I)*A + (1 + 29*I)*B)*Log[Cos[c + d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c +
 d*x)]]])*Sec[c + d*x]*(Cos[3*c] + I*Sin[3*c])*Sqrt[Sin[2*(c + d*x)]] + (2
*(Cos[3*d*x] - I*Sin[3*d*x])*((9*A + (33*I)*B)*Cos[c + d*x] + 21*(A + (7*I
)*B)*Cos[3*(c + d*x)] + (2*I)*(19*A + (97*I)*B + (19*A + (145*I)*B)*Cos[2*
(c + d*x)])*Sin[c + d*x])*Tan[c + d*x])/3))/(32*d*(A*Cos[c + d*x] + B*Sin[
c + d*x])*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.09, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.639$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4078, 27, 3042, 4011, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{7/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)(7a(iA-B)+a(A+13iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)^2}dx}{6a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan^{\frac{5}{2}}(c+dx)(7a(iA-B)+a(A+13iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{12a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan(c+dx)^{5/2}(7a(iA-B)+a(A+13iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{12a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{-\frac{\int -\frac{2\tan^{\frac{3}{2}}(c+dx)\left(5a^2(2A+5iB)-a^2(4iA-31B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(5a^2(2A+5iB)-a^2(4iA-31B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\int \frac{\tan(c+dx)^{3/2}\left(5a^2(2A+5iB)-a^2(4iA-31B)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \; 4078$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{\int 3\sqrt{\tan(c+dx)}\left(7(iA-4B)a^3+5(A+6iB)\tan(c+dx)a^3\right)dx}{2a^2}}{2a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\int \sqrt{\tan(c+dx)}\left(7(iA-4B)a^3+5(A+6iB)\tan(c+dx)a^3\right)dx}{2a^2}}{2a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\int \sqrt{\tan(c+dx)}\left(7(iA-4B)a^3+5(A+6iB)\tan(c+dx)a^3\right)dx}{2a^2}}{2a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \; 4011$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\int \frac{7a^3(iA-4B)\tan(c+dx)-5a^3(A+6iB)}{\sqrt{\tan(c+dx)}}dx+\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}\right)}{2a^2}}{2a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\int\frac{7a^3(iA-4B)\tan(c+dx)-5a^3(A+6iB)}{\sqrt{\tan(c+dx)}}dx+\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}\right)}{2a^2}}{2a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$\downarrow$ 4017

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{2\int-\frac{a^3(5(A+6iB)-7(iA-4B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}\right)}{2a^2}}{2a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$\downarrow$ 25

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}-\frac{2\int\frac{a^3(5(A+6iB)-7(iA-4B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}\right)}{2a^2}}{2a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a^3\int\frac{5(A+6iB)-7(iA-4B)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}\right)}{2a^2}}{2a^2}-\frac{a(2A+5iB)\tan^{\frac{5}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$\downarrow$ 1482

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((5-7i)A+(28+30i)B)\int\frac{\tan(\cdots}{\tan^2\cdots}\cdots\right.}{d}\cdots\right.}{2a^2}}{2a^2}\cdots}{12a^2}$$

$\downarrow$ 1476

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((5-7i)A+(28+30i)B)\left(\frac{1}{2}\int\cdots\right.\right.}{d}\cdots}{2a^2}}{2a^2}}{12a^2}$$

$\downarrow$ 1082

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((5-7i)A+(28+30i)B)\left(\frac{\int\cdots}{\cdots}\right.\right.}{d}\cdots}{2a^2}}{2a^2}}{12a^2}$$

$\downarrow$ 217

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((5-7i)A+(28+30i)B)\left(\frac{\arctan\cdots}{\cdots}\right.\right.}{d}\cdots}{2a^2}}{2a^2}}{12a^2}$$

$\downarrow$ 1479

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{\cdots}}{\tan(c\cdots}}{\cdots}\right.\right.}{\cdots}\cdots}{\cdots}}{2a^2}}{\cdots}$$

$\downarrow$ 25

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-$$

$$\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\right.}{\tan(c+dx)\ldots}}{\ldots}\right.\right.}{2a^2\ldots}\right.}{2a^2}$$

$$\ldots 12a^2 \ldots$$

$\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-$$

$$\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\ldots}{\tan(c+dx\ldots}\right.\right.}{2a\ldots}\right.}{2a^2}$$

$$\ldots 12\ldots$$

$\downarrow$ 1103

$$\frac{(-B+iA)\tan^{\frac{7}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-$$

$$\frac{7a^2(-4B+iA)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))}-\frac{3\left(\frac{10a^3(A+6iB)\sqrt{\tan(c+dx)}}{d}-\frac{2a^3\left(\frac{1}{2}((5-7i)A+(28+30i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\left(\frac{1}{2}+\ldots\right.\right.}{2a^2}\right.}{2a^2}$$

$$\ldots 12a^2$$

input
```
Int[(Tan[c + d*x]^(7/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I*A - B)*Tan[c + d*x]^(7/2))/(6*d*(a + I*a*Tan[c + d*x])^3) - (-((a*(2*A
 + (5*I)*B)*Tan[c + d*x]^(5/2))/(d*(a + I*a*Tan[c + d*x])^2)) + ((-3*((-2*
a^3*((((5 - 7*I)*A + (28 + 30*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + (1/2 +
 I/2)*((6 + I)*A + (1 + 29*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]/(2*Sqrt[2]))))/d + (10*a^3*(A + (6*I)*B)*Sqrt[Tan[c + d*x]])/d))/(2*a^2
) + (7*a^2*(I*A - 4*B)*Tan[c + d*x]^(3/2))/(d*(a + I*a*Tan[c + d*x])))/(2*
a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.58

method	result
derivativedivides	$\frac{2iB\sqrt{\tan(dx+c)}-\frac{i\left(\frac{-i(9iA-20B)\tan(dx+c)^{\frac{5}{2}}+\left(-\frac{38iA}{3}+\frac{98B}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(-14iB-5A)\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^3}-\frac{2(29iB+6A)\arctan}{\sqrt{2}-}\right.}{8}}{d\,a^3}$
default	$\frac{2iB\sqrt{\tan(dx+c)}-\frac{i\left(\frac{-i(9iA-20B)\tan(dx+c)^{\frac{5}{2}}+\left(-\frac{38iA}{3}+\frac{98B}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(-14iB-5A)\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^3}-\frac{2(29iB+6A)\arctan}{\sqrt{2}-}\right.}{8}}{d\,a^3}$

input
```
int(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^3*(2*I*B*tan(d*x+c)^(1/2)-1/8*I*((-I*(9*I*A-20*B)*tan(d*x+c)^(5/2)+(
-38/3*I*A+98/3*B)*tan(d*x+c)^(3/2)+(-5*A-14*I*B)*tan(d*x+c)^(1/2))/(-I+tan
(d*x+c))^3-2*(29*I*B+6*A)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2
^(1/2)-I*2^(1/2))))+4*(1/16*I*A+1/16*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d
*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(233) = 466$.

Time = 0.12 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.21

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
-1/96*(3*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c
)*log(-2*((I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^
2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a
^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*(
(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) - (A
- I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + 3*a^3*d*sqrt
((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*
((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2)) +
 6*I*A - 29*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 3*a^3*d*sqrt((36*I*A^2 - 34
8*A*B - 841*I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*((a^3*d*e^(2*I*d
*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2)) - 6*I*A + 29*B)*e
^(-2*I*d*x - 2*I*c)/(a^3*d)) - 2*(2*(10*A + 73*I*B)*e^(6*I*d*x + 6*I*c) +
(14*A + 41*I*B)*e^(4*I*d*x + 4*I*c) - (5*A + 8*I*B)*e^(2*I*d*x + 2*I*c) +
A + I*B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(
-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(7/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.49

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{3\sqrt{2}(-(6i-6)\ A+(29i+29)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}(-(i+1)\ A+(i-1)}{}$$

input
```
integrate(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*(3*sqrt(2)*(-(6*I - 6)*A + (29*I + 29)*B)*arctan((1/2*I + 1/2)*sqrt(
2)*sqrt(tan(d*x + c))) + 3*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-(1/2*I
 - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - 96*I*B*sqrt(tan(d*x + c)) - 2*(-27*I
*A*tan(d*x + c)^(5/2) + 60*B*tan(d*x + c)^(5/2) - 38*A*tan(d*x + c)^(3/2)
- 98*I*B*tan(d*x + c)^(3/2) + 15*I*A*sqrt(tan(d*x + c)) - 42*B*sqrt(tan(d*
x + c)))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.27

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \mathrm{atan}\left(\frac{8\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,9{}\mathrm{i}}{64\,a^6\,d^2}}}{3\,A}\right)\,\sqrt{\frac{A^2\,9{}\mathrm{i}}{64\,a^6\,d^2}}\,2{}\mathrm{i}$$
$$+\mathrm{atan}\left(\frac{16\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,1{}\mathrm{i}}{256\,a^6\,d^2}}}{A}\right)\,\sqrt{-\frac{A^2\,1{}\mathrm{i}}{256\,a^6\,d^2}}\,2{}\mathrm{i}$$
$$+\mathrm{atan}\left(\frac{a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,1{}\mathrm{i}}{256\,a^6\,d^2}}\,16{}\mathrm{i}}{B}\right)\,\sqrt{\frac{B^2\,1{}\mathrm{i}}{256\,a^6\,d^2}}\,2{}\mathrm{i}$$
$$-\mathrm{atan}\left(\frac{a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,841{}\mathrm{i}}{256\,a^6\,d^2}}\,16{}\mathrm{i}}{29\,B}\right)\,\sqrt{-\frac{B^2\,841{}\mathrm{i}}{256\,a^6\,d^2}}\,2{}\mathrm{i}$$
$$+\frac{\frac{5\,A\,\sqrt{\tan(c+d\,x)}}{8\,a^3\,d}-\frac{9\,A\,\tan(c+d\,x)^{5/2}}{8\,a^3\,d}+\frac{A\,\tan(c+d\,x)^{3/2}\,19{}\mathrm{i}}{12\,a^3\,d}}{-{\tan\left(c+d\,x\right)}^3\,1{}\mathrm{i}-3\,{\tan\left(c+d\,x\right)}^2+\tan\left(c+d\,x\right)\,3{}\mathrm{i}+1}$$
$$-\frac{\frac{49\,B\,\tan(c+d\,x)^{3/2}}{12\,a^3\,d}-\frac{B\,\sqrt{\tan(c+d\,x)}\,7{}\mathrm{i}}{4\,a^3\,d}+\frac{B\,\tan(c+d\,x)^{5/2}\,5{}\mathrm{i}}{2\,a^3\,d}}{-{\tan\left(c+d\,x\right)}^3\,1{}\mathrm{i}-3\,{\tan\left(c+d\,x\right)}^2+\tan\left(c+d\,x\right)\,3{}\mathrm{i}+1}+\frac{B\,\sqrt{\tan\left(c+d\,x\right)}\,2{}\mathrm{i}}{a^3\,d}$$

input
```
int((tan(c + d*x)^(7/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
atan((8*a^3*d*tan(c + d*x)^(1/2)*((A^2*9i)/(64*a^6*d^2))^(1/2))/(3*A))*((A
^2*9i)/(64*a^6*d^2))^(1/2)*2i + atan((16*a^3*d*tan(c + d*x)^(1/2)*(-(A^2*1
i)/(256*a^6*d^2))^(1/2))/A)*(-(A^2*1i)/(256*a^6*d^2))^(1/2)*2i + atan((a^3
*d*tan(c + d*x)^(1/2)*((B^2*1i)/(256*a^6*d^2))^(1/2)*16i)/B)*((B^2*1i)/(25
6*a^6*d^2))^(1/2)*2i - atan((a^3*d*tan(c + d*x)^(1/2)*(-(B^2*841i)/(256*a^
6*d^2))^(1/2)*16i)/(29*B))*(-(B^2*841i)/(256*a^6*d^2))^(1/2)*2i + ((5*A*ta
n(c + d*x)^(1/2))/(8*a^3*d) + (A*tan(c + d*x)^(3/2)*19i)/(12*a^3*d) - (9*A
*tan(c + d*x)^(5/2))/(8*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(
c + d*x)^3*1i + 1) - ((49*B*tan(c + d*x)^(3/2))/(12*a^3*d) - (B*tan(c + d*
x)^(1/2)*7i)/(4*a^3*d) + (B*tan(c + d*x)^(5/2)*5i)/(2*a^3*d))/(tan(c + d*x
)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*1i + 1) + (B*tan(c + d*x)^(1/2)*2
i)/(a^3*d)
```

Reduce [F]

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^4}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^3}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)a}{a^3}$$

input
```
int(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int((sqrt(tan(c + d*x))*tan(c + d*x)**4)/(tan(c + d*x)**3*i + 3*tan(c
 + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*b + int((sqrt(tan(c + d*x))*tan(c +
d*x)**3)/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)
*a))/a**3
```

3.148 $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 259

$$\begin{aligned}\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx &= \frac{(2A+(5-7i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} \\ &\quad - \frac{(2A+(5-7i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} \\ &\quad + \frac{(2A-(5+7i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{(iA-B)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3} \\ &\quad + \frac{(A+4iB)\tan^{\frac{3}{2}}(c+dx)}{12ad(a+ia\tan(c+dx))^2} + \frac{5B\sqrt{\tan(c+dx)}}{8d\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

output
```
-1/32*(2*A+(5-7*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a^3/d-1/
32*(2*A+(5-7*I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a^3/d+1/32*(
2*A-(5+7*I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/a^
3/d+1/6*(I*A-B)*tan(d*x+c)^(5/2)/d/(a+I*a*tan(d*x+c))^3+1/12*(A+4*I*B)*tan
(d*x+c)^(3/2)/a/d/(a+I*a*tan(d*x+c))^2+5/8*B*tan(d*x+c)^(1/2)/d/(a^3+I*a^3
*tan(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 4.91 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.98

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sec^2(c+dx)(\cos(dx)+i\sin(dx))^3\left(\left((2A+(5-7i)B)\arcsin(\cos(c+dx)-\sin(c+dx))+(1-i)((1+\right.\right.}{}$$

input
```
Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^3,x]
```

output
```
(Sec[c + d*x]^2*(Cos[d*x] + I*Sin[d*x])^3*(((2*A + (5 - 7*I)*B)*ArcSin[Cos
[c + d*x] - Sin[c + d*x]] + (1 - I)*((1 + I)*A + (1 - 6*I)*B)*Log[Cos[c +
d*x] + Sin[c + d*x] + Sqrt[Sin[2*(c + d*x)]]])*Sec[c + d*x]*(Cos[3*c] + I*
Sin[3*c])*Sqrt[Sin[2*(c + d*x)]] + (4*(Cos[3*d*x] - I*Sin[3*d*x])*Sin[c +
d*x]*((3*I)*A - 6*B + 3*((-I)*A + 7*B)*Cos[2*(c + d*x)] + (A + (19*I)*B)*S
in[2*(c + d*x)]))/3)*(A + B*Tan[c + d*x]))/(32*d*(A*Cos[c + d*x] + B*Sin[c
 + d*x])*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.10, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4078, 25, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(c+dx)^{5/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)(5a(iA-B)-a(A-11iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)^2}dx}{6a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)(5a(iA-B)-a(A-11iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{12a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan(c+dx)^{3/2}(5a(iA-B)-a(A-11iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{12a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{-\frac{\int -\frac{6\sqrt{\tan(c+dx)}\left((A+4iB)a^2+(iA+6B)\tan(c+dx)a^2\right)}{i\tan(c+dx)a+a}dx}{4a^2}-\frac{a(A+4iB)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\int \frac{\sqrt{\tan(c+dx)}\left((A+4iB)a^2+(iA+6B)\tan(c+dx)a^2\right)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(A+4iB)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\int \frac{\sqrt{\tan(c+dx)}\left((A+4iB)a^2+(iA+6B)\tan(c+dx)a^2\right)}{i\tan(c+dx)a+a}dx}{2a^2}-\frac{a(A+4iB)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(-\frac{\int -\frac{5Ba^3+(2A-7iB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{5a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(A+4iB)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{\int \frac{5Ba^3+(2A-7iB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{5a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(A+4iB)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{\int\frac{5Ba^3+(2A-7iB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{5a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(A+4iB)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{\int\frac{a^3(5B+(2A-7iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{a^2d}-\frac{5a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(A+4iB)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{a\int\frac{5B+(2A-7iB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{5a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(A+4iB)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \text{1482}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{a\left(\frac{1}{2}(2A+(5-7i)B)\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(2A-(5+7i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{5a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(A+4iB)\tan^{\frac{3}{2}}(c+dx)}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{a\left(\frac{1}{2}(2A+(5-7i)B)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}(2A-(5+7i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}\cdots}{d}\cdots\right)}{2a^2}\cdots}{12a^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{3\left(\frac{a\left(\frac{1}{2}(2A+(5-7i)B)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}(2A-(5+7i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}\right.}{2a^2}}{12a^2}$$

$\downarrow$ 217

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{3\left(\frac{a\left(\frac{1}{2}(2A+(5-7i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(2A-(5+7i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{5a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}}{12a^2}$$

$\downarrow$ 1479

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{3\left(\frac{a\left(\frac{1}{2}(2A+(5-7i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(2A-(5+7i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int}{}\right.}{d}\right.}{2a^2}}{12a^2}$$

$\downarrow$ 25

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{3\left(\frac{a\left(\frac{1}{2}(2A+(5-7i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(2A-(5+7i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{}{\tan}}{}\right.}{d}\right.}{2a^2}}{12a^2}$$

$\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{3\left(\frac{a\left(\frac{1}{2}(2A+(5-7i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(2A-(5+7i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \cdots\right.\right.}{d}\cdots}{2a^2}\cdots}{12a^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{3\left(\frac{a\left(\frac{1}{2}(2A+(5-7i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(2A-(5+7i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\cdots\right)}{\cdots}\right.\right.}{d}\cdots}{2a^2}\cdots}{12a^2}$$

input
```
Int[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I*A - B)*Tan[c + d*x]^(5/2))/(6*d*(a + I*a*Tan[c + d*x])^3) - (-((a*(A +
 (4*I)*B)*Tan[c + d*x]^(3/2))/(d*(a + I*a*Tan[c + d*x])^2)) + (3*((a*(((2*
A + (5 - 7*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcT
an[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 - ((2*A - (5 + 7*I)*B)*(-1/
2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d - (5*a^2*B*Sqr
t[Tan[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))))/(2*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{-\frac{i\left(\frac{-i(9iB+2A)\tan(dx+c)^{\frac{5}{2}}+\left(-\frac{38iB}{3}-\frac{2A}{3}\right)\tan(dx+c)^{\frac{3}{2}}-5B\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^3}-\frac{2(iA+6B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{8}+\frac{4\left(-\frac{A}{16}+\frac{i}{1}\right.}{}}{d\,a^3}$
default	$\frac{-\frac{i\left(\frac{-i(9iB+2A)\tan(dx+c)^{\frac{5}{2}}+\left(-\frac{38iB}{3}-\frac{2A}{3}\right)\tan(dx+c)^{\frac{3}{2}}-5B\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^3}-\frac{2(iA+6B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{8}+\frac{4\left(-\frac{A}{16}+\frac{i}{1}\right.}{}}{d\,a^3}$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^3*(-1/8*I*((-I*(9*I*B+2*A)*tan(d*x+c)^(5/2)+(-38/3*I*B-2/3*A)*tan(d*
x+c)^(3/2)-5*B*tan(d*x+c)^(1/2))/(-I+tan(d*x+c))^3-2*(I*A+6*B)/(2^(1/2)-I*
2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))+4*(-1/16*A+1/16*I
*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 681 vs. 2(204) = 408.

Time = 0.12 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.63

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
1/96*(3*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*
log(2*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) + (A
 - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*sqr
t((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*((a^3*d*e^
(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) - (A - I*B)*e^(2*I*d
*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + 3*a^3*d*sqrt((-I*A^2 - 12*A
*B + 36*I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*((a^3*d*e^(2*I*d*x +
 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
1))*sqrt((-I*A^2 - 12*A*B + 36*I*B^2)/(a^6*d^2)) + A - 6*I*B)*e^(-2*I*d*x
- 2*I*c)/(a^3*d)) - 3*a^3*d*sqrt((-I*A^2 - 12*A*B + 36*I*B^2)/(a^6*d^2))*e
^(6*I*d*x + 6*I*c)*log(-1/8*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 12*A*B
+ 36*I*B^2)/(a^6*d^2)) - A + 6*I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 2*(2*(
I*A - 10*B)*e^(6*I*d*x + 6*I*c) - (I*A + 14*B)*e^(4*I*d*x + 4*I*c) - (2*I*
A - 5*B)*e^(2*I*d*x + 2*I*c) + I*A - B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.50

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{3\sqrt{2}((i+1)\ A-(6i-6)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}(-(i-1)\ A-(i+1)\ B)\,a}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*(3*sqrt(2)*((I + 1)*A - (6*I - 6)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sq
rt(tan(d*x + c))) + 3*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-(1/2*I - 1/
2)*sqrt(2)*sqrt(tan(d*x + c))) + 2*(6*A*tan(d*x + c)^(5/2) + 27*I*B*tan(d*
x + c)^(5/2) - 2*I*A*tan(d*x + c)^(3/2) + 38*B*tan(d*x + c)^(3/2) - 15*I*B
*sqrt(tan(d*x + c)))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.19

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{5\,B\,\sqrt{\tan(c+d\,x)}}{8\,a^3\,d}-\frac{9\,B\,\tan(c+d\,x)^{5/2}}{8\,a^3\,d}+\frac{B\,\tan(c+d\,x)^{3/2}\,19\mathrm{i}}{12\,a^3\,d}}{-\tan\left(c+d\,x\right)^3\,1\mathrm{i}-3\,\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1}$$
$$+\frac{\frac{A\,\tan(c+d\,x)^{3/2}}{12\,a^3\,d}+\frac{A\,\tan(c+d\,x)^{5/2}\,1\mathrm{i}}{4\,a^3\,d}}{-\tan\left(c+d\,x\right)^3\,1\mathrm{i}-3\,\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1}$$
$$-\frac{(-1)^{1/4}\,A\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{8\,a^3\,d}$$
$$+\frac{(-1)^{1/4}\,A\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{8\,a^3\,d}$$
$$+\mathrm{atan}\left(\frac{8\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,9\mathrm{i}}{64\,a^6\,d^2}}}{3\,B}\right)\,\sqrt{\frac{B^2\,9\mathrm{i}}{64\,a^6\,d^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{16\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1\mathrm{i}}{256\,a^6\,d^2}}}{B}\right)\,\sqrt{-}$$

input
```
int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
atan((8*a^3*d*tan(c + d*x)^(1/2)*((B^2*9i)/(64*a^6*d^2))^(1/2))/(3*B))*((B
^2*9i)/(64*a^6*d^2))^(1/2)*2i + atan((16*a^3*d*tan(c + d*x)^(1/2)*(-(B^2*1
i)/(256*a^6*d^2))^(1/2))/B)*(-(B^2*1i)/(256*a^6*d^2))^(1/2)*2i + ((5*B*tan
(c + d*x)^(1/2))/(8*a^3*d) + (B*tan(c + d*x)^(3/2)*19i)/(12*a^3*d) - (9*B*
tan(c + d*x)^(5/2))/(8*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c
 + d*x)^3*1i + 1) + ((A*tan(c + d*x)^(3/2))/(12*a^3*d) + (A*tan(c + d*x)^(
5/2)*1i)/(4*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*1
i + 1) - ((-1)^(1/4)*A*atan((-1)^(1/4)*tan(c + d*x)^(1/2)))/(8*a^3*d) + ((
-1)^(1/4)*A*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/(8*a^3*d)
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^3}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)a}{a^3}$$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int((sqrt(tan(c + d*x))*tan(c + d*x)**3)/(tan(c + d*x)**3*i + 3*tan(c
 + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*b + int((sqrt(tan(c + d*x))*tan(c +
d*x)**2)/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)
*a))/a**3
```

3.149 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 261

$$\begin{aligned}\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx &= \frac{((1+i)A+2B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} \\ &\quad - \frac{((1+i)A+2B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} \\ &\quad + \frac{((-1+i)A+2B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3} \\ &\quad + \frac{iB\sqrt{\tan(c+dx)}}{4ad(a+ia\tan(c+dx))^2} + \frac{(A-2iB)\sqrt{\tan(c+dx)}}{8d\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

output

```
-1/32*((1+I)*A+2*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a^3/d-1/32
*((1+I)*A+2*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a^3/d+1/32*((-1+
I)*A+2*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/a^3/d+1
/6*(I*A-B)*tan(d*x+c)^(3/2)/d/(a+I*a*tan(d*x+c))^3+1/4*I*B*tan(d*x+c)^(1/2
)/a/d/(a+I*a*tan(d*x+c))^2+1/8*(A-2*I*B)*tan(d*x+c)^(1/2)/d/(a^3+I*a^3*tan
(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.78 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.70

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{3\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^3(c+dx)(\cos(3(c+dx))+i\sin(3(c+dx)))-3\sqrt[4]{-1}}{}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^3,x]
```

output
```
(3*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]
^3*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - 3*(-1)^(1/4)*B*ArcTanh[(-1)^(
3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^3*(Cos[3*(c + d*x)] + I*Sin[3*(c + d
*x)]) - I*Sqrt[Tan[c + d*x]]*(-I + 3*Tan[c + d*x])*((-3*I)*A + (A - (2*I)*
B)*Tan[c + d*x]))/(24*a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.10, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4079, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 4078}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int\frac{3\sqrt{\tan(c+dx)}(a(iA-B)-a(A-3iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)^2}dx}{6a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int\frac{\sqrt{\tan(c+dx)}(a(iA-B)-a(A-3iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\int\frac{\sqrt{\tan(c+dx)}(a(iA-B)-a(A-3iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{4a^2}$$

$\Big\downarrow$ 4078

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{-\frac{\int-\frac{2\left(iBa^2+(2iA+3B)\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{4a^2}-\frac{iaB\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\int\frac{iBa^2+(2iA+3B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{2a^2}-\frac{iaB\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\int\frac{iBa^2+(2iA+3B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{2a^2}-\frac{iaB\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 4079

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{\int\frac{Aa^3+(iA+2B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{a^2(A-2iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}}{2a^2}-\frac{iaB\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{\int\frac{Aa^3+(iA+2B)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{a^2(A-2iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}}{2a^2}-\frac{iaB\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 4017

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{\int \frac{a^3(A+(iA+2B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{a^2d}-\frac{a^2(A-2iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}}{2a^2}-\frac{iaB\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{a\int \frac{A+(iA+2B)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{a^2(A-2iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}}{2a^2}-\frac{iaB\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 1482

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{a\left(\frac{1}{2}(-2B+(1-i)A)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(2B+(1+i)A)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{a^2(A-2iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}}{2a^2}-\frac{iaB\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{4a^2}$$

$\Big\downarrow$ 1476

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{a\left(\frac{1}{2}(-2B+(1-i)A)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(2B+(1+i)A)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)\right)}{d}\cdots}{2a^2}\cdots}{4a^2}$$

$\Big\downarrow$ 1082

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{a\left(\frac{1}{2}(-2B+(1-i)A)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(2B+(1+i)A)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}-\cdots}{2a^2}\cdots}{4a^2}$$

$\Big\downarrow$ 217

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\frac{a\left(\frac{1}{2}(-2B+(1-i)A)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(2B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}-\frac{a^2(A-2iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}}{2a^2}\cdots}{4a^2}$$

$\Big\downarrow$ 1479

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{a\left(\frac{1}{2}(-2B+(1-i)A)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(2B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\text{ta}\cdots}\right.}{\sqrt{2}}\right.}{d}}{2a^2}}{4a^2}$$

↓ 25

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{a\left(\frac{1}{2}(-2B+(1-i)A)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(2B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+\cdots}\right.}{\sqrt{2}}\right.}{d}}{2a^2}}{4a^2}$$

↓ 27

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{a\left(\frac{1}{2}(-2B+(1-i)A)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}(2B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c-\cdots}\right.}{\sqrt{2}}\right.}{d}}{2a^2}}{4a^2}$$

↓ 1103

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{a\left(\frac{1}{2}(2B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(-2B+(1-i)A)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\cdots}\right.}{2\sqrt{2}}\right.}{d}}{2a^2}}{4a^2}$$

input

```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I*A - B)*Tan[c + d*x]^(3/2))/(6*d*(a + I*a*Tan[c + d*x])^3) - (((-I)*a*B
*Sqrt[Tan[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^2) + ((a*((((1 + I)*A + 2*B
)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*
Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + (((1 - I)*A - 2*B)*(-1/2*Log[1 - Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d - (a^2*(A - (2*I)*B)*Sqrt[Tan[c
 + d*x]])/(d*(a + I*a*Tan[c + d*x])))/(2*a^2))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.58

method	result
derivativedivides	$\frac{\frac{-i(-2iB+A)\tan(dx+c)^{\frac{5}{2}}+\left(-\frac{10A}{3}+\frac{2iB}{3}\right)\tan(dx+c)^{\frac{3}{2}}+iA\sqrt{\tan(dx+c)}}{8(-i+\tan(dx+c))^3}-\frac{B\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{4\left(-\frac{iA}{16}-\frac{B}{16}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{d\,a^3}$
default	$\frac{\frac{-i(-2iB+A)\tan(dx+c)^{\frac{5}{2}}+\left(-\frac{10A}{3}+\frac{2iB}{3}\right)\tan(dx+c)^{\frac{3}{2}}+iA\sqrt{\tan(dx+c)}}{8(-i+\tan(dx+c))^3}-\frac{B\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{4\left(-\frac{iA}{16}-\frac{B}{16}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{d\,a^3}$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^3*(1/8*(-I*(A-2*I*B)*tan(d*x+c)^(5/2)+(-10/3*A+2/3*I*B)*tan(d*x+c)^(
3/2)+I*A*tan(d*x+c)^(1/2))/(-I+tan(d*x+c))^3-1/4*B/(2^(1/2)-I*2^(1/2))*arc
tan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))+4*(-1/16*I*A-1/16*B)/(2^(1/2)+
I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 635 vs. 2(204) = 408.

Time = 0.13 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.43

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
1/96*(3*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)
*log(-2*((I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2
)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^
3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*((
-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) - (A -
 I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + 24*a^3*d*sqrt
(-1/64*I*B^2/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*(8*(a^3*d*e^(2*I*d*x +
 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(-1/64*I*B^2/(a^6*d^2)) + B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 24*a^
3*d*sqrt(-1/64*I*B^2/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*(8*(a^3*d*e^(
2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(-1/64*I*B^2/(a^6*d^2)) - B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
) + 2*(2*(2*A - I*B)*e^(6*I*d*x + 6*I*c) + (4*A + I*B)*e^(4*I*d*x + 4*I*c)
 - (A - 2*I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt((-I*e^(2*I*d*x + 2*I*c)
 + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.47

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{(3i+3)\sqrt{2}B\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)+3\sqrt{2}((i+1)\ A-(i-1)\ B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\right.}{48\,a^3 d}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*((3*I + 3)*sqrt(2)*B*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(tan(d*x + c))
) + 3*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(t
an(d*x + c))) - 2*(-3*I*A*tan(d*x + c)^(5/2) - 6*B*tan(d*x + c)^(5/2) - 10
*A*tan(d*x + c)^(3/2) + 2*I*B*tan(d*x + c)^(3/2) + 3*I*A*sqrt(tan(d*x + c)
))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.92

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{A\sqrt{\tan(c+dx)}}{8a^3d} - \frac{A\tan(c+dx)^{5/2}}{8a^3d} + \frac{A\tan(c+dx)^{3/2}\,5i}{12a^3d}}{-\tan(c+dx)^3\,1i - 3\tan(c+dx)^2 + \tan(c+dx)\,3i + 1}$$
$$+ \frac{\frac{B\tan(c+dx)^{3/2}}{12a^3d} + \frac{B\tan(c+dx)^{5/2}\,1i}{4a^3d}}{-\tan(c+dx)^3\,1i - 3\tan(c+dx)^2 + \tan(c+dx)\,3i + 1}$$
$$- \frac{(-1)^{1/4}B\,\mathrm{atan}\left((-1)^{1/4}\sqrt{\tan(c+dx)}\right)}{8a^3d}$$
$$+ \frac{(-1)^{1/4}B\,\mathrm{atanh}\left((-1)^{1/4}\sqrt{\tan(c+dx)}\right)}{8a^3d}$$
$$- \frac{\sqrt{-\frac{1}{256}i}\,A\,\mathrm{atan}\left(16\sqrt{-\frac{1}{256}i}\sqrt{\tan(c+dx)}\right)\,2i}{a^3d}$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
((A*tan(c + d*x)^(1/2))/(8*a^3*d) + (A*tan(c + d*x)^(3/2)*5i)/(12*a^3*d) -
 (A*tan(c + d*x)^(5/2))/(8*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - t
an(c + d*x)^3*1i + 1) + ((B*tan(c + d*x)^(3/2))/(12*a^3*d) + (B*tan(c + d*
x)^(5/2)*1i)/(4*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)
^3*1i + 1) - ((-1i/256)^(1/2)*A*atan(16*(-1i/256)^(1/2)*tan(c + d*x)^(1/2)
)*2i)/(a^3*d) - ((-1)^(1/4)*B*atan((-1)^(1/4)*tan(c + d*x)^(1/2)))/(8*a^3*
d) + ((-1)^(1/4)*B*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/(8*a^3*d)
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)b - \left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)a}{a^3}$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)**3*i + 3*tan(c
 + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*b + int((sqrt(tan(c + d*x))*tan(c +
d*x))/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*a)
)/a**3
```

3.150 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 267

$$\begin{aligned}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx &= \frac{\left(\frac{1}{16}+\frac{i}{16}\right)((1+i)A+B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^3d} \\ &\quad-\frac{\left(\frac{1}{16}+\frac{i}{16}\right)((1+i)A+B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^3d} \\ &\quad-\frac{(2iA+(1-i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{16\sqrt{2}a^3d}+\frac{(iA-B)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3} \\ &\quad+\frac{(iA+2B)\sqrt{\tan(c+dx)}}{12ad(a+ia\tan(c+dx))^2}+\frac{B\sqrt{\tan(c+dx)}}{8d\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

output
```
(-1/32-1/32*I)*((1+I)*A+B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a^3
/d-(1/32+1/32*I)*((1+I)*A+B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a^
3/d-1/32*(2*I*A+(1-I)*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*
2^(1/2)/a^3/d+1/6*(I*A-B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^3+1/12*(I*
A+2*B)*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^2+1/8*B*tan(d*x+c)^(1/2)/d/
(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{3(-1)^{3/4}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^3(c+dx)(\cos(3(c+dx))+i\sin(3(c+dx)))-3\sqrt[4]{}}{}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^3,x]
```

output
```
(3*(-1)^(3/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]
^3*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - 3*(-1)^(1/4)*A*ArcTanh[(-1)^(
3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^3*(Cos[3*(c + d*x)] + I*Sin[3*(c + d
*x)]) - I*Sqrt[Tan[c + d*x]]*(-3*I + Tan[c + d*x])*(2*A - I*B + 3*B*Tan[c
+ d*x]))/(24*a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.09, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4078, 27, 3042, 4079, 27, 3042, 4079, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\int\frac{a(iA-B)-a(5A-7iB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^2}dx}{6a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\int\frac{a(iA-B)-a(5A-7iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^2}dx}{12a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\int\frac{a(iA-B)-a(5A-7iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^2}dx}{12a^2}$$

$\Big\downarrow$ 4079

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{\int\frac{6\left(ia^2A-a^2(A-2iB)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{4a^2}-\frac{a(2B+iA)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\int\frac{ia^2A-a^2(A-2iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{2a^2}-\frac{a(2B+iA)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\int\frac{ia^2A-a^2(A-2iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{2a^2}-\frac{a(2B+iA)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$\Big\downarrow$ 4079

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{\int\frac{(2iA+B)a^3+iB\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(2B+iA)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{\int\frac{(2iA+B)a^3+iB\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(2B+iA)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$\Big\downarrow$ 4017

$$\frac{\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{\int \frac{a^3(2iA+B+iB\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{a^2d}-\frac{a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(2B+iA)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}}{}$$

$\downarrow$ 27

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{a\int \frac{2iA+B+iB\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(2B+iA)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}$$

$\downarrow$ 1482

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{a\left(\frac{1}{2}(2iA+(1-i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}-\frac{a(2B+iA)\sqrt{\tan(c+}}{d(a+ia\tan(c+dx)}}{12a^2}$$

$\downarrow$ 1476

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{a\left(\frac{1}{2}(2iA+(1-i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}}\right.\right.}{d}}{2a^2}}{12a^2}$$

$\downarrow$ 1082

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{\frac{3\left(\frac{a\left(\frac{1}{2}(2iA+(1-i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right.}{\sqrt{2}}\right.\right.}{d}}{2a^2}}{12a^2}$$

$\downarrow$ 217

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{3\left(\frac{\frac{a\left(\frac{1}{2}(2iA+(1-i)B)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}-\frac{a^2B\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx)\cdots}}{2a^2}\cdots\right.}{12a^2}$$

$$\Big\downarrow \ 1479$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{3\left(\frac{\frac{a\left(\frac{1}{2}(2iA+(1-i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\arctan\left(\cdots\right.}{\cdots}\right.\right.}{d}\cdots}{2a^2}\cdots\right.}{12a^2}$$

$$\Big\downarrow \ 25$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{3\left(\frac{\frac{a\left(\frac{1}{2}(2iA+(1-i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{\cdots}\right.\right.}{d}\cdots}{2a^2}\cdots\right.}{12a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}-\frac{3\left(\frac{\frac{a\left(\frac{1}{2}(2iA+(1-i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{\cdots}\right.\right.}{d}\cdots}{2a^2}\cdots\right.}{12a^2}$$

$$\Big\downarrow \ 1103$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3} - \frac{3\left(\frac{\frac{a\left(\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(2iA+(1-i)B)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+d\cdots\right.}{\cdots}\right.\right.}{d}\cdots}{2a^2}\right.\cdots}{12a^2}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((I*A - B)*Sqrt[Tan[c + d*x]])/(6*d*(a + I*a*Tan[c + d*x])^3) - (-((a*(I*A
 + 2*B)*Sqrt[Tan[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^2)) + (3*((a*((1/2 +
 I/2)*((1 + I)*A + B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) +
 ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + (((2*I)*A + (1 - I)*B)*
(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d - (a^2*B*S
qrt[Tan[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))))/(2*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{-iB\tan(dx+c)^{\frac{5}{2}}+\left(-\frac{10B}{3}-\frac{2iA}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(iB-2A)\sqrt{\tan(dx+c)}}{8(-i+\tan(dx+c))^3}-\frac{A\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{4\left(\frac{A}{16}-\frac{iB}{16}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{d\,a^3}$
default	$\frac{\frac{-iB\tan(dx+c)^{\frac{5}{2}}+\left(-\frac{10B}{3}-\frac{2iA}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(iB-2A)\sqrt{\tan(dx+c)}}{8(-i+\tan(dx+c))^3}-\frac{A\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{4\left(\frac{A}{16}-\frac{iB}{16}\right)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}}{d\,a^3}$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^3*(1/8*(-I*B*tan(d*x+c)^(5/2)+(-10/3*B-2/3*I*A)*tan(d*x+c)^(3/2)+(I*
B-2*A)*tan(d*x+c)^(1/2))/(-I+tan(d*x+c))^3-1/4*A/(2^(1/2)-I*2^(1/2))*arcta
n(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))+4*(1/16*A-1/16*I*B)/(2^(1/2)+I*2
^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(206) = 412.

Time = 0.11 (sec) , antiderivative size = 631, normalized size of antiderivative = 2.36

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
-1/96*(3*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)
*log(2*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) + (
A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*sq
rt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*((a^3*d*e
^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) - (A - I*B)*e^(2*I*
d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 24*a^3*d*sqrt(-1/64*I*A^2/
(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*(8*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3
*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-1/6
4*I*A^2/(a^6*d^2)) + A)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) + 24*a^3*d*sqrt(-1/6
4*I*A^2/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*(8*(a^3*d*e^(2*I*d*x + 2*I
*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(-1/64*I*A^2/(a^6*d^2)) - A)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) + 2*(2*(-I*
A - 2*B)*e^(6*I*d*x + 6*I*c) - (5*I*A + 4*B)*e^(4*I*d*x + 4*I*c) - (4*I*A
- B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) + 1)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{(3i+3)\ \sqrt{2}A\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}((i-1)\ A+(i+1)\ B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\right.}{48\,a^3d}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*((3*I + 3)*sqrt(2)*A*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(tan(d*x + c))
) + 3*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(t
an(d*x + c))) - 2*(-3*I*B*tan(d*x + c)^(5/2) - 2*I*A*tan(d*x + c)^(3/2) -
10*B*tan(d*x + c)^(3/2) - 6*A*sqrt(tan(d*x + c)) + 3*I*B*sqrt(tan(d*x + c)
))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{B\sqrt{\tan(c+dx)}}{8a^3d} - \frac{B\tan(c+dx)^{5/2}}{8a^3d} + \frac{B\tan(c+dx)^{3/2}\,5\mathrm{i}}{12a^3d}}{-\tan(c+dx)^3\,1\mathrm{i} - 3\tan(c+dx)^2 + \tan(c+dx)\,3\mathrm{i} + 1}$$
$$+ \frac{-\frac{A\tan(c+dx)^{3/2}}{12a^3d} + \frac{A\sqrt{\tan(c+dx)}\,1\mathrm{i}}{4a^3d}}{-\tan(c+dx)^3\,1\mathrm{i} - 3\tan(c+dx)^2 + \tan(c+dx)\,3\mathrm{i} + 1}$$
$$- \frac{(-1)^{1/4}A\,\mathrm{atan}\left((-1)^{1/4}\sqrt{\tan(c+dx)}\right)}{8a^3d}$$
$$- \frac{(-1)^{1/4}A\,\mathrm{atanh}\left((-1)^{1/4}\sqrt{\tan(c+dx)}\right)}{8a^3d}$$
$$- \frac{\sqrt{-\frac{1}{256}\mathrm{i}}\,B\,\mathrm{atan}\left(16\sqrt{-\frac{1}{256}\mathrm{i}}\sqrt{\tan(c+dx)}\right)\,2\mathrm{i}}{a^3d}$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
((B*tan(c + d*x)^(1/2))/(8*a^3*d) + (B*tan(c + d*x)^(3/2)*5i)/(12*a^3*d) -
 (B*tan(c + d*x)^(5/2))/(8*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - t
an(c + d*x)^3*1i + 1) + ((A*tan(c + d*x)^(1/2)*1i)/(4*a^3*d) - (A*tan(c +
d*x)^(3/2))/(12*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)
^3*1i + 1) - ((-1)^(1/4)*A*atan((-1)^(1/4)*tan(c + d*x)^(1/2)))/(8*a^3*d)
- ((-1)^(1/4)*A*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/(8*a^3*d) - ((-1i/25
6)^(1/2)*B*atan(16*(-1i/256)^(1/2)*tan(c + d*x)^(1/2))*2i)/(a^3*d)
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-2\sqrt{\tan(dx+c)}\,ai + \left(\int -\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4-3\tan(dx+c)^3 i-3\tan(dx+c)^2+\tan(dx+c)i}dx\right)ad - \left(\int -\frac{\sqrt{\tan(dx+c)}\,\tan}{\tan(dx+c)^3-3\tan(dx+c)}\right.}{}$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 2*sqrt(tan(c + d*x))*a*i + int(( - sqrt(tan(c + d*x)))/(tan(c + d*x)**
4 - 3*tan(c + d*x)**3*i - 3*tan(c + d*x)**2 + tan(c + d*x)*i),x)*a*d - int
(( - sqrt(tan(c + d*x))*tan(c + d*x)**4)/(tan(c + d*x)**3 - 3*tan(c + d*x)
**2*i - 3*tan(c + d*x) + i),x)*a*d*i - 3*int(( - sqrt(tan(c + d*x))*tan(c
+ d*x)**3)/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x)
*a*d + 2*int(( - sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)**3 - 3*
tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x)*a*d*i - 2*int(( - sqrt(tan(c +
d*x))*tan(c + d*x))/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x
) + i),x)*a*d - 3*int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)**3*i
 + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*b*d)/(3*a**3*d)
```

3.151 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 265

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}\,dx \\
&= -\frac{((7-5i)A-2iB)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} \\
&\quad+\frac{((7-5i)A-2iB)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} \\
&\quad+\frac{((7+5i)A-2iB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{16\sqrt{2}a^3d}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3} \\
&\quad+\frac{(4A+iB)\sqrt{\tan(c+dx)}}{12ad(a+ia\tan(c+dx))^2}+\frac{5A\sqrt{\tan(c+dx)}}{8d\,(a^3+ia^3\tan(c+dx))}
\end{aligned}$$

output
```
1/32*((7-5*I)*A-2*I*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a^3/d+1
/32*((7-5*I)*A-2*I*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/a^3/d+1/3
2*((7+5*I)*A-2*I*B)*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/
2)/a^3/d+1/6*(A+I*B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^3+1/12*(4*A+I*B
)*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^2+5/8*A*tan(d*x+c)^(1/2)/d/(a^3+
I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 2.81 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.63

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{3\sqrt[4]{-1}(-6A+iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\frac{-3\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^3(c+dx)(\cos(3(}{}}{24a^3d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^
3),x]
```

output
```
(3*(-1)^(1/4)*(-6*A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (-3*(-
1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^3*(C
os[3*(c + d*x)] + I*Sin[3*(c + d*x)]) + Sqrt[Tan[c + d*x]]*((27*I)*A - 6*B
 - 2*(19*A + I*B)*Tan[c + d*x] - (15*I)*A*Tan[c + d*x]^2))/(-I + Tan[c + d
*x])^3)/(24*a^3*d)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.09, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(11A-iB)-5a(iA-B)\tan(c+dx)}{2\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a(11A-iB)-5a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^2}dx}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{a(11A-iB)-5a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^2}dx}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\frac{\int \frac{6\left(a^2(6A-iB)-a^2(4iA-B)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{4a^2}+\frac{a(4A+iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\int \frac{a^2(6A-iB)-a^2(4iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{a(4A+iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3\int \frac{a^2(6A-iB)-a^2(4iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{a(4A+iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\frac{3\left(\frac{\int \frac{a^3(7A-2iB)-5ia^3A\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{2a^2}+\frac{5a^2A\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}+\frac{a(4A+iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{3\left(\frac{\int \frac{a^3(7A-2iB)-5ia^3A\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{2a^2}+\frac{5a^2A\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}+\frac{a(4A+iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 4017}$$

$$\frac{\frac{3\left(\frac{\int \frac{a^3(-5i\tan(c+dx)A+7A-2iB)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{a^2d}+\frac{5a^2A\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}+\frac{a(4A+iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 27

$$\frac{\frac{3\left(\frac{a\int \frac{-5i\tan(c+dx)A+7A-2iB}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{5a^2A\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}+\frac{a(4A+iB)\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))^2}}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 1482

$$\frac{\frac{3\left(\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((7-5i)A-2iB)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{5a^2A\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right)}{2a^2}+\frac{a(4A+iB)\sqrt{\tan(c+d}}{d(a+ia\tan(c+dx))^2}}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 1476

$$\frac{\frac{3\left(\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((7-5i)A-2iB)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+}\right)\right)}{d}\right)}{2a^2}}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 1082

$$\frac{\frac{3\left(\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((7-5i)A-2iB)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}\right)}{2a^2}}{12a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 217

$$\frac{3\left(\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}((7-5i)A-2iB)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}+\frac{5a^2A\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}\right.}{2a^2}$$

$$12a^2$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 1479$$

$$\frac{3\left(\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((7-5i)A-2iB)\left(\frac{\arctan\left(\sqrt{\right.}{}\right.}{}\right.}{d}\right.}{2a^2}$$

$$12a^2$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{3\left(\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((7-5i)A-2iB)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\mathrm{ta}}\right.}{\sqrt{2}}\right.}{d}\right.}{2a^2}$$

$$12a^2$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}((7-5i)A-2iB)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}{}\right.}{d}\right.}{2a^2}$$

$$12a^2$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{3\left(\frac{5a^2A\sqrt{\tan(c+dx)}}{d(a+ia\tan(c+dx))}+\frac{a\left(\frac{1}{2}((7-5i)A-2iB)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((7+5i)A-2iB)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}\right.}{2\sqrt{2}}\right.\right.}{d}\right.}{2a^2}}{12a^2}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{6d(a+ia\tan(c+dx))^3}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
((A + I*B)*Sqrt[Tan[c + d*x]])/(6*d*(a + I*a*Tan[c + d*x])^3) + ((a*(4*A +
 I*B)*Sqrt[Tan[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^2) + (3*((a*((((7 - 5*
I)*A - (2*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + (((7 + 5*I)*A - (2*I)*B)*(
-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d + (5*a^2*A*
Sqrt[Tan[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))))/(2*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{-\frac{5iA\tan(dx+c)^{\frac{5}{2}}+\left(\frac{38A}{3}+\frac{2iB}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(-9iA+2B)\sqrt{\tan(dx+c)}}{8(-i+\tan(dx+c))^3}-\frac{(6iA+B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{4\left(\frac{iA}{16}+\frac{B}{16}\right)\arctan\cdots}{\sqrt{2}+\cdots}}{d\,a^3}$
default	$\frac{-\frac{5iA\tan(dx+c)^{\frac{5}{2}}+\left(\frac{38A}{3}+\frac{2iB}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(-9iA+2B)\sqrt{\tan(dx+c)}}{8(-i+\tan(dx+c))^3}-\frac{(6iA+B)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{4\left(\frac{iA}{16}+\frac{B}{16}\right)\arctan\cdots}{\sqrt{2}+\cdots}}{d\,a^3}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^3*(-1/8*(5*I*A*tan(d*x+c)^(5/2)+(38/3*A+2/3*I*B)*tan(d*x+c)^(3/2)+(-
9*I*A+2*B)*tan(d*x+c)^(1/2))/(-I+tan(d*x+c))^3-1/4*(6*I*A+B)/(2^(1/2)-I*2^
(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))+4*(1/16*I*A+1/16*B)/
(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(206) = 412.

Time = 0.10 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.57

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
-1/96*(3*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c
)*log(-2*((I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^
2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a
^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*(
(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) - (A
- I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*sqrt
((36*I*A^2 + 12*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*((a^3*
d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((36*I*A^2 + 12*A*B - I*B^2)/(a^6*d^2)) + 6*I*A + B)
*e^(-2*I*d*x - 2*I*c)/(a^3*d)) + 3*a^3*d*sqrt((36*I*A^2 + 12*A*B - I*B^2)/
(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*
d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((36*I
*A^2 + 12*A*B - I*B^2)/(a^6*d^2)) - 6*I*A - B)*e^(-2*I*d*x - 2*I*c)/(a^3*d
)) - 2*(2*(10*A + I*B)*e^(6*I*d*x + 6*I*c) + (26*A + 5*I*B)*e^(4*I*d*x + 4
*I*c) + (7*A + 4*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*sqrt((-I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + ia\tan(c + dx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.49

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{3\sqrt{2}((6i-6)\ A+(i+1)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}(-(i+1)\ A+(i-1)\ B)\,a$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*(3*sqrt(2)*((6*I - 6)*A + (I + 1)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sq
rt(tan(d*x + c))) + 3*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-(1/2*I - 1/
2)*sqrt(2)*sqrt(tan(d*x + c))) - 2*(-15*I*A*tan(d*x + c)^(5/2) - 38*A*tan(
d*x + c)^(3/2) - 2*I*B*tan(d*x + c)^(3/2) + 27*I*A*sqrt(tan(d*x + c)) - 6*
B*sqrt(tan(d*x + c)))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.16

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{9\,A\,\sqrt{\tan(c+d\,x)}}{8\,a^3\,d}-\frac{5\,A\,\tan(c+d\,x)^{5/2}}{8\,a^3\,d}+\frac{A\,\tan(c+d\,x)^{3/2}\,19\mathrm{i}}{12\,a^3\,d}}{-\tan\left(c+d\,x\right)^3\,1\mathrm{i}-3\,\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1}$$
$$+\frac{-\frac{B\,\tan(c+d\,x)^{3/2}}{12\,a^3\,d}+\frac{B\,\sqrt{\tan(c+d\,x)}\,1\mathrm{i}}{4\,a^3\,d}}{-\tan\left(c+d\,x\right)^3\,1\mathrm{i}-3\,\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1}$$
$$-\frac{(-1)^{1/4}\,B\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{8\,a^3\,d}$$
$$-\frac{(-1)^{1/4}\,B\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(c+d\,x\right)}\right)}{8\,a^3\,d}$$
$$-\mathrm{atan}\left(\frac{8\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,9\mathrm{i}}{64\,a^6\,d^2}}}{3\,A}\right)\,\sqrt{\frac{A^2\,9\mathrm{i}}{64\,a^6\,d^2}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{16\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,1\mathrm{i}}{256\,a^6\,d^2}}}{A}\right)\,\sqrt{-}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
atan((16*a^3*d*tan(c + d*x)^(1/2)*(-(A^2*1i)/(256*a^6*d^2))^(1/2))/A)*(-(A
^2*1i)/(256*a^6*d^2))^(1/2)*2i - atan((8*a^3*d*tan(c + d*x)^(1/2)*((A^2*9i
)/(64*a^6*d^2))^(1/2))/(3*A))*((A^2*9i)/(64*a^6*d^2))^(1/2)*2i + ((9*A*tan
(c + d*x)^(1/2))/(8*a^3*d) + (A*tan(c + d*x)^(3/2)*19i)/(12*a^3*d) - (5*A*
tan(c + d*x)^(5/2))/(8*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c
 + d*x)^3*1i + 1) + ((B*tan(c + d*x)^(1/2)*1i)/(4*a^3*d) - (B*tan(c + d*x)
^(3/2))/(12*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3*1
i + 1) - ((-1)^(1/4)*B*atan((-1)^(1/4)*tan(c + d*x)^(1/2)))/(8*a^3*d) - ((
-1)^(1/4)*B*atanh((-1)^(1/4)*tan(c + d*x)^(1/2)))/(8*a^3*d)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-2\sqrt{\tan(dx+c)}\,bi+3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4-3\tan(dx+c)^3i-3\tan(dx+c)^2+\tan(dx+c)i}dx\right)adi-\left(\int \frac{\sqrt{\tan(}}{\tan(dx+c)^4-3\tan(dx+c)^3i-}\right.}{}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 2*sqrt(tan(c + d*x))*b*i + 3*int(sqrt(tan(c + d*x))/(tan(c + d*x)**4 -
 3*tan(c + d*x)**3*i - 3*tan(c + d*x)**2 + tan(c + d*x)*i),x)*a*d*i - int(
sqrt(tan(c + d*x))/(tan(c + d*x)**4 - 3*tan(c + d*x)**3*i - 3*tan(c + d*x)
**2 + tan(c + d*x)*i),x)*b*d + int((sqrt(tan(c + d*x))*tan(c + d*x)**4)/(t
an(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x)*b*d*i + 3*in
t((sqrt(tan(c + d*x))*tan(c + d*x)**3)/(tan(c + d*x)**3 - 3*tan(c + d*x)**
2*i - 3*tan(c + d*x) + i),x)*b*d - 2*int((sqrt(tan(c + d*x))*tan(c + d*x)*
*2)/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x)*b*d*i
+ 2*int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)**3 - 3*tan(c + d*x
)**2*i - 3*tan(c + d*x) + i),x)*b*d)/(3*a**3*d)
```

3.152 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 310

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx &= \frac{((30+28i)A-(7-5i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{16\sqrt{2}a^3d} \\ &- \frac{\left(\frac{1}{16}-\frac{i}{16}\right)((1+29i)A-(6+i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^3d} \\ &+ \frac{\left(\frac{1}{16}-\frac{i}{16}\right)((29+i)A+(1+6i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}a^3d} - \frac{5(6A+iB)}{8a^3d\sqrt{\tan(c+dx)}} \\ &+ \frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3} + \frac{5A+2iB}{12ad\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2} \\ &+ \frac{7(4A+iB)}{24d\sqrt{\tan(c+dx)}\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

output
```
-1/32*((30+28*I)*A+(-7+5*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)
/a^3/d+(-1/32+1/32*I)*((1+29*I)*A-(6+I)*B)*arctan(1+2^(1/2)*tan(d*x+c)^(1/
2))*2^(1/2)/a^3/d+(1/32-1/32*I)*((29+I)*A+(1+6*I)*B)*arctanh(2^(1/2)*tan(d
*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/a^3/d-5/8*(6*A+I*B)/a^3/d/tan(d*x+c)^(
1/2)+1/6*(A+I*B)/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3+1/12*(5*A+2*I*B)/
a/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2+7/24*(4*A+I*B)/d/tan(d*x+c)^(1/2
)/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.95 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.63

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sec^3(c+dx)\,(2i\cos(c+dx)(7A+4iB+(35A+11iB)\cos(2(c+dx))+(33iA-9B)\sin(2(c+dx)))+}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^
3),x]
```

output
```
(Sec[c + d*x]^3*((2*I)*Cos[c + d*x]*(7*A + (4*I)*B + (35*A + (11*I)*B)*Cos
[2*(c + d*x)] + ((33*I)*A - 9*B)*Sin[2*(c + d*x)]) + 6*((-29*I)*A + 6*B)*H
ypergeometric2F1[-1/2, 1, 1/2, (-I)*Tan[c + d*x]]*(Cos[3*(c + d*x)] + I*Si
n[3*(c + d*x)]) + 6*(A - I*B)*Hypergeometric2F1[-1/2, 1, 1/2, I*Tan[c + d*
x]]*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])))/(48*a^3*d*Sqrt[Tan[c + d*
x]]*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.09, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.639$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4012, 25, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(13A+iB)-7a(iA-B)\tan(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(13A+iB)-7a(iA-B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^2}dx}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(13A+iB)-7a(iA-B)\tan(c+dx)}{\tan(c+dx)^{3/2}(i\tan(c+dx)a+a)^2}dx}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int \frac{2\left(a^2(31A+4iB)-5a^2(5iA-2B)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)}dx}{4a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a^2(31A+4iB)-5a^2(5iA-2B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(31A+4iB)-5a^2(5iA-2B)\tan(c+dx)}{\tan(c+dx)^{3/2}(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\frac{\int \frac{3\left(5a^3(6A+iB)-7a^3(4iA-B)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{7a^2(4A+iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}}{2a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{5a^3(6A+iB)-7a^3(4iA-B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{7a^2(4A+iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}}{2a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{3\int \frac{5a^3(6A+iB)-7a^3(4iA-B)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx}{2a^2}+\frac{7a^2(4A+iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}}{2a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\frac{3\left(\int -\frac{7(4iA-B)a^3+5(6A+iB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx-\frac{10a^3(6A+iB)}{d\sqrt{\tan(c+dx)}}\right)}{2a^2}+\frac{7a^2(4A+iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}}{2a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{3\left(-\int \frac{7(4iA-B)a^3+5(6A+iB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx-\frac{10a^3(6A+iB)}{d\sqrt{\tan(c+dx)}}\right)}{2a^2}+\frac{7a^2(4A+iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}}{2a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{3\left(-\int \frac{7(4iA-B)a^3+5(6A+iB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx-\frac{10a^3(6A+iB)}{d\sqrt{\tan(c+dx)}}\right)}{2a^2}+\frac{7a^2(4A+iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}}{2a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \ 4017$$

$$\frac{\frac{\frac{3\left(-\frac{2\int \frac{a^3(7(4iA-B)+5(6A+iB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{10a^3(6A+iB)}{d\sqrt{\tan(c+dx)}}\right)}{2a^2}+\frac{7a^2(4A+iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}}{2a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3\left(-\frac{2a^3\int \frac{7(4iA-B)+5(6A+iB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{10a^3(6A+iB)}{d\sqrt{\tan(c+dx)}}\right)}{2a^2}+\frac{7a^2(4A+iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}}{2a^2}+\frac{a(5A+2iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^2}}{12a^2}+\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \ 1482$$

$$\frac{\frac{3\left(-\frac{2a^3\left(\frac{1}{2}((30+28i)A-(7-5i)B)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{10a^3(6A+iB)}{d\sqrt{\tan(c+dx)}}\right)}{2a^2}+\frac{7\ldots}{d\sqrt{\tan(c+\ldots}}}{2a^2}}{12a^2}$$

$$\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{3\left(-\frac{2a^3\left(\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1\ldots}{d}\ldots}{2a^2}\ldots}{2a^2}}{12a^2}$$

$$\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{3\left(-\frac{2a^3\left(\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\int \frac{1-\tan\ldots}{\tan^2(\ldots}\ldots}{d}\ldots}{2a^2}\ldots}{2a^2}}{12a^2}$$

$$\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{3\left(-\frac{2a^3\left(\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}\ldots}{2a^2}\ldots}{2a^2}}{12a^2}$$

$$\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \ 1479$$

$$3\left(-\frac{2a^3\left(\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d}}}{2\sqrt{2}}\right.\right.}{d}\right.$$

$$\overline{2a^2}$$

$$\overline{2a^2}$$

$$\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$3\left(-\frac{2a^3\left(\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+}}{2\sqrt{2}}\right.\right.}{d}\right.$$

$$\overline{2a^2}$$

$$\overline{2a^2}$$

$$\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$3\left(-\frac{2a^3\left(\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+}}{2\sqrt{2}}\right.\right.}{d}\right.$$

$$\overline{2a^2}$$

$$\overline{2a^2}$$

$$\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\frac{7a^2(4A+iB)}{d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))}+\frac{3\left(-\frac{2a^3\left(\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\cdots\right.}{d}\cdots\right.}{2a^2}\cdots}{2a^2}\cdots}{12a^2}$$

$$\frac{A+iB}{6d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^3}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
(A + I*B)/(6*d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + ((a*(5*A + (
2*I)*B))/(d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^2) + ((3*((-2*a^3*((
((30 + 28*I)*A - (7 - 5*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sq
rt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 - (1/2 - I/2)*
((29 + I)*A + (1 + 6*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*
Sqrt[2]))))/d - (10*a^3*(6*A + I*B))/(d*Sqrt[Tan[c + d*x]])))/(2*a^2) + (7
*a^2*(4*A + I*B))/(d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])))/(2*a^2))/
(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{i(14iA-5B)\tan(dx+c)^{\frac{5}{2}}+\left(\frac{98iA}{3}-\frac{38B}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(9iB+20A)\sqrt{\tan(dx+c)}}{8(-i+\tan(dx+c))^{3}}-\frac{(6iB+29A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{4\left(-\frac{A}{16}+\right.}{}}{d\,a^{3}}$
default	$\frac{\frac{i(14iA-5B)\tan(dx+c)^{\frac{5}{2}}+\left(\frac{98iA}{3}-\frac{38B}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(9iB+20A)\sqrt{\tan(dx+c)}}{8(-i+\tan(dx+c))^{3}}-\frac{(6iB+29A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{4\left(\sqrt{2}-i\sqrt{2}\right)}+\frac{4\left(-\frac{A}{16}+\right.}{}}{d\,a^{3}}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^3*(1/8*(I*(14*I*A-5*B)*tan(d*x+c)^(5/2)+(98/3*I*A-38/3*B)*tan(d*x+c)
^(3/2)+(20*A+9*I*B)*tan(d*x+c)^(1/2))/(-I+tan(d*x+c))^3-1/4*(6*I*B+29*A)/(
2^(1/2)-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))+4*(-1/16
*A+1/16*I*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)+I*2^(1
/2)))-2*A/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(235) = 470$.

Time = 0.11 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.53

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia\tan(c + dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
1/96*(3*(a^3*d*e^(8*I*d*x + 8*I*c) - a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((I*A^
2 + 2*A*B - I*B^2)/(a^6*d^2))*log(2*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*s
qrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*A^2 +
2*A*B - I*B^2)/(a^6*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2
*I*c)/(I*A + B)) - 3*(a^3*d*e^(8*I*d*x + 8*I*c) - a^3*d*e^(6*I*d*x + 6*I*c
))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*log(-2*((a^3*d*e^(2*I*d*x + 2*I
*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e
^(-2*I*d*x - 2*I*c)/(I*A + B)) + 3*(a^3*d*e^(8*I*d*x + 8*I*c) - a^3*d*e^(6
*I*d*x + 6*I*c))*sqrt((-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2))*log(1/8
*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt((-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2))
 + 29*A + 6*I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 3*(a^3*d*e^(8*I*d*x + 8*I
*c) - a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((-841*I*A^2 + 348*A*B + 36*I*B^2)/(a
^6*d^2))*log(-1/8*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-841*I*A^2 + 348*A*B + 36*
I*B^2)/(a^6*d^2)) - 29*A - 6*I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 2*(2*(73
*I*A - 10*B)*e^(8*I*d*x + 8*I*c) + 3*(35*I*A - 2*B)*e^(6*I*d*x + 6*I*c) -
(49*I*A - 19*B)*e^(4*I*d*x + 4*I*c) + 3*(-3*I*A + 2*B)*e^(2*I*d*x + 2*I*c)
 - I*A + B)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real -I
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.49

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{3\sqrt{2}(-(29i+29)\,A-(6i-6)\,B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)-3\sqrt{2}(-(i-1)\,A-(i+1)\,B}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
1/48*(3*sqrt(2)*(-(29*I + 29)*A - (6*I - 6)*B)*arctan((1/2*I + 1/2)*sqrt(2
)*sqrt(tan(d*x + c))) - 3*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-(1/2*I
- 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - 96*A/sqrt(tan(d*x + c)) - 2*(42*A*tan
(d*x + c)^(5/2) + 15*I*B*tan(d*x + c)^(5/2) - 98*I*A*tan(d*x + c)^(3/2) +
38*B*tan(d*x + c)^(3/2) - 60*A*sqrt(tan(d*x + c)) - 27*I*B*sqrt(tan(d*x +
c)))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.25

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$$
$$= 2\,\mathrm{atanh}\left(\frac{16\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,1\mathrm{i}}{256\,a^6\,d^2}}}{A}\right)\,\sqrt{\frac{A^2\,1\mathrm{i}}{256\,a^6\,d^2}}$$
$$+ 2\,\mathrm{atanh}\left(\frac{16\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,841\mathrm{i}}{256\,a^6\,d^2}}}{29\,A}\right)\,\sqrt{-\frac{A^2\,841\mathrm{i}}{256\,a^6\,d^2}}$$
$$-\mathrm{atan}\left(\frac{8\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,9\mathrm{i}}{64\,a^6\,d^2}}}{3\,B}\right)\,\sqrt{\frac{B^2\,9\mathrm{i}}{64\,a^6\,d^2}}\,2\mathrm{i}$$
$$+\mathrm{atan}\left(\frac{16\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,1\mathrm{i}}{256\,a^6\,d^2}}}{B}\right)\,\sqrt{-\frac{B^2\,1\mathrm{i}}{256\,a^6\,d^2}}\,2\mathrm{i}$$
$$-\frac{\frac{2\,A}{a^3\,d}+\frac{A\,\tan\left(c+d\,x\right)\,17\mathrm{i}}{2\,a^3\,d}-\frac{121\,A\,{\tan\left(c+d\,x\right)}^2}{12\,a^3\,d}-\frac{A\,{\tan\left(c+d\,x\right)}^3\,15\mathrm{i}}{4\,a^3\,d}}{\sqrt{\tan\left(c+d\,x\right)}+{\tan\left(c+d\,x\right)}^{3/2}\,3\mathrm{i}-3\,{\tan\left(c+d\,x\right)}^{5/2}-{\tan\left(c+d\,x\right)}^{7/2}\,1\mathrm{i}}$$
$$+\frac{\frac{9\,B\,\sqrt{\tan\left(c+d\,x\right)}}{8\,a^3\,d}-\frac{5\,B\,{\tan\left(c+d\,x\right)}^{5/2}}{8\,a^3\,d}+\frac{B\,{\tan\left(c+d\,x\right)}^{3/2}\,19\mathrm{i}}{12\,a^3\,d}}{-{\tan\left(c+d\,x\right)}^3\,1\mathrm{i}-3\,{\tan\left(c+d\,x\right)}^2+\tan\left(c+d\,x\right)\,3\mathrm{i}+1}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
2*atanh((16*a^3*d*tan(c + d*x)^(1/2)*((A^2*1i)/(256*a^6*d^2))^(1/2))/A)*((
A^2*1i)/(256*a^6*d^2))^(1/2) + 2*atanh((16*a^3*d*tan(c + d*x)^(1/2)*(-(A^2
*841i)/(256*a^6*d^2))^(1/2))/(29*A))*(-(A^2*841i)/(256*a^6*d^2))^(1/2) - a
tan((8*a^3*d*tan(c + d*x)^(1/2)*((B^2*9i)/(64*a^6*d^2))^(1/2))/(3*B))*((B^
2*9i)/(64*a^6*d^2))^(1/2)*2i + atan((16*a^3*d*tan(c + d*x)^(1/2)*(-(B^2*1i
)/(256*a^6*d^2))^(1/2))/B)*(-(B^2*1i)/(256*a^6*d^2))^(1/2)*2i - ((2*A)/(a^
3*d) + (A*tan(c + d*x)*17i)/(2*a^3*d) - (121*A*tan(c + d*x)^2)/(12*a^3*d)
- (A*tan(c + d*x)^3*15i)/(4*a^3*d))/(tan(c + d*x)^(1/2) + tan(c + d*x)^(3/
2)*3i - 3*tan(c + d*x)^(5/2) - tan(c + d*x)^(7/2)*1i) + ((9*B*tan(c + d*x)
^(1/2))/(8*a^3*d) + (B*tan(c + d*x)^(3/2)*19i)/(12*a^3*d) - (5*B*tan(c + d
*x)^(5/2))/(8*a^3*d))/(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c + d*x)^3
*1i + 1)
```

Reduce [F]

$$\int \frac{A + B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - 4*sqrt(tan(c + d*x))*tan(c + d*x)*b + 6*sqrt(tan(c + d*x))*b*i + 9*int
(sqrt(tan(c + d*x))/(tan(c + d*x)**5 - 3*tan(c + d*x)**4*i - 3*tan(c + d*x
)**3 + tan(c + d*x)**2*i),x)*tan(c + d*x)*a*d*i - 3*int(sqrt(tan(c + d*x))
/(tan(c + d*x)**5 - 3*tan(c + d*x)**4*i - 3*tan(c + d*x)**3 + tan(c + d*x)
**2*i),x)*tan(c + d*x)*b*d - 2*int(sqrt(tan(c + d*x))/(tan(c + d*x)**4*i +
 3*tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - tan(c + d*x)),x)*tan(c + d*x)*b
*d + 2*int((sqrt(tan(c + d*x))*tan(c + d*x)**4)/(tan(c + d*x)**3*i + 3*tan
(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)*b*d*i + 3*int((sqrt(t
an(c + d*x))*tan(c + d*x)**3)/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*t
an(c + d*x) + i),x)*tan(c + d*x)*b*d*i + 6*int((sqrt(tan(c + d*x))*tan(c +
 d*x)**3)/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x
)*tan(c + d*x)*b*d + 9*int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d
*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x)*tan(c + d*x)*b*d - 4
*int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)**3*i + 3*tan(c + d
*x)**2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)*b*d*i - 6*int((sqrt(tan(c +
 d*x))*tan(c + d*x))/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*
x) + i),x)*tan(c + d*x)*b*d*i + 4*int((sqrt(tan(c + d*x))*tan(c + d*x))/(t
an(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*
x)*b*d)/(9*tan(c + d*x)*a**3*d)
```

3.153 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 343

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx \\
&= \frac{\left(\frac{1}{16}-\frac{i}{16}\right)((76+i)A+(1+29i)B)\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^3d} \\
&\quad -\frac{\left(\frac{1}{16}-\frac{i}{16}\right)((76+i)A+(1+29i)B)\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}a^3d} \\
&\quad -\frac{\left(\frac{1}{16}-\frac{i}{16}\right)((1+76i)A-(29+i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}a^3d}-\frac{7(11A+4iB)}{24a^3d\tan^{\frac{3}{2}}(c+dx)} \\
&\quad +\frac{15(5iA-2B)}{8a^3d\sqrt{\tan(c+dx)}}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3} \\
&\quad +\frac{2A+iB}{4ad\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}+\frac{3(5A+2iB)}{8d\tan^{\frac{3}{2}}(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}
\end{aligned}$$

output
```
(-1/32+1/32*I)*((76+I)*A+(1+29*I)*B)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2
^(1/2)/a^3/d+(-1/32+1/32*I)*((76+I)*A+(1+29*I)*B)*arctan(1+2^(1/2)*tan(d*x
+c)^(1/2))*2^(1/2)/a^3/d+(-1/32+1/32*I)*((1+76*I)*A-(29+I)*B)*arctanh(2^(1
/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/a^3/d-7/24*(11*A+4*I*B)/a^3/d
/tan(d*x+c)^(3/2)+15/8*(5*I*A-2*B)/a^3/d/tan(d*x+c)^(1/2)+1/6*(A+I*B)/d/ta
n(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3+1/4*(2*A+I*B)/a/d/tan(d*x+c)^(3/2)/(a+
I*a*tan(d*x+c))^2+3/8*(5*A+2*I*B)/d/tan(d*x+c)^(3/2)/(a^3+I*a^3*tan(d*x+c)
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.99 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.57

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sec^3(c+dx)\,(2i\cos(c+dx)(8A+5iB+(53A+23iB)\cos(2(c+dx))+(51iA-21B)\sin(2(c+dx)))\ldots}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^
3),x]
```

output
```
(Sec[c + d*x]^3*((2*I)*Cos[c + d*x]*(8*A + (5*I)*B + (53*A + (23*I)*B)*Cos
[2*(c + d*x)] + ((51*I)*A - 21*B)*Sin[2*(c + d*x)]) + 2*((-76*I)*A + 29*B)
*Hypergeometric2F1[-3/2, 1, -1/2, (-I)*Tan[c + d*x]]*(Cos[3*(c + d*x)] + I
*Sin[3*(c + d*x)]) + 2*(A - I*B)*Hypergeometric2F1[-3/2, 1, -1/2, I*Tan[c
+ d*x]]*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])))/(48*a^3*d*Tan[c + d*x
]^(3/2)*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.07, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 3042, 4012, 25, 3042, 4012, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{3(a(5A+iB)-3a(iA-B)\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(5A+iB)-3a(iA-B)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)^2}dx}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(5A+iB)-3a(iA-B)\tan(c+dx)}{\tan(c+dx)^{5/2}(i\tan(c+dx)a+a)^2}dx}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int \frac{2\left(a^2(16A+5iB)-7a^2(2iA-B)\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)}dx}{4a^2}+\frac{a(2A+iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a^2(16A+5iB)-7a^2(2iA-B)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{a(2A+iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(16A+5iB)-7a^2(2iA-B)\tan(c+dx)}{\tan(c+dx)^{5/2}(i\tan(c+dx)a+a)}dx}{2a^2}+\frac{a(2A+iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\frac{\int \frac{7a^3(11A+4iB)-15a^3(5iA-2B)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}dx}{2a^2}+\frac{3a^2(5A+2iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}}{2a^2}+\frac{a(2A+iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{7a^3(11A+4iB)-15a^3(5iA-2B)\tan(c+dx)}{\tan(c+dx)^{5/2}}dx}{2a^2}+\frac{3a^2(5A+2iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}}{2a^2}+\frac{a(2A+iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\frac{\int -\frac{15(5iA-2B)a^3+7(11A+4iB)\tan(c+dx)a^3}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{14a^3(11A+4iB)}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{3a^2(5A+2iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}}{2a^2}+\frac{a(2A+iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{-\int \frac{15(5iA-2B)a^3+7(11A+4iB)\tan(c+dx)a^3}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{14a^3(11A+4iB)}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{3a^2(5A+2iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}}{2a^2}+\frac{a(2A+iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{-\int \frac{15(5iA-2B)a^3+7(11A+4iB)\tan(c+dx)a^3}{\tan(c+dx)^{3/2}}dx-\frac{14a^3(11A+4iB)}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{3a^2(5A+2iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}}{2a^2}+\frac{a(2A+iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\frac{-\int \frac{7a^3(11A+4iB)-15a^3(5iA-2B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{14a^3(11A+4iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{30a^3(-2B+5iA)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{3a^2(5A+2iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}}{2a^2}+\frac{a(2A+iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{-\int \frac{7a^3(11A+4iB)-15a^3(5iA-2B)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{14a^3(11A+4iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{30a^3(-2B+5iA)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{3a^2(5A+2iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}}{2a^2}+\frac{a(2A+iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}}{4a^2}+\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 4017

$$\frac{\frac{-\frac{\frac{2\int \frac{a^3(7(11A+4iB)-15(5iA-2B)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{14a^3(11A+4iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{30a^3(-2B+5iA)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{3a^2(5A+2iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}}{2a^2}+\frac{a(2A+i}{d\tan^{\frac{3}{2}}(c+dx)(a+i}}{4a^2}$$

$$\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{\frac{2a^3\int \frac{7(11A+4iB)-15(5iA-2B)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{14a^3(11A+4iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{30a^3(-2B+5iA)}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{3a^2(5A+2iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}}{2a^2}+\frac{a(2A+iB}{d\tan^{\frac{3}{2}}(c+dx)(a+ia}}{4a^2}$$

$$\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 1482$$

$$\frac{\frac{-\frac{\frac{2a^3\left(\frac{1}{2}((77+75i)A-(30-28i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)((76+i)A+(1+29i)B)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{14a^3(11A+4iB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{30a^3(-2B}{d\sqrt{\tan(c}}}{2a^2}}{2a^2}}{4a^2}$$

$$\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{-\frac{\frac{2a^3\left(\frac{1}{2}((77+75i)A-(30-28i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)((76+i)A+(1+29i)B)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{}{\tan(c+}\right.\right.}{d}}{2a^2}}{2a^2}}{4a^2}$$

$$\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$$\Big\downarrow \; 1082$$

$$-\frac{\frac{\frac{2a^3\left(\frac{1}{2}((77+75i)A-(30-28i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)((76+i)A+(1+29i)B)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\int \frac{1}{-\tan(c+dx)-1}\right.\right.}{d}}{2a^2}}{2a^2}}{4a^2}$$

$$\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 217

$$-\frac{\frac{\frac{2a^3\left(\frac{1}{2}((77+75i)A-(30-28i)B)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)((76+i)A+(1+29i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}}{2a^2}}{2a^2}}{4a^2}$$

$$\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 1479

$$-\frac{\frac{\frac{2a^3\left(\frac{1}{2}((77+75i)A-(30-28i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)((76+i)A+\right.}{d}}{2a^2}}{2a^2}}{}$$

$$\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 25

$$-\frac{\frac{\frac{2a^3\left(\frac{1}{2}((77+75i)A-(30-28i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)((76+i)A+(1+2\right.}{d}}{2a^2}}{2a^2}}{}$$

$$\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 27

$$-\frac{\frac{\frac{2a^3\left(\frac{1}{2}((77+75i)A-(30-28i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)((76+i)A+(1+\cdots}{d}}{2a^2}}{2a^2}}{}$$

$$\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

$\Big\downarrow$ 1103

$$\frac{\frac{3a^2(5A+2iB)}{d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}+\frac{-\frac{2a^3\left(\left(\frac{1}{2}-\frac{i}{2}\right)((76+i)A+(1+29i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((77+75i)A-(30-28i)B\cdots}{d}}{2a^2}}{2a^2}}{4\cdots}$$

$$\frac{A+iB}{6d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
(A + I*B)/(6*d*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3) + ((a*(2*A + I
*B))/(d*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2) + (((-2*a^3*((1/2 - I
/2)*((76 + I)*A + (1 + 29*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/
Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + (((77 + 75*I)
*A - (30 - 28*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]
)))/2))/d - (14*a^3*(11*A + (4*I)*B))/(3*d*Tan[c + d*x]^(3/2)) + (30*a^3*(
(5*I)*A - 2*B))/(d*Sqrt[Tan[c + d*x]]))/(2*a^2) + (3*a^2*(5*A + (2*I)*B))/
(d*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])))/(2*a^2))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{-\frac{i\left(\frac{i(27iA-14B)\tan(dx+c)^{\frac{5}{2}}+\left(\frac{182iA}{3}-\frac{98B}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(20iB+35A)\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^{3}}-\frac{2(29iB+76A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{8}}{d\,a^{3}}$
default	$\frac{-\frac{i\left(\frac{i(27iA-14B)\tan(dx+c)^{\frac{5}{2}}+\left(\frac{182iA}{3}-\frac{98B}{3}\right)\tan(dx+c)^{\frac{3}{2}}+(20iB+35A)\sqrt{\tan(dx+c)}}{(-i+\tan(dx+c))^{3}}-\frac{2(29iB+76A)\arctan\left(\frac{2\sqrt{\tan(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}\right)}{8}}{d\,a^{3}}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/d/a^3*(-1/8*I*((I*(27*I*A-14*B)*tan(d*x+c)^(5/2)+(182/3*I*A-98/3*B)*tan(
d*x+c)^(3/2)+(35*A+20*I*B)*tan(d*x+c)^(1/2))/(-I+tan(d*x+c))^3-2*(29*I*B+7
6*A)/(2^(1/2)-I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))+4
*(-1/16*I*A-1/16*B)/(2^(1/2)+I*2^(1/2))*arctan(2*tan(d*x+c)^(1/2)/(2^(1/2)
+I*2^(1/2)))-2/3*A/tan(d*x+c)^(3/2)-2*(-3*I*A+B)/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 875 vs. 2(258) = 516.

Time = 0.14 (sec) , antiderivative size = 875, normalized size of antiderivative = 2.55

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
1/96*(3*(a^3*d*e^(10*I*d*x + 10*I*c) - 2*a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d
*e^(6*I*d*x + 6*I*c))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*log(-2*((I*
a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) - (A - I*
B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*(a^3*d*e^(10*I
*d*x + 10*I*c) - 2*a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*c))*
sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*log(-2*((-I*a^3*d*e^(2*I*d*x + 2*
I*c) - I*a^3*d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c
))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*(a^3*d*e^(10*I*d*x + 10*I*c) - 2*a^
3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((5776*I*A^2 - 44
08*A*B - 841*I*B^2)/(a^6*d^2))*log(-1/8*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*
d)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((5776
*I*A^2 - 4408*A*B - 841*I*B^2)/(a^6*d^2)) + 76*I*A - 29*B)*e^(-2*I*d*x - 2
*I*c)/(a^3*d)) + 3*(a^3*d*e^(10*I*d*x + 10*I*c) - 2*a^3*d*e^(8*I*d*x + 8*I
*c) + a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((5776*I*A^2 - 4408*A*B - 841*I*B^2)/
(a^6*d^2))*log(1/8*((a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt((-I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((5776*I*A^2 - 4408*A*B - 8
41*I*B^2)/(a^6*d^2)) - 76*I*A + 29*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 2*(2
*(174*A + 73*I*B)*e^(10*I*d*x + 10*I*c) - (144*A + 41*I*B)*e^(8*I*d*x +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.50

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{3\sqrt{2}(-(76i-76)\ A+(29i+29)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}((i+1)\ A-(i-1$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*(3*sqrt(2)*(-(76*I - 76)*A + (29*I + 29)*B)*arctan((1/2*I + 1/2)*sqr
t(2)*sqrt(tan(d*x + c))) + 3*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-(1/2*
I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) + 2*(-225*I*A*tan(d*x + c)^4 + 90*B*t
an(d*x + c)^4 - 598*A*tan(d*x + c)^3 - 242*I*B*tan(d*x + c)^3 + 489*I*A*ta
n(d*x + c)^2 - 204*B*tan(d*x + c)^2 + 96*A*tan(d*x + c) + 48*I*B*tan(d*x +
 c) + 16*I*A)/(tan(d*x + c)^(3/2) - I*sqrt(tan(d*x + c)))^3)/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.24

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$$
$$= -\mathrm{atan}\left(\frac{16\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{A^2\,1\mathrm{i}}{256\,a^6\,d^2}}}{A}\right)\,\sqrt{-\frac{A^2\,1\mathrm{i}}{256\,a^6\,d^2}}\,2\mathrm{i}$$
$$+\mathrm{atan}\left(\frac{4\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{A^2\,361\mathrm{i}}{16\,a^6\,d^2}}}{19\,A}\right)\,\sqrt{\frac{A^2\,361\mathrm{i}}{16\,a^6\,d^2}}\,2\mathrm{i}$$
$$-\frac{\frac{A\,2\mathrm{i}}{3\,a^3\,d}+\frac{4\,A\tan(c+d\,x)}{a^3\,d}+\frac{A\tan(c+d\,x)^2\,163\mathrm{i}}{8\,a^3\,d}-\frac{299\,A\tan(c+d\,x)^3}{12\,a^3\,d}-\frac{A\tan(c+d\,x)^4\,75\mathrm{i}}{8\,a^3\,d}}{\tan\left(c+d\,x\right)^{9/2}-3\tan\left(c+d\,x\right)^{5/2}+\tan\left(c+d\,x\right)^{3/2}\,1\mathrm{i}-\tan\left(c+d\,x\right)^{7/2}\,3\mathrm{i}}$$
$$+2\,\mathrm{atanh}\left(\frac{16\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{\frac{B^2\,1\mathrm{i}}{256\,a^6\,d^2}}}{B}\right)\,\sqrt{\frac{B^2\,1\mathrm{i}}{256\,a^6\,d^2}}+2\,\mathrm{atanh}\left(\frac{16\,a^3\,d\,\sqrt{\tan\left(c+d\,x\right)}\,\sqrt{-\frac{B^2\,841\mathrm{i}}{256\,a^6\,d^2}}}{29\,B}\right.$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
atan((4*a^3*d*tan(c + d*x)^(1/2)*((A^2*361i)/(16*a^6*d^2))^(1/2))/(19*A))*
((A^2*361i)/(16*a^6*d^2))^(1/2)*2i - atan((16*a^3*d*tan(c + d*x)^(1/2)*(-(
A^2*1i)/(256*a^6*d^2))^(1/2))/A)*(-(A^2*1i)/(256*a^6*d^2))^(1/2)*2i - ((A*
2i)/(3*a^3*d) + (4*A*tan(c + d*x))/(a^3*d) + (A*tan(c + d*x)^2*163i)/(8*a^
3*d) - (299*A*tan(c + d*x)^3)/(12*a^3*d) - (A*tan(c + d*x)^4*75i)/(8*a^3*d
))/(tan(c + d*x)^(3/2)*1i - 3*tan(c + d*x)^(5/2) - tan(c + d*x)^(7/2)*3i +
 tan(c + d*x)^(9/2)) + 2*atanh((16*a^3*d*tan(c + d*x)^(1/2)*((B^2*1i)/(256
*a^6*d^2))^(1/2))/B)*((B^2*1i)/(256*a^6*d^2))^(1/2) + 2*atanh((16*a^3*d*ta
n(c + d*x)^(1/2)*(-(B^2*841i)/(256*a^6*d^2))^(1/2))/(29*B))*(-(B^2*841i)/(
256*a^6*d^2))^(1/2) - ((2*B)/(a^3*d) + (B*tan(c + d*x)*17i)/(2*a^3*d) - (1
21*B*tan(c + d*x)^2)/(12*a^3*d) - (B*tan(c + d*x)^3*15i)/(4*a^3*d))/(tan(c
 + d*x)^(1/2) + tan(c + d*x)^(3/2)*3i - 3*tan(c + d*x)^(5/2) - tan(c + d*x
)^(7/2)*1i)
```

Reduce [F]

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia\tan(c + dx))^3} \, dx = \text{Too large to display}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
(4*sqrt(tan(c + d*x))*tan(c + d*x)**2*b*i + 2*sqrt(tan(c + d*x))*b*i + 9*i
nt(sqrt(tan(c + d*x))/(tan(c + d*x)**6 - 3*tan(c + d*x)**5*i - 3*tan(c + d
*x)**4 + tan(c + d*x)**3*i),x)*tan(c + d*x)**2*a*d*i - 3*int(sqrt(tan(c +
d*x))/(tan(c + d*x)**6 - 3*tan(c + d*x)**5*i - 3*tan(c + d*x)**4 + tan(c +
 d*x)**3*i),x)*tan(c + d*x)**2*b*d + 6*int(sqrt(tan(c + d*x))/(tan(c + d*x
)**4 - 3*tan(c + d*x)**3*i - 3*tan(c + d*x)**2 + tan(c + d*x)*i),x)*tan(c
+ d*x)**2*b*d + 2*int(sqrt(tan(c + d*x))/(tan(c + d*x)**4*i + 3*tan(c + d*
x)**3 - 3*tan(c + d*x)**2*i - tan(c + d*x)),x)*tan(c + d*x)**2*b*d*i + 2*i
nt((sqrt(tan(c + d*x))*tan(c + d*x)**4)/(tan(c + d*x)**3*i + 3*tan(c + d*x
)**2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*b*d - 6*int((sqrt(tan(c +
d*x))*tan(c + d*x)**3)/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c +
d*x)*i - 1),x)*tan(c + d*x)**2*b*d*i + 3*int((sqrt(tan(c + d*x))*tan(c + d
*x)**2)/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x)*ta
n(c + d*x)**2*b*d*i - 4*int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c +
d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*tan(c + d*x)**2*b
*d + 9*int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)**3 - 3*tan(c +
d*x)**2*i - 3*tan(c + d*x) + i),x)*tan(c + d*x)**2*b*d - 4*int((sqrt(tan(c
 + d*x))*tan(c + d*x))/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c +
d*x)*i - 1),x)*tan(c + d*x)**2*b*d*i)/(9*tan(c + d*x)**2*a**3*d)
```

3.154 $\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 200

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{(-1)^{3/4}\sqrt{a}(4iA+7B)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d} + \frac{(1+i)\sqrt{a}(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

output

```
1/4*(-1)^(3/4)*a^(1/2)*(4*I*A+7*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1
/2)/(a+I*a*tan(d*x+c))^(1/2))/d+(1+I)*a^(1/2)*(I*A+B)*arctanh((1+I)*a^(1/2
)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+1/4*(4*A-I*B)*tan(d*x+c)^(1
/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/2*B*tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(
1/2)/d
```

Mathematica [A] (verified)

Time = 5.59 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.04

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{-\sqrt[4]{-1}a(4iA+7B)\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{1+i\tan(c+dx)}+\frac{4\sqrt{2}(iA+B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{}}{4d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x
]),x]
```

output
```
(-((-1)^(1/4)*a*((4*I)*A + 7*B)*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqr
t[1 + I*Tan[c + d*x]]) + (4*Sqrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Ta
n[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*Tan[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]] + a*Tan[c + d*x]*(-I + Tan[c + d*x])*((4*I)*A + B + (2*I)
*B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]])/(4*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4080, 27, 3042, 4080, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4080}$$

$$\frac{\int -\frac{1}{2}\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(3aB-a(4A-iB)\tan(c+dx))dx}{2a}+\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$$\Big\downarrow \; 27$$

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(3aB-a(4A-iB)\tan(c+dx))dx}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(3aB-a(4A-iB)\tan(c+dx))dx}{4a}$$

$$\Big\downarrow \; 4080$$

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((4A-iB)a^2+(4iA+7B)\tan(c+dx)a^2\right)}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{a(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((4A-iB)a^2+(4iA+7B)\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx}{2a}-\frac{a(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{4a}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((4A-iB)a^2+(4iA+7B)\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx}{2a}-\frac{a(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{4a}$$

$$\Big\downarrow \; 4084$$

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\frac{8a^2(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a(4A-7iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}-\frac{a(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{4a}$$

$\downarrow$ 3042

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\frac{8a^2(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a(4A-7iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}-\frac{a(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{4a}$$

$\downarrow$ 4027

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\frac{-\frac{16ia^4(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-a(4A-7iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}-\frac{a(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{4a}$$

$\downarrow$ 218

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\frac{\frac{(8-8i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-a(4A-7iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}-\frac{a(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{4a}$$

$\downarrow$ 4082

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\frac{\frac{(8-8i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^3(4A-7iB)\int\frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}}{2a}-\frac{a(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{4a}$$

$\downarrow$ 65

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\frac{\frac{(8-8i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(4A-7iB)\int\frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{2a}-\frac{a(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{4a}$$

$\downarrow$ 216

$$\frac{B\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\frac{\frac{2\sqrt[4]{-1}a^{5/2}(4A-7iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(8-8i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{2a}-\frac{a(4A-iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{4a}$$

input
```
Int[Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(2*d) - (((2*(-1)^(1/4)*
a^(5/2)*(4*A - (7*I)*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqr
t[a + I*a*Tan[c + d*x]]])/d + ((8 - 8*I)*a^(5/2)*(A - I*B)*ArcTanh[((1 + I
)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/(2*a) - (a*(
4*A - I*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d)/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(159) = 318$.

Time = 0.43 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.50

method	result
parts	$\frac{A\left(i\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{ia}\,a+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{-i}}{2d\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan}}$
derivativedivides	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(4A\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{ia}\,a-6}{}$
default	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(4A\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{ia}\,a-6}{}$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
1/2*A/d*(I*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c
)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a+2*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+ln(1/2*(2*I*a*tan(d*x+c
)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-
I*a)^(1/2))*(a*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^(1/2)/(-I*a)^(1/2)/(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)+1/8*B/d*tan(d*x+c)^(1/2)*(a*(
1+I*tan(d*x+c)))^(1/2)*(4*I*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)
*a-4*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*tan(d*x+c)*a+6*I*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)-4*(-I
*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+7
*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^
(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+7*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)
)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(I-tan(d*x+c))/(-I*a)^
(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(148) = 296$.

Time = 0.12 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.86

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
-1/8*(4*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*
a/d^2)*log((I*sqrt(2)*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*e^(I*d*x + I*
c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)
))*e^(-I*d*x - I*c)/(I*A + B)) - 4*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqr
t(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*log((-I*sqrt(2)*d*sqrt(-(I*A^2 + 2*A*B -
 I*B^2)*a/d^2)*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) +
I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 2*sqrt(2)*((4
*A - 3*I*B)*e^(3*I*d*x + 3*I*c) + (4*A + I*B)*e^(I*d*x + I*c))*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) + 1)) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt((-16*I*A^2 - 56*A*B + 49*I*B^
2)*a/d^2)*log((sqrt(2)*((4*I*A + 7*B)*e^(2*I*d*x + 2*I*c) + 4*I*A + 7*B)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) + 1)) + 2*I*d*sqrt((-16*I*A^2 - 56*A*B + 49*I*B^2)*a/d^2)*e^
(I*d*x + I*c))*e^(-I*d*x - I*c)/(4*I*A + 7*B)) + (d*e^(2*I*d*x + 2*I*c) +
d)*sqrt((-16*I*A^2 - 56*A*B + 49*I*B^2)*a/d^2)*log((sqrt(2)*((4*I*A + 7*B)
*e^(2*I*d*x + 2*I*c) + 4*I*A + 7*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*I*d*sqrt((-16
*I*A^2 - 56*A*B + 49*I*B^2)*a/d^2)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(4...
```

Sympy [F]

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}(A+B\tan\left(c+dx\right))\tan^{\frac{3}{2}}\left(c+dx\right)dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*tan(c + d*x)**(
3/2), x)
```

Maxima [F]

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)\sqrt{i\,a\tan(dx+c)+a}\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^(3/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
x)
```

output
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
 x)
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\sqrt{a}\left(\left(\int\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^2\,dx\right)b+\left(\int\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)\,dx\right)a\right)$$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x
)*b + int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a)
```

3.155 $\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 152

$$\begin{aligned}\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx &= -\frac{(-1)^{3/4}\sqrt{a}(2A-iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\\ &\quad -\frac{(1+i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\\ &\quad +\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}\end{aligned}$$

output

```
-(-1)^(3/4)*a^(1/2)*(2*A-I*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(
a+I*a*tan(d*x+c))^(1/2))/d-(1+I)*a^(1/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan
(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+B*tan(d*x+c)^(1/2)*(a+I*a*tan(d*
x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 3.44 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.24

$$\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{\sqrt[4]{-1}a(2A-iB)\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{1+i\tan(c+dx)} + \frac{aB(1+i\tan(c+dx))\tan(c+dx)-\sqrt{2}(A-iB)\mathrm{ar}\ldots}{\ldots}}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x
]),x]
```

output
```
((-1)^(1/4)*a*(2*A - I*B)*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[1 +
I*Tan[c + d*x]] + (a*B*(1 + I*Tan[c + d*x])*Tan[c + d*x] - Sqrt[2]*(A - I*
B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sq
rt[I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[Tan[c + d*x]])/(d*Sq
rt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 4080, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4080}$$

$$\frac{\int -\frac{\sqrt{i\tan(c+dx)a+a}(aB-a(2A-iB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx}{a} + \frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(aB-a(2A-iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(aB-a(2A-iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{2a}$$

$$\Big\downarrow \ 4084$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-(B+2iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}$$

$$\Big\downarrow \ 3042$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-(B+2iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}$$

$$\Big\downarrow \ 4027$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{-\frac{4ia^3(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-(B+2iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}$$

$$\Big\downarrow \ 218$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-(B+2iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}$$

$$\Big\downarrow \ 4082$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2(B+2iA)\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}}{2a}$$

$$\Big\downarrow \text{65}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(B+2iA)\int\frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{2a}$$

$$\Big\downarrow \text{216}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{2\sqrt[4]{-1}a^{3/2}(B+2iA)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{2a}$$

input
```
Int[Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*((2*(-1)^(1/4)*a^(3/2)*((2*I)*A + B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[
Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((2 - 2*I)*a^(3/2)*(I*A +
B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]
])/d)/a + (B*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(123) = 246.

Time = 0.27 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.80

method	result
parts	$-\frac{A\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a\left(i\sqrt{2}\,\sqrt{ia}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)-\sqrt{2}\,\mathrm{t}$
derivativedivides	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,\left(iB\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan$
default	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,\left(iB\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan$

input

```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output

```
-1/2*A/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(I*2^(1/2)*(I*a)^(1
/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3
*a*tan(d*x+c))/(tan(d*x+c)+I))-2^(1/2)*tan(d*x+c)*(I*a)^(1/2)*ln((2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(
tan(d*x+c)+I))+2*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*tan(d*x+c)+2*(-I*a)^(1/
2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^
(1/2)+a)/(I*a)^(1/2)))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(
I-tan(d*x+c))/(-I*a)^(1/2)+1/2*B/d*(I*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(
I*a)^(1/2)*a+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1
/2)+ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)
^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2))*(a*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+
c)^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(114) = 228.

Time = 0.12 (sec) , antiderivative size = 673, normalized size of antiderivative = 4.43

$$\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
1/2*(2*sqrt(2)*B*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) - sqrt(2)*d*sqrt(-(-
I*A^2 - 2*A*B + I*B^2)*a/d^2)*log((sqrt(2)*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2
)*a/d^2)*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A +
B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) + sqrt(2)*d*sqrt(-(-I
*A^2 - 2*A*B + I*B^2)*a/d^2)*log(-(sqrt(2)*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2
)*a/d^2)*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A +
B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) + d*sqrt((4*I*A^2 + 4
*A*B - I*B^2)*a/d^2)*log((sqrt(2)*((2*I*A + B)*e^(2*I*d*x + 2*I*c) + 2*I*A
 + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1)) + 2*d*sqrt((4*I*A^2 + 4*A*B - I*B^2)*a/d^2)*e^(
I*d*x + I*c))*e^(-I*d*x - I*c)/(2*I*A + B)) - d*sqrt((4*I*A^2 + 4*A*B - I*
B^2)*a/d^2)*log((sqrt(2)*((2*I*A + B)*e^(2*I*d*x + 2*I*c) + 2*I*A + B)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1)) - 2*d*sqrt((4*I*A^2 + 4*A*B - I*B^2)*a/d^2)*e^(I*d*x + I
*c))*e^(-I*d*x - I*c)/(2*I*A + B)))/d
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}(A+B\tan\left(c+dx\right))\sqrt{\tan\left(c+dx\right)}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*sqrt(tan(c + d*
x)), x)
```

Maxima [F]

$$\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (B\tan\left(dx+c\right)+A)\sqrt{i\,a\tan\left(dx+c\right)+a}\sqrt{\tan\left(dx+c\right)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*sqrt(tan(d*x + c
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 22.34 (sec) , antiderivative size = 2225, normalized size of antiderivative = 14.64

$$\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
x)
```

output
```
- ((B*tan(c + d*x)^(3/2)*2i)/(d*((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^
3) + (2*B*tan(c + d*x)^(1/2))/(a*d*((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2
))))/(tan(c + d*x)^2/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^4 - 1/a^2 +
 (tan(c + d*x)*2i)/(a*((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2)) - ((-a
)^(1/2)*atan((A^4*(-a)^(21/2)*tan(c + d*x)^(1/2)*(7168 - 7168i))/(((a + a*
tan(c + d*x)*1i)^(1/2) - a^(1/2))*(A^4*a^10*3584i + B^4*a^10*512i - 4096*A
*B^3*a^10 + 10240*A^3*B*a^10 - A^2*B^2*a^10*10240i - (3584*A^4*a^11*tan(c
+ d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 - (512*B^4*a^11*tan(c
+ d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + (10240*A^2*B^2*a^11*
tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 - (A*B^3*a^11*ta
n(c + d*x)*4096i)/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + (A^3*B*a^1
1*tan(c + d*x)*10240i)/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2)) + (B^
4*(-a)^(21/2)*tan(c + d*x)^(1/2)*(1024 - 1024i))/(((a + a*tan(c + d*x)*1i)
^(1/2) - a^(1/2))*(A^4*a^10*3584i + B^4*a^10*512i - 4096*A*B^3*a^10 + 1024
0*A^3*B*a^10 - A^2*B^2*a^10*10240i - (3584*A^4*a^11*tan(c + d*x))/((a + a*
tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 - (512*B^4*a^11*tan(c + d*x))/((a + a*
tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + (10240*A^2*B^2*a^11*tan(c + d*x))/((
a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 - (A*B^3*a^11*tan(c + d*x)*4096i
)/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + (A^3*B*a^11*tan(c + d*x)*1
0240i)/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2)) + (A*B^3*(-a)^(21/...
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \frac{\sqrt{a}\left(-2\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,ai+\left(\int\frac{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\tan(dx+c)}dx\right)adi+2\left(\int\sqrt{\tan(dx+c)}\right.\right.}{}$$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a*i + int((sqrt
(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x),x)*a*d*i + 2*int(sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i + int(sqrt(
tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*b*d))/d
```

3.156 $\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 112

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{2(-1)^{3/4}\sqrt{a}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{(1-i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

output

```
-2*(-1)^(3/4)*a^(1/2)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*
tan(d*x+c))^(1/2))/d+(1-I)*a^(1/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c
)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{a\left(\frac{\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{\sqrt{ia\tan(c+dx)}}+\frac{2\sqrt[4]{-1}B\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{1+i\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

input
```
Integrate[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d
*x]],x]
```

output
```
(a*((Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/Sqrt[I*a*Tan[c + d*x]] + (2*(-1)^(1/
4)*B*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[1 + I*Tan[c + d*x]])/Sqrt
[a + I*a*Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \; 4084$$

$$(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{iB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{iB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{4027}$$

$$\frac{iB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2ia^2(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{iB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}+\frac{(1-i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{4082}$$

$$\frac{iaB\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{(1-i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{65}$$

$$\frac{2iaB\int \frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}+\frac{(1-i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{(1-i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2(-1)^{3/4}\sqrt{a}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

input

```
Int[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x
]
```

output

```
(-2*(-1)^(3/4)*Sqrt[a]*B*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sq
rt[a + I*a*Tan[c + d*x]]])/d + ((1 - I)*Sqrt[a]*(A - I*B)*ArcTanh[((1 + I)
*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(89) = 178$.

Time = 0.27 (sec) , antiderivative size = 469, normalized size of antiderivative = 4.19

method	result
parts	$-\frac{iA\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}}{2d\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}} - \frac{B\sqrt{\tan(}}{}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}\,a\left(iA\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(d\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}\,a\left(iA\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(d\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/2*I*A/d*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c
)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^(1/2)*(a*(1+I*t
an(d*x+c)))^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-1/2*B
/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(I*2^(1/2)*(I*a)^(1/2)*ln
((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan
(d*x+c))/(tan(d*x+c)+I))-2^(1/2)*tan(d*x+c)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*
a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*
x+c)+I))+2*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*tan(d*x+c)+2*(-I*a)^(1/2)*ln(
1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+
a)/(I*a)^(1/2)))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(I-tan(
d*x+c))/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(82) = 164$.

Time = 0.11 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.81

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, al
gorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*log((I*sqrt(2)*d*sqrt(-(I
*A^2 + 2*A*B - I*B^2)*a/d^2)*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d
*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) -
 1/2*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*log((-I*sqrt(2)*d*sqrt(-
(I*A^2 + 2*A*B - I*B^2)*a/d^2)*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I
*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B))
 - 1/2*sqrt(4*I*B^2*a/d^2)*log((sqrt(2)*(B*e^(2*I*d*x + 2*I*c) + B)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1)) + I*sqrt(4*I*B^2*a/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
/B) + 1/2*sqrt(4*I*B^2*a/d^2)*log((sqrt(2)*(B*e^(2*I*d*x + 2*I*c) + B)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1)) - I*sqrt(4*I*B^2*a/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)/B)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}(A+B\tan\left(c+dx\right))}{\sqrt{\tan\left(c+dx\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))/sqrt(tan(c + d*
x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\sqrt{i\,a\tan{(dx+c)}+a}}{\sqrt{\tan{(dx+c)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)/sqrt(tan(d*x + c
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.32

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{B\sqrt{a}\,\ln\left(\frac{\sqrt{a}\,\sqrt{\tan(c+d\,x)}\,(2-2\mathrm{i})}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}-\sqrt{a}}-\frac{a\tan(c+d\,x)}{\left(\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}-\sqrt{a}\right)^2}+1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{d}$$
$$+\frac{\sqrt{2}\,B\sqrt{a}\,\ln\left(\sqrt{2}\,(1-\mathrm{i})+\frac{2\sqrt{a}\,\sqrt{\tan(c+d\,x)}}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}-\sqrt{a}}\right)\,(1+1\mathrm{i})}{d}$$
$$-\frac{\sqrt{\frac{1}{2}\mathrm{i}}\,B\sqrt{a}\,\ln\left(-\frac{a\tan(c+d\,x)}{\left(\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}-\sqrt{a}\right)^2}+\frac{2\,(-1)^{3/4}\,\sqrt{2}\,\sqrt{a}\,\sqrt{\tan(c+d\,x)}}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}-\sqrt{a}}+1\mathrm{i}\right)}{d}$$
$$-\frac{\sqrt{4\mathrm{i}}\,B\sqrt{a}\,\ln\left((-1)^{3/4}+\frac{\sqrt{a}\,\sqrt{\tan(c+d\,x)}}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}-\sqrt{a}}\right)}{d}$$
$$+\frac{2\sqrt{\frac{1}{2}\mathrm{i}}\,A\sqrt{-a}\,\mathrm{atanh}\left(\frac{2\sqrt{\frac{1}{2}\mathrm{i}}\,\sqrt{-a}\,\sqrt{\tan(c+d\,x)}\,\left(\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}-\sqrt{a}\right)}{a\tan(c+d\,x)-a\,1\mathrm{i}+\sqrt{a}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,1\mathrm{i}}\right)}{d}$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(1/2
),x)
```

output
```
(B*a^(1/2)*log((a^(1/2)*tan(c + d*x)^(1/2)*(2 - 2i))/((a + a*tan(c + d*x)*
1i)^(1/2) - a^(1/2)) - (a*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a
^(1/2))^2 + 1i)*(1/2 + 1i/2))/d + (2^(1/2)*B*a^(1/2)*log(2^(1/2)*(1 - 1i)
+ (2*a^(1/2)*tan(c + d*x)^(1/2))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))
)*(1 + 1i))/d - ((1i/2)^(1/2)*B*a^(1/2)*log((2*(-1)^(3/4)*2^(1/2)*a^(1/2)*
tan(c + d*x)^(1/2))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2)) - (a*tan(c +
 d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + 1i))/d - (4i^(1/2)*B*
a^(1/2)*log((-1)^(3/4) + (a^(1/2)*tan(c + d*x)^(1/2))/((a + a*tan(c + d*x)
*1i)^(1/2) - a^(1/2))))/d + (2*(1i/2)^(1/2)*A*(-a)^(1/2)*atanh((2*(1i/2)^(
1/2)*(-a)^(1/2)*tan(c + d*x)^(1/2)*((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2
)))/(a*tan(c + d*x) - a*1i + a^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*1i)))/d
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(-2\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,bi+\left(\int \frac{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\tan(dx+c)}dx\right)ad+\left(\int \frac{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx}}{\tan(dx+c)}\right.\right.}{d}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*b*i + int((sqrt
(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x),x)*a*d + int((sqrt(t
an(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x),x)*b*d*i + 2*int(sqrt(
tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*b*d*i))/d
```

3.157 $\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 90

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{(1+i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

output
```
(1+I)*a^(1/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*
x+c))^(1/2))/d-2*A*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{ia\tan(c+dx)}-2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

input
```
Integrate[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
3/2),x]
```

output
```
(Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*T
an[c + d*x]]]*Sqrt[I*a*Tan[c + d*x]] - 2*A*Sqrt[a + I*a*Tan[c + d*x]])/(d*
Sqrt[Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 4081}$$

$$\frac{2\int \frac{a(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{ 4027}$$

$$-\frac{2ia^2(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow 218$$

$$\frac{(1-i)\sqrt{a}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x
]
```

output
```
((1 - I)*Sqrt[a]*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sq
rt[a + I*a*Tan[c + d*x]]])/d - (2*A*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Ta
n[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(75) = 150$.

Time = 0.20 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.09

method	result
parts	$\frac{A\left(\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{2}\,\tan(dx+c)a-4\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{-}\right.}{2d\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(iB\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^2+iA\sqrt{2}\,\ln\left(\right.\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(iB\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^2+iA\sqrt{2}\,\ln\left(\right.\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/2*A/d*(ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-
I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*tan(d*x+c)*a-4*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))*(a*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/
2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/tan(d*x+c)^(1/2)-1/2*I*B/d*2^(1/2
)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a
*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)
/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(68) = 136$.

Time = 0.10 (sec) , antiderivative size = 435, normalized size of antiderivative = 4.83

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a}{d^2}}\log\left(\frac{\left(\sqrt{2}d\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a}{d^2}}e^{(i\,dx+i\,c)}+\sqrt{2}((i\,A+B)e^{(2i\,dx+2i\,c)}+i\,A+B)\right.}{i\,A+B}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, al
gorithm="fricas")
```

output
```
1/2*(sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/
d^2)*log((sqrt(2)*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*e^(I*d*x + I*c)
+ sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*
e^(-I*d*x - I*c)/(I*A + B)) - sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(-
I*A^2 - 2*A*B + I*B^2)*a/d^2)*log(-(sqrt(2)*d*sqrt(-(-I*A^2 - 2*A*B + I*B^
2)*a/d^2)*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A +
 B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 4*sqrt(2)*(I*A*e^(
3*I*d*x + 3*I*c) + I*A*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(2*I*d*
x + 2*I*c) - d)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\left(A+B\tan\left(c+dx\right)\right)}{\tan^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))/tan(c + d*x)**(
3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{\tan(c+d\,x)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(3/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(3/2
), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\left(-2\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,bi+\left(\int\frac{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\tan(dx+c)^2}dx\right)\tan(dx+c)\,ad-\left(\int\frac{\sqrt{\tan}}{}\right.}{\tan(dx+c)\,d}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*b*i + int((sqrt
(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)*a
*d - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*
tan(c + d*x)*b*d*i))/(tan(c + d*x)*d)
```

3.158 $\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 135

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{(1-i)\sqrt{a}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(iA+3B)\sqrt{a+ia\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}}$$

output
```
(-1+I)*a^(1/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d
*x+c))^(1/2))/d-2/3*A*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)-2/3*(I*A
+3*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{3\sqrt{2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\tan(c+dx)\sqrt{ia\tan(c+dx)}+2\sqrt{a+ia\tan(c+dx)}(-A+(-i}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
5/2),x]
```

output
```
(3*Sqrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a
*Tan[c + d*x]]]*Tan[c + d*x]*Sqrt[I*a*Tan[c + d*x]] + 2*Sqrt[a + I*a*Tan[c
 + d*x]]*(-A + ((-I)*A - 3*B)*Tan[c + d*x]))/(3*d*Tan[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$
$$\Big\downarrow \text{4081}$$
$$\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+3B)-2aA\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+3B)-2aA\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+3B)-2aA\tan(c+dx))}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 4081$$

$$\frac{\frac{2\int -\frac{3a^2(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{-3a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3B+iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{-3a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3B+iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 4027$$

$$\frac{\frac{6ia^3(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a(3B+iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 218$$

$$\frac{-\frac{(3-3i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a(3B+iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x
]
```

output
```
(-2*A*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) + (((-3 + 3*I)*
a^(3/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*
a*Tan[c + d*x]]])/d - (2*a*(I*A + 3*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt
[Tan[c + d*x]]))/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(110) = 220$.

Time = 0.21 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.62

method	result
parts	$\frac{A\left(3i\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)^2 a-4i\tan(dx+c)\sqrt{-ia}\,\sqrt{a\tan(dx+c}}{6d\tan(dx+c)^{\frac{3}{2}}\sqrt{a\tan(dx+c)(1+i\tan(dx+c}}$
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\left(3iA\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^3-3iB\sqrt{}\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\left(3iA\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^3-3iB\sqrt{}\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/6*A/d/tan(d*x+c)^(3/2)*(3*I*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*tan(d*x+c
)^2*a-4*I*tan(d*x+c)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-4*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))*(a*(1+I*tan(d*x+c)))^(
1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)+1/2*B/d*(ln((2*2^(
1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c)
)/(tan(d*x+c)+I))*2^(1/2)*tan(d*x+c)*a-4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*(-I*a)^(1/2))*(a*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)/tan(d*x+c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(101) = 202$.

Time = 0.08 (sec) , antiderivative size = 483, normalized size of antiderivative = 3.58

$$\int \frac{\sqrt{a + ia\tan(c + dx)}(A + B\tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)}\,dx =$$

$$-\frac{3\sqrt{2}\left(de^{(4i\,dx+4i\,c)} - 2\,de^{(2i\,dx+2i\,c)} + d\right)\sqrt{-\frac{(i\,A^2+2\,AB-i\,B^2)a}{d^2}}\log\left(\frac{\left(i\sqrt{2}d\sqrt{-\frac{(i\,A^2+2\,AB-i\,B^2)a}{d^2}}e^{(i\,dx+i\,c)}+\sqrt{2}((i\,A+\right.}{}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, al
gorithm="fricas")
```

output
```
-1/6*(3*sqrt(2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt
(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*log((I*sqrt(2)*d*sqrt(-(I*A^2 + 2*A*B - I
*B^2)*a/d^2)*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*
A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(2)*(d*e^
(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I*A^2 + 2*A*B - I*
B^2)*a/d^2)*log((-I*sqrt(2)*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*e^(I*d*
x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 4*sqrt(2)*((2*A - 3*I*B)*e^(5*I*d*x
 + 5*I*c) + 2*A*e^(3*I*d*x + 3*I*c) + 3*I*B*e^(I*d*x + I*c))*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \frac{\sqrt{a + ia\tan(c + dx)}(A + B\tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c + dx\right) - i\right)}(A + B\tan\left(c + dx\right))}{\tan^{\frac{5}{2}}\left(c + dx\right)}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))/tan(c + d*x)**(
5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + ia\tan(c + dx)}(A + B\tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + ia\tan(c + dx)}(A + B\tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{\tan\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(5/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(5/2
), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\left(4\sqrt{\tan\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)ai-2\sqrt{\tan\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,a+3\left(\int\right.\right.}{3\tan\left(dx+\right.}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
(sqrt(a)*(4*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i -
 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a + 3*int((sqrt(tan(c + d*x
))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**2*a*d*i + 3*
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c
 + d*x)**2*b*d))/(3*tan(c + d*x)**2*d)
```

3.159 $\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 178

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{(1+i)\sqrt{a}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2(iA+5B)\sqrt{a+ia\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{15d\sqrt{\tan(c+dx)}}$$

output
```
(-1-I)*a^(1/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d
*x+c))^(1/2))/d-2/5*A*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)-2/15*(I*
A+5*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)+2/15*(13*A-5*I*B)*(a+I*
a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 4.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx =$$

$$-\frac{i\left(\frac{15\sqrt{2}a(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\tan^3(c+dx)}{\sqrt{ia\tan(c+dx)}}+2\sqrt{a+ia\tan(c+dx)}(-3iA+(A-5iB)\tan(c+dx}{15d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
7/2),x]
```

output
```
((-1/15*I)*((15*Sqrt[2]*a*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]
])/Sqrt[a + I*a*Tan[c + d*x]]]*Tan[c + d*x]^3)/Sqrt[I*a*Tan[c + d*x]] + 2*
Sqrt[a + I*a*Tan[c + d*x]]*((-3*I)*A + (A - (5*I)*B)*Tan[c + d*x] + ((13*I
)*A + 5*B)*Tan[c + d*x]^2)))/(d*Tan[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 4081, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 4081}$$

$$\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+5B)-4aA\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+5B)-4aA\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+5B)-4aA\tan(c+dx))}{\tan(c+dx)^{5/2}}dx}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4081

$$\frac{\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((13A-5iB)a^2+2(iA+5B)\tan(c+dx)a^2\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((13A-5iB)a^2+2(iA+5B)\tan(c+dx)a^2\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((13A-5iB)a^2+2(iA+5B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4081

$$\frac{-\frac{\frac{2\int \frac{15a^3(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{15a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{15a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4027$$

$$\frac{-\frac{-\frac{30ia^4(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{\frac{(15-15i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

input

```
Int[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x
]
```

output
```
(-2*A*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) + ((-2*a*(I*A +
 5*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) - (((15 - 15*I)
*a^(5/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]])/d - (2*a^2*(13*A - (5*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])
/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(145) = 290.

Time = 0.21 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.12

method	result
parts	$-\frac{A\sqrt{a(1+i\tan(dx+c))}\left(15i\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)^{3}a-15\sqrt{2}\,\ln}{}$
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\left(15iB\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^{4}+15iA}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\left(15iB\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^{4}+15iA}{}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/30*A/d*(a*(1+I*tan(d*x+c)))^(1/2)*(15*I*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1
/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+
I))*tan(d*x+c)^3*a-15*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*tan(d*x+c)^4*a-56
*I*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)+52*(-I*
a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3+12*I*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-16*tan(d*x+c)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/tan(d*x+c)^(5/2)/(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)/(I-tan(d*x+c))/(-I*a)^(1/2)+1/6*B/d/tan(d*x+c)^(3/2)*(3*I*2
^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*
a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*tan(d*x+c)^2*a-4*I*tan(d*x+c)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)*(-I*a)^(1/2))*(a*(1+I*tan(d*x+c)))^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x
+c)))^(1/2)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(134) = 268$.

Time = 0.09 (sec) , antiderivative size = 543, normalized size of antiderivative = 3.05

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, al
gorithm="fricas")
```

output
```
-1/30*(15*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e
^(2*I*d*x + 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*log((sqrt(2)
*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*e^(I*d*x + I*c) + sqrt(2)*((I*A +
 B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/
(I*A + B)) - 15*sqrt(2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) +
 3*d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*log(-(
sqrt(2)*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*e^(I*d*x + I*c) - sqrt(2)*
((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x
- I*c)/(I*A + B)) + 4*sqrt(2)*((-17*I*A - 10*B)*e^(7*I*d*x + 7*I*c) + 3*I*
A*e^(5*I*d*x + 5*I*c) + 5*(I*A + 2*B)*e^(3*I*d*x + 3*I*c) - 15*I*A*e^(I*d*
x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x +
4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}{\tan(c+d\,x)^{7/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(7/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(7/2
), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\left(-4\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^2\,a+20\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
(sqrt(a)*( - 4*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2
*a + 20*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i -
2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 10*sqrt(t
an(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 6*sqrt(tan(c + d*x)
)*sqrt(tan(c + d*x)*i + 1)*a - 15*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x
)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**3*a*d + 15*int((sqrt(tan(c + d*
x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**3*b*d*i))/(
15*tan(c + d*x)**3*d)
```

3.160 $\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 221

$$\begin{aligned}
&\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx \\
&= -\frac{(1+i)\sqrt{a}(iA+B)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} \\
&\quad -\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{2(iA+7B)\sqrt{a+ia\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} \\
&\quad +\frac{2(31A-7iB)\sqrt{a+ia\tan(c+dx)}}{105d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(43iA+91B)\sqrt{a+ia\tan(c+dx)}}{105d\sqrt{\tan(c+dx)}}
\end{aligned}$$

output
```
(-1-I)*a^(1/2)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d
*x+c))^(1/2))/d-2/7*A*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(7/2)-2/35*(I*
A+7*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)+2/105*(31*A-7*I*B)*(a+I
*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)+2/105*(43*I*A+91*B)*(a+I*a*tan(d*x
+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 4.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$
$$=\frac{\sqrt{2}a(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{d\sqrt{ia\tan(c+dx)}}$$
$$-\frac{2\sqrt{a+ia\tan(c+dx)}(15A+3(iA+7B)\tan(c+dx)+(-31A+7iB)\tan^2(c+dx)+(-43iA-91B\ldots}{105d\tan^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
9/2),x]
```

output
```
(Sqrt[2]*a*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a
*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/(d*Sqrt[I*a*Tan[c + d*x]]) - (2*Sqrt[a
 + I*a*Tan[c + d*x]]*(15*A + 3*(I*A + 7*B)*Tan[c + d*x] + (-31*A + (7*I)*B
)*Tan[c + d*x]^2 + ((-43*I)*A - 91*B)*Tan[c + d*x]^3))/(105*d*Tan[c + d*x]
^(7/2))
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.395$, Rules used = {3042, 4081, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{4081}$$

$$\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+7B)-6aA\tan(c+dx))}{2\tan^{\frac{7}{2}}(c+dx)}dx}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+7B)-6aA\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+7B)-6aA\tan(c+dx))}{\tan(c+dx)^{7/2}}dx}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$\frac{\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((31A-7iB)a^2+4(iA+7B)\tan(c+dx)a^2\right)}{2\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((31A-7iB)a^2+4(iA+7B)\tan(c+dx)a^2\right)}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((31A-7iB)a^2+4(iA+7B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{5/2}}dx}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$-\frac{\frac{\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(43iA+91B)-2a^3(31A-7iB)\tan(c+dx)\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^2(31A-7iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(43iA+91B)-2a^3(31A-7iB)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^2(31A-7iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(43iA+91B)-2a^3(31A-7iB)\tan(c+dx)\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a^2(31A-7iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \ 4081$$

$$-\frac{\frac{\frac{\frac{2\int -\frac{105a^4(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^3(91B+43iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(31A-7iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\frac{-105a^3(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(91B+43iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(31A-7iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\frac{-105a^3(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(91B+43iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(31A-7iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4027

$$\frac{-\frac{\frac{\frac{210ia^5(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^3(91B+43iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(31A-7iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 218

$$\frac{-\frac{\frac{-\frac{(105-105i)a^{7/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(91B+43iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(31A-7iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7B+iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{7a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

input

```
Int[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/2),x
]
```

output
```
(-2*A*Sqrt[a + I*a*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(7/2)) + ((-2*a*(I*A +
 7*B)*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) - ((-2*a^2*(31*
A - (7*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) + (((-10
5 + 105*I)*a^(7/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/
Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^3*((43*I)*A + 91*B)*Sqrt[a + I*a*Tan
[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*a))/(7*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(180) = 360$.

Time = 0.22 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.85

method	result
parts	$-\frac{A\left(105i\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)^{4}a-172i\tan(dx+c)^{3}\sqrt{-ia}\,\sqrt{a\,\mathrm{t}}}{}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\left(-105iB\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^{4}-364B}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\left(-105iB\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^{4}-364B}{}$

input

```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x,method=_R
ETURNVERBOSE)
```

output

```
-1/210*A/d*(105*I*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*tan(d*x+c)^4*a-172*I*
tan(d*x+c)^3*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+12*I*(-I*a
)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-124*tan(d*x+c)^2*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)+60*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)*(-I*a)^(1/2))*(a*(1+I*tan(d*x+c)))^(1/2)/tan(d*x+c)^(7/2
)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)-1/30*B/d*(a*(1+I*tan(
d*x+c)))^(1/2)*(15*I*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*tan(d*x+c)^3*a-15*
2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I
*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*tan(d*x+c)^4*a-56*I*tan(d*x+c)^2*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)+52*(-I*a)^(1/2)*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3+12*I*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)*(-I*a)^(1/2)-16*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-
I*a)^(1/2))/tan(d*x+c)^(5/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I-tan(
d*x+c))/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(167) = 334$.

Time = 0.11 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{a + ia\tan(c + dx)}(A + B\tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input

```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, al
gorithm="fricas")
```

output

```
1/210*(105*sqrt(2)*(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*
e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I*A^2 + 2*A*B -
I*B^2)*a/d^2)*log((I*sqrt(2)*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*e^(I*d
*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 105*sqrt(2)*(d*e^(8*I*d*x + 8*I*c)
 - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*
I*c) + d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*log((-I*sqrt(2)*d*sqrt(-(I*
A^2 + 2*A*B - I*B^2)*a/d^2)*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*
x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(I*A + B)) -
4*sqrt(2)*((92*A - 119*I*B)*e^(9*I*d*x + 9*I*c) - 20*(A - 7*I*B)*e^(7*I*d*
x + 7*I*c) + 14*(2*A + I*B)*e^(5*I*d*x + 5*I*c) + 140*(A - I*B)*e^(3*I*d*x
 + 3*I*c) + 105*I*B*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(8*I*d*x +
 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d
*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{{\tan\left(c+d\,x\right)}^{9/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(9/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/tan(c + d*x)^(9/2
), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\left(-124\sqrt{\tan\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)^{3}ai-28\sqrt{\tan\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\mathrm{t}\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)
```

output
```
(sqrt(a)*( - 124*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*
*3*a*i - 28*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*b
+ 62*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*a - 14*sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i - 6*sqrt(tan
(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 42*sqrt(tan(c + d*x
))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 30*sqrt(tan(c + d*x))*sqrt(ta
n(c + d*x)*i + 1)*a - 105*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)
)/tan(c + d*x)**2,x)*tan(c + d*x)**4*a*d*i - 105*int((sqrt(tan(c + d*x))*s
qrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**4*b*d))/(105*tan
(c + d*x)**4*d)
```

3.161 $\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 248

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{(-1)^{3/4}a^{3/2}(22iA+23B)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{8d} + \frac{(2+2i)a^{3/2}(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{a(10A-9iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} + \frac{a(6iA+7B)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d} + \frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

output
```
1/8*(-1)^(3/4)*a^(3/2)*(22*I*A+23*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^
(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+(2+2*I)*a^(3/2)*(I*A+B)*arctanh((1+I)*a^
(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+1/8*a*(10*A-9*I*B)*tan(
d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/12*a*(6*I*A+7*B)*tan(d*x+c)^(3/2
)*(a+I*a*tan(d*x+c))^(1/2)/d+1/3*I*a*B*tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))
^(1/2)/d
```

Mathematica [A] (verified)

Time = 6.73 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.67

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{\frac{3a(2A-iB)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{4d} + -\frac{i\left(-\frac{3}{4}ia^3(2A-iB)-\frac{3}{4}a^3(6iA+7B)\right)\left(-\frac{2i\sqrt{2}a\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} + \frac{2ia^3 \cdots}{\cdots}\right.}{a}}{\cdots}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
(B*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2))/(3*d) + ((3*a*(2*A - I
*B)*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2))/(4*d) + (((-I)*(((-3*
I)/4)*a^3*(2*A - I*B) - (3*a^3*((6*I)*A + 7*B))/4)*(((-2*I)*Sqrt[2]*a*ArcT
anh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*
Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]) + ((2*I)*a^(3/2)*ArcSinh[Sqrt[I*a*Ta
n[c + d*x]]/Sqrt[a]]*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[I*a*Tan[c + d*x]])/(d*S
qrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/a - (((3*I)/4)*a^3*((6*I)*
A + 7*B)*(-(((-1)^(3/4)*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[a + I*
a*Tan[c + d*x]])/Sqrt[1 + I*Tan[c + d*x]]) + Sqrt[Tan[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]]))/d)/(2*a))/(3*a)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.447$, Rules used = {3042, 4077, 27, 3042, 4080, 27, 3042, 4080, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{3}\int \frac{1}{2}\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}(a(6A-5iB)+a(6iA+7B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}(a(6A-5iB)+a(6iA+7B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \tan(c+dx)^{3/2}\sqrt{i\tan(c+dx)a+a}(a(6A-5iB)+a(6iA+7B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{4080}$$

$$\frac{1}{6}\left(\frac{\int -\frac{3}{2}\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left(a^2(6iA+7B)-a^2(10A-9iB)\tan(c+dx)\right)dx}{2a}+\frac{a(7B+6iA)\tan\right.$$
$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\int\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left(a^2(6iA+7B)-a^2\right.}{4a}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\int\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left(a^2(6iA+7B)-a^2\right.}{4a}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 4080

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((10A-9iB)a^3+(22iA+23B)\tan(c+dx)a^3\right)}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{a^2(10\right.}{4a}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((10A-9iB)a^3+(22iA+23B)\tan(c+dx)a^3\right)}{\sqrt{\tan(c+dx)}}dx}{2a}-\frac{a^2(10\right.}{4a}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((10A-9iB)a^3+(22iA+23B)\tan(c+dx)a^3\right)}{\sqrt{\tan(c+dx)}}dx}{2a}-\frac{a^2(10}{}\right.}{4a}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 4084

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{32a^3(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a^2(22A-23iB)\int\frac{(a-ia\tan(c+dx))}{\sqrt{\tan(}}}{2a}\right.}{4a}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{32a^3(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a^2(22A-23iB)\int\frac{(a-ia\tan(c+dx))}{\sqrt{\tan(}}}{2a}\right.}{4a}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 4027

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{-a^2(22A-23iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{64ia^5(A-iB)\int}{}}{2a}\right.}{}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 218

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\frac{(32-32i)a^{7/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-a^2(22A-23iB)}{2a}\right.}{}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 4082

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\frac{(32-32i)a^{7/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^4(22A-23iB)\int}{}}{2a}\right.}{}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 65

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\frac{(32-32i)a^{7/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^4(22A-23iB)}{}}{2a}\right.}{}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

$\downarrow$ 216

$$\frac{1}{6}\left(\frac{a(7B+6iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{3\left(\frac{\frac{2\sqrt[4]{-1}a^{7/2}(22A-23iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(32-32i)a^{7/2}(\cdots}{\cdots}}{2a}\cdots\right.}{\cdots}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

input

```
Int[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output

```
((I/3)*a*B*Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d + ((a*((6*I)*A
 + 7*B)*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(2*d) - (3*(((2*(-1
)^(1/4)*a^(7/2)*(22*A - (23*I)*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c +
d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((32 - 32*I)*a^(7/2)*(A - I*B)*Arc
Tanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/
(2*a) - (a^2*(10*A - (9*I)*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]
])/d))/(4*a))/6
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(198) = 396$.

Time = 0.29 (sec) , antiderivative size = 652, normalized size of antiderivative = 2.63

method	result
derivativedivides	$\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a\left(16iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^2+24iA\sqrt{a\tan(dx+c}\right.}{}$
default	$\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a\left(16iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^2+24iA\sqrt{a\tan(dx+c}\right.}{}$
parts	$-\frac{A\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a\left(-4i\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)-11\ln\left(\frac{2ia\tan(dx-}{}\right.\right.}{}$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
1/48/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(16*I*B*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+24*I*A*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+27*I
*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^
(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-54*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+28*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-24*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(
1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c)
)/(tan(d*x+c)+I))*a-30*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a+60*A*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-48*I*ln(1/2*(2*I*a*ta
n(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2
))*(-I*a)^(1/2)*a+24*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-48
*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1
/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/
(I*a)^(1/2)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(184) = 368$.

Time = 0.11 (sec) , antiderivative size = 911, normalized size of antiderivative = 3.67

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
1/48*(48*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*
I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log((I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B -
I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I
*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a))
 - 48*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*c
) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log((-I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*
B^2)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c
) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) +
 2*sqrt(2)*(7*(6*A - 7*I*B)*a*e^(5*I*d*x + 5*I*c) + 2*(30*A - 19*I*B)*a*e^
(3*I*d*x + 3*I*c) + 3*(6*A - 7*I*B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)
) + 3*sqrt((-484*I*A^2 - 1012*A*B + 529*I*B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*
I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*((22*I*A + 23*B)*a*e^(2*I
*d*x + 2*I*c) + (22*I*A + 23*B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + 2*I*sqrt((-484*I
*A^2 - 1012*A*B + 529*I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/
((22*I*A + 23*B)*a)) - 3*sqrt((-484*I*A^2 - 1012*A*B + 529*I*B^2)*a^3/d^2)
*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*((2...
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(
3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeUnable to con
vert to r
```

Mupad [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \tan(c+d\,x)^{3/2}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
x)
```

output
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
 x)
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a\left(\left(\int\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^{3}dx\right)bi+\left(\int\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^{2}dx\right)ai+\left(\int\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^{2}dx\right)b+\left(\int\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)\,dx\right)a\right)$$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3
,x)*b*i + int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,
x)*a*i + int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x
)*b + int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a)
```

3.162 $\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 204

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{(-1)^{3/4}a^{3/2}(12A-11iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d} - \frac{(2+2i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{a(4iA+5B)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

output
```
-1/4*(-1)^(3/4)*a^(3/2)*(12*A-11*I*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)
^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d-(2+2*I)*a^(3/2)*(A-I*B)*arctanh((1+I)*a
^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+1/4*a*(4*I*A+5*B)*tan(
d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/2*I*a*B*tan(d*x+c)^(3/2)*(a+I*a*
tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 3.00 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.39

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{a\left(\frac{a(4A-3iB)\left((-1)^{3/4}\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)-\sqrt{1+i\tan(c+dx)}\sqrt{\tan(c+dx)}\right)(-i+\tan(c+dx))}{\sqrt{1+i\tan(c+dx)}} - 2\right.}{}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
(a*((a*(4*A - (3*I)*B)*((-1)^(3/4)*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]
- Sqrt[1 + I*Tan[c + d*x]]*Sqrt[Tan[c + d*x]])*(-I + Tan[c + d*x]))/Sqrt[1
 + I*Tan[c + d*x]] - 2*a*B*Sqrt[Tan[c + d*x]]*(-I + Tan[c + d*x])^2 + (8*(
A - I*B)*Sqrt[I*a*Tan[c + d*x]]*(Sqrt[a]*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sq
rt[a]]*Sqrt[1 + I*Tan[c + d*x]] - Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c
+ d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[a + I*a*Tan[c + d*x]]))/Sqrt[Tan
[c + d*x]]))/(4*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4077, 27, 3042, 4080, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{2}\int \frac{1}{2}\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(a(4A-3iB)+a(4iA+5B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\Big\downarrow$ 27

$$\frac{1}{4}\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(a(4A-3iB)+a(4iA+5B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(a(4A-3iB)+a(4iA+5B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\Big\downarrow$ 4080

$$\frac{1}{4}\left(\frac{\int -\frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(4iA+5B)-a^2(12A-11iB)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}}dx}{a}+\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}\right)+\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\Big\downarrow$ 27

$$\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(4iA+5B)-a^2(12A-11iB)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{2a}\right)+\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(4iA+5B)-a^2(12A-11iB)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{2a}\right)+\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\Big\downarrow$ 4084

$$\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{16a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a(11B+12iA)\int\frac{(a-}{}}{2a}\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{16a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a(11B+12iA)\int\frac{(a-}{}}{2a}\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\Big\downarrow$ 4027

$$\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{-\frac{32ia^4(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-a(11B+}{2a}\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\Big\downarrow$ 218

$$\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(16-16i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-a(11B+12}{2a}\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\Big\downarrow$ 4082

$$\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(16-16i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^3(11B+12iA}{}}{2a}\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\downarrow$ 65

$$\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(16-16i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(11B+12i\ldots}{\ldots}}{2a}\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

$\downarrow$ 216

$$\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{2\sqrt[4]{-1}a^{5/2}(11B+12iA)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(16-16i)a^5\ldots}{\ldots}}{2a}\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
((I/2)*a*B*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/d + (-1/2*((2*(-
1)^(1/4)*a^(5/2)*((12*I)*A + 11*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((16 - 16*I)*a^(5/2)*(I*A + B)*Ar
cTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)
/a + (a*((4*I)*A + 5*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d)/
4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(162) = 324$.

Time = 0.23 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.77

method	result
derivativedivides	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a\left(-4iB\sqrt{-ia}\,\sqrt{ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\tan(dx+c)+4iA\ln\left(\frac{2ia\tan(d\cdots}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a\left(-4iB\sqrt{-ia}\,\sqrt{ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\tan(dx+c)+4iA\ln\left(\frac{2ia\tan(d\cdots}{\cdots}\right.\right.}{\cdots}$
parts	$\frac{A\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a\left(3i\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a+2i\sqrt{a\tan(d\cdots}\right.}{\cdots}$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/8/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(-4*I*B*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+4*I*A*ln(1/2*
(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(
I*a)^(1/2))*a*(-I*a)^(1/2)-8*I*A*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)-4*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))
*a+5*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a-10*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+8*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)-4
*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-8*ln(1/2*(2*I*a*tan(d*
x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a
*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1
/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 829 vs. $2(150) = 300$.

Time = 0.11 (sec) , antiderivative size = 829, normalized size of antiderivative = 4.06

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
1/8*(8*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I
*c) + d)*log((sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x +
 I*c) + sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - 8*sqrt(2)*sqrt(-(-I*A^2 -
 2*A*B + I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(-(sqrt(2)*sqrt(-(
-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) - sqrt(2)*((-I*A - B)*a
*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c
)/((-I*A - B)*a)) + 2*sqrt(2)*((4*I*A + 7*B)*a*e^(3*I*d*x + 3*I*c) + (4*I*
A + 3*B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt((144*I*A^2 + 264*A
*B - 121*I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*((12*I*A
 + 11*B)*a*e^(2*I*d*x + 2*I*c) + (12*I*A + 11*B)*a)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) +
 2*sqrt((144*I*A^2 + 264*A*B - 121*I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-
I*d*x - I*c)/((12*I*A + 11*B)*a)) - sqrt((144*I*A^2 + 264*A*B - 121*I*B^2)
*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*((12*I*A + 11*B)*a*e^(2
*I*d*x + 2*I*c) + (12*I*A + 11*B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*sqrt((144...
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}(A+B\tan(c+dx))\sqrt{\tan(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*sqrt(tan(c +
 d*x)), x)
```

Maxima [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sqrt{\tan(dx+c)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x +
 c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \sqrt{\tan(c+d\,x)}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
x)
```

output
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
 x)
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a}\,a\left(-2\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,ai+\left(\int\frac{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\tan(dx+c)}dx\right)adi+$$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
(sqrt(a)*a*( - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a*i + int((sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x),x)*a*d*i + int(sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*b*d*i + 3*int(
sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i + int(sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*b*d))/d
```

3.163 $\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 156

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{(-1)^{3/4}a^{3/2}(2iA+3B)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{(2-2i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
-(-1)^(3/4)*a^(3/2)*(2*I*A+3*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)
/(a+I*a*tan(d*x+c))^(1/2))/d+(2-2*I)*a^(3/2)*(A-I*B)*arctanh((1+I)*a^(1/2)
*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+I*a*B*tan(d*x+c)^(1/2)*(a+I*
a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.65

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{aB\left(\sqrt[4]{-1}\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)+i\sqrt{1+i\tan(c+dx)}\sqrt{\tan(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{1+i\tan(c+dx)}}$$
$$+\frac{2a(iA+B)\sqrt{ia\tan(c+dx)}\left(\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{ia\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+i\tan(c+dx)}-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{a+ia\tan(c+dx)}\right)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c +
 d*x]],x]
```

output
```
(a*B*((-1)^(1/4)*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]] + I*Sqrt[1 + I*Tan
[c + d*x]]*Sqrt[Tan[c + d*x]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[1 + I*T
an[c + d*x]]) + (2*a*(I*A + B)*Sqrt[I*a*Tan[c + d*x]]*(Sqrt[a]*ArcSinh[Sqr
t[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[1 + I*Tan[c + d*x]] - Sqrt[2]*ArcTanh[(S
qrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[a + I*a*Ta
n[c + d*x]]))/(d*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {3042, 4077, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4077}$$

$$\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2A-iB)+a(2iA+3B)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2A-iB)+a(2iA+3B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2A-iB)+a(2iA+3B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4084}$$

$$\frac{1}{2}\left(4a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-(2A-3iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx\right)+\frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(4a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-(2A-3iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx\right)+\frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4027}$$

$$\frac{1}{2}\left(-\frac{8ia^3(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-(2A-3iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}\right.$$
$$\frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{(4-4i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-(2A-3iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}\right.$$

$$\frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4082}$$

$$\frac{1}{2}\left(\frac{(4-4i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2(2A-3iB)\int\frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}\right)$$

$$\frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{65}$$

$$\frac{1}{2}\left(\frac{(4-4i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(2A-3iB)\int\frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}\right)+$$

$$\frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{2\sqrt[4]{-1}a^{3/2}(2A-3iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(4-4i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\right)+$$

$$\frac{iaB\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]]
,x]
```

output
```
((2*(-1)^(1/4)*a^(3/2)*(2*A - (3*I)*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan
[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((4 - 4*I)*a^(3/2)*(A - I*B)*
ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/
d)/2 + (I*a*B*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(126) = 252$.

Time = 0.23 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.12

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}\,a\left(-iB\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a+2iB\sqrt{ia}\,\sqrt{-}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}\,a\left(-iB\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a+2iB\sqrt{ia}\,\sqrt{-}\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}\,a^2\left(i\sqrt{2}\,\sqrt{ia}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)+4i\ln\left(\right.\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/2/d*(a*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^(1/2)*a*(-I*B*ln(1/2*(2*I*a*ta
n(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2
))*(-I*a)^(1/2)*a+2*I*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)+I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+2*A*ln(1
/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a
)/(I*a)^(1/2))*(-I*a)^(1/2)*a+2*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)-(I*a)^
(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+2*ln(1/2*(2*I*a*tan(d*x+c)+2*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)
^(1/2))/(I*a)^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 743 vs. 2(116) = 232.

Time = 0.10 (sec) , antiderivative size = 743, normalized size of antiderivative = 4.76

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, al
gorithm="fricas")
```

output
```
1/2*(2*I*sqrt(2)*B*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) - 2*sqrt(2)*sqrt
(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*d*log((I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B -
 I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*
I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)
) + 2*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*d*log((-I*sqrt(2)*sqr
t(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B
)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x -
I*c)/((-I*A - B)*a)) - sqrt((-4*I*A^2 - 12*A*B + 9*I*B^2)*a^3/d^2)*d*log((
sqrt(2)*((2*I*A + 3*B)*a*e^(2*I*d*x + 2*I*c) + (2*I*A + 3*B)*a)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*
I*c) + 1)) + 2*I*sqrt((-4*I*A^2 - 12*A*B + 9*I*B^2)*a^3/d^2)*d*e^(I*d*x +
I*c))*e^(-I*d*x - I*c)/((2*I*A + 3*B)*a)) + sqrt((-4*I*A^2 - 12*A*B + 9*I*
B^2)*a^3/d^2)*d*log((sqrt(2)*((2*I*A + 3*B)*a*e^(2*I*d*x + 2*I*c) + (2*I*A
 + 3*B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*I*sqrt((-4*I*A^2 - 12*A*B + 9*I*B^2)*a
^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/((2*I*A + 3*B)*a)))/d
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx=\int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\,(A+B\tan{(c+dx)})}{\sqrt{\tan{(c+dx)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))/sqrt(tan(c +
 d*x)), x)
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)/sqrt(tan(d*x +
 c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}{\sqrt{\tan\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(1/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(1/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{\sqrt{a}\,a\Big(2\sqrt{\tan\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,a-2\sqrt{\tan}}{}$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(sqrt(a)*a*(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a - 2*sqrt(tan(c
 + d*x))*sqrt(tan(c + d*x)*i + 1)*b*i + int((sqrt(tan(c + d*x))*sqrt(tan(c
 + d*x)*i + 1))/tan(c + d*x),x)*b*d*i - 2*int(sqrt(tan(c + d*x))*sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x),x)*a*d + 3*int(sqrt(tan(c + d*x))*sqrt(tan(c
+ d*x)*i + 1)*tan(c + d*x),x)*b*d*i))/d
```

3.164 $\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 146

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt[4]{-1}a^{3/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{(2+2i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

output

```
2*(-1)^(1/4)*a^(3/2)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*t
an(d*x+c))^(1/2))/d+(2+2*I)*a^(3/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+
c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d-2*a*A*(a+I*a*tan(d*x+c))^(1/2)/d/tan(
d*x+c)^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 301 vs. 2(146) = 292.

Time = 4.86 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.06

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\left(\sqrt[4]{-1}aA\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)(1+i\tan(c\right.}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(3/2),x]
```

output
```
(2*a*((-1)^(1/4)*a*A*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*(1 + I*Tan[c +
 d*x])*Sqrt[Tan[c + d*x]]*Sqrt[I*a*Tan[c + d*x]] + a^(3/2)*(A - I*B)*ArcSi
nh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Tan[c + d*x]*(-I + Tan[c + d*x]) + a*Sq
rt[1 + I*Tan[c + d*x]]*(A*(-1 - I*Tan[c + d*x])*Sqrt[I*a*Tan[c + d*x]] + S
qrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan
[c + d*x]]]*Tan[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])))/(d*Sqrt[1 + I*Tan[c
 + d*x]]*Sqrt[Tan[c + d*x]]*Sqrt[I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*
x]])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 4076, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$2\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2iA+B)+iaB\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2iA+B)+iaB\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2iA+B)+iaB\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4084}$$

$$2a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-B\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$2a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-B\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4027}$$

$$-\frac{4ia^3(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-B\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{218}$$

$$-B\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4082}$$

$$-\frac{a^2B\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{65}$$

$$-\frac{2a^2B\int \frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}+\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{2\sqrt[4]{-1}a^{3/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2)
,x]
```

output
```
(2*(-1)^(1/4)*a^(3/2)*B*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqr
t[a + I*a*Tan[c + d*x]]])/d + ((2 - 2*I)*a^(3/2)*(I*A + B)*ArcTanh[((1 + I
)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a*A*Sqrt
[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(118) = 236$.

Time = 0.25 (sec) , antiderivative size = 521, normalized size of antiderivative = 3.57

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\,a\left(4iA\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a\tan(dx+c)-i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\right.\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\,a\left(4iA\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a\tan(dx+c)-i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\right.\right.}{}$
parts	$-\frac{A\sqrt{a(1+i\tan(dx+c))}\,a\left(i\sqrt{ia}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)a+\sqrt{2}\,\sqrt{i}\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/2/d*(a*(1+I*tan(d*x+c)))^(1/2)*a*(4*I*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*
a*tan(d*x+c)-I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+
c)+2*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I
*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)+2*I*ln(1/2*(2*I*a*tan(
d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))
*(-I*a)^(1/2)*a*tan(d*x+c)-(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I)
)*a*tan(d*x+c)-4*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a
)^(1/2)-2*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c))/tan(d*x+c)^(1/2)/(
I*a)^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(110) = 220$.

Time = 0.10 (sec) , antiderivative size = 748, normalized size of antiderivative = 5.12

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, al
gorithm="fricas")
```

output
```
-1/2*(2*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*
I*c) - d)*log((sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x
+ I*c) + sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - 2*sqrt(2)*sqrt(-(-I*A^2
- 2*A*B + I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(-(sqrt(2)*sqrt(-
(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) - sqrt(2)*((-I*A - B)*
a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*
c)/((-I*A - B)*a)) + 4*sqrt(2)*(I*A*a*e^(3*I*d*x + 3*I*c) + I*A*a*e^(I*d*x
 + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(-4*I*B^2*a^3/d^2)*(d*e^(2*I*d*x + 2*I
*c) - d)*log((sqrt(2)*(B*a*e^(2*I*d*x + 2*I*c) + B*a)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))
 + sqrt(-4*I*B^2*a^3/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(B*a)) - sqr
t(-4*I*B^2*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log((sqrt(2)*(B*a*e^(2*I*d
*x + 2*I*c) + B*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - sqrt(-4*I*B^2*a^3/d^2)*d*e^(I*d*
x + I*c))*e^(-I*d*x - I*c)/(B*a)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\,(A+B\tan{(c+dx)})}{\tan^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))/tan(c + d*x)
**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(3/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(3/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{a}\,a\left(\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
(2*sqrt(a)*a*(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b +
 sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a + int((sqrt(tan(c + d*x))*s
qrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)*a*d - int(sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*tan(c + d*x)*b*d))/(t
an(c + d*x)*d)
```

3.165 $\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 137

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{(2-2i)a^{3/2}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2a(4iA+3B)\sqrt{a+ia\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}}$$

output
```
(-2+2*I)*a^(3/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan
(d*x+c))^(1/2))/d-2/3*a*A*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)-2/3*
a*(4*I*A+3*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 5.44 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.86

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\left(-A(a+ia\tan(c+dx))^{3/2}+3a(iA+B)\tan(c+\ldots\right.}{\ldots}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(5/2),x]
```

output
```
(2*(-(A*(a + I*a*Tan[c + d*x])^(3/2)) + 3*a*(I*A + B)*Tan[c + d*x]*(Sqrt[2
]*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqr
t[I*a*Tan[c + d*x]] - (Sqrt[a + I*a*Tan[c + d*x]]*(Sqrt[a]*Sqrt[1 + I*Tan[
c + d*x]] - (-1)^(1/4)*Sqrt[a]*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt
[Tan[c + d*x]] + ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[I*a*Tan[c +
d*x]]))/(Sqrt[a]*Sqrt[1 + I*Tan[c + d*x]]))))/(3*d*Tan[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {3042, 4076, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{3}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{\tan(c+dx)^{3/2}}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$\frac{1}{3}\left(\frac{2\int -\frac{3a^2(A-iB)\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-6a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-6a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4027}$$

$$\frac{1}{3}\left(\frac{12ia^3(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(-\frac{(6-6i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2)
,x]
```

output
```
(-2*a*A*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) + (((-6 + 6*I
)*a^(3/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a +
I*a*Tan[c + d*x]]])/d - (2*a*((4*I)*A + 3*B)*Sqrt[a + I*a*Tan[c + d*x]])/(
d*Sqrt[Tan[c + d*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(112) = 224$.

Time = 0.22 (sec) , antiderivative size = 618, normalized size of antiderivative = 4.51

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a\left(-12iB\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a\tan(dx+c)^2+3i\sqrt{ia}\,\sqrt{}\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a\left(-12iB\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a\tan(dx+c)^2+3i\sqrt{ia}\,\sqrt{}\right.}{}$
parts	$-\frac{A\sqrt{a(1+i\tan(dx+c))}\,a\left(3i\sqrt{ia}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)^2a+12i\right.}{}$

input

```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_R
ETURNVERBOSE)
```

output

```
-1/6/d*(a*(1+I*tan(d*x+c)))^(1/2)*a/tan(d*x+c)^(3/2)*(-12*I*B*ln(1/2*(2*I*
a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^
(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2+3*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)
*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(t
an(d*x+c)+I))*a*tan(d*x+c)^2+16*I*A*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+12*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a
*tan(d*x+c)^2+6*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2-3*(I*a)^(
1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+12*B*(I*a)^(1/2)*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+6*ln(1/2*(2*
I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a
)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2+4*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/(I*a)^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(103) = 206.

Time = 0.09 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.82

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{3\sqrt{2}\sqrt{-\frac{(iA^2+2AB-iB^2)a^3}{d^2}}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, al
gorithm="fricas")
```

output
```
1/3*(3*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*
c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log((I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*
B^2)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c
) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) -
 3*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*c) -
 2*d*e^(2*I*d*x + 2*I*c) + d)*log((-I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2
)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c) +
 (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) + 2*
sqrt(2)*((5*A - 3*I*B)*a*e^(5*I*d*x + 5*I*c) + 2*A*a*e^(3*I*d*x + 3*I*c) -
 3*(A - I*B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(4*I*d*x + 4*I*
c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(ia(\tan(c+dx)-i))^{\frac{3}{2}}\,(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))/tan(c + d*x)
**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\ (a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}{\tan(c+d\,x)^{5/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(5/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(5/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{a}\,a\Big(2\sqrt{\tan(dx+c)}\ \sqrt{\tan(dx+c)\,i+1}\ \tan(dx+$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
(2*sqrt(a)*a*(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a
*i + 3*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - sqrt(t
an(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a + 3*int((sqrt(tan(c + d*x))*sqrt(t
an(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**2*a*d*i + 3*int((sqrt
(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**
2*b*d))/(3*tan(c + d*x)**2*d)
```

3.166 $\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 181

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{(2+2i)a^{3/2}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2a(6iA+5B)\sqrt{a+ia\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{4a(9A-10iB)\sqrt{a+ia\tan(c+dx)}}{15d\sqrt{\tan(c+dx)}}$$

output
```
(-2-2*I)*a^(3/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan
(d*x+c))^(1/2))/d-2/5*a*A*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)-2/15
*a*(6*I*A+5*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)+4/15*a*(9*A-10*
I*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 4.98 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.78

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\Big(3aA(-i+\tan(c+dx))^2+a(3iA+5B)\tan(c+$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(7/2),x]
```

output
```
(2*a*(3*a*A*(-I + Tan[c + d*x])^2 + a*((3*I)*A + 5*B)*Tan[c + d*x]*(-I + T
an[c + d*x])^2 + (15*(A - I*B)*Tan[c + d*x]^2*(Sqrt[a]*ArcSinh[Sqrt[I*a*Ta
n[c + d*x]]/Sqrt[a]]*(1 + I*Tan[c + d*x])*Sqrt[I*a*Tan[c + d*x]] - (-1)^(3
/4)*a*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[Tan[c + d*x]]*(-I + Tan[
c + d*x]) + Sqrt[1 + I*Tan[c + d*x]]*(a + I*a*Tan[c + d*x] - Sqrt[2]*ArcTa
nh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*T
an[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/Sqrt[1 + I*Tan[c + d*x]]))/(15*
d*Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 4076, 27, 3042, 4081, 25, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{5}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(6iA+5B)-a(4A-5iB)\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(6iA+5B)-a(4A-5iB)\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(6iA+5B)-a(4A-5iB)\tan(c+dx))}{\tan(c+dx)^{5/2}}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$\frac{1}{5}\left(\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}((9A-10iB)a^2+(6iA+5B)\tan(c+dx)a^2)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5B+6iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(-\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}((9A-10iB)a^2+(6iA+5B)\tan(c+dx)a^2)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5B+6iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}((9A-10iB)a^2+(6iA+5B)\tan(c+dx)a^2)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a(5B+6iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$\frac{1}{5}\left(-\frac{2\left(\frac{2\int \frac{15a^3(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^2(9A-10iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)}{3a}-\frac{2a(5B+6iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(-\frac{2\left(15a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(9A-10iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)}{3a}-\frac{2a(5B+6iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-\frac{2\left(15a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(9A-10iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)}{3a}-\frac{2a(5B+6iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4027}$$

$$\frac{1}{5}\left(-\frac{2\left(-\frac{30ia^4(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^2(9A-10iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)}{3a}-\frac{2a(5B+6iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{5}\left(-\frac{2\left(\frac{(15-15i)a^{5/2}(B+iA)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(9A-10iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)}{3a}-\frac{2a(5B+6iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2)
,x]
```

output
```
(-2*a*A*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) + ((-2*a*((6*
I)*A + 5*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) - (2*(((1
5 - 15*I)*a^(5/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/S
qrt[a + I*a*Tan[c + d*x]]])/d - (2*a^2*(9*A - (10*I)*B)*Sqrt[a + I*a*Tan[c
 + d*x]])/(d*Sqrt[Tan[c + d*x]])))/(3*a))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 704 vs. $2(148) = 296$.

Time = 0.24 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.90

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a\left(-72A\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^{2}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+60iA\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan\cdots}}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a\left(-72A\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^{2}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+60iA\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan\cdots}}{\cdots}\right.\right.}{\cdots}$
parts	$\frac{A\sqrt{a(1+i\tan(dx+c))}\,a\left(5i\sqrt{ia}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^{3}+5\sqrt{ia}\,\cdots\right.}{\cdots}$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/30/d*(a*(1+I*tan(d*x+c)))^(1/2)*a/tan(d*x+c)^(5/2)*(-72*A*(I*a)^(1/2)*(
-I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+60*I*A*ln(1
/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a
)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3-15*I*(I*a)^(1/2)*2^(1/2)*ln((2*
2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x
+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+80*I*B*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*
x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+60*B*ln(1/2*(2*I*a*tan(d*x+c)
+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a
)^(1/2)*a*tan(d*x+c)^3-15*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a
*tan(d*x+c)^3+30*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c
)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3+24*I*A*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)-3
0*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(
1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3+20*B*(I*a)^(1/2)*(-I*a)^(
1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+12*A*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(137) = 274$.

Time = 0.12 (sec) , antiderivative size = 593, normalized size of antiderivative = 3.28

$$\int \frac{(a + ia\tan(c + dx))^{3/2}(A + B\tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, al
gorithm="fricas")
```

output
```
1/15*(15*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(6*I*d*x + 6
*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log((sqrt(2
)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I
*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*
d*x - I*c)/((-I*A - B)*a)) - 15*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3
/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x +
2*I*c) - d)*log(-(sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d
*x + I*c) - sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - 2*sqrt(2)*((-27*I*A -
 25*B)*a*e^(7*I*d*x + 7*I*c) + 3*(I*A + 5*B)*a*e^(5*I*d*x + 5*I*c) + 5*(3*
I*A + 5*B)*a*e^(3*I*d*x + 3*I*c) + 15*(-I*A - B)*a*e^(I*d*x + I*c))*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(
2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(c + dx))^{3/2}(A + B\tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}{\tan\left(c+d\,x\right)^{7/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(7/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(7/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{a}\,a\left(-12\sqrt{\tan\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(d\right.\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
(2*sqrt(a)*a*( - 12*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*
x)**2*a + 10*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b
*i - 6*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 5*sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 3*sqrt(tan(c +
d*x))*sqrt(tan(c + d*x)*i + 1)*a - 15*int((sqrt(tan(c + d*x))*sqrt(tan(c +
 d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**3*a*d + 15*int((sqrt(tan(c
+ d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**3*b*d*i
))/(15*tan(c + d*x)**3*d)
```

3.167 $\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 225

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx &= -\frac{(2+2i)a^{3/2}(iA+B)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} \\ &- \frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{2a(8iA+7B)\sqrt{a+ia\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} \\ &+ \frac{4a(19A-21iB)\sqrt{a+ia\tan(c+dx)}}{105d\tan^{\frac{3}{2}}(c+dx)} + \frac{4a(67iA+63B)\sqrt{a+ia\tan(c+dx)}}{105d\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
(-2-2*I)*a^(3/2)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan
(d*x+c))^(1/2))/d-2/7*a*A*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(7/2)-2/35
*a*(8*I*A+7*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)+4/105*a*(19*A-2
1*I*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)+4/105*a*(67*I*A+63*B)*(
a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 4.68 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.58

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a\Big(15aA(-i+\tan(c+dx))^2+3a(3iA+7B)\tan(c\cdots}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(9/2),x]
```

output
```
(2*a*(15*a*A*(-I + Tan[c + d*x])^2 + 3*a*((3*I)*A + 7*B)*Tan[c + d*x]*(-I
+ Tan[c + d*x])^2 - a*(29*A - (21*I)*B)*Tan[c + d*x]^2*(-I + Tan[c + d*x])
^2 - (105*(A - I*B)*Tan[c + d*x]^3*(-((-1)^(1/4)*a*ArcSinh[(-1)^(1/4)*Sqrt
[Tan[c + d*x]]]*Sqrt[Tan[c + d*x]]*(-I + Tan[c + d*x])) + Sqrt[a]*ArcSinh[
Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[I*a*Tan[c + d*x]]*(-I + Tan[c + d*x])
 + Sqrt[1 + I*Tan[c + d*x]]*(a*(-I + Tan[c + d*x]) + I*Sqrt[2]*ArcTanh[(Sq
rt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/Sqrt[1 + I*Tan[c + d*x]]))/(105*d*Tan
[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.395$, Rules used = {3042, 4076, 27, 3042, 4081, 25, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{7}\int \frac{\sqrt{i\tan(c+dx)a+a}\,(a(8iA+7B)-a(6A-7iB)\tan(c+dx))}{2\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{\sqrt{i\tan(c+dx)a+a}\,(a(8iA+7B)-a(6A-7iB)\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{\sqrt{i\tan(c+dx)a+a}\,(a(8iA+7B)-a(6A-7iB)\tan(c+dx))}{\tan(c+dx)^{7/2}}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$\frac{1}{7}\left(\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\,((19A-21iB)a^2+2(8iA+7B)\tan(c+dx)a^2)}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2a(7B+8iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{7}\left(-\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}\,((19A-21iB)a^2+2(8iA+7B)\tan(c+dx)a^2)}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2a(7B+8iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(-\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}\left((19A-21iB)a^2+2(8iA+7B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{5/2}}dx}{5a}-\frac{2a(7B+8iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4081

$$\frac{1}{7}\left(-\frac{2\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(67iA+63B)-2a^3(19A-21iB)\tan(c+dx)\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{5a}-\frac{2a(7B+8iA)\sqrt{a}}{5d\tan^{\frac{5}{2}}}\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(-\frac{2\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(67iA+63B)-2a^3(19A-21iB)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{5a}-\frac{2a(7B+8iA)\sqrt{a}}{5d\tan^{\frac{5}{2}}(}\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(-\frac{2\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(67iA+63B)-2a^3(19A-21iB)\tan(c+dx)\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{5a}-\frac{2a(7B+8iA)\sqrt{a}}{5d\tan^{\frac{5}{2}}(}\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\Big\downarrow$ 4081

$$\frac{1}{7}\left(-\frac{2\left(\frac{\frac{2\int -\frac{105a^4(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^3(63B+67iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{5a}-\frac{2a(7B+8iA}{5d\,t}\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(-\frac{2\left(\frac{-105a^3(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(63B+67iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{5a}-\frac{2a(7B+8iA}{5d\,t}\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(-\frac{2\left(\frac{-105a^3(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(63B+67iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{5a}-\frac{2a(7B+8iA}{5d\,t}\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4027

$$\frac{1}{7}\left(-\frac{2\left(\frac{\frac{210ia^5(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^3(63B+67iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{5a}-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{7}\left(-\frac{2\left(\frac{-\frac{(105-105i)a^{7/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(63B+67iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)}{5a}-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)$$

input
```
Int[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/2)
,x]
```

output
```
(-2*a*A*Sqrt[a + I*a*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(7/2)) + ((-2*a*((8*
I)*A + 7*B)*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) - (2*((-2
*a^2*(19*A - (21*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)
) + (((-105 + 105*I)*a^(7/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^3*((67*I)*A + 63*B)*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a)))/(5*a))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(184) = 368$.

Time = 0.29 (sec) , antiderivative size = 794, normalized size of antiderivative = 3.53

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\, a\left(504B\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^3\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-420iB\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan}}{}\right.\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\, a\left(504B\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^3\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-420iB\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan}}{}\right.\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(dx+c))}\, a\left(105i\sqrt{ia}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^4-105\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/210/d*(a*(1+I*tan(d*x+c)))^(1/2)*a/tan(d*x+c)^(7/2)*(504*B*(I*a)^(1/2)*(
-I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-420*I*B*ln(
1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+
a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^4+536*I*A*(I*a)^(1/2)*(-I*a)^(1/
2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+152*A*(I*a)^(1/2)*(-
I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-168*I*B*(I*a
)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+42
0*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)
^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^4-105*(I*a)^(1/2)*2^(1/2)
*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*
tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-96*I*A*(I*a)^(1/2)*(-I*a)^(1/2)
*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+210*I*ln(1/2*(2*I*a*tan(
d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))
*(-I*a)^(1/2)*a*tan(d*x+c)^4+210*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x
+c)^4+105*I*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-
84*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*
x+c)-60*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/
(I*a)^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(171) = 342$.

Time = 0.12 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.84

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, al
gorithm="fricas")
```

output
```
-1/105*(105*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(8*I*d*x +
 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d
*x + 2*I*c) + d)*log((I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*d*e
^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - 105*sqrt(2)*sqrt
(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x
 + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)*log((-I
*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2
)*((-I*A - B)*a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*
e^(-I*d*x - I*c)/((-I*A - B)*a)) + 2*sqrt(2)*((211*A - 189*I*B)*a*e^(9*I*d
*x + 9*I*c) - 10*(16*A - 21*I*B)*a*e^(7*I*d*x + 7*I*c) + 14*(A + 6*I*B)*a*
e^(5*I*d*x + 5*I*c) + 70*(4*A - 3*I*B)*a*e^(3*I*d*x + 3*I*c) - 105*(A - I*
B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(8*I*d*x + 8*I*c) - 4*d*e
^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}{\tan\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(9/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(9/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\sqrt{a}\,a\Big(-76\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(d$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)
```

output
```
(2*sqrt(a)*a*( - 76*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*
x)**3*a*i - 84*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3
*b + 38*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*a - 42
*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i - 24*sqrt
(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 21*sqrt(tan(c +
 d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 15*sqrt(tan(c + d*x))*sqr
t(tan(c + d*x)*i + 1)*a - 105*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i
+ 1))/tan(c + d*x)**2,x)*tan(c + d*x)**4*a*d*i - 105*int((sqrt(tan(c + d*x
))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**4*b*d))/(105
*tan(c + d*x)**4*d)
```

3.168 $\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 269

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \frac{(2+2i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} - \frac{2a(10iA+9B)\sqrt{a+ia\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{4a(11A-12iB)\sqrt{a+ia\tan(c+dx)}}{105d\tan^{\frac{5}{2}}(c+dx)} + \frac{4a(61iA+57B)\sqrt{a+ia\tan(c+dx)}}{315d\tan^{\frac{3}{2}}(c+dx)} - \frac{4a(193A-201iB)\sqrt{a+ia\tan(c+dx)}}{315d\sqrt{\tan(c+dx)}}$$

output
```
(2+2*I)*a^(3/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))/d-2/9*a*A*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(9/2)-2/63*
a*(10*I*A+9*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(7/2)+4/105*a*(11*A-1
2*I*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)+4/315*a*(61*I*A+57*B)*(
a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)-4/315*a*(193*A-201*I*B)*(a+I*a*
tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 14.60 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.90

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \frac{a\left(\frac{2520(A-iB)e^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)}{\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}}\right.}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(11/2),x]
```

output
```
(a*((2520*(A - I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x))
/Sqrt[-1 + E^((2*I)*(c + d*x))]])/(E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((2*
I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]) + (Csc[c + d*x]^4*(-1197*A + (
1134*I)*B + 12*(117*A - (134*I)*B)*Cos[2*(c + d*x)] + (-487*A + (474*I)*B)
*Cos[4*(c + d*x)] + (144*I)*A*Sin[2*(c + d*x)] + 138*B*Sin[2*(c + d*x)] -
(172*I)*A*Sin[4*(c + d*x)] - 159*B*Sin[4*(c + d*x)]))/Sqrt[Tan[c + d*x]])*
Sqrt[a + I*a*Tan[c + d*x]])/(1260*d)
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.12, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4076, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{11/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{9}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(10iA+9B)-a(8A-9iB)\tan(c+dx))}{2\tan^{\frac{9}{2}}(c+dx)}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(10iA+9B)-a(8A-9iB)\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(10iA+9B)-a(8A-9iB)\tan(c+dx))}{\tan(c+dx)^{9/2}}dx-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$\frac{1}{9}\left(\frac{2\int -\frac{3\sqrt{i\tan(c+dx)a+a}((11A-12iB)a^2+(10iA+9B)\tan(c+dx)a^2)}{\tan^{\frac{7}{2}}(c+dx)}dx}{7a}-\frac{2a(9B+10iA)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(-\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}((11A-12iB)a^2+(10iA+9B)\tan(c+dx)a^2)}{\tan^{\frac{7}{2}}(c+dx)}dx}{7a}-\frac{2a(9B+10iA)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(-\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}\left((11A-12iB)a^2+(10iA+9B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{7/2}}dx}{7a}-\frac{2a(9B+10iA)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 4081

$$\frac{1}{9}\left(-\frac{6\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(61iA+57B)-4a^3(11A-12iB)\tan(c+dx)\right)}{2\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2a^2(11A-12iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}\right)}{7a}-\frac{2a(9B+10iA)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(-\frac{6\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(61iA+57B)-4a^3(11A-12iB)\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2a^2(11A-12iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}\right)}{7a}-\frac{2a(9B+10iA)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}\left(-\frac{6\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(61iA+57B)-4a^3(11A-12iB)\tan(c+dx)\right)}{\tan(c+dx)^{5/2}}dx}{5a}-\frac{2a^2(11A-12iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}\right)}{7a}-\frac{2a(9B+10iA)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)-\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 4081

$$\frac{1}{9}\left(-\frac{6\left(\frac{\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((193A-201iB)a^4+2(61iA+57B)\tan(c+dx)a^4\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^3(57B+61iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a^2(11A-12iB)\sqrt{a+ia\tan}}{5d\tan^{\frac{5}{2}}(c+dx)}\right.}{7a}\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{9}\left(-\frac{6\left(\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((193A-201iB)a^4+2(61iA+57B)\tan(c+dx)a^4\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^3(57B+61iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a^2(11A-12iB)\sqrt{a+ia\tan(}}{5d\tan^{\frac{5}{2}}(c+dx)}\right.}{7a}\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(-\frac{6\left(\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((193A-201iB)a^4+2(61iA+57B)\tan(c+dx)a^4\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a^3(57B+61iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a^2(11A-12iB)\sqrt{a+ia\tan(}}{5d\tan^{\frac{5}{2}}(c+dx)}\right.}{7a}\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 4081$$

$$\frac{1}{9}\left(-\frac{6\left(-\frac{-\frac{2\int \frac{315a^5(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^4(193A-201iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^3(57B+61iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a^2(11A-12iB)\sqrt{a+i\cdots}}{5d\tan^{\frac{5}{2}}(c+d\cdots}\cdots\right.}{7a}\cdots\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(-\frac{6\left(-\frac{-\frac{315a^4(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^4(193A-201iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^3(57B+61iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a^2(11A-12iB)\sqrt{a+ia\cdots}}{5d\tan^{\frac{5}{2}}(c+dx\cdots}\cdots\right.}{7a}\cdots\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(-\frac{6\left(-\frac{-\frac{315a^4(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^4(193A-201iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^3(57B+61iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a^2(11A-12iB)\sqrt{a+ia\cdots}}{5d\tan^{\frac{5}{2}}(c+dx\cdots}\cdots\right.}{7a}\cdots\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 4027$$

$$\frac{1}{9}\left(-\frac{6\left(-\frac{-\frac{-\frac{630 i a^6 (B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^4(193A-201iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^3(57B+61iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a\cdots}{\cdots}\right)}{7a}\cdots\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \; 218$$

$$\frac{1}{9}\left(-\frac{6\left(-\frac{-\frac{2a^3(57B+61iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{(315-315i)a^{9/2}(B+iA)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^4(193A-201iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}}{5a}-\frac{2a^2\cdots}{\cdots}\right)}{7a}\cdots\right.$$

$$\frac{2aA\sqrt{a+ia\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(11/2
),x]
```

output
```
(-2*a*A*Sqrt[a + I*a*Tan[c + d*x]])/(9*d*Tan[c + d*x]^(9/2)) + ((-2*a*((10
*I)*A + 9*B)*Sqrt[a + I*a*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(7/2)) - (6*((-
2*a^2*(11*A - (12*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2
)) + ((-2*a^3*((61*I)*A + 57*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d
*x]^(3/2)) - (((315 - 315*I)*a^(9/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sq
rt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^4*(193*A - (201*I)
*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*a)))/(7*
a))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(220) = 440$.

Time = 0.24 (sec) , antiderivative size = 885, normalized size of antiderivative = 3.29

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\,a\left(-1544A\tan(dx+c)^4\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{-ia}-288iB\sqrt{a\tan(dx+c)(1+i\tan\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\,a\left(-1544A\tan(dx+c)^4\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{-ia}-288iB\sqrt{a\tan(dx+c)(1+i\tan\right.}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x,method=_
RETURNVERBOSE)
```

output
```
1/630/d*(a*(1+I*tan(d*x+c)))^(1/2)*a/tan(d*x+c)^(9/2)*(-1544*A*tan(d*x+c)^
4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-288*I*B*t
an(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)
-315*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5+456
*B*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)+488*I*A*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2
)*(-I*a)^(1/2)+1260*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^5-200*I
*A*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/
2)-315*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5+264
*A*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)+1260*I*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^5-630*ln(1/2*(2
*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*
a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^5+630*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2
)*a*tan(d*x+c)^5+1608*I*B*tan(d*x+c)^4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)*(I*a)^(1/2)*(-I*a)^(1/2)-180*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(205) = 410$.

Time = 0.11 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.61

$$\int \frac{(a + ia\tan(c + dx))^{3/2}(A + B\tan(c + dx))}{\tan^{\frac{11}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, a
lgorithm="fricas")
```

output
```
-1/315*(315*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(10*I*d*x
 + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(
4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)*log((sqrt(2)*sqrt(-(-I*A^2
 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a*e^(2*
I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I
*A - B)*a)) - 315*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(10
*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10
*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)*log(-(sqrt(2)*sqrt(-
(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*e^(I*d*x + I*c) - sqrt(2)*((-I*A - B)*
a*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*
c)/((-I*A - B)*a)) + 2*sqrt(2)*((659*I*A + 633*B)*a*e^(11*I*d*x + 11*I*c)
+ 7*(-127*I*A - 159*B)*a*e^(9*I*d*x + 9*I*c) + 18*(47*I*A + 29*B)*a*e^(7*I
*d*x + 7*I*c) + 42*(27*I*A + 19*B)*a*e^(5*I*d*x + 5*I*c) + 105*(-9*I*A - 1
1*B)*a*e^(3*I*d*x + 3*I*c) + 315*(I*A + B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)))/(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I
*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(c + dx))^{3/2}(A + B\tan(c + dx))}{\tan^{\frac{11}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, a
lgorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}{\tan\left(c+d\,x\right)^{11/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(11/
2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/tan(c + d*x)^(11/
2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\sqrt{a}\,a\left(-176\sqrt{\tan\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(\right.\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x)
```

output
```
(2*sqrt(a)*a*( - 176*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d
*x)**4*a + 192*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4
*b*i - 88*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*a*i
- 96*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*b + 66*sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*a - 72*sqrt(tan(
c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i - 50*sqrt(tan(c + d
*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 45*sqrt(tan(c + d*x))*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 35*sqrt(tan(c + d*x))*sqrt(tan(c +
d*x)*i + 1)*a - 315*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(
c + d*x)**3,x)*tan(c + d*x)**5*a*d*i - 315*int((sqrt(tan(c + d*x))*sqrt(ta
n(c + d*x)*i + 1))/tan(c + d*x)**3,x)*tan(c + d*x)**5*b*d))/(315*tan(c + d
*x)**5*d)
```

3.169 $\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 298

$$\begin{aligned}\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A&\\+B\tan(c+dx))\,dx&=\frac{3(-1)^{3/4}a^{5/2}(120iA+121B)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{64d}\\&+\frac{(4+4i)a^{5/2}(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\\&+\frac{a^2(152A-149iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{64d}\\&+\frac{a^2(104iA+107B)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{96d}\\&-\frac{a^2(8A-11iB)\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{24d}\\&+\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}\end{aligned}$$

output
```
3/64*(-1)^(3/4)*a^(5/2)*(120*I*A+121*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+
c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+(4+4*I)*a^(5/2)*(I*A+B)*arctanh((1+I)
*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+1/64*a^2*(152*A-149*
I*B)*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/96*a^2*(104*I*A+107*B)*
tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d-1/24*a^2*(8*A-11*I*B)*tan(d*x+
c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/4*I*a*B*tan(d*x+c)^(5/2)*(a+I*a*tan(
d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 6.78 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.89

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}{4d} + \frac{\frac{a(8A-5iB)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}{6d} + \frac{-\frac{ia^4(40iA+43B)\sqrt{a+ia\tan(c+dx)}\left(-\frac{3}{4}(-1)^{3/4}\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)+\frac{5}{4}\sqrt{1+i\tan(c+dx)}\cdots\right.}{4d\sqrt{1+i\tan(c+dx)}}\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
(B*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2))/(4*d) + ((a*(8*A - (5*
I)*B)*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2))/(6*d) + (((-1/4*I)*
a^4*((40*I)*A + 43*B)*Sqrt[a + I*a*Tan[c + d*x]]*((-3*(-1)^(3/4)*ArcSinh[(
-1)^(1/4)*Sqrt[Tan[c + d*x]]])/4 + (5*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[Tan[c
+ d*x]])/4 + (I/2)*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(3/2)))/(d*Sqrt[1
 + I*Tan[c + d*x]]) + (a*((-1/4*I)*a^3*(8*A - (5*I)*B) - (a^3*((40*I)*A +
43*B))/4)*(((-4*I)*Sqrt[2]*a*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt
[a + I*a*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/Sqrt[I*a*Tan[c + d*x]] + ((4*I
)*a^(3/2)*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[1 + I*Tan[c + d*x]]
*Sqrt[Tan[c + d*x]])/(Sqrt[I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) +
 I*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + (I*Sqrt[a]*ArcSinh[Sqrt
[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
/(Sqrt[1 + I*Tan[c + d*x]]*Sqrt[I*a*Tan[c + d*x]]))))/d)/(3*a))/(4*a)
```

Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.07, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4080, 27, 3042, 4080, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{4}\int \frac{1}{2}\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(8A-5iB)+a(8iA+11B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(8A-5iB)+a(8iA+11B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \tan(c+dx)^{3/2}(i\tan(c+dx)a+a)^{3/2}(a(8A-5iB)+a(8iA+11B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{8}\left(\frac{1}{3}\int \frac{1}{2}\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}\left((88A-85iB)a^2+(104iA+107B)\tan(c+dx)a^2\right)dx-\frac{a^2(8A-1\right.$$
$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{6}\int \tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}\left((88A-85iB)a^2+(104iA+107B)\tan(c+dx)a^2\right)dx-\frac{a^2(8A-11i}{}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{8}\left(\frac{1}{6}\int \tan(c+dx)^{3/2}\sqrt{i\tan(c+dx)a+a}\left((88A-85iB)a^2+(104iA+107B)\tan(c+dx)a^2\right)dx-\frac{a^2(8A-11}{}\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \textbf{4080}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\int -\frac{3}{2}\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left(a^3(104iA+107B)-a^3(152A-149iB)\tan(c+dx)\right)dx}{2a}+\frac{a^2(1}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\int\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left(a^3(104iA\right.}{4a}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\int\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left(a^3(104iA\right.}{4a}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \textbf{4080}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((152A-149iB)a^4+3(120iA+121B)\tan(c+c\right.}{2\sqrt{\tan(c+dx)}}}{a}\right.}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((152A-149iB)a^4+3(120iA+121B)\tan(c+c\right.}{\sqrt{\tan(c+dx)}}}{2a}\right.}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((152A-149iB)a^4+3(120iA+121B)\tan(c+c\right.}{\sqrt{\tan(c+dx)}}}{2a}\right.}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\downarrow$ 4084

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{512a^4(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-3a^3(120A-121iB)\int}{2a}\right.}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{512a^4(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-3a^3(120A-121iB)\int}{2a}\right.}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\downarrow$ 4027

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{-3a^3(120A-121iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}\right.}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\downarrow$ 218

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\frac{(512-512i)a^{9/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-3}{2a}\right.}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$\downarrow$ 4082

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\frac{(512-512i)a^{9/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{3}{2a}\right.}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{65}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\frac{(512-512i)a^{9/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{6}{2a}\right.}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{a^2(107B+104iA)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{3\left(\frac{\frac{6\sqrt[4]{-1}a^{9/2}(120A-121iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-}{2a}\right.}{}\right.\right.$$

$$\frac{iaB\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{4d}$$

input
```
Int[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
((I/4)*a*B*Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2))/d + (-1/3*(a^2
*(8*A - (11*I)*B)*Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d + ((a^2
*((104*I)*A + 107*B)*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(2*d)
- (3*(((6*(-1)^(1/4)*a^(9/2)*(120*A - (121*I)*B)*ArcTan[((-1)^(3/4)*Sqrt[a
]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((512 - 512*I)*a^(9
/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Ta
n[c + d*x]]])/d)/(2*a) - (a^3*(152*A - (149*I)*B)*Sqrt[Tan[c + d*x]]*Sqrt[
a + I*a*Tan[c + d*x]])/d))/(4*a))/6)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 741 vs. 2(240) = 480.

Time = 0.32 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.49

method	result
derivativedivides	$\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-96B\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^3\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-128A\sqrt{ia}\,\sqrt{-ia}\,\mathrm{t}\right.}{}$
default	$\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-96B\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^3\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-128A\sqrt{ia}\,\sqrt{-ia}\,\mathrm{t}\right.}{}$
parts	Expression too large to display

input

```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output

```
1/384/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(-96*B*(I*a)^(1/2)
*(-I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-128*A*(I*
a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+2
72*I*B*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*
a)^(1/2)+416*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^
(1/2)*tan(d*x+c)+447*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-894*I*B*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+428*B*(I*a)^(1/2)*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-384*I*(I*a)^
(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-456*A*ln(1/2*(2*I*a*tan(d*x+c
)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*
a)^(1/2)*a+912*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^
(1/2)-768*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a+384*(I*a)^(1/2)*2^(1/2)*ln(-
(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan
(d*x+c))/(tan(d*x+c)+I))*a-768*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2))/(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(224) = 448$.

Time = 0.13 (sec) , antiderivative size = 1017, normalized size of antiderivative = 3.41

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
1/384*(768*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(6*I*d*x +
6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log((I*sqr
t(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((
-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*
e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 768*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*
B^2)*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*
I*d*x + 2*I*c) + d)*log((-I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)
*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A -
 B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) + 2*sqrt
(2)*(13*(56*A - 65*I*B)*a^2*e^(7*I*d*x + 7*I*c) + 3*(504*A - 425*I*B)*a^2*
e^(5*I*d*x + 5*I*c) + (1096*A - 1135*I*B)*a^2*e^(3*I*d*x + 3*I*c) + 3*(104
*A - 107*I*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 9*sqrt(-(14400*I
*A^2 + 29040*A*B - 14641*I*B^2)*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4
*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*((-120*I*A - 1
21*B)*a^2*e^(2*I*d*x + 2*I*c) + (-120*I*A - 121*B)*a^2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1
)) + 2*I*sqrt(-(14400*I*A^2 + 29040*A*B - 14641*I*B^2)*a^5/d^2)*d*e^(I*...
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A + B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^(
3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^{\frac{3}{2}}(c+dx)(a + ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeWarning, need to choose a branch for the root of a polynomial
 with par
```

Mupad [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
x)
```

output
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
 x)
```

Reduce [F]

$$\begin{aligned}&\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a^{2}\Bigg(-\left(\int \sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^{4}dx\right)b\\&-\left(\int \sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^{3}dx\right)a\\&+2\left(\int \sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^{3}dx\right)bi\\&+2\left(\int \sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^{2}dx\right)ai\\&+\left(\int \sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^{2}dx\right)b\\&+\left(\int \sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)\,dx\right)a\Bigg)\end{aligned}$$

input

```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output

```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d
*x)**4,x)*b - int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)
**3,x)*a + 2*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*
*3,x)*b*i + 2*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)
**2,x)*a*i + int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*
*2,x)*b + int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*
a)
```

3.170 $\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 252

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{(-1)^{3/4}a^{5/2}(46A-45iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{8d} - \frac{(4+4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{a^2(18iA+19B)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{a^2(2A-3iB)\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

output
```
-1/8*(-1)^(3/4)*a^(5/2)*(46*A-45*I*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)
^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d-(4+4*I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a
^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+1/8*a^2*(18*I*A+19*B)*
tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d-1/4*a^2*(2*A-3*I*B)*tan(d*x+c)
^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/3*I*a*B*tan(d*x+c)^(3/2)*(a+I*a*tan(d*
x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 6.60 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.00

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\frac{B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}{3d}$$

$$+\frac{\frac{ia^3(6A-5iB)\sqrt{a+ia\tan(c+dx)}\left(-\frac{3}{4}(-1)^{3/4}\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)+\frac{5}{4}\sqrt{1+i\tan(c+dx)}\sqrt{\tan(c+dx)}+\frac{1}{2}i\sqrt{1+i\tan(c+dx)}\tan^{\frac{3}{2}}(c+}{2d\sqrt{1+i\tan(c+dx)}}}{}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
(B*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2))/(3*d) + (((I/2)*a^3*(6
*A - (5*I)*B)*Sqrt[a + I*a*Tan[c + d*x]]*((-3*(-1)^(3/4)*ArcSinh[(-1)^(1/4
)*Sqrt[Tan[c + d*x]]])/4 + (5*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[Tan[c + d*x]])
/4 + (I/2)*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(3/2)))/(d*Sqrt[1 + I*Tan
[c + d*x]]) + (a*((a^2*(6*A - (5*I)*B))/2 - (I/2)*a^2*B)*(((-4*I)*Sqrt[2]*
a*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqr
t[Tan[c + d*x]])/Sqrt[I*a*Tan[c + d*x]] + ((4*I)*a^(3/2)*ArcSinh[Sqrt[I*a*
Tan[c + d*x]]/Sqrt[a]]*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[
I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + I*Sqrt[Tan[c + d*x]]*Sqrt[
a + I*a*Tan[c + d*x]] + (I*Sqrt[a]*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]
*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[1 + I*Tan[c + d*x]]*
Sqrt[I*a*Tan[c + d*x]])))/d)/(3*a)
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.447$, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4080, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{3}\int \frac{3}{2}\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}(a(2A-iB)+a(2iA+3B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}(a(2A-iB)+a(2iA+3B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}(a(2A-iB)+a(2iA+3B)\tan(c+dx))dx+\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{2}\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left((14A-13iB)a^2+(18iA+19B)\tan(c+dx)a^2\right)dx-\frac{a^2(2A-3iI\right.$$
$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left((14A-13iB)a^2+(18iA+19B)\tan(c+dx)a^2\right)dx-\frac{a^2(2A-3iB)\right.$$
$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left((14A-13iB)a^2+(18iA+19B)\tan(c+dx)a^2\right)dx-\frac{a^2(2A-3iB)}{}\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 4080$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int -\frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(18iA+19B)-a^3(46A-45iB)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}}dx}{a}+\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c}}{d}\right.\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(18iA+19B)-a^3(46A-45iB)\tan(c+dx\right.}{\sqrt{\tan(c+dx)}}}{2a}\right.\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(18iA+19B)-a^3(46A-45iB)\tan(c+dx\right.}{\sqrt{\tan(c+dx)}}}{2a}\right.\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 4084$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{64a^3(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a^2(45B+46i}{2a}\right.\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{64a^3(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a^2(45B+46i\right.\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4027

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{-a^2(45B+46iA)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}d}{2a}\right.\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 218

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(64-64i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-a^2(4}{2a}\right.\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4082

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(64-64i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^4(45}{}}{2a}\right.\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 65

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(64-64i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^4(4}{}}{2a}\right.\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 216

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{2\sqrt[4]{-1}a^{7/2}(45B+46iA)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(6}{}}{2a}\right.\right.$$

$$\frac{iaB\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
((I/3)*a*B*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2))/d + (-1/2*(a^2
*(2*A - (3*I)*B)*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/d + (-1/2*
((2*(-1)^(1/4)*a^(7/2)*((46*I)*A + 45*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[T
an[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((64 - 64*I)*a^(7/2)*(I*A +
 B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]
]])/d)/a + (a^2*((18*I)*A + 19*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c +
d*x]])/d)/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(202) = 404$.

Time = 0.28 (sec) , antiderivative size = 653, normalized size of antiderivative = 2.59

method	result
derivativedivides	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a^2\left(16B\sqrt{-ia}\,\sqrt{ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\tan(dx+c)^2-52iB\sqrt{-ia}\,\sqrt{ia}\right.}{}$
default	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a^2\left(16B\sqrt{-ia}\,\sqrt{ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\tan(dx+c)^2-52iB\sqrt{-ia}\,\sqrt{ia}\right.}{}$
parts	$\frac{A\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a^2\left(8i\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{ia}\,a+18i\right.}{}$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/48/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(16*B*(-I*a)^(1/2)
*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-52*I*B*(-I
*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+54*
I*A*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a-108*I*A*(-I*a)^(1/2)*(I*a)^(1/2)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+24*A*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-48*I*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*
(I*a)^(1/2)*2^(1/2)*a+57*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a-114*B*(-I*a)^
(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+96*I*(-I*a)^(1/2)*
ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/
2)+a)/(I*a)^(1/2))*a-48*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a
-96*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)
^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2))/(-I*a)^(1/2)/(I*a)^(1/2)/(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(188) = 376$.

Time = 0.14 (sec) , antiderivative size = 932, normalized size of antiderivative = 3.70

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
1/48*(96*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4
*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*sqrt(-(-I*A^2 - 2*A*B +
I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2
*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)
*a^2)) - 96*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(4*I*d*x
+ 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-(sqrt(2)*sqrt(-(-I*A^2 - 2*A*
B + I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) - sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x
 + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A
- B)*a^2)) + 2*sqrt(2)*((66*I*A + 91*B)*a^2*e^(5*I*d*x + 5*I*c) - 2*(-54*I
*A - 49*B)*a^2*e^(3*I*d*x + 3*I*c) - 3*(-14*I*A - 13*B)*a^2*e^(I*d*x + I*c
))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) + 1)) + 3*sqrt((2116*I*A^2 + 4140*A*B - 2025*I*B^2)*a^5/
d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*((
46*I*A + 45*B)*a^2*e^(2*I*d*x + 2*I*c) + (46*I*A + 45*B)*a^2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)) + 2*sqrt((2116*I*A^2 + 4140*A*B - 2025*I*B^2)*a^5/d^2)*d*e^(I*d*x
 + I*c))*e^(-I*d*x - I*c)/((46*I*A + 45*B)*a^2)) - 3*sqrt((2116*I*A^2 + 41
40*A*B - 2025*I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x +...
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(ia\tan(dx+c)+a)^{\frac{5}{2}}\sqrt{\tan(dx+c)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(5/2)*sqrt(tan(d*x +
 c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeWarning, need to choose a branch for the root of a polynomial
 with par
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \sqrt{\tan\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
x)
```

output
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
 x)
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a}\,a^2\left(-2\sqrt{\tan\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,ai+\left(\int\frac{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}}{\tan(dx+c)}dx\right)adi$$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
(sqrt(a)*a**2*( - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a*i + int(
(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x),x)*a*d*i - int(
sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3,x)*b*d - int(s
qrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*a*d + 2*int(
sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*b*d*i + 4*i
nt(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i + int
(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*b*d))/d
```

3.171 $\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 206

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{(-1)^{3/4}a^{5/2}(20iA+23B)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d} + \frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{a^2(4A-7iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

output

```
-1/4*(-1)^(3/4)*a^(5/2)*(20*I*A+23*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)
^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+(4-4*I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a
^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d-1/4*a^2*(4*A-7*I*B)*ta
n(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/2*I*a*B*tan(d*x+c)^(1/2)*(a+I*
a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 8.13 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.48

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx =$$
$$-\frac{ia^2\left(-3\sqrt[4]{-1}aB\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}(-i+\tan(c+dx))-20i\sqrt{a}(A-iB)\text{arcsin}\right.}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c +
 d*x]],x]
```

output
```
((-1/4*I)*a^2*(-3*(-1)^(1/4)*a*B*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sq
rt[Tan[c + d*x]]*(-I + Tan[c + d*x]) - (20*I)*Sqrt[a]*(A - I*B)*ArcSinh[Sq
rt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[I*a*Tan[c + d*x]]*(-I + Tan[c + d*x]) +
 Sqrt[1 + I*Tan[c + d*x]]*(16*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*
Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*Tan[c + d*x]]*Sqrt[a +
 I*a*Tan[c + d*x]] + a*Tan[c + d*x]*(-I + Tan[c + d*x])*(4*A - (9*I)*B + 2
*B*Tan[c + d*x]))))/(d*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]*Sqrt[a
+ I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4A-iB)+a(4iA+7B)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4A-iB)+a(4iA+7B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4A-iB)+a(4iA+7B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{4}\left(\int \frac{\sqrt{i\tan(c+dx)a+a}(3(4A-3iB)a^2+(20iA+23B)\tan(c+dx)a^2)}{2\sqrt{\tan(c+dx)}}dx-\frac{a^2(4A-7iB)\sqrt{\tan(c+dx)}\sqrt{c}}{d}\right.$$

$$\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}(3(4A-3iB)a^2+(20iA+23B)\tan(c+dx)a^2)}{\sqrt{\tan(c+dx)}}dx-\frac{a^2(4A-7iB)\sqrt{\tan(c+dx)}}{d}\right.$$

$$\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}(3(4A-3iB)a^2+(20iA+23B)\tan(c+dx)a^2)}{\sqrt{\tan(c+dx)}}dx-\frac{a^2(4A-7iB)\sqrt{\tan(c+dx)}}{d}\right.$$

$$\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{4084}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(32a^2(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a(20A-23iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx\right.\right.$$
$$\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}\left(32a^2(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a(20A-23iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx\right.\right.$$
$$\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 4027

$$\frac{1}{4}\left(\frac{1}{2}\left(-\frac{64ia^4(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-a(20A-23iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+\right.\right.}{\sqrt{\tan(c+dx)}}$$
$$\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 218

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{(32-32i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-a(20A-23iB)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+\right.\right.}{\sqrt{\tan(c+dx)}}$$
$$\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 4082

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{(32-32i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^3(20A-23iB)\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(}{d}\right.\right.$$
$$\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 65

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{(32-32i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(20A-23iB)\int\frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}\right.\right.$$

$$\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(\frac{2\sqrt[4]{-1}a^{5/2}(20A-23iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(32-32i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\right.\right.$$

$$\frac{iaB\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]]
,x]
```

output
```
((I/2)*a*B*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2))/d + (((2*(-1)^
(1/4)*a^(5/2)*(20*A - (23*I)*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*
x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((32 - 32*I)*a^(5/2)*(A - I*B)*ArcTa
nh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/2
- (a^2*(4*A - (7*I)*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(164) = 328$.

Time = 0.23 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.74

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}\,a^2\left(-9iB\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a+18iB\sqrt{ia}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}\,a^2\left(-9iB\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a+18iB\sqrt{ia}\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}\,a^2\left(2i\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{ia}\,a-2\sqrt{a}\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/8/d*(a*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^(1/2)*a^2*(-9*I*B*ln(1/2*(2*I*
a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^
(1/2))*(-I*a)^(1/2)*a+18*I*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)-4*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*tan(d*x+c)+8*I*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a
+12*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*
a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-8*A*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-8*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a
)^(1/2)*a+16*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)+16*ln(1/2*(2*I*a*tan(d*x+
c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(
-I*a)^(1/2))/(I*a)^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 849 vs. $2(152) = 304$.

Time = 0.12 (sec) , antiderivative size = 849, normalized size of antiderivative = 4.12

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, al
gorithm="fricas")
```

output
```
-1/8*(16*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*
I*c) + d)*log((I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*d*e^(I*d*x
 + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 16*sqrt(2)*sqrt(-
(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log((-I*sqrt(
2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I
*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^
(-I*d*x - I*c)/((-I*A - B)*a^2)) + 2*sqrt(2)*((4*A - 11*I*B)*a^2*e^(3*I*d*
x + 3*I*c) + (4*A - 7*I*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + sqr
t((-400*I*A^2 - 920*A*B + 529*I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*
log((sqrt(2)*((20*I*A + 23*B)*a^2*e^(2*I*d*x + 2*I*c) + (20*I*A + 23*B)*a^
2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) + 1)) + 2*I*sqrt((-400*I*A^2 - 920*A*B + 529*I*B^2)*a^5/
d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/((20*I*A + 23*B)*a^2)) - sqrt((-4
00*I*A^2 - 920*A*B + 529*I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log((
sqrt(2)*((20*I*A + 23*B)*a^2*e^(2*I*d*x + 2*I*c) + (20*I*A + 23*B)*a^2)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(c + dx))^{5/2}(A + B\tan(c + dx))}{\sqrt{\tan(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeWarning, need to choose a branch for the root of a polynomial
 with par
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}{\sqrt{\tan(c+d\,x)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(1/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(1/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{\sqrt{a}\,a^2\Big(4\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,a-2\sqrt{\mathrm{ta}}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(sqrt(a)*a**2*(4*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a - 2*sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*i + 1)*b*i - int((sqrt(tan(c + d*x))*sqrt(ta
n(c + d*x)*i + 1))/tan(c + d*x),x)*a*d + int((sqrt(tan(c + d*x))*sqrt(tan(
c + d*x)*i + 1))/tan(c + d*x),x)*b*d*i - int(sqrt(tan(c + d*x))*sqrt(tan(c
 + d*x)*i + 1)*tan(c + d*x)**2,x)*b*d - 5*int(sqrt(tan(c + d*x))*sqrt(tan(
c + d*x)*i + 1)*tan(c + d*x),x)*a*d + 4*int(sqrt(tan(c + d*x))*sqrt(tan(c
+ d*x)*i + 1)*tan(c + d*x),x)*b*d*i))/d
```

3.172 $\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 196

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{(-1)^{3/4}a^{5/2}(2A-5iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{(4+4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} + \frac{a^2(2iA-B)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

output

```
(-1)^(3/4)*a^(5/2)*(2*A-5*I*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/
(a+I*a*tan(d*x+c))^(1/2))/d+(4+4*I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*
tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d+a^2*(2*I*A-B)*tan(d*x+c)^(1/2
)*(a+I*a*tan(d*x+c))^(1/2)/d-2*a*A*(a+I*a*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(
1/2)
```

Mathematica [A] (verified)

Time = 5.28 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.47

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{ia^2\left(-3\sqrt[4]{-1}aA\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}(-i+\tan(c+dx))+5\sqrt{a}(A-iB)\text{arcsinh}\left(\right.}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(3/2),x]
```

output
```
((-I)*a^2*(-3*(-1)^(1/4)*a*A*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[T
an[c + d*x]]*(-I + Tan[c + d*x]) + 5*Sqrt[a]*(A - I*B)*ArcSinh[Sqrt[I*a*Ta
n[c + d*x]]/Sqrt[a]]*Sqrt[I*a*Tan[c + d*x]]*(-I + Tan[c + d*x]) + Sqrt[1 +
 I*Tan[c + d*x]]*(4*Sqrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*
x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c
 + d*x]] + a*(-I + Tan[c + d*x])*(2*A + B*Tan[c + d*x]))))/(d*Sqrt[1 + I*T
an[c + d*x]]*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4076, 27, 3042, 4077, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$2\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4iA+B)+a(2A+iB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4iA+B)+a(2A+iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4iA+B)+a(2A+iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4077}$$

$$\int \frac{\sqrt{i\tan(c+dx)a+a}\,(3a^2(2iA+B)-a^2(2A-5iB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx+\frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}\,(3a^2(2iA+B)-a^2(2A-5iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}\,(3a^2(2iA+B)-a^2(2A-5iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4084}$$

$$\frac{1}{2}\left(8a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx - a(5B+2iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx\right)+$$
$$\frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(8a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx - a(5B+2iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx\right)+$$
$$\frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 4027

$$\frac{1}{2}\left(-\frac{16ia^4(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d} - a(5B+2iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+}}{\sqrt{\tan(c+dx)}}\right.$$
$$\frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 218

$$\frac{1}{2}\left(\frac{(8-8i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - a(5B+2iA)\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+}}{\sqrt{\tan(c+dx)}}\right.$$
$$\frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 4082

$$\frac{1}{2}\left(\frac{(8-8i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{a^3(5B+2iA)\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}\right)$$
$$\frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 65

$$\frac{1}{2}\left(\frac{(8-8i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(5B+2iA)\int\frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}\right)+$$
$$\frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(\frac{2\sqrt[4]{-1}a^{5/2}(5B+2iA)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(8-8i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\right)+$$
$$\frac{a^2(-B+2iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2)
,x]
```

output
```
((2*(-1)^(1/4)*a^(5/2)*((2*I)*A + 5*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan
[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((8 - 8*I)*a^(5/2)*(I*A + B)*
ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/
d)/2 + (a^2*((2*I)*A - B)*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
 - (2*a*A*(a + I*a*Tan[c + d*x])^(3/2))/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(161) = 322$.

Time = 0.21 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.88

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(6iA\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a\tan(dx+c)-2i\sqrt{ia}\,\sqrt{2}\,\ln(\ldots}{\ldots}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(6iA\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a\tan(dx+c)-2i\sqrt{ia}\,\sqrt{2}\,\ln(\ldots}{\ldots}$
parts	$\frac{A\left(-i\sqrt{ia}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)a-i\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(d\ldots}}{2\ldots}\right.\right.}{\ldots}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/2/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(6*I*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2
)*a*tan(d*x+c)-2*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(
d*x+c)+3*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)-2*B*(I*a)^(1/2)*(-
I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+4*I*ln(1/2*(2*
I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a
)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)-2*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(
-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan
(d*x+c)+I))*a*tan(d*x+c)-4*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(
1/2)*(-I*a)^(1/2)-4*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c))/tan(d*x+
c)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(150) = 300$.

Time = 0.10 (sec) , antiderivative size = 857, normalized size of antiderivative = 4.37

$$\int \frac{(a + ia\tan(c + dx))^{5/2}(A + B\tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, al
gorithm="fricas")
```

output
```
-1/2*(4*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*
I*c) - d)*log((sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*d*e^(I*d*x
+ I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 4*sqrt(2)*sqrt(-(-
I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(-(sqrt(2)*
sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) - sqrt(2)*((-I*A
 - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-
I*d*x - I*c)/((-I*A - B)*a^2)) + 2*sqrt(2)*((2*I*A + B)*a^2*e^(3*I*d*x + 3
*I*c) + (2*I*A - B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt((4*I*
A^2 + 20*A*B - 25*I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log((sqrt(2)
*((2*I*A + 5*B)*a^2*e^(2*I*d*x + 2*I*c) + (2*I*A + 5*B)*a^2)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1)) + 2*sqrt((4*I*A^2 + 20*A*B - 25*I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c))
*e^(-I*d*x - I*c)/((2*I*A + 5*B)*a^2)) - sqrt((4*I*A^2 + 20*A*B - 25*I*B^2
)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log((sqrt(2)*((2*I*A + 5*B)*a^2*e^(
2*I*d*x + 2*I*c) + (2*I*A + 5*B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - 2*sqrt((4*...
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}\,(A+B\tan{(c+dx)})}{\tan^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(5/2)*(A + B*tan(c + d*x))/tan(c + d*x)
**(3/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeWarning, need to choose a branch for the root of a polynomial
 with par
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(3/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(3/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\,a^2\Big(2\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,ai+4\sqrt{\mathrm{ta}}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
(sqrt(a)*a**2*(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a*i + 4*sqrt(
tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*b + int((sqrt(tan(c + d*x))*sqrt(ta
n(c + d*x)*i + 1))/tan(c + d*x)**2,x)*a*d + int((sqrt(tan(c + d*x))*sqrt(t
an(c + d*x)*i + 1))/tan(c + d*x),x)*a*d*i - int((sqrt(tan(c + d*x))*sqrt(t
an(c + d*x)*i + 1))/tan(c + d*x),x)*b*d - 2*int(sqrt(tan(c + d*x))*sqrt(ta
n(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i - 5*int(sqrt(tan(c + d*x))*sqrt(ta
n(c + d*x)*i + 1)*tan(c + d*x),x)*b*d))/d
```

3.173 $\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 190

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(-1)^{3/4}a^{5/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2(2iA+B)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

output
```
2*(-1)^(3/4)*a^(5/2)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*t
an(d*x+c))^(1/2))/d+(-4+4*I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x
+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d-2*a^2*(2*I*A+B)*(a+I*a*tan(d*x+c))^(
1/2)/d/tan(d*x+c)^(1/2)-2/3*a*A*(a+I*a*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(3/2
)
```

Mathematica [A] (verified)

Time = 6.74 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.63

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx =$$

$$-\frac{ia^2\left(-3(-1)^{3/4}a(5A-3iB)\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)(-i+\tan(c+dx))-\frac{15i(iA+B)\text{ar}}{}\right.}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(5/2),x]
```

output
```
((-1/3*I)*a^2*(-3*(-1)^(3/4)*a*(5*A - (3*I)*B)*ArcSinh[(-1)^(1/4)*Sqrt[Tan
[c + d*x]]]*Tan[c + d*x]^(3/2)*(-I + Tan[c + d*x]) - ((15*I)*(I*A + B)*Arc
Sinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*(I*a*Tan[c + d*x])^(3/2)*(-I + Tan[c
+ d*x]))/Sqrt[a] + 2*Sqrt[1 + I*Tan[c + d*x]]*((6*Sqrt[2]*(I*A + B)*ArcTan
h[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*(I*a*Tan[c
+ d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/a + a*(-I + Tan[c + d*x])*(A + (
(7*I)*A + 3*B)*Tan[c + d*x]))))/(d*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(
3/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{3}\int \frac{3(i\tan(c+dx)a+a)^{3/2}(a(2iA+B)+iaB\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(2iA+B)+iaB\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(2iA+B)+iaB\tan(c+dx))}{\tan(c+dx)^{3/2}}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4076}$$

$$2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)}{2\sqrt{\tan(c+dx)}}dx-\frac{2a^2(B+2iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{\sqrt{i\tan(c+dx)a+a}\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(B+2iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sqrt{i\tan(c+dx)a+a}\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(B+2iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4084}$$

$$-4a^2(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-iaB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(B+2iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-4a^2(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-iaB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(B+2iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4027}$$

$$\frac{8ia^4(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-iaB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(B+2iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{218}$$

$$-iaB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(B+2iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4082}$$

$$-\frac{ia^3B\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(B+2iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 65$$

$$-\frac{2ia^3B\int \frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(B+2iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 216$$

$$-\frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{2(-1)^{3/4}a^{5/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(B+2iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2)
,x]
```

output
```
(2*(-1)^(3/4)*a^(5/2)*B*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqr
t[a + I*a*Tan[c + d*x]]])/d - ((4 - 4*I)*a^(5/2)*(A - I*B)*ArcTanh[((1 + I
)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^2*((2*
I)*A + B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]) - (2*a*A*(a +
 I*a*Tan[c + d*x])^(3/2))/(3*d*Tan[c + d*x]^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(154) = 308$.

Time = 0.23 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.25

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-9iB\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a\tan(dx+c)^2+3i\sqrt{ia}\,\sqrt{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-9iB\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right)\sqrt{-ia}\,a\tan(dx+c)^2+3i\sqrt{ia}\,\sqrt{}$
parts	$-\frac{A\sqrt{a(1+i\tan(dx+c))}\,a^2\left(3i\sqrt{ia}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)^2a-3\sqrt{}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_R
ETURNVERBOSE)
```

output

```
-1/3/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2/tan(d*x+c)^(3/2)*(-9*I*B*ln(1/2*(2*I
*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)
^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2+3*I*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(ta
n(d*x+c)+I))*tan(d*x+c)^2*a+14*I*A*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+12*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*
tan(d*x+c)^2-3*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*tan(d*x+c)^2
*a+6*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I
*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2+6*B*(I*a)^(1/2)*(-I*
a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+6*ln(1/2*(2*I*a*
tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1
/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2+2*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
*(I*a)^(1/2)*(-I*a)^(1/2))/(I*a)^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(144) = 288$.

Time = 0.13 (sec) , antiderivative size = 846, normalized size of antiderivative = 4.45

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, al
gorithm="fricas")
```

output
```
1/6*(12*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I
*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log((I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I
*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*
I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*
a^2)) - 12*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(4*I*d*x +
4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log((-I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B
 - I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x
+ 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A -
 B)*a^2)) + 4*sqrt(2)*((8*A - 3*I*B)*a^2*e^(5*I*d*x + 5*I*c) + 2*A*a^2*e^(
3*I*d*x + 3*I*c) - 3*(2*A - I*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))
 + 3*sqrt(4*I*B^2*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c
) + d)*log((sqrt(2)*(B*a^2*e^(2*I*d*x + 2*I*c) + B*a^2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1
)) + I*sqrt(4*I*B^2*a^5/d^2)*d*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(B*a^2))
- 3*sqrt(4*I*B^2*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c)
 + d)*log((sqrt(2)*(B*a^2*e^(2*I*d*x + 2*I*c) + B*a^2)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +...
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(ia(\tan(c+dx)-i))^{\frac{5}{2}}\,(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(5/2)*(A + B*tan(c + d*x))/tan(c + d*x)
**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeWarning, need to choose a branch for the root of a polynomial
 with par
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}{\tan(c+d\,x)^{5/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(5/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(5/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{a}\,a^2\Big(3\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
(2*sqrt(a)*a**2*(3*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x
)**2*b*i + 5*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i
+ 3*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - sqrt(tan(
c + d*x))*sqrt(tan(c + d*x)*i + 1)*a + 6*int((sqrt(tan(c + d*x))*sqrt(tan(
c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**2*a*d*i + 3*int((sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**2*b
*d - 3*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*tan
(c + d*x)**2*b*d*i))/(3*tan(c + d*x)**2*d)
```

3.174 $\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 185

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{(4+4i)a^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2(8iA+5B)\sqrt{a+ia\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{2a^2(38A-35iB)\sqrt{a+ia\tan(c+dx)}}{15d\sqrt{\tan(c+dx)}} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

output

```
(-4-4*I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan
(d*x+c))^(1/2))/d-2/15*a^2*(8*I*A+5*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+
c)^(3/2)+2/15*a^2*(38*A-35*I*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2
)-2/5*a*A*(a+I*a*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(5/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 447 vs. 2(185) = 370.

Time = 7.13 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.42

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{2A(a+ia\tan(c+dx))^{5/2}}{5d\tan^{\frac{5}{2}}(c+dx)} + (iA+B)\left(\frac{4i\sqrt{2}a^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{4ia^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{ia\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+i\tan(c}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan}}\right.$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(7/2),x]
```

output
```
(-2*A*(a + I*a*Tan[c + d*x])^(5/2))/(5*d*Tan[c + d*x]^(5/2)) + (I*A + B)*(
((4*I)*Sqrt[2]*a^2*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*T
an[c + d*x]]]*Sqrt[I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]) - ((4*I)*a^(5
/2)*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[
I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (5*
(-1)^(3/4)*a^2*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[a + I*a*Tan[c +
 d*x]])/(d*Sqrt[1 + I*Tan[c + d*x]]) - (2*a^2*Sqrt[a + I*a*Tan[c + d*x]])/
(3*d*Tan[c + d*x]^(3/2)) - (((14*I)/3)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*
Sqrt[Tan[c + d*x]]) - (I*a^(3/2)*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*S
qrt[I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[1 + I*Tan[c + d*
x]]*Sqrt[Tan[c + d*x]]))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$\big\downarrow$ 3042

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$\big\downarrow$ 4076

$$\frac{2}{5}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(8iA+5B)-a(2A-5iB)\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(8iA+5B)-a(2A-5iB)\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(8iA+5B)-a(2A-5iB)\tan(c+dx))}{\tan(c+dx)^{5/2}}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 4076

$$\frac{1}{5}\left(\frac{2}{3}\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((38A-35iB)a^2+(22iA+25B)\tan(c+dx)a^2\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a^2(5B+8iA)\sqrt{a+ia\,\text{ta}}}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{5}\left(-\frac{1}{3}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((38A-35iB)a^2+(22iA+25B)\tan(c+dx)a^2\right)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a^2(5B+8iA)\sqrt{a+ia\,\text{ta}}}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(-\frac{1}{3}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((38A-35iB)a^2+(22iA+25B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{3/2}}dx-\frac{2a^2(5B+8iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4081$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^2(38A-35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2\int \frac{30a^3(iA+B)\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}\right)-\frac{2a^2(5B+8iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^2(38A-35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-60a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2a^2(5B+8iA)\sqrt{a}}{3d\tan^{\frac{3}{2}}(}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^2(38A-35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-60a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx\right)-\frac{2a^2(5B+8iA)\sqrt{a}}{3d\tan^{\frac{3}{2}}(}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4027$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{120ia^4(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}+\frac{2a^2(38A-35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a^2(5B}{}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 218$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^2(38A-35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{(60-60i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\right)-\frac{2a^2(5.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2)
,x]
```

output
```
(-2*a*A*(a + I*a*Tan[c + d*x])^(3/2))/(5*d*Tan[c + d*x]^(5/2)) + ((-2*a^2*
((8*I)*A + 5*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) + (((
-60 + 60*I)*a^(5/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])
/Sqrt[a + I*a*Tan[c + d*x]]])/d + (2*a^2*(38*A - (35*I)*B)*Sqrt[a + I*a*Ta
n[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(152) = 304$.

Time = 0.22 (sec) , antiderivative size = 709, normalized size of antiderivative = 3.83

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-76A\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+60iA\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a}}{}\right.\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-76A\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+60iA\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a}}{}\right.\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(dx+c))}\,a^2\left(15i\sqrt{ia}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^3+15\sqrt{}\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/15/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2/tan(d*x+c)^(5/2)*(-76*A*(I*a)^(1/2)
*(-I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+60*I*A*ln
(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)
+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3-15*I*(I*a)^(1/2)*2^(1/2)*ln(-
(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan
(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+70*I*B*(I*a)^(1/2)*(-I*a)^(1/2)*ta
n(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+60*B*ln(1/2*(2*I*a*tan(d*
x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(
-I*a)^(1/2)*a*tan(d*x+c)^3+30*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+
c)^3-15*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+22
*I*A*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)-30*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(
I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3+10*B*(I*a)^(1/2)*(-
I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+6*A*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/(I*a)^(1/2)/(-I*a)^(
1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(141) = 282$.

Time = 0.13 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.29

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, al
gorithm="fricas")
```

output
```
2/15*(15*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(6*I*d*x + 6
*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log((sqrt(2
)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I
*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^
(-I*d*x - I*c)/((-I*A - B)*a^2)) - 15*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^
2)*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*
d*x + 2*I*c) - d)*log(-(sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*d*
e^(I*d*x + I*c) - sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)
*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 2*sqrt(2)
*(2*(-13*I*A - 10*B)*a^2*e^(7*I*d*x + 7*I*c) + 3*(3*I*A + 5*B)*a^2*e^(5*I*
d*x + 5*I*c) + 20*(I*A + B)*a^2*e^(3*I*d*x + 3*I*c) + 15*(-I*A - B)*a^2*e^
(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d
*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeWarning, need to choose a branch for the root of a polynomial
 with par
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}{\tan\left(c+d\,x\right)^{7/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(7/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(7/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{a}\,a^2\Big(-22\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(c$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
(2*sqrt(a)*a**2*( - 22*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c +
 d*x)**2*a + 25*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**
2*b*i - 11*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i -
5*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 3*sqrt(tan(
c + d*x))*sqrt(tan(c + d*x)*i + 1)*a - 30*int((sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**3*a*d + 30*int((sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**3*b
*d*i))/(15*tan(c + d*x)**3*d)
```

3.175 $\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 231

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -\frac{(4+4i)a^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2(10iA+7B)\sqrt{a+ia\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{105d\tan^{\frac{3}{2}}(c+dx)} + \frac{4a^2(130iA+133B)\sqrt{a+ia\tan(c+dx)}}{105d\sqrt{\tan(c+dx)}} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

output

```
(-4-4*I)*a^(5/2)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan
(d*x+c))^(1/2))/d-2/35*a^2*(10*I*A+7*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x
+c)^(5/2)+2/105*a^2*(80*A-77*I*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3
/2)+4/105*a^2*(130*I*A+133*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-
2/7*a*A*(a+I*a*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 16.50 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.57

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \frac{4\sqrt{2}e^{-2ic}\sqrt{e^{idx}}\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}\left(e^{i(c+dx)}\sqrt{-1+e^{2i(c+}\right.}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(9/2),x]
```

output
```
(4*Sqrt[2]*Sqrt[E^(I*d*x)]*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/(1 + E^(
(2*I)*(c + d*x)))]*(E^(I*(c + d*x))*Sqrt[-1 + E^((2*I)*(c + d*x))]*((7*I)*
B*(-15 + 50*E^((2*I)*(c + d*x)) - 61*E^((4*I)*(c + d*x)) + 26*E^((6*I)*(c
+ d*x))) - 5*A*(-21 + 70*E^((2*I)*(c + d*x)) - 77*E^((4*I)*(c + d*x)) + 40
*E^((6*I)*(c + d*x)))) + 105*(A - I*B)*(-1 + E^((2*I)*(c + d*x)))^4*Log[E^
(I*(c + d*x)) + Sqrt[-1 + E^((2*I)*(c + d*x))]])*(a + I*a*Tan[c + d*x])^(5
/2)*(A + B*Tan[c + d*x]))/(105*d*E^((2*I)*c)*(-1 + E^((2*I)*(c + d*x)))^(9
/2)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sec[c + d*x]^(7/2)*(Co
s[d*x] + I*Sin[d*x])^(5/2)*(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4081, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{7}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(10iA+7B)-a(4A-7iB)\tan(c+dx))}{2\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(10iA+7B)-a(4A-7iB)\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(10iA+7B)-a(4A-7iB)\tan(c+dx))}{\tan(c+dx)^{7/2}}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{7}\left(\frac{2}{5}\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((80A-77iB)a^2+3(20iA+21B)\tan(c+dx)a^2\right)}{2\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2a^2(7B+10iA)\sqrt{a+ia}}{5d\tan^{\frac{5}{2}}(c+d}\right.$$
$$-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(-\frac{1}{5}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((80A-77iB)a^2+3(20iA+21B)\tan(c+dx)a^2\right)}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2a^2(7B+10iA)\sqrt{a+ia}}{5d\tan^{\frac{5}{2}}(c+d}\right.$$
$$-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(-\frac{1}{5}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((80A-77iB)a^2+3(20iA+21B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{5/2}}dx-\frac{2a^2(7B+10iA)\sqrt{a+ia}}{5d\tan^{\frac{5}{2}}(c+d}\right.$$
$$-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\int\frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(130iA+133B)-a^3(80A-77iB)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}\right)-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\int\frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(130iA+133B)-a^3(80A-77iB)\tan(c+dx)\right)}{\tan(c+dx)^{3/2}}dx}{3a}\right)-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{ 4081}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\left(\frac{2\int-\frac{105a^4(A-iB)\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^3(133B+130iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right.}{3a}\right.\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\left(-210a^3(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(133B+130iA)\sqrt{a+ia}}{d\sqrt{\tan(c+dx)}}\right.}{3a}\right.\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\left(-210a^3(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(133B+130iA)\sqrt{a+ia}}{d\sqrt{\tan(c+dx)}}\right.}{3a}\right.\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4027}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\left(\frac{420ia^5(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^3(133B+130iA\ldots}{d\sqrt{\tan\ldots}}\right.}{3a}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \ 218$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\left(-\frac{(210-210i)a^{7/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(133B+1\ldots}{d\ldots}\right.}{3a}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/2)
,x]
```

output
```
(-2*a*A*(a + I*a*Tan[c + d*x])^(3/2))/(7*d*Tan[c + d*x]^(7/2)) + ((-2*a^2*
((10*I)*A + 7*B)*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) + ((
2*a^2*(80*A - (77*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2
)) - (2*(((-210 + 210*I)*a^(7/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[T
an[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^3*((130*I)*A + 133*B)*
Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])))/(3*a))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(190) = 380$.

Time = 0.27 (sec) , antiderivative size = 798, normalized size of antiderivative = 3.45

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(532B\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^3\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+520iA\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^3\sqrt{}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(532B\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^3\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+520iA\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)^3\sqrt{}\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(21i\sqrt{ia}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^4+84i\,\mathrm{l}\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/105/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2/tan(d*x+c)^(7/2)*(532*B*(I*a)^(1/2)
*(-I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+520*I*A*(
I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
+105*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+160
*A*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)-90*I*A*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(
-I*a)^(1/2)+420*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^4-154*I*B*(
I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
-105*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+210*I
*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1
/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^4+210*ln(1/2*(2*I*a*tan(d*x+
c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I
*a)^(1/2)*a*tan(d*x+c)^4-420*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x
+c)^4-42*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*
tan(d*x+c)-30*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(
1/2))/(I*a)^(1/2)/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(177) = 354$.

Time = 0.09 (sec) , antiderivative size = 657, normalized size of antiderivative = 2.84

$$\int \frac{(a + ia\tan(c + dx))^{5/2}(A + B\tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, al
gorithm="fricas")
```

output
```
-2/105*(105*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(8*I*d*x +
 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d
*x + 2*I*c) + d)*log((I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*d*e
^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*
a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(
e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 105*sqrt(2
)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6
*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)*l
og((-I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) +
sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) + 2*sqrt(2)*(2*(100*A - 91*I*
B)*a^2*e^(9*I*d*x + 9*I*c) - 5*(37*A - 49*I*B)*a^2*e^(7*I*d*x + 7*I*c) - 7
*(5*A - 11*I*B)*a^2*e^(5*I*d*x + 5*I*c) + 245*(A - I*B)*a^2*e^(3*I*d*x + 3
*I*c) - 105*(A - I*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(8*I
*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^
(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeWarning, need to choose a branch for the root of a polynomial
 with par
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}{\tan\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(9/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(9/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx=\frac{2\sqrt{a}\,a^2\left(-160\sqrt{\tan\left(dx+c\right)}\,\sqrt{\tan\left(dx+c\right)i+1}\,\tan\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)
```

output
```
(2*sqrt(a)*a**2*( - 160*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c
+ d*x)**3*a*i - 154*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*
x)**3*b + 80*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*a
 - 77*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i - 45
*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 21*sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 15*sqrt(tan(c + d*x)
)*sqrt(tan(c + d*x)*i + 1)*a - 210*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*
x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**4*a*d*i - 210*int((sqrt(tan(c
+ d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**4*b*d))
/(105*tan(c + d*x)**4*d)
```

3.176 $\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 277

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \frac{(4+4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{2a^2(4iA+3B)\sqrt{a+ia\tan(c+dx)}}{21d\tan^{\frac{7}{2}}(c+dx)} + \frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{105d\tan^{\frac{5}{2}}(c+dx)} + \frac{8a^2(59iA+60B)\sqrt{a+ia\tan(c+dx)}}{315d\tan^{\frac{3}{2}}(c+dx)} - \frac{8a^2(197A-195iB)\sqrt{a+ia\tan(c+dx)}}{315d\sqrt{\tan(c+dx)}} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

output
```
(4+4*I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))/d-2/21*a^2*(4*I*A+3*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c
)^(7/2)+2/105*a^2*(46*A-45*I*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2
)+8/315*a^2*(59*I*A+60*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)-8/31
5*a^2*(197*A-195*I*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-2/9*a*A*
(a+I*a*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(9/2)
```

Mathematica [A] (verified)

Time = 16.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.89

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \frac{a^2\left(\frac{1260(A-iB)e^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\text{arctanh}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right)}{\sqrt{-\frac{i\left(-1+e^{2i(c+dx)}\right)}{1+e^{2i(c+dx)}}}}\right.}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(11/2),x]
```

output
```
(a^2*((1260*(A - I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))]*ArcTanh[E^(I*(c + d*x
))/Sqrt[-1 + E^((2*I)*(c + d*x))]])/(E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((
2*I)*(c + d*x))))/(1 + E^((2*I)*(c + d*x)))]) + (Csc[2*(c + d*x)]^2*(-2331
*A + (2205*I)*B + 12*(251*A - (260*I)*B)*Cos[2*(c + d*x)] + (-961*A + (915
*I)*B)*Cos[4*(c + d*x)] + (282*I)*A*Sin[2*(c + d*x)] + 390*B*Sin[2*(c + d*
x)] - (331*I)*A*Sin[4*(c + d*x)] - 285*B*Sin[4*(c + d*x)]))/Tan[c + d*x]^(
5/2))*Sqrt[a + I*a*Tan[c + d*x]])/(315*d)
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.09, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{11/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{9}\int \frac{3(i\tan(c+dx)a+a)^{3/2}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{2\tan^{\frac{9}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{\tan(c+dx)^{9/2}}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{3}\left(\frac{2}{7}\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((46A-45iB)a^2+(38iA+39B)\tan(c+dx)a^2\right)}{2\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2a^2(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{1}{7}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((46A-45iB)a^2+(38iA+39B)\tan(c+dx)a^2\right)}{\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2a^2(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-\frac{1}{7}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((46A-45iB)a^2+(38iA+39B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{7/2}}dx-\frac{2a^2(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right)-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2\int\frac{2\sqrt{i\tan(c+dx)a+a}(a^3(59iA+60B)-a^3(46A-45iB)\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}\right)-\ldots\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\int\frac{\sqrt{i\tan(c+dx)a+a}(a^3(59iA+60B)-a^3(46A-45iB)\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}\right)-\ldots\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\int\frac{\sqrt{i\tan(c+dx)a+a}(a^3(59iA+60B)-a^3(46A-45iB)\tan(c+dx))}{\tan(c+dx)^{5/2}}dx}{5a}\right)-\ldots\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 4081

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(\frac{2\int-\frac{\sqrt{i\tan(c+dx)a+a}\left((197A-195iB)a^4+2(59iA+60B)\tan(c+dx)a^4\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^3(\ldots}{\ldots}\right.}{5a}\ldots\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((197A-195iB)a^4+2(59iA+60B)\tan(c+dx)a^4\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^3(6\cdots}{\cdots}\right.}{5a}\cdots\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((197A-195iB)a^4+2(59iA+60B)\tan(c+dx)a^4\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a^3(6\cdots}{\cdots}\right.}{5a}\cdots\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 4081

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{\frac{2\int \frac{315a^5(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^4(197A-195iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\cdots\right.}{5a}\cdots\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{315a^4(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^4(197A-195iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\cdots\right.}{5a}\cdots\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{315a^4(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^4(197A-195iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\right.}{5a}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 4027

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{-\frac{630ia^6(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^4(197A-195iB)}{d\sqrt{\tan}}}{3a}\right.}{5a}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 218

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{2a^3(60B+59iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{(315-315i)a^{9/2}(B+iA)\text{arctanh}\left(\frac{(1}{}\right.}{d}\right.}{5a}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(11/2
),x]
```

output
```
(-2*a*A*(a + I*a*Tan[c + d*x])^(3/2))/(9*d*Tan[c + d*x]^(9/2)) + ((-2*a^2*
((4*I)*A + 3*B)*Sqrt[a + I*a*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(7/2)) + ((2
*a^2*(46*A - (45*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)
) - (4*((-2*a^3*((59*I)*A + 60*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c +
 d*x]^(3/2)) - (((315 - 315*I)*a^(9/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*
Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^4*(197*A - (195*
I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a)))/(5*a))/7
)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 886 vs. 2(228) = 456.

Time = 0.25 (sec) , antiderivative size = 887, normalized size of antiderivative = 3.20

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-1576A\tan(dx+c)^4\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\sqrt{ia}\sqrt{-ia}+1560iB\tan(dx+c)^4\sqrt{a\tan(\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-1576A\tan(dx+c)^4\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\sqrt{ia}\sqrt{-ia}+1560iB\tan(dx+c)^4\sqrt{a\tan(\right.}{}$
parts	Expression too large to display

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x,method=_
RETURNVERBOSE)
```

output
```
1/315/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2/tan(d*x+c)^(9/2)*(-1576*A*tan(d*x+c
)^4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+1560*I*
B*tan(d*x+c)^4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1
/2)-190*I*A*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(
-I*a)^(1/2)+480*B*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)+630*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^5+1
260*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*
a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^5+1260*I*A*ln(1/2*(2*I*
a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^
(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^5-315*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)
*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(t
an(d*x+c)+I))*a*tan(d*x+c)^5+276*A*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+472*I*A*tan(d*x+c)^3*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-630*ln(1/2*(2*I*a*tan(d*x+c
)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*
a)^(1/2)*a*tan(d*x+c)^5-270*I*B*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-315*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(
tan(d*x+c)+I))*a*tan(d*x+c)^5-90*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(213) = 426.

Time = 0.12 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.61

$$\int \frac{(a + ia\tan(c + dx))^{5/2}(A + B\tan(c + dx))}{\tan^{\frac{11}{2}}(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, a
lgorithm="fricas")
```

output
```
-2/315*(315*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(10*I*d*x
 + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(
4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)*log((sqrt(2)*sqrt(-(-I*A^2
 - 2*A*B + I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(
2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
(-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/
((-I*A - B)*a^2)) - 315*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d
*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c
) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) - d)*log(-(sqrt(2)*
sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) - sqrt(2)*((-I*A
 - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-
I*d*x - I*c)/((-I*A - B)*a^2)) + 2*sqrt(2)*(2*(323*I*A + 300*B)*a^2*e^(11*
I*d*x + 11*I*c) + 77*(-13*I*A - 15*B)*a^2*e^(9*I*d*x + 9*I*c) + 18*(38*I*A
 + 25*B)*a^2*e^(7*I*d*x + 7*I*c) + 42*(23*I*A + 20*B)*a^2*e^(5*I*d*x + 5*I
*c) + 1050*(-I*A - B)*a^2*e^(3*I*d*x + 3*I*c) + 315*(I*A + B)*a^2*e^(I*d*x
 + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1)))/(d*e^(10*I*d*x + 10*I*c) - 5*d*e^(8*I*d*x +
 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) - 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I
*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(c + dx))^{5/2}(A + B\tan(c + dx))}{\tan^{\frac{11}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, a
lgorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeWarning, need to choose a branch for the root of a polynomial
 with par
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}{\tan(c+d\,x)^{11/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(11/
2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(11/
2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\sqrt{a}\,a^2\Big(-368\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x)
```

output
```
(2*sqrt(a)*a**2*( - 368*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c
+ d*x)**4*a + 360*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)
**4*b*i - 184*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*
a*i - 180*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*b +
138*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*a - 135*sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i - 95*sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 45*sqrt(tan(c + d*
x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 35*sqrt(tan(c + d*x))*sqrt(t
an(c + d*x)*i + 1)*a - 630*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1
))/tan(c + d*x)**3,x)*tan(c + d*x)**5*a*d*i - 630*int((sqrt(tan(c + d*x))*
sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**3,x)*tan(c + d*x)**5*b*d))/(315*ta
n(c + d*x)**5*d)
```

3.177 $\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 323

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx =& -\frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} \\ &- \frac{2a^2(14iA+11B)\sqrt{a+ia\tan(c+dx)}}{99d\tan^{\frac{9}{2}}(c+dx)} \\ &+ \frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{693d\tan^{\frac{7}{2}}(c+dx)} \\ &+ \frac{4a^2(250iA+253B)\sqrt{a+ia\tan(c+dx)}}{1155d\tan^{\frac{5}{2}}(c+dx)} \\ &- \frac{8a^2(655A-649iB)\sqrt{a+ia\tan(c+dx)}}{3465d\tan^{\frac{3}{2}}(c+dx)} \\ &- \frac{8a^2(2155iA+2167B)\sqrt{a+ia\tan(c+dx)}}{3465d\sqrt{\tan(c+dx)}} - \frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}\end{aligned}$$

output
```
(-4+4*I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan
(d*x+c))^(1/2))/d-2/99*a^2*(14*I*A+11*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*
x+c)^(9/2)+2/693*a^2*(212*A-209*I*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)
^(7/2)+4/1155*a^2*(250*I*A+253*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5
/2)-8/3465*a^2*(655*A-649*I*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)
-8/3465*a^2*(2155*I*A+2167*B)*(a+I*a*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)-
2/11*a*A*(a+I*a*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(11/2)
```

Mathematica [A] (verified)

Time = 18.22 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.02

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \frac{4\sqrt{2}a^2(iA+B)e^{-i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\mathrm{ar}}{d\sqrt{-\frac{i(-1+e^{2i(c+dx)})}{1+e^{2i(c+dx)}}}}$$
$$-\frac{a^2\csc^3(c+dx)\sec^2(c+dx)(66(95A-47iB)\cos(c+dx)+(-5225A+6743iB)\cos(3(c+dx))+3995A}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]
^(13/2),x]
```

output
```
(4*Sqrt[2]*a^2*(I*A + B)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[(a*E^((2*I)*(
c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^
((2*I)*(c + d*x))]])/(d*E^(I*(c + d*x))*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x
))))/(1 + E^((2*I)*(c + d*x)))]) - (a^2*Csc[c + d*x]^3*Sec[c + d*x]^2*(66*
(95*A - (47*I)*B)*Cos[c + d*x] + (-5225*A + (6743*I)*B)*Cos[3*(c + d*x)] +
 3995*A*Cos[5*(c + d*x)] - (3641*I)*B*Cos[5*(c + d*x)] + (84810*I)*A*Sin[c
 + d*x] + 84414*B*Sin[c + d*x] - (42185*I)*A*Sin[3*(c + d*x)] - 43703*B*Si
n[3*(c + d*x)] + (10925*I)*A*Sin[5*(c + d*x)] + 10571*B*Sin[5*(c + d*x)])*
Sqrt[a + I*a*Tan[c + d*x]])/(27720*d*Tan[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.10, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.553$, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4081, 27, 3042, 4081, 25, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{13/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{11}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(14iA+11B)-a(8A-11iB)\tan(c+dx))}{2\tan^{\frac{11}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(14iA+11B)-a(8A-11iB)\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(14iA+11B)-a(8A-11iB)\tan(c+dx))}{\tan(c+dx)^{11/2}}dx-\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{11}\left(\frac{2}{9}\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((212A-209iB)a^2+(184iA+187B)\tan(c+dx)a^2\right)}{2\tan^{\frac{9}{2}}(c+dx)}dx-\frac{2a^2(11B+14iA)\sqrt{}}{9d\tan^{\frac{9}{2}}(}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\left(-\frac{1}{9}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((212A-209iB)a^2+(184iA+187B)\tan(c+dx)a^2\right)}{\tan^{\frac{9}{2}}(c+dx)}dx-\frac{2a^2(11B+14iA)\sqrt{}}{9d\tan^{\frac{9}{2}}(}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(-\frac{1}{9}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((212A-209iB)a^2+(184iA+187B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{9/2}}dx-\frac{2a^2(11B+14iA)\sqrt{}}{9d\tan^{\frac{9}{2}}(}\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{2\int \frac{3\sqrt{i\tan(c+dx)a+a}\left(a^3(250iA+253B)-a^3(212A-209iB)\tan(c+dx)\right)}{\tan^{\frac{7}{2}}(c+dx)}d}{7a}\right.\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(250iA+253B)-a^3(212A-209iB)\tan(c+dx)\right)}{\tan^{\frac{7}{2}}(c+dx)}dx}{7a}\right.\right.$$
$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\int\frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(250iA+253B)-a^3(212A-209iB)\tan(c+dx)\right)}{\tan(c+dx)^{7/2}}dx}{7a}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\big\downarrow$ 4081

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(\frac{2\int-\frac{\sqrt{i\tan(c+dx)a+a}\left((655A-649iB)a^4+2(250iA+253B)\tan(c+dx)a^4\right)}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\right.}{7a}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\big\downarrow$ 25

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(-\frac{2\int\frac{\sqrt{i\tan(c+dx)a+a}\left((655A-649iB)a^4+2(250iA+253B)\tan(c+dx)a^4\right)}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\right.}{7a}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(-\frac{2\int\frac{\sqrt{i\tan(c+dx)a+a}\left((655A-649iB)a^4+2(250iA+253B)\tan(c+dx)a^4\right)}{\tan(c+dx)^{5/2}}dx}{5a}-\right.}{7a}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\big\downarrow$ 4081

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(-\frac{2\left(\frac{2\int\frac{\sqrt{i\tan(c+dx)a+a}\left(a^5(2155iA+2167B)-2a^5(655A-649iB)\tan(c+dx)\right)}{2\tan^{\frac{3}{2}}(c+dx)}}{3a}\right.}{5a}\right.}{}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(-\frac{2\left(\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left(a^5(2155iA+2167B)-2a^5(655A-649iB)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}}{3a}\right.}{5a}\right.}{}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(-\frac{2\left(\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left(a^5(2155iA+2167B)-2a^5(655A-649iB)\tan(c+dx)\right)}{\tan(c+dx)^{3/2}}}{3a}\right.}{5a}\right.}{}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\Big\downarrow$ 4081

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(-\frac{2\left(\frac{\frac{2\int-\frac{3465a^6(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^5(2167B+2155iA)\sqrt{a+ia\tan}}{d\sqrt{\tan(c+dx)}}}{3a}\right.}{5a}\right.}{}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(-\frac{2\left(\frac{-3465a^5(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^5(2167B+2155iA)\sqrt{a+ia\tan}}{d\sqrt{\tan(c+dx)}}}{3a}\right.}{5a}\right.}{}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(-\frac{2\left(\frac{-3465a^5(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^5(2167B+2155iA)\sqrt{a+ia\tan}}{d\sqrt{\tan(c+dx)}}}{3a}\right.}{5a}\right.}{}\right.\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\Big\downarrow$ 4027

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(-\frac{2\left(\frac{\frac{6930ia^7(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{3a}-\frac{2a^5(2167B\cdots}{\cdots}\right)}{5a}\cdots\right)}{\cdots}\right)\cdots\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 218

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2a^2(212A-209iB)\sqrt{a+ia\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{6\left(-\frac{2a^3(253B+250iA)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2\left(-\frac{(3465-3465i)a^{11/2}(A-i\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)\cdots\right.$$

$$\frac{2aA(a+ia\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(13/2
),x]
```

output
```
(-2*a*A*(a + I*a*Tan[c + d*x])^(3/2))/(11*d*Tan[c + d*x]^(11/2)) + ((-2*a^
2*((14*I)*A + 11*B)*Sqrt[a + I*a*Tan[c + d*x]])/(9*d*Tan[c + d*x]^(9/2)) +
 ((2*a^2*(212*A - (209*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(7*d*Tan[c + d*x]
^(7/2)) - (6*((-2*a^3*((250*I)*A + 253*B)*Sqrt[a + I*a*Tan[c + d*x]])/(5*d
*Tan[c + d*x]^(5/2)) - (2*((-2*a^4*(655*A - (649*I)*B)*Sqrt[a + I*a*Tan[c
+ d*x]])/(3*d*Tan[c + d*x]^(3/2)) + (((-3465 + 3465*I)*a^(11/2)*(A - I*B)*
ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/
d - (2*a^5*((2155*I)*A + 2167*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c
 + d*x]]))/(3*a)))/(5*a)))/(7*a))/9)/11
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(266) = 532$.

Time = 0.23 (sec) , antiderivative size = 976, normalized size of antiderivative = 3.02

method	result	size
derivativedivides	Expression too large to display	976
default	Expression too large to display	976
parts	Expression too large to display	1045

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x,method=_
RETURNVERBOSE)
```

output
```
-1/3465/d*(a*(1+I*tan(d*x+c)))^(1/2)*a^2/tan(d*x+c)^(11/2)*(17336*B*tan(d*
x+c)^5*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-5192
*I*B*tan(d*x+c)^4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)
^(1/2)+6930*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^6+5240*A*tan(d*
x+c)^4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+3465
*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^6+13860*A
*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1
/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^6+17240*I*A*tan(d*x+c)^5*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)-3465*(I*a)^(1/
2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^6-3036*B*(I*a)^(1/2)*
(-I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-13860*I*B*
ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/
2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^6+6930*ln(1/2*(2*I*a*tan(d*x+
c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I
*a)^(1/2)*a*tan(d*x+c)^6-2120*A*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+2090*I*B*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+1610*I*A*tan(d*x+c)*(a*tan...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 771 vs. 2(249) = 498.

Time = 0.10 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.39

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x, a
lgorithm="fricas")
```

output
```
2/3465*(3465*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(12*I*d*x
 + 12*I*c) - 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) - 20*d*e
^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) - 6*d*e^(2*I*d*x + 2*I*c) +
d)*log((I*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c)
 + sqrt(2)*((-I*A - B)*a^2*e^(2*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 3465*sqrt(2)*sqrt(-(I*A^
2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(12*I*d*x + 12*I*c) - 6*d*e^(10*I*d*x + 1
0*I*c) + 15*d*e^(8*I*d*x + 8*I*c) - 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I
*d*x + 4*I*c) - 6*d*e^(2*I*d*x + 2*I*c) + d)*log((-I*sqrt(2)*sqrt(-(I*A^2
+ 2*A*B - I*B^2)*a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*((-I*A - B)*a^2*e^(2
*I*d*x + 2*I*c) + (-I*A - B)*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((
-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(
(-I*A - B)*a^2)) + 2*sqrt(2)*(2*(3730*A - 3553*I*B)*a^2*e^(13*I*d*x + 13*I
*c) - 9*(1805*A - 2013*I*B)*a^2*e^(11*I*d*x + 11*I*c) + 55*(397*A - 337*I*
B)*a^2*e^(9*I*d*x + 9*I*c) + 66*(95*A - 47*I*B)*a^2*e^(7*I*d*x + 7*I*c) -
1386*(15*A - 16*I*B)*a^2*e^(5*I*d*x + 5*I*c) + 15015*(A - I*B)*a^2*e^(3*I*
d*x + 3*I*c) - 3465*(A - I*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(
d*e^(12*I*d*x + 12*I*c) - 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x, a
lgorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeWarning, need to choose a branch for the root of a polynomial
 with par
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}{\tan(c+d\,x)^{13/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(13/
2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c + d*x)^(13/
2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \frac{2\sqrt{a}\,a^2\left(3392\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x)
```

output
```
(2*sqrt(a)*a**2*(3392*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c +
d*x)**5*a*i + 3344*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x
)**5*b - 1696*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4*
a + 1672*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4*b*i -
 1272*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*a*i - 12
54*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*b + 1060*sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*a - 1045*sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i - 805*sqrt(tan(c
+ d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 385*sqrt(tan(c + d*x))
*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 315*sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*i + 1)*a - 6930*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)
)/tan(c + d*x)**4,x)*tan(c + d*x)**6*a*d*i - 6930*int((sqrt(tan(c + d*x))*
sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**4,x)*tan(c + d*x)**6*b*d))/(3465*t
an(c + d*x)**6*d)
```

3.178 $\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 190

$$\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx=\frac{2(-1)^{3/4}a^{5/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(2+2i)a^{3/2}(2a+3ib)B\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a(a+3ib)B\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

output

```
2*(-1)^(3/4)*a^(5/2)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*t
an(d*x+c))^(1/2))/d+(2+2*I)*a^(3/2)*(2*a+3*I*b)*B*arctanh((1+I)*a^(1/2)*ta
n(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d-2*a*(a+3*I*b)*B*(a+I*a*tan(d*x+
c))^(1/2)/d/tan(d*x+c)^(1/2)-b*B*(a+I*a*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(3/
2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 566 vs. $2(190) = 380$.

Time = 10.33 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.98

$$\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{bB(a+ia\tan(c+dx))^{5/2}}{ad\tan^{\frac{3}{2}}(c+dx)}$$

$$+ \frac{2\left(-\frac{3(2a+5ib)B(a+ia\tan(c+dx))^{5/2}}{2d\sqrt{\tan(c+dx)}} + \frac{2\left(\frac{3ia^3(2a+5ib)B\sqrt{a+ia\tan(c+dx)}\left(-\frac{3}{4}(-1)^{3/4}\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)+\frac{5}{4}\sqrt{1+i\tan(c+dx)}\sqrt{\ta}\right.}{2d\sqrt{1+i\tan(c+dx)}}\right.}{}\right.}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Tan[c + d*x]))/
Tan[c + d*x]^(5/2),x]
```

output
```
-((b*B*(a + I*a*Tan[c + d*x])^(5/2))/(a*d*Tan[c + d*x]^(3/2))) + (2*((-3*(
2*a + (5*I)*b)*B*(a + I*a*Tan[c + d*x])^(5/2))/(2*d*Sqrt[Tan[c + d*x]]) +
(2*((((3*I)/2)*a^3*(2*a + (5*I)*b)*B*Sqrt[a + I*a*Tan[c + d*x]]*((-3*(-1)^
(3/4)*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]])/4 + (5*Sqrt[1 + I*Tan[c + d*
x]]*Sqrt[Tan[c + d*x]])/4 + (I/2)*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(3
/2)))/(d*Sqrt[1 + I*Tan[c + d*x]]) + (a*(((3*I)/8)*a^2*((10*I)*a - 23*b)*B
 + (3*a^2*(2*a + (5*I)*b)*B)/2)*(((-4*I)*Sqrt[2]*a*ArcTanh[(Sqrt[2]*Sqrt[I
*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/Sqrt[I*a
*Tan[c + d*x]] + ((4*I)*a^(3/2)*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sq
rt[1 + I*Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a*Tan[c + d*x]]*Sqrt[a
+ I*a*Tan[c + d*x]]) + I*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + (
I*Sqrt[a]*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[Tan[c + d*x]]*Sqrt[
a + I*a*Tan[c + d*x]])/(Sqrt[1 + I*Tan[c + d*x]]*Sqrt[I*a*Tan[c + d*x]])))
/d))/a))/(3*a)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4076, 27, 3042, 4076, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{3}\int \frac{3(i\tan(c+dx)a+a)^{3/2}((a+3ib)B+ia\tan(c+dx)B)}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(i\tan(c+dx)a+a)^{3/2}((a+3ib)B+ia\tan(c+dx)B)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(i\tan(c+dx)a+a)^{3/2}((a+3ib)B+ia\tan(c+dx)B)}{\tan(c+dx)^{3/2}}dx-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4076}$$

$$2\int \frac{\sqrt{i\tan(c+dx)a+a}\,(3a(ia-2b)B-a^2B\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aB(a+3ib)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{i\tan(c+dx)a+a}\,(3a(ia-2b)B-a^2B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aB(a+3ib)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{i\tan(c+dx)a+a}\,(3a(ia-2b)B-a^2B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aB(a+3ib)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4084}$$

$$2aB(-3b+2ia)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-iaB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aB(a+3ib)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$2aB(-3b+2ia)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-iaB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aB(a+3ib)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4027}$$

$$-\frac{4ia^3B(-3b+2ia)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-iaB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aB(a+3ib)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{218}$$

$$-iaB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{(2-2i)a^{3/2}B(-3b+2ia)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aB(a+3ib)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4082}$$

$$-\frac{ia^3B\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{(2-2i)a^{3/2}B(-3b+2ia)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aB(a+3ib)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{65}$$

$$-\frac{2ia^3B\int \frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}+\frac{(2-2i)a^{3/2}B(-3b+2ia)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aB(a+3ib)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2(-1)^{3/4}a^{5/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(2-2i)a^{3/2}B(-3b+2ia)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{bB(a+ia\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aB(a+3ib)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[((a + I*a*Tan[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Tan[c + d*x]))/Tan[c
+ d*x]^(5/2),x]
```

output
```
(2*(-1)^(3/4)*a^(5/2)*B*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqr
t[a + I*a*Tan[c + d*x]]])/d + ((2 - 2*I)*a^(3/2)*((2*I)*a - 3*b)*B*ArcTanh
[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*
a*(a + (3*I)*b)*B*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]) - (b*
B*(a + I*a*Tan[c + d*x])^(3/2))/(d*Tan[c + d*x]^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(156) = 312$.

Time = 0.34 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.26

method	result
derivativedivides	$\frac{aB\left(-i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^2+6i\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{}}{}\right.\right.}{}$
default	$\frac{aB\left(-i\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^2+6i\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{}}{}\right.\right.}{}$
parts	$\frac{B\left(-i\sqrt{ia}\,\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)a-i\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(d}}{2}\right.\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/2/d*a*B*(-I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c
)^2+6*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(
I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a^2*tan(d*x+c)^2+2*I*ln(1/2*(2*I*a
*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(
1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2-14*I*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*b-(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/
2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/
(tan(d*x+c)+I))*a*tan(d*x+c)^2-12*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*b*tan(
d*x+c)^2-2*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2-4*(I*a)^(1/2)*(-
I*a)^(1/2)*a*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-2*b*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))*(a*(1+I*tan(d*x+c)
))^(1/2)/tan(d*x+c)^(3/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2
)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(146) = 292$.

Time = 0.10 (sec) , antiderivative size = 900, normalized size of antiderivative = 4.74

$$\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/
2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(
-(-4*I*B^2*a^5 + 12*B^2*a^4*b + 9*I*B^2*a^3*b^2)/d^2)*log((sqrt(2)*d*sqrt(
-(-4*I*B^2*a^5 + 12*B^2*a^4*b + 9*I*B^2*a^3*b^2)/d^2)*e^(I*d*x + I*c) + sq
rt(2)*(2*I*B*a^2 - 3*B*a*b + (2*I*B*a^2 - 3*B*a*b)*e^(2*I*d*x + 2*I*c))*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(2*I*B*a^2 - 3*B*a*b)) - 2*sqrt(2)*(d
*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-4*I*B^2*a^5 +
12*B^2*a^4*b + 9*I*B^2*a^3*b^2)/d^2)*log(-(sqrt(2)*d*sqrt(-(-4*I*B^2*a^5 +
 12*B^2*a^4*b + 9*I*B^2*a^3*b^2)/d^2)*e^(I*d*x + I*c) - sqrt(2)*(2*I*B*a^2
 - 3*B*a*b + (2*I*B*a^2 - 3*B*a*b)*e^(2*I*d*x + 2*I*c))*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1
)))*e^(-I*d*x - I*c)/(2*I*B*a^2 - 3*B*a*b)) + 4*sqrt(2)*(B*a*b*e^(3*I*d*x
+ 3*I*c) - (I*B*a^2 - 4*B*a*b)*e^(5*I*d*x + 5*I*c) - (-I*B*a^2 + 3*B*a*b)*
e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + sqrt(4*I*B^2*a^5/d^2)*(d*e^(4*I*d*
x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*(B*a^2*e^(2*I*d*x +
 2*I*c) + B*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + I*sqrt(4*I*B^2*a^5/d^2)*d*e^(I*d*x
 + I*c))*e^(-I*d*x - I*c)/(B*a^2)) - sqrt(4*I*B^2*a^5/d^2)*(d*e^(4*I*d*x +
 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log((sqrt(2)*(B*a^2*e^(2*I*d*x +...
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{B\left(\int \frac{2a^3\sqrt{ia\tan(c+dx)+a}}{\tan^{\frac{3}{2}}(c+dx)}\,dx + \int \left(-2a^3\sqrt{ia\tan(c+\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)**(
5/2),x)
```

output
```
B*(Integral(2*a**3*sqrt(I*a*tan(c + d*x) + a)/tan(c + d*x)**(3/2), x) + In
tegral(-2*a**3*sqrt(I*a*tan(c + d*x) + a)*sqrt(tan(c + d*x)), x) + Integra
l(4*I*a**3*sqrt(I*a*tan(c + d*x) + a)/sqrt(tan(c + d*x)), x) + Integral(3*
a**2*b*sqrt(I*a*tan(c + d*x) + a)/tan(c + d*x)**(5/2), x) + Integral(-3*a*
*2*b*sqrt(I*a*tan(c + d*x) + a)/sqrt(tan(c + d*x)), x) + Integral(6*I*a**2
*b*sqrt(I*a*tan(c + d*x) + a)/tan(c + d*x)**(3/2), x))/(2*a)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/
2),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeWarning, need to choose a branch for the root of a polynomial
 with par
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(B\tan(c+d\,x)+\frac{3\,B\,b}{2\,a}\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/}}{\tan\left(c+d\,x\right)^{5/2}}$$

input
```
int(((B*tan(c + d*x) + (3*B*b)/(2*a))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c
 + d*x)^(5/2),x)
```

output
```
int(((B*tan(c + d*x) + (3*B*b)/(2*a))*(a + a*tan(c + d*x)*1i)^(5/2))/tan(c
 + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{a}\,ab\Big(2\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
(sqrt(a)*a*b*(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**
2*a*i + 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a + 5*s
qrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b*i - sqrt(tan(c +
 d*x))*sqrt(tan(c + d*x)*i + 1)*b + 2*int((sqrt(tan(c + d*x))*sqrt(tan(c +
 d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**2*a*d + 6*int((sqrt(tan(c +
 d*x))*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x)**2,x)*tan(c + d*x)**2*b*d*i
 - 2*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*tan(c
 + d*x)**2*a*d*i))/(tan(c + d*x)**2*d)
```

3.179 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 205

$$\begin{aligned}
&\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx \\
&= \frac{(-1)^{3/4}(2iA-B)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d} \\
&\quad + \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d} + \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad}
\end{aligned}$$

output
```
(-1)^(3/4)*(2*I*A-B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan
(d*x+c))^(1/2))/a^(1/2)/d+(1/2+1/2*I)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*
x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(1/2)/d+(I*A-B)*tan(d*x+c)^(3/2)/d/
(a+I*a*tan(d*x+c))^(1/2)-(A+2*I*B)*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/
2)/a/d
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.87

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d} - \frac{(-1)^{3/4}(2A+iB)\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{1+i\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\sqrt{\tan(c+dx)}(-A-2iB+B\tan(c+dx))}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d
*x]],x]
```

output
```
((1/2 + I/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]])/(Sqrt[a]*d) - ((-1)^(3/4)*(2*A + I*B)*ArcSinh[(-1)^
(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[1 + I*Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c
+ d*x]]) + (Sqrt[Tan[c + d*x]]*(-A - (2*I)*B + B*Tan[c + d*x]))/(d*Sqrt[a
+ I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4078, 27, 3042, 4080, 25, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \frac{1}{2}\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(3a(iA-B)+2a(A+2iB)\tan(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(3a(iA-B)+2a(A+2iB)\tan(c+dx))dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(3a(iA-B)+2a(A+2iB)\tan(c+dx))dx}{2a^2}$$

$$\Big\downarrow \text{4080}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{\int -\frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(A+2iB)-a^2(2iA-B)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a}+\frac{2a(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}}{2a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(A+2iB)-a^2(2iA-B)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a}}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(A+2iB)-a^2(2iA-B)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a}}{2a^2}$$

$\Big\downarrow$ 4084

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{a(2A+iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a^2(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{a(2A+iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a^2(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}}{2a^2}$$

$\Big\downarrow$ 4027

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{2ia^4(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}+a(2A+iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a}}{\sqrt{\tan(c+dx)}}}{a}}{2a^2}$$

$\Big\downarrow$ 218

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{a(2A+iB)\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{(1-i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx}}{\sqrt{a+ia\tan(c+dx)}}\right.}{d}}{a}}{2a^2}$$

$\Big\downarrow$ 4082

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{2a(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{a^3(2A+iB)\int\frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{(1-i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c}}{\sqrt{a+ia\tan(c+}}\right.}{d}}{a}}{2a^2}$$

$\Big\downarrow$ 65

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\frac{2a(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{\frac{2a^3(2A+iB)\int \frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d} - \frac{(1-i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{a}}{2a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\frac{2a(A+2iB)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{-\frac{2\sqrt[4]{-1}a^{5/2}(2A+iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d} - \frac{(1-i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{a}}{2a^2}$$

input
```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x
]
```

output
```
((I*A - B)*Tan[c + d*x]^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (-(((-2*(-
1)^(1/4)*a^(5/2)*(2*A + I*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]]
)/Sqrt[a + I*a*Tan[c + d*x]]])/d - ((1 - I)*a^(5/2)*(A - I*B)*ArcTanh[((1
+ I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/a) + (2*a
*(A + (2*I)*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d)/(2*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(165) = 330$.

Time = 0.36 (sec) , antiderivative size = 1135, normalized size of antiderivative = 5.54

method	result	size
derivativedivides	Expression too large to display	1135
default	Expression too large to display	1135
parts	Expression too large to display	1199

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/4/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a*(I*A*2^(1/2)*ln((2*2^(
1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c)
)/(tan(d*x+c)+I))*(I*a)^(1/2)*a*tan(d*x+c)^2-I*A*2^(1/2)*ln((2*2^(1/2)*(-I
*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d
*x+c)+I))*(I*a)^(1/2)*a+B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a
*tan(d*x+c)^2-8*I*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)-2*I*B*ln(
1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+
a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-4*I*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+2*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1
/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+
I))*(I*a)^(1/2)*a*tan(d*x+c)+2*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d
*x+c)^2+8*I*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)+4*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I
*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2-B*2^(1/2)*ln((2*2^(1
/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))
/(tan(d*x+c)+I))*(I*a)^(1/2)*a+4*I*A*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-2*I*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(155) = 310$.

Time = 0.20 (sec) , antiderivative size = 807, normalized size of antiderivative = 3.94

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="fricas")
```

output
```
-1/4*(sqrt(2)*a*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x + I*c)*l
og((I*sqrt(2)*a*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x + I*c) +
 sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(
4*I*A + 4*B)) - sqrt(2)*a*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*
x + I*c)*log((-I*sqrt(2)*a*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d
*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)))/(4*I*A + 4*B)) - a*d*sqrt((-4*I*A^2 + 4*A*B + I*B^2)/(a*d^2))*e^
(I*d*x + I*c)*log(104/605*(2*sqrt(2)*((2*I*A - B)*e^(3*I*d*x + 3*I*c) + (2
*I*A - B)*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + (3*I*a*d*e^(2*I*d*x + 2*
I*c) - I*a*d)*sqrt((-4*I*A^2 + 4*A*B + I*B^2)/(a*d^2)))/((2*I*A - B)*e^(2*
I*d*x + 2*I*c) + 2*I*A - B)) + a*d*sqrt((-4*I*A^2 + 4*A*B + I*B^2)/(a*d^2)
)*e^(I*d*x + I*c)*log(104/605*(2*sqrt(2)*((2*I*A - B)*e^(3*I*d*x + 3*I*c)
 + (2*I*A - B)*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + (-3*I*a*d*e^(2*I*d*x
 + 2*I*c) + I*a*d)*sqrt((-4*I*A^2 + 4*A*B + I*B^2)/(a*d^2)))/((2*I*A - B)*
e^(2*I*d*x + 2*I*c) + 2*I*A - B)) + 2*sqrt(2)*((A + 3*I*B)*e^(2*I*d*x + 2*
I*c) + A + I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x +...
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan^{\frac{3}{2}}{(c+dx)}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(3/2)/sqrt(I*a*(tan(c + d*x) -
 I)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2
),x)
```

output
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2
), x)
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2
*a - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i - 2
*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 6*sqrt(tan(c
 + d*x))*sqrt(tan(c + d*x)*i + 1)*a - 6*sqrt(tan(c + d*x))*sqrt(tan(c + d*
x)*i + 1)*b*i + 2*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c +
 d*x)**3)/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*a*d + int((sqrt(tan(c +
 d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3)/(tan(c + d*x)**2 + 1),x)*
tan(c + d*x)**2*b*d*i + 2*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)
*tan(c + d*x)**3)/(tan(c + d*x)**2 + 1),x)*a*d + int((sqrt(tan(c + d*x))*s
qrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3)/(tan(c + d*x)**2 + 1),x)*b*d*i +
3*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**3 + tan
(c + d*x)),x)*tan(c + d*x)**2*a*d + 3*int((sqrt(tan(c + d*x))*sqrt(tan(c +
 d*x)*i + 1))/(tan(c + d*x)**3 + tan(c + d*x)),x)*tan(c + d*x)**2*b*d*i +
3*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**3 + tan
(c + d*x)),x)*a*d + 3*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(t
an(c + d*x)**3 + tan(c + d*x)),x)*b*d*i + 3*int((sqrt(tan(c + d*x))*sqrt(t
an(c + d*x)*i + 1))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*a*d*i + 3*int
((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**2 + 1),x)*a*
d*i))/(a*d*(tan(c + d*x)**2 + 1))
```

3.180 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 156

$$\begin{aligned}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{2\sqrt[4]{-1}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}\\ &\quad-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}+\frac{(iA-B)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
-2*(-1)^(1/4)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(1/2)/d-(1/2+1/2*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^
(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(1/2)/d+(I*A-B)*tan(d*x+c)^(1/2)/d/(a+I*
a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}$$
$$+\frac{\left(-2\sqrt[4]{-1}B\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{1+i\tan(c+dx)}+(A+iB)\sqrt{\tan(c+dx)}\right)\sqrt{a+ia\tan}}{ad(-i+\tan(c+dx))}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d
*x]],x]
```

output
```
((-1/2 - I/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[
a + I*a*Tan[c + d*x]]])/(Sqrt[a]*d) + ((-2*(-1)^(1/4)*B*ArcSinh[(-1)^(1/4)
*Sqrt[Tan[c + d*x]]]*Sqrt[1 + I*Tan[c + d*x]] + (A + I*B)*Sqrt[Tan[c + d*x
]])*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 4078, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}(a(iA-B)+2iaB\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx}{a^2}$$

$\downarrow$ 27

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}(a(iA-B)+2iaB\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{2a^2}$$

$\downarrow$ 3042

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}(a(iA-B)+2iaB\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{2a^2}$$

$\downarrow$ 4084

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-2B\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a^2}$$

$\downarrow$ 3042

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-2B\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a^2}$$

$\downarrow$ 4027

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{-\frac{2ia^3(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-2B\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a^2}$$

$\downarrow$ 218

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-2B\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a^2}$$

$\downarrow$ 4082

$$\frac{\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2B\int\frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}}{2a^2}}{}$$

$\downarrow$ 65

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{4a^2B\int\frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{2a^2}$$

$\downarrow$ 216

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{4\sqrt[4]{-1}a^{3/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{2a^2}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x
]
```

output
```
-1/2*((4*(-1)^(1/4)*a^(3/2)*B*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]
])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((1 - I)*a^(3/2)*(I*A + B)*ArcTanh[((1
 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/a^2 + ((
I*A - B)*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(123) = 246$.

Time = 0.23 (sec) , antiderivative size = 900, normalized size of antiderivative = 5.77

method	result
derivativedivides	$\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(iB\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(d$
default	$\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(iB\sqrt{ia}\,\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(d$
parts	$A\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}\left(2i\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{2}\,\tan(dx+c)a-$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/4/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a*(I*B*(I*a)^(1/2)*2^(1/
2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-
3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+2*I*A*(I*a)^(1/2)*2^(1/2)*l
n(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*
tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-A*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(
1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c)
)/(tan(d*x+c)+I))*a*tan(d*x+c)^2-I*B*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(
-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan
(d*x+c)+I))*a-8*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)+4*I*B*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+2*B
*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a*tan(d*x+c)+4*B*ln(1/2*
(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(
I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2-4*I*A*(I*a)^(1/2)*(-I*a)^(1/2)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*
(I*a)^(1/2)*a+4*A*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)*tan(d*x+c)-4*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a+4*B*(-I*a)^(1/2...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 718 vs. 2(116) = 232.

Time = 0.20 (sec) , antiderivative size = 718, normalized size of antiderivative = 4.60

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="fricas")
```

output
```
-1/4*(sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d*x + I*c)*
log((sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d*x + I*c) +
 sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(
4*I*A + 4*B)) - sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d
*x + I*c)*log(-(sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d
*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)))/(4*I*A + 4*B)) - a*d*sqrt(-4*I*B^2/(a*d^2))*e^(I*d*x + I*c)*log(
52/605*(4*sqrt(2)*(B*e^(3*I*d*x + 3*I*c) + B*e^(I*d*x + I*c))*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) + 1)) + (3*a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(-4*I*B^2/(a*d^2)))/(B*e^
(2*I*d*x + 2*I*c) + B)) + a*d*sqrt(-4*I*B^2/(a*d^2))*e^(I*d*x + I*c)*log(5
2/605*(4*sqrt(2)*(B*e^(3*I*d*x + 3*I*c) + B*e^(I*d*x + I*c))*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1)) - (3*a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(-4*I*B^2/(a*d^2)))/(B*e^(
2*I*d*x + 2*I*c) + B)) + 2*sqrt(2)*((-I*A + B)*e^(2*I*d*x + 2*I*c) - I*A +
 B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\sqrt{\tan(c+dx)}}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(tan(c + d*x))/sqrt(I*a*(tan(c + d*x) -
I)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeWarning, need
 to choos
```

Mupad [B] (verification not implemented)

Time = 22.72 (sec) , antiderivative size = 4040, normalized size of antiderivative = 25.90

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2
),x)
```

output
```
-(A*a^(5/2)*tan(c + d*x)^(1/2)*4i + 4*A*a^(5/2)*tan(c + d*x)^(3/2) - 4*B*a
^(5/2)*tan(c + d*x)^(1/2) + B*a^(5/2)*tan(c + d*x)^(3/2)*4i + A*a^(3/2)*ta
n(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)*4i - 4*B*a^(3/2)*tan(c + d*x)^(1/
2)*(a + a*tan(c + d*x)*1i) + (1i/8)^(1/2)*A*(-a)^(5/2)*atanh(((1i/8)^(1/2)
*(-a)^(15/2)*a^(17/2)*tan(c + d*x)^(1/2)*4i - 4*(1i/8)^(1/2)*(-a)^(31/2)*t
an(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2) - (1i/8)^(1/2)*(-a)^(31/2)
*tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*8i + 4*(1i/8)^(1/2)*(-a)
^(15/2)*a^(17/2)*tan(c + d*x)^(3/2) + (1i/8)^(1/2)*(-a)^(15/2)*a^(15/2)*ta
n(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)*4i)/(a^15*(a + a*tan(c + d*x)*1i)
 - 2*a^(31/2)*(a + a*tan(c + d*x)*1i)^(1/2) + a^16 + a^16*tan(c + d*x)^2))
*8i + 8*(1i/8)^(1/2)*B*(-a)^(5/2)*atanh(((1i/8)^(1/2)*(-a)^(15/2)*a^(17/2)
*tan(c + d*x)^(1/2)*4i - 4*(1i/8)^(1/2)*(-a)^(31/2)*tan(c + d*x)^(3/2)*(a
+ a*tan(c + d*x)*1i)^(1/2) - (1i/8)^(1/2)*(-a)^(31/2)*tan(c + d*x)^(1/2)*(
a + a*tan(c + d*x)*1i)^(1/2)*8i + 4*(1i/8)^(1/2)*(-a)^(15/2)*a^(17/2)*tan(
c + d*x)^(3/2) + (1i/8)^(1/2)*(-a)^(15/2)*a^(15/2)*tan(c + d*x)^(1/2)*(a +
 a*tan(c + d*x)*1i)*4i)/(a^15*(a + a*tan(c + d*x)*1i) - 2*a^(31/2)*(a + a*
tan(c + d*x)*1i)^(1/2) + a^16 + a^16*tan(c + d*x)^2)) - A*a^2*tan(c + d*x)
^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)*8i - 4*A*a^2*tan(c + d*x)^(3/2)*(a +
a*tan(c + d*x)*1i)^(1/2) + 8*B*a^2*tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*
1i)^(1/2) - B*a^2*tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)*4i +...
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{2\sqrt{a}\left(-5\sqrt{\tan\left(dx+c\right)}\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(dx+c\right)^{2}b+3\sqrt{\tan\left(dx+c\right)}\sqrt{\tan\left(dx+c\right)i+1}\,\tan\left(\right.\right.}{}$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*( - 5*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*
*2*b + 3*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a + 6*sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b*i + 4*sqrt(tan(c
+ d*x))*sqrt(tan(c + d*x)*i + 1)*a*i - 18*sqrt(tan(c + d*x))*sqrt(tan(c +
d*x)*i + 1)*b + 5*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c +
 d*x)**3)/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*b*d + 5*int((sqrt(tan(c
 + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3)/(tan(c + d*x)**2 + 1),x
)*b*d - 2*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)*
*3 + tan(c + d*x)),x)*tan(c + d*x)**2*a*d*i + 9*int((sqrt(tan(c + d*x))*sq
rt(tan(c + d*x)*i + 1))/(tan(c + d*x)**3 + tan(c + d*x)),x)*tan(c + d*x)**
2*b*d - 2*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)*
*3 + tan(c + d*x)),x)*a*d*i + 9*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*
i + 1))/(tan(c + d*x)**3 + tan(c + d*x)),x)*b*d))/(5*a*d*(tan(c + d*x)**2
+ 1))
```

3.181 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 99

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(\frac{1}{2}-\frac{i}{2}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d} + \frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

output
```
(1/2-1/2*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(1/2)/d+(A+I*B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\tan(c+dx)}\left(\frac{\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{ia\tan(c+dx)}}+\frac{2(A+iB)}{\sqrt{a+ia\tan(c+dx)}}\right)}{2d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*
x]]),x]
```

output
```
(Sqrt[Tan[c + d*x]]*((Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c +
d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/Sqrt[I*a*Tan[c + d*x]] + (2*(A + I*B))
/Sqrt[a + I*a*Tan[c + d*x]]))/(2*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4027}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{ia(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x
]
```

output
```
((1/2 - I/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]])/(Sqrt[a]*d) + ((A + I*B)*Sqrt[Tan[c + d*x]])/(d*Sqr
t[a + I*a*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(79) = 158$.

Time = 0.24 (sec) , antiderivative size = 639, normalized size of antiderivative = 6.45

method	result
derivativedivides	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(iA\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx-\right.}{}$
default	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(iA\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx-\right.}{}$
parts	$-\frac{A\sqrt{a(1+i\tan(dx+c))}\,\sqrt{\tan(dx+c)}\left(i\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)^2\right.}{}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/4/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(I*A*2^(1/2)*ln(-(-2*2^
(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c
))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-2*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I
))*a*tan(d*x+c)+B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*
tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-I*A*
2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+4*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+2*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*
a*tan(d*x+c)+4*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-B*2^
(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I
*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-4*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)*tan(d*x+c)+4*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c
)))^(1/2))/a/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)/(I-tan(d*x
+c))^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(73) = 146$.

Time = 0.11 (sec) , antiderivative size = 416, normalized size of antiderivative = 4.20

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(\sqrt{2}ad\sqrt{-\frac{iA^2+2AB-iB^2}{ad^2}}e^{(idx+ic)}\log\left(\frac{i\sqrt{2}ad\sqrt{-\frac{iA^2+2AB-iB^2}{ad^2}}e^{(idx+ic)}+\sqrt{2}\left((iA+B)e^{(2idx+2ic)}+iA+B\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+}}}{4iA+4B}\right.\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="fricas")
```

output
```
1/4*(sqrt(2)*a*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x + I*c)*lo
g((I*sqrt(2)*a*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x + I*c) +
sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4
*I*A + 4*B)) - sqrt(2)*a*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x
 + I*c)*log((-I*sqrt(2)*a*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*
x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) + 1)))/(4*I*A + 4*B)) + 2*sqrt(2)*((A + I*B)*e^(2*I*d*x + 2*I*c) + A + I
*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}\sqrt{a + ia\tan(c + dx)}}\,dx = \int \frac{A + B\tan{(c + dx)}}{\sqrt{ia(\tan{(c + dx)} - i)}\sqrt{\tan{(c + dx)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(sqrt(I*a*(tan(c + d*x) - I))*sqrt(tan(c + d
*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}\sqrt{a + ia\tan(c + dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}\sqrt{a + ia\tan(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.30

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{B\ln\left(\frac{\sqrt{a}\sqrt{\tan(c+dx)}\,(2-2\mathrm{i})}{\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}}-\frac{a\tan(c+dx)}{\left(\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}\right)^2}+1\mathrm{i}\right)\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{\sqrt{a}\,d}$$
$$+\frac{A\sqrt{\tan(c+dx)}\,2\mathrm{i}}{\left(\sqrt{a+a\tan(c+dx)\ 1\mathrm{i}}-\sqrt{a}\right)\left(d\,1\mathrm{i}-\frac{a\,d\tan(c+dx)}{\left(\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}\right)^2}\right)}$$
$$-\frac{2B\sqrt{\tan(c+dx)}}{\left(\sqrt{a+a\tan(c+dx)\ 1\mathrm{i}}-\sqrt{a}\right)\left(d\,1\mathrm{i}-\frac{a\,d\tan(c+dx)}{\left(\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}\right)^2}\right)}$$
$$-\frac{\sqrt{\frac{1}{8}\mathrm{i}}\,B\ln\left(-\frac{a\tan(c+dx)}{\left(\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}\right)^2}+\frac{2(-1)^{3/4}\sqrt{2}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}}+1\mathrm{i}\right)}{\sqrt{a}\,d}$$
$$+\frac{2\sqrt{\frac{1}{8}\mathrm{i}}\,A\,\mathrm{atanh}\left(\frac{32\sqrt{\frac{1}{8}\mathrm{i}}\,A^2(-a)^{9/2}\sqrt{\tan(c+dx)}}{\left(A^2a^4\,4\mathrm{i}-\frac{4A^2a^5\tan(c+dx)}{\left(\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}\right)^2}\right)\left(\sqrt{a+a\tan(c+dx)\,1\mathrm{i}}-\sqrt{a}\right)}\right)}{\sqrt{-a}\,d}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)
),x)
```

output
```
(B*log((a^(1/2)*tan(c + d*x)^(1/2)*(2 - 2i))/((a + a*tan(c + d*x)*1i)^(1/2
) - a^(1/2)) - (a*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^
2 + 1i)*(1/4 + 1i/4))/(a^(1/2)*d) + (A*tan(c + d*x)^(1/2)*2i)/(((a + a*tan
(c + d*x)*1i)^(1/2) - a^(1/2))*(d*1i - (a*d*tan(c + d*x))/((a + a*tan(c +
d*x)*1i)^(1/2) - a^(1/2))^2)) - (2*B*tan(c + d*x)^(1/2))/(((a + a*tan(c +
d*x)*1i)^(1/2) - a^(1/2))*(d*1i - (a*d*tan(c + d*x))/((a + a*tan(c + d*x)*
1i)^(1/2) - a^(1/2))^2)) - ((1i/8)^(1/2)*B*log((2*(-1)^(3/4)*2^(1/2)*a^(1/
2)*tan(c + d*x)^(1/2))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2)) - (a*tan(
c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2))^2 + 1i))/(a^(1/2)*d) +
 (2*(1i/8)^(1/2)*A*atanh((32*(1i/8)^(1/2)*A^2*(-a)^(9/2)*tan(c + d*x)^(1/2
))/((A^2*a^4*4i - (4*A^2*a^5*tan(c + d*x))/((a + a*tan(c + d*x)*1i)^(1/2)
- a^(1/2))^2)*((a + a*tan(c + d*x)*1i)^(1/2) - a^(1/2)))))/((-a)^(1/2)*d)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{a}\left(-2\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)\,ai-\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(c\right.}{}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*( - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*
a*i - sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b + sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a + 2*int((sqrt(tan(c + d*x))*sqrt(ta
n(c + d*x)*i + 1))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*a*d*i + 2*int(
(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**2 + 1),x)*tan
(c + d*x)**2*b*d + 2*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(ta
n(c + d*x)**2 + 1),x)*a*d*i + 2*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*
i + 1))/(tan(c + d*x)**2 + 1),x)*b*d))/(a*d*(tan(c + d*x)**2 + 1))
```

3.182 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 143

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d} + \frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{(3A+iB)\sqrt{a+ia\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}$$

output
```
(1/2+1/2*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(1/2)/d+(A+I*B)/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-(
3*A+I*B)*(a+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\frac{\sqrt{2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\tan(c+dx)}{\sqrt{ia\tan(c+dx)}}+\frac{-4A+2(-3iA+B)\tan(c+dx)}{\sqrt{a+ia\tan(c+dx)}}}{2d\sqrt{\tan(c+dx)}}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*
x]]),x]
```

output
```
((Sqrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]]*Tan[c + d*x])/Sqrt[I*a*Tan[c + d*x]] + (-4*A + 2*((-3*I)*A
+ B)*Tan[c + d*x])/Sqrt[a + I*a*Tan[c + d*x]])/(2*d*Sqrt[Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {3042, 4079, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(3A+iB)-2a(iA-B)\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx}{a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(3A+iB)-2a(iA-B)\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(3A+iB)-2a(iA-B)\tan(c+dx))}{\tan(c+dx)^{3/2}}dx}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 4081$$

$$\frac{\frac{2\int \frac{a^2(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3A+iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3A+iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3A+iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 4027$$

$$\frac{-\frac{2ia^3(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a(3A+iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{(1-i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a(3A+iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x
]
```

output
```
(A + I*B)/(d*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((1 - I)*a^
(3/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]])/d - (2*a*(3*A + I*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[T
an[c + d*x]]))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(117) = 234$.

Time = 0.26 (sec) , antiderivative size = 701, normalized size of antiderivative = 4.90

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\left(iB\sqrt{2}\ln\left(-\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^3+2iA\sqrt{2}\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\left(iB\sqrt{2}\ln\left(-\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^3+2iA\sqrt{2}\right.}{}$
parts	$-\frac{A\sqrt{a(1+i\tan(dx+c))}\left(2i\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)^2a-\ln\left(\frac{2\sqrt{2}\sqrt{}}{}\right.\right.}{}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/4/d*(a*(1+I*tan(d*x+c)))^(1/2)*(I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I)
)*a*tan(d*x+c)^3+2*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2
-A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-I*B*2^(1/2)*ln(-(-2
*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*
x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+4*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+2*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*
tan(d*x+c)^2-20*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan
(d*x+c)+A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+12*A*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+4*B*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-8*A*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2))/a/tan(d*x+c)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x
+c)))^(1/2)/(I-tan(d*x+c))^2/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(109) = 218$.

Time = 0.17 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.31

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)\sqrt{a + ia\tan(c + dx)}}\, dx$$

$$= \frac{\sqrt{2}\left(ade^{(3i\,dx+3i\,c)} - ade^{(i\,dx+i\,c)}\right)\sqrt{-\frac{-i\,A^2-2\,AB+i\,B^2}{ad^2}}\log\left(\frac{\sqrt{2}ad\sqrt{-\frac{-i\,A^2-2\,AB+i\,B^2}{ad^2}}e^{(i\,dx+i\,c)}+\sqrt{2}((i\,A+B)e^{(2i\,dx+2i\,c)}+\cdots}{4i\,A+4\,B}\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="fricas")
```

output
```
1/4*(sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(-(-I*A^2
 - 2*A*B + I*B^2)/(a*d^2))*log((sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)
/(a*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A +
 B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c
) - a*d*e^(I*d*x + I*c))*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*log(-(sqr
t(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d*x + I*c) - sqrt(2)
*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A +
4*B)) - 2*sqrt(2)*((5*I*A - B)*e^(4*I*d*x + 4*I*c) + 4*I*A*e^(2*I*d*x + 2*
I*c) - I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*
d*x + I*c))
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{A+B\tan{(c+dx)}}{\sqrt{ia(\tan{(c+dx)}-i)}\tan^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**
(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)\sqrt{a + ia\tan(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)\sqrt{a + ia\tan(c + dx)}}\,dx = \int \frac{A + B\tan(c + d\,x)}{{\tan\left(c + d\,x\right)}^{3/2}\,\sqrt{a + a\tan\left(c + d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)\sqrt{a + ia\tan(c + dx)}}\,dx$$
$$= \frac{2\sqrt{a}\left(-8\sqrt{\tan(dx + c)}\,\sqrt{\tan(dx + c)\,i + 1}\,\tan(dx + c)^2\,a - 2\sqrt{\tan(dx + c)}\,\sqrt{\tan(dx + c)\,i + 1}\,\tan(\right.}{}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*( - 8*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*
*2*a - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i -
 4*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i + sqrt(tan
(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 5*sqrt(tan(c + d*x))*
sqrt(tan(c + d*x)*i + 1)*a + 2*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i
 + 1))/(tan(c + d*x)**3 + tan(c + d*x)),x)*tan(c + d*x)**3*a*d*i + 2*int((
sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**3 + tan(c + d*
x)),x)*tan(c + d*x)**3*b*d + 2*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i
 + 1))/(tan(c + d*x)**3 + tan(c + d*x)),x)*tan(c + d*x)*a*d*i + 2*int((sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**3 + tan(c + d*x))
,x)*tan(c + d*x)*b*d))/(5*tan(c + d*x)*a*d*(tan(c + d*x)**2 + 1))
```

3.183 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 191

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(iA+B)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d} + \frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{(7iA-9B)\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}}$$

output

```
(1/2+1/2*I)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(1/2)/d+(A+I*B)/d/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2)-1
/3*(5*A+3*I*B)*(a+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(3/2)+1/3*(7*I*A-9*
B)*(a+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{-\frac{3\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\tan^2(c+dx)}{\sqrt{ia\tan(c+dx)}}-\frac{2(2A+(-2iA+6B)\tan(c+dx)+(7A+9iB)\tan^2(c+dx))}{\sqrt{a+ia\tan(c+dx)}}}{6d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*
x]]),x]
```

output
```
((-3*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]]*Tan[c + d*x]^2)/Sqrt[I*a*Tan[c + d*x]] - (2*(2*A + ((-2*
I)*A + 6*B)*Tan[c + d*x] + (7*A + (9*I)*B)*Tan[c + d*x]^2))/Sqrt[a + I*a*T
an[c + d*x]])/(6*d*Tan[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 4079, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}\sqrt{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(5A+3iB)-4a(iA-B)\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)}dx}{a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(5A+3iB)-4a(iA-B)\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(5A+3iB)-4a(iA-B)\tan(c+dx))}{\tan(c+dx)^{5/2}}dx}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4081}$$

$$\frac{\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((7iA-9B)a^2+2(5A+3iB)\tan(c+dx)a^2\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((7iA-9B)a^2+2(5A+3iB)\tan(c+dx)a^2\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((7iA-9B)a^2+2(5A+3iB)\tan(c+dx)a^2\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4081}$$

$$\frac{-\frac{\frac{2\int \frac{3a^3(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^2(-9B+7iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{3a^2(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(-9B+7iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{3a^2(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(-9B+7iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4027}$$

$$\frac{-\frac{-\frac{6ia^4(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^2(-9B+7iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\frac{(3-3i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(-9B+7iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x
]
```

output
```
(A + I*B)/(d*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((-2*a*(5*A
+ (3*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) - (((3 - 3
*I)*a^(5/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ I*a*Tan[c + d*x]]])/d - (2*a^2*((7*I)*A - 9*B)*Sqrt[a + I*a*Tan[c + d*x]
])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(155) = 310$.

Time = 0.24 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.91

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(-6iB\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^3-36B\sqrt{}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(-6iB\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^3-36B\sqrt{}\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(dx+c))}\left(3i\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)^4a-3i\sqrt{2}\,\ln\left(2\sqrt{}\right.\right.}{}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/12/d*(a*(1+I*tan(d*x+c)))^(1/2)/a/tan(d*x+c)^(3/2)*(-6*I*B*2^(1/2)*ln(-(
-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(
d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-36*B*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+3*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I
))*a*tan(d*x+c)^4+3*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+
36*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+60*I*
B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+28*I*A*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+6*A*2^(1/2)
*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*
a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-3*I*A*2^(1/2)*ln(-(-2*2^(1/2)
*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(t
an(d*x+c)+I))*a*tan(d*x+c)^2-3*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*ta
n(d*x+c)^2+24*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x
+c)+8*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2))/(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)/(I-tan(d*x+c))^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(145) = 290$.

Time = 0.13 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.80

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx =$$

$$-\frac{3\sqrt{2}\left(ade^{(5i\,dx+5i\,c)}-2\,ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)\sqrt{-\frac{i\,A^2+2\,AB-i\,B^2}{ad^2}}\log\left(\frac{i\sqrt{2}ad\sqrt{-\frac{i\,A^2+2\,AB-i\,B^2}{ad^2}}e^{(i\,dx+i\,c)}+\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="fricas")
```

output
```
-1/12*(3*sqrt(2)*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*
d*e^(I*d*x + I*c))*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*log((I*sqrt(2)*a
*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A
+ B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) -
 3*sqrt(2)*(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I
*d*x + I*c))*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*log((-I*sqrt(2)*a*d*sq
rt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*
e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + 2*sq
rt(2)*((7*A + 15*I*B)*e^(6*I*d*x + 6*I*c) - (11*A + 3*I*B)*e^(4*I*d*x + 4*
I*c) - 15*(A + I*B)*e^(2*I*d*x + 2*I*c) + 3*A + 3*I*B)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)
))/(a*d*e^(5*I*d*x + 5*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I
*c))
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{A+B\tan{(c+dx)}}{\sqrt{ia(\tan{(c+dx)}-i)}\tan^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(sqrt(I*a*(tan(c + d*x) - I))*tan(c + d*x)**
(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{5/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{2\sqrt{a}\left(8\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(dx+c)^3\,ai-4\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)\,i+1}\,\tan(\right.}{}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(a)*(8*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*
a*i - 4*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*a + 2*
sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 3*sqrt(tan(
c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - sqrt(tan(c + d*x))*sqr
t(tan(c + d*x)*i + 1)*a - 6*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i +
1))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**4*a*d*i - 6*int((sqrt(tan(c + d
*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**4*b*
d - 6*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**2 +
 1),x)*tan(c + d*x)**2*a*d*i - 6*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)
*i + 1))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*b*d))/(3*tan(c + d*x)**2
*a*d*(tan(c + d*x)**2 + 1))
```

3.184 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 237

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d} \\ &\quad+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}-\frac{(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)} \\ &\quad+\frac{(23iA-25B)\sqrt{a+ia\tan(c+dx)}}{15ad\tan^{\frac{3}{2}}(c+dx)}+\frac{(61A+35iB)\sqrt{a+ia\tan(c+dx)}}{15ad\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
(-1/2-1/2*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x
+c))^(1/2))/a^(1/2)/d+(A+I*B)/d/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(1/2)-
1/5*(7*A+5*I*B)*(a+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(5/2)+1/15*(23*I*A
-25*B)*(a+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(3/2)+1/15*(61*A+35*I*B)*(a
+I*a*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.73

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\frac{15\sqrt{2}a(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\tan^4(c+dx)}{(ia\tan(c+dx))^{3/2}} + \frac{2(-6A+2i(A+5iB)\tan(c+dx)+2(19A+5iB)\tan^2(c+dx)+(61iA-35B)\tan^3(c}{\sqrt{a+ia\tan(c+dx)}}}{30d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*
x]]),x]
```

output
```
((15*Sqrt[2]*a*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a +
 I*a*Tan[c + d*x]]]*Tan[c + d*x]^4)/(I*a*Tan[c + d*x])^(3/2) + (2*(-6*A +
(2*I)*(A + (5*I)*B)*Tan[c + d*x] + 2*(19*A + (5*I)*B)*Tan[c + d*x]^2 + ((6
1*I)*A - 35*B)*Tan[c + d*x]^3))/Sqrt[a + I*a*Tan[c + d*x]])/(30*d*Tan[c +
d*x]^(5/2))
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.395$, Rules used = {3042, 4079, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{7/2}\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(7A+5iB)-6a(iA-B)\tan(c+dx))}{2\tan^{\frac{7}{2}}(c+dx)}dx}{a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(7A+5iB)-6a(iA-B)\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(7A+5iB)-6a(iA-B)\tan(c+dx))}{\tan(c+dx)^{7/2}}dx}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4081}$$

$$\frac{\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((23iA-25B)a^2+4(7A+5iB)\tan(c+dx)a^2\right)}{2\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((23iA-25B)a^2+4(7A+5iB)\tan(c+dx)a^2\right)}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((23iA-25B)a^2+4(7A+5iB)\tan(c+dx)a^2\right)}{\tan(c+dx)^{5/2}}dx}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4081}$$

$$\frac{-\frac{\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(61A+35iB)-2a^3(23iA-25B)\tan(c+dx)\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^2(-25B+23iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(61A+35iB)-2a^3(23iA-25B)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^2(-25B+23iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(61A+35iB)-2a^3(23iA-25B)\tan(c+dx)\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a^2(-25B+23iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 4081

$$\frac{-\frac{\frac{\frac{2\int \frac{15a^4(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^3(61A+35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(-25B+23iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\frac{15a^3(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(61A+35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(-25B+23iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\frac{15a^3(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(61A+35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(-25B+23iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 4027

$$\frac{-\frac{\frac{-\frac{30ia^5(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^3(61A+35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(-25B+23iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

$\Big\downarrow$ 218

$$\frac{-\frac{\frac{\frac{(15-15i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(61A+35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(-25B+23iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2a(7A+5iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}}{2a^2}+\frac{A+iB}{d\tan^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x
]
```

output
```
(A + I*B)/(d*Tan[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((-2*a*(7*A
+ (5*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) - ((-2*a^2
*((23*I)*A - 25*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) +
(((15 - 15*I)*a^(7/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]
])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^3*(61*A + (35*I)*B)*Sqrt[a + I*a*
Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*a))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(193) = 386$.

Time = 0.25 (sec) , antiderivative size = 821, normalized size of antiderivative = 3.46

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(15iB\sqrt{2}\ln\left(-\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^5+30iA\sqrt{2}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\left(15iB\sqrt{2}\ln\left(-\frac{-2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c)^5+30iA\sqrt{2}\cdots\right.}{\cdots}$
parts	$\frac{A\sqrt{a(1+i\tan(dx+c))}\left(30i\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\tan(dx+c)^4a-15\ln\left(\frac{2\sqrt{2}\cdots}{\cdots}\right.\right.}{\cdots}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/60/d*(a*(1+I*tan(d*x+c)))^(1/2)*(15*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(
1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)
+I))*a*tan(d*x+c)^5+30*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+
c)^4-15*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5-15*I*B*2^(1/
2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-
3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+140*I*B*(a*tan(d*x+c)*(1+I*
tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^4+30*B*2^(1/2)*ln(-(-2*2^(1/2)*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(ta
n(d*x+c)+I))*a*tan(d*x+c)^4-396*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*
(-I*a)^(1/2)*tan(d*x+c)^3+15*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(
d*x+c)^3+244*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+
c)^4+180*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3
+16*I*A*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-144*
A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+40*B*(-I
*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+24*A*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2))/a/tan(d*x+c)^(5/2)/(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)/(I-tan(d*x+c))^2/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(181) = 362$.

Time = 0.12 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.51

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{7}{2}}(c + dx)\sqrt{a + ia\tan(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="fricas")
```

output
```
-1/60*(15*sqrt(2)*(a*d*e^(7*I*d*x + 7*I*c) - 3*a*d*e^(5*I*d*x + 5*I*c) + 3
*a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(-(-I*A^2 - 2*A*B + I*
B^2)/(a*d^2))*log((sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(
I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) + 1)))/(4*I*A + 4*B)) - 15*sqrt(2)*(a*d*e^(7*I*d*x + 7*I*c) - 3*a*d*
e^(5*I*d*x + 5*I*c) + 3*a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqr
t(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*log(-(sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A
*B + I*B^2)/(a*d^2))*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I
*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + 2*sqrt(2)*((-103*I*A
 + 35*B)*e^(8*I*d*x + 8*I*c) + 6*(17*I*A - 15*B)*e^(6*I*d*x + 6*I*c) + 20*
(2*I*A - B)*e^(4*I*d*x + 4*I*c) + 30*(-5*I*A + 3*B)*e^(2*I*d*x + 2*I*c) +
15*I*A - 15*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(a*d*e^(7*I*d*x + 7*I*c) - 3*a*d*e^(5
*I*d*x + 5*I*c) + 3*a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{7}{2}}(c + dx)\sqrt{a + ia\tan(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{7}{2}}(c + dx)\sqrt{a + ia\tan(c + dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{7/2}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{7}{2}}(c + dx)\sqrt{a + ia\tan(c + dx)}}\,dx$$
$$= \frac{2\sqrt{a}\left(32\sqrt{\tan(dx + c)}\,\sqrt{\tan(dx + c)\,i + 1}\,\tan(dx + c)^4 a + 16\sqrt{\tan(dx + c)}\,\sqrt{\tan(dx + c)\,i + 1}\,\tan\right.}{}$$

input

```
int((A+B*tan(d*x+c))/tan(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output

```
(2*sqrt(a)*(32*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4
*a + 16*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4*b*i +
16*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*a*i - 8*sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*b + 20*sqrt(tan(c
 + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*a + 10*sqrt(tan(c + d*x)
)*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i + 4*sqrt(tan(c + d*x))*sqrt
(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i - 5*sqrt(tan(c + d*x))*sqrt(tan(c +
d*x)*i + 1)*tan(c + d*x)*b - 3*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)
*a - 6*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**3
+ tan(c + d*x)),x)*tan(c + d*x)**5*a*d*i - 6*int((sqrt(tan(c + d*x))*sqrt(
tan(c + d*x)*i + 1))/(tan(c + d*x)**3 + tan(c + d*x)),x)*tan(c + d*x)**5*b
*d - 6*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**3
+ tan(c + d*x)),x)*tan(c + d*x)**3*a*d*i - 6*int((sqrt(tan(c + d*x))*sqrt(
tan(c + d*x)*i + 1))/(tan(c + d*x)**3 + tan(c + d*x)),x)*tan(c + d*x)**3*b
*d))/(15*tan(c + d*x)**3*a*d*(tan(c + d*x)**2 + 1))
```

3.185 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 203

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2(-1)^{3/4}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{(A+3iB)\sqrt{\tan(c+dx)}}{2ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*(-1)^(3/4)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c
))^(1/2))/a^(3/2)/d+(1/4+1/4*I)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(
1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(3/2)/d+1/3*(I*A-B)*tan(d*x+c)^(3/2)/d/(a
+I*a*tan(d*x+c))^(3/2)+1/2*(A+3*I*B)*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c
))^(1/2)
```

Mathematica [A] (verified)

Time = 2.98 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.88

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{\sqrt{a+ia\tan(c+dx)}\left(12\sqrt[4]{-1}B\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)(1+i\tan(c+dx))^{3/2}+\sqrt{\tan(c+dx)}(-3}{6a^2d(-i+\tan(c+dx))^2}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^(3/2),x]
```

output
```
((1/4 + I/4)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]])/(a^(3/2)*d) + (Sqrt[a + I*a*Tan[c + d*x]]*(12*(-1)^
(1/4)*B*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*(1 + I*Tan[c + d*x])^(3/2)
+ Sqrt[Tan[c + d*x]]*(-3*(A + (3*I)*B) + ((-5*I)*A + 11*B)*Tan[c + d*x])))
/(6*a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int\frac{3\sqrt{\tan(c+dx)}(a(iA-B)+2iaB\tan(c+dx))}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int\frac{\sqrt{\tan(c+dx)}(a(iA-B)+2iaB\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int\frac{\sqrt{\tan(c+dx)}(a(iA-B)+2iaB\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}$$

$\Big\downarrow$ 4078

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{-\frac{\int-\frac{\sqrt{i\tan(c+dx)a+a}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{2\sqrt{\tan(c+dx)}}dx}{a^2}-\frac{a(A+3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{a(A+3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{a(A+3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$\Big\downarrow$ 4084

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{a^2(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+4iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{a(A+3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{a^2(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+4iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{a(A+3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \text{4027}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{4iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2ia^4(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{2a^2}-\frac{a(A+3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{4iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{(1-i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{2a^2}-\frac{a(A+3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \text{4082}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{4ia^3B\int\frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{(1-i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{2a^2}-\frac{a(A+3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \text{65}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{8ia^3B\int\frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}+\frac{(1-i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{2a^2}-\frac{a(A+3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{(1-i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{8(-1)^{3/4}a^{5/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{2a^2}-\frac{a(A+3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

input
```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2)
,x]
```

output
```
((I*A - B)*Tan[c + d*x]^(3/2))/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) - (((-8*
(-1)^(3/4)*a^(5/2)*B*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]])/d + ((1 - I)*a^(5/2)*(A - I*B)*ArcTanh[((1 + I)*Sqr
t[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/(2*a^2) - (a*(A +
 (3*I)*B)*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(160) = 320$.

Time = 0.29 (sec) , antiderivative size = 1223, normalized size of antiderivative = 6.02

method	result	size
derivativedivides	Expression too large to display	1223
default	Expression too large to display	1223
parts	Expression too large to display	1229

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/24*I/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(-9*I*B*(I*a)^(1/
2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+20*A*(I*a)^(1/2)*(-I*
a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-72*I*B*ln(1/2*
(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(
I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^2-3*A*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^
(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c
))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+24*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a
+44*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan
(d*x+c)^2+3*I*B*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x
+c)^3+9*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a*tan(d*x+c)^2+
24*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a
)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a*tan(d*x+c)^3-3*I*A*(I*a)^(1/2)*2^(1
/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a
-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-36*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+9*A*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I
*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(ta...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(149) = 298$.

Time = 0.17 (sec) , antiderivative size = 758, normalized size of antiderivative = 3.73

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="fricas")
```

output
```
-1/12*(3*sqrt(1/2)*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2))*e^(3*I*d
*x + 3*I*c)*log((2*I*sqrt(1/2)*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3*d^
2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - 3*sqrt(1/2)*a^2*d*sqrt((-I*A^2 - 2*A*
B + I*B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log((-2*I*sqrt(1/2)*a^2*d*sqrt((
-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^
(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + 3*a^2*
d*sqrt(4*I*B^2/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(52/605*(4*sqrt(2)*(B*e^(
3*I*d*x + 3*I*c) + B*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - (3*I*a^2*d*e^
(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(4*I*B^2/(a^3*d^2)))/(B*e^(2*I*d*x + 2*I*
c) + B)) - 3*a^2*d*sqrt(4*I*B^2/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(52/605*
(4*sqrt(2)*(B*e^(3*I*d*x + 3*I*c) + B*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)
) - (-3*I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(4*I*B^2/(a^3*d^2)))/(B
*e^(2*I*d*x + 2*I*c) + B)) - sqrt(2)*(2*(2*A + 5*I*B)*e^(4*I*d*x + 4*I*c)
+ 3*(A + 3*I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^...
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(3/2)/(I*a*(tan(c + d*x) - I))
**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeWarning, need
 to choos
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{(a+a\tan(c+d\,x)\;1\mathrm{i})^{3/2}}\,dx$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2
),x)
```

output
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2
), x)
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-16\left(\int -\frac{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{16\tan(dx+c)^3 i+16\tan(dx+c)^2+16\tan(dx+c)i+16}dx\right)a-\left(\right.}{}$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 16*int(( - sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c
+ d*x))/(16*tan(c + d*x)**3*i + 16*tan(c + d*x)**2 + 16*tan(c + d*x)*i + 1
6),x)*a - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3
)/(tan(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*b*i - int(
(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c + d*x
)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*a*i + int((sqrt(tan(c +
d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c + d*x)**3*i + tan(c
 + d*x)**2 + tan(c + d*x)*i + 1),x)*b))/a**2
```

3.186 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 150

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{(iA-B)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{(iA+5B)\sqrt{\tan(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
(-1/4-1/4*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x
+c))^(1/2))/a^(3/2)/d+1/3*(I*A-B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(3
/2)+1/6*(I*A+5*B)*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\tan(c+dx)}\left(-\frac{3i\sqrt{2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{ia\tan(c+dx)}} + \frac{6(A-iB)+}{(-i+\tan(c+}\right.}{12ad}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^(3/2),x]
```

output
```
(Sqrt[Tan[c + d*x]]*(((-3*I)*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*T
an[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/Sqrt[I*a*Tan[c + d*x]] + (6*(A
- I*B) + 2*(I*A + 5*B)*Tan[c + d*x])/((-I + Tan[c + d*x])*Sqrt[a + I*a*Tan
[c + d*x]])))/(12*a*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {3042, 4078, 27, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 4078$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{a(iA-B)-2a(A-2iB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{a(iA-B)-2a(A-2iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{a(iA-B)-2a(A-2iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}$$

$\Big\downarrow$ 4079

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int \frac{3a^2(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}-\frac{a(5B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}$$

$\Big\downarrow$ 27

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{3}{2}(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{a(5B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}$$

$\Big\downarrow$ 3042

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{3}{2}(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{a(5B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}$$

$\Big\downarrow$ 4027

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{-\frac{3ia^2(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{a(5B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}$$

$\Big\downarrow$ 218

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\left(\frac{3}{2}-\frac{3i}{2}\right)\sqrt{a}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a(5B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2)
,x]
```

output
```
((I*A - B)*Sqrt[Tan[c + d*x]])/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) - (((3/2
 - (3*I)/2)*Sqrt[a]*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])
/Sqrt[a + I*a*Tan[c + d*x]]])/d - (a*(I*A + 5*B)*Sqrt[Tan[c + d*x]])/(d*Sq
rt[a + I*a*Tan[c + d*x]]))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(120) = 240$.

Time = 0.22 (sec) , antiderivative size = 868, normalized size of antiderivative = 5.79

method	result
derivativedivides	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,\left(-16iA\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\tan(dx+c)+4A\sqrt{a\tan(dx+c)(1+\right.}{}$
default	$-\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,\left(-16iA\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\tan(dx+c)+4A\sqrt{a\tan(dx+c)(1+\right.}{}$
parts	Expression too large to display

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/24/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(-16*I*A*(-I*a)^(1
/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+4*A*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+9*I*A*2^(1/2)*ln(-(-2*2^(1/
2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/
(tan(d*x+c)+I))*a*tan(d*x+c)^2-3*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*
tan(d*x+c)^3+3*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-3*I
*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+9*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*
a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*
x+c)+I))*a*tan(d*x+c)^2-20*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)*tan(d*x+c)^2+12*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)+9*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c
)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-9*I*B*2^(1/2)*l
n(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*
tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-3*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a
)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x
+c)+I))*a-32*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+
c)-12*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2))/(a*tan(d*x+...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(110) = 220$.

Time = 0.12 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.95

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{\left(3\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{iA^2+2AB-iB^2}{a^3d^2}}e^{(3idx+3ic)}\log\left(\frac{2\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{iA^2+2AB-iB^2}{a^3d^2}}e^{(idx+ic)}+\sqrt{2}((iA+B)e^{(2idx+2ic)}+iA+B)\sqrt{\frac{a}{e^{(2idx+2\ldots}}}}{4iA+4B}\right.\right.}{\ldots}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="fricas")
```

output
```
-1/12*(3*sqrt(1/2)*a^2*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^3*d^2))*e^(3*I*d*
x + 3*I*c)*log((2*sqrt(1/2)*a^2*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^3*d^2))*
e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1)))/(4*I*A + 4*B)) - 3*sqrt(1/2)*a^2*d*sqrt((I*A^2 + 2*A*B - I
*B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-(2*sqrt(1/2)*a^2*d*sqrt((I*A^2 +
 2*A*B - I*B^2)/(a^3*d^2))*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d*x
 + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + sqrt(2)*(2*(-I
*A - 2*B)*e^(4*I*d*x + 4*I*c) + 3*(-I*A - B)*e^(2*I*d*x + 2*I*c) - I*A + B
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) + 1)))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan(c+dx))\sqrt{\tan(c+dx)}}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(tan(c + d*x))/(I*a*(tan(c + d*x) - I))*
*(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeWarning, need
 to choos
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\tan\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)}{{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2
),x)
```

output
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2
), x)
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(16\left(\int -\frac{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)}{16\tan(dx+c)^3i+16\tan(dx+c)^2+16\tan(dx+c)i+16}dx\right)ai - 16\right.}{}$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(16*int(( - sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d
*x))/(16*tan(c + d*x)**3*i + 16*tan(c + d*x)**2 + 16*tan(c + d*x)*i + 16),
x)*a*i - 16*int(( - sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*
x))/(16*tan(c + d*x)**3*i + 16*tan(c + d*x)**2 + 16*tan(c + d*x)*i + 16),x
)*b - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(t
an(c + d*x)**3*i + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*b*i + int((sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**3*i + tan(c + d*x
)**2 + tan(c + d*x)*i + 1),x)*a))/a**2
```

3.187 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 148

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\frac{1}{4}-\frac{i}{4}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{(7A+iB)\sqrt{\tan(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}}$$

output
```
(1/4-1/4*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(3/2)/d+1/3*(A+I*B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(3/
2)+1/6*(7*A+I*B)*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.03

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\tan(c+dx)}\left(\frac{3\sqrt{2}a^2(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{ia\tan(c+dx)}} + \frac{4a^3(\ldots}{(a+ia\tan\ldots}\right.}{12a^3d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^
(3/2)),x]
```

output
```
(Sqrt[Tan[c + d*x]]*((3*Sqrt[2]*a^2*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Ta
n[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/Sqrt[I*a*Tan[c + d*x]] + (4*a^3*
(A + I*B))/(a + I*a*Tan[c + d*x])^(3/2) + (2*a^2*(7*A + I*B))/Sqrt[a + I*a
*Tan[c + d*x]]))/(12*a^3*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(5A-iB)-2a(iA-B)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(5A-iB)-2a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(5A-iB)-2a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int \frac{3a^2(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}+\frac{a(7A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3}{2}(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a(7A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{2}(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a(7A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4027}$$

$$\frac{\frac{a(7A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{3ia^2(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\left(\frac{3}{2}-\frac{3i}{2}\right)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{a(7A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2))
,x]
```

output
```
((A + I*B)*Sqrt[Tan[c + d*x]])/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((3/2
 - (3*I)/2)*Sqrt[a]*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])
/Sqrt[a + I*a*Tan[c + d*x]]])/d + (a*(7*A + I*B)*Sqrt[Tan[c + d*x]])/(d*Sq
rt[a + I*a*Tan[c + d*x]]))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(118) = 236$.

Time $= 0.24$ (sec) , antiderivative size $= 868$, normalized size of antiderivative $= 5.86$

method	result
derivativedivides	$\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(3iA\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c\right.}{}$
default	$\frac{\sqrt{\tan(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(3iA\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right)a\tan(dx+c\right.}{}$
parts	Expression too large to display

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/24/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(3*I*A*2^(1/2)*ln(-(-2*
2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x
+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-9*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(
1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)
+I))*a*tan(d*x+c)^2+3*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^
3-9*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+28*I*A*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+9*A*2^(1/2)*ln(-(-2
*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*
x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+3*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^
(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c
)+I))*a+16*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+
c)-9*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-4*B*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-36*I*A*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-3*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2
)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I)
)*a+64*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)+12*
B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/a^2/(a*tan(d*x+c)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(110) = 220.

Time = 0.13 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.98

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(3\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{-iA^2-2AB+iB^2}{a^3d^2}}e^{(3idx+3ic)}\log\left(\frac{2i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{-iA^2-2A}{a^3d}}}{}\right.\right.}{}$$

input

```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="fricas")
```

output

```
1/12*(3*sqrt(1/2)*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2))*e^(3*I*d*
x + 3*I*c)*log((2*I*sqrt(1/2)*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2
))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - 3*sqrt(1/2)*a^2*d*sqrt((-I*A^2 - 2*A*B
 + I*B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log((-2*I*sqrt(1/2)*a^2*d*sqrt((-
I*A^2 - 2*A*B + I*B^2)/(a^3*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(
2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + sqrt(2)
*(2*(4*A + I*B)*e^(4*I*d*x + 4*I*c) + 3*(3*A + I*B)*e^(2*I*d*x + 2*I*c) +
A + I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sqrt{\tan{(c+dx)}}}\,dx$$

input

```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/((I*a*(tan(c + d*x) - I))**(3/2)*sqrt(tan(c
+ d*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + ia\tan(c + dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + ia\tan(c + dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\tan\left(c+d\,x\right)}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*
i + 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b + sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*i + 1)*a + sqrt(tan(c + d*x))*sqrt(tan(c + d
*x)*i + 1)*b*i - 20*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c
 + d*x))/(8*tan(c + d*x)**3*i + 8*tan(c + d*x)**2 + 8*tan(c + d*x)*i + 8),
x)*tan(c + d*x)**2*a*d - 36*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i +
1)*tan(c + d*x))/(8*tan(c + d*x)**3*i + 8*tan(c + d*x)**2 + 8*tan(c + d*x)
*i + 8),x)*tan(c + d*x)**2*b*d*i - 20*int((sqrt(tan(c + d*x))*sqrt(tan(c +
 d*x)*i + 1)*tan(c + d*x))/(8*tan(c + d*x)**3*i + 8*tan(c + d*x)**2 + 8*ta
n(c + d*x)*i + 8),x)*a*d - 36*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i
+ 1)*tan(c + d*x))/(8*tan(c + d*x)**3*i + 8*tan(c + d*x)**2 + 8*tan(c + d*
x)*i + 8),x)*b*d*i + 12*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/
(8*tan(c + d*x)**4*i + 8*tan(c + d*x)**3 + 8*tan(c + d*x)**2*i + 8*tan(c +
 d*x)),x)*tan(c + d*x)**2*a*d - 4*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x
)*i + 1))/(8*tan(c + d*x)**4*i + 8*tan(c + d*x)**3 + 8*tan(c + d*x)**2*i +
 8*tan(c + d*x)),x)*tan(c + d*x)**2*b*d*i + 12*int((sqrt(tan(c + d*x))*sqr
t(tan(c + d*x)*i + 1))/(8*tan(c + d*x)**4*i + 8*tan(c + d*x)**3 + 8*tan(c
+ d*x)**2*i + 8*tan(c + d*x)),x)*a*d - 4*int((sqrt(tan(c + d*x))*sqrt(tan(
c + d*x)*i + 1))/(8*tan(c + d*x)**4*i + 8*tan(c + d*x)**3 + 8*tan(c + d*x)
**2*i + 8*tan(c + d*x)),x)*b*d*i))/(2*a**2*d*(tan(c + d*x)**2 + 1))
```

3.188 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 194

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}+\frac{11A+5iB}{6ad\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{6a^2d\sqrt{\tan(c+dx)}}$$

output

```
(1/4+1/4*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(3/2)/d+1/3*(A+I*B)/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/
2)+1/6*(11*A+5*I*B)/a/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/6*(25*
A+7*I*B)*(a+I*a*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.86

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\frac{3\sqrt{2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\tan(c+dx)}{\sqrt{ia\tan(c+dx)}} - \frac{2i(-12A+(-39iA+9B)}{(-i+\tan(c}}{12ad\sqrt{\tan(c+dx)}}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^
(3/2)),x]
```

output
```
((3*Sqrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*
a*Tan[c + d*x]]]*Tan[c + d*x])/Sqrt[I*a*Tan[c + d*x]] - ((2*I)*(-12*A + ((
-39*I)*A + 9*B)*Tan[c + d*x] + (25*A + (7*I)*B)*Tan[c + d*x]^2))/((-I + Ta
n[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]))/(12*a*d*Sqrt[Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\int \frac{a(7A+iB)-4a(iA-B)\tan(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{3a^2} + \frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a(7A+iB)-4a(iA-B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(7A+iB)-4a(iA-B)\tan(c+dx)}{\tan(c+dx)^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 4079

$$\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(25A+7iB)-2a^2(11iA-5B)\tan(c+dx)\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(25A+7iB)-2a^2(11iA-5B)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(25A+7iB)-2a^2(11iA-5B)\tan(c+dx)\right)}{\tan(c+dx)^{3/2}}dx}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 4081

$$\frac{\frac{\frac{2\int \frac{3a^3(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^2(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{3a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4027}$$

$$\frac{\frac{-\frac{6ia^4(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^2(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{(3-3i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2))
,x]
```

output
```
(A + I*B)/(3*d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((a*(11*
A + (5*I)*B))/(d*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((3 - 3
*I)*a^(5/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ I*a*Tan[c + d*x]]])/d - (2*a^2*(25*A + (7*I)*B)*Sqrt[a + I*a*Tan[c + d*x
]])/(d*Sqrt[Tan[c + d*x]]))/(2*a^2))/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(156) = 312$.

Time = 0.23 (sec) , antiderivative size = 931, normalized size of antiderivative = 4.80

method	result	size
derivativedivides	Expression too large to display	931
default	Expression too large to display	931
parts	Expression too large to display	967

input

```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_R
ETURNVERBOSE)
```

output

```
1/24/d*(a*(1+I*tan(d*x+c)))^(1/2)*(3*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1
/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+
I))*a*tan(d*x+c)^4+9*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)
^3-3*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-9*I*B*2^(1/2)*l
n(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*
tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+28*I*B*(-I*a)^(1/2)*(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3+9*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)
^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+
c)+I))*a*tan(d*x+c)^3-3*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x
+c)-256*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^
2+9*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+100*A*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3-36*I*B*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)-3*B*2^(1/2)*ln(-(-2*2^(1/
2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/
(tan(d*x+c)+I))*a*tan(d*x+c)+64*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*tan(d*x+c)^2+48*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(146) = 292.

Time = 0.12 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.63

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{3\sqrt{\frac{1}{2}}\left(a^2de^{(5i\,dx+5i\,c)}-a^2de^{(3i\,dx+3i\,c)}\right)\sqrt{\frac{i\,A^2+2\,AB-i\,B^2}{a^3d^2}}\log\left(\frac{2\sqrt{}}{}\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="fricas")
```

output
```
1/12*(3*sqrt(1/2)*(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c))*
sqrt((I*A^2 + 2*A*B - I*B^2)/(a^3*d^2))*log((2*sqrt(1/2)*a^2*d*sqrt((I*A^2
 + 2*A*B - I*B^2)/(a^3*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d
*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - 3*sqrt(1/2)*
(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(3*I*d*x + 3*I*c))*sqrt((I*A^2 + 2*A*
B - I*B^2)/(a^3*d^2))*log(-(2*sqrt(1/2)*a^2*d*sqrt((I*A^2 + 2*A*B - I*B^2)
/(a^3*d^2))*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A
 + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - sqrt(2)*(2*(19*I*A - 4*B)*e^(
6*I*d*x + 6*I*c) - (-25*I*A + B)*e^(4*I*d*x + 4*I*c) + 2*(-7*I*A + 4*B)*e^
(2*I*d*x + 2*I*c) - I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(a^2*d*e^(5*I*d*x + 5*I
*c) - a^2*d*e^(3*I*d*x + 3*I*c))
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\tan^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/((I*a*(tan(c + d*x) - I))**(3/2)*tan(c + d*x
)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia\tan(c + dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia\tan(c + dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx=anti$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
anti
```

3.189 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 240

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} + \frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{5A+3iB}{2ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{(21A+11iB)\sqrt{a+ia\tan(c+dx)}}{6a^2d\tan^{\frac{3}{2}}(c+dx)} + \frac{(39iA-25B)\sqrt{a+ia\tan(c+dx)}}{6a^2d\sqrt{\tan(c+dx)}}$$

output

```
(1/4+1/4*I)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(3/2)/d+1/3*(A+I*B)/d/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/
2)+1/2*(5*A+3*I*B)/a/d/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2)-1/6*(21*A
+11*I*B)*(a+I*a*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(3/2)+1/6*(39*I*A-25*B)
*(a+I*a*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.77

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{-\frac{3\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\tan^2(c+dx)}{\sqrt{ia\tan(c+dx)}} + \frac{8iA+24(A+iB)\tan(\cdots}{\cdots}}{12ad\tan^{\frac{3}{2}}(c+dx\cdots}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^
(3/2)),x]
```

output
```
((-3*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]]*Tan[c + d*x]^2)/Sqrt[I*a*Tan[c + d*x]] + ((8*I)*A + 24*(
A + I*B)*Tan[c + d*x] + ((114*I)*A - 78*B)*Tan[c + d*x]^2 - 2*(39*A + (25*
I)*B)*Tan[c + d*x]^3)/((-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]))/(1
2*a*d*Tan[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.395, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{3(a(3A+iB)-2a(iA-B)\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{3a^2} + \frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\int \frac{a(3A+iB)-2a(iA-B)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{a(3A+iB)-2a(iA-B)\tan(c+dx)}{\tan(c+dx)^{5/2}\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 4079

$$\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(21A+11iB)-4a^2(5iA-3B)\tan(c+dx)\right)}{2\tan^{\frac{5}{2}}(c+dx)}dx}{a^2}+\frac{a(5A+3iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(21A+11iB)-4a^2(5iA-3B)\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}dx}{2a^2}+\frac{a(5A+3iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(21A+11iB)-4a^2(5iA-3B)\tan(c+dx)\right)}{\tan(c+dx)^{5/2}}dx}{2a^2}+\frac{a(5A+3iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 4081

$$\frac{\frac{\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((39iA-25B)a^3+2(21A+11iB)\tan(c+dx)a^3\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^2(21A+11iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a(5A+3iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((39iA-25B)a^3+2(21A+11iB)\tan(c+dx)a^3\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^2(21A+11iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a(5A+3iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((39iA-25B)a^3+2(21A+11iB)\tan(c+dx)a^3\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a^2(21A+11iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a(5A+3iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 4081

$$\frac{\frac{-\frac{\frac{2\int \frac{3a^4(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^3(-25B+39iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(21A+11iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a(5A+3iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{3a^3(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(-25B+39iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(21A+11iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a(5A+3iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{-\frac{3a^3(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(-25B+39iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(21A+11iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a(5A+3iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4027$$

$$\frac{-\frac{-\frac{-\frac{6ia^5(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^3(-25B+39iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(21A+11iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a(5A-}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{}}}{2a^2}$$

$$\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{-\frac{2a^2(21A+11iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{\frac{(3-3i)a^{7/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(-25B+39iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}}{2a^2}+\frac{a(5A+3i}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+}}}{2a^2}$$

$$\frac{A+iB}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2))
,x]
```

output
```
(A + I*B)/(3*d*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((a*(5*A
 + (3*I)*B))/(d*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((-2*a^2*
(21*A + (11*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) - (
((3 - 3*I)*a^(7/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/
Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^3*((39*I)*A - 25*B)*Sqrt[a + I*a*Tan
[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(2*a^2))/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1011 vs. $2(194) = 388$.

Time = 0.24 (sec) , antiderivative size = 1012, normalized size of antiderivative = 4.22

method	result	size
derivativedivides	Expression too large to display	1012
default	Expression too large to display	1012
parts	Expression too large to display	1052

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/24/d*(a*(1+I*tan(d*x+c)))^(1/2)/a^2/tan(d*x+c)^(3/2)*(-9*I*B*2^(1/2)*ln
(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*t
an(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-100*B*(-I*a)^(1/2)*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^4-9*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)
^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+
c)+I))*a*tan(d*x+c)^3+3*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c
)^5+384*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3+
256*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+3*
I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+9*A*2^(1/2)*ln(-(-
2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d
*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+204*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-48*I*B*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)*(-I*a)^(1/2)-276*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)*tan(d*x+c)^2-9*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(
d*x+c)^3-32*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c
)+3*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5-3*A*2^(1/2)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(182) = 364$.

Time = 0.12 (sec) , antiderivative size = 569, normalized size of antiderivative = 2.37

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia\tan(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="fricas")
```

output
```
-1/12*(3*sqrt(1/2)*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c
) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2))*lo
g((2*I*sqrt(1/2)*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2))*e^(I*d*x +
 I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
 1)))/(4*I*A + 4*B)) - 3*sqrt(1/2)*(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^
(5*I*d*x + 5*I*c) + a^2*d*e^(3*I*d*x + 3*I*c))*sqrt((-I*A^2 - 2*A*B + I*B^
2)/(a^3*d^2))*log((-2*I*sqrt(1/2)*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3
*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + sqrt(2)*(2*(26*A + 19*I*B)*e^(8*I*
d*x + 8*I*c) - (35*A + 13*I*B)*e^(6*I*d*x + 6*I*c) - 3*(23*A + 13*I*B)*e^(
4*I*d*x + 4*I*c) + (19*A + 13*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1)))/(a^2*d*e^(7*I*d*x + 7*I*c) - 2*a^2*d*e^(5*I*d*x + 5*I*c) + a
^2*d*e^(3*I*d*x + 3*I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia\tan(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{5/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*(2912*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3
*a*i + 1120*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*b
- 1456*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*a + 560
*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*b*i + 1820*sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*a*i + 700*sqrt(tan(
c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*b - 546*sqrt(tan(c + d*x))
*sqrt(tan(c + d*x)*i + 1)*a + 418*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i +
 1)*b*i - 455*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(tan(c + d
*x)**6*i + tan(c + d*x)**5 + tan(c + d*x)**4*i + tan(c + d*x)**3),x)*tan(c
 + d*x)**4*a*d + 315*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1))/(ta
n(c + d*x)**6*i + tan(c + d*x)**5 + tan(c + d*x)**4*i + tan(c + d*x)**3),x
)*tan(c + d*x)**4*b*d*i - 455*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i
+ 1))/(tan(c + d*x)**6*i + tan(c + d*x)**5 + tan(c + d*x)**4*i + tan(c + d
*x)**3),x)*tan(c + d*x)**2*a*d + 315*int((sqrt(tan(c + d*x))*sqrt(tan(c +
d*x)*i + 1))/(tan(c + d*x)**6*i + tan(c + d*x)**5 + tan(c + d*x)**4*i + ta
n(c + d*x)**3),x)*tan(c + d*x)**2*b*d*i - 1183*int((sqrt(tan(c + d*x))*sqr
t(tan(c + d*x)*i + 1))/(tan(c + d*x)**4*i + tan(c + d*x)**3 + tan(c + d*x)
**2*i + tan(c + d*x)),x)*tan(c + d*x)**4*a*d + 819*int((sqrt(tan(c + d*x))
*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**4*i + tan(c + d*x)**3 + tan(c +
d*x)**2*i + tan(c + d*x)),x)*tan(c + d*x)**4*b*d*i - 1183*int((sqrt(tan...
```

3.190 $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 249

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\sqrt[4]{-1}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{\left(\frac{1}{8}+\frac{i}{8}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(iA-B)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{6ad(a+ia\tan(c+dx))^{3/2}} - \frac{(iA-7B)\sqrt{\tan(c+dx)}}{4a^2d\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*(-1)^(1/4)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c
))^(1/2))/a^(5/2)/d+(1/8+1/8*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(
1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(5/2)/d+1/5*(I*A-B)*tan(d*x+c)^(5/2)/d/(a
+I*a*tan(d*x+c))^(5/2)+1/6*(A+3*I*B)*tan(d*x+c)^(3/2)/a/d/(a+I*a*tan(d*x+c
))^(3/2)-1/4*(I*A-7*B)*tan(d*x+c)^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 3.82 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.19

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{240\sqrt[4]{-1}\sqrt{a}B\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sec^3(c+dx)(-i\cos$$

input
```
Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^(5/2),x]
```

output
```
(240*(-1)^(1/4)*Sqrt[a]*B*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d
*x]^3*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]) - (1 + I)*Sec[c + d*x]^2*
Sqrt[1 + I*Tan[c + d*x]]*((-1 - I)*Sqrt[a]*(-11*A - (21*I)*B + 2*(13*A + (
63*I)*B)*Cos[2*(c + d*x)] + (20*I)*(A + (6*I)*B)*Sin[2*(c + d*x)])*Sqrt[Ta
n[c + d*x]] + 15*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sq
rt[a + I*a*Tan[c + d*x]]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*Sqrt[a +
 I*a*Tan[c + d*x]]))/(120*a^(5/2)*d*Sqrt[1 + I*Tan[c + d*x]]*(-I + Tan[c +
 d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.447$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4078, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{5/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{5\tan^{\frac{3}{2}}(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan(c+dx)^{3/2}(a(iA-B)+2iaB\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{2a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{\int -\frac{3\sqrt{\tan(c+dx)}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{\sqrt{\tan(c+dx)}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{\sqrt{\tan(c+dx)}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((iA-7B)a^3+8iB\tan(c+dx)a^3\right)}{2\sqrt{\tan(c+dx)}}dx}{a^2}}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((iA-7B)a^3+8iB\tan(c+dx)a^3\right)}{\sqrt{\tan(c+dx)}}dx}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}$$

↓ 3042

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((iA-7B)a^3+8iB\tan(c+dx)a^3\right)}{\sqrt{\tan(c+dx)}}dx}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}$$

↓ 4084

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a^3(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-8a^2B\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}$$

↓ 3042

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a^3(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-8a^2B\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}$$

↓ 4027

$$\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{-8a^2B\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2ia^5(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}}{2a^2}$$

↓ 218

$$\frac{\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-8a^2B\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c+\ldots}{3d(a+ia\tan(c+dx))\ldots}}{2a^2}$$

$\Big\downarrow$ 4082

$$\frac{\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{8a^4B\int\frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}\ldots}{3d(a+ia\tan(c+\ldots}}{2a^2}$$

$\Big\downarrow$ 65

$$\frac{\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{16a^4B\int\frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(\ldots}{3d(a+ia\tan(c+d\ldots}}{2a^2}$$

$\Big\downarrow$ 216

$$\frac{\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{16\sqrt[4]{-1}a^{7/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{2a^2}}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}\ldots}{3d(a+ia\tan(c+\ldots}}{2a^2}$$

input
```
Int[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2)
,x]
```

output
```
((I*A - B)*Tan[c + d*x]^(5/2))/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) - (-1/3*
(a*(A + (3*I)*B)*Tan[c + d*x]^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (-
1/2*((16*(-1)^(1/4)*a^(7/2)*B*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]
])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((1 - I)*a^(7/2)*(I*A + B)*ArcTanh[((1
 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/a^2 + (a
^2*(I*A - 7*B)*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a^2)
)/(2*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1529 vs. $2(198) = 396$.

Time = 0.30 (sec) , antiderivative size = 1530, normalized size of antiderivative = 6.14

method	result	size
parts	Expression too large to display	1530
derivativedivides	Expression too large to display	1542
default	Expression too large to display	1542

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/240*A/d*(a*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^(1/2)/a^3*(148*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3+60*I*2^(1/2)*ln((2*2
^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+
c))/(tan(d*x+c)+I))*tan(d*x+c)^3*a-15*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*t
an(d*x+c)^4*a-60*I*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*tan(d*x+c)*a-308*I*(
-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+90*ln((2*2^
(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c
))/(tan(d*x+c)+I))*2^(1/2)*tan(d*x+c)^2*a+60*I*(-I*a)^(1/2)*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)-15*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a-220*tan(d*
x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)/(I-tan(d*x+c))^4-1/80*B/d*(a*(1+I*tan(d*
x+c)))^(1/2)*tan(d*x+c)^(1/2)*(5*I*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a
)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x
+c)+I))*tan(d*x+c)^4*a-30*I*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))
*tan(d*x+c)^2*a-320*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*tan(d*x+c)^3*a+19...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(185) = 370$.

Time = 0.18 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.12

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
1/120*(15*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(5*I*d
*x + 5*I*c)*log((2*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))
*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) + 1)))/(4*I*A + 4*B)) - 15*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B -
 I*B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-(2*sqrt(1/2)*a^3*d*sqrt((I*A^2
 + 2*A*B - I*B^2)/(a^5*d^2))*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d
*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - 30*a^3*d*sqr
t(-4*I*B^2/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(52/605*(4*sqrt(2)*(B*e^(3*I*
d*x + 3*I*c) + B*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((
-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) + (3*a^3*d*e^(2*I*d
*x + 2*I*c) - a^3*d)*sqrt(-4*I*B^2/(a^5*d^2)))/(B*e^(2*I*d*x + 2*I*c) + B)
) + 30*a^3*d*sqrt(-4*I*B^2/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(52/605*(4*sq
rt(2)*(B*e^(3*I*d*x + 3*I*c) + B*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)) - (
3*a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt(-4*I*B^2/(a^5*d^2)))/(B*e^(2*I*d
*x + 2*I*c) + B)) + sqrt(2)*((-23*I*A + 123*B)*e^(6*I*d*x + 6*I*c) - 6*(2*
I*A - 17*B)*e^(4*I*d*x + 4*I*c) - 2*(-4*I*A + 9*B)*e^(2*I*d*x + 2*I*c) - 3
*I*A + 3*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I...
```

Sympy [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{(A+B\tan{(c+dx)})\tan^{\frac{5}{2}}{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(5/2)/(I*a*(tan(c + d*x) - I))
**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeWarning, need
 to choos
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

input
```
int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2
),x)
```

output
```
int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2
), x)
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^3}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}\right.}{}$$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int((sqrt(tan(c + d*x))*tan(c + d*x)**3)/(sqrt(tan(c + d*x)*i + 1)*ta
n(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d
*x)*i + 1)),x)*b + int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(sqrt(tan(c +
d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i -
sqrt(tan(c + d*x)*i + 1)),x)*a))/(sqrt(a)*a**2)
```

3.191 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 194

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(\frac{1}{8}+\frac{i}{8}\right)(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(iA-B)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(A+11iB)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{(13A-37iB)\sqrt{\tan(c+dx)}}{60a^2d\sqrt{a+ia\tan(c+dx)}}$$

output
```
(1/8+1/8*I)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(5/2)/d+1/5*(I*A-B)*tan(d*x+c)^(3/2)/d/(a+I*a*tan(d*x+c))^(5/
2)+1/30*(A+11*I*B)*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)+1/60*(13*
A-37*I*B)*tan(d*x+c)^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.15

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\sec^2(c+dx)\sqrt{\tan(c+dx)}\Big(2(A+11iB+2(7A-13iB)\cos(2(c+dx))+20(iA+B)\sin(2(c+dx)))\sqrt{\ \ }}{120a^2d\sqrt{ia\tan(c+dx)}(-i}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^(5/2),x]
```

output
```
-1/120*(Sec[c + d*x]^2*Sqrt[Tan[c + d*x]]*(2*(A + (11*I)*B + 2*(7*A - (13*
I)*B)*Cos[2*(c + d*x)] + 20*(I*A + B)*Sin[2*(c + d*x)])*Sqrt[I*a*Tan[c + d
*x]] - 15*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[
a + I*a*Tan[c + d*x]]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*Sqrt[a + I*
a*Tan[c + d*x]]))/(a^2*d*Sqrt[I*a*Tan[c + d*x]]*(-I + Tan[c + d*x])^2*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4078, 27, 3042, 4078, 27, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\sqrt{\tan(c+dx)}(3a(iA-B)-2a(A-4iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\sqrt{\tan(c+dx)}(3a(iA-B)-2a(A-4iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\sqrt{\tan(c+dx)}(3a(iA-B)-2a(A-4iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{\int -\frac{(A+11iB)a^2+2(7iA+13B)\tan(c+dx)a^2}{2\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}-\frac{a(A+11iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{(A+11iB)a^2+2(7iA+13B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}-\frac{a(A+11iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{(A+11iB)a^2+2(7iA+13B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}-\frac{a(A+11iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{4079}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{\int \frac{15a^3(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}-\frac{a^2(13A-37iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(A+11iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{15}{2}a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{a^2(13A-37iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(A+11iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{15}{2}a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{a^2(13A-37iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(A+11iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{4027}$$

$$\frac{\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{15ia^3(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{a^2(13A-37iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(A+11iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\left(\frac{15}{2}-\frac{15i}{2}\right)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2(13A-37iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(A+11iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

input
```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2)
,x]
```

output
```
((I*A - B)*Tan[c + d*x]^(3/2))/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) - (-1/3*
(a*(A + (11*I)*B)*Sqrt[Tan[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (
((15/2 - (15*I)/2)*a^(3/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (a^2*(13*A - (37*I)*B)*Sqrt[Tan[c
 + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]))/(6*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1095 vs. $2(156) = 312$.

Time = 0.22 (sec) , antiderivative size = 1096, normalized size of antiderivative = 5.65

method	result	size
derivativedivides	Expression too large to display	1096
default	Expression too large to display	1096
parts	Expression too large to display	1143

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/240/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(15*I*A*2^(1/2)*ln
(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*t
an(d*x+c))/(tan(d*x+c)+I))*a-148*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(
-I*a)^(1/2)*tan(d*x+c)^3-60*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*
a)^(1/2)+15*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-212*A*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+308*I*B*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+60*I*B*2^(1/2)
*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*
a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+60*A*2^(1/2)*ln(-(-2*2^(1/2)*(-
I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(
d*x+c)+I))*a*tan(d*x+c)^3-52*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I
*a)^(1/2)*tan(d*x+c)^3-90*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d
*x+c)^2-90*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+15*I*A*2^
(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I
*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+220*I*A*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)-60*A*2^(1/2)*ln(-(-2*2^(1/2)
*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(144) = 288$.

Time = 0.12 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.37

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{-iA^2-2AB+iB^2}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(\frac{2i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{-iA^2-2AB+iB^2}{a^5d^2}}e^{(i\,dx+i\,c)}+\sqrt{2}\left((iA+B)e^{(2i\,dx+2i\,c)}+iA+B\right)\sqrt{\frac{\cdots}{e^{(2\cdots}}}}{4iA+4B}\right.\right.}{\cdots}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/120*(15*sqrt(1/2)*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*e^(5*I
*d*x + 5*I*c)*log((2*I*sqrt(1/2)*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^5*
d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - 15*sqrt(1/2)*a^3*d*sqrt((-I*A^2 - 2
*A*B + I*B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log((-2*I*sqrt(1/2)*a^3*d*sqr
t((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)
*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - sqr
t(2)*((17*A - 23*I*B)*e^(6*I*d*x + 6*I*c) + 6*(3*A - 2*I*B)*e^(4*I*d*x + 4
*I*c) - 2*(A - 4*I*B)*e^(2*I*d*x + 2*I*c) - 3*A - 3*I*B)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) +
1)))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(3/2)/(I*a*(tan(c + d*x) - I))
**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeWarning, need
 to choos
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2
),x)
```

output
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2
), x)
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}\cdots}\right.}{}$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(sqrt(tan(c + d*x)*i + 1)*ta
n(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d
*x)*i + 1)),x)*b + int((sqrt(tan(c + d*x))*tan(c + d*x))/(sqrt(tan(c + d*x
)*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqr
t(tan(c + d*x)*i + 1)),x)*a))/(sqrt(a)*a**2)
```

3.192 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 196

$$\begin{aligned}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx =& -\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(iA-B)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{(3iA+7B)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} - \frac{(3iA-13B)\sqrt{\tan(c+dx)}}{60a^2d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output
```
(-1/8-1/8*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x
+c))^(1/2))/a^(5/2)/d+1/5*(I*A-B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(5
/2)+1/30*(3*I*A+7*B)*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)-1/60*(3
*I*A-13*B)*tan(d*x+c)^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^2(c+dx)\sqrt{\tan(c+dx)}\Big(-2i(9A-iB+2(3A-7iB)\cos(2$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^(5/2),x]
```

output
```
(Sec[c + d*x]^2*Sqrt[Tan[c + d*x]]*((-2*I)*(9*A - I*B + 2*(3*A - (7*I)*B)*
Cos[2*(c + d*x)] + 20*B*Sin[2*(c + d*x)])*Sqrt[I*a*Tan[c + d*x]] + 15*Sqrt
[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c
+ d*x]]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]
]))/(120*a^2*d*Sqrt[I*a*Tan[c + d*x]]*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*T
an[c + d*x]])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 4078, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{a(iA-B)-2a(2A-3iB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{a(iA-B)-2a(2A-3iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{a(iA-B)-2a(2A-3iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}$$

$$\Big\downarrow \; 4079$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{a^2(9iA+B)-2a^2(3A-7iB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{a^2(9iA+B)-2a^2(3A-7iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{a^2(9iA+B)-2a^2(3A-7iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \; 4079$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{\int \frac{15a^3(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}+\frac{a^2(-13B+3iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{15}{2}a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(-13B+3iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{15}{2}a(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(-13B+3iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}}{}$$

$$\Big\downarrow \text{ 4027}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-13B+3iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{15ia^3(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{6a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{\left(\frac{15}{2}-\frac{15i}{2}\right)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{a^2(-13B+3iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2)
,x]
```

output
```
((I*A - B)*Sqrt[Tan[c + d*x]])/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) - (-1/3*
(a*((3*I)*A + 7*B)*Sqrt[Tan[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) +
(((15/2 - (15*I)/2)*a^(3/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c
+ d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + (a^2*((3*I)*A - 13*B)*Sqrt[Tan[c
 + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]))/(6*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] && !GtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1095 vs. $2(158) = 316$.

Time = 0.23 (sec) , antiderivative size = 1096, normalized size of antiderivative = 5.59

method	result	size
derivativedivides	Expression too large to display	1096
default	Expression too large to display	1096
parts	Expression too large to display	1143

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/240/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(15*I*B*2^(1/2)*ln
(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*t
an(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-12*A*(-I*a)^(1/2)*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3+60*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)
^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+
c)+I))*a*tan(d*x+c)^3-15*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+
c)^4-212*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2
-60*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-52*I*B*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+60*B*2^(1/2)*ln(-(-
2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d
*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+15*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a
)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x
+c)+I))*a-90*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+90*A*
2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+220*I*B*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+12*I*A*(a*tan(d*x+c)*(1+I*
tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2-60*B*2^(1/2)*ln(-(-2*2^(1/...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(146) = 292.

Time = 0.15 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{iA^2+2AB-iB^2}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(\frac{2\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{iA^2+2AB-iB^2}{a^5d^2}}e^{(i\,dx+i\,c)}+\sqrt{2}\left((iA+B)e^{(2i\,dx+2i\,c)}+iA+B\right)\sqrt{\frac{a}{e^{(2i\,dx+}}}}{4iA+4B}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/120*(15*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(5*I*
d*x + 5*I*c)*log((2*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2)
)*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) + 1)))/(4*I*A + 4*B)) - 15*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B
- I*B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-(2*sqrt(1/2)*a^3*d*sqrt((I*A^
2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*
d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - sqrt(2)*((3
*I*A + 17*B)*e^(6*I*d*x + 6*I*c) - 6*(-2*I*A - 3*B)*e^(4*I*d*x + 4*I*c) -
2*(-6*I*A + B)*e^(2*I*d*x + 2*I*c) + 3*I*A - 3*B)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))*e^
(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\sqrt{\tan(c+dx)}}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(tan(c + d*x))/(I*a*(tan(c + d*x) - I))*
*(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeWarning, need
 to choos
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\tan\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)}{{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2
),x)
```

output
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2
), x)
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\sqrt{\tan(dx+c)}}{\sqrt{\tan(dx+c)i+1}\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\tan(dx+c)i-\sqrt{\tan(dx+c)i+}}\right.}{}$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(sqrt(tan(c + d*x))/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*
sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a +
 int((sqrt(tan(c + d*x))*tan(c + d*x))/(sqrt(tan(c + d*x)*i + 1)*tan(c + d
*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i +
 1)),x)*b))/(sqrt(a)*a**2)
```

3.193 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 194

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(\frac{1}{8}-\frac{i}{8}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}} + \frac{(13A+3iB)\sqrt{\tan(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}} + \frac{(67A-3iB)\sqrt{\tan(c+dx)}}{60a^2d\sqrt{a+ia\tan(c+dx)}}$$

output
```
(1/8-1/8*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(5/2)/d+1/5*(A+I*B)*tan(d*x+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(5/
2)+1/30*(13*A+3*I*B)*tan(d*x+c)^(1/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)+1/60*(6
7*A-3*I*B)*tan(d*x+c)^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.25

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\tan(c+dx)}\left(24a^5(A+iB)\sqrt{ia\tan(c+dx)}-(a+ia\tan\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^
(5/2)),x]
```

output
```
(Sqrt[Tan[c + d*x]]*(24*a^5*(A + I*B)*Sqrt[I*a*Tan[c + d*x]] - (a + I*a*Ta
n[c + d*x])*(-4*a^4*(13*A + (3*I)*B)*Sqrt[I*a*Tan[c + d*x]] + (a + I*a*Tan
[c + d*x])*(2*a^3*(-67*A + (3*I)*B)*Sqrt[I*a*Tan[c + d*x]] - 15*Sqrt[2]*a^
3*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c +
d*x]]]*Sqrt[a + I*a*Tan[c + d*x]]))))/(120*a^5*d*Sqrt[I*a*Tan[c + d*x]]*(a
 + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(9A-iB)-4a(iA-B)\tan(c+dx)}{2\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(9A-iB)-4a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(9A-iB)-4a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 4079$$

$$\frac{\frac{\int \frac{a^2(41A-9iB)-2a^2(13iA-3B)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{a^2(41A-9iB)-2a^2(13iA-3B)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\int \frac{a^2(41A-9iB)-2a^2(13iA-3B)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 4079$$

$$\frac{\frac{\frac{\int \frac{15a^3(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}+\frac{a^2(67A-3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{15}{2}a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(67A-3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{15}{2}a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(67A-3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 4027$$

$$\frac{\frac{\frac{a^2(67A-3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{15ia^3(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\left(\frac{15}{2}-\frac{15i}{2}\right)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{a^2(67A-3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2))
,x]
```

output
```
((A + I*B)*Sqrt[Tan[c + d*x]])/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((a*(1
3*A + (3*I)*B)*Sqrt[Tan[c + d*x]])/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((
(15/2 - (15*I)/2)*a^(3/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c +
d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + (a^2*(67*A - (3*I)*B)*Sqrt[Tan[c +
 d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]))/(6*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1095 vs. $2(156) = 312$.

Time = 0.25 (sec) , antiderivative size = 1096, normalized size of antiderivative = 5.65

method	result	size
derivativedivides	Expression too large to display	1096
default	Expression too large to display	1096
parts	Expression too large to display	1143

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/240/d*tan(d*x+c)^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(15*I*A*2^(1/2)*ln(-(
-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(
d*x+c))/(tan(d*x+c)+I))*a-60*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I
*a)^(1/2)+15*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-12*I*B*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+60*I*B*2^(
1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*
a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+60*A*2^(1/2)*ln(-(-2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(
tan(d*x+c)+I))*a*tan(d*x+c)^3+268*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(-I*a)^(1/2)*tan(d*x+c)^3-90*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*
tan(d*x+c)^2-90*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*
tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+12*B
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+15*I*A*2^
(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I
*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-1060*I*A*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)-60*A*2^(1/2)*ln(-(-2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(
tan(d*x+c)+I))*a*tan(d*x+c)+908*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(146) = 292$.

Time = 0.12 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.37

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + ia\tan(c + dx))^{5/2}}\,dx = \frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{-iA^2-2AB+iB^2}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(\frac{2i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{-iA^2-2}{a^5}}}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
1/120*(15*sqrt(1/2)*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*e^(5*I*
d*x + 5*I*c)*log((2*I*sqrt(1/2)*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^5*d
^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - 15*sqrt(1/2)*a^3*d*sqrt((-I*A^2 - 2*
A*B + I*B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log((-2*I*sqrt(1/2)*a^3*d*sqrt
((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*
e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + sqrt
(2)*((83*A + 3*I*B)*e^(6*I*d*x + 6*I*c) + 6*(17*A + 2*I*B)*e^(4*I*d*x + 4*
I*c) + 2*(11*A + 6*I*B)*e^(2*I*d*x + 2*I*c) + 3*A + 3*I*B)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
+ 1)))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + ia\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + ia\tan(c + dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\tan(c+d\,x)}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\tan(dx+c)}{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+}}\right.}{}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(tan(c + d*x)/(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c +
 d*x)**2 - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i -
sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)),x)*b + int(1/(sqrt(tan(c + d*
x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x))*sqrt(t
an(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i
 + 1)),x)*a))/(sqrt(a)*a**2)
```

3.194 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 240

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{\left(\frac{1}{8}+\frac{i}{8}\right)(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}\\
&+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}+\frac{17A+7iB}{30ad\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\\
&+\frac{151A+41iB}{60a^2d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{(317A+67iB)\sqrt{a+ia\tan(c+dx)}}{60a^3d\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
(1/8+1/8*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(5/2)/d+1/5*(A+I*B)/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/
2)+1/30*(17*A+7*I*B)/a/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2)+1/60*(1
51*A+41*I*B)/a^2/d/tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/60*(317*A+6
7*I*B)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.67 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^2(c+dx)\left(\frac{15\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)(-i\cos(2(c+dx))+}{\sqrt{ia\tan(c+dx)}}\right.}{12(}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^
(5/2)),x]
```

output
```
(Sec[c + d*x]^2*((15*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d
*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)
])*Tan[c + d*x])/Sqrt[I*a*Tan[c + d*x]] + ((2*I)*((340*I)*A - 80*B + (149*
A + (19*I)*B)*Tan[c + d*x] + Cos[2*(c + d*x)]*((-460*I)*A + 80*B + (466*A
+ (86*I)*B)*Tan[c + d*x])))/Sqrt[a + I*a*Tan[c + d*x]]))/(120*a^2*d*Sqrt[T
an[c + d*x]]*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.395$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(11A+iB)-6a(iA-B)\tan(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(11A+iB)-6a(iA-B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(11A+iB)-6a(iA-B)\tan(c+dx)}{\tan(c+dx)^{3/2}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int \frac{a^2(83A+13iB)-4a^2(17iA-7B)\tan(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a^2(83A+13iB)-4a^2(17iA-7B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(83A+13iB)-4a^2(17iA-7B)\tan(c+dx)}{\tan(c+dx)^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(317A+67iB)-2a^3(151iA-41B)\tan(c+dx)\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(317A+67iB)-2a^3(151iA-41B)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(317A+67iB)-2a^3(151iA-41B)\tan(c+dx)\right)}{\tan(c+dx)^{3/2}}dx}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 4081$$

$$\frac{\frac{\frac{\frac{2\int \frac{15a^4(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^3(317A+67iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{15a^3(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(317A+67iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{15a^3(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(317A+67iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 4027}$$

$$\frac{\frac{-\frac{\frac{30ia^5(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^3(317A+67iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{\frac{(15-15i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(317A+67iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2))
,x]
```

output
```
(A + I*B)/(5*d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)) + ((a*(17*
A + (7*I)*B))/(3*d*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((a^
2*(151*A + (41*I)*B))/(d*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) +
(((15 - 15*I)*a^(7/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]
])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^3*(317*A + (67*I)*B)*Sqrt[a + I*a
*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(2*a^2))/(6*a^2))/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1157 vs. 2(194) = 388.

Time = 0.24 (sec) , antiderivative size = 1158, normalized size of antiderivative = 4.82

method	result	size
derivativedivides	Expression too large to display	1158
default	Expression too large to display	1158
parts	Expression too large to display	1188

input

```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output

```
-1/240/d*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(15*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-
I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(
d*x+c)+I))*a*tan(d*x+c)^5+1268*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I
*a)^(1/2)*tan(d*x+c)^4-1060*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*
a)^(1/2)*tan(d*x+c)^2-15*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+
c)^5+908*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3
+15*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-4468*I*A*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+60*B*2^(1/2)*ln(-
(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan
(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-5660*A*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+268*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^4-60*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(
1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)
+I))*a*tan(d*x+c)^2+90*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)
^3+60*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+2940*I*A*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)-60*B*2^(1/2)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(182) = 364$.

Time = 0.13 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.20

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + ia\tan(c + dx))^{5/2}}\,dx = \frac{15\sqrt{\frac{1}{2}}\left(a^3de^{(7i\,dx+7i\,c)} - a^3de^{(5i\,dx+5i\,c)}\right)\sqrt{\frac{iA^2+2\,AB-iB^2}{a^5d^2}}\log\left(\frac{2}{-}\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
1/120*(15*sqrt(1/2)*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c)
)*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*log((2*sqrt(1/2)*a^3*d*sqrt((I*A
^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I
*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) - 15*sqrt(1/
2)*(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x + 5*I*c))*sqrt((I*A^2 + 2
*A*B - I*B^2)/(a^5*d^2))*log(-(2*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B - I*B
^2)/(a^5*d^2))*e^(I*d*x + I*c) - sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) +
I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + sqrt(2)*((-463*I*A + 83*B)
*e^(8*I*d*x + 8*I*c) + (-269*I*A + 19*B)*e^(6*I*d*x + 6*I*c) - 20*(-11*I*A
 + 4*B)*e^(4*I*d*x + 4*I*c) + (29*I*A - 19*B)*e^(2*I*d*x + 2*I*c) + 3*I*A
- 3*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) + 1)))/(a^3*d*e^(7*I*d*x + 7*I*c) - a^3*d*e^(5*I*d*x
 + 5*I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{-\left(\int \frac{1}{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^3-2\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(1/(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3 -
2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*i - sqrt(tan
(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)),x)*a + int(1/(sqrt(tan(c
 + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x))*s
qrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x))*sqrt(tan(c + d
*x)*i + 1)),x)*b))/(sqrt(a)*a**2)
```

3.195 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 286

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(\frac{1}{8}+\frac{i}{8}\right)(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} + \frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}} + \frac{21A+11iB}{30ad\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}} + \frac{89A+39iB}{20a^2d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}} - \frac{(361A+151iB)\sqrt{a+ia\tan(c+dx)}}{60a^3d\tan^{\frac{3}{2}}(c+dx)} + \frac{(707iA-317B)\sqrt{a+ia\tan(c+dx)}}{60a^3d\sqrt{\tan(c+dx)}}$$

output
```
(1/8+1/8*I)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))/a^(5/2)/d+1/5*(A+I*B)/d/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/
2)+1/30*(21*A+11*I*B)/a/d/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2)+1/20*(
89*A+39*I*B)/a^2/d/tan(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(1/2)-1/60*(361*A+1
51*I*B)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d/tan(d*x+c)^(3/2)+1/60*(707*I*A-317*
B)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 4.76 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.91

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{i\sec^2(c+dx)\left(\frac{15\sqrt{2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)(\cos(2(c+dx))+i\sin(2(c+dx)))\tan^2(c+dx)}{\sqrt{ia\tan(c+dx)}}+\frac{\sec^2(c+dx)(-174iA+84B+}{}\right.}{120a^2d\tan^{\frac{3}{2}}(c+d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^
(5/2)),x]
```

output
```
((-1/120*I)*Sec[c + d*x]^2*((15*Sqrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*
a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*(Cos[2*(c + d*x)] + I*Sin[2*(
c + d*x)])*Tan[c + d*x]^2)/Sqrt[I*a*Tan[c + d*x]] + (Sec[c + d*x]^2*((-174
*I)*A + 84*B + ((747*I)*A - 317*B)*Cos[2*(c + d*x)] + ((-493*I)*A + 233*B)
*Cos[4*(c + d*x)] - 780*A*Sin[2*(c + d*x)] - (340*I)*B*Sin[2*(c + d*x)] +
490*A*Sin[4*(c + d*x)] + (230*I)*B*Sin[4*(c + d*x)]))/Sqrt[a + I*a*Tan[c +
 d*x]]))/(a^2*d*Tan[c + d*x]^(3/2)*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.07, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(13A+3iB)-8a(iA-B)\tan(c+dx)}{2\tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}+\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(13A+3iB)-8a(iA-B)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(13A+3iB)-8a(iA-B)\tan(c+dx)}{\tan(c+dx)^{5/2}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int \frac{3\left(a^2(47A+17iB)-2a^2(21iA-11B)\tan(c+dx)\right)}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{a(21A+11iB)}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a^2(47A+17iB)-2a^2(21iA-11B)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}+\frac{a(21A+11iB)}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2(47A+17iB)-2a^2(21iA-11B)\tan(c+dx)}{\tan(c+dx)^{5/2}\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}+\frac{a(21A+11iB)}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(361A+151iB)-4a^3(89iA-39B)\tan(c+dx)\right)}{2\tan^{\frac{5}{2}}(c+dx)}dx}{a^2}+\frac{a^2(89A+39iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(21A+11iB)}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(361A+151iB)-4a^3(89iA-39B)\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}dx}{2a^2}+\frac{a^2(89A+39iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(21A+11iB)}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(361A+151iB)-4a^3(89iA-39B)\tan(c+dx)\right)}{\tan(c+dx)^{5/2}}dx}{2a^2}+\frac{a^2(89A+39iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{a(21A+11iB)}{3d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 4081

$$\frac{\frac{\frac{\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((707iA-317B)a^4+2(361A+151iB)\tan(c+dx)a^4\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^3(361A+151iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a^2(89A+39iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{\cdots}{3d\,\mathrm{t}\cdots}}{10a^2}+\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((707iA-317B)a^4+2(361A+151iB)\tan(c+dx)a^4\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^3(361A+151iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a^2(89A+39iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{\cdots}{3d\,\mathrm{ta}\cdots}}{10a^2}$$

$$\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{-\frac{-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((707iA-317B)a^4+2(361A+151iB)\tan(c+dx)a^4\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a^3(361A+151iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a^2(89A+39iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{\cdots}{3d\,\mathrm{ta}\cdots}}{10a^2}$$

$$\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 4081$$

$$\frac{\frac{-\frac{-\frac{\frac{2\int\frac{15a^5(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^4(-317B+707iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^3(361A+151iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a^2(89A+39iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\cdots}{10a^2}$$

$$\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-\frac{-\frac{15a^4(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^4(-317B+707iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^3(361A+151iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a^2(89A+39iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{\cdots}{3d\cdots}}{10a^2}$$

$$\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{-\frac{\frac{15a^4(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^4(-317B+707iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^3(361A+151iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a^2(89A+39iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}}{2a^2}+\frac{}{3d}}{10a^2}$$

$$\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 4027

$$\frac{\frac{-\frac{-\frac{\frac{30ia^6(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^4(-317B+707iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^3(361A+151iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{a^2(89}{d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}}{10a^2}$$

$$\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 218

$$\frac{\frac{\frac{a^2(89A+39iB)}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}+\frac{-\frac{2a^3(361A+151iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{\frac{(15-15i)a^{9/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^4(-317B+707iA)\sqrt{a}}{d\sqrt{\tan(c+d}}}{3a}}{2a^2}}{2a^2}}{10a^2}$$

$$\frac{A+iB}{5d\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2))
,x]
```

output
```
(A + I*B)/(5*d*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)) + ((a*(21*
A + (11*I)*B))/(3*d*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((a
^2*(89*A + (39*I)*B))/(d*Tan[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) +
((-2*a^3*(361*A + (151*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]
^(3/2)) - (((15 - 15*I)*a^(9/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^4*((707*I)*A - 317*B)*S
qrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(2*a^2))/(2*a^2)
)/(10*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1238 vs. $2(232) = 464$.

Time = 0.25 (sec) , antiderivative size = 1239, normalized size of antiderivative = 4.33

method	result	size
derivativedivides	Expression too large to display	1239
default	Expression too large to display	1239
parts	Expression too large to display	1273

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/240/d*(a*(1+I*tan(d*x+c)))^(1/2)*(-60*I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)
^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+
c)+I))*a*tan(d*x+c)^5-2940*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a
)^(1/2)*tan(d*x+c)^2+15*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c
)^6+2828*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)
^5-12260*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)
^3+60*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5+4468*I*B*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^4+15*I*A*2^(1/2)*
ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a
*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-90*B*2^(1/2)*ln(-(-2*2^(1/2)*(
-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan
(d*x+c)+I))*a*tan(d*x+c)^4-1268*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-
I*a)^(1/2)*tan(d*x+c)^5-90*I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(
d*x+c)^4+640*I*A*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(
1/2)-60*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+9868*A*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^4+15*I*A*2^(1/...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(218) = 436.

Time = 0.13 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.06

$$\int \frac{A + B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/120*(15*sqrt(1/2)*(a^3*d*e^(9*I*d*x + 9*I*c) - 2*a^3*d*e^(7*I*d*x + 7*I
*c) + a^3*d*e^(5*I*d*x + 5*I*c))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*
log((2*I*sqrt(1/2)*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*e^(I*d*x
 + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A + B)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 + 1)))/(4*I*A + 4*B)) - 15*sqrt(1/2)*(a^3*d*e^(9*I*d*x + 9*I*c) - 2*a^3*d
*e^(7*I*d*x + 7*I*c) + a^3*d*e^(5*I*d*x + 5*I*c))*sqrt((-I*A^2 - 2*A*B + I
*B^2)/(a^5*d^2))*log((-2*I*sqrt(1/2)*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(
a^5*d^2))*e^(I*d*x + I*c) + sqrt(2)*((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A +
 B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) + 1)))/(4*I*A + 4*B)) + sqrt(2)*((983*A + 463*I*B)*e^(1
0*I*d*x + 10*I*c) - 2*(272*A + 97*I*B)*e^(8*I*d*x + 8*I*c) - 3*(393*A + 16
3*I*B)*e^(6*I*d*x + 6*I*c) + (381*A + 191*I*B)*e^(4*I*d*x + 4*I*c) + 2*(18
*A + 13*I*B)*e^(2*I*d*x + 2*I*c) + 3*A + 3*I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1)))/(a^3
*d*e^(9*I*d*x + 9*I*c) - 2*a^3*d*e^(7*I*d*x + 7*I*c) + a^3*d*e^(5*I*d*x +
5*I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + ia\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{5/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{-\left(\int \frac{1}{\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^4-2\sqrt{\tan(dx+c)}\,\sqrt{\tan(dx+c)i+1}}\right.}{}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(1/(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**4 -
2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3*i - sqrt(tan
(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2),x)*a + int(1/(sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3 - 2*sqrt(tan(c + d*x)
)*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2*i - sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*i + 1)*tan(c + d*x)),x)*b))/(sqrt(a)*a**2)
```

3.196 $\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 201

$$\begin{aligned}\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx &= -\frac{\sqrt[3]{a}(A-iB)x}{2\ 2^{2/3}}-\frac{\sqrt{3}\sqrt[3]{a}(iA+B)\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2^{2/3}d}\\&\quad+\frac{\sqrt[3]{a}(iA+B)\log(\cos(c+dx))}{2\ 2^{2/3}d}\\&\quad+\frac{3\sqrt[3]{a}(iA+B)\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\ 2^{2/3}d}+\frac{3B\sqrt[3]{a+ia\tan(c+dx)}}{d}\end{aligned}$$

output
```
-1/4*a^(1/3)*(A-I*B)*x*2^(1/3)-1/2*3^(1/2)*a^(1/3)*(I*A+B)*arctan(1/3*(a^(
1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))*3^(1/2)/a^(1/3))*2^(1/3)/d+1/4*a^(1
/3)*(I*A+B)*ln(cos(d*x+c))*2^(1/3)/d+3/4*a^(1/3)*(I*A+B)*ln(2^(1/3)*a^(1/3
)-(a+I*a*tan(d*x+c))^(1/3))*2^(1/3)/d+3*B*(a+I*a*tan(d*x+c))^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

$$\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \frac{-i\sqrt[3]{2}\sqrt[3]{a}(A-iB)\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right.}{4}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(1/3)*(A + B*Tan[c + d*x]),x]
```

output
```
((-I)*2^(1/3)*a^(1/3)*(A - I*B)*(2*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + I*a*T
an[c + d*x])^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[2^(1/3)*a^(1/3) - (a + I*a*T
an[c + d*x])^(1/3)] + Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + I*a*Tan[c
 + d*x])^(1/3) + (a + I*a*Tan[c + d*x])^(2/3)]) + 12*B*(a + I*a*Tan[c + d*
x])^(1/3))/(4*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.69, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4010, 3042, 3962, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4010$$

$$(A-iB)\int \sqrt[3]{i\tan(c+dx)a+a}dx+\frac{3B\sqrt[3]{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$(A-iB)\int \sqrt[3]{i\tan(c+dx)a+a}dx+\frac{3B\sqrt[3]{a+ia\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 3962$$

$$\frac{3B\sqrt[3]{a+ia\tan(c+dx)}}{d}-\frac{ia(A-iB)\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \; 69$$

$$\frac{3B\sqrt[3]{a+ia\tan(c+dx)}}{d}-$$

$$\frac{ia(A-iB)\left(\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)}d\sqrt[3]{i\tan(c+dx)a+a}}{2\ 2^{2/3}a^{2/3}}+\frac{3\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+}}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{d}$$

$$\Big\downarrow \; 16$$

$$\frac{3B\sqrt[3]{a+ia\tan(c+dx)}}{d}-$$

$$\frac{ia(A-iB)\left(\frac{3\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a-i}{2\ 2}\right.}{d}$$

$$\Big\downarrow \; 1082$$

$$\frac{3B\sqrt[3]{a+ia\tan(c+dx)}}{d}-$$

$$\frac{ia(A-iB)\left(-\frac{3\int \frac{1}{a^2\tan^2(c+dx)-3}d\left(i2^{2/3}a^{2/3}\tan(c+dx)+1\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a-ia\tan(c+dx))}{2\ 2^{2/3}a^{2/3}}\right)}{d}$$

$$\Big\downarrow \; 217$$

$$\frac{3B\sqrt[3]{a+ia\tan(c+dx)}}{d}-$$

$$\frac{ia(A-iB)\left(\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a-ia\tan(c+dx))}{2\ 2^{2/3}a^{2/3}}\right)}{d}$$

input

```
Int[(a + I*a*Tan[c + d*x])^(1/3)*(A + B*Tan[c + d*x]),x]
```

output
```
((-I)*a*(A - I*B)*((I*Sqrt[3]*ArcTanh[(a*Tan[c + d*x])/Sqrt[3]])/(2^(2/3)*
a^(2/3)) - (3*Log[2^(1/3)*a^(1/3) - I*a*Tan[c + d*x]])/(2*2^(2/3)*a^(2/3))
 + Log[a - I*a*Tan[c + d*x]]/(2*2^(2/3)*a^(2/3))))/d + (3*B*(a + I*a*Tan[c
 + d*x])^(1/3))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{3i\left(-iB(a+ia\tan(dx+c))^{\frac{1}{3}}+\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}\right.}{12a^{\frac{2}{3}}}\right.\right.}{d}$
default	$\frac{3i\left(-iB(a+ia\tan(dx+c))^{\frac{1}{3}}+\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}\right.}{12a^{\frac{2}{3}}}\right.\right.}{d}$
parts	$\frac{3iAa\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\sqrt{3}\arctan\left(\right.}{}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3*I/d*(-I*B*(a+I*a*tan(d*x+c))^(1/3)+(1/6*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*
x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(2
/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))-1/6*2^(1/3)/
a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/
3)+1)))*a*(A-I*B))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(146) = 292$.

Time = 0.09 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.01

$$\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{6 \cdot 2^{\frac{1}{3}} B\left(\frac{a}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{2}{3}i\,dx+\frac{2}{3}i\,c\right)} + \left(\frac{1}{4}\right)^{\frac{1}{3}}\left(-i\sqrt{3}d-d\right)\left(\frac{(-iA^3-3A^2B+3iAB^2+B^3)a}{d^3}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}(iA+B)\left(\frac{}{e^{(2i\,dx}}\right.}{}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(6*2^(1/3)*B*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*
c) + (1/4)^(1/3)*(-I*sqrt(3)*d - d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*
a/d^3)^(1/3)*log((2^(1/3)*(I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^
(2/3*I*d*x + 2/3*I*c) + (1/4)^(1/3)*(I*sqrt(3)*d + d)*((-I*A^3 - 3*A^2*B +
 3*I*A*B^2 + B^3)*a/d^3)^(1/3))/(I*A + B)) + (1/4)^(1/3)*(I*sqrt(3)*d - d)
*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a/d^3)^(1/3)*log((2^(1/3)*(I*A + B)
*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (1/4)^(1/3)
*(-I*sqrt(3)*d + d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a/d^3)^(1/3))/(I
*A + B)) + 2*(1/4)^(1/3)*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a/d^3)^(1
/3)*log((2^(1/3)*(I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*
x + 2/3*I*c) - 2*(1/4)^(1/3)*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a/d^3
)^(1/3))/(I*A + B)))/d
```

Sympy [F]

$$\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int \sqrt[3]{ia\,(\tan{(c+dx)}-i)}(A+B\tan{(c+dx)})\;dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/3)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(1/3)*(A + B*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

$$\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{i\left(2\sqrt{3}2^{\frac{1}{3}}(A-i\,B)a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}(A-i\,B)a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\,a\,\mathrm{ta}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*I*(2*sqrt(3)*2^(1/3)*(A - I*B)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^
(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 2^(1/3)*(A - I*
B)*a^(4/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1
/3) + (I*a*tan(d*x + c) + a)^(2/3)) - 2*2^(1/3)*(A - I*B)*a^(4/3)*log(-2^(
1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)) + 12*I*(I*a*tan(d*x + c) + a)
^(1/3)*B*a)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.27

$$\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$

$$-\frac{2i\,\sqrt{3}2^{\frac{1}{3}}Aa^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)+2\,\sqrt{3}2^{\frac{1}{3}}Ba^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a\right.}{6\,a^{\frac{1}{3}}}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(2*I*sqrt(3)*2^(1/3)*A*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^
(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 2*sqrt(3)*2^(1/3)*B*a^(
1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a
)^(1/3))/a^(1/3)) + I*2^(1/3)*A*a^(1/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a
*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) + 2^(1/3)
*B*a^(1/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1
/3) + (I*a*tan(d*x + c) + a)^(2/3)) - 2*2^(1/3)*(I*A*a + B*a)*log(-2^(1/3)
*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3))/a^(2/3) - 12*(I*a*tan(d*x + c) +
a)^(1/3)*B)/d
```

Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.82

$$\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{3\,B\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d} + \frac{2^{1/3}\,B\,a^{1/3}\,\ln\left({\left(a\,\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)\right)}^{1/3}-2^{1/3}\,a^{1/3}\right)}{2\,d} - \frac{{\left(\frac{1}{4}{}\mathrm{i}\right)}^{1/3}\,A\,a^{1/3}\,\ln\left(A\,a\,d^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}\,9{}\mathrm{i}+18\,{\left(\frac{1}{4}{}\mathrm{i}\right)}^{1/3}\,A\,a^{4/3}\,d^2\right)}{d} - \frac{{\left(\frac{1}{4}{}\mathrm{i}\right)}^{1/3}\,A\,a^{1/3}\,\ln\left(A\,a\,d^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}\,9{}\mathrm{i}+18\,{\left(\frac{1}{4}{}\mathrm{i}\right)}^{1/3}\,A\,a^{4/3}\,d^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{d} + \frac{{\left(\frac{1}{4}{}\mathrm{i}\right)}^{1/3}\,A\,a^{1/3}\,\ln\left(A\,a\,d^2\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}\,9{}\mathrm{i}-18\,{\left(\frac{1}{4}{}\mathrm{i}\right)}^{1/3}\,A\,a^{4/3}\,d^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{d} + \frac{4^{2/3}\,B\,a^{1/3}\,\ln\left(\frac{9\,B\,a\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}-\frac{9\,2^{1/3}\,B\,a^{4/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,d}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{4\,d} - \frac{4^{2/3}\,B\,a^{1/3}\,\ln\left(\frac{9\,B\,a\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{1/3}}{d}+\frac{9\,2^{1/3}\,B\,a^{4/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,d}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{4\,d}$$

input
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/3),x)
```

output
```
(3*B*(a + a*tan(c + d*x)*1i)^(1/3))/d + (2^(1/3)*B*a^(1/3)*log((a*(tan(c +
 d*x)*1i + 1))^(1/3) - 2^(1/3)*a^(1/3)))/(2*d) - ((1i/4)^(1/3)*A*a^(1/3)*l
og(A*a*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*9i + 18*(1i/4)^(1/3)*A*a^(4/3)*d^
2))/d - ((1i/4)^(1/3)*A*a^(1/3)*log(A*a*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*
9i + 18*(1i/4)^(1/3)*A*a^(4/3)*d^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2
 - 1/2))/d + ((1i/4)^(1/3)*A*a^(1/3)*log(A*a*d^2*(a + a*tan(c + d*x)*1i)^(
1/3)*9i - 18*(1i/4)^(1/3)*A*a^(4/3)*d^2*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*
1i)/2 + 1/2))/d + (4^(2/3)*B*a^(1/3)*log((9*B*a*(a + a*tan(c + d*x)*1i)^(1
/3))/d - (9*2^(1/3)*B*a^(4/3)*(3^(1/2)*1i - 1))/(2*d))*((3^(1/2)*1i)/2 - 1
/2))/(4*d) - (4^(2/3)*B*a^(1/3)*log((9*B*a*(a + a*tan(c + d*x)*1i)^(1/3))/
d + (9*2^(1/3)*B*a^(4/3)*(3^(1/2)*1i + 1))/(2*d))*((3^(1/2)*1i)/2 + 1/2))/
(4*d)
```

Reduce [F]

$$\int \sqrt[3]{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= a^{\frac{1}{3}}\left(\left(\int (\tan(dx+c)\,i+1)^{\frac{1}{3}}\,dx\right)a+\left(\int (\tan(dx+c)\,i+1)^{\frac{1}{3}}\tan(dx+c)\,dx\right)b\right)$$

input
```
int((a+I*a*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x)
```

output
```
a**(1/3)*(int((tan(c + d*x)*i + 1)**(1/3),x)*a + int((tan(c + d*x)*i + 1)*
*(1/3)*tan(c + d*x),x)*b)
```

3.197 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 270

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \frac{a^{2/3}(A-iB)x}{2\sqrt[3]{2}} - \frac{\sqrt{3}a^{2/3}(iA+B)\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{2}d} - \frac{a^{2/3}(iA+B)\log(\cos(c+dx))}{2\sqrt[3]{2}d} - \frac{3a^{2/3}(iA+B)\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{2}d} - \frac{9B(a+ia\tan(c+dx))^{2/3}}{8d} + \frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d} - \frac{3(4iA+B)(a+ia\tan(c+dx))^{5/3}}{20ad}$$

output
```
1/4*a^(2/3)*(A-I*B)*x*2^(2/3)-1/2*3^(1/2)*a^(2/3)*(I*A+B)*arctan(1/3*(a^(1
/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))*3^(1/2)/a^(1/3))*2^(2/3)/d-1/4*a^(2/
3)*(I*A+B)*ln(cos(d*x+c))*2^(2/3)/d-3/4*a^(2/3)*(I*A+B)*ln(2^(1/3)*a^(1/3)
-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/d-9/8*B*(a+I*a*tan(d*x+c))^(2/3)/d+3/8*
B*tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)/d-3/20*(4*I*A+B)*(a+I*a*tan(d*x+c)
)^(5/3)/a/d
```

Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.73

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx =$$

$$-\frac{10\ 2^{2/3}a^{2/3}(iA+B)\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\log(i+\tan(c+dx))+3\log\left(\sqrt[3]{2}\sqrt[3]{\right.}\right.}{}$$

input
```
Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x])
,x]
```

output
```
-1/40*(10*2^(2/3)*a^(2/3)*(I*A + B)*(2*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + I
*a*Tan[c + d*x])^(1/3))/a^(1/3))/Sqrt[3]] - Log[I + Tan[c + d*x]] + 3*Log[
2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)]) + 45*B*(a + I*a*Tan[c + d
*x])^(2/3) - 15*B*Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(2/3) + (6*((4*I)*
A + B)*(a + I*a*Tan[c + d*x])^(5/3))/a)/d
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.80, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 4080, 27, 3042, 4075, 3042, 4010, 3042, 3962, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^2(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4080}$$

$$\frac{3\int -\frac{2}{3}\tan(c+dx)(i\tan(c+dx)a+a)^{2/3}(3aB-a(4A-iB)\tan(c+dx))dx}{8a}+\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{\int \tan(c+dx)(i\tan(c+dx)a+a)^{2/3}(3aB-a(4A-iB)\tan(c+dx))dx}{4a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{\int \tan(c+dx)(i\tan(c+dx)a+a)^{2/3}(3aB-a(4A-iB)\tan(c+dx))dx}{4a}$$

$$\Big\downarrow \text{ 4075}$$

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{\int (i\tan(c+dx)a+a)^{2/3}(a(4A-iB)+3aB\tan(c+dx))dx+\frac{3(B+4iA)(a+ia\tan(c+dx))^{5/3}}{5d}}{4a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{\int (i\tan(c+dx)a+a)^{2/3}(a(4A-iB)+3aB\tan(c+dx))dx+\frac{3(B+4iA)(a+ia\tan(c+dx))^{5/3}}{5d}}{4a}$$

$$\Big\downarrow \text{ 4010}$$

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{4a(A-iB)\int (i\tan(c+dx)a+a)^{2/3}dx+\frac{3(B+4iA)(a+ia\tan(c+dx))^{5/3}}{5d}+\frac{9aB(a+ia\tan(c+dx))^{2/3}}{2d}}{4a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{4a(A-iB)\int(i\tan(c+dx)a+a)^{2/3}dx+\frac{3(B+4iA)(a+ia\tan(c+dx))^{5/3}}{5d}+\frac{9aB(a+ia\tan(c+dx))^{2/3}}{2d}}{4a}$$

$\big\downarrow$ 3962

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{-\frac{4ia^2(A-iB)\int\frac{1}{(a-ia\tan(c+dx))\sqrt[3]{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}+\frac{3(B+4iA)(a+ia\tan(c+dx))^{5/3}}{5d}+\frac{9aB(a+ia\tan(c+dx))^{2/3}}{2d}}{4a}$$

$\big\downarrow$ 67

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{-\frac{4ia^2(A-iB)\left(-\frac{3}{2}\int\frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}+\frac{3\int\frac{1}{\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)}d\sqrt[3]{i\tan(c+d}}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{d}}{4a}$$

$\big\downarrow$ 16

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{-\frac{4ia^2(A-iB)\left(-\frac{3}{2}\int\frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c}{2\sqrt[3]{2}\sqrt[3]{}}\right.}{d}}{4a}$$

$\big\downarrow$ 1082

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{-\frac{4ia^2(A-iB)\left(\frac{3\int\frac{1}{a^2\tan^2(c+dx)-3}d\left(i2^{2/3}a^{2/3}\tan(c+dx)+1\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}+\frac{3(B+4iA)(a+ia\tan(c-}{5d}}{4a}$$

$\big\downarrow$ 217

$$\frac{3B\tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}}{8d}-\frac{-\frac{4ia^2(A-iB)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}+\frac{3(B+4iA)(a+ia\tan(c+dx))^{5/3}}{5d}+\frac{9c}{}}{4a}$$

input
```
Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x]
```

output
```
(3*B*Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^(2/3))/(8*d) - (((-4*I)*a^2*(A
- I*B)*(((-I)*Sqrt[3]*ArcTanh[(a*Tan[c + d*x])/Sqrt[3]])/(2^(1/3)*a^(1/3))
 - (3*Log[2^(1/3)*a^(1/3) - I*a*Tan[c + d*x]])/(2*2^(1/3)*a^(1/3)) + Log[a
 - I*a*Tan[c + d*x]]/(2*2^(1/3)*a^(1/3))))/d + (9*a*B*(a + I*a*Tan[c + d*x
])^(2/3))/(2*d) + (3*((4*I)*A + B)*(a + I*a*Tan[c + d*x])^(5/3))/(5*d))/(4
*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{3i\left(\frac{iB(a+ia\tan(dx+c))^{\frac{8}{3}}}{8}-\frac{iBa(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}-\frac{Aa(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}+\frac{ia^2B(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}-\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(d\cdots\right.}{6a^{\cdots}}\cdots\right)\cdots\right)}{\cdots}$
default	$\frac{3i\left(\frac{iB(a+ia\tan(dx+c))^{\frac{8}{3}}}{8}-\frac{iBa(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}-\frac{Aa(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}+\frac{ia^2B(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}-\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(d\cdots\right.}{6a^{\cdots}}\cdots\right)\cdots\right)}{\cdots}$
parts	$\frac{3iA\left(-\frac{(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}-\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2\cdots\right.}{12a^{\frac{1}{3}}}\cdots\right)\cdots\right)}{da\cdots}$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
3*I/d/a^2*(1/8*I*B*(a+I*a*tan(d*x+c))^(8/3)-1/5*I*B*a*(a+I*a*tan(d*x+c))^(
5/3)-1/5*A*a*(a+I*a*tan(d*x+c))^(5/3)+1/2*I*a^2*B*(a+I*a*tan(d*x+c))^(2/3)
-(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^
(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c
))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(
2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*a^3*(A-I*B))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(201) = 402$.

Time = 0.11 (sec) , antiderivative size = 660, normalized size of antiderivative = 2.44

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/10*(3*2^(2/3)*(2*(2*I*A + 3*B)*e^(4*I*d*x + 4*I*c) + 2*(2*I*A + 3*B)*e^
(2*I*d*x + 2*I*c) + 5*B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(4/3*I*d*x
+ 4/3*I*c) - 10*(1/2)^(1/3)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*
c) + d)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(
A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x +
2/3*I*c) + 2*(1/2)^(2/3)*d^2*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)
^(2/3))/((A^2 - 2*I*A*B - B^2)*a)) + 5*(1/2)^(1/3)*((-I*sqrt(3)*d + d)*e^(
4*I*d*x + 4*I*c) + 2*(-I*sqrt(3)*d + d)*e^(2*I*d*x + 2*I*c) - I*sqrt(3)*d
+ d)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2
 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3
*I*c) - (1/2)^(2/3)*(I*sqrt(3)*d^2 + d^2)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 -
B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)) + 5*(1/2)^(1/3)*((I*sqrt(3
)*d + d)*e^(4*I*d*x + 4*I*c) + 2*(I*sqrt(3)*d + d)*e^(2*I*d*x + 2*I*c) + I
*sqrt(3)*d + d)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(1/3)*log((2
^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*
I*d*x + 2/3*I*c) - (1/2)^(2/3)*(-I*sqrt(3)*d^2 + d^2)*((I*A^3 + 3*A^2*B -
3*I*A*B^2 - B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)))/(d*e^(4*I*d*x
 + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx=\int (ia(\tan(c+dx)-i))^{\frac{2}{3}}(A+B\tan(c+dx))\tan^2(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(2/3)*(A + B*tan(c + d*x))*tan(c + d*x)
**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.78

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx=-\frac{i\left(20\sqrt{3}2^{\frac{2}{3}}(A-iB)a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)-10\cdot 2^{\frac{2}{3}}(A-iB)a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(i\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/40*I*(20*sqrt(3)*2^(2/3)*(A - I*B)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*
(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 10*2^(2/3)*(
A - I*B)*a^(11/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/
3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) + 20*2^(2/3)*(A - I*B)*a^(11/3)
*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)) - 15*I*(I*a*tan(d*x
+ c) + a)^(8/3)*B*a + 24*(I*a*tan(d*x + c) + a)^(5/3)*(A + I*B)*a^2 - 60*I
*(I*a*tan(d*x + c) + a)^(2/3)*B*a^3)/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.23

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx =$$

$$-\frac{20i\sqrt{3}2^{\frac{2}{3}}Aa^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)+20\sqrt{3}2^{\frac{2}{3}}Ba^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)\right.}{6a^{\frac{1}{3}}}\right.}{}$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-1/40*(20*I*sqrt(3)*2^(2/3)*A*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*
a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 20*sqrt(3)*2^(2/3)*B*
a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c)
+ a)^(1/3))/a^(1/3)) - 10*I*2^(2/3)*A*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3
)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) - 1
0*2^(2/3)*B*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(
1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) + 3*(5*(I*a*tan(d*x + c) + a)
^(8/3)*B*a^14 + 8*I*(I*a*tan(d*x + c) + a)^(5/3)*A*a^15 - 8*(I*a*tan(d*x +
 c) + a)^(5/3)*B*a^15 + 20*(I*a*tan(d*x + c) + a)^(2/3)*B*a^16)/a^16 + 20*
I*2^(1/3)*(2^(1/3)*A*a^(58/3) - I*2^(1/3)*B*a^(58/3))*log(-2^(1/3)*a^(1/3)
 + (I*a*tan(d*x + c) + a)^(1/3))/a^(56/3))/d
```

Mupad [B] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.61

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(2/3),x)
```

output
```
(3*B*(a + a*tan(c + d*x)*1i)^(5/3))/(5*a*d) - (A*(a + a*tan(c + d*x)*1i)^(
5/3)*3i)/(5*a*d) - (3*B*(a + a*tan(c + d*x)*1i)^(2/3))/(2*d) - (3*B*(a + a
*tan(c + d*x)*1i)^(8/3))/(8*a^2*d) - (2^(2/3)*B*a^(2/3)*log((a*(tan(c + d*
x)*1i + 1))^(1/3) - 2^(1/3)*a^(1/3)))/(2*d) + ((1i/2)^(1/3)*A*a^(2/3)*log(
(a*(tan(c + d*x)*1i + 1))^(1/3) + (-1)^(1/3)*2^(1/3)*a^(1/3)))/d + ((1i/2)
^(1/3)*A*a^(2/3)*log(((-1)^(1/3)*2^(1/3)*a^(1/3))/2 - (a*(tan(c + d*x)*1i
+ 1))^(1/3) + ((-1)^(5/6)*2^(1/3)*3^(1/2)*a^(1/3))/2)*((3^(1/2)*1i)/2 - 1/
2))/d - (2^(2/3)*B*a^(2/3)*log((9*B^2*a^2*(a + a*tan(c + d*x)*1i)^(1/3))/d
^2 - (9*2^(1/3)*B^2*a^(7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/d^2)*((3^(1/2)*1i)/2
 - 1/2))/(2*d) + (2^(2/3)*B*a^(2/3)*log((9*B^2*a^2*(a + a*tan(c + d*x)*1i)
^(1/3))/d^2 - (9*2^(1/3)*B^2*a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/d^2)*((3^(1
/2)*1i)/2 + 1/2))/(2*d) - ((1i/2)^(1/3)*A*a^(2/3)*log((a*(tan(c + d*x)*1i
+ 1))^(1/3) - ((-1)^(1/3)*2^(1/3)*a^(1/3))/2 + ((-1)^(5/6)*2^(1/3)*3^(1/2)
*a^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/d
```

Reduce [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = a^{\frac{2}{3}}\left(\left(\int (\tan(dx+c)\,i+1)^{\frac{2}{3}}\tan(dx+c)^3\,dx\right)b+\left(\int (\tan(dx+c)\,i+1)^{\frac{2}{3}}\tan(dx+c)^2\,dx\right)a\right)$$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
a**(2/3)*(int((tan(c + d*x)*i + 1)**(2/3)*tan(c + d*x)**3,x)*b + int((tan(
c + d*x)*i + 1)**(2/3)*tan(c + d*x)**2,x)*a)
```

3.198 $\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 232

$$\begin{aligned}\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx &= \frac{a^{2/3}(iA+B)x}{2\sqrt[3]{2}} \\ &+ \frac{\sqrt{3}a^{2/3}(A-iB)\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{2}d} \\ &+ \frac{a^{2/3}(A-iB)\log(\cos(c+dx))}{2\sqrt[3]{2}d} \\ &+ \frac{3a^{2/3}(A-iB)\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{2}d} \\ &+ \frac{3A(a+ia\tan(c+dx))^{2/3}}{2d} - \frac{3iB(a+ia\tan(c+dx))^{5/3}}{5ad}\end{aligned}$$

output

```
1/4*a^(2/3)*(I*A+B)*x*2^(2/3)+1/2*3^(1/2)*a^(2/3)*(A-I*B)*arctan(1/3*(a^(1
/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))*3^(1/2)/a^(1/3))*2^(2/3)/d+1/4*a^(2/
3)*(A-I*B)*ln(cos(d*x+c))*2^(2/3)/d+3/4*a^(2/3)*(A-I*B)*ln(2^(1/3)*a^(1/3)
-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/d+3/2*A*(a+I*a*tan(d*x+c))^(2/3)/d-3/5*
I*B*(a+I*a*tan(d*x+c))^(5/3)/a/d
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.71

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \frac{5\,2^{2/3}a^{5/3}(A-iB)\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\log(i+\tan(c+d$$

input
```
Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x
]
```

output
```
(5*2^(2/3)*a^(5/3)*(A - I*B)*(2*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + I*a*Tan[
c + d*x])^(1/3))/a^(1/3))/Sqrt[3]] - Log[I + Tan[c + d*x]] + 3*Log[2^(1/3)
*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)]) + 30*a*A*(a + I*a*Tan[c + d*x])^
(2/3) - (12*I)*B*(a + I*a*Tan[c + d*x])^(5/3))/(20*a*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.74, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4075, 3042, 4010, 3042, 3962, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int (i\tan(c+dx)a+a)^{2/3}(A\tan(c+dx)-B)dx-\frac{3iB(a+ia\tan(c+dx))^{5/3}}{5ad}$$

$\downarrow$ 3042

$$\int (i\tan(c+dx)a+a)^{2/3}(A\tan(c+dx)-B)dx-\frac{3iB(a+ia\tan(c+dx))^{5/3}}{5ad}$$

$\downarrow$ 4010

$$-(B+iA)\int (i\tan(c+dx)a+a)^{2/3}dx+\frac{3A(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{3iB(a+ia\tan(c+dx))^{5/3}}{5ad}$$

$\downarrow$ 3042

$$-(B+iA)\int (i\tan(c+dx)a+a)^{2/3}dx+\frac{3A(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{3iB(a+ia\tan(c+dx))^{5/3}}{5ad}$$

$\downarrow$ 3962

$$\frac{ia(B+iA)\int \frac{1}{(a-ia\tan(c+dx))\sqrt[3]{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}+\frac{3A(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{3iB(a+ia\tan(c+dx))^{5/3}}{5ad}$$

$\downarrow$ 67

$$\frac{ia(B+iA)\left(-\frac{3}{2}\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}+\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)}d\sqrt[3]{i\tan}}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{d}$$
$$\frac{3A(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{3iB(a+ia\tan(c+dx))^{5/3}}{5ad}$$

$\downarrow$ 16

$$\frac{ia(B+iA)\left(-\frac{3}{2}\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\right.}{d}$$
$$\frac{3A(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{3iB(a+ia\tan(c+dx))^{5/3}}{5ad}$$

$\downarrow$ 1082

$$\frac{ia(B+iA)\left(\frac{3\int \frac{1}{a^2\tan^2(c+dx)-3}d\left(i2^{2/3}a^{2/3}\tan(c+dx)+1\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}+$$
$$\frac{3A(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{3iB(a+ia\tan(c+dx))^{5/3}}{5ad}$$

$$\Big\downarrow \ 217$$

$$\frac{ia(B+iA)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}+$$
$$\frac{3A(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{3iB(a+ia\tan(c+dx))^{5/3}}{5ad}$$

input
```
Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x]
```

output
```
(I*a*(I*A + B)*(((-I)*Sqrt[3]*ArcTanh[(a*Tan[c + d*x])/Sqrt[3]])/(2^(1/3)*
a^(1/3)) - (3*Log[2^(1/3)*a^(1/3) - I*a*Tan[c + d*x]])/(2*2^(1/3)*a^(1/3))
 + Log[a - I*a*Tan[c + d*x]]/(2*2^(1/3)*a^(1/3))))/d + (3*A*(a + I*a*Tan[c
 + d*x])^(2/3))/(2*d) - (((3*I)/5)*B*(a + I*a*Tan[c + d*x])^(5/3))/(a*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\frac{3iB(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}+\frac{3aA(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+3\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a\right.}{12a}\right.}{ad}$
default	$\frac{-\frac{3iB(a+ia\tan(dx+c))^{\frac{5}{3}}}{5}+\frac{3aA(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+3\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a\right.}{12a}\right.}{ad}$
parts	$A\left(\frac{3(a+ia\tan(dx+c))^{\frac{2}{3}}}{2d}+\frac{a^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{2d}-\frac{a^{\frac{2}{3}}2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+i\right.}{4d}\right.$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
3/d/a*(-1/5*I*B*(a+I*a*tan(d*x+c))^(5/3)+1/2*a*A*(a+I*a*tan(d*x+c))^(2/3)+
(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(
2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c)
)^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(2
^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*a^2*(A-I*B))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(168) = 336$.

Time = 0.10 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.46

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/10*(3*2^(2/3)*((5*A - 4*I*B)*e^(2*I*d*x + 2*I*c) + 5*A)*(a/(e^(2*I*d*x +
 2*I*c) + 1))^(2/3)*e^(4/3*I*d*x + 4/3*I*c) + 10*(1/2)^(1/3)*(d*e^(2*I*d*x
 + 2*I*c) + d)*((A^3 - 3*I*A^2*B - 3*A*B^2 + I*B^3)*a^2/d^3)^(1/3)*log((2^
(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I
*d*x + 2/3*I*c) - 2*(1/2)^(2/3)*d^2*((A^3 - 3*I*A^2*B - 3*A*B^2 + I*B^3)*a
^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)) - 5*(1/2)^(1/3)*((I*sqrt(3)*d +
d)*e^(2*I*d*x + 2*I*c) + I*sqrt(3)*d + d)*((A^3 - 3*I*A^2*B - 3*A*B^2 + I*
B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x +
2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (1/2)^(2/3)*(I*sqrt(3)*d^2 -
d^2)*((A^3 - 3*I*A^2*B - 3*A*B^2 + I*B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B
- B^2)*a)) - 5*(1/2)^(1/3)*((-I*sqrt(3)*d + d)*e^(2*I*d*x + 2*I*c) - I*sqr
t(3)*d + d)*((A^3 - 3*I*A^2*B - 3*A*B^2 + I*B^3)*a^2/d^3)^(1/3)*log((2^(1/
3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*
x + 2/3*I*c) - (1/2)^(2/3)*(-I*sqrt(3)*d^2 - d^2)*((A^3 - 3*I*A^2*B - 3*A*
B^2 + I*B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)))/(d*e^(2*I*d*x + 2
*I*c) + d)
```

Sympy [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{2}{3}}\,(A+B\tan(c+dx))\tan(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(2/3)*(A + B*tan(c + d*x))*tan(c + d*x)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.81

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \frac{10\sqrt{3}2^{\frac{2}{3}}(A-iB)a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-5\cdot 2^{\frac{2}{3}}(A-iB)a^{\frac{8}{3}}\log$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/20*(10*sqrt(3)*2^(2/3)*(A - I*B)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 5*2^(2/3)*(A - I
*B)*a^(8/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(
1/3) + (I*a*tan(d*x + c) + a)^(2/3)) + 10*2^(2/3)*(A - I*B)*a^(8/3)*log(-2
^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)) - 12*I*(I*a*tan(d*x + c) +
a)^(5/3)*B*a + 30*(I*a*tan(d*x + c) + a)^(2/3)*A*a^2)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.27

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \frac{10\sqrt{3}2^{\frac{2}{3}}Aa^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-10i\sqrt{3}2^{\frac{2}{3}}Ba^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2\right.}{}\right.$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
1/20*(10*sqrt(3)*2^(2/3)*A*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(
1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 10*I*sqrt(3)*2^(2/3)*B*a
^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) +
 a)^(1/3))/a^(1/3)) - 5*2^(2/3)*A*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I
*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) + 5*I*2
^(2/3)*B*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3
)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) - 6*(2*I*(I*a*tan(d*x + c) + a)^
(5/3)*B*a^4 - 5*(I*a*tan(d*x + c) + a)^(2/3)*A*a^5)/a^5 + 10*2^(1/3)*(2^(1
/3)*A*a^(22/3) - I*2^(1/3)*B*a^(22/3))*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x
 + c) + a)^(1/3))/a^(20/3))/d
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.68

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \frac{3\,A\,\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)^{2/3}}{2\,d} - \frac{B\,\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)^{5/3}\,3\mathrm{i}}{5\,a\,d} + \frac{2^{2/3}\,A\,a^{2/3}\,\ln\left(\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{2\,d} + \frac{\left(\frac{1}{2}\mathrm{i}\right)^{1/3}\,B\,a^{2/3}\,\ln\left(\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}+(-1)^{1/3}\,2^{1/3}\,a^{1/3}\right)}{d} + \frac{\left(\frac{1}{2}\mathrm{i}\right)^{1/3}\,B\,a^{2/3}\,\ln\left(\frac{(-1)^{1/3}\,2^{1/3}\,a^{1/3}}{2}-\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}+\frac{(-1)^{5/6}\,2^{1/3}\,\sqrt{3}\,a^{1/3}}{2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d} + \frac{2^{2/3}\,A\,a^{2/3}\,\ln\left(\frac{9\,A^2\,a^2\,\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}}{d^2}-\frac{9\,2^{1/3}\,A^2\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} - \frac{2^{2/3}\,A\,a^{2/3}\,\ln\left(\frac{9\,A^2\,a^2\,\left(a+a\,\tan(c+d\,x)\,1\mathrm{i}\right)^{1/3}}{d^2}-\frac{9\,2^{1/3}\,A^2\,a^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} - \frac{\left(\frac{1}{2}\mathrm{i}\right)^{1/3}\,B\,a^{2/3}\,\ln\left(\left(a\,\left(1+\tan(c+d\,x)\,1\mathrm{i}\right)\right)^{1/3}-\frac{(-1)^{1/3}\,2^{1/3}\,a^{1/3}}{2}+\frac{(-1)^{5/6}\,2^{1/3}\,\sqrt{3}\,a^{1/3}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(2/3),x)
```

output
```
(3*A*(a + a*tan(c + d*x)*1i)^(2/3))/(2*d) - (B*(a + a*tan(c + d*x)*1i)^(5/
3)*3i)/(5*a*d) + (2^(2/3)*A*a^(2/3)*log((a*(tan(c + d*x)*1i + 1))^(1/3) -
2^(1/3)*a^(1/3)))/(2*d) + ((1i/2)^(1/3)*B*a^(2/3)*log((a*(tan(c + d*x)*1i
+ 1))^(1/3) + (-1)^(1/3)*2^(1/3)*a^(1/3)))/d + ((1i/2)^(1/3)*B*a^(2/3)*log
(((-1)^(1/3)*2^(1/3)*a^(1/3))/2 - (a*(tan(c + d*x)*1i + 1))^(1/3) + ((-1)^
(5/6)*2^(1/3)*3^(1/2)*a^(1/3))/2)*((3^(1/2)*1i)/2 - 1/2))/d + (2^(2/3)*A*a
^(2/3)*log((9*A^2*a^2*(a + a*tan(c + d*x)*1i)^(1/3))/d^2 - (9*2^(1/3)*A^2*
a^(7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/d^2)*((3^(1/2)*1i)/2 - 1/2))/(2*d) - (2^
(2/3)*A*a^(2/3)*log((9*A^2*a^2*(a + a*tan(c + d*x)*1i)^(1/3))/d^2 - (9*2^(
1/3)*A^2*a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/d^2)*((3^(1/2)*1i)/2 + 1/2))/(2
*d) - ((1i/2)^(1/3)*B*a^(2/3)*log((a*(tan(c + d*x)*1i + 1))^(1/3) - ((-1)^
(1/3)*2^(1/3)*a^(1/3))/2 + ((-1)^(5/6)*2^(1/3)*3^(1/2)*a^(1/3))/2)*((3^(1/
2)*1i)/2 + 1/2))/d
```

Reduce [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = a^{\frac{2}{3}}\left(\left(\int (\tan(dx+c)\,i+1)^{\frac{2}{3}}\tan(dx+c)^2\,dx\right)b+\left(\int (\tan(dx+c)\,i+1)^{\frac{2}{3}}\tan(dx+c)\,dx\right)a\right)$$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
a**(2/3)*(int((tan(c + d*x)*i + 1)**(2/3)*tan(c + d*x)**2,x)*b + int((tan(
c + d*x)*i + 1)**(2/3)*tan(c + d*x),x)*a)
```

3.199 $\int (a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 202

$$\int (a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = -\frac{a^{2/3}(A-iB)x}{2\sqrt[3]{2}} + \frac{\sqrt{3}a^{2/3}(iA+B)\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{2}d} + \frac{a^{2/3}(iA+B)\log(\cos(c+dx))}{2\sqrt[3]{2}d} + \frac{3a^{2/3}(iA+B)\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{2}d} + \frac{3B(a+ia\tan(c+dx))^{2/3}}{2d}$$

output
```
-1/4*a^(2/3)*(A-I*B)*x*2^(2/3)+1/2*3^(1/2)*a^(2/3)*(I*A+B)*arctan(1/3*(a^(
1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))*3^(1/2)/a^(1/3))*2^(2/3)/d+1/4*a^(2
/3)*(I*A+B)*ln(cos(d*x+c))*2^(2/3)/d+3/4*a^(2/3)*(I*A+B)*ln(2^(1/3)*a^(1/3
)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/d+3/2*B*(a+I*a*tan(d*x+c))^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int (a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \frac{2^{2/3}a^{2/3}(iA+B)\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\log(i+\tan(c+dx)\cdots}{4d}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x]
```

output
```
(2^(2/3)*a^(2/3)*(I*A + B)*(2*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + I*a*Tan[c
+ d*x])^(1/3))/a^(1/3))/Sqrt[3]] - Log[I + Tan[c + d*x]] + 3*Log[2^(1/3)*a
^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)]) + 6*B*(a + I*a*Tan[c + d*x])^(2/3)
)/(4*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4010, 3042, 3962, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4010}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^{2/3}dx+\frac{3B(a+ia\tan(c+dx))^{2/3}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^{2/3}dx+\frac{3B(a+ia\tan(c+dx))^{2/3}}{2d}$$

$$\Big\downarrow \text{3962}$$

$$\frac{3B(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{ia(A-iB)\int \frac{1}{(a-ia\tan(c+dx))\sqrt[3]{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{67}$$

$$\frac{3B(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{ia(A-iB)\left(-\frac{3}{2}\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}+\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)}d\sqrt[3]{i\tan}}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{3B(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{ia(A-iB)\left(-\frac{3}{2}\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\right.}{d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3B(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{ia(A-iB)\left(\frac{3\int \frac{1}{a^2\tan^2(c+dx)-3}d(i2^{2/3}a^{2/3}\tan(c+dx)+1)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{3B(a+ia\tan(c+dx))^{2/3}}{2d}-\frac{ia(A-iB)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}$$

input
```
Int[(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x]
```

output
```
((-I)*a*(A - I*B)*(((-I)*Sqrt[3]*ArcTanh[(a*Tan[c + d*x])/Sqrt[3]])/(2^(1/
3)*a^(1/3)) - (3*Log[2^(1/3)*a^(1/3) - I*a*Tan[c + d*x]])/(2*2^(1/3)*a^(1/
3)) + Log[a - I*a*Tan[c + d*x]]/(2*2^(1/3)*a^(1/3))))/d + (3*B*(a + I*a*Ta
n[c + d*x])^(2/3))/(2*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{3i\left(-\frac{iB(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2\right.}{12a^{\frac{1}{3}}}\right.\right.}{d}$
default	$\frac{3i\left(-\frac{iB(a+ia\tan(dx+c))^{\frac{2}{3}}}{2}+\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2\right.}{12a^{\frac{1}{3}}}\right.\right.}{d}$
parts	$\frac{3iAa\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\right.}{}\right.}{d}$

input
```
int((a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
3*I/d*(-1/2*I*B*(a+I*a*tan(d*x+c))^(2/3)+(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*ta
n(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c)
)^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1
/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))
^(1/3)+1)))*a*(A-I*B))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 489 vs. 2(146) = 292.

Time = 0.08 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.42

$$\int (a + ia\tan(c + dx))^{2/3}(A + B\tan(c + dx))\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(3*2^(2/3)*B*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(4/3*I*d*x + 4/3*I*
c) + 2*(1/2)^(1/3)*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a^2/d^3)^(1/3)*
log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e
^(2/3*I*d*x + 2/3*I*c) + 2*(1/2)^(2/3)*d^2*((-I*A^3 - 3*A^2*B + 3*I*A*B^2
+ B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)) + (1/2)^(1/3)*(I*sqrt(3)
*d - d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*
(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x +
 2/3*I*c) - (1/2)^(2/3)*(I*sqrt(3)*d^2 + d^2)*((-I*A^3 - 3*A^2*B + 3*I*A*B
^2 + B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)) + (1/2)^(1/3)*(-I*sqr
t(3)*d - d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)*a^2/d^3)^(1/3)*log((2^(1
/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d
*x + 2/3*I*c) - (1/2)^(2/3)*(-I*sqrt(3)*d^2 + d^2)*((-I*A^3 - 3*A^2*B + 3*
I*A*B^2 + B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)))/d
```

Sympy [F]

$$\int (a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx=\int (ia(\tan{(c+dx)}-i))^{\frac{2}{3}}\,(A+B\tan{(c+dx)})\,\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(2/3)*(A + B*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.83

$$\int (a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx=\frac{i\left(2\sqrt{3}2^{\frac{2}{3}}(A-iB)a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}(A-iB)a^{\frac{5}{3}}\log\left(\right.$$

input
```
integrate((a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*I*(2*sqrt(3)*2^(2/3)*(A - I*B)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*(A - I*B
)*a^(5/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/
3) + (I*a*tan(d*x + c) + a)^(2/3)) + 2*2^(2/3)*(A - I*B)*a^(5/3)*log(-2^(1
/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)) - 6*I*(I*a*tan(d*x + c) + a)^(
2/3)*B*a)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.32

$$\int (a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx =$$

$$-\frac{-2i\sqrt{3}2^{\frac{2}{3}}Aa^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-2\sqrt{3}2^{\frac{2}{3}}Ba^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(i\,a\tan(dx+c)+a)^{\frac{1}{3}}\right.}{6\,a^{\frac{1}{3}}}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(-2*I*sqrt(3)*2^(2/3)*A*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a
^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2*sqrt(3)*2^(2/3)*B*a^
(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) +
a)^(1/3))/a^(1/3)) + I*2^(2/3)*A*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*
a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) + 2^(2/3
)*B*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(
1/3) + (I*a*tan(d*x + c) + a)^(2/3)) - 6*(I*a*tan(d*x + c) + a)^(2/3)*B -
2*I*2^(1/3)*(2^(1/3)*A*a^(4/3) - I*2^(1/3)*B*a^(4/3))*log(-2^(1/3)*a^(1/3)
 + (I*a*tan(d*x + c) + a)^(1/3))/a^(2/3))/d
```

Mupad [B] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.82

$$\int (a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \frac{3\,B\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{2/3}}{2\,d}$$
$$+\frac{2^{2/3}\,B\,a^{2/3}\,\ln\left(\left(a\,(1+\tan(c+d\,x)\ 1\mathrm{i})\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{2\,d}$$
$$-\frac{\left(\frac{1}{2}\mathrm{i}\right)^{1/3}\,A\,a^{2/3}\,\ln\left(\left(a\,(1+\tan(c+d\,x)\ 1\mathrm{i})\right)^{1/3}+(-1)^{1/3}\,2^{1/3}\,a^{1/3}\right)}{d}$$
$$-\frac{\left(\frac{1}{2}\mathrm{i}\right)^{1/3}\,A\,a^{2/3}\,\ln\left(\frac{(-1)^{1/3}\,2^{1/3}\,a^{1/3}}{2}-\left(a\,(1+\tan(c+d\,x)\ 1\mathrm{i})\right)^{1/3}+\frac{(-1)^{5/6}\,2^{1/3}\,\sqrt{3}\,a^{1/3}}{2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}$$
$$+\frac{2^{2/3}\,B\,a^{2/3}\,\ln\left(\frac{9\,B^2\,a^2\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{1/3}}{d^2}-\frac{9\,2^{1/3}\,B^2\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$-\frac{2^{2/3}\,B\,a^{2/3}\,\ln\left(\frac{9\,B^2\,a^2\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{1/3}}{d^2}-\frac{9\,2^{1/3}\,B^2\,a^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{d^2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$+\frac{\left(\frac{1}{2}\mathrm{i}\right)^{1/3}\,A\,a^{2/3}\,\ln\left(\left(a\,(1+\tan(c+d\,x)\ 1\mathrm{i})\right)^{1/3}-\frac{(-1)^{1/3}\,2^{1/3}\,a^{1/3}}{2}+\frac{(-1)^{5/6}\,2^{1/3}\,\sqrt{3}\,a^{1/3}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d}$$

input
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(2/3),x)
```

output
```
(3*B*(a + a*tan(c + d*x)*1i)^(2/3))/(2*d) + (2^(2/3)*B*a^(2/3)*log((a*(tan
(c + d*x)*1i + 1))^(1/3) - 2^(1/3)*a^(1/3)))/(2*d) - ((1i/2)^(1/3)*A*a^(2/
3)*log((a*(tan(c + d*x)*1i + 1))^(1/3) + (-1)^(1/3)*2^(1/3)*a^(1/3)))/d -
((1i/2)^(1/3)*A*a^(2/3)*log(((-1)^(1/3)*2^(1/3)*a^(1/3))/2 - (a*(tan(c + d
*x)*1i + 1))^(1/3) + ((-1)^(5/6)*2^(1/3)*3^(1/2)*a^(1/3))/2)*((3^(1/2)*1i)
/2 - 1/2))/d + (2^(2/3)*B*a^(2/3)*log((9*B^2*a^2*(a + a*tan(c + d*x)*1i)^(
1/3))/d^2 - (9*2^(1/3)*B^2*a^(7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/d^2)*((3^(1/2
)*1i)/2 - 1/2))/(2*d) - (2^(2/3)*B*a^(2/3)*log((9*B^2*a^2*(a + a*tan(c + d
*x)*1i)^(1/3))/d^2 - (9*2^(1/3)*B^2*a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/d^2)
*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((1i/2)^(1/3)*A*a^(2/3)*log((a*(tan(c + d
*x)*1i + 1))^(1/3) - ((-1)^(1/3)*2^(1/3)*a^(1/3))/2 + ((-1)^(5/6)*2^(1/3)*
3^(1/2)*a^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/d
```

Reduce [F]

$$\int (a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = a^{\frac{2}{3}}\left(\left(\int (\tan (dx+c)\,i+1)^{\frac{2}{3}}\,dx\right)a + \left(\int (\tan (dx+c)\,i+1)^{\frac{2}{3}}\tan (dx+c)\,dx\right)b\right)$$

input
```
int((a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
a**(2/3)*(int((tan(c + d*x)*i + 1)**(2/3),x)*a + int((tan(c + d*x)*i + 1)*
*(2/3)*tan(c + d*x),x)*b)
```

3.200 $\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 289

$$\begin{aligned}
\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx &= \\
&-\frac{a^{2/3}(iA+B)x}{2\sqrt[3]{2}}+\frac{\sqrt{3}a^{2/3}A\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{d} \\
&-\frac{\sqrt{3}a^{2/3}(A-iB)\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{2}d} \\
&-\frac{a^{2/3}(A-iB)\log(\cos(c+dx))}{2\sqrt[3]{2}d}-\frac{a^{2/3}A\log(\tan(c+dx))}{2d} \\
&+\frac{3a^{2/3}A\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2d} \\
&-\frac{3a^{2/3}(A-iB)\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{2}d}
\end{aligned}$$

output
```
-1/4*a^(2/3)*(I*A+B)*x*2^(2/3)+3^(1/2)*a^(2/3)*A*arctan(1/3*(a^(1/3)+2*(a+
I*a*tan(d*x+c))^(1/3))*3^(1/2)/a^(1/3))/d-1/2*3^(1/2)*a^(2/3)*(A-I*B)*arct
an(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))*3^(1/2)/a^(1/3))*2^(2/3)
/d-1/4*a^(2/3)*(A-I*B)*ln(cos(d*x+c))*2^(2/3)/d-1/2*a^(2/3)*A*ln(tan(d*x+c
))/d+3/2*a^(2/3)*A*ln(a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))/d-3/4*a^(2/3)*(A-I
*B)*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))^(1/3))*2^(2/3)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.75 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.44

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \frac{3\left(\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left((A-iB)\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},\frac{e^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)-2A\,\text{Hypergeon}\right.}{2\sqrt[3]{2}d}$$

input
```
Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x
]
```

output
```
(3*((a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*((A - I*B)*Hy
pergeometric2F1[2/3, 1, 5/3, E^((2*I)*(c + d*x))/(1 + E^((2*I)*(c + d*x)))
] - 2*A*Hypergeometric2F1[2/3, 1, 5/3, (2*E^((2*I)*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))]))/(2*2^(1/3)*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.75, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {3042, 4083, 3042, 3962, 67, 16, 1082, 217, 4082, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4083}$$

$$(B+iA)\int (i\tan(c+dx)a+a)^{2/3}dx+\frac{A\int \cot(c+dx)(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(B+iA)\int (i\tan(c+dx)a+a)^{2/3}dx+\frac{A\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}{\tan(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3962}$$

$$\frac{A\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}{\tan(c+dx)}dx}{a}-\frac{ia(B+iA)\int \frac{1}{(a-ia\tan(c+dx))\sqrt[3]{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{67}$$

$$\frac{A\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}{\tan(c+dx)}dx}{a}-\frac{ia(B+iA)\left(-\frac{3}{2}\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}+\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)}d\sqrt[3]{i\tan}}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{A\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}{\tan(c+dx)}dx}{a}-\frac{ia(B+iA)\left(-\frac{3}{2}\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\right.}{d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{A\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}{\tan(c+dx)}dx}{a}-\frac{ia(B+iA)\left(\frac{3\int \frac{1}{a^2\tan^2(c+dx)-3}d(i2^{2/3}a^{2/3}\tan(c+dx)+1)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}$$

$$\Big\downarrow \; 217$$

$$\frac{A\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}{\tan(c+dx)}dx}{a}-\frac{ia(B+iA)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}$$

$$\Big\downarrow \; 4082$$

$$\frac{aA\int \frac{\cot(c+dx)}{\sqrt[3]{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{ia(B+iA)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}$$

$$\Big\downarrow \; 67$$

$$\frac{aA\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{i\tan(c+dx)a+a}\sqrt[3]{a}+(i\tan(c+dx)a+a)^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{i\tan(c+dx)a+a}}d}{2\sqrt[3]{a}}\right.}{d}$$
$$-\frac{ia(B+iA)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}$$

$$\Big\downarrow \; 16$$

$$\frac{aA\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{i\tan(c+dx)a+a}\sqrt[3]{a}+(i\tan(c+dx)a+a)^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}-\frac{\log(\tan(c+dx))}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a}\right.}{2}\right.}{d}$$
$$-\frac{ia(B+iA)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}$$

$\big\downarrow$ 1082

$$\frac{aA\left(-\frac{3\int \frac{1}{-(i\tan(c+dx)a+a)^{2/3}-3}d\left(\frac{2\sqrt[3]{i\tan(c+dx)a+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}-\frac{\log(\tan(c+dx))}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{a}}\right)}{d}-\frac{ia(B+iA)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}$$

$\big\downarrow$ 217

$$\frac{aA\left(\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{\log(\tan(c+dx))}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{a}}\right)}{d}-\frac{ia(B+iA)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}$$

input
```
Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x]
```

output
```
((-I)*a*(I*A + B)*(((-I)*Sqrt[3]*ArcTanh[(a*Tan[c + d*x])/Sqrt[3]])/(2^(1/
3)*a^(1/3)) - (3*Log[2^(1/3)*a^(1/3) - I*a*Tan[c + d*x]])/(2*2^(1/3)*a^(1/
3)) + Log[a - I*a*Tan[c + d*x]]/(2*2^(1/3)*a^(1/3))))/d + (a*A*((Sqrt[3]*A
rcTan[(1 + (2*(a + I*a*Tan[c + d*x])^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - L
og[Tan[c + d*x]]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + I*a*Tan[c + d*x])^(1/
3)])/(2*a^(1/3))))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86

method	result
derivativedivides	$3a\left(\left(\left(\frac{2^{\frac{2}{3}}\ln\left(\left(a+ia\tan\left(dx+c\right)\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left(\left(a+ia\tan\left(dx+c\right)\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(a+ia\tan\left(dx+c\right)\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\cdots\right.}{\cdots}\right.\right.\right.$
default	$3a\left(\left(\left(\frac{2^{\frac{2}{3}}\ln\left(\left(a+ia\tan\left(dx+c\right)\right)^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left(\left(a+ia\tan\left(dx+c\right)\right)^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}\left(a+ia\tan\left(dx+c\right)\right)^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\cdots\right.}{\cdots}\right.\right.\right.$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
3/d*a*((1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1
/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan
(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(
1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*(-A+I*B)+(1/3/a^(1/3)*
ln((a+I*a*tan(d*x+c))^(1/3)-a^(1/3))-1/6/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/
3)+a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/
3*3^(1/2)*(2/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*A)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 711 vs. $2(211) = 422$.

Time = 0.10 (sec) , antiderivative size = 711, normalized size of antiderivative = 2.46

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/2*(1/2)^(1/3)*(-I*sqrt(3) - 1)*(-(A^3 - 3*I*A^2*B - 3*A*B^2 + I*B^3)*a^2
/d^3)^(1/3)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) +
 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (1/2)^(2/3)*(I*sqrt(3)*d^2 - d^2)*(-(
A^3 - 3*I*A^2*B - 3*A*B^2 + I*B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*
a)) + 1/2*(1/2)^(1/3)*(I*sqrt(3) - 1)*(-(A^3 - 3*I*A^2*B - 3*A*B^2 + I*B^3
)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I
*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (1/2)^(2/3)*(-I*sqrt(3)*d^2 - d^
2)*(-(A^3 - 3*I*A^2*B - 3*A*B^2 + I*B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B -
 B^2)*a)) + 1/2*(A^3*a^2/d^3)^(1/3)*(I*sqrt(3) - 1)*log(1/2*(2*2^(1/3)*A^2
*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (I*sqrt(3
)*d^2 + d^2)*(A^3*a^2/d^3)^(2/3))/(A^2*a)) + 1/2*(A^3*a^2/d^3)^(1/3)*(-I*s
qrt(3) - 1)*log(1/2*(2*2^(1/3)*A^2*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e
^(2/3*I*d*x + 2/3*I*c) + (-I*sqrt(3)*d^2 + d^2)*(A^3*a^2/d^3)^(2/3))/(A^2*
a)) + (1/2)^(1/3)*(-(A^3 - 3*I*A^2*B - 3*A*B^2 + I*B^3)*a^2/d^3)^(1/3)*log
((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2
/3*I*d*x + 2/3*I*c) - 2*(1/2)^(2/3)*d^2*(-(A^3 - 3*I*A^2*B - 3*A*B^2 + I*B
^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)) + (A^3*a^2/d^3)^(1/3)*log((
2^(1/3)*A^2*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c)
- (A^3*a^2/d^3)^(2/3)*d^2)/(A^2*a))
```

Sympy [F]

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{2}{3}}(A+B\tan(c+dx))\cot(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(2/3)*(A + B*tan(c + d*x))*cot(c + d*x)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.87

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = -\frac{2\sqrt{3}2^{\frac{2}{3}}(A-iB)a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(ia\tan(dx+c)+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}(A-iB)a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(ia\tan(dx\right.}{}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/4*(2*sqrt(3)*2^(2/3)*(A - I*B)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1
/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*(A - I*B)
*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3
) + (I*a*tan(d*x + c) + a)^(2/3)) + 2*2^(2/3)*(A - I*B)*a^(2/3)*log(-2^(1/
3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)) - 4*sqrt(3)*A*a^(2/3)*arctan(1/
3*sqrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3)) + 2*A*a^(2/3
)*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a*tan(d*x + c) + a)^(1/3)*a^(1/3)
+ a^(2/3)) - 4*A*a^(2/3)*log((I*a*tan(d*x + c) + a)^(1/3) - a^(1/3)))/d
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.23

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\sqrt{3}2^{\frac{2}{3}}Aa^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)-2i\sqrt{3}2^{\frac{2}{3}}Ba^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{}$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/4*(2*sqrt(3)*2^(2/3)*A*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1
/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2*I*sqrt(3)*2^(2/3)*B*a^(
2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a
)^(1/3))/a^(1/3)) - 2^(2/3)*A*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*t
an(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) + I*2^(2/3)
*B*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1
/3) + (I*a*tan(d*x + c) + a)^(2/3)) - 4*sqrt(3)*A*a^(2/3)*arctan(1/3*sqrt(
3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3)) + 2*A*a^(2/3)*log((
I*a*tan(d*x + c) + a)^(2/3) + (I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + a^(2/
3)) - 4*A*a^(2/3)*log((I*a*tan(d*x + c) + a)^(1/3) - a^(1/3)) + 2*2^(1/3)*
(2^(1/3)*A*a^(4/3) - I*2^(1/3)*B*a^(4/3))*log(-2^(1/3)*a^(1/3) + (I*a*tan(
d*x + c) + a)^(1/3))/a^(2/3))/d
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 1761, normalized size of antiderivative = 6.09

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(2/3),x)
```

output
```
log(- (486*d^3*(3*A*B^2*a^9 - B^3*a^9*1i + A^2*B*a^9*3i) - (1458*a^7*d^6*(
(A^3*a^2)/d^3)^(2/3) + 243*d*(a + a*tan(c + d*x)*1i)^(1/3)*(B^2*a^8*d^3 -
5*A^2*a^8*d^3 + A*B*a^8*d^3*2i))*((A^3*a^2)/d^3)^(1/3))*((A^3*a^2)/d^3)^(2
/3) - 243*d*(a + a*tan(c + d*x)*1i)^(1/3)*(A^5*a^10 - A^4*B*a^10*4i + A^2*
B^3*a^10*2i - 5*A^3*B^2*a^10))*((A^3*a^2)/d^3)^(1/3) + log(- (486*d^3*(3*A
*B^2*a^9 - B^3*a^9*1i + A^2*B*a^9*3i) - (1458*a^7*d^6*(-(A^3*a^2 + B^3*a^2
*1i - 3*A*B^2*a^2 - A^2*B*a^2*3i)/(2*d^3))^(2/3) + 243*d*(a + a*tan(c + d*
x)*1i)^(1/3)*(B^2*a^8*d^3 - 5*A^2*a^8*d^3 + A*B*a^8*d^3*2i))*(-(A^3*a^2 +
B^3*a^2*1i - 3*A*B^2*a^2 - A^2*B*a^2*3i)/(2*d^3))^(1/3))*(-(A^3*a^2 + B^3*
a^2*1i - 3*A*B^2*a^2 - A^2*B*a^2*3i)/(2*d^3))^(2/3) - 243*d*(a + a*tan(c +
 d*x)*1i)^(1/3)*(A^5*a^10 - A^4*B*a^10*4i + A^2*B^3*a^10*2i - 5*A^3*B^2*a^
10))*(-(A^3*a^2 + B^3*a^2*1i - 3*A*B^2*a^2 - A^2*B*a^2*3i)/(2*d^3))^(1/3)
+ (log(- ((3^(1/2)*1i - 1)^2*(486*d^3*(3*A*B^2*a^9 - B^3*a^9*1i + A^2*B*a^
9*3i) - ((3^(1/2)*1i - 1)*(243*d*(a + a*tan(c + d*x)*1i)^(1/3)*(B^2*a^8*d^
3 - 5*A^2*a^8*d^3 + A*B*a^8*d^3*2i) + (729*a^7*d^6*(3^(1/2)*1i - 1)^2*((A^
3*a^2)/d^3)^(2/3))/2)*((A^3*a^2)/d^3)^(1/3))/2)*((A^3*a^2)/d^3)^(2/3))/4 -
 243*d*(a + a*tan(c + d*x)*1i)^(1/3)*(A^5*a^10 - A^4*B*a^10*4i + A^2*B^3*a
^10*2i - 5*A^3*B^2*a^10))*(3^(1/2)*1i - 1)*((A^3*a^2)/d^3)^(1/3))/2 - (log
(- ((3^(1/2)*1i + 1)^2*(486*d^3*(3*A*B^2*a^9 - B^3*a^9*1i + A^2*B*a^9*3i)
+ ((3^(1/2)*1i + 1)*(243*d*(a + a*tan(c + d*x)*1i)^(1/3)*(B^2*a^8*d^3 -...
```

Reduce [F]

$$\int \cot(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = a^{\frac{2}{3}}\left(\left(\int (\tan(dx+c)\,i+1)^{\frac{2}{3}}\cot(dx+c)\tan(dx+c)\,dx\right)b+\left(\int (\tan(dx+c)\,i+1)^{\frac{2}{3}}\cot(dx+c)\,dx\right)a\right)$$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
a**(2/3)*(int((tan(c + d*x)*i + 1)**(2/3)*cot(c + d*x)*tan(c + d*x),x)*b +
 int((tan(c + d*x)*i + 1)**(2/3)*cot(c + d*x),x)*a)
```

3.201 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 342

$$\begin{aligned}
\int \cot^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx &= \frac{a^{2/3}(A-iB)x}{2\sqrt[3]{2}} \\
&+ \frac{a^{2/3}(2iA+3B)\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}d} \\
&- \frac{\sqrt{3}a^{2/3}(iA+B)\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{2}d} \\
&- \frac{a^{2/3}(iA+B)\log(\cos(c+dx))}{2\sqrt[3]{2}d} - \frac{a^{2/3}(2iA+3B)\log(\tan(c+dx))}{6d} \\
&+ \frac{a^{2/3}(2iA+3B)\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2d} \\
&- \frac{3a^{2/3}(iA+B)\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{2}d} \\
&- \frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}
\end{aligned}$$

output
```
1/4*a^(2/3)*(A-I*B)*x*2^(2/3)+1/3*a^(2/3)*(2*I*A+3*B)*arctan(1/3*(a^(1/3)+
2*(a+I*a*tan(d*x+c))^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/d-1/2*3^(1/2)*a^(2/3)
*(I*A+B)*arctan(1/3*(a^(1/3)+2^(2/3)*(a+I*a*tan(d*x+c))^(1/3))*3^(1/2)/a^(
1/3))*2^(2/3)/d-1/4*a^(2/3)*(I*A+B)*ln(cos(d*x+c))*2^(2/3)/d-1/6*a^(2/3)*(
2*I*A+3*B)*ln(tan(d*x+c))/d+1/2*a^(2/3)*(2*I*A+3*B)*ln(a^(1/3)-(a+I*a*tan(
d*x+c))^(1/3))/d-3/4*a^(2/3)*(I*A+B)*ln(2^(1/3)*a^(1/3)-(a+I*a*tan(d*x+c))
^(1/3))*2^(2/3)/d-A*cot(d*x+c)*(a+I*a*tan(d*x+c))^(2/3)/d
```

Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.73

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \frac{2a^2(2iA+3B)\left(2\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)-\log(\tan(c+dx))+3\,\mathrm{l}\cdots}{\cdots}$$

input
```
Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x])
,x]
```

output
```
(2*a^2*((2*I)*A + 3*B)*(2*Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + I*a*Tan[c + d*x
])^(1/3))/(Sqrt[3]*a^(1/3))] - Log[Tan[c + d*x]] + 3*Log[a^(1/3) - (a + I*
a*Tan[c + d*x])^(1/3)]) - (3*I)*2^(2/3)*a^2*(A - I*B)*(2*Sqrt[3]*ArcTan[(1
 + (2^(2/3)*(a + I*a*Tan[c + d*x])^(1/3))/a^(1/3))/Sqrt[3]] - Log[I + Tan[
c + d*x]] + 3*Log[2^(1/3)*a^(1/3) - (a + I*a*Tan[c + d*x])^(1/3)]) - 12*a^
(4/3)*A*Cot[c + d*x]*(a + I*a*Tan[c + d*x])^(2/3))/(12*a^(4/3)*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.78, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4081, 27, 3042, 4083, 3042, 3962, 67, 16, 1082, 217, 4082, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4081}$$

$$\frac{\int \frac{1}{3}\cot(c+dx)(i\tan(c+dx)a+a)^{2/3}(a(2iA+3B)-aA\tan(c+dx))dx}{a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cot(c+dx)(i\tan(c+dx)a+a)^{2/3}(a(2iA+3B)-aA\tan(c+dx))dx}{3a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(i\tan(c+dx)a+a)^{2/3}(a(2iA+3B)-aA\tan(c+dx))}{\tan(c+dx)}dx}{3a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$$\Big\downarrow \text{4083}$$

$$\frac{(3B+2iA)\int \cot(c+dx)(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}dx-3a(A-iB)\int (i\tan(c+dx)a+a)^{2/3}d}{3a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3B+2iA)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}{\tan(c+dx)}dx-3a(A-iB)\int (i\tan(c+dx)a+a)^{2/3}dx}{3a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$$\Big\downarrow \text{3962}$$

$$\frac{\frac{3ia^2(A-iB)\int \frac{1}{(a-ia\tan(c+dx))\sqrt[3]{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}+(3B+2iA)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}{\tan(c+dx)}dx}{3a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$$\Big\downarrow \text{67}$$

$$\frac{\frac{3ia^2(A-iB)\left(-\frac{3}{2}\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}+\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)}d\sqrt[3]{i\tan(c+dx)}}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{d}}{3a}$$

$$\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{3ia^2(A-iB)\left(-\frac{3}{2}\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{d}}{3a}$$

$$\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{3ia^2(A-iB)\left(\frac{3\int \frac{1}{a^2\tan^2(c+dx)-3}d\left(i2^{2/3}a^{2/3}\tan(c+dx)+1\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{d}+(3B+2iA)\int \frac{(a-ia\,t}{}}{3a}$$

$$\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{(3B+2iA)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}{\tan(c+dx)}dx+\frac{3ia^2(A-iB)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\ldots}{d}}{3a}$$

$$\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$\downarrow$ 4082

$$\frac{\frac{a^2(3B+2iA)\int \frac{\cot(c+dx)}{\sqrt[3]{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{3ia^2(A-iB)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-\ldots}{2\ldots}\right.}{d}}{3a}$$

$$\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$\downarrow$ 67

$$\frac{\frac{a^2(3B+2iA)\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{i\tan(c+dx)a+a}\sqrt[3]{a}+(i\tan(c+dx)a+a)^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}-\frac{3\int\frac{1}{\sqrt[3]{a}-\sqrt[3]{i\tan(c+dx)a+a}\ldots}}{2\sqrt[3]{a}}\right.}{d}}{\ldots}$$

$$\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$\downarrow$ 16

$$\frac{\frac{a^2(3B+2iA)\left(\frac{3}{2}\int \frac{1}{a^{2/3}+\sqrt[3]{i\tan(c+dx)a+a}\sqrt[3]{a}+(i\tan(c+dx)a+a)^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}-\frac{\log(\tan(c+dx))}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+\ldots}\right.}{2\sqrt[3]{\ldots}}\right.}{d}}{3a}$$

$$\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$\downarrow$ 1082

$$\frac{\frac{a^2(3B+2iA)\left(-\frac{3\int \frac{1}{-(i\tan(c+dx)a+a)^{2/3}-3}d\left(\frac{2\sqrt[3]{i\tan(c+dx)a+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}-\frac{\log(\tan(c+dx))}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{a}}\right)}{d}+}{3a}$$

$$\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{a^2(3B+2iA)\left(\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{\log(\tan(c+dx))}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{2\sqrt[3]{a}}\right)}{d}+\frac{3ia^2(A-iB)\left(-\frac{i\sqrt{}}{}\right.}{}}{3a}$$

$$\frac{A\cot(c+dx)(a+ia\tan(c+dx))^{2/3}}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x]
```

output
```
(((3*I)*a^2*(A - I*B)*(((-I)*Sqrt[3]*ArcTanh[(a*Tan[c + d*x])/Sqrt[3]])/(2
^(1/3)*a^(1/3)) - (3*Log[2^(1/3)*a^(1/3) - I*a*Tan[c + d*x]])/(2*2^(1/3)*a
^(1/3)) + Log[a - I*a*Tan[c + d*x]]/(2*2^(1/3)*a^(1/3))))/d + (a^2*((2*I)*
A + 3*B)*((Sqrt[3]*ArcTan[(1 + (2*(a + I*a*Tan[c + d*x])^(1/3))/a^(1/3))/S
qrt[3]])/a^(1/3) - Log[Tan[c + d*x]]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + I
*a*Tan[c + d*x])^(1/3)])/(2*a^(1/3))))/d)/(3*a) - (A*Cot[c + d*x]*(a + I*a
*Tan[c + d*x])^(2/3))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{3ia^2\left(-\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan}{}\right.}{a}\right.}{}$
default	$\frac{3ia^2\left(-\frac{\left(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan}{}\right.}{a}\right.}{}$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
3*I/d*a^2*(-(1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/
3))-1/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*
a*tan(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/
3*3^(1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*(A-I*B)/a+1/a*(1/
3*I*A*(a+I*a*tan(d*x+c))^(2/3)/a/tan(d*x+c)+(2/3*A-I*B)*(1/3/a^(1/3)*ln((a
+I*a*tan(d*x+c))^(1/3)-a^(1/3))-1/6/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+a^
(1/3)*(a+I*a*tan(d*x+c))^(1/3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/3*3^(
1/2)*(2/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(255) = 510$.

Time = 0.15 (sec) , antiderivative size = 1096, normalized size of antiderivative = 3.20

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/6*(6*2^(2/3)*(I*A*e^(2*I*d*x + 2*I*c) + I*A)*(a/(e^(2*I*d*x + 2*I*c) +
1))^(2/3)*e^(4/3*I*d*x + 4/3*I*c) - 6*(1/2)^(1/3)*(d*e^(2*I*d*x + 2*I*c) -
 d)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2
- 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*
I*c) + 2*(1/2)^(2/3)*d^2*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(2/
3))/((A^2 - 2*I*A*B - B^2)*a)) - 3*(1/2)^(1/3)*((I*sqrt(3)*d - d)*e^(2*I*d
*x + 2*I*c) - I*sqrt(3)*d + d)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^
3)^(1/3)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1)
)^(1/3)*e^(2/3*I*d*x + 2/3*I*c) - (1/2)^(2/3)*(I*sqrt(3)*d^2 + d^2)*((I*A^
3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a))
- 3*(1/2)^(1/3)*((-I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) + I*sqrt(3)*d + d)
*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3)*a^2/d^3)^(1/3)*log((2^(1/3)*(A^2 - 2
*I*A*B - B^2)*a*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c
) - (1/2)^(2/3)*(-I*sqrt(3)*d^2 + d^2)*((I*A^3 + 3*A^2*B - 3*I*A*B^2 - B^3
)*a^2/d^3)^(2/3))/((A^2 - 2*I*A*B - B^2)*a)) - ((-I*sqrt(3)*d - d)*e^(2*I*
d*x + 2*I*c) + I*sqrt(3)*d + d)*((-8*I*A^3 - 36*A^2*B + 54*I*A*B^2 + 27*B^
3)*a^2/d^3)^(1/3)*log(1/2*(2*2^(1/3)*(4*A^2 - 12*I*A*B - 9*B^2)*a*(a/(e^(2
*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (I*sqrt(3)*d^2 - d^2
)*((-8*I*A^3 - 36*A^2*B + 54*I*A*B^2 + 27*B^3)*a^2/d^3)^(2/3))/((4*A^2 - 1
2*I*A*B - 9*B^2)*a)) - ((I*sqrt(3)*d - d)*e^(2*I*d*x + 2*I*c) - I*sqrt(...
```

Sympy [F]

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{2}{3}}(A+B\tan(c+dx))\cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(2/3)*(A + B*tan(c + d*x))*cot(c + d*x)
**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.87

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx =$$

$$-\frac{i\left(\frac{6\sqrt{3}2^{\frac{2}{3}}(A-iB)\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{3\cdot 2^{\frac{2}{3}}(A-iB)\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(ia\tan(dx+c)+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(ia\tan(dx+c)+\cdots\right.}{a^{\frac{1}{3}}}\cdots\right.}{\cdots}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/12*I*(6*sqrt(3)*2^(2/3)*(A - I*B)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a
^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(1/3) - 3*2^(2/3)*(A -
 I*B)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) +
 (I*a*tan(d*x + c) + a)^(2/3))/a^(1/3) + 6*2^(2/3)*(A - I*B)*log(-2^(1/3)*
a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3))/a^(1/3) - 4*sqrt(3)*(2*A - 3*I*B)*
arctan(1/3*sqrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) + 2*(2*A - 3*I*B)*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a*tan(d*x + c
) + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) - 4*(2*A - 3*I*B)*log((I*a*tan(d*x
 + c) + a)^(1/3) - a^(1/3))/a^(1/3) - 12*I*(I*a*tan(d*x + c) + a)^(2/3)*A/
(a*tan(d*x + c)))*a/d
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.37

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-1/12*(6*I*sqrt(3)*2^(2/3)*A*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a
^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 6*sqrt(3)*2^(2/3)*B*a^
(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) +
a)^(1/3))/a^(1/3)) - 3*I*2^(2/3)*A*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(
I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) - 3*2^
(2/3)*B*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)
*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3)) - 8*I*sqrt(3)*A*a^(2/3)*arctan(1/
3*sqrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a^(1/3)) - 12*sqrt(3)
*B*a^(2/3)*arctan(1/3*sqrt(3)*(2*(I*a*tan(d*x + c) + a)^(1/3) + a^(1/3))/a
^(1/3)) + 4*I*A*a^(2/3)*log((I*a*tan(d*x + c) + a)^(2/3) + (I*a*tan(d*x +
c) + a)^(1/3)*a^(1/3) + a^(2/3)) + 6*B*a^(2/3)*log((I*a*tan(d*x + c) + a)^
(2/3) + (I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + a^(2/3)) + 6*I*2^(1/3)*(2^(
1/3)*A*a^(4/3) - I*2^(1/3)*B*a^(4/3))*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x
+ c) + a)^(1/3))/a^(2/3) + 4*(-2*I*A*a^(4/3) - 3*B*a^(4/3))*log((I*a*tan(d
*x + c) + a)^(1/3) - a^(1/3))/a^(2/3) + 12*(I*a*tan(d*x + c) + a)^(2/3)*A/
tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 5825, normalized size of antiderivative = 17.03

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(2/3),x)
```

output
```
log((18*d^3*(A^3*a^9*19i - A*B^2*a^9*27i + 45*A^2*B*a^9) - (1458*a^7*d^6*(
-((d^3*((((594*A^3*a^12 + 1458*A*B^2*a^12)*1i)/d^3 + (1458*B^3*a^12 + 486*
A^2*B*a^12)/d^3)^2 - 11664*a^10*((432*A^6*a^14 - 1458*B^6*a^14 + 15066*A^2
*B^4*a^14 - 10044*A^4*B^2*a^14)/d^6 - ((7290*A*B^5*a^14 + 3240*A^5*B*a^14
- 16470*A^3*B^3*a^14)*1i)/d^6))^(1/2)*1i - 594*A^3*a^12 + B^3*a^12*1458i -
 1458*A*B^2*a^12 + A^2*B*a^12*486i)*1i)/(5832*a^10*d^3))^(2/3) - 9*d*(a +
a*tan(c + d*x)*1i)^(1/3)*(135*B^2*a^8*d^3 - 75*A^2*a^8*d^3 + A*B*a^8*d^3*1
98i))*(-((d^3*((((594*A^3*a^12 + 1458*A*B^2*a^12)*1i)/d^3 + (1458*B^3*a^12
 + 486*A^2*B*a^12)/d^3)^2 - 11664*a^10*((432*A^6*a^14 - 1458*B^6*a^14 + 15
066*A^2*B^4*a^14 - 10044*A^4*B^2*a^14)/d^6 - ((7290*A*B^5*a^14 + 3240*A^5*
B*a^14 - 16470*A^3*B^3*a^14)*1i)/d^6))^(1/2)*1i - 594*A^3*a^12 + B^3*a^12*
1458i - 1458*A*B^2*a^12 + A^2*B*a^12*486i)*1i)/(5832*a^10*d^3))^(1/3))*(-(
(d^3*((((594*A^3*a^12 + 1458*A*B^2*a^12)*1i)/d^3 + (1458*B^3*a^12 + 486*A^
2*B*a^12)/d^3)^2 - 11664*a^10*((432*A^6*a^14 - 1458*B^6*a^14 + 15066*A^2*B
^4*a^14 - 10044*A^4*B^2*a^14)/d^6 - ((7290*A*B^5*a^14 + 3240*A^5*B*a^14 -
16470*A^3*B^3*a^14)*1i)/d^6))^(1/2)*1i - 594*A^3*a^12 + B^3*a^12*1458i - 1
458*A*B^2*a^12 + A^2*B*a^12*486i)*1i)/(5832*a^10*d^3))^(2/3) + 9*d*(a + a*
tan(c + d*x)*1i)^(1/3)*(A^5*a^10*16i + 27*B^5*a^10 + A*B^4*a^10*126i + 92*
A^4*B*a^10 - 231*A^2*B^3*a^10 - A^3*B^2*a^10*208i))*(-((d^3*((((594*A^3*a^
12 + 1458*A*B^2*a^12)*1i)/d^3 + (1458*B^3*a^12 + 486*A^2*B*a^12)/d^3)^2...
```

Reduce [F]

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = a^{\frac{2}{3}}\left(\left(\int (\tan(dx+c)i+1)^{\frac{2}{3}}\cot(dx+c)^2\tan(dx+c)\,dx\right)b+\left(\int (\tan(dx+c)i+1)^{\frac{2}{3}}\cot(dx+c)^2\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
a**(2/3)*(int((tan(c + d*x)*i + 1)**(2/3)*cot(c + d*x)**2*tan(c + d*x),x)*
b + int((tan(c + d*x)*i + 1)**(2/3)*cot(c + d*x)**2,x)*a)
```

3.202 $\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 213

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = -\frac{(A-iB)x}{4\sqrt[3]{2}\sqrt[3]{a}} + \frac{\sqrt{3}(iA+B)\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{2}\sqrt[3]{a}d} + \frac{(iA+B)\log(\cos(c+dx))}{4\sqrt[3]{2}\sqrt[3]{a}d} + \frac{3(iA+B)\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\sqrt[3]{2}\sqrt[3]{a}d} + \frac{3(iA-B)}{2d\sqrt[3]{a+ia\tan(c+dx)}}$$

output
```
-1/8*(A-I*B)*x*2^(2/3)/a^(1/3)+1/4*3^(1/2)*(I*A+B)*arctan(1/3*(a^(1/3)+2^(
2/3)*(a+I*a*tan(d*x+c))^(1/3))*3^(1/2)/a^(1/3))*2^(2/3)/a^(1/3)/d+1/8*(I*A
+B)*ln(cos(d*x+c))*2^(2/3)/a^(1/3)/d+3/8*(I*A+B)*ln(2^(1/3)*a^(1/3)-(a+I*a
*tan(d*x+c))^(1/3))*2^(2/3)/a^(1/3)/d+3/2*(I*A-B)/d/(a+I*a*tan(d*x+c))^(1/
3)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.70

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\frac{2^{2/3}(iA+B)\left(2\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)-\log(i+\tan(c+dx))+3\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)\right)}{\sqrt[3]{a}}}{8d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(1/3),x]
```

output
```
((2^(2/3)*(I*A + B)*(2*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*a*Tan[c +
d*x])^(1/3))/(Sqrt[3]*a^(1/3))] - Log[I + Tan[c + d*x]] + 3*Log[2^(1/3)*a^
(1/3) - (a + I*a*Tan[c + d*x])^(1/3)]))/a^(1/3) + ((12*I)*(A + I*B))/(a +
I*a*Tan[c + d*x])^(1/3))/(8*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4009, 3042, 3962, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}dx$$
$$\Big\downarrow \text{ 4009}$$
$$\frac{(A-iB)\int(i\tan(c+dx)a+a)^{2/3}dx}{2a}+\frac{3(-B+iA)}{2d\sqrt[3]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int (i\tan(c+dx)a+a)^{2/3}dx}{2a}+\frac{3(-B+iA)}{2d\sqrt[3]{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{3962}$$

$$\frac{3(-B+iA)}{2d\sqrt[3]{a+ia\tan(c+dx)}}-\frac{i(A-iB)\int \frac{1}{(a-ia\tan(c+dx))\sqrt[3]{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{2d}$$

$$\Big\downarrow \text{67}$$

$$\frac{3(-B+iA)}{2d\sqrt[3]{a+ia\tan(c+dx)}}-$$

$$\frac{i(A-iB)\left(-\frac{3}{2}\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}+\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)}d\sqrt[3]{i\tan(}}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{2d}$$

$$\Big\downarrow \text{16}$$

$$\frac{3(-B+iA)}{2d\sqrt[3]{a+ia\tan(c+dx)}}-$$

$$\frac{i(A-iB)\left(-\frac{3}{2}\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\text{lo}}{}\right.}{2d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3(-B+iA)}{2d\sqrt[3]{a+ia\tan(c+dx)}}-$$

$$\frac{i(A-iB)\left(\frac{3\int \frac{1}{a^2\tan^2(c+dx)-3}d(i2^{2/3}a^{2/3}\tan(c+dx)+1)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{2d}$$

$$\Big\downarrow \text{217}$$

$$\frac{3(-B+iA)}{2d\sqrt[3]{a+ia\tan(c+dx)}}-$$

$$\frac{i(A-iB)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\sqrt[3]{2}\sqrt[3]{a}}+\frac{\log(a-ia\tan(c+dx))}{2\sqrt[3]{2}\sqrt[3]{a}}\right)}{2d}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(1/3),x]
```

output
```
((-1/2*I)*(A - I*B)*(((-I)*Sqrt[3]*ArcTanh[(a*Tan[c + d*x])/Sqrt[3]])/(2^(
1/3)*a^(1/3)) - (3*Log[2^(1/3)*a^(1/3) - I*a*Tan[c + d*x]])/(2*2^(1/3)*a^(
1/3)) + Log[a - I*a*Tan[c + d*x]]/(2*2^(1/3)*a^(1/3))))/d + (3*(I*A - B))/
(2*d*(a + I*a*Tan[c + d*x])^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{3i\Big(\Big(\Big(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\Big(}{}}{d}$
default	$\frac{3i\Big(\Big(\Big(\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\Big(}{}}{d}$
parts	$\frac{3iAa\Big(\frac{\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{2^{\frac{2}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{1}{3}}}+\frac{\sqrt{3}\,2^{\frac{2}{3}}\arctan\Big(}{}}{2a}}{d}$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3*I/d*((1/6*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1
/12*2^(2/3)/a^(1/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan
(d*x+c))^(1/3)+2^(2/3)*a^(2/3))+1/6*3^(1/2)*2^(2/3)/a^(1/3)*arctan(1/3*3^(
1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*(-1/2*I*B+1/2*A)-(-1/2
*A-1/2*I*B)/(a+I*a*tan(d*x+c))^(1/3))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(154) = 308$.

Time = 0.09 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.57

$$\int \frac{A + B\tan(c + dx)}{\sqrt[3]{a + ia\tan(c + dx)}}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
1/4*(2*(1/2)^(1/3)*a*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a*d^3))^(1/3
)*e^(2*I*d*x + 2*I*c)*log((2*(1/2)^(2/3)*a*d^2*((-I*A^3 - 3*A^2*B + 3*I*A*
B^2 + B^3)/(a*d^3))^(2/3) + 2^(1/3)*(A^2 - 2*I*A*B - B^2)*(a/(e^(2*I*d*x +
 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c))/(A^2 - 2*I*A*B - B^2)) - (1/2
)^(1/3)*(I*sqrt(3)*a*d + a*d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a*d^3
))^(1/3)*e^(2*I*d*x + 2*I*c)*log((2^(1/3)*(A^2 - 2*I*A*B - B^2)*(a/(e^(2*I
*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c) + (1/2)^(2/3)*(I*sqrt(3)
*a*d^2 - a*d^2)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a*d^3))^(2/3))/(A^2
 - 2*I*A*B - B^2)) - (1/2)^(1/3)*(-I*sqrt(3)*a*d + a*d)*((-I*A^3 - 3*A^2*B
 + 3*I*A*B^2 + B^3)/(a*d^3))^(1/3)*e^(2*I*d*x + 2*I*c)*log((2^(1/3)*(A^2 -
 2*I*A*B - B^2)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c
) + (1/2)^(2/3)*(-I*sqrt(3)*a*d^2 - a*d^2)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2
+ B^3)/(a*d^3))^(2/3))/(A^2 - 2*I*A*B - B^2)) - 3*2^(2/3)*((-I*A + B)*e^(2
*I*d*x + 2*I*c) - I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(4/3*I*d*
x + 4/3*I*c))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = \int \frac{A+B\tan(c+dx)}{\sqrt[3]{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/3),x)
```

output
```
Integral((A + B*tan(c + d*x))/(I*a*(tan(c + d*x) - I))**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.81

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = \frac{i\left(2\sqrt{3}2^{\frac{2}{3}}(A-iB)a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)-2^{\frac{2}{3}}(A-iB)a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(ia\tan(\right.\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
1/8*I*(2*sqrt(3)*2^(2/3)*(A - I*B)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) - 2^(2/3)*(A - I*B
)*a^(2/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/
3) + (I*a*tan(d*x + c) + a)^(2/3)) + 2*2^(2/3)*(A - I*B)*a^(2/3)*log(-2^(1
/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)) + 12*(A + I*B)*a/(I*a*tan(d*x
+ c) + a)^(1/3))/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.27

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx = -\frac{-\frac{2i\sqrt{3}2^{\frac{2}{3}}A\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{2\sqrt{3}2^{\frac{2}{3}}B\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}+\frac{i\cdot 2^{\frac{2}{3}}A\log\left(2^{\frac{2}{3}}\right.}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
-1/8*(-2*I*sqrt(3)*2^(2/3)*A*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(1/3) - 2*sqrt(3)*2^(2/3)*B*ar
ctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3)
)/a^(1/3))/a^(1/3) + I*2^(2/3)*A*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*
x + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3))/a^(1/3) + 2^(2/3
)*B*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (
I*a*tan(d*x + c) + a)^(2/3))/a^(1/3) - 2*I*2^(1/3)*(2^(1/3)*A*a^(1/3) - I*
2^(1/3)*B*a^(1/3))*log(-2^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3))/a^
(2/3) + 12*(-I*A + B)/(I*a*tan(d*x + c) + a)^(1/3))/d
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.80

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{A\,3\mathrm{i}}{2\,d\,(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{1/3}} - \frac{3\,B}{2\,d\,(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{1/3}}$$
$$- \frac{\left(\frac{1}{16}\mathrm{i}\right)^{1/3}A\,\ln\left((a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}+(-1)^{1/3}\,2^{1/3}\,a^{1/3}\right)}{a^{1/3}\,d}$$
$$+ \frac{4^{1/3}\,B\,\ln\left(18\,B^2\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}-9\,4^{2/3}\,B^2\,a^{1/3}\,d\right)}{4\,a^{1/3}\,d}$$
$$+ \frac{4^{1/3}\,B\,\ln\left(18\,B^2\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}-9\,4^{2/3}\,B^2\,a^{1/3}\,d\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,a^{1/3}\,d}$$
$$- \frac{4^{1/3}\,B\,\ln\left(18\,B^2\,d\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{1/3}-9\,4^{2/3}\,B^2\,a^{1/3}\,d\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,a^{1/3}\,d}$$
$$- \frac{\left(\frac{1}{16}\mathrm{i}\right)^{1/3}A\,\ln\left(\frac{(-1)^{1/3}\,2^{1/3}\,a^{1/3}}{2}-(a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}+\frac{(-1)^{5/6}\,2^{1/3}\,\sqrt{3}\,a^{1/3}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}\,d}$$
$$+ \frac{\left(\frac{1}{16}\mathrm{i}\right)^{1/3}A\,\ln\left((a\,(1+\tan(c+d\,x)\,1\mathrm{i}))^{1/3}-\frac{(-1)^{1/3}\,2^{1/3}\,a^{1/3}}{2}+\frac{(-1)^{5/6}\,2^{1/3}\,\sqrt{3}\,a^{1/3}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{1/3}\,d}$$

input
```
int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i)^(1/3),x)
```

output
```
(A*3i)/(2*d*(a + a*tan(c + d*x)*1i)^(1/3)) - (3*B)/(2*d*(a + a*tan(c + d*x
)*1i)^(1/3)) - ((1i/16)^(1/3)*A*log((a*(tan(c + d*x)*1i + 1))^(1/3) + (-1)
^(1/3)*2^(1/3)*a^(1/3)))/(a^(1/3)*d) + (4^(1/3)*B*log(18*B^2*d*(a + a*tan(
c + d*x)*1i)^(1/3) - 9*4^(2/3)*B^2*a^(1/3)*d))/(4*a^(1/3)*d) + (4^(1/3)*B*
log(18*B^2*d*(a + a*tan(c + d*x)*1i)^(1/3) - 9*4^(2/3)*B^2*a^(1/3)*d*((3^(
1/2)*1i)/2 - 1/2)^2)*((3^(1/2)*1i)/2 - 1/2))/(4*a^(1/3)*d) - (4^(1/3)*B*lo
g(18*B^2*d*(a + a*tan(c + d*x)*1i)^(1/3) - 9*4^(2/3)*B^2*a^(1/3)*d*((3^(1/
2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i)/2 + 1/2))/(4*a^(1/3)*d) - ((1i/16)^(1/3)*
A*log(((-1)^(1/3)*2^(1/3)*a^(1/3))/2 - (a*(tan(c + d*x)*1i + 1))^(1/3) + (
(-1)^(5/6)*2^(1/3)*3^(1/2)*a^(1/3))/2)*((3^(1/2)*1i)/2 - 1/2))/(a^(1/3)*d)
 + ((1i/16)^(1/3)*A*log((a*(tan(c + d*x)*1i + 1))^(1/3) - ((-1)^(1/3)*2^(1
/3)*a^(1/3))/2 + ((-1)^(5/6)*2^(1/3)*3^(1/2)*a^(1/3))/2)*((3^(1/2)*1i)/2 +
 1/2))/(a^(1/3)*d)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx=\frac{\left(\int\frac{\tan(dx+c)}{(\tan(dx+c)i+1)^{\frac{1}{3}}}dx\right)b+\left(\int\frac{1}{(\tan(dx+c)i+1)^{\frac{1}{3}}}dx\right)a}{a^{\frac{1}{3}}}$$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/3),x)
```

output
```
(int(tan(c + d*x)/(tan(c + d*x)*i + 1)**(1/3),x)*b + int(1/(tan(c + d*x)*i
 + 1)**(1/3),x)*a)/a**(1/3)
```

3.203 $\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 213

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{2/3}}\,dx = -\frac{(A-iB)x}{4\ 2^{2/3}a^{2/3}} - \frac{\sqrt{3}(iA+B)\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\ 2^{2/3}a^{2/3}d} + \frac{(iA+B)\log(\cos(c+dx))}{4\ 2^{2/3}a^{2/3}d} + \frac{3(iA+B)\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+ia\tan(c+dx)}\right)}{4\ 2^{2/3}a^{2/3}d} + \frac{3(iA-B)}{4d(a+ia\tan(c+dx))^{2/3}}$$

output
```
-1/8*(A-I*B)*x*2^(1/3)/a^(2/3)-1/4*3^(1/2)*(I*A+B)*arctan(1/3*(a^(1/3)+2^(
2/3)*(a+I*a*tan(d*x+c))^(1/3))*3^(1/2)/a^(1/3))*2^(1/3)/a^(2/3)/d+1/8*(I*A
+B)*ln(cos(d*x+c))*2^(1/3)/a^(2/3)/d+3/8*(I*A+B)*ln(2^(1/3)*a^(1/3)-(a+I*a
*tan(d*x+c))^(1/3))*2^(1/3)/a^(2/3)/d+3/4*(I*A-B)/d/(a+I*a*tan(d*x+c))^(2/
3)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{2/3}}\,dx = -\frac{\frac{i\sqrt[3]{2}(A-iB)\left(2\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a+ia\tan(c+dx)}}{\sqrt{3}\sqrt[3]{a}}\right)-2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+\cdots}\right.}{}}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(2/3),x]
```

output
```
(((-I)*2^(1/3)*(A - I*B)*(2*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + I*a*Tan
[c + d*x])^(1/3))/(Sqrt[3]*a^(1/3))] - 2*Log[2^(1/3)*a^(1/3) - (a + I*a*Ta
n[c + d*x])^(1/3)] + Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + I*a*Tan[c
+ d*x])^(1/3) + (a + I*a*Tan[c + d*x])^(2/3)]))/a^(2/3) + ((6*I)*(A + I*B)
)/(a + I*a*Tan[c + d*x])^(2/3))/(8*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4009, 3042, 3962, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{4009}$$

$$\frac{(A-iB)\int \sqrt[3]{i\tan(c+dx)a+a}dx}{2a}+\frac{3(-B+iA)}{4d(a+ia\tan(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int \sqrt[3]{i\tan(c+dx)a+a}dx}{2a}+\frac{3(-B+iA)}{4d(a+ia\tan(c+dx))^{2/3}}$$

$\big\downarrow$ 3962

$$\frac{3(-B+iA)}{4d(a+ia\tan(c+dx))^{2/3}}-\frac{i(A-iB)\int \frac{1}{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^{2/3}}d(ia\tan(c+dx))}{2d}$$

$\big\downarrow$ 69

$$\frac{3(-B+iA)}{4d(a+ia\tan(c+dx))^{2/3}}-\frac{i(A-iB)\left(\frac{3\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)}d\sqrt[3]{i\tan(c+dx)a+a}}{2\ 2^{2/3}a^{2/3}}+\frac{3\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}}{2\sqrt[3]{2}\sqrt[3]{a}}\right.}{2d}$$

$\big\downarrow$ 16

$$\frac{3(-B+iA)}{4d(a+ia\tan(c+dx))^{2/3}}-\frac{i(A-iB)\left(\frac{3\int \frac{1}{-a^2\tan^2(c+dx)+i\sqrt[3]{2}a^{4/3}\tan(c+dx)+2^{2/3}a^{2/3}}d\sqrt[3]{i\tan(c+dx)a+a}}{2\sqrt[3]{2}\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a-ia}{2\ 2^{2}}\right.}{2d}$$

$\big\downarrow$ 1082

$$\frac{3(-B+iA)}{4d(a+ia\tan(c+dx))^{2/3}}-\frac{i(A-iB)\left(-\frac{3\int \frac{1}{a^2\tan^2(c+dx)-3}d(i2^{2/3}a^{2/3}\tan(c+dx)+1)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a-ia\tan(c+dx))}{2\ 2^{2/3}a^{2/3}}\right)}{2d}$$

$\big\downarrow$ 217

$$\frac{3(-B+iA)}{4d(a+ia\tan(c+dx))^{2/3}}-\frac{i(A-iB)\left(\frac{i\sqrt{3}\text{arctanh}\left(\frac{a\tan(c+dx)}{\sqrt{3}}\right)}{2^{2/3}a^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{a}-ia\tan(c+dx)\right)}{2\ 2^{2/3}a^{2/3}}+\frac{\log(a-ia\tan(c+dx))}{2\ 2^{2/3}a^{2/3}}\right)}{2d}$$

input

```
Int[(A + B*Tan[c + d*x])/(a + I*a*Tan[c + d*x])^(2/3),x]
```

output
```
((-1/2*I)*(A - I*B)*((I*Sqrt[3]*ArcTanh[(a*Tan[c + d*x])/Sqrt[3]])/(2^(2/3
)*a^(2/3)) - (3*Log[2^(1/3)*a^(1/3) - I*a*Tan[c + d*x]])/(2*2^(2/3)*a^(2/3
)) + Log[a - I*a*Tan[c + d*x]]/(2*2^(2/3)*a^(2/3))))/d + (3*(I*A - B))/(4*
d*(a + I*a*Tan[c + d*x])^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{3i\left(\left(\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\sqrt{3}\arctan\left(\ldots\right.}{\ldots}\right.\right.\right.}{d}$
default	$\frac{3i\left(\left(\left(\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\sqrt{3}\arctan\left(\ldots\right.}{\ldots}\right.\right.\right.}{d}$
parts	$\frac{3iAa\left(\frac{\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{1}{3}}-2^{\frac{1}{3}}a^{\frac{1}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\ln\left((a+ia\tan(dx+c))^{\frac{2}{3}}+2^{\frac{1}{3}}a^{\frac{1}{3}}(a+ia\tan(dx+c))^{\frac{1}{3}}+2^{\frac{2}{3}}a^{\frac{2}{3}}\right)}{12a^{\frac{2}{3}}}-\frac{2^{\frac{1}{3}}\sqrt{3}\arctan\left(\ldots\right.}{\ldots}}{2a}\right.}{d}$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3*I/d*((1/6*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(1/3)-2^(1/3)*a^(1/3))-1
/12*2^(1/3)/a^(2/3)*ln((a+I*a*tan(d*x+c))^(2/3)+2^(1/3)*a^(1/3)*(a+I*a*tan
(d*x+c))^(1/3)+2^(2/3)*a^(2/3))-1/6*2^(1/3)/a^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2^(2/3)/a^(1/3)*(a+I*a*tan(d*x+c))^(1/3)+1)))*(-1/2*I*B+1/2*A)-1/2*(
-1/2*A-1/2*I*B)/(a+I*a*tan(d*x+c))^(2/3))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(154) = 308$.

Time = 0.11 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.31

$$\int \frac{A + B\tan(c + dx)}{(a + ia\tan(c + dx))^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
1/8*(4*(1/4)^(1/3)*a*d*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1
/3)*e^(2*I*d*x + 2*I*c)*log(-(2*(1/4)^(1/3)*a*d*((-I*A^3 - 3*A^2*B + 3*I*A
*B^2 + B^3)/(a^2*d^3))^(1/3) - 2^(1/3)*(I*A + B)*(a/(e^(2*I*d*x + 2*I*c) +
 1))^(1/3)*e^(2/3*I*d*x + 2/3*I*c))/(I*A + B)) - 2*(1/4)^(1/3)*(-I*sqrt(3)
*a*d + a*d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1/3)*e^(2*I*
d*x + 2*I*c)*log((2^(1/3)*(I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^
(2/3*I*d*x + 2/3*I*c) - (1/4)^(1/3)*(I*sqrt(3)*a*d - a*d)*((-I*A^3 - 3*A^2
*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1/3))/(I*A + B)) - 2*(1/4)^(1/3)*(I*sqrt
(3)*a*d + a*d)*((-I*A^3 - 3*A^2*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1/3)*e^(2
*I*d*x + 2*I*c)*log((2^(1/3)*(I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)
*e^(2/3*I*d*x + 2/3*I*c) - (1/4)^(1/3)*(-I*sqrt(3)*a*d - a*d)*((-I*A^3 - 3
*A^2*B + 3*I*A*B^2 + B^3)/(a^2*d^3))^(1/3))/(I*A + B)) - 3*2^(1/3)*((-I*A
+ B)*e^(2*I*d*x + 2*I*c) - I*A + B)*(a/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^
(2/3*I*d*x + 2/3*I*c))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{2/3}}\,dx = \int \frac{A+B\tan{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*tan(c + d*x))/(I*a*(tan(c + d*x) - I))**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.80

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{2/3}}\,dx = -\frac{i\left(2\sqrt{3}2^{\frac{1}{3}}(A-iB)a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)+2^{\frac{1}{3}}(A-iB)a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}(ia\tan(\right.\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
-1/8*I*(2*sqrt(3)*2^(1/3)*(A - I*B)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^
(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3)) + 2^(1/3)*(A - I*
B)*a^(1/3)*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1
/3) + (I*a*tan(d*x + c) + a)^(2/3)) - 2*2^(1/3)*(A - I*B)*a^(1/3)*log(-2^(
1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(1/3)) - 6*(A + I*B)*a/(I*a*tan(d*x
+ c) + a)^(2/3))/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.21

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{2/3}}\,dx = -\frac{\frac{2i\sqrt{3}2^{\frac{1}{3}}A\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{2\sqrt{3}2^{\frac{1}{3}}B\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2(ia\tan(dx+c)+a)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{i\cdot 2^{\frac{1}{3}}A\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2\right.}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
-1/8*(2*I*sqrt(3)*2^(1/3)*A*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) +
2*(I*a*tan(d*x + c) + a)^(1/3))/a^(1/3))/a^(2/3) + 2*sqrt(3)*2^(1/3)*B*arc
tan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3) + 2*(I*a*tan(d*x + c) + a)^(1/3))
/a^(1/3))/a^(2/3) + I*2^(1/3)*A*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x
 + c) + a)^(1/3)*a^(1/3) + (I*a*tan(d*x + c) + a)^(2/3))/a^(2/3) + 2^(1/3)
*B*log(2^(2/3)*a^(2/3) + 2^(1/3)*(I*a*tan(d*x + c) + a)^(1/3)*a^(1/3) + (I
*a*tan(d*x + c) + a)^(2/3))/a^(2/3) - 2*I*2^(1/3)*(A - I*B)*log(-2^(1/3)*a
^(1/3) + (I*a*tan(d*x + c) + a)^(1/3))/a^(2/3) + 6*(-I*A + B)/(I*a*tan(d*x
 + c) + a)^(2/3))/d
```

Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.83

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{2/3}}\,dx = \frac{A\,3\mathrm{i}}{4\,d\,(a+a\tan{(c+d\,x)}\;1\mathrm{i})^{2/3}} - \frac{3\,B}{4\,d\,(a+a\tan{(c+d\,x)}\;1\mathrm{i})^{2/3}} - \frac{\left(\frac{1}{32}\mathrm{i}\right)^{1/3}A\,\ln\left(A\,d^2\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{1/3}\,36\mathrm{i}+144\,\left(\frac{1}{32}\mathrm{i}\right)^{1/3}A\,a^{1/3}\,d^2\right)}{a^{2/3}\,d} + \frac{2^{1/3}\,B\,\ln\left(36\,B\,d^2\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{1/3}-36\,2^{1/3}\,B\,a^{1/3}\,d^2\right)}{4\,a^{2/3}\,d} - \frac{\left(\frac{1}{32}\mathrm{i}\right)^{1/3}A\,\ln\left(A\,d^2\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{1/3}\,36\mathrm{i}+144\,\left(\frac{1}{32}\mathrm{i}\right)^{1/3}A\,a^{1/3}\,d^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{2/3}\,d} + \frac{\left(\frac{1}{32}\mathrm{i}\right)^{1/3}A\,\ln\left(A\,d^2\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{1/3}\,36\mathrm{i}-144\,\left(\frac{1}{32}\mathrm{i}\right)^{1/3}A\,a^{1/3}\,d^2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{2/3}\,d} + \frac{2^{1/3}\,B\,\ln\left(36\,B\,d^2\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{1/3}-36\,2^{1/3}\,B\,a^{1/3}\,d^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,a^{2/3}\,d} - \frac{2^{1/3}\,B\,\ln\left(36\,B\,d^2\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{1/3}+36\,2^{1/3}\,B\,a^{1/3}\,d^2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{4\,a^{2/3}\,d}$$

input
```
int((A + B*tan(c + d*x))/(a + a*tan(c + d*x)*1i)^(2/3),x)
```

output
```
(A*3i)/(4*d*(a + a*tan(c + d*x)*1i)^(2/3)) - (3*B)/(4*d*(a + a*tan(c + d*x
)*1i)^(2/3)) - ((1i/32)^(1/3)*A*log(A*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*36
i + 144*(1i/32)^(1/3)*A*a^(1/3)*d^2))/(a^(2/3)*d) + (2^(1/3)*B*log(36*B*d^
2*(a + a*tan(c + d*x)*1i)^(1/3) - 36*2^(1/3)*B*a^(1/3)*d^2))/(4*a^(2/3)*d)
 - ((1i/32)^(1/3)*A*log(A*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*36i + 144*(1i/
32)^(1/3)*A*a^(1/3)*d^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(a
^(2/3)*d) + ((1i/32)^(1/3)*A*log(A*d^2*(a + a*tan(c + d*x)*1i)^(1/3)*36i -
 144*(1i/32)^(1/3)*A*a^(1/3)*d^2*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 +
 1/2))/(a^(2/3)*d) + (2^(1/3)*B*log(36*B*d^2*(a + a*tan(c + d*x)*1i)^(1/3)
 - 36*2^(1/3)*B*a^(1/3)*d^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2)
)/(4*a^(2/3)*d) - (2^(1/3)*B*log(36*B*d^2*(a + a*tan(c + d*x)*1i)^(1/3) +
36*2^(1/3)*B*a^(1/3)*d^2*((3^(1/2)*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(
4*a^(2/3)*d)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{(a+ia\tan(c+dx))^{2/3}}\,dx = \frac{\left(\int \frac{\tan(dx+c)}{(\tan(dx+c)i+1)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{1}{(\tan(dx+c)i+1)^{\frac{2}{3}}}dx\right)a}{a^{\frac{2}{3}}}$$

input
```
int((A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(2/3),x)
```

output
```
(int(tan(c + d*x)/(tan(c + d*x)*i + 1)**(2/3),x)*b + int(1/(tan(c + d*x)*i
 + 1)**(2/3),x)*a)/a**(2/3)
```

3.204 $\int \tan^m(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 290

$$\begin{aligned}\int \tan^m(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \\ &-\frac{2a^4(A(64+60m+19m^2+2m^3)-iB(67+60m+19m^2+2m^3))\tan^{1+m}(c+dx)}{d(1+m)(2+m)(3+m)(4+m)} \\ &+\frac{8a^4(A-iB)\,\text{Hypergeometric2F1}(1,1+m,2+m,i\tan(c+dx))\tan^{1+m}(c+dx)}{d(1+m)} \\ &+\frac{iaB\tan^{1+m}(c+dx)(a+ia\tan(c+dx))^3}{d(4+m)} \\ &-\frac{(A(4+m)-iB(7+m))\tan^{1+m}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{d(3+m)(4+m)} \\ &-\frac{2(A(4+m)^2-iB(19+8m+m^2))\tan^{1+m}(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d(2+m)(3+m)(4+m)}\end{aligned}$$

output
```
-2*a^4*(A*(2*m^3+19*m^2+60*m+64)-I*B*(2*m^3+19*m^2+60*m+67))*tan(d*x+c)^(1
+m)/d/(1+m)/(2+m)/(3+m)/(4+m)+8*a^4*(A-I*B)*hypergeom([1, 1+m],[2+m],I*tan
(d*x+c))*tan(d*x+c)^(1+m)/d/(1+m)+I*a*B*tan(d*x+c)^(1+m)*(a+I*a*tan(d*x+c)
)^3/d/(4+m)-(A*(4+m)-I*B*(7+m))*tan(d*x+c)^(1+m)*(a^2+I*a^2*tan(d*x+c))^2/
d/(3+m)/(4+m)-2*(A*(4+m)^2-I*B*(m^2+8*m+19))*tan(d*x+c)^(1+m)*(a^4+I*a^4*t
an(d*x+c))/d/(2+m)/(3+m)/(4+m)
```

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.60

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{a^4\tan^{1+m}(c+dx)\left(\frac{(A-iB)(-7(6+5m+m^2)+8(6+5m+m^2)\text{ Hypergeometric2F1}(1,1+m,2+m,i\tan(c+dx))-4i(3+4m+m^2)\tan(c+}{(1+m)(2+m)(3+m)}\right.}{d}$$

input
```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(a^4*Tan[c + d*x]^(1 + m)*(((A - I*B)*(-7*(6 + 5*m + m^2) + 8*(6 + 5*m + m
^2)*Hypergeometric2F1[1, 1 + m, 2 + m, I*Tan[c + d*x]] - (4*I)*(3 + 4*m +
m^2)*Tan[c + d*x] + (2 + 3*m + m^2)*Tan[c + d*x]^2))/((1 + m)*(2 + m)*(3 +
 m)) + I*B*((1 + m)^(-1) + ((3*I)*Tan[c + d*x])/(2 + m) - (3*Tan[c + d*x]^
2)/(3 + m) - (I*Tan[c + d*x]^3)/(4 + m))))/d
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4077, 25, 3042, 4077, 27, 3042, 4077, 3042, 4075, 3042, 4020, 27, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^4\tan^m(c+dx)(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \ 3042$$
$$\int (a+ia\tan(c+dx))^4\tan(c+dx)^m(A+B\tan(c+dx))dx$$
$$\Big\downarrow \ 4077$$

$$\frac{\int -\tan^m(c+dx)(i\tan(c+dx)a+a)^3(a(iB(m+1)-A(m+4))-a(iA(m+4)+B(m+7))\tan(c+dx))dx}{m+4} - \frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)}$$

$\downarrow$ 25

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} - \frac{\int \tan^m(c+dx)(i\tan(c+dx)a+a)^3(a(iB(m+1)-A(m+4))-a(iA(m+4)+B(m+7))\tan(c+dx))dx}{m+4}$$

$\downarrow$ 3042

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} - \frac{\int \tan(c+dx)^m(i\tan(c+dx)a+a)^3(a(iB(m+1)-A(m+4))-a(iA(m+4)+B(m+7))\tan(c+dx))dx}{m+4}$$

$\downarrow$ 4077

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} - \frac{\frac{\int 2\tan^m(c+dx)(i\tan(c+dx)a+a)^2\left(a^2\left(iB\left(m^2+6m+5\right)-A\left(m^2+6m+8\right)\right)-a^2\left(iA(m+4)^2+B\left(m^2+8m+19\right)\right)\tan(c+dx)\right)dx}{m+3}+\frac{(A(m+4)-iB(}{}}{m+4}$$

$\downarrow$ 27

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} - \frac{\frac{2\int \tan^m(c+dx)(i\tan(c+dx)a+a)^2\left(a^2\left(iB\left(m^2+6m+5\right)-A\left(m^2+6m+8\right)\right)-a^2\left(iA(m+4)^2+B\left(m^2+8m+19\right)\right)\tan(c+dx)\right)dx}{m+3}+\frac{(A(m+4)-iB(}{}}{m+4}$$

$\downarrow$ 3042

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} - \frac{\frac{2\int \tan(c+dx)^m(i\tan(c+dx)a+a)^2\left(a^2\left(iB\left(m^2+6m+5\right)-A\left(m^2+6m+8\right)\right)-a^2\left(iA(m+4)^2+B\left(m^2+8m+19\right)\right)\tan(c+dx)\right)dx}{m+3}+\frac{(A(m+4)-iB(}{}}{m+4}$$

$\downarrow$ 4077

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} - \frac{\frac{2\left(\frac{\int \tan^m(c+dx)(i\tan(c+dx)a+a)\left(a^3\left(iB\left(2m^3+17m^2+44m+29\right)-A\left(2m^3+17m^2+44m+32\right)\right)-a^3\left(iA\left(2m^3+19m^2+60m+64\right)+B\left(2m^3+19m^2+60m+67\right)\right)\tan\right.}{m+2}\right.}{m+3}}{m+}$$

$\downarrow$ 3042

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} -$$

$$\frac{2\left(\frac{\int \tan(c+dx)^m(i\tan(c+dx)a+a)\left(a^3\left(iB\left(2m^3+17m^2+44m+29\right)-A\left(2m^3+17m^2+44m+32\right)\right)-a^3\left(iA\left(2m^3+19m^2+60m+64\right)+B\left(2m^3+19m^2+60m+67\right)\right)\tan\right.}{m+2}}{m+3}}{m+}$$

$\downarrow$ 4075

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} -$$

$$\frac{2\left(\frac{\int \tan^m(c+dx)\left(-4(A-iB)(m+2)(m+3)(m+4)a^4-4(iA+B)(m+2)(m+3)(m+4)\tan(c+dx)a^4\right)dx+\frac{a^4\left(A\left(2m^3+19m^2+60m+64\right)-iB\left(2m^3+19m^2+60m+67\right.\right.}{d(m+1)}}{m+2}}{m+3}}{m}$$

$\downarrow$ 3042

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} -$$

$$\frac{2\left(\frac{\int \tan(c+dx)^m\left(-4(A-iB)(m+2)(m+3)(m+4)a^4-4(iA+B)(m+2)(m+3)(m+4)\tan(c+dx)a^4\right)dx+\frac{a^4\left(A\left(2m^3+19m^2+60m+64\right)-iB\left(2m^3+19m^2+60m+67\right.\right.}{d(m+1)}}{m+2}}{m+3}}{m}$$

$\downarrow$ 4020

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} -$$

$$2\left(\frac{\frac{16ia^8(m+2)^2(m+3)^2(m+4)^2(A-iB)^2\int\frac{\tan^m(c+dx)}{4a^4(m+2)(m+3)(m+4)\left(4(iA+B)^2(m+2)(m+3)(m+4)a^4+4(A-iB)(iA+B)(m+2)(m+3)(m+4)\tan(c+dx)a^4\right)}d\left(-\right.}{d}}{m+2}\right.$$

$\downarrow$ 27

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} -$$

$$2\left(\frac{\frac{ia^4 4^{1-m}(m+2)(m+3)(m+4)(A-iB)^2\int\frac{4^m\tan^m(c+dx)}{4(iA+B)^2(m+2)(m+3)(m+4)a^4+4(A-iB)(iA+B)(m+2)(m+3)(m+4)\tan(c+dx)a^4}d\left(-4a^4(iA+B)(m+2)(m+3)\right.}{d}}{m+2}\right.$$

$\downarrow$ 74

$$\frac{iaB(a+ia\tan(c+dx))^3\tan^{m+1}(c+dx)}{d(m+4)} -$$

$$\frac{2\left(\frac{\frac{a^4\left(A\left(2m^3+19m^2+60m+64\right)-iB\left(2m^3+19m^2+60m+67\right)\right)\tan^{m+1}(c+dx)}{d(m+1)}-\frac{4ia^4(m+2)(m+3)(m+4)(A-iB)^2\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,m+1\right.}{d(m+1)(B+iA)}}{m+2}\right.}{m+3}$$

input
```
Int[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(I*a*B*Tan[c + d*x]^(1 + m)*(a + I*a*Tan[c + d*x])^3)/(d*(4 + m)) - (((A*(
4 + m) - I*B*(7 + m))*Tan[c + d*x]^(1 + m)*(a^2 + I*a^2*Tan[c + d*x])^2)/(
d*(3 + m)) + (2*(((A*(4 + m)^2 - I*B*(19 + 8*m + m^2))*Tan[c + d*x]^(1 + m
)*(a^4 + I*a^4*Tan[c + d*x]))/(d*(2 + m)) + ((a^4*(A*(64 + 60*m + 19*m^2 +
 2*m^3) - I*B*(67 + 60*m + 19*m^2 + 2*m^3))*Tan[c + d*x]^(1 + m))/(d*(1 +
m)) - ((4*I)*a^4*(A - I*B)^2*(2 + m)*(3 + m)*(4 + m)*Hypergeometric2F1[1,
1 + m, 2 + m, ((I*A + B)*Tan[c + d*x])/(A - I*B)]*Tan[c + d*x]^(1 + m))/((
I*A + B)*d*(1 + m)))/(2 + m)))/(3 + m))/(4 + m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [F]

$$\int \tan (dx+c)^m (a+ia\tan (dx+c))^4 (A+B\tan (dx+c))\,dx$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^4\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral(16*((A - I*B)*a^4*e^(10*I*d*x + 10*I*c) + (A + I*B)*a^4*e^(8*I*d*
x + 8*I*c))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m/(e^
(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6*I*c) + 10*
e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = a^4\left(\int A\tan^m(c+dx)\,dx + \int\left(-6A\tan^2(c+dx)\tan^m(c+dx)\right)dx + \int A\tan^4(c+dx)\tan^m(c+dx)\,dx + \int B\tan(c+dx)\tan^m(c+dx)\,dx + \int\left(-6B\tan^3(c+dx)\tan^m(c+dx)\right)dx + \int B\tan^5(c+dx)\tan^m(c+dx)\,dx + \int 4iA\tan(c+dx)\tan^m(c+dx)\,dx + \int\left(-4iA\tan^3(c+dx)\tan^m(c+dx)\right)dx + \int 4iB\tan^2(c+dx)\tan^m(c+dx)\,dx + \int\left(-4iB\tan^4(c+dx)\tan^m(c+dx)\right)dx\right)$$

input
```
integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
a**4*(Integral(A*tan(c + d*x)**m, x) + Integral(-6*A*tan(c + d*x)**2*tan(c
 + d*x)**m, x) + Integral(A*tan(c + d*x)**4*tan(c + d*x)**m, x) + Integral
(B*tan(c + d*x)*tan(c + d*x)**m, x) + Integral(-6*B*tan(c + d*x)**3*tan(c
+ d*x)**m, x) + Integral(B*tan(c + d*x)**5*tan(c + d*x)**m, x) + Integral(
4*I*A*tan(c + d*x)*tan(c + d*x)**m, x) + Integral(-4*I*A*tan(c + d*x)**3*t
an(c + d*x)**m, x) + Integral(4*I*B*tan(c + d*x)**2*tan(c + d*x)**m, x) +
Integral(-4*I*B*tan(c + d*x)**4*tan(c + d*x)**m, x))
```

Maxima [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^4\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^4*tan(d*x + c)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,4]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})^4\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output

```
(a**4*(tan(c + d*x)**m*tan(c + d*x)**4*b*m**2 + 2*tan(c + d*x)**m*tan(c +
d*x)**4*b*m - 4*tan(c + d*x)**m*tan(c + d*x)**2*a*i*m**2 - 16*tan(c + d*x)
**m*tan(c + d*x)**2*a*i*m - 7*tan(c + d*x)**m*tan(c + d*x)**2*b*m**2 - 28*
tan(c + d*x)**m*tan(c + d*x)**2*b*m + 8*tan(c + d*x)**m*a*i*m**2 + 48*tan(
c + d*x)**m*a*i*m + 64*tan(c + d*x)**m*a*i + 8*tan(c + d*x)**m*b*m**2 + 48
*tan(c + d*x)**m*b*m + 64*tan(c + d*x)**m*b + int(tan(c + d*x)**m,x)*a*d*m
**3 + 6*int(tan(c + d*x)**m,x)*a*d*m**2 + 8*int(tan(c + d*x)**m,x)*a*d*m -
 8*int(tan(c + d*x)**m/tan(c + d*x),x)*a*d*i*m**3 - 48*int(tan(c + d*x)**m
/tan(c + d*x),x)*a*d*i*m**2 - 64*int(tan(c + d*x)**m/tan(c + d*x),x)*a*d*i
*m - 8*int(tan(c + d*x)**m/tan(c + d*x),x)*b*d*m**3 - 48*int(tan(c + d*x)*
*m/tan(c + d*x),x)*b*d*m**2 - 64*int(tan(c + d*x)**m/tan(c + d*x),x)*b*d*m
 + int(tan(c + d*x)**m*tan(c + d*x)**4,x)*a*d*m**3 + 6*int(tan(c + d*x)**m
*tan(c + d*x)**4,x)*a*d*m**2 + 8*int(tan(c + d*x)**m*tan(c + d*x)**4,x)*a*
d*m - 4*int(tan(c + d*x)**m*tan(c + d*x)**4,x)*b*d*i*m**3 - 24*int(tan(c +
 d*x)**m*tan(c + d*x)**4,x)*b*d*i*m**2 - 32*int(tan(c + d*x)**m*tan(c + d*
x)**4,x)*b*d*i*m - 6*int(tan(c + d*x)**m*tan(c + d*x)**2,x)*a*d*m**3 - 36*
int(tan(c + d*x)**m*tan(c + d*x)**2,x)*a*d*m**2 - 48*int(tan(c + d*x)**m*t
an(c + d*x)**2,x)*a*d*m + 4*int(tan(c + d*x)**m*tan(c + d*x)**2,x)*b*d*i*m
**3 + 24*int(tan(c + d*x)**m*tan(c + d*x)**2,x)*b*d*i*m**2 + 32*int(tan(c
+ d*x)**m*tan(c + d*x)**2,x)*b*d*i*m))/(d*m*(m**2 + 6*m + 8))
```

3.205 $\int \tan^m(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 205

$$\begin{aligned}&\int \tan^m(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx\\&=-\frac{a^3(A(15+11m+2m^2)-iB(17+11m+2m^2))\tan^{1+m}(c+dx)}{d(1+m)(2+m)(3+m)}\\&\quad+\frac{4a^3(A-iB)\,\text{Hypergeometric2F1}(1,1+m,2+m,i\tan(c+dx))\tan^{1+m}(c+dx)}{d(1+m)}\\&\quad+\frac{iaB\tan^{1+m}(c+dx)(a+ia\tan(c+dx))^2}{d(3+m)}\\&\quad-\frac{(A(3+m)-iB(5+m))\tan^{1+m}(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{d(2+m)(3+m)}\end{aligned}$$

output

```
-a^3*(A*(2*m^2+11*m+15)-I*B*(2*m^2+11*m+17))*tan(d*x+c)^(1+m)/d/(1+m)/(2+m
)/(3+m)+4*a^3*(A-I*B)*hypergeom([1, 1+m],[2+m],I*tan(d*x+c))*tan(d*x+c)^(1
+m)/d/(1+m)+I*a*B*tan(d*x+c)^(1+m)*(a+I*a*tan(d*x+c))^2/d/(3+m)-(A*(3+m)-I
*B*(5+m))*tan(d*x+c)^(1+m)*(a^3+I*a^3*tan(d*x+c))/d/(2+m)/(3+m)
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3\tan^{1+m}(c+dx)\left(\frac{(A-iB)(-3(2+m)+4(2+m)\,\text{Hypergeometric2F1}(1,1+m,2+m,i\tan(c+dx))-i(1+m)\tan(c+dx))}{(1+m)(2+m)}+iB\left(\frac{1}{1+m}\right.\right.}{d}$$

input
```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(a^3*Tan[c + d*x]^(1 + m)*(((A - I*B)*(-3*(2 + m) + 4*(2 + m)*Hypergeometr
ic2F1[1, 1 + m, 2 + m, I*Tan[c + d*x]] - I*(1 + m)*Tan[c + d*x]))/((1 + m)
*(2 + m)) + I*B*((1 + m)^(-1) + ((2*I)*Tan[c + d*x])/(2 + m) - Tan[c + d*x
]^2/(3 + m))))/d
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4077, 25, 3042, 4077, 3042, 4075, 3042, 4020, 27, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^3\tan^m(c+dx)(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^3\tan(c+dx)^m(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{\int -\tan^m(c+dx)(i\tan(c+dx)a+a)^2(a(iB(m+1)-A(m+3))-a(iA(m+3)+B(m+5))\tan(c+dx))dx}{m+3}-$$
$$\frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}$$

$\downarrow$ 25

$$\frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}-\frac{\int \tan^m(c+dx)(i\tan(c+dx)a+a)^2(a(iB(m+1)-A(m+3))-a(iA(m+3)+B(m+5))\tan(c+dx))dx}{m+3}$$

$\downarrow$ 3042

$$\frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}-\frac{\int \tan(c+dx)^m(i\tan(c+dx)a+a)^2(a(iB(m+1)-A(m+3))-a(iA(m+3)+B(m+5))\tan(c+dx))dx}{m+3}$$

$\downarrow$ 4077

$$\frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}-\frac{\frac{\int \tan^m(c+dx)(i\tan(c+dx)a+a)\left(a^2\left(iB\left(2m^2+9m+7\right)-A\left(2m^2+9m+9\right)\right)-a^2\left(iA\left(2m^2+11m+15\right)+B\left(2m^2+11m+17\right)\right)\tan(c+dx)\right)dx}{m+2}+\frac{(A(\cdots}{}}{m+3}$$

$\downarrow$ 3042

$$\frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}-\frac{\frac{\int \tan(c+dx)^m(i\tan(c+dx)a+a)\left(a^2\left(iB\left(2m^2+9m+7\right)-A\left(2m^2+9m+9\right)\right)-a^2\left(iA\left(2m^2+11m+15\right)+B\left(2m^2+11m+17\right)\right)\tan(c+dx)\right)dx}{m+2}+\frac{(A(\cdots}{}}{m+3}$$

$\downarrow$ 4075

$$\frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}-\frac{\frac{\int \tan^m(c+dx)\left(-4(A-iB)(m+2)(m+3)a^3-4(iA+B)\left(m^2+5m+6\right)\tan(c+dx)a^3\right)dx+\frac{a^3\left(A\left(2m^2+11m+15\right)-iB\left(2m^2+11m+17\right)\right)\tan^{m+1}(c+dx)}{d(m+1)}}{m+2}-\cdots}{m+3}$$

$\downarrow$ 3042

$$\frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}-\frac{\frac{\int \tan(c+dx)^m\left(-4(A-iB)(m+2)(m+3)a^3-4(iA+B)\left(m^2+5m+6\right)\tan(c+dx)a^3\right)dx+\frac{a^3\left(A\left(2m^2+11m+15\right)-iB\left(2m^2+11m+17\right)\right)\tan^{m+1}(c+dx)}{d(m+1)}}{m+2}-\cdots}{m+3}$$

$\downarrow$ 4020

$$\frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}-\frac{\frac{\frac{16ia^6(m+2)^2(m+3)^2(A-iB)^2\int \frac{\left(\frac{(m+2)(m+3)\tan(c+dx)}{m^2+5m+6}\right)^m}{4a^3\left(4(iA+B)^2\left(m^2+5m+6\right)^2a^3+4(A-iB)(iA+B)(m+2)^2(m+3)^2\tan(c+dx)a^3\right)}d\left(-4a^3(iA+B)(m+2)(m+3)\tan(c+dx)\right)}{d}+}{m+2}}{m+3}$$

$\Big\downarrow$ 27

$$\frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}-\frac{\frac{\frac{ia^3 4^{1-m}(m+2)^2(m+3)^2(A-iB)^2\int \frac{4^m\left(\frac{(m+2)(m+3)\tan(c+dx)}{m^2+5m+6}\right)^m}{4(iA+B)^2\left(m^2+5m+6\right)^2a^3+4(A-iB)(iA+B)(m+2)^2(m+3)^2\tan(c+dx)a^3}d\left(-4a^3(iA+B)(m+2)(m+3)\tan(c+dx)\right)}{d}+a^3}{m+2}}{m+3}$$

$\Big\downarrow$ 74

$$\frac{iaB(a+ia\tan(c+dx))^2\tan^{m+1}(c+dx)}{d(m+3)}-\frac{\frac{\frac{a^3\left(A\left(2m^2+11m+15\right)-iB\left(2m^2+11m+17\right)\right)\tan^{m+1}(c+dx)}{d(m+1)}-\frac{4ia^3(m+2)^2(m+3)^2(A-iB)^2\left(\frac{(m+2)(m+3)\tan(c+dx)}{m^2+5m+6}\right)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2\right.}{d(m+1)\left(m^2+5m+6\right)(B+iA)}}{m+2}}{m+3}$$

input
```
Int[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(I*a*B*Tan[c + d*x]^(1 + m)*(a + I*a*Tan[c + d*x])^2)/(d*(3 + m)) - (((A*(
3 + m) - I*B*(5 + m))*Tan[c + d*x]^(1 + m)*(a^3 + I*a^3*Tan[c + d*x]))/(d*
(2 + m)) + ((a^3*(A*(15 + 11*m + 2*m^2) - I*B*(17 + 11*m + 2*m^2))*Tan[c +
 d*x]^(1 + m))/(d*(1 + m)) - ((4*I)*a^3*(A - I*B)^2*(2 + m)^2*(3 + m)^2*Hy
pergeometric2F1[1, 1 + m, 2 + m, ((I*A + B)*Tan[c + d*x])/(A - I*B)]*(((2
+ m)*(3 + m)*Tan[c + d*x])/(6 + 5*m + m^2))^(1 + m))/((I*A + B)*d*(1 + m)*
(6 + 5*m + m^2)))/(2 + m))/(3 + m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [F]

$$\int \tan(dx+c)^m (a+ia\tan(dx+c))^3 (A+B\tan(dx+c))\,dx$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^3\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral(8*((A - I*B)*a^3*e^(8*I*d*x + 8*I*c) + (A + I*B)*a^3*e^(6*I*d*x +
 6*I*c))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m/(e^(8*
I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*
d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= -ia^3\left(\int iA\tan^m(c+dx)\,dx + \int (-3A\tan(c+dx)\tan^m(c+dx))\,dx + \int A\tan^3(c+dx)\tan^m(c+dx)\,dx + \int (-3B\tan^2(c+dx)\tan^m(c+dx))\,dx + \int B\tan^4(c+dx)\tan^m(c+dx)\,dx + \int (-3iA\tan^2(c+dx)\tan^m(c+dx))\,dx + \int iB\tan(c+dx)\tan^m(c+dx)\,dx + \int (-3iB\tan^3(c+dx)\tan^m(c+dx))\,dx\right)$$

input
```
integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
-I*a**3*(Integral(I*A*tan(c + d*x)**m, x) + Integral(-3*A*tan(c + d*x)*tan
(c + d*x)**m, x) + Integral(A*tan(c + d*x)**3*tan(c + d*x)**m, x) + Integr
al(-3*B*tan(c + d*x)**2*tan(c + d*x)**m, x) + Integral(B*tan(c + d*x)**4*t
an(c + d*x)**m, x) + Integral(-3*I*A*tan(c + d*x)**2*tan(c + d*x)**m, x) +
 Integral(I*B*tan(c + d*x)*tan(c + d*x)**m, x) + Integral(-3*I*B*tan(c + d
*x)**3*tan(c + d*x)**m, x))
```

Maxima [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^3\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^3*tan(d*x + c)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,3]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^3\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3\left(-\tan(dx+c)^m\tan(dx+c)^2 aim - 3\tan(dx+c)^m\tan(dx+c)^2 bm + 4\tan(dx+c)^m aim + 8\tan\right.}{}$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*( - tan(c + d*x)**m*tan(c + d*x)**2*a*i*m - 3*tan(c + d*x)**m*tan(c
+ d*x)**2*b*m + 4*tan(c + d*x)**m*a*i*m + 8*tan(c + d*x)**m*a*i + 4*tan(c
+ d*x)**m*b*m + 8*tan(c + d*x)**m*b + int(tan(c + d*x)**m,x)*a*d*m**2 + 2*
int(tan(c + d*x)**m,x)*a*d*m - 4*int(tan(c + d*x)**m/tan(c + d*x),x)*a*d*i
*m**2 - 8*int(tan(c + d*x)**m/tan(c + d*x),x)*a*d*i*m - 4*int(tan(c + d*x)
**m/tan(c + d*x),x)*b*d*m**2 - 8*int(tan(c + d*x)**m/tan(c + d*x),x)*b*d*m
 - int(tan(c + d*x)**m*tan(c + d*x)**4,x)*b*d*i*m**2 - 2*int(tan(c + d*x)*
*m*tan(c + d*x)**4,x)*b*d*i*m - 3*int(tan(c + d*x)**m*tan(c + d*x)**2,x)*a
*d*m**2 - 6*int(tan(c + d*x)**m*tan(c + d*x)**2,x)*a*d*m + 3*int(tan(c + d
*x)**m*tan(c + d*x)**2,x)*b*d*i*m**2 + 6*int(tan(c + d*x)**m*tan(c + d*x)*
*2,x)*b*d*i*m))/(d*m*(m + 2))
```

3.206 $\int \tan^m(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 132

$$\begin{aligned}
&\int \tan^m(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx \\
&= \frac{ia^2(B+(iA+B)(2+m))\tan^{1+m}(c+dx)}{d(1+m)(2+m)} \\
&\quad + \frac{2a^2(A-iB)\,\text{Hypergeometric2F1}(1,1+m,2+m,i\tan(c+dx))\tan^{1+m}(c+dx)}{d(1+m)} \\
&\quad + \frac{iB\tan^{1+m}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d(2+m)}
\end{aligned}$$

output
```
I*a^2*(B+(I*A+B)*(2+m))*tan(d*x+c)^(1+m)/d/(1+m)/(2+m)+2*a^2*(A-I*B)*hyper
geom([1, 1+m],[2+m],I*tan(d*x+c))*tan(d*x+c)^(1+m)/d/(1+m)+I*B*tan(d*x+c)^
(1+m)*(a^2+I*a^2*tan(d*x+c))/d/(2+m)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{a^2\tan^{1+m}(c+dx)((A-2iB)(2+m)-2(A-iB)(2+m)\,\text{Hypergeometric2F1}(1,1+m,2+m,i\tan$$
$$d(1+m)(2+m)$$

input
```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
-((a^2*Tan[c + d*x]^(1 + m)*((A - (2*I)*B)*(2 + m) - 2*(A - I*B)*(2 + m)*H
ypergeometric2F1[1, 1 + m, 2 + m, I*Tan[c + d*x]] + B*(1 + m)*Tan[c + d*x]
))/(d*(1 + m)*(2 + m)))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4077, 25, 3042, 4075, 3042, 4020, 27, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^2\tan^m(c+dx)(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^2\tan(c+dx)^m(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{\int -\tan^m(c+dx)(i\tan(c+dx)a+a)(a(iB(m+1)-A(m+2))-a(B+(iA+B)(m+2))\tan(c+dx))dx}{m+2}+\frac{iB\left(a^2+ia^2\tan(c+dx)\right)\tan^{m+1}(c+dx)}{d(m+2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{iB\left(a^2+ia^2\tan(c+dx)\right)\tan^{m+1}(c+dx)}{d(m+2)}-\frac{\int \tan^m(c+dx)(i\tan(c+dx)a+a)(a(iB(m+1)-A(m+2))-a(B+(iA+B)(m+2))\tan(c+dx))dx}{m+2}$$

$\big\downarrow$ 3042

$$\frac{iB\left(a^2+ia^2\tan(c+dx)\right)\tan^{m+1}(c+dx)}{d(m+2)}-\frac{\int \tan(c+dx)^m(i\tan(c+dx)a+a)(a(iB(m+1)-A(m+2))-a(B+(iA+B)(m+2))\tan(c+dx))dx}{m+2}$$

$\big\downarrow$ 4075

$$\frac{iB\left(a^2+ia^2\tan(c+dx)\right)\tan^{m+1}(c+dx)}{d(m+2)}-\frac{\int \tan^m(c+dx)\left(-2(A-iB)(m+2)a^2-2(iA+B)(m+2)\tan(c+dx)a^2\right)dx-\frac{ia^2(B+(m+2)(B+iA))\tan^{m+1}(c+dx}{d(m+1)}}{m+2}$$

$\big\downarrow$ 3042

$$\frac{iB\left(a^2+ia^2\tan(c+dx)\right)\tan^{m+1}(c+dx)}{d(m+2)}-\frac{\int \tan(c+dx)^m\left(-2(A-iB)(m+2)a^2-2(iA+B)(m+2)\tan(c+dx)a^2\right)dx-\frac{ia^2(B+(m+2)(B+iA))\tan^{m+1}(c+dx}{d(m+1)}}{m+2}$$

$\big\downarrow$ 4020

$$\frac{iB\left(a^2+ia^2\tan(c+dx)\right)\tan^{m+1}(c+dx)}{d(m+2)}-\frac{\frac{4ia^4(m+2)^2(A-iB)^2\int\frac{\tan^m(c+dx)}{2a^2(m+2)\left(2(iA+B)^2(m+2)a^2+2(A-iB)(iA+B)(m+2)\tan(c+dx)a^2\right)}d\left(-2a^2(iA+B)(m+2)\tan(c+dx)\right)}{d}-\frac{ia^2(B+(m+2)(}{d}}{m+2}$$

$\big\downarrow$ 27

$$\frac{iB\left(a^2+ia^2\tan(c+dx)\right)\tan^{m+1}(c+dx)}{d(m+2)}-\frac{\frac{ia^2 2^{1-m}(m+2)(A-iB)^2\int\frac{2^m\tan^m(c+dx)}{2(iA+B)^2(m+2)a^2+2(A-iB)(iA+B)(m+2)\tan(c+dx)a^2}d\left(-2a^2(iA+B)(m+2)\tan(c+dx)\right)}{d}-\frac{ia^2(B+(m+2)(B+iA))}{d(m+1)}}{m+2}$$

$\big\downarrow$ 74

$$\frac{iB\left(a^2+ia^2\tan(c+dx)\right)\tan^{m+1}(c+dx)}{d(m+2)}-\frac{-\frac{2ia^2(m+2)(A-iB)^2\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{(iA+B)\tan(c+dx)}{A-iB}\right)}{d(m+1)(B+iA)}-\frac{ia^2(B+(m+2)(B+iA))\tan^{m+1}(c+dx)}{d(m+1)}}{m+2}$$

input
```
Int[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(I*B*Tan[c + d*x]^(1 + m)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(2 + m)) - (((-I)
*a^2*(B + (I*A + B)*(2 + m))*Tan[c + d*x]^(1 + m))/(d*(1 + m)) - ((2*I)*a^
2*(A - I*B)^2*(2 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, ((I*A + B)*Tan[c
+ d*x])/(A - I*B)]*Tan[c + d*x]^(1 + m))/((I*A + B)*d*(1 + m)))/(2 + m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [F]

$$\int \tan(dx+c)^{m} (a+ia\tan(dx+c))^{2} (A+B\tan(dx+c))\, dx$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^{m}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\, dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{2}\tan(dx+c)^{m}\, dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral(4*((A - I*B)*a^2*e^(6*I*d*x + 6*I*c) + (A + I*B)*a^2*e^(4*I*d*x +
 4*I*c))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m/(e^(6*
I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -a^2\left(\int(-A\tan^m(c+dx))\,dx+\int A\tan^2(c+dx)\tan^m(c+dx)\,dx + \int(-B\tan(c+dx)\tan^m(c+dx))\,dx+\int B\tan^3(c+dx)\tan^m(c+dx)\,dx + \int(-2iA\tan(c+dx)\tan^m(c+dx))\,dx + \int(-2iB\tan^2(c+dx)\tan^m(c+dx))\,dx\right)$$

input
```
integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
-a**2*(Integral(-A*tan(c + d*x)**m, x) + Integral(A*tan(c + d*x)**2*tan(c
+ d*x)**m, x) + Integral(-B*tan(c + d*x)*tan(c + d*x)**m, x) + Integral(B*
tan(c + d*x)**3*tan(c + d*x)**m, x) + Integral(-2*I*A*tan(c + d*x)*tan(c +
 d*x)**m, x) + Integral(-2*I*B*tan(c + d*x)**2*tan(c + d*x)**m, x))
```

Maxima [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \int(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^2\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^2*tan(d*x + c)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^2\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{a^2\left(-\tan\left(dx+c\right)^m\tan\left(dx+c\right)^2bm+2\tan\left(dx+c\right)^maim+4\tan\left(dx+c\right)^mai+2\tan\left(dx+c\right)^mbm+\right.}{}$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*( - tan(c + d*x)**m*tan(c + d*x)**2*b*m + 2*tan(c + d*x)**m*a*i*m +
4*tan(c + d*x)**m*a*i + 2*tan(c + d*x)**m*b*m + 4*tan(c + d*x)**m*b + int(
tan(c + d*x)**m,x)*a*d*m**2 + 2*int(tan(c + d*x)**m,x)*a*d*m - 2*int(tan(c
 + d*x)**m/tan(c + d*x),x)*a*d*i*m**2 - 4*int(tan(c + d*x)**m/tan(c + d*x)
,x)*a*d*i*m - 2*int(tan(c + d*x)**m/tan(c + d*x),x)*b*d*m**2 - 4*int(tan(c
 + d*x)**m/tan(c + d*x),x)*b*d*m - int(tan(c + d*x)**m*tan(c + d*x)**2,x)*
a*d*m**2 - 2*int(tan(c + d*x)**m*tan(c + d*x)**2,x)*a*d*m + 2*int(tan(c +
d*x)**m*tan(c + d*x)**2,x)*b*d*i*m**2 + 4*int(tan(c + d*x)**m*tan(c + d*x)
**2,x)*b*d*i*m))/(d*m*(m + 2))
```

3.207 $\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 70

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{iaB\tan^{1+m}(c+dx)}{d(1+m)} + \frac{a(A-iB)\,\text{Hypergeometric2F1}(1,1+m,2+m,i\tan(c+dx))\tan^{1+m}(c+dx)}{d(1+m)}$$

output
```
I*a*B*tan(d*x+c)^(1+m)/d/(1+m)+a*(A-I*B)*hypergeom([1, 1+m],[2+m],I*tan(d*
x+c))*tan(d*x+c)^(1+m)/d/(1+m)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{a(iB+(A-iB)\,\text{Hypergeometric2F1}(1,1+m,2+m,i\tan(c+dx)))\tan^{1+m}(c+dx)}{d(1+m)}$$

input
```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(a*(I*B + (A - I*B)*Hypergeometric2F1[1, 1 + m, 2 + m, I*Tan[c + d*x]])*Ta
n[c + d*x]^(1 + m))/(d*(1 + m))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4075, 3042, 4020, 27, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + ia\tan(c + dx))\tan^m(c + dx)(A + B\tan(c + dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a + ia\tan(c + dx))\tan(c + dx)^m(A + B\tan(c + dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int \tan^m(c + dx)(a(A - iB) + a(iA + B)\tan(c + dx))dx + \frac{iaB\tan^{m+1}(c + dx)}{d(m + 1)}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c + dx)^m(a(A - iB) + a(iA + B)\tan(c + dx))dx + \frac{iaB\tan^{m+1}(c + dx)}{d(m + 1)}$$

$$\Big\downarrow \text{4020}$$

$$\frac{ia^2(A - iB)^2\int \frac{\tan^m(c+dx)}{a(a(iA+B)^2+a(A-iB)\tan(c+dx)(iA+B))}d(a(iA + B)\tan(c + dx))}{d} + \frac{iaB\tan^{m+1}(c + dx)}{d(m + 1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{ia(A-iB)^2 \int \frac{\tan^m(c+dx)}{a(iA+B)^2+a(A-iB)\tan(c+dx)(iA+B)}d(a(iA+B)\tan(c+dx))}{d}+$$
$$\frac{iaB\tan^{m+1}(c+dx)}{d(m+1)}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{ia(A-iB)^2\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{(iA+B)\tan(c+dx)}{A-iB}\right)}{d(m+1)(B+iA)}+$$
$$\frac{iaB\tan^{m+1}(c+dx)}{d(m+1)}$$

input

```
Int[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output

```
(I*a*B*Tan[c + d*x]^(1 + m))/(d*(1 + m)) + (I*a*(A - I*B)^2*Hypergeometric
2F1[1, 1 + m, 2 + m, ((I*A + B)*Tan[c + d*x])/(A - I*B)]*Tan[c + d*x]^(1 +
 m))/((I*A + B)*d*(1 + m))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Maple [F]

$$\int \tan (dx+c)^m (a+ia\tan (dx+c)) (A+B\tan (dx+c)) dx$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx)) dx = \int (B\tan (dx+c)+A)(i a\tan (dx+c)+a)\tan (dx+c)^m dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral(2*((A - I*B)*a*e^(4*I*d*x + 4*I*c) + (A + I*B)*a*e^(2*I*d*x + 2*I
*c))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m/(e^(4*I*d*
x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= ia\left(\int(-iA\tan^m(c+dx))\,dx+\int A\tan(c+dx)\tan^m(c+dx)\,dx\right.$$
$$\left.+\int B\tan^2(c+dx)\tan^m(c+dx)\,dx+\int(-iB\tan(c+dx)\tan^m(c+dx))\,dx\right)$$

input
```
integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*A*tan(c + d*x)**m, x) + Integral(A*tan(c + d*x)*tan(c + d
*x)**m, x) + Integral(B*tan(c + d*x)**2*tan(c + d*x)**m, x) + Integral(-I*
B*tan(c + d*x)*tan(c + d*x)**m, x))
```

Maxima [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\int(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)*tan(d*x + c)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \int \tan(c+d\,x)^m\,\left(A+B\tan(c+d\,x)\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{a\left(\tan(dx+c)^m ai + \tan(dx+c)^m b + \left(\int \tan(dx+c)^m\,dx\right) adm - \left(\int \frac{\tan(dx+c)^m}{\tan(dx+c)}dx\right) adim - \left(\int \frac{\tan(dx\right.}{\tan(dx}\right.}{dm}$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*(tan(c + d*x)**m*a*i + tan(c + d*x)**m*b + int(tan(c + d*x)**m,x)*a*d*m
 - int(tan(c + d*x)**m/tan(c + d*x),x)*a*d*i*m - int(tan(c + d*x)**m/tan(c
 + d*x),x)*b*d*m + int(tan(c + d*x)**m*tan(c + d*x)**2,x)*b*d*i*m))/(d*m)
```

3.208 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 168

$$\begin{aligned}\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx &= \frac{(A(1-m)-iB(1+m))\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{2ad(1+m)}\\ &+ \frac{(iA-B)m\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)}{2ad(2+m)}\\ &+ \frac{(A+iB)\tan^{1+m}(c+dx)}{2d(a+ia\tan(c+dx))}\end{aligned}$$

output
```
1/2*(A*(1-m)-I*B*(1+m))*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2
)*tan(d*x+c)^(1+m)/a/d/(1+m)+1/2*(I*A-B)*m*hypergeom([1, 1+1/2*m],[2+1/2*m
],-tan(d*x+c)^2)*tan(d*x+c)^(2+m)/a/d/(2+m)+1/2*(A+I*B)*tan(d*x+c)^(1+m)/d
/(a+I*a*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.82

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\tan^{1+m}(c+dx)\left(-\frac{(A(-1+m)+iB(1+m))\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)}{1+m}+\frac{i(A+iB)m\,\text{Hypergeometric2F1}\left(1,\frac{2+}{2}\right.}{2+n}\right.}{2ad}$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
(Tan[c + d*x]^(1 + m)*(-(((A*(-1 + m) + I*B*(1 + m))*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2])/(1 + m)) + (I*(A + I*B)*m*Hypergeo
metric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(2 + m)
+ ((-I)*A + B)/(-I + Tan[c + d*x])))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4079, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \; 4079$$
$$\frac{\int \tan^m(c+dx)(a(-mA+A-iB(m+1))+a(iA-B)m\tan(c+dx))dx}{2a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan(c+dx)^m(a(-mA+A-iB(m+1))+a(iA-B)m\tan(c+dx))dx}{2a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{4021}$$

$$\frac{am(-B+iA)\int \tan^{m+1}(c+dx)dx+a(A(-m)+A-iB(m+1))\int \tan^m(c+dx)dx}{2a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(A(-m)+A-iB(m+1))\int \tan(c+dx)^m dx+am(-B+iA)\int \tan(c+dx)^{m+1}dx}{2a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\frac{am(-B+iA)\int \frac{\tan^{m+1}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{a(A(-m)+A-iB(m+1))\int \frac{\tan^m(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}}{2a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{2d(a+ia\tan(c+dx))}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{a(A(-m)+A-iB(m+1))\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}+\frac{am(-B+iA)\tan^{m+2}(c+dx)\,\text{Hypergeometric2F}}{d(m+2)}}{2a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{2d(a+ia\tan(c+dx))}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
((A + I*B)*Tan[c + d*x]^(1 + m))/(2*d*(a + I*a*Tan[c + d*x])) + ((a*(A - A
*m - I*B*(1 + m))*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]
^2]*Tan[c + d*x]^(1 + m))/(d*(1 + m)) + (a*(I*A - B)*m*Hypergeometric2F1[1
, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m))/(d*(2 + m))
)/(2*a^2)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [F]

$$\int \frac{\tan (dx + c)^m (A + B \tan (dx + c))}{a + ia \tan (dx + c)} dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

Fricas [F]

$$\int \frac{\tan^m(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx = \int \frac{(B \tan (dx + c) + A) \tan (dx + c)^m}{i\,a \tan (dx + c) + a} dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral(1/2*((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*((-I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(-2*I*d*x - 2*I*c)/a, x)
```

Sympy [F]

$$\int \frac{\tan^m(c + dx)(A + B \tan(c + dx))}{a + ia \tan(c + dx)} dx = -\frac{i\left(\int \frac{A \tan^m (c+dx)}{\tan (c+dx)-i}\, dx + \int \frac{B \tan (c+dx) \tan^m (c+dx)}{\tan (c+dx)-i}\, dx\right)}{a}$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*(Integral(A*tan(c + d*x)**m/(tan(c + d*x) - I), x) + Integral(B*tan(c +
 d*x)*tan(c + d*x)**m/(tan(c + d*x) - I), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$=\frac{-\tan(dx+c)^m ai-\left(\int -\frac{\tan(dx+c)^m}{\tan(dx+c)^2-\tan(dx+c)i}dx\right)adm-\left(\int -\frac{\tan(dx+c)^m\tan(dx+c)^2}{\tan(dx+c)-i}dx\right)adim-\left(\int -\frac{\tan(d\ldots}{\ldots}\right.}{adm}$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
( - tan(c + d*x)**m*a*i - int(( - tan(c + d*x)**m)/(tan(c + d*x)**2 - tan(
c + d*x)*i),x)*a*d*m - int(( - tan(c + d*x)**m*tan(c + d*x)**2)/(tan(c + d
*x) - i),x)*a*d*i*m - int(( - tan(c + d*x)**m*tan(c + d*x))/(tan(c + d*x)
- i),x)*a*d*m + int((tan(c + d*x)**m*tan(c + d*x))/(tan(c + d*x)*i + 1),x)
*b*d*m)/(a*d*m)
```

3.209 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 226

$$\begin{aligned}\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx &= \frac{(1-m)(A(1-m)-iB(1+m))\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{4a^2d(1+m)}\\
&+\frac{(A(2-m)-iBm)\tan^{1+m}(c+dx)}{4a^2d(1+i\tan(c+dx))}\\
&+\frac{m(iA(2-m)+Bm)\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)}{4a^2d(2+m)}\\
&+\frac{(A+iB)\tan^{1+m}(c+dx)}{4d(a+ia\tan(c+dx))^2}\end{aligned}$$

output

```
1/4*(1-m)*(A*(1-m)-I*B*(1+m))*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*
x+c)^2)*tan(d*x+c)^(1+m)/a^2/d/(1+m)+1/4*(A*(2-m)-I*B*m)*tan(d*x+c)^(1+m)/
a^2/d/(1+I*tan(d*x+c))+1/4*m*(I*A*(2-m)+B*m)*hypergeom([1, 1+1/2*m],[2+1/2
*m],-tan(d*x+c)^2)*tan(d*x+c)^(2+m)/a^2/d/(2+m)+1/4*(A+I*B)*tan(d*x+c)^(1+
m)/d/(a+I*a*tan(d*x+c))^2
```

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.77

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\tan^{1+m}(c+dx)\left(\frac{2a(-1+m)(A(-1+m)+iB(1+m))\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)}{1+m}+\frac{2am(-iA(-2+m)+Bm)\,\text{Hy}}{}\right.}{8a^3d}$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x
]
```

output
```
(Tan[c + d*x]^(1 + m)*((2*a*(-1 + m)*(A*(-1 + m) + I*B*(1 + m))*Hypergeome
tric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2])/(1 + m) + (2*a*m*((-I)*
A*(-2 + m) + B*m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]
^2]*Tan[c + d*x])/(2 + m) - (2*a*(A + I*B))/(-I + Tan[c + d*x])^2 + ((2*I)
*a*A*(-2 + m) - 2*a*B*m)/(-I + Tan[c + d*x])))/(8*a^3*d)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4079, 3042, 4079, 27, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{4079}$$
$$\frac{\int \frac{\tan^m(c+dx)(a(A(3-m)-iB(m+1))-a(iA-B)(1-m)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\tan(c+dx)^m (a(A(3-m)-iB(m+1))-a(iA-B)(1-m)\tan(c+dx))}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int 2\tan^m(c+dx)\left((1-m)(A(1-m)-iB(m+1))a^2+m(iA(2-m)+Bm)\tan(c+dx)a^2\right)dx}{2a^2}+\frac{(A(2-m)-iBm)\tan^{m+1}(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \tan^m(c+dx)\left((1-m)(A(1-m)-iB(m+1))a^2+m(iA(2-m)+Bm)\tan(c+dx)a^2\right)dx}{a^2}+\frac{(A(2-m)-iBm)\tan^{m+1}(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \tan(c+dx)^m\left((1-m)(A(1-m)-iB(m+1))a^2+m(iA(2-m)+Bm)\tan(c+dx)a^2\right)dx}{a^2}+\frac{(A(2-m)-iBm)\tan^{m+1}(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{4021}$$

$$\frac{\frac{a^2m(Bm+iA(2-m))\int \tan^{m+1}(c+dx)dx+a^2(1-m)(A(1-m)-iB(m+1))\int \tan^m(c+dx)dx}{a^2}+\frac{(A(2-m)-iBm)\tan^{m+1}(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2(1-m)(A(1-m)-iB(m+1))\int \tan(c+dx)^m dx+a^2m(Bm+iA(2-m))\int \tan(c+dx)^{m+1}dx}{a^2}+\frac{(A(2-m)-iBm)\tan^{m+1}(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\frac{\frac{a^2 m(Bm+iA(2-m))\int \frac{\tan^{m+1}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{a^2(1-m)(A(1-m)-iB(m+1))\int \frac{\tan^m(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}}{a^2}+\frac{(A(2-m)-iBm)\tan^{m+1}(c+dx)}{d(1+i\tan(c+dx))}}{4a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

$\Big\downarrow$ 278

$$\frac{\frac{\frac{a^2(1-m)(A(1-m)-iB(m+1))\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}+\frac{a^2 m(Bm+iA(2-m))\tan^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(1,\right.}{d(m+2)}}{a^2}}{4a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((A + I*B)*Tan[c + d*x]^(1 + m))/(4*d*(a + I*a*Tan[c + d*x])^2) + (((A*(2
- m) - I*B*m)*Tan[c + d*x]^(1 + m))/(d*(1 + I*Tan[c + d*x])) + ((a^2*(1 -
m)*(A*(1 - m) - I*B*(1 + m))*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -T
an[c + d*x]^2]*Tan[c + d*x]^(1 + m))/(d*(1 + m)) + (a^2*m*(I*A*(2 - m) + B
*m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]^(2 + m))/(d*(2 + m)))/a^2)/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{(a+ia\tan (dx+c))^2}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(i\,a\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"fricas")
```

output
```
integral(1/4*((A - I*B)*e^(4*I*d*x + 4*I*c) + 2*A*e^(2*I*d*x + 2*I*c) + A
+ I*B)*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(-4*I*
d*x - 4*I*c)/a^2, x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= -\frac{\int \frac{A\tan^m(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx + \int \frac{B\tan(c+dx)\tan^m(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-(Integral(A*tan(c + d*x)**m/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)
+ Integral(B*tan(c + d*x)*tan(c + d*x)**m/(tan(c + d*x)**2 - 2*I*tan(c + d
*x) - 1), x))/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algorithm=
"giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{-\left(\int \frac{\tan(dx+c)^m}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a-\left(\int \frac{\tan(dx+c)^m\tan(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b}{a^2}$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(tan(c + d*x)**m/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a + i
nt((tan(c + d*x)**m*tan(c + d*x))/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1)
,x)*b))/a**2
```

3.210 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 308

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(1-m)\left(iB(3+m-2m^2)-A(3-7m+2m^2)\right)\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\text{t}}{24a^3d(1+m)}$$
$$+\frac{(2-m)m(B+iA(5-2m)+2Bm)\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c+d}{24a^3d(2+m)}$$
$$+\frac{(A+iB)\tan^{1+m}(c+dx)}{6d(a+ia\tan(c+dx))^3}+\frac{(iB(1-2m)+A(7-2m))\tan^{1+m}(c+dx)}{24ad(a+ia\tan(c+dx))^2}$$
$$+\frac{(2-m)(A(5-2m)-i(B+2Bm))\tan^{1+m}(c+dx)}{24d\left(a^3+ia^3\tan(c+dx)\right)}$$

output
```
-1/24*(1-m)*(I*B*(-2*m^2+m+3)-A*(2*m^2-7*m+3))*hypergeom([1, 1/2+1/2*m],[3
/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/a^3/d/(1+m)+1/24*(2-m)*m*(B+I*A*
(5-2*m)+2*B*m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^
(2+m)/a^3/d/(2+m)+1/6*(A+I*B)*tan(d*x+c)^(1+m)/d/(a+I*a*tan(d*x+c))^3+1/24
*(I*B*(1-2*m)+A*(7-2*m))*tan(d*x+c)^(1+m)/a/d/(a+I*a*tan(d*x+c))^2+1/24*(2
-m)*(A*(5-2*m)-I*(2*B*m+B))*tan(d*x+c)^(1+m)/d/(a^3+I*a^3*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.74

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\tan^{1+m}(c+dx)\left(\frac{(-1+m)(iB(3+m-2m^2)+A(-3+7m-2m^2))\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)}{1+m}-\frac{(-2+m)m(B+2\cdots}{\cdots}\right)}{\cdots}$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x
]
```

output
```
(Tan[c + d*x]^(1 + m)*(((-1 + m)*(I*B*(3 + m - 2*m^2) + A*(-3 + 7*m - 2*m^
2))*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2])/(1 + m) -
 ((-2 + m)*m*(B + 2*B*m - I*A*(-5 + 2*m))*Hypergeometric2F1[1, (2 + m)/2,
(4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(2 + m) + ((4*I)*(A + I*B))/(-I
+ Tan[c + d*x])^3 + (A*(-7 + 2*m) + I*B*(-1 + 2*m))/(-I + Tan[c + d*x])^2
+ ((-2 + m)*(B + 2*B*m - I*A*(-5 + 2*m)))/(-I + Tan[c + d*x])))/(24*a^3*d)
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {3042, 4079, 3042, 4079, 25, 3042, 4079, 27, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{\int \frac{\tan^m(c+dx)(a(A(5-m)-iB(m+1))-a(iA-B)(2-m)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{\tan(c+dx)^m(a(A(5-m)-iB(m+1))-a(iA-B)(2-m)\tan(c+dx))}{(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 4079

$$\frac{\frac{\int -\frac{\tan^m(c+dx)\left(a^2\left(iB\left(-2m^2+3m+5\right)-A\left(2m^2-9m+13\right)\right)-a^2(B(1-2m)-iA(7-2m))(1-m)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}+\frac{a(A(7-2m)+iB(1-2m))\tan^{m+1}(c+d}{4d(a+ia\tan(c+dx))^2}}{6a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 25

$$\frac{\frac{a(A(7-2m)+iB(1-2m))\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int \frac{\tan^m(c+dx)\left(a^2\left(iB\left(-2m^2+3m+5\right)-A\left(2m^2-9m+13\right)\right)-a^2(B(1-2m)-iA(7-2m))(1-m)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}}{6a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 3042

$$\frac{\frac{a(A(7-2m)+iB(1-2m))\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)^m\left(a^2\left(iB\left(-2m^2+3m+5\right)-A\left(2m^2-9m+13\right)\right)-a^2(B(1-2m)-iA(7-2m))(1-m)\tan(c+dx)\right)}{i\tan(c+dx)a+a}dx}{4a^2}}{6a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 4079

$$\frac{\frac{a(A(7-2m)+iB(1-2m))\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int 2\tan^m(c+dx)\left(a^3(1-m)\left(iB\left(-2m^2+m+3\right)-A\left(2m^2-7m+3\right)\right)-a^3(2-m)m(2mB+B+iA(5-2m))\tan(c+d}{2a^2}}{4a^2}}{6a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\big\downarrow$ 27

$$\frac{\frac{a(A(7-2m)+iB(1-2m))\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int \tan^m(c+dx)\left(a^3(1-m)\left(iB\left(-2m^2+m+3\right)-A\left(2m^2-7m+3\right)\right)-a^3(2-m)m(2mB+B+iA(5-2m))\tan(c+dx}{a^2}}{4a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{a(A(7-2m)+iB(1-2m))\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\int \tan(c+dx)^m\left(a^3(1-m)\left(iB\left(-2m^2+m+3\right)-A\left(2m^2-7m+3\right)\right)-a^3(2-m)m(2mB+B+iA(5-2m))\tan(c+dx}{a^2}}{4a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\downarrow$ 4021

$$\frac{\frac{a(A(7-2m)+iB(1-2m))\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{a^3(1-m)\left(-A\left(2m^2-7m+3\right)+iB\left(-2m^2+m+3\right)\right)\int \tan^m(c+dx)dx-a^3(2-m)m(iA(5-2m)+2Bm+B)\int \tan^m}{a^2}}{4a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{a(A(7-2m)+iB(1-2m))\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{a^3(1-m)\left(-A\left(2m^2-7m+3\right)+iB\left(-2m^2+m+3\right)\right)\int \tan(c+dx)^m dx-a^3(2-m)m(iA(5-2m)+2Bm+B)\int \tan(}{a^2}}{4a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\downarrow$ 3957

$$\frac{\frac{a(A(7-2m)+iB(1-2m))\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2}-\frac{\frac{\frac{a^3(1-m)\left(-A\left(2m^2-7m+3\right)+iB\left(-2m^2+m+3\right)\right)\int \frac{\tan^m(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{a^3(2-m)m(iA(5-2m)+2B}{}}{a^2}}{4a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

$\downarrow$ 278

$$\frac{\frac{a(A(7-2m)+iB(1-2m))\tan^{m+1}(c+dx)}{4d(a+ia\tan(c+dx))^2} - \frac{\frac{a^3(1-m)\left(-A\left(2m^2-7m+3\right)+iB\left(-2m^2+m+3\right)\right)\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(\right.}{d(m+1)}}{a}}{6}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{6d(a+ia\tan(c+dx))^3}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((A + I*B)*Tan[c + d*x]^(1 + m))/(6*d*(a + I*a*Tan[c + d*x])^3) + ((a*(I*B
*(1 - 2*m) + A*(7 - 2*m))*Tan[c + d*x]^(1 + m))/(4*d*(a + I*a*Tan[c + d*x]
)^2) - (-((a^2*(2 - m)*(A*(5 - 2*m) - I*(B + 2*B*m))*Tan[c + d*x]^(1 + m))
/(d*(a + I*a*Tan[c + d*x]))) + ((a^3*(1 - m)*(I*B*(3 + m - 2*m^2) - A*(3 -
 7*m + 2*m^2))*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]
*Tan[c + d*x]^(1 + m))/(d*(1 + m)) - (a^3*(2 - m)*m*(B + I*A*(5 - 2*m) + 2
*B*m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c +
d*x]^(2 + m))/(d*(2 + m)))/a^2)/(4*a^2))/(6*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [F]

$$\int \frac{\tan(dx+c)^m (A + B\tan(dx+c))}{(a + ia\tan(dx+c))^3} dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(i\,a\tan(dx+c)+a)^3}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"fricas")
```

output
```
integral(1/8*((A - I*B)*e^(6*I*d*x + 6*I*c) + (3*A - I*B)*e^(4*I*d*x + 4*I
*c) + (3*A + I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*((-I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(-6*I*d*x - 6*I*c)/a^3, x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\left(\int \frac{A\tan^m(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx + \int \frac{B\tan(c+dx)\tan^m(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx\right)}{a^3}$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*(Integral(A*tan(c + d*x)**m/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*t
an(c + d*x) + I), x) + Integral(B*tan(c + d*x)*tan(c + d*x)**m/(tan(c + d*
x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c + d*x) + I), x))/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algorithm=
"giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,3]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{(a+a\tan(c+d\,x)\,1\mathrm{i})^3}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{-\tan(dx+c)^m ai + \left(\int -\frac{\tan(dx+c)^m}{\tan(dx+c)^4-3\tan(dx+c)^3 i-3\tan(dx+c)^2+\tan(dx+c)i}dx\right)adm - \left(\int -\frac{\tan(dx+c)^m \tan\cdots}{\tan(dx+c)^3-3\tan(dx+c\cdots}\cdots\right.}{}$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - tan(c + d*x)**m*a*i + int(( - tan(c + d*x)**m)/(tan(c + d*x)**4 - 3*ta
n(c + d*x)**3*i - 3*tan(c + d*x)**2 + tan(c + d*x)*i),x)*a*d*m - int(( - t
an(c + d*x)**m*tan(c + d*x)**4)/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3
*tan(c + d*x) + i),x)*a*d*i*m - 3*int(( - tan(c + d*x)**m*tan(c + d*x)**3)
/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x)*a*d*m + 2
*int(( - tan(c + d*x)**m*tan(c + d*x)**2)/(tan(c + d*x)**3 - 3*tan(c + d*x
)**2*i - 3*tan(c + d*x) + i),x)*a*d*i*m - 2*int(( - tan(c + d*x)**m*tan(c
+ d*x))/(tan(c + d*x)**3 - 3*tan(c + d*x)**2*i - 3*tan(c + d*x) + i),x)*a*
d*m - 3*int((tan(c + d*x)**m*tan(c + d*x))/(tan(c + d*x)**3*i + 3*tan(c +
d*x)**2 - 3*tan(c + d*x)*i - 1),x)*b*d*m)/(3*a**3*d*m)
```

3.211 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 386

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{(3-4m+m^2)\,(iB(1-m^2)-A(1-4m+m^2))\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\,\text{t}}{48a^4d(1+m)}$$
$$-\frac{(iB(1+3m-m^2)-A(13-7m+m^2))\tan^{1+m}(c+dx)}{48a^4d(1+i\tan(c+dx))^2}$$
$$-\frac{(2-m)\,(iB(2+2m-m^2)-A(8-6m+m^2))\tan^{1+m}(c+dx)}{48a^4d(1+i\tan(c+dx))}$$
$$+\frac{(2-m)m(B(2+2m-m^2)+iA(8-6m+m^2))\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\,\text{t}}{48a^4d(2+m)}$$
$$+\frac{(A+iB)\tan^{1+m}(c+dx)}{8d(a+ia\tan(c+dx))^4}+\frac{(iB(1-m)+A(5-m))\tan^{1+m}(c+dx)}{24ad(a+ia\tan(c+dx))^3}$$

output
```
-1/48*(m^2-4*m+3)*(I*B*(-m^2+1)-A*(m^2-4*m+1))*hypergeom([1, 1/2+1/2*m],[3
/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/a^4/d/(1+m)-1/48*(I*B*(-m^2+3*m+
1)-A*(m^2-7*m+13))*tan(d*x+c)^(1+m)/a^4/d/(1+I*tan(d*x+c))^2-1/48*(2-m)*(I
*B*(-m^2+2*m+2)-A*(m^2-6*m+8))*tan(d*x+c)^(1+m)/a^4/d/(1+I*tan(d*x+c))+1/4
8*(2-m)*m*(B*(-m^2+2*m+2)+I*A*(m^2-6*m+8))*hypergeom([1, 1+1/2*m],[2+1/2*m
],-tan(d*x+c)^2)*tan(d*x+c)^(2+m)/a^4/d/(2+m)+1/8*(A+I*B)*tan(d*x+c)^(1+m)
/d/(a+I*a*tan(d*x+c))^4+1/24*(I*B*(1-m)+A*(5-m))*tan(d*x+c)^(1+m)/a/d/(a+I
*a*tan(d*x+c))^3
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.72

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\tan^{1+m}(c+dx)\left(\frac{(3-4m+m^2)(iB(-1+m^2)+A(1-4m+m^2))\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)}{1+m}+\frac{(-2+m)m(-iA(8\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x
]
```

output
```
(Tan[c + d*x]^(1 + m)*(((3 - 4*m + m^2)*(I*B*(-1 + m^2) + A*(1 - 4*m + m^2
))*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2])/(1 + m) +
((-2 + m)*m*((-I)*A*(8 - 6*m + m^2) + B*(-2 - 2*m + m^2))*Hypergeometric2F
1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(2 + m) + (6*(A
+ I*B))/(-I + Tan[c + d*x])^4 + (2*((-I)*A*(-5 + m) + B*(-1 + m)))/(-I + T
an[c + d*x])^3 - (A*(13 - 7*m + m^2) + I*B*(-1 - 3*m + m^2))/(-I + Tan[c +
 d*x])^2 + ((-2 + m)*(B*(2 + 2*m - m^2) + I*A*(8 - 6*m + m^2)))/(-I + Tan[
c + d*x])))/(48*a^4*d)
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4079, 3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{\tan^m(c+dx)(a(A(7-m)-iB(m+1))-a(iA-B)(3-m)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{8a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\tan(c+dx)^m(a(A(7-m)-iB(m+1))-a(iA-B)(3-m)\tan(c+dx))}{(i\tan(c+dx)a+a)^3}dx}{8a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int -\frac{2\tan^m(c+dx)\left(a^2\left(iB\left(-m^2+3m+4\right)-A\left(m^2-7m+16\right)\right)-a^2(B(1-m)-iA(5-m))(2-m)\tan(c+dx)\right)}{(i\tan(c+dx)a+a)^2}dx}{6a^2}+\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}}{8a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\int \frac{\tan^m(c+dx)\left(a^2\left(iB\left(-m^2+3m+4\right)-A\left(m^2-7m+16\right)\right)-a^2(B(1-m)-iA(5-m))(2-m)\tan(c+dx)\right)}{(i\tan(c+dx)a+a)^2}dx}{3a^2}}{8a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\int\frac{\tan(c+dx)^m\left(a^2\left(iB\left(-m^2+3m+4\right)-A\left(m^2-7m+16\right)\right)-a^2(B(1-m)-iA(5-m))(2-m)\tan(c+dx)\right)}{(i\tan(c+dx)a+a)^2}dx}{3a^2}}{8a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 4079

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\frac{\int-\frac{2\tan^m(c+dx)\left(a^3\left(A\left(-m^3+8m^2-20m+19\right)-iB\left(m^3-4m^2+2m+7\right)\right)-a^3(1-m)\left(B\left(-m^2+3m+1\right)+iA\left(m^2\right.\right.\right.}{i\tan(c+dx)a+a}}{4a^2}}{3a^2}}{8a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\frac{\left(-A\left(m^2-7m+13\right)+iB\left(-m^2+3m+1\right)\right)\tan^{m+1}(c+dx)}{2d(1+i\tan(c+dx))^2}-\frac{\int\frac{\tan^m(c+dx)\left(a^3\left(A\left(-m^3+8m^2-20m+19\right)-iB\left(m\right.\right.\right.}{}}{}}{3a^2}}{8a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\frac{\left(-A\left(m^2-7m+13\right)+iB\left(-m^2+3m+1\right)\right)\tan^{m+1}(c+dx)}{2d(1+i\tan(c+dx))^2}-\frac{\int\frac{\tan(c+dx)^m\left(a^3\left(A\left(-m^3+8m^2-20m+19\right)-iB\left(m\right.\right.\right.}{}}{}}{3a^2}}{8a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 4079

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\left(-A\left(m^2-7m+13\right)+iB\left(-m^2+3m+1\right)\right)\tan^{m+1}(c+dx)}{2d(1+i\tan(c+dx))^2}-\frac{\int-2\tan^m(c+dx)\left(a^4\left(m^2-4m+3\right)\left(iB\left(1-m^2\right)-A\left(\right.\right.\right.}{}}{8a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\frac{\left(-A\left(m^2-7m+13\right)+iB\left(-m^2+3m+1\right)\right)\tan^{m+1}(c+dx)}{2d(1+i\tan(c+dx))^2}-\frac{-\int\tan^m(c+dx)\left(a^4\left(m^2-4m+3\right)\left(iB\left(1-m^2\right)-A\left(m\cdots\right.\right.\right.}{\cdots}}{\cdots}}{8a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 3042

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\frac{\left(-A\left(m^2-7m+13\right)+iB\left(-m^2+3m+1\right)\right)\tan^{m+1}(c+dx)}{2d(1+i\tan(c+dx))^2}-\frac{-\int\tan(c+dx)^m\left(a^4\left(m^2-4m+3\right)\left(iB\left(1-m^2\right)-A\left(m\cdots\right.\right.\right.}{\cdots}}{\cdots}}{8a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 4021

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\frac{\left(-A\left(m^2-7m+13\right)+iB\left(-m^2+3m+1\right)\right)\tan^{m+1}(c+dx)}{2d(1+i\tan(c+dx))^2}-\frac{a^4\left(m^2-4m+3\right)\left(-A\left(m^2-4m+1\right)+iB\left(1-m^2\right)\right)\int\cdots}{\cdots}}{\cdots}}{8a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 3042

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\frac{\left(-A\left(m^2-7m+13\right)+iB\left(-m^2+3m+1\right)\right)\tan^{m+1}(c+dx)}{2d(1+i\tan(c+dx))^2}-\frac{a^4\left(m^2-4m+3\right)\left(-A\left(m^2-4m+1\right)+iB\left(1-m^2\right)\right)\int\cdots}{\cdots}}{\cdots}}{8a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 3957

$$\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\frac{\left(-A\left(m^2-7m+13\right)+iB\left(-m^2+3m+1\right)\right)\tan^{m+1}(c+dx)}{2d(1+i\tan(c+dx))^2}-\frac{\frac{a^4\left(m^2-4m+3\right)\left(-A\left(m^2-4m+1\right)+iB\left(1-m^2\right)\right)\int\cdots}{d}}{\cdots}}{\cdots}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

$\downarrow$ 278

$$\frac{\frac{a(A(5-m)+iB(1-m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^3}-\frac{\frac{\left(-A\left(m^2-7m+13\right)+iB\left(-m^2+3m+1\right)\right)\tan^{m+1}(c+dx)}{2d(1+i\tan(c+dx))^2}-\frac{-\frac{a^3(2-m)\left(-A\left(m^2-6m+8\right)+iB\left(-m^2+2m+2\right)\right)\tan\cdots}{d(a+ia\tan(c+dx))}\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{8d(a+ia\tan(c+dx))^4}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^4,x]
```

output
```
((A + I*B)*Tan[c + d*x]^(1 + m))/(8*d*(a + I*a*Tan[c + d*x])^4) + ((a*(I*B
*(1 - m) + A*(5 - m))*Tan[c + d*x]^(1 + m))/(3*d*(a + I*a*Tan[c + d*x])^3)
 - (((I*B*(1 + 3*m - m^2) - A*(13 - 7*m + m^2))*Tan[c + d*x]^(1 + m))/(2*d
*(1 + I*Tan[c + d*x])^2) - (-((a^3*(2 - m)*(I*B*(2 + 2*m - m^2) - A*(8 - 6
*m + m^2))*Tan[c + d*x]^(1 + m))/(d*(a + I*a*Tan[c + d*x]))) - ((a^4*(3 -
4*m + m^2)*(I*B*(1 - m^2) - A*(1 - 4*m + m^2))*Hypergeometric2F1[1, (1 + m
)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(1 + m))/(d*(1 + m)) - (a^4*
(2 - m)*m*(B*(2 + 2*m - m^2) + I*A*(8 - 6*m + m^2))*Hypergeometric2F1[1, (
2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m))/(d*(2 + m)))/a
^2)/(2*a^2))/(3*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{(a+ia\tan (dx+c))^4}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(i\,a\tan(dx+c)+a)^4}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"fricas")
```

output
```
integral(1/16*((A - I*B)*e^(8*I*d*x + 8*I*c) + 2*(2*A - I*B)*e^(6*I*d*x +
6*I*c) + 6*A*e^(4*I*d*x + 4*I*c) + 2*(2*A + I*B)*e^(2*I*d*x + 2*I*c) + A +
 I*B)*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(-8*I*d
*x - 8*I*c)/a^4, x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{A\tan^m(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx + \int \frac{B\tan(c+dx)\tan^m(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**4,x)
```

output
```
(Integral(A*tan(c + d*x)**m/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan
(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x) + Integral(B*tan(c + d*x)*tan(c +
 d*x)**m/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c + d*x)**2 + 4*I*
tan(c + d*x) + 1), x))/a**4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x, algorithm=
"giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,4]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^4,x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^4, x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(\int \frac{\tan(dx+c)^m}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c) i+1}dx\right)a+\left(\int \frac{\tan(dx+c)^m\tan(dx+c)}{\tan(dx+c)^4-4\tan(dx+c)^3 i-6\tan(dx+c)^2+4\tan(dx+c) i+$$

$$a^4$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^4,x)
```

output
```
(int(tan(c + d*x)**m/(tan(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*
x)**2 + 4*tan(c + d*x)*i + 1),x)*a + int((tan(c + d*x)**m*tan(c + d*x))/(t
an(c + d*x)**4 - 4*tan(c + d*x)**3*i - 6*tan(c + d*x)**2 + 4*tan(c + d*x)*
i + 1),x)*b)/a**4
```

3.212 $\int \tan^m(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 315

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =$$
$$-\frac{4a^2(iA+B)\operatorname{AppellF1}\left(\frac{1}{2},-m,1,\frac{3}{2},1+i\tan(c+dx),\frac{1}{2}(1+i\tan(c+dx))\right)(-i\tan(c+dx))^{-m}\tan^m(c+}{d}$$
$$+\frac{2a^2(2B(19+17m+4m^2)+iA(35+34m+8m^2))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+i\tan(c+dx)\right)(-}{d(3+2m)(5+2m)}$$
$$+\frac{2a^2(2iB(4+m)-A(5+2m))\tan^{1+m}(c+dx)\sqrt{a+ia\tan(c+dx)}}{d(3+2m)(5+2m)}$$
$$+\frac{2iaB\tan^{1+m}(c+dx)(a+ia\tan(c+dx))^{3/2}}{d(5+2m)}$$

output
```
-4*a^2*(I*A+B)*AppellF1(1/2,-m,1,3/2,1+I*tan(d*x+c),1/2+1/2*I*tan(d*x+c))*
tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)/d/((-I*tan(d*x+c))^m)+2*a^2*(2*B*(4*
m^2+17*m+19)+I*A*(8*m^2+34*m+35))*hypergeom([1/2, -m],[3/2],1+I*tan(d*x+c)
)*tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)/d/(3+2*m)/(5+2*m)/((-I*tan(d*x+c))
^m)+2*a^2*(2*I*B*(4+m)-A*(5+2*m))*tan(d*x+c)^(1+m)*(a+I*a*tan(d*x+c))^(1/2
)/d/(3+2*m)/(5+2*m)+2*I*a*B*tan(d*x+c)^(1+m)*(a+I*a*tan(d*x+c))^(3/2)/d/(5
+2*m)
```

Mathematica [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \tan^m(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x])
, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.63 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4077, 27, 3042, 4077, 27, 3042, 4084, 3042, 4047, 25, 27, 152, 150, 4082, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{5/2}\tan^m(c+dx)(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(c+dx))^{5/2}\tan(c+dx)^m(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4077}$$

$$\frac{2\int -\frac{1}{2}\tan^m(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(2iB(m+1)-A(2m+5))-a(2B(m+4)+iA(2m+5))\tan(c-}{2m+5}$$
$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\int \tan^m(c+dx)(i\tan(c+dx)a+a)^{3/2}(a(2iB(m+1)-A(2m+5))-a(2B(m+4)+iA(2m+5))\tan(c+dx)}{2m+5}$$

$\downarrow$ 3042

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\int \tan(c+dx)^m(i\tan(c+dx)a+a)^{3/2}(a(2iB(m+1)-A(2m+5))-a(2B(m+4)+iA(2m+5))\tan(c+dx)}{2m+5}$$

$\downarrow$ 4077

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\frac{2\int \frac{1}{2}\tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^2\left(2iB\left(4m^2+15m+11\right)-A\left(8m^2+30m+25\right)\right)-a^2\left(2B\left(4m^2+17m+19\right)+iA\left(8m^2+34m+35\right)\right)\tan(c+dx)\right)}{2m+3}}{2m+5}$$

$\downarrow$ 27

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\frac{\int \tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}\left(a^2\left(2iB\left(4m^2+15m+11\right)-A\left(8m^2+30m+25\right)\right)-a^2\left(2B\left(4m^2+17m+19\right)+iA\left(8m^2+34m+35\right)\right)\tan(c+dx)\right)dx}{2m+3}}{2m+5}$$

$\downarrow$ 3042

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\frac{\int \tan(c+dx)^m\sqrt{i\tan(c+dx)a+a}\left(a^2\left(2iB\left(4m^2+15m+11\right)-A\left(8m^2+30m+25\right)\right)-a^2\left(2B\left(4m^2+17m+19\right)+iA\left(8m^2+34m+35\right)\right)\tan(c+dx)\right)dx}{2m+3}}{2m+5}$$

$\downarrow$ 4084

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\frac{-4a^2\left(4m^2+16m+15\right)(A-iB)\int \tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}dx-a\left(-A\left(8m^2+34m+35\right)+2iB\left(4m^2+17m+19\right)\right)\int \tan^m(c+dx)(a-ia\tan(c+}{2m+3}}{2m+5}$$

$\downarrow$ 3042

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\frac{-4a^2\left(4m^2+16m+15\right)(A-iB)\int \tan(c+dx)^m\sqrt{i\tan(c+dx)a+a}dx-a\left(-A\left(8m^2+34m+35\right)+2iB\left(4m^2+17m+19\right)\right)\int \tan(c+dx)^m(a-ia\tan(c+}{2m+3}}{2m+5}$$

$$\Big\downarrow \text{4047}$$

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{-\frac{\frac{4ia^4(4m^2+16m+15)(A-iB)\int -\frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}-a(-A(8m^2+34m+35)+2iB(4m^2+17m+19))\int \tan(c+dx)^m}{2m+3}}{2m+5}$$

$$\Big\downarrow \text{25}$$

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\frac{\frac{4ia^4(4m^2+16m+15)(A-iB)\int \frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}-a(-A(8m^2+34m+35)+2iB(4m^2+17m+19))\int \tan(c+dx)^m(a-}{2m+3}}{2m+5}$$

$$\Big\downarrow \text{27}$$

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\frac{\frac{4ia^3(4m^2+16m+15)(A-iB)\int \frac{\tan^m(c+dx)}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}-a(-A(8m^2+34m+35)+2iB(4m^2+17m+19))\int \tan(c+dx)^m(a-i}{2m+3}}{2m+5}$$

$$\Big\downarrow \text{152}$$

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\frac{4ia^3(4m^2+16m+15)(A-iB)\sqrt{1+i\tan(c+dx)}\int \frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)+1}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d\sqrt{a+ia\tan(c+dx)}}-a(-A(8m^2+34m+35)+2iB(4m^2+17m+19))\int \tan(}{2m+3}}{2m+5}$$

$$\Big\downarrow \text{150}$$

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\frac{-a(-A(8m^2+34m+35)+2iB(4m^2+17m+19))\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-\frac{4a^3(4m^2+16m+15)(A-iB)\sqrt{1+i\tan(c+dx)}}{}}{2m+3}}{2m+5}$$

$$\Big\downarrow \text{4082}$$

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)}-\frac{\frac{-\frac{a^3(-A(8m^2+34m+35)+2iB(4m^2+17m+19))\int \frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{4a^3(4m^2+16m+15)(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\operatorname{AppellF1}(m-}{d(m+1)\sqrt{a+ia\tan(c+dx)}}}{2m+3}}{2m+5}$$

$\downarrow$ 77

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)} - \frac{-\frac{\frac{a^3\left(-A\left(8m^2+34m+35\right)+2iB\left(4m^2+17m+19\right)\right)\tan^m(c+dx)(-i\tan(c+dx))^{-m}\int\frac{(-i\tan(c+dx))^m}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{4a^3\left(4m^2+16m+15\right)(A-iB)\sqrt{1+i\tan(c+}}{}}{2m+3}}{2m+5}$$

$\downarrow$ 75

$$\frac{2iaB(a+ia\tan(c+dx))^{3/2}\tan^{m+1}(c+dx)}{d(2m+5)} - \frac{\frac{2ia^2\left(-A\left(8m^2+34m+35\right)+2iB\left(4m^2+17m+19\right)\right)\sqrt{a+ia\tan(c+dx)}(-i\tan(c+dx))^{-m}\tan^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},i\tan(c+dx)+1\right)}{d}-\frac{4a^3\left(4m^2+\right.}{}}{2m+3}$$

input
```
Int[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((2*I)*a*B*Tan[c + d*x]^(1 + m)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(5 + 2*m)
) - ((-2*a^2*((2*I)*B*(4 + m) - A*(5 + 2*m))*Tan[c + d*x]^(1 + m)*Sqrt[a +
 I*a*Tan[c + d*x]])/(d*(3 + 2*m)) + ((-4*a^3*(A - I*B)*(15 + 16*m + 4*m^2)
*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1
+ I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[a + I*a*Tan[c + d*
x]]) + ((2*I)*a^2*((2*I)*B*(19 + 17*m + 4*m^2) - A*(35 + 34*m + 8*m^2))*Hy
pergeometric2F1[1/2, -m, 3/2, 1 + I*Tan[c + d*x]]*Tan[c + d*x]^m*Sqrt[a +
I*a*Tan[c + d*x]])/(d*((-I)*Tan[c + d*x])^m))/(3 + 2*m))/(5 + 2*m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \tan (dx+c)^m (a+ia\tan (dx+c))^{\frac{5}{2}} (A+B\tan (dx+c))\,dx$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
integral(4*sqrt(2)*((A - I*B)*a^2*e^(7*I*d*x + 7*I*c) + (A + I*B)*a^2*e^(5
*I*d*x + 5*I*c))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^
m*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x +
4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^m
, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a}\,a^2\left(-2\tan\left(dx+c\right)^m\sqrt{\tan\left(dx+c\right)i+1}\,ai+2\left(\int\frac{\tan(dx+c)^m\sqrt{\tan(dx+c)i+1}}{\tan(dx+c)}dx\right)adi\right.}{}$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
(sqrt(a)*a**2*( - 2*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i + 2*int((
tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x),x)*a*d*i*m - int(ta
n(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**3,x)*b*d - int(tan(c
+ d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*a*d + 2*int(tan(c +
d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*b*d*i + 2*int(tan(c +
d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i*m + 3*int(tan(c + d
*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i + int(tan(c + d*x)**
m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*b*d))/d
```

3.213 $\int \tan^m(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 224

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{2a(iA+B)\operatorname{AppellF1}\left(\frac{1}{2},-m,1,\frac{3}{2},1+i\tan(c+dx),\frac{1}{2}(1+i\tan(c+dx))\right)(-i\tan(c+dx))^{-m}\tan^m(c+}{d}$$

$$+\frac{2a(B+(iA+B)(3+2m))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+i\tan(c+dx)\right)(-i\tan(c+dx))^{-m}\tan^m(c}{d(3+2m)}$$

$$+\frac{2iaB\tan^{1+m}(c+dx)\sqrt{a+ia\tan(c+dx)}}{d(3+2m)}$$

output
```
-2*a*(I*A+B)*AppellF1(1/2,-m,1,3/2,1+I*tan(d*x+c),1/2+1/2*I*tan(d*x+c))*ta
n(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)/d/((-I*tan(d*x+c))^m)+2*a*(B+(I*A+B)*(
3+2*m))*hypergeom([1/2, -m],[3/2],1+I*tan(d*x+c))*tan(d*x+c)^m*(a+I*a*tan(
d*x+c))^(1/2)/d/(3+2*m)/((-I*tan(d*x+c))^m)+2*I*a*B*tan(d*x+c)^(1+m)*(a+I*
a*tan(d*x+c))^(1/2)/d/(3+2*m)
```

Mathematica [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \tan^m(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x])
,x]
```

output
```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x])
, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4077, 27, 3042, 4084, 3042, 4047, 25, 27, 152, 150, 4082, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^{3/2}\tan^m(c+dx)(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^{3/2}\tan(c+dx)^m(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4077}$$

$$\frac{2\int -\frac{1}{2}\tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}(a(2iB(m+1)-A(2m+3))-a(B+(iA+B)(2m+3))\tan(c+dx)}{2m+3}$$
$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{\int \tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}(a(2iB(m+1)-A(2m+3))-a(B+(iA+B)(2m+3))\tan(c+dx))dx}{2m+3}$$

$\downarrow$ 3042

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{\int \tan(c+dx)^m\sqrt{i\tan(c+dx)a+a}(a(2iB(m+1)-A(2m+3))-a(B+(iA+B)(2m+3))\tan(c+dx))dx}{2m+3}$$

$\downarrow$ 4084

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{-2a(2m+3)(A-iB)\int \tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}dx-i(B+(2m+3)(B+iA))\int \tan^m(c+dx)(a-i}{2m+3}$$

$\downarrow$ 3042

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{-2a(2m+3)(A-iB)\int \tan(c+dx)^m\sqrt{i\tan(c+dx)a+a}dx-i(B+(2m+3)(B+iA))\int \tan(c+dx)^m(a-i}{2m+3}$$

$\downarrow$ 4047

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{-\frac{2ia^3(2m+3)(A-iB)\int -\frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}-i(B+(2m+3)(B+iA))\int \tan(c+dx)^m(a-ia}{2m+3}$$

$\downarrow$ 25

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{\frac{2ia^3(2m+3)(A-iB)\int \frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}-i(B+(2m+3)(B+iA))\int \tan(c+dx)^m(a-ia\tan}{2m+3}$$

$\downarrow$ 27

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{\frac{2ia^2(2m+3)(A-iB)\int \frac{\tan^m(c+dx)}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}-i(B+(2m+3)(B+iA))\int \tan(c+dx)^m(a-ia\tan(}{2m+3}$$

$$\downarrow \text{152}$$

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{\frac{2ia^2(2m+3)(A-iB)\sqrt{1+i\tan(c+dx)}\int\frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)+1}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d\sqrt{a+ia\tan(c+dx)}}-i(B+(2m+3)(B+iA))\int\tan(c+dx)^{m}}{2m+3}$$

$$\downarrow \text{150}$$

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{-i(B+(2m+3)(B+iA))\int\tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-\frac{2a^2(2m+3)(A-iB)\sqrt{1+i\tan}}{}}{2m+3}$$

$$\downarrow \text{4082}$$

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{-\frac{ia^2(B+(2m+3)(B+iA))\int\frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{2a^2(2m+3)(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\text{AppellF1}\left(m+1,\frac{1}{2},1,m+2,-\right.}{d(m+1)\sqrt{a+ia\tan(c+dx)}}}{2m+3}$$

$$\downarrow \text{77}$$

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{-\frac{ia^2(B+(2m+3)(B+iA))\tan^m(c+dx)(-i\tan(c+dx))^{-m}\int\frac{(-i\tan(c+dx))^m}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{2a^2(2m+3)(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+}{d(m+1)}}{2m+3}$$

$$\downarrow \text{75}$$

$$\frac{2iaB\sqrt{a+ia\tan(c+dx)}\tan^{m+1}(c+dx)}{d(2m+3)}-\frac{-\frac{2a^2(2m+3)(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\text{AppellF1}\left(m+1,\frac{1}{2},1,m+2,-i\tan(c+dx),i\tan(c+dx)\right)}{d(m+1)\sqrt{a+ia\tan(c+dx)}}-\frac{2a(B+(2m+3)(B+iA))\sqrt{a+i}}{}}{2m+3}$$

input
```
Int[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((2*I)*a*B*Tan[c + d*x]^(1 + m)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(3 + 2*m))
- ((-2*a^2*(A - I*B)*(3 + 2*m)*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)*Tan[c +
 d*x], I*Tan[c + d*x]]*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m))/(d*(
1 + m)*Sqrt[a + I*a*Tan[c + d*x]]) - (2*a*(B + (I*A + B)*(3 + 2*m))*Hyperg
eometric2F1[1/2, -m, 3/2, 1 + I*Tan[c + d*x]]*Tan[c + d*x]^m*Sqrt[a + I*a*
Tan[c + d*x]])/(d*((-I)*Tan[c + d*x])^m))/(3 + 2*m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \tan (dx+c)^{m} (a+ia\tan (dx+c))^{\frac{3}{2}} (A+B\tan (dx+c))\, dx$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\, dx = \int (B\tan (dx+c)+A)(i\, a\tan (dx+c)+a)^{\frac{3}{2}} \tan (dx+c)^{m}\, dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
integral(2*sqrt(2)*((A - I*B)*a*e^(5*I*d*x + 5*I*c) + (A + I*B)*a*e^(3*I*d
*x + 3*I*c))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*
c) + 1), x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\, dx = \int (ia(\tan (c+dx)-i))^{\frac{3}{2}} (A+B\tan (c+dx)) \tan^{m} (c+dx)\, dx$$

input
```
integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*tan(c + d*x)
**m, x)
```

Maxima [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^m
, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \tan(c+d\,x)^m\,(A + B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a}\,a\left(-2\tan\left(dx+c\right)^m\sqrt{\tan\left(dx+c\right)i+1}\,ai+2\left(\int\frac{\tan(dx+c)^m\sqrt{\tan(dx+c)i+1}}{\tan(dx+c)}dx\right)adim$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
(sqrt(a)*a*( - 2*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i + 2*int((tan
(c + d*x)**m*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x),x)*a*d*i*m + int(tan(c
 + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2,x)*b*d*i + 2*int(tan(c
 + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i*m + 2*int(tan(c
+ d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i + int(tan(c + d*x
)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*b*d))/d
```

3.214 $\int \tan^m(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 158

$$\int \tan^m(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{(iA+B)\operatorname{AppellF1}\left(\frac{1}{2},-m,1,\frac{3}{2},1+i\tan(c+dx),\frac{1}{2}(1+i\tan(c+dx))\right)(-i\tan(c+dx))^{-m}\tan^m(c+\ldots}{d} + \frac{2B\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+i\tan(c+dx)\right)(-i\tan(c+dx))^{-m}\tan^m(c+dx)\sqrt{a+ia\tan(c\ldots}}{d}$$

output
```
-(I*A+B)*AppellF1(1/2,-m,1,3/2,1+I*tan(d*x+c),1/2+1/2*I*tan(d*x+c))*tan(d*
x+c)^m*(a+I*a*tan(d*x+c))^(1/2)/d/((-I*tan(d*x+c))^m)+2*B*hypergeom([1/2,
-m],[3/2],1+I*tan(d*x+c))*tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)/d/((-I*tan
(d*x+c))^m)
```

Mathematica [F]

$$\int \tan^m(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \tan^m(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^m*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x
]
```

output
```
Integrate[Tan[c + d*x]^m*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),
x]
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 4084, 3042, 4047, 25, 27, 152, 150, 4082, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(c+dx)}\tan^m(c+dx)(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(c+dx)}\tan(c+dx)^m(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4084}$$

$$(A-iB)\int \tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}dx+\frac{iB\int \tan^m(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\int \tan(c+dx)^m\sqrt{i\tan(c+dx)a+a}dx+\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}$$

$$\Big\downarrow \text{4047}$$

$$\frac{ia^2(A-iB)\int -\frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}+\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}-\frac{ia^2(A-iB)\int \frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}-\frac{ia(A-iB)\int \frac{\tan^m(c+dx)}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{152}$$

$$\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}-\frac{ia(A-iB)\sqrt{1+i\tan(c+dx)}\int \frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)+1}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx}{a}+\frac{a(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\operatorname{AppellF1}\left(m+1,\frac{1}{2},1,m+2,-i\tan(c+dx),i\tan(c+dx)\right)}{d(m+1)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{4082}$$

$$\frac{iaB\int \frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{a(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\operatorname{AppellF1}\left(m+1,\frac{1}{2},1,m+2,-i\tan(c+dx),i\tan(c+dx)\right)}{d(m+1)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \text{77}$$

$$\frac{iaB\tan^m(c+dx)(-i\tan(c+dx))^{-m}\int\frac{(-i\tan(c+dx))^m}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{a(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\operatorname{AppellF1}\left(m+1,\frac{1}{2},1,m+2,-i\tan(c+dx),i\tan(c+dx)\right)}{d(m+1)\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 75$$

$$\frac{a(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\operatorname{AppellF1}\left(m+1,\frac{1}{2},1,m+2,-i\tan(c+dx),i\tan(c+dx)\right)}{d(m+1)\sqrt{a+ia\tan(c+dx)}}+\frac{2B\sqrt{a+ia\tan(c+dx)}\tan^m(c+dx)(-i\tan(c+dx))^{-m}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},i\tan(c+dx)+1\right)}{d}$$

input
```
Int[Tan[c + d*x]^m*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(a*(A - I*B)*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)*Tan[c + d*x], I*Tan[c + d
*x]]*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[a + I*
a*Tan[c + d*x]]) + (2*B*Hypergeometric2F1[1/2, -m, 3/2, 1 + I*Tan[c + d*x]
]*Tan[c + d*x]^m*Sqrt[a + I*a*Tan[c + d*x]])/(d*((-I)*Tan[c + d*x])^m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \tan(dx+c)^m \sqrt{a+ia\tan(dx+c)}\,(A+B\tan(dx+c))\,dx$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)\sqrt{i\,a\tan(dx+c)+a}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
integral(sqrt(2)*((A - I*B)*e^(3*I*d*x + 3*I*c) + (A + I*B)*e^(I*d*x + I*c
))*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \tan^m(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{ia(\tan(c+dx)-i)}(A+B\tan(c+dx))\tan^m(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*tan(c + d*x)**m
, x)
```

Maxima [F]

$$\int \tan^m(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)\sqrt{i\,a\tan(dx+c)+a}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*tan(d*x + c)^m,
x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \tan^m(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{\sqrt{a}\left(-2\tan(dx+c)^m\sqrt{\tan(dx+c)\,i+1}\,ai+2\left(\int\frac{\tan(dx+c)^m\sqrt{\tan(dx+c)i+1}}{\tan(dx+c)}dx\right)adim+2\left(\int\tan(dx+c)\right.}{}$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
(sqrt(a)*( - 2*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i + 2*int((tan(c
 + d*x)**m*sqrt(tan(c + d*x)*i + 1))/tan(c + d*x),x)*a*d*i*m + 2*int(tan(c
 + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i*m + int(tan(c +
d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*a*d*i + int(tan(c + d*x)*
*m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x),x)*b*d))/d
```

3.215 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 216

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(A+iB)\tan^{1+m}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(iA+B)\,\text{AppellF1}\left(\frac{1}{2},-m,1,\frac{3}{2},1+i\tan(c+dx),\frac{1}{2}(1+i\tan(c+dx))\right)(-i\tan(c+dx))^{-m}\tan^m(c+ \cdots}{2ad} + \frac{(iA-B)(1+2m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+i\tan(c+dx)\right)(-i\tan(c+dx))^{-m}\tan^m(c+dx \cdots}{ad}$$

output

```
(A+I*B)*tan(d*x+c)^(1+m)/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*(I*A+B)*AppellF1(1
/2,-m,1,3/2,1+I*tan(d*x+c),1/2+1/2*I*tan(d*x+c))*tan(d*x+c)^m*(a+I*a*tan(d
*x+c))^(1/2)/a/d/((-I*tan(d*x+c))^m)+(I*A-B)*(1+2*m)*hypergeom([1/2, -m],[
3/2],1+I*tan(d*x+c))*tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)/a/d/((-I*tan(d*
x+c))^m)
```

Mathematica [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]]
,x]
```

output
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]]
, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.05 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4079, 27, 3042, 4084, 3042, 4047, 25, 27, 152, 150, 4082, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \; 4079$$

$$\frac{\int -\frac{1}{2}\tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}(2a(Am+iB(m+1))-a(iA-B)(2m+1)\tan(c+dx))dx}{a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}(2a(Am+iB(m+1))-a(iA-B)(2m+1)\tan(c+dx))dx}{2a^2}$$

$\big\downarrow$ 3042

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \tan(c+dx)^m\sqrt{i\tan(c+dx)a+a}(2a(Am+iB(m+1))-a(iA-B)(2m+1)\tan(c+dx))dx}{2a^2}$$

$\big\downarrow$ 4084

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{(2m+1)(A+iB)\int \tan^m(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-a(A-iB)\int \tan^m(c+dx)\sqrt{i\,\text{ta}}}{2a^2}$$

$\big\downarrow$ 3042

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{(2m+1)(A+iB)\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-a(A-iB)\int \tan(c+dx)^m\sqrt{i\,\text{ta}}}{2a^2}$$

$\big\downarrow$ 4047

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{(2m+1)(A+iB)\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-\frac{ia^3(A-iB)\int -\frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c}}}{d}}{2a^2}$$

$\big\downarrow$ 25

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{ia^3(A-iB)\int \frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}+(2m+1)(A+iB)\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\,\text{ta}}}{2a^2}$$

$\big\downarrow$ 27

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{ia^2(A-iB)\int \frac{\tan^m(c+dx)}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}+(2m+1)(A+iB)\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\,\text{tan}}}{2a^2}$$

$$\Big\downarrow \text{152}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{ia^2(A-iB)\sqrt{1+i\tan(c+dx)}\int\frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)+1}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d\sqrt{a+ia\tan(c+dx)}}+(2m+1)(A+iB)\int\tan(c+dx)^m(a-ia\tan(c+}{2a^2}$$

$$\Big\downarrow \text{150}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{(2m+1)(A+iB)\int\tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-\frac{a^2(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+}{d(m+}}{2a^2}$$

$$\Big\downarrow \text{4082}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{a^2(2m+1)(A+iB)\int\frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{a^2(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\text{AppellF1}(m+1,\frac{1}{2},1,m+2,-i\tan(c+dx),i\tan(}{d(m+1)\sqrt{a+ia\tan(c+dx)}}}{2a^2}$$

$$\Big\downarrow \text{77}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{a^2(2m+1)(A+iB)(-i\tan(c+dx))^{-m}\tan^m(c+dx)\int\frac{(-i\tan(c+dx))^m}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{a^2(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\text{AppellF1}(m+}{d(m+1)\sqrt{a+ia\tan(c+}}}{2a^2}$$

$$\Big\downarrow \text{75}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{-\frac{a^2(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\text{AppellF1}(m+1,\frac{1}{2},1,m+2,-i\tan(c+dx),i\tan(c+dx))}{d(m+1)\sqrt{a+ia\tan(c+dx)}}-\frac{2ia(2m+1)(A+iB)\sqrt{a+ia\tan(c+dx)}\text{ta}}{}}{2a^2}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x]
```

output
```
((A + I*B)*Tan[c + d*x]^(1 + m))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (-((a^2*
(A - I*B)*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)*Tan[c + d*x], I*Tan[c + d*x]
]*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[a + I*a*T
an[c + d*x]])) - ((2*I)*a*(A + I*B)*(1 + 2*m)*Hypergeometric2F1[1/2, -m, 3
/2, 1 + I*Tan[c + d*x]]*Tan[c + d*x]^m*Sqrt[a + I*a*Tan[c + d*x]])/(d*((-I
)*Tan[c + d*x])^m))/(2*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{\sqrt{a+ia\tan (dx+c)}}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(B\tan (dx+c)+A)\tan (dx+c)^m}{\sqrt{i\,a\tan (dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="fricas")
```

output
```
integral(1/2*sqrt(2)*((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*((-I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*e^(-I*d*x - I*c)/a, x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(A+B\tan (c+dx))\tan^m (c+dx)}{\sqrt{ia\,(\tan (c+dx)-i)}}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/sqrt(I*a*(tan(c + d*x) - I))
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, algori
thm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2), x
)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(-2\tan(dx+c)^m\sqrt{\tan(dx+c)\,i+1}\,ai-\left(\int\frac{\tan(dx+c)^m\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2}{\tan(dx+c)^2+1}dx\right)\tan(dx+c)^2\,bdi\right.}{}$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - 2*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i - int((tan(c +
 d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2)/(tan(c + d*x)**2 + 1),x
)*tan(c + d*x)**2*b*d*i - int((tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*ta
n(c + d*x)**2)/(tan(c + d*x)**2 + 1),x)*b*d*i + 2*int((tan(c + d*x)**m*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)*
*2*a*d*i*m - 4*int((tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))
/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)**2*a*d*i + int((tan(c + d*x)**m*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*tan(c + d*x)*
*2*b*d + 2*int((tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(ta
n(c + d*x)**2 + 1),x)*a*d*i*m - 4*int((tan(c + d*x)**m*sqrt(tan(c + d*x)*i
 + 1)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*a*d*i + int((tan(c + d*x)**m*
sqrt(tan(c + d*x)*i + 1)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*b*d + 2*in
t((tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1))/(tan(c + d*x)**3 + tan(c + d*
x)),x)*tan(c + d*x)**2*a*d*i*m + 2*int((tan(c + d*x)**m*sqrt(tan(c + d*x)*
i + 1))/(tan(c + d*x)**3 + tan(c + d*x)),x)*a*d*i*m))/(a*d*(tan(c + d*x)**
2 + 1))
```

3.216 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 284

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{(A+iB)\tan^{1+m}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$
$$+\frac{(A(5-4m)-i(B+4Bm))\tan^{1+m}(c+dx)}{6ad\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{(iA+B)\,\text{AppellF1}\left(\frac{1}{2},-m,1,\frac{3}{2},1+i\tan(c+dx),\frac{1}{2}(1+i\tan(c+dx))\right)(-i\tan(c+dx))^{-m}\tan^m(c+d}{4a^2d}$$
$$+\frac{(1+2m)(B+iA(5-4m)+4Bm)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+i\tan(c+dx)\right)(-i\tan(c+dx))^{-}}{6a^2d}$$

output
```
1/3*(A+I*B)*tan(d*x+c)^(1+m)/d/(a+I*a*tan(d*x+c))^(3/2)+1/6*(A*(5-4*m)-I*(
4*B*m+B))*tan(d*x+c)^(1+m)/a/d/(a+I*a*tan(d*x+c))^(1/2)-1/4*(I*A+B)*Appell
F1(1/2,-m,1,3/2,1+I*tan(d*x+c),1/2+1/2*I*tan(d*x+c))*tan(d*x+c)^m*(a+I*a*t
an(d*x+c))^(1/2)/a^2/d/((-I*tan(d*x+c))^m)+1/6*(1+2*m)*(B+I*A*(5-4*m)+4*B*
m)*hypergeom([1/2, -m],[3/2],1+I*tan(d*x+c))*tan(d*x+c)^m*(a+I*a*tan(d*x+c
))^(1/2)/a^2/d/((-I*tan(d*x+c))^m)
```

Mathematica [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/
2),x]
```

output
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/
2), x]
```

Rubi [A] (warning: unable to verify)

Time = 1.57 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4084, 3042, 4047, 25, 27, 152, 150, 4082, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{\tan^m(c+dx)(2a(A(2-m)-iB(m+1))-a(iA-B)(1-2m)\tan(c+dx))}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\tan^m(c+dx)(2a(A(2-m)-iB(m+1))-a(iA-B)(1-2m)\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\tan(c+dx)^m(2a(A(2-m)-iB(m+1))-a(iA-B)(1-2m)\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int -\frac{1}{2}\tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}\left(2a^2\left(iB\left(-4m^2-3m+1\right)+A\left(-4m^2+3m+1\right)\right)-a^2(2m+1)(4mB+B+iA(5-4m))\tan(c+dx)\right)dx}{a^2}+\frac{a(A(5}{}}{6a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}\left(2a^2\left(iB\left(-4m^2-3m+1\right)+A\left(-4m^2+3m+1\right)\right)-a^2(2m+1)(4mB}{2a^2}}{6a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \tan(c+dx)^m\sqrt{i\tan(c+dx)a+a}\left(2a^2\left(iB\left(-4m^2-3m+1\right)+A\left(-4m^2+3m+1\right)\right)-a^2(2m+1)(4mB}{2a^2}}{6a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4084}$$

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a(2m+1)(A(5-4m)-i(4Bm+B))\int \tan^m(c+dx)(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-3a^2(A-}{2a^2}}{6a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a(2m+1)(A(5-4m)-i(4Bm+B))\int \tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-3a^2(A-}{2a^2}}{6a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4047}$$

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a(2m+1)(A(5-4m)-i(4Bm+B))\int\tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-\frac{3ia^4(A-}{}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{3ia^4(A-iB)\int\frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}+a(2m+1)(A(5-4m)-i(4Bm+B))}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{3ia^3(A-iB)\int\frac{\tan^m(c+dx)}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}+a(2m+1)(A(5-4m)-i(4Bm+B))\int}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 152

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{3ia^3(A-iB)\sqrt{1+i\tan(c+dx)}\int\frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)+1}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d\sqrt{a+ia\tan(c+dx)}}+a(2m+1)(A(5-4m)-}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 150

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a(2m+1)(A(5-4m)-i(4Bm+B))\int\tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}dx-\frac{3a^3(A-}{}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 4082

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{a^3(2m+1)(A(5-4m)-i(4Bm+B))\int\frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{3a^3(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c}{d(r}}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 77

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{a^3(2m+1)(A(5-4m)-i(4Bm+B))(-i\tan(c+dx))^{-m}\tan^m(c+dx)\int\frac{(-i\tan(c+dx))^m}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-3a^3(A}{2a^2}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

$\downarrow$ 75

$$\frac{\frac{a(A(5-4m)-i(4Bm+B))\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}-\frac{-\frac{3a^3(A-iB)\sqrt{1+i\tan(c+dx)}\tan^{m+1}(c+dx)\operatorname{AppellF1}\left(m+1,\frac{1}{2},1,m+2,-i\tan(c+dx),i\tan(c+dx)\right)}{d(m+1)\sqrt{a+ia\tan(c+dx)}}-\frac{2ia^2(2}{}}{}}{6a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2),x]
```

output
```
((A + I*B)*Tan[c + d*x]^(1 + m))/(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((a*
(A*(5 - 4*m) - I*(B + 4*B*m))*Tan[c + d*x]^(1 + m))/(d*Sqrt[a + I*a*Tan[c
+ d*x]]) - ((-3*a^3*(A - I*B)*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)*Tan[c +
d*x], I*Tan[c + d*x]]*Sqrt[1 + I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m))/(d*(1
 + m)*Sqrt[a + I*a*Tan[c + d*x]]) - ((2*I)*a^2*(1 + 2*m)*(A*(5 - 4*m) - I*
(B + 4*B*m))*Hypergeometric2F1[1/2, -m, 3/2, 1 + I*Tan[c + d*x]]*Tan[c + d
*x]^m*Sqrt[a + I*a*Tan[c + d*x]])/(d*((-I)*Tan[c + d*x])^m))/(2*a^2))/(6*a
^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{(a+ia\tan (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
integral(1/4*sqrt(2)*((A - I*B)*e^(4*I*d*x + 4*I*c) + 2*A*e^(2*I*d*x + 2*I
*c) + A + I*B)*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-3*I*d*x - 3*I*c)/a^2, x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^m(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/(I*a*(tan(c + d*x) - I))**(3
/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2), x
)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{too large to display}$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - 1024*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i*m**17 - 281
6*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i*m**16 + 5504*tan(c + d*x)**
m*sqrt(tan(c + d*x)*i + 1)*a*i*m**15 + 22080*tan(c + d*x)**m*sqrt(tan(c +
d*x)*i + 1)*a*i*m**14 - 1440*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i*
m**13 - 58512*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i*m**12 - 29368*t
an(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i*m**11 + 71692*tan(c + d*x)**m*
sqrt(tan(c + d*x)*i + 1)*a*i*m**10 + 60710*tan(c + d*x)**m*sqrt(tan(c + d*
x)*i + 1)*a*i*m**9 - 41880*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i*m*
*8 - 52956*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i*m**7 + 8436*tan(c
+ d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i*m**6 + 22976*tan(c + d*x)**m*sqrt(t
an(c + d*x)*i + 1)*a*i*m**5 + 1900*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1
)*a*i*m**4 - 4780*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i*m**3 - 1008
*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*a*i*m**2 + 378*tan(c + d*x)**m*s
qrt(tan(c + d*x)*i + 1)*a*i*m + 108*tan(c + d*x)**m*sqrt(tan(c + d*x)*i +
1)*a*i + 512*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*b*m**17 + 2048*tan(c
 + d*x)**m*sqrt(tan(c + d*x)*i + 1)*b*m**16 - 512*tan(c + d*x)**m*sqrt(tan
(c + d*x)*i + 1)*b*m**15 - 12800*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*
b*m**14 - 14400*tan(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*b*m**13 + 18816*t
an(c + d*x)**m*sqrt(tan(c + d*x)*i + 1)*b*m**12 + 43424*tan(c + d*x)**m*sq
rt(tan(c + d*x)*i + 1)*b*m**11 + 4064*tan(c + d*x)**m*sqrt(tan(c + d*x)...
```

3.217 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 362

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{(A+iB)\tan^{1+m}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$
$$+\frac{(iB(1-4m)+A(11-4m))\tan^{1+m}(c+dx)}{30ad(a+ia\tan(c+dx))^{3/2}}$$
$$-\frac{(iB(13+12m-16m^2)-A(37-52m+16m^2))\tan^{1+m}(c+dx)}{60a^2d\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{(iA+B)\operatorname{AppellF1}\left(\frac{1}{2},-m,1,\frac{3}{2},1+i\tan(c+dx),\frac{1}{2}(1+i\tan(c+dx))\right)(-i\tan(c+dx))^{-m}\tan^m(c+d}{8a^3d}$$
$$+\frac{(1+2m)\left(B(13+12m-16m^2)+iA(37-52m+16m^2)\right)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1+i\tan(c+}{60a^3d}$$

output
```
1/5*(A+I*B)*tan(d*x+c)^(1+m)/d/(a+I*a*tan(d*x+c))^(5/2)+1/30*(I*B*(1-4*m)+
A*(11-4*m))*tan(d*x+c)^(1+m)/a/d/(a+I*a*tan(d*x+c))^(3/2)-1/60*(I*B*(-16*m
^2+12*m+13)-A*(16*m^2-52*m+37))*tan(d*x+c)^(1+m)/a^2/d/(a+I*a*tan(d*x+c))^
(1/2)-1/8*(I*A+B)*AppellF1(1/2,-m,1,3/2,1+I*tan(d*x+c),1/2+1/2*I*tan(d*x+c
))*tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)/a^3/d/((-I*tan(d*x+c))^m)+1/60*(1
+2*m)*(B*(-16*m^2+12*m+13)+I*A*(16*m^2-52*m+37))*hypergeom([1/2, -m],[3/2]
,1+I*tan(d*x+c))*tan(d*x+c)^m*(a+I*a*tan(d*x+c))^(1/2)/a^3/d/((-I*tan(d*x+
c))^m)
```

Mathematica [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/
2),x]
```

output
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/
2), x]
```

Rubi [A] (warning: unable to verify)

Time = 2.22 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.05, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4084, 3042, 4047, 25, 27, 152, 150, 4082, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{\tan^m(c+dx)(2a(A(4-m)-iB(m+1))-a(iA-B)(3-2m)\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\tan^m(c+dx)(2a(A(4-m)-iB(m+1))-a(iA-B)(3-2m)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\tan(c+dx)^m(2a(A(4-m)-iB(m+1))-a(iA-B)(3-2m)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4079$$

$$\frac{\frac{\int -\frac{\tan^m(c+dx)\left(2a^2\left(iB\left(-4m^2+3m+7\right)-A\left(4m^2-13m+13\right)\right)-a^2(B(1-4m)-iA(11-4m))(1-2m)\tan(c+dx)\right)}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{\tan^m(c+dx)\left(2a^2\left(iB\left(-4m^2+3m+7\right)-A\left(4m^2-13m+13\right)\right)-a^2(B(1-4m)-iA(11-4m))(1-2m)\tan(c+d}{\sqrt{i\tan(c+dx)a+a}}}{6a^2}}{10a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{\tan(c+dx)^m\left(2a^2\left(iB\left(-4m^2+3m+7\right)-A\left(4m^2-13m+13\right)\right)-a^2(B(1-4m)-iA(11-4m))(1-2m)\tan(c+d}{\sqrt{i\tan(c+dx)a+a}}}{6a^2}}{10a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4079$$

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{1}{2}\tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}\left(2a^3\left(A\left(16m^3-44m^2+11m+11\right)+iB\left(16m^3-4m^2-19m+1\right)\right)-a^3(2m+1}{a^2}}{10a^2}$$
$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int \tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}\left(2a^3\left(A\left(16m^3-44m^2+11m+11\right)+iB\left(16m^3-4m^2-19m+1\right)\right)-a^3(2m+1)\left(\cdots\right.}{2a^2}}{\cdots}}{10a^2\cdots}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int \tan(c+dx)^m\sqrt{i\tan(c+dx)a+a}\left(2a^3\left(A\left(16m^3-44m^2+11m+11\right)+iB\left(16m^3-4m^2-19m+1\right)\right)-a^3(2m+1)\left(\cdots\right.}{2a^2}}{\cdots}}{10a^2\cdots}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4084$$

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{-15a^3(A-iB)\int \tan^m(c+dx)\sqrt{i\tan(c+dx)a+a}dx-a^2(2m+1)\left(-A\left(16m^2-52m+37\right)+iB\left(-16m^2+12m+13\cdots\right.\right.}{2a^2}}{\cdots}}{10a^2\cdots}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{-15a^3(A-iB)\int \tan(c+dx)^m\sqrt{i\tan(c+dx)a+a}dx-a^2(2m+1)\left(-A\left(16m^2-52m+37\right)+iB\left(-16m^2+12m+13\cdots\right.\right.}{2a^2}}{\cdots}}{10a^2\cdots}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 4047$$

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{-\frac{15ia^5(A-iB)\int -\frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}-a^2(2m+1)\left(-A\left(16m^2-52m+37\right)\cdots\right.}{2a^2}}{\cdots}}{10a^2\cdots}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{15ia^5(A-iB)\int\frac{\tan^m(c+dx)}{a(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}-a^2(2m+1)\left(-A\left(16m^2-52m+37\right)+i}{2a^2}}{10a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{15ia^4(A-iB)\int\frac{\tan^m(c+dx)}{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}d(ia\tan(c+dx))}{d}-a^2(2m+1)\left(-A\left(16m^2-52m+37\right)+iB}{2a^2}}{10a^2}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 152

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{15ia^4(A-iB)\sqrt{1+i\tan(c+dx)}\int\frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)+1}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d\sqrt{a+ia\tan(c+dx)}}-a^2(2m+1)\left(-A\left(16m^2\right.\right.}{2a^2}}{10a}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 150

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{-a^2(2m+1)\left(-A\left(16m^2-52m+37\right)+iB\left(-16m^2+12m+13\right)\right)\int\tan(c+dx)^m(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a}}{2a}}{}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 4082

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{-\frac{a^4(2m+1)\left(-A\left(16m^2-52m+37\right)+iB\left(-16m^2+12m+13\right)\right)\int\frac{\tan^m(c+dx)}{\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-15a^4(A-iB}{2a^2}}{10a}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 77

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{-\frac{a^4(2m+1)\left(-A\left(16m^2-52m+37\right)+iB\left(-16m^2+12m+13\right)\right)\tan^m(c+dx)(-i\tan(c+dx))^{-m}\int\frac{(-i\tan(c+d}{\sqrt{i\tan(c+dx}}}{d}}{}}{}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

$\downarrow$ 75

$$\frac{\frac{a(A(11-4m)+iB(1-4m))\tan^{m+1}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{a^2\left(-A\left(16m^2-52m+37\right)+iB\left(-16m^2+12m+13\right)\right)\tan^{m+1}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia^3(2m+1)\left(-A\left(16m^2-52m+37\right)+i\right.}{}}{}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2),x]
```

output
```
((A + I*B)*Tan[c + d*x]^(1 + m))/(5*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((a*
(I*B*(1 - 4*m) + A*(11 - 4*m))*Tan[c + d*x]^(1 + m))/(3*d*(a + I*a*Tan[c +
 d*x])^(3/2)) - ((a^2*(I*B*(13 + 12*m - 16*m^2) - A*(37 - 52*m + 16*m^2))*
Tan[c + d*x]^(1 + m))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + ((-15*a^4*(A - I*B)
*AppellF1[1 + m, 1/2, 1, 2 + m, (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Sqrt[1
+ I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m))/(d*(1 + m)*Sqrt[a + I*a*Tan[c + d*
x]]) + ((2*I)*a^3*(1 + 2*m)*(I*B*(13 + 12*m - 16*m^2) - A*(37 - 52*m + 16*
m^2))*Hypergeometric2F1[1/2, -m, 3/2, 1 + I*Tan[c + d*x]]*Tan[c + d*x]^m*S
qrt[a + I*a*Tan[c + d*x]])/(d*((-I)*Tan[c + d*x])^m))/(2*a^2))/(6*a^2))/(1
0*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{\tan(dx+c)^m (A+B\tan(dx+c))}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(ia\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
integral(1/8*sqrt(2)*((A - I*B)*e^(6*I*d*x + 6*I*c) + (3*A - I*B)*e^(4*I*d
*x + 4*I*c) + (3*A + I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*((-I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(-5*I*d*x - 5*I*c)/a^3, x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^m(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/(I*a*(tan(c + d*x) - I))**(5
/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2), x
)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\tan(dx+c)^m}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}\right.}{}$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(tan(c + d*x)**m/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqr
t(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a + in
t((tan(c + d*x)**m*tan(c + d*x))/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2
 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x
)*b))/(sqrt(a)*a**2)
```

3.218 $\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 167

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{(A-iB)\,\text{AppellF1}(1+m,1-n,1,2+m,-i\tan(c+dx),i\tan(c+dx))(1+i\tan(c+dx))^{-n}\tan^{1+m}(c}{d(1+m)}$$
$$+ \frac{iB\,\text{Hypergeometric2F1}(1+m,1-n,2+m,-i\tan(c+dx))(1+i\tan(c+dx))^{-n}\tan^{1+m}(c+dx)(a}{d(1+m)}$$

output

```
(A-I*B)*AppellF1(1+m,1-n,1,2+m,-I*tan(d*x+c),I*tan(d*x+c))*tan(d*x+c)^(1+m
)*(a+I*a*tan(d*x+c))^n/d/(1+m)/((1+I*tan(d*x+c))^n)+I*B*hypergeom([1+m, 1-
n],[2+m],-I*tan(d*x+c))*tan(d*x+c)^(1+m)*(a+I*a*tan(d*x+c))^n/d/(1+m)/((1+
I*tan(d*x+c))^n)
```

Mathematica [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Integrate[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]), x]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4084, 3042, 4047, 25, 27, 152, 150, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4084}$$

$$\frac{(A-iB)\int \tan^m(c+dx)(i\tan(c+dx)a+a)^ndx+iB\int \tan^m(c+dx)(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^ndx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int \tan(c+dx)^m(i\tan(c+dx)a+a)^ndx+iB\int \tan(c+dx)^m(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^ndx}{a}$$

$$\Big\downarrow \text{4047}$$

$$\frac{ia^2(A-iB)\int -\frac{\tan^m(c+dx)(i\tan(c+dx)a+a)^{n-1}}{a(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}+\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n dx}{a}-\frac{ia^2(A-iB)\int \frac{\tan^m(c+dx)(i\tan(c+dx)a+a)^{n-1}}{a(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n dx}{a}-\frac{ia(A-iB)\int \frac{\tan^m(c+dx)(i\tan(c+dx)a+a)^{n-1}}{a-ia\tan(c+dx)}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{152}$$

$$\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n dx}{a}-\frac{i(A-iB)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\int \frac{(i\tan(c+dx)+1)^{n-1}\tan^m(c+dx)}{a-ia\tan(c+dx)}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{150}$$

$$\frac{iB\int \tan(c+dx)^m(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n dx}{a}+\frac{(A-iB)\tan^{m+1}(c+dx)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n \text{AppellF1}(m+1,1-n,1,m+2,-i\tan(c+}{d(m+1)}$$

$$\Big\downarrow \text{4082}$$

$$\frac{iaB\int \tan^m(c+dx)(i\tan(c+dx)a+a)^{n-1}d\tan(c+dx)}{d}+\frac{(A-iB)\tan^{m+1}(c+dx)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n \text{AppellF1}(m+1,1-n,1,m+2,-i\tan(c+}{d(m+1)}$$

$$\Big\downarrow \text{76}$$

$$\frac{iB(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\int (i\tan(c+dx)+1)^{n-1}\tan^m(c+dx)d\tan(c+dx)}{d}+\frac{(A-iB)\tan^{m+1}(c+dx)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n \text{AppellF1}(m+1,1-n,1,m+2,-i\tan(c+}{d(m+1)}$$

$\Big\downarrow$ 74

$$\frac{(A-iB)\tan^{m+1}(c+dx)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\,\text{AppellF1}(m+1,1-n,1,m+2,-i\tan(c+}{d(m+1)}$$
$$\frac{iB\tan^{m+1}(c+dx)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\,\text{Hypergeometric2F1}(m+1,1-n,m+2,-i\tan(c}{d(m+1)}$$

input
```
Int[Tan[c + d*x]^m*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((A - I*B)*AppellF1[1 + m, 1 - n, 1, 2 + m, (-I)*Tan[c + d*x], I*Tan[c + d
*x]]*Tan[c + d*x]^(1 + m)*(a + I*a*Tan[c + d*x])^n)/(d*(1 + m)*(1 + I*Tan[
c + d*x])^n) + (I*B*Hypergeometric2F1[1 + m, 1 - n, 2 + m, (-I)*Tan[c + d*
x]]*Tan[c + d*x]^(1 + m)*(a + I*a*Tan[c + d*x])^n)/(d*(1 + m)*(1 + I*Tan[c
 + d*x])^n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \tan (dx+c)^m (a+ia\tan (dx+c))^n (A+B\tan (dx+c))\,dx$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan (dx+c)+A)(i\,a\tan (dx+c)+a)^n \tan (dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral(((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c
)/(e^(2*I*d*x + 2*I*c) + 1))^n*((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) + 1))^m/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (ia(\tan (c+dx)-i))^n (A+B\tan (c+dx))\tan^m (c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**m*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*tan(c + d*x)**m,
 x)
```

Maxima [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^m, x)
```

Giac [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})^n\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{-\tan(dx+c)^m(\tan(dx+c)\,ai+a)^n\,ai+\left(\int\frac{\tan(dx+c)^m(\tan(dx+c)ai+a)^n}{\tan(dx+c)}dx\right)adim+\left(\int\tan(dx+c)^m\,(\tan\right.}{}$$

input
```
int(tan(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
( - tan(c + d*x)**m*(tan(c + d*x)*a*i + a)**n*a*i + int((tan(c + d*x)**m*(
tan(c + d*x)*a*i + a)**n)/tan(c + d*x),x)*a*d*i*m + int(tan(c + d*x)**m*(t
an(c + d*x)*a*i + a)**n*tan(c + d*x),x)*a*d*i*m + int(tan(c + d*x)**m*(tan
(c + d*x)*a*i + a)**n*tan(c + d*x),x)*a*d*i*n + int(tan(c + d*x)**m*(tan(c
 + d*x)*a*i + a)**n*tan(c + d*x),x)*b*d*n)/(d*n)
```

3.219 $\int \tan^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 245

$$\begin{aligned}
&\int \tan^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx\\
&=\frac{2(iBn-A(3+n))(a+ia\tan(c+dx))^n}{dn(2+n)(3+n)}\\
&+\frac{(A-iB)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^n}{2dn}\\
&-\frac{(iBn-A(3+n))\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(2+n)(3+n)}\\
&+\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(3+n)}\\
&-\frac{(An(3+n)-iB(6+3n+n^2))(a+ia\tan(c+dx))^{1+n}}{ad(1+n)(2+n)(3+n)}
\end{aligned}$$

output
```
2*(I*B*n-A*(3+n))*(a+I*a*tan(d*x+c))^n/d/n/(2+n)/(3+n)+1/2*(A-I*B)*hyperge
om([1, n],[1+n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/n-(I*B*n-A*(3
+n))*tan(d*x+c)^2*(a+I*a*tan(d*x+c))^n/d/(2+n)/(3+n)+B*tan(d*x+c)^3*(a+I*a
*tan(d*x+c))^n/d/(3+n)-(A*n*(3+n)-I*B*(n^2+3*n+6))*(a+I*a*tan(d*x+c))^(1+n
)/a/d/(1+n)/(2+n)/(3+n)
```

Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{(a+ia\tan(c+dx))^n\left(2iBn(8+5n+n^2)-2A(6+8n+5n^2+n^3)+(A-iB)\left(6+11n+6n^2+n^3\right)\mathrm{H}\right.}{}$$

input
```
Integrate[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((a + I*a*Tan[c + d*x])^n*((2*I)*B*n*(8 + 5*n + n^2) - 2*A*(6 + 8*n + 5*n^
2 + n^3) + (A - I*B)*(6 + 11*n + 6*n^2 + n^3)*Hypergeometric2F1[1, n, 1 +
n, (1 + I*Tan[c + d*x])/2] - 2*n*(I*A*n*(3 + n) + B*(6 + 3*n + n^2))*Tan[c
 + d*x] + 2*n*(1 + n)*((-I)*B*n + A*(3 + n))*Tan[c + d*x]^2 + 2*B*n*(2 + 3
*n + n^2)*Tan[c + d*x]^3))/(2*d*n*(1 + n)*(2 + n)*(3 + n))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {3042, 4080, 25, 3042, 4080, 25, 3042, 4075, 3042, 4010, 3042, 3962, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4080}$$

$$\frac{\int -\tan^2(c+dx)(i\tan(c+dx)a+a)^n(3aB+a(iBn-A(n+3))\tan(c+dx))dx}{a(n+3)}+\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)}$$

$\downarrow$ 25

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)}-\frac{\int \tan^2(c+dx)(i\tan(c+dx)a+a)^n(3aB+a(iBn-A(n+3))\tan(c+dx))dx}{a(n+3)}$$

$\downarrow$ 3042

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)}-\frac{\int \tan(c+dx)^2(i\tan(c+dx)a+a)^n(3aB+a(iBn-A(n+3))\tan(c+dx))dx}{a(n+3)}$$

$\downarrow$ 4080

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)}-\frac{\frac{\int -\tan(c+dx)(i\tan(c+dx)a+a)^n\left(2a^2(iBn-A(n+3))-a^2\left(iAn(n+3)+B\left(n^2+3n+6\right)\right)\tan(c+dx)\right)dx}{a(n+2)}+\frac{a(-A(n+3)+iBn)\tan^2(c+dx)(a+ia}{d(n+2)}}{a(n+3)}$$

$\downarrow$ 25

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)}-\frac{\frac{a(-A(n+3)+iBn)\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}-\frac{\int \tan(c+dx)(i\tan(c+dx)a+a)^n\left(2a^2(iBn-A(n+3))-a^2\left(iAn(n+3)+B\left(n^2+3n+6\right)\right)\tan}{a(n+2)}}{a(n+3)}$$

$\downarrow$ 3042

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)}-\frac{\frac{a(-A(n+3)+iBn)\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}-\frac{\int \tan(c+dx)(i\tan(c+dx)a+a)^n\left(2a^2(iBn-A(n+3))-a^2\left(iAn(n+3)+B\left(n^2+3n+6\right)\right)\tan}{a(n+2)}}{a(n+3)}$$

$\downarrow$ 4075

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)}-\frac{\frac{a(-A(n+3)+iBn)\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}-\frac{\int (i\tan(c+dx)a+a)^n\left(\left(iAn(n+3)+B\left(n^2+3n+6\right)\right)a^2+2(iBn-A(n+3))\tan(c+dx)a^2\right)dx}{a(n+2)}}{a(n+3)}$$

$\downarrow$ 3042

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)} - \frac{\frac{a(-A(n+3)+iBn)\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)} - \frac{\int (i\tan(c+dx)a+a)^n\left(\left(iAn(n+3)+B\left(n^2+3n+6\right)\right)a^2+2(iBn-A(n+3))\tan(c+dx)a^2\right)dx}{a(n+2)}}{a(n+3)}$$

$\big\downarrow$ 4010

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)} - \frac{\frac{a(-A(n+3)+iBn)\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)} - \frac{a^2\left(n^2+5n+6\right)(B+iA)\int (i\tan(c+dx)a+a)^n dx+\frac{2a^2(-A(n+3)+iBn)(a+ia\tan(c+dx))^n}{dn} - \frac{a(}{}}{a(n+2)}}{a(n+3)}$$

$\big\downarrow$ 3042

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)} - \frac{\frac{a(-A(n+3)+iBn)\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)} - \frac{a^2\left(n^2+5n+6\right)(B+iA)\int (i\tan(c+dx)a+a)^n dx+\frac{2a^2(-A(n+3)+iBn)(a+ia\tan(c+dx))^n}{dn} - \frac{a(}{}}{a(n+2)}}{a(n+3)}$$

$\big\downarrow$ 3962

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)} - \frac{\frac{a(-A(n+3)+iBn)\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)} - \frac{-\frac{ia^3\left(n^2+5n+6\right)(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a-ia\tan(c+dx)}d(ia\tan(c+dx))}{d}+\frac{2a^2(-A(n+3)+iBn)(a+ia\tan}{dn}}{a(n+2)}}{a(n+3)}$$

$\big\downarrow$ 78

$$\frac{B\tan^3(c+dx)(a+ia\tan(c+dx))^n}{d(n+3)} - \frac{\frac{a(-A(n+3)+iBn)\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)} - \frac{-\frac{ia^2\left(n^2+5n+6\right)(B+iA)(a+ia\tan(c+dx))^n\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{2dn}+2a^2}{a(n+2}}{a(n+3)}$$

input
```
Int[Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]^3*(a + I*a*Tan[c + d*x])^n)/(d*(3 + n)) - ((a*(I*B*n - A*(
3 + n))*Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^n)/(d*(2 + n)) - ((2*a^2*(I*
B*n - A*(3 + n))*(a + I*a*Tan[c + d*x])^n)/(d*n) - ((I/2)*a^2*(I*A + B)*(6
 + 5*n + n^2)*Hypergeometric2F1[1, n, 1 + n, (a + I*a*Tan[c + d*x])/(2*a)]
*(a + I*a*Tan[c + d*x])^n)/(d*n) - (a*(A*n*(3 + n) - I*B*(6 + 3*n + n^2))*
(a + I*a*Tan[c + d*x])^(1 + n))/(d*(1 + n)))/(a*(2 + n)))/(a*(3 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Maple [F]

$$\int \tan (dx+c)^3 (a+ia\tan (dx+c))^n (A+B\tan (dx+c))\, dx$$

input
```
int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\, dx$$
$$= \int (B\tan (dx+c)+A)(i\,a\tan (dx+c)+a)^n \tan (dx+c)^3\, dx$$

input
```
integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral(((I*A + B)*e^(8*I*d*x + 8*I*c) - 2*(I*A + 2*B)*e^(6*I*d*x + 6*I*c
) + 6*B*e^(4*I*d*x + 4*I*c) - 2*(-I*A + 2*B)*e^(2*I*d*x + 2*I*c) - I*A + B
)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(e^(8*I*d*x + 8*I*
c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*I*c)
 + 1), x)
```

Sympy [F]

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (ia(\tan{(c+dx)}-i))^n\,(A+B\tan{(c+dx)})\tan^3{(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**3*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*tan(c + d*x)**3,
 x)
```

Maxima [F]

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan{(dx+c)}+A)(i\,a\tan{(dx+c)}+a)^n\tan{(dx+c)}^3\,dx$$

input
```
integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Giac [F]

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan{(dx+c)}+A)(i\,a\tan{(dx+c)}+a)^n\tan{(dx+c)}^3\,dx$$

input
```
integrate(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \tan(c+d\,x)^3\,(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})^n\,dx$$

input
```
int(tan(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(tan(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \tan^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \left(\int (\tan(dx+c)\,ai+a)^n\tan(dx+c)^4\,dx\right)b + \left(\int (\tan(dx+c)\,ai+a)^n\tan(dx+c)^3\,dx\right)a$$

input
```
int(tan(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*tan(c + d*x)**4,x)*b + int((tan(c + d*x)*a*i
 + a)**n*tan(c + d*x)**3,x)*a
```

3.220 $\int \tan^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 164

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx=-\frac{2B(a+ia\tan(c+dx))^n}{dn(2+n)}+\frac{(iA+B)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^n}{2dn}+\frac{B\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(2+n)}-\frac{(Bn+iA(2+n))(a+ia\tan(c+dx))^{1+n}}{ad(1+n)(2+n)}$$

output
```
-2*B*(a+I*a*tan(d*x+c))^n/d/n/(2+n)+1/2*(I*A+B)*hypergeom([1, n],[1+n],1/2
+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/n+B*tan(d*x+c)^2*(a+I*a*tan(d*x+
c))^n/d/(2+n)-(B*n+I*A*(2+n))*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)/(2+n)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{(a+ia\tan(c+dx))^n\left(-4B-4iAn-4Bn-2iAn^2-2Bn^2+(iA+B)\left(2+3n+n^2\right)\text{Hypergeometric}\right.}{2dn(1+}$$

input
```
Integrate[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((a + I*a*Tan[c + d*x])^n*(-4*B - (4*I)*A*n - 4*B*n - (2*I)*A*n^2 - 2*B*n^
2 + (I*A + B)*(2 + 3*n + n^2)*Hypergeometric2F1[1, n, 1 + n, (1 + I*Tan[c
+ d*x])/2] + 2*n*((-I)*B*n + A*(2 + n))*Tan[c + d*x] + 2*B*n*(1 + n)*Tan[c
 + d*x]^2))/(2*d*n*(1 + n)*(2 + n))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 4080, 25, 3042, 4075, 3042, 4010, 3042, 3962, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)^2(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4080$$

$$\frac{\int -\tan(c+dx)(i\tan(c+dx)a+a)^n(2aB+a(iBn-A(n+2))\tan(c+dx))dx}{a(n+2)}+\frac{B\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}$$

$$\Big\downarrow \; 25$$

$$\frac{B\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}-\frac{\int \tan(c+dx)(i\tan(c+dx)a+a)^n(2aB+a(iBn-A(n+2))\tan(c+dx))dx}{a(n+2)}$$

$\big\downarrow$ 3042

$$\frac{B\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}-\frac{\int \tan(c+dx)(i\tan(c+dx)a+a)^n(2aB+a(iBn-A(n+2))\tan(c+dx))dx}{a(n+2)}$$

$\big\downarrow$ 4075

$$\frac{B\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}-\frac{\int (i\tan(c+dx)a+a)^n(2aB\tan(c+dx)-a(iBn-A(n+2)))dx+\frac{(Bn+iA(n+2))(a+ia\tan(c+dx))^{n+1}}{d(n+1)}}{a(n+2)}$$

$\big\downarrow$ 3042

$$\frac{B\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}-\frac{\int (i\tan(c+dx)a+a)^n(2aB\tan(c+dx)-a(iBn-A(n+2)))dx+\frac{(Bn+iA(n+2))(a+ia\tan(c+dx))^{n+1}}{d(n+1)}}{a(n+2)}$$

$\big\downarrow$ 4010

$$\frac{B\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}-\frac{a(n+2)(A-iB)\int (i\tan(c+dx)a+a)^n dx+\frac{(Bn+iA(n+2))(a+ia\tan(c+dx))^{n+1}}{d(n+1)}+\frac{2aB(a+ia\tan(c+dx))^n}{dn}}{a(n+2)}$$

$\big\downarrow$ 3042

$$\frac{B\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}-\frac{a(n+2)(A-iB)\int (i\tan(c+dx)a+a)^n dx+\frac{(Bn+iA(n+2))(a+ia\tan(c+dx))^{n+1}}{d(n+1)}+\frac{2aB(a+ia\tan(c+dx))^n}{dn}}{a(n+2)}$$

$\big\downarrow$ 3962

$$\frac{B\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)}-\frac{-\frac{ia^2(n+2)(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a-ia\tan(c+dx)}d(ia\tan(c+dx))}{d}+\frac{(Bn+iA(n+2))(a+ia\tan(c+dx))^{n+1}}{d(n+1)}+\frac{2aB(a+ia\tan(c+dx))^n}{dn}}{a(n+2)}$$

$\Big\downarrow$ 78

$$\frac{B\tan^2(c+dx)(a+ia\tan(c+dx))^n}{d(n+2)} - \frac{-\frac{ia(n+2)(A-iB)(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{2dn} + \frac{(Bn+iA(n+2))(a+ia\tan(c+dx))^{n+1}}{d(n+1)} + \frac{2aB(a+ia\,\mathrm{t}}{d}}{a(n+2)}$$

input
```
Int[Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]^2*(a + I*a*Tan[c + d*x])^n)/(d*(2 + n)) - ((2*a*B*(a + I*a
*Tan[c + d*x])^n)/(d*n) - ((I/2)*a*(A - I*B)*(2 + n)*Hypergeometric2F1[1,
n, 1 + n, (a + I*a*Tan[c + d*x])/(2*a)]*(a + I*a*Tan[c + d*x])^n)/(d*n) +
((B*n + I*A*(2 + n))*(a + I*a*Tan[c + d*x])^(1 + n))/(d*(1 + n)))/(a*(2 +
n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

Maple [F]

$$\int \tan (dx+c)^2 (a+ia\tan (dx+c))^n (A+B\tan (dx+c))\, dx$$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)^2\,dx$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral(-((A - I*B)*e^(6*I*d*x + 6*I*c) - (A - 3*I*B)*e^(4*I*d*x + 4*I*c)
 - (A + 3*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^
(2*I*d*x + 2*I*c) + 1))^n/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3
*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))\tan^2(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**2*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*tan(c + d*x)**2,
 x)
```

Maxima [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)^2\,dx$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Giac [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)^2\,dx$$

input
```
integrate(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^2\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \tan^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\left(\int (\tan(dx+c)\,ai+a)^n\tan(dx+c)^3\,dx\right)b$$
$$+\left(\int (\tan(dx+c)\,ai+a)^n\tan(dx+c)^2\,dx\right)a$$

input
```
int(tan(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*tan(c + d*x)**3,x)*b + int((tan(c + d*x)*a*i
 + a)**n*tan(c + d*x)**2,x)*a
```

3.221 $\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 111

$$\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{A(a+ia\tan(c+dx))^n}{dn} - \frac{(A-iB)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^n}{2dn} - \frac{iB(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}$$

output
```
A*(a+I*a*tan(d*x+c))^n/d/n-1/2*(A-I*B)*hypergeom([1, n],[1+n],1/2+1/2*I*ta
n(d*x+c))*(a+I*a*tan(d*x+c))^n/d/n-I*B*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{(a+ia\tan(c+dx))^n\left(-\left((A-iB)(1+n)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\tan(c+dx))\right)\right)+2(\right.}{2dn(1+n)}$$

input
```
Integrate[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((a + I*a*Tan[c + d*x])^n*(-((A - I*B)*(1 + n)*Hypergeometric2F1[1, n, 1 +
 n, (1 + I*Tan[c + d*x])/2]) + 2*(A + A*n - I*B*n + B*n*Tan[c + d*x])))/(2
*d*n*(1 + n))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 4075, 3042, 4010, 3042, 3962, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int (i\tan(c+dx)a+a)^n(A\tan(c+dx)-B)dx-\frac{iB(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

$$\Big\downarrow \text{3042}$$

$$\int (i\tan(c+dx)a+a)^n(A\tan(c+dx)-B)dx-\frac{iB(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

$$\Big\downarrow \text{4010}$$

$$-(B+iA)\int (i\tan(c+dx)a+a)^n dx+\frac{A(a+ia\tan(c+dx))^n}{dn}-\frac{iB(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

$$\Big\downarrow \text{3042}$$

$$-(B+iA)\int (i\tan(c+dx)a+a)^n dx+\frac{A(a+ia\tan(c+dx))^n}{dn}-\frac{iB(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

$$\Big\downarrow \text{3962}$$

$$\frac{ia(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a-ia\tan(c+dx)}d(ia\tan(c+dx))}{d}+\frac{A(a+ia\tan(c+dx))^n}{dn}-\frac{iB(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

$$\Big\downarrow \text{78}$$

$$\frac{i(B+iA)(a+ia\tan(c+dx))^n\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{2dn}+\frac{A(a+ia\tan(c+dx))^n}{dn}-\frac{iB(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

input
```
Int[Tan[c + d*x]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(A*(a + I*a*Tan[c + d*x])^n)/(d*n) + ((I/2)*(I*A + B)*Hypergeometric2F1[1,
 n, 1 + n, (a + I*a*Tan[c + d*x])/(2*a)]*(a + I*a*Tan[c + d*x])^n)/(d*n) -
 (I*B*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [F]

$$\int \tan(dx+c)(a+ia\tan(dx+c))^n(A+B\tan(dx+c))\,dx$$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)\,dx$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral(((-I*A - B)*e^(4*I*d*x + 4*I*c) + 2*B*e^(2*I*d*x + 2*I*c) + I*A -
 B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(e^(4*I*d*x + 4*
I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))\tan(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*tan(c + d*x), x)
```

Maxima [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)\;dx$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c), x)
```

Giac [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)\;dx$$

input
```
integrate(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)\,\left(A+B\tan(c+d\,x)\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \tan(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\left(\int \left(\tan\left(dx+c\right)ai+a\right)^n\tan\left(dx+c\right)^2dx\right)b$$
$$+\left(\int \left(\tan\left(dx+c\right)ai+a\right)^n\tan\left(dx+c\right)dx\right)a$$

input
```
int(tan(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*tan(c + d*x)**2,x)*b + int((tan(c + d*x)*a*i
 + a)**n*tan(c + d*x),x)*a
```

3.222 $\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 78

$$\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{B(a+ia\tan(c+dx))^n}{dn} - \frac{(iA+B)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^n}{2dn}$$

output
```
B*(a+I*a*tan(d*x+c))^n/d/n-1/2*(I*A+B)*hypergeom([1, n],[1+n],1/2+1/2*I*ta
n(d*x+c))*(a+I*a*tan(d*x+c))^n/d/n
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{\left(2B-(iA+B)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\tan(c+dx))\right)\right)(a+ia\tan(c+dx))^n}{2dn}$$

input
```
Integrate[(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((2*B - (I*A + B)*Hypergeometric2F1[1, n, 1 + n, (1 + I*Tan[c + d*x])/2])*
(a + I*a*Tan[c + d*x])^n)/(2*d*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4010, 3042, 3962, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4010}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^ndx+\frac{B(a+ia\tan(c+dx))^n}{dn}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\int (i\tan(c+dx)a+a)^ndx+\frac{B(a+ia\tan(c+dx))^n}{dn}$$

$$\Big\downarrow \text{3962}$$

$$\frac{B(a+ia\tan(c+dx))^n}{dn}-\frac{ia(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a-ia\tan(c+dx)}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$\frac{B(a+ia\tan(c+dx))^n}{dn}-\frac{i(A-iB)(a+ia\tan(c+dx))^n\text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{2dn}$$

input
```
Int[(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(B*(a + I*a*Tan[c + d*x])^n)/(d*n) - ((I/2)*(A - I*B)*Hypergeometric2F1[1,
 n, 1 + n, (a + I*a*Tan[c + d*x])/(2*a)]*(a + I*a*Tan[c + d*x])^n)/(d*n)
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [F]

$$\int (a + ia\tan(dx + c))^n (A + B\tan(dx + c))\, dx$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c
)/(e^(2*I*d*x + 2*I*c) + 1))^n/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x)), x)
```

Maxima [F]

$$\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int (a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{-(\tan(dx+c)\,ai+a)^n\,ai+\left(\int(\tan(dx+c)\,ai+a)^n\tan(dx+c)\,dx\right)adin+\left(\int(\tan(dx+c)\,ai+a)^n\right.}{dn}$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
( - (tan(c + d*x)*a*i + a)**n*a*i + int((tan(c + d*x)*a*i + a)**n*tan(c +
d*x),x)*a*d*i*n + int((tan(c + d*x)*a*i + a)**n*tan(c + d*x),x)*b*d*n)/(d*
n)
```

3.223 $\int \cot(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 97

$$\int \cot(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{(A-iB)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^n}{2dn} - \frac{A\,\text{Hypergeometric2F1}(1,n,1+n,1+i\tan(c+dx))(a+ia\tan(c+dx))^n}{dn}$$

output

```
1/2*(A-I*B)*hypergeom([1, n],[1+n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c)
)^n/d/n-A*hypergeom([1, n],[1+n],1+I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/n
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int \cot(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{(a+ia\tan(c+dx))^n\left((iA+B)n\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{1}{2}(1+i\tan(c+dx))\right)(-i+\tan(\right.}{}$$

input
```
Integrate[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((a + I*a*Tan[c + d*x])^n*((I*A + B)*n*Hypergeometric2F1[1, 1 + n, 2 + n,
(1 + I*Tan[c + d*x])/2]*(-I + Tan[c + d*x]) - (2*I)*(((-I)*A + B)*(1 + n)
+ 2*A*n*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + I*Tan[c + d*x]]*(-I + Tan[c
 + d*x]))))/(4*d*n*(1 + n))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 4083, 3042, 3962, 78, 4082, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4083}$$

$$(B+iA)\int (i\tan(c+dx)a+a)^ndx+\frac{A\int \cot(c+dx)(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^ndx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(B+iA)\int (i\tan(c+dx)a+a)^ndx+\frac{A\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\tan(c+dx)}dx}{a}$$

$$\Big\downarrow \text{3962}$$

$$\frac{A\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\tan(c+dx)}dx}{a}-\frac{ia(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a-ia\tan(c+dx)}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{78}$$

$$\frac{A\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\tan(c+dx)}dx}{a}-\frac{i(B+iA)(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{2dn}$$

$\Big\downarrow$ 4082

$$\frac{aA\int \cot(c+dx)(i\tan(c+dx)a+a)^{n-1}d\tan(c+dx)}{d}-\frac{i(B+iA)(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{2dn}$$

$\Big\downarrow$ 75

$$-\frac{i(B+iA)(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{2dn}-\frac{A(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}(1,n,n+1,i\tan(c+dx)+1)}{dn}$$

input
```
Int[Cot[c + d*x]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
-((A*Hypergeometric2F1[1, n, 1 + n, 1 + I*Tan[c + d*x]]*(a + I*a*Tan[c + d
*x])^n)/(d*n)) - ((I/2)*(I*A + B)*Hypergeometric2F1[1, n, 1 + n, (a + I*a*
Tan[c + d*x])/(2*a)]*(a + I*a*Tan[c + d*x])^n)/(d*n)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \cot (dx+c) (a+ia \tan (dx+c))^n (A+B \tan (dx+c)) \, dx$$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral(((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A - B)*(2*a*e^(2*I*d*x + 2*I*c
)/(e^(2*I*d*x + 2*I*c) + 1))^n/(e^(2*I*d*x + 2*I*c) - 1), x)
```

Sympy [F]

$$\int \cot(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))\cot(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*cot(c + d*x), x)
```

Maxima [F]

$$\int \cot(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c), x)
```

Giac [F]

$$\int \cot(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)\;dx$$

input
```
integrate(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)\;(A+B\tan(c+d\,x))\;(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cot(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \left(\int (\tan(dx+c)\,ai+a)^n \cot(dx+c)\tan(dx+c)\,dx\right)b + \left(\int (\tan(dx+c)\,ai+a)^n \cot(dx+c)\,dx\right)a$$

input
```
int(cot(d*x+c)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*cot(c + d*x)*tan(c + d*x),x)*b + int((tan(c
+ d*x)*a*i + a)**n*cot(c + d*x),x)*a
```

3.224 $\int \cot^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 131

$$\begin{aligned}\int \cot^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx &= -\frac{A\cot(c+dx)(a+ia\tan(c+dx))^n}{d}\\ &+\frac{(iA+B)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^n}{2dn}\\ &-\frac{(B+iAn)\,\text{Hypergeometric2F1}(1,n,1+n,1+i\tan(c+dx))(a+ia\tan(c+dx))^n}{dn}\end{aligned}$$

output
```
-A*cot(d*x+c)*(a+I*a*tan(d*x+c))^n/d+1/2*(I*A+B)*hypergeom([1, n],[1+n],1/
2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/n-(B+I*A*n)*hypergeom([1, n],[1
+n],1+I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/n
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.19

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx =$$
$$-\frac{(a+ia\tan(c+dx))^n\left(2(-iA+B)(1+n)+4Bn\,\text{Hypergeometric2F1}(1,1+n,2+n,1+i\tan(c+d$$

input
```
Integrate[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
-1/4*((a + I*a*Tan[c + d*x])^n*(2*((-I)*A + B)*(1 + n) + 4*B*n*Hypergeomet
ric2F1[1, 1 + n, 2 + n, 1 + I*Tan[c + d*x]]*(1 + I*Tan[c + d*x]) + (A - I*
B)*n*Hypergeometric2F1[1, 1 + n, 2 + n, (1 + I*Tan[c + d*x])/2]*(-I + Tan[
c + d*x]) + 4*A*n*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + I*Tan[c + d*x]]*(
-I + Tan[c + d*x])))/(d*n*(1 + n))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4081, 3042, 4083, 3042, 3962, 78, 4082, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4081}$$

$$\frac{\int \cot(c+dx)(i\tan(c+dx)a+a)^n(a(B+iAn)-aA(1-n)\tan(c+dx))dx}{a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^n}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(i\tan(c+dx)a+a)^n(a(B+iAn)-aA(1-n)\tan(c+dx))}{\tan(c+dx)}dx}{a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^n}{d}$$

$$\Big\downarrow \text{4083}$$

$$\frac{(B+iAn)\int \cot(c+dx)(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^ndx-a(A-iB)\int (i\tan(c+dx)a+a)^ndx}{a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^n}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(B+iAn)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\tan(c+dx)}dx-a(A-iB)\int (i\tan(c+dx)a+a)^ndx}{a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^n}{d}$$

$$\Big\downarrow \text{3962}$$

$$\frac{\frac{ia^2(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a-ia\tan(c+dx)}d(ia\tan(c+dx))}{d}+(B+iAn)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\tan(c+dx)}dx}{a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^n}{d}$$

$$\Big\downarrow \text{78}$$

$$\frac{(B+iAn)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\tan(c+dx)}dx+\frac{ia(A-iB)(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{2dn}}{a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^n}{d}$$

$$\Big\downarrow \text{4082}$$

$$\frac{\frac{a^2(B+iAn)\int \cot(c+dx)(i\tan(c+dx)a+a)^{n-1}d\tan(c+dx)}{d}+\frac{ia(A-iB)(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{2dn}}{a}-\frac{A\cot(c+dx)(a+ia\tan(c+dx))^n}{d}$$

$$\Big\downarrow \text{75}$$

$$\frac{\frac{ia(A-iB)(a+ia\tan(c+dx))^n \operatorname{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{2dn} - \frac{a(B+iAn)(a+ia\tan(c+dx))^n \operatorname{Hypergeometric2F1}(1,n,n+1,i\tan(c+dx)+1)}{dn}}{a}$$
$$\frac{A\cot(c+dx)(a+ia\tan(c+dx))^n}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
-((A*Cot[c + d*x]*(a + I*a*Tan[c + d*x])^n)/d) + (-((a*(B + I*A*n)*Hyperge
ometric2F1[1, n, 1 + n, 1 + I*Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*n
)) + ((I/2)*a*(A - I*B)*Hypergeometric2F1[1, n, 1 + n, (a + I*a*Tan[c + d*
x])/(2*a)]*(a + I*a*Tan[c + d*x])^n)/(d*n))/a
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \cot (dx + c)^2 (a + ia \tan (dx + c))^n (A + B \tan (dx + c))\, dx$$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral(-((A - I*B)*e^(4*I*d*x + 4*I*c) + 2*A*e^(2*I*d*x + 2*I*c) + A + I
*B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(e^(4*I*d*x + 4*
I*c) - 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))\cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*cot(c + d*x)**2,
 x)
```

Maxima [F]

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Giac [F]

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int \cot(c+d\,x)^2\,(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})^n\,dx$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cot^2(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\left(\int (\tan(dx+c)\,ai+a)^n\cot(dx+c)^2\tan(dx+c)\,dx\right)b$$
$$+\left(\int (\tan(dx+c)\,ai+a)^n\cot(dx+c)^2\,dx\right)a$$

input
```
int(cot(d*x+c)^2*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*cot(c + d*x)**2*tan(c + d*x),x)*b + int((tan
(c + d*x)*a*i + a)**n*cot(c + d*x)**2,x)*a
```

3.225 $\int \cot^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 185

$$\begin{aligned}
&\int \cot^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx \\
&= -\frac{(2B+iAn)\cot(c+dx)(a+ia\tan(c+dx))^n}{2d} - \frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d} \\
&\quad - \frac{(A-iB)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^n}{2dn} \\
&\quad - \frac{(2iBn-A(2-n+n^2))\,\text{Hypergeometric2F1}(1,n,1+n,1+i\tan(c+dx))(a+ia\tan(c+dx))^n}{2dn}
\end{aligned}$$

output

```
-1/2*(2*B+I*A*n)*cot(d*x+c)*(a+I*a*tan(d*x+c))^n/d-1/2*A*cot(d*x+c)^2*(a+I
*a*tan(d*x+c))^n/d-1/2*(A-I*B)*hypergeom([1, n],[1+n],1/2+1/2*I*tan(d*x+c)
)*(a+I*a*tan(d*x+c))^n/d/n-1/2*(2*I*B*n-A*(n^2-n+2))*hypergeom([1, n],[1+n
],1+I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/n
```

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.19

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{(a+ia\tan(c+dx))^n\left(-i(A-iB)n\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\tfrac{1}{2}(1+i\tan(c+dx))\right)(-i+\text{ta}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((a + I*a*Tan[c + d*x])^n*((-I)*(A - I*B)*n*Hypergeometric2F1[1, 1 + n, 2
+ n, (1 + I*Tan[c + d*x])/2]*(-I + Tan[c + d*x]) + 2*(A + I*B + A*n + I*B*
n + 2*A*n*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + I*Tan[c + d*x]] + 2*A*n*H
ypergeometric2F1[1, 1 + n, 2 + n, 1 + I*Tan[c + d*x]]*(1 + I*Tan[c + d*x])
 + (2*I)*A*n*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + I*Tan[c + d*x]]*Tan[c
+ d*x] - 2*B*n*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + I*Tan[c + d*x]]*(-I
+ Tan[c + d*x]))))/(4*d*n*(1 + n))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4081, 3042, 4081, 3042, 4083, 3042, 3962, 78, 4082, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4081}$$

$$\frac{\int \cot^2(c+dx)(i\tan(c+dx)a+a)^n(a(2B+iAn)-aA(2-n)\tan(c+dx))dx}{2a}-\frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(i\tan(c+dx)a+a)^n(a(2B+iAn)-aA(2-n)\tan(c+dx))}{\tan(c+dx)^2}dx}{2a}-\frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d}$$

$\Big\downarrow$ 4081

$$\frac{\frac{\int \cot(c+dx)(i\tan(c+dx)a+a)^n\left(a^2\left(2iBn-A\left(n^2-n+2\right)\right)-a^2(1-n)(2B+iAn)\tan(c+dx)\right)dx}{a}-\frac{a(2B+iAn)\cot(c+dx)(a+ia\tan(c+dx))^n}{d}}{2a}-\frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{(i\tan(c+dx)a+a)^n\left(a^2\left(2iBn-A\left(n^2-n+2\right)\right)-a^2(1-n)(2B+iAn)\tan(c+dx)\right)}{\tan(c+dx)}dx}{a}-\frac{a(2B+iAn)\cot(c+dx)(a+ia\tan(c+dx))^n}{d}}{2a}-\frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d}$$

$\Big\downarrow$ 4083

$$\frac{\frac{a\left(-A\left(n^2-n+2\right)+2iBn\right)\int \cot(c+dx)(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^ndx-2a^2(B+iA)\int (i\tan(c+dx)a+a)^ndx}{a}-\frac{a(2B+iAn)\cot(c+dx)(a}{d}}{2a}-\frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a\left(-A\left(n^2-n+2\right)+2iBn\right)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\tan(c+dx)}dx-2a^2(B+iA)\int (i\tan(c+dx)a+a)^ndx}{a}-\frac{a(2B+iAn)\cot(c+dx)(a+ia\tan(c+da}{d}}{2a}-\frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d}$$

$\Big\downarrow$ 3962

$$\frac{\frac{\frac{2ia^3(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a-ia\tan(c+dx)}d(ia\tan(c+dx))}{d}+a\left(-A\left(n^2-n+2\right)+2iBn\right)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\tan(c+dx)}dx}{a}-\frac{a(2B+iAn)\cot(c+dx)(a-}{d}}{2a}-\frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d}$$

$\big\downarrow$ 78

$$\frac{\frac{a\left(-A(n^2-n+2)+2iBn\right)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\tan(c+dx)}dx+\frac{ia^2(B+iA)(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{dn}}{a} - \frac{a(2}{}}{2a}$$

$$\frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d}$$

$\big\downarrow$ 4082

$$\frac{\frac{\frac{a^3\left(-A\left(n^2-n+2\right)+2iBn\right)\int \cot(c+dx)(i\tan(c+dx)a+a)^{n-1}d\tan(c+dx)}{d}+\frac{ia^2(B+iA)(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{dn}}{a} -}{2a}$$

$$\frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d}$$

$\big\downarrow$ 75

$$\frac{\frac{\frac{ia^2(B+iA)(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{i\tan(c+dx)a+a}{2a}\right)}{dn}-\frac{a^2\left(-A\left(n^2-n+2\right)+2iBn\right)(a+ia\tan(c+dx))^n \text{Hypergeometric2F1}(1,n,n+1,i\tan}{dn}}{a}}{2a}$$

$$\frac{A\cot^2(c+dx)(a+ia\tan(c+dx))^n}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(A*Cot[c + d*x]^2*(a + I*a*Tan[c + d*x])^n)/d + (-((a*(2*B + I*A*n)*C
ot[c + d*x]*(a + I*a*Tan[c + d*x])^n)/d) + (-((a^2*((2*I)*B*n - A*(2 - n +
 n^2))*Hypergeometric2F1[1, n, 1 + n, 1 + I*Tan[c + d*x]]*(a + I*a*Tan[c +
 d*x])^n)/(d*n)) + (I*a^2*(I*A + B)*Hypergeometric2F1[1, n, 1 + n, (a + I*
a*Tan[c + d*x])/(2*a)]*(a + I*a*Tan[c + d*x])^n)/(d*n))/a)/(2*a)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3962
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-b/d   S
ubst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4083
```
Int[(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(
A*b + a*B)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m, x], x] - Simp[(B*c - A
*d)/(b*c + a*d)   Int[(a + b*Tan[e + f*x])^m*((a - b*Tan[e + f*x])/(c + d*T
an[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c -
 a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \cot(dx+c)^3 (a+ia\tan(dx+c))^n (A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral(((-I*A - B)*e^(6*I*d*x + 6*I*c) + (-3*I*A - B)*e^(4*I*d*x + 4*I*c
) + (-3*I*A + B)*e^(2*I*d*x + 2*I*c) - I*A + B)*(2*a*e^(2*I*d*x + 2*I*c)/(
e^(2*I*d*x + 2*I*c) + 1))^n/(e^(6*I*d*x + 6*I*c) - 3*e^(4*I*d*x + 4*I*c) +
 3*e^(2*I*d*x + 2*I*c) - 1), x)
```

Sympy [F]

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))\cot^3(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**3*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*cot(c + d*x)**3,
 x)
```

Maxima [F]

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Giac [F]

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int \cot(c+d\,x)^3\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cot^3(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \left(\int (\tan(dx+c)\,ai+a)^n\cot(dx+c)^3\tan(dx+c)\,dx\right)b$$
$$+ \left(\int (\tan(dx+c)\,ai+a)^n\cot(dx+c)^3\,dx\right)a$$

input
```
int(cot(d*x+c)^3*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*cot(c + d*x)**3*tan(c + d*x),x)*b + int((tan
(c + d*x)*a*i + a)**n*cot(c + d*x)**3,x)*a
```

3.226 $\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 383

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$= -\frac{2(2iAn(5+2n)+B(15+10n+4n^2))\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(1+2n)(3+2n)(5+2n)}$$

$$+\frac{2(iA+B)\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}}{d}$$

$$-\frac{2(4Bn(9+8n+2n^2)+iA(15+36n+32n^2+8n^3))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)}{d(1+2n)(3+2n)(5+2n)}$$

$$-\frac{2(2iBn-A(5+2n))\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(3+2n)(5+2n)}$$

$$+\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(5+2n)}$$

output
```
-2*(2*I*A*n*(5+2*n)+B*(4*n^2+10*n+15))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))
^n/d/(1+2*n)/(3+2*n)/(5+2*n)+2*(I*A+B)*AppellF1(1/2,1-n,1,3/2,-I*tan(d*x+c
),I*tan(d*x+c))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/((1+I*tan(d*x+c))^
n)-2*(4*B*n*(2*n^2+8*n+9)+I*A*(8*n^3+32*n^2+36*n+15))*hypergeom([1/2, 1-n]
,[3/2],-I*tan(d*x+c))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/(1+2*n)/(3+2
*n)/(5+2*n)/((1+I*tan(d*x+c))^n)-2*(2*I*B*n-A*(5+2*n))*tan(d*x+c)^(3/2)*(a
+I*a*tan(d*x+c))^n/d/(3+2*n)/(5+2*n)+2*B*tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c
))^n/d/(5+2*n)
```

Mathematica [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
,x]
```

output
```
Integrate[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
, x]
```

Rubi [A] (warning: unable to verify)

Time = 2.04 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.10, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3042, 4080, 27, 3042, 4080, 27, 3042, 4080, 27, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{5/2}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4080$$

$$\frac{2\int -\frac{1}{2}\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^n(5aB+a(2iBn-A(2n+5))\tan(c+dx))dx}{a(2n+5)}+\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\int \tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^n(5aB+a(2iBn-A(2n+5))\tan(c+dx))dx}{a(2n+5)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\int \tan(c+dx)^{3/2}(i\tan(c+dx)a+a)^n(5aB+a(2iBn-A(2n+5))\tan(c+dx))dx}{a(2n+5)}$$

$$\Big\downarrow \; 4080$$

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2\int -\frac{1}{2}\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n\left(3a^2(2iBn-A(2n+5))-a^2\left(2iAn(2n+5)+B\left(4n^2+10n+15\right)\right)\tan(c+dx)\right)dx}{a(2n+3)}+\frac{2a(-A(2n+5)+2iBn)\,\mathrm{ta}}{d}}{a(2n+5)}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n\left(3a^2(2iBn-A(2n+5))-a^2\left(2iAn(2n+5)+B\left(4n^2+\right.\right.\right.}{a(2n+3)}}{a(2n+5)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n\left(3a^2(2iBn-A(2n+5))-a^2\left(2iAn(2n+5)+B\left(4n^2+\right.\right.\right.}{a(2n+3)}}{a(2n+5)}$$

$$\Big\downarrow \; 4080$$

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)} - \frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)} - \frac{2\int \frac{(i\tan(c+dx)a+a)^n\left(\left(2iAn(2n+5)+B\left(4n^2+10n+15\right)\right)a^3+\left(4iBn\left(2n^2+8n+9\right)-A\left(8n^3+3\cdots\right.\right.\right.}{2\sqrt{\tan(c+dx)}}}{a(2n+1)}\cdots}{a(2n+5)}$$

$\downarrow$ 27

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)} - \frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)} - \frac{\int \frac{(i\tan(c+dx)a+a)^n\left(\left(2iAn(2n+5)+B\left(4n^2+10n+15\right)\right)a^3+\left(4iBn\left(2n^2+8n+9\right)-A\left(8n^3+32\cdots\right.\right.\right.}{\sqrt{\tan(c+dx)}}}{a(2n+1)}\cdots}{a(2n+5)}$$

$\downarrow$ 3042

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)} - \frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)} - \frac{\int \frac{(i\tan(c+dx)a+a)^n\left(\left(2iAn(2n+5)+B\left(4n^2+10n+15\right)\right)a^3+\left(4iBn\left(2n^2+8n+9\right)-A\left(8n^3+32\cdots\right.\right.\right.}{\sqrt{\tan(c+dx)}}}{a(2n+1)}\cdots}{a(2n+5)}$$

$\downarrow$ 4084

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)} - \frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)} - \frac{a^3\left(8n^3+36n^2+46n+15\right)(B+iA)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx - a^2\left(4Bn\left(2n^2+8n+9\right)+iA\left(8n^3+32\cdots\right.\right.}{a(2n+1)}\cdots}{a(2n+5)}$$

$\downarrow$ 3042

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)} - \frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)} - \frac{a^3\left(8n^3+36n^2+46n+15\right)(B+iA)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx - a^2\left(4Bn\left(2n^2+8n+9\right)+iA\left(8n^3+32\cdots\right.\right.}{a(2n+1)}\cdots}{a(2n+5)}$$

$\downarrow$ 4047

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)} - \frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)} - \frac{\frac{\frac{ia^5(8n^3+36n^2+46n+15)(B+iA)\int -\frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d} - a^2(4B}{a(2n+1)}}{}}{a(2n}$$

$\downarrow$ 25

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)} - \frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)} - \frac{-\frac{ia^5(8n^3+36n^2+46n+15)(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d} - a^2(4B}{a(2n+1)}}{a(2n}$$

$\downarrow$ 27

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)} - \frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)} - \frac{-\frac{ia^4(8n^3+36n^2+46n+15)(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d} - a^2(4Bn}{a(2n+1)}}{a(2n}$$

$\downarrow$ 148

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)} - \frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)} - \frac{\frac{2a^5(8n^3+36n^2+46n+15)(B+iA)\int \frac{(a-ia^3\tan^2(c+dx))^{n-1}}{a(ia^2\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{d} - a^2(4Bn(2n^2+8}{a(2n+1)}}{a(2n+5}$$

$\downarrow$ 27

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)} - \frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)} - \frac{\frac{2a^4(8n^3+36n^2+46n+15)(B+iA)\int \frac{(a-ia^3\tan^2(c+dx))^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d} - a^2(4Bn(2n^2+8}{a(2n+1)}}{a(2n+5}$$

$\downarrow$ 334

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-$$

$$\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{2a^3\left(8n^3+36n^2+46n+15\right)(B+iA)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\int\frac{\left(1-ia^2\tan\right.}{ia^2\tan}}{d}$$

$\downarrow$ 333

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-$$

$$\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{2ia^4\left(8n^3+36n^2+46n+15\right)(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n A}{d}$$

$\downarrow$ 4082

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-$$

$$\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{2ia^4\left(8n^3+36n^2+46n+15\right)(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n A}{d}$$

$\downarrow$ 76

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-$$

$$\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{2ia^4\left(8n^3+36n^2+46n+15\right)(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n A}{d}$$

$\downarrow$ 74

$$\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-$$

$$\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{2ia^4\left(8n^3+36n^2+46n+15\right)(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n A}{d}$$

input
```
Int[Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*Tan[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^n)/(d*(5 + 2*n)) - ((2*a*((
2*I)*B*n - A*(5 + 2*n))*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n)/(d*(3
 + 2*n)) - ((-2*a^2*((2*I)*A*n*(5 + 2*n) + B*(15 + 10*n + 4*n^2))*Sqrt[Tan
[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 2*n)) + ((-2*a^3*(4*B*n*(9 +
8*n + 2*n^2) + I*A*(15 + 36*n + 32*n^2 + 8*n^3))*Hypergeometric2F1[1/2, 1
- n, 3/2, (-I)*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/
(d*(1 + I*Tan[c + d*x])^n) + ((2*I)*a^4*(I*A + B)*(15 + 46*n + 36*n^2 + 8*
n^3)*AppellF1[1/2, 1, 1 - n, 3/2, (-I)*a^2*Tan[c + d*x]^2, I*a^2*Tan[c + d
*x]^2]*Tan[c + d*x]*(a - I*a^3*Tan[c + d*x]^2)^n)/(d*(1 - I*a^2*Tan[c + d*
x]^2)^n))/(a*(1 + 2*n)))/(a*(3 + 2*n)))/(a*(5 + 2*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \tan\left(dx+c\right)^{\frac{5}{2}}\left(a+ia\tan\left(dx+c\right)\right)^{n}\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{n}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan\left(dx+c\right)+A)(i\,a\tan\left(dx+c\right)+a)^{n}\tan\left(dx+c\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
integral(-((A - I*B)*e^(6*I*d*x + 6*I*c) - (A - 3*I*B)*e^(4*I*d*x + 4*I*c)
 - (A + 3*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^
(2*I*d*x + 2*I*c) + 1))^n*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) + 1))/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x +
 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^(5/2)
, x)
```

Giac [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^(5/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int(tan(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(tan(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\left(\int \sqrt{\tan(dx+c)}\,(\tan(dx+c)\,ai+a)^n\tan(dx+c)^3\,dx\right)b$$
$$+\left(\int \sqrt{\tan(dx+c)}\,(\tan(dx+c)\,ai+a)^n\tan(dx+c)^2\,dx\right)a$$

input
```
int(tan(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x)**3,x)*b + in
t(sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x)**2,x)*a
```

3.227 $\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 291

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= -\frac{2(2iBn-A(3+2n))\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(1+2n)(3+2n)}$$
$$-\frac{2(A-iB)\,\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}}{d}$$
$$+\frac{2(2An(3+2n)-iB(3+6n+4n^2))\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)(1+i\tan(c+dx)}{d(1+2n)(3+2n)}$$
$$+\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(3+2n)}$$

output
```
-2*(2*I*B*n-A*(3+2*n))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/(1+2*n)/(3+
2*n)-2*(A-I*B)*AppellF1(1/2,1-n,1,3/2,-I*tan(d*x+c),I*tan(d*x+c))*tan(d*x+
c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/((1+I*tan(d*x+c))^n)+2*(2*A*n*(3+2*n)-I*B*
(4*n^2+6*n+3))*hypergeom([1/2, 1-n],[3/2],-I*tan(d*x+c))*tan(d*x+c)^(1/2)*
(a+I*a*tan(d*x+c))^n/d/(1+2*n)/(3+2*n)/((1+I*tan(d*x+c))^n)+2*B*tan(d*x+c)
^(3/2)*(a+I*a*tan(d*x+c))^n/d/(3+2*n)
```

Mathematica [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
,x]
```

output
```
Integrate[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.40 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.528, Rules used = {3042, 4080, 27, 3042, 4080, 27, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4080}$$

$$\frac{2\int -\frac{1}{2}\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n(3aB+a(2iBn-A(2n+3))\tan(c+dx))dx}{a(2n+3)}+\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\int\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n(3aB+a(2iBn-A(2n+3))\tan(c+dx))dx}{a(2n+3)}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\int\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n(3aB+a(2iBn-A(2n+3))\tan(c+dx))dx}{a(2n+3)}$$

$\Big\downarrow$ 4080

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2\int-\frac{(i\tan(c+dx)a+a)^n\left(a^2(2iBn-A(2n+3))-a^2\left(2iAn(2n+3)+B\left(4n^2+6n+3\right)\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}}dx}{a(2n+1)}+\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{a(2n+3)}$$

$\Big\downarrow$ 27

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\int\frac{(i\tan(c+dx)a+a)^n\left(a^2(2iBn-A(2n+3))-a^2\left(2iAn(2n+3)+B\left(4n^2+6n+3\right)\right)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}}{a(2n+3)}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\int\frac{(i\tan(c+dx)a+a)^n\left(a^2(2iBn-A(2n+3))-a^2\left(2iAn(2n+3)+B\left(4n^2+6n+3\right)\right)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}}{a(2n+3)}$$

$\Big\downarrow$ 4084

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a\left(2An(2n+3)-iB\left(4n^2+6n+3\right)\right)\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-a^2\left(4n^2+8n\right.}{a(2n+1)}}{a(2n+3)}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-$$

$$\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a\left(2An(2n+3)-iB\left(4n^2+6n+3\right)\right)\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-a^2\left(4n^2+8n\right.}{a(2n+1)}}{a(2n+3)}$$

$\big\downarrow$ 4047

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-$$

$$\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a\left(2An(2n+3)-iB\left(4n^2+6n+3\right)\right)\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-\frac{ia^4\left(4n^2+8n\right.}{}}{a(2n+1)}}{a(2n+3)}$$

$\big\downarrow$ 25

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-$$

$$\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{ia^4\left(4n^2+8n+3\right)(A-iB)\int\frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}+a(2An(2n+3)-}{a(2n+1)}}{a(2n+3)}$$

$\big\downarrow$ 27

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-$$

$$\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{ia^3\left(4n^2+8n+3\right)(A-iB)\int\frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}+a\left(2An(2n+3)-\right.}{a(2n+1)}}{a(2n+3)}$$

$\big\downarrow$ 148

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-$$

$$\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a\left(2An(2n+3)-iB\left(4n^2+6n+3\right)\right)\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-\frac{2a^4\left(4n^2+8\right.}{}}{a(2n+1)}}{a(2n+3)}$$

$\big\downarrow$ 27

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a\left(2An(2n+3)-iB\left(4n^2+6n+3\right)\right)\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3\left(4n^2+8\cdots\right.}{\cdots}}{a(2n+1)}}{a(2n+3)}$$

$\downarrow$ 334

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a\left(2An(2n+3)-iB\left(4n^2+6n+3\right)\right)\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2\left(4n^2+8\cdots\right.}{\cdots}}{a\cdots}}{a(2n+3)}$$

$\downarrow$ 333

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a\left(2An(2n+3)-iB\left(4n^2+6n+3\right)\right)\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-\frac{2ia^3\left(4n^2+8\cdots\right.}{\cdots}}{\cdots}}{a(2n+3)}$$

$\downarrow$ 4082

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{a^3\left(2An(2n+3)-iB\left(4n^2+6n+3\right)\right)\int\frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}}d\tan(c+dx)}{d}-\frac{2ia^3\left(4n^2+8n+3\right)(A\cdots}{\cdots}}{\cdots}}{a(2n+3)}$$

$\downarrow$ 76

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{a^2\left(2An(2n+3)-iB\left(4n^2+6n+3\right)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\int\frac{(i\tan(c+dx)+1)^{n\cdots}}{\sqrt{\tan(c+dx)}}\cdots}{d}\cdots}{\cdots}}{a(2n\cdots}$$

$\downarrow$ 74

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-$$

$$\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{2a^2\left(2An(2n+3)-iB\left(4n^2+6n+3\right)\right)\sqrt{\tan(c+dx)}(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\,\text{Hyperg}}{d}}{}$$

input
```
Int[Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*Tan[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n)/(d*(3 + 2*n)) - ((2*a*((
2*I)*B*n - A*(3 + 2*n))*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1
 + 2*n)) - ((2*a^2*(2*A*n*(3 + 2*n) - I*B*(3 + 6*n + 4*n^2))*Hypergeometri
c2F1[1/2, 1 - n, 3/2, (-I)*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c
 + d*x])^n)/(d*(1 + I*Tan[c + d*x])^n) - ((2*I)*a^3*(A - I*B)*(3 + 8*n + 4
*n^2)*AppellF1[1/2, 1, 1 - n, 3/2, (-I)*a^2*Tan[c + d*x]^2, I*a^2*Tan[c +
d*x]^2]*Tan[c + d*x]*(a - I*a^3*Tan[c + d*x]^2)^n)/(d*(1 - I*a^2*Tan[c + d
*x]^2)^n))/(a*(1 + 2*n)))/(a*(3 + 2*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \tan\left(dx+c\right)^{\frac{3}{2}} \left(a+ia\tan\left(dx+c\right)\right)^{n} \left(A+B\tan\left(dx+c\right)\right) dx$$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan\left(dx+c\right)+A)(i\,a\tan\left(dx+c\right)+a)^{n}\tan\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
integral(((-I*A - B)*e^(4*I*d*x + 4*I*c) + 2*B*e^(2*I*d*x + 2*I*c) + I*A -
 B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((-I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))/(e^(4*I*d*x + 4*I*c) + 2*e^(2
*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(ia\tan(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^(3/2)
, x)
```

Giac [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*tan(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \left(\int \sqrt{\tan(dx+c)}\,(\tan(dx+c)\,ai+a)^n\tan(dx+c)^2\,dx\right)b + \left(\int \sqrt{\tan(dx+c)}\,(\tan(dx+c)\,ai+a)^n\tan(dx+c)\,dx\right)a$$

input
```
int(tan(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x)**2,x)*b + in
t(sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x),x)*a
```

3.228 $\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 215

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(1+2n)}$$
$$- \frac{2(iA+B)\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}}{d}$$
$$+ \frac{2(2Bn+iA(1+2n))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}\sqrt{\tan(c}}{d(1+2n)}$$

output
```
2*B*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/(1+2*n)-2*(I*A+B)*AppellF1(1/2
,1-n,1,3/2,-I*tan(d*x+c),I*tan(d*x+c))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))
^n/d/((1+I*tan(d*x+c))^n)+2*(2*B*n+I*A*(1+2*n))*hypergeom([1/2, 1-n],[3/2]
,-I*tan(d*x+c))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/(1+2*n)/((1+I*tan(
d*x+c))^n)
```

Mathematica [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
,x]
```

output
```
Integrate[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.15, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4080, 27, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4080}$$

$$\frac{2\int -\frac{(i\tan(c+dx)a+a)^n(aB-a(2nA+A-2iBn)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx}{a(2n+1)}+\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\int \frac{(i\tan(c+dx)a+a)^n(aB-a(2nA+A-2iBn)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\int \frac{(i\tan(c+dx)a+a)^n(aB-a(2nA+A-2iBn)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}$$

$$\Big\downarrow \; 4084$$

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a(2n+1)(B+iA)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-(2Bn+iA(2n+1))\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a(2n+1)(B+iA)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-(2Bn+iA(2n+1))\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}$$

$$\Big\downarrow \; 4047$$

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{ia^3(2n+1)(B+iA)\int -\frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-(2Bn+iA(2n+1))\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}$$

$$\Big\downarrow \; 25$$

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{-\frac{ia^3(2n+1)(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-(2Bn+iA(2n+1))\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{-\frac{ia^2(2n+1)(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-(2Bn+iA(2n+1))\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}$$

$$\Big\downarrow \; 148$$

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{2a^3(2n+1)(B+iA)\int\frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{a\left(ia^2\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{d}-(2Bn+iA(2n+1))\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}$$

$\Big\downarrow$ 27

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{2a^2(2n+1)(B+iA)\int\frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-(2Bn+iA(2n+1))\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}$$

$\Big\downarrow$ 334

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{2a(2n+1)(B+iA)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\int\frac{\left(1-ia^2\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-(2Bn+iA(2n+1))\int\frac{(a-ia\tan}{}$$

$\Big\downarrow$ 333

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{2ia^2(2n+1)(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+dx),ia^2\tan^2(c+dx)\right)}{d}-(2Bn}{a(2n+1)}$$

$\Big\downarrow$ 4082

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{2ia^2(2n+1)(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+dx),ia^2\tan^2(c+dx)\right)}{d}-\frac{a^2(2B}{}}{a(2n+1)}$$

$\Big\downarrow$ 76

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{2ia^2(2n+1)(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+dx),ia^2\tan^2(c+dx)\right)}{d}-\frac{a(2Bn}{}}{a(2n+1)}$$

$\Big\downarrow$ 74

$$\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)} - \frac{\frac{2ia^2(2n+1)(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n \operatorname{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+dx),ia^2\tan^2(c+dx)\right)}{d} - \frac{2a(2B}{}}{a(2n+1)}$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 2*n)) - ((-2*a*(
2*B*n + I*A*(1 + 2*n))*Hypergeometric2F1[1/2, 1 - n, 3/2, (-I)*Tan[c + d*x
]]*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + I*Tan[c + d*x])^n)
 + ((2*I)*a^2*(I*A + B)*(1 + 2*n)*AppellF1[1/2, 1, 1 - n, 3/2, (-I)*a^2*Ta
n[c + d*x]^2, I*a^2*Tan[c + d*x]^2]*Tan[c + d*x]*(a - I*a^3*Tan[c + d*x]^2
)^n)/(d*(1 - I*a^2*Tan[c + d*x]^2)^n))/(a*(1 + 2*n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \sqrt{\tan (dx+c)}\,(a+ia\tan (dx+c))^n\,(A+B\tan (dx+c))\,dx$$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx \\ = \int (B\tan (dx+c)+A)(i\,a\tan (dx+c)+a)^n\sqrt{\tan (dx+c)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
integral(((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c
)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1))/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))\,\sqrt{\tan(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*sqrt(tan(c + d*x
)), x)
```

Maxima [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\sqrt{\tan(dx+c)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*sqrt(tan(d*x + c))
, x)
```

Giac [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\sqrt{\tan(dx+c)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*sqrt(tan(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \sqrt{\tan(c+d\,x)}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^n\,dx$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{-2\sqrt{\tan(dx+c)}\,(\tan(dx+c)\,ai+a)^n\,ai+\left(\int \frac{\sqrt{\tan(dx+c)}\,(\tan(dx+c)ai+a)^n}{\tan(dx+c)}dx\right)adi+2\left(\int \sqrt{\tan(dx+c)}\,(\right.}{}$$

input
```
int(tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
( - 2*sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n*a*i + int((sqrt(tan(c +
 d*x))*(tan(c + d*x)*a*i + a)**n)/tan(c + d*x),x)*a*d*i + 2*int(sqrt(tan(c
 + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x),x)*a*d*i*n + int(sqrt(tan(
c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x),x)*a*d*i + 2*int(sqrt(tan
(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x),x)*b*d*n)/(2*d*n)
```

3.229 $\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 158

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{2(A-iB)\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}(a}{d} + \frac{2iB\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}(a+ia\tan(}{d}$$

output
```
2*(A-I*B)*AppellF1(1/2,1-n,1,3/2,-I*tan(d*x+c),I*tan(d*x+c))*tan(d*x+c)^(1
/2)*(a+I*a*tan(d*x+c))^n/d/((1+I*tan(d*x+c))^n)+2*I*B*hypergeom([1/2, 1-n]
,[3/2],-I*tan(d*x+c))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/((1+I*tan(d*
x+c))^n)
```

Mathematica [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$=\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x
]],x]
```

output
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x
]], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4084}$$

$$(A-iB)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx+\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$(A-iB)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx+\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{4047}$$

$$\frac{ia^2(A-iB)\int -\frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}+\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{ia^2(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{ia(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}$$

$$\Big\downarrow \text{148}$$

$$\frac{2a^2(A-iB)\int \frac{(a-ia^3\tan^2(c+dx))^{n-1}}{a(ia^2\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{d}+\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a(A-iB)\int \frac{(a-ia^3\tan^2(c+dx))^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{334}$$

$$\frac{2(A-iB)\left(a-ia^3\tan^2(c+dx)\right)^n\left(1-ia^2\tan^2(c+dx)\right)^{-n}\int \frac{(1-ia^2\tan^2(c+dx))^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{333}$$

$$\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}+ \frac{2ia(A-iB)\tan(c+dx)\left(a-ia^3\tan^2(c+dx)\right)^n\left(1-ia^2\tan^2(c+dx)\right)^{-n}\text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+\right.}{d}$$

$$\Big\downarrow \text{4082}$$

$$\frac{iaB\int \frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}}d\tan(c+dx)}{d}+ \frac{2ia(A-iB)\tan(c+dx)\left(a-ia^3\tan^2(c+dx)\right)^n\left(1-ia^2\tan^2(c+dx)\right)^{-n}\text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+\right.}{d}$$

$$\Big\downarrow \text{76}$$

$$\frac{iB(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\int \frac{(i\tan(c+dx)+1)^{n-1}}{\sqrt{\tan(c+dx)}}d\tan(c+dx)}{d}+ \frac{2ia(A-iB)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+\right.}{d}$$

$$\Big\downarrow \text{74}$$

$$\frac{2ia(A-iB)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+\right.}{d} \frac{2iB\sqrt{\tan(c+dx)}(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)}{d}$$

input
```
Int[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x]
```

output
```
((2*I)*B*Hypergeometric2F1[1/2, 1 - n, 3/2, (-I)*Tan[c + d*x]]*Sqrt[Tan[c
+ d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + I*Tan[c + d*x])^n) + ((2*I)*a*(A
 - I*B)*AppellF1[1/2, 1, 1 - n, 3/2, (-I)*a^2*Tan[c + d*x]^2, I*a^2*Tan[c
+ d*x]^2]*Tan[c + d*x]*(a - I*a^3*Tan[c + d*x]^2)^n)/(d*(1 - I*a^2*Tan[c +
 d*x]^2)^n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n (A+B\tan(dx+c))}{\sqrt{\tan(dx+c)}}dx$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algori
thm="fricas")
```

output
```
integral((((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A - B)*(2*a*e^(2*I*d*x + 2*I*c
)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) + 1))/(e^(2*I*d*x + 2*I*c) - 1), x)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))/sqrt(tan(c + d*x
)), x)
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/sqrt(tan(d*x + c))
, x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/sqrt(tan(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n}{\sqrt{\tan(c+d\,x)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/tan(c + d*x)^(1/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/tan(c + d*x)^(1/2), x
)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{-2\sqrt{\tan(dx+c)}\,(\tan(dx+c)\,ai+a)^n\,bi+2\left(\int \frac{\sqrt{\tan(dx+c)}\,(\tan(dx+c)ai+a)^n}{\tan(dx+c)}dx\right)adn+\left(\int \frac{\sqrt{\tan(dx+c)}\,(\tan(dx}{\tan(dx+c)}\right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
( - 2*sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n*b*i + 2*int((sqrt(tan(c
 + d*x))*(tan(c + d*x)*a*i + a)**n)/tan(c + d*x),x)*a*d*n + int((sqrt(tan(
c + d*x))*(tan(c + d*x)*a*i + a)**n)/tan(c + d*x),x)*b*d*i + 2*int(sqrt(ta
n(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x),x)*b*d*i*n + int(sqrt(t
an(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x),x)*b*d*i)/(2*d*n)
```

3.230 $\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 194

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$
$$+\frac{2(iA+B)\,\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}}{d}$$
$$-\frac{2iA(1-2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}(a-}{d}$$

output
```
-2*A*(a+I*a*tan(d*x+c))^n/d/tan(d*x+c)^(1/2)+2*(I*A+B)*AppellF1(1/2,1-n,1,
3/2,-I*tan(d*x+c),I*tan(d*x+c))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/((
1+I*tan(d*x+c))^n)-2*I*A*(1-2*n)*hypergeom([1/2, 1-n],[3/2],-I*tan(d*x+c))
*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/((1+I*tan(d*x+c))^n)
```

Mathematica [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/
2),x]
```

output
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/
2), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.13, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4081, 27, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4081}$$

$$\frac{2\int \frac{(i\tan(c+dx)a+a)^n(a(B+2iAn)-aA(1-2n)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(i\tan(c+dx)a+a)^n(a(B+2iAn)-aA(1-2n)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{(i\tan(c+dx)a+a)^n(a(B+2iAn)-aA(1-2n)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\big\downarrow$ 4084

$$\frac{a(B+iA)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\big\downarrow$ 3042

$$\frac{a(B+iA)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\big\downarrow$ 4047

$$\frac{\frac{ia^3(B+iA)\int -\frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\big\downarrow$ 25

$$\frac{-\frac{ia^3(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\big\downarrow$ 27

$$\frac{-\frac{ia^2(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 148

$$\frac{\frac{2a^3(B+iA)\int \frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{a\left(ia^2\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{\frac{2a^2(B+iA)\int \frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 334

$$\frac{\frac{2a(B+iA)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\int \frac{\left(1-ia^2\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c-}{\sqrt{\tan(c+dx)}}}{a}$$

$$\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 333

$$\frac{\frac{2ia^2(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n \text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+dx),ia^2\tan^2(c+dx)\right)}{d}-iA(1-2n)}{a}$$

$$\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 4082

$$\frac{\frac{2ia^2(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n \text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+dx),ia^2\tan^2(c+dx)\right)}{d} - \frac{ia^2A(1-2n)\ \ldots}{\ldots}}{a}$$
$$\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 76

$$\frac{\frac{2ia^2(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n \text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+dx),ia^2\tan^2(c+dx)\right)}{d} - \frac{iaA(1-2n)(1\ \ldots}{\ldots}}{a}$$
$$\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 74

$$\frac{\frac{2ia^2(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n \text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+dx),ia^2\tan^2(c+dx)\right)}{d} - \frac{2iaA(1-2n)\ \ldots}{\ldots}}{a}$$
$$\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x]
```

output
```
(-2*A*(a + I*a*Tan[c + d*x])^n)/(d*Sqrt[Tan[c + d*x]]) + (((-2*I)*a*A*(1 -
 2*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, (-I)*Tan[c + d*x]]*Sqrt[Tan[c + d
*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + I*Tan[c + d*x])^n) + ((2*I)*a^2*(I*
A + B)*AppellF1[1/2, 1, 1 - n, 3/2, (-I)*a^2*Tan[c + d*x]^2, I*a^2*Tan[c +
 d*x]^2]*Tan[c + d*x]*(a - I*a^3*Tan[c + d*x]^2)^n)/(d*(1 - I*a^2*Tan[c +
d*x]^2)^n))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(a + ia\tan(dx + c))^n (A + B\tan(dx + c))}{\tan(dx + c)^{\frac{3}{2}}} dx$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algori
thm="fricas")
```

output
```
integral(-((A - I*B)*e^(4*I*d*x + 4*I*c) + 2*A*e^(2*I*d*x + 2*I*c) + A + I
*B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((-I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) + 1))/(e^(4*I*d*x + 4*I*c) - 2*e^(2
*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))/tan(c + d*x)**(3
/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algori
thm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/tan(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n}{\tan(c+d\,x)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/tan(c + d*x)^(3/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/tan(c + d*x)^(3/2), x
)
```

Reduce [F]

$$\int \frac{(a + ia\tan(c + dx))^n (A + B\tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)}\,dx$$
$$= \frac{4\sqrt{\tan(dx + c)}\,(\tan(dx + c)\,ai + a)^n \tan(dx + c)\,bn - 2\sqrt{\tan(dx + c)}\,(\tan(dx + c)\,ai + a)^n \tan(dx +$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
(4*sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x)*b*n - 2*sqrt(
tan(c + d*x))*(tan(c + d*x)*a*i + a)**n*tan(c + d*x)*b - 4*sqrt(tan(c + d*
x))*(tan(c + d*x)*a*i + a)**n*b*i*n + 4*int((sqrt(tan(c + d*x))*(tan(c + d
*x)*a*i + a)**n)/tan(c + d*x)**2,x)*tan(c + d*x)*a*d*n**2 - 2*int((sqrt(ta
n(c + d*x))*(tan(c + d*x)*a*i + a)**n)/tan(c + d*x)**2,x)*tan(c + d*x)*b*d
*i*n - 2*int((sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n)/tan(c + d*x),x
)*tan(c + d*x)*b*d*n + int((sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n)/
tan(c + d*x),x)*tan(c + d*x)*b*d - 4*int(sqrt(tan(c + d*x))*(tan(c + d*x)*
a*i + a)**n*tan(c + d*x),x)*tan(c + d*x)*b*d*n**2 + int(sqrt(tan(c + d*x))
*(tan(c + d*x)*a*i + a)**n*tan(c + d*x),x)*tan(c + d*x)*b*d)/(4*tan(c + d*
x)*d*n**2)
```

3.231 $\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 247

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(3B+2iAn)(a+ia\tan(c+dx))^n}{3d\sqrt{\tan(c+dx)}} - \frac{2(A-iB)\,\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}\sqrt{\tan(c+dx)}}{d} - \frac{2(1-2n)(3iB-2An)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}\sqrt{\tan(c}{3d}$$

output

```
-2/3*A*(a+I*a*tan(d*x+c))^n/d/tan(d*x+c)^(3/2)-2/3*(3*B+2*I*A*n)*(a+I*a*ta
n(d*x+c))^n/d/tan(d*x+c)^(1/2)-2*(A-I*B)*AppellF1(1/2,1-n,1,3/2,-I*tan(d*x
+c),I*tan(d*x+c))*tan(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/((1+I*tan(d*x+c)
)^n)-2/3*(1-2*n)*(3*I*B-2*A*n)*hypergeom([1/2, 1-n],[3/2],-I*tan(d*x+c))*t
an(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d/((1+I*tan(d*x+c))^n)
```

Mathematica [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/
2),x]
```

output
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/
2), x]
```

Rubi [A] (warning: unable to verify)

Time = 1.36 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4081, 27, 3042, 4081, 27, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4081}$$

$$\frac{2\int \frac{(i\tan(c+dx)a+a)^n(a(3B+2iAn)-aA(3-2n)\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(i\tan(c+dx)a+a)^n(a(3B+2iAn)-aA(3-2n)\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(i\tan(c+dx)a+a)^n(a(3B+2iAn)-aA(3-2n)\tan(c+dx))}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4081}$$

$$\frac{\frac{2\int \frac{(i\tan(c+dx)a+a)^n\left(a^2\left(6iBn-A\left(4n^2-2n+3\right)\right)-a^2(1-2n)(3B+2iAn)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(i\tan(c+dx)a+a)^n\left(a^2\left(6iBn-A\left(4n^2-2n+3\right)\right)-a^2(1-2n)(3B+2iAn)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{(i\tan(c+dx)a+a)^n\left(a^2\left(6iBn-A\left(4n^2-2n+3\right)\right)-a^2(1-2n)(3B+2iAn)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4084}$$

$$\frac{\frac{-3a^2(A-iB)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-3a^2(A-iB)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 4047

$$\frac{\frac{-\frac{3ia^4(A-iB)\int -\frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a}{d\sqrt{\tan}}}{3a}$$

$$\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{3ia^4(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia}{d\sqrt{\tan(c+}}}{3a}$$

$$\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{3ia^3(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia\,t}{d\sqrt{\tan(c+d}}}{3a}$$

$$\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 148

$$\frac{\frac{-\frac{6a^4(A-iB)\int \frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{a\left(ia^2\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{d}-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia\tan(c}{d\sqrt{\tan(c+dx)}}}{3a}$$

$$\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{\frac{-\frac{6a^3(A-iB)\int\frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-a(1-2n)(-2An+3iB)\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia\tan(c}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 334}$$

$$\frac{\frac{-\frac{6a^2(A-iB)\left(a-ia^3\tan^2(c+dx)\right)^n\left(1-ia^2\tan^2(c+dx)\right)^{-n}\int\frac{\left(1-ia^2\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-a(1-2n)(-2An+3iB)\int\frac{(a-ia\tan(c+dx))(i\tan(c+}{\sqrt{\tan(c+dx)}}}{a}}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 333}$$

$$\frac{\frac{-a(1-2n)(-2An+3iB)\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-\frac{6ia^3(A-iB)\tan(c+dx)\left(a-ia^3\tan^2(c+dx)\right)^n\left(1-ia^2\tan^2(c+dx)\right)^{-n}\operatorname{AppellF1}\left(\frac{1}{2},1,1-\right.}{d}}{a}}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4082}$$

$$\frac{\frac{-\frac{a^3(1-2n)(-2An+3iB)\int\frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}}d\tan(c+dx)}{d}-\frac{6ia^3(A-iB)\tan(c+dx)\left(a-ia^3\tan^2(c+dx)\right)^n\left(1-ia^2\tan^2(c+dx)\right)^{-n}\operatorname{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-i\right.}{d}}{a}}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 76}$$

$$\frac{\frac{-\frac{a^2(1-2n)(-2An+3iB)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\int\frac{(i\tan(c+dx)+1)^{n-1}}{\sqrt{\tan(c+dx)}}d\tan(c+dx)}{d}-\frac{6ia^3(A-iB)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan\right.}{}}{a}}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 74}$$

$$\frac{-\frac{2a^2(1-2n)(-2An+3iB)\sqrt{\tan(c+dx)}(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)}{d}-\frac{6ia^3(A-iB)\tan(c+dx)\left(1-ia^2\tan\right.}{a}}{3a}$$

$$\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x]
```

output
```
(-2*A*(a + I*a*Tan[c + d*x])^n)/(3*d*Tan[c + d*x]^(3/2)) + ((-2*a*(3*B + (
2*I)*A*n)*(a + I*a*Tan[c + d*x])^n)/(d*Sqrt[Tan[c + d*x]]) + ((-2*a^2*(1 -
 2*n)*((3*I)*B - 2*A*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, (-I)*Tan[c + d*
x]]*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + I*Tan[c + d*x])^n
) - ((6*I)*a^3*(A - I*B)*AppellF1[1/2, 1, 1 - n, 3/2, (-I)*a^2*Tan[c + d*x
]^2, I*a^2*Tan[c + d*x]^2]*Tan[c + d*x]*(a - I*a^3*Tan[c + d*x]^2)^n)/(d*(
1 - I*a^2*Tan[c + d*x]^2)^n))/a)/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n (A+B\tan(dx+c))}{\tan(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\tan(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algori
thm="fricas")
```

output
```
integral(((-I*A - B)*e^(6*I*d*x + 6*I*c) + (-3*I*A - B)*e^(4*I*d*x + 4*I*c
) + (-3*I*A + B)*e^(2*I*d*x + 2*I*c) - I*A + B)*(2*a*e^(2*I*d*x + 2*I*c)/(
e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((-I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) + 1))/(e^(6*I*d*x + 6*I*c) - 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x
 + 2*I*c) - 1), x)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))/tan(c + d*x)**(5
/2), x)
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\tan(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/tan(d*x + c)^(5/2)
, x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\tan(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/tan(d*x + c)^(5/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})^n}{\tan(c+d\,x)^{5/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/tan(c + d*x)^(5/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/tan(c + d*x)^(5/2), x
)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-2\sqrt{\tan(dx+c)}\,(\tan(dx+c)\,ai+a)^n\,bi+2\left(\int \frac{\sqrt{\tan(dx+c)}\,(\tan(dx+c)ai+a)^n}{\tan(dx+c)^3}dx\right)\tan(dx+c)^2\,adn-3\left(\int \right.}{}$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
( - 2*sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n*b*i + 2*int((sqrt(tan(c
 + d*x))*(tan(c + d*x)*a*i + a)**n)/tan(c + d*x)**3,x)*tan(c + d*x)**2*a*d
*n - 3*int((sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a)**n)/tan(c + d*x)**3,
x)*tan(c + d*x)**2*b*d*i + 2*int((sqrt(tan(c + d*x))*(tan(c + d*x)*a*i + a
)**n)/tan(c + d*x),x)*tan(c + d*x)**2*b*d*i*n - 3*int((sqrt(tan(c + d*x))*
(tan(c + d*x)*a*i + a)**n)/tan(c + d*x),x)*tan(c + d*x)**2*b*d*i)/(2*tan(c
 + d*x)**2*d*n)
```

3.232 $\int \tan^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 87

$$\int \tan^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = -((aA-bB)x)+\frac{(Ab+aB)\log(\cos(c+dx))}{d} + \frac{(aA-bB)\tan(c+dx)}{d}+\frac{(Ab+aB)\tan^2(c+dx)}{2d}+\frac{bB\tan^3(c+dx)}{3d}$$

output
```
-(A*a-B*b)*x+(A*b+B*a)*ln(cos(d*x+c))/d+(A*a-B*b)*tan(d*x+c)/d+1/2*(A*b+B*
a)*tan(d*x+c)^2/d+1/3*b*B*tan(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

$$\int \tan^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{aA\arctan(\tan(c+dx))}{d} + \frac{bB\arctan(\tan(c+dx))}{d} + \frac{Ab(2\log(\cos(c+dx))+\sec^2(c+dx))}{2d} + \frac{aB(2\log(\cos(c+dx))+\sec^2(c+dx))}{2d} + \frac{aA\tan(c+dx)}{d} - \frac{bB\tan(c+dx)}{d} + \frac{bB\tan^3(c+dx)}{3d}$$

input
```
Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-((a*A*ArcTan[Tan[c + d*x]])/d) + (b*B*ArcTan[Tan[c + d*x]])/d + (A*b*(2*L
og[Cos[c + d*x]] + Sec[c + d*x]^2))/(2*d) + (a*B*(2*Log[Cos[c + d*x]] + Se
c[c + d*x]^2))/(2*d) + (a*A*Tan[c + d*x])/d - (b*B*Tan[c + d*x])/d + (b*B*
Tan[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 4075, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^2(a+b\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4075}$$

$$\int \tan^2(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx + \frac{bB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{bB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int \tan(c+dx)(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{(aB+Ab)\tan^2(c+dx)}{2d}+\frac{bB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{(aB+Ab)\tan^2(c+dx)}{2d}+\frac{bB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4008}$$

$$-(aB+Ab)\int \tan(c+dx)dx+\frac{(aB+Ab)\tan^2(c+dx)}{2d}+\frac{(aA-bB)\tan(c+dx)}{d}-x(aA-bB)+\frac{bB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-(aB+Ab)\int \tan(c+dx)dx+\frac{(aB+Ab)\tan^2(c+dx)}{2d}+\frac{(aA-bB)\tan(c+dx)}{d}-x(aA-bB)+\frac{bB\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(aB+Ab)\tan^2(c+dx)}{2d}+\frac{(aA-bB)\tan(c+dx)}{d}+\frac{(aB+Ab)\log(\cos(c+dx))}{d}-x(aA-bB)+\frac{bB\tan^3(c+dx)}{3d}$$

input
```
Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-((a*A - b*B)*x) + ((A*b + a*B)*Log[Cos[c + d*x]])/d + ((a*A - b*B)*Tan[c
+ d*x])/d + ((A*b + a*B)*Tan[c + d*x]^2)/(2*d) + (b*B*Tan[c + d*x]^3)/(3*d
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

method	result
norman	$(-aA+Bb)\,x+\frac{(aA-Bb)\tan(dx+c)}{d}+\frac{(Ab+Ba)\tan(dx+c)^{2}}{2d}+\frac{bB\tan(dx+c)^{3}}{3d}-\frac{(Ab+Ba)\ln\left(1+\tan(d\right.}{2d}$
parts	$\frac{(Ab+Ba)\left(\frac{\tan(dx+c)^{2}}{2}-\frac{\ln\left(1+\tan(dx+c)^{2}\right)}{2}\right)}{d}+\frac{Bb\left(\frac{\tan(dx+c)^{3}}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{aA(\tan(dx+c}{}$
derivativedivides	$\frac{\frac{Bb\tan(dx+c)^{3}}{3}+\frac{Ab\tan(dx+c)^{2}}{2}+\frac{B\tan(dx+c)^{2}a}{2}+A\tan(dx+c)a-bB\tan(dx+c)+\frac{(-Ab-Ba)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-aA+}{d}$
default	$\frac{\frac{Bb\tan(dx+c)^{3}}{3}+\frac{Ab\tan(dx+c)^{2}}{2}+\frac{B\tan(dx+c)^{2}a}{2}+A\tan(dx+c)a-bB\tan(dx+c)+\frac{(-Ab-Ba)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-aA+}{d}$
parallelrisch	$-\frac{-2Bb\tan(dx+c)^{3}+6Axad-3Ab\tan(dx+c)^{2}-6Bbdx-3B\tan(dx+c)^{2}a+3A\ln\left(1+\tan(dx+c)^{2}\right)b-6A\tan(dx+c)a}{6d}$
risch	$-iAbx-iBax-Aax+Bbx-\frac{2iAbc}{d}-\frac{2iaBc}{d}+\frac{2i\left(-3iAb\,e^{4i(dx+c)}-3iBa\,e^{4i(dx+c)}+3Aa\,e^{4i(dx+c)}\right.}{}$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
(-A*a+B*b)*x+(A*a-B*b)*tan(d*x+c)/d+1/2*(A*b+B*a)*tan(d*x+c)^2/d+1/3*b*B*t
an(d*x+c)^3/d-1/2*(A*b+B*a)/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \tan^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{2\,Bb\tan\left(dx+c\right)^{3}-6\,(Aa-Bb)dx+3\,(Ba+Ab)\tan\left(dx+c\right)^{2}+3\,(Ba+Ab)\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)+6\,(A}{6\,d}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/6*(2*B*b*tan(d*x + c)^3 - 6*(A*a - B*b)*d*x + 3*(B*a + A*b)*tan(d*x + c)
^2 + 3*(B*a + A*b)*log(1/(tan(d*x + c)^2 + 1)) + 6*(A*a - B*b)*tan(d*x + c
))/d
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.56

$$\int \tan^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} -Aax + \frac{Aa\tan(c+dx)}{d} - \frac{Ab\log(\tan^2(c+dx)+1)}{2d} + \frac{Ab\tan^2(c+dx)}{2d} - \frac{Ba\log(\tan^2(c+dx)+1)}{2d} + \frac{Ba\tan^2(c+dx)}{2d} + Bbx + \\ x(A+B\tan(c))(a+b\tan(c))\tan^2(c) \end{cases}$$

input
```
integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((-A*a*x + A*a*tan(c + d*x)/d - A*b*log(tan(c + d*x)**2 + 1)/(2*d
) + A*b*tan(c + d*x)**2/(2*d) - B*a*log(tan(c + d*x)**2 + 1)/(2*d) + B*a*t
an(c + d*x)**2/(2*d) + B*b*x + B*b*tan(c + d*x)**3/(3*d) - B*b*tan(c + d*x
)/d, Ne(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c))*tan(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \tan^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,Bb\tan(dx+c)^3 + 3\,(Ba+Ab)\tan(dx+c)^2 - 6\,(Aa-Bb)(dx+c) - 3\,(Ba+Ab)\log\left(\tan(dx+c)^2\right.}{6\,d}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/6*(2*B*b*tan(d*x + c)^3 + 3*(B*a + A*b)*tan(d*x + c)^2 - 6*(A*a - B*b)*(
d*x + c) - 3*(B*a + A*b)*log(tan(d*x + c)^2 + 1) + 6*(A*a - B*b)*tan(d*x +
 c))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \tan^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Aa-Bb)(dx+c)}{d} - \frac{(Ba+Ab)\log\left(\tan(dx+c)^2+1\right)}{2d} + \frac{2Bbd^2\tan(dx+c)^3+3Bad^2\tan(dx+c)^2+3Abd^2\tan(dx+c)^2+6Aad^2\tan(dx+c)-6Bbd^2\tan}{6d^3}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
-(A*a - B*b)*(d*x + c)/d - 1/2*(B*a + A*b)*log(tan(d*x + c)^2 + 1)/d + 1/6
*(2*B*b*d^2*tan(d*x + c)^3 + 3*B*a*d^2*tan(d*x + c)^2 + 3*A*b*d^2*tan(d*x
+ c)^2 + 6*A*a*d^2*tan(d*x + c) - 6*B*b*d^2*tan(d*x + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \tan^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)\,(A\,a-B\,b)-\ln\left(\tan(c+d\,x)^2+1\right)\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)+\tan(c+d\,x)^2\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)-d\,x\,(A\,a-B}{d}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
(tan(c + d*x)*(A*a - B*b) - log(tan(c + d*x)^2 + 1)*((A*b)/2 + (B*a)/2) +
tan(c + d*x)^2*((A*b)/2 + (B*a)/2) - d*x*(A*a - B*b) + (B*b*tan(c + d*x)^3
)/3)/d
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \tan^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{-3\log\left(\tan(dx+c)^2+1\right)ab+\tan(dx+c)^3b^2+3\tan(dx+c)^2ab+3\tan(dx+c)a^2-3\tan(dx+c)}{3d}$$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
( - 3*log(tan(c + d*x)**2 + 1)*a*b + tan(c + d*x)**3*b**2 + 3*tan(c + d*x)
**2*a*b + 3*tan(c + d*x)*a**2 - 3*tan(c + d*x)*b**2 - 3*a**2*d*x + 3*b**2*
d*x)/(3*d)
```

3.233 $\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 65

$$\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = -((Ab+aB)x)-\frac{(aA-bB)\log(\cos(c+dx))}{d} + \frac{(Ab+aB)\tan(c+dx)}{d}+\frac{bB\tan^2(c+dx)}{2d}$$

output
```
-(A*b+B*a)*x-(A*a-B*b)*ln(cos(d*x+c))/d+(A*b+B*a)*tan(d*x+c)/d+1/2*b*B*tan
(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{-2(Ab+aB)\arctan(\tan(c+dx))+2(-aA+bB)\log(\cos(c+dx))+bB\sec^2(c+dx)+2(Ab+aB)\text{ta}}{2d}$$

input
```
Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*(A*b + a*B)*ArcTan[Tan[c + d*x]] + 2*(-(a*A) + b*B)*Log[Cos[c + d*x]]
+ b*B*Sec[c + d*x]^2 + 2*(A*b + a*B)*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4075, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int \tan(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{bB\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{bB\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4008}$$

$$(aA-bB)\int \tan(c+dx)dx+\frac{(aB+Ab)\tan(c+dx)}{d}-x(aB+Ab)+\frac{bB\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$(aA-bB)\int \tan(c+dx)dx+\frac{(aB+Ab)\tan(c+dx)}{d}-x(aB+Ab)+\frac{bB\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(aB+Ab)\tan(c+dx)}{d}-\frac{(aA-bB)\log(\cos(c+dx))}{d}-x(aB+Ab)+\frac{bB\tan^2(c+dx)}{2d}$$

input
```
Int[Tan[c + d*x]*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-((A*b + a*B)*x) - ((a*A - b*B)*Log[Cos[c + d*x]])/d + ((A*b + a*B)*Tan[c
+ d*x])/d + (b*B*Tan[c + d*x]^2)/(2*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

method	result
norman	$(-Ab-Ba)\,x+\frac{(Ab+Ba)\tan(dx+c)}{d}+\frac{bB\tan(dx+c)^2}{2d}+\frac{(aA-Bb)\ln\left(1+\tan(dx+c)^2\right)}{2d}$
derivativedivides	$\frac{\frac{B\tan(dx+c)^2 b}{2}+A\tan(dx+c)b+B\tan(dx+c)a+\frac{(aA-Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Ab-Ba)\arctan(\tan(dx+c))}{d}$
default	$\frac{\frac{B\tan(dx+c)^2 b}{2}+A\tan(dx+c)b+B\tan(dx+c)a+\frac{(aA-Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Ab-Ba)\arctan(\tan(dx+c))}{d}$
parts	$\frac{(Ab+Ba)(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{aA\ln\left(1+\tan(dx+c)^2\right)}{2d}+\frac{Bb\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}$
parallelrisch	$\frac{-2Abdx-2Bxad+B\tan(dx+c)^2 b+A\ln\left(1+\tan(dx+c)^2\right)a+2A\tan(dx+c)b-B\ln\left(1+\tan(dx+c)^2\right)b+2B\tan(dx+c)a}{2d}$
risch	$-Abx-Bax+iAax-iBbx+\frac{2iaAc}{d}-\frac{2iBbc}{d}+\frac{2i\left(-iBb\,e^{2i(dx+c)}+Ab\,e^{2i(dx+c)}+Ba\,e^{2i(dx+c)}+A\right.}{d\left(e^{2i(dx+c)}+1\right)^2}$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(-A*b-B*a)*x+(A*b+B*a)*tan(d*x+c)/d+1/2*b*B*tan(d*x+c)^2/d+1/2*(A*a-B*b)/d
*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{Bb\tan{(dx+c)}^2-2\,(Ba+Ab)dx-(Aa-Bb)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)+2\,(Ba+Ab)\tan{(dx+c)}}{2\,d}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="frica
s")
```

output
```
1/2*(B*b*tan(d*x + c)^2 - 2*(B*a + A*b)*d*x - (A*a - B*b)*log(1/(tan(d*x +
 c)^2 + 1)) + 2*(B*a + A*b)*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\begin{cases}\frac{Aa\log\left(\tan^2(c+dx)+1\right)}{2d}-Abx+\frac{Ab\tan(c+dx)}{d}-Bax+\frac{Ba\tan(c+dx)}{d}-\frac{Bb\log\left(\tan^2(c+dx)+1\right)}{2d}+\frac{Bb\tan^2(c+dx)}{2d} & \text{for } d\neq 0\\ x(A+B\tan(c))\,(a+b\tan(c))\tan(c) & \text{otherwise}\end{cases}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((A*a*log(tan(c + d*x)**2 + 1)/(2*d) - A*b*x + A*b*tan(c + d*x)/d
 - B*a*x + B*a*tan(c + d*x)/d - B*b*log(tan(c + d*x)**2 + 1)/(2*d) + B*b*t
an(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c))*tan(c),
True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{Bb\tan(dx+c)^2-2\,(Ba+Ab)(dx+c)+(Aa-Bb)\log\left(\tan(dx+c)^2+1\right)+2\,(Ba+Ab)\tan(dx+c)}{2\,d}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxim
a")
```

output
```
1/2*(B*b*tan(d*x + c)^2 - 2*(B*a + A*b)*(d*x + c) + (A*a - B*b)*log(tan(d*
x + c)^2 + 1) + 2*(B*a + A*b)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Ba+Ab)(dx+c)}{d}+\frac{(Aa-Bb)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}+\frac{Bbd\tan\left(dx+c\right)^2+2\,Bad\tan\left(dx+c\right)+2\,Abd\tan\left(dx+c\right)}{2\,d^2}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac"
)
```

output
```
-(B*a + A*b)*(d*x + c)/d + 1/2*(A*a - B*b)*log(tan(d*x + c)^2 + 1)/d + 1/2
*(B*b*d*tan(d*x + c)^2 + 2*B*a*d*tan(d*x + c) + 2*A*b*d*tan(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)\,\left(A\,b+B\,a\right)+\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{A\,a}{2}-\frac{B\,b}{2}\right)-d\,x\,\left(A\,b+B\,a\right)+\frac{B\,b\tan(c+d\,x)^2}{2}}{d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
(tan(c + d*x)*(A*b + B*a) + log(tan(c + d*x)^2 + 1)*((A*a)/2 - (B*b)/2) -
d*x*(A*b + B*a) + (B*b*tan(c + d*x)^2)/2)/d
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \tan(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{\log\left(\tan\left(dx+c\right)^2+1\right)a^2-\log\left(\tan\left(dx+c\right)^2+1\right)b^2+\tan\left(dx+c\right)^2b^2+4\tan\left(dx+c\right)ab-4abdx}{2d}$$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(log(tan(c + d*x)**2 + 1)*a**2 - log(tan(c + d*x)**2 + 1)*b**2 + tan(c + d
*x)**2*b**2 + 4*tan(c + d*x)*a*b - 4*a*b*d*x)/(2*d)
```

3.234 $\int (a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int (a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = (aA-bB)x-\frac{(Ab+aB)\log(\cos(c+dx))}{d}+\frac{bB\tan(c+dx)}{d}$$

output
```
(A*a-B*b)*x-(A*b+B*a)*ln(cos(d*x+c))/d+b*B*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int (a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = aAx-\frac{bB\arctan(\tan(c+dx))}{d}-\frac{Ab\log(\cos(c+dx))}{d}-\frac{aB\log(\cos(c+dx))}{d}+\frac{bB\tan(c+dx)}{d}$$

input
```
Integrate[(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
a*A*x - (b*B*ArcTan[Tan[c + d*x]])/d - (A*b*Log[Cos[c + d*x]])/d - (a*B*Lo
g[Cos[c + d*x]])/d + (b*B*Tan[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4008$$

$$(aB+Ab)\int \tan(c+dx)dx+x(aA-bB)+\frac{bB\tan(c+dx)}{d}$$

$$\Big\downarrow \; 3042$$

$$(aB+Ab)\int \tan(c+dx)dx+x(aA-bB)+\frac{bB\tan(c+dx)}{d}$$

$$\Big\downarrow \; 3956$$

$$-\frac{(aB+Ab)\log(\cos(c+dx))}{d}+x(aA-bB)+\frac{bB\tan(c+dx)}{d}$$

input
```
Int[(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(a*A - b*B)*x - ((A*b + a*B)*Log[Cos[c + d*x]])/d + (b*B*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

method	result
norman	$(aA-Bb)\,x+\frac{bB\tan(dx+c)}{d}+\frac{(Ab+Ba)\ln\left(1+\tan(dx+c)^{2}\right)}{2d}$
derivativedivides	$\frac{bB\tan(dx+c)+\frac{(Ab+Ba)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(aA-Bb)\arctan(\tan(dx+c))}{d}$
default	$\frac{bB\tan(dx+c)+\frac{(Ab+Ba)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(aA-Bb)\arctan(\tan(dx+c))}{d}$
parts	$Aax+\frac{(Ab+Ba)\ln\left(1+\tan(dx+c)^{2}\right)}{2d}+\frac{Bb(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$
parallelrisch	$\frac{2Axad-2Bbdx+A\ln\left(1+\tan(dx+c)^{2}\right)b+B\ln\left(1+\tan(dx+c)^{2}\right)a+2bB\tan(dx+c)}{2d}$
risch	$iAbx+iBax+Aax-Bbx+\frac{2iAbc}{d}+\frac{2iaBc}{d}+\frac{2iBb}{d\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{2i(dx+c)}+1\right)Ab}{d}-\frac{a\ln(e^{2i(d}$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(A*a-B*b)*x+b*B*tan(d*x+c)/d+1/2*(A*b+B*a)/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int (a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,(Aa-Bb)dx+2\,Bb\tan\left(dx+c\right)-(Ba+Ab)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*(A*a - B*b)*d*x + 2*B*b*tan(d*x + c) - (B*a + A*b)*log(1/(tan(d*x +
 c)^2 + 1)))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int (a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} Aax+\frac{Ab\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ba\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-Bbx+\frac{Bb\tan\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x(A+B\tan\left(c\right))\left(a+b\tan\left(c\right)\right) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((A*a*x + A*b*log(tan(c + d*x)**2 + 1)/(2*d) + B*a*log(tan(c + d*
x)**2 + 1)/(2*d) - B*b*x + B*b*tan(c + d*x)/d, Ne(d, 0)), (x*(A + B*tan(c)
)*(a + b*tan(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int (a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{2\,Bb\tan\left(dx+c\right)+2\,(Aa-Bb)(dx+c)+(Ba+Ab)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*B*b*tan(d*x + c) + 2*(A*a - B*b)*(d*x + c) + (B*a + A*b)*log(tan(d*
x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int (a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{Bb\tan\left(dx+c\right)}{d}+\frac{(Aa-Bb)(dx+c)}{d}+\frac{(Ba+Ab)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
B*b*tan(d*x + c)/d + (A*a - B*b)*(d*x + c)/d + 1/2*(B*a + A*b)*log(tan(d*x
 + c)^2 + 1)/d
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int (a + b\tan(c + dx))(A + B\tan(c + dx))\,dx = \frac{B\,b\tan(c + d\,x) + \frac{A\,b\,\ln\left(\tan(c+d\,x)^2+1\right)}{2} + \frac{B\,a\,\ln\left(\tan(c+d\,x)^2+1\right)}{2} + A\,a\,d\,x - B\,b\,d\,x}{d}$$

input
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
(B*b*tan(c + d*x) + (A*b*log(tan(c + d*x)^2 + 1))/2 + (B*a*log(tan(c + d*x
)^2 + 1))/2 + A*a*d*x - B*b*d*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (a + b\tan(c + dx))(A + B\tan(c + dx))\,dx = \frac{\log\left(\tan(dx + c)^2 + 1\right)ab + \tan(dx + c)\,b^2 + a^2dx - b^2dx}{d}$$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(log(tan(c + d*x)**2 + 1)*a*b + tan(c + d*x)*b**2 + a**2*d*x - b**2*d*x)/d
```

3.235 $\int \cot(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 37

$$\int \cot(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = (Ab+aB)x - \frac{bB\log(\cos(c+dx))}{d} + \frac{aA\log(\sin(c+dx))}{d}$$

output
```
(A*b+B*a)*x-b*B*ln(cos(d*x+c))/d+a*A*ln(sin(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \cot(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = Abx + aBx - \frac{bB\log(\cos(c+dx))}{d} + \frac{aA\log(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
A*b*x + a*B*x - (b*B*Log[Cos[c + d*x]])/d + (a*A*Log[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {3042, 4072, 3042, 3956, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4072}$$

$$\int \cot(c+dx)(aA+(Ab+aB)\tan(c+dx))dx+bB\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aA+(Ab+aB)\tan(c+dx)}{\tan(c+dx)}dx+bB\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3956}$$

$$\int \frac{aA+(Ab+aB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{bB\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{4014}$$

$$aA\int \cot(c+dx)dx+x(aB+Ab)-\frac{bB\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{3042}$$

$$aA\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+x(aB+Ab)-\frac{bB\log(\cos(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$-aA\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+x(aB+Ab)-\frac{bB\log(\cos(c+dx))}{d}$$

$$\downarrow \text{3956}$$

$$x(aB+Ab)+\frac{aA\log(-\sin(c+dx))}{d}-\frac{bB\log(\cos(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(A*b + a*B)*x - (b*B*Log[Cos[c + d*x]])/d + (a*A*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

method	result	size
parallelrisch	$\frac{(-aA+Bb)\ln\left(\sec(dx+c)^2\right)+2aA\ln(\tan(dx+c))+2dx(Ab+Ba)}{2d}$	47
norman	$(Ab+Ba)\,x+\frac{aA\ln(\tan(dx+c))}{d}-\frac{(aA-Bb)\ln\left(1+\tan(dx+c)^2\right)}{2d}$	48
derivativedivides	$\frac{\frac{(-aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Ab+Ba)\arctan(\tan(dx+c))+aA\ln(\tan(dx+c))}{d}$	52
default	$\frac{\frac{(-aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Ab+Ba)\arctan(\tan(dx+c))+aA\ln(\tan(dx+c))}{d}$	52
risch	$Abx+Bax-iAax+iBbx-\frac{2iaAc}{d}+\frac{2iBbc}{d}+\frac{aA\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)Bb}{d}$	77

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*((-A*a+B*b)*ln(sec(d*x+c)^2)+2*a*A*ln(tan(d*x+c))+2*d*x*(A*b+B*a))/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \cot(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{2\,(Ba+Ab)dx+Aa\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)-Bb\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="frica
s")
```

output
```
1/2*(2*(B*a + A*b)*d*x + A*a*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - B*
b*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(34) = 68$.

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \cot(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} -\frac{Aa\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{Aa\log\left(\tan\left(c+dx\right)\right)}{d} + Abx + Bax + \frac{Bb\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} & \text{for } d \neq 0 \\ x(A+B\tan\left(c\right))\left(a+b\tan\left(c\right)\right)\cot\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((-A*a*log(tan(c + d*x)**2 + 1)/(2*d) + A*a*log(tan(c + d*x))/d +
 A*b*x + B*a*x + B*b*log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*(A + B*
tan(c))*(a + b*tan(c))*cot(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \cot(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,Aa\log\left(\tan\left(dx+c\right)\right) + 2\,(Ba+Ab)(dx+c) - (Aa-Bb)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="maxim
a")
```

output
```
1/2*(2*A*a*log(tan(d*x + c)) + 2*(B*a + A*b)*(d*x + c) - (A*a - B*b)*log(t
an(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.49

$$\int \cot(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{Aa\log\left(|\tan\left(dx+c\right)|\right)}{d}+\frac{(Ba+Ab)(dx+c)}{d}-\frac{(Aa-Bb)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="giac"
)
```

output
```
A*a*log(abs(tan(d*x + c)))/d + (B*a + A*b)*(d*x + c)/d - 1/2*(A*a - B*b)*l
og(tan(d*x + c)^2 + 1)/d
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.86

$$\int \cot(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{A\,a\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)}{2\,d}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
(log(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)*1i)/(2*d) - (log(tan(c + d*x
) - 1i)*(A + B*1i)*(a + b*1i))/(2*d) + (A*a*log(tan(c + d*x)))/d
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.59

$$\int \cot(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)a^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)b^2-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b^2-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d}$$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
( - log(tan((c + d*x)/2)**2 + 1)*a**2 + log(tan((c + d*x)/2)**2 + 1)*b**2
- log(tan((c + d*x)/2) - 1)*b**2 - log(tan((c + d*x)/2) + 1)*b**2 + log(ta
n((c + d*x)/2))*a**2 + 2*a*b*d*x)/d
```

3.236 $\int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 43

$$\int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = -((aA-bB)x)-\frac{aA\cot(c+dx)}{d}+\frac{(Ab+aB)\log(\sin(c+dx))}{d}$$

output
```
-(A*a-B*b)*x-a*A*cot(d*x+c)/d+(A*b+B*a)*ln(sin(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = bBx-\frac{aA\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}+\frac{Ab\log(\sin(c+dx))}{d}+\frac{aB\log(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
b*B*x - (a*A*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2]
)/d + (A*b*Log[Sin[c + d*x]])/d + (a*B*Log[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 4074, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{ 4074}$$

$$\int \cot(c+dx)(Ab+aB-(aA-bB)\tan(c+dx))dx-\frac{aA\cot(c+dx)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{aA\cot(c+dx)}{d}$$

$$\Big\downarrow \text{ 4014}$$

$$(aB+Ab)\int \cot(c+dx)dx-(x(aA-bB))-\frac{aA\cot(c+dx)}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$(aB+Ab)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-(x(aA-bB))-\frac{aA\cot(c+dx)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$-(aB+Ab)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-(x(aA-bB))-\frac{aA\cot(c+dx)}{d}$$

$$\Big\downarrow \ 3956$$

$$\frac{(aB+Ab)\log(-\sin(c+dx))}{d}-(x(aA-bB))-\frac{aA\cot(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-((a*A - b*B)*x) - (a*A*Cot[c + d*x])/d + ((A*b + a*B)*Log[-Sin[c + d*x]])
/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

method	result
parallelrisch	$\frac{(-Ab-Ba)\ln\left(\sec(dx+c)^2\right)+(2Ab+2Ba)\ln(\tan(dx+c))-2aA\cot(dx+c)-2dx(aA-Bb)}{2d}$
derivativedivides	$\frac{\frac{(-Ab-Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-aA+Bb)\arctan(\tan(dx+c))+(Ab+Ba)\ln(\tan(dx+c))-\frac{aA}{\tan(dx+c)}}{d}$
default	$\frac{\frac{(-Ab-Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-aA+Bb)\arctan(\tan(dx+c))+(Ab+Ba)\ln(\tan(dx+c))-\frac{aA}{\tan(dx+c)}}{d}$
norman	$\frac{(-aA+Bb)x\tan(dx+c)-\frac{aA}{d}}{\tan(dx+c)}+\frac{(Ab+Ba)\ln(\tan(dx+c))}{d}-\frac{(Ab+Ba)\ln\left(1+\tan(dx+c)^2\right)}{2d}$
risch	$-iAbx-iBax-Aax+Bbx-\frac{2iAbc}{d}-\frac{2iaBc}{d}-\frac{2iaA}{d\left(e^{2i(dx+c)}-1\right)}+\frac{\ln\left(e^{2i(dx+c)}-1\right)Ab}{d}+\frac{a\ln(e^{2}$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/2*((-A*b-B*a)*ln(sec(d*x+c)^2)+(2*A*b+2*B*a)*ln(tan(d*x+c))-2*a*A*cot(d*
x+c)-2*d*x*(A*a-B*b))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{2\,(Aa-Bb)dx\tan\left(dx+c\right)-(Ba+Ab)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)+2\,Aa}{2\,d\tan\left(dx+c\right)}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/2*(2*(A*a - B*b)*d*x*tan(d*x + c) - (B*a + A*b)*log(tan(d*x + c)^2/(tan
(d*x + c)^2 + 1))*tan(d*x + c) + 2*A*a)/(d*tan(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(36) = 72$.

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.81

$$\int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \tilde{\infty}Aax \\ x(A+B\tan\left(c\right))\left(a+b\tan\left(c\right)\right)\cot^2\left(c\right) \\ \tilde{\infty}Aax \\ -Aax-\frac{Aa}{d\tan\left(c+dx\right)}-\frac{Ab\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ab\log\left(\tan\left(c+dx\right)\right)}{d}-\frac{Ba\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ba\log\left(\tan\left(c+dx\right)\right)}{d}+Bb \end{cases}$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c
))*cot(c)**2, Eq(d, 0)), (zoo*A*a*x, Eq(c, -d*x)), (-A*a*x - A*a/(d*tan(c
+ d*x)) - A*b*log(tan(c + d*x)**2 + 1)/(2*d) + A*b*log(tan(c + d*x))/d - B
*a*log(tan(c + d*x)**2 + 1)/(2*d) + B*a*log(tan(c + d*x))/d + B*b*x, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{2\,(Aa-Bb)(dx+c)+(Ba+Ab)\log\left(\tan\left(dx+c\right)^2+1\right)-2\,(Ba+Ab)\log\left(\tan\left(dx+c\right)\right)+\frac{2\,Aa}{\tan(dx+c)}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/2*(2*(A*a - B*b)*(d*x + c) + (B*a + A*b)*log(tan(d*x + c)^2 + 1) - 2*(B
*a + A*b)*log(tan(d*x + c)) + 2*A*a/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\begin{aligned}\int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx &= -\frac{(Aa-Bb)(dx+c)}{d}-\frac{(Ba+Ab)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}\\ &\quad+\frac{(Ba+Ab)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}-\frac{Aa}{d\tan\left(dx+c\right)}\end{aligned}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
-(A*a - B*b)*(d*x + c)/d - 1/2*(B*a + A*b)*log(tan(d*x + c)^2 + 1)/d + (B*
a + A*b)*log(abs(tan(d*x + c)))/d - A*a/(d*tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{\ln\left(\tan(c+d\,x)\right)\,\left(A\,b+B\,a\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)}{2\,d}-\frac{A\,a\cot(c+d\,x)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
(log(tan(c + d*x))*(A*b + B*a))/d + (log(tan(c + d*x) - 1i)*(A + B*1i)*(a
+ b*1i)*1i)/(2*d) - (log(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b))/(2*d) -
 (A*a*cot(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int \cot^2(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{-\cos\left(dx+c\right)a^2-2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+c\right)ab+2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sin\left(dx+c\right)ab-\sin(}{\sin\left(dx+c\right)d}$$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
( - cos(c + d*x)*a**2 - 2*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a*b +
2*log(tan((c + d*x)/2))*sin(c + d*x)*a*b - sin(c + d*x)*a**2*d*x + sin(c +
 d*x)*b**2*d*x)/(sin(c + d*x)*d)
```

3.237 $\int \cot^3(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 66

$$\int \cot^3(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = -((Ab+aB)x) - \frac{(Ab+aB)\cot(c+dx)}{d} - \frac{aA\cot^2(c+dx)}{2d} - \frac{(aA-bB)\log(\sin(c+dx))}{d}$$

output
```
-(A*b+B*a)*x-(A*b+B*a)*cot(d*x+c)/d-1/2*a*A*cot(d*x+c)^2/d-(A*a-B*b)*ln(si
n(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.62

$$\int \cot^3(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{aA\csc^2(c+dx)}{2d} - \frac{Ab\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d} - \frac{aB\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d} - \frac{aA\log(\sin(c+dx))}{d} + \frac{bB\log(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(a*A*Csc[c + d*x]^2)/d - (A*b*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1,
 1/2, -Tan[c + d*x]^2])/d - (a*B*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1
/2, -Tan[c + d*x]^2])/d - (a*A*Log[Sin[c + d*x]])/d + (b*B*Log[Sin[c + d*x
]])/d
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 4074, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$
$$\Big\downarrow \text{4074}$$
$$\int \cot^2(c+dx)(Ab+aB-(aA-bB)\tan(c+dx))dx - \frac{aA\cot^2(c+dx)}{2d}$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4012}$$

$$\int -\cot(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx-\frac{(aB+Ab)\cot(c+dx)}{d}-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx-\frac{(aB+Ab)\cot(c+dx)}{d}-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{(aB+Ab)\cot(c+dx)}{d}-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4014}$$

$$-(aA-bB)\int \cot(c+dx)dx-\frac{(aB+Ab)\cot(c+dx)}{d}-x(aB+Ab)-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-(aA-bB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{(aB+Ab)\cot(c+dx)}{d}-x(aB+Ab)-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{25}$$

$$(aA-bB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{(aB+Ab)\cot(c+dx)}{d}-x(aB+Ab)-\frac{aA\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(aB+Ab)\cot(c+dx)}{d}-\frac{(aA-bB)\log(-\sin(c+dx))}{d}-x(aB+Ab)-\frac{aA\cot^2(c+dx)}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-((A*b + a*B)*x) - ((A*b + a*B)*Cot[c + d*x])/d - (a*A*Cot[c + d*x]^2)/(2*
d) - ((a*A - b*B)*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{\frac{(aA-Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Ab-Ba)\arctan(\tan(dx+c))-\frac{Ab+Ba}{\tan(dx+c)}+(-aA+Bb)\ln(\tan(dx+c))-\frac{aA}{2\tan(dx+c)^2}}{d}$
default	$\frac{\frac{(aA-Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Ab-Ba)\arctan(\tan(dx+c))-\frac{Ab+Ba}{\tan(dx+c)}+(-aA+Bb)\ln(\tan(dx+c))-\frac{aA}{2\tan(dx+c)^2}}{d}$
parallelrisch	$-\frac{aA\left(-\ln\left(\sec(dx+c)^2\right)+2\ln(\tan(dx+c))\right)-Bb\left(-\ln\left(\sec(dx+c)^2\right)+2\ln(\tan(dx+c))\right)+2Abdx+2Bxad+2A\cot(dx}{2d}$
norman	$\frac{(-Ab-Ba)x\tan(dx+c)^2-\frac{aA}{2d}-\frac{(Ab+Ba)\tan(dx+c)}{d}}{\tan(dx+c)^2}-\frac{(aA-Bb)\ln(\tan(dx+c))}{d}+\frac{(aA-Bb)\ln\left(1+\tan(dx+c)^2\right)}{2d}$
risch	$-Abx-Bax+iAax-iBbx+\frac{2iaAc}{d}-\frac{2iBbc}{d}-\frac{2i\left(iAa\,e^{2i(dx+c)}+Ab\,e^{2i(dx+c)}+Ba\,e^{2i(dx+c)}-Ab-\right.}{d\left(e^{2i(dx+c)}-1\right)^2}$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/2*(A*a-B*b)*ln(1+tan(d*x+c)^2)+(-A*b-B*a)*arctan(tan(d*x+c))-(A*b+B
*a)/tan(d*x+c)+(-A*a+B*b)*ln(tan(d*x+c))-1/2*a*A/tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.44

$$\int\cot^3(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx=$$
$$-\frac{(Aa-Bb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+(2(Ba+Ab)dx+Aa)\tan(dx+c)^2+Aa+2(Ba+Ab}{2\,d\tan(dx+c)^2}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/2*((A*a - B*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2
+ (2*(B*a + A*b)*d*x + A*a)*tan(d*x + c)^2 + A*a + 2*(B*a + A*b)*tan(d*x +
 c))/(d*tan(d*x + c)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(56) = 112$.

Time = 0.75 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.24

$$\int \cot^3(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \begin{cases} \tilde{\infty}Aax \\ x(A+B\tan(c))\,(a+b\tan(c))\cot^3(c) \\ \tilde{\infty}Aax \\ \frac{Aa\log(\tan^2(c+dx)+1)}{2d} - \frac{Aa\log(\tan(c+dx))}{d} - \frac{Aa}{2d\tan^2(c+dx)} - Abx - \frac{Ab}{d\tan(c+dx)} - Bax - \frac{Ba}{d\tan(c+dx)} - \frac{Bb\log(\tan^2 \end{cases}$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c
))*cot(c)**3, Eq(d, 0)), (zoo*A*a*x, Eq(c, -d*x)), (A*a*log(tan(c + d*x)**
2 + 1)/(2*d) - A*a*log(tan(c + d*x))/d - A*a/(2*d*tan(c + d*x)**2) - A*b*x
 - A*b/(d*tan(c + d*x)) - B*a*x - B*a/(d*tan(c + d*x)) - B*b*log(tan(c + d
*x)**2 + 1)/(2*d) + B*b*log(tan(c + d*x))/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.30

$$\int \cot^3(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{2\,(Ba+Ab)(dx+c) - (Aa-Bb)\log\left(\tan(dx+c)^2+1\right) + 2\,(Aa-Bb)\log\left(\tan(dx+c)\right) + \frac{Aa+2\,(Ba}{\tan}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/2*(2*(B*a + A*b)*(d*x + c) - (A*a - B*b)*log(tan(d*x + c)^2 + 1) + 2*(A
*a - B*b)*log(tan(d*x + c)) + (A*a + 2*(B*a + A*b)*tan(d*x + c))/tan(d*x +
 c)^2)/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.44

$$\int \cot^3(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{(Ba+Ab)(dx+c)}{d}+\frac{(Aa-Bb)\log\left(\tan(dx+c)^2+1\right)}{2d} - \frac{(Aa-Bb)\log(|\tan(dx+c)|)}{d}-\frac{Aa+2(Ba+Ab)\tan(dx+c)}{2d\tan(dx+c)^2}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
-(B*a + A*b)*(d*x + c)/d + 1/2*(A*a - B*b)*log(tan(d*x + c)^2 + 1)/d - (A*
a - B*b)*log(abs(tan(d*x + c)))/d - 1/2*(A*a + 2*(B*a + A*b)*tan(d*x + c))
/(d*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.64

$$\int \cot^3(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{\ln(\tan(c+d\,x))\,(A\,a-B\,b)}{d}-\frac{\cot(c+d\,x)^2\,\left(\frac{A\,a}{2}+\tan(c+d\,x)\,(A\,b+B\,a)\right)}{d} + \frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(A+B\,1\mathrm{i})\,(a+b\,1\mathrm{i})}{2\,d} - \frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(A-B\,1\mathrm{i})\,(b+a\,1\mathrm{i})\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
(log(tan(c + d*x) - 1i)*(A + B*1i)*(a + b*1i))/(2*d) - (cot(c + d*x)^2*((A
*a)/2 + tan(c + d*x)*(A*b + B*a)))/d - (log(tan(c + d*x))*(A*a - B*b))/d -
 (log(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)*1i)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.44

$$\int \cot^3(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{-8\cos(dx+c)\sin(dx+c)\,ab+4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)^2a^2-4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\mathrm{si}}{}$$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
( - 8*cos(c + d*x)*sin(c + d*x)*a*b + 4*log(tan((c + d*x)/2)**2 + 1)*sin(c
 + d*x)**2*a**2 - 4*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*b**2 - 4*
log(tan((c + d*x)/2))*sin(c + d*x)**2*a**2 + 4*log(tan((c + d*x)/2))*sin(c
 + d*x)**2*b**2 + sin(c + d*x)**2*a**2 - 8*sin(c + d*x)**2*a*b*d*x - 2*a**
2)/(4*sin(c + d*x)**2*d)
```

3.238 $\int \cot^4(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 87

$$\int \cot^4(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = (aA-bB)x+\frac{(aA-bB)\cot(c+dx)}{d}-\frac{(Ab+aB)\cot^2(c+dx)}{2d}-\frac{aA\cot^3(c+dx)}{3d}-\frac{(Ab+aB)\log(\sin(c+dx))}{d}$$

output
```
(A*a-B*b)*x+(A*a-B*b)*cot(d*x+c)/d-1/2*(A*b+B*a)*cot(d*x+c)^2/d-1/3*a*A*co
t(d*x+c)^3/d-(A*b+B*a)*ln(sin(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

$$\int \cot^4(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{Ab\csc^2(c+dx)}{2d} - \frac{aB\csc^2(c+dx)}{2d} - \frac{aA\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d} - \frac{bB\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d} - \frac{Ab\log(\sin(c+dx))}{d} - \frac{aB\log(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(A*b*Csc[c + d*x]^2)/d - (a*B*Csc[c + d*x]^2)/(2*d) - (a*A*Cot[c + d*
x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) - (b*B*Cot[c
 + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d - (A*b*Log[Sin
[c + d*x]])/d - (a*B*Log[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {3042, 4074, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4074}$$

$$\int \cot^3(c+dx)(Ab+aB-(aA-bB)\tan(c+dx))dx-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan(c+dx)^3}dx-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 4012

$$\int -\cot^2(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx-\frac{(aB+Ab)\cot^2(c+dx)}{2d}-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 25

$$-\int \cot^2(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx-\frac{(aB+Ab)\cot^2(c+dx)}{2d}-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$-\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{(aB+Ab)\cot^2(c+dx)}{2d}-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 4012

$$-\int \cot(c+dx)(Ab+aB-(aA-bB)\tan(c+dx))dx-\frac{(aB+Ab)\cot^2(c+dx)}{2d}+\frac{(aA-bB)\cot(c+dx)}{d}-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$-\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{(aB+Ab)\cot^2(c+dx)}{2d}+\frac{(aA-bB)\cot(c+dx)}{d}-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 4014

$$-(aB+Ab)\int \cot(c+dx)dx-\frac{(aB+Ab)\cot^2(c+dx)}{2d}+\frac{(aA-bB)\cot(c+dx)}{d}+x(aA-bB)-\frac{aA\cot^3(c+dx)}{3d}$$

$\Big\downarrow$ 3042

$$-(aB+Ab)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{(aB+Ab)\cot^2(c+dx)}{2d}+\frac{(aA-bB)\cot(c+dx)}{d}+x(aA-bB)-\frac{aA\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{25}$$

$$(aB+Ab)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{(aB+Ab)\cot^2(c+dx)}{2d}+\frac{(aA-bB)\cot(c+dx)}{d}+x(aA-bB)-\frac{aA\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(aB+Ab)\cot^2(c+dx)}{2d}+\frac{(aA-bB)\cot(c+dx)}{d}-\frac{(aB+Ab)\log(-\sin(c+dx))}{d}+x(aA-bB)-\frac{aA\cot^3(c+dx)}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(a*A - b*B)*x + ((a*A - b*B)*Cot[c + d*x])/d - ((A*b + a*B)*Cot[c + d*x]^2
)/(2*d) - (a*A*Cot[c + d*x]^3)/(3*d) - ((A*b + a*B)*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{(Ab+Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(aA-Bb)\arctan(\tan(dx+c))+(-Ab-Ba)\ln(\tan(dx+c))-\frac{Ab+Ba}{2\tan(dx+c)^2}-\frac{-aA+Bb}{\tan(dx+c)}-\frac{}{3\tan}}{d}$
default	$\frac{\frac{(Ab+Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(aA-Bb)\arctan(\tan(dx+c))+(-Ab-Ba)\ln(\tan(dx+c))-\frac{Ab+Ba}{2\tan(dx+c)^2}-\frac{-aA+Bb}{\tan(dx+c)}-\frac{}{3\tan}}{d}$
norman	$\frac{\frac{(aA-Bb)\tan(dx+c)^2}{d}+(aA-Bb)x\tan(dx+c)^3-\frac{aA}{3d}-\frac{(Ab+Ba)\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{(Ab+Ba)\ln(\tan(dx+c))}{d}+\frac{(Ab+Ba)\ln(}{}$
parallelrisch	$\frac{-3Ab\left(-\ln\left(\sec(dx+c)^2\right)+2\ln(\tan(dx+c))\right)-3Ba\left(-\ln\left(\sec(dx+c)^2\right)+2\ln(\tan(dx+c))\right)-2A\cot(dx+c)^3a-3A\cot(}{6d}$
risch	$iAbx+iBax+Aax-Bbx+\frac{2iAbc}{d}+\frac{2iaBc}{d}-\frac{2i(3iAb\,e^{4i(dx+c)}+3iBa\,e^{4i(dx+c)}-6Aa\,e^{4i(dx+c)}+3I}{}$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/2*(A*b+B*a)*ln(1+tan(d*x+c)^2)+(A*a-B*b)*arctan(tan(d*x+c))+(-A*b-B
*a)*ln(tan(d*x+c))-1/2*(A*b+B*a)/tan(d*x+c)^2-(-A*a+B*b)/tan(d*x+c)-1/3*a*
A/tan(d*x+c)^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \cot^4(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{3\,(Ba+Ab)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3-3\,(2\,(Aa-Bb)dx-Ba-Ab)\tan\left(dx+c\right)^3-6\,(Aa-}{6\,d\tan\left(dx+c\right)^3}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/6*(3*(B*a + A*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^
3 - 3*(2*(A*a - B*b)*d*x - B*a - A*b)*tan(d*x + c)^3 - 6*(A*a - B*b)*tan(d
*x + c)^2 + 2*A*a + 3*(B*a + A*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(75) = 150.

Time = 0.85 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.05

$$\int \cot^4(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\begin{cases}\tilde{\infty}Aax\\ x(A+B\tan(c))(a+b\tan(c))\cot^4(c)\\ \tilde{\infty}Aax\\ Aax+\frac{Aa}{d\tan(c+dx)}-\frac{Aa}{3d\tan^3(c+dx)}+\frac{Ab\log(\tan^2(c+dx)+1)}{2d}-\frac{Ab\log(\tan(c+dx))}{d}-\frac{Ab}{2d\tan^2(c+dx)}+\frac{Ba\log(\tan^2(c+dx)\cdot}{2d}\end{cases}$$

input
```
integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c
))*cot(c)**4, Eq(d, 0)), (zoo*A*a*x, Eq(c, -d*x)), (A*a*x + A*a/(d*tan(c +
 d*x)) - A*a/(3*d*tan(c + d*x)**3) + A*b*log(tan(c + d*x)**2 + 1)/(2*d) -
A*b*log(tan(c + d*x))/d - A*b/(2*d*tan(c + d*x)**2) + B*a*log(tan(c + d*x)
**2 + 1)/(2*d) - B*a*log(tan(c + d*x))/d - B*a/(2*d*tan(c + d*x)**2) - B*b
*x - B*b/(d*tan(c + d*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \cot^4(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{6\,(Aa-Bb)(dx+c)+3\,(Ba+Ab)\log\left(\tan\left(dx+c\right)^2+1\right)-6\,(Ba+Ab)\log\left(\tan\left(dx+c\right)\right)+\frac{6\,(Aa-Bb)}{}}{6\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/6*(6*(A*a - B*b)*(d*x + c) + 3*(B*a + A*b)*log(tan(d*x + c)^2 + 1) - 6*(
B*a + A*b)*log(tan(d*x + c)) + (6*(A*a - B*b)*tan(d*x + c)^2 - 2*A*a - 3*(
B*a + A*b)*tan(d*x + c))/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

$$\begin{aligned}\int \cot^4(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx &= \frac{(Aa-Bb)(dx+c)}{d}+\frac{(Ba+Ab)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}\\ &\quad-\frac{(Ba+Ab)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}\\ &\quad+\frac{6\,(Aa-Bb)\tan\left(dx+c\right)^2-2\,Aa-3\,(Ba+Ab)\tan\left(dx+c\right)}{6\,d\tan\left(dx+c\right)^3}\end{aligned}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
(A*a - B*b)*(d*x + c)/d + 1/2*(B*a + A*b)*log(tan(d*x + c)^2 + 1)/d - (B*a
 + A*b)*log(abs(tan(d*x + c)))/d + 1/6*(6*(A*a - B*b)*tan(d*x + c)^2 - 2*A
*a - 3*(B*a + A*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.46

$$\int \cot^4(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= -\frac{\cot(c+d\,x)^3\,\left((B\,b-A\,a)\,\tan(c+d\,x)^2+\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)\,\tan(c+d\,x)+\frac{A\,a}{3}\right)}{d} - \frac{\ln\left(\tan(c+d\,x)\right)\,\left(A\,b+B\,a\right)}{d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)}{2\,d}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
(log(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b))/(2*d) - (log(tan(c + d*x))*
(A*b + B*a))/d - (log(tan(c + d*x) - 1i)*(A + B*1i)*(a + b*1i)*1i)/(2*d) -
 (cot(c + d*x)^3*((A*a)/3 + tan(c + d*x)*((A*b)/2 + (B*a)/2) - tan(c + d*x
)^2*(A*a - B*b)))/d
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.87

$$\int \cot^4(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{8\cos(dx+c)\sin(dx+c)^2a^2-6\cos(dx+c)\sin(dx+c)^2b^2-2\cos(dx+c)a^2+12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}{}$$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(8*cos(c + d*x)*sin(c + d*x)**2*a**2 - 6*cos(c + d*x)*sin(c + d*x)**2*b**2
 - 2*cos(c + d*x)*a**2 + 12*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*a
*b - 12*log(tan((c + d*x)/2))*sin(c + d*x)**3*a*b + 6*sin(c + d*x)**3*a**2
*d*x + 3*sin(c + d*x)**3*a*b - 6*sin(c + d*x)**3*b**2*d*x - 6*sin(c + d*x)
*a*b)/(6*sin(c + d*x)**3*d)
```

3.239 $\int \cot^5(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 108

$$\int \cot^5(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = (Ab+aB)x+\frac{(Ab+aB)\cot(c+dx)}{d}+\frac{(aA-bB)\cot^2(c+dx)}{2d} - \frac{(Ab+aB)\cot^3(c+dx)}{3d}-\frac{aA\cot^4(c+dx)}{4d}+\frac{(aA-bB)\log(\sin(c+dx))}{d}$$

output
```
(A*b+B*a)*x+(A*b+B*a)*cot(d*x+c)/d+1/2*(A*a-B*b)*cot(d*x+c)^2/d-1/3*(A*b+B
*a)*cot(d*x+c)^3/d-1/4*a*A*cot(d*x+c)^4/d+(A*a-B*b)*ln(sin(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.35

$$\int \cot^5(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{aA\csc^2(c+dx)}{d}-\frac{bB\csc^2(c+dx)}{2d}-\frac{aA\csc^4(c+dx)}{4d}$$
$$-\frac{Ab\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}$$
$$-\frac{aB\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}$$
$$+\frac{aA\log(\sin(c+dx))}{d}-\frac{bB\log(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(a*A*Csc[c + d*x]^2)/d - (b*B*Csc[c + d*x]^2)/(2*d) - (a*A*Csc[c + d*x]^4)
/(4*d) - (A*b*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x
]^2])/(3*d) - (a*B*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c
+ d*x]^2])/(3*d) + (a*A*Log[Sin[c + d*x]])/d - (b*B*Log[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {3042, 4074, 3042, 4012, 25, 3042, 4012, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{ 4074}$$

$$\int \cot^4(c+dx)(Ab+aB-(aA-bB)\tan(c+dx))dx-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan(c+dx)^4}dx-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \ 4012$$

$$\int -\cot^3(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \ 25$$

$$-\int \cot^3(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \ 3042$$

$$-\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan(c+dx)^3}dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \ 4012$$

$$-\int \cot^2(c+dx)(Ab+aB-(aA-bB)\tan(c+dx))dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}+\frac{(aA-bB)\cot^2(c+dx)}{2d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \ 3042$$

$$-\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}+\frac{(aA-bB)\cot^2(c+dx)}{2d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \ 4012$$

$$-\int -\cot(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}+\frac{(aA-bB)\cot^2(c+dx)}{2d}+\frac{(aB+Ab)\cot(c+dx)}{d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \ 25$$

$$\int \cot(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}+\frac{(aA-bB)\cot^2(c+dx)}{2d}+\frac{(aB+Ab)\cot(c+dx)}{d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan(c+dx)}dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}+\frac{(aA-bB)\cot^2(c+dx)}{2d}+\frac{(aB+Ab)\cot(c+dx)}{d}-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{4014}$$

$$(aA-bB)\int \cot(c+dx)dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}+\frac{(aA-bB)\cot^2(c+dx)}{2d}+\frac{(aB+Ab)\cot(c+dx)}{d}+x(aB+Ab)-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$(aA-bB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}+\frac{(aA-bB)\cot^2(c+dx)}{2d}+\frac{(aB+Ab)\cot(c+dx)}{d}+x(aB+Ab)-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{25}$$

$$-(aA-bB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{(aB+Ab)\cot^3(c+dx)}{3d}+\frac{(aA-bB)\cot^2(c+dx)}{2d}+\frac{(aB+Ab)\cot(c+dx)}{d}+x(aB+Ab)-\frac{aA\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{(aB+Ab)\cot^3(c+dx)}{3d}+\frac{(aA-bB)\cot^2(c+dx)}{2d}+\frac{(aB+Ab)\cot(c+dx)}{d}+\frac{(aA-bB)\log(-\sin(c+dx))}{d}+x(aB+Ab)-\frac{aA\cot^4(c+dx)}{4d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(A*b + a*B)*x + ((A*b + a*B)*Cot[c + d*x])/d + ((a*A - b*B)*Cot[c + d*x]^2
)/(2*d) - ((A*b + a*B)*Cot[c + d*x]^3)/(3*d) - (a*A*Cot[c + d*x]^4)/(4*d)
+ ((a*A - b*B)*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{(-aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Ab+Ba)\arctan(\tan(dx+c))-\frac{-Ab-Ba}{\tan(dx+c)}-\frac{Ab+Ba}{3\tan(dx+c)^3}-\frac{-aA+Bb}{2\tan(dx+c)^2}+(aA-Bb)\ln(\tan}{d}$
default	$\frac{\frac{(-aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Ab+Ba)\arctan(\tan(dx+c))-\frac{-Ab-Ba}{\tan(dx+c)}-\frac{Ab+Ba}{3\tan(dx+c)^3}-\frac{-aA+Bb}{2\tan(dx+c)^2}+(aA-Bb)\ln(\tan}{d}$
norman	$\frac{\frac{(Ab+Ba)\tan(dx+c)^3}{d}+(Ab+Ba)x\tan(dx+c)^4-\frac{aA}{4d}-\frac{(Ab+Ba)\tan(dx+c)}{3d}+\frac{(aA-Bb)\tan(dx+c)^2}{2d}}{\tan(dx+c)^4}+\frac{(aA-Bb)\ln(\tan(dx-}{d}$
parallelrisch	$-\frac{-6aA\left(-\ln\left(\sec(dx+c)^2\right)+2\ln(\tan(dx+c))\right)+6Bb\left(-\ln\left(\sec(dx+c)^2\right)+2\ln(\tan(dx+c))\right)+3A\cot(dx+c)^4a+4A\,c}{}$
risch	$Abx+Bax-iAax+iBbx-\frac{2iaAc}{d}+\frac{2iBbc}{d}-\frac{2\left(-6iAb\,e^{6i(dx+c)}-6iBa\,e^{6i(dx+c)}+6Aa\,e^{6i(dx+c)}-3\right.}{}$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/2*(-A*a+B*b)*ln(1+tan(d*x+c)^2)+(A*b+B*a)*arctan(tan(d*x+c))-(-A*b-
B*a)/tan(d*x+c)-1/3*(A*b+B*a)/tan(d*x+c)^3-1/2*(-A*a+B*b)/tan(d*x+c)^2+(A*
a-B*b)*ln(tan(d*x+c))-1/4*a*A/tan(d*x+c)^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.28

$$\int \cot^5(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{6\,(Aa-Bb)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+3\,(4\,(Ba+Ab)dx+3\,Aa-2\,Bb)\tan\left(dx+c\right)^4+12\,(Ba}{12\,d\tan\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/12*(6*(A*a - B*b)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^
4 + 3*(4*(B*a + A*b)*d*x + 3*A*a - 2*B*b)*tan(d*x + c)^4 + 12*(B*a + A*b)*
tan(d*x + c)^3 + 6*(A*a - B*b)*tan(d*x + c)^2 - 3*A*a - 4*(B*a + A*b)*tan(
d*x + c))/(d*tan(d*x + c)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(95) = 190$.

Time = 1.36 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.94

$$\int \cot^5(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \tilde{\infty}Aax \\ x(A+B\tan(c))\,(a+b\tan(c))\cot^5(c) \\ \tilde{\infty}Aax \\ -\frac{Aa\log(\tan^2(c+dx)+1)}{2d} + \frac{Aa\log(\tan(c+dx))}{d} + \frac{Aa}{2d\tan^2(c+dx)} - \frac{Aa}{4d\tan^4(c+dx)} + Abx + \frac{Ab}{d\tan(c+dx)} - \frac{Ab}{3d\tan^3(c+dx)} + \end{cases}$$

input
```
integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c
))*cot(c)**5, Eq(d, 0)), (zoo*A*a*x, Eq(c, -d*x)), (-A*a*log(tan(c + d*x)*
*2 + 1)/(2*d) + A*a*log(tan(c + d*x))/d + A*a/(2*d*tan(c + d*x)**2) - A*a/
(4*d*tan(c + d*x)**4) + A*b*x + A*b/(d*tan(c + d*x)) - A*b/(3*d*tan(c + d*
x)**3) + B*a*x + B*a/(d*tan(c + d*x)) - B*a/(3*d*tan(c + d*x)**3) + B*b*lo
g(tan(c + d*x)**2 + 1)/(2*d) - B*b*log(tan(c + d*x))/d - B*b/(2*d*tan(c +
d*x)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \cot^5(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{12\,(Ba+Ab)(dx+c)-6\,(Aa-Bb)\log\left(\tan\left(dx+c\right)^2+1\right)+12\,(Aa-Bb)\log\left(\tan\left(dx+c\right)\right)+\frac{12\,(Ba+}{}}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/12*(12*(B*a + A*b)*(d*x + c) - 6*(A*a - B*b)*log(tan(d*x + c)^2 + 1) + 1
2*(A*a - B*b)*log(tan(d*x + c)) + (12*(B*a + A*b)*tan(d*x + c)^3 + 6*(A*a
- B*b)*tan(d*x + c)^2 - 3*A*a - 4*(B*a + A*b)*tan(d*x + c))/tan(d*x + c)^4
)/d
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19

$$\int \cot^5(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx=\frac{(Ba+Ab)(dx+c)}{d}$$
$$-\frac{(Aa-Bb)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}+\frac{(Aa-Bb)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$
$$+\frac{12\,(Ba+Ab)\tan\left(dx+c\right)^3+6\,(Aa-Bb)\tan\left(dx+c\right)^2-3\,Aa-4\,(Ba+Ab)\tan\left(dx+c\right)}{12\,d\tan\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
(B*a + A*b)*(d*x + c)/d - 1/2*(A*a - B*b)*log(tan(d*x + c)^2 + 1)/d + (A*a
 - B*b)*log(abs(tan(d*x + c)))/d + 1/12*(12*(B*a + A*b)*tan(d*x + c)^3 + 6
*(A*a - B*b)*tan(d*x + c)^2 - 3*A*a - 4*(B*a + A*b)*tan(d*x + c))/(d*tan(d
*x + c)^4)
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.34

$$\int \cot^5(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{\ln(\tan(c+dx))\,(A\,a-B\,b)}{d} - \frac{\cot(c+dx)^4\left((-A\,b-B\,a)\,\tan(c+dx)^3+\left(\frac{B\,b}{2}-\frac{A\,a}{2}\right)\,\tan(c+dx)^2+\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,\tan(c+dx)+\frac{A\,a}{4}\right)}{d} - \frac{\ln(\tan(c+dx)-\mathrm{i})\,(A+B\,1\mathrm{i})\,(a+b\,1\mathrm{i})}{2\,d} + \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(A-B\,1\mathrm{i})\,(b+a\,1\mathrm{i})\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
(log(tan(c + d*x))*(A*a - B*b))/d - (cot(c + d*x)^4*((A*a)/4 + tan(c + d*x
)*((A*b)/3 + (B*a)/3) - tan(c + d*x)^3*(A*b + B*a) - tan(c + d*x)^2*((A*a)
/2 - (B*b)/2)))/d - (log(tan(c + d*x) - 1i)*(A + B*1i)*(a + b*1i))/(2*d) +
 (log(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)*1i)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.03

$$\int \cot^5(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{256\cos(dx+c)\sin(dx+c)^3ab-64\cos(dx+c)\sin(dx+c)ab-96\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)}{}$$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(256*cos(c + d*x)*sin(c + d*x)**3*a*b - 64*cos(c + d*x)*sin(c + d*x)*a*b -
 96*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**4*a**2 + 96*log(tan((c + d*
x)/2)**2 + 1)*sin(c + d*x)**4*b**2 + 96*log(tan((c + d*x)/2))*sin(c + d*x)
**4*a**2 - 96*log(tan((c + d*x)/2))*sin(c + d*x)**4*b**2 - 39*sin(c + d*x)
**4*a**2 + 192*sin(c + d*x)**4*a*b*d*x + 24*sin(c + d*x)**4*b**2 + 96*sin(
c + d*x)**2*a**2 - 48*sin(c + d*x)**2*b**2 - 24*a**2)/(96*sin(c + d*x)**4*
d)
```

3.240 $\int \tan^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 148

$$\begin{aligned}&\int \tan^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx\\&=-\left(\left(a^2A-Ab^2-2abB\right)x\right)+\frac{(2aAb+a^2B-b^2B)\log(\cos(c+dx))}{d}\\&\quad-\frac{b(Ab+aB)\tan(c+dx)}{d}-\frac{B(a+b\tan(c+dx))^2}{2d}\\&\quad+\frac{(4Ab-aB)(a+b\tan(c+dx))^3}{12b^2d}+\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}\end{aligned}$$

output
```
-(A*a^2-A*b^2-2*B*a*b)*x+(2*A*a*b+B*a^2-B*b^2)*ln(cos(d*x+c))/d-b*(A*b+B*a
)*tan(d*x+c)/d-1/2*B*(a+b*tan(d*x+c))^2/d+1/12*(4*A*b-B*a)*(a+b*tan(d*x+c)
)^3/b^2/d+1/4*B*tan(d*x+c)*(a+b*tan(d*x+c))^3/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.49

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}$$
$$+ \frac{\frac{(4Ab-aB)(a+b\tan(c+dx))^3}{3bd} + \frac{2\left((Ab-aB)\left(i(a+ib)^2\log(i-\tan(c+dx))-i(a-ib)^2\log(i+\tan(c+dx))-2b^2\tan(c+dx)\right)-B\left((ia-b)^3\log(i-\right.}{d}}{4b}$$

input
```
Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(4*b*d) + (((4*A*b - a*B)*(a + b*T
an[c + d*x])^3)/(3*b*d) + (2*((A*b - a*B)*(I*(a + I*b)^2*Log[I - Tan[c + d
*x]] - I*(a - I*b)^2*Log[I + Tan[c + d*x]] - 2*b^2*Tan[c + d*x]) - B*((I*a
 - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*a*b^
2*Tan[c + d*x] + b^3*Tan[c + d*x]^2)))/d)/(4*b)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {3042, 4090, 25, 3042, 4113, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -(a+b\tan(c+dx))^2\left(-\left((4Ab-aB)\tan^2(c+dx)\right)+4bB\tan(c+dx)+aB\right)dx}{4b}+\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int (a+b\tan(c+dx))^2\left(-\left((4Ab-aB)\tan^2(c+dx)\right)+4bB\tan(c+dx)+aB\right)dx}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int (a+b\tan(c+dx))^2\left(-\left((4Ab-aB)\tan(c+dx)^2\right)+4bB\tan(c+dx)+aB\right)dx}{4b}$$

$$\Big\downarrow \text{ 4113}$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int (a+b\tan(c+dx))^2(4Ab+4B\tan(c+dx)b)dx-\frac{(4Ab-aB)(a+b\tan(c+dx))^3}{3bd}}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int (a+b\tan(c+dx))^2(4Ab+4B\tan(c+dx)b)dx-\frac{(4Ab-aB)(a+b\tan(c+dx))^3}{3bd}}{4b}$$

$$\Big\downarrow \text{ 4011}$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int (a+b\tan(c+dx))(4b(aA-bB)+4b(Ab+aB)\tan(c+dx))dx-\frac{(4Ab-aB)(a+b\tan(c+dx))^3}{3bd}+\frac{2bB(a+b\tan(c+dx))^2}{d}}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{\int (a+b\tan(c+dx))(4b(aA-bB)+4b(Ab+aB)\tan(c+dx))dx-\frac{(4Ab-aB)(a+b\tan(c+dx))^3}{3bd}+\frac{2bB(a+b\tan(c+dx))^2}{d}}{4b}$$

$$\Big\downarrow \text{ 4008}$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{4b(a^2B+2aAb-b^2B)\int\tan(c+dx)dx+4bx(a^2A-2abB-Ab^2)+\frac{4b^2(aB+Ab)\tan(c+dx)}{d}-\frac{(4Ab-aB)(a+b\tan(c+d}{3bd}}{4b}$$

$\big\downarrow$ 3042

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{4b(a^2B+2aAb-b^2B)\int\tan(c+dx)dx+4bx(a^2A-2abB-Ab^2)+\frac{4b^2(aB+Ab)\tan(c+dx)}{d}-\frac{(4Ab-aB)(a+b\tan(c+d}{3bd}}{4b}$$

$\big\downarrow$ 3956

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^3}{4bd}-\frac{-\frac{4b(a^2B+2aAb-b^2B)\log(\cos(c+dx))}{d}+4bx(a^2A-2abB-Ab^2)+\frac{4b^2(aB+Ab)\tan(c+dx)}{d}-\frac{(4Ab-aB)(a+b\tan(c+dx))^3}{3bd}+\frac{2bl}{}}{4b}$$

input
```
Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]*(a + b*Tan[c + d*x])^3)/(4*b*d) - (4*b*(a^2*A - A*b^2 - 2*
a*b*B)*x - (4*b*(2*a*A*b + a^2*B - b^2*B)*Log[Cos[c + d*x]])/d + (4*b^2*(A
*b + a*B)*Tan[c + d*x])/d + (2*b*B*(a + b*Tan[c + d*x])^2)/d - ((4*A*b - a
*B)*(a + b*Tan[c + d*x])^3)/(3*b*d))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

method	result
parts	$\frac{(A b^2+2Bab)\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{(2Aab+B a^2)\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+$
norman	$(-A a^2+A b^2+2Bab)x+\frac{(A a^2-A b^2-2Bab)\tan(dx+c)}{d}+\frac{(2Aab+B a^2-B b^2)\tan(dx+c)^2}{2d}+\frac{B b^2 t}{}$
derivativedivides	$\frac{\frac{B b^2\tan(dx+c)^4}{4}+\frac{A b^2\tan(dx+c)^3}{3}+\frac{2Bab\tan(dx+c)^3}{3}+Aab\tan(dx+c)^2+\frac{B\tan(dx+c)^2 a^2}{2}-\frac{B b^2\tan(dx+c)^2}{2}+A\tan(dx+c}{}$
default	$\frac{\frac{B b^2\tan(dx+c)^4}{4}+\frac{A b^2\tan(dx+c)^3}{3}+\frac{2Bab\tan(dx+c)^3}{3}+Aab\tan(dx+c)^2+\frac{B\tan(dx+c)^2 a^2}{2}-\frac{B b^2\tan(dx+c)^2}{2}+A\tan(dx+c}{}$
parallelrisch	$-\frac{-3B b^2\tan(dx+c)^4-4A b^2\tan(dx+c)^3-8Bab\tan(dx+c)^3+12Ax a^2d-12A b^2dx-12Aab\tan(dx+c)^2-24Babdx-6}{}$
risch	$\frac{2iB b^2c}{d}-iB a^2x-2iAabx-A a^2x+A b^2x+2Babx-\frac{4iAabc}{d}+iB b^2x-\frac{2iB a^2c}{d}+\frac{-16}{}$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
(A*b^2+2*B*a*b)/d*(1/3*tan(d*x+c)^3-tan(d*x+c)+arctan(tan(d*x+c)))+(2*A*a*
b+B*a^2)/d*(1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)^2))+A*a^2/d*(tan(d*x+c)-a
rctan(tan(d*x+c)))+B*b^2/d*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2+1/2*ln(1+tan
(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{3\,Bb^2\tan{(dx+c)}^4+4\,(2\,Bab+Ab^2)\tan{(dx+c)}^3-12\,(Aa^2-2\,Bab-Ab^2)dx+6\,(Ba^2+2\,Aab-B}{12\,d}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/12*(3*B*b^2*tan(d*x + c)^4 + 4*(2*B*a*b + A*b^2)*tan(d*x + c)^3 - 12*(A*
a^2 - 2*B*a*b - A*b^2)*d*x + 6*(B*a^2 + 2*A*a*b - B*b^2)*tan(d*x + c)^2 +
6*(B*a^2 + 2*A*a*b - B*b^2)*log(1/(tan(d*x + c)^2 + 1)) + 12*(A*a^2 - 2*B*
a*b - A*b^2)*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.66

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\begin{cases} -Aa^2x+\frac{Aa^2\tan(c+dx)}{d}-\frac{Aab\log(\tan^2(c+dx)+1)}{d}+\frac{Aab\tan^2(c+dx)}{d}+Ab^2x+\frac{Ab^2\tan^3(c+dx)}{3d}-\frac{Ab^2\tan(c+dx)}{d}-\frac{Ba \\ x(A+B\tan(c))\,(a+b\tan(c))^2\tan^2(c) \end{cases}$$

input
```
integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((-A*a**2*x + A*a**2*tan(c + d*x)/d - A*a*b*log(tan(c + d*x)**2 +
 1)/d + A*a*b*tan(c + d*x)**2/d + A*b**2*x + A*b**2*tan(c + d*x)**3/(3*d)
- A*b**2*tan(c + d*x)/d - B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**2*t
an(c + d*x)**2/(2*d) + 2*B*a*b*x + 2*B*a*b*tan(c + d*x)**3/(3*d) - 2*B*a*b
*tan(c + d*x)/d + B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*b**2*tan(c + d
*x)**4/(4*d) - B*b**2*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*tan(c))*
(a + b*tan(c))**2*tan(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{3\,Bb^2\tan(dx+c)^4+4\,(2\,Bab+Ab^2)\tan(dx+c)^3+6\,(Ba^2+2\,Aab-Bb^2)\tan(dx+c)^2-12\,(Aa^2-}{}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/12*(3*B*b^2*tan(d*x + c)^4 + 4*(2*B*a*b + A*b^2)*tan(d*x + c)^3 + 6*(B*a
^2 + 2*A*a*b - B*b^2)*tan(d*x + c)^2 - 12*(A*a^2 - 2*B*a*b - A*b^2)*(d*x +
 c) - 6*(B*a^2 + 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1) + 12*(A*a^2 - 2*
B*a*b - A*b^2)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.43

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Aa^2-2\,Bab-Ab^2)(dx+c)}{d} - \frac{(Ba^2+2\,Aab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$
$$+ \frac{3\,Bb^2d^3\tan\left(dx+c\right)^4+8\,Babd^3\tan\left(dx+c\right)^3+4\,Ab^2d^3\tan\left(dx+c\right)^3+6\,Ba^2d^3\tan\left(dx+c\right)^2+12\,A}{}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c)/d - 1/2*(B*a^2 + 2*A*a*b - B*b^2)*log
(tan(d*x + c)^2 + 1)/d + 1/12*(3*B*b^2*d^3*tan(d*x + c)^4 + 8*B*a*b*d^3*ta
n(d*x + c)^3 + 4*A*b^2*d^3*tan(d*x + c)^3 + 6*B*a^2*d^3*tan(d*x + c)^2 + 1
2*A*a*b*d^3*tan(d*x + c)^2 - 6*B*b^2*d^3*tan(d*x + c)^2 + 12*A*a^2*d^3*tan
(d*x + c) - 24*B*a*b*d^3*tan(d*x + c) - 12*A*b^2*d^3*tan(d*x + c))/d^4
```

Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.02

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= x\left(-A\,a^2+2\,B\,a\,b+A\,b^2\right)+\frac{\tan(c+d\,x)^3\,\left(\frac{A\,b^2}{3}+\frac{2\,B\,a\,b}{3}\right)}{d} - \frac{\tan(c+d\,x)\,\left(-A\,a^2+2\,B\,a\,b+A\,b^2\right)}{d} - \frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{B\,a^2}{2}+A\,a\,b-\frac{B\,b^2}{2}\right)}{d} + \frac{\tan(c+d\,x)^2\,\left(\frac{B\,a^2}{2}+A\,a\,b-\frac{B\,b^2}{2}\right)}{d}+\frac{B\,b^2\tan(c+d\,x)^4}{4\,d}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
x*(A*b^2 - A*a^2 + 2*B*a*b) + (tan(c + d*x)^3*((A*b^2)/3 + (2*B*a*b)/3))/d
 - (tan(c + d*x)*(A*b^2 - A*a^2 + 2*B*a*b))/d - (log(tan(c + d*x)^2 + 1)*(
(B*a^2)/2 - (B*b^2)/2 + A*a*b))/d + (tan(c + d*x)^2*((B*a^2)/2 - (B*b^2)/2
 + A*a*b))/d + (B*b^2*tan(c + d*x)^4)/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{-6\log\left(\tan(dx+c)^2+1\right)a^2b+2\log\left(\tan(dx+c)^2+1\right)b^3+\tan(dx+c)^4\,b^3+4\tan(dx+c)^3\,a\,b^2+6}{4d}$$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
( - 6*log(tan(c + d*x)**2 + 1)*a**2*b + 2*log(tan(c + d*x)**2 + 1)*b**3 +
tan(c + d*x)**4*b**3 + 4*tan(c + d*x)**3*a*b**2 + 6*tan(c + d*x)**2*a**2*b
 - 2*tan(c + d*x)**2*b**3 + 4*tan(c + d*x)*a**3 - 12*tan(c + d*x)*a*b**2 -
 4*a**3*d*x + 12*a*b**2*d*x)/(4*d)
```

3.241 $\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 112

$$\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\left(\left(2aAb+a^2B-b^2B\right)x\right)-\frac{(a^2A-Ab^2-2abB)\log(\cos(c+dx))}{d} + \frac{b(aA-bB)\tan(c+dx)}{d}+\frac{A(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3bd}$$

output
```
-(2*A*a*b+B*a^2-B*b^2)*x-(A*a^2-A*b^2-2*B*a*b)*ln(cos(d*x+c))/d+b*(A*a-B*b
)*tan(d*x+c)/d+1/2*A*(a+b*tan(d*x+c))^2/d+1/3*B*(a+b*tan(d*x+c))^3/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.54

$$\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{2B(a+b\tan(c+dx))^3+3(aA+bB)\left(i((a+ib)^2\log(i-\tan(c+dx))-(a-ib)^2\log(i+\tan(c+dx))\right)}{}$$

input
```
Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*(a + b*Tan[c + d*x])^3 + 3*(a*A + b*B)*(I*((a + I*b)^2*Log[I - Tan[c
+ d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x]]) - 2*b^2*Tan[c + d*x]) + 3*A*(
(I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*
a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2))/(6*b*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 4075, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^2dx+\frac{B(a+b\tan(c+dx))^3}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^2dx+\frac{B(a+b\tan(c+dx))^3}{3bd}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{A(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{A(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3bd}$$

$\Big\downarrow$ 4008

$$\left(a^2A-2abB-Ab^2\right)\int \tan(c+dx)dx-x\left(a^2B+2aAb-b^2B\right)+\frac{b(aA-bB)\tan(c+dx)}{d}+\frac{A(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3bd}$$

$\Big\downarrow$ 3042

$$\left(a^2A-2abB-Ab^2\right)\int \tan(c+dx)dx-x\left(a^2B+2aAb-b^2B\right)+\frac{b(aA-bB)\tan(c+dx)}{d}+\frac{A(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3bd}$$

$\Big\downarrow$ 3956

$$-\frac{\left(a^2A-2abB-Ab^2\right)\log(\cos(c+dx))}{d}-x\left(a^2B+2aAb-b^2B\right)+\frac{b(aA-bB)\tan(c+dx)}{d}+\frac{A(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3bd}$$

input
```
Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
-((2*a*A*b + a^2*B - b^2*B)*x) - ((a^2*A - A*b^2 - 2*a*b*B)*Log[Cos[c + d*
x]])/d + (b*(a*A - b*B)*Tan[c + d*x])/d + (A*(a + b*Tan[c + d*x])^2)/(2*d)
 + (B*(a + b*Tan[c + d*x])^3)/(3*b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

method	result
norman	$(-2Aab-B\,a^2+B\,b^2)\,x+\frac{(2Aab+B\,a^2-B\,b^2)\tan(dx+c)}{d}+\frac{B\,b^2\tan(dx+c)^3}{3d}+\frac{b(Ab+2Ba)\tan(dx+}{2d}$
parts	$\frac{(2Aab+B\,a^2)(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{(A\,b^2+2Bab)\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{A\ln\left(1+\tan(\right.}{2d}$
derivativedivides	$\frac{\frac{B\tan(dx+c)^3b^2}{3}+\frac{A\tan(dx+c)^2b^2}{2}+B\tan(dx+c)^2ab+2A\tan(dx+c)ab+B\tan(dx+c)a^2-B\tan(dx+c)b^2+\frac{(A\,a^2-A\,b^2-}{}}{d}$
default	$\frac{\frac{B\tan(dx+c)^3b^2}{3}+\frac{A\tan(dx+c)^2b^2}{2}+B\tan(dx+c)^2ab+2A\tan(dx+c)ab+B\tan(dx+c)a^2-B\tan(dx+c)b^2+\frac{(A\,a^2-A\,b^2-}{}}{d}$
parallelrisch	$\frac{2B\tan(dx+c)^3b^2-12Aabdx+3A\tan(dx+c)^2b^2-6Bx\,a^2d+6B\,b^2dx+6B\tan(dx+c)^2ab+3A\ln\left(1+\tan(dx+c)^2\right)a^2-3}{6d}$
risch	$-2Aabx-B\,a^2x+B\,b^2x+\frac{2iA\,a^2c}{d}+iA\,a^2x-\frac{4iBabc}{d}-iA\,b^2x+\frac{2i(-3iA\,b^2e^{4i(dx+c)}-6iBa}{}$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
(-2*A*a*b-B*a^2+B*b^2)*x+(2*A*a*b+B*a^2-B*b^2)/d*tan(d*x+c)+1/3*B*b^2/d*ta
n(d*x+c)^3+1/2*b*(A*b+2*B*a)/d*tan(d*x+c)^2+1/2*(A*a^2-A*b^2-2*B*a*b)/d*ln
(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{2\,Bb^2\tan\left(dx+c\right)^3-6\left(Ba^2+2\,Aab-Bb^2\right)dx+3\left(2\,Bab+Ab^2\right)\tan\left(dx+c\right)^2-3\left(Aa^2-2\,Bab-Ab\right.}{6\,d}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/6*(2*B*b^2*tan(d*x + c)^3 - 6*(B*a^2 + 2*A*a*b - B*b^2)*d*x + 3*(2*B*a*b
 + A*b^2)*tan(d*x + c)^2 - 3*(A*a^2 - 2*B*a*b - A*b^2)*log(1/(tan(d*x + c)
^2 + 1)) + 6*(B*a^2 + 2*A*a*b - B*b^2)*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.71

$$\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\begin{cases}\frac{Aa^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-2Aabx+\frac{2Aab\tan\left(c+dx\right)}{d}-\frac{Ab^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ab^2\tan^2\left(c+dx\right)}{2d}-Ba^2x+\frac{Ba^2\tan\left(c+d\right.}{d} \\ x(A+B\tan\left(c\right))\left(a+b\tan\left(c\right)\right)^2\tan\left(c\right)\end{cases}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((A*a**2*log(tan(c + d*x)**2 + 1)/(2*d) - 2*A*a*b*x + 2*A*a*b*tan
(c + d*x)/d - A*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + A*b**2*tan(c + d*x)*
*2/(2*d) - B*a**2*x + B*a**2*tan(c + d*x)/d - B*a*b*log(tan(c + d*x)**2 +
1)/d + B*a*b*tan(c + d*x)**2/d + B*b**2*x + B*b**2*tan(c + d*x)**3/(3*d) -
 B*b**2*tan(c + d*x)/d, Ne(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c))**2*tan
(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

$$\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,Bb^2\tan\left(dx+c\right)^3+3\,(2\,Bab+Ab^2)\tan\left(dx+c\right)^2-6\,(Ba^2+2\,Aab-Bb^2)(dx+c)+3\,(Aa^2-2\,Bab}{6\,d}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/6*(2*B*b^2*tan(d*x + c)^3 + 3*(2*B*a*b + A*b^2)*tan(d*x + c)^2 - 6*(B*a^
2 + 2*A*a*b - B*b^2)*(d*x + c) + 3*(A*a^2 - 2*B*a*b - A*b^2)*log(tan(d*x +
 c)^2 + 1) + 6*(B*a^2 + 2*A*a*b - B*b^2)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.44

$$\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Ba^2+2\,Aab-Bb^2)(dx+c)}{d}+\frac{(Aa^2-2\,Bab-Ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$
$$+\frac{2\,Bb^2d^2\tan\left(dx+c\right)^3+6\,Babd^2\tan\left(dx+c\right)^2+3\,Ab^2d^2\tan\left(dx+c\right)^2+6\,Ba^2d^2\tan\left(dx+c\right)+12\,Aa}{6\,d^3}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
-(B*a^2 + 2*A*a*b - B*b^2)*(d*x + c)/d + 1/2*(A*a^2 - 2*B*a*b - A*b^2)*log
(tan(d*x + c)^2 + 1)/d + 1/6*(2*B*b^2*d^2*tan(d*x + c)^3 + 6*B*a*b*d^2*tan
(d*x + c)^2 + 3*A*b^2*d^2*tan(d*x + c)^2 + 6*B*a^2*d^2*tan(d*x + c) + 12*A
*a*b*d^2*tan(d*x + c) - 6*B*b^2*d^2*tan(d*x + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{\tan(c+d\,x)^2\,\left(\frac{A\,b^2}{2}+B\,a\,b\right)}{d} - x\,\left(B\,a^2+2\,A\,a\,b-B\,b^2\right) + \frac{\tan(c+d\,x)\,\left(B\,a^2+2\,A\,a\,b-B\,b^2\right)}{d} - \frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(-\frac{A\,a^2}{2}+B\,a\,b+\frac{A\,b^2}{2}\right)}{d} + \frac{B\,b^2\tan(c+d\,x)^3}{3\,d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
(tan(c + d*x)^2*((A*b^2)/2 + B*a*b))/d - x*(B*a^2 - B*b^2 + 2*A*a*b) + (ta
n(c + d*x)*(B*a^2 - B*b^2 + 2*A*a*b))/d - (log(tan(c + d*x)^2 + 1)*((A*b^2
)/2 - (A*a^2)/2 + B*a*b))/d + (B*b^2*tan(c + d*x)^3)/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \tan(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{3\log\left(\tan(dx+c)^2+1\right)a^3 - 9\log\left(\tan(dx+c)^2+1\right)a\,b^2 + 2\tan(dx+c)^3 b^3 + 9\tan(dx+c)^2 a\,b^2 + 18}{6d}$$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(3*log(tan(c + d*x)**2 + 1)*a**3 - 9*log(tan(c + d*x)**2 + 1)*a*b**2 + 2*t
an(c + d*x)**3*b**3 + 9*tan(c + d*x)**2*a*b**2 + 18*tan(c + d*x)*a**2*b -
6*tan(c + d*x)*b**3 - 18*a**2*b*d*x + 6*b**3*d*x)/(6*d)
```

3.242 $\int (a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\begin{aligned}\int (a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx &= \left(a^2A-Ab^2-2abB\right)x-\frac{\left(2aAb+a^2B-b^2B\right)\log(\cos(c+dx))}{d}\\&\quad+\frac{b(Ab+aB)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}\end{aligned}$$

output
```
(A*a^2-A*b^2-2*B*a*b)*x-(2*A*a*b+B*a^2-B*b^2)*ln(cos(d*x+c))/d+b*(A*b+B*a)
*tan(d*x+c)/d+1/2*B*(a+b*tan(d*x+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int (a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{(a+ib)^2(-iA+B)\log(i-\tan(c+dx))+(a-ib)^2(iA+B)\log(i+\tan(c+dx))+2b(Ab+2aB)\tan(}{2d}$$

input
```
Integrate[(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
((a + I*b)^2*((-I)*A + B)*Log[I - Tan[c + d*x]] + (a - I*b)^2*(I*A + B)*Lo
g[I + Tan[c + d*x]] + 2*b*(A*b + 2*a*B)*Tan[c + d*x] + b^2*B*Tan[c + d*x]^
2)/(2*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{B(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{B(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4008}$$

$$(a^2B+2aAb-b^2B)\int \tan(c+dx)dx+x(a^2A-2abB-Ab^2)+\frac{b(aB+Ab)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^2B+2aAb-b^2B\right)\int \tan(c+dx)dx+x\left(a^2A-2abB-Ab^2\right)+\frac{b(aB+Ab)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{\left(a^2B+2aAb-b^2B\right)\log(\cos(c+dx))}{d}+x\left(a^2A-2abB-Ab^2\right)+\frac{b(aB+Ab)\tan(c+dx)}{d}+\frac{B(a+b\tan(c+dx))^2}{2d}$$

input
```
Int[(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(a^2*A - A*b^2 - 2*a*b*B)*x - ((2*a*A*b + a^2*B - b^2*B)*Log[Cos[c + d*x]]
)/d + (b*(A*b + a*B)*Tan[c + d*x])/d + (B*(a + b*Tan[c + d*x])^2)/(2*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result
norman	$(A a^2 - A b^2 - 2Bab) x + \frac{b(Ab+2Ba)\tan(dx+c)}{d} + \frac{B b^2 \tan(dx+c)^2}{2d} + \frac{(2Aab+B a^2-B b^2)\ln\left(1+\tan(\right.}{2d}$
derivativedivides	$\frac{\frac{B b^2 \tan(dx+c)^2}{2} + A b^2 \tan(dx+c) + 2Bab\tan(dx+c) + \frac{(2Aab+B a^2-B b^2)\ln\left(1+\tan(dx+c)^2\right)}{2} + (A a^2 - A b^2 - 2Bab)\,\mathrm{arctan}}{d}$
default	$\frac{\frac{B b^2 \tan(dx+c)^2}{2} + A b^2 \tan(dx+c) + 2Bab\tan(dx+c) + \frac{(2Aab+B a^2-B b^2)\ln\left(1+\tan(dx+c)^2\right)}{2} + (A a^2 - A b^2 - 2Bab)\,\mathrm{arctan}}{d}$
parts	$A a^2 x + \frac{(A b^2+2Bab)(\tan(dx+c)-\arctan(\tan(dx+c)))}{d} + \frac{(2Aab+B a^2)\ln\left(1+\tan(dx+c)^2\right)}{2d} + \frac{B b^2 \left(\frac{\tan(dx+c}{2}\right.}{}$
parallelrisch	$\frac{2Ax a^2 d - 2A b^2 dx - 4Babdx + B b^2 \tan(dx+c)^2 + 2A\ln\left(1+\tan(dx+c)^2\right)ab + 2A b^2\tan(dx+c) + B\ln\left(1+\tan(dx+c)^2\right)a^2}{2d}$
risch	$-\frac{2iB b^2 c}{d} + 2iAabx + iB a^2 x + A a^2 x - A b^2 x - 2Babx - iB b^2 x + \frac{2ib\left(Ab\,e^{2i(dx+c)} + 2Ba\,e^{2}\right.}{d\left(e^{2}\right.}$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(A*a^2-A*b^2-2*B*a*b)*x+b*(A*b+2*B*a)/d*tan(d*x+c)+1/2*B*b^2/d*tan(d*x+c)^
2+1/2*(2*A*a*b+B*a^2-B*b^2)/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int (a + b\tan(c+dx))^2 (A + B\tan(c+dx))\,dx$$
$$= \frac{Bb^2\tan(dx+c)^2 + 2(Aa^2 - 2Bab - Ab^2)dx - (Ba^2 + 2Aab - Bb^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right) + 2(2Bab + A}{2d}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(B*b^2*tan(d*x + c)^2 + 2*(A*a^2 - 2*B*a*b - A*b^2)*d*x - (B*a^2 + 2*A
*a*b - B*b^2)*log(1/(tan(d*x + c)^2 + 1)) + 2*(2*B*a*b + A*b^2)*tan(d*x +
c))/d
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.64

$$\int (a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \begin{cases} Aa^2x + \frac{Aab\log(\tan^2(c+dx)+1)}{d} - Ab^2x + \frac{Ab^2\tan(c+dx)}{d} + \frac{Ba^2\log(\tan^2(c+dx)+1)}{2d} - 2Babx + \frac{2Bab\tan(c+dx)}{d} - \frac{Bb}{} \\ x(A+B\tan(c))\,(a+b\tan(c))^2 \end{cases}$$

input
```
integrate((a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((A*a**2*x + A*a*b*log(tan(c + d*x)**2 + 1)/d - A*b**2*x + A*b**2
*tan(c + d*x)/d + B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) - 2*B*a*b*x + 2*B*
a*b*tan(c + d*x)/d - B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*b**2*tan(c
+ d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c))**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int (a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{Bb^2\tan(dx+c)^2 + 2\,(Aa^2 - 2\,Bab - Ab^2)(dx+c) + (Ba^2 + 2\,Aab - Bb^2)\log\left(\tan(dx+c)^2+1\right) + 2\,(}{2\,d}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(B*b^2*tan(d*x + c)^2 + 2*(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c) + (B*a^2
 + 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1) + 2*(2*B*a*b + A*b^2)*tan(d*x
+ c))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21

$$\int (a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{(Aa^2-2\,Bab-Ab^2)(dx+c)}{d}+\frac{(Ba^2+2\,Aab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}+\frac{Bb^2d\tan\left(dx+c\right)^2+4\,Babd\tan\left(dx+c\right)+2\,Ab^2d\tan\left(dx+c\right)}{2\,d^2}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c)/d + 1/2*(B*a^2 + 2*A*a*b - B*b^2)*log(
tan(d*x + c)^2 + 1)/d + 1/2*(B*b^2*d*tan(d*x + c)^2 + 4*B*a*b*d*tan(d*x +
c) + 2*A*b^2*d*tan(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05

$$\int (a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{B\,a^2}{2}+A\,a\,b-\frac{B\,b^2}{2}\right)}{d}-x\,\left(-A\,a^2+2\,B\,a\,b+A\,b^2\right)+\frac{\tan(c+d\,x)\,\left(A\,b^2+2\,B\,a\,b\right)}{d}+\frac{B\,b^2\tan(c+d\,x)^2}{2\,d}$$

input
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
(log(tan(c + d*x)^2 + 1)*((B*a^2)/2 - (B*b^2)/2 + A*a*b))/d - x*(A*b^2 - A
*a^2 + 2*B*a*b) + (tan(c + d*x)*(A*b^2 + 2*B*a*b))/d + (B*b^2*tan(c + d*x)
^2)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int (a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{3\log\left(\tan(dx+c)^2+1\right)a^2b-\log\left(\tan(dx+c)^2+1\right)b^3+\tan(dx+c)^2b^3+6\tan(dx+c)\,ab^2+2a^3dx}{2d}$$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(3*log(tan(c + d*x)**2 + 1)*a**2*b - log(tan(c + d*x)**2 + 1)*b**3 + tan(c
 + d*x)**2*b**3 + 6*tan(c + d*x)*a*b**2 + 2*a**3*d*x - 6*a*b**2*d*x)/(2*d)
```

3.243 $\int \cot(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 70

$$\int \cot(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = (2aAb+a^2B-b^2B)\,x-\frac{b(Ab+2aB)\log(\cos(c+dx))}{d}+\frac{a^2A\log(\sin(c+dx))}{d}+\frac{b^2B\tan(c+dx)}{d}$$

output
```
(2*A*a*b+B*a^2-B*b^2)*x-b*(A*b+2*B*a)*ln(cos(d*x+c))/d+a^2*A*ln(sin(d*x+c)
)/d+b^2*B*tan(d*x+c)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.33

$$\int \cot(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{(a+ib)^2(A+iB)\log(i-\tan(c+dx))-2a^2A\log(\tan(c+dx))+(a-ib)^2(A-iB)\log(i+\tan(c+$$
$$2d$$

input
```
Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*((a + I*b)^2*(A + I*B)*Log[I - Tan[c + d*x]] - 2*a^2*A*Log[Tan[c + d*
x]] + (a - I*b)^2*(A - I*B)*Log[I + Tan[c + d*x]] - 2*b*B*(a + b*Tan[c + d
*x]))/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 4089, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4089}$$

$$\int \cot(c+dx)\left(Aa^2+b(Ab+2aB)\tan^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{b^2B\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Aa^2+b(Ab+2aB)\tan(c+dx)^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{b^2B\tan(c+dx)}{d}$$

$$\Big\downarrow \text{4107}$$

$$a^2A\int \cot(c+dx)dx+b(2aB+Ab)\int \tan(c+dx)dx+x\left(a^2B+2aAb-b^2B\right)+\frac{b^2B\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a^2A\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+b(2aB+Ab)\int \tan(c+dx)dx+x\left(a^2B+2aAb-b^2B\right)+\frac{b^2B\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$a^2(-A)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+b(2aB+Ab)\int \tan(c+dx)dx+x\left(a^2B+2aAb-b^2B\right)+\frac{b^2B\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 3956}$$

$$x\left(a^2B+2aAb-b^2B\right)+\frac{a^2A\log(-\sin(c+dx))}{d}-\frac{b(2aB+Ab)\log(\cos(c+dx))}{d}+\frac{b^2B\tan(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(2*a*A*b + a^2*B - b^2*B)*x - (b*(A*b + 2*a*B)*Log[Cos[c + d*x]])/d + (a^2
*A*Log[-Sin[c + d*x]])/d + (b^2*B*Tan[c + d*x])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4089
```
Int[(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (
f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b^2
*B*(Tan[e + f*x]/(d*f)), x] + Simp[1/d   Int[(a^2*A*d - b^2*B*c + (2*a*A*b
+ B*(a^2 - b^2))*d*Tan[e + f*x] + (A*b^2*d - b*B*(b*c - 2*a*d))*Tan[e + f*x
]^2)/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{\left(-A\,a^{2}+A\,b^{2}+2Bab\right)\ln\left(\sec(dx+c)^{2}\right)+2A\,a^{2}\ln(\tan(dx+c))+2B\tan(dx+c)b^{2}+4xd\left(Aab+\frac{1}{2}B\,a^{2}-\frac{1}{2}B\,b^{2}\right)}{2d}$
norman	$\left(2Aab+B\,a^{2}-B\,b^{2}\right)x+\frac{b^{2}B\tan(dx+c)}{d}+\frac{A\,a^{2}\ln(\tan(dx+c))}{d}-\frac{\left(A\,a^{2}-A\,b^{2}-2Bab\right)\ln\left(1+\tan(dx+c\right.}{2d}$
derivativedivides	$-\frac{\frac{\left(A\,a^{2}-A\,b^{2}-2Bab\right)\ln\left(\cot(dx+c)^{2}+1\right)}{2}+\left(2Aab+B\,a^{2}-B\,b^{2}\right)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)-\frac{B\,b^{2}}{\cot(dx+c)}+b(Ab+2Ba)\ln(\cot(}{d}$
default	$-\frac{\frac{\left(A\,a^{2}-A\,b^{2}-2Bab\right)\ln\left(\cot(dx+c)^{2}+1\right)}{2}+\left(2Aab+B\,a^{2}-B\,b^{2}\right)\left(\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)-\frac{B\,b^{2}}{\cot(dx+c)}+b(Ab+2Ba)\ln(\cot(}{d}$
risch	$2Aabx+B\,a^{2}x-B\,b^{2}x+\frac{2iB\,b^{2}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2iA\,a^{2}c}{d}+\frac{4iBabc}{d}-iA\,a^{2}x+iA\,b^{2}x+2iBabx$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/2*((-A*a^2+A*b^2+2*B*a*b)*ln(sec(d*x+c)^2)+2*A*a^2*ln(tan(d*x+c))+2*B*ta
n(d*x+c)*b^2+4*x*d*(A*a*b+1/2*B*a^2-1/2*B*b^2))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \cot(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{Aa^2\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+2\,Bb^2\tan(dx+c)+2\,(Ba^2+2\,Aab-Bb^2)dx-(2\,Bab+Ab^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/2*(A*a^2*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + 2*B*b^2*tan(d*x + c)
 + 2*(B*a^2 + 2*A*a*b - B*b^2)*d*x - (2*B*a*b + A*b^2)*log(1/(tan(d*x + c)
^2 + 1)))/d
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.84

$$\int \cot(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \begin{cases} -\frac{Aa^2\log(\tan^2(c+dx)+1)}{2d}+\frac{Aa^2\log(\tan(c+dx))}{d}+2Aabx+\frac{Ab^2\log(\tan^2(c+dx)+1)}{2d}+Ba^2x+\frac{Bab\log(\tan^2(c+dx)+1)}{d} - \cdots \\ x(A+B\tan(c))\,(a+b\tan(c))^2\cot(c) \end{cases}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((-A*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + A*a**2*log(tan(c + d*x
))/d + 2*A*a*b*x + A*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**2*x + B*a*
b*log(tan(c + d*x)**2 + 1)/d - B*b**2*x + B*b**2*tan(c + d*x)/d, Ne(d, 0))
, (x*(A + B*tan(c))*(a + b*tan(c))**2*cot(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

$$\int \cot(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{2\,Aa^2\log\left(\tan\left(dx+c\right)\right)+2\,Bb^2\tan\left(dx+c\right)+2\,(Ba^2+2\,Aab-Bb^2)(dx+c)-(Aa^2-2\,Bab-Ab^2)\,\mathrm{l}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/2*(2*A*a^2*log(tan(d*x + c)) + 2*B*b^2*tan(d*x + c) + 2*(B*a^2 + 2*A*a*b
 - B*b^2)*(d*x + c) - (A*a^2 - 2*B*a*b - A*b^2)*log(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int \cot(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{Aa^2\log\left(|\tan\left(dx+c\right)|\right)}{d}+\frac{Bb^2\tan\left(dx+c\right)}{d}+\frac{(Ba^2+2\,Aab-Bb^2)(dx+c)}{d}-\frac{(Aa^2-2\,Bab-Ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
A*a^2*log(abs(tan(d*x + c)))/d + B*b^2*tan(d*x + c)/d + (B*a^2 + 2*A*a*b -
 B*b^2)*(d*x + c)/d - 1/2*(A*a^2 - 2*B*a*b - A*b^2)*log(tan(d*x + c)^2 + 1
)/d
```

Mupad [B] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int \cot(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{A\,a^2\,\ln(\tan(c+d\,x))}{d} + \frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(A-B\,1\mathrm{i})\,(b+a\,1\mathrm{i})^2}{2\,d} + \frac{B\,b^2\tan(c+d\,x)}{d} + \frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(A+B\,1\mathrm{i})\,(-b+a\,1\mathrm{i})^2}{2\,d}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
(A*a^2*log(tan(c + d*x)))/d + (log(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b
)^2)/(2*d) + (B*b^2*tan(c + d*x))/d + (log(tan(c + d*x) - 1i)*(A + B*1i)*(
a*1i - b)^2)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.41

$$\int \cot(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{-\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)a^3+3\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)a\,b^2-3\cos(dx+c)\,\mathrm{lo}}{}$$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
( - cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*a**3 + 3*cos(c + d*x)*log(ta
n((c + d*x)/2)**2 + 1)*a*b**2 - 3*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*a
*b**2 - 3*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**2 + cos(c + d*x)*log
(tan((c + d*x)/2))*a**3 + 3*cos(c + d*x)*a**2*b*d*x - cos(c + d*x)*b**3*d*
x + sin(c + d*x)*b**3)/(cos(c + d*x)*d)
```

3.244 $\int \cot^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 72

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\left(\left(a^2A-Ab^2-2abB\right)x\right)-\frac{a^2A\cot(c+dx)}{d} - \frac{b^2B\log(\cos(c+dx))}{d}+\frac{a(2Ab+aB)\log(\sin(c+dx))}{d}$$

output

```
-(A*a^2-A*b^2-2*B*a*b)*x-a^2*A*cot(d*x+c)/d-b^2*B*ln(cos(d*x+c))/d+a*(2*A*
b+B*a)*ln(sin(d*x+c))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{-2a^2A\cot(c+dx)+i(a+ib)^2(A+iB)\log(i-\tan(c+dx))+2a(2Ab+aB)\log(\tan(c+dx))-(a-}{2d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a^2*A*Cot[c + d*x] + I*(a + I*b)^2*(A + I*B)*Log[I - Tan[c + d*x]] + 2
*a*(2*A*b + a*B)*Log[Tan[c + d*x]] - (a - I*b)^2*(I*A + B)*Log[I + Tan[c +
 d*x]])/(2*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3042, 4087, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4087}$$

$$\int \cot(c+dx)\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)dx-\frac{a^2A\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\tan(c+dx)}dx-\frac{a^2A\cot(c+dx)}{d}$$

$$\Big\downarrow \text{4107}$$

$$a(aB+2Ab)\int \cot(c+dx)dx+b^2B\int \tan(c+dx)dx-x\left(a^2A-2abB-Ab^2\right)-\frac{a^2A\cot(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$a(aB+2Ab)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+b^2B\int \tan(c+dx)dx-x\left(a^2A-2abB-Ab^2\right)-\frac{a^2A\cot(c+dx)}{d}$$

$\Big\downarrow$ 25

$$-a(aB+2Ab)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+b^2B\int \tan(c+dx)dx-x\left(a^2A-2abB-Ab^2\right)-\frac{a^2A\cot(c+dx)}{d}$$

$\Big\downarrow$ 3956

$$-x\left(a^2A-2abB-Ab^2\right)-\frac{a^2A\cot(c+dx)}{d}+\frac{a(aB+2Ab)\log(-\sin(c+dx))}{d}-\frac{b^2B\log(\cos(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
-((a^2*A - A*b^2 - 2*a*b*B)*x) - (a^2*A*Cot[c + d*x])/d - (b^2*B*Log[Cos[c
 + d*x]])/d + (a*(2*A*b + a*B)*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{A\,b^2(dx+c)-B\,b^2\ln(\cos(dx+c))+2Aab\ln(\sin(dx+c))+2Bab(dx+c)+A\,a^2(-\cot(dx+c)-dx-c)+B\,a^2\ln(\sin(dx+c))}{d}$
default	$\frac{A\,b^2(dx+c)-B\,b^2\ln(\cos(dx+c))+2Aab\ln(\sin(dx+c))+2Bab(dx+c)+A\,a^2(-\cot(dx+c)-dx-c)+B\,a^2\ln(\sin(dx+c))}{d}$
parallelrisch	$\frac{(-2Aab-B\,a^2+B\,b^2)\ln\left(\sec(dx+c)^2\right)+(4Aab+2B\,a^2)\ln(\tan(dx+c))-2A\,a^2\cot(dx+c)-2dx\left(A\,a^2-A\,b^2-2Bab\right)}{2d}$
norman	$\frac{(-A\,a^2+A\,b^2+2Bab)x\tan(dx+c)-\frac{A\,a^2}{d}}{\tan(dx+c)}+\frac{a(2Ab+Ba)\ln(\tan(dx+c))}{d}-\frac{(2Aab+B\,a^2-B\,b^2)\ln\left(1+\tan(dx+c)^2\right)}{2d}$
risch	$-2iAabx-\frac{2iA\,a^2}{d\left(e^{2i(dx+c)}-1\right)}-iB\,a^2x-A\,a^2x+A\,b^2x+2Babx+iB\,b^2x-\frac{4iAabc}{d}+\frac{2iB\,b}{d}$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(A*b^2*(d*x+c)-B*b^2*ln(cos(d*x+c))+2*A*a*b*ln(sin(d*x+c))+2*B*a*b*(d*
x+c)+A*a^2*(-cot(d*x+c)-d*x-c)+B*a^2*ln(sin(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{Bb^2\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan(dx+c)+2\left(Aa^2-2\,Bab-Ab^2\right)dx\tan(dx+c)+2\,Aa^2-\left(Ba^2+2\,Aab\right)}{2\,d\tan(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/2*(B*b^2*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c) + 2*(A*a^2 - 2*B*a*b
- A*b^2)*d*x*tan(d*x + c) + 2*A*a^2 - (B*a^2 + 2*A*a*b)*log(tan(d*x + c)^2
/(tan(d*x + c)^2 + 1))*tan(d*x + c))/(d*tan(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(66) = 132$.

Time = 0.59 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.32

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$= \begin{cases} \tilde{\infty}Aa^2x \\ x(A+B\tan(c))\,(a+b\tan(c))^2\cot^2(c) \\ \tilde{\infty}Aa^2x \\ -Aa^2x-\frac{Aa^2}{d\tan(c+dx)}-\frac{Aab\log(\tan^2(c+dx)+1)}{d}+\frac{2Aab\log(\tan(c+dx))}{d}+Ab^2x-\frac{Ba^2\log(\tan^2(c+dx)+1)}{2d}+\frac{Ba^2\log(\text{ta}}{d} \end{cases}$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**2*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**2*cot(c)**2, Eq(d, 0)), (zoo*A*a**2*x, Eq(c, -d*x)), (-A*a**2*x - A
*a**2/(d*tan(c + d*x)) - A*a*b*log(tan(c + d*x)**2 + 1)/d + 2*A*a*b*log(ta
n(c + d*x))/d + A*b**2*x - B*a**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**2*
log(tan(c + d*x))/d + 2*B*a*b*x + B*b**2*log(tan(c + d*x)**2 + 1)/(2*d), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.29

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{2\,(Aa^2-2\,Bab-Ab^2)(dx+c)+(Ba^2+2\,Aab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)-2\,(Ba^2+2\,Aab)\log}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/2*(2*(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c) + (B*a^2 + 2*A*a*b - B*b^2)*lo
g(tan(d*x + c)^2 + 1) - 2*(B*a^2 + 2*A*a*b)*log(tan(d*x + c)) + 2*A*a^2/ta
n(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{(Aa^2-2\,Bab-Ab^2)(dx+c)}{d}-\frac{(Ba^2+2\,Aab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}+\frac{(Ba^2+2\,Aab)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}-\frac{Aa^2}{d\tan\left(dx+c\right)}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c)/d - 1/2*(B*a^2 + 2*A*a*b - B*b^2)*log
(tan(d*x + c)^2 + 1)/d + (B*a^2 + 2*A*a*b)*log(abs(tan(d*x + c)))/d - A*a^
2/(d*tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{\ln(\tan(c+d\,x))\,\left(B\,a^2+2\,A\,b\,a\right)}{d}-\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,\left(-B+A\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^2}{2\,d}+\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,\left(B+A\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^2}{2\,d}-\frac{A\,a^2\cot(c+d\,x)}{d}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
(log(tan(c + d*x))*(B*a^2 + 2*A*a*b))/d - (log(tan(c + d*x) - 1i)*(A*1i -
B)*(a*1i - b)^2)/(2*d) + (log(tan(c + d*x) + 1i)*(A*1i + B)*(a*1i + b)^2)/
(2*d) - (A*a^2*cot(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.35

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{-\cos(dx+c)\,a^3-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)\,a^2b+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)\,b^3-}{}$$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
( - cos(c + d*x)*a**3 - 3*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a**2*b
 + log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*b**3 - log(tan((c + d*x)/2) -
 1)*sin(c + d*x)*b**3 - log(tan((c + d*x)/2) + 1)*sin(c + d*x)*b**3 + 3*lo
g(tan((c + d*x)/2))*sin(c + d*x)*a**2*b - sin(c + d*x)*a**3*d*x + 3*sin(c
+ d*x)*a*b**2*d*x)/(sin(c + d*x)*d)
```

3.245 $\int \cot^3(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 88

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \left(b^2B-a(2Ab+aB)\right)x-\frac{a(2Ab+aB)\cot(c+dx)}{d} -\frac{a^2A\cot^2(c+dx)}{2d}-\frac{(a^2A-Ab^2-2abB)\log(\sin(c+dx))}{d}$$

output
```
(B*b^2-a*(2*A*b+B*a))*x-a*(2*A*b+B*a)*cot(d*x+c)/d-1/2*a^2*A*cot(d*x+c)^2/
d-(A*a^2-A*b^2-2*B*a*b)*ln(sin(d*x+c))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.40

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{-2a(2Ab+aB)\cot(c+dx)-a^2A\cot^2(c+dx)+(a+ib)^2(A+iB)\log(i-\tan(c+dx))-2(a^2A-Ab}{2d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a*(2*A*b + a*B)*Cot[c + d*x] - a^2*A*Cot[c + d*x]^2 + (a + I*b)^2*(A +
 I*B)*Log[I - Tan[c + d*x]] - 2*(a^2*A - A*b^2 - 2*a*b*B)*Log[Tan[c + d*x]
] + (a - I*b)^2*(A - I*B)*Log[I + Tan[c + d*x]])/(2*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 4087, 3042, 4111, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4087}$$

$$\int \cot^2(c+dx)\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)dx-\frac{a^2A\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\tan(c+dx)^2}dx-\frac{a^2A\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{4111}$$

$$\int -\cot(c+dx)\left(Aa^2-2bBa-Ab^2-\left(b^2B-a(2Ab+aB)\right)\tan(c+dx)\right)dx-\frac{a^2A\cot^2(c+dx)}{2d}-\frac{a(aB+2Ab)\cot(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(c+dx)\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx-\frac{a^2A\cot^2(c+dx)}{2d}-\frac{a(aB+2Ab)\cot(c+dx)}{d}$$

$\big\downarrow$ 3042

$$-\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a^2A\cot^2(c+dx)}{2d}-\frac{a(aB+2Ab)\cot(c+dx)}{d}$$

$\big\downarrow$ 4014

$$-\left(a^2A-2abB-Ab^2\right)\int \cot(c+dx)dx-x\left(a^2B+2aAb-b^2B\right)-\frac{a^2A\cot^2(c+dx)}{2d}-\frac{a(aB+2Ab)\cot(c+dx)}{d}$$

$\big\downarrow$ 3042

$$-\left(a^2A-2abB-Ab^2\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-x\left(a^2B+2aAb-b^2B\right)-\frac{a^2A\cot^2(c+dx)}{2d}-\frac{a(aB+2Ab)\cot(c+dx)}{d}$$

$\big\downarrow$ 25

$$\left(a^2A-2abB-Ab^2\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-x\left(a^2B+2aAb-b^2B\right)-\frac{a^2A\cot^2(c+dx)}{2d}-\frac{a(aB+2Ab)\cot(c+dx)}{d}$$

$\big\downarrow$ 3956

$$-\frac{\left(a^2A-2abB-Ab^2\right)\log(-\sin(c+dx))}{d}-x\left(a^2B+2aAb-b^2B\right)-\frac{a^2A\cot^2(c+dx)}{2d}-\frac{a(aB+2Ab)\cot(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
-((2*a*A*b + a^2*B - b^2*B)*x) - (a*(2*A*b + a*B)*Cot[c + d*x])/d - (a^2*A
*Cot[c + d*x]^2)/(2*d) - ((a^2*A - A*b^2 - 2*a*b*B)*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{A\,b^2\ln(\sin(dx+c))+B\,b^2(dx+c)+2Aab(-\cot(dx+c)-dx-c)+2Bab\ln(\sin(dx+c))+A\,a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c}{d}$
default	$\frac{A\,b^2\ln(\sin(dx+c))+B\,b^2(dx+c)+2Aab(-\cot(dx+c)-dx-c)+2Bab\ln(\sin(dx+c))+A\,a^2\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c}{d}$
parallelrisch	$\frac{(A\,a^2-A\,b^2-2Bab)\ln\left(\sec(dx+c)^2\right)+(-2A\,a^2+2A\,b^2+4Bab)\ln(\tan(dx+c))-A\cot(dx+c)^2a^2+(-4Aab-2B\,a^2)\cot}{2d}$
norman	$\frac{(-2Aab-B\,a^2+B\,b^2)x\tan(dx+c)^2-\frac{A\,a^2}{2d}-\frac{a(2Ab+Ba)\tan(dx+c)}{d}}{\tan(dx+c)^2}-\frac{(A\,a^2-A\,b^2-2Bab)\ln(\tan(dx+c))}{d}+\frac{(A\,a^2-}{}$
risch	$-2Aabx-B\,a^2x+B\,b^2x+\frac{2iA\,a^2c}{d}+iA\,a^2x-\frac{4iBabc}{d}-\frac{2iA\,b^2c}{d}-iA\,b^2x-\frac{2ia\left(iAa\,e^{2i(dx+}\right.}{}$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(A*b^2*ln(sin(d*x+c))+B*b^2*(d*x+c)+2*A*a*b*(-cot(d*x+c)-d*x-c)+2*B*a*
b*ln(sin(d*x+c))+A*a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+B*a^2*(-cot(d*x+
c)-d*x-c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{(Aa^2-2\,Bab-Ab^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+Aa^2+(Aa^2+2\,(Ba^2+2\,Aab-Bb^2)dx)\tan(}{2\,d\tan\left(dx+c\right)^2}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/2*((A*a^2 - 2*B*a*b - A*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*t
an(d*x + c)^2 + A*a^2 + (A*a^2 + 2*(B*a^2 + 2*A*a*b - B*b^2)*d*x)*tan(d*x
+ c)^2 + 2*(B*a^2 + 2*A*a*b)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(78) = 156$.

Time = 0.81 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.43

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \tilde{\infty} Aa^2x \\ x(A+B\tan(c))\,(a+b\tan(c))^2\cot^3(c) \\ \tilde{\infty} Aa^2x \\ \frac{Aa^2\log(\tan^2(c+dx)+1)}{2d} - \frac{Aa^2\log(\tan(c+dx))}{d} - \frac{Aa^2}{2d\tan^2(c+dx)} - 2Aabx - \frac{2Aab}{d\tan(c+dx)} - \frac{Ab^2\log(\tan^2(c+dx)+1)}{2d} + \frac{Ab^2}{} \end{cases}$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**2*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**2*cot(c)**3, Eq(d, 0)), (zoo*A*a**2*x, Eq(c, -d*x)), (A*a**2*log(ta
n(c + d*x)**2 + 1)/(2*d) - A*a**2*log(tan(c + d*x))/d - A*a**2/(2*d*tan(c
+ d*x)**2) - 2*A*a*b*x - 2*A*a*b/(d*tan(c + d*x)) - A*b**2*log(tan(c + d*x
)**2 + 1)/(2*d) + A*b**2*log(tan(c + d*x))/d - B*a**2*x - B*a**2/(d*tan(c
+ d*x)) - B*a*b*log(tan(c + d*x)**2 + 1)/d + 2*B*a*b*log(tan(c + d*x))/d +
 B*b**2*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2(Ba^2+2Aab-Bb^2)(dx+c) - (Aa^2-2Bab-Ab^2)\log\left(\tan(dx+c)^2+1\right) + 2(Aa^2-2Bab-A}{2d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/2*(2*(B*a^2 + 2*A*a*b - B*b^2)*(d*x + c) - (A*a^2 - 2*B*a*b - A*b^2)*lo
g(tan(d*x + c)^2 + 1) + 2*(A*a^2 - 2*B*a*b - A*b^2)*log(tan(d*x + c)) + (A
*a^2 + 2*(B*a^2 + 2*A*a*b)*tan(d*x + c))/tan(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.47

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Ba^2+2\,Aab-Bb^2)(dx+c)}{d} + \frac{(Aa^2-2\,Bab-Ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d} - \frac{(Aa^2-2\,Bab-Ab^2)\log\left(|\tan\left(dx+c\right)|\right)}{d} - \frac{Aa^2+2\left(Ba^2+2\,Aab\right)\tan\left(dx+c\right)}{2\,d\tan\left(dx+c\right)^2}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-(B*a^2 + 2*A*a*b - B*b^2)*(d*x + c)/d + 1/2*(A*a^2 - 2*B*a*b - A*b^2)*log
(tan(d*x + c)^2 + 1)/d - (A*a^2 - 2*B*a*b - A*b^2)*log(abs(tan(d*x + c)))/
d - 1/2*(A*a^2 + 2*(B*a^2 + 2*A*a*b)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{\ln(\tan(c+d\,x))\,\left(-A\,a^2+2\,B\,a\,b+A\,b^2\right)}{d} - \frac{\cot(c+d\,x)^2\,\left(\frac{A\,a^2}{2}+\tan(c+d\,x)\,\left(B\,a^2+2\,A\,b\,a\right)\right)}{d} - \frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^2}{2\,d} - \frac{\ln(\tan(c+d\,x)-\mathrm{i})\,\left(A+B\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^2}{2\,d}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
(log(tan(c + d*x))*(A*b^2 - A*a^2 + 2*B*a*b))/d - (cot(c + d*x)^2*((A*a^2)
/2 + tan(c + d*x)*(B*a^2 + 2*A*a*b)))/d - (log(tan(c + d*x) + 1i)*(A - B*1
i)*(a*1i + b)^2)/(2*d) - (log(tan(c + d*x) - 1i)*(A + B*1i)*(a*1i - b)^2)/
(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.07

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{-12\cos(dx+c)\sin(dx+c)\,a^2b+4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)^2\,a^3-12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right.}{}$$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
( - 12*cos(c + d*x)*sin(c + d*x)*a**2*b + 4*log(tan((c + d*x)/2)**2 + 1)*s
in(c + d*x)**2*a**3 - 12*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a*b*
*2 - 4*log(tan((c + d*x)/2))*sin(c + d*x)**2*a**3 + 12*log(tan((c + d*x)/2
))*sin(c + d*x)**2*a*b**2 + sin(c + d*x)**2*a**3 - 12*sin(c + d*x)**2*a**2
*b*d*x + 4*sin(c + d*x)**2*b**3*d*x - 2*a**3)/(4*sin(c + d*x)**2*d)
```

3.246 $\int \cot^4(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 118

$$\begin{aligned}\int \cot^4(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx &= \left(a^2A-Ab^2-2abB\right)x+\frac{(a^2A-Ab^2-2abB)\cot(c+dx)}{d}\\ &\quad-\frac{a(2Ab+aB)\cot^2(c+dx)}{2d}-\frac{a^2A\cot^3(c+dx)}{3d}\\ &\quad+\frac{(b^2B-a(2Ab+aB))\log(\sin(c+dx))}{d}\end{aligned}$$

output

```
(A*a^2-A*b^2-2*B*a*b)*x+(A*a^2-A*b^2-2*B*a*b)*cot(d*x+c)/d-1/2*a*(2*A*b+B*
a)*cot(d*x+c)^2/d-1/3*a^2*A*cot(d*x+c)^3/d+(B*b^2-a*(2*A*b+B*a))*ln(sin(d*
x+c))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.29

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{6(a^2A-Ab^2-2abB)\cot(c+dx)-3a(2Ab+aB)\cot^2(c+dx)-2a^2A\cot^3(c+dx)+3(a+ib)^2(-iA$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(6*(a^2*A - A*b^2 - 2*a*b*B)*Cot[c + d*x] - 3*a*(2*A*b + a*B)*Cot[c + d*x]
^2 - 2*a^2*A*Cot[c + d*x]^3 + 3*(a + I*b)^2*((-I)*A + B)*Log[I - Tan[c + d
*x]] - 6*(2*a*A*b + a^2*B - b^2*B)*Log[Tan[c + d*x]] + 3*(a - I*b)^2*(I*A
+ B)*Log[I + Tan[c + d*x]])/(6*d)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4087, 3042, 4111, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$
$$\Big\downarrow \text{4087}$$
$$\int \cot^3(c+dx)\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)dx-\frac{a^2A\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\tan(c+dx)^3}dx-\frac{a^2A\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \; 4111$$

$$\int -\cot^2(c+dx)\left(Aa^2-2bBa-Ab^2-\left(b^2B-a(2Ab+aB)\right)\tan(c+dx)\right)dx-\frac{a^2A\cot^3(c+dx)}{3d}-\frac{a(aB+2Ab)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \; 25$$

$$-\int \cot^2(c+dx)\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx-\frac{a^2A\cot^3(c+dx)}{3d}-\frac{a(aB+2Ab)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \; 3042$$

$$-\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{a^2A\cot^3(c+dx)}{3d}-\frac{a(aB+2Ab)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \; 4012$$

$$-\int \cot(c+dx)\left(Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot(c+dx)}{d}-\frac{a^2A\cot^3(c+dx)}{3d}-\frac{a(aB+2Ab)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \; 3042$$

$$-\int \frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot(c+dx)}{d}-\frac{a^2A\cot^3(c+dx)}{3d}-\frac{a(aB+2Ab)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \; 4014$$

$$-\left(a^2B+2aAb-b^2B\right)\int \cot(c+dx)dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot(c+dx)}{d}+x\left(a^2A-2abB-Ab^2\right)-\frac{a^2A\cot^3(c+dx)}{3d}-\frac{a(aB+2Ab)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \; 3042$$

$$-\left(a^2B+2aAb-b^2B\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot(c+dx)}{d}+x\left(a^2A-2abB-Ab^2\right)-\frac{a^2A\cot^3(c+dx)}{3d}-\frac{a(aB+2Ab)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 25}$$

$$\left(a^2B+2aAb-b^2B\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot(c+dx)}{d}+x\left(a^2A-2abB-Ab^2\right)-\frac{a^2A\cot^3(c+dx)}{3d}-\frac{a(aB+2Ab)\cot^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\left(a^2A-2abB-Ab^2\right)\cot(c+dx)}{d}-\frac{\left(a^2B+2aAb-b^2B\right)\log(-\sin(c+dx))}{d}+x\left(a^2A-2abB-Ab^2\right)-\frac{a^2A\cot^3(c+dx)}{3d}-\frac{a(aB+2Ab)\cot^2(c+dx)}{2d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(a^2*A - A*b^2 - 2*a*b*B)*x + ((a^2*A - A*b^2 - 2*a*b*B)*Cot[c + d*x])/d -
 (a*(2*A*b + a*B)*Cot[c + d*x]^2)/(2*d) - (a^2*A*Cot[c + d*x]^3)/(3*d) - (
(2*a*A*b + a^2*B - b^2*B)*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{A\,b^2(-\cot(dx+c)-dx-c)+B\,b^2\ln(\sin(dx+c))+2Aab\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+2Bab(-\cot(dx+c)-dx-c)+A}{d}$
default	$\frac{A\,b^2(-\cot(dx+c)-dx-c)+B\,b^2\ln(\sin(dx+c))+2Aab\left(-\frac{\cot(dx+c)^2}{2}-\ln(\sin(dx+c))\right)+2Bab(-\cot(dx+c)-dx-c)+A}{d}$
parallelrisch	$\frac{3(2Aab+B\,a^2-B\,b^2)\ln\left(\sec(dx+c)^2\right)+6(-2Aab-B\,a^2+B\,b^2)\ln(\tan(dx+c))-2A\cot(dx+c)^3a^2+3(-2Aab-B\,a^2)\,c}{6d}$
norman	$\frac{\frac{\left(A\,a^2-A\,b^2-2Bab\right)\tan(dx+c)^2}{d}+(A\,a^2-A\,b^2-2Bab)x\tan(dx+c)^3-\frac{A\,a^2}{3d}-\frac{a(2Ab+Ba)\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{(2Aab+B\,a^2-B\,b}{d}$
risch	$2iAabx+iB\,a^2x-\frac{2i\left(6iAab\,e^{4i(dx+c)}+3iB\,a^2e^{4i(dx+c)}-6A\,a^2e^{4i(dx+c)}+3A\,b^2e^{4i(dx+c)}+6Bab\,e^{4i(dx+c)}-6\right.}{3}$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(A*b^2*(-cot(d*x+c)-d*x-c)+B*b^2*ln(sin(d*x+c))+2*A*a*b*(-1/2*cot(d*x+
c)^2-ln(sin(d*x+c)))+2*B*a*b*(-cot(d*x+c)-d*x-c)+A*a^2*(-1/3*cot(d*x+c)^3+
cot(d*x+c)+d*x+c)+B*a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.33

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{3\,(Ba^2+2\,Aab-Bb^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^3+3\,(Ba^2+2\,Aab-2\,(Aa^2-2\,Bab-Ab^2)dx)}{6\,d\tan\left(dx+\right.}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/6*(3*(B*a^2 + 2*A*a*b - B*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))
*tan(d*x + c)^3 + 3*(B*a^2 + 2*A*a*b - 2*(A*a^2 - 2*B*a*b - A*b^2)*d*x)*ta
n(d*x + c)^3 + 2*A*a^2 - 6*(A*a^2 - 2*B*a*b - A*b^2)*tan(d*x + c)^2 + 3*(B
*a^2 + 2*A*a*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(107) = 214$.

Time = 1.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.20

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \tilde{\infty} A a^2 x \\ x(A+B\tan(c))(a+b\tan(c))^2\cot^4(c) \\ \tilde{\infty} A a^2 x \\ A a^2 x + \frac{A a^2}{d\tan(c+dx)} - \frac{A a^2}{3d\tan^3(c+dx)} + \frac{A a b\log(\tan^2(c+dx)+1)}{d} - \frac{2 A a b\log(\tan(c+dx))}{d} - \frac{A a b}{d\tan^2(c+dx)} - A b^2 x - \frac{A}{d\tan} \end{cases}$$

input
```
integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**2*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**2*cot(c)**4, Eq(d, 0)), (zoo*A*a**2*x, Eq(c, -d*x)), (A*a**2*x + A*
a**2/(d*tan(c + d*x)) - A*a**2/(3*d*tan(c + d*x)**3) + A*a*b*log(tan(c + d
*x)**2 + 1)/d - 2*A*a*b*log(tan(c + d*x))/d - A*a*b/(d*tan(c + d*x)**2) -
A*b**2*x - A*b**2/(d*tan(c + d*x)) + B*a**2*log(tan(c + d*x)**2 + 1)/(2*d)
 - B*a**2*log(tan(c + d*x))/d - B*a**2/(2*d*tan(c + d*x)**2) - 2*B*a*b*x -
 2*B*a*b/(d*tan(c + d*x)) - B*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + B*b**2
*log(tan(c + d*x))/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.26

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{6\,(Aa^2-2\,Bab-Ab^2)(dx+c)+3\,(Ba^2+2\,Aab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)-6\,(Ba^2+2\,Aab-Bb}{6\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/6*(6*(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c) + 3*(B*a^2 + 2*A*a*b - B*b^2)*l
og(tan(d*x + c)^2 + 1) - 6*(B*a^2 + 2*A*a*b - B*b^2)*log(tan(d*x + c)) - (
2*A*a^2 - 6*(A*a^2 - 2*B*a*b - A*b^2)*tan(d*x + c)^2 + 3*(B*a^2 + 2*A*a*b)
*tan(d*x + c))/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.32

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{(Aa^2-2\,Bab-Ab^2)(dx+c)}{d}+\frac{(Ba^2+2\,Aab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$
$$-\frac{(Ba^2+2\,Aab-Bb^2)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$
$$-\frac{2\,Aa^2-6\,(Aa^2-2\,Bab-Ab^2)\tan\left(dx+c\right)^2+3\,(Ba^2+2\,Aab)\tan\left(dx+c\right)}{6\,d\tan\left(dx+c\right)^3}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
(A*a^2 - 2*B*a*b - A*b^2)*(d*x + c)/d + 1/2*(B*a^2 + 2*A*a*b - B*b^2)*log(
tan(d*x + c)^2 + 1)/d - (B*a^2 + 2*A*a*b - B*b^2)*log(abs(tan(d*x + c)))/d
 - 1/6*(2*A*a^2 - 6*(A*a^2 - 2*B*a*b - A*b^2)*tan(d*x + c)^2 + 3*(B*a^2 +
2*A*a*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.32

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{\cot(c+d\,x)^3\,\left(\frac{A\,a^2}{3}+\tan(c+d\,x)^2\,\left(-A\,a^2+2\,B\,a\,b+A\,b^2\right)+\tan(c+d\,x)\,\left(\frac{B\,a^2}{2}+A\,b\,a\right)\right)}{d}$$
$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,a^2+2\,A\,a\,b-B\,b^2\right)}{d}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^2}{2\,d}$$
$$-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^2}{2\,d}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
(log(tan(c + d*x) - 1i)*(A*1i - B)*(a*1i - b)^2)/(2*d) - (log(tan(c + d*x)
)*(B*a^2 - B*b^2 + 2*A*a*b))/d - (cot(c + d*x)^3*((A*a^2)/3 + tan(c + d*x)
^2*(A*b^2 - A*a^2 + 2*B*a*b) + tan(c + d*x)*((B*a^2)/2 + A*a*b)))/d - (log
(tan(c + d*x) + 1i)*(A*1i + B)*(a*1i + b)^2)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.89

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{16\cos(dx+c)\sin(dx+c)^2a^3-36\cos(dx+c)\sin(dx+c)^2ab^2-4\cos(dx+c)a^3+36\log\left(\tan\left(\frac{dx}{2}+\right.\right.}{}$$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(16*cos(c + d*x)*sin(c + d*x)**2*a**3 - 36*cos(c + d*x)*sin(c + d*x)**2*a*
b**2 - 4*cos(c + d*x)*a**3 + 36*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*
*3*a**2*b - 12*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*b**3 - 36*log(
tan((c + d*x)/2))*sin(c + d*x)**3*a**2*b + 12*log(tan((c + d*x)/2))*sin(c
+ d*x)**3*b**3 + 12*sin(c + d*x)**3*a**3*d*x + 9*sin(c + d*x)**3*a**2*b -
36*sin(c + d*x)**3*a*b**2*d*x - 18*sin(c + d*x)*a**2*b)/(12*sin(c + d*x)**
3*d)
```

3.247 $\int \cot^5(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 151

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \left(2aAb+a^2B-b^2B\right)x-\frac{(b^2B-a(2Ab+aB))\cot(c+dx)}{d} + \frac{(a^2A-Ab^2-2abB)\cot^2(c+dx)}{2d}-\frac{a(2Ab+aB)\cot^3(c+dx)}{3d} - \frac{a^2A\cot^4(c+dx)}{4d}+\frac{(a^2A-Ab^2-2abB)\log(\sin(c+dx))}{d}$$

output
```
(2*A*a*b+B*a^2-B*b^2)*x-(B*b^2-a*(2*A*b+B*a))*cot(d*x+c)/d+1/2*(A*a^2-A*b^
2-2*B*a*b)*cot(d*x+c)^2/d-1/3*a*(2*A*b+B*a)*cot(d*x+c)^3/d-1/4*a^2*A*cot(d
*x+c)^4/d+(A*a^2-A*b^2-2*B*a*b)*ln(sin(d*x+c))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{12(2aAb+a^2B-b^2B)\cot(c+dx)+6(a^2A-Ab^2-2abB)\cot^2(c+dx)-4a(2Ab+aB)\cot^3(c+dx)}{}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(12*(2*a*A*b + a^2*B - b^2*B)*Cot[c + d*x] + 6*(a^2*A - A*b^2 - 2*a*b*B)*C
ot[c + d*x]^2 - 4*a*(2*A*b + a*B)*Cot[c + d*x]^3 - 3*a^2*A*Cot[c + d*x]^4
- 6*((a + I*b)^2*(A + I*B)*Log[I - Tan[c + d*x]] + (-2*a^2*A + 2*A*b^2 + 4
*a*b*B)*Log[Tan[c + d*x]] + (a - I*b)^2*(A - I*B)*Log[I + Tan[c + d*x]]))/
(12*d)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4087, 3042, 4111, 25, 3042, 4012, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{4087}$$

$$\int \cot^4(c+dx)\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)dx-\frac{a^2A\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\tan(c+dx)^4}dx-\frac{a^2A\cot^4(c+dx)}{4d}$$

$$\Big\downarrow \text{4111}$$

$$\int -\cot^3(c+dx)\left(Aa^2-2bBa-Ab^2-\left(b^2B-a(2Ab+aB)\right)\tan(c+dx)\right)dx-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot^3(c+dx)\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan(c+dx)^3}dx-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4012}$$

$$-\int \cot^2(c+dx)\left(Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot^2(c+dx)}{2d}-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\tan(c+dx)^2}dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot^2(c+dx)}{2d}-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \text{4012}$$

$$-\int -\cot(c+dx)\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2B+2aAb-b^2B\right)\cot(c+dx)}{d}-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \textbf{25}$$

$$\int \cot(c+dx)\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2B+2aAb-b^2B\right)\cot(c+dx)}{d}-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2B+2aAb-b^2B\right)\cot(c+dx)}{d}-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \textbf{4014}$$

$$\left(a^2A-2abB-Ab^2\right)\int \cot(c+dx)dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2B+2aAb-b^2B\right)\cot(c+dx)}{d}+x\left(a^2B+2aAb-b^2B\right)-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \textbf{3042}$$

$$\left(a^2A-2abB-Ab^2\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2B+2aAb-b^2B\right)\cot(c+dx)}{d}+x\left(a^2B+2aAb-b^2B\right)-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \textbf{25}$$

$$-\left(a^2A-2abB-Ab^2\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{\left(a^2A-2abB-Ab^2\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2B+2aAb-b^2B\right)\cot(c+dx)}{d}+x\left(a^2B+2aAb-b^2B\right)-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

$$\Big\downarrow \; 3956$$

$$\frac{\left(a^2A-2abB-Ab^2\right)\cot^2(c+dx)}{2d}+\frac{\left(a^2B+2aAb-b^2B\right)\cot(c+dx)}{d}+\frac{\left(a^2A-2abB-Ab^2\right)\log(-\sin(c+dx))}{d}+x\left(a^2B+2aAb-b^2B\right)-\frac{a^2A\cot^4(c+dx)}{4d}-\frac{a(aB+2Ab)\cot^3(c+dx)}{3d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(2*a*A*b + a^2*B - b^2*B)*x + ((2*a*A*b + a^2*B - b^2*B)*Cot[c + d*x])/d +
 ((a^2*A - A*b^2 - 2*a*b*B)*Cot[c + d*x]^2)/(2*d) - (a*(2*A*b + a*B)*Cot[c
 + d*x]^3)/(3*d) - (a^2*A*Cot[c + d*x]^4)/(4*d) + ((a^2*A - A*b^2 - 2*a*b*
B)*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{A\,b^{2}\left(-\frac{\cot(dx+c)^{2}}{2}-\ln(\sin(dx+c))\right)+B\,b^{2}(-\cot(dx+c)-dx-c)+2Aab\left(-\frac{\cot(dx+c)^{3}}{3}+\cot(dx+c)+dx+c\right)+2Bab\left(-\right.}{}$
default	$\frac{A\,b^{2}\left(-\frac{\cot(dx+c)^{2}}{2}-\ln(\sin(dx+c))\right)+B\,b^{2}(-\cot(dx+c)-dx-c)+2Aab\left(-\frac{\cot(dx+c)^{3}}{3}+\cot(dx+c)+dx+c\right)+2Bab\left(-\right.}{}$
parallelrisch	$\frac{6(-A\,a^{2}+A\,b^{2}+2Bab)\ln\left(\sec(dx+c)^{2}\right)+12(A\,a^{2}-A\,b^{2}-2Bab)\ln(\tan(dx+c))-3A\cot(dx+c)^{4}a^{2}+4(-2Aab-B\,a^{2})}{12d}$
norman	$\frac{\frac{(2Aab+B\,a^{2}-B\,b^{2})\tan(dx+c)^{3}}{d}+(2Aab+B\,a^{2}-B\,b^{2})x\tan(dx+c)^{4}-\frac{A\,a^{2}}{4d}+\frac{(A\,a^{2}-A\,b^{2}-2Bab)\tan(dx+c)^{2}}{2d}-\frac{a(2Ab+Ba)}{3d}}{\tan(dx+c)^{4}}$
risch	$2Aabx+B\,a^{2}x-B\,b^{2}x-\frac{2iA\,a^{2}c}{d}-iA\,a^{2}x+\frac{2iA\,b^{2}c}{d}+iA\,b^{2}x-\frac{2i\left(6iBab\,e^{2i(dx+c)}-6iA\,b^{2}e^{4i}\right.}{}$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(A*b^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+B*b^2*(-cot(d*x+c)-d*x-c)+2*
A*a*b*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*B*a*b*(-1/2*cot(d*x+c)^2-ln(s
in(d*x+c)))+A*a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+B*a^
2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.26

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{6\left(Aa^2-2\,Bab-Ab^2\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+3\left(3\,Aa^2-4\,Bab-2\,Ab^2+4\left(Ba^2+2\,Aab-\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/12*(6*(A*a^2 - 2*B*a*b - A*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))
*tan(d*x + c)^4 + 3*(3*A*a^2 - 4*B*a*b - 2*A*b^2 + 4*(B*a^2 + 2*A*a*b - B*
b^2)*d*x)*tan(d*x + c)^4 + 12*(B*a^2 + 2*A*a*b - B*b^2)*tan(d*x + c)^3 - 3
*A*a^2 + 6*(A*a^2 - 2*B*a*b - A*b^2)*tan(d*x + c)^2 - 4*(B*a^2 + 2*A*a*b)*
tan(d*x + c))/(d*tan(d*x + c)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(136) = 272$.

Time = 2.07 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.07

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \tilde{\infty}Aa^2x \\ x(A+B\tan(c))(a+b\tan(c))^2\cot^5(c) \\ \tilde{\infty}Aa^2x \\ -\frac{Aa^2\log(\tan^2(c+dx)+1)}{2d} + \frac{Aa^2\log(\tan(c+dx))}{d} + \frac{Aa^2}{2d\tan^2(c+dx)} - \frac{Aa^2}{4d\tan^4(c+dx)} + 2Aabx + \frac{2Aab}{d\tan(c+dx)} - \frac{2Aab}{3d\tan^3(c+d}\end{cases}$$

input
```
integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**2*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**2*cot(c)**5, Eq(d, 0)), (zoo*A*a**2*x, Eq(c, -d*x)), (-A*a**2*log(t
an(c + d*x)**2 + 1)/(2*d) + A*a**2*log(tan(c + d*x))/d + A*a**2/(2*d*tan(c
 + d*x)**2) - A*a**2/(4*d*tan(c + d*x)**4) + 2*A*a*b*x + 2*A*a*b/(d*tan(c
+ d*x)) - 2*A*a*b/(3*d*tan(c + d*x)**3) + A*b**2*log(tan(c + d*x)**2 + 1)/
(2*d) - A*b**2*log(tan(c + d*x))/d - A*b**2/(2*d*tan(c + d*x)**2) + B*a**2
*x + B*a**2/(d*tan(c + d*x)) - B*a**2/(3*d*tan(c + d*x)**3) + B*a*b*log(ta
n(c + d*x)**2 + 1)/d - 2*B*a*b*log(tan(c + d*x))/d - B*a*b/(d*tan(c + d*x)
**2) - B*b**2*x - B*b**2/(d*tan(c + d*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.16

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{12\,(Ba^2+2\,Aab-Bb^2)(dx+c)-6\,(Aa^2-2\,Bab-Ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)+12\,(Aa^2-2\,Bab-}{12}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/12*(12*(B*a^2 + 2*A*a*b - B*b^2)*(d*x + c) - 6*(A*a^2 - 2*B*a*b - A*b^2)
*log(tan(d*x + c)^2 + 1) + 12*(A*a^2 - 2*B*a*b - A*b^2)*log(tan(d*x + c))
+ (12*(B*a^2 + 2*A*a*b - B*b^2)*tan(d*x + c)^3 - 3*A*a^2 + 6*(A*a^2 - 2*B*
a*b - A*b^2)*tan(d*x + c)^2 - 4*(B*a^2 + 2*A*a*b)*tan(d*x + c))/tan(d*x +
c)^4)/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.21

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{(Ba^2+2\,Aab-Bb^2)(dx+c)}{d} - \frac{(Aa^2-2\,Bab-Ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$
$$+ \frac{(Aa^2-2\,Bab-Ab^2)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$
$$+ \frac{12\,(Ba^2+2\,Aab-Bb^2)\tan\left(dx+c\right)^3-3\,Aa^2+6\,(Aa^2-2\,Bab-Ab^2)\tan\left(dx+c\right)^2-4\,(Ba^2+2\,A}{12\,d\tan\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
(B*a^2 + 2*A*a*b - B*b^2)*(d*x + c)/d - 1/2*(A*a^2 - 2*B*a*b - A*b^2)*log(
tan(d*x + c)^2 + 1)/d + (A*a^2 - 2*B*a*b - A*b^2)*log(abs(tan(d*x + c)))/d
 + 1/12*(12*(B*a^2 + 2*A*a*b - B*b^2)*tan(d*x + c)^3 - 3*A*a^2 + 6*(A*a^2
- 2*B*a*b - A*b^2)*tan(d*x + c)^2 - 4*(B*a^2 + 2*A*a*b)*tan(d*x + c))/(d*t
an(d*x + c)^4)
```

Mupad [B] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.21

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{\cot(c+d\,x)^4\left(\frac{A\,a^2}{4}+\tan(c+d\,x)^2\left(-\frac{A\,a^2}{2}+B\,a\,b+\frac{A\,b^2}{2}\right)-\tan(c+d\,x)^3\left(B\,a^2+2\,A\,a\,b-B\,b^2\right)+\mathrm{t}\right.}{d} - \frac{\ln\left(\tan(c+d\,x)\right)\,\left(-A\,a^2+2\,B\,a\,b+A\,b^2\right)}{d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^2}{2\,d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^2}{2\,d}$$

input
```
int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
(log(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)^2)/(2*d) - (log(tan(c + d*x)
)*(A*b^2 - A*a^2 + 2*B*a*b))/d - (cot(c + d*x)^4*((A*a^2)/4 + tan(c + d*x)
^2*((A*b^2)/2 - (A*a^2)/2 + B*a*b) - tan(c + d*x)^3*(B*a^2 - B*b^2 + 2*A*a
*b) + tan(c + d*x)*((B*a^2)/3 + (2*A*a*b)/3)))/d + (log(tan(c + d*x) - 1i)
*(A + B*1i)*(a*1i - b)^2)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.74

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$= \frac{128\cos(dx+c)\sin(dx+c)^3 a^2b - 32\cos(dx+c)\sin(dx+c)^3 b^3 - 32\cos(dx+c)\sin(dx+c)a^2b - 32}{}$$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(128*cos(c + d*x)*sin(c + d*x)**3*a**2*b - 32*cos(c + d*x)*sin(c + d*x)**3
*b**3 - 32*cos(c + d*x)*sin(c + d*x)*a**2*b - 32*log(tan((c + d*x)/2)**2 +
 1)*sin(c + d*x)**4*a**3 + 96*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**4
*a*b**2 + 32*log(tan((c + d*x)/2))*sin(c + d*x)**4*a**3 - 96*log(tan((c +
d*x)/2))*sin(c + d*x)**4*a*b**2 - 13*sin(c + d*x)**4*a**3 + 96*sin(c + d*x
)**4*a**2*b*d*x + 24*sin(c + d*x)**4*a*b**2 - 32*sin(c + d*x)**4*b**3*d*x
+ 32*sin(c + d*x)**2*a**3 - 48*sin(c + d*x)**2*a*b**2 - 8*a**3)/(32*sin(c
+ d*x)**4*d)
```

3.248 $\int \tan^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 201

$$\begin{aligned}\int \tan^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= -\left(\left(a^3A-3aAb^2-3a^2bB+b^3B\right)x\right)\\ &\quad+\frac{(3a^2Ab-Ab^3+a^3B-3ab^2B)\log(\cos(c+dx))}{d}\\ &\quad-\frac{b(2aAb+a^2B-b^2B)\tan(c+dx)}{d}-\frac{(Ab+aB)(a+b\tan(c+dx))^2}{2d}\\ &\quad-\frac{B(a+b\tan(c+dx))^3}{3d}+\frac{(5Ab-aB)(a+b\tan(c+dx))^4}{20b^2d}\\ &\quad+\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}\end{aligned}$$

output
```
-(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*x+(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*ln(
cos(d*x+c))/d-b*(2*A*a*b+B*a^2-B*b^2)*tan(d*x+c)/d-1/2*(A*b+B*a)*(a+b*tan(
d*x+c))^2/d-1/3*B*(a+b*tan(d*x+c))^3/d+1/20*(5*A*b-B*a)*(a+b*tan(d*x+c))^4
/b^2/d+1/5*B*tan(d*x+c)*(a+b*tan(d*x+c))^4/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.20

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{\frac{3(5Ab-aB)(a+b\tan(c+dx))^4}{b}+12B\tan(c+dx)(a+b\tan(c+dx))^4-30(Ab-aB)\left((ia-b)^3\log(i-\tan(c+\right.}{}$$

input
```
Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
((3*(5*A*b - a*B)*(a + b*Tan[c + d*x])^4)/b + 12*B*Tan[c + d*x]*(a + b*Tan
[c + d*x])^4 - 30*(A*b - a*B)*((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a +
b)^3*Log[I + Tan[c + d*x]] + 6*a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2) +
10*B*((3*I)*(a + I*b)^4*Log[I - Tan[c + d*x]] - (3*I)*(a - I*b)^4*Log[I +
Tan[c + d*x]] + 6*b^2*(-6*a^2 + b^2)*Tan[c + d*x] - 12*a*b^3*Tan[c + d*x]^
2 - 2*b^4*Tan[c + d*x]^3))/(60*b*d)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4090, 25, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)^2(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4090$$

$$\frac{\int -(a+b\tan(c+dx))^3\left(-\left((5Ab-aB)\tan^2(c+dx)\right)+5bB\tan(c+dx)+aB\right)dx}{5b}+\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}$$

$$\Big\downarrow \; 25$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-\frac{\int (a+b\tan(c+dx))^3\left(-\left((5Ab-aB)\tan^2(c+dx)\right)+5bB\tan(c+dx)+aB\right)dx}{5b}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-\frac{\int (a+b\tan(c+dx))^3\left(-\left((5Ab-aB)\tan(c+dx)^2\right)+5bB\tan(c+dx)+aB\right)dx}{5b}$$

$$\Big\downarrow \; 4113$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-\frac{\int (a+b\tan(c+dx))^3(5Ab+5B\tan(c+dx)b)dx-\frac{(5Ab-aB)(a+b\tan(c+dx))^4}{4bd}}{5b}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-\frac{\int (a+b\tan(c+dx))^3(5Ab+5B\tan(c+dx)b)dx-\frac{(5Ab-aB)(a+b\tan(c+dx))^4}{4bd}}{5b}$$

$$\Big\downarrow \; 4011$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-\frac{\int (a+b\tan(c+dx))^2(5b(aA-bB)+5b(Ab+aB)\tan(c+dx))dx-\frac{(5Ab-aB)(a+b\tan(c+dx))^4}{4bd}+\frac{5bB(a+b\tan(c+dx))}{3d}}{5b}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-\frac{\int (a+b\tan(c+dx))^2(5b(aA-bB)+5b(Ab+aB)\tan(c+dx))dx-\frac{(5Ab-aB)(a+b\tan(c+dx))^4}{4bd}+\frac{5bB(a+b\tan(c+dx))}{3d}}{5b}$$

$$\Big\downarrow \; 4011$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-$$
$$\frac{\int(a+b\tan(c+dx))\left(5b\left(Aa^2-2bBa-Ab^2\right)+5b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx-\frac{(5Ab-aB)(a+b\tan(c+dx))}{4bd}}{5b}$$

↓ 3042

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-$$
$$\frac{\int(a+b\tan(c+dx))\left(5b\left(Aa^2-2bBa-Ab^2\right)+5b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx-\frac{(5Ab-aB)(a+b\tan(c+dx))}{4bd}}{5b}$$

↓ 4008

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-$$
$$\frac{5b\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int\tan(c+dx)dx+\frac{5b^2\left(a^2B+2aAb-b^2B\right)\tan(c+dx)}{d}+5bx\left(a^3A-3a^2bB-3aAb^2+\right.}{5b}$$

↓ 3042

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-$$
$$\frac{5b\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int\tan(c+dx)dx+\frac{5b^2\left(a^2B+2aAb-b^2B\right)\tan(c+dx)}{d}+5bx\left(a^3A-3a^2bB-3aAb^2+\right.}{5b}$$

↓ 3956

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^4}{5bd}-$$
$$\frac{\frac{5b^2\left(a^2B+2aAb-b^2B\right)\tan(c+dx)}{d}-\frac{5b\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\log(\cos(c+dx))}{d}+5bx\left(a^3A-3a^2bB-3aAb^2+b^3B\right)-\frac{(5Ab-}{}}{5b}$$

input
```
Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]*(a + b*Tan[c + d*x])^4)/(5*b*d) - (5*b*(a^3*A - 3*a*A*b^2
- 3*a^2*b*B + b^3*B)*x - (5*b*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Log[
Cos[c + d*x]])/d + (5*b^2*(2*a*A*b + a^2*B - b^2*B)*Tan[c + d*x])/d + (5*b
*(A*b + a*B)*(a + b*Tan[c + d*x])^2)/(2*d) + (5*b*B*(a + b*Tan[c + d*x])^3
)/(3*d) - ((5*A*b - a*B)*(a + b*Tan[c + d*x])^4)/(4*b*d))/(5*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.03

method	result
parts	$\frac{(A b^3+3Ba b^2)\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}+\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{(3Aa b^2+3B a^2b)\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\text{arcta}\right.}{d}$
norman	$(-A a^3+3Aa b^2+3B a^2b-B b^3) x+\frac{(A a^3-3Aa b^2-3B a^2b+B b^3)\tan(dx+c)}{d}+\frac{(3A a^2b-A b^3+B}{}$
derivativedivides	$\frac{B b^3\tan(dx+c)^5}{5}+\frac{A b^3\tan(dx+c)^4}{4}+\frac{3Ba b^2\tan(dx+c)^4}{4}+Aa b^2\tan(dx+c)^3+B a^2b\tan(dx+c)^3-\frac{B b^3\tan(dx+c)^3}{3}+\frac{3A a^2b}{}$
default	$\frac{B b^3\tan(dx+c)^5}{5}+\frac{A b^3\tan(dx+c)^4}{4}+\frac{3Ba b^2\tan(dx+c)^4}{4}+Aa b^2\tan(dx+c)^3+B a^2b\tan(dx+c)^3-\frac{B b^3\tan(dx+c)^3}{3}+\frac{3A a^2b}{}$
parallelrisch	$-\frac{-12B b^3\tan(dx+c)^5-15A b^3\tan(dx+c)^4-45Ba b^2\tan(dx+c)^4-60Aa b^2\tan(dx+c)^3-60B a^2b\tan(dx+c)^3+20B b}{}$
risch	$-A a^3x+3Aa b^2x+3B a^2bx-B b^3x+\frac{6iBa b^2c}{d}+\frac{2i\left(-60Aa b^2-60B a^2b+45B b^3 e^{8i(dx+c)}+60A a\right.}{}$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
(A*b^3+3*B*a*b^2)/d*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2+1/2*ln(1+tan(d*x+c)
^2))+(3*A*a*b^2+3*B*a^2*b)/d*(1/3*tan(d*x+c)^3-tan(d*x+c)+arctan(tan(d*x+c
)))+(3*A*a^2*b+B*a^3)/d*(1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)^2))+A*a^3/d*
(tan(d*x+c)-arctan(tan(d*x+c)))+B*b^3/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3
+tan(d*x+c)-arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.06

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{12\,Bb^3\tan\left(dx+c\right)^5+15\left(3\,Bab^2+Ab^3\right)\tan\left(dx+c\right)^4+20\left(3\,Ba^2b+3\,Aab^2-Bb^3\right)\tan\left(dx+c\right)^3-60}{}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/60*(12*B*b^3*tan(d*x + c)^5 + 15*(3*B*a*b^2 + A*b^3)*tan(d*x + c)^4 + 20
*(3*B*a^2*b + 3*A*a*b^2 - B*b^3)*tan(d*x + c)^3 - 60*(A*a^3 - 3*B*a^2*b -
3*A*a*b^2 + B*b^3)*d*x + 30*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*tan(d*
x + c)^2 + 30*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*log(1/(tan(d*x + c)^
2 + 1)) + 60*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*tan(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(185) = 370.

Time = 0.19 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.91

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} -Aa^3x+\frac{Aa^3\tan\left(c+dx\right)}{d}-\frac{3Aa^2b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{3Aa^2b\tan^2\left(c+dx\right)}{2d}+3Aab^2x+\frac{Aab^2\tan^3\left(c+dx\right)}{d}-\frac{3Aab^2\tan\left(c\right.}{d} \\ x(A+B\tan\left(c\right))\left(a+b\tan\left(c\right)\right)^3\tan^2\left(c\right) \end{cases}$$

input
```
integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((-A*a**3*x + A*a**3*tan(c + d*x)/d - 3*A*a**2*b*log(tan(c + d*x)
**2 + 1)/(2*d) + 3*A*a**2*b*tan(c + d*x)**2/(2*d) + 3*A*a*b**2*x + A*a*b**
2*tan(c + d*x)**3/d - 3*A*a*b**2*tan(c + d*x)/d + A*b**3*log(tan(c + d*x)*
*2 + 1)/(2*d) + A*b**3*tan(c + d*x)**4/(4*d) - A*b**3*tan(c + d*x)**2/(2*d
) - B*a**3*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**3*tan(c + d*x)**2/(2*d) +
 3*B*a**2*b*x + B*a**2*b*tan(c + d*x)**3/d - 3*B*a**2*b*tan(c + d*x)/d + 3
*B*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + 3*B*a*b**2*tan(c + d*x)**4/(4*d
) - 3*B*a*b**2*tan(c + d*x)**2/(2*d) - B*b**3*x + B*b**3*tan(c + d*x)**5/(
5*d) - B*b**3*tan(c + d*x)**3/(3*d) + B*b**3*tan(c + d*x)/d, Ne(d, 0)), (x
*(A + B*tan(c))*(a + b*tan(c))**3*tan(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.06

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{12\,Bb^3\tan\left(dx+c\right)^5+15\left(3\,Bab^2+Ab^3\right)\tan\left(dx+c\right)^4+20\left(3\,Ba^2b+3\,Aab^2-Bb^3\right)\tan\left(dx+c\right)^3+30}{}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/60*(12*B*b^3*tan(d*x + c)^5 + 15*(3*B*a*b^2 + A*b^3)*tan(d*x + c)^4 + 20
*(3*B*a^2*b + 3*A*a*b^2 - B*b^3)*tan(d*x + c)^3 + 30*(B*a^3 + 3*A*a^2*b -
3*B*a*b^2 - A*b^3)*tan(d*x + c)^2 - 60*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*
b^3)*(d*x + c) - 30*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*log(tan(d*x +
c)^2 + 1) + 60*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.60

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Aa^3 - 3\,Ba^2b - 3\,Aab^2 + Bb^3)(dx+c)}{d}$$
$$- \frac{(Ba^3 + 3\,Aa^2b - 3\,Bab^2 - Ab^3)\log\left(\tan\left(dx+c\right)^2 + 1\right)}{2\,d}$$
$$+ \frac{12\,Bb^3d^4\tan\left(dx+c\right)^5 + 45\,Bab^2d^4\tan\left(dx+c\right)^4 + 15\,Ab^3d^4\tan\left(dx+c\right)^4 + 60\,Ba^2bd^4\tan\left(dx+c\right)^3 +}{}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*(d*x + c)/d - 1/2*(B*a^3 + 3*A*a^
2*b - 3*B*a*b^2 - A*b^3)*log(tan(d*x + c)^2 + 1)/d + 1/60*(12*B*b^3*d^4*ta
n(d*x + c)^5 + 45*B*a*b^2*d^4*tan(d*x + c)^4 + 15*A*b^3*d^4*tan(d*x + c)^4
 + 60*B*a^2*b*d^4*tan(d*x + c)^3 + 60*A*a*b^2*d^4*tan(d*x + c)^3 - 20*B*b^
3*d^4*tan(d*x + c)^3 + 30*B*a^3*d^4*tan(d*x + c)^2 + 90*A*a^2*b*d^4*tan(d*
x + c)^2 - 90*B*a*b^2*d^4*tan(d*x + c)^2 - 30*A*b^3*d^4*tan(d*x + c)^2 + 6
0*A*a^3*d^4*tan(d*x + c) - 180*B*a^2*b*d^4*tan(d*x + c) - 180*A*a*b^2*d^4*
tan(d*x + c) + 60*B*b^3*d^4*tan(d*x + c))/d^5
```

Mupad [B] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.08

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)\,\left(A\,a^3+B\,b^3-3\,a\,b\,(A\,b+B\,a)\right)}{d} - \frac{\tan(c+d\,x)^3\,\left(\frac{B\,b^3}{3}-a\,b\,(A\,b+B\,a)\right)}{d} - x\,\left(A\,a^3-3\,B\,a^2\,b-3\,A\,a\,b^2+B\,b^3\right) + \frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(-\frac{B\,a^3}{2}-\frac{3\,A\,a^2\,b}{2}+\frac{3\,B\,a\,b^2}{2}+\frac{A\,b^3}{2}\right)}{d} + \frac{\tan(c+d\,x)^4\,\left(\frac{A\,b^3}{4}+\frac{3\,B\,a\,b^2}{4}\right)}{d} - \frac{\tan(c+d\,x)^2\,\left(-\frac{B\,a^3}{2}-\frac{3\,A\,a^2\,b}{2}+\frac{3\,B\,a\,b^2}{2}+\frac{A\,b^3}{2}\right)}{d} + \frac{B\,b^3\,\tan(c+d\,x)^5}{5\,d}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
(tan(c + d*x)*(A*a^3 + B*b^3 - 3*a*b*(A*b + B*a)))/d - (tan(c + d*x)^3*((B
*b^3)/3 - a*b*(A*b + B*a)))/d - x*(A*a^3 + B*b^3 - 3*A*a*b^2 - 3*B*a^2*b)
+ (log(tan(c + d*x)^2 + 1)*((A*b^3)/2 - (B*a^3)/2 - (3*A*a^2*b)/2 + (3*B*a
*b^2)/2))/d + (tan(c + d*x)^4*((A*b^3)/4 + (3*B*a*b^2)/4))/d - (tan(c + d*
x)^2*((A*b^3)/2 - (B*a^3)/2 - (3*A*a^2*b)/2 + (3*B*a*b^2)/2))/d + (B*b^3*t
an(c + d*x)^5)/(5*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.92

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{-30\log\left(\tan\left(dx+c\right)^2+1\right)a^3b+30\log\left(\tan\left(dx+c\right)^2+1\right)a\,b^3+3\tan\left(dx+c\right)^5b^4+15\tan\left(dx+c\right)^4a}{}$$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
( - 30*log(tan(c + d*x)**2 + 1)*a**3*b + 30*log(tan(c + d*x)**2 + 1)*a*b**
3 + 3*tan(c + d*x)**5*b**4 + 15*tan(c + d*x)**4*a*b**3 + 30*tan(c + d*x)**
3*a**2*b**2 - 5*tan(c + d*x)**3*b**4 + 30*tan(c + d*x)**2*a**3*b - 30*tan(
c + d*x)**2*a*b**3 + 15*tan(c + d*x)*a**4 - 90*tan(c + d*x)*a**2*b**2 + 15
*tan(c + d*x)*b**4 - 15*a**4*d*x + 90*a**2*b**2*d*x - 15*b**4*d*x)/(15*d)
```

3.249 $\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 165

$$
\begin{aligned}
\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx
&= -\left(\left(3a^2Ab-Ab^3+a^3B-3ab^2B\right)x\right) \\
&\quad - \frac{(a^3A-3aAb^2-3a^2bB+b^3B)\log(\cos(c+dx))}{d} \\
&\quad + \frac{b(a^2A-Ab^2-2abB)\tan(c+dx)}{d} + \frac{(aA-bB)(a+b\tan(c+dx))^2}{2d} \\
&\quad + \frac{A(a+b\tan(c+dx))^3}{3d} + \frac{B(a+b\tan(c+dx))^4}{4bd}
\end{aligned}
$$

output
```
-(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*x-(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*ln(
cos(d*x+c))/d+b*(A*a^2-A*b^2-2*B*a*b)*tan(d*x+c)/d+1/2*(A*a-B*b)*(a+b*tan(
d*x+c))^2/d+1/3*A*(a+b*tan(d*x+c))^3/d+1/4*B*(a+b*tan(d*x+c))^4/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.27

$$\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{-6iA(a+ib)^4\log(i-\tan(c+dx))+6iA(a-ib)^4\log(i+\tan(c+dx))-12Ab^2(-6a^2+b^2)\tan(c+dx}{}$$

input
```
Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
((-6*I)*A*(a + I*b)^4*Log[I - Tan[c + d*x]] + (6*I)*A*(a - I*b)^4*Log[I +
Tan[c + d*x]] - 12*A*b^2*(-6*a^2 + b^2)*Tan[c + d*x] + 24*a*A*b^3*Tan[c +
d*x]^2 + 4*A*b^4*Tan[c + d*x]^3 + 3*B*(a + b*Tan[c + d*x])^4 - 6*(a*A + b*
B)*((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]]
+ 6*a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2))/(12*b*d)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 4075, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{4075}$$
$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^3dx+\frac{B(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 3042$$

$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^3dx+\frac{B(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 4011$$

$$\int (a+b\tan(c+dx))^2(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{A(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\tan(c+dx))^2(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{A(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 4011$$

$$\int (a+b\tan(c+dx))\left(-Ba^2-2Aba+b^2B+\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)dx+\frac{(aA-bB)(a+b\tan(c+dx))^2}{2d}+\frac{A(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\tan(c+dx))\left(-Ba^2-2Aba+b^2B+\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)dx+\frac{(aA-bB)(a+b\tan(c+dx))^2}{2d}+\frac{A(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 4008$$

$$\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\int \tan(c+dx)dx+\frac{b\left(a^2A-2abB-Ab^2\right)\tan(c+dx)}{d}-x\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)+\frac{(aA-bB)(a+b\tan(c+dx))^2}{2d}+\frac{A(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \; 3042$$

$$\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\int \tan(c+dx)dx+\frac{b\left(a^2A-2abB-Ab^2\right)\tan(c+dx)}{d}-x\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)+\frac{(aA-bB)(a+b\tan(c+dx))^2}{2d}+\frac{A(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4bd}$$

$$\Big\downarrow \text{3956}$$

$$\frac{b(a^2A-2abB-Ab^2)\tan(c+dx)}{d}-\frac{(a^3A-3a^2bB-3aAb^2+b^3B)\log(\cos(c+dx))}{d}-x(a^3B+3a^2Ab-3ab^2B-Ab^3)+\frac{(aA-bB)(a+b\tan(c+dx))^2}{2d}+\frac{A(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4bd}$$

input
```
Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
-((3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*x) - ((a^3*A - 3*a*A*b^2 - 3*a^2
*b*B + b^3*B)*Log[Cos[c + d*x]])/d + (b*(a^2*A - A*b^2 - 2*a*b*B)*Tan[c +
d*x])/d + ((a*A - b*B)*(a + b*Tan[c + d*x])^2)/(2*d) + (A*(a + b*Tan[c + d
*x])^3)/(3*d) + (B*(a + b*Tan[c + d*x])^4)/(4*b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.09

method	result
norman	$(-3A a^2 b + A b^3 - B a^3 + 3B a b^2) x + \frac{(3A a^2 b - A b^3 + B a^3 - 3B a b^2)\tan(dx+c)}{d} + \frac{B b^3 \tan(dx+c)^4}{4d}$
parts	$\frac{(A b^3 + 3B a b^2)\left(\frac{\tan(dx+c)^3}{3} - \tan(dx+c) + \arctan(\tan(dx+c))\right)}{d} + \frac{(3A a^2 b + B a^3)(\tan(dx+c) - \arctan(\tan(dx+c)))}{d}$
derivativedivides	$\frac{\frac{B\tan(dx+c)^4 b^3}{4} + \frac{A\tan(dx+c)^3 b^3}{3} + B\tan(dx+c)^3 a b^2 + \frac{3A\tan(dx+c)^2 a b^2}{2} + \frac{3B\tan(dx+c)^2 a^2 b}{2} - \frac{B\tan(dx+c)^2 b^3}{2} + 3A\tan(}{}$
default	$\frac{\frac{B\tan(dx+c)^4 b^3}{4} + \frac{A\tan(dx+c)^3 b^3}{3} + B\tan(dx+c)^3 a b^2 + \frac{3A\tan(dx+c)^2 a b^2}{2} + \frac{3B\tan(dx+c)^2 a^2 b}{2} - \frac{B\tan(dx+c)^2 b^3}{2} + 3A\tan(}{}$
parallelrisch	$3B\tan(dx+c)^4 b^3 + 4A\tan(dx+c)^3 b^3 + 12B\tan(dx+c)^3 a b^2 - 36A a^2 b d x + 12A b^3 d x + 18A\tan(dx+c)^2 a b^2 - 12B x a^3 d +$
risch	$\frac{2iB b^3 c}{d} + \frac{2i(-12B a b^2 + 9A a^2 b - 4A b^3 - 9iA a b^2 e^{6i(dx+c)} - 9iA a b^2 e^{2i(dx+c)} - 9iB a^2 b e^{2i(dx+c)} - 18iA a b^2 e^{4i(dx+c)}}{}$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
(-3*A*a^2*b+A*b^3-B*a^3+3*B*a*b^2)*x+(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)/d*t
an(d*x+c)+1/4*B*b^3/d*tan(d*x+c)^4+1/2*b*(3*A*a*b+3*B*a^2-B*b^2)/d*tan(d*x
+c)^2+1/3*b^2*(A*b+3*B*a)/d*tan(d*x+c)^3+1/2*(A*a^3-3*A*a*b^2-3*B*a^2*b+B*
b^3)/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{3\,Bb^3\tan\left(dx+c\right)^4+4\,(3\,Bab^2+Ab^3)\tan\left(dx+c\right)^3-12\,(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)dx+6\,(3\,Ba^2b}{}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/12*(3*B*b^3*tan(d*x + c)^4 + 4*(3*B*a*b^2 + A*b^3)*tan(d*x + c)^3 - 12*(
B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*d*x + 6*(3*B*a^2*b + 3*A*a*b^2 - B*
b^3)*tan(d*x + c)^2 - 6*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*log(1/(tan
(d*x + c)^2 + 1)) + 12*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*tan(d*x + c
))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(151) = 302.

Time = 0.16 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.88

$$\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \frac{Aa^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} - 3Aa^2bx + \frac{3Aa^2b\tan\left(c+dx\right)}{d} - \frac{3Aab^2\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{3Aab^2\tan^2\left(c+dx\right)}{2d} + Ab^3x + \frac{Ab^3\tan}{} \\ x(A+B\tan\left(c\right))\left(a+b\tan\left(c\right)\right)^3\tan\left(c\right) \end{cases}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((A*a**3*log(tan(c + d*x)**2 + 1)/(2*d) - 3*A*a**2*b*x + 3*A*a**2
*b*tan(c + d*x)/d - 3*A*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) + 3*A*a*b**2
*tan(c + d*x)**2/(2*d) + A*b**3*x + A*b**3*tan(c + d*x)**3/(3*d) - A*b**3*
tan(c + d*x)/d - B*a**3*x + B*a**3*tan(c + d*x)/d - 3*B*a**2*b*log(tan(c +
 d*x)**2 + 1)/(2*d) + 3*B*a**2*b*tan(c + d*x)**2/(2*d) + 3*B*a*b**2*x + B*
a*b**2*tan(c + d*x)**3/d - 3*B*a*b**2*tan(c + d*x)/d + B*b**3*log(tan(c +
d*x)**2 + 1)/(2*d) + B*b**3*tan(c + d*x)**4/(4*d) - B*b**3*tan(c + d*x)**2
/(2*d), Ne(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c))**3*tan(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.08

$$\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{3\,Bb^3\tan\left(dx+c\right)^4+4\,(3\,Bab^2+Ab^3)\tan\left(dx+c\right)^3+6\,(3\,Ba^2b+3\,Aab^2-Bb^3)\tan\left(dx+c\right)^2-12\,(B}{}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/12*(3*B*b^3*tan(d*x + c)^4 + 4*(3*B*a*b^2 + A*b^3)*tan(d*x + c)^3 + 6*(3
*B*a^2*b + 3*A*a*b^2 - B*b^3)*tan(d*x + c)^2 - 12*(B*a^3 + 3*A*a^2*b - 3*B
*a*b^2 - A*b^3)*(d*x + c) + 6*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*log(
tan(d*x + c)^2 + 1) + 12*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*tan(d*x +
 c))/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.52

$$\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)(dx+c)}{d}$$
$$+\frac{(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$
$$+\frac{3\,Bb^3d^3\tan\left(dx+c\right)^4+12\,Bab^2d^3\tan\left(dx+c\right)^3+4\,Ab^3d^3\tan\left(dx+c\right)^3+18\,Ba^2bd^3\tan\left(dx+c\right)^2+}{}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
-(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*(d*x + c)/d + 1/2*(A*a^3 - 3*B*a^
2*b - 3*A*a*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1)/d + 1/12*(3*B*b^3*d^3*tan
(d*x + c)^4 + 12*B*a*b^2*d^3*tan(d*x + c)^3 + 4*A*b^3*d^3*tan(d*x + c)^3 +
 18*B*a^2*b*d^3*tan(d*x + c)^2 + 18*A*a*b^2*d^3*tan(d*x + c)^2 - 6*B*b^3*d
^3*tan(d*x + c)^2 + 12*B*a^3*d^3*tan(d*x + c) + 36*A*a^2*b*d^3*tan(d*x + c
) - 36*B*a*b^2*d^3*tan(d*x + c) - 12*A*b^3*d^3*tan(d*x + c))/d^4
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.10

$$\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= x\left(-B\,a^3-3\,A\,a^2\,b+3\,B\,a\,b^2+A\,b^3\right)-\frac{\tan(c+d\,x)^2\left(\frac{B\,b^3}{2}-\frac{3\,a\,b\,(A\,b+B\,a)}{2}\right)}{d}$$
$$-\frac{\tan(c+d\,x)\,\left(-B\,a^3-3\,A\,a^2\,b+3\,B\,a\,b^2+A\,b^3\right)}{d}$$
$$+\frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{A\,a^3}{2}-\frac{3\,B\,a^2\,b}{2}-\frac{3\,A\,a\,b^2}{2}+\frac{B\,b^3}{2}\right)}{d}$$
$$+\frac{\tan(c+d\,x)^3\,\left(\frac{A\,b^3}{3}+B\,a\,b^2\right)}{d}+\frac{B\,b^3\tan(c+d\,x)^4}{4\,d}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
x*(A*b^3 - B*a^3 - 3*A*a^2*b + 3*B*a*b^2) - (tan(c + d*x)^2*((B*b^3)/2 - (
3*a*b*(A*b + B*a))/2))/d - (tan(c + d*x)*(A*b^3 - B*a^3 - 3*A*a^2*b + 3*B*
a*b^2))/d + (log(tan(c + d*x)^2 + 1)*((A*a^3)/2 + (B*b^3)/2 - (3*A*a*b^2)/
2 - (3*B*a^2*b)/2))/d + (tan(c + d*x)^3*((A*b^3)/3 + B*a*b^2))/d + (B*b^3*
tan(c + d*x)^4)/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.93

$$\int \tan(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{6\log\left(\tan\left(dx+c\right)^2+1\right)a^4-36\log\left(\tan\left(dx+c\right)^2+1\right)a^2b^2+6\log\left(\tan\left(dx+c\right)^2+1\right)b^4+3\tan\left(dx+\right.}{}$$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(6*log(tan(c + d*x)**2 + 1)*a**4 - 36*log(tan(c + d*x)**2 + 1)*a**2*b**2 +
 6*log(tan(c + d*x)**2 + 1)*b**4 + 3*tan(c + d*x)**4*b**4 + 16*tan(c + d*x
)**3*a*b**3 + 36*tan(c + d*x)**2*a**2*b**2 - 6*tan(c + d*x)**2*b**4 + 48*t
an(c + d*x)*a**3*b - 48*tan(c + d*x)*a*b**3 - 48*a**3*b*d*x + 48*a*b**3*d*
x)/(12*d)
```

3.250 $\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\begin{aligned}
&\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\
&= \left(a^3A-3aAb^2-3a^2bB+b^3B\right)x-\frac{(3a^2Ab-Ab^3+a^3B-3ab^2B)\log(\cos(c+dx))}{d} \\
&\quad+\frac{b(2aAb+a^2B-b^2B)\tan(c+dx)}{d} \\
&\quad+\frac{(Ab+aB)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}
\end{aligned}$$

output
```
(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*x-(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*ln(c
os(d*x+c))/d+b*(2*A*a*b+B*a^2-B*b^2)*tan(d*x+c)/d+1/2*(A*b+B*a)*(a+b*tan(d
*x+c))^2/d+1/3*B*(a+b*tan(d*x+c))^3/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

$$\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{3(a+ib)^3(-iA+B)\log(i-\tan(c+dx))+3(a-ib)^3(iA+B)\log(i+\tan(c+dx))+6b(3aAb+3a^2B}{6d}$$

input
```
Integrate[(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(3*(a + I*b)^3*((-I)*A + B)*Log[I - Tan[c + d*x]] + 3*(a - I*b)^3*(I*A + B
)*Log[I + Tan[c + d*x]] + 6*b*(3*a*A*b + 3*a^2*B - b^2*B)*Tan[c + d*x] + 3
*b^2*(A*b + 3*a*B)*Tan[c + d*x]^2 + 2*b^3*B*Tan[c + d*x]^3)/(6*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))^2(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{B(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{B(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 4011

$$\int (a+b\tan(c+dx))\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{(aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3042

$$\int (a+b\tan(c+dx))\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{(aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 4008

$$\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \tan(c+dx)dx+\frac{b\left(a^2B+2aAb-b^2B\right)\tan(c+dx)}{d}+x\left(a^3A-3a^2bB-3aAb^2+b^3B\right)+\frac{(aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3042

$$\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \tan(c+dx)dx+\frac{b\left(a^2B+2aAb-b^2B\right)\tan(c+dx)}{d}+x\left(a^3A-3a^2bB-3aAb^2+b^3B\right)+\frac{(aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3956

$$\frac{b\left(a^2B+2aAb-b^2B\right)\tan(c+dx)}{d}-\frac{\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\log(\cos(c+dx))}{d}+x\left(a^3A-3a^2bB-3aAb^2+b^3B\right)+\frac{(aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{B(a+b\tan(c+dx))^3}{3d}$$

input
```
Int[(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*x - ((3*a^2*A*b - A*b^3 + a^3*B -
3*a*b^2*B)*Log[Cos[c + d*x]])/d + (b*(2*a*A*b + a^2*B - b^2*B)*Tan[c + d*x
])/d + ((A*b + a*B)*(a + b*Tan[c + d*x])^2)/(2*d) + (B*(a + b*Tan[c + d*x]
)^3)/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

method	result
norman	$\left(A a^{3}-3 A a b^{2}-3 B a^{2} b+B b^{3}\right) x+\frac{b\left(3 A a b+3 B a^{2}-B b^{2}\right) \tan (d x+c)}{d}+\frac{B b^{3} \tan (d x+c)^{3}}{3 d}+\frac{b^{2}(A b-$
parts	$A a^{3} x+\frac{\left(A b^{3}+3 B a b^{2}\right)\left(\frac{\tan (d x+c)^{2}}{2}-\frac{\ln \left(1+\tan (d x+c)^{2}\right)}{2}\right)}{d}+\frac{\left(3 A a b^{2}+3 B a^{2} b\right)(\tan (d x+c)-\arctan (\tan (d x+c)))}{d}$
derivativedivides	$\frac{\frac{B b^{3} \tan (d x+c)^{3}}{3}+\frac{A b^{3} \tan (d x+c)^{2}}{2}+\frac{3 B a b^{2} \tan (d x+c)^{2}}{2}+3 A a b^{2} \tan (d x+c)+3 B a^{2} b \tan (d x+c)-B b^{3} \tan (d x+c)+\frac{(3 A a^{2}}{}}{d}$
default	$\frac{\frac{B b^{3} \tan (d x+c)^{3}}{3}+\frac{A b^{3} \tan (d x+c)^{2}}{2}+\frac{3 B a b^{2} \tan (d x+c)^{2}}{2}+3 A a b^{2} \tan (d x+c)+3 B a^{2} b \tan (d x+c)-B b^{3} \tan (d x+c)+\frac{(3 A a^{2}}{}}{d}$
parallelrisch	$\frac{2 B b^{3} \tan (d x+c)^{3}+6 A x a^{3} d-18 A a b^{2} d x+3 A b^{3} \tan (d x+c)^{2}-18 B a^{2} b d x+6 B b^{3} d x+9 B a b^{2} \tan (d x+c)^{2}+9 A \ln (1+\tan}{}$
risch	$A a^{3} x-3 A a b^{2} x-3 B a^{2} b x+B b^{3} x-\frac{2 i A b^{3} c}{d}-\frac{6 i B a b^{2} c}{d}+3 i A a^{2} b x+\frac{2 i B a^{3} c}{d}+\frac{6 i A a^{2} b c}{d}$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*x+b*(3*A*a*b+3*B*a^2-B*b^2)/d*tan(d*x+c)
+1/3*B*b^3/d*tan(d*x+c)^3+1/2*b^2*(A*b+3*B*a)/d*tan(d*x+c)^2+1/2*(3*A*a^2*
b-A*b^3+B*a^3-3*B*a*b^2)/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,Bb^3\tan\left(dx+c\right)^3+6\,(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3)dx+3\,(3\,Bab^2+Ab^3)\tan\left(dx+c\right)^2-3\,(Ba^3+}{6\,d}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*B*b^3*tan(d*x + c)^3 + 6*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*d*
x + 3*(3*B*a*b^2 + A*b^3)*tan(d*x + c)^2 - 3*(B*a^3 + 3*A*a^2*b - 3*B*a*b^
2 - A*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 6*(3*B*a^2*b + 3*A*a*b^2 - B*b^3)
*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.71

$$\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} Aa^3x+\frac{3Aa^2b\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}-3Aab^2x+\frac{3Aab^2\tan\left(c+dx\right)}{d}-\frac{Ab^3\log\left(\tan^2\left(c+dx\right)+1\right)}{2d}+\frac{Ab^3\tan^2\left(c+dx\right)}{2d}+\frac{Ba^3\log}{} \\ x(A+B\tan\left(c\right))\left(a+b\tan\left(c\right)\right)^3 \end{cases}$$

input
```
integrate((a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((A*a**3*x + 3*A*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) - 3*A*a*b*
*2*x + 3*A*a*b**2*tan(c + d*x)/d - A*b**3*log(tan(c + d*x)**2 + 1)/(2*d) +
 A*b**3*tan(c + d*x)**2/(2*d) + B*a**3*log(tan(c + d*x)**2 + 1)/(2*d) - 3*
B*a**2*b*x + 3*B*a**2*b*tan(c + d*x)/d - 3*B*a*b**2*log(tan(c + d*x)**2 +
1)/(2*d) + 3*B*a*b**2*tan(c + d*x)**2/(2*d) + B*b**3*x + B*b**3*tan(c + d*
x)**3/(3*d) - B*b**3*tan(c + d*x)/d, Ne(d, 0)), (x*(A + B*tan(c))*(a + b*t
an(c))**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02

$$\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,Bb^3\tan{(dx+c)}^3+3\,(3\,Bab^2+Ab^3)\tan{(dx+c)}^2+6\,(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3)(dx+c)+3\,(B}{6\,d}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(2*B*b^3*tan(d*x + c)^3 + 3*(3*B*a*b^2 + A*b^3)*tan(d*x + c)^2 + 6*(A*
a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*(d*x + c) + 3*(B*a^3 + 3*A*a^2*b - 3*
B*a*b^2 - A*b^3)*log(tan(d*x + c)^2 + 1) + 6*(3*B*a^2*b + 3*A*a*b^2 - B*b^
3)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.30

$$\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3)(dx+c)}{d}$$
$$+ \frac{(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)\log\left(\tan{(dx+c)}^2+1\right)}{2\,d}$$
$$+ \frac{2\,Bb^3d^2\tan{(dx+c)}^3+9\,Bab^2d^2\tan{(dx+c)}^2+3\,Ab^3d^2\tan{(dx+c)}^2+18\,Ba^2bd^2\tan{(dx+c)}+18}{6\,d^3}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*(d*x + c)/d + 1/2*(B*a^3 + 3*A*a^2
*b - 3*B*a*b^2 - A*b^3)*log(tan(d*x + c)^2 + 1)/d + 1/6*(2*B*b^3*d^2*tan(d
*x + c)^3 + 9*B*a*b^2*d^2*tan(d*x + c)^2 + 3*A*b^3*d^2*tan(d*x + c)^2 + 18
*B*a^2*b*d^2*tan(d*x + c) + 18*A*a*b^2*d^2*tan(d*x + c) - 6*B*b^3*d^2*tan(
d*x + c))/d^3
```

Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01

$$\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= x\left(A\,a^3-3\,B\,a^2\,b-3\,A\,a\,b^2+B\,b^3\right) - \frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(-\frac{B\,a^3}{2}-\frac{3\,A\,a^2\,b}{2}+\frac{3\,B\,a\,b^2}{2}+\frac{A\,b^3}{2}\right)}{d} + \frac{\tan(c+d\,x)^2\,\left(\frac{A\,b^3}{2}+\frac{3\,B\,a\,b^2}{2}\right)}{d} - \frac{\tan(c+d\,x)\,\left(B\,b^3-3\,a\,b\,(A\,b+B\,a)\right)}{d} + \frac{B\,b^3\tan(c+d\,x)^3}{3\,d}$$

input
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
x*(A*a^3 + B*b^3 - 3*A*a*b^2 - 3*B*a^2*b) - (log(tan(c + d*x)^2 + 1)*((A*b
^3)/2 - (B*a^3)/2 - (3*A*a^2*b)/2 + (3*B*a*b^2)/2))/d + (tan(c + d*x)^2*((
A*b^3)/2 + (3*B*a*b^2)/2))/d - (tan(c + d*x)*(B*b^3 - 3*a*b*(A*b + B*a)))/
d + (B*b^3*tan(c + d*x)^3)/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int (a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{6\log\left(\tan\left(dx+c\right)^2+1\right)a^3b-6\log\left(\tan\left(dx+c\right)^2+1\right)a\,b^3+\tan\left(dx+c\right)^3b^4+6\tan\left(dx+c\right)^2a\,b^3+18}{3d}$$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(6*log(tan(c + d*x)**2 + 1)*a**3*b - 6*log(tan(c + d*x)**2 + 1)*a*b**3 + t
an(c + d*x)**3*b**4 + 6*tan(c + d*x)**2*a*b**3 + 18*tan(c + d*x)*a**2*b**2
 - 3*tan(c + d*x)*b**4 + 3*a**4*d*x - 18*a**2*b**2*d*x + 3*b**4*d*x)/(3*d)
```

3.251 $\int \cot(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 117

$$\begin{aligned}\int \cot(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= \left(3a^2Ab-Ab^3+a^3B-3ab^2B\right)x-\frac{b(3aAb+3a^2B-b^2B)\log(\cos(c+dx))}{d}\\ &\quad+\frac{a^3A\log(\sin(c+dx))}{d}+\frac{b^2(Ab+2aB)\tan(c+dx)}{d}+\frac{bB(a+b\tan(c+dx))^2}{2d}\end{aligned}$$

output
```
(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*x-b*(3*A*a*b+3*B*a^2-B*b^2)*ln(cos(d*x+c
))/d+a^3*A*ln(sin(d*x+c))/d+b^2*(A*b+2*B*a)*tan(d*x+c)/d+1/2*b*B*(a+b*tan(
d*x+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98

$$\int \cot(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{-(a+ib)^3(A+iB)\log(i-\tan(c+dx))+2a^3A\log(\tan(c+dx))-(a-ib)^3(A-iB)\log(i+\tan(c+d}{2d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(-((a + I*b)^3*(A + I*B)*Log[I - Tan[c + d*x]]) + 2*a^3*A*Log[Tan[c + d*x]
] - (a - I*b)^3*(A - I*B)*Log[I + Tan[c + d*x]] + 2*b^2*(A*b + 2*a*B)*Tan[
c + d*x] + b*B*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {3042, 4090, 27, 3042, 4120, 25, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{1}{2}\int 2\cot(c+dx)(a+b\tan(c+dx))\left(Aa^2+b(Ab+2aB)\tan^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{bB(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(Aa^2+b(Ab+2aB)\tan^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{bB(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))\left(Aa^2+b(Ab+2aB)\tan(c+dx)^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{\tan(c+dx)}dx+\frac{bB(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4120}$$

$$-\int -\cot(c+dx)\left(Aa^3+b\left(3Ba^2+3Aba-b^2B\right)\tan^2(c+dx)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{b^2(2aB+Ab)\tan(c+dx)}{d}+\frac{bB(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{25}$$

$$\int \cot(c+dx)\left(Aa^3+b\left(3Ba^2+3Aba-b^2B\right)\tan^2(c+dx)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{b^2(2aB+Ab)\tan(c+dx)}{d}+\frac{bB(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Aa^3+b\left(3Ba^2+3Aba-b^2B\right)\tan(c+dx)^2+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{b^2(2aB+Ab)\tan(c+dx)}{d}+\frac{bB(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4107}$$

$$a^3A\int \cot(c+dx)dx+b\left(3a^2B+3aAb-b^2B\right)\int \tan(c+dx)dx+x\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)+\frac{b^2(2aB+Ab)\tan(c+dx)}{d}+\frac{bB(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$a^3 A \int -\tan\left(c+dx+\frac{\pi}{2}\right)dx + b\left(3a^2 B + 3aAb - b^2 B\right)\int \tan(c+dx)dx + x\left(a^3 B + 3a^2 Ab - 3ab^2 B - Ab^3\right) + \frac{b^2(2aB+Ab)\tan(c+dx)}{d} + \frac{bB(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \; 25$$

$$a^3(-A)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx + b\left(3a^2 B + 3aAb - b^2 B\right)\int \tan(c+dx)dx + x\left(a^3 B + 3a^2 Ab - 3ab^2 B - Ab^3\right) + \frac{b^2(2aB+Ab)\tan(c+dx)}{d} + \frac{bB(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \; 3956$$

$$\frac{a^3 A\log(-\sin(c+dx))}{d} - \frac{b\left(3a^2 B + 3aAb - b^2 B\right)\log(\cos(c+dx))}{d} + x\left(a^3 B + 3a^2 Ab - 3ab^2 B - Ab^3\right) + \frac{b^2(2aB+Ab)\tan(c+dx)}{d} + \frac{bB(a+b\tan(c+dx))^2}{2d}$$

input
```
Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*x - (b*(3*a*A*b + 3*a^2*B - b^2*B)
*Log[Cos[c + d*x]])/d + (a^3*A*Log[-Sin[c + d*x]])/d + (b^2*(A*b + 2*a*B)*
Tan[c + d*x])/d + (b*B*(a + b*Tan[c + d*x])^2)/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{(-A a^3+3Aa b^2+3B a^2b-B b^3) \ln\left(\sec(dx+c)^2\right)+2A a^3 \ln(\tan(dx+c))+B \tan(dx+c)^2b^3+(2A b^3+6Ba b^2) \tan(dx+c}{2d}$
norman	$(3A a^2b-A b^3+B a^3-3Ba b^2) x+\frac{b^2(Ab+3Ba) \tan(dx+c)}{d}+\frac{B b^3 \tan(dx+c)^2}{2d}+\frac{A a^3 \ln(\tan(dx+}{d}$
derivativedivides	$\frac{\frac{B \tan(dx+c)^2b^3}{2}+A \tan(dx+c)b^3+3B \tan(dx+c)a b^2+\frac{(-A a^3+3Aa b^2+3B a^2b-B b^3) \ln\left(1+\tan(dx+c)^2\right)}{2}+(3A a^2b-A b^3-}{d}$
default	$\frac{\frac{B \tan(dx+c)^2b^3}{2}+A \tan(dx+c)b^3+3B \tan(dx+c)a b^2+\frac{(-A a^3+3Aa b^2+3B a^2b-B b^3) \ln\left(1+\tan(dx+c)^2\right)}{2}+(3A a^2b-A b^3-}{d}$
risch	$\frac{2ib^2\left(-iBb\,e^{2i(dx+c)}+Ab\,e^{2i(dx+c)}+3Ba\,e^{2i(dx+c)}+Ab+3Ba\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-iB b^3x-\frac{2iB b^3c}{d}-\frac{2iA a^3c}{d}+3A a^2bx-$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/2*((-A*a^3+3*A*a*b^2+3*B*a^2*b-B*b^3)*ln(sec(d*x+c)^2)+2*A*a^3*ln(tan(d*
x+c))+B*tan(d*x+c)^2*b^3+(2*A*b^3+6*B*a*b^2)*tan(d*x+c)+6*x*(A*a^2*b-1/3*A
*b^3+1/3*B*a^3-B*a*b^2)*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int \cot(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{Bb^3\tan{(dx+c)}^2+Aa^3\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+2\,(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)dx-(3\,Ba^2b+3\,Aab^2-}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/2*(B*b^3*tan(d*x + c)^2 + A*a^3*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))
 + 2*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*d*x - (3*B*a^2*b + 3*A*a*b^2
- B*b^3)*log(1/(tan(d*x + c)^2 + 1)) + 2*(3*B*a*b^2 + A*b^3)*tan(d*x + c))
/d
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.74

$$\int \cot(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\begin{cases} -\frac{Aa^3\log(\tan^2(c+dx)+1)}{2d}+\frac{Aa^3\log(\tan(c+dx))}{d}+3Aa^2bx+\frac{3Aab^2\log(\tan^2(c+dx)+1)}{2d}-Ab^3x+\frac{Ab^3\tan(c+dx)}{d}+Ba \\ x(A+B\tan(c))(a+b\tan(c))^3\cot(c) \end{cases}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((-A*a**3*log(tan(c + d*x)**2 + 1)/(2*d) + A*a**3*log(tan(c + d*x
))/d + 3*A*a**2*b*x + 3*A*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - A*b**3*x
 + A*b**3*tan(c + d*x)/d + B*a**3*x + 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/
(2*d) - 3*B*a*b**2*x + 3*B*a*b**2*tan(c + d*x)/d - B*b**3*log(tan(c + d*x)
**2 + 1)/(2*d) + B*b**3*tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*tan(c)
)*(a + b*tan(c))**3*cot(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \cot(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{Bb^3\tan(dx+c)^2+2\,Aa^3\log(\tan(dx+c))+2\,(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)(dx+c)-(Aa^3-3\,Ba}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/2*(B*b^3*tan(d*x + c)^2 + 2*A*a^3*log(tan(d*x + c)) + 2*(B*a^3 + 3*A*a^2
*b - 3*B*a*b^2 - A*b^3)*(d*x + c) - (A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3
)*log(tan(d*x + c)^2 + 1) + 2*(3*B*a*b^2 + A*b^3)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \cot(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{Aa^3\log\left(|\tan\left(dx+c\right)|\right)}{d} + \frac{(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)(dx+c)}{d} - \frac{(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d} + \frac{Bb^3d\tan\left(dx+c\right)^2+6\,Bab^2d\tan\left(dx+c\right)+2\,Ab^3d\tan\left(dx+c\right)}{2\,d^2}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
A*a^3*log(abs(tan(d*x + c)))/d + (B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*(
d*x + c)/d - 1/2*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*log(tan(d*x + c)^
2 + 1)/d + 1/2*(B*b^3*d*tan(d*x + c)^2 + 6*B*a*b^2*d*tan(d*x + c) + 2*A*b^
3*d*tan(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \cot(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)\,\left(A\,b^3+3\,B\,a\,b^2\right)}{d} + \frac{A\,a^3\,\ln\left(\tan(c+d\,x)\right)}{d} + \frac{B\,b^3\tan(c+d\,x)^2}{2\,d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
(tan(c + d*x)*(A*b^3 + 3*B*a*b^2))/d + (A*a^3*log(tan(c + d*x)))/d - (log(
tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)^3*1i)/(2*d) - (log(tan(c + d*x) -
 1i)*(A + B*1i)*(a*1i - b)^3*1i)/(2*d) + (B*b^3*tan(c + d*x)^2)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.92

$$\int \cot(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{-8\cos(dx+c)\sin(dx+c)\,a\,b^3 - 2\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)^2\,a^4 + 12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}{}$$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
( - 8*cos(c + d*x)*sin(c + d*x)*a*b**3 - 2*log(tan((c + d*x)/2)**2 + 1)*si
n(c + d*x)**2*a**4 + 12*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**2*
b**2 - 2*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*b**4 + 2*log(tan((c
+ d*x)/2)**2 + 1)*a**4 - 12*log(tan((c + d*x)/2)**2 + 1)*a**2*b**2 + 2*log
(tan((c + d*x)/2)**2 + 1)*b**4 - 12*log(tan((c + d*x)/2) - 1)*sin(c + d*x)
**2*a**2*b**2 + 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*b**4 + 12*log(
tan((c + d*x)/2) - 1)*a**2*b**2 - 2*log(tan((c + d*x)/2) - 1)*b**4 - 12*lo
g(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a**2*b**2 + 2*log(tan((c + d*x)/2)
 + 1)*sin(c + d*x)**2*b**4 + 12*log(tan((c + d*x)/2) + 1)*a**2*b**2 - 2*lo
g(tan((c + d*x)/2) + 1)*b**4 + 2*log(tan((c + d*x)/2))*sin(c + d*x)**2*a**
4 - 2*log(tan((c + d*x)/2))*a**4 + 8*sin(c + d*x)**2*a**3*b*d*x - 8*sin(c
+ d*x)**2*a*b**3*d*x - sin(c + d*x)**2*b**4 - 8*a**3*b*d*x + 8*a*b**3*d*x)
/(2*d*(sin(c + d*x)**2 - 1))
```

3.252 $\int \cot^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 119

$$\begin{aligned}
&\int \cot^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\
&= -\left(\left(a^3A-3aAb^2-3a^2bB+b^3B\right)x\right) \\
&\quad -\frac{b^2(Ab+3aB)\log(\cos(c+dx))}{d}+\frac{a^2(3Ab+aB)\log(\sin(c+dx))}{d} \\
&\quad +\frac{b^2(aA+bB)\tan(c+dx)}{d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^2}{d}
\end{aligned}$$

output

```
-(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*x-b^2*(A*b+3*B*a)*ln(cos(d*x+c))/d+a^2*
(3*A*b+B*a)*ln(sin(d*x+c))/d+b^2*(A*a+B*b)*tan(d*x+c)/d-a*A*cot(d*x+c)*(a+
b*tan(d*x+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{-2a^3A\cot(c+dx)+i(a+ib)^3(A+iB)\log(i-\tan(c+dx))+2a^2(3Ab+aB)\log(\tan(c+dx))+(ia-}{2d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a^3*A*Cot[c + d*x] + I*(a + I*b)^3*(A + I*B)*Log[I - Tan[c + d*x]] + 2
*a^2*(3*A*b + a*B)*Log[Tan[c + d*x]] + (I*a + b)^3*(A - I*B)*Log[I + Tan[c
 + d*x]] + 2*b^3*B*Tan[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 4088, 3042, 4120, 25, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4088}$$

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(b(aA+bB)\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+aB)\right)dx-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))\left(b(aA+bB)\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+aB)\right)}{\tan(c+dx)}dx-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{4120}$$

$$-\int -\cot(c+dx)\left((3Ab+aB)a^2+b^2(Ab+3aB)\tan^2(c+dx)-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{b^2(aA+bB)\tan(c+dx)}{d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{25}$$

$$\int \cot(c+dx)\left((3Ab+aB)a^2+b^2(Ab+3aB)\tan^2(c+dx)-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{b^2(aA+bB)\tan(c+dx)}{d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(3Ab+aB)a^2+b^2(Ab+3aB)\tan(c+dx)^2-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{b^2(aA+bB)\tan(c+dx)}{d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{4107}$$

$$a^2(aB+3Ab)\int \cot(c+dx)dx+b^2(3aB+Ab)\int \tan(c+dx)dx-x\left(a^3A-3a^2bB-3aAb^2+b^3B\right)+\frac{b^2(aA+bB)\tan(c+dx)}{d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$a^2(aB+3Ab)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+b^2(3aB+Ab)\int \tan(c+dx)dx-x\left(a^3A-3a^2bB-3aAb^2+b^3B\right)+\frac{b^2(aA+bB)\tan(c+dx)}{d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{25}$$

$$-\left(a^2(aB+3Ab)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx\right)+b^2(3aB+Ab)\int \tan(c+dx)dx-x\left(a^3A-3a^2bB-3aAb^2+b^3B\right)+\frac{b^2(aA+bB)\tan(c+dx)}{d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

$\big\downarrow$ 3956

$$\frac{a^2(aB+3Ab)\log(-\sin(c+dx))}{d}-x\left(a^3A-3a^2bB-3aAb^2+b^3B\right)+\frac{b^2(aA+bB)\tan(c+dx)}{d}-\frac{b^2(3aB+Ab)\log(\cos(c+dx))}{d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^2}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
-((a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*x) - (b^2*(A*b + 3*a*B)*Log[Cos[
c + d*x]])/d + (a^2*(3*A*b + a*B)*Log[-Sin[c + d*x]])/d + (b^2*(a*A + b*B)
*Tan[c + d*x])/d - (a*A*Cot[c + d*x]*(a + b*Tan[c + d*x])^2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{(-3Aa^2b+Ab^3-Ba^3+3Bab^2)\ln(\sec(dx+c)^2)+(6Aa^2b+2Ba^3)\ln(\tan(dx+c))-2A\cot(dx+c)a^3+2Bb^3\tan(dx+c}{2d}$
derivativedivides	$\frac{Bb^3\tan(dx+c)+\frac{(-3Aa^2b+Ab^3-Ba^3+3Bab^2)\ln(1+\tan(dx+c)^2)}{2}+(-Aa^3+3Aab^2+3Ba^2b-Bb^3)\arctan(\tan(dx+c)}{d}$
default	$\frac{Bb^3\tan(dx+c)+\frac{(-3Aa^2b+Ab^3-Ba^3+3Bab^2)\ln(1+\tan(dx+c)^2)}{2}+(-Aa^3+3Aab^2+3Ba^2b-Bb^3)\arctan(\tan(dx+c)}{d}$
norman	$\frac{(-Aa^3+3Aab^2+3Ba^2b-Bb^3)x\tan(dx+c)+\frac{Bb^3\tan(dx+c)^2}{d}-\frac{Aa^3}{d}}{\tan(dx+c)}+\frac{a^2(3Ab+Ba)\ln(\tan(dx+c))}{d}-\frac{(3Aa^2b-A}{}$
risch	$-Aa^3x+3Aab^2x+3Ba^2bx-Bb^3x-\frac{2i(Aa^3e^{2i(dx+c)}-Bb^3e^{2i(dx+c)}+Aa^3+Bb^3)}{d(e^{2i(dx+c)}-1)(e^{2i(dx+c)}+1)}+\frac{2iAb^3c}{d}+$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/2*((-3*A*a^2*b+A*b^3-B*a^3+3*B*a*b^2)*ln(sec(d*x+c)^2)+(6*A*a^2*b+2*B*a^
3)*ln(tan(d*x+c))-2*A*cot(d*x+c)*a^3+2*B*b^3*tan(d*x+c)-2*d*x*(A*a^3-3*A*a
*b^2-3*B*a^2*b+B*b^3))/d
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

$$\int\cot^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{2\,Bb^3\tan(dx+c)^2-2\,Aa^3-2\,(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3)dx\tan(dx+c)+(Ba^3+3\,Aa^2b)\log\left(\frac{}{\mathrm{ta}}\right.}{2\,d\tan(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/2*(2*B*b^3*tan(d*x + c)^2 - 2*A*a^3 - 2*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 +
 B*b^3)*d*x*tan(d*x + c) + (B*a^3 + 3*A*a^2*b)*log(tan(d*x + c)^2/(tan(d*x
 + c)^2 + 1))*tan(d*x + c) - (3*B*a*b^2 + A*b^3)*log(1/(tan(d*x + c)^2 + 1
))*tan(d*x + c))/(d*tan(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.87

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= \begin{cases} \tilde{\infty} A a^3 x \\ x(A+B\tan(c))\,(a+b\tan(c))^3\cot^2(c) \\ \tilde{\infty} A a^3 x \\ -Aa^3x - \frac{Aa^3}{d\tan(c+dx)} - \frac{3Aa^2b\log(\tan^2(c+dx)+1)}{2d} + \frac{3Aa^2b\log(\tan(c+dx))}{d} + 3Aab^2x + \frac{Ab^3\log(\tan^2(c+dx)+1)}{2d} - \frac{Ba^3}{} \end{cases}$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**3*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**3*cot(c)**2, Eq(d, 0)), (zoo*A*a**3*x, Eq(c, -d*x)), (-A*a**3*x - A
*a**3/(d*tan(c + d*x)) - 3*A*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*A*a
**2*b*log(tan(c + d*x))/d + 3*A*a*b**2*x + A*b**3*log(tan(c + d*x)**2 + 1)
/(2*d) - B*a**3*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**3*log(tan(c + d*x))/
d + 3*B*a**2*b*x + 3*B*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - B*b**3*x +
B*b**3*tan(c + d*x)/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= \frac{2\,Bb^3\tan(dx+c) - \frac{2\,Aa^3}{\tan(dx+c)} - 2\,(Aa^3 - 3\,Ba^2b - 3\,Aab^2 + Bb^3)(dx+c) - (Ba^3 + 3\,Aa^2b - 3\,Bab^2 -}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/2*(2*B*b^3*tan(d*x + c) - 2*A*a^3/tan(d*x + c) - 2*(A*a^3 - 3*B*a^2*b -
3*A*a*b^2 + B*b^3)*(d*x + c) - (B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*log
(tan(d*x + c)^2 + 1) + 2*(B*a^3 + 3*A*a^2*b)*log(tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{Bb^3\tan(dx+c)}{d}-\frac{Aa^3}{d\tan(dx+c)}-\frac{(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3)(dx+c)}{d}-\frac{(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)\log\left(\tan(dx+c)^2+1\right)}{2\,d}+\frac{(Ba^3+3\,Aa^2b)\log(|\tan(dx+c)|)}{d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
B*b^3*tan(d*x + c)/d - A*a^3/(d*tan(d*x + c)) - (A*a^3 - 3*B*a^2*b - 3*A*a
*b^2 + B*b^3)*(d*x + c)/d - 1/2*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*lo
g(tan(d*x + c)^2 + 1)/d + (B*a^3 + 3*A*a^2*b)*log(abs(tan(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{\ln(\tan(c+d\,x))\,\left(B\,a^3+3\,A\,b\,a^2\right)}{d}-\frac{A\,a^3\cot(c+d\,x)}{d}+\frac{B\,b^3\tan(c+d\,x)}{d}+\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(A+B\,1\mathrm{i})\,(a+b\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}-\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(A-B\,1\mathrm{i})\,(a-b\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
(log(tan(c + d*x))*(B*a^3 + 3*A*a^2*b))/d + (log(tan(c + d*x) - 1i)*(A + B
*1i)*(a + b*1i)^3*1i)/(2*d) - (log(tan(c + d*x) + 1i)*(A - B*1i)*(a - b*1i
)^3*1i)/(2*d) - (A*a^3*cot(c + d*x))/d + (B*b^3*tan(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.20

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{-4\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)\,a^3b+4\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)}{}$$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
( - 4*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a**3*b + 4*co
s(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a*b**3 - 4*cos(c + d*
x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*a*b**3 - 4*cos(c + d*x)*log(tan(
(c + d*x)/2) + 1)*sin(c + d*x)*a*b**3 + 4*cos(c + d*x)*log(tan((c + d*x)/2
))*sin(c + d*x)*a**3*b - cos(c + d*x)*sin(c + d*x)*a**4*d*x + 6*cos(c + d*
x)*sin(c + d*x)*a**2*b**2*d*x - cos(c + d*x)*sin(c + d*x)*b**4*d*x + sin(c
 + d*x)**2*a**4 + sin(c + d*x)**2*b**4 - a**4)/(cos(c + d*x)*sin(c + d*x)*
d)
```

3.253 $\int \cot^3(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 127

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\left(\left(3a^2Ab-Ab^3+a^3B-3ab^2B\right)x\right)-\frac{a^2(2Ab+aB)\cot(c+dx)}{d} - \frac{b^3B\log(\cos(c+dx))}{d}-\frac{a(a^2A-3Ab^2-3abB)\log(\sin(c+dx))}{d} - \frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

output
```
-(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*x-a^2*(2*A*b+B*a)*cot(d*x+c)/d-b^3*B*ln
(cos(d*x+c))/d-a*(A*a^2-3*A*b^2-3*B*a*b)*ln(sin(d*x+c))/d-1/2*a*A*cot(d*x+
c)^2*(a+b*tan(d*x+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{-2a^2(3Ab+aB)\cot(c+dx)-a^3A\cot^2(c+dx)+(a+ib)^3(A+iB)\log(i-\tan(c+dx))-2a(a^2A-}{2d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a^2*(3*A*b + a*B)*Cot[c + d*x] - a^3*A*Cot[c + d*x]^2 + (a + I*b)^3*(A
 + I*B)*Log[I - Tan[c + d*x]] - 2*a*(a^2*A - 3*A*b^2 - 3*a*b*B)*Log[Tan[c
+ d*x]] + (a - I*b)^3*(A - I*B)*Log[I + Tan[c + d*x]])/(2*d)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4088, 27, 3042, 4118, 25, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{2}\int 2\cot^2(c+dx)(a+b\tan(c+dx))\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)dx-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot^2(c+dx)(a+b\tan(c+dx))\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)dx-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))\left(b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)}{\tan(c+dx)^2}dx-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4118}$$

$$\int -\cot(c+dx)\left(-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx-\frac{a^2(aB+2Ab)\cot(c+dx)}{d}-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(c+dx)\left(-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx-\frac{a^2(aB+2Ab)\cot(c+dx)}{d}-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{-B\tan(c+dx)^2b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan(c+dx)}dx-\frac{a^2(aB+2Ab)\cot(c+dx)}{d}-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4107}$$

$$-a\left(a^2A-3abB-3Ab^2\right)\int \cot(c+dx)dx+b^3B\int \tan(c+dx)dx-\frac{a^2(aB+2Ab)\cot(c+dx)}{d}-x\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-a\left(a^2A-3abB-3Ab^2\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+b^3B\int \tan(c+dx)dx-\frac{a^2(aB+2Ab)\cot(c+dx)}{d}-x\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 25

$$a\left(a^2A-3abB-3Ab^2\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+b^3B\int \tan(c+dx)dx-\frac{a^2(aB+2Ab)\cot(c+dx)}{d}-x\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}$$

$\Big\downarrow$ 3956

$$-\frac{a\left(a^2A-3abB-3Ab^2\right)\log(-\sin(c+dx))}{d}-\frac{a^2(aB+2Ab)\cot(c+dx)}{d}-x\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{b^3B\log(\cos(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
-((3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*x) - (a^2*(2*A*b + a*B)*Cot[c +
d*x])/d - (b^3*B*Log[Cos[c + d*x]])/d - (a*(a^2*A - 3*A*b^2 - 3*a*b*B)*Log
[-Sin[c + d*x]])/d - (a*A*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{(A a^3-3Aa b^2-3B a^2b+B b^3) \ln\left(\sec(dx+c)^2\right)+(-2A a^3+6Aa b^2+6B a^2b) \ln(\tan(dx+c))-A \cot(dx+c)^2 a^3+(-6A a}{2d}$
derivativedivides	$\frac{\frac{\left(A a^3-3Aa b^2-3B a^2b+B b^3\right) \ln\left(1+\tan(dx+c)^2\right)}{2}+(-3A a^2b+A b^3-B a^3+3Ba b^2) \arctan(\tan(dx+c))-\frac{A a^3}{2 \tan(dx+c)^2}-\frac{a^2}{t}}{d}$
default	$\frac{\frac{\left(A a^3-3Aa b^2-3B a^2b+B b^3\right) \ln\left(1+\tan(dx+c)^2\right)}{2}+(-3A a^2b+A b^3-B a^3+3Ba b^2) \arctan(\tan(dx+c))-\frac{A a^3}{2 \tan(dx+c)^2}-\frac{a^2}{t}}{d}$
norman	$\frac{(-3A a^2b+A b^3-B a^3+3Ba b^2)x \tan(dx+c)^2-\frac{A a^3}{2d}-\frac{a^2(3Ab+Ba)\tan(dx+c)}{d}}{\tan(dx+c)^2}+\frac{(A a^3-3Aa b^2-3B a^2b+B b^3) \ln\left(1+\right.}{2d}$
risch	$-\frac{6iAa b^2c}{d}+\frac{2iB b^3c}{d}+iB b^3x-\frac{2ia^2\left(iAa\, e^{2i(dx+c)}+3Ab\, e^{2i(dx+c)}+Ba\, e^{2i(dx+c)}-3Ab-Ba\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-3A a^2bx$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/2*((A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*ln(sec(d*x+c)^2)+(-2*A*a^3+6*A*a*b^
2+6*B*a^2*b)*ln(tan(d*x+c))-A*cot(d*x+c)^2*a^3+(-6*A*a^2*b-2*B*a^3)*cot(d*
x+c)-6*x*(A*a^2*b-1/3*A*b^3+1/3*B*a^3-B*a*b^2)*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.28

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{Bb^3\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+Aa^3+(Aa^3-3\,Ba^2b-3\,Aab^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2}{2\,d\tan(dx}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/2*(B*b^3*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 + A*a^3 + (A*a^3 -
3*B*a^2*b - 3*A*a*b^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x +
c)^2 + (A*a^3 + 2*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*d*x)*tan(d*x + c
)^2 + 2*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(121) = 242$.

Time = 1.05 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.06

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \tilde{\infty}Aa^3x \\ x(A+B\tan(c))\,(a+b\tan(c))^3\cot^3(c) \\ \tilde{\infty}Aa^3x \\ \frac{Aa^3\log(\tan^2(c+dx)+1)}{2d} - \frac{Aa^3\log(\tan(c+dx))}{d} - \frac{Aa^3}{2d\tan^2(c+dx)} - 3Aa^2bx - \frac{3Aa^2b}{d\tan(c+dx)} - \frac{3Aab^2\log(\tan^2(c+dx)+1)}{2d} + 3 \end{cases}$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**3*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**3*cot(c)**3, Eq(d, 0)), (zoo*A*a**3*x, Eq(c, -d*x)), (A*a**3*log(ta
n(c + d*x)**2 + 1)/(2*d) - A*a**3*log(tan(c + d*x))/d - A*a**3/(2*d*tan(c
+ d*x)**2) - 3*A*a**2*b*x - 3*A*a**2*b/(d*tan(c + d*x)) - 3*A*a*b**2*log(t
an(c + d*x)**2 + 1)/(2*d) + 3*A*a*b**2*log(tan(c + d*x))/d + A*b**3*x - B*
a**3*x - B*a**3/(d*tan(c + d*x)) - 3*B*a**2*b*log(tan(c + d*x)**2 + 1)/(2*
d) + 3*B*a**2*b*log(tan(c + d*x))/d + 3*B*a*b**2*x + B*b**3*log(tan(c + d*
x)**2 + 1)/(2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.12

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3\right)(dx+c)-\left(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)+}{2\,d}$$

input

```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output

```
-1/2*(2*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*(d*x + c) - (A*a^3 - 3*B*a
^2*b - 3*A*a*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1) + 2*(A*a^3 - 3*B*a^2*b -
 3*A*a*b^2)*log(tan(d*x + c)) + (A*a^3 + 2*(B*a^3 + 3*A*a^2*b)*tan(d*x + c
))/tan(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \cot^3(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\ = -\frac{\left(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3\right)(dx+c)}{d} \\ + \frac{\left(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d} \\ - \frac{\left(Aa^3-3\,Ba^2b-3\,Aab^2\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d} \\ - \frac{Aa^3+2\left(Ba^3+3\,Aa^2b\right)\tan\left(dx+c\right)}{2\,d\tan\left(dx+c\right)^2}\end{aligned}$$

input

```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*(d*x + c)/d + 1/2*(A*a^3 - 3*B*a^
2*b - 3*A*a*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1)/d - (A*a^3 - 3*B*a^2*b -
3*A*a*b^2)*log(abs(tan(d*x + c)))/d - 1/2*(A*a^3 + 2*(B*a^3 + 3*A*a^2*b)*t
an(d*x + c))/(d*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{\ln(\tan(c+d\,x))\,\left(-A\,a^3+3\,B\,a^2\,b+3\,A\,a\,b^2\right)}{d} - \frac{\cot(c+d\,x)^2\,\left(\tan(c+d\,x)\,\left(B\,a^3+3\,A\,b\,a^2\right)+\frac{A\,a^3}{2}\right)}{d} + \frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d} + \frac{\ln(\tan(c+d\,x)-\mathrm{i})\,\left(A+B\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
(log(tan(c + d*x))*(3*A*a*b^2 - A*a^3 + 3*B*a^2*b))/d - (cot(c + d*x)^2*(t
an(c + d*x)*(B*a^3 + 3*A*a^2*b) + (A*a^3)/2))/d + (log(tan(c + d*x) + 1i)*
(A - B*1i)*(a*1i + b)^3*1i)/(2*d) + (log(tan(c + d*x) - 1i)*(A + B*1i)*(a*
1i - b)^3*1i)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 12.17 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.08

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{-16\cos(dx+c)\sin(dx+c)\,a^3b+4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)^2\,a^4-24\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right.}{}$$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
( - 16*cos(c + d*x)*sin(c + d*x)*a**3*b + 4*log(tan((c + d*x)/2)**2 + 1)*s
in(c + d*x)**2*a**4 - 24*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**2
*b**2 + 4*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*b**4 - 4*log(tan((c
 + d*x)/2) - 1)*sin(c + d*x)**2*b**4 - 4*log(tan((c + d*x)/2) + 1)*sin(c +
 d*x)**2*b**4 - 4*log(tan((c + d*x)/2))*sin(c + d*x)**2*a**4 + 24*log(tan(
(c + d*x)/2))*sin(c + d*x)**2*a**2*b**2 + sin(c + d*x)**2*a**4 - 16*sin(c
+ d*x)**2*a**3*b*d*x + 16*sin(c + d*x)**2*a*b**3*d*x - 2*a**4)/(4*sin(c +
d*x)**2*d)
```

3.254 $\int \cot^4(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 154

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \left(a^3A-3aAb^2-3a^2bB+b^3B\right)x+\frac{a(3a^2A-8Ab^2-9abB)\cot(c+dx)}{3d} - \frac{a^2(5Ab+3aB)\cot^2(c+dx)}{6d}-\frac{(3a^2Ab-Ab^3+a^3B-3ab^2B)\log(\sin(c+dx))}{d} - \frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

output
```
(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*x+1/3*a*(3*A*a^2-8*A*b^2-9*B*a*b)*cot(d*
x+c)/d-1/6*a^2*(5*A*b+3*B*a)*cot(d*x+c)^2/d-(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b
^2)*ln(sin(d*x+c))/d-1/3*a*A*cot(d*x+c)^3*(a+b*tan(d*x+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.06

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{6a(a^2A-3Ab^2-3abB)\cot(c+dx)-3a^2(3Ab+aB)\cot^2(c+dx)-2a^3A\cot^3(c+dx)+3(a+ib)^3(-}{}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(6*a*(a^2*A - 3*A*b^2 - 3*a*b*B)*Cot[c + d*x] - 3*a^2*(3*A*b + a*B)*Cot[c
+ d*x]^2 - 2*a^3*A*Cot[c + d*x]^3 + 3*(a + I*b)^3*((-I)*A + B)*Log[I - Tan
[c + d*x]] - 6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Log[Tan[c + d*x]] +
 3*(a - I*b)^3*(I*A + B)*Log[I + Tan[c + d*x]])/(6*d)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4088, 3042, 4118, 25, 3042, 4111, 27, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{3}\int \cot^3(c+dx)(a+b\tan(c+dx))\left(-b(aA-3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+3aB)\right)dx-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\int \frac{(a+b\tan(c+dx))\left(-b(aA-3bB)\tan(c+dx)^2-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+3aB)\right)}{\tan(c+dx)^3}dx-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 4118

$$\frac{1}{3}\left(\int -\cot^2(c+dx)\left(b^2(aA-3bB)\tan^2(c+dx)+3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(3Aa^2-9b\right.\right.\right.$$
$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 25

$$\frac{1}{3}\left(-\int \cot^2(c+dx)\left(b^2(aA-3bB)\tan^2(c+dx)+3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(3Aa^2-9b\right.\right.\right.$$
$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(-\int \frac{b^2(aA-3bB)\tan(c+dx)^2+3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(3Aa^2-9bBa-8Ab^2\right)}{\tan(c+dx)^2}d\right.$$
$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 4111

$$\frac{1}{3}\left(-\int 3\cot(c+dx)\left(Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{a(3a^2\right.$$
$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(-3\int \cot(c+dx)\left(Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{a(3a^2A\right.$$
$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(-3\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{a\left(3a^2A-9abB-8A\right.}{d}\right.$$
$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 4014

$$\frac{1}{3}\left(-3\left(\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \cot(c+dx)dx-x\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\right)+\frac{a\left(3a^2A-9abB\right.}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(-3\left(\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-x\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\right)+\frac{a(3a^2}{}\right.$$
$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 25

$$\frac{1}{3}\left(-3\left(-\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\left(x\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\right)\right)+\right.$$
$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

$\Big\downarrow$ 3956

$$\frac{1}{3}\left(\frac{a\left(3a^2A-9abB-8Ab^2\right)\cot(c+dx)}{d}-\frac{a^2(3aB+5Ab)\cot^2(c+dx)}{2d}-3\left(\frac{\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\log}{d}\right.\right.$$
$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^2}{3d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
((a*(3*a^2*A - 8*A*b^2 - 9*a*b*B)*Cot[c + d*x])/d - (a^2*(5*A*b + 3*a*B)*C
ot[c + d*x]^2)/(2*d) - 3*(-((a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*x) + (
(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Log[-Sin[c + d*x]])/d))/3 - (a*A*C
ot[c + d*x]^3*(a + b*Tan[c + d*x])^2)/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{3(3Aa^2b-Ab^3+Ba^3-3Bab^2)\ln\left(\sec(dx+c)^2\right)+6(-3Aa^2b+Ab^3-Ba^3+3Bab^2)\ln(\tan(dx+c))-2A\cot(dx+c)^3a^3}{6d}$
derivativedivides	$\frac{\frac{(3Aa^2b-Ab^3+Ba^3-3Bab^2)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Aa^3-3Aab^2-3Ba^2b+Bb^3)\arctan(\tan(dx+c))+(-3Aa^2b+Ab^3-}{d}$
default	$\frac{\frac{(3Aa^2b-Ab^3+Ba^3-3Bab^2)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Aa^3-3Aab^2-3Ba^2b+Bb^3)\arctan(\tan(dx+c))+(-3Aa^2b+Ab^3-}{d}$
norman	$\frac{(Aa^3-3Aab^2-3Ba^2b+Bb^3)x\tan(dx+c)^3+\frac{a\left(Aa^2-3Ab^2-3Bab\right)\tan(dx+c)^2}{d}-\frac{Aa^3}{3d}-\frac{a^2(3Ab+Ba)\tan(dx+c)}{2d}}{\tan(dx+c)^3}-\frac{(3A}{}$
risch	$Aa^3x-3Aab^2x-3Ba^2bx+Bb^3x-\frac{2iAb^3c}{d}+3iAa^2bx+\frac{2iBa^3c}{d}-3iBab^2x-\frac{6iBab}{d}$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/6*(3*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*ln(sec(d*x+c)^2)+6*(-3*A*a^2*b+A*
b^3-B*a^3+3*B*a*b^2)*ln(tan(d*x+c))-2*A*cot(d*x+c)^3*a^3+3*(-3*A*a^2*b-B*a
^3)*cot(d*x+c)^2+6*a*cot(d*x+c)*(A*a^2-3*A*b^2-3*B*a*b)+6*d*x*(A*a^3-3*A*a
*b^2-3*B*a^2*b+B*b^3))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.18

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{3\left(Ba^3+3Aa^2b-3Bab^2-Ab^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^3+2Aa^3+3\left(Ba^3+3Aa^2b-2\left(Aa^3\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/6*(3*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*log(tan(d*x + c)^2/(tan(d*
x + c)^2 + 1))*tan(d*x + c)^3 + 2*A*a^3 + 3*(B*a^3 + 3*A*a^2*b - 2*(A*a^3
- 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*d*x)*tan(d*x + c)^3 - 6*(A*a^3 - 3*B*a^2*
b - 3*A*a*b^2)*tan(d*x + c)^2 + 3*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/(d*tan
(d*x + c)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(150) = 300$.

Time = 2.06 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.16

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \tilde{\infty} A a^3 x \\ x(A+B\tan(c))\,(a+b\tan(c))^3\cot^4(c) \\ \tilde{\infty} A a^3 x \\ Aa^3x + \frac{Aa^3}{d\tan(c+dx)} - \frac{Aa^3}{3d\tan^3(c+dx)} + \frac{3Aa^2b\log(\tan^2(c+dx)+1)}{2d} - \frac{3Aa^2b\log(\tan(c+dx))}{d} - \frac{3Aa^2b}{2d\tan^2(c+dx)} - 3Aab^2x - \end{cases}$$

input
```
integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**3*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**3*cot(c)**4, Eq(d, 0)), (zoo*A*a**3*x, Eq(c, -d*x)), (A*a**3*x + A*
a**3/(d*tan(c + d*x)) - A*a**3/(3*d*tan(c + d*x)**3) + 3*A*a**2*b*log(tan(
c + d*x)**2 + 1)/(2*d) - 3*A*a**2*b*log(tan(c + d*x))/d - 3*A*a**2*b/(2*d*
tan(c + d*x)**2) - 3*A*a*b**2*x - 3*A*a*b**2/(d*tan(c + d*x)) - A*b**3*log
(tan(c + d*x)**2 + 1)/(2*d) + A*b**3*log(tan(c + d*x))/d + B*a**3*log(tan(
c + d*x)**2 + 1)/(2*d) - B*a**3*log(tan(c + d*x))/d - B*a**3/(2*d*tan(c +
d*x)**2) - 3*B*a**2*b*x - 3*B*a**2*b/(d*tan(c + d*x)) - 3*B*a*b**2*log(tan
(c + d*x)**2 + 1)/(2*d) + 3*B*a*b**2*log(tan(c + d*x))/d + B*b**3*x, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{6\left(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3\right)(dx+c)+3\left(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)-}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/6*(6*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*(d*x + c) + 3*(B*a^3 + 3*A*
a^2*b - 3*B*a*b^2 - A*b^3)*log(tan(d*x + c)^2 + 1) - 6*(B*a^3 + 3*A*a^2*b
- 3*B*a*b^2 - A*b^3)*log(tan(d*x + c)) - (2*A*a^3 - 6*(A*a^3 - 3*B*a^2*b -
 3*A*a*b^2)*tan(d*x + c)^2 + 3*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/tan(d*x +
 c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \cot^4(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= \frac{\left(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3\right)(dx+c)}{d} \\ &\quad+\frac{\left(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d} \\ &\quad-\frac{\left(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d} \\ &\quad-\frac{2\,Aa^3-6\left(Aa^3-3\,Ba^2b-3\,Aab^2\right)\tan\left(dx+c\right)^2+3\left(Ba^3+3\,Aa^2b\right)\tan\left(dx+c\right)}{6\,d\tan\left(dx+c\right)^3}\end{aligned}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*(d*x + c)/d + 1/2*(B*a^3 + 3*A*a^2
*b - 3*B*a*b^2 - A*b^3)*log(tan(d*x + c)^2 + 1)/d - (B*a^3 + 3*A*a^2*b - 3
*B*a*b^2 - A*b^3)*log(abs(tan(d*x + c)))/d - 1/6*(2*A*a^3 - 6*(A*a^3 - 3*B
*a^2*b - 3*A*a*b^2)*tan(d*x + c)^2 + 3*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/(
d*tan(d*x + c)^3)
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{\ln(\tan(c+dx))\left(-B\,a^3-3\,A\,a^2\,b+3\,B\,a\,b^2+A\,b^3\right)}{d} - \frac{\cot(c+dx)^3\left(\tan(c+dx)\left(\frac{B\,a^3}{2}+\frac{3\,A\,b\,a^2}{2}\right)+\frac{A\,a^3}{3}+\tan(c+dx)^2\left(-A\,a^3+3\,B\,a^2\,b+3\,A\,a\,b^2\right)\right)}{d} - \frac{\ln(\tan(c+dx)-\mathrm{i})\,(A+B\,1\mathrm{i})\,(a+b\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d} + \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(A-B\,1\mathrm{i})\,(a-b\,1\mathrm{i})^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
(log(tan(c + d*x))*(A*b^3 - B*a^3 - 3*A*a^2*b + 3*B*a*b^2))/d - (cot(c + d
*x)^3*(tan(c + d*x)*((B*a^3)/2 + (3*A*a^2*b)/2) + (A*a^3)/3 + tan(c + d*x)
^2*(3*A*a*b^2 - A*a^3 + 3*B*a^2*b)))/d - (log(tan(c + d*x) - 1i)*(A + B*1i
)*(a + b*1i)^3*1i)/(2*d) + (log(tan(c + d*x) + 1i)*(A - B*1i)*(a - b*1i)^3
*1i)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 19.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.58

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= \frac{4\cos(dx+c)\sin(dx+c)^2 a^4 - 18\cos(dx+c)\sin(dx+c)^2 a^2b^2 - \cos(dx+c)a^4 + 12\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(4*cos(c + d*x)*sin(c + d*x)**2*a**4 - 18*cos(c + d*x)*sin(c + d*x)**2*a**
2*b**2 - cos(c + d*x)*a**4 + 12*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*
*3*a**3*b - 12*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*a*b**3 - 12*lo
g(tan((c + d*x)/2))*sin(c + d*x)**3*a**3*b + 12*log(tan((c + d*x)/2))*sin(
c + d*x)**3*a*b**3 + 3*sin(c + d*x)**3*a**4*d*x + 3*sin(c + d*x)**3*a**3*b
 - 18*sin(c + d*x)**3*a**2*b**2*d*x + 3*sin(c + d*x)**3*b**4*d*x - 6*sin(c
 + d*x)*a**3*b)/(3*sin(c + d*x)**3*d)
```

3.255 $\int \cot^5(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 191

$$\begin{aligned}
&\int \cot^5(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\
&= \left(3a^2Ab-Ab^3+a^3B-3ab^2B\right)x+\frac{(3a^2Ab-Ab^3+a^3B-3ab^2B)\cot(c+dx)}{d} \\
&\quad+\frac{a(2a^2A-5Ab^2-6abB)\cot^2(c+dx)}{4d}-\frac{a^2(3Ab+2aB)\cot^3(c+dx)}{6d} \\
&\quad+\frac{(a^3A-3aAb^2-3a^2bB+b^3B)\log(\sin(c+dx))}{d} \\
&\quad-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}
\end{aligned}$$

output
```
(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*x+(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*cot(
d*x+c)/d+1/4*a*(2*A*a^2-5*A*b^2-6*B*a*b)*cot(d*x+c)^2/d-1/6*a^2*(3*A*b+2*B
*a)*cot(d*x+c)^3/d+(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*ln(sin(d*x+c))/d-1/4*
a*A*cot(d*x+c)^4*(a+b*tan(d*x+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.04

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{12(3a^2Ab-Ab^3+a^3B-3ab^2B)\cot(c+dx)+6a(a^2A-3Ab^2-3abB)\cot^2(c+dx)-4a^2(3Ab+aB)}{}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(12*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Cot[c + d*x] + 6*a*(a^2*A - 3*
A*b^2 - 3*a*b*B)*Cot[c + d*x]^2 - 4*a^2*(3*A*b + a*B)*Cot[c + d*x]^3 - 3*a
^3*A*Cot[c + d*x]^4 - 6*(a + I*b)^3*(A + I*B)*Log[I - Tan[c + d*x]] + 12*(
a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*Log[Tan[c + d*x]] - 6*(a - I*b)^3*(
A - I*B)*Log[I + Tan[c + d*x]])/(12*d)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {3042, 4088, 27, 3042, 4118, 25, 3042, 4111, 27, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{4}\int 2\cot^4(c+dx)(a+b\tan(c+dx))\left(-b(aA-2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)dx-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{2}\int \cot^4(c+dx)(a+b\tan(c+dx))\left(-b(aA-2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)dx-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\int \frac{(a+b\tan(c+dx))\left(-b(aA-2bB)\tan(c+dx)^2-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)}{\tan(c+dx)^4}dx-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\Big\downarrow$ 4118

$$\frac{1}{2}\left(\int -\cot^3(c+dx)\left(b^2(aA-2bB)\tan^2(c+dx)+2\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(2Aa^2-6b\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(-\int \cot^3(c+dx)\left(b^2(aA-2bB)\tan^2(c+dx)+2\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(2Aa^2-6b\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\left(-\int \frac{b^2(aA-2bB)\tan(c+dx)^2+2\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(2Aa^2-6bBa-5Ab^2\right)}{\tan(c+dx)^3}d\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\Big\downarrow$ 4111

$$\frac{1}{2}\left(-\int 2\cot^2(c+dx)\left(Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{a(2a^2\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{2}\left(-2\int \cot^2(c+dx)\left(Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{a(2a^2\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{2}\left(-2\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\tan(c+dx)^2}dx+\frac{a(2a^2A-6abB-5A}{2d}\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \textbf{4012}$$

$$\frac{1}{2}\left(-2\left(\int -\cot(c+dx)\left(Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx-\frac{(a^3}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{1}{2}\left(-2\left(-\int \cot(c+dx)\left(Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx-\frac{(a^3}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{2}\left(-2\left(-\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan(c+dx)}dx-\frac{(a^3B+3a^2Ab-}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$$\Big\downarrow \textbf{4014}$$

$$\frac{1}{2}\left(-2\left(-\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\int\cot(c+dx)dx-\frac{\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\cot(c+dx)}{d}-(x(\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\left(-2\left(-\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\cot(c+d\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(-2\left(\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\cot(c+\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

$\Big\downarrow$ 3956

$$\frac{1}{2}\left(\frac{a\left(2a^2A-6abB-5Ab^2\right)\cot^2(c+dx)}{2d}-\frac{a^2(2aB+3Ab)\cot^3(c+dx)}{3d}-2\left(-\frac{\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)}{d}\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^2}{4d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
((a*(2*a^2*A - 5*A*b^2 - 6*a*b*B)*Cot[c + d*x]^2)/(2*d) - (a^2*(3*A*b + 2*
a*B)*Cot[c + d*x]^3)/(3*d) - 2*(-((3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*
x) - ((3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Cot[c + d*x])/d - ((a^3*A -
3*a*A*b^2 - 3*a^2*b*B + b^3*B)*Log[-Sin[c + d*x]])/d))/2 - (a*A*Cot[c + d*
x]^4*(a + b*Tan[c + d*x])^2)/(4*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{6(-A\,a^3+3Aa\,b^2+3B\,a^2b-B\,b^3)\ln\left(\sec(dx+c)^2\right)+12(A\,a^3-3Aa\,b^2-3B\,a^2b+B\,b^3)\ln(\tan(dx+c))-3A\cot(dx+c)^4a}{}$
derivativedivides	$\frac{\frac{\left(-A\,a^3+3Aa\,b^2+3B\,a^2b-B\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+(3A\,a^2b-A\,b^3+B\,a^3-3Ba\,b^2)\arctan(\tan(dx+c))-\frac{-3A\,a^2b+A\,b^3-B}{\tan(dx+}}{d}$
default	$\frac{\frac{\left(-A\,a^3+3Aa\,b^2+3B\,a^2b-B\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+(3A\,a^2b-A\,b^3+B\,a^3-3Ba\,b^2)\arctan(\tan(dx+c))-\frac{-3A\,a^2b+A\,b^3-B}{\tan(dx+}}{d}$
norman	$\frac{\frac{\left(3A\,a^2b-A\,b^3+B\,a^3-3Ba\,b^2\right)\tan(dx+c)^3}{d}+(3A\,a^2b-A\,b^3+B\,a^3-3Ba\,b^2)x\tan(dx+c)^4-\frac{A\,a^3}{4d}+\frac{a\left(A\,a^2-3A\,b^2-3Bab\right)\tan}{2d}}{\tan(dx+c)^4}$
risch	$\frac{6iB\,a^2bc}{d}-iB\,b^3x-\frac{2iA\,a^3c}{d}-\frac{2i\left(-9Ba\,b^2+12A\,a^2b-3A\,b^3+9iAa\,b^2\mathrm{e}^{6i(dx+c)}+9iAa\,b^2\mathrm{e}^{2i(dx+c)}+9iB\,a^2b\,\mathrm{e}^{2i}\right.}{}$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/12*(6*(-A*a^3+3*A*a*b^2+3*B*a^2*b-B*b^3)*ln(sec(d*x+c)^2)+12*(A*a^3-3*A*
a*b^2-3*B*a^2*b+B*b^3)*ln(tan(d*x+c))-3*A*cot(d*x+c)^4*a^3+4*(-3*A*a^2*b-B
*a^3)*cot(d*x+c)^3+6*a*cot(d*x+c)^2*(A*a^2-3*A*b^2-3*B*a*b)+12*cot(d*x+c)*
(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)+36*x*(A*a^2*b-1/3*A*b^3+1/3*B*a^3-B*a*b^
2)*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{6\left(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+3\left(3\,Aa^3-6\,Ba^2b-6\,Aab^2+4\,(Ba\right.}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/12*(6*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*log(tan(d*x + c)^2/(tan(d*
x + c)^2 + 1))*tan(d*x + c)^4 + 3*(3*A*a^3 - 6*B*a^2*b - 6*A*a*b^2 + 4*(B*
a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*d*x)*tan(d*x + c)^4 - 3*A*a^3 + 12*(B
*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*tan(d*x + c)^3 + 6*(A*a^3 - 3*B*a^2*
b - 3*A*a*b^2)*tan(d*x + c)^2 - 4*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/(d*tan
(d*x + c)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(187) = 374$.

Time = 2.68 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.09

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= \begin{cases} \tilde{\infty}Aa^3x \\ x(A+B\tan(c))\,(a+b\tan(c))^3\cot^5(c) \\ \tilde{\infty}Aa^3x \\ -\frac{Aa^3\log(\tan^2(c+dx)+1)}{2d} + \frac{Aa^3\log(\tan(c+dx))}{d} + \frac{Aa^3}{2d\tan^2(c+dx)} - \frac{Aa^3}{4d\tan^4(c+dx)} + 3Aa^2bx + \frac{3Aa^2b}{d\tan(c+dx)} - \frac{Aa^2b}{d\tan^3(c+\end{cases}$$

input
```
integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**3*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**3*cot(c)**5, Eq(d, 0)), (zoo*A*a**3*x, Eq(c, -d*x)), (-A*a**3*log(t
an(c + d*x)**2 + 1)/(2*d) + A*a**3*log(tan(c + d*x))/d + A*a**3/(2*d*tan(c
 + d*x)**2) - A*a**3/(4*d*tan(c + d*x)**4) + 3*A*a**2*b*x + 3*A*a**2*b/(d*
tan(c + d*x)) - A*a**2*b/(d*tan(c + d*x)**3) + 3*A*a*b**2*log(tan(c + d*x)
**2 + 1)/(2*d) - 3*A*a*b**2*log(tan(c + d*x))/d - 3*A*a*b**2/(2*d*tan(c +
d*x)**2) - A*b**3*x - A*b**3/(d*tan(c + d*x)) + B*a**3*x + B*a**3/(d*tan(c
 + d*x)) - B*a**3/(3*d*tan(c + d*x)**3) + 3*B*a**2*b*log(tan(c + d*x)**2 +
 1)/(2*d) - 3*B*a**2*b*log(tan(c + d*x))/d - 3*B*a**2*b/(2*d*tan(c + d*x)*
*2) - 3*B*a*b**2*x - 3*B*a*b**2/(d*tan(c + d*x)) - B*b**3*log(tan(c + d*x)
**2 + 1)/(2*d) + B*b**3*log(tan(c + d*x))/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.13

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{12\,(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)(dx+c)-6\,(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)+\cdots}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/12*(12*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*(d*x + c) - 6*(A*a^3 - 3*
B*a^2*b - 3*A*a*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1) + 12*(A*a^3 - 3*B*a^2
*b - 3*A*a*b^2 + B*b^3)*log(tan(d*x + c)) - (3*A*a^3 - 12*(B*a^3 + 3*A*a^2
*b - 3*B*a*b^2 - A*b^3)*tan(d*x + c)^3 - 6*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2)
*tan(d*x + c)^2 + 4*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/tan(d*x + c)^4)/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.16

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)(dx+c)}{d} - \frac{(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3)\log\left(\tan(dx+c)^2+1\right)}{2\,d} + \frac{(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3)\log\left(|\tan(dx+c)|\right)}{d} - \frac{3\,Aa^3-12\,(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)\tan(dx+c)^3-6\,(Aa^3-3\,Ba^2b-3\,Aab^2)\tan(dx+c)^2+\cdots}{12\,d\tan(dx+c)^4}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*(d*x + c)/d - 1/2*(A*a^3 - 3*B*a^2
*b - 3*A*a*b^2 + B*b^3)*log(tan(d*x + c)^2 + 1)/d + (A*a^3 - 3*B*a^2*b - 3
*A*a*b^2 + B*b^3)*log(abs(tan(d*x + c)))/d - 1/12*(3*A*a^3 - 12*(B*a^3 + 3
*A*a^2*b - 3*B*a*b^2 - A*b^3)*tan(d*x + c)^3 - 6*(A*a^3 - 3*B*a^2*b - 3*A*
a*b^2)*tan(d*x + c)^2 + 4*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/(d*tan(d*x + c
)^4)
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.07

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{\ln(\tan(c+d\,x))\,\left(A\,a^3-3\,B\,a^2\,b-3\,A\,a\,b^2+B\,b^3\right)}{d}$$
$$-\frac{\cot(c+d\,x)^4\left(\tan(c+d\,x)\,\left(\frac{B\,a^3}{3}+A\,b\,a^2\right)+\frac{A\,a^3}{4}+\tan(c+d\,x)^2\left(-\frac{A\,a^3}{2}+\frac{3\,B\,a^2\,b}{2}+\frac{3\,A\,a\,b^2}{2}\right)+\tan(c\right.}{d}$$
$$-\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$
$$-\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,\left(A+B\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
(log(tan(c + d*x))*(A*a^3 + B*b^3 - 3*A*a*b^2 - 3*B*a^2*b))/d - (cot(c + d
*x)^4*(tan(c + d*x)*((B*a^3)/3 + A*a^2*b) + (A*a^3)/4 + tan(c + d*x)^2*((3
*A*a*b^2)/2 - (A*a^3)/2 + (3*B*a^2*b)/2) + tan(c + d*x)^3*(A*b^3 - B*a^3 -
 3*A*a^2*b + 3*B*a*b^2)))/d - (log(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b
)^3*1i)/(2*d) - (log(tan(c + d*x) - 1i)*(A + B*1i)*(a*1i - b)^3*1i)/(2*d)
```

Reduce [F]

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^5\,(a+\tan(dx+c)\,b)^3\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

3.256 $\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 233

$$\begin{aligned}\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= -\left(\left(a^3A-3aAb^2-3a^2bB+b^3B\right)x\right)-\frac{(a^3A-3aAb^2-3a^2bB+b^3B)\cot(c+dx)}{d}\\ &\quad+\frac{(3a^2Ab-Ab^3+a^3B-3ab^2B)\cot^2(c+dx)}{2d}\\ &\quad+\frac{a(5a^2A-12Ab^2-15abB)\cot^3(c+dx)}{15d}-\frac{a^2(7Ab+5aB)\cot^4(c+dx)}{20d}\\ &\quad+\frac{(3a^2Ab-Ab^3+a^3B-3ab^2B)\log(\sin(c+dx))}{d}\\ &\quad-\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}\end{aligned}$$

output

```
-(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*x-(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*cot
(d*x+c)/d+1/2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*cot(d*x+c)^2/d+1/15*a*(5*A
*a^2-12*A*b^2-15*B*a*b)*cot(d*x+c)^3/d-1/20*a^2*(7*A*b+5*B*a)*cot(d*x+c)^4
/d+(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*ln(sin(d*x+c))/d-1/5*a*A*cot(d*x+c)^5
*(a+b*tan(d*x+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.02

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{-60(a^3A-3aAb^2-3a^2bB+b^3B)\cot(c+dx)+30(3a^2Ab-Ab^3+a^3B-3ab^2B)\cot^2(c+dx)+20a(}{}$$

input
```
Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(-60*(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*Cot[c + d*x] + 30*(3*a^2*A*b
- A*b^3 + a^3*B - 3*a*b^2*B)*Cot[c + d*x]^2 + 20*a*(a^2*A - 3*A*b^2 - 3*a*
b*B)*Cot[c + d*x]^3 - 15*a^2*(3*A*b + a*B)*Cot[c + d*x]^4 - 12*a^3*A*Cot[c
 + d*x]^5 + (30*I)*(a + I*b)^3*(A + I*B)*Log[I - Tan[c + d*x]] + 60*(3*a^2
*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Log[Tan[c + d*x]] + 30*(I*a + b)^3*(A -
I*B)*Log[I + Tan[c + d*x]])/(60*d)
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {3042, 4088, 3042, 4118, 25, 3042, 4111, 27, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^6}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{5}\int \cot^5(c+dx)(a+b\tan(c+ dx))\left(-b(3aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+5aB)\right)dx-\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\int \frac{(a+b\tan(c+dx))\left(-b(3aA-5bB)\tan(c+dx)^2-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+5aB)\right)}{\tan(c+dx)^5}dx$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 4118

$$\frac{1}{5}\left(\int -\cot^4(c+dx)\left(b^2(3aA-5bB)\tan^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(5Aa^2-1\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 25

$$\frac{1}{5}\left(-\int \cot^4(c+dx)\left(b^2(3aA-5bB)\tan^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(5Aa^2-1\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(-\int \frac{b^2(3aA-5bB)\tan(c+dx)^2+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(5Aa^2-15bBa-12Ab\right.}{\tan(c+dx)^4}\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 4111

$$\frac{1}{5}\left(-\int 5\cot^3(c+dx)\left(Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{a\left(5a^2\right.}{}\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\big\downarrow$ 27

$$\frac{1}{5}\left(-5\int \cot^3(c+dx)\left(Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{a\left(5a^2\right.}{}\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-5\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\tan(c+dx)^3}dx+\frac{a\left(5a^2A-15abB-1\right.}{3a}\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 4012

$$\frac{1}{5}\left(-5\left(\int -\cot^2(c+dx)\left(Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx-\frac{(a}{}\right.\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 25

$$\frac{1}{5}\left(-5\left(-\int \cot^2(c+dx)\left(Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx-\frac{(a}{}\right.\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-5\left(-\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan(c+dx)^2}dx-\frac{\left(a^3B+3a^2Ab-\right.}{}\right.\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 4012

$$\frac{1}{5}\left(-5\left(-\int \cot(c+dx)\left(Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx-\frac{(a^3}{}\right.\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-5\left(-\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\tan(c+dx)}dx-\frac{\left(a^3B+3a^2Ab-\right.}{}\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 4014

$$\frac{1}{5}\left(-5\left(-\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \cot(c+dx)dx-\frac{\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\cot^2(c+dx)}{2d}+\frac{(a^3}{}\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-5\left(-\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\cot^2(c+}{2d}\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 25

$$\frac{1}{5}\left(-5\left(\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\cot^2(c+}{2d}\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

$\downarrow$ 3956

$$\frac{1}{5}\left(\frac{a\left(5a^2A-15abB-12Ab^2\right)\cot^3(c+dx)}{3d}-\frac{a^2(5aB+7Ab)\cot^4(c+dx)}{4d}-5\left(-\frac{\left(a^3B+3a^2Ab-3ab^2B-Ab\right.}{2d}\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^2}{5d}$$

input
```
Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
((a*(5*a^2*A - 12*A*b^2 - 15*a*b*B)*Cot[c + d*x]^3)/(3*d) - (a^2*(7*A*b +
5*a*B)*Cot[c + d*x]^4)/(4*d) - 5*((a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*
x + ((a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*Cot[c + d*x])/d - ((3*a^2*A*b
 - A*b^3 + a^3*B - 3*a*b^2*B)*Cot[c + d*x]^2)/(2*d) - ((3*a^2*A*b - A*b^3
+ a^3*B - 3*a*b^2*B)*Log[-Sin[c + d*x]])/d))/5 - (a*A*Cot[c + d*x]^5*(a +
b*Tan[c + d*x])^2)/(5*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{\left(-90 A\,a^{2} b+30 A\,b^{3}-30 B\,a^{3}+90 B a\,b^{2}\right) \ln\left(\sec\left(d x+c\right)^{2}\right)+\left(180 A\,a^{2} b-60 A\,b^{3}+60 B\,a^{3}-180 B a\,b^{2}\right) \ln\left(\tan\left(d x+c\right)\right)-12 A}{}$
derivativedivides	$\frac{\frac{\left(-3 A\,a^{2} b+A\,b^{3}-B\,a^{3}+3 B a\,b^{2}\right) \ln\left(1+\tan\left(d x+c\right)^{2}\right)}{2}+\left(-A\,a^{3}+3 A a\,b^{2}+3 B\,a^{2} b-B\,b^{3}\right) \arctan\left(\tan\left(d x+c\right)\right)-\frac{-3 A\,a^{2} b+A\,b^{3}-}{2 \tan\left(d x\right.}}{}$
default	$\frac{\frac{\left(-3 A\,a^{2} b+A\,b^{3}-B\,a^{3}+3 B a\,b^{2}\right) \ln\left(1+\tan\left(d x+c\right)^{2}\right)}{2}+\left(-A\,a^{3}+3 A a\,b^{2}+3 B\,a^{2} b-B\,b^{3}\right) \arctan\left(\tan\left(d x+c\right)\right)-\frac{-3 A\,a^{2} b+A\,b^{3}-}{2 \tan\left(d x\right.}}{}$
norman	$\frac{\left(-A\,a^{3}+3 A a\,b^{2}+3 B\,a^{2} b-B\,b^{3}\right) x \tan\left(d x+c\right)^{5}-\frac{A\,a^{3}}{5 d}+\frac{\left(3 A\,a^{2} b-A\,b^{3}+B\,a^{3}-3 B a\,b^{2}\right) \tan\left(d x+c\right)^{3}}{2 d}-\frac{\left(A\,a^{3}-3 A a\,b^{2}-3 B\,a^{2} b+\right.}{d}}{\tan\left(d x+c\right)^{5}}$
risch	$-A\,a^{3} x+3 A a\,b^{2} x+3 B\,a^{2} b x-B\,b^{3} x-\frac{2 i\left(-60 A a\,b^{2}-60 B\,a^{2} b+15 B\,b^{3} {\mathrm e}^{8 i\left(d x+c\right)}-90 A\,a^{3} {\mathrm e}^{6 i\left(d x+c\right)}-\right.}{}$

input
```
int(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/60*((-90*A*a^2*b+30*A*b^3-30*B*a^3+90*B*a*b^2)*ln(sec(d*x+c)^2)+(180*A*a
^2*b-60*A*b^3+60*B*a^3-180*B*a*b^2)*ln(tan(d*x+c))-12*A*cot(d*x+c)^5*a^3+(
-45*A*a^2*b-15*B*a^3)*cot(d*x+c)^4+20*a*cot(d*x+c)^3*(A*a^2-3*A*b^2-3*B*a*
b)+(90*A*a^2*b-30*A*b^3+30*B*a^3-90*B*a*b^2)*cot(d*x+c)^2+(-60*A*a^3+180*A
*a*b^2+180*B*a^2*b-60*B*b^3)*cot(d*x+c)-60*d*x*(A*a^3-3*A*a*b^2-3*B*a^2*b+
B*b^3))/d
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.14

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{30\left(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^5+15\left(3\,Ba^3+9\,Aa^2b-6\,Bab^2-2\,A\right.}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/60*(30*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*log(tan(d*x + c)^2/(tan(d
*x + c)^2 + 1))*tan(d*x + c)^5 + 15*(3*B*a^3 + 9*A*a^2*b - 6*B*a*b^2 - 2*A
*b^3 - 4*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*d*x)*tan(d*x + c)^5 - 60*
(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*tan(d*x + c)^4 - 12*A*a^3 + 30*(B*
a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*tan(d*x + c)^3 + 20*(A*a^3 - 3*B*a^2*
b - 3*A*a*b^2)*tan(d*x + c)^2 - 15*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/(d*ta
n(d*x + c)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(231) = 462.

Time = 4.00 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.02

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= \begin{cases} \tilde{\infty} A a^3 x \\ x(A+B\tan(c))(a+b\tan(c))^3\cot^6(c) \\ \tilde{\infty} A a^3 x \\ -Aa^3x - \frac{Aa^3}{d\tan(c+dx)} + \frac{Aa^3}{3d\tan^3(c+dx)} - \frac{Aa^3}{5d\tan^5(c+dx)} - \frac{3Aa^2b\log(\tan^2(c+dx)+1)}{2d} + \frac{3Aa^2b\log(\tan(c+dx))}{d} + \frac{3Aa^2}{2d\tan^2(c} \end{cases}$$

input
```
integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**3*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**3*cot(c)**6, Eq(d, 0)), (zoo*A*a**3*x, Eq(c, -d*x)), (-A*a**3*x - A
*a**3/(d*tan(c + d*x)) + A*a**3/(3*d*tan(c + d*x)**3) - A*a**3/(5*d*tan(c
+ d*x)**5) - 3*A*a**2*b*log(tan(c + d*x)**2 + 1)/(2*d) + 3*A*a**2*b*log(ta
n(c + d*x))/d + 3*A*a**2*b/(2*d*tan(c + d*x)**2) - 3*A*a**2*b/(4*d*tan(c +
 d*x)**4) + 3*A*a*b**2*x + 3*A*a*b**2/(d*tan(c + d*x)) - A*a*b**2/(d*tan(c
 + d*x)**3) + A*b**3*log(tan(c + d*x)**2 + 1)/(2*d) - A*b**3*log(tan(c + d
*x))/d - A*b**3/(2*d*tan(c + d*x)**2) - B*a**3*log(tan(c + d*x)**2 + 1)/(2
*d) + B*a**3*log(tan(c + d*x))/d + B*a**3/(2*d*tan(c + d*x)**2) - B*a**3/(
4*d*tan(c + d*x)**4) + 3*B*a**2*b*x + 3*B*a**2*b/(d*tan(c + d*x)) - B*a**2
*b/(d*tan(c + d*x)**3) + 3*B*a*b**2*log(tan(c + d*x)**2 + 1)/(2*d) - 3*B*a
*b**2*log(tan(c + d*x))/d - 3*B*a*b**2/(2*d*tan(c + d*x)**2) - B*b**3*x -
B*b**3/(d*tan(c + d*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.07

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{60\left(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3\right)(dx+c)+30\left(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3\right)\log\left(\tan\left(dx+c\right)^2+\right.}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/60*(60*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*(d*x + c) + 30*(B*a^3 +
3*A*a^2*b - 3*B*a*b^2 - A*b^3)*log(tan(d*x + c)^2 + 1) - 60*(B*a^3 + 3*A*a
^2*b - 3*B*a*b^2 - A*b^3)*log(tan(d*x + c)) + (60*(A*a^3 - 3*B*a^2*b - 3*A
*a*b^2 + B*b^3)*tan(d*x + c)^4 + 12*A*a^3 - 30*(B*a^3 + 3*A*a^2*b - 3*B*a*
b^2 - A*b^3)*tan(d*x + c)^3 - 20*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2)*tan(d*x +
 c)^2 + 15*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.11

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= -\frac{\left(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3\right)(dx+c)}{d}$$

$$-\frac{\left(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

$$+\frac{\left(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3\right)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$

$$-\frac{60\left(Aa^3-3\,Ba^2b-3\,Aab^2+Bb^3\right)\tan\left(dx+c\right)^4+12\,Aa^3-30\left(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3\right)\tan\left(d\right.}{60\,d\tan\left(dx+c\right)^5}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-(A*a^3 - 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*(d*x + c)/d - 1/2*(B*a^3 + 3*A*a^
2*b - 3*B*a*b^2 - A*b^3)*log(tan(d*x + c)^2 + 1)/d + (B*a^3 + 3*A*a^2*b -
3*B*a*b^2 - A*b^3)*log(abs(tan(d*x + c)))/d - 1/60*(60*(A*a^3 - 3*B*a^2*b
- 3*A*a*b^2 + B*b^3)*tan(d*x + c)^4 + 12*A*a^3 - 30*(B*a^3 + 3*A*a^2*b - 3
*B*a*b^2 - A*b^3)*tan(d*x + c)^3 - 20*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2)*tan(
d*x + c)^2 + 15*(B*a^3 + 3*A*a^2*b)*tan(d*x + c))/(d*tan(d*x + c)^5)
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{\cot(c+d\,x)^5\left(\tan(c+d\,x)\,\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right)+\frac{A\,a^3}{5}+\tan(c+d\,x)^2\,\left(-\frac{A\,a^3}{3}+B\,a^2\,b+A\,a\,b^2\right)+\tan(c\right.}{d}$$
$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(-B\,a^3-3\,A\,a^2\,b+3\,B\,a\,b^2+A\,b^3\right)}{d}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$
$$-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)\,\left(a-b\,1\mathrm{i}\right)^3\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^6*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
(log(tan(c + d*x) - 1i)*(A + B*1i)*(a + b*1i)^3*1i)/(2*d) - (log(tan(c + d
*x))*(A*b^3 - B*a^3 - 3*A*a^2*b + 3*B*a*b^2))/d - (cot(c + d*x)^5*(tan(c +
 d*x)*((B*a^3)/4 + (3*A*a^2*b)/4) + (A*a^3)/5 + tan(c + d*x)^2*(A*a*b^2 -
(A*a^3)/3 + B*a^2*b) + tan(c + d*x)^4*(A*a^3 + B*b^3 - 3*A*a*b^2 - 3*B*a^2
*b) + tan(c + d*x)^3*((A*b^3)/2 - (B*a^3)/2 - (3*A*a^2*b)/2 + (3*B*a*b^2)/
2)))/d - (log(tan(c + d*x) + 1i)*(A - B*1i)*(a - b*1i)^3*1i)/(2*d)
```

Reduce [F]

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \int \cot(dx+c)^6(a+\tan(dx+c)\,b)^3(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^6*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

3.257 $\int \tan^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 263

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx = -\left(\left(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B\right)x\right) + \frac{(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B)\log(\cos(c+dx))}{d} - \frac{b(3a^2Ab-Ab^3+a^3B-3ab^2B)\tan(c+dx)}{d} - \frac{(2aAb+a^2B-b^2B)\,(a+b\tan(c+dx))^2}{2d} - \frac{(Ab+aB)(a+b\tan(c+dx))^3}{3d} - \frac{B(a+b\tan(c+dx))^4}{4d} + \frac{(6Ab-aB)(a+b\tan(c+dx))^5}{30b^2d} + \frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}$$

output
```
-(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*x+(4*A*a^3*b-4*A*a*b^3+B*a^
4-6*B*a^2*b^2+B*b^4)*ln(cos(d*x+c))/d-b*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*
tan(d*x+c)/d-1/2*(2*A*a*b+B*a^2-B*b^2)*(a+b*tan(d*x+c))^2/d-1/3*(A*b+B*a)*
(a+b*tan(d*x+c))^3/d-1/4*B*(a+b*tan(d*x+c))^4/d+1/30*(6*A*b-B*a)*(a+b*tan(
d*x+c))^5/b^2/d+1/6*B*tan(d*x+c)*(a+b*tan(d*x+c))^5/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.10

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{\frac{2(6Ab-aB)(a+b\tan(c+dx))^5}{b} + 10B\tan(c+dx)(a+b\tan(c+dx))^5 + 10(Ab-aB)\left(3i(a+ib)^4\log(i-\tan(c\right.}{}$$

input
```
Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
((2*(6*A*b - a*B)*(a + b*Tan[c + d*x])^5)/b + 10*B*Tan[c + d*x]*(a + b*Tan
[c + d*x])^5 + 10*(A*b - a*B)*((3*I)*(a + I*b)^4*Log[I - Tan[c + d*x]] - (
3*I)*(a - I*b)^4*Log[I + Tan[c + d*x]] + 6*b^2*(-6*a^2 + b^2)*Tan[c + d*x]
 - 12*a*b^3*Tan[c + d*x]^2 - 2*b^4*Tan[c + d*x]^3) + 5*B*((6*I)*(a + I*b)^
5*Log[I - Tan[c + d*x]] - 6*(I*a + b)^5*Log[I + Tan[c + d*x]] - 60*a*b^2*(
2*a^2 - b^2)*Tan[c + d*x] + 6*b^3*(-10*a^2 + b^2)*Tan[c + d*x]^2 - 20*a*b^
4*Tan[c + d*x]^3 - 3*b^5*Tan[c + d*x]^4))/(60*b*d)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4090, 25, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a+b\tan(c+dx))^4(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -(a+b\tan(c+dx))^4 \left(-\left((6Ab-aB)\tan^2(c+dx)\right)+6bB\tan(c+dx)+aB\right)dx}{6b}+\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}$$

$$\Big\downarrow \; 25$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-\frac{\int (a+b\tan(c+dx))^4 \left(-\left((6Ab-aB)\tan^2(c+dx)\right)+6bB\tan(c+dx)+aB\right)dx}{6b}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-\frac{\int (a+b\tan(c+dx))^4 \left(-\left((6Ab-aB)\tan(c+dx)^2\right)+6bB\tan(c+dx)+aB\right)dx}{6b}$$

$$\Big\downarrow \; 4113$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-\frac{\int (a+b\tan(c+dx))^4(6Ab+6B\tan(c+dx)b)dx-\frac{(6Ab-aB)(a+b\tan(c+dx))^5}{5bd}}{6b}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-\frac{\int (a+b\tan(c+dx))^4(6Ab+6B\tan(c+dx)b)dx-\frac{(6Ab-aB)(a+b\tan(c+dx))^5}{5bd}}{6b}$$

$$\Big\downarrow \; 4011$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-\frac{\int (a+b\tan(c+dx))^3(6b(aA-bB)+6b(Ab+aB)\tan(c+dx))dx-\frac{(6Ab-aB)(a+b\tan(c+dx))^5}{5bd}+\frac{3bB(a+b\tan(c+dx))}{2d}}{6b}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-\frac{\int (a+b\tan(c+dx))^3(6b(aA-bB)+6b(Ab+aB)\tan(c+dx))dx-\frac{(6Ab-aB)(a+b\tan(c+dx))^5}{5bd}+\frac{3bB(a+b\tan(c+dx))}{2d}}{6b}$$

$$\Big\downarrow \; 4011$$

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-$$
$$\frac{\int(a+b\tan(c+dx))^2\left(6b\left(Aa^2-2bBa-Ab^2\right)+6b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx-\frac{(6Ab-aB)(a+b\tan(c+dx}{5bd}}{6b}$$

$\downarrow$ 3042

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-$$
$$\frac{\int(a+b\tan(c+dx))^2\left(6b\left(Aa^2-2bBa-Ab^2\right)+6b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx-\frac{(6Ab-aB)(a+b\tan(c+dx}{5bd}}{6b}$$

$\downarrow$ 4011

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-$$
$$\frac{\int(a+b\tan(c+dx))\left(6b\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+6b\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{3}{}}{6b}$$

$\downarrow$ 3042

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-$$
$$\frac{\int(a+b\tan(c+dx))\left(6b\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+6b\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{3}{}}{6b}$$

$\downarrow$ 4008

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-$$
$$6b\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\int\tan(c+dx)dx+\frac{3b\left(a^2B+2aAb-b^2B\right)(a+b\tan(c+dx))^2}{d}+\frac{6b^2\left(a^3B+3a^2Ab-\right.}{}$$

$\downarrow$ 3042

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-$$
$$6b\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\int\tan(c+dx)dx+\frac{3b\left(a^2B+2aAb-b^2B\right)(a+b\tan(c+dx))^2}{d}+\frac{6b^2\left(a^3B+3a^2Ab-\right.}{}$$

$\downarrow$ 3956

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^5}{6bd}-$$
$$\frac{3b\left(a^2B+2aAb-b^2B\right)(a+b\tan(c+dx))^2}{d}+\frac{6b^2\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\tan(c+dx)}{d}-\frac{6b\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\log(\cos(c+d}{d}$$

input
```
Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]*(a + b*Tan[c + d*x])^5)/(6*b*d) - (6*b*(a^4*A - 6*a^2*A*b^
2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*x - (6*b*(4*a^3*A*b - 4*a*A*b^3 + a^4*B
 - 6*a^2*b^2*B + b^4*B)*Log[Cos[c + d*x]])/d + (6*b^2*(3*a^2*A*b - A*b^3 +
 a^3*B - 3*a*b^2*B)*Tan[c + d*x])/d + (3*b*(2*a*A*b + a^2*B - b^2*B)*(a +
b*Tan[c + d*x])^2)/d + (2*b*(A*b + a*B)*(a + b*Tan[c + d*x])^3)/d + (3*b*B
*(a + b*Tan[c + d*x])^4)/(2*d) - ((6*A*b - a*B)*(a + b*Tan[c + d*x])^5)/(5
*b*d))/(6*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.04

method	result
parts	$\frac{(A b^4+4B a b^3)\left(\frac{\tan(dx+c)^5}{5}-\frac{\tan(dx+c)^3}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{(4Aa b^3+6B a^2b^2)\left(\frac{\tan(dx+c)^4}{4}-\tan\right.}{d}$
norman	$(-A a^4+6A a^2b^2-A b^4+4B a^3b-4Ba b^3)x+\frac{(A a^4-6A a^2b^2+A b^4-4B a^3b+4Ba b^3)\tan(dx+}{d}$
derivativedivides	$\frac{B b^4\tan(dx+c)^6}{6}+\frac{A b^4\tan(dx+c)^5}{5}+\frac{4Ba b^3\tan(dx+c)^5}{5}+Aa b^3\tan(dx+c)^4+\frac{3B a^2b^2\tan(dx+c)^4}{2}-\frac{B b^4\tan(dx+c)^4}{4}+2A a^2$
default	$\frac{B b^4\tan(dx+c)^6}{6}+\frac{A b^4\tan(dx+c)^5}{5}+\frac{4Ba b^3\tan(dx+c)^5}{5}+Aa b^3\tan(dx+c)^4+\frac{3B a^2b^2\tan(dx+c)^4}{2}-\frac{B b^4\tan(dx+c)^4}{4}+2A a^2$
parallelrisch	$-\frac{60Ax a^4d+120A\ln(1+\tan(dx+c)^2)a^3b-120A\ln(1+\tan(dx+c)^2)a b^3-180B\ln(1+\tan(dx+c)^2)a^2b^2-48Ba b^3\tan}{}$
risch	Expression too large to display

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
(A*b^4+4*B*a*b^3)/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-arctan(t
an(d*x+c)))+(4*A*a*b^3+6*B*a^2*b^2)/d*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2+1
/2*ln(1+tan(d*x+c)^2))+(6*A*a^2*b^2+4*B*a^3*b)/d*(1/3*tan(d*x+c)^3-tan(d*x
+c)+arctan(tan(d*x+c)))+(4*A*a^3*b+B*a^4)/d*(1/2*tan(d*x+c)^2-1/2*ln(1+tan
(d*x+c)^2))+A*a^4/d*(tan(d*x+c)-arctan(tan(d*x+c)))+B*b^4/d*(1/6*tan(d*x+c
)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.10

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{10\,Bb^4\tan(dx+c)^6+12\,(4\,Bab^3+Ab^4)\tan(dx+c)^5+15\,(6\,Ba^2b^2+4\,Aab^3-Bb^4)\tan(dx+c)^4+2}{}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/60*(10*B*b^4*tan(d*x + c)^6 + 12*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^5 + 15
*(6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*tan(d*x + c)^4 + 20*(4*B*a^3*b + 6*A*a^
2*b^2 - 4*B*a*b^3 - A*b^4)*tan(d*x + c)^3 - 60*(A*a^4 - 4*B*a^3*b - 6*A*a^
2*b^2 + 4*B*a*b^3 + A*b^4)*d*x + 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A
*a*b^3 + B*b^4)*tan(d*x + c)^2 + 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A
*a*b^3 + B*b^4)*log(1/(tan(d*x + c)^2 + 1)) + 60*(A*a^4 - 4*B*a^3*b - 6*A*
a^2*b^2 + 4*B*a*b^3 + A*b^4)*tan(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(248) = 496.

Time = 0.27 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.04

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} -Aa^4x+\frac{Aa^4\tan(c+dx)}{d}-\frac{2Aa^3b\log(\tan^2(c+dx)+1)}{d}+\frac{2Aa^3b\tan^2(c+dx)}{d}+6Aa^2b^2x+\frac{2Aa^2b^2\tan^3(c+dx)}{d}-\frac{6Aa^2b^2\tan}{} \\ x(A+B\tan(c))\,(a+b\tan(c))^4\tan^2(c) \end{cases}$$

input
```
integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((-A*a**4*x + A*a**4*tan(c + d*x)/d - 2*A*a**3*b*log(tan(c + d*x)
**2 + 1)/d + 2*A*a**3*b*tan(c + d*x)**2/d + 6*A*a**2*b**2*x + 2*A*a**2*b**
2*tan(c + d*x)**3/d - 6*A*a**2*b**2*tan(c + d*x)/d + 2*A*a*b**3*log(tan(c
+ d*x)**2 + 1)/d + A*a*b**3*tan(c + d*x)**4/d - 2*A*a*b**3*tan(c + d*x)**2
/d - A*b**4*x + A*b**4*tan(c + d*x)**5/(5*d) - A*b**4*tan(c + d*x)**3/(3*d
) + A*b**4*tan(c + d*x)/d - B*a**4*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**4
*tan(c + d*x)**2/(2*d) + 4*B*a**3*b*x + 4*B*a**3*b*tan(c + d*x)**3/(3*d) -
 4*B*a**3*b*tan(c + d*x)/d + 3*B*a**2*b**2*log(tan(c + d*x)**2 + 1)/d + 3*
B*a**2*b**2*tan(c + d*x)**4/(2*d) - 3*B*a**2*b**2*tan(c + d*x)**2/d - 4*B*
a*b**3*x + 4*B*a*b**3*tan(c + d*x)**5/(5*d) - 4*B*a*b**3*tan(c + d*x)**3/(
3*d) + 4*B*a*b**3*tan(c + d*x)/d - B*b**4*log(tan(c + d*x)**2 + 1)/(2*d) +
 B*b**4*tan(c + d*x)**6/(6*d) - B*b**4*tan(c + d*x)**4/(4*d) + B*b**4*tan(
c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*tan(c))*(a + b*tan(c))**4*tan(c)**
2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.10

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{10\,Bb^4\tan{(dx+c)}^6+12\,(4\,Bab^3+Ab^4)\tan{(dx+c)}^5+15\,(6\,Ba^2b^2+4\,Aab^3-Bb^4)\tan{(dx+c)}^4+2}{}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/60*(10*B*b^4*tan(d*x + c)^6 + 12*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^5 + 15
*(6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*tan(d*x + c)^4 + 20*(4*B*a^3*b + 6*A*a^
2*b^2 - 4*B*a*b^3 - A*b^4)*tan(d*x + c)^3 + 30*(B*a^4 + 4*A*a^3*b - 6*B*a^
2*b^2 - 4*A*a*b^3 + B*b^4)*tan(d*x + c)^2 - 60*(A*a^4 - 4*B*a^3*b - 6*A*a^
2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) - 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2
 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1) + 60*(A*a^4 - 4*B*a^3*b - 6*
A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.71

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)(dx+c)}{d}$$
$$-\frac{(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$
$$+\frac{10\,Bb^4d^5\tan\left(dx+c\right)^6+48\,Bab^3d^5\tan\left(dx+c\right)^5+12\,Ab^4d^5\tan\left(dx+c\right)^5+90\,Ba^2b^2d^5\tan\left(dx+c\right)^4}{}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c)/d - 1/2*(
B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 +
1)/d + 1/60*(10*B*b^4*d^5*tan(d*x + c)^6 + 48*B*a*b^3*d^5*tan(d*x + c)^5 +
 12*A*b^4*d^5*tan(d*x + c)^5 + 90*B*a^2*b^2*d^5*tan(d*x + c)^4 + 60*A*a*b^
3*d^5*tan(d*x + c)^4 - 15*B*b^4*d^5*tan(d*x + c)^4 + 80*B*a^3*b*d^5*tan(d*
x + c)^3 + 120*A*a^2*b^2*d^5*tan(d*x + c)^3 - 80*B*a*b^3*d^5*tan(d*x + c)^
3 - 20*A*b^4*d^5*tan(d*x + c)^3 + 30*B*a^4*d^5*tan(d*x + c)^2 + 120*A*a^3*
b*d^5*tan(d*x + c)^2 - 180*B*a^2*b^2*d^5*tan(d*x + c)^2 - 120*A*a*b^3*d^5*
tan(d*x + c)^2 + 30*B*b^4*d^5*tan(d*x + c)^2 + 60*A*a^4*d^5*tan(d*x + c) -
 240*B*a^3*b*d^5*tan(d*x + c) - 360*A*a^2*b^2*d^5*tan(d*x + c) + 240*B*a*b
^3*d^5*tan(d*x + c) + 60*A*b^4*d^5*tan(d*x + c))/d^6
```

Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.14

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(c+d\,x)\,\left(A\,a^4+A\,b^4+4\,B\,a\,b^3-2\,a^2\,b\,(3\,A\,b+2\,B\,a)\right)}{d} - \frac{\tan(c+d\,x)^3\,\left(\frac{A\,b^4}{3}+\frac{4\,B\,a\,b^3}{3}-\frac{2\,a^2\,b\,(3\,A\,b+2\,B\,a)}{3}\right)}{d} - x\,\left(A\,a^4-4\,B\,a^3\,b-6\,A\,a^2\,b^2+4\,B\,a\,b^3+A\,b^4\right) + \frac{\tan(c+d\,x)^5\,\left(\frac{A\,b^4}{5}+\frac{4\,B\,a\,b^3}{5}\right)}{d} - \frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{B\,a^4}{2}+2\,A\,a^3\,b-3\,B\,a^2\,b^2-2\,A\,a\,b^3+\frac{B\,b^4}{2}\right)}{d} - \frac{\tan(c+d\,x)^4\,\left(\frac{B\,b^4}{4}-\frac{a\,b^2\,(2\,A\,b+3\,B\,a)}{2}\right)}{d} + \frac{\tan(c+d\,x)^2\,\left(\frac{B\,a^4}{2}+\frac{B\,b^4}{2}+2\,A\,a^3\,b-a\,b^2\,(2\,A\,b+3\,B\,a)\right)}{d} + \frac{B\,b^4\,\tan(c+d\,x)^6}{6\,d}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
(tan(c + d*x)*(A*a^4 + A*b^4 + 4*B*a*b^3 - 2*a^2*b*(3*A*b + 2*B*a)))/d - (
tan(c + d*x)^3*((A*b^4)/3 + (4*B*a*b^3)/3 - (2*a^2*b*(3*A*b + 2*B*a))/3))/
d - x*(A*a^4 + A*b^4 - 6*A*a^2*b^2 + 4*B*a*b^3 - 4*B*a^3*b) + (tan(c + d*x
)^5*((A*b^4)/5 + (4*B*a*b^3)/5))/d - (log(tan(c + d*x)^2 + 1)*((B*a^4)/2 +
 (B*b^4)/2 - 3*B*a^2*b^2 - 2*A*a*b^3 + 2*A*a^3*b))/d - (tan(c + d*x)^4*((B
*b^4)/4 - (a*b^2*(2*A*b + 3*B*a))/2))/d + (tan(c + d*x)^2*((B*a^4)/2 + (B*
b^4)/2 + 2*A*a^3*b - a*b^2*(2*A*b + 3*B*a)))/d + (B*b^4*tan(c + d*x)^6)/(6
*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.95

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{-30\log\left(\tan(dx+c)^2+1\right)a^4b+60\log\left(\tan(dx+c)^2+1\right)a^2b^3-6\log\left(\tan(dx+c)^2+1\right)b^5+2\tan(dx}{}$$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
( - 30*log(tan(c + d*x)**2 + 1)*a**4*b + 60*log(tan(c + d*x)**2 + 1)*a**2*
b**3 - 6*log(tan(c + d*x)**2 + 1)*b**5 + 2*tan(c + d*x)**6*b**5 + 12*tan(c
 + d*x)**5*a*b**4 + 30*tan(c + d*x)**4*a**2*b**3 - 3*tan(c + d*x)**4*b**5
+ 40*tan(c + d*x)**3*a**3*b**2 - 20*tan(c + d*x)**3*a*b**4 + 30*tan(c + d*
x)**2*a**4*b - 60*tan(c + d*x)**2*a**2*b**3 + 6*tan(c + d*x)**2*b**5 + 12*
tan(c + d*x)*a**5 - 120*tan(c + d*x)*a**3*b**2 + 60*tan(c + d*x)*a*b**4 -
12*a**5*d*x + 120*a**3*b**2*d*x - 60*a*b**4*d*x)/(12*d)
```

3.258 $\int \tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 226

$$\begin{aligned}\int \tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx &= -\left(\left(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B\right)x\right)\\ &\quad-\frac{(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B)\log(\cos(c+dx))}{d}\\ &\quad+\frac{b(a^3A-3aAb^2-3a^2bB+b^3B)\tan(c+dx)}{d}\\ &\quad+\frac{(a^2A-Ab^2-2abB)\,(a+b\tan(c+dx))^2}{2d}+\frac{(aA-bB)(a+b\tan(c+dx))^3}{3d}\\ &\quad+\frac{A(a+b\tan(c+dx))^4}{4d}+\frac{B(a+b\tan(c+dx))^5}{5bd}\end{aligned}$$

output

```
-(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*x-(A*a^4-6*A*a^2*b^2+A*b^4-
4*B*a^3*b+4*B*a*b^3)*ln(cos(d*x+c))/d+b*(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*
tan(d*x+c)/d+1/2*(A*a^2-A*b^2-2*B*a*b)*(a+b*tan(d*x+c))^2/d+1/3*(A*a-B*b)*
(a+b*tan(d*x+c))^3/d+1/4*A*(a+b*tan(d*x+c))^4/d+1/5*B*(a+b*tan(d*x+c))^5/b
/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14

$$\int \tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{12B(a+b\tan(c+dx))^5+10(aA+bB)\left(3i(a+ib)^4\log(i-\tan(c+dx))-3i(a-ib)^4\log(i+\tan(c+\right.}{}$$

input
```
Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(12*B*(a + b*Tan[c + d*x])^5 + 10*(a*A + b*B)*((3*I)*(a + I*b)^4*Log[I - T
an[c + d*x]] - (3*I)*(a - I*b)^4*Log[I + Tan[c + d*x]] + 6*b^2*(-6*a^2 + b
^2)*Tan[c + d*x] - 12*a*b^3*Tan[c + d*x]^2 - 2*b^4*Tan[c + d*x]^3) - 5*A*(
(6*I)*(a + I*b)^5*Log[I - Tan[c + d*x]] - 6*(I*a + b)^5*Log[I + Tan[c + d*
x]] - 60*a*b^2*(2*a^2 - b^2)*Tan[c + d*x] + 6*b^3*(-10*a^2 + b^2)*Tan[c +
d*x]^2 - 20*a*b^4*Tan[c + d*x]^3 - 3*b^5*Tan[c + d*x]^4))/(60*b*d)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {3042, 4075, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{4075}$$
$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^4dx+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^4dx+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))^3(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{A(a+b\tan(c+dx))^4}{4d}+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^3(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{A(a+b\tan(c+dx))^4}{4d}+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))^2\left(-Ba^2-2Aba+b^2B+\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)dx+\frac{(aA-bB)(a+b\tan(c+dx))^3}{3d}+\frac{A(a+b\tan(c+dx))^4}{4d}+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2\left(-Ba^2-2Aba+b^2B+\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)dx+\frac{(aA-bB)(a+b\tan(c+dx))^3}{3d}+\frac{A(a+b\tan(c+dx))^4}{4d}+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))\left(-Ba^3-3Aba^2+3b^2Ba+Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{\left(a^2A-2abB-Ab^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{(aA-bB)(a+b\tan(c+dx))^3}{3d}+\frac{A(a+b\tan(c+dx))^4}{4d}+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))\left(-Ba^3-3Aba^2+3b^2Ba+Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{\left(a^2A-2abB-Ab^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{(aA-bB)(a+b\tan(c+dx))^3}{3d}+\frac{A(a+b\tan(c+dx))^4}{4d}+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

$$\Big\downarrow \text{4008}$$

$$\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)\int \tan(c+dx)dx+\frac{\left(a^2A-2abB-Ab^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{b\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\tan(c+dx)}{d}-x\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)+\frac{(aA-bB)(a+b\tan(c+dx))^3}{3d}+\frac{A(a+b\tan(c+dx))^4}{4d}+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)\int \tan(c+dx)dx+\frac{\left(a^2A-2abB-Ab^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{b\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\tan(c+dx)}{d}-x\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)+\frac{(aA-bB)(a+b\tan(c+dx))^3}{3d}+\frac{A(a+b\tan(c+dx))^4}{4d}+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\left(a^2A-2abB-Ab^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{b\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\tan(c+dx)}{d}-\frac{\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)\log(\cos(c+dx))}{d}-x\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)+\frac{(aA-bB)(a+b\tan(c+dx))^3}{3d}+\frac{A(a+b\tan(c+dx))^4}{4d}+\frac{B(a+b\tan(c+dx))^5}{5bd}$$

input
```
Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-((4*a^3*A*b - 4*a*A*b^3 + a^4*B - 6*a^2*b^2*B + b^4*B)*x) - ((a^4*A - 6*a
^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*Log[Cos[c + d*x]])/d + (b*(a^3*A
 - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*Tan[c + d*x])/d + ((a^2*A - A*b^2 - 2*a*
b*B)*(a + b*Tan[c + d*x])^2)/(2*d) + ((a*A - b*B)*(a + b*Tan[c + d*x])^3)/
(3*d) + (A*(a + b*Tan[c + d*x])^4)/(4*d) + (B*(a + b*Tan[c + d*x])^5)/(5*b
*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.07

method	result
parts	$\frac{\left(4Aa\,b^3+6B\,a^2b^2\right)\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{\left(4A\,a^3b+B\,a^4\right)\left(\tan(dx+c)-\arctan(\tan(dx+c\right.}{d}$
norman	$\left(-4A\,a^3b+4Aa\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)x+\frac{\left(4A\,a^3b-4Aa\,b^3+B\,a^4-6B\,a^2b^2+B\,b^4\right)\tan(dx+}{d}$
derivativedivides	$\frac{\frac{B\tan(dx+c)^5b^4}{5}+\frac{A\tan(dx+c)^4b^4}{4}+B\tan(dx+c)^4a\,b^3+\frac{4A\tan(dx+c)^3a\,b^3}{3}+2B\tan(dx+c)^3a^2b^2-\frac{B\tan(dx+c)^3b^4}{3}+3A\,ta}{}$
default	$\frac{\frac{B\tan(dx+c)^5b^4}{5}+\frac{A\tan(dx+c)^4b^4}{4}+B\tan(dx+c)^4a\,b^3+\frac{4A\tan(dx+c)^3a\,b^3}{3}+2B\tan(dx+c)^3a^2b^2-\frac{B\tan(dx+c)^3b^4}{3}+3A\,ta}{}$
parallelrisch	$\frac{30A\ln\left(1+\tan(dx+c)^2\right)b^4-60B\,b^4dx-60Bx\,a^4d+30A\ln\left(1+\tan(dx+c)^2\right)a^4+60B\tan(dx+c)a^4-240A\,a^3bdx+240A}{}$
risch	$\frac{2iA\,b^4c}{d}-\frac{a^4\ln\left(e^{2i(dx+c)}+1\right)A}{d}+\frac{6\ln\left(e^{2i(dx+c)}+1\right)A\,a^2b^2}{d}+\frac{4\ln\left(e^{2i(dx+c)}+1\right)B\,a^3b}{d}-\frac{4\ln\left(e^{2i(dx+c)}+1\right)Ba}{d}$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
(4*A*a*b^3+6*B*a^2*b^2)/d*(1/3*tan(d*x+c)^3-tan(d*x+c)+arctan(tan(d*x+c)))
+(4*A*a^3*b+B*a^4)/d*(tan(d*x+c)-arctan(tan(d*x+c)))+(6*A*a^2*b^2+4*B*a^3*
b)/d*(1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)^2))+(A*b^4+4*B*a*b^3)/d*(1/4*ta
n(d*x+c)^4-1/2*tan(d*x+c)^2+1/2*ln(1+tan(d*x+c)^2))+1/2*A*a^4/d*ln(1+tan(d
*x+c)^2)+B*b^4/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-arctan(tan(
d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.08

$$\int\tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$=\frac{12\,Bb^4\tan\left(dx+c\right)^5+15\left(4\,Bab^3+Ab^4\right)\tan\left(dx+c\right)^4+20\left(6\,Ba^2b^2+4\,Aab^3-Bb^4\right)\tan\left(dx+c\right)^3-6}{}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/60*(12*B*b^4*tan(d*x + c)^5 + 15*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^4 + 20
*(6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*tan(d*x + c)^3 - 60*(B*a^4 + 4*A*a^3*b
- 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*d*x + 30*(4*B*a^3*b + 6*A*a^2*b^2 - 4*B
*a*b^3 - A*b^4)*tan(d*x + c)^2 - 30*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B
*a*b^3 + A*b^4)*log(1/(tan(d*x + c)^2 + 1)) + 60*(B*a^4 + 4*A*a^3*b - 6*B*
a^2*b^2 - 4*A*a*b^3 + B*b^4)*tan(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(214) = 428$.

Time = 0.20 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.93

$$\int \tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \frac{Aa^4\log\left(\tan^2(c+dx)+1\right)}{2d} - 4Aa^3bx + \frac{4Aa^3b\tan(c+dx)}{d} - \frac{3Aa^2b^2\log\left(\tan^2(c+dx)+1\right)}{d} + \frac{3Aa^2b^2\tan^2(c+dx)}{d} + 4Aab^3x + \\ x(A+B\tan(c))(a+b\tan(c))^4\tan(c) \end{cases}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((A*a**4*log(tan(c + d*x)**2 + 1)/(2*d) - 4*A*a**3*b*x + 4*A*a**3
*b*tan(c + d*x)/d - 3*A*a**2*b**2*log(tan(c + d*x)**2 + 1)/d + 3*A*a**2*b*
*2*tan(c + d*x)**2/d + 4*A*a*b**3*x + 4*A*a*b**3*tan(c + d*x)**3/(3*d) - 4
*A*a*b**3*tan(c + d*x)/d + A*b**4*log(tan(c + d*x)**2 + 1)/(2*d) + A*b**4*
tan(c + d*x)**4/(4*d) - A*b**4*tan(c + d*x)**2/(2*d) - B*a**4*x + B*a**4*t
an(c + d*x)/d - 2*B*a**3*b*log(tan(c + d*x)**2 + 1)/d + 2*B*a**3*b*tan(c +
 d*x)**2/d + 6*B*a**2*b**2*x + 2*B*a**2*b**2*tan(c + d*x)**3/d - 6*B*a**2*
b**2*tan(c + d*x)/d + 2*B*a*b**3*log(tan(c + d*x)**2 + 1)/d + B*a*b**3*tan
(c + d*x)**4/d - 2*B*a*b**3*tan(c + d*x)**2/d - B*b**4*x + B*b**4*tan(c +
d*x)**5/(5*d) - B*b**4*tan(c + d*x)**3/(3*d) + B*b**4*tan(c + d*x)/d, Ne(d
, 0)), (x*(A + B*tan(c))*(a + b*tan(c))**4*tan(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.09

$$\int \tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{12\,Bb^4\tan\left(dx+c\right)^5+15\left(4\,Bab^3+Ab^4\right)\tan\left(dx+c\right)^4+20\left(6\,Ba^2b^2+4\,Aab^3-Bb^4\right)\tan\left(dx+c\right)^3+3$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/60*(12*B*b^4*tan(d*x + c)^5 + 15*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^4 + 20
*(6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*tan(d*x + c)^3 + 30*(4*B*a^3*b + 6*A*a^
2*b^2 - 4*B*a*b^3 - A*b^4)*tan(d*x + c)^2 - 60*(B*a^4 + 4*A*a^3*b - 6*B*a^
2*b^2 - 4*A*a*b^3 + B*b^4)*(d*x + c) + 30*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2
 + 4*B*a*b^3 + A*b^4)*log(tan(d*x + c)^2 + 1) + 60*(B*a^4 + 4*A*a^3*b - 6*
B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.60

$$\int \tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)(dx+c)}{d}$$
$$+\frac{(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$
$$+\frac{12\,Bb^4d^4\tan\left(dx+c\right)^5+60\,Bab^3d^4\tan\left(dx+c\right)^4+15\,Ab^4d^4\tan\left(dx+c\right)^4+120\,Ba^2b^2d^4\tan\left(dx+c\right)$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
-(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*(d*x + c)/d + 1/2*(
A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(tan(d*x + c)^2 +
1)/d + 1/60*(12*B*b^4*d^4*tan(d*x + c)^5 + 60*B*a*b^3*d^4*tan(d*x + c)^4 +
 15*A*b^4*d^4*tan(d*x + c)^4 + 120*B*a^2*b^2*d^4*tan(d*x + c)^3 + 80*A*a*b
^3*d^4*tan(d*x + c)^3 - 20*B*b^4*d^4*tan(d*x + c)^3 + 120*B*a^3*b*d^4*tan(
d*x + c)^2 + 180*A*a^2*b^2*d^4*tan(d*x + c)^2 - 120*B*a*b^3*d^4*tan(d*x +
c)^2 - 30*A*b^4*d^4*tan(d*x + c)^2 + 60*B*a^4*d^4*tan(d*x + c) + 240*A*a^3
*b*d^4*tan(d*x + c) - 360*B*a^2*b^2*d^4*tan(d*x + c) - 240*A*a*b^3*d^4*tan
(d*x + c) + 60*B*b^4*d^4*tan(d*x + c))/d^5
```

Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.11

$$\begin{aligned}
&\int \tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx \\
&= \frac{\tan(c+d\,x)\,\left(B\,a^4+B\,b^4+4\,A\,a^3\,b-2\,a\,b^2\,(2\,A\,b+3\,B\,a)\right)}{d} \\
&\quad - \frac{\tan(c+d\,x)^2\,\left(\frac{A\,b^4}{2}+2\,B\,a\,b^3-a^2\,b\,(3\,A\,b+2\,B\,a)\right)}{d} \\
&\quad - x\,\left(B\,a^4+4\,A\,a^3\,b-6\,B\,a^2\,b^2-4\,A\,a\,b^3+B\,b^4\right)+\frac{\tan(c+d\,x)^4\,\left(\frac{A\,b^4}{4}+B\,a\,b^3\right)}{d} \\
&\quad + \frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{A\,a^4}{2}-2\,B\,a^3\,b-3\,A\,a^2\,b^2+2\,B\,a\,b^3+\frac{A\,b^4}{2}\right)}{d} \\
&\quad - \frac{\tan(c+d\,x)^3\,\left(\frac{B\,b^4}{3}-\frac{2\,a\,b^2\,(2\,A\,b+3\,B\,a)}{3}\right)}{d}+\frac{B\,b^4\,\tan(c+d\,x)^5}{5\,d}
\end{aligned}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
(tan(c + d*x)*(B*a^4 + B*b^4 + 4*A*a^3*b - 2*a*b^2*(2*A*b + 3*B*a)))/d - (
tan(c + d*x)^2*((A*b^4)/2 + 2*B*a*b^3 - a^2*b*(3*A*b + 2*B*a)))/d - x*(B*a
^4 + B*b^4 - 6*B*a^2*b^2 - 4*A*a*b^3 + 4*A*a^3*b) + (tan(c + d*x)^4*((A*b^
4)/4 + B*a*b^3))/d + (log(tan(c + d*x)^2 + 1)*((A*a^4)/2 + (A*b^4)/2 - 3*A
*a^2*b^2 + 2*B*a*b^3 - 2*B*a^3*b))/d - (tan(c + d*x)^3*((B*b^4)/3 - (2*a*b
^2*(2*A*b + 3*B*a))/3))/d + (B*b^4*tan(c + d*x)^5)/(5*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.91

$$\int \tan(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{30\log\left(\tan\left(dx+c\right)^2+1\right)a^5-300\log\left(\tan\left(dx+c\right)^2+1\right)a^3b^2+150\log\left(\tan\left(dx+c\right)^2+1\right)a\,b^4+12\tan}{}$$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(30*log(tan(c + d*x)**2 + 1)*a**5 - 300*log(tan(c + d*x)**2 + 1)*a**3*b**2
 + 150*log(tan(c + d*x)**2 + 1)*a*b**4 + 12*tan(c + d*x)**5*b**5 + 75*tan(
c + d*x)**4*a*b**4 + 200*tan(c + d*x)**3*a**2*b**3 - 20*tan(c + d*x)**3*b*
*5 + 300*tan(c + d*x)**2*a**3*b**2 - 150*tan(c + d*x)**2*a*b**4 + 300*tan(
c + d*x)*a**4*b - 600*tan(c + d*x)*a**2*b**3 + 60*tan(c + d*x)*b**5 - 300*
a**4*b*d*x + 600*a**2*b**3*d*x - 60*b**5*d*x)/(60*d)
```

3.259 $\int (a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 202

$$\begin{aligned}
&\int (a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx \\
&= \left(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B\right)x \\
&\quad -\frac{(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B)\log(\cos(c+dx))}{d} \\
&\quad +\frac{b(3a^2Ab-Ab^3+a^3B-3ab^2B)\tan(c+dx)}{d} \\
&\quad +\frac{(2aAb+a^2B-b^2B)\,(a+b\tan(c+dx))^2}{2d} \\
&\quad +\frac{(Ab+aB)(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4d}
\end{aligned}$$

output

```
(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*x-(4*A*a^3*b-4*A*a*b^3+B*a^4
-6*B*a^2*b^2+B*b^4)*ln(cos(d*x+c))/d+b*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*t
an(d*x+c)/d+1/2*(2*A*a*b+B*a^2-B*b^2)*(a+b*tan(d*x+c))^2/d+1/3*(A*b+B*a)*(
a+b*tan(d*x+c))^3/d+1/4*B*(a+b*tan(d*x+c))^4/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.19

$$\int (a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{-2(Ab-aB)\left(3i(a+ib)^4\log(i-\tan(c+dx))-3i(a-ib)^4\log(i+\tan(c+dx))+6b^2(-6a^2+b^2)\tan(\right.}{}$$

input
```
Integrate[(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*(A*b - a*B)*((3*I)*(a + I*b)^4*Log[I - Tan[c + d*x]] - (3*I)*(a - I*b)
^4*Log[I + Tan[c + d*x]] + 6*b^2*(-6*a^2 + b^2)*Tan[c + d*x] - 12*a*b^3*Ta
n[c + d*x]^2 - 2*b^4*Tan[c + d*x]^3) + B*(6*((-I)*a + b)^5*Log[I - Tan[c +
 d*x]] + 6*(I*a + b)^5*Log[I + Tan[c + d*x]] + 60*a*b^2*(2*a^2 - b^2)*Tan[
c + d*x] - 6*b^3*(-10*a^2 + b^2)*Tan[c + d*x]^2 + 20*a*b^4*Tan[c + d*x]^3
+ 3*b^5*Tan[c + d*x]^4))/(12*b*d)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4011, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \; 3042$$
$$\int (a+b\tan(c+dx))^4(A+B\tan(c+dx))dx$$
$$\Big\downarrow \; 4011$$
$$\int (a+b\tan(c+dx))^3(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{B(a+b\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^3(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{B(a+b\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))^2\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{(aB+Ab)(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{(aB+Ab)(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))\left(Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{\left(a^2B+2aAb-b^2B\right)(a+b\tan(c+dx))^2}{2d}+\frac{(aB+Ab)(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))\left(Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{\left(a^2B+2aAb-b^2B\right)(a+b\tan(c+dx))^2}{2d}+\frac{(aB+Ab)(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{4008}$$

$$\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\int \tan(c+dx)dx+\frac{\left(a^2B+2aAb-b^2B\right)(a+b\tan(c+dx))^2}{2d}+\frac{b\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\tan(c+dx)}{d}+x\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)+\frac{(aB+Ab)(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\int \tan(c+dx)dx+\frac{\left(a^2B+2aAb-b^2B\right)(a+b\tan(c+dx))^2}{2d}+\frac{b\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\tan(c+dx)}{d}+x\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)+\frac{(aB+Ab)(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\left(a^2B+2aAb-b^2B\right)(a+b\tan(c+dx))^2}{2d}+\frac{b\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\tan(c+dx)}{d}-\frac{\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\log(\cos(c+dx))}{d}+x\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)+\frac{(aB+Ab)(a+b\tan(c+dx))^3}{3d}+\frac{B(a+b\tan(c+dx))^4}{4d}$$

input
```
Int[(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*x - ((4*a^3*A*b - 4*
a*A*b^3 + a^4*B - 6*a^2*b^2*B + b^4*B)*Log[Cos[c + d*x]])/d + (b*(3*a^2*A*
b - A*b^3 + a^3*B - 3*a*b^2*B)*Tan[c + d*x])/d + ((2*a*A*b + a^2*B - b^2*B
)*(a + b*Tan[c + d*x])^2)/(2*d) + ((A*b + a*B)*(a + b*Tan[c + d*x])^3)/(3*
d) + (B*(a + b*Tan[c + d*x])^4)/(4*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

method	result
norman	$\left(A a^4-6A a^2b^2+A b^4-4B a^3b+4Ba b^3\right)x+\frac{b\left(6A a^2b-A b^3+4B a^3-4Ba b^2\right)\tan(dx+c)}{d}+\frac{B b}{}$
parts	$A a^4x+\frac{\left(A b^4+4Ba b^3\right)\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{\left(6A a^2b^2+4B a^3b\right)(\tan(dx+c)-\mathrm{arcta}}{d}$
derivativedivides	$\frac{\frac{B b^4\tan(dx+c)^4}{4}+\frac{A b^4\tan(dx+c)^3}{3}+\frac{4Ba b^3\tan(dx+c)^3}{3}+2Aa b^3\tan(dx+c)^2+3B a^2b^2\tan(dx+c)^2-\frac{B b^4\tan(dx+c)^2}{2}+6A}{}$
default	$\frac{\frac{B b^4\tan(dx+c)^4}{4}+\frac{A b^4\tan(dx+c)^3}{3}+\frac{4Ba b^3\tan(dx+c)^3}{3}+2Aa b^3\tan(dx+c)^2+3B a^2b^2\tan(dx+c)^2-\frac{B b^4\tan(dx+c)^2}{2}+6A}{}$
parallelrisch	$\frac{3B b^4\tan(dx+c)^4+4A b^4\tan(dx+c)^3+16Ba b^3\tan(dx+c)^3+12Ax a^4d-72A a^2b^2dx+12A b^4dx+24Aa b^3\tan(dx+c)^2}{}$
risch	$\frac{8iA a^3bc}{d}+\frac{2iB b^4c}{d}+iB a^4x-6iB a^2b^2x-4iAa b^3x+\frac{2iB a^4c}{d}-\frac{a^4\ln\left(e^{2i(dx+c)}+1\right)B}{d}-\frac{\ln(e^{2i(}}{}$

input
```
int((a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*x+b*(6*A*a^2*b-A*b^3+4*B*a^3
-4*B*a*b^2)/d*tan(d*x+c)+1/4*B*b^4/d*tan(d*x+c)^4+1/2*b^2*(4*A*a*b+6*B*a^2
-B*b^2)/d*tan(d*x+c)^2+1/3*b^3*(A*b+4*B*a)/d*tan(d*x+c)^3+1/2*(4*A*a^3*b-4
*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

$$\int (a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{3\,Bb^4\tan\left(dx+c\right)^4+4\left(4\,Bab^3+Ab^4\right)\tan\left(dx+c\right)^3+12\left(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4\right)dx+\cdots}{}$$

input
```
integrate((a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*B*b^4*tan(d*x + c)^4 + 4*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^3 + 12*(
A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*d*x + 6*(6*B*a^2*b^2
+ 4*A*a*b^3 - B*b^4)*tan(d*x + c)^2 - 6*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 -
 4*A*a*b^3 + B*b^4)*log(1/(tan(d*x + c)^2 + 1)) + 12*(4*B*a^3*b + 6*A*a^2*
b^2 - 4*B*a*b^3 - A*b^4)*tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.72

$$\int (a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\begin{cases} Aa^4x+\frac{2Aa^3b\log\left(\tan^2\left(c+dx\right)+1\right)}{d}-6Aa^2b^2x+\frac{6Aa^2b^2\tan\left(c+dx\right)}{d}-\frac{2Aab^3\log\left(\tan^2\left(c+dx\right)+1\right)}{d}+\frac{2Aab^3\tan^2\left(c+dx\right)}{d}+A\cdots \\ x(A+B\tan\left(c\right))\left(a+b\tan\left(c\right)\right)^4 \end{cases}$$

input
```
integrate((a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((A*a**4*x + 2*A*a**3*b*log(tan(c + d*x)**2 + 1)/d - 6*A*a**2*b**
2*x + 6*A*a**2*b**2*tan(c + d*x)/d - 2*A*a*b**3*log(tan(c + d*x)**2 + 1)/d
 + 2*A*a*b**3*tan(c + d*x)**2/d + A*b**4*x + A*b**4*tan(c + d*x)**3/(3*d)
- A*b**4*tan(c + d*x)/d + B*a**4*log(tan(c + d*x)**2 + 1)/(2*d) - 4*B*a**3
*b*x + 4*B*a**3*b*tan(c + d*x)/d - 3*B*a**2*b**2*log(tan(c + d*x)**2 + 1)/
d + 3*B*a**2*b**2*tan(c + d*x)**2/d + 4*B*a*b**3*x + 4*B*a*b**3*tan(c + d*
x)**3/(3*d) - 4*B*a*b**3*tan(c + d*x)/d + B*b**4*log(tan(c + d*x)**2 + 1)/
(2*d) + B*b**4*tan(c + d*x)**4/(4*d) - B*b**4*tan(c + d*x)**2/(2*d), Ne(d,
 0)), (x*(A + B*tan(c))*(a + b*tan(c))**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int (a + b\tan(c + dx))^4 (A + B\tan(c + dx))\,dx$$
$$= \frac{3\,Bb^4\tan{(dx+c)}^4 + 4\,(4\,Bab^3 + Ab^4)\tan{(dx+c)}^3 + 6\,(6\,Ba^2b^2 + 4\,Aab^3 - Bb^4)\tan{(dx+c)}^2 + 12\,(A}{}$$

input
```
integrate((a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(3*B*b^4*tan(d*x + c)^4 + 4*(4*B*a*b^3 + A*b^4)*tan(d*x + c)^3 + 6*(6
*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*tan(d*x + c)^2 + 12*(A*a^4 - 4*B*a^3*b - 6
*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) + 6*(B*a^4 + 4*A*a^3*b - 6*B*a^2
*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1) + 12*(4*B*a^3*b + 6*A*a^
2*b^2 - 4*B*a*b^3 - A*b^4)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.35

$$\int (a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)(dx+c)}{d} + \frac{(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d} + \frac{3\,Bb^4d^3\tan\left(dx+c\right)^4+16\,Bab^3d^3\tan\left(dx+c\right)^3+4\,Ab^4d^3\tan\left(dx+c\right)^3+36\,Ba^2b^2d^3\tan\left(dx+c\right)^2+}{}$$

input
```
integrate((a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c)/d + 1/2*(B
*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1
)/d + 1/12*(3*B*b^4*d^3*tan(d*x + c)^4 + 16*B*a*b^3*d^3*tan(d*x + c)^3 + 4
*A*b^4*d^3*tan(d*x + c)^3 + 36*B*a^2*b^2*d^3*tan(d*x + c)^2 + 24*A*a*b^3*d
^3*tan(d*x + c)^2 - 6*B*b^4*d^3*tan(d*x + c)^2 + 48*B*a^3*b*d^3*tan(d*x +
c) + 72*A*a^2*b^2*d^3*tan(d*x + c) - 48*B*a*b^3*d^3*tan(d*x + c) - 12*A*b^
4*d^3*tan(d*x + c))/d^4
```

Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int (a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= x\,\left(A\,a^4-4\,B\,a^3\,b-6\,A\,a^2\,b^2+4\,B\,a\,b^3+A\,b^4\right) - \frac{\tan(c+d\,x)\,\left(A\,b^4+4\,B\,a\,b^3-2\,a^2\,b\,(3\,A\,b+2\,B\,a)\right)}{d} + \frac{\tan(c+d\,x)^3\,\left(\frac{A\,b^4}{3}+\frac{4\,B\,a\,b^3}{3}\right)}{d} + \frac{\ln\left(\tan(c+d\,x)^2+1\right)\,\left(\frac{B\,a^4}{2}+2\,A\,a^3\,b-3\,B\,a^2\,b^2-2\,A\,a\,b^3+\frac{B\,b^4}{2}\right)}{d} - \frac{\tan(c+d\,x)^2\,\left(\frac{B\,b^4}{2}-a\,b^2\,(2\,A\,b+3\,B\,a)\right)}{d} + \frac{B\,b^4\,\tan(c+d\,x)^4}{4\,d}$$

input
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
x*(A*a^4 + A*b^4 - 6*A*a^2*b^2 + 4*B*a*b^3 - 4*B*a^3*b) - (tan(c + d*x)*(A
*b^4 + 4*B*a*b^3 - 2*a^2*b*(3*A*b + 2*B*a)))/d + (tan(c + d*x)^3*((A*b^4)/
3 + (4*B*a*b^3)/3))/d + (log(tan(c + d*x)^2 + 1)*((B*a^4)/2 + (B*b^4)/2 -
3*B*a^2*b^2 - 2*A*a*b^3 + 2*A*a^3*b))/d - (tan(c + d*x)^2*((B*b^4)/2 - a*b
^2*(2*A*b + 3*B*a)))/d + (B*b^4*tan(c + d*x)^4)/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.82

$$\int (a + b\tan(c + dx))^4 (A + B\tan(c + dx))\,dx$$
$$= \frac{30\log\left(\tan(dx + c)^2 + 1\right) a^4 b - 60\log\left(\tan(dx + c)^2 + 1\right) a^2 b^3 + 6\log\left(\tan(dx + c)^2 + 1\right) b^5 + 3\tan(dx +}{}$$

input
```
int((a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(30*log(tan(c + d*x)**2 + 1)*a**4*b - 60*log(tan(c + d*x)**2 + 1)*a**2*b**
3 + 6*log(tan(c + d*x)**2 + 1)*b**5 + 3*tan(c + d*x)**4*b**5 + 20*tan(c +
d*x)**3*a*b**4 + 60*tan(c + d*x)**2*a**2*b**3 - 6*tan(c + d*x)**2*b**5 + 1
20*tan(c + d*x)*a**3*b**2 - 60*tan(c + d*x)*a*b**4 + 12*a**5*d*x - 120*a**
3*b**2*d*x + 60*a*b**4*d*x)/(12*d)
```

3.260 $\int \cot(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 172

$$\begin{aligned}\int \cot(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx &= \left(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B\right)x \\ &\quad -\frac{b(6a^2Ab-Ab^3+4a^3B-4ab^2B)\log(\cos(c+dx))}{d} \\ &\quad +\frac{a^4A\log(\sin(c+dx))}{d}+\frac{b^2(3aAb+3a^2B-b^2B)\tan(c+dx)}{d} \\ &\quad +\frac{b(Ab+2aB)(a+b\tan(c+dx))^2}{2d}+\frac{bB(a+b\tan(c+dx))^3}{3d}\end{aligned}$$

output
```
(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*x-b*(6*A*a^2*b-A*b^3+4*B*a^3
-4*B*a*b^2)*ln(cos(d*x+c))/d+a^4*A*ln(sin(d*x+c))/d+b^2*(3*A*a*b+3*B*a^2-B
*b^2)*tan(d*x+c)/d+1/2*b*(A*b+2*B*a)*(a+b*tan(d*x+c))^2/d+1/3*b*B*(a+b*tan
(d*x+c))^3/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87

$$\int \cot(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{-3(a+ib)^4(A+iB)\log(i-\tan(c+dx))+6a^4A\log(\tan(c+dx))-3(a-ib)^4(A-iB)\log(i+\tan(c+dx))}{}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(-3*(a + I*b)^4*(A + I*B)*Log[I - Tan[c + d*x]] + 6*a^4*A*Log[Tan[c + d*x]
] - 3*(a - I*b)^4*(A - I*B)*Log[I + Tan[c + d*x]] + 6*b^2*(3*a*A*b + 3*a^2
*B - b^2*B)*Tan[c + d*x] + 3*b*(A*b + 2*a*B)*(a + b*Tan[c + d*x])^2 + 2*b*
B*(a + b*Tan[c + d*x])^3)/(6*d)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4120, 25, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{1}{3}\int 3\cot(c+dx)(a+b\tan(c+dx))^2\left(Aa^2+b(Ab+2aB)\tan^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 27

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(Aa^2+b(Ab+2aB)\tan^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3042

$$\int \frac{(a+b\tan(c+dx))^2\left(Aa^2+b(Ab+2aB)\tan(c+dx)^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{\tan(c+dx)}dx+\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 4130

$$\frac{1}{2}\int 2\cot(c+dx)(a+b\tan(c+dx))\left(Aa^3+b\left(3Ba^2+3Aba-b^2B\right)\tan^2(c+dx)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{b(2aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 27

$$\int \cot(c+dx)(a+b\tan(c+dx))\left(Aa^3+b\left(3Ba^2+3Aba-b^2B\right)\tan^2(c+dx)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{b(2aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3042

$$\int \frac{(a+b\tan(c+dx))\left(Aa^3+b\left(3Ba^2+3Aba-b^2B\right)\tan(c+dx)^2+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx\right.}{\tan(c+dx)}$$

$$\frac{b(2aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 4120

$$-\int -\cot(c+$$
$$dx)\left(Aa^4+b\left(4Ba^3+6Aba^2-4b^2Ba-Ab^3\right)\tan^2(c+dx)+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\tan(c+$$
$$\frac{b^2\left(3a^2B+3aAb-b^2B\right)\tan(c+dx)}{d}+\frac{b(2aB+Ab)(a+b\tan(c+dx))^2}{2d}+$$
$$\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{25}$$

$$\int \cot(c+$$
$$dx)\left(Aa^4+b\left(4Ba^3+6Aba^2-4b^2Ba-Ab^3\right)\tan^2(c+dx)+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\tan(c+$$
$$\frac{b^2\left(3a^2B+3aAb-b^2B\right)\tan(c+dx)}{d}+\frac{b(2aB+Ab)(a+b\tan(c+dx))^2}{2d}+$$
$$\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Aa^4+b\left(4Ba^3+6Aba^2-4b^2Ba-Ab^3\right)\tan(c+dx)^2+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\tan(c+dx}{\tan(c+dx)}$$
$$\frac{b^2\left(3a^2B+3aAb-b^2B\right)\tan(c+dx)}{d}+\frac{b(2aB+Ab)(a+b\tan(c+dx))^2}{2d}+$$
$$\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4107}$$

$$a^4A\int \cot(c+dx)dx+b\left(4a^3B+6a^2Ab-4ab^2B-Ab^3\right)\int \tan(c+dx)dx+$$
$$\frac{b^2\left(3a^2B+3aAb-b^2B\right)\tan(c+dx)}{d}+x\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)+$$
$$\frac{b(2aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a^4A\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+b\left(4a^3B+6a^2Ab-4ab^2B-Ab^3\right)\int \tan(c+dx)dx+$$
$$\frac{b^2\left(3a^2B+3aAb-b^2B\right)\tan(c+dx)}{d}+x\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)+$$
$$\frac{b(2aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{25}$$

$$a^4(-A)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+b\left(4a^3B+6a^2Ab-4ab^2B-Ab^3\right)\int \tan(c+dx)dx+\frac{b^2\left(3a^2B+3aAb-b^2B\right)\tan(c+dx)}{d}+x\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)+\frac{b(2aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{bB(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{a^4A\log(-\sin(c+dx))}{d}+\frac{b^2\left(3a^2B+3aAb-b^2B\right)\tan(c+dx)}{d}-\frac{b\left(4a^3B+6a^2Ab-4ab^2B-Ab^3\right)\log(\cos(c+dx))}{d}+x\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)+\frac{b(2aB+Ab)(a+b\tan(c+dx))^2}{2d}+\frac{bB(a+b\tan(c+dx))^3}{3d}$$

input

```
Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output

```
(4*a^3*A*b - 4*a*A*b^3 + a^4*B - 6*a^2*b^2*B + b^4*B)*x - (b*(6*a^2*A*b -
A*b^3 + 4*a^3*B - 4*a*b^2*B)*Log[Cos[c + d*x]])/d + (a^4*A*Log[-Sin[c + d*
x]])/d + (b^2*(3*a*A*b + 3*a^2*B - b^2*B)*Tan[c + d*x])/d + (b*(A*b + 2*a*
B)*(a + b*Tan[c + d*x])^2)/(2*d) + (b*B*(a + b*Tan[c + d*x])^3)/(3*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{\left(-3A\,a^4+18A\,a^2b^2-3A\,b^4+12B\,a^3b-12Ba\,b^3\right)\ln\left(\sec(dx+c)^2\right)+6A\,a^4\ln(\tan(dx+c))+2B\tan(dx+c)^3b^4+(3A\,b^4+1}{6d}$
norman	$\left(4A\,a^3b-4Aa\,b^3+B\,a^4-6B\,a^2b^2+B\,b^4\right)x+\frac{b^2\left(4Aab+6B\,a^2-B\,b^2\right)\tan(dx+c)}{d}+\frac{B\,b^4\tan(dx}{3d}$
derivativedivides	$\frac{\frac{B\tan(dx+c)^3b^4}{3}+\frac{A\tan(dx+c)^2b^4}{2}+2B\tan(dx+c)^2a\,b^3+4A\tan(dx+c)a\,b^3+6B\tan(dx+c)a^2b^2-B\tan(dx+c)b^4+\frac{(-A}{}}{}$
default	$\frac{\frac{B\tan(dx+c)^3b^4}{3}+\frac{A\tan(dx+c)^2b^4}{2}+2B\tan(dx+c)^2a\,b^3+4A\tan(dx+c)a\,b^3+6B\tan(dx+c)a^2b^2-B\tan(dx+c)b^4+\frac{(-A}{}}{}$
risch	$\frac{12iA\,a^2b^2c}{d}+6iA\,a^2b^2x-4iBa\,b^3x+4iB\,a^3bx+\frac{2ib^2\left(-3iA\,b^2e^{4i(dx+c)}-12iBab\,e^{4i(dx+c)}+12Aab\,e^{4}\right.}{}$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/6*((-3*A*a^4+18*A*a^2*b^2-3*A*b^4+12*B*a^3*b-12*B*a*b^3)*ln(sec(d*x+c)^2
)+6*A*a^4*ln(tan(d*x+c))+2*B*tan(d*x+c)^3*b^4+(3*A*b^4+12*B*a*b^3)*tan(d*x
+c)^2+(24*A*a*b^3+36*B*a^2*b^2-6*B*b^4)*tan(d*x+c)+24*(A*a^3*b-A*a*b^3+1/4
*B*a^4-3/2*B*a^2*b^2+1/4*B*b^4)*x*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08

$$\int \cot(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{2\,Bb^4\tan\left(dx+c\right)^3+3\,Aa^4\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)+6\left(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4\right)dx+3\,(4\,B}{}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/6*(2*B*b^4*tan(d*x + c)^3 + 3*A*a^4*log(tan(d*x + c)^2/(tan(d*x + c)^2 +
 1)) + 6*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*d*x + 3*(4*
B*a*b^3 + A*b^4)*tan(d*x + c)^2 - 3*(4*B*a^3*b + 6*A*a^2*b^2 - 4*B*a*b^3 -
 A*b^4)*log(1/(tan(d*x + c)^2 + 1)) + 6*(6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*
tan(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.69

$$\int \cot(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} -\frac{Aa^4\log(\tan^2(c+dx)+1)}{2d} + \frac{Aa^4\log(\tan(c+dx))}{d} + 4Aa^3bx + \frac{3Aa^2b^2\log(\tan^2(c+dx)+1)}{d} - 4Aab^3x + \frac{4Aab^3\tan(c+dx)}{d} \\ x(A+B\tan(c))(a+b\tan(c))^4\cot(c) \end{cases}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((-A*a**4*log(tan(c + d*x)**2 + 1)/(2*d) + A*a**4*log(tan(c + d*x
))/d + 4*A*a**3*b*x + 3*A*a**2*b**2*log(tan(c + d*x)**2 + 1)/d - 4*A*a*b**
3*x + 4*A*a*b**3*tan(c + d*x)/d - A*b**4*log(tan(c + d*x)**2 + 1)/(2*d) +
A*b**4*tan(c + d*x)**2/(2*d) + B*a**4*x + 2*B*a**3*b*log(tan(c + d*x)**2 +
 1)/d - 6*B*a**2*b**2*x + 6*B*a**2*b**2*tan(c + d*x)/d - 2*B*a*b**3*log(ta
n(c + d*x)**2 + 1)/d + 2*B*a*b**3*tan(c + d*x)**2/d + B*b**4*x + B*b**4*ta
n(c + d*x)**3/(3*d) - B*b**4*tan(c + d*x)/d, Ne(d, 0)), (x*(A + B*tan(c))*
(a + b*tan(c))**4*cot(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int \cot(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,Bb^4\tan(dx+c)^3 + 6\,Aa^4\log(\tan(dx+c)) + 3\,(4\,Bab^3 + Ab^4)\tan(dx+c)^2 + 6\,(Ba^4 + 4\,Aa^3b - 6\,B}{}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/6*(2*B*b^4*tan(d*x + c)^3 + 6*A*a^4*log(tan(d*x + c)) + 3*(4*B*a*b^3 + A
*b^4)*tan(d*x + c)^2 + 6*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*
b^4)*(d*x + c) - 3*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*l
og(tan(d*x + c)^2 + 1) + 6*(6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*tan(d*x + c))
/d
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.26

$$\int \cot(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \frac{Aa^4\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d} + \frac{(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)(dx+c)}{d} - \frac{(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d} + \frac{2\,Bb^4d^2\tan\left(dx+c\right)^3+12\,Bab^3d^2\tan\left(dx+c\right)^2+3\,Ab^4d^2\tan\left(dx+c\right)^2+36\,Ba^2b^2d^2\tan\left(dx+c\right)+2}{6\,d^3}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
A*a^4*log(abs(tan(d*x + c)))/d + (B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*
b^3 + B*b^4)*(d*x + c)/d - 1/2*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^
3 + A*b^4)*log(tan(d*x + c)^2 + 1)/d + 1/6*(2*B*b^4*d^2*tan(d*x + c)^3 + 1
2*B*a*b^3*d^2*tan(d*x + c)^2 + 3*A*b^4*d^2*tan(d*x + c)^2 + 36*B*a^2*b^2*d
^2*tan(d*x + c) + 24*A*a*b^3*d^2*tan(d*x + c) - 6*B*b^4*d^2*tan(d*x + c))/
d^3
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \cot(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{\tan(c+d\,x)^2\,\left(\frac{A\,b^4}{2}+2\,B\,a\,b^3\right)}{d}-\frac{\tan(c+d\,x)\,\left(B\,b^4-2\,a\,b^2\,(2\,A\,b+3\,B\,a)\right)}{d}+\frac{A\,a^4\,\ln\left(\tan(c+d\,x)\right)}{d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^4}{2\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^4}{2\,d}+\frac{B\,b^4\,\tan(c+d\,x)^3}{3\,d}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
(tan(c + d*x)^2*((A*b^4)/2 + 2*B*a*b^3))/d - (tan(c + d*x)*(B*b^4 - 2*a*b^
2*(2*A*b + 3*B*a)))/d + (A*a^4*log(tan(c + d*x)))/d - (log(tan(c + d*x) +
1i)*(A - B*1i)*(a*1i + b)^4)/(2*d) - (log(tan(c + d*x) - 1i)*(A + B*1i)*(a
*1i - b)^4)/(2*d) + (B*b^4*tan(c + d*x)^3)/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 674, normalized size of antiderivative = 3.92

$$\int \cot(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
( - 6*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**5 + 60*
cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**3*b**2 - 30*c
os(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a*b**4 + 6*cos(c
+ d*x)*log(tan((c + d*x)/2)**2 + 1)*a**5 - 60*cos(c + d*x)*log(tan((c + d*
x)/2)**2 + 1)*a**3*b**2 + 30*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*a*b
**4 - 60*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a**3*b**2
+ 30*cos(c + d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**4 + 60*co
s(c + d*x)*log(tan((c + d*x)/2) - 1)*a**3*b**2 - 30*cos(c + d*x)*log(tan((
c + d*x)/2) - 1)*a*b**4 - 60*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c
+ d*x)**2*a**3*b**2 + 30*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*sin(c + d*
x)**2*a*b**4 + 60*cos(c + d*x)*log(tan((c + d*x)/2) + 1)*a**3*b**2 - 30*co
s(c + d*x)*log(tan((c + d*x)/2) + 1)*a*b**4 + 6*cos(c + d*x)*log(tan((c +
d*x)/2))*sin(c + d*x)**2*a**5 - 6*cos(c + d*x)*log(tan((c + d*x)/2))*a**5
+ 30*cos(c + d*x)*sin(c + d*x)**2*a**4*b*d*x - 60*cos(c + d*x)*sin(c + d*x
)**2*a**2*b**3*d*x - 15*cos(c + d*x)*sin(c + d*x)**2*a*b**4 + 6*cos(c + d*
x)*sin(c + d*x)**2*b**5*d*x - 30*cos(c + d*x)*a**4*b*d*x + 60*cos(c + d*x)
*a**2*b**3*d*x - 6*cos(c + d*x)*b**5*d*x + 60*sin(c + d*x)**3*a**2*b**3 -
8*sin(c + d*x)**3*b**5 - 60*sin(c + d*x)*a**2*b**3 + 6*sin(c + d*x)*b**5)/
(6*cos(c + d*x)*d*(sin(c + d*x)**2 - 1))
```

3.261 $\int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 175

$$\begin{aligned}\int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx\\ &= -\left(\left(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B\right)x\right)\\ &\quad-\frac{b^2(4aAb+6a^2B-b^2B)\log(\cos(c+dx))}{d}\\ &\quad+\frac{a^3(4Ab+aB)\log(\sin(c+dx))}{d}+\frac{b^2(a^2A+Ab^2+3abB)\tan(c+dx)}{d}\\ &\quad+\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}\end{aligned}$$

output
```
-(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*x-b^2*(4*A*a*b+6*B*a^2-B*b^
2)*ln(cos(d*x+c))/d+a^3*(4*A*b+B*a)*ln(sin(d*x+c))/d+b^2*(A*a^2+A*b^2+3*B*
a*b)*tan(d*x+c)/d+1/2*b*(2*A*a+B*b)*(a+b*tan(d*x+c))^2/d-a*A*cot(d*x+c)*(a
+b*tan(d*x+c))^3/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.77

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{-2a^4A\cot(c+dx)+i(a+ib)^4(A+iB)\log(i-\tan(c+dx))+2a^3(4Ab+aB)\log(\tan(c+dx))-(a-}{2d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a^4*A*Cot[c + d*x] + I*(a + I*b)^4*(A + I*B)*Log[I - Tan[c + d*x]] + 2
*a^3*(4*A*b + a*B)*Log[Tan[c + d*x]] - (a - I*b)^4*(I*A + B)*Log[I + Tan[c
 + d*x]] + 2*b^3*(A*b + 4*a*B)*Tan[c + d*x] + b^4*B*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4088, 3042, 4130, 27, 3042, 4120, 25, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4088}$$

$$\int \cot(c+dx)(a+b\tan(c+dx))^2\left(b(2aA+bB)\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+aB)\right)dx-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2\left(b(2aA+bB)\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+aB)\right)}{\tan(c+dx)}dx-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{4130}$$

$$\frac{1}{2}\int 2\cot(c+dx)(a+b\tan(c+dx))\left((4Ab+aB)a^2+b\left(Aa^2+3bBa+Ab^2\right)\tan^2(c+dx)-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{27}$$

$$\int \cot(c+dx)(a+b\tan(c+dx))\left((4Ab+aB)a^2+b\left(Aa^2+3bBa+Ab^2\right)\tan^2(c+dx)-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)dx+\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))\left((4Ab+aB)a^2+b\left(Aa^2+3bBa+Ab^2\right)\tan(c+dx)^2-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\text{ta}\right.}{\tan(c+dx)}$$
$$\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{4120}$$

$$-\int -\cot(c+dx)\left((4Ab+aB)a^3+b^2\left(6Ba^2+4Aba-b^2B\right)\tan^2(c+dx)-\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\tan(c+\right.$$
$$\frac{b^2\left(a^2A+3abB+Ab^2\right)\tan(c+dx)}{d}+\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \text{25}$$

$$\int \cot(c+dx)\left((4Ab+aB)a^3+b^2\left(6Ba^2+4Aba-b^2B\right)\tan^2(c+dx)-\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\tan(c+\right.$$
$$\frac{b^2\left(a^2A+3abB+Ab^2\right)\tan(c+dx)}{d}+\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(4Ab+aB)a^3+b^2\left(6Ba^2+4Aba-b^2B\right)\tan(c+dx)^2-\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\tan(c+dx)}{\tan(c+dx)}dx+\frac{b^2\left(a^2A+3abB+Ab^2\right)\tan(c+dx)}{d}+\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 4107$$

$$a^3(aB+4Ab)\int\cot(c+dx)dx+b^2\left(6a^2B+4aAb-b^2B\right)\int\tan(c+dx)dx+\frac{b^2\left(a^2A+3abB+Ab^2\right)\tan(c+dx)}{d}-x\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)+\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 3042$$

$$a^3(aB+4Ab)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx+b^2\left(6a^2B+4aAb-b^2B\right)\int\tan(c+dx)dx+\frac{b^2\left(a^2A+3abB+Ab^2\right)\tan(c+dx)}{d}-x\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)+\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 25$$

$$-\left(a^3(aB+4Ab)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx\right)+b^2\left(6a^2B+4aAb-b^2B\right)\int\tan(c+dx)dx+\frac{b^2\left(a^2A+3abB+Ab^2\right)\tan(c+dx)}{d}-x\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)+\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

$$\Big\downarrow \; 3956$$

$$\frac{a^3(aB+4Ab)\log(-\sin(c+dx))}{d}+\frac{b^2\left(a^2A+3abB+Ab^2\right)\tan(c+dx)}{d}-\frac{b^2\left(6a^2B+4aAb-b^2B\right)\log(\cos(c+dx))}{d}-x\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)+\frac{b(2aA+bB)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^3}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-((a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*x) - (b^2*(4*a*A*b
 + 6*a^2*B - b^2*B)*Log[Cos[c + d*x]])/d + (a^3*(4*A*b + a*B)*Log[-Sin[c +
 d*x]])/d + (b^2*(a^2*A + A*b^2 + 3*a*b*B)*Tan[c + d*x])/d + (b*(2*a*A + b
*B)*(a + b*Tan[c + d*x])^2)/(2*d) - (a*A*Cot[c + d*x]*(a + b*Tan[c + d*x])
^3)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

method	result
parallelrisch	$\frac{\left(-4A\,a^{3}b+4Aa\,b^{3}-B\,a^{4}+6B\,a^{2}b^{2}-B\,b^{4}\right)\ln\left(\sec(dx+c)^{2}\right)+\left(8A\,a^{3}b+2B\,a^{4}\right)\ln(\tan(dx+c))+B\,b^{4}\tan(dx+c)^{2}+(2A\,b}{2d}$
derivativedivides	$\frac{\frac{B\,b^{4}\tan(dx+c)^{2}}{2}+A\,b^{4}\tan(dx+c)+4Ba\,b^{3}\tan(dx+c)+\frac{\left(-4A\,a^{3}b+4Aa\,b^{3}-B\,a^{4}+6B\,a^{2}b^{2}-B\,b^{4}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-A}{d}$
default	$\frac{\frac{B\,b^{4}\tan(dx+c)^{2}}{2}+A\,b^{4}\tan(dx+c)+4Ba\,b^{3}\tan(dx+c)+\frac{\left(-4A\,a^{3}b+4Aa\,b^{3}-B\,a^{4}+6B\,a^{2}b^{2}-B\,b^{4}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-A}{d}$
norman	$\frac{\left(-A\,a^{4}+6A\,a^{2}b^{2}-A\,b^{4}+4B\,a^{3}b-4Ba\,b^{3}\right)x\tan(dx+c)+\frac{b^{3}(Ab+4Ba)\tan(dx+c)^{2}}{d}-\frac{A\,a^{4}}{d}+\frac{B\,b^{4}\tan(dx+c)^{3}}{2d}}{\tan(dx+c)}+\frac{a^{3}(4Ab+}{}$
risch	$\frac{8iAa\,b^{3}c}{d}-\frac{2iB\,b^{4}c}{d}-iB\,a^{4}x+\frac{12iB\,a^{2}b^{2}c}{d}+\frac{a^{4}\ln\left(e^{2i(dx+c)}-1\right)B}{d}+\frac{\ln\left(e^{2i(dx+c)}+1\right)B\,b^{4}}{d}+6iB\,a^{2}b^{2}$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/2*((-4*A*a^3*b+4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*ln(sec(d*x+c)^2)+(8*A*
a^3*b+2*B*a^4)*ln(tan(d*x+c))+B*b^4*tan(d*x+c)^2+(2*A*b^4+8*B*a*b^3)*tan(d
*x+c)-2*A*cot(d*x+c)*a^4-2*d*x*(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^
3))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.10

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{Bb^4\tan(dx+c)^3-2\,Aa^4+\left(Ba^4+4\,Aa^3b\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)-\left(6\,Ba^2b^2+4\,Aab^3-Bb^4\right)}{}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/2*(B*b^4*tan(d*x + c)^3 - 2*A*a^4 + (B*a^4 + 4*A*a^3*b)*log(tan(d*x + c)
^2/(tan(d*x + c)^2 + 1))*tan(d*x + c) - (6*B*a^2*b^2 + 4*A*a*b^3 - B*b^4)*
log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c) + 2*(4*B*a*b^3 + A*b^4)*tan(d*x +
 c)^2 + (B*b^4 - 2*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*d
*x)*tan(d*x + c))/(d*tan(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.65

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\begin{cases} \tilde{\infty}Aa^4x \\ x(A+B\tan(c))\,(a+b\tan(c))^4\cot^2(c) \\ \tilde{\infty}Aa^4x \\ -Aa^4x-\frac{Aa^4}{d\tan(c+dx)}-\frac{2Aa^3b\log(\tan^2(c+dx)+1)}{d}+\frac{4Aa^3b\log(\tan(c+dx))}{d}+6Aa^2b^2x+\frac{2Aab^3\log(\tan^2(c+dx)+1)}{d}-A \end{cases}$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**4*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**4*cot(c)**2, Eq(d, 0)), (zoo*A*a**4*x, Eq(c, -d*x)), (-A*a**4*x - A
*a**4/(d*tan(c + d*x)) - 2*A*a**3*b*log(tan(c + d*x)**2 + 1)/d + 4*A*a**3*
b*log(tan(c + d*x))/d + 6*A*a**2*b**2*x + 2*A*a*b**3*log(tan(c + d*x)**2 +
 1)/d - A*b**4*x + A*b**4*tan(c + d*x)/d - B*a**4*log(tan(c + d*x)**2 + 1)
/(2*d) + B*a**4*log(tan(c + d*x))/d + 4*B*a**3*b*x + 3*B*a**2*b**2*log(tan
(c + d*x)**2 + 1)/d - 4*B*a*b**3*x + 4*B*a*b**3*tan(c + d*x)/d - B*b**4*lo
g(tan(c + d*x)**2 + 1)/(2*d) + B*b**4*tan(c + d*x)**2/(2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.94

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{Bb^4\tan(dx+c)^2 - \frac{2Aa^4}{\tan(dx+c)} - 2(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4)(dx+c) - (Ba^4 + 4Aa^3b -}{}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/2*(B*b^4*tan(d*x + c)^2 - 2*A*a^4/tan(d*x + c) - 2*(A*a^4 - 4*B*a^3*b -
6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) - (B*a^4 + 4*A*a^3*b - 6*B*a^2*
b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1) + 2*(B*a^4 + 4*A*a^3*b)*l
og(tan(d*x + c)) + 2*(4*B*a*b^3 + A*b^4)*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{Aa^4}{d\tan(dx+c)} - \frac{(Aa^4 - 4Ba^3b - 6Aa^2b^2 + 4Bab^3 + Ab^4)(dx+c)}{d}$$
$$- \frac{(Ba^4 + 4Aa^3b - 6Ba^2b^2 - 4Aab^3 + Bb^4)\log\left(\tan(dx+c)^2 + 1\right)}{2d}$$
$$+ \frac{(Ba^4 + 4Aa^3b)\log(|\tan(dx+c)|)}{d}$$
$$+ \frac{Bb^4d\tan(dx+c)^2 + 8Bab^3d\tan(dx+c) + 2Ab^4d\tan(dx+c)}{2d^2}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-A*a^4/(d*tan(d*x + c)) - (A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A
*b^4)*(d*x + c)/d - 1/2*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b
^4)*log(tan(d*x + c)^2 + 1)/d + (B*a^4 + 4*A*a^3*b)*log(abs(tan(d*x + c)))
/d + 1/2*(B*b^4*d*tan(d*x + c)^2 + 8*B*a*b^3*d*tan(d*x + c) + 2*A*b^4*d*ta
n(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx &= \frac{\tan(c+d\,x)\,\left(A\,b^4+4\,B\,a\,b^3\right)}{d}+\frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,a^4+4\,A\,b\,a^3\right)}{d}\\&+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)\,{\left(-b+a\,1\mathrm{i}\right)}^4}{2\,d}\\&-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)\,{\left(b+a\,1\mathrm{i}\right)}^4}{2\,d}\\&-\frac{A\,a^4\cot(c+d\,x)}{d}+\frac{B\,b^4\tan(c+d\,x)^2}{2\,d}\end{aligned}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
(tan(c + d*x)*(A*b^4 + 4*B*a*b^3))/d + (log(tan(c + d*x))*(B*a^4 + 4*A*a^3
*b))/d + (log(tan(c + d*x) - 1i)*(A*1i - B)*(a*1i - b)^4)/(2*d) - (log(tan
(c + d*x) + 1i)*(A*1i + B)*(a*1i + b)^4)/(2*d) - (A*a^4*cot(c + d*x))/d +
(B*b^4*tan(c + d*x)^2)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 595, normalized size of antiderivative = 3.40

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
( - 2*cos(c + d*x)*sin(c + d*x)**2*a**5 - 10*cos(c + d*x)*sin(c + d*x)**2*
a*b**4 + 2*cos(c + d*x)*a**5 - 10*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x
)**3*a**4*b + 20*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*a**2*b**3 -
2*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*b**5 + 10*log(tan((c + d*x)
/2)**2 + 1)*sin(c + d*x)*a**4*b - 20*log(tan((c + d*x)/2)**2 + 1)*sin(c +
d*x)*a**2*b**3 + 2*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*b**5 - 20*log
(tan((c + d*x)/2) - 1)*sin(c + d*x)**3*a**2*b**3 + 2*log(tan((c + d*x)/2)
- 1)*sin(c + d*x)**3*b**5 + 20*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*a**2
*b**3 - 2*log(tan((c + d*x)/2) - 1)*sin(c + d*x)*b**5 - 20*log(tan((c + d*
x)/2) + 1)*sin(c + d*x)**3*a**2*b**3 + 2*log(tan((c + d*x)/2) + 1)*sin(c +
 d*x)**3*b**5 + 20*log(tan((c + d*x)/2) + 1)*sin(c + d*x)*a**2*b**3 - 2*lo
g(tan((c + d*x)/2) + 1)*sin(c + d*x)*b**5 + 10*log(tan((c + d*x)/2))*sin(c
 + d*x)**3*a**4*b - 10*log(tan((c + d*x)/2))*sin(c + d*x)*a**4*b - 2*sin(c
 + d*x)**3*a**5*d*x + 20*sin(c + d*x)**3*a**3*b**2*d*x - 10*sin(c + d*x)**
3*a*b**4*d*x - sin(c + d*x)**3*b**5 + 2*sin(c + d*x)*a**5*d*x - 20*sin(c +
 d*x)*a**3*b**2*d*x + 10*sin(c + d*x)*a*b**4*d*x)/(2*sin(c + d*x)*d*(sin(c
 + d*x)**2 - 1))
```

3.262 $\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 186

$$\begin{aligned}
&\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx \\
&= -\left(\left(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B\right)x\right)-\frac{b^3(Ab+4aB)\log(\cos(c+dx))}{d} \\
&\quad -\frac{a^2(a^2A-6Ab^2-4abB)\log(\sin(c+dx))}{d}+\frac{b^2(3aAb+a^2B+b^2B)\tan(c+dx)}{d} \\
&\quad -\frac{a(5Ab+2aB)\cot(c+dx)(a+b\tan(c+dx))^2}{2d} \\
&\quad -\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}
\end{aligned}$$

output
```
-(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*x-b^3*(A*b+4*B*a)*ln(cos(d*
x+c))/d-a^2*(A*a^2-6*A*b^2-4*B*a*b)*ln(sin(d*x+c))/d+b^2*(3*A*a*b+B*a^2+B*
b^2)*tan(d*x+c)/d-1/2*a*(5*A*b+2*B*a)*cot(d*x+c)*(a+b*tan(d*x+c))^2/d-1/2*
a*A*cot(d*x+c)^2*(a+b*tan(d*x+c))^3/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.75

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{-2a^3(4Ab+aB)\cot(c+dx)-a^4A\cot^2(c+dx)+(a+ib)^4(A+iB)\log(i-\tan(c+dx))-2a^2(a^2A-}{2d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a^3*(4*A*b + a*B)*Cot[c + d*x] - a^4*A*Cot[c + d*x]^2 + (a + I*b)^4*(A
 + I*B)*Log[I - Tan[c + d*x]] - 2*a^2*(a^2*A - 6*A*b^2 - 4*a*b*B)*Log[Tan[
c + d*x]] + (a - I*b)^4*(A - I*B)*Log[I + Tan[c + d*x]] + 2*b^4*B*Tan[c +
d*x])/(2*d)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4088, 3042, 4128, 27, 3042, 4120, 25, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{2}\int \cot^2(c+dx)(a+b\tan(c+dx))^2\left(b(aA+2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+2aB)\right)dx-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\int \frac{(a+b\tan(c+dx))^2\left(b(aA+2bB)\tan(c+dx)^2-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+2aB)\right)}{\tan(c+dx)^2}dx-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\Big\downarrow$ 4128

$$\frac{1}{2}\left(\int -2\cot(c+dx)(a+b\tan(c+dx))\left(-b\left(Ba^2+3Aba+b^2B\right)\tan^2(c+dx)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-2\int \cot(c+dx)(a+b\tan(c+dx))\left(-b\left(Ba^2+3Aba+b^2B\right)\tan^2(c+dx)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\left(-2\int \frac{(a+b\tan(c+dx))\left(-b\left(Ba^2+3Aba+b^2B\right)\tan(c+dx)^2+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+d\right.}{\tan(c+dx)}\right.$$
$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\Big\downarrow$ 4120

$$\frac{1}{2}\left(-2\left(-\int -\cot(c+dx)\left(-\left((Ab+4aB)\tan^2(c+dx)b^3\right)+a^2\left(Aa^2-4bBa-6Ab^2\right)+\left(Ba^4+4Aba^3-6b^2B\right.\right.\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(-2\left(\int \cot(c+dx)\left(-\left((Ab+4aB)\tan^2(c+dx)b^3\right)+a^2\left(Aa^2-4bBa-6Ab^2\right)+\left(Ba^4+4Aba^3-6b^2Ba^2-\right.\right.\right.\right.$$

$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(-2\left(\int \frac{-\left((Ab+4aB)\tan(c+dx)^2b^3\right)+a^2\left(Aa^2-4bBa-6Ab^2\right)+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right.}{\tan(c+dx)}\right.\right.$$

$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\big\downarrow$ 4107

$$\frac{1}{2}\left(-2\left(a^2\left(a^2A-4abB-6Ab^2\right)\int \cot(c+dx)dx-\left(b^3(4aB+Ab)\int \tan(c+dx)dx\right)-\frac{b^2\left(a^2B+3aAb+b^2B\right.}{d}\right.\right.$$

$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(-2\left(a^2\left(a^2A-4abB-6Ab^2\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\left(b^3(4aB+Ab)\int \tan(c+dx)dx\right)-\frac{b^2\left(a^2B+3a\right.}{}\right.\right.$$

$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(-2\left(-a^2\left(a^2A-4abB-6Ab^2\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\left(b^3(4aB+Ab)\int \tan(c+dx)dx\right)-\frac{b^2\left(a^2B\right.}{}\right.\right.$$

$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

$\big\downarrow$ 3956

$$\frac{1}{2}\left(-2\left(-\frac{b^2\left(a^2B+3aAb+b^2B\right)\tan(c+dx)}{d}+\frac{a^2\left(a^2A-4abB-6Ab^2\right)\log(-\sin(c+dx))}{d}+x\left(a^4B+4a^3Ab\right.\right.\right.$$

$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^3}{2d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(a*A*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^3)/d + (-((a*(5*A*b + 2*a*B)
*Cot[c + d*x]*(a + b*Tan[c + d*x])^2)/d) - 2*((4*a^3*A*b - 4*a*A*b^3 + a^4
*B - 6*a^2*b^2*B + b^4*B)*x + (b^3*(A*b + 4*a*B)*Log[Cos[c + d*x]])/d + (a
^2*(a^2*A - 6*A*b^2 - 4*a*b*B)*Log[-Sin[c + d*x]])/d - (b^2*(3*a*A*b + a^2
*B + b^2*B)*Tan[c + d*x])/d))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{\left(A\,a^{4}-6A\,a^{2}b^{2}+A\,b^{4}-4B\,a^{3}b+4Ba\,b^{3}\right)\ln\left(\sec(dx+c)^{2}\right)+\left(-2A\,a^{4}+12A\,a^{2}b^{2}+8B\,a^{3}b\right)\ln(\tan(dx+c))-A\cot(dx+c)}{2d}$
derivativedivides	$\frac{B\tan(dx+c)b^{4}+\frac{\left(A\,a^{4}-6A\,a^{2}b^{2}+A\,b^{4}-4B\,a^{3}b+4Ba\,b^{3}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(-4A\,a^{3}b+4Aa\,b^{3}-B\,a^{4}+6B\,a^{2}b^{2}-B\,b^{4}\right)\text{ar}}{d}$
default	$\frac{B\tan(dx+c)b^{4}+\frac{\left(A\,a^{4}-6A\,a^{2}b^{2}+A\,b^{4}-4B\,a^{3}b+4Ba\,b^{3}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(-4A\,a^{3}b+4Aa\,b^{3}-B\,a^{4}+6B\,a^{2}b^{2}-B\,b^{4}\right)\text{ar}}{d}$
norman	$\frac{\left(-4A\,a^{3}b+4Aa\,b^{3}-B\,a^{4}+6B\,a^{2}b^{2}-B\,b^{4}\right)x\tan(dx+c)^{2}+\frac{B\,b^{4}\tan(dx+c)^{3}}{d}-\frac{A\,a^{4}}{2d}-\frac{a^{3}(4Ab+Ba)\tan(dx+c)}{d}}{\tan(dx+c)^{2}}+\frac{(A\,a^{4}-6A}{}$
risch	$-\frac{A\,a^{4}\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}-\frac{4\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)Ba\,b^{3}}{d}-\frac{8iB\,a^{3}bc}{d}-6iA\,a^{2}b^{2}x+4iBa\,b^{3}x-4iB\,a^{3}bx$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/2*((A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*ln(sec(d*x+c)^2)+(-2*A*
a^4+12*A*a^2*b^2+8*B*a^3*b)*ln(tan(d*x+c))-A*cot(d*x+c)^2*a^4+(-8*A*a^3*b-
2*B*a^4)*cot(d*x+c)+2*B*tan(d*x+c)*b^4-8*(A*a^3*b-A*a*b^3+1/4*B*a^4-3/2*B*
a^2*b^2+1/4*B*b^4)*x*d)/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.07

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\frac{2\,Bb^{4}\tan\left(dx+c\right)^{3}-Aa^{4}-\left(Aa^{4}-4\,Ba^{3}b-6\,Aa^{2}b^{2}\right)\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right)\tan\left(dx+c\right)^{2}-\left(4\,Bab^{3}+Ab^{4}\right.}{}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/2*(2*B*b^4*tan(d*x + c)^3 - A*a^4 - (A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*lo
g(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 - (4*B*a*b^3 + A*b^4
)*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 - (A*a^4 + 2*(B*a^4 + 4*A*a^3
*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*d*x)*tan(d*x + c)^2 - 2*(B*a^4 + 4*A
*a^3*b)*tan(d*x + c))/(d*tan(d*x + c)^2)
```

Sympy [A] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.66

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\begin{cases} \tilde{\infty}Aa^4x \\ x(A+B\tan(c))(a+b\tan(c))^4\cot^3(c) \\ \tilde{\infty}Aa^4x \\ \frac{Aa^4\log(\tan^2(c+dx)+1)}{2d} - \frac{Aa^4\log(\tan(c+dx))}{d} - \frac{Aa^4}{2d\tan^2(c+dx)} - 4Aa^3bx - \frac{4Aa^3b}{d\tan(c+dx)} - \frac{3Aa^2b^2\log(\tan^2(c+dx)+1)}{d} + \end{cases}$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**4*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**4*cot(c)**3, Eq(d, 0)), (zoo*A*a**4*x, Eq(c, -d*x)), (A*a**4*log(ta
n(c + d*x)**2 + 1)/(2*d) - A*a**4*log(tan(c + d*x))/d - A*a**4/(2*d*tan(c
+ d*x)**2) - 4*A*a**3*b*x - 4*A*a**3*b/(d*tan(c + d*x)) - 3*A*a**2*b**2*lo
g(tan(c + d*x)**2 + 1)/d + 6*A*a**2*b**2*log(tan(c + d*x))/d + 4*A*a*b**3*
x + A*b**4*log(tan(c + d*x)**2 + 1)/(2*d) - B*a**4*x - B*a**4/(d*tan(c + d
*x)) - 2*B*a**3*b*log(tan(c + d*x)**2 + 1)/d + 4*B*a**3*b*log(tan(c + d*x)
)/d + 6*B*a**2*b**2*x + 2*B*a*b**3*log(tan(c + d*x)**2 + 1)/d - B*b**4*x +
 B*b**4*tan(c + d*x)/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.93

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{2\,Bb^4\tan(dx+c) - 2\,(Ba^4 + 4\,Aa^3b - 6\,Ba^2b^2 - 4\,Aab^3 + Bb^4)(dx+c) + (Aa^4 - 4\,Ba^3b - 6\,Aa^2b^2 +}{}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/2*(2*B*b^4*tan(d*x + c) - 2*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3
 + B*b^4)*(d*x + c) + (A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4
)*log(tan(d*x + c)^2 + 1) - 2*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*log(tan(d*
x + c)) - (A*a^4 + 2*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/tan(d*x + c)^2)/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.99

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{Bb^4\tan(dx+c)}{d} - \frac{(Ba^4 + 4\,Aa^3b - 6\,Ba^2b^2 - 4\,Aab^3 + Bb^4)(dx+c)}{d}$$
$$+ \frac{(Aa^4 - 4\,Ba^3b - 6\,Aa^2b^2 + 4\,Bab^3 + Ab^4)\log\left(\tan(dx+c)^2 + 1\right)}{2\,d}$$
$$- \frac{(Aa^4 - 4\,Ba^3b - 6\,Aa^2b^2)\log(|\tan(dx+c)|)}{d}$$
$$- \frac{Aa^4 + 2\,(Ba^4 + 4\,Aa^3b)\tan(dx+c)}{2\,d\tan(dx+c)^2}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
B*b^4*tan(d*x + c)/d - (B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^
4)*(d*x + c)/d + 1/2*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)
*log(tan(d*x + c)^2 + 1)/d - (A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*log(abs(tan
(d*x + c)))/d - 1/2*(A*a^4 + 2*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/(d*tan(d*
x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.80

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{\ln(\tan(c+dx))\,(-A\,a^4+4\,B\,a^3\,b+6\,A\,a^2\,b^2)}{d} - \frac{\cot(c+dx)^2\left(\tan(c+dx)\,(B\,a^4+4\,A\,b\,a^3)+\frac{A\,a^4}{2}\right)}{d} + \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(A-B\,1\mathrm{i})\,(b+a\,1\mathrm{i})^4}{2\,d} + \frac{B\,b^4\tan(c+dx)}{d} + \frac{\ln(\tan(c+dx)-\mathrm{i})\,(A+B\,1\mathrm{i})\,(-b+a\,1\mathrm{i})^4}{2\,d}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
(log(tan(c + d*x))*(6*A*a^2*b^2 - A*a^4 + 4*B*a^3*b))/d - (cot(c + d*x)^2*
(tan(c + d*x)*(B*a^4 + 4*A*a^3*b) + (A*a^4)/2))/d + (log(tan(c + d*x) + 1i
)*(A - B*1i)*(a*1i + b)^4)/(2*d) + (B*b^4*tan(c + d*x))/d + (log(tan(c + d
*x) - 1i)*(A + B*1i)*(a*1i - b)^4)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.07

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{8\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)^2a^5 - 80\cos(dx+c)\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+$$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(8*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**5 - 80*cos
(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**3*b**2 + 40*cos(
c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a*b**4 - 40*cos(c +
d*x)*log(tan((c + d*x)/2) - 1)*sin(c + d*x)**2*a*b**4 - 40*cos(c + d*x)*lo
g(tan((c + d*x)/2) + 1)*sin(c + d*x)**2*a*b**4 - 8*cos(c + d*x)*log(tan((c
 + d*x)/2))*sin(c + d*x)**2*a**5 + 80*cos(c + d*x)*log(tan((c + d*x)/2))*s
in(c + d*x)**2*a**3*b**2 + cos(c + d*x)*sin(c + d*x)**2*a**5 - 40*cos(c +
d*x)*sin(c + d*x)**2*a**4*b*d*x + 80*cos(c + d*x)*sin(c + d*x)**2*a**2*b**
3*d*x - 8*cos(c + d*x)*sin(c + d*x)**2*b**5*d*x - 4*cos(c + d*x)*a**5 + 40
*sin(c + d*x)**3*a**4*b + 8*sin(c + d*x)**3*b**5 - 40*sin(c + d*x)*a**4*b)
/(8*cos(c + d*x)*sin(c + d*x)**2*d)
```

3.263 $\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 187

$$\begin{aligned}\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx &= \left(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B\right)x+\frac{a^2(a^2A-3Ab^2-3abB)\cot(c+dx)}{d}\\ &\quad-\frac{b^4B\log(\cos(c+dx))}{d}-\frac{a(4a^2Ab-4Ab^3+a^3B-6ab^2B)\log(\sin(c+dx))}{d}\\ &\quad-\frac{a(2Ab+aB)\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}\\ &\quad-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}\end{aligned}$$

output
```
(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*x+a^2*(A*a^2-3*A*b^2-3*B*a*b
)*cot(d*x+c)/d-b^4*B*ln(cos(d*x+c))/d-a*(4*A*a^2*b-4*A*b^3+B*a^3-6*B*a*b^2
)*ln(sin(d*x+c))/d-1/2*a*(2*A*b+B*a)*cot(d*x+c)^2*(a+b*tan(d*x+c))^2/d-1/3
*a*A*cot(d*x+c)^3*(a+b*tan(d*x+c))^3/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.89

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{6a^2(a^2A-6Ab^2-4abB)\cot(c+dx)-3a^3(4Ab+aB)\cot^2(c+dx)-2a^4A\cot^3(c+dx)+3(a+ib)^4(\cdots}{\cdots}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(6*a^2*(a^2*A - 6*A*b^2 - 4*a*b*B)*Cot[c + d*x] - 3*a^3*(4*A*b + a*B)*Cot[
c + d*x]^2 - 2*a^4*A*Cot[c + d*x]^3 + 3*(a + I*b)^4*((-I)*A + B)*Log[I - T
an[c + d*x]] - 6*a*(4*a^2*A*b - 4*A*b^3 + a^3*B - 6*a*b^2*B)*Log[Tan[c + d
*x]] + 3*(a - I*b)^4*(I*A + B)*Log[I + Tan[c + d*x]])/(6*d)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4118, 3042, 4107, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{3}\int 3\cot^3(c+dx)(a+b\tan(c+dx))^2\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)dx-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 27

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^2\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)dx-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3042

$$\int \frac{(a+b\tan(c+dx))^2\left(b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)}{\tan(c+dx)^3}dx-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 4128

$$\frac{1}{2}\int -2\cot^2(c+dx)(a+b\tan(c+dx))\left(-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx-\frac{a(aB+2Ab)\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 27

$$-\int \cot^2(c+dx)(a+b\tan(c+dx))\left(-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx-\frac{a(aB+2Ab)\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3042

$$-\int \frac{(a+b\tan(c+dx))\left(-B\tan(c+dx)^2b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c-\right.}{\tan(c+dx)^2}\; \frac{a(aB+2Ab)\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 4118

$$-\int \cot(c+dx)\left(-B\tan^2(c+dx)b^4+a\left(Ba^3+4Aba^2-6b^2Ba-4Ab^3\right)-\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\tan(c+\right.$$

$$\frac{a^2\left(a^2A-3abB-3Ab^2\right)\cot(c+dx)}{d}-\frac{a(aB+2Ab)\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3042

$$-\int \frac{-B\tan(c+dx)^2b^4+a\left(Ba^3+4Aba^2-6b^2Ba-4Ab^3\right)-\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\tan(c-}{\tan(c+dx)}$$

$$\frac{a^2\left(a^2A-3abB-3Ab^2\right)\cot(c+dx)}{d}-\frac{a(aB+2Ab)\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 4107

$$-a\left(a^3B+4a^2Ab-6ab^2B-4Ab^3\right)\int\cot(c+dx)dx+b^4B\int\tan(c+dx)dx+\frac{a^2\left(a^2A-3abB-3Ab^2\right)\cot(c+dx)}{d}+x\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)-\frac{a(aB+2Ab)\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3042

$$-a\left(a^3B+4a^2Ab-6ab^2B-4Ab^3\right)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx+b^4B\int\tan(c+dx)dx+\frac{a^2\left(a^2A-3abB-3Ab^2\right)\cot(c+dx)}{d}+x\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)-\frac{a(aB+2Ab)\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 25

$$a\left(a^3B+4a^2Ab-6ab^2B-4Ab^3\right)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+b^4B\int\tan(c+dx)dx+\frac{a^2\left(a^2A-3abB-3Ab^2\right)\cot(c+dx)}{d}+x\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)-\frac{a(aB+2Ab)\cot^2(c+dx)(a+b\tan(c+dx))^2}{2d}-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^3}{3d}$$

$\Big\downarrow$ 3956

$$\frac{a^2(a^2A - 3abB - 3Ab^2)\cot(c+dx)}{d} - \frac{a(a^3B + 4a^2Ab - 6ab^2B - 4Ab^3)\log(-\sin(c+dx))}{d} + x(a^4A - 4a^3bB - 6a^2Ab^2 + 4ab^3B + Ab^4) - \frac{a(aB + 2Ab)\cot^2(c+dx)(a + b\tan(c+dx))^2}{2d} - \frac{aA\cot^3(c+dx)(a + b\tan(c+dx))^3}{3d} - \frac{b^4B\log(\cos(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*x + (a^2*(a^2*A - 3*
A*b^2 - 3*a*b*B)*Cot[c + d*x])/d - (b^4*B*Log[Cos[c + d*x]])/d - (a*(4*a^2
*A*b - 4*A*b^3 + a^3*B - 6*a*b^2*B)*Log[-Sin[c + d*x]])/d - (a*(2*A*b + a*
B)*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^2)/(2*d) - (a*A*Cot[c + d*x]^3*(a +
 b*Tan[c + d*x])^3)/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4107
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[B*x, x] + (Simp[A   Int[1/Tan[
e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x], x]) /; FreeQ[{e, f, A, B,
 C}, x] && NeQ[A, C]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{(4Aa^3b-4Aab^3+Ba^4-6Ba^2b^2+Bb^4)\ln(1+\tan(dx+c)^2)}{2}+(Aa^4-6Aa^2b^2+Ab^4-4Ba^3b+4Bab^3)\arctan(\tan(dx+c))-}{d}$
default	$\frac{\frac{(4Aa^3b-4Aab^3+Ba^4-6Ba^2b^2+Bb^4)\ln(1+\tan(dx+c)^2)}{2}+(Aa^4-6Aa^2b^2+Ab^4-4Ba^3b+4Bab^3)\arctan(\tan(dx+c))-}{d}$
parallelrisch	$3(4Aa^3b-4Aab^3+Ba^4-6Ba^2b^2+Bb^4)\ln(\sec(dx+c)^2)+6(-4Aa^3b+4Aab^3-Ba^4+6Ba^2b^2)\ln(\tan(dx+c))-2A$
norman	$\frac{(Aa^4-6Aa^2b^2+Ab^4-4Ba^3b+4Bab^3)x\tan(dx+c)^3+\frac{a^2(Aa^2-6Ab^2-4Bab)\tan(dx+c)^2}{d}-\frac{Aa^4}{3d}-\frac{a^3(4Ab+Ba)\tan(dx+}{2d}}{\tan(dx+c)^3}$
risch	$\frac{8iAa^3bc}{d}+iBa^4x-6iBa^2b^2x-\frac{8iAab^3c}{d}-4iAab^3x-\frac{a^4\ln(e^{2i(dx+c)}-1)B}{d}-\frac{\ln(e^{2i(dx+c)}+1)B}{d}$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/2*(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*ln(1+tan(d*x+c)^2)+
(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*arctan(tan(d*x+c))-1/3*A*a^4
/tan(d*x+c)^3-1/2*a^3*(4*A*b+B*a)/tan(d*x+c)^2-a*(4*A*a^2*b-4*A*b^3+B*a^3-
6*B*a*b^2)*ln(tan(d*x+c))+a^2*(A*a^2-6*A*b^2-4*B*a*b)/tan(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.19

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$

$$-\frac{3\,Bb^4\log\left(\frac{1}{\tan(dx+c)^2+1}\right)\tan(dx+c)^3+2\,Aa^4+3\left(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/6*(3*B*b^4*log(1/(tan(d*x + c)^2 + 1))*tan(d*x + c)^3 + 2*A*a^4 + 3*(B*
a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)*log(tan(d*x + c)^2/(tan(d*x + c
)^2 + 1))*tan(d*x + c)^3 + 3*(B*a^4 + 4*A*a^3*b - 2*(A*a^4 - 4*B*a^3*b - 6
*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*d*x)*tan(d*x + c)^3 - 6*(A*a^4 - 4*B*a^3*b
 - 6*A*a^2*b^2)*tan(d*x + c)^2 + 3*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/(d*ta
n(d*x + c)^3)
```

Sympy [A] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.97

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \tilde{\infty} A a^4 x \\ x(A+B\tan{(c)})\,(a+b\tan{(c)})^4\cot^4{(c)} \\ \tilde{\infty} A a^4 x \\ Aa^4x + \frac{Aa^4}{d\tan{(c+dx)}} - \frac{Aa^4}{3d\tan^3{(c+dx)}} + \frac{2Aa^3b\log{(\tan^2{(c+dx)}+1)}}{d} - \frac{4Aa^3b\log{(\tan{(c+dx)})}}{d} - \frac{2Aa^3b}{d\tan^2{(c+dx)}} - 6Aa^2b^2x - \end{cases}$$

input
```
integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**4*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**4*cot(c)**4, Eq(d, 0)), (zoo*A*a**4*x, Eq(c, -d*x)), (A*a**4*x + A*
a**4/(d*tan(c + d*x)) - A*a**4/(3*d*tan(c + d*x)**3) + 2*A*a**3*b*log(tan(
c + d*x)**2 + 1)/d - 4*A*a**3*b*log(tan(c + d*x))/d - 2*A*a**3*b/(d*tan(c
+ d*x)**2) - 6*A*a**2*b**2*x - 6*A*a**2*b**2/(d*tan(c + d*x)) - 2*A*a*b**3
*log(tan(c + d*x)**2 + 1)/d + 4*A*a*b**3*log(tan(c + d*x))/d + A*b**4*x +
B*a**4*log(tan(c + d*x)**2 + 1)/(2*d) - B*a**4*log(tan(c + d*x))/d - B*a**
4/(2*d*tan(c + d*x)**2) - 4*B*a**3*b*x - 4*B*a**3*b/(d*tan(c + d*x)) - 3*B
*a**2*b**2*log(tan(c + d*x)**2 + 1)/d + 6*B*a**2*b**2*log(tan(c + d*x))/d
+ 4*B*a*b**3*x + B*b**4*log(tan(c + d*x)**2 + 1)/(2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.08

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{6\,(Aa^4 - 4\,Ba^3b - 6\,Aa^2b^2 + 4\,Bab^3 + Ab^4)(dx+c) + 3\,(Ba^4 + 4\,Aa^3b - 6\,Ba^2b^2 - 4\,Aab^3 + Bb^4)\log\left(\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/6*(6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c) + 3
*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2
+ 1) - 6*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)*log(tan(d*x + c)) -
 (2*A*a^4 - 6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan(d*x + c)^2 + 3*(B*a^4
+ 4*A*a^3*b)*tan(d*x + c))/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{(Aa^4 - 4\,Ba^3b - 6\,Aa^2b^2 + 4\,Bab^3 + Ab^4)(dx+c)}{d} + \frac{(Ba^4 + 4\,Aa^3b - 6\,Ba^2b^2 - 4\,Aab^3 + Bb^4)\log\left(\tan(dx+c)^2 + 1\right)}{2\,d} - \frac{(Ba^4 + 4\,Aa^3b - 6\,Ba^2b^2 - 4\,Aab^3)\log\left(|\tan(dx+c)|\right)}{d} - \frac{2\,Aa^4 - 6\,(Aa^4 - 4\,Ba^3b - 6\,Aa^2b^2)\tan(dx+c)^2 + 3\,(Ba^4 + 4\,Aa^3b)\tan(dx+c)}{6\,d\tan(dx+c)^3}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c)/d + 1/2*(B
*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1
)/d - (B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)*log(abs(tan(d*x + c)))
/d - 1/6*(2*A*a^4 - 6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan(d*x + c)^2 + 3
*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/(d*tan(d*x + c)^3)
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,a^4+4\,A\,a^3\,b-6\,B\,a^2\,b^2-4\,A\,a\,b^3\right)}{d}$$
$$-\frac{\cot(c+d\,x)^3\,\left(\tan(c+d\,x)\,\left(\frac{B\,a^4}{2}+2\,A\,b\,a^3\right)+\frac{A\,a^4}{3}+\tan(c+d\,x)^2\,\left(-A\,a^4+4\,B\,a^3\,b+6\,A\,a^2\,b^2\right)\right)}{d}$$
$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)\,{\left(-b+a\,1\mathrm{i}\right)}^4}{2\,d}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)\,{\left(b+a\,1\mathrm{i}\right)}^4}{2\,d}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
(log(tan(c + d*x) + 1i)*(A*1i + B)*(a*1i + b)^4)/(2*d) - (cot(c + d*x)^3*(
tan(c + d*x)*((B*a^4)/2 + 2*A*a^3*b) + (A*a^4)/3 + tan(c + d*x)^2*(6*A*a^2
*b^2 - A*a^4 + 4*B*a^3*b)))/d - (log(tan(c + d*x) - 1i)*(A*1i - B)*(a*1i -
 b)^4)/(2*d) - (log(tan(c + d*x))*(B*a^4 - 6*B*a^2*b^2 - 4*A*a*b^3 + 4*A*a
^3*b))/d
```

Reduce [F]

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^4\,(a+\tan(dx+c)\,b)^4\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

3.264 $\int \cot^5(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 225

$$\begin{aligned}
&\int \cot^5(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx\\
&=\left(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B\right)x\\
&\quad+\frac{a(24a^2Ab-19Ab^3+6a^3B-34ab^2B)\cot(c+dx)}{6d}\\
&\quad+\frac{a^2(6a^2A-13Ab^2-16abB)\cot^2(c+dx)}{12d}\\
&\quad+\frac{(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B)\log(\sin(c+dx))}{d}\\
&\quad-\frac{a(7Ab+4aB)\cot^3(c+dx)(a+b\tan(c+dx))^2}{12d}\\
&\quad-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}
\end{aligned}$$

output
```
(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*x+1/6*a*(24*A*a^2*b-19*A*b^3
+6*B*a^3-34*B*a*b^2)*cot(d*x+c)/d+1/12*a^2*(6*A*a^2-13*A*b^2-16*B*a*b)*cot
(d*x+c)^2/d+(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*ln(sin(d*x+c))/d
-1/12*a*(7*A*b+4*B*a)*cot(d*x+c)^3*(a+b*tan(d*x+c))^2/d-1/4*a*A*cot(d*x+c)
^4*(a+b*tan(d*x+c))^3/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.94

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{12a(4a^2Ab-4Ab^3+a^3B-6ab^2B)\cot(c+dx)+6a^2(a^2A-6Ab^2-4abB)\cot^2(c+dx)-4a^3(4Ab+a}{}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(12*a*(4*a^2*A*b - 4*A*b^3 + a^3*B - 6*a*b^2*B)*Cot[c + d*x] + 6*a^2*(a^2*
A - 6*A*b^2 - 4*a*b*B)*Cot[c + d*x]^2 - 4*a^3*(4*A*b + a*B)*Cot[c + d*x]^3
 - 3*a^4*A*Cot[c + d*x]^4 - 6*(a + I*b)^4*(A + I*B)*Log[I - Tan[c + d*x]]
+ 12*(a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*Log[Tan[c + d*x
]] - 6*(a - I*b)^4*(A - I*B)*Log[I + Tan[c + d*x]])/(12*d)
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4088, 3042, 4128, 27, 3042, 4118, 3042, 4111, 27, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^5}dx$$
$$\Big\downarrow \text{4088}$$

$$\frac{1}{4}\int \cot^4(c+dx)(a+b\tan(c+dx))^2\left(-b(aA-4bB)\tan^2(c+dx)-4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+4aB)\right)dx-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\int \frac{(a+b\tan(c+dx))^2\left(-b(aA-4bB)\tan(c+dx)^2-4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+4aB)\right)}{\tan(c+dx)^4}dx-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$\downarrow$ 4128

$$\frac{1}{4}\left(\frac{1}{3}\int -2\cot^3(c+dx)(a+b\tan(c+dx))\left(b\left(2Ba^2+5Aba-6b^2B\right)\tan^2(c+dx)+6\left(Ba^3+3Aba^2-3b^2Ba-\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{2}{3}\int \cot^3(c+dx)(a+b\tan(c+dx))\left(b\left(2Ba^2+5Aba-6b^2B\right)\tan^2(c+dx)+6\left(Ba^3+3Aba^2-3b^2Ba-\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(-\frac{2}{3}\int \frac{(a+b\tan(c+dx))\left(b\left(2Ba^2+5Aba-6b^2B\right)\tan(c+dx)^2+6\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+\right.}{\tan(c+dx)^3}\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$\downarrow$ 4118

$$\frac{1}{4}\left(-\frac{2}{3}\left(\int \cot^2(c+dx)\left(b^2\left(2Ba^2+5Aba-6b^2B\right)\tan^2(c+dx)-6\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\text{ta}\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(-\frac{2}{3}\left(\int \frac{b^2\left(2Ba^2+5Aba-6b^2B\right)\tan(c+dx)^2-6\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\tan(c+dx)+a}{\tan(c+dx)^2}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4111}$$

$$\frac{1}{4}\left(-\frac{2}{3}\left(\int -6\cot(c+dx)\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\right.\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{2}{3}\left(-6\int \cot(c+dx)\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\right.\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(-\frac{2}{3}\left(-6\int \frac{Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\tan(c+dx)}{\tan(c+dx)}d\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{4014}$$

$$\frac{1}{4}\left(-\frac{2}{3}\left(-6\left(\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)\int \cot(c+dx)dx+x\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+\right.\right.\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(-\frac{2}{3}\left(-6\left(\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+x\left(a^4B+4a^3Ab-6a^2b^2B-\right.\right.\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(-\frac{2}{3}\left(-6\left(x\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)-\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)\int\tan\left(\frac{1}{2}(\right.\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{4}\left(-\frac{2}{3}\left(-\frac{a^2\left(6a^2A-16abB-13Ab^2\right)\cot^2(c+dx)}{2d}-\frac{a\left(6a^3B+24a^2Ab-34ab^2B-19Ab^3\right)\cot(c+dx)}{d}-6\left(\right.\right.\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^3}{4d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-1/4*(a*A*Cot[c + d*x]^4*(a + b*Tan[c + d*x])^3)/d + ((-2*(-((a*(24*a^2*A*
b - 19*A*b^3 + 6*a^3*B - 34*a*b^2*B)*Cot[c + d*x])/d) - (a^2*(6*a^2*A - 13
*A*b^2 - 16*a*b*B)*Cot[c + d*x]^2)/(2*d) - 6*((4*a^3*A*b - 4*a*A*b^3 + a^4
*B - 6*a^2*b^2*B + b^4*B)*x + ((a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b*B +
4*a*b^3*B)*Log[-Sin[c + d*x]])/d)))/3 - (a*(7*A*b + 4*a*B)*Cot[c + d*x]^3*
(a + b*Tan[c + d*x])^2)/(3*d))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\frac{\left(-A a^4+6A a^2b^2-A b^4+4B a^3b-4Ba b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(4A a^3b-4Aa b^3+B a^4-6B a^2b^2+B b^4\right)\arctan(\tan(dx+c))}{}$
default	$\frac{\frac{\left(-A a^4+6A a^2b^2-A b^4+4B a^3b-4Ba b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(4A a^3b-4Aa b^3+B a^4-6B a^2b^2+B b^4\right)\arctan(\tan(dx+c))}{}$
parallelrisch	$\frac{6\left(-A a^4+6A a^2b^2-A b^4+4B a^3b-4Ba b^3\right)\ln\left(\sec(dx+c)^2\right)+12\left(A a^4-6A a^2b^2+A b^4-4B a^3b+4Ba b^3\right)\ln(\tan(dx+c}{}$
norman	$\frac{\left(4A a^3b-4Aa b^3+B a^4-6B a^2b^2+B b^4\right)x\tan(dx+c)^4+\frac{a\left(4A a^2b-4A b^3+B a^3-6Ba b^2\right)\tan(dx+c)^3}{d}-\frac{A a^4}{4d}+\frac{a^2\left(A a^2-6A\right.}{}}{\tan(dx+c)^4}$
risch	$-\frac{8iBa b^3c}{d}+\frac{8iB a^3bc}{d}-\frac{2iA b^4c}{d}+\frac{12iA a^2b^2c}{d}-\frac{4ia\left(-9Ba b^2+8A a^2b-6A b^3+9iAa b^2e^{6i(dx+c)}+9iAa b^2e^{2i}\right.}{}$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/2*(-A*a^4+6*A*a^2*b^2-A*b^4+4*B*a^3*b-4*B*a*b^3)*ln(1+tan(d*x+c)^2)
+(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*arctan(tan(d*x+c))+(A*a^4-6
*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*ln(tan(d*x+c))-1/4*A*a^4/tan(d*x+c)^
4-1/3*a^3*(4*A*b+B*a)/tan(d*x+c)^3+a*(4*A*a^2*b-4*A*b^3+B*a^3-6*B*a*b^2)/t
an(d*x+c)+1/2*a^2*(A*a^2-6*A*b^2-4*B*a*b)/tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.11

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{6\left(Aa^4-4Ba^3b-6Aa^2b^2+4Bab^3+Ab^4\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^4-3Aa^4+3\left(3Aa^4-8Ba^3b\right.}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/12*(6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(tan(d*x
+ c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4 - 3*A*a^4 + 3*(3*A*a^4 - 8*B*a
^3*b - 12*A*a^2*b^2 + 4*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b
^4)*d*x)*tan(d*x + c)^4 + 12*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)
*tan(d*x + c)^3 + 6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan(d*x + c)^2 - 4*(
B*a^4 + 4*A*a^3*b)*tan(d*x + c))/(d*tan(d*x + c)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(226) = 452.

Time = 3.91 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.04

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \tilde{\infty}Aa^4x \\ x(A+B\tan(c))\,(a+b\tan(c))^4\cot^5(c) \\ \tilde{\infty}Aa^4x \\ -\frac{Aa^4\log(\tan^2(c+dx)+1)}{2d}+\frac{Aa^4\log(\tan(c+dx))}{d}+\frac{Aa^4}{2d\tan^2(c+dx)}-\frac{Aa^4}{4d\tan^4(c+dx)}+4Aa^3bx+\frac{4Aa^3b}{d\tan(c+dx)}-\frac{4Aa^3b}{3d\tan^3(c+}\end{cases}$$

input
```
integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**4*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**4*cot(c)**5, Eq(d, 0)), (zoo*A*a**4*x, Eq(c, -d*x)), (-A*a**4*log(t
an(c + d*x)**2 + 1)/(2*d) + A*a**4*log(tan(c + d*x))/d + A*a**4/(2*d*tan(c
 + d*x)**2) - A*a**4/(4*d*tan(c + d*x)**4) + 4*A*a**3*b*x + 4*A*a**3*b/(d*
tan(c + d*x)) - 4*A*a**3*b/(3*d*tan(c + d*x)**3) + 3*A*a**2*b**2*log(tan(c
 + d*x)**2 + 1)/d - 6*A*a**2*b**2*log(tan(c + d*x))/d - 3*A*a**2*b**2/(d*t
an(c + d*x)**2) - 4*A*a*b**3*x - 4*A*a*b**3/(d*tan(c + d*x)) - A*b**4*log(
tan(c + d*x)**2 + 1)/(2*d) + A*b**4*log(tan(c + d*x))/d + B*a**4*x + B*a**
4/(d*tan(c + d*x)) - B*a**4/(3*d*tan(c + d*x)**3) + 2*B*a**3*b*log(tan(c +
 d*x)**2 + 1)/d - 4*B*a**3*b*log(tan(c + d*x))/d - 2*B*a**3*b/(d*tan(c + d
*x)**2) - 6*B*a**2*b**2*x - 6*B*a**2*b**2/(d*tan(c + d*x)) - 2*B*a*b**3*lo
g(tan(c + d*x)**2 + 1)/d + 4*B*a*b**3*log(tan(c + d*x))/d + B*b**4*x, True
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.09

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{12\,(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)(dx+c)-6\,(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)\log}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/12*(12*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*(d*x + c) -
 6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(tan(d*x + c)^
2 + 1) + 12*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(tan(
d*x + c)) - (3*A*a^4 - 12*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)*ta
n(d*x + c)^3 - 6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan(d*x + c)^2 + 4*(B*a
^4 + 4*A*a^3*b)*tan(d*x + c))/tan(d*x + c)^4)/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.12

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)(dx+c)}{d} - \frac{(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)\log\left(\tan(dx+c)^2+1\right)}{2\,d} + \frac{(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)\log\left(|\tan(dx+c)|\right)}{d} - \frac{3\,Aa^4-12\,(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3)\tan(dx+c)^3-6\,(Aa^4-4\,Ba^3b-6\,Aa^2b^2)\tan(dx+}{12\,d\tan(dx+c)^4}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*(d*x + c)/d - 1/2*(A
*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(tan(d*x + c)^2 + 1
)/d + (A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(abs(tan(d*
x + c)))/d - 1/12*(3*A*a^4 - 12*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b
^3)*tan(d*x + c)^3 - 6*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan(d*x + c)^2 +
4*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/(d*tan(d*x + c)^4)
```

Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.97

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{\ln(\tan(c+d\,x))\,\left(A\,a^4-4\,B\,a^3\,b-6\,A\,a^2\,b^2+4\,B\,a\,b^3+A\,b^4\right)}{d} - \frac{\cot(c+d\,x)^4\left(\tan(c+d\,x)\,\left(\frac{B\,a^4}{3}+\frac{4\,A\,b\,a^3}{3}\right)+\frac{A\,a^4}{4}-\tan(c+d\,x)^3\,\left(B\,a^4+4\,A\,a^3\,b-6\,B\,a^2\,b^2-4\,A\,a\right.\right.}{d} - \frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^4}{2\,d} - \frac{\ln(\tan(c+d\,x)-\mathrm{i})\,\left(A+B\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^4}{2\,d}$$

input
```
int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
(log(tan(c + d*x))*(A*a^4 + A*b^4 - 6*A*a^2*b^2 + 4*B*a*b^3 - 4*B*a^3*b))/
d - (cot(c + d*x)^4*(tan(c + d*x)*((B*a^4)/3 + (4*A*a^3*b)/3) + (A*a^4)/4
- tan(c + d*x)^3*(B*a^4 - 6*B*a^2*b^2 - 4*A*a*b^3 + 4*A*a^3*b) + tan(c + d
*x)^2*(3*A*a^2*b^2 - (A*a^4)/2 + 2*B*a^3*b)))/d - (log(tan(c + d*x) + 1i)*
(A - B*1i)*(a*1i + b)^4)/(2*d) - (log(tan(c + d*x) - 1i)*(A + B*1i)*(a*1i
- b)^4)/(2*d)
```

Reduce [F]

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^5(a+\tan(dx+c)\,b)^4(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

3.265 $\int \cot^6(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 273

$$
\begin{aligned}
&\int \cot^6(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx \\
&= -\left(\left(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B\right)x\right) \\
&\quad -\frac{(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B)\cot(c+dx)}{d} \\
&\quad +\frac{a(40a^2Ab-28Ab^3+10a^3B-55ab^2B)\cot^2(c+dx)}{20d} \\
&\quad +\frac{a^2(10a^2A-18Ab^2-25abB)\cot^3(c+dx)}{30d} \\
&\quad +\frac{(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B)\log(\sin(c+dx))}{d} \\
&\quad -\frac{a(8Ab+5aB)\cot^4(c+dx)(a+b\tan(c+dx))^2}{20d} \\
&\quad -\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}
\end{aligned}
$$

output
```
-(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*x-(A*a^4-6*A*a^2*b^2+A*b^4-
4*B*a^3*b+4*B*a*b^3)*cot(d*x+c)/d+1/20*a*(40*A*a^2*b-28*A*b^3+10*B*a^3-55*
B*a*b^2)*cot(d*x+c)^2/d+1/30*a^2*(10*A*a^2-18*A*b^2-25*B*a*b)*cot(d*x+c)^3
/d+(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*ln(sin(d*x+c))/d-1/20*a*(
8*A*b+5*B*a)*cot(d*x+c)^4*(a+b*tan(d*x+c))^2/d-1/5*a*A*cot(d*x+c)^5*(a+b*t
an(d*x+c))^3/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.94

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{-60(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B)\cot(c+dx)+30a(4a^2Ab-4Ab^3+a^3B-6ab^2B)\cot^2(c+}{}$$

input
```
Integrate[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(-60*(a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*Cot[c + d*x] +
30*a*(4*a^2*A*b - 4*A*b^3 + a^3*B - 6*a*b^2*B)*Cot[c + d*x]^2 + 20*a^2*(a^
2*A - 6*A*b^2 - 4*a*b*B)*Cot[c + d*x]^3 - 15*a^3*(4*A*b + a*B)*Cot[c + d*x
]^4 - 12*a^4*A*Cot[c + d*x]^5 + (30*I)*(a + I*b)^4*(A + I*B)*Log[I - Tan[c
 + d*x]] + 60*(4*a^3*A*b - 4*a*A*b^3 + a^4*B - 6*a^2*b^2*B + b^4*B)*Log[Ta
n[c + d*x]] - 30*(a - I*b)^4*(I*A + B)*Log[I + Tan[c + d*x]])/(60*d)
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {3042, 4088, 3042, 4128, 27, 3042, 4118, 3042, 4111, 27, 3042, 4012, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$\big\downarrow$ 3042

$$\int \frac{(a+b\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^6}dx$$

$\big\downarrow$ 4088

$$\frac{1}{5}\int \cot^5(c+dx)(a+b\tan(c+dx))^2\left(-b(2aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+5aB)\right)dx-\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\int \frac{(a+b\tan(c+dx))^2\left(-b(2aA-5bB)\tan(c+dx)^2-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+5aB)\right)}{\tan(c+dx)^5} d$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 4128

$$\frac{1}{5}\left(\frac{1}{4}\int -2\cot^4(c+dx)(a+b\tan(c+dx))\left(b\left(5Ba^2+12Aba-10b^2B\right)\tan^2(c+dx)+10\left(Ba^3+3Aba^2-3b^2B\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 27

$$\frac{1}{5}\left(-\frac{1}{2}\int \cot^4(c+dx)(a+b\tan(c+dx))\left(b\left(5Ba^2+12Aba-10b^2B\right)\tan^2(c+dx)+10\left(Ba^3+3Aba^2-3b^2Ba\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(-\frac{1}{2}\int \frac{(a+b\tan(c+dx))\left(b\left(5Ba^2+12Aba-10b^2B\right)\tan(c+dx)^2+10\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(\right.}{\tan(c+dx)^4}\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 4118

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{a^2\left(10a^2A-25abB-18Ab^2\right)\cot^3(c+dx)}{3d}-\int\cot^3(c+dx)\left(b^2\left(5Ba^2+12Aba-10b^2B\right)\tan^2(c+dx)-\right.\right.\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{a^2\left(10a^2A-25abB-18Ab^2\right)\cot^3(c+dx)}{3d}-\int\frac{b^2\left(5Ba^2+12Aba-10b^2B\right)\tan(c+dx)^2-10\left(Aa^4-4b\right.}{}\right.\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \text{4111}$$

$$\frac{1}{5}\left(\frac{1}{2}\left(-\int-10\cot^2(c+dx)\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4\right.\right.\right.\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{2}\left(10\int\cot^2(c+dx)\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\text{ta}\right.\right.\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{2}\left(10\int\frac{Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\tan(c+dx)}{\tan(c+dx)^2}dx\right.\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \text{4012}$$

$$\frac{1}{5}\left(\frac{1}{2}\left(10\left(\int\cot(c+dx)\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B-\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\text{t}\right.\right.\right.\right.$$

$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{2}\left(10\left(\int \frac{Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B-\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\tan(c+dx)}{\tan(c+dx)}dx\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 4014

$$\frac{1}{5}\left(\frac{1}{2}\left(10\left(\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\int \cot(c+dx)dx-\frac{\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab\right.}{d}\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(\frac{1}{2}\left(10\left(\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{\left(a^4A-4a^3bB-6a^2Ab^2+4a\right.}{d}\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 25

$$\frac{1}{5}\left(\frac{1}{2}\left(10\left(-\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{\left(a^4A-4a^3bB-6a^2Ab^2\right.}{}\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

$\big\downarrow$ 3956

$$\frac{1}{5}\left(\frac{1}{2}\left(\frac{a^2\left(10a^2A-25abB-18Ab^2\right)\cot^3(c+dx)}{3d}+\frac{a\left(10a^3B+40a^2Ab-55ab^2B-28Ab^3\right)\cot^2(c+dx)}{2d}+10\left(\right.\right.\right.$$
$$\frac{aA\cot^5(c+dx)(a+b\tan(c+dx))^3}{5d}$$

input
```
Int[Cot[c + d*x]^6*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-1/5*(a*A*Cot[c + d*x]^5*(a + b*Tan[c + d*x])^3)/d + (((a*(40*a^2*A*b - 28
*A*b^3 + 10*a^3*B - 55*a*b^2*B)*Cot[c + d*x]^2)/(2*d) + (a^2*(10*a^2*A - 1
8*A*b^2 - 25*a*b*B)*Cot[c + d*x]^3)/(3*d) + 10*(-((a^4*A - 6*a^2*A*b^2 + A
*b^4 - 4*a^3*b*B + 4*a*b^3*B)*x) - ((a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b
*B + 4*a*b^3*B)*Cot[c + d*x])/d + ((4*a^3*A*b - 4*a*A*b^3 + a^4*B - 6*a^2*
b^2*B + b^4*B)*Log[-Sin[c + d*x]])/d))/2 - (a*(8*A*b + 5*a*B)*Cot[c + d*x]
^4*(a + b*Tan[c + d*x])^2)/(4*d))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{(-120A\,a^3b+120Aa\,b^3-30B\,a^4+180B\,a^2b^2-30B\,b^4)\ln\left(\sec(dx+c)^2\right)+(240A\,a^3b-240Aa\,b^3+60B\,a^4-360B\,a^2b^2+6}{}$
derivativedivides	$\frac{\frac{\left(-4A\,a^3b+4Aa\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-A\,a^4+6A\,a^2b^2-A\,b^4+4B\,a^3b-4Ba\,b^3)\arctan(\tan(dx+c}{}$
default	$\frac{\frac{\left(-4A\,a^3b+4Aa\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-A\,a^4+6A\,a^2b^2-A\,b^4+4B\,a^3b-4Ba\,b^3)\arctan(\tan(dx+c}{}$
norman	$\frac{(-A\,a^4+6A\,a^2b^2-A\,b^4+4B\,a^3b-4Ba\,b^3)x\tan(dx+c)^5-\frac{A\,a^4}{5d}-\frac{\left(A\,a^4-6A\,a^2b^2+A\,b^4-4B\,a^3b+4Ba\,b^3\right)\tan(dx+c)^4}{d}+\frac{a(4}{}}{\tan(dx+c)^5}$
risch	$-\frac{8iA\,a^3bc}{d}-\frac{2iB\,b^4c}{d}-iB\,a^4x+\frac{\ln(e^{2i(dx+c)}-1)B\,b^4}{d}+\frac{a^4\ln(e^{2i(dx+c)}-1)B}{d}+\frac{12iB\,a^2b^2c}{d}+6iB\,a^2$

input
```
int(cot(d*x+c)^6*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/60*((-120*A*a^3*b+120*A*a*b^3-30*B*a^4+180*B*a^2*b^2-30*B*b^4)*ln(sec(d*
x+c)^2)+(240*A*a^3*b-240*A*a*b^3+60*B*a^4-360*B*a^2*b^2+60*B*b^4)*ln(tan(d
*x+c))-12*A*cot(d*x+c)^5*a^4+(-60*A*a^3*b-15*B*a^4)*cot(d*x+c)^4+20*a^2*co
t(d*x+c)^3*(A*a^2-6*A*b^2-4*B*a*b)+(120*A*a^3*b-120*A*a*b^3+30*B*a^4-180*B
*a^2*b^2)*cot(d*x+c)^2+(-60*A*a^4+360*A*a^2*b^2-60*A*b^4+240*B*a^3*b-240*B
*a*b^3)*cot(d*x+c)-60*d*x*(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3))/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.10

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{30\,(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^5+15\,(3\,Ba^4+12\,Aa^3b-1}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/60*(30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x
 + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^5 + 15*(3*B*a^4 + 12*A*a^3*b -
12*B*a^2*b^2 - 8*A*a*b^3 - 4*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3
+ A*b^4)*d*x)*tan(d*x + c)^5 - 12*A*a^4 - 60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*
b^2 + 4*B*a*b^3 + A*b^4)*tan(d*x + c)^4 + 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*
b^2 - 4*A*a*b^3)*tan(d*x + c)^3 + 20*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan
(d*x + c)^2 - 15*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/(d*tan(d*x + c)^5)
```

Sympy [A] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.00

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \begin{cases} \tilde{\infty}Aa^4x \\ x(A+B\tan{(c)})\,(a+b\tan{(c)})^4\cot^6{(c)} \\ \tilde{\infty}Aa^4x \\ -Aa^4x-\frac{Aa^4}{d\tan{(c+dx)}}+\frac{Aa^4}{3d\tan^3{(c+dx)}}-\frac{Aa^4}{5d\tan^5{(c+dx)}}-\frac{2Aa^3b\log{(\tan^2{(c+dx)}+1)}}{d}+\frac{4Aa^3b\log{(\tan{(c+dx)})}}{d}+\frac{2Aa^3}{d\tan^2{(c-}} \end{cases}$$

input
```
integrate(cot(d*x+c)**6*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**4*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**4*cot(c)**6, Eq(d, 0)), (zoo*A*a**4*x, Eq(c, -d*x)), (-A*a**4*x - A
*a**4/(d*tan(c + d*x)) + A*a**4/(3*d*tan(c + d*x)**3) - A*a**4/(5*d*tan(c
+ d*x)**5) - 2*A*a**3*b*log(tan(c + d*x)**2 + 1)/d + 4*A*a**3*b*log(tan(c
+ d*x))/d + 2*A*a**3*b/(d*tan(c + d*x)**2) - A*a**3*b/(d*tan(c + d*x)**4)
+ 6*A*a**2*b**2*x + 6*A*a**2*b**2/(d*tan(c + d*x)) - 2*A*a**2*b**2/(d*tan(
c + d*x)**3) + 2*A*a*b**3*log(tan(c + d*x)**2 + 1)/d - 4*A*a*b**3*log(tan(
c + d*x))/d - 2*A*a*b**3/(d*tan(c + d*x)**2) - A*b**4*x - A*b**4/(d*tan(c
+ d*x)) - B*a**4*log(tan(c + d*x)**2 + 1)/(2*d) + B*a**4*log(tan(c + d*x))
/d + B*a**4/(2*d*tan(c + d*x)**2) - B*a**4/(4*d*tan(c + d*x)**4) + 4*B*a**
3*b*x + 4*B*a**3*b/(d*tan(c + d*x)) - 4*B*a**3*b/(3*d*tan(c + d*x)**3) + 3
*B*a**2*b**2*log(tan(c + d*x)**2 + 1)/d - 6*B*a**2*b**2*log(tan(c + d*x))/
d - 3*B*a**2*b**2/(d*tan(c + d*x)**2) - 4*B*a*b**3*x - 4*B*a*b**3/(d*tan(c
 + d*x)) - B*b**4*log(tan(c + d*x)**2 + 1)/(2*d) + B*b**4*log(tan(c + d*x)
)/d, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.06

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$

$$-\frac{60\,(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)(dx+c)+30\,(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/60*(60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c)
+ 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c
)^2 + 1) - 60*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(ta
n(d*x + c)) + (12*A*a^4 + 60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3
+ A*b^4)*tan(d*x + c)^4 - 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)
*tan(d*x + c)^3 - 20*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan(d*x + c)^2 + 15
*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.09

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)(dx+c)}{d}$$
$$-\frac{(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$
$$+\frac{(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{d}$$
$$-\frac{12\,Aa^4+60\,(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)\tan\left(dx+c\right)^4-30\,(Ba^4+4\,Aa^3b-6\,Ba^2b^2-}{60\,d\tan(}$$

input
```
integrate(cot(d*x+c)^6*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*(d*x + c)/d - 1/2*(
B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 +
1)/d + (B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(abs(tan(d
*x + c)))/d - 1/60*(12*A*a^4 + 60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a
*b^3 + A*b^4)*tan(d*x + c)^4 - 30*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a
*b^3)*tan(d*x + c)^3 - 20*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan(d*x + c)^2
 + 15*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/(d*tan(d*x + c)^5)
```

Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.96

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{\ln(\tan(c+dx))\,(B\,a^4+4\,A\,a^3\,b-6\,B\,a^2\,b^2-4\,A\,a\,b^3+B\,b^4)}{d}$$
$$- \frac{\cot(c+dx)^5\left(\tan(c+dx)\,\left(\frac{B\,a^4}{4}+A\,b\,a^3\right)+\frac{A\,a^4}{5}-\tan(c+dx)^3\left(\frac{B\,a^4}{2}+2\,A\,a^3\,b-3\,B\,a^2\,b^2-2\,A\,a\right.\right.}{}$$
$$+ \frac{\ln(\tan(c+dx)-\mathrm{i})\,(-B+A\,1\mathrm{i})\,(-b+a\,1\mathrm{i})^4}{2\,d}$$
$$- \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(B+A\,1\mathrm{i})\,(b+a\,1\mathrm{i})^4}{2\,d}$$

input
```
int(cot(c + d*x)^6*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
(log(tan(c + d*x))*(B*a^4 + B*b^4 - 6*B*a^2*b^2 - 4*A*a*b^3 + 4*A*a^3*b))/
d - (cot(c + d*x)^5*(tan(c + d*x)*((B*a^4)/4 + A*a^3*b) + (A*a^4)/5 - tan(
c + d*x)^3*((B*a^4)/2 - 3*B*a^2*b^2 - 2*A*a*b^3 + 2*A*a^3*b) + tan(c + d*x
)^2*(2*A*a^2*b^2 - (A*a^4)/3 + (4*B*a^3*b)/3) + tan(c + d*x)^4*(A*a^4 + A*
b^4 - 6*A*a^2*b^2 + 4*B*a*b^3 - 4*B*a^3*b)))/d + (log(tan(c + d*x) - 1i)*(
A*1i - B)*(a*1i - b)^4)/(2*d) - (log(tan(c + d*x) + 1i)*(A*1i + B)*(a*1i +
 b)^4)/(2*d)
```

Reduce [F]

$$\int \cot^6(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \int \cot(dx+c)^6\,(a+\tan(dx+c)\,b)^4\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^6*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^6*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

3.266 $\int \cot^7(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 323

$$
\begin{aligned}
&\int \cot^7(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx \\
&= -\left(\left(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B\right)x\right) \\
&\quad-\frac{(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B)\cot(c+dx)}{d} \\
&\quad-\frac{(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B)\cot^2(c+dx)}{2d} \\
&\quad+\frac{a(20a^2Ab-13Ab^3+5a^3B-27ab^2B)\cot^3(c+dx)}{15d} \\
&\quad+\frac{a^2(5a^2A-8Ab^2-12abB)\cot^4(c+dx)}{20d} \\
&\quad-\frac{(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B)\log(\sin(c+dx))}{d} \\
&\quad-\frac{a(3Ab+2aB)\cot^5(c+dx)(a+b\tan(c+dx))^2}{10d} \\
&\quad-\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}
\end{aligned}
$$

output
```
-(4*A*a^3*b-4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*x-(4*A*a^3*b-4*A*a*b^3+B*a^
4-6*B*a^2*b^2+B*b^4)*cot(d*x+c)/d-1/2*(A*a^4-6*A*a^2*b^2+A*b^4-4*B*a^3*b+4
*B*a*b^3)*cot(d*x+c)^2/d+1/15*a*(20*A*a^2*b-13*A*b^3+5*B*a^3-27*B*a*b^2)*c
ot(d*x+c)^3/d+1/20*a^2*(5*A*a^2-8*A*b^2-12*B*a*b)*cot(d*x+c)^4/d-(A*a^4-6*
A*a^2*b^2+A*b^4-4*B*a^3*b+4*B*a*b^3)*ln(sin(d*x+c))/d-1/10*a*(3*A*b+2*B*a)
*cot(d*x+c)^5*(a+b*tan(d*x+c))^2/d-1/6*a*A*cot(d*x+c)^6*(a+b*tan(d*x+c))^3
/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.93

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{-60(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B)\cot(c+dx)-30(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B)}{}$$

input
```
Integrate[Cot[c + d*x]^7*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(-60*(4*a^3*A*b - 4*a*A*b^3 + a^4*B - 6*a^2*b^2*B + b^4*B)*Cot[c + d*x] -
30*(a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*Cot[c + d*x]^2 +
20*a*(4*a^2*A*b - 4*A*b^3 + a^3*B - 6*a*b^2*B)*Cot[c + d*x]^3 + 15*a^2*(a^
2*A - 6*A*b^2 - 4*a*b*B)*Cot[c + d*x]^4 - 12*a^3*(4*A*b + a*B)*Cot[c + d*x
]^5 - 10*a^4*A*Cot[c + d*x]^6 + 30*(a + I*b)^4*(A + I*B)*Log[I - Tan[c + d
*x]] - 60*(a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*Log[Tan[c
+ d*x]] + 30*(a - I*b)^4*(A - I*B)*Log[I + Tan[c + d*x]])/(60*d)
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.05, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.677$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4118, 3042, 4111, 27, 3042, 4012, 3042, 4012, 25, 3042, 4014, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^4(A+B\tan(c+dx))}{\tan(c+dx)^7}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{6}\int 3\cot^6(c+dx)(a+b\tan(c+dx))^2\left(-b(aA-2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)dx-\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \cot^6(c+dx)(a+b\tan(c+dx))^2\left(-b(aA-2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)dx-\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{(a+b\tan(c+dx))^2\left(-b(aA-2bB)\tan(c+dx)^2-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)}{\tan(c+dx)^6}dx - \frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{4128}$$

$$\frac{1}{2}\left(\frac{1}{5}\int -2\cot^5(c+dx)(a+b\tan(c+dx))\left(b\left(3Ba^2+7Aba-5b^2B\right)\tan^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{2}{5}\int \cot^5(c+dx)(a+b\tan(c+dx))\left(b\left(3Ba^2+7Aba-5b^2B\right)\tan^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{2}{5}\int \frac{(a+b\tan(c+dx))\left(b\left(3Ba^2+7Aba-5b^2B\right)\tan(c+dx)^2+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+\right.}{\tan(c+dx)^5}\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\downarrow$ 4118

$$\frac{1}{2}\left(-\frac{2}{5}\left(\int \cot^4(c+dx)\left(b^2\left(3Ba^2+7Aba-5b^2B\right)\tan^2(c+dx)-5\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\mathrm{ta}\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-\frac{2}{5}\left(\int \frac{b^2\left(3Ba^2+7Aba-5b^2B\right)\tan(c+dx)^2-5\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\tan(c+dx)+a}{\tan(c+dx)^4}\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\downarrow$ 4111

$$\frac{1}{2}\left(-\frac{2}{5}\left(\int -5\cot^3(c+dx)\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(-\frac{2}{5}\left(-5\int \cot^3(c+dx)\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-\frac{2}{5}\left(-5\int \frac{Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\tan(c+dx)}{\tan(c+dx)^3}d\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\downarrow$ 4012

$$\frac{1}{2}\left(-\frac{2}{5}\left(-5\left(\int \cot^2(c+dx)\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B-\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab\right.\right.\right.\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(-\frac{2}{5}\left(-5\left(\int \frac{Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B-\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\right)\tan(c+dx)}{\tan(c+dx)^2}\right.\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\big\downarrow$ 4012

$$\frac{1}{2}\left(-\frac{2}{5}\left(-5\left(\int -\cot(c+dx)\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4\right.\right.\right.\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(-\frac{2}{5}\left(-5\left(-\int \cot(c+dx)\left(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4\right.\right.\right.\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(-\frac{2}{5}\left(-5\left(-\int \frac{Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4+\left(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\right)\tan(c+}{\tan(c+dx)}\right.\right.\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\big\downarrow$ 4014

$$\frac{1}{2}\left(-\frac{2}{5}\left(-5\left(-\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)\int \cot(c+dx)dx-\frac{\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B\right.}{2d}\right.\right.\right.$$

$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(-\frac{2}{5}\left(-5\left(-(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx-\frac{(a^4A-4a^3bB-6a^2Ab^2-\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(-5\left((a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx-\frac{(a^4A-4a^3bB-6a^2Ab^2\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

$$\Big\downarrow \; 3956$$

$$\frac{1}{2}\left(-\frac{2}{5}\left(-\frac{a^2(5a^2A-12abB-8Ab^2)\cot^4(c+dx)}{4d}-\frac{a(5a^3B+20a^2Ab-27ab^2B-13Ab^3)\cot^3(c+dx)}{3d}-5\left(\right.\right.\right.$$
$$\frac{aA\cot^6(c+dx)(a+b\tan(c+dx))^3}{6d}$$

input
```
Int[Cot[c + d*x]^7*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
-1/6*(a*A*Cot[c + d*x]^6*(a + b*Tan[c + d*x])^3)/d + ((-2*(-1/3*(a*(20*a^2
*A*b - 13*A*b^3 + 5*a^3*B - 27*a*b^2*B)*Cot[c + d*x]^3)/d - (a^2*(5*a^2*A
- 8*A*b^2 - 12*a*b*B)*Cot[c + d*x]^4)/(4*d) - 5*(-((4*a^3*A*b - 4*a*A*b^3
+ a^4*B - 6*a^2*b^2*B + b^4*B)*x) - ((4*a^3*A*b - 4*a*A*b^3 + a^4*B - 6*a^
2*b^2*B + b^4*B)*Cot[c + d*x])/d - ((a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b
*B + 4*a*b^3*B)*Cot[c + d*x]^2)/(2*d) - ((a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*
a^3*b*B + 4*a*b^3*B)*Log[-Sin[c + d*x]])/d)))/5 - (a*(3*A*b + 2*a*B)*Cot[c
 + d*x]^5*(a + b*Tan[c + d*x])^2)/(5*d))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.02

method	result
parallelrisch	$\frac{\left(30A\,a^4-180A\,a^2b^2+30A\,b^4-120B\,a^3b+120Ba\,b^3\right)\ln\left(\sec(dx+c)^2\right)+\left(-60A\,a^4+360A\,a^2b^2-60A\,b^4+240B\,a^3b-240\right.}{}$
derivativedivides	$\frac{\frac{\left(A\,a^4-6A\,a^2b^2+A\,b^4-4B\,a^3b+4Ba\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-4A\,a^3b+4Aa\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)\arctan(\tan(dx+c))}{}$
default	$\frac{\frac{\left(A\,a^4-6A\,a^2b^2+A\,b^4-4B\,a^3b+4Ba\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-4A\,a^3b+4Aa\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)\arctan(\tan(dx+c))}{}$
norman	$\frac{\left(-4A\,a^3b+4Aa\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)x\tan(dx+c)^6-\frac{A\,a^4}{6d}-\frac{\left(4A\,a^3b-4Aa\,b^3+B\,a^4-6B\,a^2b^2+B\,b^4\right)\tan(dx+c)^5}{d}-\frac{(A}{}}{}$
risch	Expression too large to display

input
```
int(cot(d*x+c)^7*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/60*((30*A*a^4-180*A*a^2*b^2+30*A*b^4-120*B*a^3*b+120*B*a*b^3)*ln(sec(d*x
+c)^2)+(-60*A*a^4+360*A*a^2*b^2-60*A*b^4+240*B*a^3*b-240*B*a*b^3)*ln(tan(d
*x+c))-10*A*cot(d*x+c)^6*a^4+(-48*A*a^3*b-12*B*a^4)*cot(d*x+c)^5+15*a^2*co
t(d*x+c)^4*(A*a^2-6*A*b^2-4*B*a*b)+(80*A*a^3*b-80*A*a*b^3+20*B*a^4-120*B*a
^2*b^2)*cot(d*x+c)^3+(-30*A*a^4+180*A*a^2*b^2-30*A*b^4+120*B*a^3*b-120*B*a
*b^3)*cot(d*x+c)^2+(-240*A*a^3*b+240*A*a*b^3-60*B*a^4+360*B*a^2*b^2-60*B*b
^4)*cot(d*x+c)-240*(A*a^3*b-A*a*b^3+1/4*B*a^4-3/2*B*a^2*b^2+1/4*B*b^4)*x*d
)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.08

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$
$$-\frac{30\left(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4\right)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^6+5\left(11\,Aa^4-36\,Ba^3b-\right.}{}$$

input
```
integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/60*(30*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(tan(d*
x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^6 + 5*(11*A*a^4 - 36*B*a^3*b -
 54*A*a^2*b^2 + 24*B*a*b^3 + 6*A*b^4 + 12*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2
 - 4*A*a*b^3 + B*b^4)*d*x)*tan(d*x + c)^6 + 60*(B*a^4 + 4*A*a^3*b - 6*B*a^
2*b^2 - 4*A*a*b^3 + B*b^4)*tan(d*x + c)^5 + 10*A*a^4 + 30*(A*a^4 - 4*B*a^3
*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*tan(d*x + c)^4 - 20*(B*a^4 + 4*A*a^3
*b - 6*B*a^2*b^2 - 4*A*a*b^3)*tan(d*x + c)^3 - 15*(A*a^4 - 4*B*a^3*b - 6*A
*a^2*b^2)*tan(d*x + c)^2 + 12*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/(d*tan(d*x
 + c)^6)
```

Sympy [A] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.99

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**7*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*a**4*x, Eq(c, 0) & Eq(d, 0)), (x*(A + B*tan(c))*(a + b*ta
n(c))**4*cot(c)**7, Eq(d, 0)), (zoo*A*a**4*x, Eq(c, -d*x)), (A*a**4*log(ta
n(c + d*x)**2 + 1)/(2*d) - A*a**4*log(tan(c + d*x))/d - A*a**4/(2*d*tan(c
+ d*x)**2) + A*a**4/(4*d*tan(c + d*x)**4) - A*a**4/(6*d*tan(c + d*x)**6) -
 4*A*a**3*b*x - 4*A*a**3*b/(d*tan(c + d*x)) + 4*A*a**3*b/(3*d*tan(c + d*x)
**3) - 4*A*a**3*b/(5*d*tan(c + d*x)**5) - 3*A*a**2*b**2*log(tan(c + d*x)**
2 + 1)/d + 6*A*a**2*b**2*log(tan(c + d*x))/d + 3*A*a**2*b**2/(d*tan(c + d*
x)**2) - 3*A*a**2*b**2/(2*d*tan(c + d*x)**4) + 4*A*a*b**3*x + 4*A*a*b**3/(
d*tan(c + d*x)) - 4*A*a*b**3/(3*d*tan(c + d*x)**3) + A*b**4*log(tan(c + d*
x)**2 + 1)/(2*d) - A*b**4*log(tan(c + d*x))/d - A*b**4/(2*d*tan(c + d*x)**
2) - B*a**4*x - B*a**4/(d*tan(c + d*x)) + B*a**4/(3*d*tan(c + d*x)**3) - B
*a**4/(5*d*tan(c + d*x)**5) - 2*B*a**3*b*log(tan(c + d*x)**2 + 1)/d + 4*B*
a**3*b*log(tan(c + d*x))/d + 2*B*a**3*b/(d*tan(c + d*x)**2) - B*a**3*b/(d*
tan(c + d*x)**4) + 6*B*a**2*b**2*x + 6*B*a**2*b**2/(d*tan(c + d*x)) - 2*B*
a**2*b**2/(d*tan(c + d*x)**3) + 2*B*a*b**3*log(tan(c + d*x)**2 + 1)/d - 4*
B*a*b**3*log(tan(c + d*x))/d - 2*B*a*b**3/(d*tan(c + d*x)**2) - B*b**4*x -
 B*b**4/(d*tan(c + d*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.03

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$

$$-\frac{60\,(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)(dx+c)-30\,(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)}{}$$

input
```
integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/60*(60*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*(d*x + c)
- 30*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(tan(d*x + c
)^2 + 1) + 60*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(ta
n(d*x + c)) + (60*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*ta
n(d*x + c)^5 + 10*A*a^4 + 30*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3
+ A*b^4)*tan(d*x + c)^4 - 20*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3)
*tan(d*x + c)^3 - 15*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan(d*x + c)^2 + 12
*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/tan(d*x + c)^6)/d
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.06

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= -\frac{(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)(dx+c)}{d}$$
$$+\frac{(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$
$$-\frac{(Aa^4-4\,Ba^3b-6\,Aa^2b^2+4\,Bab^3+Ab^4)\log(|\tan(dx+c)|)}{d}$$
$$-\frac{60\,(Ba^4+4\,Aa^3b-6\,Ba^2b^2-4\,Aab^3+Bb^4)\tan(dx+c)^5+10\,Aa^4+30\,(Aa^4-4\,Ba^3b-6\,Aa^2b^2+}{}$$

input
```
integrate(cot(d*x+c)^7*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*(d*x + c)/d + 1/2*(
A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(tan(d*x + c)^2 +
1)/d - (A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*log(abs(tan(d
*x + c)))/d - 1/60*(60*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a*b^3 + B*b^
4)*tan(d*x + c)^5 + 10*A*a^4 + 30*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2 + 4*B*a
*b^3 + A*b^4)*tan(d*x + c)^4 - 20*(B*a^4 + 4*A*a^3*b - 6*B*a^2*b^2 - 4*A*a
*b^3)*tan(d*x + c)^3 - 15*(A*a^4 - 4*B*a^3*b - 6*A*a^2*b^2)*tan(d*x + c)^2
 + 12*(B*a^4 + 4*A*a^3*b)*tan(d*x + c))/(d*tan(d*x + c)^6)
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.95

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$

$$-\frac{\cot(c+d\,x)^6\left(\tan(c+d\,x)\,\left(\frac{B\,a^4}{5}+\frac{4\,A\,b\,a^3}{5}\right)+\frac{A\,a^4}{6}-\tan(c+d\,x)^3\left(\frac{B\,a^4}{3}+\frac{4\,A\,a^3\,b}{3}-2\,B\,a^2\,b^2-\frac{4\,A\,a\,b^3}{3}\right.}{}$$

$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(A\,a^4-4\,B\,a^3\,b-6\,A\,a^2\,b^2+4\,B\,a\,b^3+A\,b^4\right)}{d}$$

$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)\,\left(b+a\,1\mathrm{i}\right)^4}{2\,d}$$

$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)\,\left(-b+a\,1\mathrm{i}\right)^4}{2\,d}$$

input
```
int(cot(c + d*x)^7*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
(log(tan(c + d*x) + 1i)*(A - B*1i)*(a*1i + b)^4)/(2*d) - (log(tan(c + d*x)
)*(A*a^4 + A*b^4 - 6*A*a^2*b^2 + 4*B*a*b^3 - 4*B*a^3*b))/d - (cot(c + d*x)
^6*(tan(c + d*x)*((B*a^4)/5 + (4*A*a^3*b)/5) + (A*a^4)/6 - tan(c + d*x)^3*
((B*a^4)/3 - 2*B*a^2*b^2 - (4*A*a*b^3)/3 + (4*A*a^3*b)/3) + tan(c + d*x)^2
*((3*A*a^2*b^2)/2 - (A*a^4)/4 + B*a^3*b) + tan(c + d*x)^4*((A*a^4)/2 + (A*
b^4)/2 - 3*A*a^2*b^2 + 2*B*a*b^3 - 2*B*a^3*b) + tan(c + d*x)^5*(B*a^4 + B*
b^4 - 6*B*a^2*b^2 - 4*A*a*b^3 + 4*A*a^3*b)))/d + (log(tan(c + d*x) - 1i)*(
A + B*1i)*(a*1i - b)^4)/(2*d)
```

Reduce [F]

$$\int \cot^7(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\int \cot\left(dx+c\right)^7\left(a+\tan\left(dx+c\right)b\right)^4\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(cot(d*x+c)^7*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^7*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

3.267 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 127

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(Ab-aB)x}{a^2+b^2}+\frac{(aA+bB)\log(\cos(c+dx))}{(a^2+b^2)\,d} - \frac{a^3(Ab-aB)\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)\,d} + \frac{(Ab-aB)\tan(c+dx)}{b^2d}+\frac{B\tan^2(c+dx)}{2bd}$$

output
```
-(A*b-B*a)*x/(a^2+b^2)+(A*a+B*b)*ln(cos(d*x+c))/(a^2+b^2)/d-a^3*(A*b-B*a)*
ln(a+b*tan(d*x+c))/b^3/(a^2+b^2)/d+(A*b-B*a)*tan(d*x+c)/b^2/d+1/2*B*tan(d*
x+c)^2/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.09

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{-\frac{b(A+iB)\log(i-\tan(c+dx))}{a+ib}-\frac{b(A-iB)\log(i+\tan(c+dx))}{a-ib}+\frac{2a^3(-Ab+aB)\log(a+b\tan(c+dx))}{b^2(a^2+b^2)}+\frac{2(Ab-aB)\tan(c+dx)}{b}+B\tan^2}{2bd}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(-((b*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b)) - (b*(A - I*B)*Log[I + T
an[c + d*x]])/(a - I*b) + (2*a^3*(-(A*b) + a*B)*Log[a + b*Tan[c + d*x]])/(
b^2*(a^2 + b^2)) + (2*(A*b - a*B)*Tan[c + d*x])/b + B*Tan[c + d*x]^2)/(2*b
*d)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4090, 27, 3042, 4130, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -\frac{2\tan(c+dx)\left(-\left((Ab-aB)\tan^2(c+dx)\right)+bB\tan(c+dx)+aB\right)}{a+b\tan(c+dx)}dx}{2b}+\frac{B\tan^2(c+dx)}{2bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{B\tan^2(c+dx)}{2bd}-\frac{\int \frac{\tan(c+dx)\left(-\left((Ab-aB)\tan^2(c+dx)\right)+bB\tan(c+dx)+aB\right)}{a+b\tan(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\tan^2(c+dx)}{2bd}-\frac{\int \frac{\tan(c+dx)\left(-\left((Ab-aB)\tan(c+dx)^2\right)+bB\tan(c+dx)+aB\right)}{a+b\tan(c+dx)}dx}{b}$$

$$\Big\downarrow \text{4130}$$

$$\frac{B\tan^2(c+dx)}{2bd}-\frac{\frac{\int \frac{A\tan(c+dx)b^2+\left(-Ba^2+Aba+b^2B\right)\tan^2(c+dx)+a(Ab-aB)}{a+b\tan(c+dx)}dx}{b}-\frac{(Ab-aB)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\tan^2(c+dx)}{2bd}-\frac{\frac{\int \frac{A\tan(c+dx)b^2+\left(-Ba^2+Aba+b^2B\right)\tan(c+dx)^2+a(Ab-aB)}{a+b\tan(c+dx)}dx}{b}-\frac{(Ab-aB)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \text{4109}$$

$$\frac{B\tan^2(c+dx)}{2bd}-\frac{\frac{\frac{b^2(aA+bB)\int \tan(c+dx)dx}{a^2+b^2}+\frac{a^3(Ab-aB)\int \frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2x(Ab-aB)}{a^2+b^2}}{b}-\frac{(Ab-aB)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\tan^2(c+dx)}{2bd}-\frac{\frac{\frac{b^2(aA+bB)\int \tan(c+dx)dx}{a^2+b^2}+\frac{a^3(Ab-aB)\int \frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2x(Ab-aB)}{a^2+b^2}}{b}-\frac{(Ab-aB)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{B\tan^2(c+dx)}{2bd}-\frac{\frac{\frac{a^3(Ab-aB)\int \frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b^2(aA+bB)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^2x(Ab-aB)}{a^2+b^2}}{b}-\frac{(Ab-aB)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \text{4100}$$

$$\frac{B\tan^2(c+dx)}{2bd}-\frac{\frac{\frac{a^3(Ab-aB)\int \frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd\left(a^2+b^2\right)}-\frac{b^2(aA+bB)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^2x(Ab-aB)}{a^2+b^2}}{b}-\frac{(Ab-aB)\tan(c+dx)}{bd}}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{B\tan^2(c+dx)}{2bd}-\frac{\frac{-\frac{b^2(aA+bB)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^2x(Ab-aB)}{a^2+b^2}+\frac{a^3(Ab-aB)\log(a+b\tan(c+dx))}{bd\left(a^2+b^2\right)}}{b}-\frac{(Ab-aB)\tan(c+dx)}{bd}}{b}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]^2)/(2*b*d) - (((b^2*(A*b - a*B)*x)/(a^2 + b^2) - (b^2*(a*A
 + b*B)*Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a^3*(A*b - a*B)*Log[a + b*Ta
n[c + d*x]])/(b*(a^2 + b^2)*d))/b - ((A*b - a*B)*Tan[c + d*x])/(b*d))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\frac{B\tan(dx+c)^2 b}{2}+A\tan(dx+c)b-B\tan(dx+c)a}{b^2}+\frac{\frac{(-aA-Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Ab+Ba)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^3(Ab-Ba)\ln(\cdots}{b^3\left(a^2\cdots\right.}}{d}$
default	$\frac{\frac{\frac{B\tan(dx+c)^2 b}{2}+A\tan(dx+c)b-B\tan(dx+c)a}{b^2}+\frac{\frac{(-aA-Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-Ab+Ba)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^3(Ab-Ba)\ln(\cdots}{b^3\left(a^2\cdots\right.}}{d}$
norman	$\frac{(Ab-Ba)\tan(dx+c)}{b^2 d}-\frac{(Ab-Ba)x}{a^2+b^2}+\frac{B\tan(dx+c)^2}{2bd}-\frac{(aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}-\frac{a^3(Ab-Ba)\ln(a+b\tan\cdots}{b^3(a^2+b^2)d}$
parallelrisch	$-\frac{2A b^4 dx-2Ba b^3 dx-B a^2 b^2\tan(dx+c)^2-B b^4\tan(dx+c)^2+A\ln\left(1+\tan(dx+c)^2\right)a b^3+2A\ln(a+b\tan(dx+c))a^3 b-\cdots}{\cdots}$
risch	$-\frac{xB}{ib-a}+\frac{2ia^3Ax}{b^2(a^2+b^2)}-\frac{2ia^4Bx}{b^3(a^2+b^2)}+\frac{2ia^3Ac}{b^2d(a^2+b^2)}-\frac{2iAac}{b^2d}+\frac{2iBa^2c}{b^3d}-\frac{2ia^4Bc}{b^3d(a^2+b^2)}-\frac{ixA}{ib-a}-\frac{2iAax}{b^2}+\cdots$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/b^2*(1/2*B*tan(d*x+c)^2*b+A*tan(d*x+c)*b-B*tan(d*x+c)*a)+1/(a^2+b^2
)*(1/2*(-A*a-B*b)*ln(1+tan(d*x+c)^2)+(-A*b+B*a)*arctan(tan(d*x+c)))-1/b^3*
a^3*(A*b-B*a)/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.50

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{2\,(Bab^3-Ab^4)dx+(Ba^2b^2+Bb^4)\tan{(dx+c)}^2+(Ba^4-Aa^3b)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(B}{2\,(a^2b^3+b^5)}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/2*(2*(B*a*b^3 - A*b^4)*d*x + (B*a^2*b^2 + B*b^4)*tan(d*x + c)^2 + (B*a^4
 - A*a^3*b)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x +
 c)^2 + 1)) - (B*a^4 - A*a^3*b - A*a*b^3 - B*b^4)*log(1/(tan(d*x + c)^2 +
1)) - 2*(B*a^3*b - A*a^2*b^2 + B*a*b^3 - A*b^4)*tan(d*x + c))/((a^2*b^3 +
b^5)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 1297, normalized size of antiderivative = 10.21

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(A + B*tan(c))*tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0))
, ((-A*log(tan(c + d*x)**2 + 1)/(2*d) + A*tan(c + d*x)**2/(2*d) + B*x + B*
tan(c + d*x)**3/(3*d) - B*tan(c + d*x)/d)/a, Eq(b, 0)), (-3*A*d*x*tan(c +
d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*A*d*x/(2*b*d*tan(c + d*x) - 2*I*
b*d) + I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I
*b*d) + A*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*A*ta
n(c + d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*A/(2*b*d*tan(c + d*x) - 2
*I*b*d) - 3*I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - 3*B*d*x/
(2*b*d*tan(c + d*x) - 2*I*b*d) - 2*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)
/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*I*B*log(tan(c + d*x)**2 + 1)/(2*b*d*ta
n(c + d*x) - 2*I*b*d) + B*tan(c + d*x)**3/(2*b*d*tan(c + d*x) - 2*I*b*d) +
 I*B*tan(c + d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*B/(2*b*d*tan(c +
 d*x) - 2*I*b*d), Eq(a, -I*b)), (-3*A*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)
 + 2*I*b*d) - 3*I*A*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*A*log(tan(c + d
*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + A*log(tan(c + d*
x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*A*tan(c + d*x)**2/(2*b*d*tan
(c + d*x) + 2*I*b*d) + 3*A/(2*b*d*tan(c + d*x) + 2*I*b*d) + 3*I*B*d*x*tan(
c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*B*d*x/(2*b*d*tan(c + d*x) + 2*
I*b*d) - 2*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2
*I*b*d) - 2*I*B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d)...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=\frac{\frac{2\,(Ba-Ab)(dx+c)}{a^2+b^2}+\frac{2\,(Ba^4-Aa^3b)\log(b\tan(dx+c)+a)}{a^2b^3+b^5}-\frac{(Aa+Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{Bb\tan(dx+c)^2-2\,(Ba-Ab)\tan(dx+c)}{b^2}}{2\,d}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/2*(2*(B*a - A*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a^4 - A*a^3*b)*log(b*tan(d
*x + c) + a)/(a^2*b^3 + b^5) - (A*a + B*b)*log(tan(d*x + c)^2 + 1)/(a^2 +
b^2) + (B*b*tan(d*x + c)^2 - 2*(B*a - A*b)*tan(d*x + c))/b^2)/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.15

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{(Ba-Ab)(dx+c)}{a^2d+b^2d} - \frac{(Aa+Bb)\log\left(\tan(dx+c)^2+1\right)}{2\,(a^2d+b^2d)} + \frac{(Ba^4-Aa^3b)\log\left(|b\tan(dx+c)+a|\right)}{a^2b^3d+b^5d} + \frac{Bbd\tan(dx+c)^2-2\,Bad\tan(dx+c)+2\,Abd\tan(dx+c)}{2\,b^2d^2}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
(B*a - A*b)*(d*x + c)/(a^2*d + b^2*d) - 1/2*(A*a + B*b)*log(tan(d*x + c)^2
 + 1)/(a^2*d + b^2*d) + (B*a^4 - A*a^3*b)*log(abs(b*tan(d*x + c) + a))/(a^
2*b^3*d + b^5*d) + 1/2*(B*b*d*tan(d*x + c)^2 - 2*B*a*d*tan(d*x + c) + 2*A*
b*d*tan(d*x + c))/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\tan(c+d\,x)\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(-b+a\,1\mathrm{i}\right)} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,a^4-A\,a^3\,b\right)}{d\,\left(a^2\,b^3+b^5\right)} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)} + \frac{B\tan(c+d\,x)^2}{2\,b\,d}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
(tan(c + d*x)*(A/b - (B*a)/b^2))/d - (log(tan(c + d*x) - 1i)*(A*1i - B))/(
2*d*(a*1i - b)) + (log(a + b*tan(c + d*x))*(B*a^4 - A*a^3*b))/(d*(b^5 + a^
2*b^3)) - (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a - b*1i)) + (B*tan(c
+ d*x)^2)/(2*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{-\log\left(\tan\left(dx+c\right)^2+1\right)+\tan\left(dx+c\right)^2}{2d}$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
( - log(tan(c + d*x)**2 + 1) + tan(c + d*x)**2)/(2*d)
```

3.268 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 101

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(aA+bB)x}{a^2+b^2} - \frac{(Ab-aB)\log(\cos(c+dx))}{(a^2+b^2)\,d} + \frac{a^2(Ab-aB)\log(a+b\tan(c+dx))}{b^2\,(a^2+b^2)\,d} + \frac{B\tan(c+dx)}{bd}$$

output
```
-(A*a+B*b)*x/(a^2+b^2)-(A*b-B*a)*ln(cos(d*x+c))/(a^2+b^2)/d+a^2*(A*b-B*a)*
ln(a+b*tan(d*x+c))/b^2/(a^2+b^2)/d+B*tan(d*x+c)/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\frac{i(A+iB)\log(i-\tan(c+dx))}{a+ib} - \frac{(iA+B)\log(i+\tan(c+dx))}{a-ib} + \frac{2a^2(Ab-aB)\log(a+b\tan(c+dx))}{b^2(a^2+b^2)} + \frac{2B\tan(c+dx)}{b}}{2d}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
((I*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b) - ((I*A + B)*Log[I + Tan[c
+ d*x]])/(a - I*b) + (2*a^2*(A*b - a*B)*Log[a + b*Tan[c + d*x]])/(b^2*(a^2
 + b^2)) + (2*B*Tan[c + d*x])/b)/(2*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 4089, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4089}$$

$$\frac{\int -\frac{-((Ab-aB)\tan^2(c+dx))+bB\tan(c+dx)+aB}{a+b\tan(c+dx)}dx}{b}+\frac{B\tan(c+dx)}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{B\tan(c+dx)}{bd}-\frac{\int \frac{-((Ab-aB)\tan^2(c+dx))+bB\tan(c+dx)+aB}{a+b\tan(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\tan(c+dx)}{bd}-\frac{\int \frac{-((Ab-aB)\tan(c+dx)^2)+bB\tan(c+dx)+aB}{a+b\tan(c+dx)}dx}{b}$$

$$\Big\downarrow \text{4109}$$

$$\frac{B\tan(c+dx)}{bd}-\frac{-\frac{a^2(Ab-aB)\int \frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b(Ab-aB)\int \tan(c+dx)dx}{a^2+b^2}+\frac{bx(aA+bB)}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\tan(c+dx)}{bd}-\frac{-\frac{a^2(Ab-aB)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b(Ab-aB)\int\tan(c+dx)dx}{a^2+b^2}+\frac{bx(aA+bB)}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{B\tan(c+dx)}{bd}-\frac{-\frac{a^2(Ab-aB)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b(Ab-aB)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(aA+bB)}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{4100}$$

$$\frac{B\tan(c+dx)}{bd}-\frac{-\frac{a^2(Ab-aB)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd(a^2+b^2)}+\frac{b(Ab-aB)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(aA+bB)}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{B\tan(c+dx)}{bd}-\frac{-\frac{a^2(Ab-aB)\log(a+b\tan(c+dx))}{bd(a^2+b^2)}+\frac{b(Ab-aB)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(aA+bB)}{a^2+b^2}}{b}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-(((b*(a*A + b*B)*x)/(a^2 + b^2) + (b*(A*b - a*B)*Log[Cos[c + d*x]])/((a^2
 + b^2)*d) - (a^2*(A*b - a*B)*Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d))/
b) + (B*Tan[c + d*x])/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4089
```
Int[(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (
f_.)*(x_)]))/((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b^2
*B*(Tan[e + f*x]/(d*f)), x] + Simp[1/d   Int[(a^2*A*d - b^2*B*c + (2*a*A*b
+ B*(a^2 - b^2))*d*Tan[e + f*x] + (A*b^2*d - b*B*(b*c - 2*a*d))*Tan[e + f*x
]^2)/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)B}{b}+\frac{\frac{(Ab-Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-aA-Bb)\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^2(Ab-Ba)\ln(a+b\tan(dx+c))}{b^2\left(a^2+b^2\right)}}{d}$
default	$\frac{\frac{\tan(dx+c)B}{b}+\frac{\frac{(Ab-Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-aA-Bb)\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^2(Ab-Ba)\ln(a+b\tan(dx+c))}{b^2\left(a^2+b^2\right)}}{d}$
norman	$\frac{B\tan(dx+c)}{bd}-\frac{(aA+Bb)x}{a^2+b^2}+\frac{a^2(Ab-Ba)\ln(a+b\tan(dx+c))}{b^2(a^2+b^2)d}+\frac{(Ab-Ba)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}$
parallelrisch	$\frac{-2Aa\,b^2dx-2B\,b^3dx+A\ln\left(1+\tan(dx+c)^2\right)b^3+2A\ln(a+b\tan(dx+c))a^2b-B\ln\left(1+\tan(dx+c)^2\right)a\,b^2-2B\ln(a+b\tan}{2d(a^2+b^2)b^2}$
risch	$-\frac{ixB}{ib-a}+\frac{xA}{ib-a}+\frac{2iAx}{b}+\frac{2iAc}{bd}-\frac{2iaBx}{b^2}-\frac{2iBac}{b^2d}-\frac{2ia^2Ax}{(a^2+b^2)b}-\frac{2ia^2Ac}{d(a^2+b^2)b}+\frac{2ia^3Bx}{b^2(a^2+b^2)}+\frac{2ia^3Bc}{d(a^2+b^2)}$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(tan(d*x+c)*B/b+1/(a^2+b^2)*(1/2*(A*b-B*a)*ln(1+tan(d*x+c)^2)+(-A*a-B*
b)*arctan(tan(d*x+c)))+1/b^2*a^2*(A*b-B*a)/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{2\,(Aab^2+Bb^3)dx+(Ba^3-Aa^2b)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(Ba^3-Aa^2b+Bab^2-Ab^3)\log}{2\,(a^2b^2+b^4)d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/2*(2*(A*a*b^2 + B*b^3)*d*x + (B*a^3 - A*a^2*b)*log((b^2*tan(d*x + c)^2
+ 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - (B*a^3 - A*a^2*b + B*a
*b^2 - A*b^3)*log(1/(tan(d*x + c)^2 + 1)) - 2*(B*a^2*b + B*b^3)*tan(d*x +
c))/((a^2*b^2 + b^4)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 1015, normalized size of antiderivative = 10.05

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(A + B*tan(c))*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (
(-A*x + A*tan(c + d*x)/d - B*log(tan(c + d*x)**2 + 1)/(2*d) + B*tan(c + d*
x)**2/(2*d))/a, Eq(b, 0)), (I*A*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I
*b*d) + A*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) + A*log(tan(c + d*x)**2 + 1)*
tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*A*log(tan(c + d*x)**2 + 1)
/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*A/(2*b*d*tan(c + d*x) - 2*I*b*d) - 3*B
*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*B*d*x/(2*b*d*tan(c
+ d*x) - 2*I*b*d) + I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c
 + d*x) - 2*I*b*d) + B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*
b*d) + 2*B*tan(c + d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*B/(2*b*d*tan
(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (-I*A*d*x*tan(c + d*x)/(2*b*d*tan(c +
d*x) + 2*I*b*d) + A*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) + A*log(tan(c + d*x
)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*A*log(tan(c + d*
x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*A/(2*b*d*tan(c + d*x) + 2*I*
b*d) - 3*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*I*B*d*x/(2*
b*d*tan(c + d*x) + 2*I*b*d) - I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2
*b*d*tan(c + d*x) + 2*I*b*d) + B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d
*x) + 2*I*b*d) + 2*B*tan(c + d*x)**2/(2*b*d*tan(c + d*x) + 2*I*b*d) + 3*B/
(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(A + B*tan(c))*tan(c)**2/(
a + b*tan(c)), Eq(d, 0)), (2*A*a**2*b*log(a/b + tan(c + d*x))/(2*a**2*b...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=-\frac{\frac{2\,(Aa+Bb)(dx+c)}{a^2+b^2}+\frac{2\,(Ba^3-Aa^2b)\log(b\tan(dx+c)+a)}{a^2b^2+b^4}+\frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,B\tan(dx+c)}{b}}{2\,d}$$

input

```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="max
ima")
```

output

```
-1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a^3 - A*a^2*b)*log(b*tan(
d*x + c) + a)/(a^2*b^2 + b^4) + (B*a - A*b)*log(tan(d*x + c)^2 + 1)/(a^2 +
 b^2) - 2*B*tan(d*x + c)/b)/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.18

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx=-\frac{(Aa+Bb)(dx+c)}{a^2d+b^2d}-\frac{(Ba-Ab)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,(a^2d+b^2d)}-\frac{(Ba^3-Aa^2b)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^2b^2d+b^4d}+\frac{B\tan\left(dx+c\right)}{bd}$$

input

```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="gia
c")
```

output

```
-(A*a + B*b)*(d*x + c)/(a^2*d + b^2*d) - 1/2*(B*a - A*b)*log(tan(d*x + c)^
2 + 1)/(a^2*d + b^2*d) - (B*a^3 - A*a^2*b)*log(abs(b*tan(d*x + c) + a))/(a
^2*b^2*d + b^4*d) + B*tan(d*x + c)/(b*d)
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.16

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\tan(c+d\,x)}{b\,d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,a^3-A\,a^2\,b\right)}{d\,\left(a^2\,b^2+b^4\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
(log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a*1i + b)) - (log(a + b*tan(c +
d*x))*(B*a^3 - A*a^2*b))/(d*(b^4 + a^2*b^2)) + (B*tan(c + d*x))/(b*d) + (l
og(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(a + b*1i))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.15

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\tan\left(dx+c\right)-dx}{d}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(tan(c + d*x) - d*x)/d
```

3.269 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 80

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{(Ab-aB)x}{a^2+b^2} - \frac{B\log(\cos(c+dx))}{bd} - \frac{a(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{b(a^2+b^2)d}$$

output
```
(A*b-B*a)*x/(a^2+b^2)-B*ln(cos(d*x+c))/b/d-a*(A*b-B*a)*ln(a*cos(d*x+c)+b*s
in(d*x+c))/b/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{(a-ib)b(A+iB)\log(i-\tan(c+dx)) + (a+ib)b(A-iB)\log(i+\tan(c+dx)) + 2a(-Ab+aB)\log(a}{2b(a^2+b^2)d}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
((a - I*b)*b*(A + I*B)*Log[I - Tan[c + d*x]] + (a + I*b)*b*(A - I*B)*Log[I
 + Tan[c + d*x]] + 2*a*(-(A*b) + a*B)*Log[a + b*Tan[c + d*x]])/(2*b*(a^2 +
 b^2)*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 4072, 27, 3042, 3956, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4072}$$

$$\frac{\int \frac{(Ab-aB)\tan(c+dx)}{a+b\tan(c+dx)}dx}{b}+\frac{B\int \tan(c+dx)dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{(Ab-aB)\int \frac{\tan(c+dx)}{a+b\tan(c+dx)}dx}{b}+\frac{B\int \tan(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{\tan(c+dx)}{a+b\tan(c+dx)}dx}{b}+\frac{B\int \tan(c+dx)dx}{b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(Ab-aB)\int \frac{\tan(c+dx)}{a+b\tan(c+dx)}dx}{b}-\frac{B\log(\cos(c+dx))}{bd}$$

$$\Big\downarrow \text{4014}$$

$$\frac{(Ab-aB)\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)}{b}-\frac{B\log(\cos(c+dx))}{bd}$$

$\Big\downarrow$ 3042

$$\frac{(Ab-aB)\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)}{b}-\frac{B\log(\cos(c+dx))}{bd}$$

$\Big\downarrow$ 4013

$$\frac{(Ab-aB)\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)}\right)}{b}-\frac{B\log(\cos(c+dx))}{bd}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-((B*Log[Cos[c + d*x]])/(b*d)) + ((A*b - a*B)*((b*x)/(a^2 + b^2) - (a*Log[
a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4072
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)]))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B*(d/
b)   Int[Tan[e + f*x], x], x] + Simp[1/b   Int[Simp[A*b*c + (A*b*d + B*(b*c
 - a*d))*Tan[e + f*x], x]/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, A, B}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{\frac{(aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Ab-Ba)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a(Ab-Ba)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)b}}{d}$
default	$\frac{\frac{\frac{(aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Ab-Ba)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a(Ab-Ba)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)b}}{d}$
norman	$\frac{(Ab-Ba)x}{a^2+b^2}+\frac{(aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}-\frac{a(Ab-Ba)\ln(a+b\tan(dx+c))}{(a^2+b^2)db}$
parallelrisch	$\frac{2A\,b^2dx-2Babdx+A\ln\left(1+\tan(dx+c)^2\right)ab-2A\ln(a+b\tan(dx+c))ab+B\ln\left(1+\tan(dx+c)^2\right)b^2+2B\ln(a+b\tan(dx+}{2(a^2+b^2)db}$
risch	$\frac{xB}{ib-a}+\frac{ixA}{ib-a}+\frac{2iaAx}{a^2+b^2}+\frac{2iaAc}{d(a^2+b^2)}-\frac{2ia^2Bx}{b(a^2+b^2)}-\frac{2ia^2Bc}{bd(a^2+b^2)}+\frac{2iBx}{b}+\frac{2iBc}{bd}-\frac{a\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)A}{d(a^2+b^2)}$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)*(1/2*(A*a+B*b)*ln(1+tan(d*x+c)^2)+(A*b-B*a)*arctan(tan(d*
x+c)))-a*(A*b-B*a)/(a^2+b^2)/b*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{2\,(Bab-Ab^2)dx-(Ba^2-Aab)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)+(Ba^2+Bb^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,(a^2b+b^3)d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="frica
s")
```

output
```
-1/2*(2*(B*a*b - A*b^2)*d*x - (B*a^2 - A*a*b)*log((b^2*tan(d*x + c)^2 + 2*
a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) + (B*a^2 + B*b^2)*log(1/(tan
(d*x + c)^2 + 1)))/((a^2*b + b^3)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 700, normalized size of antiderivative = 8.75

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(A + B*tan(c)), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((A*log(
tan(c + d*x)**2 + 1)/(2*d) - B*x + B*tan(c + d*x)/d)/a, Eq(b, 0)), (A*d*x*
tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*A*d*x/(2*b*d*tan(c + d*x)
- 2*I*b*d) - A/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*B*d*x*tan(c + d*x)/(2*b*
d*tan(c + d*x) - 2*I*b*d) + B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*log(t
an(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*B*log(
tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*B/(2*b*d*tan(c + d
*x) - 2*I*b*d), Eq(a, -I*b)), (A*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*
I*b*d) + I*A*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - A/(2*b*d*tan(c + d*x) +
2*I*b*d) - I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*d*x/(2*
b*d*tan(c + d*x) + 2*I*b*d) + B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b
*d*tan(c + d*x) + 2*I*b*d) + I*B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d
*x) + 2*I*b*d) + I*B/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(A +
B*tan(c))*tan(c)/(a + b*tan(c)), Eq(d, 0)), (-2*A*a*b*log(a/b + tan(c + d*
x))/(2*a**2*b*d + 2*b**3*d) + A*a*b*log(tan(c + d*x)**2 + 1)/(2*a**2*b*d +
 2*b**3*d) + 2*A*b**2*d*x/(2*a**2*b*d + 2*b**3*d) + 2*B*a**2*log(a/b + tan
(c + d*x))/(2*a**2*b*d + 2*b**3*d) - 2*B*a*b*d*x/(2*a**2*b*d + 2*b**3*d) +
 B*b**2*log(tan(c + d*x)**2 + 1)/(2*a**2*b*d + 2*b**3*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{\frac{2\,(Ba-Ab)(dx+c)}{a^2+b^2} - \frac{2\,(Ba^2-Aab)\log(b\tan(dx+c)+a)}{a^2b+b^3} - \frac{(Aa+Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxim
a")
```

output
```
-1/2*(2*(B*a - A*b)*(d*x + c)/(a^2 + b^2) - 2*(B*a^2 - A*a*b)*log(b*tan(d*
x + c) + a)/(a^2*b + b^3) - (A*a + B*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2
))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(Ba-Ab)(dx+c)}{a^2d+b^2d} + \frac{(Aa+Bb)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^2d+b^2d\right)} + \frac{(Ba^2-Aab)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^2bd+b^3d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="giac"
)
```

output
```
-(B*a - A*b)*(d*x + c)/(a^2*d + b^2*d) + 1/2*(A*a + B*b)*log(tan(d*x + c)^
2 + 1)/(a^2*d + b^2*d) + (B*a^2 - A*a*b)*log(abs(b*tan(d*x + c) + a))/(a^2
*b*d + b^3*d)
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(-b+a\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)} - \frac{a\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(A\,b-B\,a\right)}{b\,d\,\left(a^2+b^2\right)}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
(log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(a*1i - b)) + (log(tan(c + d*x) +
 1i)*(A - B*1i))/(2*d*(a - b*1i)) - (a*log(a + b*tan(c + d*x))*(A*b - B*a)
)/(b*d*(a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.20

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx=\frac{\log\left(\tan\left(dx+c\right)^2+1\right)}{2d}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
log(tan(c + d*x)**2 + 1)/(2*d)
```

3.270 $\int \frac{A+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 58

$$\int \frac{A+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(aA+bB)x}{a^2+b^2} + \frac{(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

output
```
(A*a+B*b)*x/(a^2+b^2)+(A*b-B*a)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{A+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2(aA+bB)\arctan(\tan(c+dx)) - (Ab-aB)\left(\log\left(\sec^2(c+dx)\right) - 2\log(a+b\tan(c+dx))\right)}{2\left(a^2+b^2\right)d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x]),x]
```

output
```
(2*(a*A + b*B)*ArcTan[Tan[c + d*x]] - (A*b - a*B)*(Log[Sec[c + d*x]^2] - 2
*Log[a + b*Tan[c + d*x]]))/(2*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4014}$$

$$\frac{(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(aA+bB)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(aA+bB)}{a^2+b^2}$$

$$\Big\downarrow \text{4013}$$

$$\frac{(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)}+\frac{x(aA+bB)}{a^2+b^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x]),x]
```

output
```
((a*A + b*B)*x)/(a^2 + b^2) + ((A*b - a*B)*Log[a*Cos[c + d*x] + b*Sin[c +
d*x]])/((a^2 + b^2)*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{\frac{(-Ab+Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(aA+Bb)\arctan(\tan(dx+c))}{a^2+b^2}+\frac{(Ab-Ba)\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$
default	$\frac{\frac{\frac{(-Ab+Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(aA+Bb)\arctan(\tan(dx+c))}{a^2+b^2}+\frac{(Ab-Ba)\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$
norman	$\frac{(aA+Bb)x}{a^2+b^2}+\frac{(Ab-Ba)\ln(a+b\tan(dx+c))}{d(a^2+b^2)}-\frac{(Ab-Ba)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}$
parallelrisch	$-\frac{-2Axad-2Bbdx+A\ln\left(1+\tan(dx+c)^2\right)b-2A\ln(a+b\tan(dx+c))b-B\ln\left(1+\tan(dx+c)^2\right)a+2B\ln(a+b\tan(dx+c\ldots}{2d(a^2+b^2)}$
risch	$\frac{ixB}{ib-a}-\frac{xA}{ib-a}-\frac{2iAbx}{a^2+b^2}+\frac{2iBxa}{a^2+b^2}-\frac{2iAbc}{d(a^2+b^2)}+\frac{2iBac}{d(a^2+b^2)}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)Ab}{d(a^2+b^2)}-\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a\ldots}\right.}{d(a^2+b^2)}$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)*(1/2*(-A*b+B*a)*ln(1+tan(d*x+c)^2)+(A*a+B*b)*arctan(tan(d
*x+c)))+(A*b-B*a)/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{A+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\,(Aa+Bb)dx-(Ba-Ab)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2\,(a^2+b^2)d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*(A*a + B*b)*d*x - (B*a - A*b)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d
*x + c) + a^2)/(tan(d*x + c)^2 + 1)))/((a^2 + b^2)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 524, normalized size of antiderivative = 9.03

$$\int \frac{A+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x(A+B\tan{(c)})}{\tan{(c)}} \\ \frac{Ax+\frac{B\log{\left(\tan^2{(c+dx)}+1\right)}}{2d}}{a} \\ \frac{iAdx\tan{(c+dx)}}{2bd\tan{(c+dx)}-2ibd}+\frac{Adx}{2bd\tan{(c+dx)}-2ibd}+\frac{iA}{2bd\tan{(c+dx)}-2ibd}+\frac{Bdx\tan{(c+dx)}}{2bd\tan{(c+dx)}-2ibd}-\frac{iBdx}{2bd\tan{(c+dx)}-2ibd}-\frac{B}{2bd\tan{(c+} \\ -\frac{iAdx\tan{(c+dx)}}{2bd\tan{(c+dx)}+2ibd}+\frac{Adx}{2bd\tan{(c+dx)}+2ibd}-\frac{iA}{2bd\tan{(c+dx)}+2ibd}+\frac{Bdx\tan{(c+dx)}}{2bd\tan{(c+dx)}+2ibd}+\frac{iBdx}{2bd\tan{(c+dx)}+2ibd}-\frac{}{2bd\tan{(c}} \\ \frac{x(A+B\tan{(c)})}{a+b\tan{(c)}} \\ \frac{2Aadx}{2a^2d+2b^2d}+\frac{2Ab\log{\left(\frac{a}{b}+\tan{(c+dx)}\right)}}{2a^2d+2b^2d}-\frac{Ab\log{\left(\tan^2{(c+dx)}+1\right)}}{2a^2d+2b^2d}-\frac{2Ba\log{\left(\frac{a}{b}+\tan{(c+dx)}\right)}}{2a^2d+2b^2d}+\frac{Ba\log{\left(\tan^2{(c+dx)}+1\right)}}{2a^2d+2b^2d}+\frac{2Bbd}{2a^2d+2} \end{cases}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(A + B*tan(c))/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (
(A*x + B*log(tan(c + d*x)**2 + 1)/(2*d))/a, Eq(b, 0)), (I*A*d*x*tan(c + d*
x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + A*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) +
 I*A/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*
x) - 2*I*b*d) - I*B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) - B/(2*b*d*tan(c +
d*x) - 2*I*b*d), Eq(a, -I*b)), (-I*A*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)
+ 2*I*b*d) + A*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*A/(2*b*d*tan(c + d*x
) + 2*I*b*d) + B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*B*d*x
/(2*b*d*tan(c + d*x) + 2*I*b*d) - B/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a,
I*b)), (x*(A + B*tan(c))/(a + b*tan(c)), Eq(d, 0)), (2*A*a*d*x/(2*a**2*d +
 2*b**2*d) + 2*A*b*log(a/b + tan(c + d*x))/(2*a**2*d + 2*b**2*d) - A*b*log
(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*d) - 2*B*a*log(a/b + tan(c + d*x)
)/(2*a**2*d + 2*b**2*d) + B*a*log(tan(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*
d) + 2*B*b*d*x/(2*a**2*d + 2*b**2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int \frac{A + B\tan(c + dx)}{a + b\tan(c + dx)}\,dx = \frac{\frac{2(Aa+Bb)(dx+c)}{a^2+b^2} - \frac{2(Ba-Ab)\log(b\tan(dx+c)+a)}{a^2+b^2} + \frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) - 2*(B*a - A*b)*log(b*tan(d*x + c
) + a)/(a^2 + b^2) + (B*a - A*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.72

$$\int \frac{A+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(Aa+Bb)(dx+c)}{a^2d+b^2d} + \frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{2\,(a^2d+b^2d)} - \frac{(Bab-Ab^2)\log\left(|b\tan(dx+c)+a|\right)}{a^2bd+b^3d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(A*a + B*b)*(d*x + c)/(a^2*d + b^2*d) + 1/2*(B*a - A*b)*log(tan(d*x + c)^2
 + 1)/(a^2*d + b^2*d) - (B*a*b - A*b^2)*log(abs(b*tan(d*x + c) + a))/(a^2*
b*d + b^3*d)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.60

$$\int \frac{A+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(A\,b-B\,a\right)}{d\,\left(a^2+b^2\right)} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int((A + B*tan(c + d*x))/(a + b*tan(c + d*x)),x)
```

output
```
(log(a + b*tan(c + d*x))*(A*b - B*a))/(d*(a^2 + b^2)) - (log(tan(c + d*x)
+ 1i)*(A - B*1i))/(2*d*(a*1i + b)) - (log(tan(c + d*x) - 1i)*(A*1i - B))/(
2*d*(a + b*1i))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.02

$$\int \frac{A + B\tan(c + dx)}{a + b\tan(c + dx)}\,dx = x$$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
x
```

3.271 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 80

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(Ab-aB)x}{a^2+b^2}+\frac{A\log(\sin(c+dx))}{ad}-\frac{b(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{a(a^2+b^2)\,d}$$

output
```
-(A*b-B*a)*x/(a^2+b^2)+A*ln(sin(d*x+c))/a/d-b*(A*b-B*a)*ln(a*cos(d*x+c)+b*
sin(d*x+c))/a/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{\frac{(A+iB)\log(i-\tan(c+dx))}{a+ib}-\frac{2A\log(\tan(c+dx))}{a}+\frac{(A-iB)\log(i+\tan(c+dx))}{a-ib}+\frac{2b(Ab-aB)\log(a+b\tan(c+dx))}{a(a^2+b^2)}}{2d}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-1/2*(((A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b) - (2*A*Log[Tan[c + d*x]]
)/a + ((A - I*B)*Log[I + Tan[c + d*x]])/(a - I*b) + (2*b*(A*b - a*B)*Log[a
 + b*Tan[c + d*x]])/(a*(a^2 + b^2)))/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3042, 4094, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4094}$$

$$-\frac{b(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{A\int \cot(c+dx)dx}{a}-\frac{x(Ab-aB)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{A\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{x(Ab-aB)}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{A\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{x(Ab-aB)}{a^2+b^2}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{x(Ab-aB)}{a^2+b^2}+\frac{A\log(-\sin(c+dx))}{ad}$$

$$\Big\downarrow \text{4013}$$

$$-\frac{b(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}-\frac{x(Ab-aB)}{a^2+b^2}+\frac{A\log(-\sin(c+dx))}{ad}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-(((A*b - a*B)*x)/(a^2 + b^2)) + (A*Log[-Sin[c + d*x]])/(a*d) - (b*(A*b -
a*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4094
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(B*(b*
c + a*d) + A*(a*c - b*d))*(x/((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[b*((A*b
 - a*B)/((b*c - a*d)*(a^2 + b^2)))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
+ f*x]), x], x] + Simp[d*((B*c - A*d)/((b*c - a*d)*(c^2 + d^2)))   Int[(d -
 c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

method	result
parallelrisch	$\frac{(-2A\,b^{2}+2Bab)\ln(a+b\tan(dx+c))+(-A\,a^{2}-Bab)\ln\left(\sec(dx+c)^{2}\right)+2A(a^{2}+b^{2})\ln(\tan(dx+c))-2adx(Ab-Ba)}{2(a^{2}+b^{2})ad}$
derivativedivides	$\frac{\frac{\frac{(-aA-Bb)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-Ab+Ba)\arctan(\tan(dx+c))}{a^{2}+b^{2}}+\frac{A\ln(\tan(dx+c))}{a}-\frac{(Ab-Ba)b\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)a}}{d}$
default	$\frac{\frac{\frac{(-aA-Bb)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+(-Ab+Ba)\arctan(\tan(dx+c))}{a^{2}+b^{2}}+\frac{A\ln(\tan(dx+c))}{a}-\frac{(Ab-Ba)b\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)a}}{d}$
norman	$-\frac{(Ab-Ba)x}{a^{2}+b^{2}}+\frac{A\ln(\tan(dx+c))}{da}-\frac{(aA+Bb)\ln\left(1+\tan(dx+c)^{2}\right)}{2d(a^{2}+b^{2})}-\frac{(Ab-Ba)b\ln(a+b\tan(dx+c))}{(a^{2}+b^{2})ad}$
risch	$-\frac{xB}{ib-a}-\frac{ixA}{ib-a}-\frac{2iAx}{a}-\frac{2iAc}{ad}+\frac{2ib^{2}Ax}{a(a^{2}+b^{2})}+\frac{2ib^{2}Ac}{ad(a^{2}+b^{2})}-\frac{2ibBx}{a^{2}+b^{2}}-\frac{2ibBc}{d(a^{2}+b^{2})}+\frac{A\ln(e^{2i(dx+c)}-1)}{ad}-$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*((-2*A*b^2+2*B*a*b)*ln(a+b*tan(d*x+c))+(-A*a^2-B*a*b)*ln(sec(d*x+c)^2)
+2*A*(a^2+b^2)*ln(tan(d*x+c))-2*a*d*x*(A*b-B*a))/(a^2+b^2)/a/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int\frac{\cot(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=\frac{2\left(Ba^{2}-Aab\right)dx+\left(Aa^{2}+Ab^{2}\right)\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right)+\left(Bab-Ab^{2}\right)\log\left(\frac{b^{2}\tan(dx+c)^{2}+2\,ab\tan(dx+c)+a^{2}}{\tan(dx+c)^{2}+1}\right)}{2\left(a^{3}+ab^{2}\right)d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="frica
s")
```

output
```
1/2*(2*(B*a^2 - A*a*b)*d*x + (A*a^2 + A*b^2)*log(tan(d*x + c)^2/(tan(d*x +
 c)^2 + 1)) + (B*a*b - A*b^2)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
 + a^2)/(tan(d*x + c)^2 + 1)))/((a^3 + a*b^2)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 952, normalized size of antiderivative = 11.90

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*x*(A + B*tan(c))*cot(c)/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d,
 0)), ((-A*log(tan(c + d*x)**2 + 1)/(2*d) + A*log(tan(c + d*x))/d + B*x)/a
, Eq(b, 0)), ((-A*x - A/(d*tan(c + d*x)) - B*log(tan(c + d*x)**2 + 1)/(2*d
) + B*log(tan(c + d*x))/d)/b, Eq(a, 0)), (A*d*x*tan(c + d*x)/(2*b*d*tan(c
+ d*x) - 2*I*b*d) - I*A*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*A*log(tan(c
 + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - A*log(tan(c
+ d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*I*A*log(tan(c + d*x))*ta
n(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*A*log(tan(c + d*x))/(2*b*d*t
an(c + d*x) - 2*I*b*d) + A/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*B*d*x*tan(c
+ d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*
d) + I*B/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (A*d*x*tan(c + d*x)
/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*A*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) +
 I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d)
- A*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2*I*A*log(ta
n(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*A*log(tan(c +
d*x))/(2*b*d*tan(c + d*x) + 2*I*b*d) + A/(2*b*d*tan(c + d*x) + 2*I*b*d) -
I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*d*x/(2*b*d*tan(c +
 d*x) + 2*I*b*d) - I*B/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(A
+ B*tan(c))*cot(c)/(a + b*tan(c)), Eq(d, 0)), (-A*a**2*log(tan(c + d*x)**2
 + 1)/(2*a**3*d + 2*a*b**2*d) + 2*A*a**2*log(tan(c + d*x))/(2*a**3*d + ...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.34

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=\frac{\frac{2\,(Ba-Ab)(dx+c)}{a^2+b^2}+\frac{2\,(Bab-Ab^2)\log(b\tan(dx+c)+a)}{a^3+ab^2}-\frac{(Aa+Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,A\log(\tan(dx+c))}{a}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxim
a")
```

output
```
1/2*(2*(B*a - A*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a*b - A*b^2)*log(b*tan(d*x
 + c) + a)/(a^3 + a*b^2) - (A*a + B*b)*log(tan(d*x + c)^2 + 1)/(a^2 + b^2)
 + 2*A*log(tan(d*x + c))/a)/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx=\frac{(Ba-Ab)(dx+c)}{a^2d+b^2d}-\frac{(Aa+Bb)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^2d+b^2d\right)}+\frac{(Bab^2-Ab^3)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^3bd+ab^3d}+\frac{A\log\left(\left|\tan\left(dx+c\right)\right|\right)}{ad}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="giac"
)
```

output
```
(B*a - A*b)*(d*x + c)/(a^2*d + b^2*d) - 1/2*(A*a + B*b)*log(tan(d*x + c)^2
 + 1)/(a^2*d + b^2*d) + (B*a*b^2 - A*b^3)*log(abs(b*tan(d*x + c) + a))/(a^
3*b*d + a*b^3*d) + A*log(abs(tan(d*x + c)))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.44

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{A\,\ln\left(\tan(c+d\,x)\right)}{a\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(-b+a\,1\mathrm{i}\right)} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)} - \frac{b\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(A\,b-B\,a\right)}{a\,d\,\left(a^2+b^2\right)}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
(A*log(tan(c + d*x)))/(a*d) - (log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(a*
1i - b)) - (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a - b*1i)) - (b*log(a
 + b*tan(c + d*x))*(A*b - B*a))/(a*d*(a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.39

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
( - log(tan((c + d*x)/2)**2 + 1) + log(tan((c + d*x)/2)))/d
```

3.272 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 103

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(aA+bB)x}{a^2+b^2}-\frac{A\cot(c+dx)}{ad}-\frac{(Ab-aB)\log(\sin(c+dx))}{a^2d}+\frac{b^2(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2(a^2+b^2)d}$$

output
```
-(A*a+B*b)*x/(a^2+b^2)-A*cot(d*x+c)/a/d-(A*b-B*a)*ln(sin(d*x+c))/a^2/d+b^2
*(A*b-B*a)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^2/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{-\frac{2A\cot(c+dx)}{a}+\frac{i(A+iB)\log(i-\tan(c+dx))}{a+ib}+\frac{2(-Ab+aB)\log(\tan(c+dx))}{a^2}-\frac{(iA+B)\log(i+\tan(c+dx))}{a-ib}+\frac{2b^2(Ab-aB)\log(a+b\tan\ldots}{a^2(a^2+b^2)}}{2d}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
((-2*A*Cot[c + d*x])/a + (I*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b) + (
2*(-(A*b) + a*B)*Log[Tan[c + d*x]])/a^2 - ((I*A + B)*Log[I + Tan[c + d*x]]
)/(a - I*b) + (2*b^2*(A*b - a*B)*Log[a + b*Tan[c + d*x]])/(a^2*(a^2 + b^2)
))/(2*d)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 4092, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{\int \frac{\cot(c+dx)\left(Ab\tan^2(c+dx)+aA\tan(c+dx)+Ab-aB\right)}{a+b\tan(c+dx)}dx}{a}-\frac{A\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{Ab\tan(c+dx)^2+aA\tan(c+dx)+Ab-aB}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a}-\frac{A\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{4134}$$

$$-\frac{-\frac{b^2(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{(Ab-aB)\int \cot(c+dx)dx}{a}+\frac{ax(aA+bB)}{a^2+b^2}}{a}-\frac{A\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{b^2(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{(Ab-aB)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\frac{ax(aA+bB)}{a^2+b^2}}{a}-\frac{A\cot(c+dx)}{ad}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{b^2(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{(Ab-aB)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}+\frac{ax(aA+bB)}{a^2+b^2}}{a}-\frac{A\cot(c+dx)}{ad}$$

$$\Big\downarrow \ 3956$$

$$-\frac{-\frac{b^2(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{ax(aA+bB)}{a^2+b^2}+\frac{(Ab-aB)\log(-\sin(c+dx))}{ad}}{a}-\frac{A\cot(c+dx)}{ad}$$

$$\Big\downarrow \ 4013$$

$$-\frac{-\frac{b^2(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{ad(a^2+b^2)}+\frac{ax(aA+bB)}{a^2+b^2}+\frac{(Ab-aB)\log(-\sin(c+dx))}{ad}}{a}-\frac{A\cot(c+dx)}{ad}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-((A*Cot[c + d*x])/(a*d)) - ((a*(a*A + b*B)*x)/(a^2 + b^2) + ((A*b - a*B)*
Log[-Sin[c + d*x]])/(a*d) - (b^2*(A*b - a*B)*Log[a*Cos[c + d*x] + b*Sin[c
+ d*x]])/(a*(a^2 + b^2)*d))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.18

method	result
parallelrisch	$\frac{(2Ab^3-2Bab^2)\ln(a+b\tan(dx+c))+(Aa^2b-Ba^3)\ln\left(\sec(dx+c)^2\right)-2(a^2+b^2)(Ab-Ba)\ln(\tan(dx+c))-2a(A(a^2+}{2a^2d(a^2+b^2)}$
derivativedivides	$\frac{\frac{\frac{(Ab-Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-aA-Bb)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{A}{a\tan(dx+c)}+\frac{(-Ab+Ba)\ln(\tan(dx+c))}{a^2}+\frac{(Ab-Ba)b^2\ln(a+b\tan(}{\left(a^2+b^2\right)a^2}}{d}$
default	$\frac{\frac{\frac{(Ab-Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(-aA-Bb)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{A}{a\tan(dx+c)}+\frac{(-Ab+Ba)\ln(\tan(dx+c))}{a^2}+\frac{(Ab-Ba)b^2\ln(a+b\tan(}{\left(a^2+b^2\right)a^2}}{d}$
norman	$\frac{-\frac{A}{ad}-\frac{(aA+Bb)x\tan(dx+c)}{a^2+b^2}}{\tan(dx+c)}+\frac{(Ab-Ba)b^2\ln(a+b\tan(dx+c))}{a^2d(a^2+b^2)}-\frac{(Ab-Ba)\ln(\tan(dx+c))}{a^2d}+\frac{(Ab-Ba)\ln\left(1+\tan(\right.}{2d(a^2+b^2)}$
risch	$-\frac{ixB}{ib-a}+\frac{xA}{ib-a}+\frac{2iAbx}{a^2}+\frac{2iAbc}{a^2d}-\frac{2ixB}{a}-\frac{2iBc}{ad}-\frac{2ib^3Ax}{a^2(a^2+b^2)}-\frac{2ib^3Ac}{a^2d(a^2+b^2)}+\frac{2ib^2Bx}{a(a^2+b^2)}+\frac{2ib^2Bc}{ad(a^2+b}$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/2*((2*A*b^3-2*B*a*b^2)*ln(a+b*tan(d*x+c))+(A*a^2*b-B*a^3)*ln(sec(d*x+c)^
2)-2*(a^2+b^2)*(A*b-B*a)*ln(tan(d*x+c))-2*a*(A*(a^2+b^2)*cot(d*x+c)+a*d*x*
(A*a+B*b)))/a^2/d/(a^2+b^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.72

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{2\,Aa^3+2\,Aab^2+2\,(Aa^3+Ba^2b)dx\tan(dx+c)-(Ba^3-Aa^2b+Bab^2-Ab^3)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan}{2\,(a^4+a^2b^2)d\tan(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/2*(2*A*a^3 + 2*A*a*b^2 + 2*(A*a^3 + B*a^2*b)*d*x*tan(d*x + c) - (B*a^3
- A*a^2*b + B*a*b^2 - A*b^3)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(
d*x + c) + (B*a*b^2 - A*b^3)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
+ a^2)/(tan(d*x + c)^2 + 1))*tan(d*x + c))/((a^4 + a^2*b^2)*d*tan(d*x + c)
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 2071, normalized size of antiderivative = 20.11

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((-A*x - A
/(d*tan(c + d*x)) - B*log(tan(c + d*x)**2 + 1)/(2*d) + B*log(tan(c + d*x))
/d)/a, Eq(b, 0)), ((A*log(tan(c + d*x)**2 + 1)/(2*d) - A*log(tan(c + d*x))
/d - A/(2*d*tan(c + d*x)**2) - B*x - B/(d*tan(c + d*x)))/b, Eq(a, 0)), (-3
*A*d*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - 3*
I*A*d*x*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - I*A*
log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*
tan(c + d*x)) + A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a*d*tan(c + d*x
)**2 + 2*I*a*d*tan(c + d*x)) + 2*I*A*log(tan(c + d*x))*tan(c + d*x)**2/(2*
a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - 2*A*log(tan(c + d*x))*tan(c
+ d*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - 3*A*tan(c + d*x)/(
2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - 2*I*A/(2*a*d*tan(c + d*x)*
*2 + 2*I*a*d*tan(c + d*x)) + I*B*d*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**
2 + 2*I*a*d*tan(c + d*x)) - B*d*x*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*
I*a*d*tan(c + d*x)) - B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*a*d*ta
n(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - I*B*log(tan(c + d*x)**2 + 1)*tan(c
 + d*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) + 2*B*log(tan(c + d
*x))*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) + 2*I*
B*log(tan(c + d*x))*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c +
d*x)) + I*B*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x))...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{\frac{2(Aa+Bb)(dx+c)}{a^2+b^2}+\frac{2(Bab^2-Ab^3)\log(b\tan(dx+c)+a)}{a^4+a^2b^2}+\frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2(Ba-Ab)\log(\tan(dx+c))}{a^2}+\frac{2A}{a\tan(dx+c)}}{2d}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/2*(2*(A*a + B*b)*(d*x + c)/(a^2 + b^2) + 2*(B*a*b^2 - A*b^3)*log(b*tan(
d*x + c) + a)/(a^4 + a^2*b^2) + (B*a - A*b)*log(tan(d*x + c)^2 + 1)/(a^2 +
 b^2) - 2*(B*a - A*b)*log(tan(d*x + c))/a^2 + 2*A/(a*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = &-\frac{(Aa+Bb)(dx+c)}{a^2d+b^2d} \\ &-\frac{(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{2(a^2d+b^2d)} \\ &-\frac{(Bab^3-Ab^4)\log(|b\tan(dx+c)+a|)}{a^4bd+a^2b^3d} \\ &+\frac{(Ba-Ab)\log(|\tan(dx+c)|)}{a^2d} \\ &-\frac{A}{ad\tan(dx+c)}\end{aligned}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
-(A*a + B*b)*(d*x + c)/(a^2*d + b^2*d) - 1/2*(B*a - A*b)*log(tan(d*x + c)^
2 + 1)/(a^2*d + b^2*d) - (B*a*b^3 - A*b^4)*log(abs(b*tan(d*x + c) + a))/(a
^4*b*d + a^2*b^3*d) + (B*a - A*b)*log(abs(tan(d*x + c)))/(a^2*d) - A/(a*d*
tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.36

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(A\,b^3-B\,a\,b^2\right)}{d\,\left(a^4+a^2\,b^2\right)} - \frac{\ln\left(\tan(c+d\,x)\right)\,\left(A\,b-B\,a\right)}{a^2\,d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)} - \frac{A\cot(c+d\,x)}{a\,d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
(log(a + b*tan(c + d*x))*(A*b^3 - B*a*b^2))/(d*(a^4 + a^2*b^2)) - (log(tan
(c + d*x))*(A*b - B*a))/(a^2*d) + (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d
*(a*1i + b)) - (A*cot(c + d*x))/(a*d) + (log(tan(c + d*x) - 1i)*(A*1i - B)
)/(2*d*(a + b*1i))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.17

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{-\cot\left(dx+c\right)-dx}{d}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
( - (cot(c + d*x) + d*x))/d
```

3.273 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 137

$$\begin{aligned}\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx &= \frac{(Ab-aB)x}{a^2+b^2}+\frac{(Ab-aB)\cot(c+dx)}{a^2d}-\frac{A\cot^2(c+dx)}{2ad}\\ &\quad-\frac{(a^2A-Ab^2+abB)\log(\sin(c+dx))}{a^3d}\\ &\quad-\frac{b^3(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3(a^2+b^2)\,d}\end{aligned}$$

output
```
(A*b-B*a)*x/(a^2+b^2)+(A*b-B*a)*cot(d*x+c)/a^2/d-1/2*A*cot(d*x+c)^2/a/d-(A
*a^2-A*b^2+B*a*b)*ln(sin(d*x+c))/a^3/d-b^3*(A*b-B*a)*ln(a*cos(d*x+c)+b*sin
(d*x+c))/a^3/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=\frac{\frac{2(Ab-aB)\cot(c+dx)}{a^2}-\frac{A\cot^2(c+dx)}{a}+\frac{(A+iB)\log(i-\tan(c+dx))}{a+ib}-\frac{2(a^2A-Ab^2+abB)\log(\tan(c+dx))}{a^3}+\frac{(A-iB)\log(i+\tan(c+dx))}{a-ib}}{2d}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
((2*(A*b - a*B)*Cot[c + d*x])/a^2 - (A*Cot[c + d*x]^2)/a + ((A + I*B)*Log[
I - Tan[c + d*x]])/(a + I*b) - (2*(a^2*A - A*b^2 + a*b*B)*Log[Tan[c + d*x]
])/a^3 + ((A - I*B)*Log[I + Tan[c + d*x]])/(a - I*b) + (2*b^3*(-(A*b) + a*
B)*Log[a + b*Tan[c + d*x]])/(a^3*(a^2 + b^2)))/(2*d)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4092, 27, 3042, 4132, 25, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+b\tan(c+dx))}dx$$
$$\Big\downarrow \text{4092}$$
$$-\frac{\int \frac{2\cot^2(c+dx)\left(Ab\tan^2(c+dx)+aA\tan(c+dx)+Ab-aB\right)}{a+b\tan(c+dx)}dx}{2a}-\frac{A\cot^2(c+dx)}{2ad}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\cot^2(c+dx)\left(Ab\tan^2(c+dx)+aA\tan(c+dx)+Ab-aB\right)}{a+b\tan(c+dx)}dx}{a}-\frac{A\cot^2(c+dx)}{2ad}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{Ab\tan(c+dx)^2+aA\tan(c+dx)+Ab-aB}{\tan(c+dx)^2(a+b\tan(c+dx))}dx}{a}-\frac{A\cot^2(c+dx)}{2ad}$$

$$\Big\downarrow \; 4132$$

$$-\frac{-\frac{\int -\frac{\cot(c+dx)\left(Aa^2+B\tan(c+dx)a^2+bBa-Ab^2-b(Ab-aB)\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx}{a}-\frac{(Ab-aB)\cot(c+dx)}{ad}}{a}-\frac{A\cot^2(c+dx)}{2ad}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int \frac{\cot(c+dx)\left(Aa^2+B\tan(c+dx)a^2+bBa-Ab^2-b(Ab-aB)\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx}{a}-\frac{(Ab-aB)\cot(c+dx)}{ad}}{a}-\frac{A\cot^2(c+dx)}{2ad}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{Aa^2+B\tan(c+dx)a^2+bBa-Ab^2-b(Ab-aB)\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a}-\frac{(Ab-aB)\cot(c+dx)}{ad}}{a}-\frac{A\cot^2(c+dx)}{2ad}$$

$$\Big\downarrow \; 4134$$

$$-\frac{\frac{\frac{\left(a^2A+abB-Ab^2\right)\int \cot(c+dx)dx}{a}+\frac{b^3(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2x(Ab-aB)}{a^2+b^2}}{a}-\frac{(Ab-aB)\cot(c+dx)}{ad}}{a}-\frac{A\cot^2(c+dx)}{2ad}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\left(a^2A+abB-Ab^2\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\frac{b^3(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2x(Ab-aB)}{a^2+b^2}}{a}-\frac{(Ab-aB)\cot(c+dx)}{ad}}{a}-\frac{A\cot^2(c+dx)}{2ad}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{-\frac{\left(a^2A+abB-Ab^2\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}+\frac{b^3(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2x(Ab-aB)}{a^2+b^2}}{a}-\frac{(Ab-aB)\cot(c+dx)}{ad}}{a}-\frac{A\cot^2(c+dx)}{2ad}$$

$\Big\downarrow$ 3956

$$-\frac{\frac{\frac{b^3(Ab-aB)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{\left(a^2A+abB-Ab^2\right)\log(-\sin(c+dx))}{ad}-\frac{a^2x(Ab-aB)}{a^2+b^2}}{a}-\frac{(Ab-aB)\cot(c+dx)}{ad}}{a}-\frac{A\cot^2(c+dx)}{2ad}$$

$\Big\downarrow$ 4013

$$-\frac{\frac{\frac{\left(a^2A+abB-Ab^2\right)\log(-\sin(c+dx))}{ad}-\frac{a^2x(Ab-aB)}{a^2+b^2}+\frac{b^3(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}}{a}-\frac{(Ab-aB)\cot(c+dx)}{ad}}{a}-\frac{A\cot^2(c+dx)}{2ad}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-1/2*(A*Cot[c + d*x]^2)/(a*d) - (-(((A*b - a*B)*Cot[c + d*x])/(a*d)) + (-(
(a^2*(A*b - a*B)*x)/(a^2 + b^2)) + ((a^2*A - A*b^2 + a*b*B)*Log[-Sin[c + d
*x]])/(a*d) + (b^3*(A*b - a*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a
^2 + b^2)*d))/a)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\frac{(aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Ab-Ba)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{A}{2a\tan(dx+c)^2}-\frac{-Ab+Ba}{a^2\tan(dx+c)}+\frac{\left(-A\,a^2+A\,b^2-Bab\right)\ln(\tan(dx+c))}{a^3}$
default	$\frac{\frac{\frac{(aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2}+(Ab-Ba)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{A}{2a\tan(dx+c)^2}-\frac{-Ab+Ba}{a^2\tan(dx+c)}+\frac{\left(-A\,a^2+A\,b^2-Bab\right)\ln(\tan(dx+c)}{a^3}$
parallelrisch	$\frac{\left(-2A\,b^4+2Ba\,b^3\right)\ln(a+b\tan(dx+c))+\left(A\,a^4+B\,a^3b\right)\ln\left(\sec(dx+c)^2\right)+\left(-2A\,a^4+2A\,b^4-2B\,a^3b-2Ba\,b^3\right)\ln(\tan(dx+c)}{2(a^2+b^2)a^3d}$
norman	$\frac{\frac{(Ab-Ba)\tan(dx+c)}{a^2d}+\frac{(Ab-Ba)x\tan(dx+c)^2}{a^2+b^2}-\frac{A}{2ad}}{\tan(dx+c)^2}+\frac{(aA+Bb)\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}-\frac{\left(A\,a^2-A\,b^2+Bab\right)\ln(\tan(dx+c)}{a^3d}$
risch	$\frac{xB}{ib-a}+\frac{ixA}{ib-a}-\frac{2ib^3Bx}{a^2(a^2+b^2)}-\frac{2i\left(iAa\,e^{2i(dx+c)}-Ab\,e^{2i(dx+c)}+Ba\,e^{2i(dx+c)}+Ab-Ba\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}-\frac{2ib^3Bc}{a^2d(a^2+b^2)}-\frac{2iA\,b}{a^3}$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/(a^2+b^2)*(1/2*(A*a+B*b)*ln(1+tan(d*x+c)^2)+(A*b-B*a)*arctan(tan(d*
x+c)))-1/2/a*A/tan(d*x+c)^2-(-A*b+B*a)/a^2/tan(d*x+c)+1/a^3*(-A*a^2+A*b^2-
B*a*b)*ln(tan(d*x+c))-(A*b-B*a)*b^3/(a^2+b^2)/a^3*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.71

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{Aa^4 + Aa^2b^2 + (Aa^4 + Ba^3b + Bab^3 - Ab^4)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2 - (Bab^3 - Ab^4)\log\left(\frac{b^2\tan\cdots}{\cdots}\right)\cdots}{\cdots}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/2*(A*a^4 + A*a^2*b^2 + (A*a^4 + B*a^3*b + B*a*b^3 - A*b^4)*log(tan(d*x
+ c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 - (B*a*b^3 - A*b^4)*log((b^2*t
an(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))*tan(d*x +
c)^2 + (A*a^4 + A*a^2*b^2 + 2*(B*a^4 - A*a^3*b)*d*x)*tan(d*x + c)^2 + 2*(B
*a^4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)*tan(d*x + c))/((a^5 + a^3*b^2)*d*tan
(d*x + c)^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 2599, normalized size of antiderivative = 18.97

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((A*log(ta
n(c + d*x)**2 + 1)/(2*d) - A*log(tan(c + d*x))/d - A/(2*d*tan(c + d*x)**2)
 - B*x - B/(d*tan(c + d*x)))/a, Eq(b, 0)), ((A*x + A/(d*tan(c + d*x)) - A/
(3*d*tan(c + d*x)**3) + B*log(tan(c + d*x)**2 + 1)/(2*d) - B*log(tan(c + d
*x))/d - B/(2*d*tan(c + d*x)**2))/b, Eq(a, 0)), (-3*I*A*d*x*tan(c + d*x)**
3/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) + 3*A*d*x*tan(c + d*x)
**2/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) + 2*A*log(tan(c + d*
x)**2 + 1)*tan(c + d*x)**3/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**
2) + 2*I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**3
 + 2*I*a*d*tan(c + d*x)**2) - 4*A*log(tan(c + d*x))*tan(c + d*x)**3/(2*a*d
*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - 4*I*A*log(tan(c + d*x))*tan(
c + d*x)**2/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - 3*I*A*tan(
c + d*x)**2/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) + A*tan(c +
d*x)/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - I*A/(2*a*d*tan(c
+ d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - 3*B*d*x*tan(c + d*x)**3/(2*a*d*tan(
c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - 3*I*B*d*x*tan(c + d*x)**2/(2*a*d*
tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - I*B*log(tan(c + d*x)**2 + 1)*
tan(c + d*x)**3/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) + B*log(
tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(
c + d*x)**2) + 2*I*B*log(tan(c + d*x))*tan(c + d*x)**3/(2*a*d*tan(c + d...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.15

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx =$$

$$-\frac{\frac{2\,(Ba-Ab)(dx+c)}{a^2+b^2}-\frac{2\,(Bab^3-Ab^4)\log(b\tan(dx+c)+a)}{a^5+a^3b^2}-\frac{(Aa+Bb)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}+\frac{2\,(Aa^2+Bab-Ab^2)\log(\tan(dx+c))}{a^3}+\cdots}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/2*(2*(B*a - A*b)*(d*x + c)/(a^2 + b^2) - 2*(B*a*b^3 - A*b^4)*log(b*tan(
d*x + c) + a)/(a^5 + a^3*b^2) - (A*a + B*b)*log(tan(d*x + c)^2 + 1)/(a^2 +
 b^2) + 2*(A*a^2 + B*a*b - A*b^2)*log(tan(d*x + c))/a^3 + (A*a + 2*(B*a -
A*b)*tan(d*x + c))/(a^2*tan(d*x + c)^2))/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.30

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(Ba-Ab)(dx+c)}{a^2d+b^2d} + \frac{(Aa+Bb)\log\left(\tan(dx+c)^2+1\right)}{2\left(a^2d+b^2d\right)} + \frac{\left(Bab^4-Ab^5\right)\log\left(|b\tan(dx+c)+a|\right)}{a^5bd+a^3b^3d} - \frac{\left(Aa^2+Bab-Ab^2\right)\log\left(|\tan(dx+c)|\right)}{a^3d} - \frac{Aa^2+2\left(Ba^2-Aab\right)\tan(dx+c)}{2\,a^3d\tan(dx+c)^2}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
-(B*a - A*b)*(d*x + c)/(a^2*d + b^2*d) + 1/2*(A*a + B*b)*log(tan(d*x + c)^
2 + 1)/(a^2*d + b^2*d) + (B*a*b^4 - A*b^5)*log(abs(b*tan(d*x + c) + a))/(a
^5*b*d + a^3*b^3*d) - (A*a^2 + B*a*b - A*b^2)*log(abs(tan(d*x + c)))/(a^3*
d) - 1/2*(A*a^2 + 2*(B*a^2 - A*a*b)*tan(d*x + c))/(a^3*d*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{\cot(c+d\,x)^2\left(\frac{A}{2\,a}-\frac{\tan(c+d\,x)\,(A\,b-B\,a)}{a^2}\right)}{d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,(-B+A\,1\mathrm{i})}{2\,d\,(-b+a\,1\mathrm{i})} - \frac{\ln\left(\tan(c+d\,x)\right)\,\left(A\,a^2+B\,a\,b-A\,b^2\right)}{a^3\,d} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(A\,b^4-B\,a\,b^3\right)}{d\,\left(a^5+a^3\,b^2\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,(A-B\,1\mathrm{i})}{2\,d\,(a-b\,1\mathrm{i})}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
(log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(a*1i - b)) - (cot(c + d*x)^2*(A/
(2*a) - (tan(c + d*x)*(A*b - B*a))/a^2))/d - (log(tan(c + d*x))*(A*a^2 - A
*b^2 + B*a*b))/(a^3*d) - (log(a + b*tan(c + d*x))*(A*b^4 - B*a*b^3))/(d*(a
^5 + a^3*b^2)) + (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a - b*1i))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.49

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+c\right)^2-4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sin\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2}{4\sin\left(dx+c\right)^2d}$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(4*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2 - 4*log(tan((c + d*x)/2))*
sin(c + d*x)**2 + sin(c + d*x)**2 - 2)/(4*sin(c + d*x)**2*d)
```

3.274 $\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 169

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{(aA+bB)x}{a^2+b^2}+\frac{(a^2A-Ab^2+abB)\cot(c+dx)}{a^3d}+\frac{(Ab-aB)\cot^2(c+dx)}{2a^2d} - \frac{A\cot^3(c+dx)}{3ad}+\frac{(a^2-b^2)(Ab-aB)\log(\sin(c+dx))}{a^4d} + \frac{b^4(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4(a^2+b^2)d}$$

output

```
(A*a+B*b)*x/(a^2+b^2)+(A*a^2-A*b^2+B*a*b)*cot(d*x+c)/a^3/d+1/2*(A*b-B*a)*c
ot(d*x+c)^2/a^2/d-1/3*A*cot(d*x+c)^3/a/d+(a^2-b^2)*(A*b-B*a)*ln(sin(d*x+c)
)/a^4/d+b^4*(A*b-B*a)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^4/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.15

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{6(a^2A-Ab^2+abB)\cot(c+dx)}{a^3} + \frac{3(Ab-aB)\cot^2(c+dx)}{a^2} - \frac{2A\cot^3(c+dx)}{a} + \frac{3(-iA+B)\log(i-\tan(c+dx))}{a+ib} + \frac{6(a-b)(a+b)(Ab-aB)\log(}{a^4}}{6d}$$

input
```
Integrate[(Cot[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
((6*(a^2*A - A*b^2 + a*b*B)*Cot[c + d*x])/a^3 + (3*(A*b - a*B)*Cot[c + d*x
]^2)/a^2 - (2*A*Cot[c + d*x]^3)/a + (3*((-I)*A + B)*Log[I - Tan[c + d*x]])
/(a + I*b) + (6*(a - b)*(a + b)*(A*b - a*B)*Log[Tan[c + d*x]])/a^4 + (3*(I
*A + B)*Log[I + Tan[c + d*x]])/(a - I*b) + (6*b^4*(A*b - a*B)*Log[a + b*Ta
n[c + d*x]])/(a^4*(a^2 + b^2)))/(6*d)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^4(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 4092}$$

$$-\frac{\int \frac{3\cot^3(c+dx)(Ab\tan^2(c+dx)+aA\tan(c+dx)+Ab-aB)}{a+b\tan(c+dx)}dx}{3a} - \frac{A\cot^3(c+dx)}{3ad}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\cot^3(c+dx)\left(Ab\tan^2(c+dx)+aA\tan(c+dx)+Ab-aB\right)}{a+b\tan(c+dx)}dx}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{Ab\tan(c+dx)^2+aA\tan(c+dx)+Ab-aB}{\tan(c+dx)^3(a+b\tan(c+dx))}dx}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$$\Big\downarrow \; 4132$$

$$-\frac{-\frac{\int -\frac{2\cot^2(c+dx)\left(Aa^2+B\tan(c+dx)a^2+bBa-Ab^2-b(Ab-aB)\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx}{2a}-\frac{(Ab-aB)\cot^2(c+dx)}{2ad}}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\cot^2(c+dx)\left(Aa^2+B\tan(c+dx)a^2+bBa-Ab^2-b(Ab-aB)\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx}{a}-\frac{(Ab-aB)\cot^2(c+dx)}{2ad}}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{Aa^2+B\tan(c+dx)a^2+bBa-Ab^2-b(Ab-aB)\tan(c+dx)^2}{\tan(c+dx)^2(a+b\tan(c+dx))}dx}{a}-\frac{(Ab-aB)\cot^2(c+dx)}{2ad}}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\frac{-\frac{\int \frac{\cot(c+dx)\left(A\tan(c+dx)a^3+b\left(Aa^2+bBa-Ab^2\right)\tan^2(c+dx)+\left(a^2-b^2\right)(Ab-aB)\right)}{a+b\tan(c+dx)}dx}{a}-\frac{\left(a^2A+abB-Ab^2\right)\cot(c+dx)}{ad}}{a}-\frac{(Ab-aB)\cot^2(c+dx)}{2ad}}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{-\frac{\int \frac{A\tan(c+dx)a^3+b\left(Aa^2+bBa-Ab^2\right)\tan(c+dx)^2+\left(a^2-b^2\right)(Ab-aB)}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a}-\frac{\left(a^2A+abB-Ab^2\right)\cot(c+dx)}{ad}}{a}-\frac{(Ab-aB)\cot^2(c+dx)}{2ad}}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$$\Big\downarrow \; 4134$$

$$-\frac{\frac{-\frac{\frac{(a^2-b^2)(Ab-aB)\int\cot(c+dx)dx}{a}+\frac{b^4(Ab-aB)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{a^3x(aA+bB)}{a^2+b^2}}{a}-\frac{(a^2A+abB-Ab^2)\cot(c+dx)}{ad}}{a}-\frac{(Ab-aB)\cot^2(c+dx)}{2ad}}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{-\frac{\frac{(a^2-b^2)(Ab-aB)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\frac{b^4(Ab-aB)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{a^3x(aA+bB)}{a^2+b^2}}{a}-\frac{(a^2A+abB-Ab^2)\cot(c+dx)}{ad}}{a}-\frac{(Ab-aB)\cot^2(c+dx)}{2ad}}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$\Big\downarrow$ 25

$$-\frac{\frac{-\frac{-\frac{(a^2-b^2)(Ab-aB)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}+\frac{b^4(Ab-aB)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{a^3x(aA+bB)}{a^2+b^2}}{a}-\frac{(a^2A+abB-Ab^2)\cot(c+dx)}{ad}}{a}-\frac{(Ab-aB)\cot^2(c+dx)}{2ad}}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$\Big\downarrow$ 3956

$$-\frac{\frac{-\frac{\frac{b^4(Ab-aB)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{(a^2-b^2)(Ab-aB)\log(-\sin(c+dx))}{ad}+\frac{a^3x(aA+bB)}{a^2+b^2}}{a}-\frac{(a^2A+abB-Ab^2)\cot(c+dx)}{ad}}{a}-\frac{(Ab-aB)\cot^2(c+dx)}{2ad}}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

$\Big\downarrow$ 4013

$$-\frac{\frac{-\frac{(a^2A+abB-Ab^2)\cot(c+dx)}{ad}-\frac{\frac{(a^2-b^2)(Ab-aB)\log(-\sin(c+dx))}{ad}+\frac{b^4(Ab-aB)\log(a\cos(c+dx)+b\sin(c+dx))}{ad(a^2+b^2)}+\frac{a^3x(aA+bB)}{a^2+b^2}}{a}}{a}-\frac{(Ab-aB)\cot^2(c+dx)}{2ad}}{a}-\frac{A\cot^3(c+dx)}{3ad}$$

input
```
Int[(Cot[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-1/3*(A*Cot[c + d*x]^3)/(a*d) - (-1/2*((A*b - a*B)*Cot[c + d*x]^2)/(a*d) +
 (-(((a^2*A - A*b^2 + a*b*B)*Cot[c + d*x])/(a*d)) - ((a^3*(a*A + b*B)*x)/(
a^2 + b^2) + ((a^2 - b^2)*(A*b - a*B)*Log[-Sin[c + d*x]])/(a*d) + (b^4*(A*
b - a*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/a)/a)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{\frac{(-Ab+Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(aA+Bb)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{-A\,a^2+A\,b^2-Bab}{a^3\tan(dx+c)}+\frac{\left(A\,a^2b-A\,b^3-B\,a^3+Ba\,b^2\right)\ln(\tan(dx+c))}{a^4}-}{d}$
default	$\frac{\frac{\frac{(-Ab+Ba)\ln\left(1+\tan(dx+c)^2\right)}{2}+(aA+Bb)\arctan(\tan(dx+c))}{a^2+b^2}-\frac{-A\,a^2+A\,b^2-Bab}{a^3\tan(dx+c)}+\frac{\left(A\,a^2b-A\,b^3-B\,a^3+Ba\,b^2\right)\ln(\tan(dx+c))}{a^4}-}{d}$
parallelrisch	$\frac{(6A\,b^5-6Ba\,b^4)\ln(a+b\tan(dx+c))+(-3A\,a^4b+3B\,a^5)\ln\left(\sec(dx+c)^2\right)+(6A\,a^4b-6A\,b^5-6B\,a^5+6Ba\,b^4)\ln(\tan(dx}{6(a}$
norman	$\frac{\frac{(aA+Bb)x\tan(dx+c)^3}{a^2+b^2}+\frac{\left(A\,a^2-A\,b^2+Bab\right)\tan(dx+c)^2}{a^3d}-\frac{A}{3ad}+\frac{(Ab-Ba)\tan(dx+c)}{2a^2d}}{\tan(dx+c)^3}+\frac{(Ab-Ba)\left(a^2-b^2\right)\ln(\tan(dx+c))}{a^4d}-$
risch	$-\frac{2ib^5Ac}{(a^2+b^2)a^4d}-\frac{xA}{ib-a}-\frac{2iB\,b^2c}{a^3d}+\frac{2iA\,b^3c}{a^4d}+\frac{2ib^4Bc}{(a^2+b^2)a^3d}-\frac{2iAbx}{a^2}+\frac{2ixB}{a}+\frac{2iA\,b^3x}{a^4}-\frac{2iAbc}{a^2d}+\frac{ixB}{ib-a}$

input
```
int(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/(a^2+b^2)*(1/2*(-A*b+B*a)*ln(1+tan(d*x+c)^2)+(A*a+B*b)*arctan(tan(d
*x+c)))-(-A*a^2+A*b^2-B*a*b)/a^3/tan(d*x+c)+1/a^4*(A*a^2*b-A*b^3-B*a^3+B*a
*b^2)*ln(tan(d*x+c))-1/3/a*A/tan(d*x+c)^3-1/2*(-A*b+B*a)/a^2/tan(d*x+c)^2+
(A*b-B*a)*b^4/(a^2+b^2)/a^4*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.73

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx =$$

$$-\frac{2\,Aa^5+2\,Aa^3b^2+3\,(Ba^5-Aa^4b-Bab^4+Ab^5)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^3+3\,(Bab^4-Ab^5)\log}{}$$

input
```
integrate(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/6*(2*A*a^5 + 2*A*a^3*b^2 + 3*(B*a^5 - A*a^4*b - B*a*b^4 + A*b^5)*log(ta
n(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^3 + 3*(B*a*b^4 - A*b^5)*lo
g((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))*ta
n(d*x + c)^3 + 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*(A*a^5 + B*a
^4*b)*d*x)*tan(d*x + c)^3 - 6*(A*a^5 + B*a^4*b + B*a^2*b^3 - A*a*b^4)*tan(
d*x + c)^2 + 3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3)*tan(d*x + c))/((a
^6 + a^4*b^2)*d*tan(d*x + c)^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.67 (sec) , antiderivative size = 3016, normalized size of antiderivative = 17.85

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*A*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), ((A*x + A/
(d*tan(c + d*x)) - A/(3*d*tan(c + d*x)**3) + B*log(tan(c + d*x)**2 + 1)/(2
*d) - B*log(tan(c + d*x))/d - B/(2*d*tan(c + d*x)**2))/a, Eq(b, 0)), ((-A*
log(tan(c + d*x)**2 + 1)/(2*d) + A*log(tan(c + d*x))/d + A/(2*d*tan(c + d*
x)**2) - A/(4*d*tan(c + d*x)**4) + B*x + B/(d*tan(c + d*x)) - B/(3*d*tan(c
 + d*x)**3))/b, Eq(a, 0)), (15*A*d*x*tan(c + d*x)**4/(6*a*d*tan(c + d*x)**
4 + 6*I*a*d*tan(c + d*x)**3) + 15*I*A*d*x*tan(c + d*x)**3/(6*a*d*tan(c + d
*x)**4 + 6*I*a*d*tan(c + d*x)**3) + 6*I*A*log(tan(c + d*x)**2 + 1)*tan(c +
 d*x)**4/(6*a*d*tan(c + d*x)**4 + 6*I*a*d*tan(c + d*x)**3) - 6*A*log(tan(c
 + d*x)**2 + 1)*tan(c + d*x)**3/(6*a*d*tan(c + d*x)**4 + 6*I*a*d*tan(c + d
*x)**3) - 12*I*A*log(tan(c + d*x))*tan(c + d*x)**4/(6*a*d*tan(c + d*x)**4
+ 6*I*a*d*tan(c + d*x)**3) + 12*A*log(tan(c + d*x))*tan(c + d*x)**3/(6*a*d
*tan(c + d*x)**4 + 6*I*a*d*tan(c + d*x)**3) + 15*A*tan(c + d*x)**3/(6*a*d*
tan(c + d*x)**4 + 6*I*a*d*tan(c + d*x)**3) + 9*I*A*tan(c + d*x)**2/(6*a*d*
tan(c + d*x)**4 + 6*I*a*d*tan(c + d*x)**3) + A*tan(c + d*x)/(6*a*d*tan(c +
 d*x)**4 + 6*I*a*d*tan(c + d*x)**3) - 2*I*A/(6*a*d*tan(c + d*x)**4 + 6*I*a
*d*tan(c + d*x)**3) - 9*I*B*d*x*tan(c + d*x)**4/(6*a*d*tan(c + d*x)**4 + 6
*I*a*d*tan(c + d*x)**3) + 9*B*d*x*tan(c + d*x)**3/(6*a*d*tan(c + d*x)**4 +
 6*I*a*d*tan(c + d*x)**3) + 6*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(
6*a*d*tan(c + d*x)**4 + 6*I*a*d*tan(c + d*x)**3) + 6*I*B*log(tan(c + d*...
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.18

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{6\,(Aa+Bb)(dx+c)}{a^2+b^2} - \frac{6\,(Bab^4-Ab^5)\log(b\tan(dx+c)+a)}{a^6+a^4b^2} + \frac{3\,(Ba-Ab)\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} - \frac{6\,(Ba^3-Aa^2b-Bab^2+Ab^3)\log(\tan(dx+c)}{a^4}}{6\,d}$$

input
```
integrate(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
1/6*(6*(A*a + B*b)*(d*x + c)/(a^2 + b^2) - 6*(B*a*b^4 - A*b^5)*log(b*tan(d
*x + c) + a)/(a^6 + a^4*b^2) + 3*(B*a - A*b)*log(tan(d*x + c)^2 + 1)/(a^2
+ b^2) - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*log(tan(d*x + c))/a^4 - (2*
A*a^2 - 6*(A*a^2 + B*a*b - A*b^2)*tan(d*x + c)^2 + 3*(B*a^2 - A*a*b)*tan(d
*x + c))/(a^3*tan(d*x + c)^3))/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.30

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{(Aa+Bb)(dx+c)}{a^2d+b^2d} + \frac{(Ba-Ab)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^2d+b^2d\right)} - \frac{(Bab^5-Ab^6)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^6bd+a^4b^3d} - \frac{(Ba^3-Aa^2b-Bab^2+Ab^3)\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^4d} - \frac{2\,Aa^3-6\left(Aa^3+Ba^2b-Aab^2\right)\tan\left(dx+c\right)^2+3\left(Ba^3-Aa^2b\right)\tan\left(dx+c\right)}{6\,a^4d\tan\left(dx+c\right)^3}$$

input
```
integrate(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
(A*a + B*b)*(d*x + c)/(a^2*d + b^2*d) + 1/2*(B*a - A*b)*log(tan(d*x + c)^2
 + 1)/(a^2*d + b^2*d) - (B*a*b^5 - A*b^6)*log(abs(b*tan(d*x + c) + a))/(a^
6*b*d + a^4*b^3*d) - (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*log(abs(tan(d*x +
 c)))/(a^4*d) - 1/6*(2*A*a^3 - 6*(A*a^3 + B*a^2*b - A*a*b^2)*tan(d*x + c)^
2 + 3*(B*a^3 - A*a^2*b)*tan(d*x + c))/(a^4*d*tan(d*x + c)^3)
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.23

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\cot(c+d\,x)^3\,\left(\frac{\tan(c+d\,x)^2\,\left(A\,a^2+B\,a\,b-A\,b^2\right)}{a^3}-\frac{A}{3\,a}+\frac{\tan(c+d\,x)\,\left(A\,b-B\,a\right)}{2\,a^2}\right)}{d} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(A\,b^5-B\,a\,b^4\right)}{d\,\left(a^6+a^4\,b^2\right)} - \frac{\ln\left(\tan(c+d\,x)\right)\,\left(B\,a^3-A\,a^2\,b-B\,a\,b^2+A\,b^3\right)}{a^4\,d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int((cot(c + d*x)^4*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
(cot(c + d*x)^3*((tan(c + d*x)^2*(A*a^2 - A*b^2 + B*a*b))/a^3 - A/(3*a) +
(tan(c + d*x)*(A*b - B*a))/(2*a^2)))/d + (log(a + b*tan(c + d*x))*(A*b^5 -
 B*a*b^4))/(d*(a^6 + a^4*b^2)) - (log(tan(c + d*x))*(A*b^3 + B*a^3 - A*a^2
*b - B*a*b^2))/(a^4*d) - (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a*1i +
b)) - (log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(a + b*1i))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.17

$$\int \frac{\cot^4(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{-\cot(dx+c)^3+3\cot(dx+c)+3dx}{3d}$$

input
```
int(cot(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
( - cot(c + d*x)**3 + 3*cot(c + d*x) + 3*d*x)/(3*d)
```

3.275 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 208

$$\begin{aligned}
&\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx \\
&= -\frac{(2aAb-a^2B+b^2B)\,x}{(a^2+b^2)^2}+\frac{(a^2A-Ab^2+2abB)\log(\cos(c+dx))}{(a^2+b^2)^2\,d} \\
&\quad+\frac{a^2(a^2Ab+3Ab^3-2a^3B-4ab^2B)\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)^2\,d} \\
&\quad-\frac{(aAb-2a^2B-b^2B)\tan(c+dx)}{b^2\,(a^2+b^2)\,d}+\frac{a(Ab-aB)\tan^2(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
-(2*A*a*b-B*a^2+B*b^2)*x/(a^2+b^2)^2+(A*a^2-A*b^2+2*B*a*b)*ln(cos(d*x+c))/
(a^2+b^2)^2/d+a^2*(A*a^2*b+3*A*b^3-2*B*a^3-4*B*a*b^2)*ln(a+b*tan(d*x+c))/b
^3/(a^2+b^2)^2/d-(A*a*b-2*B*a^2-B*b^2)*tan(d*x+c)/b^2/(a^2+b^2)/d+a*(A*b-B
*a)*tan(d*x+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.33 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.93

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{(A+iB)\log(i-\tan(c+dx))}{(a+ib)^2}+\frac{(A-iB)\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2a^2(-a^2Ab-3Ab^3+2a^3B+4ab^2B)\log(a+b\tan(c+dx))}{b^3(a^2+b^2)^2}+\frac{2a^2(-aAb+2a\ldots}{b^3(a^2+b^2)(a+b\ldots}}{2d}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
-1/2*(((A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b)^2 + ((A - I*B)*Log[I + T
an[c + d*x]])/(a - I*b)^2 + (2*a^2*(-(a^2*A*b) - 3*A*b^3 + 2*a^3*B + 4*a*b
^2*B)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)^2) + (2*a^2*(-(a*A*b) + 2*
a^2*B + b^2*B))/(b^3*(a^2 + b^2)*(a + b*Tan[c + d*x])) - (2*B*Tan[c + d*x]
^2)/(b*(a + b*Tan[c + d*x])))/d
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4088, 25, 3042, 4130, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{\int -\frac{\tan(c+dx)\left(\left(-2Ba^2+Aba-b^2B\right)\tan^2(c+dx)-b(Ab-aB)\tan(c+dx)+2a(Ab-aB)\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-\frac{\int \frac{\tan(c+dx)\left(\left(-2Ba^2+Aba-b^2B\right)\tan^2(c+dx)-b(Ab-aB)\tan(c+dx)+2a(Ab-aB)\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-\frac{\int \frac{\tan(c+dx)\left(\left(-2Ba^2+Aba-b^2B\right)\tan(c+dx)^2-b(Ab-aB)\tan(c+dx)+2a(Ab-aB)\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 4130$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-\frac{\frac{\int -\frac{-\left((aA+bB)\tan(c+dx)b^2\right)+\left(a^2+b^2\right)(Ab-2aB)\tan^2(c+dx)+a\left(-2Ba^2+Aba-b^2B\right)}{a+b\tan(c+dx)}dx}{b}+\frac{\left(-2a^2B+aAb-b^2B\right)\tan(c+dx)}{bd}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-\frac{\frac{\left(-2a^2B+aAb-b^2B\right)\tan(c+dx)}{bd}-\frac{\int \frac{-\left((aA+bB)\tan(c+dx)b^2\right)+\left(a^2+b^2\right)(Ab-2aB)\tan^2(c+dx)+a\left(-2Ba^2+Aba-b^2B\right)}{a+b\tan(c+dx)}dx}{b}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-\frac{\frac{\left(-2a^2B+aAb-b^2B\right)\tan(c+dx)}{bd}-\frac{\int \frac{-\left((aA+bB)\tan(c+dx)b^2\right)+\left(a^2+b^2\right)(Ab-2aB)\tan(c+dx)^2+a\left(-2Ba^2+Aba-b^2B\right)}{a+b\tan(c+dx)}dx}{b}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 4109$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{(-2a^2B+aAb-b^2B)\tan(c+dx)}{bd}-\frac{-\frac{b^2(a^2A+2abB-Ab^2)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^2(-2a^3B+a^2Ab-4ab^2B+3Ab^3)\int\frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b^2x(a^2(-B)+2aAb\cdots}{a^2+b^2}}{b}}{b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{(-2a^2B+aAb-b^2B)\tan(c+dx)}{bd}-\frac{-\frac{b^2(a^2A+2abB-Ab^2)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^2(-2a^3B+a^2Ab-4ab^2B+3Ab^3)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b^2x(a^2(-B)+2aAb\cdots}{a^2+b^2}}{b}}{b(a^2+b^2)}$$

$\downarrow$ 3956

$$\frac{a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{(-2a^2B+aAb-b^2B)\tan(c+dx)}{bd}-\frac{\frac{a^2(-2a^3B+a^2Ab-4ab^2B+3Ab^3)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2(a^2A+2abB-Ab^2)\log(\cos(c+dx))}{d(a^2+b^2)}-\frac{b^2x(a^2(-B)+2aAb+b\cdots}{a^2+b^2}}{b}}{b(a^2+b^2)}$$

$\downarrow$ 4100

$$\frac{a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{(-2a^2B+aAb-b^2B)\tan(c+dx)}{bd}-\frac{\frac{a^2(-2a^3B+a^2Ab-4ab^2B+3Ab^3)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd(a^2+b^2)}+\frac{b^2(a^2A+2abB-Ab^2)\log(\cos(c+dx))}{d(a^2+b^2)}-\frac{b^2x(a^2(-\cdots}{\cdots}}{b}}{b(a^2+b^2)}$$

$\downarrow$ 16

$$\frac{a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{(-2a^2B+aAb-b^2B)\tan(c+dx)}{bd}-\frac{\frac{b^2(a^2A+2abB-Ab^2)\log(\cos(c+dx))}{d(a^2+b^2)}-\frac{b^2x(a^2(-B)+2aAb+b^2B)}{a^2+b^2}+\frac{a^2(-2a^3B+a^2Ab-4ab^2B+3Ab^3)\log(a+b\tan(c+\cdots}{bd(a^2+b^2)}}{b}}{b(a^2+b^2)}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
(a*(A*b - a*B)*Tan[c + d*x]^2)/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - (-
((-((b^2*(2*a*A*b - a^2*B + b^2*B)*x)/(a^2 + b^2)) + (b^2*(a^2*A - A*b^2 +
 2*a*b*B)*Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a^2*(a^2*A*b + 3*A*b^3 - 2
*a^3*B - 4*a*b^2*B)*Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d))/b) + ((a*A
*b - 2*a^2*B - b^2*B)*Tan[c + d*x])/(b*d))/(b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)B}{b^2}+\frac{\frac{\left(-A\,a^2+A\,b^2-2Bab\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-2Aab+B\,a^2-B\,b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{a^2\left(A\,a^2b+3A\,b^3-2B\,a^3-4}{b^3\left(a^2+\right.}}{d}$
default	$\frac{\frac{\tan(dx+c)B}{b^2}+\frac{\frac{\left(-A\,a^2+A\,b^2-2Bab\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-2Aab+B\,a^2-B\,b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{a^2\left(A\,a^2b+3A\,b^3-2B\,a^3-4}{b^3\left(a^2+\right.}}{d}$
norman	$\frac{\frac{B\tan(dx+c)^2}{bd}+\frac{\left(A\,a^2b-2B\,a^3-Ba\,b^2\right)a}{d\,b^3\left(a^2+b^2\right)}-\frac{a\left(2Aab-B\,a^2+B\,b^2\right)x}{a^4+2a^2b^2+b^4}-\frac{b\left(2Aab-B\,a^2+B\,b^2\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4}}{a+b\tan(dx+c)}+\frac{a^2\left(A\,a^2b+3A\,b^3-2\right.}{(a}$
parallelrisch	$-\frac{4Ax\tan(dx+c)a\,b^5d-2Bx\tan(dx+c)a^2b^4d-4B\tan(dx+c)^2a^2b^4-A\ln\left(1+\tan(dx+c)^2\right)\tan(dx+c)b^6+A\ln\left(1+\tan\right.}{}$
risch	$-\frac{xB}{2iab-a^2+b^2}+\frac{2iAc}{b^2d}-\frac{2ia^4Ax}{b^2(a^4+2a^2b^2+b^4)}-\frac{4iBac}{b^3d}-\frac{2ia^4Ac}{b^2d(a^4+2a^2b^2+b^4)}-\frac{6ia^2Ax}{a^4+2a^2b^2+b^4}-\frac{6ia^2Ac}{d(a^4+2a^2b^2}$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(tan(d*x+c)*B/b^2+1/(a^2+b^2)^2*(1/2*(-A*a^2+A*b^2-2*B*a*b)*ln(1+tan(d
*x+c)^2)+(-2*A*a*b+B*a^2-B*b^2)*arctan(tan(d*x+c)))+1/b^3*a^2*(A*a^2*b+3*A
*b^3-2*B*a^3-4*B*a*b^2)/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/b^3*a^3*(A*b-B*a)
/(a^2+b^2)/(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(209) = 418$.

Time = 0.13 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.09

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{2\,Ba^4b^2-2\,Aa^3b^3-2\,(Ba^3b^3-2\,Aa^2b^4-Bab^5)dx-2\,(Ba^4b^2+2\,Ba^2b^4+Bb^6)\tan{(dx+c)}^2+(2\,B}{}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
-1/2*(2*B*a^4*b^2 - 2*A*a^3*b^3 - 2*(B*a^3*b^3 - 2*A*a^2*b^4 - B*a*b^5)*d*
x - 2*(B*a^4*b^2 + 2*B*a^2*b^4 + B*b^6)*tan(d*x + c)^2 + (2*B*a^6 - A*a^5*
b + 4*B*a^4*b^2 - 3*A*a^3*b^3 + (2*B*a^5*b - A*a^4*b^2 + 4*B*a^3*b^3 - 3*A
*a^2*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2
)/(tan(d*x + c)^2 + 1)) - (2*B*a^6 - A*a^5*b + 4*B*a^4*b^2 - 2*A*a^3*b^3 +
 2*B*a^2*b^4 - A*a*b^5 + (2*B*a^5*b - A*a^4*b^2 + 4*B*a^3*b^3 - 2*A*a^2*b^
4 + 2*B*a*b^5 - A*b^6)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 2*(2*B*
a^5*b - A*a^4*b^2 + 2*B*a^3*b^3 + B*a*b^5 + (B*a^2*b^4 - 2*A*a*b^5 - B*b^6
)*d*x)*tan(d*x + c))/((a^4*b^4 + 2*a^2*b^6 + b^8)*d*tan(d*x + c) + (a^5*b^
3 + 2*a^3*b^5 + a*b^7)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 4534, normalized size of antiderivative = 21.80

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*(A + B*tan(c))*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (
(-A*log(tan(c + d*x)**2 + 1)/(2*d) + A*tan(c + d*x)**2/(2*d) + B*x + B*tan
(c + d*x)**3/(3*d) - B*tan(c + d*x)/d)/a**2, Eq(b, 0)), (3*I*A*d*x*tan(c +
 d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) +
 6*A*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x)
- 4*b**2*d) - 3*I*A*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x
) - 4*b**2*d) + 2*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan
(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*A*log(tan(c + d*x
)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x)
 - 4*b**2*d) - 2*A*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 - 8*
I*b**2*d*tan(c + d*x) - 4*b**2*d) - 5*I*A*tan(c + d*x)/(4*b**2*d*tan(c + d
*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*A/(4*b**2*d*tan(c + d*x)*
*2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 9*B*d*x*tan(c + d*x)**2/(4*b**2
*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 18*I*B*d*x*tan(
c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) +
 9*B*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) +
 4*I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2
- 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 8*B*log(tan(c + d*x)**2 + 1)*tan(c
 + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) -
4*I*B*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*t...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.06

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{2\,(Ba^2-2\,Aab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4} - \frac{2\,(2\,Ba^5-Aa^4b+4\,Ba^3b^2-3\,Aa^2b^3)\log(b\tan(dx+c)+a)}{a^4b^3+2\,a^2b^5+b^7} - \frac{(Aa^2+2\,Bab-Ab^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{a^3}{}}{2\,d}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="m
axima")
```

output
```
1/2*(2*(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(2*
B*a^5 - A*a^4*b + 4*B*a^3*b^2 - 3*A*a^2*b^3)*log(b*tan(d*x + c) + a)/(a^4*
b^3 + 2*a^2*b^5 + b^7) - (A*a^2 + 2*B*a*b - A*b^2)*log(tan(d*x + c)^2 + 1)
/(a^4 + 2*a^2*b^2 + b^4) - 2*(B*a^4 - A*a^3*b)/(a^3*b^3 + a*b^5 + (a^2*b^4
 + b^6)*tan(d*x + c)) + 2*B*tan(d*x + c)/b^2)/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.16

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{(Ba^2-2\,Aab-Bb^2)(dx+c)}{a^4d+2\,a^2b^2d+b^4d} - \frac{(Aa^2+2\,Bab-Ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^4d+2\,a^2b^2d+b^4d\right)} - \frac{(2\,Ba^5-Aa^4b+4\,Ba^3b^2-3\,Aa^2b^3)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^4b^3d+2\,a^2b^5d+b^7d} + \frac{B\tan\left(dx+c\right)}{b^2d} - \frac{Ba^6-Aa^5b+Ba^4b^2-Aa^3b^3}{(a^2+b^2)^2(b\tan\left(dx+c\right)+a)b^3d}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="g
iac")
```

output
```
(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4*d + 2*a^2*b^2*d + b^4*d) - 1/2*(A
*a^2 + 2*B*a*b - A*b^2)*log(tan(d*x + c)^2 + 1)/(a^4*d + 2*a^2*b^2*d + b^4
*d) - (2*B*a^5 - A*a^4*b + 4*B*a^3*b^2 - 3*A*a^2*b^3)*log(abs(b*tan(d*x +
c) + a))/(a^4*b^3*d + 2*a^2*b^5*d + b^7*d) + B*tan(d*x + c)/(b^2*d) - (B*a
^6 - A*a^5*b + B*a^4*b^2 - A*a^3*b^3)/((a^2 + b^2)^2*(b*tan(d*x + c) + a)*
b^3*d)
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.01

$$
\begin{aligned}
&\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx \\
&= \frac{B\tan(c+d\,x)}{b^2\,d} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(2\,B\,a^5-A\,a^4\,b+4\,B\,a^3\,b^2-3\,A\,a^2\,b^3\right)}{d\,\left(a^4\,b^3+2\,a^2\,b^5+b^7\right)} \\
&\quad - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)} \\
&\quad - \frac{a^2\,\left(B\,a^2-A\,a\,b\right)}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^3+a\,b^2\right)\,\left(a^2+b^2\right)}
\end{aligned}
$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(B*tan(c + d*x))/(b^2*d) - (log(a + b*tan(c + d*x))*(2*B*a^5 - 3*A*a^2*b^3
 + 4*B*a^3*b^2 - A*a^4*b))/(d*(b^7 + 2*a^2*b^5 + a^4*b^3)) - (log(tan(c +
d*x) - 1i)*(A + B*1i))/(2*d*(a*b*2i + a^2 - b^2)) - (log(tan(c + d*x) + 1i
)*(A*1i + B))/(2*d*(2*a*b + a^2*1i - b^2*1i)) - (a^2*(B*a^2 - A*a*b))/(b*d
*(a*b^2 + b^3*tan(c + d*x))*(a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.39

$$
\begin{aligned}
&\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx \\
&= \frac{-\log\left(\tan\left(dx+c\right)^2+1\right)a\,b^2-2\log(a+\tan\left(dx+c\right)b)\,a^3+2\tan\left(dx+c\right)a^2b+2\tan\left(dx+c\right)b^3-2b^3}{2b^2d\left(a^2+b^2\right)}
\end{aligned}
$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
( - log(tan(c + d*x)**2 + 1)*a*b**2 - 2*log(tan(c + d*x)*b + a)*a**3 + 2*t
an(c + d*x)*a**2*b + 2*tan(c + d*x)*b**3 - 2*b**3*d*x)/(2*b**2*d*(a**2 + b
**2))
```

3.276 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 157

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{(a^2A-Ab^2+2abB)\,x}{(a^2+b^2)^2}-\frac{(2aAb-a^2B+b^2B)\log(\cos(c+dx))}{(a^2+b^2)^2\,d} -\frac{a(2Ab^3-a(a^2+3b^2)\,B)\log(a+b\tan(c+dx))}{b^2\,(a^2+b^2)^2\,d}-\frac{a^2(Ab-aB)}{b^2\,(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
-(A*a^2-A*b^2+2*B*a*b)*x/(a^2+b^2)^2-(2*A*a*b-B*a^2+B*b^2)*ln(cos(d*x+c))/
(a^2+b^2)^2/d-a*(2*A*b^3-a*(a^2+3*b^2)*B)*ln(a+b*tan(d*x+c))/b^2/(a^2+b^2)
^2/d-a^2*(A*b-B*a)/b^2/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{i(A+iB)\log(i-\tan(c+dx))}{(a+ib)^2} - \frac{i(A-iB)\log(i+\tan(c+dx))}{(a-ib)^2} + \frac{2a\left(\left(-2Ab+a\left(3+\frac{a^2}{b^2}\right)B\right)\log(a+b\tan(c+dx))+\frac{a(a^2+b^2)(-Ab+aB)}{b^2(a+b\tan(c+dx))}\right)}{(a^2+b^2)^2}}{2d}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
((I*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*(A - I*B)*Log[I + Ta
n[c + d*x]])/(a - I*b)^2 + (2*a*((-2*A*b + a*(3 + a^2/b^2)*B)*Log[a + b*Ta
n[c + d*x]] + (a*(a^2 + b^2)*(-(A*b) + a*B))/(b^2*(a + b*Tan[c + d*x]))))/
(a^2 + b^2)^2)/(2*d)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 4087, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4087}$$

$$\frac{\int -\frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{a+b\tan(c+dx)}dx}{b\,(a^2+b^2)} - \frac{a^2(Ab-aB)}{b^2d\,(a^2+b^2)\,(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{a+b\tan(c+dx)}dx}{b(a^2+b^2)}-\frac{a^2(Ab-aB)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 3042

$$-\frac{\int \frac{-((a^2+b^2)B\tan(c+dx)^2)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{a+b\tan(c+dx)}dx}{b(a^2+b^2)}-\frac{a^2(Ab-aB)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 4109

$$-\frac{-\frac{b(a^2(-B)+2aAb+b^2B)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a(2Ab^3-aB(a^2+3b^2))\int\frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{bx(a^2A+2abB-Ab^2)}{a^2+b^2}}{b(a^2+b^2)}-\frac{a^2(Ab-aB)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 3042

$$-\frac{-\frac{b(a^2(-B)+2aAb+b^2B)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a(2Ab^3-aB(a^2+3b^2))\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{bx(a^2A+2abB-Ab^2)}{a^2+b^2}}{b(a^2+b^2)}-\frac{a^2(Ab-aB)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 3956

$$-\frac{\frac{a(2Ab^3-aB(a^2+3b^2))\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b(a^2(-B)+2aAb+b^2B)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(a^2A+2abB-Ab^2)}{a^2+b^2}}{b(a^2+b^2)}-\frac{a^2(Ab-aB)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 4100

$$-\frac{\frac{a(2Ab^3-aB(a^2+3b^2))\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd(a^2+b^2)}+\frac{b(a^2(-B)+2aAb+b^2B)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(a^2A+2abB-Ab^2)}{a^2+b^2}}{b(a^2+b^2)}-\frac{a^2(Ab-aB)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}$$

$\downarrow$ 16

$$-\frac{a^2(Ab-aB)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{b(a^2(-B)+2aAb+b^2B)\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{bx(a^2A+2abB-Ab^2)}{a^2+b^2}+\frac{a(2Ab^3-aB(a^2+3b^2))\log(a+b\tan(c+dx))}{bd(a^2+b^2)}}{b(a^2+b^2)}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
-(((b*(a^2*A - A*b^2 + 2*a*b*B)*x)/(a^2 + b^2) + (b*(2*a*A*b - a^2*B + b^2
*B)*Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a*(2*A*b^3 - a*(a^2 + 3*b^2)*B)*
Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d))/(b*(a^2 + b^2))) - (a^2*(A*b -
 a*B))/(b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\frac{\left(2Aab-B\,a^2+B\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-A\,a^2+A\,b^2-2Bab\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{a^2(Ab-Ba)}{b^2\left(a^2+b^2\right)(a+b\tan(dx+c))}-\frac{a\left(2A\,b^3-B\,a\right.}{}}{d}$
default	$\frac{\frac{\frac{\left(2Aab-B\,a^2+B\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-A\,a^2+A\,b^2-2Bab\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{a^2(Ab-Ba)}{b^2\left(a^2+b^2\right)(a+b\tan(dx+c))}-\frac{a\left(2A\,b^3-B\,a\right.}{}}{d}$
norman	$\frac{-\frac{a\left(A\,a^2-A\,b^2+2Bab\right)x}{a^4+2a^2b^2+b^4}-\frac{b\left(A\,a^2-A\,b^2+2Bab\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{\left(Aab-B\,a^2\right)a}{d\,b^2\left(a^2+b^2\right)}}{a+b\tan(dx+c)}+\frac{\left(2Aab-B\,a^2+B\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2d\left(a^4+2a^2b^2+b^4\right)}$
parallelrisch	$\frac{2A\ln\left(1+\tan(dx+c)^2\right)\tan(dx+c)a\,b^4-4A\ln(a+b\tan(dx+c))\tan(dx+c)a\,b^4-B\ln\left(1+\tan(dx+c)^2\right)\tan(dx+c)a^2b^3+}{}$
risch	$-\frac{2ia^4Bx}{(a^4+2a^2b^2+b^4)b^2}+\frac{xA}{2iab-a^2+b^2}-\frac{6ia^2Bx}{a^4+2a^2b^2+b^4}+\frac{2iBc}{d\,b^2}-\frac{ixB}{2iab-a^2+b^2}-\frac{2ia^3B}{(ib+a)db(-ib+a)^2\left(-ib\,e^{2i(dx}\right.}$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^2*(1/2*(2*A*a*b-B*a^2+B*b^2)*ln(1+tan(d*x+c)^2)+(-A*a^2+A
*b^2-2*B*a*b)*arctan(tan(d*x+c)))-a^2*(A*b-B*a)/b^2/(a^2+b^2)/(a+b*tan(d*x
+c))-a*(2*A*b^3-B*a^3-3*B*a*b^2)/(a^2+b^2)^2/b^2*ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(155) = 310.

Time = 0.14 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.98

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2\,Ba^3b^2 - 2\,Aa^2b^3 - 2\,(Aa^3b^2 + 2\,Ba^2b^3 - Aab^4)dx + (Ba^5 + 3\,Ba^3b^2 - 2\,Aa^2b^3 + (Ba^4b + 3\,Ba^2b^3 - 2}{}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
1/2*(2*B*a^3*b^2 - 2*A*a^2*b^3 - 2*(A*a^3*b^2 + 2*B*a^2*b^3 - A*a*b^4)*d*x
 + (B*a^5 + 3*B*a^3*b^2 - 2*A*a^2*b^3 + (B*a^4*b + 3*B*a^2*b^3 - 2*A*a*b^4
)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d
*x + c)^2 + 1)) - (B*a^5 + 2*B*a^3*b^2 + B*a*b^4 + (B*a^4*b + 2*B*a^2*b^3
+ B*b^5)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 2*(B*a^4*b - A*a^3*b^
2 + (A*a^2*b^3 + 2*B*a*b^4 - A*b^5)*d*x)*tan(d*x + c))/((a^4*b^3 + 2*a^2*b
^5 + b^7)*d*tan(d*x + c) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 3485, normalized size of antiderivative = 22.20

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*(A + B*tan(c)), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-A*x +
 A*tan(c + d*x)/d - B*log(tan(c + d*x)**2 + 1)/(2*d) + B*tan(c + d*x)**2/(
2*d))/a**2, Eq(b, 0)), (A*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 -
8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*A*d*x*tan(c + d*x)/(4*b**2*d*tan
(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - A*d*x/(4*b**2*d*tan(c
 + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*A*tan(c + d*x)/(4*b**
2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*A/(4*b**2*
d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 3*I*B*d*x*tan(c
+ d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d)
+ 6*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x)
 - 4*b**2*d) - 3*I*B*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*
x) - 4*b**2*d) + 2*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(4*b**2*d*ta
n(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*I*B*log(tan(c + d*
x)**2 + 1)*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x
) - 4*b**2*d) - 2*B*log(tan(c + d*x)**2 + 1)/(4*b**2*d*tan(c + d*x)**2 - 8
*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 5*I*B*tan(c + d*x)/(4*b**2*d*tan(c +
d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 4*B/(4*b**2*d*tan(c + d*x)
**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (A*d*x*tan(c + d*
x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*
I*A*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.25

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{\frac{2\,(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(Ba^4+3\,Ba^2b^2-2\,Aab^3)\log(b\tan(dx+c)+a)}{a^4b^2+2\,a^2b^4+b^6}+\frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{\cdots}{a^3b^2+ab^{\cdots}}}{2\,d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="m
axima")
```

output
```
-1/2*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(B
*a^4 + 3*B*a^2*b^2 - 2*A*a*b^3)*log(b*tan(d*x + c) + a)/(a^4*b^2 + 2*a^2*b
^4 + b^6) + (B*a^2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2
*b^2 + b^4) - 2*(B*a^3 - A*a^2*b)/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(d
*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.38

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4d+2\,a^2b^2d+b^4d} - \frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,(a^4d+2\,a^2b^2d+b^4d)} + \frac{(Ba^4+3\,Ba^2b^2-2\,Aab^3)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^4b^2d+2\,a^2b^4d+b^6d} + \frac{Ba^5-Aa^4b+Ba^3b^2-Aa^2b^3}{(a^2+b^2)^2(b\tan\left(dx+c\right)+a)b^2d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="g
iac")
```

output
```
-(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4*d + 2*a^2*b^2*d + b^4*d) - 1/2*(
B*a^2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4*d + 2*a^2*b^2*d + b^
4*d) + (B*a^4 + 3*B*a^2*b^2 - 2*A*a*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4
*b^2*d + 2*a^2*b^4*d + b^6*d) + (B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3)/
((a^2 + b^2)^2*(b*tan(d*x + c) + a)*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.05

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)} - \frac{a^2\,\left(A\,b-B\,a\right)}{b^2\,d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)} + \frac{a\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,a^3+3\,B\,a\,b^2-2\,A\,b^3\right)}{b^2\,d\,{\left(a^2+b^2\right)}^2}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(log(tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(a*b*2i - a^2 + b^2)) + (log(tan(
c + d*x) - 1i)*(A + B*1i))/(2*d*(2*a*b - a^2*1i + b^2*1i)) - (a^2*(A*b - B
*a))/(b^2*d*(a^2 + b^2)*(a + b*tan(c + d*x))) + (a*log(a + b*tan(c + d*x))
*(B*a^3 - 2*A*b^3 + 3*B*a*b^2))/(b^2*d*(a^2 + b^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.35

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\log\left(\tan\left(dx+c\right)^2+1\right)b^2+2\log\left(a+\tan\left(dx+c\right)b\right)a^2-2abdx}{2bd\left(a^2+b^2\right)}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(log(tan(c + d*x)**2 + 1)*b**2 + 2*log(tan(c + d*x)*b + a)*a**2 - 2*a*b*d*
x)/(2*b*d*(a**2 + b**2))
```

3.277 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 115

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2} dx = \frac{(2aAb-a^2B+b^2B)\,x}{(a^2+b^2)^2} - \frac{(a^2A-Ab^2+2abB)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} + \frac{a(Ab-aB)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
(2*A*a*b-B*a^2+B*b^2)*x/(a^2+b^2)^2-(A*a^2-A*b^2+2*B*a*b)*ln(a*cos(d*x+c)+
b*sin(d*x+c))/(a^2+b^2)^2/d+a*(A*b-B*a)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.22

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2} dx = \frac{\frac{(A+iB)\log(i-\tan(c+dx))}{(a+ib)^2} + \frac{(A-iB)\log(i+\tan(c+dx))}{(a-ib)^2} + \frac{2\left((-a^2A+Ab^2-2abB)\log(a+b\tan(c+dx)) - \frac{a\left(a^2+b^2\right)(-Ab+aB)}{b(a+b\tan(c+dx))}\right)}{(a^2+b^2)^2}}{2d}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
(((A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b)^2 + ((A - I*B)*Log[I + Tan[c
+ d*x]])/(a - I*b)^2 + (2*((-(a^2*A) + A*b^2 - 2*a*b*B)*Log[a + b*Tan[c +
d*x]] - (a*(a^2 + b^2)*(-(A*b) + a*B))/(b*(a + b*Tan[c + d*x]))))/(a^2 + b
^2)^2)/(2*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3042, 4074, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4074}$$

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\frac{x\left(a^2(-B)+2aAb+b^2B\right)}{a^2+b^2}-\frac{\left(a^2A+2abB-Ab^2\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{x(a^2(-B)+2aAb+b^2B)}{a^2+b^2}-\frac{(a^2A+2abB-Ab^2)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4013}$$

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{\frac{x(a^2(-B)+2aAb+b^2B)}{a^2+b^2}-\frac{(a^2A+2abB-Ab^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)}}{a^2+b^2}$$

input

```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output

```
(((2*a*A*b - a^2*B + b^2*B)*x)/(a^2 + b^2) - ((a^2*A - A*b^2 + 2*a*b*B)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2 + b^2) + (a*(A*b
 - a*B))/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{\frac{(A a^2-A b^2+2Bab)\ln(1+\tan(dx+c)^2)}{2}+(2Aab-B a^2+B b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{a(Ab-Ba)}{(a^2+b^2)b(a+b\tan(dx+c))}-\frac{(A a^2-A b^2+2B}{(a}}{d}$
default	$\frac{\frac{\frac{(A a^2-A b^2+2Bab)\ln(1+\tan(dx+c)^2)}{2}+(2Aab-B a^2+B b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{a(Ab-Ba)}{(a^2+b^2)b(a+b\tan(dx+c))}-\frac{(A a^2-A b^2+2B}{(a}}{d}$
norman	$\frac{\frac{a(2Aab-B a^2+B b^2)x}{a^4+2a^2b^2+b^4}+\frac{b(2Aab-B a^2+B b^2)x\tan(dx+c)}{a^4+2a^2b^2+b^4}+\frac{a(Ab-Ba)}{(a^2+b^2)db}}{a+b\tan(dx+c)}+\frac{(A a^2-A b^2+2Bab)\ln(1+\tan(dx+c)^2)}{2d(a^4+2a^2b^2+b^4)}-($
parallelrisch	$2A a^3b-2B a^2b^2+2Aa b^3+4Ax\tan(dx+c)a b^3d-2Bx\tan(dx+c)a^2b^2d+A\ln(1+\tan(dx+c)^2)a^3b-A\ln(1+\tan(dx+c$
risch	$\frac{xB}{2iab-a^2+b^2}+\frac{ixA}{2iab-a^2+b^2}+\frac{2ia^2Ax}{a^4+2a^2b^2+b^4}-\frac{2iAb^2x}{a^4+2a^2b^2+b^4}+\frac{4iBabx}{a^4+2a^2b^2+b^4}+\frac{2ia^2Ac}{d(a^4+2a^2b^2+b^4)}-\frac{}{d(a^4}$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/(a^2+b^2)^2*(1/2*(A*a^2-A*b^2+2*B*a*b)*ln(1+tan(d*x+c)^2)+(2*A*a*b-
B*a^2+B*b^2)*arctan(tan(d*x+c)))+a*(A*b-B*a)/(a^2+b^2)/b/(a+b*tan(d*x+c))-
(A*a^2-A*b^2+2*B*a*b)/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.92

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,Ba^2b-2\,Aab^2+2\,(Ba^3-2\,Aa^2b-Bab^2)dx+(Aa^3+2\,Ba^2b-Aab^2+(Aa^2b+2\,Bab^2-Ab^3)\tan}{2\,((a^4b+2\,a^2b^3+b^5)d\tan(d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fri
cas")
```

output
```
-1/2*(2*B*a^2*b - 2*A*a*b^2 + 2*(B*a^3 - 2*A*a^2*b - B*a*b^2)*d*x + (A*a^3
 + 2*B*a^2*b - A*a*b^2 + (A*a^2*b + 2*B*a*b^2 - A*b^3)*tan(d*x + c))*log((
b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(
B*a^3 - A*a^2*b - (B*a^2*b - 2*A*a*b^2 - B*b^3)*d*x)*tan(d*x + c))/((a^4*b
 + 2*a^2*b^3 + b^5)*d*tan(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 2987, normalized size of antiderivative = 25.97

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*(A + B*tan(c))/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (
(A*log(tan(c + d*x)**2 + 1)/(2*d) - B*x + B*tan(c + d*x)/d)/a**2, Eq(b, 0)
), (I*A*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c +
 d*x) - 4*b**2*d) + 2*A*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b
**2*d*tan(c + d*x) - 4*b**2*d) - I*A*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b
**2*d*tan(c + d*x) - 4*b**2*d) + I*A*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**
2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + B*d*x*tan(c + d*x)**2/(4*b**2*d*
tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*B*d*x*tan(c +
d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - B*d
*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - 3*B*t
an(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d
) + 2*I*B/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d),
 Eq(a, -I*b)), (-I*A*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b
**2*d*tan(c + d*x) - 4*b**2*d) + 2*A*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*
x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*A*d*x/(4*b**2*d*tan(c + d*
x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*A*tan(c + d*x)/(4*b**2*d*t
an(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + B*d*x*tan(c + d*x)*
*2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*I*B
*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*
b**2*d) - B*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\,(Ba^2-2\,Aab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Aa^2+2\,Bab-Ab^2)\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}-\frac{(Aa^2+2\,Bab-Ab^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(Ba}{a^3b+ab^3+(a^2b}}{2\,d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="max
ima")
```

output
```
-1/2*(2*(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(A
*a^2 + 2*B*a*b - A*b^2)*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) -
(A*a^2 + 2*B*a*b - A*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4)
+ 2*(B*a^2 - A*a*b)/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.85

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= -\frac{(Ba^2-2\,Aab-Bb^2)(dx+c)}{a^4d+2\,a^2b^2d+b^4d} + \frac{(Aa^2+2\,Bab-Ab^2)\log\left(\tan(dx+c)^2+1\right)}{2\,(a^4d+2\,a^2b^2d+b^4d)}$$
$$-\frac{(Aa^2b+2\,Bab^2-Ab^3)\log(|b\tan(dx+c)+a|)}{a^4bd+2\,a^2b^3d+b^5d}$$
$$-\frac{Ba^4-Aa^3b+Ba^2b^2-Aab^3}{(a^2+b^2)^2(b\tan(dx+c)+a)bd}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="gia
c")
```

output
```
-(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(
A*a^2 + 2*B*a*b - A*b^2)*log(tan(d*x + c)^2 + 1)/(a^4*d + 2*a^2*b^2*d + b^
4*d) - (A*a^2*b + 2*B*a*b^2 - A*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b*d
 + 2*a^2*b^3*d + b^5*d) - (B*a^4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)/((a^2 +
b^2)^2*(b*tan(d*x + c) + a)*b*d)
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.42

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{a\,(A\,b-B\,a)}{b\,d\,(a^2+b^2)\,(a+b\tan(c+d\,x))}$$
$$+\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(A+B\,1\mathrm{i})}{2\,d\,(a^2+a\,b\,2\mathrm{i}-b^2)}$$
$$+\frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(B+A\,1\mathrm{i})}{2\,d\,(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i})}$$
$$-\frac{\ln(a+b\tan(c+d\,x))\,\left(\frac{A}{a^2+b^2}-\frac{2\,b\,(A\,b-B\,a)}{(a^2+b^2)^2}\right)}{d}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(log(tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(a*b*2i + a^2 - b^2)) - (log(a +
b*tan(c + d*x))*(A/(a^2 + b^2) - (2*b*(A*b - B*a))/(a^2 + b^2)^2))/d + (lo
g(tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(2*a*b + a^2*1i - b^2*1i)) + (a*(A*b
 - B*a))/(b*d*(a^2 + b^2)*(a + b*tan(c + d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.41

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\log\left(\tan\left(dx+c\right)^2+1\right)a-2\log(a+\tan\left(dx+c\right)b)\,a+2bdx}{2d\left(a^2+b^2\right)}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(log(tan(c + d*x)**2 + 1)*a - 2*log(tan(c + d*x)*b + a)*a + 2*b*d*x)/(2*d*
(a**2 + b**2))
```

3.278 $\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{(a^2A-Ab^2+2abB)\,x}{(a^2+b^2)^2} + \frac{(2aAb-a^2B+b^2B)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} - \frac{Ab-aB}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
(A*a^2-A*b^2+2*B*a*b)*x/(a^2+b^2)^2+(2*A*a*b-B*a^2+B*b^2)*ln(a*cos(d*x+c)+
b*sin(d*x+c))/(a^2+b^2)^2/d-(A*b-B*a)/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.70 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.71

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{B((-ia-b)\log(i-\tan(c+dx))+i(a+ib)\log(i+\tan(c+dx))+2b\log(a+b\tan(c+dx)))}{a^2+b^2} - (Ab-aB)\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^2} - \frac{i\log(i+\tan(c}{(a-ib)^2}\right.}{2bd}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^2,x]
```

output
```
((B*(((-I)*a - b)*Log[I - Tan[c + d*x]] + I*(a + I*b)*Log[I + Tan[c + d*x]
] + 2*b*Log[a + b*Tan[c + d*x]]))/(a^2 + b^2) - (A*b - a*B)*((I*Log[I - Ta
n[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (2*b*(-
2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a + b*Tan[c + d*x])))/(a^2 + b^
2)^2))/(2*b*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{Ab-aB}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{Ab-aB}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\frac{(a^2(-B)+2aAb+b^2B)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(a^2A+2abB-Ab^2)}{a^2+b^2}}{a^2+b^2}-\frac{Ab-aB}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(a^2(-B)+2aAb+b^2B)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(a^2A+2abB-Ab^2)}{a^2+b^2}}{a^2+b^2}-\frac{Ab-aB}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4013}$$

$$\frac{\frac{(a^2(-B)+2aAb+b^2B)\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)}+\frac{x(a^2A+2abB-Ab^2)}{a^2+b^2}}{a^2+b^2}-\frac{Ab-aB}{d(a^2+b^2)(a+b\tan(c+dx))}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^2,x]
```

output
```
(((a^2*A - A*b^2 + 2*a*b*B)*x)/(a^2 + b^2) + ((2*a*A*b - a^2*B + b^2*B)*Lo
g[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2 + b^2) - (A*b -
a*B)/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\frac{\frac{\left(-2Aab+B\,a^2-B\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(A\,a^2-A\,b^2+2Bab\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{Ab-Ba}{\left(a^2+b^2\right)(a+b\tan(dx+c))}+\frac{\left(2Aab-B\,a^2+B\right.}{\left(\right.}}{d}$
default	$\frac{\frac{\frac{\left(-2Aab+B\,a^2-B\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(A\,a^2-A\,b^2+2Bab\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}-\frac{Ab-Ba}{\left(a^2+b^2\right)(a+b\tan(dx+c))}+\frac{\left(2Aab-B\,a^2+B\right.}{\left(\right.}}{d}$
norman	$\frac{\frac{a\left(A\,a^2-A\,b^2+2Bab\right)x}{a^4+2a^2b^2+b^4}+\frac{b\left(A\,a^2-A\,b^2+2Bab\right)x\tan(dx+c)}{a^4+2a^2b^2+b^4}+\frac{(Ab-Ba)b\tan(dx+c)}{ad\left(a^2+b^2\right)}}{a+b\tan(dx+c)}+\frac{\left(2Aab-B\,a^2+B\,b^2\right)\ln(a+b\tan(dx+c}{d(a^4+2a^2b^2+b^4)}$
parallelrisch	$-\frac{2Ax\tan(dx+c)a\,b^3d-4Bx\tan(dx+c)a^2b^2d-2Ax\,a^4d+2A\ln\left(1+\tan(dx+c)^2\right)a^3b+B\ln\left(1+\tan(dx+c)^2\right)a^2b^2-2A}{}$
risch	$\frac{ixB}{2iab-a^2+b^2}-\frac{xA}{2iab-a^2+b^2}-\frac{4iabAx}{a^4+2a^2b^2+b^4}+\frac{2ia^2Bx}{a^4+2a^2b^2+b^4}-\frac{2iB\,b^2x}{a^4+2a^2b^2+b^4}-\frac{4iabAc}{(a^4+2a^2b^2+b^4)d}+\frac{}{(a^4+}$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^2*(1/2*(-2*A*a*b+B*a^2-B*b^2)*ln(1+tan(d*x+c)^2)+(A*a^2-A
*b^2+2*B*a*b)*arctan(tan(d*x+c)))-(A*b-B*a)/(a^2+b^2)/(a+b*tan(d*x+c))+(2*
A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.00

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2\,Bab^2-2\,Ab^3+2\,(Aa^3+2\,Ba^2b-Aab^2)dx-(Ba^3-2\,Aa^2b-Bab^2+(Ba^2b-2\,Aab^2-Bb^3)\tan\,(d}{2\,((a^4b+2\,a^2b^3+b^5)d\tan\,(dx}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(2*B*a*b^2 - 2*A*b^3 + 2*(A*a^3 + 2*B*a^2*b - A*a*b^2)*d*x - (B*a^3 -
2*A*a^2*b - B*a*b^2 + (B*a^2*b - 2*A*a*b^2 - B*b^3)*tan(d*x + c))*log((b^2
*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(B*a
^2*b - A*a*b^2 - (A*a^2*b + 2*B*a*b^2 - A*b^3)*d*x)*tan(d*x + c))/((a^4*b
+ 2*a^2*b^3 + b^5)*d*tan(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 2878, normalized size of antiderivative = 25.93

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*(A + B*tan(c))/tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0))
, ((A*x + B*log(tan(c + d*x)**2 + 1)/(2*d))/a**2, Eq(b, 0)), (-A*d*x*tan(c
 + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d)
 + 2*I*A*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d
*x) - 4*b**2*d) + A*d*x/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x
) - 4*b**2*d) - A*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(
c + d*x) - 4*b**2*d) + 2*I*A/(4*b**2*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c
+ d*x) - 4*b**2*d) + I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 - 8
*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*B*d*x*tan(c + d*x)/(4*b**2*d*tan(c
+ d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) - I*B*d*x/(4*b**2*d*tan(c
+ d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*B*tan(c + d*x)/(4*b**2
*d*tan(c + d*x)**2 - 8*I*b**2*d*tan(c + d*x) - 4*b**2*d), Eq(a, -I*b)), (-
A*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x)
- 4*b**2*d) - 2*I*A*d*x*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*
d*tan(c + d*x) - 4*b**2*d) + A*d*x/(4*b**2*d*tan(c + d*x)**2 + 8*I*b**2*d*
tan(c + d*x) - 4*b**2*d) - A*tan(c + d*x)/(4*b**2*d*tan(c + d*x)**2 + 8*I*
b**2*d*tan(c + d*x) - 4*b**2*d) - 2*I*A/(4*b**2*d*tan(c + d*x)**2 + 8*I*b*
*2*d*tan(c + d*x) - 4*b**2*d) - I*B*d*x*tan(c + d*x)**2/(4*b**2*d*tan(c +
d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + 2*B*d*x*tan(c + d*x)/(4*b*
*2*d*tan(c + d*x)**2 + 8*I*b**2*d*tan(c + d*x) - 4*b**2*d) + I*B*d*x/(4...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.59

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} - \frac{2\,(Ba^2-2\,Aab-Bb^2)\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4} + \frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(Ba-A}{a^3+ab^2+(a^2b+b^3}}{2\,d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(B*
a^2 - 2*A*a*b - B*b^2)*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) + (
B*a^2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) +
 2*(B*a - A*b)/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.85

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4d+2\,a^2b^2d+b^4d} + \frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^4d+2\,a^2b^2d+b^4d\right)} - \frac{(Ba^2b-2\,Aab^2-Bb^3)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^4bd+2\,a^2b^3d+b^5d} + \frac{Ba^3-Aa^2b+Bab^2-Ab^3}{(a^2+b^2)^2(b\tan\left(dx+c\right)+a)d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(B
*a^2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4*d + 2*a^2*b^2*d + b^4
*d) - (B*a^2*b - 2*A*a*b^2 - B*b^3)*log(abs(b*tan(d*x + c) + a))/(a^4*b*d
+ 2*a^2*b^3*d + b^5*d) + (B*a^3 - A*a^2*b + B*a*b^2 - A*b^3)/((a^2 + b^2)^
2*(b*tan(d*x + c) + a)*d)
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.38

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(-B\,a^2+2\,A\,a\,b+B\,b^2\right)}{d\,{\left(a^2+b^2\right)}^2} - \frac{A\,b-B\,a}{d\,\left(a^2+b^2\right)\,\left(a+b\tan\left(c+d\,x\right)\right)} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{2\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)}$$

input
```
int((A + B*tan(c + d*x))/(a + b*tan(c + d*x))^2,x)
```

output
```
(log(a + b*tan(c + d*x))*(B*b^2 - B*a^2 + 2*A*a*b))/(d*(a^2 + b^2)^2) - (A
*b - B*a)/(d*(a^2 + b^2)*(a + b*tan(c + d*x))) - (log(tan(c + d*x) + 1i)*(
A*1i + B))/(2*d*(a*b*2i - a^2 + b^2)) - (log(tan(c + d*x) - 1i)*(A + B*1i)
)/(2*d*(2*a*b - a^2*1i + b^2*1i))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.43

$$\int \frac{A + B\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{-\log\left(\tan(dx+c)^2+1\right)b + 2\log(a+\tan(dx+c)\,b)\,b + 2adx}{2d\,(a^2+b^2)}$$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
( - log(tan(c + d*x)**2 + 1)*b + 2*log(tan(c + d*x)*b + a)*b + 2*a*d*x)/(2
*d*(a**2 + b**2))
```

3.279 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 137

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{(2aAb-a^2B+b^2B)\,x}{(a^2+b^2)^2}+\frac{A\log(\sin(c+dx))}{a^2d} - \frac{b(3a^2Ab+Ab^3-2a^3B)\log(a\cos(c+dx)+b\sin(c+dx))}{a^2\,(a^2+b^2)^2\,d} + \frac{b(Ab-aB)}{a\,(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
-(2*A*a*b-B*a^2+B*b^2)*x/(a^2+b^2)^2+A*ln(sin(d*x+c))/a^2/d-b*(3*A*a^2*b+A
*b^3-2*B*a^3)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^2/(a^2+b^2)^2/d+b*(A*b-B*a)/
a/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.34

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{-\frac{a(a-ib)(A+iB)\log(i-\tan(c+dx))}{2(a+ib)} + \frac{A(a^2+b^2)\log(\tan(c+dx))}{a} - \frac{a(a+ib)(A-iB)\log(i+\tan(c+dx))}{2(a-ib)} + \frac{b(-3a^2Ab-Ab^3+2a^3B)\log(a\cdots}{a(a^2+b^2)}}{a\left(a^2+b^2\right)d}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-1/2*(a*(a - I*b)*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b) + (A*(a^2 +
b^2)*Log[Tan[c + d*x]])/a - (a*(a + I*b)*(A - I*B)*Log[I + Tan[c + d*x]])/
(2*(a - I*b)) + (b*(-3*a^2*A*b - A*b^3 + 2*a^3*B)*Log[a + b*Tan[c + d*x]])
/(a*(a^2 + b^2)) + (b*(A*b - a*B))/(a + b*Tan[c + d*x]))/(a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 4092, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{4092}$$
$$\frac{\int \frac{\cot(c+dx)\left(b(Ab-aB)\tan^2(c+dx)-a(Ab-aB)\tan(c+dx)+A\left(a^2+b^2\right)\right)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(Ab-aB)\tan(c+dx)^2-a(Ab-aB)\tan(c+dx)+A(a^2+b^2)}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4134}$$

$$\frac{\frac{A(a^2+b^2)\int \cot(c+dx)dx}{a}-\frac{b(-2a^3B+3a^2Ab+Ab^3)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{ax(a^2(-B)+2aAb+b^2B)}{a^2+b^2}}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{A(a^2+b^2)\int -\tan(c+dx+\frac{\pi}{2})dx}{a}-\frac{b(-2a^3B+3a^2Ab+Ab^3)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{ax(a^2(-B)+2aAb+b^2B)}{a^2+b^2}}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{A(a^2+b^2)\int \tan(\frac{1}{2}(2c+\pi)+dx)dx}{a}-\frac{b(-2a^3B+3a^2Ab+Ab^3)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{ax(a^2(-B)+2aAb+b^2B)}{a^2+b^2}}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3956}$$

$$\frac{-\frac{b(-2a^3B+3a^2Ab+Ab^3)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{ax(a^2(-B)+2aAb+b^2B)}{a^2+b^2}+\frac{A(a^2+b^2)\log(-\sin(c+dx))}{ad}}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4013}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{-\frac{ax(a^2(-B)+2aAb+b^2B)}{a^2+b^2}+\frac{A(a^2+b^2)\log(-\sin(c+dx))}{ad}-\frac{b(-2a^3B+3a^2Ab+Ab^3)\log(a\cos(c+dx)+b\sin(c+dx))}{ad(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
(-((a*(2*a*A*b - a^2*B + b^2*B)*x)/(a^2 + b^2)) + (A*(a^2 + b^2)*Log[-Sin[
c + d*x]])/(a*d) - (b*(3*a^2*A*b + A*b^3 - 2*a^3*B)*Log[a*Cos[c + d*x] + b
*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/(a*(a^2 + b^2)) + (b*(A*b - a*B))/(a*(a
^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{\frac{(-A a^2+A b^2-2Bab)\ln(1+\tan(dx+c)^2)}{2}+(-2Aab+B a^2-B b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{A\ln(\tan(dx+c))}{a^2}-\frac{b(3A a^2b+A b^3-2B a^3}{(a^2+b^2}}{d}$
default	$\frac{\frac{\frac{(-A a^2+A b^2-2Bab)\ln(1+\tan(dx+c)^2)}{2}+(-2Aab+B a^2-B b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^2}+\frac{A\ln(\tan(dx+c))}{a^2}-\frac{b(3A a^2b+A b^3-2B a^3}{(a^2+b^2}}{d}$
parallelrisch	$\frac{-6b(a+b\tan(dx+c))(A a^2b+\frac{1}{3}A b^3-\frac{2}{3}B a^3)\ln(a+b\tan(dx+c))-a^2(a+b\tan(dx+c))(A a^2-A b^2+2Bab)\ln(\sec(dx+}{}$
norman	$\frac{-\frac{a(2Aab-B a^2+B b^2)x}{a^4+2a^2b^2+b^4}-\frac{b(2Aab-B a^2+B b^2)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{(A b^2-Bab)b\tan(dx+c)}{d a^2(a^2+b^2)}}{a+b\tan(dx+c)}+\frac{A\ln(\tan(dx+c))}{a^2d}-\frac{(A a^2-A b}{}$
risch	$-\frac{xB}{2iab-a^2+b^2}-\frac{4iBabx}{a^4+2a^2b^2+b^4}+\frac{6iA b^2x}{a^4+2a^2b^2+b^4}+\frac{2ib^4Ac}{a^2d(a^4+2a^2b^2+b^4)}-\frac{ixA}{2iab-a^2+b^2}-\frac{4iBabc}{d(a^4+2a^2b^2+b^4)}$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/(a^2+b^2)^2*(1/2*(-A*a^2+A*b^2-2*B*a*b)*ln(1+tan(d*x+c)^2)+(-2*A*a*
b+B*a^2-B*b^2)*arctan(tan(d*x+c)))+1/a^2*A*ln(tan(d*x+c))-b*(3*A*a^2*b+A*b
^3-2*B*a^3)/(a^2+b^2)^2/a^2*ln(a+b*tan(d*x+c))+(A*b-B*a)*b/(a^2+b^2)/a/(a+
b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(137) = 274$.

Time = 0.12 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.36

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{2\,Ba^2b^3 - 2\,Aab^4 - 2\,(Ba^5 - 2\,Aa^4b - Ba^3b^2)dx - (Aa^5 + 2\,Aa^3b^2 + Aab^4 + (Aa^4b + 2\,Aa^2b^3 + Ab^5)}{}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fri
cas")
```

output
```
-1/2*(2*B*a^2*b^3 - 2*A*a*b^4 - 2*(B*a^5 - 2*A*a^4*b - B*a^3*b^2)*d*x - (A
*a^5 + 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b + 2*A*a^2*b^3 + A*b^5)*tan(d*x + c
))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - (2*B*a^4*b - 3*A*a^3*b^2 - A
*a*b^4 + (2*B*a^3*b^2 - 3*A*a^2*b^3 - A*b^5)*tan(d*x + c))*log((b^2*tan(d*
x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(B*a^3*b^2
- A*a^2*b^3 + (B*a^4*b - 2*A*a^3*b^2 - B*a^2*b^3)*d*x)*tan(d*x + c))/((a^6
*b + 2*a^4*b^3 + a^2*b^5)*d*tan(d*x + c) + (a^7 + 2*a^5*b^2 + a^3*b^4)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 4488, normalized size of antiderivative = 32.76

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*(A + B*tan(c))*cot(c)/tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq
(d, 0)), ((-A*log(tan(c + d*x)**2 + 1)/(2*d) + A*log(tan(c + d*x))/d + B*x
)/a**2, Eq(b, 0)), ((A*log(tan(c + d*x)**2 + 1)/(2*d) - A*log(tan(c + d*x)
)/d - A/(2*d*tan(c + d*x)**2) - B*x - B/(d*tan(c + d*x)))/b**2, Eq(a, 0)),
 (3*I*A*d*x*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c +
 d*x) - 4*a**2*d) - 6*A*d*x*tan(c + d*x)/(4*a**2*d*tan(c + d*x)**2 + 8*I*a
**2*d*tan(c + d*x) - 4*a**2*d) - 3*I*A*d*x/(4*a**2*d*tan(c + d*x)**2 + 8*I
*a**2*d*tan(c + d*x) - 4*a**2*d) - 2*A*log(tan(c + d*x)**2 + 1)*tan(c + d*
x)**2/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) - 4*
I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(4*a**2*d*tan(c + d*x)**2 + 8*I*
a**2*d*tan(c + d*x) - 4*a**2*d) + 2*A*log(tan(c + d*x)**2 + 1)/(4*a**2*d*t
an(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) + 4*A*log(tan(c + d*x
))*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4
*a**2*d) + 8*I*A*log(tan(c + d*x))*tan(c + d*x)/(4*a**2*d*tan(c + d*x)**2
+ 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) - 4*A*log(tan(c + d*x))/(4*a**2*d*ta
n(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) + 3*I*A*tan(c + d*x)/(
4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) - 4*A/(4*a*
*2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) + B*d*x*tan(c +
 d*x)**2/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + d*x) - 4*a**2*d) +
 2*I*B*d*x*tan(c + d*x)/(4*a**2*d*tan(c + d*x)**2 + 8*I*a**2*d*tan(c + ...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.52

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{2\,(Ba^2-2\,Aab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(2\,Ba^3b-3\,Aa^2b^2-Ab^4)\log(b\tan(dx+c)+a)}{a^6+2\,a^4b^2+a^2b^4} - \frac{(Aa^2+2\,Bab-Ab^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{2\,(}{a^4+a^2b^2+(a}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="max
ima")
```

output
```
1/2*(2*(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(2*
B*a^3*b - 3*A*a^2*b^2 - A*b^4)*log(b*tan(d*x + c) + a)/(a^6 + 2*a^4*b^2 +
a^2*b^4) - (A*a^2 + 2*B*a*b - A*b^2)*log(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*
b^2 + b^4) - 2*(B*a*b - A*b^2)/(a^4 + a^2*b^2 + (a^3*b + a*b^3)*tan(d*x +
c)) + 2*A*log(tan(d*x + c))/a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.74

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{(Ba^2-2\,Aab-Bb^2)(dx+c)}{a^4d+2\,a^2b^2d+b^4d} - \frac{(Aa^2+2\,Bab-Ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,(a^4d+2\,a^2b^2d+b^4d)} + \frac{(2\,Ba^3b^2-3\,Aa^2b^3-Ab^5)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^6bd+2\,a^4b^3d+a^2b^5d} + \frac{A\log\left(|\tan\left(dx+c\right)|\right)}{a^2d} - \frac{Ba^4b-Aa^3b^2+Ba^2b^3-Aab^4}{(a^2+b^2)^2(b\tan\left(dx+c\right)+a)a^2d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="gia
c")
```

output
```
(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4*d + 2*a^2*b^2*d + b^4*d) - 1/2*(A
*a^2 + 2*B*a*b - A*b^2)*log(tan(d*x + c)^2 + 1)/(a^4*d + 2*a^2*b^2*d + b^4
*d) + (2*B*a^3*b^2 - 3*A*a^2*b^3 - A*b^5)*log(abs(b*tan(d*x + c) + a))/(a^
6*b*d + 2*a^4*b^3*d + a^2*b^5*d) + A*log(abs(tan(d*x + c)))/(a^2*d) - (B*a
^4*b - A*a^3*b^2 + B*a^2*b^3 - A*a*b^4)/((a^2 + b^2)^2*(b*tan(d*x + c) + a
)*a^2*d)
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{A\ln(\tan(c+dx))}{a^2 d} - \frac{\ln(\tan(c+dx)-\mathrm{i})\,(A+B\,1\mathrm{i})}{2\,d\,(a^2+a\,b\,2\mathrm{i}-b^2)}$$
$$- \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(B+A\,1\mathrm{i})}{2\,d\,(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i})} + \frac{A\,b^2-B\,a\,b}{a\,d\,(a^2+b^2)\,(a+b\tan(c+dx))}$$
$$- \frac{b\ln(a+b\tan(c+dx))\,(-2\,B\,a^3+3\,A\,a^2\,b+A\,b^3)}{a^2\,d\,(a^2+b^2)^2}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(A*log(tan(c + d*x)))/(a^2*d) - (log(tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(
a*b*2i + a^2 - b^2)) - (log(tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(2*a*b + a
^2*1i - b^2*1i)) + (A*b^2 - B*a*b)/(a*d*(a^2 + b^2)*(a + b*tan(c + d*x)))
- (b*log(a + b*tan(c + d*x))*(A*b^3 - 2*B*a^3 + 3*A*a^2*b))/(a^2*d*(a^2 +
b^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)a^2 - \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a - 2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b - a\right)b^2 + \log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2 + }{ad\,(a^2+b^2)}$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
( - log(tan((c + d*x)/2)**2 + 1)*a**2 - log(tan((c + d*x)/2)**2*a - 2*tan(
(c + d*x)/2)*b - a)*b**2 + log(tan((c + d*x)/2))*a**2 + log(tan((c + d*x)/
2))*b**2 - a*b*d*x)/(a*d*(a**2 + b**2))
```

3.280 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 192

$$\begin{aligned}\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx &= -\frac{(a^2A-Ab^2+2abB)\,x}{(a^2+b^2)^2}-\frac{(2Ab-aB)\log(\sin(c+dx))}{a^3d}\\ &\quad+\frac{b^2(4a^2Ab+2Ab^3-3a^3B-ab^2B)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3\,(a^2+b^2)^2\,d}\\ &\quad-\frac{b(a^2A+2Ab^2-abB)}{a^2\,(a^2+b^2)\,d(a+b\tan(c+dx))}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(A*a^2-A*b^2+2*B*a*b)*x/(a^2+b^2)^2-(2*A*b-B*a)*ln(sin(d*x+c))/a^3/d+b^2*
(4*A*a^2*b+2*A*b^3-3*B*a^3-B*a*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^3/(a^2
+b^2)^2/d-b*(A*a^2+2*A*b^2-B*a*b)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))-A*cot(d
*x+c)/a/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{-\frac{2A\cot(c+dx)}{a^2} + \frac{i(A+iB)\log(i-\tan(c+dx))}{(a+ib)^2} + \frac{2(-2Ab+aB)\log(\tan(c+dx))}{a^3} - \frac{(iA+B)\log(i+\tan(c+dx))}{(a-ib)^2} - \frac{2b^2(-4a^2Ab-2Ab^3+3a}{a^3}}{2d}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
((-2*A*Cot[c + d*x])/a^2 + (I*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b)^2
 + (2*(-2*A*b + a*B)*Log[Tan[c + d*x]])/a^3 - ((I*A + B)*Log[I + Tan[c + d
*x]])/(a - I*b)^2 - (2*b^2*(-4*a^2*A*b - 2*A*b^3 + 3*a^3*B + a*b^2*B)*Log[
a + b*Tan[c + d*x]])/(a^3*(a^2 + b^2)^2) + (2*b^2*(-(A*b) + a*B))/(a^2*(a^
2 + b^2)*(a + b*Tan[c + d*x])))/(2*d)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 4092, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{4092}$$
$$-\frac{\int \frac{\cot(c+dx)\left(2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB\right)}{(a+b\tan(c+dx))^2}dx}{a} - \frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{2Ab\tan(c+dx)^2+aA\tan(c+dx)+2Ab-aB}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 4132

$$-\frac{\frac{\int \frac{\cot(c+dx)\left((aA+bB)\tan(c+dx)a^2+b\left(Aa^2-bBa+2Ab^2\right)\tan^2(c+dx)+\left(a^2+b^2\right)(2Ab-aB)\right)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{b(a^2A-abB+2Ab^2)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{(aA+bB)\tan(c+dx)a^2+b\left(Aa^2-bBa+2Ab^2\right)\tan(c+dx)^2+\left(a^2+b^2\right)(2Ab-aB)}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a(a^2+b^2)}+\frac{b(a^2A-abB+2Ab^2)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 4134

$$-\frac{\frac{\frac{\left(a^2+b^2\right)(2Ab-aB)\int\cot(c+dx)dx}{a}-\frac{b^2\left(-3a^3B+4a^2Ab-ab^2B+2Ab^3\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^2x\left(a^2A+2abB-Ab^2\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(a^2A-abB+2Ab^2)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\left(a^2+b^2\right)(2Ab-aB)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b^2\left(-3a^3B+4a^2Ab-ab^2B+2Ab^3\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^2x\left(a^2A+2abB-Ab^2\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(a^2A-abB+2Ab^2)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\Big\downarrow$ 25

$$-\frac{\frac{-\frac{\left(a^2+b^2\right)(2Ab-aB)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{b^2\left(-3a^3B+4a^2Ab-ab^2B+2Ab^3\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^2x\left(a^2A+2abB-Ab^2\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(a^2A-abB+2Ab^2)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\downarrow$ 3956

$$-\frac{\frac{-\frac{b^2\left(-3a^3B+4a^2Ab-ab^2B+2Ab^3\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{\left(a^2+b^2\right)(2Ab-aB)\log(-\sin(c+dx))}{ad}+\frac{a^2x\left(a^2A+2abB-Ab^2\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(a^2A-abB+2Ab^2)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

$\downarrow$ 4013

$$-\frac{\frac{b(a^2A-abB+2Ab^2)}{ad(a^2+b^2)(a+b\tan(c+dx))}+\frac{\frac{\left(a^2+b^2\right)(2Ab-aB)\log(-\sin(c+dx))}{ad}+\frac{a^2x\left(a^2A+2abB-Ab^2\right)}{a^2+b^2}-\frac{b^2\left(-3a^3B+4a^2Ab-ab^2B+2Ab^3\right)\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}}{a(a^2+b^2)}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
-((A*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x]))) - (((a^2*(a^2*A - A*b^2 + 2
*a*b*B)*x)/(a^2 + b^2) + ((a^2 + b^2)*(2*A*b - a*B)*Log[-Sin[c + d*x]])/(a
*d) - (b^2*(4*a^2*A*b + 2*A*b^3 - 3*a^3*B - a*b^2*B)*Log[a*Cos[c + d*x] +
b*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/(a*(a^2 + b^2)) + (b*(a^2*A + 2*A*b^2
- a*b*B))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{\frac{\frac{(2Aab-B\,a^2+B\,b^2)\ln(1+\tan(dx+c)^2)}{2}+(-A\,a^2+A\,b^2-2Bab)\arctan(\tan(dx+c))}{(a^2+b^2)^2}-\frac{A}{a^2\tan(dx+c)}+\frac{(-2Ab+Ba)\ln(\tan(dx+c))}{a^3}+}{d}$
default	$\frac{\frac{\frac{(2Aab-B\,a^2+B\,b^2)\ln(1+\tan(dx+c)^2)}{2}+(-A\,a^2+A\,b^2-2Bab)\arctan(\tan(dx+c))}{(a^2+b^2)^2}-\frac{A}{a^2\tan(dx+c)}+\frac{(-2Ab+Ba)\ln(\tan(dx+c))}{a^3}+}{d}$
parallelrisch	$4(A\,a^2b+\frac{1}{2}A\,b^3-\frac{3}{4}B\,a^3-\frac{1}{4}Ba\,b^2)b^2(a+b\tan(dx+c))\ln(a+b\tan(dx+c))+a^3(Aab-\frac{1}{2}B\,a^2+\frac{1}{2}B\,b^2)(a+b\tan(dx+c))\,\mathrm{l}$
norman	$\frac{\frac{(A\,a^2b+2A\,b^3-Ba\,b^2)b\tan(dx+c)^2}{d\,a^3(a^2+b^2)}-\frac{A}{ad}-\frac{a(A\,a^2-A\,b^2+2Bab)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{b(A\,a^2-A\,b^2+2Bab)x\tan(dx+c)^2}{a^4+2a^2b^2+b^4}}{\tan(dx+c)(a+b\tan(dx+c))}+\frac{b^2(4A\,a^2}{}$
risch	$\frac{4iAbc}{a^3d}+\frac{xA}{2iab-a^2+b^2}+\frac{6iB\,b^2x}{a^4+2a^2b^2+b^4}-\frac{8ib^3Ac}{(a^4+2a^2b^2+b^4)ad}+\frac{2ib^4Bx}{(a^4+2a^2b^2+b^4)a^2}+\frac{6iB\,b^2c}{d(a^4+2a^2b^2+b^4)}+\frac{4iA}{a}$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^2*(1/2*(2*A*a*b-B*a^2+B*b^2)*ln(1+tan(d*x+c)^2)+(-A*a^2+A
*b^2-2*B*a*b)*arctan(tan(d*x+c)))-1/a^2*A/tan(d*x+c)+(-2*A*b+B*a)/a^3*ln(t
an(d*x+c))+b^2*(4*A*a^2*b+2*A*b^3-3*B*a^3-B*a*b^2)/(a^2+b^2)^2/a^3*ln(a+b*
tan(d*x+c))-(A*b-B*a)*b^2/(a^2+b^2)/a^2/(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(190) = 380$.

Time = 0.13 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.42

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{2\,Aa^6+4\,Aa^4b^2+2\,Aa^2b^4+2\,(Ba^3b^3-Aa^2b^4+(Aa^5b+2\,Ba^4b^2-Aa^3b^3)dx)\tan(dx+c)^2-((Ba}{}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
-1/2*(2*A*a^6 + 4*A*a^4*b^2 + 2*A*a^2*b^4 + 2*(B*a^3*b^3 - A*a^2*b^4 + (A*
a^5*b + 2*B*a^4*b^2 - A*a^3*b^3)*d*x)*tan(d*x + c)^2 - ((B*a^5*b - 2*A*a^4
*b^2 + 2*B*a^3*b^3 - 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*tan(d*x + c)^2 + (B*
a^6 - 2*A*a^5*b + 2*B*a^4*b^2 - 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*b^5)*tan(d
*x + c))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + ((3*B*a^3*b^3 - 4*A*a^
2*b^4 + B*a*b^5 - 2*A*b^6)*tan(d*x + c)^2 + (3*B*a^4*b^2 - 4*A*a^3*b^3 + B
*a^2*b^4 - 2*A*a*b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*
x + c) + a^2)/(tan(d*x + c)^2 + 1)) + 2*(A*a^5*b + 2*A*a^3*b^3 - B*a^2*b^4
 + 2*A*a*b^5 + (A*a^6 + 2*B*a^5*b - A*a^4*b^2)*d*x)*tan(d*x + c))/((a^7*b
+ 2*a^5*b^3 + a^3*b^5)*d*tan(d*x + c)^2 + (a^8 + 2*a^6*b^2 + a^4*b^4)*d*ta
n(d*x + c))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.72 (sec) , antiderivative size = 8145, normalized size of antiderivative = 42.42

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise(((-A*x - A/(d*tan(c + d*x)) - B*log(tan(c + d*x)**2 + 1)/(2*d) +
 B*log(tan(c + d*x))/d)/a**2, Eq(b, 0)), ((A*x + A/(d*tan(c + d*x)) - A/(3
*d*tan(c + d*x)**3) + B*log(tan(c + d*x)**2 + 1)/(2*d) - B*log(tan(c + d*x
))/d - B/(2*d*tan(c + d*x)**2))/b**2, Eq(a, 0)), (-9*A*d*x*tan(c + d*x)**3
/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c +
 d*x)) - 18*I*A*d*x*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d
*tan(c + d*x)**2 - 4*a**2*d*tan(c + d*x)) + 9*A*d*x*tan(c + d*x)/(4*a**2*d
*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c + d*x)) - 4
*I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(4*a**2*d*tan(c + d*x)**3 +
8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c + d*x)) + 8*A*log(tan(c + d*x)
**2 + 1)*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*
x)**2 - 4*a**2*d*tan(c + d*x)) + 4*I*A*log(tan(c + d*x)**2 + 1)*tan(c + d*
x)/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c
 + d*x)) + 8*I*A*log(tan(c + d*x))*tan(c + d*x)**3/(4*a**2*d*tan(c + d*x)*
*3 + 8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c + d*x)) - 16*A*log(tan(c
+ d*x))*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x
)**2 - 4*a**2*d*tan(c + d*x)) - 8*I*A*log(tan(c + d*x))*tan(c + d*x)/(4*a*
*2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x)**2 - 4*a**2*d*tan(c + d*x))
 - 9*A*tan(c + d*x)**2/(4*a**2*d*tan(c + d*x)**3 + 8*I*a**2*d*tan(c + d*x)
**2 - 4*a**2*d*tan(c + d*x)) - 14*I*A*tan(c + d*x)/(4*a**2*d*tan(c + d*...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.36

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\,(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(3\,Ba^3b^2-4\,Aa^2b^3+Bab^4-2\,Ab^5)\log(b\tan(dx+c)+a)}{a^7+2\,a^5b^2+a^3b^4}+\frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="m
axima")
```

output
```
-1/2*(2*(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) + 2*(3
*B*a^3*b^2 - 4*A*a^2*b^3 + B*a*b^4 - 2*A*b^5)*log(b*tan(d*x + c) + a)/(a^7
 + 2*a^5*b^2 + a^3*b^4) + (B*a^2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1
)/(a^4 + 2*a^2*b^2 + b^4) + 2*(A*a^3 + A*a*b^2 + (A*a^2*b - B*a*b^2 + 2*A*
b^3)*tan(d*x + c))/((a^4*b + a^2*b^3)*tan(d*x + c)^2 + (a^5 + a^3*b^2)*tan
(d*x + c)) - 2*(B*a - 2*A*b)*log(tan(d*x + c))/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.54

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{(Aa^2+2\,Bab-Ab^2)(dx+c)}{a^4d+2\,a^2b^2d+b^4d} - \frac{(Ba^2-2\,Aab-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^4d+2\,a^2b^2d+b^4d\right)} - \frac{(3\,Ba^3b^3-4\,Aa^2b^4+Bab^5-2\,Ab^6)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^7bd+2\,a^5b^3d+a^3b^5d} + \frac{(Ba-2\,Ab)\log\left(|\tan\left(dx+c\right)|\right)}{a^3d} - \frac{Aa^5+2\,Aa^3b^2+Aab^4+(Aa^4b-Ba^3b^2+3\,Aa^2b^3-Bab^4+2\,Ab^5)\tan\left(dx+c\right)}{(a^2+b^2)^2(b\tan\left(dx+c\right)+a)a^2d\tan\left(dx+c\right)}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="g
iac")
```

output
```
-(A*a^2 + 2*B*a*b - A*b^2)*(d*x + c)/(a^4*d + 2*a^2*b^2*d + b^4*d) - 1/2*(
B*a^2 - 2*A*a*b - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^4*d + 2*a^2*b^2*d + b^
4*d) - (3*B*a^3*b^3 - 4*A*a^2*b^4 + B*a*b^5 - 2*A*b^6)*log(abs(b*tan(d*x +
 c) + a))/(a^7*b*d + 2*a^5*b^3*d + a^3*b^5*d) + (B*a - 2*A*b)*log(abs(tan(
d*x + c)))/(a^3*d) - (A*a^5 + 2*A*a^3*b^2 + A*a*b^4 + (A*a^4*b - B*a^3*b^2
 + 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5)*tan(d*x + c))/((a^2 + b^2)^2*(b*tan(d*
x + c) + a)*a^2*d*tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.20

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{b^2\ln(a+b\tan(c+dx))\left(-3Ba^3+4Aa^2b-Bab^2+2Ab^3\right)}{a^3d\left(a^2+b^2\right)^2} - \frac{\ln(\tan(c+dx))\,(2Ab-Ba)}{a^3d} + \frac{\ln(\tan(c+dx)+1\mathrm{i})\,(B+A\,1\mathrm{i})}{2d\,(-a^2+ab\,2\mathrm{i}+b^2)} + \frac{\ln(\tan(c+dx)-\mathrm{i})\,(A+B\,1\mathrm{i})}{2d\,(-a^2\,1\mathrm{i}+2ab+b^2\,1\mathrm{i})} - \frac{\frac{A}{a}+\frac{\tan(c+dx)\,(Aa^2b-Bab^2+2Ab^3)}{a^2\,(a^2+b^2)}}{d\left(b\tan(c+dx)^2+a\tan(c+dx)\right)}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(log(tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(a*b*2i - a^2 + b^2)) - (log(tan(
c + d*x))*(2*A*b - B*a))/(a^3*d) - (A/a + (tan(c + d*x)*(2*A*b^3 + A*a^2*b
 - B*a*b^2))/(a^2*(a^2 + b^2)))/(d*(a*tan(c + d*x) + b*tan(c + d*x)^2)) +
(log(tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(2*a*b - a^2*1i + b^2*1i)) + (b^2
*log(a + b*tan(c + d*x))*(2*A*b^3 - 3*B*a^3 + 4*A*a^2*b - B*a*b^2))/(a^3*d
*(a^2 + b^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{-\cos(dx+c)\,a^3-\cos(dx+c)\,ab^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)\,a^2b+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\right.}{\mathrm{s}}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
( - cos(c + d*x)*a**3 - cos(c + d*x)*a*b**2 + log(tan((c + d*x)/2)**2 + 1)
*sin(c + d*x)*a**2*b + log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b -
a)*sin(c + d*x)*b**3 - log(tan((c + d*x)/2))*sin(c + d*x)*a**2*b - log(tan
((c + d*x)/2))*sin(c + d*x)*b**3 - sin(c + d*x)*a**3*d*x)/(sin(c + d*x)*a*
*2*d*(a**2 + b**2))
```

3.281 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 250

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{(2aAb-a^2B+b^2B)\,x}{(a^2+b^2)^2} - \frac{(a^2A-3Ab^2+2abB)\log(\sin(c+dx))}{a^4d} - \frac{b^3(5a^2Ab+3Ab^3-4a^3B-2ab^2B)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4(a^2+b^2)^2d} + \frac{b(2a^2Ab+3Ab^3-a^3B-2ab^2B)}{a^3(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{(3Ab-2aB)\cot(c+dx)}{2a^2d(a+b\tan(c+dx))} - \frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

output
```
(2*A*a*b-B*a^2+B*b^2)*x/(a^2+b^2)^2-(A*a^2-3*A*b^2+2*B*a*b)*ln(sin(d*x+c))
/a^4/d-b^3*(5*A*a^2*b+3*A*b^3-4*B*a^3-2*B*a*b^2)*ln(a*cos(d*x+c)+b*sin(d*x
+c))/a^4/(a^2+b^2)^2/d+b*(2*A*a^2*b+3*A*b^3-B*a^3-2*B*a*b^2)/a^3/(a^2+b^2)
/d/(a+b*tan(d*x+c))+1/2*(3*A*b-2*B*a)*cot(d*x+c)/a^2/d/(a+b*tan(d*x+c))-1/
2*A*cot(d*x+c)^2/a/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.41 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.88

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{-\frac{2(-2Ab+aB)\cot(c+dx)}{a^3} - \frac{A\cot^2(c+dx)}{a^2} + \frac{(A+iB)\log(i-\tan(c+dx))}{(a+ib)^2} - \frac{2(a^2A-3Ab^2+2abB)\log(\tan(c+dx))}{a^4} + \frac{(A-iB)\log(i+\tan(c+dx))}{(a-ib)^2}}{2d}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
((-2*(-2*A*b + a*B)*Cot[c + d*x])/a^3 - (A*Cot[c + d*x]^2)/a^2 + ((A + I*B
)*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (2*(a^2*A - 3*A*b^2 + 2*a*b*B)*Log[
Tan[c + d*x]])/a^4 + ((A - I*B)*Log[I + Tan[c + d*x]])/(a - I*b)^2 + (2*b^
3*(-5*a^2*A*b - 3*A*b^3 + 4*a^3*B + 2*a*b^2*B)*Log[a + b*Tan[c + d*x]])/(a
^4*(a^2 + b^2)^2) + (2*b^3*(A*b - a*B))/(a^3*(a^2 + b^2)*(a + b*Tan[c + d*
x])))/(2*d)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4092, 3042, 4132, 27, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{4092}$$

$$-\frac{\int \frac{\cot^2(c+dx)(3Ab\tan^2(c+dx)+2aA\tan(c+dx)+3Ab-2aB)}{(a+b\tan(c+dx))^2}dx}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{3Ab\tan(c+dx)^2+2aA\tan(c+dx)+3Ab-2aB}{\tan(c+dx)^2(a+b\tan(c+dx))^2}dx}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{-\frac{\int -\frac{2\cot(c+dx)\left(Aa^2+B\tan(c+dx)a^2+2bBa-3Ab^2-b(3Ab-2aB)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}dx}{a}-\frac{(3Ab-2aB)\cot(c+dx)}{ad(a+b\tan(c+dx))}}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{2\int \frac{\cot(c+dx)\left(Aa^2+B\tan(c+dx)a^2+2bBa-3Ab^2-b(3Ab-2aB)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}dx}{a}-\frac{(3Ab-2aB)\cot(c+dx)}{ad(a+b\tan(c+dx))}}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2\int \frac{Aa^2+B\tan(c+dx)a^2+2bBa-3Ab^2-b(3Ab-2aB)\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a}-\frac{(3Ab-2aB)\cot(c+dx)}{ad(a+b\tan(c+dx))}}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{2\left(\frac{\int \frac{\cot(c+dx)\left(-\left((Ab-aB)\tan(c+dx)a^3\right)-b\left(-Ba^3+2Aba^2-2b^2Ba+3Ab^3\right)\tan^2(c+dx)+\left(a^2+b^2\right)\left(Aa^2+2bBa-3Ab^2\right)\right)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{b\left(a^3(-B)+2a^2Ab-2ab^2B+\cdots\right.}{ad\left(a^2+b^2\right)(a+b\tan(c+\cdots}\cdots}{a}\cdots}{2a}\cdots$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(\frac{\int \frac{-\left((Ab-aB)\tan(c+dx)a^3\right)-b\left(-Ba^3+2Aba^2-2b^2Ba+3Ab^3\right)\tan(c+dx)^2+\left(a^2+b^2\right)\left(Aa^2+2bBa-3Ab^2\right)}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a\left(a^2+b^2\right)}-\frac{b\left(a^3(-B)+2a^2Ab-2ab^2B+3Ab^3\right)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}\right)}{a} - \ldots}{2a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$\downarrow$ 4134

$$-\frac{2\left(\frac{\frac{\left(a^2+b^2\right)\left(a^2A+2abB-3Ab^2\right)\int\cot(c+dx)dx}{a}+\frac{b^3\left(-4a^3B+5a^2Ab-2ab^2B+3Ab^3\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^3x\left(a^2(-B)+2aAb+b^2B\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}-\frac{b\left(a^3(-B)+2a^2Ab\ldots\right.}{ad\left(a^2+b^2\right)(a+\ldots}\right.}{a}}{2a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$\downarrow$ 3042

$$-\frac{2\left(\frac{\frac{\left(a^2+b^2\right)\left(a^2A+2abB-3Ab^2\right)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\frac{b^3\left(-4a^3B+5a^2Ab-2ab^2B+3Ab^3\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^3x\left(a^2(-B)+2aAb+b^2B\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}-\frac{b\left(a^3(-B)\ldots\right.}{ad\left(a^2+\ldots\right.}\right.}{a}}{2a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$\downarrow$ 25

$$-\frac{2\left(\frac{-\frac{\left(a^2+b^2\right)\left(a^2A+2abB-3Ab^2\right)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}+\frac{b^3\left(-4a^3B+5a^2Ab-2ab^2B+3Ab^3\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^3x\left(a^2(-B)+2aAb+b^2B\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}-\frac{b\left(a^3(-\ldots\right.}{ad\left(\ldots\right.}\right.}{a}}{2a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$\downarrow$ 3956

$$-\frac{\frac{2\left(\frac{\frac{b^3\left(-4a^3B+5a^2Ab-2ab^2B+3Ab^3\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{\left(a^2+b^2\right)\left(a^2A+2abB-3Ab^2\right)\log(-\sin(c+dx))}{ad}-\frac{a^3x\left(a^2(-B)+2aAb+b^2B\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}-\frac{b\left(a^3(-B)+2a^2 \cdots\right.}{ad\left(a^2+b^2\right)(\cdots}\right.}{a}}{2a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4013}$$

$$-\frac{\frac{2\left(\frac{\frac{\left(a^2+b^2\right)\left(a^2A+2abB-3Ab^2\right)\log(-\sin(c+dx))}{ad}-\frac{a^3x\left(a^2(-B)+2aAb+b^2B\right)}{a^2+b^2}+\frac{b^3\left(-4a^3B+5a^2Ab-2ab^2B+3Ab^3\right)\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}-\frac{b\left(a^3(\cdots\right.}{ad \cdots}\right.}{a}}{2a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
-1/2*(A*Cot[c + d*x]^2)/(a*d*(a + b*Tan[c + d*x])) - (-(((3*A*b - 2*a*B)*C
ot[c + d*x])/(a*d*(a + b*Tan[c + d*x]))) + (2*((-((a^3*(2*a*A*b - a^2*B +
b^2*B)*x)/(a^2 + b^2)) + ((a^2 + b^2)*(a^2*A - 3*A*b^2 + 2*a*b*B)*Log[-Sin
[c + d*x]])/(a*d) + (b^3*(5*a^2*A*b + 3*A*b^3 - 4*a^3*B - 2*a*b^2*B)*Log[a
*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/(a*(a^2 + b^2)) - (b*(
2*a^2*A*b + 3*A*b^3 - a^3*B - 2*a*b^2*B))/(a*(a^2 + b^2)*d*(a + b*Tan[c +
d*x]))))/a)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\frac{\left(A\,a^{2}-A\,b^{2}+2Bab\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(2Aab-B\,a^{2}+B\,b^{2}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{2}}-\frac{A}{2a^{2}\tan(dx+c)^{2}}-\frac{-2Ab+Ba}{a^{3}\tan(dx+c)}+\frac{\left(-A\,a^{2}+3A \cdots}{\cdots}}{d}$
default	$\frac{\frac{\frac{\left(A\,a^{2}-A\,b^{2}+2Bab\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(2Aab-B\,a^{2}+B\,b^{2}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{2}}-\frac{A}{2a^{2}\tan(dx+c)^{2}}-\frac{-2Ab+Ba}{a^{3}\tan(dx+c)}+\frac{\left(-A\,a^{2}+3A \cdots}{\cdots}}{d}$
parallelrisch	$\frac{-10b^{3}\left(A\,a^{2}b+\frac{3}{5}A\,b^{3}-\frac{4}{5}B\,a^{3}-\frac{2}{5}Ba\,b^{2}\right)(a+b\tan(dx+c))\ln(a+b\tan(dx+c))+a^{4}(a+b\tan(dx+c))\left(A\,a^{2}-A\,b^{2}+2Bab\right) \cdots}{\cdots}$
norman	$\frac{\frac{a\left(2Aab-B\,a^{2}+B\,b^{2}\right)x\tan(dx+c)^{2}}{a^{4}+2a^{2}b^{2}+b^{4}}+\frac{b\left(2Aab-B\,a^{2}+B\,b^{2}\right)x\tan(dx+c)^{3}}{a^{4}+2a^{2}b^{2}+b^{4}}-\frac{A}{2ad}+\frac{(3Ab-2Ba)\tan(dx+c)}{2a^{2}d}-\frac{\left(2A\,a^{2}b^{2}+3A\,b^{4}-B\,a^{3} \cdots}{d\,a^{4}\left(a \cdots\right)}}{\tan(dx+c)^{2}(a+b\tan(dx+c))}$
risch	$\frac{xB}{2iab-a^{2}+b^{2}}+\frac{6ib^{6}Ac}{a^{4}d(a^{4}+2a^{2}b^{2}+b^{4})}+\frac{6ib^{6}Ax}{a^{4}(a^{4}+2a^{2}b^{2}+b^{4})}+\frac{4iBbc}{a^{3}d}-\frac{8ib^{3}Bc}{ad(a^{4}+2a^{2}b^{2}+b^{4})}-\frac{4ib^{5}Bx}{a^{3}(a^{4}+2a^{2}b^{2}+b^{4})} \cdots$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^2*(1/2*(A*a^2-A*b^2+2*B*a*b)*ln(1+tan(d*x+c)^2)+(2*A*a*b-
B*a^2+B*b^2)*arctan(tan(d*x+c)))-1/2/a^2*A/tan(d*x+c)^2-(-2*A*b+B*a)/a^3/t
an(d*x+c)+(-A*a^2+3*A*b^2-2*B*a*b)/a^4*ln(tan(d*x+c))-b^3*(5*A*a^2*b+3*A*b
^3-4*B*a^3-2*B*a*b^2)/(a^2+b^2)^2/a^4*ln(a+b*tan(d*x+c))+(A*b-B*a)*b^3/(a^
2+b^2)/a^3/(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(246) = 492$.

Time = 0.16 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.36

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{Aa^7 + 2\,Aa^5b^2 + Aa^3b^4 + (Aa^6b + 2\,Aa^4b^3 - 2\,Ba^3b^4 + 3\,Aa^2b^5 + 2\,(Ba^6b - 2\,Aa^5b^2 - Ba^4b^3)dx)\tan}{}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
-1/2*(A*a^7 + 2*A*a^5*b^2 + A*a^3*b^4 + (A*a^6*b + 2*A*a^4*b^3 - 2*B*a^3*b
^4 + 3*A*a^2*b^5 + 2*(B*a^6*b - 2*A*a^5*b^2 - B*a^4*b^3)*d*x)*tan(d*x + c)
^3 + (A*a^7 + 2*B*a^6*b - 2*A*a^5*b^2 + 4*B*a^4*b^3 - 7*A*a^3*b^4 + 4*B*a^
2*b^5 - 6*A*a*b^6 + 2*(B*a^7 - 2*A*a^6*b - B*a^5*b^2)*d*x)*tan(d*x + c)^2
+ ((A*a^6*b + 2*B*a^5*b^2 - A*a^4*b^3 + 4*B*a^3*b^4 - 5*A*a^2*b^5 + 2*B*a*
b^6 - 3*A*b^7)*tan(d*x + c)^3 + (A*a^7 + 2*B*a^6*b - A*a^5*b^2 + 4*B*a^4*b
^3 - 5*A*a^3*b^4 + 2*B*a^2*b^5 - 3*A*a*b^6)*tan(d*x + c)^2)*log(tan(d*x +
c)^2/(tan(d*x + c)^2 + 1)) - ((4*B*a^3*b^4 - 5*A*a^2*b^5 + 2*B*a*b^6 - 3*A
*b^7)*tan(d*x + c)^3 + (4*B*a^4*b^3 - 5*A*a^3*b^4 + 2*B*a^2*b^5 - 3*A*a*b^
6)*tan(d*x + c)^2)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(ta
n(d*x + c)^2 + 1)) + (2*B*a^7 - 3*A*a^6*b + 4*B*a^5*b^2 - 6*A*a^4*b^3 + 2*
B*a^3*b^4 - 3*A*a^2*b^5)*tan(d*x + c))/((a^8*b + 2*a^6*b^3 + a^4*b^5)*d*ta
n(d*x + c)^3 + (a^9 + 2*a^7*b^2 + a^5*b^4)*d*tan(d*x + c)^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.14 (sec) , antiderivative size = 9896, normalized size of antiderivative = 39.58

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise(((A*log(tan(c + d*x)**2 + 1)/(2*d) - A*log(tan(c + d*x))/d - A/(
2*d*tan(c + d*x)**2) - B*x - B/(d*tan(c + d*x)))/a**2, Eq(b, 0)), ((-A*log
(tan(c + d*x)**2 + 1)/(2*d) + A*log(tan(c + d*x))/d + A/(2*d*tan(c + d*x)*
*2) - A/(4*d*tan(c + d*x)**4) + B*x + B/(d*tan(c + d*x)) - B/(3*d*tan(c +
d*x)**3))/b**2, Eq(a, 0)), (-15*I*A*d*x*tan(c + d*x)**4/(4*a**2*d*tan(c +
d*x)**4 + 8*I*a**2*d*tan(c + d*x)**3 - 4*a**2*d*tan(c + d*x)**2) + 30*A*d*
x*tan(c + d*x)**3/(4*a**2*d*tan(c + d*x)**4 + 8*I*a**2*d*tan(c + d*x)**3 -
 4*a**2*d*tan(c + d*x)**2) + 15*I*A*d*x*tan(c + d*x)**2/(4*a**2*d*tan(c +
d*x)**4 + 8*I*a**2*d*tan(c + d*x)**3 - 4*a**2*d*tan(c + d*x)**2) + 8*A*log
(tan(c + d*x)**2 + 1)*tan(c + d*x)**4/(4*a**2*d*tan(c + d*x)**4 + 8*I*a**2
*d*tan(c + d*x)**3 - 4*a**2*d*tan(c + d*x)**2) + 16*I*A*log(tan(c + d*x)**
2 + 1)*tan(c + d*x)**3/(4*a**2*d*tan(c + d*x)**4 + 8*I*a**2*d*tan(c + d*x)
**3 - 4*a**2*d*tan(c + d*x)**2) - 8*A*log(tan(c + d*x)**2 + 1)*tan(c + d*x
)**2/(4*a**2*d*tan(c + d*x)**4 + 8*I*a**2*d*tan(c + d*x)**3 - 4*a**2*d*tan
(c + d*x)**2) - 16*A*log(tan(c + d*x))*tan(c + d*x)**4/(4*a**2*d*tan(c + d
*x)**4 + 8*I*a**2*d*tan(c + d*x)**3 - 4*a**2*d*tan(c + d*x)**2) - 32*I*A*l
og(tan(c + d*x))*tan(c + d*x)**3/(4*a**2*d*tan(c + d*x)**4 + 8*I*a**2*d*ta
n(c + d*x)**3 - 4*a**2*d*tan(c + d*x)**2) + 16*A*log(tan(c + d*x))*tan(c +
 d*x)**2/(4*a**2*d*tan(c + d*x)**4 + 8*I*a**2*d*tan(c + d*x)**3 - 4*a**2*d
*tan(c + d*x)**2) - 15*I*A*tan(c + d*x)**3/(4*a**2*d*tan(c + d*x)**4 + ...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.30

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\,(Ba^2-2\,Aab-Bb^2)(dx+c)}{a^4+2\,a^2b^2+b^4} - \frac{2\,(4\,Ba^3b^3-5\,Aa^2b^4+2\,Bab^5-3\,Ab^6)\log(b\tan(dx+c)+a)}{a^8+2\,a^6b^2+a^4b^4} - \frac{(Aa^2+2\,Bab-Ab^2)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} +}{}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="m
axima")
```

output
```
-1/2*(2*(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(4
*B*a^3*b^3 - 5*A*a^2*b^4 + 2*B*a*b^5 - 3*A*b^6)*log(b*tan(d*x + c) + a)/(a
^8 + 2*a^6*b^2 + a^4*b^4) - (A*a^2 + 2*B*a*b - A*b^2)*log(tan(d*x + c)^2 +
 1)/(a^4 + 2*a^2*b^2 + b^4) + (A*a^4 + A*a^2*b^2 + 2*(B*a^3*b - 2*A*a^2*b^
2 + 2*B*a*b^3 - 3*A*b^4)*tan(d*x + c)^2 + (2*B*a^4 - 3*A*a^3*b + 2*B*a^2*b
^2 - 3*A*a*b^3)*tan(d*x + c))/((a^5*b + a^3*b^3)*tan(d*x + c)^3 + (a^6 + a
^4*b^2)*tan(d*x + c)^2) + 2*(A*a^2 + 2*B*a*b - 3*A*b^2)*log(tan(d*x + c))/
a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.52

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= -\frac{(Ba^2-2\,Aab-Bb^2)(dx+c)}{a^4d+2\,a^2b^2d+b^4d} + \frac{(Aa^2+2\,Bab-Ab^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,(a^4d+2\,a^2b^2d+b^4d)}$$
$$+ \frac{(4\,Ba^3b^4-5\,Aa^2b^5+2\,Bab^6-3\,Ab^7)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^8bd+2\,a^6b^3d+a^4b^5d}$$
$$- \frac{(Aa^2+2\,Bab-3\,Ab^2)\log\left(|\tan\left(dx+c\right)|\right)}{a^4d}$$
$$- \frac{Aa^7+2\,Aa^5b^2+Aa^3b^4+2\left(Ba^6b-2\,Aa^5b^2+3\,Ba^4b^3-5\,Aa^3b^4+2\,Ba^2b^5-3\,Aab^6\right)\tan\left(dx+c\right)^2 +}{2\left(a^2+b^2\right)^2(b\tan\left(dx+c\right)+a)a^4d\tan}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="g
iac")
```

output
```
-(B*a^2 - 2*A*a*b - B*b^2)*(d*x + c)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(
A*a^2 + 2*B*a*b - A*b^2)*log(tan(d*x + c)^2 + 1)/(a^4*d + 2*a^2*b^2*d + b^
4*d) + (4*B*a^3*b^4 - 5*A*a^2*b^5 + 2*B*a*b^6 - 3*A*b^7)*log(abs(b*tan(d*x
 + c) + a))/(a^8*b*d + 2*a^6*b^3*d + a^4*b^5*d) - (A*a^2 + 2*B*a*b - 3*A*b
^2)*log(abs(tan(d*x + c)))/(a^4*d) - 1/2*(A*a^7 + 2*A*a^5*b^2 + A*a^3*b^4
+ 2*(B*a^6*b - 2*A*a^5*b^2 + 3*B*a^4*b^3 - 5*A*a^3*b^4 + 2*B*a^2*b^5 - 3*A
*a*b^6)*tan(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b + 4*B*a^5*b^2 - 6*A*a^4*b^3
+ 2*B*a^3*b^4 - 3*A*a^2*b^5)*tan(d*x + c))/((a^2 + b^2)^2*(b*tan(d*x + c)
+ a)*a^4*d*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 8.19 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.14

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{\tan(c+dx)\,(3\,A\,b-2\,B\,a)}{2\,a^2}-\frac{A}{2\,a}+\frac{\tan(c+dx)^2\,\left(-B\,a^3\,b+2\,A\,a^2\,b^2-2\,B\,a\,b^3+3\,A\,b^4\right)}{a^3\,(a^2+b^2)}}{d\,\left(b\tan\left(c+d\,x\right)^3+a\tan\left(c+d\,x\right)^2\right)}$$
$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(A\,a^2+2\,B\,a\,b-3\,A\,b^2\right)}{a^4\,d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)}{2\,d\,\left(a^2+a\,b\,2\mathrm{i}-b^2\right)}$$
$$-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(-4\,B\,a^3\,b^3+5\,A\,a^2\,b^4-2\,B\,a\,b^5+3\,A\,b^6\right)}{d\,\left(a^8+2\,a^6\,b^2+a^4\,b^4\right)}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
((tan(c + d*x)*(3*A*b - 2*B*a))/(2*a^2) - A/(2*a) + (tan(c + d*x)^2*(3*A*b
^4 + 2*A*a^2*b^2 - 2*B*a*b^3 - B*a^3*b))/(a^3*(a^2 + b^2)))/(d*(a*tan(c +
d*x)^2 + b*tan(c + d*x)^3)) - (log(tan(c + d*x))*(A*a^2 - 3*A*b^2 + 2*B*a*
b))/(a^4*d) + (log(tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(a*b*2i + a^2 - b^2
)) - (log(a + b*tan(c + d*x))*(3*A*b^6 + 5*A*a^2*b^4 - 4*B*a^3*b^3 - 2*B*a
*b^5))/(d*(a^8 + a^4*b^4 + 2*a^6*b^2)) + (log(tan(c + d*x) + 1i)*(A*1i + B
))/(2*d*(2*a*b + a^2*1i - b^2*1i))
```

Reduce [B] (verification not implemented)

Time = 30.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.94

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{4\cos\left(dx+c\right)\sin\left(dx+c\right)a^3b+4\cos\left(dx+c\right)\sin\left(dx+c\right)a\,b^3+4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin\left(dx+c\right)^2 a}{}$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(4*cos(c + d*x)*sin(c + d*x)*a**3*b + 4*cos(c + d*x)*sin(c + d*x)*a*b**3 +
 4*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**4 - 4*log(tan((c + d*x)
/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*b**4 - 4*log(tan((c +
 d*x)/2))*sin(c + d*x)**2*a**4 + 4*log(tan((c + d*x)/2))*sin(c + d*x)**2*b
**4 + sin(c + d*x)**2*a**4 + 4*sin(c + d*x)**2*a**3*b*d*x + sin(c + d*x)**
2*a**2*b**2 - 2*a**4 - 2*a**2*b**2)/(4*sin(c + d*x)**2*a**3*d*(a**2 + b**2
))
```

3.282 $\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 331

$$\begin{aligned}\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx &= \frac{(a^3A-3aAb^2+3a^2bB-b^3B)\,x}{(a^2+b^2)^3}+\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\log(\cos(c+dx))}{(a^2+b^2)^3\,d}\\&+\frac{a^2(a^4Ab+3a^2Ab^3+6Ab^5-3a^5B-9a^3b^2B-10ab^4B)\log(a+b\tan(c+dx))}{b^4\,(a^2+b^2)^3\,d}\\&-\frac{(a^3Ab+3aAb^3-3a^4B-6a^2b^2B-b^4B)\tan(c+dx)}{b^3\,(a^2+b^2)^2\,d}\\&+\frac{a(Ab-aB)\tan^3(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}\\&+\frac{a(a^2Ab+5Ab^3-3a^3B-7ab^2B)\tan^2(c+dx)}{2b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output

```
(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*x/(a^2+b^2)^3+(3*A*a^2*b-A*b^3-B*a^3+3*B
*a*b^2)*ln(cos(d*x+c))/(a^2+b^2)^3/d+a^2*(A*a^4*b+3*A*a^2*b^3+6*A*b^5-3*B*
a^5-9*B*a^3*b^2-10*B*a*b^4)*ln(a+b*tan(d*x+c))/b^4/(a^2+b^2)^3/d-(A*a^3*b+
3*A*a*b^3-3*B*a^4-6*B*a^2*b^2-B*b^4)*tan(d*x+c)/b^3/(a^2+b^2)^2/d+1/2*a*(A
*b-B*a)*tan(d*x+c)^3/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/2*a*(A*a^2*b+5*A*b
^3-3*B*a^3-7*B*a*b^2)*tan(d*x+c)^2/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.52 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.83

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(A+iB)\log(i-\tan(c+dx))}{(-ia+b)^3}+\frac{(A-iB)\log(i+\tan(c+dx))}{(ia+b)^3}+\frac{2a^2(a^4Ab+3a^2Ab^3+6Ab^5-3a^5B-9a^3b^2B-10ab^4B)\log(a+b\tan(c+dx))}{b^4(a^2+b^2)^3}+\frac{}{b^{}}}{2d}$$

input
```
Integrate[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(((A + I*B)*Log[I - Tan[c + d*x]])/((-I)*a + b)^3 + ((A - I*B)*Log[I + Tan
[c + d*x]])/(I*a + b)^3 + (2*a^2*(a^4*A*b + 3*a^2*A*b^3 + 6*A*b^5 - 3*a^5*
B - 9*a^3*b^2*B - 10*a*b^4*B)*Log[a + b*Tan[c + d*x]])/(b^4*(a^2 + b^2)^3)
 + (a^3*(-(a*A*b) + 3*a^2*B + 2*b^2*B))/(b^4*(a^2 + b^2)*(a + b*Tan[c + d*
x])^2) + (2*B*Tan[c + d*x]^3)/(b*(a + b*Tan[c + d*x])^2) - (2*a^2*(-2*a^3*
A*b - 4*a*A*b^3 + 6*a^4*B + 11*a^2*b^2*B + 3*b^4*B))/(b^4*(a^2 + b^2)^2*(a
 + b*Tan[c + d*x])))/(2*d)
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4088, 25, 3042, 4128, 27, 3042, 4130, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\tan(c+dx)^4(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{\int -\frac{\tan^2(c+dx)\left(\left(-3Ba^2+Aba-2b^2B\right)\tan^2(c+dx)-2b(Ab-aB)\tan(c+dx)+3a(Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan^2(c+dx)\left(\left(-3Ba^2+Aba-2b^2B\right)\tan^2(c+dx)-2b(Ab-aB)\tan(c+dx)+3a(Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)^2\left(\left(-3Ba^2+Aba-2b^2B\right)\tan(c+dx)^2-2b(Ab-aB)\tan(c+dx)+3a(Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4128}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\int \frac{2\tan(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(-3Ba^4+Aba^3-6b^2Ba^2+3Ab^3a-b^4B\right)\tan^2(c+dx)+a\left(-3Ba^3+Aba^2-7b^2Ba+5Ab^3\right)\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}-\frac{a\left(-3a^3B+a\cdots\right.}{bd(\cdots}}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{2\int \frac{\tan(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(-3Ba^4+Aba^3-6b^2Ba^2+3Ab^3a-b^4B\right)\tan^2(c+dx)+a\left(-3Ba^3+Aba^2-7b^2Ba+5Ab^3\right)\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}-\frac{a\left(-3a^3B+a\cdots\right.}{bd(\cdots}}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{2\int \frac{\tan(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(-3Ba^4+Aba^3-6b^2Ba^2+3Ab^3a-b^4B\right)\tan(c+dx)^2+a\left(-3Ba^3+Aba^2-7b^2Ba+5Ab^3\right)\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}-\frac{a\left(-3a^3B+a\cdots\right.}{bd(\cdots}}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4130}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\frac{\int -\frac{-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)b^3\right)+\left(a^2+b^2\right)^2(Ab-3aB)\tan^2(c+dx)+a\left(-3Ba^4+Aba^3-6b^2Ba^2+3Ab^3a-b^4B\right)}{a+b\tan(c+dx)}dx}{b}+\frac{\left(-3a^4B+a^3Ab-6a^2b^2B+3aA\right.}{bd}}{b(a^2+b^2)}\right.}{2b(a^2+b^2)}$$

$\downarrow$ 25

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\frac{\left(-3a^4B+a^3Ab-6a^2b^2B+3aAb^3-b^4B\right)\tan(c+dx)}{bd}-\frac{\int \frac{-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)b^3\right)+\left(a^2+b^2\right)^2(Ab-3aB)\tan^2(c+dx)+a\left(-3Ba^4+Aba^3-6b^2Ba\right.}{a+b\tan(c+dx)}}{b}}{b(a^2+b^2)}\right.}{2b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\frac{\left(-3a^4B+a^3Ab-6a^2b^2B+3aAb^3-b^4B\right)\tan(c+dx)}{bd}-\frac{\int \frac{-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)b^3\right)+\left(a^2+b^2\right)^2(Ab-3aB)\tan(c+dx)^2+a\left(-3Ba^4+Aba^3-6b^2Ba\right.}{a+b\tan(c+dx)}}{b}}{b(a^2+b^2)}\right.}{2b(a^2+b^2)}$$

$\downarrow$ 4109

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\frac{\left(-3a^4B+a^3Ab-6a^2b^2B+3aAb^3-b^4B\right)\tan(c+dx)}{bd}-\frac{-\frac{b^3\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^2\left(-3a^5B+a^4Ab-9a^3b^2B+3a^2Ab^3-10ab^4B+\right.}{a^2+b^2}}{b}}{b(a^2+b^2)}\right.}{2b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\frac{\left(-3a^4B+a^3Ab-6a^2b^2B+3aAb^3-b^4B\right)\tan(c+dx)}{bd}-\frac{-\frac{b^3\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^2\left(-3a^5B+a^4Ab-9a^3b^2B+3a^2Ab^3-10ab^4B+\right.}{a^2+b^2}}{b}}{b(a^2+b^2)}\right.}{2b(a^2+b^2)}$$

$\downarrow$ 3956

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\left(-3a^4B+a^3Ab-6a^2b^2B+3aAb^3-b^4B\right)\tan(c+dx)}{bd}-\frac{\frac{a^2\left(-3a^5B+a^4Ab-9a^3b^2B+3a^2Ab^3-10ab^4B+6Ab^5\right)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^3\left(a^3(-B)+3a^2Ab+3ab\cdots\right)}{d\left(a^2\cdots\right)}}{b}\cdots\right)}{b(a^2+b^2)}\cdots}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 4100$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\left(-3a^4B+a^3Ab-6a^2b^2B+3aAb^3-b^4B\right)\tan(c+dx)}{bd}-\frac{\frac{a^2\left(-3a^5B+a^4Ab-9a^3b^2B+3a^2Ab^3-10ab^4B+6Ab^5\right)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd\left(a^2+b^2\right)}+\frac{b^3\left(a^3(-B)+\cdots\right)}{\cdots}}{b}\cdots\right)}{b(a^2+b^2)}\cdots}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 16$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{2\left(\frac{\left(-3a^4B+a^3Ab-6a^2b^2B+3aAb^3-b^4B\right)\tan(c+dx)}{bd}-\frac{\frac{b^3\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^3x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}+\frac{a^2\left(-3a^5B+a^4\cdots\right)}{\cdots}}{b}\cdots\right)}{b(a^2+b^2)}\cdots}{2b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(a*(A*b - a*B)*Tan[c + d*x]^3)/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
- (-((a*(a^2*A*b + 5*A*b^3 - 3*a^3*B - 7*a*b^2*B)*Tan[c + d*x]^2)/(b*(a^2
+ b^2)*d*(a + b*Tan[c + d*x]))) + (2*(-(((b^3*(a^3*A - 3*a*A*b^2 + 3*a^2*b
*B - b^3*B)*x)/(a^2 + b^2) + (b^3*(3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*
Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a^2*(a^4*A*b + 3*a^2*A*b^3 + 6*A*b^5
 - 3*a^5*B - 9*a^3*b^2*B - 10*a*b^4*B)*Log[a + b*Tan[c + d*x]])/(b*(a^2 +
b^2)*d))/b) + ((a^3*A*b + 3*a*A*b^3 - 3*a^4*B - 6*a^2*b^2*B - b^4*B)*Tan[c
 + d*x])/(b*d)))/(b*(a^2 + b^2)))/(2*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)B}{b^3}+\frac{\frac{\left(-3A\,a^2b+A\,b^3+B\,a^3-3Ba\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(A\,a^3-3Aa\,b^2+3B\,a^2b-B\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a^2\left(A\,a^4\right.}{}}{}$
default	$\frac{\frac{\tan(dx+c)B}{b^3}+\frac{\frac{\left(-3A\,a^2b+A\,b^3+B\,a^3-3Ba\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(A\,a^3-3Aa\,b^2+3B\,a^2b-B\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a^2\left(A\,a^4\right.}{}}{}$
norman	$\frac{\frac{B\tan(dx+c)^3}{db}+\frac{\left(A\,a^3-3Aa\,b^2+3B\,a^2b-B\,b^3\right)a^2x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{b^2\left(A\,a^3-3Aa\,b^2+3B\,a^2b-B\,b^3\right)x\tan(dx+c)^2}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{a\left(2A\,a^4b+4A\,a^2b^3-6B\,a^5\right.}{d\,b^3\left(a^4\right.}}{(a+b\tan}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(tan(d*x+c)*B/b^3+1/(a^2+b^2)^3*(1/2*(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2
)*ln(1+tan(d*x+c)^2)+(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*arctan(tan(d*x+c)))
+1/b^4*a^2*(A*a^4*b+3*A*a^2*b^3+6*A*b^5-3*B*a^5-9*B*a^3*b^2-10*B*a*b^4)/(a
^2+b^2)^3*ln(a+b*tan(d*x+c))-1/2/b^4*a^4*(A*b-B*a)/(a^2+b^2)/(a+b*tan(d*x+
c))^2+1/b^4*a^3*(2*A*a^2*b+4*A*b^3-3*B*a^3-5*B*a*b^2)/(a^2+b^2)^2/(a+b*tan
(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(328) = 656$.

Time = 0.17 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.69

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/2*(3*B*a^7*b^2 - A*a^6*b^3 + 9*B*a^5*b^4 - 7*A*a^4*b^5 - 2*(B*a^6*b^3 +
 3*B*a^4*b^5 + 3*B*a^2*b^7 + B*b^9)*tan(d*x + c)^3 - 2*(A*a^5*b^4 + 3*B*a^
4*b^5 - 3*A*a^3*b^6 - B*a^2*b^7)*d*x - (9*B*a^7*b^2 - 3*A*a^6*b^3 + 23*B*a
^5*b^4 - 9*A*a^4*b^5 + 12*B*a^3*b^6 + 4*B*a*b^8 + 2*(A*a^3*b^6 + 3*B*a^2*b
^7 - 3*A*a*b^8 - B*b^9)*d*x)*tan(d*x + c)^2 + (3*B*a^9 - A*a^8*b + 9*B*a^7
*b^2 - 3*A*a^6*b^3 + 10*B*a^5*b^4 - 6*A*a^4*b^5 + (3*B*a^7*b^2 - A*a^6*b^3
 + 9*B*a^5*b^4 - 3*A*a^4*b^5 + 10*B*a^3*b^6 - 6*A*a^2*b^7)*tan(d*x + c)^2
+ 2*(3*B*a^8*b - A*a^7*b^2 + 9*B*a^6*b^3 - 3*A*a^5*b^4 + 10*B*a^4*b^5 - 6*
A*a^3*b^6)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^
2)/(tan(d*x + c)^2 + 1)) - (3*B*a^9 - A*a^8*b + 9*B*a^7*b^2 - 3*A*a^6*b^3
+ 9*B*a^5*b^4 - 3*A*a^4*b^5 + 3*B*a^3*b^6 - A*a^2*b^7 + (3*B*a^7*b^2 - A*a
^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 + 3*B*a*b^8
 - A*b^9)*tan(d*x + c)^2 + 2*(3*B*a^8*b - A*a^7*b^2 + 9*B*a^6*b^3 - 3*A*a^
5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 + 3*B*a^2*b^7 - A*a*b^8)*tan(d*x + c))*l
og(1/(tan(d*x + c)^2 + 1)) - 2*(3*B*a^8*b - A*a^7*b^2 + 6*B*a^6*b^3 - 3*A*
a^5*b^4 - 2*B*a^4*b^5 + 4*A*a^3*b^6 + B*a^2*b^7 + 2*(A*a^4*b^5 + 3*B*a^3*b
^6 - 3*A*a^2*b^7 - B*a*b^8)*d*x)*tan(d*x + c))/((a^6*b^6 + 3*a^4*b^8 + 3*a
^2*b^10 + b^12)*d*tan(d*x + c)^2 + 2*(a^7*b^5 + 3*a^5*b^7 + 3*a^3*b^9 + a*
b^11)*d*tan(d*x + c) + (a^8*b^4 + 3*a^6*b^6 + 3*a^4*b^8 + a^2*b^10)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)**4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.18

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{2\,(3\,Ba^7-Aa^6b+9\,Ba^5b^2-3\,Aa^4b^3+10\,Ba^3b^4-6\,Aa^2b^5)\log(b\tan(dx+c)+a)}{a^6b^4+3\,a^4b^6+3\,a^2b^8+b^{10}} + \frac{(Ba^3-3\,Aa^2b-3}{a^6+}}{2}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="m
axima")
```

output
```
1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - 2*(3*B*a^7 - A*a^6*b + 9*B*a^5*b^2 - 3*A*a^4*b^3 + 10
*B*a^3*b^4 - 6*A*a^2*b^5)*log(b*tan(d*x + c) + a)/(a^6*b^4 + 3*a^4*b^6 + 3
*a^2*b^8 + b^10) + (B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan(d*x + c
)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (5*B*a^7 - 3*A*a^6*b + 9*B*
a^5*b^2 - 7*A*a^4*b^3 + 2*(3*B*a^6*b - 2*A*a^5*b^2 + 5*B*a^4*b^3 - 4*A*a^3
*b^4)*tan(d*x + c))/(a^6*b^4 + 2*a^4*b^6 + a^2*b^8 + (a^4*b^6 + 2*a^2*b^8
+ b^10)*tan(d*x + c)^2 + 2*(a^5*b^5 + 2*a^3*b^7 + a*b^9)*tan(d*x + c)) + 2
*B*tan(d*x + c)/b^3)/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.16

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)(dx+c)}{a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d}$$
$$+ \frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log\left(\tan(dx+c)^2+1\right)}{2\,(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d)}$$
$$- \frac{(3\,Ba^7-Aa^6b+9\,Ba^5b^2-3\,Aa^4b^3+10\,Ba^3b^4-6\,Aa^2b^5)\log\left(|b\tan(dx+c)+a|\right)}{a^6b^4d+3\,a^4b^6d+3\,a^2b^8d+b^{10}d}$$
$$+ \frac{B\tan(dx+c)}{b^3d}$$
$$- \frac{5\,Ba^9-3\,Aa^8b+14\,Ba^7b^2-10\,Aa^6b^3+9\,Ba^5b^4-7\,Aa^4b^5+2\,(3\,Ba^8b-2\,Aa^7b^2+8\,Ba^6b^3-6\,Aa^5}{2\,(a^2+b^2)^3(b\tan(dx+c)+a)^2b^4d}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="g
iac")
```

output
```
(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6*d + 3*a^4*b^2*d + 3
*a^2*b^4*d + b^6*d) + 1/2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan(
d*x + c)^2 + 1)/(a^6*d + 3*a^4*b^2*d + 3*a^2*b^4*d + b^6*d) - (3*B*a^7 - A
*a^6*b + 9*B*a^5*b^2 - 3*A*a^4*b^3 + 10*B*a^3*b^4 - 6*A*a^2*b^5)*log(abs(b
*tan(d*x + c) + a))/(a^6*b^4*d + 3*a^4*b^6*d + 3*a^2*b^8*d + b^10*d) + B*t
an(d*x + c)/(b^3*d) - 1/2*(5*B*a^9 - 3*A*a^8*b + 14*B*a^7*b^2 - 10*A*a^6*b
^3 + 9*B*a^5*b^4 - 7*A*a^4*b^5 + 2*(3*B*a^8*b - 2*A*a^7*b^2 + 8*B*a^6*b^3
- 6*A*a^5*b^4 + 5*B*a^4*b^5 - 4*A*a^3*b^6)*tan(d*x + c))/((a^2 + b^2)^3*(b
*tan(d*x + c) + a)^2*b^4*d)
```

Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.01

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \frac{B\tan(c+d\,x)}{b^3\,d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)} - \frac{\frac{5\,B\,a^7-3\,A\,a^6\,b+9\,B\,a^5\,b^2-7\,A\,a^4\,b^3}{2\,b\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(3\,B\,a^6-2\,A\,a^5\,b+5\,B\,a^4\,b^2-4\,A\,a^3\,b^3\right)}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(a^2\,b^3+2\,a\,b^4\,\tan\left(c+d\,x\right)+b^5\,{\tan\left(c+d\,x\right)}^2\right)} + \frac{a^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-3\,B\,a^5+A\,a^4\,b-9\,B\,a^3\,b^2+3\,A\,a^2\,b^3-10\,B\,a\,b^4+6\,A\,b^5\right)}{b^4\,d\,{\left(a^2+b^2\right)}^3}$$

input
```
int((tan(c + d*x)^4*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
(log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1
i)) - ((5*B*a^7 - 7*A*a^4*b^3 + 9*B*a^5*b^2 - 3*A*a^6*b)/(2*b*(a^4 + b^4 +
 2*a^2*b^2)) + (tan(c + d*x)*(3*B*a^6 - 4*A*a^3*b^3 + 5*B*a^4*b^2 - 2*A*a^
5*b))/(a^4 + b^4 + 2*a^2*b^2))/(d*(a^2*b^3 + b^5*tan(c + d*x)^2 + 2*a*b^4*
tan(c + d*x))) + (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a*b^2*3i - 3*a^
2*b - a^3*1i + b^3)) + (B*tan(c + d*x))/(b^3*d) + (a^2*log(a + b*tan(c + d
*x))*(6*A*b^5 - 3*B*a^5 + 3*A*a^2*b^3 - 9*B*a^3*b^2 + A*a^4*b - 10*B*a*b^4
))/(b^4*d*(a^2 + b^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.94

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{-\log\left(\tan(dx+c)^2+1\right)\tan(dx+c)\,ab^5-\log\left(\tan(dx+c)^2+1\right)a^2b^4-2\log(a+\tan(dx+c)\,b)\tan(\ldots}{}$$

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
( - log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a*b**5 - log(tan(c + d*x)**2 + 1
)*a**2*b**4 - 2*log(tan(c + d*x)*b + a)*tan(c + d*x)*a**5*b - 4*log(tan(c
+ d*x)*b + a)*tan(c + d*x)*a**3*b**3 - 2*log(tan(c + d*x)*b + a)*a**6 - 4*
log(tan(c + d*x)*b + a)*a**4*b**2 + tan(c + d*x)**2*a**4*b**2 + 2*tan(c +
d*x)**2*a**2*b**4 + tan(c + d*x)**2*b**6 + 2*tan(c + d*x)*a**5*b + 3*tan(c
 + d*x)*a**3*b**3 + tan(c + d*x)*a**2*b**4*d*x + tan(c + d*x)*a*b**5 - tan
(c + d*x)*b**6*d*x + a**3*b**3*d*x - a*b**5*d*x)/(b**3*d*(tan(c + d*x)*a**
4*b + 2*tan(c + d*x)*a**2*b**3 + tan(c + d*x)*b**5 + a**5 + 2*a**3*b**2 +
a*b**4))
```

3.283 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 250

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\,x}{(a^2+b^2)^3} + \frac{(a^3A-3aAb^2+3a^2bB-b^3B)\log(\cos(c+dx))}{(a^2+b^2)^3\,d} + \frac{a(a^2Ab^3-3Ab^5+a^5B+3a^3b^2B+6ab^4B)\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)^3\,d} + \frac{a(Ab-aB)\tan^2(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{a^2(2Ab^3-a(a^2+3b^2)\,B)}{b^3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

output

```
-(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*x/(a^2+b^2)^3+(A*a^3-3*A*a*b^2+3*B*a^2*
b-B*b^3)*ln(cos(d*x+c))/(a^2+b^2)^3/d+a*(A*a^2*b^3-3*A*b^5+B*a^5+3*B*a^3*b
^2+6*B*a*b^4)*ln(a+b*tan(d*x+c))/b^3/(a^2+b^2)^3/d+1/2*a*(A*b-B*a)*tan(d*x
+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-a^2*(2*A*b^3-a*(a^2+3*b^2)*B)/b^3/(
a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.60 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.89

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3} \, dx = \frac{-\frac{(A+iB)\log(i-\tan(c+dx))}{(a+ib)^3} - \frac{(A-iB)\log(i+\tan(c+dx))}{(a-ib)^3} + \frac{2a(a^2Ab^3-3Ab^5+a^5B+3a^3b^2B+6ab^4B)\log(a+b\tan(c+dx))}{b^3(a^2+b^2)^3} + \frac{a^3}{b^3(a^2+b^2)}}{2d}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(-(((A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b)^3) - ((A - I*B)*Log[I + Tan
[c + d*x]])/(a - I*b)^3 + (2*a*(a^2*A*b^3 - 3*A*b^5 + a^5*B + 3*a^3*b^2*B
+ 6*a*b^4*B)*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)^3) + (a^3*(A*b - a*
B))/(b^3*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) + (2*a^2*(-(a^2*A*b) - 3*A*b^
3 + 2*a^3*B + 4*a*b^2*B))/(b^3*(a^2 + b^2)^2*(a + b*Tan[c + d*x])))/(2*d)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4088, 27, 3042, 4118, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{\int -\frac{2\tan(c+dx)\left(-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)\left(-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)\left(-\left(\left(a^2+b^2\right)B\tan(c+dx)^2\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 4118}$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\int \frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx)+a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)}{a+b\tan(c+dx)}dx}{b(a^2+b^2)}+\frac{a^2\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\int \frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan(c+dx)^2+a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)}{a+b\tan(c+dx)}dx}{b(a^2+b^2)}+\frac{a^2\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 4109}$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{b^2\left(a^3A+3a^2bB-3aAb^2-b^3B\right)\int\tan(c+dx)dx}{a^2+b^2}-\frac{a\left(a^5B+3a^3b^2B+a^2Ab^3+6ab^4B-3Ab^5\right)\int\frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2x\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{a^2+b^2}}{b(a^2+b^2)}+\frac{a^2\cdots}{b^2d(\cdots}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}-\frac{\frac{\frac{b^2\left(a^3A+3a^2bB-3aAb^2-b^3B\right)\int\tan(c+dx)dx}{a^2+b^2}-\frac{a\left(a^5B+3a^3b^2B+a^2Ab^3+6ab^4B-3Ab^5\right)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2x\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{a^2+b^2}}{b(a^2+b^2)}+\frac{a^2}{b^2d(}}{b\,(a^2+b^2)}$$

↓ 3956

$$\frac{a(Ab-aB)\tan^2(c+dx)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}-\frac{\frac{-\frac{a\left(a^5B+3a^3b^2B+a^2Ab^3+6ab^4B-3Ab^5\right)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b^2\left(a^3A+3a^2bB-3aAb^2-b^3B\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^2x\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{a^2+b^2}}{b(a^2+b^2)}+\frac{a}{b^2}}{b\,(a^2+b^2)}$$

↓ 4100

$$\frac{a(Ab-aB)\tan^2(c+dx)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}-\frac{\frac{-\frac{a\left(a^5B+3a^3b^2B+a^2Ab^3+6ab^4B-3Ab^5\right)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd\left(a^2+b^2\right)}-\frac{b^2\left(a^3A+3a^2bB-3aAb^2-b^3B\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^2x\left(a^3(-B)+3a^2Ab+3ab^2B-A\right.}{a^2+b^2}}{b(a^2+b^2)}}{b\,(a^2+b^2)}$$

↓ 16

$$\frac{a(Ab-aB)\tan^2(c+dx)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}-\frac{\frac{a^2\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{b^2d(a^2+b^2)(a+b\tan(c+dx))}+\frac{-\frac{b^2\left(a^3A+3a^2bB-3aAb^2-b^3B\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^2x\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{a^2+b^2}-\frac{a\left(a^5B+3a^3b^2B+a^2Ab^3+6ab^4B\right.}{bd\left(a^2+\right.}}{b(a^2+b^2)}}{b\,(a^2+b^2)}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(a*(A*b - a*B)*Tan[c + d*x]^2)/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
- (((b^2*(3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*x)/(a^2 + b^2) - (b^2*(a^
3*A - 3*a*A*b^2 + 3*a^2*b*B - b^3*B)*Log[Cos[c + d*x]])/((a^2 + b^2)*d) -
(a*(a^2*A*b^3 - 3*A*b^5 + a^5*B + 3*a^3*b^2*B + 6*a*b^4*B)*Log[a + b*Tan[c
 + d*x]])/(b*(a^2 + b^2)*d))/(b*(a^2 + b^2)) + (a^2*(2*A*b^3 - a*(a^2 + 3*
b^2)*B))/(b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{\frac{\left(-A a^{3}+3 A a b^{2}-3 B a^{2} b+B b^{3}\right) \ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(-3 A a^{2} b+A b^{3}+B a^{3}-3 B a b^{2}\right) \arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{a\left(A a^{2} b^{3}-3 A b^{5}+B a \cdots\right.}{\cdots}}{d}$
default	$\frac{\frac{\frac{\left(-A a^{3}+3 A a b^{2}-3 B a^{2} b+B b^{3}\right) \ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(-3 A a^{2} b+A b^{3}+B a^{3}-3 B a b^{2}\right) \arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}+\frac{a\left(A a^{2} b^{3}-3 A b^{5}+B a \cdots\right.}{\cdots}}{d}$
norman	$\frac{-\frac{a^{2}\left(A a^{3} b+5 A a b^{3}-3 B a^{4}-7 B a^{2} b^{2}\right)}{2 d b^{3}\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{\left(3 A a^{2} b-A b^{3}-B a^{3}+3 B a b^{2}\right) a^{2} x}{\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}-\frac{b^{2}\left(3 A a^{2} b-A b^{3}-B a^{3}+3 B a b^{2}\right) x \tan(dx+c)^{2}}{\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}-\frac{a\left(\cdots\right.}{\cdots}}{(a+b\tan(dx+c))^{2}}$
parallelrisch	Expression too large to display
risch	$-\frac{xB}{3ia^{2}b-ib^{3}-a^{3}+3a b^{2}}-\frac{2ia^{3}Ac}{d(a^{6}+3b^{2}a^{4}+3b^{4}a^{2}+b^{6})}+\frac{2iBx}{b^{3}}-\frac{6ia^{4}Bc}{bd(a^{6}+3b^{2}a^{4}+3b^{4}a^{2}+b^{6})}-\frac{2ia^{6}Bc}{b^{3}d(a^{6}+3b^{2}a^{4}+3b^{4}\cdots}$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(-A*a^3+3*A*a*b^2-3*B*a^2*b+B*b^3)*ln(1+tan(d*x+c)
^2)+(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*arctan(tan(d*x+c)))+a*(A*a^2*b^3-3*
A*b^5+B*a^5+3*B*a^3*b^2+6*B*a*b^4)/(a^2+b^2)^3/b^3*ln(a+b*tan(d*x+c))-a^2*
(A*a^2*b+3*A*b^3-2*B*a^3-4*B*a*b^2)/b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+1/2*a
^3*(A*b-B*a)/b^3/(a^2+b^2)/(a+b*tan(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(245) = 490$.

Time = 0.20 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.66

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \frac{Ba^6b^2 + Aa^5b^3 + 7\,Ba^4b^4 - 5\,Aa^3b^5 + 2\,(Ba^5b^3 - 3\,Aa^4b^4 - 3\,Ba^3b^5 + Aa^2b^6)dx - (3\,Ba^6b^2 - Aa^5b^3 +}{}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/2*(B*a^6*b^2 + A*a^5*b^3 + 7*B*a^4*b^4 - 5*A*a^3*b^5 + 2*(B*a^5*b^3 - 3*
A*a^4*b^4 - 3*B*a^3*b^5 + A*a^2*b^6)*d*x - (3*B*a^6*b^2 - A*a^5*b^3 + 9*B*
a^4*b^4 - 7*A*a^3*b^5 - 2*(B*a^3*b^5 - 3*A*a^2*b^6 - 3*B*a*b^7 + A*b^8)*d*
x)*tan(d*x + c)^2 + (B*a^8 + 3*B*a^6*b^2 + A*a^5*b^3 + 6*B*a^4*b^4 - 3*A*a
^3*b^5 + (B*a^6*b^2 + 3*B*a^4*b^4 + A*a^3*b^5 + 6*B*a^2*b^6 - 3*A*a*b^7)*t
an(d*x + c)^2 + 2*(B*a^7*b + 3*B*a^5*b^3 + A*a^4*b^4 + 6*B*a^3*b^5 - 3*A*a
^2*b^6)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) - (B*a^8 + 3*B*a^6*b^2 + 3*B*a^4*b^4 + B*a^2*b^6 + (
B*a^6*b^2 + 3*B*a^4*b^4 + 3*B*a^2*b^6 + B*b^8)*tan(d*x + c)^2 + 2*(B*a^7*b
 + 3*B*a^5*b^3 + 3*B*a^3*b^5 + B*a*b^7)*tan(d*x + c))*log(1/(tan(d*x + c)^
2 + 1)) - 2*(B*a^7*b + 3*B*a^5*b^3 - 3*A*a^4*b^4 - 4*B*a^3*b^5 + 3*A*a^2*b
^6 - 2*(B*a^4*b^4 - 3*A*a^3*b^5 - 3*B*a^2*b^6 + A*a*b^7)*d*x)*tan(d*x + c)
)/((a^6*b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*d*tan(d*x + c)^2 + 2*(a^7*b^4
+ 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*d*tan(d*x + c) + (a^8*b^3 + 3*a^6*b^5 +
3*a^4*b^7 + a^2*b^9)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.46

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\,(Ba^6+3\,Ba^4b^2+Aa^3b^3+6\,Ba^2b^4-3\,Aab^5)\log(b\tan(dx+c)+a)}{a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9} - \frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)}{a^6+3\,a^4b^2+3\,a^2}}{2\,d}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="m
axima")
```

output
```
1/2*(2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + 2*(B*a^6 + 3*B*a^4*b^2 + A*a^3*b^3 + 6*B*a^2*b^4 - 3*
A*a*b^5)*log(b*tan(d*x + c) + a)/(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9) -
 (A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6) + (3*B*a^6 - A*a^5*b + 7*B*a^4*b^2 - 5*A*a^3*b^
3 + 2*(2*B*a^5*b - A*a^4*b^2 + 4*B*a^3*b^3 - 3*A*a^2*b^4)*tan(d*x + c))/(a
^6*b^3 + 2*a^4*b^5 + a^2*b^7 + (a^4*b^5 + 2*a^2*b^7 + b^9)*tan(d*x + c)^2
+ 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.44

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)(dx+c)}{a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d}$$
$$-\frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d\right)}$$
$$+\frac{(Ba^6+3\,Ba^4b^2+Aa^3b^3+6\,Ba^2b^4-3\,Aab^5)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^6b^3d+3\,a^4b^5d+3\,a^2b^7d+b^9d}$$
$$+\frac{2\left(2\,Ba^7-Aa^6b+6\,Ba^5b^2-4\,Aa^4b^3+4\,Ba^3b^4-3\,Aa^2b^5\right)\tan\left(dx+c\right)+\frac{3\,Ba^8-Aa^7b+10\,Ba^6b^2-6\,Aa^5b^3+7}{b}}{2\left(a^2+b^2\right)^3\left(b\tan\left(dx+c\right)+a\right)^2b^2d}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="g
iac")
```

output
```
(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*(d*x + c)/(a^6*d + 3*a^4*b^2*d + 3
*a^2*b^4*d + b^6*d) - 1/2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(tan(
d*x + c)^2 + 1)/(a^6*d + 3*a^4*b^2*d + 3*a^2*b^4*d + b^6*d) + (B*a^6 + 3*B
*a^4*b^2 + A*a^3*b^3 + 6*B*a^2*b^4 - 3*A*a*b^5)*log(abs(b*tan(d*x + c) + a
))/(a^6*b^3*d + 3*a^4*b^5*d + 3*a^2*b^7*d + b^9*d) + 1/2*(2*(2*B*a^7 - A*a
^6*b + 6*B*a^5*b^2 - 4*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5)*tan(d*x + c)
 + (3*B*a^8 - A*a^7*b + 10*B*a^6*b^2 - 6*A*a^5*b^3 + 7*B*a^4*b^4 - 5*A*a^3
*b^5)/b)/((a^2 + b^2)^3*(b*tan(d*x + c) + a)^2*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.23

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$=\frac{\frac{3\,B\,a^6-A\,a^5\,b+7\,B\,a^4\,b^2-5\,A\,a^3\,b^3}{2\,b^3\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{a^2\,\tan(c+d\,x)\,\left(-2\,B\,a^3+A\,a^2\,b-4\,B\,a\,b^2+3\,A\,b^3\right)}{b^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}$$
$$+\frac{a\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,a^5+3\,B\,a^3\,b^2+A\,a^2\,b^3+6\,B\,a\,b^4-3\,A\,b^5\right)}{b^3\,d\,\left(a^2+b^2\right)^3}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
((3*B*a^6 - 5*A*a^3*b^3 + 7*B*a^4*b^2 - A*a^5*b)/(2*b^3*(a^4 + b^4 + 2*a^2
*b^2)) - (a^2*tan(c + d*x)*(3*A*b^3 - 2*B*a^3 + A*a^2*b - 4*B*a*b^2))/(b^2
*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*
x))) + (log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(a*b^2*3i + 3*a^2*b - a^3*
1i - b^3)) + (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(3*a*b^2 + a^2*b*3i
- a^3 - b^3*1i)) + (a*log(a + b*tan(c + d*x))*(B*a^5 - 3*A*b^5 + A*a^2*b^3
 + 3*B*a^3*b^2 + 6*B*a*b^4))/(b^3*d*(a^2 + b^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.11

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \frac{-\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)a^2b^3+\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)b^5-\log\left(\tan\left(dx+c\right)^2+\right.}{}$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
( - log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a**2*b**3 + log(tan(c + d*x)**2
+ 1)*tan(c + d*x)*b**5 - log(tan(c + d*x)**2 + 1)*a**3*b**2 + log(tan(c +
d*x)**2 + 1)*a*b**4 + 2*log(tan(c + d*x)*b + a)*tan(c + d*x)*a**4*b + 6*lo
g(tan(c + d*x)*b + a)*tan(c + d*x)*a**2*b**3 + 2*log(tan(c + d*x)*b + a)*a
**5 + 6*log(tan(c + d*x)*b + a)*a**3*b**2 - 2*tan(c + d*x)*a**4*b - 2*tan(
c + d*x)*a**2*b**3 - 4*tan(c + d*x)*a*b**4*d*x - 4*a**2*b**3*d*x)/(2*b**2*
d*(tan(c + d*x)*a**4*b + 2*tan(c + d*x)*a**2*b**3 + tan(c + d*x)*b**5 + a*
*5 + 2*a**3*b**2 + a*b**4))
```

3.284 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 189

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{(a^3A-3aAb^2+3a^2bB-b^3B)\,x}{(a^2+b^2)^3} - \frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} - \frac{a^2(Ab-aB)}{2b^2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{a(2Ab^3-a(a^2+3b^2)\,B)}{b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

output
```
-(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*x/(a^2+b^2)^3-(3*A*a^2*b-A*b^3-B*a^3+3*
B*a*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d-1/2*a^2*(A*b-B*a)/b^2
/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+a*(2*A*b^3-a*(a^2+3*b^2)*B)/b^2/(a^2+b^2)^
2/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.52

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{-\frac{Ab+aB}{b(a+b\tan(c+dx))^2} - \frac{2B\tan(c+dx)}{(a+b\tan(c+dx))^2} + B\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^2} - \frac{i\log(i+\tan(c+dx))}{(a-ib)^2} + \frac{2b\left(-2a\log(a+b\tan(c+dx))+\frac{a^2+b^2}{a+b\tan(c+}\right.}{(a^2+b^2)^2}\right.}{}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(-((A*b + a*B)/(b*(a + b*Tan[c + d*x])^2)) - (2*B*Tan[c + d*x])/(a + b*Tan
[c + d*x])^2 + B*((I*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c
 + d*x]])/(a - I*b)^2 + (2*b*(-2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(
a + b*Tan[c + d*x])))/(a^2 + b^2)^2) + (A*b - a*B)*((I*Log[I - Tan[c + d*x
]])/(a + I*b)^3 - Log[I + Tan[c + d*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)
*Log[a + b*Tan[c + d*x]] + ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[c + d*x])
)/(a + b*Tan[c + d*x])^2))/(a^2 + b^2)^3))/(2*b*d)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 4087, 25, 3042, 4111, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4087}$$

$$\frac{\int -\frac{-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{(a+b\tan(c+dx))^2}dx}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{(a+b\tan(c+dx))^2}dx}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-\left(\left(a^2+b^2\right)B\tan(c+dx)^2\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{(a+b\tan(c+dx))^2}dx}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{4111}$$

$$-\frac{\frac{\int \frac{b\left(Aa^2+2bBa-Ab^2\right)-b\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{b\left(Aa^2+2bBa-Ab^2\right)-b\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{4014}$$

$$-\frac{\frac{\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{bx\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a^2+b^2}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{bx\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a^2+b^2}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \textcolor{blue}{4013}$$

$$-\frac{a^2(Ab-aB)}{2b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{bx\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a^2+b^2}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)(a+b\tan(c+dx))}}{b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*(a^2*(A*b - a*B))/(b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (((b*(
a^3*A - 3*a*A*b^2 + 3*a^2*b*B - b^3*B)*x)/(a^2 + b^2) + (b*(3*a^2*A*b - A*
b^3 - a^3*B + 3*a*b^2*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2
)*d))/(a^2 + b^2) - (a*(2*A*b^3 - a*(a^2 + 3*b^2)*B))/(b*(a^2 + b^2)*d*(a
+ b*Tan[c + d*x])))/(b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\frac{\left(3A\,a^2b-A\,b^3-B\,a^3+3Ba\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-A\,a^3+3Aa\,b^2-3B\,a^2b+B\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{a^2(Ab-Ba)}{2b^2\left(a^2+b^2\right)(a+b\tan(d}}{d}$
default	$\frac{\frac{\frac{\left(3A\,a^2b-A\,b^3-B\,a^3+3Ba\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-A\,a^3+3Aa\,b^2-3B\,a^2b+B\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{a^2(Ab-Ba)}{2b^2\left(a^2+b^2\right)(a+b\tan(d}}{d}$
norman	$\frac{-\frac{\left(2Aa\,b^3-B\,a^4-3B\,a^2b^2\right)\tan(dx+c)^2}{2ad\left(a^4+2a^2b^2+b^4\right)}-\frac{a\left(A\,a^3-Aa\,b^2+2B\,a^2b\right)}{2db\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(A\,a^3-3Aa\,b^2+3B\,a^2b-B\,b^3\right)a^2x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{b^2\left(A\,a^3-3Aa\,b^2+3B\,a\right.}{\left(a^4+2a^2b^2+\right.}}{(a+b\tan(dx+c))^2}$
risch	$-\frac{ixB}{3ia^2b-ib^3-a^3+3a\,b^2}+\frac{xA}{3ia^2b-ib^3-a^3+3a\,b^2}+\frac{6iA\,a^2bx}{a^6+3b^2a^4+3b^4a^2+b^6}-\frac{2iA\,b^3x}{a^6+3b^2a^4+3b^4a^2+b^6}-\frac{2iB}{a^6+3b^2a^4}$
parallelrisch	$\frac{12Ax\tan(dx+c)a^2b^5d-12Bx\tan(dx+c)a^3b^4d+4Bx\tan(dx+c)a\,b^6d-2Ax\tan(dx+c)^2a^3b^4d+6Ax\tan(dx+c)^2a\,b^6d-}{}$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*ln(1+tan(d*x+c)^
2)+(-A*a^3+3*A*a*b^2-3*B*a^2*b+B*b^3)*arctan(tan(d*x+c)))-1/2*a^2*(A*b-B*a
)/b^2/(a^2+b^2)/(a+b*tan(d*x+c))^2-(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)/(a^2+
b^2)^3*ln(a+b*tan(d*x+c))+a*(2*A*b^3-B*a^3-3*B*a*b^2)/(a^2+b^2)^2/b^2/(a+b
*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(184) = 368$.

Time = 0.12 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.53

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{Ba^5 - 3\,Aa^4b - 5\,Ba^3b^2 + 3\,Aa^2b^3 - 2\,(Aa^5 + 3\,Ba^4b - 3\,Aa^3b^2 - Ba^2b^3)dx + (Ba^5 + Aa^4b + 7\,Ba^3b^2}{}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/2*(B*a^5 - 3*A*a^4*b - 5*B*a^3*b^2 + 3*A*a^2*b^3 - 2*(A*a^5 + 3*B*a^4*b
- 3*A*a^3*b^2 - B*a^2*b^3)*d*x + (B*a^5 + A*a^4*b + 7*B*a^3*b^2 - 5*A*a^2*
b^3 - 2*(A*a^3*b^2 + 3*B*a^2*b^3 - 3*A*a*b^4 - B*b^5)*d*x)*tan(d*x + c)^2
+ (B*a^5 - 3*A*a^4*b - 3*B*a^3*b^2 + A*a^2*b^3 + (B*a^3*b^2 - 3*A*a^2*b^3
- 3*B*a*b^4 + A*b^5)*tan(d*x + c)^2 + 2*(B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2*b
^3 + A*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) +
 a^2)/(tan(d*x + c)^2 + 1)) + 2*(A*a^5 + 3*B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2
*b^3 + 2*A*a*b^4 - 2*(A*a^4*b + 3*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4)*d*x)*
tan(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c)^2 +
2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a^6*b^
2 + 3*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.76

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{2\,(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log\left(\tan(d\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{2\,d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) - 2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(b*tan(
d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (B*a^3 - 3*A*a^2*b - 3
*B*a*b^2 + A*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + (B*a^5 + A*a^4*b + 5*B*a^3*b^2 - 3*A*a^2*b^3 + 2*(B*a^4*b + 3*B*a^2*
b^3 - 2*A*a*b^4)*tan(d*x + c))/(a^6*b^2 + 2*a^4*b^4 + a^2*b^6 + (a^4*b^4 +
 2*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*tan(d*x
 + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.76

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)(dx+c)}{a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d} - \frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d\right)} + \frac{(Ba^3b-3\,Aa^2b^2-3\,Bab^3+Ab^4)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^6bd+3\,a^4b^3d+3\,a^2b^5d+b^7d} - \frac{Ba^7+Aa^6b+6\,Ba^5b^2-2\,Aa^4b^3+5\,Ba^3b^4-3\,Aa^2b^5+2\left(Ba^6b+4\,Ba^4b^3-2\,Aa^3b^4+3\,Ba^2b^5-2\,A\right.}{2\left(a^2+b^2\right)^3\left(b\tan\left(dx+c\right)+a\right)^2b^2d}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6*d + 3*a^4*b^2*d +
3*a^2*b^4*d + b^6*d) - 1/2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan
(d*x + c)^2 + 1)/(a^6*d + 3*a^4*b^2*d + 3*a^2*b^4*d + b^6*d) + (B*a^3*b -
3*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b*d + 3
*a^4*b^3*d + 3*a^2*b^5*d + b^7*d) - 1/2*(B*a^7 + A*a^6*b + 6*B*a^5*b^2 - 2
*A*a^4*b^3 + 5*B*a^3*b^4 - 3*A*a^2*b^5 + 2*(B*a^6*b + 4*B*a^4*b^3 - 2*A*a^
3*b^4 + 3*B*a^2*b^5 - 2*A*a*b^6)*tan(d*x + c))/((a^2 + b^2)^3*(b*tan(d*x +
 c) + a)^2*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.48

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,a^3-3\,A\,a^2\,b-3\,B\,a\,b^2+A\,b^3\right)}{d\,{\left(a^2+b^2\right)}^3} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)} - \frac{\frac{a\,\left(B\,a^4+A\,a^3\,b+5\,B\,a^2\,b^2-3\,A\,a\,b^3\right)}{2\,b^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(B\,a^4+3\,B\,a^2\,b^2-2\,A\,a\,b^3\right)}{b\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2+2\,a\,b\,\tan\left(c+d\,x\right)+b^2\,\tan\left(c+d\,x\right)^2\right)}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
(log(a + b*tan(c + d*x))*(A*b^3 + B*a^3 - 3*A*a^2*b - 3*B*a*b^2))/(d*(a^2
+ b^2)^3) - (log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(3*a*b^2 - a^2*b*3i -
 a^3 + b^3*1i)) - (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a*b^2*3i - 3*a
^2*b - a^3*1i + b^3)) - ((a*(B*a^4 + 5*B*a^2*b^2 - 3*A*a*b^3 + A*a^3*b))/(
2*b^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)*(B*a^4 + 3*B*a^2*b^2 - 2*A*
a*b^3))/(b*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*
tan(c + d*x)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.05

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\log\left(\tan(dx+c)^2+1\right)\tan(dx+c)\,a\,b^2+\log\left(\tan(dx+c)^2+1\right)a^2b-2\log(a+\tan(dx+c)\,b)\tan(dx+\cdots}{d\left(\tan(dx+c)\,a^4b+2\,\mathrm{t}\cdots\right.}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
(log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a*b**2 + log(tan(c + d*x)**2 + 1)*a
**2*b - 2*log(tan(c + d*x)*b + a)*tan(c + d*x)*a*b**2 - 2*log(tan(c + d*x)
*b + a)*a**2*b + tan(c + d*x)*a**3 - tan(c + d*x)*a**2*b*d*x + tan(c + d*x
)*a*b**2 + tan(c + d*x)*b**3*d*x - a**3*d*x + a*b**2*d*x)/(d*(tan(c + d*x)
*a**4*b + 2*tan(c + d*x)*a**2*b**3 + tan(c + d*x)*b**5 + a**5 + 2*a**3*b**
2 + a*b**4))
```

3.285 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 179

$$\begin{aligned}\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx &= \frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\,x}{(a^2+b^2)^3} \\ &\quad - \frac{(a^3A-3aAb^2+3a^2bB-b^3B)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} \\ &\quad + \frac{a(Ab-aB)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{a^2A-Ab^2+2abB}{(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*x/(a^2+b^2)^3-(A*a^3-3*A*a*b^2+3*B*a^2*b
-B*b^3)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d+1/2*a*(A*b-B*a)/b/(a^2
+b^2)/d/(a+b*tan(d*x+c))^2+(A*a^2-A*b^2+2*B*a*b)/(a^2+b^2)^2/d/(a+b*tan(d*
x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.05

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(A+iB)\log(i-\tan(c+dx))}{(a+ib)^3}+\frac{(A-iB)\log(i+\tan(c+dx))}{(a-ib)^3}-\frac{2(a^3A-3aAb^2+3a^2bB-b^3B)\log(a+b\tan(c+dx))}{(a^2+b^2)^3}+\frac{a(Ab-aB)}{b(a^2+b^2)(a+b\tan(c+dx)}}{2d}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(((A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b)^3 + ((A - I*B)*Log[I + Tan[c
+ d*x]])/(a - I*b)^3 - (2*(a^3*A - 3*a*A*b^2 + 3*a^2*b*B - b^3*B)*Log[a +
b*Tan[c + d*x]])/(a^2 + b^2)^3 + (a*(A*b - a*B))/(b*(a^2 + b^2)*(a + b*Tan
[c + d*x])^2) + (2*(a^2*A - A*b^2 + 2*a*b*B))/((a^2 + b^2)^2*(a + b*Tan[c
+ d*x])))/(2*d)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {3042, 4074, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \text{4074}$$
$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}+\frac{a(Ab-aB)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}+\frac{a(Ab-aB)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\int \frac{-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{a^2A+2abB-Ab^2}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a(Ab-aB)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{a^2A+2abB-Ab^2}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a(Ab-aB)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4014

$$\frac{\frac{\frac{x\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{a^2+b^2}-\frac{\left(a^3A+3a^2bB-3aAb^2-b^3B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a^2A+2abB-Ab^2}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a(Ab-aB)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{x\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{a^2+b^2}-\frac{\left(a^3A+3a^2bB-3aAb^2-b^3B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a^2A+2abB-Ab^2}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a(Ab-aB)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4013

$$\frac{a(Ab-aB)}{2bd\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{\frac{a^2A+2abB-Ab^2}{d(a^2+b^2)(a+b\tan(c+dx))}+\frac{\frac{x\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{a^2+b^2}-\frac{\left(a^3A+3a^2bB-3aAb^2-b^3B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}}{a^2+b^2}}{a^2+b^2}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(a*(A*b - a*B))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((((3*a^2*A*b
 - A*b^3 - a^3*B + 3*a*b^2*B)*x)/(a^2 + b^2) - ((a^3*A - 3*a*A*b^2 + 3*a^2
*b*B - b^3*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2
+ b^2) + (a^2*A - A*b^2 + 2*a*b*B)/((a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(
a^2 + b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{\frac{\left(A a^3-3Aa b^2+3B a^2b-B b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(3A a^2b-A b^3-B a^3+3Ba b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a(Ab-Ba)}{2\left(a^2+b^2\right)b(a+b\tan(dx+\cdots}}{d}$
default	$\frac{\frac{\frac{\left(A a^3-3Aa b^2+3B a^2b-B b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(3A a^2b-A b^3-B a^3+3Ba b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{a(Ab-Ba)}{2\left(a^2+b^2\right)b(a+b\tan(dx+\cdots}}{d}$
norman	$\frac{\frac{\left(A a^2b^2-A b^4+2Ba b^3\right)\tan(dx+c)}{db\left(a^4+2a^2b^2+b^4\right)}+\frac{\left(3A a^2b-A b^3-B a^3+3Ba b^2\right)a^2x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{b^2\left(3A a^2b-A b^3-B a^3+3Ba b^2\right)x\tan(dx+c)^2}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{a(3A\cdots}{}}{(a+b\tan(dx+c))^2}$
risch	$\frac{xB}{3ia^2b-ib^3-a^3+3a b^2}+\frac{ixA}{3ia^2b-ib^3-a^3+3a b^2}+\frac{2ia^3Ax}{a^6+3b^2a^4+3b^4a^2+b^6}-\frac{6ia b^2Ax}{a^6+3b^2a^4+3b^4a^2+b^6}+\frac{6ia^2b\cdots}{a^6+3b^2a^4+3\cdots}$
parallelrisch	$\frac{-2A\ln(a+b\tan(dx+c))\tan(dx+c)^2a^3b^4+6A\ln(a+b\tan(dx+c))\tan(dx+c)^2a b^6-6B\ln(a+b\tan(dx+c))\tan(dx+c)^2\cdots}{}$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*ln(1+tan(d*x+c)^
2)+(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*arctan(tan(d*x+c)))+1/2*a*(A*b-B*a)/(
a^2+b^2)/b/(a+b*tan(d*x+c))^2+(A*a^2-A*b^2+2*B*a*b)/(a^2+b^2)^2/(a+b*tan(d
*x+c))-(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)/(a^2+b^2)^3*ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(176) = 352$.

Time = 0.12 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.73

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{3\,Ba^4b - 5\,Aa^3b^2 - 3\,Ba^2b^3 + Aab^4 + 2\,(Ba^5 - 3\,Aa^4b - 3\,Ba^3b^2 + Aa^2b^3)dx - (Ba^4b - 3\,Aa^3b^2 - 5}{}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="fri
cas")
```

output
```
-1/2*(3*B*a^4*b - 5*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4 + 2*(B*a^5 - 3*A*a^4
*b - 3*B*a^3*b^2 + A*a^2*b^3)*d*x - (B*a^4*b - 3*A*a^3*b^2 - 5*B*a^2*b^3 +
 3*A*a*b^4 - 2*(B*a^3*b^2 - 3*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*d*x)*tan(d*x
+ c)^2 + (A*a^5 + 3*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + (A*a^3*b^2 + 3*B*a
^2*b^3 - 3*A*a*b^4 - B*b^5)*tan(d*x + c)^2 + 2*(A*a^4*b + 3*B*a^3*b^2 - 3*
A*a^2*b^3 - B*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x
 + c) + a^2)/(tan(d*x + c)^2 + 1)) - 2*(B*a^5 - 2*A*a^4*b - 3*B*a^3*b^2 +
3*A*a^2*b^3 + 2*B*a*b^4 - A*b^5 - 2*(B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2*b^3 +
 A*a*b^4)*d*x)*tan(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*ta
n(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) +
(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.84

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{2\,(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)\log\left(\tan(d\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{2\,d}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="max
ima")
```

output
```
-1/2*(2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + 2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(b*tan(
d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (A*a^3 + 3*B*a^2*b - 3
*A*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + (B*a^4 - 3*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3 - 2*(A*a^2*b^2 + 2*B*a*b^
3 - A*b^4)*tan(d*x + c))/(a^6*b + 2*a^4*b^3 + a^2*b^5 + (a^4*b^3 + 2*a^2*b
^5 + b^7)*tan(d*x + c)^2 + 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*tan(d*x + c)))/
d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.80

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)(dx+c)}{a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d} + \frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d\right)} - \frac{(Aa^3b+3\,Ba^2b^2-3\,Aab^3-Bb^4)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^6bd+3\,a^4b^3d+3\,a^2b^5d+b^7d} - \frac{Ba^6-3\,Aa^5b-2\,Ba^4b^2-2\,Aa^3b^3-3\,Ba^2b^4+Aab^5-2\left(Aa^4b^2+2\,Ba^3b^3+2\,Bab^5-Ab^6\right)\tan\left(dx-\right.}{2\left(a^2+b^2\right)^3\left(b\tan\left(dx+c\right)+a\right)^2bd}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="gia
c")
```

output
```
-(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*(d*x + c)/(a^6*d + 3*a^4*b^2*d +
3*a^2*b^4*d + b^6*d) + 1/2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(tan
(d*x + c)^2 + 1)/(a^6*d + 3*a^4*b^2*d + 3*a^2*b^4*d + b^6*d) - (A*a^3*b +
3*B*a^2*b^2 - 3*A*a*b^3 - B*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b*d + 3
*a^4*b^3*d + 3*a^2*b^5*d + b^7*d) - 1/2*(B*a^6 - 3*A*a^5*b - 2*B*a^4*b^2 -
 2*A*a^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 - 2*(A*a^4*b^2 + 2*B*a^3*b^3 + 2*B*a*
b^5 - A*b^6)*tan(d*x + c))/((a^2 + b^2)^3*(b*tan(d*x + c) + a)^2*b*d)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.58

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{\tan(c+d\,x)\,\left(A\,a^2\,b+2\,B\,a\,b^2-A\,b^3\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{B\,a^4-3\,A\,a^3\,b-3\,B\,a^2\,b^2+A\,a\,b^3}{2\,b\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{A\,a+3\,B\,b}{{\left(a^2+b^2\right)}^2}-\frac{4\,b^2\,\left(A\,a+B\,b\right)}{{\left(a^2+b^2\right)}^3}\right)}{d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1{}\mathrm{i}\right)}{2\,d\,\left(-a^3\,1{}\mathrm{i}+3\,a^2\,b+a\,b^2\,3{}\mathrm{i}-b^3\right)} - \frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(A-B\,1{}\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3{}\mathrm{i}+3\,a\,b^2-b^3\,1{}\mathrm{i}\right)}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
((tan(c + d*x)*(A*a^2*b - A*b^3 + 2*B*a*b^2))/(a^4 + b^4 + 2*a^2*b^2) - (B
*a^4 - 3*B*a^2*b^2 + A*a*b^3 - 3*A*a^3*b)/(2*b*(a^4 + b^4 + 2*a^2*b^2)))/(
d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) - (log(a + b*tan(c + d*
x))*((A*a + 3*B*b)/(a^2 + b^2)^2 - (4*b^2*(A*a + B*b))/(a^2 + b^2)^3))/d -
 (log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(a*b^2*3i + 3*a^2*b - a^3*1i - b
^3)) - (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(3*a*b^2 + a^2*b*3i - a^3
- b^3*1i))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.45

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)a^2b-\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)b^3+\log\left(\tan\left(dx+c\right)^2+1\right)}{}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
(log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a**2*b - log(tan(c + d*x)**2 + 1)*t
an(c + d*x)*b**3 + log(tan(c + d*x)**2 + 1)*a**3 - log(tan(c + d*x)**2 + 1
)*a*b**2 - 2*log(tan(c + d*x)*b + a)*tan(c + d*x)*a**2*b + 2*log(tan(c + d
*x)*b + a)*tan(c + d*x)*b**3 - 2*log(tan(c + d*x)*b + a)*a**3 + 2*log(tan(
c + d*x)*b + a)*a*b**2 - 2*tan(c + d*x)*a**2*b + 4*tan(c + d*x)*a*b**2*d*x
 - 2*tan(c + d*x)*b**3 + 4*a**2*b*d*x)/(2*d*(tan(c + d*x)*a**4*b + 2*tan(c
 + d*x)*a**2*b**3 + tan(c + d*x)*b**5 + a**5 + 2*a**3*b**2 + a*b**4))
```

3.286 $\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 175

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{(a^3A-3aAb^2+3a^2bB-b^3B)\,x}{(a^2+b^2)^3} \\ &\quad+\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} \\ &\quad-\frac{Ab-aB}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}-\frac{2aAb-a^2B+b^2B}{(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*x/(a^2+b^2)^3+(3*A*a^2*b-A*b^3-B*a^3+3*B
*a*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d-1/2*(A*b-B*a)/(a^2+b^2
)/d/(a+b*tan(d*x+c))^2-(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/d/(a+b*tan(d*x+c)
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.80 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.39

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{B\left(\frac{i\log(i-\tan(c+dx))}{(a+ib)^2}-\frac{i\log(i+\tan(c+dx))}{(a-ib)^2}+\frac{2b\left(-2a\log(a+b\tan(c+dx))+\frac{a^2+b^2}{a+b\tan(c+dx)}\right)}{(a^2+b^2)^2}\right)+(Ab-aB)\left(\frac{i\log(i-\tan(c+\cdots}{(a+ib)^3}\right.}{2bd}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*(B*((I*Log[I - Tan[c + d*x]])/(a + I*b)^2 - (I*Log[I + Tan[c + d*x]])
/(a - I*b)^2 + (2*b*(-2*a*Log[a + b*Tan[c + d*x]] + (a^2 + b^2)/(a + b*Tan
[c + d*x])))/(a^2 + b^2)^2) + (A*b - a*B)*((I*Log[I - Tan[c + d*x]])/(a +
I*b)^3 - Log[I + Tan[c + d*x]]/(I*a + b)^3 + (b*((-6*a^2 + 2*b^2)*Log[a +
b*Tan[c + d*x]] + ((a^2 + b^2)*(5*a^2 + b^2 + 4*a*b*Tan[c + d*x]))/(a + b*
Tan[c + d*x])^2))/(a^2 + b^2)^3))/(b*d)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4012, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 4012$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 4012$$

$$\frac{\frac{\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 4014$$

$$\frac{\frac{\frac{\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a^2+b^2}-\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a^2+b^2}-\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 4013$$

$$\frac{\frac{\frac{\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a^2+b^2}-\frac{a^2(-B)+2aAb+b^2B}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{Ab-aB}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*(A*b - a*B)/((a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((((a^3*A - 3*a*
A*b^2 + 3*a^2*b*B - b^3*B)*x)/(a^2 + b^2) + ((3*a^2*A*b - A*b^3 - a^3*B +
3*a*b^2*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2 + b
^2) - (2*a*A*b - a^2*B + b^2*B)/((a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(a^2
 + b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{\frac{\left(-3Aa^2b+Ab^3+Ba^3-3Bab^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(Aa^3-3Aab^2+3Ba^2b-Bb^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{\left(3Aa^2b-Ab^3-Ba^3+3\right.}{\left(a^2\right.}}{d}$
default	$\frac{\frac{\frac{\left(-3Aa^2b+Ab^3+Ba^3-3Bab^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(Aa^3-3Aab^2+3Ba^2b-Bb^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{\left(3Aa^2b-Ab^3-Ba^3+3\right.}{\left(a^2\right.}}{d}$
norman	$\frac{\frac{\left(Aa^3-3Aab^2+3Ba^2b-Bb^3\right)a^2x}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{b^2\left(Aa^3-3Aab^2+3Ba^2b-Bb^3\right)x\tan(dx+c)^2}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}-\frac{3Aa^2b^2+Ab^4-2Ba^3b}{2bd\left(a^4+2a^2b^2+b^4\right)}+\frac{b\left(2Aab^2-Ba^2b+\right.}{2da\left(a^4+2\right.}}{(a+b\tan(dx+c))^2}$
risch	$\frac{ixB}{3ia^2b-ib^3-a^3+3ab^2}-\frac{xA}{3ia^2b-ib^3-a^3+3ab^2}-\frac{6iAa^2bx}{a^6+3b^2a^4+3b^4a^2+b^6}+\frac{2iAb^3x}{a^6+3b^2a^4+3b^4a^2+b^6}+\frac{2iBx}{a^6+3b^2a^4+3}$
parallelrisch	$-\frac{-6A\ln(a+b\tan(dx+c))\tan(dx+c)^2a^3b^4+2A\ln(a+b\tan(dx+c))\tan(dx+c)^2ab^6+2B\ln(a+b\tan(dx+c))\tan(dx+}{}$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*ln(1+tan(d*x+c)
^2)+(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*arctan(tan(d*x+c)))+(3*A*a^2*b-A*b^3
-B*a^3+3*B*a*b^2)/(a^2+b^2)^3*ln(a+b*tan(d*x+c))-1/2*(A*b-B*a)/(a^2+b^2)/(
a+b*tan(d*x+c))^2-(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2)^2/(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(171) = 342.

Time = 0.11 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.75

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$=\frac{5\,Ba^3b^2-7\,Aa^2b^3-Bab^4-Ab^5+2\left(Aa^5+3\,Ba^4b-3\,Aa^3b^2-Ba^2b^3\right)dx-\left(3\,Ba^3b^2-5\,Aa^2b^3-3\,B\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(5*B*a^3*b^2 - 7*A*a^2*b^3 - B*a*b^4 - A*b^5 + 2*(A*a^5 + 3*B*a^4*b -
3*A*a^3*b^2 - B*a^2*b^3)*d*x - (3*B*a^3*b^2 - 5*A*a^2*b^3 - 3*B*a*b^4 + A*
b^5 - 2*(A*a^3*b^2 + 3*B*a^2*b^3 - 3*A*a*b^4 - B*b^5)*d*x)*tan(d*x + c)^2
- (B*a^5 - 3*A*a^4*b - 3*B*a^3*b^2 + A*a^2*b^3 + (B*a^3*b^2 - 3*A*a^2*b^3
- 3*B*a*b^4 + A*b^5)*tan(d*x + c)^2 + 2*(B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2*b
^3 + A*a*b^4)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) +
 a^2)/(tan(d*x + c)^2 + 1)) - 2*(2*B*a^4*b - 3*A*a^3*b^2 - 3*B*a^2*b^3 + 3
*A*a*b^4 + B*b^5 - 2*(A*a^4*b + 3*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4)*d*x)*
tan(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*tan(d*x + c)^2 +
2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*tan(d*x + c) + (a^8 + 3*a^6*b^
2 + 3*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.83

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{2\left(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{2\left(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3\right)\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3\right)\log\left(\tan(dx+c\right.}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}}{2\,d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) - 2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(b*tan(d
*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (B*a^3 - 3*A*a^2*b - 3*
B*a*b^2 + A*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) + (3*B*a^3 - 5*A*a^2*b - B*a*b^2 - A*b^3 + 2*(B*a^2*b - 2*A*a*b^2 - B*b
^3)*tan(d*x + c))/(a^6 + 2*a^4*b^2 + a^2*b^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)
*tan(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.80

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)(dx+c)}{a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d}$$
$$+\frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d\right)}$$
$$-\frac{(Ba^3b-3\,Aa^2b^2-3\,Bab^3+Ab^4)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^6bd+3\,a^4b^3d+3\,a^2b^5d+b^7d}$$
$$+\frac{3\,Ba^5-5\,Aa^4b+2\,Ba^3b^2-6\,Aa^2b^3-Bab^4-Ab^5+2\left(Ba^4b-2\,Aa^3b^2-2\,Aab^4-Bb^5\right)\tan\left(dx+c\right.}{2\left(a^2+b^2\right)^3\left(b\tan\left(dx+c\right)+a\right)^2d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6*d + 3*a^4*b^2*d + 3
*a^2*b^4*d + b^6*d) + 1/2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan(
d*x + c)^2 + 1)/(a^6*d + 3*a^4*b^2*d + 3*a^2*b^4*d + b^6*d) - (B*a^3*b - 3
*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b*d + 3*
a^4*b^3*d + 3*a^2*b^5*d + b^7*d) + 1/2*(3*B*a^5 - 5*A*a^4*b + 2*B*a^3*b^2
- 6*A*a^2*b^3 - B*a*b^4 - A*b^5 + 2*(B*a^4*b - 2*A*a^3*b^2 - 2*A*a*b^4 - B
*b^5)*tan(d*x + c))/((a^2 + b^2)^3*(b*tan(d*x + c) + a)^2*d)
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.59

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{3\,A\,b-B\,a}{{\left(a^2+b^2\right)}^2}-\frac{4\,b^2\,\left(A\,b-B\,a\right)}{{\left(a^2+b^2\right)}^3}\right)}{d} - \frac{\frac{-3\,B\,a^3+5\,A\,a^2\,b+B\,a\,b^2+A\,b^3}{2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)\,\left(-B\,a^2\,b+2\,A\,a\,b^2+B\,b^3\right)}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$

input
```
int((A + B*tan(c + d*x))/(a + b*tan(c + d*x))^3,x)
```

output
```
(log(a + b*tan(c + d*x))*((3*A*b - B*a)/(a^2 + b^2)^2 - (4*b^2*(A*b - B*a)
)/(a^2 + b^2)^3))/d - ((A*b^3 - 3*B*a^3 + 5*A*a^2*b + B*a*b^2)/(2*(a^4 + b
^4 + 2*a^2*b^2)) + (tan(c + d*x)*(B*b^3 + 2*A*a*b^2 - B*a^2*b))/(a^4 + b^4
 + 2*a^2*b^2))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) + (log(
tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) +
 (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b
^3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.21

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{-\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)a^2b^2-\log\left(\tan\left(dx+c\right)^2+1\right)a^3b+2\log(a+\tan\left(dx+c\right)b)\tan(d}{ad\left(\tan\left(dx+c\right)a^4b+\right.}$$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
( - log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a**2*b**2 - log(tan(c + d*x)**2
+ 1)*a**3*b + 2*log(tan(c + d*x)*b + a)*tan(c + d*x)*a**2*b**2 + 2*log(tan
(c + d*x)*b + a)*a**3*b + tan(c + d*x)*a**3*b*d*x + tan(c + d*x)*a**2*b**2
 - tan(c + d*x)*a*b**3*d*x + tan(c + d*x)*b**4 + a**4*d*x - a**2*b**2*d*x)
/(a*d*(tan(c + d*x)*a**4*b + 2*tan(c + d*x)*a**2*b**3 + tan(c + d*x)*b**5
+ a**5 + 2*a**3*b**2 + a*b**4))
```

3.287 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 215

$$\begin{aligned}
&\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx \\
&= -\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\,x}{(a^2+b^2)^3}+\frac{A\log(\sin(c+dx))}{a^3d} \\
&\quad -\frac{b(6a^4Ab+3a^2Ab^3+Ab^5-3a^5B+a^3b^2B)\log(a\cos(c+dx)+b\sin(c+dx))}{a^3\left(a^2+b^2\right)^3d} \\
&\quad +\frac{b(Ab-aB)}{2a\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{b(3a^2Ab+Ab^3-2a^3B)}{a^2\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
-(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*x/(a^2+b^2)^3+A*ln(sin(d*x+c))/a^3/d-b*
(6*A*a^4*b+3*A*a^2*b^3+A*b^5-3*B*a^5+B*a^3*b^2)*ln(a*cos(d*x+c)+b*sin(d*x+
c))/a^3/(a^2+b^2)^3/d+1/2*b*(A*b-B*a)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+b*(
3*A*a^2*b+A*b^3-2*B*a^3)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.55 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.18

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{-\frac{a(a-ib)(A+iB)\log(i-\tan(c+dx))}{(a+ib)^2}+\frac{2A(a^2+b^2)\log(\tan(c+dx))}{a^2}-\frac{a(a+ib)(A-iB)\log(i+\tan(c+dx))}{(a-ib)^2}-\frac{2b(6a^4Ab+3a^2Ab^3+Ab^5-3a}{a^2(a}}{2a\left(a^2+b^2\right)d}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(-((a*(a - I*b)*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b)^2) + (2*A*(a^2
+ b^2)*Log[Tan[c + d*x]])/a^2 - (a*(a + I*b)*(A - I*B)*Log[I + Tan[c + d*x
]])/(a - I*b)^2 - (2*b*(6*a^4*A*b + 3*a^2*A*b^3 + A*b^5 - 3*a^5*B + a^3*b^
2*B)*Log[a + b*Tan[c + d*x]])/(a^2*(a^2 + b^2)^2) + (b*(A*b - a*B))/(a + b
*Tan[c + d*x])^2 + (4*a*b*(A*b - a*B))/((a^2 + b^2)*(a + b*Tan[c + d*x]))
+ (2*A*b^2)/(a^2 + a*b*Tan[c + d*x]))/(2*a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {3042, 4092, 27, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{\int \frac{2\cot(c+dx)\left(b(Ab-aB)\tan^2(c+dx)-a(Ab-aB)\tan(c+dx)+A\left(a^2+b^2\right)\right)}{(a+b\tan(c+dx))^2}dx}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\int \frac{\cot(c+dx)\left(b(Ab-aB)\tan^2(c+dx)-a(Ab-aB)\tan(c+dx)+A\left(a^2+b^2\right)\right)}{(a+b\tan(c+dx))^2}dx}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\int \frac{b(Ab-aB)\tan(c+dx)^2-a(Ab-aB)\tan(c+dx)+A\left(a^2+b^2\right)}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 4132

$$\frac{\frac{\int \frac{\cot(c+dx)\left(-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2\right)+A\left(a^2+b^2\right)^2+b\left(-2Ba^3+3Aba^2+Ab^3\right)\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2\right)+A\left(a^2+b^2\right)^2+b\left(-2Ba^3+3Aba^2+Ab^3\right)\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a(a^2+b^2)}+\frac{b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)(a+b\tan(c+dx))}}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 4134

$$\frac{\frac{\frac{A\left(a^2+b^2\right)^2\int\cot(c+dx)dx}{a}-\frac{b\left(-3a^5B+6a^4Ab+a^3b^2B+3a^2Ab^3+Ab^5\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2x\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(-2a^3B+3a^2Ab+}{ad(a^2+b^2)(a+b\tan(}}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{A(a^2+b^2)^2\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b(-3a^5B+6a^4Ab+a^3b^2B+3a^2Ab^3+Ab^5)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{a^2x(a^3(-B)+3a^2Ab+3ab^2B-Ab^3)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(-2a^3B+3a\cdots}{ad(a^2+b^2)(a+\cdots}}{a(a^2+b^2)}$$

$$\frac{b(Ab-aB)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{-\frac{A(a^2+b^2)^2\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{b(-3a^5B+6a^4Ab+a^3b^2B+3a^2Ab^3+Ab^5)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{a^2x(a^3(-B)+3a^2Ab+3ab^2B-Ab^3)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(-2a^3B\cdots}{ad(a^2+b^2)\cdots}}{a(a^2+b^2)}$$

$$\frac{b(Ab-aB)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3956

$$\frac{\frac{-\frac{b(-3a^5B+6a^4Ab+a^3b^2B+3a^2Ab^3+Ab^5)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{A(a^2+b^2)^2\log(-\sin(c+dx))}{ad}-\frac{a^2x(a^3(-B)+3a^2Ab+3ab^2B-Ab^3)}{a^2+b^2}}{a(a^2+b^2)}+\frac{b(-2a^3B+3a^2\cdots}{ad(a^2+b^2)(a+b\,t\cdots}}{a(a^2+b^2)}$$

$$\frac{b(Ab-aB)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4013

$$\frac{b(Ab-aB)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}+$$

$$\frac{\frac{b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)(a+b\tan(c+dx))}+\frac{\frac{A(a^2+b^2)^2\log(-\sin(c+dx))}{ad}-\frac{a^2x(a^3(-B)+3a^2Ab+3ab^2B-Ab^3)}{a^2+b^2}-\frac{b(-3a^5B+6a^4Ab+a^3b^2B+3a^2Ab^3+Ab^5)\log(a\cos(c\cdots}{ad(a^2+b^2)}}{a(a^2+b^2)}}{a(a^2+b^2)}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(A*b - a*B))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((-((a^2*(3*a
^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*x)/(a^2 + b^2)) + (A*(a^2 + b^2)^2*Log
[-Sin[c + d*x]])/(a*d) - (b*(6*a^4*A*b + 3*a^2*A*b^3 + A*b^5 - 3*a^5*B + a
^3*b^2*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/(a*(a^2
 + b^2)) + (b*(3*a^2*A*b + A*b^3 - 2*a^3*B))/(a*(a^2 + b^2)*d*(a + b*Tan[c
 + d*x])))/(a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{\frac{(-A\,a^3+3Aa\,b^2-3B\,a^2b+B\,b^3)\ln(1+\tan(dx+c)^2)}{2}+(-3A\,a^2b+A\,b^3+B\,a^3-3Ba\,b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^3}+\frac{A\ln(\tan(dx+c))}{a^3}+\frac{}{(a}}{d}$
default	$\frac{\frac{\frac{(-A\,a^3+3Aa\,b^2-3B\,a^2b+B\,b^3)\ln(1+\tan(dx+c)^2)}{2}+(-3A\,a^2b+A\,b^3+B\,a^3-3Ba\,b^2)\arctan(\tan(dx+c))}{(a^2+b^2)^3}+\frac{A\ln(\tan(dx+c))}{a^3}+\frac{}{(a}}{d}$
parallelrisch	$-12(A\,a^4b+\frac{1}{2}A\,a^2b^3+\frac{1}{6}A\,b^5-\frac{1}{2}B\,a^5+\frac{1}{6}B\,a^3b^2)b(a+b\tan(dx+c))^2\ln(a+b\tan(dx+c))-a^3(a+b\tan(dx+c))^2(A\,a^3-$
norman	$\frac{-\frac{(3A\,a^2b-A\,b^3-B\,a^3+3Ba\,b^2)a^2x}{(a^4+2a^2b^2+b^4)(a^2+b^2)}-\frac{b(4A\,a^2b^2+2A\,b^4-3B\,a^3b-Ba\,b^3)\tan(dx+c)}{d\,a^2(a^4+2a^2b^2+b^4)}-\frac{b^2(3A\,a^2b-A\,b^3-B\,a^3+3Ba\,b^2)x\tan(dx}{(a^4+2a^2b^2+b^4)(a^2+b^2)}}{(a+b\tan(dx+c))^2}$
risch	$-\frac{xB}{3ia^2b-ib^3-a^3+3a\,b^2}-\frac{6ia^2bBx}{a^6+3b^2a^4+3b^4a^2+b^6}+\frac{6ib^4Ac}{ad(a^6+3b^2a^4+3b^4a^2+b^6)}-\frac{2iAc}{a^3d}-\frac{2iAx}{a^3}+\frac{2ib}{a^3d(a^6+3b^2}$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(-A*a^3+3*A*a*b^2-3*B*a^2*b+B*b^3)*ln(1+tan(d*x+c)
^2)+(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*arctan(tan(d*x+c)))+1/a^3*A*ln(tan(
d*x+c))+b*(3*A*a^2*b+A*b^3-2*B*a^3)/(a^2+b^2)^2/a^2/(a+b*tan(d*x+c))-b*(6*
A*a^4*b+3*A*a^2*b^3+A*b^5-3*B*a^5+B*a^3*b^2)/(a^2+b^2)^3/a^3*ln(a+b*tan(d*
x+c))+1/2*(A*b-B*a)*b/(a^2+b^2)/a/(a+b*tan(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(213) = 426$.

Time = 0.14 (sec) , antiderivative size = 683, normalized size of antiderivative = 3.18

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx =$$
$$-\frac{7\,Ba^5b^3-9\,Aa^4b^4+Ba^3b^5-3\,Aa^2b^6-2\,(Ba^8-3\,Aa^7b-3\,Ba^6b^2+Aa^5b^3)dx-(5\,Ba^5b^3-7\,Aa^4b^4}{}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="fri
cas")
```

output
```
-1/2*(7*B*a^5*b^3 - 9*A*a^4*b^4 + B*a^3*b^5 - 3*A*a^2*b^6 - 2*(B*a^8 - 3*A
*a^7*b - 3*B*a^6*b^2 + A*a^5*b^3)*d*x - (5*B*a^5*b^3 - 7*A*a^4*b^4 - B*a^3
*b^5 - A*a^2*b^6 + 2*(B*a^6*b^2 - 3*A*a^5*b^3 - 3*B*a^4*b^4 + A*a^3*b^5)*d
*x)*tan(d*x + c)^2 - (A*a^8 + 3*A*a^6*b^2 + 3*A*a^4*b^4 + A*a^2*b^6 + (A*a
^6*b^2 + 3*A*a^4*b^4 + 3*A*a^2*b^6 + A*b^8)*tan(d*x + c)^2 + 2*(A*a^7*b +
3*A*a^5*b^3 + 3*A*a^3*b^5 + A*a*b^7)*tan(d*x + c))*log(tan(d*x + c)^2/(tan
(d*x + c)^2 + 1)) - (3*B*a^7*b - 6*A*a^6*b^2 - B*a^5*b^3 - 3*A*a^4*b^4 - A
*a^2*b^6 + (3*B*a^5*b^3 - 6*A*a^4*b^4 - B*a^3*b^5 - 3*A*a^2*b^6 - A*b^8)*t
an(d*x + c)^2 + 2*(3*B*a^6*b^2 - 6*A*a^5*b^3 - B*a^4*b^4 - 3*A*a^3*b^5 - A
*a*b^7)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/
(tan(d*x + c)^2 + 1)) - 2*(3*B*a^6*b^2 - 4*A*a^5*b^3 - 3*B*a^4*b^4 + 3*A*a
^3*b^5 + A*a*b^7 + 2*(B*a^7*b - 3*A*a^6*b^2 - 3*B*a^5*b^3 + A*a^4*b^4)*d*x
)*tan(d*x + c))/((a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8)*d*tan(d*x + c
)^2 + 2*(a^10*b + 3*a^8*b^3 + 3*a^6*b^5 + a^4*b^7)*d*tan(d*x + c) + (a^11
+ 3*a^9*b^2 + 3*a^7*b^4 + a^5*b^6)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.73

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\,(3\,Ba^5b-6\,Aa^4b^2-Ba^3b^3-3\,Aa^2b^4-Ab^6)\log(b\tan(dx+c)+a)}{a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6} - \frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)\,l}{a^6+3\,a^4b^2+3\,a^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="max
ima")
```

output
```
1/2*(2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6) + 2*(3*B*a^5*b - 6*A*a^4*b^2 - B*a^3*b^3 - 3*A*a^2*b^4
- A*b^6)*log(b*tan(d*x + c) + a)/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6) -
 (A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6) - (5*B*a^4*b - 7*A*a^3*b^2 + B*a^2*b^3 - 3*A*a*
b^4 + 2*(2*B*a^3*b^2 - 3*A*a^2*b^3 - A*b^5)*tan(d*x + c))/(a^8 + 2*a^6*b^2
 + a^4*b^4 + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*tan(d*x + c)^2 + 2*(a^7*b + 2
*a^5*b^3 + a^3*b^5)*tan(d*x + c)) + 2*A*log(tan(d*x + c))/a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.73

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)(dx+c)}{a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d} - \frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d\right)} + \frac{(3\,Ba^5b^2-6\,Aa^4b^3-Ba^3b^4-3\,Aa^2b^5-Ab^7)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^9bd+3\,a^7b^3d+3\,a^5b^5d+a^3b^7d} + \frac{A\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^3d} - \frac{5\,Ba^7b-7\,Aa^6b^2+6\,Ba^5b^3-10\,Aa^4b^4+Ba^3b^5-3\,Aa^2b^6+2\left(2\,Ba^6b^2-3\,Aa^5b^3+2\,Ba^4b^4-4\,Aa^3\right.}{2\left(a^2+b^2\right)^3\left(b\tan\left(dx+c\right)+a\right)^2a^3d}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="gia
c")
```

output
```
(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*(d*x + c)/(a^6*d + 3*a^4*b^2*d + 3
*a^2*b^4*d + b^6*d) - 1/2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(tan(
d*x + c)^2 + 1)/(a^6*d + 3*a^4*b^2*d + 3*a^2*b^4*d + b^6*d) + (3*B*a^5*b^2
 - 6*A*a^4*b^3 - B*a^3*b^4 - 3*A*a^2*b^5 - A*b^7)*log(abs(b*tan(d*x + c) +
 a))/(a^9*b*d + 3*a^7*b^3*d + 3*a^5*b^5*d + a^3*b^7*d) + A*log(abs(tan(d*x
 + c)))/(a^3*d) - 1/2*(5*B*a^7*b - 7*A*a^6*b^2 + 6*B*a^5*b^3 - 10*A*a^4*b^
4 + B*a^3*b^5 - 3*A*a^2*b^6 + 2*(2*B*a^6*b^2 - 3*A*a^5*b^3 + 2*B*a^4*b^4 -
 4*A*a^3*b^5 - A*a*b^7)*tan(d*x + c))/((a^2 + b^2)^3*(b*tan(d*x + c) + a)^
2*a^3*d)
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{-5\,B\,a^3\,b+7\,A\,a^2\,b^2-B\,a\,b^3+3\,A\,b^4}{2\,a\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)\,\left(-2\,B\,a^3\,b^2+3\,A\,a^2\,b^3+A\,b^5\right)}{a^2\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)}+\frac{A\,\ln\left(\tan(c+d\,x)\right)}{a^3\,d}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}$$
$$-\frac{b\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-3\,B\,a^5+6\,A\,a^4\,b+B\,a^3\,b^2+3\,A\,a^2\,b^3+A\,b^5\right)}{a^3\,d\,\left(a^2+b^2\right)^3}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
((3*A*b^4 + 7*A*a^2*b^2 - B*a*b^3 - 5*B*a^3*b)/(2*a*(a^4 + b^4 + 2*a^2*b^2
)) + (tan(c + d*x)*(A*b^5 + 3*A*a^2*b^3 - 2*B*a^3*b^2))/(a^2*(a^4 + b^4 +
2*a^2*b^2)))/(d*(a^2 + b^2*tan(c + d*x)^2 + 2*a*b*tan(c + d*x))) + (A*log(
tan(c + d*x)))/(a^3*d) + (log(tan(c + d*x) - 1i)*(A*1i - B))/(2*d*(a*b^2*3
i + 3*a^2*b - a^3*1i - b^3)) + (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(3
*a*b^2 + a^2*b*3i - a^3 - b^3*1i)) - (b*log(a + b*tan(c + d*x))*(A*b^5 - 3
*B*a^5 + 3*A*a^2*b^3 + B*a^3*b^2 + 6*A*a^4*b))/(a^3*d*(a^2 + b^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.52

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
( - cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*a**5 + cos(c + d*x)*log(tan(
(c + d*x)/2)**2 + 1)*a**3*b**2 - 3*cos(c + d*x)*log(tan((c + d*x)/2)**2*a
- 2*tan((c + d*x)/2)*b - a)*a**3*b**2 - cos(c + d*x)*log(tan((c + d*x)/2)*
*2*a - 2*tan((c + d*x)/2)*b - a)*a*b**4 + cos(c + d*x)*log(tan((c + d*x)/2
))*a**5 + 2*cos(c + d*x)*log(tan((c + d*x)/2))*a**3*b**2 + cos(c + d*x)*lo
g(tan((c + d*x)/2))*a*b**4 - 2*cos(c + d*x)*a**4*b*d*x + cos(c + d*x)*a**3
*b**2 + cos(c + d*x)*a*b**4 - log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a*
*4*b + log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a**2*b**3 - 3*log(tan((c
+ d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)*a**2*b**3 - log(ta
n((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)*b**5 + log(ta
n((c + d*x)/2))*sin(c + d*x)*a**4*b + 2*log(tan((c + d*x)/2))*sin(c + d*x)
*a**2*b**3 + log(tan((c + d*x)/2))*sin(c + d*x)*b**5 - 2*sin(c + d*x)*a**3
*b**2*d*x)/(a**2*d*(cos(c + d*x)*a**5 + 2*cos(c + d*x)*a**3*b**2 + cos(c +
 d*x)*a*b**4 + sin(c + d*x)*a**4*b + 2*sin(c + d*x)*a**2*b**3 + sin(c + d*
x)*b**5))
```

3.288 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 287

$$\begin{aligned}\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx &= -\frac{(a^3A-3aAb^2+3a^2bB-b^3B)\,x}{(a^2+b^2)^3}-\frac{(3Ab-aB)\log(\sin(c+dx))}{a^4d}\\ &+\frac{b^2(10a^4Ab+9a^2Ab^3+3Ab^5-6a^5B-3a^3b^2B-ab^4B)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4\,(a^2+b^2)^3\,d}\\ &-\frac{b(2a^2A+3Ab^2-abB)}{2a^2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}\\ &-\frac{b(a^4A+6a^2Ab^2+3Ab^4-3a^3bB-ab^3B)}{a^3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*x/(a^2+b^2)^3-(3*A*b-B*a)*ln(sin(d*x+c)
)/a^4/d+b^2*(10*A*a^4*b+9*A*a^2*b^3+3*A*b^5-6*B*a^5-3*B*a^3*b^2-B*a*b^4)*l
n(a*cos(d*x+c)+b*sin(d*x+c))/a^4/(a^2+b^2)^3/d-1/2*b*(2*A*a^2+3*A*b^2-B*a*
b)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-A*cot(d*x+c)/a/d/(a+b*tan(d*x+c))^2-
b*(A*a^4+6*A*a^2*b^2+3*A*b^4-3*B*a^3*b-B*a*b^3)/a^3/(a^2+b^2)^2/d/(a+b*tan
(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.28 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx \\
&= -\frac{A\cot(c+dx)}{a^3d}+\frac{(A+iB)\log(i-\tan(c+dx))}{2(ia-b)^3d} \\
&\quad -\frac{(3Ab-aB)\log(\tan(c+dx))}{a^4d}-\frac{(iA+B)\log(i+\tan(c+dx))}{2(a-ib)^3d} \\
&\quad +\frac{b^2(10a^4Ab+9a^2Ab^3+3Ab^5-6a^5B-3a^3b^2B-ab^4B)\log(a+b\tan(c+dx))}{a^4\left(a^2+b^2\right)^3d} \\
&\quad -\frac{b^2(Ab-aB)}{2a^2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}-\frac{b^2(4a^2Ab+2Ab^3-3a^3B-ab^2B)}{a^3\left(a^2+b^2\right)^2d(a+b\tan(c+dx))}
\end{aligned}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
-((A*Cot[c + d*x])/(a^3*d)) + ((A + I*B)*Log[I - Tan[c + d*x]])/(2*(I*a -
b)^3*d) - ((3*A*b - a*B)*Log[Tan[c + d*x]])/(a^4*d) - ((I*A + B)*Log[I + T
an[c + d*x]])/(2*(a - I*b)^3*d) + (b^2*(10*a^4*A*b + 9*a^2*A*b^3 + 3*A*b^5
 - 6*a^5*B - 3*a^3*b^2*B - a*b^4*B)*Log[a + b*Tan[c + d*x]])/(a^4*(a^2 + b
^2)^3*d) - (b^2*(A*b - a*B))/(2*a^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
- (b^2*(4*a^2*A*b + 2*A*b^3 - 3*a^3*B - a*b^2*B))/(a^3*(a^2 + b^2)^2*d*(a
+ b*Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4092, 3042, 4132, 27, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{\int \frac{\cot(c+dx)(3Ab\tan^2(c+dx)+aA\tan(c+dx)+3Ab-aB)}{(a+b\tan(c+dx))^3}dx}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{3Ab\tan(c+dx)^2+aA\tan(c+dx)+3Ab-aB}{\tan(c+dx)(a+b\tan(c+dx))^3}dx}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{\int \frac{2\cot(c+dx)\left((aA+bB)\tan(c+dx)a^2+b\left(2Aa^2-bBa+3Ab^2\right)\tan^2(c+dx)+\left(a^2+b^2\right)(3Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{2a(a^2+b^2)}+\frac{b(2a^2A-abB+3Ab^2)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\cot(c+dx)\left((aA+bB)\tan(c+dx)a^2+b\left(2Aa^2-bBa+3Ab^2\right)\tan^2(c+dx)+\left(a^2+b^2\right)(3Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{a(a^2+b^2)}+\frac{b(2a^2A-abB+3Ab^2)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{(aA+bB)\tan(c+dx)a^2+b\left(2Aa^2-bBa+3Ab^2\right)\tan(c+dx)^2+\left(a^2+b^2\right)(3Ab-aB)}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a(a^2+b^2)}+\frac{b(2a^2A-abB+3Ab^2)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+b\left(Aa^4-3bBa^3+6Ab^2a^2-b^3Ba+3Ab^4\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2(3Ab-aB)\right)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{b\left(a^4A-3a^3bB+6a^2Ab^2-a\cdots\right)}{ad\left(a^2+b^2\right)(a+b\tan(c\cdots)}}{a(a^2+b^2)}\cdots}{a}$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+b\left(Aa^4-3bBa^3+6Ab^2a^2-b^3Ba+3Ab^4\right)\tan(c+dx)^2+\left(a^2+b^2\right)^2(3Ab-aB)}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a\left(a^2+b^2\right)}+\frac{b\left(a^4A-3a^3bB+6a^2Ab^2-ab^3B+3Ab^4\right)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a(a^2+b^2)}\cdots}{a}$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4134

$$-\frac{\frac{\frac{\frac{\left(a^2+b^2\right)^2(3Ab-aB)\int\cot(c+dx)dx}{a}-\frac{b^2\left(-6a^5B+10a^4Ab-3a^3b^2B+9a^2Ab^3-ab^4B+3Ab^5\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^3x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}+\frac{b\left(a^4A\cdots\right)}{\cdots}}{a(a^2+b^2)}\cdots}{a}$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{\left(a^2+b^2\right)^2(3Ab-aB)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b^2\left(-6a^5B+10a^4Ab-3a^3b^2B+9a^2Ab^3-ab^4B+3Ab^5\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^3x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}\cdots}{a(a^2+b^2)}\cdots}{a}$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{-\frac{\left(a^2+b^2\right)^2(3Ab-aB)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{b^2\left(-6a^5B+10a^4Ab-3a^3b^2B+9a^2Ab^3-ab^4B+3Ab^5\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^3x\left(a^3A+3a^2bB-3aAb^2-b^3\cdots\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}\cdots}{a(a^2+b^2)}\cdots}{a}$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\downarrow$ 3956

$$-\frac{\frac{\frac{-\frac{b^2\left(-6a^5B+10a^4Ab-3a^3b^2B+9a^2Ab^3-ab^4B+3Ab^5\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{\left(a^2+b^2\right)^2(3Ab-aB)\log(-\sin(c+dx))}{ad}+\frac{a^3x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}+\frac{b(\cdots}{\cdots}}{a(a^2+b^2)}}{a}$$
$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

$\downarrow$ 4013

$$-\frac{\frac{b(2a^2A-abB+3Ab^2)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}+\frac{\frac{b\left(a^4A-3a^3bB+6a^2Ab^2-ab^3B+3Ab^4\right)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{\frac{\left(a^2+b^2\right)^2(3Ab-aB)\log(-\sin(c+dx))}{ad}+\frac{a^3x\left(a^3A+3a^2bB-3aAb^2-b^3B\right)}{a^2+b^2}-\cdots}{a\cdots}}{a(a^2+b^2)}}{a}$$
$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^2}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
-((A*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x])^2)) - ((b*(2*a^2*A + 3*A*b^2
 - a*b*B))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (((a^3*(a^3*A - 3*a
*A*b^2 + 3*a^2*b*B - b^3*B)*x)/(a^2 + b^2) + ((a^2 + b^2)^2*(3*A*b - a*B)*
Log[-Sin[c + d*x]])/(a*d) - (b^2*(10*a^4*A*b + 9*a^2*A*b^3 + 3*A*b^5 - 6*a
^5*B - 3*a^3*b^2*B - a*b^4*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^
2 + b^2)*d))/(a*(a^2 + b^2)) + (b*(a^4*A + 6*a^2*A*b^2 + 3*A*b^4 - 3*a^3*b
*B - a*b^3*B))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(a*(a^2 + b^2)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\frac{\left(3A\,a^2b-A\,b^3-B\,a^3+3Ba\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-A\,a^3+3Aa\,b^2-3B\,a^2b+B\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{A}{a^3\tan(dx+c)}+\frac{(-3Ab+}{}}{}$
default	$\frac{\frac{\frac{\left(3A\,a^2b-A\,b^3-B\,a^3+3Ba\,b^2\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-A\,a^3+3Aa\,b^2-3B\,a^2b+B\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{A}{a^3\tan(dx+c)}+\frac{(-3Ab+}{}}{}$
parallelrisch	$\frac{20\left(A\,a^4b+\frac{9}{10}A\,a^2b^3+\frac{3}{10}A\,b^5-\frac{3}{5}B\,a^5-\frac{3}{10}B\,a^3b^2-\frac{1}{10}Ba\,b^4\right)b^2(a+b\tan(dx+c))^2\ln(a+b\tan(dx+c))+3a^4\left(A\,a^2b-\frac{1}{3}A\right.}{}$
norman	$\frac{\frac{b\left(3A\,a^4b+11A\,a^2b^3+6A\,b^5-4B\,a^3b^2-2Ba\,b^4\right)\tan(dx+c)^2}{d\,a^3\left(a^4+2a^2b^2+b^4\right)}-\frac{A}{ad}-\frac{b^2\left(A\,a^3-3Aa\,b^2+3B\,a^2b-B\,b^3\right)x\tan(dx+c)^3}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)}+\frac{b^2\left(4A\,a^4b+17\right.}{}}{\tan(dx-}$
risch	Expression too large to display

input

```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output

```
1/d*(1/(a^2+b^2)^3*(1/2*(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*ln(1+tan(d*x+c)^
2)+(-A*a^3+3*A*a*b^2-3*B*a^2*b+B*b^3)*arctan(tan(d*x+c)))-1/a^3*A/tan(d*x+
c)+(-3*A*b+B*a)/a^4*ln(tan(d*x+c))-b^2*(4*A*a^2*b+2*A*b^3-3*B*a^3-B*a*b^2)
/(a^2+b^2)^2/a^3/(a+b*tan(d*x+c))+b^2*(10*A*a^4*b+9*A*a^2*b^3+3*A*b^5-6*B*
a^5-3*B*a^3*b^2-B*a*b^4)/(a^2+b^2)^3/a^4*ln(a+b*tan(d*x+c))-1/2*(A*b-B*a)*
b^2/(a^2+b^2)/a^2/(a+b*tan(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(283) = 566$.

Time = 0.24 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.20

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/2*(2*A*a^9 + 6*A*a^7*b^2 + 6*A*a^5*b^4 + 2*A*a^3*b^6 + (7*B*a^5*b^4 - 9
*A*a^4*b^5 + B*a^3*b^6 - 3*A*a^2*b^7 + 2*(A*a^7*b^2 + 3*B*a^6*b^3 - 3*A*a^
5*b^4 - B*a^4*b^5)*d*x)*tan(d*x + c)^3 + 2*(A*a^7*b^2 + 4*B*a^6*b^3 - 2*A*
a^5*b^4 - 3*B*a^4*b^5 + 6*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8 + 2*(A*a^8*b +
 3*B*a^7*b^2 - 3*A*a^6*b^3 - B*a^5*b^4)*d*x)*tan(d*x + c)^2 - ((B*a^7*b^2
- 3*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 + B*
a*b^8 - 3*A*b^9)*tan(d*x + c)^3 + 2*(B*a^8*b - 3*A*a^7*b^2 + 3*B*a^6*b^3 -
 9*A*a^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 + B*a^2*b^7 - 3*A*a*b^8)*tan(d*x
+ c)^2 + (B*a^9 - 3*A*a^8*b + 3*B*a^7*b^2 - 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*
A*a^4*b^5 + B*a^3*b^6 - 3*A*a^2*b^7)*tan(d*x + c))*log(tan(d*x + c)^2/(tan
(d*x + c)^2 + 1)) + ((6*B*a^5*b^4 - 10*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b
^7 + B*a*b^8 - 3*A*b^9)*tan(d*x + c)^3 + 2*(6*B*a^6*b^3 - 10*A*a^5*b^4 + 3
*B*a^4*b^5 - 9*A*a^3*b^6 + B*a^2*b^7 - 3*A*a*b^8)*tan(d*x + c)^2 + (6*B*a^
7*b^2 - 10*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 + B*a^3*b^6 - 3*A*a^2*b^7
)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d
*x + c)^2 + 1)) + (4*A*a^8*b + 12*A*a^6*b^3 - 9*B*a^5*b^4 + 23*A*a^4*b^5 -
 3*B*a^3*b^6 + 9*A*a^2*b^7 + 2*(A*a^9 + 3*B*a^8*b - 3*A*a^7*b^2 - B*a^6*b^
3)*d*x)*tan(d*x + c))/((a^10*b^2 + 3*a^8*b^4 + 3*a^6*b^6 + a^4*b^8)*d*tan(
d*x + c)^3 + 2*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*d*tan(d*x + c)^2
 + (a^12 + 3*a^10*b^2 + 3*a^8*b^4 + a^6*b^6)*d*tan(d*x + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.58

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{2\,(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(6\,Ba^5b^2-10\,Aa^4b^3+3\,Ba^3b^4-9\,Aa^2b^5+Bab^6-3\,Ab^7)\log(b\tan(dx+c)+a)}{a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6}+\frac{(Ba^3-3\,Aa^2b\cdots}{\cdots}}{\cdots}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/2*(2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + 2*(6*B*a^5*b^2 - 10*A*a^4*b^3 + 3*B*a^3*b^4 - 9*A*a^
2*b^5 + B*a*b^6 - 3*A*b^7)*log(b*tan(d*x + c) + a)/(a^10 + 3*a^8*b^2 + 3*a
^6*b^4 + a^4*b^6) + (B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan(d*x +
c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*A*a^6 + 4*A*a^4*b^2 + 2
*A*a^2*b^4 + 2*(A*a^4*b^2 - 3*B*a^3*b^3 + 6*A*a^2*b^4 - B*a*b^5 + 3*A*b^6)
*tan(d*x + c)^2 + (4*A*a^5*b - 7*B*a^4*b^2 + 17*A*a^3*b^3 - 3*B*a^2*b^4 +
9*A*a*b^5)*tan(d*x + c))/((a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*tan(d*x + c)^3 +
 2*(a^8*b + 2*a^6*b^3 + a^4*b^5)*tan(d*x + c)^2 + (a^9 + 2*a^7*b^2 + a^5*b
^4)*tan(d*x + c)) - 2*(B*a - 3*A*b)*log(tan(d*x + c))/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.62

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)(dx+c)}{a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d}$$
$$-\frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d\right)}$$
$$-\frac{(6\,Ba^5b^3-10\,Aa^4b^4+3\,Ba^3b^5-9\,Aa^2b^6+Bab^7-3\,Ab^8)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^{10}bd+3\,a^8b^3d+3\,a^6b^5d+a^4b^7d}$$
$$+\frac{(Ba-3\,Ab)\log\left(|\tan\left(dx+c\right)|\right)}{a^4d}$$
$$-\frac{2\,Aa^9+6\,Aa^7b^2+6\,Aa^5b^4+2\,Aa^3b^6+2\left(Aa^7b^2-3\,Ba^6b^3+7\,Aa^5b^4-4\,Ba^4b^5+9\,Aa^3b^6-Ba^2b^7+\right.}{2\left(a^2+b^2\right)^3(b\tan(}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*(d*x + c)/(a^6*d + 3*a^4*b^2*d +
3*a^2*b^4*d + b^6*d) - 1/2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*log(tan
(d*x + c)^2 + 1)/(a^6*d + 3*a^4*b^2*d + 3*a^2*b^4*d + b^6*d) - (6*B*a^5*b^
3 - 10*A*a^4*b^4 + 3*B*a^3*b^5 - 9*A*a^2*b^6 + B*a*b^7 - 3*A*b^8)*log(abs(
b*tan(d*x + c) + a))/(a^10*b*d + 3*a^8*b^3*d + 3*a^6*b^5*d + a^4*b^7*d) +
(B*a - 3*A*b)*log(abs(tan(d*x + c)))/(a^4*d) - 1/2*(2*A*a^9 + 6*A*a^7*b^2
+ 6*A*a^5*b^4 + 2*A*a^3*b^6 + 2*(A*a^7*b^2 - 3*B*a^6*b^3 + 7*A*a^5*b^4 - 4
*B*a^4*b^5 + 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*tan(d*x + c)^2 + (4*A*a^
8*b - 7*B*a^7*b^2 + 21*A*a^6*b^3 - 10*B*a^5*b^4 + 26*A*a^4*b^5 - 3*B*a^3*b
^6 + 9*A*a^2*b^7)*tan(d*x + c))/((a^2 + b^2)^3*(b*tan(d*x + c) + a)^2*a^4*
d*tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.32

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{b^2 \ln(a+b\tan(c+dx)) (-6Ba^5+10Aa^4b-3Ba^3b^2+9Aa^2b^3-Bab^4+3Ab^5)}{a^4 d (a^2+b^2)^3} - \frac{\ln(\tan(c+dx)-i) (-B+A\,1i)}{2d(-a^3-a^2b\,3i+3ab^2+b^3\,1i)} - \frac{\ln(\tan(c+dx)) (3Ab-Ba)}{a^4 d} - \frac{\ln(\tan(c+dx)+1i) (A-B\,1i)}{2d(-a^3\,1i-3a^2b+ab^2\,3i+b^3)} - \frac{\frac{A}{a} + \frac{\tan(c+dx)^2 (Aa^4b^2-3Ba^3b^3+6Aa^2b^4-Bab^5+3Ab^6)}{a^3(a^4+2a^2b^2+b^4)} + \frac{\tan(c+dx)(4Aa^4b-7Ba^3b^2+17Aa^2b^3-3Bab^4+9Ab^5)}{2a^2(a^4+2a^2b^2+b^4)}}{d\left(a^2\tan(c+dx)+2ab\tan(c+dx)^2+b^2\tan(c+dx)^3\right)}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
(b^2*log(a + b*tan(c + d*x))*(3*A*b^5 - 6*B*a^5 + 9*A*a^2*b^3 - 3*B*a^3*b^
2 + 10*A*a^4*b - B*a*b^4))/(a^4*d*(a^2 + b^2)^3) - (log(tan(c + d*x) - 1i)
*(A*1i - B))/(2*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) - (log(tan(c + d*x)
)*(3*A*b - B*a))/(a^4*d) - (log(tan(c + d*x) + 1i)*(A - B*1i))/(2*d*(a*b^2
*3i - 3*a^2*b - a^3*1i + b^3)) - (A/a + (tan(c + d*x)^2*(3*A*b^6 + 6*A*a^2
*b^4 + A*a^4*b^2 - 3*B*a^3*b^3 - B*a*b^5))/(a^3*(a^4 + b^4 + 2*a^2*b^2)) +
 (tan(c + d*x)*(9*A*b^5 + 17*A*a^2*b^3 - 7*B*a^3*b^2 + 4*A*a^4*b - 3*B*a*b
^4))/(2*a^2*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2*tan(c + d*x) + b^2*tan(c + d
*x)^3 + 2*a*b*tan(c + d*x)^2))
```

Reduce [B] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 726, normalized size of antiderivative = 2.53

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
(4*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a**5*b**2 + 8*co
s(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d
*x)*a**3*b**4 + 4*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)
/2)*b - a)*sin(c + d*x)*a*b**6 - 4*cos(c + d*x)*log(tan((c + d*x)/2))*sin(
c + d*x)*a**5*b**2 - 8*cos(c + d*x)*log(tan((c + d*x)/2))*sin(c + d*x)*a**
3*b**4 - 4*cos(c + d*x)*log(tan((c + d*x)/2))*sin(c + d*x)*a*b**6 - cos(c
+ d*x)*sin(c + d*x)*a**7 - 2*cos(c + d*x)*sin(c + d*x)*a**6*b*d*x - 4*cos(
c + d*x)*sin(c + d*x)*a**5*b**2 + 2*cos(c + d*x)*sin(c + d*x)*a**4*b**3*d*
x - 7*cos(c + d*x)*sin(c + d*x)*a**3*b**4 - 4*cos(c + d*x)*sin(c + d*x)*a*
b**6 + 4*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**4*b**3 + 8*log(ta
n((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*a**2*b**5
+ 4*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*
b**7 - 4*log(tan((c + d*x)/2))*sin(c + d*x)**2*a**4*b**3 - 8*log(tan((c +
d*x)/2))*sin(c + d*x)**2*a**2*b**5 - 4*log(tan((c + d*x)/2))*sin(c + d*x)*
*2*b**7 + sin(c + d*x)**2*a**6*b - 2*sin(c + d*x)**2*a**5*b**2*d*x + 2*sin
(c + d*x)**2*a**4*b**3 + 2*sin(c + d*x)**2*a**3*b**4*d*x + sin(c + d*x)**2
*a**2*b**5 - 2*a**6*b - 4*a**4*b**3 - 2*a**2*b**5)/(2*sin(c + d*x)*a**3*b*
d*(cos(c + d*x)*a**5 + 2*cos(c + d*x)*a**3*b**2 + cos(c + d*x)*a*b**4 + si
n(c + d*x)*a**4*b + 2*sin(c + d*x)*a**2*b**3 + sin(c + d*x)*b**5))
```

3.289 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 352

$$\begin{aligned}\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx &= \frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\,x}{(a^2+b^2)^3}-\frac{(a^2A-6Ab^2+3abB)\log(\sin(c+dx))}{a^5d}\\ &-\frac{b^3(15a^4Ab+17a^2Ab^3+6Ab^5-10a^5B-9a^3b^2B-3ab^4B)\log(a\cos(c+dx)+b\sin(c+dx))}{a^5(a^2+b^2)^3d}\\ &+\frac{b(5a^2Ab+6Ab^3-2a^3B-3ab^2B)}{2a^3(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{(2Ab-aB)\cot(c+dx)}{a^2d(a+b\tan(c+dx))^2}\\ &-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}+\frac{b(3a^4Ab+11a^2Ab^3+6Ab^5-a^5B-6a^3b^2B-3ab^4B)}{a^4(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*x/(a^2+b^2)^3-(A*a^2-6*A*b^2+3*B*a*b)*ln
(sin(d*x+c))/a^5/d-b^3*(15*A*a^4*b+17*A*a^2*b^3+6*A*b^5-10*B*a^5-9*B*a^3*b
^2-3*B*a*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^5/(a^2+b^2)^3/d+1/2*b*(5*A*a
^2*b+6*A*b^3-2*B*a^3-3*B*a*b^2)/a^3/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+(2*A*b-
B*a)*cot(d*x+c)/a^2/d/(a+b*tan(d*x+c))^2-1/2*A*cot(d*x+c)^2/a/d/(a+b*tan(d
*x+c))^2+b*(3*A*a^4*b+11*A*a^2*b^3+6*A*b^5-B*a^5-6*B*a^3*b^2-3*B*a*b^4)/a^
4/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.91

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{(3Ab-aB)\cot(c+dx)}{a^4d} - \frac{A\cot^2(c+dx)}{2a^3d} + \frac{(A+iB)\log(i-\tan(c+dx))}{2(a+ib)^3d}$$
$$- \frac{(a^2A-6Ab^2+3abB)\log(\tan(c+dx))}{a^5d} + \frac{(A-iB)\log(i+\tan(c+dx))}{2(a-ib)^3d}$$
$$- \frac{b^3(15a^4Ab+17a^2Ab^3+6Ab^5-10a^5B-9a^3b^2B-3ab^4B)\log(a+b\tan(c+dx))}{a^5(a^2+b^2)^3d}$$
$$+ \frac{b^3(Ab-aB)}{2a^3(a^2+b^2)d(a+b\tan(c+dx))^2} + \frac{b^3(5a^2Ab+3Ab^3-4a^3B-2ab^2B)}{a^4(a^2+b^2)^2d(a+b\tan(c+dx))}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
((3*A*b - a*B)*Cot[c + d*x])/(a^4*d) - (A*Cot[c + d*x]^2)/(2*a^3*d) + ((A
+ I*B)*Log[I - Tan[c + d*x]])/(2*(a + I*b)^3*d) - ((a^2*A - 6*A*b^2 + 3*a*
b*B)*Log[Tan[c + d*x]])/(a^5*d) + ((A - I*B)*Log[I + Tan[c + d*x]])/(2*(a
- I*b)^3*d) - (b^3*(15*a^4*A*b + 17*a^2*A*b^3 + 6*A*b^5 - 10*a^5*B - 9*a^3
*b^2*B - 3*a*b^4*B)*Log[a + b*Tan[c + d*x]])/(a^5*(a^2 + b^2)^3*d) + (b^3*
(A*b - a*B))/(2*a^3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (b^3*(5*a^2*A*
b + 3*A*b^3 - 4*a^3*B - 2*a*b^2*B))/(a^4*(a^2 + b^2)^2*d*(a + b*Tan[c + d*
x]))
```

Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.15, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {3042, 4092, 27, 3042, 4132, 25, 3042, 4132, 27, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 4092}$$

$$-\frac{\int \frac{2\cot^2(c+dx)(2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB)}{(a+b\tan(c+dx))^3}dx}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\cot^2(c+dx)(2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB)}{(a+b\tan(c+dx))^3}dx}{a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{2Ab\tan(c+dx)^2+aA\tan(c+dx)+2Ab-aB}{\tan(c+dx)^2(a+b\tan(c+dx))^3}dx}{a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 4132}$$

$$-\frac{-\frac{\int -\frac{\cot(c+dx)\left(Aa^2+B\tan(c+dx)a^2+3bBa-6Ab^2-3b(2Ab-aB)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}dx}{a}-\frac{(2Ab-aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^2}}{a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int \frac{\cot(c+dx)\left(Aa^2+B\tan(c+dx)a^2+3bBa-6Ab^2-3b(2Ab-aB)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^3}dx}{a}-\frac{(2Ab-aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^2}}{a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\int \frac{Aa^2+B\tan(c+dx)a^2+3bBa-6Ab^2-3b(2Ab-aB)\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))^3}dx}{a}-\frac{(2Ab-aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^2}}{a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 4132}$$

$$-\frac{\frac{\frac{\frac{\int \frac{2\cot(c+dx)\left(-\left((Ab-aB)\tan(c+dx)a^3\right)-b\left(-2Ba^3+5Aba^2-3b^2Ba+6Ab^3\right)\tan^2(c+dx)+\left(a^2+b^2\right)\left(Aa^2+3bBa-6Ab^2\right)\right)}{(a+b\tan(c+dx))^2}dx}{2a\left(a^2+b^2\right)}-\frac{b\left(-2a^3B+5a^2Ab-3ab^2B+6\cdots\right.}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx\cdots}}{a}\cdots}{a}\cdots$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(-\left((Ab-aB)\tan(c+dx)a^3\right)-b\left(-2Ba^3+5Aba^2-3b^2Ba+6Ab^3\right)\tan^2(c+dx)+\left(a^2+b^2\right)\left(Aa^2+3bBa-6Ab^2\right)\right)}{(a+b\tan(c+dx))^2}dx}{a\left(a^2+b^2\right)}-\frac{b\left(-2a^3B+5a^2Ab-3ab^2B+6A\cdots\right.}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx)\cdots}}{a}\cdots}{a}\cdots$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\frac{\int \frac{-\left((Ab-aB)\tan(c+dx)a^3\right)-b\left(-2Ba^3+5Aba^2-3b^2Ba+6Ab^3\right)\tan(c+dx)^2+\left(a^2+b^2\right)\left(Aa^2+3bBa-6Ab^2\right)}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a\left(a^2+b^2\right)}-\frac{b\left(-2a^3B+5a^2Ab-3ab^2B+6Ab^3\right)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}}{a}-\frac{(2A\cdots}{ad(\cdots}}{a}\cdots$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\frac{\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^4\right)-b\left(-Ba^5+3Aba^4-6b^2Ba^3+11Ab^3a^2-3b^4Ba+6Ab^5\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2\left(Aa^2+3bBa-6Ab^2\right)\right)}{a+b\tan(c+dx)}dx\cdots}{a\left(a^2+b^2\right)}\cdots}{a\left(a^2+b^2\right)}\cdots}{a}\cdots}{a}\cdots$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\frac{\frac{\int \frac{-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^4\right)-b\left(-Ba^5+3Aba^4-6b^2Ba^3+11Ab^3a^2-3b^4Ba+6Ab^5\right)\tan(c+dx)^2+\left(a^2+b^2\right)^2\left(Aa^2+3bBa-6Ab^2\right)}{\tan(c+dx)(a+b\tan(c+dx))}dx-\frac{b\left(a^5(-B)\cdots\right.}{\cdots}}{a\left(a^2+b^2\right)}\cdots}{a\left(a^2+b^2\right)}\cdots}{a}\cdots}{a}\cdots$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4134

$$-\frac{\frac{\frac{\frac{\frac{(a^2+b^2)^2(a^2A+3abB-6Ab^2)\int\cot(c+dx)dx}{a}+\frac{b^3(-10a^5B+15a^4Ab-9a^3b^2B+17a^2Ab^3-3ab^4B+6Ab^5)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{a^4x(a^3(-B)+3a^2Ab+3ab^2\ldots}{a^2+b^2}}{a(a^2+b^2)}\ldots}{a(a^2+b^2)}\ldots}{a}\ldots}{a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{\frac{(a^2+b^2)^2(a^2A+3abB-6Ab^2)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\frac{b^3(-10a^5B+15a^4Ab-9a^3b^2B+17a^2Ab^3-3ab^4B+6Ab^5)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{a^4x(a^3(-B)+3a^2A\ldots}{a^2+b\ldots}}{a(a^2+b^2)}\ldots}{a(a^2+b^2)}\ldots}{a}\ldots}{a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{\frac{-\frac{(a^2+b^2)^2(a^2A+3abB-6Ab^2)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}+\frac{b^3(-10a^5B+15a^4Ab-9a^3b^2B+17a^2Ab^3-3ab^4B+6Ab^5)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{a^4x(a^3(-B)+3a\ldots}{a^2\ldots}}{a(a^2+b^2)}\ldots}{a(a^2+b^2)}\ldots}{a}\ldots}{a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3956

$$-\frac{\frac{\frac{\frac{\frac{b^3(-10a^5B+15a^4Ab-9a^3b^2B+17a^2Ab^3-3ab^4B+6Ab^5)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{(a^2+b^2)^2(a^2A+3abB-6Ab^2)\log(-\sin(c+dx))}{ad}-\frac{a^4x(a^3(-B)+3a^2Ab+3a\ldots}{a^2+b^2}}{a(a^2+b^2)}\ldots}{a(a^2+b^2)}\ldots}{a}\ldots}{a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4013

$$-\frac{\frac{\frac{\frac{\left(a^2+b^2\right)^2\left(a^2A+3abB-6Ab^2\right)\log(-\sin(c+dx))}{ad}-\frac{a^4x\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{a^2+b^2}+\frac{b^3\left(-10a^5B+15a^4Ab-9a^3b^2B+17a^2Ab^3-3ab^4B+6Ab^5\right)\log(a\cos}{ad\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}}{a}}{a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^2}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*(A*Cot[c + d*x]^2)/(a*d*(a + b*Tan[c + d*x])^2) - (-(((2*A*b - a*B)*C
ot[c + d*x])/(a*d*(a + b*Tan[c + d*x])^2)) + (-1/2*(b*(5*a^2*A*b + 6*A*b^3
 - 2*a^3*B - 3*a*b^2*B))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((-((a
^4*(3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*x)/(a^2 + b^2)) + ((a^2 + b^2)^
2*(a^2*A - 6*A*b^2 + 3*a*b*B)*Log[-Sin[c + d*x]])/(a*d) + (b^3*(15*a^4*A*b
 + 17*a^2*A*b^3 + 6*A*b^5 - 10*a^5*B - 9*a^3*b^2*B - 3*a*b^4*B)*Log[a*Cos[
c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/(a*(a^2 + b^2)) - (b*(3*a^4
*A*b + 11*a^2*A*b^3 + 6*A*b^5 - a^5*B - 6*a^3*b^2*B - 3*a*b^4*B))/(a*(a^2
+ b^2)*d*(a + b*Tan[c + d*x])))/(a*(a^2 + b^2)))/a)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\left(A\,a^{3}-3Aa\,b^{2}+3B\,a^{2}b-B\,b^{3}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(3A\,a^{2}b-A\,b^{3}-B\,a^{3}+3Ba\,b^{2}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}-\frac{A}{2a^{3}\tan(dx+c)^{2}}-\frac{-3Ab}{a^{4}\tan(}$
default	$\frac{\frac{\left(A\,a^{3}-3Aa\,b^{2}+3B\,a^{2}b-B\,b^{3}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(3A\,a^{2}b-A\,b^{3}-B\,a^{3}+3Ba\,b^{2}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{3}}-\frac{A}{2a^{3}\tan(dx+c)^{2}}-\frac{-3Ab}{a^{4}\tan(}$
parallelrisch	$-30b^{3}\left(A\,a^{4}b+\frac{17}{15}A\,a^{2}b^{3}+\frac{2}{5}A\,b^{5}-\frac{2}{3}B\,a^{5}-\frac{3}{5}B\,a^{3}b^{2}-\frac{1}{5}Ba\,b^{4}\right)(a+b\tan(dx+c))^{2}\ln(a+b\tan(dx+c))+a^{5}(a+b\tan(dx+$
norman	$\frac{\frac{(2Ab-Ba)\tan(dx+c)}{a^{2}d}+\frac{b^{2}\left(3A\,a^{2}b-A\,b^{3}-B\,a^{3}+3Ba\,b^{2}\right)x\tan(dx+c)^{4}}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}+\frac{\left(3A\,a^{2}b-A\,b^{3}-B\,a^{3}+3Ba\,b^{2}\right)a^{2}x\tan(dx+c)^{2}}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)}-\frac{A}{2ad}-}{}$
risch	Expression too large to display

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*ln(1+tan(d*x+c)^
2)+(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*arctan(tan(d*x+c)))-1/2/a^3*A/tan(d*x
+c)^2-(-3*A*b+B*a)/a^4/tan(d*x+c)+(-A*a^2+6*A*b^2-3*B*a*b)/a^5*ln(tan(d*x+
c))+b^3*(5*A*a^2*b+3*A*b^3-4*B*a^3-2*B*a*b^2)/(a^2+b^2)^2/a^4/(a+b*tan(d*x
+c))-b^3*(15*A*a^4*b+17*A*a^2*b^3+6*A*b^5-10*B*a^5-9*B*a^3*b^2-3*B*a*b^4)/
(a^2+b^2)^3/a^5*ln(a+b*tan(d*x+c))+1/2*(A*b-B*a)*b^3/(a^2+b^2)/a^3/(a+b*ta
n(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(346) = 692$.

Time = 0.25 (sec) , antiderivative size = 1065, normalized size of antiderivative = 3.03

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/2*(A*a^10 + 3*A*a^8*b^2 + 3*A*a^6*b^4 + A*a^4*b^6 + (A*a^8*b^2 + 3*A*a^
6*b^4 - 9*B*a^5*b^5 + 14*A*a^4*b^6 - 3*B*a^3*b^7 + 6*A*a^2*b^8 + 2*(B*a^8*
b^2 - 3*A*a^7*b^3 - 3*B*a^6*b^4 + A*a^5*b^5)*d*x)*tan(d*x + c)^4 + 2*(A*a^
9*b + B*a^8*b^2 - 2*B*a^6*b^4 + 6*B*a^4*b^6 - 11*A*a^3*b^7 + 3*B*a^2*b^8 -
 6*A*a*b^9 + 2*(B*a^9*b - 3*A*a^8*b^2 - 3*B*a^7*b^3 + A*a^6*b^4)*d*x)*tan(
d*x + c)^3 + (A*a^10 + 4*B*a^9*b - 8*A*a^8*b^2 + 12*B*a^7*b^3 - 30*A*a^6*b
^4 + 23*B*a^5*b^5 - 45*A*a^4*b^6 + 9*B*a^3*b^7 - 18*A*a^2*b^8 + 2*(B*a^10
- 3*A*a^9*b - 3*B*a^8*b^2 + A*a^7*b^3)*d*x)*tan(d*x + c)^2 + ((A*a^8*b^2 +
 3*B*a^7*b^3 - 3*A*a^6*b^4 + 9*B*a^5*b^5 - 15*A*a^4*b^6 + 9*B*a^3*b^7 - 17
*A*a^2*b^8 + 3*B*a*b^9 - 6*A*b^10)*tan(d*x + c)^4 + 2*(A*a^9*b + 3*B*a^8*b
^2 - 3*A*a^7*b^3 + 9*B*a^6*b^4 - 15*A*a^5*b^5 + 9*B*a^4*b^6 - 17*A*a^3*b^7
 + 3*B*a^2*b^8 - 6*A*a*b^9)*tan(d*x + c)^3 + (A*a^10 + 3*B*a^9*b - 3*A*a^8
*b^2 + 9*B*a^7*b^3 - 15*A*a^6*b^4 + 9*B*a^5*b^5 - 17*A*a^4*b^6 + 3*B*a^3*b
^7 - 6*A*a^2*b^8)*tan(d*x + c)^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))
 - ((10*B*a^5*b^5 - 15*A*a^4*b^6 + 9*B*a^3*b^7 - 17*A*a^2*b^8 + 3*B*a*b^9
- 6*A*b^10)*tan(d*x + c)^4 + 2*(10*B*a^6*b^4 - 15*A*a^5*b^5 + 9*B*a^4*b^6
- 17*A*a^3*b^7 + 3*B*a^2*b^8 - 6*A*a*b^9)*tan(d*x + c)^3 + (10*B*a^7*b^3 -
 15*A*a^6*b^4 + 9*B*a^5*b^5 - 17*A*a^4*b^6 + 3*B*a^3*b^7 - 6*A*a^2*b^8)*ta
n(d*x + c)^2)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x
 + c)^2 + 1)) + 2*(B*a^10 - 2*A*a^9*b + 3*B*a^8*b^2 - 6*A*a^7*b^3 + 3*B...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.54

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{2\,(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{2\,(10\,Ba^5b^3-15\,Aa^4b^4+9\,Ba^3b^5-17\,Aa^2b^6+3\,Bab^7-6\,Ab^8)\log(b\tan(dx+c)+a)}{a^{11}+3\,a^9b^2+3\,a^7b^4+a^5b^6}-\frac{(Aa^3+3\,B}{}}{}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/2*(2*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*(d*x + c)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) - 2*(10*B*a^5*b^3 - 15*A*a^4*b^4 + 9*B*a^3*b^5 - 17*A*
a^2*b^6 + 3*B*a*b^7 - 6*A*b^8)*log(b*tan(d*x + c) + a)/(a^11 + 3*a^9*b^2 +
 3*a^7*b^4 + a^5*b^6) - (A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(tan(d*
x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (A*a^7 + 2*A*a^5*b^2 +
 A*a^3*b^4 + 2*(B*a^5*b^2 - 3*A*a^4*b^3 + 6*B*a^3*b^4 - 11*A*a^2*b^5 + 3*B
*a*b^6 - 6*A*b^7)*tan(d*x + c)^3 + (4*B*a^6*b - 11*A*a^5*b^2 + 17*B*a^4*b^
3 - 33*A*a^3*b^4 + 9*B*a^2*b^5 - 18*A*a*b^6)*tan(d*x + c)^2 + 2*(B*a^7 - 2
*A*a^6*b + 2*B*a^5*b^2 - 4*A*a^4*b^3 + B*a^3*b^4 - 2*A*a^2*b^5)*tan(d*x +
c))/((a^8*b^2 + 2*a^6*b^4 + a^4*b^6)*tan(d*x + c)^4 + 2*(a^9*b + 2*a^7*b^3
 + a^5*b^5)*tan(d*x + c)^3 + (a^10 + 2*a^8*b^2 + a^6*b^4)*tan(d*x + c)^2)
+ 2*(A*a^2 + 3*B*a*b - 6*A*b^2)*log(tan(d*x + c))/a^5)/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.60

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)(dx+c)}{a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d}$$
$$+\frac{(Aa^3+3\,Ba^2b-3\,Aab^2-Bb^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,(a^6d+3\,a^4b^2d+3\,a^2b^4d+b^6d)}$$
$$+\frac{(10\,Ba^5b^4-15\,Aa^4b^5+9\,Ba^3b^6-17\,Aa^2b^7+3\,Bab^8-6\,Ab^9)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^{11}bd+3\,a^9b^3d+3\,a^7b^5d+a^5b^7d}$$
$$-\frac{(Aa^2+3\,Bab-6\,Ab^2)\log\left(|\tan\left(dx+c\right)|\right)}{a^5d}$$
$$-\frac{Aa^9+3\,Aa^7b^2+3\,Aa^5b^4+Aa^3b^6+2\,(Ba^7b^2-3\,Aa^6b^3+7\,Ba^5b^4-14\,Aa^4b^5+9\,Ba^3b^6-17\,Aa^2b^7-}{}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*(d*x + c)/(a^6*d + 3*a^4*b^2*d +
3*a^2*b^4*d + b^6*d) + 1/2*(A*a^3 + 3*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(tan
(d*x + c)^2 + 1)/(a^6*d + 3*a^4*b^2*d + 3*a^2*b^4*d + b^6*d) + (10*B*a^5*b
^4 - 15*A*a^4*b^5 + 9*B*a^3*b^6 - 17*A*a^2*b^7 + 3*B*a*b^8 - 6*A*b^9)*log(
abs(b*tan(d*x + c) + a))/(a^11*b*d + 3*a^9*b^3*d + 3*a^7*b^5*d + a^5*b^7*d
) - (A*a^2 + 3*B*a*b - 6*A*b^2)*log(abs(tan(d*x + c)))/(a^5*d) - 1/2*(A*a^
9 + 3*A*a^7*b^2 + 3*A*a^5*b^4 + A*a^3*b^6 + 2*(B*a^7*b^2 - 3*A*a^6*b^3 + 7
*B*a^5*b^4 - 14*A*a^4*b^5 + 9*B*a^3*b^6 - 17*A*a^2*b^7 + 3*B*a*b^8 - 6*A*b
^9)*tan(d*x + c)^3 + (4*B*a^8*b - 11*A*a^7*b^2 + 21*B*a^6*b^3 - 44*A*a^5*b
^4 + 26*B*a^4*b^5 - 51*A*a^3*b^6 + 9*B*a^2*b^7 - 18*A*a*b^8)*tan(d*x + c)^
2 + 2*(B*a^9 - 2*A*a^8*b + 3*B*a^7*b^2 - 6*A*a^6*b^3 + 3*B*a^5*b^4 - 6*A*a
^4*b^5 + B*a^3*b^6 - 2*A*a^2*b^7)*tan(d*x + c))/((a^2 + b^2)^3*(b*tan(d*x
+ c) + a)^2*a^4*d*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.23

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{\tan(c+d\,x)\,(2\,A\,b-B\,a)}{a^2}-\frac{A}{2\,a}+\frac{\tan(c+d\,x)^3\,\left(-B\,a^5\,b^2+3\,A\,a^4\,b^3-6\,B\,a^3\,b^4+11\,A\,a^2\,b^5-3\,B\,a\,b^6+6\,A\,b^7\right)}{a^4\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)^2\,\left(-4\,B\,a^5\,b+11\,A\cdots\right.}{\cdots}}{d\,\left(a^2\tan\left(c+d\,x\right)^2+2\,a\,b\tan\left(c+d\,x\right)^3+b^2\tan\left(c+d\,x\right)^4\right)}$$
$$+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{A}{a^3}-\frac{A\,a+3\,B\,b}{{\left(a^2+b^2\right)}^2}-\frac{6\,A\,b^2}{a^5}+\frac{3\,B\,b}{a^4}+\frac{4\,b^2\,\left(A\,a+B\,b\right)}{{\left(a^2+b^2\right)}^3}\right)}{d}$$
$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-B+A\,1\mathrm{i}\right)}{2\,d\,\left(-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3\right)}-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(A\,a^2+3\,B\,a\,b-6\,A\,b^2\right)}{a^5\,d}$$
$$-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(A-B\,1\mathrm{i}\right)}{2\,d\,\left(-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}\right)}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
((tan(c + d*x)*(2*A*b - B*a))/a^2 - A/(2*a) + (tan(c + d*x)^3*(6*A*b^7 + 1
1*A*a^2*b^5 + 3*A*a^4*b^3 - 6*B*a^3*b^4 - B*a^5*b^2 - 3*B*a*b^6))/(a^4*(a^
4 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^2*(18*A*b^6 + 33*A*a^2*b^4 + 11*A*a^
4*b^2 - 17*B*a^3*b^3 - 9*B*a*b^5 - 4*B*a^5*b))/(2*a^3*(a^4 + b^4 + 2*a^2*b
^2)))/(d*(a^2*tan(c + d*x)^2 + b^2*tan(c + d*x)^4 + 2*a*b*tan(c + d*x)^3))
 + (log(a + b*tan(c + d*x))*(A/a^3 - (A*a + 3*B*b)/(a^2 + b^2)^2 - (6*A*b^
2)/a^5 + (3*B*b)/a^4 + (4*b^2*(A*a + B*b))/(a^2 + b^2)^3))/d - (log(tan(c
+ d*x) - 1i)*(A*1i - B))/(2*d*(a*b^2*3i + 3*a^2*b - a^3*1i - b^3)) - (log(
tan(c + d*x))*(A*a^2 - 6*A*b^2 + 3*B*a*b))/(a^5*d) - (log(tan(c + d*x) + 1
i)*(A - B*1i))/(2*d*(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i))
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx=\int \frac{\cot(dx+c)^3\,(A+B\tan(dx+c))}{(a+\tan(dx+c)\,b)^3}dx$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

3.290 $\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 351

$$\begin{aligned}
&\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx \\
&= \frac{(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)\,x}{(a^2+b^2)^4} \\
&\quad+\frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)\log(\cos(c+dx))}{(a^2+b^2)^4\,d} \\
&\quad+\frac{a(4a^2Ab^5-4Ab^7+a^7B+4a^5b^2B+5a^3b^4B+10ab^6B)\log(a+b\tan(c+dx))}{b^4\,(a^2+b^2)^4\,d} \\
&\quad+\frac{a(Ab-aB)\tan^3(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}+\frac{a(2Ab^3-a(a^2+3b^2)\,B)\tan^2(c+dx)}{2b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} \\
&\quad+\frac{a^2(a^2Ab^3-3Ab^5+a^5B+3a^3b^2B+6ab^4B)}{b^4\,(a^2+b^2)^3\,d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*x/(a^2+b^2)^4+(4*A*a^3*b-4*A
*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*ln(cos(d*x+c))/(a^2+b^2)^4/d+a*(4*A*a^2*b^
5-4*A*b^7+B*a^7+4*B*a^5*b^2+5*B*a^3*b^4+10*B*a*b^6)*ln(a+b*tan(d*x+c))/b^4
/(a^2+b^2)^4/d+1/3*a*(A*b-B*a)*tan(d*x+c)^3/b/(a^2+b^2)/d/(a+b*tan(d*x+c))
^3+1/2*a*(2*A*b^3-a*(a^2+3*b^2)*B)*tan(d*x+c)^2/b^2/(a^2+b^2)^2/d/(a+b*tan
(d*x+c))^2+a^2*(A*a^2*b^3-3*A*b^5+B*a^5+3*B*a^3*b^2+6*B*a*b^4)/b^4/(a^2+b^
2)^3/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.34 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.87

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{3(-iA+B)\log(i-\tan(c+dx))}{(a+ib)^4}+\frac{3(iA+B)\log(i+\tan(c+dx))}{(a-ib)^4}+\frac{6a(4a^2Ab^5-4Ab^7+a^7B+4a^5b^2B+5a^3b^4B+10ab^6B)\log(a+b\tan(c+dx))}{b^4(a^2+b^2)^4}}{6d}$$

input
```
Integrate[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
((3*((-I)*A + B)*Log[I - Tan[c + d*x]])/(a + I*b)^4 + (3*(I*A + B)*Log[I +
 Tan[c + d*x]])/(a - I*b)^4 + (6*a*(4*a^2*A*b^5 - 4*A*b^7 + a^7*B + 4*a^5*
b^2*B + 5*a^3*b^4*B + 10*a*b^6*B)*Log[a + b*Tan[c + d*x]])/(b^4*(a^2 + b^2
)^4) + (2*a^4*(-(A*b) + a*B))/(b^4*(a^2 + b^2)*(a + b*Tan[c + d*x])^3) + (
3*a^3*(2*a^2*A*b + 4*A*b^3 - 3*a^3*B - 5*a*b^2*B))/(b^4*(a^2 + b^2)^2*(a +
 b*Tan[c + d*x])^2) + (6*a^2*(-(a^4*A*b) - 3*a^2*A*b^3 - 6*A*b^5 + 3*a^5*B
 + 9*a^3*b^2*B + 10*a*b^4*B))/(b^4*(a^2 + b^2)^3*(a + b*Tan[c + d*x])))/(6
*d)
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4118, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^4(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{\int -\frac{3\tan^2(c+dx)\left(-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)\right)}{(a+b\tan(c+dx))^3}dx}{3b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\int \frac{\tan^2(c+dx)\left(-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)\right)}{(a+b\tan(c+dx))^3}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\int \frac{\tan(c+dx)^2\left(-\left(\left(a^2+b^2\right)B\tan(c+dx)^2\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)\right)}{(a+b\tan(c+dx))^3}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4128}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\frac{\int \frac{2\tan(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx)+a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)\right)}{(a+b\tan(c+dx))^2}dx}{2b(a^2+b^2)}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\tan^2(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\frac{\int \frac{\tan(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx)+a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)\right)}{(a+b\tan(c+dx))^2}dx}{b(a^2+b^2)}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\tan^2(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\frac{\int \frac{\tan(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan(c+dx)^2+a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)\right)}{(a+b\tan(c+dx))^2}dx}{b(a^2+b^2)}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\tan^2(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4118}$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\frac{\frac{\int -\frac{-\left(\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)b^3\right)+\left(a^2+b^2\right)^3B\tan^2(c+dx)+a\left(Ba^5+3b^2Ba^3+Ab^3a^2+6b^4Ba-3Ab^5\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}-\frac{a^2\left(a^5B+3a^3b^2B+a^2Ab^3+6ab^4B\right.}{b^2d\left(a^2+b^2\right)(a+b\tan(c+dx}}{b(a^2+b^2)}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 25

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\frac{-\frac{\int \frac{-\left(\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)b^3\right)+\left(a^2+b^2\right)^3B\tan^2(c+dx)+a\left(Ba^5+3b^2Ba^3+Ab^3a^2+6b^4Ba-3Ab^5\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}-\frac{a^2\left(a^5B+3a^3b^2B+a^2Ab^3+6ab^4\right.}{b^2d\left(a^2+b^2\right)(a+b\tan(c+d}}{b(a^2+b^2)}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\frac{-\frac{\int \frac{-\left(\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)b^3\right)+\left(a^2+b^2\right)^3B\tan(c+dx)^2+a\left(Ba^5+3b^2Ba^3+Ab^3a^2+6b^4Ba-3Ab^5\right)}{a+b\tan(c+dx)}dx}{b\left(a^2+b^2\right)}-\frac{a^2\left(a^5B+3a^3b^2B+a^2Ab^3+6ab^4\right.}{b^2d\left(a^2+b^2\right)(a+b\tan(c+d}}{b(a^2+b^2)}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 4109

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\frac{-\frac{-\frac{b^3\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a\left(a^7B+4a^5b^2B+5a^3b^4B+4a^2Ab^5+10ab^6B-4Ab^7\right)\int\frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^3x\left(a^4A+4a^3bB-\right.}{}}{b\left(a^2+b^2\right)}}{b(a^2+b^2)}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\frac{-\frac{-\frac{b^3\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\int\tan(c+dx)dx}{a^2+b^2}+\frac{a\left(a^7B+4a^5b^2B+5a^3b^4B+4a^2Ab^5+10ab^6B-4Ab^7\right)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^3x\left(a^4A+4a^3bB-\right.}{}}{b\left(a^2+b^2\right)}}{b(a^2+b^2)}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 3956

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-$$

$$-\frac{\frac{a\left(a^7B+4a^5b^2B+5a^3b^4B+4a^2Ab^5+10ab^6B-4Ab^7\right)\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^3\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^3x\left(a^4A+4a^3bB-6\right.}{a^2}}{b\left(a^2+b^2\right)}$$

$$\overline{\quad b(a^2+b^2)\quad}$$

$$\overline{\quad b\left(a^2+b^2\right)\quad}$$

$$\Big\downarrow \; 4100$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-$$

$$-\frac{\frac{a\left(a^7B+4a^5b^2B+5a^3b^4B+4a^2Ab^5+10ab^6B-4Ab^7\right)\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd\left(a^2+b^2\right)}+\frac{b^3\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^3x\left(a^4\right.}{}}{b\left(a^2+b^2\right)}$$

$$\overline{\quad b(a^2+b^2)\quad}$$

$$\overline{\quad b\left(a^2+b^2\right)\quad}$$

$$\Big\downarrow \; 16$$

$$\frac{a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-$$

$$-\frac{a^2\left(a^5B+3a^3b^2B+a^2Ab^3+6ab^4B-3Ab^5\right)}{b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{b^3\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\log(\cos(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^3x\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)}{a^2+b^2}+\frac{a\left(a^7\right.}{}}{b\left(a^2+b^2\right)}$$

$$\overline{\quad b(a^2+b^2)\quad}$$

$$\overline{\quad b\left(a^2+b^2\right)\quad}$$

input
```
Int[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
(a*(A*b - a*B)*Tan[c + d*x]^3)/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3)
 - (-1/2*(a*(2*A*b^3 - a*(a^2 + 3*b^2)*B)*Tan[c + d*x]^2)/(b*(a^2 + b^2)*d*
(a + b*Tan[c + d*x])^2) + (-(((b^3*(a^4*A - 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*
B - 4*a*b^3*B)*x)/(a^2 + b^2) + (b^3*(4*a^3*A*b - 4*a*A*b^3 - a^4*B + 6*a^
2*b^2*B - b^4*B)*Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a*(4*a^2*A*b^5 - 4*
A*b^7 + a^7*B + 4*a^5*b^2*B + 5*a^3*b^4*B + 10*a*b^6*B)*Log[a + b*Tan[c +
d*x]])/(b*(a^2 + b^2)*d))/(b*(a^2 + b^2))) - (a^2*(a^2*A*b^3 - 3*A*b^5 + a
^5*B + 3*a^3*b^2*B + 6*a*b^4*B))/(b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))
/(b*(a^2 + b^2)))/(b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{\left(-4A\,a^3b+4Aa\,b^3+B\,a^4-6B\,a^2b^2+B\,b^4\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4Ba\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{a\left(4\cdots\right.}{\cdots}$
default	$\frac{\frac{\left(-4A\,a^3b+4Aa\,b^3+B\,a^4-6B\,a^2b^2+B\,b^4\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4Ba\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{a\left(4\cdots\right.}{\cdots}$
norman	$\frac{\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4Ba\,b^3\right)a^3x}{\left(a^6+3b^2a^4+3b^4a^2+b^6\right)\left(a^2+b^2\right)}+\frac{b^3\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4Ba\,b^3\right)x\tan(dx+c)^3}{\left(a^6+3b^2a^4+3b^4a^2+b^6\right)\left(a^2+b^2\right)}-\frac{a^3\left(2A\,a^5b+4A\,a^3b^3+26\cdots\right.}{6d\,b^4\left(a\cdots\right.}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^4*(1/2*(-4*A*a^3*b+4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*ln(
1+tan(d*x+c)^2)+(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*arctan(tan(d
*x+c)))+a*(4*A*a^2*b^5-4*A*b^7+B*a^7+4*B*a^5*b^2+5*B*a^3*b^4+10*B*a*b^6)/(
a^2+b^2)^4/b^4*ln(a+b*tan(d*x+c))-a^2*(A*a^4*b+3*A*a^2*b^3+6*A*b^5-3*B*a^5
-9*B*a^3*b^2-10*B*a*b^4)/b^4/(a^2+b^2)^3/(a+b*tan(d*x+c))-1/3*a^4*(A*b-B*a
)/b^4/(a^2+b^2)/(a+b*tan(d*x+c))^3+1/2*a^3*(2*A*a^2*b+4*A*b^3-3*B*a^3-5*B*
a*b^2)/b^4/(a^2+b^2)^2/(a+b*tan(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1113 vs. $2(346) = 692$.

Time = 0.20 (sec) , antiderivative size = 1113, normalized size of antiderivative = 3.17

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
1/6*(3*B*a^9*b^2 + 6*B*a^7*b^4 + 18*A*a^6*b^5 + 47*B*a^5*b^6 - 26*A*a^4*b^
7 - (11*B*a^8*b^3 - 2*A*a^7*b^4 + 42*B*a^6*b^5 - 6*A*a^5*b^6 + 75*B*a^4*b^
7 - 48*A*a^3*b^8 - 6*(A*a^4*b^7 + 4*B*a^3*b^8 - 6*A*a^2*b^9 - 4*B*a*b^10 +
 A*b^11)*d*x)*tan(d*x + c)^3 + 6*(A*a^7*b^4 + 4*B*a^6*b^5 - 6*A*a^5*b^6 -
4*B*a^4*b^7 + A*a^3*b^8)*d*x - 3*(5*B*a^9*b^2 + 18*B*a^7*b^4 + 2*A*a^6*b^5
 + 37*B*a^5*b^6 - 30*A*a^4*b^7 - 20*B*a^3*b^8 + 12*A*a^2*b^9 - 6*(A*a^5*b^
6 + 4*B*a^4*b^7 - 6*A*a^3*b^8 - 4*B*a^2*b^9 + A*a*b^10)*d*x)*tan(d*x + c)^
2 + 3*(B*a^11 + 4*B*a^9*b^2 + 5*B*a^7*b^4 + 4*A*a^6*b^5 + 10*B*a^5*b^6 - 4
*A*a^4*b^7 + (B*a^8*b^3 + 4*B*a^6*b^5 + 5*B*a^4*b^7 + 4*A*a^3*b^8 + 10*B*a
^2*b^9 - 4*A*a*b^10)*tan(d*x + c)^3 + 3*(B*a^9*b^2 + 4*B*a^7*b^4 + 5*B*a^5
*b^6 + 4*A*a^4*b^7 + 10*B*a^3*b^8 - 4*A*a^2*b^9)*tan(d*x + c)^2 + 3*(B*a^1
0*b + 4*B*a^8*b^3 + 5*B*a^6*b^5 + 4*A*a^5*b^6 + 10*B*a^4*b^7 - 4*A*a^3*b^8
)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d
*x + c)^2 + 1)) - 3*(B*a^11 + 4*B*a^9*b^2 + 6*B*a^7*b^4 + 4*B*a^5*b^6 + B*
a^3*b^8 + (B*a^8*b^3 + 4*B*a^6*b^5 + 6*B*a^4*b^7 + 4*B*a^2*b^9 + B*b^11)*t
an(d*x + c)^3 + 3*(B*a^9*b^2 + 4*B*a^7*b^4 + 6*B*a^5*b^6 + 4*B*a^3*b^8 + B
*a*b^10)*tan(d*x + c)^2 + 3*(B*a^10*b + 4*B*a^8*b^3 + 6*B*a^6*b^5 + 4*B*a^
4*b^7 + B*a^2*b^9)*tan(d*x + c))*log(1/(tan(d*x + c)^2 + 1)) - 3*(2*B*a^10
*b + 5*B*a^8*b^3 + 2*A*a^7*b^4 + 12*B*a^6*b^5 - 22*A*a^5*b^6 - 35*B*a^4*b^
7 + 20*A*a^3*b^8 - 6*(A*a^6*b^5 + 4*B*a^5*b^6 - 6*A*a^4*b^7 - 4*B*a^3*b...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)**4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.66

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{6\,(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{6\,(Ba^8+4\,Ba^6b^2+5\,Ba^4b^4+4\,Aa^3b^5+10\,Ba^2b^6-4\,Aab^7)\log(b\tan(dx+c)+a)}{a^8b^4+4\,a^6b^6+6\,a^4b^8+4\,a^2b^{10}+b^{12}} + \frac{3\,(Ba^4-}{}}{}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="m
axima")
```

output
```
1/6*(6*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 6*(B*a^8 + 4*B*a^6*b^2 + 5*
B*a^4*b^4 + 4*A*a^3*b^5 + 10*B*a^2*b^6 - 4*A*a*b^7)*log(b*tan(d*x + c) + a
)/(a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12) + 3*(B*a^4 - 4*A*a
^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (11*B*a^9 - 2*A*a^8*b + 34*B*a^7*b
^2 - 4*A*a^6*b^3 + 47*B*a^5*b^4 - 26*A*a^4*b^5 + 6*(3*B*a^7*b^2 - A*a^6*b^
3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 + 10*B*a^3*b^6 - 6*A*a^2*b^7)*tan(d*x + c)^2
 + 3*(9*B*a^8*b - 2*A*a^7*b^2 + 28*B*a^6*b^3 - 6*A*a^5*b^4 + 35*B*a^4*b^5
- 20*A*a^3*b^6)*tan(d*x + c))/(a^9*b^4 + 3*a^7*b^6 + 3*a^5*b^8 + a^3*b^10
+ (a^6*b^7 + 3*a^4*b^9 + 3*a^2*b^11 + b^13)*tan(d*x + c)^3 + 3*(a^7*b^6 +
3*a^5*b^8 + 3*a^3*b^10 + a*b^12)*tan(d*x + c)^2 + 3*(a^8*b^5 + 3*a^6*b^7 +
 3*a^4*b^9 + a^2*b^11)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.52

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)(dx+c)}{a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d}$$
$$+\frac{(Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\left(a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d\right)}$$
$$+\frac{(Ba^8+4\,Ba^6b^2+5\,Ba^4b^4+4\,Aa^3b^5+10\,Ba^2b^6-4\,Aab^7)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^8b^4d+4\,a^6b^6d+6\,a^4b^8d+4\,a^2b^{10}d+b^{12}d}$$
$$+\frac{6\left(3\,Ba^9b-Aa^8b^2+12\,Ba^7b^3-4\,Aa^6b^4+19\,Ba^5b^5-9\,Aa^4b^6+10\,Ba^3b^7-6\,Aa^2b^8\right)\tan\left(dx+c\right)^2+}{}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="g
iac")
```

output
```
(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^8*d + 4
*a^6*b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + 1/2*(B*a^4 - 4*A*a^3*b -
 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1)/(a^8*d + 4*a^6*b
^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + (B*a^8 + 4*B*a^6*b^2 + 5*B*a^4
*b^4 + 4*A*a^3*b^5 + 10*B*a^2*b^6 - 4*A*a*b^7)*log(abs(b*tan(d*x + c) + a)
)/(a^8*b^4*d + 4*a^6*b^6*d + 6*a^4*b^8*d + 4*a^2*b^10*d + b^12*d) + 1/6*(6
*(3*B*a^9*b - A*a^8*b^2 + 12*B*a^7*b^3 - 4*A*a^6*b^4 + 19*B*a^5*b^5 - 9*A*
a^4*b^6 + 10*B*a^3*b^7 - 6*A*a^2*b^8)*tan(d*x + c)^2 + 3*(9*B*a^10 - 2*A*a
^9*b + 37*B*a^8*b^2 - 8*A*a^7*b^3 + 63*B*a^6*b^4 - 26*A*a^5*b^5 + 35*B*a^4
*b^6 - 20*A*a^3*b^7)*tan(d*x + c) + (11*B*a^11 - 2*A*a^10*b + 45*B*a^9*b^2
 - 6*A*a^8*b^3 + 81*B*a^7*b^4 - 30*A*a^6*b^5 + 47*B*a^5*b^6 - 26*A*a^4*b^7
)/b)/((a^2 + b^2)^4*(b*tan(d*x + c) + a)^3*b^3*d)
```

Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.38

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$

$$= \frac{\frac{11\,B\,a^9-2\,A\,a^8\,b+34\,B\,a^7\,b^2-4\,A\,a^6\,b^3+47\,B\,a^5\,b^4-26\,A\,a^4\,b^5}{6\,b^4\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^2\,\left(3\,B\,a^7-A\,a^6\,b+9\,B\,a^5\,b^2-3\,A\,a^4\,b^3+10\,B\,a^3\,b^4-6\,A\,a^2\,b^5\right)}{b^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+}{d\,\left(a^3+3\,a^2\,b\,\tan\left(c+d\,x\right)+3\,a\,b^2\,{\tan\left(c+d\,x\right)}^2+b^3\,\tan\left(\right.\right.}$$

$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)}{2\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$

$$+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{2\,d\,\left(a^4-a^3\,b\,4\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4\mathrm{i}+b^4\right)}$$

$$+\frac{a\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(B\,a^7+4\,B\,a^5\,b^2+5\,B\,a^3\,b^4+4\,A\,a^2\,b^5+10\,B\,a\,b^6-4\,A\,b^7\right)}{b^4\,d\,{\left(a^2+b^2\right)}^4}$$

input
```
int((tan(c + d*x)^4*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^4,x)
```

output
```
((11*B*a^9 - 26*A*a^4*b^5 - 4*A*a^6*b^3 + 47*B*a^5*b^4 + 34*B*a^7*b^2 - 2*
A*a^8*b)/(6*b^4*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^2*(3*
B*a^7 - 6*A*a^2*b^5 - 3*A*a^4*b^3 + 10*B*a^3*b^4 + 9*B*a^5*b^2 - A*a^6*b))
/(b^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)*(9*B*a^8 - 20*A
*a^3*b^5 - 6*A*a^5*b^3 + 35*B*a^4*b^4 + 28*B*a^6*b^2 - 2*A*a^7*b))/(2*b^3*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3 + b^3*tan(c + d*x)^3 + 3*a*b
^2*tan(c + d*x)^2 + 3*a^2*b*tan(c + d*x))) + (log(tan(c + d*x) - 1i)*(A +
B*1i))/(2*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)) + (log(tan
(c + d*x) + 1i)*(A*1i + B))/(2*d*(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*
b^2)) + (a*log(a + b*tan(c + d*x))*(B*a^7 - 4*A*b^7 + 4*A*a^2*b^5 + 5*B*a^
3*b^4 + 4*B*a^5*b^2 + 10*B*a*b^6))/(b^4*d*(a^2 + b^2)^4)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.84

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

output

```
( - 3*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2*a**2*b**6 + log(tan(c + d*x
)**2 + 1)*tan(c + d*x)**2*b**8 - 6*log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a
**3*b**5 + 2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a*b**7 - 3*log(tan(c +
d*x)**2 + 1)*a**4*b**4 + log(tan(c + d*x)**2 + 1)*a**2*b**6 + 2*log(tan(c
+ d*x)*b + a)*tan(c + d*x)**2*a**6*b**2 + 6*log(tan(c + d*x)*b + a)*tan(c
+ d*x)**2*a**4*b**4 + 12*log(tan(c + d*x)*b + a)*tan(c + d*x)**2*a**2*b**6
 + 4*log(tan(c + d*x)*b + a)*tan(c + d*x)*a**7*b + 12*log(tan(c + d*x)*b +
 a)*tan(c + d*x)*a**5*b**3 + 24*log(tan(c + d*x)*b + a)*tan(c + d*x)*a**3*
b**5 + 2*log(tan(c + d*x)*b + a)*a**8 + 6*log(tan(c + d*x)*b + a)*a**6*b**
2 + 12*log(tan(c + d*x)*b + a)*a**4*b**4 - 2*tan(c + d*x)**2*a**6*b**2 - 6
*tan(c + d*x)**2*a**4*b**4 + 2*tan(c + d*x)**2*a**3*b**5*d*x - 4*tan(c + d
*x)**2*a**2*b**6 - 6*tan(c + d*x)**2*a*b**7*d*x + 4*tan(c + d*x)*a**4*b**4
*d*x - 12*tan(c + d*x)*a**2*b**6*d*x + a**8 + 4*a**6*b**2 + 2*a**5*b**3*d*
x + 3*a**4*b**4 - 6*a**3*b**5*d*x)/(2*b**3*d*(tan(c + d*x)**2*a**6*b**2 +
3*tan(c + d*x)**2*a**4*b**4 + 3*tan(c + d*x)**2*a**2*b**6 + tan(c + d*x)**
2*b**8 + 2*tan(c + d*x)*a**7*b + 6*tan(c + d*x)*a**5*b**3 + 6*tan(c + d*x)
*a**3*b**5 + 2*tan(c + d*x)*a*b**7 + a**8 + 3*a**6*b**2 + 3*a**4*b**4 + a*
*2*b**6))
```

3.291 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 298

$$\begin{aligned}\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx &= -\frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)\,x}{(a^2+b^2)^4} \\ &\quad+\frac{(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} \\ &\quad+\frac{a(Ab-aB)\tan^2(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}+\frac{a^2(a^2Ab-5Ab^3+2a^3B+8ab^2B)}{6b^3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} \\ &\quad-\frac{a(a^4Ab+5a^2Ab^3-8Ab^5+2a^5B+7a^3b^2B+17ab^4B)}{3b^3\,(a^2+b^2)^3\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*x/(a^2+b^2)^4+(A*a^4-6*A*a^
2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4
/d+1/3*a*(A*b-B*a)*tan(d*x+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3+1/6*a^2*(
A*a^2*b-5*A*b^3+2*B*a^3+8*B*a*b^2)/b^3/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2-1/
3*a*(A*a^4*b+5*A*a^2*b^3-8*A*b^5+2*B*a^5+7*B*a^3*b^2+17*B*a*b^4)/b^3/(a^2+
b^2)^3/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.22 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.56

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = -\frac{B\tan^2(c+dx)}{bd(a+b\tan(c+dx))^3}$$

$$-\frac{-\frac{(-Ab-2aB)\tan(c+dx)}{2bd(a+b\tan(c+dx))^3}-\frac{-\frac{aAb+2a^2B-2b^2B}{3bd(a+b\tan(c+dx))^3}+\frac{\left(6aAb^3+6b^4B\right)\left(-\frac{i\log(i-\tan(c+dx))}{2(a+ib)^4}+\frac{i\log(i+\tan(c+dx))}{2(a-ib)^4}+\frac{4a(a-b)b(a+b)\log(a+b\tan(c+d}{\left(a^2+b^2\right)^4}\right.}{}}{}}{}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
-((B*Tan[c + d*x]^2)/(b*d*(a + b*Tan[c + d*x])^3)) - (-1/2*((-(A*b) - 2*a*
B)*Tan[c + d*x])/(b*d*(a + b*Tan[c + d*x])^3) - (-1/3*(a*A*b + 2*a^2*B - 2
*b^2*B)/(b*d*(a + b*Tan[c + d*x])^3) + (((6*a*A*b^3 + 6*b^4*B)*(((-1/2*I)*
Log[I - Tan[c + d*x]])/(a + I*b)^4 + ((I/2)*Log[I + Tan[c + d*x]])/(a - I*
b)^4 + (4*a*(a - b)*b*(a + b)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^4 - b/(
3*(a^2 + b^2)*(a + b*Tan[c + d*x])^3) - (a*b)/((a^2 + b^2)^2*(a + b*Tan[c
+ d*x])^2) - (b*(3*a^2 - b^2))/((a^2 + b^2)^3*(a + b*Tan[c + d*x]))))/b -
6*A*b^2*(-1/2*Log[I - Tan[c + d*x]]/(I*a - b)^3 + Log[I + Tan[c + d*x]]/(2
*(I*a + b)^3) + (b*(3*a^2 - b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 -
b/(2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) - (2*a*b)/((a^2 + b^2)^2*(a + b*T
an[c + d*x]))))/(3*b*d))/(2*b))/b
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4088, 25, 3042, 4118, 25, 3042, 4111, 27, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \; 4088$$

$$\frac{\int -\frac{\tan(c+dx)\left(-\left(\left(2Ba^2+Aba+3b^2B\right)\tan^2(c+dx)\right)-3b(Ab-aB)\tan(c+dx)+2a(Ab-aB)\right)}{(a+b\tan(c+dx))^3}dx}{3b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\int \frac{\tan(c+dx)\left(-\left(\left(2Ba^2+Aba+3b^2B\right)\tan^2(c+dx)\right)-3b(Ab-aB)\tan(c+dx)+2a(Ab-aB)\right)}{(a+b\tan(c+dx))^3}dx}{3b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\int \frac{\tan(c+dx)\left(-\left(\left(2Ba^2+Aba+3b^2B\right)\tan(c+dx)^2\right)-3b(Ab-aB)\tan(c+dx)+2a(Ab-aB)\right)}{(a+b\tan(c+dx))^3}dx}{3b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 4118$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{\frac{\int -\frac{-3\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(a^2+b^2\right)\left(2Ba^2+Aba+3b^2B\right)\tan^2(c+dx)+a\left(2Ba^3+Aba^2+8b^2Ba-5Ab^3\right)}{(a+b\tan(c+dx))^2}dx}{b(a^2+b^2)}-\frac{a^2\left(2a^3B+a^2Ab+8ab^2B-5Ab^3\right)}{2b^2d(a^2+b^2)(a+b\tan(c+dx))^2}}{3b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{-\frac{\int \frac{-3\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(a^2+b^2\right)\left(2Ba^2+Aba+3b^2B\right)\tan^2(c+dx)+a\left(2Ba^3+Aba^2+8b^2Ba-5Ab^3\right)}{(a+b\tan(c+dx))^2}dx}{b(a^2+b^2)}-\frac{a^2\left(2a^3B+a^2Ab+8ab^2B-5Ab^3\right)}{2b^2d(a^2+b^2)(a+b\tan(c+dx))^2}}{3b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{-\frac{\int\frac{-3\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(a^2+b^2\right)\left(2Ba^2+Aba+3b^2B\right)\tan(c+dx)^2+a\left(2Ba^3+Aba^2+8b^2Ba-5Ab^3\right)}{(a+b\tan(c+dx))^2}dx}{b(a^2+b^2)}-\frac{a^2\left(2a^3B+a^2Ab+8ab^2B-5Ab^3\right.}{2b^2d(a^2+b^2)(a+b\tan(c+dx))^2}}{3b\left(a^2+b^2\right)}$$

$\big\downarrow$ 4111

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{-\frac{\frac{\int-\frac{3\left(\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)b^2+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)b^2\right)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a\left(2a^5B+a^4Ab+7a^3b^2B+5a^2Ab^3+17ab^4B-8Ab^5\right)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}}{b(a^2+b^2)}-\frac{a^2\left(2a^3B\right.}{2b^2d(a^2}}{3b\left(a^2+b^2\right)}$$

$\big\downarrow$ 27

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{-\frac{-\frac{3\int\frac{\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)b^2+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)b^2}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a\left(2a^5B+a^4Ab+7a^3b^2B+5a^2Ab^3+17ab^4B-8Ab^5\right)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}}{b(a^2+b^2)}-\frac{a^2\left(2a^3B+\right.}{2b^2d(a^2+b}}{3b\left(a^2+b^2\right)}$$

$\big\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{-\frac{-\frac{3\int\frac{\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)b^2+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)b^2}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a\left(2a^5B+a^4Ab+7a^3b^2B+5a^2Ab^3+17ab^4B-8Ab^5\right)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}}{b(a^2+b^2)}-\frac{a^2\left(2a^3B+\right.}{2b^2d(a^2+b}}{3b\left(a^2+b^2\right)}$$

$\big\downarrow$ 4014

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{-\frac{-\frac{3\left(\frac{b^2x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)}{a^2+b^2}-\frac{b^2\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\left(2a^5B+a^4Ab+7a^3b^2B+5a^2Ab^3+17\right.}{bd\left(a^2+b^2\right)(a+b\tan(c+dx}}{b(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\big\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{-\frac{-\frac{3\left(\frac{b^2x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)}{a^2+b^2}-\frac{b^2\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\left(2a^5B+a^4Ab+7a^3b^2B+5a^2Ab^3+17\right.}{bd\left(a^2+b^2\right)(a+b\tan(c+dx}}{b(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4013}$$

$$\frac{a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-\frac{-\frac{a^2\left(2a^3B+a^2Ab+8ab^2B-5Ab^3\right)}{2b^2d(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{3\left(\frac{b^2x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)}{a^2+b^2}-\frac{b^2\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\log(a\cos(c+dx)+b\sin}{d\left(a^2+b^2\right)}\right.}{a^2+b^2}}{b(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

input

```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output

```
(a*(A*b - a*B)*Tan[c + d*x]^2)/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3)
- (-1/2*(a^2*(a^2*A*b - 5*A*b^3 + 2*a^3*B + 8*a*b^2*B))/(b^2*(a^2 + b^2)*d
*(a + b*Tan[c + d*x])^2) - ((-3*((b^2*(4*a^3*A*b - 4*a*A*b^3 - a^4*B + 6*a
^2*b^2*B - b^4*B)*x)/(a^2 + b^2) - (b^2*(a^4*A - 6*a^2*A*b^2 + A*b^4 + 4*a
^3*b*B - 4*a*b^3*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))
)/(a^2 + b^2) - (a*(a^4*A*b + 5*a^2*A*b^3 - 8*A*b^5 + 2*a^5*B + 7*a^3*b^2*
B + 17*a*b^4*B))/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(b*(a^2 + b^2)))/
(3*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\frac{\left(-A\,a^4+6A\,a^2b^2-A\,b^4-4B\,a^3b+4Ba\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-4A\,a^3b+4Aa\,b^3+B\,a^4-6B\,a^2b^2+B\,b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}+$
default	$\frac{\frac{\left(-A\,a^4+6A\,a^2b^2-A\,b^4-4B\,a^3b+4Ba\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-4A\,a^3b+4Aa\,b^3+B\,a^4-6B\,a^2b^2+B\,b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}+$
norman	$\frac{\left(A\,a^6b+8A\,a^4b^3-9A\,a^2b^5+2a^7B+6B\,a^5b^2+20B\,a^3b^4\right)\tan(dx+c)^3}{6a^2d\left(a^6+3b^2a^4+3b^4a^2+b^6\right)}+\frac{\left(A\,a^6+6A\,a^4b^2-3A\,a^2b^4+8B\,a^3b^3\right)\tan(dx+c)^2}{2ad\left(a^6+3b^2a^4+3b^4a^2+b^6\right)}-$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^4*(1/2*(-A*a^4+6*A*a^2*b^2-A*b^4-4*B*a^3*b+4*B*a*b^3)*ln(
1+tan(d*x+c)^2)+(-4*A*a^3*b+4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*arctan(tan(
d*x+c)))+(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)/(a^2+b^2)^4*ln(a+b*
tan(d*x+c))-a*(A*a^2*b^3-3*A*b^5+B*a^5+3*B*a^3*b^2+6*B*a*b^4)/(a^2+b^2)^3/
b^3/(a+b*tan(d*x+c))-1/2*a^2*(A*a^2*b+3*A*b^3-2*B*a^3-4*B*a*b^2)/b^3/(a^2+
b^2)^2/(a+b*tan(d*x+c))^2+1/3*a^3*(A*b-B*a)/b^3/(a^2+b^2)/(a+b*tan(d*x+c))
^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(289) = 578$.

Time = 0.11 (sec) , antiderivative size = 813, normalized size of antiderivative = 2.73

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
1/6*(3*A*a^7 + 18*B*a^6*b - 30*A*a^5*b^2 - 26*B*a^4*b^3 + 11*A*a^3*b^4 + (
2*B*a^7 + A*a^6*b + 6*B*a^5*b^2 + 18*A*a^4*b^3 + 48*B*a^3*b^4 - 27*A*a^2*b
^5 + 6*(B*a^4*b^3 - 4*A*a^3*b^4 - 6*B*a^2*b^5 + 4*A*a*b^6 + B*b^7)*d*x)*ta
n(d*x + c)^3 + 6*(B*a^7 - 4*A*a^6*b - 6*B*a^5*b^2 + 4*A*a^4*b^3 + B*a^3*b^
4)*d*x + 3*(A*a^7 - 2*B*a^6*b + 16*A*a^5*b^2 + 30*B*a^4*b^3 - 23*A*a^3*b^4
 - 12*B*a^2*b^5 + 6*A*a*b^6 + 6*(B*a^5*b^2 - 4*A*a^4*b^3 - 6*B*a^3*b^4 + 4
*A*a^2*b^5 + B*a*b^6)*d*x)*tan(d*x + c)^2 + 3*(A*a^7 + 4*B*a^6*b - 6*A*a^5
*b^2 - 4*B*a^4*b^3 + A*a^3*b^4 + (A*a^4*b^3 + 4*B*a^3*b^4 - 6*A*a^2*b^5 -
4*B*a*b^6 + A*b^7)*tan(d*x + c)^3 + 3*(A*a^5*b^2 + 4*B*a^4*b^3 - 6*A*a^3*b
^4 - 4*B*a^2*b^5 + A*a*b^6)*tan(d*x + c)^2 + 3*(A*a^6*b + 4*B*a^5*b^2 - 6*
A*a^4*b^3 - 4*B*a^3*b^4 + A*a^2*b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2
 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 3*(2*B*a^7 - 9*A*a^6*
b - 22*B*a^5*b^2 + 26*A*a^4*b^3 + 20*B*a^3*b^4 - 9*A*a^2*b^5 - 6*(B*a^6*b
- 4*A*a^5*b^2 - 6*B*a^4*b^3 + 4*A*a^3*b^4 + B*a^2*b^5)*d*x)*tan(d*x + c))/
((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*tan(d*x + c)^3 + 3
*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*tan(d*x + c)^2 +
 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*d*tan(d*x + c) +
 (a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.85

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$

$$= \frac{\frac{6\,(Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{6\,(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{3\,(Aa^4+4\,Ba^3b-6\,Aa^2b}{a^8+4\,a^6}}{}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="m
axima")
```

output
```
1/6*(6*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*(d*x + c)/(a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 6*(A*a^4 + 4*B*a^3*b - 6*A*
a^2*b^2 - 4*B*a*b^3 + A*b^4)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8) - 3*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^
3 + A*b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8) - (2*B*a^8 + A*a^7*b + 4*B*a^6*b^2 + 14*A*a^5*b^3 + 26*B*a^4*b^4
- 11*A*a^3*b^5 + 6*(B*a^6*b^2 + 3*B*a^4*b^4 + A*a^3*b^5 + 6*B*a^2*b^6 - 3*
A*a*b^7)*tan(d*x + c)^2 + 3*(2*B*a^7*b + A*a^6*b^2 + 6*B*a^5*b^3 + 8*A*a^4
*b^4 + 20*B*a^3*b^5 - 9*A*a^2*b^6)*tan(d*x + c))/(a^9*b^3 + 3*a^7*b^5 + 3*
a^5*b^7 + a^3*b^9 + (a^6*b^6 + 3*a^4*b^8 + 3*a^2*b^10 + b^12)*tan(d*x + c)
^3 + 3*(a^7*b^5 + 3*a^5*b^7 + 3*a^3*b^9 + a*b^11)*tan(d*x + c)^2 + 3*(a^8*
b^4 + 3*a^6*b^6 + 3*a^4*b^8 + a^2*b^10)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.69

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4} dx$$
$$= \frac{(Ba^4 - 4Aa^3b - 6Ba^2b^2 + 4Aab^3 + Bb^4)(dx+c)}{a^8d + 4a^6b^2d + 6a^4b^4d + 4a^2b^6d + b^8d}$$
$$- \frac{(Aa^4 + 4Ba^3b - 6Aa^2b^2 - 4Bab^3 + Ab^4)\log\left(\tan(dx+c)^2 + 1\right)}{2(a^8d + 4a^6b^2d + 6a^4b^4d + 4a^2b^6d + b^8d)}$$
$$+ \frac{(Aa^4b + 4Ba^3b^2 - 6Aa^2b^3 - 4Bab^4 + Ab^5)\log(|b\tan(dx+c) + a|)}{a^8bd + 4a^6b^3d + 6a^4b^5d + 4a^2b^7d + b^9d}$$
$$- \frac{2Ba^{10} + Aa^9b + 6Ba^8b^2 + 15Aa^7b^3 + 30Ba^6b^4 + 3Aa^5b^5 + 26Ba^4b^6 - 11Aa^3b^7 + 6(Ba^8b^2 + 4Ba}{}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="g
iac")
```

output
```
(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*(d*x + c)/(a^8*d + 4
*a^6*b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) - 1/2*(A*a^4 + 4*B*a^3*b -
 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*log(tan(d*x + c)^2 + 1)/(a^8*d + 4*a^6*b
^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + (A*a^4*b + 4*B*a^3*b^2 - 6*A*a
^2*b^3 - 4*B*a*b^4 + A*b^5)*log(abs(b*tan(d*x + c) + a))/(a^8*b*d + 4*a^6*
b^3*d + 6*a^4*b^5*d + 4*a^2*b^7*d + b^9*d) - 1/6*(2*B*a^10 + A*a^9*b + 6*B
*a^8*b^2 + 15*A*a^7*b^3 + 30*B*a^6*b^4 + 3*A*a^5*b^5 + 26*B*a^4*b^6 - 11*A
*a^3*b^7 + 6*(B*a^8*b^2 + 4*B*a^6*b^4 + A*a^5*b^5 + 9*B*a^4*b^6 - 2*A*a^3*
b^7 + 6*B*a^2*b^8 - 3*A*a*b^9)*tan(d*x + c)^2 + 3*(2*B*a^9*b + A*a^8*b^2 +
 8*B*a^7*b^3 + 9*A*a^6*b^4 + 26*B*a^5*b^5 - A*a^4*b^6 + 20*B*a^3*b^7 - 9*A
*a^2*b^8)*tan(d*x + c))/((a^2 + b^2)^4*(b*tan(d*x + c) + a)^3*b^3*d)
```

Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.58

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\ln\left(a+b\tan(c+d\,x)\right)\left(\frac{A}{\left(a^2+b^2\right)^2}-\frac{4\,b\,(2\,A\,b-B\,a)}{\left(a^2+b^2\right)^3}+\frac{8\,b^3\,(A\,b-B\,a)}{\left(a^2+b^2\right)^4}\right)}{d}$$
$$-\frac{\frac{a^2\,\left(2\,B\,a^6+A\,a^5\,b+4\,B\,a^4\,b^2+14\,A\,a^3\,b^3+26\,B\,a^2\,b^4-11\,A\,a\,b^5\right)}{6\,b^3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)^2\,\left(B\,a^6+3\,B\,a^4\,b^2+A\,a^3\,b^3+6\,B\,a^2\,b^4-3\,A\,a\,b^5\right)}{b\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{a\tan(c}{}}{d\,\left(a^3+3\,a^2\,b\tan\left(c+d\,x\right)+3\,a\,b^2\tan\left(c+d\,x\right)^2+b^3\tan\left(c+\right.\right.}$$
$$-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{2\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)}{2\,d\,\left(a^4+a^3\,b\,4\mathrm{i}-6\,a^2\,b^2-a\,b^3\,4\mathrm{i}+b^4\right)}$$

input

```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^4,x)
```

output

```
(log(a + b*tan(c + d*x))*(A/(a^2 + b^2)^2 - (4*b*(2*A*b - B*a))/(a^2 + b^2
)^3 + (8*b^3*(A*b - B*a))/(a^2 + b^2)^4))/d - ((a^2*(2*B*a^6 + 14*A*a^3*b^
3 + 26*B*a^2*b^4 + 4*B*a^4*b^2 - 11*A*a*b^5 + A*a^5*b))/(6*b^3*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^2*(B*a^6 + A*a^3*b^3 + 6*B*a^2*b
^4 + 3*B*a^4*b^2 - 3*A*a*b^5))/(b*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (
a*tan(c + d*x)*(2*B*a^6 + 8*A*a^3*b^3 + 20*B*a^2*b^4 + 6*B*a^4*b^2 - 9*A*a
*b^5 + A*a^5*b))/(2*b^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3 + b^
3*tan(c + d*x)^3 + 3*a*b^2*tan(c + d*x)^2 + 3*a^2*b*tan(c + d*x))) - (log(
tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i -
 a^2*b^2*6i)) - (log(tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(a^3*b*4i - a*b^3
*4i + a^4 + b^4 - 6*a^2*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.85

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{-\log\left(\tan(dx+c)^2+1\right)\tan(dx+c)^2 a^3b^2+3\log\left(\tan(dx+c)^2+1\right)\tan(dx+c)^2 a\,b^4-2\log\left(\tan(dx\right.}{}$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

output
```
( - log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2*a**3*b**2 + 3*log(tan(c + d*x
)**2 + 1)*tan(c + d*x)**2*a*b**4 - 2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)
*a**4*b + 6*log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a**2*b**3 - log(tan(c +
d*x)**2 + 1)*a**5 + 3*log(tan(c + d*x)**2 + 1)*a**3*b**2 + 2*log(tan(c + d
*x)*b + a)*tan(c + d*x)**2*a**3*b**2 - 6*log(tan(c + d*x)*b + a)*tan(c + d
*x)**2*a*b**4 + 4*log(tan(c + d*x)*b + a)*tan(c + d*x)*a**4*b - 12*log(tan
(c + d*x)*b + a)*tan(c + d*x)*a**2*b**3 + 2*log(tan(c + d*x)*b + a)*a**5 -
 6*log(tan(c + d*x)*b + a)*a**3*b**2 + tan(c + d*x)**2*a**5 + 4*tan(c + d*
x)**2*a**3*b**2 - 6*tan(c + d*x)**2*a**2*b**3*d*x + 3*tan(c + d*x)**2*a*b*
*4 + 2*tan(c + d*x)**2*b**5*d*x - 12*tan(c + d*x)*a**3*b**2*d*x + 4*tan(c
+ d*x)*a*b**4*d*x - 2*a**5 - 6*a**4*b*d*x - 2*a**3*b**2 + 2*a**2*b**3*d*x)
/(2*d*(tan(c + d*x)**2*a**6*b**2 + 3*tan(c + d*x)**2*a**4*b**4 + 3*tan(c +
 d*x)**2*a**2*b**6 + tan(c + d*x)**2*b**8 + 2*tan(c + d*x)*a**7*b + 6*tan(
c + d*x)*a**5*b**3 + 6*tan(c + d*x)*a**3*b**5 + 2*tan(c + d*x)*a*b**7 + a*
*8 + 3*a**6*b**2 + 3*a**4*b**4 + a**2*b**6))
```

3.292 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 261

$$\begin{aligned}\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx &= -\frac{(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)\,x}{(a^2+b^2)^4}\\ &\quad-\frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}\\ &\quad-\frac{a^2(Ab-aB)}{3b^2\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}\\ &\quad+\frac{a(2Ab^3-a(a^2+3b^2)\,B)}{2b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2}+\frac{3a^2Ab-Ab^3-a^3B+3ab^2B}{(a^2+b^2)^3\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*x/(a^2+b^2)^4-(4*A*a^3*b-4*
A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4
/d-1/3*a^2*(A*b-B*a)/b^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^3+1/2*a*(2*A*b^3-a*(
a^2+3*b^2)*B)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+(3*A*a^2*b-A*b^3-B*a^3+
3*B*a*b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.22 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.57

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = -\frac{B\tan(c+dx)}{2bd(a+b\tan(c+dx))^3}$$

$$-\frac{\frac{2Ab+aB}{3bd(a+b\tan(c+dx))^3}+\frac{\frac{\left(6Ab^3-6ab^2B\right)\left(-\frac{i\log(i-\tan(c+dx))}{2(a+ib)^4}+\frac{i\log(i+\tan(c+dx))}{2(a-ib)^4}+\frac{4a(a-b)b(a+b)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4}-\frac{b}{3\left(a^2+b^2\right)(a+b\tan(c+dx)}\right.}{b}}{}}{}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/2*(B*Tan[c + d*x])/(b*d*(a + b*Tan[c + d*x])^3) - ((2*A*b + a*B)/(3*b*d
*(a + b*Tan[c + d*x])^3) + (((6*A*b^3 - 6*a*b^2*B)*(((-1/2*I)*Log[I - Tan[
c + d*x]])/(a + I*b)^4 + ((I/2)*Log[I + Tan[c + d*x]])/(a - I*b)^4 + (4*a*
(a - b)*b*(a + b)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^4 - b/(3*(a^2 + b^2
)*(a + b*Tan[c + d*x])^3) - (a*b)/((a^2 + b^2)^2*(a + b*Tan[c + d*x])^2) -
 (b*(3*a^2 - b^2))/((a^2 + b^2)^3*(a + b*Tan[c + d*x]))))/b + 6*b*B*(-1/2*
Log[I - Tan[c + d*x]]/(I*a - b)^3 + Log[I + Tan[c + d*x]]/(2*(I*a + b)^3)
+ (b*(3*a^2 - b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 - b/(2*(a^2 + b^
2)*(a + b*Tan[c + d*x])^2) - (2*a*b)/((a^2 + b^2)^2*(a + b*Tan[c + d*x])))
)/(3*b*d))/(2*b)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4087, 25, 3042, 4111, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{ 4087}$$

$$\frac{\int -\frac{-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{(a+b\tan(c+dx))^3}dx}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{(a+b\tan(c+dx))^3}dx}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{-\left(\left(a^2+b^2\right)B\tan(c+dx)^2\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{(a+b\tan(c+dx))^3}dx}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 4111}$$

$$-\frac{\frac{\int \frac{b\left(Aa^2+2bBa-Ab^2\right)-b\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\int \frac{b\left(Aa^2+2bBa-Ab^2\right)-b\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 4012}$$

$$-\frac{\frac{\frac{\int \frac{b\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-b\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}}{b\left(a^2+b^2\right)}-\frac{a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\frac{\int \frac{b\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-b\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-}{b\left(a^2+b^2\right)}$$

$$\frac{a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 4014

$$-\frac{\frac{\frac{\frac{b\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{bx\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)}{a^2+b^2}}{a^2+b^2}-\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{a(2A\cdots}{2bd(a^2\cdots}}{b\left(a^2+b^2\right)}$$

$$\frac{a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{b\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{bx\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)}{a^2+b^2}}{a^2+b^2}-\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{a(2A\cdots}{2bd(a^2\cdots}}{b\left(a^2+b^2\right)}$$

$$\frac{a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 4013

$$-\frac{a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}-$$

$$\frac{\frac{\frac{\frac{b\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{bx\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)}{a^2+b^2}}{a^2+b^2}-\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{\cdots}{2\cdots}}{b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/3*(a^2*(A*b - a*B))/(b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) - (-1/2*
(a*(2*A*b^3 - a*(a^2 + 3*b^2)*B))/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
 + (((b*(a^4*A - 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B - 4*a*b^3*B)*x)/(a^2 + b^
2) + (b*(4*a^3*A*b - 4*a*A*b^3 - a^4*B + 6*a^2*b^2*B - b^4*B)*Log[a*Cos[c
+ d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2 + b^2) - (b*(3*a^2*A*b - A
*b^3 - a^3*B + 3*a*b^2*B))/((a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(a^2 + b^
2))/(b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{\left(4A\,a^{3}b-4Aa\,b^{3}-B\,a^{4}+6B\,a^{2}b^{2}-B\,b^{4}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(-A\,a^{4}+6A\,a^{2}b^{2}-A\,b^{4}-4B\,a^{3}b+4Ba\,b^{3}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{4}}-\frac{}{3b^{2}}$
default	$\frac{\frac{\left(4A\,a^{3}b-4Aa\,b^{3}-B\,a^{4}+6B\,a^{2}b^{2}-B\,b^{4}\right)\ln\left(1+\tan(dx+c)^{2}\right)}{2}+\left(-A\,a^{4}+6A\,a^{2}b^{2}-A\,b^{4}-4B\,a^{3}b+4Ba\,b^{3}\right)\arctan(\tan(dx+c))}{\left(a^{2}+b^{2}\right)^{4}}-\frac{}{3b^{2}}$
norman	$-\frac{\left(8A\,a^{3}b^{3}-B\,a^{6}-6B\,a^{4}b^{2}+3B\,a^{2}b^{4}\right)\tan(dx+c)^{2}}{2ad\left(a^{6}+3b^{2}a^{4}+3b^{4}a^{2}+b^{6}\right)}-\frac{a\left(A\,a^{5}-3A\,a^{3}b^{2}+3B\,a^{4}b-B\,a^{2}b^{3}\right)}{3bd\left(a^{6}+3b^{2}a^{4}+3b^{4}a^{2}+b^{6}\right)}-\frac{\left(A\,a^{4}-6A\,a^{2}b^{2}+A\,b^{4}+4B\,a^{3}b-4Ba\,b\right.}{\left(a^{6}+3b^{2}a^{4}+3b^{4}a^{2}+b^{6}\right)\left(a^{2}+b^{2}\right.}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^4*(1/2*(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*ln(1
+tan(d*x+c)^2)+(-A*a^4+6*A*a^2*b^2-A*b^4-4*B*a^3*b+4*B*a*b^3)*arctan(tan(d
*x+c)))-1/3*a^2*(A*b-B*a)/b^2/(a^2+b^2)/(a+b*tan(d*x+c))^3+(3*A*a^2*b-A*b^
3-B*a^3+3*B*a*b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))-(4*A*a^3*b-4*A*a*b^3-B*a^4
+6*B*a^2*b^2-B*b^4)/(a^2+b^2)^4*ln(a+b*tan(d*x+c))+1/2*a*(2*A*b^3-B*a^3-3*
B*a*b^2)/(a^2+b^2)^2/b^2/(a+b*tan(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. $2(253) = 506$.

Time = 0.12 (sec) , antiderivative size = 836, normalized size of antiderivative = 3.20

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
1/6*(3*B*a^7 - 12*A*a^6*b - 30*B*a^5*b^2 + 30*A*a^4*b^3 + 11*B*a^3*b^4 - 2
*A*a^2*b^5 + (B*a^6*b + 2*A*a^5*b^2 + 18*B*a^4*b^3 - 30*A*a^3*b^4 - 27*B*a
^2*b^5 + 12*A*a*b^6 - 6*(A*a^4*b^3 + 4*B*a^3*b^4 - 6*A*a^2*b^5 - 4*B*a*b^6
 + A*b^7)*d*x)*tan(d*x + c)^3 - 6*(A*a^7 + 4*B*a^6*b - 6*A*a^5*b^2 - 4*B*a
^4*b^3 + A*a^3*b^4)*d*x + 3*(B*a^7 + 2*A*a^6*b + 16*B*a^5*b^2 - 24*A*a^4*b
^3 - 23*B*a^3*b^4 + 16*A*a^2*b^5 + 6*B*a*b^6 - 2*A*b^7 - 6*(A*a^5*b^2 + 4*
B*a^4*b^3 - 6*A*a^3*b^4 - 4*B*a^2*b^5 + A*a*b^6)*d*x)*tan(d*x + c)^2 + 3*(
B*a^7 - 4*A*a^6*b - 6*B*a^5*b^2 + 4*A*a^4*b^3 + B*a^3*b^4 + (B*a^4*b^3 - 4
*A*a^3*b^4 - 6*B*a^2*b^5 + 4*A*a*b^6 + B*b^7)*tan(d*x + c)^3 + 3*(B*a^5*b^
2 - 4*A*a^4*b^3 - 6*B*a^3*b^4 + 4*A*a^2*b^5 + B*a*b^6)*tan(d*x + c)^2 + 3*
(B*a^6*b - 4*A*a^5*b^2 - 6*B*a^4*b^3 + 4*A*a^3*b^4 + B*a^2*b^5)*tan(d*x +
c))*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 +
1)) + 3*(2*A*a^7 + 9*B*a^6*b - 16*A*a^5*b^2 - 26*B*a^4*b^3 + 24*A*a^3*b^4
+ 9*B*a^2*b^5 - 2*A*a*b^6 - 6*(A*a^6*b + 4*B*a^5*b^2 - 6*A*a^4*b^3 - 4*B*a
^3*b^4 + A*a^2*b^5)*d*x)*tan(d*x + c))/((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 +
 4*a^2*b^9 + b^11)*d*tan(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 +
 4*a^3*b^8 + a*b^10)*d*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5
+ 4*a^4*b^7 + a^2*b^9)*d*tan(d*x + c) + (a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*
a^5*b^6 + a^3*b^8)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(253) = 506$.

Time = 0.13 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.02

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx =$$

$$-\frac{\frac{6\,(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{6\,(Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{3\,(Ba^4-4\,Aa^3b-6\,B}{a^8+4}}{}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="m
axima")
```

output
```
-1/6*(6*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(B*a^4 - 4*A*a^3*b - 6*B
*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b
^3 + B*b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8) + (B*a^7 + 2*A*a^6*b + 14*B*a^5*b^2 - 20*A*a^4*b^3 - 11*B*a^3*b^
4 + 2*A*a^2*b^5 + 6*(B*a^3*b^4 - 3*A*a^2*b^5 - 3*B*a*b^6 + A*b^7)*tan(d*x
+ c)^2 + 3*(B*a^6*b + 8*B*a^4*b^3 - 14*A*a^3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6
)*tan(d*x + c))/(a^9*b^2 + 3*a^7*b^4 + 3*a^5*b^6 + a^3*b^8 + (a^6*b^5 + 3*
a^4*b^7 + 3*a^2*b^9 + b^11)*tan(d*x + c)^3 + 3*(a^7*b^4 + 3*a^5*b^6 + 3*a^
3*b^8 + a*b^10)*tan(d*x + c)^2 + 3*(a^8*b^3 + 3*a^6*b^5 + 3*a^4*b^7 + a^2*
b^9)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.84

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= -\frac{(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)(dx+c)}{a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d}$$
$$-\frac{(Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,(a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d)}$$
$$+\frac{(Ba^4b-4\,Aa^3b^2-6\,Ba^2b^3+4\,Aab^4+Bb^5)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^8bd+4\,a^6b^3d+6\,a^4b^5d+4\,a^2b^7d+b^9d}$$
$$-\frac{Ba^9+2\,Aa^8b+15\,Ba^7b^2-18\,Aa^6b^3+3\,Ba^5b^4-18\,Aa^4b^5-11\,Ba^3b^6+2\,Aa^2b^7+6\,(Ba^5b^4-3\,Aa^4}{}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="g
iac")
```

output
```
-(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^8*d +
4*a^6*b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) - 1/2*(B*a^4 - 4*A*a^3*b
- 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1)/(a^8*d + 4*a^6*
b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + (B*a^4*b - 4*A*a^3*b^2 - 6*B*
a^2*b^3 + 4*A*a*b^4 + B*b^5)*log(abs(b*tan(d*x + c) + a))/(a^8*b*d + 4*a^6
*b^3*d + 6*a^4*b^5*d + 4*a^2*b^7*d + b^9*d) - 1/6*(B*a^9 + 2*A*a^8*b + 15*
B*a^7*b^2 - 18*A*a^6*b^3 + 3*B*a^5*b^4 - 18*A*a^4*b^5 - 11*B*a^3*b^6 + 2*A
*a^2*b^7 + 6*(B*a^5*b^4 - 3*A*a^4*b^5 - 2*B*a^3*b^6 - 2*A*a^2*b^7 - 3*B*a*
b^8 + A*b^9)*tan(d*x + c)^2 + 3*(B*a^8*b + 9*B*a^6*b^3 - 14*A*a^5*b^4 - B*
a^4*b^5 - 12*A*a^3*b^6 - 9*B*a^2*b^7 + 2*A*a*b^8)*tan(d*x + c))/((a^2 + b^
2)^4*(b*tan(d*x + c) + a)^3*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.71

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{B}{{\left(a^2+b^2\right)}^2}-\frac{4\,b\,\left(A\,a+2\,B\,b\right)}{{\left(a^2+b^2\right)}^3}+\frac{8\,b^3\,\left(A\,a+B\,b\right)}{{\left(a^2+b^2\right)}^4}\right)}{d}$$
$$-\frac{\frac{{\mathrm{tan}\left(c+d\,x\right)}^2\,\left(B\,a^3\,b^2-3\,A\,a^2\,b^3-3\,B\,a\,b^4+A\,b^5\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{a\,\left(B\,a^6+2\,A\,a^5\,b+14\,B\,a^4\,b^2-20\,A\,a^3\,b^3-11\,B\,a^2\,b^4+2\,A\,a\,b^5\right)}{6\,b^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\mathrm{tan}\left(c+d\,x\right)\,\left(B\,a^6+\cdots\right)}{2\,b\cdots}}{d\,\left(a^3+3\,a^2\,b\,\mathrm{tan}\left(c+d\,x\right)+3\,a\,b^2\,{\mathrm{tan}\left(c+d\,x\right)}^2+b^3\,{\mathrm{tan}\left(c+d\,x\right)}^3\right)}$$
$$-\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)-\mathrm{i}\right)\,\left(A+B\,1{}\mathrm{i}\right)}{2\,d\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}$$
$$-\frac{\ln\left(\mathrm{tan}\left(c+d\,x\right)+1{}\mathrm{i}\right)\,\left(B+A\,1{}\mathrm{i}\right)}{2\,d\,\left(a^4-a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4{}\mathrm{i}+b^4\right)}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^4,x)
```

output
```
(log(a + b*tan(c + d*x))*(B/(a^2 + b^2)^2 - (4*b*(A*a + 2*B*b))/(a^2 + b^2
)^3 + (8*b^3*(A*a + B*b))/(a^2 + b^2)^4))/d - ((tan(c + d*x)^2*(A*b^5 - 3*
A*a^2*b^3 + B*a^3*b^2 - 3*B*a*b^4))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) +
(a*(B*a^6 - 20*A*a^3*b^3 - 11*B*a^2*b^4 + 14*B*a^4*b^2 + 2*A*a*b^5 + 2*A*a
^5*b))/(6*b^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)*(B*a^6
- 14*A*a^3*b^3 - 9*B*a^2*b^4 + 8*B*a^4*b^2 + 2*A*a*b^5))/(2*b*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3 + b^3*tan(c + d*x)^3 + 3*a*b^2*tan(c + d
*x)^2 + 3*a^2*b*tan(c + d*x))) - (log(tan(c + d*x) - 1i)*(A + B*1i))/(2*d*
(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)) - (log(tan(c + d*x) +
1i)*(A*1i + B))/(2*d*(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.10

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \frac{3\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)^2a^2b^4-\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)^2b^6+6\log\left(\tan\left(dx+c\right)\right.}{}$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

output
```
(3*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2*a**2*b**4 - log(tan(c + d*x)**
2 + 1)*tan(c + d*x)**2*b**6 + 6*log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a**3
*b**3 - 2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a*b**5 + 3*log(tan(c + d*x
)**2 + 1)*a**4*b**2 - log(tan(c + d*x)**2 + 1)*a**2*b**4 - 6*log(tan(c + d
*x)*b + a)*tan(c + d*x)**2*a**2*b**4 + 2*log(tan(c + d*x)*b + a)*tan(c + d
*x)**2*b**6 - 12*log(tan(c + d*x)*b + a)*tan(c + d*x)*a**3*b**3 + 4*log(ta
n(c + d*x)*b + a)*tan(c + d*x)*a*b**5 - 6*log(tan(c + d*x)*b + a)*a**4*b**
2 + 2*log(tan(c + d*x)*b + a)*a**2*b**4 - 2*tan(c + d*x)**2*a**3*b**3*d*x
- 2*tan(c + d*x)**2*a**2*b**4 + 6*tan(c + d*x)**2*a*b**5*d*x - 2*tan(c + d
*x)**2*b**6 - 4*tan(c + d*x)*a**4*b**2*d*x + 12*tan(c + d*x)*a**2*b**4*d*x
 - a**6 - 2*a**5*b*d*x + 6*a**3*b**3*d*x + a**2*b**4)/(2*b*d*(tan(c + d*x)
**2*a**6*b**2 + 3*tan(c + d*x)**2*a**4*b**4 + 3*tan(c + d*x)**2*a**2*b**6
+ tan(c + d*x)**2*b**8 + 2*tan(c + d*x)*a**7*b + 6*tan(c + d*x)*a**5*b**3
+ 6*tan(c + d*x)*a**3*b**5 + 2*tan(c + d*x)*a*b**7 + a**8 + 3*a**6*b**2 +
3*a**4*b**4 + a**2*b**6))
```

3.293 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 250

$$\begin{aligned}
&\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx \\
&= \frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)\,x}{(a^2+b^2)^4} \\
&\quad - \frac{(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} \\
&\quad + \frac{a(Ab-aB)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} \\
&\quad + \frac{a^2A-Ab^2+2abB}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{a^3A-3aAb^2+3a^2bB-b^3B}{(a^2+b^2)^3\,d(a+b\tan(c+dx))}
\end{aligned}$$

output

```
(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*x/(a^2+b^2)^4-(A*a^4-6*A*a^2
*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/
d+1/3*a*(A*b-B*a)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3+1/2*(A*a^2-A*b^2+2*B*a*
b)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)/(a^2
+b^2)^3/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.99

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{3(A+iB)\log(i-\tan(c+dx))}{(a+ib)^4} + \frac{3(A-iB)\log(i+\tan(c+dx))}{(a-ib)^4} - \frac{6(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)\log(a+b\tan(c+dx))}{(a^2+b^2)^4} + \frac{2a(Ab}{b(a^2+b^2)(a+}}{6d}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
((3*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*b)^4 + (3*(A - I*B)*Log[I + Ta
n[c + d*x]])/(a - I*b)^4 - (6*(a^4*A - 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B - 4
*a*b^3*B)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^4 + (2*a*(A*b - a*B))/(b*(a
^2 + b^2)*(a + b*Tan[c + d*x])^3) + (3*(a^2*A - A*b^2 + 2*a*b*B))/((a^2 +
b^2)^2*(a + b*Tan[c + d*x])^2) + (6*(a^3*A - 3*a*A*b^2 + 3*a^2*b*B - b^3*B
))/((a^2 + b^2)^3*(a + b*Tan[c + d*x])))/(6*d)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 4074, 3042, 4012, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{4074}$$

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{(a+b\tan(c+dx))^3}dx}{a^2+b^2}+\frac{a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{(a+b\tan(c+dx))^3}dx}{a^2+b^2}+\frac{a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \textbf{4012}$$

$$\frac{\frac{\int \frac{-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}+\frac{a^2A+2abB-Ab^2}{2d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\int \frac{-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}+\frac{a^2A+2abB-Ab^2}{2d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \textbf{4012}$$

$$\frac{\frac{\frac{\int \frac{-Ba^3+3Aba^2+3b^2Ba-Ab^3+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{a^3A+3a^2bB-3aAb^2-b^3B}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a^2A+2abB-Ab^2}{2d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\frac{\int \frac{-Ba^3+3Aba^2+3b^2Ba-Ab^3+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{a^3A+3a^2bB-3aAb^2-b^3B}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a^2A+2abB-Ab^2}{2d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}+\frac{a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \textbf{4014}$$

$$\frac{\frac{\frac{\frac{x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)}{a^2+b^2}-\frac{\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a^3A+3a^2bB-3aAb^2-b^3B}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a^2A+2abB-}{2d(a^2+b^2)(a+b\,\mathrm{ta}}}{a^2+b^2}$$

$$\frac{a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\frac{x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)}{a^2+b^2}-\frac{\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a^3A+3a^2bB-3aAb^2-b^3B}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}+\frac{a^2A+2abB-}{2d(a^2+b^2)(a+b\,\mathrm{ta}}}{a^2+b^2}$$

$$\frac{a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 4013

$$\frac{a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^3}+$$

$$\frac{\frac{a^2A+2abB-Ab^2}{2d(a^2+b^2)(a+b\tan(c+dx))^2}+\frac{\frac{a^3A+3a^2bB-3aAb^2-b^3B}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{\frac{x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)}{a^2+b^2}-\frac{\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\log(a\cos}{d\left(a^2+b^2\right)}}{a^2+b^2}}{a^2+b^2}}{a^2+b^2}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
(a*(A*b - a*B))/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) + ((a^2*A - A*b
^2 + 2*a*b*B)/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((((4*a^3*A*b - 4
*a*A*b^3 - a^4*B + 6*a^2*b^2*B - b^4*B)*x)/(a^2 + b^2) - ((a^4*A - 6*a^2*A
*b^2 + A*b^4 + 4*a^3*b*B - 4*a*b^3*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/((a^2 + b^2)*d))/(a^2 + b^2) + (a^3*A - 3*a*A*b^2 + 3*a^2*b*B - b^3*B)/(
(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(a^2 + b^2))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4B a\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(4A\,a^3b-4A a\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{}{3\left(a^2\right.}$
default	$\frac{\frac{\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4B a\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(4A\,a^3b-4A a\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{}{3\left(a^2\right.}$
norman	$\frac{\left(4A\,a^3b-4A a\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)a^3x}{\left(a^6+3b^2a^4+3b^4a^2+b^6\right)\left(a^2+b^2\right)}+\frac{b^3\left(4A\,a^3b-4A a\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)x\tan(dx+c)^3}{\left(a^6+3b^2a^4+3b^4a^2+b^6\right)\left(a^2+b^2\right)}+\frac{a\left(9A\,a^4b^2-8A\,a^2b^4-A\right.}{6b^2\left(a^6+3b^2a^4\right.}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/(a^2+b^2)^4*(1/2*(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*ln(1
+tan(d*x+c)^2)+(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*arctan(tan(d*
x+c)))+1/3*a*(A*b-B*a)/(a^2+b^2)/b/(a+b*tan(d*x+c))^3+1/2*(A*a^2-A*b^2+2*B
*a*b)/(a^2+b^2)^2/(a+b*tan(d*x+c))^2+(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)/(a^
2+b^2)^3/(a+b*tan(d*x+c))-(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)/(a
^2+b^2)^4*ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 838 vs. 2(245) = 490.

Time = 0.14 (sec) , antiderivative size = 838, normalized size of antiderivative = 3.35

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="fri
cas")
```

output
```
-1/6*(12*B*a^6*b - 27*A*a^5*b^2 - 30*B*a^4*b^3 + 18*A*a^3*b^4 + 2*B*a^2*b^
5 + A*a*b^6 - (2*B*a^5*b^2 - 11*A*a^4*b^3 - 30*B*a^3*b^4 + 30*A*a^2*b^5 +
12*B*a*b^6 - 3*A*b^7 - 6*(B*a^4*b^3 - 4*A*a^3*b^4 - 6*B*a^2*b^5 + 4*A*a*b^
6 + B*b^7)*d*x)*tan(d*x + c)^3 + 6*(B*a^7 - 4*A*a^6*b - 6*B*a^5*b^2 + 4*A*
a^4*b^3 + B*a^3*b^4)*d*x - 3*(2*B*a^6*b - 9*A*a^5*b^2 - 24*B*a^4*b^3 + 26*
A*a^3*b^4 + 16*B*a^2*b^5 - 9*A*a*b^6 - 2*B*b^7 - 6*(B*a^5*b^2 - 4*A*a^4*b^
3 - 6*B*a^3*b^4 + 4*A*a^2*b^5 + B*a*b^6)*d*x)*tan(d*x + c)^2 + 3*(A*a^7 +
4*B*a^6*b - 6*A*a^5*b^2 - 4*B*a^4*b^3 + A*a^3*b^4 + (A*a^4*b^3 + 4*B*a^3*b
^4 - 6*A*a^2*b^5 - 4*B*a*b^6 + A*b^7)*tan(d*x + c)^3 + 3*(A*a^5*b^2 + 4*B*
a^4*b^3 - 6*A*a^3*b^4 - 4*B*a^2*b^5 + A*a*b^6)*tan(d*x + c)^2 + 3*(A*a^6*b
 + 4*B*a^5*b^2 - 6*A*a^4*b^3 - 4*B*a^3*b^4 + A*a^2*b^5)*tan(d*x + c))*log(
(b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 3*
(2*B*a^7 - 6*A*a^6*b - 16*B*a^5*b^2 + 23*A*a^4*b^3 + 24*B*a^3*b^4 - 16*A*a
^2*b^5 - 2*B*a*b^6 - A*b^7 - 6*(B*a^6*b - 4*A*a^5*b^2 - 6*B*a^4*b^3 + 4*A*
a^3*b^4 + B*a^2*b^5)*d*x)*tan(d*x + c))/((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7
+ 4*a^2*b^9 + b^11)*d*tan(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6
+ 4*a^3*b^8 + a*b^10)*d*tan(d*x + c)^2 + 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5
 + 4*a^4*b^7 + a^2*b^9)*d*tan(d*x + c) + (a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4
*a^5*b^6 + a^3*b^8)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(245) = 490$.

Time = 0.12 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.09

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx =$$

$$-\frac{\frac{6\,(Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{6\,(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{3\,(Aa^4+4\,Ba^3b-6\,A}{a^8+4}}{}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="max
ima")
```

output
```
-1/6*(6*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*(d*x + c)/(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 6*(A*a^4 + 4*B*a^3*b - 6*A
*a^2*b^2 - 4*B*a*b^3 + A*b^4)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6
*a^4*b^4 + 4*a^2*b^6 + b^8) - 3*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b
^3 + A*b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8) + (2*B*a^6 - 11*A*a^5*b - 20*B*a^4*b^2 + 14*A*a^3*b^3 + 2*B*a^2*
b^4 + A*a*b^5 - 6*(A*a^3*b^3 + 3*B*a^2*b^4 - 3*A*a*b^5 - B*b^6)*tan(d*x +
c)^2 - 3*(5*A*a^4*b^2 + 14*B*a^3*b^3 - 12*A*a^2*b^4 - 2*B*a*b^5 - A*b^6)*t
an(d*x + c))/(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7 + (a^6*b^4 + 3*a^4*b
^6 + 3*a^2*b^8 + b^10)*tan(d*x + c)^3 + 3*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7
 + a*b^9)*tan(d*x + c)^2 + 3*(a^8*b^2 + 3*a^6*b^4 + 3*a^4*b^6 + a^2*b^8)*t
an(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.92

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= -\frac{(Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)(dx+c)}{a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d}$$
$$+\frac{(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,(a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d)}$$
$$-\frac{(Aa^4b+4\,Ba^3b^2-6\,Aa^2b^3-4\,Bab^4+Ab^5)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^8bd+4\,a^6b^3d+6\,a^4b^5d+4\,a^2b^7d+b^9d}$$
$$-\frac{2\,Ba^8-11\,Aa^7b-18\,Ba^6b^2+3\,Aa^5b^3-18\,Ba^4b^4+15\,Aa^3b^5+2\,Ba^2b^6+Aab^7-6\,(Aa^5b^3+3\,Ba^4b}{}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="gia
c")
```

output
```
-(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*(d*x + c)/(a^8*d +
4*a^6*b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + 1/2*(A*a^4 + 4*B*a^3*b
- 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*log(tan(d*x + c)^2 + 1)/(a^8*d + 4*a^6*
b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) - (A*a^4*b + 4*B*a^3*b^2 - 6*A*
a^2*b^3 - 4*B*a*b^4 + A*b^5)*log(abs(b*tan(d*x + c) + a))/(a^8*b*d + 4*a^6
*b^3*d + 6*a^4*b^5*d + 4*a^2*b^7*d + b^9*d) - 1/6*(2*B*a^8 - 11*A*a^7*b -
18*B*a^6*b^2 + 3*A*a^5*b^3 - 18*B*a^4*b^4 + 15*A*a^3*b^5 + 2*B*a^2*b^6 + A
*a*b^7 - 6*(A*a^5*b^3 + 3*B*a^4*b^4 - 2*A*a^3*b^5 + 2*B*a^2*b^6 - 3*A*a*b^
7 - B*b^8)*tan(d*x + c)^2 - 3*(5*A*a^6*b^2 + 14*B*a^5*b^3 - 7*A*a^4*b^4 +
12*B*a^3*b^5 - 13*A*a^2*b^6 - 2*B*a*b^7 - A*b^8)*tan(d*x + c))/((a^2 + b^2
)^4*(b*tan(d*x + c) + a)^3*b*d)
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.79

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx =$$
$$-\frac{\frac{\tan(c+d\,x)\,\left(-5\,A\,a^4\,b-14\,B\,a^3\,b^2+12\,A\,a^2\,b^3+2\,B\,a\,b^4+A\,b^5\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{2\,B\,a^6-11\,A\,a^5\,b-20\,B\,a^4\,b^2+14\,A\,a^3\,b^3+2\,B\,a^2\,b^4+A\,a\,b^5}{6\,b\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d}{}}{d\,\left(a^3+3\,a^2\,b\tan\left(c+d\,x\right)+3\,a\,b^2\tan\left(c+d\,x\right)^2+b^3\tan\left(c+d\,x\right)^3\right)}$$
$$-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{A}{{\left(a^2+b^2\right)}^2}-\frac{4\,b\,\left(2\,A\,b-B\,a\right)}{{\left(a^2+b^2\right)}^3}+\frac{8\,b^3\,\left(A\,b-B\,a\right)}{{\left(a^2+b^2\right)}^4}\right)}{d}$$
$$+\frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(B+A\,1{}\mathrm{i}\right)}{2\,d\,\left(a^4\,1{}\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}-4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}$$
$$+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1{}\mathrm{i}\right)}{2\,d\,\left(a^4+a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2-a\,b^3\,4{}\mathrm{i}+b^4\right)}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^4,x)
```

output
```
(log(tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4
*1i - a^2*b^2*6i)) - (log(a + b*tan(c + d*x))*(A/(a^2 + b^2)^2 - (4*b*(2*A
*b - B*a))/(a^2 + b^2)^3 + (8*b^3*(A*b - B*a))/(a^2 + b^2)^4))/d - ((tan(c
 + d*x)*(A*b^5 + 12*A*a^2*b^3 - 14*B*a^3*b^2 - 5*A*a^4*b + 2*B*a*b^4))/(2*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*B*a^6 + 14*A*a^3*b^3 + 2*B*a^2*b
^4 - 20*B*a^4*b^2 + A*a*b^5 - 11*A*a^5*b)/(6*b*(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2)) + (tan(c + d*x)^2*(B*b^5 - A*a^3*b^2 - 3*B*a^2*b^3 + 3*A*a*b^4))
/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(d*(a^3 + b^3*tan(c + d*x)^3 + 3*a*b
^2*tan(c + d*x)^2 + 3*a^2*b*tan(c + d*x))) + (log(tan(c + d*x) - 1i)*(A +
B*1i))/(2*d*(a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.18

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$=\frac{\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)^2a^4b^2-3\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)^2a^2b^4+2\log\left(\tan\left(dx+\right.\right.}{}$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

output
```
(log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2*a**4*b**2 - 3*log(tan(c + d*x)**
2 + 1)*tan(c + d*x)**2*a**2*b**4 + 2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)
*a**5*b - 6*log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a**3*b**3 + log(tan(c +
d*x)**2 + 1)*a**6 - 3*log(tan(c + d*x)**2 + 1)*a**4*b**2 - 2*log(tan(c + d
*x)*b + a)*tan(c + d*x)**2*a**4*b**2 + 6*log(tan(c + d*x)*b + a)*tan(c + d
*x)**2*a**2*b**4 - 4*log(tan(c + d*x)*b + a)*tan(c + d*x)*a**5*b + 12*log(
tan(c + d*x)*b + a)*tan(c + d*x)*a**3*b**3 - 2*log(tan(c + d*x)*b + a)*a**
6 + 6*log(tan(c + d*x)*b + a)*a**4*b**2 - tan(c + d*x)**2*a**4*b**2 + 6*ta
n(c + d*x)**2*a**3*b**3*d*x - 2*tan(c + d*x)**2*a*b**5*d*x + tan(c + d*x)*
*2*b**6 + 12*tan(c + d*x)*a**4*b**2*d*x - 4*tan(c + d*x)*a**2*b**4*d*x + 2
*a**6 + 6*a**5*b*d*x + 2*a**4*b**2 - 2*a**3*b**3*d*x)/(2*a*d*(tan(c + d*x)
**2*a**6*b**2 + 3*tan(c + d*x)**2*a**4*b**4 + 3*tan(c + d*x)**2*a**2*b**6
+ tan(c + d*x)**2*b**8 + 2*tan(c + d*x)*a**7*b + 6*tan(c + d*x)*a**5*b**3
+ 6*tan(c + d*x)*a**3*b**5 + 2*tan(c + d*x)*a*b**7 + a**8 + 3*a**6*b**2 +
3*a**4*b**4 + a**2*b**6))
```

3.294 $\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 247

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx &= \frac{(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)\,x}{(a^2+b^2)^4} \\ &\quad+\frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} \\ &\quad-\frac{Ab-aB}{3\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} \\ &\quad-\frac{2aAb-a^2B+b^2B}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2}-\frac{3a^2Ab-Ab^3-a^3B+3ab^2B}{(a^2+b^2)^3\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*x/(a^2+b^2)^4+(4*A*a^3*b-4*A
*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^4/
d-1/3*(A*b-B*a)/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-1/2*(2*A*a*b-B*a^2+B*b^2)/(
a^2+b^2)^2/d/(a+b*tan(d*x+c))^2-(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)/(a^2+b^2
)^3/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.21 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.32

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx =$$
$$-\frac{(Ab-aB)\left(\frac{3i\log(i-\tan(c+dx))}{(a+ib)^4}-\frac{3i\log(i+\tan(c+dx))}{(a-ib)^4}-\frac{24a(a-b)b(a+b)\log(a+b\tan(c+dx))}{(a^2+b^2)^4}+\frac{2b}{(a^2+b^2)(a+b\tan(c+dx))^3}+\cdots\right.}{6bd}$$
$$-\frac{B\left(\frac{\log(i-\tan(c+dx))}{(ia-b)^3}-\frac{\log(i+\tan(c+dx))}{(ia+b)^3}-\frac{2b(3a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^3}+\frac{b}{(a^2+b^2)(a+b\tan(c+dx))^2}+\frac{4ab}{(a^2+b^2)^2(a+b\tan(c-\cdots}\right.}{2bd}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/6*((A*b - a*B)*(((3*I)*Log[I - Tan[c + d*x]])/(a + I*b)^4 - ((3*I)*Log[
I + Tan[c + d*x]])/(a - I*b)^4 - (24*a*(a - b)*b*(a + b)*Log[a + b*Tan[c +
 d*x]])/(a^2 + b^2)^4 + (2*b)/((a^2 + b^2)*(a + b*Tan[c + d*x])^3) + (6*a*
b)/((a^2 + b^2)^2*(a + b*Tan[c + d*x])^2) + (6*b*(3*a^2 - b^2))/((a^2 + b^
2)^3*(a + b*Tan[c + d*x]))))/(b*d) - (B*(Log[I - Tan[c + d*x]]/(I*a - b)^3
 - Log[I + Tan[c + d*x]]/(I*a + b)^3 - (2*b*(3*a^2 - b^2)*Log[a + b*Tan[c
+ d*x]])/(a^2 + b^2)^3 + b/((a^2 + b^2)*(a + b*Tan[c + d*x])^2) + (4*a*b)/
((a^2 + b^2)^2*(a + b*Tan[c + d*x]))))/(2*b*d)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 4012, 3042, 4012, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^4}dx$$

$\downarrow$ 4012

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{(a+b\tan(c+dx))^3}dx}{a^2+b^2}-\frac{Ab-aB}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{(a+b\tan(c+dx))^3}dx}{a^2+b^2}-\frac{Ab-aB}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 4012

$$\frac{\frac{\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{a^2(-B)+2aAb+b^2B}{2d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{Ab-aB}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{a^2(-B)+2aAb+b^2B}{2d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{Ab-aB}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 4012

$$\frac{\frac{\frac{\int \frac{Aa^3+3bBa^2-3Ab^2a-b^3B-\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a^3(-B)+3a^2Ab+3ab^2B-Ab^3}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{a^2(-B)+2aAb+b^2B}{2d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{Ab-aB}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{Aa^3+3bBa^2-3Ab^2a-b^3B-\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{a^3(-B)+3a^2Ab+3ab^2B-Ab^3}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{a^2(-B)+2aAb+b^2B}{2d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{Ab-aB}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 4014

$$\frac{\frac{\frac{\frac{\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)}{a^2+b^2}}{a^2+b^2}-\frac{a^3(-B)+3a^2Ab+3ab^2B-Ab^3}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{a^2(-B)+2}{2d(a^2+b^2)(a+}}{a^2+b^2}$$

$$\frac{Ab-aB}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\frac{\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)}{a^2+b^2}}{a^2+b^2}-\frac{a^3(-B)+3a^2Ab+3ab^2B-Ab^3}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{a^2(-B)+2}{2d(a^2+b^2)(a+}}{a^2+b^2}$$

$$\frac{Ab-aB}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 4013

$$\frac{\frac{\frac{\frac{\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{x\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)}{a^2+b^2}}{a^2+b^2}-\frac{a^3(-B)+3a^2Ab+3ab^2B-Ab^3}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{a^2}{2d(a^2}}{a^2+b^2}$$

$$\frac{Ab-aB}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/3*(A*b - a*B)/((a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) + (-1/2*(2*a*A*b -
 a^2*B + b^2*B)/((a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((((a^4*A - 6*a^2
*A*b^2 + A*b^4 + 4*a^3*b*B - 4*a*b^3*B)*x)/(a^2 + b^2) + ((4*a^3*A*b - 4*a
*A*b^3 - a^4*B + 6*a^2*b^2*B - b^4*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/((a^2 + b^2)*d))/(a^2 + b^2) - (3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)/(
(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(a^2 + b^2))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{\left(-4A\,a^3b+4Aa\,b^3+B\,a^4-6B\,a^2b^2+B\,b^4\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4Ba\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{3A\,c}{\left(a^2\right.}$
default	$\frac{\frac{\left(-4A\,a^3b+4Aa\,b^3+B\,a^4-6B\,a^2b^2+B\,b^4\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4Ba\,b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{3A\,c}{\left(a^2\right.}$
norman	$\frac{\frac{\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4Ba\,b^3\right)a^3x}{\left(a^6+3b^2a^4+3b^4a^2+b^6\right)\left(a^2+b^2\right)}+\frac{b^3\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4Ba\,b^3\right)x\tan(dx+c)^3}{\left(a^6+3b^2a^4+3b^4a^2+b^6\right)\left(a^2+b^2\right)}-\frac{20A\,a^4b^3+6A\,a^2b^5+2A}{6b^2d\left(a^6+3b\right.}}{}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^4*(1/2*(-4*A*a^3*b+4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*ln(
1+tan(d*x+c)^2)+(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*arctan(tan(d
*x+c)))-(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)/(a^2+b^2)^3/(a+b*tan(d*x+c))+(4*
A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)/(a^2+b^2)^4*ln(a+b*tan(d*x+c))-
1/3*(A*b-B*a)/(a^2+b^2)/(a+b*tan(d*x+c))^3-1/2*(2*A*a*b-B*a^2+B*b^2)/(a^2+
b^2)^2/(a+b*tan(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(239) = 478$.

Time = 0.15 (sec) , antiderivative size = 815, normalized size of antiderivative = 3.30

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/6*(27*B*a^5*b^2 - 48*A*a^4*b^3 - 18*B*a^3*b^4 - 6*A*a^2*b^5 - B*a*b^6 -
2*A*b^7 - (11*B*a^4*b^3 - 26*A*a^3*b^4 - 30*B*a^2*b^5 + 18*A*a*b^6 + 3*B*b
^7 - 6*(A*a^4*b^3 + 4*B*a^3*b^4 - 6*A*a^2*b^5 - 4*B*a*b^6 + A*b^7)*d*x)*ta
n(d*x + c)^3 + 6*(A*a^7 + 4*B*a^6*b - 6*A*a^5*b^2 - 4*B*a^4*b^3 + A*a^3*b^
4)*d*x - 3*(9*B*a^5*b^2 - 20*A*a^4*b^3 - 26*B*a^3*b^4 + 22*A*a^2*b^5 + 9*B
*a*b^6 - 2*A*b^7 - 6*(A*a^5*b^2 + 4*B*a^4*b^3 - 6*A*a^3*b^4 - 4*B*a^2*b^5
+ A*a*b^6)*d*x)*tan(d*x + c)^2 - 3*(B*a^7 - 4*A*a^6*b - 6*B*a^5*b^2 + 4*A*
a^4*b^3 + B*a^3*b^4 + (B*a^4*b^3 - 4*A*a^3*b^4 - 6*B*a^2*b^5 + 4*A*a*b^6 +
 B*b^7)*tan(d*x + c)^3 + 3*(B*a^5*b^2 - 4*A*a^4*b^3 - 6*B*a^3*b^4 + 4*A*a^
2*b^5 + B*a*b^6)*tan(d*x + c)^2 + 3*(B*a^6*b - 4*A*a^5*b^2 - 6*B*a^4*b^3 +
 4*A*a^3*b^4 + B*a^2*b^5)*tan(d*x + c))*log((b^2*tan(d*x + c)^2 + 2*a*b*ta
n(d*x + c) + a^2)/(tan(d*x + c)^2 + 1)) - 3*(6*B*a^6*b - 12*A*a^5*b^2 - 23
*B*a^4*b^3 + 30*A*a^3*b^4 + 16*B*a^2*b^5 - 2*A*a*b^6 + B*b^7 - 6*(A*a^6*b
+ 4*B*a^5*b^2 - 6*A*a^4*b^3 - 4*B*a^3*b^4 + A*a^2*b^5)*d*x)*tan(d*x + c))/
((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*tan(d*x + c)^3 + 3
*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*tan(d*x + c)^2 +
 3*(a^10*b + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*d*tan(d*x + c) +
 (a^11 + 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{(a + b\tan(c + dx))^4}\, dx = \text{Exception raised: AttributeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(239) = 478.

Time = 0.13 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.08

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{\frac{6\,(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{6\,(Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{3\,(Ba^4-4\,Aa^3b-6\,Ba^2b}{a^8+4\,a^6}}{}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/6*(6*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(B*a^4 - 4*A*a^3*b - 6*B*
a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^
3 + B*b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8) + (11*B*a^5 - 26*A*a^4*b - 14*B*a^3*b^2 - 4*A*a^2*b^3 - B*a*b^4 -
 2*A*b^5 + 6*(B*a^3*b^2 - 3*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*tan(d*x + c)^2
+ 3*(5*B*a^4*b - 14*A*a^3*b^2 - 12*B*a^2*b^3 + 2*A*a*b^4 - B*b^5)*tan(d*x
+ c))/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6 + (a^6*b^3 + 3*a^4*b^5 + 3*a^
2*b^7 + b^9)*tan(d*x + c)^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*
tan(d*x + c)^2 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(d*x + c))
)/d
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.91

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)(dx+c)}{a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d}$$
$$+\frac{(Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)\log\left(\tan(dx+c)^2+1\right)}{2\,(a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d)}$$
$$-\frac{(Ba^4b-4\,Aa^3b^2-6\,Ba^2b^3+4\,Aab^4+Bb^5)\log\left(|b\tan(dx+c)+a|\right)}{a^8bd+4\,a^6b^3d+6\,a^4b^5d+4\,a^2b^7d+b^9d}$$
$$+\frac{11\,Ba^7-26\,Aa^6b-3\,Ba^5b^2-30\,Aa^4b^3-15\,Ba^3b^4-6\,Aa^2b^5-Bab^6-2\,Ab^7+6\,(Ba^5b^2-3\,Aa^4b^3}{}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="giac")
```

output
```
(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^8*d + 4
*a^6*b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + 1/2*(B*a^4 - 4*A*a^3*b -
 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1)/(a^8*d + 4*a^6*b
^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) - (B*a^4*b - 4*A*a^3*b^2 - 6*B*a
^2*b^3 + 4*A*a*b^4 + B*b^5)*log(abs(b*tan(d*x + c) + a))/(a^8*b*d + 4*a^6*
b^3*d + 6*a^4*b^5*d + 4*a^2*b^7*d + b^9*d) + 1/6*(11*B*a^7 - 26*A*a^6*b -
3*B*a^5*b^2 - 30*A*a^4*b^3 - 15*B*a^3*b^4 - 6*A*a^2*b^5 - B*a*b^6 - 2*A*b^
7 + 6*(B*a^5*b^2 - 3*A*a^4*b^3 - 2*B*a^3*b^4 - 2*A*a^2*b^5 - 3*B*a*b^6 + A
*b^7)*tan(d*x + c)^2 + 3*(5*B*a^6*b - 14*A*a^5*b^2 - 7*B*a^4*b^3 - 12*A*a^
3*b^4 - 13*B*a^2*b^5 + 2*A*a*b^6 - B*b^7)*tan(d*x + c))/((a^2 + b^2)^4*(b*
tan(d*x + c) + a)^3*d)
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.79

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx = -\frac{\frac{-11\,B\,a^5+26\,A\,a^4\,b+14\,B\,a^3\,b^2+4\,A\,a^2\,b^3+B\,a\,b^4+2\,A\,b^5}{6\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{\tan(c+d\,x)\,\left(-5\,B\,a^4\,b+14\,A\,a^3\,b^2+12\,B\,a^2\,b^3-2\,A\,a\,b^4+B\,b^5\right)}{2\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}-\frac{\tan(c+d\,x}{}}{d\,\left(a^3+3\,a^2\,b\tan\left(c+d\,x\right)+3\,a\,b^2\tan\left(c+d\,x\right)^2+b^3\tan\left(c+d\,x\right)^3\right)} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{B}{{\left(a^2+b^2\right)}^2}-\frac{4\,b\,(A\,a+2\,B\,b)}{{\left(a^2+b^2\right)}^3}+\frac{8\,b^3\,(A\,a+B\,b)}{{\left(a^2+b^2\right)}^4}\right)}{d} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)}{2\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{2\,d\,\left(a^4-a^3\,b\,4\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4\mathrm{i}+b^4\right)}$$

input
```
int((A + B*tan(c + d*x))/(a + b*tan(c + d*x))^4,x)
```

output
```
(log(tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4
*1i - a^2*b^2*6i)) - (log(a + b*tan(c + d*x))*(B/(a^2 + b^2)^2 - (4*b*(A*a
 + 2*B*b))/(a^2 + b^2)^3 + (8*b^3*(A*a + B*b))/(a^2 + b^2)^4))/d - ((2*A*b
^5 - 11*B*a^5 + 4*A*a^2*b^3 + 14*B*a^3*b^2 + 26*A*a^4*b + B*a*b^4)/(6*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)*(B*b^5 + 14*A*a^3*b^2 + 1
2*B*a^2*b^3 - 2*A*a*b^4 - 5*B*a^4*b))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)) - (tan(c + d*x)^2*(A*b^5 - 3*A*a^2*b^3 + B*a^3*b^2 - 3*B*a*b^4))/(a^6
+ b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(d*(a^3 + b^3*tan(c + d*x)^3 + 3*a*b^2*tan
(c + d*x)^2 + 3*a^2*b*tan(c + d*x))) + (log(tan(c + d*x) + 1i)*(A*1i + B))
/(2*d*(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.20

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^4}\,dx = \frac{-3\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)^2a^2b^3+\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)^2b^5-6\log\left(\tan\left(dx+\right.\right.}{}$$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

output
```
( - 3*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2*a**2*b**3 + log(tan(c + d*x
)**2 + 1)*tan(c + d*x)**2*b**5 - 6*log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a
**3*b**2 + 2*log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a*b**4 - 3*log(tan(c +
d*x)**2 + 1)*a**4*b + log(tan(c + d*x)**2 + 1)*a**2*b**3 + 6*log(tan(c + d
*x)*b + a)*tan(c + d*x)**2*a**2*b**3 - 2*log(tan(c + d*x)*b + a)*tan(c + d
*x)**2*b**5 + 12*log(tan(c + d*x)*b + a)*tan(c + d*x)*a**3*b**2 - 4*log(ta
n(c + d*x)*b + a)*tan(c + d*x)*a*b**4 + 6*log(tan(c + d*x)*b + a)*a**4*b -
 2*log(tan(c + d*x)*b + a)*a**2*b**3 + 2*tan(c + d*x)**2*a**3*b**2*d*x + 2
*tan(c + d*x)**2*a**2*b**3 - 6*tan(c + d*x)**2*a*b**4*d*x + 2*tan(c + d*x)
**2*b**5 + 4*tan(c + d*x)*a**4*b*d*x - 12*tan(c + d*x)*a**2*b**3*d*x + 2*a
**5*d*x - 3*a**4*b - 6*a**3*b**2*d*x - 4*a**2*b**3 - b**5)/(2*d*(tan(c + d
*x)**2*a**6*b**2 + 3*tan(c + d*x)**2*a**4*b**4 + 3*tan(c + d*x)**2*a**2*b*
*6 + tan(c + d*x)**2*b**8 + 2*tan(c + d*x)*a**7*b + 6*tan(c + d*x)*a**5*b*
*3 + 6*tan(c + d*x)*a**3*b**5 + 2*tan(c + d*x)*a*b**7 + a**8 + 3*a**6*b**2
 + 3*a**4*b**4 + a**2*b**6))
```

3.295 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 302

$$\begin{aligned}
&\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx \\
&= -\frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)\,x}{(a^2+b^2)^4}+\frac{A\log(\sin(c+dx))}{a^4d} \\
&\quad -\frac{b(10a^6Ab+5a^4Ab^3+4a^2Ab^5+Ab^7-4a^7B+4a^5b^2B)\log(a\cos(c+dx)+b\sin(c+dx))}{a^4\,(a^2+b^2)^4\,d} \\
&\quad +\frac{b(Ab-aB)}{3a\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}+\frac{b(3a^2Ab+Ab^3-2a^3B)}{2a^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} \\
&\quad +\frac{b(6a^4Ab+3a^2Ab^3+Ab^5-3a^5B+a^3b^2B)}{a^3\,(a^2+b^2)^3\,d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
-(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*x/(a^2+b^2)^4+A*ln(sin(d*x+
c))/a^4/d-b*(10*A*a^6*b+5*A*a^4*b^3+4*A*a^2*b^5+A*b^7-4*B*a^7+4*B*a^5*b^2)
*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^4/(a^2+b^2)^4/d+1/3*b*(A*b-B*a)/a/(a^2+b^
2)/d/(a+b*tan(d*x+c))^3+1/2*b*(3*A*a^2*b+A*b^3-2*B*a^3)/a^2/(a^2+b^2)^2/d/
(a+b*tan(d*x+c))^2+b*(6*A*a^4*b+3*A*a^2*b^3+A*b^5-3*B*a^5+B*a^3*b^2)/a^3/(
a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.02

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{3\left(-a^4(a-ib)^4(A+iB)\log(i-\tan(c+dx))+2A\left(a^2+b^2\right)^4\log(\tan(c+dx))-a^4(a+ib)^4(A-iB)\log(i+\tan(c+dx))-2b\left(10a^6Ab+5a^4Ab^3+4a^2Ab^5-\right.}{a^2\left(a^2+b^2\right)^2}}{6a^2\left(a^2\right.}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
((3*(-(a^4*(a - I*b)^4*(A + I*B)*Log[I - Tan[c + d*x]]) + 2*A*(a^2 + b^2)^
4*Log[Tan[c + d*x]] - a^4*(a + I*b)^4*(A - I*B)*Log[I + Tan[c + d*x]] - 2*
b*(10*a^6*A*b + 5*a^4*A*b^3 + 4*a^2*A*b^5 + A*b^7 - 4*a^7*B + 4*a^5*b^2*B)
*Log[a + b*Tan[c + d*x]]))/(a^2*(a^2 + b^2)^2) + (2*a*b*(a^2 + b^2)*(A*b -
 a*B))/(a + b*Tan[c + d*x])^3 + (3*b*(3*a^2*A*b + A*b^3 - 2*a^3*B))/(a + b
*Tan[c + d*x])^2 + (6*b*(6*a^4*A*b + 3*a^2*A*b^3 + A*b^5 - 3*a^5*B + a^3*b
^2*B))/(a*(a^2 + b^2)*(a + b*Tan[c + d*x])))/(6*a^2*(a^2 + b^2)^2*d)
```

Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+b\tan(c+dx))^4}dx$$
$$\Big\downarrow \text{4092}$$

$$\frac{\int \frac{3\cot(c+dx)(b(Ab-aB)\tan^2(c+dx)-a(Ab-aB)\tan(c+dx)+A(a^2+b^2))}{(a+b\tan(c+dx))^3}dx}{3a(a^2+b^2)}+\frac{b(Ab-aB)}{3ad(a^2+b^2)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cot(c+dx)(b(Ab-aB)\tan^2(c+dx)-a(Ab-aB)\tan(c+dx)+A(a^2+b^2))}{(a+b\tan(c+dx))^3}dx}{a(a^2+b^2)}+\frac{b(Ab-aB)}{3ad(a^2+b^2)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(Ab-aB)\tan(c+dx)^2-a(Ab-aB)\tan(c+dx)+A(a^2+b^2)}{\tan(c+dx)(a+b\tan(c+dx))^3}dx}{a(a^2+b^2)}+\frac{b(Ab-aB)}{3ad(a^2+b^2)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{4132}$$

$$\frac{\frac{\int \frac{2\cot(c+dx)\left(-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2\right)+A\left(a^2+b^2\right)^2+b\left(-2Ba^3+3Aba^2+Ab^3\right)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}dx}{2a(a^2+b^2)}+\frac{b(-2a^3B+3a^2Ab+Ab^3)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}}{a(a^2+b^2)}+\frac{b(Ab-aB)}{3ad(a^2+b^2)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\cot(c+dx)\left(-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2\right)+A\left(a^2+b^2\right)^2+b\left(-2Ba^3+3Aba^2+Ab^3\right)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^2}dx}{a(a^2+b^2)}+\frac{b(-2a^3B+3a^2Ab+Ab^3)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}}{a(a^2+b^2)}+\frac{b(Ab-aB)}{3ad(a^2+b^2)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2\right)+A\left(a^2+b^2\right)^2+b\left(-2Ba^3+3Aba^2+Ab^3\right)\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a(a^2+b^2)}+\frac{b(-2a^3B+3a^2Ab+Ab^3)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}}{a(a^2+b^2)}+\frac{b(Ab-aB)}{3ad(a^2+b^2)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{4132}$$

$$\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(-\left(\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)a^3\right)+A\left(a^2+b^2\right)^3+b\left(-3Ba^5+6Aba^4+b^2Ba^3+3Ab^3a^2+Ab^5\right)\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{b\left(-3a^5B+6a^4Ab+a\right.}{ad\left(a^2+b^2\right)(a}}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{-\left(\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)a^3\right)+A\left(a^2+b^2\right)^3+b\left(-3Ba^5+6Aba^4+b^2Ba^3+3Ab^3a^2+Ab^5\right)\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a\left(a^2+b^2\right)}+\frac{b\left(-3a^5B+6a^4Ab+a^3b^2B+3a^2Ab\right.}{ad\left(a^2+b^2\right)(a+b\tan(c+dx}}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 4134

$$\frac{\frac{\frac{\frac{A\left(a^2+b^2\right)^3\int \cot(c+dx)dx}{a}-\frac{b\left(-4a^7B+10a^6Ab+4a^5b^2B+5a^4Ab^3+4a^2Ab^5+Ab^7\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^3x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}+\frac{b\left(-\right.}{}}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\frac{A\left(a^2+b^2\right)^3\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b\left(-4a^7B+10a^6Ab+4a^5b^2B+5a^4Ab^3+4a^2Ab^5+Ab^7\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^3x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\downarrow$ 25

$$\frac{\frac{\frac{-\frac{A\left(a^2+b^2\right)^3\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{b\left(-4a^7B+10a^6Ab+4a^5b^2B+5a^4Ab^3+4a^2Ab^5+Ab^7\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^3x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4\right.}{a^2+b^2}}{a\left(a^2+b^2\right)}}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{-\frac{b\left(-4a^7B+10a^6Ab+4a^5b^2B+5a^4Ab^3+4a^2Ab^5+Ab^7\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{A\left(a^2+b^2\right)^3\log(-\sin(c+dx))}{ad}-\frac{a^3x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)}{a^2+b^2}}{a\left(a^2+b^2\right)}+}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 4013$$

$$\frac{b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}+$$

$$\frac{\frac{b\left(-2a^3B+3a^2Ab+Ab^3\right)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}+\frac{\frac{b\left(-3a^5B+6a^4Ab+a^3b^2B+3a^2Ab^3+Ab^5\right)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{\frac{A\left(a^2+b^2\right)^3\log(-\sin(c+dx))}{ad}-\frac{a^3x\left(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B\right)}{a^2+b^2}}{}}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(A*b - a*B))/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) + ((b*(3*a^2*A*
b + A*b^3 - 2*a^3*B))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((-((a^
3*(4*a^3*A*b - 4*a*A*b^3 - a^4*B + 6*a^2*b^2*B - b^4*B)*x)/(a^2 + b^2)) +
(A*(a^2 + b^2)^3*Log[-Sin[c + d*x]])/(a*d) - (b*(10*a^6*A*b + 5*a^4*A*b^3
+ 4*a^2*A*b^5 + A*b^7 - 4*a^7*B + 4*a^5*b^2*B)*Log[a*Cos[c + d*x] + b*Sin[
c + d*x]])/(a*(a^2 + b^2)*d))/(a*(a^2 + b^2)) + (b*(6*a^4*A*b + 3*a^2*A*b^
3 + A*b^5 - 3*a^5*B + a^3*b^2*B))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/
(a*(a^2 + b^2)))/(a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\left(-A a^4+6A a^2b^2-A b^4-4B a^3b+4B a b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-4A a^3b+4A a b^3+B a^4-6B a^2b^2+B b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}+A$
default	$\frac{\frac{\left(-A a^4+6A a^2b^2-A b^4-4B a^3b+4B a b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-4A a^3b+4A a b^3+B a^4-6B a^2b^2+B b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}+A$
parallelrisch	$-20b(a+b\tan(dx+c))^3\left(A a^6b+\frac{1}{2}A a^4b^3+\frac{2}{5}A a^2b^5+\frac{1}{10}A b^7-\frac{2}{5}a^7B+\frac{2}{5}B a^5b^2\right)\ln(a+b\tan(dx+c))-a^4(a+b\tan(dx+c$
norman	$-\frac{\left(4A a^3b-4A a b^3-B a^4+6B a^2b^2-B b^4\right)a^3x}{\left(a^6+3b^2a^4+3b^4a^2+b^6\right)\left(a^2+b^2\right)}-\frac{b^3\left(4A a^3b-4A a b^3-B a^4+6B a^2b^2-B b^4\right)x\tan(dx+c)^3}{\left(a^6+3b^2a^4+3b^4a^2+b^6\right)\left(a^2+b^2\right)}-\frac{b\left(10A a^4b^2+9A a^2b^4\right.}{d a}$
risch	Expression too large to display

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBOSE
)
```

output
```
1/d*(1/(a^2+b^2)^4*(1/2*(-A*a^4+6*A*a^2*b^2-A*b^4-4*B*a^3*b+4*B*a*b^3)*ln(
1+tan(d*x+c)^2)+(-4*A*a^3*b+4*A*a*b^3+B*a^4-6*B*a^2*b^2+B*b^4)*arctan(tan(
d*x+c)))+1/a^4*A*ln(tan(d*x+c))+1/2*b*(3*A*a^2*b+A*b^3-2*B*a^3)/(a^2+b^2)^
2/a^2/(a+b*tan(d*x+c))^2+b*(6*A*a^4*b+3*A*a^2*b^3+A*b^5-3*B*a^5+B*a^3*b^2)
/(a^2+b^2)^3/a^3/(a+b*tan(d*x+c))-b*(10*A*a^6*b+5*A*a^4*b^3+4*A*a^2*b^5+A*
b^7-4*B*a^7+4*B*a^5*b^2)/(a^2+b^2)^4/a^4*ln(a+b*tan(d*x+c))+1/3*(A*b-B*a)*
b/(a^2+b^2)/a/(a+b*tan(d*x+c))^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(299) = 598$.

Time = 0.20 (sec) , antiderivative size = 1126, normalized size of antiderivative = 3.73

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="fri
cas")
```

output
```
-1/6*(48*B*a^8*b^3 - 75*A*a^7*b^4 + 6*B*a^6*b^5 - 42*A*a^5*b^6 + 2*B*a^4*b
^7 - 11*A*a^3*b^8 - (26*B*a^7*b^4 - 47*A*a^6*b^5 - 18*B*a^5*b^6 - 6*A*a^4*
b^7 - 3*A*a^2*b^9 + 6*(B*a^8*b^3 - 4*A*a^7*b^4 - 6*B*a^6*b^5 + 4*A*a^5*b^6
 + B*a^4*b^7)*d*x)*tan(d*x + c)^3 - 6*(B*a^11 - 4*A*a^10*b - 6*B*a^9*b^2 +
 4*A*a^8*b^3 + B*a^7*b^4)*d*x - 3*(20*B*a^8*b^3 - 35*A*a^7*b^4 - 22*B*a^6*
b^5 + 12*A*a^5*b^6 + 2*B*a^4*b^7 + 5*A*a^3*b^8 + 2*A*a*b^10 + 6*(B*a^9*b^2
 - 4*A*a^8*b^3 - 6*B*a^7*b^4 + 4*A*a^6*b^5 + B*a^5*b^6)*d*x)*tan(d*x + c)^
2 - 3*(A*a^11 + 4*A*a^9*b^2 + 6*A*a^7*b^4 + 4*A*a^5*b^6 + A*a^3*b^8 + (A*a
^8*b^3 + 4*A*a^6*b^5 + 6*A*a^4*b^7 + 4*A*a^2*b^9 + A*b^11)*tan(d*x + c)^3
+ 3*(A*a^9*b^2 + 4*A*a^7*b^4 + 6*A*a^5*b^6 + 4*A*a^3*b^8 + A*a*b^10)*tan(d
*x + c)^2 + 3*(A*a^10*b + 4*A*a^8*b^3 + 6*A*a^6*b^5 + 4*A*a^4*b^7 + A*a^2*
b^9)*tan(d*x + c))*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) - 3*(4*B*a^10*
b - 10*A*a^9*b^2 - 4*B*a^8*b^3 - 5*A*a^7*b^4 - 4*A*a^5*b^6 - A*a^3*b^8 + (
4*B*a^7*b^4 - 10*A*a^6*b^5 - 4*B*a^5*b^6 - 5*A*a^4*b^7 - 4*A*a^2*b^9 - A*b
^11)*tan(d*x + c)^3 + 3*(4*B*a^8*b^3 - 10*A*a^7*b^4 - 4*B*a^6*b^5 - 5*A*a^
5*b^6 - 4*A*a^3*b^8 - A*a*b^10)*tan(d*x + c)^2 + 3*(4*B*a^9*b^2 - 10*A*a^8
*b^3 - 4*B*a^7*b^4 - 5*A*a^6*b^5 - 4*A*a^4*b^7 - A*a^2*b^9)*tan(d*x + c))*
log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))
- 3*(12*B*a^9*b^2 - 20*A*a^8*b^3 - 30*B*a^7*b^4 + 37*A*a^6*b^5 + 2*B*a^5*b
^6 + 18*A*a^4*b^7 + 5*A*a^2*b^9 + 6*(B*a^10*b - 4*A*a^9*b^2 - 6*B*a^8*b...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.92

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$

$$= \frac{\frac{6\,(Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{6\,(4\,Ba^7b-10\,Aa^6b^2-4\,Ba^5b^3-5\,Aa^4b^4-4\,Aa^2b^6-Ab^8)\log(b\tan(dx+c)+a)}{a^{12}+4\,a^{10}b^2+6\,a^8b^4+4\,a^6b^6+a^4b^8} - \frac{3\,(Aa^4+}{}}{}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="max
ima")
```

output
```
1/6*(6*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*(d*x + c)/(a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 6*(4*B*a^7*b - 10*A*a^6*b^2
 - 4*B*a^5*b^3 - 5*A*a^4*b^4 - 4*A*a^2*b^6 - A*b^8)*log(b*tan(d*x + c) + a
)/(a^12 + 4*a^10*b^2 + 6*a^8*b^4 + 4*a^6*b^6 + a^4*b^8) - 3*(A*a^4 + 4*B*a
^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (26*B*a^7*b - 47*A*a^6*b^2 + 4*B*a
^5*b^3 - 34*A*a^4*b^4 + 2*B*a^3*b^5 - 11*A*a^2*b^6 + 6*(3*B*a^5*b^3 - 6*A*
a^4*b^4 - B*a^3*b^5 - 3*A*a^2*b^6 - A*b^8)*tan(d*x + c)^2 + 3*(14*B*a^6*b^
2 - 27*A*a^5*b^3 - 2*B*a^4*b^4 - 16*A*a^3*b^5 - 5*A*a*b^7)*tan(d*x + c))/(
a^12 + 3*a^10*b^2 + 3*a^8*b^4 + a^6*b^6 + (a^9*b^3 + 3*a^7*b^5 + 3*a^5*b^7
 + a^3*b^9)*tan(d*x + c)^3 + 3*(a^10*b^2 + 3*a^8*b^4 + 3*a^6*b^6 + a^4*b^8
)*tan(d*x + c)^2 + 3*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*tan(d*x +
c)) + 6*A*log(tan(d*x + c))/a^4)/d
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.78

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{(Ba^4 - 4\,Aa^3b - 6\,Ba^2b^2 + 4\,Aab^3 + Bb^4)(dx+c)}{a^8d + 4\,a^6b^2d + 6\,a^4b^4d + 4\,a^2b^6d + b^8d}$$
$$- \frac{(Aa^4 + 4\,Ba^3b - 6\,Aa^2b^2 - 4\,Bab^3 + Ab^4)\log\left(\tan\left(dx+c\right)^2 + 1\right)}{2\left(a^8d + 4\,a^6b^2d + 6\,a^4b^4d + 4\,a^2b^6d + b^8d\right)}$$
$$+ \frac{(4\,Ba^7b^2 - 10\,Aa^6b^3 - 4\,Ba^5b^4 - 5\,Aa^4b^5 - 4\,Aa^2b^7 - Ab^9)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^{12}bd + 4\,a^{10}b^3d + 6\,a^8b^5d + 4\,a^6b^7d + a^4b^9d}$$
$$+ \frac{A\log\left(\left|\tan\left(dx+c\right)\right|\right)}{a^4d}$$
$$- \frac{26\,Ba^{10}b - 47\,Aa^9b^2 + 30\,Ba^8b^3 - 81\,Aa^7b^4 + 6\,Ba^6b^5 - 45\,Aa^5b^6 + 2\,Ba^4b^7 - 11\,Aa^3b^8 + 6\,(3\,Ba^8b^3}{}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="gia
c")
```

output
```
(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*(d*x + c)/(a^8*d + 4
*a^6*b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) - 1/2*(A*a^4 + 4*B*a^3*b -
 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*log(tan(d*x + c)^2 + 1)/(a^8*d + 4*a^6*b
^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + (4*B*a^7*b^2 - 10*A*a^6*b^3 -
4*B*a^5*b^4 - 5*A*a^4*b^5 - 4*A*a^2*b^7 - A*b^9)*log(abs(b*tan(d*x + c) +
a))/(a^12*b*d + 4*a^10*b^3*d + 6*a^8*b^5*d + 4*a^6*b^7*d + a^4*b^9*d) + A*
log(abs(tan(d*x + c)))/(a^4*d) - 1/6*(26*B*a^10*b - 47*A*a^9*b^2 + 30*B*a^
8*b^3 - 81*A*a^7*b^4 + 6*B*a^6*b^5 - 45*A*a^5*b^6 + 2*B*a^4*b^7 - 11*A*a^3
*b^8 + 6*(3*B*a^8*b^3 - 6*A*a^7*b^4 + 2*B*a^6*b^5 - 9*A*a^5*b^6 - B*a^4*b^
7 - 4*A*a^3*b^8 - A*a*b^10)*tan(d*x + c)^2 + 3*(14*B*a^9*b^2 - 27*A*a^8*b^
3 + 12*B*a^7*b^4 - 43*A*a^6*b^5 - 2*B*a^5*b^6 - 21*A*a^4*b^7 - 5*A*a^2*b^9
)*tan(d*x + c))/((a^2 + b^2)^4*(b*tan(d*x + c) + a)^3*a^4*d)
```

Mupad [B] (verification not implemented)

Time = 7.28 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.60

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{-26\,B\,a^5\,b+47\,A\,a^4\,b^2-4\,B\,a^3\,b^3+34\,A\,a^2\,b^4-2\,B\,a\,b^5+11\,A\,b^6}{6\,a\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{\tan(c+d\,x)^2\,(-3\,B\,a^5\,b^3+6\,A\,a^4\,b^4+B\,a^3\,b^5+3\,A\,a^2\,b^6+A\,b^8)}{a^3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{\tan(c+d}{}}{d\,\left(a^3+3\,a^2\,b\tan\left(c+d\,x\right)+3\,a\,b^2\tan\left(c+d\,x\right)^2+b^3\tan\left(c+d\,x\right.\right.}$$
$$+\frac{A\,\ln\left(\tan(c+d\,x)\right)}{a^4\,d}-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{2\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)}{2\,d\,\left(a^4+a^3\,b\,4\mathrm{i}-6\,a^2\,b^2-a\,b^3\,4\mathrm{i}+b^4\right)}$$
$$-\frac{b\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-4\,B\,a^7+10\,A\,a^6\,b+4\,B\,a^5\,b^2+5\,A\,a^4\,b^3+4\,A\,a^2\,b^5+A\,b^7\right)}{a^4\,d\,\left(a^2+b^2\right)^4}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^4,x)
```

output
```
((11*A*b^6 + 34*A*a^2*b^4 + 47*A*a^4*b^2 - 4*B*a^3*b^3 - 2*B*a*b^5 - 26*B*
a^5*b)/(6*a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^2*(A*b^8
+ 3*A*a^2*b^6 + 6*A*a^4*b^4 + B*a^3*b^5 - 3*B*a^5*b^3))/(a^3*(a^6 + b^6 +
3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)*(5*A*b^7 + 16*A*a^2*b^5 + 27*A*a^4
*b^3 + 2*B*a^3*b^4 - 14*B*a^5*b^2))/(2*a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2)))/(d*(a^3 + b^3*tan(c + d*x)^3 + 3*a*b^2*tan(c + d*x)^2 + 3*a^2*b*tan
(c + d*x))) + (A*log(tan(c + d*x)))/(a^4*d) - (log(tan(c + d*x) + 1i)*(A*1
i + B))/(2*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)) - (log(ta
n(c + d*x) - 1i)*(A + B*1i))/(2*d*(a^3*b*4i - a*b^3*4i + a^4 + b^4 - 6*a^2
*b^2)) - (b*log(a + b*tan(c + d*x))*(A*b^7 - 4*B*a^7 + 4*A*a^2*b^5 + 5*A*a
^4*b^3 + 4*B*a^5*b^2 + 10*A*a^6*b))/(a^4*d*(a^2 + b^2)^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1290, normalized size of antiderivative = 4.27

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

output
```
( - 4*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a**7*b + 12*c
os(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a**5*b**3 - 24*cos(c
 + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)
*a**5*b**3 - 12*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2
)*b - a)*sin(c + d*x)*a**3*b**5 - 4*cos(c + d*x)*log(tan((c + d*x)/2)**2*a
 - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)*a*b**7 + 4*cos(c + d*x)*log(tan(
(c + d*x)/2))*sin(c + d*x)*a**7*b + 12*cos(c + d*x)*log(tan((c + d*x)/2))*
sin(c + d*x)*a**5*b**3 + 12*cos(c + d*x)*log(tan((c + d*x)/2))*sin(c + d*x
)*a**3*b**5 + 4*cos(c + d*x)*log(tan((c + d*x)/2))*sin(c + d*x)*a*b**7 - 1
2*cos(c + d*x)*sin(c + d*x)*a**6*b**2*d*x + 4*cos(c + d*x)*sin(c + d*x)*a*
*4*b**4*d*x + 2*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**8 - 8*log(
tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**6*b**2 + 6*log(tan((c + d*x)/2
)**2 + 1)*sin(c + d*x)**2*a**4*b**4 - 2*log(tan((c + d*x)/2)**2 + 1)*a**8
+ 6*log(tan((c + d*x)/2)**2 + 1)*a**6*b**2 + 12*log(tan((c + d*x)/2)**2*a
- 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*a**6*b**2 - 6*log(tan((c + d*x
)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*a**4*b**4 - 4*log(ta
n((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*a**2*b**6
- 2*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*
b**8 - 12*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*a**6*b**2
- 6*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*a**4*b**4 - 2...
```

3.296 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 399

$$\begin{aligned}
&\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx \\
&= -\frac{(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)\,x}{(a^2+b^2)^4} - \frac{(4Ab-aB)\log(\sin(c+dx))}{a^5d} \\
&\quad + \frac{b^2(20a^6Ab+24a^4Ab^3+16a^2Ab^5+4Ab^7-10a^7B-5a^5b^2B-4a^3b^4B-ab^6B)\log(a\cos(c+dx)+b\,s}{a^5\,(a^2+b^2)^4\,d} \\
&\quad - \frac{b(3a^2A+4Ab^2-abB)}{3a^2\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} - \frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3} \\
&\quad - \frac{b(2a^4A+8a^2Ab^2+4Ab^4-3a^3bB-ab^3B)}{2a^3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} \\
&\quad - \frac{b(a^6A+13a^4Ab^2+12a^2Ab^4+4Ab^6-6a^5bB-3a^3b^3B-ab^5B)}{a^4\,(a^2+b^2)^3\,d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
-(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*x/(a^2+b^2)^4-(4*A*b-B*a)*l
n(sin(d*x+c))/a^5/d+b^2*(20*A*a^6*b+24*A*a^4*b^3+16*A*a^2*b^5+4*A*b^7-10*B
*a^7-5*B*a^5*b^2-4*B*a^3*b^4-B*a*b^6)*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^5/(a
^2+b^2)^4/d-1/3*b*(3*A*a^2+4*A*b^2-B*a*b)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))
^3-A*cot(d*x+c)/a/d/(a+b*tan(d*x+c))^3-1/2*b*(2*A*a^4+8*A*a^2*b^2+4*A*b^4-
3*B*a^3*b-B*a*b^3)/a^3/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2-b*(A*a^6+13*A*a^4*
b^2+12*A*a^2*b^4+4*A*b^6-6*B*a^5*b-3*B*a^3*b^3-B*a*b^5)/a^4/(a^2+b^2)^3/d/
(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.36 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.89

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{-\frac{6A\cot(c+dx)}{a^4} + \frac{3i(A+iB)\log(i-\tan(c+dx))}{(a+ib)^4} + \frac{6(-4Ab+aB)\log(\tan(c+dx))}{a^5} - \frac{3(iA+B)\log(i+\tan(c+dx))}{(a-ib)^4} - \frac{6b^2(-20a^6Ab-24a^4A}{}}{}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
((-6*A*Cot[c + d*x])/a^4 + ((3*I)*(A + I*B)*Log[I - Tan[c + d*x]])/(a + I*
b)^4 + (6*(-4*A*b + a*B)*Log[Tan[c + d*x]])/a^5 - (3*(I*A + B)*Log[I + Tan
[c + d*x]])/(a - I*b)^4 - (6*b^2*(-20*a^6*A*b - 24*a^4*A*b^3 - 16*a^2*A*b^
5 - 4*A*b^7 + 10*a^7*B + 5*a^5*b^2*B + 4*a^3*b^4*B + a*b^6*B)*Log[a + b*Ta
n[c + d*x]])/(a^5*(a^2 + b^2)^4) + (2*b^2*(-(A*b) + a*B))/(a^2*(a^2 + b^2)
*(a + b*Tan[c + d*x])^3) + (3*b^2*(-4*a^2*A*b - 2*A*b^3 + 3*a^3*B + a*b^2*
B))/(a^3*(a^2 + b^2)^2*(a + b*Tan[c + d*x])^2) + (6*b^2*(-10*a^4*A*b - 9*a
^2*A*b^3 - 3*A*b^5 + 6*a^5*B + 3*a^3*b^2*B + a*b^4*B))/(a^4*(a^2 + b^2)^3*
(a + b*Tan[c + d*x])))/(6*d)
```

Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 4092, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \ 4092$$

$$-\frac{\int \frac{\cot(c+dx)\left(4Ab\tan^2(c+dx)+aA\tan(c+dx)+4Ab-aB\right)}{(a+b\tan(c+dx))^4}dx}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{4Ab\tan(c+dx)^2+aA\tan(c+dx)+4Ab-aB}{\tan(c+dx)(a+b\tan(c+dx))^4}dx}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \ 4132$$

$$-\frac{\frac{\int \frac{3\cot(c+dx)\left((aA+bB)\tan(c+dx)a^2+b\left(3Aa^2-bBa+4Ab^2\right)\tan^2(c+dx)+\left(a^2+b^2\right)(4Ab-aB)\right)}{(a+b\tan(c+dx))^3}dx}{3a(a^2+b^2)}+\frac{b(3a^2A-abB+4Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^3}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{\cot(c+dx)\left((aA+bB)\tan(c+dx)a^2+b\left(3Aa^2-bBa+4Ab^2\right)\tan^2(c+dx)+\left(a^2+b^2\right)(4Ab-aB)\right)}{(a+b\tan(c+dx))^3}dx}{a(a^2+b^2)}+\frac{b(3a^2A-abB+4Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^3}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\int \frac{(aA+bB)\tan(c+dx)a^2+b\left(3Aa^2-bBa+4Ab^2\right)\tan(c+dx)^2+\left(a^2+b^2\right)(4Ab-aB)}{\tan(c+dx)(a+b\tan(c+dx))^3}dx}{a(a^2+b^2)}+\frac{b(3a^2A-abB+4Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^3}}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\frac{\frac{\int \frac{2\cot(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+b\left(2Aa^4-3bBa^3+8Ab^2a^2-b^3Ba+4Ab^4\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2(4Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{2a\left(a^2+b^2\right)}+\frac{b\left(2a^4A-3a^3bB+8a^2Ab^2\cdots\right.}{2ad\left(a^2+b^2\right)(a+b\tan\cdots}}{a(a^2+b^2)}\cdots}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+b\left(2Aa^4-3bBa^3+8Ab^2a^2-b^3Ba+4Ab^4\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2(4Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{a\left(a^2+b^2\right)}+\frac{b\left(2a^4A-3a^3bB+8a^2Ab^2-\cdots\right.}{2ad\left(a^2+b^2\right)(a+b\tan\cdots}}{a(a^2+b^2)}\cdots}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\int \frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+b\left(2Aa^4-3bBa^3+8Ab^2a^2-b^3Ba+4Ab^4\right)\tan(c+dx)^2+\left(a^2+b^2\right)^2(4Ab-aB)}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a\left(a^2+b^2\right)}+\frac{b\left(2a^4A-3a^3bB+8a^2Ab^2-ab^3B+4Ab^4\cdots\right.}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}}{a(a^2+b^2)}\cdots}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)a^4+b\left(Aa^6-6bBa^5+13Ab^2a^4-3b^3Ba^3+12Ab^4a^2-b^5Ba+4Ab^6\right)\tan^2(c+dx)+\left(a^2+b^2\right)^3(4Ab-aB)\right)}{a+b\tan(c+dx)}d\cdots}{a\left(a^2+b^2\right)}\cdots}{a\left(a^2+b^2\right)}\cdots}{a(a^2+b^2)}\cdots}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\frac{\int \frac{\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)a^4+b\left(Aa^6-6bBa^5+13Ab^2a^4-3b^3Ba^3+12Ab^4a^2-b^5Ba+4Ab^6\right)\tan(c+dx)^2+\left(a^2+b^2\right)^3(4Ab-aB)}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a\left(a^2+b^2\right)}+\frac{b\left(a^6A-6\cdots\right.}{\cdots}}{a\left(a^2+b^2\right)}\cdots}{a\left(a^2+b^2\right)}\cdots}{a(a^2+b^2)}\cdots}{a}\cdots$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 4134

$$-\frac{\frac{\frac{\frac{\left(a^2+b^2\right)^3(4Ab-aB)\int\cot(c+dx)dx}{a}-\frac{b^2\left(-10a^7B+20a^6Ab-5a^5b^2B+24a^4Ab^3-4a^3b^4B+16a^2Ab^5-ab^6B+4Ab^7\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^4x\left(a^4A+4a^3bB-\cdots\right.}{a\cdots}}{a\left(a^2+b^2\right)}\cdots}{a\left(a^2+b^2\right)}\cdots}{a(a^2+b^2)}\cdots}{\cdots}$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{\left(a^2+b^2\right)^3(4Ab-aB)\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{b^2\left(-10a^7B+20a^6Ab-5a^5b^2B+24a^4Ab^3-4a^3b^4B+16a^2Ab^5-ab^6B+4Ab^7\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^4x\left(a^4A+4\cdots\right.}{\cdots}}{a\left(a^2+b^2\right)}\cdots}{a\left(a^2+b^2\right)}\cdots}{a(a^2+b^2)}\cdots}{\cdots}$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{-\frac{\left(a^2+b^2\right)^3(4Ab-aB)\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{b^2\left(-10a^7B+20a^6Ab-5a^5b^2B+24a^4Ab^3-4a^3b^4B+16a^2Ab^5-ab^6B+4Ab^7\right)\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{a^4x\left(a^4\cdots\right.}{\cdots}}{a\left(a^2+b^2\right)}\cdots}{a\left(a^2+b^2\right)}\cdots}{a(a^2+b^2)}\cdots$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 3956

$$-\frac{\frac{\frac{\frac{-\frac{b^2\left(-10a^7B+20a^6Ab-5a^5b^2B+24a^4Ab^3-4a^3b^4B+16a^2Ab^5-ab^6B+4Ab^7\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{\left(a^2+b^2\right)^3(4Ab-aB)\log(-\sin(c+dx))}{ad}+\frac{a^4x\left(a^4A+4a^3\cdots\right.}{\cdots}}{a\left(a^2+b^2\right)}\cdots}{a\left(a^2+b^2\right)}\cdots}{a(a^2+b^2)}\cdots}{\cdots}$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{4013}$$

$$-\frac{b(3a^2A-abB+4Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^3}+\frac{\frac{b\left(2a^4A-3a^3bB+8a^2Ab^2-ab^3B+4Ab^4\right)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{\frac{b\left(a^6A-6a^5bB+13a^4Ab^2-3a^3b^3B+12a^2Ab^4-ab^5B+4Ab^6\right)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{\left(a^2+b^2\right)^3\left(\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^3}$$

input

```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output

```
-((A*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x])^3)) - ((b*(3*a^2*A + 4*A*b^2
 - a*b*B))/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) + ((b*(2*a^4*A + 8*a^
2*A*b^2 + 4*A*b^4 - 3*a^3*b*B - a*b^3*B))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c
 + d*x])^2) + (((a^4*(a^4*A - 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B - 4*a*b^3*B)*
x)/(a^2 + b^2) + ((a^2 + b^2)^3*(4*A*b - a*B)*Log[-Sin[c + d*x]])/(a*d) -
(b^2*(20*a^6*A*b + 24*a^4*A*b^3 + 16*a^2*A*b^5 + 4*A*b^7 - 10*a^7*B - 5*a^
5*b^2*B - 4*a^3*b^4*B - a*b^6*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*
(a^2 + b^2)*d))/(a*(a^2 + b^2)) + (b*(a^6*A + 13*a^4*A*b^2 + 12*a^2*A*b^4
 + 4*A*b^6 - 6*a^5*b*B - 3*a^3*b^3*B - a*b^5*B))/(a*(a^2 + b^2)*d*(a + b*Ta
n[c + d*x])))/(a*(a^2 + b^2)))/(a*(a^2 + b^2)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\frac{\left(4A a^3 b-4Aa b^3-B a^4+6B a^2 b^2-B b^4\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-A a^4+6A a^2 b^2-A b^4-4B a^3 b+4Ba b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{a^4 \mathrm{t}}{}}{}$
default	$\frac{\frac{\frac{\left(4A a^3 b-4Aa b^3-B a^4+6B a^2 b^2-B b^4\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(-A a^4+6A a^2 b^2-A b^4-4B a^3 b+4Ba b^3\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{a^4 \mathrm{t}}{}}{}$
norman	$\frac{\frac{b\left(6A a^6 b+33A a^4 b^3+35A a^2 b^5+12A b^7-10B a^5 b^2-9B a^3 b^4-3Ba b^6\right)\tan(dx+c)^2}{d a^3\left(a^6+3b^2 a^4+3b^4 a^2+b^6\right)}-\frac{A}{ad}-\frac{b^3\left(A a^4-6A a^2 b^2+A b^4+4B a^3 b-4Ba\right.}{\left(a^6+3b^2 a^4+3b^4 a^2+b^6\right)\left(a^2\right.}}{}$
parallelrisch	Expression too large to display
risch	Expression too large to display

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^4*(1/2*(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*ln(1
+tan(d*x+c)^2)+(-A*a^4+6*A*a^2*b^2-A*b^4-4*B*a^3*b+4*B*a*b^3)*arctan(tan(d
*x+c)))-1/a^4*A/tan(d*x+c)+(-4*A*b+B*a)/a^5*ln(tan(d*x+c))-1/2*b^2*(4*A*a^
2*b+2*A*b^3-3*B*a^3-B*a*b^2)/(a^2+b^2)^2/a^3/(a+b*tan(d*x+c))^2-b^2*(10*A*
a^4*b+9*A*a^2*b^3+3*A*b^5-6*B*a^5-3*B*a^3*b^2-B*a*b^4)/(a^2+b^2)^3/a^4/(a+
b*tan(d*x+c))+b^2*(20*A*a^6*b+24*A*a^4*b^3+16*A*a^2*b^5+4*A*b^7-10*B*a^7-5
*B*a^5*b^2-4*B*a^3*b^4-B*a*b^6)/(a^2+b^2)^4/a^5*ln(a+b*tan(d*x+c))-1/3*(A*
b-B*a)*b^2/(a^2+b^2)/a^2/(a+b*tan(d*x+c))^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1510 vs. $2(393) = 786$.

Time = 0.34 (sec) , antiderivative size = 1510, normalized size of antiderivative = 3.78

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="f
ricas")
```

output

```
-1/6*(6*A*a^12 + 24*A*a^10*b^2 + 36*A*a^8*b^4 + 24*A*a^6*b^6 + 6*A*a^4*b^8
 + (47*B*a^7*b^5 - 74*A*a^6*b^6 + 6*B*a^5*b^7 - 42*A*a^4*b^8 + 3*B*a^3*b^9
 - 12*A*a^2*b^10 + 6*(A*a^9*b^3 + 4*B*a^8*b^4 - 6*A*a^7*b^5 - 4*B*a^6*b^6
+ A*a^5*b^7)*d*x)*tan(d*x + c)^4 + 3*(2*A*a^9*b^3 + 35*B*a^8*b^4 - 46*A*a^
7*b^5 - 12*B*a^6*b^6 + 8*A*a^5*b^7 - 5*B*a^4*b^8 + 20*A*a^3*b^9 - 2*B*a^2*
b^10 + 8*A*a*b^11 + 6*(A*a^10*b^2 + 4*B*a^9*b^3 - 6*A*a^8*b^4 - 4*B*a^7*b^
5 + A*a^6*b^6)*d*x)*tan(d*x + c)^3 + 3*(6*A*a^10*b^2 + 20*B*a^9*b^3 - 6*A*
a^8*b^4 - 37*B*a^7*b^5 + 80*A*a^6*b^6 - 18*B*a^5*b^7 + 68*A*a^4*b^8 - 5*B*
a^3*b^9 + 20*A*a^2*b^10 + 6*(A*a^11*b + 4*B*a^10*b^2 - 6*A*a^9*b^3 - 4*B*a
^8*b^4 + A*a^7*b^5)*d*x)*tan(d*x + c)^2 - 3*((B*a^9*b^3 - 4*A*a^8*b^4 + 4*
B*a^7*b^5 - 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 + 4*B*a^3*b^9 - 16*A
*a^2*b^10 + B*a*b^11 - 4*A*b^12)*tan(d*x + c)^4 + 3*(B*a^10*b^2 - 4*A*a^9*
b^3 + 4*B*a^8*b^4 - 16*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 + 4*B*a^4*b^
8 - 16*A*a^3*b^9 + B*a^2*b^10 - 4*A*a*b^11)*tan(d*x + c)^3 + 3*(B*a^11*b -
 4*A*a^10*b^2 + 4*B*a^9*b^3 - 16*A*a^8*b^4 + 6*B*a^7*b^5 - 24*A*a^6*b^6 +
4*B*a^5*b^7 - 16*A*a^4*b^8 + B*a^3*b^9 - 4*A*a^2*b^10)*tan(d*x + c)^2 + (B
*a^12 - 4*A*a^11*b + 4*B*a^10*b^2 - 16*A*a^9*b^3 + 6*B*a^8*b^4 - 24*A*a^7*
b^5 + 4*B*a^6*b^6 - 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*tan(d*x + c))*
log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1)) + 3*((10*B*a^7*b^5 - 20*A*a^6*b^6
 + 5*B*a^5*b^7 - 24*A*a^4*b^8 + 4*B*a^3*b^9 - 16*A*a^2*b^10 + B*a*b^11 ...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

input

```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

output

```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 698, normalized size of antiderivative = 1.75

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx =$$

$$-\frac{\frac{6\,(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{6\,(10\,Ba^7b^2-20\,Aa^6b^3+5\,Ba^5b^4-24\,Aa^4b^5+4\,Ba^3b^6-16\,Aa^2b^7+Bab^8-4\,Ab^9)\log(b\,t}{a^{13}+4\,a^{11}b^2+6\,a^9b^4+4\,a^7b^6+a^5b^8}}{}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="m
axima")
```

output
```
-1/6*(6*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 6*(10*B*a^7*b^2 - 20*A*a^6
*b^3 + 5*B*a^5*b^4 - 24*A*a^4*b^5 + 4*B*a^3*b^6 - 16*A*a^2*b^7 + B*a*b^8 -
 4*A*b^9)*log(b*tan(d*x + c) + a)/(a^13 + 4*a^11*b^2 + 6*a^9*b^4 + 4*a^7*b
^6 + a^5*b^8) + 3*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*lo
g(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (6
*A*a^9 + 18*A*a^7*b^2 + 18*A*a^5*b^4 + 6*A*a^3*b^6 + 6*(A*a^6*b^3 - 6*B*a^
5*b^4 + 13*A*a^4*b^5 - 3*B*a^3*b^6 + 12*A*a^2*b^7 - B*a*b^8 + 4*A*b^9)*tan
(d*x + c)^3 + 3*(6*A*a^7*b^2 - 27*B*a^6*b^3 + 62*A*a^5*b^4 - 16*B*a^4*b^5
+ 60*A*a^3*b^6 - 5*B*a^2*b^7 + 20*A*a*b^8)*tan(d*x + c)^2 + (18*A*a^8*b -
47*B*a^7*b^2 + 128*A*a^6*b^3 - 34*B*a^5*b^4 + 130*A*a^4*b^5 - 11*B*a^3*b^6
 + 44*A*a^2*b^7)*tan(d*x + c))/((a^10*b^3 + 3*a^8*b^5 + 3*a^6*b^7 + a^4*b^
9)*tan(d*x + c)^4 + 3*(a^11*b^2 + 3*a^9*b^4 + 3*a^7*b^6 + a^5*b^8)*tan(d*x
 + c)^3 + 3*(a^12*b + 3*a^10*b^3 + 3*a^8*b^5 + a^6*b^7)*tan(d*x + c)^2 + (
a^13 + 3*a^11*b^2 + 3*a^9*b^4 + a^7*b^6)*tan(d*x + c)) - 6*(B*a - 4*A*b)*l
og(tan(d*x + c))/a^5)/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.67

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= -\frac{(Aa^4+4\,Ba^3b-6\,Aa^2b^2-4\,Bab^3+Ab^4)(dx+c)}{a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d}$$
$$-\frac{(Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)\log\left(\tan(dx+c)^2+1\right)}{2\,(a^8d+4\,a^6b^2d+6\,a^4b^4d+4\,a^2b^6d+b^8d)}$$
$$-\frac{(10\,Ba^7b^3-20\,Aa^6b^4+5\,Ba^5b^5-24\,Aa^4b^6+4\,Ba^3b^7-16\,Aa^2b^8+Bab^9-4\,Ab^{10})\log\left(|b\tan(dx+c\right.}{a^{13}bd+4\,a^{11}b^3d+6\,a^9b^5d+4\,a^7b^7d+a^5b^9d}$$
$$+\frac{(Ba-4\,Ab)\log\left(|\tan(dx+c)|\right)}{a^5d}$$
$$-\frac{6\,Aa^{12}+24\,Aa^{10}b^2+36\,Aa^8b^4+24\,Aa^6b^6+6\,Aa^4b^8+6\,(Aa^9b^3-6\,Ba^8b^4+14\,Aa^7b^5-9\,Ba^6b^6+25}{}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="g
iac")
```

output
```
-(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*(d*x + c)/(a^8*d +
4*a^6*b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) - 1/2*(B*a^4 - 4*A*a^3*b
- 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(tan(d*x + c)^2 + 1)/(a^8*d + 4*a^6*
b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) - (10*B*a^7*b^3 - 20*A*a^6*b^4
+ 5*B*a^5*b^5 - 24*A*a^4*b^6 + 4*B*a^3*b^7 - 16*A*a^2*b^8 + B*a*b^9 - 4*A*
b^10)*log(abs(b*tan(d*x + c) + a))/(a^13*b*d + 4*a^11*b^3*d + 6*a^9*b^5*d
+ 4*a^7*b^7*d + a^5*b^9*d) + (B*a - 4*A*b)*log(abs(tan(d*x + c)))/(a^5*d)
- 1/6*(6*A*a^12 + 24*A*a^10*b^2 + 36*A*a^8*b^4 + 24*A*a^6*b^6 + 6*A*a^4*b^
8 + 6*(A*a^9*b^3 - 6*B*a^8*b^4 + 14*A*a^7*b^5 - 9*B*a^6*b^6 + 25*A*a^5*b^7
 - 4*B*a^4*b^8 + 16*A*a^3*b^9 - B*a^2*b^10 + 4*A*a*b^11)*tan(d*x + c)^3 +
3*(6*A*a^10*b^2 - 27*B*a^9*b^3 + 68*A*a^8*b^4 - 43*B*a^7*b^5 + 122*A*a^6*b
^6 - 21*B*a^5*b^7 + 80*A*a^4*b^8 - 5*B*a^3*b^9 + 20*A*a^2*b^10)*tan(d*x +
c)^2 + (18*A*a^11*b - 47*B*a^10*b^2 + 146*A*a^9*b^3 - 81*B*a^8*b^4 + 258*A
*a^7*b^5 - 45*B*a^6*b^6 + 174*A*a^5*b^7 - 11*B*a^4*b^8 + 44*A*a^3*b^9)*tan
(d*x + c))/((a^2 + b^2)^4*(b*tan(d*x + c) + a)^3*a^5*d*tan(d*x + c))
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.44

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{b^2\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(-10\,B\,a^7+20\,A\,a^6\,b-5\,B\,a^5\,b^2+24\,A\,a^4\,b^3-4\,B\,a^3\,b^4+16\,A\,a^2\,b^5-B\,a\,b^6\right.}{a^5\,d\,{\left(a^2+b^2\right)}^4}$$
$$-\frac{\ln\left(\tan(c+d\,x)\right)\,\left(4\,A\,b-B\,a\right)}{a^5\,d}-\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(A+B\,1\mathrm{i}\right)}{2\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B+A\,1\mathrm{i}\right)}{2\,d\,\left(a^4-a^3\,b\,4\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4\mathrm{i}+b^4\right)}$$
$$-\frac{\frac{A}{a}+\frac{\tan(c+d\,x)^3\,\left(A\,a^6\,b^3-6\,B\,a^5\,b^4+13\,A\,a^4\,b^5-3\,B\,a^3\,b^6+12\,A\,a^2\,b^7-B\,a\,b^8+4\,A\,b^9\right)}{a^4\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{\tan(c+d\,x)^2\,\left(6\,A\,a^6\,b^2-27\,B\,a^5\,b^3+62\,A\,a^4\,b^4\right.}{2\,a^3\,\left(a^6+3\,a^4\,b^2\right.}}{d\,\left(a^3\,\tan\left(c+d\,x\right)+3\,a^2\,b\,\tan\left(c+d\,x\right)^2\right.}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^4,x)
```

output
```
(b^2*log(a + b*tan(c + d*x))*(4*A*b^7 - 10*B*a^7 + 16*A*a^2*b^5 + 24*A*a^4
*b^3 - 4*B*a^3*b^4 - 5*B*a^5*b^2 + 20*A*a^6*b - B*a*b^6))/(a^5*d*(a^2 + b^
2)^4) - (log(tan(c + d*x))*(4*A*b - B*a))/(a^5*d) - (log(tan(c + d*x) - 1i
)*(A + B*1i))/(2*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i - a^2*b^2*6i)) - (
log(tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(a*b^3*4i - a^3*b*4i + a^4 + b^4 -
 6*a^2*b^2)) - (A/a + (tan(c + d*x)^3*(4*A*b^9 + 12*A*a^2*b^7 + 13*A*a^4*b
^5 + A*a^6*b^3 - 3*B*a^3*b^6 - 6*B*a^5*b^4 - B*a*b^8))/(a^4*(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^2*(20*A*b^8 + 60*A*a^2*b^6 + 62*A*a
^4*b^4 + 6*A*a^6*b^2 - 16*B*a^3*b^5 - 27*B*a^5*b^3 - 5*B*a*b^7))/(2*a^3*(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)*(44*A*b^7 + 130*A*a^2*b
^5 + 128*A*a^4*b^3 - 34*B*a^3*b^4 - 47*B*a^5*b^2 + 18*A*a^6*b - 11*B*a*b^6
))/(6*a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3*tan(c + d*x) + b^3
*tan(c + d*x)^4 + 3*a^2*b*tan(c + d*x)^2 + 3*a*b^2*tan(c + d*x)^3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1617, normalized size of antiderivative = 4.05

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

output
```
(48*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**7*b**3 -
16*cos(c + d*x)*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**5*b**5 + 1
60*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(
c + d*x)**2*a**5*b**5 + 144*cos(c + d*x)*log(tan((c + d*x)/2)**2*a - 2*tan
((c + d*x)/2)*b - a)*sin(c + d*x)**2*a**3*b**7 + 48*cos(c + d*x)*log(tan((
c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*a*b**9 - 48*c
os(c + d*x)*log(tan((c + d*x)/2))*sin(c + d*x)**2*a**7*b**3 - 144*cos(c +
d*x)*log(tan((c + d*x)/2))*sin(c + d*x)**2*a**5*b**5 - 144*cos(c + d*x)*lo
g(tan((c + d*x)/2))*sin(c + d*x)**2*a**3*b**7 - 48*cos(c + d*x)*log(tan((c
 + d*x)/2))*sin(c + d*x)**2*a*b**9 + 2*cos(c + d*x)*sin(c + d*x)**2*a**9*b
 - 16*cos(c + d*x)*sin(c + d*x)**2*a**8*b**2*d*x + 6*cos(c + d*x)*sin(c +
d*x)**2*a**7*b**3 + 48*cos(c + d*x)*sin(c + d*x)**2*a**6*b**4*d*x + 6*cos(
c + d*x)*sin(c + d*x)**2*a**5*b**5 + 2*cos(c + d*x)*sin(c + d*x)**2*a**3*b
**7 - 8*cos(c + d*x)*a**9*b - 24*cos(c + d*x)*a**7*b**3 - 24*cos(c + d*x)*
a**5*b**5 - 8*cos(c + d*x)*a**3*b**7 - 24*log(tan((c + d*x)/2)**2 + 1)*sin
(c + d*x)**3*a**8*b**2 + 32*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*a
**6*b**4 - 8*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**3*a**4*b**6 + 24*l
og(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)*a**8*b**2 - 8*log(tan((c + d*x)/2
)**2 + 1)*sin(c + d*x)*a**6*b**4 - 80*log(tan((c + d*x)/2)**2*a - 2*tan((c
 + d*x)/2)*b - a)*sin(c + d*x)**3*a**6*b**4 + 8*log(tan((c + d*x)/2)**2...
```

3.297 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 477

$$\begin{aligned}
&\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx \\
&= \frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)\,x}{(a^2+b^2)^4}-\frac{(a^2A-10Ab^2+4abB)\log(\sin(c+dx))}{a^6d} \\
&\quad -\frac{b^3(35a^6Ab+56a^4Ab^3+39a^2Ab^5+10Ab^7-20a^7B-24a^5b^2B-16a^3b^4B-4ab^6B)\log(a\cos(c+dx)}{a^6\left(a^2+b^2\right)^4d} \\
&\quad +\frac{b(9a^2Ab+10Ab^3-3a^3B-4ab^2B)}{3a^3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}+\frac{(5Ab-2aB)\cot(c+dx)}{2a^2d(a+b\tan(c+dx))^3} \\
&\quad -\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}+\frac{b(7a^4Ab+19a^2Ab^3+10Ab^5-2a^5B-8a^3b^2B-4ab^4B)}{2a^4\left(a^2+b^2\right)^2d(a+b\tan(c+dx))^2} \\
&\quad +\frac{b(4a^6Ab+27a^4Ab^3+29a^2Ab^5+10Ab^7-a^7B-13a^5b^2B-12a^3b^4B-4ab^6B)}{a^5\left(a^2+b^2\right)^3d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*x/(a^2+b^2)^4-(A*a^2-10*A*b^
2+4*B*a*b)*ln(sin(d*x+c))/a^6/d-b^3*(35*A*a^6*b+56*A*a^4*b^3+39*A*a^2*b^5+
10*A*b^7-20*B*a^7-24*B*a^5*b^2-16*B*a^3*b^4-4*B*a*b^6)*ln(a*cos(d*x+c)+b*s
in(d*x+c))/a^6/(a^2+b^2)^4/d+1/3*b*(9*A*a^2*b+10*A*b^3-3*B*a^3-4*B*a*b^2)/
a^3/(a^2+b^2)/d/(a+b*tan(d*x+c))^3+1/2*(5*A*b-2*B*a)*cot(d*x+c)/a^2/d/(a+b
*tan(d*x+c))^3-1/2*A*cot(d*x+c)^2/a/d/(a+b*tan(d*x+c))^3+1/2*b*(7*A*a^4*b+
19*A*a^2*b^3+10*A*b^5-2*B*a^5-8*B*a^3*b^2-4*B*a*b^4)/a^4/(a^2+b^2)^2/d/(a+
b*tan(d*x+c))^2+b*(4*A*a^6*b+27*A*a^4*b^3+29*A*a^2*b^5+10*A*b^7-B*a^7-13*B
*a^5*b^2-12*B*a^3*b^4-4*B*a*b^6)/a^5/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.48 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.87

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{(4Ab-aB)\cot(c+dx)}{a^5d} - \frac{A\cot^2(c+dx)}{2a^4d} + \frac{(A+iB)\log(i-\tan(c+dx))}{2(a+ib)^4d}$$
$$- \frac{(a^2A-10Ab^2+4abB)\log(\tan(c+dx))}{a^6d} + \frac{(A-iB)\log(i+\tan(c+dx))}{2(a-ib)^4d}$$
$$- \frac{b^3(35a^6Ab+56a^4Ab^3+39a^2Ab^5+10Ab^7-20a^7B-24a^5b^2B-16a^3b^4B-4ab^6B)\log(a+b\tan(c+}{a^6(a^2+b^2)^4d}$$
$$+ \frac{b^3(Ab-aB)}{3a^3(a^2+b^2)d(a+b\tan(c+dx))^3} + \frac{b^3(5a^2Ab+3Ab^3-4a^3B-2ab^2B)}{2a^4(a^2+b^2)^2d(a+b\tan(c+dx))^2}$$
$$+ \frac{b^3(15a^4Ab+17a^2Ab^3+6Ab^5-10a^5B-9a^3b^2B-3ab^4B)}{a^5(a^2+b^2)^3d(a+b\tan(c+dx))}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
((4*A*b - a*B)*Cot[c + d*x])/(a^5*d) - (A*Cot[c + d*x]^2)/(2*a^4*d) + ((A
+ I*B)*Log[I - Tan[c + d*x]])/(2*(a + I*b)^4*d) - ((a^2*A - 10*A*b^2 + 4*a
*b*B)*Log[Tan[c + d*x]])/(a^6*d) + ((A - I*B)*Log[I + Tan[c + d*x]])/(2*(a
 - I*b)^4*d) - (b^3*(35*a^6*A*b + 56*a^4*A*b^3 + 39*a^2*A*b^5 + 10*A*b^7 -
 20*a^7*B - 24*a^5*b^2*B - 16*a^3*b^4*B - 4*a*b^6*B)*Log[a + b*Tan[c + d*x
]])/(a^6*(a^2 + b^2)^4*d) + (b^3*(A*b - a*B))/(3*a^3*(a^2 + b^2)*d*(a + b*
Tan[c + d*x])^3) + (b^3*(5*a^2*A*b + 3*A*b^3 - 4*a^3*B - 2*a*b^2*B))/(2*a^
4*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) + (b^3*(15*a^4*A*b + 17*a^2*A*b^
3 + 6*A*b^5 - 10*a^5*B - 9*a^3*b^2*B - 3*a*b^4*B))/(a^5*(a^2 + b^2)^3*d*(a
 + b*Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 3.17 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.14, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$, Rules used = {3042, 4092, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{\int \frac{\cot^2(c+dx)(5Ab\tan^2(c+dx)+2aA\tan(c+dx)+5Ab-2aB)}{(a+b\tan(c+dx))^4}dx}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{5Ab\tan(c+dx)^2+2aA\tan(c+dx)+5Ab-2aB}{\tan(c+dx)^2(a+b\tan(c+dx))^4}dx}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{-\frac{\int -\frac{2\cot(c+dx)\left(Aa^2+B\tan(c+dx)a^2+4bBa-10Ab^2-2b(5Ab-2aB)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^4}dx}{a}-\frac{(5Ab-2aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^3}}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 27

$$-\frac{\frac{2\int \frac{\cot(c+dx)\left(Aa^2+B\tan(c+dx)a^2+4bBa-10Ab^2-2b(5Ab-2aB)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^4}dx}{a}-\frac{(5Ab-2aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^3}}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2\int \frac{Aa^2+B\tan(c+dx)a^2+4bBa-10Ab^2-2b(5Ab-2aB)\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))^4}dx}{a}-\frac{(5Ab-2aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^3}}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 4132

$$-\frac{\frac{2\left(\frac{\int \frac{3\cot(c+dx)\left(-\left((Ab-aB)\tan(c+dx)a^3\right)-b\left(-3Ba^3+9Aba^2-4b^2Ba+10Ab^3\right)\tan^2(c+dx)+\left(a^2+b^2\right)\left(Aa^2+4bBa-10Ab^2\right)\right)}{(a+b\tan(c+dx))^3}dx}{3a\left(a^2+b^2\right)}-\frac{b\left(-3a^3B+9a^2Ab-4ab\right.}{3ad\left(a^2+b^2\right)(a+b\tan\right.}}{a}}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 27

$$-\frac{\frac{2\left(\frac{\int \frac{\cot(c+dx)\left(-\left((Ab-aB)\tan(c+dx)a^3\right)-b\left(-3Ba^3+9Aba^2-4b^2Ba+10Ab^3\right)\tan^2(c+dx)+\left(a^2+b^2\right)\left(Aa^2+4bBa-10Ab^2\right)\right)}{(a+b\tan(c+dx))^3}dx}{a\left(a^2+b^2\right)}-\frac{b\left(-3a^3B+9a^2Ab-4ab^2\right.}{3ad\left(a^2+b^2\right)(a+b\tan(\right.}}{a}}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2\left(\frac{\int \frac{-\left(\left(Ab-aB\right)\tan(c+dx)a^3\right)-b\left(-3Ba^3+9Aba^2-4b^2Ba+10Ab^3\right)\tan(c+dx)^2+\left(a^2+b^2\right)\left(Aa^2+4bBa-10Ab^2\right)}{\tan(c+dx)(a+b\tan(c+dx))^3}dx}{a\left(a^2+b^2\right)}-\frac{b\left(-3a^3B+9a^2Ab-4ab^2B+10Ab^3\right)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}\right)}{a}}{2a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\frac{2\left(\frac{\frac{\int \frac{2\cot(c+dx)\left(-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^4\right)-b\left(-2Ba^5+7Aba^4-8b^2Ba^3+19Ab^3a^2-4b^4Ba+10Ab^5\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2\left(Aa^2+4bBa-10A \cdots\right.\right.}{(a+b\tan(c+dx))^2}}{2a\left(a^2+b^2\right)} \cdots}{a\left(a^2+b^2\right)} \cdots\right.}{a}}{2a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{2\left(\frac{\frac{\int \frac{\cot(c+dx)\left(-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^4\right)-b\left(-2Ba^5+7Aba^4-8b^2Ba^3+19Ab^3a^2-4b^4Ba+10Ab^5\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2\left(Aa^2+4bBa-10Ab \cdots\right.\right.}{(a+b\tan(c+dx))^2}}{a\left(a^2+b^2\right)} \cdots}{a\left(a^2+b^2\right)} \cdots\right.}{a}}{2a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{2\left(\frac{\frac{\int \frac{-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^4\right)-b\left(-2Ba^5+7Aba^4-8b^2Ba^3+19Ab^3a^2-4b^4Ba+10Ab^5\right)\tan(c+dx)^2+\left(a^2+b^2\right)^2\left(Aa^2+4bBa-10Ab^2\right)}{\tan(c+dx)(a+b\tan(c+dx))^2}dx}{a\left(a^2+b^2\right)}-\frac{b\left(- \cdots\right.}{\cdots}}{a\left(a^2+b^2\right)} \cdots\right.}{a}}{2a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \; 4132$$

$$-\frac{2\left(\frac{\frac{\int \frac{\cot(c+dx)\left(-\left(\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)a^5\right)-b\left(-Ba^7+4Aba^6-13b^2Ba^5+27Ab^3a^4-12b^4Ba^3+29Ab^5a^2-4b^6Ba+10Ab^7\right)\tan^2(c+dx)\cdots}{a+b\tan(c+dx)}}{a\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}\cdots\right.}{\cdots}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{2\left(\frac{\frac{\int \frac{-\left(\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)a^5\right)-b\left(-Ba^7+4Aba^6-13b^2Ba^5+27Ab^3a^4-12b^4Ba^3+29Ab^5a^2-4b^6Ba+10Ab^7\right)\tan(c+dx)^2+\left(a^2+b^2\right)^3\cdots}{\tan(c+dx)(a+b\tan(c+dx))}}{a\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}\cdots\right.}{\cdots}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 4134

$$-\frac{2\left(\frac{\frac{\frac{\left(a^2+b^2\right)^3\left(a^2A+4abB-10Ab^2\right)\int \cot(c+dx)dx}{a}+\frac{b^3\left(-20a^7B+35a^6Ab-24a^5b^2B+56a^4Ab^3-16a^3b^4B+39a^2Ab^5-4ab^6B+10Ab^7\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\cdots}{a\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}\cdots\right.}{\cdots}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$-\frac{2\left(\frac{\frac{\frac{\left(a^2+b^2\right)^3\left(a^2A+4abB-10Ab^2\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\frac{b^3\left(-20a^7B+35a^6Ab-24a^5b^2B+56a^4Ab^3-16a^3b^4B+39a^2Ab^5-4ab^6B+10Ab^7\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}\right.}{}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\downarrow$ 25

$$-\frac{2\left(\frac{\frac{-\frac{\left(a^2+b^2\right)^3\left(a^2A+4abB-10Ab^2\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}+\frac{b^3\left(-20a^7B+35a^6Ab-24a^5b^2B+56a^4Ab^3-16a^3b^4B+39a^2Ab^5-4ab^6B+10Ab^7\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}\right.}{}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\downarrow$ 3956

$$-\frac{2\left(\frac{\frac{\frac{b^3\left(-20a^7B+35a^6Ab-24a^5b^2B+56a^4Ab^3-16a^3b^4B+39a^2Ab^5-4ab^6B+10Ab^7\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{\left(a^2+b^2\right)^3\left(a^2A+4abB-10Ab^2\right)\log(-\sin(c+dx)}{ad}}{a\left(a^2+b^2\right)}}{a\left(a^2+b^2\right)}\right.}{}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

$\downarrow$ 4013

$$-\frac{2\left(\frac{\frac{\frac{(a^2+b^2)^3(a^2A+4abB-10Ab^2)\log(-\sin(c+dx))}{ad}-\frac{a^5x(a^4(-B)+4a^3Ab+6a^2b^2B-4aAb^3-b^4B)}{a^2+b^2}+\frac{b^3(-20a^7B+35a^6Ab-24a^5b^2B+56a^4Ab^3-16a^3b^4B}{ad}}{a(a^2+b^2)}}{a(a^2+b^2)}\right.}{}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^3}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
-1/2*(A*Cot[c + d*x]^2)/(a*d*(a + b*Tan[c + d*x])^3) - (-(((5*A*b - 2*a*B)
*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x])^3)) + (2*(-1/3*(b*(9*a^2*A*b + 10
*A*b^3 - 3*a^3*B - 4*a*b^2*B))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) +
(-1/2*(b*(7*a^4*A*b + 19*a^2*A*b^3 + 10*A*b^5 - 2*a^5*B - 8*a^3*b^2*B - 4*
a*b^4*B))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + ((-((a^5*(4*a^3*A*b -
 4*a*A*b^3 - a^4*B + 6*a^2*b^2*B - b^4*B)*x)/(a^2 + b^2)) + ((a^2 + b^2)^3
*(a^2*A - 10*A*b^2 + 4*a*b*B)*Log[-Sin[c + d*x]])/(a*d) + (b^3*(35*a^6*A*b
 + 56*a^4*A*b^3 + 39*a^2*A*b^5 + 10*A*b^7 - 20*a^7*B - 24*a^5*b^2*B - 16*a
^3*b^4*B - 4*a*b^6*B)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)
*d))/(a*(a^2 + b^2)) - (b*(4*a^6*A*b + 27*a^4*A*b^3 + 29*a^2*A*b^5 + 10*A*
b^7 - a^7*B - 13*a^5*b^2*B - 12*a^3*b^4*B - 4*a*b^6*B))/(a*(a^2 + b^2)*d*(
a + b*Tan[c + d*x])))/(a*(a^2 + b^2)))/(a*(a^2 + b^2))))/a)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4Ba\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(4A\,a^3b-4Aa\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{\ }{2a^4\,\mathrm{ta}\cdots}$
default	$\frac{\frac{\left(A\,a^4-6A\,a^2b^2+A\,b^4+4B\,a^3b-4Ba\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(4A\,a^3b-4Aa\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{\ }{2a^4\,\mathrm{ta}\cdots}$
parallelrisch	$-70\left(A\,a^6b+\frac{8}{5}A\,a^4b^3+\frac{39}{35}A\,a^2b^5+\frac{2}{7}A\,b^7-\frac{4}{7}a^7B-\frac{24}{35}B\,a^5b^2-\frac{16}{35}B\,a^3b^4-\frac{4}{35}Ba\,b^6\right)b^3(a+b\tan(dx+c))^3\ln(a+b\tan(dx\cdots$
norman	$\frac{b^3\left(4A\,a^3b-4Aa\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)x\tan(dx+c)^5}{\left(a^6+3b^2a^4+3b^4a^2+b^6\right)\left(a^2+b^2\right)}+\frac{\left(4A\,a^3b-4Aa\,b^3-B\,a^4+6B\,a^2b^2-B\,b^4\right)a^3x\tan(dx+c)^2}{\left(a^6+3b^2a^4+3b^4a^2+b^6\right)\left(a^2+b^2\right)}-\frac{A}{2ad}+\frac{(5Ab\cdots}{}$
risch	Expression too large to display

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/d*(1/(a^2+b^2)^4*(1/2*(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*ln(1
+tan(d*x+c)^2)+(4*A*a^3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*arctan(tan(d*
x+c)))-1/2/a^4*A/tan(d*x+c)^2-(-4*A*b+B*a)/a^5/tan(d*x+c)+(-A*a^2+10*A*b^2
-4*B*a*b)/a^6*ln(tan(d*x+c))+1/2*b^3*(5*A*a^2*b+3*A*b^3-4*B*a^3-2*B*a*b^2)
/(a^2+b^2)^2/a^4/(a+b*tan(d*x+c))^2+b^3*(15*A*a^4*b+17*A*a^2*b^3+6*A*b^5-1
0*B*a^5-9*B*a^3*b^2-3*B*a*b^4)/(a^2+b^2)^3/a^5/(a+b*tan(d*x+c))-b^3*(35*A*
a^6*b+56*A*a^4*b^3+39*A*a^2*b^5+10*A*b^7-20*B*a^7-24*B*a^5*b^2-16*B*a^3*b^
4-4*B*a*b^6)/(a^2+b^2)^4/a^6*ln(a+b*tan(d*x+c))+1/3*(A*b-B*a)*b^3/(a^2+b^2
)/a^3/(a+b*tan(d*x+c))^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1732 vs. $2(467) = 934$.

Time = 0.31 (sec) , antiderivative size = 1732, normalized size of antiderivative = 3.63

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
-1/6*(3*A*a^13 + 12*A*a^11*b^2 + 18*A*a^9*b^4 + 12*A*a^7*b^6 + 3*A*a^5*b^8
 + (3*A*a^10*b^3 + 12*A*a^8*b^5 - 74*B*a^7*b^6 + 125*A*a^6*b^7 - 42*B*a^5*
b^8 + 102*A*a^4*b^9 - 12*B*a^3*b^10 + 30*A*a^2*b^11 + 6*(B*a^10*b^3 - 4*A*
a^9*b^4 - 6*B*a^8*b^5 + 4*A*a^7*b^6 + B*a^6*b^7)*d*x)*tan(d*x + c)^5 + 3*(
3*A*a^11*b^2 + 2*B*a^10*b^3 + 4*A*a^9*b^4 - 46*B*a^8*b^5 + 63*A*a^7*b^6 +
8*B*a^6*b^7 - 10*A*a^5*b^8 + 20*B*a^4*b^9 - 48*A*a^3*b^10 + 8*B*a^2*b^11 -
 20*A*a*b^12 + 6*(B*a^11*b^2 - 4*A*a^10*b^3 - 6*B*a^9*b^4 + 4*A*a^8*b^5 +
B*a^7*b^6)*d*x)*tan(d*x + c)^4 + 3*(3*A*a^12*b + 6*B*a^11*b^2 - 11*A*a^10*
b^3 - 6*B*a^9*b^4 - 32*A*a^8*b^5 + 80*B*a^7*b^6 - 177*A*a^6*b^7 + 68*B*a^5
*b^8 - 165*A*a^4*b^9 + 20*B*a^3*b^10 - 50*A*a^2*b^11 + 6*(B*a^12*b - 4*A*a
^11*b^2 - 6*B*a^10*b^3 + 4*A*a^9*b^4 + B*a^8*b^5)*d*x)*tan(d*x + c)^3 + (3
*A*a^13 + 18*B*a^12*b - 51*A*a^11*b^2 + 72*B*a^10*b^3 - 234*A*a^9*b^4 + 21
6*B*a^8*b^5 - 513*A*a^7*b^6 + 162*B*a^6*b^7 - 399*A*a^5*b^8 + 44*B*a^4*b^9
 - 110*A*a^3*b^10 + 6*(B*a^13 - 4*A*a^12*b - 6*B*a^11*b^2 + 4*A*a^10*b^3 +
 B*a^9*b^4)*d*x)*tan(d*x + c)^2 + 3*((A*a^10*b^3 + 4*B*a^9*b^4 - 6*A*a^8*b
^5 + 16*B*a^7*b^6 - 34*A*a^6*b^7 + 24*B*a^5*b^8 - 56*A*a^4*b^9 + 16*B*a^3*
b^10 - 39*A*a^2*b^11 + 4*B*a*b^12 - 10*A*b^13)*tan(d*x + c)^5 + 3*(A*a^11*
b^2 + 4*B*a^10*b^3 - 6*A*a^9*b^4 + 16*B*a^8*b^5 - 34*A*a^7*b^6 + 24*B*a^6*
b^7 - 56*A*a^5*b^8 + 16*B*a^4*b^9 - 39*A*a^3*b^10 + 4*B*a^2*b^11 - 10*A*a*
b^12)*tan(d*x + c)^4 + 3*(A*a^12*b + 4*B*a^11*b^2 - 6*A*a^10*b^3 + 16*B...
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 815, normalized size of antiderivative = 1.71

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="m
axima")
```

output
```
-1/6*(6*(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*(d*x + c)/(a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(20*B*a^7*b^3 - 35*A*a^6
*b^4 + 24*B*a^5*b^5 - 56*A*a^4*b^6 + 16*B*a^3*b^7 - 39*A*a^2*b^8 + 4*B*a*b
^9 - 10*A*b^10)*log(b*tan(d*x + c) + a)/(a^14 + 4*a^12*b^2 + 6*a^10*b^4 +
4*a^8*b^6 + a^6*b^8) - 3*(A*a^4 + 4*B*a^3*b - 6*A*a^2*b^2 - 4*B*a*b^3 + A*
b^4)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^
8) + (3*A*a^10 + 9*A*a^8*b^2 + 9*A*a^6*b^4 + 3*A*a^4*b^6 + 6*(B*a^7*b^3 -
4*A*a^6*b^4 + 13*B*a^5*b^5 - 27*A*a^4*b^6 + 12*B*a^3*b^7 - 29*A*a^2*b^8 +
4*B*a*b^9 - 10*A*b^10)*tan(d*x + c)^4 + 3*(6*B*a^8*b^2 - 23*A*a^7*b^3 + 62
*B*a^6*b^4 - 134*A*a^5*b^5 + 60*B*a^4*b^6 - 145*A*a^3*b^7 + 20*B*a^2*b^8 -
 50*A*a*b^9)*tan(d*x + c)^3 + (18*B*a^9*b - 63*A*a^8*b^2 + 128*B*a^7*b^3 -
 296*A*a^6*b^4 + 130*B*a^5*b^5 - 319*A*a^4*b^6 + 44*B*a^3*b^7 - 110*A*a^2*
b^8)*tan(d*x + c)^2 + 3*(2*B*a^10 - 5*A*a^9*b + 6*B*a^8*b^2 - 15*A*a^7*b^3
 + 6*B*a^6*b^4 - 15*A*a^5*b^5 + 2*B*a^4*b^6 - 5*A*a^3*b^7)*tan(d*x + c))/(
(a^11*b^3 + 3*a^9*b^5 + 3*a^7*b^7 + a^5*b^9)*tan(d*x + c)^5 + 3*(a^12*b^2
+ 3*a^10*b^4 + 3*a^8*b^6 + a^6*b^8)*tan(d*x + c)^4 + 3*(a^13*b + 3*a^11*b^
3 + 3*a^9*b^5 + a^7*b^7)*tan(d*x + c)^3 + (a^14 + 3*a^12*b^2 + 3*a^10*b^4
+ a^8*b^6)*tan(d*x + c)^2) + 6*(A*a^2 + 4*B*a*b - 10*A*b^2)*log(tan(d*x +
c))/a^6)/d
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 798, normalized size of antiderivative = 1.67

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="g
iac")
```

output
```
-(B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*(d*x + c)/(a^8*d +
4*a^6*b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + 1/2*(A*a^4 + 4*B*a^3*b
- 6*A*a^2*b^2 - 4*B*a*b^3 + A*b^4)*log(tan(d*x + c)^2 + 1)/(a^8*d + 4*a^6*
b^2*d + 6*a^4*b^4*d + 4*a^2*b^6*d + b^8*d) + (20*B*a^7*b^4 - 35*A*a^6*b^5
+ 24*B*a^5*b^6 - 56*A*a^4*b^7 + 16*B*a^3*b^8 - 39*A*a^2*b^9 + 4*B*a*b^10 -
 10*A*b^11)*log(abs(b*tan(d*x + c) + a))/(a^14*b*d + 4*a^12*b^3*d + 6*a^10
*b^5*d + 4*a^8*b^7*d + a^6*b^9*d) - (A*a^2 + 4*B*a*b - 10*A*b^2)*log(abs(t
an(d*x + c)))/(a^6*d) - 1/6*(3*A*a^13 + 12*A*a^11*b^2 + 18*A*a^9*b^4 + 12*
A*a^7*b^6 + 3*A*a^5*b^8 + 6*(B*a^10*b^3 - 4*A*a^9*b^4 + 14*B*a^8*b^5 - 31*
A*a^7*b^6 + 25*B*a^6*b^7 - 56*A*a^5*b^8 + 16*B*a^4*b^9 - 39*A*a^3*b^10 + 4
*B*a^2*b^11 - 10*A*a*b^12)*tan(d*x + c)^4 + 3*(6*B*a^11*b^2 - 23*A*a^10*b^
3 + 68*B*a^9*b^4 - 157*A*a^8*b^5 + 122*B*a^7*b^6 - 279*A*a^6*b^7 + 80*B*a^
5*b^8 - 195*A*a^4*b^9 + 20*B*a^3*b^10 - 50*A*a^2*b^11)*tan(d*x + c)^3 + (1
8*B*a^12*b - 63*A*a^11*b^2 + 146*B*a^10*b^3 - 359*A*a^9*b^4 + 258*B*a^8*b^
5 - 615*A*a^7*b^6 + 174*B*a^6*b^7 - 429*A*a^5*b^8 + 44*B*a^4*b^9 - 110*A*a
^3*b^10)*tan(d*x + c)^2 + 3*(2*B*a^13 - 5*A*a^12*b + 8*B*a^11*b^2 - 20*A*a
^10*b^3 + 12*B*a^9*b^4 - 30*A*a^8*b^5 + 8*B*a^7*b^6 - 20*A*a^6*b^7 + 2*B*a
^5*b^8 - 5*A*a^4*b^9)*tan(d*x + c))/((a^2 + b^2)^4*(b*tan(d*x + c) + a)^3*
a^6*d*tan(d*x + c)^2)
```

Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.39

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \frac{\frac{\tan(c+d\,x)\,(5\,A\,b-2\,B\,a)}{2\,a^2} - \frac{A}{2\,a} + \frac{\tan(c+d\,x)^4\,\left(-B\,a^7\,b^3+4\,A\,a^6\,b^4-13\,B\,a^5\,b^5+27\,A\,a^4\,b^6-12\,B\,a^3\,b^7+29\,A\,a^2\,b^8-4\,B\,a\,b^9+10\,A\,b^{10}\right)}{a^5\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)} + }{d\,\left(a^3\tan(c+\right.}$$
$$- \frac{\ln(\tan(c+d\,x))\,\left(A\,a^2+4\,B\,a\,b-10\,A\,b^2\right)}{a^6\,d}$$
$$+ \frac{\ln(\tan(c+d\,x)+1\mathrm{i})\,(B+A\,1\mathrm{i})}{2\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$
$$+ \frac{\ln(\tan(c+d\,x)-\mathrm{i})\,(A+B\,1\mathrm{i})}{2\,d\,\left(a^4+a^3\,b\,4\mathrm{i}-6\,a^2\,b^2-a\,b^3\,4\mathrm{i}+b^4\right)}$$
$$- \frac{\ln(a+b\tan(c+d\,x))\,\left(-20\,B\,a^7\,b^3+35\,A\,a^6\,b^4-24\,B\,a^5\,b^5+56\,A\,a^4\,b^6-16\,B\,a^3\,b^7+39\,A\,a^2\,b^8-4\right.}{d\,\left(a^{14}+4\,a^{12}\,b^2+6\,a^{10}\,b^4+4\,a^8\,b^6+a^6\,b^8\right)}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^4,x)
```

output
```
((tan(c + d*x)*(5*A*b - 2*B*a))/(2*a^2) - A/(2*a) + (tan(c + d*x)^4*(10*A*
b^10 + 29*A*a^2*b^8 + 27*A*a^4*b^6 + 4*A*a^6*b^4 - 12*B*a^3*b^7 - 13*B*a^5
*b^5 - B*a^7*b^3 - 4*B*a*b^9))/(a^5*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) +
 (tan(c + d*x)^3*(50*A*b^9 + 145*A*a^2*b^7 + 134*A*a^4*b^5 + 23*A*a^6*b^3
- 60*B*a^3*b^6 - 62*B*a^5*b^4 - 6*B*a^7*b^2 - 20*B*a*b^8))/(2*a^4*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c + d*x)^2*(110*A*b^8 + 319*A*a^2*b^6
+ 296*A*a^4*b^4 + 63*A*a^6*b^2 - 130*B*a^3*b^5 - 128*B*a^5*b^3 - 44*B*a*b^
7 - 18*B*a^7*b))/(6*a^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^3*tan(
c + d*x)^2 + b^3*tan(c + d*x)^5 + 3*a^2*b*tan(c + d*x)^3 + 3*a*b^2*tan(c +
 d*x)^4)) - (log(tan(c + d*x))*(A*a^2 - 10*A*b^2 + 4*B*a*b))/(a^6*d) + (lo
g(tan(c + d*x) + 1i)*(A*1i + B))/(2*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i
 - a^2*b^2*6i)) + (log(tan(c + d*x) - 1i)*(A + B*1i))/(2*d*(a^3*b*4i - a*b
^3*4i + a^4 + b^4 - 6*a^2*b^2)) - (log(a + b*tan(c + d*x))*(10*A*b^10 + 39
*A*a^2*b^8 + 56*A*a^4*b^6 + 35*A*a^6*b^4 - 16*B*a^3*b^7 - 24*B*a^5*b^5 - 2
0*B*a^7*b^3 - 4*B*a*b^9))/(d*(a^14 + a^6*b^8 + 4*a^8*b^6 + 6*a^10*b^4 + 4*
a^12*b^2))
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \int \frac{\cot(dx+c)^3\,(A+B\tan(dx+c))}{(a+\tan(dx+c)\,b)^4}dx$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

output
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

3.298 $\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 29

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\log(\cos(c+dx))}{d}+\frac{B\tan^2(c+dx)}{2d}$$

output
```
B*ln(cos(d*x+c))/d+1/2*B*tan(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B(2\log(\cos(c+dx))+\sec^2(c+dx))}{2d}$$

input
```
Integrate[(Tan[c + d*x]^3*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
(B*(2*Log[Cos[c + d*x]] + Sec[c + d*x]^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2011, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \tan^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \tan(c+dx)^3dx$$

$$\Big\downarrow \text{3954}$$

$$B\left(\frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$B\left(\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(Tan[c + d*x]^3*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
B*(Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{B\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}$	30
default	$\frac{B\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}$	30
parallelrisch	$-\frac{-B\tan(dx+c)^2+B\ln\left(1+\tan(dx+c)^2\right)}{2d}$	31
norman	$\frac{B\tan(dx+c)^2}{2d}-\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d}$	33
risch	$-iBx-\frac{2iBc}{d}+\frac{2B\,e^{2i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{B\ln\left(e^{2i(dx+c)}+1\right)}{d}$	60

input
```
int(tan(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*B*(1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)^2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\tan{(dx+c)}^2 + B\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate(tan(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/2*(B*tan(d*x + c)^2 + B*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \begin{cases} -\frac{B\log{(\tan^2{(c+dx)}+1)}}{2d} + \frac{B\tan^2{(c+dx)}}{2d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\tan{(c)})\tan^3{(c)}}{a+b\tan{(c)}} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*tan(c + d*x)**2/(2*d), Ne
(d, 0)), (x*(B*a + B*b*tan(c))*tan(c)**3/(a + b*tan(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\tan{(dx+c)}^2 - B\log\left(\tan{(dx+c)}^2+1\right)}{2\,d}$$

input
```
integrate(tan(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/2*(B*tan(d*x + c)^2 - B*log(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{1}{2}\,B\left(\frac{\tan{(dx+c)}^2}{d} - \frac{\log\left(\tan{(dx+c)}^2+1\right)}{d}\right)$$

input
```
integrate(tan(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
1/2*B*(tan(d*x + c)^2/d - log(tan(d*x + c)^2 + 1)/d)
```

Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\left(\ln\left(\tan(c+d\,x)^2+1\right) - \tan(c+d\,x)^2\right)}{2\,d}$$

input
```
int((tan(c + d*x)^3*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
-(B*(log(tan(c + d*x)^2 + 1) - tan(c + d*x)^2))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{b\left(-\log\left(\tan\left(dx+c\right)^2+1\right)+\tan\left(dx+c\right)^2\right)}{2d}$$

input
```
int(tan(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(b*( - log(tan(c + d*x)**2 + 1) + tan(c + d*x)**2))/(2*d)
```

3.299 $\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 16

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -Bx+\frac{B\tan(c+dx)}{d}$$

output
```
-B*x+B*tan(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = B\left(-\frac{\arctan(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}\right)$$

input
```
Integrate[(Tan[c + d*x]^2*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
B*(-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2011, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \tan^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \tan(c+dx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$B\left(\frac{\tan(c+dx)}{d}-\int 1dx\right)$$

$$\Big\downarrow \text{24}$$

$$B\left(\frac{\tan(c+dx)}{d}-x\right)$$

input
```
Int[(Tan[c + d*x]^2*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
B*(-x + Tan[c + d*x]/d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
norman	$-Bx+\frac{B\tan(dx+c)}{d}$	17
parallelrisch	$-\frac{Bdx-B\tan(dx+c)}{d}$	20
derivativedivides	$\frac{B(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$	22
default	$\frac{B(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$	22
risch	$-Bx+\frac{2iB}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}$	26

input
```
int(tan(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-B*x+B*tan(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{Bdx-B\tan(dx+c)}{d}$$

input
```
integrate(tan(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-(B*d*x - B*tan(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \begin{cases} -Bx+\frac{B\tan(c+dx)}{d} & \text{for } d\neq 0 \\ \frac{x(Ba+Bb\tan(c))\tan^2(c)}{a+b\tan(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((-B*x + B*tan(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*tan(c))*tan(c
)**2/(a + b*tan(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(dx+c)B-B\tan(dx+c)}{d}$$

input
```
integrate(tan(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-((d*x + c)*B - B*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -B\left(\frac{dx+c}{d}-\frac{\tan{(dx+c)}}{d}\right)$$

input
```
integrate(tan(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-B*((d*x + c)/d - tan(d*x + c)/d)
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\tan(c+d\,x)}{d}-B\,x$$

input
```
int((tan(c + d*x)^2*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
(B*tan(c + d*x))/d - B*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{b(\tan{(dx+c)}-dx)}{d}$$

input
```
int(tan(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output

```
(b*(tan(c + d*x) - d*x))/d
```

3.300 $\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 13

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\log(\cos(c+dx))}{d}$$

output
```
-B*ln(cos(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\log(\cos(c+dx))}{d}$$

input
```
Integrate[(Tan[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-((B*Log[Cos[c + d*x]])/d)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2011, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \tan(c+dx)dx$$

$$\Big\downarrow \text{3956}$$

$$-\frac{B\log(\cos(c+dx))}{d}$$

input
```
Int[(Tan[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-((B*Log[Cos[c + d*x]])/d)
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (warning: unable to verify)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d}$	18
default	$\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d}$	18
norman	$\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d}$	18
parallelrisch	$\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d}$	18
risch	$iBx+\frac{2iBc}{d}-\frac{B\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	33

input
```
int(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/2*B/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/2*B*log(1/(tan(d*x + c)^2 + 1))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.85

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \begin{cases} \frac{B\log\left(\tan^2(c+dx)+1\right)}{2d} & \text{for } d\neq 0 \\ \frac{x(Ba+Bb\tan(c))\tan(c)}{a+b\tan(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((B*log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*(B*a + B*b*tan(
c))*tan(c)/(a + b*tan(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

input
```
integrate(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
1/2*B*log(tan(d*x + c)^2 + 1)/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

input
```
integrate(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
1/2*B*log(tan(d*x + c)^2 + 1)/d
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\,\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}$$

input
```
int((tan(c + d*x)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
(B*log(tan(c + d*x)^2 + 1))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\log\left(\tan\left(dx+c\right)^2+1\right)b}{2d}$$

input
```
int(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(log(tan(c + d*x)**2 + 1)*b)/(2*d)
```

3.301 $\int \frac{aB+bB\tan(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 3

$$\int \frac{aB+bB\tan(c+dx)}{a+b\tan(c+dx)}\,dx=Bx$$

output
```
B*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{aB+bB\tan(c+dx)}{a+b\tan(c+dx)}\,dx=Bx$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(a + b*Tan[c + d*x]),x]
```

output
```
B*x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2011, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\tan(c + dx)}{a + b\tan(c + dx)}\,dx$$

$$\Big\downarrow \; 2011$$

$$B\int 1dx$$

$$\Big\downarrow \; 24$$

$$Bx$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(a + b*Tan[c + d*x]),x]
```

output
```
B*x
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	Bx	4
norman	Bx	4
risch	Bx	4
derivativedivides	$\frac{B\arctan(\tan(dx+c))}{d}$	13
orering	$\frac{x(Ba+bB\tan(dx+c))}{a+b\tan(dx+c)}$	28

input
```
int((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
B*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{aB+bB\tan(c+dx)}{a+b\tan(c+dx)}\,dx = Bx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
B*x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int \frac{aB+bB\tan(c+dx)}{a+b\tan(c+dx)}\,dx = Bx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
B*x
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 3.33

$$\int \frac{aB + bB\tan(c + dx)}{a + b\tan(c + dx)}\,dx = \frac{(dx + c)B}{d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)*B/d
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 3.33

$$\int \frac{aB + bB\tan(c + dx)}{a + b\tan(c + dx)}\,dx = \frac{(dx + c)B}{d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(d*x + c)*B/d
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{aB + bB\tan(c+dx)}{a+b\tan(c+dx)}\,dx = B\,x$$

input
```
int((B*a + B*b*tan(c + d*x))/(a + b*tan(c + d*x)),x)
```

output
```
B*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{aB + bB\tan(c+dx)}{a+b\tan(c+dx)}\,dx = bx$$

input
```
int((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
b*x
```

3.302 $\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 12

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\log(\sin(c+dx))}{d}$$

output
```
B*ln(sin(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\log(\sin(c+dx))}{d}$$

input
```
Integrate[(Cot[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(B*Log[Sin[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2011, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \cot(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-B\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{B\log(-\sin(c+dx))}{d}$$

input
```
Int[(Cot[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(B*Log[-Sin[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$-\frac{B\ln\left(\cot\left(dx+c\right)^{2}+1\right)}{2d}$	18
default	$-\frac{B\ln\left(\cot\left(dx+c\right)^{2}+1\right)}{2d}$	18
parallelrisch	$\frac{B\left(\ln\left(\tan\left(dx+c\right)\right)+\ln\left(\frac{1}{\sqrt{\sec\left(dx+c\right)^{2}}}\right)\right)}{d}$	25
norman	$\frac{B\ln\left(\tan\left(dx+c\right)\right)}{d}-\frac{B\ln\left(1+\tan\left(dx+c\right)^{2}\right)}{2d}$	31
risch	$-iBx-\frac{2iBc}{d}+\frac{B\ln\left(e^{2i(dx+c)}-1\right)}{d}$	32

input
```
int(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-1/2/d*B*ln(cot(d*x+c)^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\log\left(-\frac{1}{2}\,\cos\left(2\,dx+2\,c\right)+\frac{1}{2}\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/2*B*log(-1/2*cos(2*d*x + 2*c) + 1/2)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(10) = 20.

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.08

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \begin{cases} -\frac{B\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{B\log\left(\tan\left(c+dx\right)\right)}{d} & \text{for}\ d\neq 0 \\ \frac{x(Ba+Bb\tan\left(c\right))\cot\left(c\right)}{a+b\tan\left(c\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((-B*log(tan(c + d*x)**2 + 1)/(2*d) + B*log(tan(c + d*x))/d, Ne(d
, 0)), (x*(B*a + B*b*tan(c))*cot(c)/(a + b*tan(c)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\log\left(\tan\left(dx+c\right)^2+1\right)-2\,B\log\left(\tan\left(dx+c\right)\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/2*(B*log(tan(d*x + c)^2 + 1) - 2*B*log(tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{1}{2}\,B\left(\frac{\log\left(\tan\left(dx+c\right)^2+1\right)}{d}-\frac{\log\left(\tan\left(dx+c\right)^2\right)}{d}\right)$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
-1/2*B*(log(tan(d*x + c)^2 + 1)/d - log(tan(d*x + c)^2)/d)
```

Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\left(\ln\left(\tan(c+d\,x)^2+1\right)-2\,\ln\left(\tan(c+d\,x)\right)\right)}{2\,d}$$

input
```
int((cot(c + d*x)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
-(B*(log(tan(c + d*x)^2 + 1) - 2*log(tan(c + d*x))))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.67

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{b\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{d}$$

input
```
int(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(b*( - log(tan((c + d*x)/2)**2 + 1) + log(tan((c + d*x)/2))))/d
```

3.303 $\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 17

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -Bx - \frac{B\cot(c+dx)}{d}$$

output
```
-B*x-B*cot(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}$$

input
```
Integrate[(Cot[c + d*x]^2*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
-((B*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2])/d)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{2011, 3042, 3954, 24\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \cot^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3954}$$

$$B\left(-\int 1dx-\frac{\cot(c+dx)}{d}\right)$$

$$\Big\downarrow \text{24}$$

$$B\left(-\frac{\cot(c+dx)}{d}-x\right)$$

input
```
Int[(Cot[c + d*x]^2*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
B*(-x - Cot[c + d*x]/d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$-\frac{B(dx+\cot(dx+c))}{d}$	17
risch	$-Bx-\frac{2iB}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)}$	26
derivativedivides	$\frac{B\left(-\cot(dx+c)+\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{d}$	27
default	$\frac{B\left(-\cot(dx+c)+\frac{\pi}{2}-\operatorname{arccot}(\cot(dx+c))\right)}{d}$	27
norman	$\frac{-\frac{B}{d}-Bx\tan(dx+c)}{\tan(dx+c)}$	27

input
```
int(cot(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-1/d*B*(d*x+cot(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{Bdx\sin(2\,dx+2\,c)+B\cos(2\,dx+2\,c)+B}{d\sin(2\,dx+2\,c)}$$

input
```
integrate(cot(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-(B*d*x*sin(2*d*x + 2*c) + B*cos(2*d*x + 2*c) + B)/(d*sin(2*d*x + 2*c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \begin{cases} -Bx - \frac{B\cot(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\tan(c))\cot^2(c)}{a+b\tan(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(d*x+c)**2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((-B*x - B*cot(c + d*x)/d, Ne(d, 0)), (x*(B*a + B*b*tan(c))*cot(c
)**2/(a + b*tan(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(dx+c)B+\frac{B}{\tan(dx+c)}}{d}$$

input
```
integrate(cot(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-((d*x + c)*B + B/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -B\left(\frac{dx+c}{d}+\frac{1}{d\tan\left(dx+c\right)}\right)$$

input
```
integrate(cot(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
-B*((d*x + c)/d + 1/(d*tan(d*x + c)))
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\left(\cot(c+d\,x)+d\,x\right)}{d}$$

input
```
int((cot(c + d*x)^2*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
-(B*(cot(c + d*x) + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{b(\cot(dx+c)+dx)}{d}$$

input
```
int(cot(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
( - b*(cot(c + d*x) + d*x))/d
```

3.304 $\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 30

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\cot^2(c+dx)}{2d}-\frac{B\log(\sin(c+dx))}{d}$$

output
```
-1/2*B*cot(d*x+c)^2/d-B*ln(sin(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = B\left(-\frac{\csc^2(c+dx)}{2d}-\frac{\log(\sin(c+dx))}{d}\right)$$

input
```
Integrate[(Cot[c + d*x]^3*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
B*(-1/2*Csc[c + d*x]^2/d - Log[Sin[c + d*x]]/d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2011, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \cot^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-B\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3dx$$

$$\Big\downarrow \text{3954}$$

$$-B\left(\frac{\cot^2(c+dx)}{2d}-\int -\cot(c+dx)dx\right)$$

$$\Big\downarrow \text{25}$$

$$-B\left(\int \cot(c+dx)dx+\frac{\cot^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$-B\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{25}$$

$$-B\left(\frac{\cot^2(c+dx)}{2d}-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$-B\left(\frac{\cot^2(c+dx)}{2d}+\frac{\log(-\sin(c+dx))}{d}\right)$$

input
```
Int[(Cot[c + d*x]^3*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-(B*(Cot[c + d*x]^2/(2*d) + Log[-Sin[c + d*x]]/d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{B\left(-\frac{\cot(dx+c)^2}{2}+\frac{\ln\left(\cot(dx+c)^2+1\right)}{2}\right)}{d}$	30
default	$\frac{B\left(-\frac{\cot(dx+c)^2}{2}+\frac{\ln\left(\cot(dx+c)^2+1\right)}{2}\right)}{d}$	30
parallelrisch	$-\frac{B\left(2\ln(\tan(dx+c))-\ln\left(\sec(dx+c)^2\right)+\cot(dx+c)^2\right)}{2d}$	36
norman	$-\frac{B}{2d\tan(dx+c)^2}-\frac{B\ln(\tan(dx+c))}{d}+\frac{B\ln\left(1+\tan(dx+c)^2\right)}{2d}$	46
risch	$iBx+\frac{2iBc}{d}+\frac{2B\,\mathrm{e}^{2i(dx+c)}}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}-\frac{B\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}$	61

input
```
int(cot(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*B*(-1/2*cot(d*x+c)^2+1/2*ln(cot(d*x+c)^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(B\cos(2\,dx+2\,c)-B)\log\left(-\frac{1}{2}\cos(2\,dx+2\,c)+\frac{1}{2}\right)-2\,B}{2\,(d\cos(2\,dx+2\,c)-d)}$$

input
```
integrate(cot(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-1/2*((B*cos(2*d*x + 2*c) - B)*log(-1/2*cos(2*d*x + 2*c) + 1/2) - 2*B)/(d*
cos(2*d*x + 2*c) - d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.

Time = 1.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.53

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \begin{cases} \tilde{\infty}Bx & \text{for } c=0 \wedge d=0 \\ \frac{x(Ba+Bb\tan(c))\cot^3(c)}{a+b\tan(c)} & \text{for } d=0 \\ \tilde{\infty}Bx & \text{for } c=-dx \\ \frac{B\log(\tan^2(c+dx)+1)}{2d} - \frac{B\log(\tan(c+dx))}{d} - \frac{B}{2d\tan^2(c+dx)} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(d*x+c)**3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((zoo*B*x, Eq(c, 0) & Eq(d, 0)), (x*(B*a + B*b*tan(c))*cot(c)**3/
(a + b*tan(c)), Eq(d, 0)), (zoo*B*x, Eq(c, -d*x)), (B*log(tan(c + d*x)**2
+ 1)/(2*d) - B*log(tan(c + d*x))/d - B/(2*d*tan(c + d*x)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\log\left(\tan(dx+c)^2+1\right) - 2B\log(\tan(dx+c)) - \frac{B}{\tan(dx+c)^2}}{2d}$$

input
```
integrate(cot(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/2*(B*log(tan(d*x + c)^2 + 1) - 2*B*log(tan(d*x + c)) - B/tan(d*x + c)^2)
/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.83

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{1}{2}\,B\left(\frac{\log\left(\tan\left(dx+c\right)^2+1\right)}{d}-\frac{\log\left(\tan\left(dx+c\right)^2\right)}{d}+\frac{\tan\left(dx+c\right)^2-1}{d\tan\left(dx+c\right)^2}\right)$$

input
```
integrate(cot(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
1/2*B*(log(tan(d*x + c)^2 + 1)/d - log(tan(d*x + c)^2)/d + (tan(d*x + c)^2
 - 1)/(d*tan(d*x + c)^2))
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= -\frac{B\,\left(\cot(c+d\,x)^2-\ln\left(\tan(c+d\,x)^2+1\right)+2\,\ln\left(\tan(c+d\,x)\right)\right)}{2\,d}$$

input
```
int((cot(c + d*x)^3*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
-(B*(2*log(tan(c + d*x)) - log(tan(c + d*x)^2 + 1) + cot(c + d*x)^2))/(2*d
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{b\left(4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)^2-4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sin(dx+c)^2+\sin(dx+c)^2-2\right)}{4\sin(dx+c)^2 d}$$

input
```
int(cot(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(b*(4*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2 - 4*log(tan((c + d*x)/2
))*sin(c + d*x)**2 + sin(c + d*x)**2 - 2))/(4*sin(c + d*x)**2*d)
```

3.305 $\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 31

$$\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = Bx+\frac{B\cot(c+dx)}{d}-\frac{B\cot^3(c+dx)}{3d}$$

output
```
B*x+B*cot(d*x+c)/d-1/3*B*cot(d*x+c)^3/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}$$

input
```
Integrate[(Cot[c + d*x]^4*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
-1/3*(B*Cot[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/
d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2011, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \cot^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \tan\left(c+dx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3954}$$

$$B\left(-\int \cot^2(c+dx)dx-\frac{\cot^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(-\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{\cot^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3954}$$

$$B\left(\int 1dx-\frac{\cot^3(c+dx)}{3d}+\frac{\cot(c+dx)}{d}\right)$$

$$\Big\downarrow \text{24}$$

$$B\left(-\frac{\cot^3(c+dx)}{3d}+\frac{\cot(c+dx)}{d}+x\right)$$

input
```
Int[(Cot[c + d*x]^4*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
B*(x + Cot[c + d*x]/d - Cot[c + d*x]^3/(3*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$-\frac{B\left(\cot(dx+c)^{3}-3dx-3\cot(dx+c)\right)}{3d}$	28
derivativedivides	$\frac{B\left(-\frac{\cot(dx+c)^{3}}{3}+\cot(dx+c)-\frac{\pi}{2}+\operatorname{arccot}(\cot(dx+c))\right)}{d}$	33
default	$\frac{B\left(-\frac{\cot(dx+c)^{3}}{3}+\cot(dx+c)-\frac{\pi}{2}+\operatorname{arccot}(\cot(dx+c))\right)}{d}$	33
norman	$\frac{Bx\tan(dx+c)^{3}+\frac{B\tan(dx+c)^{2}}{d}-\frac{B}{3d}}{\tan(dx+c)^{3}}$	41
risch	$Bx+\frac{4iB\left(3\,e^{4i(dx+c)}-3\,e^{2i(dx+c)}+2\right)}{3d\left(e^{2i(dx+c)}-1\right)^{3}}$	49

input
```
int(cot(d*x+c)^4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-1/3*B*(cot(d*x+c)^3-3*d*x-3*cot(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.90

$$\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{4\,B\cos\left(2\,dx+2\,c\right)^{2}+2\,B\cos\left(2\,dx+2\,c\right)+3\,\left(Bdx\cos\left(2\,dx+2\,c\right)-Bdx\right)\sin\left(2\,dx+2\,c\right)-2\,B}{3\,\left(d\cos\left(2\,dx+2\,c\right)-d\right)\sin\left(2\,dx+2\,c\right)}$$

input
```
integrate(cot(d*x+c)^4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/3*(4*B*cos(2*d*x + 2*c)^2 + 2*B*cos(2*d*x + 2*c) + 3*(B*d*x*cos(2*d*x +
2*c) - B*d*x)*sin(2*d*x + 2*c) - 2*B)/((d*cos(2*d*x + 2*c) - d)*sin(2*d*x
+ 2*c))
```

Sympy [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \begin{cases} Bx - \frac{B\cot^3(c+dx)}{3d} + \frac{B\cot(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x(Ba+Bb\tan(c))\cot^4(c)}{a+b\tan(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(d*x+c)**4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((B*x - B*cot(c + d*x)**3/(3*d) + B*cot(c + d*x)/d, Ne(d, 0)), (x
*(B*a + B*b*tan(c))*cot(c)**4/(a + b*tan(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{3\,(dx+c)B + \frac{3\,B\tan(dx+c)^2 - B}{\tan(dx+c)^3}}{3\,d}$$

input
```
integrate(cot(d*x+c)^4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/3*(3*(d*x + c)*B + (3*B*tan(d*x + c)^2 - B)/tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{1}{3}\,B\left(\frac{3\,(dx+c)}{d} + \frac{3\,\tan(dx+c)^2 - 1}{d\tan(dx+c)^3}\right)$$

input
```
integrate(cot(d*x+c)^4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
1/3*B*(3*(d*x + c)/d + (3*tan(d*x + c)^2 - 1)/(d*tan(d*x + c)^3))
```

Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = B\,x-\frac{\frac{B}{3}-B\tan(c+d\,x)^2}{d\tan\left(c+d\,x\right)^3}$$

input
```
int((cot(c + d*x)^4*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
B*x - (B/3 - B*tan(c + d*x)^2)/(d*tan(c + d*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\cot^4(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{b\left(-\cot\left(dx+c\right)^3+3\cot\left(dx+c\right)+3dx\right)}{3d}$$

input
```
int(cot(d*x+c)^4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(b*( - cot(c + d*x)**3 + 3*cot(c + d*x) + 3*d*x))/(3*d)
```

3.306 $\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 102

$$\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{aBx}{a^2+b^2}+\frac{bB\log(\cos(c+dx))}{(a^2+b^2)\,d}+\frac{a^4B\log(a+b\tan(c+dx))}{b^3\,(a^2+b^2)\,d}-\frac{aB\tan(c+dx)}{b^2d}+\frac{B\tan^2(c+dx)}{2bd}$$

output

```
a*B*x/(a^2+b^2)+b*B*ln(cos(d*x+c))/(a^2+b^2)/d+a^4*B*ln(a+b*tan(d*x+c))/b^
3/(a^2+b^2)/d-a*B*tan(d*x+c)/b^2/d+1/2*B*tan(d*x+c)^2/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{B\left(\frac{\log(i-\tan(c+dx))}{ia-b}-\frac{\log(i+\tan(c+dx))}{ia+b}+\frac{2a^4\log(a+b\tan(c+dx))}{b^3(a^2+b^2)}-\frac{2a\tan(c+dx)}{b^2}+\frac{\tan^2(c+dx)}{b}\right)}{2d}$$

input
```
Integrate[(Tan[c + d*x]^4*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2
,x]
```

output
```
(B*(Log[I - Tan[c + d*x]]/(I*a - b) - Log[I + Tan[c + d*x]]/(I*a + b) + (2
*a^4*Log[a + b*Tan[c + d*x]])/(b^3*(a^2 + b^2)) - (2*a*Tan[c + d*x])/b^2 +
 Tan[c + d*x]^2/b))/(2*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {2011, 3042, 4049, 27, 3042, 4130, 25, 3042, 4110, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\tan^4(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\tan(c+dx)^4}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4049}$$

$$B\left(\frac{\int -\frac{2\tan(c+dx)\left(a\tan^2(c+dx)+b\tan(c+dx)+a\right)}{a+b\tan(c+dx)}dx}{2b}+\frac{\tan^2(c+dx)}{2bd}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{\tan^2(c+dx)}{2bd}-\frac{\int \frac{\tan(c+dx)\left(a\tan^2(c+dx)+b\tan(c+dx)+a\right)}{a+b\tan(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\tan^2(c+dx)}{2bd}-\frac{\int\frac{\tan(c+dx)(a\tan(c+dx)^2+b\tan(c+dx)+a)}{a+b\tan(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{4130}$$

$$B\left(\frac{\tan^2(c+dx)}{2bd}-\frac{\frac{\int-\frac{a^2+(a^2-b^2)\tan^2(c+dx)}{a+b\tan(c+dx)}dx}{b}+\frac{a\tan(c+dx)}{bd}}{b}\right)$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{\tan^2(c+dx)}{2bd}-\frac{\frac{a\tan(c+dx)}{bd}-\frac{\int\frac{a^2+(a^2-b^2)\tan^2(c+dx)}{a+b\tan(c+dx)}dx}{b}}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\tan^2(c+dx)}{2bd}-\frac{\frac{a\tan(c+dx)}{bd}-\frac{\int\frac{a^2+(a^2-b^2)\tan(c+dx)^2}{a+b\tan(c+dx)}dx}{b}}{b}\right)$$

$$\Big\downarrow \text{4110}$$

$$B\left(\frac{\tan^2(c+dx)}{2bd}-\frac{\frac{a\tan(c+dx)}{bd}-\frac{-\frac{b^3\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^4\int\frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{ab^2x}{a^2+b^2}}{b}}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\tan^2(c+dx)}{2bd}-\frac{\frac{a\tan(c+dx)}{bd}-\frac{-\frac{b^3\int\tan(c+dx)dx}{a^2+b^2}+\frac{a^4\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{ab^2x}{a^2+b^2}}{b}}{b}\right)$$

$$\Big\downarrow \text{3956}$$

$$B\left(\frac{\tan^2(c+dx)}{2bd}-\frac{\frac{a\tan(c+dx)}{bd}-\frac{\frac{a^4\int\frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{ab^2x}{a^2+b^2}+\frac{b^3\log(\cos(c+dx))}{d(a^2+b^2)}}{b}}{b}\right)$$

$$\Big\downarrow \text{4100}$$

$$B\left(\frac{\tan^2(c+dx)}{2bd}-\frac{\frac{a\tan(c+dx)}{bd}-\frac{\frac{a^4\int\frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd(a^2+b^2)}+\frac{ab^2x}{a^2+b^2}+\frac{b^3\log(\cos(c+dx))}{d(a^2+b^2)}}{b}}{b}\right)$$

$$\Big\downarrow \text{16}$$

$$B\left(\frac{\tan^2(c+dx)}{2bd}-\frac{\frac{a\tan(c+dx)}{bd}-\frac{\frac{ab^2x}{a^2+b^2}+\frac{b^3\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{a^4\log(a+b\tan(c+dx))}{bd(a^2+b^2)}}{b}}{b}\right)$$

input
```
Int[(Tan[c + d*x]^4*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
B*(Tan[c + d*x]^2/(2*b*d) - (-(((a*b^2*x)/(a^2 + b^2) + (b^3*Log[Cos[c + d
*x]])/((a^2 + b^2)*d) + (a^4*Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d))/b
) + (a*Tan[c + d*x])/(b*d))/b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4110
```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[a*(A - C)*(x/(a^2 + b^2)), x] + (Simp[(a^2*C +
A*b^2)/(a^2 + b^2)   Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x]
- Simp[b*((A - C)/(a^2 + b^2))   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b,
e, f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{B\left(-\frac{-\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{b^2}+\frac{-\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^4\ln(a+b\tan(dx+c))}{b^3\left(a^2+b^2\right)}\right)}{d}$
default	$\frac{B\left(-\frac{-\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{b^2}+\frac{-\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^4\ln(a+b\tan(dx+c))}{b^3\left(a^2+b^2\right)}\right)}{d}$
parallelrisch	$-\frac{-2Ba\,b^3dx-B\,a^2b^2\tan(dx+c)^2-B\,b^4\tan(dx+c)^2+B\ln\left(1+\tan(dx+c)^2\right)b^4-2B\ln(a+b\tan(dx+c))a^4+2B\,a^3b\tan}{2b^3d(a^2+b^2)}$
norman	$\frac{\frac{B\,a^2x}{a^2+b^2}+\frac{B\,a^3}{d\,b^3}+\frac{bBax\tan(dx+c)}{a^2+b^2}+\frac{B\tan(dx+c)^3}{2d}-\frac{Ba\tan(dx+c)^2}{2db}}{a+b\tan(dx+c)}+\frac{a^4B\ln(a+b\tan(dx+c))}{b^3(a^2+b^2)d}-\frac{Bb\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}$
risch	$-\frac{xB}{ib-a}+\frac{2iB\,a^2x}{b^3}+\frac{2iB\,a^2c}{b^3d}-\frac{2iBx}{b}-\frac{2iBc}{bd}-\frac{2ia^4Bx}{b^3(a^2+b^2)}-\frac{2ia^4Bc}{b^3d(a^2+b^2)}+\frac{2B\left(-ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c}\right.}{d\,b^2\left(e^{2i(dx+c)}+1\right)^2}$

input
```
int(tan(d*x+c)^4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*B*(-1/b^2*(-1/2*b*tan(d*x+c)^2+a*tan(d*x+c))+1/(a^2+b^2)*(-1/2*b*ln(1+
tan(d*x+c)^2)+a*arctan(tan(d*x+c)))+1/b^3*a^4/(a^2+b^2)*ln(a+b*tan(d*x+c))
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.41

$$\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2\,Bab^3dx + Ba^4\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right) + (Ba^2b^2+Bb^4)\tan(dx+c)^2 - (Ba^4-Bb^4)\log\left(\frac{}{\tan}\right.}{2\,(a^2b^3+b^5)d}$$

input
```
integrate(tan(d*x+c)^4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
1/2*(2*B*a*b^3*d*x + B*a^4*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) +
a^2)/(tan(d*x + c)^2 + 1)) + (B*a^2*b^2 + B*b^4)*tan(d*x + c)^2 - (B*a^4 -
 B*b^4)*log(1/(tan(d*x + c)^2 + 1)) - 2*(B*a^3*b + B*a*b^3)*tan(d*x + c))/
((a^2*b^3 + b^5)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 782, normalized size of antiderivative = 7.67

$$\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)**4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*B*x*tan(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*(x + tan
(c + d*x)**3/(3*d) - tan(c + d*x)/d)/a, Eq(b, 0)), (-3*I*B*d*x*tan(c + d*x
)/(2*b*d*tan(c + d*x) - 2*I*b*d) - 3*B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d)
- 2*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d)
 + 2*I*B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*tan(c
 + d*x)**3/(2*b*d*tan(c + d*x) - 2*I*b*d) + I*B*tan(c + d*x)**2/(2*b*d*tan
(c + d*x) - 2*I*b*d) - B*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 4*I
*B/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (3*I*B*d*x*tan(c + d*x)/(
2*b*d*tan(c + d*x) + 2*I*b*d) - 3*B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - 2
*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) -
2*I*B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*tan(c +
d*x)**3/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*B*tan(c + d*x)**2/(2*b*d*tan(c
+ d*x) + 2*I*b*d) - B*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 4*I*B/
(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(B*a + B*b*tan(c))*tan(c)*
*4/(a + b*tan(c))**2, Eq(d, 0)), (2*B*a**4*log(a/b + tan(c + d*x))/(2*a**2
*b**3*d + 2*b**5*d) - 2*B*a**3*b*tan(c + d*x)/(2*a**2*b**3*d + 2*b**5*d) +
 B*a**2*b**2*tan(c + d*x)**2/(2*a**2*b**3*d + 2*b**5*d) + 2*B*a*b**3*d*x/(
2*a**2*b**3*d + 2*b**5*d) - 2*B*a*b**3*tan(c + d*x)/(2*a**2*b**3*d + 2*b**
5*d) - B*b**4*log(tan(c + d*x)**2 + 1)/(2*a**2*b**3*d + 2*b**5*d) + B*b**4
*tan(c + d*x)**2/(2*a**2*b**3*d + 2*b**5*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,Ba^4\log\left(b\tan(dx+c)+a\right)}{a^2b^3+b^5} + \frac{2\,(dx+c)Ba}{a^2+b^2} - \frac{Bb\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} + \frac{Bb\tan(dx+c)^2-2\,Ba\tan(dx+c)}{b^2}}{2\,d}$$

input
```
integrate(tan(d*x+c)^4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
1/2*(2*B*a^4*log(b*tan(d*x + c) + a)/(a^2*b^3 + b^5) + 2*(d*x + c)*B*a/(a^
2 + b^2) - B*b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + (B*b*tan(d*x + c)^2 -
 2*B*a*tan(d*x + c))/b^2)/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{1}{2}\left(\frac{2\,a^4\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^2b^3d+b^5d}+\frac{2\left(dx+c\right)a}{a^2d+b^2d}-\frac{b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2d+b^2d}+\frac{bd\tan\left(dx+c\right)^2-2\,ad\tan\left(dx+c\right)}{b^2d^2}\right.$$

input
```
integrate(tan(d*x+c)^4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
1/2*(2*a^4*log(abs(b*tan(d*x + c) + a))/(a^2*b^3*d + b^5*d) + 2*(d*x + c)*
a/(a^2*d + b^2*d) - b*log(tan(d*x + c)^2 + 1)/(a^2*d + b^2*d) + (b*d*tan(d
*x + c)^2 - 2*a*d*tan(d*x + c))/(b^2*d^2))*B
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx=\frac{B\tan(c+d\,x)^2}{2\,b\,d}-\frac{B\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}-\frac{B\,a\tan(c+d\,x)}{b^2\,d}+\frac{B\,a^4\,\ln\left(a+b\tan(c+d\,x)\right)}{b^3\,d\,\left(a^2+b^2\right)}-\frac{B\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int((tan(c + d*x)^4*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(B*tan(c + d*x)^2)/(2*b*d) - (B*log(tan(c + d*x) + 1i))/(2*d*(a*1i + b)) -
 (B*log(tan(c + d*x) - 1i)*1i)/(2*d*(a + b*1i)) - (B*a*tan(c + d*x))/(b^2*
d) + (B*a^4*log(a + b*tan(c + d*x)))/(b^3*d*(a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int \frac{\tan^4(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{-\log\left(\tan\left(dx+c\right)^2+1\right)b^4+2\log(a+\tan\left(dx+c\right)b)\,a^4+\tan\left(dx+c\right)^2a^2b^2+\tan\left(dx+c\right)^2b^4-2\tan}{2b^2d\left(a^2+b^2\right)}$$

input
```
int(tan(d*x+c)^4*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
( - log(tan(c + d*x)**2 + 1)*b**4 + 2*log(tan(c + d*x)*b + a)*a**4 + tan(c
 + d*x)**2*a**2*b**2 + tan(c + d*x)**2*b**4 - 2*tan(c + d*x)*a**3*b - 2*ta
n(c + d*x)*a*b**3 + 2*a*b**3*d*x)/(2*b**2*d*(a**2 + b**2))
```

3.307 $\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2} dx$

Optimal result

Integrand size = 34, antiderivative size = 83

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2} dx = -\frac{bBx}{a^2+b^2}+\frac{aB\log(\cos(c+dx))}{(a^2+b^2)d} - \frac{a^3B\log(a+b\tan(c+dx))}{b^2(a^2+b^2)d} + \frac{B\tan(c+dx)}{bd}$$

output
```
-b*B*x/(a^2+b^2)+a*B*ln(cos(d*x+c))/(a^2+b^2)/d-a^3*B*ln(a+b*tan(d*x+c))/b
^2/(a^2+b^2)/d+B*tan(d*x+c)/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2} dx = -\frac{B\left(\frac{\log(i-\tan(c+dx))}{a+ib}+\frac{\log(i+\tan(c+dx))}{a-ib}+\frac{2a^3\log(a+b\tan(c+dx))}{b^2(a^2+b^2)}-\frac{2\tan(c+dx)}{b}\right)}{2d}$$

input
```
Integrate[(Tan[c + d*x]^3*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2
,x]
```

output
```
-1/2*(B*(Log[I - Tan[c + d*x]]/(a + I*b) + Log[I + Tan[c + d*x]]/(a - I*b)
 + (2*a^3*Log[a + b*Tan[c + d*x]])/(b^2*(a^2 + b^2)) - (2*Tan[c + d*x])/b)
)/d
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2011, 3042, 4049, 25, 3042, 4109, 3042, 3956, 4100, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\tan^3(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\tan(c+dx)^3}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4049}$$

$$B\left(\frac{\int -\frac{a\tan^2(c+dx)+b\tan(c+dx)+a}{a+b\tan(c+dx)}dx}{b}+\frac{\tan(c+dx)}{bd}\right)$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{\tan(c+dx)}{bd}-\frac{\int \frac{a\tan^2(c+dx)+b\tan(c+dx)+a}{a+b\tan(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\tan(c+dx)}{bd}-\frac{\int \frac{a\tan(c+dx)^2+b\tan(c+dx)+a}{a+b\tan(c+dx)}dx}{b}\right)$$

$$\Big\downarrow \text{4109}$$

$$B\left(\frac{\tan(c+dx)}{bd}-\frac{\frac{ab\int \tan(c+dx)dx}{a^2+b^2}+\frac{a^3\int \frac{\tan^2(c+dx)+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2x}{a^2+b^2}}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\tan(c+dx)}{bd}-\frac{\frac{ab\int \tan(c+dx)dx}{a^2+b^2}+\frac{a^3\int \frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{b^2x}{a^2+b^2}}{b}\right)$$

$$\Big\downarrow \text{3956}$$

$$B\left(\frac{\tan(c+dx)}{bd}-\frac{\frac{a^3\int \frac{\tan(c+dx)^2+1}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{ab\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{b^2x}{a^2+b^2}}{b}\right)$$

$$\Big\downarrow \text{4100}$$

$$B\left(\frac{\tan(c+dx)}{bd}-\frac{\frac{a^3\int \frac{1}{a+b\tan(c+dx)}d(b\tan(c+dx))}{bd(a^2+b^2)}-\frac{ab\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{b^2x}{a^2+b^2}}{b}\right)$$

$$\Big\downarrow \text{16}$$

$$B\left(\frac{\tan(c+dx)}{bd}-\frac{-\frac{ab\log(\cos(c+dx))}{d(a^2+b^2)}+\frac{b^2x}{a^2+b^2}+\frac{a^3\log(a+b\tan(c+dx))}{bd(a^2+b^2)}}{b}\right)$$

input

```
Int[(Tan[c + d*x]^3*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output

```
B*(-(((b^2*x)/(a^2 + b^2) - (a*b*Log[Cos[c + d*x]])/((a^2 + b^2)*d) + (a^3
*Log[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d))/b) + Tan[c + d*x]/(b*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4100
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[A/(b*f)   Subst[Int[(a + x)^m, x], x, b*
Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

rule 4109
```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(
1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Simp[(A*b - a*B - b*C)/(
a^2 + b^2)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &
& NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C
, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{B\left(\frac{\tan(dx+c)}{b}+\frac{-\frac{a\ln\left(1+\tan(dx+c)^2\right)}{2}-b\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^3\ln(a+b\tan(dx+c))}{b^2\left(a^2+b^2\right)}\right)}{d}$
default	$\frac{B\left(\frac{\tan(dx+c)}{b}+\frac{-\frac{a\ln\left(1+\tan(dx+c)^2\right)}{2}-b\arctan(\tan(dx+c))}{a^2+b^2}-\frac{a^3\ln(a+b\tan(dx+c))}{b^2\left(a^2+b^2\right)}\right)}{d}$
parallelrisch	$-\frac{2B\,b^3dx+B\ln\left(1+\tan(dx+c)^2\right)a\,b^2+2B\ln(a+b\tan(dx+c))a^3-2B\,a^2b\tan(dx+c)-2B\,b^3\tan(dx+c)}{2(a^2+b^2)b^2d}$
norman	$\frac{\frac{B\tan(dx+c)^2}{d}-\frac{B\,a^2}{d\,b^2}-\frac{bBax}{a^2+b^2}-\frac{b^2Bx\tan(dx+c)}{a^2+b^2}}{a+b\tan(dx+c)}-\frac{Ba\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}-\frac{a^3B\ln(a+b\tan(dx+c))}{b^2(a^2+b^2)d}$
risch	$-\frac{ixB}{ib-a}-\frac{2iaBx}{b^2}-\frac{2iBac}{b^2d}+\frac{2ia^3Bx}{b^2(a^2+b^2)}+\frac{2ia^3Bc}{d(a^2+b^2)b^2}+\frac{2iB}{db(e^{2i(dx+c)}+1)}+\frac{\ln(e^{2i(dx+c)}+1)Ba}{b^2d}-\frac{a^3\ln(\ldots}{}$

input
```
int(tan(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*B*(tan(d*x+c)/b+1/(a^2+b^2)*(-1/2*a*ln(1+tan(d*x+c)^2)-b*arctan(tan(d*
x+c)))-1/b^2*a^3/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.43

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{2\,Bb^3dx + Ba^3\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right) - (Ba^3+Bab^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right) - 2\,(Ba^2b+Bb^3)}{2\,(a^2b^2+b^4)d}$$

input
```
integrate(tan(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
-1/2*(2*B*b^3*d*x + B*a^3*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a
^2)/(tan(d*x + c)^2 + 1)) - (B*a^3 + B*a*b^2)*log(1/(tan(d*x + c)^2 + 1))
- 2*(B*a^2*b + B*b^3)*tan(d*x + c))/((a^2*b^2 + b^4)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 660, normalized size of antiderivative = 7.95

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)**3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*B*x*tan(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*(-log(ta
n(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d))/a, Eq(b, 0)), (-3*B*d*x*
tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 3*I*B*d*x/(2*b*d*tan(c + d*x
) - 2*I*b*d) + I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*
x) - 2*I*b*d) + B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d)
+ 2*B*tan(c + d*x)**2/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*I*B*tan(c + d*x)/
(2*b*d*tan(c + d*x) - 2*I*b*d) + 5*B/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a,
 -I*b)), (-3*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) - 3*I*B*d*x
/(2*b*d*tan(c + d*x) + 2*I*b*d) - I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x
)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c
 + d*x) + 2*I*b*d) + 2*B*tan(c + d*x)**2/(2*b*d*tan(c + d*x) + 2*I*b*d) -
2*I*B*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 5*B/(2*b*d*tan(c + d*x
) + 2*I*b*d), Eq(a, I*b)), (x*(B*a + B*b*tan(c))*tan(c)**3/(a + b*tan(c))*
*2, Eq(d, 0)), (-2*B*a**3*log(a/b + tan(c + d*x))/(2*a**2*b**2*d + 2*b**4*
d) + 2*B*a**2*b*tan(c + d*x)/(2*a**2*b**2*d + 2*b**4*d) - B*a*b**2*log(tan
(c + d*x)**2 + 1)/(2*a**2*b**2*d + 2*b**4*d) - 2*B*b**3*d*x/(2*a**2*b**2*d
 + 2*b**4*d) + 2*B*b**3*tan(c + d*x)/(2*a**2*b**2*d + 2*b**4*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\,Ba^3\log(b\tan(dx+c)+a)}{a^2b^2+b^4}+\frac{2\,(dx+c)Bb}{a^2+b^2}+\frac{Ba\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,B\tan(dx+c)}{b}}{2\,d}$$

input
```
integrate(tan(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
-1/2*(2*B*a^3*log(b*tan(d*x + c) + a)/(a^2*b^2 + b^4) + 2*(d*x + c)*B*b/(a
^2 + b^2) + B*a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*B*tan(d*x + c)/b)/
d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{2}\left(\frac{2\,a^3\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^2b^2d+b^4d}+\frac{2\,(dx+c)b}{a^2d+b^2d}+\frac{a\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2d+b^2d}-\frac{2\,\tan\left(dx+c\right)}{bd}\right)B$$

input
```
integrate(tan(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
-1/2*(2*a^3*log(abs(b*tan(d*x + c) + a))/(a^2*b^2*d + b^4*d) + 2*(d*x + c)
*b/(a^2*d + b^2*d) + a*log(tan(d*x + c)^2 + 1)/(a^2*d + b^2*d) - 2*tan(d*x
 + c)/(b*d))*B
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{B\tan(c+d\,x)}{b\,d}-\frac{B\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)} - \frac{B\,a^3\,\ln\left(a+b\tan(c+d\,x)\right)}{b^2\,d\,\left(a^2+b^2\right)} - \frac{B\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}$$

input
```
int((tan(c + d*x)^3*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(B*tan(c + d*x))/(b*d) - (B*log(tan(c + d*x) + 1i))/(2*d*(a - b*1i)) - (B*
log(tan(c + d*x) - 1i)*1i)/(2*d*(a*1i - b)) - (B*a^3*log(a + b*tan(c + d*x
)))/(b^2*d*(a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{\tan^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{-\log\left(\tan\left(dx+c\right)^2+1\right)a\,b^2-2\log\left(a+\tan\left(dx+c\right)b\right)a^3+2\tan\left(dx+c\right)a^2b+2\tan\left(dx+c\right)b^3-2b^3}{2bd\left(a^2+b^2\right)}$$

input
```
int(tan(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
( - log(tan(c + d*x)**2 + 1)*a*b**2 - 2*log(tan(c + d*x)*b + a)*a**3 + 2*t
an(c + d*x)*a**2*b + 2*tan(c + d*x)*b**3 - 2*b**3*d*x)/(2*b*d*(a**2 + b**2
))
```

3.308 $\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 81

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{aBx}{b^2}+\frac{a^3Bx}{b^2(a^2+b^2)}-\frac{B\log(\cos(c+dx))}{bd}+\frac{a^2B\log(a\cos(c+dx)+b\sin(c+dx))}{b(a^2+b^2)\,d}$$

output
```
-a*B*x/b^2+a^3*B*x/b^2/(a^2+b^2)-B*ln(cos(d*x+c))/b/d+a^2*B*ln(a*cos(d*x+c
)+b*sin(d*x+c))/b/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{B(b(ia+b)\log(i-\tan(c+dx))+b(-ia+b)\log(i+\tan(c+dx))+2a^2\log(a+b\tan(c+dx)))}{2b(a^2+b^2)\,d}$$

input
```
Integrate[(Tan[c + d*x]^2*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2
,x]
```

output
```
(B*(b*(I*a + b)*Log[I - Tan[c + d*x]] + b*((-I)*a + b)*Log[I + Tan[c + d*x
]] + 2*a^2*Log[a + b*Tan[c + d*x]]))/(2*b*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2011, 3042, 4024, 3042, 3956, 3965, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\tan^2(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\tan(c+dx)^2}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4024}$$

$$B\left(\frac{a^2\int \frac{1}{a+b\tan(c+dx)}dx}{b^2}+\frac{\int \tan(c+dx)dx}{b}-\frac{ax}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{a^2\int \frac{1}{a+b\tan(c+dx)}dx}{b^2}+\frac{\int \tan(c+dx)dx}{b}-\frac{ax}{b^2}\right)$$

$$\Big\downarrow \text{3956}$$

$$B\left(\frac{a^2\int \frac{1}{a+b\tan(c+dx)}dx}{b^2}-\frac{ax}{b^2}-\frac{\log(\cos(c+dx))}{bd}\right)$$

$$\Big\downarrow \text{3965}$$

$$B\left(\frac{a^2\left(\frac{b\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{b^2}-\frac{ax}{b^2}-\frac{\log(\cos(c+dx))}{bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{a^2\left(\frac{b\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{b^2}-\frac{ax}{b^2}-\frac{\log(\cos(c+dx))}{bd}\right)$$

$$\Big\downarrow \text{4013}$$

$$B\left(\frac{a^2\left(\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)}+\frac{ax}{a^2+b^2}\right)}{b^2}-\frac{ax}{b^2}-\frac{\log(\cos(c+dx))}{bd}\right)$$

input
```
Int[(Tan[c + d*x]^2*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
B*(-((a*x)/b^2) - Log[Cos[c + d*x]]/(b*d) + (a^2*((a*x)/(a^2 + b^2) + (b*L
og[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d)))/b^2)
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4024
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Simp[d^2/b   I
nt[Tan[e + f*x], x], x] + Simp[(b*c - a*d)^2/b^2   Int[1/(a + b*Tan[e + f*x
]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 + b^2, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

method	result	size
parallelrisch	$\frac{-2Babdx+B\ln\left(1+\tan(dx+c)^2\right)b^2+2B\ln(a+b\tan(dx+c))a^2}{2bd(a^2+b^2)}$	59
derivativedivides	$\frac{B\left(\frac{\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2}-a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^2\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)b}\right)}{d}$	69
default	$\frac{B\left(\frac{\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2}-a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{a^2\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)b}\right)}{d}$	69
norman	$\frac{-\frac{B\,a^2x}{a^2+b^2}-\frac{bBax\tan(dx+c)}{a^2+b^2}}{a+b\tan(dx+c)}+\frac{B\,a^2\ln(a+b\tan(dx+c))}{bd(a^2+b^2)}+\frac{Bb\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}$	111
risch	$\frac{xB}{ib-a}+\frac{2iBx}{b}+\frac{2iBc}{bd}-\frac{2ia^2Bx}{b(a^2+b^2)}-\frac{2ia^2Bc}{bd(a^2+b^2)}-\frac{\ln(\mathrm{e}^{2i(dx+c)}+1)B}{bd}+\frac{a^2\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)B}{bd(a^2+b^2)}$	147

input
```
int(tan(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/2*(-2*B*a*b*d*x+B*ln(1+tan(d*x+c)^2)*b^2+2*B*ln(a+b*tan(d*x+c))*a^2)/b/d
/(a^2+b^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= -\frac{2\,Babdx - Ba^2\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right) + (Ba^2+Bb^2)\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,(a^2b+b^3)d}$$

input
```
integrate(tan(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
-1/2*(2*B*a*b*d*x - B*a^2*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a
^2)/(tan(d*x + c)^2 + 1)) + (B*a^2 + B*b^2)*log(1/(tan(d*x + c)^2 + 1)))/(
(a^2*b + b^3)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 442, normalized size of antiderivative = 5.46

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \begin{cases} \tilde{\infty}Bx\tan{(c)} \\ \frac{B\left(-x+\frac{\tan{(c+dx)}}{d}\right)}{a} \\ \frac{iBdx\tan{(c+dx)}}{2bd\tan{(c+dx)}-2ibd} + \frac{Bdx}{2bd\tan{(c+dx)}-2ibd} + \frac{B\log{(\tan^2{(c+dx)}+1)}\tan{(c+dx)}}{2bd\tan{(c+dx)}-2ibd} - \frac{iB\log{(\tan^2{(c+dx)}+1)}}{2bd\tan{(c+dx)}-2ibd} - \frac{iB}{2bd\tan{(c+dx)}-2ibd} \\ -\frac{iBdx\tan{(c+dx)}}{2bd\tan{(c+dx)}+2ibd} + \frac{Bdx}{2bd\tan{(c+dx)}+2ibd} + \frac{B\log{(\tan^2{(c+dx)}+1)}\tan{(c+dx)}}{2bd\tan{(c+dx)}+2ibd} + \frac{iB\log{(\tan^2{(c+dx)}+1)}}{2bd\tan{(c+dx)}+2ibd} + \frac{iB}{2bd\tan{(c+dx)}+2i} \\ \frac{x(Ba+Bb\tan{(c)})\tan^2{(c)}}{(a+b\tan{(c)})^2} \\ \frac{2Ba^2\log{(\frac{a}{b}+\tan{(c+dx)})}}{2a^2bd+2b^3d} - \frac{2Babdx}{2a^2bd+2b^3d} + \frac{Bb^2\log{(\tan^2{(c+dx)}+1)}}{2a^2bd+2b^3d} \end{cases}$$

input
```
integrate(tan(d*x+c)**2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*B*x*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*(-x + tan(c
 + d*x)/d)/a, Eq(b, 0)), (I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b
*d) + B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*log(tan(c + d*x)**2 + 1)*ta
n(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*B*log(tan(c + d*x)**2 + 1)/(
2*b*d*tan(c + d*x) - 2*I*b*d) - I*B/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a,
-I*b)), (-I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*d*x/(2*b
*d*tan(c + d*x) + 2*I*b*d) + B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*
d*tan(c + d*x) + 2*I*b*d) + I*B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*
x) + 2*I*b*d) + I*B/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*(B*a +
 B*b*tan(c))*tan(c)**2/(a + b*tan(c))**2, Eq(d, 0)), (2*B*a**2*log(a/b + t
an(c + d*x))/(2*a**2*b*d + 2*b**3*d) - 2*B*a*b*d*x/(2*a**2*b*d + 2*b**3*d)
 + B*b**2*log(tan(c + d*x)**2 + 1)/(2*a**2*b*d + 2*b**3*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,Ba^2\log(b\tan(dx+c)+a)}{a^2b+b^3} - \frac{2\,(dx+c)Ba}{a^2+b^2} + \frac{Bb\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(tan(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
1/2*(2*B*a^2*log(b*tan(d*x + c) + a)/(a^2*b + b^3) - 2*(d*x + c)*B*a/(a^2
+ b^2) + B*b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{1}{2}\left(\frac{2\,a^2\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^2bd+b^3d}-\frac{2\,(dx+c)a}{a^2d+b^2d}+\frac{b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2d+b^2d}\right)B$$

input
```
integrate(tan(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
1/2*(2*a^2*log(abs(b*tan(d*x + c) + a))/(a^2*b*d + b^3*d) - 2*(d*x + c)*a/
(a^2*d + b^2*d) + b*log(tan(d*x + c)^2 + 1)/(a^2*d + b^2*d))*B
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx=\frac{B\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}+\frac{B\,a^2\,\ln\left(a+b\tan(c+d\,x)\right)}{b\,d\,\left(a^2+b^2\right)}+\frac{B\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int((tan(c + d*x)^2*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(B*log(tan(c + d*x) - 1i)*1i)/(2*d*(a + b*1i)) + (B*log(tan(c + d*x) + 1i)
)/(2*d*(a*1i + b)) + (B*a^2*log(a + b*tan(c + d*x)))/(b*d*(a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

$$\int \frac{\tan^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\log\left(\tan\left(dx+c\right)^2+1\right)b^2+2\log(a+\tan\left(dx+c\right)b)\,a^2-2abdx}{2d\left(a^2+b^2\right)}$$

input
```
int(tan(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(log(tan(c + d*x)**2 + 1)*b**2 + 2*log(tan(c + d*x)*b + a)*a**2 - 2*a*b*d*
x)/(2*d*(a**2 + b**2))
```

3.309 $\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 48

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{bBx}{a^2+b^2}-\frac{aB\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

output
```
b*B*x/(a^2+b^2)-a*B*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{B(2(-ia+b)(c+dx)+2ia\arctan(\tan(c+dx))-a\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right))}{2\,(a^2+b^2)\,d}$$

input
```
Integrate[(Tan[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x
]
```

output
```
(B*(2*((-I)*a + b)*(c + d*x) + (2*I)*a*ArcTan[Tan[c + d*x]] - a*Log[(a*Cos
[c + d*x] + b*Sin[c + d*x])^2]))/(2*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = $\{2011, 3042, 4014, 3042, 4013\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\tan(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\tan(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4014}$$

$$B\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{4013}$$

$$B\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)}\right)$$

input
```
Int[(Tan[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
B*((b*x)/(a^2 + b^2) - (a*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^
2)*d))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$\frac{2Bbdx+B\ln\left(1+\tan(dx+c)^{2}\right)a-2B\ln(a+b\tan(dx+c))a}{2d(a^{2}+b^{2})}$	51
derivativedivides	$\frac{B\left(\frac{\frac{a\ln\left(1+\tan(dx+c)^{2}\right)}{2}+b\arctan(\tan(dx+c))}{a^{2}+b^{2}}-\frac{a\ln(a+b\tan(dx+c))}{a^{2}+b^{2}}\right)}{d}$	64
default	$\frac{B\left(\frac{\frac{a\ln\left(1+\tan(dx+c)^{2}\right)}{2}+b\arctan(\tan(dx+c))}{a^{2}+b^{2}}-\frac{a\ln(a+b\tan(dx+c))}{a^{2}+b^{2}}\right)}{d}$	64
risch	$\frac{ixB}{ib-a}+\frac{2iBxa}{a^{2}+b^{2}}+\frac{2iBac}{d(a^{2}+b^{2})}-\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)Ba}{d(a^{2}+b^{2})}$	95
norman	$\frac{\frac{bBax}{a^{2}+b^{2}}+\frac{b^{2}Bx\tan(dx+c)}{a^{2}+b^{2}}}{a+b\tan(dx+c)}+\frac{Ba\ln\left(1+\tan(dx+c)^{2}\right)}{2d(a^{2}+b^{2})}-\frac{Ba\ln(a+b\tan(dx+c))}{d(a^{2}+b^{2})}$	105

input
```
int(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVER
BOSE)
```

output
```
1/2*(2*B*b*d*x+B*ln(1+tan(d*x+c)^2)*a-2*B*ln(a+b*tan(d*x+c))*a)/d/(a^2+b^2
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{2\,Bbdx-Ba\log\left(\frac{b^{2}\tan(dx+c)^{2}+2\,ab\tan(dx+c)+a^{2}}{\tan(dx+c)^{2}+1}\right)}{2\,(a^{2}+b^{2})d}$$

input
```
integrate(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm=
"fricas")
```

output
```
1/2*(2*B*b*d*x - B*a*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(
tan(d*x + c)^2 + 1)))/((a^2 + b^2)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 282, normalized size of antiderivative = 5.88

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \begin{cases} \tilde{\infty}Bx & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{B\log\left(\tan^2(c+dx)+1\right)}{2ad} & \text{for } b=0 \\ \frac{Bdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd} - \frac{iBdx}{2bd\tan(c+dx)-2ibd} - \frac{B}{2bd\tan(c+dx)-2ibd} & \text{for } a=-ib \\ \frac{Bdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd} + \frac{iBdx}{2bd\tan(c+dx)+2ibd} - \frac{B}{2bd\tan(c+dx)+2ibd} & \text{for } a=ib \\ \frac{x(Ba+Bb\tan(c))\tan(c)}{(a+b\tan(c))^2} & \text{for } d=0 \\ -\frac{2Ba\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2d+2b^2d} + \frac{Ba\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d} + \frac{2Bbdx}{2a^2d+2b^2d} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*B*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*log(tan(c + d*x)**
2 + 1)/(2*a*d), Eq(b, 0)), (B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b
*d) - I*B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) - B/(2*b*d*tan(c + d*x) - 2*I
*b*d), Eq(a, -I*b)), (B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) +
I*B*d*x/(2*b*d*tan(c + d*x) + 2*I*b*d) - B/(2*b*d*tan(c + d*x) + 2*I*b*d),
 Eq(a, I*b)), (x*(B*a + B*b*tan(c))*tan(c)/(a + b*tan(c))**2, Eq(d, 0)), (
-2*B*a*log(a/b + tan(c + d*x))/(2*a**2*d + 2*b**2*d) + B*a*log(tan(c + d*x
)**2 + 1)/(2*a**2*d + 2*b**2*d) + 2*B*b*d*x/(2*a**2*d + 2*b**2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.48

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,(dx+c)Bb}{a^2+b^2} - \frac{2\,Ba\log(b\tan(dx+c)+a)}{a^2+b^2} + \frac{Ba\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm=
"maxima")
```

output
```
1/2*(2*(d*x + c)*B*b/(a^2 + b^2) - 2*B*a*log(b*tan(d*x + c) + a)/(a^2 + b^
2) + B*a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.71

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{2}\left(\frac{2\,ab\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^2bd+b^3d} - \frac{2\,(dx+c)b}{a^2d+b^2d} - \frac{a\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2d+b^2d}\right)B$$

input
```
integrate(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm=
"giac")
```

output
```
-1/2*(2*a*b*log(abs(b*tan(d*x + c) + a))/(a^2*b*d + b^3*d) - 2*(d*x + c)*b
/(a^2*d + b^2*d) - a*log(tan(d*x + c)^2 + 1)/(a^2*d + b^2*d))*B
```

Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{B\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)} - \frac{B\,a\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2+b^2\right)} + \frac{B\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}$$

input
```
int((tan(c + d*x)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(B*log(tan(c + d*x) + 1i))/(2*d*(a - b*1i)) + (B*log(tan(c + d*x) - 1i)*1i
)/(2*d*(a*1i - b)) - (B*a*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\tan(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{b\left(\log\left(\tan\left(dx+c\right)^2+1\right)a-2\log\left(a+\tan\left(dx+c\right)b\right)a+2bdx\right)}{2d\left(a^2+b^2\right)}$$

input
```
int(tan(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(b*(log(tan(c + d*x)**2 + 1)*a - 2*log(tan(c + d*x)*b + a)*a + 2*b*d*x))/(
2*d*(a**2 + b**2))
```

3.310 $\int \frac{aB+bB\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \frac{aB+bB\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{aBx}{a^2+b^2}+\frac{bB\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

output
```
a*B*x/(a^2+b^2)+b*B*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int \frac{aB+bB\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{B((-ia-b)\log(i-\tan(c+dx))+i(a+ib)\log(i+\tan(c+dx))+2b\log(a+b\tan(c+dx)))}{2\,(a^2+b^2)\,d}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(a + b*Tan[c + d*x])^2,x]
```

output
```
(B*(((-I)*a - b)*Log[I - Tan[c + d*x]] + I*(a + I*b)*Log[I + Tan[c + d*x]]
 + 2*b*Log[a + b*Tan[c + d*x]]))/(2*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2011, 3042, 3965, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3965}$$

$$B\left(\frac{b\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{b\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)$$

$$\Big\downarrow \text{4013}$$

$$B\left(\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)}+\frac{ax}{a^2+b^2}\right)$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(a + b*Tan[c + d*x])^2,x]
```

output
```
B*((a*x)/(a^2 + b^2) + (b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^
2)*d))
```

Defintions of rubi rules used

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$-\frac{-2Bxad+B\ln\left(1+\tan(dx+c)^{2}\right)b-2Bb\ln(a+b\tan(dx+c))}{2d(a^{2}+b^{2})}$	51
derivativedivides	$\frac{B\left(\frac{-\frac{b\ln\left(1+\tan(dx+c)^{2}\right)}{2}+a\arctan(\tan(dx+c))}{a^{2}+b^{2}}+\frac{b\ln(a+b\tan(dx+c))}{a^{2}+b^{2}}\right)}{d}$	63
default	$\frac{B\left(\frac{-\frac{b\ln\left(1+\tan(dx+c)^{2}\right)}{2}+a\arctan(\tan(dx+c))}{a^{2}+b^{2}}+\frac{b\ln(a+b\tan(dx+c))}{a^{2}+b^{2}}\right)}{d}$	63
risch	$-\frac{xB}{ib-a}-\frac{2ibBx}{a^{2}+b^{2}}-\frac{2ibBc}{d(a^{2}+b^{2})}+\frac{b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)B}{d(a^{2}+b^{2})}$	93
norman	$\frac{\frac{Ba^{2}x}{a^{2}+b^{2}}+\frac{bBax\tan(dx+c)}{a^{2}+b^{2}}}{a+b\tan(dx+c)}+\frac{Bb\ln(a+b\tan(dx+c))}{d(a^{2}+b^{2})}-\frac{Bb\ln\left(1+\tan(dx+c)^{2}\right)}{2d(a^{2}+b^{2})}$	104

input
```
int((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*B*x*a*d+B*ln(1+tan(d*x+c)^2)*b-2*B*b*ln(a+b*tan(d*x+c)))/d/(a^2+b
^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \frac{aB + bB\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{2\,Badx + Bb\log\left(\frac{b^2\tan(dx+c)^2 + 2\,ab\tan(dx+c) + a^2}{\tan(dx+c)^2 + 1}\right)}{2\,(a^2+b^2)d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(2*B*a*d*x + B*b*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(
tan(d*x + c)^2 + 1)))/((a^2 + b^2)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.79

$$\int \frac{aB + bB\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \begin{cases} \frac{\tilde{\infty}Bx}{\tan(c)} & \text{for}\ a=0 \wedge b=0 \wedge d=0 \\ \frac{Bx}{a} & \text{for}\ b=0 \\ \frac{iBdx\tan(c+dx)}{2bd\tan(c+dx)-2ibd} + \frac{Bdx}{2bd\tan(c+dx)-2ibd} + \frac{iB}{2bd\tan(c+dx)-2ibd} & \text{for}\ a=-ib \\ -\frac{iBdx\tan(c+dx)}{2bd\tan(c+dx)+2ibd} + \frac{Bdx}{2bd\tan(c+dx)+2ibd} - \frac{iB}{2bd\tan(c+dx)+2ibd} & \text{for}\ a=ib \\ \frac{x(Ba+Bb\tan(c))}{(a+b\tan(c))^2} & \text{for}\ d=0 \\ \frac{2Badx}{2a^2d+2b^2d} + \frac{2Bb\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^2d+2b^2d} - \frac{Bb\log\left(\tan^2(c+dx)+1\right)}{2a^2d+2b^2d} & \text{otherwise} \end{cases}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*B*x/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*x/a, Eq(b,
0)), (I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + B*d*x/(2*b*d*t
an(c + d*x) - 2*I*b*d) + I*B/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)),
 (-I*B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + B*d*x/(2*b*d*tan(
c + d*x) + 2*I*b*d) - I*B/(2*b*d*tan(c + d*x) + 2*I*b*d), Eq(a, I*b)), (x*
(B*a + B*b*tan(c))/(a + b*tan(c))**2, Eq(d, 0)), (2*B*a*d*x/(2*a**2*d + 2*
b**2*d) + 2*B*b*log(a/b + tan(c + d*x))/(2*a**2*d + 2*b**2*d) - B*b*log(ta
n(c + d*x)**2 + 1)/(2*a**2*d + 2*b**2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \frac{aB + bB\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,(dx+c)Ba}{a^2+b^2} + \frac{2\,Bb\log(b\tan(dx+c)+a)}{a^2+b^2} - \frac{Bb\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(2*(d*x + c)*B*a/(a^2 + b^2) + 2*B*b*log(b*tan(d*x + c) + a)/(a^2 + b^
2) - B*b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.77

$$\int \frac{aB + bB\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{1}{2}\left(\frac{2\,b^2\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^2bd+b^3d} + \frac{2\,(dx+c)a}{a^2d+b^2d} - \frac{b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2d+b^2d}\right)B$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(2*b^2*log(abs(b*tan(d*x + c) + a))/(a^2*b*d + b^3*d) + 2*(d*x + c)*a/
(a^2*d + b^2*d) - b*log(tan(d*x + c)^2 + 1)/(a^2*d + b^2*d))*B
```

Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^2}\,dx = \frac{B\,b\,\ln\left(a + b\tan(c + d\,x)\right)}{d\,\left(a^2 + b^2\right)} - \frac{B\,\ln\left(\tan(c + d\,x) + 1\mathrm{i}\right)}{2\,d\,\left(b + a\,1\mathrm{i}\right)} - \frac{B\,\ln\left(\tan(c + d\,x) - \mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a + b\,1\mathrm{i}\right)}$$

input
```
int((B*a + B*b*tan(c + d*x))/(a + b*tan(c + d*x))^2,x)
```

output
```
(B*b*log(a + b*tan(c + d*x)))/(d*(a^2 + b^2)) - (B*log(tan(c + d*x) + 1i))
/(2*d*(a*1i + b)) - (B*log(tan(c + d*x) - 1i)*1i)/(2*d*(a + b*1i))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^2}\,dx = \frac{b\left(-\log\left(\tan\left(dx + c\right)^2 + 1\right)b + 2\log\left(a + \tan\left(dx + c\right)b\right)b + 2adx\right)}{2d\left(a^2 + b^2\right)}$$

input
```
int((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(b*( - log(tan(c + d*x)**2 + 1)*b + 2*log(tan(c + d*x)*b + a)*b + 2*a*d*x)
)/(2*d*(a**2 + b**2))
```

3.311 $\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 69

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{bBx}{a^2+b^2}+\frac{B\log(\sin(c+dx))}{ad} - \frac{b^2B\log(a\cos(c+dx)+b\sin(c+dx))}{a\,(a^2+b^2)\,d}$$

output
```
-b*B*x/(a^2+b^2)+B*ln(sin(d*x+c))/a/d-b^2*B*ln(a*cos(d*x+c)+b*sin(d*x+c))/
a/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{B(a(a+ib)\log(i-\cot(c+dx))+a(a-ib)\log(i+\cot(c+dx))+2b^2\log(b+a\cot(c+dx)))}{2a\,(a^2+b^2)\,d}$$

input
```
Integrate[(Cot[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x
]
```

output
```
-1/2*(B*(a*(a + I*b)*Log[I - Cot[c + d*x]] + a*(a - I*b)*Log[I + Cot[c + d
*x]] + 2*b^2*Log[b + a*Cot[c + d*x]]))/(a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2011, 3042, 4054, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\cot(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\tan(c+dx)(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4054}$$

$$B\left(-\frac{b^2\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{\int \cot(c+dx)dx}{a}-\frac{bx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(-\frac{b^2\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}-\frac{bx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{25}$$

$$B\left(-\frac{b^2\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}-\frac{bx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3956}$$

$$B\left(-\frac{b^2\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{bx}{a^2+b^2}+\frac{\log(-\sin(c+dx))}{ad}\right)$$

$$\Big\downarrow \text{4013}$$

$$B\left(-\frac{b^2\log(a\cos(c+dx)+b\sin(c+dx))}{ad\left(a^2+b^2\right)}-\frac{bx}{a^2+b^2}+\frac{\log(-\sin(c+dx))}{ad}\right)$$

input

```
Int[(Cot[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output

```
B*(-((b*x)/(a^2 + b^2)) + Log[-Sin[c + d*x]]/(a*d) - (b^2*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2011

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4054
```
Int[1/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f
_.)*(x_)])), x_Symbol] :> Simp[(a*c - b*d)*(x/((a^2 + b^2)*(c^2 + d^2))), x
] + (Simp[b^2/((b*c - a*d)*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*T
an[e + f*x]), x], x] - Simp[d^2/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[
e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

method	result	siz
parallelrisch	$\frac{B\left(a^{2}\left(-\ln\left(\sec(dx+c)^{2}\right)+2\ln(\tan(dx+c))\right)+2b^{2}(\ln(\tan(dx+c))-\ln(a+b\tan(dx+c)))-2abdx\right)}{2ad(a^{2}+b^{2})}$	77
derivativedivides	$\frac{B\left(\frac{-\frac{a\ln\left(1+\tan(dx+c)^{2}\right)}{2}-b\arctan(\tan(dx+c))}{a^{2}+b^{2}}+\frac{\ln(\tan(dx+c))}{a}-\frac{b^{2}\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)a}\right)}{d}$	81
default	$\frac{B\left(\frac{-\frac{a\ln\left(1+\tan(dx+c)^{2}\right)}{2}-b\arctan(\tan(dx+c))}{a^{2}+b^{2}}+\frac{\ln(\tan(dx+c))}{a}-\frac{b^{2}\ln(a+b\tan(dx+c))}{\left(a^{2}+b^{2}\right)a}\right)}{d}$	81
norman	$\frac{-\frac{bBax}{a^{2}+b^{2}}-\frac{b^{2}Bx\tan(dx+c)}{a^{2}+b^{2}}}{a+b\tan(dx+c)}+\frac{B\ln(\tan(dx+c))}{ad}-\frac{Ba\ln\left(1+\tan(dx+c)^{2}\right)}{2d(a^{2}+b^{2})}-\frac{B\,b^{2}\ln(a+b\tan(dx+c))}{ad(a^{2}+b^{2})}$	127
risch	$-\frac{ixB}{ib-a}-\frac{2ixB}{a}-\frac{2iBc}{ad}+\frac{2ib^{2}Bx}{a(a^{2}+b^{2})}+\frac{2ib^{2}Bc}{ad(a^{2}+b^{2})}+\frac{\ln\left(e^{2i(dx+c)}-1\right)B}{ad}-\frac{b^{2}\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)B}{ad(a^{2}+b^{2})}$	149

input
```
int(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNVER
BOSE)
```

output
```
1/2*B*(a^2*(-ln(sec(d*x+c)^2)+2*ln(tan(d*x+c)))+2*b^2*(ln(tan(d*x+c))-ln(a
+b*tan(d*x+c)))-2*a*b*d*x)/a/d/(a^2+b^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.51

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{2\,Babdx+Bb^2\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)-(Ba^2+Bb^2)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)}{2\,(a^3+ab^2)d}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm=
"fricas")
```

output
```
-1/2*(2*B*a*b*d*x + B*b^2*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a
^2)/(tan(d*x + c)^2 + 1)) - (B*a^2 + B*b^2)*log(tan(d*x + c)^2/(tan(d*x +
c)^2 + 1)))/((a^3 + a*b^2)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 672, normalized size of antiderivative = 9.74

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*B*x*cot(c)/tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*(-lo
g(tan(c + d*x)**2 + 1)/(2*d) + log(tan(c + d*x))/d)/a, Eq(b, 0)), (B*(-x -
 1/(d*tan(c + d*x)))/b, Eq(a, 0)), (B*d*x*tan(c + d*x)/(2*b*d*tan(c + d*x)
 - 2*I*b*d) - I*B*d*x/(2*b*d*tan(c + d*x) - 2*I*b*d) - I*B*log(tan(c + d*x
)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) - B*log(tan(c + d*x)
**2 + 1)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*I*B*log(tan(c + d*x))*tan(c +
d*x)/(2*b*d*tan(c + d*x) - 2*I*b*d) + 2*B*log(tan(c + d*x))/(2*b*d*tan(c +
 d*x) - 2*I*b*d) + B/(2*b*d*tan(c + d*x) - 2*I*b*d), Eq(a, -I*b)), (B*d*x*
tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + I*B*d*x/(2*b*d*tan(c + d*x)
+ 2*I*b*d) + I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*b*d*tan(c + d*x)
 + 2*I*b*d) - B*log(tan(c + d*x)**2 + 1)/(2*b*d*tan(c + d*x) + 2*I*b*d) -
2*I*B*log(tan(c + d*x))*tan(c + d*x)/(2*b*d*tan(c + d*x) + 2*I*b*d) + 2*B*
log(tan(c + d*x))/(2*b*d*tan(c + d*x) + 2*I*b*d) + B/(2*b*d*tan(c + d*x) +
 2*I*b*d), Eq(a, I*b)), (x*(B*a + B*b*tan(c))*cot(c)/(a + b*tan(c))**2, Eq
(d, 0)), (-B*a**2*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2*a*b**2*d) + 2*B*a
**2*log(tan(c + d*x))/(2*a**3*d + 2*a*b**2*d) - 2*B*a*b*d*x/(2*a**3*d + 2*
a*b**2*d) - 2*B*b**2*log(a/b + tan(c + d*x))/(2*a**3*d + 2*a*b**2*d) + 2*B
*b**2*log(tan(c + d*x))/(2*a**3*d + 2*a*b**2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\,Bb^2\log(b\tan(dx+c)+a)}{a^3+ab^2}+\frac{2\,(dx+c)Bb}{a^2+b^2}+\frac{Ba\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,B\log(\tan(dx+c))}{a}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm=
"maxima")
```

output
```
-1/2*(2*B*b^2*log(b*tan(d*x + c) + a)/(a^3 + a*b^2) + 2*(d*x + c)*B*b/(a^2
 + b^2) + B*a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*B*log(tan(d*x + c))/
a)/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{2}\left(\frac{2\,b^3\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^3bd+ab^3d}+\frac{2\left(dx+c\right)b}{a^2d+b^2d}+\frac{a\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2d+b^2d}-\frac{2\,\log\left(|\tan\left(dx+c\right)|\right)}{ad}\right)$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm=
"giac")
```

output
```
-1/2*(2*b^3*log(abs(b*tan(d*x + c) + a))/(a^3*b*d + a*b^3*d) + 2*(d*x + c)
*b/(a^2*d + b^2*d) + a*log(tan(d*x + c)^2 + 1)/(a^2*d + b^2*d) - 2*log(abs
(tan(d*x + c)))/(a*d))*B
```

Mupad [B] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{B\,\ln\left(\tan(c+d\,x)\right)}{a\,d} - \frac{B\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)} - \frac{B\,b^2\,\ln\left(a+b\tan(c+d\,x)\right)}{a\,d\,\left(a^2+b^2\right)} - \frac{B\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}$$

input
```
int((cot(c + d*x)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(B*log(tan(c + d*x)))/(a*d) - (B*log(tan(c + d*x) + 1i))/(2*d*(a - b*1i))
- (B*log(tan(c + d*x) - 1i)*1i)/(2*d*(a*1i - b)) - (B*b^2*log(a + b*tan(c
+ d*x)))/(a*d*(a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.54

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{b\left(-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)a^2-\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b-a\right)b^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{ad\,(a^2+b^2)}$$

input
```
int(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(b*( - log(tan((c + d*x)/2)**2 + 1)*a**2 - log(tan((c + d*x)/2)**2*a - 2*t
an((c + d*x)/2)*b - a)*b**2 + log(tan((c + d*x)/2))*a**2 + log(tan((c + d*
x)/2))*b**2 - a*b*d*x))/(a*d*(a**2 + b**2))
```

3.312 $\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 85

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{aBx}{a^2+b^2}-\frac{B\cot(c+dx)}{ad}-\frac{bB\log(\sin(c+dx))}{a^2d}+\frac{b^3B\log(a\cos(c+dx)+b\sin(c+dx))}{a^2(a^2+b^2)\,d}$$

output
```
-a*B*x/(a^2+b^2)-B*cot(d*x+c)/a/d-b*B*ln(sin(d*x+c))/a^2/d+b^3*B*ln(a*cos(
d*x+c)+b*sin(d*x+c))/a^2/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{B\left(\frac{\cot(c+dx)}{a}-\frac{\log(i-\cot(c+dx))}{2(ia+b)}+\frac{\log(i+\cot(c+dx))}{2(ia-b)}-\frac{b^3\log(b+a\cot(c+dx))}{a^2(a^2+b^2)}\right)}{d}$$

input
```
Integrate[(Cot[c + d*x]^2*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2
,x]
```

output
```
-((B*(Cot[c + d*x]/a - Log[I - Cot[c + d*x]]/(2*(I*a + b)) + Log[I + Cot[c
 + d*x]]/(2*(I*a - b)) - (b^3*Log[b + a*Cot[c + d*x]])/(a^2*(a^2 + b^2))))
/d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2011, 3042, 4052, 3042, 4134, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\cot^2(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\tan(c+dx)^2(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4052}$$

$$B\left(-\frac{\int \frac{\cot(c+dx)\left(b\tan^2(c+dx)+a\tan(c+dx)+b\right)}{a+b\tan(c+dx)}dx}{a}-\frac{\cot(c+dx)}{ad}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(-\frac{\int \frac{b\tan(c+dx)^2+a\tan(c+dx)+b}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a}-\frac{\cot(c+dx)}{ad}\right)$$

$$\Big\downarrow \text{4134}$$

$$B\left(-\frac{-\frac{b^3\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{b\int\cot(c+dx)dx}{a}+\frac{a^2x}{a^2+b^2}}{a}-\frac{\cot(c+dx)}{ad}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$B\left(-\frac{-\frac{b^3\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{b\int-\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\frac{a^2x}{a^2+b^2}}{a}-\frac{\cot(c+dx)}{ad}\right)$$

$$\Big\downarrow \text{ 25}$$

$$B\left(-\frac{-\frac{b^3\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}-\frac{b\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}+\frac{a^2x}{a^2+b^2}}{a}-\frac{\cot(c+dx)}{ad}\right)$$

$$\Big\downarrow \text{ 3956}$$

$$B\left(-\frac{-\frac{b^3\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{a^2x}{a^2+b^2}+\frac{b\log(-\sin(c+dx))}{ad}}{a}-\frac{\cot(c+dx)}{ad}\right)$$

$$\Big\downarrow \text{ 4013}$$

$$B\left(-\frac{\frac{a^2x}{a^2+b^2}-\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{ad(a^2+b^2)}+\frac{b\log(-\sin(c+dx))}{ad}}{a}-\frac{\cot(c+dx)}{ad}\right)$$

input
```
Int[(Cot[c + d*x]^2*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
B*(-(Cot[c + d*x]/(a*d)) - ((a^2*x)/(a^2 + b^2) + (b*Log[-Sin[c + d*x]])/(
a*d) - (b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4134
```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Sim
p[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d - c*Tan[e + f*
x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{B\left(\frac{\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2}-a\arctan(\tan(dx+c))}{a^2+b^2}-\frac{1}{a\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2}+\frac{b^3\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)a^2}\right)}{d}$
default	$\frac{B\left(\frac{\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2}-a\arctan(\tan(dx+c))}{a^2+b^2}-\frac{1}{a\tan(dx+c)}-\frac{b\ln(\tan(dx+c))}{a^2}+\frac{b^3\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)a^2}\right)}{d}$
parallelrisch	$-\frac{B\left(2x\,a^3d+a^2b\left(-\ln\left(\sec(dx+c)^2\right)+2\ln(\tan(dx+c))\right)+2b^3(\ln(\tan(dx+c))-\ln(a+b\tan(dx+c)))+2\cot(dx+c)a(a^2\right.}{2(a^2+b^2)a^2d}$
norman	$\frac{\frac{B\,b^2\tan(dx+c)^2}{d\,a^2}-\frac{B}{d}-\frac{B\,a^2x\tan(dx+c)}{a^2+b^2}-\frac{bBax\tan(dx+c)^2}{a^2+b^2}}{\tan(dx+c)(a+b\tan(dx+c))}+\frac{B\,b^3\ln(a+b\tan(dx+c))}{(a^2+b^2)a^2d}-\frac{Bb\ln(\tan(dx+c))}{a^2d}+\frac{Bb\ln\left(\right.}{}$
risch	$\frac{xB}{ib-a}+\frac{2iBbx}{a^2}+\frac{2iBbc}{a^2d}-\frac{2ib^3Bx}{a^2(a^2+b^2)}-\frac{2ib^3Bc}{a^2d(a^2+b^2)}-\frac{2iB}{da\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)Bb}{a^2d}+\frac{b^3\ln\left(\mathrm{e}^{2i}\right.}{a^2}$

input
```
int(cot(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*B*(1/(a^2+b^2)*(1/2*b*ln(1+tan(d*x+c)^2)-a*arctan(tan(d*x+c)))-1/a/tan
(d*x+c)-1/a^2*b*ln(tan(d*x+c))+b^3/(a^2+b^2)/a^2*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.73

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,Ba^3dx\tan{(dx+c)} - Bb^3\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)\tan{(dx+c)} + 2\,Ba^3 + 2\,Bab^2 + (Ba^2b \cdots}{2\,(a^4+a^2b^2)d\tan{(dx+c)}}$$

input
```
integrate(cot(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
-1/2*(2*B*a^3*d*x*tan(d*x + c) - B*b^3*log((b^2*tan(d*x + c)^2 + 2*a*b*tan
(d*x + c) + a^2)/(tan(d*x + c)^2 + 1))*tan(d*x + c) + 2*B*a^3 + 2*B*a*b^2
+ (B*a^2*b + B*b^3)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c))
/((a^4 + a^2*b^2)*d*tan(d*x + c))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.90 (sec) , antiderivative size = 1137, normalized size of antiderivative = 13.38

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*B*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), (B*(-x - c
ot(c + d*x)/d)/a, Eq(b, 0)), (B*(log(tan(c + d*x)**2 + 1)/(2*d) - log(tan(
c + d*x))/d - 1/(2*d*tan(c + d*x)**2))/b, Eq(a, 0)), (-3*B*d*x*tan(c + d*x
)**2/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - 3*I*B*d*x*tan(c + d*
x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - I*B*log(tan(c + d*x)**
2 + 1)*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c + d*x)) + B*
log(tan(c + d*x)**2 + 1)*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan
(c + d*x)) + 2*I*B*log(tan(c + d*x))*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**
2 + 2*I*a*d*tan(c + d*x)) - 2*B*log(tan(c + d*x))*tan(c + d*x)/(2*a*d*tan(
c + d*x)**2 + 2*I*a*d*tan(c + d*x)) - 3*B*tan(c + d*x)/(2*a*d*tan(c + d*x)
**2 + 2*I*a*d*tan(c + d*x)) - 2*I*B/(2*a*d*tan(c + d*x)**2 + 2*I*a*d*tan(c
 + d*x)), Eq(b, -I*a)), (-3*B*d*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 -
 2*I*a*d*tan(c + d*x)) + 3*I*B*d*x*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 - 2
*I*a*d*tan(c + d*x)) + I*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**2/(2*a*d
*tan(c + d*x)**2 - 2*I*a*d*tan(c + d*x)) + B*log(tan(c + d*x)**2 + 1)*tan(
c + d*x)/(2*a*d*tan(c + d*x)**2 - 2*I*a*d*tan(c + d*x)) - 2*I*B*log(tan(c
+ d*x))*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**2 - 2*I*a*d*tan(c + d*x)) - 2
*B*log(tan(c + d*x))*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 - 2*I*a*d*tan(c +
 d*x)) - 3*B*tan(c + d*x)/(2*a*d*tan(c + d*x)**2 - 2*I*a*d*tan(c + d*x)) +
 2*I*B/(2*a*d*tan(c + d*x)**2 - 2*I*a*d*tan(c + d*x)), Eq(b, I*a)), (zo...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,Bb^3\log(b\tan(dx+c)+a)}{a^4+a^2b^2}-\frac{2\,(dx+c)Ba}{a^2+b^2}+\frac{Bb\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}-\frac{2\,Bb\log(\tan(dx+c))}{a^2}-\frac{2\,B}{a\tan(dx+c)}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
1/2*(2*B*b^3*log(b*tan(d*x + c) + a)/(a^4 + a^2*b^2) - 2*(d*x + c)*B*a/(a^
2 + b^2) + B*b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) - 2*B*b*log(tan(d*x + c
))/a^2 - 2*B/(a*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{1}{2}\left(\frac{2\,b^4\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^4bd+a^2b^3d}-\frac{2\,(dx+c)a}{a^2d+b^2d}+\frac{b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2d+b^2d}-\frac{2\,b\log\left(|\tan\left(dx+c\right)|\right)}{a^2d}-\right.$$

input
```
integrate(cot(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
1/2*(2*b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b*d + a^2*b^3*d) - 2*(d*x + c
)*a/(a^2*d + b^2*d) + b*log(tan(d*x + c)^2 + 1)/(a^2*d + b^2*d) - 2*b*log(
abs(tan(d*x + c)))/(a^2*d) - 2/(a*d*tan(d*x + c)))*B
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.33

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx=\frac{B\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(b+a\,1\mathrm{i}\right)}-\frac{B\cot(c+d\,x)}{a\,d}-\frac{B\,b\,\ln\left(\tan(c+d\,x)\right)}{a^2\,d}+\frac{B\,b^3\,\ln\left(a+b\tan(c+d\,x)\right)}{a^2\,d\,\left(a^2+b^2\right)}+\frac{B\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(a+b\,1\mathrm{i}\right)}$$

input
```
int((cot(c + d*x)^2*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(B*log(tan(c + d*x) - 1i)*1i)/(2*d*(a + b*1i)) + (B*log(tan(c + d*x) + 1i)
)/(2*d*(a*1i + b)) - (B*cot(c + d*x))/(a*d) - (B*b*log(tan(c + d*x)))/(a^2
*d) + (B*b^3*log(a + b*tan(c + d*x)))/(a^2*d*(a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.00

$$\int \frac{\cot^2(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{b\left(-\cos(dx+c)\,a^3-\cos(dx+c)\,a\,b^2+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)\,a^2b+\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a\right.\right.}{}$$

input
```
int(cot(d*x+c)^2*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(b*( - cos(c + d*x)*a**3 - cos(c + d*x)*a*b**2 + log(tan((c + d*x)/2)**2 +
 1)*sin(c + d*x)*a**2*b + log(tan((c + d*x)/2)**2*a - 2*tan((c + d*x)/2)*b
 - a)*sin(c + d*x)*b**3 - log(tan((c + d*x)/2))*sin(c + d*x)*a**2*b - log(
tan((c + d*x)/2))*sin(c + d*x)*b**3 - sin(c + d*x)*a**3*d*x))/(sin(c + d*x
)*a**2*d*(a**2 + b**2))
```

3.313 $\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 112

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{bBx}{a^2+b^2}+\frac{bB\cot(c+dx)}{a^2d}-\frac{B\cot^2(c+dx)}{2ad}-\frac{(a^2-b^2)\,B\log(\sin(c+dx))}{a^3d}-\frac{b^4B\log(a\cos(c+dx)+b\sin(c+dx))}{a^3\,(a^2+b^2)\,d}$$

output
```
b*B*x/(a^2+b^2)+b*B*cot(d*x+c)/a^2/d-1/2*B*cot(d*x+c)^2/a/d-(a^2-b^2)*B*ln
(sin(d*x+c))/a^3/d-b^4*B*ln(a*cos(d*x+c)+b*sin(d*x+c))/a^3/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{B\left(-\frac{2b\cot(c+dx)}{a^2}+\frac{\cot^2(c+dx)}{a}-\frac{\log(i-\cot(c+dx))}{a-ib}-\frac{\log(i+\cot(c+dx))}{a+ib}+\frac{2b^4\log(b+a\cot(c+dx))}{a^3(a^2+b^2)}\right)}{2d}$$

input
```
Integrate[(Cot[c + d*x]^3*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2
,x]
```

output
```
-1/2*(B*((-2*b*Cot[c + d*x])/a^2 + Cot[c + d*x]^2/a - Log[I - Cot[c + d*x]
]/(a - I*b) - Log[I + Cot[c + d*x]]/(a + I*b) + (2*b^4*Log[b + a*Cot[c + d
*x]])/(a^3*(a^2 + b^2))))/d
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {2011, 3042, 4052, 27, 3042, 4132, 25, 3042, 4135, 3042, 25, 3956, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\cot^3(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\tan(c+dx)^3(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4052}$$

$$B\left(-\frac{\int \frac{2\cot^2(c+dx)\left(b\tan^2(c+dx)+a\tan(c+dx)+b\right)}{a+b\tan(c+dx)}dx}{2a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(-\frac{\int \frac{\cot^2(c+dx)\left(b\tan^2(c+dx)+a\tan(c+dx)+b\right)}{a+b\tan(c+dx)}dx}{a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(-\frac{\int \frac{b\tan(c+dx)^2+a\tan(c+dx)+b}{\tan(c+dx)^2(a+b\tan(c+dx))}dx}{a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

$\Big\downarrow$ 4132

$$B\left(-\frac{-\frac{\int -\frac{\cot(c+dx)\left(a^2-b^2-b^2\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx}{a}-\frac{b\cot(c+dx)}{ad}}{a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

$\Big\downarrow$ 25

$$B\left(-\frac{\frac{\int \frac{\cot(c+dx)\left(a^2-b^2-b^2\tan^2(c+dx)\right)}{a+b\tan(c+dx)}dx}{a}-\frac{b\cot(c+dx)}{ad}}{a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

$\Big\downarrow$ 3042

$$B\left(-\frac{\frac{\int \frac{a^2-b^2-b^2\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))}dx}{a}-\frac{b\cot(c+dx)}{ad}}{a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

$\Big\downarrow$ 4135

$$B\left(-\frac{\frac{\frac{\left(a^2-b^2\right)\int \cot(c+dx)dx}{a}+\frac{b^4\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2bx}{a^2+b^2}}{a}-\frac{b\cot(c+dx)}{ad}}{a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

$\Big\downarrow$ 3042

$$B\left(-\frac{\frac{\frac{\left(a^2-b^2\right)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{a}+\frac{b^4\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2bx}{a^2+b^2}}{a}-\frac{b\cot(c+dx)}{ad}}{a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

$\Big\downarrow$ 25

$$B\left(-\frac{\frac{-\frac{\left(a^2-b^2\right)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{a}+\frac{b^4\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}-\frac{a^2bx}{a^2+b^2}}{a}-\frac{b\cot(c+dx)}{ad}}{a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

$\Big\downarrow$ 3956

$$B\left(-\frac{\frac{\frac{b^4\int\frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a(a^2+b^2)}+\frac{(a^2-b^2)\log(-\sin(c+dx))}{ad}-\frac{a^2bx}{a^2+b^2}}{a}-\frac{b\cot(c+dx)}{ad}}{a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

$$\Big\downarrow \text{4013}$$

$$B\left(-\frac{\frac{\frac{(a^2-b^2)\log(-\sin(c+dx))}{ad}-\frac{a^2bx}{a^2+b^2}+\frac{b^4\log(a\cos(c+dx)+b\sin(c+dx))}{ad(a^2+b^2)}}{a}-\frac{b\cot(c+dx)}{ad}}{a}-\frac{\cot^2(c+dx)}{2ad}\right)$$

input
```
Int[(Cot[c + d*x]^3*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
B*(-1/2*Cot[c + d*x]^2/(a*d) - (-((b*Cot[c + d*x])/(a*d)) + (-((a^2*b*x)/(
a^2 + b^2)) + ((a^2 - b^2)*Log[-Sin[c + d*x]])/(a*d) + (b^4*Log[a*Cos[c +
d*x] + b*Sin[c + d*x]])/(a*(a^2 + b^2)*d))/a)/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4135
```
Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(a*(
A*c - c*C) - b*(A*d - C*d))*(x/((a^2 + b^2)*(c^2 + d^2))), x] + (Simp[(A*b^
2 + a^2*C)/((b*c - a*d)*(a^2 + b^2))   Int[(b - a*Tan[e + f*x])/(a + b*Tan[
e + f*x]), x], x] - Simp[(c^2*C + A*d^2)/((b*c - a*d)*(c^2 + d^2))   Int[(d
 - c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f
, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{B\left(\frac{\frac{a\ln\left(1+\tan(dx+c)^2\right)}{2}+b\arctan(\tan(dx+c))}{a^2+b^2}-\frac{1}{2a\tan(dx+c)^2}+\frac{\left(-a^2+b^2\right)\ln(\tan(dx+c))}{a^3}+\frac{b}{a^2\tan(dx+c)}-\frac{b^4\ln(a+b\tan(dx+c}{\left(a^2+b^2\right)a^3}\right.}{d}$
default	$\frac{B\left(\frac{\frac{a\ln\left(1+\tan(dx+c)^2\right)}{2}+b\arctan(\tan(dx+c))}{a^2+b^2}-\frac{1}{2a\tan(dx+c)^2}+\frac{\left(-a^2+b^2\right)\ln(\tan(dx+c))}{a^3}+\frac{b}{a^2\tan(dx+c)}-\frac{b^4\ln(a+b\tan(dx+c}{\left(a^2+b^2\right)a^3}\right.}{d}$
parallelrisch	$-\frac{B\left(-2b^4(\ln(\tan(dx+c))-\ln(a+b\tan(dx+c)))+a^4\left(-\ln\left(\sec(dx+c)^2\right)+2\ln(\tan(dx+c))\right)-2x\,a^3bd+\cot(dx+c)^2a^4\right.}{2(a^2+b^2)a^3d}$
norman	$\frac{\frac{b^2Bx\tan(dx+c)^3}{a^2+b^2}+\frac{B\,b^2\tan(dx+c)^2}{d\,a^2}+\frac{bBax\tan(dx+c)^2}{a^2+b^2}-\frac{B}{2d}+\frac{Bb\tan(dx+c)}{2ad}}{\tan(dx+c)^2(a+b\tan(dx+c))}+\frac{Ba\ln\left(1+\tan(dx+c)^2\right)}{2d(a^2+b^2)}-\frac{(a^2-b^2)B\ln(}{a^3}$
risch	$\frac{ixB}{ib-a}+\frac{2ixB}{a}+\frac{2iBc}{ad}-\frac{2iB\,b^2x}{a^3}-\frac{2iB\,b^2c}{a^3d}+\frac{2ib^4Bx}{(a^2+b^2)a^3}+\frac{2ib^4Bc}{(a^2+b^2)a^3d}+\frac{2iB\left(-ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}\right.}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}$

input
```
int(cot(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*B*(1/(a^2+b^2)*(1/2*a*ln(1+tan(d*x+c)^2)+b*arctan(tan(d*x+c)))-1/2/a/t
an(d*x+c)^2+(-a^2+b^2)/a^3*ln(tan(d*x+c))+1/a^2*b/tan(d*x+c)-b^4/(a^2+b^2)
/a^3*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.71

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{Bb^4\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)\tan(dx+c)^2+Ba^4+Ba^2b^2+(Ba^4-Bb^4)\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\mathrm{t}}{2\,(a^5+a^3b^2)d\tan(d}$$

input
```
integrate(cot(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
-1/2*(B*b^4*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2)/(tan(d*x +
 c)^2 + 1))*tan(d*x + c)^2 + B*a^4 + B*a^2*b^2 + (B*a^4 - B*b^4)*log(tan(d
*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^2 - (2*B*a^3*b*d*x - B*a^4 -
B*a^2*b^2)*tan(d*x + c)^2 - 2*(B*a^3*b + B*a*b^3)*tan(d*x + c))/((a^5 + a^
3*b^2)*d*tan(d*x + c)^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 1401, normalized size of antiderivative = 12.51

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)**3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*B*x, Eq(a, 0) & Eq(b, 0) & Eq(c, 0) & Eq(d, 0)), (B*(log(ta
n(c + d*x)**2 + 1)/(2*d) - log(tan(c + d*x))/d - 1/(2*d*tan(c + d*x)**2))/
a, Eq(b, 0)), (B*(x + 1/(d*tan(c + d*x)) - 1/(3*d*tan(c + d*x)**3))/b, Eq(
a, 0)), (-3*I*B*d*x*tan(c + d*x)**3/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c
 + d*x)**2) + 3*B*d*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan
(c + d*x)**2) + 2*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(2*a*d*tan(c
+ d*x)**3 + 2*I*a*d*tan(c + d*x)**2) + 2*I*B*log(tan(c + d*x)**2 + 1)*tan(
c + d*x)**2/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2) - 4*B*log(ta
n(c + d*x))*tan(c + d*x)**3/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)*
*2) - 4*I*B*log(tan(c + d*x))*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**3 + 2*I
*a*d*tan(c + d*x)**2) - 3*I*B*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**3 + 2*I
*a*d*tan(c + d*x)**2) + B*tan(c + d*x)/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*ta
n(c + d*x)**2) - I*B/(2*a*d*tan(c + d*x)**3 + 2*I*a*d*tan(c + d*x)**2), Eq
(b, -I*a)), (3*I*B*d*x*tan(c + d*x)**3/(2*a*d*tan(c + d*x)**3 - 2*I*a*d*ta
n(c + d*x)**2) + 3*B*d*x*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**3 - 2*I*a*d*
tan(c + d*x)**2) + 2*B*log(tan(c + d*x)**2 + 1)*tan(c + d*x)**3/(2*a*d*tan
(c + d*x)**3 - 2*I*a*d*tan(c + d*x)**2) - 2*I*B*log(tan(c + d*x)**2 + 1)*t
an(c + d*x)**2/(2*a*d*tan(c + d*x)**3 - 2*I*a*d*tan(c + d*x)**2) - 4*B*log
(tan(c + d*x))*tan(c + d*x)**3/(2*a*d*tan(c + d*x)**3 - 2*I*a*d*tan(c + d*
x)**2) + 4*I*B*log(tan(c + d*x))*tan(c + d*x)**2/(2*a*d*tan(c + d*x)**3...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.16

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\,Bb^4\log(b\tan(dx+c)+a)}{a^5+a^3b^2} - \frac{2\,(dx+c)Bb}{a^2+b^2} - \frac{Ba\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2} + \frac{2\,(Ba^2-Bb^2)\log(\tan(dx+c))}{a^3} - \frac{2\,Bb\tan(dx+c)-Ba}{a^2\tan(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
-1/2*(2*B*b^4*log(b*tan(d*x + c) + a)/(a^5 + a^3*b^2) - 2*(d*x + c)*B*b/(a
^2 + b^2) - B*a*log(tan(d*x + c)^2 + 1)/(a^2 + b^2) + 2*(B*a^2 - B*b^2)*lo
g(tan(d*x + c))/a^3 - (2*B*b*tan(d*x + c) - B*a)/(a^2*tan(d*x + c)^2))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{2}\left(\frac{2\,b^5\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^5bd+a^3b^3d} - \frac{2\,(dx+c)b}{a^2d+b^2d} - \frac{a\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2d+b^2d} + \frac{2\,(a^2-b^2)\log\left(|\tan\left(dx\right.\right.}{a^3d}\right.$$

input
```
integrate(cot(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
-1/2*(2*b^5*log(abs(b*tan(d*x + c) + a))/(a^5*b*d + a^3*b^3*d) - 2*(d*x +
c)*b/(a^2*d + b^2*d) - a*log(tan(d*x + c)^2 + 1)/(a^2*d + b^2*d) + 2*(a^2
- b^2)*log(abs(tan(d*x + c)))/(a^3*d) - (2*a*b*tan(d*x + c) - a^2)/(a^3*d*
tan(d*x + c)^2))*B
```

Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{\cot(c+d\,x)^2\,\left(\frac{B}{2\,a}-\frac{B\,b\tan(c+d\,x)}{a^2}\right)}{d} + \frac{B\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)}{2\,d\,\left(a-b\,1\mathrm{i}\right)} - \frac{B\,\ln\left(\tan(c+d\,x)\right)\,\left(a^2-b^2\right)}{a^3\,d} - \frac{B\,b^4\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^5+a^3\,b^2\right)} + \frac{B\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{2\,d\,\left(-b+a\,1\mathrm{i}\right)}$$

input
```
int((cot(c + d*x)^3*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(B*log(tan(c + d*x) + 1i))/(2*d*(a - b*1i)) - (cot(c + d*x)^2*(B/(2*a) - (
B*b*tan(c + d*x))/a^2))/d + (B*log(tan(c + d*x) - 1i)*1i)/(2*d*(a*1i - b))
 - (B*log(tan(c + d*x))*(a^2 - b^2))/(a^3*d) - (B*b^4*log(a + b*tan(c + d*
x)))/(d*(a^5 + a^3*b^2))
```

Reduce [B] (verification not implemented)

Time = 30.00 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.10

$$\int \frac{\cot^3(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{b\left(4\cos(dx+c)\sin(dx+c)\,a^3b+4\cos(dx+c)\sin(dx+c)\,a\,b^3+4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\sin(dx+c)\right.}{}$$

input
```
int(cot(d*x+c)^3*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(b*(4*cos(c + d*x)*sin(c + d*x)*a**3*b + 4*cos(c + d*x)*sin(c + d*x)*a*b**
3 + 4*log(tan((c + d*x)/2)**2 + 1)*sin(c + d*x)**2*a**4 - 4*log(tan((c + d
*x)/2)**2*a - 2*tan((c + d*x)/2)*b - a)*sin(c + d*x)**2*b**4 - 4*log(tan((
c + d*x)/2))*sin(c + d*x)**2*a**4 + 4*log(tan((c + d*x)/2))*sin(c + d*x)**
2*b**4 + sin(c + d*x)**2*a**4 + 4*sin(c + d*x)**2*a**3*b*d*x + sin(c + d*x
)**2*a**2*b**2 - 2*a**4 - 2*a**2*b**2))/(4*sin(c + d*x)**2*a**3*d*(a**2 +
b**2))
```

3.314 $\int \frac{3+\tan(c+dx)}{2-\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{3+\tan(c+dx)}{2-\tan(c+dx)}\,dx = x - \frac{\log(2\cos(c+dx)-\sin(c+dx))}{d}$$

output
```
x-ln(2*cos(d*x+c)-sin(d*x+c))/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \frac{3+\tan(c+dx)}{2-\tan(c+dx)}\,dx = \frac{\arctan(\tan(c+dx))}{d} + \frac{\log\left(5-4(2-\tan(c+dx))+(2-\tan(c+dx))^2\right)}{2d} - \frac{\log(2-\tan(c+dx))}{d}$$

input
```
Integrate[(3 + Tan[c + d*x])/(2 - Tan[c + d*x]),x]
```

output
```
ArcTan[Tan[c + d*x]]/d + Log[5 - 4*(2 - Tan[c + d*x]) + (2 - Tan[c + d*x])
^2]/(2*d) - Log[2 - Tan[c + d*x]]/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4014, 25, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)+3}{2-\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)+3}{2-\tan(c+dx)}dx$$

$$\Big\downarrow \text{4014}$$

$$x-\int -\frac{2\tan(c+dx)+1}{2-\tan(c+dx)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{2\tan(c+dx)+1}{2-\tan(c+dx)}dx+x$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{2\tan(c+dx)+1}{2-\tan(c+dx)}dx+x$$

$$\Big\downarrow \text{4013}$$

$$x-\frac{\log(2\cos(c+dx)-\sin(c+dx))}{d}$$

input
```
Int[(3 + Tan[c + d*x])/(2 - Tan[c + d*x]),x]
```

output
```
x - Log[2*Cos[c + d*x] - Sin[c + d*x]]/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result	size
norman	$x-\frac{\ln \left(-2+\tan \left(d x+c\right)\right)}{d}+\frac{\ln \left(1+\tan \left(d x+c\right)^{2}\right)}{2 d}$	33
risch	$i x+x+\frac{2 i c}{d}-\frac{\ln \left({\mathrm e}^{2 i \left(d x+c\right)}+\frac{3}{5}-\frac{4 i}{5}\right)}{d}$	33
parallelrisch	$-\frac{-2 d x+2 \ln \left(-2+\tan \left(d x+c\right)\right)-\ln \left(1+\tan \left(d x+c\right)^{2}\right)}{2 d}$	35
derivativedivides	$\frac{\frac{\ln \left(1+\tan \left(d x+c\right)^{2}\right)}{2}+\arctan \left(\tan \left(d x+c\right)\right)-\ln \left(-2+\tan \left(d x+c\right)\right)}{d}$	37
default	$\frac{\frac{\ln \left(1+\tan \left(d x+c\right)^{2}\right)}{2}+\arctan \left(\tan \left(d x+c\right)\right)-\ln \left(-2+\tan \left(d x+c\right)\right)}{d}$	37

input
```
int((3+tan(d*x+c))/(2-tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
x-1/d*ln(-2+tan(d*x+c))+1/2/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{3+\tan(c+dx)}{2-\tan(c+dx)}\,dx = \frac{2\,dx-\log\left(\frac{\tan(dx+c)^2-4\,\tan(dx+c)+4}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate((3+tan(d*x+c))/(2-tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*d*x - log((tan(d*x + c)^2 - 4*tan(d*x + c) + 4)/(tan(d*x + c)^2 + 1
)))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(19) = 38$.

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{3+\tan(c+dx)}{2-\tan(c+dx)}\,dx = \begin{cases} x-\frac{\log(\tan(c+dx)-2)}{d}+\frac{\log(\tan^2(c+dx)+1)}{2d} & \text{for } d\neq 0 \\ \frac{x(\tan(c)+3)}{2-\tan(c)} & \text{otherwise} \end{cases}$$

input
```
integrate((3+tan(d*x+c))/(2-tan(d*x+c)),x)
```

output
```
Piecewise((x - log(tan(c + d*x) - 2)/d + log(tan(c + d*x)**2 + 1)/(2*d), N
e(d, 0)), (x*(tan(c) + 3)/(2 - tan(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{3+\tan(c+dx)}{2-\tan(c+dx)}\,dx = \frac{2\,dx+2\,c+\log\left(\tan\left(dx+c\right)^2+1\right)-2\,\log\left(\tan\left(dx+c\right)-2\right)}{2\,d}$$

input
```
integrate((3+tan(d*x+c))/(2-tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*d*x + 2*c + log(tan(d*x + c)^2 + 1) - 2*log(tan(d*x + c) - 2))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{3+\tan(c+dx)}{2-\tan(c+dx)}\,dx = \frac{dx+c}{d}+\frac{\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}-\frac{\log\left(\left|\tan\left(dx+c\right)-2\right|\right)}{d}$$

input
```
integrate((3+tan(d*x+c))/(2-tan(d*x+c)),x, algorithm="giac")
```

output
```
(d*x + c)/d + 1/2*log(tan(d*x + c)^2 + 1)/d - log(abs(tan(d*x + c) - 2))/d
```

Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{3+\tan(c+dx)}{2-\tan(c+dx)}\,dx = -\frac{\ln\left(\tan(c+d\,x)-2\right)}{d}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{d} + \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{d}$$

input
```
int(-(tan(c + d*x) + 3)/(tan(c + d*x) - 2),x)
```

output
```
(log(tan(c + d*x) - 1i)*(1/2 - 1i/2))/d - log(tan(c + d*x) - 2)/d + (log(t
an(c + d*x) + 1i)*(1/2 + 1i/2))/d
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{3+\tan(c+dx)}{2-\tan(c+dx)}\,dx = \frac{\log\left(\tan\left(dx+c\right)^{2}+1\right)-2\log(\tan\left(dx+c\right)-2)+2dx}{2d}$$

input
```
int((3+tan(d*x+c))/(2-tan(d*x+c)),x)
```

output
```
(log(tan(c + d*x)**2 + 1) - 2*log(tan(c + d*x) - 2) + 2*d*x)/(2*d)
```

3.315 $\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 58

$$\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{2bBx}{a^2+b^2}-\frac{\left(a-\frac{b^2}{a}\right)B\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

output
```
2*b*B*x/(a^2+b^2)-(a-b^2/a)*B*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx =\frac{B(4ab\arctan(\tan(c+dx))+(a^2-b^2)\left(\log\left(\sec^2(c+dx)\right)-2\log(a+b\tan(c+dx))\right))}{2a\,(a^2+b^2)\,d}$$

input
```
Integrate[((b*B)/a + B*Tan[c + d*x])/(a + b*Tan[c + d*x]),x]
```

output
```
(B*(4*a*b*ArcTan[Tan[c + d*x]] + (a^2 - b^2)*(Log[Sec[c + d*x]^2] - 2*Log[
a + b*Tan[c + d*x]])))/(2*a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 4014}$$

$$\frac{2bBx}{a^2+b^2}-\frac{B\left(a-\frac{b^2}{a}\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2bBx}{a^2+b^2}-\frac{B\left(a-\frac{b^2}{a}\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 4013}$$

$$\frac{2bBx}{a^2+b^2}-\frac{B\left(a-\frac{b^2}{a}\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)}$$

input
```
Int[((b*B)/a + B*Tan[c + d*x])/(a + b*Tan[c + d*x]),x]
```

output
```
(2*b*B*x)/(a^2 + b^2) - ((a - b^2/a)*B*Log[a*Cos[c + d*x] + b*Sin[c + d*x]
])/((a^2 + b^2)*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{B\left(\frac{\frac{(a^2-b^2)\ln\left(1+\tan(dx+c)^2\right)}{2}+2ab\arctan(\tan(dx+c))}{a^2+b^2}-\frac{(a^2-b^2)\ln(a+b\tan(dx+c))}{a^2+b^2}\right)}{da}$
default	$\frac{B\left(\frac{\frac{(a^2-b^2)\ln\left(1+\tan(dx+c)^2\right)}{2}+2ab\arctan(\tan(dx+c))}{a^2+b^2}-\frac{(a^2-b^2)\ln(a+b\tan(dx+c))}{a^2+b^2}\right)}{da}$
norman	$\frac{2bBx}{a^2+b^2}+\frac{B(a^2-b^2)\ln\left(1+\tan(dx+c)^2\right)}{2ad(a^2+b^2)}-\frac{B(a^2-b^2)\ln(a+b\tan(dx+c))}{ad(a^2+b^2)}$
parallelrisch	$\frac{4Babdx+B\ln\left(1+\tan(dx+c)^2\right)a^2-B\ln\left(1+\tan(dx+c)^2\right)b^2-2B\ln(a+b\tan(dx+c))a^2+2B\,b^2\ln(a+b\tan(dx+c))}{2ad(a^2+b^2)}$
risch	$-\frac{xBb}{(ib-a)a}+\frac{ixB}{ib-a}+\frac{2iBxa}{a^2+b^2}-\frac{2ib^2Bx}{a(a^2+b^2)}+\frac{2iBac}{d(a^2+b^2)}-\frac{2ib^2Bc}{ad(a^2+b^2)}-\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)Ba}{d(a^2+b^2)}+\frac{b^2\ln\left(e^{2i\right.}}{a}$

input
```
int((b*B/a+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*B*(1/(a^2+b^2)*(1/2*(a^2-b^2)*ln(1+tan(d*x+c)^2)+2*a*b*arctan(tan(d*
x+c)))-(a^2-b^2)/(a^2+b^2)*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx=\frac{4\,Babdx-(Ba^2-Bb^2)\log\left(\frac{b^2\tan(dx+c)^2+2\,ab\tan(dx+c)+a^2}{\tan(dx+c)^2+1}\right)}{2\,(a^3+ab^2)d}$$

input
```
integrate((b*B/a+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(4*B*a*b*d*x - (B*a^2 - B*b^2)*log((b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x
 + c) + a^2)/(tan(d*x + c)^2 + 1)))/((a^3 + a*b^2)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 235, normalized size of antiderivative = 4.05

$$\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$=\begin{cases}\text{NaN}\\ \frac{B\log\left(\tan^2(c+dx)+1\right)}{2ad}\\ -\frac{B}{bd\tan(c+dx)-ibd}\\ -\frac{B}{bd\tan(c+dx)+ibd}\\ \frac{x\left(B\tan(c)+\frac{Bb}{a}\right)}{a+b\tan(c)}\\ -\frac{2Ba^2\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^3d+2ab^2d}+\frac{Ba^2\log\left(\tan^2(c+dx)+1\right)}{2a^3d+2ab^2d}+\frac{4Babdx}{2a^3d+2ab^2d}+\frac{2Bb^2\log\left(\frac{a}{b}+\tan(c+dx)\right)}{2a^3d+2ab^2d}-\frac{Bb^2\log\left(\tan^2(c+dx)+1\right)}{2a^3d+2ab^2d}\end{cases}$$

input
```
integrate((b*B/a+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Piecewise((nan, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*log(tan(c + d*x)**2 +
1)/(2*a*d), Eq(b, 0)), (-B/(b*d*tan(c + d*x) - I*b*d), Eq(a, -I*b)), (-B/(
b*d*tan(c + d*x) + I*b*d), Eq(a, I*b)), (x*(B*tan(c) + B*b/a)/(a + b*tan(c
)), Eq(d, 0)), (-2*B*a**2*log(a/b + tan(c + d*x))/(2*a**3*d + 2*a*b**2*d)
+ B*a**2*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2*a*b**2*d) + 4*B*a*b*d*x/(2
*a**3*d + 2*a*b**2*d) + 2*B*b**2*log(a/b + tan(c + d*x))/(2*a**3*d + 2*a*b
**2*d) - B*b**2*log(tan(c + d*x)**2 + 1)/(2*a**3*d + 2*a*b**2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{4\,(dx+c)Bb}{a^2+b^2}-\frac{2\,(Ba^2-Bb^2)\log(b\tan(dx+c)+a)}{a^3+ab^2}+\frac{(Ba^2-Bb^2)\log\left(\tan(dx+c)^2+1\right)}{a^3+ab^2}}{2\,d}$$

input
```
integrate((b*B/a+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(4*(d*x + c)*B*b/(a^2 + b^2) - 2*(B*a^2 - B*b^2)*log(b*tan(d*x + c) +
a)/(a^3 + a*b^2) + (B*a^2 - B*b^2)*log(tan(d*x + c)^2 + 1)/(a^3 + a*b^2))/
d
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

$$\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\,(dx+c)Bb}{a^2d+b^2d}+\frac{(Ba^2-Bb^2)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,(a^3d+ab^2d)} - \frac{(Ba^2b-Bb^3)\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^3bd+ab^3d}$$

input
```
integrate((b*B/a+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
2*(d*x + c)*B*b/(a^2*d + b^2*d) + 1/2*(B*a^2 - B*b^2)*log(tan(d*x + c)^2 +
 1)/(a^3*d + a*b^2*d) - (B*a^2*b - B*b^3)*log(abs(b*tan(d*x + c) + a))/(a^
3*b*d + a*b^3*d)
```

Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.93

$$\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(B\,b+B\,a\,1\mathrm{i}\right)}{2\,d\,\left(a\,b-a^2\,1\mathrm{i}\right)} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(B\,a+B\,b\,1\mathrm{i}\right)}{2\,d\,\left(-a^2+a\,b\,1\mathrm{i}\right)} - \frac{B\,\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^2-b^2\right)}{a\,d\,\left(a^2+b^2\right)}$$

input
```
int((B*tan(c + d*x) + (B*b)/a)/(a + b*tan(c + d*x)),x)
```

output
```
- (log(tan(c + d*x) - 1i)*(B*a*1i + B*b))/(2*d*(a*b - a^2*1i)) - (log(tan(
c + d*x) + 1i)*(B*a + B*b*1i))/(2*d*(a*b*1i - a^2)) - (B*log(a + b*tan(c +
 d*x))*(a^2 - b^2))/(a*d*(a^2 + b^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int \frac{\frac{bB}{a}+B\tan(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{b\left(\log\left(\tan\left(dx+c\right)^2+1\right)a^2-\log\left(\tan\left(dx+c\right)^2+1\right)b^2-2\log\left(a+\tan\left(dx+c\right)b\right)a^2+2\log\left(a+\tan\left(d\right.\right.\right.}{2ad\left(a^2+b^2\right)}$$

input
```
int((b*B/a+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(b*(log(tan(c + d*x)**2 + 1)*a**2 - log(tan(c + d*x)**2 + 1)*b**2 - 2*log(
tan(c + d*x)*b + a)*a**2 + 2*log(tan(c + d*x)*b + a)*b**2 + 4*a*b*d*x))/(2
*a*d*(a**2 + b**2))
```

3.316 $\int \frac{a+b\tan(c+dx)}{(b+a\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \frac{a+b\tan(c+dx)}{(b+a\tan(c+dx))^2}\,dx = -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^2} + \frac{b(3a^2-b^2)\log(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^2\,d} - \frac{a^2-b^2}{(a^2+b^2)\,d(b+a\tan(c+dx))}$$

output

```
-a*(a^2-3*b^2)*x/(a^2+b^2)^2+b*(3*a^2-b^2)*ln(b*cos(d*x+c)+a*sin(d*x+c))/(
a^2+b^2)^2/d-(a^2-b^2)/(a^2+b^2)/d/(b+a*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.85

$$\int \frac{a+b\tan(c+dx)}{(b+a\tan(c+dx))^2}\,dx = \frac{\frac{b(-((a+ib)\log(i-\tan(c+dx)))-(a-ib)\log(i+\tan(c+dx))+2a\log(b+a\tan(c+dx)))}{a^2+b^2} + (a-b)(a+b)\left(\frac{i\log(i-\tan(c+dx))}{(a-ib)^2} - \frac{i\log(i+}{(a}\right.}{2ad}$$

input
```
Integrate[(a + b*Tan[c + d*x])/(b + a*Tan[c + d*x])^2,x]
```

output
```
((b*(-((a + I*b)*Log[I - Tan[c + d*x]]) - (a - I*b)*Log[I + Tan[c + d*x]]
+ 2*a*Log[b + a*Tan[c + d*x]]))/(a^2 + b^2) + (a - b)*(a + b)*((I*Log[I -
Tan[c + d*x]])/(a - I*b)^2 - (I*Log[I + Tan[c + d*x]])/(a + I*b)^2 + (2*a*
(2*b*Log[b + a*Tan[c + d*x]] - (a^2 + b^2)/(b + a*Tan[c + d*x])))/(a^2 + b
^2)^2))/(2*a*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(c+dx)}{(a\tan(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(c+dx)}{(a\tan(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int \frac{2ab-(a^2-b^2)\tan(c+dx)}{b+a\tan(c+dx)}dx}{a^2+b^2}-\frac{a^2-b^2}{d(a^2+b^2)(a\tan(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2ab-(a^2-b^2)\tan(c+dx)}{b+a\tan(c+dx)}dx}{a^2+b^2}-\frac{a^2-b^2}{d(a^2+b^2)(a\tan(c+dx)+b)}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\frac{b(3a^2-b^2)\int \frac{a-b\tan(c+dx)}{b+a\tan(c+dx)}dx}{a^2+b^2}-\frac{ax(a^2-3b^2)}{a^2+b^2}}{a^2+b^2}-\frac{a^2-b^2}{d(a^2+b^2)(a\tan(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b(3a^2-b^2)\int \frac{a-b\tan(c+dx)}{b+a\tan(c+dx)}dx}{a^2+b^2}-\frac{ax(a^2-3b^2)}{a^2+b^2}}{a^2+b^2}-\frac{a^2-b^2}{d(a^2+b^2)(a\tan(c+dx)+b)}$$

$\downarrow$ 4013

$$\frac{\frac{b(3a^2-b^2)\log(a\sin(c+dx)+b\cos(c+dx))}{d(a^2+b^2)}-\frac{ax(a^2-3b^2)}{a^2+b^2}}{a^2+b^2}-\frac{a^2-b^2}{d(a^2+b^2)(a\tan(c+dx)+b)}$$

input

```
Int[(a + b*Tan[c + d*x])/(b + a*Tan[c + d*x])^2,x]
```

output

```
(-((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)) + (b*(3*a^2 - b^2)*Log[b*Cos[c + d*x]
+ a*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2 + b^2) - (a^2 - b^2)/((a^2 + b^2)
*d*(b + a*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{-\frac{a^2-b^2}{(a^2+b^2)(b+a\tan(dx+c))}+\frac{b(3a^2-b^2)\ln(b+a\tan(dx+c))}{(a^2+b^2)^2}+\frac{\frac{(-3a^2b+b^3)\ln(1+\tan(dx+c)^2)}{2}+(-a^3+3a\,b^2)\arctan(\tan(dx+c)}{(a^2+b^2)^2}}{d}$
default	$\frac{-\frac{a^2-b^2}{(a^2+b^2)(b+a\tan(dx+c))}+\frac{b(3a^2-b^2)\ln(b+a\tan(dx+c))}{(a^2+b^2)^2}+\frac{\frac{(-3a^2b+b^3)\ln(1+\tan(dx+c)^2)}{2}+(-a^3+3a\,b^2)\arctan(\tan(dx+c)}{(a^2+b^2)^2}}{d}$
norman	$\frac{\frac{(a^2-b^2)a\tan(dx+c)}{bd(a^2+b^2)}-\frac{a^2(a^2-3b^2)x\tan(dx+c)}{a^4+2a^2b^2+b^4}-\frac{ba(a^2-3b^2)x}{a^4+2a^2b^2+b^4}}{b+a\tan(dx+c)}+\frac{b(3a^2-b^2)\ln(b+a\tan(dx+c))}{d(a^4+2a^2b^2+b^4)}-\frac{b(3a^2-b^2)\ln(1+}{2d(a^4+2a^2b}$
parallelrisch	$-\frac{2x\tan(dx+c)a^4bd-6x\tan(dx+c)a^2b^3d+3\ln(1+\tan(dx+c)^2)\tan(dx+c)a^3b^2-\ln(1+\tan(dx+c)^2)\tan(dx+c)a\,b^4}{}$
risch	$\frac{ixb}{2iab+a^2-b^2}-\frac{xa}{2iab+a^2-b^2}-\frac{6ib\,a^2x}{a^4+2a^2b^2+b^4}+\frac{2ib^3x}{a^4+2a^2b^2+b^4}-\frac{6ib\,a^2c}{d(a^4+2a^2b^2+b^4)}+\frac{2ib^3c}{d(a^4+2a^2b^2+b^4)}-$

input
```
int((a+b*tan(d*x+c))/(b+a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(a^2-b^2)/(a^2+b^2)/(b+a*tan(d*x+c))+b*(3*a^2-b^2)/(a^2+b^2)^2*ln(b+
a*tan(d*x+c))+1/(a^2+b^2)^2*(1/2*(-3*a^2*b+b^3)*ln(1+tan(d*x+c)^2)+(-a^3+3
*a*b^2)*arctan(tan(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.89

$$\int \frac{a+b\tan(c+dx)}{(b+a\tan(c+dx))^2}\,dx = -\frac{2\,a^4-2\,a^2b^2+2\,(a^3b-3\,ab^3)dx-(3\,a^2b^2-b^4+(3\,a^3b-ab^3)\tan{(dx+c)})\log\left(\frac{a^2\tan(dx+c)^2+2\,ab\tan(dx}{\tan(dx+c)^2+1}\right.}{2\,((a^5+2\,a^3b^2+ab^4)d\tan{(dx+c)}+(a^4b+2\,a^2b^3+}$$

input
```
integrate((a+b*tan(d*x+c))/(b+a*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^4 - 2*a^2*b^2 + 2*(a^3*b - 3*a*b^3)*d*x - (3*a^2*b^2 - b^4 + (3*
a^3*b - a*b^3)*tan(d*x + c))*log((a^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c)
+ b^2)/(tan(d*x + c)^2 + 1)) - 2*(a^3*b - a*b^3 - (a^4 - 3*a^2*b^2)*d*x)*t
an(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d*tan(d*x + c) + (a^4*b + 2*a^2*b^
3 + b^5)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 1348, normalized size of antiderivative = 13.35

$$\int \frac{a+b\tan(c+dx)}{(b+a\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))/(b+a*tan(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*tan(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c + d*x
)**2 + 1)/(2*b*d), Eq(a, 0)), (I/(2*a*d*tan(c + d*x)**2 - 4*I*a*d*tan(c +
d*x) - 2*a*d), Eq(b, -I*a)), (-I/(2*a*d*tan(c + d*x)**2 + 4*I*a*d*tan(c +
d*x) - 2*a*d), Eq(b, I*a)), (x*(a + b*tan(c))/(a*tan(c) + b)**2, Eq(d, 0))
, (-2*a**4*d*x*tan(c + d*x)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b
**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) -
 2*a**4/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b**2*d*tan(c + d*x) +
 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) - 2*a**3*b*d*x/(2*a**
5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d
 + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) + 6*a**3*b*log(tan(c + d*x) + b/a)*
tan(c + d*x)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a**3*b**2*d*tan(c + d
*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5*d) - 3*a**3*b*log(t
an(c + d*x)**2 + 1)*tan(c + d*x)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4*a
**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b**5
*d) + 6*a**2*b**2*d*x*tan(c + d*x)/(2*a**5*d*tan(c + d*x) + 2*a**4*b*d + 4
*a**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*x) + 2*b*
*5*d) + 6*a**2*b**2*log(tan(c + d*x) + b/a)/(2*a**5*d*tan(c + d*x) + 2*a**
4*b*d + 4*a**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4*d*tan(c + d*
x) + 2*b**5*d) - 3*a**2*b**2*log(tan(c + d*x)**2 + 1)/(2*a**5*d*tan(c + d*
x) + 2*a**4*b*d + 4*a**3*b**2*d*tan(c + d*x) + 4*a**2*b**3*d + 2*a*b**4...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.59

$$\int \frac{a+b\tan(c+dx)}{(b+a\tan(c+dx))^2}\,dx = -\frac{\frac{2\,(a^3-3\,ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} - \frac{2\,(3\,a^2b-b^3)\log(a\tan(dx+c)+b)}{a^4+2\,a^2b^2+b^4} + \frac{(3\,a^2b-b^3)\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\,(a^2-b^2)}{a^2b+b^3+(a^3+ab^2)\tan(dx+c)}}{2\,d}$$

input
```
integrate((a+b*tan(d*x+c))/(b+a*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^2*b - b
^3)*log(a*tan(d*x + c) + b)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)*log(
tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^2 - b^2)/(a^2*b + b^3 +
 (a^3 + a*b^2)*tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.69

$$\int \frac{a+b\tan(c+dx)}{(b+a\tan(c+dx))^2}\,dx = -\frac{(a^3-3\,ab^2)(dx+c)}{a^4d+2\,a^2b^2d+b^4d} - \frac{(3\,a^2b-b^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,(a^4d+2\,a^2b^2d+b^4d)} + \frac{(3\,a^3b-ab^3)\log\left(|a\tan\left(dx+c\right)+b|\right)}{a^5d+2\,a^3b^2d+ab^4d} - \frac{a^4-b^4}{(a^2+b^2)^2(a\tan\left(dx+c\right)+b)d}$$

input
```
integrate((a+b*tan(d*x+c))/(b+a*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-(a^3 - 3*a*b^2)*(d*x + c)/(a^4*d + 2*a^2*b^2*d + b^4*d) - 1/2*(3*a^2*b -
b^3)*log(tan(d*x + c)^2 + 1)/(a^4*d + 2*a^2*b^2*d + b^4*d) + (3*a^3*b - a*
b^3)*log(abs(a*tan(d*x + c) + b))/(a^5*d + 2*a^3*b^2*d + a*b^4*d) - (a^4 -
 b^4)/((a^2 + b^2)^2*(a*tan(d*x + c) + b)*d)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.50

$$\int \frac{a+b\tan(c+dx)}{(b+a\tan(c+dx))^2}\,dx = \frac{b\,\ln\left(b+a\tan(c+d\,x)\right)\,\left(3\,a^2-b^2\right)}{d\,{\left(a^2+b^2\right)}^2} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a-b\,1\mathrm{i}\right)}{2\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)} - \frac{a^2-b^2}{d\,\left(a^2+b^2\right)\,\left(b+a\tan\left(c+d\,x\right)\right)} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a+b\,1\mathrm{i}\right)}{2\,d\,\left(a^2\,1\mathrm{i}+2\,a\,b-b^2\,1\mathrm{i}\right)}$$

input
```
int((a + b*tan(c + d*x))/(b + a*tan(c + d*x))^2,x)
```

output
```
(b*log(b + a*tan(c + d*x))*(3*a^2 - b^2))/(d*(a^2 + b^2)^2) - (log(tan(c +
 d*x) + 1i)*(a - b*1i))/(2*d*(2*a*b - a^2*1i + b^2*1i)) - (a^2 - b^2)/(d*(
a^2 + b^2)*(b + a*tan(c + d*x))) - (log(tan(c + d*x) - 1i)*(a + b*1i))/(2*
d*(2*a*b + a^2*1i - b^2*1i))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.95

$$\int \frac{a+b\tan(c+dx)}{(b+a\tan(c+dx))^2}\,dx$$
$$= \frac{-3\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)a^3b^2+\log\left(\tan\left(dx+c\right)^2+1\right)\tan\left(dx+c\right)a\,b^4-3\log\left(\tan\left(dx+\right.\right.}{}$$

input
```
int((a+b*tan(d*x+c))/(b+a*tan(d*x+c))^2,x)
```

output
```
( - 3*log(tan(c + d*x)**2 + 1)*tan(c + d*x)*a**3*b**2 + log(tan(c + d*x)**
2 + 1)*tan(c + d*x)*a*b**4 - 3*log(tan(c + d*x)**2 + 1)*a**2*b**3 + log(ta
n(c + d*x)**2 + 1)*b**5 + 6*log(tan(c + d*x)*a + b)*tan(c + d*x)*a**3*b**2
 - 2*log(tan(c + d*x)*a + b)*tan(c + d*x)*a*b**4 + 6*log(tan(c + d*x)*a +
b)*a**2*b**3 - 2*log(tan(c + d*x)*a + b)*b**5 + 2*tan(c + d*x)*a**5 - 2*ta
n(c + d*x)*a**4*b*d*x + 6*tan(c + d*x)*a**2*b**3*d*x - 2*tan(c + d*x)*a*b*
*4 - 2*a**3*b**2*d*x + 6*a*b**4*d*x)/(2*b*d*(tan(c + d*x)*a**5 + 2*tan(c +
 d*x)*a**3*b**2 + tan(c + d*x)*a*b**4 + a**4*b + 2*a**2*b**3 + b**5))
```

3.317 $\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 233

$$\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a-ib}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{\sqrt{a+ib}(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2A\sqrt{a+b\tan(c+dx)}}{d} - \frac{2(14aAb-8a^2B+35b^2B)\,(a+b\tan(c+dx))^{3/2}}{105b^3d} + \frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{35b^2d} + \frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}$$

output
```
(a-I*b)^(1/2)*(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I
*b)^(1/2)*(A+I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2*A*(a+b
*tan(d*x+c))^(1/2)/d-2/105*(14*A*a*b-8*B*a^2+35*B*b^2)*(a+b*tan(d*x+c))^(3
/2)/b^3/d+2/35*(7*A*b-4*B*a)*tan(d*x+c)*(a+b*tan(d*x+c))^(3/2)/b^2/d+2/7*B
*tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)/b/d
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.91

$$\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{\sqrt{a-ib}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$
$$+\frac{2\sqrt{a+b\tan(c+dx)}(-14a^2Ab-105Ab^3+8a^3B-35ab^2B-b(-7aAb+4a^2B+35b^2B)\tan(c+dx}{105b^3d}$$

input
```
Integrate[Tan[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(Sqrt[a - I*b]*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/
d + (Sqrt[a + I*b]*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b
]])/d + (2*Sqrt[a + b*Tan[c + d*x]]*(-14*a^2*A*b - 105*A*b^3 + 8*a^3*B - 3
5*a*b^2*B - b*(-7*a*A*b + 4*a^2*B + 35*b^2*B)*Tan[c + d*x] + 3*b^2*(7*A*b
+ a*B)*Tan[c + d*x]^2 + 15*b^3*B*Tan[c + d*x]^3))/(105*b^3*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.515, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4113, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)^3\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{4090}$$

$$\frac{2\int -\frac{1}{2}\tan(c+dx)\sqrt{a+b\tan(c+dx)}\left(-\left((7Ab-4aB)\tan^2(c+dx)\right)+7bB\tan(c+dx)+4aB\right)dx}{7b}+\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}\left(-\left((7Ab-4aB)\tan^2(c+dx)\right)+7bB\tan(c+dx)+4aB\right)dx}{7b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}\left(-\left((7Ab-4aB)\tan(c+dx)^2\right)+7bB\tan(c+dx)+4aB\right)dx}{7b}$$

$$\Big\downarrow \; 4130$$

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{\frac{2\int \frac{1}{2}\sqrt{a+b\tan(c+dx)}\left(35A\tan(c+dx)b^2+\left(-8Ba^2+14Aba+35b^2B\right)\tan^2(c+dx)+2a(7Ab-4aB)\right)dx}{5b}-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+da}{5bd}}{7b}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{\frac{\int \sqrt{a+b\tan(c+dx)}\left(35A\tan(c+dx)b^2+\left(-8Ba^2+14Aba+35b^2B\right)\tan^2(c+dx)+2a(7Ab-4aB)\right)dx}{5b}-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^3}{5bd}}{7b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{\frac{\int \sqrt{a+b\tan(c+dx)}\left(35A\tan(c+dx)b^2+\left(-8Ba^2+14Aba+35b^2B\right)\tan(c+dx)^2+2a(7Ab-4aB)\right)dx}{5b}-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^3}{5bd}}{7b}$$

$$\Big\downarrow \; 4113$$

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{\frac{\int \sqrt{a+b\tan(c+dx)}\left(35Ab^2\tan(c+dx)-35b^2B\right)dx+\frac{2\left(-8a^2B+14aAb+35b^2B\right)(a+b\tan(c+dx))^{3/2}}{3bd}}{5b}-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}}{7b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{\frac{\int\sqrt{a+b\tan(c+dx)}(35Ab^2\tan(c+dx)-35b^2B)dx+\frac{2\left(-8a^2B+14aAb+35b^2B\right)(a+b\tan(c+dx))^{3/2}}{3bd}}{5b}-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}}{7b}$$

$\downarrow$ 4011

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{\frac{\int\frac{35b^2(aA-bB)\tan(c+dx)-35b^2(Ab+aB)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(-8a^2B+14aAb+35b^2B\right)(a+b\tan(c+dx))^{3/2}}{3bd}+\frac{70Ab^2\sqrt{a+b\tan(c+dx)}}{d}}{5b}-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}}{7b}$$

$\downarrow$ 3042

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{\frac{\int\frac{35b^2(aA-bB)\tan(c+dx)-35b^2(Ab+aB)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(-8a^2B+14aAb+35b^2B\right)(a+b\tan(c+dx))^{3/2}}{3bd}+\frac{70Ab^2\sqrt{a+b\tan(c+dx)}}{d}}{5b}-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}}{7b}$$

$\downarrow$ 4022

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}+\frac{\frac{35}{2}b^2(-b+ia)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{35}{2}b^2(b+ia)(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(-8a^2 \cdots\right.}{\cdots}}{5b}}{7b}$$

$\downarrow$ 3042

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}+\frac{\frac{35}{2}b^2(-b+ia)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{35}{2}b^2(b+ia)(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(-8a^2 \cdots\right.}{\cdots}}{5b}}{7b}$$

$\downarrow$ 4020

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd}-\frac{-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}+\frac{-\frac{35ib^2(b+ia)(A-iB)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{35ib^2(-b+ia)(A+iB)\int-\cdots}{\cdots}}{\cdots}}{7b}$$

$\downarrow$ 25

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd} - \frac{-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd} + \frac{\frac{35ib^2(b+ia)(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d} + \frac{35ib^2(-b+ia)(A+iB)\int \frac{}{(i\tan}}{}}{7b}$$

$\Big\downarrow$ 73

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd} - \frac{-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd} + \frac{\frac{35b(-b+ia)(A+iB)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d} - \frac{35b(b+ia)(A-iB)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{i}{}}}{d}}{5b}}{7b}$$

$\Big\downarrow$ 221

$$\frac{2B\tan^2(c+dx)(a+b\tan(c+dx))^{3/2}}{7bd} - \frac{-\frac{2(7Ab-4aB)\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd} + \frac{\frac{2(-8a^2B+14aAb+35b^2B)(a+b\tan(c+dx))^{3/2}}{3bd} - \frac{35b^2(b+ia)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} + \frac{35b^2(-b+}{}}{5b}}{7b}$$

input
```
Int[Tan[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2))/(7*b*d) - ((-2*(7*A*b - 4*
a*B)*Tan[c + d*x]*(a + b*Tan[c + d*x])^(3/2))/(5*b*d) + ((-35*b^2*(I*a + b
)*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + (35*(I
*a - b)*b^2*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d
) + (70*A*b^2*Sqrt[a + b*Tan[c + d*x]])/d + (2*(14*a*A*b - 8*a^2*B + 35*b^
2*B)*(a + b*Tan[c + d*x])^(3/2))/(3*b*d))/(5*b))/(7*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(201) = 402$.

Time = 0.45 (sec) , antiderivative size = 937, normalized size of antiderivative = 4.02

method	result
parts	$\frac{2A\left(\frac{(a+b\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{a(a+b\tan(dx+c))^{\frac{3}{2}}}{3}-\sqrt{a+b\tan(dx+c)}\,b^2-b^2\left(\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2 \cdots}\right.}{8}\right.\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(tan(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
2*A/d/b^2*(1/5*(a+b*tan(d*x+c))^(5/2)-1/3*a*(a+b*tan(d*x+c))^(3/2)-(a+b*ta
n(d*x+c))^(1/2)*b^2-b^2*(1/8*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*tan(d*x
+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))+1
/2*(-a+(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))
-1/8*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2
)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+1/2*(a-(a^2+b^2)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)
^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))+B*(2/7/d/b^3*(a+b*tan(
d*x+c))^(7/2)-4/5/d/b^3*a*(a+b*tan(d*x+c))^(5/2)+2/3/d/b^3*a^2*(a+b*tan(d*
x+c))^(3/2)-2/3/d/b*(a+b*tan(d*x+c))^(3/2)-1/4/d/b*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x
+c)-a-(a^2+b^2)^(1/2))+1/4/d/b*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/
2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-
(a^2+b^2)^(1/2))-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^
(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+
1/4/d/b*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))
^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))-1/4/d/b*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+1/d*b/(2*(a^2+b^2)^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1312 vs. $2(195) = 390$.

Time = 0.11 (sec) , antiderivative size = 1312, normalized size of antiderivative = 5.63

$$\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/210*(105*b^3*d*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A
*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(
2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (B*d^3*sqr
t(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d
^4) + (2*A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B
^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2
- B^2)*a)/d^2)) - 105*b^3*d*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*
(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d
^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) -
(B*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B
^4)*b^2)/d^4) + (2*A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b + d^2*sqrt
(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^
4) - (A^2 - B^2)*a)/d^2)) - 105*b^3*d*sqrt(-(2*A*B*b - d^2*sqrt(-(4*A^2*B^
2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 -
 B^2)*a)/d^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x +
c) + a) + (B*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A
^2*B^2 + B^4)*b^2)/d^4) - (2*A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b
- d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^
4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)) + 105*b^3*d*sqrt(-(2*A*B*b - d^2*sqrt(
-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/...
```

Sympy [F]

$$\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}\tan^3{(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**3*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*tan(c + d*x)**3, x)
```

Maxima [F]

$$\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan{(dx+c)}+A)\sqrt{b\tan{(dx+c)}+a}\tan{(dx+c)}^3\,dx$$

input
```
integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,24,9]%%%}+%%%{10,[0,22,9]%%%}+%%%{45,[0,20,9]%%%}
+%%%{120,
```

Mupad [B] (verification not implemented)

Time = 57.80 (sec) , antiderivative size = 1093, normalized size of antiderivative = 4.69

$$\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
atan((d^3*((16*(B^2*b^4 - B^2*a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 + (
16*a*b^2*((-B^4*b^2*d^4)^(1/2) + B^2*a*d^2)*(a + b*tan(c + d*x))^(1/2))/d^
4)*(-((-B^4*b^2*d^4)^(1/2) + B^2*a*d^2)/(4*d^4))^(1/2)*1i)/(8*(B^3*b^5 + B
^3*a^2*b^3)))*(-((-B^4*b^2*d^4)^(1/2) + B^2*a*d^2)/(4*d^4))^(1/2)*2i - ((2
*B*(a^2 + b^2))/(3*b^3*d) - (4*B*a^2)/(3*b^3*d))*(a + b*tan(c + d*x))^(3/2
) + atan((d^3*((16*(B^2*b^4 - B^2*a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2
 - (16*a*b^2*((-B^4*b^2*d^4)^(1/2) - B^2*a*d^2)*(a + b*tan(c + d*x))^(1/2)
)/d^4)*(((-B^4*b^2*d^4)^(1/2) - B^2*a*d^2)/(4*d^4))^(1/2)*1i)/(8*(B^3*b^5
+ B^3*a^2*b^3)))*(((-B^4*b^2*d^4)^(1/2) - B^2*a*d^2)/(4*d^4))^(1/2)*2i - (
a + b*tan(c + d*x))^(1/2)*(2*a*((2*B*(a^2 + b^2))/(b^3*d) - (4*B*a^2)/(b^3
*d)) + (8*B*a^3)/(b^3*d) - (4*B*a*(a^2 + b^2))/(b^3*d)) - atan((A^2*b^4*((
-A^4*b^2*d^4)^(1/2)/(4*d^4) + (A^2*a)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^
(1/2)*32i)/((16*A*b^4*(-A^4*b^2*d^4)^(1/2))/d^3 + (16*A*a^2*b^2*(-A^4*b^2*
d^4)^(1/2))/d^3) + (a*b^2*((-A^4*b^2*d^4)^(1/2)/(4*d^4) + (A^2*a)/(4*d^2))
^(1/2)*(a + b*tan(c + d*x))^(1/2)*(-A^4*b^2*d^4)^(1/2)*32i)/((16*A*b^4*(-A
^4*b^2*d^4)^(1/2))/d + (16*A*a^2*b^2*(-A^4*b^2*d^4)^(1/2))/d))*(((-A^4*b^2
*d^4)^(1/2) + A^2*a*d^2)/(4*d^4))^(1/2)*2i + atan((A^2*b^4*((A^2*a)/(4*d^2
) - (-A^4*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((
16*A*b^4*(-A^4*b^2*d^4)^(1/2))/d^3 + (16*A*a^2*b^2*(-A^4*b^2*d^4)^(1/2))/d
^3) - (a*b^2*((A^2*a)/(4*d^2) - (-A^4*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*(a ...
```

Reduce [F]

$$\int \tan^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^4\,dx\right)b+\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)a$$

input
```
int(tan(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4,x)*b + int(sqrt(tan(c + d*x)*
b + a)*tan(c + d*x)**3,x)*a
```

3.318 $\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 186

$$\begin{aligned}\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx &= \frac{\sqrt{a-ib}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} \\ &\quad - \frac{\sqrt{a+ib}(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2B\sqrt{a+b\tan(c+dx)}}{d} \\ &\quad + \frac{2(5Ab-2aB)(a+b\tan(c+dx))^{3/2}}{15b^2d} + \frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}\end{aligned}$$

output
```
(a-I*b)^(1/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I
*b)^(1/2)*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2*B*(a+b
*tan(d*x+c))^(1/2)/d+2/15*(5*A*b-2*B*a)*(a+b*tan(d*x+c))^(3/2)/b^2/d+2/5*B
*tan(d*x+c)*(a+b*tan(d*x+c))^(3/2)/b/d
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

$$\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{15\sqrt{a-ib}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+15\sqrt{a+ib}(-iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+\frac{2\sqrt{a+b\tan(}}{}}{15d}$$

input
```
Integrate[Tan[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(15*Sqrt[a - I*b]*(I*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]
] + 15*Sqrt[a + I*b]*((-I)*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a
+ I*b]] + (2*Sqrt[a + b*Tan[c + d*x]]*(5*a*A*b - 2*a^2*B - 15*b^2*B + b*(5
*A*b + a*B)*Tan[c + d*x] + 3*b^2*B*Tan[c + d*x]^2))/b^2)/(15*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.424, Rules used = {3042, 4090, 27, 3042, 4113, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2\int -\frac{1}{2}\sqrt{a+b\tan(c+dx)}\left(-\left((5Ab-2aB)\tan^2(c+dx)\right)+5bB\tan(c+dx)+2aB\right)dx}{5b}+\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\int\sqrt{a+b\tan(c+dx)}\left(-\left((5Ab-2aB)\tan^2(c+dx)\right)+5bB\tan(c+dx)+2aB\right)dx}{5b}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\int\sqrt{a+b\tan(c+dx)}\left(-\left((5Ab-2aB)\tan(c+dx)^2\right)+5bB\tan(c+dx)+2aB\right)dx}{5b}$$

$\Big\downarrow$ 4113

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\int\sqrt{a+b\tan(c+dx)}(5Ab+5B\tan(c+dx)b)dx-\frac{2(5Ab-2aB)(a+b\tan(c+dx))^{3/2}}{3bd}}{5b}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\int\sqrt{a+b\tan(c+dx)}(5Ab+5B\tan(c+dx)b)dx-\frac{2(5Ab-2aB)(a+b\tan(c+dx))^{3/2}}{3bd}}{5b}$$

$\Big\downarrow$ 4011

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\int\frac{5b(aA-bB)+5b(Ab+aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(5Ab-2aB)(a+b\tan(c+dx))^{3/2}}{3bd}+\frac{10bB\sqrt{a+b\tan(c+dx)}}{d}}{5b}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\int\frac{5b(aA-bB)+5b(Ab+aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(5Ab-2aB)(a+b\tan(c+dx))^{3/2}}{3bd}+\frac{10bB\sqrt{a+b\tan(c+dx)}}{d}}{5b}$$

$\Big\downarrow$ 4022

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\frac{5}{2}b(a+ib)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{5}{2}b(a-ib)(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(5Ab-2aB)(a+b\tan(c+dx))^{3/2}}{3bd}+}{5b}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\frac{5}{2}b(a+ib)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{5}{2}b(a-ib)(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(5Ab-2aB)(a+b\tan(c+dx))^{3/2}}{3bd}+}{5b}$$

$$\Big\downarrow \text{4020}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\frac{5ib(a-ib)(A-iB)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{5ib(a+ib)(A+iB)\int-\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-}{5b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{-\frac{5ib(a-ib)(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{5ib(a+ib)(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-}{5b}$$

$$\Big\downarrow \text{73}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\frac{5(a+ib)(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}+\frac{5(a-ib)(A-iB)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}-\frac{2(5Ab-2aB)(a+b\tan}{3bd}}{5b}$$

$$\Big\downarrow \text{221}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{3/2}}{5bd}-\frac{\frac{5b\sqrt{a-ib}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{5b\sqrt{a+ib}(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}-\frac{2(5Ab-2aB)(a+b\tan(c+dx))^{3/2}}{3bd}+\frac{10bB\sqrt{a+b\tan(c+dx)}}{d}}{5b}$$

input
```
Int[Tan[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*Tan[c + d*x]*(a + b*Tan[c + d*x])^(3/2))/(5*b*d) - ((5*Sqrt[a - I*b]*
b*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + (5*Sqrt[a + I*b]*b*(A
+ I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d + (10*b*B*Sqrt[a + b*Tan[c +
d*x]])/d - (2*(5*A*b - 2*a*B)*(a + b*Tan[c + d*x])^(3/2))/(3*b*d))/(5*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(158) = 316$.

Time = 0.16 (sec) , antiderivative size = 873, normalized size of antiderivative = 4.69

method	result
parts	$A\left(\frac{2(a+b\tan(dx+c))^{\frac{3}{2}}}{3db}+\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,a\ln\left(\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\tan(dx+c)-a-\sqrt{a^2+b^2}\right)}{4db}-\right.$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
A*(2/3/d/b*(a+b*tan(d*x+c))^(3/2)+1/4/d/b*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*
ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^
2+b^2)^(1/2))-1/4/d/b*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*ln((a+
b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)
^(1/2))+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a
)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4/d/b*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+1/4/d/b*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*(a^2+b^2)^(1/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2)))+2*B/d/b^2*(1/5*(a+b*tan(d*x+c))^(5/2)-1/3*a*(a+b*ta
n(d*x+c))^(3/2)-(a+b*tan(d*x+c))^(1/2)*b^2-b^2*(1/8*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+
c)-a-(a^2+b^2)^(1/2))+1/2*(-a+(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2))-1/8*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c)+
a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+1/
2*(a-(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x
+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(152) = 304$.

Time = 0.11 (sec) , antiderivative size = 1265, normalized size of antiderivative = 6.80

$$\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/30*(15*b^2*d*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3
)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A
^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (A*d^3*sqrt(-(
4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)
- (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^
2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2
)*a)/d^2)) - 15*b^2*d*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B
- A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log
(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) - (A*d^3*
sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2
)/d^4) - (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2
*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^
2 - B^2)*a)/d^2)) - 15*b^2*d*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*
(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d
^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) +
(A*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B
^4)*b^2)/d^4) + (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b - d^2*sqrt(
-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4
) - (A^2 - B^2)*a)/d^2)) + 15*b^2*d*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a
^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 -...
```

Sympy [F]

$$\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan(c+dx))\sqrt{a+b\tan(c+dx)}\tan^2(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*tan(c + d*x)**2, x)
```

Maxima [F]

$$\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\tan(dx+c)^2\,dx$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,19,7]%%%}+%%%{8,[0,17,7]%%%}+%%%{28,[0,15,7]%%%}+
%%%{56,[0
```

Mupad [B] (verification not implemented)

Time = 19.96 (sec) , antiderivative size = 938, normalized size of antiderivative = 5.04

$$\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
atanh((d^3*((16*(A^2*b^4 - A^2*a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 +
(16*a*b^2*((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)*(a + b*tan(c + d*x))^(1/2))/d
^4)*(-((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)/d^4)^(1/2))/(16*(A^3*b^5 + A^3*a^
2*b^3)))*(-((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)/d^4)^(1/2) - ((2*B*(a^2 + b^
2))/(b^2*d) - (2*B*a^2)/(b^2*d))*(a + b*tan(c + d*x))^(1/2) + atanh((d^3*(
(16*(A^2*b^4 - A^2*a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 - (16*a*b^2*((
-A^4*b^2*d^4)^(1/2) - A^2*a*d^2)*(a + b*tan(c + d*x))^(1/2))/d^4)*(((-A^4*
b^2*d^4)^(1/2) - A^2*a*d^2)/d^4)^(1/2))/(16*(A^3*b^5 + A^3*a^2*b^3)))*(((-
A^4*b^2*d^4)^(1/2) - A^2*a*d^2)/d^4)^(1/2) - atan((B^2*b^4*((-B^4*b^2*d^4)
^(1/2)/(4*d^4) + (B^2*a)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((
16*B*b^4*(-B^4*b^2*d^4)^(1/2))/d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d
^3) + (a*b^2*((-B^4*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a)/(4*d^2))^(1/2)*(a + b
*tan(c + d*x))^(1/2)*(-B^4*b^2*d^4)^(1/2)*32i)/((16*B*b^4*(-B^4*b^2*d^4)^(
1/2))/d + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d))*(((-B^4*b^2*d^4)^(1/2) +
 B^2*a*d^2)/(4*d^4))^(1/2)*2i + atan((B^2*b^4*((B^2*a)/(4*d^2) - (-B^4*b^2
*d^4)^(1/2)/(4*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((16*B*b^4*(-B^
4*b^2*d^4)^(1/2))/d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d^3) - (a*b^2*
((B^2*a)/(4*d^2) - (-B^4*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*(a + b*tan(c + d*x)
)^(1/2)*(-B^4*b^2*d^4)^(1/2)*32i)/((16*B*b^4*(-B^4*b^2*d^4)^(1/2))/d + (16
*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d))*(-((-B^4*b^2*d^4)^(1/2) - B^2*a*d^...
```

Reduce [F]

$$\int \tan^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)b + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)a$$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3,x)*b + int(sqrt(tan(c + d*x)*
b + a)*tan(c + d*x)**2,x)*a
```

3.319 $\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 146

$$\begin{aligned}\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx &= -\frac{\sqrt{a-ib}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}\\ &\quad-\frac{\sqrt{a+ib}(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}\\ &\quad+\frac{2A\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}\end{aligned}$$

output
```
-(a-I*b)^(1/2)*(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+
I*b)^(1/2)*(A+I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*A*(a+
b*tan(d*x+c))^(1/2)/d+2/3*B*(a+b*tan(d*x+c))^(3/2)/b/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.96

$$\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \frac{-3\sqrt{a-ib}b(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-3\sqrt{a+ib}b(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)+2\sqrt{a+b}}{3bd}$$

input
```
Integrate[Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(-3*Sqrt[a - I*b]*b*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*
b]] - 3*Sqrt[a + I*b]*b*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a
+ I*b]] + 2*Sqrt[a + b*Tan[c + d*x]]*(3*A*b + a*B + b*B*Tan[c + d*x]))/(3*
b*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4075, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{4075}$$
$$\int (A\tan(c+dx)-B)\sqrt{a+b\tan(c+dx)}dx+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}$$
$$\Big\downarrow \text{3042}$$

$$\int (A\tan(c+dx)-B)\sqrt{a+b\tan(c+dx)}dx+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 4011

$$\int \frac{-Ab-aB+(aA-bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2A\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 3042

$$\int \frac{-Ab-aB+(aA-bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2A\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 4022

$$\frac{1}{2}(-b+ia)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(b+ia)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2A\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}(-b+ia)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(b+ia)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2A\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 4020

$$-\frac{i(b+ia)(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(-b+ia)(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2A\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 25

$$\frac{i(b+ia)(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(-b+ia)(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2A\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}$$

$\Big\downarrow$ 73

$$\frac{(-b+ia)(A+iB)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}-\frac{(b+ia)(A-iB)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{2A\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(b+ia)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{(-b+ia)(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}+\frac{2A\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3bd}$$

input
```
Int[Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
-(((I*a + b)*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*
d)) + ((I*a - b)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I
*b]*d) + (2*A*Sqrt[a + b*Tan[c + d*x]])/d + (2*B*(a + b*Tan[c + d*x])^(3/2
))/(3*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. 2(122) = 244.

Time = 0.14 (sec) , antiderivative size = 829, normalized size of antiderivative = 5.68

method	result
parts	$A\left(2\sqrt{a+b\tan(dx+c)}+\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\tan(dx+c)-a-\sqrt{a^2+b^2}\right)}{4}+\frac{\left(-a+\sqrt{a^2+b^2}\right)\text{ar}}{}\right.$
derivativedivides	$\frac{2B(a+b\tan(dx+c))^{\frac{3}{2}}}{3bd}+\frac{2A\sqrt{a+b\tan(dx+c)}}{d}-\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)}{4d}$
default	$\frac{2B(a+b\tan(dx+c))^{\frac{3}{2}}}{3bd}+\frac{2A\sqrt{a+b\tan(dx+c)}}{d}-\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)}{4d}$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
A/d*(2*(a+b*tan(d*x+c))^(1/2)+1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*ta
n(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/
2))+(-a+(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2
)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
)-1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/
2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+(a-(a^2+b^2)^(1/2))/(2*(
a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1
/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+B*(2/3/d/b*(a+b*tan(d*x+c)
)^(3/2)+1/4/d/b*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a*ln((a+b*tan(d*x+c))^(1/2)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))-1/4/d/b*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))+1/d*b/(2*(a^2+b^2)
^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))
^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4/d/b*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)*a*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)+(a^2+b^2)^(1/2))+1/4/d/b*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*l
n(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2
+b^2)^(1/2))-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c)
)^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1228 vs. $2(116) = 232$.

Time = 0.10 (sec) , antiderivative size = 1228, normalized size of antiderivative = 8.41

$$\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/6*(3*b*d*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a
*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*
B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (B*d^3*sqrt(-(4*A
^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) + (
2*A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2
+ 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*
a)/d^2)) - 3*b*d*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*
B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2
*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) - (B*d^3*sqrt
(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^
4) + (2*A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^
2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 -
 B^2)*a)/d^2)) - 3*b*d*sqrt(-(2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*
B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*l
og(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (B*d^
3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b
^2)/d^4) - (2*A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b - d^2*sqrt(-(4*
A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) -
(A^2 - B^2)*a)/d^2)) + 3*b*d*sqrt(-(2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4
*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*...
```

Sympy [F]

$$\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}\tan{(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*tan(c + d*x), x)
```

Maxima [F]

$$\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\tan(dx+c)\,dx$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 864, normalized size of antiderivative = 5.92

$$\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
atanh((d^3*((16*(B^2*b^4 - B^2*a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 +
(16*a*b^2*((-B^4*b^2*d^4)^(1/2) + B^2*a*d^2)*(a + b*tan(c + d*x))^(1/2))/d
^4)*(-((-B^4*b^2*d^4)^(1/2) + B^2*a*d^2)/d^4)^(1/2))/(16*(B^3*b^5 + B^3*a^
2*b^3)))*(-((-B^4*b^2*d^4)^(1/2) + B^2*a*d^2)/d^4)^(1/2) + atanh((d^3*((16
*(B^2*b^4 - B^2*a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 - (16*a*b^2*((-B^
4*b^2*d^4)^(1/2) - B^2*a*d^2)*(a + b*tan(c + d*x))^(1/2))/d^4)*(((-B^4*b^2
*d^4)^(1/2) - B^2*a*d^2)/d^4)^(1/2))/(16*(B^3*b^5 + B^3*a^2*b^3)))*(((-B^4
*b^2*d^4)^(1/2) - B^2*a*d^2)/d^4)^(1/2) - 2*atanh((32*A^2*b^4*((-A^4*b^2*d
^4)^(1/2)/(4*d^4) + (A^2*a)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((1
6*A*b^4*(-A^4*b^2*d^4)^(1/2))/d^3 + (16*A*a^2*b^2*(-A^4*b^2*d^4)^(1/2))/d^
3) + (32*a*b^2*((-A^4*b^2*d^4)^(1/2)/(4*d^4) + (A^2*a)/(4*d^2))^(1/2)*(a +
 b*tan(c + d*x))^(1/2)*(-A^4*b^2*d^4)^(1/2))/((16*A*b^4*(-A^4*b^2*d^4)^(1/
2))/d + (16*A*a^2*b^2*(-A^4*b^2*d^4)^(1/2))/d))*(((-A^4*b^2*d^4)^(1/2) + A
^2*a*d^2)/(4*d^4))^(1/2) + 2*atanh((32*A^2*b^4*((A^2*a)/(4*d^2) - (-A^4*b^
2*d^4)^(1/2)/(4*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*A*b^4*(-A^4*b
^2*d^4)^(1/2))/d^3 + (16*A*a^2*b^2*(-A^4*b^2*d^4)^(1/2))/d^3) - (32*a*b^2*
((A^2*a)/(4*d^2) - (-A^4*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*(a + b*tan(c + d*x)
)^(1/2)*(-A^4*b^2*d^4)^(1/2))/((16*A*b^4*(-A^4*b^2*d^4)^(1/2))/d + (16*A*a
^2*b^2*(-A^4*b^2*d^4)^(1/2))/d))*(-((-A^4*b^2*d^4)^(1/2) - A^2*a*d^2)/(4*d
^4))^(1/2) + (2*A*(a + b*tan(c + d*x))^(1/2))/d + (2*B*(a + b*tan(c + d...
```

Reduce [F]

$$\int \tan(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)b + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)a$$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*b + int(sqrt(tan(c + d*x)*
b + a)*tan(c + d*x),x)*a
```

3.320 $\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{\sqrt{a-ib}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{\sqrt{a+ib}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2B\sqrt{a+b\tan(c+dx)}}{d}$$

output
```
-(a-I*b)^(1/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+
I*b)^(1/2)*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*B*(a+
b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{-i\sqrt{a-ib}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right) + i\sqrt{a+ib}(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right) + 2B\sqrt{a+b\,t}}{d}$$

input
```
Integrate[Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
((-I)*Sqrt[a - I*b]*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*
b]] + I*Sqrt[a + I*b]*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a +
I*b]] + 2*B*Sqrt[a + b*Tan[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2B\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2B\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(a+ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2B\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(a+ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2B\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 4020

$$\frac{i(a-ib)(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a+ib)(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2B\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 25

$$-\frac{i(a-ib)(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(a+ib)(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2B\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 73

$$\frac{(a+ib)(A+iB)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a-ib)(A-iB)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{2B\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 221

$$\frac{\sqrt{a-ib}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2B\sqrt{a+b\tan(c+dx)}}{d}$$

input
```
Int[Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(Sqrt[a - I*b]*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + (Sqrt[a +
 I*b]*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d + (2*B*Sqrt[a + b*Ta
n[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(102) = 204$.

Time = 0.12 (sec) , antiderivative size = 812, normalized size of antiderivative = 6.66

method	result
parts	$-\frac{\ln\left(\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\tan(dx+c)-a-\sqrt{a^2+b^2}\right)A\sqrt{2\sqrt{a^2+b^2}+2a}\,a}{4db}+\frac{\ln\left(\sqrt{a+b\tan(dx+c)}\,\sqrt{2\right.}{}$
derivativedivides	$\frac{2B\sqrt{a+b\tan(dx+c)}}{d}-\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)A\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}{4db}$
default	$\frac{2B\sqrt{a+b\tan(dx+c)}}{d}-\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)A\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}{4db}$

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x
+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/d/b*ln((a+b*t
an(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1
/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/d*b/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1
/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A+1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d
*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a-1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
(a^2+b^2)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x
+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A
+B/d*(2*(a+b*tan(d*x+c))^(1/2)+1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*t
an(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1
/2))+(-a+(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
))-1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1
/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+(a-(a^2+b^2)^(1/2))/(2*
(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(
1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(96) = 192$.

Time = 0.11 (sec) , antiderivative size = 1199, normalized size of antiderivative = 9.83

$$\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(d*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b +
 (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B +
A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (A*d^3*sqrt(-(4*A^2*B
^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (2*A*
B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(
A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^
2)) - d*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b +
 (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B +
A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) - (A*d^3*sqrt(-(4*A^2*B
^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (2*A*
B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(
A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^
2)) - d*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b +
 (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B +
A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (A*d^3*sqrt(-(4*A^2*B
^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) + (2*A*
B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(
A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^
2)) + d*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b +
 (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B...
```

Sympy [F]

$$\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 845, normalized size of antiderivative = 6.93

$$\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
2*atanh((32*B^2*b^4*((B^2*a)/(4*d^2) - (-B^4*b^2*d^4)^(1/2)/(4*d^4))^(1/2)
*(a + b*tan(c + d*x))^(1/2))/((16*B*b^4*(-B^4*b^2*d^4)^(1/2))/d^3 + (16*B*
a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d^3) - (32*a*b^2*((B^2*a)/(4*d^2) - (-B^4*b^
2*d^4)^(1/2)/(4*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(-B^4*b^2*d^4)^(1/2
))/((16*B*b^4*(-B^4*b^2*d^4)^(1/2))/d + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2)
)/d))*(-((-B^4*b^2*d^4)^(1/2) - B^2*a*d^2)/(4*d^4))^(1/2) - atanh((d^3*((1
6*(A^2*b^4 - A^2*a^2*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 - (16*a*b^2*((-A
^4*b^2*d^4)^(1/2) - A^2*a*d^2)*(a + b*tan(c + d*x))^(1/2))/d^4)*(((-A^4*b^
2*d^4)^(1/2) - A^2*a*d^2)/d^4)^(1/2))/(16*(A^3*b^5 + A^3*a^2*b^3)))*(((-A^
4*b^2*d^4)^(1/2) - A^2*a*d^2)/d^4)^(1/2) - 2*atanh((32*B^2*b^4*((-B^4*b^2*
d^4)^(1/2)/(4*d^4) + (B^2*a)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((
16*B*b^4*(-B^4*b^2*d^4)^(1/2))/d^3 + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d
^3) + (32*a*b^2*((-B^4*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a)/(4*d^2))^(1/2)*(a
+ b*tan(c + d*x))^(1/2)*(-B^4*b^2*d^4)^(1/2))/((16*B*b^4*(-B^4*b^2*d^4)^(1
/2))/d + (16*B*a^2*b^2*(-B^4*b^2*d^4)^(1/2))/d))*(((-B^4*b^2*d^4)^(1/2) +
B^2*a*d^2)/(4*d^4))^(1/2) - atanh((d^3*((16*(A^2*b^4 - A^2*a^2*b^2)*(a + b
*tan(c + d*x))^(1/2))/d^2 + (16*a*b^2*((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)*(
a + b*tan(c + d*x))^(1/2))/d^4)*(-((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)/d^4)^
(1/2))/(16*(A^3*b^5 + A^3*a^2*b^3)))*(-((-A^4*b^2*d^4)^(1/2) + A^2*a*d^2)/
d^4)^(1/2) + (2*B*(a + b*tan(c + d*x))^(1/2))/d
```

Reduce [F]

$$\int \sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\left(\int \sqrt{a+\tan(dx+c)\,b}dx\right)a+\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)b$$

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a),x)*a + int(sqrt(tan(c + d*x)*b + a)*tan(c + d
*x),x)*b
```

3.321 $\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 131

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{2\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{\sqrt{a-ib}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}$$

output
```
-2*a^(1/2)*A*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d+(a-I*b)^(1/2)*(A-I*
B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I*b)^(1/2)*(A+I*B)*a
rctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.67

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)-\frac{\left(A\left(b^2+a\sqrt{-b^2}\right)+b\left(a-\sqrt{-b^2}\right)B\right)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}}+\frac{\left(A\left(b^2-a\sqrt{-b^2}\right)+b\left(a-\right.\right.}{}}{d}$$

input
```
Integrate[Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
-((2*Sqrt[a]*A*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] - ((A*(b^2 + a*Sq
rt[-b^2]) + b*(a - Sqrt[-b^2])*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a
- Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) + ((A*(b^2 - a*Sqrt[-b^2
]) + b*(a + Sqrt[-b^2])*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + Sqrt[
-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]))/d)
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4095, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4095}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+aA\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+aA\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4022}$$

$$-\frac{1}{2}(a+ib)(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+aA\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}(a+ib)(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+aA\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(a-ib)(B+iA)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(a+ib)(-B+iA)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+aA\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{i(a-ib)(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a+ib)(-B+iA)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+aA\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{73}$$

$$-\frac{(a+ib)(-B+iA)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a-ib)(B+iA)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+aA\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 221$$

$$aA\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{\sqrt{a-ib}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}$$

$$\Big\downarrow \; 4117$$

$$\frac{aA\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{\sqrt{a-ib}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}$$

$$\Big\downarrow \; 73$$

$$\frac{2aA\int \frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{\sqrt{a-ib}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{a-ib}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}-\frac{2\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}$$

input
```
Int[Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(Sqrt[a - I*b]*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d - (Sqrt[a +
 I*b]*(I*A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d - (2*Sqrt[a]*A*ArcTa
nh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4095
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1
/(a^2 + b^2)   Int[Simp[A*(a*c + b*d) + B*(b*c - a*d) - (A*(b*c - a*d) - B*
(a*c + b*d))*Tan[e + f*x], x]/Sqrt[c + d*Tan[e + f*x]], x], x] - Simp[(b*c
- a*d)*((B*a - A*b)/(a^2 + b^2))   Int[(1 + Tan[e + f*x]^2)/((a + b*Tan[e +
 f*x])*Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(107) = 214$.

Time = 0.14 (sec) , antiderivative size = 976, normalized size of antiderivative = 7.45

method	result
derivativedivides	$\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)A\sqrt{2\sqrt{a^2+b^2}+2a}}{4d} - \frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan\right.}{}$
default	$\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)A\sqrt{2\sqrt{a^2+b^2}+2a}}{4d} - \frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan\right.}{}$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/4/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/4/d/b*ln(b*tan(d*x+c
)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*
B*(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b*ln(b*tan(d*x+c)+a+
(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*
(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))*A*(a^2+b^2)^(1/2)-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(
a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2))*A*a+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c
))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B-1
/4/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-
a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b*ln((a+b*tan(d*x
+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B
*(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/4/d/b*ln((a+b*tan(d*x+c))
^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*
(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2))*A*(a^2+b^2)^(1/2)+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(
a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1215 vs. $2(101) = 202$.

Time = 0.36 (sec) , antiderivative size = 2449, normalized size of antiderivative = 18.69

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[-1/2*(d*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b
 + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B
+ A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (B*d^3*sqrt(-(4*A^2
*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) + (2*
A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 +
4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)
/d^2)) - d*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a
*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*
B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) - (B*d^3*sqrt(-(4*A
^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) + (
2*A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2
+ 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*
a)/d^2)) - d*sqrt(-(2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)
*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^
3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (B*d^3*sqrt(-(4
*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) -
 (2*A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^
2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2
)*a)/d^2)) + d*sqrt(-(2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^
3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(...
```

Sympy [F]

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}\cot{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*cot(c + d*x), x)
```

Maxima [F]

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 9785, normalized size of antiderivative = 74.69

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
(A*a^(1/2)*atan(((A*a^(1/2)*((32*(a + b*tan(c + d*x))^(1/2)*(A^4*b^12 + B^
4*b^12 + 2*A^2*B^2*b^12 + 3*A^4*a^4*b^8 + 2*B^4*a^2*b^10 + B^4*a^4*b^8 + 6
*A^2*B^2*a^2*b^10 - 8*A^3*B*a^3*b^9))/d^4 + (A*a^(1/2)*((32*(3*A^3*a^2*b^1
0*d^2 + 3*A^3*a^4*b^8*d^2 + B^3*a^3*b^9*d^2 + B^3*a*b^11*d^2 - 15*A^2*B*a*
b^11*d^2 - 9*A*B^2*a^2*b^10*d^2 - 9*A*B^2*a^4*b^8*d^2 - 15*A^2*B*a^3*b^9*d
^2))/d^5 + (A*a^(1/2)*((32*(a + b*tan(c + d*x))^(1/2)*(10*B^2*a^3*b^8*d^2
- 18*A^2*a^3*b^8*d^2 + 16*A*B*b^11*d^2 - 6*A^2*a*b^10*d^2 + 6*B^2*a*b^10*d
^2 + 24*A*B*a^2*b^9*d^2))/d^4 - (A*a^(1/2)*((32*(12*A*a*b^10*d^4 + 12*A*a^
3*b^8*d^4))/d^5 - (32*A*a^(1/2)*(16*b^10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(
c + d*x))^(1/2))/d^5))/d))/d))/d)*1i)/d + (A*a^(1/2)*((32*(a + b*tan(c + d
*x))^(1/2)*(A^4*b^12 + B^4*b^12 + 2*A^2*B^2*b^12 + 3*A^4*a^4*b^8 + 2*B^4*a
^2*b^10 + B^4*a^4*b^8 + 6*A^2*B^2*a^2*b^10 - 8*A^3*B*a^3*b^9))/d^4 - (A*a^
(1/2)*((32*(3*A^3*a^2*b^10*d^2 + 3*A^3*a^4*b^8*d^2 + B^3*a^3*b^9*d^2 + B^3
*a*b^11*d^2 - 15*A^2*B*a*b^11*d^2 - 9*A*B^2*a^2*b^10*d^2 - 9*A*B^2*a^4*b^8
*d^2 - 15*A^2*B*a^3*b^9*d^2))/d^5 - (A*a^(1/2)*((32*(a + b*tan(c + d*x))^(
1/2)*(10*B^2*a^3*b^8*d^2 - 18*A^2*a^3*b^8*d^2 + 16*A*B*b^11*d^2 - 6*A^2*a*
b^10*d^2 + 6*B^2*a*b^10*d^2 + 24*A*B*a^2*b^9*d^2))/d^4 + (A*a^(1/2)*((32*(
12*A*a*b^10*d^4 + 12*A*a^3*b^8*d^4))/d^5 + (32*A*a^(1/2)*(16*b^10*d^4 + 24
*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/d^5))/d))/d))/d)*1i)/d)/((64*(A^
5*a*b^12 + A^5*a^3*b^10 + A^2*B^3*a^2*b^11 + A^2*B^3*a^4*b^9 + 3*A^3*B^...
```

Reduce [F]

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)\tan(dx+c)\,dx\right)b + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)\,dx\right)a$$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*cot(c + d*x)*tan(c + d*x),x)*b + int(sqrt(tan
(c + d*x)*b + a)*cot(c + d*x),x)*a
```

3.322 $\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 167

$$\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{(Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+\frac{\sqrt{a-ib}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{\sqrt{a+ib}(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

output
```
-(A*b+2*B*a)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d+(a-I*b)^(1/
2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*b)^(1/2)*(
I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-A*cot(d*x+c)*(a+b*t
an(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.41

$$\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{-\frac{(Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\frac{\left(A\left(b^2+a\sqrt{-b^2}\right)+b\left(a-\sqrt{-b^2}\right)B\right)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{a-\sqrt{-b^2}}} + \frac{\left(A\left(b^2-a\sqrt{-b^2}\right)+b\left(a+\sqrt{-b^2}\right)B\right)\cdots}{\sqrt{a+\cdots}}}{b}}{d}$$

input

```
Integrate[Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output

```
(-(((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a]) + ((
(A*(b^2 + a*Sqrt[-b^2]) + b*(a - Sqrt[-b^2])*B)*ArcTanh[Sqrt[a + b*Tan[c +
 d*x]]/Sqrt[a - Sqrt[-b^2]]])/Sqrt[a - Sqrt[-b^2]] + ((A*(b^2 - a*Sqrt[-b^
2]) + b*(a + Sqrt[-b^2])*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + Sqrt
[-b^2]]])/Sqrt[a + Sqrt[-b^2]] - A*b*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]]
)/b)/d
```

Rubi [A] (warning: unable to verify)

Time = 1.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 4091, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4091}$$

$$-\int -\frac{\cot(c+dx)\left(-Ab\tan^2(c+dx)-2(aA-bB)\tan(c+dx)+Ab+2aB\right)}{2\sqrt{a+b\tan(c+dx)}}dx-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{\cot(c+dx)\left(-Ab\tan^2(c+dx)-2(aA-bB)\tan(c+dx)+Ab+2aB\right)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{-Ab\tan(c+dx)^2-2(aA-bB)\tan(c+dx)+Ab+2aB}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 4136}$$

$$\frac{1}{2}\left(\int -\frac{2(aA-bB+(Ab+aB)\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx+(2aB+Ab)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx\right)-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left((2aB+Ab)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-2\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx\right)-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left((2aB+Ab)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx\right)-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 4022}$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left((2aB+Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{1}{2}(a+ib)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-\right.\right.$$

$\downarrow$ 3042

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left((2aB+Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{1}{2}(a+ib)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-\right.\right.$$

$\downarrow$ 4020

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left((2aB+Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{i(a-ib)(A-iB)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan\right.\right.}{2d}$$

$\downarrow$ 25

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left((2aB+Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{i(a+ib)(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan\right.\right.}{2d}$$

$\downarrow$ 73

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left((2aB+Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{(a+ib)(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+\right.\right.}{bd}$$

$\downarrow$ 221

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left((2aB+Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{\sqrt{a-ib}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}(A+\right.\right.$$

$\downarrow$ 4117

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(\frac{(2aB+Ab)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}-2\left(\frac{\sqrt{a-ib}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}(A+iB)\,\mathrm{arc}}{d}\right.\right.$$

$$\Big\downarrow \text{73}$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(\frac{2(2aB+Ab)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}-2\left(\frac{\sqrt{a-ib}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}(A-}{}\right.\right.$$

$$\Big\downarrow \text{221}$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(-\frac{2(2aB+Ab)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-2\left(\frac{\sqrt{a-ib}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{\sqrt{a+ib}(A+iB)\,\mathrm{arcta}}{d}\right.\right.$$

input
```
Int[Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*((Sqrt[a - I*b]*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + (Sqr
t[a + I*b]*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d) - (2*(A*b + 2*
a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d))/2 - (A*Cot[c
+ d*x]*Sqrt[a + b*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(141) = 282$.

Time = 0.16 (sec) , antiderivative size = 1029, normalized size of antiderivative = 6.16

method	result	size
derivativedivides	Expression too large to display	1029
default	Expression too large to display	1029

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/4/
d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/d*ln(b*tan(d*x+c)+
a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*
(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))*B*(a^2+b^2)^(1/2)-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2
*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2))*A-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))
^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a-1
/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c
)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+1/4/d
/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-
(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/4/d*ln((a+b*tan(d*x+c
))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*
(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2))*B*(a^2+b^2)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2
*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. $2(135) = 270$.

Time = 0.73 (sec) , antiderivative size = 2576, normalized size of antiderivative = 15.43

$$\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
[1/2*(a*d*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b
 + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B
+ A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (A*d^3*sqrt(-(4*A^2
*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (2*
A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4
*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/
d^2))*tan(d*x + c) - a*d*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3
*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*
log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) - (A*d
^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*
b^2)/d^4) - (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b + d^2*sqrt(-(4*
A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) -
(A^2 - B^2)*a)/d^2))*tan(d*x + c) - a*d*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B
^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2
- B^2)*a)/d^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x +
 c) + a) + (A*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*
A^2*B^2 + B^4)*b^2)/d^4) + (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b
- d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^
4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2))*tan(d*x + c) + a*d*sqrt((2*A*B*b - d^2
*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)...
```

Sympy [F]

$$\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}\cot^2{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*cot(c + d*x)**2, x)
```

Maxima [F]

$$\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 10987, normalized size of antiderivative = 65.79

$$\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
(atan(((((16*(a + b*tan(c + d*x))^(1/2)*(3*A^4*b^12 + 2*B^4*b^12 + 3*A^2*B
^2*b^12 + 3*A^4*a^2*b^10 + 2*A^4*a^4*b^8 + 6*B^4*a^4*b^8 + 29*A^2*B^2*a^2*
b^10 - 4*A*B^3*a*b^11 + 8*A^3*B*a*b^11 + 20*A*B^3*a^3*b^9 - 4*A^3*B*a^3*b^
9))/d^4 + ((A*b + 2*B*a)*((8*(12*B^3*a^2*b^10*d^2 - 20*A^3*a^3*b^9*d^2 + 1
2*B^3*a^4*b^8*d^2 + 28*A^2*B*b^12*d^2 - 20*A^3*a*b^11*d^2 + 60*A*B^2*a*b^1
1*d^2 + 60*A*B^2*a^3*b^9*d^2 - 8*A^2*B*a^2*b^10*d^2 - 36*A^2*B*a^4*b^8*d^2
))/d^5 - (((16*(a + b*tan(c + d*x))^(1/2)*(36*B^2*a^3*b^8*d^2 - 20*A^2*a^3
*b^8*d^2 + 32*A*B*b^11*d^2 - 8*A^2*a*b^10*d^2 + 12*B^2*a*b^10*d^2 + 64*A*B
*a^2*b^9*d^2))/d^4 + ((A*b + 2*B*a)*((8*(32*A*b^11*d^4 + 48*B*a*b^10*d^4 +
 32*A*a^2*b^9*d^4 + 48*B*a^3*b^8*d^4))/d^5 - (8*(A*b + 2*B*a)*(32*b^10*d^4
 + 48*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/(a^(1/2)*d^5)))/(2*a^(1/2)*
d))*(A*b + 2*B*a))/(2*a^(1/2)*d)))/(2*a^(1/2)*d))*(A*b + 2*B*a)*1i)/(2*a^(
1/2)*d) + (((16*(a + b*tan(c + d*x))^(1/2)*(3*A^4*b^12 + 2*B^4*b^12 + 3*A^
2*B^2*b^12 + 3*A^4*a^2*b^10 + 2*A^4*a^4*b^8 + 6*B^4*a^4*b^8 + 29*A^2*B^2*a
^2*b^10 - 4*A*B^3*a*b^11 + 8*A^3*B*a*b^11 + 20*A*B^3*a^3*b^9 - 4*A^3*B*a^3
*b^9))/d^4 - ((A*b + 2*B*a)*((8*(12*B^3*a^2*b^10*d^2 - 20*A^3*a^3*b^9*d^2
+ 12*B^3*a^4*b^8*d^2 + 28*A^2*B*b^12*d^2 - 20*A^3*a*b^11*d^2 + 60*A*B^2*a*
b^11*d^2 + 60*A*B^2*a^3*b^9*d^2 - 8*A^2*B*a^2*b^10*d^2 - 36*A^2*B*a^4*b^8*
d^2))/d^5 + (((16*(a + b*tan(c + d*x))^(1/2)*(36*B^2*a^3*b^8*d^2 - 20*A^2*
a^3*b^8*d^2 + 32*A*B*b^11*d^2 - 8*A^2*a*b^10*d^2 + 12*B^2*a*b^10*d^2 + ...
```

Reduce [F]

$$\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^2\sqrt{a+\tan(dx+c)\,b}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

3.323 $\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 219

$$\begin{aligned}\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx &= \frac{(8a^2A+Ab^2-4abB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} \\ &\quad - \frac{\sqrt{a-ib}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} \\ &\quad - \frac{\sqrt{a+ib}(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} \\ &\quad - \frac{(Ab+4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4ad} - \frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}\end{aligned}$$

output

```
1/4*(8*A*a^2+A*b^2-4*B*a*b)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2
)/d-(a-I*b)^(1/2)*(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-
(a+I*b)^(1/2)*(A+I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-1/4*
(A*b+4*B*a)*cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)/a/d-1/2*A*cot(d*x+c)^2*(a+b*
tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 3.21 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.24

$$\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{\frac{(8a^2A+Ab^2-4abB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\frac{4\left(-aAb+Ab\sqrt{-b^2}+b^2B+a\sqrt{-b^2}B\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{a-\sqrt{-b^2}}}-\frac{4\left(aAb+Ab\sqrt{-b^2}-b^2B+\cdots\right.}{\cdots}}{\cdots}}{4d}$$

input
```
Integrate[Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(((8*a^2*A + A*b^2 - 4*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/a
^(3/2) + ((4*(-(a*A*b) + A*b*Sqrt[-b^2] + b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[
Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/Sqrt[a - Sqrt[-b^2]] - (4*
(a*A*b + A*b*Sqrt[-b^2] - b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Tan[c
 + d*x]]/Sqrt[a + Sqrt[-b^2]]])/Sqrt[a + Sqrt[-b^2]] - (b*Cot[c + d*x]*(A*
b + 4*a*B + 2*a*A*Cot[c + d*x])*Sqrt[a + b*Tan[c + d*x]])/a)/b)/(4*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.81 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.576, Rules used = {3042, 4091, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$
$$\Big\downarrow \text{4091}$$

$$-\frac{1}{2}\int -\frac{\cot^2(c+dx)\left(-3Ab\tan^2(c+dx)-4(aA-bB)\tan(c+dx)+Ab+4aB\right)}{2\sqrt{a+b\tan(c+dx)}}dx-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 27

$$\frac{1}{4}\int \frac{\cot^2(c+dx)\left(-3Ab\tan^2(c+dx)-4(aA-bB)\tan(c+dx)+Ab+4aB\right)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\int \frac{-3Ab\tan(c+dx)^2-4(aA-bB)\tan(c+dx)+Ab+4aB}{\tan(c+dx)^2\sqrt{a+b\tan(c+dx)}}dx-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 4132

$$\frac{1}{4}\left(-\frac{\int \frac{\cot(c+dx)\left(8Aa^2-4bBa+8(Ab+aB)\tan(c+dx)a+Ab^2+b(Ab+4aB)\tan^2(c+dx)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan}}{ad}\right.$$
$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{\int \frac{\cot(c+dx)\left(8Aa^2-4bBa+8(Ab+aB)\tan(c+dx)a+Ab^2+b(Ab+4aB)\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan}}{ad}\right.$$
$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(-\frac{\int \frac{8Aa^2-4bBa+8(Ab+aB)\tan(c+dx)a+Ab^2+b(Ab+4aB)\tan(c+dx)^2}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}\right)$$
$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 4136

$$\frac{1}{4}\left(-\frac{\left(8a^2A-4abB+Ab^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+\int \frac{8(a(Ab+aB)-a(aA-bB)\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(4aB+Ab)\cot(c\right.$$

$$\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow 27$$

$$\frac{1}{4}\left(-\frac{\left(8a^2A-4abB+Ab^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+8\int \frac{a(Ab+aB)-a(aA-bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(4aB+Ab)\cot(c\right.$$

$$\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow 3042$$

$$\frac{1}{4}\left(-\frac{\left(8a^2A-4abB+Ab^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\int \frac{a(Ab+aB)-a(aA-bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(4aB+Ab)\cot(c\right.$$

$$\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow 4022$$

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$

$$\frac{1}{4}\left(-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{\left(8a^2A-4abB+Ab^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(\frac{1}{2}a(a\right.\right.$$

$$\Big\downarrow 3042$$

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$

$$\frac{1}{4}\left(-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{\left(8a^2A-4abB+Ab^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(\frac{1}{2}a(a\right.\right.$$

$$\Big\downarrow 4020$$

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{\left(8a^2A-4abB+Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(\frac{ia(a-\cdots}{\cdots}\right.}{\cdots}\right.$$

↓ 25

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{\left(8a^2A-4abB+Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(-\frac{ia(\cdots}{\cdots}\right.}{\cdots}\right.$$

↓ 73

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{\left(8a^2A-4abB+Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(\frac{a(a-\cdots}{\cdots}\right.}{\cdots}\right.$$

↓ 221

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{\left(8a^2A-4abB+Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(\frac{a\sqrt{a-\cdots}}{\cdots}\right.}{2a\cdots}\right.$$

↓ 4117

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{\frac{\left(8a^2A-4abB+Ab^2\right)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+8\left(\frac{a\sqrt{a-ib}(B+iA)\,a\cdots}{\cdots}\right.}{2a}\right.$$

↓ 73

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$
$$\frac{1}{4}\left(-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{\frac{2(8a^2A-4abB+Ab^2)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+8\left(\frac{a\sqrt{a-ib}\right.}{2a}\right.$$

$\downarrow$ 221

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$
$$\frac{1}{4}\left(-\frac{(4aB+Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{-\frac{2(8a^2A-4abB+Ab^2)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+8\left(\frac{a\sqrt{a-ib}(B+iA\right.}{2a}\right.$$

input
```
Int[Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(A*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/d + (-1/2*(8*((a*Sqrt[a -
 I*b]*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d - (a*Sqrt[a + I*b]*(
I*A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d) - (2*(8*a^2*A + A*b^2 - 4*
a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d))/a - ((A*b +
 4*a*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(a*d))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1144 vs. $2(185) = 370$.

Time = 0.17 (sec) , antiderivative size = 1145, normalized size of antiderivative = 5.23

method	result	size
derivativedivides	Expression too large to display	1145
default	Expression too large to display	1145

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/4/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b*ln(b*tan(d*x+
c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))
*B*(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/4/d/b*ln(b*tan(d*x+c)+a
+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2
*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2))*A*(a^2+b^2)^(1/2)+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*
(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))*A*a-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+
c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B+
1/4/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)
-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/4/d/b*ln((a+b*tan(d*
x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*
B*(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b*ln((a+b*tan(d*x+c)
)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2
*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))*A*(a^2+b^2)^(1/2)-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*
(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1335 vs. $2(179) = 358$.

Time = 2.93 (sec) , antiderivative size = 2688, normalized size of antiderivative = 12.27

$$\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
[1/8*(4*a^2*d*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3
)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A
^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (B*d^3*sqrt(-(
4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4)
+ (2*A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a
^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^
2)*a)/d^2))*tan(d*x + c)^2 - 4*a^2*d*sqrt(-(2*A*B*b + d^2*sqrt(-(4*A^2*B^2
*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 -
B^2)*a)/d^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c
) + a) - (B*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^
2*B^2 + B^4)*b^2)/d^4) + (2*A^2*B*a + (A^3 - A*B^2)*b)*d)*sqrt(-(2*A*B*b +
 d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4
)*b^2)/d^4) - (A^2 - B^2)*a)/d^2))*tan(d*x + c)^2 - 4*a^2*d*sqrt(-(2*A*B*b
 - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B
^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4
)*b)*sqrt(b*tan(d*x + c) + a) + (B*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A
*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (2*A^2*B*a + (A^3 - A*B^2)
*b)*d)*sqrt(-(2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b +
 (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2))*tan(d*x + c)^2 +
 4*a^2*d*sqrt(-(2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*...
```

Sympy [F]

$$\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan(c+dx))\sqrt{a+b\tan(c+dx)}\cot^3(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*cot(c + d*x)**3, x)
```

Maxima [F]

$$\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 14195, normalized size of antiderivative = 64.82

$$\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
(atan(((((((2*A^3*b^14*d^2 + 2*A^3*a^2*b^12*d^2 - 96*A^3*a^4*b^10*d^2 - 96
*A^3*a^6*b^8*d^2 - 160*B^3*a^3*b^11*d^2 - 160*B^3*a^5*b^9*d^2 + 48*A^2*B*a
*b^13*d^2 - 192*A*B^2*a^2*b^12*d^2 + 96*A*B^2*a^4*b^10*d^2 + 288*A*B^2*a^6
*b^8*d^2 + 528*A^2*B*a^3*b^11*d^2 + 480*A^2*B*a^5*b^9*d^2)/(8*a^2*d^5) + (
((((64*A*a*b^12*d^4 + 448*A*a^3*b^10*d^4 + 384*A*a^5*b^8*d^4 - 256*B*a^2*b
^11*d^4 - 256*B*a^4*b^9*d^4)/(8*a^2*d^5) - ((512*a^2*b^10*d^4 + 768*a^4*b^
8*d^4)*(a + b*tan(c + d*x))^(1/2)*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b
^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^(1/2))/(64*a^5*d^5))*(
64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b
- 8*A*B*a^4*b^3)^(1/2))/(8*a^3*d) - ((a + b*tan(c + d*x))^(1/2)*(128*B^2*a
^3*b^10*d^2 - 576*A^2*a^5*b^8*d^2 - 256*A^2*a^3*b^10*d^2 + 320*B^2*a^5*b^8
*d^2 - 4*A^2*a*b^12*d^2 + 544*A*B*a^2*b^11*d^2 + 1024*A*B*a^4*b^9*d^2))/(8
*a^2*d^4))*(64*A^2*a^7 + A^2*a^3*b^4 + 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 6
4*A*B*a^6*b - 8*A*B*a^4*b^3)^(1/2))/(8*a^3*d))*(64*A^2*a^7 + A^2*a^3*b^4 +
 16*A^2*a^5*b^2 + 16*B^2*a^5*b^2 - 64*A*B*a^6*b - 8*A*B*a^4*b^3)^(1/2))/(8
*a^3*d) - ((a + b*tan(c + d*x))^(1/2)*(A^2*B^2*b^14 - A^4*b^14 + 17*A^4*a^
2*b^12 + 16*A^4*a^4*b^10 + 96*A^4*a^6*b^8 + 48*B^4*a^2*b^12 + 48*B^4*a^4*b
^10 + 32*B^4*a^6*b^8 + 95*A^2*B^2*a^2*b^12 + 448*A^2*B^2*a^4*b^10 - 8*A*B^
3*a*b^13 + 4*A^3*B*a*b^13 - 120*A*B^3*a^3*b^11 + 64*A*B^3*a^5*b^9 - 8*A^3*
B*a^3*b^11 - 320*A^3*B*a^5*b^9))/(8*a^2*d^4))*(64*A^2*a^7 + A^2*a^3*b^4...
```

Reduce [F]

$$\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^3\sqrt{a+\tan(dx+c)\,b}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

3.324 $\int \cot^4(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 279

$$
\begin{aligned}
\int \cot^4(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx
&= \frac{(8a^2Ab-Ab^3+16a^3B+2ab^2B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8a^{5/2}d} \\
&\quad - \frac{\sqrt{a-ib}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} \\
&\quad + \frac{\sqrt{a+ib}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} \\
&\quad + \frac{(8a^2A+Ab^2-2abB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{8a^2d} \\
&\quad - \frac{(Ab+6aB)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{12ad} - \frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}
\end{aligned}
$$

output
```
1/8*(8*A*a^2*b-A*b^3+16*B*a^3+2*B*a*b^2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^
(1/2))/a^(5/2)/d-(a-I*b)^(1/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I
*b)^(1/2))/d+(a+I*b)^(1/2)*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^
(1/2))/d+1/8*(8*A*a^2+A*b^2-2*B*a*b)*cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)/a^2
/d-1/12*(A*b+6*B*a)*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)/a/d-1/3*A*cot(d*x+
c)^3*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 566 vs. $2(279) = 558$.

Time = 6.30 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.03

$$\int \cot^4(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{2b^4\left(\frac{5A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{16a^{5/2}b}+\frac{(Ab+aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}b^4}-\frac{3(Ab+aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8a^{5/2}b^2}-\frac{(aA-bB}{}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^4*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b^4*((5*A*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(16*a^(5/2)*b) + (
(A*b + a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*b^4) - (3*
(A*b + a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(8*a^(5/2)*b^2) - (
(a*A - b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(2*a^(3/2)*b^3) + (
(a*A*b - A*b*Sqrt[-b^2] - b^2*B - a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Tan[c
 + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(2*b^4*Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) -
((a*A*b + A*b*Sqrt[-b^2] - b^2*B + a*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Tan[
c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(2*(-b^2)^(5/2)*Sqrt[a + Sqrt[-b^2]]) - (
5*A*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(16*a^2*b^2) + (3*(A*b + a*B)*C
ot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(8*a^2*b^3) + ((a*A - b*B)*Cot[c + d
*x]*Sqrt[a + b*Tan[c + d*x]])/(2*a*b^4) + (5*A*Cot[c + d*x]^2*Sqrt[a + b*T
an[c + d*x]])/(24*a*b^3) - ((A*b + a*B)*Cot[c + d*x]^2*Sqrt[a + b*Tan[c +
d*x]])/(4*a*b^4) - (A*Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]])/(6*b^4)))/d
```

Rubi [A] (warning: unable to verify)

Time = 2.45 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.03, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4091, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4091}$$

$$-\frac{1}{3}\int -\frac{\cot^3(c+dx)\left(-5Ab\tan^2(c+dx)-6(aA-bB)\tan(c+dx)+Ab+6aB\right)}{2\sqrt{a+b\tan(c+dx)}}dx-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{\cot^3(c+dx)\left(-5Ab\tan^2(c+dx)-6(aA-bB)\tan(c+dx)+Ab+6aB\right)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{-5Ab\tan(c+dx)^2-6(aA-bB)\tan(c+dx)+Ab+6aB}{\tan(c+dx)^3\sqrt{a+b\tan(c+dx)}}dx-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{4132}$$

$$\frac{1}{6}\left(-\frac{\int \frac{3\cot^2(c+dx)(8Aa^2-2bBa+8(Ab+aB)\tan(c+dx)a+Ab^2+b(Ab+6aB)\tan^2(c+dx))}{2\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b}}{2ad}\right.$$
$$\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{6}\left(-\frac{3\int \frac{\cot^2(c+dx)(8Aa^2-2bBa+8(Ab+aB)\tan(c+dx)a+Ab^2+b(Ab+6aB)\tan^2(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx}{4a}-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b}}{2ad}\right.$$

$$\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\big\downarrow \text{3042}$$

$$\frac{1}{6}\left(-\frac{3\int \frac{8Aa^2-2bBa+8(Ab+aB)\tan(c+dx)a+Ab^2+b(Ab+6aB)\tan(c+dx)^2}{\tan(c+dx)^2\sqrt{a+b\tan(c+dx)}}dx}{4a}-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx}}{2ad}\right.$$

$$\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\big\downarrow \text{4132}$$

$$\frac{1}{6}\left(-\frac{3\left(-\frac{\int -\frac{\cot(c+dx)\left(16Ba^3+8Aba^2-16(aA-bB)\tan(c+dx)a^2+2b^2Ba-Ab^3-b\left(8Aa^2-2bBa+Ab^2\right)\tan^2(c+dx)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{(8a^2A-2abB+Ab^2)\cot(c}{ad}\right.}{4a}\right.$$

$$\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\big\downarrow \text{27}$$

$$\frac{1}{6}\left(-\frac{3\left(\frac{\int \frac{\cot(c+dx)\left(16Ba^3+8Aba^2-16(aA-bB)\tan(c+dx)a^2+2b^2Ba-Ab^3-b\left(8Aa^2-2bBa+Ab^2\right)\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(8a^2A-2abB+Ab^2)\cot(c+dx}{ad}\right.}{4a}\right.$$

$$\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\big\downarrow \text{3042}$$

$$\frac{1}{6}\left(-\frac{3\left(\frac{\int \frac{16Ba^3+8Aba^2-16(aA-bB)\tan(c+dx)a^2+2b^2Ba-Ab^3-b\left(8Aa^2-2bBa+Ab^2\right)\tan(c+dx)^2}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{\left(8a^2A-2abB+Ab^2\right)\cot(c+dx)\sqrt{a+b\tan}}{ad}\right.}{4a}\right.$$

$$\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$\downarrow$ 4136

$$\frac{1}{6}\left(-\frac{3\left(\frac{\int -\frac{16\left((aA-bB)a^2+(Ab+aB)\tan(c+dx)a^2\right)}{\sqrt{a+b\tan(c+dx)}}dx+\left(16a^3B+8a^2Ab+2ab^2B-Ab^3\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{\left(8a^2A-2abB+Ab^2\right)\,c}{}\right.}{4a}\right.$$

$$\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\left(-\frac{3\left(\frac{\left(16a^3B+8a^2Ab+2ab^2B-Ab^3\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-16\int \frac{(aA-bB)a^2+(Ab+aB)\tan(c+dx)a^2}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{\left(8a^2A-2abB+Ab^2\right)\cot(}{ad}\right.}{4a}\right.$$

$$\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(-\frac{3\left(\frac{\left(16a^3B+8a^2Ab+2ab^2B-Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-16\int \frac{(aA-bB)a^2+(Ab+aB)\tan(c+dx)a^2}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{\left(8a^2A-2abB+Ab^2\right)\cot(}{ad}\right.}{4a}\right.$$

$$\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$\downarrow$ 4022

$$-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$
$$\frac{1}{6}\left(-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{3\left(-\frac{(8a^2A-2abB+Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(16a^3B+8a^2Ab}{}\right.}{}\right.$$

$\downarrow$ 3042

$$-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$
$$\frac{1}{6}\left(-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{3\left(-\frac{(8a^2A-2abB+Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(16a^3B+8a^2Ab}{}\right.}{}\right.$$

$\downarrow$ 4020

$$-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$
$$\frac{1}{6}\left(-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{3\left(-\frac{(8a^2A-2abB+Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(16a^3B+8a^2Ab}{}\right.}{}\right.$$

$\downarrow$ 25

$$-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$
$$\frac{1}{6}\left(-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{3\left(-\frac{(8a^2A-2abB+Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(16a^3B+8a^2Ab}{}\right.}{}\right.$$

$$\Big\downarrow \; 73$$

$$-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{3\left(-\frac{(8a^2A-2abB+Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(16a^3B+8a^2Ab}{\quad}\right.}{\quad}\right.$$

$$\Big\downarrow \; 221$$

$$-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{3\left(-\frac{(8a^2A-2abB+Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(16a^3B+8a^2Ab}{\quad}\right.}{\quad}\right.$$

$$\Big\downarrow \; 4117$$

$$-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{3\left(-\frac{(8a^2A-2abB+Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{\frac{(16a^3B+8a^2Ab+}{\quad}}{\quad}\right.}{\quad}\right.$$

$$\Big\downarrow \; 73$$

$$-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{3\left(-\frac{(8a^2A-2abB+Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{2\left(16a^3B+8a^2Ab\right.}{}\right.}{}\right.$$

$\Big\downarrow$ 221

$$-\frac{A\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(-\frac{(6aB+Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{3\left(-\frac{(8a^2A-2abB+Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{-\frac{2\left(16a^3B+8a^2A\right.}{}}{}\right.}{}\right.$$

input
```
Int[Cot[c + d*x]^4*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
-1/3*(A*Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]])/d + (-1/2*((A*b + 6*a*B)*
Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/(a*d) - (3*((-16*((a^2*Sqrt[a - I
*b]*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + (a^2*Sqrt[a + I*b]*(
A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d) - (2*(8*a^2*A*b - A*b^3 +
16*a^3*B + 2*a*b^2*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*
d))/(2*a) - ((8*a^2*A + A*b^2 - 2*a*b*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d
*x]])/(a*d)))/(4*a))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. $2(241) = 482$.

Time = 0.16 (sec) , antiderivative size = 1295, normalized size of antiderivative = 4.64

method	result	size
derivativedivides	Expression too large to display	1295
default	Expression too large to display	1295

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+1/4
/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/4/d*ln(b*tan(d*x+c)
+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2
*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2))*B*(a^2+b^2)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((
2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2))*A+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c)
)^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a+
1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+
c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/4/
d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a
-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/d*ln((a+b*tan(d*x+
c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2
*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))*B*(a^2+b^2)^(1/2)-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((
2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(235) = 470$.

Time = 8.87 (sec) , antiderivative size = 2766, normalized size of antiderivative = 9.91

$$\int \cot^4(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
[-1/48*(24*a^3*d*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B
^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*
(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + (A*d^3*sqrt(
-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4
) - (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^2*
a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B
^2)*a)/d^2))*tan(d*x + c)^3 - 24*a^3*d*sqrt((2*A*B*b + d^2*sqrt(-(4*A^2*B^
2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 -
 B^2)*a)/d^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4)*b)*sqrt(b*tan(d*x +
c) + a) - (A*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A
^2*B^2 + B^4)*b^2)/d^4) - (2*A*B^2*a + (A^2*B - B^3)*b)*d)*sqrt((2*A*B*b +
 d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4
)*b^2)/d^4) - (A^2 - B^2)*a)/d^2))*tan(d*x + c)^3 - 24*a^3*d*sqrt((2*A*B*b
 - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B
^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2)*log(-(2*(A^3*B + A*B^3)*a + (A^4 - B^4
)*b)*sqrt(b*tan(d*x + c) + a) + (A*d^3*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A
*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) + (2*A*B^2*a + (A^2*B - B^3)
*b)*d)*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a*b +
(A^4 - 2*A^2*B^2 + B^4)*b^2)/d^4) - (A^2 - B^2)*a)/d^2))*tan(d*x + c)^3 +
24*a^3*d*sqrt((2*A*B*b - d^2*sqrt(-(4*A^2*B^2*a^2 + 4*(A^3*B - A*B^3)*a...
```

Sympy [F]

$$\int \cot^4(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}\cot^4{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*cot(c + d*x)**4, x)
```

Maxima [F]

$$\int \cot^4(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (B\tan{(dx+c)}+A)\sqrt{b\tan{(dx+c)}+a}\cot{(dx+c)}^4\,dx$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^4(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 16796, normalized size of antiderivative = 60.20

$$\int \cot^4(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
atan((((((((224*A*a^4*b^11*d^4 - 32*A*a^2*b^13*d^4 + 256*A*a^6*b^9*d^4 + 64
*B*a^3*b^12*d^4 + 448*B*a^5*b^10*d^4 + 384*B*a^7*b^8*d^4)/(a^4*d^5) - ((20
48*a^4*b^10*d^4 + 3072*a^6*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*((B^2*a)/(4
*d^2) - (A^2*a)/(4*d^2) - (2*A^2*B^2*b^2*d^4 - B^4*b^2*d^4 - 4*A^2*B^2*a^2
*d^4 - A^4*b^2*d^4 + 4*A*B^3*a*b*d^4 - 4*A^3*B*a*b*d^4)^(1/2)/(4*d^4) + (A
*B*b)/(2*d^2))^(1/2))/(4*a^4*d^4))*((B^2*a)/(4*d^2) - (A^2*a)/(4*d^2) - (2
*A^2*B^2*b^2*d^4 - B^4*b^2*d^4 - 4*A^2*B^2*a^2*d^4 - A^4*b^2*d^4 + 4*A*B^3
*a*b*d^4 - 4*A^3*B*a*b*d^4)^(1/2)/(4*d^4) + (A*B*b)/(2*d^2))^(1/2) + ((a +
 b*tan(c + d*x))^(1/2)*(16*B^2*a^3*b^12*d^2 - 512*A^2*a^5*b^10*d^2 - 1280*
A^2*a^7*b^8*d^2 - 64*A^2*a^3*b^12*d^2 + 1024*B^2*a^5*b^10*d^2 + 2304*B^2*a
^7*b^8*d^2 + 4*A^2*a*b^14*d^2 - 16*A*B*a^2*b^13*d^2 + 2048*A*B*a^4*b^11*d^
2 + 4096*A*B*a^6*b^9*d^2))/(4*a^4*d^4))*((B^2*a)/(4*d^2) - (A^2*a)/(4*d^2)
 - (2*A^2*B^2*b^2*d^4 - B^4*b^2*d^4 - 4*A^2*B^2*a^2*d^4 - A^4*b^2*d^4 + 4*
A*B^3*a*b*d^4 - 4*A^3*B*a*b*d^4)^(1/2)/(4*d^4) + (A*B*b)/(2*d^2))^(1/2) -
(16*A^3*a^3*b^13*d^2 - 144*A^3*a^5*b^11*d^2 - 160*A^3*a^7*b^9*d^2 - 2*B^3*
a^2*b^14*d^2 - 2*B^3*a^4*b^12*d^2 + 96*B^3*a^6*b^10*d^2 + 96*B^3*a^8*b^8*d
^2 - (A^2*B*b^16*d^2)/2 + 2*A*B^2*a*b^15*d^2 + 50*A*B^2*a^3*b^13*d^2 + 528
*A*B^2*a^5*b^11*d^2 + 480*A*B^2*a^7*b^9*d^2 - (49*A^2*B*a^2*b^14*d^2)/2 +
168*A^2*B*a^4*b^12*d^2 - 96*A^2*B*a^6*b^10*d^2 - 288*A^2*B*a^8*b^8*d^2)/(a
^4*d^5))*((B^2*a)/(4*d^2) - (A^2*a)/(4*d^2) - (2*A^2*B^2*b^2*d^4 - B^4*...
```

Reduce [F]

$$\int \cot^4(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot\left(dx+c\right)^4\sqrt{a+\tan\left(dx+c\right)b}\,\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

3.325 $\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 214

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{(a-ib)^{3/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2(Ab+aB)\sqrt{a+b\tan(c+dx)}}{d} - \frac{2B(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{35b^2d} + \frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}$$

output
```
(a-I*b)^(3/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I
*b)^(3/2)*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2*(A*b+B
*a)*(a+b*tan(d*x+c))^(1/2)/d-2/3*B*(a+b*tan(d*x+c))^(3/2)/d+2/35*(7*A*b-2*
B*a)*(a+b*tan(d*x+c))^(5/2)/b^2/d+2/7*B*tan(d*x+c)*(a+b*tan(d*x+c))^(5/2)/
b/d
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.18

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{\frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{b}+10B\tan(c+dx)(a+b\tan(c+dx))^{5/2}+\frac{35}{3}b(A-iB)\Big(3\cdots}{}$$

input
```
Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
((2*(7*A*b - 2*a*B)*(a + b*Tan[c + d*x])^(5/2))/b + 10*B*Tan[c + d*x]*(a +
 b*Tan[c + d*x])^(5/2) + (35*b*(A - I*B)*(3*Sqrt[a - I*b]*(I*a + b)*ArcTan
h[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - I*Sqrt[a + b*Tan[c + d*x]]*(4*
a - (3*I)*b + b*Tan[c + d*x])))/3 + (35*b*(A + I*B)*(3*Sqrt[a + I*b]*((-I)
*a + b)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + I*Sqrt[a + b*Tan
[c + d*x]]*(4*a + (3*I)*b + b*Tan[c + d*x])))/3)/(35*b*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 4090, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2\int -\frac{1}{2}(a+b\tan(c+dx))^{3/2}\left(-\left((7Ab-2aB)\tan^2(c+dx)\right)+7bB\tan(c+dx)+2aB\right)dx}{7b}+\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}$$

$\Big\downarrow$ 27

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\int (a+b\tan(c+dx))^{3/2}\left(-\left((7Ab-2aB)\tan^2(c+dx)\right)+7bB\tan(c+dx)+2aB\right)dx}{7b}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\int (a+b\tan(c+dx))^{3/2}\left(-\left((7Ab-2aB)\tan(c+dx)^2\right)+7bB\tan(c+dx)+2aB\right)dx}{7b}$$

$\Big\downarrow$ 4113

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\int (a+b\tan(c+dx))^{3/2}(7Ab+7B\tan(c+dx)b)dx-\frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{5bd}}{7b}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\int (a+b\tan(c+dx))^{3/2}(7Ab+7B\tan(c+dx)b)dx-\frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{5bd}}{7b}$$

$\Big\downarrow$ 4011

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\int \sqrt{a+b\tan(c+dx)}(7b(aA-bB)+7b(Ab+aB)\tan(c+dx))dx-\frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{5bd}+\frac{14bB(a+b\tan(c-}{3d}}{7b}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\int \sqrt{a+b\tan(c+dx)}(7b(aA-bB)+7b(Ab+aB)\tan(c+dx))dx-\frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{5bd}+\frac{14bB(a+b\tan(c-}{3d}}{7b}$$

$\Big\downarrow$ 4011

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\int \frac{7b(Aa^2-2bBa-Ab^2)+7b(Ba^2+2Aba-b^2B)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{5bd}+\frac{14b(aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{14bB}{}}{7b}$$

$\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\int \frac{7b(Aa^2-2bBa-Ab^2)+7b(Ba^2+2Aba-b^2B)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{5bd}+\frac{14b(aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{14bB}{}}{7b}$$

$\downarrow$ 4022

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\frac{7}{2}b(a+ib)^2(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{7}{2}b(a-ib)^2(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{5bd}}{7b}$$

$\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\frac{7}{2}b(a+ib)^2(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{7}{2}b(a-ib)^2(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{5bd}}{7b}$$

$\downarrow$ 4020

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\frac{7ib(a-ib)^2(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{7ib(a+ib)^2(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}}{7b}$$

$\downarrow$ 25

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{-\frac{7ib(a-ib)^2(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{7ib(a+ib)^2(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-}{7b}$$

$\downarrow$ 73

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd}-\frac{\frac{7(a+ib)^2(A+iB)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}+\frac{7(a-ib)^2(A-iB)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}-\frac{2(7Ab-2aB)(a+b}{5bd}}{7b}$$

$$\downarrow 221$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{5/2}}{7bd} - \frac{\frac{7b(a-ib)^{3/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d} + \frac{7b(a+ib)^{3/2}(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d} - \frac{2(7Ab-2aB)(a+b\tan(c+dx))^{5/2}}{5bd} + \frac{14b(aB+Ab)\sqrt{a+}}{d}}{7b}$$

input
```
Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*Tan[c + d*x]*(a + b*Tan[c + d*x])^(5/2))/(7*b*d) - ((7*(a - I*b)^(3/2
)*b*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + (7*(a + I*b)^(3/2)*b
*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d + (14*b*(A*b + a*B)*Sqrt[
a + b*Tan[c + d*x]])/d + (14*b*B*(a + b*Tan[c + d*x])^(3/2))/(3*d) - (2*(7
*A*b - 2*a*B)*(a + b*Tan[c + d*x])^(5/2))/(5*b*d))/(7*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. $2(182) = 364$.

Time = 0.22 (sec) , antiderivative size = 1697, normalized size of antiderivative = 7.93

method	result	size
parts	Expression too large to display	1697
derivativedivides	Expression too large to display	1729
default	Expression too large to display	1729

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
A*(2/5/b/d*(a+b*tan(d*x+c))^(5/2)-2*b*(a+b*tan(d*x+c))^(1/2)/d-1/4/b/d*ln(
b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b
^2)^(1/2))*(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/b/d*ln(b*ta
n(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^
(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4*b/d*ln(b*tan(d*x+c)+a-(a+b*ta
n(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)+b/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*
x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*
(a^2+b^2)^(1/2)-2*b/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x
+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a
+1/4/b/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)+(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/4
/b/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+1/4*b/d*ln(b*tan(d*x+
c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+b/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2
*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2))*(a^2+b^2)^(1/2)-2*b/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*
(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))*a)+B*(2/7/d/b^2*(a+b*tan(d*x+c))^(7/2)-2/5/d/b^2*a*(a+b*tan(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3148 vs. $2(176) = 352$.

Time = 0.32 (sec) , antiderivative size = 3148, normalized size of antiderivative = 14.71

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan(c+dx))\,(a+b\tan(c+dx))^{\frac{3}{2}}\tan^2(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**2,
 x)
```

Maxima [F]

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}(A + B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^2\,dx$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^2,
x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,24,9]%%%}+%%%{10,[0,22,9]%%%}+%%%{45,[0,20,9]%%%}
+%%%{120,
```

Mupad [B] (verification not implemented)

Time = 72.16 (sec) , antiderivative size = 2993, normalized size of antiderivative = 13.99

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
log((16*A^3*a*b^3*(a^2 + b^2)^2)/d^3 - (((16*b^2*(((-A^4*b^2*d^4*(3*a^2 -
b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b
 + a*d*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*
d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2)))/d + (16*A^2*b^2*(a + b*tan(c
+ d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2)*(((-A^4*b^2*d^4*(3*a^2 - b^2)^
2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2))/2)*((6*A^4*a^2*b^4*d
^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2)/(4*d^4) - (A^2*a^3)/(4*d^2) +
(3*A^2*a*b^2)/(4*d^2))^(1/2) - log((16*A^3*a*b^3*(a^2 + b^2)^2)/d^3 - (((1
6*b^2*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*
d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b - a*d*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(
1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2
)))/d - (16*A^2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^
2)*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2
)/d^4)^(1/2))/2)*(-((6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^
(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/(4*d^4))^(1/2) - log((16*A^3*a*b^3*
(a^2 + b^2)^2)/d^3 - (((16*b^2*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^
2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b - a*d*(((-A^4*b^2
*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a
 + b*tan(c + d*x))^(1/2)))/d - (16*A^2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^4
 + b^4 - 6*a^2*b^2))/d^2)*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*...
```

Reduce [F]

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^4\,dx\right)b^2 + 2\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)ab + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)a^2$$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4,x)*b**2 + 2*int(sqrt(tan(c +
d*x)*b + a)*tan(c + d*x)**3,x)*a*b + int(sqrt(tan(c + d*x)*b + a)*tan(c +
d*x)**2,x)*a**2
```

3.326 $\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 175

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{(a-ib)^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2(aA-bB)\sqrt{a+b\tan(c+dx)}}{d} + \frac{2A(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

output
```
-(a-I*b)^(3/2)*(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+
I*b)^(3/2)*(A+I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*(A*a-
B*b)*(a+b*tan(d*x+c))^(1/2)/d+2/3*A*(a+b*tan(d*x+c))^(3/2)/d+2/5*B*(a+b*ta
n(d*x+c))^(5/2)/b/d
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{\frac{6B(a+b\tan(c+dx))^{5/2}}{b} + 5(A-iB)\left(-3(a-ib)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right) + \sqrt{a+b\tan(c+dx)}\right.}{}$$

input
```
Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((6*B*(a + b*Tan[c + d*x])^(5/2))/b + 5*(A - I*B)*(-3*(a - I*b)^(3/2)*ArcT
anh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + Sqrt[a + b*Tan[c + d*x]]*(4*
a - (3*I)*b + b*Tan[c + d*x])) + 5*(A + I*B)*(-3*(a + I*b)^(3/2)*ArcTanh[S
qrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + Sqrt[a + b*Tan[c + d*x]]*(4*a + (
3*I)*b + b*Tan[c + d*x])))/(15*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4075, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^{3/2}dx + \frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^{3/2}dx+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{4011}$$

$$\int \sqrt{a+b\tan(c+dx)}(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{2A(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan(c+dx)}(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{2A(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{-Ba^2-2Aba+b^2B+(Aa^2-2bBa-Ab^2)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(aA-bB)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2A(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-Ba^2-2Aba+b^2B+(Aa^2-2bBa-Ab^2)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(aA-bB)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2A(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(a+ib)^2(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(a-ib)^2(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(aA-bB)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2A(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(a+ib)^2(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(a-ib)^2(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(aA-bB)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2A(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{4020}$$

$$-\frac{i(a-ib)^2(B+iA)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a+ib)^2(-B+iA)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2(aA-bB)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2A(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{i(a-ib)^2(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(a+ib)^2(-B+iA)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2(aA-bB)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2A(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a+ib)^2(-B+iA)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}-\frac{(a-ib)^2(B+iA)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{2(aA-bB)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2A(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(a-ib)^{3/2}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3/2}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2(aA-bB)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2A(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5bd}$$

input

```
Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output

```
-(((a - I*b)^(3/2)*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d) + ((a
 + I*b)^(3/2)*(I*A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d + (2*(a*A - b
*B)*Sqrt[a + b*Tan[c + d*x]])/d + (2*A*(a + b*Tan[c + d*x])^(3/2))/(3*d) +
 (2*B*(a + b*Tan[c + d*x])^(5/2))/(5*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1667 vs. $2(147) = 294$.

Time = 0.15 (sec) , antiderivative size = 1668, normalized size of antiderivative = 9.53

method	result	size
parts	Expression too large to display	1668
derivativedivides	Expression too large to display	1686
default	Expression too large to display	1686

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
A*(2/3/d*(a+b*tan(d*x+c))^(3/2)+2/d*(a+b*tan(d*x+c))^(1/2)*a-1/4/d*ln((a+b
*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^
(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+1/2/d*ln((a+b*tan(d*x
+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((
2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2))*(a^2+b^2)+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
))*(a^2+b^2)^(1/2)*a-2/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2
)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
)*a^2+1/4/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/
2/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)*a+2/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*
arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2))*a^2)+B*(2/5/b/d*(a+b*tan(d*x+c))^(5/2)-2*b*(a+b*ta...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3102 vs. $2(141) = 282$.

Time = 0.37 (sec) , antiderivative size = 3102, normalized size of antiderivative = 17.73

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^{\frac{3}{2}}\tan{(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*tan(c + d*x), x)
```

Maxima [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A + B\tan(c+dx))\,dx = \int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^{\frac{3}{2}}\tan{(dx+c)}\;dx$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*tan(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,19,7]%%%}+%%%{8,[0,17,7]%%%}+%%%{28,[0,15,7]%%%}+
%%%{56,[0
```

Mupad [B] (verification not implemented)

Time = 28.91 (sec) , antiderivative size = 2868, normalized size of antiderivative = 16.39

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
log((16*B^3*a*b^3*(a^2 + b^2)^2)/d^3 - (((16*b^2*(((-B^4*b^2*d^4*(3*a^2 -
b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(B*b^3 + B*a^2*b
 + a*d*(((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*
d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2)))/d + (16*B^2*b^2*(a + b*tan(c
+ d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2)*(((-B^4*b^2*d^4*(3*a^2 - b^2)^
2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^(1/2))/2)*((6*B^4*a^2*b^4*d
^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^(1/2)/(4*d^4) - (B^2*a^3)/(4*d^2) +
(3*B^2*a*b^2)/(4*d^2))^(1/2) - log((16*B^3*a*b^3*(a^2 + b^2)^2)/d^3 - (((1
6*b^2*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*
d^2)/d^4)^(1/2)*(B*b^3 + B*a^2*b - a*d*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(
1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2
)))/d - (16*B^2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^
2)*(-((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2
)/d^4)^(1/2))/2)*(-((6*B^4*a^2*b^4*d^4 - B^4*b^6*d^4 - 9*B^4*a^4*b^2*d^4)^
(1/2) + B^2*a^3*d^2 - 3*B^2*a*b^2*d^2)/(4*d^4))^(1/2) - log((16*B^3*a*b^3*
(a^2 + b^2)^2)/d^3 - (((16*b^2*(((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^
2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(B*b^3 + B*a^2*b - a*d*(((-B^4*b^2
*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*a^3*d^2 + 3*B^2*a*b^2*d^2)/d^4)^(1/2)*(a
 + b*tan(c + d*x))^(1/2)))/d - (16*B^2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^4
 + b^4 - 6*a^2*b^2))/d^2)*(((-B^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - B^2*...
```

Reduce [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)b^2 + 2\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)ab + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)a^2$$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3,x)*b**2 + 2*int(sqrt(tan(c +
d*x)*b + a)*tan(c + d*x)**2,x)*a*b + int(sqrt(tan(c + d*x)*b + a)*tan(c +
d*x),x)*a**2
```

3.327 $\int (a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 150

$$\int (a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{(a-ib)^{3/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{3/2}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2(Ab+aB)\sqrt{a+b\tan(c+dx)}}{d} + \frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

output
```
-(a-I*b)^(3/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+
I*b)^(3/2)*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*(A*b+
B*a)*(a+b*tan(d*x+c))^(1/2)/d+2/3*B*(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int (a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{-3i(a-ib)^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+3i(a+ib)^{3/2}(A+iB)\text{arctanh}\left(\frac{\sqrt{a\cdots}}{}\right.}{3d}$$

input
```
Integrate[(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((-3*I)*(a - I*b)^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a
- I*b]] + (3*I)*(a + I*b)^(3/2)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]
/Sqrt[a + I*b]] + 2*Sqrt[a + b*Tan[c + d*x]]*(3*A*b + 4*a*B + b*B*Tan[c +
d*x]))/(3*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4011$$

$$\int \sqrt{a+b\tan(c+dx)}(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a+b\tan(c+dx)}(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(a-ib)^2(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^2(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(a-ib)^2(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^2(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(a-ib)^2(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a+ib)^2(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2(aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i(a-ib)^2(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(a+ib)^2(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2(aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 73$$

$$\frac{(a-ib)^2(A-iB)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a+ib)^2(A+iB)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{2(aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 221$$

$$\frac{(a-ib)^{3/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3/2}(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2(aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2B(a+b\tan(c+dx))^{3/2}}{3d}$$

input
```
Int[(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((a - I*b)^(3/2)*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + ((a + I
*b)^(3/2)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d + (2*(A*b + a*B)
*Sqrt[a + b*Tan[c + d*x]])/d + (2*B*(a + b*Tan[c + d*x])^(3/2))/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(126) = 252$.

Time = 0.14 (sec) , antiderivative size = 1657, normalized size of antiderivative = 11.05

method	result	size
parts	Expression too large to display	1657
derivativedivides	Expression too large to display	1665
default	Expression too large to display	1665

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
A*(2*b*(a+b*tan(d*x+c))^(1/2)/d+1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*(a^2+b^2)^(1/2)*a-1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*a^2+1/4/d*b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d
*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)-2/d*b/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*
x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-1/4/b/d*ln(b*tan(d*x+c)+a+(a
+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(a^2+b
^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/b/d*ln(b*tan(d*x+c)+a+(a+b*t
an(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^2-1/4*b/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)-b/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)+2
*b/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^
2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a)+B*(2/3/d*(a+b*t
an(d*x+c))^(3/2)+2/d*(a+b*tan(d*x+c))^(1/2)*a-1/4/d*ln((a+b*tan(d*x+c))...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3058 vs. $2(120) = 240$.

Time = 0.29 (sec) , antiderivative size = 3058, normalized size of antiderivative = 20.39

$$\int (a + b\tan(c + dx))^{3/2}(A + B\tan(c + dx))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int (a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [B] (verification not implemented)

Time = 16.25 (sec) , antiderivative size = 2823, normalized size of antiderivative = 18.82

$$\int (a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
log((((16*b^2*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2
*a*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b - a*d*(((-A^4*b^2*d^4*(3*a^2 - b^2
)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x)
)^(1/2)))/d - (16*A^2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^
2))/d^2)*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^
2*d^2)/d^4)^(1/2))/2 - (16*A^3*a*b^3*(a^2 + b^2)^2)/d^3)*((6*A^4*a^2*b^4*d
^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2)/(4*d^4) - (A^2*a^3)/(4*d^2) +
(3*A^2*a*b^2)/(4*d^2))^(1/2) - log((((16*b^2*(((-A^4*b^2*d^4*(3*a^2 - b^2)
^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(A*b^3 + A*a^2*b + a
*d*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)
/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2)))/d + (16*A^2*b^2*(a + b*tan(c + d*
x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2)*(((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(
1/2) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/d^4)^(1/2))/2 - (16*A^3*a*b^3*(a^2 +
 b^2)^2)/d^3)*(((6*A^4*a^2*b^4*d^4 - A^4*b^6*d^4 - 9*A^4*a^4*b^2*d^4)^(1/2
) - A^2*a^3*d^2 + 3*A^2*a*b^2*d^2)/(4*d^4))^(1/2) - log((((16*b^2*(-((-A^4
*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2
)*(A*b^3 + A*a^2*b + a*d*(-((-A^4*b^2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3
*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2)))/d + (16*A^
2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2)*(-((-A^4*b^
2*d^4*(3*a^2 - b^2)^2)^(1/2) + A^2*a^3*d^2 - 3*A^2*a*b^2*d^2)/d^4)^(1/2...
```

Reduce [F]

$$\int (a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{a+\tan(dx+c)\,b}dx\right)a^2 + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)b^2 + 2\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)ab$$

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a),x)*a**2 + int(sqrt(tan(c + d*x)*b + a)*tan(c
+ d*x)**2,x)*b**2 + 2*int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a*b
```

3.328 $\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 152

$$\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{2a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{(a-ib)^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3/2}(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

output
```
-2*a^(3/2)*A*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d+(a-I*b)^(3/2)*(A-I*
B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I*b)^(3/2)*(A+I*B)*a
rctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*b*B*(a+b*tan(d*x+c))^(1/2
)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

$$\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{-2a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)+(a-ib)^{3/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+(a}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a^(3/2)*A*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] + (a - I*b)^(3/2)*
(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + (a + I*b)^(3/2
)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*b*B*Sqrt[a
 + b*Tan[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 1.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4090, 27, 3042, 4136, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4090}$$

$$2\int \frac{\cot(c+dx)\left(Aa^2+b(Ab+2aB)\tan^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{2\sqrt{a+b\tan(c+dx)}}dx+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 27

$$\int \frac{\cot(c+dx)\left(Aa^2+b(Ab+2aB)\tan^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 3042

$$\int \frac{Aa^2+b(Ab+2aB)\tan(c+dx)^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 4136

$$\int \frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+a^2A\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 3042

$$\int \frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+a^2A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 4022

$$a^2A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(a+ib)^2(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)^2(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 3042

$$a^2A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(a+ib)^2(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)^2(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 4020

$$a^2A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{i(a-ib)^2(B+iA)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(a+ib)^2(-B+iA)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{25}$$

$$a^2A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{i(a-ib)^2(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a+ib)^2(-B+iA)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{73}$$

$$a^2A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{(a+ib)^2(-B+iA)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a-ib)^2(B+iA)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{221}$$

$$a^2A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{(a-ib)^{3/2}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4117}$$

$$\frac{a^2A\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{(a-ib)^{3/2}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{73}$$

$$\frac{2a^2A\int \frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a-ib)^{3/2}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{(a-ib)^{3/2}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{3/2}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2bB\sqrt{a+b\tan(c+dx)}}{d}$$

input
```
Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((a - I*b)^(3/2)*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d - ((a + I
*b)^(3/2)*(I*A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d - (2*a^(3/2)*A*A
rcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d + (2*b*B*Sqrt[a + b*Tan[c + d*
x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1653 vs. $2(126) = 252$.

Time = 0.14 (sec) , antiderivative size = 1654, normalized size of antiderivative = 10.88

method	result	size
derivativedivides	Expression too large to display	1654
default	Expression too large to display	1654

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
2*b*B*(a+b*tan(d*x+c))^(1/2)/d+1/4/d*b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)+1/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c)
)^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A-1/
d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A-1/4/d*b*ln(b*ta
n(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^
(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arc
tan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^
(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)*a-1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d
*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arct
an((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(
1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)*a+1/4/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a
^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/2/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*a-1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3072 vs. $2(120) = 240$.

Time = 2.41 (sec) , antiderivative size = 6162, normalized size of antiderivative = 40.54

$$\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^{\frac{3}{2}}\cot{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*cot(c + d*x), x)
```

Maxima [F]

$$\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}(A + B\tan(c+dx))\,dx = \int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^{\frac{3}{2}}\cot{(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,13,4]%%%}+%%%{6,[0,11,4]%%%}+%%%{15,[0,9,4]%%%}+%
%%{20,[0,
```

Mupad [B] (verification not implemented)

Time = 6.95 (sec) , antiderivative size = 20255, normalized size of antiderivative = 133.26

$$\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
(2*B*b*(a + b*tan(c + d*x))^(1/2))/d - atan((((((32*(4*B*a*b^11*d^4 + 12*
A*a^2*b^10*d^4 + 12*A*a^4*b^8*d^4 + 4*B*a^3*b^9*d^4))/d^5 - (32*(16*b^10*d
^4 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(-(((8*A^2*a^3*d^2 - 8*B^2
*a^3*d^2 + 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 - 48*A*B*a
^2*b*d^2)^2/64 - d^4*(A^4*a^6 + A^4*b^6 + B^4*a^6 + B^4*b^6 + 2*A^2*B^2*a^
6 + 2*A^2*B^2*b^6 + 3*A^4*a^2*b^4 + 3*A^4*a^4*b^2 + 3*B^4*a^2*b^4 + 3*B^4*
a^4*b^2 + 6*A^2*B^2*a^2*b^4 + 6*A^2*B^2*a^4*b^2))^(1/2) - A^2*a^3*d^2 + B^
2*a^3*d^2 - 2*A*B*b^3*d^2 + 3*A^2*a*b^2*d^2 - 3*B^2*a*b^2*d^2 + 6*A*B*a^2*
b*d^2)/(4*d^4))^(1/2))/d^4)*(-(((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 + 16*A*B*b^
3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 - 48*A*B*a^2*b*d^2)^2/64 - d^4
*(A^4*a^6 + A^4*b^6 + B^4*a^6 + B^4*b^6 + 2*A^2*B^2*a^6 + 2*A^2*B^2*b^6 +
3*A^4*a^2*b^4 + 3*A^4*a^4*b^2 + 3*B^4*a^2*b^4 + 3*B^4*a^4*b^2 + 6*A^2*B^2*
a^2*b^4 + 6*A^2*B^2*a^4*b^2))^(1/2) - A^2*a^3*d^2 + B^2*a^3*d^2 - 2*A*B*b^
3*d^2 + 3*A^2*a*b^2*d^2 - 3*B^2*a*b^2*d^2 + 6*A*B*a^2*b*d^2)/(4*d^4))^(1/2
) - (32*(a + b*tan(c + d*x))^(1/2)*(28*A^2*a^3*b^10*d^2 - 18*A^2*a^5*b^8*d
^2 - 28*B^2*a^3*b^10*d^2 + 10*B^2*a^5*b^8*d^2 - 16*A*B*b^13*d^2 + 22*A^2*a
*b^12*d^2 - 22*B^2*a*b^12*d^2 + 16*A*B*a^2*b^11*d^2 + 64*A*B*a^4*b^9*d^2))
/d^4)*(-(((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 + 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d
^2 + 24*B^2*a*b^2*d^2 - 48*A*B*a^2*b*d^2)^2/64 - d^4*(A^4*a^6 + A^4*b^6 +
B^4*a^6 + B^4*b^6 + 2*A^2*B^2*a^6 + 2*A^2*B^2*b^6 + 3*A^4*a^2*b^4 + 3*A...
```

Reduce [F]

$$\int \cot(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)\tan(dx+c)^2\,dx\right)b^2 + 2\left(\int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)\tan(dx+c)\,dx\right)ab + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)\,dx\right)a^2$$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*cot(c + d*x)*tan(c + d*x)**2,x)*b**2 + 2*int(
sqrt(tan(c + d*x)*b + a)*cot(c + d*x)*tan(c + d*x),x)*a*b + int(sqrt(tan(c
 + d*x)*b + a)*cot(c + d*x),x)*a**2
```

3.329 $\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 169

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{\sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{(a-ib)^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{3/2}(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

output
```
-a^(1/2)*(3*A*b+2*B*a)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d+(a-I*b)^(
3/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*b)^(3/2)
*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-a*A*cot(d*x+c)*(a
+b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.67

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{-\sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)+(a-ib)^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+}}{\sqrt{a-ib}}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
(-(Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]]) + (a
 - I*b)^(3/2)*(I*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] -
I*a*A*Sqrt[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + A*Sq
rt[a + I*b]*b*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + a*Sqrt[a +
 I*b]*B*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + I*Sqrt[a + I*b]*
b*B*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - a*A*Cot[c + d*x]*Sqr
t[a + b*Tan[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 1.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 4088, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4088}$$

$$\int \frac{\cot(c+dx)\left(-b(aA-2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)}{2\sqrt{a+b\tan(c+dx)}}dx-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$\downarrow$ 27

$$\frac{1}{2}\int \frac{\cot(c+dx)\left(-b(aA-2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\int \frac{-b(aA-2bB)\tan(c+dx)^2-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$\downarrow$ 4136

$$\frac{1}{2}\left(\int -\frac{2\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx+a(2aB+3Ab)\int \frac{\cot(c+dx)\left(\tan^2(c+\right.}{\sqrt{a+b\tan(c+}}\right.$$
$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(a(2aB+3Ab)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-2\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c}{\sqrt{a+b\tan(c+dx)}}\right.$$
$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(a(2aB+3Ab)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c}{\sqrt{a+b\tan(c+dx)}}\right.$$
$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}$$

$\downarrow$ 4022

$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(a(2aB+3Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{1}{2}(a-ib)^2(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(\right.\right.$$

$\downarrow$ 3042

$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(a(2aB+3Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{1}{2}(a-ib)^2(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(\right.\right.$$

$\downarrow$ 4020

$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(a(2aB+3Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{i(a-ib)^2(A-iB)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d}{2d}\right.\right.$$

$\downarrow$ 25

$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(a(2aB+3Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{i(a+ib)^2(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(}{2d}\right.\right.$$

$\downarrow$ 73

$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(a(2aB+3Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{(a-ib)^2(A-iB)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c}}{bd}\right.\right.$$

$\downarrow$ 221

$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(a(2aB+3Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{(a-ib)^{3/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3}}{}\right.\right.$$

$\downarrow$ 4117

$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(\frac{a(2aB+3Ab)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}-2\left(\frac{(a-ib)^{3/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3/2}(A+\right.\right.$$

$\downarrow$ 73

$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(\frac{2a(2aB+3Ab)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}-2\left(\frac{(a-ib)^{3/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib\right.\right.$$

$\downarrow$ 221

$$-\frac{aA\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+$$
$$\frac{1}{2}\left(-\frac{2\sqrt{a}(2aB+3Ab)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}-2\left(\frac{(a-ib)^{3/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{3/2}(A+\right.\right.$$

input
```
Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*(((a - I*b)^(3/2)*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + ((
a + I*b)^(3/2)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d) - (2*Sqrt[
a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d)/2 - (a*A*
Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1688 vs. $2(143) = 286$.

Time = 0.15 (sec) , antiderivative size = 1689, normalized size of antiderivative = 9.99

method	result	size
derivativedivides	Expression too large to display	1689
default	Expression too large to display	1689

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+
c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+1/4/d*ln((a+b*ta
n(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/
2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/2/d*ln((a+b*tan(d*x+
c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan
(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/
2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)+2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta
n(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2))*A*a+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))
*B*a^2-2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2
)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a-1/d/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^
(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2-1/4/d/b*ln((a+b*tan
(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2
))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-1/d/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2
))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1/2)*a+1/4/d/b*ln(b*tan(d*x
+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3113 vs. $2(137) = 274$.

Time = 3.20 (sec) , antiderivative size = 6245, normalized size of antiderivative = 36.95

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan(c+dx))\,(a+b\tan(c+dx))^{\frac{3}{2}}\cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*cot(c + d*x)**2,
 x)
```

Maxima [F]

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^2,
x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 21319, normalized size of antiderivative = 126.15

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
(a^(1/2)*atan(((a^(1/2)*((16*(a + b*tan(c + d*x))^(1/2)*(2*A^4*b^16 + 2*B^
4*b^16 + 4*A^2*B^2*b^16 - A^4*a^2*b^14 + 66*A^4*a^4*b^12 - A^4*a^6*b^10 +
2*A^4*a^8*b^8 + 8*B^4*a^2*b^14 + 16*B^4*a^4*b^12 - 16*B^4*a^6*b^10 + 6*B^4
*a^8*b^8 + 25*A^2*B^2*a^2*b^14 - 130*A^2*B^2*a^4*b^12 + 145*A^2*B^2*a^6*b^
10 + 12*A*B^3*a^3*b^13 - 104*A*B^3*a^5*b^11 + 44*A*B^3*a^7*b^9 - 84*A^3*B*
a^3*b^13 + 144*A^3*B*a^5*b^11 - 12*A^3*B*a^7*b^9))/d^4 + (a^(1/2)*(3*A*b +
 2*B*a)*((8*(100*A^3*a^2*b^13*d^2 + 44*A^3*a^4*b^11*d^2 - 56*A^3*a^6*b^9*d
^2 - 92*B^3*a^3*b^12*d^2 - 84*B^3*a^5*b^10*d^2 + 12*B^3*a^7*b^8*d^2 + 4*B^
3*a*b^14*d^2 - 92*A^2*B*a*b^14*d^2 - 216*A*B^2*a^2*b^13*d^2 - 48*A*B^2*a^4
*b^11*d^2 + 168*A*B^2*a^6*b^9*d^2 + 208*A^2*B*a^3*b^12*d^2 + 264*A^2*B*a^5
*b^10*d^2 - 36*A^2*B*a^7*b^8*d^2))/d^5 - (a^(1/2)*(3*A*b + 2*B*a)*((16*(a
+ b*tan(c + d*x))^(1/2)*(92*A^2*a^3*b^10*d^2 - 20*A^2*a^5*b^8*d^2 - 56*B^2
*a^3*b^10*d^2 + 36*B^2*a^5*b^8*d^2 - 32*A*B*b^13*d^2 + 44*A^2*a*b^12*d^2 -
 44*B^2*a*b^12*d^2 + 32*A*B*a^2*b^11*d^2 + 176*A*B*a^4*b^9*d^2))/d^4 + (a^
(1/2)*(3*A*b + 2*B*a)*((8*(80*A*a*b^11*d^4 + 80*A*a^3*b^9*d^4 + 48*B*a^2*b
^10*d^4 + 48*B*a^4*b^8*d^4))/d^5 - (8*a^(1/2)*(3*A*b + 2*B*a)*(32*b^10*d^4
 + 48*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2))/d^5))/(2*d)))/(2*d)))/(2*d)
)*(3*A*b + 2*B*a)*1i)/(2*d) + (a^(1/2)*((16*(a + b*tan(c + d*x))^(1/2)*(2*
A^4*b^16 + 2*B^4*b^16 + 4*A^2*B^2*b^16 - A^4*a^2*b^14 + 66*A^4*a^4*b^12 -
A^4*a^6*b^10 + 2*A^4*a^8*b^8 + 8*B^4*a^2*b^14 + 16*B^4*a^4*b^12 - 16*B^...
```

Reduce [F]

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \cot(dx+c)^2\left(a+\tan(dx+c)\,b\right)^{\frac{3}{2}}\left(A+B\tan(dx+c)\right)dx$$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

3.330 $\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 219

$$\begin{aligned}\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}(A&+B\tan(c+dx))\,dx = \frac{(8a^2A-3Ab^2-12abB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}d}\\&-\frac{(a-ib)^{3/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}\\&-\frac{(a+ib)^{3/2}(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}\\&-\frac{(5Ab+4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d}\\&-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}\end{aligned}$$

output
```
1/4*(8*A*a^2-3*A*b^2-12*B*a*b)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(
1/2)/d-(a-I*b)^(3/2)*(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))
/d-(a+I*b)^(3/2)*(A+I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-1
/4*(5*A*b+4*B*a)*cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)/d-1/2*a*A*cot(d*x+c)^2*
(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.89

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{(8a^2A-3Ab^2-12abB)\,\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)-\sqrt{a}\Big(4(a-ib)^{3/2}(A-iB)\text{arctanh}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
((8*a^2*A - 3*A*b^2 - 12*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]]
- Sqrt[a]*(4*(a - I*b)^(3/2)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sq
rt[a - I*b]] + 4*(a + I*b)^(3/2)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]
]/Sqrt[a + I*b]] + Cot[c + d*x]*(5*A*b + 4*a*B + 2*a*A*Cot[c + d*x])*Sqrt[
a + b*Tan[c + d*x]]))/(4*Sqrt[a]*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.88 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 4088, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{2}\int \frac{\cot^2(c+dx)\left(-b(3aA-4bB)\tan^2(c+dx)-4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+4aB)\right)}{2\sqrt{a+b\tan(c+dx)}}dx-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 27

$$\frac{1}{4}\int \frac{\cot^2(c+dx)\left(-b(3aA-4bB)\tan^2(c+dx)-4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+4aB)\right)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\int \frac{-b(3aA-4bB)\tan(c+dx)^2-4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+4aB)}{\tan(c+dx)^2\sqrt{a+b\tan(c+dx)}}dx-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 4132

$$\frac{1}{4}\left(-\frac{\int \frac{\cot(c+dx)\left(ab(5Ab+4aB)\tan^2(c+dx)+8a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(8Aa^2-12bBa-3Ab^2\right)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{(4aB+5Ab)\cot(c+\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{\int \frac{\cot(c+dx)\left(ab(5Ab+4aB)\tan^2(c+dx)+8a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(8Aa^2-12bBa-3Ab^2\right)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(4aB+5Ab)\cot(c+\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(-\frac{\int \frac{ab(5Ab+4aB)\tan(c+dx)^2+8a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(8Aa^2-12bBa-3Ab^2\right)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+}}{d}\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{4136}$$

$$\frac{1}{4}\left(-\frac{\int \frac{8\left(a\left(Ba^2+2Aba-b^2B\right)-a\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx+a\left(8a^2A-12abB-3Ab^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{8\int \frac{a\left(Ba^2+2Aba-b^2B\right)-a\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+a\left(8a^2A-12abB-3Ab^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(-\frac{8\int \frac{a\left(Ba^2+2Aba-b^2B\right)-a\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+a\left(8a^2A-12abB-3Ab^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{2a}-\right.$$

$$\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{4022}$$

$$-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$

$$\frac{1}{4}\left(-\frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{a\left(8a^2A-12abB-3Ab^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\Big(}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$

$$\frac{1}{4}\left(-\frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{a\left(8a^2A-12abB-3Ab^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\Big(}{}\right.$$

$$\Big\downarrow \text{4020}$$

$$-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$

$$\frac{1}{4}\left(-\frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{a\left(8a^2A-12abB-3Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\Big(\right.$$

$\downarrow$ 25

$$-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$

$$\frac{1}{4}\left(-\frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{a\left(8a^2A-12abB-3Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\Big(\right.$$

$\downarrow$ 73

$$-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$

$$\frac{1}{4}\left(-\frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{a\left(8a^2A-12abB-3Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\Big(\right.$$

$\downarrow$ 221

$$-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$

$$\frac{1}{4}\left(-\frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{a\left(8a^2A-12abB-3Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\Big(\right.$$

$\downarrow$ 4117

$$-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$

$$\frac{1}{4}\left(-\frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{\frac{a\left(8a^2A-12abB-3Ab^2\right)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+8\Big(\frac{a(a-ib)^{3/2}(}{}}{2\ldots}\right.$$

$\downarrow$ 73

$$-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$
$$\frac{1}{4}\left(-\frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{\frac{2a(8a^2A-12abB-3Ab^2)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+8\left(\frac{a(\right.}{}}{}\right.$$

$\downarrow$ 221

$$-\frac{aA\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$
$$\frac{1}{4}\left(-\frac{(4aB+5Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{-\frac{2\sqrt{a}(8a^2A-12abB-3Ab^2)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+8\left(\frac{a(a-ib)}{}\right.}{}\right.$$

input
```
Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(a*A*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/d + (-1/2*(8*((a*(a - I
*b)^(3/2)*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d - (a*(a + I*b)^(
3/2)*(I*A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d) - (2*Sqrt[a]*(8*a^2*
A - 3*A*b^2 - 12*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d)/a -
((5*A*b + 4*a*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/d)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(185) = 370$.

Time = 0.16 (sec) , antiderivative size = 1805, normalized size of antiderivative = 8.24

method	result	size
derivativedivides	Expression too large to display	1805
default	Expression too large to display	1805

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
2*a^(3/2)*A*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d-3/d*b*a^(1/2)*arctan
h((a+b*tan(d*x+c))^(1/2)/a^(1/2))*B+3/4/d/tan(d*x+c)^2*(a+b*tan(d*x+c))^(1
/2)*A*a-5/4/d/tan(d*x+c)^2*A*(a+b*tan(d*x+c))^(3/2)-1/4/d/b*ln((a+b*tan(d*
x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*
B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a+1/4/d/b*ln(b*tan(d*x+c)+
a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-1/d/b/tan(d*x+c)^2*B*(a+b*
tan(d*x+c))^(3/2)*a+1/d/b/tan(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*B*a^2-3/4/d*
b^2/a^(1/2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))*A+1/d/(2*(a^2+b^2)^(1/
2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/
2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)*a-1/d/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1
/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)*a+2/d*b/(2*(a^2+b^2)
^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))
^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a+1/4/d/b*ln((a+b*tan(d*x+c))^(1/
2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*a^2-1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*
(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2))*B*(a^2+b^2)^(1/2)-1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/
2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3167 vs. $2(179) = 358$.

Time = 5.91 (sec) , antiderivative size = 6354, normalized size of antiderivative = 29.01

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan(c+dx))\,(a+b\tan(c+dx))^{\frac{3}{2}}\cot^3(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*cot(c + d*x)**3,
 x)
```

Maxima [F]

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}(A + B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^3,
x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(c+dx)(a + b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 23016, normalized size of antiderivative = 105.10

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
atan(((((932*A^3*a^3*b^12*d^2 + 1344*A^3*a^5*b^10*d^2 - 192*A^3*a^7*b^8*d^
2 + 1600*B^3*a^2*b^13*d^2 + 704*B^3*a^4*b^11*d^2 - 896*B^3*a^6*b^9*d^2 + 3
48*A^2*B*b^15*d^2 - 604*A^3*a*b^14*d^2 + 1760*A*B^2*a*b^14*d^2 - 3232*A*B^
2*a^3*b^12*d^2 - 4416*A*B^2*a^5*b^10*d^2 + 576*A*B^2*a^7*b^8*d^2 - 3780*A^
2*B*a^2*b^13*d^2 - 1440*A^2*B*a^4*b^11*d^2 + 2688*A^2*B*a^6*b^9*d^2)/(2*d^
5) - (((384*A*b^12*d^4 + 1280*B*a*b^11*d^4 - 384*A*a^2*b^10*d^4 - 768*A*a^
4*b^8*d^4 + 1280*B*a^3*b^9*d^4)/(2*d^5) + ((512*b^10*d^4 + 768*a^2*b^8*d^4
)*(a + b*tan(c + d*x))^(1/2)*((((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 + 16*A*B*b^
3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 - 48*A*B*a^2*b*d^2)^2/64 - d^4
*(A^4*a^6 + A^4*b^6 + B^4*a^6 + B^4*b^6 + 2*A^2*B^2*a^6 + 2*A^2*B^2*b^6 +
3*A^4*a^2*b^4 + 3*A^4*a^4*b^2 + 3*B^4*a^2*b^4 + 3*B^4*a^4*b^2 + 6*A^2*B^2*
a^2*b^4 + 6*A^2*B^2*a^4*b^2))^(1/2) + A^2*a^3*d^2 - B^2*a^3*d^2 + 2*A*B*b^
3*d^2 - 3*A^2*a*b^2*d^2 + 3*B^2*a*b^2*d^2 - 6*A*B*a^2*b*d^2)/(4*d^4))^(1/2
))/d^4)*((((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 + 16*A*B*b^3*d^2 - 24*A^2*a*b^2*
d^2 + 24*B^2*a*b^2*d^2 - 48*A*B*a^2*b*d^2)^2/64 - d^4*(A^4*a^6 + A^4*b^6 +
 B^4*a^6 + B^4*b^6 + 2*A^2*B^2*a^6 + 2*A^2*B^2*b^6 + 3*A^4*a^2*b^4 + 3*A^4
*a^4*b^2 + 3*B^4*a^2*b^4 + 3*B^4*a^4*b^2 + 6*A^2*B^2*a^2*b^4 + 6*A^2*B^2*a
^4*b^2))^(1/2) + A^2*a^3*d^2 - B^2*a^3*d^2 + 2*A*B*b^3*d^2 - 3*A^2*a*b^2*d
^2 + 3*B^2*a*b^2*d^2 - 6*A*B*a^2*b*d^2)/(4*d^4))^(1/2) + ((a + b*tan(c + d
*x))^(1/2)*(1088*A^2*a^3*b^10*d^2 - 576*A^2*a^5*b^8*d^2 - 1472*B^2*a^3*...
```

Reduce [F]

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \cot(dx+c)^3(a+\tan(dx+c)\,b)^{\frac{3}{2}}(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

3.331 $\int \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 278

$$\begin{aligned}\int \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}(A&+B\tan(c+dx))\,dx = \frac{(24a^2Ab+Ab^3+16a^3B-6ab^2B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8a^{3/2}d}\\
&-\frac{(a-ib)^{3/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}\\
&+\frac{(a+ib)^{3/2}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}\\
&+\frac{(8a^2A-Ab^2-10abB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{8ad}\\
&-\frac{(7Ab+6aB)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{12d}-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}\end{aligned}$$

output
```
1/8*(24*A*a^2*b+A*b^3+16*B*a^3-6*B*a*b^2)*arctanh((a+b*tan(d*x+c))^(1/2)/a
^(1/2))/a^(3/2)/d-(a-I*b)^(3/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-
I*b)^(1/2))/d+(a+I*b)^(3/2)*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)
^(1/2))/d+1/8*(8*A*a^2-A*b^2-10*B*a*b)*cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)/a
/d-1/12*(7*A*b+6*B*a)*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)/d-1/3*a*A*cot(d*
x+c)^3*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 3.85 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.87

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{3(24a^2Ab+Ab^3+16a^3B-6ab^2B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt{a}\left(-24ia(a-ib)^{3/2}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
(3*(24*a^2*A*b + A*b^3 + 16*a^3*B - 6*a*b^2*B)*ArcTanh[Sqrt[a + b*Tan[c +
d*x]]/Sqrt[a]] + Sqrt[a]*((-24*I)*a*(a - I*b)^(3/2)*(A - I*B)*ArcTanh[Sqrt
[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + (24*I)*a*(a + I*b)^(3/2)*(A + I*B)*A
rcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] - Cot[c + d*x]*(-24*a^2*A +
 3*A*b^2 + 30*a*b*B + 2*a*(7*A*b + 6*a*B)*Cot[c + d*x] + 8*a^2*A*Cot[c + d
*x]^2)*Sqrt[a + b*Tan[c + d*x]]))/(24*a^(3/2)*d)
```

Rubi [A] (warning: unable to verify)

Time = 2.37 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.01, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4088, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{3}\int \frac{\cot^3(c+dx)\left(-b(5aA-6bB)\tan^2(c+dx)-6\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+6aB)\right)}{2\sqrt{a+b\tan(c+dx)}}dx-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\int \frac{\cot^3(c+dx)\left(-b(5aA-6bB)\tan^2(c+dx)-6\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+6aB)\right)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \frac{-b(5aA-6bB)\tan(c+dx)^2-6\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+6aB)}{\tan(c+dx)^3\sqrt{a+b\tan(c+dx)}}dx-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{4132}$$

$$\frac{1}{6}\left(-\frac{\int \frac{3\cot^2(c+dx)\left(ab(7Ab+6aB)\tan^2(c+dx)+8a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(8Aa^2-10bBa-Ab^2\right)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(6aB+7Ab)\cot^2(c\right.$$
$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(-\frac{3\int \frac{\cot^2(c+dx)\left(ab(7Ab+6aB)\tan^2(c+dx)+8a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(8Aa^2-10bBa-Ab^2\right)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{4a}-\frac{(6aB+7Ab)\cot^2(c\right.$$
$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(-\frac{3\int \frac{ab(7Ab+6aB)\tan(c+dx)^2+8a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(8Aa^2-10bBa-Ab^2\right)}{\tan(c+dx)^2\sqrt{a+b\tan(c+dx)}}dx}{4a}-\frac{(6aB+7Ab)\cot^2(c+dx)\sqrt{a}}{2d}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \; 4132$$

$$\frac{1}{6}\left(-\frac{3\left(-\frac{\int -\frac{\cot(c+dx)\left(-16\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-b\left(8Aa^2-10bBa-Ab^2\right)\tan^2(c+dx)a+\left(16Ba^3+24Aba^2-6b^2Ba+Ab^3\right)a\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{(8a^2A-}{}\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(-\frac{3\left(\frac{\int \frac{\cot(c+dx)\left(-16\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-b\left(8Aa^2-10bBa-Ab^2\right)\tan^2(c+dx)a+\left(16Ba^3+24Aba^2-6b^2Ba+Ab^3\right)a\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(8a^2A-10ab}{}\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{6}\left(-\frac{3\left(\frac{\int \frac{-16\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-b\left(8Aa^2-10bBa-Ab^2\right)\tan(c+dx)^2a+\left(16Ba^3+24Aba^2-6b^2Ba+Ab^3\right)a}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(8a^2A-10abB-Ab^2)\,\mathrm{co}}{}\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$$\Big\downarrow \; 4136$$

$$\frac{1}{6}\left(-\frac{3\left(\frac{\int -\frac{16\left(\left(Aa^2-2bBa-Ab^2\right)a^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^2\right)}{\sqrt{a+b\tan(c+dx)}}dx+a\left(16a^3B+24a^2Ab-6ab^2B+Ab^3\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a} - \cdots\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$\downarrow$ 27

$$\frac{1}{6}\left(-\frac{3\left(\frac{a\left(16a^3B+24a^2Ab-6ab^2B+Ab^3\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-16\int \frac{\left(Aa^2-2bBa-Ab^2\right)a^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^2}{\sqrt{a+b\tan(c+dx)}}dx}{2a} - \frac{(8a^2\cdots}{}\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{6}\left(-\frac{3\left(\frac{a\left(16a^3B+24a^2Ab-6ab^2B+Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-16\int \frac{\left(Aa^2-2bBa-Ab^2\right)a^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^2}{\sqrt{a+b\tan(c+dx)}}dx}{2a} - \frac{(8a^2\cdots}{}\right.}{4a}\right.$$

$$\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

$\downarrow$ 4022

$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+7Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{\left(8a^2A-10abB-Ab^2\right)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{a\left(16a^3B+24\cdots\right.}{}\right.}{}\right.$$

$\downarrow$ 3042

$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+7Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{(8a^2A-10abB-Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{a(16a^3B+24}{}\right.}{}\right.$$

$\downarrow$ 4020

$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+7Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{(8a^2A-10abB-Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{a(16a^3B+24}{}\right.}{}\right.$$

$\downarrow$ 25

$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+7Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{(8a^2A-10abB-Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{a(16a^3B+24}{}\right.}{}\right.$$

$\downarrow$ 73

$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+7Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{(8a^2A-10abB-Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{a\left(16a^3B+24\right.}{}\right.}{}\right.$$

$\downarrow$ 221

$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+7Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{(8a^2A-10abB-Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{a\left(16a^3B+24\right.}{}\right.}{}\right.$$

$\downarrow$ 4117

$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+7Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{(8a^2A-10abB-Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{\frac{a\left(16a^3B+24a\right.}{}}{}\right.}{}\right.$$

$\downarrow$ 73

$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+7Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{(8a^2A-10abB-Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{\frac{2a\left(16a^3B+24\right.}{}}{}\right.}{}\right.$$

$$\Big\downarrow \text{221}$$

$$-\frac{aA\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+$$

$$\frac{1}{6}\left(-\frac{(6aB+7Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{(8a^2A-10abB-Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{-\frac{2\sqrt{a}\left(16a^3B\right.}{}}{}\right.}{}\right.$$

input
```
Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-1/3*(a*A*Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]])/d + (-1/2*((7*A*b + 6*a
*B)*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/d - (3*((-16*((a^2*(a - I*b)^
(3/2)*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + (a^2*(a + I*b)^(3/
2)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d) - (2*Sqrt[a]*(24*a^2*A
*b + A*b^3 + 16*a^3*B - 6*a*b^2*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a
]])/d)/(2*a) - ((8*a^2*A - A*b^2 - 10*a*b*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c
 + d*x]])/d))/(4*a))/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1984 vs. $2(240) = 480$.

Time = 0.15 (sec) , antiderivative size = 1985, normalized size of antiderivative = 7.14

method	result	size
derivativedivides	Expression too large to display	1985
default	Expression too large to display	1985

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+
c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-1/
4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a-1/3/
d/tan(d*x+c)^3*A*(a+b*tan(d*x+c))^(3/2)+2/d*a^(3/2)*arctanh((a+b*tan(d*x+c
))^(1/2)/a^(1/2))*B-1/2/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+
1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2-1/4/d*ln((a+b
*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^
(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+1/2/d*ln((a+b*tan(d
*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))
*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta
n(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2))*B*a^2+1/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2))*B+1/4/d*b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b
*tan(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/4/d*b*ln(
b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b
^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d*ln(b*tan(d*x+c)+a+(a+b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3215 vs. 2(234) = 468.

Time = 14.72 (sec) , antiderivative size = 6449, normalized size of antiderivative = 23.20

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan(c+dx))(a+b\tan(c+dx))^{\frac{3}{2}}\cot^4(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*cot(c + d*x)**4,
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 25789, normalized size of antiderivative = 92.77

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
atan((((((256*A*a*b^13*d^4 + 5376*A*a^3*b^11*d^4 + 5120*A*a^5*b^9*d^4 - 1
536*B*a^2*b^12*d^4 + 1536*B*a^4*b^10*d^4 + 3072*B*a^6*b^8*d^4)/(8*a^2*d^5)
 - ((2048*a^2*b^10*d^4 + 3072*a^4*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(-((
(8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 + 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^
2*a*b^2*d^2 - 48*A*B*a^2*b*d^2)^2/64 - d^4*(A^4*a^6 + A^4*b^6 + B^4*a^6 +
B^4*b^6 + 2*A^2*B^2*a^6 + 2*A^2*B^2*b^6 + 3*A^4*a^2*b^4 + 3*A^4*a^4*b^2 +
3*B^4*a^2*b^4 + 3*B^4*a^4*b^2 + 6*A^2*B^2*a^2*b^4 + 6*A^2*B^2*a^4*b^2))^(1
/2) + A^2*a^3*d^2 - B^2*a^3*d^2 + 2*A*B*b^3*d^2 - 3*A^2*a*b^2*d^2 + 3*B^2*
a*b^2*d^2 - 6*A*B*a^2*b*d^2)/(4*d^4))^(1/2))/(4*a^2*d^4))*(-(((8*A^2*a^3*d
^2 - 8*B^2*a^3*d^2 + 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2
- 48*A*B*a^2*b*d^2)^2/64 - d^4*(A^4*a^6 + A^4*b^6 + B^4*a^6 + B^4*b^6 + 2*
A^2*B^2*a^6 + 2*A^2*B^2*b^6 + 3*A^4*a^2*b^4 + 3*A^4*a^4*b^2 + 3*B^4*a^2*b^
4 + 3*B^4*a^4*b^2 + 6*A^2*B^2*a^2*b^4 + 6*A^2*B^2*a^4*b^2))^(1/2) + A^2*a^
3*d^2 - B^2*a^3*d^2 + 2*A*B*b^3*d^2 - 3*A^2*a*b^2*d^2 + 3*B^2*a*b^2*d^2 -
6*A*B*a^2*b*d^2)/(4*d^4))^(1/2) + ((a + b*tan(c + d*x))^(1/2)*(3008*A^2*a^
3*b^12*d^2 + 5888*A^2*a^5*b^10*d^2 - 1280*A^2*a^7*b^8*d^2 - 2672*B^2*a^3*b
^12*d^2 - 4352*B^2*a^5*b^10*d^2 + 2304*B^2*a^7*b^8*d^2 + 4*A^2*a*b^14*d^2
- 2096*A*B*a^2*b^13*d^2 + 1024*A*B*a^4*b^11*d^2 + 11264*A*B*a^6*b^9*d^2))/
(4*a^2*d^4))*(-(((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 + 16*A*B*b^3*d^2 - 24*A^2*
a*b^2*d^2 + 24*B^2*a*b^2*d^2 - 48*A*B*a^2*b*d^2)^2/64 - d^4*(A^4*a^6 + ...
```

Reduce [F]

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \cot(dx+c)^4\,(a+\tan(dx+c)\,b)^{\frac{3}{2}}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

3.332 $\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 252

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{(a-ib)^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\tan(c+dx)}}{d} - \frac{2(Ab+aB)(a+b\tan(c+dx))^{3/2}}{3d} - \frac{2B(a+b\tan(c+dx))^{5/2}}{5d} + \frac{2(9Ab-2aB)(a+b\tan(c+dx))^{7/2}}{63b^2d} + \frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}$$

output
```
(a-I*b)^(5/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I
*b)^(5/2)*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2*(2*A*a
*b+B*a^2-B*b^2)*(a+b*tan(d*x+c))^(1/2)/d-2/3*(A*b+B*a)*(a+b*tan(d*x+c))^(3
/2)/d-2/5*B*(a+b*tan(d*x+c))^(5/2)/d+2/63*(9*A*b-2*B*a)*(a+b*tan(d*x+c))^(
7/2)/b^2/d+2/9*B*tan(d*x+c)*(a+b*tan(d*x+c))^(7/2)/b/d
```

Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.17

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\frac{2(9Ab-2aB)(a+b\tan(c+dx))^{7/2}}{b}+14B\tan(c+dx)(a+b\tan(c+dx))^{7/2}-\frac{63}{2}ib(A-iB)\left(\frac{2}{5}\right.}{}$$

input
```
Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
((2*(9*A*b - 2*a*B)*(a + b*Tan[c + d*x])^(7/2))/b + 14*B*Tan[c + d*x]*(a +
 b*Tan[c + d*x])^(7/2) - ((63*I)/2)*b*(A - I*B)*((2*(a + b*Tan[c + d*x])^(
5/2))/5 + (2*(a - I*b)*(-3*(a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]
]/Sqrt[a - I*b]] + Sqrt[a + b*Tan[c + d*x]]*(4*a - (3*I)*b + b*Tan[c + d*x
])))/3) + ((63*I)/2)*b*(A + I*B)*((2*(a + b*Tan[c + d*x])^(5/2))/5 + (2*(a
 + I*b)*(-3*(a + I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]
] + Sqrt[a + b*Tan[c + d*x]]*(4*a + (3*I)*b + b*Tan[c + d*x])))/3))/(63*b*
d)
```

Rubi [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4090, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2\int -\frac{1}{2}(a+b\tan(c+dx))^{5/2}\left(-\left((9Ab-2aB)\tan^2(c+dx)\right)+9bB\tan(c+dx)+2aB\right)dx}{9b}+\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}-\frac{\int(a+b\tan(c+dx))^{5/2}\left(-\left((9Ab-2aB)\tan^2(c+dx)\right)+9bB\tan(c+dx)+2aB\right)dx}{9b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}-\frac{\int(a+b\tan(c+dx))^{5/2}\left(-\left((9Ab-2aB)\tan(c+dx)^2\right)+9bB\tan(c+dx)+2aB\right)dx}{9b}$$

$$\Big\downarrow \text{4113}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}-\frac{\int(a+b\tan(c+dx))^{5/2}(9Ab+9B\tan(c+dx)b)dx-\frac{2(9Ab-2aB)(a+b\tan(c+dx))^{7/2}}{7bd}}{9b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}-\frac{\int(a+b\tan(c+dx))^{5/2}(9Ab+9B\tan(c+dx)b)dx-\frac{2(9Ab-2aB)(a+b\tan(c+dx))^{7/2}}{7bd}}{9b}$$

$$\Big\downarrow \text{4011}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}-\frac{\int(a+b\tan(c+dx))^{3/2}(9b(aA-bB)+9b(Ab+aB)\tan(c+dx))dx-\frac{2(9Ab-2aB)(a+b\tan(c+dx))^{7/2}}{7bd}+\frac{18bB(a+b\tan}{5d}}{9b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}-\frac{\int(a+b\tan(c+dx))^{3/2}(9b(aA-bB)+9b(Ab+aB)\tan(c+dx))dx-\frac{2(9Ab-2aB)(a+b\tan(c+dx))^{7/2}}{7bd}+\frac{18bB(a+b\tan}{5d}}{9b}$$

$$\Big\downarrow \text{4011}$$

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd} - \frac{\int \sqrt{a+b\tan(c+dx)}\left(9b\left(Aa^2-2bBa-Ab^2\right)+9b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx - \frac{2(9Ab-2aB)(a+b\tan(c+dx}{7bd}}{9b}$$

$\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd} - \frac{\int \sqrt{a+b\tan(c+dx)}\left(9b\left(Aa^2-2bBa-Ab^2\right)+9b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx - \frac{2(9Ab-2aB)(a+b\tan(c+dx}{7bd}}{9b}$$

$\downarrow$ 4011

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd} - \frac{\int \frac{9b(Aa^3-3bBa^2-3Ab^2a+b^3B)+9b(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx + \frac{18b(a^2B+2aAb-b^2B)\sqrt{a+b\tan(c+dx)}}{d} - 2(9Ab-2aB)(}{9b}$$

$\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd} - \frac{\int \frac{9b(Aa^3-3bBa^2-3Ab^2a+b^3B)+9b(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx + \frac{18b(a^2B+2aAb-b^2B)\sqrt{a+b\tan(c+dx)}}{d} - 2(9Ab-2aB)(}{9b}$$

$\downarrow$ 4022

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd} - \frac{\frac{9}{2}b(a+ib)^3(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx + \frac{9}{2}b(a-ib)^3(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx + \frac{18b(a^2B+2aAb-b^2B)\sqrt{a+b\tan(c}{d}}{9b}$$

$\downarrow$ 3042

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd} - \frac{\frac{9}{2}b(a+ib)^3(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx + \frac{9}{2}b(a-ib)^3(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx + \frac{18b(a^2B+2aAb-b^2B)\sqrt{a+b\tan(c}{d}}{9b}$$

$\downarrow$ 4020

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd} - \frac{\frac{9ib(a-ib)^3(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d} - \frac{9ib(a+ib)^3(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}}{}$$

$\downarrow$ 25

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}-$$
$$-\frac{9ib(a-ib)^3(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{9ib(a+ib)^3(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+$$

$\Big\downarrow$ 73

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}-$$
$$\frac{\frac{9(a+ib)^3(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}+\frac{9(a-ib)^3(A-iB)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}+\frac{18b(a^2B+2aAb-}{}}{9b}$$

$\Big\downarrow$ 221

$$\frac{2B\tan(c+dx)(a+b\tan(c+dx))^{7/2}}{9bd}-$$
$$\frac{\frac{18b(a^2B+2aAb-b^2B)\sqrt{a+b\tan(c+dx)}}{d}+\frac{9b(a-ib)^{5/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{9b(a+ib)^{5/2}(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}-\frac{2(9Ab-2a}{}}{9b}$$

input
```
Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(2*B*Tan[c + d*x]*(a + b*Tan[c + d*x])^(7/2))/(9*b*d) - ((9*(a - I*b)^(5/2
)*b*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + (9*(a + I*b)^(5/2)*b
*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d + (18*b*(2*a*A*b + a^2*B
- b^2*B)*Sqrt[a + b*Tan[c + d*x]])/d + (6*b*(A*b + a*B)*(a + b*Tan[c + d*x
])^(3/2))/d + (18*b*B*(a + b*Tan[c + d*x])^(5/2))/(5*d) - (2*(9*A*b - 2*a*
B)*(a + b*Tan[c + d*x])^(7/2))/(7*b*d))/(9*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2437 vs. $2(216) = 432$.

Time = 0.23 (sec) , antiderivative size = 2438, normalized size of antiderivative = 9.67

method	result	size
parts	Expression too large to display	2438
derivativedivides	Expression too large to display	2469
default	Expression too large to display	2469

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
A*(2/7/b/d*(a+b*tan(d*x+c))^(7/2)-2/3*b*(a+b*tan(d*x+c))^(3/2)/d-4*b/d*(a+
b*tan(d*x+c))^(1/2)*a-1/4/b/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
(a^2+b^2)^(1/2)*a^2+1/4*b/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a
^2+b^2)^(1/2)+1/4/b/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-3/4*
b/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a
-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-2*b/d/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2)
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)*a+3*b/d/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2
))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-b^3/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*
arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2))+1/4/b/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(
a^2+b^2)^(1/2)*a^2-1/4*b/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^
2+b^2)^(1/2)-1/4/b/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+3/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4963 vs. $2(210) = 420$.

Time = 0.71 (sec) , antiderivative size = 4963, normalized size of antiderivative = 19.69

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^{\frac{5}{2}}\tan^2{(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(5/2)*tan(c + d*x)**2,
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,29,11]%%%}+%%%{12,[0,27,11]%%%}+%%%{66,[0,25,11]%
%%}+%%%{2
```

Mupad [B] (verification not implemented)

Time = 179.32 (sec) , antiderivative size = 4139, normalized size of antiderivative = 16.42

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
log((8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3 - ((((((-B^4*b^2*d^4*(5*
a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^
2*a*b^4*d^2)/d^4)^(1/2)*(32*B*a^4*b^2 - 32*B*b^6 + 32*a*b^2*d*(((-B^4*b^2*
d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2
 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2)))/(2*d) - (16*B^
2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^
2)*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*
B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2))/2)*((20*B^4*a^2*b^8*d^4 - B
^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2*d
^4)^(1/2)/(4*d^4) + (B^2*a^5)/(4*d^2) - (5*B^2*a^3*b^2)/(2*d^2) + (5*B^2*a
*b^4)/(4*d^2))^(1/2) - (2*a*(2*a*((2*B*(a^2 + b^2))/(b^2*d) - (2*B*a^2)/(b
^2*d)) + (2*B*a^3)/(b^2*d) - (2*B*a*(a^2 + b^2))/(b^2*d)) - ((2*B*(a^2 + b
^2))/(b^2*d) - (2*B*a^2)/(b^2*d))*(a^2 + b^2))*(a + b*tan(c + d*x))^(1/2)
- log((((((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2
 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(32*B*b^6 - 32*B*a^4*b
^2 + 32*a*b^2*d*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*
a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*tan(c +
d*x))^(1/2)))/(2*d) - (16*B^2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^6 - b^6 +
15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^
2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/...
```

Reduce [F]

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^5\,dx\right)b^3 + 3\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^4\,dx\right)a\,b^2 + 3\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)a^2b + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)a^3$$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**5,x)*b**3 + 3*int(sqrt(tan(c +
d*x)*b + a)*tan(c + d*x)**4,x)*a*b**2 + 3*int(sqrt(tan(c + d*x)*b + a)*tan
(c + d*x)**3,x)*a**2*b + int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*a
**3
```

3.333 $\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 213

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{(a-ib)^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2(a^2A-Ab^2-2abB)\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2A(a+b\tan(c+dx))^{5/2}}{5d} + \frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

output
```
-(a-I*b)^(5/2)*(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+
I*b)^(5/2)*(A+I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*(A*a^
2-A*b^2-2*B*a*b)*(a+b*tan(d*x+c))^(1/2)/d+2/3*(A*a-B*b)*(a+b*tan(d*x+c))^(
3/2)/d+2/5*A*(a+b*tan(d*x+c))^(5/2)/d+2/7*B*(a+b*tan(d*x+c))^(7/2)/b/d
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.21

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\frac{4B(a+b\tan(c+dx))^{7/2}}{b} - 7i(iA+B)\left(\frac{2}{5}(a+b\tan(c+dx))^{5/2}+\frac{2}{3}(a-ib)\left(-3(a-ib)^{3/2}a\right.\right.}{}$$

input
```
Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((4*B*(a + b*Tan[c + d*x])^(7/2))/b - (7*I)*(I*A + B)*((2*(a + b*Tan[c + d
*x])^(5/2))/5 + (2*(a - I*b)*(-3*(a - I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c
+ d*x]]/Sqrt[a - I*b]] + Sqrt[a + b*Tan[c + d*x]]*(4*a - (3*I)*b + b*Tan[c
 + d*x])))/3) - (7*I)*(I*A - B)*((2*(a + b*Tan[c + d*x])^(5/2))/5 + (2*(a
+ I*b)*(-3*(a + I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]
 + Sqrt[a + b*Tan[c + d*x]]*(4*a + (3*I)*b + b*Tan[c + d*x])))/3))/(14*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4075, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^{5/2}dx + \frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^{5/2}dx+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))^{3/2}(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^{3/2}(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{4011}$$

$$\int \sqrt{a+b\tan(c+dx)}\left(-Ba^2-2Aba+b^2B+\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)dx+\frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan(c+dx)}\left(-Ba^2-2Aba+b^2B+\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)dx+\frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{-Ba^3-3Aba^2+3b^2Ba+Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(a^2A-2abB-Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-Ba^3-3Aba^2+3b^2Ba+Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(a^2A-2abB-Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{4022}$$

$$-\frac{1}{2}(-b+ia)^3(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(b+ia)^3(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(a^2A-2abB-Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}(-b+ia)^3(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(b+ia)^3(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(a^2A-2abB-Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(b+ia)^3(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(-b+ia)^3(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2\left(a^2A-2abB-Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i(b+ia)^3(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(-b+ia)^3(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2\left(a^2A-2abB-Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(-b+ia)^3(A+iB)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(b+ia)^3(A-iB)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{2\left(a^2A-2abB-Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

$$\Big\downarrow \; 221$$

$$\frac{2\left(a^2A-2abB-Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{(b+ia)^3(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{(-b+ia)^3(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}+\frac{2(aA-bB)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2A(a+b\tan(c+dx))^{5/2}}{5d}+\frac{2B(a+b\tan(c+dx))^{7/2}}{7bd}$$

input
```
Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((I*a + b)^3*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*
d) - ((I*a - b)^3*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a +
I*b]*d) + (2*(a^2*A - A*b^2 - 2*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/d + (2*(a
*A - b*B)*(a + b*Tan[c + d*x])^(3/2))/(3*d) + (2*A*(a + b*Tan[c + d*x])^(5
/2))/(5*d) + (2*B*(a + b*Tan[c + d*x])^(7/2))/(7*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2411 vs. $2(181) = 362$.

Time = 0.16 (sec) , antiderivative size = 2412, normalized size of antiderivative = 11.32

method	result	size
parts	Expression too large to display	2412
derivativedivides	Expression too large to display	2426
default	Expression too large to display	2426

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2
*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a^3+1/d/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+
2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a^3+3/d*b^2/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2
))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a-3/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2))*A*a-1/2/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*(a^2+b^2)^(1/2)*a+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b
^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2))*A*(a^2+b^2)^(1/2)*a^2-1/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2
*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))*A*(a^2+b^2)^(1/2)+1/2/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2
)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*(a^2+b^2)^(1/2)*a-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*
tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2))*A*(a^2+b^2)^(1/2)*a^2+1/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((
2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2))*A*(a^2+b^2)^(1/2)-3/4/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4916 vs. $2(175) = 350$.

Time = 0.69 (sec) , antiderivative size = 4916, normalized size of antiderivative = 23.08

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan(c+dx))(a+b\tan(c+dx))^{\frac{5}{2}}\tan(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(5/2)*tan(c + d*x), x)
```

Maxima [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A + B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)\,dx$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*tan(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,24,9]%%%}+%%%{10,[0,22,9]%%%}+%%%{45,[0,20,9]%%%}
+%%%{120,
```

Mupad [B] (verification not implemented)

Time = 82.18 (sec) , antiderivative size = 3932, normalized size of antiderivative = 18.46

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output

```
log(- ((((((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + A^2*a^5*d^2
 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(32*A*b^6 - 32*A*a^4*b
^2 + 32*a*b^2*d*(((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + A^2*
a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*tan(c +
d*x))^(1/2)))/(2*d) - (16*A^2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^6 - b^6 +
15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^
2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^(1/2))
/2 - (8*A^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*((20*A^4*a^2*b^8*d^4 -
 A^4*b^10*d^4 - 110*A^4*a^4*b^6*d^4 + 100*A^4*a^6*b^4*d^4 - 25*A^4*a^8*b^2
*d^4)^(1/2)/(4*d^4) + (A^2*a^5)/(4*d^2) - (5*A^2*a^3*b^2)/(2*d^2) + (5*A^2
*a*b^4)/(4*d^2))^(1/2) - log((((((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)
^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4)^(1/2)
*(32*A*a^4*b^2 - 32*A*b^6 + 32*a*b^2*d*(((-A^4*b^2*d^4*(5*a^4 + b^4 - 10*a
^2*b^2)^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2*a*b^4*d^2)/d^4
)^(1/2)*(a + b*tan(c + d*x))^(1/2)))/(2*d) - (16*A^2*b^2*(a + b*tan(c + d*
x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((-A^4*b^2*d^4*(5*a
^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 5*A^2
*a*b^4*d^2)/d^4)^(1/2))/2 - (8*A^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)
*(((20*A^4*a^2*b^8*d^4 - A^4*b^10*d^4 - 110*A^4*a^4*b^6*d^4 + 100*A^4*a^6*
b^4*d^4 - 25*A^4*a^8*b^2*d^4)^(1/2) + A^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 ...
```

Reduce [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^4\,dx\right)b^3 + 3\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)a\,b^2 + 3\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)a^2 b + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)a^3$$

input

```
int(tan(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output

```
int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4,x)*b**3 + 3*int(sqrt(tan(c +
d*x)*b + a)*tan(c + d*x)**3,x)*a*b**2 + 3*int(sqrt(tan(c + d*x)*b + a)*tan
(c + d*x)**2,x)*a**2*b + int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a**3
```

3.334 $\int (a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 188

$$\int (a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{(a-ib)^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{2(2aAb+a^2B-b^2B)\sqrt{a+b\tan(c+dx)}}{d} + \frac{2(Ab+aB)(a+b\tan(c+dx))^{3/2}}{3d} + \frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

output
```
-(a-I*b)^(5/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+
I*b)^(5/2)*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*(2*A*
a*b+B*a^2-B*b^2)*(a+b*tan(d*x+c))^(1/2)/d+2/3*(A*b+B*a)*(a+b*tan(d*x+c))^(
3/2)/d+2/5*B*(a+b*tan(d*x+c))^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.24

$$\int (a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{i\left((A-iB)\left(\frac{2}{5}(a+b\tan(c+dx))^{5/2}+\frac{2}{3}(a-ib)\left(-3(a-ib)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+d}}{\sqrt{a-ib}}\right.\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((I/2)*((A - I*B)*((2*(a + b*Tan[c + d*x])^(5/2))/5 + (2*(a - I*b)*(-3*(a
- I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] + Sqrt[a + b*
Tan[c + d*x]]*(4*a - (3*I)*b + b*Tan[c + d*x])))/3) - (A + I*B)*((2*(a + b
*Tan[c + d*x])^(5/2))/5 + (2*(a + I*b)*(-3*(a + I*b)^(3/2)*ArcTanh[Sqrt[a
+ b*Tan[c + d*x]]/Sqrt[a + I*b]] + Sqrt[a + b*Tan[c + d*x]]*(4*a + (3*I)*b
 + b*Tan[c + d*x])))/3)))/d
```

Rubi [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 4011, 3042, 4011, 3042, 4011, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))^{3/2}(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^{3/2}(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\downarrow \text{4011}$$

$$\int \sqrt{a+b\tan(c+dx)}\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{2(aB+Ab)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\downarrow \text{3042}$$

$$\int \sqrt{a+b\tan(c+dx)}\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{2(aB+Ab)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\downarrow \text{4011}$$

$$\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aB+Ab)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\downarrow \text{3042}$$

$$\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aB+Ab)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\downarrow \text{4022}$$

$$\frac{1}{2}(a-ib)^3(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^3(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aB+Ab)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\downarrow \text{3042}$$

$$\frac{1}{2}(a-ib)^3(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^3(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aB+Ab)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(a-ib)^3(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a+ib)^3(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aB+Ab)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i(a-ib)^3(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(a+ib)^3(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aB+Ab)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{73}$$

$$\frac{(a-ib)^3(A-iB)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a+ib)^3(A+iB)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2(aB+Ab)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(a^2B+2aAb-b^2B\right)\sqrt{a+b\tan(c+dx)}}{d}+\frac{(a-ib)^{5/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{5/2}(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2(aB+Ab)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{2B(a+b\tan(c+dx))^{5/2}}{5d}$$

input
```
Int[(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((a - I*b)^(5/2)*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + ((a + I
*b)^(5/2)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d + (2*(2*a*A*b +
a^2*B - b^2*B)*Sqrt[a + b*Tan[c + d*x]])/d + (2*(A*b + a*B)*(a + b*Tan[c +
 d*x])^(3/2))/(3*d) + (2*B*(a + b*Tan[c + d*x])^(5/2))/(5*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2391 vs. $2(160) = 320$.

Time = 0.13 (sec) , antiderivative size = 2392, normalized size of antiderivative = 12.72

method	result	size
parts	Expression too large to display	2392
derivativedivides	Expression too large to display	2405
default	Expression too large to display	2405

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
A*(2/3*b*(a+b*tan(d*x+c))^(3/2)/d+4*b/d*(a+b*tan(d*x+c))^(1/2)*a+1/4/b/d*l
n((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2
+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-1/4*b/d*ln(
(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b
^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-1/4/b/d*ln((a+b*t
an(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1
/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+3/4*b/d*ln((a+b*tan(d*x+c))^(1/2)*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)*a+2*b/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2
)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
)*(a^2+b^2)^(1/2)*a-3*b/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
))*a^2+b^3/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)
^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4/b/d*ln
(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+
b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+1/4*b/d*ln(b
*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^
2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+1/4/b/d*ln(b*tan(d
*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/
2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-3/4*b/d*ln(b*tan(d*x+c)+a+(a+b*ta...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4855 vs. $2(154) = 308$.

Time = 0.70 (sec) , antiderivative size = 4855, normalized size of antiderivative = 25.82

$$\int (a + b\tan(c + dx))^{5/2}(A + B\tan(c + dx))\, dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int (a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,19,7]%%%}+%%%{8,[0,17,7]%%%}+%%%{28,[0,15,7]%%%}+
%%%{56,[0
```

Mupad [B] (verification not implemented)

Time = 33.63 (sec) , antiderivative size = 3863, normalized size of antiderivative = 20.55

$$\int (a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
log(- ((((((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2
 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(32*B*b^6 - 32*B*a^4*b
^2 + 32*a*b^2*d*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*
a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)*(a + b*tan(c +
d*x))^(1/2)))/(2*d) - (16*B^2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^6 - b^6 +
15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^
2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2))
/2 - (8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)*((20*B^4*a^2*b^8*d^4 -
 B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*b^4*d^4 - 25*B^4*a^8*b^2
*d^4)^(1/2)/(4*d^4) + (B^2*a^5)/(4*d^2) - (5*B^2*a^3*b^2)/(2*d^2) + (5*B^2
*a*b^4)/(4*d^2))^(1/2) - log((((((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)
^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4)^(1/2)
*(32*B*a^4*b^2 - 32*B*b^6 + 32*a*b^2*d*(((-B^4*b^2*d^4*(5*a^4 + b^4 - 10*a
^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2*a*b^4*d^2)/d^4
)^(1/2)*(a + b*tan(c + d*x))^(1/2)))/(2*d) - (16*B^2*b^2*(a + b*tan(c + d*
x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((-B^4*b^2*d^4*(5*a
^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 + 5*B^2
*a*b^4*d^2)/d^4)^(1/2))/2 - (8*B^3*a*b^2*(a^2 - 3*b^2)*(a^2 + b^2)^3)/d^3)
*(((20*B^4*a^2*b^8*d^4 - B^4*b^10*d^4 - 110*B^4*a^4*b^6*d^4 + 100*B^4*a^6*
b^4*d^4 - 25*B^4*a^8*b^2*d^4)^(1/2) + B^2*a^5*d^2 - 10*B^2*a^3*b^2*d^2 ...
```

Reduce [F]

$$\int (a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{a+\tan(dx+c)\,b}dx\right)a^3 + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)a\,b^2 + 3\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)a^2b$$

input

```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output

```
int(sqrt(tan(c + d*x)*b + a),x)*a**3 + int(sqrt(tan(c + d*x)*b + a)*tan(c
+ d*x)**3,x)*b**3 + 3*int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*a*b*
*2 + 3*int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a**2*b
```

3.335 $\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 182

$$\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{2a^{5/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{(a-ib)^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{5/2}(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}+\frac{2b(Ab+2aB)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

output
```
-2*a^(5/2)*A*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d+(a-I*b)^(5/2)*(A-I*
B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I*b)^(5/2)*(A+I*B)*a
rctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+2*b*(A*b+2*B*a)*(a+b*tan(d*
x+c))^(1/2)/d+2/3*b*B*(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.97

$$\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{2\left(-3a^{5/2}A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)+\frac{3}{2}(a-ib)^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)+\dots\right.}{\dots}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(2*(-3*a^(5/2)*A*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]] + (3*(a - I*b)^
(5/2)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/2 + (3*(a
 + I*b)^(5/2)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/2
 + 3*b*(A*b + 2*a*B)*Sqrt[a + b*Tan[c + d*x]] + b*B*(a + b*Tan[c + d*x])^(
3/2)))/(3*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.69 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4136, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2}{3}\int \frac{3}{2}\cot(c+dx)\sqrt{a+b\tan(c+dx)}\left(Aa^2+b(Ab+2aB)\tan^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}\left(Aa^2+b(Ab+2aB)\tan^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\int \frac{\sqrt{a+b\tan(c+dx)}\left(Aa^2+b(Ab+2aB)\tan(c+dx)^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{\tan(c+dx)}dx+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4130

$$2\int \frac{\cot(c+dx)\left(Aa^3+b\left(3Ba^2+3Aba-b^2B\right)\tan^2(c+dx)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{2\sqrt{a+b\tan(c+dx)}}dx+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\int \frac{\cot(c+dx)\left(Aa^3+b\left(3Ba^2+3Aba-b^2B\right)\tan^2(c+dx)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\int \frac{Aa^3+b\left(3Ba^2+3Aba-b^2B\right)\tan(c+dx)^2+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4136

$$a^3A\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a^3A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4022}$$

$$a^3A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(a+ib)^3(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)^3(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$a^3A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(a+ib)^3(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)^3(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4020}$$

$$a^3A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{i(a-ib)^3(B+iA)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(a+ib)^3(-B+iA)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{25}$$

$$a^3A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{i(a-ib)^3(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a+ib)^3(-B+iA)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \textbf{73}$$

$$a^3A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{(a+ib)^3(-B+iA)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a-ib)^3(B+iA)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \textbf{221}$$

$$a^3A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{(a-ib)^{5/2}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \textbf{4117}$$

$$\frac{a^3A\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{(a-ib)^{5/2}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{2a^3A\int \frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a-ib)^{5/2}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \; 221$$

$$-\frac{2a^{5/2}A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{(a-ib)^{5/2}(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d}+\frac{2b(2aB+Ab)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2bB(a+b\tan(c+dx))^{3/2}}{3d}$$

input
```
Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
((a - I*b)^(5/2)*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d - ((a + I
*b)^(5/2)*(I*A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d - (2*a^(5/2)*A*A
rcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d + (2*b*(A*b + 2*a*B)*Sqrt[a +
b*Tan[c + d*x]])/d + (2*b*B*(a + b*Tan[c + d*x])^(3/2))/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2372 vs. $2(152) = 304$.

Time = 0.15 (sec) , antiderivative size = 2373, normalized size of antiderivative = 13.04

method	result	size
derivativedivides	Expression too large to display	2373
default	Expression too large to display	2373

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2
+1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x
+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2
-2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(
a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1/2
)*a+2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1
/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(
1/2)*a+3/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2
)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2-1/4/
d*b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a
-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+3/4/d*b*
ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^
2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d*b^2/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2)
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)-3/d*b^2/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1
/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a-3/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2))*B*a^2-1/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arct...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4869 vs. $2(146) = 292$.

Time = 10.51 (sec) , antiderivative size = 9754, normalized size of antiderivative = 53.59

$$\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}(A + B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,18,6]%%%}+%%%{8,[0,16,6]%%%}+%%%{28,[0,14,6]%%%}+
%%%{56,[0
```

Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 29441, normalized size of antiderivative = 161.76

$$\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
((2*A*b^2 - 2*B*a*b)/d + (6*B*a*b)/d)*(a + b*tan(c + d*x))^(1/2) - atan(((
((32*(3*A^3*a^2*b^16*d^2 + 48*A^3*a^4*b^14*d^2 - 30*A^3*a^6*b^12*d^2 - 72*
A^3*a^8*b^10*d^2 + 3*A^3*a^10*b^8*d^2 + 6*B^3*a^5*b^13*d^2 + 8*B^3*a^7*b^1
1*d^2 + 3*B^3*a^9*b^9*d^2 - B^3*a*b^17*d^2 - A^2*B*a*b^17*d^2 + 3*A*B^2*a^
2*b^16*d^2 - 32*A*B^2*a^4*b^14*d^2 + 46*A*B^2*a^6*b^12*d^2 + 72*A*B^2*a^8*
b^10*d^2 - 9*A*B^2*a^10*b^8*d^2 - 16*A^2*B*a^3*b^15*d^2 + 150*A^2*B*a^5*b^
13*d^2 + 96*A^2*B*a^7*b^11*d^2 - 69*A^2*B*a^9*b^9*d^2))/d^5 - (((32*(4*A*a
*b^12*d^4 + 16*A*a^3*b^10*d^4 + 12*A*a^5*b^8*d^4 + 8*B*a^2*b^11*d^4 + 8*B*
a^4*b^9*d^4))/d^5 - (32*(16*b^10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x)
)^(1/2)*((((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - 80*B^2*a^
3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - 160*A*B
*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(A^4*a^10 + A^4*b^10 + B^4*a^1
0 + B^4*b^10 + 2*A^2*B^2*a^10 + 2*A^2*B^2*b^10 + 5*A^4*a^2*b^8 + 10*A^4*a^
4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^4*a^2*b^8 + 10*B^4*a^4*b^6 +
10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a^2*b^8 + 20*A^2*B^2*a^4*b^6 +
 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^(1/2) + A^2*a^5*d^2 - B^2*a^5*d
^2 - 10*A^2*a^3*b^2*d^2 + 10*B^2*a^3*b^2*d^2 - 2*A*B*b^5*d^2 + 5*A^2*a*b^4
*d^2 - 5*B^2*a*b^4*d^2 + 20*A*B*a^2*b^3*d^2 - 10*A*B*a^4*b*d^2)/(4*d^4))^(
1/2))/d^4)*((((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - 80*B^2
*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - 1...
```

Reduce [F]

$$\int \cot(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)\tan(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)\tan(dx+c)^2\,dx\right)a\,b^2 + 3\left(\int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)\tan(dx+c)\,dx\right)a^2b + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)\,dx\right)a^3$$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*cot(c + d*x)*tan(c + d*x)**3,x)*b**3 + 3*int(
sqrt(tan(c + d*x)*b + a)*cot(c + d*x)*tan(c + d*x)**2,x)*a*b**2 + 3*int(sq
rt(tan(c + d*x)*b + a)*cot(c + d*x)*tan(c + d*x),x)*a**2*b + int(sqrt(tan(
c + d*x)*b + a)*cot(c + d*x),x)*a**3
```

3.336 $\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 196

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{a^{3/2}(5Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{(a-ib)^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{b(aA+2bB)\sqrt{a+b\tan(c+dx)}}{d} - \frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}$$

output
```
-a^(3/2)*(5*A*b+2*B*a)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/d+(a-I*b)^(
5/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I*b)^(5/2)
*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d+b*(A*a+2*B*b)*(a+
b*tan(d*x+c))^(1/2)/d-a*A*cot(d*x+c)*(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 400 vs. 2(196) = 392.

Time = 0.73 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.04

$$\int \cot^2(c+dx)(a + b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{2bB\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d} + 2\left(-\frac{b(Ab+4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d} - 2\left(-\frac{-\frac{a^{5/2}(5Ab+2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4d} + \frac{i\sqrt{a-ib}\left(\frac{1}{4}\right.}{}}{}\right.\right.$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*b*B*Cot[c + d*x]*(a + b*Tan[c + d*x])^(3/2))/d + 2*(-((b*(A*b + 4*a*B)*
Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/d) - 2*(-((-1/4*(a^(5/2)*(5*A*b + 2
*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d + (I*Sqrt[a - I*b]*((I/
4)*a*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B) - (a*(a^3*A - 3*a*A*b^2 - 3*a
^2*b*B + b^3*B))/4)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((-a
+ I*b)*d) - (I*Sqrt[a + I*b]*((-1/4*I)*a*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*
b^2*B) - (a*(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B))/4)*ArcTanh[Sqrt[a + b
*Tan[c + d*x]]/Sqrt[a + I*b]])/((-a - I*b)*d))/a) + ((a^2*A - 2*A*b^2 - 6*
a*b*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(4*d)))
```

Rubi [A] (warning: unable to verify)

Time = 1.78 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.95, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 4088, 27, 3042, 4130, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4088}$$

$$\int \frac{1}{2}\cot(c+dx)\sqrt{a+b\tan(c+dx)}\left(b(aA+2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+2aB)\right)dx-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \cot(c+dx)\sqrt{a+b\tan(c+dx)}\left(b(aA+2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+2aB)\right)dx-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(aA+2bB)\tan(c+dx)^2-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+2aB)\right)}{\tan(c+dx)}dx-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{4130}$$

$$\frac{1}{2}\left(2\int \frac{\cot(c+dx)\left((5Ab+2aB)a^2-b\left(Aa^2-6bBa-2Ab^2\right)\tan^2(c+dx)-2\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)t\right.}{2\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int \frac{\cot(c+dx)\left((5Ab+2aB)a^2-b\left(Aa^2-6bBa-2Ab^2\right)\tan^2(c+dx)-2\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan\right.}{\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{(5Ab+2aB)a^2-b\left(Aa^2-6bBa-2Ab^2\right)\tan(c+dx)^2-2\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx\right.$$
$$\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}$$

$\Big\downarrow$ 4136

$$\frac{1}{2}\left(a^2(2aB+5Ab)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+\int -\frac{2\left(Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2\right.\right.}{\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(a^2(2aB+5Ab)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-2\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-\right.}{\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\left(a^2(2aB+5Ab)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-\right.}{\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}$$

$\Big\downarrow$ 4022

$$-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}+$$
$$\frac{1}{2}\left(a^2(2aB+5Ab)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{1}{2}(a-ib)^3(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}\right.\right.$$

$\Big\downarrow$ 3042

$$-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}+$$
$$\frac{1}{2}\left(a^2(2aB+5Ab)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{1}{2}(a-ib)^3(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}\right.\right.$$

$\Big\downarrow$ 4020

$$-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}+\frac{1}{2}\left(a^2(2aB+5Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{i(a-ib)^3(A-iB)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}}{2d}\right.\right.$$

$\downarrow$ 25

$$-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}+\frac{1}{2}\left(a^2(2aB+5Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{i(a+ib)^3(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d}{2d}\right.\right.$$

$\downarrow$ 73

$$-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}+\frac{1}{2}\left(a^2(2aB+5Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{(a-ib)^3(A-iB)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(}}{bd}\right.\right.$$

$\downarrow$ 221

$$-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}+\frac{1}{2}\left(a^2(2aB+5Ab)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-2\left(\frac{(a-ib)^{5/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib}{}\right.\right.$$

$\downarrow$ 4117

$$-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}+\frac{1}{2}\left(\frac{a^2(2aB+5Ab)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}-2\left(\frac{(a-ib)^{5/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{5/2}(A}{}\right.\right.$$

$\downarrow$ 73

$$-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}+\frac{1}{2}\left(\frac{2a^2(2aB+5Ab)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}-2\left(\frac{(a-ib)^{5/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+i}{}\right.\right.$$

$\downarrow$ 221

$$-\frac{aA\cot(c+dx)(a+b\tan(c+dx))^{3/2}}{d}+$$
$$\frac{1}{2}\left(-\frac{2a^{3/2}(2aB+5Ab)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{d}-2\left(\frac{(a-ib)^{5/2}(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d}+\frac{(a+ib)^{5/2}(A-\right.\right.$$

input
```
Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-((a*A*Cot[c + d*x]*(a + b*Tan[c + d*x])^(3/2))/d) + (-2*(((a - I*b)^(5/2)
*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + ((a + I*b)^(5/2)*(A + I
*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d) - (2*a^(3/2)*(5*A*b + 2*a*B)*Ar
cTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d + (2*b*(a*A + 2*b*B)*Sqrt[a + b
*Tan[c + d*x]])/d)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2389 vs. $2(168) = 336$.

Time = 0.16 (sec) , antiderivative size = 2390, normalized size of antiderivative = 12.19

method	result	size
derivativedivides	Expression too large to display	2390
default	Expression too large to display	2390

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+
2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)
*a-1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d
*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a
^2-2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1
/2)*a+2/d*b^2*(a+b*tan(d*x+c))^(1/2)*B-2/d*a^(5/2)*arctanh((a+b*tan(d*x+c)
)^(1/2)/a^(1/2))*B-1/d*a^2*A*(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)-5/d*b*A*a^(
3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))-1/d/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a
^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1/2)*a^2-3/d*b/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a^2+3/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2))*B*a-1/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*
(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))*B*(a^2+b^2)^(1/2)+1/4/d*b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4897 vs. $2(162) = 324$.

Time = 12.98 (sec) , antiderivative size = 9812, normalized size of antiderivative = 50.06

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^2,
x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,17,7]%%%}+%%%{8,[0,15,7]%%%}+%%%{28,[0,13,7]%%%}+
%%%{56,[0
```

Mupad [B] (verification not implemented)

Time = 7.22 (sec) , antiderivative size = 31186, normalized size of antiderivative = 159.11

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(2*B*b^2*(a + b*tan(c + d*x))^(1/2))/d - atan(((((8*(176*A^3*a^5*b^13*d^2
- 400*A^3*a^3*b^15*d^2 + 488*A^3*a^7*b^11*d^2 - 92*A^3*a^9*b^9*d^2 + 12*B^
3*a^2*b^16*d^2 + 192*B^3*a^4*b^14*d^2 - 120*B^3*a^6*b^12*d^2 - 288*B^3*a^8
*b^10*d^2 + 12*B^3*a^10*b^8*d^2 + 4*A^3*a*b^17*d^2 + 4*A*B^2*a*b^17*d^2 +
464*A*B^2*a^3*b^15*d^2 - 920*A*B^2*a^5*b^13*d^2 - 1104*A*B^2*a^7*b^11*d^2
+ 276*A*B^2*a^9*b^9*d^2 + 172*A^2*B*a^2*b^16*d^2 - 1468*A^2*B*a^4*b^14*d^2
 - 776*A^2*B*a^6*b^12*d^2 + 828*A^2*B*a^8*b^10*d^2 - 36*A^2*B*a^10*b^8*d^2
))/d^5 - (((8*(16*B*a*b^12*d^4 + 128*A*a^2*b^11*d^4 + 128*A*a^4*b^9*d^4 +
64*B*a^3*b^10*d^4 + 48*B*a^5*b^8*d^4))/d^5 - (16*(32*b^10*d^4 + 48*a^2*b^8
*d^4)*(a + b*tan(c + d*x))^(1/2)*((((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^
2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 4
0*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(A^4*
a^10 + A^4*b^10 + B^4*a^10 + B^4*b^10 + 2*A^2*B^2*a^10 + 2*A^2*B^2*b^10 +
5*A^4*a^2*b^8 + 10*A^4*a^4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^4*a^
2*b^8 + 10*B^4*a^4*b^6 + 10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a^2*b
^8 + 20*A^2*B^2*a^4*b^6 + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^(1/2)
- A^2*a^5*d^2 + B^2*a^5*d^2 + 10*A^2*a^3*b^2*d^2 - 10*B^2*a^3*b^2*d^2 + 2*
A*B*b^5*d^2 - 5*A^2*a*b^4*d^2 + 5*B^2*a*b^4*d^2 - 20*A*B*a^2*b^3*d^2 + 10*
A*B*a^4*b*d^2)/(4*d^4))^(1/2))/d^4)*((((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80
*A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d...
```

Reduce [F]

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \cot(dx+c)^2\,(a+\tan(dx+c)\,b)^{\frac{5}{2}}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.337 $\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 220

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{a}(8a^2A-15Ab^2-20abB)\,\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4d} - \frac{(a-ib)^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} - \frac{(a+ib)^{5/2}(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} - \frac{a(7Ab+4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4d} - \frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}$$

output
```
1/4*a^(1/2)*(8*A*a^2-15*A*b^2-20*B*a*b)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(
1/2))/d-(a-I*b)^(5/2)*(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2)
)/d-(a+I*b)^(5/2)*(A+I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-
1/4*a*(7*A*b+4*B*a)*cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)/d-1/2*a*A*cot(d*x+c)
^2*(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 448 vs. 2(220) = 440.

Time = 1.63 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.04

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{-\sqrt{a}(8a^2A-15Ab^2-20abB)\,\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)+4(a-ib)^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-}{}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
-1/4*(-(Sqrt[a]*(8*a^2*A - 15*A*b^2 - 20*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c +
 d*x]]/Sqrt[a]]) + 4*(a - I*b)^(5/2)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c +
d*x]]/Sqrt[a - I*b]] + 4*a^2*A*Sqrt[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*
x]]/Sqrt[a + I*b]] + (8*I)*a*A*Sqrt[a + I*b]*b*ArcTanh[Sqrt[a + b*Tan[c +
d*x]]/Sqrt[a + I*b]] - 4*A*Sqrt[a + I*b]*b^2*ArcTanh[Sqrt[a + b*Tan[c + d*
x]]/Sqrt[a + I*b]] + (4*I)*a^2*Sqrt[a + I*b]*B*ArcTanh[Sqrt[a + b*Tan[c +
d*x]]/Sqrt[a + I*b]] - 8*a*Sqrt[a + I*b]*b*B*ArcTanh[Sqrt[a + b*Tan[c + d*
x]]/Sqrt[a + I*b]] - (4*I)*Sqrt[a + I*b]*b^2*B*ArcTanh[Sqrt[a + b*Tan[c +
d*x]]/Sqrt[a + I*b]] + 9*a*A*b*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]] + 4*a
^2*B*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]] + 2*a^2*A*Cot[c + d*x]^2*Sqrt[a
 + b*Tan[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 1.87 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.97, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{1}{2}\int \frac{1}{2}\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}\left(-b(aA-4bB)\tan^2(c+dx)-4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+4aB)\right)dx- \frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \cot^2(c+dx)\sqrt{a+b\tan(c+dx)}\left(-b(aA-4bB)\tan^2(c+dx)-4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+4aB)\right)dx- \frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(aA-4bB)\tan(c+dx)^2-4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+4aB)\right)}{\tan(c+dx)^2}dx \frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{4128}$$

$$\frac{1}{4}\left(\int -\frac{\cot(c+dx)\left(b\left(4Ba^2+9Aba-8b^2B\right)\tan^2(c+dx)+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a(8\right.}{2\sqrt{a+b\tan(c+dx)}} \right. \frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{\cot(c+dx)\left(b\left(4Ba^2+9Aba-8b^2B\right)\tan^2(c+dx)+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\right.}{\sqrt{a+b\tan(c+dx)}} \right. \frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{b(4Ba^2+9Aba-8b^2B)\tan(c+dx)^2+8(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)+a(8Aa^2-20b}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}$$

$\downarrow$ 4136

$$\frac{1}{4}\left(\frac{1}{2}\left(-a(8a^2A-20abB-15Ab^2)\int\frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-\int\frac{8(Ba^3+3Aba^2-3b^2Ba-Ab^3-}{\sqrt{a}}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}$$

$\downarrow$ 27

$$\frac{1}{4}\left(\frac{1}{2}\left(-a(8a^2A-20abB-15Ab^2)\int\frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-8\int\frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-}{\sqrt{a}}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}\left(-a(8a^2A-20abB-15Ab^2)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-8\int\frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-}{\sqrt{a}}\right.\right.$$
$$\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}$$

$\downarrow$ 4022

$$-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}+$$
$$\frac{1}{4}\left(-\frac{a(4aB+7Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{1}{2}\left(-a(8a^2A-20abB-15Ab^2)\int\frac{\tan(c+dx)^2+}{\tan(c+dx)\sqrt{a+b\tan}}\right.\right.$$

$\downarrow$ 3042

$$-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}+$$
$$\frac{1}{4}\left(-\frac{a(4aB+7Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{1}{2}\left(-a(8a^2A-20abB-15Ab^2)\int\frac{\tan(c+dx)^2+}{\tan(c+dx)\sqrt{a+b\tan}}\right.\right.$$

$\downarrow$ 4020

$$-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}+$$
$$\frac{1}{4}\left(-\frac{a(4aB+7Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{1}{2}\left(-a\left(8a^2A-20abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+}{\tan(c+dx)\sqrt{a+b\tan}}\right.\right.$$

$\downarrow$ 25

$$-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}+$$
$$\frac{1}{4}\left(-\frac{a(4aB+7Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{1}{2}\left(-a\left(8a^2A-20abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+}{\tan(c+dx)\sqrt{a+b\tan}}\right.\right.$$

$\downarrow$ 73

$$-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}+$$
$$\frac{1}{4}\left(-\frac{a(4aB+7Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{1}{2}\left(-a\left(8a^2A-20abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+}{\tan(c+dx)\sqrt{a+b\tan}}\right.\right.$$

$\downarrow$ 221

$$-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}+$$
$$\frac{1}{4}\left(-\frac{a(4aB+7Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{1}{2}\left(-a\left(8a^2A-20abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+}{\tan(c+dx)\sqrt{a+b\tan}}\right.\right.$$

$\downarrow$ 4117

$$-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}+$$
$$\frac{1}{4}\left(-\frac{a(4aB+7Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{1}{2}\left(-\frac{a\left(8a^2A-20abB-15Ab^2\right)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c-}{d}\right.\right.$$

$\downarrow$ 73

$$-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}+$$
$$\frac{1}{4}\left(-\frac{a(4aB+7Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{1}{2}\left(-\frac{2a\left(8a^2A-20abB-15Ab^2\right)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+}}{bd}\right.\right.$$

$\downarrow$ 221

$$-\frac{aA\cot^2(c+dx)(a+b\tan(c+dx))^{3/2}}{2d}+$$
$$\frac{1}{4}\left(-\frac{a(4aB+7Ab)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}+\frac{1}{2}\left(\frac{2\sqrt{a}\left(8a^2A-20abB-15Ab^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right.}{d}\right.\right.$$

input
```
Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(a*A*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(3/2))/d + ((-8*(((a - I*b)^
(5/2)*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d - ((a + I*b)^(5/2)*(
I*A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d) + (2*Sqrt[a]*(8*a^2*A - 15
*A*b^2 - 20*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d)/2 - (a*(7
*A*b + 4*a*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/d)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2487 vs. $2(186) = 372$.

Time = 0.17 (sec) , antiderivative size = 2488, normalized size of antiderivative = 11.31

method	result	size
derivativedivides	Expression too large to display	2488
default	Expression too large to display	2488

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-
1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+
c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+
2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1/2)
*a-2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1
/2)*a-3/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2+1/4/d
*b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-
(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-3/4/d*b*l
n((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2
+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d*b^2/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)+3/d*b^2/(2*(a^2+b^2)^(1/
2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/
2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a+3/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2))*B*a^2+1/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4932 vs. $2(180) = 360$.

Time = 15.97 (sec) , antiderivative size = 9885, normalized size of antiderivative = 44.93

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 7.35 (sec) , antiderivative size = 32561, normalized size of antiderivative = 148.00

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
atan(-((((3708*A^3*a^2*b^16*d^2 - 6912*A^3*a^4*b^14*d^2 - 5820*A^3*a^6*b^1
2*d^2 + 4608*A^3*a^8*b^10*d^2 - 192*A^3*a^10*b^8*d^2 - 6400*B^3*a^3*b^15*d
^2 + 2816*B^3*a^5*b^13*d^2 + 7808*B^3*a^7*b^11*d^2 - 1472*B^3*a^9*b^9*d^2
+ 64*B^3*a*b^17*d^2 - 1856*A^2*B*a*b^17*d^2 - 7552*A*B^2*a^2*b^16*d^2 + 23
488*A*B^2*a^4*b^14*d^2 + 16256*A*B^2*a^6*b^12*d^2 - 14208*A*B^2*a^8*b^10*d
^2 + 576*A*B^2*a^10*b^8*d^2 + 20504*A^2*B*a^3*b^15*d^2 - 5000*A^2*B*a^5*b^
13*d^2 - 22944*A^2*B*a^7*b^11*d^2 + 4416*A^2*B*a^9*b^9*d^2)/(2*d^5) - (((1
664*A*a*b^12*d^4 + 896*A*a^3*b^10*d^4 - 768*A*a^5*b^8*d^4 + 2048*B*a^2*b^1
1*d^4 + 2048*B*a^4*b^9*d^4)/(2*d^5) - ((512*b^10*d^4 + 768*a^2*b^8*d^4)*(a
 + b*tan(c + d*x))^(1/2)*(((((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3*b^
2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*B^2*a*
b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(A^4*a^10 + A
^4*b^10 + B^4*a^10 + B^4*b^10 + 2*A^2*B^2*a^10 + 2*A^2*B^2*b^10 + 5*A^4*a^
2*b^8 + 10*A^4*a^4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^4*a^2*b^8 +
10*B^4*a^4*b^6 + 10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a^2*b^8 + 20*
A^2*B^2*a^4*b^6 + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^(1/2) + A^2*a^
5*d^2 - B^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 10*B^2*a^3*b^2*d^2 - 2*A*B*b^5*
d^2 + 5*A^2*a*b^4*d^2 - 5*B^2*a*b^4*d^2 + 20*A*B*a^2*b^3*d^2 - 10*A*B*a^4*
b*d^2)/(4*d^4))^(1/2))/d^4)*(((((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 80*A^2*a^3
*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2 + 40*...
```

Reduce [F]

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \cot(dx+c)^3\,(a+\tan(dx+c)\,b)^{\frac{5}{2}}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.338 $\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 277

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{(40a^2Ab-5Ab^3+16a^3B-30ab^2B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8\sqrt{a}d} - \frac{(a-ib)^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{5/2}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d} + \frac{(8a^2A-11Ab^2-18abB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{8d} - \frac{a(3Ab+2aB)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{4d} - \frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

output
```
1/8*(40*A*a^2*b-5*A*b^3+16*B*a^3-30*B*a*b^2)*arctanh((a+b*tan(d*x+c))^(1/2
)/a^(1/2))/a^(1/2)/d-(a-I*b)^(5/2)*(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/
(a-I*b)^(1/2))/d+(a+I*b)^(5/2)*(I*A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I
*b)^(1/2))/d+1/8*(8*A*a^2-11*A*b^2-18*B*a*b)*cot(d*x+c)*(a+b*tan(d*x+c))^(
1/2)/d-1/4*a*(3*A*b+2*B*a)*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)/d-1/3*a*A*c
ot(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 35.01 (sec) , antiderivative size = 40192, normalized size of antiderivative = 145.10

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 2.34 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^4}dx$$

$\downarrow$ 4088

$$\frac{1}{3}\int \frac{3}{2}\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}\left(-b(aA-2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)dx-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\frac{1}{2}\int \cot^3(c+dx)\sqrt{a+b\tan(c+dx)}\left(-b(aA-2bB)\tan^2(c+dx)-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)dx-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{2}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(aA-2bB)\tan(c+dx)^2-2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(3Ab+2aB)\right)}{\tan(c+dx)^3}dx-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4128

$$\frac{1}{2}\left(\frac{1}{2}\int -\frac{\cot^2(c+dx)\left(b\left(6Ba^2+13Aba-8b^2B\right)\tan^2(c+dx)+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)-\right.}{2\sqrt{a+b\tan(c+dx)}}\right.-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{\cot^2(c+dx)\left(b\left(6Ba^2+13Aba-8b^2B\right)\tan^2(c+dx)+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)-\right.}{\sqrt{a+b\tan(c+dx)}}\right.-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{b\left(6Ba^2+13Aba-8b^2B\right)\tan(c+dx)^2+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(8Aa^2-18\right.}{\tan(c+dx)^2\sqrt{a+b\tan(c+dx)}}\right.$$

$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4132

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int -\frac{\cot(c+dx)\left(-ab\left(8Aa^2-18bBa-11Ab^2\right)\tan^2(c+dx)-16a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)+a\left(16Ba^3+40Aba^2-30b^2Ba-5\right.\right.}{2\sqrt{a+b\tan(c+dx)}}}{a}\right.\right.$$

$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\left(8a^2A-18abB-11Ab^2\right)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int \cot(c+dx)\left(-ab\left(8Aa^2-18bBa-11Ab^2\right)\tan^2(c+dx)-\right.}{}\right.\right.$$

$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\left(8a^2A-18abB-11Ab^2\right)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int \frac{-ab\left(8Aa^2-18bBa-11Ab^2\right)\tan(c+dx)^2-16a\left(Aa^3-3\right.}{\tan(c-}}{}\right.\right.$$

$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4136

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\left(8a^2A-18abB-11Ab^2\right)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int -\frac{16\left(a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+a\left(Ba^3+3Aba^2-\right.\right.}{\sqrt{a+b\tan(c+dx)}}}{}\right.\right.$$

$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\left(8a^2A-18abB-11Ab^2\right)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{a\left(16a^3B+40a^2Ab-30ab^2B-5Ab^3\right)\int \frac{\cot(}{}}{}\right.\right.$$

$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\left(8a^2A-18abB-11Ab^2\right)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}-\frac{a\left(16a^3B+40a^2Ab-30ab^2B-5Ab^3\right)\int \frac{}{\tan}}{}\right.\right.$$

$$\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4022

$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+$$

$$\frac{1}{2}\left(-\frac{a(2aB+3Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(\frac{\left(8a^2A-18abB-11Ab^2\right)\cot(c+dx)\sqrt{a+b\tan(c+}}{d}\right.\right.$$

$\downarrow$ 3042

$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+$$

$$\frac{1}{2}\left(-\frac{a(2aB+3Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(\frac{\left(8a^2A-18abB-11Ab^2\right)\cot(c+dx)\sqrt{a+b\tan(c+}}{d}\right.\right.$$

$\downarrow$ 4020

$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+$$

$$\frac{1}{2}\left(-\frac{a(2aB+3Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(\frac{\left(8a^2A-18abB-11Ab^2\right)\cot(c+dx)\sqrt{a+b\tan(c+}}{d}\right.\right.$$

$\downarrow$ 25

$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+$$

$$\frac{1}{2}\left(-\frac{a(2aB+3Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(\frac{(8a^2A-18abB-11Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+\right.\right.$$

$\downarrow$ 73

$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+$$

$$\frac{1}{2}\left(-\frac{a(2aB+3Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(\frac{(8a^2A-18abB-11Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+\right.\right.$$

$\downarrow$ 221

$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+$$

$$\frac{1}{2}\left(-\frac{a(2aB+3Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(\frac{(8a^2A-18abB-11Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+\right.\right.$$

$\downarrow$ 4117

$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+$$

$$\frac{1}{2}\left(-\frac{a(2aB+3Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(\frac{(8a^2A-18abB-11Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+\right.\right.$$

$\downarrow$ 73

$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+$$

$$\frac{1}{2}\left(-\frac{a(2aB+3Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(\frac{(8a^2A-18abB-11Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+\right.\right.$$

$$\Big\downarrow \text{221}$$

$$-\frac{aA\cot^3(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+$$

$$\frac{1}{2}\left(-\frac{a(2aB+3Ab)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(\frac{(8a^2A-18abB-11Ab^2)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{d}\right.\right.$$

input
```
Int[Cot[c + d*x]^4*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-1/3*(a*A*Cot[c + d*x]^3*(a + b*Tan[c + d*x])^(3/2))/d + (-1/2*(a*(3*A*b +
 2*a*B)*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/d + (-1/2*(-16*((a*(a - I
*b)^(5/2)*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d + (a*(a + I*b)^(
5/2)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d) - (2*Sqrt[a]*(40*a^2
*A*b - 5*A*b^3 + 16*a^3*B - 30*a*b^2*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/S
qrt[a]])/d)/a + ((8*a^2*A - 11*A*b^2 - 18*a*b*B)*Cot[c + d*x]*Sqrt[a + b*T
an[c + d*x]])/d)/4)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2667 vs. $2(239) = 478$.

Time = 0.16 (sec) , antiderivative size = 2668, normalized size of antiderivative = 9.63

method	result	size
derivativedivides	Expression too large to display	2668
default	Expression too large to display	2668

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)+(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2
-2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(
a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2
)*a+1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(
d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*
a^2+2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1
/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(
1/2)*a-11/8/d/tan(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*A+2/d*a^(5/2)*arctanh((a
+b*tan(d*x+c))^(1/2)/a^(1/2))*B+5/d*b*A*a^(3/2)*arctanh((a+b*tan(d*x+c))^(
1/2)/a^(1/2))-5/8/d*b^3/a^(1/2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))*A-
15/4/d*b^2*a^(1/2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))*B+5/3/d/tan(d*x
+c)^3*(a+b*tan(d*x+c))^(3/2)*A*a-5/8/d/tan(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)
*A*a^2+1/d/b^2/tan(d*x+c)^3*(a+b*tan(d*x+c))^(5/2)*A*a^2-9/4/d/b/tan(d*x+c
)^3*(a+b*tan(d*x+c))^(5/2)*B*a-2/d/b^2/tan(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)
*A*a^3+4/d/b/tan(d*x+c)^3*(a+b*tan(d*x+c))^(3/2)*B*a^2+1/d/b^2/tan(d*x+c)^
3*(a+b*tan(d*x+c))^(1/2)*A*a^4-7/4/d/b/tan(d*x+c)^3*(a+b*tan(d*x+c))^(1/2)
*B*a^3+1/d/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(
1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^
(1/2)*a^2+3/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4971 vs. $2(233) = 466$.

Time = 24.74 (sec) , antiderivative size = 9962, normalized size of antiderivative = 35.96

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**4*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 33949, normalized size of antiderivative = 122.56

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(atan((((((a + b*tan(c + d*x))^(1/2)*(153*A^4*b^20 + 128*B^4*b^20 + 231*A^
2*B^2*b^20 - 7*A^4*a^2*b^18 + 9895*A^4*a^4*b^16 - 27465*A^4*a^6*b^14 + 263
20*A^4*a^8*b^12 - 832*A^4*a^10*b^10 + 128*A^4*a^12*b^8 - 132*B^4*a^2*b^18
+ 16380*B^4*a^4*b^16 - 25596*B^4*a^6*b^14 + 21060*B^4*a^8*b^12 - 4032*B^4*
a^10*b^10 + 384*B^4*a^12*b^8 + 6811*A^2*B^2*a^2*b^18 - 61315*A^2*B^2*a^4*b
^16 + 184661*A^2*B^2*a^6*b^14 - 121620*A^2*B^2*a^8*b^12 + 23296*A^2*B^2*a^
10*b^10 - 300*A*B^3*a*b^19 + 600*A^3*B*a*b^19 + 17860*A*B^3*a^3*b^17 - 917
00*A*B^3*a^5*b^15 + 110172*A*B^3*a^7*b^13 - 43520*A*B^3*a^9*b^11 + 4352*A*
B^3*a^11*b^9 - 12860*A^3*B*a^3*b^17 + 79680*A^3*B*a^5*b^15 - 126700*A^3*B*
a^7*b^13 + 40960*A^3*B*a^9*b^11 - 1280*A^3*B*a^11*b^9))/(64*d^4) + (((3225
*A^3*a^3*b^15*d^2 - 1088*A^3*a^5*b^13*d^2 - 3984*A^3*a^7*b^11*d^2 + 736*A^
3*a^9*b^9*d^2 + 1854*B^3*a^2*b^16*d^2 - 3456*B^3*a^4*b^14*d^2 - 2910*B^3*a
^6*b^12*d^2 + 2304*B^3*a^8*b^10*d^2 - 96*B^3*a^10*b^8*d^2 + (295*A^2*B*b^1
8*d^2)/2 - 407*A^3*a*b^17*d^2 + 1178*A*B^2*a*b^17*d^2 - 10572*A*B^2*a^3*b^
15*d^2 + 1930*A*B^2*a^5*b^13*d^2 + 11472*A*B^2*a^7*b^11*d^2 - 2208*A*B^2*a
^9*b^9*d^2 - 4716*A^2*B*a^2*b^16*d^2 + (22193*A^2*B*a^4*b^14*d^2)/2 + 8568
*A^2*B*a^6*b^12*d^2 - 7104*A^2*B*a^8*b^10*d^2 + 288*A^2*B*a^10*b^8*d^2)/(1
6*d^5) + ((((a + b*tan(c + d*x))^(1/2)*(320*A^2*a^3*b^12*d^2 - 19456*A^2*a
^5*b^10*d^2 + 1280*A^2*a^7*b^8*d^2 - 2320*B^2*a^3*b^12*d^2 + 16896*B^2*a^5
*b^10*d^2 - 2304*B^2*a^7*b^8*d^2 - 2048*A*B*b^15*d^2 + 4764*A^2*a*b^14*...
```

Reduce [F]

$$\int \cot^4(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \cot(dx+c)^4(a+\tan(dx+c)\,b)^{\frac{5}{2}}(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^4*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.339 $\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 342

$$\begin{aligned}\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =\\ -\frac{(128a^4A-240a^2Ab^2-5Ab^4-320a^3bB+40ab^3B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{64a^{3/2}d}\\ +\frac{(a-ib)^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}\\ +\frac{(a+ib)^{5/2}(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}\\ +\frac{(144a^2Ab-5Ab^3+64a^3B-88ab^2B)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{64ad}\\ +\frac{(48a^2A-59Ab^2-104abB)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{96d}\\ -\frac{a(11Ab+8aB)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{24d}\\ -\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}\end{aligned}$$

output
```
-1/64*(128*A*a^4-240*A*a^2*b^2-5*A*b^4-320*B*a^3*b+40*B*a*b^3)*arctanh((a+
b*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+(a-I*b)^(5/2)*(A-I*B)*arctanh((a+b*
tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d+(a+I*b)^(5/2)*(A+I*B)*arctanh((a+b*tan(
d*x+c))^(1/2)/(a+I*b)^(1/2))/d+1/64*(144*A*a^2*b-5*A*b^3+64*B*a^3-88*B*a*b
^2)*cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)/a/d+1/96*(48*A*a^2-59*A*b^2-104*B*a*
b)*cot(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)/d-1/24*a*(11*A*b+8*B*a)*cot(d*x+c)^
3*(a+b*tan(d*x+c))^(1/2)/d-1/4*a*A*cot(d*x+c)^4*(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 35.79 (sec) , antiderivative size = 43365, normalized size of antiderivative = 126.80

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 3.09 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.04, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.758$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^5}dx$$

$\Big\downarrow$ 4088

$$\frac{1}{4}\int \frac{1}{2}\cot^4(c+$$
$$dx)\sqrt{a+b\tan(c+dx)}\left(-b(5aA-8bB)\tan^2(c+dx)-8\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(11Ab+8aB)\right)dx$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\int \cot^4(c+$$
$$dx)\sqrt{a+b\tan(c+dx)}\left(-b(5aA-8bB)\tan^2(c+dx)-8\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(11Ab+8aB)\right)dx$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(5aA-8bB)\tan(c+dx)^2-8\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(11Ab+8aB)\right)}{\tan(c+dx)^4}d$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 4128

$$\frac{1}{8}\left(\frac{1}{3}\int -\frac{\cot^3(c+dx)\left(b\left(40Ba^2+85Aba-48b^2B\right)\tan^2(c+dx)+48\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+d\right.}{2\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(-\frac{1}{6}\int \frac{\cot^3(c+dx)\left(b\left(40Ba^2+85Aba-48b^2B\right)\tan^2(c+dx)+48\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+d\right.}{\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(-\frac{1}{6}\int \frac{b\left(40Ba^2+85Aba-48b^2B\right)\tan(c+dx)^2+48\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(48Aa^2\right.}{\tan(c+dx)^3\sqrt{a+b\tan(c+dx)}}\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 4132

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\int -\frac{3\cot^2(c+dx)\left(-ab\left(48Aa^2-104bBa-59Ab^2\right)\tan^2(c+dx)-64a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)+a\left(64Ba^3+144Aba^2-88b^2\right.\right.}{2\sqrt{a+b\tan(c+dx)}}}{2a}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\int \frac{\cot^2(c+dx)\left(-ab\left(48Aa^2-104bBa-59Ab^2\right)\tan^2\right.}{}}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\int \frac{-ab\left(48Aa^2-104bBa-59Ab^2\right)\tan(c+dx)^2-64a}{}}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 4132

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{\int \frac{\cot(c+dx)\left(128Aa^5-320bBa^4-240Ab^2a^3+40b^3\right.}{}}{}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{\int \frac{\cot(c+dx)\left(128\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right.}{}}{}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{\int \frac{128\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)a^2+}{}}{}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 4136

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{\int \frac{128\left(a^2\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-a^2\left(Aa^3-\right.\right.}{\sqrt{a+b\tan(c+dx)}}}{}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{128\int \frac{a^2\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-a^2\left(Aa^3-\right.}{\sqrt{a+b\tan(c+}}}{}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$\Big\downarrow$ 3042

$$\frac{1}{8}\left(\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}-\frac{3\left(-\frac{128\int\frac{a^2\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-a^2\left(Aa^3-\right.}{\sqrt{a+b\tan(c+d}}}{}\right.}{}\right.\right.$$

$$\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}$$

$$\Big\downarrow \text{4022}$$

$$-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}+$$

$$\frac{1}{8}\left(-\frac{a(8aB+11Ab)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan}}{2d}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}+$$

$$\frac{1}{8}\left(-\frac{a(8aB+11Ab)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan}}{2d}\right.\right.$$

$$\Big\downarrow \text{4020}$$

$$-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}+$$

$$\frac{1}{8}\left(-\frac{a(8aB+11Ab)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan}}{2d}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}+$$

$$\frac{1}{8}\left(-\frac{a(8aB+11Ab)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan}}{2d}\right.\right.$$

$\downarrow$ 73

$$-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}+$$

$$\frac{1}{8}\left(-\frac{a(8aB+11Ab)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan}}{2d}\right.\right.$$

$\downarrow$ 221

$$-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}+$$

$$\frac{1}{8}\left(-\frac{a(8aB+11Ab)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(\frac{\left(48a^2A-104abB-59Ab^2\right)\cot^2(c+dx)\sqrt{a+b\tan}}{2d}\right.\right.$$

$\downarrow$ 4117

$$-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}+$$

$$\frac{1}{8}\left(-\frac{a(8aB+11Ab)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(\frac{(48a^2A-104abB-59Ab^2)\cot^2(c+dx)\sqrt{a+b\tan}}{2d}\right.\right.$$

↓ 73

$$-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}+$$

$$\frac{1}{8}\left(-\frac{a(8aB+11Ab)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(\frac{(48a^2A-104abB-59Ab^2)\cot^2(c+dx)\sqrt{a+b\tan}}{2d}\right.\right.$$

↓ 221

$$-\frac{aA\cot^4(c+dx)(a+b\tan(c+dx))^{3/2}}{4d}+$$

$$\frac{1}{8}\left(-\frac{a(8aB+11Ab)\cot^3(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}+\frac{1}{6}\left(\frac{(48a^2A-104abB-59Ab^2)\cot^2(c+dx)\sqrt{a+b\tan}}{2d}\right.\right.$$

input
```
Int[Cot[c + d*x]^5*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
-1/4*(a*A*Cot[c + d*x]^4*(a + b*Tan[c + d*x])^(3/2))/d + (-1/3*(a*(11*A*b
+ 8*a*B)*Cot[c + d*x]^3*Sqrt[a + b*Tan[c + d*x]])/d + (((48*a^2*A - 59*A*b
^2 - 104*a*b*B)*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/(2*d) - (3*(-1/2*
(128*((a^2*(a - I*b)^(5/2)*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/d
 - (a^2*(a + I*b)^(5/2)*(I*A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/d) -
 (2*Sqrt[a]*(128*a^4*A - 240*a^2*A*b^2 - 5*A*b^4 - 320*a^3*b*B + 40*a*b^3*
B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/d)/a - ((144*a^2*A*b - 5*A*b
^3 + 64*a^3*B - 88*a*b^2*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/d))/(4*
a))/6)/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2913 vs. $2(300) = 600$.

Time = 0.18 (sec) , antiderivative size = 2914, normalized size of antiderivative = 8.52

method	result	size
derivativedivides	Expression too large to display	2914
default	Expression too large to display	2914

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2
+1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x
+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2
-2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(
a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(1/2
)*a+2/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1
/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*(a^2+b^2)^(
1/2)*a+3/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2
)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2-1/4/
d*b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a
-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)+3/4/d*b*
ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^
2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d*b^2/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2)
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*(a^2+b^2)^(1/2)-3/d*b^2/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1
/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a-3/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+
b^2)^(1/2)-2*a)^(1/2))*B*a^2-1/d*b^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arct...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5042 vs. $2(294) = 588$.

Time = 53.34 (sec) , antiderivative size = 10104, normalized size of antiderivative = 29.54

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**5*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^5*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.23 (sec) , antiderivative size = 36736, normalized size of antiderivative = 107.42

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^5*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
atan((((((10240*A*a*b^14*d^4 + 436224*A*a^3*b^12*d^4 + 229376*A*a^5*b^10*
d^4 - 196608*A*a^7*b^8*d^4 - 81920*B*a^2*b^13*d^4 + 442368*B*a^4*b^11*d^4
+ 524288*B*a^6*b^9*d^4)/(512*a^2*d^5) - ((131072*a^2*b^10*d^4 + 196608*a^4
*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*((((8*B^2*a^5*d^2 - 8*A^2*a^5*d^2 + 8
0*A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*A^2*a*b^4*d^2
 + 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/64 - d^4*(
A^4*a^10 + A^4*b^10 + B^4*a^10 + B^4*b^10 + 2*A^2*B^2*a^10 + 2*A^2*B^2*b^1
0 + 5*A^4*a^2*b^8 + 10*A^4*a^4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a^8*b^2 + 5*B^
4*a^2*b^8 + 10*B^4*a^4*b^6 + 10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 + 10*A^2*B^2*a
^2*b^8 + 20*A^2*B^2*a^4*b^6 + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2*a^8*b^2))^(1
/2) + A^2*a^5*d^2 - B^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 10*B^2*a^3*b^2*d^2
- 2*A*B*b^5*d^2 + 5*A^2*a*b^4*d^2 - 5*B^2*a*b^4*d^2 + 20*A*B*a^2*b^3*d^2 -
 10*A*B*a^4*b*d^2)/(4*d^4))^(1/2))/(256*a^2*d^4))*((((8*B^2*a^5*d^2 - 8*A^
2*a^5*d^2 + 80*A^2*a^3*b^2*d^2 - 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 - 40*
A^2*a*b^4*d^2 + 40*B^2*a*b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)
^2/64 - d^4*(A^4*a^10 + A^4*b^10 + B^4*a^10 + B^4*b^10 + 2*A^2*B^2*a^10 +
2*A^2*B^2*b^10 + 5*A^4*a^2*b^8 + 10*A^4*a^4*b^6 + 10*A^4*a^6*b^4 + 5*A^4*a
^8*b^2 + 5*B^4*a^2*b^8 + 10*B^4*a^4*b^6 + 10*B^4*a^6*b^4 + 5*B^4*a^8*b^2 +
 10*A^2*B^2*a^2*b^8 + 20*A^2*B^2*a^4*b^6 + 20*A^2*B^2*a^6*b^4 + 10*A^2*B^2
*a^8*b^2))^(1/2) + A^2*a^5*d^2 - B^2*a^5*d^2 - 10*A^2*a^3*b^2*d^2 + 10*...
```

Reduce [F]

$$\int \cot^5(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \cot(dx+c)^5 (a+\tan(dx+c)\,b)^{\frac{5}{2}} (A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^5*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.340 $\int (-a+b\tan(c+dx))(a+b\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 151

$$\int (-a+b\tan(c+dx))(a+b\tan(c+dx))^{5/2}\,dx=\frac{(ia-b)(a-ib)^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{d}-\frac{(a+ib)^{5/2}(ia+b)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{d}-\frac{2b(a^2+b^2)\sqrt{a+b\tan(c+dx)}}{d}+\frac{2b(a+b\tan(c+dx))^{5/2}}{5d}$$

output
```
(I*a-b)*(a-I*b)^(5/2)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/d-(a+I
*b)^(5/2)*(I*a+b)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/d-2*b*(a^2
+b^2)*(a+b*tan(d*x+c))^(1/2)/d+2/5*b*(a+b*tan(d*x+c))^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.28

$$\int (-a+b\tan(c+dx))(a+b\tan(c+dx))^{5/2}\,dx = \frac{\cos(c+dx)(a-b\tan(c+dx))\left(5i(a-ib)^{5/2}(a+ib)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-5i(a-ib)(a\right.}{5d(a\cos}$$

input
```
Integrate[(-a + b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]*(a - b*Tan[c + d*x])*((5*I)*(a - I*b)^(5/2)*(a + I*b)*ArcTan
h[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - (5*I)*(a - I*b)*(a + I*b)^(5/2
)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*b*Sqrt[a + b*Tan[c +
 d*x]]*(-4*a^2 - 5*b^2 + 2*a*b*Tan[c + d*x] + b^2*Tan[c + d*x]^2)))/(5*d*(
a*Cos[c + d*x] - b*Sin[c + d*x]))
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4011, 27, 3042, 3963, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx)-a)(a+b\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (b\tan(c+dx)-a)(a+b\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{ 4011}$$

$$\int \left(-a^2-b^2\right)(a+b\tan(c+dx))^{3/2}dx+\frac{2b(a+b\tan(c+dx))^{5/2}}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2b(a+b\tan(c+dx))^{5/2}}{5d}-\left(a^2+b^2\right)\int(a+b\tan(c+dx))^{3/2}dx$$

$\big\downarrow$ 3042

$$\frac{2b(a+b\tan(c+dx))^{5/2}}{5d}-\left(a^2+b^2\right)\int(a+b\tan(c+dx))^{3/2}dx$$

$\big\downarrow$ 3963

$$\frac{2b(a+b\tan(c+dx))^{5/2}}{5d}-\left(a^2+b^2\right)\left(\int\frac{a^2+2b\tan(c+dx)a-b^2}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2b\sqrt{a+b\tan(c+dx)}}{d}\right)$$

$\big\downarrow$ 3042

$$\frac{2b(a+b\tan(c+dx))^{5/2}}{5d}-\left(a^2+b^2\right)\left(\int\frac{a^2+2b\tan(c+dx)a-b^2}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2b\sqrt{a+b\tan(c+dx)}}{d}\right)$$

$\big\downarrow$ 4022

$$\frac{2b(a+b\tan(c+dx))^{5/2}}{5d}-\left(a^2+b^2\right)\left(\frac{1}{2}(a-ib)^2\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^2\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2b\sqrt{a+b\tan(c+dx)}}{d}\right.$$

$\big\downarrow$ 3042

$$\frac{2b(a+b\tan(c+dx))^{5/2}}{5d}-\left(a^2+b^2\right)\left(\frac{1}{2}(a-ib)^2\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^2\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2b\sqrt{a+b\tan(c+dx)}}{d}\right.$$

$\big\downarrow$ 4020

$$\frac{2b(a+b\tan(c+dx))^{5/2}}{5d}-\left(a^2+b^2\right)\left(\frac{i(a-ib)^2\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a+ib)^2\int-\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx}}}{2d}\right.$$

$\big\downarrow$ 25

$$\frac{2b(a+b\tan(c+dx))^{5/2}}{5d} - (a^2+b^2)\left(-\frac{i(a-ib)^2\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d} + \frac{i(a+ib)^2\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d\right.$$

$\downarrow$ 73

$$\frac{2b(a+b\tan(c+dx))^{5/2}}{5d} - (a^2+b^2)\left(\frac{(a-ib)^2\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd} + \frac{(a+ib)^2\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}\right.$$

$\downarrow$ 221

$$\frac{2b(a+b\tan(c+dx))^{5/2}}{5d} - (a^2+b^2)\left(\frac{(a-ib)^{3/2}\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d} + \frac{(a+ib)^{3/2}\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d} + \frac{2b\sqrt{a+b\tan(c+dx)}}{d}\right)$$

input
```
Int[(-a + b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
(2*b*(a + b*Tan[c + d*x])^(5/2))/(5*d) - (a^2 + b^2)*(((a - I*b)^(3/2)*Arc
Tan[Tan[c + d*x]/Sqrt[a - I*b]])/d + ((a + I*b)^(3/2)*ArcTan[Tan[c + d*x]/
Sqrt[a + I*b]])/d + (2*b*Sqrt[a + b*Tan[c + d*x]])/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3963
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. $2(127) = 254$.

Time = 0.11 (sec) , antiderivative size = 1375, normalized size of antiderivative = 9.11

method	result	size
derivativedivides	Expression too large to display	1375
default	Expression too large to display	1375
parts	Expression too large to display	2386

input
```
int((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*b*(a+b*tan(d*x+c))^(5/2)/d-2/d*b*(a+b*tan(d*x+c))^(1/2)*a^2-2/d*b^3*(a
+b*tan(d*x+c))^(1/2)+1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)*a^3+1/4/d*b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)*a-1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+1/4
/d*b^3*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2
))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)*a^2+1/d*b^3/(2*(a^2+b^2)
^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)
^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*(a^2+b^2)^(1/2)-2/d*b/(2*(a^2+b^2)^
(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^
(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-2/d*b^3/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(
a^2+b^2)^(1/2)-2*a)^(1/2))*a-1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*a^3-1/4/d*b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)*(2*(a^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(121) = 242$.

Time = 0.11 (sec) , antiderivative size = 1472, normalized size of antiderivative = 9.75

$$\int (-a + b\tan(c + dx))(a + b\tan(c + dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/10*(5*d*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 + d^2*sqrt(-(9*a^12*
b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^1
4)/d^4))/d^2)*log(-(3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - a^2*b
^9 - b^11)*sqrt(b*tan(d*x + c) + a) + (a*d^3*sqrt(-(9*a^12*b^2 + 30*a^10*b
^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4) + (3*a^
6*b^2 + 5*a^4*b^4 + a^2*b^6 - b^8)*d)*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3
*a*b^6 + d^2*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*
a^4*b^10 - 2*a^2*b^12 + b^14)/d^4))/d^2)) - 5*d*sqrt(-(a^7 - a^5*b^2 - 5*a
^3*b^4 - 3*a*b^6 + d^2*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^
6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4))/d^2)*log(-(3*a^10*b + 11*a^8
*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - a^2*b^9 - b^11)*sqrt(b*tan(d*x + c) + a) -
 (a*d^3*sqrt(-(9*a^12*b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b
^10 - 2*a^2*b^12 + b^14)/d^4) + (3*a^6*b^2 + 5*a^4*b^4 + a^2*b^6 - b^8)*d)
*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 + d^2*sqrt(-(9*a^12*b^2 + 30*a
^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4))/d
^2)) - 5*d*sqrt(-(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6 - d^2*sqrt(-(9*a^12*
b^2 + 30*a^10*b^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^1
4)/d^4))/d^2)*log(-(3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - a^2*b
^9 - b^11)*sqrt(b*tan(d*x + c) + a) + (a*d^3*sqrt(-(9*a^12*b^2 + 30*a^10*b
^4 + 31*a^8*b^6 + 4*a^6*b^8 - 9*a^4*b^10 - 2*a^2*b^12 + b^14)/d^4) - (3...
```

Sympy [F]

$$\int(-a+b\tan(c+dx))(a+b\tan(c+dx))^{5/2}\,dx = -\int a^{3}\sqrt{a+b\tan{(c+dx)}}\,dx - \int\left(-b^{3}\sqrt{a+b\tan{(c+dx)}}\tan^{3}{(c+dx)}\right)dx - \int\left(-ab^{2}\sqrt{a+b\tan{(c+dx)}}\tan^{2}{(c+dx)}\right)dx - \int a^{2}b\sqrt{a+b\tan{(c+dx)}}\tan{(c+dx)}\,dx$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))**(5/2),x)
```

output
```
-Integral(a**3*sqrt(a + b*tan(c + d*x)), x) - Integral(-b**3*sqrt(a + b*ta
n(c + d*x))*tan(c + d*x)**3, x) - Integral(-a*b**2*sqrt(a + b*tan(c + d*x)
)*tan(c + d*x)**2, x) - Integral(a**2*b*sqrt(a + b*tan(c + d*x))*tan(c + d
*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int(-a+b\tan(c+dx))(a+b\tan(c+dx))^{5/2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int (-a+b\tan(c+dx))(a+b\tan(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,19,7]%%%}+%%%{8,[0,17,7]%%%}+%%%{28,[0,15,7]%%%}+
%%%{56,[0
```

Mupad [B] (verification not implemented)

Time = 26.92 (sec) , antiderivative size = 3441, normalized size of antiderivative = 22.79

$$\int (-a+b\tan(c+dx))(a+b\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
int(-(a + b*tan(c + d*x))^(5/2)*(a - b*tan(c + d*x)),x)
```

output
```
log((8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 - ((((((-a^4*b^2*d^4*(5*a^
4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 10*a^5*b^2*d^2)
/d^4)^(1/2)*(64*a^2*b^5 + 64*a^4*b^3 + 32*a*b^2*d*(((-a^4*b^2*d^4*(5*a^4 +
 b^4 - 10*a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 10*a^5*b^2*d^2)/d^
4)^(1/2)*(a + b*tan(c + d*x))^(1/2)))/(2*d) + (16*a^2*b^2*(a + b*tan(c + d
*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2))/d^2)*(((-a^4*b^2*d^4*(5*
a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) - a^7*d^2 - 5*a^3*b^4*d^2 + 10*a^5*b^2*d^
2)/d^4)^(1/2))/2)*((20*a^6*b^8*d^4 - a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*
a^10*b^4*d^4 - 25*a^12*b^2*d^4)^(1/2)/(4*d^4) - a^7/(4*d^2) - (5*a^3*b^4)/
(4*d^2) + (5*a^5*b^2)/(2*d^2))^(1/2) - log((8*a^3*b^3*(3*a^2 - b^2)*(a^2 +
 b^2)^3)/d^3 - ((((-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a
^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(64*a^2*b^5 + 64*a^4*b
^3 - 32*a*b^2*d*(-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2) + a^7
*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/
2)))/(2*d) - (16*a^2*b^2*(a + b*tan(c + d*x))^(1/2)*(a^6 - b^6 + 15*a^2*b^
4 - 15*a^4*b^2))/d^2)*(-((-a^4*b^2*d^4*(5*a^4 + b^4 - 10*a^2*b^2)^2)^(1/2)
 + a^7*d^2 + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/d^4)^(1/2))/2)*(-(a^7*d^2 + (
20*a^6*b^8*d^4 - a^4*b^10*d^4 - 110*a^8*b^6*d^4 + 100*a^10*b^4*d^4 - 25*a^
12*b^2*d^4)^(1/2) + 5*a^3*b^4*d^2 - 10*a^5*b^2*d^2)/(4*d^4))^(1/2) - log((
8*a^3*b^3*(3*a^2 - b^2)*(a^2 + b^2)^3)/d^3 - ((((((-a^4*b^2*d^4*(5*a^4 ...
```

Reduce [F]

$$\int (-a+b\tan(c+dx))(a+b\tan(c+dx))^{5/2}\,dx = -\left(\int \sqrt{a+\tan(dx+c)\,b}dx\right)a^3 + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)b^3 + \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)a\,b^2 - \left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)a^2b$$

input
```
int((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(5/2),x)
```

output
```
 - int(sqrt(tan(c + d*x)*b + a),x)*a**3 + int(sqrt(tan(c + d*x)*b + a)*tan
(c + d*x)**3,x)*b**3 + int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*a*b
**2 - int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a**2*b
```

3.341 $\int (-a+b\tan(c+dx))(a+b\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 310

$$\int (-a+b\tan(c+dx))(a+b\tan(c+dx))^{3/2}\,dx = \frac{b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}{a+\sqrt{a^2+b^2}+b\tan(c+dx)}\right)}{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} - \frac{b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} + \frac{b(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} + \frac{2b(a+b\tan(c+dx))^{3/2}}{3d}$$

output
```
1/2*b*(a^2+b^2)*arctanh(2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))
^(1/2)/(a+(a^2+b^2)^(1/2)+b*tan(d*x+c)))*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)
/d-1/2*b*(a^2+b^2)*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x
+c))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)/d
+1/2*b*(a^2+b^2)*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)+2^(1/2)*(a+b*tan(d*x+c
))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)/d+2
/3*b*(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.59

$$\int (-a+b\tan(c+dx))(a+b\tan(c+dx))^{3/2}\,dx = \frac{(a-b\tan(c+dx))\left(3i\sqrt{a-ib}(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)\cos(c+dx)-3i\sqrt{a+ib}(a^2\right.}{3d(a\cos(c+}$$

input
```
Integrate[(-a + b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
((a - b*Tan[c + d*x])*((3*I)*Sqrt[a - I*b]*(a^2 + b^2)*ArcTanh[Sqrt[a + b*
Tan[c + d*x]]/Sqrt[a - I*b]]*Cos[c + d*x] - (3*I)*Sqrt[a + I*b]*(a^2 + b^2
)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]*Cos[c + d*x] + 2*b*(a*Co
s[c + d*x] + b*Sin[c + d*x])*Sqrt[a + b*Tan[c + d*x]]))/(3*d*(a*Cos[c + d*
x] - b*Sin[c + d*x]))
```

Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.39, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {3042, 4011, 27, 3042, 3966, 483, 1449, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx)-a)(a+b\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx)-a)(a+b\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{4011}$$

$$\int \left(-a^2-b^2\right)\sqrt{a+b\tan(c+dx)}dx+\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \ 27$$

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-\left(a^2+b^2\right)\int \sqrt{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-\left(a^2+b^2\right)\int \sqrt{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \ 3966$$

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-\frac{b\left(a^2+b^2\right)\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \ 483$$

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-\frac{2b\left(a^2+b^2\right)\int \frac{b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)-2ab^2\tan^2(c+dx)+a^2+b^2}d\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 1449$$

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-$$

$$\frac{2b\left(a^2+b^2\right)\left(\frac{\int \frac{\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b\sqrt{a+\sqrt{a^2+b^2}}\tan(c+dx)+\sqrt{a^2+b^2}}d\sqrt{a+b\tan(c+dx)}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}-\frac{\int \frac{\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b\sqrt{a+\sqrt{a^2+b^2}}\tan(c+dx)+\sqrt{a^2+b^2}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.}{d}$$

$$\Big\downarrow \ 1142$$

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-$$

$$2b\left(a^2+b^2\right)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}+\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a\right.}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{}}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$$\Big\downarrow \ 25$$

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-$$

$$2b\left(a^2+b^2\right)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\right.}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a}}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$\Big\downarrow$ 27

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-$$

$$2b(a^2+b^2)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int\frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\frac{\int\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+}}}}{\sqrt{2}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$\Big\downarrow$ 1083

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-$$

$$2b(a^2+b^2)\left(\frac{-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\int\frac{1}{2\left(a-\sqrt{a^2+b^2}\right)-b^2\tan^2(c+dx)}d\left(2\sqrt{a+b\tan(c+dx)}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\right)-\frac{\int\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+}}}{}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$\Big\downarrow$ 219

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-$$

$$2b(a^2+b^2)\left(\frac{-\frac{\int\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\tan(c+dx)}-\sqrt{2}\sqrt{\sqrt{}}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{a-\sqrt{a^2+b^2}}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$\Big\downarrow$ 1103

$$\frac{2b(a+b\tan(c+dx))^{3/2}}{3d}-$$

$$2b(a^2+b^2)\left(\frac{\frac{1}{2}\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+b^2\tan^2(c+dx)\right)-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\tan(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2+b}}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{a-\sqrt{a^2+b^2}}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

input
```
Int[(-a + b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*b*(a^2 + b^2)*((-((Sqrt[a + Sqrt[a^2 + b^2]]*ArcTanh[(-(Sqrt[2]*Sqrt[a
 + Sqrt[a^2 + b^2]]) + 2*Sqrt[a + b*Tan[c + d*x]])/(Sqrt[2]*Sqrt[a - Sqrt[
a^2 + b^2]])])/Sqrt[a - Sqrt[a^2 + b^2]]) + Log[Sqrt[a^2 + b^2] + b^2*Tan[
c + d*x]^2 - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]]/2
)/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]) - ((Sqrt[a + Sqrt[a^2 + b^2]]*ArcT
anh[(Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]] + 2*Sqrt[a + b*Tan[c + d*x]])/(Sqrt
[2]*Sqrt[a - Sqrt[a^2 + b^2]])])/Sqrt[a - Sqrt[a^2 + b^2]] + Log[Sqrt[a^2
+ b^2] + b^2*Tan[c + d*x]^2 + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b
*Tan[c + d*x]]]/2)/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]])))/d + (2*b*(a + b
*Tan[c + d*x])^(3/2))/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1449
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Simp[1/(2*c*r)   Int[x^(m - 1)/(q + r*x + x^2), x],
 x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m,
3] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. $2(256) = 512$.

Time = 0.11 (sec) , antiderivative size = 986, normalized size of antiderivative = 3.18

method	result
derivativedivides	$\frac{2b(a+b\tan(dx+c))^{\frac{3}{2}}}{3d} - \frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)\sqrt{2\sqrt{a^2+b^2}+2a}\,a^3}{4bd} - \frac{b\ln}{}$
default	$\frac{2b(a+b\tan(dx+c))^{\frac{3}{2}}}{3d} - \frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)\sqrt{2\sqrt{a^2+b^2}+2a}\,a^3}{4bd} - \frac{b\ln}{}$
parts	Expression too large to display

input
```
int((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*b*(a+b*tan(d*x+c))^(3/2)/d-1/4/b/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^
(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)*a^3-1/4*b/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-b/d/(2*
(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(
1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/4/b/d*ln(b*tan(d*x+c
)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2+1/4*b/d*ln(b*tan(d*x+c)+
a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2
*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)-b^3/d/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(
a^2+b^2)^(1/2)-2*a)^(1/2))+1/4/b/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)*a^3+1/4*b/d*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b
*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+b/d/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*
x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/4/b/d*ln((a+b*tan(d*x+c)
)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a^2-1/4*b/d*ln((a+b*tan(d*x+c))^
(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 956 vs. $2(258) = 516$.

Time = 0.10 (sec) , antiderivative size = 956, normalized size of antiderivative = 3.08

$$\int (-a + b\tan(c + dx))(a + b\tan(c + dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(3*d*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 + d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 +
 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*log(d^3*sqrt(-(a^5 + 2*a^3*b^2 +
 a*b^4 + d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^
4))/d^2)*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4) +
 (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sqrt(b*tan(d*x + c) + a)) - 3*d*sqr
t(-(a^5 + 2*a^3*b^2 + a*b^4 + d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 +
 4*a^2*b^8 + b^10)/d^4))/d^2)*log(-d^3*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 + d^
2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*sq
rt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4) + (a^6*b + 3
*a^4*b^3 + 3*a^2*b^5 + b^7)*sqrt(b*tan(d*x + c) + a)) - 3*d*sqrt(-(a^5 + 2
*a^3*b^2 + a*b^4 - d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8
+ b^10)/d^4))/d^2)*log(d^3*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 - d^2*sqrt(-(a^8
*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*sqrt(-(a^8*b^2
 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4) + (a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*sqrt(b*tan(d*x + c) + a)) + 3*d*sqrt(-(a^5 + 2*a^3*b^2 + a
*b^4 - d^2*sqrt(-(a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4)
)/d^2)*log(-d^3*sqrt(-(a^5 + 2*a^3*b^2 + a*b^4 - d^2*sqrt(-(a^8*b^2 + 4*a^
6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4))/d^2)*sqrt(-(a^8*b^2 + 4*a^6*b^
4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)/d^4) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7)*sqrt(b*tan(d*x + c) + a)) + 4*(b^2*tan(d*x + c) + a*b)*sqrt(b*tan(...
```

Sympy [F]

$$\int (-a + b\tan(c + dx))(a + b\tan(c + dx))^{3/2}\, dx = -\int a^2\sqrt{a + b\tan(c + dx)}\, dx - \int \left(-b^2\sqrt{a + b\tan(c + dx)}\tan^2(c + dx)\right) dx$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))**(3/2),x)
```

output
```
-Integral(a**2*sqrt(a + b*tan(c + d*x)), x) - Integral(-b**2*sqrt(a + b*ta
n(c + d*x))*tan(c + d*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (-a + b\tan(c + dx))(a + b\tan(c + dx))^{3/2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int (-a + b\tan(c + dx))(a + b\tan(c + dx))^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [B] (verification not implemented)

Time = 14.99 (sec) , antiderivative size = 2529, normalized size of antiderivative = 8.16

$$\int(-a+b\tan(c+dx))(a+b\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
int(-(a + b*tan(c + d*x))^(3/2)*(a - b*tan(c + d*x)),x)
```

output
```
log((((16*b^4*(a + b*tan(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 + (1
6*a*b^2*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^
4)^(1/2)*(a^2*b + b^3 - d*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2
 + a^3*b^2*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2)))/d)*(((-b^6*d^4*(3*
a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/2))/2 - (8*b^5*(a
^2 - b^2)*(a^2 + b^2)^2)/d^3)*((6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^
(1/2)/(4*d^4) - (3*a*b^4)/(4*d^2) + (a^3*b^2)/(4*d^2))^(1/2) - log(- (((16
*b^4*(a + b*tan(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*(
-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^(1/2)
*(a^2*b + b^3 + d*(-((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*
b^2*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2)))/d)*(-((-b^6*d^4*(3*a^2 -
b^2)^2)^(1/2) + 3*a*b^4*d^2 - a^3*b^2*d^2)/d^4)^(1/2))/2 - (8*b^5*(a^2 - b
^2)*(a^2 + b^2)^2)/d^3)*(-((6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^(1/2
) + 3*a*b^4*d^2 - a^3*b^2*d^2)/(4*d^4))^(1/2) - log(- (((16*b^4*(a + b*tan
(c + d*x))^(1/2)*(a^4 + b^4 - 6*a^2*b^2))/d^2 - (16*a*b^2*(((-b^6*d^4*(3*a
^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/2)*(a^2*b + b^3 +
d*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/
2)*(a + b*tan(c + d*x))^(1/2)))/d)*(((-b^6*d^4*(3*a^2 - b^2)^2)^(1/2) - 3*
a*b^4*d^2 + a^3*b^2*d^2)/d^4)^(1/2))/2 - (8*b^5*(a^2 - b^2)*(a^2 + b^2)^2)
/d^3)*(((6*a^2*b^8*d^4 - b^10*d^4 - 9*a^4*b^6*d^4)^(1/2) - 3*a*b^4*d^2 ...
```

Reduce [F]

$$\int (-a + b\tan(c + dx))(a + b\tan(c + dx))^{3/2}\,dx =$$
$$-\left(\int \sqrt{a + \tan(dx + c)\,b}dx\right)a^2 + \left(\int \sqrt{a + \tan(dx + c)\,b}\,\tan(dx + c)^2\,dx\right)b^2$$

input
```
int((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(3/2),x)
```

output
```
 - int(sqrt(tan(c + d*x)*b + a),x)*a**2 + int(sqrt(tan(c + d*x)*b + a)*tan
(c + d*x)**2,x)*b**2
```

3.342 $\int (-a+b\tan(c+dx))\sqrt{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 321

$$\begin{aligned}\int (-a+b\tan(c+dx))\sqrt{a+b\tan(c+dx)}\,dx &= -\frac{b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}{a+\sqrt{a^2+b^2}+b\tan(c+dx)}\right)}{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d}\\ &\quad-\frac{b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d}\\ &\quad+\frac{b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d}+\frac{2b\sqrt{a+b\tan(c+dx)}}{d}\end{aligned}$$

output
```
-1/2*b*(a^2+b^2)^(1/2)*arctanh(2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(
d*x+c))^(1/2)/(a+(a^2+b^2)^(1/2)+b*tan(d*x+c)))*2^(1/2)/(a+(a^2+b^2)^(1/2)
)^(1/2)/d-1/2*b*(a^2+b^2)^(1/2)*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)-2^(1/2)
*(a+b*tan(d*x+c))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*2^(1/2)/(a-(a^2+b^2)^(
1/2))^(1/2)/d+1/2*b*(a^2+b^2)^(1/2)*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)+2^(
1/2)*(a+b*tan(d*x+c))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*2^(1/2)/(a-(a^2+b^
2)^(1/2))^(1/2)/d+2*b*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.49

$$\int (-a+b\tan(c+dx))\sqrt{a+b\tan(c+dx)}\,dx$$
$$= \frac{\cos(c+dx)(a-b\tan(c+dx))\left(i\sqrt{a-ib}(a+ib)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-i(a-ib)\sqrt{a+ib}\text{arctanh}\left(\frac{\sqrt{}}{}\right.\right.}{d(a\cos(c+dx)-b\sin(c+dx))}$$

input
```
Integrate[(-a + b*Tan[c + d*x])*Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
(Cos[c + d*x]*(a - b*Tan[c + d*x])*(I*Sqrt[a - I*b]*(a + I*b)*ArcTanh[Sqrt
[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - I*(a - I*b)*Sqrt[a + I*b]*ArcTanh[Sq
rt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]] + 2*b*Sqrt[a + b*Tan[c + d*x]]))/(d*
(a*Cos[c + d*x] - b*Sin[c + d*x]))
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.41, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.481, Rules used = {3042, 4011, 27, 3042, 3966, 484, 1407, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx)-a)\sqrt{a+b\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (b\tan(c+dx)-a)\sqrt{a+b\tan(c+dx)}dx$$
$$\Big\downarrow \text{4011}$$
$$\int \frac{-a^2-b^2}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2b\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-\left(a^2+b^2\right)\int \frac{1}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-\left(a^2+b^2\right)\int \frac{1}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3966$$

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-\frac{b\left(a^2+b^2\right)\int \frac{1}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)b^2+b^2\right)}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 484$$

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-\frac{2b\left(a^2+b^2\right)\int \frac{1}{b^4\tan^4(c+dx)-2ab^2\tan^2(c+dx)+a^2+b^2}d\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 1407$$

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-\frac{2b\left(a^2+b^2\right)\left(\frac{\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}+\frac{\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b \cdots}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b \cdots}}}\right.}{d}$$

$$\Big\downarrow \; 1142$$

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-\frac{2b\left(a^2+b^2\right)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a \cdots}\right.}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\cdots}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-\frac{2b\left(a^2+b^2\right)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b \cdots}\right.}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a \cdots}}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.}{\cdots}$$

$\downarrow$ 27

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-$$

$$2b(a^2+b^2)\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int\frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}+\frac{\int\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+}}}}{\sqrt{2}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$\downarrow$ 1083

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-$$

$$2b(a^2+b^2)\left(\frac{\frac{\int\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\int\frac{1}{2\left(a-\sqrt{a^2+b^2}\right)-b^2\tan^2(c+dx)}d\Big(2}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$\downarrow$ 219

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-$$

$$2b(a^2+b^2)\left(\frac{\frac{\int\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\tan(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2}}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right.}{\sqrt{a-\sqrt{a^2+b^2}}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$\downarrow$ 1103

$$\frac{2b\sqrt{a+b\tan(c+dx)}}{d}-$$

$$2b(a^2+b^2)\left(\frac{-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\tan(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{a-\sqrt{a^2+b^2}}}-\frac{1}{2}\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+b^2\tan^2(\right.}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

input

```
Int[(-a + b*Tan[c + d*x])*Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
(-2*b*(a^2 + b^2)*((-((Sqrt[a + Sqrt[a^2 + b^2]]*ArcTanh[(-(Sqrt[2]*Sqrt[a
 + Sqrt[a^2 + b^2]]) + 2*Sqrt[a + b*Tan[c + d*x]])/(Sqrt[2]*Sqrt[a - Sqrt[
a^2 + b^2]])])/Sqrt[a - Sqrt[a^2 + b^2]]) - Log[Sqrt[a^2 + b^2] + b^2*Tan[
c + d*x]^2 - Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]]/2
)/(2*Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a + Sqrt[a^2 + b^2]]) + (-((Sqrt[a + Sqr
t[a^2 + b^2]]*ArcTanh[(Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]] + 2*Sqrt[a + b*Ta
n[c + d*x]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]])])/Sqrt[a - Sqrt[a^2 + b^2
]]) + Log[Sqrt[a^2 + b^2] + b^2*Tan[c + d*x]^2 + Sqrt[2]*Sqrt[a + Sqrt[a^2
 + b^2]]*Sqrt[a + b*Tan[c + d*x]]]/2)/(2*Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a +
Sqrt[a^2 + b^2]])))/d + (2*b*Sqrt[a + b*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(262) = 524$.

Time = 0.11 (sec) , antiderivative size = 814, normalized size of antiderivative = 2.54

method	result
parts	$\frac{b\left(2\sqrt{a+b\tan(dx+c)}+\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}-b\tan(dx+c)-a-\sqrt{a^2+b^2}\right)}{4}+\frac{\left(-a+\sqrt{a^2+b^2}\right)\text{arc}}{}\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b/d*(2*(a+b*tan(d*x+c))^(1/2)+1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*ta
n(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/
2))+(-a+(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2
)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
)-1/4*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/
2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+(a-(a^2+b^2)^(1/2))/(2*(
a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1
/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/d/b*ln((a+b*tan(d*x+c)
)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*(a^2+b^2)^(1/2)*a+1/d*b/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(
a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a
)^(1/2))*a-1/4/b/d*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+1/4/b/d
*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a
^2+b^2)^(1/2))*(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-b/d/(2*(a^2
+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)
+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(264) = 528$.

Time = 0.10 (sec) , antiderivative size = 719, normalized size of antiderivative = 2.24

$$\int (-a + b\tan(c+dx))\sqrt{a+b\tan(c+dx)}\,dx =$$

$$-\frac{d\sqrt{-\frac{a^3+ab^2+d^2\sqrt{-\frac{a^4b^2+2a^2b^4+b^6}{d^4}}}{d^2}}\log\left(\left(a^4b+2a^2b^3+b^5\right)\sqrt{b\tan(dx+c)+a}+\left(ad^3\sqrt{-\frac{a^4b^2+2a^2b^4+b^6}{d^4}}+\right.\right.}{}$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(d*sqrt(-(a^3 + a*b^2 + d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d
^2)*log((a^4*b + 2*a^2*b^3 + b^5)*sqrt(b*tan(d*x + c) + a) + (a*d^3*sqrt(-
(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4) + (a^2*b^2 + b^4)*d)*sqrt(-(a^3 + a*b^2 +
 d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)) - d*sqrt(-(a^3 + a*b^2
+ d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)*log((a^4*b + 2*a^2*b^3
+ b^5)*sqrt(b*tan(d*x + c) + a) - (a*d^3*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)
/d^4) + (a^2*b^2 + b^4)*d)*sqrt(-(a^3 + a*b^2 + d^2*sqrt(-(a^4*b^2 + 2*a^2
*b^4 + b^6)/d^4))/d^2)) - d*sqrt(-(a^3 + a*b^2 - d^2*sqrt(-(a^4*b^2 + 2*a^
2*b^4 + b^6)/d^4))/d^2)*log((a^4*b + 2*a^2*b^3 + b^5)*sqrt(b*tan(d*x + c)
+ a) + (a*d^3*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4) - (a^2*b^2 + b^4)*d)*
sqrt(-(a^3 + a*b^2 - d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)) + d
*sqrt(-(a^3 + a*b^2 - d^2*sqrt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)*log
((a^4*b + 2*a^2*b^3 + b^5)*sqrt(b*tan(d*x + c) + a) - (a*d^3*sqrt(-(a^4*b^
2 + 2*a^2*b^4 + b^6)/d^4) - (a^2*b^2 + b^4)*d)*sqrt(-(a^3 + a*b^2 - d^2*sq
rt(-(a^4*b^2 + 2*a^2*b^4 + b^6)/d^4))/d^2)) - 4*sqrt(b*tan(d*x + c) + a)*b
)/d
```

Sympy [F]

$$\int (-a + b\tan(c+dx))\sqrt{a + b\tan(c+dx)}\,dx$$
$$= -\int a\sqrt{a + b\tan{(c+dx)}}\,dx - \int \left(-b\sqrt{a + b\tan{(c+dx)}}\tan{(c+dx)}\right)\,dx$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))**(1/2),x)
```

output
```
-Integral(a*sqrt(a + b*tan(c + d*x)), x) - Integral(-b*sqrt(a + b*tan(c +
d*x))*tan(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (-a + b\tan(c + dx))\sqrt{a + b\tan(c + dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int (-a + b\tan(c + dx))\sqrt{a + b\tan(c + dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.81

$$\int (-a+b\tan(c+dx))\sqrt{a+b\tan(c+dx)}\,dx$$
$$=\operatorname{atanh}\left(\frac{d^3\left(\frac{16\,(a^2\,b^4-a^4\,b^2)\,\sqrt{a+b\tan(c+d\,x)}}{d^2}+\frac{16\,a\,b^2\,(a^3+1\mathrm{i}\,b\,a^2)\,\sqrt{a+b\tan(c+d\,x)}}{d^2}\right)\sqrt{-\frac{a^3+1\mathrm{i}\,b\,a^2}{d^2}}}{16\,(a^5\,b^3+a^3\,b^5)}\right)\sqrt{-\frac{a^3+1\mathrm{i}\,b\,a^2}{d^2}}$$
$$+\operatorname{atanh}\left(\frac{d^3\,\sqrt{\frac{-a^3+a^2\,b\,1\mathrm{i}}{d^2}}\left(\frac{16\,(a^2\,b^4-a^4\,b^2)\,\sqrt{a+b\tan(c+d\,x)}}{d^2}-\frac{16\,a\,b^2\,(-a^3+a^2\,b\,1\mathrm{i})\,\sqrt{a+b\tan(c+d\,x)}}{d^2}\right)}{16\,(a^5\,b^3+a^3\,b^5)}\right)\sqrt{\frac{-a^3+a^2\,b\,1}{d^2}}$$
$$+\frac{2\,b\,\sqrt{a+b\tan(c+d\,x)}}{d}-\operatorname{atan}\left(\frac{b^6\,\sqrt{\frac{a\,b^2}{4\,d^2}-\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}\,32\mathrm{i}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}\right.$$
$$\left.+\frac{32\,a\,b^5\,\sqrt{\frac{a\,b^2}{4\,d^2}-\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}\right)\sqrt{\frac{a\,b^2-b^3\,1\mathrm{i}}{4\,d^2}}\,2\mathrm{i}$$
$$+\operatorname{atan}\left(\frac{b^6\,\sqrt{\frac{a\,b^2}{4\,d^2}+\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}\,32\mathrm{i}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}\right.$$
$$\left.-\frac{32\,a\,b^5\,\sqrt{\frac{a\,b^2}{4\,d^2}+\frac{b^3\,1\mathrm{i}}{4\,d^2}}\,\sqrt{a+b\tan(c+d\,x)}}{\frac{b^8\,16\mathrm{i}}{d}+\frac{a^2\,b^6\,16\mathrm{i}}{d}}\right)\sqrt{\frac{b^3\,1\mathrm{i}+a\,b^2}{4\,d^2}}\,2\mathrm{i}$$

input
```
int(-(a + b*tan(c + d*x))^(1/2)*(a - b*tan(c + d*x)),x)
```

output
```
atan((b^6*((b^3*1i)/(4*d^2) + (a*b^2)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^
(1/2)*32i)/((b^8*16i)/d + (a^2*b^6*16i)/d) - (32*a*b^5*((b^3*1i)/(4*d^2) +
 (a*b^2)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((b^8*16i)/d + (a^2*b^
6*16i)/d))*((a*b^2 + b^3*1i)/(4*d^2))^(1/2)*2i - atan((b^6*((a*b^2)/(4*d^2
) - (b^3*1i)/(4*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((b^8*16i)/d +
 (a^2*b^6*16i)/d) + (32*a*b^5*((a*b^2)/(4*d^2) - (b^3*1i)/(4*d^2))^(1/2)*(
a + b*tan(c + d*x))^(1/2))/((b^8*16i)/d + (a^2*b^6*16i)/d))*((a*b^2 - b^3*
1i)/(4*d^2))^(1/2)*2i + atanh((d^3*((16*(a^2*b^4 - a^4*b^2)*(a + b*tan(c +
 d*x))^(1/2))/d^2 + (16*a*b^2*(a^2*b*1i + a^3)*(a + b*tan(c + d*x))^(1/2))
/d^2)*(-(a^2*b*1i + a^3)/d^2)^(1/2))/(16*(a^3*b^5 + a^5*b^3)))*(-(a^2*b*1i
 + a^3)/d^2)^(1/2) + atanh((d^3*((a^2*b*1i - a^3)/d^2)^(1/2)*((16*(a^2*b^4
 - a^4*b^2)*(a + b*tan(c + d*x))^(1/2))/d^2 - (16*a*b^2*(a^2*b*1i - a^3)*(
a + b*tan(c + d*x))^(1/2))/d^2))/(16*(a^3*b^5 + a^5*b^3)))*((a^2*b*1i - a^
3)/d^2)^(1/2) + (2*b*(a + b*tan(c + d*x))^(1/2))/d
```

Reduce [F]

$$\int (-a+b\tan(c+dx))\sqrt{a+b\tan(c+dx)}\,dx$$
$$= -\left(\int \sqrt{a+\tan(dx+c)\,b}dx\right)a+\left(\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)b$$

input
```
int((-a+b*tan(d*x+c))*(a+b*tan(d*x+c))^(1/2),x)
```

output
```
 - int(sqrt(tan(c + d*x)*b + a),x)*a + int(sqrt(tan(c + d*x)*b + a)*tan(c
+ d*x),x)*b
```

3.343 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 213

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} + \frac{(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d} - \frac{2(10aAb-8a^2B+15b^2B)\sqrt{a+b\tan(c+dx)}}{15b^3d} + \frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{15b^2d} + \frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}$$

output
```
(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(1/2)/d+(A+I
*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(1/2)/d-2/15*(10
*A*a*b-8*B*a^2+15*B*b^2)*(a+b*tan(d*x+c))^(1/2)/b^3/d+2/15*(5*A*b-4*B*a)*t
an(d*x+c)*(a+b*tan(d*x+c))^(1/2)/b^2/d+2/5*B*tan(d*x+c)^2*(a+b*tan(d*x+c))
^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 2.75 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.80

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$= \frac{\frac{15(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \frac{15(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}} + \frac{2\sqrt{a+b\tan(c+dx)}(-10aAb+8a^2B-15b^2B+b(5Ab-}{b^3}}{15d}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x
]
```

output
```
((15*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I
*b] + (15*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt[
a + I*b] + (2*Sqrt[a + b*Tan[c + d*x]]*(-10*a*A*b + 8*a^2*B - 15*b^2*B + b
*(5*A*b - 4*a*B)*Tan[c + d*x] + 3*b^2*B*Tan[c + d*x]^2))/b^3)/(15*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.18 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$
$$\Big\downarrow \text{4090}$$

$$\frac{2\int -\frac{\tan(c+dx)(-((5Ab-4aB)\tan^2(c+dx))+5bB\tan(c+dx)+4aB)}{2\sqrt{a+b\tan(c+dx)}}dx}{5b}+\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{\int \frac{\tan(c+dx)(-((5Ab-4aB)\tan^2(c+dx))+5bB\tan(c+dx)+4aB)}{\sqrt{a+b\tan(c+dx)}}dx}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{\int \frac{\tan(c+dx)(-((5Ab-4aB)\tan(c+dx)^2)+5bB\tan(c+dx)+4aB)}{\sqrt{a+b\tan(c+dx)}}dx}{5b}$$

$$\Big\downarrow \text{4130}$$

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{\frac{2\int \frac{15A\tan(c+dx)b^2+\left(-8Ba^2+10Aba+15b^2B\right)\tan^2(c+dx)+2a(5Ab-4aB)}{2\sqrt{a+b\tan(c+dx)}}dx}{3b}-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{\frac{\int \frac{15A\tan(c+dx)b^2+\left(-8Ba^2+10Aba+15b^2B\right)\tan^2(c+dx)+2a(5Ab-4aB)}{\sqrt{a+b\tan(c+dx)}}dx}{3b}-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{\frac{\int \frac{15A\tan(c+dx)b^2+\left(-8Ba^2+10Aba+15b^2B\right)\tan(c+dx)^2+2a(5Ab-4aB)}{\sqrt{a+b\tan(c+dx)}}dx}{3b}-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}}{5b}$$

$$\Big\downarrow \text{4113}$$

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{\frac{\int \frac{15Ab^2\tan(c+dx)-15b^2B}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(-8a^2B+10aAb+15b^2B\right)\sqrt{a+b\tan(c+dx)}}{bd}}{3b}-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{\frac{\int\frac{15Ab^2\tan(c+dx)-15b^2B}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(-8a^2B+10aAb+15b^2B\right)\sqrt{a+b\tan(c+dx)}}{bd}}{3b}-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}}{5b}$$

$\Big\downarrow$ 4022

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}+\frac{\frac{15}{2}b^2(-B+iA)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{15}{2}b^2(B+iA)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(-8a^2B+10aAb+15b^2B\right)}{bd}}{3b}}{5b}$$

$\Big\downarrow$ 3042

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}+\frac{\frac{15}{2}b^2(-B+iA)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{15}{2}b^2(B+iA)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(-8a^2B+10aAb+15b^2B\right)}{bd}}{3b}}{5b}$$

$\Big\downarrow$ 4020

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}+\frac{-\frac{15ib^2(B+iA)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{15ib^2(-B+iA)\int-\frac{}{(i\tan(c+dx)+1)}}{2d}}{3b}}{5b}$$

$\Big\downarrow$ 25

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}+\frac{\frac{15ib^2(B+iA)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{15ib^2(-B+iA)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b}}}{2d}}{3b}}{5b}$$

$\Big\downarrow$ 73

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd}-\frac{-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}+\frac{\frac{15b(-B+iA)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}-\frac{15b(B+iA)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(}}{d}}{3b}}{5b}$$

$\Big\downarrow$ 221

$$\frac{2B\tan^2(c+dx)\sqrt{a+b\tan(c+dx)}}{5bd} - \frac{-\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd} + \frac{\frac{2\left(-8a^2B+10aAb+15b^2B\right)\sqrt{a+b\tan(c+dx)}}{bd} - \frac{15b^2(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} + \frac{15b^2(-B+iA)\arctan\left(\mathrm{t}\cdots\right.}{d\sqrt{a+ib}}}{3b}}{5b}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
(2*B*Tan[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/(5*b*d) - ((-2*(5*A*b - 4*a*
B)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(3*b*d) + ((-15*b^2*(I*A + B)*Ar
cTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + (15*b^2*(I*A - B)*Ar
cTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d) + (2*(10*a*A*b - 8*a^2
*B + 15*b^2*B)*Sqrt[a + b*Tan[c + d*x]])/(b*d))/(3*b))/(5*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2021 vs. $2(183) = 366$.

Time = 0.25 (sec) , antiderivative size = 2022, normalized size of antiderivative = 9.49

method	result	size
parts	Expression too large to display	2022
derivativedivides	Expression too large to display	4107
default	Expression too large to display	4107

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
2*A/d/b^2*(1/3*(a+b*tan(d*x+c))^(3/2)-(a+b*tan(d*x+c))^(1/2)*a-b^2*(1/4/(a
^2+b^2)^(1/2)*(1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*tan(d*x+c))^(1/2)
*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))+2*(a-(a^2+b
^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(
1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/4/(a^2+b^
2)^(1/2)*(-1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*
x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-a+(a^2+b^2)
^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*
(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))))+B*(2/5/d/b^3
*(a+b*tan(d*x+c))^(5/2)-4/3/d/b^3*a*(a+b*tan(d*x+c))^(3/2)+2/d/b^3*a^2*(a+
b*tan(d*x+c))^(1/2)-2/d/b*(a+b*tan(d*x+c))^(1/2)-1/4/d/b/(a^2+b^2)*ln((a+b
*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^
(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d*b/(a^2+b^2)*ln((a+b*tan(d*x
+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b/(a^2+b^2)^(3/2)*ln((a+b*tan(d*x+c))^(
1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2
+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/d*b/(a^2+b^2)^(3/2)*ln((a+b*tan(d*x+c))^(1/
2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b
^2)^(1/2)+2*a)^(1/2)*a+1/d/b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1767 vs. $2(177) = 354$.

Time = 0.14 (sec) , antiderivative size = 1767, normalized size of antiderivative = 8.30

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
-1/30*(15*b^3*d*sqrt(((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*
B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2
*A*B*b + (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^4
 - B^4)*b)*sqrt(b*tan(d*x + c) + a) + ((B*a^3 - A*a^2*b + B*a*b^2 - A*b^3)
*d^3*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4
)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (2*A^2*B*a^2 - (A^3 - 3*A*B^2)*a*b
 - (A^2*B - B^3)*b^2)*d)*sqrt(((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A
^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*
d^4)) + 2*A*B*b + (A^2 - B^2)*a)/((a^2 + b^2)*d^2))) - 15*b^3*d*sqrt(((a^2
 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^
2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B^2)*a)/((
a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^4 - B^4)*b)*sqrt(b*tan(d*x
+ c) + a) - ((B*a^3 - A*a^2*b + B*a*b^2 - A*b^3)*d^3*sqrt(-(4*A^2*B^2*a^2
- 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 +
 b^4)*d^4)) - (2*A^2*B*a^2 - (A^3 - 3*A*B^2)*a*b - (A^2*B - B^3)*b^2)*d)*s
qrt(((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 -
 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B
^2)*a)/((a^2 + b^2)*d^2))) - 15*b^3*d*sqrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*
B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a
^2*b^2 + b^4)*d^4)) - 2*A*B*b - (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((...
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx=\int \frac{(A+B\tan{(c+dx)})\tan^3{(c+dx)}}{\sqrt{a+b\tan{(c+dx)}}}\,dx$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**3/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^3}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^3/sqrt(b*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,19,7]%%%}+%%%{8,[0,17,7]%%%}+%%%{28,[0,15,7]%%%}+
%%%{56,[0
```

Mupad [B] (verification not implemented)

Time = 11.19 (sec) , antiderivative size = 3054, normalized size of antiderivative = 14.34

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
atan((B^2*b^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d
^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*32i)/((16*B^
3*a*b^3*d^3)/(a^2*d^4 + b^2*d^4) - (4*B*b^3*d^2*(-16*B^4*b^2*d^4)^(1/2))/(
a^2*d^5 + b^2*d^5)) + (a*b^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d
^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/
2)*(-16*B^4*b^2*d^4)^(1/2)*8i)/((16*B^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (
4*B*b^5*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*B^3*a^3*b^3
*d^5)/(a^2*d^4 + b^2*d^4) - (4*B*a^2*b^3*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2
*d^5 + b^2*d^5)) - (B^2*a^2*b^2*d^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4
+ b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*
x))^(1/2)*32i)/((16*B^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*B*b^5*d^4*(-16
*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*B^3*a^3*b^3*d^5)/(a^2*d^4 +
 b^2*d^4) - (4*B*a^2*b^3*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5))
)*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*
d^4 + b^2*d^4)))^(1/2)*2i - atan((A^2*b^2*((A^2*a*d^2)/(4*(a^2*d^4 + b^2*d
^4)) - (-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(
c + d*x))^(1/2)*32i)/((16*A^3*b^2)/d - (16*A^3*a^2*b^2*d^3)/(a^2*d^4 + b^2
*d^4) + (4*A*a*b^2*d^2*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (a*
b^2*((A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)) - (-16*A^4*b^2*d^4)^(1/2)/(16*(a^
2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(-16*A^4*b^2*d^4)^(...
```

Reduce [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3,x)
```

3.344 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 166

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} - \frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d} + \frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{3b^2d} + \frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}$$

output
```
(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(1/2)/d-(I*A
-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(1/2)/d+2/3*(3*A
*b-2*B*a)*(a+b*tan(d*x+c))^(1/2)/b^2/d+2/3*B*tan(d*x+c)*(a+b*tan(d*x+c))^(
1/2)/b/d
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.84

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$=\frac{\frac{3(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}+\frac{3(-iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}+\frac{2\sqrt{a+b\tan(c+dx)}(3Ab-2aB+bB\tan(c+dx))}{b^2}}{3d}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x
]
```

output
```
((3*(I*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*
b] + (3*((-I)*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt
[a + I*b] + (2*Sqrt[a + b*Tan[c + d*x]]*(3*A*b - 2*a*B + b*B*Tan[c + d*x])
)/b^2)/(3*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4090, 27, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$
$$\Big\downarrow \text{4090}$$
$$\frac{2\int -\frac{-((3Ab-2aB)\tan^2(c+dx))+3bB\tan(c+dx)+2aB}{2\sqrt{a+b\tan(c+dx)}}dx}{3b}+\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int\frac{-((3Ab-2aB)\tan^2(c+dx))+3bB\tan(c+dx)+2aB}{\sqrt{a+b\tan(c+dx)}}dx}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int\frac{-((3Ab-2aB)\tan(c+dx)^2)+3bB\tan(c+dx)+2aB}{\sqrt{a+b\tan(c+dx)}}dx}{3b}$$

$$\Big\downarrow \; 4113$$

$$\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int\frac{3Ab+3B\tan(c+dx)b}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{bd}}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int\frac{3Ab+3B\tan(c+dx)b}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{bd}}{3b}$$

$$\Big\downarrow \; 4022$$

$$\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\frac{3}{2}b(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{3}{2}b(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{bd}}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\frac{3}{2}b(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{3}{2}b(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{bd}}{3b}$$

$$\Big\downarrow \; 4020$$

$$\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\frac{3ib(A-iB)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{3ib(A+iB)\int-\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{2(3Ab-2aB}{}}{3b}$$

$$\Big\downarrow \; 25$$

$$\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{-\frac{3ib(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{3ib(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{2(3Ab-2aB)}{}}{3b}$$

$$\Big\downarrow \text{73}$$

$$\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\frac{3(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}+\frac{3(A-iB)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}-\frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{bd}}{3b}$$

$$\Big\downarrow \text{221}$$

$$\frac{2B\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\frac{3b(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{3b(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}-\frac{2(3Ab-2aB)\sqrt{a+b\tan(c+dx)}}{bd}}{3b}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
(2*B*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(3*b*d) - ((3*b*(A - I*B)*ArcT
an[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + (3*b*(A + I*B)*ArcTan[
Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d) - (2*(3*A*b - 2*a*B)*Sqrt[a
 + b*Tan[c + d*x]])/(b*d))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1957 vs. $2(140) = 280$.

Time = 0.17 (sec) , antiderivative size = 1958, normalized size of antiderivative = 11.80

method	result	size
parts	Expression too large to display	1958
derivativedivides	Expression too large to display	4040
default	Expression too large to display	4040

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
A*(2/d/b*(a+b*tan(d*x+c))^(1/2)+1/4/d/b/(a^2+b^2)*ln((a+b*tan(d*x+c))^(1/2
)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^2+1/4/d*b/(a^2+b^2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)
+2*a)^(1/2)-1/4/d/b/(a^2+b^2)^(3/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*a^3-1/4/d*b/(a^2+b^2)^(3/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*a-1/d/b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+
b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2))*a^2-1/d*b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a
^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2))+1/d/b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2))*a^4+3/d*b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*
(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2))*a^2+2/d*b^3/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan
(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/
2)-2*a)^(1/2))-1/4/d/b/(a^2+b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1725 vs. $2(134) = 268$.

Time = 0.13 (sec) , antiderivative size = 1725, normalized size of antiderivative = 10.39

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
1/6*(3*b^2*d*sqrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^
3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A
*B*b + (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^4 -
 B^4)*b)*sqrt(b*tan(d*x + c) + a) + ((A*a^3 + B*a^2*b + A*a*b^2 + B*b^3)*d
^3*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*
b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + (2*A*B^2*a^2 - (3*A^2*B - B^3)*a*b +
 (A^3 - A*B^2)*b^2)*d)*sqrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^
3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d
^4)) + 2*A*B*b + (A^2 - B^2)*a)/((a^2 + b^2)*d^2))) - 3*b^2*d*sqrt(-((a^2
+ b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2
 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B^2)*a)/((a
^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^4 - B^4)*b)*sqrt(b*tan(d*x +
 c) + a) - ((A*a^3 + B*a^2*b + A*a*b^2 + B*b^3)*d^3*sqrt(-(4*A^2*B^2*a^2 -
 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 +
b^4)*d^4)) + (2*A*B^2*a^2 - (3*A^2*B - B^3)*a*b + (A^3 - A*B^2)*b^2)*d)*sq
rt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 -
 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B
^2)*a)/((a^2 + b^2)*d^2))) - 3*b^2*d*sqrt(((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^
2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2
*b^2 + b^4)*d^4)) - 2*A*B*b - (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*...
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan^2{(c+dx)}}{\sqrt{a+b\tan{(c+dx)}}}\,dx$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**2/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\tan{(dx+c)}^2}{\sqrt{b\tan{(dx+c)}+a}}\,dx$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^2/sqrt(b*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [B] (verification not implemented)

Time = 7.34 (sec) , antiderivative size = 2981, normalized size of antiderivative = 17.96

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
2*atanh((32*A^2*b^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A
^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*
A^3*a*b^3*d^3)/(a^2*d^4 + b^2*d^4) - (4*A*b^3*d^2*(-16*A^4*b^2*d^4)^(1/2))
/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b
^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))
^(1/2)*(-16*A^4*b^2*d^4)^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) -
(4*A*b^5*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^
3*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^
2*d^5 + b^2*d^5)) - (32*A^2*a^2*b^2*d^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*
d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c
+ d*x))^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*A*b^5*d^4*(-16
*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^4 +
 b^2*d^4) - (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5))
)*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*
d^4 + b^2*d^4)))^(1/2) - atan((a*b^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4
 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d
*x))^(1/2)*(-16*B^4*b^2*d^4)^(1/2)*8i)/((16*B^3*a^2*b^4*d^5)/(a^2*d^4 + b^
2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/(a^2*d^4 +
 b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)
+ (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (B^2*b...
```

Reduce [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)
```

3.345 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 124

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} - \frac{(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d} + \frac{2B\sqrt{a+b\tan(c+dx)}}{bd}$$

output
```
-(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(1/2)/d-(A+
I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(1/2)/d+2*B*(a+
b*tan(d*x+c))^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}+\frac{(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}-\frac{2B\sqrt{a+b\tan(c+dx)}}{b}}{d}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
-((((A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*
b] + ((A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt[a +
I*b] - (2*B*Sqrt[a + b*Tan[c + d*x]])/b)/d)
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 4075, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4075}$$

$$\int \frac{A\tan(c+dx)-B}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2B\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\tan(c+dx)-B}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2B\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2B\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2B\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{4020}$$

$$-\frac{i(B+iA)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(-B+iA)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2B\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{i(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(-B+iA)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{2B\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{73}$$

$$\frac{(-B+iA)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}-\frac{(B+iA)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{2B\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}+\frac{2B\sqrt{a+b\tan(c+dx)}}{bd}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
-(((I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)) + ((I*
A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d) + (2*B*Sqrt[a
 + b*Tan[c + d*x]])/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1918 vs. $2(104) = 208$.

Time = 0.13 (sec) , antiderivative size = 1919, normalized size of antiderivative = 15.48

method	result	size
parts	Expression too large to display	1919
derivativedivides	Expression too large to display	3997
default	Expression too large to display	3997

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
A/d*(1/2/(a^2+b^2)^(1/2)*(1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*tan(d*
x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))+
2*(a-(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(
1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+
1/2/(a^2+b^2)^(1/2)*(-1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c)+a+
(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-
a+(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c)
)^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))+B*
(2/d/b*(a+b*tan(d*x+c))^(1/2)+1/4/d/b/(a^2+b^2)*ln((a+b*tan(d*x+c))^(1/2)*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)*a^2+1/4/d*b/(a^2+b^2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)-1/4/d/b/(a^2+b^2)^(3/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)*a^3-1/4/d*b/(a^2+b^2)^(3/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/
2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)*a-1/d/b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^
2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
))*a^2-1/d*b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2
+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1706 vs. $2(98) = 196$.

Time = 0.12 (sec) , antiderivative size = 1706, normalized size of antiderivative = 13.76

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
1/2*(b*d*sqrt(((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*
b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b
+ (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^4 - B^4)
*b)*sqrt(b*tan(d*x + c) + a) + ((B*a^3 - A*a^2*b + B*a*b^2 - A*b^3)*d^3*sq
rt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/
((a^4 + 2*a^2*b^2 + b^4)*d^4)) - (2*A^2*B*a^2 - (A^3 - 3*A*B^2)*a*b - (A^2
*B - B^3)*b^2)*d)*sqrt(((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B -
A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) +
 2*A*B*b + (A^2 - B^2)*a)/((a^2 + b^2)*d^2))) - b*d*sqrt(((a^2 + b^2)*d^2*
sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2
)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B^2)*a)/((a^2 + b^2)*d
^2))*log((2*(A^3*B + A*B^3)*a - (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) -
((B*a^3 - A*a^2*b + B*a*b^2 - A*b^3)*d^3*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B -
 A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4))
- (2*A^2*B*a^2 - (A^3 - 3*A*B^2)*a*b - (A^2*B - B^3)*b^2)*d)*sqrt(((a^2 +
b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 +
 B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B^2)*a)/((a^2
 + b^2)*d^2))) - b*d*sqrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*
B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4
)) - 2*A*B*b - (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)...
```

Sympy [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx=\int \frac{(A+B\tan{(c+dx)})\tan{(c+dx)}}{\sqrt{a+b\tan{(c+dx)}}}\,dx$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan\left(dx+c\right)+A)\tan\left(dx+c\right)}{\sqrt{b\tan\left(dx+c\right)+a}}\,dx$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)/sqrt(b*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 2930, normalized size of antiderivative = 23.63

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
2*atanh((32*B^2*b^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (B
^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*
B^3*a*b^3*d^3)/(a^2*d^4 + b^2*d^4) - (4*B*b^3*d^2*(-16*B^4*b^2*d^4)^(1/2))
/(a^2*d^5 + b^2*d^5)) + (8*a*b^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b
^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))
^(1/2)*(-16*B^4*b^2*d^4)^(1/2))/((16*B^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) -
(4*B*b^5*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*B^3*a^3*b^
3*d^5)/(a^2*d^4 + b^2*d^4) - (4*B*a^2*b^3*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^
2*d^5 + b^2*d^5)) - (32*B^2*a^2*b^2*d^2*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*
d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c
+ d*x))^(1/2))/((16*B^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) - (4*B*b^5*d^4*(-16
*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*B^3*a^3*b^3*d^5)/(a^2*d^4 +
 b^2*d^4) - (4*B*a^2*b^3*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5))
)*((-16*B^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a*d^2)/(4*(a^2*
d^4 + b^2*d^4)))^(1/2) - 2*atanh((8*a*b^2*((-16*A^4*b^2*d^4)^(1/2)/(16*(a^
2*d^4 + b^2*d^4)) + (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(
c + d*x))^(1/2)*(-16*A^4*b^2*d^4)^(1/2))/((16*A^3*a^2*b^4*d^5)/(a^2*d^4 +
b^2*d^4) - 16*A^3*a^2*b^2*d - 16*A^3*b^4*d + (16*A^3*a^4*b^2*d^5)/(a^2*d^4
 + b^2*d^4) + (4*A*a^3*b^2*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5
) + (4*A*a*b^4*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*...
```

Reduce [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx=\int \sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)
```

3.346 $\int \frac{A+B\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 102

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} + \frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

output
```
-(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(1/2)/d+(I*
A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{i\left(-\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \frac{(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}\right)}{d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
(I*(-(((A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a -
 I*b]) + ((A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt[
a + I*b]))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{i(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{i(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}$$

$$\Big\downarrow \text{73}$$

$$\frac{(A+iB)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(A-iB)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{221}$$

$$\frac{(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

input
```
Int[(A + B*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
((A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + ((A + I
*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1893 vs. $2(84) = 168$.

Time = 0.49 (sec) , antiderivative size = 1894, normalized size of antiderivative = 18.57

method	result	size
parts	Expression too large to display	1894
derivativedivides	Expression too large to display	3976
default	Expression too large to display	3976

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/4/d/b/(a^2+b^2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/
d*b/(a^2+b^2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+
2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b/(a^2+b^2
)^(3/2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/d*b/(a^2+b^2)^
(3/2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d/b/(a^2+b^2)^(1/2)/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^2
)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/d*b/(a^2+b^2)^(1/
2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+
b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/d/b/(a^2+b^2)^(3/2
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b
^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4+3/d*b/(a^2+b^2)^(
3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^
2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+2/d*b^3/(a^2+b
^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/4/d/b/(a^2+
b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+1/4/d*b/(a^2+b^2)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1669 vs. $2(79) = 158$.

Time = 0.12 (sec) , antiderivative size = 1669, normalized size of antiderivative = 16.36

$$\int \frac{A + B\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output

```
-1/2*sqrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b +
 (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (
A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*a - (A^4 - B^4)*b)
*sqrt(b*tan(d*x + c) + a) + ((A*a^3 + B*a^2*b + A*a*b^2 + B*b^3)*d^3*sqrt(
-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a
^4 + 2*a^2*b^2 + b^4)*d^4)) + (2*A*B^2*a^2 - (3*A^2*B - B^3)*a*b + (A^3 -
A*B^2)*b^2)*d)*sqrt(-((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*
B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2
*A*B*b + (A^2 - B^2)*a)/((a^2 + b^2)*d^2))) + 1/2*sqrt(-((a^2 + b^2)*d^2*s
qrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)
/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B^2)*a)/((a^2 + b^2)*d^
2))*log((2*(A^3*B + A*B^3)*a - (A^4 - B^4)*b)*sqrt(b*tan(d*x + c) + a) - (
(A*a^3 + B*a^2*b + A*a*b^2 + B*b^3)*d^3*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B -
A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) +
 (2*A*B^2*a^2 - (3*A^2*B - B^3)*a*b + (A^3 - A*B^2)*b^2)*d)*sqrt(-((a^2 +
b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B - A*B^3)*a*b + (A^4 - 2*A^2*B^2 +
 B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*A*B*b + (A^2 - B^2)*a)/((a^2
 + b^2)*d^2))) + 1/2*sqrt(((a^2 + b^2)*d^2*sqrt(-(4*A^2*B^2*a^2 - 4*(A^3*B
 - A*B^3)*a*b + (A^4 - 2*A^2*B^2 + B^4)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^4)
) - 2*A*B*b - (A^2 - B^2)*a)/((a^2 + b^2)*d^2))*log((2*(A^3*B + A*B^3)*...
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\, dx = \int \frac{A + B\tan{(c + dx)}}{\sqrt{a + b\tan{(c + dx)}}}\, dx$$

input

```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output

```
Integral((A + B*tan(c + d*x))/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 2909, normalized size of antiderivative = 28.52

$$\int \frac{A + B\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
2*atanh((8*a*b^2*(- (-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^
2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(-16*A^
4*b^2*d^4)^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*b^5*d^4*(
-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2*d^
4 + b^2*d^4) + (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^
5)) - (32*A^2*b^2*(- (-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A
^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*
A^3*a*b^3*d^3)/(a^2*d^4 + b^2*d^4) + (4*A*b^3*d^2*(-16*A^4*b^2*d^4)^(1/2))
/(a^2*d^5 + b^2*d^5)) + (32*A^2*a^2*b^2*d^2*(- (-16*A^4*b^2*d^4)^(1/2)/(16
*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*
tan(c + d*x))^(1/2))/((16*A^3*a*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (4*A*b^5*d^
4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (16*A^3*a^3*b^3*d^5)/(a^2
*d^4 + b^2*d^4) + (4*A*a^2*b^3*d^4*(-16*A^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2
*d^5)))*(- (-16*A^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (A^2*a*d^2)/
(4*(a^2*d^4 + b^2*d^4)))^(1/2) - 2*atanh((8*a*b^2*((-16*B^4*b^2*d^4)^(1/2)
/(16*(a^2*d^4 + b^2*d^4)) + (B^2*a*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a
+ b*tan(c + d*x))^(1/2)*(-16*B^4*b^2*d^4)^(1/2))/((16*B^3*a^2*b^4*d^5)/(a^
2*d^4 + b^2*d^4) - 16*B^3*a^2*b^2*d - 16*B^3*b^4*d + (16*B^3*a^4*b^2*d^5)/
(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^2*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 +
 b^2*d^5) + (4*B*a*b^4*d^4*(-16*B^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx=\int \sqrt{a+\tan(dx+c)\,b}dx$$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a),x)
```

3.347 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 131

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} + \frac{(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

output

```
-2*A*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d+(A-I*B)*arctanh((a+
b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(1/2)/d+(A+I*B)*arctanh((a+b*ta
n(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.30

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{\frac{2A\,\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\frac{\left(-Ab+\sqrt{-b^2}B\right)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{a-\sqrt{-b^2}}}-\frac{\left(Ab+\sqrt{-b^2}B\right)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{a+\sqrt{-b^2}}}}{b}}{d}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
-(((2*A*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] + (((-(A*b) + S
qrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/Sqrt[
a - Sqrt[-b^2]] - ((A*b + Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/S
qrt[a + Sqrt[-b^2]]])/Sqrt[a + Sqrt[-b^2]])/b)/d)
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4096, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4096}$$

$$\int \frac{B-A\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+A\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B-A\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4022}$$

$$-\frac{1}{2}(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+ A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+ A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(B+iA)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+ \frac{i(-B+iA)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+ A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{i(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}- \frac{i(-B+iA)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+ A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{73}$$

$$-\frac{(-B+iA)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+ \frac{(B+iA)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{221}$$

$$A\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

$$\Big\downarrow \text{4117}$$

$$\frac{A\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2A\int \frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

$$\Big\downarrow \text{221}$$

$$\frac{(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
((I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) - ((I*A -
 B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d) - (2*A*ArcTanh[S
qrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4096
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)])^(n_))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*Simp[a*A + b*B - (A*b - a*B)*Tan
[e + f*x], x], x], x] + Simp[b*((A*b - a*B)/(a^2 + b^2))   Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4001 vs. $2(107) = 214$.

Time = 0.15 (sec) , antiderivative size = 4002, normalized size of antiderivative = 30.55

method	result	size
derivativedivides	Expression too large to display	4002
default	Expression too large to display	4002

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2*A*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d-1/4/d/b/(a^2+b^2)^(
3/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/4/d*b/(a^2+b^2)^(
3/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d/b/(a^2+b^2)^(1/2)
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^
2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2-1/d/b/(a^2+b^2)^
(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^4-1/4/d/b^2/(
a^2+b^2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x
+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/d*b^2/(a^2+b^
2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a-3/d*b/(a^2
+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^
(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2-1/d/b
^2*(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2
)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a+
1/d/b^2/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))
*A*a^3+1/d/b^2/(a^2+b^2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1718 vs. $2(101) = 202$.

Time = 0.78 (sec) , antiderivative size = 3455, normalized size of antiderivative = 26.37

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)/sqrt(b*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regula
r value [
```

Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 7099, normalized size of antiderivative = 54.19

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
- atan(((((32*(12*A^2*B*b^9*d^2 + 3*A^3*a*b^8*d^2 - 9*A*B^2*a*b^8*d^2))/d^
5 - (((32*(16*A*b^10*d^4 - 4*B*a*b^9*d^4 + 12*A*a^2*b^8*d^4))/d^5 - (32*(1
6*b^10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(-(((8*A^2*a*d^2 -
 8*B^2*a*d^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 + 16*b^2*d^4)*(A^4 + 2*A^2*
B^2 + B^4))^(1/2) - 4*A^2*a*d^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(16*(a^2*d^4
+ b^2*d^4)))^(1/2))/d^4)*(-(((8*A^2*a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2)^2/
4 - (16*a^2*d^4 + 16*b^2*d^4)*(A^4 + 2*A^2*B^2 + B^4))^(1/2) - 4*A^2*a*d^2
 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2) + (32*(a + b
*tan(c + d*x))^(1/2)*(16*A*B*b^9*d^2 + 18*A^2*a*b^8*d^2 - 10*B^2*a*b^8*d^2
))/d^4)*(-(((8*A^2*a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 +
 16*b^2*d^4)*(A^4 + 2*A^2*B^2 + B^4))^(1/2) - 4*A^2*a*d^2 + 4*B^2*a*d^2 -
8*A*B*b*d^2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2))*(-(((8*A^2*a*d^2 - 8*B^2*a*d
^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 + 16*b^2*d^4)*(A^4 + 2*A^2*B^2 + B^4)
)^(1/2) - 4*A^2*a*d^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(16*(a^2*d^4 + b^2*d^4)
))^(1/2) + (32*(3*A^4*b^8 + B^4*b^8)*(a + b*tan(c + d*x))^(1/2))/d^4)*(-((
(8*A^2*a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 + 16*b^2*d^4)
*(A^4 + 2*A^2*B^2 + B^4))^(1/2) - 4*A^2*a*d^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)
/(16*(a^2*d^4 + b^2*d^4)))^(1/2)*1i - (((32*(12*A^2*B*b^9*d^2 + 3*A^3*a*b^
8*d^2 - 9*A*B^2*a*b^8*d^2))/d^5 - (((32*(16*A*b^10*d^4 - 4*B*a*b^9*d^4 + 1
2*A*a^2*b^8*d^4))/d^5 + (32*(16*b^10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(c...
```

Reduce [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)\,dx$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*cot(c + d*x),x)
```

3.348 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 169

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{(Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} - \frac{(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d} - \frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}$$

output
```
(A*b-2*B*a)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+(I*A+B)*arct
anh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(1/2)/d-(I*A-B)*arctanh(
(a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(1/2)/d-A*cot(d*x+c)*(a+b*ta
n(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.19

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$= \frac{\frac{b(Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\left(A\sqrt{-b^2}+bB\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{a-\sqrt{-b^2}}} - \frac{\left(A\sqrt{-b^2}-bB\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+\sqrt{-b^2}}}\right)}{\sqrt{a+\sqrt{-b^2}}}}{bd}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x
]
```

output
```
((b*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/a^(3/2) + ((A
*Sqrt[-b^2] + b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])
/Sqrt[a - Sqrt[-b^2]] - ((A*Sqrt[-b^2] - b*B)*ArcTanh[Sqrt[a + b*Tan[c + d
*x]]/Sqrt[a + Sqrt[-b^2]]])/Sqrt[a + Sqrt[-b^2]] - (A*b*Cot[c + d*x]*Sqrt[
a + b*Tan[c + d*x]])/a)/(b*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 4092, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2\sqrt{a+b\tan(c+dx)}}dx$$
$$\Big\downarrow \text{4092}$$

$$-\frac{\int \frac{\cot(c+dx)\left(Ab\tan^2(c+dx)+2aA\tan(c+dx)+Ab-2aB\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cot(c+dx)\left(Ab\tan^2(c+dx)+2aA\tan(c+dx)+Ab-2aB\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{Ab\tan(c+dx)^2+2aA\tan(c+dx)+Ab-2aB}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{4136}$$

$$-\frac{\int \frac{2(aA+aB\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx+(Ab-2aB)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{aA+aB\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+(Ab-2aB)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{aA+aB\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+(Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}$$

$$\Big\downarrow \text{4022}$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{(Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{1}{2}a(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx\right)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{(Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{1}{2}a(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx\right)}{2a}$$

$$\Big\downarrow \ 4020$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{(Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{ia(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{ia(A+iB)\int -\frac{}{(i\tan(c+}}{}\right.}{2a}$$

$$\Big\downarrow \ 25$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{(Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{ia(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{ia(A-iB)\int \frac{}{(1-i\tan(c}}{}\right.}{2a}$$

$$\Big\downarrow \ 73$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{(Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{a(A+iB)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{a(A-iB)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+}}{bd}\right.}{2a}$$

$$\Big\downarrow \ 221$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{(Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{a(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{a(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}\right)}{2a}$$

$$\Big\downarrow \ 4117$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{\frac{(Ab-2aB)\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+2\left(\frac{a(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{a(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}\right)}{2a}$$

$$\Big\downarrow \ 73$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{\frac{2(Ab-2aB)\int \frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+2\left(\frac{a(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{a(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}\right)}{2a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{A\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}-\frac{-\frac{2(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+2\left(\frac{a(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{a(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}\right)}{2a}$$

input

```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output

```
-1/2*(2*((a*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d
) + (a*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)) -
(2*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d))/a
 - (A*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(a*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4056 vs. $2(143) = 286$.

Time = 0.17 (sec) , antiderivative size = 4057, normalized size of antiderivative = 24.01

method	result	size
derivativedivides	Expression too large to display	4057
default	Expression too large to display	4057

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/4/d/(a^2+b^2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b
*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/d/(a^
2+b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-2/d/(a^2+b^2)/(2*(a
^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/
2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2+1/d/b^2/(2*(a^2+b^2)^(
1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(
1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2-2*B*arctanh((a+b*tan(d*x+c))^(1
/2)/a^(1/2))/a^(1/2)/d-1/d*A/a*(a+b*tan(d*x+c))^(1/2)/tan(d*x+c)+1/d*b/a^(
3/2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))*A-1/4/d/b^2*ln(b*tan(d*x+c)+a
+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d*b/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*
a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A-2/d*b^3/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A+1/4/d*b^2/(a^2+b^2)^(3/2)*ln(b*tan(d*x+
c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))
*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*b^2/(a^2+b^2)/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*
(a^2+b^2)^(1/2)-2*a)^(1/2))*B-1/4/d*b/(a^2+b^2)*ln(b*tan(d*x+c)+a+(a+b*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. $2(139) = 278$.

Time = 2.00 (sec) , antiderivative size = 3579, normalized size of antiderivative = 21.18

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\cot^2{(c+dx)}}{\sqrt{a+b\tan{(c+dx)}}}\,dx$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**2/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\cot{(dx+c)}^2}{\sqrt{b\tan{(dx+c)}+a}}\,dx$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^2/sqrt(b*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regula
r value [
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 9790, normalized size of antiderivative = 57.93

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
atan(((((((8*(32*A*a*b^11*d^4 + 16*A*a^3*b^9*d^4 - 64*B*a^2*b^10*d^4 - 48*
B*a^4*b^8*d^4))/(a^2*d^5) - (16*(32*a^2*b^10*d^4 + 48*a^4*b^8*d^4)*(a + b*
tan(c + d*x))^(1/2)*((((8*A^2*a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2)^2/4 - (1
6*a^2*d^4 + 16*b^2*d^4)*(A^4 + 2*A^2*B^2 + B^4))^(1/2) - 4*A^2*a*d^2 + 4*B
^2*a*d^2 - 8*A*B*b*d^2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2))/(a^2*d^4))*((((8*
A^2*a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 + 16*b^2*d^4)*(A
^4 + 2*A^2*B^2 + B^4))^(1/2) - 4*A^2*a*d^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(1
6*(a^2*d^4 + b^2*d^4)))^(1/2) - (16*(a + b*tan(c + d*x))^(1/2)*(20*A^2*a^3
*b^8*d^2 - 36*B^2*a^3*b^8*d^2 - 4*A^2*a*b^10*d^2 + 48*A*B*a^2*b^9*d^2))/(a
^2*d^4))*((((8*A^2*a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 +
 16*b^2*d^4)*(A^4 + 2*A^2*B^2 + B^4))^(1/2) - 4*A^2*a*d^2 + 4*B^2*a*d^2 -
8*A*B*b*d^2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2) + (8*(4*A^3*b^11*d^2 + 16*A^3
*a^2*b^9*d^2 + 12*B^3*a^3*b^8*d^2 + 12*A^2*B*a*b^10*d^2 - 48*A*B^2*a^2*b^9
*d^2 - 36*A^2*B*a^3*b^8*d^2))/(a^2*d^5))*((((8*A^2*a*d^2 - 8*B^2*a*d^2 + 1
6*A*B*b*d^2)^2/4 - (16*a^2*d^4 + 16*b^2*d^4)*(A^4 + 2*A^2*B^2 + B^4))^(1/2
) - 4*A^2*a*d^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(16*(a^2*d^4 + b^2*d^4)))^(1/
2) - (16*(a + b*tan(c + d*x))^(1/2)*(A^2*B^2*b^10 - A^4*b^10 + 2*A^4*a^2*b
^8 + 6*B^4*a^2*b^8 - 4*A*B^3*a*b^9 + 4*A^3*B*a*b^9))/(a^2*d^4))*((((8*A^2*
a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 + 16*b^2*d^4)*(A^4 +
 2*A^2*B^2 + B^4))^(1/2) - 4*A^2*a*d^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(16...
```

Reduce [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)^2\,dx$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*cot(c + d*x)**2,x)
```

3.349 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 224

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{(8a^2A-3Ab^2+4abB)\,\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}d} - \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} - \frac{(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d} + \frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{4a^2d} - \frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

output
```
1/4*(8*A*a^2-3*A*b^2+4*B*a*b)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(5
/2)/d-(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(1/2)/
d-(A+I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(1/2)/d+1/
4*(3*A*b-4*B*a)*cot(d*x+c)*(a+b*tan(d*x+c))^(1/2)/a^2/d-1/2*A*cot(d*x+c)^2
*(a+b*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 6.20 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.61

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= \frac{2b^3\left(\frac{A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}b^3} - \frac{3A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{8a^{5/2}b} + \frac{B\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{2a^{3/2}b^2} - \frac{\left(A\sqrt{-b^2}+bB\right)\text{arctan}}{2b^3\sqrt{-b^2}\sqrt{}}\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x
]
```

output
```
(2*b^3*((A*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*b^3) - (3*A
*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(8*a^(5/2)*b) + (B*ArcTanh[Sqr
t[a + b*Tan[c + d*x]]/Sqrt[a]])/(2*a^(3/2)*b^2) - ((A*Sqrt[-b^2] + b*B)*Ar
cTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(2*b^3*Sqrt[-b^2]*Sq
rt[a - Sqrt[-b^2]]) - (b*(A*Sqrt[-b^2] - b*B)*ArcTanh[Sqrt[a + b*Tan[c + d
*x]]/Sqrt[a + Sqrt[-b^2]]])/(2*(-b^2)^(5/2)*Sqrt[a + Sqrt[-b^2]]) + (3*A*C
ot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(8*a^2*b^2) - (B*Cot[c + d*x]*Sqrt[a
 + b*Tan[c + d*x]])/(2*a*b^3) - (A*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]]
)/(4*a*b^3)))/d
```

Rubi [A] (warning: unable to verify)

Time = 1.72 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{\int \frac{\cot^2(c+dx)(3Ab\tan^2(c+dx)+4aA\tan(c+dx)+3Ab-4aB)}{2\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cot^2(c+dx)(3Ab\tan^2(c+dx)+4aA\tan(c+dx)+3Ab-4aB)}{\sqrt{a+b\tan(c+dx)}}dx}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{3Ab\tan(c+dx)^2+4aA\tan(c+dx)+3Ab-4aB}{\tan(c+dx)^2\sqrt{a+b\tan(c+dx)}}dx}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{-\frac{\int -\frac{\cot(c+dx)\left(8Aa^2+8B\tan(c+dx)a^2+4bBa-3Ab^2-b(3Ab-4aB)\tan^2(c+dx)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\cot(c+dx)\left(8Aa^2+8B\tan(c+dx)a^2+4bBa-3Ab^2-b(3Ab-4aB)\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{8Aa^2+8B\tan(c+dx)a^2+4bBa-3Ab^2-b(3Ab-4aB)\tan(c+dx)^2}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

$$\Big\downarrow \text{4136}$$

$$-\frac{\frac{\frac{(8a^2A+4abB-3Ab^2)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+\int \frac{8\left(a^2B-a^2A\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{(8a^2A+4abB-3Ab^2)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+8\int \frac{a^2B-a^2A\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{(8a^2A+4abB-3Ab^2)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\int \frac{a^2B-a^2A\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{2a}-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}}{4a}-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}$$

$\downarrow$ 4022

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(8a^2A+4abB-3Ab^2)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(\frac{1}{2}a^2(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a^2(-\right.}{2a}}{4a}$$

$\downarrow$ 3042

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(8a^2A+4abB-3Ab^2)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(\frac{1}{2}a^2(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a^2(-\right.}{2a}}{4a}$$

$\downarrow$ 4020

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(8a^2A+4abB-3Ab^2)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(\frac{ia^2(B+iA)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+\right.}}{2d}}{2a}}{4a}$$

$\downarrow$ 25

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(8a^2A+4abB-3Ab^2)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(-\frac{ia^2(B+iA)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+}}{2d}\right.}{2a}}{4a}$$

$\downarrow$ 73

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(8a^2A+4abB-3Ab^2)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(\frac{a^2(B+iA)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan}}{bd}\right.}{2a}}{4a}$$

$\downarrow$ 221

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{(8a^2A+4abB-3Ab^2)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+8\left(\frac{a^2(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a^2(-B+iA}{}\right.}{2a}}{4a}$$

$\downarrow$ 4117

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{\frac{(8a^2A+4abB-3Ab^2)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+8\left(\frac{a^2(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a^2(-B+iA)a}{d}\right.}{2a}}{4a}$$

$\downarrow$ 73

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{\frac{2(8a^2A+4abB-3Ab^2)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+8\left(\frac{a^2(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a^2(-}{}\right.}{2a}}{4a}$$

$\downarrow$ 221

$$-\frac{A\cot^2(c+dx)\sqrt{a+b\tan(c+dx)}}{2ad}-\frac{-\frac{(3Ab-4aB)\cot(c+dx)\sqrt{a+b\tan(c+dx)}}{ad}+\frac{-\frac{2(8a^2A+4abB-3Ab^2)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+8\left(\frac{a^2(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a^2(-B+i}{}\right.}{2a}}{4a}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
-1/2*(A*Cot[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]])/(a*d) - ((8*((a^2*(I*A +
B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) - (a^2*(I*A - B)*
ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)) - (2*(8*a^2*A - 3*A
*b^2 + 4*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d))/(2
*a) - ((3*A*b - 4*a*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(a*d))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4174 vs. $2(190) = 380$.

Time = 0.16 (sec) , antiderivative size = 4175, normalized size of antiderivative = 18.64

method	result	size
derivativedivides	Expression too large to display	4175
default	Expression too large to display	4175

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
2*A*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d+3/4/d/tan(d*x+c)^2/a
^2*(a+b*tan(d*x+c))^(3/2)*A-5/4/d/tan(d*x+c)^2/a*(a+b*tan(d*x+c))^(1/2)*A+
1/d/b/tan(d*x+c)^2*(a+b*tan(d*x+c))^(1/2)*B-3/4/d*b^2/a^(5/2)*arctanh((a+b
*tan(d*x+c))^(1/2)/a^(1/2))*A+1/d*b/a^(3/2)*arctanh((a+b*tan(d*x+c))^(1/2)
/a^(1/2))*B-1/d/b/tan(d*x+c)^2/a*(a+b*tan(d*x+c))^(3/2)*B+1/4/d/b/(a^2+b^2
)^(3/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/d*b/(a^2+b^2
)^(3/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/b/(a^2+b^2)^(1
/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2+1/d/b/(a^2+b^
2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^4+1/4/d/b^
2/(a^2+b^2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(
d*x+c)-a-(a^2+b^2)^(1/2))*A*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/d*b^2/(a^2
+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^
(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*A*a+3/d*b/(
a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*
a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2+1/
d/b^2*(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(184) = 368$.

Time = 7.17 (sec) , antiderivative size = 3685, normalized size of antiderivative = 16.45

$$\int \frac{\cot^3(c + dx)(A + B\tan(c + dx))}{\sqrt{a + b\tan(c + dx)}}\, dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\cot^3{(c+dx)}}{\sqrt{a+b\tan{(c+dx)}}}\,dx$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**3/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\cot{(dx+c)}^3}{\sqrt{b\tan{(dx+c)}+a}}\,dx$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^3/sqrt(b*tan(d*x + c) + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 13182, normalized size of antiderivative = 58.85

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
- (((5*A*b^2 - 4*B*a*b)*(a + b*tan(c + d*x))^(1/2))/(4*a) - ((3*A*b^2 - 4*
B*a*b)*(a + b*tan(c + d*x))^(3/2))/(4*a^2))/(d*(a + b*tan(c + d*x))^2 + a^
2*d - 2*a*d*(a + b*tan(c + d*x))) - atan((((((((640*A*a^4*b^10*d^4 - 384*A*
a^2*b^12*d^4 + 768*A*a^6*b^8*d^4 + 512*B*a^3*b^11*d^4 + 256*B*a^5*b^9*d^4)
/(2*a^4*d^5) + ((512*a^4*b^10*d^4 + 768*a^6*b^8*d^4)*(a + b*tan(c + d*x))^
(1/2)*(-(((8*A^2*a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 + 1
6*b^2*d^4)*(A^4 + 2*A^2*B^2 + B^4))^(1/2) - 4*A^2*a*d^2 + 4*B^2*a*d^2 - 8*
A*B*b*d^2)/(16*(a^2*d^4 + b^2*d^4)))^(1/2))/(a^4*d^4))*(-(((8*A^2*a*d^2 -
8*B^2*a*d^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 + 16*b^2*d^4)*(A^4 + 2*A^2*B
^2 + B^4))^(1/2) - 4*A^2*a*d^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(16*(a^2*d^4 +
 b^2*d^4)))^(1/2) - ((a + b*tan(c + d*x))^(1/2)*(576*A^2*a^5*b^8*d^2 - 192
*A^2*a^3*b^10*d^2 + 64*B^2*a^3*b^10*d^2 - 320*B^2*a^5*b^8*d^2 + 36*A^2*a*b
^12*d^2 - 96*A*B*a^2*b^11*d^2 + 768*A*B*a^4*b^9*d^2))/(a^4*d^4))*(-(((8*A^
2*a*d^2 - 8*B^2*a*d^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 + 16*b^2*d^4)*(A^4
 + 2*A^2*B^2 + B^4))^(1/2) - 4*A^2*a*d^2 + 4*B^2*a*d^2 - 8*A*B*b*d^2)/(16*
(a^2*d^4 + b^2*d^4)))^(1/2) + (64*B^3*a^2*b^11*d^2 - 192*A^3*a^5*b^8*d^2 +
 256*B^3*a^4*b^9*d^2 + 36*A^2*B*b^13*d^2 + 36*A^3*a*b^12*d^2 - 96*A*B^2*a*
b^12*d^2 - 384*A*B^2*a^3*b^10*d^2 + 576*A*B^2*a^5*b^8*d^2 + 96*A^2*B*a^2*b
^11*d^2 - 768*A^2*B*a^4*b^9*d^2)/(2*a^4*d^5))*(-(((8*A^2*a*d^2 - 8*B^2*a*d
^2 + 16*A*B*b*d^2)^2/4 - (16*a^2*d^4 + 16*b^2*d^4)*(A^4 + 2*A^2*B^2 + B...
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)^3\,dx$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*cot(c + d*x)**3,x)
```

3.350 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 264

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} + \frac{(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} + \frac{2a(Ab-aB)\tan^2(c+dx)}{b(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{2(6a^2Ab+3Ab^3-8a^3B-5ab^2B)\sqrt{a+b\tan(c+dx)}}{3b^3(a^2+b^2)\,d} - \frac{2(3aAb-4a^2B-b^2B)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3b^2(a^2+b^2)\,d}$$

output
```
(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+(A+I
*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+2*a*(A*b
-B*a)*tan(d*x+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)+2/3*(6*A*a^2*b+3*A
*b^3-8*B*a^3-5*B*a*b^2)*(a+b*tan(d*x+c))^(1/2)/b^3/(a^2+b^2)/d-2/3*(3*A*a*
b-4*B*a^2-B*b^2)*tan(d*x+c)*(a+b*tan(d*x+c))^(1/2)/b^2/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.43 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.14

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{3iA\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}\right)+\frac{2(6aAb}{b^2\sqrt{a}}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2)
,x]
```

output
```
((3*I)*A*(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] -
ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b]) + (2*(6*a*A
*b - 8*a^2*B + 3*b^2*B))/(b^2*Sqrt[a + b*Tan[c + d*x]]) + ((3*I)*(a*A + b*
B)*((a + I*b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a - I*
b)] - (a - I*b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a +
I*b)]))/((a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]]) + (2*(3*A*b - 4*a*B)*Tan[c
+ d*x])/(b*Sqrt[a + b*Tan[c + d*x]]) + (2*B*Tan[c + d*x]^2)/Sqrt[a + b*Tan
[c + d*x]])/(3*b*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.46 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4088, 27, 3042, 4130, 27, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 4088$$

$$\frac{2\int -\frac{\tan(c+dx)\left(\left(-4Ba^2+3Aba-b^2B\right)\tan^2(c+dx)-b(Ab-aB)\tan(c+dx)+4a(Ab-aB)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}+\frac{2a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{\tan(c+dx)\left(\left(-4Ba^2+3Aba-b^2B\right)\tan^2(c+dx)-b(Ab-aB)\tan(c+dx)+4a(Ab-aB)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{\tan(c+dx)\left(\left(-4Ba^2+3Aba-b^2B\right)\tan(c+dx)^2-b(Ab-aB)\tan(c+dx)+4a(Ab-aB)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 4130$$

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2\int -\frac{-3(aA+bB)\tan(c+dx)b^2+\left(-8Ba^3+6Aba^2-5b^2Ba+3Ab^3\right)\tan^2(c+dx)+2a\left(-4Ba^2+3Aba-b^2B\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{3b}+\frac{2\left(-4a^2B+3aAb-b^2B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2\left(-4a^2B+3aAb-b^2B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int \frac{-3(aA+bB)\tan(c+dx)b^2+\left(-8Ba^3+6Aba^2-5b^2Ba+3Ab^3\right)\tan^2(c+dx)+2a\left(-4Ba^2+3Aba-b^2B\right)}{\sqrt{a+b\tan(c+dx)}}dx}{3b}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2\left(-4a^2B+3aAb-b^2B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int \frac{-3(aA+bB)\tan(c+dx)b^2+\left(-8Ba^3+6Aba^2-5b^2Ba+3Ab^3\right)\tan(c+dx)^2+2a\left(-4Ba^2+3Aba-b^2B\right)}{\sqrt{a+b\tan(c+dx)}}dx}{3b}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 4113$$

$$\frac{\frac{2a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2(-4a^2B+3aAb-b^2B)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int\frac{-3(Ab-aB)b^2-3(aA+bB)\tan(c+dx)b^2}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(-8a^3B+6a^2Ab-5ab^2B+3Ab^3)\sqrt{a+b\tan(c+dx)}}{bd}}{3b}}{b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{\frac{2a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2(-4a^2B+3aAb-b^2B)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int\frac{-3(Ab-aB)b^2-3(aA+bB)\tan(c+dx)b^2}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(-8a^3B+6a^2Ab-5ab^2B+3Ab^3)\sqrt{a+b\tan(c+dx)}}{bd}}{3b}}{b(a^2+b^2)}$$

$\downarrow$ 4022

$$\frac{\frac{2a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2(-4a^2B+3aAb-b^2B)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{-\frac{3}{2}b^2(b+ia)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{3}{2}b^2(a+ib)(B+iA)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(\cdots}{}}{3b}}{b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{\frac{2a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2(-4a^2B+3aAb-b^2B)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{-\frac{3}{2}b^2(b+ia)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{3}{2}b^2(a+ib)(B+iA)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(\cdots}{}}{3b}}{b(a^2+b^2)}$$

$\downarrow$ 4020

$$\frac{\frac{2a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2(-4a^2B+3aAb-b^2B)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\frac{3ib^2(a+ib)(B+iA)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{3ib^2(b+ia)(A+iB)\int-\cdots}{}}{}}{b(a^2+b^2)}$$

$\downarrow$ 25

$$\frac{\frac{2a(Ab-aB)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2(-4a^2B+3aAb-b^2B)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{-\frac{3ib^2(a+ib)(B+iA)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{3ib^2(b+ia)(A+iB)\int\frac{\cdots}{(i\cdots}}{}}{}}{b(a^2+b^2)}$$

$\downarrow$ 73

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2\left(-4a^2B+3aAb-b^2B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{-\frac{3b(b+ia)(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}+\frac{3b(a+ib)(B+iA)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}\cdots}}{d}}{3b}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 221

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2\left(-4a^2B+3aAb-b^2B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\frac{2\left(-8a^3B+6a^2Ab-5ab^2B+3Ab^3\right)\sqrt{a+b\tan(c+dx)}}{bd}+\frac{3b^2(a+ib)(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\cdots}{3b}}{b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
(2*a*(A*b - a*B)*Tan[c + d*x]^2)/(b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]
) - ((2*(3*a*A*b - 4*a^2*B - b^2*B)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]])
/(3*b*d) - ((3*(a + I*b)*b^2*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])
/(Sqrt[a - I*b]*d) - (3*b^2*(I*a + b)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a
 + I*b]])/(Sqrt[a + I*b]*d) + (2*(6*a^2*A*b + 3*A*b^3 - 8*a^3*B - 5*a*b^2*
B)*Sqrt[a + b*Tan[c + d*x]])/(b*d))/(3*b))/(b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3750 vs. $2(236) = 472$.

Time = 0.24 (sec) , antiderivative size = 3751, normalized size of antiderivative = 14.21

method	result	size
parts	Expression too large to display	3751
derivativedivides	Expression too large to display	8025
default	Expression too large to display	8025

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
A*(2/d/b^2*(a+b*tan(d*x+c))^(1/2)+1/4/d/(a^2+b^2)^2*ln(b*tan(d*x+c)+a-(a+b
*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+
b^2)^(1/2)+2*a)^(1/2)*a^2+1/4/d*b^2/(a^2+b^2)^2*ln(b*tan(d*x+c)+a-(a+b*tan
(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-1/2/d/(a^2+b^2)^(5/2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))
^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*a^3-1/2/d*b^2/(a^2+b^2)^(5/2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))
^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2
*a)^(1/2)*a+1/d/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a
+b*tan(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a
)^(1/2))*a^2-1/d/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*
tan(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2))*a^3+1/d*b^2/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*
(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))-1/d*b^2/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+
b*tan(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2))*a-2/d*b^2/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*
(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2))*a^2-2/d*b^4/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arcta
n((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4421 vs. $2(229) = 458$.

Time = 0.58 (sec) , antiderivative size = 4421, normalized size of antiderivative = 16.75

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^3(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**3/(a + b*tan(c + d*x))**(3/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^3}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^3/(b*tan(d*x + c) + a)^(3/2),
x)
```

Mupad [B] (verification not implemented)

Time = 19.11 (sec) , antiderivative size = 5811, normalized size of antiderivative = 22.01

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
(log(24*A^3*a^3*b^6*d^2 - ((((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*
A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*
d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((((96*A^4*a^2*b^4*d^4 - 16*A
^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2
)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*tan(c
+ d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^
7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 - 32*A*b^12*d^4 - 96*A*a
^2*b^10*d^4 - 64*A*a^4*b^8*d^4 + 64*A*a^6*b^6*d^4 + 96*A*a^8*b^4*d^4 + 32*
A*a^10*b^2*d^4))/4 + (a + b*tan(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*
a^2*b^8*d^3 - 32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3))*(((96*A^4*a^2*b^4*
d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2
*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4
+ 24*A^3*a^5*b^4*d^2 + 8*A^3*a^7*b^2*d^2 + 8*A^3*a*b^8*d^2)*(((96*A^4*a^2*
b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12
*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)
)/4 + (log(24*A^3*a^3*b^6*d^2 - ((((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4
 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4
 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((-((96*A^4*a^2*b^4*d^
4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a
*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a...
```

Reduce [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3}{a+\tan(dx+c)\,b}dx$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3)/(tan(c + d*x)*b + a),x)
```

3.351 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}} dx$

Optimal result

Integrand size = 33, antiderivative size = 167

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}} dx = \frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} - \frac{2a^2(Ab-aB)}{b^2(a^2+b^2)d\sqrt{a+b\tan(c+dx)}} + \frac{2B\sqrt{a+b\tan(c+dx)}}{b^2d}$$

output
```
(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-(I*A
-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-2*a^2*(A
*b-B*a)/b^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)+2*B*(a+b*tan(d*x+c))^(1/2)/
b^2/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.91 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.49

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{iB\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}\right) + \frac{-2Ab}{b\sqrt{a+b\tan}}}{}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2)
,x]
```

output
```
(I*B*(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] - ArcT
anh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b]) + (-2*A*b + 4*a
*B)/(b*Sqrt[a + b*Tan[c + d*x]]) + ((A*b - a*B)*(((-I)*a + b)*Hypergeometr
ic2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a - I*b)] + (I*a + b)*Hypergeome
tric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a + I*b)]))/((a^2 + b^2)*Sqrt[
a + b*Tan[c + d*x]]) + (2*B*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]])/(b*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4087, 25, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4087}$$

$$\frac{\int -\frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2a^2(Ab-aB)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}$$

$\big\downarrow$ 25

$$-\frac{\int \frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2a^2(Ab-aB)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}$$

$\big\downarrow$ 3042

$$-\frac{\int \frac{-((a^2+b^2)B\tan(c+dx)^2)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2a^2(Ab-aB)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}$$

$\big\downarrow$ 4113

$$-\frac{\int \frac{b(aA+bB)-b(Ab-aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2B(a^2+b^2)\sqrt{a+b\tan(c+dx)}}{bd}}{b(a^2+b^2)}-\frac{2a^2(Ab-aB)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}$$

$\big\downarrow$ 3042

$$-\frac{\int \frac{b(aA+bB)-b(Ab-aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2B(a^2+b^2)\sqrt{a+b\tan(c+dx)}}{bd}}{b(a^2+b^2)}-\frac{2a^2(Ab-aB)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}$$

$\big\downarrow$ 4022

$$-\frac{2a^2(Ab-aB)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{1}{2}b(a-ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}b(a+ib)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2B(a^2+b^2)\sqrt{a+b\tan(c+dx)}}{bd}}{b(a^2+b^2)}$$

$\big\downarrow$ 3042

$$-\frac{2a^2(Ab-aB)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{1}{2}b(a-ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}b(a+ib)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2B(a^2+b^2)\sqrt{a+b\tan(c+dx)}}{bd}}{b(a^2+b^2)}$$

$\big\downarrow$ 4020

$$-\frac{2a^2(Ab-aB)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{ib(a+ib)(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{ib(a-ib)(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-2}{b(a^2+b^2)}$$

$$\Big\downarrow \ 25$$

$$-\frac{2a^2(Ab-aB)}{b^2d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{ib(a+ib)(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{ib(a-ib)(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-2B}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 73$$

$$-\frac{2a^2(Ab-aB)}{b^2d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{(a-ib)(A+iB)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}+\frac{(a+ib)(A-iB)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}-\frac{2B\left(a^2+b^2\right)\sqrt{a+b\tan(}}{bd}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 221$$

$$-\frac{2a^2(Ab-aB)}{b^2d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2B\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}{bd}+\frac{b(a+ib)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{b(a-ib)(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}}{b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*a^2*(A*b - a*B))/(b^2*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]) - (((a +
 I*b)*b*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) +
((a - I*b)*b*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*
d) - (2*(a^2 + b^2)*B*Sqrt[a + b*Tan[c + d*x]])/(b*d))/(b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3708 vs. $2(145) = 290$.

Time = 0.16 (sec) , antiderivative size = 3709, normalized size of antiderivative = 22.21

method	result	size
parts	Expression too large to display	3709
derivativedivides	Expression too large to display	7982
default	Expression too large to display	7982

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
A*(1/4/b/d/(a^2+b^2)^2*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4
*b/d/(a^2+b^2)^2*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b
*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/4/b/d/(a^
2+b^2)^(5/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan
(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+1/4*b^3/d/(a^
2+b^2)^(5/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan
(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/b/d/(a^2+b^2)^(
3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2
*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-b/d/(a^2+b^2)^
(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-b/d/(a^2+b^2)^2
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+
b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/b/d/(a^2+b^2)^(5
/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*
(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5-b^3/d/(a^2+b^2)
^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(
a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+3*b^3/d/(a^2+b^2)^(5
/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*
(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a+4*b/d/(a^2+b^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4379 vs. 2(139) = 278.

Time = 0.66 (sec) , antiderivative size = 4379, normalized size of antiderivative = 26.22

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan^2{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**2/(a + b*tan(c + d*x))**(3/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 11.20 (sec) , antiderivative size = 5768, normalized size of antiderivative = 34.54

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
(log((((a + b*tan(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 -
32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) + ((((96*A^4*a^2*b^4*d^4 - 16*A^4
*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/
(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(64*A*a*b^11*d^
4 - ((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) -
 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*
a^4*b^2*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^
10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2
*d^5))/4 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*
A*a^9*b^3*d^4))/4)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^
2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^
2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 - 8*A^3*b^9*d^2 - 24*A^3*a^2*b^7*d^2
- 24*A^3*a^4*b^5*d^2 - 8*A^3*a^6*b^3*d^2)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b
^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a
^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + (log((((a +
b*tan(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b
^4*d^3 - 16*A^2*a^8*b^2*d^3) + ((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 -
144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 +
b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(64*A*a*b^11*d^4 - ((-((96
*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*...
```

Reduce [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2}{a+\tan(dx+c)\,b}dx$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)
```

3.352 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 141

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} + \frac{2a(Ab-aB)}{b\,(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
-(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-(A+
I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+2*a*(A*
b-B*a)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.62

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{\frac{b\left(A\left(b^2-a\sqrt{-b^2}\right)-b\left(a+\sqrt{-b^2}\right)B\right)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{-b^2}}}\right)}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}} - \frac{b\left(A\left(b^2+a\sqrt{-b^2}\right)\right.}{}}{b\,(a^2+b^2)\,d}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x
]
```

output
```
((b*(A*(b^2 - a*Sqrt[-b^2]) - b*(a + Sqrt[-b^2])*B)*ArcTanh[Sqrt[a + b*Tan
[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - (b*(
A*(b^2 + a*Sqrt[-b^2]) + b*(-a + Sqrt[-b^2])*B)*ArcTanh[Sqrt[a + b*Tan[c +
 d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + (2*a*(A*
b - a*B))/Sqrt[a + b*Tan[c + d*x]])/(b*(a^2 + b^2)*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3042, 4074, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4074}$$

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}+\frac{2a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}+\frac{2a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{4022}$$

$$\frac{2a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(b+ia)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(-b+ia)(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(b+ia)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(-b+ia)(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{4020}$$

$$\frac{2a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{i(-b+ia)(A-iB)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(b+ia)(A+iB)\int-\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{2a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{i(-b+ia)(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(b+ia)(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}}{a^2+b^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{2a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(b+ia)(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}-\frac{(-b+ia)(A-iB)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}}{a^2+b^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{2a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(b+ia)(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}-\frac{(-b+ia)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}}{a^2+b^2}$$

input
```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
(-(((I*a - b)*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]
*d)) + ((I*a + b)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a +
I*b]*d))/(a^2 + b^2) + (2*a*(A*b - a*B))/(b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c
 + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3683 vs. $2(121) = 242$.

Time = 0.14 (sec) , antiderivative size = 3684, normalized size of antiderivative = 26.13

method	result	size
parts	Expression too large to display	3684
derivativedivides	Expression too large to display	7956
default	Expression too large to display	7956

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
A*(-1/4/d/(a^2+b^2)^2*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/
d*b^2/(a^2+b^2)^2*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/2/d/(a^2+b
^2)^(5/2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/2/d*b^2/(a^2+b
^2)^(5/2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d/(a^2+b^2)^(3/2
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b
^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/d/(a^2+b^2)^2/(
2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^2)
^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-1/d*b^2/(a^2+b^2)^(3
/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/d*b^2/(a^2+b^2)^2
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^
2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a+2/d*b^2/(a^2+b^2)^(5
/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+2/d*b^4/(a^2+b^
2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2
*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/4/d/(a^2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4329 vs. $2(115) = 230$.

Time = 0.56 (sec) , antiderivative size = 4329, normalized size of antiderivative = 30.70

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)/(a + b*tan(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\tan{(dx+c)}}{(b\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)/(b*tan(d*x + c) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[3,9,3]%%%}+%%%{4,[3,7,3]%%%}+%%%{6,[3,5,3]%%%}+%%%{
4,[3,3,3]
```

Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 5742, normalized size of antiderivative = 40.72

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
(log(- ((((((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1
/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4
 + 3*a^4*b^2*d^4))^(1/2)*(32*A*b^12*d^4 + ((((96*A^4*a^2*b^4*d^4 - 16*A^4*
b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(
a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a + b*tan(c + d
*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b
^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 96*A*a^2*b^10*d^4 + 64*A*
a^4*b^8*d^4 - 64*A*a^6*b^6*d^4 - 96*A*a^8*b^4*d^4 - 32*A*a^10*b^2*d^4))/4
+ (a + b*tan(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^
2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3))*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^
4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^
4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 - 24*A^3*a^3*b^6*d^
2 - 24*A^3*a^5*b^4*d^2 - 8*A^3*a^7*b^2*d^2 - 8*A^3*a*b^8*d^2)*(((96*A^4*a^
2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 -
12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/
2))/4 + (log(- ((((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^
2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^
2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(32*A*b^12*d^4 + ((-((96*A^4*a^2*b^4*d^4
 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*
b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(a ...
```

Reduce [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)}{a+\tan(dx+c)\,b}dx$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*x)*b + a),x)
```

3.353 $\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} + \frac{(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} - \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
-(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+(I*
A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-2*(A*b-
B*a)/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{(A-iB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan(c+dx)}{a-ib}\right)}{a-ib} - \frac{(A+iB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+}{}\right.}{a+ib}\right.}{d\sqrt{a+b\tan(c+dx)}}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
(I*(((A - I*B)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a - I
*b)])/(a - I*b) - ((A + I*B)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c
+ d*x])/(a + I*b)])/(a + I*b)))/(d*Sqrt[a + b*Tan[c + d*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4012, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}-\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}-\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{4022}$$

$$-\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(a-ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(a-ib)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \ 4020$$

$$-\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{i(a+ib)(A-iB)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a-ib)(A+iB)\int-\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}}{a^2+b^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{i(a-ib)(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{i(a+ib)(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}}{a^2+b^2}$$

$$\Big\downarrow \ 73$$

$$-\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(a-ib)(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a+ib)(A-iB)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}}{a^2+b^2}$$

$$\Big\downarrow \ 221$$

$$-\frac{2(Ab-aB)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(a+ib)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{(a-ib)(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}}{a^2+b^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
(((a + I*b)*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d
) + ((a - I*b)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b
]*d))/(a^2 + b^2) - (2*(A*b - a*B))/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]
])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3678 vs. $2(118) = 236$.

Time = 0.13 (sec) , antiderivative size = 3679, normalized size of antiderivative = 26.66

method	result	size
parts	Expression too large to display	3679
derivativedivides	Expression too large to display	7951
default	Expression too large to display	7951

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(-1/4/b/d/(a^2+b^2)^2*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/
4*b/d/(a^2+b^2)^2*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/4/b/d/(a
^2+b^2)^(5/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*ta
n(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-1/4*b^3/d/(a
^2+b^2)^(5/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*ta
n(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/b/d/(a^2+b^2)^
(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+b/d/(a^2+b^2)
^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)
-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a+b/d/(a^2+b^2)^
2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a
+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/b/d/(a^2+b^2)^(
5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2
*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5+b^3/d/(a^2+b^2
)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*
(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-3*b^3/d/(a^2+b^2)^(
5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2
*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-4*b/d/(a^2+b^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4318 vs. $2(113) = 226$.

Time = 0.55 (sec) , antiderivative size = 4318, normalized size of antiderivative = 31.29

$$\int \frac{A + B\tan(c + dx)}{(a + b\tan(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{(a + b\tan(c + dx))^{3/2}}\,dx = \int \frac{A + B\tan{(c + dx)}}{(a + b\tan{(c + dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(a + b*tan(c + d*x))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{(a + b\tan(c + dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[3,9,3]%%%}+%%%{4,[3,7,3]%%%}+%%%{6,[3,5,3]%%%}+%%%{
4,[3,3,3]
```

Mupad [B] (verification not implemented)

Time = 9.77 (sec) , antiderivative size = 5737, normalized size of antiderivative = 41.57

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
(log((((a + b*tan(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 -
32*A^2*a^6*b^4*d^3 - 16*A^2*a^8*b^2*d^3) - ((((96*A^4*a^2*b^4*d^4 - 16*A^4
*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/
(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((((96*A^4*a^2
*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 1
2*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2
)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b
^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5))/4 + 64*A*a*
b^11*d^4 + 256*A*a^3*b^9*d^4 + 384*A*a^5*b^7*d^4 + 256*A*a^7*b^5*d^4 + 64*
A*a^9*b^3*d^4))/4)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 - 144*A^4*a^4*b^
2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^
2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + 8*A^3*b^9*d^2 + 24*A^3*a^2*b^7*d^2
+ 24*A^3*a^4*b^5*d^2 + 8*A^3*a^6*b^3*d^2)*(((96*A^4*a^2*b^4*d^4 - 16*A^4*b
^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) - 4*A^2*a^3*d^2 + 12*A^2*a*b^2*d^2)/(a
^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 + (log((((a +
b*tan(c + d*x))^(1/2)*(16*A^2*b^10*d^3 + 32*A^2*a^2*b^8*d^3 - 32*A^2*a^6*b
^4*d^3 - 16*A^2*a^8*b^2*d^3) - ((-((96*A^4*a^2*b^4*d^4 - 16*A^4*b^6*d^4 -
144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a*b^2*d^2)/(a^6*d^4 +
b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(((-((96*A^4*a^2*b^4*d^4 -
 16*A^4*b^6*d^4 - 144*A^4*a^4*b^2*d^4)^(1/2) + 4*A^2*a^3*d^2 - 12*A^2*a...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx=\frac{2\sqrt{a+\tan(dx+c)\,b}-\left(\int\frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{a+\tan(dx+c)b}dx\right)bd}{bd}$$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(tan(c + d*x)*b + a) - int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**
2)/(tan(c + d*x)*b + a),x)*b*d)/(b*d)
```

3.354 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 171

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{2A\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}+\frac{2b(Ab-aB)}{a(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
-2*A*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+(A-I*B)*arctanh((a+
b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+(A+I*B)*arctanh((a+b*ta
n(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+2*b*(A*b-B*a)/a/(a^2+b^2)/d
/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{-\frac{2A(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{a(a+ib)(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+}}{\sqrt{a-ib}}\right.}{\sqrt{a-ib}}}{a\left(a^2+b^2\right.}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x
]
```

output
```
((-2*A*(a^2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] + (a
*(a + I*b)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt
[a - I*b] + (a*(a - I*b)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a
 + I*b]])/Sqrt[a + I*b] + (2*b*(A*b - a*B))/Sqrt[a + b*Tan[c + d*x]])/(a*(
a^2 + b^2)*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 4092, 27, 3042, 4136, 25, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4092}$$

$$\frac{2\int \frac{\cot(c+dx)\left(b(Ab-aB)\tan^2(c+dx)-a(Ab-aB)\tan(c+dx)+A\left(a^2+b^2\right)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cot(c+dx)\left(b(Ab-aB)\tan^2(c+dx)-a(Ab-aB)\tan(c+dx)+A\left(a^2+b^2\right)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(Ab-aB)\tan(c+dx)^2-a(Ab-aB)\tan(c+dx)+A\left(a^2+b^2\right)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{4136}$$

$$\frac{A\left(a^2+b^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+\int -\frac{a(Ab-aB)+a(aA+bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{A\left(a^2+b^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-\int \frac{a(Ab-aB)+a(aA+bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\left(a^2+b^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\int \frac{a(Ab-aB)+a(aA+bB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{4022}$$

$$\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{A\left(a^2+b^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a(b+ia)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a(-b+ia)(A-iB)\int \frac{i\tan(}{\sqrt{a+b}}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+A\left(a^2+b^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a(b+ia)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a(-b+ia)(A-iB)\int\frac{i\tan(}{\sqrt{a+b}}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4020

$$\frac{\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+A\left(a^2+b^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{ia(-b+ia)(A-iB)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{ia(b+ia)(A+iB)\int}{}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+A\left(a^2+b^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{ia(-b+ia)(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{ia(b+ia)(A+iB)\int}{}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 73

$$\frac{\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+A\left(a^2+b^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{a(b+ia)(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{a(-b+ia)(A-iB)\int\frac{}{\frac{i\tan^2(c}{b}}}{}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 221

$$\frac{\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+A\left(a^2+b^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{a(-b+ia)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a(b+ia)(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4117

$$\frac{\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{A\left(a^2+b^2\right)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{a(-b+ia)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a(b+ia)(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 73

$$\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{2A\left(a^2+b^2\right)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{a(-b+ia)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a(b+ia)(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{2b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2A\left(a^2+b^2\right)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+\frac{a(-b+ia)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a(b+ia)(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}}{a\left(a^2+b^2\right)}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
((a*(I*a - b)*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]
*d) - (a*(I*a + b)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a +
 I*b]*d) - (2*A*(a^2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sq
rt[a]*d))/(a*(a^2 + b^2)) + (2*b*(A*b - a*B))/(a*(a^2 + b^2)*d*Sqrt[a + b*
Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7981 vs. $2(145) = 290$.

Time = 0.15 (sec) , antiderivative size = 7982, normalized size of antiderivative = 46.68

method	result	size
derivativedivides	Expression too large to display	7982
default	Expression too large to display	7982

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4399 vs. $2(139) = 278$.

Time = 3.95 (sec) , antiderivative size = 8817, normalized size of antiderivative = 51.56

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\cot{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)/(a + b*tan(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\cot{(dx+c)}}{(b\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)/(b*tan(d*x + c) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[4,13,4]%%%}+%%%{6,[4,11,4]%%%}+%%%{15,[4,9,4]%%%}+%
%%{20,[4,
```

Mupad [B] (verification not implemented)

Time = 10.47 (sec) , antiderivative size = 26139, normalized size of antiderivative = 152.86

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
atan(-(((-(((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 - 16*A*B*b^3*d^2 - 24*A^2*a*b^2
*d^2 + 24*B^2*a*b^2*d^2 + 48*A*B*a^2*b*d^2)^2/4 - (A^4 + 2*A^2*B^2 + B^4)*
(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 4*A^2
*a^3*d^2 + 4*B^2*a^3*d^2 + 8*A*B*b^3*d^2 + 12*A^2*a*b^2*d^2 - 12*B^2*a*b^2
*d^2 - 24*A*B*a^2*b*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^
2*d^4)))^(1/2)*(((a + b*tan(c + d*x))^(1/2)*(256*A^2*a^8*b^26*d^7 + 1472*A
^2*a^10*b^24*d^7 + 3712*A^2*a^12*b^22*d^7 + 6272*A^2*a^14*b^20*d^7 + 9856*
A^2*a^16*b^18*d^7 + 14336*A^2*a^18*b^16*d^7 + 15232*A^2*a^20*b^14*d^7 + 10
112*A^2*a^22*b^12*d^7 + 3712*A^2*a^24*b^10*d^7 + 576*A^2*a^26*b^8*d^7 + 83
2*B^2*a^10*b^24*d^7 + 5504*B^2*a^12*b^22*d^7 + 15232*B^2*a^14*b^20*d^7 + 2
2400*B^2*a^16*b^18*d^7 + 17920*B^2*a^18*b^16*d^7 + 6272*B^2*a^20*b^14*d^7
 - 896*B^2*a^22*b^12*d^7 - 1408*B^2*a^24*b^10*d^7 - 320*B^2*a^26*b^8*d^7 -
512*A*B*a^9*b^25*d^7 - 1792*A*B*a^11*b^23*d^7 + 1792*A*B*a^13*b^21*d^7 + 1
9712*A*B*a^15*b^19*d^7 + 44800*A*B*a^17*b^17*d^7 + 51968*A*B*a^19*b^15*d^7
 + 34048*A*B*a^21*b^13*d^7 + 12032*A*B*a^23*b^11*d^7 + 1792*A*B*a^25*b^9*d
^7) + (-(((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 - 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d
^2 + 24*B^2*a*b^2*d^2 + 48*A*B*a^2*b*d^2)^2/4 - (A^4 + 2*A^2*B^2 + B^4)*(1
6*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 4*A^2*a
^3*d^2 + 4*B^2*a^3*d^2 + 8*A*B*b^3*d^2 + 12*A^2*a*b^2*d^2 - 12*B^2*a*b^2*d
^2 - 24*A*B*a^2*b*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b...
```

Reduce [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)}{a+\tan(dx+c)\,b}dx$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*cot(c + d*x))/(tan(c + d*x)*b + a),x)
```

3.355 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 219

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{(3Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} - \frac{b(a^2A+3Ab^2-2abB)}{a^2(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} - \frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

output
```
(3*A*b-2*B*a)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+(I*A+B)*ar
ctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-(I*A-B)*arctan
h((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-b*(A*a^2+3*A*b^2-2
*B*a*b)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)-A*cot(d*x+c)/a/d/(a+b*tan(d
*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.70 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.95

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{\frac{(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + a^2\left(\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx}}{\sqrt{a-ib}}\right.}{(a-ib)^{3/2}}\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2)
,x]
```

output
```
(((3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] + a^2
*(((I*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/(a - I*b)^(3
/2) + (((-I)*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/(a +
I*b)^(3/2)) - (b*(a^2*A + 3*A*b^2 - 2*a*b*B))/((a^2 + b^2)*Sqrt[a + b*Tan[
c + d*x]]) - (a*A*Cot[c + d*x])/Sqrt[a + b*Tan[c + d*x]])/(a^2*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.81 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.16, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.576, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4092}$$

$$-\frac{\int \frac{\cot(c+dx)(3Ab\tan^2(c+dx)+2aA\tan(c+dx)+3Ab-2aB)}{2(a+b\tan(c+dx))^{3/2}}dx}{a} - \frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\cot(c+dx)\left(3Ab\tan^2(c+dx)+2aA\tan(c+dx)+3Ab-2aB\right)}{(a+b\tan(c+dx))^{3/2}}dx}{2a}-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{3Ab\tan(c+dx)^2+2aA\tan(c+dx)+3Ab-2aB}{\tan(c+dx)(a+b\tan(c+dx))^{3/2}}dx}{2a}-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\frac{2\int \frac{\cot(c+dx)\left(2(aA+bB)\tan(c+dx)a^2+b\left(Aa^2-2bBa+3Ab^2\right)\tan^2(c+dx)+\left(a^2+b^2\right)(3Ab-2aB)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{2a}-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\cot(c+dx)\left(2(aA+bB)\tan(c+dx)a^2+b\left(Aa^2-2bBa+3Ab^2\right)\tan^2(c+dx)+\left(a^2+b^2\right)(3Ab-2aB)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{2a}-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{2(aA+bB)\tan(c+dx)a^2+b\left(Aa^2-2bBa+3Ab^2\right)\tan(c+dx)^2+\left(a^2+b^2\right)(3Ab-2aB)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{2a}-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 4136$$

$$-\frac{\frac{(a^2+b^2)(3Ab-2aB)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+\int \frac{2\left(a^2(aA+bB)-a^2(Ab-aB)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{2a}-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{(a^2+b^2)(3Ab-2aB)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+2\int \frac{a^2(aA+bB)-a^2(Ab-aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{2a}-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{(a^2+b^2)(3Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\int \frac{a^2(aA+bB)-a^2(Ab-aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{2a}-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}$$

$\downarrow$ 4022

$$-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{(a^2+b^2)(3Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{1}{2}a^2(a-ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a^2(a+ib)(A-\right.}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 3042

$$-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{(a^2+b^2)(3Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{1}{2}a^2(a-ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a^2(a+ib)(A-\right.}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 4020

$$-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{(a^2+b^2)(3Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{ia^2(a+ib)(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan\right.}{2d}}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 25

$$-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{(a^2+b^2)(3Ab-2aB)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{ia^2(a-ib)(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan\cdots}{2d}\cdots\right.}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 73

$$-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{(a^2+b^2)(3Ab-2aB)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{a^2(a-ib)(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}\cdots\right.}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 221

$$-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{(a^2+b^2)(3Ab-2aB)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{a^2(a+ib)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{a^2(a-ib)(A+iB)\,\mathrm{arct}\cdots}{d\sqrt{a+\cdots}}\right.}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 4117

$$-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(a^2+b^2)(3Ab-2aB)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+2\left(\frac{a^2(a+ib)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{a^2(a-ib)(A+iB)\arctan\cdots}{d\sqrt{a+ib}}\right.}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 73

$$-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{2(a^2+b^2)(3Ab-2aB)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+2\left(\frac{a^2(a+ib)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{a^2(a-ib)(A+i\cdots}{\cdots}\right.}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 221

$$-\frac{A\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-2abB+3Ab^2)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2(a^2+b^2)(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+2\left(\frac{a^2(a+ib)(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{a^2(a-ib)(A+iB)\,\text{arc}}{d\sqrt{a+}}\right.}{a(a^2+b^2)}}{2a}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
-((A*Cot[c + d*x])/(a*d*Sqrt[a + b*Tan[c + d*x]])) - ((2*((a^2*(a + I*b)*(
A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + (a^2*(a -
 I*b)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)) - (
2*(a^2 + b^2)*(3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(
Sqrt[a]*d))/(a*(a^2 + b^2)) + (2*b*(a^2*A + 3*A*b^2 - 2*a*b*B))/(a*(a^2 +
b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8042 vs. $2(191) = 382$.

Time = 0.17 (sec) , antiderivative size = 8043, normalized size of antiderivative = 36.73

method	result	size
derivativedivides	Expression too large to display	8043
default	Expression too large to display	8043

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4527 vs. 2(186) = 372.

Time = 16.56 (sec) , antiderivative size = 9072, normalized size of antiderivative = 41.42

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\cot^2{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**2/(a + b*tan(c + d*x))**(3/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[3,17,7]%%%}+%%%{8,[3,15,7]%%%}+%%%{28,[3,13,7]%%%}+
%%%{56,[3
```

Mupad [B] (verification not implemented)

Time = 7.42 (sec) , antiderivative size = 38368, normalized size of antiderivative = 175.20

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
((2*(A*b^3 - B*a*b^2))/(a*b^2 + a^3) - ((a + b*tan(c + d*x))*(3*A*b^3 + A*
a^2*b - 2*B*a*b^2))/(a*(a*b^2 + a^3)))/(d*(a + b*tan(c + d*x))^(3/2) - a*d
*(a + b*tan(c + d*x))^(1/2)) + atan(-((((((8*A^2*a^3*d^2 - 8*B^2*a^3*d^2 -
 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 + 48*A*B*a^2*b*d^2)^
2/4 - (A^4 + 2*A^2*B^2 + B^4)*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 +
48*a^4*b^2*d^4))^(1/2) - 4*A^2*a^3*d^2 + 4*B^2*a^3*d^2 + 8*A*B*b^3*d^2 + 1
2*A^2*a*b^2*d^2 - 12*B^2*a*b^2*d^2 - 24*A*B*a^2*b*d^2)/(16*(a^6*d^4 + b^6*
d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(((a + b*tan(c + d*x))^(1/2)*
(576*A^2*a^15*b^28*d^7 + 5184*A^2*a^17*b^26*d^7 + 21568*A^2*a^19*b^24*d^7
+ 53888*A^2*a^21*b^22*d^7 + 87808*A^2*a^23*b^20*d^7 + 94976*A^2*a^25*b^18*
d^7 + 66304*A^2*a^27*b^16*d^7 + 27008*A^2*a^29*b^14*d^7 + 4288*A^2*a^31*b^
12*d^7 - 832*A^2*a^33*b^10*d^7 - 320*A^2*a^35*b^8*d^7 + 256*B^2*a^17*b^26*
d^7 + 1472*B^2*a^19*b^24*d^7 + 3712*B^2*a^21*b^22*d^7 + 6272*B^2*a^23*b^20
*d^7 + 9856*B^2*a^25*b^18*d^7 + 14336*B^2*a^27*b^16*d^7 + 15232*B^2*a^29*b
^14*d^7 + 10112*B^2*a^31*b^12*d^7 + 3712*B^2*a^33*b^10*d^7 + 576*B^2*a^35*
b^8*d^7 - 768*A*B*a^16*b^27*d^7 - 6400*A*B*a^18*b^25*d^7 - 25856*A*B*a^20*
b^23*d^7 - 66304*A*B*a^22*b^21*d^7 - 116480*A*B*a^24*b^19*d^7 - 141568*A*B
*a^26*b^17*d^7 - 116480*A*B*a^28*b^15*d^7 - 61696*A*B*a^30*b^13*d^7 - 1894
4*A*B*a^32*b^11*d^7 - 2560*A*B*a^34*b^9*d^7) - ((((8*A^2*a^3*d^2 - 8*B^2*a
^3*d^2 - 16*A*B*b^3*d^2 - 24*A^2*a*b^2*d^2 + 24*B^2*a*b^2*d^2 + 48*A*B*...
```

Reduce [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)^2}{a+\tan(dx+c)\,b}dx$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*cot(c + d*x)**2)/(tan(c + d*x)*b + a),x)
```

3.356 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 285

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{(8a^2A-15Ab^2+12abB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}d} - \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} + \frac{b(7a^2Ab+15Ab^3-4a^3B-12ab^2B)}{4a^3(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}} + \frac{(5Ab-4aB)\cot(c+dx)}{4a^2d\sqrt{a+b\tan(c+dx)}} - \frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

output
```
1/4*(8*A*a^2-15*A*b^2+12*B*a*b)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^
(7/2)/d-(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2
)/d-(A+I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+
1/4*b*(7*A*a^2*b+15*A*b^3-4*B*a^3-12*B*a*b^2)/a^3/(a^2+b^2)/d/(a+b*tan(d*x
+c))^(1/2)+1/4*(5*A*b-4*B*a)*cot(d*x+c)/a^2/d/(a+b*tan(d*x+c))^(1/2)-1/2*A
*cot(d*x+c)^2/a/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 6.18 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.44

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$$-\frac{-\frac{(5Ab-4aB)\cot(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}} - \frac{2\left(\frac{(a^2+b^2)(8a^2A-15Ab^2+12abB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}d} + \frac{i\sqrt{a-ib}\left(a^3(Ab-aB)-ia^3(aA+bB)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}}{}\right)}{(-a+ib)d}\right.}{a\left(a^2+b^2\right)}}{2a}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2)
,x]
```

output
```
-1/2*(A*Cot[c + d*x]^2)/(a*d*Sqrt[a + b*Tan[c + d*x]]) - (-1/2*((5*A*b - 4
*a*B)*Cot[c + d*x])/(a*d*Sqrt[a + b*Tan[c + d*x]]) - ((2*(((a^2 + b^2)*(8*
a^2*A - 15*A*b^2 + 12*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(4
*Sqrt[a]*d) + (I*Sqrt[a - I*b]*(a^3*(A*b - a*B) - I*a^3*(a*A + b*B))*ArcTa
nh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/((-a + I*b)*d) - (I*Sqrt[a + I
*b]*(a^3*(A*b - a*B) + I*a^3*(a*A + b*B))*ArcTanh[Sqrt[a + b*Tan[c + d*x]]
/Sqrt[a + I*b]])/((-a - I*b)*d)))/(a*(a^2 + b^2)) + (2*((b^2*(-8*a^2*A + 1
5*A*b^2 - 12*a*b*B))/4 - a*(-2*a^2*b*B - (3*a*b*(5*A*b - 4*a*B))/4)))/(a*(
a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/a)/(2*a)
```

Rubi [A] (warning: unable to verify)

Time = 2.38 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.14, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{\int \frac{\cot^2(c+dx)\left(5Ab\tan^2(c+dx)+4aA\tan(c+dx)+5Ab-4aB\right)}{2(a+b\tan(c+dx))^{3/2}}dx}{2a}-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cot^2(c+dx)\left(5Ab\tan^2(c+dx)+4aA\tan(c+dx)+5Ab-4aB\right)}{(a+b\tan(c+dx))^{3/2}}dx}{4a}-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{5Ab\tan(c+dx)^2+4aA\tan(c+dx)+5Ab-4aB}{\tan(c+dx)^2(a+b\tan(c+dx))^{3/2}}dx}{4a}-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{-\frac{\int -\frac{\cot(c+dx)\left(8Aa^2+8B\tan(c+dx)a^2+12bBa-15Ab^2-3b(5Ab-4aB)\tan^2(c+dx)\right)}{2(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}}{4a}-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\cot(c+dx)\left(8Aa^2+8B\tan(c+dx)a^2+12bBa-15Ab^2-3b(5Ab-4aB)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^{3/2}}dx}{2a}-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}}{4a}-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{8Aa^2+8B\tan(c+dx)a^2+12bBa-15Ab^2-3b(5Ab-4aB)\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))^{3/2}}dx}{2a}-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}}{4a}-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{\frac{2\int \frac{\cot(c+dx)\left(-8(Ab-aB)\tan(c+dx)a^3-b\left(-4Ba^3+7Aba^2-12b^2Ba+15Ab^3\right)\tan^2(c+dx)+\left(a^2+b^2\right)\left(8Aa^2+12bBa-15Ab^2\right)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-4a^3B+7a^2Ab-12ab\ldots\right.}{ad\left(a^2+b^2\right)\sqrt{a+b\tan\ldots}}}{2a}\ldots}{4a}$$

$$\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(-8(Ab-aB)\tan(c+dx)a^3-b\left(-4Ba^3+7Aba^2-12b^2Ba+15Ab^3\right)\tan^2(c+dx)+\left(a^2+b^2\right)\left(8Aa^2+12bBa-15Ab^2\right)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2\ldots\right.}{ad\left(a^2+b^2\right)\sqrt{a+b\tan\ldots}}}{2a}\ldots}{4a}$$

$$\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{-8(Ab-aB)\tan(c+dx)a^3-b\left(-4Ba^3+7Aba^2-12b^2Ba+15Ab^3\right)\tan(c+dx)^2+\left(a^2+b^2\right)\left(8Aa^2+12bBa-15Ab^2\right)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{2a}\ldots}{4a}$$

$$\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$\downarrow$ 4136

$$-\frac{\frac{\frac{\int -\frac{8\left((Ab-aB)a^3+(aA+bB)\tan(c+dx)a^3\right)}{\sqrt{a+b\tan(c+dx)}}dx+\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{2a}-\ldots}{4a}$$

$$\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-8\int \frac{(Ab-aB)a^3+(aA+bB)\tan(c+dx)a^3}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{2a}-\frac{(5\ldots}{\ldots}}{4a}$$

$$\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-8\int\frac{(Ab-aB)a^3+(aA+bB)\tan(c+dx)a^3}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{2a}-(5\ldots}{4a}$$

$$\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$\downarrow$ 4022

$$-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}-$$

$$\frac{-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-8\left(\frac{1}{2}a^3(b+ia)(A+\ldots\right.}{a\left(a^2+b^2\right)}}{2a}}{4a}$$

$\downarrow$ 3042

$$-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}-$$

$$\frac{-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-8\left(\frac{1}{2}a^3(b+ia)(A+\ldots\right.}{a\left(a^2+b^2\right)}}{2a}}{4a}$$

$\downarrow$ 4020

$$-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}-$$

$$\frac{-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-8\left(-\frac{ia^3(-b+ia)(\ldots}{\ldots}\right.}{\ldots}}{\ldots}}{4a}$$

$\downarrow$ 25

$$-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}-$$

$$\frac{-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-8\left(\frac{ia^3(-b+ia)(A\ldots}{\ldots}\right.}{\ldots}}{\ldots}}{4a}$$

$\downarrow$ 73

$$-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-8\left(\frac{a^3(b+ia)(A+\cdots}{\cdots}\right.}{\cdots}}{2a}}{4a}$$

$\downarrow$ 221

$$-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-8\left(\frac{a^3(b+ia)(A+\cdots}{\cdots}\right.}{a\left(a^2+b^2\right)}}{2a}}{4a}$$

$\downarrow$ 4117

$$-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}-8\left(\frac{a^3(b+ia)(A+i\cdots}{\cdots}\right.}{a\left(a^2+b^2\right)}}{2a}}{4a}$$

$\downarrow$ 73

$$-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{2\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}-8\left(\frac{a^3(b\cdots}{\cdots}\right.}{a\left(a^2+b^2\cdots\right.}}{2a}}{4a}$$

$\downarrow$ 221

$$-\frac{A\cot^2(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{(5Ab-4aB)\cot(c+dx)}{ad\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-4a^3B+7a^2Ab-12ab^2B+15Ab^3\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2\left(a^2+b^2\right)\left(8a^2A+12abB-15Ab^2\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}-8\left(\frac{a^3(b+ia)(\cdots}{\cdots}\right.}{a\left(a^2+b^2\right)}}{2a}}{4a}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
-1/2*(A*Cot[c + d*x]^2)/(a*d*Sqrt[a + b*Tan[c + d*x]]) - (-(((5*A*b - 4*a*
B)*Cot[c + d*x])/(a*d*Sqrt[a + b*Tan[c + d*x]])) + ((-8*(-((a^3*(I*a - b)*
(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)) + (a^3*(I
*a + b)*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)) -
 (2*(a^2 + b^2)*(8*a^2*A - 15*A*b^2 + 12*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c +
 d*x]]/Sqrt[a]])/(Sqrt[a]*d))/(a*(a^2 + b^2)) - (2*b*(7*a^2*A*b + 15*A*b^3
 - 4*a^3*B - 12*a*b^2*B))/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(2*a
))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8163 vs. $2(247) = 494$.

Time = 0.17 (sec) , antiderivative size = 8164, normalized size of antiderivative = 28.65

method	result	size
derivativedivides	Expression too large to display	8164
default	Expression too large to display	8164

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4598 vs. 2(241) = 482.

Time = 40.93 (sec) , antiderivative size = 9216, normalized size of antiderivative = 32.34

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan(c+dx))\cot^3(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**3/(a + b*tan(c + d*x))**(3/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{-1,[6,21,6]%%%}+%%%{-10,[6,19,6]%%%}+%%%{-45,[6,17,6]%
%%}+%%%{-
```

Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 42371, normalized size of antiderivative = 148.67

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
((2*(A*b^4 - B*a*b^3))/(a*(a^2 + b^2)) + ((a + b*tan(c + d*x))^2*(15*A*b^4
 + 7*A*a^2*b^2 - 12*B*a*b^3 - 4*B*a^3*b))/(4*a^3*(a^2 + b^2)) - ((a + b*ta
n(c + d*x))*(25*A*b^4 + 9*A*a^2*b^2 - 20*B*a*b^3 - 4*B*a^3*b))/(4*a^2*(a^2
 + b^2)))/(d*(a + b*tan(c + d*x))^(5/2) - 2*a*d*(a + b*tan(c + d*x))^(3/2)
 + a^2*d*(a + b*tan(c + d*x))^(1/2)) + atan((((a + b*tan(c + d*x))^(1/2)*(
704643072*A^4*a^29*b^20*d^5 - 290979840*A^4*a^23*b^26*d^5 - 465043456*A^4*
a^25*b^24*d^5 - 37224448*A^4*a^27*b^22*d^5 - 58982400*A^4*a^21*b^28*d^5 +
767033344*A^4*a^31*b^18*d^5 + 238551040*A^4*a^33*b^16*d^5 + 1572864*A^4*a^
35*b^14*d^5 + 92536832*A^4*a^37*b^12*d^5 + 96468992*A^4*a^39*b^10*d^5 + 25
165824*A^4*a^41*b^8*d^5 + 37748736*B^4*a^23*b^26*d^5 + 226492416*B^4*a^25*
b^24*d^5 + 536870912*B^4*a^27*b^22*d^5 + 587202560*B^4*a^29*b^20*d^5 + 176
160768*B^4*a^31*b^18*d^5 - 234881024*B^4*a^33*b^16*d^5 - 234881024*B^4*a^3
5*b^14*d^5 - 50331648*B^4*a^37*b^12*d^5 + 20971520*B^4*a^39*b^10*d^5 + 838
8608*B^4*a^41*b^8*d^5 - 94371840*A*B^3*a^22*b^27*d^5 - 364904448*A*B^3*a^2
4*b^25*d^5 + 37748736*A*B^3*a^26*b^23*d^5 + 2554331136*A*B^3*a^28*b^21*d^5
 + 5989466112*A*B^3*a^30*b^19*d^5 + 6606028800*A*B^3*a^32*b^17*d^5 + 37874
56512*A*B^3*a^34*b^15*d^5 + 918552576*A*B^3*a^36*b^13*d^5 - 56623104*A*B^3
*a^38*b^11*d^5 - 50331648*A*B^3*a^40*b^9*d^5 + 330301440*A^3*B*a^22*b^27*d
^5 + 1915748352*A^3*B*a^24*b^25*d^5 + 4279238656*A^3*B*a^26*b^23*d^5 + 405
9037696*A^3*B*a^28*b^21*d^5 + 154140672*A^3*B*a^30*b^19*d^5 - 282591232...
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\cot(dx+c)^3\,(A+B\tan(dx+c))}{(a+\tan(dx+c)\,b)^{\frac{3}{2}}}dx$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

3.357 $\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 371

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} + \frac{2a(Ab-aB)\tan^3(c+dx)}{3b(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2a(a^2Ab+3Ab^3-2a^3B-4ab^2B)\tan^2(c+dx)}{b^2(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} + \frac{2(8a^4Ab+17a^2Ab^3+3Ab^5-16a^5B-30a^3b^2B-8ab^4B)\sqrt{a+b\tan(c+dx)}}{3b^4(a^2+b^2)^2\,d} - \frac{2(4a^3Ab+10aAb^3-8a^4B-15a^2b^2B-b^4B)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3b^3(a^2+b^2)^2\,d}$$

output

```
-(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+(I*
A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+2/3*a*(
A*b-B*a)*tan(d*x+c)^3/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)+2*a*(A*a^2*b+3*
A*b^3-2*B*a^3-4*B*a*b^2)*tan(d*x+c)^2/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(
1/2)+2/3*(8*A*a^4*b+17*A*a^2*b^3+3*A*b^5-16*B*a^5-30*B*a^3*b^2-8*B*a*b^4)*
(a+b*tan(d*x+c))^(1/2)/b^4/(a^2+b^2)^2/d-2/3*(4*A*a^3*b+10*A*a*b^3-8*B*a^4
-15*B*a^2*b^2-B*b^4)*tan(d*x+c)*(a+b*tan(d*x+c))^(1/2)/b^3/(a^2+b^2)^2/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.20 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.21

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{2B\tan^3(c+dx)}{3bd(a+b\tan(c+dx))^{3/2}}$$

$$+\frac{2\left(\frac{3(Ab-2aB)\tan^2(c+dx)}{bd(a+b\tan(c+dx))^{3/2}}+\frac{2\left(\frac{3\left(4aAb-8a^2B+b^2B\right)\tan(c+dx)}{2bd(a+b\tan(c+dx))^{3/2}}-\frac{3\left(-\frac{2\left(8a^2Ab+Ab^3-16a^3B+2ab^2B\right)}{3b(a+b\tan(c+dx))^{3/2}}+\frac{2\left(\left(-\frac{3Ab^5}{2}+\frac{3}{2}ab^4B\right)\left(-\frac{\text{Hypergeometric2}\cdots}{3(ia+\cdots}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2)
,x]
```

output
```
(2*B*Tan[c + d*x]^3)/(3*b*d*(a + b*Tan[c + d*x])^(3/2)) + (2*((3*(A*b - 2*
a*B)*Tan[c + d*x]^2)/(b*d*(a + b*Tan[c + d*x])^(3/2)) + (2*((3*(4*a*A*b -
8*a^2*B + b^2*B)*Tan[c + d*x])/(2*b*d*(a + b*Tan[c + d*x])^(3/2)) - (3*((-
2*(8*a^2*A*b + A*b^3 - 16*a^3*B + 2*a*b^2*B))/(3*b*(a + b*Tan[c + d*x])^(3
/2)) + (2*((((-3*A*b^5)/2 + (3*a*b^4*B)/2)*(-1/3*Hypergeometric2F1[-3/2, 1
, -1/2, (a + b*Tan[c + d*x])/(a - I*b)]/((I*a + b)*(a + b*Tan[c + d*x])^(3
/2)) + Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a + I*b)]/(3
*(I*a - b)*(a + b*Tan[c + d*x])^(3/2))))/b - (3*b^3*B*(-(Hypergeometric2F1
[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a - I*b)]/((I*a + b)*Sqrt[a + b*Tan[c
 + d*x]])) + Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a + I*b
)]/((I*a - b)*Sqrt[a + b*Tan[c + d*x]])))/2))/(3*b)))/(4*b*d)))/b))/(3*b)
```

Rubi [A] (warning: unable to verify)

Time = 2.24 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.06, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4130, 27, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^4(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2\int -\frac{3\tan^2(c+dx)\left(\left(-2Ba^2+Aba-b^2B\right)\tan^2(c+dx)-b(Ab-aB)\tan(c+dx)+2a(Ab-aB)\right)}{2(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}+\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int\frac{\tan^2(c+dx)\left(\left(-2Ba^2+Aba-b^2B\right)\tan^2(c+dx)-b(Ab-aB)\tan(c+dx)+2a(Ab-aB)\right)}{(a+b\tan(c+dx))^{3/2}}dx}{b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int\frac{\tan(c+dx)^2\left(\left(-2Ba^2+Aba-b^2B\right)\tan(c+dx)^2-b(Ab-aB)\tan(c+dx)+2a(Ab-aB)\right)}{(a+b\tan(c+dx))^{3/2}}dx}{b\left(a^2+b^2\right)}$$

$\downarrow$ 4128

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2\int\frac{\tan(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(-8Ba^4+4Aba^3-15b^2Ba^2+10Ab^3a-b^4B\right)\tan^2(c+dx)+4a\left(-2Ba^3+Aba^2-4b^2Ba+3Ab^3\right)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2a(-2a}{}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 27

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\tan(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(-8Ba^4+4Aba^3-15b^2Ba^2+10Ab^3a-b^4B\right)\tan^2(c+dx)+4a\left(-2Ba^3+Aba^2-4b^2Ba+3Ab^3\right)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2a(-2a^3}{}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\tan(c+dx)\left(\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(-8Ba^4+4Aba^3-15b^2Ba^2+10Ab^3a-b^4B\right)\tan(c+dx)^2+4a\left(-2Ba^3+Aba^2-4b^2Ba+3Ab^3\right)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2a(-2a^3}{}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 4130

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\frac{2\int-\frac{-3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)b^3+\left(-16Ba^5+8Aba^4-30b^2Ba^3+17Ab^3a^2-8b^4Ba+3Ab^5\right)\tan^2(c+dx)+2a\left(-8Ba^4+4Aba^3-15b^2Ba^2+10Ab^3a-b^4B\right)}{2\sqrt{a+b\tan(c+dx)}}}{3b}}{b(a^2+b^2)}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 27

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-$$

$$\frac{\frac{2\left(-8a^4B+4a^3Ab-15a^2b^2B+10aAb^3-b^4B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int\frac{-3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)b^3+\left(-16Ba^5+8Aba^4-30b^2Ba^3+17Ab^3a^2-8b^4\right.}{\sqrt{a+b\tan(c+}}}{3b}}{b(a^2+b^2)}$$

$$b\left(a^2+b^2\right)$$

$\downarrow$ 3042

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-$$

$$\frac{\frac{2\left(-8a^4B+4a^3Ab-15a^2b^2B+10aAb^3-b^4B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int\frac{-3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)b^3+\left(-16Ba^5+8Aba^4-30b^2Ba^3+17Ab^3a^2-8b^4\right.}{\sqrt{a+b\tan(c+}}}{3b}}{b(a^2+b^2)}$$

$$b\left(a^2+b^2\right)$$

$\downarrow$ 4113

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-$$

$$\frac{\frac{2\left(-8a^4B+4a^3Ab-15a^2b^2B+10aAb^3-b^4B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int\frac{3b^3\left(Aa^2+2bBa-Ab^2\right)-3b^3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(-16a^5B+8a^4A\right.}{}}{3b}}{b(a^2+b^2)}$$

$$b\left(a^2+b^2\right)$$

$\downarrow$ 3042

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-$$

$$\frac{\frac{2\left(-8a^4B+4a^3Ab-15a^2b^2B+10aAb^3-b^4B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\int\frac{3b^3\left(Aa^2+2bBa-Ab^2\right)-3b^3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(-16a^5B+8a^4A\right.}{}}{3b}}{b(a^2+b^2)}$$

$$b\left(a^2+b^2\right)$$

$\downarrow$ 4022

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-$$

$$\frac{-\frac{2a\left(-2a^3B+a^2Ab-4ab^2B+3Ab^3\right)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{2\left(-8a^4B+4a^3Ab-15a^2b^2B+10aAb^3-b^4B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\frac{3}{2}b^3(a-ib)^2(A+iB)\int\frac{1-}{\sqrt{a}}}{}}{}}{b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{2a\left(-2a^3B+a^2Ab-4ab^2B+3Ab^3\right)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{2\left(-8a^4B+4a^3Ab-15a^2b^2B+10aAb^3-b^4B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{\frac{3}{2}b^3(a-ib)^2(A+iB)\int\ldots}{\ldots}$$

$$b\left(a^2+b^2\right)$$

$\big\downarrow$ 4020

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{2a\left(-2a^3B+a^2Ab-4ab^2B+3Ab^3\right)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{2\left(-8a^4B+4a^3Ab-15a^2b^2B+10aAb^3-b^4B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{3ib^3(a+ib)^2(A-iB)\int-\ldots}{\ldots}$$

$\big\downarrow$ 25

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{2a\left(-2a^3B+a^2Ab-4ab^2B+3Ab^3\right)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{2\left(-8a^4B+4a^3Ab-15a^2b^2B+10aAb^3-b^4B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{-3ib^3(a+ib)^2(A-iB)\int\ldots}{\ldots}$$

$\big\downarrow$ 73

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{2a\left(-2a^3B+a^2Ab-4ab^2B+3Ab^3\right)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{2\left(-8a^4B+4a^3Ab-15a^2b^2B+10aAb^3-b^4B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{3b^2(a-ib)^2(A+iB)\int\ldots}{\ldots}$$

$\big\downarrow$ 221

$$\frac{2a(Ab-aB)\tan^3(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{2a\left(-2a^3B+a^2Ab-4ab^2B+3Ab^3\right)\tan^2(c+dx)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{2\left(-8a^4B+4a^3Ab-15a^2b^2B+10aAb^3-b^4B\right)\tan(c+dx)\sqrt{a+b\tan(c+dx)}}{3bd}-\frac{2\left(-16a^5B+8a^4Ab-30a^3\ldots\right.}{\ldots}$$

$$b\left(a^2+b^2\right)$$

input
```
Int[(Tan[c + d*x]^4*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
(2*a*(A*b - a*B)*Tan[c + d*x]^3)/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(
3/2)) - ((-2*a*(a^2*A*b + 3*A*b^3 - 2*a^3*B - 4*a*b^2*B)*Tan[c + d*x]^2)/(
b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]) + ((2*(4*a^3*A*b + 10*a*A*b^3 -
8*a^4*B - 15*a^2*b^2*B - b^4*B)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]])/(3*
b*d) - ((3*(a + I*b)^2*b^3*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(
Sqrt[a - I*b]*d) + (3*(a - I*b)^2*b^3*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a
 + I*b]])/(Sqrt[a + I*b]*d) + (2*(8*a^4*A*b + 17*a^2*A*b^3 + 3*A*b^5 - 16*
a^5*B - 30*a^3*b^2*B - 8*a*b^4*B)*Sqrt[a + b*Tan[c + d*x]])/(b*d))/(3*b))/
(b*(a^2 + b^2)))/(b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4591 vs. $2(339) = 678$.

Time = 0.27 (sec) , antiderivative size = 4592, normalized size of antiderivative = 12.38

method	result	size
parts	Expression too large to display	4592
derivativedivides	Expression too large to display	12953
default	Expression too large to display	12953

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
A*(8/d/b*a^3/(a^2+b^2)^2/(a+b*tan(d*x+c))^(1/2)-2/3/d/b^3*a^4/(a^2+b^2)/(a
+b*tan(d*x+c))^(3/2)+4/d/b^3*a^5/(a^2+b^2)^2/(a+b*tan(d*x+c))^(1/2)+1/d*b^
3/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))
^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-2/d*b
^5/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c)
)^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/d*
b^3/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/
2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+2/d
*b^5/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1
/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/
4/d*b^3/(a^2+b^2)^3*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d*b^3/
(a^2+b^2)^3*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(
d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+2/d/b^3*(a+b*tan(d
*x+c))^(1/2)-2/d*b/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+
b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2))*a^3+2/d*b/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2))*a^3-2/d*b^3/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+
b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7358 vs. $2(334) = 668$.

Time = 2.07 (sec) , antiderivative size = 7358, normalized size of antiderivative = 19.83

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan^4{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**4/(a + b*tan(c + d*x))**(5/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 40.45 (sec) , antiderivative size = 9547, normalized size of antiderivative = 25.73

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^4*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(log((((a + b*tan(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3
+ 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 +
320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) - ((((320*A^4*a^2*b^8*d^4 - 16
*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*
b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a
^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a
^8*b^2*d^4))^(1/2)*(((((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a
^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5
*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2
*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a + b*
tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5
+ 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^1
3*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64
*a^21*b^2*d^5))/4 - 32*A*b^21*d^4 - 160*A*a^2*b^19*d^4 - 128*A*a^4*b^17*d^
4 + 896*A*a^6*b^15*d^4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^4 + 4480
*A*a^12*b^9*d^4 + 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a^18*b^3
*d^4))/4)*(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4
+ 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A
^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 +
10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 + 96*A^3*a^3...
```

Reduce [F]

$$\int \frac{\tan^4(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^4}{\tan(dx+c)^2\,b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(tan(d*x+c)^4*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4)/(tan(c + d*x)**2*b**2 + 2*t
an(c + d*x)*a*b + a**2),x)
```

3.358 $\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 261

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} + \frac{2a(Ab-aB)\tan^2(c+dx)}{3b\,(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2a^2(a^2Ab+7Ab^3-4a^3B-10ab^2B)}{3b^3\,(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}} - \frac{2(aAb-4a^2B-3b^2B)\sqrt{a+b\tan(c+dx)}}{3b^3\,(a^2+b^2)\,d}$$

output
```
(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+(A+I
*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+2/3*a*(A
*b-B*a)*tan(d*x+c)^2/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)-2/3*a^2*(A*a^2*b
+7*A*b^3-4*B*a^3-10*B*a*b^2)/b^3/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)-2/3*
(A*a*b-4*B*a^2-3*B*b^2)*(a+b*tan(d*x+c))^(1/2)/b^3/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.36 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.18

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{-2(a-ib)(a+ib)\left(-2aAb+8a^2B+b^2B\right)-b^2(aA+bB)\left(i(a+ib)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a-}{}\right.\right.}{}$$

input
```
Integrate[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2)
,x]
```

output
```
-1/3*(-2*(a - I*b)*(a + I*b)*(-2*a*A*b + 8*a^2*B + b^2*B) - b^2*(a*A + b*B
)*(I*(a + I*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a -
I*b)] - (I*a + b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a
 + I*b)]) - 6*(a - I*b)*(a + I*b)*b*(-(A*b) + 4*a*B)*Tan[c + d*x] - 6*(a -
 I*b)*(a + I*b)*b^2*B*Tan[c + d*x]^2 + 3*A*b^2*(I*(a + I*b)*Hypergeometric
2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a - I*b)] - (I*a + b)*Hypergeometr
ic2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]))
/(b^3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 1.56 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 4088, 27, 3042, 4118, 3042, 4113, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(c+dx)^3(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$\Big\downarrow$ 4088

$$\frac{2\int -\frac{\tan(c+dx)\left(\left(-4Ba^2+Aba-3b^2B\right)\tan^2(c+dx)-3b(Ab-aB)\tan(c+dx)+4a(Ab-aB)\right)}{2(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}+\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int \frac{\tan(c+dx)\left(\left(-4Ba^2+Aba-3b^2B\right)\tan^2(c+dx)-3b(Ab-aB)\tan(c+dx)+4a(Ab-aB)\right)}{(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int \frac{\tan(c+dx)\left(\left(-4Ba^2+Aba-3b^2B\right)\tan(c+dx)^2-3b(Ab-aB)\tan(c+dx)+4a(Ab-aB)\right)}{(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4118

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int \frac{3\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(a^2+b^2\right)\left(-4Ba^2+Aba-3b^2B\right)\tan^2(c+dx)+a\left(-4Ba^3+Aba^2-10b^2Ba+7Ab^3\right)}{\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}+\frac{2a^2\left(-4a^3B+a^2Ab-10ab^2B+7\right.}{b^2d\left(a^2+b^2\right)\sqrt{a+b\tan(c+d}}}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int \frac{3\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(a^2+b^2\right)\left(-4Ba^2+Aba-3b^2B\right)\tan(c+dx)^2+a\left(-4Ba^3+Aba^2-10b^2Ba+7Ab^3\right)}{\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}+\frac{2a^2\left(-4a^3B+a^2Ab-10ab^2B+7\right.}{b^2d\left(a^2+b^2\right)\sqrt{a+b\tan(c+d}}}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4113

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int \frac{3\left(-Ba^2+2Aba+b^2B\right)b^2+3\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2\left(a^2+b^2\right)\left(-4a^2B+aAb-3b^2B\right)\sqrt{a+b\tan(c+dx)}}{bd}}{b\left(a^2+b^2\right)}+\frac{2a^2\left(-4a^3B+a^2Ab-10ab^2B+7A\right.}{b^2d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{3(-Ba^2+2Aba+b^2B)b^2+3(Aa^2+2bBa-Ab^2)\tan(c+dx)b^2}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(a^2+b^2)(-4a^2B+aAb-3b^2B)\sqrt{a+b\tan(c+dx)}}{bd}}{b(a^2+b^2)}+\frac{2a^2(-4a^3B+a^2Ab-10ab^2B+7A}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3b(a^2+b^2)}$$

$\downarrow$ 4022

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2a^2(-4a^3B+a^2Ab-10ab^2B+7Ab^3)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{3}{2}b^2(a-ib)^2(-B+iA)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{3}{2}b^2(a+ib)^2(B+iA)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(a^2+b^2)(-4a^2B}{}}{b(a^2+b^2)}}{3b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2a^2(-4a^3B+a^2Ab-10ab^2B+7Ab^3)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{3}{2}b^2(a-ib)^2(-B+iA)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{3}{2}b^2(a+ib)^2(B+iA)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx+\frac{2(a^2+b^2)(-4a^2B}{}}{b(a^2+b^2)}}{3b(a^2+b^2)}$$

$\downarrow$ 4020

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2a^2(-4a^3B+a^2Ab-10ab^2B+7Ab^3)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{3ib^2(a+ib)^2(B+iA)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{3ib^2(a-ib)^2(-B+iA)\int-\frac{}{(i\tan(c+da}}{2}}{b(a^2+b^2)}}{3b(a^2+b^2)}$$

$\downarrow$ 25

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2a^2(-4a^3B+a^2Ab-10ab^2B+7Ab^3)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{3ib^2(a+ib)^2(B+iA)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{3ib^2(a-ib)^2(-B+iA)\int\frac{1}{(i\tan(c+dx)+1)}}{2d}}{b(a^2+b^2)}}{3b(a^2+b^2)}$$

$\downarrow$ 73

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2a^2(-4a^3B+a^2Ab-10ab^2B+7Ab^3)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{3b(a-ib)^2(-B+iA)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}-\frac{3b(a+ib)^2(B+iA)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+}}{d}}{b(a^2+b^2)}}{3b(a^2+b^2)}$$

$$\Big\downarrow \; 221$$

$$\frac{2a(Ab-aB)\tan^2(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2a^2\left(-4a^3B+a^2Ab-10ab^2B+7Ab^3\right)}{b^2d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{2\left(a^2+b^2\right)\left(-4a^2B+aAb-3b^2B\right)\sqrt{a+b\tan(c+dx)}}{bd}-\frac{3b^2(a+ib)^2(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{3b^2(a-ib)^2(-B+i\cdots}{d\cdots}}{b(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
(2*a*(A*b - a*B)*Tan[c + d*x]^2)/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(
3/2)) - ((2*a^2*(a^2*A*b + 7*A*b^3 - 4*a^3*B - 10*a*b^2*B))/(b^2*(a^2 + b^
2)*d*Sqrt[a + b*Tan[c + d*x]]) + ((-3*(a + I*b)^2*b^2*(I*A + B)*ArcTan[Tan
[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + (3*(a - I*b)^2*b^2*(I*A - B)
*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d) + (2*(a^2 + b^2)*(a
*A*b - 4*a^2*B - 3*b^2*B)*Sqrt[a + b*Tan[c + d*x]])/(b*d))/(b*(a^2 + b^2))
)/(3*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4118

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4542 vs. $2(231) = 462$.

Time = 0.19 (sec) , antiderivative size = 4543, normalized size of antiderivative = 17.41

method	result	size
parts	Expression too large to display	4543
derivativedivides	Expression too large to display	12907
default	Expression too large to display	12907

input

```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
A*(1/2/d*b^2/(a^2+b^2)^(7/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a
^2+2/d*b^2/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*ta
n(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2))*a-6/d*b^2/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b
*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2))*a^3-5/d*b^4/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2
*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2))*a-1/2/d*b^2/(a^2+b^2)^3*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/
2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^
(1/2)*a+1/d*b^4/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*t
an(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2))-2/d/b^2*a^4/(a^2+b^2)^2/(a+b*tan(d*x+c))^(1/2)-1/4/d*b^4/(a^2+b^2)^(7
/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-6/d*a^2/(a^2+b^2)^2/(a+b*t
an(d*x+c))^(1/2)+2/d/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(
(2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2
)-2*a)^(1/2))*a^3-1/d/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan
((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/
2)-2*a)^(1/2))*a^5-3/4/d/(a^2+b^2)^(7/2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7284 vs. $2(227) = 454$.

Time = 2.10 (sec) , antiderivative size = 7284, normalized size of antiderivative = 27.91

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^3(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**3/(a + b*tan(c + d*x))**(5/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^3}{(b\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^3/(b*tan(d*x + c) + a)^(5/2),
x)
```

Mupad [B] (verification not implemented)

Time = 30.29 (sec) , antiderivative size = 9498, normalized size of antiderivative = 36.39

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(log(((((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 16
00*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a
^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a
^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*((a + b*tan(c + d*x))^
(1/2)*(320*B^2*a^4*b^14*d^3 - 16*B^2*b^18*d^3 + 1024*B^2*a^6*b^12*d^3 + 14
40*B^2*a^8*b^10*d^3 + 1024*B^2*a^10*b^8*d^3 + 320*B^2*a^12*b^6*d^3 - 16*B^
2*a^16*b^2*d^3) - ((((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4
*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d
^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b
^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(((((320*
B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a^6*b^
4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d^2 -
20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 +
10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64*a*b^
22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 13440*
a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*b^8*d
^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 - 32*B*b^2
1*d^4 - 160*B*a^2*b^19*d^4 - 128*B*a^4*b^17*d^4 + 896*B*a^6*b^15*d^4 + 313
6*B*a^8*b^13*d^4 + 4928*B*a^10*b^11*d^4 + 4480*B*a^12*b^9*d^4 + 2432*B*a^1
4*b^7*d^4 + 736*B*a^16*b^5*d^4 + 96*B*a^18*b^3*d^4))/4))/4 + 96*B^3*a^3...
```

Reduce [F]

$$\int \frac{\tan^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3}{\tan(dx+c)^2 b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(tan(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3)/(tan(c + d*x)**2*b**2 + 2*t
an(c + d*x)*a*b + a**2),x)
```

3.359 $\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 198

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{(iA-B)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} - \frac{2a^2(Ab-aB)}{3b^2(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2a(2Ab^3-a(a^2+3b^2)\,B)}{b^2(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-(I*A
-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d-2/3*a^2*
(A*b-B*a)/b^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)+2*a*(2*A*b^3-a*(a^2+3*b^2
)*B)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.73 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.31

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{2(a-ib)(a+ib)(Ab+2aB)+b(Ab-aB)\left(i(a+ib)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan(c+dx)}{a-ib}\right)-(\cdots}{\cdots}$$

input
```
Integrate[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2)
,x]
```

output
```
-1/3*(2*(a - I*b)*(a + I*b)*(A*b + 2*a*B) + b*(A*b - a*B)*(I*(a + I*b)*Hyp
ergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a - I*b)] - (I*a + b)*
Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*x])/(a + I*b)]) + 6*(a -
 I*b)*(a + I*b)*b*B*Tan[c + d*x] + 3*b*B*(I*(a + I*b)*Hypergeometric2F1[-1
/2, 1, 1/2, (a + b*Tan[c + d*x])/(a - I*b)] - (I*a + b)*Hypergeometric2F1[
-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]))/(b^2*
(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 1.10 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4087, 25, 3042, 4111, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^2(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4087}$$

$$\frac{\int -\frac{-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{(a+b\tan(c+dx))^{3/2}}dx}{b\left(a^2+b^2\right)}-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{(a+b\tan(c+dx))^{3/2}}dx}{b\left(a^2+b^2\right)}-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-\left(\left(a^2+b^2\right)B\tan(c+dx)^2\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{(a+b\tan(c+dx))^{3/2}}dx}{b\left(a^2+b^2\right)}-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4111}$$

$$-\frac{\frac{\int \frac{b\left(Aa^2+2bBa-Ab^2\right)-b\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{b\left(Aa^2+2bBa-Ab^2\right)-b\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4022}$$

$$-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}b(a-ib)^2(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}b(a+ib)^2(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}b(a-ib)^2(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}b(a+ib)^2(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 4020}$$

$$-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{ib(a+ib)^2(A-iB)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{ib(a-ib)^2(A+iB)\int-\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}}{2d}}{a^2+b^2}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{ib(a-ib)^2(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{ib(a+ib)^2(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}}{2d}}{a^2+b^2}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(a-ib)^2(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}+\frac{(a+ib)^2(A-iB)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{d}}{a^2+b^2}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{2a^2(Ab-aB)}{3b^2d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{b(a-ib)^2(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}+\frac{b(a+ib)^2(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}}{a^2+b^2}}{b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*a^2*(A*b - a*B))/(3*b^2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - ((
((a + I*b)^2*b*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b
]*d) + ((a - I*b)^2*b*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[
a + I*b]*d))/(a^2 + b^2) - (2*a*(2*A*b^3 - a*(a^2 + 3*b^2)*B))/(b*(a^2 + b
^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4486 vs. $2(174) = 348$.

Time = 0.16 (sec) , antiderivative size = 4487, normalized size of antiderivative = 22.66

method	result	size
parts	Expression too large to display	4487
derivativedivides	Expression too large to display	12849
default	Expression too large to display	12849

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
A*(-1/d*b^3/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*t
an(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2))+2/d*b^5/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*
tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2))+1/d*b^3/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2))-2/d*b^5/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a
^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2))+1/4/d*b^3/(a^2+b^2)^3*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
1/4/d*b^3/(a^2+b^2)^3*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1
/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+2/d*b/(a
^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-2/d*b/(a^
2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+2/d*b^3/(a
^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a+1/4/d/b/(a^
2+b^2)^3*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x
+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4-2/d*b^3/(a^2+b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7218 vs. $2(168) = 336$.

Time = 2.09 (sec) , antiderivative size = 7218, normalized size of antiderivative = 36.45

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan^2{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**2/(a + b*tan(c + d*x))**(5/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[6,14,5]%%%}+%%%{6,[6,12,5]%%%}+%%%{15,[6,10,5]%%%}+
%%%{20,[6
```

Mupad [B] (verification not implemented)

Time = 21.53 (sec) , antiderivative size = 9468, normalized size of antiderivative = 47.82

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(log((((a + b*tan(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3
+ 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 +
320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) + ((((320*A^4*a^2*b^8*d^4 - 16
*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*
b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a
^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a
^8*b^2*d^4))^(1/2)*(896*A*a^6*b^15*d^4 - 32*A*b^21*d^4 - 160*A*a^2*b^19*d^
4 - 128*A*a^4*b^17*d^4 - ((((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*
A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^
2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 +
5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a
 + b*tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18
*d^5 + 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 1344
0*a^13*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5
 + 64*a^21*b^2*d^5))/4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^4 + 4480
*A*a^12*b^9*d^4 + 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a^18*b^3
*d^4))/4)*(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4
+ 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A
^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 +
10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 - 96*A^3*a^3...
```

Reduce [F]

$$\int \frac{\tan^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2}{\tan(dx+c)^2\,b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(tan(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)**2*b**2 + 2*t
an(c + d*x)*a*b + a**2),x)
```

3.360 $\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 188

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{(A+iB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} + \frac{2a(Ab-aB)}{3b(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2(a^2A-Ab^2+2abB)}{(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
-(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-(A+
I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+2/3*a*(
A*b-B*a)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)+2*(A*a^2-A*b^2+2*B*a*b)/(a^2
+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.73

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{\frac{3b\left(-a^2\left(A\sqrt{-b^2}+bB\right)+b^2\left(A\sqrt{-b^2}+bB\right)+2ab\left(Ab-\sqrt{-b^2}B\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+}}{\sqrt{a-\sqrt{-b}}}\right.}{\sqrt{-b^2}\sqrt{a-\sqrt{-b^2}}}}{}$$

input
```
Integrate[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x
]
```

output
```
((3*b*(-(a^2*(A*Sqrt[-b^2] + b*B)) + b^2*(A*Sqrt[-b^2] + b*B) + 2*a*b*(A*b
 - Sqrt[-b^2]*B))*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - Sqrt[-b^2]]])/
(Sqrt[-b^2]*Sqrt[a - Sqrt[-b^2]]) - (3*b*(2*a*A*b^2 + a^2*A*Sqrt[-b^2] + A
*(-b^2)^(3/2) - a^2*b*B + b^3*B + 2*a*b*Sqrt[-b^2]*B)*ArcTanh[Sqrt[a + b*T
an[c + d*x]]/Sqrt[a + Sqrt[-b^2]]])/(Sqrt[-b^2]*Sqrt[a + Sqrt[-b^2]]) + (2
*a*(a^2 + b^2)*(A*b - a*B))/(a + b*Tan[c + d*x])^(3/2) + (6*b*(a^2*A - A*b
^2 + 2*a*b*B))/Sqrt[a + b*Tan[c + d*x]])/(3*b*(a^2 + b^2)^2*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4074, 3042, 4012, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4074}$$

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}dx}{a^2+b^2}+\frac{2a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}dx}{a^2+b^2}+\frac{2a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 4012

$$\frac{\frac{\int \frac{-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}+\frac{2\left(a^2A+2abB-Ab^2\right)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a^2+b^2}+\frac{2a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}+\frac{2\left(a^2A+2abB-Ab^2\right)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a^2+b^2}+\frac{2a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\Big\downarrow$ 4022

$$\frac{2a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2\left(a^2A+2abB-Ab^2\right)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(a-ib)^2(-B+iA)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(a+ib)^2(B+iA)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}}{a^2+b^2}$$

$\Big\downarrow$ 3042

$$\frac{2a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2\left(a^2A+2abB-Ab^2\right)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(a-ib)^2(-B+iA)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(a+ib)^2(B+iA)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}}{a^2+b^2}$$

$\Big\downarrow$ 4020

$$\frac{2a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2\left(a^2A+2abB-Ab^2\right)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{i(a-ib)^2(-B+iA)\int-\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{i(a+ib)^2(B+iA)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(}}}{2d}}{a^2+b^2}}{a^2+b^2}$$

$\Big\downarrow$ 25

$$\frac{2a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2(a^2A+2abB-Ab^2)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{i(a-ib)^2(-B+iA)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{i(a+ib)^2(B+iA)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx}}}{2d}}{a^2+b^2}}{a^2+b^2}$$

$\Big\downarrow$ 73

$$\frac{2a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2(a^2A+2abB-Ab^2)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(a-ib)^2(-B+iA)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}-\frac{(a+ib)^2(B+iA)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}}{a^2+b^2}}{a^2+b^2}$$

$\Big\downarrow$ 221

$$\frac{2a(Ab-aB)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2(a^2A+2abB-Ab^2)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(a-ib)^2(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}-\frac{(a+ib)^2(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}}{a^2+b^2}}{a^2+b^2}$$

input

```
Int[(Tan[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x]
```

output

```
(2*a*(A*b - a*B))/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + ((-(((a
 + I*b)^2*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d))
 + ((a - I*b)^2*(I*A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*
b]*d))/(a^2 + b^2) + (2*(a^2*A - A*b^2 + 2*a*b*B))/((a^2 + b^2)*d*Sqrt[a +
 b*Tan[c + d*x]]))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4476 vs. $2(164) = 328$.

Time = 0.15 (sec) , antiderivative size = 4477, normalized size of antiderivative = 23.81

method	result	size
parts	Expression too large to display	4477
derivativedivides	Expression too large to display	12841
default	Expression too large to display	12841

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
A*(-1/2/d*b^2/(a^2+b^2)^(7/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*
(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*
a^2-2/d*b^2/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*t
an(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2))*a+6/d*b^2/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+
b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2))*a^3+5/d*b^4/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((
2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)
-2*a)^(1/2))*a+1/2/d*b^2/(a^2+b^2)^3*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1
/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a-1/d*b^4/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*
tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2))+1/4/d*b^4/(a^2+b^2)^(7/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(
2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2
)+2/d*a^2/(a^2+b^2)^2/(a+b*tan(d*x+c))^(1/2)-2/d/(a^2+b^2)^2/(a+b*tan(d*x+
c))^(1/2)*b^2+2/3/d*a/(a^2+b^2)/(a+b*tan(d*x+c))^(3/2)-2/d/(a^2+b^2)^(5/2)
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^
2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+1/d/(a^2+b^2)^(7/2
)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b
^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^5+3/4/d/(a^2+b^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7209 vs. $2(158) = 316$.

Time = 2.29 (sec) , antiderivative size = 7209, normalized size of antiderivative = 38.35

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\tan(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)/(a + b*tan(c + d*x))**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[6,14,5]%%%}+%%%{6,[6,12,5]%%%}+%%%{15,[6,10,5]%%%}+
%%%{20,[6
```

Mupad [B] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 9464, normalized size of antiderivative = 50.34

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(log(((((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 16
00*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a
^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a
^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*((a + b*tan(c + d*x))^
(1/2)*(320*B^2*a^4*b^14*d^3 - 16*B^2*b^18*d^3 + 1024*B^2*a^6*b^12*d^3 + 14
40*B^2*a^8*b^10*d^3 + 1024*B^2*a^10*b^8*d^3 + 320*B^2*a^12*b^6*d^3 - 16*B^
2*a^16*b^2*d^3) + ((((320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4
*b^6*d^4 + 1600*B^4*a^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d
^2 + 40*B^2*a^3*b^2*d^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b
^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(896*B*a^
6*b^15*d^4 - 32*B*b^21*d^4 - 160*B*a^2*b^19*d^4 - 128*B*a^4*b^17*d^4 - (((
(320*B^4*a^2*b^8*d^4 - 16*B^4*b^10*d^4 - 1760*B^4*a^4*b^6*d^4 + 1600*B^4*a
^6*b^4*d^4 - 400*B^4*a^8*b^2*d^4)^(1/2) - 4*B^2*a^5*d^2 + 40*B^2*a^3*b^2*d
^2 - 20*B^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d
^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(64
*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 + 1
3440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^15*
b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 + 313
6*B*a^8*b^13*d^4 + 4928*B*a^10*b^11*d^4 + 4480*B*a^12*b^9*d^4 + 2432*B*a^1
4*b^7*d^4 + 736*B*a^16*b^5*d^4 + 96*B*a^18*b^3*d^4))/4))/4 - 96*B^3*a^3...
```

Reduce [F]

$$\int \frac{\tan(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)}{\tan(dx+c)^2 b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(tan(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*x)**2*b**2 + 2*tan(
c + d*x)*a*b + a**2),x)
```

3.361 $\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 185

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} - \frac{2(Ab-aB)}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} - \frac{2(2aAb-a^2B+b^2B)}{\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}}$$

output
```
-(I*A+B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+(I*
A-B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d-2/3*(A*
b-B*a)/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)-2*(2*A*a*b-B*a^2+B*b^2)/(a^2+b^2
)^2/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.62

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{i\left(-\frac{(A-iB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan(c+dx)}{a-ib}\right)}{a-ib}+\frac{(A+iB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan(c+dx)}{a+ib}\right)}{a+ib}\right)}{3d(a+b\tan(c+dx))^{3/2}}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/3*I)*(-(((A - I*B)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tan[c + d*
x])/(a - I*b)])/(a - I*b)) + ((A + I*B)*Hypergeometric2F1[-3/2, 1, -1/2, (
a + b*Tan[c + d*x])/(a + I*b)])/(a + I*b)))/(d*(a + b*Tan[c + d*x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4012, 3042, 4012, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}dx}{a^2+b^2}-\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}dx}{a^2+b^2}-\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4012$$

$$\frac{\frac{\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}-\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a^2+b^2}-\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}-\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a^2+b^2}-\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4022$$

$$-\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(a-ib)^2(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^2(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(a-ib)^2(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^2(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 4020$$

$$-\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2\left(a^2(-B)+2aAb+b^2B\right)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{i(a+ib)^2(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a-ib)^2(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+}}}{2d}}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2(a^2(-B)+2aAb+b^2B)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{i(a-ib)^2(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{i(a+ib)^2(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}}{2d}}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2(a^2(-B)+2aAb+b^2B)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(a-ib)^2(A+iB)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a+ib)^2(A-iB)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{2(Ab-aB)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2(a^2(-B)+2aAb+b^2B)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(a-ib)^2(A+iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}+\frac{(a+ib)^2(A-iB)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}}{a^2+b^2}}{a^2+b^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*(A*b - a*B))/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + ((((a + I*
b)^2*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + ((a
 - I*b)^2*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d))
/(a^2 + b^2) - (2*(2*a*A*b - a^2*B + b^2*B))/((a^2 + b^2)*d*Sqrt[a + b*Tan
[c + d*x]]))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4471 vs. $2(161) = 322$.

Time = 0.14 (sec) , antiderivative size = 4472, normalized size of antiderivative = 24.17

method	result	size
parts	Expression too large to display	4472
derivativedivides	Expression too large to display	12836
default	Expression too large to display	12836

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/d*b^3/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*ta
n(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2))-2/d*b^5/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*t
an(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1
/2))-1/d*b^3/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2
+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(
1/2))+2/d*b^5/(a^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2))-1/4/d*b^3/(a^2+b^2)^3*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1
/4/d*b^3/(a^2+b^2)^3*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2/d*b/(a^
2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2
*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+2/d*b/(a^2
+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2
)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-2/d*b^3/(a^
2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2
*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-1/4/d/b/(a^2
+b^2)^3*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+
c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+2/d*b^3/(a^2+b^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7181 vs. $2(156) = 312$.

Time = 2.11 (sec) , antiderivative size = 7181, normalized size of antiderivative = 38.82

$$\int \frac{A + B\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\,dx = \int \frac{A + B\tan(c + dx)}{(a + b\tan(c + dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(a + b*tan(c + d*x))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[6,14,5]%%%}+%%%{6,[6,12,5]%%%}+%%%{15,[6,10,5]%%%}+
%%%{20,[6
```

Mupad [B] (verification not implemented)

Time = 21.15 (sec) , antiderivative size = 9457, normalized size of antiderivative = 51.12

$$\int \frac{A + B\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(log((((a + b*tan(c + d*x))^(1/2)*(320*A^2*a^4*b^14*d^3 - 16*A^2*b^18*d^3
+ 1024*A^2*a^6*b^12*d^3 + 1440*A^2*a^8*b^10*d^3 + 1024*A^2*a^10*b^8*d^3 +
320*A^2*a^12*b^6*d^3 - 16*A^2*a^16*b^2*d^3) - ((((320*A^4*a^2*b^8*d^4 - 16
*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*
b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a
^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a
^8*b^2*d^4))^(1/2)*(((((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a
^4*b^6*d^4 + 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5
*d^2 + 40*A^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2
*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a + b*
tan(c + d*x))^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5
+ 7680*a^7*b^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^1
3*b^10*d^5 + 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64
*a^21*b^2*d^5))/4 - 32*A*b^21*d^4 - 160*A*a^2*b^19*d^4 - 128*A*a^4*b^17*d^
4 + 896*A*a^6*b^15*d^4 + 3136*A*a^8*b^13*d^4 + 4928*A*a^10*b^11*d^4 + 4480
*A*a^12*b^9*d^4 + 2432*A*a^14*b^7*d^4 + 736*A*a^16*b^5*d^4 + 96*A*a^18*b^3
*d^4))/4)*(((320*A^4*a^2*b^8*d^4 - 16*A^4*b^10*d^4 - 1760*A^4*a^4*b^6*d^4
+ 1600*A^4*a^6*b^4*d^4 - 400*A^4*a^8*b^2*d^4)^(1/2) - 4*A^2*a^5*d^2 + 40*A
^2*a^3*b^2*d^2 - 20*A^2*a*b^4*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 +
10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2))/4 + 96*A^3*a^3...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{-2\sqrt{a+\tan(dx+c)\,b}-\left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{\tan(dx+c)^2b^2+2\tan(dx+c)ab+a^2}dx\right)\tan(dx+c)\,b^2d}{bd\,(a+\tan(dx+c)\,b)}$$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
( - 2*sqrt(tan(c + d*x)*b + a) - int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x
)**2)/(tan(c + d*x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2),x)*tan(c + d*x)*b
**2*d - int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)**2*b*
*2 + 2*tan(c + d*x)*a*b + a**2),x)*a*b*d)/(b*d*(tan(c + d*x)*b + a))
```

3.362 $\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 224

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} + \frac{(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} + \frac{2b(Ab-aB)}{3a(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2b(3a^2Ab+Ab^3-2a^3B)}{a^2(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
-2*A*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(5/2)/d+(A-I*B)*arctanh((a+
b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d+(A+I*B)*arctanh((a+b*ta
n(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+2/3*b*(A*b-B*a)/a/(a^2+b^2)
/d/(a+b*tan(d*x+c))^(3/2)+2*b*(3*A*a^2*b+A*b^3-2*B*a^3)/a^2/(a^2+b^2)^2/d/
(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 3.51 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.08

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{2\left(-\frac{3A(a^2+b^2)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{3a(a+ib)(A-iB)\text{arctanh}\left(\frac{\sqrt{a+b}}{\sqrt{}}\right.}{2(a-ib)^{3/2}}\right.}{}$$

input
```
Integrate[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x
]
```

output
```
(2*((-3*A*(a^2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/a^(3/2) +
 (3*a*(a + I*b)*(A - I*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])
/(2*(a - I*b)^(3/2)) + (3*a*(a - I*b)*(A + I*B)*ArcTanh[Sqrt[a + b*Tan[c +
 d*x]]/Sqrt[a + I*b]])/(2*(a + I*b)^(3/2)) + (b*(A*b - a*B))/(a + b*Tan[c
+ d*x])^(3/2) + (3*b*(3*a^2*A*b + A*b^3 - 2*a^3*B))/(a*(a^2 + b^2)*Sqrt[a
+ b*Tan[c + d*x]])))/(3*a*(a^2 + b^2)*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.95 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.21, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.613$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4136, 25, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{2\int \frac{3\cot(c+dx)\left(b(Ab-aB)\tan^2(c+dx)-a(Ab-aB)\tan(c+dx)+A\left(a^2+b^2\right)\right)}{2(a+b\tan(c+dx))^{3/2}}dx}{3a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\int \frac{\cot(c+dx)\left(b(Ab-aB)\tan^2(c+dx)-a(Ab-aB)\tan(c+dx)+A\left(a^2+b^2\right)\right)}{(a+b\tan(c+dx))^{3/2}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\int \frac{b(Ab-aB)\tan(c+dx)^2-a(Ab-aB)\tan(c+dx)+A\left(a^2+b^2\right)}{\tan(c+dx)(a+b\tan(c+dx))^{3/2}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 4132

$$\frac{\frac{2\int \frac{\cot(c+dx)\left(-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2\right)+A\left(a^2+b^2\right)^2+b\left(-2Ba^3+3Aba^2+Ab^3\right)\tan^2(c+dx)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\cot(c+dx)\left(-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2\right)+A\left(a^2+b^2\right)^2+b\left(-2Ba^3+3Aba^2+Ab^3\right)\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{-\left(\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2\right)+A\left(a^2+b^2\right)^2+b\left(-2Ba^3+3Aba^2+Ab^3\right)\tan(c+dx)^2}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 4136

$$\frac{\frac{\int -\frac{\left(-Ba^2+2Aba+b^2B\right)a^2+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^2}{\sqrt{a+b\tan(c+dx)}}dx+A\left(a^2+b^2\right)^2\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^{3/2}}$$

$\big\downarrow$ 25

$$\frac{\frac{A\left(a^2+b^2\right)^2\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-\int \frac{\left(-Ba^2+2Aba+b^2B\right)a^2+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^2}{\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{A\left(a^2+b^2\right)^2\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\int \frac{\left(-Ba^2+2Aba+b^2B\right)a^2+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^2}{\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^{3/2}}$$

$\big\downarrow$ 4022

$$\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^{3/2}}+\frac{\frac{2b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{A\left(a^2+b^2\right)^2\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a^2(a-ib)^2(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a^2(a+ib)^2(B+iA)\int -}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 3042

$$\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^{3/2}}+\frac{\frac{2b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{A\left(a^2+b^2\right)^2\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a^2(a-ib)^2(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a^2(a+ib)^2(B+iA)\int -}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$\big\downarrow$ 4020

$$\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-2a^3B+3a^2Ab+Ab^3\right)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{A\left(a^2+b^2\right)^2\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{ia^2(a-ib)^2(-B+iA)\int-\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}\cdots}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 25

$$\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-2a^3B+3a^2Ab+Ab^3\right)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{A\left(a^2+b^2\right)^2\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{ia^2(a-ib)^2(-B+iA)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\cdots}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 73

$$\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-2a^3B+3a^2Ab+Ab^3\right)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{A\left(a^2+b^2\right)^2\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{a^2(a-ib)^2(-B+iA)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{a^2(a+ib)\cdots}{\cdots}}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 221

$$\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-2a^3B+3a^2Ab+Ab^3\right)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{A\left(a^2+b^2\right)^2\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{a^2(a-ib)^2(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}+\frac{a^2(a+ib)^2(B+iA)\arctan\left(\frac{\tan(c\cdots}{\sqrt{a\cdots}}\right)}{d\sqrt{a-ib}}\cdots}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 4117

$$\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-2a^3B+3a^2Ab+Ab^3\right)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{A\left(a^2+b^2\right)^2\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}-\frac{a^2(a-ib)^2(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}+\frac{a^2(a+ib)^2(B+iA)\arctan\left(\frac{\tan(c+\cdots}{\sqrt{a-\cdots}}\right)}{d\sqrt{a-ib}}\cdots}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$\downarrow$ 73

$$\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-2a^3B+3a^2Ab+Ab^3\right)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{2A\left(a^2+b^2\right)^2\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}-\frac{a^2(a-ib)^2(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}+\frac{a^2(a+ib)^2(B+iA)\,\mathrm{arcta}\cdots}{d\sqrt{a-ib}}\cdots}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{2b(Ab-aB)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b(-2a^3B+3a^2Ab+Ab^3)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{a^2(a-ib)^2(-B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}+\frac{a^2(a+ib)^2(B+iA)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{2A\left(a^2+b^2\right)^2\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}}{a(a^2+b^2)}}{a\left(a^2+b^2\right)}$$

input
```
Int[(Cot[c + d*x]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
(2*b*(A*b - a*B))/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + (((a^2*
(a + I*b)^2*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d
) - (a^2*(a - I*b)^2*(I*A - B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a
 + I*b]*d) - (2*A*(a^2 + b^2)^2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])
/(Sqrt[a]*d))/(a*(a^2 + b^2)) + (2*b*(3*a^2*A*b + A*b^3 - 2*a^3*B))/(a*(a^
2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 12869 vs. $2(194) = 388$.

Time = 0.15 (sec) , antiderivative size = 12870, normalized size of antiderivative = 57.46

method	result	size
derivativedivides	Expression too large to display	12870
default	Expression too large to display	12870

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7339 vs. $2(189) = 378$.

Time = 18.89 (sec) , antiderivative size = 14697, normalized size of antiderivative = 65.61

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm=
"fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\cot(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)/(a + b*tan(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)}{(b\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)/(b*tan(d*x + c) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[8,18,6]%%%}+%%%{8,[8,16,6]%%%}+%%%{28,[8,14,6]%%%}+
%%%{56,[8
```

Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 45681, normalized size of antiderivative = 203.93

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
atan(-(((-(((8*A^2*a^5*d^2 - 8*B^2*a^5*d^2 - 80*A^2*a^3*b^2*d^2 + 80*B^2*a
^3*b^2*d^2 + 16*A*B*b^5*d^2 + 40*A^2*a*b^4*d^2 - 40*B^2*a*b^4*d^2 - 160*A*
B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/4 - (A^4 + 2*A^2*B^2 + B^4)*(16*a^10*d
^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 + 80
*a^8*b^2*d^4))^(1/2) - 4*A^2*a^5*d^2 + 4*B^2*a^5*d^2 + 40*A^2*a^3*b^2*d^2
- 40*B^2*a^3*b^2*d^2 - 8*A*B*b^5*d^2 - 20*A^2*a*b^4*d^2 + 20*B^2*a*b^4*d^2
 + 80*A*B*a^2*b^3*d^2 - 40*A*B*a^4*b*d^2)/(16*(a^10*d^4 + b^10*d^4 + 5*a^2
*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)))^(1/2)*(((a +
 b*tan(c + d*x))^(1/2)*(256*A^2*a^15*b^44*d^7 + 4608*A^2*a^17*b^42*d^7 + 4
0512*A^2*a^19*b^40*d^7 + 224768*A^2*a^21*b^38*d^7 + 864768*A^2*a^23*b^36*d
^7 + 2419200*A^2*a^25*b^34*d^7 + 5055232*A^2*a^27*b^32*d^7 + 8007168*A^2*a
^29*b^30*d^7 + 9664512*A^2*a^31*b^28*d^7 + 8859136*A^2*a^33*b^26*d^7 + 609
5232*A^2*a^35*b^24*d^7 + 3095040*A^2*a^37*b^22*d^7 + 1164800*A^2*a^39*b^20
*d^7 + 376320*A^2*a^41*b^18*d^7 + 154368*A^2*a^43*b^16*d^7 + 76288*A^2*a^4
5*b^14*d^7 + 28416*A^2*a^47*b^12*d^7 + 6144*A^2*a^49*b^10*d^7 + 576*A^2*a^
51*b^8*d^7 - 1344*B^2*a^19*b^40*d^7 - 15872*B^2*a^21*b^38*d^7 - 81408*B^2*
a^23*b^36*d^7 - 225792*B^2*a^25*b^34*d^7 - 302848*B^2*a^27*b^32*d^7 + 1397
76*B^2*a^29*b^30*d^7 + 1537536*B^2*a^31*b^28*d^7 + 3587584*B^2*a^33*b^26*d
^7 + 5106816*B^2*a^35*b^24*d^7 + 5051904*B^2*a^37*b^22*d^7 + 3587584*B^2*a
^39*b^20*d^7 + 1817088*B^2*a^41*b^18*d^7 + 628992*B^2*a^43*b^16*d^7 + 1...
```

Reduce [F]

$$\int \frac{\cot(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)}{\tan(dx+c)^2\,b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(cot(d*x+c)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*cot(c + d*x))/(tan(c + d*x)**2*b**2 + 2*tan(
c + d*x)*a*b + a**2),x)
```

3.363 $\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 289

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(5Ab-2aB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}d} + \frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} - \frac{b(3a^2A+5Ab^2-2abB)}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}} - \frac{b(a^4A+10a^2Ab^2+5Ab^4-6a^3bB-2ab^3B)}{a^3(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
(5*A*b-2*B*a)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(7/2)/d+(I*A+B)*ar
ctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-(I*A-B)*arctan
h((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d-1/3*b*(3*A*a^2+5*A
*b^2-2*B*a*b)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)-A*cot(d*x+c)/a/d/(a+b
*tan(d*x+c))^(3/2)-b*(A*a^4+10*A*a^2*b^2+5*A*b^4-6*B*a^3*b-2*B*a*b^3)/a^3/
(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.06

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{\frac{b(-3a^2A-5Ab^2+2abB)}{(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{3aA\cot(c+dx)}{(a+b\tan(c+dx))^{3/2}}+\frac{3\left(\frac{(a^2+b^2)^2(5Ab-2aB)\operatorname{arcta}}{\sqrt{a}}\right.}{}}{}$$

input
```
Integrate[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2)
,x]
```

output
```
((b*(-3*a^2*A - 5*A*b^2 + 2*a*b*B))/((a^2 + b^2)*(a + b*Tan[c + d*x])^(3/2
)) - (3*a*A*Cot[c + d*x])/(a + b*Tan[c + d*x])^(3/2) + (3*(((a^2 + b^2)^2*
(5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] + (a^3*
(a + I*b)^2*(I*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqr
t[a - I*b] + (a^3*(a - I*b)^2*((-I)*A + B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]
]/Sqrt[a + I*b]])/Sqrt[a + I*b] - (b*(a^4*A + 10*a^2*A*b^2 + 5*A*b^4 - 6*a
^3*b*B - 2*a*b^3*B))/Sqrt[a + b*Tan[c + d*x]]))/(a*(a^2 + b^2)^2))/(3*a^2*
d)
```

Rubi [A] (warning: unable to verify)

Time = 2.49 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.19, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^2(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \ 4092$$

$$-\frac{\int \frac{\cot(c+dx)(5Ab\tan^2(c+dx)+2aA\tan(c+dx)+5Ab-2aB)}{2(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{\cot(c+dx)(5Ab\tan^2(c+dx)+2aA\tan(c+dx)+5Ab-2aB)}{(a+b\tan(c+dx))^{5/2}}dx}{2a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{5Ab\tan(c+dx)^2+2aA\tan(c+dx)+5Ab-2aB}{\tan(c+dx)(a+b\tan(c+dx))^{5/2}}dx}{2a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4132$$

$$-\frac{\frac{2\int \frac{3\cot(c+dx)\left(2(aA+bB)\tan(c+dx)a^2+b\left(3Aa^2-2bBa+5Ab^2\right)\tan^2(c+dx)+\left(a^2+b^2\right)(5Ab-2aB)\right)}{2(a+b\tan(c+dx))^{3/2}}dx}{3a(a^2+b^2)}+\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}}{2a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{\cot(c+dx)\left(2(aA+bB)\tan(c+dx)a^2+b\left(3Aa^2-2bBa+5Ab^2\right)\tan^2(c+dx)+\left(a^2+b^2\right)(5Ab-2aB)\right)}{(a+b\tan(c+dx))^{3/2}}dx}{a(a^2+b^2)}+\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}}{2a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{\int \frac{2(aA+bB)\tan(c+dx)a^2+b\left(3Aa^2-2bBa+5Ab^2\right)\tan(c+dx)^2+\left(a^2+b^2\right)(5Ab-2aB)}{\tan(c+dx)(a+b\tan(c+dx))^{3/2}}dx}{a(a^2+b^2)}+\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}}{2a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \ 4132$$

$$-\frac{\frac{\frac{2\int \frac{\cot(c+dx)\left(2\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+b\left(Aa^4-6bBa^3+10Ab^2a^2-2b^3Ba+5Ab^4\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2(5Ab-2aB)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(a^4A-6a^3bB+10a^2\cdots\right.}{ad\left(a^2+b^2\right)\sqrt{\cdots}}}{a(a^2+b^2)}}{2a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(2\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+b\left(Aa^4-6bBa^3+10Ab^2a^2-2b^3Ba+5Ab^4\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2(5Ab-2aB)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(a^4A-6a^3bB+10a^2\cdots\right.}{ad\left(a^2+b^2\right)\sqrt{a+\cdots}}}{a(a^2+b^2)}}{2a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{2\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+b\left(Aa^4-6bBa^3+10Ab^2a^2-2b^3Ba+5Ab^4\right)\tan(c+dx)^2+\left(a^2+b^2\right)^2(5Ab-2aB)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3B+\cdots\right.}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{a(a^2+b^2)}}{2a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 4136

$$-\frac{\frac{\frac{\left(a^2+b^2\right)^2(5Ab-2aB)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+\int \frac{2\left(a^3\left(Aa^2+2bBa-Ab^2\right)-a^3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2a\cdots\right.}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c\cdots}}}{a(a^2+b^2)}}{2a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\left(a^2+b^2\right)^2(5Ab-2aB)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+2\int \frac{a^3\left(Aa^2+2bBa-Ab^2\right)-a^3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3\cdots\right.}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+d\cdots}}}{a(a^2+b^2)}}{2a}-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\left(a^2+b^2\right)^2(5Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\int \frac{a^3\left(Aa^2+2bBa-Ab^2\right)-a^3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3\cdots\right.}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+\cdots}}}{a(a^2+b^2)}\cdots}{2a}$$

$$\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 4022

$$-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3B+5Ab^4\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2(5Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{1}{2}a^3(a-ib)^2(A\cdots\right.}{a\left(a^2\cdots\right.}}{a(a^2+b^2)}\cdots}{2a}$$

$\downarrow$ 3042

$$-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3B+5Ab^4\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2(5Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{1}{2}a^3(a-ib)^2(A\cdots\right.}{a\left(a^2\cdots\right.}}{a(a^2+b^2)}\cdots}{2a}$$

$\downarrow$ 4020

$$-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3B+5Ab^4\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2(5Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{ia^3(a+ib)^2(A\cdots}{\cdots}\right.}{\cdots}}{\cdots}}{2a}$$

$\downarrow$ 25

$$-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3B+5Ab^4\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2(5Ab-2aB)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{ia^3(a-ib)^2(A\cdots}{\cdots}\right.}{\cdots}}{\cdots}}{2a}$$

$\downarrow$ 73

$$-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3B+5Ab^4\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2(5Ab-2aB)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{a^3(a-ib)^2(A+\cdots}{\cdots}\right.}{a(a^2+b\cdots}}{2a}$$

$\downarrow$ 221

$$-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3B+5Ab^4\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2(5Ab-2aB)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+2\left(\frac{a^3(a+ib)^2(A-\cdots}{a\left(a^2+b\cdots\right.}\right.}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 4117

$$-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3B+5Ab^4\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{\left(a^2+b^2\right)^2(5Ab-2aB)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+2\left(\frac{a^3(a+ib)^2(A-\cdots}{a\left(a^2+b\cdots\right.}\right.}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 73

$$-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3B+5Ab^4\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{2\left(a^2+b^2\right)^2(5Ab-2aB)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+2\left(\frac{a^3(a\cdots}{a\left(\cdots\right.}\right.}{a(a^2+b^2)}}{2a}$$

$\downarrow$ 221

$$-\frac{A\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-2abB+5Ab^2)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(a^4A-6a^3bB+10a^2Ab^2-2ab^3B+5Ab^4\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2\left(a^2+b^2\right)^2(5Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+2\left(\frac{a^3(a+ib)^2(\cdots}{a\left(a^2\cdots\right.}\right.}{a(a^2+b^2)}}{2a}$$

input
```
Int[(Cot[c + d*x]^2*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
-((A*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x])^(3/2))) - ((2*b*(3*a^2*A + 5*
A*b^2 - 2*a*b*B))/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + ((2*((a
^3*(a + I*b)^2*(A - I*B)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b
]*d) + (a^3*(a - I*b)^2*(A + I*B)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqr
t[a + I*b]*d)) - (2*(a^2 + b^2)^2*(5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*Tan[c
 + d*x]]/Sqrt[a]])/(Sqrt[a]*d))/(a*(a^2 + b^2)) + (2*b*(a^4*A + 10*a^2*A*b
^2 + 5*A*b^4 - 6*a^3*b*B - 2*a*b^3*B))/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c +
 d*x]]))/(a*(a^2 + b^2)))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 12967 vs. $2(257) = 514$.

Time = 0.18 (sec) , antiderivative size = 12968, normalized size of antiderivative = 44.87

method	result	size
derivativedivides	Expression too large to display	12968
default	Expression too large to display	12968

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7530 vs. 2(252) = 504.

Time = 54.90 (sec) , antiderivative size = 15079, normalized size of antiderivative = 52.18

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\cot^2(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**2/(a + b*tan(c + d*x))**(5/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[9,22,7]%%%}+%%%{10,[9,20,7]%%%}+%%%{45,[9,18,7]%%%}
+%%%{120,
```

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 67465, normalized size of antiderivative = 233.44

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^2*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
((2*(a + b*tan(c + d*x))*(5*A*b^5 + 11*A*a^2*b^3 - 8*B*a^3*b^2 - 2*B*a*b^4
))/(3*(a*b^2 + a^3)^2) + (2*(A*b^3 - B*a*b^2))/(3*a*(a^2 + b^2)) - ((a + b
*tan(c + d*x))^2*(5*A*b^5 + 10*A*a^2*b^3 - 6*B*a^3*b^2 + A*a^4*b - 2*B*a*b
^4))/(a^3*(a^2 + b^2)^2))/(d*(a + b*tan(c + d*x))^(5/2) - a*d*(a + b*tan(c
 + d*x))^(3/2)) + atan((((((8*A^2*a^5*d^2 - 8*B^2*a^5*d^2 - 80*A^2*a^3*b^
2*d^2 + 80*B^2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 + 40*A^2*a*b^4*d^2 - 40*B^2*a*
b^4*d^2 - 160*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/4 - (A^4 + 2*A^2*B^2 +
 B^4)*(16*a^10*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*
a^6*b^4*d^4 + 80*a^8*b^2*d^4))^(1/2) - 4*A^2*a^5*d^2 + 4*B^2*a^5*d^2 + 40*
A^2*a^3*b^2*d^2 - 40*B^2*a^3*b^2*d^2 - 8*A*B*b^5*d^2 - 20*A^2*a*b^4*d^2 +
20*B^2*a*b^4*d^2 + 80*A*B*a^2*b^3*d^2 - 40*A*B*a^4*b*d^2)/(16*(a^10*d^4 +
b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4
)))^(1/2)*((((((8*A^2*a^5*d^2 - 8*B^2*a^5*d^2 - 80*A^2*a^3*b^2*d^2 + 80*B^
2*a^3*b^2*d^2 + 16*A*B*b^5*d^2 + 40*A^2*a*b^4*d^2 - 40*B^2*a*b^4*d^2 - 160
*A*B*a^2*b^3*d^2 + 80*A*B*a^4*b*d^2)^2/4 - (A^4 + 2*A^2*B^2 + B^4)*(16*a^1
0*d^4 + 16*b^10*d^4 + 80*a^2*b^8*d^4 + 160*a^4*b^6*d^4 + 160*a^6*b^4*d^4 +
 80*a^8*b^2*d^4))^(1/2) - 4*A^2*a^5*d^2 + 4*B^2*a^5*d^2 + 40*A^2*a^3*b^2*d
^2 - 40*B^2*a^3*b^2*d^2 - 8*A*B*b^5*d^2 - 20*A^2*a*b^4*d^2 + 20*B^2*a*b^4*
d^2 + 80*A*B*a^2*b^3*d^2 - 40*A*B*a^4*b*d^2)/(16*(a^10*d^4 + b^10*d^4 + 5*
a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)))^(1/2)*...
```

Reduce [F]

$$\int \frac{\cot^2(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)^2}{\tan(dx+c)^2\,b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(cot(d*x+c)^2*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*cot(c + d*x)**2)/(tan(c + d*x)**2*b**2 + 2*t
an(c + d*x)*a*b + a**2),x)
```

3.364 $\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 364

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(8a^2A-35Ab^2+20abB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{9/2}d} - \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{(A+iB)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} + \frac{b(27a^2Ab+35Ab^3-12a^3B-20ab^2B)}{12a^3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{(7Ab-4aB)\cot(c+dx)}{4a^2d(a+b\tan(c+dx))^{3/2}} - \frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}} + \frac{b(11a^4Ab+62a^2Ab^3+35Ab^5-4a^5B-40a^3b^2B-20ab^4B)}{4a^4(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
1/4*(8*A*a^2-35*A*b^2+20*B*a*b)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^
(9/2)/d-(A-I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2
)/d-(A+I*B)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+
1/12*b*(27*A*a^2*b+35*A*b^3-12*B*a^3-20*B*a*b^2)/a^3/(a^2+b^2)/d/(a+b*tan(
d*x+c))^(3/2)+1/4*(7*A*b-4*B*a)*cot(d*x+c)/a^2/d/(a+b*tan(d*x+c))^(3/2)-1/
2*A*cot(d*x+c)^2/a/d/(a+b*tan(d*x+c))^(3/2)+1/4*b*(11*A*a^4*b+62*A*a^2*b^3
+35*A*b^5-4*B*a^5-40*B*a^3*b^2-20*B*a*b^4)/a^4/(a^2+b^2)^2/d/(a+b*tan(d*x+
c))^(1/2)
```

Mathematica [A] (verified)

Time = 4.77 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.02

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{\frac{b(27a^2Ab+35Ab^3-12a^3B-20ab^2B)}{a^2(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{3(7Ab-4aB)\cot(c+dx)}{a(a+b\tan(c+dx))^{3/2}}-\frac{6A\cot^2(c+dx)}{(a+b\tan(c+dx))^{3}}}{}$$

input
```
Integrate[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2)
,x]
```

output
```
((b*(27*a^2*A*b + 35*A*b^3 - 12*a^3*B - 20*a*b^2*B))/(a^2*(a^2 + b^2)*(a +
 b*Tan[c + d*x])^(3/2)) + (3*(7*A*b - 4*a*B)*Cot[c + d*x])/(a*(a + b*Tan[c
 + d*x])^(3/2)) - (6*A*Cot[c + d*x]^2)/(a + b*Tan[c + d*x])^(3/2) + (3*(((
a^2 + b^2)^2*(8*a^2*A - 35*A*b^2 + 20*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*
x]]/Sqrt[a]])/Sqrt[a] - (4*a^4*(a + I*b)^2*(A - I*B)*ArcTanh[Sqrt[a + b*Ta
n[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*b] - (4*a^4*(a - I*b)^2*(A + I*B)*A
rcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt[a + I*b] + (b*(11*a^4
*A*b + 62*a^2*A*b^3 + 35*A*b^5 - 4*a^5*B - 40*a^3*b^2*B - 20*a*b^4*B))/Sqr
t[a + b*Tan[c + d*x]]))/(a^3*(a^2 + b^2)^2))/(12*a*d)
```

Rubi [A] (warning: unable to verify)

Time = 3.14 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.16, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.758$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^3(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 4092$$

$$-\frac{\int \frac{\cot^2(c+dx)(7Ab\tan^2(c+dx)+4aA\tan(c+dx)+7Ab-4aB)}{2(a+b\tan(c+dx))^{5/2}}dx}{2a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{\cot^2(c+dx)(7Ab\tan^2(c+dx)+4aA\tan(c+dx)+7Ab-4aB)}{(a+b\tan(c+dx))^{5/2}}dx}{4a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{7Ab\tan(c+dx)^2+4aA\tan(c+dx)+7Ab-4aB}{\tan(c+dx)^2(a+b\tan(c+dx))^{5/2}}dx}{4a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4132$$

$$-\frac{-\frac{\int -\frac{\cot(c+dx)\left(8Aa^2+8B\tan(c+dx)a^2+20bBa-35Ab^2-5b(7Ab-4aB)\tan^2(c+dx)\right)}{2(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}}{4a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{\cot(c+dx)\left(8Aa^2+8B\tan(c+dx)a^2+20bBa-35Ab^2-5b(7Ab-4aB)\tan^2(c+dx)\right)}{(a+b\tan(c+dx))^{5/2}}dx}{2a}-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}}{4a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{8Aa^2+8B\tan(c+dx)a^2+20bBa-35Ab^2-5b(7Ab-4aB)\tan(c+dx)^2}{\tan(c+dx)(a+b\tan(c+dx))^{5/2}}dx}{2a}-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}}{4a}-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\frac{\frac{2\int \frac{3\cot(c+dx)\left(-8(Ab-aB)\tan(c+dx)a^3-b\left(-12Ba^3+27Aba^2-20b^2Ba+35Ab^3\right)\tan^2(c+dx)+\left(a^2+b^2\right)\left(8Aa^2+20bBa-35Ab^2\right)\right)}{2(a+b\tan(c+dx))^{3/2}}dx}{3a\left(a^2+b^2\right)}-\frac{2b\left(-12a^3B+27a^2Ab\right.}{3ad\left(a^2+b^2\right)(a+b}}{2a}}{4a}$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(-8(Ab-aB)\tan(c+dx)a^3-b\left(-12Ba^3+27Aba^2-20b^2Ba+35Ab^3\right)\tan^2(c+dx)+\left(a^2+b^2\right)\left(8Aa^2+20bBa-35Ab^2\right)\right)}{(a+b\tan(c+dx))^{3/2}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-12a^3B+27a^2Ab-2\ldots\right.}{3ad\left(a^2+b^2\right)(a+b\tan\ldots}}{2a}}{4a}\ldots$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{-8(Ab-aB)\tan(c+dx)a^3-b\left(-12Ba^3+27Aba^2-20b^2Ba+35Ab^3\right)\tan(c+dx)^2+\left(a^2+b^2\right)\left(8Aa^2+20bBa-35Ab^2\right)}{\tan(c+dx)(a+b\tan(c+dx))^{3/2}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-12a^3B+27a^2Ab-20ab^2B+35A\ldots\right.}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}}{2a}}{4a}\ldots$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 4132

$$-\frac{\frac{\frac{\frac{2\int \frac{\cot(c+dx)\left(-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^4-b\left(-4Ba^5+11Aba^4-40b^2Ba^3+62Ab^3a^2-20b^4Ba+35Ab^5\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2\left(8Aa^2+20bBa-35\ldots\right.\right.}{2\sqrt{a+b\tan(c+dx)}}\ldots}{a\left(a^2+b^2\right)}\ldots}{a\left(a^2+b^2\right)}\ldots}{2a}\ldots}{4a}\ldots$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\frac{\int \frac{\cot(c+dx)\left(-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^4-b\left(-4Ba^5+11Aba^4-40b^2Ba^3+62Ab^3a^2-20b^4Ba+35Ab^5\right)\tan^2(c+dx)+\left(a^2+b^2\right)^2\left(8Aa^2+20bBa-35A\ldots\right.\right.}{\sqrt{a+b\tan(c+dx)}}\ldots}{a\left(a^2+b^2\right)}\ldots}{a\left(a^2+b^2\right)}\ldots}{2a}\ldots}{4a}\ldots$$

$$\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$\int \frac{-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^4-b\left(-4Ba^5+11Aba^4-40b^2Ba^3+62Ab^3a^2-20b^4Ba+35Ab^5\right)\tan(c+dx)^2+\left(a^2+b^2\right)^2\left(8Aa^2+20bBa-35Ab^2\right)}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx - \frac{2b(\cdots}{\cdots}$$

$$a\left(a^2+b^2\right) \qquad a\left(a^2+b^2\right) \qquad 2a \qquad 4a$$

$$-\ \cdots \qquad \frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$$\downarrow \ 4136$$

$$\left(a^2+b^2\right)^2\left(8a^2A+20abB-35Ab^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+\int -\frac{8\left(\left(-Ba^2+2Aba+b^2B\right)a^4+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^4\right)}{\sqrt{a+b\tan(c+dx)}}dx - \frac{2b\left(-4a^5B+11a^4\cdots\right.}{a\cdots}$$

$$a\left(a^2+b^2\right) \qquad a\left(a^2+b^2\right) \qquad 2a \qquad 4a$$

$$-\ \cdots \qquad \frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$$\downarrow \ 27$$

$$\left(a^2+b^2\right)^2\left(8a^2A+20abB-35Ab^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-8\int \frac{\left(-Ba^2+2Aba+b^2B\right)a^4+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^4}{\sqrt{a+b\tan(c+dx)}}dx - \frac{2b\left(-4a^5B+11a^4Ab\cdots\right.}{ad\left(a\cdots\right.}$$

$$a\left(a^2+b^2\right) \qquad a\left(a^2+b^2\right) \qquad 2a \qquad 4a$$

$$-\ \cdots \qquad \frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$$\downarrow \ 3042$$

$$\left(a^2+b^2\right)^2\left(8a^2A+20abB-35Ab^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-8\int \frac{\left(-Ba^2+2Aba+b^2B\right)a^4+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^4}{\sqrt{a+b\tan(c+dx)}}dx - \frac{2b\left(-4a^5B+11a^4Ab\cdots\right.}{ad\left(a\cdots\right.}$$

$$a\left(a^2+b^2\right) \qquad a\left(a^2+b^2\right) \qquad 2a \qquad 4a$$

$$-\ \cdots \qquad \frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}$$

$$\downarrow \ 4022$$

$$-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-12a^3B+27a^2Ab-20ab^2B+35Ab^3\right)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-4a^5B+11a^4Ab-40a^3b^2B+62a^2Ab^3-20ab^4B+35Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2\left(8a^2A+\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{4a}$$

$\downarrow$ 3042

$$-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-12a^3B+27a^2Ab-20ab^2B+35Ab^3\right)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-4a^5B+11a^4Ab-40a^3b^2B+62a^2Ab^3-20ab^4B+35Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2\left(8a^2A+\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{4a}$$

$\downarrow$ 4020

$$-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-12a^3B+27a^2Ab-20ab^2B+35Ab^3\right)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-4a^5B+11a^4Ab-40a^3b^2B+62a^2Ab^3-20ab^4B+35Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2\left(8a^2A+\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 25

$$-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-12a^3B+27a^2Ab-20ab^2B+35Ab^3\right)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-4a^5B+11a^4Ab-40a^3b^2B+62a^2Ab^3-20ab^4B+35Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2\left(8a^2A+\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 73

$$-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-12a^3B+27a^2Ab-20ab^2B+35Ab^3\right)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-4a^5B+11a^4Ab-40a^3b^2B+62a^2Ab^3-20ab^4B+35Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)^2\left(8a^2A+\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 221

$$-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-12a^3B+27a^2Ab-20ab^2B+35Ab^3)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-4a^5B+11a^4Ab-40a^3b^2B+62a^2Ab^3-20ab^4B+35Ab^5)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{(a^2+b^2)^2(8a^2A+}{}}{}}{4a}$$

$\downarrow$ 4117

$$-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-12a^3B+27a^2Ab-20ab^2B+35Ab^3)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-4a^5B+11a^4Ab-40a^3b^2B+62a^2Ab^3-20ab^4B+35Ab^5)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{(a^2+b^2)^2(8a^2A-}{}}{}}{4a}$$

$\downarrow$ 73

$$-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-12a^3B+27a^2Ab-20ab^2B+35Ab^3)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-4a^5B+11a^4Ab-40a^3b^2B+62a^2Ab^3-20ab^4B+35Ab^5)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{2(a^2+b^2)^2(8a^2A}{}}{}}{4a}$$

$\downarrow$ 221

$$-\frac{A\cot^2(c+dx)}{2ad(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{(7Ab-4aB)\cot(c+dx)}{ad(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-12a^3B+27a^2Ab-20ab^2B+35Ab^3)}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-4a^5B+11a^4Ab-40a^3b^2B+62a^2Ab^3-20ab^4B+35Ab^5)}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2(a^2+b^2)^2(8a^2}{}}{}}{}}{4a}$$

input
```
Int[(Cot[c + d*x]^3*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
-1/2*(A*Cot[c + d*x]^2)/(a*d*(a + b*Tan[c + d*x])^(3/2)) - (-(((7*A*b - 4*
a*B)*Cot[c + d*x])/(a*d*(a + b*Tan[c + d*x])^(3/2))) + ((-2*b*(27*a^2*A*b
+ 35*A*b^3 - 12*a^3*B - 20*a*b^2*B))/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x
])^(3/2)) + ((-8*(-((a^4*(a + I*b)^2*(I*A + B)*ArcTan[Tan[c + d*x]/Sqrt[a
- I*b]])/(Sqrt[a - I*b]*d)) + (a^4*(a - I*b)^2*(I*A - B)*ArcTan[Tan[c + d*
x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)) - (2*(a^2 + b^2)^2*(8*a^2*A - 35*A*b
^2 + 20*a*b*B)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d))/(a*
(a^2 + b^2)) - (2*b*(11*a^4*A*b + 62*a^2*A*b^3 + 35*A*b^5 - 4*a^5*B - 40*a
^3*b^2*B - 20*a*b^4*B))/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(a*(a^
2 + b^2)))/(2*a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4092

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13091 vs. $2(322) = 644$.

Time = 0.18 (sec) , antiderivative size = 13092, normalized size of antiderivative = 35.97

method	result	size
derivativedivides	Expression too large to display	13092
default	Expression too large to display	13092

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7651 vs. 2(316) = 632.

Time = 104.84 (sec) , antiderivative size = 15322, normalized size of antiderivative = 42.09

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\cot^3(c+dx)}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**3/(a + b*tan(c + d*x))**(5/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[10,26,8]%%%}+%%%{12,[10,24,8]%%%}+%%%{66,[10,22,8]%
%%}+%%%{2
```

Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 71314, normalized size of antiderivative = 195.92

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)^3*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(((a + b*tan(c + d*x))^3*(35*A*b^6 + 62*A*a^2*b^4 + 11*A*a^4*b^2 - 40*B*a^
3*b^3 - 20*B*a*b^5 - 4*B*a^5*b))/(4*(a^8 + a^4*b^4 + 2*a^6*b^2)) - ((a + b
*tan(c + d*x))^2*(175*A*b^6 + 310*A*a^2*b^4 + 39*A*a^4*b^2 - 208*B*a^3*b^3
 - 100*B*a*b^5 - 12*B*a^5*b))/(12*(a^7 + a^3*b^4 + 2*a^5*b^2)) + (2*(A*b^4
 - B*a*b^3))/(3*a*(a^2 + b^2)) + (2*(a + b*tan(c + d*x))*(7*A*b^6 + 13*A*a
^2*b^4 - 10*B*a^3*b^3 - 4*B*a*b^5))/(3*a^2*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a
 + b*tan(c + d*x))^(7/2) - 2*a*d*(a + b*tan(c + d*x))^(5/2) + a^2*d*(a + b
*tan(c + d*x))^(3/2)) + atan((((a + b*tan(c + d*x))^(1/2)*(321126400*A^4*a
^28*b^44*d^5 + 2422210560*A^4*a^30*b^42*d^5 + 1411383296*A^4*a^32*b^40*d^5
 - 54989422592*A^4*a^34*b^38*d^5 - 325864914944*A^4*a^36*b^36*d^5 - 101129
4928896*A^4*a^38*b^34*d^5 - 2054783238144*A^4*a^40*b^32*d^5 - 292349070540
8*A^4*a^42*b^30*d^5 - 2962565365760*A^4*a^44*b^28*d^5 - 2094150975488*A^4*
a^46*b^26*d^5 - 943762440192*A^4*a^48*b^24*d^5 - 175655354368*A^4*a^50*b^2
2*d^5 + 74523344896*A^4*a^52*b^20*d^5 + 62081990656*A^4*a^54*b^18*d^5 + 17
307795456*A^4*a^56*b^16*d^5 + 1629487104*A^4*a^58*b^14*d^5 + 44302336*A^4*
a^60*b^12*d^5 + 104857600*A^4*a^62*b^10*d^5 + 25165824*A^4*a^64*b^8*d^5 -
104857600*B^4*a^30*b^42*d^5 - 838860800*B^4*a^32*b^40*d^5 - 838860800*B^4*
a^34*b^38*d^5 + 17624465408*B^4*a^36*b^36*d^5 + 114621939712*B^4*a^38*b^34
*d^5 + 382445027328*B^4*a^40*b^32*d^5 + 842753114112*B^4*a^42*b^30*d^5 + 1
327925035008*B^4*a^44*b^28*d^5 + 1546246946816*B^4*a^46*b^26*d^5 + 1344...
```

Reduce [F]

$$\int \frac{\cot^3(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\cot(dx+c)^3\,(A+B\tan(dx+c))}{(a+\tan(dx+c)\,b)^{\frac{5}{2}}}dx$$

input
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int(cot(d*x+c)^3*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

3.365 $\int \frac{aB+bB\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 271

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{bB\text{arctanh}\left(\frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}{a+\sqrt{a^2+b^2}+b\tan(c+dx)}\right)}{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}d} + \frac{bB\text{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{bB\text{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}d}$$

output
```
-1/2*b*B*arctanh(2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)/
(a+(a^2+b^2)^(1/2)+b*tan(d*x+c)))*2^(1/2)/(a+(a^2+b^2)^(1/2))^(1/2)/d+1/2*
b*B*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a-
(a^2+b^2)^(1/2))^(1/2))*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)/d-1/2*b*B*arctan
h(((a+(a^2+b^2)^(1/2))^(1/2)+2^(1/2)*(a+b*tan(d*x+c))^(1/2))/(a-(a^2+b^2)^
(1/2))^(1/2))*2^(1/2)/(a-(a^2+b^2)^(1/2))^(1/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.32

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{iB\left(\sqrt{a-ib}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-\sqrt{a+ib}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)\right)}{d}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
((-I)*B*(Sqrt[a - I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - S
qrt[a + I*b]*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.49, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2011, 3042, 3966, 483, 1449, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \sqrt{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \sqrt{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3966}$$

$$\frac{bB\int \frac{\sqrt{a+b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 483$$

$$\frac{2bB\int \frac{b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)-2ab^2\tan^2(c+dx)+a^2+b^2}d\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 1449$$

$$\frac{2bB\left(\frac{\int \frac{\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b\sqrt{a+\sqrt{a^2+b^2}}\tan(c+dx)+\sqrt{a^2+b^2}}d\sqrt{a+b\tan(c+dx)}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}-\frac{\int \frac{\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b\sqrt{a+\sqrt{a^2+b^2}}\tan(c+dx)+\sqrt{a^2+b^2}}d\sqrt{a+b\,\mathrm{t}\cdots}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots\right.}{d}$$

$$\Big\downarrow \; 1142$$

$$\frac{2bB\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}+\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+d\cdots}\right.}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a-\cdots}}\cdots}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots\right.}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{2bB\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}\cdots\right.}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cdots}}\cdots}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots\right.}{\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{2bB\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\,\mathrm{ta}\cdots}}\cdots}{\sqrt{2}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots\right.}{\cdots}$$

$$\Big\downarrow \; 1083$$

$$2bB\left(\frac{-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{2\left(a-\sqrt{a^2+b^2}\right)-b^2\tan^2(c+dx)}d\left(2\sqrt{a+b\tan(c+dx)}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\right)-\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+d}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{}}}{\sqrt{2}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$\downarrow$ 219

$$2bB\left(\frac{-\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\tan(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{a-\sqrt{a^2+b^2}}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$$\overline{\qquad d}$$

$\downarrow$ 1103

$$2bB\left(\frac{\frac{1}{2}\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+b^2\tan^2(c+dx)\right)-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\tan(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{a-\sqrt{a^2+b^2}}}}{2\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}-\right.$$

$$\overline{\qquad d}$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
(2*b*B*((-((Sqrt[a + Sqrt[a^2 + b^2]]*ArcTanh[(-(Sqrt[2]*Sqrt[a + Sqrt[a^2
 + b^2]]) + 2*Sqrt[a + b*Tan[c + d*x]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]
)])/Sqrt[a - Sqrt[a^2 + b^2]]) + Log[Sqrt[a^2 + b^2] + b^2*Tan[c + d*x]^2
- Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]]/2)/(2*Sqrt[2
]*Sqrt[a + Sqrt[a^2 + b^2]]) - ((Sqrt[a + Sqrt[a^2 + b^2]]*ArcTanh[(Sqrt[2
]*Sqrt[a + Sqrt[a^2 + b^2]] + 2*Sqrt[a + b*Tan[c + d*x]])/(Sqrt[2]*Sqrt[a
- Sqrt[a^2 + b^2]])])/Sqrt[a - Sqrt[a^2 + b^2]] + Log[Sqrt[a^2 + b^2] + b^
2*Tan[c + d*x]^2 + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*
x]]]/2)/(2*Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]])))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 483
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[2*d
Subst[Int[x^2/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]], x
] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1449
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q =
Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*r)   Int[x^(m - 1)/(q
 - r*x + x^2), x], x] - Simp[1/(2*c*r)   Int[x^(m - 1)/(q + r*x + x^2), x],
 x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 1] && LtQ[m,
3] && NegQ[b^2 - 4*a*c]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(220) = 440.

Time = 0.16 (sec) , antiderivative size = 662, normalized size of antiderivative = 2.44

method	result
derivativedivides	$\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)B\sqrt{2\sqrt{a^2+b^2}+2a}\,a}{4db} - \frac{B\,a^2\arctan\left(\frac{2\sqrt{a+b\tan(dx}}{\sqrt{2\sqrt{}}}\right.}{db\sqrt{2\sqrt{a^2+}}}$
default	$\frac{\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)B\sqrt{2\sqrt{a^2+b^2}+2a}\,a}{4db} - \frac{B\,a^2\arctan\left(\frac{2\sqrt{a+b\tan(dx}}{\sqrt{2\sqrt{}}}\right.}{db\sqrt{2\sqrt{a^2+}}}$
parts	Expression too large to display

input
```
int((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/b*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d*B/b*a^2/(2*(a^
2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2
)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4/d/b*ln(b*tan(d*x+c)+a+(a+
b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(a^2+
b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/d*B/b*(a^2+b^2)/(2*(a^2+b^2)^(1
/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1
/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(
a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(
1/2)+2*a)^(1/2)*a+1/d*B/b*a^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^
2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2))+1/4/d/b*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*t
an(d*x+c)-a-(a^2+b^2)^(1/2))*B*(a^2+b^2)^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)-1/d*B/b*(a^2+b^2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1
/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.47

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{1}{2}\sqrt{-\frac{B^2a+\sqrt{-\frac{B^4b^2}{d^4}}d^2}{d^2}}\log\left(\sqrt{b\tan\left(dx+c\right)+a}B^3b+\sqrt{-\frac{B^4b^2}{d^4}}d^3\sqrt{-\frac{B^2a+\sqrt{-\frac{B^4b^2}{d^4}}d^2}{d^2}}\right) + \frac{1}{2}\sqrt{-\frac{B^2a+\sqrt{-\frac{B^4b^2}{d^4}}d^2}{d^2}}\log\left(\sqrt{b\tan\left(dx+c\right)+a}B^3b-\sqrt{-\frac{B^4b^2}{d^4}}d^3\sqrt{-\frac{B^2a+\sqrt{-\frac{B^4b^2}{d^4}}d^2}{d^2}}\right) + \frac{1}{2}\sqrt{-\frac{B^2a-\sqrt{-\frac{B^4b^2}{d^4}}d^2}{d^2}}\log\left(\sqrt{b\tan\left(dx+c\right)+a}B^3b+\sqrt{-\frac{B^4b^2}{d^4}}d^3\sqrt{-\frac{B^2a-\sqrt{-\frac{B^4b^2}{d^4}}d^2}{d^2}}\right) - \frac{1}{2}\sqrt{-\frac{B^2a-\sqrt{-\frac{B^4b^2}{d^4}}d^2}{d^2}}\log\left(\sqrt{b\tan\left(dx+c\right)+a}B^3b-\sqrt{-\frac{B^4b^2}{d^4}}d^3\sqrt{-\frac{B^2a-\sqrt{-\frac{B^4b^2}{d^4}}d^2}{d^2}}\right)$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas
")
```

output
```
-1/2*sqrt(-(B^2*a + sqrt(-B^4*b^2/d^4)*d^2)/d^2)*log(sqrt(b*tan(d*x + c) +
 a)*B^3*b + sqrt(-B^4*b^2/d^4)*d^3*sqrt(-(B^2*a + sqrt(-B^4*b^2/d^4)*d^2)/
d^2)) + 1/2*sqrt(-(B^2*a + sqrt(-B^4*b^2/d^4)*d^2)/d^2)*log(sqrt(b*tan(d*x
 + c) + a)*B^3*b - sqrt(-B^4*b^2/d^4)*d^3*sqrt(-(B^2*a + sqrt(-B^4*b^2/d^4
)*d^2)/d^2)) + 1/2*sqrt(-(B^2*a - sqrt(-B^4*b^2/d^4)*d^2)/d^2)*log(sqrt(b*
tan(d*x + c) + a)*B^3*b + sqrt(-B^4*b^2/d^4)*d^3*sqrt(-(B^2*a - sqrt(-B^4*
b^2/d^4)*d^2)/d^2)) - 1/2*sqrt(-(B^2*a - sqrt(-B^4*b^2/d^4)*d^2)/d^2)*log(
sqrt(b*tan(d*x + c) + a)*B^3*b - sqrt(-B^4*b^2/d^4)*d^3*sqrt(-(B^2*a - sqr
t(-B^4*b^2/d^4)*d^2)/d^2))
```

Sympy [F]

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = B\int \sqrt{a + b\tan(c + dx)}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
B*Integral(sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\, dx = \text{Timed out}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 3033, normalized size of antiderivative = 11.19

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\, dx = \text{Too large to display}$$

input
```
int((B*a + B*b*tan(c + d*x))/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
2*atanh((8*a*b^2*(a + b*tan(c + d*x))^(1/2)*(- (-16*B^4*a^4*b^2*d^4)^(1/2)
/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(
-16*B^4*a^4*b^2*d^4)^(1/2))/((16*B^3*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (1
6*B^3*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) + (4*B*a^3*b^3*d^4*(-16*B^4*a^4*b^2
*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) + (4*B*a*b^5*d^4*(-16*B^4*a^4*b^2*d^4)^(1
/2))/(a^2*d^5 + b^2*d^5)) - (32*B^2*a^2*b^2*(a + b*tan(c + d*x))^(1/2)*(-
(-16*B^4*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a^3*d^2)/(4*(a
^2*d^4 + b^2*d^4)))^(1/2))/((16*B^3*a^4*b^3*d^3)/(a^2*d^4 + b^2*d^4) + (4*
B*a*b^3*d^2*(-16*B^4*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) + (32*B^2*a^
4*b^2*d^2*(a + b*tan(c + d*x))^(1/2)*(- (-16*B^4*a^4*b^2*d^4)^(1/2)/(16*(a
^2*d^4 + b^2*d^4)) - (B^2*a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2))/((16*B^
3*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*B^3*a^6*b^3*d^5)/(a^2*d^4 + b^2*d
^4) + (4*B*a^3*b^3*d^4*(-16*B^4*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) +
(4*B*a*b^5*d^4*(-16*B^4*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*(- (-16*
B^4*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (B^2*a^3*d^2)/(4*(a^2*d^
4 + b^2*d^4)))^(1/2) - 2*atanh((8*a*b^2*((-16*B^4*b^6*d^4)^(1/2)/(16*(a^2*
d^4 + b^2*d^4)) + (B^2*a*b^2*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*ta
n(c + d*x))^(1/2)*(-16*B^4*b^6*d^4)^(1/2))/((16*B^3*a^2*b^7*d^5)/(a^2*d^4
+ b^2*d^4) - 16*B^3*a^2*b^5*d - 16*B^3*b^7*d + (16*B^3*a^4*b^5*d^5)/(a^2*d
^4 + b^2*d^4) + (4*B*a^3*b^3*d^4*(-16*B^4*b^6*d^4)^(1/2))/(a^2*d^5 + b^...
```

Reduce [F]

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \left(\int \sqrt{a + \tan\left(dx + c\right)b}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a),x)*b
```

3.366 $\int \frac{aB+bB\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 303

$$\int \frac{aB+bB\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{bB\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}{a+\sqrt{a^2+b^2}+b\tan(c+dx)}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a+\sqrt{a^2+b^2}}d} + \frac{bB\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}d} - \frac{bB\operatorname{arctanh}\left(\frac{\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{2}\sqrt{a+b\tan(c+dx)}}{\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{2}\sqrt{a^2+b^2}\sqrt{a-\sqrt{a^2+b^2}}d}$$

output
```
1/2*b*B*arctanh(2^(1/2)*(a+(a^2+b^2)^(1/2))^(1/2)*(a+b*tan(d*x+c))^(1/2)/(
a+(a^2+b^2)^(1/2)+b*tan(d*x+c)))*2^(1/2)/(a^2+b^2)^(1/2)/(a+(a^2+b^2)^(1/2
))^(1/2)/d+1/2*b*B*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)-2^(1/2)*(a+b*tan(d*x
+c))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*2^(1/2)/(a^2+b^2)^(1/2)/(a-(a^2+b^2
)^(1/2))^(1/2)/d-1/2*b*B*arctanh(((a+(a^2+b^2)^(1/2))^(1/2)+2^(1/2)*(a+b*t
an(d*x+c))^(1/2))/(a-(a^2+b^2)^(1/2))^(1/2))*2^(1/2)/(a^2+b^2)^(1/2)/(a-(a
^2+b^2)^(1/2))^(1/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.29

$$\int \frac{aB+bB\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{iB\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}\right)}{d}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*B*(ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] - A
rcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b]))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2011, 3042, 3966, 484, 1407, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{1}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{1}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3966}$$

$$\frac{bB\int \frac{1}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$

$\downarrow$ 484

$$\frac{2bB\int \frac{1}{b^4\tan^4(c+dx)-2ab^2\tan^2(c+dx)+a^2+b^2}d\sqrt{a+b\tan(c+dx)}}{d}$$

$\downarrow$ 1407

$$\frac{2bB\left(\frac{\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}+\frac{\int \frac{\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}+\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}+\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}\cdots}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots\right.}{d}$$

$\downarrow$ 1142

$$2bB\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\frac{1}{2}\int -\frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+d\cdots}\right.}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a-\cdots}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots\right.$$

$\downarrow$ 25

$$2bB\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\left(\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\cdots}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots\right.$$

$\downarrow$ 27

$$2bB\left(\frac{\frac{\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}+\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan\cdots}}}{\sqrt{2}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\cdots\right.$$

$\downarrow$ 1083

$$2bB\left(\frac{\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\int \frac{1}{2\left(a-\sqrt{a^2+b^2}\right)-b^2\tan^2(c+dx)}d\left(2\sqrt{a+b\tan\right.}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

$\Big\downarrow$ 219

$$2bB\left(\frac{\frac{\int \frac{\sqrt{a+\sqrt{a^2+b^2}}-\sqrt{2}\sqrt{a+b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{a^2+b^2}-\sqrt{2}\sqrt{a+\sqrt{a^2+b^2}}\sqrt{a+b\tan(c+dx)}}d\sqrt{a+b\tan(c+dx)}}{\sqrt{2}}-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\tan(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{a-\sqrt{a^2+b^2}}}}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

d

$\Big\downarrow$ 1103

$$2bB\left(\frac{-\frac{\sqrt{\sqrt{a^2+b^2}+a}\,\text{arctanh}\left(\frac{2\sqrt{a+b\tan(c+dx)}-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}}{\sqrt{2}\sqrt{a-\sqrt{a^2+b^2}}}\right)}{\sqrt{a-\sqrt{a^2+b^2}}}-\frac{1}{2}\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+b^2}+a}\sqrt{a+b\tan(c+dx)}+\sqrt{a^2+b^2}+b^2\tan^2(c+dx)\right)}{2\sqrt{2}\sqrt{a^2+b^2}\sqrt{\sqrt{a^2+b^2}+a}}\right.$$

d

input
```
Int[(a*B + b*B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
(2*b*B*((-((Sqrt[a + Sqrt[a^2 + b^2]]*ArcTanh[(-(Sqrt[2]*Sqrt[a + Sqrt[a^2
 + b^2]]) + 2*Sqrt[a + b*Tan[c + d*x]])/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]]
)])/Sqrt[a - Sqrt[a^2 + b^2]]) - Log[Sqrt[a^2 + b^2] + b^2*Tan[c + d*x]^2
- Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sqrt[a + b*Tan[c + d*x]]]/2)/(2*Sqrt[2
]*Sqrt[a^2 + b^2]*Sqrt[a + Sqrt[a^2 + b^2]]) + (-((Sqrt[a + Sqrt[a^2 + b^2
]]*ArcTanh[(Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]] + 2*Sqrt[a + b*Tan[c + d*x]]
)/(Sqrt[2]*Sqrt[a - Sqrt[a^2 + b^2]])])/Sqrt[a - Sqrt[a^2 + b^2]]) + Log[S
qrt[a^2 + b^2] + b^2*Tan[c + d*x]^2 + Sqrt[2]*Sqrt[a + Sqrt[a^2 + b^2]]*Sq
rt[a + b*Tan[c + d*x]]]/2)/(2*Sqrt[2]*Sqrt[a^2 + b^2]*Sqrt[a + Sqrt[a^2 +
b^2]])))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 484
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[2*
d   Subst[Int[1/(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4), x], x, Sqrt[c + d*x]],
 x] /; FreeQ[{a, b, c, d}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3966
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Su
bst[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d, n}, x] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1574 vs. $2(247) = 494$.

Time = 0.12 (sec) , antiderivative size = 1575, normalized size of antiderivative = 5.20

method	result	size
derivativedivides	Expression too large to display	1575
default	Expression too large to display	1575
parts	Expression too large to display	3681

input
```
int((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/b/(a^2+b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2+1/4/d*
b/(a^2+b^2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-1/4/d/b/(a^2+b^2
)^(3/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3-1/4/d*b/(a^2+b^2
)^(3/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)+(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+3/d*b/(a^2+b^2)^(3
/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2
+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2+2/d*b^3/(a^2+
b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+
(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B-1/d/b/(a^2
+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^2-1/d*b
/(a^2+b^2)^(1/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^
(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B+1/d/
b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))
^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*B*a^4
-1/4/d/b/(a^2+b^2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*B*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. $2(249) = 498$.

Time = 0.10 (sec) , antiderivative size = 805, normalized size of antiderivative = 2.66

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas
")
```

output
```
1/2*sqrt(-((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^2 +
B^2*a)/((a^2 + b^2)*d^2))*log(sqrt(b*tan(d*x + c) + a)*B^3*b + (B^2*b^2*d
+ (a^3 + a*b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^3)*sqrt(-((
a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^2 + B^2*a)/((a^2
 + b^2)*d^2))) - 1/2*sqrt(-((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 +
b^4)*d^4))*d^2 + B^2*a)/((a^2 + b^2)*d^2))*log(sqrt(b*tan(d*x + c) + a)*B^
3*b - (B^2*b^2*d + (a^3 + a*b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^
4))*d^3)*sqrt(-((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d
^2 + B^2*a)/((a^2 + b^2)*d^2))) + 1/2*sqrt(((a^2 + b^2)*sqrt(-B^4*b^2/((a^
4 + 2*a^2*b^2 + b^4)*d^4))*d^2 - B^2*a)/((a^2 + b^2)*d^2))*log(sqrt(b*tan(
d*x + c) + a)*B^3*b + (B^2*b^2*d - (a^3 + a*b^2)*sqrt(-B^4*b^2/((a^4 + 2*a
^2*b^2 + b^4)*d^4))*d^3)*sqrt(((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2
 + b^4)*d^4))*d^2 - B^2*a)/((a^2 + b^2)*d^2))) - 1/2*sqrt(((a^2 + b^2)*sqr
t(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^2 - B^2*a)/((a^2 + b^2)*d^2))*
log(sqrt(b*tan(d*x + c) + a)*B^3*b - (B^2*b^2*d - (a^3 + a*b^2)*sqrt(-B^4*
b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^3)*sqrt(((a^2 + b^2)*sqrt(-B^4*b^2/((
a^4 + 2*a^2*b^2 + b^4)*d^4))*d^2 - B^2*a)/((a^2 + b^2)*d^2)))
```

Sympy [F]

$$\int \frac{aB + bB\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{a+b\tan{(c+dx)}}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
B*Integral(1/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{aB + bB\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 8.01 (sec) , antiderivative size = 6453, normalized size of antiderivative = 21.30

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((B*a + B*b*tan(c + d*x))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
log(((a + b*tan(c + d*x))^(1/2)*(16*B^2*a^2*b^10*d^3 + 32*B^2*a^4*b^8*d^3
- 32*B^2*a^8*b^4*d^3 - 16*B^2*a^10*b^2*d^3) - ((((8*B^2*a^5*d^2 - 24*B^2*a
^3*b^2*d^2)^2/4 - B^4*a^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a
^4*b^2*d^4))^(1/2) - 4*B^2*a^5*d^2 + 12*B^2*a^3*b^2*d^2)/(16*(a^6*d^4 + b^
6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*((a + b*tan(c + d*x))^(1/2)
*((((8*B^2*a^5*d^2 - 24*B^2*a^3*b^2*d^2)^2/4 - B^4*a^4*(16*a^6*d^4 + 16*b^
6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 4*B^2*a^5*d^2 + 12*B^2*a
^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2
)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^5*b^8*d^5 + 640*a^7*b^6*d^5 +
320*a^9*b^4*d^5 + 64*a^11*b^2*d^5) + 64*B*a^2*b^11*d^4 + 256*B*a^4*b^9*d^4
 + 384*B*a^6*b^7*d^4 + 256*B*a^8*b^5*d^4 + 64*B*a^10*b^3*d^4))*((((8*B^2*a
^5*d^2 - 24*B^2*a^3*b^2*d^2)^2/4 - B^4*a^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a
^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 4*B^2*a^5*d^2 + 12*B^2*a^3*b^2*d^2)/
(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + 8*B^3*a^
3*b^9*d^2 + 24*B^3*a^5*b^7*d^2 + 24*B^3*a^7*b^5*d^2 + 8*B^3*a^9*b^3*d^2)*(
(((8*B^2*a^5*d^2 - 24*B^2*a^3*b^2*d^2)^2/4 - B^4*a^4*(16*a^6*d^4 + 16*b^6*
d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 4*B^2*a^5*d^2 + 12*B^2*a^3
*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)
- log(8*B^3*a^3*b^9*d^2 - (-((96*B^4*a^6*b^4*d^4 - 16*B^4*a^4*b^6*d^4 - 14
4*B^4*a^8*b^2*d^4)^(1/2) + 4*B^2*a^5*d^2 - 12*B^2*a^3*b^2*d^2)/(16*a^6*...
```

Reduce [F]

$$\int \frac{aB + bB\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+\tan(dx+c)\,b} - \left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{a+\tan(dx+c)b}dx\right)bd}{d}$$

input
```
int((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(tan(c + d*x)*b + a) - int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**
2)/(tan(c + d*x)*b + a),x)*b*d)/d
```

3.367 $\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 119

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{2B\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{B\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d} + \frac{B\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

output
```
-2*B*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d+B*arctanh((a+b*tan(
d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(1/2)/d+B*arctanh((a+b*tan(d*x+c))^(1
/2)/(a+I*b)^(1/2))/(a+I*b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.94

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{B\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}} + \text{arcta}\right.}{d}$$

input
```
Integrate[(Cot[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/
2),x]
```

output
```
(B*((-2*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] + ArcTanh[Sqrt[
a + b*Tan[c + d*x]]/Sqrt[a - I*b]]/Sqrt[a - I*b] + ArcTanh[Sqrt[a + b*Tan[
c + d*x]]/Sqrt[a + I*b]]/Sqrt[a + I*b]))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2011, 3042, 4057, 25, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4057}$$

$$B\left(\int -\frac{\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx\right)$$

$$\Big\downarrow \text{25}$$

$$B\left(\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-\int \frac{\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx - \int \frac{\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx\right)$$

$\Big\downarrow$ 4022

$$B\left(\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx - \frac{1}{2}i\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx + \frac{1}{2}i\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx\right)$$

$\Big\downarrow$ 3042

$$B\left(-\frac{1}{2}i\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx + \frac{1}{2}i\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx + \int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx\right)$$

$\Big\downarrow$ 4020

$$B\left(\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx - \frac{\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d} - \frac{\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{}}}{}\right.$$

$\Big\downarrow$ 25

$$B\left(\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx + \frac{\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d} + \frac{\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+}}}{}\right.$$

$\Big\downarrow$ 73

$$B\left(-\frac{i\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd} + \frac{i\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd} + \int \frac{\tan(c+}{\tan(c+dx)\sqrt{a}}\right.$$

$\Big\downarrow$ 221

$$B\left(\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx + \frac{i\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} - \frac{i\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}\right)$$

$\Big\downarrow$ 4117

$$B\left(\frac{\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d} + \frac{i\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}} - \frac{i\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}\right)$$

$\Big\downarrow$ 73

$$B\left(\frac{2\int \frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{i\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{i\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}\right)$$

$$\Big\downarrow \text{221}$$

$$B\left(\frac{i\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{i\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}\right)$$

input
```
Int[(Cot[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
B*((I*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) - (I*ArcTan[Ta
n[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d) - (2*ArcTanh[Sqrt[a + b*Tan[c
 + d*x]]/Sqrt[a]])/(Sqrt[a]*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4057
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[(a + b*Tan[e + f*x])^m
*(c - d*Tan[e + f*x]), x], x] + Simp[d^2/(c^2 + d^2)   Int[(a + b*Tan[e + f
*x])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(97) = 194$.

Time = 0.19 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.14

method	result
default	$2Bb^2\left(\frac{\frac{\frac{\sqrt{2\sqrt{a^2+b^2}+2a}\,\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)}{2}+\frac{2\left(a-\sqrt{a^2+b^2}\right)\arctan\left(\frac{2\sqrt{a+b\tan(dx+c)}+\sqrt{2\sqrt{a}}}{\sqrt{2\sqrt{a^2+b^2}-2a}}\right.}{\sqrt{2\sqrt{a^2+b^2}-2a}}}{4\sqrt{a^2+b^2}}}{}\right.$

input
```
int(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
2*B/d*b^2*(1/b^2*(1/4/(a^2+b^2)^(1/2)*(1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*l
n(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2
+b^2)^(1/2))+2*(a-(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2
*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-
2*a)^(1/2)))+1/4/(a^2+b^2)^(1/2)*(-1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a
+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2
)^(1/2))+2*(-a+(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(
a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a
)^(1/2))))-1/b^2/a^(1/2)*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(93) = 186$.

Time = 0.12 (sec) , antiderivative size = 1671, normalized size of antiderivative = 14.04

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algori
thm="fricas")
```

output
```
[-1/2*(a*d*sqrt(((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*
d^2 + B^2*a)/((a^2 + b^2)*d^2))*log(sqrt(b*tan(d*x + c) + a)*B^3 + ((a^2 +
 b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^3 - B^2*a*d)*sqrt(((a
^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^2 + B^2*a)/((a^2
+ b^2)*d^2))) - a*d*sqrt(((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^
4)*d^4))*d^2 + B^2*a)/((a^2 + b^2)*d^2))*log(sqrt(b*tan(d*x + c) + a)*B^3
- ((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^3 - B^2*a*d)
*sqrt(((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^2 + B^2*
a)/((a^2 + b^2)*d^2))) - a*d*sqrt(-((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^
2*b^2 + b^4)*d^4))*d^2 - B^2*a)/((a^2 + b^2)*d^2))*log(sqrt(b*tan(d*x + c)
 + a)*B^3 + ((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^3
+ B^2*a*d)*sqrt(-((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))
*d^2 - B^2*a)/((a^2 + b^2)*d^2))) + a*d*sqrt(-((a^2 + b^2)*sqrt(-B^4*b^2/(
(a^4 + 2*a^2*b^2 + b^4)*d^4))*d^2 - B^2*a)/((a^2 + b^2)*d^2))*log(sqrt(b*t
an(d*x + c) + a)*B^3 - ((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)
*d^4))*d^3 + B^2*a*d)*sqrt(-((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 +
 b^4)*d^4))*d^2 - B^2*a)/((a^2 + b^2)*d^2))) - 2*B*sqrt(a)*log((b*tan(d*x
+ c) - 2*sqrt(b*tan(d*x + c) + a)*sqrt(a) + 2*a)/tan(d*x + c)))/(a*d), -1/
2*(a*d*sqrt(((a^2 + b^2)*sqrt(-B^4*b^2/((a^4 + 2*a^2*b^2 + b^4)*d^4))*d^2
+ B^2*a)/((a^2 + b^2)*d^2))*log(sqrt(b*tan(d*x + c) + a)*B^3 + ((a^2 + ...
```

Sympy [F]

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = B\int \frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
B*Integral(cot(c + d*x)/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algori
thm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch fo
r the root of a polynomial with parameters. This might be wrong.Non regula
r value [
```

Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 2142, normalized size of antiderivative = 18.00

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
- atan(((((((32*(16*B*b^10*d^2 + 12*B*a^2*b^8*d^2))/d^3 - (32*(16*b^10*d^4
 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(B^2/(4*(a*d^2 - b*d^2*1i)))
^(1/2))/d^4)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (576*B^2*a*b^8*(a + b*ta
n(c + d*x))^(1/2))/d^2)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(1/2) - (96*B^3*a*b^8
)/d^3)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(1/2) - (96*B^4*b^8*(a + b*tan(c + d*x
))^(1/2))/d^4)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(1/2)*1i - (((((32*(16*B*b^10*
d^2 + 12*B*a^2*b^8*d^2))/d^3 + (32*(16*b^10*d^4 + 24*a^2*b^8*d^4)*(a + b*t
an(c + d*x))^(1/2)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(1/2))/d^4)*(B^2/(4*(a*d^2
 - b*d^2*1i)))^(1/2) - (576*B^2*a*b^8*(a + b*tan(c + d*x))^(1/2))/d^2)*(B^
2/(4*(a*d^2 - b*d^2*1i)))^(1/2) - (96*B^3*a*b^8)/d^3)*(B^2/(4*(a*d^2 - b*d
^2*1i)))^(1/2) + (96*B^4*b^8*(a + b*tan(c + d*x))^(1/2))/d^4)*(B^2/(4*(a*d
^2 - b*d^2*1i)))^(1/2)*1i)/((((((32*(16*B*b^10*d^2 + 12*B*a^2*b^8*d^2))/d^
3 - (32*(16*b^10*d^4 + 24*a^2*b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(B^2/(4*
(a*d^2 - b*d^2*1i)))^(1/2))/d^4)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (576
*B^2*a*b^8*(a + b*tan(c + d*x))^(1/2))/d^2)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(
1/2) - (96*B^3*a*b^8)/d^3)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(1/2) - (96*B^4*b^
8*(a + b*tan(c + d*x))^(1/2))/d^4)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(1/2) + ((
(((32*(16*B*b^10*d^2 + 12*B*a^2*b^8*d^2))/d^3 + (32*(16*b^10*d^4 + 24*a^2*
b^8*d^4)*(a + b*tan(c + d*x))^(1/2)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(1/2))/d^
4)*(B^2/(4*(a*d^2 - b*d^2*1i)))^(1/2) - (576*B^2*a*b^8*(a + b*tan(c + d...
```

Reduce [F]

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)}{a+\tan(dx+c)\,b}dx\right)b$$

input
```
int(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*cot(c + d*x))/(tan(c + d*x)*b + a),x)*b
```

3.368 $\int \frac{aB+bB\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{aB+bB\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx=-\frac{iB\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{iB\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}-\frac{2bB}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
-I*B*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+I*B*arc
tanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d-2*b*B/(a^2+b^2)
/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

$$\int \frac{aB+bB\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx=\frac{B\left(i(a+ib)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan(c+dx)}{a-ib}\right)+(-ia-b)\,\text{Hyper}\right.}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
(B*(I*(a + I*b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/(a -
I*b)] + ((-I)*a - b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan[c + d*x])/
(a + I*b)]))/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2011, 3042, 3964, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3964}$$

$$B\left(\frac{\int \frac{a-b\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}-\frac{2b}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\int \frac{a-b\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}-\frac{2b}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{4022}$$

$$B\left(-\frac{2b}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(a-ib)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}\right)$$

$\Big\downarrow$ 3042

$$B\left(-\frac{2b}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(a-ib)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}\right)$$

$\Big\downarrow$ 4020

$$B\left(-\frac{2b}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{i(a+ib)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a-ib)\int-\frac{1}{(i\tan(c+dx)+1)\sqrt{a+\ldots}}}{2d}}{a^2+b^2}\right.$$

$\Big\downarrow$ 25

$$B\left(-\frac{2b}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{i(a-ib)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{i(a+ib)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\,\mathrm{t}\ldots}}}{2d}}{a^2+b^2}\right.$$

$\Big\downarrow$ 73

$$B\left(-\frac{2b}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(a-ib)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a+ib)\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan\ldots}}{bd}}{a^2+b^2}\right.$$

$\Big\downarrow$ 221

$$B\left(-\frac{2b}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{(a+ib)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}+\frac{(a-ib)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}}{a^2+b^2}\right)$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
B*((((a + I*b)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + ((a
 - I*b)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d))/(a^2 + b^2)
 - (2*b)/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1954 vs. $2(103) = 206$.

Time = 0.09 (sec) , antiderivative size = 1955, normalized size of antiderivative = 15.89

method	result	size
derivativedivides	Expression too large to display	1955
default	Expression too large to display	1955
parts	Expression too large to display	4474

input
```
int((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*B/b/(a^2+b^2)^2*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/
d*B*b/(a^2+b^2)^2*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/4/d*B/b/
(a^2+b^2)^(5/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^4+1/4/d*B*b^
3/(a^2+b^2)^(5/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+4/d*B*b/(a^2
+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)
+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+3/d*B*b
^3/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c)
)^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-1/
d*B/b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x
+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a
^3-1/d*B*b/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*ta
n(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/
2))*a-1/d*B*b/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan
(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2
))*a^2+1/d*B/b/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+
b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2174 vs. $2(97) = 194$.

Time = 0.12 (sec) , antiderivative size = 2174, normalized size of antiderivative = 17.67

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas
")
```

output
```
-1/2*(4*sqrt(b*tan(d*x + c) + a)*B*b + ((a^2*b + b^3)*d*tan(d*x + c) + (a^
3 + a*b^2)*d)*sqrt(-(B^2*a^3 - 3*B^2*a*b^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*d^2*sqrt(-(9*B^4*a^4*b^2 - 6*B^4*a^2*b^4 + B^4*b^6)/((a^12 + 6*a^10
*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))*log(-(3*B^3*a^2*b - B^3*b^3)*sqrt
(b*tan(d*x + c) + a) + ((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^3*sqrt(-(9*B
^4*a^4*b^2 - 6*B^4*a^2*b^4 + B^4*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 2
0*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + 2*(3*B^2*a^3*b^2 - B^2
*a*b^4)*d)*sqrt(-(B^2*a^3 - 3*B^2*a*b^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*d^2*sqrt(-(9*B^4*a^4*b^2 - 6*B^4*a^2*b^4 + B^4*b^6)/((a^12 + 6*a^10*b^
2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))) - ((a^2*b + b^3)*d*tan(d*x + c) + (
a^3 + a*b^2)*d)*sqrt(-(B^2*a^3 - 3*B^2*a*b^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*d^2*sqrt(-(9*B^4*a^4*b^2 - 6*B^4*a^2*b^4 + B^4*b^6)/((a^12 + 6*a^
10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/
((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))*log(-(3*B^3*a^2*b - B^3*b^3)*sq
rt(b*tan(d*x + c) + a) - ((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d^3*sqrt(-(9
*B^4*a^4*b^2 - 6*B^4*a^2*b^4 + B^4*b^6)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 +
 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + 2*(3*B^2*a^3*b^2 - B
^2*a*b^4)*d)*sqrt(-(B^2*a^3 - 3*B^2*a*b^2 + (a^6 + 3*a^4*b^2 + 3*a^2*b^...
```

Sympy [F]

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\, dx = B\int \frac{1}{a\sqrt{a + b\tan{(c + dx)}} + b\sqrt{a + b\tan{(c + dx)}}\tan{(c + dx)}}\, dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
B*Integral(1/(a*sqrt(a + b*tan(c + d*x)) + b*sqrt(a + b*tan(c + d*x))*tan(
c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[3,9,3]%%%}+%%%{4,[3,7,3]%%%}+%%%{6,[3,5,3]%%%}+%%%{
4,[3,3,3]
```

Mupad [B] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 9618, normalized size of antiderivative = 78.20

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((B*a + B*b*tan(c + d*x))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(log(16*B^3*a^4*b^15*d^2 - ((((320*B^4*a^6*b^8*d^4 - 16*B^4*a^4*b^10*d^4 -
 1760*B^4*a^8*b^6*d^4 + 1600*B^4*a^10*b^4*d^4 - 400*B^4*a^12*b^2*d^4)^(1/2
) - 4*B^2*a^7*d^2 - 20*B^2*a^3*b^4*d^2 + 40*B^2*a^5*b^2*d^2)/(a^10*d^4 + b
^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)
)^(1/2)*(((((320*B^4*a^6*b^8*d^4 - 16*B^4*a^4*b^10*d^4 - 1760*B^4*a^8*b^6*
d^4 + 1600*B^4*a^10*b^4*d^4 - 400*B^4*a^12*b^2*d^4)^(1/2) - 4*B^2*a^7*d^2
- 20*B^2*a^3*b^4*d^2 + 40*B^2*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^
8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(((((320*B
^4*a^6*b^8*d^4 - 16*B^4*a^4*b^10*d^4 - 1760*B^4*a^8*b^6*d^4 + 1600*B^4*a^1
0*b^4*d^4 - 400*B^4*a^12*b^2*d^4)^(1/2) - 4*B^2*a^7*d^2 - 20*B^2*a^3*b^4*d
^2 + 40*B^2*a^5*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6
*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(a + b*tan(c + d*x))^(1/2)*(
64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b^16*d^5 +
 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 + 7680*a^1
5*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^5))/4 - 3
2*B*a*b^21*d^4 - 160*B*a^3*b^19*d^4 - 128*B*a^5*b^17*d^4 + 896*B*a^7*b^15*
d^4 + 3136*B*a^9*b^13*d^4 + 4928*B*a^11*b^11*d^4 + 4480*B*a^13*b^9*d^4 + 2
432*B*a^15*b^7*d^4 + 736*B*a^17*b^5*d^4 + 96*B*a^19*b^3*d^4))/4 - (a + b*t
an(c + d*x))^(1/2)*(320*B^2*a^6*b^14*d^3 - 16*B^2*a^2*b^18*d^3 + 1024*B^2*
a^8*b^12*d^3 + 1440*B^2*a^10*b^10*d^3 + 1024*B^2*a^12*b^8*d^3 + 320*B^2...
```

Reduce [F]

$$\int \frac{aB + bB\tan(c + dx)}{(a + b\tan(c + dx))^{5/2}}\,dx = \frac{-2\sqrt{a+\tan(dx+c)\,b} - \left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{\tan(dx+c)^2 b^2 + 2\tan(dx+c)ab + a^2}dx\right)\tan(dx+c)\,b^2 d}{d\,(a+\tan(dx+c)\,b)}$$

input
```
int((B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
( - 2*sqrt(tan(c + d*x)*b + a) - int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x
)**2)/(tan(c + d*x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2),x)*tan(c + d*x)*b
**2*d - int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)**2*b*
*2 + 2*tan(c + d*x)*a*b + a**2),x)*a*b*d)/(d*(tan(c + d*x)*b + a))
```

3.369 $\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 154

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{2B\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d}+\frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d}+\frac{2b^2B}{a\left(a^2+b^2\right)d\sqrt{a+b\tan(c+dx)}}$$

output
```
-2*B*arctanh((a+b*tan(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+B*arctanh((a+b*tan(
d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d+B*arctanh((a+b*tan(d*x+c))^(1
/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+2*b^2*B/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{B\left(-\frac{2(a^2+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{a(a+ib)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}\right.}{a\left(a^2+b^2\right)}$$

input
```
Integrate[(Cot[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/
2),x]
```

output
```
(B*((-2*(a^2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/Sqrt[a] + (
a*(a + I*b)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/Sqrt[a - I*b]
 + (a*(a - I*b)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/Sqrt[a +
I*b] + (2*b^2)/Sqrt[a + b*Tan[c + d*x]]))/(a*(a^2 + b^2)*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2011, 3042, 4052, 27, 3042, 4136, 25, 3042, 4022, 3042, 4020, 25, 73, 221, 4117, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\cot(c+dx)}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\tan(c+dx)(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4052}$$

$$B\left(\frac{2\int \frac{\cot(c+dx)\left(a^2-b\tan(c+dx)a+b^2+b^2\tan^2(c+dx)\right)}{2\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$\downarrow$ 27

$$B\left(\frac{\int \frac{\cot(c+dx)\left(a^2-b\tan(c+dx)a+b^2+b^2\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$\downarrow$ 3042

$$B\left(\frac{\int \frac{a^2-b\tan(c+dx)a+b^2+b^2\tan(c+dx)^2}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$\downarrow$ 4136

$$B\left(\frac{\left(a^2+b^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx+\int -\frac{\tan(c+dx)a^2+ba}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$\downarrow$ 25

$$B\left(\frac{\left(a^2+b^2\right)\int \frac{\cot(c+dx)\left(\tan^2(c+dx)+1\right)}{\sqrt{a+b\tan(c+dx)}}dx-\int \frac{\tan(c+dx)a^2+ba}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$\downarrow$ 3042

$$B\left(\frac{\left(a^2+b^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\int \frac{\tan(c+dx)a^2+ba}{\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$\downarrow$ 4022

$$B\left(\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\int \frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a(b+ia)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}}{a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 3042

$$B\left(\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a(b+ia)\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}}{a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 4020

$$B\left(\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{ia(-b+ia)\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}}{2d}}{a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 25

$$B\left(\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{ia(-b+ia)\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}}{2d}}{a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 73

$$B\left(\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{a(b+ia)\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan}}{bd}}{a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 221

$$B\left(\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\left(a^2+b^2\right)\int\frac{\tan(c+dx)^2+1}{\tan(c+dx)\sqrt{a+b\tan(c+dx)}}dx+\frac{a(-b+ia)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a(b+ia)}{}}{a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 4117

$$B\left(\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{\left(a^2+b^2\right)\int\frac{\cot(c+dx)}{\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{a(-b+ia)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a(b+ia)\arctan\left(\right.}{d\sqrt{a+}}}{a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 73

$$B\left(\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{2\left(a^2+b^2\right)\int\frac{1}{\frac{a+b\tan(c+dx)}{b}-\frac{a}{b}}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{a(-b+ia)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a(b+ia}{}}{a\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{ 221}$$

$$B\left(\frac{2b^2}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2\left(a^2+b^2\right)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+\frac{a(-b+ia)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{a(b+ia)\arcta}{d\sqrt{a}}}{a\left(a^2+b^2\right)}\right.$$

input
```
Int[(Cot[c + d*x]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
B*(((a*(I*a - b)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) - (
a*(I*a + b)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d) - (2*(a^
2 + b^2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d))/(a*(a^2 +
 b^2)) + (2*b^2)/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1778 vs. $2(130) = 260$.

Time = 0.18 (sec) , antiderivative size = 1779, normalized size of antiderivative = 11.55

method	result	size
default	Expression too large to display	1779

input
```
int(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
B*(-1/4/d/(a^2+b^2)^2*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/
d*b^2/(a^2+b^2)^2*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1
/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/2/d/(a^2+b
^2)^(5/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/2/d*b^2/(a^2+b
^2)^(5/2)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-2/d*b^4/(a^2+b^2)^
(5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a
^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/d/(a^2+b^2)^(3/
2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+
b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/d/(a^2+b^2)^2/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2
)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+1/d*b^2/(a^2+b^2)^(
3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^
2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/d*b^2/(a^2+b^2)^
2/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b
^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a-2/d*b^2/(a^2+b^2)^(
5/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^
2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2+1/4/d/(a^2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2278 vs. $2(126) = 252$.

Time = 0.19 (sec) , antiderivative size = 4575, normalized size of antiderivative = 29.71

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algori
thm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = B\int \frac{\cot(c+dx)}{a\sqrt{a+b\tan(c+dx)}+b\sqrt{a+b\tan(c+dx)}\tan(c+dx}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
B*Integral(cot(c + d*x)/(a*sqrt(a + b*tan(c + d*x)) + b*sqrt(a + b*tan(c +
 d*x))*tan(c + d*x)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algori
thm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[4,13,4]%%%}+%%%{6,[4,11,4]%%%}+%%%{15,[4,9,4]%%%}+%
%%{20,[4,
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 7172, normalized size of antiderivative = 46.57

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((cot(c + d*x)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(log(((((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2)
+ 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3
*a^4*b^2*d^4))^(1/2)*(((((((96*B^4*a^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*
a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 - 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4
+ 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2)*(512*B*a^8*b^28*d^8 - ((((96*B^4*a
^2*b^4*d^4 - 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 -
 12*B^2*a*b^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1
/2)*(a + b*tan(c + d*x))^(1/2)*(512*a^9*b^28*d^9 + 5376*a^11*b^26*d^9 + 25
344*a^13*b^24*d^9 + 70656*a^15*b^22*d^9 + 129024*a^17*b^20*d^9 + 161280*a^
19*b^18*d^9 + 139776*a^21*b^16*d^9 + 82944*a^23*b^14*d^9 + 32256*a^25*b^12
*d^9 + 7424*a^27*b^10*d^9 + 768*a^29*b^8*d^9))/4 + 5248*B*a^10*b^26*d^8 +
23936*B*a^12*b^24*d^8 + 64000*B*a^14*b^22*d^8 + 111104*B*a^16*b^20*d^8 + 1
30816*B*a^18*b^18*d^8 + 105728*B*a^20*b^16*d^8 + 57856*B*a^22*b^14*d^8 + 2
0480*B*a^24*b^12*d^8 + 4224*B*a^26*b^10*d^8 + 384*B*a^28*b^8*d^8))/4 + (a
+ b*tan(c + d*x))^(1/2)*(256*B^2*a^8*b^26*d^7 + 1472*B^2*a^10*b^24*d^7 + 3
712*B^2*a^12*b^22*d^7 + 6272*B^2*a^14*b^20*d^7 + 9856*B^2*a^16*b^18*d^7 +
14336*B^2*a^18*b^16*d^7 + 15232*B^2*a^20*b^14*d^7 + 10112*B^2*a^22*b^12*d^
7 + 3712*B^2*a^24*b^10*d^7 + 576*B^2*a^26*b^8*d^7))*(((96*B^4*a^2*b^4*d^4
- 16*B^4*b^6*d^4 - 144*B^4*a^4*b^2*d^4)^(1/2) + 4*B^2*a^3*d^2 - 12*B^2*a*b
^2*d^2)/(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4))^(1/2))/4 -...
```

Reduce [F]

$$\int \frac{\cot(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \left(\int \frac{\sqrt{a+\tan(dx+c)\,b}\,\cot(dx+c)}{\tan(dx+c)^2 b^2+2\tan(dx+c)\,ab+a^2}dx\right)b$$

input
```
int(cot(d*x+c)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*cot(c + d*x))/(tan(c + d*x)**2*b**2 + 2*tan(
c + d*x)*a*b + a**2),x)*b
```

3.370 $\int \frac{-a+b\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 102

$$\int \frac{-a+b\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx=\frac{(ia-b)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}d}-\frac{(ia+b)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

output
```
(I*a-b)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(1/2)/d-(I*a
+b)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int \frac{-a+b\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx=\frac{i\left((a+ib)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)-(a-ib)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)\right)}{\sqrt{a-ib}\sqrt{a+ib}d}$$

input
```
Integrate[(-a + b*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
(I*((a + I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]] - (a -
 I*b)^(3/2)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]]))/(Sqrt[a - I*
b]*Sqrt[a + I*b]*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b\tan(c+dx)-a}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b\tan(c+dx)-a}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4022}$$

$$-\frac{1}{2}(a-ib)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(a+ib)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}(a-ib)\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(a+ib)\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(a-ib)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{i(a+ib)\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{i(a+ib)\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a-ib)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}$$

$$\Big\downarrow \ 73$$

$$-\frac{(a-ib)\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}-\frac{(a+ib)\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \ 221$$

$$-\frac{(a+ib)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}-\frac{(a-ib)\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

input
```
Int[(-a + b*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
-(((a + I*b)*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)) - ((a
- I*b)*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1894 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 1895, normalized size of antiderivative = 18.58

method	result	size
parts	Expression too large to display	1895
derivativedivides	Expression too large to display	1905
default	Expression too large to display	1905

input
```
int((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b/d*(1/2/(a^2+b^2)^(1/2)*(1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln((a+b*tan(d*
x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))+
2*(a-(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(
1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+
1/2/(a^2+b^2)^(1/2)*(-1/2*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*ln(b*tan(d*x+c)+a+
(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))+2*(-
a+(a^2+b^2)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c)
)^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))))-a*
(-1/4/d/b/(a^2+b^2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^
(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-1/4/d*
b/(a^2+b^2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d/b/(a^2+b^2)^
(3/2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/
2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+1/4/d*b/(a^2+b^2)^(3
/2)*ln(b*tan(d*x+c)+a-(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-1/d/b/(a^2+b^2)^(1/2)/(2
*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^2)^
(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/d*b/(a^2+b^2)^(1/2)
/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)-(2*(a^2+b^
2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+1/d/b/(a^2+b^2)^(3/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(78) = 156$.

Time = 0.10 (sec) , antiderivative size = 1115, normalized size of antiderivative = 10.93

$$\int \frac{-a + b\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a
^2*b^2 + b^4)*d^4)) + a^3 - 3*a*b^2)/((a^2 + b^2)*d^2))*log(-(3*a^4*b + 2*
a^2*b^3 - b^5)*sqrt(b*tan(d*x + c) + a) + ((a^4 - b^4)*d^3*sqrt(-(9*a^4*b^
2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*(3*a^3*b^2 - a*b^4
)*d)*sqrt(-((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*
a^2*b^2 + b^4)*d^4)) + a^3 - 3*a*b^2)/((a^2 + b^2)*d^2))) - 1/2*sqrt(-((a^
2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*
d^4)) + a^3 - 3*a*b^2)/((a^2 + b^2)*d^2))*log(-(3*a^4*b + 2*a^2*b^3 - b^5)
*sqrt(b*tan(d*x + c) + a) - ((a^4 - b^4)*d^3*sqrt(-(9*a^4*b^2 - 6*a^2*b^4
+ b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) + 2*(3*a^3*b^2 - a*b^4)*d)*sqrt(-((a
^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)
*d^4)) + a^3 - 3*a*b^2)/((a^2 + b^2)*d^2))) - 1/2*sqrt(((a^2 + b^2)*d^2*sq
rt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - a^3 + 3
*a*b^2)/((a^2 + b^2)*d^2))*log(-(3*a^4*b + 2*a^2*b^3 - b^5)*sqrt(b*tan(d*x
 + c) + a) + ((a^4 - b^4)*d^3*sqrt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 +
2*a^2*b^2 + b^4)*d^4)) - 2*(3*a^3*b^2 - a*b^4)*d)*sqrt(((a^2 + b^2)*d^2*sq
rt(-(9*a^4*b^2 - 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - a^3 + 3
*a*b^2)/((a^2 + b^2)*d^2))) + 1/2*sqrt(((a^2 + b^2)*d^2*sqrt(-(9*a^4*b^2 -
 6*a^2*b^4 + b^6)/((a^4 + 2*a^2*b^2 + b^4)*d^4)) - a^3 + 3*a*b^2)/((a^2 +
b^2)*d^2))*log(-(3*a^4*b + 2*a^2*b^3 - b^5)*sqrt(b*tan(d*x + c) + a) - ...
```

Sympy [F]

$$\int \frac{-a+b\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx=-\int \frac{a}{\sqrt{a+b\tan{(c+dx)}}}\,dx-\int \left(-\frac{b\tan{(c+dx)}}{\sqrt{a+b\tan{(c+dx)}}}\right)dx$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
-Integral(a/sqrt(a + b*tan(c + d*x)), x) - Integral(-b*tan(c + d*x)/sqrt(a
 + b*tan(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{-a + b\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-1)]

Timed out.

$$\int \frac{-a + b\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Timed out}$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 2731, normalized size of antiderivative = 26.77

$$\int \frac{-a + b\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Too large to display}$$

input
```
int(-(a - b*tan(c + d*x))/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
2*atanh((32*a^2*b^2*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) - (a
^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*a^
4*b^3*d^3)/(a^2*d^4 + b^2*d^4) - (4*a*b^3*d^2*(-16*a^4*b^2*d^4)^(1/2))/(a^
2*d^5 + b^2*d^5)) + (8*a*b^2*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d
^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2)
*(-16*a^4*b^2*d^4)^(1/2))/((16*a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*
b^3*d^5)/(a^2*d^4 + b^2*d^4) - (4*a^3*b^3*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^
2*d^5 + b^2*d^5) - (4*a*b^5*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^
5)) - (32*a^4*b^2*d^2*((-16*a^4*b^2*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) -
(a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*
a^4*b^5*d^5)/(a^2*d^4 + b^2*d^4) + (16*a^6*b^3*d^5)/(a^2*d^4 + b^2*d^4) -
(4*a^3*b^3*d^4*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5) - (4*a*b^5*d^4
*(-16*a^4*b^2*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)))*((-16*a^4*b^2*d^4)^(1/2)/(
16*(a^2*d^4 + b^2*d^4)) - (a^3*d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2) - 2*ata
nh((32*a^2*b^4*d^2*((-16*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*
d^2)/(4*(a^2*d^4 + b^2*d^4)))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((16*a^2*b
^7*d^5)/(a^2*d^4 + b^2*d^4) - 16*a^2*b^5*d - 16*b^7*d + (16*a^4*b^5*d^5)/(
a^2*d^4 + b^2*d^4) + (4*a*b^5*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)
 + (4*a^3*b^3*d^4*(-16*b^6*d^4)^(1/2))/(a^2*d^5 + b^2*d^5)) - (32*b^4*((-1
6*b^6*d^4)^(1/2)/(16*(a^2*d^4 + b^2*d^4)) + (a*b^2*d^2)/(4*(a^2*d^4 + b...
```

Reduce [F]

$$\int \frac{-a+b\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{-2\sqrt{a+\tan(dx+c)\,b}\,a+\left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{a+\tan(dx+c)b}dx\right)abd+\left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)}{a+\tan(dx+c)b}dx\right)b^2d}{bd}$$

input
```
int((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
( - 2*sqrt(tan(c + d*x)*b + a)*a + int((sqrt(tan(c + d*x)*b + a)*tan(c + d
*x)**2)/(tan(c + d*x)*b + a),x)*a*b*d + int((sqrt(tan(c + d*x)*b + a)*tan(
c + d*x))/(tan(c + d*x)*b + a),x)*b**2*d)/(b*d)
```

3.371 $\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{(ia-b)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{3/2}d} - \frac{(ia+b)\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{3/2}d} + \frac{4ab}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
(I*a-b)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(3/2)/d-(I*a
+b)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(3/2)/d+4*a*b/(a
^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.17

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{i\cos(c+dx)\left((a+ib)^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tan(c+dx)}{a-ib}\right)-(a-ib)^2\,\text{Hypergeometric2F1}\left(-\right.\right.}{(a-ib)(a+ib)d(a\cos(c+dx)-b\sin(c+dx))\sqrt{a+b\tan(c+d}}$$

input
```
Integrate[(-a + b*Tan[c + d*x])/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
((-I)*Cos[c + d*x]*((a + I*b)^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tan
[c + d*x])/(a - I*b)] - (a - I*b)^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b
*Tan[c + d*x])/(a + I*b)])*(a - b*Tan[c + d*x]))/((a - I*b)*(a + I*b)*d*(a
*Cos[c + d*x] - b*Sin[c + d*x])*Sqrt[a + b*Tan[c + d*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {3042, 4012, 25, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b\tan(c+dx)-a}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b\tan(c+dx)-a}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int -\frac{a^2-2b\tan(c+dx)a-b^2}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}+\frac{4ab}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4ab}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{a^2-2b\tan(c+dx)a-b^2}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4ab}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{a^2-2b\tan(c+dx)a-b^2}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{4022}$$

$$\frac{4ab}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{1}{2}(a-ib)^2\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^2\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}$$

$\big\downarrow$ 3042

$$\frac{4ab}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{1}{2}(a-ib)^2\int\frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^2\int\frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}$$

$\big\downarrow$ 4020

$$\frac{4ab}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{i(a+ib)^2\int-\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a-ib)^2\int-\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}}{a^2+b^2}$$

$\big\downarrow$ 25

$$\frac{4ab}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{i(a-ib)^2\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{i(a+ib)^2\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}}{a^2+b^2}$$

$\big\downarrow$ 73

$$\frac{4ab}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{(a-ib)^2\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}+\frac{(a+ib)^2\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}}{a^2+b^2}$$

$\big\downarrow$ 221

$$\frac{4ab}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\frac{(a-ib)^2\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}+\frac{(a+ib)^2\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}}{a^2+b^2}$$

input
```
Int[(-a + b*Tan[c + d*x])/(a + b*Tan[c + d*x])^(3/2),x]
```

output
```
-((((a + I*b)^2*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + ((
a - I*b)^2*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d))/(a^2 + b
^2)) + (4*a*b)/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2290 vs. $2(112) = 224$.

Time = 0.11 (sec) , antiderivative size = 2291, normalized size of antiderivative = 17.36

method	result	size
derivativedivides	Expression too large to display	2291
default	Expression too large to display	2291
parts	Expression too large to display	3680

input
```
int((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
1/d*b^3/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)
^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))
-1/d*b^3/(a^2+b^2)^(3/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(
d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)
)+4*a*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^(1/2)+4/d*b/(a^2+b^2)^(5/2)/(2*(a^2+b
^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+
c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^4-2/d*b/(a^2+b^2)^2/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c
))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3+2/d*b^5/(a^2+b^2)^(5/2)/(2*(a
^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/
2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))-1/4/d*b^3/(a^2+b^2)^2*ln((a+
b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)
^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+1/4/d*b^3/(a^2+b^2)^2*ln(b*tan(d*x+c
)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)+(a^2+b^2)^(1/2))*
(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2/d*b^5/(a^2+b^2)^(5/2)/(2*(a^2+b^2)^(1/2)-2
*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/
(2*(a^2+b^2)^(1/2)-2*a)^(1/2))+2/d*b^3/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)
^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*
(a^2+b^2)^(1/2)-2*a)^(1/2))*a-2/d*b^3/(a^2+b^2)^2/(2*(a^2+b^2)^(1/2)-2*a)^
(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2320 vs. $2(106) = 212$.

Time = 0.13 (sec) , antiderivative size = 2320, normalized size of antiderivative = 17.58

$$\int \frac{-a + b\tan(c + dx)}{(a + b\tan(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(8*sqrt(b*tan(d*x + c) + a)*a*b - ((a^2*b + b^3)*d*tan(d*x + c) + (a^3
 + a*b^2)*d)*sqrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4 + (a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*d^2*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
+ b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2
*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))*log((5*a^6
*b - 5*a^4*b^3 - 9*a^2*b^5 + b^7)*sqrt(b*tan(d*x + c) + a) + ((a^9 - 6*a^5
*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^3*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*
b^6 - 20*a^2*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 1
5*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)) + (15*a^6*b^2 - 35*a^4*b^4 + 13*a^2*b
^6 - b^8)*d)*sqrt(-(a^5 - 10*a^3*b^2 + 5*a*b^4 + (a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*d^2*sqrt(-(25*a^8*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8
+ b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 15*a^4*b^8 + 6*a^2
*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2))) + ((a^2*b
 + b^3)*d*tan(d*x + c) + (a^3 + a*b^2)*d)*sqrt(-(a^5 - 10*a^3*b^2 + 5*a*b^
4 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d^2*sqrt(-(25*a^8*b^2 - 100*a^6*b^
4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 + 6*a^10*b^2 + 15*a^8*b^4 + 20
*a^6*b^6 + 15*a^4*b^8 + 6*a^2*b^10 + b^12)*d^4)))/((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*d^2))*log((5*a^6*b - 5*a^4*b^3 - 9*a^2*b^5 + b^7)*sqrt(b*tan(
d*x + c) + a) - ((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d^3*sqrt(-(25*a^8
*b^2 - 100*a^6*b^4 + 110*a^4*b^6 - 20*a^2*b^8 + b^10)/((a^12 + 6*a^10*b...
```

Sympy [F]

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = -\int \frac{a}{a\sqrt{a+b\tan{(c+dx)}}+b\sqrt{a+b\tan{(c+dx)}}\tan{(c+dx)}}\,dx - \int \left(-\frac{b\tan{(c+dx)}}{a\sqrt{a+b\tan{(c+dx)}}+b\sqrt{a+b\tan{(c+dx)}}\tan{(c+dx)}}\right)dx$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
-Integral(a/(a*sqrt(a + b*tan(c + d*x)) + b*sqrt(a + b*tan(c + d*x))*tan(c
 + d*x)), x) - Integral(-b*tan(c + d*x)/(a*sqrt(a + b*tan(c + d*x)) + b*sq
rt(a + b*tan(c + d*x))*tan(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{-a + b\tan(c + dx)}{(a + b\tan(c + dx))^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-a + b\tan(c + dx)}{(a + b\tan(c + dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[3,9,3]%%%}+%%%{4,[3,7,3]%%%}+%%%{6,[3,5,3]%%%}+%%%{
4,[3,3,3]
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 5475, normalized size of antiderivative = 41.48

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(-(a - b*tan(c + d*x))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
log(- (((((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^
4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3*b^2*d^2
)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(32*b^13
*d^4 + 96*a^2*b^11*d^4 + 64*a^4*b^9*d^4 - 64*a^6*b^7*d^4 - 96*a^8*b^5*d^4
- 32*a^10*b^3*d^4 + (a + b*tan(c + d*x))^(1/2)*((((24*a*b^4*d^2 - 8*a^3*b^
2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^
4))^(1/2) - 12*a*b^4*d^2 + 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b
^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(64*a*b^12*d^5 + 320*a^3*b^10*d^5 + 640*a^
5*b^8*d^5 + 640*a^7*b^6*d^5 + 320*a^9*b^4*d^5 + 64*a^11*b^2*d^5)) + (a + b
*tan(c + d*x))^(1/2)*(16*b^12*d^3 + 32*a^2*b^10*d^3 - 32*a^6*b^6*d^3 - 16*
a^8*b^4*d^3))*((((24*a*b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16
*b^6*d^4 + 48*a^2*b^4*d^4 + 48*a^4*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3*
b^2*d^2)/(16*(a^6*d^4 + b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) -
 8*a*b^11*d^2 - 24*a^3*b^9*d^2 - 24*a^5*b^7*d^2 - 8*a^7*b^5*d^2)*((((24*a*
b^4*d^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d
^4 + 48*a^4*b^2*d^4))^(1/2) - 12*a*b^4*d^2 + 4*a^3*b^2*d^2)/(16*(a^6*d^4 +
 b^6*d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2) + log(- ((-(((24*a*b^4*d
^2 - 8*a^3*b^2*d^2)^2/4 - b^4*(16*a^6*d^4 + 16*b^6*d^4 + 48*a^2*b^4*d^4 +
48*a^4*b^2*d^4))^(1/2) + 12*a*b^4*d^2 - 4*a^3*b^2*d^2)/(16*(a^6*d^4 + b^6*
d^4 + 3*a^2*b^4*d^4 + 3*a^4*b^2*d^4)))^(1/2)*(32*b^13*d^4 + 96*a^2*b^11...
```

Reduce [F]

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+\tan(dx+c)\,b}\,a+\left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{\tan(dx+c)^2b^2+2\tan(dx+c)ab+a^2}dx\right)\tan(dx+c)\,a\,b^2$$

input
```
int((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output

```
(2*sqrt(tan(c + d*x)*b + a)*a + int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)
**2)/(tan(c + d*x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2),x)*tan(c + d*x)*a*
b**2*d + int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)**2*b
**2 + 2*tan(c + d*x)*a*b + a**2),x)*a**2*b*d + int((sqrt(tan(c + d*x)*b +
a)*tan(c + d*x))/(tan(c + d*x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2),x)*tan
(c + d*x)*b**3*d + int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*
x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2),x)*a*b**2*d)/(b*d*(tan(c + d*x)*b
+ a))
```

3.372 $\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 174

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(ia-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{(a-ib)^{5/2}d} - \frac{(ia+b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{(a+ib)^{5/2}d} + \frac{4ab}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2b(3a^2-b^2)}{(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
(I*a-b)*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(5/2)/d-(I*a
+b)*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(5/2)/d+4/3*a*b/
(a^2+b^2)/d/(a+b*tan(d*x+c))^(3/2)+2*b*(3*a^2-b^2)/(a^2+b^2)^2/d/(a+b*tan(
d*x+c))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{i\cos(c+dx)\left((a+ib)^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tan(c+dx)}{a-ib}\right)-(a-ib)^2\,\text{Hypergeometric2F1}\left(\right.\right.}{3(a-ib)(a+ib)d(a\cos(c+dx)-b\sin(c+dx))(a+b\tan(c+d}$$

input
```
Integrate[(-a + b*Tan[c + d*x])/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
((-1/3*I)*Cos[c + d*x]*((a + I*b)^2*Hypergeometric2F1[-3/2, 1, -1/2, (a +
b*Tan[c + d*x])/(a - I*b)] - (a - I*b)^2*Hypergeometric2F1[-3/2, 1, -1/2,
(a + b*Tan[c + d*x])/(a + I*b)])*(a - b*Tan[c + d*x]))/((a - I*b)*(a + I*b
)*d*(a*Cos[c + d*x] - b*Sin[c + d*x])*(a + b*Tan[c + d*x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4012, 25, 3042, 4012, 3042, 4022, 3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b\tan(c+dx)-a}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b\tan(c+dx)-a}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4012}$$

$$\frac{\int -\frac{a^2-2b\tan(c+dx)a-b^2}{(a+b\tan(c+dx))^{3/2}}dx}{a^2+b^2}+\frac{4ab}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{4ab}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int \frac{a^2-2b\tan(c+dx)a-b^2}{(a+b\tan(c+dx))^{3/2}}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{4ab}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int \frac{a^2-2b\tan(c+dx)a-b^2}{(a+b\tan(c+dx))^{3/2}}dx}{a^2+b^2}$$

$$\Big\downarrow \; 4012$$

$$\frac{4ab}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int \frac{a\left(a^2-3b^2\right)-b\left(3a^2-b^2\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}-\frac{2b\left(3a^2-b^2\right)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{4ab}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int \frac{a\left(a^2-3b^2\right)-b\left(3a^2-b^2\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}-\frac{2b\left(3a^2-b^2\right)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{a^2+b^2}$$

$$\Big\downarrow \; 4022$$

$$\frac{4ab}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2b\left(3a^2-b^2\right)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(a-ib)^3\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^3\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{4ab}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2b\left(3a^2-b^2\right)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}(a-ib)^3\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a+ib)^3\int \frac{i\tan(c+dx)+1}{\sqrt{a+b\tan(c+dx)}}dx}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 4020$$

$$\frac{4ab}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2b\left(3a^2-b^2\right)}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{i(a+ib)^3\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a-ib)^3\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+\cdots}{2d}}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{4ab}{3d(a^2+b^2)(a+b\tan(c+dx))^{3/2}} - \frac{-\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}} + \frac{\frac{i(a-ib)^3\int\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d} - \frac{i(a+ib)^3\int\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}}{a^2+b^2}}{a^2+b^2}}{}$$

$$\Big\downarrow \; 73$$

$$\frac{4ab}{3d(a^2+b^2)(a+b\tan(c+dx))^{3/2}} - \frac{-\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}} + \frac{\frac{(a-ib)^3\int\frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd} + \frac{(a+ib)^3\int\frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}}{a^2+b^2}}{a^2+b^2}$$

$$\Big\downarrow \; 221$$

$$\frac{4ab}{3d(a^2+b^2)(a+b\tan(c+dx))^{3/2}} - \frac{-\frac{2b(3a^2-b^2)}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}} + \frac{\frac{(a-ib)^3\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}} + \frac{(a+ib)^3\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}}{a^2+b^2}}{a^2+b^2}$$

input
```
Int[(-a + b*Tan[c + d*x])/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
(4*a*b)/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - ((((a + I*b)^3*ArcT
an[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d) + ((a - I*b)^3*ArcTan[Ta
n[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d))/(a^2 + b^2) - (2*b*(3*a^2 -
b^2))/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3054 vs. $2(150) = 300$.

Time = 0.11 (sec) , antiderivative size = 3055, normalized size of antiderivative = 17.56

method	result	size
derivativedivides	Expression too large to display	3055
default	Expression too large to display	3055
parts	Expression too large to display	4473

input
```
int((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-3/4/d*b^3/(a^2+b^2)^3*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(
1/2)-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-2/d*b
^3/(a^2+b^2)^3/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*
a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^2-1/4/
d/b/(a^2+b^2)^3*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2
)+2*a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^5+3/4/d*b^3/
(a^2+b^2)^3*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)+(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+1/d/b/(a^2+b^2)^
(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^7-5/d*b^3/(a^2+
b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a)^(
1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a^3-1/4/d/b/
(a^2+b^2)^(7/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*
tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^6+5/4/d*b^3/
(a^2+b^2)^(7/2)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*
tan(d*x+c)-a-(a^2+b^2)^(1/2))*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^2-7/d*b^5/(a
^2+b^2)^(7/2)/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan(((2*(a^2+b^2)^(1/2)+2*a
)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2))*a+6/d*b/(
a^2+b^2)^2/(a+b*tan(d*x+c))^(1/2)*a^2-1/4/d*b^5/(a^2+b^2)^(7/2)*ln((a+b*ta
n(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-b*tan(d*x+c)-a-(a^2+b^2)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3695 vs. $2(144) = 288$.

Time = 0.18 (sec) , antiderivative size = 3695, normalized size of antiderivative = 21.24

$$\int \frac{-a + b\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx =$$
$$-\int \frac{a}{a^2\sqrt{a+b\tan{(c+dx)}}+2ab\sqrt{a+b\tan{(c+dx)}}\tan{(c+dx)}+b^2\sqrt{a+b\tan{(c+dx)}}\tan^2{(c+dx)}}$$
$$-\int \left(-\frac{b\tan{(c+dx)}}{a^2\sqrt{a+b\tan{(c+dx)}}+2ab\sqrt{a+b\tan{(c+dx)}}\tan{(c+dx)}+b^2\sqrt{a+b\tan{(c+dx)}}\tan^2{(c+dx)}}\right.$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
-Integral(a/(a**2*sqrt(a + b*tan(c + d*x)) + 2*a*b*sqrt(a + b*tan(c + d*x)
)*tan(c + d*x) + b**2*sqrt(a + b*tan(c + d*x))*tan(c + d*x)**2), x) - Inte
gral(-b*tan(c + d*x)/(a**2*sqrt(a + b*tan(c + d*x)) + 2*a*b*sqrt(a + b*tan
(c + d*x))*tan(c + d*x) + b**2*sqrt(a + b*tan(c + d*x))*tan(c + d*x)**2),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [F(-2)]

Exception generated.

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[6,14,5]%%%}+%%%{6,[6,12,5]%%%}+%%%{15,[6,10,5]%%%}+
%%%{20,[6
```

Mupad [B] (verification not implemented)

Time = 18.99 (sec) , antiderivative size = 8437, normalized size of antiderivative = 48.49

$$\int \frac{-a+b\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(-(a - b*tan(c + d*x))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
(log(((((-(4*a^7*d^2 + (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d
^4 + 1600*a^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5
*b^2*d^2)/(a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b
^4*d^4 + 5*a^8*b^2*d^4))^(1/2)*(896*a^7*b^15*d^4 - 32*a*b^21*d^4 - 160*a^3
*b^19*d^4 - 128*a^5*b^17*d^4 - ((a + b*tan(c + d*x))^(1/2)*(-(4*a^7*d^2 +
(320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a^10*b^4*d^4
- 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(a^10*d^4 + b
^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*a^8*b^2*d^4)
)^(1/2)*(64*a*b^22*d^5 + 640*a^3*b^20*d^5 + 2880*a^5*b^18*d^5 + 7680*a^7*b
^16*d^5 + 13440*a^9*b^14*d^5 + 16128*a^11*b^12*d^5 + 13440*a^13*b^10*d^5 +
 7680*a^15*b^8*d^5 + 2880*a^17*b^6*d^5 + 640*a^19*b^4*d^5 + 64*a^21*b^2*d^
5))/4 + 3136*a^9*b^13*d^4 + 4928*a^11*b^11*d^4 + 4480*a^13*b^9*d^4 + 2432*
a^15*b^7*d^4 + 736*a^17*b^5*d^4 + 96*a^19*b^3*d^4))/4 + (a + b*tan(c + d*x
))^(1/2)*(320*a^6*b^14*d^3 - 16*a^2*b^18*d^3 + 1024*a^8*b^12*d^3 + 1440*a^
10*b^10*d^3 + 1024*a^12*b^8*d^3 + 320*a^14*b^6*d^3 - 16*a^18*b^2*d^3))*(-(
4*a^7*d^2 + (320*a^6*b^8*d^4 - 16*a^4*b^10*d^4 - 1760*a^8*b^6*d^4 + 1600*a
^10*b^4*d^4 - 400*a^12*b^2*d^4)^(1/2) + 20*a^3*b^4*d^2 - 40*a^5*b^2*d^2)/(
a^10*d^4 + b^10*d^4 + 5*a^2*b^8*d^4 + 10*a^4*b^6*d^4 + 10*a^6*b^4*d^4 + 5*
a^8*b^2*d^4))^(1/2))/4 - 16*a^4*b^15*d^2 - 96*a^6*b^13*d^2 - 240*a^8*b^11*
d^2 - 320*a^10*b^9*d^2 - 240*a^12*b^7*d^2 - 96*a^14*b^5*d^2 - 16*a^16*b...
```

Reduce [F]

$$\int \frac{-a + b\tan(c+dx)}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{2\sqrt{a+\tan(dx+c)\,b}\,a + 3\left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{\tan(dx+c)^3b^3+3\tan(dx+c)^2ab^2+3\tan(dx+c)a^2b+a^3}dx\right)$$

input
```
int((-a+b*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
(2*sqrt(tan(c + d*x)*b + a)*a + 3*int((sqrt(tan(c + d*x)*b + a)*tan(c + d*
x)**2)/(tan(c + d*x)**3*b**3 + 3*tan(c + d*x)**2*a*b**2 + 3*tan(c + d*x)*a
**2*b + a**3),x)*tan(c + d*x)**2*a*b**3*d + 6*int((sqrt(tan(c + d*x)*b + a
)*tan(c + d*x)**2)/(tan(c + d*x)**3*b**3 + 3*tan(c + d*x)**2*a*b**2 + 3*ta
n(c + d*x)*a**2*b + a**3),x)*tan(c + d*x)*a**2*b**2*d + 3*int((sqrt(tan(c
+ d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)**3*b**3 + 3*tan(c + d*x)**2*a
*b**2 + 3*tan(c + d*x)*a**2*b + a**3),x)*a**3*b*d + 3*int((sqrt(tan(c + d*
x)*b + a)*tan(c + d*x))/(tan(c + d*x)**3*b**3 + 3*tan(c + d*x)**2*a*b**2 +
 3*tan(c + d*x)*a**2*b + a**3),x)*tan(c + d*x)**2*b**4*d + 6*int((sqrt(tan
(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*x)**3*b**3 + 3*tan(c + d*x)**2*a
*b**2 + 3*tan(c + d*x)*a**2*b + a**3),x)*tan(c + d*x)*a*b**3*d + 3*int((sq
rt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*x)**3*b**3 + 3*tan(c + d*x
)**2*a*b**2 + 3*tan(c + d*x)*a**2*b + a**3),x)*a**2*b**2*d)/(3*b*d*(tan(c
+ d*x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2))
```

3.373 $\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{2i\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}$$

output
```
-2*I*arctanh((a+b*tan(d*x+c))^(1/2)/(a-I*b)^(1/2))/(a-I*b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{2i\operatorname{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}\,d}$$

input
```
Integrate[(1 + I*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
((-2*I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4020}$$

$$\frac{i\int -\frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{i\int \frac{1}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d(i\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\int \frac{1}{\frac{i\tan^2(c+dx)}{b}+\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2\arctan\left(\frac{\tan(c+dx)}{\sqrt{a-ib}}\right)}{d\sqrt{a-ib}}$$

input
```
Int[(1 + I*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
(2*ArcTan[Tan[c + d*x]/Sqrt[a - I*b]])/(Sqrt[a - I*b]*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(36) = 72$.

Time = 0.20 (sec) , antiderivative size = 729, normalized size of antiderivative = 16.20

method	result
derivativedivides	$\frac{\frac{\left(-i\sqrt{a^2+b^2}-ia+b\right)\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)}{2}+\frac{2\left(-i\sqrt{2\sqrt{a^2+b^2}+2a}\,a+\sqrt{2\sqrt{a^2+b^2}+2a}\cdots\right.}{\cdots}}{\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\cdots}$
default	$\frac{\frac{\left(-i\sqrt{a^2+b^2}-ia+b\right)\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)}{2}+\frac{2\left(-i\sqrt{2\sqrt{a^2+b^2}+2a}\,a+\sqrt{2\sqrt{a^2+b^2}+2a}\cdots\right.}{\cdots}}{\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}\cdots}$
parts	Expression too large to display

input
```
int((1+I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)*(1/2*(-I*(a^2+b^2)^(1
/2)-I*a+b)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)+(a^2+b^2)^(1/2))+2*(-I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a+(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*b-1/2*(-I*(a^2+b^2)^(1/2)-I*a+b)*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(2
*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))+1/(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^2+b^2)*(-1/2*(-2*I
*(a^2+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*b^2+(a^2+b^2)^(1/
2)*a*b+a^2*b+b^3)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)-
b*tan(d*x+c)-a-(a^2+b^2)^(1/2))+2*(I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^
2)^(1/2)*a^2+I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+I*(2*(a^2+b^2)^(1/2)+2*a)
^(1/2)*a*b^2-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b-(2*(a^2+b^2
)^(1/2)+2*a)^(1/2)*a^2*b-(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^3+1/2*(-2*I*(a^2+
b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*b^2+(a^2+b^2)^(1/2)*a*b
+a^2*b+b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*a
rctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^2
)^(1/2)-2*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(33) = 66.

Time = 0.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 5.53

$$\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$= \frac{1}{4}\sqrt{-\frac{4i}{(i\,a+b)d^2}}\log\left(\left(\left((i\,a+b)de^{(2i\,dx+2i\,c)}+(i\,a+b)d\right)\sqrt{\frac{(a-i\,b)e^{(2i\,dx+2i\,c)}+a+i\,b}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{4i}{(i\,a+b)d^2}}\right.\right.$$
$$-\frac{1}{4}\sqrt{-\frac{4i}{(i\,a+b)d^2}}\log\left(\left(\left((-i\,a-b)de^{(2i\,dx+2i\,c)}+(-i\,a-b)d\right)\sqrt{\frac{(a-i\,b)e^{(2i\,dx+2i\,c)}+a+i\,b}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{}{(i\,a}}\right.\right.$$

input
```
integrate((1+I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(-4*I/((I*a + b)*d^2))*log((((I*a + b)*d*e^(2*I*d*x + 2*I*c) + (I*
a + b)*d)*sqrt(((a - I*b)*e^(2*I*d*x + 2*I*c) + a + I*b)/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(-4*I/((I*a + b)*d^2)) + 2*(a - I*b)*e^(2*I*d*x + 2*I*c) + 2
*a)*e^(-2*I*d*x - 2*I*c)) - 1/4*sqrt(-4*I/((I*a + b)*d^2))*log((((-I*a - b
)*d*e^(2*I*d*x + 2*I*c) + (-I*a - b)*d)*sqrt(((a - I*b)*e^(2*I*d*x + 2*I*c
) + a + I*b)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-4*I/((I*a + b)*d^2)) + 2*(a
- I*b)*e^(2*I*d*x + 2*I*c) + 2*a)*e^(-2*I*d*x - 2*I*c))
```

Sympy [F]

$$\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = i\left(\int\left(-\frac{i}{\sqrt{a+b\tan(c+dx)}}\right)dx + \int\frac{\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx\right)$$

input
```
integrate((1+I*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
I*(Integral(-I/sqrt(a + b*tan(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(a
 + b*tan(c + d*x)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6480 vs. $2(33) = 66$.

Time = 0.35 (sec) , antiderivative size = 6480, normalized size of antiderivative = 144.00

$$\int \frac{1 + i\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((1+I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(2*a^2 + 2*b^2)*sqrt(a + sqrt(a^2 + b^2))*(2*arctan2((b^2*cos(2*
d*x + 2*c) - a*b*sin(2*d*x + 2*c) + ((a^4 + 2*a^2*b^2 + b^4)*cos(2*d*x + 2
*c)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*d*x + 2*c)^4 + a^4 + 2*a^2*b^2 + b^4
 + 4*(a^4 + a^2*b^2)*cos(2*d*x + 2*c)^3 + 4*(a^3*b + a*b^3)*sin(2*d*x + 2*
c)^3 + 2*(3*a^4 + 2*a^2*b^2 - b^4)*cos(2*d*x + 2*c)^2 + 2*(a^4 + 2*a^2*b^2
 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*d*x + 2*c)^2 + 2*(a^4 + a^2*b^2)*co
s(2*d*x + 2*c))*sin(2*d*x + 2*c)^2 + 4*(a^4 + a^2*b^2)*cos(2*d*x + 2*c) +
4*(a^3*b + a*b^3 + (a^3*b + a*b^3)*cos(2*d*x + 2*c)^2 + 2*(a^3*b + a*b^3)*
cos(2*d*x + 2*c))*sin(2*d*x + 2*c))^(1/4)*b*sin(1/2*arctan2(-2*(a*b*cos(2*
d*x + 2*c)^2 - a*b*sin(2*d*x + 2*c)^2 + a*b*cos(2*d*x + 2*c) - (a^2 + (a^2
 - b^2)*cos(2*d*x + 2*c))*sin(2*d*x + 2*c))/b^2, (2*a^2*cos(2*d*x + 2*c) +
 (a^2 - b^2)*cos(2*d*x + 2*c)^2 - (a^2 - b^2)*sin(2*d*x + 2*c)^2 + a^2 + b
^2 + 2*(2*a*b*cos(2*d*x + 2*c) + a*b)*sin(2*d*x + 2*c))/b^2)))/b^2, -(a*co
s(2*d*x + 2*c) + b*sin(2*d*x + 2*c) - ((a^4 + 2*a^2*b^2 + b^4)*cos(2*d*x +
 2*c)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*d*x + 2*c)^4 + a^4 + 2*a^2*b^2 + b
^4 + 4*(a^4 + a^2*b^2)*cos(2*d*x + 2*c)^3 + 4*(a^3*b + a*b^3)*sin(2*d*x +
2*c)^3 + 2*(3*a^4 + 2*a^2*b^2 - b^4)*cos(2*d*x + 2*c)^2 + 2*(a^4 + 2*a^2*b
^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*d*x + 2*c)^2 + 2*(a^4 + a^2*b^2)*
cos(2*d*x + 2*c))*sin(2*d*x + 2*c)^2 + 4*(a^4 + a^2*b^2)*cos(2*d*x + 2*c)
+ 4*(a^3*b + a*b^3 + (a^3*b + a*b^3)*cos(2*d*x + 2*c)^2 + 2*(a^3*b + a*...
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(33) = 66$.

Time = 0.53 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.53

$$\int \frac{1 + i\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx$$
$$= \frac{2i\sqrt{2}\arctan\left(\frac{2\left(\sqrt{b\tan(dx+c)+a}a-\sqrt{a^2+b^2}\sqrt{b\tan(dx+c)+a}\right)}{\sqrt{2}a\sqrt{-a+\sqrt{a^2+b^2}}-i\sqrt{2}\sqrt{-a+\sqrt{a^2+b^2}}b-\sqrt{2}\sqrt{a^2+b^2}\sqrt{-a+\sqrt{a^2+b^2}}}\right)}{\sqrt{-a+\sqrt{a^2+b^2}}d\left(-\frac{ib}{a-\sqrt{a^2+b^2}}+1\right)}$$

input
```
integrate((1+I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*I*sqrt(2)*arctan(2*(sqrt(b*tan(d*x + c) + a)*a - sqrt(a^2 + b^2)*sqrt(b*
tan(d*x + c) + a))/(sqrt(2)*a*sqrt(-a + sqrt(a^2 + b^2)) - I*sqrt(2)*sqrt(
-a + sqrt(a^2 + b^2))*b - sqrt(2)*sqrt(a^2 + b^2)*sqrt(-a + sqrt(a^2 + b^2
))))/(sqrt(-a + sqrt(a^2 + b^2))*d*(-I*b/(a - sqrt(a^2 + b^2)) + 1))
```

Mupad [B] (verification not implemented)

Time = 5.68 (sec) , antiderivative size = 1410, normalized size of antiderivative = 31.33

$$\int \frac{1 + i\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)*1i + 1)/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
2*atanh((32*b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2
))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((a^2*b^2*d^2*64i)/(4*a^2*d^3 + 4*b^2
*d^3) - (b^2*16i)/d + (64*a*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) - (128*a^2*b
^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a +
 b*tan(c + d*x))^(1/2))/((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a^2
*b^2*64i)/d - (b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a
^4*b^2*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*
b^2*d^3)) + (a*b^3*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2*d^2 + 4*b^2*
d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*128i)/((a^2*b^4*d^2*256i)/(4*a^2*d^
3 + 4*b^2*d^3) - (a^2*b^2*64i)/d - (b^4*64i)/d + (256*a^3*b^3*d^2)/(4*a^2*
d^3 + 4*b^2*d^3) + (a^4*b^2*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (256*a*b^5
*d^2)/(4*a^2*d^3 + 4*b^2*d^3)))*(-(a - b*1i)/(4*a^2*d^2 + 4*b^2*d^2))^(1/2
) + (log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*tan(c + d*x))^(1/2)*1i + 1)*
(-1/(a*d^2 - b*d^2*1i))^(1/2))/2 - log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a +
b*tan(c + d*x))^(1/2) + 1i)*(-1/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (log(16*b^
2*(a + b*tan(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2) - (16*
a*b^2*(a + b*tan(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1i))^(1/2
))/2 - log(16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*tan(c + d*
x))^(1/2) + (16*a*b^2*(a + b*tan(c + d*x))^(1/2))/(a - b*1i))*(-1/(4*(a*d^
2 - b*d^2*1i)))^(1/2) + 2*atanh((32*b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2)...
```

Reduce [F]

$$\int \frac{1+i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{2\sqrt{a+\tan(dx+c)\,b}-\left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{a+\tan(dx+c)b}dx\right)bd+\left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)}{a+\tan(dx+c)b}dx\right)bdi}{bd}$$

input
```
int((1+I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(tan(c + d*x)*b + a) - int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**
2)/(tan(c + d*x)*b + a),x)*b*d + int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x
))/(tan(c + d*x)*b + a),x)*b*d*i)/(b*d)
```

3.374 $\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{2i\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

output
```
2*I*arctanh((a+b*tan(d*x+c))^(1/2)/(a+I*b)^(1/2))/(a+I*b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{2i\text{arctanh}\left(\frac{\sqrt{a+b\tan(c+dx)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}d}$$

input
```
Integrate[(1 - I*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
((2*I)*ArcTanh[Sqrt[a + b*Tan[c + d*x]]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3042, 4020, 25, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-i\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4020}$$

$$-\frac{i\int -\frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{i\int \frac{1}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{d}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{-\frac{i\tan^2(c+dx)}{b}-\frac{ia}{b}+1}d\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\arctan\left(\frac{\tan(c+dx)}{\sqrt{a+ib}}\right)}{d\sqrt{a+ib}}$$

input
```
Int[(1 - I*Tan[c + d*x])/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
(2*ArcTan[Tan[c + d*x]/Sqrt[a + I*b]])/(Sqrt[a + I*b]*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 741, normalized size of antiderivative = 16.47

method	result
derivativedivides	$-\frac{\frac{\left(-i\sqrt{a^2+b^2}-ia-b\right)\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)}{2}+\frac{2\left(-i\sqrt{2\sqrt{a^2+b^2}+2a}\,a-\sqrt{2\sqrt{a^2+b^2}+2}\right.}{}}{\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}$
default	$-\frac{\frac{\left(-i\sqrt{a^2+b^2}-ia-b\right)\ln\left(b\tan(dx+c)+a+\sqrt{a+b\tan(dx+c)}\,\sqrt{2\sqrt{a^2+b^2}+2a}+\sqrt{a^2+b^2}\right)}{2}+\frac{2\left(-i\sqrt{2\sqrt{a^2+b^2}+2a}\,a-\sqrt{2\sqrt{a^2+b^2}+2}\right.}{}}{\sqrt{2\sqrt{a^2+b^2}+2a}\,\sqrt{a^2+b^2}}$
parts	Expression too large to display

input
```
int((1-I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(2*(a^2+b^2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)*(1/2*(-I*(a^2+b^2)^(
1/2)-I*a-b)*ln(b*tan(d*x+c)+a+(a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2)+(a^2+b^2)^(1/2))+2*(-I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a-(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*b-1/2*(-I*(a^2+b^2)^(1/2)-I*a-b)*(2*(a^2+b^2)^(1/2)+2*
a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*arctan((2*(a+b*tan(d*x+c))^(1/2)+(
2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)))-1/(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)/(a^2+b^2)^(1/2)/((a^2+b^2)^(1/2)*a+a^2+b^2)*(-1/2*(-2*
I*(a^2+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*b^2-(a^2+b^2)^(1
/2)*a*b-a^2*b-b^3)*ln((a+b*tan(d*x+c))^(1/2)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)
-b*tan(d*x+c)-a-(a^2+b^2)^(1/2))+2*(I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b
^2)^(1/2)*a^2+I*(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*a^3+I*(2*(a^2+b^2)^(1/2)+2*a
)^(1/2)*a*b^2+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*(a^2+b^2)^(1/2)*a*b+(2*(a^2+b^
2)^(1/2)+2*a)^(1/2)*a^2*b+(2*(a^2+b^2)^(1/2)+2*a)^(1/2)*b^3+1/2*(-2*I*(a^2
+b^2)^(1/2)*a^2-I*(a^2+b^2)^(1/2)*b^2-2*I*a^3-2*I*a*b^2-(a^2+b^2)^(1/2)*a*
b-a^2*b-b^3)*(2*(a^2+b^2)^(1/2)+2*a)^(1/2))/(2*(a^2+b^2)^(1/2)-2*a)^(1/2)*
arctan(((2*(a^2+b^2)^(1/2)+2*a)^(1/2)-2*(a+b*tan(d*x+c))^(1/2))/(2*(a^2+b^
2)^(1/2)-2*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(33) = 66$.

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 5.93

$$\int \frac{1 - i\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\, dx =$$

$$-\frac{1}{4}\sqrt{\frac{4i}{(-i\,a+b)d^2}}\log\left(\frac{\left(\left((i\,a-b)de^{(2i\,dx+2i\,c)}+(i\,a-b)d\right)\sqrt{\frac{(a-i\,b)e^{(2i\,dx+2i\,c)}+a+i\,b}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{4i}{(-i\,a+b)d^2}}+2\,ae^{(2i\,\cdots}}{(-i\,a+b)d}\right.$$

$$+\frac{1}{4}\sqrt{\frac{4i}{(-i\,a+b)d^2}}\log\left(\frac{\left(\left((-i\,a+b)de^{(2i\,dx+2i\,c)}+(-i\,a+b)d\right)\sqrt{\frac{(a-i\,b)e^{(2i\,dx+2i\,c)}+a+i\,b}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{4i}{(-i\,a+b)d^2}}+2\,a\cdots}{(-i\,a+b)d}\right.$$

input
```
integrate((1-I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(4*I/((-I*a + b)*d^2))*log((((I*a - b)*d*e^(2*I*d*x + 2*I*c) + (I
*a - b)*d)*sqrt(((a - I*b)*e^(2*I*d*x + 2*I*c) + a + I*b)/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt(4*I/((-I*a + b)*d^2)) + 2*a*e^(2*I*d*x + 2*I*c) + 2*a + 2*
I*b)*e^(-2*I*d*x - 2*I*c)/((-I*a + b)*d)) + 1/4*sqrt(4*I/((-I*a + b)*d^2))
*log((((-I*a + b)*d*e^(2*I*d*x + 2*I*c) + (-I*a + b)*d)*sqrt(((a - I*b)*e^
(2*I*d*x + 2*I*c) + a + I*b)/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(4*I/((-I*a +
b)*d^2)) + 2*a*e^(2*I*d*x + 2*I*c) + 2*a + 2*I*b)*e^(-2*I*d*x - 2*I*c)/((-
I*a + b)*d))
```

Sympy [F]

$$\int \frac{1 - i\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\, dx = -i\left(\int \frac{i}{\sqrt{a + b\tan(c + dx)}}\, dx + \int \frac{\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\, dx\right)$$

input
```
integrate((1-I*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
-I*(Integral(I/sqrt(a + b*tan(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(a
 + b*tan(c + d*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1 - i\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((1-I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(33) = 66.

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.53

$$\int \frac{1 - i\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\, dx = -\frac{2i\sqrt{2}\arctan\left(\frac{2\left(\sqrt{b\tan(dx+c)+a}\,a-\sqrt{a^2+b^2}\sqrt{b\tan(dx+c)+a}\right)}{\sqrt{2}a\sqrt{-a+\sqrt{a^2+b^2}}+i\sqrt{2}\sqrt{-a+\sqrt{a^2+b^2}}\,b-\sqrt{2}\sqrt{a^2+b^2}\sqrt{-a+\sqrt{a^2+b^2}}}\right)}{\sqrt{-a+\sqrt{a^2+b^2}}\,d\left(\frac{ib}{a-\sqrt{a^2+b^2}}+1\right)}$$

input
```
integrate((1-I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*I*sqrt(2)*arctan(2*(sqrt(b*tan(d*x + c) + a)*a - sqrt(a^2 + b^2)*sqrt(b
*tan(d*x + c) + a))/(sqrt(2)*a*sqrt(-a + sqrt(a^2 + b^2)) + I*sqrt(2)*sqrt
(-a + sqrt(a^2 + b^2))*b - sqrt(2)*sqrt(a^2 + b^2)*sqrt(-a + sqrt(a^2 + b^
2))))/(sqrt(-a + sqrt(a^2 + b^2))*d*(I*b/(a - sqrt(a^2 + b^2)) + 1))
```

Mupad [B] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 1410, normalized size of antiderivative = 31.33

$$\int \frac{1 - i\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\, dx = \text{Too large to display}$$

input
```
int(-(tan(c + d*x)*1i - 1)/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
(log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*tan(c + d*x))^(1/2) + 1i)*(-1/(a
*d^2 - b*d^2*1i))^(1/2))/2 - 2*atanh((32*b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^
2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((a^2*b^
2*d^2*64i)/(4*a^2*d^3 + 4*b^2*d^3) - (b^2*16i)/d + (64*a*b^3*d^2)/(4*a^2*d
^3 + 4*b^2*d^3)) - (128*a^2*b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2) - a/(4*a^2
*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2))/((a^2*b^4*d^2*256i)/(
4*a^2*d^3 + 4*b^2*d^3) - (a^2*b^2*64i)/d - (b^4*64i)/d + (256*a^3*b^3*d^2)
/(4*a^2*d^3 + 4*b^2*d^3) + (a^4*b^2*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) + (2
56*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)) + (a*b^3*((b*1i)/(4*a^2*d^2 + 4*b^2
*d^2) - a/(4*a^2*d^2 + 4*b^2*d^2))^(1/2)*(a + b*tan(c + d*x))^(1/2)*128i)/
((a^2*b^4*d^2*256i)/(4*a^2*d^3 + 4*b^2*d^3) - (a^2*b^2*64i)/d - (b^4*64i)/
d + (256*a^3*b^3*d^2)/(4*a^2*d^3 + 4*b^2*d^3) + (a^4*b^2*d^2*256i)/(4*a^2*
d^3 + 4*b^2*d^3) + (256*a*b^5*d^2)/(4*a^2*d^3 + 4*b^2*d^3)))*(-(a - b*1i)/
(4*a^2*d^2 + 4*b^2*d^2))^(1/2) - log(d*(-1/(d^2*(a - b*1i)))^(1/2)*(a + b*
tan(c + d*x))^(1/2)*1i + 1)*(-1/(4*(a*d^2 - b*d^2*1i)))^(1/2) + (log(16*b^
2*(a + b*tan(c + d*x))^(1/2) + 16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2) - (16*
a*b^2*(a + b*tan(c + d*x))^(1/2))/(a - b*1i))*(-1/(a*d^2 - b*d^2*1i))^(1/2
))/2 - log(16*b^3*d*(-1/(d^2*(a - b*1i)))^(1/2) - 16*b^2*(a + b*tan(c + d*
x))^(1/2) + (16*a*b^2*(a + b*tan(c + d*x))^(1/2))/(a - b*1i))*(-1/(4*(a*d^
2 - b*d^2*1i)))^(1/2) + 2*atanh((32*b^2*((b*1i)/(4*a^2*d^2 + 4*b^2*d^2)...
```

Reduce [F]

$$\int \frac{1 - i\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}}\,dx$$
$$= \frac{2\sqrt{a + \tan(dx + c)\,b} - \left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{a+\tan(dx+c)b}dx\right)bd - \left(\int \frac{\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)}{a+\tan(dx+c)b}dx\right)bdi}{bd}$$

input
```
int((1-I*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(tan(c + d*x)*b + a) - int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**
2)/(tan(c + d*x)*b + a),x)*b*d - int((sqrt(tan(c + d*x)*b + a)*tan(c + d*x
))/(tan(c + d*x)*b + a),x)*b*d*i)/(b*d)
```

3.375 $\int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = -\sqrt{2}\arctan\left(\frac{1-3\tan(x)}{\sqrt{2}\sqrt{4+3\tan(x)}}\right)$$

output
```
-2^(1/2)*arctan(1/2*(1-3*tan(x))*2^(1/2)/(4+3*tan(x))^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = -\frac{(1+3i)\operatorname{arctanh}\left(\frac{\sqrt{4+3\tan(x)}}{\sqrt{4-3i}}\right)}{\sqrt{4-3i}} - \frac{(1-3i)\operatorname{arctanh}\left(\frac{\sqrt{4+3\tan(x)}}{\sqrt{4+3i}}\right)}{\sqrt{4+3i}}$$

input
```
Integrate[(3 + Tan[x])/Sqrt[4 + 3*Tan[x]],x]
```

output
```
((-1 - 3*I)*ArcTanh[Sqrt[4 + 3*Tan[x]]/Sqrt[4 - 3*I]])/Sqrt[4 - 3*I] - ((1
 - 3*I)*ArcTanh[Sqrt[4 + 3*Tan[x]]/Sqrt[4 + 3*I]])/Sqrt[4 + 3*I]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4018, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)+3}{\sqrt{3\tan(x)+4}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)+3}{\sqrt{3\tan(x)+4}}dx$$

$$\Big\downarrow \text{4018}$$

$$-2\int \frac{1}{\frac{(1-3\tan(x))^2}{3\tan(x)+4}+2}d\frac{1-3\tan(x)}{\sqrt{3\tan(x)+4}}$$

$$\Big\downarrow \text{216}$$

$$-\sqrt{2}\arctan\left(\frac{1-3\tan(x)}{\sqrt{2}\sqrt{3\tan(x)+4}}\right)$$

input
```
Int[(3 + Tan[x])/Sqrt[4 + 3*Tan[x]],x]
```

output
```
-(Sqrt[2]*ArcTan[(1 - 3*Tan[x])/(Sqrt[2]*Sqrt[4 + 3*Tan[x]])])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4018
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*b*c*d - 4*a*d^2
+ x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0
] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.80

method	result	size
derivativedivides	$\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(x)}+3\sqrt{2}\right)\sqrt{2}}{2}\right)+\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(x)}-3\sqrt{2}\right)\sqrt{2}}{2}\right)$	54
default	$\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(x)}+3\sqrt{2}\right)\sqrt{2}}{2}\right)+\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(x)}-3\sqrt{2}\right)\sqrt{2}}{2}\right)$	54
parts	$\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(x)}+3\sqrt{2}\right)\sqrt{2}}{2}\right)+\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(x)}-3\sqrt{2}\right)\sqrt{2}}{2}\right)$	54

input
```
int((3+tan(x))/(4+3*tan(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arctan(1/2*(2*(4+3*tan(x))^(1/2)+3*2^(1/2))*2^(1/2))+2^(1/2)*arcta
n(1/2*(2*(4+3*tan(x))^(1/2)-3*2^(1/2))*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = \sqrt{2}\arctan\left(\frac{3\sqrt{2}\tan\left(x\right)-\sqrt{2}}{2\sqrt{3\,\tan\left(x\right)+4}}\right)$$

input
```
integrate((3+tan(x))/(4+3*tan(x))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(1/2*(3*sqrt(2)*tan(x) - sqrt(2))/sqrt(3*tan(x) + 4))
```

Sympy [F]

$$\int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = \int \frac{\tan(x)+3}{\sqrt{3\tan(x)+4}}\,dx$$

input
```
integrate((3+tan(x))/(4+3*tan(x))**(1/2),x)
```

output
```
Integral((tan(x) + 3)/sqrt(3*tan(x) + 4), x)
```

Maxima [F]

$$\int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = \int \frac{\tan(x)+3}{\sqrt{3\tan(x)+4}}\,dx$$

input
```
integrate((3+tan(x))/(4+3*tan(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((tan(x) + 3)/sqrt(3*tan(x) + 4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.17

$$\int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = \sqrt{2}\arctan\left(\frac{1}{250}\cdot 25^{\frac{3}{4}}\sqrt{10}\left(3\cdot 25^{\frac{1}{4}}\sqrt{10}+10\sqrt{3\tan(x)+4}\right)\right) + \sqrt{2}\arctan\left(-\frac{1}{250}\cdot 25^{\frac{3}{4}}\sqrt{10}\left(3\cdot 25^{\frac{1}{4}}\sqrt{10}-10\sqrt{3\tan(x)+4}\right)\right)$$

input
```
integrate((3+tan(x))/(4+3*tan(x))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(1/250*25^(3/4)*sqrt(10)*(3*25^(1/4)*sqrt(10) + 10*sqrt(3*ta
n(x) + 4))) + sqrt(2)*arctan(-1/250*25^(3/4)*sqrt(10)*(3*25^(1/4)*sqrt(10)
 - 10*sqrt(3*tan(x) + 4)))
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}}\,dx=\sqrt{2}\left(\operatorname{atan}\left(\sqrt{6\tan(x)+8}\left(\frac{1}{10}-\frac{3}{10}\mathrm{i}\right)\right)+\operatorname{atan}\left(\sqrt{6\tan(x)+8}\left(\frac{1}{10}+\frac{3}{10}\mathrm{i}\right)\right)\right)$$

input
```
int((tan(x) + 3)/(3*tan(x) + 4)^(1/2),x)
```

output
```
2^(1/2)*(atan((6*tan(x) + 8)^(1/2)*(1/10 - 3i/10)) + atan((6*tan(x) + 8)^(
1/2)*(1/10 + 3i/10)))
```

Reduce [F]

$$\int \frac{3+\tan(x)}{\sqrt{4+3\tan(x)}}\,dx=2\sqrt{3\tan(x)+4}-3\left(\int \frac{\sqrt{3\tan(x)+4}\,\tan(x)^2}{3\tan(x)+4}dx\right)+\int \frac{\sqrt{3\tan(x)+4}\,\tan(x)}{3\tan(x)+4}dx$$

input
```
int((3+tan(x))/(4+3*tan(x))^(1/2),x)
```

output
```
2*sqrt(3*tan(x) + 4) - 3*int((sqrt(3*tan(x) + 4)*tan(x)**2)/(3*tan(x) + 4)
,x) + int((sqrt(3*tan(x) + 4)*tan(x))/(3*tan(x) + 4),x)
```

3.376 $\int \frac{1-3\tan(x)}{\sqrt{4+3\tan(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{1-3\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = \sqrt{2}\text{arctanh}\left(\frac{3+\tan(x)}{\sqrt{2}\sqrt{4+3\tan(x)}}\right)$$

output
```
2^(1/2)*arctanh(1/2*(3+tan(x))*2^(1/2)/(4+3*tan(x))^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \frac{1-3\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = \frac{(3-i)\text{arctanh}\left(\frac{\sqrt{4+3\tan(x)}}{\sqrt{4-3i}}\right)}{\sqrt{4-3i}} + \frac{(3+i)\text{arctanh}\left(\frac{\sqrt{4+3\tan(x)}}{\sqrt{4+3i}}\right)}{\sqrt{4+3i}}$$

input
```
Integrate[(1 - 3*Tan[x])/Sqrt[4 + 3*Tan[x]],x]
```

output
```
((3 - I)*ArcTanh[Sqrt[4 + 3*Tan[x]]/Sqrt[4 - 3*I]])/Sqrt[4 - 3*I] + ((3 + 
I)*ArcTanh[Sqrt[4 + 3*Tan[x]]/Sqrt[4 + 3*I]])/Sqrt[4 + 3*I]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4018, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-3\tan(x)}{\sqrt{3\tan(x)+4}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-3\tan(x)}{\sqrt{3\tan(x)+4}}dx$$

$$\Big\downarrow \text{4018}$$

$$-18\int \frac{1}{\frac{81(\tan(x)+3)^2}{3\tan(x)+4}-162}d\frac{9(\tan(x)+3)}{\sqrt{3\tan(x)+4}}$$

$$\Big\downarrow \text{220}$$

$$\sqrt{2}\text{arctanh}\left(\frac{\tan(x)+3}{\sqrt{2}\sqrt{3\tan(x)+4}}\right)$$

input
```
Int[(1 - 3*Tan[x])/Sqrt[4 + 3*Tan[x]],x]
```

output
```
Sqrt[2]*ArcTanh[(3 + Tan[x])/(Sqrt[2]*Sqrt[4 + 3*Tan[x]])]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4018
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*b*c*d - 4*a*d^2
+ x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0
] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\ln\left(9+3\tan(x)+3\sqrt{4+3\tan(x)}\,\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(9+3\tan(x)-3\sqrt{4+3\tan(x)}\,\sqrt{2}\right)}{2}$	52
default	$\frac{\sqrt{2}\,\ln\left(9+3\tan(x)+3\sqrt{4+3\tan(x)}\,\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(9+3\tan(x)-3\sqrt{4+3\tan(x)}\,\sqrt{2}\right)}{2}$	52
parts	$\frac{\sqrt{2}\,\ln\left(9+3\tan(x)+3\sqrt{4+3\tan(x)}\,\sqrt{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(9+3\tan(x)-3\sqrt{4+3\tan(x)}\,\sqrt{2}\right)}{2}$	52

input
```
int((1-3*tan(x))/(4+3*tan(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/2)*ln(9+3*tan(x)+3*(4+3*tan(x))^(1/2)*2^(1/2))-1/2*2^(1/2)*ln(9+3
*tan(x)-3*(4+3*tan(x))^(1/2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{1-3\tan(x)}{\sqrt{4+3\tan(x)}}\,dx$$
$$=\frac{1}{2}\,\sqrt{2}\log\left(\frac{\tan(x)^2+2\left(\sqrt{2}\tan(x)+3\sqrt{2}\right)\sqrt{3\tan(x)+4}+12\tan(x)+17}{\tan(x)^2+1}\right)$$

input
```
integrate((1-3*tan(x))/(4+3*tan(x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log((tan(x)^2 + 2*(sqrt(2)*tan(x) + 3*sqrt(2))*sqrt(3*tan(x) +
 4) + 12*tan(x) + 17)/(tan(x)^2 + 1))
```

Sympy [F]

$$\int \frac{1-3\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = -\int \frac{3\tan(x)}{\sqrt{3\tan(x)+4}}\,dx - \int \left(-\frac{1}{\sqrt{3\tan(x)+4}}\right)dx$$

input
```
integrate((1-3*tan(x))/(4+3*tan(x))**(1/2),x)
```

output
```
-Integral(3*tan(x)/sqrt(3*tan(x) + 4), x) - Integral(-1/sqrt(3*tan(x) + 4)
, x)
```

Maxima [F]

$$\int \frac{1-3\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = \int -\frac{3\tan(x)-1}{\sqrt{3\tan(x)+4}}\,dx$$

input
```
integrate((1-3*tan(x))/(4+3*tan(x))^(1/2),x, algorithm="maxima")
```

output
```
-integrate((3*tan(x) - 1)/sqrt(3*tan(x) + 4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(22) = 44$.

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \frac{1-3\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = \frac{1}{2}\sqrt{2}\log\left(\frac{3}{5}\cdot 25^{\frac{1}{4}}\sqrt{10}\sqrt{3\tan(x)+4}+3\tan(x)+9\right) - \frac{1}{2}\sqrt{2}\log\left(-\frac{3}{5}\cdot 25^{\frac{1}{4}}\sqrt{10}\sqrt{3\tan(x)+4}+3\tan(x)+9\right)$$

input
```
integrate((1-3*tan(x))/(4+3*tan(x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*log(3/5*25^(1/4)*sqrt(10)*sqrt(3*tan(x) + 4) + 3*tan(x) + 9) -
 1/2*sqrt(2)*log(-3/5*25^(1/4)*sqrt(10)*sqrt(3*tan(x) + 4) + 3*tan(x) + 9)
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{1-3\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = \sqrt{2}\left(\operatorname{atan}\left(\sqrt{6\tan(x)+8}\left(\frac{1}{10}-\frac{3}{10}\mathrm{i}\right)\right)-\operatorname{atan}\left(\sqrt{6\tan(x)+8}\left(\frac{1}{10}+\frac{3}{10}\mathrm{i}\right)\right)\right)1\mathrm{i}$$

input
```
int(-(3*tan(x) - 1)/(3*tan(x) + 4)^(1/2),x)
```

output
```
2^(1/2)*(atan((6*tan(x) + 8)^(1/2)*(1/10 - 3i/10)) - atan((6*tan(x) + 8)^(
1/2)*(1/10 + 3i/10)))*1i
```

Reduce [F]

$$\int \frac{1-3\tan(x)}{\sqrt{4+3\tan(x)}}\,dx = \frac{2\sqrt{3\tan(x)+4}}{3}-\left(\int \frac{\sqrt{3\tan(x)+4}\,\tan(x)^2}{3\tan(x)+4}dx\right)-3\left(\int \frac{\sqrt{3\tan(x)+4}\,\tan(x)}{3\tan(x)+4}dx\right)$$

input
```
int((1-3*tan(x))/(4+3*tan(x))^(1/2),x)
```

output
```
(2*sqrt(3*tan(x) + 4) - 3*int((sqrt(3*tan(x) + 4)*tan(x)**2)/(3*tan(x) + 4
),x) - 9*int((sqrt(3*tan(x) + 4)*tan(x))/(3*tan(x) + 4),x))/3
```

3.377 $\int \frac{4-3\tan(a+bx)}{\sqrt{4+3\tan(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{4-3\tan(a+bx)}{\sqrt{4+3\tan(a+bx)}}\,dx = -\frac{9\arctan\left(\frac{1-3\tan(a+bx)}{\sqrt{2}\sqrt{4+3\tan(a+bx)}}\right)}{5\sqrt{2}b}+\frac{13\text{arctanh}\left(\frac{3+\tan(a+bx)}{\sqrt{2}\sqrt{4+3\tan(a+bx)}}\right)}{5\sqrt{2}b}$$

output
```
-9/10*arctan(1/2*(1-3*tan(b*x+a))*2^(1/2)/(4+3*tan(b*x+a))^(1/2))*2^(1/2)/
b+13/10*arctanh(1/2*(3+tan(b*x+a))*2^(1/2)/(4+3*tan(b*x+a))^(1/2))*2^(1/2)
/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{4-3\tan(a+bx)}{\sqrt{4+3\tan(a+bx)}}\,dx = \frac{(3-4i)\text{arctanh}\left(\frac{\sqrt{4+3\tan(a+bx)}}{\sqrt{4-3i}}\right)}{\sqrt{4-3i}b} + \frac{(3+4i)\text{arctanh}\left(\frac{\sqrt{4+3\tan(a+bx)}}{\sqrt{4+3i}}\right)}{\sqrt{4+3i}b}$$

input
```
Integrate[(4 - 3*Tan[a + b*x])/Sqrt[4 + 3*Tan[a + b*x]],x]
```

output
```
((3 - 4*I)*ArcTanh[Sqrt[4 + 3*Tan[a + b*x]]/Sqrt[4 - 3*I]])/(Sqrt[4 - 3*I]
*b) + ((3 + 4*I)*ArcTanh[Sqrt[4 + 3*Tan[a + b*x]]/Sqrt[4 + 3*I]])/(Sqrt[4
+ 3*I]*b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3042, 4019, 27, 3042, 4018, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4-3\tan(a+bx)}{\sqrt{3\tan(a+bx)+4}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{4-3\tan(a+bx)}{\sqrt{3\tan(a+bx)+4}}dx$$

$$\Big\downarrow \text{4019}$$

$$\frac{1}{10}\int \frac{9(\tan(a+bx)+3)}{\sqrt{3\tan(a+bx)+4}}dx-\frac{1}{10}\int -\frac{13(1-3\tan(a+bx))}{\sqrt{3\tan(a+bx)+4}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{13}{10}\int \frac{1-3\tan(a+bx)}{\sqrt{3\tan(a+bx)+4}}dx+\frac{9}{10}\int \frac{\tan(a+bx)+3}{\sqrt{3\tan(a+bx)+4}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{13}{10}\int \frac{1-3\tan(a+bx)}{\sqrt{3\tan(a+bx)+4}}dx+\frac{9}{10}\int \frac{\tan(a+bx)+3}{\sqrt{3\tan(a+bx)+4}}dx$$

$$\Big\downarrow \text{4018}$$

$$-\frac{9\int \frac{1}{\frac{(1-3\tan(a+bx))^2}{3\tan(a+bx)+4}+2}d\frac{1-3\tan(a+bx)}{\sqrt{3\tan(a+bx)+4}}}{5b}-\frac{117\int \frac{1}{\frac{81(\tan(a+bx)+3)^2}{3\tan(a+bx)+4}-162}d\frac{9(\tan(a+bx)+3)}{\sqrt{3\tan(a+bx)+4}}}{5b}$$

$$\Big\downarrow \; 216$$

$$-\frac{117\int \frac{1}{\frac{81(\tan(a+bx)+3)^2}{3\tan(a+bx)+4}-162}d\frac{9(\tan(a+bx)+3)}{\sqrt{3\tan(a+bx)+4}}}{5b}-\frac{9\arctan\left(\frac{1-3\tan(a+bx)}{\sqrt{2}\sqrt{3\tan(a+bx)+4}}\right)}{5\sqrt{2}b}$$

$$\Big\downarrow \; 220$$

$$\frac{13\text{arctanh}\left(\frac{\tan(a+bx)+3}{\sqrt{2}\sqrt{3\tan(a+bx)+4}}\right)}{5\sqrt{2}b}-\frac{9\arctan\left(\frac{1-3\tan(a+bx)}{\sqrt{2}\sqrt{3\tan(a+bx)+4}}\right)}{5\sqrt{2}b}$$

input
```
Int[(4 - 3*Tan[a + b*x])/Sqrt[4 + 3*Tan[a + b*x]],x]
```

output
```
(-9*ArcTan[(1 - 3*Tan[a + b*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[a + b*x]])])/(5*Sq
rt[2]*b) + (13*ArcTanh[(3 + Tan[a + b*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[a + b*x]
])])/(5*Sqrt[2]*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4018
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[-2*(d^2/f)   Subst[Int[1/(2*b*c*d - 4*a*d^2
+ x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0
] && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

rule 4019
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Simp[1/(2*q)   Int[(
a*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
 x], x] - Simp[1/(2*q)   Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f
*x])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d -
 b*(c^2 - d^2), 0] && NiceSqrtQ[a^2 + b^2]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{\frac{13\sqrt{2}\,\ln\left(9+3\tan(bx+a)+3\sqrt{4+3\tan(bx+a)}\,\sqrt{2}\right)}{20}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(bx+a)}+3\sqrt{2}\right)\sqrt{2}}{2}\right)}{10}-\frac{13\sqrt{2}\,\ln\left(9+3\tan(bx+a)-3\sqrt{4+\cdots}\right)}{20}\cdots}{b}$
default	$\frac{\frac{13\sqrt{2}\,\ln\left(9+3\tan(bx+a)+3\sqrt{4+3\tan(bx+a)}\,\sqrt{2}\right)}{20}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(bx+a)}+3\sqrt{2}\right)\sqrt{2}}{2}\right)}{10}-\frac{13\sqrt{2}\,\ln\left(9+3\tan(bx+a)-3\sqrt{4+\cdots}\right)}{20}\cdots}{b}$
parts	$-\frac{\sqrt{2}\,\ln\left(9+3\tan(bx+a)-3\sqrt{4+3\tan(bx+a)}\,\sqrt{2}\right)}{5b}+\frac{6\sqrt{2}\,\arctan\left(\frac{\left(2\sqrt{4+3\tan(bx+a)}-3\sqrt{2}\right)\sqrt{2}}{2}\right)}{5b}+\frac{\sqrt{2}\,\ln\left(9+3\tan\cdots\right)}{\cdots}$

input
```
int((4-3*tan(b*x+a))/(4+3*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(13/20*2^(1/2)*ln(9+3*tan(b*x+a)+3*(4+3*tan(b*x+a))^(1/2)*2^(1/2))+9/1
0*2^(1/2)*arctan(1/2*(2*(4+3*tan(b*x+a))^(1/2)+3*2^(1/2))*2^(1/2))-13/20*2
^(1/2)*ln(9+3*tan(b*x+a)-3*(4+3*tan(b*x+a))^(1/2)*2^(1/2))+9/10*2^(1/2)*ar
ctan(1/2*(2*(4+3*tan(b*x+a))^(1/2)-3*2^(1/2))*2^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(71) = 142$.

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.85

$$\int \frac{4-3\tan(a+bx)}{\sqrt{4+3\tan(a+bx)}}\,dx = \frac{9}{5}\sqrt{\frac{1}{2}}\sqrt{\frac{1}{b^2}}\arctan\left(2\sqrt{\frac{1}{2}}b\sqrt{\frac{1}{b^2}}\sqrt{3\tan(bx+a)+4}+3\right) + \frac{9}{5}\sqrt{\frac{1}{2}}\sqrt{\frac{1}{b^2}}\arctan\left(2\sqrt{\frac{1}{2}}b\sqrt{\frac{1}{b^2}}\sqrt{3\tan(bx+a)+4} - 3\right) + \frac{13}{10}\sqrt{\frac{1}{2}}\sqrt{\frac{1}{b^2}}\log\left(2\sqrt{\frac{1}{2}}b\sqrt{\frac{1}{b^2}}\sqrt{3\tan(bx+a)+4} + \tan(bx+a)+3\right) - \frac{13}{10}\sqrt{\frac{1}{2}}\sqrt{\frac{1}{b^2}}\log\left(-2\sqrt{\frac{1}{2}}b\sqrt{\frac{1}{b^2}}\sqrt{3\tan(bx+a)+4} + \tan(bx+a)+3\right)$$

input
```
integrate((4-3*tan(b*x+a))/(4+3*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
9/5*sqrt(1/2)*sqrt(b^(-2))*arctan(2*sqrt(1/2)*b*sqrt(b^(-2))*sqrt(3*tan(b*
x + a) + 4) + 3) + 9/5*sqrt(1/2)*sqrt(b^(-2))*arctan(2*sqrt(1/2)*b*sqrt(b^
(-2))*sqrt(3*tan(b*x + a) + 4) - 3) + 13/10*sqrt(1/2)*sqrt(b^(-2))*log(2*s
qrt(1/2)*b*sqrt(b^(-2))*sqrt(3*tan(b*x + a) + 4) + tan(b*x + a) + 3) - 13/
10*sqrt(1/2)*sqrt(b^(-2))*log(-2*sqrt(1/2)*b*sqrt(b^(-2))*sqrt(3*tan(b*x +
 a) + 4) + tan(b*x + a) + 3)
```

Sympy [F]

$$\int \frac{4-3\tan(a+bx)}{\sqrt{4+3\tan(a+bx)}}\,dx = -\int \frac{3\tan(a+bx)}{\sqrt{3\tan(a+bx)+4}}\,dx - \int \left(-\frac{4}{\sqrt{3\tan(a+bx)+4}}\right)dx$$

input
```
integrate((4-3*tan(b*x+a))/(4+3*tan(b*x+a))**(1/2),x)
```

output
```
-Integral(3*tan(a + b*x)/sqrt(3*tan(a + b*x) + 4), x) - Integral(-4/sqrt(3
*tan(a + b*x) + 4), x)
```

Maxima [F]

$$\int \frac{4-3\tan(a+bx)}{\sqrt{4+3\tan(a+bx)}}\,dx = \int -\frac{3\tan(bx+a)-4}{\sqrt{3\tan(bx+a)+4}}\,dx$$

input
```
integrate((4-3*tan(b*x+a))/(4+3*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-integrate((3*tan(b*x + a) - 4)/sqrt(3*tan(b*x + a) + 4), x)
```

Giac [F]

$$\int \frac{4-3\tan(a+bx)}{\sqrt{4+3\tan(a+bx)}}\,dx = \int -\frac{3\tan(bx+a)-4}{\sqrt{3\tan(bx+a)+4}}\,dx$$

input
```
integrate((4-3*tan(b*x+a))/(4+3*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.73

$$\int \frac{4-3\tan(a+bx)}{\sqrt{4+3\tan(a+bx)}}\,dx = \operatorname{atan}\left(\frac{b\sqrt{\frac{-\frac{16}{25}-\frac{12}{25}\mathrm{i}}{b^2}}\,\sqrt{3\tan(a+b\,x)+4}}{2}\right)\sqrt{\frac{-\frac{16}{25}-\frac{12}{25}\mathrm{i}}{b^2}}\,2\mathrm{i} - \operatorname{atan}\left(\frac{b\sqrt{\frac{-\frac{16}{25}+\frac{12}{25}\mathrm{i}}{b^2}}\,\sqrt{3\tan(a+b\,x)+4}}{2}\right)\sqrt{\frac{-\frac{16}{25}+\frac{12}{25}\mathrm{i}}{b^2}}\,2\mathrm{i} + 2\operatorname{atanh}\left(\frac{2\,b\sqrt{\frac{\frac{9}{25}-\frac{27}{100}\mathrm{i}}{b^2}}\,\sqrt{3\tan(a+b\,x)+4}}{3}\right)\sqrt{\frac{\frac{9}{25}-\frac{27}{100}\mathrm{i}}{b^2}} + 2\operatorname{atanh}\left(\frac{2\,b\sqrt{\frac{\frac{9}{25}+\frac{27}{100}\mathrm{i}}{b^2}}\,\sqrt{3\tan(a+b\,x)+4}}{3}\right)\sqrt{\frac{\frac{9}{25}+\frac{27}{100}\mathrm{i}}{b^2}}$$

input
```
int(-(3*tan(a + b*x) - 4)/(3*tan(a + b*x) + 4)^(1/2),x)
```

output
```
atan((b*((- 16/25 - 12i/25)/b^2)^(1/2)*(3*tan(a + b*x) + 4)^(1/2))/2)*((-
16/25 - 12i/25)/b^2)^(1/2)*2i - atan((b*((- 16/25 + 12i/25)/b^2)^(1/2)*(3*
tan(a + b*x) + 4)^(1/2))/2)*((- 16/25 + 12i/25)/b^2)^(1/2)*2i + 2*atanh((2
*b*((9/25 - 27i/100)/b^2)^(1/2)*(3*tan(a + b*x) + 4)^(1/2))/3)*((9/25 - 27
i/100)/b^2)^(1/2) + 2*atanh((2*b*((9/25 + 27i/100)/b^2)^(1/2)*(3*tan(a + b
*x) + 4)^(1/2))/3)*((9/25 + 27i/100)/b^2)^(1/2)
```

Reduce [F]

$$\int \frac{4-3\tan(a+bx)}{\sqrt{4+3\tan(a+bx)}}\,dx = \frac{8\sqrt{3\tan(bx+a)+4}-12\left(\int \frac{\sqrt{3\tan(bx+a)+4}\,\tan(bx+a)^2}{3\tan(bx+a)+4}dx\right)b-9\left(\int \frac{\sqrt{3\tan(bx+a)+4}\,\tan(bx+a)}{3\tan(bx+a)+4}dx\right)b}{3b}$$

input
```
int((4-3*tan(b*x+a))/(4+3*tan(b*x+a))^(1/2),x)
```

output

```
(8*sqrt(3*tan(a + b*x) + 4) - 12*int((sqrt(3*tan(a + b*x) + 4)*tan(a + b*x
)**2)/(3*tan(a + b*x) + 4),x)*b - 9*int((sqrt(3*tan(a + b*x) + 4)*tan(a +
b*x))/(3*tan(a + b*x) + 4),x)*b)/(3*b)
```

3.378 $\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 226

$$\begin{aligned}
&\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx \\
&= \frac{(a(A-B)-b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad -\frac{(a(A-B)-b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad +\frac{(b(A-B)+a(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d}-\frac{2(Ab+aB)\sqrt{\tan(c+dx)}}{d} \\
&\quad +\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(Ab+aB)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}
\end{aligned}$$

output
```
-1/2*(a*(A-B)-b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(
a*(A-B)-b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*(b*(A-B)
+a*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d-2*(A*
b+B*a)*tan(d*x+c)^(1/2)/d+2/3*(A*a-B*b)*tan(d*x+c)^(3/2)/d+2/5*(A*b+B*a)*t
an(d*x+c)^(5/2)/d+2/7*b*B*tan(d*x+c)^(7/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.67

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{-105\sqrt[4]{-1}(ia+b)(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+105(-1)^{3/4}(a+ib)(A+iB)\text{arctanh}\left((-1\right.}{}$$

input
```
Integrate[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(-105*(-1)^(1/4)*(I*a + b)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
 + 105*(-1)^(3/4)*(a + I*b)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]
]] + 2*Sqrt[Tan[c + d*x]]*(-105*(A*b + a*B) + 35*(a*A - b*B)*Tan[c + d*x]
+ 21*(A*b + a*B)*Tan[c + d*x]^2 + 15*b*B*Tan[c + d*x]^3))/(105*d)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.13, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.581, Rules used = {3042, 4075, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{5/2}(a+b\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int \tan^{\frac{5}{2}}(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{5/2}(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{4011}$$

$$\int \tan^{\frac{3}{2}}(c+dx)(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{4011}$$

$$\int \sqrt{\tan(c+dx)}(-aA+bB-(Ab+aB)\tan(c+dx))dx+\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(-aA+bB-(Ab+aB)\tan(c+dx))dx+\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{4017}$$

$$\frac{2\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 1482

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+\ldots}{d}\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 1476

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d\ldots}}\right.\right.}{d}\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 1082

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan\ldots}\right.}{\sqrt{2}}\right.\right.}{d}\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$\downarrow$ 217

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\ldots\right.\right.}{d}\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-}{d}$$

$$\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(a\cdots}{d}$$

$$\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\cdots}{d}$$

$$\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(a(A-B)-\cdots}{d}$$

$$\frac{2(aB+Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2(aA-bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{7}{2}}(c+dx)}{7d}$$

input
```
Int[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(2*(-1/2*((a*(A - B) - b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((b*(A - B
) + a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sq
rt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2
))/d - (2*(A*b + a*B)*Sqrt[Tan[c + d*x]])/d + (2*(a*A - b*B)*Tan[c + d*x]^
(3/2))/(3*d) + (2*(A*b + a*B)*Tan[c + d*x]^(5/2))/(5*d) + (2*b*B*Tan[c + d
*x]^(7/2))/(7*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{2Bb\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{2Ab\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2Ba\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2aA\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{2Bb\tan(dx+c)^{\frac{3}{2}}}{3}-2Ab\sqrt{\tan(dx+c)}-2Ba\sqrt{\tan(}$
default	$\frac{2Bb\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{2Ab\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2Ba\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2aA\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{2Bb\tan(dx+c)^{\frac{3}{2}}}{3}-2Ab\sqrt{\tan(dx+c)}-2Ba\sqrt{\tan(}$
parts	$\frac{(Ab+Ba)\left(\frac{2\tan(dx+c)^{\frac{5}{2}}}{5}-2\sqrt{\tan(dx+c)}+\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan(}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(2/7*B*b*tan(d*x+c)^(7/2)+2/5*A*b*tan(d*x+c)^(5/2)+2/5*B*a*tan(d*x+c)^
(5/2)+2/3*a*A*tan(d*x+c)^(3/2)-2/3*B*b*tan(d*x+c)^(3/2)-2*A*b*tan(d*x+c)^(
1/2)-2*B*a*tan(d*x+c)^(1/2)+1/4*(A*b+B*a)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*
tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^
(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-A*a+B
*b)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2
)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^
(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(195) = 390$.

Time = 0.10 (sec) , antiderivative size = 937, normalized size of antiderivative = 4.15

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/210*(210*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b +
 (A^2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A + B)*a + (A - B)*b)
*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)
*b^2)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2
 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2
- 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B
^2)*a^2 + (A^2 - B^2)*b^2)) + 210*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^
2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*
((A + B)*a + (A - B)*b)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*
b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*
A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((
A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)
)/(4*A*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2)) - 105*sqrt(1/2)*d*sqrt(
((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^
2)*log(2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (
A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A + B)*a - (A - B)*b -
((A + B)*a + (A - B)*b)*tan(d*x + c)) + 105*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B
 + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*log(-2*sqr
t(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B
+ B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A + B)*a - (A - B)*b - ((A + B)*...
```

Sympy [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})\tan^{\frac{5}{2}}{(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))*tan(c + d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{120\,Bb\tan{(dx+c)}^{\frac{7}{2}} + 168\,(Ba+Ab)\tan{(dx+c)}^{\frac{5}{2}} - 210\,\sqrt{2}((A-B)a-(A+B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/420*(120*B*b*tan(d*x + c)^(7/2) + 168*(B*a + A*b)*tan(d*x + c)^(5/2) - 2
10*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(ta
n(d*x + c)))) - 210*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(-1/2*sqrt(2)*(s
qrt(2) - 2*sqrt(tan(d*x + c)))) + 105*sqrt(2)*((A + B)*a + (A - B)*b)*log(
sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 105*sqrt(2)*((A + B)*a +
(A - B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 280*(A*a
- B*b)*tan(d*x + c)^(3/2) - 840*(B*a + A*b)*sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 1522, normalized size of antiderivative = 6.73

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
atan((A^2*a^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a
^4*d^4)^(1/2)/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2)*32i)/((16*A*b*(2*A^4*a^2*
b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a^3)/d + (16*A^3
*a*b^2)/d) - (A^2*b^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4
 - A^4*a^4*d^4)^(1/2)/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2)*32i)/((16*A*b*(2*
A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a^3)/d +
 (16*A^3*a*b^2)/d))*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)
/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2)*2i - atan((A^2*a^2*tan(c + d*x)^(1/2)*
((A^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)
/(4*d^4))^(1/2)*32i)/((16*A^3*a^3)/d + (16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^
4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a*b^2)/d) - (A^2*b^2*tan(c + d*x
)^(1/2)*((A^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^
4)^(1/2)/(4*d^4))^(1/2)*32i)/((16*A^3*a^3)/d + (16*A*b*(2*A^4*a^2*b^2*d^4
- A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a*b^2)/d))*((A^2*a*b)/(2
*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4))^(1/
2)*2i + atan((B^2*a^2*tan(c + d*x)^(1/2)*(- (2*B^4*a^2*b^2*d^4 - B^4*b^4*d
^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2)*32i)/((16*B*a*(
2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2))/d^3 - (16*B^3*b^3)/d
 + (16*B^3*a^2*b)/d) - (B^2*b^2*tan(c + d*x)^(1/2)*(- (2*B^4*a^2*b^2*d^4 -
 B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2)*32...
```

Reduce [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{4\sqrt{\tan(dx+c)}\,\tan(dx+c)^2\,ab - 20\sqrt{\tan(dx+c)}\,ab + 10\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)abd + 5\left(\int \sqrt{\tan(dx+c)}\right.}{5d}$$

input
```
int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(4*sqrt(tan(c + d*x))*tan(c + d*x)**2*a*b - 20*sqrt(tan(c + d*x))*a*b + 10
*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*b*d + 5*int(sqrt(tan(c + d*x))*t
an(c + d*x)**4,x)*b**2*d + 5*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*a**
2*d)/(5*d)
```

3.379 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 202

$$\begin{aligned}\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx &= \frac{(b(A-B)+a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &- \frac{(b(A-B)+a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &- \frac{(a(A-B)-b(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} \\ &+ \frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d} + \frac{2(Ab+aB)\tan^{\frac{3}{2}}(c+dx)}{3d} + \frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}\end{aligned}$$

output
```
-1/2*(b*(A-B)+a*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(
b*(A-B)+a*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(a*(A-B)
-b*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d+2*(A*
a-B*b)*tan(d*x+c)^(1/2)/d+2/3*(A*b+B*a)*tan(d*x+c)^(3/2)/d+2/5*b*B*tan(d*x
+c)^(5/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{15\sqrt[4]{-1}(a-ib)(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+15\sqrt[4]{-1}(a+ib)(A+iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\ldots}{15d}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(15*(-1)^(1/4)*(a - I*b)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] +
 15*(-1)^(1/4)*(a + I*b)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
+ 2*Sqrt[Tan[c + d*x]]*(15*(a*A - b*B) + 5*(A*b + a*B)*Tan[c + d*x] + 3*b*
B*Tan[c + d*x]^2))/(15*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {3042, 4075, 3042, 4011, 3042, 4011, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(c+dx)^{3/2}(a+b\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \ 4075$$

$$\int \tan^{\frac{3}{2}}(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{4011}$$

$$\int \sqrt{\tan(c+dx)}(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(-Ab-aB+(aA-bB)\tan(c+dx))dx+\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{-aA+bB-(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-aA+bB-(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{4017}$$

$$\frac{2\int -\frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{1482}$$

$$\frac{2\left(-\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A+B)+b(A-B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c\cdots}{d}$$
$$\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\big\downarrow$ 1476

$$\frac{2\left(-\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c\cdots}\right.\right.}{d}$$
$$\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\big\downarrow$ 1082

$$\frac{2\left(-\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cdots}\right.}{\sqrt{2}}\right.\right.}{d}$$
$$\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\big\downarrow$ 217

$$\frac{2\left(-\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\cdots\right.\right.}{d}$$
$$\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\big\downarrow$ 1479

$$\frac{2\left(-\frac{1}{2}(a(A-B)-b(A+B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)\cdots\right.}{d}$$
$$\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$\big\downarrow$ 25

$$\frac{2\left(-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(\ldots}{d}$$

$$\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+d\ldots}{d}$$

$$\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \; 1103$$

$$\frac{2\left(-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)-b(A+B))\left(\ldots\right.}{d}$$

$$\frac{2(aB+Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(aA-bB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)}{5d}$$

input

```
Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output

```
(2*(-1/2*((b*(A - B) + a*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) - ((a*(A - B
) - b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sq
rt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2
))/d + (2*(a*A - b*B)*Sqrt[Tan[c + d*x]])/d + (2*(A*b + a*B)*Tan[c + d*x]^
(3/2))/(3*d) + (2*b*B*Tan[c + d*x]^(5/2))/(5*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{2Bb\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2Ab\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{2Ba\tan(dx+c)^{\frac{3}{2}}}{3}+2aA\sqrt{\tan(dx+c)}-2\sqrt{\tan(dx+c)}\,Bb+\frac{(-aA+Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c}{\tan(dx+c}\right.\right.}{}}{}$
default	$\frac{\frac{2Bb\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2Ab\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{2Ba\tan(dx+c)^{\frac{3}{2}}}{3}+2aA\sqrt{\tan(dx+c)}-2\sqrt{\tan(dx+c)}\,Bb+\frac{(-aA+Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c}{\tan(dx+c}\right.\right.}{}}{}$
parts	$\frac{(Ab+Ba)\left(\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx-}\right.\right.}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(2/5*B*b*tan(d*x+c)^(5/2)+2/3*A*b*tan(d*x+c)^(3/2)+2/3*B*a*tan(d*x+c)^
(3/2)+2*a*A*tan(d*x+c)^(1/2)-2*tan(d*x+c)^(1/2)*B*b+1/4*(-A*a+B*b)*2^(1/2)
*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c
)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d
*x+c)^(1/2)))+1/4*(-A*b-B*a)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1
/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x
+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. 2(174) = 348.

Time = 0.10 (sec) , antiderivative size = 921, normalized size of antiderivative = 4.56

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx=\text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/30*(30*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (
A^2 - 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a - (A + B)*b)*d
*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b
^2)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 -
 B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 -
2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2
)*a^2 + (A^2 - B^2)*b^2)) + 30*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 +
 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A
 - B)*a - (A + B)*b)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b +
 (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B
 + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2
 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(
4*A*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2)) + 15*sqrt(1/2)*d*sqrt(((A^
2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*l
og(2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2
+ 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A - B)*a + (A + B)*b - ((A
- B)*a - (A + B)*b)*tan(d*x + c)) - 15*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^
2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*log(-2*sqrt(1/2
)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2
)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A - B)*a + (A + B)*b - ((A - B)*a - (...
```

Sympy [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})\tan^{\frac{3}{2}}{(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))*tan(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.04

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{24\,Bb\tan{(dx+c)}^{\frac{5}{2}} - 30\,\sqrt{2}((A+B)a+(A-B)b)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan{(dx+c)}}\right)\right) - 30\,\sqrt{2}((}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/60*(24*B*b*tan(d*x + c)^(5/2) - 30*sqrt(2)*((A + B)*a + (A - B)*b)*arcta
n(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) - 30*sqrt(2)*((A + B)*a +
(A - B)*b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 15*sqrt
(2)*((A - B)*a - (A + B)*b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c)
+ 1) + 15*sqrt(2)*((A - B)*a - (A + B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c))
+ tan(d*x + c) + 1) + 40*(B*a + A*b)*tan(d*x + c)^(3/2) + 120*(A*a - B*b)*
sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 1492, normalized size of antiderivative = 7.39

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
atan((A^2*a^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a
^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2)*32i)/((16*A^3*b^3)/d + (1
6*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3
*a^2*b)/d) - (A^2*b^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4
 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2)*32i)/((16*A^3*b^3
)/d + (16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 -
 (16*A^3*a^2*b)/d))*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)
/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2)*2i - atan((A^2*a^2*tan(c + d*x)^(1/2)*
(- (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*
b)/(2*d^2))^(1/2)*32i)/((16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4
*d^4)^(1/2))/d^3 - (16*A^3*b^3)/d + (16*A^3*a^2*b)/d) - (A^2*b^2*tan(c + d
*x)^(1/2)*(- (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4)
 - (A^2*a*b)/(2*d^2))^(1/2)*32i)/((16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4
 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*b^3)/d + (16*A^3*a^2*b)/d))*(- (2*A^4
*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2
))^(1/2)*2i - atan((B^2*a^2*tan(c + d*x)^(1/2)*((B^2*a*b)/(2*d^2) - (2*B^4
*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2)*32i)/((16*B
^3*a^3)/d + (16*B*b*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2))
/d^3 - (16*B^3*a*b^2)/d) - (B^2*b^2*tan(c + d*x)^(1/2)*((B^2*a*b)/(2*d^2)
- (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2)*...
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 b^2 + 10\sqrt{\tan(dx+c)}\,a^2 - 10\sqrt{\tan(dx+c)}\,b^2 - 5\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a^2 d +}{5d}$$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(2*sqrt(tan(c + d*x))*tan(c + d*x)**2*b**2 + 10*sqrt(tan(c + d*x))*a**2 -
10*sqrt(tan(c + d*x))*b**2 - 5*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a**2
*d + 5*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b**2*d + 10*int(sqrt(tan(c +
 d*x))*tan(c + d*x)**2,x)*a*b*d)/(5*d)
```

3.380 $\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 178

$$\begin{aligned}\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx &= -\frac{(a(A-B)-b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &+\frac{(a(A-B)-b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &-\frac{(b(A-B)+a(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} \\ &+\frac{2(Ab+aB)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}\end{aligned}$$

output
```
1/2*(a*(A-B)-b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a
*(A-B)-b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(b*(A-B)+
a*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d+2*(A*b
+B*a)*tan(d*x+c)^(1/2)/d+2/3*b*B*tan(d*x+c)^(3/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{3\sqrt[4]{-1}(ia+b)(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-3(-1)^{3/4}(a+ib)(A+iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\ldots}\right.}{3d}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(3*(-1)^(1/4)*(I*a + b)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] -
3*(-1)^(3/4)*(a + I*b)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] +
2*Sqrt[Tan[c + d*x]]*(3*A*b + 3*a*B + b*B*Tan[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4075, 3042, 4011, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int \sqrt{\tan(c+dx)}(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{-Ab-aB+(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-Ab-aB+(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{4017}$$

$$\frac{2\int -\frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{1482}$$

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+\right.}{d}$$
$$\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan\right.\right.}{d}$$
$$\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A+B\right.}{d}$$
$$\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 217

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A+B)+b(A-B))\int\frac{1}{\tan}\right.}{d}$$

$$\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 1479

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A+B)+b(A-B))\left(-\right.\right.}{d}$$

$$\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 25

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int}{}\right.\right.}{d}$$

$$\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 27

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int}{}\right.\right.}{d}$$

$$\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

$\Big\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\log}{}\right.\right.}{d}$$

$$\frac{2(aB+Ab)\sqrt{\tan(c+dx)}}{d}+\frac{2bB\tan^{\frac{3}{2}}(c+dx)}{3d}$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(2*(((a*(A - B) - b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqr
t[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 - ((b*(A - B) +
 a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[
2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/
d + (2*(A*b + a*B)*Sqrt[Tan[c + d*x]])/d + (2*b*B*Tan[c + d*x]^(3/2))/(3*d
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{2Bb\tan(dx+c)^{\frac{3}{2}}}{3}+2Ab\sqrt{\tan(dx+c)}+2Ba\sqrt{\tan(dx+c)}+\frac{(-Ab-Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{\right.}\right.}{4}}{}$
default	$\frac{\frac{2Bb\tan(dx+c)^{\frac{3}{2}}}{3}+2Ab\sqrt{\tan(dx+c)}+2Ba\sqrt{\tan(dx+c)}+\frac{(-Ab-Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{\right.}\right.}{4}}{}$
parts	$\frac{(Ab+Ba)\left(2\sqrt{\tan(dx+c)}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(d\right.}\right.}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(2/3*B*b*tan(d*x+c)^(3/2)+2*A*b*tan(d*x+c)^(1/2)+2*B*a*tan(d*x+c)^(1/2
)+1/4*(-A*b-B*a)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(
d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2
*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(A*a-B*b)*2^(1/2)*(ln((tan(d*x+c
)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*a
rctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(154) = 308$.

Time = 0.11 (sec) , antiderivative size = 901, normalized size of antiderivative = 5.06

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-1/6*(6*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A
^2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A + B)*a + (A - B)*b)*d*
sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^
2)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 -
B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2
*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2)
*a^2 + (A^2 - B^2)*b^2)) + 6*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2
*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A +
 B)*a + (A - B)*b)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (
A^2 + 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B +
 B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 -
 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*
A*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2)) - 3*sqrt(1/2)*d*sqrt(((A^2 +
 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*log(
2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2
*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A + B)*a - (A - B)*b - ((A + B
)*a + (A - B)*b)*tan(d*x + c)) + 3*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a
^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*log(-2*sqrt(1/2)*d*
sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^
2)/d^2)*sqrt(tan(d*x + c)) - (A + B)*a - (A - B)*b - ((A + B)*a + (A - ...
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})\,\sqrt{\tan{(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))*sqrt(tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{8\,Bb\tan{(dx+c)}^{\frac{3}{2}} + 6\,\sqrt{2}((A-B)a-(A+B)b)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan{(dx+c)}}\right)\right) + 6\,\sqrt{2}((A-$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/12*(8*B*b*tan(d*x + c)^(3/2) + 6*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(
1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 6*sqrt(2)*((A - B)*a - (A
+ B)*b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 3*sqrt(2)*
((A + B)*a + (A - B)*b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)
 + 3*sqrt(2)*((A + B)*a + (A - B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan
(d*x + c) + 1) + 24*(B*a + A*b)*sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 1456, normalized size of antiderivative = 8.18

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
2*atanh((32*A^2*a^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 -
 A^4*a^4*d^4)^(1/2)/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2))/((16*A*b*(2*A^4*a^
2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a^3)/d + (16*A
^3*a*b^2)/d) - (32*A^2*b^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^
4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2))/((16*A*b*(2
*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a^3)/d
+ (16*A^3*a*b^2)/d))*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2
)/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2) - 2*atanh((32*A^2*a^2*tan(c + d*x)^(1
/2)*((A^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(
1/2)/(4*d^4))^(1/2))/((16*A^3*a^3)/d + (16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^
4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a*b^2)/d) - (32*A^2*b^2*tan(c +
d*x)^(1/2)*((A^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4
*d^4)^(1/2)/(4*d^4))^(1/2))/((16*A^3*a^3)/d + (16*A*b*(2*A^4*a^2*b^2*d^4 -
 A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*a*b^2)/d))*((A^2*a*b)/(2*
d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2
) - atan((B^2*a^2*tan(c + d*x)^(1/2)*(- (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 -
 B^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2)*32i)/((16*B*a*(2*B^
4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2))/d^3 - (16*B^3*b^3)/d + (
16*B^3*a^2*b)/d) - (B^2*b^2*tan(c + d*x)^(1/2)*(- (2*B^4*a^2*b^2*d^4 - B^4
*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2)*32i)/(...
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{4\sqrt{\tan(dx+c)}\,ab-2\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)abd+\left(\int\sqrt{\tan(dx+c)}dx\right)a^2d+\left(\int\sqrt{\tan(dx+c)}\,\tan(dx+\right.}{d}$$

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(4*sqrt(tan(c + d*x))*a*b - 2*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*b*d
 + int(sqrt(tan(c + d*x)),x)*a**2*d + int(sqrt(tan(c + d*x))*tan(c + d*x)*
*2,x)*b**2*d)/d
```

3.381 $\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 152

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{(b(A-B)+a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{(b(A-B)+a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{(a(A-B)-b(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{2bB\sqrt{\tan(c+dx)}}{d}$$

output

```
1/2*(b*(A-B)+a*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*(b
*(A-B)+a*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a*(A-B)-
b*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d+2*b*B*
tan(d*x+c)^(1/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx =$$
$$-\frac{\sqrt[4]{-1}(a-ib)(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt[4]{-1}(a+ib)(A+iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\tan\right.}}{d}$$

input
```
Integrate[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x
]
```

output
```
-(((-1)^(1/4)*(a - I*b)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] +
(-1)^(1/4)*(a + I*b)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - 2*
b*B*Sqrt[Tan[c + d*x]])/d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4075, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4075}$$

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 3042

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 4017

$$\frac{2\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 1482

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A+B)+b(A-B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+\right.}{d}$$
$$\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 1476

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d}}\right.\right.}{d}$$
$$\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 1082

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan\right.}{\sqrt{2}}\right.\right.}{d}$$
$$\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 217

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}$$
$$\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 1479

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\right.}{d}$$

$$\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

$\downarrow$ 25

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(a\right.}{d}$$

$$\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right.\right.}{d}$$

$$\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\mathrm{lo}}{-}\right.\right.}{d}$$

$$\frac{2bB\sqrt{\tan(c+dx)}}{d}$$

input
```
Int[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x]
```

output
```
(2*(((b*(A - B) + a*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqr
t[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((a*(A - B) -
 b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[
2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/
d + (2*b*B*Sqrt[Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{2\sqrt{\tan(dx+c)}\,Bb+\frac{(aA-Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(d}\right.\right.}{4}}{d}$
default	$\frac{2\sqrt{\tan(dx+c)}\,Bb+\frac{(aA-Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(d}\right.\right.}{4}}{d}$
parts	$\frac{(Ab+Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4d}$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(2*tan(d*x+c)^(1/2)*B*b+1/4*(A*a-B*b)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*
tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^
(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(A*b+B*
a)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)
*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(
1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(133) = 266$.

Time = 0.10 (sec) , antiderivative size = 886, normalized size of antiderivative = 5.83

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
-1/2*(2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A
^2 - 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a - (A + B)*b)*d*
sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^
2)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 -
B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2
*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2)
*a^2 + (A^2 - B^2)*b^2)) + 2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2
*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A -
 B)*a - (A + B)*b)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (
A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B +
 B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 -
 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*
A*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2)) + sqrt(1/2)*d*sqrt(((A^2 - 2
*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*log(2*
sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A
*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A - B)*a + (A + B)*b - ((A - B)*
a - (A + B)*b)*tan(d*x + c)) - sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 -
 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*log(-2*sqrt(1/2)*d*sqrt
(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d
^2)*sqrt(tan(d*x + c)) - (A - B)*a + (A + B)*b - ((A - B)*a - (A + B)*b...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})}{\sqrt{\tan{(c+dx)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))/sqrt(tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{2\sqrt{2}((A+B)a+(A-B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((A+B)a+(A-B)b)\operatorname{arc}}{}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
1/4*(2*sqrt(2)*((A + B)*a + (A - B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqr
t(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a + (A - B)*b)*arctan(-1/2*sqrt(2)*
(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((A - B)*a - (A + B)*b)*log(sq
rt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A - B)*a - (A + B
)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*B*b*sqrt(tan(
d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 1420, normalized size of antiderivative = 9.34

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x)))/tan(c + d*x)^(1/2),x)
```

output
```
2*atanh((32*A^2*a^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 -
 A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2))/((16*A^3*b^3)/d +
(16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3 - (16*A
^3*a^2*b)/d) - (32*A^2*b^2*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^
4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2))/((16*A^3*b^
3)/d + (16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2))/d^3
- (16*A^3*a^2*b)/d))*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2
)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2) - 2*atanh((32*A^2*a^2*tan(c + d*x)^(1
/2)*(- (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^
2*a*b)/(2*d^2))^(1/2))/((16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4
*d^4)^(1/2))/d^3 - (16*A^3*b^3)/d + (16*A^3*a^2*b)/d) - (32*A^2*b^2*tan(c
+ d*x)^(1/2)*(- (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d
^4) - (A^2*a*b)/(2*d^2))^(1/2))/((16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4
- A^4*a^4*d^4)^(1/2))/d^3 - (16*A^3*b^3)/d + (16*A^3*a^2*b)/d))*(- (2*A^4*
a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2)
)^(1/2) - 2*atanh((32*B^2*a^2*tan(c + d*x)^(1/2)*((B^2*a*b)/(2*d^2) - (2*B
^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2))/((16*B^3
*a^3)/d + (16*B*b*(2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2))/d
^3 - (16*B^3*a*b^2)/d) - (32*B^2*b^2*tan(c + d*x)^(1/2)*((B^2*a*b)/(2*d^2)
 - (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2)...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{2\sqrt{\tan(dx+c)}\,b^2+\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a^2d-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)b^2d+2\left(\int \sqrt{\tan(dx+c)}dx\right)abd}{d}$$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*b**2 + int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a**2*d
 - int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b**2*d + 2*int(sqrt(tan(c + d*x)
),x)*a*b*d)/d
```

3.382 $\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 153

$$\begin{aligned}\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx &= \frac{(a(A-B)-b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad - \frac{(a(A-B)-b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad + \frac{(b(A-B)+a(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{2aA}{d\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
-1/2*(a*(A-B)-b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(
a*(A-B)-b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*(b*(A-B)
+a*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d-2*a*A
/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{-2\sqrt{2}(a(A-B)-b(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)+}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x
]
```

output
```
-1/4*(-2*Sqrt[2]*(a*(A - B) - b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + Sqrt[2]*(b*(A - B) + a*
(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) + (8*a*A)/Sqrt[Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4074, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4074}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 4017$$

$$\frac{2\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 1482$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+\right.}{d}$$

$$\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 1476$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d}}\right.\right.}{d}$$

$$\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 1082$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan\right.}{\sqrt{2}}\right.\right.}{d}$$

$$\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 217$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}$$

$$\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 1479$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-}{d}$$

$$\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 25

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(a}{d}$$

$$\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right.}{d}$$

$$\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(a(A-B)-\right.}{d}$$

$$\frac{2aA}{d\sqrt{\tan(c+dx)}}$$

input

```
Int[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x]
```

output

```
(2*(-1/2*((a*(A - B) - b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((b*(A - B
) + a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sq
rt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2
))/d - (2*a*A)/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{-\frac{2aA}{\sqrt{\tan(dx+c)}}+\frac{(Ab+Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right.}{4}}{d}$
default	$\frac{-\frac{2aA}{\sqrt{\tan(dx+c)}}+\frac{(Ab+Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right.}{4}}{d}$
parts	$\frac{(Ab+Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4d}$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(-2*a*A/tan(d*x+c)^(1/2)+1/4*(A*b+B*a)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)
*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2
^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-A*a+
B*b)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/
2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2
^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(134) = 268$.

Time = 0.10 (sec) , antiderivative size = 916, normalized size of antiderivative = 5.99

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm=
"fricas")
```

output
```
1/2*(2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^
2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A + B)*a + (A - B)*b)*d*s
qrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2
)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B
^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*
(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2)*
a^2 + (A^2 - B^2)*b^2))*tan(d*x + c) + 2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B +
B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqr
t(1/2)*((A + B)*a + (A - B)*b)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 -
B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A
^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*
sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^
2)/d^2))/(4*A*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2))*tan(d*x + c) - s
qrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*
B + B^2)*b^2)/d^2)*log(2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^
2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A + B)*
a - (A - B)*b - ((A + B)*a + (A - B)*b)*tan(d*x + c))*tan(d*x + c) + sqrt(
1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B +
B^2)*b^2)/d^2)*log(-2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 -
 B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A + B)*...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))/tan(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\sqrt{2}((A-B)a-(A+B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((A-B)a-(A+B)b)}{}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm=
"maxima")
```

output
```
-1/4*(2*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sq
rt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(-1/2*sqrt(2)
*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((A + B)*a + (A - B)*b)*log(s
qrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((A + B)*a + (A -
B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*A*a/sqrt(tan
(d*x + c)))/d
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorithm=
"giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 1420, normalized size of antiderivative = 9.28

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x)))/tan(c + d*x)^(3/2),x)
```

output
```
2*atanh((32*A^2*a^2*d^3*tan(c + d*x)^(1/2)*((A^2*a*b)/(2*d^2) - (2*A^4*a^2
*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2))/(16*A*b*(2*A^4
*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2) + 16*A^3*a^3*d^2 - 16*A^3*
a*b^2*d^2) - (32*A^2*b^2*d^3*tan(c + d*x)^(1/2)*((A^2*a*b)/(2*d^2) - (2*A^
4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2))/(16*A*b*(
2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2) + 16*A^3*a^3*d^2 - 16
*A^3*a*b^2*d^2))*((A^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A
^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2) - 2*atanh((32*A^2*a^2*d^3*tan(c + d*x)^(1
/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) + (A^2*
a*b)/(2*d^2))^(1/2))/(16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^
4)^(1/2) - 16*A^3*a^3*d^2 + 16*A^3*a*b^2*d^2) - (32*A^2*b^2*d^3*tan(c + d*
x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) +
(A^2*a*b)/(2*d^2))^(1/2))/(16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a
^4*d^4)^(1/2) - 16*A^3*a^3*d^2 + 16*A^3*a*b^2*d^2))*((2*A^4*a^2*b^2*d^4 -
A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2) - 2*at
anh((32*B^2*a^2*tan(c + d*x)^(1/2)*(- (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B
^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2))/((16*B*a*(2*B^4*a^2*
b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2))/d^3 - (16*B^3*b^3)/d + (16*B^3
*a^2*b)/d) - (32*B^2*b^2*tan(c + d*x)^(1/2)*(- (2*B^4*a^2*b^2*d^4 - B^4*b^
4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2))/((16*B*a...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)a^2 + 2\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)ab + \left(\int \sqrt{\tan(dx+c)}dx\right)b^2$$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a**2 + 2*int(sqrt(tan(c + d*x))/
tan(c + d*x),x)*a*b + int(sqrt(tan(c + d*x)),x)*b**2
```

3.383 $\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 177

$$\begin{aligned}\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx &= \frac{(b(A-B)+a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad - \frac{(b(A-B)+a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad - \frac{(a(A-B)-b(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad - \frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(Ab+aB)}{d\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
-1/2*(b*(A-B)+a*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(
b*(A-B)+a*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(a*(A-B)
-b*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d-2/3*a
*A/d/tan(d*x+c)^(3/2)-2*(A*b+B*a)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{6\sqrt{2}(b(A-B)+a(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)+3\sqrt{2}}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x
]
```

output
```
(6*Sqrt[2]*(b*(A - B) + a*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
 - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + 3*Sqrt[2]*(a*(A - B) - b*(A +
 B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) - (8*a*A)/Tan[c + d*x]^(3/2) - (24*(A
*b + a*B))/Sqrt[Tan[c + d*x]])/(12*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4074, 3042, 4012, 25, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$
$$\Big\downarrow \text{4074}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4012

$$\int -\frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 25

$$-\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$-\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4017

$$-\frac{2\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1482

$$-\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A+B)+b(A-B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c-}{d}$$
$$\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1476

$$-\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+}}\right.\right.}{d}$$
$$\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1082

$$-\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{t\right.}}{\sqrt{2}}\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 217

$$-\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1479

$$-\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+d\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1103

$$-\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a(A-B)-b(A+B))\left(\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x]
```

output
```
(-2*(((b*(A - B) + a*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sq
rt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((a*(A - B)
- b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt
[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))
/d - (2*a*A)/(3*d*Tan[c + d*x]^(3/2)) - (2*(A*b + a*B))/(d*Sqrt[Tan[c + d*
x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{(-aA+Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-Ab-Ba\cdots}{\cdots}}{\cdots}$
default	$\frac{\frac{(-aA+Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(-Ab-Ba\cdots}{\cdots}}{\cdots}$
parts	$\frac{(Ab+Ba)\left(-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}-\frac{\cdots}{\sqrt{\tan(\cdots}}\cdots\right.}{d}$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(1/4*(-A*a+B*b)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(t
an(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-A*b-B*a)*2^(1/2)*(ln((tan(d
*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))
+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)
))-2*(A*b+B*a)/tan(d*x+c)^(1/2)-2/3*a*A/tan(d*x+c)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(153) = 306$.

Time = 0.10 (sec) , antiderivative size = 944, normalized size of antiderivative = 5.33

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
1/6*(6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^
2 - 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a - (A + B)*b)*d*s
qrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2
)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B
^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*
(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2)*
a^2 + (A^2 - B^2)*b^2))*tan(d*x + c)^2 + 6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B
+ B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*arctan((2*s
qrt(1/2)*((A - B)*a - (A + B)*b)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2
- B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt((
(A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2
)*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*
b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2))*tan(d*x + c)^2
 + 3*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2
+ 2*A*B + B^2)*b^2)/d^2)*log(2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 -
 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A
 - B)*a + (A + B)*b - ((A - B)*a - (A + B)*b)*tan(d*x + c))*tan(d*x + c)^2
 - 3*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2
+ 2*A*B + B^2)*b^2)/d^2)*log(-2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2
- 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) ...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))/tan(c + d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{6\sqrt{2}((A+B)a+(A-B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+6\sqrt{2}((A+B)a+(A-B)b)\ldots}{}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
-1/12*(6*sqrt(2)*((A + B)*a + (A - B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*s
qrt(tan(d*x + c)))) + 6*sqrt(2)*((A + B)*a + (A - B)*b)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + 3*sqrt(2)*((A - B)*a - (A + B)*b)*lo
g(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)*((A - B)*a -
(A + B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*(A*a +
3*(B*a + A*b)*tan(d*x + c))/tan(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)}{\tan(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 1448, normalized size of antiderivative = 8.18

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x)))/tan(c + d*x)^(5/2),x)
```

output
```
2*atanh((32*A^2*a^2*d^3*tan(c + d*x)^(1/2)*(- (2*A^4*a^2*b^2*d^4 - A^4*b^4
*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2))/(16*A*a*(2*A
^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2) - 16*A^3*b^3*d^2 + 16*A^
3*a^2*b*d^2) - (32*A^2*b^2*d^3*tan(c + d*x)^(1/2)*(- (2*A^4*a^2*b^2*d^4 -
A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2))/(16*A
*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2) - 16*A^3*b^3*d^2
+ 16*A^3*a^2*b*d^2))*(- (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1
/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2) - 2*atanh((32*A^2*a^2*d^3*tan(c + d
*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) -
 (A^2*a*b)/(2*d^2))^(1/2))/(16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*
a^4*d^4)^(1/2) + 16*A^3*b^3*d^2 - 16*A^3*a^2*b*d^2) - (32*A^2*b^2*d^3*tan(
c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d
^4) - (A^2*a*b)/(2*d^2))^(1/2))/(16*A*a*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 -
 A^4*a^4*d^4)^(1/2) + 16*A^3*b^3*d^2 - 16*A^3*a^2*b*d^2))*((2*A^4*a^2*b^2*
d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) - (A^2*a*b)/(2*d^2))^(1/2)
+ 2*atanh((32*B^2*a^2*d^3*tan(c + d*x)^(1/2)*((B^2*a*b)/(2*d^2) - (2*B^4*a
^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2))/(16*B*b*(2*B
^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2) + 16*B^3*a^3*d^2 - 16*B^
3*a*b^2*d^2) - (32*B^2*b^2*d^3*tan(c + d*x)^(1/2)*((B^2*a*b)/(2*d^2) - (2*
B^4*a^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2))/(16*...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)a^2 + 2\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)ab + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)b^2$$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*a**2 + 2*int(sqrt(tan(c + d*x))/
tan(c + d*x)**2,x)*a*b + int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b**2
```

3.384 $\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 203

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{(a(A-B)-b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{(a(A-B)-b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(b(A-B)+a(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2(Ab+aB)}{3d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}$$

output
```
1/2*(a*(A-B)-b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a
*(A-B)-b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(b*(A-B)+
a*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d-2/5*a*
A/d/tan(d*x+c)^(5/2)-2/3*(A*b+B*a)/d/tan(d*x+c)^(3/2)+2*(A*a-B*b)/d/tan(d*
x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{30\sqrt{2}(a(A-B)-b(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)-}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x
]
```

output
```
-1/60*(30*Sqrt[2]*(a*(A - B) - b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
 d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) - 15*Sqrt[2]*(b*(A - B)
+ a*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) + (24*a*A)/Tan[c + d*x]^(5/2)
 + (40*(A*b + a*B))/Tan[c + d*x]^(3/2) - (120*(a*A - b*B))/Sqrt[Tan[c + d*
x]])/d
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {3042, 4074, 3042, 4012, 25, 3042, 4012, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \; 4074$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan(c+dx)^{5/2}}dx-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4012

$$\int -\frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 25

$$-\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$-\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx-\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4012

$$-\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$-\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4017

$$-\frac{2\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 1482

$$-\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c-}{d}$$

$$\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 1476

$$-\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+}}\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 1082

$$-\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{t}\right.}{\sqrt{2}}\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 217

$$-\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 1479

$$-\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)\right.}{d}$$

$$\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(\right.}{d}$$

$$\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+d\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\big\downarrow$ 1103

$$-\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(a(A-B)\right.}{d}$$

$$\frac{2(aB+Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(aA-bB)}{d\sqrt{\tan(c+dx)}}-\frac{2aA}{5d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x]
```

output
```
(-2*(-1/2*((a*(A - B) - b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]
]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((b*(A -
B) + a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/S
qrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/
2))/d - (2*a*A)/(5*d*Tan[c + d*x]^(5/2)) - (2*(A*b + a*B))/(3*d*Tan[c + d*
x]^(3/2)) + (2*(a*A - b*B))/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{(-Ab-Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(aA-Bb)}{}}{}$
default	$\frac{\frac{(-Ab-Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}+\frac{(aA-Bb)}{}}{}$
parts	$\frac{(Ab+Ba)\left(-\frac{2}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(d}\right.\right.}{4}\right.}{d}$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(1/4*(-A*b-B*a)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(t
an(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(A*a-B*b)*2^(1/2)*(ln((tan(d*
x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+
2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
)-2/3*(A*b+B*a)/tan(d*x+c)^(3/2)-2*(-A*a+B*b)/tan(d*x+c)^(1/2)-2/5*a*A/tan
(d*x+c)^(5/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(175) = 350.

Time = 0.11 (sec) , antiderivative size = 961, normalized size of antiderivative = 4.73

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm=
"fricas")
```

output
```
-1/30*(30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b +
(A^2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A + B)*a + (A - B)*b)*
d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*
b^2)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2
- B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 -
 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^
2)*a^2 + (A^2 - B^2)*b^2))*tan(d*x + c)^3 + 30*sqrt(1/2)*d*sqrt(((A^2 - 2*
A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*arctan(
(2*sqrt(1/2)*((A + B)*a + (A - B)*b)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(
A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sq
rt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)
/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B
^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2))*tan(d*x +
c)^3 - 15*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b +
(A^2 - 2*A*B + B^2)*b^2)/d^2)*log(2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*
a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c))
 - (A + B)*a - (A - B)*b - ((A + B)*a + (A - B)*b)*tan(d*x + c))*tan(d*x +
 c)^3 + 15*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b +
 (A^2 - 2*A*B + B^2)*b^2)/d^2)*log(-2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2
)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x ...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))/tan(c + d*x)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{30\sqrt{2}((A-B)a-(A+B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+30\sqrt{2}((A-B)a-(A+B)b)}{}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm=
"maxima")
```

output
```
1/60*(30*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*s
qrt(tan(d*x + c)))) + 30*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(-1/2*sqrt(
2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 15*sqrt(2)*((A + B)*a + (A - B)*b)*
log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 15*sqrt(2)*((A + B)*a
 + (A - B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*(15*
(A*a - B*b)*tan(d*x + c)^2 - 3*A*a - 5*(B*a + A*b)*tan(d*x + c))/tan(d*x +
 c)^(5/2))/d
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)}{\tan(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorithm=
"giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 1473, normalized size of antiderivative = 7.26

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x)))/tan(c + d*x)^(7/2),x)
```

output
```
2*atanh((32*A^2*a^2*d^3*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d
^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2))/(16*A*b*(2*A^4
*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2) - 16*A^3*a^3*d^2 + 16*A^3*
a*b^2*d^2) - (32*A^2*b^2*d^3*tan(c + d*x)^(1/2)*((2*A^4*a^2*b^2*d^4 - A^4*
b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4) + (A^2*a*b)/(2*d^2))^(1/2))/(16*A*b*(
2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2) - 16*A^3*a^3*d^2 + 16
*A^3*a*b^2*d^2))*((2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4
*d^4) + (A^2*a*b)/(2*d^2))^(1/2) - 2*atanh((32*A^2*a^2*d^3*tan(c + d*x)^(1
/2)*((A^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(
1/2)/(4*d^4))^(1/2))/(16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^
4)^(1/2) + 16*A^3*a^3*d^2 - 16*A^3*a*b^2*d^2) - (32*A^2*b^2*d^3*tan(c + d*
x)^(1/2)*((A^2*a*b)/(2*d^2) - (2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d
^4)^(1/2)/(4*d^4))^(1/2))/(16*A*b*(2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a
^4*d^4)^(1/2) + 16*A^3*a^3*d^2 - 16*A^3*a*b^2*d^2))*((A^2*a*b)/(2*d^2) - (
2*A^4*a^2*b^2*d^4 - A^4*b^4*d^4 - A^4*a^4*d^4)^(1/2)/(4*d^4))^(1/2) + 2*at
anh((32*B^2*a^2*d^3*tan(c + d*x)^(1/2)*(- (2*B^4*a^2*b^2*d^4 - B^4*b^4*d^4
 - B^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2))/(16*B*a*(2*B^4*a
^2*b^2*d^4 - B^4*b^4*d^4 - B^4*a^4*d^4)^(1/2) - 16*B^3*b^3*d^2 + 16*B^3*a^
2*b*d^2) - (32*B^2*b^2*d^3*tan(c + d*x)^(1/2)*(- (2*B^4*a^2*b^2*d^4 - B^4*
b^4*d^4 - B^4*a^4*d^4)^(1/2)/(4*d^4) - (B^2*a*b)/(2*d^2))^(1/2))/(16*B*...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)a^2 + 2\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)ab$$
$$+ \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)b^2$$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x)**4,x)*a**2 + 2*int(sqrt(tan(c + d*x))/
tan(c + d*x)**3,x)*a*b + int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*b**2
```

3.385 $\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 330

$$\begin{aligned}
&\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx \\
&= \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad-\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad+\frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad-\frac{2(2aAb+a^2B-b^2B)\sqrt{\tan(c+dx)}}{d}+\frac{2(a^2A-Ab^2-2abB)\tan^{\frac{3}{2}}(c+dx)}{3d} \\
&\quad+\frac{2(2aAb+a^2B-b^2B)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{2b(9Ab+11aB)\tan^{\frac{7}{2}}(c+dx)}{63d} \\
&\quad+\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}
\end{aligned}$$

output
```
-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
*2^(1/2)/d-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))*2^(1/2)/d+1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctanh(2^(1/2)*
tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d-2*(2*A*a*b+B*a^2-B*b^2)*tan(d*x
+c)^(1/2)/d+2/3*(A*a^2-A*b^2-2*B*a*b)*tan(d*x+c)^(3/2)/d+2/5*(2*A*a*b+B*a^
2-B*b^2)*tan(d*x+c)^(5/2)/d+2/63*b*(9*A*b+11*B*a)*tan(d*x+c)^(7/2)/d+2/9*b
*B*tan(d*x+c)^(7/2)*(a+b*tan(d*x+c))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.23 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.62

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$= \frac{-315\sqrt[4]{-1}(a-ib)^2(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+315(-1)^{3/4}(a+ib)^2(A+iB)\text{arctanh}\left((-\right.}{}$$

input
```
Integrate[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x
]
```

output
```
(-315*(-1)^(1/4)*(a - I*b)^2*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]
]] + 315*(-1)^(3/4)*(a + I*b)^2*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c +
d*x]]] + 2*Sqrt[Tan[c + d*x]]*(-315*(2*a*A*b + a^2*B - b^2*B) + 105*(a^2*A
 - A*b^2 - 2*a*b*B)*Tan[c + d*x] + 63*(2*a*A*b + a^2*B - b^2*B)*Tan[c + d*
x]^2 + 45*b*(A*b + 2*a*B)*Tan[c + d*x]^3 + 35*b^2*B*Tan[c + d*x]^4))/(315*
d)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.06, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4090, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{5/2}(a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2}{9}\int \frac{1}{2}\tan^{\frac{5}{2}}(c+dx)\left(b(9Ab+11aB)\tan^2(c+dx)+9\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(9aA-7bB)\right)dx+\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \tan^{\frac{5}{2}}(c+dx)\left(b(9Ab+11aB)\tan^2(c+dx)+9\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(9aA-7bB)\right)dx+\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \tan(c+dx)^{5/2}\left(b(9Ab+11aB)\tan(c+dx)^2+9\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(9aA-7bB)\right)dx+\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$$\Big\downarrow \text{4113}$$

$$\frac{1}{9}\left(\int \tan^{\frac{5}{2}}(c+dx)\left(9\left(Aa^2-2bBa-Ab^2\right)+9\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{2b(11aB+9Ab)\tan^{\frac{7}{2}}(c+\right.$$
$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\int \tan(c+dx)^{5/2}\left(9\left(Aa^2-2bBa-Ab^2\right)+9\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{2b(11aB+9Ab)\tan^{\frac{7}{2}}(c\right.$$
$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$\downarrow$ 4011

$$\frac{1}{9}\left(\int \tan^{\frac{3}{2}}(c+dx)\left(9\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)-9\left(Ba^2+2Aba-b^2B\right)\right)dx+\frac{18\left(a^2B+2aAb-b^2B\right)t}{5d}\right.$$
$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\int \tan(c+dx)^{3/2}\left(9\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)-9\left(Ba^2+2Aba-b^2B\right)\right)dx+\frac{18\left(a^2B+2aAb-b^2B\right)}{5d}\right.$$
$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$\downarrow$ 4011

$$\frac{1}{9}\left(\int \sqrt{\tan(c+dx)}\left(-9\left(Aa^2-2bBa-Ab^2\right)-9\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{18\left(a^2B+2aAb-b^2B\right.}{5d}\right.$$
$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\int \sqrt{\tan(c+dx)}\left(-9\left(Aa^2-2bBa-Ab^2\right)-9\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{18\left(a^2B+2aAb-b^2B\right.}{5d}\right.$$
$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$\downarrow$ 4011

$$\frac{1}{9}\left(\int \frac{9\left(Ba^2+2Aba-b^2B\right)-9\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{18\left(a^2B+2aAb-b^2B\right)\tan^{\frac{5}{2}}(c+dx)}{5d}+\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{9}\left(\int \frac{9\left(Ba^2+2Aba-b^2B\right)-9\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{18\left(a^2B+2aAb-b^2B\right)\tan^{\frac{5}{2}}(c+dx)}{5d}+\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$$\Big\downarrow \; 4017$$

$$\frac{1}{9}\left(\frac{2\int \frac{9\left(Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{18\left(a^2B+2aAb-b^2B\right)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{6\left(a^2\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(\frac{18\int \frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{18\left(a^2B+2aAb-b^2B\right)\tan^{\frac{5}{2}}(c+dx)}{5d}+\frac{6\left(a^2A\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$$\Big\downarrow \; 1482$$

$$\frac{1}{9}\left(\frac{18\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$$\Big\downarrow \; 1476$$

$$\frac{1}{9}\left(\frac{18\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$\Big\downarrow$ 1082

$$\frac{1}{9}\left(\frac{18\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$\Big\downarrow$ 217

$$\frac{1}{9}\left(\frac{18\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$\Big\downarrow$ 1479

$$\frac{1}{9}\left(\frac{18\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx}\right.}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(}}}{2\sqrt{}}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$\Big\downarrow$ 25

$$\frac{1}{9}\left(\frac{18\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}\cdots}}{2\sqrt{2}}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{9}\left(\frac{18\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}\cdots}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c\cdots}}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{9}\left(\frac{18\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+\cdots\right.}{2\sqrt{2}}\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))}{9d}$$

input
```
Int[Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b*B*Tan[c + d*x]^(7/2)*(a + b*Tan[c + d*x]))/(9*d) + ((18*(-1/2*((a^2*(
A - B) - b^2*(A - B) - 2*a*b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((2*a*
b*(A - B) + a^2*(A + B) - b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
 + d*x]]/(2*Sqrt[2])))/2))/d - (18*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Tan[c +
d*x]])/d + (6*(a^2*A - A*b^2 - 2*a*b*B)*Tan[c + d*x]^(3/2))/d + (18*(2*a*A
*b + a^2*B - b^2*B)*Tan[c + d*x]^(5/2))/(5*d) + (2*b*(9*A*b + 11*a*B)*Tan[
c + d*x]^(7/2))/(7*d))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{2B\,b^{2}\tan(dx+c)^{\frac{9}{2}}}{9}+\frac{2A\,b^{2}\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{4Bab\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{4Aab\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2B\,a^{2}\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{2B\,b^{2}\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2A\,a$
default	$\frac{2B\,b^{2}\tan(dx+c)^{\frac{9}{2}}}{9}+\frac{2A\,b^{2}\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{4Bab\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{4Aab\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2B\,a^{2}\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{2B\,b^{2}\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2A\,a$
parts	$\frac{(A\,b^{2}+2Bab)\left(\frac{2\tan(dx+c)^{\frac{7}{2}}}{7}-\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\,\mathrm{arct}}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2/9*B*b^2*tan(d*x+c)^(9/2)+2/7*A*b^2*tan(d*x+c)^(7/2)+4/7*B*a*b*tan(d
*x+c)^(7/2)+4/5*A*a*b*tan(d*x+c)^(5/2)+2/5*B*a^2*tan(d*x+c)^(5/2)-2/5*B*b^
2*tan(d*x+c)^(5/2)+2/3*A*a^2*tan(d*x+c)^(3/2)-2/3*A*b^2*tan(d*x+c)^(3/2)-4
/3*B*a*b*tan(d*x+c)^(3/2)-4*A*a*b*tan(d*x+c)^(1/2)-2*B*a^2*tan(d*x+c)^(1/2
)+2*tan(d*x+c)^(1/2)*B*b^2+1/4*(2*A*a*b+B*a^2-B*b^2)*2^(1/2)*(ln((tan(d*x+
c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*
arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+
1/4*(-A*a^2+A*b^2+2*B*a*b)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2
)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. $2(295) = 590$.

Time = 0.11 (sec) , antiderivative size = 1546, normalized size of antiderivative = 4.68

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/630*(630*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*
b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B
^2)*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*
b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*
B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2)*sqr
t(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b
 + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2
)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 +
 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2
))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 -
(A^2 - B^2)*b^4)) + 630*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2
 - B^2)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2
 - 2*A*B + B^2)*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*
b - (A + B)*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2
*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b
^4)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 -
 B^2)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 +
 2*A*B + B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*
b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B
^2)*b^4)/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - ...
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(5/2)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{280\,Bb^2\tan(dx+c)^{\frac{9}{2}} + 360\,(2\,Bab + Ab^2)\tan(dx+c)^{\frac{7}{2}} + 504\,(Ba^2 + 2\,Aab - Bb^2)\tan(dx+c)^{\frac{5}{2}} - 63}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/1260*(280*B*b^2*tan(d*x + c)^(9/2) + 360*(2*B*a*b + A*b^2)*tan(d*x + c)^
(7/2) + 504*(B*a^2 + 2*A*a*b - B*b^2)*tan(d*x + c)^(5/2) - 630*sqrt(2)*((A
 - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*s
qrt(tan(d*x + c)))) - 630*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b
^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + 315*sqrt(2)*((
A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) +
 tan(d*x + c) + 1) - 315*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^
2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 840*(A*a^2 - 2*B*
a*b - A*b^2)*tan(d*x + c)^(3/2) - 2520*(B*a^2 + 2*A*a*b - B*b^2)*sqrt(tan(
d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.80 (sec) , antiderivative size = 3914, normalized size of antiderivative = 11.86

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
atan((B^2*a^4*tan(c + d*x)^(1/2)*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 -
B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)
/(4*d^4) - (B^2*a^3*b)/d^2)^(1/2)*32i)/((16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^
4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/
d^3 - (192*B^3*a^3*b^3)/d - (16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 -
B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^3 + (32*B^
3*a*b^5)/d + (32*B^3*a^5*b)/d) + (B^2*b^4*tan(c + d*x)^(1/2)*((B^2*a*b^3)/
d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4
 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4) - (B^2*a^3*b)/d^2)^(1/2)*32i)/((16*B*
a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 +
 12*B^4*a^6*b^2*d^4)^(1/2))/d^3 - (192*B^3*a^3*b^3)/d - (16*B*b^2*(12*B^4*
a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*
b^2*d^4)^(1/2))/d^3 + (32*B^3*a*b^5)/d + (32*B^3*a^5*b)/d) - (B^2*a^2*b^2*
tan(c + d*x)^(1/2)*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 -
B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4) - (B^
2*a^3*b)/d^2)^(1/2)*192i)/((16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B
^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^3 - (192*B^
3*a^3*b^3)/d - (16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 -
 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2))/d^3 + (32*B^3*a*b^5)/d +
(32*B^3*a^5*b)/d))*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4...
```

Reduce [F]

$$\int \tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{10\sqrt{\tan(dx+c)}\,\tan(dx+c)^4 b^3+54\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 a^2b-18\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{}$$

input
```
int(tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(10*sqrt(tan(c + d*x))*tan(c + d*x)**4*b**3 + 54*sqrt(tan(c + d*x))*tan(c
+ d*x)**2*a**2*b - 18*sqrt(tan(c + d*x))*tan(c + d*x)**2*b**3 - 270*sqrt(t
an(c + d*x))*a**2*b + 90*sqrt(tan(c + d*x))*b**3 + 135*int(sqrt(tan(c + d*
x))/tan(c + d*x),x)*a**2*b*d - 45*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b
**3*d + 135*int(sqrt(tan(c + d*x))*tan(c + d*x)**4,x)*a*b**2*d + 45*int(sq
rt(tan(c + d*x))*tan(c + d*x)**2,x)*a**3*d)/(45*d)
```

3.386 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 295

$$\begin{aligned}\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx &= \frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &- \frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &- \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} \\ &+ \frac{2(a^2A-Ab^2-2abB)\sqrt{\tan(c+dx)}}{d} + \frac{2(2aAb+a^2B-b^2B)\tan^{\frac{3}{2}}(c+dx)}{3d} \\ &+ \frac{2b(7Ab+9aB)\tan^{\frac{5}{2}}(c+dx)}{35d} + \frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}\end{aligned}$$

output
```
-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
*2^(1/2)/d-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))*2^(1/2)/d-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctanh(2^(1/2)*
tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d+2*(A*a^2-A*b^2-2*B*a*b)*tan(d*x
+c)^(1/2)/d+2/3*(2*A*a*b+B*a^2-B*b^2)*tan(d*x+c)^(3/2)/d+2/35*b*(7*A*b+9*B
*a)*tan(d*x+c)^(5/2)/d+2/7*b*B*tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.60

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{105\sqrt[4]{-1}(a-ib)^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+105\sqrt[4]{-1}(a+ib)^2(A+iB)\text{arctanh}\left((-1)^{3/4}\right.}{}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x
]
```

output
```
(105*(-1)^(1/4)*(a - I*b)^2*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]
] + 105*(-1)^(1/4)*(a + I*b)^2*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d
*x]]] + 2*Sqrt[Tan[c + d*x]]*(105*(a^2*A - A*b^2 - 2*a*b*B) + 35*(2*a*A*b
+ a^2*B - b^2*B)*Tan[c + d*x] + 21*b*(A*b + 2*a*B)*Tan[c + d*x]^2 + 15*b^2
*B*Tan[c + d*x]^3))/(105*d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.07, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 4090, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}(a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2}{7}\int \frac{1}{2}\tan^{\frac{3}{2}}(c+dx)\left(b(7Ab+9aB)\tan^2(c+dx)+7\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(7aA-5bB)\right)dx+\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \tan^{\frac{3}{2}}(c+dx)\left(b(7Ab+9aB)\tan^2(c+dx)+7\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(7aA-5bB)\right)dx+\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \tan(c+dx)^{3/2}\left(b(7Ab+9aB)\tan(c+dx)^2+7\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(7aA-5bB)\right)dx+\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{4113}$$

$$\frac{1}{7}\left(\int \tan^{\frac{3}{2}}(c+dx)\left(7\left(Aa^2-2bBa-Ab^2\right)+7\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{2b(9aB+7Ab)\tan^{\frac{5}{2}}(c+}{5d}\right.$$
$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\int \tan(c+dx)^{3/2}\left(7\left(Aa^2-2bBa-Ab^2\right)+7\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{2b(9aB+7Ab)\tan^{\frac{5}{2}}(c}{5d}\right.$$
$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{4011}$$

$$\frac{1}{7}\left(\int \sqrt{\tan(c+dx)}\left(7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)-7\left(Ba^2+2Aba-b^2B\right)\right)dx+\frac{14\left(a^2B+2aAb-b^2B\right)}{3d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\int \sqrt{\tan(c+dx)}\left(7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)-7\left(Ba^2+2Aba-b^2B\right)\right)dx+\frac{14\left(a^2B+2aAb-b^2B\right)}{3d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{4011}$$

$$\frac{1}{7}\left(\int \frac{-7\left(Aa^2-2bBa-Ab^2\right)-7\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{14\left(a^2B+2aAb-b^2B\right)\tan^{\frac{3}{2}}(c+dx)}{3d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\int \frac{-7\left(Aa^2-2bBa-Ab^2\right)-7\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{14\left(a^2B+2aAb-b^2B\right)\tan^{\frac{3}{2}}(c+dx)}{3d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{4017}$$

$$\frac{1}{7}\left(\frac{2\int -\frac{7\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{14\left(a^2B+2aAb-b^2B\right)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{14}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(-\frac{14\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{14\left(a^2B+2aAb-b^2B\right)\tan^{\frac{3}{2}}(c+dx)}{3d}+\frac{14(}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow 1482$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B)\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow 1476$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B)\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow 1082$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow 217$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$$\Big\downarrow 1479$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c\right.}{\tan(c+dx)+\sqrt{2}\sqrt{\tan}}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$\downarrow$ 25

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right.}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+d}}}{2\sqrt{2}}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$\downarrow$ 27

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+d}}{\tan(c+dx)+\sqrt{2}\sqrt{\tan}}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

$\downarrow$ 1103

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(a^2\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}{7d}$$

input
```
Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b*B*Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x]))/(7*d) + ((-14*(((2*a*b*(A
- B) + a^2*(A + B) - b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((a^2*(A
 - B) - b^2*(A - B) - 2*a*b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2])))/2))/d + (14*(a^2*A - A*b^2 - 2*a*b*B)*Sqrt[Tan[c + d*
x]])/d + (14*(2*a*A*b + a^2*B - b^2*B)*Tan[c + d*x]^(3/2))/(3*d) + (2*b*(7
*A*b + 9*a*B)*Tan[c + d*x]^(5/2))/(5*d))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{2B\,b^2\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{2A\,b^2\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{4Bab\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{4Aab\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{2B\,a^2\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{2B\,b^2\tan(dx+c)^{\frac{3}{2}}}{3}+2A\,a \cdots}{}$
default	$\frac{\frac{2B\,b^2\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{2A\,b^2\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{4Bab\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{4Aab\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{2B\,a^2\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{2B\,b^2\tan(dx+c)^{\frac{3}{2}}}{3}+2A\,a \cdots}{}$
parts	$\frac{(A\,b^2+2Bab)\left(\frac{2\tan(dx+c)^{\frac{5}{2}}}{5}-2\sqrt{\tan(dx+c)}+\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\,\text{arc}\cdots}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2/7*B*b^2*tan(d*x+c)^(7/2)+2/5*A*b^2*tan(d*x+c)^(5/2)+4/5*B*a*b*tan(d
*x+c)^(5/2)+4/3*A*a*b*tan(d*x+c)^(3/2)+2/3*B*a^2*tan(d*x+c)^(3/2)-2/3*B*b^
2*tan(d*x+c)^(3/2)+2*A*a^2*tan(d*x+c)^(1/2)-2*tan(d*x+c)^(1/2)*A*b^2-4*tan
(d*x+c)^(1/2)*B*a*b+1/4*(-A*a^2+A*b^2+2*B*a*b)*2^(1/2)*(ln((tan(d*x+c)+2^(
1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan
(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-
2*A*a*b-B*a^2+B*b^2)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(
tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2
))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1527 vs. $2(263) = 526$.

Time = 0.13 (sec) , antiderivative size = 1527, normalized size of antiderivative = 5.18

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/210*(210*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b
 + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^
2)*b^4)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^
2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B
+ B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4)/d^2)*sqrt(
tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b +
2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*
b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6
*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))
/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A
^2 - B^2)*b^4)) + 210*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 -
 B^2)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 +
 2*A*B + B^2)*b^4)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a^2 - 2*(A + B)*a*b -
 (A - B)*b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(A
^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4)
/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^
2)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*
A*B + B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b +
 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)
*b^4)/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2...
```

Sympy [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \int (A+B\tan(c+dx))\,(a+b\tan(c+dx))^2\tan^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2*tan(c + d*x)**(3/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.02

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{120\,Bb^2\tan(dx+c)^{\frac{7}{2}} + 168\,(2\,Bab+Ab^2)\tan(dx+c)^{\frac{5}{2}} - 210\,\sqrt{2}((A+B)a^2 + 2\,(A-B)ab - (A+B}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/420*(120*B*b^2*tan(d*x + c)^(7/2) + 168*(2*B*a*b + A*b^2)*tan(d*x + c)^(
5/2) - 210*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2*
sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) - 210*sqrt(2)*((A + B)*a^2 + 2*(
A - B)*a*b - (A + B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x +
c)))) - 105*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*log(sqrt(2
)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 105*sqrt(2)*((A - B)*a^2 - 2*(A
 + B)*a*b - (A - B)*b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) +
1) + 280*(B*a^2 + 2*A*a*b - B*b^2)*tan(d*x + c)^(3/2) + 840*(A*a^2 - 2*B*a
*b - A*b^2)*sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 15.72 (sec) , antiderivative size = 3869, normalized size of antiderivative = 13.12

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
atan((B^2*a^4*tan(c + d*x)^(1/2)*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*
a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4) - (B^2*a*
b^3)/d^2 + (B^2*a^3*b)/d^2)^(1/2)*32i)/((16*B^3*b^6)/d - (16*B^3*a^6)/d -
(112*B^3*a^2*b^4)/d + (112*B^3*a^4*b^2)/d + (32*B*a*b*(12*B^4*a^2*b^6*d^4
- B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/
2))/d^3) + (B^2*b^4*tan(c + d*x)^(1/2)*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4
- B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4) - (
B^2*a*b^3)/d^2 + (B^2*a^3*b)/d^2)^(1/2)*32i)/((16*B^3*b^6)/d - (16*B^3*a^6
)/d - (112*B^3*a^2*b^4)/d + (112*B^3*a^4*b^2)/d + (32*B*a*b*(12*B^4*a^2*b^
6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^
4)^(1/2))/d^3) - (B^2*a^2*b^2*tan(c + d*x)^(1/2)*((12*B^4*a^2*b^6*d^4 - B^
4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(
4*d^4) - (B^2*a*b^3)/d^2 + (B^2*a^3*b)/d^2)^(1/2)*192i)/((16*B^3*b^6)/d -
(16*B^3*a^6)/d - (112*B^3*a^2*b^4)/d + (112*B^3*a^4*b^2)/d + (32*B*a*b*(12
*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4
*a^6*b^2*d^4)^(1/2))/d^3))*((12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^
4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4) - (B^2*a*b^3)/d
^2 + (B^2*a^3*b)/d^2)^(1/2)*2i - atan((B^2*a^4*tan(c + d*x)^(1/2)*((B^2*a^
3*b)/d^2 - (B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d
^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*32i)...
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{6\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 a\,b^2 + 10\sqrt{\tan(dx+c)}\,a^3 - 30\sqrt{\tan(dx+c)}\,a\,b^2 - 5\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right) a^3 \cdots}{\cdots}$$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(6*sqrt(tan(c + d*x))*tan(c + d*x)**2*a*b**2 + 10*sqrt(tan(c + d*x))*a**3
- 30*sqrt(tan(c + d*x))*a*b**2 - 5*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*
a**3*d + 15*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*b**2*d + 5*int(sqrt(t
an(c + d*x))*tan(c + d*x)**4,x)*b**3*d + 15*int(sqrt(tan(c + d*x))*tan(c +
 d*x)**2,x)*a**2*b*d)/(5*d)
```

3.387 $\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 263

$$
\begin{aligned}
&\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx \\
&= -\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad+\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad-\frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad+\frac{2(2aAb+a^2B-b^2B)\sqrt{\tan(c+dx)}}{d}+\frac{2b(5Ab+7aB)\tan^{\frac{3}{2}}(c+dx)}{15d} \\
&\quad+\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}
\end{aligned}
$$

output
```
1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*
2^(1/2)/d+1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))*2^(1/2)/d-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctanh(2^(1/2)*t
an(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d+2*(2*A*a*b+B*a^2-B*b^2)*tan(d*x+
c)^(1/2)/d+2/15*b*(5*A*b+7*B*a)*tan(d*x+c)^(3/2)/d+2/5*b*B*tan(d*x+c)^(3/2
)*(a+b*tan(d*x+c))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.57

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{15\sqrt[4]{-1}(a-ib)^2(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-15(-1)^{3/4}(a+ib)^2(A+iB)\text{arctanh}\left((-1)^{3/4}\right.}{}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x
]
```

output
```
(15*(-1)^(1/4)*(a - I*b)^2*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
 - 15*(-1)^(3/4)*(a + I*b)^2*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x
]]] + 2*Sqrt[Tan[c + d*x]]*(15*(2*a*A*b + a^2*B - b^2*B) + 5*b*(A*b + 2*a*
B)*Tan[c + d*x] + 3*b^2*B*Tan[c + d*x]^2))/(15*d)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.08, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4090, 27, 3042, 4113, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{\tan(c+dx)}\left(b(5Ab+7aB)\tan^2(c+dx)+5\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(5aA-3bB)\right)dx+\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{\tan(c+dx)}\left(b(5Ab+7aB)\tan^2(c+dx)+5\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(5aA-3bB)\right)dx+\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \sqrt{\tan(c+dx)}\left(b(5Ab+7aB)\tan(c+dx)^2+5\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(5aA-3bB)\right)dx+\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$$\Big\downarrow \text{4113}$$

$$\frac{1}{5}\left(\int \sqrt{\tan(c+dx)}\left(5\left(Aa^2-2bBa-Ab^2\right)+5\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{2b(7aB+5Ab)\tan^{\frac{3}{2}}(c+\right.$$
$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\int \sqrt{\tan(c+dx)}\left(5\left(Aa^2-2bBa-Ab^2\right)+5\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)dx+\frac{2b(7aB+5Ab)\tan^{\frac{3}{2}}(c+\right.$$
$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$$\Big\downarrow \text{4011}$$

$$\frac{1}{5}\left(\int \frac{5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)-5\left(Ba^2+2Aba-b^2B\right)}{\sqrt{\tan(c+dx)}}dx+\frac{10\left(a^2B+2aAb-b^2B\right)\sqrt{\tan(c+dx)}}{d}+\right.$$
$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\int \frac{5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)-5\left(Ba^2+2Aba-b^2B\right)}{\sqrt{\tan(c+dx)}}dx+\frac{10\left(a^2B+2aAb-b^2B\right)\sqrt{\tan(c+dx)}}{d}+\right.$$
$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\downarrow$ 4017

$$\frac{1}{5}\left(\frac{2\int -\frac{5\left(Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{10\left(a^2B+2aAb-b^2B\right)\sqrt{\tan(c+dx)}}{d}+\frac{2\right.$$
$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\downarrow$ 27

$$\frac{1}{5}\left(-\frac{10\int \frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{10\left(a^2B+2aAb-b^2B\right)\sqrt{\tan(c+dx)}}{d}+\frac{2b(\right.$$
$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\downarrow$ 1482

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)\right.\right.}{d}\right.$$
$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\downarrow$ 1476

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\big\downarrow$ 1082

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\big\downarrow$ 217

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\big\downarrow$ 1479

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c\right.}{\tan(c+dx)+\sqrt{2}\sqrt{ta}}}{}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\big\downarrow$ 25

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+d}}}{2\sqrt{2}}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\downarrow$ 27

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx}}{\tan(c+dx)+\sqrt{2}\sqrt{\tan}}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

$\downarrow$ 1103

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx}\right.}{2\sqrt{2}}\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}{5d}$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b*B*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]))/(5*d) + ((-10*(-1/2*((a^2*
(A - B) - b^2*(A - B) - 2*a*b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((2*a
*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c +
 d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[
c + d*x]]/(2*Sqrt[2])))/2))/d + (10*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Tan[c +
 d*x]])/d + (2*b*(5*A*b + 7*a*B)*Tan[c + d*x]^(3/2))/(3*d))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{2Bb^2\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2Ab^2\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{4Bab\tan(dx+c)^{\frac{3}{2}}}{3}+4Aab\sqrt{\tan(dx+c)}+2Ba^2\sqrt{\tan(dx+c)}-2\sqrt{\tan(dx+c)}\,Bb^2-$
default	$\frac{2Bb^2\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2Ab^2\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{4Bab\tan(dx+c)^{\frac{3}{2}}}{3}+4Aab\sqrt{\tan(dx+c)}+2Ba^2\sqrt{\tan(dx+c)}-2\sqrt{\tan(dx+c)}\,Bb^2-$
parts	$(Ab^2+2Bab)\left(\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan\right.\right.}{4}\right.$ / d

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2/5*B*b^2*tan(d*x+c)^(5/2)+2/3*A*b^2*tan(d*x+c)^(3/2)+4/3*B*a*b*tan(d
*x+c)^(3/2)+4*A*a*b*tan(d*x+c)^(1/2)+2*B*a^2*tan(d*x+c)^(1/2)-2*tan(d*x+c)
^(1/2)*B*b^2+1/4*(-2*A*a*b-B*a^2+B*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*ta
n(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(A*a^2-A*
b^2-2*B*a*b)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+
c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arc
tan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1492 vs. $2(235) = 470$.

Time = 0.15 (sec) , antiderivative size = 1492, normalized size of antiderivative = 5.67

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/30*(30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b +
 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)
*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2
)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B +
 B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2)*sqrt(t
an(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2
*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b
^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*
A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))/
(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A^
2 - B^2)*b^4)) + 30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B
^2)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2
*A*B + B^2)*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*b -
(A + B)*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^
2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/
d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2
)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A
*B + B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b +
2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*
b^4)/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)...
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^2\,\sqrt{\tan{(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2*sqrt(tan(c + d*x)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.05

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{24\,Bb^2\tan{(dx+c)}^{\frac{5}{2}} + 30\,\sqrt{2}((A-B)a^2 - 2\,(A+B)ab - (A-B)b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2\,\sqrt{\tan{(dx}}\right.\right.}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/60*(24*B*b^2*tan(d*x + c)^(5/2) + 30*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*
b - (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 30
*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*arctan(-1/2*sqrt(2)*(
sqrt(2) - 2*sqrt(tan(d*x + c)))) - 15*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b
 - (A + B)*b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 15*sq
rt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(-sqrt(2)*sqrt(tan(d*
x + c)) + tan(d*x + c) + 1) + 40*(2*B*a*b + A*b^2)*tan(d*x + c)^(3/2) + 12
0*(B*a^2 + 2*A*a*b - B*b^2)*sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 10.06 (sec) , antiderivative size = 3825, normalized size of antiderivative = 14.54

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
atan((A^2*a^4*tan(c + d*x)^(1/2)*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2 - (12*
A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*
a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*32i)/((16*A^3*a^6)/d - (16*A^3*b^6)/d +
(112*A^3*a^2*b^4)/d - (112*A^3*a^4*b^2)/d + (32*A*a*b*(12*A^4*a^2*b^6*d^4
- A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^(1/
2))/d^3) + (A^2*b^4*tan(c + d*x)^(1/2)*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2
- (12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 1
2*A^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*32i)/((16*A^3*a^6)/d - (16*A^3*b^6
)/d + (112*A^3*a^2*b^4)/d - (112*A^3*a^4*b^2)/d + (32*A*a*b*(12*A^4*a^2*b^
6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^
4)^(1/2))/d^3) - (A^2*a^2*b^2*tan(c + d*x)^(1/2)*((A^2*a^3*b)/d^2 - (A^2*a
*b^3)/d^2 - (12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b
^4*d^4 + 12*A^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*192i)/((16*A^3*a^6)/d -
(16*A^3*b^6)/d + (112*A^3*a^2*b^4)/d - (112*A^3*a^4*b^2)/d + (32*A*a*b*(12
*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4
*a^6*b^2*d^4)^(1/2))/d^3))*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2 - (12*A^4*a^
2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^
2*d^4)^(1/2)/(4*d^4))^(1/2)*2i - atan((B^2*a^4*tan(c + d*x)^(1/2)*((12*B^4
*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6
*b^2*d^4)^(1/2)/(4*d^4) + (B^2*a*b^3)/d^2 - (B^2*a^3*b)/d^2)^(1/2)*32i)...
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 b^3 + 30\sqrt{\tan(dx+c)}\,a^2b - 10\sqrt{\tan(dx+c)}\,b^3 - 15\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a^2b}{5}$$

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(2*sqrt(tan(c + d*x))*tan(c + d*x)**2*b**3 + 30*sqrt(tan(c + d*x))*a**2*b
- 10*sqrt(tan(c + d*x))*b**3 - 15*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a
**2*b*d + 5*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b**3*d + 5*int(sqrt(tan
(c + d*x)),x)*a**3*d + 15*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*a*b**2
*d)/(5*d)
```

3.388 $\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 228

$$\begin{aligned}\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx &= -\frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}\\ &+\frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}\\ &+\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d}\\ &+\frac{2b(3Ab+5aB)\sqrt{\tan(c+dx)}}{3d}+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}\end{aligned}$$

output
```
1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*
2^(1/2)/d+1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))*2^(1/2)/d+1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctanh(2^(1/2)*t
an(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d+2/3*b*(3*A*b+5*B*a)*tan(d*x+c)^(
1/2)/d+2/3*b*B*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.52

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{-3\sqrt[4]{-1}(a-ib)^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-3\sqrt[4]{-1}(a+ib)^2(A+iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\right.}}{3d}$$

input
```
Integrate[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]]
,x]
```

output
```
(-3*(-1)^(1/4)*(a - I*b)^2*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
 - 3*(-1)^(1/4)*(a + I*b)^2*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]
]] + 2*b*Sqrt[Tan[c + d*x]]*(3*A*b + 6*a*B + b*B*Tan[c + d*x]))/(3*d)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {3042, 4090, 27, 3042, 4113, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2}{3}\int \frac{b(3Ab+5aB)\tan^2(c+dx)+3\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(3aA-bB)}{2\sqrt{\tan(c+dx)}}dx+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\int \frac{b(3Ab+5aB)\tan^2(c+dx)+3\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(3aA-bB)}{\sqrt{\tan(c+dx)}}dx+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\int \frac{b(3Ab+5aB)\tan(c+dx)^2+3\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(3aA-bB)}{\sqrt{\tan(c+dx)}}dx+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$\Big\downarrow$ 4113

$$\frac{1}{3}\left(\int \frac{3\left(Aa^2-2bBa-Ab^2\right)+3\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2b(5aB+3Ab)\sqrt{\tan(c+dx)}}{d}\right)+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\int \frac{3\left(Aa^2-2bBa-Ab^2\right)+3\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2b(5aB+3Ab)\sqrt{\tan(c+dx)}}{d}\right)+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$\Big\downarrow$ 4017

$$\frac{1}{3}\left(\frac{2\int \frac{3\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2b(5aB+3Ab)\sqrt{\tan(c+dx)}}{d}\right)+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{6\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2b(5aB+3Ab)\sqrt{\tan(c+dx)}}{d}\right)+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$$\Big\downarrow \text{1482}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-\right.\right.}{d}\right.$$
$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-\right.\right.}{d}\right.$$
$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-\right.\right.}{d}\right.$$
$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-\right.\right.}{d}\right.$$
$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}\right.}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c}}}{2\sqrt{2}}\right.\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$\downarrow$ 25

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+}}{2\sqrt{2}}\right.\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+}}\right.\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

$\downarrow$ 1103

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(a^2(A\right.\right.}{d}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}{3d}$$

input
```
Int[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x]
```

output
```
((6*(((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sq
rt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2
]))/2 + ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*(-1/2*Log[1 - Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d + (2*b*(3*A*b + 5*a*B)*Sqrt[Tan
[c + d*x]])/d)/3 + (2*b*B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]))/(3*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{2B\,b^2\tan(dx+c)^{\frac{3}{2}}}{3}+2\sqrt{\tan(dx+c)}\,A\,b^2+4\sqrt{\tan(dx+c)}\,Bab+\frac{\left(A\,a^2-A\,b^2-2Bab\right)\sqrt{2}\,\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+\right.}{}}{}$
default	$\frac{\frac{2B\,b^2\tan(dx+c)^{\frac{3}{2}}}{3}+2\sqrt{\tan(dx+c)}\,A\,b^2+4\sqrt{\tan(dx+c)}\,Bab+\frac{\left(A\,a^2-A\,b^2-2Bab\right)\sqrt{2}\,\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+\right.}{}}{}$
parts	$\frac{\left(A\,b^2+2Bab\right)\left(2\sqrt{\tan(dx+c)}-\frac{\sqrt{2}\,\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\mathrm{ta}}\right.\right.}{4}\right.}{d}$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2/3*B*b^2*tan(d*x+c)^(3/2)+2*tan(d*x+c)^(1/2)*A*b^2+4*tan(d*x+c)^(1/2
)*B*a*b+1/4*(A*a^2-A*b^2-2*B*a*b)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+
c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*ta
n(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(2*A*a*b+B*a^2-
B*b^2)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(
1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1
+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. 2(203) = 406.

Time = 0.12 (sec) , antiderivative size = 1472, normalized size of antiderivative = 6.46

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-1/6*(6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b +
2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*
b^4)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*
d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B
^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4)/d^2)*sqrt(tan
(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(
A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4
)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*
B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))/(8
*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A^2
- B^2)*b^4)) + 6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)
*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*
B + B^2)*b^4)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a^2 - 2*(A + B)*a*b - (A -
 B)*b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(A^2 -
6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4)/d^2)
*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^
3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B +
 B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A
^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)
/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^2}{\sqrt{\tan{(c+dx)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**2*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2/sqrt(tan(c + d*x)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{8\,Bb^2\tan\left(dx+c\right)^{\frac{3}{2}} + 6\,\sqrt{2}((A+B)a^2 + 2\,(A-B)ab - (A+B)b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan\left(dx+\right.}\right.\right.}{}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
1/12*(8*B*b^2*tan(d*x + c)^(3/2) + 6*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b
- (A + B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 6*sq
rt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*arctan(-1/2*sqrt(2)*(sqr
t(2) - 2*sqrt(tan(d*x + c)))) + 3*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (
A - B)*b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)
*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*log(-sqrt(2)*sqrt(tan(d*x + c
)) + tan(d*x + c) + 1) + 24*(2*B*a*b + A*b^2)*sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.62 (sec) , antiderivative size = 3773, normalized size of antiderivative = 16.55

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2)/tan(c + d*x)^(1/2),x)
```

output
```
atan((B^2*a^4*tan(c + d*x)^(1/2)*((B^2*a^3*b)/d^2 - (B^2*a*b^3)/d^2 - (12*
B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*
a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*32i)/((16*B^3*a^6)/d - (16*B^3*b^6)/d +
(112*B^3*a^2*b^4)/d - (112*B^3*a^4*b^2)/d + (32*B*a*b*(12*B^4*a^2*b^6*d^4
- B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/
2))/d^3) + (B^2*b^4*tan(c + d*x)^(1/2)*((B^2*a^3*b)/d^2 - (B^2*a*b^3)/d^2
- (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 1
2*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*32i)/((16*B^3*a^6)/d - (16*B^3*b^6
)/d + (112*B^3*a^2*b^4)/d - (112*B^3*a^4*b^2)/d + (32*B*a*b*(12*B^4*a^2*b^
6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^
4)^(1/2))/d^3) - (B^2*a^2*b^2*tan(c + d*x)^(1/2)*((B^2*a^3*b)/d^2 - (B^2*a
*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b
^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2)*192i)/((16*B^3*a^6)/d -
(16*B^3*b^6)/d + (112*B^3*a^2*b^4)/d - (112*B^3*a^4*b^2)/d + (32*B*a*b*(12
*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4
*a^6*b^2*d^4)^(1/2))/d^3))*((B^2*a^3*b)/d^2 - (B^2*a*b^3)/d^2 - (12*B^4*a^
2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^
2*d^4)^(1/2)/(4*d^4))^(1/2)*2i - atan((B^2*a^4*tan(c + d*x)^(1/2)*((12*B^4
*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6
*b^2*d^4)^(1/2)/(4*d^4) - (B^2*a*b^3)/d^2 + (B^2*a^3*b)/d^2)^(1/2)*32i)...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{6\sqrt{\tan(dx+c)}\,a\,b^2 + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a^3d - 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a\,b^2d + 3\left(\int \sqrt{\tan(dx+c)}dx\right)a^2bd +}{d}$$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(6*sqrt(tan(c + d*x))*a*b**2 + int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a**3
*d - 3*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*b**2*d + 3*int(sqrt(tan(c
+ d*x)),x)*a**2*b*d + int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*b**3*d)/d
```

3.389 $\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 212

$$\begin{aligned}\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx &= \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad - \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad + \frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad - \frac{2a^2A}{d\sqrt{\tan(c+dx)}} + \frac{2b^2B\sqrt{\tan(c+dx)}}{d}\end{aligned}$$

output

```
-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
*2^(1/2)/d-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))*2^(1/2)/d+1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctanh(2^(1/2)*
tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d-2*a^2*A/d/tan(d*x+c)^(1/2)+2*b^
2*B*tan(d*x+c)^(1/2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.63 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{-8b(Ab+3aB)-8(a^2A-Ab^2-2abB)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\tan^2(c+dx)\right)-\sqrt{2}(2aAb+a}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2)
,x]
```

output
```
(-8*b*(A*b + 3*a*B) - 8*(a^2*A - A*b^2 - 2*a*b*B)*Hypergeometric2F1[-1/4,
1, 3/4, -Tan[c + d*x]^2] - Sqrt[2]*(2*a*A*b + a^2*B - b^2*B)*(2*ArcTan[1 -
 Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] +
Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x]] + 8*b*B*(a + b*Tan[c + d
*x]))/(4*d*Sqrt[Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 4087, 3042, 4113, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \; 4087$$

$$\int \frac{b^2 B \tan^2(c+dx) - \left(Aa^2 - 2bBa - Ab^2\right)\tan(c+dx) + a(2Ab+aB)}{\sqrt{\tan(c+dx)}}dx - \frac{2a^2 A}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{b^2 B \tan(c+dx)^2 - \left(Aa^2 - 2bBa - Ab^2\right)\tan(c+dx) + a(2Ab+aB)}{\sqrt{\tan(c+dx)}}dx - \frac{2a^2 A}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 4113$$

$$\int \frac{Ba^2 + 2Aba - b^2 B - \left(Aa^2 - 2bBa - Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx - \frac{2a^2 A}{d\sqrt{\tan(c+dx)}} + \frac{2b^2 B\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{Ba^2 + 2Aba - b^2 B - \left(Aa^2 - 2bBa - Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx - \frac{2a^2 A}{d\sqrt{\tan(c+dx)}} + \frac{2b^2 B\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 4017$$

$$\frac{2\int \frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d} - \frac{2a^2 A}{d\sqrt{\tan(c+dx)}} + \frac{2b^2 B\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 1482$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B) + 2ab(A-B) - b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)} - \frac{1}{2}\left(a^2(A-B) - 2ab(A+B) - b^2(A\right.}{d} \frac{2a^2 A}{d\sqrt{\tan(c+dx)}} + \frac{2b^2 B\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 1476$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B) + 2ab(A-B) - b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)} - \frac{1}{2}\left(a^2(A-B) - 2ab(A+B) - b^2(A\right.}{d} \frac{2a^2 A}{d\sqrt{\tan(c+dx)}} + \frac{2b^2 B\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 1082$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(\right.\right.}{d}$$
$$\frac{2a^2A}{d\sqrt{\tan(c+dx)}}+\frac{2b^2B\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 217$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(\right.\right.}{d}$$
$$\frac{2a^2A}{d\sqrt{\tan(c+dx)}}+\frac{2b^2B\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 1479$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+}}{2\sqrt{2}}\right.\right.}{}$$
$$\frac{2a^2A}{d\sqrt{\tan(c+dx)}}+\frac{2b^2B\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{}}{2\sqrt{2}}\right.\right.}{d}$$
$$\frac{2a^2A}{d\sqrt{\tan(c+dx)}}+\frac{2b^2B\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}-}\right.\right.}{}$$
$$\frac{2a^2A}{d\sqrt{\tan(c+dx)}}+\frac{2b^2B\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)\right.}{d}$$

$$\frac{2a^2A}{d\sqrt{\tan(c+dx)}}+\frac{2b^2B\sqrt{\tan(c+dx)}}{d}$$

input

```
Int[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x]
```

output

```
(2*(-1/2*((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*(-(ArcTan[1 - Sqrt[2
]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sq
rt[2])) + ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(-1/2*Log[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c
 + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d - (2*a^2*A)/(d*Sqrt[Tan[c + d
*x]]) + (2*b^2*B*Sqrt[Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{2\sqrt{\tan(dx+c)}\,B\,b^2-\frac{2A\,a^2}{\sqrt{\tan(dx+c)}}+\frac{\left(2Aab+B\,a^2-B\,b^2\right)\sqrt{2}\,\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\cdots\right)}{4}}{\cdots}$
default	$\frac{2\sqrt{\tan(dx+c)}\,B\,b^2-\frac{2A\,a^2}{\sqrt{\tan(dx+c)}}+\frac{\left(2Aab+B\,a^2-B\,b^2\right)\sqrt{2}\,\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\cdots\right)}{4}}{\cdots}$
parts	$\frac{\left(A\,b^2+2Bab\right)\sqrt{2}\,\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\cdots\right)}{4d}$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2*tan(d*x+c)^(1/2)*B*b^2-2*A*a^2/tan(d*x+c)^(1/2)+1/4*(2*A*a*b+B*a^2-
B*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(
1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1
+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-A*a^2+A*b^2+2*B*a*b)*2^(1/2)*(ln((tan(d*
x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+
2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1490 vs. $2(191) = 382$.

Time = 0.13 (sec) , antiderivative size = 1490, normalized size of antiderivative = 7.03

$$\int \frac{(a + b\tan(c + dx))^2(A + B\tan(c + dx))}{\tan^{\frac{3}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
-1/2*(2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b +
2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*
b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)
*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B +
B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2)*sqrt(ta
n(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*
(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^
4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A
*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))/(
8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A^2
 - B^2)*b^4))*tan(d*x + c) + 2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 -
 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3
 + (A^2 - 2*A*B + B^2)*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A
- B)*a*b - (A + B)*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^
3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B +
 B^2)*b^4)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4
*(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 +
 (A^2 + 2*A*B + B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^
2)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*
A*B + B^2)*b^4)/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^2}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**2*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2/tan(c + d*x)**(3/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{8\,Bb^2\sqrt{\tan\left(dx+c\right)} - 2\sqrt{2}((A-B)a^2 - 2\,(A+B)ab - (A-B)b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+\right.}\right.\right.}{}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
1/4*(8*B*b^2*sqrt(tan(d*x + c)) - 2*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b -
 (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) - 2*sqr
t(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt
(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A +
 B)*b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A
+ B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) +
tan(d*x + c) + 1) - 8*A*a^2/sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 3749, normalized size of antiderivative = 17.68

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2)/tan(c + d*x)^(3/2),x)
```

output
```
2*atanh((32*A^2*a^4*d^3*tan(c + d*x)^(1/2)*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/
d^2 - (12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4
 + 12*A^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2))/(16*A^3*a^6*d^2 - 16*A^3*b^6*
d^2 + 112*A^3*a^2*b^4*d^2 - 112*A^3*a^4*b^2*d^2 + 32*A*a*b*(12*A^4*a^2*b^6
*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4
)^(1/2)) + (32*A^2*b^4*d^3*tan(c + d*x)^(1/2)*((A^2*a^3*b)/d^2 - (A^2*a*b^
3)/d^2 - (12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*
d^4 + 12*A^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2))/(16*A^3*a^6*d^2 - 16*A^3*b
^6*d^2 + 112*A^3*a^2*b^4*d^2 - 112*A^3*a^4*b^2*d^2 + 32*A*a*b*(12*A^4*a^2*
b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*
d^4)^(1/2)) - (192*A^2*a^2*b^2*d^3*tan(c + d*x)^(1/2)*((A^2*a^3*b)/d^2 - (
A^2*a*b^3)/d^2 - (12*A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*
a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2))/(16*A^3*a^6*d^2 -
16*A^3*b^6*d^2 + 112*A^3*a^2*b^4*d^2 - 112*A^3*a^4*b^2*d^2 + 32*A*a*b*(12*
A^4*a^2*b^6*d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*
a^6*b^2*d^4)^(1/2)))*((A^2*a^3*b)/d^2 - (A^2*a*b^3)/d^2 - (12*A^4*a^2*b^6*
d^4 - A^4*b^8*d^4 - A^4*a^8*d^4 - 38*A^4*a^4*b^4*d^4 + 12*A^4*a^6*b^2*d^4)
^(1/2)/(4*d^4))^(1/2) - 2*atanh((32*B^2*a^4*tan(c + d*x)^(1/2)*((12*B^4*a^
2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^
2*d^4)^(1/2)/(4*d^4) + (B^2*a*b^3)/d^2 - (B^2*a^3*b)/d^2)^(1/2))/((16*B...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2\sqrt{\tan(dx+c)}\,b^3 + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)a^3d + 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a^2bd - \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)b^3d + 3\left(\int \sqrt{\mathrm{ta}\right.}{d}$$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*b**3 + int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a**
3*d + 3*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a**2*b*d - int(sqrt(tan(c +
 d*x))/tan(c + d*x),x)*b**3*d + 3*int(sqrt(tan(c + d*x)),x)*a*b**2*d)/d
```

3.390 $\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 218

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2a(2Ab+aB)}{d\sqrt{\tan(c+dx)}}$$

output

```
-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
*2^(1/2)/d-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))*2^(1/2)/d-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctanh(2^(1/2)*
tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d-2/3*a^2*A/d/tan(d*x+c)^(3/2)-2*
a*(2*A*b+B*a)/d/tan(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.55

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2(-a^2A+Ab^2+2abB)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\tan^2(c+dx)\right)-6(2aAb+a^2B-b^2B)\,\text{Hyperg}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2)
,x]
```

output
```
(2*(-(a^2*A) + A*b^2 + 2*a*b*B)*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d
*x]^2] - 6*(2*a*A*b + a^2*B - b^2*B)*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[
c + d*x]^2]*Tan[c + d*x] - 2*b*(A*b + 2*a*B + 3*b*B*Tan[c + d*x]))/(3*d*Ta
n[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4087, 3042, 4111, 25, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$
$$\Big\downarrow \text{ 4087}$$

$$\int \frac{b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\int \frac{b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\tan(c+dx)^{3/2}}dx-\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4111

$$\int -\frac{Aa^2-2bBa-Ab^2-\left(b^2B-a(2Ab+aB)\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 25

$$-\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 4017

$$-\frac{2\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 1482

$$-\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2\right.}{d}$$
$$\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 1476

$$-\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2\right.}{d}$$

$$\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 1082

$$-\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2\right.}{d}$$

$$\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 217

$$-\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2\right.}{d}$$

$$\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 1479

$$-\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right.}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx}}}{2\sqrt{2}}\right.\right.}{}$$

$$\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d}{2\sqrt{2}}\right.\right.}{}$$

$$\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}\cdots}\right.\right.}{\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(a^2(A-\cdots\right.\right.}{d}$$

$$\frac{2a^2A}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x]
```

output
```
(-2*(((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sq
rt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2
]))/2 + ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*(-1/2*Log[1 - Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d - (2*a^2*A)/(3*d*Tan[c + d*x]^(
3/2)) - (2*a*(2*A*b + a*B))/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\left(-A\,a^2+A\,b^2+2Bab\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4}+}{}$
default	$\frac{\frac{\left(-A\,a^2+A\,b^2+2Bab\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4}+}{}$
parts	$\frac{\left(A\,b^2+2Bab\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right.}{4d}$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(1/4*(-A*a^2+A*b^2+2*B*a*b)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)
^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(
d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(-2*A*a*b-B*a^2+B
*b^2)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1
/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+
2^(1/2)*tan(d*x+c)^(1/2)))-2/3*A*a^2/tan(d*x+c)^(3/2)-2*a*(2*A*b+B*a)/tan(
d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1514 vs. $2(194) = 388$.

Time = 0.11 (sec) , antiderivative size = 1514, normalized size of antiderivative = 6.94

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
1/6*(6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2
*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b
^4)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*d
*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B^
2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4)/d^2)*sqrt(tan(
d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(A
^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4)
/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B
 + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))/(8*
A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A^2 -
 B^2)*b^4))*tan(d*x + c)^2 + 6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 +
 4*(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3
 + (A^2 + 2*A*B + B^2)*b^4)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a^2 - 2*(A +
 B)*a*b - (A - B)*b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3
*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B +
B^2)*b^4)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*
(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 +
(A^2 + 2*A*B + B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2
)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A
*B + B^2)*b^4)/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^2}{\tan^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**2*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2/tan(c + d*x)**(5/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx =$$
$$-\frac{6\sqrt{2}((A+B)a^2+2(A-B)ab-(A+B)b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+6\sqrt{2}((A+B)a^2+\ldots}{}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
-1/12*(6*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 6*sqrt(2)*((A + B)*a^2 + 2*(A -
B)*a*b - (A + B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))
) + 3*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*log(sqrt(2)*sqrt
(tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*
b - (A - B)*b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*(
A*a^2 + 3*(B*a^2 + 2*A*a*b)*tan(d*x + c))/tan(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^2}{\tan(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 7.27 (sec) , antiderivative size = 3745, normalized size of antiderivative = 17.18

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2)/tan(c + d*x)^(5/2),x)
```

output
```
2*atanh((32*B^2*a^4*d^3*tan(c + d*x)^(1/2)*((B^2*a^3*b)/d^2 - (B^2*a*b^3)/
d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4
 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2))/(16*B^3*a^6*d^2 - 16*B^3*b^6*
d^2 + 32*B*a*b*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^
4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2) + 112*B^3*a^2*b^4*d^2 - 112*B^3*a^4*
b^2*d^2) + (32*B^2*b^4*d^3*tan(c + d*x)^(1/2)*((B^2*a^3*b)/d^2 - (B^2*a*b^
3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*
d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2))/(16*B^3*a^6*d^2 - 16*B^3*b
^6*d^2 + 32*B*a*b*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4
*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2) + 112*B^3*a^2*b^4*d^2 - 112*B^3*a
^4*b^2*d^2) - (192*B^2*a^2*b^2*d^3*tan(c + d*x)^(1/2)*((B^2*a^3*b)/d^2 - (
B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*
a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4))^(1/2))/(16*B^3*a^6*d^2 -
16*B^3*b^6*d^2 + 32*B*a*b*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4
- 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2) + 112*B^3*a^2*b^4*d^2 - 1
12*B^3*a^4*b^2*d^2))*((B^2*a^3*b)/d^2 - (B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*
d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)
^(1/2)/(4*d^4))^(1/2) - 2*atanh((32*B^2*a^4*d^3*tan(c + d*x)^(1/2)*((12*B^
4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^
6*b^2*d^4)^(1/2)/(4*d^4) - (B^2*a*b^3)/d^2 + (B^2*a^3*b)/d^2)^(1/2))/(1...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)a^3 + 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)a^2b$$
$$+ 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a\,b^2 + \left(\int \sqrt{\tan(dx+c)}dx\right)b^3$$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*a**3 + 3*int(sqrt(tan(c + d*x))/
tan(c + d*x)**2,x)*a**2*b + 3*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*b**
2 + int(sqrt(tan(c + d*x)),x)*b**3
```

3.391 $\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 254

$$\begin{aligned}\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx &= -\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad+\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad-\frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} \\ &\quad-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(2Ab+aB)}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2(a^2A-Ab^2-2abB)}{d\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*
2^(1/2)/d+1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))*2^(1/2)/d-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctanh(2^(1/2)*t
an(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/d-2/5*a^2*A/d/tan(d*x+c)^(5/2)-2/3
*a*(2*A*b+B*a)/d/tan(d*x+c)^(3/2)+2*(A*a^2-A*b^2-2*B*a*b)/d/tan(d*x+c)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.47

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2\left((-3a^2A+3Ab^2+6abB)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\tan^2(c+dx)\right)-5(2aAb+a^2B-b^2B)\,\text{H}\right.}{15d\tan^{\frac{5}{2}}(c+}$$

input
```
Integrate[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2)
,x]
```

output
```
(2*((-3*a^2*A + 3*A*b^2 + 6*a*b*B)*Hypergeometric2F1[-5/4, 1, -1/4, -Tan[c
 + d*x]^2] - 5*(2*a*A*b + a^2*B - b^2*B)*Hypergeometric2F1[-3/4, 1, 1/4, -
Tan[c + d*x]^2]*Tan[c + d*x] - b*(3*A*b + 6*a*B + 5*b*B*Tan[c + d*x])))/(1
5*d*Tan[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 4087, 3042, 4111, 25, 3042, 4012, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{4087}$$

$$\int \frac{b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\tan(c+dx)^{5/2}}dx-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4111}$$

$$\int -\frac{Aa^2-2bBa-Ab^2-\left(b^2B-a(2Ab+aB)\right)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\tan(c+dx)^{3/2}}dx-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4012}$$

$$-\int \frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2\left(a^2A-2abB-Ab^2\right)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2\left(a^2A-2abB-Ab^2\right)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4017}$$

$$-\frac{2\int \frac{Ba^2+2Aba-b^2B-(Aa^2-2bBa-Ab^2)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2(a^2A-2abB-Ab^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1482

$$-\frac{2\left(\frac{1}{2}(a^2(A+B)+2ab(A-B)-b^2(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a^2(A-B)-2ab(A+B)-b^2\right.}{d}$$
$$\frac{2(a^2A-2abB-Ab^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1476

$$-\frac{2\left(\frac{1}{2}(a^2(A+B)+2ab(A-B)-b^2(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a^2(A-B)-2ab(A+B)-b^2\right.}{d}$$
$$\frac{2(a^2A-2abB-Ab^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1082

$$-\frac{2\left(\frac{1}{2}(a^2(A+B)+2ab(A-B)-b^2(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a^2(A-B)-2ab(A+B)-b^2\right.}{d}$$
$$\frac{2(a^2A-2abB-Ab^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 217

$$-\frac{2\left(\frac{1}{2}(a^2(A+B)+2ab(A-B)-b^2(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a^2(A-B)-2ab(A+B)-b^2\right.}{d}$$
$$\frac{2(a^2A-2abB-Ab^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1479

$$-\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right.}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+d}}}{2\sqrt{2}}\right.\right.}{}$$

$$\frac{2(a^2A-2abB-Ab^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d}{2\sqrt{2}}\right.\right.}{}$$

$$\frac{2(a^2A-2abB-Ab^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+d}}\right.\right.}{}$$

$$\frac{2(a^2A-2abB-Ab^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1103

$$-\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)\right.}{d}$$

$$\frac{2(a^2A-2abB-Ab^2)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2A}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2a(aB+2Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x]
```

output
```
(-2*(-1/2*((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*(-(ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/S
qrt[2])) + ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(-1/2*Log[1 - Sqrt
[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d - (2*a^2*A)/(5*d*Tan[c + d*x
]^(5/2)) - (2*a*(2*A*b + a*B))/(3*d*Tan[c + d*x]^(3/2)) + (2*(a^2*A - A*b^
2 - 2*a*b*B))/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{\left(-2Aab-B\,a^2+B\,b^2\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4}+}{}$
default	$\frac{\frac{\left(-2Aab-B\,a^2+B\,b^2\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4}+}{}$
parts	$\frac{\left(A\,b^2+2Bab\right)\left(-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4}-\right.}{d}$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(1/4*(-2*A*a*b-B*a^2+B*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)
^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(
d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(A*a^2-A*b^2-2*B*
a*b)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/
2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2
^(1/2)*tan(d*x+c)^(1/2)))-2*(-A*a^2+A*b^2+2*B*a*b)/tan(d*x+c)^(1/2)-2/5*A*
a^2/tan(d*x+c)^(5/2)-2/3*a*(2*A*b+B*a)/tan(d*x+c)^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. $2(226) = 452$.

Time = 0.14 (sec) , antiderivative size = 1534, normalized size of antiderivative = 6.04

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
1/30*(30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b +
 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)
*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2
)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B +
 B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2)*sqrt(t
an(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2
*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b
^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*
A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))/
(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A^
2 - B^2)*b^4))*tan(d*x + c)^3 + 30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a
^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a
*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2
*(A - B)*a*b - (A + B)*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2
)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A
*B + B^2)*b^4)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4
 + 4*(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b
^3 + (A^2 + 2*A*B + B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2
- B^2)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2
- 2*A*B + B^2)*b^4)/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 ...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^2}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**2*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2/tan(c + d*x)**(7/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{30\sqrt{2}((A-B)a^2-2(A+B)ab-(A-B)b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+30\sqrt{2}((A-B}{}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
1/60*(30*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 30*sqrt(2)*((A - B)*a^2 - 2*(A +
 B)*a*b - (A - B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))
)) - 15*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)*sq
rt(tan(d*x + c)) + tan(d*x + c) + 1) + 15*sqrt(2)*((A + B)*a^2 + 2*(A - B)
*a*b - (A + B)*b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) -
8*(3*A*a^2 - 15*(A*a^2 - 2*B*a*b - A*b^2)*tan(d*x + c)^2 + 5*(B*a^2 + 2*A*
a*b)*tan(d*x + c))/tan(d*x + c)^(5/2))/d
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^2}{\tan(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 10.47 (sec) , antiderivative size = 3782, normalized size of antiderivative = 14.89

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2)/tan(c + d*x)^(7/2),x)
```

output
```
2*atanh((32*B^2*a^4*d^3*tan(c + d*x)^(1/2)*((B^2*a*b^3)/d^2 - (12*B^4*a^2*
b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*
d^4)^(1/2)/(4*d^4) - (B^2*a^3*b)/d^2)^(1/2))/(16*B*a^2*(12*B^4*a^2*b^6*d^4
 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1
/2) - 16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^
4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2) - 192*B^3*a^3*b^3*d^2 + 32*B^3*a*b^5
*d^2 + 32*B^3*a^5*b*d^2) + (32*B^2*b^4*d^3*tan(c + d*x)^(1/2)*((B^2*a*b^3)
/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^
4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4) - (B^2*a^3*b)/d^2)^(1/2))/(16*B*a^2*
(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*
B^4*a^6*b^2*d^4)^(1/2) - 16*B*b^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*
a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2) - 192*B^3*a^3*b^3
*d^2 + 32*B^3*a*b^5*d^2 + 32*B^3*a^5*b*d^2) - (192*B^2*a^2*b^2*d^3*tan(c +
 d*x)^(1/2)*((B^2*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8
*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2)/(4*d^4) - (B^2*a^3*b
)/d^2)^(1/2))/(16*B*a^2*(12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 -
38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)^(1/2) - 16*B*b^2*(12*B^4*a^2*b^6*
d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*a^4*b^4*d^4 + 12*B^4*a^6*b^2*d^4)
^(1/2) - 192*B^3*a^3*b^3*d^2 + 32*B^3*a*b^5*d^2 + 32*B^3*a^5*b*d^2))*((B^2
*a*b^3)/d^2 - (12*B^4*a^2*b^6*d^4 - B^4*b^8*d^4 - B^4*a^8*d^4 - 38*B^4*...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)a^3 + 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)a^2b + 3\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)a\,b^2 + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)b^3$$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x)**4,x)*a**3 + 3*int(sqrt(tan(c + d*x))/
tan(c + d*x)**3,x)*a**2*b + 3*int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a*
b**2 + int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b**3
```

3.392 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 388

$$
\begin{aligned}
&\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\
&= \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad -\frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad -\frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} \\
&\quad +\frac{2(a^3A-3aAb^2-3a^2bB+b^3B)\sqrt{\tan(c+dx)}}{d} \\
&\quad +\frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\tan^{\frac{3}{2}}(c+dx)}{3d} \\
&\quad +\frac{2b(27aAb+22a^2B-9b^2B)\tan^{\frac{5}{2}}(c+dx)}{45d} \\
&\quad +\frac{2b^2(9Ab+13aB)\tan^{\frac{7}{2}}(c+dx)}{63d}+\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}
\end{aligned}
$$

output
```
-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))*2^(1/2)/d-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*
(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(a^3*(A-B)-3*a*b^2
*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*
x+c)))*2^(1/2)/d+2*(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*tan(d*x+c)^(1/2)/d+2/
3*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*tan(d*x+c)^(3/2)/d+2/45*b*(27*A*a*b+22
*B*a^2-9*B*b^2)*tan(d*x+c)^(5/2)/d+2/63*b^2*(9*A*b+13*B*a)*tan(d*x+c)^(7/2
)/d+2/9*b*B*tan(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.57

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2\Big(7b(27aAb+22a^2B-9b^2B)\tan^{\frac{5}{2}}(c+dx)+5b^2(9Ab+13aB)\tan^{\frac{7}{2}}(c+dx)+35bB\tan^{\frac{5}{2}}(c+dx)(a+\cdots}{\cdots}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*(7*b*(27*a*A*b + 22*a^2*B - 9*b^2*B)*Tan[c + d*x]^(5/2) + 5*b^2*(9*A*b
+ 13*a*B)*Tan[c + d*x]^(7/2) + 35*b*B*Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*
x])^2 + (105*(a - I*b)^3*(I*A + B)*(-3*(-1)^(3/4)*ArcTan[(-1)^(3/4)*Sqrt[T
an[c + d*x]]] + Sqrt[Tan[c + d*x]]*(-3*I + Tan[c + d*x])))/2 + (105*(a + I
*b)^3*((-I)*A + B)*(3*(-1)^(3/4)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] +
Sqrt[Tan[c + d*x]]*(3*I + Tan[c + d*x])))/2))/(315*d)
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.02, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.697$, Rules used = {3042, 4090, 27, 3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2}{9}\int \frac{1}{2}\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))\left(b(9Ab+13aB)\tan^2(c+dx)+9\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(9aA-5bB)\right)dx+\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))\left(b(9Ab+13aB)\tan^2(c+dx)+9\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(9aA-5bB)\right)dx+\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \tan(c+dx)^{3/2}(a+b\tan(c+dx))\left(b(9Ab+13aB)\tan(c+dx)^2+9\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(9aA-5bB)\right)dx+\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$$\Big\downarrow \text{4120}$$

$$\frac{1}{9}\left(\frac{2b^2(13aB+9Ab)\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2}{7}\int -\frac{7}{2}\tan^{\frac{3}{2}}(c+dx)\left((9aA-5bB)a^2+b\left(22Ba^2+27Aba-9b^2B\right)\tan^2(\right.\right.$$
$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(\int \tan^{\frac{3}{2}}(c+dx)\left((9aA-5bB)a^2+b\left(22Ba^2+27Aba-9b^2B\right)\tan^2(c+dx)+9\left(Ba^3+3Aba^2-3b^2Ba-Ab\right.\right.\right.$$
$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\int \tan(c+dx)^{3/2}\left((9aA-5bB)a^2+b\left(22Ba^2+27Aba-9b^2B\right)\tan(c+dx)^2+9\left(Ba^3+3Aba^2-3b^2Ba-A\right.\right.\right.$$
$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$
$$\Big\downarrow \text{4113}$$

$$\frac{1}{9}\left(\int \tan^{\frac{3}{2}}(c+dx)\left(9\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+9\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{2b(2}{}\right.$$
$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\int \tan(c+dx)^{3/2}\left(9\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+9\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{2b(}{}\right.$$
$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$
$$\Big\downarrow \text{4011}$$

$$\frac{1}{9}\left(\int \sqrt{\tan(c+dx)}\left(9\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)-9\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\right)dx+\frac{2b(}{}\right.$$
$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(\int \sqrt{\tan(c+dx)}\left(9\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)-9\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\right)dx+\frac{2b(\cdots}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\downarrow$ 4011

$$\frac{1}{9}\left(\int \frac{-9\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)-9\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2b\left(22a^2B+27aA\cdots\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(\int \frac{-9\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)-9\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2b\left(22a^2B+27aA\cdots\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\downarrow$ 4017

$$\frac{1}{9}\left(\frac{2\int -\frac{9\left(Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2b\left(22a^2B+27aAb-9b^2B\right)\cdots}{5d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\downarrow$ 27

$$\frac{1}{9}\left(-\frac{18\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2b\left(22a^2B+27aAb-9b^2B\right)\mathrm{t}\cdots}{5d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\downarrow$ 1482

$$\frac{1}{9}\left(-\frac{18\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+\cdots\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\Big\downarrow$ 1476

$$\frac{1}{9}\left(-\frac{18\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+B\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\Big\downarrow$ 1082

$$\frac{1}{9}\left(-\frac{18\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\Big\downarrow$ 217

$$\frac{1}{9}\left(-\frac{18\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\Big\downarrow$ 1479

$$\frac{1}{9}\left(-\frac{18\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\Big\downarrow$ 25

$$\frac{1}{9}\left(-\frac{18\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{}{\tan}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\downarrow$ 27

$$\frac{1}{9}\left(-\frac{18\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

$\downarrow$ 1103

$$\frac{1}{9}\left(\frac{2b\left(22a^2B+27aAb-9b^2B\right)\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{18\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}{9d}$$

input
```
Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b*B*Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2)/(9*d) + ((-18*(((3*a^2*b
*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*(-(ArcTan[1 - Sqrt
[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/
Sqrt[2]))/2 + ((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A +
 B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Lo
g[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d + (18
*(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*Sqrt[Tan[c + d*x]])/d + (6*(3*a^2
*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Tan[c + d*x]^(3/2))/d + (2*b*(27*a*A*b +
 22*a^2*B - 9*b^2*B)*Tan[c + d*x]^(5/2))/(5*d) + (2*b^2*(9*A*b + 13*a*B)*T
an[c + d*x]^(7/2))/(7*d))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{2Bb^3\tan(dx+c)^{\frac{9}{2}}}{9}+\frac{2Ab^3\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{6Bab^2\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{6Aab^2\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{6Ba^2b\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{2Bb^3\tan(dx+c)^{\frac{5}{2}}}{5}+$
default	$\frac{2Bb^3\tan(dx+c)^{\frac{9}{2}}}{9}+\frac{2Ab^3\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{6Bab^2\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{6Aab^2\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{6Ba^2b\tan(dx+c)^{\frac{5}{2}}}{5}-\frac{2Bb^3\tan(dx+c)^{\frac{5}{2}}}{5}+$
parts	$\frac{(Ab^3+3Bab^2)\left(\frac{2\tan(dx+c)^{\frac{7}{2}}}{7}-\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\,\mathrm{ar}\right.}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2/9*B*b^3*tan(d*x+c)^(9/2)+2/7*A*b^3*tan(d*x+c)^(7/2)+6/7*B*a*b^2*tan
(d*x+c)^(7/2)+6/5*A*a*b^2*tan(d*x+c)^(5/2)+6/5*B*a^2*b*tan(d*x+c)^(5/2)-2/
5*B*b^3*tan(d*x+c)^(5/2)+2*A*a^2*b*tan(d*x+c)^(3/2)-2/3*A*b^3*tan(d*x+c)^(
3/2)+2/3*B*a^3*tan(d*x+c)^(3/2)-2*B*a*b^2*tan(d*x+c)^(3/2)+2*A*a^3*tan(d*x
+c)^(1/2)-6*tan(d*x+c)^(1/2)*A*a*b^2-6*tan(d*x+c)^(1/2)*B*a^2*b+2*tan(d*x+
c)^(1/2)*B*b^3+1/4*(-A*a^3+3*A*a*b^2+3*B*a^2*b-B*b^3)*2^(1/2)*(ln((tan(d*x
+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2
*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))
+1/4*(-3*A*a^2*b+A*b^3-B*a^3+3*B*a*b^2)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*ta
n(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2124 vs. $2(352) = 704$.

Time = 0.15 (sec) , antiderivative size = 2124, normalized size of antiderivative = 5.47

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/630*(630*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*
b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*
A*B + B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*a
rctan(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A
+ B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 -
 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a
^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x
+ c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 -
 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a
^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 -
2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 +
20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*
b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A
*B*a*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b
^4 + (A^2 - B^2)*b^6)) + 630*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6
*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b
^3 + 3*(A^2 + 10*A*B + B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B +
 B^2)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A
 - B)*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)
*a^5*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A...
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.03

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{280\,Bb^3\tan\left(dx+c\right)^{\frac{9}{2}} + 360\,(3\,Bab^2 + Ab^3)\tan\left(dx+c\right)^{\frac{7}{2}} + 504\,(3\,Ba^2b + 3\,Aab^2 - Bb^3)\tan\left(dx+c\right)^{\frac{5}{2}}}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/1260*(280*B*b^3*tan(d*x + c)^(9/2) + 360*(3*B*a*b^2 + A*b^3)*tan(d*x + c
)^(7/2) + 504*(3*B*a^2*b + 3*A*a*b^2 - B*b^3)*tan(d*x + c)^(5/2) - 630*sqr
t(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*arcta
n(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) - 630*sqrt(2)*((A + B)*a^3
 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*arctan(-1/2*sqrt(2)*(s
qrt(2) - 2*sqrt(tan(d*x + c)))) - 315*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2
*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d
*x + c) + 1) + 315*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^
2 + (A + B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 840
*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*tan(d*x + c)^(3/2) + 2520*(A*a^3
- 3*B*a^2*b - 3*A*a*b^2 + B*b^3)*sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 31.72 (sec) , antiderivative size = 6774, normalized size of antiderivative = 17.46

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
tan(c + d*x)^(1/2)*((2*A*a^3)/d - (6*A*a*b^2)/d) - tan(c + d*x)^(3/2)*((2*
A*b^3)/(3*d) - (2*A*a^2*b)/d) + tan(c + d*x)^(3/2)*((2*B*a^3)/(3*d) - (2*B
*a*b^2)/d) + tan(c + d*x)^(1/2)*((2*B*b^3)/d - (6*B*a^2*b)/d) - tan(c + d*
x)^(5/2)*((2*B*b^3)/(5*d) - (6*B*a^2*b)/(5*d)) - atan((((8*(4*A*a^3*d^2 -
12*A*a*b^2*d^2)*((5*A^2*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4
 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*
b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2)/(4*d^4) - (3*A^2*a*b^5)/(2*d^2) - (3*
A^2*a^5*b)/(2*d^2))^(1/2))/d^3 - (16*tan(c + d*x)^(1/2)*(A^2*a^6 - A^2*b^6
 + 15*A^2*a^2*b^4 - 15*A^2*a^4*b^2))/d^2)*((5*A^2*a^3*b^3)/d^2 - (30*A^4*a
^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*
a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2)/(4*d^4) - (
3*A^2*a*b^5)/(2*d^2) - (3*A^2*a^5*b)/(2*d^2))^(1/2)*1i - ((8*(4*A*a^3*d^2
- 12*A*a*b^2*d^2)*((5*A^2*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^4*b^12*d
^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^
8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2)/(4*d^4) - (3*A^2*a*b^5)/(2*d^2) - (
3*A^2*a^5*b)/(2*d^2))^(1/2))/d^3 + (16*tan(c + d*x)^(1/2)*(A^2*a^6 - A^2*b
^6 + 15*A^2*a^2*b^4 - 15*A^2*a^4*b^2))/d^2)*((5*A^2*a^3*b^3)/d^2 - (30*A^4
*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^
4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2)/(4*d^4) -
 (3*A^2*a*b^5)/(2*d^2) - (3*A^2*a^5*b)/(2*d^2))^(1/2)*1i)/((16*(3*A^3*a...
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= \frac{10\sqrt{\tan(dx+c)}\,\tan(dx+c)^4 b^4 + 108\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 a^2b^2 - 18\sqrt{\tan(dx+c)}\,\tan(dx+}{}$$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(10*sqrt(tan(c + d*x))*tan(c + d*x)**4*b**4 + 108*sqrt(tan(c + d*x))*tan(c
 + d*x)**2*a**2*b**2 - 18*sqrt(tan(c + d*x))*tan(c + d*x)**2*b**4 + 90*sqr
t(tan(c + d*x))*a**4 - 540*sqrt(tan(c + d*x))*a**2*b**2 + 90*sqrt(tan(c +
d*x))*b**4 - 45*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a**4*d + 270*int(sq
rt(tan(c + d*x))/tan(c + d*x),x)*a**2*b**2*d - 45*int(sqrt(tan(c + d*x))/t
an(c + d*x),x)*b**4*d + 180*int(sqrt(tan(c + d*x))*tan(c + d*x)**4,x)*a*b*
*3*d + 180*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*a**3*b*d)/(45*d)
```

3.393 $\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 345

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\sqrt{\tan(c+dx)}}{d} + \frac{2b(21aAb+18a^2B-7b^2B)\tan^{\frac{3}{2}}(c+dx)}{21d} + \frac{2b^2(7Ab+11aB)\tan^{\frac{5}{2}}(c+dx)}{35d} + \frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

output
```
1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctan(-1+2^(1/2)*ta
n(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(
A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(3*a^2*b*(A-B)-b^3*
(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x
+c)))*2^(1/2)/d+2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*tan(d*x+c)^(1/2)/d+2/2
1*b*(21*A*a*b+18*B*a^2-7*B*b^2)*tan(d*x+c)^(3/2)/d+2/35*b^2*(7*A*b+11*B*a)
*tan(d*x+c)^(5/2)/d+2/7*b*B*tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.57

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2\left(\frac{105}{2}(a-ib)^3(iA+B)\left(\sqrt[4]{-1}\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}\right)+\frac{105}{2}(a+ib)^3(-iA+\right.}{}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*((105*(a - I*b)^3*(I*A + B)*((-1)^(1/4)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c +
d*x]]] + Sqrt[Tan[c + d*x]]))/2 + (105*(a + I*b)^3*((-I)*A + B)*((-1)^(1/4
)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + Sqrt[Tan[c + d*x]]))/2 + 5*b*(2
1*a*A*b + 18*a^2*B - 7*b^2*B)*Tan[c + d*x]^(3/2) + 3*b^2*(7*A*b + 11*a*B)*
Tan[c + d*x]^(5/2) + 15*b*B*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2))/(1
05*d)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.03, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 4090, 27, 3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2}{7}\int \frac{1}{2}\sqrt{\tan(c+dx)}(a+b\tan(c+dx))\left(b(7Ab+11aB)\tan^2(c+dx)+7\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(7aA-3bB)\right)dx+\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))\left(b(7Ab+11aB)\tan^2(c+dx)+7\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(7aA-3bB)\right)dx+\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))\left(b(7Ab+11aB)\tan(c+dx)^2+7\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(7aA-3bB)\right)dx+\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{4120}$$

$$\frac{1}{7}\left(\frac{2b^2(11aB+7Ab)\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{2}{5}\int -\frac{5}{2}\sqrt{\tan(c+dx)}\left((7aA-3bB)a^2+b\left(18Ba^2+21Aba-7b^2B\right)\tan^2(\right.\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$\big\downarrow$ 27

$$\frac{1}{7}\left(\int \sqrt{\tan(c+dx)}\left((7aA-3bB)a^2+b\left(18Ba^2+21Aba-7b^2B\right)\tan^2(c+dx)+7\left(Ba^3+3Aba^2-3b^2Ba-A\right.\right.\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\int \sqrt{\tan(c+dx)}\left((7aA-3bB)a^2+b\left(18Ba^2+21Aba-7b^2B\right)\tan(c+dx)^2+7\left(Ba^3+3Aba^2-3b^2Ba-A\right.\right.\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$\big\downarrow$ 4113

$$\frac{1}{7}\left(\int \sqrt{\tan(c+dx)}\left(7\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{2b(}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\int \sqrt{\tan(c+dx)}\left(7\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)dx+\frac{2b(}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$\big\downarrow$ 4011

$$\frac{1}{7}\left(\int \frac{7\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)-7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)}{\sqrt{\tan(c+dx)}}dx+\frac{2b\left(18a^2B+21aAb\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\int \frac{7\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)-7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)}{\sqrt{\tan(c+dx)}}dx+\frac{2b\left(18a^2B+21aAb\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{4017}$$

$$\frac{1}{7}\left(\frac{2\int -\frac{7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2b\left(18a^2B+21aAb-7b^2B\right)}{3d}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(-\frac{14\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2b\left(18a^2B+21aAb-7b^2B\right)\text{t}}{3d}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{1482}$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{d}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int -}{}\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\right.\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

$\downarrow$ 1103

$$\frac{1}{7}\left(\frac{2b\left(18a^2B+21aAb-7b^2B\right)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\right.}{}\right.$$

$$\frac{2bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}{7d}$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b*B*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2)/(7*d) + ((-14*(-1/2*((a^
3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*(-(ArcTan[1 -
 Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*
x]]]/Sqrt[2])) + ((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(
A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] +
 Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d +
(14*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Tan[c + d*x]])/d + (2*b*(
21*a*A*b + 18*a^2*B - 7*b^2*B)*Tan[c + d*x]^(3/2))/(3*d) + (2*b^2*(7*A*b +
 11*a*B)*Tan[c + d*x]^(5/2))/(5*d))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{2Bb^3\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{2Ab^3\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{6Bab^2\tan(dx+c)^{\frac{5}{2}}}{5}+2Aab^2\tan(dx+c)^{\frac{3}{2}}+2Ba^2b\tan(dx+c)^{\frac{3}{2}}-\frac{2Bb^3\tan(dx+c)^{\frac{3}{2}}}{3}}{}$
default	$\frac{\frac{2Bb^3\tan(dx+c)^{\frac{7}{2}}}{7}+\frac{2Ab^3\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{6Bab^2\tan(dx+c)^{\frac{5}{2}}}{5}+2Aab^2\tan(dx+c)^{\frac{3}{2}}+2Ba^2b\tan(dx+c)^{\frac{3}{2}}-\frac{2Bb^3\tan(dx+c)^{\frac{3}{2}}}{3}}{}$
parts	$\frac{(Ab^3+3Bab^2)\left(\frac{2\tan(dx+c)^{\frac{5}{2}}}{5}-2\sqrt{\tan(dx+c)}+\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2a\right.}{4}\right.}{d}$

input

```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output

```
1/d*(2/7*B*b^3*tan(d*x+c)^(7/2)+2/5*A*b^3*tan(d*x+c)^(5/2)+6/5*B*a*b^2*tan
(d*x+c)^(5/2)+2*A*a*b^2*tan(d*x+c)^(3/2)+2*B*a^2*b*tan(d*x+c)^(3/2)-2/3*B*
b^3*tan(d*x+c)^(3/2)+6*A*a^2*b*tan(d*x+c)^(1/2)-2*tan(d*x+c)^(1/2)*A*b^3+2
*B*a^3*tan(d*x+c)^(1/2)-6*tan(d*x+c)^(1/2)*B*a*b^2+1/4*(-3*A*a^2*b+A*b^3-B
*a^3+3*B*a*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d
*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*
arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)
*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*t
an(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/
2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(313) = 626$.

Time = 0.14 (sec) , antiderivative size = 2094, normalized size of antiderivative = 6.07

$$\int\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx=\text{Too large to display}$$

input

```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/210*(210*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b
 + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A
*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*ar
ctan(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A -
 B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 +
10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^
2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x +
 c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 -
10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^
2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2
*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 2
0*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b
^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*
B*a*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^
4 + (A^2 - B^2)*b^6)) + 210*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*
(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^
3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B +
B^2)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A
+ B)*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*
a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^...
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan(c+dx))\,(a+b\tan(c+dx))^3\,\sqrt{\tan(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3*sqrt(tan(c + d*x)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.05

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{120\,Bb^3\tan(dx+c)^{\frac{7}{2}} + 168\,(3\,Bab^2 + Ab^3)\tan(dx+c)^{\frac{5}{2}} + 210\,\sqrt{2}((A-B)a^3 - 3\,(A+B)a^2b - 3\,(A \cdots}{\cdots}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/420*(120*B*b^3*tan(d*x + c)^(7/2) + 168*(3*B*a*b^2 + A*b^3)*tan(d*x + c)
^(5/2) + 210*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A
 + B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 210*sqrt
(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 105*sqrt(2)*((A + B)*a^3
 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*log(sqrt(2)*sqrt(tan(d
*x + c)) + tan(d*x + c) + 1) + 105*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b
- 3*(A + B)*a*b^2 - (A - B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x
 + c) + 1) + 280*(3*B*a^2*b + 3*A*a*b^2 - B*b^3)*tan(d*x + c)^(3/2) + 840*
(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 - A*b^3)*sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 18.92 (sec) , antiderivative size = 6716, normalized size of antiderivative = 19.47

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
tan(c + d*x)^(1/2)*((2*B*a^3)/d - (6*B*a*b^2)/d) - tan(c + d*x)^(1/2)*((2*
A*b^3)/d - (6*A*a^2*b)/d) - tan(c + d*x)^(3/2)*((2*B*b^3)/(3*d) - (2*B*a^2
*b)/d) - atan((((8*(4*A*b^3*d^2 - 12*A*a^2*b*d^2)*((3*A^2*a*b^5)/(2*d^2) -
 (5*A^2*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4
- 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4
*a^10*b^2*d^4)^(1/2)/(4*d^4) + (3*A^2*a^5*b)/(2*d^2))^(1/2))/d^3 - (16*tan
(c + d*x)^(1/2)*(A^2*a^6 - A^2*b^6 + 15*A^2*a^2*b^4 - 15*A^2*a^4*b^2))/d^2
)*((3*A^2*a*b^5)/(2*d^2) - (5*A^2*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^
4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 25
5*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2)/(4*d^4) + (3*A^2*a^5*b)/(2*
d^2))^(1/2)*1i - ((8*(4*A*b^3*d^2 - 12*A*a^2*b*d^2)*((3*A^2*a*b^5)/(2*d^2)
 - (5*A^2*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^
4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A
^4*a^10*b^2*d^4)^(1/2)/(4*d^4) + (3*A^2*a^5*b)/(2*d^2))^(1/2))/d^3 + (16*t
an(c + d*x)^(1/2)*(A^2*a^6 - A^2*b^6 + 15*A^2*a^2*b^4 - 15*A^2*a^4*b^2))/d
^2)*((3*A^2*a*b^5)/(2*d^2) - (5*A^2*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 -
A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 -
255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2)/(4*d^4) + (3*A^2*a^5*b)/(
2*d^2))^(1/2)*1i)/(((8*(4*A*b^3*d^2 - 12*A*a^2*b*d^2)*((3*A^2*a*b^5)/(2*d^
2) - (5*A^2*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^...
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{8\sqrt{\tan(dx+c)}\,\tan(dx+c)^2\,a\,b^3 + 40\sqrt{\tan(dx+c)}\,a^3 b - 40\sqrt{\tan(dx+c)}\,a\,b^3 - 20\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)}{}$$

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(8*sqrt(tan(c + d*x))*tan(c + d*x)**2*a*b**3 + 40*sqrt(tan(c + d*x))*a**3*
b - 40*sqrt(tan(c + d*x))*a*b**3 - 20*int(sqrt(tan(c + d*x))/tan(c + d*x),
x)*a**3*b*d + 20*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*b**3*d + 5*int(s
qrt(tan(c + d*x)),x)*a**4*d + 5*int(sqrt(tan(c + d*x))*tan(c + d*x)**4,x)*
b**4*d + 30*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*a**2*b**2*d)/(5*d)
```

3.394 $\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 304

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}$$
$$+\frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d}$$
$$+\frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d}$$
$$+\frac{2b(15aAb+14a^2B-5b^2B)\sqrt{\tan(c+dx)}}{5d}$$
$$+\frac{2b^2(5Ab+9aB)\tan^{\frac{3}{2}}(c+dx)}{15d}+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

output
```
1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctan(-1+2^(1/2)*ta
n(d*x+c)^(1/2))*2^(1/2)/d+1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(
A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a^3*(A-B)-3*a*b^2*
(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x
+c)))*2^(1/2)/d+2/5*b*(15*A*a*b+14*B*a^2-5*B*b^2)*tan(d*x+c)^(1/2)/d+2/15*
b^2*(5*A*b+9*B*a)*tan(d*x+c)^(3/2)/d+2/5*b*B*tan(d*x+c)^(1/2)*(a+b*tan(d*x
+c))^2/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.50

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{-15\sqrt[4]{-1}(a-ib)^3(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-15\sqrt[4]{-1}(a+ib)^3(A+iB)\text{arctanh}\left((-1)^{3/\cdots}\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]]
,x]
```

output
```
(-15*(-1)^(1/4)*(a - I*b)^3*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]
] - 15*(-1)^(1/4)*(a + I*b)^3*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*
x]]] + 2*b*Sqrt[Tan[c + d*x]]*(15*(3*a*A*b + 3*a^2*B - b^2*B) + 5*b*(A*b +
 3*a*B)*Tan[c + d*x] + 3*b^2*B*Tan[c + d*x]^2))/(15*d)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 4090, 27, 3042, 4120, 27, 3042, 4113, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \textbf{4090}$$

$$\frac{2}{5}\int \frac{(a+b\tan(c+dx))\left(b(5Ab+9aB)\tan^2(c+dx)+5\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(5aA-bB)\right)}{2\sqrt{\tan(c+dx)}}dx+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{5}\int \frac{(a+b\tan(c+dx))\left(b(5Ab+9aB)\tan^2(c+dx)+5\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(5aA-bB)\right)}{\sqrt{\tan(c+dx)}}dx+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{5}\int \frac{(a+b\tan(c+dx))\left(b(5Ab+9aB)\tan(c+dx)^2+5\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(5aA-bB)\right)}{\sqrt{\tan(c+dx)}}dx+\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \textbf{4120}$$

$$\frac{1}{5}\left(\frac{2b^2(9aB+5Ab)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2}{3}\int -\frac{3\left((5aA-bB)a^2+b\left(14Ba^2+15Aba-5b^2B\right)\tan^2(c+dx)+5\left(Ba^3\right.\right.}{2\sqrt{\tan(c+dx)}}\right.$$
$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{5}\left(\int \frac{(5aA-bB)a^2+b\left(14Ba^2+15Aba-5b^2B\right)\tan^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}\right.$$
$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{5}\left(\int \frac{(5aA-bB)a^2+b\left(14Ba^2+15Aba-5b^2B\right)\tan(c+dx)^2+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}d\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \; 4113$$

$$\frac{1}{5}\left(\int \frac{5\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2b\left(14a^2B+15aAb\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(\int \frac{5\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2b\left(14a^2B+15aAb\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \; 4017$$

$$\frac{1}{5}\left(\frac{2\int \frac{5\left(Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2b\left(14a^2B+15aAb-5b^2B\right)\sqrt{}}{d}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}\left(\frac{10\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2b\left(14a^2B+15aAb-5b^2B\right)\sqrt{t}}{d}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \; 1482$$

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+B)\right.\right.}{d}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+B)\right.\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+B\right.\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+B\right.\right.}{d}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int}{}\right.\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{}{\tan(}}{}\right.\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{}{\mathrm{ta}}\right.\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{1}{5}\left(\frac{2b(14a^2B+15aAb-5b^2B)\sqrt{\tan(c+dx)}}{d}+\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\right.}{}\right.$$

$$\frac{2bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}{5d}$$

input
```
Int[((a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x]
```

output
```
(2*b*B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2)/(5*d) + ((10*(((3*a^2*b*
(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*(-(ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/S
qrt[2]))/2 + ((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A +
B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/d + (2*b
*(15*a*A*b + 14*a^2*B - 5*b^2*B)*Sqrt[Tan[c + d*x]])/d + (2*b^2*(5*A*b + 9
*a*B)*Tan[c + d*x]^(3/2))/(3*d))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
 !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{2Bb^3\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2Ab^3\tan(dx+c)^{\frac{3}{2}}}{3}+2Bab^2\tan(dx+c)^{\frac{3}{2}}+6\sqrt{\tan(dx+c)}\,Aab^2+6\sqrt{\tan(dx+c)}\,Ba^2b-2\sqrt{\tan(dx+c)}}{}$
default	$\frac{\frac{2Bb^3\tan(dx+c)^{\frac{5}{2}}}{5}+\frac{2Ab^3\tan(dx+c)^{\frac{3}{2}}}{3}+2Bab^2\tan(dx+c)^{\frac{3}{2}}+6\sqrt{\tan(dx+c)}\,Aab^2+6\sqrt{\tan(dx+c)}\,Ba^2b-2\sqrt{\tan(dx+c)}}{}$
parts	$\frac{(Ab^3+3Bab^2)\left(\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{t}\right.}{4}\right.}{d}$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2/5*B*b^3*tan(d*x+c)^(5/2)+2/3*A*b^3*tan(d*x+c)^(3/2)+2*B*a*b^2*tan(d
*x+c)^(3/2)+6*tan(d*x+c)^(1/2)*A*a*b^2+6*tan(d*x+c)^(1/2)*B*a^2*b-2*tan(d*
x+c)^(1/2)*B*b^3+1/4*(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*2^(1/2)*(ln((tan(d*
x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+
2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
)+1/4*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*ta
n(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2051 vs. $2(275) = 550$.

Time = 0.15 (sec) , antiderivative size = 2051, normalized size of antiderivative = 6.75

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input

```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="fricas")
```

output

```
1/30*(30*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b +
 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B
 + B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*arct
an(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B
)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10
*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*
b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x + c
)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10
*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*
b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2*A
*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*
(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5
 + (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*B*
a*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^4
+ (A^2 - B^2)*b^6)) + 30*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^
2 - B^2)*a^5*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 +
 3*(A^2 + 10*A*B + B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2
)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B
)*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5
*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 +...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^3}{\sqrt{\tan(c+dx)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**3*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3/sqrt(tan(c + d*x)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{24\,Bb^3\tan(dx+c)^{\frac{5}{2}} + 30\,\sqrt{2}((A+B)a^3 + 3\,(A-B)a^2b - 3\,(A+B)ab^2 - (A-B)b^3)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{\right.\right.}{}}{}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
1/60*(24*B*b^3*tan(d*x + c)^(5/2) + 30*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^
2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(
tan(d*x + c)))) + 30*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*
b^2 - (A - B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) +
 15*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3
)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 15*sqrt(2)*((A - B)
*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*log(-sqrt(2)*sqrt(
tan(d*x + c)) + tan(d*x + c) + 1) + 40*(3*B*a*b^2 + A*b^3)*tan(d*x + c)^(3
/2) + 120*(3*B*a^2*b + 3*A*a*b^2 - B*b^3)*sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 6657, normalized size of antiderivative = 21.90

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3)/tan(c + d*x)^(1/2),x)
```

output
```
atan((((8*(4*A*a^3*d^2 - 12*A*a*b^2*d^2)*((5*A^2*a^3*b^3)/d^2 - (30*A^4*a^
2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a
^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2)/(4*d^4) - (3
*A^2*a*b^5)/(2*d^2) - (3*A^2*a^5*b)/(2*d^2))^(1/2))/d^3 - (16*tan(c + d*x)
^(1/2)*(A^2*a^6 - A^2*b^6 + 15*A^2*a^2*b^4 - 15*A^2*a^4*b^2))/d^2)*((5*A^2
*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A
^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b
^2*d^4)^(1/2)/(4*d^4) - (3*A^2*a*b^5)/(2*d^2) - (3*A^2*a^5*b)/(2*d^2))^(1/
2)*1i - ((8*(4*A*a^3*d^2 - 12*A*a*b^2*d^2)*((5*A^2*a^3*b^3)/d^2 - (30*A^4*
a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4
*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2)/(4*d^4) -
(3*A^2*a*b^5)/(2*d^2) - (3*A^2*a^5*b)/(2*d^2))^(1/2))/d^3 + (16*tan(c + d*
x)^(1/2)*(A^2*a^6 - A^2*b^6 + 15*A^2*a^2*b^4 - 15*A^2*a^4*b^2))/d^2)*((5*A
^2*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255
*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10
*b^2*d^4)^(1/2)/(4*d^4) - (3*A^2*a*b^5)/(2*d^2) - (3*A^2*a^5*b)/(2*d^2))^(
1/2)*1i)/((16*(3*A^3*a^8*b - A^3*b^9 + 6*A^3*a^4*b^5 + 8*A^3*a^6*b^3))/d^3
 + ((8*(4*A*a^3*d^2 - 12*A*a*b^2*d^2)*((5*A^2*a^3*b^3)/d^2 - (30*A^4*a^2*b
^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*
b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2)/(4*d^4) - (3...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\tan(dx+c)}\,\tan(dx+c)^2 b^4 + 60\sqrt{\tan(dx+c)}\,a^2b^2 - 10\sqrt{\tan(dx+c)}\,b^4 + 5\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right) a^4 d$$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*tan(c + d*x)**2*b**4 + 60*sqrt(tan(c + d*x))*a**2*b*
*2 - 10*sqrt(tan(c + d*x))*b**4 + 5*int(sqrt(tan(c + d*x))/tan(c + d*x),x)
*a**4*d - 30*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a**2*b**2*d + 5*int(sq
rt(tan(c + d*x))/tan(c + d*x),x)*b**4*d + 20*int(sqrt(tan(c + d*x)),x)*a**
3*b*d + 20*int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*a*b**3*d)/(5*d)
```

3.395 $\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 297

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{2b(2a^2A+Ab^2+3abB)\sqrt{\tan(c+dx)}}{d} + \frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d} - \frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

output
```
-1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))*2^(1/2)/d-1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*
(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*(3*a^2*b*(A-B)-b^3
*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*
x+c)))*2^(1/2)/d+2*b*(2*A*a^2+A*b^2+3*B*a*b)*tan(d*x+c)^(1/2)/d+2/3*b^2*(3
*A*a+B*b)*tan(d*x+c)^(3/2)/d-2*a*A*(a+b*tan(d*x+c))^2/d/tan(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.77 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{8b(-12aAb-17a^2B+3b^2B)-3\Big(8(a^3A-3aAb^2-3a^2bB+b^3B)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},-\tan\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2)
,x]
```

output
```
(8*b*(-12*a*A*b - 17*a^2*B + 3*b^2*B) - 3*(8*(a^3*A - 3*a*A*b^2 - 3*a^2*b*
B + b^3*B)*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2] + Sqrt[2]*(3*a
^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*(2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]])*Sqrt[Tan[c + d*x]]) + 8*b*(3*A*b + 7*a*B)*(a + b*Tan[c + d*x]) + 8*b
*B*(a + b*Tan[c + d*x])^2)/(12*d*Sqrt[Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4088, 27, 3042, 4120, 27, 3042, 4113, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 4088}$$

$$2\int \frac{(a+b\tan(c+dx))\left(b(3aA+bB)\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+aB)\right)}{2\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(a+b\tan(c+dx))\left(b(3aA+bB)\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+aB)\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))\left(b(3aA+bB)\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(5Ab+aB)\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{ 4120}$$

$$-\frac{2}{3}\int -\frac{3\left((5Ab+aB)a^2+b\left(2Aa^2+3bBa+Ab^2\right)\tan^2(c+dx)-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}}$$

$$\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 27

$$\int \frac{(5Ab+aB)a^2+b\left(2Aa^2+3bBa+Ab^2\right)\tan^2(c+dx)-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 3042

$$\int \frac{(5Ab+aB)a^2+b\left(2Aa^2+3bBa+Ab^2\right)\tan(c+dx)^2-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 4113

$$\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2b\left(2a^2A+3abB+Ab^2\right)\sqrt{\tan(c+dx)}}{d}+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 3042

$$\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2b\left(2a^2A+3abB+Ab^2\right)\sqrt{\tan(c+dx)}}{d}+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$\downarrow$ 4017

$$\frac{2\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-(Aa^3-3bBa^2-3Ab^2a+b^3B)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2b(2a^2A+3abB+Ab^2)\sqrt{\tan(c+dx)}}{d}+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 1482

$$\frac{2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-B)-3a\right.}{d}$$
$$\frac{2b(2a^2A+3abB+Ab^2)\sqrt{\tan(c+dx)}}{d}+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 1476

$$2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-B)-3a\right.$$
$$\frac{2b(2a^2A+3abB+Ab^2)\sqrt{\tan(c+dx)}}{d}+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 1082

$$2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-B)-3a\right.$$
$$\frac{2b(2a^2A+3abB+Ab^2)\sqrt{\tan(c+dx)}}{d}+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 217

$$\frac{2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-B)-3a\right.\right.}{d}$$

$$\frac{2b\left(2a^2A+3abB+Ab^2\right)\sqrt{\tan(c+dx)}}{d}+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{}}{\tan(}}{}\right.\right.$$

$$\frac{2b\left(2a^2A+3abB+Ab^2\right)\sqrt{\tan(c+dx)}}{d}+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\right.}{\tan(c+dx)-}}{}\right.\right.$$

$$\frac{2b\left(2a^2A+3abB+Ab^2\right)\sqrt{\tan(c+dx)}}{d}+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{}}{\tan(c+d}\right.\right.$$

$$\frac{2b\left(2a^2A+3abB+Ab^2\right)\sqrt{\tan(c+dx)}}{d}+\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2b(2a^2A+3abB+Ab^2)\sqrt{\tan(c+dx)}}{d}+$$

$$\frac{2\left(\frac{1}{2}(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\right.}{2\sqrt{}}\right.\right.}{}$$

$$\frac{2b^2(3aA+bB)\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA(a+b\tan(c+dx))^2}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[((a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x]
```

output
```
(2*(-1/2*((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*
(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/Sqrt[2])) + ((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B
) - 3*a*b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2
])))/2))/d + (2*b*(2*a^2*A + A*b^2 + 3*a*b*B)*Sqrt[Tan[c + d*x]])/d + (2*b
^2*(3*a*A + b*B)*Tan[c + d*x]^(3/2))/(3*d) - (2*a*A*(a + b*Tan[c + d*x])^2
)/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{2B\,b^3\tan(dx+c)^{\frac{3}{2}}}{3}+2\sqrt{\tan(dx+c)}\,A\,b^3+6\sqrt{\tan(dx+c)}\,Ba\,b^2-\frac{2A\,a^3}{\sqrt{\tan(dx+c)}}+\frac{\left(3A\,a^2b-A\,b^3+B\,a^3-3Ba\,b^2\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx}{\tan(dx}\right.\right.}{}}{}$
default	$\frac{\frac{2B\,b^3\tan(dx+c)^{\frac{3}{2}}}{3}+2\sqrt{\tan(dx+c)}\,A\,b^3+6\sqrt{\tan(dx+c)}\,Ba\,b^2-\frac{2A\,a^3}{\sqrt{\tan(dx+c)}}+\frac{\left(3A\,a^2b-A\,b^3+B\,a^3-3Ba\,b^2\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx}{\tan(dx}\right.\right.}{}}{}$
parts	$\frac{(A\,b^3+3Ba\,b^2)\left(2\sqrt{\tan(dx+c)}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\right.}{4}\right.}{d}$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2/3*B*b^3*tan(d*x+c)^(3/2)+2*tan(d*x+c)^(1/2)*A*b^3+6*tan(d*x+c)^(1/2
)*B*a*b^2-2*A*a^3/tan(d*x+c)^(1/2)+1/4*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*2
^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan
(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)
*tan(d*x+c)^(1/2)))+1/4*(-A*a^3+3*A*a*b^2+3*B*a^2*b-B*b^3)*2^(1/2)*(ln((ta
n(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+
1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1
/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2074 vs. $2(272) = 544$.

Time = 0.13 (sec) , antiderivative size = 2074, normalized size of antiderivative = 6.98

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
-1/6*(6*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b +
3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B
+ B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*arcta
n(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)
*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*
A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b
^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x + c)
) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*
A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*b
^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2*A*
B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(
A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5
+ (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*B*a
*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^4 +
 (A^2 - B^2)*b^6))*tan(d*x + c) + 6*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*
a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2
)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 +
2*A*B + B^2)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a^2*b
 - 3*(A + B)*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2
 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^3}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**3*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3/tan(c + d*x)**(3/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{8\,Bb^3\tan(dx+c)^{\frac{3}{2}} - \frac{24\,Aa^3}{\sqrt{\tan(dx+c)}} - 6\sqrt{2}((A-B)a^3 - 3\,(A+B)a^2b - 3\,(A-B)ab^2 + (A+B)b^3)\,\mathrm{arcta}}{}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
1/12*(8*B*b^3*tan(d*x + c)^(3/2) - 24*A*a^3/sqrt(tan(d*x + c)) - 6*sqrt(2)
*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*arctan(1/
2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) - 6*sqrt(2)*((A - B)*a^3 - 3*(
A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2)
 - 2*sqrt(tan(d*x + c)))) + 3*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(
A + B)*a*b^2 - (A - B)*b^3)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c)
+ 1) - 3*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B
)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 24*(3*B*a*b^2
 + A*b^3)*sqrt(tan(d*x + c)))/d
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^3}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3/tan(d*x + c)^(3/2),
x)
```

Mupad [B] (verification not implemented)

Time = 7.53 (sec) , antiderivative size = 7108, normalized size of antiderivative = 23.93

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3)/tan(c + d*x)^(3/2),x)
```

output
```
atan((((8*(4*B*a^3*d^2 - 12*B*a*b^2*d^2)*((5*B^2*a^3*b^3)/d^2 - (30*B^4*a^
2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a
^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^(1/2)/(4*d^4) - (3
*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^(1/2))/d^3 - (16*tan(c + d*x)
^(1/2)*(B^2*a^6 - B^2*b^6 + 15*B^2*a^2*b^4 - 15*B^2*a^4*b^2))/d^2)*((5*B^2
*a^3*b^3)/d^2 - (30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B
^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b
^2*d^4)^(1/2)/(4*d^4) - (3*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^(1/
2)*1i - ((8*(4*B*a^3*d^2 - 12*B*a*b^2*d^2)*((5*B^2*a^3*b^3)/d^2 - (30*B^4*
a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4
*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^(1/2)/(4*d^4) -
(3*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^(1/2))/d^3 + (16*tan(c + d*
x)^(1/2)*(B^2*a^6 - B^2*b^6 + 15*B^2*a^2*b^4 - 15*B^2*a^4*b^2))/d^2)*((5*B
^2*a^3*b^3)/d^2 - (30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255
*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10
*b^2*d^4)^(1/2)/(4*d^4) - (3*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^(
1/2)*1i)/(((8*(4*B*a^3*d^2 - 12*B*a*b^2*d^2)*((5*B^2*a^3*b^3)/d^2 - (30*B^
4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B
^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^(1/2)/(4*d^4)
- (3*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^(1/2))/d^3 - (16*tan(c...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{8\sqrt{\tan(dx+c)}\,a\,b^3 + \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)a^4d + 4\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a^3bd - 4\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a\,b^3d + 6\Big(\int}{d}$$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
(8*sqrt(tan(c + d*x))*a*b**3 + int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a
**4*d + 4*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a**3*b*d - 4*int(sqrt(tan
(c + d*x))/tan(c + d*x),x)*a*b**3*d + 6*int(sqrt(tan(c + d*x)),x)*a**2*b**
2*d + int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*b**4*d)/d
```

3.396 $\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 297

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{2a^2(7Ab+3aB)}{3d\sqrt{\tan(c+dx)}} + \frac{2b^2(aA+3bB)\sqrt{\tan(c+dx)}}{3d} - \frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

output
```
-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))*2^(1/2)/d-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*
(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(a^3*(A-B)-3*a*b^2
*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*
x+c)))*2^(1/2)/d-2/3*a^2*(7*A*b+3*B*a)/d/tan(d*x+c)^(1/2)+2/3*b^2*(A*a+3*B
*b)*tan(d*x+c)^(1/2)/d-2/3*a*A*(a+b*tan(d*x+c))^2/d/tan(d*x+c)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.86 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.56

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left((a^3A-3aAb^2-3a^2bB+b^3B)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},1,\frac{1}{4},-\tan^2(c+dx)\right)+3(3a^2Ab-Ab^3+\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2)
,x]
```

output
```
(-2*((a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*Hypergeometric2F1[-3/4, 1, 1/
4, -Tan[c + d*x]^2] + 3*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Hypergeome
tric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]*Tan[c + d*x] + b*(3*a*A*b + 3*a^2*B
 - b^2*B + 3*b*(A*b + 3*a*B)*Tan[c + d*x] - 3*b^2*B*Tan[c + d*x]^2)))/(3*d
*Tan[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.02, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 4088, 27, 3042, 4118, 25, 3042, 4113, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 4088}$$

$$\frac{2}{3}\int \frac{(a+b\tan(c+dx))\left(b(aA+3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+3aB)\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\int \frac{(a+b\tan(c+dx))\left(b(aA+3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+3aB)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\int \frac{(a+b\tan(c+dx))\left(b(aA+3bB)\tan(c+dx)^2-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(7Ab+3aB)\right)}{\tan(c+dx)^{3/2}}dx-\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4118

$$\frac{1}{3}\left(\int -\frac{-b^2(aA+3bB)\tan^2(c+dx)+3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(3Aa^2-9bBa-10Ab^2\right.}{\sqrt{\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 25

$$\frac{1}{3}\left(-\int \frac{-b^2(aA+3bB)\tan^2(c+dx)+3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(3Aa^2-9bBa-10Ab^2\right.}{\sqrt{\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(-\int \frac{-b^2(aA+3bB)\tan(c+dx)^2+3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(3Aa^2-9bBa-10Ab^2\right.}{\sqrt{\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4113}$$

$$\frac{1}{3}\left(-\int \frac{3\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(3aB+7Ab)}{d\sqrt{\tan(c+dx)}}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-\int \frac{3\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(3aB+7Ab)}{d\sqrt{\tan(c+dx)}}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4017}$$

$$\frac{1}{3}\left(-\frac{2\int \frac{3\left(Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^2(3aB+7Ab)}{d\sqrt{\tan(c+dx)}}+\frac{2b^2(aA-\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{6\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2a^2(3aB+7Ab)}{d\sqrt{\tan(c+dx)}}+\frac{2b^2(aA+3\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1482}$$

$$\frac{1}{3}\left(-\frac{6\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+B\right.\right.}{d}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{3}\left(-\frac{6\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+B\right.\right.}{}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{3}\left(-\frac{6\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+B\right.\right.}{}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{3}\left(-\frac{6\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A+B\right.\right.}{d}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{1}{3}\left(-\frac{6\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\right.\right.}{}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(-\frac{6\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{}{\tan}}{}\right.\right.}{}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(-\frac{6\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{}{\mathrm{t}}\right.\right.}{}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{3}\left(-\frac{2a^2(3aB+7Ab)}{d\sqrt{\tan(c+dx)}}-\frac{6\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x]
```

output
```
((-6*(((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*(-(
ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]]]/Sqrt[2]))/2 + ((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A
+ B) + b^3*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]
)))/2))/d - (2*a^2*(7*A*b + 3*a*B))/(d*Sqrt[Tan[c + d*x]]) + (2*b^2*(a*A +
 3*b*B)*Sqrt[Tan[c + d*x]])/d)/3 - (2*a*A*(a + b*Tan[c + d*x])^2)/(3*d*Tan
[c + d*x]^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2\sqrt{\tan(dx+c)}\,B\,b^3+\frac{\left(-A\,a^3+3Aa\,b^2+3B\,a^2b-B\,b^3\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+\cdots}{4}}{\cdots}$
default	$\frac{2\sqrt{\tan(dx+c)}\,B\,b^3+\frac{\left(-A\,a^3+3Aa\,b^2+3B\,a^2b-B\,b^3\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+\cdots}{4}}{\cdots}$
parts	$\frac{\left(A\,b^3+3Ba\,b^2\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+\cdots}\right.\right.}{4d}$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2*tan(d*x+c)^(1/2)*B*b^3+1/4*(-A*a^3+3*A*a*b^2+3*B*a^2*b-B*b^3)*2^(1/
2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x
+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2)))+1/4*(-3*A*a^2*b+A*b^3-B*a^3+3*B*a*b^2)*2^(1/2)*(ln((tan(d*
x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+
2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
)-2/3*A*a^3/tan(d*x+c)^(3/2)-2*a^2*(3*A*b+B*a)/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2077 vs. $2(267) = 534$.

Time = 0.14 (sec) , antiderivative size = 2077, normalized size of antiderivative = 6.99

$$\int \frac{(a + b\tan(c + dx))^3 (A + B\tan(c + dx))}{\tan^{\frac{5}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorith
m="fricas")
```

output
```
-1/6*(6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b +
3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B
+ B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*arcta
n(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)
*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*
A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*b
^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x + c)
) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*
A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*b
^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2*A*
B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(
A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5
+ (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*B*a
*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^4 +
 (A^2 - B^2)*b^6))*tan(d*x + c)^2 + 6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2
)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B
^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2
- 2*A*B + B^2)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2
*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A
^2 - B^2)*a^5*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^3}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**3*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3/tan(c + d*x)**(5/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{24\,Bb^3\sqrt{\tan(dx+c)} - 6\sqrt{2}((A+B)a^3 + 3\,(A-B)a^2b - 3\,(A+B)ab^2 - (A-B)b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{\right.\right.}{}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorith
m="maxima")
```

output
```
1/12*(24*B*b^3*sqrt(tan(d*x + c)) - 6*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2
*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(t
an(d*x + c)))) - 6*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^
2 - (A - B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 3
*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*l
og(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 3*sqrt(2)*((A - B)*a^3
 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*log(-sqrt(2)*sqrt(tan(
d*x + c)) + tan(d*x + c) + 1) - 8*(A*a^3 + 3*(B*a^3 + 3*A*a^2*b)*tan(d*x +
 c))/tan(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$=\int \frac{(B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^3}{\tan{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3/tan(d*x + c)^(5/2),
x)
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 7578, normalized size of antiderivative = 25.52

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3)/tan(c + d*x)^(5/2),x)
```

output
```
2*atanh((32*A^2*a^6*d^3*tan(c + d*x)^(1/2)*((5*A^2*a^3*b^3)/d^2 - (30*A^4*
a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4
*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2)/(4*d^4) -
(3*A^2*a*b^5)/(2*d^2) - (3*A^2*a^5*b)/(2*d^2))^(1/2))/(16*A*a^3*(30*A^4*a^
2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a
^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4)^(1/2) + 16*A^3*b^9
*d^2 - 288*A^3*a^2*b^7*d^2 + 960*A^3*a^4*b^5*d^2 - 736*A^3*a^6*b^3*d^2 - 4
8*A*a*b^2*(30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a^4
*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d^4
)^(1/2) + 48*A^3*a^8*b*d^2) - (32*A^2*b^6*d^3*tan(c + d*x)^(1/2)*((5*A^2*a
^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4
*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2
*d^4)^(1/2)/(4*d^4) - (3*A^2*a*b^5)/(2*d^2) - (3*A^2*a^5*b)/(2*d^2))^(1/2)
)/(16*A*a^3*(30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*a^12*d^4 - 255*A^4*a
^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4 + 30*A^4*a^10*b^2*d
^4)^(1/2) + 16*A^3*b^9*d^2 - 288*A^3*a^2*b^7*d^2 + 960*A^3*a^4*b^5*d^2 - 7
36*A^3*a^6*b^3*d^2 - 48*A*a*b^2*(30*A^4*a^2*b^10*d^4 - A^4*b^12*d^4 - A^4*
a^12*d^4 - 255*A^4*a^4*b^8*d^4 + 452*A^4*a^6*b^6*d^4 - 255*A^4*a^8*b^4*d^4
 + 30*A^4*a^10*b^2*d^4)^(1/2) + 48*A^3*a^8*b*d^2) + (480*A^2*a^2*b^4*d^3*t
an(c + d*x)^(1/2)*((5*A^2*a^3*b^3)/d^2 - (30*A^4*a^2*b^10*d^4 - A^4*b^1...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2\sqrt{\tan(dx+c)}\,b^4+\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)a^4d+4\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)a^3bd+6\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a^2b^2d-\left(\int \frac{\sqrt{}}{}\right.}{d}$$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*b**4 + int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*a**
4*d + 4*int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a**3*b*d + 6*int(sqrt(ta
n(c + d*x))/tan(c + d*x),x)*a**2*b**2*d - int(sqrt(tan(c + d*x))/tan(c + d
*x),x)*b**4*d + 4*int(sqrt(tan(c + d*x)),x)*a*b**3*d)/d
```

3.397 $\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 304

$$\begin{aligned}\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx =& -\frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &+\frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} \\ &-\frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} \\ &-\frac{2a^2(9Ab+5aB)}{15d\tan^{\frac{3}{2}}(c+dx)}+\frac{2a(5a^2A-14Ab^2-15abB)}{5d\sqrt{\tan(c+dx)}}-\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}\end{aligned}$$

output
```
1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctan(-1+2^(1/2)*ta
n(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(
A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*(3*a^2*b*(A-B)-b^3*
(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x
+c)))*2^(1/2)/d-2/15*a^2*(9*A*b+5*B*a)/d/tan(d*x+c)^(3/2)+2/5*a*(5*A*a^2-1
4*A*b^2-15*B*a*b)/d/tan(d*x+c)^(1/2)-2/5*a*A*(a+b*tan(d*x+c))^2/d/tan(d*x+
c)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.93 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.55

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx =$$
$$-\frac{2\left(3(a^3A-3aAb^2-3a^2bB+b^3B)\,\text{Hypergeometric2F1}\left(-\frac{5}{4},1,-\frac{1}{4},-\tan^2(c+dx)\right)+5(3a^2Ab-Ab^3\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2)
,x]
```

output
```
(-2*(3*(a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*Hypergeometric2F1[-5/4, 1,
-1/4, -Tan[c + d*x]^2] + 5*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Hyperge
ometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2]*Tan[c + d*x] + b*(9*a*A*b + 9*a^
2*B - 3*b^2*B + 5*b*(A*b + 3*a*B)*Tan[c + d*x] + 15*b^2*B*Tan[c + d*x]^2))
)/(15*d*Tan[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.576, Rules used = {3042, 4088, 27, 3042, 4118, 25, 3042, 4111, 27, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 4088}$$

$$\frac{2}{5}\int \frac{(a+b\tan(c+dx))\left(-b(aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(9Ab+5aB)\right)}{2\tan^{\frac{5}{2}}(c+dx)}dx$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{5}\int \frac{(a+b\tan(c+dx))\left(-b(aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(9Ab+5aB)\right)}{\tan^{\frac{5}{2}}(c+dx)}dx$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\int \frac{(a+b\tan(c+dx))\left(-b(aA-5bB)\tan(c+dx)^2-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(9Ab+5aB)\right)}{\tan(c+dx)^{5/2}}dx$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4118

$$\frac{1}{5}\left(\int -\frac{b^2(aA-5bB)\tan^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(5Aa^2-15bBa-14Ab^2\right)}{\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 25

$$\frac{1}{5}\left(-\int \frac{b^2(aA-5bB)\tan^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(5Aa^2-15bBa-14Ab^2\right)}{\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-\int \frac{b^2(aA-5bB)\tan(c+dx)^2+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(5Aa^2-15bBa-14Ab^2\right)}{\tan(c+dx)^{3/2}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4111}$$

$$\frac{1}{5}\left(-\int \frac{5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx+\frac{2a\left(5a^2A-15abB\right.}{d\sqrt{\tan(c+\right.}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(-5\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2a\left(5a^2A-15abB-\right.}{d\sqrt{\tan(c+dx}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-5\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx+\frac{2a\left(5a^2A-15abB-\right.}{d\sqrt{\tan(c+dx}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4017}$$

$$\frac{1}{5}\left(-\frac{10\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2a\left(5a^2A-15abB-14Ab^2\right)}{d\sqrt{\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{1482}$$

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{d}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 1082

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 217

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{d}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 1479

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\right.\right.}{}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 25

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \ldots}{\ldots}\right.\right.}{\ldots}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \ldots\right.\right.}{\ldots}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 1103

$$\frac{1}{5}\left(\frac{2a\left(5a^2A-15abB-14Ab^2\right)}{d\sqrt{\tan(c+dx)}}-\frac{2a^2(5aB+9Ab)}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A\ldots\right.\right.}{\ldots}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^2}{5d\tan^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x]
```

output
```
((-10*(-1/2*((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B
))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A
+ B) - 3*a*b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqr
t[2])))/2))/d - (2*a^2*(9*A*b + 5*a*B))/(3*d*Tan[c + d*x]^(3/2)) + (2*a*(5
*a^2*A - 14*A*b^2 - 15*a*b*B))/(d*Sqrt[Tan[c + d*x]]))/5 - (2*a*A*(a + b*T
an[c + d*x])^2)/(5*d*Tan[c + d*x]^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{\left(-3A\,a^2b+A\,b^3-B\,a^3+3Ba\,b^2\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(\right.\right.}{4}}{}$
default	$\frac{\frac{\left(-3A\,a^2b+A\,b^3-B\,a^3+3Ba\,b^2\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(\right.\right.}{4}}{}$
parts	$\frac{\left(A\,b^3+3Ba\,b^2\right)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+\right.\right.}{4d}$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(1/4*(-3*A*a^2*b+A*b^3-B*a^3+3*B*a*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2
)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+
2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/4*(A*a^
3-3*A*a*b^2-3*B*a^2*b+B*b^3)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1
/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x
+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-2/5*A*a^3/tan(d*x+c)^(5/
2)-2/3*a^2*(3*A*b+B*a)/tan(d*x+c)^(3/2)+2*a*(A*a^2-3*A*b^2-3*B*a*b)/tan(d*
x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2099 vs. $2(274) = 548$.

Time = 0.17 (sec) , antiderivative size = 2099, normalized size of antiderivative = 6.90

$$\int \frac{(a + b\tan(c + dx))^3(A + B\tan(c + dx))}{\tan^{\frac{7}{2}}(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorith
m="fricas")
```

output
```
1/30*(30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b +
 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B
 + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*arct
an(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B
)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10
*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*
b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x + c
)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10
*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*
b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2*A
*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*
(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5
 + (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*B*
a*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^4
+ (A^2 - B^2)*b^6))*tan(d*x + c)^3 + 30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B
^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 -
 B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^
2 + 2*A*B + B^2)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a
^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*
(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^3}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**3*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3/tan(c + d*x)**(7/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{30\sqrt{2}((A-B)a^3-3(A+B)a^2b-3(A-B)ab^2+(A+B)b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)}{}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorith
m="maxima")
```

output
```
1/60*(30*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B
)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 30*sqrt(2)*(
(A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - 15*sqrt(2)*((A + B)*a^3 + 3*(
A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*log(sqrt(2)*sqrt(tan(d*x + c
)) + tan(d*x + c) + 1) + 15*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A
+ B)*a*b^2 - (A - B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) +
 1) - 8*(3*A*a^3 - 15*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2)*tan(d*x + c)^2 + 5*(
B*a^3 + 3*A*a^2*b)*tan(d*x + c))/tan(d*x + c)^(5/2))/d
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^3}{\tan(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algorith
m="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 7591, normalized size of antiderivative = 24.97

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3)/tan(c + d*x)^(7/2),x)
```

output
```
2*atanh((32*B^2*a^6*d^3*tan(c + d*x)^(1/2)*((5*B^2*a^3*b^3)/d^2 - (30*B^4*
a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4
*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^(1/2)/(4*d^4) -
(3*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^(1/2))/(16*B^3*b^9*d^2 + 16
*B*a^3*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^
8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^(
1/2) - 288*B^3*a^2*b^7*d^2 + 960*B^3*a^4*b^5*d^2 - 736*B^3*a^6*b^3*d^2 + 4
8*B^3*a^8*b*d^2 - 48*B*a*b^2*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^1
2*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 +
30*B^4*a^10*b^2*d^4)^(1/2)) - (32*B^2*b^6*d^3*tan(c + d*x)^(1/2)*((5*B^2*a
^3*b^3)/d^2 - (30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4
*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2
*d^4)^(1/2)/(4*d^4) - (3*B^2*a*b^5)/(2*d^2) - (3*B^2*a^5*b)/(2*d^2))^(1/2)
)/(16*B^3*b^9*d^2 + 16*B*a^3*(30*B^4*a^2*b^10*d^4 - B^4*b^12*d^4 - B^4*a^1
2*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 - 255*B^4*a^8*b^4*d^4 +
30*B^4*a^10*b^2*d^4)^(1/2) - 288*B^3*a^2*b^7*d^2 + 960*B^3*a^4*b^5*d^2 - 7
36*B^3*a^6*b^3*d^2 + 48*B^3*a^8*b*d^2 - 48*B*a*b^2*(30*B^4*a^2*b^10*d^4 -
B^4*b^12*d^4 - B^4*a^12*d^4 - 255*B^4*a^4*b^8*d^4 + 452*B^4*a^6*b^6*d^4 -
255*B^4*a^8*b^4*d^4 + 30*B^4*a^10*b^2*d^4)^(1/2)) + (480*B^2*a^2*b^4*d^3*t
an(c + d*x)^(1/2)*((5*B^2*a^3*b^3)/d^2 - (30*B^4*a^2*b^10*d^4 - B^4*b^1...
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4}dx\right)a^4 + 4\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)a^3b + 6\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)a^2b^2 + 4\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)a\,b^3 + \left(\int \sqrt{\tan(dx+c)}dx\right)b^4$$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output

```
int(sqrt(tan(c + d*x))/tan(c + d*x)**4,x)*a**4 + 4*int(sqrt(tan(c + d*x))/
tan(c + d*x)**3,x)*a**3*b + 6*int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*a*
*2*b**2 + 4*int(sqrt(tan(c + d*x))/tan(c + d*x),x)*a*b**3 + int(sqrt(tan(c
 + d*x)),x)*b**4
```

3.398 $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 264

$$\begin{aligned}\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx &= \frac{(a(A-B)+b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\\ &\quad-\frac{(a(A-B)+b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\\ &\quad-\frac{2a^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}\,(a^2+b^2)\,d}\\ &\quad-\frac{(b(A-B)-a(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\\ &\quad+\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{b^2d}+\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}\end{aligned}$$

output
```
-1/2*(a*(A-B)+b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^
2)/d-1/2*(a*(A-B)+b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2
+b^2)/d-2*a^(5/2)*(A*b-B*a)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(5/
2)/(a^2+b^2)/d-1/2*(b*(A-B)-a*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+t
an(d*x+c)))*2^(1/2)/(a^2+b^2)/d+2*(A*b-B*a)*tan(d*x+c)^(1/2)/b^2/d+2/3*B*t
an(d*x+c)^(3/2)/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.71

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{-3\sqrt[4]{-1}(a+ib)b^{5/2}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+6a^{5/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{}$$

input
```
Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
(-3*(-1)^(1/4)*(a + I*b)*b^(5/2)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c +
d*x]]] + 6*a^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt
[a]] + 3*(-1)^(1/4)*b^(5/2)*(I*a + b)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Ta
n[c + d*x]]] + 2*Sqrt[b]*(a^2 + b^2)*Sqrt[Tan[c + d*x]]*(3*A*b - 3*a*B + b
*B*Tan[c + d*x]))/(3*b^(5/2)*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.09, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4136, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{5/2}(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{2\int -\frac{3\sqrt{\tan(c+dx)}(-((Ab-aB)\tan^2(c+dx))+bB\tan(c+dx)+aB)}{2(a+b\tan(c+dx))}dx}{3b}+\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\int \frac{\sqrt{\tan(c+dx)}(-((Ab-aB)\tan^2(c+dx))+bB\tan(c+dx)+aB)}{a+b\tan(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\int \frac{\sqrt{\tan(c+dx)}(-((Ab-aB)\tan(c+dx)^2)+bB\tan(c+dx)+aB)}{a+b\tan(c+dx)}dx}{b}$$

$$\Big\downarrow \text{4130}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{2\int \frac{A\tan(c+dx)b^2+\left(-Ba^2+Aba+b^2B\right)\tan^2(c+dx)+a(Ab-aB)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{bd}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{\int \frac{A\tan(c+dx)b^2+\left(-Ba^2+Aba+b^2B\right)\tan^2(c+dx)+a(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{\int \frac{A\tan(c+dx)b^2+\left(-Ba^2+Aba+b^2B\right)\tan(c+dx)^2+a(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{bd}}{b}$$

$$\Big\downarrow \text{4136}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{\frac{\int \frac{(Ab-aB)b^2+(aA+bB)\tan(c+dx)b^2}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{a^3(Ab-aB)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{bd}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{\frac{\int\frac{(Ab-aB)b^2+(aA+bB)\tan(c+dx)b^2}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{a^3(Ab-aB)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{bd}}{b}$$

$$\Big\downarrow \text{ 4017}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{\frac{2\int\frac{b^2(Ab-aB+(aA+bB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{a^3(Ab-aB)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{bd}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{\frac{2b^2\int\frac{Ab-aB+(aA+bB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{a^3(Ab-aB)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{bd}}{b}$$

$$\Big\downarrow \text{ 1482}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{\frac{2b^2\left(\frac{1}{2}(b(A-B)-a(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d(a^2+b^2)}+\frac{a^3(Ab-aB)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}}{a^2+b^2}}{b}}{b}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{\frac{2b^2\left(\frac{1}{2}(b(A-B)-a(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c}}\right.\right.}{d(a^2+b^2)}}{b}}{b}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-\frac{\frac{\frac{2b^2\left(\frac{1}{2}(b(A-B)-a(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+d}\right.}{\sqrt{2}}\right.\right.}{d(a^2+b^2)}}{b}}{b}$$

$\downarrow$ 217

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-$$

$$\frac{\frac{\frac{2b^2\left(\frac{1}{2}(b(A-B)-a(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d\left(a^2+b^2\right)}+\frac{a^3(Ab-aB)}{}}{b}}{b}$$

$\downarrow$ 1479

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-$$

$$\frac{\frac{\frac{2b^2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\text{ar}}{}\right.}{d\left(a^2+b^2\right)}}{b}}{b}$$

$\downarrow$ 25

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-$$

$$\frac{\frac{\frac{2b^2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan(}{}\right.}{d\left(a^2+b^2\right)}}{b}}{b}$$

$\downarrow$ 27

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-$$

$$\frac{\frac{\frac{2b^2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\text{arctan}}{}\right.}{d\left(a^2+b^2\right)}}{b}}{b}$$

$\downarrow$ 1103

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd}-$$

$$\frac{\frac{\frac{a^3(Ab-aB)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B)-a(A+B)\right.}{d\left(a^2+b^2\right)}}{b}}{b}$$

$\downarrow$ 4117

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd} - \frac{\frac{\frac{a^3(Ab-aB)\int \frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d(a^2+b^2)} + \frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B)\cdots}{d(a^2+b^2)}}{b}}{b}$$

↓ 73

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd} - \frac{\frac{\frac{2a^3(Ab-aB)\int \frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)} + \frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B)-a(A+B))\cdots}{d(a^2+b^2)}}{b}}{b}$$

↓ 218

$$\frac{2B\tan^{\frac{3}{2}}(c+dx)}{3bd} - \frac{\frac{\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}} - \frac{\log(\tan\cdots}{}\right.\cdots}{d(a^2+b^2)}}{b}}{b}$$

input
```
Int[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-((((2*a^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(
Sqrt[b]*(a^2 + b^2)*d) + (2*b^2*(((a*(A - B) + b*(A + B))*(-(ArcTan[1 - Sq
rt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
]/Sqrt[2]))/2 + ((b*(A - B) - a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan
[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/b - (2*(A*b - a*B)*Sqrt[Tan[
c + d*x]])/(b*d))/b) + (2*B*Tan[c + d*x]^(3/2))/(3*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\frac{2Bb\tan(dx+c)^{\frac{3}{2}}}{3}+2Ab\sqrt{\tan(dx+c)}-2Ba\sqrt{\tan(dx+c)}}{b^2}-\frac{2a^3(Ab-Ba)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{b^2\left(a^2+b^2\right)\sqrt{ab}}+\frac{(-Ab+Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{}}{\tan(dx+c)-\sqrt{}}\right.\right.}{}}{}$
default	$\frac{\frac{\frac{2Bb\tan(dx+c)^{\frac{3}{2}}}{3}+2Ab\sqrt{\tan(dx+c)}-2Ba\sqrt{\tan(dx+c)}}{b^2}-\frac{2a^3(Ab-Ba)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{b^2\left(a^2+b^2\right)\sqrt{ab}}+\frac{(-Ab+Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{}}{\tan(dx+c)-\sqrt{}}\right.\right.}{}}{}$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(2/b^2*(1/3*B*b*tan(d*x+c)^(3/2)+A*b*tan(d*x+c)^(1/2)-B*a*tan(d*x+c)^(
1/2))-2/b^2*a^3*(A*b-B*a)/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/
(a*b)^(1/2))+2/(a^2+b^2)*(1/8*(-A*b+B*a)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*t
an(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(
1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-A*a-B*
b)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)
*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(
1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(230) = 460$.

Time = 20.06 (sec) , antiderivative size = 2674, normalized size of antiderivative = 10.13

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[-1/6*(6*sqrt(1/2)*(a^2*b^2 + b^4)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^
2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arc
tan(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 -
B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((
A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^
4 + 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A
 + B)*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)
*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*
x + c)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + 6*sqrt(1/2)*(a
^2*b^2 + b^4)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 +
 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^
2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*
A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2*A*B + B^2)*a
^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)
*d^2)) - 2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A + B)*a*b^2 - (A - B
)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B
+ B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(4*A*B*a*b
+ (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) - 3*sqrt(1/2)*(a^2*b^2 + b^4)*d*sqrt
(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/(
(a^4 + 2*a^2*b^2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 ...
```

Sympy [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx=\int \frac{(A+B\tan{(c+dx)})\tan^{\frac{5}{2}}{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(5/2)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.98

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{24\,(Ba^4-Aa^3b)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2b^2+b^4)\sqrt{ab}} - \frac{3\left(2\sqrt{2}((A-B)a+(A+B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((A-B)a+(A+B)b)\operatorname{arcta}\right.}{}}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/12*(24*(B*a^4 - A*a^3*b)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2*b^
2 + b^4)*sqrt(a*b)) - 3*(2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(1/2*sqrt
(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a + (A + B)*b)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((A + B)*a
 - (A - B)*b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)
*((A + B)*a - (A - B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) +
1))/(a^2 + b^2) + 8*(B*b*tan(d*x + c)^(3/2) - 3*(B*a - A*b)*sqrt(tan(d*x +
 c)))/b^2)/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 10.64 (sec) , antiderivative size = 16441, normalized size of antiderivative = 62.28

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
atan((((((32*(4*A*a*b^8*d^4 + 8*A*a^3*b^6*d^4 + 4*A*a^5*b^4*d^4))/(b*d^5)
 - (32*tan(c + d*x)^(1/2)*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4
*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*
a^2*b^2*d^4)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a
^6*b^4*d^4))/(b*d^4))*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4
 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*
b^2*d^4)))^(1/2) + (32*tan(c + d*x)^(1/2)*(4*A^2*a^3*b^5*d^2 + 2*A^2*a^5*b
^3*d^2 - 14*A^2*a*b^7*d^2 + 16*A^2*a^7*b*d^2))/(b*d^4))*(((64*A^4*a^2*b^2*
d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^
2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) - (32*(A^3*a^2*b^5*d^2
- 15*A^3*a^4*b^3*d^2 + 12*A^3*a^6*b*d^2))/(b*d^5))*(((64*A^4*a^2*b^2*d^4 -
 A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(1
6*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) + (32*tan(c + d*x)^(1/2)*(2*
A^4*a^6 - A^4*b^6))/(b*d^4))*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*
b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 +
 2*a^2*b^2*d^4)))^(1/2)*1i - (((((32*(4*A*a*b^8*d^4 + 8*A*a^3*b^6*d^4 + 4*
A*a^5*b^4*d^4))/(b*d^5) + (32*tan(c + d*x)^(1/2)*(((64*A^4*a^2*b^2*d^4 - A
^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*
(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4
- 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))/(b*d^4))*(((64*A^4*a^2*b^2*d^4 - A^...
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \sqrt{\tan(dx+c)}\,\tan(dx+c)^2\,dx$$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)
```

3.399 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 237

$$\begin{aligned}\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx &= -\frac{(b(A-B)-a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad+\frac{(b(A-B)-a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad+\frac{2a^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}\,(a^2+b^2)\,d} \\ &\quad-\frac{(a(A-B)+b(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}+\frac{2B\sqrt{\tan(c+dx)}}{bd}\end{aligned}$$

output

```
1/2*(b*(A-B)-a*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2
)/d+1/2*(b*(A-B)-a*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+
b^2)/d+2*a^(3/2)*(A*b-B*a)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(3/2
)/(a^2+b^2)/d-1/2*(a*(A-B)+b*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+ta
n(d*x+c)))*2^(1/2)/(a^2+b^2)/d+2*B*tan(d*x+c)^(1/2)/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.70

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\sqrt[4]{-1}(a+ib)b^{3/2}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-2a^{3/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)+}{b^{3/2}\left(a^2+b^2\right)}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
((-1)^(1/4)*(a + I*b)*b^(3/2)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x
]]] - 2*a^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]
] + (-1)^(1/4)*(a - I*b)*b^(3/2)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c +
 d*x]]] + 2*Sqrt[b]*(a^2 + b^2)*B*Sqrt[Tan[c + d*x]])/(b^(3/2)*(a^2 + b^2)
*d)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.07, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 4090, 27, 3042, 4136, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$
$$\Big\downarrow \text{4090}$$

$$\frac{2\int -\frac{-((Ab-aB)\tan^2(c+dx))+bB\tan(c+dx)+aB}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}+\frac{2B\sqrt{\tan(c+dx)}}{bd}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\int \frac{-((Ab-aB)\tan^2(c+dx))+bB\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\int \frac{-((Ab-aB)\tan(c+dx)^2)+bB\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}$$

$$\Big\downarrow \; 4136$$

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{\int \frac{b(aA+bB)-b(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{a^2(Ab-aB)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{\int \frac{b(aA+bB)-b(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{a^2(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}$$

$$\Big\downarrow \; 4017$$

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2\int \frac{b(aA+bB-(Ab-aB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}-\frac{a^2(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}-\frac{a^2(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}$$

$$\Big\downarrow \; 1482$$

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d(a^2+b^2)}-\frac{a^2(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}$$

$$\Big\downarrow \; 1476$$

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{}{\tan(c+dx)}\right.\right.}{d(a^2+b^2)}}{b}$$

$\downarrow$ 1082

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{}\right.}{\sqrt{2}}\right.\right.}{d(a^2+b^2)}}{b}$$

$\downarrow$ 217

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d(a^2+b^2)}-\frac{a}{}}{b}$$

$\downarrow$ 1479

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B\right.}{d(a^2+b^2)}}{b}$$

$\downarrow$ 25

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\right.\right.}{d(a^2+b^2)}}{b}$$

$\downarrow$ 27

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}(b(A-B)-a(A+B)}{d(a^2+b^2)}}{b}$$

$\downarrow$ 1103

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}}\right.}{d(a^2+b^2)}}{b}$$

$\downarrow$ 4117

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}}\right.}{d(a^2+b^2)}}{b}$$

$\downarrow$ 73

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}}\right.}{d(a^2+b^2)}}{b}$$

$\downarrow$ 218

$$\frac{2B\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}}\right.}{d(a^2+b^2)}}{b}$$

input
```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
-(((-2*a^(3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(
Sqrt[b]*(a^2 + b^2)*d) + (2*b*(-1/2*((b*(A - B) - a*(A + B))*(-(ArcTan[1 -
 Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*
x]]]/Sqrt[2])) + ((a*(A - B) + b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c
 + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/b) + (2*B*Sqrt[Tan[c + d*x]
])/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{2\sqrt{\tan(dx+c)}\,B}{b}+\frac{2a^{2}(Ab-Ba)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{b\left(a^{2}+b^{2}\right)\sqrt{ab}}+\frac{(-aA-Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cdots}\right.\right.}{4}}{\cdots}$
default	$\frac{\frac{2\sqrt{\tan(dx+c)}\,B}{b}+\frac{2a^{2}(Ab-Ba)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{b\left(a^{2}+b^{2}\right)\sqrt{ab}}+\frac{(-aA-Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cdots}\right.\right.}{4}}{\cdots}$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(2*tan(d*x+c)^(1/2)*B/b+2/b*a^2*(A*b-B*a)/(a^2+b^2)/(a*b)^(1/2)*arctan
(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))+2/(a^2+b^2)*(1/8*(-A*a-B*b)*2^(1/2)*(ln((
tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2
)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^
(1/2)))+1/8*(A*b-B*a)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/
(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/
2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1267 vs. $2(207) = 414$.

Time = 8.26 (sec) , antiderivative size = 2560, normalized size of antiderivative = 10.80

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[-1/2*(2*sqrt(1/2)*(a^2*b + b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2
- B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arcta
n(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^
2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^
2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4
+ 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A -
 B)*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a
*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x
+ c)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + 2*sqrt(1/2)*(a^2
*b + b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A
*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 +
b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B
+ B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2*A*B + B^2)*a^2 +
 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2
)) - 2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A - B)*a*b^2 + (A + B)*b^
3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^
2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(4*A*B*a*b + (A
^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + sqrt(1/2)*(a^2*b + b^3)*d*sqrt(((A^2 -
 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 - 2*A*B ...
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan^{\frac{3}{2}}{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(3/2)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.99

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{\frac{8\,(Ba^3-Aa^2b)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2b+b^3)\sqrt{ab}}-\frac{8\,B\sqrt{\tan(dx+c)}}{b}+\frac{2\,\sqrt{2}((A+B)a-(A-B)b)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan(dx+c)}\right)\right)+2\,\sqrt{2}((A+}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/4*(8*(B*a^3 - A*a^2*b)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2*b +
 b^3)*sqrt(a*b)) - 8*B*sqrt(tan(d*x + c))/b + (2*sqrt(2)*((A + B)*a - (A -
 B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((
A + B)*a - (A - B)*b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))
) + sqrt(2)*((A - B)*a + (A + B)*b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d
*x + c) + 1) - sqrt(2)*((A - B)*a + (A + B)*b)*log(-sqrt(2)*sqrt(tan(d*x +
 c)) + tan(d*x + c) + 1))/(a^2 + b^2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 15701, normalized size of antiderivative = 66.25

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
atan(((((((32*(4*B*a*b^8*d^4 + 8*B*a^3*b^6*d^4 + 4*B*a^5*b^4*d^4))/(b*d^5)
 - (32*tan(c + d*x)^(1/2)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4
*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*
a^2*b^2*d^4)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a
^6*b^4*d^4))/(b*d^4))*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4
 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*
b^2*d^4)))^(1/2) + (32*tan(c + d*x)^(1/2)*(4*B^2*a^3*b^5*d^2 + 2*B^2*a^5*b
^3*d^2 - 14*B^2*a*b^7*d^2 + 16*B^2*a^7*b*d^2))/(b*d^4))*(((64*B^4*a^2*b^2*
d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^
2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) - (32*(B^3*a^2*b^5*d^2
- 15*B^3*a^4*b^3*d^2 + 12*B^3*a^6*b*d^2))/(b*d^5))*(((64*B^4*a^2*b^2*d^4 -
 B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(1
6*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) + (32*tan(c + d*x)^(1/2)*(2*
B^4*a^6 - B^4*b^6))/(b*d^4))*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*
b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 +
 2*a^2*b^2*d^4)))^(1/2)*1i - (((((32*(4*B*a*b^8*d^4 + 8*B*a^3*b^6*d^4 + 4*
B*a^5*b^4*d^4))/(b*d^5) + (32*tan(c + d*x)^(1/2)*(((64*B^4*a^2*b^2*d^4 - B
^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*
(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4
- 16*a^4*b^6*d^4 - 16*a^6*b^4*d^4))/(b*d^4))*(((64*B^4*a^2*b^2*d^4 - B^...
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{2\sqrt{\tan(dx+c)} - \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)d}{d}$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(2*sqrt(tan(c + d*x)) - int(sqrt(tan(c + d*x))/tan(c + d*x),x)*d)/d
```

3.400 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 216

$$\begin{aligned}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx &= -\frac{(a(A-B)+b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\\ &\quad+\frac{(a(A-B)+b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\\ &\quad-\frac{2\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}\,(a^2+b^2)\,d}\\ &\quad+\frac{(b(A-B)-a(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\end{aligned}$$

output
```
1/2*(a*(A-B)+b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2
)/d+1/2*(a*(A-B)+b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+
b^2)/d-2*a^(1/2)*(A*b-B*a)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(1/2
)/(a^2+b^2)/d+1/2*(b*(A-B)-a*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+ta
n(d*x+c)))*2^(1/2)/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx =$$

$$-\frac{2\sqrt{2}(a(A-B)+b(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)+ \cdots}{\cdots}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
-1/4*(2*Sqrt[2]*(a*(A - B) + b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (8*Sqrt[a]*(A*b - a*B)*A
rcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[b] - Sqrt[2]*(b*(-A + B)
 + a*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/((a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4095, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \ 4095$$

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{a(Ab-aB)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{a(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\Big\downarrow$ 4017

$$\frac{2\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{a(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\Big\downarrow$ 1482

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+}\right.}{d\left(a^2+b^2\right)}$$
$$\frac{a(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\Big\downarrow$ 1476

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d}}\right.\right.}{d\left(a^2+b^2\right)}$$
$$\frac{a(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\Big\downarrow$ 1082

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}$$
$$\frac{a(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\Big\downarrow$ 217

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\big\downarrow$ 1479

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\big\downarrow$ 25

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(a\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\big\downarrow$ 27

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\big\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B)-a(A+B))\left(\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$$\Big\downarrow \; 4117$$

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B)-a(A+B))\Bigg(\frac{\mathrm{lo}}{-}}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int\frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 73$$

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B)-a(A+B))\Bigg(\frac{\mathrm{lo}}{-}}{d\left(a^2+b^2\right)}$$

$$\frac{2a(Ab-aB)\int\frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B)-a(A+B))\Bigg(\frac{\mathrm{lo}}{-}}{d\left(a^2+b^2\right)}$$

$$\frac{2\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}d\left(a^2+b^2\right)}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(-2*Sqrt[a]*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqr
t[b]*(a^2 + b^2)*d) + (2*(((a*(A - B) + b*(A + B))*(-(ArcTan[1 - Sqrt[2]*S
qrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[
2]))/2 + ((b*(A - B) - a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4095
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1
/(a^2 + b^2)   Int[Simp[A*(a*c + b*d) + B*(b*c - a*d) - (A*(b*c - a*d) - B*
(a*c + b*d))*Tan[e + f*x], x]/Sqrt[c + d*Tan[e + f*x]], x], x] - Simp[(b*c
- a*d)*((B*a - A*b)/(a^2 + b^2))   Int[(1 + Tan[e + f*x]^2)/((a + b*Tan[e +
 f*x])*Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-\frac{2a(Ab-Ba)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\,\mathrm{arct}}{4}}{}$
default	$\frac{-\frac{2a(Ab-Ba)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\,\mathrm{arct}}{4}}{}$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(-2*a*(A*b-B*a)/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^
(1/2))+2/(a^2+b^2)*(1/8*(A*b-B*a)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+
c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*ta
n(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(A*a+B*b)*2^(1/
2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x
+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. 2(189) = 378.

Time = 2.72 (sec) , antiderivative size = 2508, normalized size of antiderivative = 11.61

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[1/2*(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B
^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan((
(a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*
a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 -
 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A + B)
*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b
+ (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c
)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + 2*sqrt(1/2)*(a^2 +
b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B +
B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 + b^4)*
d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2
)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A
^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) -
2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A + B)*a*b^2 - (A - B)*b^3)*d*
sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^
2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(4*A*B*a*b + (A^2 -
B^2)*a^2 - (A^2 - B^2)*b^2)) - sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 + 2*A*B
+ B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^
2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*...
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{\tan{(c+dx)}}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(tan(c + d*x))/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{8\,(Ba^2-Aab)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}} + \frac{2\sqrt{2}((A-B)a+(A+B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((A-B)a+(A+B)b)\arctan\left(-\frac{1}{2}\right.}{}}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/4*(8*(B*a^2 - A*a*b)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2 + b^2)
*sqrt(a*b)) + (2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(1/2*sqrt(2)*(sqrt(
2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((A + B)*a - (A - B)
*b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((A + B)*
a - (A - B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2 +
 b^2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 15090, normalized size of antiderivative = 69.86

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
atan(((((32*(13*A^3*a^2*b^4*d^2 + A^3*a^4*b^2*d^2))/d^5 + (((32*(12*A*a*b^
7*d^4 + 24*A*a^3*b^5*d^4 + 12*A*a^5*b^3*d^4))/d^5 - (32*tan(c + d*x)^(1/2)
*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(
1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(16*
b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(((64*A^
4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*
A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) + (32*tan(c +
 d*x)^(1/2)*(20*A^2*a^3*b^4*d^2 + 2*A^2*a^5*b^2*d^2 - 14*A^2*a*b^6*d^2))/d
^4)*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4)
)^(1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2))*
(((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1
/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) - (32
*tan(c + d*x)^(1/2)*(A^4*b^5 - 2*A^4*a^2*b^3))/d^4)*(((64*A^4*a^2*b^2*d^4
- A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(
16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*1i - (((32*(13*A^3*a^2*b^4*
d^2 + A^3*a^4*b^2*d^2))/d^5 + (((32*(12*A*a*b^7*d^4 + 24*A*a^3*b^5*d^4 + 1
2*A*a^5*b^3*d^4))/d^5 + (32*tan(c + d*x)^(1/2)*(((64*A^4*a^2*b^2*d^4 - A^4
*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*(a
^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 1
6*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a...
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \sqrt{\tan(dx+c)}dx$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)),x)
```

3.401 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 217

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx &= \frac{(b(A-B)-a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\\ &\quad-\frac{(b(A-B)-a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\\ &\quad+\frac{2\sqrt{b}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,(a^2+b^2)\,d}\\ &\quad+\frac{(a(A-B)+b(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\end{aligned}$$

output
```
-1/2*(b*(A-B)-a*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^
2)/d-1/2*(b*(A-B)-a*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2
+b^2)/d+2*b^(1/2)*(A*b-B*a)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(1/
2)/(a^2+b^2)/d+1/2*(a*(A-B)+b*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+t
an(d*x+c)))*2^(1/2)/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx =$$

$$-\frac{2\sqrt{2}(b(-A+B)+a(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)+}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])),x
]
```

output
```
-1/4*(2*Sqrt[2]*(b*(-A + B) + a*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (8*Sqrt[b]*(-(A*b) + a*
B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[a] + Sqrt[2]*(a*(A -
 B) + b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/((a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4096, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \; 4096$$

$$\frac{b(Ab-aB)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\big\downarrow$ 4017

$$\frac{2\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{b(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\big\downarrow$ 1482

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+\right.}{d(a^2+b^2)}$$
$$\frac{b(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\big\downarrow$ 1476

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d}}\right.\right.}{d(a^2+b^2)}$$
$$\frac{b(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\big\downarrow$ 1082

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}}\right.\right.}{d(a^2+b^2)}$$
$$\frac{b(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\big\downarrow$ 217

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\downarrow$ 1479

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\downarrow$ 25

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(b(\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}$$

$\downarrow$ 1103

$$\frac{b(Ab-aB)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+$$

$$\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-\right.}{d\left(a^2+b^2\right)}$$

$\downarrow$ 4117

$$\frac{b(Ab-aB)\int \frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d(a^2+b^2)}+\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-\cdots}{d(a^2+b^2)}$$

$$\Big\downarrow \color{blue}{73}$$

$$\frac{2b(Ab-aB)\int \frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-\cdots}{d(a^2+b^2)}$$

$$\Big\downarrow \color{blue}{218}$$

$$\frac{2\sqrt{b}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d(a^2+b^2)}+\frac{2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-\cdots}{d(a^2+b^2)}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])),x]
```

output
```
(2*Sqrt[b]*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt
[a]*(a^2 + b^2)*d) + (2*(-1/2*((b*(A - B) - a*(A + B))*(-(ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/S
qrt[2])) + ((a*(A - B) + b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x
]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4096
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)])^(n_))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*Simp[a*A + b*B - (A*b - a*B)*Tan
[e + f*x], x], x], x] + Simp[b*((A*b - a*B)/(a^2 + b^2))   Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{2(Ab-Ba)b\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\left(a^2+b^2\right)\sqrt{ab}}+\frac{(aA+Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan}{4}}{}$
default	$\frac{\frac{2(Ab-Ba)b\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\left(a^2+b^2\right)\sqrt{ab}}+\frac{(aA+Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan}{4}}{}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(2*(A*b-B*a)*b/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(
1/2))+2/(a^2+b^2)*(1/8*(A*a+B*b)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c
)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-A*b+B*a)*2^(1/
2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x
+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(190) = 380$.

Time = 3.87 (sec) , antiderivative size = 2488, normalized size of antiderivative = 11.47

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[1/2*(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B
^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan((
(a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*
a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 -
 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A - B)
*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b
+ (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c
)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + 2*sqrt(1/2)*(a^2 +
b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B +
B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 + b^4)*
d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2
)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A
^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) -
2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A - B)*a*b^2 + (A + B)*b^3)*d*
sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^
2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(4*A*B*a*b + (A^2 -
B^2)*a^2 - (A^2 - B^2)*b^2)) + sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 - 2*A*B
+ B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^
2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*...
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))}\, dx = \int \frac{A + B\tan{(c + dx)}}{(a + b\tan{(c + dx)})\sqrt{\tan{(c + dx)}}}\, dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))*sqrt(tan(c + d*x))), x
)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx = -\frac{\frac{8\,(Bab-Ab^2)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}} - \frac{2\sqrt{2}((A+B)a-(A-B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((A+B)a-(A-B)b)\arctan\left(\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/4*(8*(B*a*b - A*b^2)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2 + b^2
)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(1/2*sqrt(2)*(sqrt
(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((A - B)*a + (A + B
)*b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A - B)
*a + (A + B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^2
+ b^2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 14816, normalized size of antiderivative = 68.28

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))),x)
```

output
```
atan((((((32*(13*B^3*a^2*b^4*d^2 + B^3*a^4*b^2*d^2))/d^5 + (((32*(12*B*a*b^
7*d^4 + 24*B*a^3*b^5*d^4 + 12*B*a^5*b^3*d^4))/d^5 - (32*tan(c + d*x)^(1/2)
*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(
1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(16*
b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(((64*B^
4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*
B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) + (32*tan(c +
 d*x)^(1/2)*(20*B^2*a^3*b^4*d^2 + 2*B^2*a^5*b^2*d^2 - 14*B^2*a*b^6*d^2))/d
^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4)
)^(1/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2))*
(((64*B^4*a^2*b^2*d^4 - B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1
/2) - 8*B^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) - (32
*tan(c + d*x)^(1/2)*(B^4*b^5 - 2*B^4*a^2*b^3))/d^4)*(((64*B^4*a^2*b^2*d^4
- B^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(
16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*1i - (((32*(13*B^3*a^2*b^4*
d^2 + B^3*a^4*b^2*d^2))/d^5 + (((32*(12*B*a*b^7*d^4 + 24*B*a^3*b^5*d^4 + 1
2*B*a^5*b^3*d^4))/d^5 + (32*tan(c + d*x)^(1/2)*(((64*B^4*a^2*b^2*d^4 - B^4
*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b*d^2)/(16*(a
^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 1
6*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*(((64*B^4*a^2*b^2*d^4 - B^4*(16*a...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx = \int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}\,dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x),x)
```

3.402 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 236

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx &= \frac{(a(A-B)+b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad -\frac{(a(A-B)+b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad -\frac{2b^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}\,(a^2+b^2)\,d} \\ &\quad -\frac{(b(A-B)-a(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{2A}{ad\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
-1/2*(a*(A-B)+b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^
2)/d-1/2*(a*(A-B)+b*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2
+b^2)/d-2*b^(3/2)*(A*b-B*a)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(3/
2)/(a^2+b^2)/d-1/2*(b*(A-B)-a*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+t
an(d*x+c)))*2^(1/2)/(a^2+b^2)/d-2*A/a/d/tan(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$$
$$= \frac{\frac{2b^{3/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2+b^2)} + \frac{\sqrt[4]{-1}a\left((-ia+b)(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+(ia+b)(A+iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\right.}\right.}{a^2+b^2}}{ad}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x
]
```

output
```
((2*b^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(
Sqrt[a]*(a^2 + b^2)) + ((-1)^(1/4)*a*(((-I)*a + b)*(A - I*B)*ArcTan[(-1)^(
3/4)*Sqrt[Tan[c + d*x]]] + (I*a + b)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan
[c + d*x]]]))/(a^2 + b^2) - (2*A)/Sqrt[Tan[c + d*x]])/(a*d)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.08, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 4092, 27, 3042, 4136, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{2\int \frac{Ab\tan^2(c+dx)+aA\tan(c+dx)+Ab-aB}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{Ab\tan^2(c+dx)+aA\tan(c+dx)+Ab-aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{Ab\tan(c+dx)^2+aA\tan(c+dx)+Ab-aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \ 4136$$

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{a(Ab-aB)+a(aA+bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{a(Ab-aB)+a(aA+bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \ 4017$$

$$-\frac{\frac{2\int \frac{a(Ab-aB+(aA+bB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{b^2(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{2a\int \frac{Ab-aB+(aA+bB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{b^2(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \ 1482$$

$$-\frac{\frac{2a\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d(a^2+b^2)}+\frac{b^2(Ab-aB)\int \frac{\tan(c}{\sqrt{\tan(c+dx}}}{a^2+b^2}}{a}$$

$$\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$\downarrow$ 1476

$$-\frac{\frac{2a\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{}{\tan(c+\cdots}\right.\right.}{d(a^2+b^2)}}{a}$$

$$\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$\downarrow$ 1082

$$-\frac{\frac{2a\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{\cdots}\right.}{\sqrt{2}}\right.\right.}{d(a^2+b^2)}}{a}$$

$$\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$\downarrow$ 217

$$-\frac{\frac{2a\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d(a^2+b^2)}+\cdots}{a}$$

$$\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$\downarrow$ 1479

$$-\frac{\frac{2a\left(\frac{1}{2}(b(A-B)-a(A+B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(a(A-B)+b(A-\cdots\right.}{d(a^2+b^2)}}{a}$$

$$\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$\downarrow$ 25

$$-\frac{\frac{2a\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(a(A-B)+b(A+B))\cdots\right.}{d(a^2+b^2)}}{a}$$

$$\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{2a\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}(a(A-B)+b(A+ \cdots}{d(a^2+b^2)}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 1103

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B \cdots}{d(a^2+b^2 \cdots}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 4117

$$-\frac{\frac{b^2(Ab-aB)\int \frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d(a^2+b^2)}+\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+ \cdots}{\cdots}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 73

$$-\frac{\frac{2b^2(Ab-aB)\int \frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B \cdots}{d(a^2+b^2 \cdots}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

$\Big\downarrow$ 218

$$-\frac{\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}} \cdots}{d(a^2+b^2)}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]
```

output
```
-(((2*b^(3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(S
qrt[a]*(a^2 + b^2)*d) + (2*a*(((a*(A - B) + b*(A + B))*(-(ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/S
qrt[2]))/2 + ((b*(A - B) - a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/a) - (2*A)/(a*d*Sqrt[Tan[c + d*
x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.12

method	result
derivativedivides	$-\frac{2(Ab-Ba)b^2\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{a\left(a^2+b^2\right)\sqrt{ab}}-\frac{2A}{a\sqrt{\tan(dx+c)}}+\frac{(-Ab+Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\right.}\right.}{4}$
default	$-\frac{2(Ab-Ba)b^2\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{a\left(a^2+b^2\right)\sqrt{ab}}-\frac{2A}{a\sqrt{\tan(dx+c)}}+\frac{(-Ab+Ba)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\right.}\right.}{4}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(-2/a*(A*b-B*a)*b^2/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a
*b)^(1/2))-2/a*A/tan(d*x+c)^(1/2)+2/(a^2+b^2)*(1/8*(-A*b+B*a)*2^(1/2)*(ln(
(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/
2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)
^(1/2)))+1/8*(-A*a-B*b)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1
)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(
1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1309 vs. $2(206) = 412$.

Time $= 11.31$ (sec) , antiderivative size $= 2648$, normalized size of antiderivative $= 11.22$

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[-1/2*(2*sqrt(1/2)*(a^3 + a*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2
- B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arcta
n(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^
2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^
2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4
+ 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A +
 B)*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a
*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x
+ c)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2))*tan(d*x + c) + 2*s
qrt(1/2)*(a^3 + a*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b
 + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 +
 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b +
(A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2*A*B
 + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b
^2 + b^4)*d^2)) - 2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A + B)*a*b^2
 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2
 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(4
*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2))*tan(d*x + c) - sqrt(1/2)*(a
^3 + a*b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2
*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + ...
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx=\int \frac{A+B\tan{(c+dx)}}{(a+b\tan{(c+dx)})\tan^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))*tan(c + d*x)**(3/2)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{\frac{8\,(Bab^2-Ab^3)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^3+ab^2)\sqrt{ab}} - \frac{2\,\sqrt{2}((A-B)a+(A+B)b)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan(dx+c)}\right)\right)+2\,\sqrt{2}((A-B)a+(A+B)b)\arctan\left(-\right.}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/4*(8*(B*a*b^2 - A*b^3)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^3 + a*
b^2)*sqrt(a*b)) - (2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(1/2*sqrt(2)*(s
qrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((A + B)*a - (A
- B)*b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((A +
 B)*a - (A - B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a
^2 + b^2) - 8*A/(a*sqrt(tan(d*x + c))))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 15318, normalized size of antiderivative = 64.91

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))),x)
```

output
```
atan(((tan(c + d*x)^(1/2)*(64*A^4*a^7*b^7*d^5 - 32*A^4*a^9*b^5*d^5) + (((6
4*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2)
- 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*((tan(c +
 d*x)^(1/2)*(512*A^2*a^8*b^8*d^7 - 448*A^2*a^10*b^6*d^7 + 128*A^2*a^12*b^4
*d^7 + 64*A^2*a^14*b^2*d^7) - (((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16
*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4
+ 2*a^2*b^2*d^4)))^(1/2)*(tan(c + d*x)^(1/2)*(((64*A^4*a^2*b^2*d^4 - A^4*(
16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*(a^4
*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(512*a^9*b^9*d^9 + 512*a^11*b^7*d^
9 - 512*a^13*b^5*d^9 - 512*a^15*b^3*d^9) - 512*A*a^8*b^9*d^8 - 640*A*a^10*
b^7*d^8 + 256*A*a^12*b^5*d^8 + 384*A*a^14*b^3*d^8))*(((64*A^4*a^2*b^2*d^4
- A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(
16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) + 128*A^3*a^7*b^8*d^6 - 32*
A^3*a^11*b^4*d^6 - 32*A^3*a^13*b^2*d^6))*(((64*A^4*a^2*b^2*d^4 - A^4*(16*a
^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4
 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*1i + (tan(c + d*x)^(1/2)*(64*A^4*a^7*b
^7*d^5 - 32*A^4*a^9*b^5*d^5) + (((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 1
6*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4
 + 2*a^2*b^2*d^4)))^(1/2)*((tan(c + d*x)^(1/2)*(512*A^2*a^8*b^8*d^7 - 448*
A^2*a^10*b^6*d^7 + 128*A^2*a^12*b^4*d^7 + 64*A^2*a^14*b^2*d^7) - (((64*...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}\,dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)
```

3.403 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 265

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx &= -\frac{(b(A-B)-a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad+\frac{(b(A-B)-a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad+\frac{2b^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}\,(a^2+b^2)\,d} \\ &\quad-\frac{(a(A-B)+b(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{2(Ab-aB)}{a^2d\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
1/2*(b*(A-B)-a*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2
)/d+1/2*(b*(A-B)-a*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+
b^2)/d+2*b^(5/2)*(A*b-B*a)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(5/2
)/(a^2+b^2)/d-1/2*(a*(A-B)+b*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+ta
n(d*x+c)))*2^(1/2)/(a^2+b^2)/d-2/3*A/a/d/tan(d*x+c)^(3/2)+2*(A*b-B*a)/a^2/
d/tan(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.66

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$$
$$= \frac{\frac{3\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{a-ib} + \frac{6b^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}(a^2+b^2)} + \frac{3\sqrt[4]{-1}(A+iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\tan(c+}\right.}{a+ib}}{3d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x
]
```

output
```
((3*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/(a - I*b)
+ (6*b^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^
(5/2)*(a^2 + b^2)) + (3*(-1)^(1/4)*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c
 + d*x]]])/(a + I*b) - (2*(a*A + (-3*A*b + 3*a*B)*Tan[c + d*x]))/(a^2*Tan[
c + d*x]^(3/2)))/(3*d)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.09, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4136, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 4092}$$

$$-\frac{2\int \frac{3\left(Ab\tan^2(c+dx)+aA\tan(c+dx)+Ab-aB\right)}{2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{Ab\tan^2(c+dx)+aA\tan(c+dx)+Ab-aB}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{Ab\tan(c+dx)^2+aA\tan(c+dx)+Ab-aB}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))}dx}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4132}$$

$$-\frac{-\frac{2\int -\frac{Aa^2+B\tan(c+dx)a^2+bBa-Ab^2-b(Ab-aB)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2(Ab-aB)}{ad\sqrt{\tan(c+dx)}}}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\int \frac{Aa^2+B\tan(c+dx)a^2+bBa-Ab^2-b(Ab-aB)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2(Ab-aB)}{ad\sqrt{\tan(c+dx)}}}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\int \frac{Aa^2+B\tan(c+dx)a^2+bBa-Ab^2-b(Ab-aB)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2(Ab-aB)}{ad\sqrt{\tan(c+dx)}}}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4136}$$

$$-\frac{\frac{\frac{\int \frac{a^2(aA+bB)-a^2(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b^3(Ab-aB)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2(Ab-aB)}{ad\sqrt{\tan(c+dx)}}}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{a^2(aA+bB)-a^2(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b^3(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2(Ab-aB)}{ad\sqrt{\tan(c+dx)}}}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4017

$$-\frac{\frac{\frac{2\int \frac{a^2(aA+bB-(Ab-aB)\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{b^3(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2(Ab-aB)}{ad\sqrt{\tan(c+dx)}}}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{2a^2\int \frac{aA+bB-(Ab-aB)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{b^3(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2(Ab-aB)}{ad\sqrt{\tan(c+dx)}}}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1482

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d\left(a^2+b^2\right)}-\frac{b^3(Ab-aB)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+d}{a^2+b^2}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1476

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan}}\right.\right.}{d\left(a^2+b^2\right)}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c}\right.}{\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d\left(a^2+b^2\right)}-\frac{b^3(Ab-a}{}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\right.\right.}{d\left(a^2+b^2\right)}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\text{arcta}}{}\right.}{d\left(a^2+b^2\right)}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$-\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\text{arc}}{}\right.}{d\left(a^2+b^2\right)}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1103

$$-\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right.}{\sqrt{2}}\right.}{d\left(a^2+b^2\right)}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4117

$$-\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right.}{\sqrt{2}}\right.}{d\left(a^2+b^2\right)}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 73

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.}{d\left(a^2+b^2\right)}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 218

$$-\frac{\frac{\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.}{d\left(a^2+b^2\right)}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]
```

output
```
-((((-2*b^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/
(Sqrt[a]*(a^2 + b^2)*d) + (2*a^2*(-1/2*((b*(A - B) - a*(A + B))*(-(ArcTan[
1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]]]/Sqrt[2])) + ((a*(A - B) + b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] +
 Tan[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/a - (2*(A*b - a*B))/(a*d
*Sqrt[Tan[c + d*x]]))/a) - (2*A)/(3*a*d*Tan[c + d*x]^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{2b^3(Ab-Ba)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{a^2\left(a^2+b^2\right)\sqrt{ab}}+\frac{(-aA-Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\,\mathrm{arct}}{4}}{}$
default	$\frac{\frac{2b^3(Ab-Ba)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{a^2\left(a^2+b^2\right)\sqrt{ab}}+\frac{(-aA-Bb)\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\,\mathrm{arct}}{4}}{}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(2/a^2*b^3*(A*b-B*a)/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(
a*b)^(1/2))+2/(a^2+b^2)*(1/8*(-A*a-B*b)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*ta
n(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(A*b-B*a)
*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*t
an(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/
2)*tan(d*x+c)^(1/2))))-2/3/a*A/tan(d*x+c)^(3/2)-2/a^2*(-A*b+B*a)/tan(d*x+c
)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1358 vs. $2(231) = 462$.

Time = 25.08 (sec) , antiderivative size = 2746, normalized size of antiderivative = 10.36

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[-1/6*(6*sqrt(1/2)*(a^4 + a^2*b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^
2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arc
tan(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 -
B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((
A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^
4 + 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A
 - B)*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)
*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*
x + c)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2))*tan(d*x + c)^2 +
 6*sqrt(1/2)*(a^4 + a^2*b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^
2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-(
(a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*
a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 -
 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2)) - 2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A - B)
*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b
+ (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c
)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2))*tan(d*x + c)^2 + 3*sq
rt(1/2)*(a^4 + a^2*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*
b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(2*sqrt(...
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx=\int \frac{A+B\tan{(c+dx)}}{(a+b\tan{(c+dx)})\tan^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))*tan(c + d*x)**(5/2)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.97

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = -\frac{\frac{24\,(Bab^3-Ab^4)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^4+a^2b^2)\sqrt{ab}}+\frac{3\left(2\sqrt{2}((A+B)a-(A-B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}((A+B)a-(A-B)b)\,\mathrm{ar}}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/12*(24*(B*a*b^3 - A*b^4)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^4 +
 a^2*b^2)*sqrt(a*b)) + 3*(2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(1/2*sqr
t(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a - (A - B)*b)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((A - B)*
a + (A + B)*b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2
)*((A - B)*a + (A + B)*b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) +
 1))/(a^2 + b^2) + 8*(A*a + 3*(B*a - A*b)*tan(d*x + c))/(a^2*tan(d*x + c)^
(3/2)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 10.63 (sec) , antiderivative size = 16111, normalized size of antiderivative = 60.80

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))),x)
```

output
```
atan(((tan(c + d*x)^(1/2)*(64*A^4*a^14*b^9*d^5 + 32*A^4*a^18*b^5*d^5) + (-
((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/
2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(((-((
64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2)
 - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(tan(c +
 d*x)^(1/2)*(-((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2
*b^2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4))
)^(1/2)*(512*a^18*b^9*d^9 + 512*a^20*b^7*d^9 - 512*a^22*b^5*d^9 - 512*a^24
*b^3*d^9) - 512*A*a^16*b^10*d^8 - 512*A*a^18*b^8*d^8 + 384*A*a^20*b^6*d^8
+ 256*A*a^22*b^4*d^8 - 128*A*a^24*b^2*d^8) - tan(c + d*x)^(1/2)*(512*A^2*a
^15*b^10*d^7 + 448*A^2*a^19*b^6*d^7 - 128*A^2*a^21*b^4*d^7 - 64*A^2*a^23*b
^2*d^7))*(-((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^
2*d^4))^(1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(
1/2) + 384*A^3*a^15*b^9*d^6 - 32*A^3*a^19*b^5*d^6 - 32*A^3*a^21*b^3*d^6))*
(-((64*A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(
1/2) - 8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*1i +
 (tan(c + d*x)^(1/2)*(64*A^4*a^14*b^9*d^5 + 32*A^4*a^18*b^5*d^5) + (-((64*
A^4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) -
8*A^2*a*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(((-((64*A^
4*a^2*b^2*d^4 - A^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) -...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)
```

3.404 $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 364

$$\begin{aligned}
&\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx \\
&= \frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad + \frac{a^{3/2}(a^2Ab+5Ab^3-3a^3B-7ab^2B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{5/2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{(aAb-3a^2B-2b^2B)\sqrt{\tan(c+dx)}}{b^2\,(a^2+b^2)\,d} + \frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
*2^(1/2)/(a^2+b^2)^2/d-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d+a^(3/2)*(A*a^2*b+5*A*b^3-3*B*a
^3-7*B*a*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(5/2)/(a^2+b^2)^2
/d-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/
(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)^2/d-(A*a*b-3*B*a^2-2*B*b^2)*tan(d*x+c)^(
1/2)/b^2/(a^2+b^2)/d+a*(A*b-B*a)*tan(d*x+c)^(3/2)/b/(a^2+b^2)/d/(a+b*tan(d
*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.76

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2\left((-Ab+3aB)\sqrt{\tan(c+dx)}+\frac{(a^2Ab+2Ab^3-3a^3B-4ab^2B)\sqrt{\tan(c+dx)}}{2(a^2+b^2)}+bB\tan^{\frac{3}{2}}(c+dx)-\frac{\left(\sqrt[4]{-1}(a+ib)^2b^{5/2}\right.}{}\right.}{}$$

input
```
Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2
,x]
```

output
```
(2*((-(A*b) + 3*a*B)*Sqrt[Tan[c + d*x]] + ((a^2*A*b + 2*A*b^3 - 3*a^3*B -
4*a*b^2*B)*Sqrt[Tan[c + d*x]])/(2*(a^2 + b^2)) + b*B*Tan[c + d*x]^(3/2) -
(((-1)^(1/4)*(a + I*b)^2*b^(5/2)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c +
d*x]]] + a^(3/2)*(-(a^2*A*b) - 5*A*b^3 + 3*a^3*B + 7*a*b^2*B)*ArcTan[(Sqrt
[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] + (-1)^(3/4)*b^(5/2)*(I*a + b)^2*(A + I*B
)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])*(a + b*Tan[c + d*x]))/(2*Sqrt[b]
*(a^2 + b^2)^2)))/(b^2*d*(a + b*Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.697$, Rules used = {3042, 4088, 27, 3042, 4130, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{5/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{\int -\frac{\sqrt{\tan(c+dx)}\left(\left(-3Ba^2+Aba-2b^2B\right)\tan^2(c+dx)-2b(Ab-aB)\tan(c+dx)+3a(Ab-aB)\right)}{2(a+b\tan(c+dx))}dx}{b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\int \frac{\sqrt{\tan(c+dx)}\left(\left(-3Ba^2+Aba-2b^2B\right)\tan^2(c+dx)-2b(Ab-aB)\tan(c+dx)+3a(Ab-aB)\right)}{a+b\tan(c+dx)}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\int \frac{\sqrt{\tan(c+dx)}\left(\left(-3Ba^2+Aba-2b^2B\right)\tan(c+dx)^2-2b(Ab-aB)\tan(c+dx)+3a(Ab-aB)\right)}{a+b\tan(c+dx)}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4130}$$

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{2\int -\frac{-2(aA+bB)\tan(c+dx)b^2+\left(-3Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\tan^2(c+dx)+a\left(-3Ba^2+Aba-2b^2B\right)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}+\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}}{2b(a^2+b^2)}$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\int \frac{-2(aA+bB)\tan(c+dx)b^2+\left(-3Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\tan^2(c+dx)+a\left(-3Ba^2+Aba-2b^2B\right)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}}{2b(a^2+b^2)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\int \frac{-2(aA+bB)\tan(c+dx)b^2+\left(-3Ba^3+Aba^2-4b^2Ba+2Ab^3\right)\tan(c+dx)^2+a\left(-3Ba^2+Aba-2b^2B\right)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}}{2b(a^2+b^2)}$$

$\Big\downarrow$ 4136

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{\int -\frac{2\left(\left(-Ba^2+2Aba+b^2B\right)b^2+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2\right)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int \frac{\cdots}{\sqrt{\tan(\cdots}}}{a^2+b^2}}{b}}{2b(a^2+b^2)}$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\int \frac{\left(-Ba^2+2Aba+b^2B\right)b^2+\left(Aa^2+2bBa-\cdots\right.}{\sqrt{\tan(c+dx)}}}{a^2+b^2}}{b}}{2b(a^2+b^2)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\int \frac{\left(-Ba^2+2Aba+b^2B\right)b^2+\left(Aa^2+2bBa-\cdots\right.}{\sqrt{\tan(c+dx)}}}{a^2+b^2}}{b}}{2b(a^2+b^2)}$$

$\Big\downarrow$ 4017

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{4\int\frac{b^2\left(-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-\right.\right.}{\tan^2(c+dx)+1}}{d\left(a^2+b^2\right)}}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{4b^2\int\frac{-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-A\right.}{\tan^2(c+dx)+1}}{d\left(a^2+b^2\right)}}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 1482

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{4b^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+\right.\right.}{}}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{4b^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+\right.\right.}{}}{}}{}$$

$\Big\downarrow$ 1082

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{4b^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+\right.\right.}{}}{}}{2b\,(a}$$

$\Big\downarrow$ 217

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-$$

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{4b^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+\right.\right.}{}}{}$$

$$2b\left(a^2+b^2\right)$$

$\downarrow$ 1479

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-$$

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{4b^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+\right.\right.}{}}{}$$

$\downarrow$ 25

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-$$

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{4b^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+\right.\right.}{}}{}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-$$

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{4b^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+\right.\right.}{}}{}$$

$\downarrow$ 1103

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-$$

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\right.}{}}{}$$

$\Big\downarrow$ 4117

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{\frac{2(-3a^2B+aAb-2b^2B)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2(-3a^3B+a^2Ab-7ab^2B+5Ab^3)\int\frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d(a^2+b^2)}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 73

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{\frac{2(-3a^2B+aAb-2b^2B)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2a^2(-3a^3B+a^2Ab-7ab^2B+5Ab^3)\int\frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 218

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{bd\,(a^2+b^2)\,(a+b\tan(c+dx))}-\frac{\frac{2(-3a^2B+aAb-2b^2B)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2a^{3/2}(-3a^3B+a^2Ab-7ab^2B+5Ab^3)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}d(a^2+b^2)}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\text{arc}\cdots}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
-1/2*(-(((2*a^(3/2)*(a^2*A*b + 5*A*b^3 - 3*a^3*B - 7*a*b^2*B)*ArcTan[(Sqrt
[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[b]*(a^2 + b^2)*d) - (4*b^2*(((a^2*
(A - B) - b^2*(A - B) + 2*a*b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((2
*a*b*(A - B) - a^2*(A + B) + b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c
 + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/b) + (2*(a*A*b - 3*a^2*B -
2*b^2*B)*Sqrt[Tan[c + d*x]])/(b*d))/(b*(a^2 + b^2)) + (a*(A*b - a*B)*Tan[c
 + d*x]^(3/2))/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{2\sqrt{\tan(dx+c)}\,B}{b^2}+\frac{2a^2\left(\frac{\left(-\frac{1}{2}A\,a^2b-\frac{1}{2}A\,b^3+\frac{1}{2}B\,a^3+\frac{1}{2}B\,a\,b^2\right)\sqrt{\tan(dx+c)}}{a+b\tan(dx+c)}+\frac{\left(A\,a^2b+5A\,b^3-3B\,a^3-7B\,a\,b^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c}}{\sqrt{ab}}\right.}{2\sqrt{ab}}\right.}{b^2\left(a^2+b^2\right)^2}}{}$
default	$\frac{\frac{2\sqrt{\tan(dx+c)}\,B}{b^2}+\frac{2a^2\left(\frac{\left(-\frac{1}{2}A\,a^2b-\frac{1}{2}A\,b^3+\frac{1}{2}B\,a^3+\frac{1}{2}B\,a\,b^2\right)\sqrt{\tan(dx+c)}}{a+b\tan(dx+c)}+\frac{\left(A\,a^2b+5A\,b^3-3B\,a^3-7B\,a\,b^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c}}{\sqrt{ab}}\right.}{2\sqrt{ab}}\right.}{b^2\left(a^2+b^2\right)^2}}{}$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2*tan(d*x+c)^(1/2)*B/b^2+2*a^2/b^2/(a^2+b^2)^2*((-1/2*A*a^2*b-1/2*A*b
^3+1/2*B*a^3+1/2*B*a*b^2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))+1/2*(A*a^2*b+5
*A*b^3-3*B*a^3-7*B*a*b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2
)))+2/(a^2+b^2)^2*(1/8*(-2*A*a*b+B*a^2-B*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1
/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(
1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-A
*a^2+A*b^2-2*B*a*b)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(t
an(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2436 vs. $2(335) = 670$.

Time = 33.24 (sec) , antiderivative size = 4897, normalized size of antiderivative = 13.45

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan^{\frac{5}{2}}{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(tan(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(5/2)/(a + b*tan(c + d*x))**2,
 x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.04

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{4\,(3\,Ba^5-Aa^4b+7\,Ba^3b^2-5\,Aa^2b^3)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^4b^2+2\,a^2b^4+b^6)\sqrt{ab}}-\frac{4\,(Ba^3-Aa^2b)\sqrt{\tan(dx+c)}}{a^3b^2+ab^4+(a^2b^3+b^5)\tan(dx+c)}+\frac{2\,\sqrt{2}((A-B)a^2+2\,(A+B)ab-(A-B)}{}}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
-1/4*(4*(3*B*a^5 - A*a^4*b + 7*B*a^3*b^2 - 5*A*a^2*b^3)*arctan(b*sqrt(tan(
d*x + c))/sqrt(a*b))/((a^4*b^2 + 2*a^2*b^4 + b^6)*sqrt(a*b)) - 4*(B*a^3 -
A*a^2*b)*sqrt(tan(d*x + c))/(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*tan(d*x + c
)) + (2*sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sqr
t(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a^2 + 2*(A + B
)*a*b - (A - B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
 - sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)*sqrt(ta
n(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (
A + B)*b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 + 2*
a^2*b^2 + b^4) - 8*B*sqrt(tan(d*x + c))/b^2)/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 35.53 (sec) , antiderivative size = 18313, normalized size of antiderivative = 50.31

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(log(((((((((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^
4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B^2*a^3*b*d
^2)/(d^4*(a^2 + b^2)^4))^(1/2) - (128*B*a*b^2*(7*a^4 + b^4 + 8*a^2*b^2))/d
)*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B
^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (64*B^2*a*tan(c + d*x)^(1/2)
*(18*a^10 - 15*b^10 + 17*a^2*b^8 - a^4*b^6 + 97*a^6*b^4 + 84*a^8*b^2))/(b^
2*d^2*(a^2 + b^2)^2))*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*
B^2*a*b^3*d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (32*B^3*
a^4*(127*a^2*b^6 - 112*b^8 - 9*a^8 + 173*a^4*b^4 + 21*a^6*b^2))/(b^3*d^3*(
a^2 + b^2)^3))*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*b
^3*d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (16*B^4*tan(c +
 d*x)^(1/2)*(9*a^12 + 2*b^12 + 4*a^2*b^10 + 2*a^4*b^8 - 49*a^6*b^6 + 7*a^8
*b^4 + 33*a^10*b^2))/(b^3*d^4*(a^2 + b^2)^4))*((4*(-B^4*d^4*(a^4 + b^4 - 6
*a^2*b^2)^2)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)
^4))^(1/2))/4 - (16*B^5*a^3*b^2*(3*a^2 + 7*b^2))/(d^5*(a^2 + b^2)^4))*(((1
92*B^4*a^2*b^6*d^4 - 16*B^4*b^8*d^4 - 16*B^4*a^8*d^4 - 608*B^4*a^4*b^4*d^4
 + 192*B^4*a^6*b^2*d^4)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B^2*a^3*b*d^2)/(a^8*
d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(1/2))/4 +
 (log(((((((((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(-(4*(-
B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a*b^3*d^2 - 16*B^2*a^...
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{a+\tan(dx+c)\,b}dx$$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)
```

3.405 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 317

$$\begin{aligned}
\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx
&= -\frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{\sqrt{a}(a^2Ab-3Ab^3+a^3B+5ab^2B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}\,(a^2+b^2)^2\,d} \\
&\quad-\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{b\,(a^2+b^2)\,d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*
2^(1/2)/(a^2+b^2)^2/d+1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d+a^(1/2)*(A*a^2*b-3*A*b^3+B*a^3+
5*B*a*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(3/2)/(a^2+b^2)^2/d-
1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+
tan(d*x+c)))*2^(1/2)/(a^2+b^2)^2/d+a*(A*b-B*a)*tan(d*x+c)^(1/2)/b/(a^2+b^2
)/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.73

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{\sqrt{a}(a^2Ab-3Ab^3+a^3B+5ab^2B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt[4]{-1}b^{3/2}\left((a+ib)^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+(a-ib)^2(A+iB)\text{arcta}\right.}{\sqrt{b}(a^2+b^2)^2}}{bd}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2
,x]
```

output
```
((Sqrt[a]*(a^2*A*b - 3*A*b^3 + a^3*B + 5*a*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[Tan
[c + d*x]])/Sqrt[a]] + (-1)^(1/4)*b^(3/2)*((a + I*b)^2*(A - I*B)*ArcTan[(-
1)^(3/4)*Sqrt[Tan[c + d*x]]] + (a - I*b)^2*(A + I*B)*ArcTanh[(-1)^(3/4)*Sq
rt[Tan[c + d*x]]]))/(Sqrt[b]*(a^2 + b^2)^2) - (2*B*Sqrt[Tan[c + d*x]])/(a
+ b*Tan[c + d*x]) + ((a*A*b + a^2*B + 2*b^2*B)*Sqrt[Tan[c + d*x]])/((a^2 +
 b^2)*(a + b*Tan[c + d*x])))/(b*d)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.06, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 4088, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 4088}$$

$$\frac{\int -\frac{-((Ba^2+Aba+2b^2B)\tan^2(c+dx))-2b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b(a^2+b^2)}+\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\int \frac{-((Ba^2+Aba+2b^2B)\tan^2(c+dx))-2b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2b(a^2+b^2)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\int \frac{-((Ba^2+Aba+2b^2B)\tan(c+dx)^2)-2b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2b(a^2+b^2)}$$

$$\Big\downarrow \text{ 4136}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{\int \frac{2\left(b\left(Aa^2+2bBa-Ab^2\right)-b\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{2\int \frac{b\left(Aa^2+2bBa-Ab^2\right)-b\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{2\int \frac{b\left(Aa^2+2bBa-Ab^2\right)-b\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4017

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{4\int \frac{b\left(Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{4b\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 1482

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d\left(a^2+b^2\right)}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-$$
$$\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d}}\right.\right.}{d(a^2+b^2)}$$
$$2b\left(a^2+\right.$$

$\downarrow$ 1082

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-$$
$$\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan}\right.}{\sqrt{2}}\right.\right.}{d(a^2+b^2)}$$
$$2b\left(a^2+b^2\right)$$

$\downarrow$ 217

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-$$
$$\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{d(a^2+b^2)}$$
$$2b\left(a^2+b^2\right)$$

$\downarrow$ 1479

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-$$
$$\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\right.\right.}{d(a^2+b^2)}$$

$\downarrow$ 25

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-$$
$$\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(\right.\right.\right.}{d(a^2+b^2)}$$
$$2a$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-$$
$$4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}(-($$
$$d(a^2+b^2)$$

$\Big\downarrow$ 1103

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-$$
$$4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)\right.$$
$$d(a^2+b^2)$$
$$2b\left(a^2+b\right.$$

$\Big\downarrow$ 4117

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-$$
$$4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)\right.$$
$$d(a^2+b^2)$$
$$2b\left(a^2\right.$$

$\Big\downarrow$ 73

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-$$
$$4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)\right.$$
$$d(a^2+b^2)$$
$$2b\left(a^2+b\right.$$

$\Big\downarrow$ 218

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}-$$
$$4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)\right.$$
$$d(a^2+b^2)$$
$$2b\left(a^2+b^2\right.$$

input
```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
-1/2*((-2*Sqrt[a]*(a^2*A*b - 3*A*b^3 + a^3*B + 5*a*b^2*B)*ArcTan[(Sqrt[b]*
Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[b]*(a^2 + b^2)*d) + (4*b*(-1/2*((2*a*b
*(A - B) - a^2*(A + B) + b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((a^2*
(A - B) - b^2*(A - B) + 2*a*b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
 + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/(b*(a^2 + b^2)) + (a*(A*b - a*
B)*Sqrt[Tan[c + d*x]])/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{2a\left(\frac{\left(A\,a^2b+A\,b^3-B\,a^3-Ba\,b^2\right)\sqrt{\tan(dx+c)}}{2b(a+b\tan(dx+c))}+\frac{\left(A\,a^2b-3A\,b^3+B\,a^3+5Ba\,b^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{\left(-A\,a^2+A\,b^2-2Bab\right)}{}$
default	$\frac{2a\left(\frac{\left(A\,a^2b+A\,b^3-B\,a^3-Ba\,b^2\right)\sqrt{\tan(dx+c)}}{2b(a+b\tan(dx+c))}+\frac{\left(A\,a^2b-3A\,b^3+B\,a^3+5Ba\,b^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{\left(-A\,a^2+A\,b^2-2Bab\right)}{}$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2*a/(a^2+b^2)^2*(1/2*(A*a^2*b+A*b^3-B*a^3-B*a*b^2)/b*tan(d*x+c)^(1/2)
/(a+b*tan(d*x+c))+1/2*(A*a^2*b-3*A*b^3+B*a^3+5*B*a*b^2)/b/(a*b)^(1/2)*arct
an(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^2*(1/8*(-A*a^2+A*b^2-2*B*a
*b)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2
)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^
(1/2)*tan(d*x+c)^(1/2)))+1/8*(2*A*a*b-B*a^2+B*b^2)*2^(1/2)*(ln((tan(d*x+c)
-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(288) = 576$.

Time = 18.42 (sec) , antiderivative size = 4800, normalized size of antiderivative = 15.14

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{(A+B\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(3/2)/(a + b*tan(c + d*x))**2,
 x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.12

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{4\left(Ba^4+Aa^3b+5\,Ba^2b^2-3\,Aab^3\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^4b+2\,a^2b^3+b^5)\sqrt{ab}} - \frac{4\left(Ba^2-Aab\right)\sqrt{\tan(dx+c)}}{a^3b+ab^3+(a^2b^2+b^4)\tan(dx+c)} - \frac{2\sqrt{2}\left((A+B)a^2-2\,(A-B)ab-(A+B)b^2\right)\mathrm{arc}}{}}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
1/4*(4*(B*a^4 + A*a^3*b + 5*B*a^2*b^2 - 3*A*a*b^3)*arctan(b*sqrt(tan(d*x +
 c))/sqrt(a*b))/((a^4*b + 2*a^2*b^3 + b^5)*sqrt(a*b)) - 4*(B*a^2 - A*a*b)*
sqrt(tan(d*x + c))/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(d*x + c)) - (2*sqr
t(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(
2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A
+ B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*
((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*log(sqrt(2)*sqrt(tan(d*x + c))
 + tan(d*x + c) + 1) - sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)
*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b
^4))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 32.15 (sec) , antiderivative size = 17579, normalized size of antiderivative = 55.45

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(log((((((((((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-A^
4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*A^2*a*b^3*d^2 + 16*A^2*a^3*b*d
^2)/(d^4*(a^2 + b^2)^4))^(1/2) - (128*A*a*b^2*(5*b^4 - a^4 + 4*a^2*b^2))/d
)*((4*(-A^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*A^2*a*b^3*d^2 + 16*A
^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (64*A^2*a*b^2*tan(c + d*x)^(
1/2)*(a^6 - 15*b^6 + 35*a^2*b^4 - 13*a^4*b^2))/(d^2*(a^2 + b^2)^2))*((4*(-
A^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*A^2*a*b^3*d^2 + 16*A^2*a^3*b
*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (32*A^3*a^2*b*(a^6 - 39*b^6 + 43*a^2
*b^4 - 13*a^4*b^2))/(d^3*(a^2 + b^2)^3))*((4*(-A^4*d^4*(a^4 + b^4 - 6*a^2*
b^2)^2)^(1/2) - 16*A^2*a*b^3*d^2 + 16*A^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^
(1/2))/4 - (16*A^4*b*tan(c + d*x)^(1/2)*(a^8 + 2*b^8 - 5*a^2*b^6 + 17*a^4*
b^4 - 7*a^6*b^2))/(d^4*(a^2 + b^2)^4))*((4*(-A^4*d^4*(a^4 + b^4 - 6*a^2*b^
2)^2)^(1/2) - 16*A^2*a*b^3*d^2 + 16*A^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1
/2))/4 + (16*A^5*a*b^4*(a^2 - 3*b^2))/(d^5*(a^2 + b^2)^4))*(((192*A^4*a^2*
b^6*d^4 - 16*A^4*b^8*d^4 - 16*A^4*a^8*d^4 - 608*A^4*a^4*b^4*d^4 + 192*A^4*
a^6*b^2*d^4)^(1/2) - 16*A^2*a*b^3*d^2 + 16*A^2*a^3*b*d^2)/(a^8*d^4 + b^8*d
^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(1/2))/4 + (log((((((
(((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(-(4*(-A^4*d^4*(a^
4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*A^2*a*b^3*d^2 - 16*A^2*a^3*b*d^2)/(d^4*
(a^2 + b^2)^4))^(1/2) - (128*A*a*b^2*(5*b^4 - a^4 + 4*a^2*b^2))/d)*(-(4...
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)}{a+\tan(dx+c)\,b}dx$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)*b + a),x)
```

3.406 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 318

$$\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx
&= -\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad-\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*
2^(1/2)/(a^2+b^2)^2/d+1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d-(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^
2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)/(a^2+b^2)^2/d+
1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+
tan(d*x+c)))*2^(1/2)/(a^2+b^2)^2/d-(A*b-B*a)*tan(d*x+c)^(1/2)/(a^2+b^2)/d/
(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{\sqrt{a}(-3a^2Ab+Ab^3+a^3B-3ab^2B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}(a^2+b^2)}+\frac{\sqrt[4]{-1}a\left((a+ib)^2(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+(a-ib)^2(-iA+B)\text{ar}}{a^2+b^2}}{a\left(a^2+b^2\right)d}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2
,x]
```

output
```
((Sqrt[a]*(-3*a^2*A*b + A*b^3 + a^3*B - 3*a*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a]])/(Sqrt[b]*(a^2 + b^2)) + ((-1)^(1/4)*a*((a + I*b)^2*
(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (a - I*b)^2*((-I)*A + B)
*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2) + (-(A*b) + a*B)*Sqr
t[Tan[c + d*x]] + (b*(A*b - a*B)*Tan[c + d*x]^(3/2))/(a + b*Tan[c + d*x]))
/(a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.05, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 4091, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4091}$$

$$-\frac{\int -\frac{-b(Ab-aB)\tan^2(c+dx)+2b(aA+bB)\tan(c+dx)+b(Ab-aB)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b(a^2+b^2)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-b(Ab-aB)\tan^2(c+dx)+2b(aA+bB)\tan(c+dx)+b(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2b(a^2+b^2)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-b(Ab-aB)\tan(c+dx)^2+2b(aA+bB)\tan(c+dx)+b(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2b(a^2+b^2)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4136}$$

$$\frac{\frac{\int \frac{2\left(b\left(-Ba^2+2Aba+b^2B\right)+b\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2b(a^2+b^2)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{b\left(-Ba^2+2Aba+b^2B\right)+b\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\int \frac{b\left(-Ba^2+2Aba+b^2B\right)+b\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 4017

$$\frac{\frac{4\int \frac{b\left(-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}-\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 27

$$\frac{\frac{4b\int \frac{-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}-\frac{b\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 1482

$$\frac{\frac{4b\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right.}{d(a^2+b^2)}}{2b\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 1476

$$\frac{4b\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d}}\right.\right.}{d(a^2+b^2)}$$

$$2b\,(a^2+$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}$$

$\downarrow$ 1082

$$\frac{4b\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\mathrm{ta}}\right.}{\sqrt{2}}\right.\right.}{d(a^2+b^2)}$$

$$2b\,(a^2+b^2)$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}$$

$\downarrow$ 217

$$\frac{4b\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{d(a^2+b^2)}$$

$$2b\,(a^2+b^2)$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}$$

$\downarrow$ 1479

$$\frac{4b\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\right.}{d(a^2+b^2)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}$$

$\downarrow$ 25

$$\frac{4b\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(a^2\right.\right.}{d(a^2+b^2)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}$$

$\downarrow$ 27

$$\frac{4b\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}}{d(a^2+b^2)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 1103

$$\frac{\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\mathrm{lo}}{-}\right.}{d(a^2+b^2)}}{2b\left(a^2+\right.}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 4117

$$\frac{\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\mathrm{lo}}{-}\right.}{d(a^2+b^2)}}{2b\left(a\right.}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 73

$$\frac{\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\mathrm{lo}}{-}\right.}{d(a^2+b^2)}}{2b\left(a^2+\right.}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 218

$$\frac{\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\mathrm{lo}}{-}\right.}{d(a^2+b^2)}}{2b\left(a^2+b\right.}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
((-2*Sqrt[b]*(3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[
Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)*d) + (4*b*(((a^2*(A - B) - b
^2*(A - B) + 2*a*b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt
[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((2*a*b*(A - B
) - a^2*(A + B) + b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] +
Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/(2*b*(a^2 + b^2)) - ((A*b - a*B)*Sqrt[
Tan[c + d*x]])/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.04

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{1}{2}A a^2 b+\frac{1}{2}A b^3-\frac{1}{2}B a^3-\frac{1}{2}B a b^2\right)\sqrt{\tan(dx+c)}}{a+b\tan(dx+c)}+\frac{\left(3A a^2 b-A b^3-B a^3+3B a b^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{(2Aab-B a^2-\cdots}{\cdots}$
default	$-\frac{2\left(\frac{\left(\frac{1}{2}A a^2 b+\frac{1}{2}A b^3-\frac{1}{2}B a^3-\frac{1}{2}B a b^2\right)\sqrt{\tan(dx+c)}}{a+b\tan(dx+c)}+\frac{\left(3A a^2 b-A b^3-B a^3+3B a b^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{(2Aab-B a^2-\cdots}{\cdots}$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(-2/(a^2+b^2)^2*((1/2*A*a^2*b+1/2*A*b^3-1/2*B*a^3-1/2*B*a*b^2)*tan(d*x
+c)^(1/2)/(a+b*tan(d*x+c))+1/2*(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)/(a*b)^(1/
2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^2*(1/8*(2*A*a*b-B*a
^2+B*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-
2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan
(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(A*a^2-A*b^2+2*B*a*b)*2^(1/2)*(ln((tan(
d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)
)+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2
)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2395 vs. $2(285) = 570$.

Time = 13.56 (sec) , antiderivative size = 4816, normalized size of antiderivative = 15.14

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{\tan{(c+dx)}}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(tan(c + d*x))/(a + b*tan(c + d*x))**2,
x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{4\,(Ba^3-3\,Aa^2b-3\,Bab^2+Ab^3)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{ab}} + \frac{4\,(Ba-Ab)\sqrt{\tan(dx+c)}}{a^3+ab^2+(a^2b+b^3)\tan(dx+c)} + \frac{2\,\sqrt{2}((A-B)a^2+2\,(A+B)ab-(A-B)b^2)\arctan\left(\right.}{}}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
1/4*(4*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*arctan(b*sqrt(tan(d*x + c))
/sqrt(a*b))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a*b)) + 4*(B*a - A*b)*sqrt(tan(d
*x + c))/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)) + (2*sqrt(2)*((A - B)*
a^2 + 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(ta
n(d*x + c)))) + 2*sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*arct
an(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((A + B)*a^2 -
 2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c
) + 1) + sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*log(-sqrt(2)*
sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 32.11 (sec) , antiderivative size = 17089, normalized size of antiderivative = 53.74

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
(log((16*B^5*a*b^4*(a^2 - 3*b^2))/(d^5*(a^2 + b^2)^4) - ((((((((128*B*a*b
^2*(5*b^4 - a^4 + 4*a^2*b^2))/d - 128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(
a^2 + b^2)^2*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*b^3
*d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))*((4*(-B^4*d^4*(a^4 +
b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2
 + b^2)^4))^(1/2))/4 - (64*B^2*a*b^2*tan(c + d*x)^(1/2)*(a^6 - 15*b^6 + 35
*a^2*b^4 - 13*a^4*b^2))/(d^2*(a^2 + b^2)^2))*((4*(-B^4*d^4*(a^4 + b^4 - 6*
a^2*b^2)^2)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^
4))^(1/2))/4 - (32*B^3*a^2*b*(a^6 - 39*b^6 + 43*a^2*b^4 - 13*a^4*b^2))/(d^
3*(a^2 + b^2)^3))*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*
a*b^3*d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (16*B^4*b*ta
n(c + d*x)^(1/2)*(a^8 + 2*b^8 - 5*a^2*b^6 + 17*a^4*b^4 - 7*a^6*b^2))/(d^4*
(a^2 + b^2)^4))*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*
b^3*d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4)*(((192*B^4*a^2*
b^6*d^4 - 16*B^4*b^8*d^4 - 16*B^4*a^8*d^4 - 608*B^4*a^4*b^4*d^4 + 192*B^4*
a^6*b^2*d^4)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B^2*a^3*b*d^2)/(a^8*d^4 + b^8*d
^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(1/2))/4 + (log((16*B
^5*a*b^4*(a^2 - 3*b^2))/(d^5*(a^2 + b^2)^4) - ((((((((128*B*a*b^2*(5*b^4
- a^4 + 4*a^2*b^2))/d - 128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)
^2*(-(4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a*b^3*d^2 -...
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2\sqrt{\tan(dx+c)}-\left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2b+\tan(dx+c)a}dx\right)ad-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{a+\tan(dx+c)b}dx\right)bd-\left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)}{a+\tan(dx+c)b}dx\right)ad}{bd}$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(2*sqrt(tan(c + d*x)) - int(sqrt(tan(c + d*x))/(tan(c + d*x)**2*b + tan(c
+ d*x)*a),x)*a*d - int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)*
b + a),x)*b*d - int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)*b + a)
,x)*a*d)/(b*d)
```

3.407 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 316

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx &= \frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\ &\quad - \frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\ &\quad + \frac{\sqrt{b}(5a^2Ab+Ab^3-3a^3B+ab^2B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}\,(a^2+b^2)^2\,d} \\ &\quad + \frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\ &\quad + \frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{a\,(a^2+b^2)\,d(a+b\tan(c+dx))}\end{aligned}$$

output
```
-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
*2^(1/2)/(a^2+b^2)^2/d-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d+b^(1/2)*(5*A*a^2*b+A*b^3-3*B*a
^3+B*a*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(3/2)/(a^2+b^2)^2/d
+1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1
+tan(d*x+c)))*2^(1/2)/(a^2+b^2)^2/d+b*(A*b-B*a)*tan(d*x+c)^(1/2)/a/(a^2+b^
2)/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.65

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{\sqrt{b}(5a^2Ab+Ab^3-3a^3B+ab^2B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}(a^2+b^2)} + \frac{\sqrt[4]{-1}\left(-a(a+ib)^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-a(a-ib)^2(A+iB)\mathrm{arc}\right.}{a^2+b^2}}{a\,(a^2+b^2)\,d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2)
,x]
```

output
```
((Sqrt[b]*(5*a^2*A*b + A*b^3 - 3*a^3*B + a*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[Tan
[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)) + ((-1)^(1/4)*(-(a*(a + I*b)^2
*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]) - a*(a - I*b)^2*(A + I*B
)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2) + (b*(A*b - a*B)*Sq
rt[Tan[c + d*x]])/(a + b*Tan[c + d*x]))/(a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.06, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.606, Rules used = {3042, 4092, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{\int \frac{2Aa^2+bBa-2(Ab-aB)\tan(c+dx)a+Ab^2+b(Ab-aB)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Aa^2+bBa-2(Ab-aB)\tan(c+dx)a+Ab^2+b(Ab-aB)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2Aa^2+bBa-2(Ab-aB)\tan(c+dx)a+Ab^2+b(Ab-aB)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4136}$$

$$\frac{\frac{\int \frac{2\left(a\left(Aa^2+2bBa-Ab^2\right)-a\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{a\left(Aa^2+2bBa-Ab^2\right)-a\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\int \frac{a\left(Aa^2+2bBa-Ab^2\right)-a\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 4017$$

$$\frac{\frac{4\int \frac{a\left(Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{4a\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 1482$$

$$\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx}}{d(a^2+b^2)}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 1476$$

$$\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+d}\right.\right.}{d(a^2+b^2)}$$

$$2a\left(a^2+\right.$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 1082

$$\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{t}\right.}{\sqrt{2}}\right.\right.}{d(a^2+b^2)}$$

$$2a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 217

$$\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{d(a^2+b^2)}$$

$$2a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 1479

$$\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\right.\right.}{d(a^2+b^2)}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 25

$$\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(\right.\right.\right.}{d(a^2+b^2)}$$

$$2$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 27

$$\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}(-(}{d(a^2+b^2)}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 1103

$$\frac{\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right.\right.}{}}{2a\left(a^2+\right.}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 4117

$$\frac{\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d(a^2+b^2)}+\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(\right.}{2\sqrt{2}}\right.\right.}{}}{2a\left(a\right.}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 73

$$\frac{\frac{2b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right.\right.}{}}{2a\left(a^2+\right.}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$\downarrow$ 218

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}+$$

$$\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)\right.}{d(a^2+b^2)}}{2a\left(a^2+b\right.}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2),x]
```

output
```
((2*Sqrt[b]*(5*a^2*A*b + A*b^3 - 3*a^3*B + a*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[T
an[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)*d) + (4*a*(-1/2*((2*a*b*(A -
B) - a^2*(A + B) + b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/
Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((a^2*(A - B
) - b^2*(A - B) + 2*a*b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^2)) + (b*(A*b - a*B)*S
qrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{2b\left(\frac{\left(A\,a^2b+A\,b^3-B\,a^3-Ba\,b^2\right)\sqrt{\tan(dx+c)}}{2a(a+b\tan(dx+c))}+\frac{\left(5A\,a^2b+A\,b^3-3B\,a^3+Ba\,b^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{\left(A\,a^2-A\,b^2+2Bab\right)\sqrt{}}{}$
default	$\frac{2b\left(\frac{\left(A\,a^2b+A\,b^3-B\,a^3-Ba\,b^2\right)\sqrt{\tan(dx+c)}}{2a(a+b\tan(dx+c))}+\frac{\left(5A\,a^2b+A\,b^3-3B\,a^3+Ba\,b^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{\left(a^2+b^2\right)^2}+\frac{\left(A\,a^2-A\,b^2+2Bab\right)\sqrt{}}{}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2*b/(a^2+b^2)^2*(1/2*(A*a^2*b+A*b^3-B*a^3-B*a*b^2)/a*tan(d*x+c)^(1/2)
/(a+b*tan(d*x+c))+1/2*(5*A*a^2*b+A*b^3-3*B*a^3+B*a*b^2)/a/(a*b)^(1/2)*arct
an(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^2*(1/8*(A*a^2-A*b^2+2*B*a*
b)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)
*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(
1/2)*tan(d*x+c)^(1/2)))+1/8*(-2*A*a*b+B*a^2-B*b^2)*2^(1/2)*(ln((tan(d*x+c)
-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(291) = 582$.

Time = 22.29 (sec) , antiderivative size = 4803, normalized size of antiderivative = 15.20

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = \int \frac{A+B\tan{(c+dx)}}{(a+b\tan{(c+dx)})^2\sqrt{\tan{(c+dx)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**2*sqrt(tan(c + d*x)))
, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.13

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{4\,(3\,Ba^3b-5\,Aa^2b^2-Bab^3-Ab^4)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^5+2\,a^3b^2+ab^4)\sqrt{ab}} + \frac{4\,(Bab-Ab^2)\sqrt{\tan(dx+c)}}{a^4+a^2b^2+(a^3b+ab^3)\tan(dx+c)} - \frac{2\,\sqrt{2}((A+B)a^2-2\,(A-B)ab-(A+B)b^2)}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
-1/4*(4*(3*B*a^3*b - 5*A*a^2*b^2 - B*a*b^3 - A*b^4)*arctan(b*sqrt(tan(d*x
+ c))/sqrt(a*b))/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a*b)) + 4*(B*a*b - A*b^2)
*sqrt(tan(d*x + c))/(a^4 + a^2*b^2 + (a^3*b + a*b^3)*tan(d*x + c)) - (2*sq
rt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2*sqrt(2)*(sqrt
(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A
 + B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)
*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*log(sqrt(2)*sqrt(tan(d*x + c)
) + tan(d*x + c) + 1) - sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2
)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 +
b^4))/d
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = \int \frac{B\tan{(dx+c)}+A}{(b\tan{(dx+c)}+a)^2\sqrt{\tan{(dx+c)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 32.41 (sec) , antiderivative size = 17494, normalized size of antiderivative = 55.36

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2),x)
```

output
```
(log(- ((((((((((256*B*b^3*(2*a^4 - b^4 + a^2*b^2))/d - 128*b^3*tan(c + d*x
)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)
^(1/2) + 16*B^2*a*b^3*d^2 - 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))*
((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a*b^3*d^2 - 16*B^2
*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (64*B^2*a*b^2*tan(c + d*x)^(1/
2)*(a^6 + 17*b^6 - 29*a^2*b^4 + 19*a^4*b^2))/(d^2*(a^2 + b^2)^2))*((4*(-B^
4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a*b^3*d^2 - 16*B^2*a^3*b*d
^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (32*B^3*a*b^2*(a^6 + 13*b^6 - 45*a^2*b
^4 + 39*a^4*b^2))/(d^3*(a^2 + b^2)^3))*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^
2)^2)^(1/2) + 16*B^2*a*b^3*d^2 - 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1
/2))/4 - (16*B^4*b^3*tan(c + d*x)^(1/2)*(9*a^6 - 3*b^6 + 3*a^2*b^4 - 17*a^
4*b^2))/(d^4*(a^2 + b^2)^4))*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2
) + 16*B^2*a*b^3*d^2 - 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (
8*B^5*b^3*(9*a^4 - b^4))/(d^5*(a^2 + b^2)^4))*(((192*B^4*a^2*b^6*d^4 - 16*
B^4*b^8*d^4 - 16*B^4*a^8*d^4 - 608*B^4*a^4*b^4*d^4 + 192*B^4*a^6*b^2*d^4)^
(1/2) + 16*B^2*a*b^3*d^2 - 16*B^2*a^3*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^
6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(1/2))/4 + (log(- ((((((((((256*B*b
^3*(2*a^4 - b^4 + a^2*b^2))/d - 128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^
2 + b^2)^2*(-(4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*b^3*
d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))*(-(4*(-B^4*d^4*(a^4...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2 b+\tan(dx+c)\,a}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x)
```

output
```
int(sqrt(tan(c + d*x))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)
```

3.408 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 367

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx \\
&= \frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{b^{3/2}(7a^2Ab+3Ab^3-5a^3B-ab^2B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{5/2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{2a^2A+3Ab^2-abB}{a^2\,(a^2+b^2)\,d\sqrt{\tan(c+dx)}} + \frac{b(Ab-aB)}{a\,(a^2+b^2)\,d\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}
\end{aligned}$$

output
```
-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
*2^(1/2)/(a^2+b^2)^2/d-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d-b^(3/2)*(7*A*a^2*b+3*A*b^3-5*B
*a^3-B*a*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(5/2)/(a^2+b^2)^2
/d-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/
(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)^2/d-(2*A*a^2+3*A*b^2-B*a*b)/a^2/(a^2+b^2
)/d/tan(d*x+c)^(1/2)+b*(A*b-B*a)/a/(a^2+b^2)/d/tan(d*x+c)^(1/2)/(a+b*tan(d
*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.65

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{b^{3/2}(-7a^2Ab-3Ab^3+5a^3B+ab^2B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}(a^2+b^2)} + \frac{\sqrt[4]{-1}a\left(-i(a+ib)^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+i(a-ib)^2(A+iB}{a^2+b^2}}{a\left(a^2+b^2\right)d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2)
,x]
```

output
```
((b^(3/2)*(-7*a^2*A*b - 3*A*b^3 + 5*a^3*B + a*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[
Tan[c + d*x]])/Sqrt[a]])/(a^(3/2)*(a^2 + b^2)) + ((-1)^(1/4)*a*((-I)*(a +
I*b)^2*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + I*(a - I*b)^2*(A
+ I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2) + (-2*a^2*A -
3*A*b^2 + a*b*B)/(a*Sqrt[Tan[c + d*x]]) + (b*(A*b - a*B))/(Sqrt[Tan[c + d*
x]]*(a + b*Tan[c + d*x])))/(a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.697, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{\int \frac{2Aa^2-bBa-2(Ab-aB)\tan(c+dx)a+3Ab^2+3b(Ab-aB)\tan^2(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx}{a(a^2+b^2)}+\frac{b(Ab-aB)}{ad(a^2+b^2)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Aa^2-bBa-2(Ab-aB)\tan(c+dx)a+3Ab^2+3b(Ab-aB)\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}+\frac{b(Ab-aB)}{ad(a^2+b^2)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2Aa^2-bBa-2(Ab-aB)\tan(c+dx)a+3Ab^2+3b(Ab-aB)\tan(c+dx)^2}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}+\frac{b(Ab-aB)}{ad(a^2+b^2)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{4132}$$

$$\frac{-\frac{2\int \frac{-2Ba^3+4Aba^2+2(aA+bB)\tan(c+dx)a^2-b^2Ba+3Ab^3+b\left(2Aa^2-bBa+3Ab^2\right)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2(2a^2A-abB+3Ab^2)}{ad\sqrt{\tan(c+dx)}}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{-2Ba^3+4Aba^2+2(aA+bB)\tan(c+dx)a^2-b^2Ba+3Ab^3+b\left(2Aa^2-bBa+3Ab^2\right)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2(2a^2A-abB+3Ab^2)}{ad\sqrt{\tan(c+dx)}}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\downarrow$ 3042

$$\frac{-\frac{\int \frac{-2Ba^3+4Aba^2+2(aA+bB)\tan(c+dx)a^2-b^2Ba+3Ab^3+b\left(2Aa^2-bBa+3Ab^2\right)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2(2a^2A-abB+3Ab^2)}{ad\sqrt{\tan(c+dx)}}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\downarrow$ 4136

$$\frac{-\frac{\frac{\int \frac{2\left(\left(-Ba^2+2Aba+b^2B\right)a^2+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2(2a^2A-abB+3Ab^2)}{ad\sqrt{\tan(c+dx)}}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\downarrow$ 27

$$\frac{-\frac{\frac{2\int \frac{\left(-Ba^2+2Aba+b^2B\right)a^2+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2(2a^2A-abB+3Ab^2)}{ad\sqrt{\tan(c+dx)}}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{2\int \frac{\left(-Ba^2+2Aba+b^2B\right)a^2+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2\left(2a^2A-abB-\right.}{ad\sqrt{\tan(c-}}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 4017$$

$$-\frac{\frac{\frac{4\int \frac{a^2\left(-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2\left(2a^2\right.}{ad\sqrt{}}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{4a^2\int \frac{-Ba^2+2Aba+b^2B+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2\left(2a^2A-\right.}{ad\sqrt{t}}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 1482$$

$$-\frac{\frac{\frac{4a^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d\left(a^2+b^2\right)}+\frac{b^2\left(-\right.}{}}{a}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 1476$$

$$-\frac{\frac{\frac{4a^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{}\right.\right.}{d\left(a^2+b^2\right)}}{a}}{2a\left(a\right.}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\Big\downarrow$ 1082

$$-\frac{\frac{4a^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}}{a} \quad 2a\left(a^2+b\right.$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\Big\downarrow$ 217

$$-\frac{\frac{4a^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1\right.}{}\right.\right.}{d\left(a^2+b^2\right)}}{a} \quad 2a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\Big\downarrow$ 1479

$$-\frac{\frac{4a^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\Big\downarrow$ 25

$$-\frac{\frac{4a^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(a^2(A\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\Big\downarrow$ 27

$$-\frac{\frac{4a^2\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(a^2(\right.}{d\left(a^2+b^2\right)}}{}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\downarrow$ 1103

$$-\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.}{}\right.\right.}{a}}{2a}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\downarrow$ 4117

$$-\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int\frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d\left(a^2+b^2\right)}+\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\text{ar}}{}\right.\right.}{a}}{}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\downarrow$ 73

$$-\frac{\frac{2b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int\frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\right.}{\sqrt{}}\right.\right.}{a}}{2a}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}$$

$\downarrow$ 218

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}+$$

$$-\frac{2\left(2a^2A-abB+3Ab^2\right)}{ad\sqrt{\tan(c+dx)}}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab\right.}{d\left(a^2+b^2\right)}$$

$$2a\,($$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2),x]
```

output
```
(-(((2*b^(3/2)*(7*a^2*A*b + 3*A*b^3 - 5*a^3*B - a*b^2*B)*ArcTan[(Sqrt[b]*S
qrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)*d) + (4*a^2*(((a^2*(A -
B) - b^2*(A - B) + 2*a*b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((2*a*b*
(A - B) - a^2*(A + B) + b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/a) - (2*(2*a^2*A + 3*A*b^2 - a*b
*B))/(a*d*Sqrt[Tan[c + d*x]]))/(2*a*(a^2 + b^2)) + (b*(A*b - a*B))/(a*(a^2
 + b^2)*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.96

method	result
derivativedivides	$-\frac{2b^2\left(\frac{\left(\frac{1}{2}Aa^2b+\frac{1}{2}Ab^3-\frac{1}{2}Ba^3-\frac{1}{2}Bab^2\right)\sqrt{\tan(dx+c)}}{a+b\tan(dx+c)}+\frac{\left(7Aa^2b+3Ab^3-5Ba^3-Bab^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2\left(a^2+b^2\right)^2}+\frac{\left(-2Aab+\right.}{}$
default	$-\frac{2b^2\left(\frac{\left(\frac{1}{2}Aa^2b+\frac{1}{2}Ab^3-\frac{1}{2}Ba^3-\frac{1}{2}Bab^2\right)\sqrt{\tan(dx+c)}}{a+b\tan(dx+c)}+\frac{\left(7Aa^2b+3Ab^3-5Ba^3-Bab^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^2\left(a^2+b^2\right)^2}+\frac{\left(-2Aab+\right.}{}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(-2*b^2/a^2/(a^2+b^2)^2*((1/2*A*a^2*b+1/2*A*b^3-1/2*B*a^3-1/2*B*a*b^2)
*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))+1/2*(7*A*a^2*b+3*A*b^3-5*B*a^3-B*a*b^2)
/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^2*(1/8*(-
2*A*a*b+B*a^2-B*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(
tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2
))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-A*a^2+A*b^2-2*B*a*b)*2^(1/
2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x
+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2))))-2/a^2*A/tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2481 vs. $2(334) = 668$.

Time = 36.36 (sec) , antiderivative size = 4991, normalized size of antiderivative = 13.60

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.08

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{4\,(5\,Ba^3b^2-7\,Aa^2b^3+Bab^4-3\,Ab^5)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^6+2\,a^4b^2+a^2b^4)\sqrt{ab}} - \frac{2\,\sqrt{2}((A-B)a^2+2\,(A+B)ab-(A-B)b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{\tan(dx+c)}\right)\right)\cdots}{\cdots}}{\cdots}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
1/4*(4*(5*B*a^3*b^2 - 7*A*a^2*b^3 + B*a*b^4 - 3*A*b^5)*arctan(b*sqrt(tan(d
*x + c))/sqrt(a*b))/((a^6 + 2*a^4*b^2 + a^2*b^4)*sqrt(a*b)) - (2*sqrt(2)*(
(A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2
*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b
^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((A +
B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan
(d*x + c) + 1) + sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*log(-
sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4) -
4*(2*A*a^3 + 2*A*a*b^2 + (2*A*a^2*b - B*a*b^2 + 3*A*b^3)*tan(d*x + c))/((a
^4*b + a^2*b^3)*tan(d*x + c)^(3/2) + (a^5 + a^3*b^2)*sqrt(tan(d*x + c))))/
d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 24.14 (sec) , antiderivative size = 22667, normalized size of antiderivative = 61.76

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^2),x)
```

output
```
(log((((((((((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^
4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B^2*a^3*b*d
^2)/(d^4*(a^2 + b^2)^4))^(1/2) + (128*B*b^2*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a
^4*b^2))/(a*d))*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*
b^3*d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (64*B^2*b^2*ta
n(c + d*x)^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2))/(a*d^2*
(a^2 + b^2)^2))*((4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*
b^3*d^2 + 16*B^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (32*B^3*b^5*(2
5*a^6 + b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(a^2*d^3*(a^2 + b^2)^3))*((4*(-B^4
*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B^2*a^3*b*d^
2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (16*B^4*b^5*tan(c + d*x)^(1/2)*(b^6 - 2
7*a^6 + 7*a^2*b^4 + 11*a^4*b^2))/(a^2*d^4*(a^2 + b^2)^4))*((4*(-B^4*d^4*(a
^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B^2*a^3*b*d^2)/(d^4
*(a^2 + b^2)^4))^(1/2))/4 + (16*B^5*b^6*(5*a^2 + b^2))/(a*d^5*(a^2 + b^2)^
4))*(((192*B^4*a^2*b^6*d^4 - 16*B^4*b^8*d^4 - 16*B^4*a^8*d^4 - 608*B^4*a^4
*b^4*d^4 + 192*B^4*a^6*b^2*d^4)^(1/2) - 16*B^2*a*b^3*d^2 + 16*B^2*a^3*b*d^
2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(1
/2))/4 + (log((((((((((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2
*(-(4*(-B^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a*b^3*d^2 - 16*B
^2*a^3*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2) + (128*B*b^2*(2*b^6 - a^6 + 9*...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3 b+\tan(dx+c)^2 a}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x)
```

output
```
int(sqrt(tan(c + d*x))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)
```

3.409 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 419

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx \\
&= -\frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{b^{5/2}(9a^2Ab+5Ab^3-7a^3B-3ab^2B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{7/2}\,(a^2+b^2)^2\,d} \\
&\quad-\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad-\frac{2a^2A+5Ab^2-3abB}{3a^2\,(a^2+b^2)\,d\tan^{\frac{3}{2}}(c+dx)}+\frac{4a^2Ab+5Ab^3-2a^3B-3ab^2B}{a^3\,(a^2+b^2)\,d\sqrt{\tan(c+dx)}} \\
&\quad+\frac{b(Ab-aB)}{a\,(a^2+b^2)\,d\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}
\end{aligned}$$

output
```
1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*
2^(1/2)/(a^2+b^2)^2/d+1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctan(1+2^(1/
2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d+b^(5/2)*(9*A*a^2*b+5*A*b^3-7*B*
a^3-3*B*a*b^2)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(7/2)/(a^2+b^2)^
2/d-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1/2)
/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)^2/d-1/3*(2*A*a^2+5*A*b^2-3*B*a*b)/a^2/(
a^2+b^2)/d/tan(d*x+c)^(3/2)+(4*A*a^2*b+5*A*b^3-2*B*a^3-3*B*a*b^2)/a^3/(a^2
+b^2)/d/tan(d*x+c)^(1/2)+b*(A*b-B*a)/a/(a^2+b^2)/d/tan(d*x+c)^(3/2)/(a+b*t
an(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.68

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \frac{\frac{3\left(\sqrt[4]{-1}a^{7/2}(a+ib)^2(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+b^{5/2}\left(9a^2Ab+5Ab^3-7a^3B-3ab^2B\right)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt[4]{-1}a^{7/2}(a-ib}{a^{5/2}(a^2+b^2)}}{3a\left(a^2\right.}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2)
,x]
```

output
```
((3*((-1)^(1/4)*a^(7/2)*(a + I*b)^2*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c
 + d*x]]] + b^(5/2)*(9*a^2*A*b + 5*A*b^3 - 7*a^3*B - 3*a*b^2*B)*ArcTan[(Sq
rt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] + (-1)^(1/4)*a^(7/2)*(a - I*b)^2*(A + I
*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/(a^(5/2)*(a^2 + b^2)) + (-2*a
^2*A - 5*A*b^2 + 3*a*b*B)/(a*Tan[c + d*x]^(3/2)) + (3*(4*a^2*A*b + 5*A*b^3
 - 2*a^3*B - 3*a*b^2*B))/(a^2*Sqrt[Tan[c + d*x]]) + (3*b*(A*b - a*B))/(Tan
[c + d*x]^(3/2)*(a + b*Tan[c + d*x])))/(3*a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.788$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 4092}$$

$$\frac{\int \frac{2Aa^2-3bBa-2(Ab-aB)\tan(c+dx)a+5Ab^2+5b(Ab-aB)\tan^2(c+dx)}{2\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}dx}{a(a^2+b^2)}+\frac{b(Ab-aB)}{ad(a^2+b^2)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2Aa^2-3bBa-2(Ab-aB)\tan(c+dx)a+5Ab^2+5b(Ab-aB)\tan^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}+\frac{b(Ab-aB)}{ad(a^2+b^2)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{2Aa^2-3bBa-2(Ab-aB)\tan(c+dx)a+5Ab^2+5b(Ab-aB)\tan(c+dx)^2}{\tan(c+dx)^{5/2}(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}+\frac{b(Ab-aB)}{ad(a^2+b^2)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 4132}$$

$$\frac{-\frac{2\int \frac{3\left(-2Ba^3+4Aba^2+2(aA+bB)\tan(c+dx)a^2-3b^2Ba+5Ab^3+b\left(2Aa^2-3bBa+5Ab^2\right)\tan^2(c+dx)\right)}{2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx}{3a}-\frac{2(2a^2A-3abB+5Ab^2)}{3ad\tan^{\frac{3}{2}}(c+dx)}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{-2Ba^3+4Aba^2+2(aA+bB)\tan(c+dx)a^2-3b^2Ba+5Ab^3+b\left(2Aa^2-3bBa+5Ab^2\right)\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx}{a}-\frac{2(2a^2A-3abB+5Ab^2)}{3ad\tan^{\frac{3}{2}}(c+dx)}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{-2Ba^3+4Aba^2+2(aA+bB)\tan(c+dx)a^2-3b^2Ba+5Ab^3+b\left(2Aa^2-3bBa+5Ab^2\right)\tan(c+dx)^2}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))}dx}{a}-\frac{2(2a^2A-3abB+5Ab^2)}{3ad\tan^{\frac{3}{2}}(c+dx)}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 4132$$

$$\frac{-\frac{-\frac{2\int -\frac{2Aa^4+4bBa^3-2(Ab-aB)\tan(c+dx)a^3-4Ab^2a^2+3b^3Ba-5Ab^4-b\left(-2Ba^3+4Aba^2-3b^2Ba+5Ab^3\right)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2\left(-2a^3B+4a^2Ab-3ab^2B+5Ab^3\cdots\right)}{ad\sqrt{\tan(c+dx)}}\cdots}{a}\cdots}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\frac{\int \frac{2Aa^4+4bBa^3-2(Ab-aB)\tan(c+dx)a^3-4Ab^2a^2+3b^3Ba-5Ab^4-b\left(-2Ba^3+4Aba^2-3b^2Ba+5Ab^3\right)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2\left(-2a^3B+4a^2Ab-3ab^2B+5Ab^3\right)}{ad\sqrt{\tan(c+dx)}}-\cdots}{a}\cdots}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\int \frac{2Aa^4+4bBa^3-2(Ab-aB)\tan(c+dx)a^3-4Ab^2a^2+3b^3Ba-5Ab^4-b\left(-2Ba^3+4Aba^2-3b^2Ba+5Ab^3\right)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2\left(-2a^3B+4a^2Ab-3ab^2B+5Ab^3\right)}{ad\sqrt{\tan(c+dx)}}}{a}-}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 4136

$$-\frac{\frac{\frac{\frac{\int \frac{2\left(a^3\left(Aa^2+2bBa-Ab^2\right)-a^3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b^3\left(-7a^3B+9a^2Ab-3ab^2B+5Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2\left(-2a^3B+4a^2\right.}{ad\sqrt{}}}{a}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\frac{2\int \frac{a^3\left(Aa^2+2bBa-Ab^2\right)-a^3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b^3\left(-7a^3B+9a^2Ab-3ab^2B+5Ab^3\right)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2\left(-2a^3B+4a^2A\right.}{ad\sqrt{\tan}}}{a}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{2\int \frac{a^3\left(Aa^2+2bBa-Ab^2\right)-a^3\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b^3\left(-7a^3B+9a^2Ab-3ab^2B+5Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2\left(-2a^3B+4a^2A\right.}{ad\sqrt{\tan}}}{a}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 4017

$$-\frac{\frac{\frac{4\int \frac{a^3\left(Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{b^3\left(-7a^3B+9a^2Ab-3ab^2B+5Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2\left(-2a^3\right.}{}}{a}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{4a^3\int \frac{Aa^2+2bBa-Ab^2-\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{b^3\left(-7a^3B+9a^2Ab-3ab^2B+5Ab^3\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2\left(-2a^3B-\right.}{a}}{a}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 1482

$$-\frac{\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d\left(a^2+b^2\right)}-\frac{b^3\left(-\right.}{}}{a}}{a}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 1476

$$-\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{}\right.\right.}{d\left(a^2+b^2\right)}}{a}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 1082

$$-\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}}{a}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 217

$$-\frac{\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.}{d\left(a^2+b^2\right)}}{a}}{a}$$

$2a\,(a$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 1479

$$-\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a\right.\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 25

$$-\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+\right.\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\downarrow$ 27

$$-\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\left(-\left(a^2(A\cdots\right.\right.\right.}{d\left(a^2+b^2\right)}\cdots}{\cdots}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 1103

$$-\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(\cdots\right.\right.}{d\left(a^2+b^2\right)}\cdots}{a}\cdots}{\cdots}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 4117

$$-\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(\cdots\right.\right.}{d\left(a^2+b^2\right)}\cdots}{a}\cdots}{\cdots}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 73

$$-\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(\cdots\right.\right.}{d\left(a^2+b^2\right)}\cdots}{a}\cdots}{\cdots}$$

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 218

$$\frac{b(Ab-aB)}{ad\left(a^2+b^2\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}+$$

$$-\frac{2(2a^2A-3abB+5Ab^2)}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{\frac{4a^3\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(\right.\right.}{d\left(a^2+b^2\right)}}{}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2),x]
```

output
```
(-((((-2*b^(5/2)*(9*a^2*A*b + 5*A*b^3 - 7*a^3*B - 3*a*b^2*B)*ArcTan[(Sqrt[
b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2 + b^2)*d) + (4*a^3*(-1/2*((
2*a*b*(A - B) - a^2*(A + B) + b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[
c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + (
(a^2*(A - B) - b^2*(A - B) + 2*a*b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] +
Tan[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/a - (2*(4*a^2*A*b + 5*A*b
^3 - 2*a^3*B - 3*a*b^2*B))/(a*d*Sqrt[Tan[c + d*x]]))/a) - (2*(2*a^2*A + 5*
A*b^2 - 3*a*b*B))/(3*a*d*Tan[c + d*x]^(3/2)))/(2*a*(a^2 + b^2)) + (b*(A*b
- a*B))/(a*(a^2 + b^2)*d*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{2b^3\left(\frac{\left(\frac{1}{2}Aa^2b+\frac{1}{2}Ab^3-\frac{1}{2}Ba^3-\frac{1}{2}Bab^2\right)\sqrt{\tan(dx+c)}}{a+b\tan(dx+c)}+\frac{\left(9Aa^2b+5Ab^3-7Ba^3-3Bab^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3\left(a^2+b^2\right)^2}+\frac{\left(-Aa^2+A\right.}{}}{}$
default	$\frac{\frac{2b^3\left(\frac{\left(\frac{1}{2}Aa^2b+\frac{1}{2}Ab^3-\frac{1}{2}Ba^3-\frac{1}{2}Bab^2\right)\sqrt{\tan(dx+c)}}{a+b\tan(dx+c)}+\frac{\left(9Aa^2b+5Ab^3-7Ba^3-3Bab^2\right)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3\left(a^2+b^2\right)^2}+\frac{\left(-Aa^2+A\right.}{}}{}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2*b^3/a^3/(a^2+b^2)^2*((1/2*A*a^2*b+1/2*A*b^3-1/2*B*a^3-1/2*B*a*b^2)*
tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))+1/2*(9*A*a^2*b+5*A*b^3-7*B*a^3-3*B*a*b^2
)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^2*(1/8*(
-A*a^2+A*b^2-2*B*a*b)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/
(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/
2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(2*A*a*b-B*a^2+B*b^2)*2^(1/
2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x
+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2))))-2/3/a^2*A/tan(d*x+c)^(3/2)-2*(-2*A*b+B*a)/a^3/tan(d*x+c)^
(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2578 vs. $2(386) = 772$.

Time = 69.12 (sec) , antiderivative size = 5186, normalized size of antiderivative = 12.38

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.07

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^2}\,dx = -\frac{\frac{12\,(7\,Ba^3b^3 - 9\,Aa^2b^4 + 3\,Bab^5 - 5\,Ab^6)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^7 + 2\,a^5b^2 + a^3b^4)\sqrt{ab}} + \frac{4\left(2\,Aa^4 + 2\,Aa^2b^2 + 3\,(2\,Ba^3b - 4\,Aa^2b^2 + 3\,Bab^3 - 5\,Ab^4)\tan(dx+c)^2 + 2\right.}{(a^5b + a^3b^3)\tan(dx+c)^{\frac{5}{2}} + (a^6 + a^4b^2)}}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
-1/12*(12*(7*B*a^3*b^3 - 9*A*a^2*b^4 + 3*B*a*b^5 - 5*A*b^6)*arctan(b*sqrt(
tan(d*x + c))/sqrt(a*b))/((a^7 + 2*a^5*b^2 + a^3*b^4)*sqrt(a*b)) + 4*(2*A*
a^4 + 2*A*a^2*b^2 + 3*(2*B*a^3*b - 4*A*a^2*b^2 + 3*B*a*b^3 - 5*A*b^4)*tan(
d*x + c)^2 + 2*(3*B*a^4 - 5*A*a^3*b + 3*B*a^2*b^2 - 5*A*a*b^3)*tan(d*x + c
))/((a^5*b + a^3*b^3)*tan(d*x + c)^(5/2) + (a^6 + a^4*b^2)*tan(d*x + c)^(3
/2)) + 3*(2*sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2
*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a^2 - 2*(A
 - B)*a*b - (A + B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c
)))) + sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*log(sqrt(2)*sqr
t(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b
 - (A - B)*b^2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/(a^4
+ 2*a^2*b^2 + b^4))/d
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx=\int \frac{B\tan{(dx+c)}+A}{(b\tan{(dx+c)}+a)^2\tan{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 21.20 (sec) , antiderivative size = 24620, normalized size of antiderivative = 58.76

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx=\text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2),x)
```

output
```
(log(80*A^5*a^24*b^20*d^4 - ((((192*A^4*a^2*b^6*d^4 - 16*A^4*b^8*d^4 - 16*
A^4*a^8*d^4 - 608*A^4*a^4*b^4*d^4 + 192*A^4*a^6*b^2*d^4)^(1/2) - 16*A^2*a*
b^3*d^2 + 16*A^2*a^3*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4
*d^4 + 4*a^6*b^2*d^4))^(1/2)*(tan(c + d*x)^(1/2)*(9472*A^4*a^31*b^15*d^5 -
 3040*A^4*a^23*b^23*d^5 - 9056*A^4*a^25*b^21*d^5 - 12352*A^4*a^27*b^19*d^5
 - 4256*A^4*a^29*b^17*d^5 - 400*A^4*a^21*b^25*d^5 + 13760*A^4*a^33*b^13*d^
5 + 7744*A^4*a^35*b^11*d^5 + 1968*A^4*a^37*b^9*d^5 + 224*A^4*a^39*b^7*d^5
+ 32*A^4*a^41*b^5*d^5) + ((((192*A^4*a^2*b^6*d^4 - 16*A^4*b^8*d^4 - 16*A^4
*a^8*d^4 - 608*A^4*a^4*b^4*d^4 + 192*A^4*a^6*b^2*d^4)^(1/2) - 16*A^2*a*b^3
*d^2 + 16*A^2*a^3*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^
4 + 4*a^6*b^2*d^4))^(1/2)*(12928*A^3*a^25*b^23*d^6 - 800*A^3*a^21*b^27*d^6
 - 2080*A^3*a^23*b^25*d^6 - ((tan(c + d*x)^(1/2)*(3200*A^2*a^22*b^28*d^7 +
 33920*A^2*a^24*b^26*d^7 + 158208*A^2*a^26*b^24*d^7 + 425536*A^2*a^28*b^22
*d^7 + 727296*A^2*a^30*b^20*d^7 + 820672*A^2*a^32*b^18*d^7 + 615936*A^2*a^
34*b^16*d^7 + 304256*A^2*a^36*b^14*d^7 + 98432*A^2*a^38*b^12*d^7 + 22016*A
^2*a^40*b^10*d^7 + 3072*A^2*a^42*b^8*d^7 - 704*A^2*a^44*b^6*d^7 - 512*A^2*
a^46*b^4*d^7 - 64*A^2*a^48*b^2*d^7) + ((((192*A^4*a^2*b^6*d^4 - 16*A^4*b^8
*d^4 - 16*A^4*a^8*d^4 - 608*A^4*a^4*b^4*d^4 + 192*A^4*a^6*b^2*d^4)^(1/2) -
 16*A^2*a*b^3*d^2 + 16*A^2*a^3*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 +
 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(1/2)*(1280*A*a^24*b^28*d^8 - (tan(c +...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx=\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^4 b+\tan(dx+c)^3 a}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x)
```

output
```
int(sqrt(tan(c + d*x))/(tan(c + d*x)**4*b + tan(c + d*x)**3*a),x)
```

3.410 $\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 514

$$\begin{aligned}
&\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx \\
&= \frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&\quad-\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&\quad+\frac{a^{3/2}(3a^4Ab+6a^2Ab^3+35Ab^5-15a^5B-46a^3b^2B-63ab^4B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{7/2}\,(a^2+b^2)^3\,d} \\
&\quad+\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&\quad-\frac{(3a^3Ab+11aAb^3-15a^4B-31a^2b^2B-8b^4B)\sqrt{\tan(c+dx)}}{4b^3\,(a^2+b^2)^2\,d} \\
&\quad+\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} \\
&\quad+\frac{a(a^2Ab+9Ab^3-5a^3B-13ab^2B)\tan^{\frac{3}{2}}(c+dx)}{4b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}
\end{aligned}$$

output
```
-1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d-1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A
+B)+3*a*b^2*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/
d+1/4*a^(3/2)*(3*A*a^4*b+6*A*a^2*b^3+35*A*b^5-15*B*a^5-46*B*a^3*b^2-63*B*a
*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(7/2)/(a^2+b^2)^3/d+1/2*(
a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctanh(2^(1/2)*tan(d*x+c
)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d-1/4*(3*A*a^3*b+11*A*a*b^3-15
*B*a^4-31*B*a^2*b^2-8*B*b^4)*tan(d*x+c)^(1/2)/b^3/(a^2+b^2)^2/d+1/2*a*(A*b
-B*a)*tan(d*x+c)^(5/2)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/4*a*(A*a^2*b+9*A
*b^3-5*B*a^3-13*B*a*b^2)*tan(d*x+c)^(3/2)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c
))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.53 (sec) , antiderivative size = 1034, normalized size of antiderivative = 2.01

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(Tan[c + d*x]^(7/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3
,x]
```

output
```
(Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*((-3*a^3*A*b - 13*a*A*
b^3 + 15*a^4*B + 33*a^2*b^2*B + 8*b^4*B)/(4*(a - I*b)^2*(a + I*b)^2*b^3) -
 (a^3*(-(A*b) + a*B))/(2*(a - I*b)^2*(a + I*b)^2*b*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2) + (a^3*A*b*Sin[c + d*x] + 13*a*A*b^3*Sin[c + d*x] - 5*a^4*B*
Sin[c + d*x] - 17*a^2*b^2*B*Sin[c + d*x])/(4*(a - I*b)^2*(a + I*b)^2*b^2*(
a*Cos[c + d*x] + b*Sin[c + d*x])))*Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x])
)/(d*(A*Cos[c + d*x] + B*Sin[c + d*x])*(a + b*Tan[c + d*x])^3) - (Sec[c +
d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3*(A + B*Tan[c + d*x])*((2*(-3*a^
4*A*b - 7*a^2*A*b^3 - 4*A*b^5 + 15*a^5*B + 31*a^3*b^2*B + 16*a*b^4*B)*ArcT
an[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Csc[c + d*x]*Sec[c + d*x]^3*(a +
b*Tan[c + d*x]))/(Sqrt[a]*Sqrt[b]*(b + a*Cot[c + d*x])*(1 + Tan[c + d*x]^2
)^2) + ((8*a*A*b^4 - 4*a^2*b^3*B + 4*b^5*B)*Csc[c + d*x]^2*(-8*Sqrt[a]*Sqr
t[b]*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] + Sqrt[2]*(-2*(a + b)*Ar
cTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*(a + b)*ArcTan[1 + Sqrt[2]*Sqrt[T
an[c + d*x]]] + (a - b)*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])))*Sec[c + d*x]^2*S
in[2*(c + d*x)]*(a + b*Tan[c + d*x]))/(4*(a^2 + b^2)*(b + a*Cot[c + d*x])*
(1 + Tan[c + d*x]^2)) - ((-4*a^2*A*b^3 + 4*A*b^5 - 8*a*b^4*B)*Cos[2*(c + d
*x)]*Csc[c + d*x]*((4*(a^2 - b^2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt
[a]])/(Sqrt[a]*Sqrt[b]) + Sqrt[2]*(2*(a - b)*ArcTan[1 - Sqrt[2]*Sqrt[Ta...
```

Rubi [A] (verified)

Time = 2.76 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.788$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4130, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{7/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{\int -\frac{\tan^{\frac{3}{2}}(c+dx)\left(\left(-5Ba^2+Aba-4b^2B\right)\tan^2(c+dx)-4b(Ab-aB)\tan(c+dx)+5a(Ab-aB)\right)}{2(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)\left(\left(-5Ba^2+Aba-4b^2B\right)\tan^2(c+dx)-4b(Ab-aB)\tan(c+dx)+5a(Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)^{3/2}\left(\left(-5Ba^2+Aba-4b^2B\right)\tan(c+dx)^2-4b(Ab-aB)\tan(c+dx)+5a(Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \textbf{4128}$$

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$
$$\frac{\frac{\int \frac{\sqrt{\tan(c+dx)}\left(8\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(-15Ba^4+3Aba^3-31b^2Ba^2+11Ab^3a-8b^4B\right)\tan^2(c+dx)+3a\left(-5Ba^3+Aba^2-13b^2Ba+9Ab^3\right)\right)}{2(a+b\tan(c+dx))}dx}{b\left(a^2+b^2\right)}-\frac{a(-}{}}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$
$$\frac{\frac{\int \frac{\sqrt{\tan(c+dx)}\left(8\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(-15Ba^4+3Aba^3-31b^2Ba^2+11Ab^3a-8b^4B\right)\tan^2(c+dx)+3a\left(-5Ba^3+Aba^2-13b^2Ba+9Ab^3\right)\right)}{a+b\tan(c+dx)}dx}{2b\left(a^2+b^2\right)}-\frac{a(-}{}}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$
$$\frac{\frac{\int \frac{\sqrt{\tan(c+dx)}\left(8\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2+\left(-15Ba^4+3Aba^3-31b^2Ba^2+11Ab^3a-8b^4B\right)\tan(c+dx)^2+3a\left(-5Ba^3+Aba^2-13b^2Ba+9Ab^3\right)\right)}{a+b\tan(c+dx)}dx}{2b\left(a^2+b^2\right)}-\frac{a(-}{}}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \textbf{4130}$$

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-$$

$$\frac{\frac{\frac{2\int -\frac{-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)b^3+\left(-15Ba^5+3Aba^4-31b^2Ba^3+3Ab^3a^2-24b^4Ba+8Ab^5\right)\tan^2(c+dx)+a\left(-15Ba^4+3Aba^3-31b^2Ba^2+11Ab^3a-8b^4B\right.}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-$$

$$\frac{\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\int\frac{-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)b^3+\left(-15Ba^5+3Aba^4-31b^2Ba^3+3Ab^3a^2-24b^4Ba+8Ab^5\right)t}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-$$

$$\frac{\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\int\frac{-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)b^3+\left(-15Ba^5+3Aba^4-31b^2Ba^3+3Ab^3a^2-24b^4Ba+8Ab^5\right)t}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 4136

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-$$

$$\frac{\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{\int\frac{8\left(b^3\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-b^3\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{a^2\left(-15a\right.}{}}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-$$

$$\frac{\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{8\int\frac{b^3\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-b^3\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{a^2\left(-15a^5\right.}{}}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{8\int\frac{b^3\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-b^3\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{a^2\left(-15a^5\right.}{b}$$

$$2b\left(a^2+b^2\right)$$

$$4b\left(a^2+b^2\right)$$

$\Big\downarrow$ 4017

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{16\int\frac{b^3\left(Aa^3+3bBa^2-3Ab^2a-b^3B-\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a}{b}$$

$$2b\left(a^2+b^2\right)$$

$$4b\left(a^2+b\right.$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{16b^3\int\frac{Aa^3+3bBa^2-3Ab^2a-b^3B-\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^2\left(\right.}{b}$$

$$2b\left(a^2+b^2\right)$$

$$4b\left(a^2+b^2\right.$$

$\Big\downarrow$ 1482

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(\right.\right.}{d\left(a^2+b^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(\right.\right.}{}$$

$\Big\downarrow$ 1082

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(\right.\right.\right.}{}$$

$\downarrow$ 217

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(\right.\right.\right.}{}$$

$\downarrow$ 1479

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}}{2\sqrt{2}}\right.\right.}{}$$

$\downarrow$ 25

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\right.}{2\sqrt{2}}\right.\right.}{}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\cdots}}{2\sqrt{2}}\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-15a^5B+3a^4Ab-46a^3b^2B+6a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\cdots}{\cdots}}{\cdots}$$

$$\Big\downarrow \text{ 4117}$$

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^2\left(-15a^5B+3a^4Ab-46a^3b^2B+6a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(\cdots)}{d\left(a^2+b^2\right)}\cdots}{\cdots}}{\cdots}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2a^2\left(-15a^5B+3a^4Ab-46a^3b^2B+6a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{16b\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{a(Ab-aB)\tan^{\frac{5}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)\sqrt{\tan(c+dx)}}{bd}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
Int[(Tan[c + d*x]^(7/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(a*(A*b - a*B)*Tan[c + d*x]^(5/2))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) - ((-(((2*a^(3/2)*(3*a^4*A*b + 6*a^2*A*b^3 + 35*A*b^5 - 15*a^5*B - 46*
a^3*b^2*B - 63*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqr
t[b]*(a^2 + b^2)*d) + (16*b^3*(-1/2*((3*a^2*b*(A - B) - b^3*(A - B) - a^3*
(A + B) + 3*a*b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[
2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((a^3*(A - B) - 3
*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt
[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/b) + (2*(3*a^3*A*b +
11*a*A*b^3 - 15*a^4*B - 31*a^2*b^2*B - 8*b^4*B)*Sqrt[Tan[c + d*x]])/(b*d))
/(2*b*(a^2 + b^2)) - (a*(a^2*A*b + 9*A*b^3 - 5*a^3*B - 13*a*b^2*B)*Tan[c +
 d*x]^(3/2))/(b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(4*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{2\sqrt{\tan(dx+c)}\,B}{b^3}+\frac{2a^2\left(\frac{\left(-\frac{5}{8}A\,a^4b^2-\frac{9}{4}A\,a^2b^4-\frac{13}{8}A\,b^6+\frac{9}{8}B\,a^5b+\frac{13}{4}B\,a^3b^3+\frac{17}{8}Ba\,b^5\right)\tan(dx+c)^{\frac{3}{2}}-\frac{a\left(3A\,a^4b+14A\,a^2b^3+11A\right.}{}}{(a+b\tan(dx+c))^2}\right.}{}}{}$
default	$\frac{\frac{2\sqrt{\tan(dx+c)}\,B}{b^3}+\frac{2a^2\left(\frac{\left(-\frac{5}{8}A\,a^4b^2-\frac{9}{4}A\,a^2b^4-\frac{13}{8}A\,b^6+\frac{9}{8}B\,a^5b+\frac{13}{4}B\,a^3b^3+\frac{17}{8}Ba\,b^5\right)\tan(dx+c)^{\frac{3}{2}}-\frac{a\left(3A\,a^4b+14A\,a^2b^3+11A\right.}{}}{(a+b\tan(dx+c))^2}\right.}{}}{}$

input
```
int(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2*tan(d*x+c)^(1/2)*B/b^3+2*a^2/b^3/(a^2+b^2)^3*(((-5/8*A*a^4*b^2-9/4*
A*a^2*b^4-13/8*A*b^6+9/8*B*a^5*b+13/4*B*a^3*b^3+17/8*B*a*b^5)*tan(d*x+c)^(
3/2)-1/8*a*(3*A*a^4*b+14*A*a^2*b^3+11*A*b^5-7*B*a^5-22*B*a^3*b^2-15*B*a*b^
4)*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8*(3*A*a^4*b+6*A*a^2*b^3+35*A*b^
5-15*B*a^5-46*B*a^3*b^2-63*B*a*b^4)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/
(a*b)^(1/2)))+2/(a^2+b^2)^3*(1/8*(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*2^(1/2)
*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c
)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d
*x+c)^(1/2)))+1/8*(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*2^(1/2)*(ln((tan(d*x+
c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*
arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3735 vs. 2(470) = 940.

Time = 158.04 (sec) , antiderivative size = 7496, normalized size of antiderivative = 14.58

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.11

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{(15\,Ba^7-3\,Aa^6b+46\,Ba^5b^2-6\,Aa^4b^3+63\,Ba^3b^4-35\,Aa^2b^5)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9)\sqrt{ab}} - \frac{2\sqrt{2}((A+B)a^3-3\,(A-B)a^2b-3\,(A+B)ab^2+(A-}{}}{}$$

input
```
integrate(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
-1/4*((15*B*a^7 - 3*A*a^6*b + 46*B*a^5*b^2 - 6*A*a^4*b^3 + 63*B*a^3*b^4 -
35*A*a^2*b^5)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^6*b^3 + 3*a^4*b^5
 + 3*a^2*b^7 + b^9)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b
 - 3*(A + B)*a*b^2 + (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan
(d*x + c)))) + 2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2
+ (A - B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqr
t(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(s
qrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A - B)*a^3 + 3*(
A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x +
c)) + tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((9*B*a^5*b
 - 5*A*a^4*b^2 + 17*B*a^3*b^3 - 13*A*a^2*b^4)*tan(d*x + c)^(3/2) + (7*B*a^
6 - 3*A*a^5*b + 15*B*a^4*b^2 - 11*A*a^3*b^3)*sqrt(tan(d*x + c)))/(a^6*b^3
+ 2*a^4*b^5 + a^2*b^7 + (a^4*b^5 + 2*a^2*b^7 + b^9)*tan(d*x + c)^2 + 2*(a^
5*b^4 + 2*a^3*b^6 + a*b^8)*tan(d*x + c)) - 8*B*sqrt(tan(d*x + c))/b^3)/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 62.62 (sec) , antiderivative size = 27429, normalized size of antiderivative = 53.36

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(7/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
(log((((((((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^
4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80*B^2*a^3*b^3*d^2
- 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2) - (64*B*
a*b*(15*a^4 + 2*b^4 + 41*a^2*b^2))/d)*((4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^
4 - 15*a^4*b^2)^2)^(1/2) + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*
a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2))/4 + (8*B^2*a*tan(c + d*x)^(1/2)*(22
5*a^14 - 184*b^14 + 608*a^2*b^12 - 272*a^4*b^10 + 3937*a^6*b^8 + 5804*a^8*
b^6 + 4006*a^10*b^4 + 1380*a^12*b^2))/(b^4*d^2*(a^2 + b^2)^4))*((4*(-B^4*d
^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80*B^2*a^3*b^3*d^2 - 2
4*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2))/4 - (2*B^3
*a^2*(1125*a^14 + 16*b^14 + 6112*a^2*b^12 - 17727*a^4*b^10 - 23239*a^6*b^8
 - 11174*a^8*b^6 + 2930*a^10*b^4 + 3525*a^12*b^2))/(b^4*d^3*(a^2 + b^2)^6)
)*((4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80*B^2*a^
3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2
))/4 - (B^4*tan(c + d*x)^(1/2)*(32*b^18 - 225*a^18 + 128*a^2*b^16 + 192*a^
4*b^14 - 3841*a^6*b^12 + 18050*a^8*b^10 + 26801*a^10*b^8 + 16860*a^12*b^6
+ 4049*a^14*b^4 - 30*a^16*b^2))/(b^5*d^4*(a^2 + b^2)^8))*((4*(-B^4*d^4*(a^
6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80*B^2*a^3*b^3*d^2 - 24*B^2*
a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2))/4 - (B^5*a^3*(22
5*a^12 + 504*b^12 + 872*a^2*b^10 + 4457*a^4*b^8 + 5916*a^6*b^6 + 4006*a...
```

Reduce [F]

$$\int \frac{\tan^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx=\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^3}{\tan(dx+c)^2 b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(tan(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
int((sqrt(tan(c + d*x))*tan(c + d*x)**3)/(tan(c + d*x)**2*b**2 + 2*tan(c +
 d*x)*a*b + a**2),x)
```

3.411 $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 448

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$- \frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+ \frac{\sqrt{a}(a^4Ab+18a^2Ab^3-15Ab^5+3a^5B+6a^3b^2B+35ab^4B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4b^{5/2}\,(a^2+b^2)^3\,d}$$
$$- \frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+ \frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2b\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}$$
$$- \frac{a(a^2Ab-7Ab^3+3a^3B+11ab^2B)\sqrt{\tan(c+dx)}}{4b^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

output
```
-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*
b*(A+B)-b^3*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/
d+1/4*a^(1/2)*(A*a^4*b+18*A*a^2*b^3-15*A*b^5+3*B*a^5+6*B*a^3*b^2+35*B*a*b^
4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(5/2)/(a^2+b^2)^3/d-1/2*(3*a
^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(
1/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d+1/2*a*(A*b-B*a)*tan(d*x+c)^(3/2
)/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-1/4*a*(A*a^2*b-7*A*b^3+3*B*a^3+11*B*a*b
^2)*tan(d*x+c)^(1/2)/b^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.83

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(-\left((Ab+3aB)\sqrt{\tan(c+dx)}\right)+\frac{(a^2Ab+4Ab^3+3a^3B)\sqrt{\tan(c+dx)}}{4(a^2+b^2)}-3bB\tan^{\frac{3}{2}}(c+dx)+\frac{(a+b\tan(c+dx))\left(\frac{3}{8}a^{5/2}\right.}{}\right.}{}$$

input
```
Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3
,x]
```

output
```
(2*(-((A*b + 3*a*B)*Sqrt[Tan[c + d*x]]) + ((a^2*A*b + 4*A*b^3 + 3*a^3*B)*S
qrt[Tan[c + d*x]])/(4*(a^2 + b^2)) - 3*b*B*Tan[c + d*x]^(3/2) + ((a + b*Ta
n[c + d*x])*((3*a^(5/2)*Sqrt[b]*(a^2 + b^2)*(a^3*A*b + 9*a*A*b^3 + 3*a^4*B
 + 3*a^2*b^2*B + 8*b^4*B)*Sqrt[Tan[c + d*x]])/8 + ((3*a^3*(a^4*A*b + 18*a^
2*A*b^3 - 15*A*b^5 + 3*a^5*B + 6*a^3*b^2*B + 35*a*b^4*B)*ArcTan[(Sqrt[b]*S
qrt[Tan[c + d*x]])/Sqrt[a]])/8 + (3*(-1)^(1/4)*a^(5/2)*b^(5/2)*((I*a - b)^
3*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - (I*a + b)^3*(A + I*B)*
ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))/2)*(a + b*Tan[c + d*x])))/(a^(5/2
)*Sqrt[b]*(a^2 + b^2)^3)))/(3*b^2*d*(a + b*Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.697$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{5/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{\int -\frac{\sqrt{\tan(c+dx)}\left(-\left(\left(3Ba^2+Aba+4b^2B\right)\tan^2(c+dx)\right)-4b(Ab-aB)\tan(c+dx)+3a(Ab-aB)\right)}{2(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\sqrt{\tan(c+dx)}\left(-\left(\left(3Ba^2+Aba+4b^2B\right)\tan^2(c+dx)\right)-4b(Ab-aB)\tan(c+dx)+3a(Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{\sqrt{\tan(c+dx)}\left(-\left(\left(3Ba^2+Aba+4b^2B\right)\tan(c+dx)^2\right)-4b(Ab-aB)\tan(c+dx)+3a(Ab-aB)\right)}{(a+b\tan(c+dx))^2}dx}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4128}$$

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{\int -\frac{-8(Aa^2+2bBa-Ab^2)\tan(c+dx)b^2+(3Ba^4+Aba^3+3b^2Ba^2+9Ab^3a+8b^4B)\tan^2(c+dx)+a(3Ba^3+Aba^2+11b^2Ba-7Ab^3)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b(a^2+b^2)}+\frac{a(3a^3B+a^2Ab+11ab^2 \ldots}{bd(a^2+b^2)(a \ldots}}{4b(a^2+b^2)}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\int \frac{-8(Aa^2+2bBa-Ab^2)\tan(c+dx)b^2+(3Ba^4+Aba^3+3b^2Ba^2+9Ab^3a+8b^4B)\tan^2(c+dx)+a(3Ba^3+A \ldots}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}}{2b(a^2+b^2)}}{4b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\int \frac{-8(Aa^2+2bBa-Ab^2)\tan(c+dx)b^2+(3Ba^4+Aba^3+3b^2Ba^2+9Ab^3a+8b^4B)\tan(c+dx)^2+a(3Ba^3+A \ldots}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}}{2b(a^2+b^2)}}{4b(a^2+b^2)}$$

$\downarrow$ 4136

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{\int -\frac{8((-Ba^3+3Aba^2+3b^2Ba-Ab^3)b^2+(Aa^3+3bBa^2-3Ab^2a-b^3B)\tan(c+dx)b^2)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{a(3a^5B+a^4A \ldots}{}}{2b(a^2+b^2)}}{4b(a^2+b^2)}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5)\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{8\int \frac{(-Ba^3+3 \ldots}{}}{}}{2b(a^2+b^2)}}{4b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{8\int \frac{(-Ba^3+3 \ldots}{}}{}}{2b(a^2+b^2)}}{4b(a^2+b^2)}$$

$\downarrow$ 4017

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{a\left(3a^3B+a^2Ab+11ab^2B-7Ab^3\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-16\int\frac{b^2\left(-Ba^3\cdots\right.}{\cdots}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{a\left(3a^3B+a^2Ab+11ab^2B-7Ab^3\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-16b^2\int\frac{-Ba^3\cdots}{\cdots}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 1482

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{a\left(3a^3B+a^2Ab+11ab^2B-7Ab^3\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-16b^2\left(\frac{1}{2}\left(-\left(a\cdots\right.\right.\right.}{\cdots}}{4b\left(\cdots\right.}$$

$\downarrow$ 1476

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{a\left(3a^3B+a^2Ab+11ab^2B-7Ab^3\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-16b^2\left(\frac{1}{2}\left(-\left(a\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 1082

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{a\left(3a^3B+a^2Ab+11ab^2B-7Ab^3\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-16b^2\left(\frac{1}{2}\left(-\left(a\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 217

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-$$

$$\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{16b^2\left(\frac{1}{2}\left(-\left(a\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 1479

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-$$

$$\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{16b^2\left(\frac{1}{2}\left(-\left(a\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 25

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-$$

$$\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{16b^2\left(\frac{1}{2}\left(-\left(a\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-$$

$$\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{16b^2\left(\frac{1}{2}\left(-\left(a\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 1103

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-$$

$$\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{16b^2\left(\frac{1}{2}\left(a^3(A\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \text{ 4117}$$

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d\left(a^2+b^2\right)}-\frac{16b^2}{}}{}}{}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{2a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{16b^2\left(\frac{1}{2}\left(a^3(A-\right.\right.}{}}{}}{}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{a(3a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{2\sqrt{a}\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}d\left(a^2+b^2\right)}-\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)\right.\right.}{}}{}}{}$$

input

```
Int[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output

```
(a*(A*b - a*B)*Tan[c + d*x]^(3/2))/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) - (-1/2*((2*Sqrt[a]*(a^4*A*b + 18*a^2*A*b^3 - 15*A*b^5 + 3*a^5*B + 6*a
^3*b^2*B + 35*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt
[b]*(a^2 + b^2)*d) - (16*b^2*(((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A
 + B) - b^3*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) +
ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((3*a^2*b*(A - B) - b
^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] +
Tan[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/(b*(a^2 + b^2)) + (a*(a^2
*A*b - 7*A*b^3 + 3*a^3*B + 11*a*b^2*B)*Sqrt[Tan[c + d*x]])/(b*(a^2 + b^2)*
d*(a + b*Tan[c + d*x])))/(4*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{2a\left(\frac{\frac{\left(A\,a^4b+10A\,a^2b^3+9A\,b^5-5B\,a^5-18B\,a^3b^2-13Ba\,b^4\right)\tan(dx+c)^{\frac{3}{2}}}{8b}-\frac{a\left(A\,a^4b-6A\,a^2b^3-7A\,b^5+3B\,a^5+14B\,a^3b^2+11Ba\,b^4\right)}{8b^2}}{(a+b\tan(dx+c))^2}\right.}{\left(a^2+b^2\right)^3}$
default	$\frac{2a\left(\frac{\frac{\left(A\,a^4b+10A\,a^2b^3+9A\,b^5-5B\,a^5-18B\,a^3b^2-13Ba\,b^4\right)\tan(dx+c)^{\frac{3}{2}}}{8b}-\frac{a\left(A\,a^4b-6A\,a^2b^3-7A\,b^5+3B\,a^5+14B\,a^3b^2+11Ba\,b^4\right)}{8b^2}}{(a+b\tan(dx+c))^2}\right.}{\left(a^2+b^2\right)^3}$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2*a/(a^2+b^2)^3*((1/8*(A*a^4*b+10*A*a^2*b^3+9*A*b^5-5*B*a^5-18*B*a^3*
b^2-13*B*a*b^4)/b*tan(d*x+c)^(3/2)-1/8*a*(A*a^4*b-6*A*a^2*b^3-7*A*b^5+3*B*
a^5+14*B*a^3*b^2+11*B*a*b^4)/b^2*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8*
(A*a^4*b+18*A*a^2*b^3-15*A*b^5+3*B*a^5+6*B*a^3*b^2+35*B*a*b^4)/b^2/(a*b)^(
1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^3*(1/8*(-3*A*a^2*
b+A*b^3+B*a^3-3*B*a*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+
1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^
(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-A*a^3+3*A*a*b^2-3*B*a^
2*b+B*b^3)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)
+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arcta
n(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3664 vs. $2(408) = 816$.

Time = 101.41 (sec) , antiderivative size = 7355, normalized size of antiderivative = 16.42

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.23

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(3\,Ba^6+Aa^5b+6\,Ba^4b^2+18\,Aa^3b^3+35\,Ba^2b^4-15\,Aab^5)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)\sqrt{ab}} - \frac{2\,\sqrt{2}((A-B)a^3+3\,(A+B)a^2b-3\,(A-B)ab^2-(A+B)b^3)\,\text{ar}}{}}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
1/4*((3*B*a^6 + A*a^5*b + 6*B*a^4*b^2 + 18*A*a^3*b^3 + 35*B*a^2*b^4 - 15*A
*a*b^5)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^6*b^2 + 3*a^4*b^4 + 3*a
^2*b^6 + b^8)*sqrt(a*b)) - (2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(
A - B)*a*b^2 - (A + B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x +
 c)))) + 2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A +
 B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(
(A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(sqrt(2)
*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((A + B)*a^3 - 3*(A - B)
*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) +
tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((5*B*a^4*b - A*a
^3*b^2 + 13*B*a^2*b^3 - 9*A*a*b^4)*tan(d*x + c)^(3/2) + (3*B*a^5 + A*a^4*b
 + 11*B*a^3*b^2 - 7*A*a^2*b^3)*sqrt(tan(d*x + c)))/(a^6*b^2 + 2*a^4*b^4 +
a^2*b^6 + (a^4*b^4 + 2*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^5*b^3 + 2*a^3*
b^5 + a*b^7)*tan(d*x + c)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 52.53 (sec) , antiderivative size = 26614, normalized size of antiderivative = 59.41

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
(log(((((((((((64*A*a*b^3*(11*a^2 - 13*b^2))/d + 128*b^3*tan(c + d*x)^(1/2)
*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*
b^2)^2)^(1/2) + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/
(d^4*(a^2 + b^2)^6))^(1/2))*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4
*b^2)^2)^(1/2) + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)
/(d^4*(a^2 + b^2)^6))^(1/2))/4 + (8*A^2*a*tan(c + d*x)^(1/2)*(a^10 - 184*b
^10 + 833*a^2*b^8 - 812*a^4*b^6 + 262*a^6*b^4 + 44*a^8*b^2))/(d^2*(a^2 + b
^2)^4))*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80*
A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6)
)^(1/2))/4 + (2*A^3*a^2*(5*a^10 - 1199*b^10 + 5017*a^2*b^8 - 5142*a^4*b^6
+ 1106*a^6*b^4 + 181*a^8*b^2))/(d^3*(a^2 + b^2)^6))*((4*(-A^4*d^4*(a^6 - b
^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5
*d^2 - 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2))/4 + (A^4*tan(c + d*x)
^(1/2)*(a^14 - 32*b^14 + 97*a^2*b^12 - 2082*a^4*b^10 + 3631*a^6*b^8 - 2300
*a^8*b^6 + 79*a^10*b^4 + 30*a^12*b^2))/(b*d^4*(a^2 + b^2)^8))*((4*(-A^4*d^
4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80*A^2*a^3*b^3*d^2 - 24
*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2))/4 + (A^5*a*
(a^10 - 120*b^10 + 249*a^2*b^8 - 388*a^4*b^6 + 302*a^6*b^4 + 36*a^8*b^2))/
(2*b*d^5*(a^2 + b^2)^8))*(((480*A^4*a^2*b^10*d^4 - 16*A^4*b^12*d^4 - 16*A^
4*a^12*d^4 - 4080*A^4*a^4*b^8*d^4 + 7232*A^4*a^6*b^6*d^4 - 4080*A^4*a^8...
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{\tan(dx+c)^2\,b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)**2*b**2 + 2*tan(c +
 d*x)*a*b + a**2),x)
```

3.412 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 448

$$\begin{aligned}\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = &-\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}\\
&+\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}\\
&+\frac{(3a^4Ab-26a^2Ab^3+3Ab^5+a^5B+18a^3b^2B-15ab^4B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}\left(a^2+b^2\right)^3 d}\\
&-\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d}\\
&+\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2b\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{(3a^2Ab-5Ab^3+a^3B+9ab^2B)\sqrt{\tan(c+dx)}}{4b\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))}\end{aligned}$$

output
```
1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctan(-1+2^(1/2)*ta
n(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d+1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+
B)+3*a*b^2*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d
+1/4*(3*A*a^4*b-26*A*a^2*b^3+3*A*b^5+B*a^5+18*B*a^3*b^2-15*B*a*b^4)*arctan
(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(1/2)/b^(3/2)/(a^2+b^2)^3/d-1/2*(a^3*
(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1
/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d+1/2*a*(A*b-B*a)*tan(d*x+c)^(1/2)
/b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/4*(3*A*a^2*b-5*A*b^3+B*a^3+9*B*a*b^2)*
tan(d*x+c)^(1/2)/b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.08 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.74

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{-4B\sqrt{\tan(c+dx)} + \frac{(3aAb+a^2B+4b^2B)\sqrt{\tan(c+dx)}}{a^2+b^2} - \frac{2(a+b\tan(c+dx))\left(-\frac{3}{4}a^{5/2}\sqrt{b}(a^2+b^2)(3a^2Ab-5Ab^3+a^3B+9ab^2B)\sqrt{\tan(c+dx)}\right.}{}}{}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3
,x]
```

output
```
(-4*B*Sqrt[Tan[c + d*x]] + ((3*a*A*b + a^2*B + 4*b^2*B)*Sqrt[Tan[c + d*x]]
)/(a^2 + b^2) - (2*(a + b*Tan[c + d*x])*((-3*a^(5/2)*Sqrt[b]*(a^2 + b^2)*(
3*a^2*A*b - 5*A*b^3 + a^3*B + 9*a*b^2*B)*Sqrt[Tan[c + d*x]])/4 + ((-3*a^2*
(3*a^4*A*b - 26*a^2*A*b^3 + 3*A*b^5 + a^5*B + 18*a^3*b^2*B - 15*a*b^4*B)*A
rcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/4 - 3*(-1)^(1/4)*a^(5/2)*b^(3
/2)*((a + I*b)^3*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (a - I*
b)^3*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))*(a + b*Tan[c + d*x
])))/(a^(5/2)*Sqrt[b]*(a^2 + b^2)^3))/(6*b*d*(a + b*Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.03, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.697, Rules used = {3042, 4088, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{\int -\frac{-\left(\left(Ba^2+3Aba+4b^2B\right)\tan^2(c+dx)\right)-4b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{-\left(\left(Ba^2+3Aba+4b^2B\right)\tan^2(c+dx)\right)-4b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\int \frac{-\left(\left(Ba^2+3Aba+4b^2B\right)\tan(c+dx)^2\right)-4b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4132}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{\int \frac{-a(Ba^3+3Aba^2+9b^2Ba-5Ab^3)\tan^2(c+dx)-8ab(-Ba^2+2Aba+b^2B)\tan(c+dx)+a(-Ba^3+5Aba^2+7b^2Ba-3Ab^3)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a(a^2+b^2)}-\frac{(a^3B+3a^2Ab+9ab^2B-5Ab^3}{d(a^2+b^2)(a+b\tan(c-}}{4b(a^2+b^2)}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{\int \frac{-a(Ba^3+3Aba^2+9b^2Ba-5Ab^3)\tan^2(c+dx)-8ab(-Ba^2+2Aba+b^2B)\tan(c+dx)+a(-Ba^3+5Aba^2+7b^2Ba-3Ab^3)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}-\frac{(a^3B+3a^2Ab+9ab^2B-5Ab^3}{d(a^2+b^2)(a+b\tan(c-}}{4b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{\int \frac{-a(Ba^3+3Aba^2+9b^2Ba-5Ab^3)\tan(c+dx)^2-8ab(-Ba^2+2Aba+b^2B)\tan(c+dx)+a(-Ba^3+5Aba^2+7b^2Ba-3Ab^3)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}-\frac{(a^3B+3a^2Ab+9ab^2B-5Ab^3}{d(a^2+b^2)(a+b\tan(c-}}{4b(a^2+b^2)}$$

$\downarrow$ 4136

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{\int \frac{8(ab(Aa^3+3bBa^2-3Ab^2a-b^3B)-ab(-Ba^3+3Aba^2+3b^2Ba-Ab^3)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{a(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5)\int\frac{\tan^2(c+}{\sqrt{\tan(c+dx)}(a-}}{a^2+b^2}}{2a(a^2+b^2)}}{4b(a^2+b^2)}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{8\int \frac{ab(Aa^3+3bBa^2-3Ab^2a-b^3B)-ab(-Ba^3+3Aba^2+3b^2Ba-Ab^3)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{a(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5)\int\frac{\tan^2(c+dx}{\sqrt{\tan(c+dx)}(a+b}}{a^2+b^2}}{2a(a^2+b^2)}}{4b(a^2+b^2)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{8\int \frac{ab(Aa^3+3bBa^2-3Ab^2a-b^3B)-ab(-Ba^3+3Aba^2+3b^2Ba-Ab^3)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{a(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5)\int\frac{\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b}}{a^2+b^2}}{2a(a^2+b^2)}}{4b(a^2+b^2)}$$

$\downarrow$ 4017

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{16\int\frac{ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B-\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int\frac{}{\sqrt{\tan(c}}}{a^2+b^2}}{2a(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{16ab\int\frac{Aa^3+3bBa^2-3Ab^2a-b^3B-\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int\frac{\tan}{\sqrt{\tan(c+d}}}{a^2+b^2}}{2a(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 1482

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int\frac{\tan(c+dx}{\tan^2(c+dx}}{d\left(a^2+b^2\right)}}{2a(a^2+b^2)}}{4b\left(a\right.}$$

$\downarrow$ 1476

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{1}{2}\int\frac{}{\tan(c+}\right.}{d\left(a^2+b^2\right)}}{}$$

$\downarrow$ 1082

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\int\frac{1}{-\tan(c}\right.\right.}{d\left(a^2+b^2\right)}}{2a}$$

$\downarrow$ 217

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.}{d\left(a^2+b^2\right)}}{2a(a^2+b^2)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\right.}{d\left(a^2+b^2\right)}}{}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)\right.}{d\left(a^2+b^2\right)}}{}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\Big\downarrow \text{4117}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a\right.}{d\left(a^2+b^2\right)}}{}$$

$$\Big\downarrow \text{73}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a\right.}{d\left(a^2+b^2\right)}}{}$$

$$\Big\downarrow \text{218}$$

$$\frac{a(Ab-aB)\sqrt{\tan(c+dx)}}{2bd\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a\right.}{d\left(a^2+b^2\right)}}{}$$

input

```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output

```
(a*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(2*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) - (((-2*Sqrt[a]*(3*a^4*A*b - 26*a^2*A*b^3 + 3*A*b^5 + a^5*B + 18*a^3*b
^2*B - 15*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[b]*
(a^2 + b^2)*d) + (16*a*b*(-1/2*((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A +
B) + 3*a*b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) +
 ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((a^3*(A - B) - 3*a*b^
2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^2)) - ((3*a^
2*A*b - 5*A*b^3 + a^3*B + 9*a*b^2*B)*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*d*(a
 + b*Tan[c + d*x])))/(4*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{2\left(\left(\frac{3}{8}Aa^4b-\frac{1}{4}Aa^2b^3-\frac{5}{8}Ab^5+\frac{1}{8}Ba^5+\frac{5}{4}Ba^3b^2+\frac{9}{8}Bab^4\right)\tan(dx+c)^{\frac{3}{2}}+\frac{a\left(5Aa^4b+2Aa^2b^3-3Ab^5-Ba^5+6Ba^3b^2+7Bab^4\right)\sqrt{}}{8b}}{(a+b\tan(dx+c))^2}}{\left(a^2+b^2\right)^3}$
default	$\frac{\frac{2\left(\left(\frac{3}{8}Aa^4b-\frac{1}{4}Aa^2b^3-\frac{5}{8}Ab^5+\frac{1}{8}Ba^5+\frac{5}{4}Ba^3b^2+\frac{9}{8}Bab^4\right)\tan(dx+c)^{\frac{3}{2}}+\frac{a\left(5Aa^4b+2Aa^2b^3-3Ab^5-Ba^5+6Ba^3b^2+7Bab^4\right)\sqrt{}}{8b}}{(a+b\tan(dx+c))^2}}{\left(a^2+b^2\right)^3}$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2/(a^2+b^2)^3*(((3/8*A*a^4*b-1/4*A*a^2*b^3-5/8*A*b^5+1/8*B*a^5+5/4*B*
a^3*b^2+9/8*B*a*b^4)*tan(d*x+c)^(3/2)+1/8*a*(5*A*a^4*b+2*A*a^2*b^3-3*A*b^5
-B*a^5+6*B*a^3*b^2+7*B*a*b^4)/b*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8*(
3*A*a^4*b-26*A*a^2*b^3+3*A*b^5+B*a^5+18*B*a^3*b^2-15*B*a*b^4)/b/(a*b)^(1/2
)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^3*(1/8*(-A*a^3+3*A*a
*b^2-3*B*a^2*b+B*b^3)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/
(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/
2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(3*A*a^2*b-A*b^3-B*a^3+3*B*
a*b^2)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(
1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1
+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3687 vs. $2(406) = 812$.

Time = 68.19 (sec) , antiderivative size = 7400, normalized size of antiderivative = 16.52

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.20

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(Ba^5+3Aa^4b+18Ba^3b^2-26Aa^2b^3-15Bab^4+3Ab^5)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^6b+3a^4b^3+3a^2b^5+b^7)\sqrt{ab}} - \frac{2\sqrt{2}((A+B)a^3-3(A-B)a^2b-3(A+B)ab^2+(A-B)b^3)\operatorname{arct}}{}}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
1/4*((B*a^5 + 3*A*a^4*b + 18*B*a^3*b^2 - 26*A*a^2*b^3 - 15*B*a*b^4 + 3*A*b
^5)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5
 + b^7)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)
*a*b^2 + (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
 + 2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^
3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*((A - B
)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(sqrt(2)*sqrt(
tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b
 - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*
x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((B*a^3*b + 3*A*a^2*b^2
 + 9*B*a*b^3 - 5*A*b^4)*tan(d*x + c)^(3/2) - (B*a^4 - 5*A*a^3*b - 7*B*a^2*
b^2 + 3*A*a*b^3)*sqrt(tan(d*x + c)))/(a^6*b + 2*a^4*b^3 + a^2*b^5 + (a^4*b
^3 + 2*a^2*b^5 + b^7)*tan(d*x + c)^2 + 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*tan
(d*x + c)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 52.85 (sec) , antiderivative size = 25944, normalized size of antiderivative = 57.91

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
(log(((((((((((64*B*a*b^3*(11*a^2 - 13*b^2))/d + 128*b^3*tan(c + d*x)^(1/2)
*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*
b^2)^2)^(1/2) + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/
(d^4*(a^2 + b^2)^6))^(1/2))*((4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4
*b^2)^2)^(1/2) + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)
/(d^4*(a^2 + b^2)^6))^(1/2))/4 + (8*B^2*a*tan(c + d*x)^(1/2)*(a^10 - 184*b
^10 + 833*a^2*b^8 - 812*a^4*b^6 + 262*a^6*b^4 + 44*a^8*b^2))/(d^2*(a^2 + b
^2)^4))*((4*(-B^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80*
B^2*a^3*b^3*d^2 - 24*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6)
)^(1/2))/4 + (2*B^3*a^2*(5*a^10 - 1199*b^10 + 5017*a^2*b^8 - 5142*a^4*b^6
+ 1106*a^6*b^4 + 181*a^8*b^2))/(d^3*(a^2 + b^2)^6))*((4*(-B^4*d^4*(a^6 - b
^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80*B^2*a^3*b^3*d^2 - 24*B^2*a*b^5
*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2))/4 + (B^4*tan(c + d*x)
^(1/2)*(a^14 - 32*b^14 + 97*a^2*b^12 - 2082*a^4*b^10 + 3631*a^6*b^8 - 2300
*a^8*b^6 + 79*a^10*b^4 + 30*a^12*b^2))/(b*d^4*(a^2 + b^2)^8))*((4*(-B^4*d^
4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80*B^2*a^3*b^3*d^2 - 24
*B^2*a*b^5*d^2 - 24*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2))/4 + (B^5*a*
(a^10 - 120*b^10 + 249*a^2*b^8 - 388*a^4*b^6 + 302*a^6*b^4 + 36*a^8*b^2))/
(2*b*d^5*(a^2 + b^2)^8))*(((480*B^4*a^2*b^10*d^4 - 16*B^4*b^12*d^4 - 16*B^
4*a^12*d^4 - 4080*B^4*a^4*b^8*d^4 + 7232*B^4*a^6*b^6*d^4 - 4080*B^4*a^8...
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)}{\tan(dx+c)^2\,b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)**2*b**2 + 2*tan(c + d*
x)*a*b + a**2),x)
```

3.413 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 444

$$
\begin{aligned}
\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = & -\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
& +\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
& -\frac{(15a^4Ab-18a^2Ab^3-Ab^5-3a^5B+26a^3b^2B-3ab^4B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\,(a^2+b^2)^3\,d} \\
& +\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
& -\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}-\frac{(7a^2Ab-Ab^3-3a^3B+5ab^2B)\sqrt{\tan(c+dx)}}{4a\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}
\end{aligned}
$$

output
```
1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(-1+2^(1/2)*ta
n(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d+1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b
*(A+B)-b^3*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d
-1/4*(15*A*a^4*b-18*A*a^2*b^3-A*b^5-3*B*a^5+26*B*a^3*b^2-3*B*a*b^4)*arctan
(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)/(a^2+b^2)^3/d+1/2*(3*a^
2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(1
/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d-1/2*(A*b-B*a)*tan(d*x+c)^(1/2)/(
a^2+b^2)/d/(a+b*tan(d*x+c))^2-1/4*(7*A*a^2*b-A*b^3-3*B*a^3+5*B*a*b^2)*tan(
d*x+c)^(1/2)/a/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.30 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\tan^{\frac{3}{2}}(c+dx)-(Ab-aB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))+\frac{2(a+b\tan(c+dx))\left(\frac{1}{4}a^{3/2}b^{3/2}(a^2+b^2)\right.}{}}{}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3
,x]
```

output
```
(b*(A*b - a*B)*Tan[c + d*x]^(3/2) - (A*b - a*B)*Sqrt[Tan[c + d*x]]*(a + b*
Tan[c + d*x]) + (2*(a + b*Tan[c + d*x])*((a^(3/2)*b^(3/2)*(a^2 + b^2)*(-7*
a^2*A*b + A*b^3 + 3*a^3*B - 5*a*b^2*B)*Sqrt[Tan[c + d*x]])/4 - (-1/4*(a*b*
(-15*a^4*A*b + 18*a^2*A*b^3 + A*b^5 + 3*a^5*B - 26*a^3*b^2*B + 3*a*b^4*B)*
ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]) + (-1)^(1/4)*a^(5/2)*b^(3/2)
*((I*a - b)^3*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - (I*a + b)^
3*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))*(a + b*Tan[c + d*x]))
)/(a^(3/2)*b^(3/2)*(a^2 + b^2)^2))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2)
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.05, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.697, Rules used = {3042, 4091, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \; 4091$$

$$-\frac{\int -\frac{-3b(Ab-aB)\tan^2(c+dx)+4b(aA+bB)\tan(c+dx)+b(Ab-aB)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx}{2b\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-3b(Ab-aB)\tan^2(c+dx)+4b(aA+bB)\tan(c+dx)+b(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx}{4b\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{-3b(Ab-aB)\tan(c+dx)^2+4b(aA+bB)\tan(c+dx)+b(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx}{4b\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 4132$$

$$\frac{\frac{\int \frac{-b\left(-3Ba^3+7Aba^2+5b^2Ba-Ab^3\right)\tan^2(c+dx)+8ab\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)+b\left(-5Ba^3+9Aba^2+3b^2Ba+Ab^3\right)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a(a^2+b^2)}-\frac{b(-3a^3B+7a^2Ab+5ab^2B-A}{ad(a^2+b^2)(a+b\tan(}}{4b\left(a^2+b^2\right)}$$
$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{-b\left(-3Ba^3+7Aba^2+5b^2Ba-Ab^3\right)\tan^2(c+dx)+8ab\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)+b\left(-5Ba^3+9Aba^2+3b^2Ba+Ab^3\right)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}-\frac{b(-3a^3B+7a^2Ab+5ab^2B-A\cdots}{ad(a^2+b^2)(a+b\tan(\cdots}}{4b\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{-b\left(-3Ba^3+7Aba^2+5b^2Ba-Ab^3\right)\tan(c+dx)^2+8ab\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)+b\left(-5Ba^3+9Aba^2+3b^2Ba+Ab^3\right)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}-\frac{b(-3a^3B+7a^2Ab+5ab^2B-A\cdots}{ad(a^2+b^2)(a+b\tan(\cdots}}{4b\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4136

$$\frac{\frac{\frac{\int \frac{8\left(ab\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)+ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{\tan^2(c\cdots}{\sqrt{\tan(c+dx)}(\cdots}}{a^2+b^2}}{2a(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{8\int \frac{ab\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)+ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{\tan^2(c+\cdots}{\sqrt{\tan(c+dx)}(a+\cdots}}{a^2+b^2}}{2a(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{8\int \frac{ab\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)+ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{\tan(c+d\cdots}{\sqrt{\tan(c+dx)}(a+\cdots}}{a^2+b^2}}{2a(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4017

$$\frac{\frac{\frac{16\int \frac{ab\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{}{\sqrt{\tan(}}}{a^2+b^2}}{2a(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{16ab\int \frac{-Ba^3+3Aba^2+3b^2Ba-Ab^3+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}-\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{\tan}{\sqrt{\tan(c+}}}{a^2+b^2}}{2a(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 1482

$$\frac{\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int \frac{\tan(c+dx}{\tan^2(c+d}}{d\left(a^2+b^2\right)}}{2a(a^2+b^2)}$$

$4b$ (

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 1476

$$\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{1}{2}\int \frac{}{\tan(c+}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 1082

$$\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\int \frac{1}{-\tan(c}\right.\right.}{d\left(a^2+b^2\right)}$$

2

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{d\left(a^2+b^2\right)}}{2a(a^2+b^2)}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(\ldots}}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+d\ldots}\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$$\Big\downarrow \; 1103$$

$$16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(\ldots\right.\right.$$

$$d\left(a^2+b^2\right)$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$\big\downarrow$ 4117

$$16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(\ldots\right.\right.$$

$$d\left(a^2+b^2\right)$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$\big\downarrow$ 73

$$16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(\ldots\right.\right.$$

$$d\left(a^2+b^2\right)$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$\big\downarrow$ 218

$$16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(\ldots\right.\right.$$

$$d\left(a^2+b^2\right)$$

$$\frac{(Ab-aB)\sqrt{\tan(c+dx)}}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*((A*b - a*B)*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*d*(a + b*Tan[c + d*x])^
2) + (((-2*Sqrt[b]*(15*a^4*A*b - 18*a^2*A*b^3 - A*b^5 - 3*a^5*B + 26*a^3*b
^2*B - 3*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(
a^2 + b^2)*d) + (16*a*b*(((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B)
 - b^3*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((3*a^2*b*(A - B) - b^3*(A
 - B) - a^3*(A + B) + 3*a*b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
 + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^2)) - (b*(7*a^2*
A*b - A*b^3 - 3*a^3*B + 5*a*b^2*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d*(a
 + b*Tan[c + d*x])))/(4*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{2\left(\frac{\frac{b\left(7Aa^4b+6Aa^2b^3-Ab^5-3Ba^5+2Ba^3b^2+5Bab^4\right)\tan(dx+c)^{\frac{3}{2}}}{8a}+\left(\frac{9}{8}Aa^4b+\frac{5}{4}Aa^2b^3-\frac{5}{8}Ba^5+\frac{3}{8}Bab^4+\frac{1}{8}Ab^5-\frac{1}{4}Ba^3b^2\right)}{(a+b\tan(dx+c))^2}\right.}{\left(a^2+b^2\right)^3}$
default	$-\frac{2\left(\frac{\frac{b\left(7Aa^4b+6Aa^2b^3-Ab^5-3Ba^5+2Ba^3b^2+5Bab^4\right)\tan(dx+c)^{\frac{3}{2}}}{8a}+\left(\frac{9}{8}Aa^4b+\frac{5}{4}Aa^2b^3-\frac{5}{8}Ba^5+\frac{3}{8}Bab^4+\frac{1}{8}Ab^5-\frac{1}{4}Ba^3b^2\right)}{(a+b\tan(dx+c))^2}\right.}{\left(a^2+b^2\right)^3}$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(-2/(a^2+b^2)^3*((1/8*b*(7*A*a^4*b+6*A*a^2*b^3-A*b^5-3*B*a^5+2*B*a^3*b
^2+5*B*a*b^4)/a*tan(d*x+c)^(3/2)+(9/8*A*a^4*b+5/4*A*a^2*b^3-5/8*B*a^5+3/8*
B*a*b^4+1/8*A*b^5-1/4*B*a^3*b^2)*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8*
(15*A*a^4*b-18*A*a^2*b^3-A*b^5-3*B*a^5+26*B*a^3*b^2-3*B*a*b^4)/a/(a*b)^(1/
2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^3*(1/8*(3*A*a^2*b-A
*b^3-B*a^3+3*B*a*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/
(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/
2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(A*a^3-3*A*a*b^2+3*B*a^2*b-
B*b^3)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(
1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1
+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3684 vs. 2(403) = 806.

Time = 73.75 (sec) , antiderivative size = 7394, normalized size of antiderivative = 16.65

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(3\,Ba^5-15\,Aa^4b-26\,Ba^3b^2+18\,Aa^2b^3+3\,Bab^4+Ab^5)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\sqrt{ab}} + \frac{2\sqrt{2}((A-B)a^3+3\,(A+B)a^2b-3\,(A-B)ab^2-(A+B)b^3)\,\mathrm{arct}}{}}{}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
1/4*((3*B*a^5 - 15*A*a^4*b - 26*B*a^3*b^2 + 18*A*a^2*b^3 + 3*B*a*b^4 + A*b
^5)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
 a*b^6)*sqrt(a*b)) + (2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)
*a*b^2 - (A + B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
 + 2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^
3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*((A + B
)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(sqrt(2)*sqrt(
tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b
 - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*
x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((3*B*a^3*b - 7*A*a^2*b
^2 - 5*B*a*b^3 + A*b^4)*tan(d*x + c)^(3/2) + (5*B*a^4 - 9*A*a^3*b - 3*B*a^
2*b^2 - A*a*b^3)*sqrt(tan(d*x + c)))/(a^7 + 2*a^5*b^2 + a^3*b^4 + (a^5*b^2
 + 2*a^3*b^4 + a*b^6)*tan(d*x + c)^2 + 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)*tan
(d*x + c)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 51.36 (sec) , antiderivative size = 26133, normalized size of antiderivative = 58.86

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
(log(((((((((64*A*b^3*(b^4 - 10*a^4 + 15*a^2*b^2))/(a*d) + 128*b^3*tan(c
+ d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b
^4 - 15*a^4*b^2)^2)^(1/2) + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2
*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2))*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*
b^4 - 15*a^4*b^2)^2)^(1/2) + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^
2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6))^(1/2))/4 + (8*A^2*b^2*tan(c + d*x)^(1/2)
*(8*a^10 + b^10 - 148*a^2*b^8 + 902*a^4*b^6 - 812*a^6*b^4 + 193*a^8*b^2))/
(a*d^2*(a^2 + b^2)^4))*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)
^2)^(1/2) + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(d^4
*(a^2 + b^2)^6))^(1/2))/4 - (2*A^3*b^2*(16*a^12 + b^12 - 71*a^2*b^10 - 138
2*a^4*b^8 + 5266*a^6*b^6 - 4539*a^8*b^4 + 1189*a^10*b^2))/(a^2*d^3*(a^2 +
b^2)^6))*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 80
*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^6
))^(1/2))/4 - (A^4*b^3*tan(c + d*x)^(1/2)*(2*a^2*b^10 - b^12 - 225*a^12 +
49*a^4*b^8 + 2460*a^6*b^6 - 3631*a^8*b^4 + 1922*a^10*b^2))/(a^2*d^4*(a^2 +
 b^2)^8))*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/2) + 8
0*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^
6))^(1/2))/4 + (A^5*b^3*(7*b^8 - 225*a^8 + 116*a^2*b^6 - 270*a^4*b^4 + 420
*a^6*b^2))/(2*a*d^5*(a^2 + b^2)^8))*(((480*A^4*a^2*b^10*d^4 - 16*A^4*b^12*
d^4 - 16*A^4*a^12*d^4 - 4080*A^4*a^4*b^8*d^4 + 7232*A^4*a^6*b^6*d^4 - 4...
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2 b^2 + 2\tan(dx+c)\,ab + a^2}dx$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
int(sqrt(tan(c + d*x))/(tan(c + d*x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2),
x)
```

3.414 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 448

$$
\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3}\,dx \\
&= \frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&\quad - \frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&\quad + \frac{\sqrt{b}(35a^4Ab+6a^2Ab^3+3Ab^5-15a^5B+18a^3b^2B+ab^4B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d} \\
&\quad + \frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&\quad + \frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2a\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} \\
&\quad + \frac{b(11a^2Ab+3Ab^3-7a^3B+ab^2B)\,\sqrt{\tan(c+dx)}}{4a^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}
\end{aligned}
$$

output
```
-1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d-1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A
+B)+3*a*b^2*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/
d+1/4*b^(1/2)*(35*A*a^4*b+6*A*a^2*b^3+3*A*b^5-15*B*a^5+18*B*a^3*b^2+B*a*b^
4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(5/2)/(a^2+b^2)^3/d+1/2*(a^3
*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctanh(2^(1/2)*tan(d*x+c)^(
1/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d+1/2*b*(A*b-B*a)*tan(d*x+c)^(1/2
)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/4*b*(11*A*a^2*b+3*A*b^3-7*B*a^3+B*a*b
^2)*tan(d*x+c)^(1/2)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.13 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.64

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2\left(\frac{1}{2}\sqrt{b}(35a^4Ab+6a^2Ab^3+3Ab^5-15a^5B+18a^3b^2B+ab^4B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)-2\sqrt[4]{-1}a^{5/2}\left((a+ib)^3(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+\right.\right.\right.}{a^{3/2}(a^2+b^2)^2}}{4a\,(a^2+}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^3)
,x]
```

output
```
((2*((Sqrt[b]*(35*a^4*A*b + 6*a^2*A*b^3 + 3*A*b^5 - 15*a^5*B + 18*a^3*b^2*
B + a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/2 - 2*(-1)^(1/4
)*a^(5/2)*((a + I*b)^3*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + (
a - I*b)^3*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])))/(a^(3/2)*(a
^2 + b^2)^2) + (2*b*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(a + b*Tan[c + d*x])^2
 + (b*(11*a^2*A*b + 3*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Tan[c + d*x]])/(a*(a
^2 + b^2)*(a + b*Tan[c + d*x]))))/(4*a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.697$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{\int \frac{4Aa^2+bBa-4(Ab-aB)\tan(c+dx)a+3Ab^2+3b(Ab-aB)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4Aa^2+bBa-4(Ab-aB)\tan(c+dx)a+3Ab^2+3b(Ab-aB)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx}{4a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4Aa^2+bBa-4(Ab-aB)\tan(c+dx)a+3Ab^2+3b(Ab-aB)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}dx}{4a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{4132}$$

$$\frac{\frac{\int \frac{8Aa^4+9bBa^3+3Ab^2a^2-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2+b^3Ba+3Ab^4+b\left(-7Ba^3+11Aba^2+b^2Ba+3Ab^3\right)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a(a^2+b^2)}+\frac{b(-7a^3B+11a^2Ab+ab^2\ldots}{ad(a^2+b^2)(a+\ldots}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{8Aa^4+9bBa^3+3Ab^2a^2-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2+b^3Ba+3Ab^4+b\left(-7Ba^3+11Aba^2+b^2Ba+3Ab^3\right)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}+\frac{b(-7a^3B+11a^2Ab+ab^2\ldots}{ad(a^2+b^2)(a+\ldots}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{8Aa^4+9bBa^3+3Ab^2a^2-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2+b^3Ba+3Ab^4+b\left(-7Ba^3+11Aba^2+b^2Ba+3Ab^3\right)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}+\frac{b(-7a^3B+11a^2Ab+ab^2\ldots}{ad(a^2+b^2)(a+\ldots}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4136

$$\frac{\frac{\frac{\int \frac{8\left(a^2\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-a^2\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int \frac{\tan^2(\ldots}{\sqrt{\tan(c+dx)}(\ldots}}{a^2+b^2}}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{8\int \frac{a^2\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-a^2\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int \frac{\tan^2(c+\ldots}{\sqrt{\tan(c+dx)}(a\ldots}}{a^2+b^2}}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{8\int \frac{a^2\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-a^2\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int \frac{\tan(c+d}{\sqrt{\tan(c+dx)}(a-}}{a^2+b^2}}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 4017

$$\frac{\frac{16\int \frac{a^2\left(Aa^3+3bBa^2-3Ab^2a-b^3B-\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int \frac{}{\sqrt{\tan}}}{a^2+b^2}}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{16a^2\int \frac{Aa^3+3bBa^2-3Ab^2a-b^3B-\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int \frac{t}{\sqrt{\tan(c-}}}{a^2+b^2}}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 1482

$$\frac{\frac{16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int \frac{\tan(c+da}{\tan^2(c+d}}{d\left(a^2+b^2\right)}}{2a(a^2+b^2)}$$

$$4a$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 1476

$$\frac{16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{1}{2}\int \frac{}{\tan(c+}\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 1082

$$16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int}{-\tan(c\ldots}\right.\right.$$

$$d\left(a^2+b^2\right) \qquad 2\ldots$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 217

$$16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{\ldots}\right.}{}\right.\right.$$

$$d\left(a^2+b^2\right) \qquad 2a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 1479

$$16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+d\ldots}}{2\sqrt{2}}\right.\right.$$

$$d\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 25

$$16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)-\ldots\right.$$

$$d\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 27

$$\frac{16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\log\left(\tan\ldots\right.}{}\right.\right.}{}}{}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{4117}$$

$$\frac{\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d\left(a^2+b^2\right)}+\frac{16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\right.}{}}{}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\log\left(\tan(\ldots\right.}{}\right.\right.}{}}{}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{b(Ab-aB)\sqrt{\tan(c+dx)}}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+$$

$$\frac{\frac{b\left(-7a^3B+11a^2Ab+ab^2B+3Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\ldots\right.}{}\right.\right.}{}}{}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^3),x]
```

output
```
(b*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) + (((2*Sqrt[b]*(35*a^4*A*b + 6*a^2*A*b^3 + 3*A*b^5 - 15*a^5*B + 18*a^3
*b^2*B + a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(
a^2 + b^2)*d) + (16*a^2*(-1/2*((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B
) + 3*a*b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) +
ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])) + ((a^3*(A - B) - 3*a*b^2
*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c
 + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^2)) + (b*(11*
a^2*A*b + 3*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*
d*(a + b*Tan[c + d*x])))/(4*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{2b\left(\frac{\frac{b(11A\,a^4b+14A\,a^2b^3+3A\,b^5-7B\,a^5-6B\,a^3b^2+B\,a\,b^4)\tan(dx+c)^{\frac{3}{2}}}{8a^2}+\frac{(13A\,a^4b+18A\,a^2b^3+5A\,b^5-9B\,a^5-10B\,a^3b^2-B\,a\,b^4)}{8a}}{(a+b\tan(dx+c))^2}\right.}{(a^2+b^2)^3}$
default	$\frac{2b\left(\frac{\frac{b(11A\,a^4b+14A\,a^2b^3+3A\,b^5-7B\,a^5-6B\,a^3b^2+B\,a\,b^4)\tan(dx+c)^{\frac{3}{2}}}{8a^2}+\frac{(13A\,a^4b+18A\,a^2b^3+5A\,b^5-9B\,a^5-10B\,a^3b^2-B\,a\,b^4)}{8a}}{(a+b\tan(dx+c))^2}\right.}{(a^2+b^2)^3}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2*b/(a^2+b^2)^3*((1/8*b*(11*A*a^4*b+14*A*a^2*b^3+3*A*b^5-7*B*a^5-6*B*
a^3*b^2+B*a*b^4)/a^2*tan(d*x+c)^(3/2)+1/8*(13*A*a^4*b+18*A*a^2*b^3+5*A*b^5
-9*B*a^5-10*B*a^3*b^2-B*a*b^4)/a*tan(d*x+c)^(1/2))/(a+b*tan(d*x+c))^2+1/8*
(35*A*a^4*b+6*A*a^2*b^3+3*A*b^5-15*B*a^5+18*B*a^3*b^2+B*a*b^4)/a^2/(a*b)^(
1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)^3*(1/8*(A*a^3-3*A
*a*b^2+3*B*a^2*b-B*b^3)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1
)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(
1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-3*A*a^2*b+A*b^3+B*a^3-3
*B*a*b^2)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+
2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan
(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3664 vs. $2(409) = 818$.

Time = 110.87 (sec) , antiderivative size = 7359, normalized size of antiderivative = 16.43

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}\,(a + b\tan(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.23

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}\,(a + b\tan(c + dx))^3}\,dx =$$

$$-\frac{\frac{(15\,Ba^5b - 35\,Aa^4b^2 - 18\,Ba^3b^3 - 6\,Aa^2b^4 - Bab^5 - 3\,Ab^6)\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^8 + 3\,a^6b^2 + 3\,a^4b^4 + a^2b^6)\sqrt{ab}} - \frac{2\sqrt{2}((A+B)a^3 - 3\,(A-B)a^2b - 3\,(A+B)ab^2 + (A-B)b^3}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
-1/4*((15*B*a^5*b - 35*A*a^4*b^2 - 18*B*a^3*b^3 - 6*A*a^2*b^4 - B*a*b^5 -
3*A*b^6)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^8 + 3*a^6*b^2 + 3*a^4*
b^4 + a^2*b^6)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*
(A + B)*a*b^2 + (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x
+ c)))) + 2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A
- B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*
((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(sqrt(2
)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*((A - B)*a^3 + 3*(A + B
)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) +
 tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((7*B*a^3*b^2 -
11*A*a^2*b^3 - B*a*b^4 - 3*A*b^5)*tan(d*x + c)^(3/2) + (9*B*a^4*b - 13*A*a
^3*b^2 + B*a^2*b^3 - 5*A*a*b^4)*sqrt(tan(d*x + c)))/(a^8 + 2*a^6*b^2 + a^4
*b^4 + (a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*tan(d*x + c)^2 + 2*(a^7*b + 2*a^5*b
^3 + a^3*b^5)*tan(d*x + c)))/d
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3}\,dx = \int \frac{B\tan(dx+c)+A}{(b\tan(dx+c)+a)^3\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 53.29 (sec) , antiderivative size = 26707, normalized size of antiderivative = 59.61

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^3),x)
```

output
```
((A*tan(c + d*x)^(1/2)*(5*b^4 + 13*a^2*b^2))/(4*a*(a^4 + b^4 + 2*a^2*b^2))
 + (A*b*tan(c + d*x)^(3/2)*(3*b^4 + 11*a^2*b^2))/(4*a^2*(a^4 + b^4 + 2*a^2
*b^2)))/(a^2*d + b^2*d*tan(c + d*x)^2 + 2*a*b*d*tan(c + d*x)) - ((tan(c +
d*x)^(1/2)*(B*b^3 + 9*B*a^2*b))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c + d*x
)^(3/2)*(B*b^4 - 7*B*a^2*b^2))/(4*a*(a^4 + b^4 + 2*a^2*b^2)))/(a^2*d + b^2
*d*tan(c + d*x)^2 + 2*a*b*d*tan(c + d*x)) + (log(((((((((((64*A*b^2*(3*b^6
- 2*a^6 + 3*a^2*b^4 + 22*a^4*b^2))/(a^2*d) + 128*b^3*tan(c + d*x)^(1/2)*(a
^2 - b^2)*(a^2 + b^2)^2*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2
)^2)^(1/2) - 80*A^2*a^3*b^3*d^2 + 24*A^2*a*b^5*d^2 + 24*A^2*a^5*b*d^2)/(d^
4*(a^2 + b^2)^6))^(1/2))*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^
2)^2)^(1/2) - 80*A^2*a^3*b^3*d^2 + 24*A^2*a*b^5*d^2 + 24*A^2*a^5*b*d^2)/(d
^4*(a^2 + b^2)^6))^(1/2))/4 + (8*A^2*b^2*tan(c + d*x)^(1/2)*(9*b^12 - 8*a^
12 + 36*a^2*b^10 + 430*a^4*b^8 - 188*a^6*b^6 + 1497*a^8*b^4 + 32*a^10*b^2)
)/(a^3*d^2*(a^2 + b^2)^4))*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*
b^2)^2)^(1/2) - 80*A^2*a^3*b^3*d^2 + 24*A^2*a*b^5*d^2 + 24*A^2*a^5*b*d^2)/
(d^4*(a^2 + b^2)^6))^(1/2))/4 - (2*A^3*b^3*(45*b^12 - 16*a^12 + 333*a^2*b^
10 + 146*a^4*b^8 + 1178*a^6*b^6 - 9791*a^8*b^4 + 1161*a^10*b^2))/(a^3*d^3*
(a^2 + b^2)^6))*((4*(-A^4*d^4*(a^6 - b^6 + 15*a^2*b^4 - 15*a^4*b^2)^2)^(1/
2) - 80*A^2*a^3*b^3*d^2 + 24*A^2*a*b^5*d^2 + 24*A^2*a^5*b*d^2)/(d^4*(a^2 +
 b^2)^6))^(1/2))/4 + (A^4*b^5*tan(c + d*x)^(1/2)*(18*a^2*b^10 - 9*b^12 ...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^3}\,dx$$
$$=\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3 b^2+2\tan(dx+c)^2 ab+\tan(dx+c)\,a^2}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x)
```

output
```
int(sqrt(tan(c + d*x))/(tan(c + d*x)**3*b**2 + 2*tan(c + d*x)**2*a*b + tan
(c + d*x)*a**2),x)
```

3.415 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 515

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx \\
&= \frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} \\
&\quad -\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} \\
&\quad -\frac{b^{3/2}(63a^4Ab+46a^2Ab^3+15Ab^5-35a^5B-6a^3b^2B-3ab^4B)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{4a^{7/2}\left(a^2+b^2\right)^3 d} \\
&\quad -\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)^3 d} \\
&\quad -\frac{8a^4A+31a^2Ab^2+15Ab^4-11a^3bB-3ab^3B}{4a^3\left(a^2+b^2\right)^2 d\sqrt{\tan(c+dx)}} \\
&\quad +\frac{b(Ab-aB)}{2a\left(a^2+b^2\right) d\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2} \\
&\quad +\frac{b(13a^2Ab+5Ab^3-9a^3B-ab^2B)}{4a^2\left(a^2+b^2\right)^2 d\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}
\end{aligned}$$

output
```
-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*
b*(A+B)-b^3*(A+B))*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/
d-1/4*b^(3/2)*(63*A*a^4*b+46*A*a^2*b^3+15*A*b^5-35*B*a^5-6*B*a^3*b^2-3*B*a
*b^4)*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(7/2)/(a^2+b^2)^3/d-1/2*(
3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctanh(2^(1/2)*tan(d*x+c
)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d-1/4*(8*A*a^4+31*A*a^2*b^2+15
*A*b^4-11*B*a^3*b-3*B*a*b^3)/a^3/(a^2+b^2)^2/d/tan(d*x+c)^(1/2)+1/2*b*(A*b
-B*a)/a/(a^2+b^2)/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2+1/4*b*(13*A*a^2*b+
5*A*b^3-9*B*a^3-B*a*b^2)/a^2/(a^2+b^2)^2/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.94 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.66

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{-\sqrt{a}(a^2+b^2)\left(8a^4A+31a^2Ab^2+15Ab^4-11a^3bB-3ab^3B\right)+\left(b^{3/2}\left(-63a^4Ab-46a^2Ab^3-15Ab^5+35a^5B+6a^3b^2B+3ab^4B\right)\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+d}}{\sqrt{a}}\right.\right.}{a^{5/2}(a^2+b}}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3)
,x]
```

output
```
((-(Sqrt[a]*(a^2 + b^2)*(8*a^4*A + 31*a^2*A*b^2 + 15*A*b^4 - 11*a^3*b*B -
3*a*b^3*B)) + (b^(3/2)*(-63*a^4*A*b - 46*a^2*A*b^3 - 15*A*b^5 + 35*a^5*B +
 6*a^3*b^2*B + 3*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] + 4
*(-1)^(1/4)*a^(7/2)*((I*a - b)^3*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c +
d*x]]] - (I*a + b)^3*(A + I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]))*Sq
rt[Tan[c + d*x]])/(a^(5/2)*(a^2 + b^2)^2) + (2*b*(A*b - a*B))/(a + b*Tan[c
 + d*x])^2 + (b*(13*a^2*A*b + 5*A*b^3 - 9*a^3*B - a*b^2*B))/(a*(a^2 + b^2)
*(a + b*Tan[c + d*x])))/(4*a*(a^2 + b^2)*d*Sqrt[Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 2.98 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.788$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 4092}$$

$$\frac{\int \frac{4Aa^2-bBa-4(Ab-aB)\tan(c+dx)a+5Ab^2+5b(Ab-aB)\tan^2(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}dx}{2a(a^2+b^2)}+\frac{b(Ab-aB)}{2ad(a^2+b^2)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{4Aa^2-bBa-4(Ab-aB)\tan(c+dx)a+5Ab^2+5b(Ab-aB)\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}dx}{4a(a^2+b^2)}+\frac{b(Ab-aB)}{2ad(a^2+b^2)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{4Aa^2-bBa-4(Ab-aB)\tan(c+dx)a+5Ab^2+5b(Ab-aB)\tan(c+dx)^2}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^2}dx}{4a(a^2+b^2)}+\frac{b(Ab-aB)}{2ad(a^2+b^2)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{ 4132}$$

$$\frac{\frac{\int \frac{8Aa^4-11bBa^3+31Ab^2a^2-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2-3b^3Ba+15Ab^4+3b\left(-9Ba^3+13Aba^2-b^2Ba+5Ab^3\right)\tan^2(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx}{a(a^2+b^2)}+\frac{b(-9a^3B+13a^2}{ad(a^2+b^2)\sqrt{\tan(c+}}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{8Aa^4-11bBa^3+31Ab^2a^2-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2-3b^3Ba+15Ab^4+3b\left(-9Ba^3+13Aba^2-b^2Ba+5Ab^3\right)\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}+\frac{b(-9a^3B+13a^2}{ad(a^2+b^2)\sqrt{\tan(c+}}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{8Aa^4-11bBa^3+31Ab^2a^2-8\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2-3b^3Ba+15Ab^4+3b\left(-9Ba^3+13Aba^2-b^2Ba+5Ab^3\right)\tan(c+dx)^2}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))}dx}{2a(a^2+b^2)}+\frac{b(-9a^3B+13a^2}{ad(a^2+b^2)\sqrt{\tan(c+}}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 4132

$$\frac{-\frac{\frac{2\int \frac{-8Ba^5+24Aba^4-3b^2Ba^3+8\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+31Ab^3a^2-3b^4Ba+15Ab^5+b\left(8Aa^4-11bBa^3+31Ab^2a^2-3b^3Ba+15Ab^4\right)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2}{}}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 27

$$\frac{-\frac{\frac{\int \frac{-8Ba^5+24Aba^4-3b^2Ba^3+8\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+31Ab^3a^2-3b^4Ba+15Ab^5+b\left(8Aa^4-11bBa^3+31Ab^2a^2-3b^3Ba+15Ab^4\right)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2(}{}}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{\int \frac{-8Ba^5+24Aba^4-3b^2Ba^3+8\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3+31Ab^3a^2-3b^4Ba+15Ab^5+b\left(8Aa^4-11bBa^3+31Ab^2a^2-3b^3Ba+15Ab^4\right)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-2(}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 4136

$$-\frac{\frac{\frac{\int \frac{8\left(\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)a^3+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)a^3\right)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{}{\sqrt{\tan(c}}}{a^2+b^2}}{a}}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{8\int \frac{\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)a^3+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{\tan}{\sqrt{\tan(c+}}}{a^2+b^2}}{a}}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{8\int \frac{\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)a^3+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{\tan}{\sqrt{\tan(c+}}}{a^2+b^2}}{a}}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 4017

$$-\frac{\frac{\frac{16\int \frac{a^3\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)\right)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int}{a^2+b^2}}{a}}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{16a^3\int \frac{-Ba^3+3Aba^2+3b^2Ba-Ab^3+\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int}{a^2+b^2}}{a}}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 1482

$$-\frac{\frac{16a^3\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int \frac{\tan(c+}{\tan^2(c+}\right.}{d\left(a^2+b^2\right)}}{a}}{2a}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 1476

$$-\frac{16a^3\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{1}{2}\int \frac{}{\tan(}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 1082

$$-\frac{16a^3\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{}{-\tan}\cdots}{}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 217$$

$$-\frac{16a^3\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\cdots\right.}{}\right.\right.}{d\left(a^2+b^2\right)} \cdots a$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 1479$$

$$-\frac{16a^3\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan\cdots}}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{16a^3\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+\cdots}}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{16a^3\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c\ldots}{d\left(a^2+b^2\right)}\ldots$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 1103

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\ldots\right.\right.}{\ldots}}{\ldots}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 4117

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d\left(a^2+b^2\right)}+\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(\ldots\right.\right.}{\ldots}}{\ldots}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 73

$$-\frac{\frac{2b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}+\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\mathrm{arc}\ldots}{\ldots}\right.\right.}{\ldots}}{\ldots}$$

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}$$

$\downarrow$ 218

$$\frac{b(Ab-aB)}{2ad\left(a^2+b^2\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}+$$

$$\frac{\frac{b\left(-9a^3B+13a^2Ab-ab^2B+5Ab^3\right)}{ad(a^2+b^2)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}+\frac{-\frac{2\left(8a^4A-11a^3bB+31a^2Ab^2-3ab^3B+15Ab^4\right)}{ad\sqrt{\tan(c+dx)}}-\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3),x]
```

output
```
(b*(A*b - a*B))/(2*a*(a^2 + b^2)*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])
^2) + ((-(((2*b^(3/2)*(63*a^4*A*b + 46*a^2*A*b^3 + 15*A*b^5 - 35*a^5*B - 6
*a^3*b^2*B - 3*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqr
t[a]*(a^2 + b^2)*d) + (16*a^3*(((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(
A + B) - b^3*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) +
 ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((3*a^2*b*(A - B) -
b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] +
 Tan[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/a) - (2*(8*a^4*A + 31*a^
2*A*b^2 + 15*A*b^4 - 11*a^3*b*B - 3*a*b^3*B))/(a*d*Sqrt[Tan[c + d*x]]))/(2
*a*(a^2 + b^2)) + (b*(13*a^2*A*b + 5*A*b^3 - 9*a^3*B - a*b^2*B))/(a*(a^2 +
 b^2)*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])))/(4*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.90

method	result
derivativedivides	$-\frac{2b^2\left(\frac{\left(\frac{15}{8}A\,a^4b^2+\frac{11}{4}A\,a^2b^4+\frac{7}{8}A\,b^6-\frac{11}{8}B\,a^5b-\frac{7}{4}B\,a^3b^3-\frac{3}{8}Ba\,b^5\right)\tan(dx+c)^{\frac{3}{2}}+\frac{a\left(17A\,a^4b+26A\,a^2b^3+9A\,b^5-13B\,a^5-18B\,a\right.}{8}}{(a+b\tan(dx+c))^2}\right.}{a^3\left(a^2+b^2\right)^3}$
default	$-\frac{2b^2\left(\frac{\left(\frac{15}{8}A\,a^4b^2+\frac{11}{4}A\,a^2b^4+\frac{7}{8}A\,b^6-\frac{11}{8}B\,a^5b-\frac{7}{4}B\,a^3b^3-\frac{3}{8}Ba\,b^5\right)\tan(dx+c)^{\frac{3}{2}}+\frac{a\left(17A\,a^4b+26A\,a^2b^3+9A\,b^5-13B\,a^5-18B\,a\right.}{8}}{(a+b\tan(dx+c))^2}\right.}{a^3\left(a^2+b^2\right)^3}$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(-2*b^2/a^3/(a^2+b^2)^3*(((15/8*A*a^4*b^2+11/4*A*a^2*b^4+7/8*A*b^6-11/
8*B*a^5*b-7/4*B*a^3*b^3-3/8*B*a*b^5)*tan(d*x+c)^(3/2)+1/8*a*(17*A*a^4*b+26
*A*a^2*b^3+9*A*b^5-13*B*a^5-18*B*a^3*b^2-5*B*a*b^4)*tan(d*x+c)^(1/2))/(a+b
*tan(d*x+c))^2+1/8*(63*A*a^4*b+46*A*a^2*b^3+15*A*b^5-35*B*a^5-6*B*a^3*b^2-
3*B*a*b^4)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2)))+2/(a^2+b^2)
^3*(1/8*(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)
*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2
^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*(-A*a^
3+3*A*a*b^2-3*B*a^2*b+B*b^3)*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1
/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x
+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))-2/a^3*A/tan(d*x+c)^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3780 vs. $2(470) = 940$.

Time = 173.30 (sec) , antiderivative size = 7591, normalized size of antiderivative = 14.74

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^3}\, dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.18

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
1/4*((35*B*a^5*b^2 - 63*A*a^4*b^3 + 6*B*a^3*b^4 - 46*A*a^2*b^5 + 3*B*a*b^6
 - 15*A*b^7)*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^9 + 3*a^7*b^2 + 3*
a^5*b^4 + a^3*b^6)*sqrt(a*b)) - (8*A*a^6 + 16*A*a^4*b^2 + 8*A*a^2*b^4 + (8
*A*a^4*b^2 - 11*B*a^3*b^3 + 31*A*a^2*b^4 - 3*B*a*b^5 + 15*A*b^6)*tan(d*x +
 c)^2 + (16*A*a^5*b - 13*B*a^4*b^2 + 49*A*a^3*b^3 - 5*B*a^2*b^4 + 25*A*a*b
^5)*tan(d*x + c))/((a^7*b^2 + 2*a^5*b^4 + a^3*b^6)*tan(d*x + c)^(5/2) + 2*
(a^8*b + 2*a^6*b^3 + a^4*b^5)*tan(d*x + c)^(3/2) + (a^9 + 2*a^7*b^2 + a^5*
b^4)*sqrt(tan(d*x + c))) - (2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(
A - B)*a*b^2 - (A + B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x +
 c)))) + 2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A +
 B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*(
(A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(sqrt(2)
*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*((A + B)*a^3 - 3*(A - B)
*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(-sqrt(2)*sqrt(tan(d*x + c)) +
tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6))/d
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \int \frac{B\tan\left(dx+c\right)+A}{\left(b\tan\left(dx+c\right)+a\right)^3\tan\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 48.52 (sec) , antiderivative size = 35300, normalized size of antiderivative = 68.54

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^3),x)
```

output
```
((B*tan(c + d*x)^(1/2)*(5*b^4 + 13*a^2*b^2))/(4*a*(a^4 + b^4 + 2*a^2*b^2))
 + (B*b*tan(c + d*x)^(3/2)*(3*b^4 + 11*a^2*b^2))/(4*a^2*(a^4 + b^4 + 2*a^2
*b^2)))/(a^2*d + b^2*d*tan(c + d*x)^2 + 2*a*b*d*tan(c + d*x)) - ((2*A)/a +
 (A*tan(c + d*x)^2*(15*b^6 + 31*a^2*b^4 + 8*a^4*b^2))/(4*a^3*(a^4 + b^4 +
2*a^2*b^2)) + (A*tan(c + d*x)*(16*a^4*b + 25*b^5 + 49*a^2*b^3))/(4*a^2*(a^
4 + b^4 + 2*a^2*b^2)))/(a^2*d*tan(c + d*x)^(1/2) + b^2*d*tan(c + d*x)^(5/2
) + 2*a*b*d*tan(c + d*x)^(3/2)) + (log(29491200*A^5*a^22*b^35*d^4 - ((tan(
c + d*x)^(1/2)*(7610564608*A^4*a^27*b^33*d^5 - 597688320*A^4*a^23*b^37*d^5
 - 1671430144*A^4*a^25*b^35*d^5 - 58982400*A^4*a^21*b^39*d^5 + 85774565376
*A^4*a^29*b^31*d^5 + 385487994880*A^4*a^31*b^29*d^5 + 1104303620096*A^4*a^
33*b^27*d^5 + 2240523796480*A^4*a^35*b^25*d^5 + 3345249468416*A^4*a^37*b^2
3*d^5 + 3717287903232*A^4*a^39*b^21*d^5 + 3053967114240*A^4*a^41*b^19*d^5
+ 1807474491392*A^4*a^43*b^17*d^5 + 726513221632*A^4*a^45*b^15*d^5 + 17076
8990208*A^4*a^47*b^13*d^5 + 10492051456*A^4*a^49*b^11*d^5 - 4917821440*A^4
*a^51*b^9*d^5 - 923009024*A^4*a^53*b^7*d^5 + 8388608*A^4*a^55*b^5*d^5) + (
(((480*A^4*a^2*b^10*d^4 - 16*A^4*b^12*d^4 - 16*A^4*a^12*d^4 - 4080*A^4*a^4
*b^8*d^4 + 7232*A^4*a^6*b^6*d^4 - 4080*A^4*a^8*b^4*d^4 + 480*A^4*a^10*b^2*
d^4)^(1/2) + 80*A^2*a^3*b^3*d^2 - 24*A^2*a*b^5*d^2 - 24*A^2*a^5*b*d^2)/(a^
12*d^4 + b^12*d^4 + 6*a^2*b^10*d^4 + 15*a^4*b^8*d^4 + 20*a^6*b^6*d^4 + 15*
a^8*b^4*d^4 + 6*a^10*b^2*d^4))^(1/2)*((((((480*A^4*a^2*b^10*d^4 - 16*A...
```

Reduce [F]

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^3}\,dx$$
$$= \int \frac{\sqrt{\tan(dx + c)}}{\tan(dx + c)^4 b^2 + 2\tan(dx + c)^3 ab + \tan(dx + c)^2 a^2}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x)
```

output
```
int(sqrt(tan(c + d*x))/(tan(c + d*x)**4*b**2 + 2*tan(c + d*x)**3*a*b + tan
(c + d*x)**2*a**2),x)
```

3.416 $\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 116

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{B\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{2B\tan^{\frac{3}{2}}(c+dx)}{3d}$$

output
```
-1/2*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+ta
n(d*x+c)))*2^(1/2)/d+2/3*B*tan(d*x+c)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{B\left(-3\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan(c+dx)}+3\text{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan(c+dx)}+\right.}{3d\sqrt[4]{\tan(c+dx)}}$$

input
```
Integrate[(Tan[c + d*x]^(5/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
]),x]
```

output
```
(B*(-3*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4) + 3*ArcTanh[(
-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4) + 2*Tan[c + d*x]^(7/4)))/(3*
d*Tan[c + d*x]^(1/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.35, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2011, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \tan^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \tan(c+dx)^{5/2}dx$$

$$\Big\downarrow \text{3954}$$

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\int\sqrt{\tan(c+dx)}dx\right)$$

$\Big\downarrow$ 3042

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\int\sqrt{\tan(c+dx)}dx\right)$$

$\Big\downarrow$ 3957

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{\int\frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}\right)$$

$\Big\downarrow$ 266

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2\int\frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}\right)$$

$\Big\downarrow$ 826

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2\left(\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}\right)$$

$\Big\downarrow$ 1476

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c\right.\right.}{d}\right.$$

$\Big\downarrow$ 1082

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+d}{\tan^2(c+dx)\cdot}\right.}{d}\right.$$

$\Big\downarrow$ 217

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right.}{d}\right.$$

$\Big\downarrow$ 1479

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\right.\right.}{d}\right.$$

$\Big\downarrow$ 25

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\right.\right.}{d}\right.$$

$\Big\downarrow$ 27

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right.\right.}{d}\right.$$

$\Big\downarrow$ 1103

$$B\left(\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+\right.}{2\sqrt{2}}\right.\right.}{d}\right.$$

input
```
Int[(Tan[c + d*x]^(5/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
B*((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]/(2*Sqrt[2]))/2))/d + (2*Tan[c + d*x]^(3/2))/(3*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{B\left(\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}\right)}{d}$
default	$\frac{B\left(\frac{2\tan(dx+c)^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}\right)}{d}$

input
```
int(tan(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*B*(2/3*tan(d*x+c)^(3/2)-1/4*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)
^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(
d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{6\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan(dx+c)}+1\right)+6\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan(dx+c)}-1\right)-3\sqrt{2}B\log\left(\sqrt{2}\sqrt{\right.}\right.}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/12*(6*sqrt(2)*B*arctan(sqrt(2)*sqrt(tan(d*x + c)) + 1) + 6*sqrt(2)*B*ar
ctan(sqrt(2)*sqrt(tan(d*x + c)) - 1) - 3*sqrt(2)*B*log(sqrt(2)*sqrt(tan(d*
x + c)) + tan(d*x + c) + 1) + 3*sqrt(2)*B*log(-sqrt(2)*sqrt(tan(d*x + c))
+ tan(d*x + c) + 1) - 8*B*tan(d*x + c)^(3/2))/d
```

Sympy [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = B\int \tan^{\frac{5}{2}}(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(tan(c + d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{8\,B\tan\left(dx+c\right)^{\frac{3}{2}} - 3\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right) + 2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\mathrm{t}\right.\right.\right.}}{}$$

input
```
integrate(tan(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/12*(8*B*tan(d*x + c)^(3/2) - 3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) +
2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(t
an(d*x + c)))) - sqrt(2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1
) + sqrt(2)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*B)/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 16727, normalized size of antiderivative = 144.20

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(5/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
atan((((((32*(4*B*a^2*b^8*d^4 + 8*B*a^4*b^6*d^4 + 4*B*a^6*b^4*d^4))/(b*d^
5) - (32*tan(c + d*x)^(1/2)*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 +
16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*
d^4 + 2*a^2*b^2*d^4)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^
4 - 16*a^6*b^4*d^4))/(b*d^4))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4
+ 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^
4*d^4 + 2*a^2*b^2*d^4)))^(1/2) + (32*tan(c + d*x)^(1/2)*(4*B^2*a^5*b^5*d^2
 - 14*B^2*a^3*b^7*d^2 + 2*B^2*a^7*b^3*d^2 + 16*B^2*a^9*b*d^2))/(b*d^4))*((
(64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^
(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) -
 (32*(B^3*a^5*b^5*d^2 - 15*B^3*a^7*b^3*d^2 + 12*B^3*a^9*b*d^2))/(b*d^5))*(
((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))
^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)
+ (32*tan(c + d*x)^(1/2)*(2*B^4*a^10 - B^4*a^4*b^6))/(b*d^4))*(((64*B^4*a^
6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*
B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*1i - (((((3
2*(4*B*a^2*b^8*d^4 + 8*B*a^4*b^6*d^4 + 4*B*a^6*b^4*d^4))/(b*d^5) + (32*tan
(c + d*x)^(1/2)*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 +
 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*
b^2*d^4)))^(1/2)*(16*b^10*d^4 + 16*a^2*b^8*d^4 - 16*a^4*b^6*d^4 - 16*a^...
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \left(\int \sqrt{\tan(dx+c)}\,\tan(dx+c)^2\,dx\right)b$$

input
```
int(tan(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*tan(c + d*x)**2,x)*b
```

3.417 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 115

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{B\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{2B\sqrt{\tan(c+dx)}}{d}$$

output
```
-1/2*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+ta
n(d*x+c)))*2^(1/2)/d+2*B*tan(d*x+c)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{B\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan\right.}{2\sqrt{2}}\right.}{d}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
]),x]
```

output
```
(B*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sq
rt[Tan[c + d*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d
*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*S
qrt[2]) + 2*Sqrt[Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.35, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2011, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \tan^{\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \tan(c+dx)^{3/2}dx$$

$$\Big\downarrow \text{ 3954}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\int\frac{1}{\sqrt{\tan(c+dx)}}dx\right)$$

$$\Big\downarrow \text{ 3957}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{\int\frac{1}{\sqrt{\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}\right)$$

$$\Big\downarrow \text{ 266}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\int\frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}\right)$$

$$\Big\downarrow \text{ 755}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\right.\right.}{d}\right.$$

$$\Big\downarrow \text{ 1082}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\right.}{}\right.\right.}{d}\right.$$

$$\Big\downarrow \text{ 217}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right.\right.}{d}\right.$$

$$\Big\downarrow \text{1479}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\right.}{d}\right.$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arc}}{}\right.\right.}{d}\right.$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)\right.}{d}\right.$$

$$\Big\downarrow \text{1103}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+\right.}{2\sqrt{2}}\right.\right.}{d}\right.$$

input
```
Int[(Tan[c + d*x]^(3/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
B*((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
 + d*x]]/(2*Sqrt[2]))/2))/d + (2*Sqrt[Tan[c + d*x]])/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{B\left(2\sqrt{\tan(dx+c)}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}\right)}{d}$
default	$\frac{B\left(2\sqrt{\tan(dx+c)}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}\right)}{d}$

input
```
int(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*B*(2*tan(d*x+c)^(1/2)-1/4*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(
1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*
x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{2\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan(dx+c)}+1\right)+2\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan(dx+c)}-1\right)+\sqrt{2}B\log\left(\sqrt{2}\sqrt{\tan(dx+c)}\right.}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/4*(2*sqrt(2)*B*arctan(sqrt(2)*sqrt(tan(d*x + c)) + 1) + 2*sqrt(2)*B*arc
tan(sqrt(2)*sqrt(tan(d*x + c)) - 1) + sqrt(2)*B*log(sqrt(2)*sqrt(tan(d*x +
 c)) + tan(d*x + c) + 1) - sqrt(2)*B*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan
(d*x + c) + 1) - 8*B*sqrt(tan(d*x + c)))/d
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = B\int \tan^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(tan(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{2\sqrt{2}B\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\cdots}{}$$

input
```
integrate(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/4*(2*sqrt(2)*B*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2
*sqrt(2)*B*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)
*B*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*B*log(-sqr
t(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*B*sqrt(tan(d*x + c)))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.55 (sec) , antiderivative size = 16060, normalized size of antiderivative = 139.65

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(3/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
atan(((((32*(B^3*a^2*b^7*d^2 - 15*B^3*a^4*b^5*d^2 + 12*B^3*a^6*b^3*d^2))/d
^5 - (((32*(4*B*a*b^8*d^4 + 8*B*a^3*b^6*d^4 + 4*B*a^5*b^4*d^4))/d^5 - (32*
tan(c + d*x)^(1/2)*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^
4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a
^2*b^2*d^4)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6
*b^3*d^4))/d^4)*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 +
 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*
b^2*d^4)))^(1/2) + (32*tan(c + d*x)^(1/2)*(4*B^2*a^3*b^6*d^2 + 2*B^2*a^5*b
^4*d^2 + 16*B^2*a^7*b^2*d^2 - 14*B^2*a*b^8*d^2))/d^4)*(((64*B^4*a^2*b^6*d^
4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b^
3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2))*(((64*B^4*a^2*b^6*
d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*
b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) + (32*tan(c + d*x
)^(1/2)*(B^4*b^9 - 2*B^4*a^6*b^3))/d^4)*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(1
6*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b^3*d^2)/(16*(a^
4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*1i - (((32*(B^3*a^2*b^7*d^2 - 15*
B^3*a^4*b^5*d^2 + 12*B^3*a^6*b^3*d^2))/d^5 - (((32*(4*B*a*b^8*d^4 + 8*B*a^
3*b^6*d^4 + 4*B*a^5*b^4*d^4))/d^5 + (32*tan(c + d*x)^(1/2)*(((64*B^4*a^2*b
^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2
*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(16*b^9*d^4...
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx=\frac{b\left(2\sqrt{\tan\left(dx+c\right)}-\left(\int\frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)d\right)}{d}$$

input
```
int(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(b*(2*sqrt(tan(c + d*x)) - int(sqrt(tan(c + d*x))/tan(c + d*x),x)*d))/d
```

3.418 $\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 99

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{B\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d}$$

output
```
1/2*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctan(1+2^(1/2)
*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan
(d*x+c)))*2^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=\frac{B\left(\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)-\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\right)\sqrt[4]{-\tan(c+dx)}}{d\sqrt[4]{\tan(c+dx)}}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
]),x]
```

output
```
(B*(ArcTan[(-Tan[c + d*x]^2)^(1/4)] - ArcTanh[(-Tan[c + d*x]^2)^(1/4)])*(-
Tan[c + d*x])^(1/4))/(d*Tan[c + d*x]^(1/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2011, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{2011}$$
$$B\int \sqrt{\tan(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$B\int \sqrt{\tan(c+dx)}dx$$
$$\Big\downarrow \text{3957}$$

$$\frac{B\int \frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$\Big\downarrow$ 266

$$\frac{2B\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 826

$$\frac{2B\left(\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}$$

$\Big\downarrow$ 1476

$$\frac{2B\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\int \frac{1-\tan}{\tan^2(c}\right.}{d}$$

$\Big\downarrow$ 1082

$$\frac{2B\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}$$

$\Big\downarrow$ 217

$$\frac{2B\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}$$

$\Big\downarrow$ 1479

$$\frac{2B\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+\right.}{\sqrt{2}}\right.\right.}{d}$$

$\Big\downarrow$ 25

$$\frac{2B\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right.}{\sqrt{2}}\right.\right.}{d}$$

$\Big\downarrow$ 27

$$\frac{2B\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan\ldots}\right.}{\sqrt{2}\ldots}\right.\right.}{d}$$

$\downarrow$ 1103

$$\frac{2B\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)+\sqrt{\ldots}\right.}{2\ldots}\right.\right.}{d}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(2*B*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt
[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] +
 Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]/(2*Sqrt[2]))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{B\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4d}$	90
default	$\frac{B\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4d}$	90

input
```
int(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/4/d*B*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^
(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-
1+2^(1/2)*tan(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{2\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan(dx+c)}+1\right)+2\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan(dx+c)}-1\right)-\sqrt{2}B\log\left(\sqrt{2}\sqrt{\tan\right.}{4d}$$

input
```
integrate(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/4*(2*sqrt(2)*B*arctan(sqrt(2)*sqrt(tan(d*x + c)) + 1) + 2*sqrt(2)*B*arct
an(sqrt(2)*sqrt(tan(d*x + c)) - 1) - sqrt(2)*B*log(sqrt(2)*sqrt(tan(d*x +
c)) + tan(d*x + c) + 1) + sqrt(2)*B*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(
d*x + c) + 1))/d
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = B\int \sqrt{\tan\left(c+dx\right)}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(sqrt(tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)-\sqrt{2}\mathrm{l}\right.}{4\,d}$$

input
```
integrate(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(
sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt
(tan(d*x + c)) + tan(d*x + c) + 1))*B/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 15753, normalized size of antiderivative = 159.12

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(1/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
atan(((((32*(13*B^3*a^5*b^4*d^2 + B^3*a^7*b^2*d^2))/d^5 + (((32*(12*B*a^2*
b^7*d^4 + 24*B*a^4*b^5*d^4 + 12*B*a^6*b^3*d^4))/d^5 - (32*tan(c + d*x)^(1/
2)*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d
^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1
/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*
(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4)
)^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)
 + (32*tan(c + d*x)^(1/2)*(20*B^2*a^5*b^4*d^2 - 14*B^2*a^3*b^6*d^2 + 2*B^2
*a^7*b^2*d^2))/d^4)*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d
^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*
a^2*b^2*d^4)))^(1/2))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4
*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 +
2*a^2*b^2*d^4)))^(1/2) - (32*tan(c + d*x)^(1/2)*(B^4*a^4*b^5 - 2*B^4*a^6*b
^3))/d^4)*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^
2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^
4)))^(1/2)*1i - (((32*(13*B^3*a^5*b^4*d^2 + B^3*a^7*b^2*d^2))/d^5 + (((32*
(12*B*a^2*b^7*d^4 + 24*B*a^4*b^5*d^4 + 12*B*a^6*b^3*d^4))/d^5 + (32*tan(c
+ d*x)^(1/2)*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32
*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2
*d^4)))^(1/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^...
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \left(\int \sqrt{\tan(dx+c)}dx\right)b$$

input
```
int(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x)),x)*b
```

3.419 $\int \frac{aB+bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 98

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx = -\frac{B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{B\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d}$$

output
```
1/2*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctan(1+2^(1/2)
*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan
(d*x+c)))*2^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.12

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx = \frac{B\left(-2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right.}{2\sqrt{2}d}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]
)),x]
```

output
```
(B*(-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[
Tan[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Log[
1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(2*Sqrt[2]*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2011, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{B\int \frac{1}{\sqrt{\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}$$

$\Big\downarrow$ 266

$$\frac{2B\int \frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 755

$$\frac{2B\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}$$

$\Big\downarrow$ 1476

$$\frac{2B\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+}\right.\right.}{d}$$

$\Big\downarrow$ 1082

$$\frac{2B\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}$$

$\Big\downarrow$ 217

$$\frac{2B\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}$$

$\Big\downarrow$ 1479

$$\frac{2B\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}}\right.\right.}{d}$$

$\Big\downarrow$ 25

$$\frac{2B\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}$$

$\Big\downarrow$ 27

$$\frac{2B\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(\right.}{\sqrt{2}}\right.}{d}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{2B\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{}\right.}{2\sqrt{}}\right.}{d}$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])),x]
```

output
```
(2*B*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt
[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{B\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4d}$	90
default	$\frac{B\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4d}$	90

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/4/d*B*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^
(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-
1+2^(1/2)*tan(d*x+c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx$$
$$=\frac{2\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+1\right)+2\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}-1\right)+\sqrt{2}B\log\left(\sqrt{2}\sqrt{\tan\right.\right.}{4\,d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/4*(2*sqrt(2)*B*arctan(sqrt(2)*sqrt(tan(d*x + c)) + 1) + 2*sqrt(2)*B*arct
an(sqrt(2)*sqrt(tan(d*x + c)) - 1) + sqrt(2)*B*log(sqrt(2)*sqrt(tan(d*x +
c)) + tan(d*x + c) + 1) - sqrt(2)*B*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(
d*x + c) + 1))/d
```

Sympy [F]

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))}\,dx = B\int \frac{1}{\sqrt{\tan(c + dx)}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(1/sqrt(tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))}\,dx = \frac{2\sqrt{2}B\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2\sqrt{\tan(dx + c)}\right)\right) + 2\sqrt{2}B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2\sqrt{\tan(dx + c)}\right)\right) + \sqrt{}}{4d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/4*(2*sqrt(2)*B*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*
sqrt(2)*B*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*
B*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*B*log(-sqrt
(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/d
```

Giac [F]

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx = \int \frac{Bb\tan(dx+c) + Ba}{(b\tan(dx+c)+a)\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 15437, normalized size of antiderivative = 157.52

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx = \text{Too large to display}$$

input
```
int((B*a + B*b*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))),x)
```

output
```
atan(((((32*(13*B^3*a^2*b^7*d^2 + B^3*a^4*b^5*d^2))/d^5 + (((32*(12*B*a*b^
8*d^4 + 24*B*a^3*b^6*d^4 + 12*B*a^5*b^4*d^4))/d^5 - (32*tan(c + d*x)^(1/2)
*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4
))^(1/2) - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2
)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^4)*((
(64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^
(1/2) - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) +
 (32*tan(c + d*x)^(1/2)*(20*B^2*a^3*b^6*d^2 + 2*B^2*a^5*b^4*d^2 - 14*B^2*a
*b^8*d^2))/d^4)*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 +
 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*
b^2*d^4)))^(1/2))*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4
 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^
2*b^2*d^4)))^(1/2) - (32*tan(c + d*x)^(1/2)*(B^4*b^9 - 2*B^4*a^2*b^7))/d^4
)*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^
4))^(1/2) - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/
2)*1i - (((32*(13*B^3*a^2*b^7*d^2 + B^3*a^4*b^5*d^2))/d^5 + (((32*(12*B*a*
b^8*d^4 + 24*B*a^3*b^6*d^4 + 12*B*a^5*b^4*d^4))/d^5 + (32*tan(c + d*x)^(1/
2)*(((64*B^4*a^2*b^6*d^4 - B^4*b^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d
^4))^(1/2) - 8*B^2*a*b^3*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1
/2)*(16*b^9*d^4 + 16*a^2*b^7*d^4 - 16*a^4*b^5*d^4 - 16*a^6*b^3*d^4))/d^...
```

Reduce [F]

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}\,dx = \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x),x)*b
```

3.420 $\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 114

$$\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} + \frac{B\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{2B}{d\sqrt{\tan(c+dx)}}$$

output
```
-1/2*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+ta
n(d*x+c)))*2^(1/2)/d-2*B/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$$
$$= \frac{B\left(-2-\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx}\right.}{d\sqrt{\tan(c+dx)}}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]
)),x]
```

output
```
(B*(-2 - ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4) + ArcTanh
[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4)))/(d*Sqrt[Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.36, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2011, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{1}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 3955}$$

$$B\left(-\int \sqrt{\tan(c+dx)}dx-\frac{2}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(-\int \sqrt{\tan(c+dx)}dx-\frac{2}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3957}$$

$$B\left(-\frac{\int \frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{266}$$

$$B\left(-\frac{2\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{826}$$

$$B\left(-\frac{2\left(\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{1476}$$

$$B\left(-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\int \frac{1}{\text{ta}}\right.}{d}\right.$$

$$\Big\downarrow \text{1082}$$

$$B\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx}\right.}{d}\right.$$

$$\Big\downarrow \text{217}$$

$$B\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2}{d\sqrt{\tan(c+}}\right.$$

$\downarrow$ 1479

$$B\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+}\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$\downarrow$ 25

$$B\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+d}\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$\downarrow$ 27

$$B\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.\right.}{d}\right.$$

$\downarrow$ 1103

$$B\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx\right.}{}\right.\right.}{d}\right.$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]
```

output
```
B*((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]/(2*Sqrt[2]))/2))/d - 2/(d*Sqrt[Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{B\left(-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}-\frac{2}{\sqrt{\tan(dx+c)}}\right)}{d}$
default	$\frac{B\left(-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\tan(dx+c)}\right)\right)}{4}-\frac{2}{\sqrt{\tan(dx+c)}}\right)}{d}$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*B*(-1/4*2^(1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c
)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arct
an(-1+2^(1/2)*tan(d*x+c)^(1/2)))-2/tan(d*x+c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27

$$\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = -\frac{2\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan(dx+c)}+1\right)\tan(dx+c)+2\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan(dx+c)}-1\right)\tan(dx}{}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/4*(2*sqrt(2)*B*arctan(sqrt(2)*sqrt(tan(d*x + c)) + 1)*tan(d*x + c) + 2*
sqrt(2)*B*arctan(sqrt(2)*sqrt(tan(d*x + c)) - 1)*tan(d*x + c) - sqrt(2)*B*
log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)*tan(d*x + c) + sqrt(2)*
B*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)*tan(d*x + c) + 8*B*s
qrt(tan(d*x + c)))/(d*tan(d*x + c))
```

Sympy [F]

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = B\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(tan(c + d*x)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx =$$
$$-\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{}\right.}{}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*lo
g(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sq
rt(tan(d*x + c)) + tan(d*x + c) + 1))*B + 8*B/sqrt(tan(d*x + c)))/d
```

Giac [F]

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))}\,dx = \int \frac{Bb\tan(dx + c) + Ba}{(b\tan(dx + c) + a)\tan(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 15569, normalized size of antiderivative = 136.57

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))}\,dx = \text{Too large to display}$$

input
```
int((B*a + B*b*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))),x)
```

output
```
atan(((tan(c + d*x)^(1/2)*(64*B^4*a^2*b^7*d^5 - 32*B^4*a^4*b^5*d^5) + (((6
4*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1
/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*((t
an(c + d*x)^(1/2)*(128*B^2*a^5*b^4*d^7 - 448*B^2*a^3*b^6*d^7 + 64*B^2*a^7*
b^2*d^7 + 512*B^2*a*b^8*d^7) - (((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4
 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b
^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(tan(c + d*x)^(1/2)*(((64*B^4*a^6*b^2*d^4
- B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*
d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(512*b^9*d^9 + 512*a^
2*b^7*d^9 - 512*a^4*b^5*d^9 - 512*a^6*b^3*d^9) - 512*B*b^9*d^8 - 640*B*a^2
*b^7*d^8 + 256*B*a^4*b^5*d^8 + 384*B*a^6*b^3*d^8))*(((64*B^4*a^6*b^2*d^4 -
 B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d
^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2) - 32*B^3*a^5*b^4*d^6 -
 32*B^3*a^7*b^2*d^6 + 128*B^3*a*b^8*d^6))*(((64*B^4*a^6*b^2*d^4 - B^4*a^4*
(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(
a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*1i + (tan(c + d*x)^(1/2)*(64*B^
4*a^2*b^7*d^5 - 32*B^4*a^4*b^5*d^5) + (((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*
a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*
d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*((tan(c + d*x)^(1/2)*(128*B^2*a^5*b
^4*d^7 - 448*B^2*a^3*b^6*d^7 + 64*B^2*a^7*b^2*d^7 + 512*B^2*a*b^8*d^7) ...
```

Reduce [F]

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x)**2,x)*b
```

3.421 $\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 117

$$\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{B\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}d} - \frac{2B}{3d\tan^{\frac{3}{2}}(c+dx)}$$

output
```
-1/2*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+ta
n(d*x+c)))*2^(1/2)/d-2/3*B/d/tan(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{B\left(-2+3\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\left(-\tan^2(c+dx)\right)^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\left(-\tan^2(c+\right.\right.}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x]
)),x]
```

output
```
(B*(-2 + 3*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4) + 3*Arc
Tanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4)))/(3*d*Tan[c + d*x]^
(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2011, 3042, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{1}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3955}$$

$$B\left(-\int \frac{1}{\sqrt{\tan(c+dx)}}dx - \frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\big\downarrow$ 3042

$$B\left(-\int \frac{1}{\sqrt{\tan(c+dx)}}dx - \frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\big\downarrow$ 3957

$$B\left(-\frac{\int \frac{1}{\sqrt{\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d} - \frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\big\downarrow$ 266

$$B\left(-\frac{2\int \frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d} - \frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\big\downarrow$ 755

$$B\left(-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)} + \frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d} - \frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\big\downarrow$ 1476

$$B\left(-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)} + \frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)} + \frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan}}\right.\right.}{d}\right.$$

$\big\downarrow$ 1082

$$B\left(-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)} + \frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} - \frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}\right.$$

$\big\downarrow$ 217

$$B\left(-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}-\frac{2}{3d\tan^{\frac{3}{2}}(c \cdots}\right.$$

$\downarrow$ 1479

$$B\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(\cdots}}{\sqrt{2}}\cdots\right.\right.}{d}\cdots\right.$$

$\downarrow$ 25

$$B\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}\cdots}{\sqrt{2}}\cdots\right.\right.}{d}\cdots\right.$$

$\downarrow$ 27

$$B\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\cdots}{}\cdots\right.\right.}{d}\cdots\right.$$

$\downarrow$ 1103

$$B\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx\cdots\right.}{}\cdots\right.\right.}{d}\cdots\right.$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]
```

output
```
B*((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
 + d*x]]/(2*Sqrt[2]))/2))/d - 2/(3*d*Tan[c + d*x]^(3/2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{B\left(-\frac{2}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4}\right.}{d}$
default	$\frac{B\left(-\frac{2}{3\tan(dx+c)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right)}{4}\right.}{d}$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
1/d*B*(-2/3/tan(d*x+c)^(3/2)-1/4*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c
)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx =$$
$$-\frac{6\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+1\right)\tan\left(dx+c\right)^{2}+6\sqrt{2}B\arctan\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}-1\right)\tan\left(dx\right.}{}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/12*(6*sqrt(2)*B*arctan(sqrt(2)*sqrt(tan(d*x + c)) + 1)*tan(d*x + c)^2 +
 6*sqrt(2)*B*arctan(sqrt(2)*sqrt(tan(d*x + c)) - 1)*tan(d*x + c)^2 + 3*sqr
t(2)*B*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)*tan(d*x + c)^2 -
 3*sqrt(2)*B*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)*tan(d*x +
 c)^2 + 8*B*sqrt(tan(d*x + c)))/(d*tan(d*x + c)^2)
```

Sympy [F]

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = B\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(tan(c + d*x)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = -\frac{6\sqrt{2}B\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right) + 6\sqrt{2}B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right) +$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/12*(6*sqrt(2)*B*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) +
6*sqrt(2)*B*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + 3*sqrt
(2)*B*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 3*sqrt(2)*B*log
(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + 8*B/tan(d*x + c)^(3/2))
/d
```

Giac [F]

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))}\,dx = \int \frac{Bb\tan\left(dx + c\right) + Ba}{\left(b\tan\left(dx + c\right) + a\right)\tan\left(dx + c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 16545, normalized size of antiderivative = 141.41

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))}\,dx = \text{Too large to display}$$

input
```
int((B*a + B*b*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))),x)
```

output
```
atan(((tan(c + d*x)^(1/2)*(64*B^4*a^9*b^9*d^5 + 32*B^4*a^13*b^5*d^5) - (-(
(64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^
(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(
(tan(c + d*x)^(1/2)*(512*B^2*a^8*b^10*d^7 + 448*B^2*a^12*b^6*d^7 - 128*B^2
*a^14*b^4*d^7 - 64*B^2*a^16*b^2*d^7) - (-((64*B^4*a^6*b^2*d^4 - B^4*a^4*(1
6*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^
4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(tan(c + d*x)^(1/2)*(-((64*B^4*a^
6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*
B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^2*b^2*d^4)))^(1/2)*(512*a^9*b^
9*d^9 + 512*a^11*b^7*d^9 - 512*a^13*b^5*d^9 - 512*a^15*b^3*d^9) - 512*B*a^
8*b^10*d^8 - 512*B*a^10*b^8*d^8 + 384*B*a^12*b^6*d^8 + 256*B*a^14*b^4*d^8
- 128*B*a^16*b^2*d^8))*(-((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b
^4*d^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4
+ 2*a^2*b^2*d^4)))^(1/2) - 384*B^3*a^9*b^9*d^6 + 32*B^3*a^13*b^5*d^6 + 32*
B^3*a^15*b^3*d^6))*(-((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d
^4 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*
a^2*b^2*d^4)))^(1/2)*1i + (tan(c + d*x)^(1/2)*(64*B^4*a^9*b^9*d^5 + 32*B^4
*a^13*b^5*d^5) - (-((64*B^4*a^6*b^2*d^4 - B^4*a^4*(16*a^4*d^4 + 16*b^4*d^4
 + 32*a^2*b^2*d^4))^(1/2) - 8*B^2*a^3*b*d^2)/(16*(a^4*d^4 + b^4*d^4 + 2*a^
2*b^2*d^4)))^(1/2)*((tan(c + d*x)^(1/2)*(512*B^2*a^8*b^10*d^7 + 448*B^2...
```

Reduce [F]

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^3}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))/tan(c + d*x)**3,x)*b
```

3.422 $\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 202

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{(a+b)B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a+b)B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{2a^{5/2}B\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{b^{3/2}\,(a^2+b^2)\,d} + \frac{(a-b)B\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{2B\sqrt{\tan(c+dx)}}{bd}$$

output
```
-1/2*(a+b)*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)/d-1/2*(
a+b)*B*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)/d-2*a^(5/2)*B*
arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(3/2)/(a^2+b^2)/d+1/2*(a-b)*B*a
rctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)/d+2*B*ta
n(d*x+c)^(1/2)/b/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.77

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{B\left(\sqrt[4]{-1}b^{3/2}(-ia+b)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)-2a^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)+\sqrt[4]{-1}b^{3/2}(ia+b)\right.}{b^{3/2}\left(a^2+b^2\right)d}$$

input
```
Integrate[(Tan[c + d*x]^(5/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
])^2,x]
```

output
```
(B*((-1)^(1/4)*b^(3/2)*((-I)*a + b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
- 2*a^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] + (-1)^(1/4)*b^(3
/2)*(I*a + b)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + 2*a^2*Sqrt[b]*Sqrt[
Tan[c + d*x]] + 2*b^(5/2)*Sqrt[Tan[c + d*x]]))/(b^(3/2)*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.13, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2011, 3042, 4049, 27, 3042, 4136, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{\tan^{\frac{5}{2}}(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{\tan(c+dx)^{5/2}}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4049}$$

$$B\left(\frac{2\int -\frac{a\tan^2(c+dx)+b\tan(c+dx)+a}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}+\frac{2\sqrt{\tan(c+dx)}}{bd}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\int \frac{a\tan^2(c+dx)+b\tan(c+dx)+a}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\int \frac{a\tan(c+dx)^2+b\tan(c+dx)+a}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{b}\right)$$

$$\Big\downarrow \text{4136}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{\int \frac{b^2+a\tan(c+dx)b}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{a^3\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{\int \frac{b^2+a\tan(c+dx)b}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{a^3\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}\right)$$

$$\Big\downarrow \text{4017}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2\int \frac{b(b+a\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{a^3\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\int \frac{b+a\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{a^3\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{b}\right)$$

$$\Big\downarrow \text{1482}$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a+b)\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a-b)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d(a^2+b^2)}+\frac{a^3\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+d}}{a^2+b^2}}{b}\right.$$

$$\Big\downarrow \; 1476$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a+b)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}(a\right.}{d(a^2+b^2)}}{b}\right.$$

$$\Big\downarrow \; 1082$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a+b)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+}\right.}{d(a^2+b^2)}}{b}\right.$$

$$\Big\downarrow \; 217$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d(a^2+b^2)}+\frac{a^3}{}}{b}\right.$$

$$\Big\downarrow \; 1479$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{}}{2\sqrt{2}}\right.\right.}{d(a^2+b^2)}}{b}\right.$$

$$\Big\downarrow \; 25$$

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan}}{2\sqrt{2}}\right.\right.}{d(a^2+b^2)}}{b}\right.$$

$\downarrow$ 27

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan}}{2\sqrt{2}}\right.\right.}{d(a^2+b^2)}}{b}\right.$$

$\downarrow$ 1103

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^3\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{2b\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\right.}{d(a^2+}}{b}\right.$$

$\downarrow$ 4117

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{a^3\int\frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d(a^2+b^2)}+\frac{2b\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right.}{}}{b}\right.$$

$\downarrow$ 73

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2a^3\int\frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{2b\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\right.}{d(a^2+}}{b}\right.$$

$\downarrow$ 218

$$B\left(\frac{2\sqrt{\tan(c+dx)}}{bd}-\frac{\frac{2b\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\mathrm{lo}}{}\right.}{d(a^2+b^2)}}{b}\right.$$

input
```
Int[(Tan[c + d*x]^(5/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x
]
```

output
```
B*(-(((2*a^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[b]*(a
^2 + b^2)*d) + (2*b*(((a + b)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sq
rt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 - ((a - b)*(-1
/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d)
)/b) + (2*Sqrt[Tan[c + d*x]])/(b*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4049
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Simp[1/(d*(m + n - 1))
 Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n
- 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[
e + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2
, 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || I
ntegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0]))
)
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{B\left(\frac{2\sqrt{\tan(dx+c)}}{b}-\frac{2a^3\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{b\left(a^2+b^2\right)\sqrt{ab}}+\frac{-\frac{b\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2}{4}}{}\right.}{}$
default	$\frac{B\left(\frac{2\sqrt{\tan(dx+c)}}{b}-\frac{2a^3\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{b\left(a^2+b^2\right)\sqrt{ab}}+\frac{-\frac{b\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2}{4}}{}\right.}{}$

input
```
int(tan(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/d*B*(2*tan(d*x+c)^(1/2)/b-2/b*a^3/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x
+c)^(1/2)/(a*b)^(1/2))+2/(a^2+b^2)*(-1/8*b*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)
*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2
^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/8*a*2^(1
/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*
x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*ta
n(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 922 vs. 2(173) = 346.

Time = 0.18 (sec) , antiderivative size = 1870, normalized size of antiderivative = 9.26

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
[1/2*(2*B*a^2*sqrt(-a/b)*log(-(2*b*sqrt(-a/b)*sqrt(tan(d*x + c)) - b*tan(d
*x + c) + a)/(b*tan(d*x + c) + a)) - 2*sqrt(1/2)*(a^2*b + b^3)*d*sqrt((B^2
*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(((a^4 +
2*a^2*b^2 + b^4)*d^2*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^
2 + b^4)*d^2))*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^
4)*d^2)) + 2*sqrt(1/2)*(B*a^3 - B*a^2*b + B*a*b^2 - B*b^3)*d*sqrt((B^2*a^2
 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))
/(B^2*a^2 - B^2*b^2)) - 2*sqrt(1/2)*(a^2*b + b^3)*d*sqrt((B^2*a^2 + 2*B^2*
a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 +
b^4)*d^2*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2
))*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 2
*sqrt(1/2)*(B*a^3 - B*a^2*b + B*a*b^2 - B*b^3)*d*sqrt((B^2*a^2 + 2*B^2*a*b
 + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(B^2*a^2 -
B^2*b^2)) - sqrt(1/2)*(a^2*b + b^3)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)
/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^
2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c))
 - B*a + B*b - (B*a - B*b)*tan(d*x + c)) + sqrt(1/2)*(a^2*b + b^3)*d*sqrt(
(B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-2*sqrt
(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2
 + b^4)*d^2))*sqrt(tan(d*x + c)) - B*a + B*b - (B*a - B*b)*tan(d*x + c)...
```

Sympy [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = B\int \frac{\tan^{\frac{5}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
B*Integral(tan(c + d*x)**(5/2)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.93

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{8\,Ba^3\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2b+b^3)\sqrt{ab}} + \frac{\left(2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\cdots}{a^2+b^2\cdots}}{4\,d}$$

input
```
integrate(tan(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
-1/4*(8*B*a^3*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2*b + b^3)*sqrt(a
*b)) + (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c
)))) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c
)))) - sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)
+ sqrt(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*B/(
a^2 + b^2) - 8*B*sqrt(tan(d*x + c))/b)/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 34.73 (sec) , antiderivative size = 18514, normalized size of antiderivative = 91.65

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(5/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x
)
```

output
```
(log((((((((((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^
4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a
^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2) + (768*B*a^3*b^3*(a^2 + b^2))/d)*((4*
(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B
^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (64*B^2*a^3*tan(c + d*x)^(1/
2)*(2*a^8 + 15*b^8 - 17*a^2*b^6 + 51*a^4*b^4 + 21*a^6*b^2))/(d^2*(a^2 + b^
2)^2))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3
*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (32*B^3*a^4*(4*a^
8 + b^8 - 77*a^2*b^6 + 47*a^4*b^4 + 33*a^6*b^2))/(d^3*(a^2 + b^2)^3))*((4*
(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B
^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (16*B^4*a^4*tan(c + d*x)^(1/
2)*(a^10 - 2*b^10 - 4*a^2*b^8 - 27*a^4*b^6 + 15*a^6*b^4 + 9*a^8*b^2))/(b*d
^4*(a^2 + b^2)^4))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16
*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (8*B^
5*a^7*(a^6 + 10*b^6 + 27*a^2*b^4 + 10*a^4*b^2))/(b*d^5*(a^2 + b^2)^4))*(((
192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^12*d^4 - 608*B^4*a^8*b
^4*d^4 + 192*B^4*a^10*b^2*d^4)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d
^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(
1/2))/4 + (log((((((((((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^
2*(-(4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*...
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{a+\tan(dx+c)\,b}dx\right)b$$

input
```
int(tan(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*b
```

3.423 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 186

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{(a-b)B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a-b)B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{2a^{3/2}B\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}\,(a^2+b^2)\,d} - \frac{(a+b)B\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}$$

output
```
-1/2*(a-b)*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)/d-1/2*(
a-b)*B*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)/d+2*a^(3/2)*B*
arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/b^(1/2)/(a^2+b^2)/d-1/2*(a+b)*B*a
rctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.23

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{B\left(3a\left(2\sqrt{2}\sqrt{b}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\sqrt{b}\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+8\sqrt{a}\arctan\left(\right.\right.\right.}{}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
])^2,x]
```

output
```
(B*(3*a*(2*Sqrt[2]*Sqrt[b]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt
[2]*Sqrt[b]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + 8*Sqrt[a]*ArcTan[(Sqr
t[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]] + Sqrt[2]*Sqrt[b]*Log[1 - Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]] - Sqrt[2]*Sqrt[b]*Log[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]] + Tan[c + d*x]]) + 8*b^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c
 + d*x]^2]*Tan[c + d*x]^(3/2)))/(12*Sqrt[b]*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {2011, 3042, 4056, 25, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{2011}$$
$$B\int \frac{\tan^{\frac{3}{2}}(c+dx)}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$B\int \frac{\tan(c+dx)^{3/2}}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \ 4056$$

$$B\left(\frac{a^2\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{\int -\frac{a-b\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 25$$

$$B\left(\frac{a^2\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{\int \frac{a-b\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 3042$$

$$B\left(\frac{a^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{\int \frac{a-b\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \ 4017$$

$$B\left(\frac{a^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\int \frac{a-b\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)}\right)$$

$$\Big\downarrow \ 1482$$

$$B\left(\frac{a^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\left(\frac{1}{2}(a+b)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a-b)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(\right.}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \ 1476$$

$$B\left(\frac{a^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\left(\frac{1}{2}(a+b)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a-b)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan}}\right.\right.}{d\left(a^2\right.}$$

$$\Big\downarrow \ 1082$$

$$B\left(\frac{a^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\left(\frac{1}{2}(a+b)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a-b)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{\right.}}{\sqrt{2}}\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \; 217$$

$$B\left(\frac{a^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\left(\frac{1}{2}(a+b)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a-b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \; 1479$$

$$B\left(\frac{a^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\left(\frac{1}{2}(a+b)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}\right.}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c}}}{2\sqrt{}}\right.\right.}{}\right.$$

$$\Big\downarrow \; 25$$

$$B\left(\frac{a^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+}}{2\sqrt{2}}\right.\right.}{d\left(\right.}\right.$$

$$\Big\downarrow \; 27$$

$$B\left(\frac{a^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c}}\right.\right.}{}\right.$$

$$\Big\downarrow \; 1103$$

$$B\left(\frac{a^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}-\frac{2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a+b\right.}{d\left(a^2+\right.}\right.$$

$$\Big\downarrow \; 4117$$

$$B\left(\frac{a^2\int \frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d\,(a^2+b^2)}-\frac{2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right.}{}\right.$$

$$\Big\downarrow \; 73$$

$$B\left(\frac{2a^2\int \frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)}-\frac{2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a\right.}{d\,(a^2}\right.$$

$$\Big\downarrow \; 218$$

$$B\left(\frac{2a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{b}d\,(a^2+b^2)}-\frac{2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a+b)\right.}{d\,(a^2+b^2)}\right.$$

input
```
Int[(Tan[c + d*x]^(3/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x
]
```

output
```
B*((2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[b]*(a^2
+ b^2)*d) - (2*(((a - b)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]
) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((a + b)*(-1/2*Lo
g[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4056
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[Simp[a^2*c - b^2*c +
2*a*b*d + (2*a*b*c - a^2*d + b^2*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x
]], x], x] + Simp[(b*c - a*d)^2/(c^2 + d^2)   Int[(1 + Tan[e + f*x]^2)/(Sqr
t[a + b*Tan[e + f*x]]*(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{B\left(\frac{2a^2\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{-\frac{a\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\right.}{4}}{}\right.}{d}$
default	$\frac{B\left(\frac{2a^2\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{-\frac{a\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\right.}{4}}{}\right.}{d}$

input
```
int(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/d*B*(2*a^2/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))+
2/(a^2+b^2)*(-1/8*a*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(t
an(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*b*2^(1/2)*(ln((tan(d*x+c)-2^(
1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan
(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(158) = 316.

Time = 0.14 (sec) , antiderivative size = 1784, normalized size of antiderivative = 9.59

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
[1/2*(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4
 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt((B^2*a
^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt((B^2*a^2 - 2
*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*(B*a^3 +
B*a^2*b + B*a*b^2 + B*b^3)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 +
2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(B^2*a^2 - B^2*b^2)) + 2*sqrt(1
/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 +
 b^4)*d^2))*arctan(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt((B^2*a^2 + 2*B^2*a*b
+ B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*
b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 2*sqrt(1/2)*(B*a^3 + B*a^2*b + B*a*b
^2 + B*b^3)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4
)*d^2))*sqrt(tan(d*x + c)))/(B^2*a^2 - B^2*b^2)) - sqrt(1/2)*(a^2 + b^2)*d
*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(2
*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^
2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)) + B*a + B*b + (B*a + B*b)*tan(d*x +
c)) + sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 +
 2*a^2*b^2 + b^4)*d^2))*log(-2*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 + 2*B
^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)) + B*a
+ B*b + (B*a + B*b)*tan(d*x + c)) + 2*B*a*sqrt(-a/b)*log((2*b*sqrt(-a/b)*s
qrt(tan(d*x + c)) + b*tan(d*x + c) - a)/(b*tan(d*x + c) + a)))/((a^2 + ...
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = B\int \frac{\tan^{\frac{3}{2}}(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
B*Integral(tan(c + d*x)**(3/2)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.93

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{\frac{8\,Ba^2\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}-\frac{\left(2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}(a-\cdots}{a^2+b^2}}{4\,d}$$

input
```
integrate(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
1/4*(8*B*a^2*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2 + b^2)*sqrt(a*b)
) - (2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))
) + sqrt(2)*(a + b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - s
qrt(2)*(a + b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*B/(a^2
 + b^2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 31.61 (sec) , antiderivative size = 16878, normalized size of antiderivative = 90.74

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(3/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x
)
```

output
```
(log((((((((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^
4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a
*b^5*d^2)/(d^4*(a^2 + b^2)^4))^(1/2) + (768*B*a^2*b^4*(a^2 + b^2))/d)*((4*
(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B
^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (64*B^2*a*b^2*tan(c + d*x)^(
1/2)*(2*a^8 + 15*b^8 - 17*a^2*b^6 + 51*a^4*b^4 + 21*a^6*b^2))/(d^2*(a^2 +
b^2)^2))*((4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b
^3*d^2 + 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (32*B^3*a*b^3*(
4*a^8 + b^8 - 77*a^2*b^6 + 47*a^4*b^4 + 33*a^6*b^2))/(d^3*(a^2 + b^2)^3))*
((4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 +
16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (16*B^4*b^3*tan(c + d*x)
^(1/2)*(a^10 - 2*b^10 - 4*a^2*b^8 - 27*a^4*b^6 + 15*a^6*b^4 + 9*a^8*b^2))/
(d^4*(a^2 + b^2)^4))*((4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) -
16*B^2*a^3*b^3*d^2 + 16*B^2*a*b^5*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (8*
B^5*a^2*b^4*(a^6 + 10*b^6 + 27*a^2*b^4 + 10*a^4*b^2))/(d^5*(a^2 + b^2)^4))
*(((192*B^4*a^2*b^10*d^4 - 16*B^4*b^12*d^4 - 608*B^4*a^4*b^8*d^4 + 192*B^4
*a^6*b^6*d^4 - 16*B^4*a^8*b^4*d^4)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a*b
^5*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4
))^(1/2))/4 + (log((((((((128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b
^2)^2*(-(4*(-B^4*b^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*...
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)}{a+\tan(dx+c)\,b}dx\right)b$$

input
```
int(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)*b + a),x)*b
```

3.424 $\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 184

$$\begin{aligned}\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx &= -\frac{(a+b)B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d} \\ &\quad+\frac{(a+b)B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d} \\ &\quad-\frac{2\sqrt{a}\sqrt{b}B\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\left(a^2+b^2\right)d}-\frac{(a-b)B\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\left(a^2+b^2\right)d}\end{aligned}$$

output
```
1/2*(a+b)*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)/d+1/2*(a
+b)*B*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)/d-2*a^(1/2)*b^(
1/2)*B*arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/(a^2+b^2)/d-1/2*(a-b)*B*ar
ctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{B\left(-6\sqrt{2}b\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+6\sqrt{2}b\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-24\sqrt{a}\sqrt{b}\arctan\left(\right.\right.}{}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
])^2,x]
```

output
```
(B*(-6*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 6*Sqrt[2]*b*ArcT
an[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - 24*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Sq
rt[Tan[c + d*x]])/Sqrt[a]] - 3*Sqrt[2]*b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]] + 3*Sqrt[2]*b*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
 + d*x]] + 8*a*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*Tan[c + d*x
]^(3/2)))/(12*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2011, 3042, 4055, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{2011}$$
$$B\int \frac{\sqrt{\tan(c+dx)}}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\sqrt{\tan(c+dx)}}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4055}$$

$$B\left(\frac{\int \frac{b+a\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{ab\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{\int \frac{b+a\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}-\frac{ab\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{4017}$$

$$B\left(\frac{2\int \frac{b+a\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}-\frac{ab\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{1482}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a-b)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d(a^2+b^2)}-\frac{ab\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan\ldots}}{a^2+b^2}\right.$$

$$\Big\downarrow \text{1476}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\ldots}{d(a^2+b^2)}\right.$$

$$\Big\downarrow \text{1082}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\ldots}{d(a^2+b^2)}\right.$$

$$\Big\downarrow \text{217}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{1479}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{1103}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{4117}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \; 73$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \; 218$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}\right.$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x
]
```

output
```
B*((-2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/((a^2
 + b^2)*d) + (2*(((a + b)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2
]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 - ((a - b)*(-1/2*L
og[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2
]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4055
```
Int[Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[Simp[a*c + b*d + (b*c -
 a*d)*Tan[e + f*x], x]/Sqrt[a + b*Tan[e + f*x]], x], x] - Simp[d*((b*c - a*
d)/(c^2 + d^2))   Int[(1 + Tan[e + f*x]^2)/(Sqrt[a + b*Tan[e + f*x]]*(c + d
*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{B\left(-\frac{2ab\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\left(a^2+b^2\right)\sqrt{ab}}+\frac{b\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\right.}{4}\right.}{d}$
default	$\frac{B\left(-\frac{2ab\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{\left(a^2+b^2\right)\sqrt{ab}}+\frac{b\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\right.}{4}\right.}{d}$

input
```
int(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/d*B*(-2*a*b/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))
+2/(a^2+b^2)*(1/8*b*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(t
an(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))+1/8*a*2^(1/2)*(ln((tan(d*x+c)-2^(
1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan
(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. 2(156) = 312.

Time = 0.12 (sec) , antiderivative size = 1792, normalized size of antiderivative = 9.74

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
[1/2*(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4
 + 2*a^2*b^2 + b^4)*d^2))*arctan(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt((B^2*a^
2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt((B^2*a^2 - 2*
B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*(B*a^3 - B
*a^2*b + B*a*b^2 - B*b^3)*d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(B^2*a^2 - B^2*b^2)) + 2*sqrt(1/
2)*(a^2 + b^2)*d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 +
b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt((B^2*a^2 + 2*B^2*a*b
+ B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*
b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 2*sqrt(1/2)*(B*a^3 - B*a^2*b + B*a*b
^2 - B*b^3)*d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4
)*d^2))*sqrt(tan(d*x + c)))/(B^2*a^2 - B^2*b^2)) + sqrt(1/2)*(a^2 + b^2)*d
*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(2
*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^
2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)) - B*a + B*b - (B*a - B*b)*tan(d*x +
c)) - sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 +
 2*a^2*b^2 + b^4)*d^2))*log(-2*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 - 2*B
^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)) - B*a
+ B*b - (B*a - B*b)*tan(d*x + c)) + 2*sqrt(-a*b)*B*log((b*tan(d*x + c) - a
 - 2*sqrt(-a*b)*sqrt(tan(d*x + c)))/(b*tan(d*x + c) + a))/((a^2 + b^2)...
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = B\int \frac{\sqrt{\tan(c+dx)}}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
B*Integral(sqrt(tan(c + d*x))/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{\frac{8\,Bab\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}} - \frac{\left(2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\right.}{a^2+b^2}}{4\,d}$$

input
```
integrate(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
-1/4*(8*B*a*b*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2 + b^2)*sqrt(a*b
)) - (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))
)) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))
)) - sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) +
sqrt(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*B/(a^
2 + b^2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 30.22 (sec) , antiderivative size = 17323, normalized size of antiderivative = 94.15

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(1/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x
)
```

output
```
(log(- ((((((((((256*B*a*b^3*(2*a^4 - b^4 + a^2*b^2))/d - 128*b^3*tan(c + d
*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b
^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))
^(1/2))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^
3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (64*B^2*a^3*b^2*
tan(c + d*x)^(1/2)*(a^6 + 17*b^6 - 29*a^2*b^4 + 19*a^4*b^2))/(d^2*(a^2 + b
^2)^2))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^
3*d^2 - 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (32*B^3*a^4*b^2*
(a^6 + 13*b^6 - 45*a^2*b^4 + 39*a^4*b^2))/(d^3*(a^2 + b^2)^3))*((4*(-B^4*a
^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*
b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (16*B^4*a^4*b^3*tan(c + d*x)^(1/2)*
(9*a^6 - 3*b^6 + 3*a^2*b^4 - 17*a^4*b^2))/(d^4*(a^2 + b^2)^4))*((4*(-B^4*a
^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*
b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 - (8*B^5*a^5*b^3*(9*a^4 - b^4))/(d^5*
(a^2 + b^2)^4))*(((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^12*
d^4 - 608*B^4*a^8*b^4*d^4 + 192*B^4*a^10*b^2*d^4)^(1/2) + 16*B^2*a^3*b^3*d
^2 - 16*B^2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4
+ 4*a^6*b^2*d^4))^(1/2))/4 + (log(- ((((((((((256*B*a*b^3*(2*a^4 - b^4 + a^
2*b^2))/d - 128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*(-(4*(-B^
4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^...
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2\sqrt{\tan(dx+c)} - \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2 b+\tan(dx+c)a}dx\right)ad - \left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+c)^2}{a+\tan(dx+c)b}dx\right)bd - \left(\int \frac{\sqrt{\tan(dx+c)}\,\tan(dx+}{a+\tan(dx+c)b}\right.}{d}$$

input

```
int(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output

```
(2*sqrt(tan(c + d*x)) - int(sqrt(tan(c + d*x))/(tan(c + d*x)**2*b + tan(c
+ d*x)*a),x)*a*d - int((sqrt(tan(c + d*x))*tan(c + d*x)**2)/(tan(c + d*x)*
b + a),x)*b*d - int((sqrt(tan(c + d*x))*tan(c + d*x))/(tan(c + d*x)*b + a)
,x)*a*d)/d
```

3.425 $\int \frac{aB+bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 185

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = -\frac{(a-b)B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a-b)B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{2b^{3/2}B\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,(a^2+b^2)\,d} + \frac{(a+b)B\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}$$

output
```
1/2*(a-b)*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)/d+1/2*(a
-b)*B*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)/d+2*b^(3/2)*B*a
rctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(1/2)/(a^2+b^2)/d+1/2*(a+b)*B*ar
ctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.24

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = B\left(\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}\,(a^2+b^2)\,d} - \frac{a\left(2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right) - 2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right) + \sqrt{2}\log\left(1-\sqrt{2}\sqrt{\mathrm{ta}\right.\right.}{4\,(a^2+b^2)\,d} - \frac{2b\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-\tan^2(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{3\,(a^2+b^2)\,d}\right)$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]
)^2),x]
```

output
```
B*((2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2
+ b^2)*d) - (a*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[
2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]))/(4*(a^2 + b^2)*d) - (2*b*Hypergeometric2F1[3/4, 1, 7/4,
-Tan[c + d*x]^2]*Tan[c + d*x]^(3/2))/(3*(a^2 + b^2)*d))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2011, 3042, 4057, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4057}$$

$$B\left(\frac{b^2\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{a-b\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{b^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{a-b\tan(c+dx)}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{4017}$$

$$B\left(\frac{2\int \frac{a-b\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{b^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}\right)$$

$$\Big\downarrow \text{1482}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a-b)\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d(a^2+b^2)}+\frac{b^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan}}{a^2+b^2}\right.$$

$$\Big\downarrow \text{1476}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a-b)\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{}{\tan(}\right.\right.}{d(a^2+b^2)}\right.$$

$$\Big\downarrow \text{1082}$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a-b)\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{}\right.}{\sqrt{2}}\right.\right.}{d(a^2+b^2)}\right.$$

$$\Big\downarrow \; 217$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}(a-b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \; 1479$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(a-b)\left(\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \; 25$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(a-b)\left(\frac{\arctan(}{}\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \; 27$$

$$B\left(\frac{2\left(\frac{1}{2}(a+b)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}(a-b\right.}{d\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \; 1103$$

$$B\left(\frac{b^2\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a+b\right.}{d\left(a^2+\right.}\right.$$

$$\Big\downarrow \; 4117$$

$$B\left(\frac{b^2\int \frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d(a^2+b^2)}+\frac{2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right.}{}\right.$$

$$\Big\downarrow 73$$

$$B\left(\frac{2b^2\int \frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a\right.}{d(a^2}\right.$$

$$\Big\downarrow 218$$

$$B\left(\frac{2\left(\frac{1}{2}(a-b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a+b)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}\right.\right.}{d(a^2+b^2)}\right.$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2),x
]
```

output
```
B*((2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a^2
+ b^2)*d) + (2*(((a - b)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]
) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 + ((a + b)*(-1/2*Lo
g[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]
*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4057
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)/((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[1/(c^2 + d^2)   Int[(a + b*Tan[e + f*x])^m
*(c - d*Tan[e + f*x]), x], x] + Simp[d^2/(c^2 + d^2)   Int[(a + b*Tan[e + f
*x])^m*((1 + Tan[e + f*x]^2)/(c + d*Tan[e + f*x])), x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0] &&  !IntegerQ[m]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{B\left(\frac{2b^2\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{\frac{a\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{t\right.}}{4}}{}\right.}{d}$
default	$\frac{B\left(\frac{2b^2\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{\frac{a\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{t\right.}}{4}}{}\right.}{d}$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/d*B*(2*b^2/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))+
2/(a^2+b^2)*(1/8*a*2^(1/2)*(ln((tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(ta
n(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))
+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/8*b*2^(1/2)*(ln((tan(d*x+c)-2^(1
/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(
1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. 2(158) = 316.

Time = 0.14 (sec) , antiderivative size = 1780, normalized size of antiderivative = 9.62

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
[-1/2*(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^
4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt((B^2*
a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt((B^2*a^2 -
2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*(B*a^3 +
 B*a^2*b + B*a*b^2 + B*b^3)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 +
 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(B^2*a^2 - B^2*b^2)) + 2*sqrt(
1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2
+ b^4)*d^2))*arctan(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt((B^2*a^2 + 2*B^2*a*b
 + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt((B^2*a^2 - 2*B^2*a*b + B^2
*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) - 2*sqrt(1/2)*(B*a^3 + B*a^2*b + B*a*
b^2 + B*b^3)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^
4)*d^2))*sqrt(tan(d*x + c)))/(B^2*a^2 - B^2*b^2)) - sqrt(1/2)*(a^2 + b^2)*
d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(
2*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a
^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)) + B*a + B*b + (B*a + B*b)*tan(d*x +
 c)) + sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4
+ 2*a^2*b^2 + b^4)*d^2))*log(-2*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 + 2*
B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)) + B*a
 + B*b + (B*a + B*b)*tan(d*x + c)) - 2*B*b*sqrt(-b/a)*log((2*a*sqrt(-b/a)*
sqrt(tan(d*x + c)) + b*tan(d*x + c) - a)/(b*tan(d*x + c) + a)))/((a^2 +...
```

Sympy [F]

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))^2}\,dx = B\int \frac{1}{a\sqrt{\tan(c + dx)} + b\tan^{\frac{3}{2}}(c + dx)}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**2,x)
```

output
```
B*Integral(1/(a*sqrt(tan(c + d*x)) + b*tan(c + d*x)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.93

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))^2}\,dx$$
$$= \frac{\frac{8\,Bb^2\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^2+b^2)\sqrt{ab}} + \frac{\left(2\sqrt{2}(a-b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a-b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}(a+}{a^2+b^2}}{4\,d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
1/4*(8*B*b^2*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^2 + b^2)*sqrt(a*b)
) + (2*sqrt(2)*(a - b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))
) + 2*sqrt(2)*(a - b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))
) + sqrt(2)*(a + b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - s
qrt(2)*(a + b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*B/(a^2
 + b^2))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 30.23 (sec) , antiderivative size = 16598, normalized size of antiderivative = 89.72

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int((B*a + B*b*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2),x
)
```

output
```
(log(((((((((((128*B*b^2*(2*b^6 - a^6 + 9*a^2*b^4 + 6*a^4*b^2))/d + 128*b^3
*tan(c + d*x)^(1/2)*(a^2 - b^2)*(a^2 + b^2)^2*((4*(-B^4*a^4*d^4*(a^4 + b^4
 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2
+ b^2)^4))^(1/2))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*
B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (64*B^
2*a*b^2*tan(c + d*x)^(1/2)*(2*b^8 - a^8 + 5*a^2*b^6 + 67*a^4*b^4 - a^6*b^2
))/(d^2*(a^2 + b^2)^2))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2)
 - 16*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 -
(32*B^3*a*b^5*(25*a^6 + b^6 - 13*a^2*b^4 - 85*a^4*b^2))/(d^3*(a^2 + b^2)^3
))*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2
 + 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (16*B^4*a^2*b^5*tan(c
 + d*x)^(1/2)*(b^6 - 27*a^6 + 7*a^2*b^4 + 11*a^4*b^2))/(d^4*(a^2 + b^2)^4)
)*((4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) - 16*B^2*a^3*b^3*d^2
+ 16*B^2*a^5*b*d^2)/(d^4*(a^2 + b^2)^4))^(1/2))/4 + (16*B^5*a^4*b^6*(5*a^2
 + b^2))/(d^5*(a^2 + b^2)^4))*(((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4
- 16*B^4*a^12*d^4 - 608*B^4*a^8*b^4*d^4 + 192*B^4*a^10*b^2*d^4)^(1/2) - 16
*B^2*a^3*b^3*d^2 + 16*B^2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 +
6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(1/2))/4 + (log(((((((((((128*B*b^2*(2*b^6
 - a^6 + 9*a^2*b^4 + 6*a^4*b^2))/d + 128*b^3*tan(c + d*x)^(1/2)*(a^2 - b^2)
*(a^2 + b^2)^2*(-(4*(-B^4*a^4*d^4*(a^4 + b^4 - 6*a^2*b^2)^2)^(1/2) + 16...
```

Reduce [F]

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^2}\,dx = \left(\int \frac{\sqrt{\tan(dx+c)}}{\tan(dx+c)^2 b + \tan(dx+c)\,a}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x)
```

output
```
int(sqrt(tan(c + d*x))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*b
```

3.426 $\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 202

$$\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \frac{(a+b)B\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(a+b)B\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{2b^{5/2}B\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}\,(a^2+b^2)\,d} + \frac{(a-b)B\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(c+dx)}}{1+\tan(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{2B}{ad\sqrt{\tan(c+dx)}}$$

output
```
-1/2*(a+b)*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)/d-1/2*(
a+b)*B*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)/d-2*b^(5/2)*B*
arctan(b^(1/2)*tan(d*x+c)^(1/2)/a^(1/2))/a^(3/2)/(a^2+b^2)/d+1/2*(a-b)*B*a
rctanh(2^(1/2)*tan(d*x+c)^(1/2)/(1+tan(d*x+c)))*2^(1/2)/(a^2+b^2)/d-2*B/a/
d/tan(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.65

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{B\left(-(-1)^{3/4}(a+ib)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right) - \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}} + \sqrt[4]{-1}(ia+b)\text{arctanh}\left(\right.\right.}{(a^2+b^2)\,d}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]
)^2),x]
```

output
```
(B*(-((-1)^(3/4)*(a + I*b)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]) - (2*b^(
5/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/a^(3/2) + (-1)^(1/4)*(I
*a + b)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] - (2*(a^2 + b^2))/(a*Sqrt[T
an[c + d*x]])))/((a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.13, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2011, 3042, 4052, 27, 3042, 4136, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$$
$$\Big\downarrow \text{2011}$$
$$B\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx$$
$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4052}$$

$$B\left(-\frac{2\int \frac{b\tan^2(c+dx)+a\tan(c+dx)+b}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(-\frac{\int \frac{b\tan^2(c+dx)+a\tan(c+dx)+b}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(-\frac{\int \frac{b\tan(c+dx)^2+a\tan(c+dx)+b}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a}-\frac{2}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{4136}$$

$$B\left(-\frac{\frac{\int \frac{\tan(c+dx)a^2+ba}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b^3\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(-\frac{\frac{\int \frac{\tan(c+dx)a^2+ba}{\sqrt{\tan(c+dx)}}dx}{a^2+b^2}+\frac{b^3\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{4017}$$

$$B\left(-\frac{\frac{2\int \frac{a(b+a\tan(c+dx))}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{b^3\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(-\frac{\frac{2a\int \frac{b+a\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{b^3\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{1482}$$

$$B\left(-\frac{\frac{2a\left(\frac{1}{2}(a+b)\int\frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}(a-b)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d(a^2+b^2)}+\frac{b^3\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}}{a}-\frac{2}{ad\sqrt{\tan(\ldots}}\right.$$

$\downarrow$ 1476

$$B\left(-\frac{\frac{2a\left(\frac{1}{2}(a+b)\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}(a-b)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\ldots}{d(a^2+b^2)}}{a}\right.$$

$\downarrow$ 1082

$$B\left(-\frac{\frac{2a\left(\frac{1}{2}(a+b)\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d(a^2+b^2)}+\ldots}{a}\right.$$

$\downarrow$ 217

$$B\left(-\frac{\frac{2a\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d(a^2+b^2)}+\frac{b^3\int\frac{\tan(c+dx)^2+\ldots}{\sqrt{\tan(c+dx)}(a+b\tan\ldots}}{a^2+b^2}}{a}\right.$$

$\downarrow$ 1479

$$B\left(-\frac{\frac{2a\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\ldots}{\tan(c+\ldots}}{\ldots}\right.\right.}{d(a^2+b^2)}}{a}\right.$$

$\downarrow$ 25

$$B\left(-\frac{\frac{2a\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\ldots}\right.}{\tan(c+dx)+\ldots}\ldots}{\ldots}\right.\ldots}{d(a^2+b^2)}\ldots}{a}\ldots\right.$$

$\downarrow$ 27

$$B\left(-\frac{\frac{2a\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\ldots}{\tan(c+dx\ldots}\ldots\right.\ldots}{d(a^2+b^2)}\ldots}{a}\ldots\right.$$

$\downarrow$ 1103

$$B\left(-\frac{\frac{b^3\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}dx}{a^2+b^2}+\frac{2a\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\ldots}\right.}{2\sqrt{2}}\ldots\right.\ldots}{d(a^2+b^2)}\ldots}{a}\ldots\right.$$

$\downarrow$ 4117

$$B\left(-\frac{\frac{b^3\int\frac{1}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))}d\tan(c+dx)}{d(a^2+b^2)}+\frac{2a\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(\tan(c+\ldots\right.}{\ldots}\ldots\right.\ldots}{d(a^2+b^2)}\ldots}{a}\ldots\right.$$

$\downarrow$ 73

$$B\left(-\frac{\frac{2b^3\int\frac{1}{a+b\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d(a^2+b^2)}+\frac{2a\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\ldots}\right.}{2\sqrt{2}}\ldots\right.\ldots}{d(a^2+b^2)}\ldots}{a}\ldots\right.$$

$\downarrow$ 218

$$B\left(-\frac{\frac{2a\left(\frac{1}{2}(a+b)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a-b)\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(\right.}{2\sqrt{2}}\right.}{d(a^2+b^2)}}{a}\right.$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2),x
]
```

output
```
B*(-(((2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*(a
^2 + b^2)*d) + (2*a*(((a + b)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sq
rt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]))/2 - ((a - b)*(-1
/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d)
)/a) - 2/(a*d*Sqrt[Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{B\left(-\frac{2b^3\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{a(a^2+b^2)\sqrt{ab}}-\frac{2}{a\sqrt{\tan(dx+c)}}+\frac{-\frac{b\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+}{4}$
default	$\frac{B\left(-\frac{2b^3\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{a(a^2+b^2)\sqrt{ab}}-\frac{2}{a\sqrt{\tan(dx+c)}}+\frac{-\frac{b\sqrt{2}\left(\ln\left(\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\tan(dx+c)-\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)+}{4}$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/d*B*(-2/a*b^3/(a^2+b^2)/(a*b)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2
))-2/a/tan(d*x+c)^(1/2)+2/(a^2+b^2)*(-1/8*b*2^(1/2)*(ln((tan(d*x+c)+2^(1/2
)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1))+2*arctan(1+
2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2)))-1/8*a*2^(
1/2)*(ln((tan(d*x+c)-2^(1/2)*tan(d*x+c)^(1/2)+1)/(tan(d*x+c)+2^(1/2)*tan(d
*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 956 vs. 2(173) = 346.

Time = 0.15 (sec) , antiderivative size = 1942, normalized size of antiderivative = 9.61

$$\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algo
rithm="fricas")
```

output
```
[1/2*(2*B*b^2*sqrt(-b/a)*log(-(2*a*sqrt(-b/a)*sqrt(tan(d*x + c)) - b*tan(d
*x + c) + a)/(b*tan(d*x + c) + a))*tan(d*x + c) - 2*sqrt(1/2)*(a^3 + a*b^2
)*d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*ar
ctan(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a
^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*(B*a^3 - B*a^2*b + B*a*b^2 - B*b^3)*d*
sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(t
an(d*x + c)))/(B^2*a^2 - B^2*b^2))*tan(d*x + c) - 2*sqrt(1/2)*(a^3 + a*b^2
)*d*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*ar
ctan(-((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((
a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 +
2*a^2*b^2 + b^4)*d^2)) - 2*sqrt(1/2)*(B*a^3 - B*a^2*b + B*a*b^2 - B*b^3)*d
*sqrt((B^2*a^2 + 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(
tan(d*x + c)))/(B^2*a^2 - B^2*b^2))*tan(d*x + c) - sqrt(1/2)*(a^3 + a*b^2)
*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log
(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*
a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)) - B*a + B*b - (B*a - B*b)*tan(d*x
+ c))*tan(d*x + c) + sqrt(1/2)*(a^3 + a*b^2)*d*sqrt((B^2*a^2 - 2*B^2*a*b +
 B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(-2*sqrt(1/2)*(a^2 + b^2)*d*sq
rt((B^2*a^2 - 2*B^2*a*b + B^2*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(...
```

Sympy [F]

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^2}\,dx = B\int \frac{1}{a\tan^{\frac{3}{2}}(c + dx) + b\tan^{\frac{5}{2}}(c + dx)}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**2,x)
```

output
```
B*Integral(1/(a*tan(c + d*x)**(3/2) + b*tan(c + d*x)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.93

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{8\,Bb^3\arctan\left(\frac{b\sqrt{\tan(dx+c)}}{\sqrt{ab}}\right)}{(a^3+ab^2)\sqrt{ab}} + \frac{\left(2\sqrt{2}(a+b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}(a+b)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\cdots}{a^2+b^2\cdots}}{4\,d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algo
rithm="maxima")
```

output
```
-1/4*(8*B*b^3*arctan(b*sqrt(tan(d*x + c))/sqrt(a*b))/((a^3 + a*b^2)*sqrt(a
*b)) + (2*sqrt(2)*(a + b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c
)))) + 2*sqrt(2)*(a + b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c
)))) - sqrt(2)*(a - b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1)
+ sqrt(2)*(a - b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))*B/(
a^2 + b^2) + 8*B/(a*sqrt(tan(d*x + c))))/d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{aB + bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 23.14 (sec) , antiderivative size = 22906, normalized size of antiderivative = 113.40

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^2} \, dx = \text{Too large to display}$$

input
```
int((B*a + B*b*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^2),x
)
```

output
```
(log(((((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^12*d^4 - 608*
B^4*a^8*b^4*d^4 + 192*B^4*a^10*b^2*d^4)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*B^
2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^
2*d^4))^(1/2)*(((((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^12*
d^4 - 608*B^4*a^8*b^4*d^4 + 192*B^4*a^10*b^2*d^4)^(1/2) + 16*B^2*a^3*b^3*d
^2 - 16*B^2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4
+ 4*a^6*b^2*d^4))^(1/2)*(((tan(c + d*x)^(1/2)*(1152*B^2*a^8*b^26*d^7 + 134
40*B^2*a^10*b^24*d^7 + 69056*B^2*a^12*b^22*d^7 + 202752*B^2*a^14*b^20*d^7
+ 372800*B^2*a^16*b^18*d^7 + 443136*B^2*a^18*b^16*d^7 + 337792*B^2*a^20*b^
14*d^7 + 156160*B^2*a^22*b^12*d^7 + 37632*B^2*a^24*b^10*d^7 + 3200*B^2*a^2
6*b^8*d^7 + 704*B^2*a^28*b^6*d^7 + 512*B^2*a^30*b^4*d^7 + 64*B^2*a^32*b^2*
d^7) - ((((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b^8*d^4 - 16*B^4*a^12*d^4 - 60
8*B^4*a^8*b^4*d^4 + 192*B^4*a^10*b^2*d^4)^(1/2) + 16*B^2*a^3*b^3*d^2 - 16*
B^2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6*d^4 + 6*a^4*b^4*d^4 + 4*a^6*
b^2*d^4))^(1/2)*((tan(c + d*x)^(1/2)*(((192*B^4*a^6*b^6*d^4 - 16*B^4*a^4*b
^8*d^4 - 16*B^4*a^12*d^4 - 608*B^4*a^8*b^4*d^4 + 192*B^4*a^10*b^2*d^4)^(1/
2) + 16*B^2*a^3*b^3*d^2 - 16*B^2*a^5*b*d^2)/(a^8*d^4 + b^8*d^4 + 4*a^2*b^6
*d^4 + 6*a^4*b^4*d^4 + 4*a^6*b^2*d^4))^(1/2)*(512*a^9*b^27*d^9 + 5120*a^11
*b^25*d^9 + 22528*a^13*b^23*d^9 + 56320*a^15*b^21*d^9 + 84480*a^17*b^19*d^
9 + 67584*a^19*b^17*d^9 - 67584*a^23*b^13*d^9 - 84480*a^25*b^11*d^9 - 5...
```

Reduce [F]

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^2}\,dx = \left(\int \frac{\sqrt{\tan(dx + c)}}{\tan(dx + c)^3 b + \tan(dx + c)^2 a}dx\right) b$$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x)
```

output
```
int(sqrt(tan(c + d*x))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*b
```

3.427 $\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 264

$$\begin{aligned}\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx &= \frac{\sqrt{ia-b}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} \\ &+ \frac{(4aAb-a^2B-8b^2B)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4b^{3/2}d} \\ &+ \frac{\sqrt{ia+b}(iA+B)\mathrm{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} \\ &+ \frac{(4Ab-aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4bd} \\ &+ \frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}\end{aligned}$$

output
```
(I*a-b)^(1/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+1/4*(4*A*a*b-B*a^2-8*B*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(
a+b*tan(d*x+c))^(1/2))/b^(3/2)/d+(I*a+b)^(1/2)*(I*A+B)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d+1/4*(4*A*b-B*a)*tan(d*x+c)^(
1/2)*(a+b*tan(d*x+c))^(1/2)/b/d+1/2*B*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3
/2)/b/d
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.15

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{-\sqrt{a}(-4aAb+a^2B+8b^2B)\,\mathrm{arcsinh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}+\sqrt{b}\left(-4\sqrt[4]{-1}\sqrt{-a+ib}b(iA+B)\right.}{}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x])
,x]
```

output
```
(-(Sqrt[a]*(-4*a*A*b + a^2*B + 8*b^2*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]
])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a]) + Sqrt[b]*(-4*(-1)^(1/4)*Sqrt[-a
 + I*b]*b*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/
Sqrt[a + b*Tan[c + d*x]]]*Sqrt[a + b*Tan[c + d*x]] + 4*(-1)^(3/4)*Sqrt[a +
 I*b]*b*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]]*Sqrt[a + b*Tan[c + d*x]] + Sqrt[Tan[c + d*x]]*(a +
b*Tan[c + d*x])*(4*A*b + a*B + 2*b*B*Tan[c + d*x])))/(4*b^(3/2)*d*Sqrt[a +
 b*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -\frac{\sqrt{a+b\tan(c+dx)}\left(-\left((4Ab-aB)\tan^2(c+dx)\right)+4bB\tan(c+dx)+aB\right)}{2\sqrt{\tan(c+dx)}}dx}{2b}+\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\int \frac{\sqrt{a+b\tan(c+dx)}\left(-\left((4Ab-aB)\tan^2(c+dx)\right)+4bB\tan(c+dx)+aB\right)}{\sqrt{\tan(c+dx)}}dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\int \frac{\sqrt{a+b\tan(c+dx)}\left(-\left((4Ab-aB)\tan(c+dx)^2\right)+4bB\tan(c+dx)+aB\right)}{\sqrt{\tan(c+dx)}}dx}{4b}$$

$$\Big\downarrow \text{4130}$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\int \frac{-\left(\left(-Ba^2+4Aba-8b^2B\right)\tan^2(c+dx)\right)+8b(Ab+aB)\tan(c+dx)+a(4Ab+aB)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{(4Ab-aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\frac{1}{2}\int \frac{-((-Ba^2+4Aba-8b^2B)\tan^2(c+dx))+8b(Ab+aB)\tan(c+dx)+a(4Ab+aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{(4Ab-aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}}{4b}$$

$\downarrow$ 3042

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\frac{1}{2}\int \frac{-((-Ba^2+4Aba-8b^2B)\tan(c+dx)^2)+8b(Ab+aB)\tan(c+dx)+a(4Ab+aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{(4Ab-aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}}{4b}$$

$\downarrow$ 4138

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\frac{\int \frac{-\left(\left(-Ba^2+4Aba-8b^2B\right)\tan^2(c+dx)\right)+8b(Ab+aB)\tan(c+dx)+a(4Ab+aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{2d}-\frac{(4Ab-aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}}{4b}$$

$\downarrow$ 2035

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\frac{\int \frac{-\left(\left(-Ba^2+4Aba-8b^2B\right)\tan^2(c+dx)\right)+8b(Ab+aB)\tan(c+dx)+a(4Ab+aB)}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{d}-\frac{(4Ab-aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}}{4b}$$

$\downarrow$ 2257

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\frac{\int \left(\frac{Ba^2-4Aba+8b^2B}{\sqrt{a+b\tan(c+dx)}}+\frac{8(b(aA-bB)+b(Ab+aB)\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}\right)d\sqrt{\tan(c+dx)}}{d}-\frac{(4Ab-aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}}{4b}$$

$\downarrow$ 2009

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{-\frac{(4Ab-aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}+\frac{-\frac{\left(a^2(-B)+4aAb-8b^2B\right)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}}-4b\sqrt{-b+ia}(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}}{4b}$$

input

```
Int[Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2))/(2*b*d) - ((-4*Sqrt[I*a
- b]*b*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]] - ((4*a*A*b - a^2*B - 8*b^2*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*
x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[b] - 4*b*Sqrt[I*a + b]*(I*A + B)*ArcT
anh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d - ((4*
A*b - a*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d)/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.63 (sec) , antiderivative size = 2183235, normalized size of antiderivative = 8269.83

output too large to display

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8056 vs. $2(212) = 424$.

Time = 3.33 (sec) , antiderivative size = 16118, normalized size of antiderivative = 61.05

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}\tan^{\frac{3}{2}}{(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(3/2)
, x)
```

Maxima [F]

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan{(dx+c)}+A)\sqrt{b\tan{(dx+c)}+a}\tan{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(3/2)
, x)
```

Giac [F]

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan{(dx+c)}+A)\sqrt{b\tan{(dx+c)}+a}\tan{(dx+c)}^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\sqrt{a+b\tan\left(c+d\,x\right)}\,dx$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\left(\int \sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,\tan\left(dx+c\right)^{2}dx\right)b$$
$$+\left(\int \sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,\tan\left(dx+c\right)dx\right)a$$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*b + int
(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a
```

3.428 $\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 201

$$\begin{aligned}\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx &= \frac{\sqrt{ia-b}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(2Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}d} \\ &\quad - \frac{\sqrt{ia+b}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}\end{aligned}$$

output
```
(I*a-b)^(1/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+(2*A*b+B*a)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^
(1/2))/b^(1/2)/d-(I*a+b)^(1/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1
/2)/(a+b*tan(d*x+c))^(1/2))/d+B*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.19

$$\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{\sqrt[4]{-1}\sqrt{-a+ib}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt[4]{-1}\sqrt{a+ib}(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x])
,x]
```

output
```
((-1)^(1/4)*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqr
t[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (-1)^(1/4)*Sqrt[a + I*b]*(A +
 I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]] + B*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + ((2*A*b + a*B
)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[a + b*Tan[c + d*x]])/
(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*Tan[c + d*x])/a]))/d
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 4093, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4093}$$

$$\int -\frac{-\left((2Ab+aB)\tan^2(c+dx)\right)-2(aA-bB)\tan(c+dx)+aB}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{1}{2}\int \frac{-\left((2Ab+aB)\tan^2(c+dx)\right)-2(aA-bB)\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{1}{2}\int \frac{-\left((2Ab+aB)\tan(c+dx)^2\right)-2(aA-bB)\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4138}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int \frac{-\left((2Ab+aB)\tan^2(c+dx)\right)-2(aA-bB)\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{2d}$$

$$\Big\downarrow \text{ 2035}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int \frac{-\left((2Ab+aB)\tan^2(c+dx)\right)-2(aA-bB)\tan(c+dx)+aB}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 2257}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int \left(\frac{-2Ab-aB}{\sqrt{a+b\tan(c+dx)}}+\frac{2(Ab+aB-(aA-bB)\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}\right)d\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{-\sqrt{-b+ia}(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-\frac{(aB+2Ab)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}}+\sqrt{b+ia}(A-iB)\text{arctanh}\Big(}{d}$$

input
```
Int[Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
-((-(Sqrt[I*a - b]*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]]) - ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x
]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[b] + Sqrt[I*a + b]*(A - I*B)*ArcTanh[(
Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d) + (B*Sqrt[
Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4093
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp
[1/(m + n)   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n - 1)*
Simp[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (A*b*c + a*B*c + a*A*d - b*B*d)*(m
 + n)*Tan[e + f*x] + (A*b*d*(m + n) + B*(a*d*m + b*c*n))*Tan[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[0, m, 1] && LtQ[0, n, 1]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.92 (sec) , antiderivative size = 2180690, normalized size of antiderivative = 10849.20

output too large to display

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7955 vs. $2(161) = 322$.

Time = 3.02 (sec) , antiderivative size = 15912, normalized size of antiderivative = 79.16

$$\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}\sqrt{\tan{(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*sqrt(tan(c + d*x)),
 x)
```

Maxima [F]

$$\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (B\tan{(dx+c)}+A)\sqrt{b\tan{(dx+c)}+a}\sqrt{\tan{(dx+c)}}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*sqrt(tan(d*x + c))
, x)
```

Giac [F]

$$\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\sqrt{\tan(dx+c)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*sqrt(tan(d*x + c))
, x)
```

Mupad [B] (verification not implemented)

Time = 119.55 (sec) , antiderivative size = 61200, normalized size of antiderivative = 304.48

$$\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
((2*B*a*tan(c + d*x)^(1/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (2*B*
a*b*tan(c + d*x)^(3/2))/((a + b*tan(c + d*x))^(1/2) - a^(1/2))^3)/(d + (b^
2*d*tan(c + d*x)^2)/((a + b*tan(c + d*x))^(1/2) - a^(1/2))^4 - (2*b*d*tan(
c + d*x))/((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2) - atan(((((A^2*b - A^2
*a*1i + B^2*a*1i - B^2*b + 2*A*B*a + A*B*b*2i)/(4*d^2))^(1/2)*((((A^2*b -
A^2*a*1i + B^2*a*1i - B^2*b + 2*A*B*a + A*B*b*2i)/(4*d^2))^(1/2)*((((A^2*b
 - A^2*a*1i + B^2*a*1i - B^2*b + 2*A*B*a + A*B*b*2i)/(4*d^2))^(1/2)*((((27
4877906944*(1600*a^12*b^34*d^8 - 16640*a^14*b^32*d^8 + 22784*a^16*b^30*d^8
 + 106496*a^18*b^28*d^8 + 65536*a^20*b^26*d^8))/d^8 - (274877906944*tan(c
+ d*x)*(1600*a^12*b^35*d^8 - 48000*a^14*b^33*d^8 + 155136*a^16*b^31*d^8 +
466944*a^18*b^29*d^8 + 262144*a^20*b^27*d^8))/(d^8*((a + b*tan(c + d*x))^(
1/2) - a^(1/2))^2))*((A^2*b - A^2*a*1i + B^2*a*1i - B^2*b + 2*A*B*a + A*B*
b*2i)/(4*d^2))^(1/2) - (2199023255552*tan(c + d*x)^(1/2)*(2048*A*a^20*b^27
*d^6 - 12536*A*a^16*b^31*d^6 - 3328*A*a^18*b^29*d^6 - 7160*A*a^14*b^33*d^6
 + 240*B*a^13*b^34*d^6 - 720*B*a^15*b^32*d^6 + 5696*B*a^17*b^30*d^6 + 1484
8*B*a^19*b^28*d^6 + 8192*B*a^21*b^26*d^6))/(d^7*((a + b*tan(c + d*x))^(1/2
) - a^(1/2))))*((A^2*b - A^2*a*1i + B^2*a*1i - B^2*b + 2*A*B*a + A*B*b*2i)
/(4*d^2))^(1/2) - (274877906944*(1200*A^2*a^12*b^35*d^6 - 1600*A^2*a^14*b^
33*d^6 + 272464*A^2*a^16*b^31*d^6 + 573952*A^2*a^18*b^29*d^6 + 299008*A^2*
a^20*b^27*d^6 - 1440*B^2*a^12*b^35*d^6 + 8352*B^2*a^14*b^33*d^6 - 320*B...
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}dx\right)a$$

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*b + int(sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a),x)*a
```

3.429 $\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 169

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{\sqrt{ia-b}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{2\sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{\sqrt{ia+b}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

output
```
-(I*a-b)^(1/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d+2*b^(1/2)*B*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))
^(1/2))/d-(I*a+b)^(1/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+
b*tan(d*x+c))^(1/2))/d
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{\sqrt[4]{-1}\left(\sqrt{-a+ib}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt{a+ib}(-iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right.\right.}{d}$$

input
```
Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x
]],x]
```

output
```
((-1)^(1/4)*(Sqrt[-a + I*b]*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sq
rt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + Sqrt[a + I*b]*((-I)*A + B)*A
rcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x
]]]) + (2*Sqrt[a]*Sqrt[b]*B*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*
Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 4097, 3042, 4099, 3042, 4098, 104, 216, 219, 4117, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4097}$$

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+bB\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$\downarrow$ 3042

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+bB\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$\downarrow$ 4099

$$\frac{1}{2}(a+ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+bB\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$\downarrow$ 3042

$$\frac{1}{2}(a+ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+bB\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$\downarrow$ 4098

$$\frac{(a-ib)(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{(a+ib)(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+bB\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$\downarrow$ 104

$$\frac{(a+ib)(A+iB)\int \frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(a-ib)(A-iB)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+bB\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$\downarrow$ 216

$$\frac{(a-ib)(A-iB)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+$$
$$bB\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{(a+ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}$$

$\Big\downarrow$ 219

$$bB\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{(a+ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{(a-ib)(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

$\Big\downarrow$ 4117

$$\frac{bB\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{(a+ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{(a-ib)(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

$\Big\downarrow$ 65

$$\frac{2bB\int \frac{1}{1-\frac{b\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(a+ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{(a-ib)(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

$\Big\downarrow$ 219

$$\frac{(a+ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{(a-ib)(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}+\frac{2\sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

input
```
Int[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x]
```

output
```
((a + I*b)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*Sqrt[Tan
[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + ((a - I*b)*(A - I*B)*ArcTanh[(S
qrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]
*d)
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] && !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4097
```
Int[(Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/Sqrt[(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] :> In
t[Simp[a*A - b*B + (A*b + a*B)*Tan[e + f*x], x]/(Sqrt[a + b*Tan[e + f*x]]*S
qrt[c + d*Tan[e + f*x]]), x] + Simp[b*B   Int[(1 + Tan[e + f*x]^2)/(Sqrt[a
+ b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.95 (sec) , antiderivative size = 2178123, normalized size of antiderivative = 12888.30

output too large to display

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7969 vs. $2(133) = 266$.

Time = 2.37 (sec) , antiderivative size = 15940, normalized size of antiderivative = 94.32

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}}{\sqrt{\tan{(c+dx)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))/sqrt(tan(c + d*x)),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}}{\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)/sqrt(tan(d*x + c))
, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 1141, normalized size of antiderivative = 6.75

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(1/2),x
)
```

output
```
atanh((a^(3/2)*d*tan(c + d*x)^(1/2)*(((-A^4*a^2*d^4)^(1/2) - A^2*b*d^2)/d^
4)^(1/2)*(-A^4*a^2*d^4)^(1/2) - a*d*tan(c + d*x)^(1/2)*(((-A^4*a^2*d^4)^(1
/2) - A^2*b*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2)*(-A^4*a^2*d^4)^(1/2
) + A^2*a^(3/2)*b*d^3*tan(c + d*x)^(1/2)*(((-A^4*a^2*d^4)^(1/2) - A^2*b*d^
2)/d^4)^(1/2) - A^2*a*b*d^3*tan(c + d*x)^(1/2)*(((-A^4*a^2*d^4)^(1/2) - A^
2*b*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2))/(A^3*a^3*d^2 - A*a*b*(-A^4
*a^2*d^4)^(1/2) - A*b^2*tan(c + d*x)*(-A^4*a^2*d^4)^(1/2) - A^3*a^(5/2)*d^
2*(a + b*tan(c + d*x))^(1/2) + A^3*a^2*b*d^2*tan(c + d*x) + A*a^(1/2)*b*(a
 + b*tan(c + d*x))^(1/2)*(-A^4*a^2*d^4)^(1/2)))*(((-A^4*a^2*d^4)^(1/2) - A
^2*b*d^2)/d^4)^(1/2) - atanh((a^(3/2)*d*tan(c + d*x)^(1/2)*(-((-A^4*a^2*d^
4)^(1/2) + A^2*b*d^2)/d^4)^(1/2)*(-A^4*a^2*d^4)^(1/2) - a*d*tan(c + d*x)^(
1/2)*(-((-A^4*a^2*d^4)^(1/2) + A^2*b*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^
(1/2)*(-A^4*a^2*d^4)^(1/2) - A^2*a^(3/2)*b*d^3*tan(c + d*x)^(1/2)*(-((-A^4
*a^2*d^4)^(1/2) + A^2*b*d^2)/d^4)^(1/2) + A^2*a*b*d^3*tan(c + d*x)^(1/2)*(
-((-A^4*a^2*d^4)^(1/2) + A^2*b*d^2)/d^4)^(1/2)*(a + b*tan(c + d*x))^(1/2))
/(A^3*a^3*d^2 + A*a*b*(-A^4*a^2*d^4)^(1/2) + A*b^2*tan(c + d*x)*(-A^4*a^2*
d^4)^(1/2) - A^3*a^(5/2)*d^2*(a + b*tan(c + d*x))^(1/2) + A^3*a^2*b*d^2*ta
n(c + d*x) - A*a^(1/2)*b*(a + b*tan(c + d*x))^(1/2)*(-A^4*a^2*d^4)^(1/2)))
*(-((-A^4*a^2*d^4)^(1/2) + A^2*b*d^2)/d^4)^(1/2) + atanh((2*((a^(1/2)*d*ta
n(c + d*x)^(1/2)*(((-B^4*a^2*d^4)^(1/2) + B^2*b*d^2)/d^4)^(1/2))/2 - (d...
```

Reduce [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,a-2\left(\int\frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{a+\tan(dx+c)b}dx\right)abd-\left(\int\frac{\sqrt{\tan(dx+c)}\,\sqrt{a}}{a+}\right.}{}$$

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a - 2*int((sqrt(tan(c + d*x
))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*a*b*d
 - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c +
 d*x)*b + a),x)*a**2*d + int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))
/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*a**2*d + 2*int(sqrt(tan(c + d*x))
*sqrt(tan(c + d*x)*b + a),x)*b*d)/(2*d)
```

3.430 $\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 154

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{\sqrt{ia-b}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{\sqrt{ia+b}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

output
```
-(I*a-b)^(1/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d+(I*a+b)^(1/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/
(a+b*tan(d*x+c))^(1/2))/d-2*A*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx =$$
$$-\frac{\sqrt[4]{-1}\sqrt{-a+ib}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt[4]{-1}\sqrt{a+ib}(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+i}}{\sqrt{a+b\tan}}\right.}{d}$$

input
```
Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/
2),x]
```

output
```
-(((-1)^(1/4)*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (-1)^(1/4)*Sqrt[a + I*b]*(A
 + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Ta
n[c + d*x]]] + (2*A*Sqrt[a + b*Tan[c + d*x]])/Sqrt[Tan[c + d*x]])/d)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4091, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$
$$\Big\downarrow \text{4091}$$
$$-2\int -\frac{Ab+aB-(aA-bB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4099}$$

$$-\frac{1}{2}(a+ib)(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}(a+ib)(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4098}$$

$$\frac{(a-ib)(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}-\frac{(a+ib)(-B+iA)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{(a+ib)(-B+iA)\int \frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(a-ib)(B+iA)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{(a-ib)(B+iA)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-$$
$$\frac{(a+ib)(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \ 219$$

$$-\frac{(a+ib)(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+$$
$$\frac{(a-ib)(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x]
```

output
```
-(((a + I*b)*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d)) + ((a - I*b)*(I*A + B)*ArcTanh[(Sqrt[
I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)
- (2*A*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.95 (sec) , antiderivative size = 2178365, normalized size of antiderivative = 14145.23

output too large to display

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7865 vs. $2(122) = 244$.

Time = 1.19 (sec) , antiderivative size = 7865, normalized size of antiderivative = 51.07

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(3/2)
, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\sqrt{a+b\tan\left(c+d\,x\right)}}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{-2\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,b^2+\left(\int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)b}}{\tan(dx+c)^3b+\tan(dx+c)^2a}dx\right)\tan\left(dx+c\right)a^3d-\left(\int \frac{\sqrt{\tan(dx}}{\tan(dx+}\right.}{\tan\left(dx+c\right)ad}$$

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
( - 2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*b**2 + int((sqrt(tan(c +
 d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x
)*tan(c + d*x)*a**3*d - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/
(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)*a*b**2*d + 2*int((
sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c +
d*x)*a),x)*tan(c + d*x)*a**2*b*d)/(tan(c + d*x)*a*d)
```

3.431 $\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 199

$$\begin{aligned}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\sqrt{ia-b}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} \\ &\quad + \frac{\sqrt{ia+b}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} \\ &\quad - \frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(Ab+3aB)\sqrt{a+b\tan(c+dx)}}{3ad\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
(I*a-b)^(1/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+(I*a+b)^(1/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(
a+b*tan(d*x+c))^(1/2))/d-2/3*A*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)-2
/3*(A*b+3*B*a)*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{-3\sqrt[4]{-1}\sqrt{-a+ib}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+3(-1)^{3/4}\sqrt{a+ib}(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\ldots}{\sqrt{\ldots}}\right.}{3d}$$

input
```
Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/
2),x]
```

output
```
(-3*(-1)^(1/4)*Sqrt[-a + I*b]*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 3*(-1)^(3/4)*Sqrt[a + I*b]
*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]] - (2*Sqrt[a + b*Tan[c + d*x]]*(a*A + (A*b + 3*a*B)*Tan[c +
 d*x]))/(a*Tan[c + d*x]^(3/2)))/(3*d)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {3042, 4091, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$
$$\Big\downarrow \text{4091}$$

$$-\frac{2}{3}\int -\frac{-2Ab\tan^2(c+dx)-3(aA-bB)\tan(c+dx)+Ab+3aB}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\int \frac{-2Ab\tan^2(c+dx)-3(aA-bB)\tan(c+dx)+Ab+3aB}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\int \frac{-2Ab\tan(c+dx)^2-3(aA-bB)\tan(c+dx)+Ab+3aB}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 4132$$

$$\frac{1}{3}\left(-\frac{2\int \frac{3(a(aA-bB)+a(Ab+aB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(-\frac{3\int \frac{a(aA-bB)+a(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\left(-\frac{3\int \frac{a(aA-bB)+a(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 4099$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{1}{2}a(a+ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a(a-ib)(A-\right.}{a}\right.$$

$\Big\downarrow$ 3042

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{1}{2}a(a+ib)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a(a-ib)(A-\right.}{a}\right.$$

$\Big\downarrow$ 4098

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a-ib)(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{a(a+ib)(A\right.}{a}\right.$$

$\Big\downarrow$ 104

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a+ib)(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{a(a-ib)(A-iB)\int\frac{}{1-\frac{(ia+b}{a+b}}\right.}{a}\right.$$

$\Big\downarrow$ 216

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a-ib)(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{a(a+ib)(A+iB)\arctan\left(\frac{\sqrt{}}{}\right.}{d\sqrt{-b+i}}\right.}{a}\right.$$

$\Big\downarrow$ 219

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+$$
$$\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a+ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{a(a-ib)(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{}}{\sqrt{a+b\text{ t}}}\right.}{d\sqrt{b+ia}}\right.}{a}\right.$$

input
```
Int[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x]
```

output
```
(-2*A*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) + ((-3*((a*(a + I
*b)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]]])/(Sqrt[I*a - b]*d) + (a*(a - I*b)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]
*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2
*(A*b + 3*a*B)*Sqrt[a + b*Tan[c + d*x]])/(a*d*Sqrt[Tan[c + d*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.96 (sec) , antiderivative size = 2181119, normalized size of antiderivative = 10960.40

output too large to display

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7946 vs. 2(159) = 318.

Time = 1.22 (sec) , antiderivative size = 7946, normalized size of antiderivative = 39.93

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}}{\tan^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(5/2)
, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan{(dx+c)}+A)\sqrt{b\tan{(dx+c)}+a}}{\tan{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)/tan(d*x + c)^(5/2)
, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\sqrt{a+b\tan\left(c+d\,x\right)}}{\tan\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(5/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(5/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{4\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,b-2\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,a+6\Big(\int \frac{\sqrt{\tan}}{\tan(}$$

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
(4*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*b - 2*sqrt(tan
(c + d*x))*sqrt(tan(c + d*x)*b + a)*a + 6*int((sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)*
*2*a*b*d - 3*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*
x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)**2*a**2*d + 3*int((sqrt(tan(c +
d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*ta
n(c + d*x)**2*b**2*d)/(3*tan(c + d*x)**2*d)
```

3.432 $\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 250

$$\begin{aligned}
&\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{\sqrt{ia-b}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} \\
&\quad - \frac{\sqrt{ia+b}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} \\
&\quad - \frac{2(Ab+5aB)\sqrt{a+b\tan(c+dx)}}{15ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2A+2Ab^2-5abB)\sqrt{a+b\tan(c+dx)}}{15a^2d\sqrt{\tan(c+dx)}}
\end{aligned}$$

output
```
(I*a-b)^(1/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d-(I*a+b)^(1/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(
a+b*tan(d*x+c))^(1/2))/d-2/5*A*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)-2
/15*(A*b+5*B*a)*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(3/2)+2/15*(15*A*a^2
+2*A*b^2-5*B*a*b)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{15\sqrt[4]{-1}\sqrt{-a+ib}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+15\sqrt[4]{-1}\sqrt{a+ib}(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a}\cdots}{\sqrt{a+}\cdots}\right.}{15d}$$

input
```
Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/
2),x]
```

output
```
(15*(-1)^(1/4)*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 15*(-1)^(1/4)*Sqrt[a + I*b
]*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
b*Tan[c + d*x]]] + (2*Sqrt[a + b*Tan[c + d*x]]*(-3*a^2*A - a*(A*b + 5*a*B)
*Tan[c + d*x] + (15*a^2*A + 2*A*b^2 - 5*a*b*B)*Tan[c + d*x]^2))/(a^2*Tan[c
 + d*x]^(5/2)))/(15*d)
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.16, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 4091, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{4091}$$

$$-\frac{2}{5}\int -\frac{-4Ab\tan^2(c+dx)-5(aA-bB)\tan(c+dx)+Ab+5aB}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}\int \frac{-4Ab\tan^2(c+dx)-5(aA-bB)\tan(c+dx)+Ab+5aB}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\int \frac{-4Ab\tan(c+dx)^2-5(aA-bB)\tan(c+dx)+Ab+5aB}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 4132$$

$$\frac{1}{5}\left(-\frac{2\int \frac{15Aa^2-5bBa+15(Ab+aB)\tan(c+dx)a+2Ab^2+2b(Ab+5aB)\tan^2(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{5}\left(-\frac{\int \frac{15Aa^2-5bBa+15(Ab+aB)\tan(c+dx)a+2Ab^2+2b(Ab+5aB)\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{5}\left(-\frac{\int \frac{15Aa^2-5bBa+15(Ab+aB)\tan(c+dx)a+2Ab^2+2b(Ab+5aB)\tan(c+dx)^2}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 4132$$

$$\frac{1}{5}\left(-\frac{-\frac{2\int -\frac{15\left(a^2(Ab+aB)-a^2(aA-bB)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right.$$

$$\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{5}\left(-\frac{\frac{15\int \frac{a^2(Ab+aB)-a^2(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)-$$

$$\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-\frac{\frac{15\int \frac{a^2(Ab+aB)-a^2(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)-$$

$$\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4099

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{1}{2}a^2(a-ib)(B+iA)\int \frac{i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b}}\right.}{}}{3a}\right.$$

$\downarrow$ 3042

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$
$$\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{1}{2}a^2(a-ib)(B+iA)\int\frac{i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b}}\right.}{}}{3a}\right.$$

$\downarrow$ 4098

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$
$$\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^2(a-ib)(B+iA)\int\frac{}{(1-i\tan(c+dx))\sqrt{\tan(}}}{2d}\right.}{}}{}\right.$$

$\downarrow$ 104

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$
$$\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^2(a-ib)(B+iA)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d}{d}\right.}{}}{3a}\right.$$

$\downarrow$ 216

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$
$$\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^2(a-ib)(B+iA)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d}{d}\right.}{}}{3a}\right.$$

$\downarrow$ 219

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^2(a-ib)(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan}}{\sqrt{a+b\tan(}}\right.}{d\sqrt{b+ia}}\right.}{}}{3a}\right.$$

input
```
Int[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x]
```

output
```
(-2*A*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) + ((-2*(A*b + 5*a
*B)*Sqrt[a + b*Tan[c + d*x]])/(3*a*d*Tan[c + d*x]^(3/2)) - ((15*(-((a^2*(a
 + I*b)*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/(Sqrt[I*a - b]*d)) + (a^2*(a - I*b)*(I*A + B)*ArcTanh[(Sqrt[I
*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))
/a - (2*(15*a^2*A + 2*A*b^2 - 5*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(a*d*Sqrt
[Tan[c + d*x]]))/(3*a))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.97 (sec) , antiderivative size = 2183172, normalized size of antiderivative = 8732.69

output too large to display

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7964 vs. $2(204) = 408$.

Time = 1.21 (sec) , antiderivative size = 7964, normalized size of antiderivative = 31.86

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\int \frac{(A+B\tan(c+d\,x))\ \sqrt{a+b\tan(c+d\,x)}}{\tan(c+d\,x)^{7/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(7/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(7/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$$
$$=\frac{8\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,b^2-4\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c}{}$$

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
(8*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*b**2 - 4*sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a*b - 2*sqrt(tan(c
+ d*x))*sqrt(tan(c + d*x)*b + a)*a**2 - 5*int((sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)*
*3*a**3*d + 5*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d
*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)**3*a*b**2*d - 10*int((sqrt(t
an(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a
),x)*tan(c + d*x)**3*a**2*b*d)/(5*tan(c + d*x)**3*a*d)
```

3.433 $\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 314

$$\begin{aligned}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx &= -\frac{\sqrt{ia-b}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} \\ &\quad -\frac{\sqrt{ia+b}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} \\ &\quad -\frac{2(Ab+7aB)\sqrt{a+b\tan(c+dx)}}{35ad\tan^{\frac{5}{2}}(c+dx)}+\frac{2(35a^2A+4Ab^2-7abB)\sqrt{a+b\tan(c+dx)}}{105a^2d\tan^{\frac{3}{2}}(c+dx)} \\ &\quad +\frac{2(35a^2Ab-8Ab^3+105a^3B+14ab^2B)\sqrt{a+b\tan(c+dx)}}{105a^3d\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
-(I*a-b)^(1/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d-(I*a+b)^(1/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/
(a+b*tan(d*x+c))^(1/2))/d-2/7*A*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(7/2)-
2/35*(A*b+7*B*a)*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(5/2)+2/105*(35*A*a
^2+4*A*b^2-7*B*a*b)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(3/2)+2/105*(3
5*A*a^2*b-8*A*b^3+105*B*a^3+14*B*a*b^2)*(a+b*tan(d*x+c))^(1/2)/a^3/d/tan(d
*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \frac{105\sqrt[4]{-1}\sqrt{-a+ib}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-105(-1)^{3/4}\sqrt{a+ib}(A+iB)\arctan\left(\frac{\sqrt[4]{}}{}\right.}{}$$

input
```
Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/
2),x]
```

output
```
(105*(-1)^(1/4)*Sqrt[-a + I*b]*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]
*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - 105*(-1)^(3/4)*Sqrt[a + I
*b]*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]] + (2*Sqrt[a + b*Tan[c + d*x]]*(-15*a^3*A - 3*a^2*(A*b +
 7*a*B)*Tan[c + d*x] + a*(35*a^2*A + 4*A*b^2 - 7*a*b*B)*Tan[c + d*x]^2 + (
35*a^2*A*b - 8*A*b^3 + 105*a^3*B + 14*a*b^2*B)*Tan[c + d*x]^3))/(a^3*Tan[c
 + d*x]^(7/2)))/(105*d)
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 4091, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{4091}$$

$$-\frac{2}{7}\int -\frac{-6Ab\tan^2(c+dx)-7(aA-bB)\tan(c+dx)+Ab+7aB}{2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{-6Ab\tan^2(c+dx)-7(aA-bB)\tan(c+dx)+Ab+7aB}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{-6Ab\tan(c+dx)^2-7(aA-bB)\tan(c+dx)+Ab+7aB}{\tan(c+dx)^{7/2}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4132}$$

$$\frac{1}{7}\left(-\frac{2\int \frac{35Aa^2-7bBa+35(Ab+aB)\tan(c+dx)a+4Ab^2+4b(Ab+7aB)\tan^2(c+dx)}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(-\frac{\int \frac{35Aa^2-7bBa+35(Ab+aB)\tan(c+dx)a+4Ab^2+4b(Ab+7aB)\tan^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(-\frac{\int \frac{35Aa^2-7bBa+35(Ab+aB)\tan(c+dx)a+4Ab^2+4b(Ab+7aB)\tan(c+dx)^2}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}\right)-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4132}$$

$$\frac{1}{7}\left(-\frac{-\frac{2\int -\frac{105Ba^3+35Aba^2-105(aA-bB)\tan(c+dx)a^2+14b^2Ba-8Ab^3-2b\left(35Aa^2-7bBa+4Ab^2\right)\tan^2(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(-\frac{\frac{\int \frac{105Ba^3+35Aba^2-105(aA-bB)\tan(c+dx)a^2+14b^2Ba-8Ab^3-2b\left(35Aa^2-7bBa+4Ab^2\right)\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(-\frac{\frac{\int \frac{105Ba^3+35Aba^2-105(aA-bB)\tan(c+dx)a^2+14b^2Ba-8Ab^3-2b\left(35Aa^2-7bBa+4Ab^2\right)\tan(c+dx)^2}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4132

$$\frac{1}{7}\left(-\frac{\frac{-\frac{2\int \frac{105\left((aA-bB)a^3+(Ab+aB)\tan(c+dx)a^3\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(-\frac{\frac{-\frac{105\int \frac{(aA-bB)a^3+(Ab+aB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+d}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(-\frac{\frac{-\frac{105\int \frac{(aA-bB)a^3+(Ab+aB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+d}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4099

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\,t}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 3042

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\,t}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 4098

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\tan}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$$\Big\downarrow \text{104}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\tan}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$$\Big\downarrow \text{216}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\tan}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$$\Big\downarrow \text{219}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7abB+4Ab^2)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{\ldots}}{\ldots}\right.$$

input
```
Int[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/2),x]
```

output
```
(-2*A*Sqrt[a + b*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(7/2)) + ((-2*(A*b + 7*a
*B)*Sqrt[a + b*Tan[c + d*x]])/(5*a*d*Tan[c + d*x]^(5/2)) - ((-2*(35*a^2*A
+ 4*A*b^2 - 7*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(3*a*d*Tan[c + d*x]^(3/2))
+ ((-105*((a^3*(a + I*b)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]
])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a^3*(a - I*b)*(A - I*B)
*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sq
rt[I*a + b]*d)))/a - (2*(35*a^2*A*b - 8*A*b^3 + 105*a^3*B + 14*a*b^2*B)*Sq
rt[a + b*Tan[c + d*x]])/(a*d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*a))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.00 (sec) , antiderivative size = 2185304, normalized size of antiderivative = 6959.57

output too large to display

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8039 vs. 2(262) = 524.

Time = 1.22 (sec) , antiderivative size = 8039, normalized size of antiderivative = 25.60

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}}{\tan(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)/tan(d*x + c)^(9/2)
, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \int \frac{(A+B\tan(c+d\,x))\,\sqrt{a+b\tan\left(c+d\,x\right)}}{\tan\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(9/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/tan(c + d*x)^(9/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{-140\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,a^2b+12\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(}{}$$

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)
```

output
```
( - 140*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*a**2*b
 + 12*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*b**3 + 7
0*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a**3 - 6*sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a*b**2 - 48*sqrt(
tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a**2*b - 30*sqrt(tan(c
 + d*x))*sqrt(tan(c + d*x)*b + a)*a**3 - 210*int((sqrt(tan(c + d*x))*sqrt(
tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*
x)**4*a**3*b*d + 105*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(ta
n(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)**4*a**4*d - 105*int((sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x
)*a),x)*tan(c + d*x)**4*a**2*b**2*d)/(105*tan(c + d*x)**4*a**2*d)
```

3.434 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 323

$$\begin{aligned}
\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A&\\
+B\tan(c+dx))\,dx &= \frac{(ia-b)^{3/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}\\
&+\frac{(6a^2Ab-16Ab^3-a^3B-24ab^2B)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{8b^{3/2}d}\\
&+\frac{(ia+b)^{3/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}\\
&+\frac{(6aAb-a^2B-8b^2B)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8bd}\\
&+\frac{(6Ab-aB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{12bd}\\
&+\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}
\end{aligned}$$

output
```
(I*a-b)^(3/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+1/8*(6*A*a^2*b-16*A*b^3-B*a^3-24*B*a*b^2)*arctanh(b^(1/2)*tan(
d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b^(3/2)/d+(I*a+b)^(3/2)*(A-I*B)*arcta
nh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d+1/8*(6*A*a*b-B
*a^2-8*B*b^2)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/b/d+1/12*(6*A*b-B*a)
*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)/b/d+1/3*B*tan(d*x+c)^(1/2)*(a+b*t
an(d*x+c))^(5/2)/b/d
```

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.07

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{24\sqrt[4]{-1}(-a+ib)^{3/2}b(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+24(-1)^{3/4}(a+ib)^{3/}$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
(24*(-1)^(1/4)*(-a + I*b)^(3/2)*b*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I
*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 24*(-1)^(3/4)*(a + I*b
)^(3/2)*b*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/S
qrt[a + b*Tan[c + d*x]]] - 3*(-6*a*A*b + a^2*B + 8*b^2*B)*Sqrt[Tan[c + d*x
]]*Sqrt[a + b*Tan[c + d*x]] + 2*(6*A*b - a*B)*Sqrt[Tan[c + d*x]]*(a + b*Ta
n[c + d*x])^(3/2) + 8*B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2) - (3
*Sqrt[a]*(-6*a^2*A*b + 16*A*b^3 + a^3*B + 24*a*b^2*B)*ArcSinh[(Sqrt[b]*Sqr
t[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a +
b*Tan[c + d*x]]))/(24*b*d)
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^{3/2}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -\frac{(a+b\tan(c+dx))^{3/2}\left(-\left((6Ab-aB)\tan^2(c+dx)\right)+6bB\tan(c+dx)+aB\right)}{2\sqrt{\tan(c+dx)}}dx}{3b}+\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\int \frac{(a+b\tan(c+dx))^{3/2}\left(-\left((6Ab-aB)\tan^2(c+dx)\right)+6bB\tan(c+dx)+aB\right)}{\sqrt{\tan(c+dx)}}dx}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\int \frac{(a+b\tan(c+dx))^{3/2}\left(-\left((6Ab-aB)\tan(c+dx)^2\right)+6bB\tan(c+dx)+aB\right)}{\sqrt{\tan(c+dx)}}dx}{6b}$$

$$\Big\downarrow \text{4130}$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{1}{2}\int \frac{3\sqrt{a+b\tan(c+dx)}\left(-\left(\left(-Ba^2+6Aba-8b^2B\right)\tan^2(c+dx)\right)+8b(Ab+aB)\tan(c+dx)+a(2Ab+aB)\right)}{2\sqrt{\tan(c+dx)}}dx-\frac{(6Ab-aB)\sqrt{\tan(c+dx)}(a+b\tan(}{2d}}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-$$
$$\frac{\frac{3}{4}\int\frac{\sqrt{a+b\tan(c+dx)}\left(-\left(\left(-Ba^2+6Aba-8b^2B\right)\tan^2(c+dx)\right)+8b(Ab+aB)\tan(c+dx)+a(2Ab+aB)\right)}{\sqrt{\tan(c+dx)}}dx-\frac{(6Ab-aB)\sqrt{\tan(c+dx)}(a+b\tan(c}{2d}}{6b}$$

$\downarrow$ 3042

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-$$
$$\frac{\frac{3}{4}\int\frac{\sqrt{a+b\tan(c+dx)}\left(-\left(\left(-Ba^2+6Aba-8b^2B\right)\tan(c+dx)^2\right)+8b(Ab+aB)\tan(c+dx)+a(2Ab+aB)\right)}{\sqrt{\tan(c+dx)}}dx-\frac{(6Ab-aB)\sqrt{\tan(c+dx)}(a+b\tan(c}{2d}}{6b}$$

$\downarrow$ 4130

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-$$
$$\frac{\frac{3}{4}\left(\int\frac{-\left(\left(-Ba^3+6Aba^2-24b^2Ba-16Ab^3\right)\tan^2(c+dx)\right)+16b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(Ba^2+10Aba-8b^2B\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{(a^2(-B)+6aAb-8}{}\right.}{6b}$$

$\downarrow$ 27

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-$$
$$\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{-\left(\left(-Ba^3+6Aba^2-24b^2Ba-16Ab^3\right)\tan^2(c+dx)\right)+16b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(Ba^2+10Aba-8b^2B\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{(a^2(-B)+6aAb-}{}\right.}{6b}$$

$\downarrow$ 3042

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-$$
$$\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{-\left(\left(-Ba^3+6Aba^2-24b^2Ba-16Ab^3\right)\tan(c+dx)^2\right)+16b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(Ba^2+10Aba-8b^2B\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{(a^2(-B)+6aAb-}{}\right.}{6b}$$

$\downarrow$ 4138

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-$$
$$\frac{\frac{3}{4}\left(\frac{\int\frac{-\left(\left(-Ba^3+6Aba^2-24b^2Ba-16Ab^3\right)\tan^2(c+dx)\right)+16b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(Ba^2+10Aba-8b^2B\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{2d}-\frac{(a^2(-B)+6aAb-8b^2}{}\right.}{6b}$$

$\downarrow$ 2035

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{3}{4}\left(\frac{\int\frac{-\left(\left(-Ba^3+6Aba^2-24b^2Ba-16Ab^3\right)\tan^2(c+dx)\right)+16b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(Ba^2+10Aba-8b^2B\right)}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{d}-\frac{(a^2(-B)+6aAb-8\cdots}{\cdots}\right.}{6b}$$

$\Big\downarrow$ 2257

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{3}{4}\left(\frac{\int\left(\frac{Ba^3-6Aba^2+24b^2Ba+16Ab^3}{\sqrt{a+b\tan(c+dx)}}+\frac{16\left(b\left(Aa^2-2bBa-Ab^2\right)+b\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}\right)d\sqrt{\tan(c+dx)}}{d}-\frac{(a^2(-B)+6aAb-8b^2B)\sqrt{\tan(c+\cdots}}{d}\right.}{6b}$$

$\Big\downarrow$ 2009

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{-\frac{(6Ab-aB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}+\frac{3}{4}\left(-\frac{(a^2(-B)+6aAb-8b^2B)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}+\frac{-\frac{\left(a^3(-B)+6a^2Ab-24ab^2B-\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

input
```
Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2))/(3*b*d) - (-1/2*((6*A*b
 - a*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2))/d + (3*((-8*(I*a - b
)^(3/2)*b*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*T
an[c + d*x]]] - ((6*a^2*A*b - 16*A*b^3 - a^3*B - 24*a*b^2*B)*ArcTanh[(Sqrt
[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[b] - 8*b*(I*a + b)
^(3/2)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/d - ((6*a*A*b - a^2*B - 8*b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a +
b*Tan[c + d*x]])/d))/4)/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.08 (sec) , antiderivative size = 2403184, normalized size of antiderivative = 7440.20

output too large to display

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12212 vs. 2(264) = 528.

Time = 4.97 (sec) , antiderivative size = 24426, normalized size of antiderivative = 75.62

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(3/
2), x)
```

Giac [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int(B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \tan(c+d\,x)^{3/2}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)^{3/2}\,dx$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)b^2 + 2\left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)ab + \left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)a^2$$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3,x)*b**2 +
2*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*a*b +
 int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a**2
```

3.435 $\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 260

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{(ia-b)^{3/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(12aAb+3a^2B-8b^2B)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4\sqrt{b}d} + \frac{(ia+b)^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(4Ab+5aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4d} + \frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

output
```
-(I*a-b)^(3/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d+1/4*(12*A*a*b+3*B*a^2-8*B*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/
2)/(a+b*tan(d*x+c))^(1/2))/b^(1/2)/d+(I*a+b)^(3/2)*(I*A+B)*arctanh((I*a+b)
^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d+1/4*(4*A*b+5*B*a)*tan(d*
x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d+1/2*b*B*tan(d*x+c)^(3/2)*(a+b*tan(d*x+
c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.12

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{-4\sqrt[4]{-1}(-a+ib)^{3/2}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+4\sqrt[4]{-1}(a+ib)^{3/2}(A$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
(-4*(-1)^(1/4)*(-a + I*b)^(3/2)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b
]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 4*(-1)^(1/4)*(a + I*b)^(
3/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]] + (4*A*b + 5*a*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c
+ d*x]] + 2*b*B*Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]] + (Sqrt[a]*(12
*a*A*b + 3*a^2*B - 8*b^2*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*
Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]]))/(4*d)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4090}$$

$$\frac{1}{2}\int \frac{\sqrt{\tan(c+dx)}\left(b(4Ab+5aB)\tan^2(c+dx)+4\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(4aA-3bB)\right)}{2\sqrt{a+b\tan(c+dx)}}dx+\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\int \frac{\sqrt{\tan(c+dx)}\left(b(4Ab+5aB)\tan^2(c+dx)+4\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(4aA-3bB)\right)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{4}\int \frac{\sqrt{\tan(c+dx)}\left(b(4Ab+5aB)\tan(c+dx)^2+4\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(4aA-3bB)\right)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 4130}$$

$$\frac{1}{4}\left(\frac{\int -\frac{-b\left(3Ba^2+12Aba-8b^2B\right)\tan^2(c+dx)-8b\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+ab(4Ab+5aB)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b}+\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{}}{d}\right.$$
$$\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int \frac{-b\left(3Ba^2+12Aba-8b^2B\right)\tan^2(c+dx)-8b\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{2b}\right.$$
$$\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int\frac{-b(3Ba^2+12Aba-8b^2B)\tan(c+dx)^2-8b(Aa^2-2bBa-Ab^2)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{2b}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\big\downarrow$ 4138

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int\frac{-b(3Ba^2+12Aba-8b^2B)\tan^2(c+dx)-8b(Aa^2-2bBa-Ab^2)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}}{2bd}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\big\downarrow$ 2035

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int\frac{-b(3Ba^2+12Aba-8b^2B)\tan^2(c+dx)-8b(Aa^2-2bBa-Ab^2)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}}{bd}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\big\downarrow$ 2257

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int\left(\frac{8(b(Ba^2+2Aba-b^2B)-b(Aa^2-2bBa-Ab^2)\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}-\frac{b(3Ba^2+}{\sqrt{a+b}}\right.}{bd}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}$$

$\big\downarrow$ 2009

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+$$

$$\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{-\sqrt{b}(3a^2B+12aAb-8b^2B)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+4}{}\right.$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(b*B*Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]])/(2*d) + (-((4*(I*a - b)^
(3/2)*b*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]] - Sqrt[b]*(12*a*A*b + 3*a^2*B - 8*b^2*B)*ArcTanh[(Sqrt[b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - 4*b*(I*a + b)^(3/2)*(I*A + B)*
ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(b*d
)) + ((4*A*b + 5*a*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.06 (sec) , antiderivative size = 2400957, normalized size of antiderivative = 9234.45

output too large to display

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12134 vs. 2(210) = 420.

Time = 4.47 (sec) , antiderivative size = 24274, normalized size of antiderivative = 93.36

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan(c+dx))\,(a+b\tan(c+dx))^{\frac{3}{2}}\,\sqrt{\tan(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*sqrt(tan(c + d*x
)), x)
```

Maxima [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\sqrt{\tan(dx+c)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x + c
)), x)
```

Giac [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\sqrt{\tan(dx+c)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \sqrt{\tan(c+dx)}\,(A + B\tan(c+dx))\,(a+b\tan(c+dx))^{3/2}\,dx$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)b^2 + 2\left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)ab + \left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}dx\right)a^2$$

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*b**2 +
2*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a*b + in
t(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a),x)*a**2
```

3.436 $\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 204

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = -\frac{(ia-b)^{3/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{\sqrt{b}(2Ab+3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{bB\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}$$

output
```
-(I*a-b)^(3/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d+b^(1/2)*(2*A*b+3*B*a)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*t
an(d*x+c))^(1/2))/d-(I*a+b)^(3/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)
^(1/2)/(a+b*tan(d*x+c))^(1/2))/d+b*B*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/
2)/d
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{-\sqrt[4]{-1}(-a+ib)^{3/2}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(}}{\sqrt{a+b\tan(c+dx)}}\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d
*x]],x]
```

output
```
(-((-1)^(1/4)*(-a + I*b)^(3/2)*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]
*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]) - (-1)^(3/4)*(a + I*b)^(3/
2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]] + b*B*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + (Sqr
t[a]*Sqrt[b]*(2*A*b + 3*a*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]
*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3042, 4090, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4090}$$

$$\int \frac{b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA-bB)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{bB\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\int \frac{b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA-bB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{bB\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\int \frac{b(2Ab+3aB)\tan(c+dx)^2+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA-bB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{bB\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 4138$$

$$\frac{\int \frac{b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA-bB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{2d}+\frac{bB\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 2035$$

$$\frac{\int \frac{b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA-bB)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{d}+\frac{bB\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 2257$$

$$\frac{\int \left(\frac{b(2Ab+3aB)}{\sqrt{a+b\tan(c+dx)}}+\frac{2\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right)d\sqrt{\tan(c+dx)}}{d}+\frac{bB\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{bB\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}+\frac{-\left((-b+ia)^{3/2}(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\right)+\sqrt{b}(3aB+2Ab)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-(b+ia)^{3/2}(}{d}$$

input
```
Int[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x
]
```

output
```
(-((I*a - b)^(3/2)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]]) + Sqrt[b]*(2*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (I*a + b)^(3/2)*(A - I*B)*ArcTanh
[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + (b*B*Sq
rt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.10 (sec) , antiderivative size = 2398415, normalized size of antiderivative = 11756.94

output too large to display

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12109 vs. 2(164) = 328.

Time = 4.21 (sec) , antiderivative size = 24220, normalized size of antiderivative = 118.73

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^{\frac{3}{2}}}{\sqrt{\tan{(c+dx)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)/sqrt(tan(c + d*x
)), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\tan{(dx+c)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)/sqrt(tan(d*x + c
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}}{\sqrt{\tan\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(1/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{2\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,a^2-2\Big(\int \frac{\sqrt{\tan(dx+}}{}$$

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a**2 - 2*int((sqrt(tan(c +
d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*a*
*2*b*d - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(t
an(c + d*x)*b + a),x)*a**3*d + int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b
 + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*a**3*d + 2*int(sqrt(tan(c +
 d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*b**2*d + 4*int(sqrt(tan(c
+ d*x))*sqrt(tan(c + d*x)*b + a),x)*a*b*d)/(2*d)
```

3.437 $\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 209

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{(a+ib)^2(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}d}+\frac{2b^{3/2}B\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{3/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

output
```
-(a+I*b)^2*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^
(1/2))/(I*a-b)^(1/2)/d+2*b^(3/2)*B*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*t
an(d*x+c))^(1/2))/d-(I*a+b)^(3/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)
^(1/2)/(a+b*tan(d*x+c))^(1/2))/d-2*a*A*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)
^(1/2)
```

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.99

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt[4]{-1}(-a+ib)^{3/2}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+}}{\sqrt{a+b\tan(c+dx)}}\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
3/2),x]
```

output
```
((-1)^(1/4)*(-a + I*b)^(3/2)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (-1)^(1/4)*a*A*Sqrt[a + I*b
]*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]] - (-1)^(3/4)*A*Sqrt[a + I*b]*b*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqr
t[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (-1)^(3/4)*a*Sqrt[a + I*b]*B*
ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*
x]]] + (-1)^(1/4)*Sqrt[a + I*b]*b*B*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[
Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (2*a*A*Sqrt[a + b*Tan[c + d*x]]
)/Sqrt[Tan[c + d*x]] + (2*Sqrt[a]*b^(3/2)*B*ArcSinh[(Sqrt[b]*Sqrt[Tan[c +
d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 4088, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$2\int \frac{b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4138}$$

$$\frac{\int \frac{b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}-\frac{2aA\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{2035}$$

$$\frac{2\int \frac{b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{d}-\frac{2aA\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{2257}$$

$$\frac{2\int \left(\frac{Bb^2}{\sqrt{a+b\tan(c+dx)}}+\frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}\right)d\sqrt{\tan(c+dx)}}{d}-\frac{2aA\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{2\left(\frac{1}{2}(-b+ia)^{3/2}(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-\frac{1}{2}(b+ia)^{3/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+b^{3/2}\right.}{d}$$

input
```
Int[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x
]
```

output
```
(2*(((I*a - b)^(3/2)*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/S
qrt[a + b*Tan[c + d*x]]])/2 + b^(3/2)*B*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]
])/Sqrt[a + b*Tan[c + d*x]]] - ((I*a + b)^(3/2)*(I*A + B)*ArcTanh[(Sqrt[I*
a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/2))/d - (2*a*A*Sqrt[
a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.48 (sec) , antiderivative size = 2394889, normalized size of antiderivative = 11458.80

output too large to display

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12148 vs. $2(167) = 334$.

Time = 3.94 (sec) , antiderivative size = 24297, normalized size of antiderivative = 116.25

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^{\frac{3}{2}}}{\tan^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)/tan(c + d*x)**(3
/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,ab-2\left(\int \frac{\sqrt{\tan(dx+}}{}\right.}{}$$

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a*b - 2*int((sqrt(tan(c + d
*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*a*b
**2*d - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(ta
n(c + d*x)*b + a),x)*a**2*b*d + int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*
b + a))/tan(c + d*x)**2,x)*a**2*d + int((sqrt(tan(c + d*x))*sqrt(tan(c + d
*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*a**2*b*d + int(sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*b + a),x)*b**2*d)/d
```

3.438 $\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 196

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{(ia-b)^{3/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(ia+b)^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab+3aB)\sqrt{a+b\tan(c+dx)}}{3d\sqrt{\tan(c+dx)}}$$

output
```
(I*a-b)^(3/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+(I*a+b)^(3/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(
a+b*tan(d*x+c))^(1/2))/d-2/3*a*A*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)
-2/3*(4*A*b+3*B*a)*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.21

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{3\sqrt[4]{-1}\left((-a+ib)^{3/2}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan}}{\sqrt{a+b\tan(c+d}}\right.\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
5/2),x]
```

output
```
(3*(-1)^(1/4)*((-a + I*b)^(3/2)*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b
]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + I*(a + I*b)^(3/2)*(A + I
*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]])*Tan[c + d*x]^(3/2) - 3*b*B*Sqrt[a + b*Tan[c + d*x]] + (-2*a*A +
3*b*B)*Sqrt[a + b*Tan[c + d*x]] - 2*(4*A*b + 3*a*B)*Tan[c + d*x]*Sqrt[a +
b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4088, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2}{3}\int \frac{-b(2aA-3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+3aB)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{3}\int \frac{-b(2aA-3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+3aB)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\int \frac{-b(2aA-3bB)\tan(c+dx)^2-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+3aB)}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4132

$$\frac{1}{3}\left(-\frac{2\int \frac{3\left(a\left(Aa^2-2bBa-Ab^2\right)+a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{3}\left(-\frac{3\int \frac{a\left(Aa^2-2bBa-Ab^2\right)+a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(-\frac{3\int \frac{a\left(Aa^2-2bBa-Ab^2\right)+a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4099}$$

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{1}{2}a(a-ib)^2(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a(a+ib)^2(A\right.}{a}\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{1}{2}a(a-ib)^2(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a(a+ib)^2(A\right.}{a}\right.$$

$$\Big\downarrow \text{4098}$$

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a-ib)^2(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{a(a+ib)}{}\right.}{a}\right.$$

$$\Big\downarrow \text{104}$$

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a-ib)^2(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{a(a+ib)^2(A+iB)\int\frac{}{\frac{(ia-}{a+}}}{}\right.}{a}\right.$$

$$\Big\downarrow \text{216}$$

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a-ib)^2(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{a(a+ib)^2(A+iB)\arctan}{d\sqrt{-b}}\right.}{a}\right.$$

$\downarrow$ 219

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a+ib)^2(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{a(a-ib)^2(A-iB)\text{arctanh}\left(\frac{\sqrt{b+}}{\sqrt{a}}\right.}{d\sqrt{b+ia}}\right.}{a}\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x
]
```

output
```
(-2*a*A*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) + ((-3*((a*(a +
 I*b)^2*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/(Sqrt[I*a - b]*d) + (a*(a - I*b)^2*(A - I*B)*ArcTanh[(Sqrt[I*
a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/
a - (2*(4*A*b + 3*a*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.08 (sec) , antiderivative size = 2398858, normalized size of antiderivative = 12239.07

output too large to display

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12024 vs. 2(156) = 312.

Time = 2.14 (sec) , antiderivative size = 12024, normalized size of antiderivative = 61.35

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^{\frac{3}{2}}}{\tan^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)/tan(c + d*x)**(5
/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}}{\tan(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(5/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^{3/2}}{\tan(c+d\,x)^{5/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(5/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(5/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{4\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,\tan\left(dx+c\right)a^2b -$$

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
(4*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a**2*b - 6*sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*b**3 - 2*sqrt(tan(c
+ d*x))*sqrt(tan(c + d*x)*b + a)*a**3 + 9*int((sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)*
*2*a**3*b*d - 3*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c +
 d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)**2*a*b**3*d - 3*int((sqrt(
tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*
a),x)*tan(c + d*x)**2*a**4*d + 9*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)
*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)**2*a**2*b**2
*d)/(3*tan(c + d*x)**2*a*d)
```

3.439 $\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 259

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{(a+ib)^2(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} + \frac{(ia+b)^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)} - \frac{2(6Ab+5aB)\sqrt{a+b\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2A-3Ab^2-20abB)\sqrt{a+b\tan(c+dx)}}{15ad\sqrt{\tan(c+dx)}}$$

output
```
(a+I*b)^2*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(
1/2))/(I*a-b)^(1/2)/d+(I*a+b)^(3/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+
c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d-2/5*a*A*(a+b*tan(d*x+c))^(1/2)/d/tan(d*
x+c)^(5/2)-2/15*(6*A*b+5*B*a)*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(3/2)+2/
15*(15*A*a^2-3*A*b^2-20*B*a*b)*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{-30\sqrt[4]{-1}a\left((-a+ib)^{3/2}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+i\dots}}{\sqrt{a+b\tan\dots}}\right.\right.}{\dots}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
7/2),x]
```

output
```
(-30*(-1)^(1/4)*a*((-a + I*b)^(3/2)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a +
 I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (a + I*b)^(3/2)*(A +
 I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]])*Tan[c + d*x]^(5/2) - 15*a*b*B*Sqrt[a + b*Tan[c + d*x]] - 3*a*(
4*a*A - 5*b*B)*Sqrt[a + b*Tan[c + d*x]] - 4*a*(6*A*b + 5*a*B)*Tan[c + d*x]
*Sqrt[a + b*Tan[c + d*x]] + 4*(15*a^2*A - 3*A*b^2 - 20*a*b*B)*Tan[c + d*x]
^2*Sqrt[a + b*Tan[c + d*x]])/(30*a*d*Tan[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 4088, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2}{5}\int \frac{-b(4aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(6Ab+5aB)}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int \frac{-b(4aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(6Ab+5aB)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \frac{-b(4aA-5bB)\tan(c+dx)^2-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(6Ab+5aB)}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4132}$$

$$\frac{1}{5}\left(-\frac{2\int \frac{2ab(6Ab+5aB)\tan^2(c+dx)+15a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(15Aa^2-20bBa-3Ab^2\right)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(5aB+6Ab)\sqrt{a+b\tan(}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(-\frac{\int \frac{2ab(6Ab+5aB)\tan^2(c+dx)+15a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(15Aa^2-20bBa-3Ab^2\right)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(-\frac{\int \frac{2ab(6Ab+5aB)\tan(c+dx)^2+15a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(15Aa^2-20bBa-3Ab^2\right)}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c}{3d\tan^{\frac{3}{2}}(c+dx)}\right.$$
$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4132}$$

$$\frac{1}{5}\left(-\frac{-\frac{2\int -\frac{15\left(a^2\left(Ba^2+2Aba-b^2B\right)-a^2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2(5aB+6Ab)\cdots}{3d\tan\cdots}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(-\frac{\frac{15\int \frac{a^2\left(Ba^2+2Aba-b^2B\right)-a^2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2(5aB+6Ab)\sqrt{a+\cdots}}{3d\tan^{\frac{3}{2}}(c+\cdots}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-\frac{\frac{15\int \frac{a^2\left(Ba^2+2Aba-b^2B\right)-a^2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2(5aB+6Ab)\sqrt{a+\cdots}}{3d\tan^{\frac{3}{2}}(c+\cdots}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4099}$$

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{1}{2}a^2(a-ib)^2(B+iA)\int \frac{i\tan(c\cdots}{\sqrt{\tan(c+dx)}\sqrt{\cdots}}\cdots\right.}{\cdots}}{3a}\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{1}{2}a^2(a-ib)^2(B+iA)\int\frac{i\tan(c\cdots}{\sqrt{\tan(c+dx)}\sqrt{\cdots}}\cdots\right.}{\cdots}}{3a}\cdots\right.$$

$\downarrow$ 4098

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^2(a-ib)^2(B+iA)\int\frac{\cdots}{(1-i\tan(c+dx))\sqrt{\cdots}}}{2\cdots}\cdots\right.}{\cdots}}{\cdots}\cdots\right.$$

$\downarrow$ 104

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^2(a-ib)^2(B+iA)\int\frac{1}{1-\frac{(ia+b)\tan(c+\cdots}{a+b\tan(c+dx)}}\cdots}{d}\cdots\right.}{\cdots}}{3a}\cdots\right.$$

$\downarrow$ 216

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^2(a-ib)^2(B+iA)\int\frac{1}{1-\frac{(ia+b)\tan(c+\cdots}{a+b\tan(c+dx)}}\cdots}{d}\cdots\right.}{\cdots}}{3a}\cdots\right.$$

$\downarrow$ 219

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^2(a-ib)^2(B+iA)\operatorname{arctanh}\left(\frac{\sqrt{b+ia}}{\sqrt{a+b}}\right.}{d\sqrt{b+ia}}\right.}{}}{3a}\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x
]
```

output
```
(-2*a*A*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) + ((-2*(6*A*b +
 5*a*B)*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) - ((15*(-((a^2*
(a + I*b)^2*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]])/(Sqrt[I*a - b]*d)) + (a^2*(a - I*b)^2*(I*A + B)*ArcTanh[(
Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b
]*d)))/a - (2*(15*a^2*A - 3*A*b^2 - 20*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(d
*Sqrt[Tan[c + d*x]]))/(3*a))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.05 (sec) , antiderivative size = 2400946, normalized size of antiderivative = 9270.06

output too large to display

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12093 vs. 2(211) = 422.

Time = 2.15 (sec) , antiderivative size = 12093, normalized size of antiderivative = 46.69

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}}{\tan\left(c+d\,x\right)^{7/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(7/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(7/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{44\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,\tan\left(dx+c\right)^{2}b^{2}}{}$$

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output
```
(44*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*b**2 - 22*
sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a*b - 6*sqrt(tan(
c + d*x))*sqrt(tan(c + d*x)*b + a)*a**2 - 15*int((sqrt(tan(c + d*x))*sqrt(
tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*
x)**3*a**3*d + 45*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c
 + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)**3*a*b**2*d - 45*int((sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*
x)*a),x)*tan(c + d*x)**3*a**2*b*d + 15*int((sqrt(tan(c + d*x))*sqrt(tan(c
+ d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)**3*b**
3*d)/(15*tan(c + d*x)**3*d)
```

3.440 $\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 311

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = -\frac{(ia-b)^{3/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)} - \frac{2(8Ab+7aB)\sqrt{a+b\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(35a^2A-3Ab^2-42abB)\sqrt{a+b\tan(c+dx)}}{105ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2(140a^2Ab+6Ab^3+105a^3B-21ab^2B)\sqrt{a+b\tan(c+dx)}}{105a^2d\sqrt{\tan(c+dx)}}$$

output
```
-(I*a-b)^(3/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d-(I*a+b)^(3/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/
(a+b*tan(d*x+c))^(1/2))/d-2/7*a*A*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(7/2
)-2/35*(8*A*b+7*B*a)*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(5/2)+2/105*(35*A
*a^2-3*A*b^2-42*B*a*b)*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(3/2)+2/105*(
140*A*a^2*b+6*A*b^3+105*B*a^3-21*B*a*b^2)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan
(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \frac{-35a^3bB\sqrt{a+b\tan(c+dx)}-5a^3(6aA-7bB)\sqrt{a+$$

input
```
Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
9/2),x]
```

output
```
(-35*a^3*b*B*Sqrt[a + b*Tan[c + d*x]] - 5*a^3*(6*a*A - 7*b*B)*Sqrt[a + b*T
an[c + d*x]] - 6*a^3*(8*A*b + 7*a*B)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]]
 + a*Tan[c + d*x]^2*(-105*(-1)^(3/4)*a^2*((-a + I*b)^(3/2)*(A - I*B)*ArcTa
n[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]
 + (a + I*b)^(3/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + b*Tan[c + d*x]]])*Tan[c + d*x]^(3/2) + 2*a*(35*a^2*A - 3*
A*b^2 - 42*a*b*B)*Sqrt[a + b*Tan[c + d*x]] + 2*(140*a^2*A*b + 6*A*b^3 + 10
5*a^3*B - 21*a*b^2*B)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]]))/(105*a^3*d*T
an[c + d*x]^(7/2))
```

Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 4088, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2}{7}\int \frac{-b(6aA-7bB)\tan^2(c+dx)-7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+7aB)}{2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{-b(6aA-7bB)\tan^2(c+dx)-7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+7aB)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{-b(6aA-7bB)\tan(c+dx)^2-7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+7aB)}{\tan(c+dx)^{7/2}\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4132}$$

$$\frac{1}{7}\left(-\frac{2\int\frac{4ab(8Ab+7aB)\tan^2(c+dx)+35a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(35Aa^2-42bBa-3Ab^2\right)}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(7aB+8Ab)\sqrt{a+b\tan(}{5d\tan^{\frac{5}{2}}(c+dx)}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(-\frac{\int\frac{4ab(8Ab+7aB)\tan^2(c+dx)+35a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(35Aa^2-42bBa-3Ab^2\right)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c}{5d\tan^{\frac{5}{2}}(c+dx)}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(-\frac{\int\frac{4ab(8Ab+7aB)\tan(c+dx)^2+35a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(35Aa^2-42bBa-3Ab^2\right)}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c}{5d\tan^{\frac{5}{2}}(c+dx)}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4132$$

$$\frac{1}{7}\left(-\frac{-\frac{2\int-\frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-2b\left(35Aa^2-42bBa-3Ab^2\right)\tan^2(c+dx)a+\left(105Ba^3+140Aba^2-21b^2Ba+6Ab^3\right)a}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(35a^2A-42a}{3}}{5a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(-\frac{\frac{\int \frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-2b\left(35Aa^2-42bBa-3Ab^2\right)\tan^2(c+dx)a+\left(105Ba^3+140Aba^2-21b^2Ba+6Ab^3\right)a}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(35a^2A-42abB-3}{3d\tan}}{5a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(-\frac{\frac{\int \frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-2b\left(35Aa^2-42bBa-3Ab^2\right)\tan(c+dx)^2a+\left(105Ba^3+140Aba^2-21b^2Ba+6Ab^3\right)a}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(35a^2A-42abB-3}{3d\tan}}{5a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4132

$$\frac{1}{7}\left(-\frac{\frac{-\frac{2\int \frac{105\left(\left(Aa^2-2bBa-Ab^2\right)a^3+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2(35a^2A-42ab}{3d}}{5a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(-\frac{\frac{-\frac{105\int \frac{\left(Aa^2-2bBa-Ab^2\right)a^3+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2(35a^2A-42abB-}{3d\tan}}{5a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{7}\left(-\frac{-\frac{-\frac{105\int\frac{\left(Aa^2-2bBa-Ab^2\right)a^3+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2(35a^2A-42abB-}{3d\tan}}{5a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4099

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-42abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)\sqrt{a}}{d\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 3042

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-42abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)\sqrt{a}}{d\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 4098

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-42abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)\sqrt{a}}{d\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 104

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-42abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)\sqrt{a}}{d\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 216

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-42abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)\sqrt{a}}{d\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 219

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-42abB-3Ab^2)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)\sqrt{a}}{d\sqrt{\tan(c+dx)}}}{}}{}\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/2),x
]
```

output
```
(-2*a*A*Sqrt[a + b*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(7/2)) + ((-2*(8*A*b +
 7*a*B)*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) - ((-2*(35*a^2*
A - 3*A*b^2 - 42*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2))
 + ((-105*((a^3*(a + I*b)^2*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a^3*(a - I*b)^2*(A -
 I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]
)/(Sqrt[I*a + b]*d)))/a - (2*(140*a^2*A*b + 6*A*b^3 + 105*a^3*B - 21*a*b^2
*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*a))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.95 (sec) , antiderivative size = 2403086, normalized size of antiderivative = 7726.96

output too large to display

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12128 vs. 2(259) = 518.

Time = 2.10 (sec) , antiderivative size = 12128, normalized size of antiderivative = 39.00

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}}{\tan(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)/tan(d*x + c)^(9/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}}{\tan\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(9/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(9/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx=\frac{-28\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,\tan\left(dx+c\right)^{3}a}{}$$

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)
```

output
```
( - 28*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*a**2*b
+ 36*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*b**3 + 14
*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a**3 - 18*sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a*b**2 - 18*sqrt(
tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a**2*b - 6*sqrt(tan(c
+ d*x))*sqrt(tan(c + d*x)*b + a)*a**3 - 63*int((sqrt(tan(c + d*x))*sqrt(ta
n(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)
**4*a**3*b*d + 21*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c
 + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)**4*a*b**3*d + 21*int((sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*
x)*a),x)*tan(c + d*x)**4*a**4*d - 63*int((sqrt(tan(c + d*x))*sqrt(tan(c +
d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)**4*a**2*
b**2*d)/(21*tan(c + d*x)**4*a*d)
```

3.441 $\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 382

$$\begin{aligned}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx &= \frac{(ia-b)^{3/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} \\ &- \frac{(ia+b)^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)} \\ &- \frac{2(10Ab+9aB)\sqrt{a+b\tan(c+dx)}}{63d\tan^{\frac{7}{2}}(c+dx)} + \frac{2(21a^2A-Ab^2-24abB)\sqrt{a+b\tan(c+dx)}}{105ad\tan^{\frac{5}{2}}(c+dx)} \\ &+ \frac{2(126a^2Ab+4Ab^3+105a^3B-9ab^2B)\sqrt{a+b\tan(c+dx)}}{315a^2d\tan^{\frac{3}{2}}(c+dx)} \\ &- \frac{2(315a^4A-63a^2Ab^2+8Ab^4-420a^3bB-18ab^3B)\sqrt{a+b\tan(c+dx)}}{315a^3d\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
(I*a-b)^(3/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d-(I*a+b)^(3/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(
a+b*tan(d*x+c))^(1/2))/d-2/9*a*A*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(9/2)
-2/63*(10*A*b+9*B*a)*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)^(7/2)+2/105*(21*A
*a^2-A*b^2-24*B*a*b)*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(5/2)+2/315*(12
6*A*a^2*b+4*A*b^3+105*B*a^3-9*B*a*b^2)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(d*
x+c)^(3/2)-2/315*(315*A*a^4-63*A*a^2*b^2+8*A*b^4-420*B*a^3*b-18*B*a*b^3)*(
a+b*tan(d*x+c))^(1/2)/a^3/d/tan(d*x+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 33.13 (sec) , antiderivative size = 5445, normalized size of antiderivative = 14.25

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
11/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.04 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.13, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.629$, Rules used = {3042, 4088, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{11/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2}{9}\int \frac{-b(8aA-9bB)\tan^2(c+dx)-9\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(10Ab+9aB)}{2\tan^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{-b(8aA-9bB)\tan^2(c+dx)-9\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(10Ab+9aB)}{\tan^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\int \frac{-b(8aA-9bB)\tan(c+dx)^2-9\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(10Ab+9aB)}{\tan(c+dx)^{9/2}\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4132}$$

$$\frac{1}{9}\left(-\frac{2\int \frac{3\left(2ab(10Ab+9aB)\tan^2(c+dx)+21a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(21Aa^2-24bBa-Ab^2\right)\right)}{2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{7a}-\frac{2(9aB+10Ab)\sqrt{a+b\,t}}{7d\tan^{\frac{7}{2}}(c+d}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(-\frac{3\int \frac{2ab(10Ab+9aB)\tan^2(c+dx)+21a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(21Aa^2-24bBa-Ab^2\right)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{7a}-\frac{2(9aB+10Ab)\sqrt{a+b\tan}}{7d\tan^{\frac{7}{2}}(c+dx)}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{9}\left(-\frac{3\int \frac{2ab(10Ab+9aB)\tan(c+dx)^2+21a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(21Aa^2-24bBa-Ab^2\right)}{\tan(c+dx)^{7/2}\sqrt{a+b\tan(c+dx)}}dx}{7a}-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 4132

$$\frac{1}{9}\left(-\frac{3\left(-\frac{2\int -\frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-4b\left(21Aa^2-24bBa-Ab^2\right)\tan^2(c+dx)a+\left(105Ba^3+126Aba^2-9b^2Ba+4Ab^3\right)a}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(21a^2A-24\ldots}{\ldots}\right)}{7a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{9}\left(-\frac{3\left(\frac{\int \frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-4b\left(21Aa^2-24bBa-Ab^2\right)\tan^2(c+dx)a+\left(105Ba^3+126Aba^2-9b^2Ba+4Ab^3\right)a}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(21a^2A-24abB-\ldots}{5d\tan\ldots}\right)}{7a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{9}\left(-\frac{3\left(\frac{\int \frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-4b\left(21Aa^2-24bBa-Ab^2\right)\tan(c+dx)^2a+\left(105Ba^3+126Aba^2-9b^2Ba+4Ab^3\right)a}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(21a^2A-24abB-\ldots}{5d\tan\ldots}\right)}{7a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 4132

$$\frac{1}{9}\left(-\frac{3\left(-\frac{\frac{2\int \frac{315\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3+2b\left(105Ba^3+126Aba^2-9b^2Ba+4Ab^3\right)\tan^2(c+dx)a+\left(315Aa^4-420bBa^3-63Ab^2a^2-18b^3Ba+8Ab^4\right)a}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}d}{3a}}{5a}}{7a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{9}\left(-\frac{3\left(-\frac{\frac{\int \frac{315\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3+2b\left(105Ba^3+126Aba^2-9b^2Ba+4Ab^3\right)\tan^2(c+dx)a+\left(315Aa^4-420bBa^3-63Ab^2a^2-18b^3Ba+8Ab^4\right)a}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}}{5a}}{7a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(-\frac{3\left(-\frac{-\frac{\int \frac{315\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3+2b\left(105Ba^3+126Aba^2-9b^2Ba+4Ab^3\right)\tan(c+dx)^2a+\left(315Aa^4-420bBa^3-63Ab^2a^2-18b^3Ba+8Ab^4\right)a}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}\cdots}{5a}\cdots\right)}{7a}\cdots\right)$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 4132

$$\frac{1}{9}\left(-\frac{3\left(-\frac{-\frac{-\frac{2\int -\frac{315\left(a^4\left(Ba^2+2Aba-b^2B\right)-a^4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(315a^4A-420a^3bB-63a^2Ab^2-18ab^3B+8Ab^4\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\cdots}{5a}\cdots\right)}{7a}\cdots\right)$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{9}\left(-\frac{3\left(-\frac{-\frac{315\int \frac{a^4\left(Ba^2+2Aba-b^2B\right)-a^4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(315a^4A-420a^3bB-63a^2Ab^2-18ab^3B+8Ab^4\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2\left(105a^3\right.}{}}{5a}\right.}{7a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{9}\left(-\frac{3\left(-\frac{-\frac{315\int \frac{a^4\left(Ba^2+2Aba-b^2B\right)-a^4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(315a^4A-420a^3bB-63a^2Ab^2-18ab^3B+8Ab^4\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2\left(105a^3\right.}{}}{5a}\right.}{7a}\right.$$

$$\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 4099

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21a^2A-24abB-Ab^2)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+126a^2Ab-9ab^2B+4Ab^3\right)}{3d\tan^{\frac{3}{2}}(c+dx)}}{}\right.}{}\right.$$

$\downarrow$ 3042

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21a^2A-24abB-Ab^2)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+126a^2Ab-9ab^2B+4Ab^3\right)}{3d\tan^{\frac{3}{2}}(c+dx)}}{}\right.}{}\right.$$

$\downarrow$ 4098

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21a^2A-24abB-Ab^2)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{-\frac{2(105a^3B+126a^2Ab-9ab^2B+4Ab^3)}{3d\tan^{\frac{3}{2}}(c+dx)}}{}\right.}{}\right.$$

$\downarrow$ 104

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21a^2A-24abB-Ab^2)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{-\frac{2(105a^3B+126a^2Ab-9ab^2B+4Ab^3)}{3d\tan^{\frac{3}{2}}(c+dx)}}{}\right.}{}\right.$$

$\downarrow$ 216

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21a^2A-24abB-Ab^2)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+126a^2Ab-9ab^2B+4Ab^3\right)}{3d\tan^{\frac{3}{2}}(c+dx)}}{}\right.}{}\right.$$

$\downarrow$ 219

$$-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21a^2A-24abB-Ab^2)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{-\frac{2\left(105a^3B+126a^2Ab-9ab^2B+4Ab^3\right)}{3d\tan^{\frac{3}{2}}(c+dx)}}{}\right.}{}\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(11/2),
x]
```

output
```
(-2*a*A*Sqrt[a + b*Tan[c + d*x]])/(9*d*Tan[c + d*x]^(9/2)) + ((-2*(10*A*b
+ 9*a*B)*Sqrt[a + b*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(7/2)) - (3*((-2*(21*
a^2*A - A*b^2 - 24*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2
)) + ((-2*(126*a^2*A*b + 4*A*b^3 + 105*a^3*B - 9*a*b^2*B)*Sqrt[a + b*Tan[c
 + d*x]])/(3*d*Tan[c + d*x]^(3/2)) - ((315*(-((a^4*(a + I*b)^2*(I*A - B)*A
rcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[
I*a - b]*d)) + (a^4*(a - I*b)^2*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[
c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(315*a^4*
A - 63*a^2*A*b^2 + 8*A*b^4 - 420*a^3*b*B - 18*a*b^3*B)*Sqrt[a + b*Tan[c +
d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*a)))/(7*a))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.01 (sec) , antiderivative size = 2405433, normalized size of antiderivative = 6296.95

output too large to display

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12202 vs. 2(324) = 648.

Time = 2.10 (sec) , antiderivative size = 12202, normalized size of antiderivative = 31.94

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx=\int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}}{\tan\left(c+d\,x\right)^{11/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(11/2),
x)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/tan(c + d*x)^(11/2),
 x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx=\frac{-924\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,\tan\left(dx+c\right)^{4}}{}$$

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x)
```

output
```
( - 924*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4*a**2*b
**2 + 20*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4*b**4
+ 462*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*a**3*b -
 10*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*a*b**3 + 1
26*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a**4 - 150*
sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a**2*b**2 - 19
0*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a**3*b - 70*sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a**4 + 315*int((sqrt(tan(c + d*x)
)*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan
(c + d*x)**5*a**5*d - 945*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)
)/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)**5*a**3*b**2*d +
 945*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b
+ tan(c + d*x)*a),x)*tan(c + d*x)**5*a**4*b*d - 315*int((sqrt(tan(c + d*x)
)*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c
+ d*x)**5*a**2*b**3*d)/(315*tan(c + d*x)**5*a**2*d)
```

3.442 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 397

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{(ia-b)^{5/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(40a^3Ab-320aAb^3-5a^4B-240a^2b^2B+128b^4B)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{64b^{3/2}d} - \frac{(ia+b)^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(40a^2Ab-64Ab^3-5a^3B-112ab^2B)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{64bd} + \frac{(40aAb-5a^2B-48b^2B)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{96bd} + \frac{(8Ab-aB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{24bd} + \frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}$$

output
```
-(I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d+1/64*(40*A*a^3*b-320*A*a*b^3-5*B*a^4-240*B*a^2*b^2+128*B*b^4)
*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b^(3/2)/d-(I*a+b
)^(5/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1
/2))/d+1/64*(40*A*a^2*b-64*A*b^3-5*B*a^3-112*B*a*b^2)*tan(d*x+c)^(1/2)*(a+
b*tan(d*x+c))^(1/2)/b/d+1/96*(40*A*a*b-5*B*a^2-48*B*b^2)*tan(d*x+c)^(1/2)*
(a+b*tan(d*x+c))^(3/2)/b/d+1/24*(8*A*b-B*a)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+
c))^(5/2)/b/d+1/4*B*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(7/2)/b/d
```

Mathematica [A] (verified)

Time = 3.08 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.04

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{-192\sqrt[4]{-1}(-a+ib)^{5/2}b(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+192(-1)^{3/4}(a+$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
(-192*(-1)^(1/4)*(-a + I*b)^(5/2)*b*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a +
 I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 192*(-1)^(3/4)*(a +
I*b)^(5/2)*b*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]]
)/Sqrt[a + b*Tan[c + d*x]]] - 3*(-40*a^2*A*b + 64*A*b^3 + 5*a^3*B + 112*a*
b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] - 2*(-40*a*A*b + 5*a^2*
B + 48*b^2*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2) + 8*(8*A*b - a
*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2) + 48*B*Sqrt[Tan[c + d*x]
]*(a + b*Tan[c + d*x])^(7/2) - (3*Sqrt[a]*(-40*a^3*A*b + 320*a*A*b^3 + 5*a
^4*B + 240*a^2*b^2*B - 128*b^4*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqr
t[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]]))/(1
92*b*d)
```

Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(c+dx)^{3/2}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \ 4090$$

$$\frac{\int -\frac{(a+b\tan(c+dx))^{5/2}\left(-\left((8Ab-aB)\tan^2(c+dx)\right)+8bB\tan(c+dx)+aB\right)}{2\sqrt{\tan(c+dx)}}dx}{4b}+\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}$$

$$\Big\downarrow \ 27$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\int \frac{(a+b\tan(c+dx))^{5/2}\left(-\left((8Ab-aB)\tan^2(c+dx)\right)+8bB\tan(c+dx)+aB\right)}{\sqrt{\tan(c+dx)}}dx}{8b}$$

$$\Big\downarrow \ 3042$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\int \frac{(a+b\tan(c+dx))^{5/2}\left(-\left((8Ab-aB)\tan(c+dx)^2\right)+8bB\tan(c+dx)+aB\right)}{\sqrt{\tan(c+dx)}}dx}{8b}$$

$$\Big\downarrow \ 4130$$

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{3}\int \frac{(a+b\tan(c+dx))^{3/2}\left(-\left(\left(-5Ba^2+40Aba-48b^2B\right)\tan^2(c+dx)\right)+48b(Ab+aB)\tan(c+dx)+a(8Ab+5aB)\right)}{2\sqrt{\tan(c+dx)}}dx-\frac{(8Ab-aB)\sqrt{\tan(c+dx)}(a}{3d}}{8b}$$

$\downarrow$ 27

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\int\frac{(a+b\tan(c+dx))^{3/2}(-((-5Ba^2+40Aba-48b^2B)\tan^2(c+dx))+48b(Ab+aB)\tan(c+dx)+a(8Ab+5aB))}{\sqrt{\tan(c+dx)}}dx-\frac{(8Ab-aB)\sqrt{\tan(c+dx)}(a}{3d}}{8b}$$

$\downarrow$ 3042

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\int\frac{(a+b\tan(c+dx))^{3/2}(-((-5Ba^2+40Aba-48b^2B)\tan(c+dx)^2)+48b(Ab+aB)\tan(c+dx)+a(8Ab+5aB))}{\sqrt{\tan(c+dx)}}dx-\frac{(8Ab-aB)\sqrt{\tan(c+dx)}(a}{3d}}{8b}$$

$\downarrow$ 4130

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{1}{2}\int\frac{3\sqrt{a+b\tan(c+dx)}(-((-5Ba^3+40Aba^2-112b^2Ba-64Ab^3)\tan^2(c+dx))+64b(Ba^2+2Aba-b^2B)\tan(c+dx)+a(5Ba^2+24Aba-16b^2B}{2\sqrt{\tan(c+dx)}}\right.}{8b}$$

$\downarrow$ 27

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{3}{4}\int\frac{\sqrt{a+b\tan(c+dx)}(-((-5Ba^3+40Aba^2-112b^2Ba-64Ab^3)\tan^2(c+dx))+64b(Ba^2+2Aba-b^2B)\tan(c+dx)+a(5Ba^2+24Aba-16b^2B)}{\sqrt{\tan(c+dx)}}\right.}{8b}$$

$\downarrow$ 3042

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{3}{4}\int\frac{\sqrt{a+b\tan(c+dx)}(-((-5Ba^3+40Aba^2-112b^2Ba-64Ab^3)\tan(c+dx)^2)+64b(Ba^2+2Aba-b^2B)\tan(c+dx)+a(5Ba^2+24Aba-16b^2B)}{\sqrt{\tan(c+dx)}}\right.}{8b}$$

$\downarrow$ 4130

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{1}{6}\left(\frac{3}{4}\left(\int\frac{-((-5Ba^4+40Aba^3-240b^2Ba^2-320Ab^3a+128b^4B)\tan^2(c+dx))+128b(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)+a(5Ba^3+88Aba^2-}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.\right.$$

$\downarrow$ 27

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd} - \frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{-((-5Ba^4+40Aba^3-240b^2Ba^2-320Ab^3a+128b^4B)\tan^2(c+dx))+128b(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)+a(5Ba^3+88Aba}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.\right.}{}$$

$\downarrow$ 3042

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd} - \frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{-((-5Ba^4+40Aba^3-240b^2Ba^2-320Ab^3a+128b^4B)\tan(c+dx)^2)+128b(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)+a(5Ba^3+88Aba}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.\right.}{}$$

$\downarrow$ 4138

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd} - \frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int \frac{-((-5Ba^4+40Aba^3-240b^2Ba^2-320Ab^3a+128b^4B)\tan^2(c+dx))+128b(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)+a(5Ba^3+88Aba^2-144b^2Ba-64A}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}}{2d}\right.\right.}{}$$

$\downarrow$ 2035

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd} - \frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int \frac{-((-5Ba^4+40Aba^3-240b^2Ba^2-320Ab^3a+128b^4B)\tan^2(c+dx))+128b(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)+a(5Ba^3+88Aba^2-144b^2Ba-64A}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}}{d}\right.\right.}{}$$

$\downarrow$ 2257

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd} - \frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int\left(\frac{5Ba^4-40Aba^3+240b^2Ba^2+320Ab^3a-128b^4B}{\sqrt{a+b\tan(c+dx)}}+\frac{128\left(b\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)+b\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right)d\sqrt{\tan(c+dx)}}{d}\right.\right.}{}$$

$\downarrow$ 2009

$$\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd} - \frac{-\frac{(8Ab-aB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3d} + \frac{1}{6}\left(-\frac{(-5a^2B+40aAb-48b^2B)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d} + \frac{3}{4}\left(-\frac{(-5a^3B+40a^2A\cdots}{}\right.\right.}{}$$

input
```
Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(7/2))/(4*b*d) - (-1/3*((8*A*b
- a*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2))/d + (-1/2*((40*a*A*b
 - 5*a^2*B - 48*b^2*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2))/d +
(3*((64*(I*a - b)^(5/2)*b*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x
]])/Sqrt[a + b*Tan[c + d*x]]] - ((40*a^3*A*b - 320*a*A*b^3 - 5*a^4*B - 240
*a^2*b^2*B + 128*b^4*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Ta
n[c + d*x]]])/Sqrt[b] + 64*b*(I*a + b)^(5/2)*(I*A + B)*ArcTanh[(Sqrt[I*a +
 b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d - ((40*a^2*A*b - 64*A
*b^3 - 5*a^3*B - 112*a*b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])
/d))/4)/6)/(8*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 5.22 (sec) , antiderivative size = 2656933, normalized size of antiderivative = 6692.53

output too large to display

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17728 vs. 2(334) = 668.

Time = 8.94 (sec) , antiderivative size = 35458, normalized size of antiderivative = 89.31

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^(3/
2), x)
```

Giac [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^4\,dx\right)b^3+3\left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)a\,b^2+3\left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)a^2b+\left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)a^3$$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4,x)*b**3 +
3*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3,x)*a*b**
2 + 3*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*a
**2*b + int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a*
*3
```

3.443 $\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 316

$$\begin{aligned}
&\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \\
&-\frac{(ia-b)^{5/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} \\
&+\frac{(30a^2Ab-16Ab^3+5a^3B-40ab^2B)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{8\sqrt{b}d} \\
&+\frac{(ia+b)^{5/2}(A-iB)\mathrm{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} \\
&+\frac{(14aAb+5a^2B-8b^2B)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{8d} \\
&+\frac{(2Ab+3aB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{4d} \\
&+\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}
\end{aligned}$$

output
```
-(I*a-b)^(5/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d+1/8*(30*A*a^2*b-16*A*b^3+5*B*a^3-40*B*a*b^2)*arctanh(b^(1/2)*
tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b^(1/2)/d+(I*a+b)^(5/2)*(A-I*B)*a
rctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d+1/8*(14*A*
a*b+5*B*a^2-8*B*b^2)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d+1/4*(2*A*b+
3*B*a)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)/d+1/3*b*B*tan(d*x+c)^(3/2)*
(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.09

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{24\sqrt[4]{-1}(-a+ib)^{5/2}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+24\sqrt[4]{-1}(a+ib)^{5/2}(A}{}$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
(24*(-1)^(1/4)*(-a + I*b)^(5/2)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b
]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 24*(-1)^(1/4)*(a + I*b)^
(5/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[
a + b*Tan[c + d*x]]] + 3*(14*a*A*b + 5*a^2*B - 8*b^2*B)*Sqrt[Tan[c + d*x]]
*Sqrt[a + b*Tan[c + d*x]] + 6*(2*A*b + 3*a*B)*Sqrt[Tan[c + d*x]]*(a + b*Ta
n[c + d*x])^(3/2) + 8*b*B*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2) +
(3*Sqrt[a]*(30*a^2*A*b - 16*A*b^3 + 5*a^3*B - 40*a*b^2*B)*ArcSinh[(Sqrt[b]
*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[
a + b*Tan[c + d*x]]))/(24*d)
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{1}{3}\int \frac{3}{2}\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2a\right.$$
$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA\right.$$
$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(b(2Ab+3aB)\tan(c+dx)^2+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA\right.$$
$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$$\Big\downarrow \text{4130}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{a+b\tan(c+dx)}\left(-b\left(5Ba^2+14Aba-8b^2B\right)\tan^2(c+dx)-8b\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+ab(2Ab+3aB)\right)}{2\sqrt{\tan(c+dx)}}dx}{2b}+\frac{(3aB+2Ab)\sqrt{\ }}{}\right.$$
$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b\left(5Ba^2+14Aba-8b^2B\right)\tan^2(c+dx)-8b\left(Aa^2-2\right.\right.}{\sqrt{\tan(c+dx)}}}{4b}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b\left(5Ba^2+14Aba-8b^2B\right)\tan(c+dx)^2-8b\left(Aa^2-2\right.\right.}{\sqrt{\tan(c+dx)}}}{4b}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4130

$$\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int \frac{-b\left(5Ba^3+30Aba^2-40b^2Ba-16Ab^3\right)\tan^2(c+dx)-16b\left(Aa^3-3bBa^2\right.}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan}}}{}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\frac{1}{2}\int \frac{-b\left(5Ba^3+30Aba^2-40b^2Ba-16Ab^3\right)\tan^2(c+dx)-16b\left(Aa^3-3bBa\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan}}}{}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\frac{1}{2}\int \frac{-b\left(5Ba^3+30Aba^2-40b^2Ba-16Ab^3\right)\tan(c+dx)^2-16b\left(Aa^3-3bBa\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan}}}{}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\downarrow$ 4138

$$\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int \frac{-b\left(5Ba^3+30Aba^2-40b^2Ba-16Ab^3\right)\tan^2(c+dx)-16b\left(Aa^3-3bBa^2-3Ab^2a-\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+\right.}}{2d}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 2035

$$\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int \frac{-b\left(5Ba^3+30Aba^2-40b^2Ba-16Ab^3\right)\tan^2(c+dx)-16b\left(Aa^3-3bBa^2-3Ab^2a-\right.}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right.}}{d}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 2257

$$\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int\left(\frac{16\left(b\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-b\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx\right.}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}\right.}{d}\right.$$

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

$\Big\downarrow$ 2009

$$\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+$$

$$\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{-\frac{b(5a^2B+14aAb-8b^2B)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}+\frac{-\sqrt{b}(5a^3B\right.}{}\right.$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(b*B*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2))/(3*d) + (((2*A*b + 3*a
*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2))/(2*d) - ((8*(I*a - b)^(
5/2)*b*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]] - Sqrt[b]*(30*a^2*A*b - 16*A*b^3 + 5*a^3*B - 40*a*b^2*B)*ArcTan
h[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - 8*b*(I*a + b)^(
5/2)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/d - (b*(14*a*A*b + 5*a^2*B - 8*b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a
 + b*Tan[c + d*x]])/d)/(4*b))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.88 (sec) , antiderivative size = 2657119, normalized size of antiderivative = 8408.60

output too large to display

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17621 vs. $2(260) = 520$.

Time = 7.41 (sec) , antiderivative size = 35244, normalized size of antiderivative = 111.53

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\sqrt{\tan(dx+c)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*sqrt(tan(d*x + c
)), x)
```

Giac [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\sqrt{\tan(dx+c)}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*sqrt(tan(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \sqrt{\tan(c+d\,x)}\,(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^{5/2}\,dx$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)b^3 + 3\left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)a\,b^2 + 3\left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)a^2b + \left(\int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,dx\right)a^3$$

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3,x)*b**3 +
3*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*a*b**
2 + 3*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a**2
*b + int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a),x)*a**3
```

3.444 $\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 260

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{(ia-b)^{5/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{\sqrt{b}(20aAb+15a^2B-8b^2B)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{4d} + \frac{(ia+b)^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{b(4Ab+7aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{4d} + \frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

output
```
(I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+1/4*b^(1/2)*(20*A*a*b+15*B*a^2-8*B*b^2)*arctanh(b^(1/2)*tan(d*
x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d+(I*a+b)^(5/2)*(I*A+B)*arctanh((I*a+b)
^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/d+1/4*b*(4*A*b+7*B*a)*tan(
d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d+1/2*b*B*tan(d*x+c)^(1/2)*(a+b*tan(d*
x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{4\sqrt[4]{-1}(-a+ib)^{5/2}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c}}{\sqrt{a+b\tan(c+dx)}}\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d
*x]],x]
```

output
```
(4*(-1)^(1/4)*(-a + I*b)^(5/2)*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]
*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - 4*(-1)^(3/4)*(a + I*b)^(5
/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]] + b*(4*A*b + 7*a*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c
 + d*x]] + 2*b*B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2) + (Sqrt[a]*
Sqrt[b]*(20*a*A*b + 15*a^2*B - 8*b^2*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]
])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt[a + b*Tan[c + d*x]])/(4*d)
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 4090, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{1}{2}\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(4Ab+7aB)\tan^2(c+dx)+4\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(4aA-bB)\right)}{2\sqrt{\tan(c+dx)}}dx+$$
$$\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 27

$$\frac{1}{4}\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(4Ab+7aB)\tan^2(c+dx)+4\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(4aA-bB)\right)}{\sqrt{\tan(c+dx)}}dx+$$
$$\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(4Ab+7aB)\tan(c+dx)^2+4\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(4aA-bB)\right)}{\sqrt{\tan(c+dx)}}dx+$$
$$\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 4130

$$\frac{1}{4}\left(\int \frac{b\left(15Ba^2+20Aba-8b^2B\right)\tan^2(c+dx)+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(8Aa^2-9bBa\right.}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 27

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{b\left(15Ba^2+20Aba-8b^2B\right)\tan^2(c+dx)+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(8Aa^2-9b\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{b\left(15Ba^2+20Aba-8b^2B\right)\tan(c+dx)^2+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(8Aa^2-9b\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

$\Big\downarrow$ 4138

$$\frac{1}{4}\left(\frac{\int \frac{b(15Ba^2+20Aba-8b^2B)\tan^2(c+dx)+8(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)+a(8Aa^2-9bBa-4Ab^2)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{2d}+\frac{b(7aB+\right.$$

$$\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{2035}$$

$$\frac{1}{4}\left(\frac{\int \frac{b(15Ba^2+20Aba-8b^2B)\tan^2(c+dx)+8(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)+a(8Aa^2-9bBa-4Ab^2)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{d}+\frac{b(7aB\right.$$

$$\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{2257}$$

$$\frac{1}{4}\left(\frac{\int \left(\frac{b(15Ba^2+20Aba-8b^2B)}{\sqrt{a+b\tan(c+dx)}}+\frac{8(Aa^3-3bBa^2-3Ab^2a+b^3B+(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right)d\sqrt{\tan(c+dx)}}{d}+\frac{b(7aB\right.$$

$$\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}+$$

$$\frac{1}{4}\left(\frac{b(7aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}+\frac{\sqrt{b}(15a^2B+20aAb-8b^2B)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x
]
```

output
```
(b*B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2))/(2*d) + ((4*(I*a - b)^
(5/2)*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]] + Sqrt[b]*(20*a*A*b + 15*a^2*B - 8*b^2*B)*ArcTanh[(Sqrt[b]*Sqrt[
Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 4*(I*a + b)^(5/2)*(I*A + B)*Arc
Tanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + (b*
(4*A*b + 7*a*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.23 (sec) , antiderivative size = 2652267, normalized size of antiderivative = 10201.03

output too large to display

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17603 vs. 2(210) = 420.

Time = 6.71 (sec) , antiderivative size = 35212, normalized size of antiderivative = 135.43

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^{\frac{5}{2}}}{\sqrt{\tan(c+dx)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(5/2)/sqrt(tan(c + d*x
)), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)/sqrt(tan(d*x + c
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}}{\sqrt{\tan\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(1/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{2\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,a^3-2\left(\int \frac{\sqrt{\tan(dx+\cdots}}{\cdots}\right.}{\cdots}$$

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a**3 - 2*int((sqrt(tan(c +
d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*a*
*3*b*d - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(t
an(c + d*x)*b + a),x)*a**4*d + int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b
 + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*a**4*d + 2*int(sqrt(tan(c +
 d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*b**3*d + 6*int(sqrt(tan
(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a*b**2*d + 6*int(sqrt(
tan(c + d*x))*sqrt(tan(c + d*x)*b + a),x)*a**2*b*d)/(2*d)
```

3.445 $\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 241

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{(ia-b)^{5/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{b^{3/2}(2Ab+5aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{b(2aA+bB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d} - \frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

output
```
(I*a-b)^(5/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+b^(3/2)*(2*A*b+5*B*a)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*ta
n(d*x+c))^(1/2))/d-(I*a+b)^(5/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^
(1/2)/(a+b*tan(d*x+c))^(1/2))/d+b*(2*A*a+B*b)*tan(d*x+c)^(1/2)*(a+b*tan(d*
x+c))^(1/2)/d-2*a*A*(a+b*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.49

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{(-1)^{3/4}(-a+ib)^{5/2}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan}}{\sqrt{a+b\tan(c+dx}}\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
3/2),x]
```

output
```
((-1)^(3/4)*(-a + I*b)^(5/2)*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (-1)^(1/4)*(a + I*b)^(5/2)*
(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]] - ((a - I*b)^2*(A - I*B)*Sqrt[a + b*Tan[c + d*x]])/Sqrt[Tan
[c + d*x]] - ((a + I*b)^2*(A + I*B)*Sqrt[a + b*Tan[c + d*x]])/Sqrt[Tan[c +
 d*x]] + (b*B*(a + b*Tan[c + d*x])^(3/2))/Sqrt[Tan[c + d*x]] + (b*(2*A*b +
 5*a*B)*Sqrt[a + b*Tan[c + d*x]]*((Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d
*x]])/Sqrt[a]])/Sqrt[a] - Sqrt[1 + (b*Tan[c + d*x])/a]/Sqrt[Tan[c + d*x]])
)/Sqrt[1 + (b*Tan[c + d*x])/a])/d
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 4088, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$2\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(2aA+bB)\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+aB)\right)}{2\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(2aA+bB)\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+aB)\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(2aA+bB)\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+aB)\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4130}$$

$$\int \frac{b^2(2Ab+5aB)\tan^2(c+dx)-2\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)+a\left(2Ba^2+6Aba-b^2B\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{b(2aA+bB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{b^2(2Ab+5aB)\tan^2(c+dx)-2\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)+a\left(2Ba^2+6Aba-b^2B\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{b(2aA+bB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{b^2(2Ab+5aB)\tan(c+dx)^2-2\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)+a\left(2Ba^2+6Aba-b^2B\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{b(2aA+bB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 4138$$

$$\frac{\int \frac{b^2(2Ab+5aB)\tan^2(c+dx)-2(Aa^3-3bBa^2-3Ab^2a+b^3B)\tan(c+dx)+a(2Ba^2+6Aba-b^2B)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{2d}+\frac{b(2aA+bB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 2035$$

$$\frac{\int \frac{b^2(2Ab+5aB)\tan^2(c+dx)-2(Aa^3-3bBa^2-3Ab^2a+b^3B)\tan(c+dx)+a(2Ba^2+6Aba-b^2B)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{d}+\frac{b(2aA+bB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 2257$$

$$\frac{\int \left(\frac{(2Ab+5aB)b^2}{\sqrt{a+b\tan(c+dx)}}+\frac{2(Ba^3+3Aba^2-3b^2Ba-Ab^3-(Aa^3-3bBa^2-3Ab^2a+b^3B)\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right)d\sqrt{\tan(c+dx)}}{d}+\frac{b(2aA+bB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \; 2009$$

$$\frac{(-b+ia)^{5/2}(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+b^{3/2}(5aB+2Ab)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-(b+ia)^{5/2}(A-}{d}$$
$$\frac{b(2aA+bB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{d\sqrt{\tan(c+dx)}}$$

input
```
Int[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x
]
```

output
```
((I*a - b)^(5/2)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[
a + b*Tan[c + d*x]]] + b^(3/2)*(2*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - (I*a + b)^(5/2)*(A - I*B)*ArcTanh[(S
qrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + (b*(2*a*A
+ b*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d - (2*a*A*(a + b*Tan[
c + d*x])^(3/2))/(d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.93 (sec) , antiderivative size = 2651144, normalized size of antiderivative = 11000.60

output too large to display

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17614 vs. $2(197) = 394$.

Time = 6.86 (sec) , antiderivative size = 35230, normalized size of antiderivative = 146.18

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^{\frac{5}{2}}}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(5/2)/tan(c + d*x)**(3
/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{6\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,a^2b-6\Big(\int \frac{\sqrt{\tan(dx}}{}}{}$$

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
(6*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a**2*b - 6*int((sqrt(tan(c
+ d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*
a**2*b**2*d - 3*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d
*x))/(tan(c + d*x)*b + a),x)*a**3*b*d + 2*int((sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*b + a))/tan(c + d*x)**2,x)*a**3*d + 3*int((sqrt(tan(c + d*x))*sq
rt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*a**3*b*d +
 2*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*b**3*d
+ 6*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a),x)*a*b**2*d)/(2*d)
```

3.446 $\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 240

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = -\frac{(ia-b)^{5/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}+\frac{2b^{5/2}B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2a(2Ab+aB)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

output
```
-(I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d+2*b^(5/2)*B*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))
^(1/2))/d-(I*a+b)^(5/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+
b*tan(d*x+c))^(1/2))/d-2*a*(2*A*b+B*a)*(a+b*tan(d*x+c))^(1/2)/d/tan(d*x+c)
^(1/2)-2/3*a*A*(a+b*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.87 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.74

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{iab(A+iB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{b\tan(c}{a}\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
5/2),x]
```

output
```
(I*a*b*(A + I*B)*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*Tan[c + d*x])/a)
]*Sqrt[a + b*Tan[c + d*x]] - a*b*(I*A + B)*Hypergeometric2F1[-3/2, -3/2, -
1/2, -((b*Tan[c + d*x])/a)]*Sqrt[a + b*Tan[c + d*x]] + (I*a + b)*(A - I*B)
*Sqrt[1 + (b*Tan[c + d*x])/a]*(3*(-1)^(1/4)*(-a + I*b)^(3/2)*ArcTan[((-1)^
(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c +
 d*x]^(3/2) + (I*a + (-3*a + (4*I)*b)*Tan[c + d*x])*Sqrt[a + b*Tan[c + d*x
]]) + (I*a - b)*(A + I*B)*Sqrt[1 + (b*Tan[c + d*x])/a]*(3*(-1)^(1/4)*(a +
I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]]*Tan[c + d*x]^(3/2) + (I*a + (3*a + (4*I)*b)*Tan[c + d*x])*
Sqrt[a + b*Tan[c + d*x]]))/(3*d*Tan[c + d*x]^(3/2)*Sqrt[1 + (b*Tan[c + d*x
])/a])
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$\Big\downarrow$ 4088

$$\frac{2}{3}\int \frac{3\sqrt{a+b\tan(c+dx)}\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\int \frac{\sqrt{a+b\tan(c+dx)}\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\int \frac{\sqrt{a+b\tan(c+dx)}\left(b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)\right)}{\tan(c+dx)^{3/2}}dx-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4128

$$2\int -\frac{-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2a(aB+2Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$-\int \frac{-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2a(aB+2Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$-\int \frac{-B\tan(c+dx)^2b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2a(aB+2Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4138

$$-\frac{\int \frac{-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}-\frac{2a(aB+2Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 2035

$$-\frac{2\int \frac{-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{d}-\frac{2a(aB+2Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 2257

$$-\frac{2\int \left(\frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}-\frac{b^3B}{\sqrt{a+b\tan(c+dx)}}\right)d\sqrt{\tan(c+dx)}}{d}-\frac{2a(aB+2Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 2009

$$-\frac{2\left(\frac{1}{2}(-b+ia)^{5/2}(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\frac{1}{2}(b+ia)^{5/2}(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-b^5\right.}{d}$$

$$\frac{2a(aB+2Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2aA(a+b\tan(c+dx))^{3/2}}{3d\tan^{\frac{3}{2}}(c+dx)}$$

input

```
Int[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(5/2),x
]
```

output
```
(-2*(((I*a - b)^(5/2)*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/
Sqrt[a + b*Tan[c + d*x]]])/2 - b^(5/2)*B*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x
]])/Sqrt[a + b*Tan[c + d*x]]] + ((I*a + b)^(5/2)*(I*A + B)*ArcTanh[(Sqrt[I
*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/2))/d - (2*a*(2*A*b
 + a*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]) - (2*a*A*(a + b*T
an[c + d*x])^(3/2))/(3*d*Tan[c + d*x]^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.94 (sec) , antiderivative size = 2651450, normalized size of antiderivative = 11047.71

output too large to display

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17675 vs. 2(194) = 388.

Time = 6.53 (sec) , antiderivative size = 35349, normalized size of antiderivative = 147.29

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^{\frac{5}{2}}}{\tan^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(5/2)/tan(c + d*x)**(5
/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^{\frac{5}{2}}}{\tan{(dx+c)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)/tan(d*x + c)^(5/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}}{\tan\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(5/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(5/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}\,dx=\frac{6\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,a\,b^2-6\left(\int \frac{\sqrt{\tan(dx}}{}\right.}{}$$

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
(6*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a*b**2 - 6*int((sqrt(tan(c
+ d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*
a*b**3*d - 3*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)
)/(tan(c + d*x)*b + a),x)*a**2*b**2*d + 2*int((sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*b + a))/tan(c + d*x)**3,x)*a**3*d + 6*int((sqrt(tan(c + d*x))*sq
rt(tan(c + d*x)*b + a))/tan(c + d*x)**2,x)*a**2*b*d + 3*int((sqrt(tan(c +
d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*a*
*2*b**2*d + 2*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a),x)*b**3*d)/(
2*d)
```

3.447 $\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 247

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = -\frac{(ia-b)^{5/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(ia+b)^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2a(8Ab+5aB)\sqrt{a+b\tan(c+dx)}}{15d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(15a^2A-23Ab^2-35abB)\sqrt{a+b\tan(c+dx)}}{15d\sqrt{\tan(c+dx)}} - \frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

output
```
-(I*a-b)^(5/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d+(I*a+b)^(5/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/
(a+b*tan(d*x+c))^(1/2))/d-2/15*a*(8*A*b+5*B*a)*(a+b*tan(d*x+c))^(1/2)/d/ta
n(d*x+c)^(3/2)+2/15*(15*A*a^2-23*A*b^2-35*B*a*b)*(a+b*tan(d*x+c))^(1/2)/d/
tan(d*x+c)^(1/2)-2/5*a*A*(a+b*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.30

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{60\sqrt[4]{-1}\left((-a+ib)^{5/2}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
7/2),x]
```

output
```
(60*(-1)^(1/4)*((-a + I*b)^(5/2)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*
b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (a + I*b)^(5/2)*(A + I*
B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]]])*Tan[c + d*x]^(5/2) + 15*b*(-2*A*b + a*B)*Sqrt[a + b*Tan[c + d*x]]
 - 3*(8*a^2*A - 10*A*b^2 - 15*a*b*B)*Sqrt[a + b*Tan[c + d*x]] - 4*(22*a*A*
b + 10*a^2*B - 15*b^2*B)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]] + 8*(15*a^2
*A - 23*A*b^2 - 35*a*b*B)*Tan[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]] - 60*b*B
*(a + b*Tan[c + d*x])^(3/2))/(60*d*Tan[c + d*x]^(5/2))
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.15, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 4088}$$

$$\frac{2}{5}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(2aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+5aB)\right)}{2\tan^{\frac{5}{2}}(c+dx)}dx$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{5}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(2aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+5aB)\right)}{\tan^{\frac{5}{2}}(c+dx)}dx$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{5}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(2aA-5bB)\tan(c+dx)^2-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+5aB)\right)}{\tan(c+dx)^{5/2}}dx$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textbf{4128}$$

$$\frac{1}{5}\left(\frac{2}{3}\int -\frac{b\left(10Ba^2+22Aba-15b^2B\right)\tan^2(c+dx)+15\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(15Aa^2\right.}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{5}\left(-\frac{1}{3}\int \frac{b\left(10Ba^2+22Aba-15b^2B\right)\tan^2(c+dx)+15\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(15Aa^2\right.}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{5}\left(-\frac{1}{3}\int \frac{b\left(10Ba^2+22Aba-15b^2B\right)\tan(c+dx)^2+15\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(15Aa^2\right.}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4132}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\int -\frac{15\left(a\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}+\frac{2\left(15a^2A-35abB-23Ab^2\right)\sqrt{a}}{d\sqrt{\tan(c+dx)}}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(15a^2A-35abB-23Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{15\int \frac{a\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(15a^2A-35abB-23Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{15\int \frac{a\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4099}$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2\left(15a^2A-35abB-23Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{15\left(\frac{1}{2}a(a-i\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2\left(15a^2A-35abB-23Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{15\left(\frac{1}{2}a(a-i\right.}{}\right.\right.$$

$$\Big\downarrow \text{4098}$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2(15a^2A-35abB-23Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{15\left(\frac{a(a-ib)^3(}{}\right.}{}\right.\right.$$

$\downarrow$ 104

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2(15a^2A-35abB-23Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{15\left(\frac{a(a-ib)^3(}{}\right.}{}\right.\right.$$

$\downarrow$ 216

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2(15a^2A-35abB-23Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{15\left(\frac{a(a-ib)^3(}{}\right.}{}\right.\right.$$

$\downarrow$ 219

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+$$

$$\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2(15a^2A-35abB-23Ab^2)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{15\left(\frac{a(a-ib)^3(}{}\right.}{}\right.\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(7/2),x
]
```

output
```
(-2*a*A*(a + b*Tan[c + d*x])^(3/2))/(5*d*Tan[c + d*x]^(5/2)) + ((-2*a*(8*A
*b + 5*a*B)*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) + ((-15*(-(
(a*(a + I*b)^3*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d)) + (a*(a - I*b)^3*(I*A + B)*ArcTanh[
(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a +
b]*d)))/a + (2*(15*a^2*A - 23*A*b^2 - 35*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/
(d*Sqrt[Tan[c + d*x]]))/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5392 vs. $2(207) = 414$.

Time = 12.50 (sec) , antiderivative size = 5393, normalized size of antiderivative = 21.83

method	result	size
default	Expression too large to display	5393
parts	Expression too large to display	2652111

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18802 vs. $2(201) = 402$.

Time = 4.38 (sec) , antiderivative size = 18802, normalized size of antiderivative = 76.12

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\ (a+b\tan(c+dx))^{5/2}}{\tan(c+dx)^{7/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(7/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(7/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}\,dx = \frac{64\sqrt{\tan(dx+c)}\ \sqrt{a+\tan(dx+c)\,b}\ \tan(dx+c)^2\,a^2b}{}$$

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(7/2),x)
```

output

```
(64*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a**2*b**2
- 30*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*b**4 - 32
*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a**3*b - 6*sqrt(
tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a**4 - 15*int((sqrt(tan(c + d*x))*s
qrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c
+ d*x)**3*a**5*d + 90*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(t
an(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)**3*a**3*b**2*d - 15*
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan
(c + d*x)**2*a),x)*tan(c + d*x)**3*a*b**4*d - 60*int((sqrt(tan(c + d*x))*s
qrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d
*x)**3*a**4*b*d + 60*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(ta
n(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)**3*a**2*b**3*d)/(15*tan(
c + d*x)**3*a*d)
```

3.448 $\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 309

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \frac{(ia-b)^{5/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} + \frac{(ia+b)^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2a(10Ab+7aB)\sqrt{a+b\tan(c+dx)}}{35d\tan^{\frac{5}{2}}(c+dx)} + \frac{2(35a^2A-45Ab^2-77abB)\sqrt{a+b\tan(c+dx)}}{105d\tan^{\frac{3}{2}}(c+dx)} + \frac{2(245a^2Ab-15Ab^3+105a^3B-161ab^2B)\sqrt{a+b\tan(c+dx)}}{105ad\sqrt{\tan(c+dx)}} - \frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

output
```
(I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d+(I*a+b)^(5/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(
a+b*tan(d*x+c))^(1/2))/d-2/35*a*(10*A*b+7*B*a)*(a+b*tan(d*x+c))^(1/2)/d/ta
n(d*x+c)^(5/2)+2/105*(35*A*a^2-45*A*b^2-77*B*a*b)*(a+b*tan(d*x+c))^(1/2)/d
/tan(d*x+c)^(3/2)+2/105*(245*A*a^2*b-15*A*b^3+105*B*a^3-161*B*a*b^2)*(a+b*
tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)-2/7*a*A*(a+b*tan(d*x+c))^(3/2)/d/ta
n(d*x+c)^(7/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 32.06 (sec) , antiderivative size = 5641, normalized size of antiderivative = 18.26

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
9/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.543, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2}{7}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(4aA-7bB)\tan^2(c+dx)-7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(10Ab+7aB)\right)}{2\tan^{\frac{7}{2}}(c+dx)}d$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(4aA-7bB)\tan^2(c+dx)-7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(10Ab+7aB)\right)}{\tan^{\frac{7}{2}}(c+dx)}d$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(4aA-7bB)\tan(c+dx)^2-7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(10Ab+7aB)\right)}{\tan(c+dx)^{7/2}}d$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4128}$$

$$\frac{1}{7}\left(\frac{2}{5}\int -\frac{b\left(28Ba^2+60Aba-35b^2B\right)\tan^2(c+dx)+35\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(35Aa^2\right.}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(-\frac{1}{5}\int \frac{b\left(28Ba^2+60Aba-35b^2B\right)\tan^2(c+dx)+35\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(35Aa^2\right.}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(-\frac{1}{5}\int \frac{b\left(28Ba^2+60Aba-35b^2B\right)\tan(c+dx)^2+35\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(35Aa^2\right.}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4132}$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2\int -\frac{-2ab(35Aa^2-77bBa-45Ab^2)\tan^2(c+dx)-105a(Aa^3-3bBa^2-3Ab^2a+b^3B)\tan(c+dx)+a(105Ba^3+245Aba^2-161b^2Ba-15A}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{3a}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2(35a^2A-77abB-45Ab^2)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{-2ab(35Aa^2-77bBa-45Ab^2)\tan^2(c+dx)-105a(Aa^3-3bBa^2-3}{\tan^{\frac{3}{2}}(c+dx)\sqrt{}}}{3}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2(35a^2A-77abB-45Ab^2)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{\int\frac{-2ab(35Aa^2-77bBa-45Ab^2)\tan(c+dx)^2-105a(Aa^3-3bBa^2-3}{\tan(c+dx)^{3/2}\sqrt{}}}{3}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4132$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2(35a^2A-77abB-45Ab^2)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2\int\frac{105\left(\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)a^2+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)t}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2(35a^2A-77abB-45Ab^2)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{105\int\frac{\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)a^2+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{105\int\frac{\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)a^2+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4099$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(105a^3B+}{}}{}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(105a^3B+}{}}{}\right.\right.$$

$$\Big\downarrow \; 4098$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$

$$\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(105a^3B+}{}}{}\right.\right.$$

$$\Big\downarrow \; 104$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$
$$\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(105a^3B+}{}}{}\right.\right.$$

$\downarrow$ 216

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$
$$\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(105a^3B+}{}}{}\right.\right.$$

$\downarrow$ 219

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+$$
$$\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(105a^3B+}{}}{}\right.\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(9/2),x
]
```

output
```
(-2*a*A*(a + b*Tan[c + d*x])^(3/2))/(7*d*Tan[c + d*x]^(7/2)) + ((-2*a*(10*
A*b + 7*a*B)*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) + ((2*(35*
a^2*A - 45*A*b^2 - 77*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(
3/2)) - ((-105*((a^2*(a + I*b)^3*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[
c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a^2*(a - I*b)^3
*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d
*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(245*a^2*A*b - 15*A*b^3 + 105*a^3*B - 16
1*a*b^2*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5426 vs. $2(263) = 526$.

Time = 5.86 (sec) , antiderivative size = 5427, normalized size of antiderivative = 17.56

method	result	size
default	Expression too large to display	5427
parts	Expression too large to display	2654308

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x,method=_RET
URNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18856 vs. $2(257) = 514$.

Time = 4.45 (sec) , antiderivative size = 18856, normalized size of antiderivative = 61.02

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\tan(c + dx))^{5/2}(A + B\tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + b\tan(c + dx))^{5/2}(A + B\tan(c + dx))}{\tan^{\frac{9}{2}}(c + dx)}\,dx = \int \frac{(B\tan(dx + c) + A)(b\tan(dx + c) + a)^{\frac{5}{2}}}{\tan(dx + c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)/tan(d*x + c)^(9/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}}{\tan\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(9/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(9/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}\,dx = \frac{-140\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3}{}$$

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(9/2),x)
```

output
```
( - 140*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*a**2*b
 + 488*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*b**3 +
70*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a**3 - 244*
sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a*b**2 - 132*s
qrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a**2*b - 30*sqrt(t
an(c + d*x))*sqrt(tan(c + d*x)*b + a)*a**3 - 420*int((sqrt(tan(c + d*x))*s
qrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c
+ d*x)**4*a**3*b*d + 420*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))
/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)**4*a*b**3*d + 105
*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + ta
n(c + d*x)*a),x)*tan(c + d*x)**4*a**4*d - 630*int((sqrt(tan(c + d*x))*sqrt
(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)
**4*a**2*b**2*d + 105*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(t
an(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)**4*b**4*d)/(105*tan(c +
 d*x)**4*d)
```

3.449 $\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 378

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \frac{(ia-b)^{5/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$
$$-\frac{(ia+b)^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}-\frac{2a(4Ab+3aB)\sqrt{a+b\tan(c+dx)}}{21d\tan^{\frac{7}{2}}(c+dx)}$$
$$+\frac{2(21a^2A-25Ab^2-45abB)\sqrt{a+b\tan(c+dx)}}{105d\tan^{\frac{5}{2}}(c+dx)}$$
$$+\frac{2(231a^2Ab-5Ab^3+105a^3B-135ab^2B)\sqrt{a+b\tan(c+dx)}}{315ad\tan^{\frac{3}{2}}(c+dx)}$$
$$-\frac{2(315a^4A-483a^2Ab^2-10Ab^4-735a^3bB+45ab^3B)\sqrt{a+b\tan(c+dx)}}{315a^2d\sqrt{\tan(c+dx)}}$$
$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

output
```
(I*a-b)^(5/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/d-(I*a+b)^(5/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(
a+b*tan(d*x+c))^(1/2))/d-2/21*a*(4*A*b+3*B*a)*(a+b*tan(d*x+c))^(1/2)/d/tan
(d*x+c)^(7/2)+2/105*(21*A*a^2-25*A*b^2-45*B*a*b)*(a+b*tan(d*x+c))^(1/2)/d/
tan(d*x+c)^(5/2)+2/315*(231*A*a^2*b-5*A*b^3+105*B*a^3-135*B*a*b^2)*(a+b*ta
n(d*x+c))^(1/2)/a/d/tan(d*x+c)^(3/2)-2/315*(315*A*a^4-483*A*a^2*b^2-10*A*b
^4-735*B*a^3*b+45*B*a*b^3)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(1/2)-2
/9*a*A*(a+b*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(9/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 32.18 (sec) , antiderivative size = 5725, normalized size of antiderivative = 15.15

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
11/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.01 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.13, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.629$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{11/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2}{9}\int \frac{3\sqrt{a+b\tan(c+dx)}\left(-b(2aA-3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+3aB)\right)}{2\tan^{\frac{9}{2}}(c+dx)}d$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(2aA-3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+3aB)\right)}{\tan^{\frac{9}{2}}(c+dx)}dx$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(2aA-3bB)\tan(c+dx)^2-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+3aB)\right)}{\tan(c+dx)^{9/2}}dx$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4128}$$

$$\frac{1}{3}\left(\frac{2}{7}\int -\frac{b\left(18Ba^2+38Aba-21b^2B\right)\tan^2(c+dx)+21\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(21Aa^2\right.}{2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{1}{7}\int \frac{b\left(18Ba^2+38Aba-21b^2B\right)\tan^2(c+dx)+21\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(21Aa^2\right.}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-\frac{1}{7}\int \frac{b\left(18Ba^2+38Aba-21b^2B\right)\tan(c+dx)^2+21\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a\left(21Aa^2\right.}{\tan(c+dx)^{7/2}\sqrt{a+b\tan(c+dx)}}\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 4132

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\int -\frac{-4ab\left(21Aa^2-45bBa-25Ab^2\right)\tan^2(c+dx)-105a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)+a\left(105Ba^3+231Aba^2-135b^2Ba-5Ab\right.}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{5a}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{\int \frac{-4ab\left(21Aa^2-45bBa-25Ab^2\right)\tan^2(c+dx)-105a\left(Aa^3-3bBa^2-\right.}{\tan^{\frac{5}{2}}(c+dx)\sqrt{}}}{5}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{\int \frac{-4ab\left(21Aa^2-45bBa-25Ab^2\right)\tan(c+dx)^2-105a\left(Aa^3-3bBa^2-\right.}{\tan(c+dx)^{5/2}\sqrt{}}}{5}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 4132

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2\int \frac{315\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)a^2+2b\left(105Ba^3+231Aba^2\right.}{2\tan^{\frac{3}{2}}}}{}}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{\int\frac{315\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)a^2+2b\left(105Ba^3+231Aba^2-\cdots\right.}{\tan^{\frac{3}{2}}(c-\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{\int\frac{315\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)a^2+2b\left(105Ba^3+231Aba^2-\cdots\right.}{\tan(c+dx\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 4132

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{-\frac{2\int-\frac{315\left(a^3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-a^3\left(Aa^3-3bBa^2-3Ab^2a+b\cdots\right.\right.}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}}{\cdots}}{\cdots}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{\frac{315\int\frac{a^3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-a^3\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan\cdots}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}}{\cdots}}{\cdots}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{315\int\frac{a^3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-a^3\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}$$

$$\Big\downarrow \text{4099}$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2\left(105a^3B+23\right.}{}}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2\left(105a^3B+23\right.}{}}{}\right.\right.$$

$$\Big\downarrow \text{4098}$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2\left(105a^3B+23\right.}{}}{}\right.\right.$$

$$\Big\downarrow \text{104}$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(105a^3B+23}{}}{}\right.\right.$$

$\downarrow$ 216

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(105a^3B+23}{}}{}\right.\right.$$

$\downarrow$ 219

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+$$

$$\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(105a^3B+23}{}}{}\right.\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(11/2),
x]
```

output
```
(-2*a*A*(a + b*Tan[c + d*x])^(3/2))/(9*d*Tan[c + d*x]^(9/2)) + ((-2*a*(4*A
*b + 3*a*B)*Sqrt[a + b*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(7/2)) + ((2*(21*a
^2*A - 25*A*b^2 - 45*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5
/2)) - ((-2*(231*a^2*A*b - 5*A*b^3 + 105*a^3*B - 135*a*b^2*B)*Sqrt[a + b*T
an[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) - ((315*(-((a^3*(a + I*b)^3*(I*A -
B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(S
qrt[I*a - b]*d)) + (a^3*(a - I*b)^3*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[
Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(315*
a^4*A - 483*a^2*A*b^2 - 10*A*b^4 - 735*a^3*b*B + 45*a*b^3*B)*Sqrt[a + b*Ta
n[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*a))/7)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 8.59 (sec) , antiderivative size = 15788, normalized size of antiderivative = 41.77

method	result	size
default	Expression too large to display	15788
parts	Expression too large to display	2656652

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x,method=_RE
TURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18922 vs. $2(320) = 640$.

Time = 4.41 (sec) , antiderivative size = 18922, normalized size of antiderivative = 50.06

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, alg
orithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(11/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, alg
orithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Hanged}$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(11/2),
x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(11/2),x)
```

output
```
( - 192*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4*a**2*b
**2 + 80*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4*b**4
+ 96*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*a**3*b -
40*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*a*b**3 + 18
*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a**4 - 60*sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a**2*b**2 - 40*sq
rt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a**3*b - 10*sqrt(ta
n(c + d*x))*sqrt(tan(c + d*x)*b + a)*a**4 + 45*int((sqrt(tan(c + d*x))*sqr
t(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c +
d*x)**5*a**5*d - 270*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(ta
n(c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)**5*a**3*b**2*d + 45*i
nt((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(
c + d*x)**2*a),x)*tan(c + d*x)**5*a*b**4*d + 180*int((sqrt(tan(c + d*x))*s
qrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d
*x)**5*a**4*b*d - 180*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(t
an(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)**5*a**2*b**3*d)/(45*tan
(c + d*x)**5*a*d)
```

3.450 $\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 460

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = -\frac{(ia-b)^{5/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(ia+b)^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{2a(14Ab+11aB)\sqrt{a+b\tan(c+dx)}}{99d\tan^{\frac{9}{2}}(c+dx)} + \frac{2(99a^2A-113Ab^2-209abB)\sqrt{a+b\tan(c+dx)}}{693d\tan^{\frac{7}{2}}(c+dx)} + \frac{2(495a^2Ab-5Ab^3+231a^3B-275ab^2B)\sqrt{a+b\tan(c+dx)}}{1155ad\tan^{\frac{5}{2}}(c+dx)} - \frac{2(1155a^4A-1485a^2Ab^2-20Ab^4-2541a^3bB+55ab^3B)\sqrt{a+b\tan(c+dx)}}{3465a^2d\tan^{\frac{3}{2}}(c+dx)} - \frac{2(8085a^4Ab-495a^2Ab^3+40Ab^5+3465a^5B-5313a^3b^2B-110ab^4B)\sqrt{a+b\tan(c+dx)}}{3465a^3d\sqrt{\tan(c+dx)}} - \frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

output
```
-(I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))/d-(I*a+b)^(5/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/
(a+b*tan(d*x+c))^(1/2))/d-2/99*a*(14*A*b+11*B*a)*(a+b*tan(d*x+c))^(1/2)/d/
tan(d*x+c)^(9/2)+2/693*(99*A*a^2-113*A*b^2-209*B*a*b)*(a+b*tan(d*x+c))^(1/
2)/d/tan(d*x+c)^(7/2)+2/1155*(495*A*a^2*b-5*A*b^3+231*B*a^3-275*B*a*b^2)*(
a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(5/2)-2/3465*(1155*A*a^4-1485*A*a^2*b
^2-20*A*b^4-2541*B*a^3*b+55*B*a*b^3)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+
c)^(3/2)-2/3465*(8085*A*a^4*b-495*A*a^2*b^3+40*A*b^5+3465*B*a^5-5313*B*a^3
*b^2-110*B*a*b^4)*(a+b*tan(d*x+c))^(1/2)/a^3/d/tan(d*x+c)^(1/2)-2/11*a*A*(
a+b*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(11/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 31.83 (sec) , antiderivative size = 5809, normalized size of antiderivative = 12.63

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(
13/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.64 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.11, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{13/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2}{11}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(8aA-11bB)\tan^2(c+dx)-11\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(14Ab+11a\right.}{2\tan^{\frac{11}{2}}(c+dx)}$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(8aA-11bB)\tan^2(c+dx)-11\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(14Ab+11a\right.}{\tan^{\frac{11}{2}}(c+dx)}$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{11}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(8aA-11bB)\tan(c+dx)^2-11\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(14Ab+11a\right.}{\tan(c+dx)^{11/2}}$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{4128}$$

$$\frac{1}{11}\left(\frac{2}{9}\int -\frac{b\left(88Ba^2+184Aba-99b^2B\right)\tan^2(c+dx)+99\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a(99A\right.}{2\tan^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{11}\left(-\frac{1}{9}\int \frac{b\left(88Ba^2+184Aba-99b^2B\right)\tan^2(c+dx)+99\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a(99A\right.}{\tan^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(-\frac{1}{9}\int \frac{b\left(88Ba^2+184Aba-99b^2B\right)\tan(c+dx)^2+99\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)+a(99A}{\tan(c+dx)^{9/2}\sqrt{a+b\tan(c+dx)}}\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 4132

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\int -\frac{3\left(-2ab\left(99Aa^2-209bBa-113Ab^2\right)\tan^2(c+dx)-231a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)+a\left(231Ba^3+495Aba^2-275b^2B\right.\right.}{2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{7a}\right.\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\int \frac{-2ab\left(99Aa^2-209bBa-113Ab^2\right)\tan^2(c+dx)-231a\left(Aa^3-\right.}{\tan^{\frac{7}{2}}(c}}{}\right.\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\int \frac{-2ab\left(99Aa^2-209bBa-113Ab^2\right)\tan(c+dx)^2-231a\left(Aa^3-\right.}{\tan(c+d}}{}\right.\right.$$
$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 4132

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2\int\frac{1155\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)a^2+4b\left(231Ba^3\right.}{}}{}\right.}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{\int\frac{1155\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)a^2+4b\left(231Ba^3+\right.}{}}{}\right.}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{\int\frac{1155\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)a^2+4b\left(231Ba^3+\right.}{}}{}\right.}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 4132

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{-\frac{2\int -\frac{-3465\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)a^3-2b\left(1\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.}{\cdots}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{\frac{\int \frac{-3465\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)a^3-2b\left(1155Aa\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.}{\cdots}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{\frac{\int \frac{-3465\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)a^3-2b\left(1155Aa\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.}{\cdots}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 4132

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{-\frac{2\int\frac{3465\left(\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)a^4+\left(Ba^3+3Aba^2-3b\right.\right.}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}}{}\right.}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{-\frac{3465\int\frac{\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)a^4+\left(Ba^3+3Aba^2-3b^2B\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}}{}\right.}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{-\frac{3465\int\frac{\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)a^4+\left(Ba^3+3Aba^2-3b^2B\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a}}{}\right.}{}\right.\right.$$

$$\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}$$

$$\Big\downarrow \text{4099}$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+$$

$$\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+$$

$$\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(}{}\right.}{}\right.\right.$$

$\downarrow$ 4098

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+$$

$$\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(}{}\right.}{}\right.\right.$$

$\downarrow$ 104

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+$$

$$\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\big\downarrow$ 216

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+$$

$$\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+$$

$$\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\frac{2(99a^2A-209abB-113Ab^2)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(\cdots}{}\right.}{}\right.\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(13/2),
x]
```

output
```
(-2*a*A*(a + b*Tan[c + d*x])^(3/2))/(11*d*Tan[c + d*x]^(11/2)) + ((-2*a*(1
4*A*b + 11*a*B)*Sqrt[a + b*Tan[c + d*x]])/(9*d*Tan[c + d*x]^(9/2)) + ((2*(
99*a^2*A - 113*A*b^2 - 209*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(7*d*Tan[c + d
*x]^(7/2)) - (3*((-2*(495*a^2*A*b - 5*A*b^3 + 231*a^3*B - 275*a*b^2*B)*Sqr
t[a + b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) - ((-2*(1155*a^4*A - 1485*
a^2*A*b^2 - 20*A*b^4 - 2541*a^3*b*B + 55*a*b^3*B)*Sqrt[a + b*Tan[c + d*x]]
)/(3*d*Tan[c + d*x]^(3/2)) + ((-3465*((a^4*(a + I*b)^3*(A + I*B)*ArcTan[(S
qrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]
*d) + (a^4*(a - I*b)^3*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]]
)/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(8085*a^4*A*b - 49
5*a^2*A*b^3 + 40*A*b^5 + 3465*a^5*B - 5313*a^3*b^2*B - 110*a*b^4*B)*Sqrt[a
 + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*a)))/(7*a))/9)/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.94 (sec) , antiderivative size = 2659382, normalized size of antiderivative = 5781.27

output too large to display

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18981 vs. $2(396) = 792$.

Time = 4.36 (sec) , antiderivative size = 18981, normalized size of antiderivative = 41.26

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x, alg
orithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x, alg
orithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}}{\tan\left(c+d\,x\right)^{13/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(13/2),
x)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/tan(c + d*x)^(13/2),
 x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/tan(d*x+c)^(13/2),x)
```

output

```
(11616*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**5*a**2*b*
*3 - 4480*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**5*b**5
 - 5808*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4*a**3*b
**2 + 2240*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**4*a*b
**4 + 4356*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*a**
4*b - 1680*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3*a**
2*b**3 + 990*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a
**5 - 3220*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*a**
3*b**2 - 2380*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a**
4*b - 630*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a**5 + 3465*int((sqr
t(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**4*b + tan(c + d*x
)**3*a),x)*tan(c + d*x)**6*a**6*d - 20790*int((sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*b + a))/(tan(c + d*x)**4*b + tan(c + d*x)**3*a),x)*tan(c + d*x)*
*6*a**4*b**2*d + 3465*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(t
an(c + d*x)**4*b + tan(c + d*x)**3*a),x)*tan(c + d*x)**6*a**2*b**4*d + 138
60*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b +
tan(c + d*x)**2*a),x)*tan(c + d*x)**6*a**5*b*d - 13860*int((sqrt(tan(c + d
*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*
tan(c + d*x)**6*a**3*b**3*d)/(3465*tan(c + d*x)**6*a**2*d)
```

3.451 $\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 253

$$\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{(ia-b)^{5/2}(2a-3ib)B\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{2ad} + \frac{2b^{5/2}B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d} - \frac{(2a+3ib)(ia+b)^{5/2}B\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{2ad} - \frac{2(a^2+3b^2)B\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}} - \frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

output
```
1/2*(I*a-b)^(5/2)*(2*a-3*I*b)*B*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b
*tan(d*x+c))^(1/2))/a/d+2*b^(5/2)*B*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*
tan(d*x+c))^(1/2))/d-1/2*(2*a+3*I*b)*(I*a+b)^(5/2)*B*arctanh((I*a+b)^(1/2)
*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/a/d-2*(a^2+3*b^2)*B*(a+b*tan(d*x
+c))^(1/2)/d/tan(d*x+c)^(1/2)-b*B*(a+b*tan(d*x+c))^(3/2)/d/tan(d*x+c)^(3/2
)
```

Mathematica [A] (warning: unable to verify)

Time = 3.05 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.41

$$\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{B\cos(c+dx)(3b+2a\tan(c+dx))\left(4\sqrt{a}b^{5/2}\text{arcsinl}\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Tan[c + d*x]))/Ta
n[c + d*x]^(5/2),x]
```

output
```
(B*Cos[c + d*x]*(3*b + 2*a*Tan[c + d*x])*(4*Sqrt[a]*b^(5/2)*ArcSinh[(Sqrt[
b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]
] - Sqrt[1 + (b*Tan[c + d*x])/a]*((-1)^(1/4)*(-a + I*b)^(5/2)*(2*a + (3*I)
*b)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]]*Tan[c + d*x]^(3/2) + (-1)^(1/4)*(a + I*b)^(5/2)*(2*a - (3*I)*b)*
ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*
x]]]*Tan[c + d*x]^(3/2) + 2*a*Sqrt[a + b*Tan[c + d*x]]*(a*b + (2*a^2 + 7*b
^2)*Tan[c + d*x]))))/(2*a*d*(3*b*Cos[c + d*x] + 2*a*Sin[c + d*x])*Tan[c +
d*x]^(3/2)*Sqrt[1 + (b*Tan[c + d*x])/a])
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2}{3}\int \frac{3\sqrt{a+b\tan(c+dx)}\left(2b^2B\tan^2(c+dx)+b\left(\frac{3b^2}{a}+a\right)B\tan(c+dx)+2\left(a^2+3b^2\right)B\right)}{4\tan^{\frac{3}{2}}(c+dx)}dx-\frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sqrt{a+b\tan(c+dx)}\left(2b^2B\tan^2(c+dx)+b\left(\frac{3b^2}{a}+a\right)B\tan(c+dx)+2\left(a^2+3b^2\right)B\right)}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{a+b\tan(c+dx)}\left(2b^2B\tan(c+dx)^2+b\left(\frac{3b^2}{a}+a\right)B\tan(c+dx)+2\left(a^2+3b^2\right)B\right)}{\tan(c+dx)^{3/2}}dx-\frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4128}$$

$$\frac{1}{2}\left(2\int \frac{2B\tan^2(c+dx)b^3+3\left(a^2+3b^2\right)Bb-\frac{\left(2a^4+3b^2a^2-3b^4\right)B\tan(c+dx)}{a}}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{4B\left(a^2+3b^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\int \frac{2B\tan^2(c+dx)b^3+3\left(a^2+3b^2\right)Bb-\frac{\left(2a^4+3b^2a^2-3b^4\right)B\tan(c+dx)}{a}}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{4B\left(a^2+3b^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{2B\tan(c+dx)^2b^3+3\left(a^2+3b^2\right)Bb-\frac{\left(2a^4+3b^2a^2-3b^4\right)B\tan(c+dx)}{a}}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{4B\left(a^2+3b^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)$$
$$\frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4138}$$

$$\frac{1}{2}\left(\frac{\int \frac{2B\tan^2(c+dx)b^3+3\left(a^2+3b^2\right)Bb-\frac{\left(2a^4+3b^2a^2-3b^4\right)B\tan(c+dx)}{a}}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}-\frac{4B\left(a^2+3b^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-$$
$$\frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{2035}$$

$$\frac{1}{2}\left(\frac{2\int \frac{2B\tan^2(c+dx)b^3+3\left(a^2+3b^2\right)Bb-\frac{\left(2a^4+3b^2a^2-3b^4\right)B\tan(c+dx)}{a}}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{d}-\frac{4B\left(a^2+3b^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)$$
$$\frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{2257}$$

$$\frac{1}{2}\left(\frac{2\int \left(\frac{2Bb^3}{\sqrt{a+b\tan(c+dx)}}+\frac{ab\left(3a^2+7b^2\right)B-\left(2a^4+3b^2a^2-3b^4\right)B\tan(c+dx)}{a\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}\right)d\sqrt{\tan(c+dx)}}{d}-\frac{4B\left(a^2+3b^2\right)\sqrt{a+b\tan(c\right.}}{d\sqrt{\tan(c+dx)}}\right.$$
$$\frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bB(a+b\tan(c+dx))^{3/2}}{d\tan^{\frac{3}{2}}(c+dx)}+$$
$$\frac{1}{2}\left(-\frac{4B\left(a^2+3b^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{2\left(\frac{B(2a-3ib)(-b+ia)^{5/2}\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{2a}+2b^{5/2}B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{t}}{\sqrt{a+b}}\right.\right.}{d}\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(5/2)*((3*b*B)/(2*a) + B*Tan[c + d*x]))/Tan[c +
d*x]^(5/2),x]
```

output
```
-((b*B*(a + b*Tan[c + d*x])^(3/2))/(d*Tan[c + d*x]^(3/2))) + ((2*(((I*a -
b)^(5/2)*(2*a - (3*I)*b)*B*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[
a + b*Tan[c + d*x]]])/(2*a) + 2*b^(5/2)*B*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*
x]])/Sqrt[a + b*Tan[c + d*x]]] - ((2*a + (3*I)*b)*(I*a + b)^(5/2)*B*ArcTan
h[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(2*a)))/d
- (4*(a^2 + 3*b^2)*B*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.20 (sec) , antiderivative size = 1491744, normalized size of antiderivative = 5896.22

output too large to display

input
```
int((a+b*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8171 vs. 2(205) = 410.

Time = 2.22 (sec) , antiderivative size = 16341, normalized size of antiderivative = 64.59

$$\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2)
,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx = \frac{B\left(\int \frac{2a^3\sqrt{a+b\tan(c+dx)}}{\tan^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{3b^3\sqrt{a+b\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\,dx+\int\right.}{}$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)**(5/
2),x)
```

output
```
B*(Integral(2*a**3*sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(3/2), x) + Inte
gral(3*b**3*sqrt(a + b*tan(c + d*x))/sqrt(tan(c + d*x)), x) + Integral(6*a
*b**2*sqrt(a + b*tan(c + d*x))/tan(c + d*x)**(3/2), x) + Integral(2*a*b**2
*sqrt(a + b*tan(c + d*x))*sqrt(tan(c + d*x)), x) + Integral(3*a**2*b*sqrt(
a + b*tan(c + d*x))/tan(c + d*x)**(5/2), x) + Integral(4*a**2*b*sqrt(a + b
*tan(c + d*x))/sqrt(tan(c + d*x)), x))/(2*a)
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(2\,B\tan\left(dx+c\right)+\frac{3\,Bb}{a}\right)\left(b\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}{2\,\tan\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2)
,x, algorithm="maxima")
```

output
```
1/2*integrate((2*B*tan(d*x + c) + 3*B*b/a)*(b*tan(d*x + c) + a)^(5/2)/tan(
d*x + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2)
,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx=\int \frac{\left(B\tan(c+d\,x)+\frac{3\,B\,b}{2\,a}\right)\,\left(a+b\tan(c+d\,x)\right)^{5/2}}{\tan\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((B*tan(c + d*x) + (3*B*b)/(2*a))*(a + b*tan(c + d*x))^(5/2))/tan(c +
d*x)^(5/2),x)
```

output
```
int(((B*tan(c + d*x) + (3*B*b)/(2*a))*(a + b*tan(c + d*x))^(5/2))/tan(c +
d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}\left(\frac{3bB}{2a}+B\tan(c+dx)\right)}{\tan^{\frac{5}{2}}(c+dx)}\,dx=\frac{b\left(8\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,a^2b+6\sqrt{\tan\right.}$$

input
```
int((a+b*tan(d*x+c))^(5/2)*(3/2*b*B/a+B*tan(d*x+c))/tan(d*x+c)^(5/2),x)
```

output
```
(b*(8*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a**2*b + 6*sqrt(tan(c +
d*x))*sqrt(tan(c + d*x)*b + a)*b**3 - 8*int((sqrt(tan(c + d*x))*sqrt(tan(c
 + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*a**2*b**2*d - 6*in
t((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d
*x)*b + a),x)*b**4*d - 4*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*
tan(c + d*x))/(tan(c + d*x)*b + a),x)*a**3*b*d - 3*int((sqrt(tan(c + d*x))
*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*x)*b + a),x)*a*b**3*d +
 6*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/tan(c + d*x)**3,x)*a*
*2*b*d + 4*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/tan(c + d*x)*
*2,x)*a**3*d + 12*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/tan(c
+ d*x)**2,x)*a*b**2*d + 4*int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)
)/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*a**3*b*d + 3*int((sqrt(tan(c + d
*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*a*b
**3*d + 4*int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a),x)*a*b**2*d))/(4
*a*d)
```

3.452 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 206

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} + \frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d} - \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d} + \frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}$$

output
```
-(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*
a-b)^(1/2)/d+(2*A*b-B*a)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))
^(1/2))/b^(3/2)/d-(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(
d*x+c))^(1/2))/(I*a+b)^(1/2)/d+B*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/b
/d
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.19

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$= \frac{\frac{\sqrt[4]{-1}b(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{(-1)^{3/4}b(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}+B\sqrt{\tan(c+dx)}\sqrt{c}}{bd}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x
]],x]
```

output
```
(((-1)^(1/4)*b*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*
x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + ((-1)^(3/4)*b*(A + I*B)*A
rcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x
]]])/Sqrt[a + I*b] + B*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + (Sqrt
[a]*(2*A*b - a*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (
b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]]))/(b*d)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 4090, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -\frac{-((2Ab-aB)\tan^2(c+dx))+2bB\tan(c+dx)+aB}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b}+\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\int \frac{-((2Ab-aB)\tan^2(c+dx))+2bB\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\int \frac{-((2Ab-aB)\tan(c+dx)^2)+2bB\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{2b}$$

$$\Big\downarrow \text{4138}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\int \frac{-((2Ab-aB)\tan^2(c+dx))+2bB\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{2bd}$$

$$\Big\downarrow \text{2035}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\int \frac{-((2Ab-aB)\tan^2(c+dx))+2bB\tan(c+dx)+aB}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{2257}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\int \left(\frac{aB-2Ab}{\sqrt{a+b\tan(c+dx)}}+\frac{2(Ab+B\tan(c+dx)b)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right)d\sqrt{\tan(c+dx)}}{bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\frac{b(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}-\frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}}+\frac{b(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b+ia}}}{bd}$$

input
```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
-(((b*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/Sqrt[I*a - b] - ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*
x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[b] + (b*(A - I*B)*ArcTanh[(Sqrt[I*a +
 b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[I*a + b])/(b*d)) +
 (B*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/(b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.08 (sec) , antiderivative size = 1888526, normalized size of antiderivative = 9167.60

output too large to display

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10163 vs. $2(166) = 332$.

Time = 4.60 (sec) , antiderivative size = 20328, normalized size of antiderivative = 98.68

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(3/2)/sqrt(a + b*tan(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^{\frac{3}{2}}}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^(3/2)/sqrt(b*tan(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^{\frac{3}{2}}}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^(3/2)/sqrt(b*tan(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{\sqrt{a+b\tan(c+d\,x)}}\,dx$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x
)
```

output
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)
```

3.453 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 168

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} + \frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d} - \frac{(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

output

```
(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
-b)^(1/2)/d+2*B*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b
^(1/2)/d-(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(
1/2))/(I*a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt[4]{-1}\left(-\frac{(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)+\frac{2\sqrt{a}B\,\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\dots}{\sqrt{b}\sqrt{a+b\tan\dots}}}{d}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x
]],x]
```

output
```
((-1)^(1/4)*(-(((A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]) + ((A + I*B)*ArcTan[((-1)
^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a
 + I*b]) + (2*Sqrt[a]*B*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt
[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 4097, 3042, 4099, 3042, 4098, 104, 216, 219, 4117, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 4097$$

$$\int \frac{A\tan(c+dx)-B}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+B\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\tan(c+dx)-B}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+B\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4099}$$

$$\frac{1}{2}(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+B\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+B\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4098}$$

$$-\frac{(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{(-B+iA)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+B\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{104}$$

$$\frac{(-B+iA)\int \frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{(B+iA)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+B\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{216}$$

$$-\frac{(B+iA)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+B\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}$$

$$\Big\downarrow \text{219}$$

$$B\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

$$\Big\downarrow \text{4117}$$

$$\frac{B\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

$$\Big\downarrow \text{65}$$

$$\frac{2B\int \frac{1}{1-\frac{b\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}d}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
((I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*
x]]])/(Sqrt[I*a - b]*d) + (2*B*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/(Sqrt[b]*d) - ((I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[
Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4097
```
Int[(Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/Sqrt[(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] :> In
t[Simp[a*A - b*B + (A*b + a*B)*Tan[e + f*x], x]/(Sqrt[a + b*Tan[e + f*x]]*S
qrt[c + d*Tan[e + f*x]]), x] + Simp[b*B   Int[(1 + Tan[e + f*x]^2)/(Sqrt[a
+ b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.02 (sec) , antiderivative size = 1887830, normalized size of antiderivative = 11237.08

output too large to display

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10116 vs. $2(132) = 264$.

Time = 4.00 (sec) , antiderivative size = 20234, normalized size of antiderivative = 120.44

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{\tan{(c+dx)}}}{\sqrt{a+b\tan{(c+dx)}}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(tan(c + d*x))/sqrt(a + b*tan(c + d*x)),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\sqrt{\tan{(dx+c)}}}{\sqrt{b\tan{(dx+c)}+a}}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(tan(d*x + c))/sqrt(b*tan(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{\tan(dx+c)}}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(tan(d*x + c))/sqrt(b*tan(d*x + c) + a)
, x)
```

Mupad [B] (verification not implemented)

Time = 92.98 (sec) , antiderivative size = 30600, normalized size of antiderivative = 182.14

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x
)
```

output
```
atan(((((B^2 - A^2 + A*B*2i)/(4*(a*d^2*1i + b*d^2)))^(1/2)*((((B^2 - A^2 +
 A*B*2i)/(4*(a*d^2*1i + b*d^2)))^(1/2)*((((B^2 - A^2 + A*B*2i)/(4*(a*d^2*1
i + b*d^2)))^(1/2)*((((274877906944*(1600*a^12*b^34*d^8 - 16640*a^14*b^32*
d^8 + 22784*a^16*b^30*d^8 + 106496*a^18*b^28*d^8 + 65536*a^20*b^26*d^8))/d
^8 - (274877906944*tan(c + d*x)*(1600*a^12*b^35*d^8 - 48000*a^14*b^33*d^8
+ 155136*a^16*b^31*d^8 + 466944*a^18*b^29*d^8 + 262144*a^20*b^27*d^8))/(d^
8*((a + b*tan(c + d*x))^(1/2) - a^(1/2))^2))*((B^2 - A^2 + A*B*2i)/(4*(a*d
^2*1i + b*d^2)))^(1/2) - (2199023255552*tan(c + d*x)^(1/2)*(240*A*a^13*b^3
3*d^6 + 3064*A*a^15*b^31*d^6 + 8960*A*a^17*b^29*d^6 + 6144*A*a^19*b^27*d^6
 + 6920*B*a^14*b^32*d^6 + 9472*B*a^16*b^30*d^6 - 5632*B*a^18*b^28*d^6 - 81
92*B*a^20*b^26*d^6))/(d^7*((a + b*tan(c + d*x))^(1/2) - a^(1/2))))*((B^2 -
 A^2 + A*B*2i)/(4*(a*d^2*1i + b*d^2)))^(1/2) - (274877906944*(19216*A^2*a^
14*b^31*d^6 - 1440*A^2*a^12*b^33*d^6 - 22016*A^2*a^16*b^29*d^6 - 45056*A^2
*a^18*b^27*d^6 + 1200*B^2*a^12*b^33*d^6 - 16640*B^2*a^14*b^31*d^6 + 279040
*B^2*a^16*b^29*d^6 + 561152*B^2*a^18*b^27*d^6 + 262144*B^2*a^20*b^25*d^6 +
 16480*A*B*a^13*b^32*d^6 - 25792*A*B*a^15*b^30*d^6 + 34816*A*B*a^17*b^28*d
^6 + 81920*A*B*a^19*b^26*d^6))/d^8 + (274877906944*tan(c + d*x)*(46704*A^2
*a^14*b^32*d^6 - 1440*A^2*a^12*b^34*d^6 - 137216*A^2*a^16*b^30*d^6 - 12288
0*A^2*a^18*b^28*d^6 + 65536*A^2*a^20*b^26*d^6 + 1200*B^2*a^12*b^34*d^6 - 5
2320*B^2*a^14*b^32*d^6 + 1200640*B^2*a^16*b^30*d^6 + 2306048*B^2*a^18*b...
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}dx$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a),x)
```

3.454 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 123

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} + \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

output
```
(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
-b)^(1/2)/d+(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))/(I*a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\sqrt[4]{-1}\left(-\frac{(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{(-iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)}{d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]
]),x]
```

output
```
((-1)^(1/4)*(-(((I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]) + (((-I)*A + B)*ArcTan[((
-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqr
t[a + I*b]))/d
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4099}$$

$$\frac{1}{2}(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4098}$$

$$\frac{(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}$$

$$\Big\downarrow \text{104}$$

$$\frac{(A+iB)\int \frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(A-iB)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{(A-iB)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]),x]
```

output
```
((A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*
x]]])/(Sqrt[I*a - b]*d) + ((A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.96 (sec) , antiderivative size = 1878820, normalized size of antiderivative = 15274.96

output too large to display

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9913 vs. $2(95) = 190$.

Time = 2.56 (sec) , antiderivative size = 9913, normalized size of antiderivative = 80.59

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{A+B\tan{(c+dx)}}{\sqrt{a+b\tan{(c+dx)}}\sqrt{\tan{(c+dx)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(sqrt(a + b*tan(c + d*x))*sqrt(tan(c + d*x))
), x)
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{B\tan{(dx+c)}+A}{\sqrt{b\tan{(dx+c)}+a}\sqrt{\tan{(dx+c)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/(sqrt(b*tan(d*x + c) + a)*sqrt(tan(d*x + c)
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}\sqrt{a + b\tan(c + dx)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 53.08 (sec) , antiderivative size = 8223, normalized size of antiderivative = 66.85

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}\sqrt{a + b\tan(c + dx)}}\, dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2)),x
)
```

output
```
atan(((B^5*b^9*d^7*tan(c + d*x)^(1/2)*(-((-16*B^4*a^2*d^4)^(1/2) + 4*B^2*b
*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^(1/2)*1280i)/((a + b*tan(c + d*x))^(1/2)
- a^(1/2)) + (B^3*b^10*d^9*tan(c + d*x)^(1/2)*(-((-16*B^4*a^2*d^4)^(1/2) +
 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^(3/2)*10240i)/((a + b*tan(c + d*x
))^(1/2) - a^(1/2)) + (B*b^11*d^11*tan(c + d*x)^(1/2)*(-((-16*B^4*a^2*d^4)
^(1/2) + 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^(5/2)*20480i)/((a + b*tan
(c + d*x))^(1/2) - a^(1/2)) + (B*a^10*b*d^11*tan(c + d*x)^(1/2)*(-((-16*B^
4*a^2*d^4)^(1/2) + 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^(5/2)*196608i)/
((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (B^5*a^8*b*d^7*tan(c + d*x)^(1/2)
*(-((-16*B^4*a^2*d^4)^(1/2) + 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^(1/2
)*12288i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (B*a^2*b^9*d^11*tan(c +
 d*x)^(1/2)*(-((-16*B^4*a^2*d^4)^(1/2) + 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2
*d^4))^(5/2)*274432i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) + (B*a^4*b^7*
d^11*tan(c + d*x)^(1/2)*(-((-16*B^4*a^2*d^4)^(1/2) + 4*B^2*b*d^2)/(16*a^2*
d^4 + 16*b^2*d^4))^(5/2)*897024i)/((a + b*tan(c + d*x))^(1/2) - a^(1/2)) +
 (B*a^6*b^5*d^11*tan(c + d*x)^(1/2)*(-((-16*B^4*a^2*d^4)^(1/2) + 4*B^2*b*d
^2)/(16*a^2*d^4 + 16*b^2*d^4))^(5/2)*1249280i)/((a + b*tan(c + d*x))^(1/2)
 - a^(1/2)) + (B*a^8*b^3*d^11*tan(c + d*x)^(1/2)*(-((-16*B^4*a^2*d^4)^(1/2
) + 4*B^2*b*d^2)/(16*a^2*d^4 + 16*b^2*d^4))^(5/2)*802816i)/((a + b*tan(c +
 d*x))^(1/2) - a^(1/2)) + (B^5*a^2*b^7*d^7*tan(c + d*x)^(1/2)*(-((-16*B...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \frac{2\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}-2\left(\int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)b}\,\tan(dx+c)^2}{a+\tan(dx+c)b}dx\right)bd-\left(\int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)b}}{a+\tan(dx+c)b}\right.}{2d}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a) - 2*int((sqrt(tan(c + d*x))
*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*b*d - i
nt((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*x
)*b + a),x)*a*d + int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c
 + d*x)**2*b + tan(c + d*x)*a),x)*a*d)/(2*d)
```

3.455 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 159

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}d} + \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}d} - \frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}$$

output
```
-(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*
a-b)^(1/2)/d+(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/(I*a+b)^(1/2)/d-2*A*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\frac{\sqrt[4]{-1}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}-\frac{\sqrt[4]{-1}(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}-\frac{2A\sqrt{a+b\tan(c+dx)}}{a\sqrt{\tan(c+dx)}}}{d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]
]),x]
```

output
```
(((-1)^(1/4)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]
])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] - ((-1)^(1/4)*(A + I*B)*ArcTa
n[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])
/Sqrt[a + I*b] - (2*A*Sqrt[a + b*Tan[c + d*x]])/(a*Sqrt[Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4092, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{2\int -\frac{aB-aA\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{aB-aA\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aB-aA\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}$$

$$\Big\downarrow \text{4099}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{1}{2}a(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{1}{2}a(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}$$

$$\Big\downarrow \text{4098}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{a(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}-\frac{a(-B+iA)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}}{a}$$

$$\Big\downarrow \text{104}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{a(B+iA)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{a(-B+iA)\int \frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}}{a}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{a(B+iA)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{a(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}}{a}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{a(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}-\frac{a(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}}{a}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

output
```
(-((a*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/(Sqrt[I*a - b]*d)) + (a*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[T
an[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d))/a - (2*A*Sqrt[
a + b*Tan[c + d*x]])/(a*d*Sqrt[Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.10 (sec) , antiderivative size = 1888108, normalized size of antiderivative = 11874.89

output too large to display

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10036 vs. $2(124) = 248$.

Time = 2.61 (sec) , antiderivative size = 10036, normalized size of antiderivative = 63.12

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}\,dx = \int \frac{A + B\tan{(c + dx)}}{\sqrt{a + b\tan{(c + dx)}}\tan^{\frac{3}{2}}{(c + dx)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(3/2
)), x)
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx=\int \frac{B\tan\left(dx+c\right)+A}{\sqrt{b\tan\left(dx+c\right)+a}\tan\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(3/2
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{3/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}}{\tan\left(dx+c\right)^{2}}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/tan(c + d*x)**2,x)
```

3.456 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 203

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}d} - \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}d} - \frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)} + \frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{3a^2d\sqrt{\tan(c+dx)}}$$

output
```
-(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*
a-b)^(1/2)/d-(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/(I*a+b)^(1/2)/d-2/3*A*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(3/2
)+2/3*(2*A*b-3*B*a)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.96

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \frac{\frac{3\sqrt[4]{-1}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{3(-1)^{3/4}(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}-\frac{2\sqrt{a+b\tan(c+dx)}(aA+(\cdot}{a^2\tan^{\frac{3}{2}}(\cdot}}{3d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]
]),x]
```

output
```
((3*(-1)^(1/4)*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*
x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + (3*(-1)^(3/4)*(A + I*B)*A
rcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x
]]])/Sqrt[a + I*b] - (2*Sqrt[a + b*Tan[c + d*x]]*(a*A + (-2*A*b + 3*a*B)*T
an[c + d*x]))/(a^2*Tan[c + d*x]^(3/2)))/(3*d)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4092}$$

$$-\frac{2\int \frac{2Ab\tan^2(c+dx)+3aA\tan(c+dx)+2Ab-3aB}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{2Ab\tan^2(c+dx)+3aA\tan(c+dx)+2Ab-3aB}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{2Ab\tan(c+dx)^2+3aA\tan(c+dx)+2Ab-3aB}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{-\frac{2\int -\frac{3(Aa^2+B\tan(c+dx)a^2)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3\int \frac{Aa^2+B\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\int \frac{Aa^2+B\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4099}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{1}{2}a^2(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a^2(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx\right)}{a}}{3a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{1}{2}a^2(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a^2(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx\right)}{a}}{3a}$$

$$\Big\downarrow \text{ 4098}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{a^2(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{a^2(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\cdots}}{2d}\cdots\right)}{a}}{3a}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{a^2(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{a^2(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}\right)}{a}}{3a}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{a^2(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{a^2(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}\right)}{a}}{3a}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{a^2(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{a^2(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}\right)}{a}}{3a}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

output
```
(-2*A*Sqrt[a + b*Tan[c + d*x]])/(3*a*d*Tan[c + d*x]^(3/2)) - ((3*((a^2*(A
+ I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]
)/(Sqrt[I*a - b]*d) + (a^2*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(2*A*b - 3*a*B
)*Sqrt[a + b*Tan[c + d*x]])/(a*d*Sqrt[Tan[c + d*x]]))/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.84 (sec) , antiderivative size = 1890767, normalized size of antiderivative = 9314.12

output too large to display

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10067 vs. $2(163) = 326$.

Time = 2.65 (sec) , antiderivative size = 10067, normalized size of antiderivative = 49.59

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}\, dx = \int \frac{A + B\tan(c + dx)}{\sqrt{a + b\tan(c + dx)}\tan^{\frac{5}{2}}(c + dx)}\, dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(5/2
)), x)
```

Maxima [F]

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}\, dx = \int \frac{B\tan\left(dx + c\right) + A}{\sqrt{b\tan\left(dx + c\right) + a}\tan\left(dx + c\right)^{\frac{5}{2}}}\, dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(5/2
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{5/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}}{\tan\left(dx+c\right)^{3}}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/tan(c + d*x)**3,x)
```

3.457 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 256

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx &= \frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}d}-\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}d}\\ &\quad-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}+\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{15a^2d\tan^{\frac{3}{2}}(c+dx)}\\ &\quad+\frac{2(15a^2A-8Ab^2+10abB)\sqrt{a+b\tan(c+dx)}}{15a^3d\sqrt{\tan(c+dx)}}\end{aligned}$$

output
```
(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
-b)^(1/2)/d-(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))/(I*a+b)^(1/2)/d-2/5*A*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(5/2)
+2/15*(4*A*b-5*B*a)*(a+b*tan(d*x+c))^(1/2)/a^2/d/tan(d*x+c)^(3/2)+2/15*(15
*A*a^2-8*A*b^2+10*B*a*b)*(a+b*tan(d*x+c))^(1/2)/a^3/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 3.73 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.89

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$$
$$= \frac{-\frac{15\sqrt[4]{-1}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{15\sqrt[4]{-1}(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}+\frac{2\sqrt{a+b\tan(c+dx)}(-}{}}{15d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]
]),x]
```

output
```
((-15*(-1)^(1/4)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + (15*(-1)^(1/4)*(A + I*B
)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]])/Sqrt[a + I*b] + (2*Sqrt[a + b*Tan[c + d*x]]*(-3*a^2*A - a*(-4*A*b
+ 5*a*B)*Tan[c + d*x] + (15*a^2*A - 8*A*b^2 + 10*a*b*B)*Tan[c + d*x]^2))/(
a^3*Tan[c + d*x]^(5/2)))/(15*d)
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{7/2}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{2\int \frac{4Ab\tan^2(c+dx)+5aA\tan(c+dx)+4Ab-5aB}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{4Ab\tan^2(c+dx)+5aA\tan(c+dx)+4Ab-5aB}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{4Ab\tan(c+dx)^2+5aA\tan(c+dx)+4Ab-5aB}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{-\frac{2\int -\frac{15Aa^2+15B\tan(c+dx)a^2+10bBa-8Ab^2-2b(4Ab-5aB)\tan^2(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{15Aa^2+15B\tan(c+dx)a^2+10bBa-8Ab^2-2b(4Ab-5aB)\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{15Aa^2+15B\tan(c+dx)a^2+10bBa-8Ab^2-2b(4Ab-5aB)\tan(c+dx)^2}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{-\frac{2\int -\frac{15\left(a^3B-a^3A\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{15\int \frac{a^3B-a^3A\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{15\int \frac{a^3B-a^3A\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 4099

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{1}{2}a^3(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a^3(-B+iA)\int \frac{}{\sqrt{}}\right.}{a}}{3a}}{5a}$$

$\Big\downarrow$ 3042

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{1}{2}a^3(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a^3(-B+iA)\int \frac{}{\sqrt{}}\right.}{a}}{3a}}{5a}$$

$\Big\downarrow$ 4098

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2(15a^2A+10abB-8Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^3(B+iA)\int\frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+}{2d}\right.}{3a}}{5a}$$

$\downarrow$ 104

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2(15a^2A+10abB-8Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^3(B+iA)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{a^3(-B+iA)}{}\right.}{a}}{3a}}{5a}$$

$\downarrow$ 216

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2(15a^2A+10abB-8Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^3(B+iA)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{a^3(-B+iA}{}\right.}{a}}{3a}}{5a}$$

$\downarrow$ 219

$$-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2(15a^2A+10abB-8Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{15\left(\frac{a^3(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}-\frac{a^3(-B+iA)\arctan\left(\right.}{d\sqrt{}}\right.}{a}}{3a}}{5a}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

output
```
(-2*A*Sqrt[a + b*Tan[c + d*x]])/(5*a*d*Tan[c + d*x]^(5/2)) - ((-2*(4*A*b -
 5*a*B)*Sqrt[a + b*Tan[c + d*x]])/(3*a*d*Tan[c + d*x]^(3/2)) + ((15*(-((a^
3*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d
*x]]])/(Sqrt[I*a - b]*d)) + (a^3*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan
[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(15*a^2*
A - 8*A*b^2 + 10*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(a*d*Sqrt[Tan[c + d*x]])
)/(3*a))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.08 (sec) , antiderivative size = 1892796, normalized size of antiderivative = 7393.73

output too large to display

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10125 vs. 2(210) = 420.

Time = 2.62 (sec) , antiderivative size = 10125, normalized size of antiderivative = 39.55

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{7}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{7}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(7/2)/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx=\int \frac{B\tan\left(dx+c\right)+A}{\sqrt{b\tan\left(dx+c\right)+a}\tan\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/(sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^(7/2
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx=\int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{7/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(7/2)*(a + b*tan(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx=\int \frac{\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}}{\tan\left(dx+c\right)^{4}}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(7/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/tan(c + d*x)**4,x)
```

3.458 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 219

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d}+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{3/2}d}-\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d}+\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{b(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
-(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*
a-b)^(3/2)/d+2*B*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/
b^(3/2)/d-(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^
(1/2))/(I*a+b)^(3/2)/d+2*a*(A*b-B*a)*tan(d*x+c)^(1/2)/b/(a^2+b^2)/d/(a+b*t
an(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.56

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{a}\sqrt{-a+ib}\sqrt{a+ib}(a^2+b^2)\,B\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}+\sqrt{b}\left(\sqrt[4]{-1}(a+ib)^{3/2}b(iA+B}{}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
3/2),x]
```

output
```
-((2*Sqrt[a]*Sqrt[-a + I*b]*Sqrt[a + I*b]*(a^2 + b^2)*B*ArcSinh[(Sqrt[b]*S
qrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a] + Sqrt[b]*((-1)^(
1/4)*(a + I*b)^(3/2)*b*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[a + b*Tan[c + d*x]] + Sqrt[-a
+ I*b]*(2*a*Sqrt[a + I*b]*(A*b - a*B)*Sqrt[Tan[c + d*x]] + (-1)^(1/4)*b*(I
*a + b)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]]*Sqrt[a + b*Tan[c + d*x]])))/((-a + I*b)^(3/2)*(a +
I*b)^(3/2)*b^(3/2)*d*Sqrt[a + b*Tan[c + d*x]]))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 4088, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2\int -\frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}+\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{-((a^2+b^2)B\tan(c+dx)^2)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}$$

$$\Big\downarrow \text{4138}$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{bd(a^2+b^2)}$$

$$\Big\downarrow \text{2035}$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{2\int \frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{bd(a^2+b^2)}$$

$$\Big\downarrow \text{2257}$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{2\int \left(\frac{b(aA+bB)-b(Ab-aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}-\frac{(a^2+b^2)B}{\sqrt{a+b\tan(c+dx)}}\right)d\sqrt{\tan(c+dx)}}{bd(a^2+b^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{2\left(-\frac{B(a^2+b^2)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}}+\frac{b(a-ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{2\sqrt{-b+ia}}+\frac{b(a+ib)(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+d}}{\sqrt{a+b\tan(c+dx}}\right.}{2\sqrt{b+ia}}\right.}{bd(a^2+b^2)}$$

input
```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x
]
```

output
```
(-2*(((a - I*b)*b*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]])/(2*Sqrt[I*a - b]) - ((a^2 + b^2)*B*ArcTanh[(Sqrt[b]
*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[b] + ((a + I*b)*b*(A
- I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]
])/(2*Sqrt[I*a + b])))/(b*(a^2 + b^2)*d) + (2*a*(A*b - a*B)*Sqrt[Tan[c + d
*x]])/(b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 3.21 (sec) , antiderivative size = 1560668, normalized size of antiderivative = 7126.34

output too large to display

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18781 vs. $2(176) = 352$.

Time = 10.70 (sec) , antiderivative size = 37564, normalized size of antiderivative = 171.53

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan^{\frac{3}{2}}{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(3/2)/(a + b*tan(c + d*x))**(3
/2), x)
```

Maxima [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\tan{(dx+c)}^{\frac{3}{2}}}{(b\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(3/
2), x)
```

Giac [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^{\frac{3}{2}}}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^{3/2}}\,dx$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x
)
```

output
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),
x)
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)}{a+\tan(dx+c)\,b}dx$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*
x)*b + a),x)
```

3.459 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 170

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} + \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d} - \frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
-(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*
a-b)^(3/2)/d+(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/(I*a+b)^(3/2)/d-2*(A*b-B*a)*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*ta
n(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{-\frac{\sqrt[4]{-1}a(a+ib)(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{\sqrt[4]{-1}a(a-ib)(A\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
3/2),x]
```

output
```
(-(((-1)^(1/4)*a*(a + I*b)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqr
t[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]) + ((-1)^(1/4)*
a*(a - I*b)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])
/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] + (2*b*(A*b - a*B)*Tan[c + d*x]^
(3/2))/Sqrt[a + b*Tan[c + d*x]] + 2*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]]*Sqrt
[a + b*Tan[c + d*x]])/(a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4091, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4091}$$

$$-\frac{2\int -\frac{b(Ab-aB)+b(aA+bB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b(Ab-aB)+b(aA+bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(Ab-aB)+b(aA+bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 4099

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}b(b+ia)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}b(-b+ia)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}b(b+ia)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}b(-b+ia)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4098

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{b(b+ia)(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}-\frac{b(-b+ia)(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(}{2d}}{b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 104

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{b(b+ia)(A+iB)\int \frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{b(-b+ia)(A-iB)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}}{b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 216

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{b(b+ia)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{b(-b+ia)(A-iB)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}}{b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 219

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{b(b+ia)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{b(-b+ia)(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}}{b\left(a^2+b^2\right)}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x
]
```

output
```
((b*(I*a + b)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) - ((I*a - b)*b*(A - I*B)*ArcTanh[(Sqr
t[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d
))/(b*(a^2 + b^2)) - (2*(A*b - a*B)*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*d*Sqr
t[a + b*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.96 (sec) , antiderivative size = 1559493, normalized size of antiderivative = 9173.49

output too large to display

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18532 vs. 2(138) = 276.

Time = 6.81 (sec) , antiderivative size = 18532, normalized size of antiderivative = 109.01

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{\tan{(c+dx)}}}{(a+b\tan{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(tan(c + d*x))/(a + b*tan(c + d*x))**(3/
2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{\tan(dx+c)}}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)^(3/
2), x)
```

Giac [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{\tan(dx+c)}}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(c+d\,x)}\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^{3/2}}\,dx$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x
)
```

output
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}-2\left(\int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx}}{a+\tan(dx}\right.}{}$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a) - 2*int((sqrt(tan(c + d*x))
*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*b*d - i
nt((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*x
)*b + a),x)*a*d - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c
 + d*x)**2*b + tan(c + d*x)*a),x)*a*d)/(2*b*d)
```

3.460 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 175

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} + \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d} + \frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{a(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
-b)^(3/2)/d+(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))/(I*a+b)^(3/2)/d+2*b*(A*b-B*a)*tan(d*x+c)^(1/2)/a/(a^2+b^2)/d/(a+b
*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \frac{\frac{\sqrt[4]{-1}(-ia+b)(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{\sqrt[4]{-1}(a-ib)(-i\cdots}{\cdots}}{(a^2+b^2)\,\cdots}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3
/2)),x]
```

output
```
(((-1)^(1/4)*((-I)*a + b)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + ((-1)^(1/4)*(a
 - I*b)*((-I)*A + B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/
Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] + (2*b*(A*b - a*B)*Sqrt[Tan[c + d
*x]])/(a*Sqrt[a + b*Tan[c + d*x]]))/((a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4092, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{2\int \frac{a(aA+bB)-a(Ab-aB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(aA+bB)-a(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(aA+bB)-a(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 4099

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}a(a-ib)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a(a+ib)(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}a(a-ib)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a(a+ib)(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4098

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a(a+ib)(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{a(a-ib)(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c-}{2d}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 104

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a(a-ib)(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{a(a+ib)(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 216

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a(a+ib)(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{a(a-ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}}{a\left(a^2+b^2\right)}$$

$\Big\downarrow$ 219

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a(a-ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{a(a+ib)(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}}{a\left(a^2+b^2\right)}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)),x
]
```

output
```
((a*(a - I*b)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a*(a + I*b)*(A - I*B)*ArcTanh[(Sqr
t[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d
))/(a*(a^2 + b^2)) + (2*b*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d
*Sqrt[a + b*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.51 (sec) , antiderivative size = 1559531, normalized size of antiderivative = 8911.61

output too large to display

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18563 vs. 2(143) = 286.

Time = 6.65 (sec) , antiderivative size = 18563, normalized size of antiderivative = 106.07

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{3}{2}}\sqrt{\tan{(c+dx)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**(3/2)*sqrt(tan(c + d*
x))), x)
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{B\tan\left(dx+c\right)+A}{\left(b\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\tan\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*x +
c))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\tan\left(c+d\,x\right)}\,{\left(a+b\tan\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}}{\tan(dx+c)^2\,b+\tan(dx+c)\,a}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan
(c + d*x)*a),x)
```

3.461 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 216

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} - \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d} - \frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}} - \frac{2b(a^2A+2Ab^2-abB)\sqrt{\tan(c+dx)}}{a^2(a^2+b^2)\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
-b)^(3/2)/d-(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))/(I*a+b)^(3/2)/d-2*A/a/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)-2
*b*(A*a^2+2*A*b^2-B*a*b)*tan(d*x+c)^(1/2)/a^2/(a^2+b^2)/d/(a+b*tan(d*x+c))
^(1/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.15

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \frac{\frac{\sqrt[4]{-1}\left(\frac{(a+ib)(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}-\frac{(a-ib)(A+iB)\arctan\left(\frac{\sqrt[4]{\cdots}}{\cdots}\right)}{\sqrt{a\cdots}}\right.}{a^2+b^2}\cdots}{\cdots}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3
/2)),x]
```

output
```
(((-1)^(1/4)*(((a + I*b)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[
Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] - ((a - I*b)*(A +
 I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]])/Sqrt[a + I*b]))/(a^2 + b^2) - (2*A)/(a*Sqrt[Tan[c + d*x]]*Sqrt
[a + b*Tan[c + d*x]]) - (2*b*(a^2*A + 2*A*b^2 - a*b*B)*Sqrt[Tan[c + d*x]])
/(a^2*(a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{2\int \frac{2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{2Ab\tan(c+dx)^2+aA\tan(c+dx)+2Ab-aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 4132

$$-\frac{\frac{2\int \frac{(Ab-aB)a^2+(aA+bB)\tan(c+dx)a^2}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{(Ab-aB)a^2+(aA+bB)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{(Ab-aB)a^2+(aA+bB)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 4099

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}a^2(b+ia)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a^2(-b+ia)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}}{a}$$

$\Big\downarrow$ 3042

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}a^2(b+ia)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a^2(-b+ia)(A-iB)\int\frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}}{a}$$

$\Big\downarrow$ 4098

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a^2(b+ia)(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}-\frac{a^2(-b+ia)(A-iB)\int\frac{}{(1-i\tan(c+dx)}}{}}{a(a^2+b^2)}}{a}$$

$\Big\downarrow$ 104

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a^2(b+ia)(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{a^2(-b+ia)(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+}}{\sqrt{a+b\tan}}}{d}}{a(a^2+b^2)}}{a}$$

$\Big\downarrow$ 216

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a^2(b+ia)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{a^2(-b+ia)(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}}{a(a^2+b^2)}}{a}$$

$\Big\downarrow$ 219

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a^2(b+ia)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{a^2(-b+ia)(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}}{a(a^2+b^2)}}{a}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)),x
]
```

output
```
(-2*A)/(a*d*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]) - (((a^2*(I*a + b
)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d
*x]]])/(Sqrt[I*a - b]*d) - (a^2*(I*a - b)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]
*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d))/(a*(a^2
 + b^2)) + (2*b*(a^2*A + 2*A*b^2 - a*b*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^
2)*d*Sqrt[a + b*Tan[c + d*x]]))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4053 vs. $2(186) = 372$.

Time = 9.76 (sec) , antiderivative size = 4054, normalized size of antiderivative = 18.77

method	result	size
default	Expression too large to display	4054
parts	Expression too large to display	1560457

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/d/a^2/(a^2+b^2)^(3/2)/(b+(a^2+b^2)^(1/2))^(1/2)*(4*A*(a^2+b^2)^(1/2)*(b
+(a^2+b^2)^(1/2))^(1/2)*a*b^2+4*A*(a^2+b^2)^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2
)*a^3+2*A*arctan(1/(b+(a^2+b^2)^(1/2))^(1/2)*((-b+(a^2+b^2)^(1/2))^(1/2)*(
csc(d*x+c)-cot(d*x+c))+(-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2))/(1-cos(d*x+c))*sin(d*x+c))*a^4*(-2*(a*cos(d*x+c)+b*sin(d*
x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*A*arctan(1/(b+(a^2+b^2)^(1/2))^
(1/2)*(-(-b+(a^2+b^2)^(1/2))^(1/2)*(csc(d*x+c)-cot(d*x+c))+(-2*(a*cos(d*x+
c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))/(1-cos(d*x+c))*sin(d*
x+c))*a^4*(-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/
2)-B*(a^2+b^2)^(1/2)*ln(-1/(1-cos(d*x+c))*(a*(1-cos(d*x+c))^2*csc(d*x+c)+2
*(a^2+b^2)^(1/2)*(1-cos(d*x+c))-2*(-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)*(-b+(a^2+b^2)^(1/2))^(1/2)*sin(d*x+c)-2*b*(1-co
s(d*x+c))-a*sin(d*x+c)))*(-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d
*x+c)+1)^2)^(1/2)*(-b+(a^2+b^2)^(1/2))^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*a*b
+B*(a^2+b^2)^(1/2)*ln(1/(1-cos(d*x+c))*(a*(1-cos(d*x+c))^2*csc(d*x+c)+2*(a
^2+b^2)^(1/2)*(1-cos(d*x+c))+2*(-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*(-b+(a^2+b^2)^(1/2))^(1/2)*sin(d*x+c)-2*b*(1-cos(d
*x+c))-a*sin(d*x+c)))*(-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)*(-b+(a^2+b^2)^(1/2))^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*a*b+A*
(a^2+b^2)^(1/2)*ln(1/(1-cos(d*x+c))*(a*(1-cos(d*x+c))^2*csc(d*x+c)+2*(a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18689 vs. 2(180) = 360.

Time = 6.70 (sec) , antiderivative size = 18689, normalized size of antiderivative = 86.52

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**(
3/2)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \text{Hanged}$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2)),x
)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \int \frac{\sqrt{\tan(dx + c)}\,\sqrt{a + \tan(dx + c)\,b}}{\tan(dx + c)^3\,b + \tan(dx + c)^2\,a}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan
(c + d*x)**2*a),x)
```

3.462 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 276

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{3/2}d} - \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{3/2}d} - \frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}} + \frac{2(4Ab-3aB)}{3a^2d\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}} + \frac{2b(5a^2Ab+8Ab^3-3a^3B-6ab^2B)\sqrt{\tan(c+dx)}}{3a^3(a^2+b^2)d\sqrt{a+b\tan(c+dx)}}$$

output

```
-(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*
a-b)^(3/2)/d-(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/(I*a+b)^(3/2)/d-2/3*A/a/d/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2
)+2/3*(4*A*b-3*B*a)/a^2/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)+2/3*b*(5
*A*a^2*b+8*A*b^3-3*B*a^3-6*B*a*b^2)*tan(d*x+c)^(1/2)/a^3/(a^2+b^2)/d/(a+b*
tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.08

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \frac{\frac{3\sqrt[4]{-1}a\left(\frac{(a+ib)(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{(ia+b)(A+iB)\arctan\left(\cdots\right.}{\cdots}\right.}{a^2+b^2}\cdots}{\cdots}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3
/2)),x]
```

output
```
((3*(-1)^(1/4)*a*(((a + I*b)*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + ((I*a + b)*
(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]])/Sqrt[a + I*b]))/(a^2 + b^2) - (2*A)/(Tan[c + d*x]^(3/2)*Sq
rt[a + b*Tan[c + d*x]]) + (8*A*b - 6*a*B)/(a*Sqrt[Tan[c + d*x]]*Sqrt[a + b
*Tan[c + d*x]]) + (2*b*(5*a^2*A*b + 8*A*b^3 - 3*a^3*B - 6*a*b^2*B)*Sqrt[Ta
n[c + d*x]])/(a^2*(a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]]))/(3*a*d)
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.457, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 4092$$

$$-\frac{2\int \frac{4Ab\tan^2(c+dx)+3aA\tan(c+dx)+4Ab-3aB}{2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}dx}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{4Ab\tan^2(c+dx)+3aA\tan(c+dx)+4Ab-3aB}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}dx}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{4Ab\tan(c+dx)^2+3aA\tan(c+dx)+4Ab-3aB}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^{3/2}}dx}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 4132$$

$$-\frac{-\frac{2\int -\frac{3Aa^2+3B\tan(c+dx)a^2+6bBa-8Ab^2-2b(4Ab-3aB)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{3Aa^2+3B\tan(c+dx)a^2+6bBa-8Ab^2-2b(4Ab-3aB)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{3Aa^2+3B\tan(c+dx)a^2+6bBa-8Ab^2-2b(4Ab-3aB)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \; 4132$$

$$-\frac{\frac{\frac{2\int \frac{3\left(a^3(aA+bB)-a^3(Ab-aB)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{3\int \frac{a^3(aA+bB)-a^3(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{3\int \frac{a^3(aA+bB)-a^3(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}$$

$\Big\downarrow$ 4099

$$-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}a^3(a-ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a^3(\right.}{a\left(a^2+b^2\right)}}{a}}{3a}$$

$\Big\downarrow$ 3042

$$-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}a^3(a-ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a^3(\right.}{a\left(a^2+b^2\right)}}{a}}{3a}$$

$\Big\downarrow$ 4098

$$\frac{-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^3(a+ib)(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c\ldots}}{2d}\ldots\right.}{\ldots}}{a}}{3a}$$

$\downarrow$ 104

$$\frac{-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^3(a-ib)(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}\ldots\right.}{a\left(a^2\ldots\right.}}{a}}{3a}$$

$\downarrow$ 216

$$\frac{-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^3(a+ib)(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}\ldots\right.}{a\left(a^2+b^2\right)\ldots}}{a}}{3a}$$

$\downarrow$ 219

$$\frac{-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^3(a-ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{a^3(a+ib)\ldots}{\ldots}\right.}{a\left(a^2+b^2\right)\ldots}}{a}}{3a}$$

input

```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2)),x
]
```

output
```
(-2*A)/(3*a*d*Tan[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]) - ((-2*(4*A*b -
 3*a*B))/(a*d*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]) + ((3*((a^3*(a
- I*b)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]])/(Sqrt[I*a - b]*d) + (a^3*(a + I*b)*(A - I*B)*ArcTanh[(Sqrt[I*a
 + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/(
a*(a^2 + b^2)) - (2*b*(5*a^2*A*b + 8*A*b^3 - 3*a^3*B - 6*a*b^2*B)*Sqrt[Tan
[c + d*x]])/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/a)/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4643 vs. $2(236) = 472$.

Time = 4.82 (sec) , antiderivative size = 4644, normalized size of antiderivative = 16.83

method	result	size
default	Expression too large to display	4644
parts	Expression too large to display	1562265

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/12/d/a^3/(b+(a^2+b^2)^(1/2))^(1/2)/(a^2+b^2)^(3/2)*(24*cos(d*x+c)*sin(d
*x+c)^2*B*(a^2+b^2)^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*a^4+(3*cos(d*x+c)+3)*s
in(d*x+c)^2*A*(-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*(-b+(a^2+b^2)^(1/2))^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*a^4*ln(-(a*cot
(d*x+c)*cos(d*x+c)-2*a*cot(d*x+c)+2*(-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*(-b+(a^2+b^2)^(1/2))^(1/2)*sin(d*x+c)+a*csc(d
*x+c)-a*sin(d*x+c)-2*cos(d*x+c)*(a^2+b^2)^(1/2)+2*b*cos(d*x+c)+2*(a^2+b^2)
^(1/2)-2*b)/(cos(d*x+c)-1))+(-3*cos(d*x+c)-3)*sin(d*x+c)^2*A*(-2*(a*cos(d*
x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(-b+(a^2+b^2)^(1/2))
^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*a^4*ln((a*cot(d*x+c)*cos(d*x+c)-2*a*cot(d
*x+c)-2*(-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
*(-b+(a^2+b^2)^(1/2))^(1/2)*sin(d*x+c)+a*csc(d*x+c)-a*sin(d*x+c)-2*cos(d*x
+c)*(a^2+b^2)^(1/2)+2*b*cos(d*x+c)+2*(a^2+b^2)^(1/2)-2*b)/(cos(d*x+c)-1))+
(6*cos(d*x+c)+6)*sin(d*x+c)^2*B*(a^2+b^2)^(1/2)*(-2*(a*cos(d*x+c)+b*sin(d*
x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b*arctan(1/(b+(a^2+b^2)^(1/2)
)^(1/2)*((-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2
)*sin(d*x+c)+(-b+(a^2+b^2)^(1/2))^(1/2)*cos(d*x+c)-(-b+(a^2+b^2)^(1/2))^(1
/2))/(cos(d*x+c)-1))+(6*cos(d*x+c)+6)*sin(d*x+c)^2*B*(a^2+b^2)^(1/2)*(-2*(
a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*a^3*b*arctan
(1/(b+(a^2+b^2)^(1/2))^(1/2)*((-2*(a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18754 vs. 2(227) = 454.

Time = 7.01 (sec) , antiderivative size = 18754, normalized size of antiderivative = 67.95

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}\tan^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**(
5/2)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx=\int \frac{B\tan\left(dx+c\right)+A}{\left(b\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}\tan\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^(5
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx=\int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{5/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}} \, dx = \frac{4\sqrt{\tan(dx + c)}\,\sqrt{a + \tan(dx + c)\,b}\,\tan(dx + c)\,b - 2\sqrt{\tan(\cdots}{}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
(4*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*b - 2*sqrt(tan
(c + d*x))*sqrt(tan(c + d*x)*b + a)*a - 3*int((sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)**2*
a**2*d)/(3*tan(c + d*x)**2*a**2*d)
```

3.463 $\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 282

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} + \frac{2B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{b^{5/2}d} - \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d} + \frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3b\left(a^2+b^2\right)d(a+b\tan(c+dx))^{3/2}} + \frac{2a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)\sqrt{\tan(c+dx)}}{b^2\left(a^2+b^2\right)^2 d\sqrt{a+b\tan(c+dx)}}$$

output
```
(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
-b)^(5/2)/d+2*B*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b
^(5/2)/d-(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(
1/2))/(I*a+b)^(5/2)/d+2/3*a*(A*b-B*a)*tan(d*x+c)^(3/2)/b/(a^2+b^2)/d/(a+b*
tan(d*x+c))^(3/2)+2*a*(2*A*b^3-a*(a^2+3*b^2)*B)*tan(d*x+c)^(1/2)/b^2/(a^2+
b^2)^2/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 596 vs. 2(282) = 564.

Time = 6.25 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.11

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(A-iB)\left(\frac{3\sqrt[4]{-1}\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(-a+ib)^{5/2}}+\frac{\tan^{\frac{3}{2}}(c+dx)}{(a-ib)(a+b\tan(c+dx))^{3/2}}\cdots\right)}{3d}$$

$$-\frac{(A+iB)\left(\frac{3\sqrt[4]{-1}\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{5/2}}-\frac{\tan^{\frac{3}{2}}(c+dx)}{(a+ib)(a+b\tan(c+dx))^{3/2}}-\frac{3i\sqrt{\tan(c+dx)}}{(a+ib)^2\sqrt{a+b\tan(c+dx)}}\right)}{3d}$$

$$+\frac{(iA-B)\sqrt{a+b\tan(c+dx)}\left(\frac{b^2\tan^2(c+dx)}{(a+b\tan(c+dx))^2}+\frac{3b\tan(c+dx)}{a+b\tan(c+dx)}-\frac{3\sqrt{b}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{\tan(c+dx)}}{\sqrt{a}\sqrt{1+\frac{b\tan(c+dx)}{a}}}\right)}{3b^3d\sqrt{\tan(c+dx)}}$$

$$-\frac{(iA+B)\sqrt{a+b\tan(c+dx)}\left(\frac{b^2\tan^2(c+dx)}{(a+b\tan(c+dx))^2}+\frac{3b\tan(c+dx)}{a+b\tan(c+dx)}-\frac{3\sqrt{b}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{\tan(c+dx)}}{\sqrt{a}\sqrt{1+\frac{b\tan(c+dx)}{a}}}\right)}{3b^3d\sqrt{\tan(c+dx)}}$$

input
```
Integrate[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
5/2),x]
```

output
```
((A - I*B)*((3*(-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/(-a + I*b)^(5/2) + Tan[c + d*x]^(3/2)/((a
 - I*b)*(a + b*Tan[c + d*x])^(3/2)) - ((3*I)*Sqrt[Tan[c + d*x]])/((a - I*b
)^2*Sqrt[a + b*Tan[c + d*x]])))/(3*d) - ((A + I*B)*((3*(-1)^(1/4)*ArcTan[(
(-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a
 + I*b)^(5/2) - Tan[c + d*x]^(3/2)/((a + I*b)*(a + b*Tan[c + d*x])^(3/2))
- ((3*I)*Sqrt[Tan[c + d*x]])/((a + I*b)^2*Sqrt[a + b*Tan[c + d*x]])))/(3*d
) + ((I*A - B)*Sqrt[a + b*Tan[c + d*x]]*((b^2*Tan[c + d*x]^2)/(a + b*Tan[c
 + d*x])^2 + (3*b*Tan[c + d*x])/(a + b*Tan[c + d*x]) - (3*Sqrt[b]*ArcSinh[
(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*Sqrt[1
+ (b*Tan[c + d*x])/a])))/(3*b^3*d*Sqrt[Tan[c + d*x]]) - ((I*A + B)*Sqrt[a
+ b*Tan[c + d*x]]*((b^2*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^2 + (3*b*Tan[
c + d*x])/(a + b*Tan[c + d*x]) - (3*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c +
d*x]])/Sqrt[a]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*Sqrt[1 + (b*Tan[c + d*x])/a])
))/(3*b^3*d*Sqrt[Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 4088, 27, 3042, 4128, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{5/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2\int -\frac{3\sqrt{\tan(c+dx)}\left(-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)\right)}{2(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}+\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int\frac{\sqrt{\tan(c+dx)}\left(-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)\right)}{(a+b\tan(c+dx))^{3/2}}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int\frac{\sqrt{\tan(c+dx)}\left(-\left(\left(a^2+b^2\right)B\tan(c+dx)^2\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)\right)}{(a+b\tan(c+dx))^{3/2}}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 4128}$$

$$\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2\int\frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx)+a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx)+a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan(c+dx)^2+a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 4138}$$

$$\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx)+a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{bd(a^2+b^2)}-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2035}$$

$$\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^{3/2}}-\frac{\frac{2\int\frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx)+a\left(2Ab^3-a\left(a^2+3b^2\right)B\right)}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{bd(a^2+b^2)}-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2257}$$

$$\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^{3/2}}-\frac{\frac{2\int\left(\frac{\left(-Ba^2+2Aba+b^2B\right)b^2+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}-\frac{\left(a^2+b^2\right)^2B}{\sqrt{a+b\tan(c+dx)}}\right)d\sqrt{\tan(c+dx)}}{bd(a^2+b^2)}-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^{3/2}}-\frac{-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{2\left(-\frac{B\left(a^2+b^2\right)^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}}+\frac{b^2(a-ib)^2(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{2\sqrt{-b+ia}}-\frac{b^2(a}{}\cdots\right.}{bd(a^2+b^2)}}{b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x
]
```

output
```
(2*a*(A*b - a*B)*Tan[c + d*x]^(3/2))/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x
])^(3/2)) - ((2*(((a - I*b)^2*b^2*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan
[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[I*a - b]) - ((a^2 + b^2)^2*
B*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[b]
- ((a + I*b)^2*b^2*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sq
rt[a + b*Tan[c + d*x]]])/(2*Sqrt[I*a + b])))/(b*(a^2 + b^2)*d) - (2*a*(2*A
*b^3 - a*(a^2 + 3*b^2)*B)*Sqrt[Tan[c + d*x]])/(b*(a^2 + b^2)*d*Sqrt[a + b*
Tan[c + d*x]]))/(b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 4.41 (sec) , antiderivative size = 2978162, normalized size of antiderivative = 10560.86

output too large to display

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27883 vs. $2(236) = 472$.

Time = 22.81 (sec) , antiderivative size = 55768, normalized size of antiderivative = 197.76

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan^{\frac{5}{2}}{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(5/2)/(a + b*tan(c + d*x))**(5
/2), x)
```

Maxima [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\tan{(dx+c)}^{\frac{5}{2}}}{(b\tan{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^(5/
2), x)
```

Giac [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^{\frac{5}{2}}}{(b\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^{5/2}}\,dx$$

input
```
int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x
)
```

output
```
int((tan(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2}{\tan(dx+c)^2\,b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(tan(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c +
 d*x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2),x)
```

3.464 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 244

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} + \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d} + \frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3b(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2(2a^2Ab-4Ab^3+a^3B+7ab^2B)\sqrt{\tan(c+dx)}}{3b(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
-b)^(5/2)/d+(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))/(I*a+b)^(5/2)/d+2/3*a*(A*b-B*a)*tan(d*x+c)^(1/2)/b/(a^2+b^2)/d/(a
+b*tan(d*x+c))^(3/2)+2/3*(2*A*a^2*b-4*A*b^3+B*a^3+7*B*a*b^2)*tan(d*x+c)^(1
/2)/b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.26

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{-\frac{3B\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^{3/2}} + \frac{(2aAb+a^2B+3b^2B)\sqrt{\tan(c+dx)}}{(a^2+b^2)(a+b\tan(c+dx))^{3/2}} + \frac{3\sqrt[4]{-1}b\left(\frac{(a+ib)^2(iA \cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
5/2),x]
```

output
```
((-3*B*Sqrt[Tan[c + d*x]])/(a + b*Tan[c + d*x])^(3/2) + ((2*a*A*b + a^2*B
+ 3*b^2*B)*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*(a + b*Tan[c + d*x])^(3/2)) +
(3*(-1)^(1/4)*b*(((a + I*b)^2*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + (I*(a - I*
b)^2*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/Sqrt[a + I*b]) + (2*(2*a^2*A*b - 4*A*b^3 + a^3*B + 7*
a*b^2*B)*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])/(a^2 + b^2)^2)/(3*b
*d)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.20, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4088, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 4088$$

$$\frac{2\int -\frac{-\left(\left(Ba^2+2Aba+3b^2B\right)\tan^2(c+dx)\right)-3b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}+\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int \frac{-\left(\left(Ba^2+2Aba+3b^2B\right)\tan^2(c+dx)\right)-3b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int \frac{-\left(\left(Ba^2+2Aba+3b^2B\right)\tan(c+dx)^2\right)-3b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 4132$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2\int \frac{3\left(ab\left(Aa^2+2bBa-Ab^2\right)-ab\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2\left(a^3B+2a^2Ab+7ab^2B-4Ab^3\right)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{3\int \frac{ab\left(Aa^2+2bBa-Ab^2\right)-ab\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2\left(a^3B+2a^2Ab+7ab^2B-4Ab^3\right)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{3\int \frac{ab\left(Aa^2+2bBa-Ab^2\right)-ab\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2\left(a^3B+2a^2Ab+7ab^2B-4Ab^3\right)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4099

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(a^3B+2a^2Ab+7ab^2B-4Ab^3)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}ab(a-ib)^2(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}ab(a+ib)^2(A-iB)\int\frac{i\tan(c+\ldots}{\sqrt{\tan(c+dx)}\sqrt{a\ldots}}\right.}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(a^3B+2a^2Ab+7ab^2B-4Ab^3)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}ab(a-ib)^2(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}ab(a+ib)^2(A-iB)\int\frac{i\tan(c+\ldots}{\sqrt{\tan(c+dx)}\sqrt{a\ldots}}\right.}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4098

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(a^3B+2a^2Ab+7ab^2B-4Ab^3)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{ab(a-ib)^2(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{ab(a+ib)^2(A-iB)\ldots}{\ldots}\right.}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 104

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(a^3B+2a^2Ab+7ab^2B-4Ab^3)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{ab(a-ib)^2(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{ab(a+ib)^2(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c\ldots}{a+b\tan(c+\ldots}}}{d}\right.}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 216

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(a^3B+2a^2Ab+7ab^2B-4Ab^3)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{ab(a+ib)^2(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{ab(a-ib)^2(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\ldots}{\sqrt{a+b\,t\ldots}}\right.}{d\sqrt{-b+ia}}\right.}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 219

$$\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}} - \frac{-\frac{2(a^3B+2a^2Ab+7ab^2B-4Ab^3)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}} + \frac{3\left(\frac{ab(a-ib)^2(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}} + \frac{ab(a+ib)^2(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}\right)}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

input
```
Int[(Tan[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x
]
```

output
```
(2*a*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x
])^(3/2)) - ((3*((a*(a - I*b)^2*b*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan
[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a*(a + I*b)^2*
b*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]])/(Sqrt[I*a + b]*d)))/(a*(a^2 + b^2)) - (2*(2*a^2*A*b - 4*A*b^3 + a^
3*B + 7*a*b^2*B)*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x
]]))/(3*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.42 (sec) , antiderivative size = 2975178, normalized size of antiderivative = 12193.35

output too large to display

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27576 vs. 2(204) = 408.

Time = 13.72 (sec) , antiderivative size = 27576, normalized size of antiderivative = 113.02

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\tan^{\frac{3}{2}}{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**(3/2)/(a + b*tan(c + d*x))**(5
/2), x)
```

Maxima [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\tan{(dx+c)}^{\frac{3}{2}}}{(b\tan{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(5/
2), x)
```

Giac [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^{\frac{3}{2}}}{(b\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^{5/2}}\,dx$$

input
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x
)
```

output
```
int((tan(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}-\left(\int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+ta}}{\tan(dx+c)^3b^2+2\tan(dx+c)}\right.}{ac}$$

input
```
int(tan(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a) - int((sqrt(tan(c + d*x))*s
qrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b**2 + 2*tan(c + d*x)**2*a*b + t
an(c + d*x)*a**2),x)*tan(c + d*x)*a*b*d - int((sqrt(tan(c + d*x))*sqrt(tan
(c + d*x)*b + a))/(tan(c + d*x)**3*b**2 + 2*tan(c + d*x)**2*a*b + tan(c +
d*x)*a**2),x)*a**2*d)/(a*d*(tan(c + d*x)*b + a))
```

3.465 $\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 244

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} + \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d} - \frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2a^3B+4ab^2B)\sqrt{\tan(c+dx)}}{3a(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
-(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*
a-b)^(5/2)/d+(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/(I*a+b)^(5/2)/d-2/3*(A*b-B*a)*tan(d*x+c)^(1/2)/(a^2+b^2)/d/(a+b*
tan(d*x+c))^(3/2)-2/3*(5*A*a^2*b-A*b^3-2*B*a^3+4*B*a*b^2)*tan(d*x+c)^(1/2)
/a/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{\frac{2b(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}} + \frac{6b(2aAb-a^2B+b^2B)\tan^{\frac{3}{2}}(c+dx)}{(a^2+b^2)\sqrt{a+b\tan(c+dx)}} + \frac{3\left(-\frac{\sqrt[4]{-1}a(a+}{}\right.}{}}{}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
5/2),x]
```

output
```
((2*b*(A*b - a*B)*Tan[c + d*x]^(3/2))/(a + b*Tan[c + d*x])^(3/2) + (6*b*(2
*a*A*b - a^2*B + b^2*B)*Tan[c + d*x]^(3/2))/((a^2 + b^2)*Sqrt[a + b*Tan[c
+ d*x]]) + (3*(-(((-1)^(1/4)*a*(a + I*b)^2*(A - I*B)*ArcTan[((-1)^(1/4)*Sq
rt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]
) + ((-1)^(1/4)*a*(a - I*b)^2*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] + 2*(-2*a*A*b
+ a^2*B - b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]))/(a^2 + b^2)
)/(3*a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4091, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 4091$$

$$-\frac{2\int -\frac{-2b(Ab-aB)\tan^2(c+dx)+3b(aA+bB)\tan(c+dx)+b(Ab-aB)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-2b(Ab-aB)\tan^2(c+dx)+3b(aA+bB)\tan(c+dx)+b(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{-2b(Ab-aB)\tan(c+dx)^2+3b(aA+bB)\tan(c+dx)+b(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4132$$

$$\frac{\frac{2\int \frac{3\left(ab\left(-Ba^2+2Aba+b^2B\right)+ab\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{ab\left(-Ba^2+2Aba+b^2B\right)+ab\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\int \frac{ab\left(-Ba^2+2Aba+b^2B\right)+ab\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4099$$

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}ab(a-ib)^2(-B+iA)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}ab(a+ib)^2(B+iA)\int\frac{i\tan\cdots}{\sqrt{\tan(c+d\cdots}}\cdots\right)}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}ab(a-ib)^2(-B+iA)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}ab(a+ib)^2(B+iA)\int\frac{i\tan\cdots}{\sqrt{\tan(c+d\cdots}}\cdots\right)}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\downarrow$ 4098

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{ab(a-ib)^2(-B+iA)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}-\frac{ab(a+ib)^2(B+\cdots}{\cdots}\right)}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\downarrow$ 104

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{ab(a-ib)^2(-B+iA)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{ab(a+ib)^2(B+iA)\int\frac{1}{1-\frac{(ia+b)\cdots}{a+b\tan\cdots}}\cdots}{d}\right)}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\downarrow$ 216

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{ab(a-ib)^2(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{ab(a+ib)^2(B+iA)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}\cdots}{d}\right)}{a(a^2+b^2)}}{3b\left(a^2+b^2\right)}$$

$\downarrow$ 219

$$-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{ab(a-ib)^2(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{ab(a+ib)^2(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}\right)}{a(a^2+b^2)}}{3b\,(a^2+b^2)}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x
]
```

output
```
(-2*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^
(3/2)) + ((3*((a*(a - I*b)^2*b*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c
+ d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) - (a*(a + I*b)^2*b*(
I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x
]]])/(Sqrt[I*a + b]*d)))/(a*(a^2 + b^2)) - (2*b*(5*a^2*A*b - A*b^3 - 2*a^3
*B + 4*a*b^2*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*
x]]))/(3*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.36 (sec) , antiderivative size = 2975224, normalized size of antiderivative = 12193.54

output too large to display

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27593 vs. 2(200) = 400.

Time = 13.58 (sec) , antiderivative size = 27593, normalized size of antiderivative = 113.09

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{\tan{(c+dx)}}}{(a+b\tan{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(tan(c + d*x))/(a + b*tan(c + d*x))**(5/
2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\sqrt{\tan{(dx+c)}}}{(b\tan{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)^(5/
2), x)
```

Giac [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{\tan(dx+c)}}{(b\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)^(5/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(c+dx)}\,(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$

input
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x
)
```

output
```
int((tan(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}}{\tan(dx+c)^2\,b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(tan(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b**2 +
2*tan(c + d*x)*a*b + a**2),x)
```

3.466 $\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 247

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} - \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d} + \frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3a(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} + \frac{2b(8a^2Ab+2Ab^3-5a^3B+ab^2B)\sqrt{\tan(c+dx)}}{3a^2(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
-(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*
a-b)^(5/2)/d-(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))/(I*a+b)^(5/2)/d+2/3*b*(A*b-B*a)*tan(d*x+c)^(1/2)/a/(a^2+b^2)/d/(
a+b*tan(d*x+c))^(3/2)+2/3*b*(8*A*a^2*b+2*A*b^3-5*B*a^3+B*a*b^2)*tan(d*x+c)
^(1/2)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.11

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx = \frac{-3\sqrt[4]{-1}\left(\frac{(a+ib)^2(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{i(a-ib)^2(A}{}\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5
/2)),x]
```

output
```
(-3*(-1)^(1/4)*(((a + I*b)^2*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] + (I*(a - I*b
)^2*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]])/Sqrt[a + I*b]) + (2*b*(a^2 + b^2)*(A*b - a*B)*Sqrt[Tan
[c + d*x]])/(a*(a + b*Tan[c + d*x])^(3/2)) + (2*b*(8*a^2*A*b + 2*A*b^3 - 5
*a^3*B + a*b^2*B)*Sqrt[Tan[c + d*x]])/(a^2*Sqrt[a + b*Tan[c + d*x]]))/(3*(
a^2 + b^2)^2*d)
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 4092}$$

$$\frac{2\int \frac{3Aa^2+bBa-3(Ab-aB)\tan(c+dx)a+2Ab^2+2b(Ab-aB)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a(a^2+b^2)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3Aa^2+bBa-3(Ab-aB)\tan(c+dx)a+2Ab^2+2b(Ab-aB)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a(a^2+b^2)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{3Aa^2+bBa-3(Ab-aB)\tan(c+dx)a+2Ab^2+2b(Ab-aB)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a(a^2+b^2)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4132$$

$$\frac{\frac{2\int \frac{3\left(a^2\left(Aa^2+2bBa-Ab^2\right)-a^2\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3a(a^2+b^2)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{a^2\left(Aa^2+2bBa-Ab^2\right)-a^2\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3a(a^2+b^2)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\int \frac{a^2\left(Aa^2+2bBa-Ab^2\right)-a^2\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3a(a^2+b^2)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 4099$$

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b(-5a^3B+8a^2Ab+ab^2B+2Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}a^2(a-ib)^2(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a^2(a+ib)^2(A-iB)\int\frac{i\tan(c+}{\sqrt{\tan(c+dx)}\sqrt{}}\right.}{a(a^2+b^2)}}{3a\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b(-5a^3B+8a^2Ab+ab^2B+2Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}a^2(a-ib)^2(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}a^2(a+ib)^2(A-iB)\int\frac{i\tan(c+}{\sqrt{\tan(c+dx)}\sqrt{}}\right.}{a(a^2+b^2)}}{3a\left(a^2+b^2\right)}$$

$\downarrow$ 4098

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b(-5a^3B+8a^2Ab+ab^2B+2Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^2(a-ib)^2(A+iB)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{a^2(a+ib)^2(A-iB)}{}\right.}{a(a^2+b^2)}}{3a\left(a^2+b^2\right)}$$

$\downarrow$ 104

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b(-5a^3B+8a^2Ab+ab^2B+2Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^2(a-ib)^2(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{a^2(a+ib)^2(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(}{a+b\tan(c}}}{d}\right.}{a(a^2+b^2)}}{3a\left(a^2+b^2\right)}$$

$\downarrow$ 216

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b(-5a^3B+8a^2Ab+ab^2B+2Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^2(a+ib)^2(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{a^2(a-ib)^2(A+iB)\arctan\left(\frac{\sqrt{-b+ia}}{\sqrt{a+b}}\right.}{d\sqrt{-b+ia}}\right.}{a(a^2+b^2)}}{3a\left(a^2+b^2\right)}$$

$\downarrow$ 219

$$\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^{3/2}}+\frac{\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^2(a-ib)^2(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{a^2(a+ib)^2(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}\right)}{a(a^2+b^2)}}{3a\left(a^2+b^2\right)}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)),x
]
```

output
```
(2*b*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x
])^(3/2)) + ((3*((a^2*(a - I*b)^2*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan
[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a^2*(a + I*b)^
2*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]])/(Sqrt[I*a + b]*d)))/(a*(a^2 + b^2)) + (2*b*(8*a^2*A*b + 2*A*b^3 -
5*a^3*B + a*b^2*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c +
 d*x]]))/(3*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.85 (sec) , antiderivative size = 2978185, normalized size of antiderivative = 12057.43

output too large to display

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27639 vs. 2(208) = 416.

Time = 13.56 (sec) , antiderivative size = 27639, normalized size of antiderivative = 111.90

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))^{5/2}}\, dx = \int \frac{B\tan(dx + c) + A}{(b\tan(dx + c) + a)^{\frac{5}{2}}\sqrt{\tan(dx + c)}}\, dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(5/2)*sqrt(tan(d*x +
c))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx=\int \frac{A+B\tan(c+d\,x)}{\sqrt{\tan\left(c+d\,x\right)}\,\left(a+b\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}}{\tan\left(dx+c\right)^{3}b^{2}+2\tan\left(dx+c\right)^{2}ab+\tan\left(dx+c\right)a^{2}}dx$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b**2 +
2*tan(c + d*x)**2*a*b + tan(c + d*x)*a**2),x)
```

3.467 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 301

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} - \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d} - \frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}} - \frac{2b(3a^2A+4Ab^2-abB)\sqrt{\tan(c+dx)}}{3a^2(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} - \frac{2b(3a^4A+17a^2Ab^2+8Ab^4-8a^3bB-2ab^3B)\sqrt{\tan(c+dx)}}{3a^3(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}$$

output
```
(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
-b)^(5/2)/d-(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))/(I*a+b)^(5/2)/d-2*A/a/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2)-2
/3*b*(3*A*a^2+4*A*b^2-B*a*b)*tan(d*x+c)^(1/2)/a^2/(a^2+b^2)/d/(a+b*tan(d*x
+c))^(3/2)-2/3*b*(3*A*a^4+17*A*a^2*b^2+8*A*b^4-8*B*a^3*b-2*B*a*b^3)*tan(d*
x+c)^(1/2)/a^3/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.08

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \frac{-\frac{6aA}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{2b(3a^2A+4Ab^2-abB)\sqrt{\tan(c+dx)}}{(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{3\sqrt[4]{\ldots}}{\ldots}}{\ldots}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5
/2)),x]
```

output
```
((-6*a*A)/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) - (2*b*(3*a^2*A
+ 4*A*b^2 - a*b*B)*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*(a + b*Tan[c + d*x])^(
3/2)) + (3*(-1)^(1/4)*a^3*(((a + I*b)^2*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[
-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] -
((a - I*b)^2*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]]
)/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b]) - (2*b*(3*a^4*A + 17*a^2*A*b^2
 + 8*A*b^4 - 8*a^3*b*B - 2*a*b^3*B)*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]])/(a*(a^2 + b^2)^2))/(3*a^2*d)
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.20, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{2\int \frac{4Ab\tan^2(c+dx)+aA\tan(c+dx)+4Ab-aB}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{4Ab\tan^2(c+dx)+aA\tan(c+dx)+4Ab-aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{4Ab\tan(c+dx)^2+aA\tan(c+dx)+4Ab-aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{2\int \frac{-3Ba^3+9Aba^2+3(aA+bB)\tan(c+dx)a^2-2b^2Ba+8Ab^3+2b\left(3Aa^2-bBa+4Ab^2\right)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a(a^2+b^2)}+\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{-3Ba^3+9Aba^2+3(aA+bB)\tan(c+dx)a^2-2b^2Ba+8Ab^3+2b\left(3Aa^2-bBa+4Ab^2\right)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a(a^2+b^2)}+\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{-3Ba^3+9Aba^2+3(aA+bB)\tan(c+dx)a^2-2b^2Ba+8Ab^3+2b\left(3Aa^2-bBa+4Ab^2\right)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a(a^2+b^2)}+\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{4132}$$

$$-\frac{\frac{\frac{2\int \frac{3\left(\left(-Ba^2+2Aba+b^2B\right)a^3+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{3a(a^2+b^2)}+\frac{2b(3a^2A-abB+4Ab^2}{3ad(a^2+b^2)(a+b\tan}}{a}$$

$$\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{3\int \frac{\left(-Ba^2+2Aba+b^2B\right)a^3+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{3a(a^2+b^2)}+\frac{2b(3a^2A-abB+4Ab^2)\sqrt{}}{3ad(a^2+b^2)(a+b\tan(c-}}{a}$$

$$\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{3\int \frac{\left(-Ba^2+2Aba+b^2B\right)a^3+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{3a(a^2+b^2)}+\frac{2b(3a^2A-abB+4Ab^2)\sqrt{}}{3ad(a^2+b^2)(a+b\tan(c-}}{a}$$

$$\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 4099

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-$$

$$\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}a^3(a-ib)^2(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.}{a(}}{3a(a^2+b^2)}}{a}$$

$\downarrow$ 3042

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-$$

$$\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}a^3(a-ib)^2(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.}{a(}}{3a(a^2+b^2)}}{a}$$

$\downarrow$ 4098

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^3(a-ib)^2(-B+iA)\int\frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{\cdots}}}{2d}\cdots\right)}{\cdots}}{3a(a^2+\cdots}}{a}$$

$\Big\downarrow$ 104

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^3(a-ib)^2(-B+iA)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan\cdots}}{\sqrt{a+b\,t\cdots}}}{d}\cdots\right)}{\cdots}}{3a(a^2+b^2)}}{a}$$

$\Big\downarrow$ 216

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^3(a-ib)^2(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\cdots\right)}{a\cdots}}{3a(a^2+b^2)}}{a}$$

$\Big\downarrow$ 219

$$-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{a^3(a-ib)^2(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\cdots\right)}{a\left(a^2\cdots\right.}}{3a(a^2+b^2)}}{a}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)),x
]
```

output
```
(-2*A)/(a*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) - ((2*b*(3*a^2*
A + 4*A*b^2 - a*b*B)*Sqrt[Tan[c + d*x]])/(3*a*(a^2 + b^2)*d*(a + b*Tan[c +
 d*x])^(3/2)) + ((3*((a^3*(a - I*b)^2*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) - (a^3*(a + I
*b)^2*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]])/(Sqrt[I*a + b]*d)))/(a*(a^2 + b^2)) + (2*b*(3*a^4*A + 17*a^2*A
*b^2 + 8*A*b^4 - 8*a^3*b*B - 2*a*b^3*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)
*d*Sqrt[a + b*Tan[c + d*x]]))/(3*a*(a^2 + b^2)))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.48 (sec) , antiderivative size = 2979708, normalized size of antiderivative = 9899.36

output too large to display

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27749 vs. $2(257) = 514$.

Time = 13.36 (sec) , antiderivative size = 27749, normalized size of antiderivative = 92.19

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{B\tan(dx+c)+A}{(b\tan(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^(3
/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{3/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \frac{-4\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,\tan\left(dx+c\right)b-2\sqrt{\tan}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
( - 4*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*b - 2*sqrt(
tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*a - int((sqrt(tan(c + d*x))*sqrt(ta
n(c + d*x)*b + a))/(tan(c + d*x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2),x)*t
an(c + d*x)**2*a**2*b*d - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)
)/(tan(c + d*x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2),x)*tan(c + d*x)*a**3*
d)/(tan(c + d*x)*a**2*d*(tan(c + d*x)*b + a))
```

3.468 $\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 359

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx &= \frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia-b)^{5/2}d} \\ &\quad + \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(ia+b)^{5/2}d} - \frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}} \\ &\quad + \frac{2(2Ab-aB)}{a^2d\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}} \\ &\quad + \frac{2b(7a^2Ab+8Ab^3-3a^3B-4ab^2B)\sqrt{\tan(c+dx)}}{3a^3(a^2+b^2)\,d(a+b\tan(c+dx))^{3/2}} \\ &\quad + \frac{2b(8a^4Ab+30a^2Ab^3+16Ab^5-3a^5B-17a^3b^2B-8ab^4B)\sqrt{\tan(c+dx)}}{3a^4(a^2+b^2)^2\,d\sqrt{a+b\tan(c+dx)}}\end{aligned}$$

output
```
(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a
-b)^(5/2)/d+(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))/(I*a+b)^(5/2)/d-2/3*A/a/d/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2)
+2*(2*A*b-B*a)/a^2/d/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2)+2/3*b*(7*A*a^
2*b+8*A*b^3-3*B*a^3-4*B*a*b^2)*tan(d*x+c)^(1/2)/a^3/(a^2+b^2)/d/(a+b*tan(d
*x+c))^(3/2)+2/3*b*(8*A*a^4*b+30*A*a^2*b^3+16*A*b^5-3*B*a^5-17*B*a^3*b^2-8
*B*a*b^4)*tan(d*x+c)^(1/2)/a^4/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.07

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \frac{-\frac{6A}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}+\frac{6(6Ab-3aB)}{a\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{6b(7a^2}{}}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5
/2)),x]
```

output
```
((-6*A)/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)) + (6*(6*A*b - 3*a*
B))/(a*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) + (6*b*(7*a^2*A*b +
8*A*b^3 - 3*a^3*B - 4*a*b^2*B)*Sqrt[Tan[c + d*x]])/(a^2*(a^2 + b^2)*(a + b
*Tan[c + d*x])^(3/2)) + (9*(-1)^(3/4)*a^4*(((a + I*b)^2*(A - I*B)*ArcTan[(
(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/S
qrt[-a + I*b] + ((a - I*b)^2*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sq
rt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b]) + (6*b*(8*a^4*
A*b + 30*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B - 17*a^3*b^2*B - 8*a*b^4*B)*Sqrt[T
an[c + d*x]])/Sqrt[a + b*Tan[c + d*x]])/(a^3*(a^2 + b^2)^2))/(9*a*d)
```

Rubi [A] (verified)

Time = 2.62 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.18, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 4092}$$

$$-\frac{2\int \frac{3\left(2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB\right)}{2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}dx}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{2Ab\tan(c+dx)^2+aA\tan(c+dx)+2Ab-aB}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 4132}$$

$$-\frac{-\frac{2\int -\frac{Aa^2+B\tan(c+dx)a^2+4bBa-8Ab^2-4b(2Ab-aB)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\int \frac{Aa^2+B\tan(c+dx)a^2+4bBa-8Ab^2-4b(2Ab-aB)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{\int \frac{Aa^2+B\tan(c+dx)a^2+4bBa-8Ab^2-4b(2Ab-aB)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \text{ 4132}$$

$$-\frac{\frac{\frac{2\int \frac{3Aa^4+9bBa^3-3(Ab-aB)\tan(c+dx)a^3-14Ab^2a^2+8b^3Ba-16Ab^4-2b\left(-3Ba^3+7Aba^2-4b^2Ba+8Ab^3\right)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a\left(a^2+b^2\right)}-\frac{2b\left(-3a^3B+7a^2Ab-4ab^2B+8Ab^3\right)\cdots}{3ad\left(a^2+b^2\right)(a+b\tan(c+\cdots}\cdots}{a}\cdots}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\int \frac{3Aa^4+9bBa^3-3(Ab-aB)\tan(c+dx)a^3-14Ab^2a^2+8b^3Ba-16Ab^4-2b\left(-3Ba^3+7Aba^2-4b^2Ba+8Ab^3\right)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a\left(a^2+b^2\right)}-\frac{2b\left(-3a^3B+7a^2Ab-4ab^2B+8Ab^3\right)\cdots}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx\cdots}\cdots}{a}\cdots}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\int \frac{3Aa^4+9bBa^3-3(Ab-aB)\tan(c+dx)a^3-14Ab^2a^2+8b^3Ba-16Ab^4-2b\left(-3Ba^3+7Aba^2-4b^2Ba+8Ab^3\right)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a\left(a^2+b^2\right)}-\frac{2b\left(-3a^3B+7a^2Ab-4ab^2B+8Ab^3\right)\cdots}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx\cdots}\cdots}{a}\cdots}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 4132

$$-\frac{\frac{\frac{\frac{2\int \frac{3\left(a^4\left(Aa^2+2bBa-Ab^2\right)-a^4\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4B+16Ab^5\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{3a\left(a^2+b^2\right)}-\frac{2b\left(-3a^3B+\cdots\right)}{3ad\left(\cdots\right)}\cdots}{a}\cdots}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{\frac{3\int \frac{a^4\left(Aa^2+2bBa-Ab^2\right)-a^4\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4B+16Ab^5\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{3a\left(a^2+b^2\right)}-\frac{2b\left(-3a^3B+7a^2\right.}{3ad\left(a^2+\right.}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{\frac{3\int \frac{a^4\left(Aa^2+2bBa-Ab^2\right)-a^4\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4B+16Ab^5\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{3a\left(a^2+b^2\right)}-\frac{2b\left(-3a^3B+7a^2\right.}{3ad\left(a^2+\right.}}{a}}{a}$$

$$\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$

$\downarrow$ 4099

$$-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-$$

$$\frac{-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^3B+7a^2Ab-4ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4B+16Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{}}{}}{a}$$

$\downarrow$ 3042

$$-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-$$

$$\frac{-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^3B+7a^2Ab-4ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4B+16Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{}}{}}{a}$$

$\downarrow$ 4098

$$-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^3B+7a^2Ab-4ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4B+16Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{}}{}$$

$\downarrow$ 104

$$-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^3B+7a^2Ab-4ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4B+16Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{}}{a}$$

$\downarrow$ 216

$$-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^3B+7a^2Ab-4ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4B+16Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{}}{a}$$

$\downarrow$ 219

$$-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-$$

$$-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^3B+7a^2Ab-4ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4B+16Ab^5\right)}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{}}{a}$$

input
```
Int[(A + B*Tan[c + d*x])/(Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2)),x
]
```

output
```
(-2*A)/(3*a*d*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)) - ((-2*(2*A*b
 - a*B))/(a*d*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)) + ((-2*b*(7*a
^2*A*b + 8*A*b^3 - 3*a^3*B - 4*a*b^2*B)*Sqrt[Tan[c + d*x]])/(3*a*(a^2 + b^
2)*d*(a + b*Tan[c + d*x])^(3/2)) + ((3*((a^4*(a - I*b)^2*(A + I*B)*ArcTan[
(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a -
b]*d) + (a^4*(a + I*b)^2*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x
]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/(a*(a^2 + b^2)) - (2*b*
(8*a^4*A*b + 30*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B - 17*a^3*b^2*B - 8*a*b^4*B)
*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(3*a*(a^2
 + b^2)))/a)/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.39 (sec) , antiderivative size = 2982515, normalized size of antiderivative = 8307.84

output too large to display

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27865 vs. $2(308) = 616$.

Time = 13.63 (sec) , antiderivative size = 27865, normalized size of antiderivative = 77.62

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\tan^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\tan\left(c+d\,x\right)^{5/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(tan(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \frac{16\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,\tan\left(dx+c\right)^{2}b^{2}+8\sqrt{\tan}}{}$$

input
```
int((A+B*tan(d*x+c))/tan(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
(16*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2*b**2 + 8*s
qrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)*a*b - 2*sqrt(tan(c
 + d*x))*sqrt(tan(c + d*x)*b + a)*a**2 - 3*int((sqrt(tan(c + d*x))*sqrt(ta
n(c + d*x)*b + a))/(tan(c + d*x)**3*b**2 + 2*tan(c + d*x)**2*a*b + tan(c +
 d*x)*a**2),x)*tan(c + d*x)**3*a**3*b*d - 3*int((sqrt(tan(c + d*x))*sqrt(t
an(c + d*x)*b + a))/(tan(c + d*x)**3*b**2 + 2*tan(c + d*x)**2*a*b + tan(c
+ d*x)*a**2),x)*tan(c + d*x)**2*a**4*d)/(3*tan(c + d*x)**2*a**3*d*(tan(c +
 d*x)*b + a))
```

3.469 $\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 155

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{B\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} + \frac{2B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}\,d} - \frac{B\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

output
```
-B*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(
1/2)/d+2*B*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/b^(1/2
)/d-B*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a+
b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.23

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{B\left((-1)^{3/4}\left(\frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} + \frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\ldots}{\sqrt{a+b\tan\ldots}}\right.}{\sqrt{a+ib}}\right.\right.}{d}$$

input
```
Integrate[(Tan[c + d*x]^(3/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
])^(3/2),x]
```

output
```
(B*((-1)^(3/4)*(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]]/Sqrt[-a + I*b] + ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sq
rt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b]) + (2*Sqrt[a]*Ar
cSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/
(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]])))/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2011, 3042, 4058, 614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{\tan(c+dx)^{3/2}}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4058}$$

$$\frac{B\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{614}$$

$$\frac{B\int \left(\frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B\left(-\frac{\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b+ia}}\right)}{d}$$

input
```
Int[(Tan[c + d*x]^(3/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/
2),x]
```

output
```
(B*(-(ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/
Sqrt[I*a - b]) + (2*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]])/Sqrt[b] - ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]]/Sqrt[I*a + b]))/d
```

Defintions of rubi rules used

rule 614
```
Int[((e_.)*(x_))^(m_)/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Sy
mbol] :> Simp[e^(m + 1/2)   Int[ExpandIntegrand[1/(Sqrt[e*x]*Sqrt[c + d*x])
, x^(m + 1/2)/(a + b*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[
m - 1/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4058
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
 Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.63 (sec) , antiderivative size = 944366, normalized size of antiderivative = 6092.68

output too large to display

input
```
int(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4592 vs. 2(123) = 246.

Time = 0.89 (sec) , antiderivative size = 9186, normalized size of antiderivative = 59.26

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = B\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x
)
```

output
```
B*Integral(tan(c + d*x)**(3/2)/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(Bb\tan(dx+c)+Ba)\tan(dx+c)^{\frac{3}{2}}}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*tan(d*x + c) + B*a)*tan(d*x + c)^(3/2)/(b*tan(d*x + c) + a)
^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^{3/2}\,(B\,a+B\,b\tan(c+d\,x))}{(a+b\tan\,(c+d\,x))^{3/2}}\,dx$$

input
```
int((tan(c + d*x)^(3/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/
2),x)
```

output
```
int((tan(c + d*x)^(3/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)}{a+\tan(dx+c)\,b}dx\right)b$$

input
```
int(tan(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*
x)*b + a),x)*b
```

3.470 $\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 117

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{iB\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} - \frac{iB\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

output
```
I*B*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^
(1/2)/d-I*B*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
/(I*a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt[4]{-1}B\left(-\frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)}{d}$$

input
```
Integrate[(Sqrt[Tan[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
])^(3/2),x]
```

output
```
((-1)^(1/4)*B*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]]/Sqrt[-a + I*b]) + ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b]))/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2011, 3042, 4058, 613, 104, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4058}$$

$$\frac{B\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 613}$$

$$\frac{B\left(\frac{1}{2}\int \frac{1}{\sqrt{\tan(c+dx)}(\tan(c+dx)+i)\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)-\frac{1}{2}\int \frac{1}{(i-\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)\right.}{d}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{B\left(\int \frac{1}{\frac{(a-ib)\tan(c+dx)}{a+b\tan(c+dx)}+i}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}-\int \frac{1}{i-\frac{(a+ib)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{B\left(\int \frac{1}{\frac{(a-ib)\tan(c+dx)}{a+b\tan(c+dx)}+i}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}+\frac{i\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}\right)}{d}$$

$$\Big\downarrow \text{221}$$

$$\frac{B\left(\frac{i\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}-\frac{i\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b+ia}}\right)}{d}$$

input
```
Int[(Sqrt[Tan[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/
2),x]
```

output
```
(B*((I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]
)/Sqrt[I*a - b] - (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]])/Sqrt[I*a + b]))/d
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 613
```
Int[Sqrt[(e_.)*(x_)]/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Sym
bol] :> Simp[e/(2*b)   Int[1/(Sqrt[e*x]*Sqrt[c + d*x]*(Rt[-a/b, 2] + x)), x
], x] - Simp[e/(2*b)   Int[1/(Sqrt[e*x]*Sqrt[c + d*x]*(Rt[-a/b, 2] - x)), x
], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4058
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
 Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.68 (sec) , antiderivative size = 939563, normalized size of antiderivative = 8030.45

output too large to display

input
```
int(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4509 vs. $2(89) = 178$.

Time = 0.54 (sec) , antiderivative size = 4509, normalized size of antiderivative = 38.54

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = B\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x
)
```

output
```
B*Integral(sqrt(tan(c + d*x))/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(Bb\tan(dx+c)+Ba)\sqrt{\tan(dx+c)}}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*tan(d*x + c) + B*a)*sqrt(tan(d*x + c))/(b*tan(d*x + c) + a)
^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\tan\left(c+d\,x\right)}\,\left(B\,a+B\,b\tan(c+d\,x)\right)}{\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((tan(c + d*x)^(1/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/
2),x)
```

output
```
int((tan(c + d*x)^(1/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\tan(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\tan(dx+c)}\,\sqrt{a+\tan(dx+c)\,b}-2\left(\int \frac{\sqrt{\tan(dx+c)}\,\sqrt{a+\tan}}{a+\tan(}\right.}{}$$

input
```
int(tan(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a) - 2*int((sqrt(tan(c + d*x))
*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)*b*d - i
nt((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(tan(c + d*x
)*b + a),x)*a*d - int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c
 + d*x)**2*b + tan(c + d*x)*a),x)*a*d)/(2*d)
```

3.471 $\int \frac{aB+bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 111

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx=\frac{B\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d}+\frac{B\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d}$$

output
```
B*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)^(1
/2)/d+B*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*
a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.13

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx=\frac{(-1)^{3/4}B\left(-\frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}-\frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\ldots}{\sqrt{a+b\tan(\ldots}}\right)}{\sqrt{a+ib}}\ldots\right.}{d}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]
)^(3/2)),x]
```

output
```
((-1)^(3/4)*B*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]]/Sqrt[-a + I*b]) - ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b]))/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2011, 3042, 4058, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4058}$$

$$\frac{B\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 615}$$

$$\frac{B\int \left(\frac{i}{2(i-\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{i}{2\sqrt{\tan(c+dx)}(\tan(c+dx)+i)\sqrt{a+b\tan(c+dx)}}\right)d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B\left(\frac{\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b+ia}}\right)}{d}$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2
)),x]
```

output
```
(B*(ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sq
rt[I*a - b] + ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]]/Sqrt[I*a + b]))/d
```

Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4058
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
 Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.60 (sec) , antiderivative size = 939324, normalized size of antiderivative = 8462.38

output too large to display

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4469 vs. $2(87) = 174$.

Time = 0.61 (sec) , antiderivative size = 4469, normalized size of antiderivative = 40.26

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{a+b\tan{(c+dx)}}\sqrt{\tan{(c+dx)}}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(3/2),x
)
```

output
```
B*Integral(1/(sqrt(a + b*tan(c + d*x))*sqrt(tan(c + d*x))), x)
```

Maxima [F]

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))^{3/2}}\,dx = \int \frac{Bb\tan(dx + c) + Ba}{(b\tan(dx + c) + a)^{\frac{3}{2}}\sqrt{\tan(dx + c)}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*tan(d*x + c) + B*a)/((b*tan(d*x + c) + a)^(3/2)*sqrt(tan(d*
x + c))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\tan(c + dx)}(a + b\tan(c + dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{B\,a + B\,b\tan(c+d\,x)}{\sqrt{\tan\left(c+d\,x\right)}\,\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((B*a + B*b*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2
)),x)
```

output
```
int((B*a + B*b*tan(c + d*x))/(tan(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{\tan\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}}{\tan\left(dx+c\right)^{2}b+\tan\left(dx+c\right)a}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**2*b + tan
(c + d*x)*a),x)*b
```

3.472 $\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 150

$$\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{iB\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia-b}\,d} + \frac{iB\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{ia+b}\,d} - \frac{2B\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}$$

output
```
-I*B*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))/(I*a-b)
^(1/2)/d+I*B*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
)/(I*a+b)^(1/2)/d-2*B*(a+b*tan(d*x+c))^(1/2)/a/d/tan(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \frac{B\left(\frac{\sqrt[4]{-1}\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} - \frac{\sqrt[4]{-1}\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\ldots}{\sqrt{a+b\tan\ldots}}\right)}{\sqrt{a+ib}}\ldots\right.}{d}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x]
)^(3/2)),x]
```

output
```
(B*(((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b] - ((-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqr
t[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] -
(2*Sqrt[a + b*Tan[c + d*x]])/(a*Sqrt[Tan[c + d*x]])))/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2011, 3042, 4052, 27, 3042, 4058, 613, 104, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4052}$$

$$B\left(-\frac{2\int \frac{a\sqrt{\tan(c+dx)}}{2\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$B\left(-\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(-\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}dx-\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{4058}$$

$$B\left(-\frac{\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}-\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{613}$$

$$B\left(-\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{\frac{1}{2}\int \frac{1}{\sqrt{\tan(c+dx)}(\tan(c+dx)+i)\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)-\frac{1}{2}\int \frac{1}{(i-\tan(c+dx))\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{104}$$

$$B\left(-\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{\int \frac{1}{\frac{(a-ib)\tan(c+dx)}{a+b\tan(c+dx)}+i}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}-\int \frac{1}{i-\frac{(a+ib)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}\right)$$

$$\Big\downarrow \text{218}$$

$$B\left(-\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{\int \frac{1}{\frac{(a-ib)\tan(c+dx)}{a+b\tan(c+dx)}+i}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}+\frac{i\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}}{d}\right)$$

$$\Big\downarrow \text{221}$$

$$B\left(-\frac{2\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{\frac{i\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}-\frac{i\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b+ia}}}{d}\right)$$

input

```
Int[(a*B + b*B*Tan[c + d*x])/(Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2
)),x]
```

output
```
B*(-(((I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]
]])/Sqrt[I*a - b] - (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/Sqrt[I*a + b])/d) - (2*Sqrt[a + b*Tan[c + d*x]])/(a*d*S
qrt[Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 613
```
Int[Sqrt[(e_.)*(x_)]/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Sym
bol] :> Simp[e/(2*b)   Int[1/(Sqrt[e*x]*Sqrt[c + d*x]*(Rt[-a/b, 2] + x)), x
], x] - Simp[e/(2*b)   Int[1/(Sqrt[e*x]*Sqrt[c + d*x]*(Rt[-a/b, 2] - x)), x
], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4052
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Simp[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c +
d*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
 a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Integ
erQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4058
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
 Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.71 (sec) , antiderivative size = 943933, normalized size of antiderivative = 6292.89

output too large to display

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4608 vs. $2(118) = 236$.

Time = 0.54 (sec) , antiderivative size = 4608, normalized size of antiderivative = 30.72

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{a + b\tan{(c + dx)}}\tan^{\frac{3}{2}}{(c + dx)}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x
)
```

output
```
B*Integral(1/(sqrt(a + b*tan(c + d*x))*tan(c + d*x)**(3/2)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \int \frac{B\,a + B\,b\tan(c + d\,x)}{\tan\left(c + d\,x\right)^{3/2}\left(a + b\tan\left(c + d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((B*a + B*b*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2
)),x)
```

output
```
int((B*a + B*b*tan(c + d*x))/(tan(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{aB + bB\tan(c + dx)}{\tan^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{\tan\left(dx + c\right)}\,\sqrt{a + \tan\left(dx + c\right)b}}{\tan\left(dx + c\right)^{3}b + \tan\left(dx + c\right)^{2}a}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/tan(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x))*sqrt(tan(c + d*x)*b + a))/(tan(c + d*x)**3*b + tan
(c + d*x)**2*a),x)*b
```

3.473 $\int (a+b\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 379

$$\int (a+b\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = -\frac{1}{4}(a-ib)^{2/3}(A-iB)x - \frac{1}{4}(a+ib)^{2/3}(A+iB)x + \frac{\sqrt{3}(a-ib)^{2/3}(iA+B)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt{3}(a+ib)^{2/3}(iA-$$

output

```
-1/4*(a-I*b)^(2/3)*(A-I*B)*x-1/4*(a+I*b)^(2/3)*(A+I*B)*x+1/2*3^(1/2)*(a-I*
b)^(2/3)*(I*A+B)*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/3)/(a-I*b)^(1/3))*3^(
1/2))/d-1/2*3^(1/2)*(a+I*b)^(2/3)*(I*A-B)*arctan(1/3*(1+2*(a+b*tan(d*x+c))
^(1/3)/(a+I*b)^(1/3))*3^(1/2))/d-1/4*(a+I*b)^(2/3)*(I*A-B)*ln(cos(d*x+c))/
d+1/4*(a-I*b)^(2/3)*(I*A+B)*ln(cos(d*x+c))/d+3/4*(a-I*b)^(2/3)*(I*A+B)*ln(
(a-I*b)^(1/3)-(a+b*tan(d*x+c))^(1/3))/d-3/4*(a+I*b)^(2/3)*(I*A-B)*ln((a+I*
b)^(1/3)-(a+b*tan(d*x+c))^(1/3))/d+3/2*B*(a+b*tan(d*x+c))^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.69

$$\int (a+b\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \frac{i\left((A-iB)\left((a-ib)^{2/3}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)-\log(i+\tan(c+\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x]
```

output
```
((I/4)*((A - I*B)*((a - I*b)^(2/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c
+ d*x])^(1/3))/(a - I*b)^(1/3))/Sqrt[3]] - Log[I + Tan[c + d*x]] + 3*Log[(
a - I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)]) + 3*(a + b*Tan[c + d*x])^(2/
3)) - (A + I*B)*((a + I*b)^(2/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c +
d*x])^(1/3))/(a + I*b)^(1/3))/Sqrt[3]] - Log[I - Tan[c + d*x]] + 3*Log[(a
+ I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)]) + 3*(a + b*Tan[c + d*x])^(2/3)
)))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.71, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 4011, 3042, 4022, 3042, 4020, 25, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+b\tan(c+dx))^{2/3}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4011}$$

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}dx+\frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

$\Big\downarrow$ 3042

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}dx+\frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

$\Big\downarrow$ 4022

$$\frac{1}{2}(a+ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt[3]{a+b\tan(c+dx)}}dx+\frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}(a+ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt[3]{a+b\tan(c+dx)}}dx+\frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

$\Big\downarrow$ 4020

$$\frac{i(a-ib)(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt[3]{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(a+ib)(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt[3]{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

$\Big\downarrow$ 25

$$-\frac{i(a-ib)(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt[3]{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}+\frac{i(a+ib)(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt[3]{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}+\frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

$\Big\downarrow$ 67

$$\frac{i(a-ib)(A-iB)\left(\frac{3}{2}\int \frac{1}{-\tan^2(c+dx)+(a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}-\frac{3\int \frac{1}{\sqrt[3]{a-ib}-i\tan\cdots}}{\cdots}\right.}{2d}$$

$$\frac{i(a+ib)(A+iB)\left(\frac{3}{2}\int \frac{1}{-\tan^2(c+dx)+(a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}-\frac{3\int \frac{1}{i\tan(c+dx)+\sqrt[3]{\cdots}}}{\cdots}\right.}{2d}$$

$$\frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

$\big\downarrow$ 16

$$\frac{i(a-ib)(A-iB)\left(\frac{3}{2}\int \frac{1}{-\tan^2(c+dx)+(a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}-\frac{\log(1-i\tan(c+dx\cdots}{2\sqrt[3]{a-ib}}\right.}{2d}$$

$$\frac{i(a+ib)(A+iB)\left(\frac{3}{2}\int \frac{1}{-\tan^2(c+dx)+(a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}-\frac{\log(1+i\tan(c+dx\cdots}{2\sqrt[3]{a+ib}}\right.}{2d}$$

$$\frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

$\big\downarrow$ 1082

$$\frac{i(a-ib)(A-iB)\left(-\frac{3\int \frac{1}{\tan^2(c+dx)-3}d\left(\frac{2i\tan(c+dx)}{\sqrt[3]{a-ib}}+1\right)}{\sqrt[3]{a-ib}}-\frac{\log(1-i\tan(c+dx))}{2\sqrt[3]{a-ib}}+\frac{3\log\left(\sqrt[3]{a-ib}-i\tan(c+dx)\right)}{2\sqrt[3]{a-ib}}\right)}{2d}-$$

$$\frac{i(a+ib)(A+iB)\left(-\frac{3\int \frac{1}{\tan^2(c+dx)-3}d\left(1-\frac{2i\tan(c+dx)}{\sqrt[3]{a+ib}}\right)}{\sqrt[3]{a+ib}}-\frac{\log(1+i\tan(c+dx))}{2\sqrt[3]{a+ib}}+\frac{3\log\left(\sqrt[3]{a+ib}+i\tan(c+dx)\right)}{2\sqrt[3]{a+ib}}\right)}{2d}+$$

$$\frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

$\big\downarrow$ 217

$$\frac{i(a-ib)(A-iB)\left(\frac{i\sqrt{3}\text{arctanh}\left(\frac{\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{a-ib}}-\frac{\log(1-i\tan(c+dx))}{2\sqrt[3]{a-ib}}+\frac{3\log\left(\sqrt[3]{a-ib}-i\tan(c+dx)\right)}{2\sqrt[3]{a-ib}}\right)}{2d}-\frac{i(a+ib)(A+iB)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{a+ib}}-\frac{\log(1+i\tan(c+dx))}{2\sqrt[3]{a+ib}}+\frac{3\log\left(\sqrt[3]{a+ib}+i\tan(c+dx)\right)}{2\sqrt[3]{a+ib}}\right)}{2d}+\frac{3B(a+b\tan(c+dx))^{2/3}}{2d}$$

input
```
Int[(a + b*Tan[c + d*x])^(2/3)*(A + B*Tan[c + d*x]),x]
```

output
```
((I/2)*(a - I*b)*(A - I*B)*((I*Sqrt[3]*ArcTanh[Tan[c + d*x]/Sqrt[3]])/(a -
 I*b)^(1/3) - Log[1 - I*Tan[c + d*x]]/(2*(a - I*b)^(1/3)) + (3*Log[(a - I*
b)^(1/3) - I*Tan[c + d*x]])/(2*(a - I*b)^(1/3))))/d - ((I/2)*(a + I*b)*(A
+ I*B)*(((-I)*Sqrt[3]*ArcTanh[Tan[c + d*x]/Sqrt[3]])/(a + I*b)^(1/3) - Log
[1 + I*Tan[c + d*x]]/(2*(a + I*b)^(1/3)) + (3*Log[(a + I*b)^(1/3) + I*Tan[
c + d*x]])/(2*(a + I*b)^(1/3))))/d + (3*B*(a + b*Tan[c + d*x])^(2/3))/(2*d
)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.26

method	result
derivativedivides	$\frac{\frac{3(a+b\tan(dx+c))^{\frac{2}{3}}B}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\left((Ab+Ba)\_R^4+B\left(-a^2-b^2\right)\_R\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_\right.}{\_R^5-\_R^2a}\right.}{2}}{d}$
default	$\frac{\frac{3(a+b\tan(dx+c))^{\frac{2}{3}}B}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\left((Ab+Ba)\_R^4+B\left(-a^2-b^2\right)\_R\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_\right.}{\_R^5-\_R^2a}\right.}{2}}{d}$
parts	$\frac{Ab\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\_R^2\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^3-a}\right)}{2d}+\frac{B\left(\frac{3(a+b\tan(dx+c))^{\frac{2}{3}}}{2}+\frac{\left(\_R=\mathrm{Root}\right.}{}\right.}{}$

input
```
int((a+b*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(3/2*(a+b*tan(d*x+c))^(2/3)*B+1/2*sum(((A*b+B*a)*_R^4+B*(-a^2-b^2)*_R)
/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b
^2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5405 vs. 2(281) = 562.

Time = 1.72 (sec) , antiderivative size = 5405, normalized size of antiderivative = 14.26

$$\int (a+b\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (a+b\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \int (A + B\tan(c+dx))\,(a+b\tan(c+dx))^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int (a+b\tan(c+dx))^{2/3}(A + B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int (a+b\tan(c+dx))^{2/3}(A + B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{2}{3}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 20.26 (sec) , antiderivative size = 3945, normalized size of antiderivative = 10.41

$$\int (a+b\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(2/3),x)
```

output
```
log((((2*(-B^6*a^2*b^2*d^6)^(1/2) + B^3*a^2*d^3 - B^3*b^2*d^3)/d^6)^(2/3)*
((((2*(-B^6*a^2*b^2*d^6)^(1/2) + B^3*a^2*d^3 - B^3*b^2*d^3)/d^6)^(1/3)*(19
44*a*b^4*((2*(-B^6*a^2*b^2*d^6)^(1/2) + B^3*a^2*d^3 - B^3*b^2*d^3)/d^6)^(2
/3)*(a^2 + b^2) - (1944*B^2*b^4*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/
d^2))/2 + (972*B^3*a*b^4*(3*b^4 - a^4 + 2*a^2*b^2))/d^3))/4 + (243*B^5*b^4
*(a^2 - b^2)*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^5)*(((-4*B^6*a^2*
b^2*d^6)^(1/2) + B^3*a^2*d^3 - B^3*b^2*d^3)/(8*d^6))^(1/3) + log(((-(2*(-B
^6*a^2*b^2*d^6)^(1/2) - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6)^(2/3)*(((-(2*(-B^6
*a^2*b^2*d^6)^(1/2) - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6)^(1/3)*(1944*a*b^4*(-
(2*(-B^6*a^2*b^2*d^6)^(1/2) - B^3*a^2*d^3 + B^3*b^2*d^3)/d^6)^(2/3)*(a^2 +
 b^2) - (1944*B^2*b^4*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^2))/2 +
(972*B^3*a*b^4*(3*b^4 - a^4 + 2*a^2*b^2))/d^3))/4 + (243*B^5*b^4*(a^2 - b^
2)*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^5)*(-((-4*B^6*a^2*b^2*d^6)^
(1/2) - B^3*a^2*d^3 + B^3*b^2*d^3)/(8*d^6))^(1/3) + log(((((-A^6*d^6*(a^2
- b^2)^2)^(1/2) - 2*A^3*a*b*d^3)/d^6)^(2/3)*(((1944*a*b^4*(((-A^6*d^6*(a^2
 - b^2)^2)^(1/2) - 2*A^3*a*b*d^3)/d^6)^(2/3)*(a^2 + b^2) + (1944*A^2*b^4*(
a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^2)*(((-A^6*d^6*(a^2 - b^2)^2)^(
1/2) - 2*A^3*a*b*d^3)/d^6)^(1/3))/2 + (972*A^3*b^5*(3*a^4 - b^4 + 2*a^2*b^
2))/d^3))/4 + (486*A^5*a*b^5*(a^2 + b^2)^2*(a + b*tan(c + d*x))^(1/3))/d^5
)*((2*A^6*a^2*b^2*d^6 - A^6*b^4*d^6 - A^6*a^4*d^6)^(1/2)/(8*d^6) - (A^3...
```

Reduce [F]

$$\int (a+b\tan(c+dx))^{2/3}(A+B\tan(c+dx))\,dx = \left(\int (a+\tan(dx+c)\,b)^{\frac{2}{3}}\,dx\right)a + \left(\int (a+\tan(dx+c)\,b)^{\frac{2}{3}}\tan(dx+c)\,dx\right)b$$

input
```
int((a+b*tan(d*x+c))^(2/3)*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*b + a)**(2/3),x)*a + int((tan(c + d*x)*b + a)**(2/3)*tan
(c + d*x),x)*b
```

3.474 $\int \sqrt[3]{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 377

$$\begin{aligned}
&\int \sqrt[3]{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx \\
&= -\frac{1}{4}\sqrt[3]{a-ib}(A-iB)x-\frac{1}{4}\sqrt[3]{a+ib}(A+iB)x \\
&\quad -\frac{\sqrt{3}\sqrt[3]{a-ib}(iA+B)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2d} \\
&\quad +\frac{\sqrt{3}\sqrt[3]{a+ib}(iA-B)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)}{2d} \\
&\quad -\frac{\sqrt[3]{a+ib}(iA-B)\log(\cos(c+dx))}{4d}+\frac{\sqrt[3]{a-ib}(iA+B)\log(\cos(c+dx))}{4d} \\
&\quad +\frac{3\sqrt[3]{a-ib}(iA+B)\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4d} \\
&\quad -\frac{3\sqrt[3]{a+ib}(iA-B)\log\left(\sqrt[3]{a+ib}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4d}+\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}
\end{aligned}$$

output
```
-1/4*(a-I*b)^(1/3)*(A-I*B)*x-1/4*(a+I*b)^(1/3)*(A+I*B)*x-1/2*3^(1/2)*(a-I*
b)^(1/3)*(I*A+B)*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/3)/(a-I*b)^(1/3))*3^(
1/2))/d+1/2*3^(1/2)*(a+I*b)^(1/3)*(I*A-B)*arctan(1/3*(1+2*(a+b*tan(d*x+c))
^(1/3)/(a+I*b)^(1/3))*3^(1/2))/d-1/4*(a+I*b)^(1/3)*(I*A-B)*ln(cos(d*x+c))/
d+1/4*(a-I*b)^(1/3)*(I*A+B)*ln(cos(d*x+c))/d+3/4*(a-I*b)^(1/3)*(I*A+B)*ln(
(a-I*b)^(1/3)-(a+b*tan(d*x+c))^(1/3))/d-3/4*(a+I*b)^(1/3)*(I*A-B)*ln((a+I*
b)^(1/3)-(a+b*tan(d*x+c))^(1/3))/d+3*B*(a+b*tan(d*x+c))^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{i\left((A-iB)\left(-\frac{1}{2}\sqrt[3]{a-ib}\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+\right.\right.\right.\right.}{}$$

input
```
Integrate[(a + b*Tan[c + d*x])^(1/3)*(A + B*Tan[c + d*x]),x]
```

output
```
((I/2)*((A - I*B)*(-1/2*((a - I*b)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*
Tan[c + d*x])^(1/3))/(a - I*b)^(1/3))/Sqrt[3]] - 2*Log[(a - I*b)^(1/3) - (
a + b*Tan[c + d*x])^(1/3)] + Log[(a - I*b)^(2/3) + (a - I*b)^(1/3)*(a + b*
Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)])) + 3*(a + b*Tan[c + d*x
])^(1/3)) - (A + I*B)*(-1/2*((a + I*b)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a
+ b*Tan[c + d*x])^(1/3))/(a + I*b)^(1/3))/Sqrt[3]] - 2*Log[(a + I*b)^(1/3)
 - (a + b*Tan[c + d*x])^(1/3)] + Log[(a + I*b)^(2/3) + (a + I*b)^(1/3)*(a
+ b*Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)])) + 3*(a + b*Tan[c +
 d*x])^(1/3))))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.71, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 4011, 3042, 4022, 3042, 4020, 25, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}dx+\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aA-bB+(Ab+aB)\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}dx+\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(a+ib)(A+iB)\int \frac{1-i\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}dx+\frac{1}{2}(a-ib)(A-iB)\int \frac{i\tan(c+dx)+1}{(a+b\tan(c+dx))^{2/3}}dx+\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(a+ib)(A+iB)\int \frac{1-i\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}dx+\frac{1}{2}(a-ib)(A-iB)\int \frac{i\tan(c+dx)+1}{(a+b\tan(c+dx))^{2/3}}dx+\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(a-ib)(A-iB)\int -\frac{1}{(1-i\tan(c+dx))(a+b\tan(c+dx))^{2/3}}d(i\tan(c+dx))}{2d}-\frac{i(a+ib)(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)(a+b\tan(c+dx))^{2/3}}d(-i\tan(c+dx))}{2d}+\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 25

$$-\frac{i(a-ib)(A-iB)\int \frac{1}{(1-i\tan(c+dx))(a+b\tan(c+dx))^{2/3}}d(i\tan(c+dx))}{2d}+\frac{i(a+ib)(A+iB)\int \frac{1}{(i\tan(c+dx)+1)(a+b\tan(c+dx))^{2/3}}d(-i\tan(c+dx))}{2d}+\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 69

$$\frac{i(a-ib)(A-iB)\left(-\frac{3\int \frac{1}{-\tan^2(c+dx)+(a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}}{2\sqrt[3]{a-ib}}-\frac{3\int \frac{1}{\sqrt[3]{a-ib}-i\tan \cdots}}{\cdots}\right.}{2d}$$

$$\frac{i(a+ib)(A+iB)\left(-\frac{3\int \frac{1}{-\tan^2(c+dx)+(a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}}{2\sqrt[3]{a+ib}}-\frac{3\int \frac{1}{i\tan(c+dx)+\sqrt[3]{\cdots}}}{\cdots}\right.}{2d}$$

$$\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 16

$$\frac{i(a-ib)(A-iB)\left(-\frac{3\int \frac{1}{-\tan^2(c+dx)+(a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}}{2\sqrt[3]{a-ib}}-\frac{\log(1-i\tan(c+dx)\cdots}{2(a-ib)^{2/3}}\right.}{2d}$$

$$\frac{i(a+ib)(A+iB)\left(-\frac{3\int \frac{1}{-\tan^2(c+dx)+(a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}}{2\sqrt[3]{a+ib}}-\frac{\log(1+i\tan(c+dx)\cdots}{2(a+ib)^{2/3}}\right.}{2d}$$

$$\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}$$

$\Big\downarrow$ 1082

$$\frac{i(a-ib)(A-iB)\left(\frac{3\int \frac{1}{\tan^2(c+dx)-3}d\left(\frac{2i\tan(c+dx)}{\sqrt[3]{a-ib}}+1\right)}{(a-ib)^{2/3}}-\frac{\log(1-i\tan(c+dx))}{2(a-ib)^{2/3}}+\frac{3\log\left(\sqrt[3]{a-ib}-i\tan(c+dx)\right)}{2(a-ib)^{2/3}}\right)}{2d}-\frac{i(a+ib)(A+iB)\left(\frac{3\int \frac{1}{\tan^2(c+dx)-3}d\left(1-\frac{2i\tan(c+dx)}{\sqrt[3]{a+ib}}\right)}{(a+ib)^{2/3}}-\frac{\log(1+i\tan(c+dx))}{2(a+ib)^{2/3}}+\frac{3\log\left(\sqrt[3]{a+ib}+i\tan(c+dx)\right)}{2(a+ib)^{2/3}}\right)}{2d}+\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{217}$$

$$\frac{i(a-ib)(A-iB)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{\tan(c+dx)}{\sqrt{3}}\right)}{(a-ib)^{2/3}}-\frac{\log(1-i\tan(c+dx))}{2(a-ib)^{2/3}}+\frac{3\log\left(\sqrt[3]{a-ib}-i\tan(c+dx)\right)}{2(a-ib)^{2/3}}\right)}{2d}-\frac{i(a+ib)(A+iB)\left(\frac{i\sqrt{3}\text{arctanh}\left(\frac{\tan(c+dx)}{\sqrt{3}}\right)}{(a+ib)^{2/3}}-\frac{\log(1+i\tan(c+dx))}{2(a+ib)^{2/3}}+\frac{3\log\left(\sqrt[3]{a+ib}+i\tan(c+dx)\right)}{2(a+ib)^{2/3}}\right)}{2d}+\frac{3B\sqrt[3]{a+b\tan(c+dx)}}{d}$$

input
```
Int[(a + b*Tan[c + d*x])^(1/3)*(A + B*Tan[c + d*x]),x]
```

output
```
((I/2)*(a - I*b)*(A - I*B)*(((-I)*Sqrt[3]*ArcTanh[Tan[c + d*x]/Sqrt[3]])/(
a - I*b)^(2/3) - Log[1 - I*Tan[c + d*x]]/(2*(a - I*b)^(2/3)) + (3*Log[(a -
 I*b)^(1/3) - I*Tan[c + d*x]])/(2*(a - I*b)^(2/3))))/d - ((I/2)*(a + I*b)*
(A + I*B)*((I*Sqrt[3]*ArcTanh[Tan[c + d*x]/Sqrt[3]])/(a + I*b)^(2/3) - Log
[1 + I*Tan[c + d*x]]/(2*(a + I*b)^(2/3)) + (3*Log[(a + I*b)^(1/3) + I*Tan[
c + d*x]])/(2*(a + I*b)^(2/3))))/d + (3*B*(a + b*Tan[c + d*x])^(1/3))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.26

method	result
derivativedivides	$\frac{3B(a+b\tan(dx+c))^{\frac{1}{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\left((Ab+Ba)\_R^{3}-B a^{2}-B b^{2}\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}\right.}{2}}{d}$
default	$\frac{3B(a+b\tan(dx+c))^{\frac{1}{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\left((Ab+Ba)\_R^{3}-B a^{2}-B b^{2}\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}\right.}{2}}{d}$
parts	$\frac{Ab\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\_R\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^{3}-a}\right)}{2d}+\frac{B\left(3(a+b\tan(dx+c))^{\frac{1}{3}}+\frac{\left(\sum_{\_R=\mathrm{Root}}\right.}{}\right.}{}$

input
```
int((a+b*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(3*B*(a+b*tan(d*x+c))^(1/3)+1/2*sum(((A*b+B*a)*_R^3-B*a^2-B*b^2)/(_R^5
-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b^2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2785 vs. $2(281) = 562$.

Time = 0.19 (sec) , antiderivative size = 2785, normalized size of antiderivative = 7.39

$$\int \sqrt[3]{a + b\tan(c + dx)}(A + B\tan(c + dx))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*d*(-(d^3*sqrt(-((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10
*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6) + (3*A^2
*B - B^3)*a + (A^3 - 3*A*B^2)*b)/d^3)^(1/3)*log(((A^5 - 2*A^3*B^2 - 3*A*B^
4)*a - (3*A^4*B + 2*A^2*B^3 - B^5)*b)*(b*tan(d*x + c) + a)^(1/3) + (A*d^4*
sqrt(-((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B
^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6) - ((A^3*B - 3*A*B^3)*a -
 (3*A^2*B^2 - B^4)*b)*d)*(-(d^3*sqrt(-((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 -
 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^
2)/d^6) + (3*A^2*B - B^3)*a + (A^3 - 3*A*B^2)*b)/d^3)^(1/3)) - (sqrt(-3)*d
 + d)*(-(d^3*sqrt(-((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^
3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)*b^2)/d^6) + (3*A^2*B
- B^3)*a + (A^3 - 3*A*B^2)*b)/d^3)^(1/3)*log(((A^5 - 2*A^3*B^2 - 3*A*B^4)*
a - (3*A^4*B + 2*A^2*B^3 - B^5)*b)*(b*tan(d*x + c) + a)^(1/3) + 1/2*(sqrt(
-3)*((A^3*B - 3*A*B^3)*a - (3*A^2*B^2 - B^4)*b)*d + ((A^3*B - 3*A*B^3)*a -
 (3*A^2*B^2 - B^4)*b)*d - (sqrt(-3)*A*d^4 + A*d^4)*sqrt(-((A^6 - 6*A^4*B^2
 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 -
6*A^2*B^4 + B^6)*b^2)/d^6))*(-(d^3*sqrt(-((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^
2 - 2*(3*A^5*B - 10*A^3*B^3 + 3*A*B^5)*a*b + (9*A^4*B^2 - 6*A^2*B^4 + B^6)
*b^2)/d^6) + (3*A^2*B - B^3)*a + (A^3 - 3*A*B^2)*b)/d^3)^(1/3)) + (sqrt(-3
)*d - d)*(-(d^3*sqrt(-((A^6 - 6*A^4*B^2 + 9*A^2*B^4)*a^2 - 2*(3*A^5*B -...
```

Sympy [F]

$$\int \sqrt[3]{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan{(c+dx)})\sqrt[3]{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(1/3)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 15.75 (sec) , antiderivative size = 2537, normalized size of antiderivative = 6.73

$$\int \sqrt[3]{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/3),x)
```

output
```
log(a*d^7*(((-A^6*a^2*d^6)^(1/2) - A^3*b*d^3)/d^6)^(4/3) + A*b*(a + b*tan(
c + d*x))^(1/3)*(-A^6*a^2*d^6)^(1/2) - A^4*a^2*d^3*(a + b*tan(c + d*x))^(1
/3) + 2*A^3*a*b*d^4*(((-A^6*a^2*d^6)^(1/2) - A^3*b*d^3)/d^6)^(1/3))*(((-A^
6*a^2*d^6)^(1/2) - A^3*b*d^3)/(8*d^6))^(1/3) + log(A*b*(a + b*tan(c + d*x)
)^(1/3)*(-A^6*a^2*d^6)^(1/2) + A^4*a^2*d^3*(a + b*tan(c + d*x))^(1/3) + a*
d*(-((-A^6*a^2*d^6)^(1/2) + A^3*b*d^3)/d^6)^(1/3)*(-A^6*a^2*d^6)^(1/2) - A
^3*a*b*d^4*(-((-A^6*a^2*d^6)^(1/2) + A^3*b*d^3)/d^6)^(1/3))*(-((-A^6*a^2*d
^6)^(1/2) + A^3*b*d^3)/(8*d^6))^(1/3) + log(d*(((-B^6*b^2*d^6)^(1/2) + B^3
*a*d^3)/d^6)^(1/3) - B*(a + b*tan(c + d*x))^(1/3))*(((-B^6*b^2*d^6)^(1/2)
+ B^3*a*d^3)/(8*d^6))^(1/3) + log(d*(-((-B^6*b^2*d^6)^(1/2) - B^3*a*d^3)/d
^6)^(1/3) - B*(a + b*tan(c + d*x))^(1/3))*(-((-B^6*b^2*d^6)^(1/2) - B^3*a*
d^3)/(8*d^6))^(1/3) + (log(- ((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*(1944*
a*b^4*(3^(1/2)*1i - 1)*(((-B^6*b^2*d^6)^(1/2) + B^3*a*d^3)/d^6)^(1/3)*(a^2
 + b^2) - (3888*B*a*b^4*(a^2 + b^2)*(a + b*tan(c + d*x))^(1/3))/d)*(((-B^6
*b^2*d^6)^(1/2) + B^3*a*d^3)/d^6)^(2/3))/16 - (972*B^3*b^4*(a^4 - b^4))/d^
3)*(((-B^6*b^2*d^6)^(1/2) + B^3*a*d^3)/d^6)^(1/3))/4 - (486*B^4*b^4*(a^4 -
 b^4)*(a + b*tan(c + d*x))^(1/3))/d^4)*(3^(1/2)*1i - 1)*((-B^6*b^2*d^6)^(1
/2)/(8*d^6) + (B^3*a)/(8*d^3))^(1/3))/2 - (log(- ((3^(1/2)*1i + 1)*(((3^(1
/2)*1i + 1)^2*(1944*a*b^4*(3^(1/2)*1i + 1)*(((-B^6*b^2*d^6)^(1/2) + B^3*a*
d^3)/d^6)^(1/3)*(a^2 + b^2) + (3888*B*a*b^4*(a^2 + b^2)*(a + b*tan(c + ...
```

Reduce [F]

$$\int \sqrt[3]{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \left(\int (a+\tan(dx+c)\,b)^{\frac{1}{3}}\,dx\right)a+\left(\int (a+\tan(dx+c)\,b)^{\frac{1}{3}}\tan(dx+c)\,dx\right)b$$

input
```
int((a+b*tan(d*x+c))^(1/3)*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*b + a)**(1/3),x)*a + int((tan(c + d*x)*b + a)**(1/3)*tan
(c + d*x),x)*b
```

3.475 $\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 357

$$\begin{aligned}
\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx &= -\frac{(A-iB)x}{4\sqrt[3]{a-ib}}-\frac{(A+iB)x}{4\sqrt[3]{a+ib}} \\
&+\frac{\sqrt{3}(iA+B)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2\sqrt[3]{a-ib}d} \\
&-\frac{\sqrt{3}(iA-B)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)}{2\sqrt[3]{a+ib}d} \\
&-\frac{(iA-B)\log(\cos(c+dx))}{4\sqrt[3]{a+ib}d}+\frac{(iA+B)\log(\cos(c+dx))}{4\sqrt[3]{a-ib}d} \\
&+\frac{3(iA+B)\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4\sqrt[3]{a-ib}d} \\
&-\frac{3(iA-B)\log\left(\sqrt[3]{a+ib}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4\sqrt[3]{a+ib}d}
\end{aligned}$$

output
```
-1/4*(A-I*B)*x/(a-I*b)^(1/3)-1/4*(A+I*B)*x/(a+I*b)^(1/3)+1/2*3^(1/2)*(I*A+
B)*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/3)/(a-I*b)^(1/3))*3^(1/2))/(a-I*b)^
(1/3)/d-1/2*3^(1/2)*(I*A-B)*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/3)/(a+I*b)
^(1/3))*3^(1/2))/(a+I*b)^(1/3)/d-1/4*(I*A-B)*ln(cos(d*x+c))/(a+I*b)^(1/3)/
d+1/4*(I*A+B)*ln(cos(d*x+c))/(a-I*b)^(1/3)/d+3/4*(I*A+B)*ln((a-I*b)^(1/3)-
(a+b*tan(d*x+c))^(1/3))/(a-I*b)^(1/3)/d-3/4*(I*A-B)*ln((a+I*b)^(1/3)-(a+b*
tan(d*x+c))^(1/3))/(a+I*b)^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.64

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx$$

$$= \frac{i\left(\frac{(A-iB)\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)-\log(i+\tan(c+dx))+3\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+dx)}\right)\right)}{\sqrt[3]{a-ib}}-\frac{(A\cdots}{\cdots}\right)}{4d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(1/3),x]
```

output
```
((I/4)*(((A - I*B)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(
a - I*b)^(1/3))/Sqrt[3]] - Log[I + Tan[c + d*x]] + 3*Log[(a - I*b)^(1/3) -
 (a + b*Tan[c + d*x])^(1/3)]))/(a - I*b)^(1/3) - ((A + I*B)*(2*Sqrt[3]*Arc
Tan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a + I*b)^(1/3))/Sqrt[3]] - Log[I
- Tan[c + d*x]] + 3*Log[(a + I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)]))/(a
 + I*b)^(1/3)))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.65, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4022, 3042, 4020, 25, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4022}$$

$$\frac{1}{2}(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt[3]{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{i\tan(c+dx)+1}{\sqrt[3]{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4020}$$

$$\frac{i(A-iB)\int -\frac{1}{(1-i\tan(c+dx))\sqrt[3]{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}-\frac{i(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)\sqrt[3]{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{i(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt[3]{a+b\tan(c+dx)}}d(-i\tan(c+dx))}{2d}-\frac{i(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt[3]{a+b\tan(c+dx)}}d(i\tan(c+dx))}{2d}$$

$$\Big\downarrow \text{ 67}$$

$$\frac{i(A-iB)\left(\frac{3}{2}\int \frac{1}{-\tan^2(c+dx)+(a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}-\frac{3\int \frac{1}{\sqrt[3]{a-ib}-i\tan(c+dx)}d\cdots}{2\sqrt[3]{a\cdots}}\cdots\right.}{2d}$$

$$\frac{i(A+iB)\left(\frac{3}{2}\int \frac{1}{-\tan^2(c+dx)+(a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}-\frac{3\int \frac{1}{i\tan(c+dx)+\sqrt[3]{a+ib}}d\cdots}{2\sqrt[3]{a\cdots}}\cdots\right.}{2d}$$

$$\Big\downarrow \ 16$$

$$\frac{i(A-iB)\left(\frac{3}{2}\int \frac{1}{-\tan^2(c+dx)+(a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}-\frac{\log(1-i\tan(c+dx))}{2\sqrt[3]{a-ib}}+\frac{3\log\cdots}{\cdots}\right.}{2d}$$

$$\frac{i(A+iB)\left(\frac{3}{2}\int \frac{1}{-\tan^2(c+dx)+(a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}-\frac{\log(1+i\tan(c+dx))}{2\sqrt[3]{a+ib}}+\frac{3\log\cdots}{\cdots}\right.}{2d}$$

$$\Big\downarrow \ 1082$$

$$\frac{i(A-iB)\left(-\frac{3\int \frac{1}{\tan^2(c+dx)-3}d\left(\frac{2i\tan(c+dx)}{\sqrt[3]{a-ib}}+1\right)}{\sqrt[3]{a-ib}}-\frac{\log(1-i\tan(c+dx))}{2\sqrt[3]{a-ib}}+\frac{3\log\left(\sqrt[3]{a-ib}-i\tan(c+dx)\right)}{2\sqrt[3]{a-ib}}\right)}{2d}-\frac{i(A+iB)\left(-\frac{3\int \frac{1}{\tan^2(c+dx)-3}d\left(1-\frac{2i\tan(c+dx)}{\sqrt[3]{a+ib}}\right)}{\sqrt[3]{a+ib}}-\frac{\log(1+i\tan(c+dx))}{2\sqrt[3]{a+ib}}+\frac{3\log\left(\sqrt[3]{a+ib}+i\tan(c+dx)\right)}{2\sqrt[3]{a+ib}}\right)}{2d}$$

$$\Big\downarrow \ 217$$

$$\frac{i(A-iB)\left(\frac{i\sqrt{3}\text{arctanh}\left(\frac{\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{a-ib}}-\frac{\log(1-i\tan(c+dx))}{2\sqrt[3]{a-ib}}+\frac{3\log\left(\sqrt[3]{a-ib}-i\tan(c+dx)\right)}{2\sqrt[3]{a-ib}}\right)}{2d}-\frac{i(A+iB)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt[3]{a+ib}}-\frac{\log(1+i\tan(c+dx))}{2\sqrt[3]{a+ib}}+\frac{3\log\left(\sqrt[3]{a+ib}+i\tan(c+dx)\right)}{2\sqrt[3]{a+ib}}\right)}{2d}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(1/3),x]
```

output
```
((I/2)*(A - I*B)*((I*Sqrt[3]*ArcTanh[Tan[c + d*x]/Sqrt[3]])/(a - I*b)^(1/3
) - Log[1 - I*Tan[c + d*x]]/(2*(a - I*b)^(1/3)) + (3*Log[(a - I*b)^(1/3) -
 I*Tan[c + d*x]])/(2*(a - I*b)^(1/3))))/d - ((I/2)*(A + I*B)*(((-I)*Sqrt[3
]*ArcTanh[Tan[c + d*x]/Sqrt[3]])/(a + I*b)^(1/3) - Log[1 + I*Tan[c + d*x]]
/(2*(a + I*b)^(1/3)) + (3*Log[(a + I*b)^(1/3) + I*Tan[c + d*x]])/(2*(a + I
*b)^(1/3))))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.20

method	result
derivativedivides	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\left(B\_R^4+(Ab-Ba)\_R\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2a}}{2d}$
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\left(B\_R^4+(Ab-Ba)\_R\right)\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R^5-\_R^2a}}{2d}$
parts	$\frac{Ab\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\ln\left((a+b\tan(dx+c))^{\frac{1}{3}}-\_R\right)}{\_R\left(\_R^3-a\right)}\right)}{2d}+\frac{B\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6-2a\_Z^3+a^2+b^2\right)}\frac{\ln(}{}\right.}{2d}$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*sum((B*_R^4+(A*b-B*a)*_R)/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R
),_R=RootOf(_Z^6-2*_Z^3*a+a^2+b^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4121 vs. 2(263) = 526.

Time = 0.29 (sec) , antiderivative size = 4121, normalized size of antiderivative = 11.54

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx = \int \frac{A+B\tan{(c+dx)}}{\sqrt[3]{a+b\tan{(c+dx)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/3),x)
```

output
```
Integral((A + B*tan(c + d*x))/(a + b*tan(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx = \int \frac{B\tan{(dx+c)}+A}{(b\tan{(dx+c)}+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/(b*tan(d*x + c) + a)^(1/3), x)
```

Giac [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.41

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx$$
$$= \frac{\left(\frac{A^3-3i\,A^2B-3\,AB^2+i\,B^3}{8i\,a+8\,b}\right)^{\frac{1}{3}}\log\left(-a+i\,b-(-a^2+2i\,ab+b^2)^{\frac{1}{3}}(b\tan\left(dx+c\right)+a)^{\frac{1}{3}}\right)+\left(-\frac{A^3+3i\,A^2B-3\,AB^2\cdots}{8i\,a-8\,b}\right.}{d}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
(((A^3 - 3*I*A^2*B - 3*A*B^2 + I*B^3)/(8*I*a + 8*b))^(1/3)*log(-a + I*b -
(-a^2 + 2*I*a*b + b^2)^(1/3)*(b*tan(d*x + c) + a)^(1/3)) + (-(A^3 + 3*I*A^
2*B - 3*A*B^2 - I*B^3)/(8*I*a - 8*b))^(1/3)*log(-a - I*b + (a^2 + 2*I*a*b
- b^2)^(1/3)*(b*tan(d*x + c) + a)^(1/3)))/d
```

Mupad [B] (verification not implemented)

Time = 15.90 (sec) , antiderivative size = 3228, normalized size of antiderivative = 9.04

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(a + b*tan(c + d*x))^(1/3),x)
```

output
```
log(((((-B^6*b^2*d^6)^(1/2) + B^3*a*d^3)/(d^6*(a^2 + b^2)))^(2/3)*(972*a*b
^4*(-B^6*b^2*d^6)^(1/2) - 972*B^3*b^6*d^3 + 972*B^2*b^6*d^4*(a + b*tan(c +
 d*x))^(1/3)*(((-B^6*b^2*d^6)^(1/2) + B^3*a*d^3)/(d^6*(a^2 + b^2)))^(1/3)
- 972*B^2*a^2*b^4*d^4*(a + b*tan(c + d*x))^(1/3)*(((-B^6*b^2*d^6)^(1/2) +
B^3*a*d^3)/(d^6*(a^2 + b^2)))^(1/3)))/(4*d^6) + (243*B^5*a*b^4*(a + b*tan(
c + d*x))^(1/3))/d^5)*((-64*B^6*b^2*d^6)^(1/2)/(64*(a^2*d^6 + b^2*d^6)) +
(B^3*a*d^3)/(8*(a^2*d^6 + b^2*d^6)))^(1/3) + log(((1944*a*b^4*(a^2 + b^2)*
(((-A^6*a^2*d^6)^(1/2) + A^3*b*d^3)/(d^6*(a^2 + b^2)))^(2/3) + (1944*A^2*b
^4*(a^2 - b^2)*(a + b*tan(c + d*x))^(1/3))/d^2)*((-A^6*a^2*d^6)^(1/2) + A^
3*b*d^3))/(8*d^6*(a^2 + b^2)) + (243*A^5*b^5*(a + b*tan(c + d*x))^(1/3))/d
^5)*((-64*A^6*a^2*d^6)^(1/2)/(64*(a^2*d^6 + b^2*d^6)) + (A^3*b*d^3)/(8*(a^
2*d^6 + b^2*d^6)))^(1/3) + log((243*B^5*a*b^4*(a + b*tan(c + d*x))^(1/3))/
d^5 - ((-(8*(-B^6*b^2*d^6)^(1/2) - 8*B^3*a*d^3)/(d^6*(a^2 + b^2)))^(2/3)*(
972*a*b^4*(-B^6*b^2*d^6)^(1/2) + 972*B^3*b^6*d^3 - 486*B^2*b^6*d^4*(a + b*
tan(c + d*x))^(1/3)*(-(8*(-B^6*b^2*d^6)^(1/2) - 8*B^3*a*d^3)/(d^6*(a^2 + b
^2)))^(1/3) + 486*B^2*a^2*b^4*d^4*(a + b*tan(c + d*x))^(1/3)*(-(8*(-B^6*b^
2*d^6)^(1/2) - 8*B^3*a*d^3)/(d^6*(a^2 + b^2)))^(1/3)))/(16*d^6))*((B^3*a*d
^3)/(8*(a^2*d^6 + b^2*d^6)) - (-64*B^6*b^2*d^6)^(1/2)/(64*(a^2*d^6 + b^2*d
^6)))^(1/3) + log((243*A^5*b^5*(a + b*tan(c + d*x))^(1/3))/d^5 - ((486*a*b
^4*(a^2 + b^2)*(-(8*(-A^6*a^2*d^6)^(1/2) - 8*A^3*b*d^3)/(d^6*(a^2 + b^2...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt[3]{a+b\tan(c+dx)}}\,dx = \left(\int \frac{\tan(dx+c)}{(a+\tan(dx+c)\,b)^{\frac{1}{3}}}dx\right)b + \left(\int \frac{1}{(a+\tan(dx+c)\,b)^{\frac{1}{3}}}dx\right)a$$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/3),x)
```

output
```
int(tan(c + d*x)/(tan(c + d*x)*b + a)**(1/3),x)*b + int(1/(tan(c + d*x)*b
+ a)**(1/3),x)*a
```

3.476 $\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 357

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx &= -\frac{(A-iB)x}{4(a-ib)^{2/3}}-\frac{(A+iB)x}{4(a+ib)^{2/3}}\\
&-\frac{\sqrt{3}(iA+B)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)}{2(a-ib)^{2/3}d}\\
&+\frac{\sqrt{3}(iA-B)\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a+ib}}}{\sqrt{3}}\right)}{2(a+ib)^{2/3}d}-\frac{(iA-B)\log(\cos(c+dx))}{4(a+ib)^{2/3}d}\\
&+\frac{(iA+B)\log(\cos(c+dx))}{4(a-ib)^{2/3}d}+\frac{3(iA+B)\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4(a-ib)^{2/3}d}\\
&-\frac{3(iA-B)\log\left(\sqrt[3]{a+ib}-\sqrt[3]{a+b\tan(c+dx)}\right)}{4(a+ib)^{2/3}d}\end{aligned}$$

output
```
-1/4*(A-I*B)*x/(a-I*b)^(2/3)-1/4*(A+I*B)*x/(a+I*b)^(2/3)-1/2*3^(1/2)*(I*A+
B)*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/3)/(a-I*b)^(1/3))*3^(1/2))/(a-I*b)^
(2/3)/d+1/2*3^(1/2)*(I*A-B)*arctan(1/3*(1+2*(a+b*tan(d*x+c))^(1/3)/(a+I*b)
^(1/3))*3^(1/2))/(a+I*b)^(2/3)/d-1/4*(I*A-B)*ln(cos(d*x+c))/(a+I*b)^(2/3)/
d+1/4*(I*A+B)*ln(cos(d*x+c))/(a-I*b)^(2/3)/d+3/4*(I*A+B)*ln((a-I*b)^(1/3)-
(a+b*tan(d*x+c))^(1/3))/(a-I*b)^(2/3)/d-3/4*(I*A-B)*ln((a+I*b)^(1/3)-(a+b*
tan(d*x+c))^(1/3))/(a+I*b)^(2/3)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.85

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx = \frac{i\left(-\frac{(A-iB)\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+b\tan(c+dx)}}{\sqrt[3]{a-ib}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a-ib}-\sqrt[3]{a+b\tan(\right.\right.}{(a-ib}\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(2/3),x]
```

output
```
((I/4)*(-(((A - I*B)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))
/(a - I*b)^(1/3))/Sqrt[3]] - 2*Log[(a - I*b)^(1/3) - (a + b*Tan[c + d*x])^
(1/3)] + Log[(a - I*b)^(2/3) + (a - I*b)^(1/3)*(a + b*Tan[c + d*x])^(1/3)
+ (a + b*Tan[c + d*x])^(2/3)]))/(a - I*b)^(2/3)) + ((A + I*B)*(2*Sqrt[3]*A
rcTan[(1 + (2*(a + b*Tan[c + d*x])^(1/3))/(a + I*b)^(1/3))/Sqrt[3]] - 2*Lo
g[(a + I*b)^(1/3) - (a + b*Tan[c + d*x])^(1/3)] + Log[(a + I*b)^(2/3) + (a
 + I*b)^(1/3)*(a + b*Tan[c + d*x])^(1/3) + (a + b*Tan[c + d*x])^(2/3)]))/(
a + I*b)^(2/3)))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.65, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 4022, 3042, 4020, 25, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}dx$$

$$\Big\downarrow \ 4022$$

$$\frac{1}{2}(A+iB)\int \frac{1-i\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}dx+\frac{1}{2}(A-iB)\int \frac{i\tan(c+dx)+1}{(a+b\tan(c+dx))^{2/3}}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}(A+iB)\int \frac{1-i\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}dx+\frac{1}{2}(A-iB)\int \frac{i\tan(c+dx)+1}{(a+b\tan(c+dx))^{2/3}}dx$$

$$\Big\downarrow \ 4020$$

$$\frac{i(A-iB)\int -\frac{1}{(1-i\tan(c+dx))(a+b\tan(c+dx))^{2/3}}d(i\tan(c+dx))}{2d}-\frac{i(A+iB)\int -\frac{1}{(i\tan(c+dx)+1)(a+b\tan(c+dx))^{2/3}}d(-i\tan(c+dx))}{2d}$$

$$\Big\downarrow \ 25$$

$$\frac{i(A+iB)\int \frac{1}{(i\tan(c+dx)+1)(a+b\tan(c+dx))^{2/3}}d(-i\tan(c+dx))}{2d}-\frac{i(A-iB)\int \frac{1}{(1-i\tan(c+dx))(a+b\tan(c+dx))^{2/3}}d(i\tan(c+dx))}{2d}$$

$$\Big\downarrow \ 69$$

$$\frac{i(A-iB)\left(-\frac{3\int \frac{1}{-\tan^2(c+dx)+(a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}}{2\sqrt[3]{a-ib}}-\frac{3\int \frac{1}{\sqrt[3]{a-ib}-i\tan(c+dx)}d\sqrt[3]{\cdots}}{2(a-ib)\cdots}\right.}{2d}$$

$$\frac{i(A+iB)\left(-\frac{3\int \frac{1}{-\tan^2(c+dx)+(a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}}{2\sqrt[3]{a+ib}}-\frac{3\int \frac{1}{i\tan(c+dx)+\sqrt[3]{a+ib}}d\sqrt[3]{\cdots}}{2(a+ib)\cdots}\right.}{2d}$$

$$\downarrow 16$$

$$\frac{i(A-iB)\left(-\frac{3\int \frac{1}{-\tan^2(c+dx)+(a-ib)^{2/3}+\sqrt[3]{a-ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}}{2\sqrt[3]{a-ib}}-\frac{\log(1-i\tan(c+dx))}{2(a-ib)^{2/3}}+\frac{3\log\cdots}{\cdots}\right.}{2d}$$

$$\frac{i(A+iB)\left(-\frac{3\int \frac{1}{-\tan^2(c+dx)+(a+ib)^{2/3}+\sqrt[3]{a+ib}\sqrt[3]{a+b\tan(c+dx)}}d\sqrt[3]{a+b\tan(c+dx)}}{2\sqrt[3]{a+ib}}-\frac{\log(1+i\tan(c+dx))}{2(a+ib)^{2/3}}+\frac{3\log\cdots}{\cdots}\right.}{2d}$$

$$\downarrow 1082$$

$$\frac{i(A-iB)\left(\frac{3\int \frac{1}{\tan^2(c+dx)-3}d\left(\frac{2i\tan(c+dx)}{\sqrt[3]{a-ib}}+1\right)}{(a-ib)^{2/3}}-\frac{\log(1-i\tan(c+dx))}{2(a-ib)^{2/3}}+\frac{3\log\left(\sqrt[3]{a-ib}-i\tan(c+dx)\right)}{2(a-ib)^{2/3}}\right)}{2d}-\frac{i(A+iB)\left(\frac{3\int \frac{1}{\tan^2(c+dx)-3}d\left(1-\frac{2i\tan(c+dx)}{\sqrt[3]{a+ib}}\right)}{(a+ib)^{2/3}}-\frac{\log(1+i\tan(c+dx))}{2(a+ib)^{2/3}}+\frac{3\log\left(\sqrt[3]{a+ib}+i\tan(c+dx)\right)}{2(a+ib)^{2/3}}\right)}{2d}$$

$$\downarrow 217$$

$$\frac{i(A-iB)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{\tan(c+dx)}{\sqrt{3}}\right)}{(a-ib)^{2/3}}-\frac{\log(1-i\tan(c+dx))}{2(a-ib)^{2/3}}+\frac{3\log\left(\sqrt[3]{a-ib}-i\tan(c+dx)\right)}{2(a-ib)^{2/3}}\right)}{2d}-\frac{i(A+iB)\left(\frac{i\sqrt{3}\text{arctanh}\left(\frac{\tan(c+dx)}{\sqrt{3}}\right)}{(a+ib)^{2/3}}-\frac{\log(1+i\tan(c+dx))}{2(a+ib)^{2/3}}+\frac{3\log\left(\sqrt[3]{a+ib}+i\tan(c+dx)\right)}{2(a+ib)^{2/3}}\right)}{2d}$$

input
```
Int[(A + B*Tan[c + d*x])/(a + b*Tan[c + d*x])^(2/3),x]
```

output
```
((I/2)*(A - I*B)*(((-I)*Sqrt[3]*ArcTanh[Tan[c + d*x]/Sqrt[3]])/(a - I*b)^(
2/3) - Log[1 - I*Tan[c + d*x]]/(2*(a - I*b)^(2/3)) + (3*Log[(a - I*b)^(1/3
) - I*Tan[c + d*x]])/(2*(a - I*b)^(2/3))))/d - ((I/2)*(A + I*B)*((I*Sqrt[3
]*ArcTanh[Tan[c + d*x]/Sqrt[3]])/(a + I*b)^(2/3) - Log[1 + I*Tan[c + d*x]]
/(2*(a + I*b)^(2/3)) + (3*Log[(a + I*b)^(1/3) + I*Tan[c + d*x]])/(2*(a + I
*b)^(2/3))))/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.19

method	result
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\left(B\_R^{3}+Ab-Ba\right)\ln\left(\left(a+b\tan(dx+c)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}}{2d}$
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\left(B\_R^{3}+Ab-Ba\right)\ln\left(\left(a+b\tan(dx+c)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}a}}{2d}$
parts	$\frac{Ab\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\ln\left(\left(a+b\tan(dx+c)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{2}\left(\_R^{3}-a\right)}\right)}{2d}+\frac{B\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2a\_Z^{3}+a^{2}+b^{2}\right)}\frac{\ln\left(\right.}{}\right.}{2d}$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*sum((B*_R^3+A*b-B*a)/(_R^5-_R^2*a)*ln((a+b*tan(d*x+c))^(1/3)-_R),_R=
RootOf(_Z^6-2*_Z^3*a+a^2+b^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6073 vs. $2(263) = 526$.

Time = 1.17 (sec) , antiderivative size = 6073, normalized size of antiderivative = 17.01

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx = \int \frac{A+B\tan\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(2/3),x)
```

output
```
Integral((A + B*tan(c + d*x))/(a + b*tan(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx = \int \frac{B\tan\left(dx+c\right)+A}{\left(b\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/(b*tan(d*x + c) + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx=\int \frac{B\tan\left(dx+c\right)+A}{\left(b\tan\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 18.29 (sec) , antiderivative size = 4562, normalized size of antiderivative = 12.78

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx=\text{Too large to display}$$

input
```
int((A + B*tan(c + d*x))/(a + b*tan(c + d*x))^(2/3),x)
```

output
```
log((((16*(-B^6*a^2*b^2*d^6)^(1/2) + 8*B^3*a^2*d^3 - 8*B^3*b^2*d^3)/(d^6*(
a^2 + b^2)^2))^(1/3)*(1944*a*b^4*(-B^6*a^2*b^2*d^6)^(1/2) - 1944*B^3*a*b^6
*d^3 + 243*B*b^8*d^5*(a + b*tan(c + d*x))^(1/3)*((16*(-B^6*a^2*b^2*d^6)^(1
/2) + 8*B^3*a^2*d^3 - 8*B^3*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(2/3) - 243*B*a^
4*b^4*d^5*(a + b*tan(c + d*x))^(1/3)*((16*(-B^6*a^2*b^2*d^6)^(1/2) + 8*B^3
*a^2*d^3 - 8*B^3*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(2/3)))/(4*d^6*(a^2 + b^2))
 + (486*B^4*b^4*(a + b*tan(c + d*x))^(1/3))/d^4)*((((16*B^3*a^2*d^3 - 16*B
^3*b^2*d^3)^2/4 - B^6*(64*a^4*d^6 + 64*b^4*d^6 + 128*a^2*b^2*d^6))^(1/2) +
 8*B^3*a^2*d^3 - 8*B^3*b^2*d^3)/(64*(a^4*d^6 + b^4*d^6 + 2*a^2*b^2*d^6)))^
(1/3) + log((486*B^4*b^4*(a + b*tan(c + d*x))^(1/3))/d^4 - ((-(16*(-B^6*a^
2*b^2*d^6)^(1/2) - 8*B^3*a^2*d^3 + 8*B^3*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(1/
3)*(1944*a*b^4*(-B^6*a^2*b^2*d^6)^(1/2) + 1944*B^3*a*b^6*d^3 - 243*B*b^8*d
^5*(a + b*tan(c + d*x))^(1/3)*(-(16*(-B^6*a^2*b^2*d^6)^(1/2) - 8*B^3*a^2*d
^3 + 8*B^3*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(2/3) + 243*B*a^4*b^4*d^5*(a + b*
tan(c + d*x))^(1/3)*(-(16*(-B^6*a^2*b^2*d^6)^(1/2) - 8*B^3*a^2*d^3 + 8*B^3
*b^2*d^3)/(d^6*(a^2 + b^2)^2))^(2/3)))/(4*d^6*(a^2 + b^2)))*(-(((16*B^3*a^
2*d^3 - 16*B^3*b^2*d^3)^2/4 - B^6*(64*a^4*d^6 + 64*b^4*d^6 + 128*a^2*b^2*d
^6))^(1/2) - 8*B^3*a^2*d^3 + 8*B^3*b^2*d^3)/(64*(a^4*d^6 + b^4*d^6 + 2*a^2
*b^2*d^6)))^(1/3) + log(((((1944*a*b^4*(a^2 + b^2)*((8*(-A^6*d^6*(a^2 - b^
2)^2)^(1/2) + 16*A^3*a*b*d^3)/(d^6*(a^2 + b^2)^2))^(1/3) + (7776*A*a*b^...
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{2/3}}\,dx = \left(\int \frac{\tan(dx+c)}{(a+\tan(dx+c)\,b)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{1}{(a+\tan(dx+c)\,b)^{\frac{2}{3}}}dx\right)a$$

input
```
int((A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(2/3),x)
```

output
```
int(tan(c + d*x)/(tan(c + d*x)*b + a)**(2/3),x)*b + int(1/(tan(c + d*x)*b
+ a)**(2/3),x)*a
```

3.477 $\int \frac{i-\tan(e+fx)}{\sqrt[3]{c+d\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 148

$$\int \frac{i-\tan(e+fx)}{\sqrt[3]{c+d\tan(e+fx)}}\,dx = -\frac{ix}{2\sqrt[3]{c-id}}-\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)}{\sqrt[3]{c-id}f}-\frac{\log(\cos(e+fx))}{2\sqrt[3]{c-id}f}-\frac{3\log\left(\sqrt[3]{c-id}-\sqrt[3]{c+d\tan(e+fx)}\right)}{2\sqrt[3]{c-id}f}$$

output
```
-1/2*I*x/(c-I*d)^(1/3)-3^(1/2)*arctan(1/3*(1+2*(c+d*tan(f*x+e))^(1/3)/(c-I
*d)^(1/3))*3^(1/2))/(c-I*d)^(1/3)/f-1/2*ln(cos(f*x+e))/(c-I*d)^(1/3)/f-3/2
*ln((c-I*d)^(1/3)-(c+d*tan(f*x+e))^(1/3))/(c-I*d)^(1/3)/f
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.70

$$\int \frac{i-\tan(e+fx)}{\sqrt[3]{c+d\tan(e+fx)}}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)+\log(i+\tan(e+fx))-3\log\left(\sqrt[3]{c-id}-\sqrt[3]{c+d\tan(e+f$$

$$2\sqrt[3]{c-id}f$$

input
```
Integrate[(I - Tan[e + f*x])/(c + d*Tan[e + f*x])^(1/3),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c - I*d)^(1/3))/Sq
rt[3]] + Log[I + Tan[e + f*x]] - 3*Log[(c - I*d)^(1/3) - (c + d*Tan[e + f*
x])^(1/3)])/(2*(c - I*d)^(1/3)*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4020, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-\tan(e+fx)+i}{\sqrt[3]{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-\tan(e+fx)+i}{\sqrt[3]{c+d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4020}$$

$$-\frac{i\int \frac{1}{(1-i\tan(e+fx))\sqrt[3]{c+d\tan(e+fx)}}d(-\tan(e+fx))}{f}$$

$\downarrow$ 67

$$-\frac{i\left(-\frac{3}{2}i\int \frac{1}{\tan^2(e+fx)+(c-id)^{2/3}+\sqrt[3]{c-id}\sqrt[3]{c+d\tan(e+fx)}}d\sqrt[3]{c+d\tan(e+fx)}+\frac{3i\int \frac{1}{\tan(e+fx)+\sqrt[3]{c-id}}d\sqrt[3]{c+}}{2\sqrt[3]{c-id}}\right.}{f}$$

$\downarrow$ 16

$$-\frac{i\left(-\frac{3}{2}i\int \frac{1}{\tan^2(e+fx)+(c-id)^{2/3}+\sqrt[3]{c-id}\sqrt[3]{c+d\tan(e+fx)}}d\sqrt[3]{c+d\tan(e+fx)}+\frac{i\log(\tan(e+fx)+i)}{2\sqrt[3]{c-id}}-\frac{3i\log\left(\tan(\right.}{2}\right.}{f}$$

$\downarrow$ 1082

$$-\frac{i\left(\frac{3i\int \frac{1}{-\tan^2(e+fx)-3}d\left(1-\frac{2\tan(e+fx)}{\sqrt[3]{c-id}}\right)}{\sqrt[3]{c-id}}+\frac{i\log(\tan(e+fx)+i)}{2\sqrt[3]{c-id}}-\frac{3i\log\left(\tan(e+fx)+\sqrt[3]{c-id}\right)}{2\sqrt[3]{c-id}}\right)}{f}$$

$\downarrow$ 217

$$-\frac{i\left(\frac{i\sqrt{3}\arctan\left(\frac{\tan(e+fx)}{\sqrt{3}}\right)}{\sqrt[3]{c-id}}+\frac{i\log(\tan(e+fx)+i)}{2\sqrt[3]{c-id}}-\frac{3i\log\left(\tan(e+fx)+\sqrt[3]{c-id}\right)}{2\sqrt[3]{c-id}}\right)}{f}$$

input
```
Int[(I - Tan[e + f*x])/(c + d*Tan[e + f*x])^(1/3),x]
```

output
```
((-I)*((I*Sqrt[3]*ArcTan[Tan[e + f*x]/Sqrt[3]])/(c - I*d)^(1/3) + ((I/2)*L
og[I + Tan[e + f*x]])/(c - I*d)^(1/3) - (((3*I)/2)*Log[(c - I*d)^(1/3) + T
an[e + f*x]])/(c - I*d)^(1/3)))/f
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + 
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 + 
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

method	result
derivativedivides	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3+id-c\right)}\frac{\ln\left((c+d\tan(fx+e))^{\frac{1}{3}}-\_R\right)}{\_R}}{f}$
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3+id-c\right)}\frac{\ln\left((c+d\tan(fx+e))^{\frac{1}{3}}-\_R\right)}{\_R}}{f}$
parts	$\frac{id\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2c\_Z^3+c^2+d^2\right)}\frac{\ln\left((c+d\tan(fx+e))^{\frac{1}{3}}-\_R\right)}{\_R\left(\_R^3-c\right)}\right)}{2f}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-2c\_Z^3+c^2+d^2\right)}\frac{\ln\left((c+d\tan\right.}{}}{2f}$

input
```
int((I-tan(f*x+e))/(c+d*tan(f*x+e))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/f*sum(1/_R*ln((c+d*tan(f*x+e))^(1/3)-_R),_R=RootOf(_Z^3+I*d-c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(109) = 218.

Time = 0.08 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.88

$$\int \frac{i-\tan(e+fx)}{\sqrt[3]{c+d\tan(e+fx)}}\,dx$$
$$=\frac{1}{2}\left(i\sqrt{3}-1\right)\left(-\frac{i}{(ic+d)f^3}\right)^{\frac{1}{3}}\log\left(\frac{1}{2}\left(\sqrt{3}(ic+d)f^2+(c-id)f^2\right)\left(-\frac{i}{(ic+d)f^3}\right)^{\frac{2}{3}}+\left(\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}\right)^{\frac{1}{3}}\right)+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(-\frac{i}{(ic+d)f^3}\right)^{\frac{1}{3}}\log\left(\frac{1}{2}\left(\sqrt{3}(-ic-d)f^2+(c-id)f^2\right)\left(-\frac{i}{(ic+d)f^3}\right)^{\frac{2}{3}}+\left(\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}\right)^{\frac{1}{3}}\right)+\left(-\frac{i}{(ic+d)f^3}\right)^{\frac{1}{3}}\log\left(-(c-id)f^2\left(-\frac{i}{(ic+d)f^3}\right)^{\frac{2}{3}}+\left(\frac{(c-id)e^{(2ifx+2ie)}+c+id}{e^{(2ifx+2ie)}+1}\right)^{\frac{1}{3}}\right)$$

input
```
integrate((I-tan(f*x+e))/(c+d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
1/2*(I*sqrt(3) - 1)*(-I/((I*c + d)*f^3))^(1/3)*log(1/2*(sqrt(3)*(I*c + d)*
f^2 + (c - I*d)*f^2)*(-I/((I*c + d)*f^3))^(2/3) + (((c - I*d)*e^(2*I*f*x +
 2*I*e) + c + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)) + 1/2*(-I*sqrt(3) - 1
)*(-I/((I*c + d)*f^3))^(1/3)*log(1/2*(sqrt(3)*(-I*c - d)*f^2 + (c - I*d)*f
^2)*(-I/((I*c + d)*f^3))^(2/3) + (((c - I*d)*e^(2*I*f*x + 2*I*e) + c + I*d
)/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)) + (-I/((I*c + d)*f^3))^(1/3)*log(-(c -
 I*d)*f^2*(-I/((I*c + d)*f^3))^(2/3) + (((c - I*d)*e^(2*I*f*x + 2*I*e) + c
 + I*d)/(e^(2*I*f*x + 2*I*e) + 1))^(1/3))
```

Sympy [F]

$$\int \frac{i-\tan(e+fx)}{\sqrt[3]{c+d\tan(e+fx)}}\,dx = -\int \left(-\frac{i}{\sqrt[3]{c+d\tan{(e+fx)}}}\right)dx - \int \frac{\tan{(e+fx)}}{\sqrt[3]{c+d\tan{(e+fx)}}}\,dx$$

input
```
integrate((I-tan(f*x+e))/(c+d*tan(f*x+e))**(1/3),x)
```

output
```
-Integral(-I/(c + d*tan(e + f*x))**(1/3), x) - Integral(tan(e + f*x)/(c +
d*tan(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{i-\tan(e+fx)}{\sqrt[3]{c+d\tan(e+fx)}}\,dx = \int -\frac{\tan{(fx+e)}-i}{(d\tan{(fx+e)}+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((I-tan(f*x+e))/(c+d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
-integrate((tan(f*x + e) - I)/(d*tan(f*x + e) + c)^(1/3), x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(109) = 218$.

Time = 0.42 (sec) , antiderivative size = 795, normalized size of antiderivative = 5.37

$$\int \frac{i-\tan(e+fx)}{\sqrt[3]{c+d\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((I-tan(f*x+e))/(c+d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
-(c - I*d)^(2/3)*log((d*tan(f*x + e) + c)^(1/3) - (c - I*d)^(1/3))/(c*f -
I*d*f) - (sqrt(3)*c*cos(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d) - 1/3*arctan(
d/c))^2 - sqrt(3)*c*sin(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d) - 1/3*arctan(
d/c))^2 + 2*c*cos(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d) - 1/3*arctan(d/c))*
sin(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d) - 1/3*arctan(d/c)))*arctan(1/3*sq
rt(3)*(2*(d*tan(f*x + e) + c)^(1/3) + (c - I*d)^(1/3))/(c - I*d)^(1/3))/((
c^2 + d^2)^(2/3)*f) - I*(sqrt(3)*d*cos(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d
) - 1/3*arctan(d/c))^2 - sqrt(3)*d*sin(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d
) - 1/3*arctan(d/c))^2 + 2*d*cos(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d) - 1/
3*arctan(d/c))*sin(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d) - 1/3*arctan(d/c))
)*arctan(1/3*sqrt(3)*(2*(d*tan(f*x + e) + c)^(1/3) + (c - I*d)^(1/3))/(c -
 I*d)^(1/3))/((c^2 + d^2)^(2/3)*f) - 1/2*(2*sqrt(3)*c*cos(1/6*pi*sgn(c)*sg
n(d) - 1/6*pi*sgn(d) - 1/3*arctan(d/c))*sin(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*
sgn(d) - 1/3*arctan(d/c)) - c*cos(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d) - 1
/3*arctan(d/c))^2 + c*sin(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d) - 1/3*arcta
n(d/c))^2)*log((c^2 + d^2)^(1/3)*cos(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d)
- 1/3*arctan(d/c))^2 + (c^2 + d^2)^(1/3)*sin(1/6*pi*sgn(c)*sgn(d) - 1/6*pi
*sgn(d) - 1/3*arctan(d/c))^2 + (d*tan(f*x + e) + c)^(2/3) + (d*tan(f*x + e
) + c)^(1/3)*(c - I*d)^(1/3))/((c^2 + d^2)^(2/3)*f) - 1/2*I*(2*sqrt(3)*d*c
os(1/6*pi*sgn(c)*sgn(d) - 1/6*pi*sgn(d) - 1/3*arctan(d/c))*sin(1/6*pi*s...
```

Mupad [B] (verification not implemented)

Time = 17.68 (sec) , antiderivative size = 2982, normalized size of antiderivative = 20.15

$$\int \frac{i - \tan(e + fx)}{\sqrt[3]{c + d\tan(e + fx)}}\, dx = \text{Too large to display}$$

input
```
int(-(tan(e + f*x) - 1i)/(c + d*tan(e + f*x))^(1/3),x)
```

output
```
log(d^5*f*(c + d*tan(e + f*x))^(1/3)*243i + ((1944*d^4*f^4*(c^2 - d^2)*(c
+ d*tan(e + f*x))^(1/3) - 3*2^(1/3)*c*d^4*f^6*(c^2 + d^2)*(-(11664*f^3*((c
^2*d^12*(c^2 + d^2)^2)/f^6)^(1/2) + d^9*11664i + c^2*d^7*11664i)/(d^6*f^3*
(c^2 + d^2)^2))^(2/3))*(11664*f^3*((c^2*d^12*(c^2 + d^2)^2)/f^6)^(1/2) + d
^9*11664i + c^2*d^7*11664i))/(93312*d^6*f^3*(c^2 + d^2)^2))*(-(f^3*((4*(72
9*d^8 + 729*c^2*d^6)*(46656*d^10 + 93312*c^2*d^8 + 46656*c^4*d^6))/f^6 - (
11664*d^9 + 11664*c^2*d^7)^2/f^6)^(1/2) + d^9*11664i + c^2*d^7*11664i)/(93
312*f^3*(d^10 + 2*c^2*d^8 + c^4*d^6)))^(1/3) + log(d^5*f*(c + d*tan(e + f*
x))^(1/3)*243i + ((1944*d^4*f^4*(c^2 - d^2)*(c + d*tan(e + f*x))^(1/3) - 3
*2^(1/3)*c*d^4*f^6*(c^2 + d^2)*(-(d^9*11664i - 11664*f^3*((c^2*d^12*(c^2 +
 d^2)^2)/f^6)^(1/2) + c^2*d^7*11664i)/(d^6*f^3*(c^2 + d^2)^2))^(2/3))*(d^9
*11664i - 11664*f^3*((c^2*d^12*(c^2 + d^2)^2)/f^6)^(1/2) + c^2*d^7*11664i)
)/(93312*d^6*f^3*(c^2 + d^2)^2))*(-(d^9*11664i - f^3*((4*(729*d^8 + 729*c^
2*d^6)*(46656*d^10 + 93312*c^2*d^8 + 46656*c^4*d^6))/f^6 - (11664*d^9 + 11
664*c^2*d^7)^2/f^6)^(1/2) + c^2*d^7*11664i)/(93312*f^3*(d^10 + 2*c^2*d^8 +
 c^4*d^6)))^(1/3) + (log(- ((-1/(f^3*(c - d*1i)))^(2/3)*(((-1/(f^3*(c - d*
1i)))^(1/3)*((1944*d^4*(c^2 - d^2)*(c + d*tan(e + f*x))^(1/3))/f^2 - 1944*
c*d^4*(-1/(f^3*(c - d*1i)))^(2/3)*(c^2 + d^2)))/2 - (972*d^4*(c^2 + d^2))/
f^3))/4 - (243*c*d^4*(c + d*tan(e + f*x))^(1/3))/f^5)*(-1/(c*f^3 - d*f^3*1
i))^(1/3))/2 + log(((7776*c*d^4*(c^2 + d^2)*(-1i/(8*f^3*(c*1i - d)))^(2...
```

Reduce [F]

$$\int \frac{i-\tan(e+fx)}{\sqrt[3]{c+d\tan(e+fx)}}\,dx = -\left(\int \frac{\tan(fx+e)}{(d\tan(fx+e)+c)^{\frac{1}{3}}}dx\right) + \left(\int \frac{1}{(d\tan(fx+e)+c)^{\frac{1}{3}}}dx\right)i$$

input
```
int((I-tan(f*x+e))/(c+d*tan(f*x+e))^(1/3),x)
```

output
```
 - int(tan(e + f*x)/(tan(e + f*x)*d + c)**(1/3),x) + int(1/(tan(e + f*x)*d
 + c)**(1/3),x)*i
```

3.478 $\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 299

$$\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx = -\frac{1}{4}i\sqrt[3]{c-id}x+\frac{1}{4}i\sqrt[3]{c+id}x+\frac{\sqrt{3}\sqrt[3]{c-id}\arctan\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)}{2f}+\frac{\sqrt{3}\sqrt[3]{c+id}\arctan\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c+id}}}{\sqrt{3}}\right)}{2f}-\frac{\sqrt[3]{c-id}\log(\cos(e+fx))}{4f}-\frac{\sqrt[3]{c+id}\log(\cos(e+fx))}{4f}-\frac{3\sqrt[3]{c-id}\log\left(\sqrt[3]{c-id}-\sqrt[3]{c+d\tan(e+fx)}\right)}{4f}-\frac{3\sqrt[3]{c+id}\log\left(\sqrt[3]{c+id}-\sqrt[3]{c+d\tan(e+fx)}\right)}{4f}$$

output
```
-1/4*I*(c-I*d)^(1/3)*x+1/4*I*(c+I*d)^(1/3)*x+1/2*3^(1/2)*(c-I*d)^(1/3)*arc
tan(1/3*(1+2*(c+d*tan(f*x+e))^(1/3)/(c-I*d)^(1/3))*3^(1/2))/f+1/2*3^(1/2)*
(c+I*d)^(1/3)*arctan(1/3*(1+2*(c+d*tan(f*x+e))^(1/3)/(c+I*d)^(1/3))*3^(1/2
))/f-1/4*(c-I*d)^(1/3)*ln(cos(f*x+e))/f-1/4*(c+I*d)^(1/3)*ln(cos(f*x+e))/f
-3/4*(c-I*d)^(1/3)*ln((c-I*d)^(1/3)-(c+d*tan(f*x+e))^(1/3))/f-3/4*(c+I*d)^
(1/3)*ln((c+I*d)^(1/3)-(c+d*tan(f*x+e))^(1/3))/f
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.10

$$\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx = \frac{2\sqrt{3}\sqrt[3]{c-id}\arctan\left(\frac{1+\frac{2\sqrt[3]{c+d\tan(e+fx)}}{\sqrt[3]{c-id}}}{\sqrt{3}}\right)+2\sqrt{3}\sqrt[3]{c+id}\arctan\left(\frac{1+\cdots}{\cdots}\right.}{}$$

input
```
Integrate[(d - c*Tan[e + f*x])/(c + d*Tan[e + f*x])^(2/3),x]
```

output
```
(2*Sqrt[3]*(c - I*d)^(1/3)*ArcTan[(1 + (2*(c + d*Tan[e + f*x])^(1/3))/(c -
 I*d)^(1/3))/Sqrt[3]] + 2*Sqrt[3]*(c + I*d)^(1/3)*ArcTan[(1 + (2*(c + d*Ta
n[e + f*x])^(1/3))/(c + I*d)^(1/3))/Sqrt[3]] - 2*(c - I*d)^(1/3)*Log[(c -
I*d)^(1/3) - (c + d*Tan[e + f*x])^(1/3)] - 2*(c + I*d)^(1/3)*Log[(c + I*d)
^(1/3) - (c + d*Tan[e + f*x])^(1/3)] + (c - I*d)^(1/3)*Log[(c - I*d)^(2/3)
 + (c - I*d)^(1/3)*(c + d*Tan[e + f*x])^(1/3) + (c + d*Tan[e + f*x])^(2/3)
] + (c + I*d)^(1/3)*Log[(c + I*d)^(2/3) + (c + I*d)^(1/3)*(c + d*Tan[e + f
*x])^(1/3) + (c + d*Tan[e + f*x])^(2/3)])/(4*f)
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.79, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {3042, 4022, 3042, 4020, 25, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(d+ic)\int \frac{i\tan(e+fx)+1}{(c+d\tan(e+fx))^{2/3}}dx-\frac{1}{2}(-d+ic)\int \frac{1-i\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(d+ic)\int \frac{i\tan(e+fx)+1}{(c+d\tan(e+fx))^{2/3}}dx-\frac{1}{2}(-d+ic)\int \frac{1-i\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(d+ic)\int -\frac{1}{(1-i\tan(e+fx))(c+d\tan(e+fx))^{2/3}}d(i\tan(e+fx))}{2f}+\frac{i(-d+ic)\int -\frac{1}{(i\tan(e+fx)+1)(c+d\tan(e+fx))^{2/3}}d(-i\tan(e+fx))}{2f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{i(d+ic)\int \frac{1}{(1-i\tan(e+fx))(c+d\tan(e+fx))^{2/3}}d(i\tan(e+fx))}{2f}-\frac{i(-d+ic)\int \frac{1}{(i\tan(e+fx)+1)(c+d\tan(e+fx))^{2/3}}d(-i\tan(e+fx))}{2f}$$

$$\Big\downarrow \text{69}$$

$$\frac{i(d+ic)\left(-\frac{3\int \frac{1}{-\tan^2(e+fx)+(c-id)^{2/3}+\sqrt[3]{c-id}\sqrt[3]{c+d\tan(e+fx)}}d\sqrt[3]{c+d\tan(e+fx)}}{2\sqrt[3]{c-id}}-\frac{3\int \frac{1}{\sqrt[3]{c-id}-i\tan(e+fx)}d\sqrt[3]{\cdots}}{2(c-id)^{2\cdots}}\cdots\right.}{2f}\cdots$$

$$\frac{i(-d+ic)\left(-\frac{3\int \frac{1}{-\tan^2(e+fx)+(c+id)^{2/3}+\sqrt[3]{c+id}\sqrt[3]{c+d\tan(e+fx)}}d\sqrt[3]{c+d\tan(e+fx)}}{2\sqrt[3]{c+id}}-\frac{3\int \frac{1}{i\tan(e+fx)+\sqrt[3]{c+id}}d\cdots}{2(c+i\cdots}\cdots\right.}{2f}\cdots$$

$$\Big\downarrow \text{16}$$

$$\frac{i(d+ic)\left(-\frac{3\int \frac{1}{-\tan^2(e+fx)+(c-id)^{2/3}+\sqrt[3]{c-id}\sqrt[3]{c+d\tan(e+fx)}}d\sqrt[3]{c+d\tan(e+fx)}}{2\sqrt[3]{c-id}}-\frac{\log(1-i\tan(e+fx))}{2(c-id)^{2/3}}+\frac{3\log\left(\right.}{}\right.}{2f}$$

$$\frac{i(-d+ic)\left(-\frac{3\int \frac{1}{-\tan^2(e+fx)+(c+id)^{2/3}+\sqrt[3]{c+id}\sqrt[3]{c+d\tan(e+fx)}}d\sqrt[3]{c+d\tan(e+fx)}}{2\sqrt[3]{c+id}}-\frac{\log(1+i\tan(e+fx))}{2(c+id)^{2/3}}+\frac{3\log}{}\right.}{2f}$$

$\Big\downarrow$ 1082

$$\frac{i(d+ic)\left(\frac{3\int \frac{1}{\tan^2(e+fx)-3}d\left(\frac{2i\tan(e+fx)}{\sqrt[3]{c-id}}+1\right)}{(c-id)^{2/3}}-\frac{\log(1-i\tan(e+fx))}{2(c-id)^{2/3}}+\frac{3\log\left(\sqrt[3]{c-id}-i\tan(e+fx)\right)}{2(c-id)^{2/3}}\right)}{2f}+\frac{i(-d+ic)\left(\frac{3\int \frac{1}{\tan^2(e+fx)-3}d\left(1-\frac{2i\tan(e+fx)}{\sqrt[3]{c+id}}\right)}{(c+id)^{2/3}}-\frac{\log(1+i\tan(e+fx))}{2(c+id)^{2/3}}+\frac{3\log\left(\sqrt[3]{c+id}+i\tan(e+fx)\right)}{2(c+id)^{2/3}}\right)}{2f}$$

$\Big\downarrow$ 217

$$\frac{i(d+ic)\left(-\frac{i\sqrt{3}\text{arctanh}\left(\frac{\tan(e+fx)}{\sqrt{3}}\right)}{(c-id)^{2/3}}-\frac{\log(1-i\tan(e+fx))}{2(c-id)^{2/3}}+\frac{3\log\left(\sqrt[3]{c-id}-i\tan(e+fx)\right)}{2(c-id)^{2/3}}\right)}{2f}+\frac{i(-d+ic)\left(\frac{i\sqrt{3}\text{arctanh}\left(\frac{\tan(e+fx)}{\sqrt{3}}\right)}{(c+id)^{2/3}}-\frac{\log(1+i\tan(e+fx))}{2(c+id)^{2/3}}+\frac{3\log\left(\sqrt[3]{c+id}+i\tan(e+fx)\right)}{2(c+id)^{2/3}}\right)}{2f}$$

input
```
Int[(d - c*Tan[e + f*x])/(c + d*Tan[e + f*x])^(2/3),x]
```

output
```
((I/2)*(I*c + d)*(((-I)*Sqrt[3]*ArcTanh[Tan[e + f*x]/Sqrt[3]])/(c - I*d)^(
2/3) - Log[1 - I*Tan[e + f*x]]/(2*(c - I*d)^(2/3)) + (3*Log[(c - I*d)^(1/3
) - I*Tan[e + f*x]])/(2*(c - I*d)^(2/3))))/f + ((I/2)*(I*c - d)*((I*Sqrt[3
]*ArcTanh[Tan[e + f*x]/Sqrt[3]])/(c + I*d)^(2/3) - Log[1 + I*Tan[e + f*x]]
/(2*(c + I*d)^(2/3)) + (3*Log[(c + I*d)^(1/3) + I*Tan[e + f*x]])/(2*(c + I
*d)^(2/3))))/f
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.85 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.24

method	result
derivativedivides	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2c\_Z^{3}+c^{2}+d^{2}\right)}\frac{\left(\_R^{3}c-c^{2}-d^{2}\right)\ln\left(\left(c+d\tan\left(fx+e\right)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}c}}{2f}$
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2c\_Z^{3}+c^{2}+d^{2}\right)}\frac{\left(\_R^{3}c-c^{2}-d^{2}\right)\ln\left(\left(c+d\tan\left(fx+e\right)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{5}-\_R^{2}c}}{2f}$
parts	$\frac{d^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2c\_Z^{3}+c^{2}+d^{2}\right)}\frac{\ln\left(\left(c+d\tan\left(fx+e\right)\right)^{\frac{1}{3}}-\_R\right)}{\_R^{2}\left(\_R^{3}-c\right)}\right)}{2f}-\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-2c\_Z^{3}+c^{2}+d^{2}\right)}\frac{\ln\left((c\right.}{}\right.}{2f}$

input
```
int((d-c*tan(f*x+e))/(c+d*tan(f*x+e))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*sum((_R^3*c-c^2-d^2)/(_R^5-_R^2*c)*ln((c+d*tan(f*x+e))^(1/3)-_R),_R
=RootOf(_Z^6-2*_Z^3*c+c^2+d^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(217) = 434$.

Time = 0.09 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.48

$$\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx =$$

$$-\frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{f^3\sqrt{-\frac{d^2}{f^6}}+c}{f^3}\right)^{\frac{1}{3}}\log\left(-\frac{1}{2}\left(\sqrt{-3}f+f\right)\left(-\frac{f^3\sqrt{-\frac{d^2}{f^6}}+c}{f^3}\right)^{\frac{1}{3}}\right.$$

$$\left.+\left(d\tan\left(fx+e\right)+c\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{f^3\sqrt{-\frac{d^2}{f^6}}+c}{f^3}\right)^{\frac{1}{3}}\log\left(\frac{1}{2}\left(\sqrt{-3}f-f\right)\left(-\frac{f^3\sqrt{-\frac{d^2}{f^6}}+c}{f^3}\right)^{\frac{1}{3}}\right.$$

$$\left.+\left(d\tan\left(fx+e\right)+c\right)^{\frac{1}{3}}\right)$$

$$-\frac{1}{4}\left(\sqrt{-3}+1\right)\left(\frac{f^3\sqrt{-\frac{d^2}{f^6}}-c}{f^3}\right)^{\frac{1}{3}}\log\left(-\frac{1}{2}\left(\sqrt{-3}f+f\right)\left(\frac{f^3\sqrt{-\frac{d^2}{f^6}}-c}{f^3}\right)^{\frac{1}{3}}\right.$$

$$\left.+\left(d\tan\left(fx+e\right)+c\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{4}\left(\sqrt{-3}-1\right)\left(\frac{f^3\sqrt{-\frac{d^2}{f^6}}-c}{f^3}\right)^{\frac{1}{3}}\log\left(\frac{1}{2}\left(\sqrt{-3}f-f\right)\left(\frac{f^3\sqrt{-\frac{d^2}{f^6}}-c}{f^3}\right)^{\frac{1}{3}}\right.$$

$$\left.+\left(d\tan\left(fx+e\right)+c\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{2}\left(-\frac{f^3\sqrt{-\frac{d^2}{f^6}}+c}{f^3}\right)^{\frac{1}{3}}\log\left(f\left(-\frac{f^3\sqrt{-\frac{d^2}{f^6}}+c}{f^3}\right)^{\frac{1}{3}}+\left(d\tan\left(fx+e\right)+c\right)^{\frac{1}{3}}\right)$$

$$+\frac{1}{2}\left(\frac{f^3\sqrt{-\frac{d^2}{f^6}}-c}{f^3}\right)^{\frac{1}{3}}\log\left(f\left(\frac{f^3\sqrt{-\frac{d^2}{f^6}}-c}{f^3}\right)^{\frac{1}{3}}+\left(d\tan\left(fx+e\right)+c\right)^{\frac{1}{3}}\right)$$

input
```
integrate((d-c*tan(f*x+e))/(c+d*tan(f*x+e))^(2/3),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-3) + 1)*(-(f^3*sqrt(-d^2/f^6) + c)/f^3)^(1/3)*log(-1/2*(sqrt(-
3)*f + f)*(-(f^3*sqrt(-d^2/f^6) + c)/f^3)^(1/3) + (d*tan(f*x + e) + c)^(1/
3)) + 1/4*(sqrt(-3) - 1)*(-(f^3*sqrt(-d^2/f^6) + c)/f^3)^(1/3)*log(1/2*(sq
rt(-3)*f - f)*(-(f^3*sqrt(-d^2/f^6) + c)/f^3)^(1/3) + (d*tan(f*x + e) + c)
^(1/3)) - 1/4*(sqrt(-3) + 1)*((f^3*sqrt(-d^2/f^6) - c)/f^3)^(1/3)*log(-1/2
*(sqrt(-3)*f + f)*((f^3*sqrt(-d^2/f^6) - c)/f^3)^(1/3) + (d*tan(f*x + e) +
 c)^(1/3)) + 1/4*(sqrt(-3) - 1)*((f^3*sqrt(-d^2/f^6) - c)/f^3)^(1/3)*log(1
/2*(sqrt(-3)*f - f)*((f^3*sqrt(-d^2/f^6) - c)/f^3)^(1/3) + (d*tan(f*x + e)
 + c)^(1/3)) + 1/2*(-(f^3*sqrt(-d^2/f^6) + c)/f^3)^(1/3)*log(f*(-(f^3*sqrt
(-d^2/f^6) + c)/f^3)^(1/3) + (d*tan(f*x + e) + c)^(1/3)) + 1/2*((f^3*sqrt(
-d^2/f^6) - c)/f^3)^(1/3)*log(f*((f^3*sqrt(-d^2/f^6) - c)/f^3)^(1/3) + (d*
tan(f*x + e) + c)^(1/3))
```

Sympy [F]

$$\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx = -\int \left(-\frac{d}{(c+d\tan(e+fx))^{\frac{2}{3}}}\right)dx - \int \frac{c\tan(e+fx)}{(c+d\tan(e+fx))^{\frac{2}{3}}}\,dx$$

input
```
integrate((d-c*tan(f*x+e))/(c+d*tan(f*x+e))**(2/3),x)
```

output
```
-Integral(-d/(c + d*tan(e + f*x))**(2/3), x) - Integral(c*tan(e + f*x)/(c
+ d*tan(e + f*x))**(2/3), x)
```

Maxima [F]

$$\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx = \int -\frac{c\tan{(fx+e)}-d}{(d\tan{(fx+e)}+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d-c*tan(f*x+e))/(c+d*tan(f*x+e))^(2/3),x, algorithm="maxima")
```

output
```
-integrate((c*tan(f*x + e) - d)/(d*tan(f*x + e) + c)^(2/3), x)
```

Giac [F]

$$\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx = \int -\frac{c\tan{(fx+e)}-d}{(d\tan{(fx+e)}+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d-c*tan(f*x+e))/(c+d*tan(f*x+e))^(2/3),x, algorithm="giac")
```

output
```
undef
```

Mupad [B] (verification not implemented)

Time = 17.80 (sec) , antiderivative size = 4308, normalized size of antiderivative = 14.41

$$\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx = \text{Too large to display}$$

input
```
int((d - c*tan(e + f*x))/(c + d*tan(e + f*x))^(2/3),x)
```

output
```
log(((((1944*c*d^4*(c^2 + d^2)*((8*(-d^6*f^6*(c^2 - d^2)^2)^(1/2) + 16*c*d
^4*f^3)/(f^6*(c^2 + d^2)^2))^(1/3) + (7776*c*d^6*(c + d*tan(e + f*x))^(1/3
))/f)*((8*(-d^6*f^6*(c^2 - d^2)^2)^(1/2) + 16*c*d^4*f^3)/(f^6*(c^2 + d^2)^
2))^(2/3))/16 - (972*d^8)/f^3)*((8*(-d^6*f^6*(c^2 - d^2)^2)^(1/2) + 16*c*d
^4*f^3)/(f^6*(c^2 + d^2)^2))^(1/3))/4 - (486*d^8*(c + d*tan(e + f*x))^(1/3
))/f^4)*(((256*c^2*d^8*f^6 - d^6*(64*c^4*f^6 + 64*d^4*f^6 + 128*c^2*d^2*f^
6))^(1/2) + 16*c*d^4*f^3)/(64*(c^4*f^6 + d^4*f^6 + 2*c^2*d^2*f^6)))^(1/3)
+ log(((((1944*c*d^4*(c^2 + d^2)*(-(8*(-d^6*f^6*(c^2 - d^2)^2)^(1/2) - 16*
c*d^4*f^3)/(f^6*(c^2 + d^2)^2))^(1/3) + (7776*c*d^6*(c + d*tan(e + f*x))^(
1/3))/f)*(-(8*(-d^6*f^6*(c^2 - d^2)^2)^(1/2) - 16*c*d^4*f^3)/(f^6*(c^2 + d
^2)^2))^(2/3))/16 - (972*d^8)/f^3)*(-(8*(-d^6*f^6*(c^2 - d^2)^2)^(1/2) - 1
6*c*d^4*f^3)/(f^6*(c^2 + d^2)^2))^(1/3))/4 - (486*d^8*(c + d*tan(e + f*x))
^(1/3))/f^4)*(-((256*c^2*d^8*f^6 - d^6*(64*c^4*f^6 + 64*d^4*f^6 + 128*c^2*
d^2*f^6))^(1/2) - 16*c*d^4*f^3)/(64*(c^4*f^6 + d^4*f^6 + 2*c^2*d^2*f^6)))^
(1/3) + log(- (486*c^4*d^4*(c + d*tan(e + f*x))^(1/3))/f^4 - (((16*(-c^8*d
^2*f^6)^(1/2) - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))^(1/3)*(194
4*c*d^4*(-c^8*d^2*f^6)^(1/2) + 1944*c^4*d^6*f^3 + 243*c^5*d^4*f^5*(c + d*t
an(e + f*x))^(1/3)*((16*(-c^8*d^2*f^6)^(1/2) - 8*c^5*f^3 + 8*c^3*d^2*f^3)/
(f^6*(c^2 + d^2)^2))^(2/3) - 243*c*d^8*f^5*(c + d*tan(e + f*x))^(1/3)*((16
*(-c^8*d^2*f^6)^(1/2) - 8*c^5*f^3 + 8*c^3*d^2*f^3)/(f^6*(c^2 + d^2)^2))...
```

Reduce [F]

$$\int \frac{d-c\tan(e+fx)}{(c+d\tan(e+fx))^{2/3}}\,dx = -\left(\int \frac{\tan(fx+e)}{(d\tan(fx+e)+c)^{\frac{2}{3}}}dx\right)c+\left(\int \frac{1}{(d\tan(fx+e)+c)^{\frac{2}{3}}}dx\right)d$$

input
```
int((d-c*tan(f*x+e))/(c+d*tan(f*x+e))^(2/3),x)
```

output
```
 - int(tan(e + f*x)/(tan(e + f*x)*d + c)**(2/3),x)*c + int(1/(tan(e + f*x)
*d + c)**(2/3),x)*d
```

3.479 $\int \tan^m(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 403

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx =$$

$$-\frac{b(Ab^3(12+7m+m^2)+4ab^2B(12+7m+m^2)-2a^3B(19+8m+m^2)-a^2Ab(68+37m+5m^2))\,\mathrm{ta}}{d(1+m)(3+m)(4+m)}$$

$$+\frac{(a^4A-6a^2Ab^2+Ab^4-4a^3bB+4ab^3B)\,\mathrm{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c}{d(1+m)}$$

$$+\frac{b^2(2aAb(4+m)^2-b^2B(12+7m+m^2)+a^2B(26+9m+m^2))\tan^{2+m}(c+dx)}{d(2+m)(3+m)(4+m)}$$

$$+\frac{(4a^3Ab-4aAb^3+a^4B-6a^2b^2B+b^4B)\,\mathrm{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c}{d(2+m)}$$

$$+\frac{b(Ab(4+m)+aB(7+m))\tan^{1+m}(c+dx)(a+b\tan(c+dx))^2}{d(3+m)(4+m)}$$

$$+\frac{bB\tan^{1+m}(c+dx)(a+b\tan(c+dx))^3}{d(4+m)}$$

output
```
-b*(A*b^3*(m^2+7*m+12)+4*a*b^2*B*(m^2+7*m+12)-2*a^3*B*(m^2+8*m+19)-a^2*A*b
*(5*m^2+37*m+68))*tan(d*x+c)^(1+m)/d/(1+m)/(3+m)/(4+m)+(A*a^4-6*A*a^2*b^2+
A*b^4-4*B*a^3*b+4*B*a*b^3)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c
)^2)*tan(d*x+c)^(1+m)/d/(1+m)+b^2*(2*a*A*b*(4+m)^2-b^2*B*(m^2+7*m+12)+a^2*
B*(m^2+9*m+26))*tan(d*x+c)^(2+m)/d/(2+m)/(3+m)/(4+m)+(4*A*a^3*b-4*A*a*b^3+
B*a^4-6*B*a^2*b^2+B*b^4)*hypergeom([1, 1+1/2*m],[2+1/2*m],-tan(d*x+c)^2)*t
an(d*x+c)^(2+m)/d/(2+m)+b*(A*b*(4+m)+a*B*(7+m))*tan(d*x+c)^(1+m)*(a+b*tan(
d*x+c))^2/d/(3+m)/(4+m)+b*B*tan(d*x+c)^(1+m)*(a+b*tan(d*x+c))^3/d/(4+m)
```

Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.88

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan^{1+m}(c+dx)\left(-b(2+m)\left(Ab^3(12+7m+m^2)+4ab^2B(12+7m+m^2)-2a^3B(19+8m+m^2)-a\right.\right.}{}$$

input
```
Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(Tan[c + d*x]^(1 + m)*(-(b*(2 + m)*(A*b^3*(12 + 7*m + m^2) + 4*a*b^2*B*(12
 + 7*m + m^2) - 2*a^3*B*(19 + 8*m + m^2) - a^2*A*b*(68 + 37*m + 5*m^2))) +
 (a^4*A - 6*a^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*(2 + m)*(3 + m)*(4
+ m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2] + b^2*(1
+ m)*(2*a*A*b*(4 + m)^2 - b^2*B*(12 + 7*m + m^2) + a^2*B*(26 + 9*m + m^2))
*Tan[c + d*x] + (4*a^3*A*b - 4*a*A*b^3 + a^4*B - 6*a^2*b^2*B + b^4*B)*(1 +
 m)*(3 + m)*(4 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*
x]^2]*Tan[c + d*x] + b*(1 + m)*(2 + m)*(A*b*(4 + m) + a*B*(7 + m))*(a + b*
Tan[c + d*x])^2 + b*B*(1 + m)*(2 + m)*(3 + m)*(a + b*Tan[c + d*x])^3))/(d*
(1 + m)*(2 + m)*(3 + m)*(4 + m))
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {3042, 4090, 25, 3042, 4130, 3042, 4120, 25, 3042, 4113, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m(a+b\tan(c+dx))^4(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -\tan^m(c+dx)(a+b\tan(c+dx))^2\left(-b(Ab(m+4)+aB(m+7))\tan^2(c+dx)-\left(Ba^2+2Aba-b^2B\right)(m+}{m+4}$$
$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)}$$

$$\Big\downarrow \text{25}$$

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)}-$$
$$\frac{\int \tan^m(c+dx)(a+b\tan(c+dx))^2\left(-b(Ab(m+4)+aB(m+7))\tan^2(c+dx)-\left(Ba^2+2Aba-b^2B\right)(m+4)\right.}{m+4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)}-$$
$$\frac{\int \tan(c+dx)^m(a+b\tan(c+dx))^2\left(-b(Ab(m+4)+aB(m+7))\tan(c+dx)^2-\left(Ba^2+2Aba-b^2B\right)(m+4)\right.}{m+4}$$

$$\Big\downarrow \text{4130}$$

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)}-$$
$$\frac{\frac{\int \tan^m(c+dx)(a+b\tan(c+dx))\left(-b\left(B\left(m^2+9m+26\right)a^2+2Ab(m+4)^2a-b^2B\left(m^2+7m+12\right)\right)\tan^2(c+dx)-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)(m+3)(m\right.}{m+3}}{}$$

$$\Big\downarrow \text{3042}$$

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)} -$$

$$\frac{\int \tan(c+dx)^m(a+b\tan(c+dx))\left(-b\left(B\left(m^2+9m+26\right)a^2+2Ab(m+4)^2a-b^2B\left(m^2+7m+12\right)\right)\tan(c+dx)^2-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)(m+3)(m}{m+3}$$

$\downarrow$ 4120

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)} -$$

$$-\frac{\int -\tan^m(c+dx)\Big(((m+2)(a(m+3)(bB(m+1)-aA(m+4))+b(m+1)(Ab(m+4)+aB(m+7)))a^2+b(m+2)\Big(-2B\Big(m^2+8m+19\Big)a^3-Ab\Big(5m^2+37m+68\Big)a^2+4b^2B\Big(m}{m+2}$$

$\downarrow$ 25

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)} -$$

$$\frac{\int \tan^m(c+dx)\Big(((m+2)(a(m+3)(bB(m+1)-aA(m+4))+b(m+1)(Ab(m+4)+aB(m+7)))a^2+b(m+2)\Big(-2B\Big(m^2+8m+19\Big)a^3-Ab\Big(5m^2+37m+68\Big)a^2+4b^2B\Big(m^2+7}{m+2}$$

$\downarrow$ 3042

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)} -$$

$$\frac{\int \tan(c+dx)^m\Big(((m+2)(a(m+3)(bB(m+1)-aA(m+4))+b(m+1)(Ab(m+4)+aB(m+7)))a^2+b(m+2)\Big(-2B\Big(m^2+8m+19\Big)a^3-Ab\Big(5m^2+37m+68\Big)a^2+4b^2B\Big(m^2+7}{m+2}$$

$\downarrow$ 4113

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)} -$$

$$\frac{\int \tan^m(c+dx)\Big(-\Big(\Big(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\Big)(m+2)(m+3)(m+4)\Big)-\Big(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\Big)(m+2)(m+3)\tan(c+dx)(m+4)\Big)dx+\frac{b(m}{}}{m+2}$$

$\downarrow$ 3042

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)} -$$

$$\frac{\int \tan(c+dx)^m\Big(-\Big(\Big(Aa^4-4bBa^3-6Ab^2a^2+4b^3Ba+Ab^4\Big)(m+2)(m+3)(m+4)\Big)-\Big(Ba^4+4Aba^3-6b^2Ba^2-4Ab^3a+b^4B\Big)(m+2)(m+3)\tan(c+dx)(m+4)\Big)dx+\frac{b(m}{}}{m+2}$$

$\downarrow$ 4021

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)}-$$

$$\frac{\frac{-(m+2)(m+3)(m+4)\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\int\tan^{m+1}(c+dx)dx-(m+2)(m+3)(m+4)\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)\int\tan^m(c+dx)dx+}{m+2}}{}$$

$\downarrow$ 3042

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)}-$$

$$\frac{\frac{-(m+2)(m+3)(m+4)\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)\int\tan(c+dx)^m dx-(m+2)(m+3)(m+4)\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\int\tan(c+dx)^{m+1}dx+}{m+2}}{}$$

$\downarrow$ 3957

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)}-$$

$$\frac{\frac{-\frac{(m+2)(m+3)(m+4)\left(a^4B+4a^3Ab-6a^2b^2B-4aAb^3+b^4B\right)\int\frac{\tan^{m+1}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{(m+2)(m+3)(m+4)\left(a^4A-4a^3bB-6a^2Ab^2+4ab^3B+Ab^4\right)\int\frac{\tan^m}{\tan^2(}}{d}}{m+2}}{}$$

$\downarrow$ 278

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^3}{d(m+4)}-$$

$$\frac{\frac{b(m+2)\left(-2a^3B\left(m^2+8m+19\right)-a^2Ab\left(5m^2+37m+68\right)+4ab^2B\left(m^2+7m+12\right)+Ab^3\left(m^2+7m+12\right)\right)\tan^{m+1}(c+dx)}{d(m+1)}-\frac{(m+2)(m+3)(m+4)\left(a^4A-4a^3bB-6a^2Ab\right.}{}}{}$$

input

```
Int[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^4*(A + B*Tan[c + d*x]),x]
```

output
```
(b*B*Tan[c + d*x]^(1 + m)*(a + b*Tan[c + d*x])^3)/(d*(4 + m)) - (-((b*(A*b
*(4 + m) + a*B*(7 + m))*Tan[c + d*x]^(1 + m)*(a + b*Tan[c + d*x])^2)/(d*(3
 + m))) + (-((b^2*(2*a*A*b*(4 + m)^2 - b^2*B*(12 + 7*m + m^2) + a^2*B*(26
+ 9*m + m^2))*Tan[c + d*x]^(2 + m))/(d*(2 + m))) + ((b*(2 + m)*(A*b^3*(12
+ 7*m + m^2) + 4*a*b^2*B*(12 + 7*m + m^2) - 2*a^3*B*(19 + 8*m + m^2) - a^2
*A*b*(68 + 37*m + 5*m^2))*Tan[c + d*x]^(1 + m))/(d*(1 + m)) - ((a^4*A - 6*
a^2*A*b^2 + A*b^4 - 4*a^3*b*B + 4*a*b^3*B)*(2 + m)*(3 + m)*(4 + m)*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(1 + m))
/(d*(1 + m)) - ((4*a^3*A*b - 4*a*A*b^3 + a^4*B - 6*a^2*b^2*B + b^4*B)*(3 +
 m)*(4 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Ta
n[c + d*x]^(2 + m))/d)/(2 + m))/(3 + m))/(4 + m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [F]

$$\int \tan (dx+c)^m (a+b\tan (dx+c))^4 (A+B\tan (dx+c))\, dx$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\, dx = \int (B\tan (dx+c)+A)(b\tan (dx+c)+a)^4 \tan (dx+c)^m\, dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*b^4*tan(d*x + c)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*tan(d*x + c)^
4 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*tan(d*x + c)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^
2)*tan(d*x + c)^2 + (B*a^4 + 4*A*a^3*b)*tan(d*x + c))*tan(d*x + c)^m, x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\, dx = \int (A+B\tan (c+dx)) (a+b\tan (c+dx))^4 \tan^m (c+dx)\, dx$$

input
```
integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))**4*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**4*tan(c + d*x)**m, x)
```

Maxima [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^4\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^4*tan(d*x + c)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,5]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^4dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4,x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^4, x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^4(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^4*(A+B*tan(d*x+c)),x)
```

output
```
(tan(c + d*x)**m*tan(c + d*x)**4*b**5*m**2 + 2*tan(c + d*x)**m*tan(c + d*x
)**4*b**5*m + 10*tan(c + d*x)**m*tan(c + d*x)**2*a**2*b**3*m**2 + 40*tan(c
 + d*x)**m*tan(c + d*x)**2*a**2*b**3*m - tan(c + d*x)**m*tan(c + d*x)**2*b
**5*m**2 - 4*tan(c + d*x)**m*tan(c + d*x)**2*b**5*m + 5*tan(c + d*x)**m*a*
*4*b*m**2 + 30*tan(c + d*x)**m*a**4*b*m + 40*tan(c + d*x)**m*a**4*b - 10*t
an(c + d*x)**m*a**2*b**3*m**2 - 60*tan(c + d*x)**m*a**2*b**3*m - 80*tan(c
+ d*x)**m*a**2*b**3 + tan(c + d*x)**m*b**5*m**2 + 6*tan(c + d*x)**m*b**5*m
 + 8*tan(c + d*x)**m*b**5 + int(tan(c + d*x)**m,x)*a**5*d*m**3 + 6*int(tan
(c + d*x)**m,x)*a**5*d*m**2 + 8*int(tan(c + d*x)**m,x)*a**5*d*m - 5*int(ta
n(c + d*x)**m/tan(c + d*x),x)*a**4*b*d*m**3 - 30*int(tan(c + d*x)**m/tan(c
 + d*x),x)*a**4*b*d*m**2 - 40*int(tan(c + d*x)**m/tan(c + d*x),x)*a**4*b*d
*m + 10*int(tan(c + d*x)**m/tan(c + d*x),x)*a**2*b**3*d*m**3 + 60*int(tan(
c + d*x)**m/tan(c + d*x),x)*a**2*b**3*d*m**2 + 80*int(tan(c + d*x)**m/tan(
c + d*x),x)*a**2*b**3*d*m - int(tan(c + d*x)**m/tan(c + d*x),x)*b**5*d*m**
3 - 6*int(tan(c + d*x)**m/tan(c + d*x),x)*b**5*d*m**2 - 8*int(tan(c + d*x)
**m/tan(c + d*x),x)*b**5*d*m + 5*int(tan(c + d*x)**m*tan(c + d*x)**4,x)*a*
b**4*d*m**3 + 30*int(tan(c + d*x)**m*tan(c + d*x)**4,x)*a*b**4*d*m**2 + 40
*int(tan(c + d*x)**m*tan(c + d*x)**4,x)*a*b**4*d*m + 10*int(tan(c + d*x)**
m*tan(c + d*x)**2,x)*a**3*b**2*d*m**3 + 60*int(tan(c + d*x)**m*tan(c + d*x
)**2,x)*a**3*b**2*d*m**2 + 80*int(tan(c + d*x)**m*tan(c + d*x)**2,x)*a*...
```

3.480 $\int \tan^m(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 267

$$\begin{aligned}\int \tan^m(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= \frac{b(3aAb(3+m)-b^2B(3+m)+2a^2B(4+m))\tan^{1+m}(c+dx)}{d(1+m)(3+m)}\\ &+\frac{(a^3A-3aAb^2-3a^2bB+b^3B)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}\\ &+\frac{b^2(Ab(3+m)+aB(5+m))\tan^{2+m}(c+dx)}{d(2+m)(3+m)}\\ &+\frac{(3a^2Ab-Ab^3+a^3B-3ab^2B)\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)}{d(2+m)}\\ &+\frac{bB\tan^{1+m}(c+dx)(a+b\tan(c+dx))^2}{d(3+m)}\end{aligned}$$

output
```
b*(3*a*A*b*(3+m)-b^2*B*(3+m)+2*a^2*B*(4+m))*tan(d*x+c)^(1+m)/d/(1+m)/(3+m)
+(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-t
an(d*x+c)^2)*tan(d*x+c)^(1+m)/d/(1+m)+b^2*(A*b*(3+m)+a*B*(5+m))*tan(d*x+c)
^(2+m)/d/(2+m)/(3+m)+(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*hypergeom([1, 1+1/2
*m],[2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(2+m)/d/(2+m)+b*B*tan(d*x+c)^(1+m)
*(a+b*tan(d*x+c))^2/d/(3+m)
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.87

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan^{1+m}(c+dx)\left(b(2+m)\left(3aAb(3+m)-b^2B(3+m)+2a^2B(4+m)\right)+\left(a^3A-3aAb^2-3a^2bB+b^3\right.\right.}{}$$

input
```
Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(Tan[c + d*x]^(1 + m)*(b*(2 + m)*(3*a*A*b*(3 + m) - b^2*B*(3 + m) + 2*a^2*
B*(4 + m)) + (a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*(2 + m)*(3 + m)*Hyper
geometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2] + b^2*(1 + m)*(A*b*
(3 + m) + a*B*(5 + m))*Tan[c + d*x] + (3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2
*B)*(1 + m)*(3 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*
x]^2]*Tan[c + d*x] + b*B*(1 + m)*(2 + m)*(a + b*Tan[c + d*x])^2))/(d*(1 +
m)*(2 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4090, 25, 3042, 4120, 25, 3042, 4113, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -\tan^m(c+dx)(a+b\tan(c+dx))\left(-b(Ab(m+3)+aB(m+5))\tan^2(c+dx)-\left(Ba^2+2Aba-b^2B\right)(m+3}{m+3}$$

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)}$$

$\downarrow$ 25

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)}-$$
$$\frac{\int \tan^m(c+dx)(a+b\tan(c+dx))\left(-b(Ab(m+3)+aB(m+5))\tan^2(c+dx)-\left(Ba^2+2Aba-b^2B\right)(m+3)\,t}{m+3}$$

$\downarrow$ 3042

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)}-$$
$$\frac{\int \tan(c+dx)^m(a+b\tan(c+dx))\left(-b(Ab(m+3)+aB(m+5))\tan(c+dx)^2-\left(Ba^2+2Aba-b^2B\right)(m+3)\,t}{m+3}$$

$\downarrow$ 4120

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)}-$$
$$\frac{-\frac{\int -\tan^m(c+dx)((m+2)(bB(m+1)-aA(m+3))a^2-b(m+2)(2B(m+4)a^2+3Ab(m+3)a-b^2B(m+3))\tan^2(c+dx)-(Ba^3+3Aba^2-3b^2Ba-Ab}{m+2}}{m+3}$$

$\downarrow$ 25

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)}-$$
$$\frac{\frac{\int \tan^m(c+dx)((m+2)(bB(m+1)-aA(m+3))a^2-b(m+2)(2B(m+4)a^2+3Ab(m+3)a-b^2B(m+3))\tan^2(c+dx)-(Ba^3+3Aba^2-3b^2Ba-Ab^3)(r}{m+2}}{m+3}$$

$\downarrow$ 3042

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)}-$$
$$\frac{\frac{\int \tan(c+dx)^m((m+2)(bB(m+1)-aA(m+3))a^2-b(m+2)(2B(m+4)a^2+3Ab(m+3)a-b^2B(m+3))\tan(c+dx)^2-(Ba^3+3Aba^2-3b^2Ba-Ab^3)(r}{m+2}}{m+3}$$

$\downarrow$ 4113

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)}-$$
$$\frac{\frac{\int \tan^m(c+dx)(-((Aa^3-3bBa^2-3Ab^2a+b^3B)(m+2)(m+3))-(Ba^3+3Aba^2-3b^2Ba-Ab^3)(m+2)\tan(c+dx)(m+3))dx-\frac{b(m+2)\left(2a^2B(m+4)+\right.}{}}{m+2}}{m+3}$$

$\downarrow$ 3042

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)} - \frac{\frac{\int \tan(c+dx)^m\left(-\left(\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)(m+2)(m+3)\right)-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)(m+2)\tan(c+dx)(m+3)\right)dx-\frac{b(m+2)\left(2a^2B(m+4)+\right.}{}}{m+2}}{m+3}$$

$\downarrow$ 4021

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)} - \frac{\frac{-(m+2)(m+3)\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \tan^{m+1}(c+dx)dx-(m+2)(m+3)\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\int \tan^m(c+dx)dx-\frac{b(m+2)\left(2a^2B(m\right.}{}}{m+2}}{m+3}$$

$\downarrow$ 3042

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)} - \frac{\frac{-(m+2)(m+3)\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\int \tan(c+dx)^m dx-(m+2)(m+3)\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \tan(c+dx)^{m+1}dx-\frac{b(m+2)\left(2a^2B(m\right.}{}}{m+2}}{m+3}$$

$\downarrow$ 3957

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)} - \frac{\frac{-\frac{(m+2)(m+3)\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)\int \frac{\tan^{m+1}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{(m+2)(m+3)\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\int \frac{\tan^m(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{b(m+2)}{}}{m+2}}{m+3}$$

$\downarrow$ 278

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))^2}{d(m+3)} - \frac{-\frac{b(m+2)\left(2a^2B(m+4)+3aAb(m+3)-b^2B(m+3)\right)\tan^{m+1}(c+dx)}{d(m+1)}-\frac{(m+2)(m+3)\left(a^3A-3a^2bB-3aAb^2+b^3B\right)\tan^{m+1}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+}{2}\right.}{d(m+1)}}{m+2}$$

input
```
Int[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(b*B*Tan[c + d*x]^(1 + m)*(a + b*Tan[c + d*x])^2)/(d*(3 + m)) - (-((b^2*(A
*b*(3 + m) + a*B*(5 + m))*Tan[c + d*x]^(2 + m))/(d*(2 + m))) + (-((b*(2 +
m)*(3*a*A*b*(3 + m) - b^2*B*(3 + m) + 2*a^2*B*(4 + m))*Tan[c + d*x]^(1 + m
))/(d*(1 + m))) - ((a^3*A - 3*a*A*b^2 - 3*a^2*b*B + b^3*B)*(2 + m)*(3 + m)
*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^
(1 + m))/(d*(1 + m)) - ((3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*(3 + m)*Hy
pergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2
+ m))/d)/(2 + m))/(3 + m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

Maple [F]

$$\int \tan(dx+c)^m (a+b\tan(dx+c))^3 (A+B\tan(dx+c))\,dx$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^3\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*b^3*tan(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*tan(d*x + c)^
3 + 3*(B*a^2*b + A*a*b^2)*tan(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*tan(d*x + c
))*tan(d*x + c)^m, x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan(c+dx))(a+b\tan(c+dx))^3\tan^m(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3*tan(c + d*x)**m, x)
```

Maxima [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^3\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3*tan(d*x + c)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,4]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \int \tan(c+d\,x)^m\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)^3\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3, x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{4\tan(dx+c)^m\tan(dx+c)^2\,ab^3m + 4\tan(dx+c)^m\,a^3bm + 8\tan(dx+c)^m\,a^3b - 4\tan(dx+c)^m\,ab^3r}{}$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(4*tan(c + d*x)**m*tan(c + d*x)**2*a*b**3*m + 4*tan(c + d*x)**m*a**3*b*m +
 8*tan(c + d*x)**m*a**3*b - 4*tan(c + d*x)**m*a*b**3*m - 8*tan(c + d*x)**m
*a*b**3 + int(tan(c + d*x)**m,x)*a**4*d*m**2 + 2*int(tan(c + d*x)**m,x)*a*
*4*d*m - 4*int(tan(c + d*x)**m/tan(c + d*x),x)*a**3*b*d*m**2 - 8*int(tan(c
 + d*x)**m/tan(c + d*x),x)*a**3*b*d*m + 4*int(tan(c + d*x)**m/tan(c + d*x)
,x)*a*b**3*d*m**2 + 8*int(tan(c + d*x)**m/tan(c + d*x),x)*a*b**3*d*m + int
(tan(c + d*x)**m*tan(c + d*x)**4,x)*b**4*d*m**2 + 2*int(tan(c + d*x)**m*ta
n(c + d*x)**4,x)*b**4*d*m + 6*int(tan(c + d*x)**m*tan(c + d*x)**2,x)*a**2*
b**2*d*m**2 + 12*int(tan(c + d*x)**m*tan(c + d*x)**2,x)*a**2*b**2*d*m)/(d*
m*(m + 2))
```

3.481 $\int \tan^m(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 194

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{b(Ab(2+m)+aB(3+m))\tan^{1+m}(c+dx)}{d(1+m)(2+m)} + \frac{(a^2A-Ab^2-2abB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)} + \frac{(2aAb+a^2B-b^2B)\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)}{d(2+m)} + \frac{bB\tan^{1+m}(c+dx)(a+b\tan(c+dx))}{d(2+m)}$$

output

```
b*(A*b*(2+m)+a*B*(3+m))*tan(d*x+c)^(1+m)/d/(1+m)/(2+m)+(A*a^2-A*b^2-2*B*a*
b)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/
(1+m)+(2*A*a*b+B*a^2-B*b^2)*hypergeom([1, 1+1/2*m],[2+1/2*m],-tan(d*x+c)^2
)*tan(d*x+c)^(2+m)/d/(2+m)+b*B*tan(d*x+c)^(1+m)*(a+b*tan(d*x+c))/d/(2+m)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.80

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan^{1+m}(c+dx)\left(\frac{b(Ab(2+m)+aB(3+m))}{1+m}+\frac{(a^2A-Ab^2-2abB)(2+m)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)}{1+m}+(2aAb\right.}{d(2}$$

input
```
Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(Tan[c + d*x]^(1 + m)*((b*(A*b*(2 + m) + a*B*(3 + m)))/(1 + m) + ((a^2*A -
 A*b^2 - 2*a*b*B)*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[
c + d*x]^2])/(1 + m) + (2*a*A*b + a^2*B - b^2*B)*Hypergeometric2F1[1, (2 +
 m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x] + b*B*(a + b*Tan[c + d*x])
))/(d*(2 + m))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 4090, 25, 3042, 4113, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m(a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -\tan^m(c+dx)\left(-b(Ab(m+2)+aB(m+3))\tan^2(c+dx)-\left(Ba^2+2Aba-b^2B\right)(m+2)\tan(c+dx)+a(b\right.}{m+2}$$

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))}{d(m+2)}$$

$\downarrow$ 25

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))}{d(m+2)}-$$
$$\frac{\int \tan^m(c+dx)\left(-b(Ab(m+2)+aB(m+3))\tan^2(c+dx)-\left(Ba^2+2Aba-b^2B\right)(m+2)\tan(c+dx)+a(bB\right.}{m+2}$$

$\downarrow$ 3042

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))}{d(m+2)}-$$
$$\frac{\int \tan(c+dx)^m\left(-b(Ab(m+2)+aB(m+3))\tan(c+dx)^2-\left(Ba^2+2Aba-b^2B\right)(m+2)\tan(c+dx)+a(bB\right.}{m+2}$$

$\downarrow$ 4113

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))}{d(m+2)}-$$
$$\frac{\int \tan^m(c+dx)\left(-\left(\left(Aa^2-2bBa-Ab^2\right)(m+2)\right)-\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)(m+2)\right)dx-\frac{b(aB(m+3)+}{}}{m+2}$$

$\downarrow$ 3042

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))}{d(m+2)}-$$
$$\frac{\int \tan(c+dx)^m\left(-\left(\left(Aa^2-2bBa-Ab^2\right)(m+2)\right)-\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)(m+2)\right)dx-\frac{b(aB(m+3)+}{}}{m+2}$$

$\downarrow$ 4021

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))}{d(m+2)}-$$
$$\frac{-(m+2)\left(a^2B+2aAb-b^2B\right)\int\tan^{m+1}(c+dx)dx-(m+2)\left(a^2A-2abB-Ab^2\right)\int\tan^m(c+dx)dx-\frac{b(aB(m}{}}{m+2}$$

$\downarrow$ 3042

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))}{d(m+2)}-$$
$$\frac{-(m+2)\left(a^2A-2abB-Ab^2\right)\int\tan(c+dx)^m dx-(m+2)\left(a^2B+2aAb-b^2B\right)\int\tan(c+dx)^{m+1}dx-\frac{b(aB(m}{}}{m+2}$$

$$\Big\downarrow \; 3957$$

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))}{d(m+2)} - \frac{-\frac{(m+2)(a^2B+2aAb-b^2B)\int\frac{\tan^{m+1}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d} - \frac{(m+2)(a^2A-2abB-Ab^2)\int\frac{\tan^m(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d} - \frac{b(aB(m+3)+Ab(m+2)}{d(m+1}}{m+2}$$

$$\Big\downarrow \; 278$$

$$\frac{bB\tan^{m+1}(c+dx)(a+b\tan(c+dx))}{d(m+2)} - \frac{-\frac{(m+2)(a^2A-2abB-Ab^2)\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)} - \frac{(a^2B+2aAb-b^2B)\tan^{m+2}(c+dx)\,\text{Hypergeor}}{d}}{m+2}$$

input
```
Int[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(b*B*Tan[c + d*x]^(1 + m)*(a + b*Tan[c + d*x]))/(d*(2 + m)) - (-((b*(A*b*(
2 + m) + a*B*(3 + m))*Tan[c + d*x]^(1 + m))/(d*(1 + m))) - ((a^2*A - A*b^2
 - 2*a*b*B)*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*
x]^2]*Tan[c + d*x]^(1 + m))/(d*(1 + m)) - ((2*a*A*b + a^2*B - b^2*B)*Hyper
geometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m
))/d)/(2 + m)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [F]

$$\int \tan(dx+c)^m (a+b\tan(dx+c))^2 (A+B\tan(dx+c))\,dx$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^2\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*b^2*tan(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*tan(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*tan(d*x + c))*tan(d*x + c)^m, x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \int (A+B\tan(c+dx))\,(a+b\tan(c+dx))^2\tan^m(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2*tan(c + d*x)**m, x)
```

Maxima [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^2\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^2*tan(d*x + c)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,3]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \int \tan(c+d\,x)^m\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)^2\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2, x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan(dx+c)^m\tan(dx+c)^2 b^3m+3\tan(dx+c)^m a^2bm+6\tan(dx+c)^m a^2b-\tan(dx+c)^m b^3m-2\,\mathrm{t}}{}$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(tan(c + d*x)**m*tan(c + d*x)**2*b**3*m + 3*tan(c + d*x)**m*a**2*b*m + 6*t
an(c + d*x)**m*a**2*b - tan(c + d*x)**m*b**3*m - 2*tan(c + d*x)**m*b**3 +
int(tan(c + d*x)**m,x)*a**3*d*m**2 + 2*int(tan(c + d*x)**m,x)*a**3*d*m - 3
*int(tan(c + d*x)**m/tan(c + d*x),x)*a**2*b*d*m**2 - 6*int(tan(c + d*x)**m
/tan(c + d*x),x)*a**2*b*d*m + int(tan(c + d*x)**m/tan(c + d*x),x)*b**3*d*m
**2 + 2*int(tan(c + d*x)**m/tan(c + d*x),x)*b**3*d*m + 3*int(tan(c + d*x)*
*m*tan(c + d*x)**2,x)*a*b**2*d*m**2 + 6*int(tan(c + d*x)**m*tan(c + d*x)**
2,x)*a*b**2*d*m)/(d*m*(m + 2))
```

3.482 $\int \tan^m(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 127

$$\begin{aligned}
&\int \tan^m(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx \\
&= \frac{bB\tan^{1+m}(c+dx)}{d(1+m)} \\
&\quad + \frac{(aA-bB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)} \\
&\quad + \frac{(Ab+aB)\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)}{d(2+m)}
\end{aligned}$$

output
```
b*B*tan(d*x+c)^(1+m)/d/(1+m)+(A*a-B*b)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m
],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/(1+m)+(A*b+B*a)*hypergeom([1, 1+1/2*m]
,[2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(2+m)/d/(2+m)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.85

$$\int \tan^m(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{\tan^{1+m}(c+dx)\left(\frac{bB}{1+m}+\frac{(aA-bB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)}{1+m}+\frac{(Ab+aB)\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\right.}{2+m}\right.}{d}$$

input
```
Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(Tan[c + d*x]^(1 + m)*((b*B)/(1 + m) + ((a*A - b*B)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2])/(1 + m) + ((A*b + a*B)*Hypergeometr
ic2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(2 + m)))/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3042, 4075, 3042, 4021, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m(a+b\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4075}$$

$$\int \tan^m(c+dx)(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{bB\tan^{m+1}(c+dx)}{d(m+1)}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m(aA-bB+(Ab+aB)\tan(c+dx))dx+\frac{bB\tan^{m+1}(c+dx)}{d(m+1)}$$

$$\Big\downarrow \text{4021}$$

$$(aB+Ab)\int \tan^{m+1}(c+dx)dx+(aA-bB)\int \tan^m(c+dx)dx+\frac{bB\tan^{m+1}(c+dx)}{d(m+1)}$$

$$\Big\downarrow \text{3042}$$

$$(aA-bB)\int \tan(c+dx)^m dx+(aB+Ab)\int \tan(c+dx)^{m+1}dx+\frac{bB\tan^{m+1}(c+dx)}{d(m+1)}$$

$$\Big\downarrow \text{3957}$$

$$\frac{(aB+Ab)\int \frac{\tan^{m+1}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{(aA-bB)\int \frac{\tan^m(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{bB\tan^{m+1}(c+dx)}{d(m+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{(aA-bB)\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}+\frac{(aB+Ab)\tan^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\tan^2(c+dx)\right)}{d(m+2)}+\frac{bB\tan^{m+1}(c+dx)}{d(m+1)}$$

input
```
Int[Tan[c + d*x]^m*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(b*B*Tan[c + d*x]^(1 + m))/(d*(1 + m)) + ((a*A - b*B)*Hypergeometric2F1[1,
 (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(1 + m))/(d*(1 + m))
+ ((A*b + a*B)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]
*Tan[c + d*x]^(2 + m))/(d*(2 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [F]

$$\int \tan (dx+c)^m (a+b\tan (dx+c)) (A+B\tan (dx+c)) \, dx$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
integral((B*b*tan(d*x + c)^2 + A*a + (B*a + A*b)*tan(d*x + c))*tan(d*x + c
)^m, x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan(c+dx))\,(a+b\tan(c+dx))\tan^m(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))*tan(c + d*x)**m, x)
```

Maxima [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)*tan(d*x + c)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \int \tan(c+d\,x)^m\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)), x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{2\tan(dx+c)^m ab + \left(\int \tan(dx+c)^m\,dx\right)a^2dm - 2\left(\int \frac{\tan(dx+c)^m}{\tan(dx+c)}dx\right)abdm + \left(\int \tan(dx+c)^m\tan(dx-\right.}{dm}$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(2*tan(c + d*x)**m*a*b + int(tan(c + d*x)**m,x)*a**2*d*m - 2*int(tan(c + d
*x)**m/tan(c + d*x),x)*a*b*d*m + int(tan(c + d*x)**m*tan(c + d*x)**2,x)*b*
*2*d*m)/(d*m)
```

3.483 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 185

$$\begin{aligned}\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx &= \frac{(aA+bB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{(a^2+b^2)\,d(1+m)} \\ &\quad+\frac{b(Ab-aB)\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{b\tan(c+dx)}{a}\right)\tan^{1+m}(c+dx)}{a\,(a^2+b^2)\,d(1+m)} \\ &\quad-\frac{(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)}{(a^2+b^2)\,d(2+m)}\end{aligned}$$

output
```
(A*a+B*b)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(
1+m)/(a^2+b^2)/d/(1+m)+b*(A*b-B*a)*hypergeom([1, 1+m],[2+m],-b*tan(d*x+c)/
a)*tan(d*x+c)^(1+m)/a/(a^2+b^2)/d/(1+m)-(A*b-B*a)*hypergeom([1, 1+1/2*m],[
2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(2+m)/(a^2+b^2)/d/(2+m)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.78

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=\frac{\tan^{1+m}(c+dx)\left(\left(aA+bB\right)\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)+\frac{(Ab-aB)\left(b(2+m)\text{Hypergeo}\right.}{}\right.}{\left(a^2+b^2\right)d(1+m)}$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(Tan[c + d*x]^(1 + m)*((a*A + b*B)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)
/2, -Tan[c + d*x]^2] + ((A*b - a*B)*(b*(2 + m)*Hypergeometric2F1[1, 1 + m,
 2 + m, -((b*Tan[c + d*x])/a)] - a*(1 + m)*Hypergeometric2F1[1, (2 + m)/2,
 (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]))/(a*(2 + m))))/((a^2 + b^2)*d*(
1 + m))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {3042, 4096, 3042, 4021, 3042, 3957, 278, 4117, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 4096}$$

$$\frac{\int \tan^m(c+dx)(aA+bB-(Ab-aB)\tan(c+dx))dx}{a^2+b^2}+\frac{b(Ab-aB)\int \frac{\tan^m(c+dx)(\tan^2(c+dx)+1)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \tan(c+dx)^m(aA+bB-(Ab-aB)\tan(c+dx))dx}{a^2+b^2}+\frac{b(Ab-aB)\int \frac{\tan(c+dx)^m(\tan(c+dx)^2+1)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$\Big\downarrow$ 4021

$$\frac{(aA+bB)\int \tan^m(c+dx)dx-(Ab-aB)\int \tan^{m+1}(c+dx)dx}{a^2+b^2}+\frac{b(Ab-aB)\int \frac{\tan(c+dx)^m(\tan(c+dx)^2+1)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$\Big\downarrow$ 3042

$$\frac{(aA+bB)\int \tan(c+dx)^m dx-(Ab-aB)\int \tan(c+dx)^{m+1}dx}{a^2+b^2}+\frac{b(Ab-aB)\int \frac{\tan(c+dx)^m(\tan(c+dx)^2+1)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$\Big\downarrow$ 3957

$$\frac{\frac{(aA+bB)\int \frac{\tan^m(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{(Ab-aB)\int \frac{\tan^{m+1}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}}{a^2+b^2}+\frac{b(Ab-aB)\int \frac{\tan(c+dx)^m(\tan(c+dx)^2+1)}{a+b\tan(c+dx)}dx}{a^2+b^2}$$

$\Big\downarrow$ 278

$$\frac{b(Ab-aB)\int \frac{\tan(c+dx)^m(\tan(c+dx)^2+1)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{\frac{(aA+bB)\tan^{m+1}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}-\frac{(Ab-aB)\tan^{m+2}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\text{ta}\right.}{d(m+2)}}{a^2+b^2}$$

$\Big\downarrow$ 4117

$$\frac{b(Ab-aB)\int \frac{\tan^m(c+dx)}{a+b\tan(c+dx)}d\tan(c+dx)}{d(a^2+b^2)}+\frac{\frac{(aA+bB)\tan^{m+1}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}-\frac{(Ab-aB)\tan^{m+2}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\text{ta}\right.}{d(m+2)}}{a^2+b^2}$$

$$\Big\downarrow \text{74}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{b\tan(c+dx)}{a}\right)}{ad(m+1)\left(a^2+b^2\right)}+\frac{\frac{(aA+bB)\tan^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}-\frac{(Ab-aB)\tan^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\text{ta}\right.}{d(m+2)}}{a^2+b^2}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(b*(A*b - a*B)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*Tan[c + d*x])/a)]*T
an[c + d*x]^(1 + m))/(a*(a^2 + b^2)*d*(1 + m)) + (((a*A + b*B)*Hypergeomet
ric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(1 + m))/(d*
(1 + m)) - ((A*b - a*B)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c
+ d*x]^2]*Tan[c + d*x]^(2 + m))/(d*(2 + m)))/(a^2 + b^2)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4096
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)])^(n_))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*Simp[a*A + b*B - (A*b - a*B)*Tan
[e + f*x], x], x], x] + Simp[b*((A*b - a*B)/(a^2 + b^2))   Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{a+b\tan (dx+c)}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx=\int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{b\tan(dx+c)+a}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
integral((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx=\int \frac{(A+B\tan(c+dx))\tan^m(c+dx)}{a+b\tan(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx=\int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{b\tan(dx+c)+a}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{a+b\tan\left(c+d\,x\right)}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \tan\left(dx+c\right)^m dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
int(tan(c + d*x)**m,x)
```

3.484 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 282

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{(a^2A-Ab^2+2abB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{(a^2+b^2)^2\,d(1+m)} + \frac{b(a^2Ab(2-m)-Ab^3m+ab^2B(1+m)-a^3(B-Bm))\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{b\tan}{}\right.}{a^2\,(a^2+b^2)^2\,d(1+m)} - \frac{(2aAb-a^2B+b^2B)\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)}{(a^2+b^2)^2\,d(2+m)} + \frac{b(Ab-aB)\tan^{1+m}(c+dx)}{a\,(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
(A*a^2-A*b^2+2*B*a*b)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*
tan(d*x+c)^(1+m)/(a^2+b^2)^2/d/(1+m)+b*(a^2*A*b*(2-m)-A*b^3*m+a*b^2*B*(1+m
)-a^3*(-B*m+B))*hypergeom([1, 1+m],[2+m],-b*tan(d*x+c)/a)*tan(d*x+c)^(1+m)
/a^2/(a^2+b^2)^2/d/(1+m)-(2*A*a*b-B*a^2+B*b^2)*hypergeom([1, 1+1/2*m],[2+1
/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(2+m)/(a^2+b^2)^2/d/(2+m)+b*(A*b-B*a)*tan(
d*x+c)^(1+m)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.85

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\tan^{1+m}(c+dx)\left(\frac{b(-a^2Ab(-2+m)+a^3B(-1+m)-Ab^3m+ab^2B(1+m))\,\text{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{b\tan(c+dx)}{a}\right)}{a(a^2+b^2)(1+m)}+\frac{b(Ab-}{a+b\tan}\right.}{}$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
(Tan[c + d*x]^(1 + m)*((b*(-(a^2*A*b*(-2 + m)) + a^3*B*(-1 + m) - A*b^3*m
+ a*b^2*B*(1 + m))*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*Tan[c + d*x])/a
)])/(a*(a^2 + b^2)*(1 + m)) + (b*(A*b - a*B))/(a + b*Tan[c + d*x]) + (a*((
(a^2*A - A*b^2 + 2*a*b*B)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[
c + d*x]^2])/(1 + m) + ((-2*a*A*b + a^2*B - b^2*B)*Hypergeometric2F1[1, (2
 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(2 + m)))/(a^2 + b^2)))
/(a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4092, 3042, 4136, 3042, 4021, 3042, 3957, 278, 4117, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 4092}$$

$$\frac{\int \frac{\tan^m(c+dx)\left(Aa^2+bB(m+1)a-(Ab-aB)\tan(c+dx)a-b(Ab-aB)m\tan^2(c+dx)-Ab^2m\right)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\tan(c+dx)^m\left(Aa^2+bB(m+1)a-(Ab-aB)\tan(c+dx)a-b(Ab-aB)m\tan(c+dx)^2-Ab^2m\right)}{a+b\tan(c+dx)}dx}{a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 4136}$$

$$\frac{\frac{\int \tan^m(c+dx)\left(a\left(Aa^2+2bBa-Ab^2\right)-a\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)dx}{a^2+b^2}-\frac{b\left(a^3B(1-m)-a^2Ab(2-m)-ab^2B(m+1)+Ab^3m\right)\int \frac{\tan^m(c+dx)}{a+b\,\mathrm{t}}}{a^2+b^2}}{a\left(a^2+b^2\right)}$$
$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \tan(c+dx)^m\left(a\left(Aa^2+2bBa-Ab^2\right)-a\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)dx}{a^2+b^2}-\frac{b\left(a^3B(1-m)-a^2Ab(2-m)-ab^2B(m+1)+Ab^3m\right)\int \frac{\tan(c+dx)^m}{a+b\,\mathrm{t}}}{a^2+b^2}}{a\left(a^2+b^2\right)}$$
$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 4021}$$

$$\frac{\frac{a\left(a^2A+2abB-Ab^2\right)\int \tan^m(c+dx)dx-a\left(a^2(-B)+2aAb+b^2B\right)\int \tan^{m+1}(c+dx)dx}{a^2+b^2}-\frac{b\left(a^3B(1-m)-a^2Ab(2-m)-ab^2B(m+1)+Ab^3m\right)\int \frac{\tan}{}}{a^2+b^2}}{a\left(a^2+b^2\right)}$$
$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{a\left(a^2A+2abB-Ab^2\right)\int \tan(c+dx)^m dx-a\left(a^2(-B)+2aAb+b^2B\right)\int \tan(c+dx)^{m+1}dx}{a^2+b^2}-\frac{b\left(a^3B(1-m)-a^2Ab(2-m)-ab^2B(m+1)+Ab^3m\right)\int \frac{\tan}{}}{a^2+b^2}}{a\left(a^2+b^2\right)}$$
$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 3957

$$\frac{\frac{\frac{a\left(a^2A+2abB-Ab^2\right)\int \frac{\tan^m(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{a\left(a^2(-B)+2aAb+b^2B\right)\int \frac{\tan^{m+1}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}}{a^2+b^2}-\frac{b(a^3B(1-m)-a^2Ab(2-m)-ab^2B(m+1)+}{a^2+}}{a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 278

$$\frac{\frac{\frac{a\left(a^2A+2abB-Ab^2\right)\tan^{m+1}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}-\frac{a\left(a^2(-B)+2aAb+b^2B\right)\tan^{m+2}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\right.}{d(m+2)}}{a^2+b^2}}{a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 4117

$$\frac{\frac{\frac{a\left(a^2A+2abB-Ab^2\right)\tan^{m+1}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}-\frac{a\left(a^2(-B)+2aAb+b^2B\right)\tan^{m+2}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\right.}{d(m+2)}}{a^2+b^2}}{a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}$$

$\Big\downarrow$ 74

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{ad\left(a^2+b^2\right)(a+b\tan(c+dx))}+$$

$$\frac{\frac{\frac{a\left(a^2A+2abB-Ab^2\right)\tan^{m+1}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\tan^2(c+dx)\right)}{d(m+1)}-\frac{a\left(a^2(-B)+2aAb+b^2B\right)\tan^{m+2}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\right.}{d(m+2)}}{a^2+b^2}}{a\left(a^2+\right.}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(A*b - a*B)*Tan[c + d*x]^(1 + m))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
) + (-((b*(a^3*B*(1 - m) - a^2*A*b*(2 - m) + A*b^3*m - a*b^2*B*(1 + m))*Hy
pergeometric2F1[1, 1 + m, 2 + m, -((b*Tan[c + d*x])/a)]*Tan[c + d*x]^(1 +
m))/(a*(a^2 + b^2)*d*(1 + m))) + ((a*(a^2*A - A*b^2 + 2*a*b*B)*Hypergeomet
ric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(1 + m))/(d*
(1 + m)) - (a*(2*a*A*b - a^2*B + b^2*B)*Hypergeometric2F1[1, (2 + m)/2, (4
 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2 + m))/(d*(2 + m)))/(a^2 + b^2))/
(a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{(a+b\tan (dx+c))^2}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(b\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="f
ricas")
```

output
```
integral((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b^2*tan(d*x + c)^2 + 2*a*b*t
an(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{(A+B\tan(c+dx))\tan^m(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/(a + b*tan(c + d*x))**2, x)
```

Maxima [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(b\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^2}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\tan(dx+c)^m - \left(\int \frac{\tan(dx+c)^m}{\tan(dx+c)^2b+\tan(dx+c)a}dx\right)adm - \left(\int \frac{\tan(dx+c)^m\tan(dx+c)^2}{a+\tan(dx+c)b}dx\right)bdm - \left(\int \frac{\tan(dx+c)^m\tan(dx}{a+\tan(dx+c)b}\right.}{bdm}$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
(tan(c + d*x)**m - int(tan(c + d*x)**m/(tan(c + d*x)**2*b + tan(c + d*x)*a
),x)*a*d*m - int((tan(c + d*x)**m*tan(c + d*x)**2)/(tan(c + d*x)*b + a),x)
*b*d*m - int((tan(c + d*x)**m*tan(c + d*x))/(tan(c + d*x)*b + a),x)*a*d*m)
/(b*d*m)
```

3.485 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 438

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{(a^3A-3aAb^2+3a^2bB-b^3B)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{(a^2+b^2)^3\,d(1+m)}$$
$$-\frac{b(Ab^5(1-m)m+ab^4Bm(1+m)-2a^3b^2B(3+m-m^2)+2a^2Ab^3(1+3m-m^2)-a^4Ab(6-5m+}{2a^3\,(a^2+b^2)^3\,}$$
$$-\frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c+dx)}{(a^2+b^2)^3\,d(2+m)}$$
$$+\frac{b(Ab-aB)\tan^{1+m}(c+dx)}{2a\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}$$
$$+\frac{b(Ab^3(1-m)-a^3B(3-m)+a^2Ab(5-m)+ab^2B(1+m))\tan^{1+m}(c+dx)}{2a^2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

output
```
(A*a^3-3*A*a*b^2+3*B*a^2*b-B*b^3)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-ta
n(d*x+c)^2)*tan(d*x+c)^(1+m)/(a^2+b^2)^3/d/(1+m)-1/2*b*(A*b^5*(1-m)*m+a*b^
4*B*m*(1+m)-2*a^3*b^2*B*(-m^2+m+3)+2*a^2*A*b^3*(-m^2+3*m+1)-a^4*A*b*(m^2-5
*m+6)+a^5*B*(m^2-3*m+2))*hypergeom([1, 1+m],[2+m],-b*tan(d*x+c)/a)*tan(d*x
+c)^(1+m)/a^3/(a^2+b^2)^3/d/(1+m)-(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)*hyperg
eom([1, 1+1/2*m],[2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(2+m)/(a^2+b^2)^3/d/(
2+m)+1/2*b*(A*b-B*a)*tan(d*x+c)^(1+m)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/2
*b*(A*b^3*(1-m)-a^3*B*(3-m)+a^2*A*b*(5-m)+a*b^2*B*(1+m))*tan(d*x+c)^(1+m)/
a^2/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Mathematica [A] (verified)

Time = 6.25 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.21

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \frac{b(Ab-aB)\tan^{1+m}(c+dx)}{2a\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}$$
$$+\frac{\frac{\left(-a(-2ab(Ab-aB)-ab(Ab-aB)(1-m))+b^2\left(2a^2A+Ab^2(1-m)+abB(1+m)\right)\right)\tan^{1+m}(c+dx)}{a(a^2+b^2)d(a+b\tan(c+dx))}+\frac{\left(2a^3b\left(2aAb-a^2B+b^2B\right)-a^2bm\left(Ab^3(1-m\right.\right.}{}}{}$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(A*b - a*B)*Tan[c + d*x]^(1 + m))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x
])^2) + (((-(a*(-2*a*b*(A*b - a*B) - a*b*(A*b - a*B)*(1 - m))) + b^2*(2*a^
2*A + A*b^2*(1 - m) + a*b*B*(1 + m)))*Tan[c + d*x]^(1 + m))/(a*(a^2 + b^2)
*d*(a + b*Tan[c + d*x])) + (((2*a^3*b*(2*a*A*b - a^2*B + b^2*B) - a^2*b*m*
(A*b^3*(1 - m) - a^3*B*(3 - m) + a^2*A*b*(5 - m) + a*b^2*B*(1 + m)) + b^2*
(-(a^2*b*(A*b - a*B)*(3 - m)*(1 + m)) + (a^2 - b^2*m)*(2*a^2*A + A*b^2*(1
- m) + a*b*B*(1 + m))))*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*Tan[c + d*
x])/a)]*Tan[c + d*x]^(1 + m))/(a*(a^2 + b^2)*d*(1 + m)) + ((2*a^2*(a^3*A -
 3*a*A*b^2 + 3*a^2*b*B - b^3*B)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
 -Tan[c + d*x]^2]*Tan[c + d*x]^(1 + m))/(d*(1 + m)) - (2*a^2*(3*a^2*A*b -
A*b^3 - a^3*B + 3*a*b^2*B)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan
[c + d*x]^2]*Tan[c + d*x]^(2 + m))/(d*(2 + m)))/(a^2 + b^2))/(a*(a^2 + b^2
)))/(2*a*(a^2 + b^2))
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.484, Rules used = {3042, 4092, 3042, 4132, 25, 3042, 4136, 27, 3042, 4021, 3042, 3957, 278, 4117, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{\int \frac{\tan^m(c+dx)\left(2Aa^2+bB(m+1)a-2(Ab-aB)\tan(c+dx)a+b(Ab-aB)(1-m)\tan^2(c+dx)+Ab^2(1-m)\right)}{(a+b\tan(c+dx))^2}dx}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\tan(c+dx)^m\left(2Aa^2+bB(m+1)a-2(Ab-aB)\tan(c+dx)a+b(Ab-aB)(1-m)\tan(c+dx)^2+Ab^2(1-m)\right)}{(a+b\tan(c+dx))^2}dx}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{4132}$$

$$\frac{\frac{\int -\frac{\tan^m(c+dx)\left(b(Ab-aB)(3-m)(m+1)a^2+2\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2+bm\left(-B(3-m)a^3+Ab(5-m)a^2+b^2B(m+1)a+Ab^3(1-m)\right)\tan^2(c+dx)-\left(a^2 \cdots\right.\right.}{a+b\tan(c+dx)}}{a(a^2+b^2)}\cdots}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{b(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m))\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\int\frac{\tan^m(c+dx)\left(b(Ab-aB)(3-m)(m+1)a^2+2\left(-Ba^2+2Aba+b^2B\right)\tan(c\ldots}{\ldots}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{b(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m))\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\int\frac{\tan(c+dx)^m\left(b(Ab-aB)(3-m)(m+1)a^2+2\left(-Ba^2+2Aba+b^2B\right)\tan(c\ldots}{\ldots}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 4136

$$\frac{b(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m))\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\frac{\int-2\tan^m(c+dx)\left(a^2\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-a^2\left(-Ba^3+3Aba^2+\ldots\right.\right.}{a^2+b^2}\ldots$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 27

$$\frac{b(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m))\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\frac{b\left(a^5B\left(m^2-3m+2\right)-a^4Ab\left(m^2-5m+6\right)-2a^3b^2B\left(-m^2+m+3\right)+2a^2A\ldots\right.}{\ldots}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{b(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m))\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\frac{b\left(a^5B\left(m^2-3m+2\right)-a^4Ab\left(m^2-5m+6\right)-2a^3b^2B\left(-m^2+m+3\right)+2a^2A\ldots\right.}{\ldots}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\downarrow$ 4021

$$\frac{\frac{b(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m))\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\frac{b\left(a^5B\left(m^2-3m+2\right)-a^4Ab\left(m^2-5m+6\right)-2a^3b^2B\left(-m^2+m+3\right)+2a^2A}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m))\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\frac{b\left(a^5B\left(m^2-3m+2\right)-a^4Ab\left(m^2-5m+6\right)-2a^3b^2B\left(-m^2+m+3\right)+2a^2A}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 3957

$$\frac{\frac{b(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m))\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\frac{b\left(a^5B\left(m^2-3m+2\right)-a^4Ab\left(m^2-5m+6\right)-2a^3b^2B\left(-m^2+m+3\right)+2a^2A}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 278

$$\frac{\frac{b(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m))\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\frac{b\left(a^5B\left(m^2-3m+2\right)-a^4Ab\left(m^2-5m+6\right)-2a^3b^2B\left(-m^2+m+3\right)+2a^2A}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$\Big\downarrow$ 4117

$$\frac{\frac{b(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m))\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\frac{\frac{b\left(a^5B\left(m^2-3m+2\right)-a^4Ab\left(m^2-5m+6\right)-2a^3b^2B\left(-m^2+m+3\right)+2a^2A}{d(}}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \; 74$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{2ad\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+$$

$$\frac{\frac{b\left(a^3(-B)(3-m)+a^2Ab(5-m)+ab^2B(m+1)+Ab^3(1-m)\right)\tan^{m+1}(c+dx)}{ad(a^2+b^2)(a+b\tan(c+dx))}-\frac{b\left(a^5B\left(m^2-3m+2\right)-a^4Ab\left(m^2-5m+6\right)-2a^3b^2B\left(-m^2+m+3\right)+2a^2A\cdots\right.}{\cdots}}{\cdots}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(A*b - a*B)*Tan[c + d*x]^(1 + m))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x
])^2) + ((b*(A*b^3*(1 - m) - a^3*B*(3 - m) + a^2*A*b*(5 - m) + a*b^2*B*(1
+ m))*Tan[c + d*x]^(1 + m))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - ((b*(
A*b^5*(1 - m)*m + a*b^4*B*m*(1 + m) - 2*a^3*b^2*B*(3 + m - m^2) + 2*a^2*A*
b^3*(1 + 3*m - m^2) - a^4*A*b*(6 - 5*m + m^2) + a^5*B*(2 - 3*m + m^2))*Hyp
ergeometric2F1[1, 1 + m, 2 + m, -((b*Tan[c + d*x])/a)]*Tan[c + d*x]^(1 + m
))/(a*(a^2 + b^2)*d*(1 + m)) - (2*((a^2*(a^3*A - 3*a*A*b^2 + 3*a^2*b*B - b
^3*B)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c +
d*x]^(1 + m))/(d*(1 + m)) - (a^2*(3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*H
ypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(2
 + m))/(d*(2 + m))))/(a^2 + b^2))/(a*(a^2 + b^2)))/(2*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] && !IntegerQ[2*m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{(a+b\tan (dx+c))^3}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(b\tan(dx+c)+a)^3}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
integral((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b^3*tan(d*x + c)^3 + 3*a*b^2
*tan(d*x + c)^2 + 3*a^2*b*tan(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{(A+B\tan(c+dx))\tan^m(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/(a + b*tan(c + d*x))**3, x)
```

Maxima [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(b\tan(dx+c)+a)^3}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,3]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^3}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \left(\int \frac{\tan(dx+c)^m}{\tan(dx+c)^2 b^2 m - \tan(dx+c)^2 b^2 + 2\tan(dx+c)\,abm - 2\tan(dx+c)\,ab + a^2 m - a^2}dx\right)(m-1)$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
int(tan(c + d*x)**m/(tan(c + d*x)**2*b**2*m - tan(c + d*x)**2*b**2 + 2*tan
(c + d*x)*a*b*m - 2*tan(c + d*x)*a*b + a**2*m - a**2),x)*(m - 1)
```

3.486 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4} dx$

Optimal result

Integrand size = 31, antiderivative size = 659

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4} dx = \frac{(a^4A-6a^2Ab^2+Ab^4+4a^3bB-4ab^3B)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\tan^2(c+dx)\right)\tan^{1+m}(c+}{(a^2+b^2)^4 d(1+m)}$$
$$-\frac{b(ab^6Bm(1-m^2)+3a^2Ab^5m(2-5m+m^2)+Ab^7m(2-3m+m^2)+3a^3b^4B(2+5m+2m^2-m^3)}{}$$
$$-\frac{(4a^3Ab-4aAb^3-a^4B+6a^2b^2B-b^4B)\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\tan^2(c+dx)\right)\tan^{2+m}(c}{(a^2+b^2)^4 d(2+m)}$$
$$+\frac{b(Ab-aB)\tan^{1+m}(c+dx)}{3a(a^2+b^2)d(a+b\tan(c+dx))^3}$$
$$+\frac{b(Ab^3(2-m)-a^3B(5-m)+a^2Ab(8-m)+ab^2B(1+m))\tan^{1+m}(c+dx)}{6a^2(a^2+b^2)^2 d(a+b\tan(c+dx))^2}$$
$$+\frac{b(ab^4B(1-m^2)+2a^3b^2B(7+3m-m^2)+a^4Ab(26-9m+m^2)+2a^2Ab^3(2-6m+m^2)-a^5B(11}{6a^3(a^2+b^2)^3 d(a+b\tan(c+dx))}$$

output
```
(A*a^4-6*A*a^2*b^2+A*b^4+4*B*a^3*b-4*B*a*b^3)*hypergeom([1, 1/2+1/2*m],[3/
2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/(a^2+b^2)^4/d/(1+m)-1/6*b*(a*b^6*
B*m*(-m^2+1)+3*a^2*A*b^5*m*(m^2-5*m+2)+A*b^7*m*(m^2-3*m+2)+3*a^3*b^4*B*(-m
^3+2*m^2+5*m+2)+a^7*B*(-m^3+6*m^2-11*m+6)-a^6*A*b*(-m^3+9*m^2-26*m+24)+3*a
^4*A*b^3*(m^3-7*m^2+10*m+8)-3*a^5*b^2*B*(m^3-4*m^2-m+12))*hypergeom([1, 1+
m],[2+m],-b*tan(d*x+c)/a)*tan(d*x+c)^(1+m)/a^4/(a^2+b^2)^4/d/(1+m)-(4*A*a^
3*b-4*A*a*b^3-B*a^4+6*B*a^2*b^2-B*b^4)*hypergeom([1, 1+1/2*m],[2+1/2*m],-t
an(d*x+c)^2)*tan(d*x+c)^(2+m)/(a^2+b^2)^4/d/(2+m)+1/3*b*(A*b-B*a)*tan(d*x+
c)^(1+m)/a/(a^2+b^2)/d/(a+b*tan(d*x+c))^3+1/6*b*(A*b^3*(2-m)-a^3*B*(5-m)+a
^2*A*b*(8-m)+a*b^2*B*(1+m))*tan(d*x+c)^(1+m)/a^2/(a^2+b^2)^2/d/(a+b*tan(d*
x+c))^2+1/6*b*(a*b^4*B*(-m^2+1)+2*a^3*b^2*B*(-m^2+3*m+7)+a^4*A*b*(m^2-9*m+
26)+2*a^2*A*b^3*(m^2-6*m+2)-a^5*B*(m^2-6*m+11)+A*b^5*(m^2-3*m+2))*tan(d*x+
c)^(1+m)/a^3/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1425 vs. $2(659) = 1318$.

Time = 6.29 (sec) , antiderivative size = 1425, normalized size of antiderivative = 2.16

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(A*b - a*B)*Tan[c + d*x]^(1 + m))/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x
])^3) + (((-(a*(-3*a*b*(A*b - a*B) - a*b*(A*b - a*B)*(2 - m))) + b^2*(3*a^
2*A + A*b^2*(2 - m) + a*b*B*(1 + m)))*Tan[c + d*x]^(1 + m))/(2*a*(a^2 + b^
2)*d*(a + b*Tan[c + d*x])^2) + (((b^2*(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)
) + (2*a^2 + b^2*(1 - m))*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m))) - a*(
-6*a^2*b*(2*a*A*b - a^2*B + b^2*B) - a*b*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5
 - m) + a^2*A*b*(8 - m) + a*b^2*B*(1 + m))))*Tan[c + d*x]^(1 + m))/(a*(a^2
 + b^2)*d*(a + b*Tan[c + d*x])) + (((a^2*b*(6*a^3*(2*a*A*b - a^2*B + b^2*B
) - b^2*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m) + a*b^2*B
*(1 + m)) + b*(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2*(1 - m)
)*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m)))) - a^2*m*(b^2*(-(a^2*b*(A*b -
 a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2*(1 - m))*(3*a^2*A + A*b^2*(2 - m) +
a*b*B*(1 + m))) - a*(-6*a^2*b*(2*a*A*b - a^2*B + b^2*B) - a*b*(1 - m)*(A*b
^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m) + a*b^2*B*(1 + m)))) + b^2*((
a^2 - b^2*m)*(-(a^2*b*(A*b - a*B)*(5 - m)*(1 + m)) + (2*a^2 + b^2*(1 - m))
*(3*a^2*A + A*b^2*(2 - m) + a*b*B*(1 + m))) + a*(1 + m)*(-6*a^2*b*(2*a*A*b
 - a^2*B + b^2*B) - a*b*(1 - m)*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(
8 - m) + a*b^2*B*(1 + m)))))*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*Tan[c
 + d*x])/a)]*Tan[c + d*x]^(1 + m))/(a*(a^2 + b^2)*d*(1 + m)) + ((6*a^7*A*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]...
```

Rubi [A] (verified)

Time = 4.08 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.07, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {3042, 4092, 3042, 4132, 25, 3042, 4132, 25, 3042, 4136, 27, 3042, 4021, 3042, 3957, 278, 4117, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{\int \frac{\tan^m(c+dx)\left(3Aa^2+bB(m+1)a-3(Ab-aB)\tan(c+dx)a+b(Ab-aB)(2-m)\tan^2(c+dx)+Ab^2(2-m)\right)}{(a+b\tan(c+dx))^3}dx}{3a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{\tan(c+dx)^m\left(3Aa^2+bB(m+1)a-3(Ab-aB)\tan(c+dx)a+b(Ab-aB)(2-m)\tan(c+dx)^2+Ab^2(2-m)\right)}{(a+b\tan(c+dx))^3}dx}{3a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 4132

$$\frac{\frac{\int -\frac{\tan^m(c+dx)\left(b(Ab-aB)(5-m)(m+1)a^2+6\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)a^2-b(1-m)\left(-B(5-m)a^3+Ab(8-m)a^2+b^2B(m+1)a+Ab^3(2-m)\right)\tan^2(c+dx)-\cdots}{(a+b\tan(c+dx))^2}}{2a(a^2+b^2)}\cdots}{3a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{b(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m))\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan^m(c+dx)\left(b(Ab-aB)(5-m)(m+1)a^2+6\left(-Ba^2+2Aba+b^2B\right)\tan(c\cdots\right.}{\cdots}}{\cdots}}{3a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m))\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\int \frac{\tan(c+dx)^m\left(b(Ab-aB)(5-m)(m+1)a^2+6\left(-Ba^2+2Aba+b^2B\right)\tan(c\cdots\right.}{\cdots}}{\cdots}}{3a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 4132

$$\frac{\frac{b(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m))\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{\frac{\int -\frac{\tan^m(c+dx)\left(6Aa^6+bB\left(m^3-6m^2+11m+18\right)a^5-Ab^2\left(m^3-9m^2+2\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{b(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m))\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\int\frac{\tan^m(c+dx)\left(6Aa^6+bB\left(m^3-6m^2+11m+18\right)a^5-Ab^2\left(m^3-9m^2+2\right.\right.}{}}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m))\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\int\frac{\tan(c+dx)^m\left(6Aa^6+bB\left(m^3-6m^2+11m+18\right)a^5-Ab^2\left(m^3-9m^2+2\right.\right.}{}}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$\Big\downarrow$ 4136

$$\frac{\frac{b(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m))\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{\int 6\tan^m(c+dx)\left(a^3\left(Aa^4+4bBa^3-6Ab^2a^2-4b^3Ba+Ab^4\right)-a^3\left(-Ba^4\right.\right.}{a^2+b^2}}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{b(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m))\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{6\int\tan^m(c+dx)\left(a^3\left(Aa^4+4bBa^3-6Ab^2a^2-4b^3Ba+Ab^4\right)-a^3\left(-Ba^4\right.\right.}{a^2+b^2}}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$\Big\downarrow$ 3042

$$\frac{b\left(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m)\right)\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{6\int\tan(c+dx)^m\left(a^3\left(Aa^4+4bBa^3-6Ab^2a^2-4b^3Ba+Ab^4\right)-a^3\left(-Ba^4\right.}{a^2+b^2}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$\downarrow$ 4021

$$\frac{b\left(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m)\right)\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{6\left(a^3\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\int\tan^m(c+dx)dx-a^3\left(a^4(\right.\right.}{a^2+b^2}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$\downarrow$ 3042

$$\frac{b\left(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m)\right)\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{6\left(a^3\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\int\tan(c+dx)^m dx-a^3\left(a^4(\right.\right.}{a^2+b^2}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$\downarrow$ 3957

$$\frac{b\left(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m)\right)\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{6\left(\frac{a^3\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\int\frac{\tan^m(c+dx)}{\tan^2(c+dx)+1}d\tan(c+\right.}{d}}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$\downarrow$ 278

$$\frac{\frac{b(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m))\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{6\left(\frac{a^3\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\tan^{m+1}(c+dx)\text{ Hypergeo}}{d(m+1)}\right.}{}}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{4117}$$

$$\frac{\frac{b(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m))\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{6\left(\frac{a^3\left(a^4A+4a^3bB-6a^2Ab^2-4ab^3B+Ab^4\right)\tan^{m+1}(c+dx)\text{ Hypergeo}}{d(m+1)}\right.}{}}{}}{}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{74}$$

$$\frac{b(Ab-aB)\tan^{m+1}(c+dx)}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^3}+$$

$$\frac{\frac{b(a^3(-B)(5-m)+a^2Ab(8-m)+ab^2B(m+1)+Ab^3(2-m))\tan^{m+1}(c+dx)}{2ad(a^2+b^2)(a+b\tan(c+dx))^2}-\frac{-\frac{b\left(a^5(-B)\left(m^2-6m+11\right)+a^4Ab\left(m^2-9m+26\right)+2a^3b^2B\left(-m^2+3m+7\right.\right.}{ad\left(a^2+b^2\right.}}{}}{}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^4,x]
```

output
```
(b*(A*b - a*B)*Tan[c + d*x]^(1 + m))/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x
])^3) + ((b*(A*b^3*(2 - m) - a^3*B*(5 - m) + a^2*A*b*(8 - m) + a*b^2*B*(1
+ m))*Tan[c + d*x]^(1 + m))/(2*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (
-((b*(a*b^4*B*(1 - m^2) + 2*a^3*b^2*B*(7 + 3*m - m^2) + a^4*A*b*(26 - 9*m
+ m^2) + 2*a^2*A*b^3*(2 - 6*m + m^2) - a^5*B*(11 - 6*m + m^2) + A*b^5*(2 -
 3*m + m^2))*Tan[c + d*x]^(1 + m))/(a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])))
 - (-((b*(a*b^6*B*m*(1 - m^2) + 3*a^2*A*b^5*m*(2 - 5*m + m^2) + A*b^7*m*(2
 - 3*m + m^2) + 3*a^3*b^4*B*(2 + 5*m + 2*m^2 - m^3) + a^7*B*(6 - 11*m + 6*
m^2 - m^3) - a^6*A*b*(24 - 26*m + 9*m^2 - m^3) + 3*a^4*A*b^3*(8 + 10*m - 7
*m^2 + m^3) - 3*a^5*b^2*B*(12 - m - 4*m^2 + m^3))*Hypergeometric2F1[1, 1 +
 m, 2 + m, -((b*Tan[c + d*x])/a)]*Tan[c + d*x]^(1 + m))/(a*(a^2 + b^2)*d*(
1 + m))) + (6*((a^3*(a^4*A - 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B - 4*a*b^3*B)*
Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]^(
1 + m))/(d*(1 + m)) - (a^3*(4*a^3*A*b - 4*a*A*b^3 - a^4*B + 6*a^2*b^2*B -
b^4*B)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -Tan[c + d*x]^2]*Tan[c +
 d*x]^(2 + m))/(d*(2 + m))))/(a^2 + b^2))/(a*(a^2 + b^2)))/(2*a*(a^2 + b^2
)))/(3*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4021
```
Int[((b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Tan[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Tan[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x] && NeQ[c^
2 + d^2, 0] &&  !IntegerQ[2*m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{(a+b\tan (dx+c))^4}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(b\tan(dx+c)+a)^4}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="f
ricas")
```

output
```
integral((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b^4*tan(d*x + c)^4 + 4*a*b^3
*tan(d*x + c)^3 + 6*a^2*b^2*tan(d*x + c)^2 + 4*a^3*b*tan(d*x + c) + a^4),
x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \int \frac{(A+B\tan(c+dx))\tan^m(c+dx)}{(a+b\tan(c+dx))^4}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**4,x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/(a + b*tan(c + d*x))**4, x)
```

Maxima [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(b\tan(dx+c)+a)^4}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a)^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,1,0]%%%} / %%%{1,[0,0,4]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^4}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^4,x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^4, x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^4}\,dx$$
$$= \left(\int \frac{\tan(dx+c)^m}{\tan(dx+c)^3 b^3 m - 2\tan(dx+c)^3 b^3 + 3\tan(dx+c)^2 a b^2 m - 6\tan(dx+c)^2 a b^2 + 3\tan(dx+c)\right.$$
$$-2)$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^4,x)
```

output
```
int(tan(c + d*x)**m/(tan(c + d*x)**3*b**3*m - 2*tan(c + d*x)**3*b**3 + 3*t
an(c + d*x)**2*a*b**2*m - 6*tan(c + d*x)**2*a*b**2 + 3*tan(c + d*x)*a**2*b
*m - 6*tan(c + d*x)*a**2*b + a**3*m - 2*a**3),x)*(m - 2)
```

3.487 $\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{(A-iB)\operatorname{AppellF1}\left(\frac{7}{2},-m,1,\frac{9}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a-ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right.}{7(ia+b)d} + \frac{(iA-B)\operatorname{AppellF1}\left(\frac{7}{2},-m,1,\frac{9}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a+ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}(a+b\tan(c+}{7(a+ib)d}$$

output
```
1/7*(A-I*B)*AppellF1(7/2,-m,1,9/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(a-I
*b))*tan(d*x+c)^m*(a+b*tan(d*x+c))^(7/2)/(I*a+b)/d/((-b*tan(d*x+c)/a)^m)+1
/7*(I*A-B)*AppellF1(7/2,-m,1,9/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(a+I*
b))*tan(d*x+c)^m*(a+b*tan(d*x+c))^(7/2)/(a+I*b)/d/((-b*tan(d*x+c)/a)^m)
```

Mathematica [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),
x]
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4086, 3042, 4085, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4086}$$

$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))\tan^m(c+dx)(a+b\tan(c+dx))^{5/2}dx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)\tan^m(c+dx)(a+b\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))\tan(c+dx)^m(a+b\tan(c+dx))^{5/2}dx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)\tan(c+dx)^m(a+b\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{4085}$$

$$\frac{(A-iB)\int \frac{\tan^m(c+dx)(a+b\tan(c+dx))^{5/2}}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{\tan^m(c+dx)(a+b\tan(c+dx))^{5/2}}{i\tan(c+dx)+1}d\tan(c+dx)}{2d}$$

$$\Big\downarrow \text{152}$$

$$\frac{a^2(A-iB)\sqrt{a+b\tan(c+dx)}\int \frac{\tan^m(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^{5/2}}{1-i\tan(c+dx)}d\tan(c+dx)}{2d\sqrt{\frac{b\tan(c+dx)}{a}+1}}+\frac{a^2(A+iB)\sqrt{a+b\tan(c+dx)}\int \frac{\tan^m(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^{5/2}}{i\tan(c+dx)+1}d\tan(c+dx)}{2d\sqrt{\frac{b\tan(c+dx)}{a}+1}}$$

$$\Big\downarrow \text{150}$$

$$\frac{a^2(A+iB)\tan^{m+1}(c+dx)\sqrt{a+b\tan(c+dx)}\,\text{AppellF1}\left(m+1,-\frac{5}{2},1,m+2,-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2d(m+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}+\frac{a^2(A-iB)\tan^{m+1}(c+dx)\sqrt{a+b\tan(c+dx)}\,\text{AppellF1}\left(m+1,-\frac{5}{2},1,m+2,-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)}{2d(m+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}$$

input
```
Int[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(a^2*(A + I*B)*AppellF1[1 + m, -5/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)
*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[a + b*Tan[c + d*x]])/(2*d*(1 + m)
*Sqrt[1 + (b*Tan[c + d*x])/a]) + (a^2*(A - I*B)*AppellF1[1 + m, -5/2, 1, 2
 + m, -((b*Tan[c + d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[a +
 b*Tan[c + d*x]])/(2*d*(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &
& !IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \tan(dx+c)^m (a+b\tan(dx+c))^{\frac{5}{2}} (A+B\tan(dx+c))\,dx$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
integral((B*b^2*tan(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*tan(d*x + c)^2
+ (B*a^2 + 2*A*a*b)*tan(d*x + c))*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^m,
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^m,
x)
```

Giac [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*tan(d*x + c)^m,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \left(\int \tan(dx+c)^m\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^3\,dx\right)b^3 + 3\left(\int \tan(dx+c)^m\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)a\,b^2 + 3\left(\int \tan(dx+c)^m\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)a^2b + \left(\int \tan(dx+c)^m\sqrt{a+\tan(dx+c)\,b}dx\right)a^3$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**3,x)*b**3 + 3*i
nt(tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*a*b**2 + 3*
int(tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a**2*b + int(
tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a),x)*a**3
```

3.488 $\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{(A-iB)\operatorname{AppellF1}\left(\frac{5}{2},-m,1,\frac{7}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a-ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right.}{5(ia+b)d} + \frac{(iA-B)\operatorname{AppellF1}\left(\frac{5}{2},-m,1,\frac{7}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a+ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}(a+b\tan(c+}{5(a+ib)d}$$

output
```
1/5*(A-I*B)*AppellF1(5/2,-m,1,7/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(a-I
*b))*tan(d*x+c)^m*(a+b*tan(d*x+c))^(5/2)/(I*a+b)/d/((-b*tan(d*x+c)/a)^m)+1
/5*(I*A-B)*AppellF1(5/2,-m,1,7/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(a+I*
b))*tan(d*x+c)^m*(a+b*tan(d*x+c))^(5/2)/(a+I*b)/d/((-b*tan(d*x+c)/a)^m)
```

Mathematica [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),
x]
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4086, 3042, 4085, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4086}$$

$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))\tan^m(c+dx)(a+b\tan(c+dx))^{3/2}dx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)\tan^m(c+dx)(a+b\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))\tan(c+dx)^m(a+b\tan(c+dx))^{3/2}dx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)\tan(c+dx)^m(a+b\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{4085}$$

$$\frac{(A-iB)\int \frac{\tan^m(c+dx)(a+b\tan(c+dx))^{3/2}}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{\tan^m(c+dx)(a+b\tan(c+dx))^{3/2}}{i\tan(c+dx)+1}d\tan(c+dx)}{2d}$$

$$\Big\downarrow \text{152}$$

$$\frac{a(A-iB)\sqrt{a+b\tan(c+dx)}\int \frac{\tan^m(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^{3/2}}{1-i\tan(c+dx)}d\tan(c+dx)}{2d\sqrt{\frac{b\tan(c+dx)}{a}+1}}+\frac{a(A+iB)\sqrt{a+b\tan(c+dx)}\int \frac{\tan^m(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^{3/2}}{i\tan(c+dx)+1}d\tan(c+dx)}{2d\sqrt{\frac{b\tan(c+dx)}{a}+1}}$$

$$\Big\downarrow \text{150}$$

$$\frac{a(A+iB)\tan^{m+1}(c+dx)\sqrt{a+b\tan(c+dx)}\operatorname{AppellF1}\left(m+1,-\frac{3}{2},1,m+2,-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2d(m+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}+\frac{a(A-iB)\tan^{m+1}(c+dx)\sqrt{a+b\tan(c+dx)}\operatorname{AppellF1}\left(m+1,-\frac{3}{2},1,m+2,-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)}{2d(m+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}$$

input
```
Int[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
(a*(A + I*B)*AppellF1[1 + m, -3/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*T
an[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[a + b*Tan[c + d*x]])/(2*d*(1 + m)*S
qrt[1 + (b*Tan[c + d*x])/a]) + (a*(A - I*B)*AppellF1[1 + m, -3/2, 1, 2 + m
, -((b*Tan[c + d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[a + b*T
an[c + d*x]])/(2*d*(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \tan(dx+c)^m (a+b\tan(dx+c))^{\frac{3}{2}} (A+B\tan(dx+c))\,dx$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
integral((B*b*tan(d*x + c)^2 + A*a + (B*a + A*b)*tan(d*x + c))*sqrt(b*tan(
d*x + c) + a)*tan(d*x + c)^m, x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (A+B\tan(c+dx))(a+b\tan(c+dx))^{\frac{3}{2}}\tan^m(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*tan(c + d*x)**m,
 x)
```

Maxima [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^m,
x)
```

Giac [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*tan(d*x + c)^m,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \left(\int \tan(dx+c)^m\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)^2\,dx\right)b^2 + 2\left(\int \tan(dx+c)^m\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)ab + \left(\int \tan(dx+c)^m\sqrt{a+\tan(dx+c)\,b}dx\right)a^2$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2,x)*b**2 + 2*i
nt(tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*a*b + int(tan(
c + d*x)**m*sqrt(tan(c + d*x)*b + a),x)*a**2
```

3.489 $\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{(A-iB)\,\text{AppellF1}\left(\frac{3}{2},-m,1,\frac{5}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a-ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}(a+b\tan(c+$$
$$3(ia+b)d$$
$$+\frac{(iA-B)\,\text{AppellF1}\left(\frac{3}{2},-m,1,\frac{5}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a+ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}(a+b\tan($$
$$3(a+ib)d$$

output
```
1/3*(A-I*B)*AppellF1(3/2,-m,1,5/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(a-I
*b))*tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2)/(I*a+b)/d/((-b*tan(d*x+c)/a)^m)+1
/3*(I*A-B)*AppellF1(3/2,-m,1,5/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(a+I*
b))*tan(d*x+c)^m*(a+b*tan(d*x+c))^(3/2)/(a+I*b)/d/((-b*tan(d*x+c)/a)^m)
```

Mathematica [F]

$$\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
Integrate[Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4086, 3042, 4085, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4086}$$

$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))\tan^m(c+dx)\sqrt{a+b\tan(c+dx)}dx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)\tan^m(c+dx)\sqrt{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))\tan(c+dx)^m\sqrt{a+b\tan(c+dx)}dx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)\tan(c+dx)^m\sqrt{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4085}$$

$$\frac{(A-iB)\int \frac{\tan^m(c+dx)\sqrt{a+b\tan(c+dx)}}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{\tan^m(c+dx)\sqrt{a+b\tan(c+dx)}}{i\tan(c+dx)+1}d\tan(c+dx)}{2d}$$

$\Big\downarrow$ 152

$$\frac{(A-iB)\sqrt{a+b\tan(c+dx)}\int \frac{\tan^m(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}}{1-i\tan(c+dx)}d\tan(c+dx)}{2d\sqrt{\frac{b\tan(c+dx)}{a}+1}}+\frac{(A+iB)\sqrt{a+b\tan(c+dx)}\int \frac{\tan^m(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}}{i\tan(c+dx)+1}d\tan(c+dx)}{2d\sqrt{\frac{b\tan(c+dx)}{a}+1}}$$

$\Big\downarrow$ 150

$$\frac{(A+iB)\tan^{m+1}(c+dx)\sqrt{a+b\tan(c+dx)}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},1,m+2,-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2d(m+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}+\frac{(A-iB)\tan^{m+1}(c+dx)\sqrt{a+b\tan(c+dx)}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},1,m+2,-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)}{2d(m+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}$$

input

```
Int[Tan[c + d*x]^m*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output

```
((A + I*B)*AppellF1[1 + m, -1/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*Tan
[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[a + b*Tan[c + d*x]])/(2*d*(1 + m)*Sqr
t[1 + (b*Tan[c + d*x])/a]) + ((A - I*B)*AppellF1[1 + m, -1/2, 1, 2 + m, -(
(b*Tan[c + d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[a + b*Tan[c
 + d*x]])/(2*d*(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \tan(dx+c)^m \sqrt{a+b\tan(dx+c)}\,(A+B\tan(dx+c))\,dx$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^m, x)
```

Sympy [F]

$$\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan(c+dx))\sqrt{a+b\tan(c+dx)}\tan^m(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*tan(c + d*x)**m, x)
```

Maxima [F]

$$\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^m, x)
```

Giac [F]

$$\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,\sqrt{a+b\tan(c+d\,x)}\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \tan^m(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\left(\int \tan(dx+c)^m\sqrt{a+\tan(dx+c)\,b}\,\tan(dx+c)\,dx\right)b$$
$$+\left(\int \tan(dx+c)^m\sqrt{a+\tan(dx+c)\,b}dx\right)a$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a)*tan(c + d*x),x)*b + int(tan(c
 + d*x)**m*sqrt(tan(c + d*x)*b + a),x)*a
```

3.490 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 209

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{(A-iB)\,\text{AppellF1}\left(\frac{1}{2},-m,1,\frac{3}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a-ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}\sqrt{a+b\tan(c}{(ia+b)d} + \frac{(iA-B)\,\text{AppellF1}\left(\frac{1}{2},-m,1,\frac{3}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a+ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}\sqrt{a+b\tan}{(a+ib)d}$$

output
```
(A-I*B)*AppellF1(1/2,-m,1,3/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(a-I*b))
*tan(d*x+c)^m*(a+b*tan(d*x+c))^(1/2)/(I*a+b)/d/((-b*tan(d*x+c)/a)^m)+(I*A-
B)*AppellF1(1/2,-m,1,3/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(a+I*b))*tan(
d*x+c)^m*(a+b*tan(d*x+c))^(1/2)/(a+I*b)/d/((-b*tan(d*x+c)/a)^m)
```

Mathematica [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x
]
```

output
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],
x]
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4086, 3042, 4085, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4086}$$

$$\frac{1}{2}(A+iB)\int \frac{(1-i\tan(c+dx))\tan^m(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{(i\tan(c+dx)+1)\tan^m(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB)\int \frac{(1-i\tan(c+dx))\tan(c+dx)^m}{\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{(i\tan(c+dx)+1)\tan(c+dx)^m}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4085}$$

$$\frac{(A-iB)\int \frac{\tan^m(c+dx)}{(1-i\tan(c+dx))\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{\tan^m(c+dx)}{(i\tan(c+dx)+1)\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}$$

$$\Big\downarrow \text{ 152}$$

$$\frac{(A-iB)\sqrt{\frac{b\tan(c+dx)}{a}+1}\int \frac{\tan^m(c+dx)}{(1-i\tan(c+dx))\sqrt{\frac{b\tan(c+dx)}{a}+1}}d\tan(c+dx)}{2d\sqrt{a+b\tan(c+dx)}}+\frac{(A+iB)\sqrt{\frac{b\tan(c+dx)}{a}+1}\int \frac{\tan^m(c+dx)}{(i\tan(c+dx)+1)\sqrt{\frac{b\tan(c+dx)}{a}+1}}d\tan(c+dx)}{2d\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,\text{AppellF1}\left(m+1,\frac{1}{2},1,m+2,-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2d(m+1)\sqrt{a+b\tan(c+dx)}}+\frac{(A-iB)\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,\text{AppellF1}\left(m+1,\frac{1}{2},1,m+2,-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)}{2d(m+1)\sqrt{a+b\tan(c+dx)}}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
((A + I*B)*AppellF1[1 + m, 1/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*Tan[
c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*d*(1 + m)*
Sqrt[a + b*Tan[c + d*x]]) + ((A - I*B)*AppellF1[1 + m, 1/2, 1, 2 + m, -((b
*Tan[c + d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[c
+ d*x])/a])/(2*d*(1 + m)*Sqrt[a + b*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &
& !IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{\sqrt{a+b\tan (dx+c)}}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan (dx+c)+A)\tan (dx+c)^m}{\sqrt{b\tan (dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*tan(d*x + c)^m/sqrt(b*tan(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(A+B\tan (c+dx))\tan^m (c+dx)}{\sqrt{a+b\tan (c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/sqrt(b*tan(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/sqrt(b*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{\sqrt{a+b\tan(c+d\,x)}}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \tan{(dx+c)}^m \sqrt{a+\tan{(dx+c)}\,b}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a),x)
```

3.491 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 211

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{(A-iB)\operatorname{AppellF1}\left(-\frac{1}{2},-m,1,\frac{1}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a-ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}}{(ia+b)d\sqrt{a+b\tan(c+dx)}} - \frac{(iA-B)\operatorname{AppellF1}\left(-\frac{1}{2},-m,1,\frac{1}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a+ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}}{(a+ib)d\sqrt{a+b\tan(c+dx)}}$$

output
```
-(A-I*B)*AppellF1(-1/2,-m,1,1/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(a-I*b
))*tan(d*x+c)^m/(I*a+b)/d/((-b*tan(d*x+c)/a)^m)/(a+b*tan(d*x+c))^(1/2)-(I*
A-B)*AppellF1(-1/2,-m,1,1/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(a+I*b))*t
an(d*x+c)^m/(a+I*b)/d/((-b*tan(d*x+c)/a)^m)/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx=\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2)
,x]
```

output
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2)
, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4086, 3042, 4085, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4086}$$

$$\frac{1}{2}(A+iB)\int \frac{(1-i\tan(c+dx))\tan^m(c+dx)}{(a+b\tan(c+dx))^{3/2}}dx+\frac{1}{2}(A-iB)\int \frac{(i\tan(c+dx)+1)\tan^m(c+dx)}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB)\int \frac{(1-i\tan(c+dx))\tan(c+dx)^m}{(a+b\tan(c+dx))^{3/2}}dx+\frac{1}{2}(A-iB)\int \frac{(i\tan(c+dx)+1)\tan(c+dx)^m}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4085}$$

$$\frac{(A-iB)\int \frac{\tan^m(c+dx)}{(1-i\tan(c+dx))(a+b\tan(c+dx))^{3/2}}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{\tan^m(c+dx)}{(i\tan(c+dx)+1)(a+b\tan(c+dx))^{3/2}}d\tan(c+dx)}{2d}$$

$$\Big\downarrow \text{152}$$

$$\frac{(A-iB)\sqrt{\frac{b\tan(c+dx)}{a}+1}\int \frac{\tan^m(c+dx)}{(1-i\tan(c+dx))\left(\frac{b\tan(c+dx)}{a}+1\right)^{3/2}}d\tan(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}+\frac{(A+iB)\sqrt{\frac{b\tan(c+dx)}{a}+1}\int \frac{\tan^m(c+dx)}{(i\tan(c+dx)+1)\left(\frac{b\tan(c+dx)}{a}+1\right)^{3/2}}d\tan(c+dx)}{2ad\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,\text{AppellF1}\left(m+1,\frac{3}{2},1,m+2,-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2ad(m+1)\sqrt{a+b\tan(c+dx)}}+\frac{(A-iB)\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\,\text{AppellF1}\left(m+1,\frac{3}{2},1,m+2,-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)}{2ad(m+1)\sqrt{a+b\tan(c+dx)}}$$

input

```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x]
```

output

```
((A + I*B)*AppellF1[1 + m, 3/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*Tan[
c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*a*d*(1 + m
)*Sqrt[a + b*Tan[c + d*x]]) + ((A - I*B)*AppellF1[1 + m, 3/2, 1, 2 + m, -(
(b*Tan[c + d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[
c + d*x])/a])/(2*a*d*(1 + m)*Sqrt[a + b*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{(a+b\tan (dx+c))^{\frac{3}{2}}}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan (dx+c)+A)\tan (dx+c)^m}{(b\tan (dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^m/(b^2
*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan (c+dx))\tan^m (c+dx)}{(a+b\tan (c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/(a + b*tan(c + d*x))**(3/2),
 x)
```

Maxima [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a)^(3/2),
x)
```

Giac [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^{3/2}}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{2\tan(dx+c)^m\sqrt{a+\tan(dx+c)\,b} - 4\Big(\int \frac{\tan(dx+c)^m\sqrt{a+\tan(dx+}}{2\tan(dx+c)bm+\tan(dx-}}{}$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
(2*tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a) - 4*int((tan(c + d*x)**m*sqrt(
tan(c + d*x)*b + a)*tan(c + d*x)**2)/(2*tan(c + d*x)*b*m + tan(c + d*x)*b
+ 2*a*m + a),x)*b*d*m**2 - 4*int((tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a)
*tan(c + d*x)**2)/(2*tan(c + d*x)*b*m + tan(c + d*x)*b + 2*a*m + a),x)*b*d
*m - int((tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a)*tan(c + d*x)**2)/(2*tan
(c + d*x)*b*m + tan(c + d*x)*b + 2*a*m + a),x)*b*d - 4*int((tan(c + d*x)**
m*sqrt(tan(c + d*x)*b + a)*tan(c + d*x))/(2*tan(c + d*x)*b*m + tan(c + d*x
)*b + 2*a*m + a),x)*a*d*m**2 - 2*int((tan(c + d*x)**m*sqrt(tan(c + d*x)*b
+ a)*tan(c + d*x))/(2*tan(c + d*x)*b*m + tan(c + d*x)*b + 2*a*m + a),x)*a*
d*m - 4*int((tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a))/(2*tan(c + d*x)**2*
b*m + tan(c + d*x)**2*b + 2*tan(c + d*x)*a*m + tan(c + d*x)*a),x)*a*d*m**2
 - 2*int((tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a))/(2*tan(c + d*x)**2*b*m
 + tan(c + d*x)**2*b + 2*tan(c + d*x)*a*m + tan(c + d*x)*a),x)*a*d*m)/(b*d
*(2*m + 1))
```

3.492 $\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 215

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{(A-iB)\,\text{AppellF1}\left(-\frac{3}{2},-m,1,-\frac{1}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a-ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}}{3(ia+b)d(a+b\tan(c+dx))^{3/2}} - \frac{(iA-B)\,\text{AppellF1}\left(-\frac{3}{2},-m,1,-\frac{1}{2},\frac{a+b\tan(c+dx)}{a},\frac{a+b\tan(c+dx)}{a+ib}\right)\tan^m(c+dx)\left(-\frac{b\tan(c+dx)}{a}\right)^{-m}}{3(a+ib)d(a+b\tan(c+dx))^{3/2}}$$

output
```
-1/3*(A-I*B)*AppellF1(-3/2,-m,1,-1/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/(
a-I*b))*tan(d*x+c)^m/(I*a+b)/d/((-b*tan(d*x+c)/a)^m)/(a+b*tan(d*x+c))^(3/2
)-1/3*(I*A-B)*AppellF1(-3/2,-m,1,-1/2,(a+b*tan(d*x+c))/a,(a+b*tan(d*x+c))/
(a+I*b))*tan(d*x+c)^m/(a+I*b)/d/((-b*tan(d*x+c)/a)^m)/(a+b*tan(d*x+c))^(3/
2)
```

Mathematica [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx=\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$

input
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2)
,x]
```

output
```
Integrate[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2)
, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {3042, 4086, 3042, 4085, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^m(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4086}$$

$$\frac{1}{2}(A+iB)\int \frac{(1-i\tan(c+dx))\tan^m(c+dx)}{(a+b\tan(c+dx))^{5/2}}dx+\frac{1}{2}(A-iB)\int \frac{(i\tan(c+dx)+1)\tan^m(c+dx)}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB)\int \frac{(1-i\tan(c+dx))\tan(c+dx)^m}{(a+b\tan(c+dx))^{5/2}}dx+\frac{1}{2}(A-iB)\int \frac{(i\tan(c+dx)+1)\tan(c+dx)^m}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4085}$$

$$\frac{(A-iB)\int \frac{\tan^m(c+dx)}{(1-i\tan(c+dx))(a+b\tan(c+dx))^{5/2}}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{\tan^m(c+dx)}{(i\tan(c+dx)+1)(a+b\tan(c+dx))^{5/2}}d\tan(c+dx)}{2d}$$

$$\Big\downarrow \text{152}$$

$$\frac{(A-iB)\sqrt{\frac{b\tan(c+dx)}{a}+1}\int \frac{\tan^m(c+dx)}{(1-i\tan(c+dx))\left(\frac{b\tan(c+dx)}{a}+1\right)^{5/2}}d\tan(c+dx)}{2a^2d\sqrt{a+b\tan(c+dx)}}+\frac{(A+iB)\sqrt{\frac{b\tan(c+dx)}{a}+1}\int \frac{\tan^m(c+dx)}{(i\tan(c+dx)+1)\left(\frac{b\tan(c+dx)}{a}+1\right)^{5/2}}d\tan(c+dx)}{2a^2d\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{150}$$

$$\frac{(A+iB)\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\operatorname{AppellF1}\left(m+1,\frac{5}{2},1,m+2,-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)}{2a^2d(m+1)\sqrt{a+b\tan(c+dx)}}+\frac{(A-iB)\tan^{m+1}(c+dx)\sqrt{\frac{b\tan(c+dx)}{a}+1}\operatorname{AppellF1}\left(m+1,\frac{5}{2},1,m+2,-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)}{2a^2d(m+1)\sqrt{a+b\tan(c+dx)}}$$

input
```
Int[(Tan[c + d*x]^m*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x]
```

output
```
((A + I*B)*AppellF1[1 + m, 5/2, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*Tan[
c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[1 + (b*Tan[c + d*x])/a])/(2*a^2*d*(1 +
 m)*Sqrt[a + b*Tan[c + d*x]]) + ((A - I*B)*AppellF1[1 + m, 5/2, 1, 2 + m,
-((b*Tan[c + d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*Sqrt[1 + (b*Ta
n[c + d*x])/a])/(2*a^2*d*(1 + m)*Sqrt[a + b*Tan[c + d*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \frac{\tan (dx+c)^m (A+B\tan (dx+c))}{(a+b\tan (dx+c))^{\frac{5}{2}}}dx$$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan (dx+c)+A)\tan (dx+c)^m}{(b\tan (dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*tan(d*x + c)^m/(b^3
*tan(d*x + c)^3 + 3*a*b^2*tan(d*x + c)^2 + 3*a^2*b*tan(d*x + c) + a^3), x)
```

Sympy [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan (c+dx))\tan^m (c+dx)}{(a+b\tan (c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)**m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*tan(c + d*x)**m/(a + b*tan(c + d*x))**(5/2),
 x)
```

Maxima [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(b\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a)^(5/2),
x)
```

Giac [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\tan(dx+c)^m}{(b\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*tan(d*x + c)^m/(b*tan(d*x + c) + a)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\tan(c+d\,x)^m\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^{5/2}}\,dx$$

input
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int((tan(c + d*x)^m*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\tan^m(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \left(\int \frac{\tan\left(dx+c\right)^m\sqrt{a+\tan\left(dx+\right.}}{2\tan\left(dx+c\right)^2b^2m-\tan\left(dx+c\right)^2b^2+4\tan\left(dx+c\right)abm}\right.$$

$-1)$

input
```
int(tan(d*x+c)^m*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((tan(c + d*x)**m*sqrt(tan(c + d*x)*b + a))/(2*tan(c + d*x)**2*b**2*m -
 tan(c + d*x)**2*b**2 + 4*tan(c + d*x)*a*b*m - 2*tan(c + d*x)*a*b + 2*a**2
*m - a**2),x)*(2*m - 1)
```

3.493 $\int \tan^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 185

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{(A+iB)\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{1+m}(c+dx)(a+b\tan(c+dx)}{2d(1+m)}$$
$$+ \frac{(A-iB)\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)\tan^{1+m}(c+dx)(a+b\tan(c+dx)}{2d(1+m)}$$

output
```
1/2*(A+I*B)*AppellF1(1+m,1,-n,2+m,-I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c
)^(1+m)*(a+b*tan(d*x+c))^n/d/(1+m)/(((a+b*tan(d*x+c))/a)^n)+1/2*(A-I*B)*Ap
pellF1(1+m,1,-n,2+m,I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c)^(1+m)*(a+b*ta
n(d*x+c))^n/d/(1+m)/(((a+b*tan(d*x+c))/a)^n)
```

Mathematica [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \tan^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Integrate[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]), x]
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 4086, 3042, 4085, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^m(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^m(A+B\tan(c+dx))(a+b\tan(c+dx))^ndx$$

$$\Big\downarrow \text{4086}$$

$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))\tan^m(c+dx)(a+b\tan(c+dx))^ndx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)\tan^m(c+dx)(a+b\tan(c+dx))^ndx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))\tan(c+dx)^m(a+b\tan(c+dx))^ndx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)\tan(c+dx)^m(a+b\tan(c+dx))^ndx$$

$$\Big\downarrow \text{4085}$$

$$\frac{(A-iB)\int \frac{\tan^m(c+dx)(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{\tan^m(c+dx)(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d\tan(c+dx)}{2d}$$

$\Big\downarrow$ 152

$$\frac{(A-iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int \frac{\tan^m(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int \frac{\tan^m(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{i\tan(c+dx)+1}d\tan(c+dx)}{2d}$$

$\Big\downarrow$ 150

$$\frac{(A+iB)\tan^{m+1}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\operatorname{AppellF1}\left(m+1,-n,1,m+2,-\frac{b\tan(c+dx)}{a},-\right.}{2d(m+1)}$$
$$\frac{(A-iB)\tan^{m+1}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\operatorname{AppellF1}\left(m+1,-n,1,m+2,-\frac{b\tan(c+dx)}{a},i\right.}{2d(m+1)}$$

input
```
Int[Tan[c + d*x]^m*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((A + I*B)*AppellF1[1 + m, -n, 1, 2 + m, -((b*Tan[c + d*x])/a), (-I)*Tan[c
 + d*x]]*Tan[c + d*x]^(1 + m)*(a + b*Tan[c + d*x])^n)/(2*d*(1 + m)*(1 + (b
*Tan[c + d*x])/a)^n) + ((A - I*B)*AppellF1[1 + m, -n, 1, 2 + m, -((b*Tan[c
 + d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(1 + m)*(a + b*Tan[c + d*x])^n)/
(2*d*(1 + m)*(1 + (b*Tan[c + d*x])/a)^n)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] && !IntegerQ[m] && !IntegerQ[n] && !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && !IntegerQ[m] &
&  !IntegerQ[n] && !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \tan \left(dx+c\right)^{m} \left(a+b\tan \left(dx+c\right)\right)^{n} \left(A+B\tan \left(dx+c\right)\right) dx$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^m, x)
```

Sympy [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan(c+dx))\,(a+b\tan(c+dx))^n\tan^m(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**m*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*tan(c + d*x)**m, x)
```

Maxima [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^m, x)
```

Giac [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\tan(dx+c)^m\,dx$$

input
```
integrate(tan(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^m\,(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \tan^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \left(\int \tan(dx+c)^m (a+\tan(dx+c)\,b)^n \tan(dx+c)\,dx\right) b$$
$$+ \left(\int \tan(dx+c)^m (a+\tan(dx+c)\,b)^n\,dx\right) a$$

input
```
int(tan(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(c + d*x)**m*(tan(c + d*x)*b + a)**n*tan(c + d*x),x)*b + int(tan(c
+ d*x)**m*(tan(c + d*x)*b + a)**n,x)*a
```

3.494 $\int \tan^4(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 387

$$\int \tan^4(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = -\frac{(Ab^3(2+n)(3+n)(4+n)-a(b^2B(3+n)(4+n)-2a(3aB-Ab(4+n))))\,(a+b\tan(c+dx))^{1+n}}{b^4d(1+n)(2+n)(3+n)(4+n)} + \frac{(A-iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(ia+b)d(1+n)} - \frac{(A+iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+dx))^{1+n}}{2(ia-b)d(1+n)} - \frac{(b^2B(3+n)(4+n)-2a(3aB-Ab(4+n)))\tan(c+dx)(a+b\tan(c+dx))^{1+n}}{b^3d(2+n)(3+n)(4+n)} - \frac{(3aB-Ab(4+n))\tan^2(c+dx)(a+b\tan(c+dx))^{1+n}}{b^2d(3+n)(4+n)} + \frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(4+n)}$$

output
```
-(A*b^3*(2+n)*(3+n)*(4+n)-a*(b^2*B*(3+n)*(4+n)-2*a*(3*B*a-A*b*(4+n))))*(a+
b*tan(d*x+c))^(1+n)/b^4/d/(1+n)/(2+n)/(3+n)/(4+n)+1/2*(A-I*B)*hypergeom([1
, 1+n],[2+n],(a+b*tan(d*x+c))/(a-I*b))*(a+b*tan(d*x+c))^(1+n)/(I*a+b)/d/(1
+n)-1/2*(A+I*B)*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+I*b))*(a+b*ta
n(d*x+c))^(1+n)/(I*a-b)/d/(1+n)-(b^2*B*(3+n)*(4+n)-2*a*(3*B*a-A*b*(4+n)))*
tan(d*x+c)*(a+b*tan(d*x+c))^(1+n)/b^3/d/(2+n)/(3+n)/(4+n)-(3*B*a-A*b*(4+n)
)*tan(d*x+c)^2*(a+b*tan(d*x+c))^(1+n)/b^2/d/(3+n)/(4+n)+B*tan(d*x+c)^3*(a+
b*tan(d*x+c))^(1+n)/b/d/(4+n)
```

Mathematica [A] (verified)

Time = 4.08 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.99

$$\int \tan^4(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{(a+b\tan(c+dx))^{1+n}\left(i\Big(2i(a-ib)(a+ib)\left(6a^3B-2a^2Ab(4+n)-ab^2B(3+n)(4+n)+Ab^3(2+n)\right.\right.}{}$$

input
```
Integrate[Tan[c + d*x]^4*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*(I*((2*I)*(a - I*b)*(a + I*b)*(6*a^3*B - 2*a
^2*A*b*(4 + n) - a*b^2*B*(3 + n)*(4 + n) + A*b^3*(2 + n)*(3 + n)*(4 + n))
- (a + I*b)*b^4*(A - I*B)*(24 + 26*n + 9*n^2 + n^3)*Hypergeometric2F1[1, 1
 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)] + (a - I*b)*b^4*(A + I*B)*(24
 + 26*n + 9*n^2 + n^3)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d
*x])/(a + I*b)]) + 2*(a - I*b)*(a + I*b)*b*(1 + n)*(6*a^2*B - 2*a*A*b*(4 +
 n) - b^2*B*(3 + n)*(4 + n))*Tan[c + d*x] - 2*(a - I*b)*(a + I*b)*b^2*(1 +
 n)*(2 + n)*(3*a*B - A*b*(4 + n))*Tan[c + d*x]^2 + 2*(a - I*b)*(a + I*b)*b
^3*B*(1 + n)*(2 + n)*(3 + n)*Tan[c + d*x]^3))/(2*(a - I*b)*(a + I*b)*b^4*d
*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {3042, 4090, 25, 3042, 4130, 25, 3042, 4130, 25, 3042, 4113, 3042, 4022, 3042, 4020, 25, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^4(A+B\tan(c+dx))(a+b\tan(c+dx))^ndx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -\tan^2(c+dx)(a+b\tan(c+dx))^n\left((3aB-Ab(n+4))\tan^2(c+dx)+bB(n+4)\tan(c+dx)+3aB\right)dx}{b(n+4)}+\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)}$$

$$\Big\downarrow \text{25}$$

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)}-\frac{\int \tan^2(c+dx)(a+b\tan(c+dx))^n\left((3aB-Ab(n+4))\tan^2(c+dx)+bB(n+4)\tan(c+dx)+3aB\right)dx}{b(n+4)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)}-\frac{\int \tan(c+dx)^2(a+b\tan(c+dx))^n\left((3aB-Ab(n+4))\tan(c+dx)^2+bB(n+4)\tan(c+dx)+3aB\right)dx}{b(n+4)}$$

$$\Big\downarrow \text{4130}$$

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)}-\frac{\frac{\int -\tan(c+dx)(a+b\tan(c+dx))^n\left(-A(n+3)(n+4)\tan(c+dx)b^2+\left(6Ba^2-2Ab(n+4)a-b^2B(n+3)(n+4)\right)\tan^2(c+dx)+2a(3aB-Ab(n+4))\right)dx}{b(n+3)}-}{b(n+4)}$$

$$\downarrow 25$$

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)} - \frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)} - \frac{\int \tan(c+dx)(a+b\tan(c+dx))^n(-A(n+3)(n+4)\tan(c+dx)b^2+(6Ba^2-2Ab(n+4)a-b^2B(\ldots}{b(n+3)}}{b(n+4)}$$

$$\downarrow 3042$$

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)} - \frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)} - \frac{\int \tan(c+dx)(a+b\tan(c+dx))^n(-A(n+3)(n+4)\tan(c+dx)b^2+(6Ba^2-2Ab(n+4)a-b^2B(\ldots}{b(n+3)}}{b(n+4)}$$

$$\downarrow 4130$$

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)} - \frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)} - \frac{\frac{\int -(a+b\tan(c+dx))^n(-B(n+2)(n+3)(n+4)\tan(c+dx)b^3+(6Ba^3-2Ab(n+4)a^2-b^2B(n+3)(n+4)a \ldots}{b(n+2)}}{\ldots}}{b(n+ \ldots}$$

$$\downarrow 25$$

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)} - \frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)} - \frac{\frac{\tan(c+dx)(6a^2B-2aAb(n+4)-b^2B(n+3)(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+2)} - \frac{\int (a+b\tan(c+dx))^n(-B(n \ldots}{\ldots}}{\ldots}}{b(n+ \ldots}$$

$$\downarrow 3042$$

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)} - \frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)} - \frac{\frac{\tan(c+dx)(6a^2B-2aAb(n+4)-b^2B(n+3)(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+2)} - \frac{\int (a+b\tan(c+dx))^n(-B(n \ldots}{\ldots}}{\ldots}}{b(n+ \ldots}$$

$$\downarrow 4113$$

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)} - \frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)} - \frac{\frac{\tan(c+dx)(6a^2B-2aAb(n+4)-b^2B(n+3)(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+2)} - \frac{\int (a+b\tan(c+dx))^n(-A(n \ldots}{\ldots}}{\ldots}}{b(n+4)}$$

$$\downarrow 3042$$

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)}-$$

$$\frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)\left(6a^2B-2aAb(n+4)-b^2B(n+3)(n+4)\right)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\int(a+b\tan(c+dx))^n\left(-A(n\ldots\right.}{\ldots}}{\ldots}}{b(n+4)}$$

$\downarrow$ 4022

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)}-$$

$$\frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)\left(6a^2B-2aAb(n+4)-b^2B(n+3)(n+4)\right)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{-\frac{1}{2}b^3(n+2)(n+3)(n+4)(A\ldots}{\ldots}}{\ldots}}{\ldots}$$

$\downarrow$ 3042

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)}-$$

$$\frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)\left(6a^2B-2aAb(n+4)-b^2B(n+3)(n+4)\right)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{-\frac{1}{2}b^3(n+2)(n+3)(n+4)(A\ldots}{\ldots}}{\ldots}}{\ldots}$$

$\downarrow$ 4020

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)}-$$

$$\frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)\left(6a^2B-2aAb(n+4)-b^2B(n+3)(n+4)\right)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{-\frac{ib^3(n+2)(n+3)(n+4)(A\ldots}{\ldots}}{\ldots}}{\ldots}}{\ldots}$$

$\downarrow$ 25

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)}-$$

$$\frac{\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)\left(6a^2B-2aAb(n+4)-b^2B(n+3)(n+4)\right)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\frac{ib^3(n+2)(n+3)(n+4)(A-\ldots}{\ldots}}{\ldots}}{\ldots}}{\ldots}$$

$\downarrow$ 78

$$\frac{B\tan^3(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+4)} -$$

$$\frac{\tan^2(c+dx)(3aB-Ab(n+4))(a+b\tan(c+dx))^{n+1}}{bd(n+3)} - \frac{\frac{\tan(c+dx)\left(6a^2B-2aAb(n+4)-b^2B(n+3)(n+4)\right)(a+b\tan(c+dx))^{n+1}}{bd(n+2)} - \frac{\left(6a^3B-2a^2Ab(n+4)-ab^2\right.}{}}{}$$

input
```
Int[Tan[c + d*x]^4*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]^3*(a + b*Tan[c + d*x])^(1 + n))/(b*d*(4 + n)) - (((3*a*B -
 A*b*(4 + n))*Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(b*d*(3 + n)) -
 (((6*a^2*B - 2*a*A*b*(4 + n) - b^2*B*(3 + n)*(4 + n))*Tan[c + d*x]*(a + b
*Tan[c + d*x])^(1 + n))/(b*d*(2 + n)) - (((6*a^3*B - 2*a^2*A*b*(4 + n) - a
*b^2*B*(3 + n)*(4 + n) + A*b^3*(2 + n)*(3 + n)*(4 + n))*(a + b*Tan[c + d*x
])^(1 + n))/(b*d*(1 + n)) + ((I/2)*b^3*(A - I*B)*(2 + n)*(3 + n)*(4 + n)*H
ypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)]*(a + b*T
an[c + d*x])^(1 + n))/((a - I*b)*d*(1 + n)) - ((I/2)*b^3*(A + I*B)*(2 + n)
*(3 + n)*(4 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(
a + I*b)]*(a + b*Tan[c + d*x])^(1 + n))/((a + I*b)*d*(1 + n)))/(b*(2 + n))
)/(b*(3 + n)))/(b*(4 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [F]

$$\int \tan\left(dx+c\right)^{4}\left(a+b\tan\left(dx+c\right)\right)^{n}\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(tan(d*x+c)^4*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^4*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^4(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan\left(dx+c\right)+A)(b\tan\left(dx+c\right)+a)^{n}\tan\left(dx+c\right)^{4}\,dx$$

input
```
integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*tan(d*x + c)^5 + A*tan(d*x + c)^4)*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int \tan^4(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (A+B\tan(c+dx))\,(a+b\tan(c+dx))^n\tan^4(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**4*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*tan(c + d*x)**4, x)
```

Maxima [F]

$$\int \tan^4(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\tan(dx+c)^4\,dx$$

input
```
integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Giac [F]

$$\int \tan^4(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\tan(dx+c)^4\,dx$$

input
```
integrate(tan(d*x+c)^4*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^4(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \tan(c+d\,x)^4\,(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(tan(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^4*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \tan^4(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \left(\int (a+\tan(dx+c)\,b)^n\tan(dx+c)^5\,dx\right)b + \left(\int (a+\tan(dx+c)\,b)^n\tan(dx+c)^4\,dx\right)a$$

input
```
int(tan(d*x+c)^4*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*b + a)**n*tan(c + d*x)**5,x)*b + int((tan(c + d*x)*b + a
)**n*tan(c + d*x)**4,x)*a
```

3.495 $\int \tan^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 291

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{(2a^2B-aAb(3+n)-b^2B(6+5n+n^2))\,(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)(2+n)(3+n)} + \frac{(iA+B)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(ia+b)d(1+n)} + \frac{(A+iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a+ib)d(1+n)} - \frac{(2aB-Ab(3+n))\tan(c+dx)(a+b\tan(c+dx))^{1+n}}{b^2d(2+n)(3+n)} + \frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(3+n)}$$

output
```
(2*B*a^2-a*A*b*(3+n)-b^2*B*(n^2+5*n+6))*(a+b*tan(d*x+c))^(1+n)/b^3/d/(1+n)
/(2+n)/(3+n)+1/2*(I*A+B)*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-I*b)
)*(a+b*tan(d*x+c))^(1+n)/(I*a+b)/d/(1+n)+1/2*(A+I*B)*hypergeom([1, 1+n],[2
+n],(a+b*tan(d*x+c))/(a+I*b))*(a+b*tan(d*x+c))^(1+n)/(a+I*b)/d/(1+n)-(2*B*
a-A*b*(3+n))*tan(d*x+c)*(a+b*tan(d*x+c))^(1+n)/b^2/d/(2+n)/(3+n)+B*tan(d*x
+c)^2*(a+b*tan(d*x+c))^(1+n)/b/d/(3+n)
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.97

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{(a+b\tan(c+dx))^{1+n}\left(2(a-ib)(a+ib)\left(2a^2B-aAb(3+n)-b^2B(2+n)(3+n)\right)+(a+ib)b^3(A-i\right.}{}$$

input
```
Integrate[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*(2*(a - I*b)*(a + I*b)*(2*a^2*B - a*A*b*(3 +
 n) - b^2*B*(2 + n)*(3 + n)) + (a + I*b)*b^3*(A - I*B)*(6 + 5*n + n^2)*Hyp
ergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)] + (a - I*b
)*b^3*(A + I*B)*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*
Tan[c + d*x])/(a + I*b)] - 2*(a - I*b)*(a + I*b)*b*(1 + n)*(2*a*B - A*b*(3
 + n))*Tan[c + d*x] + 2*(a - I*b)*(a + I*b)*b^2*B*(1 + n)*(2 + n)*Tan[c +
d*x]^2))/(2*(a - I*b)*(a + I*b)*b^3*d*(1 + n)*(2 + n)*(3 + n))
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 4090, 25, 3042, 4130, 25, 3042, 4113, 3042, 4022, 3042, 4020, 25, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3(A+B\tan(c+dx))(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{4090}$$

$$\frac{\int -\tan(c+dx)(a+b\tan(c+dx))^n\left((2aB-Ab(n+3))\tan^2(c+dx)+bB(n+3)\tan(c+dx)+2aB\right)dx}{b(n+3)}+\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\int \tan(c+dx)(a+b\tan(c+dx))^n\left((2aB-Ab(n+3))\tan^2(c+dx)+bB(n+3)\tan(c+dx)+2aB\right)dx}{b(n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\int \tan(c+dx)(a+b\tan(c+dx))^n\left((2aB-Ab(n+3))\tan(c+dx)^2+bB(n+3)\tan(c+dx)+2aB\right)dx}{b(n+3)}$$

$$\Big\downarrow \text{4130}$$

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\int -(a+b\tan(c+dx))^n\left(-A(n+2)(n+3)\tan(c+dx)b^2+(2Ba^2-Ab(n+3)a-b^2B(n+2)(n+3))\tan^2(c+dx)+a(2aB-Ab(n+3))\right)dx}{b(n+2)}+\frac{\tan(c+dx)(}{}}{b(n+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)(2aB-Ab(n+3))(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\int (a+b\tan(c+dx))^n\left(-A(n+2)(n+3)\tan(c+dx)b^2+(2Ba^2-Ab(n+3)a-b^2B(n+2)(n+3))\right.}{b(n+2)}}{b(n+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)(2aB-Ab(n+3))(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\int(a+b\tan(c+dx))^n\left(-A(n+2)(n+3)\tan(c+dx)b^2+\left(2Ba^2-Ab(n+3)a-b^2B(n+2)(n+3)\right)\cdots\right.}{b(n+2)}}{b(n+3)}$$

$\downarrow$ 4113

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)(2aB-Ab(n+3))(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\int(a+b\tan(c+dx))^n\left(b^2B(n+2)(n+3)-Ab^2(n+2)(n+3)\tan(c+dx)\right)dx+\frac{\left(2a^2B-aAb(n+3)-\cdots\right.}{\cdots}}{b(n+2)}}{b(n+3)}$$

$\downarrow$ 3042

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)(2aB-Ab(n+3))(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\int(a+b\tan(c+dx))^n\left(b^2B(n+2)(n+3)-Ab^2(n+2)(n+3)\tan(c+dx)\right)dx+\frac{\left(2a^2B-aAb(n+3)-\cdots\right.}{\cdots}}{b(n+2)}}{b(n+3)}$$

$\downarrow$ 4022

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)(2aB-Ab(n+3))(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{-\frac{1}{2}b^2(n+2)(n+3)(-B+iA)\int(1-i\tan(c+dx))(a+b\tan(c+dx))^n dx+\frac{1}{2}b^2(n+2)(n+3)(B+\cdots}{\cdots}}{b(n+3)}$$

$\downarrow$ 3042

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)(2aB-Ab(n+3))(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{-\frac{1}{2}b^2(n+2)(n+3)(-B+iA)\int(1-i\tan(c+dx))(a+b\tan(c+dx))^n dx+\frac{1}{2}b^2(n+2)(n+3)(B+\cdots}{\cdots}}{b(n+3)}$$

$\downarrow$ 4020

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)(2aB-Ab(n+3))(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\frac{ib^2(n+2)(n+3)(B+iA)\int-\frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d}+\frac{ib^2(n+2)(n+3)(-B+iA)\int-\frac{(a+b\tan\cdots}{i\tan(c\cdots}}{2d}\cdots}{b(n+2)}}{b(n+3)}$$

$\downarrow$ 25

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)(2aB-Ab(n+3))(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{-\frac{ib^2(n+2)(n+3)(B+iA)\int\frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d}-\frac{ib^2(n+2)(n+3)(-B+iA)\int\frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}}{2d}}{b(n+2)}}{b(n+3)}$$

$\Big\downarrow$ 78

$$\frac{B\tan^2(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+3)}-\frac{\frac{\tan(c+dx)(2aB-Ab(n+3))(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\frac{\left(2a^2B-aAb(n+3)-b^2B(n+2)(n+3)\right)(a+b\tan(c+dx))^{n+1}}{bd(n+1)}-\frac{ib^2(n+2)(n+3)(B+iA)(a+b\tan(c+dx))}{}}{}}{b(n+}$$

input
```
Int[Tan[c + d*x]^3*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(b*d*(3 + n)) - (((2*a*B -
 A*b*(3 + n))*Tan[c + d*x]*(a + b*Tan[c + d*x])^(1 + n))/(b*d*(2 + n)) - (
((2*a^2*B - a*A*b*(3 + n) - b^2*B*(2 + n)*(3 + n))*(a + b*Tan[c + d*x])^(1
 + n))/(b*d*(1 + n)) - ((I/2)*b^2*(I*A + B)*(2 + n)*(3 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)]*(a + b*Tan[c + d*x])^
(1 + n))/((a - I*b)*d*(1 + n)) - ((I/2)*b^2*(I*A - B)*(2 + n)*(3 + n)*Hype
rgeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)]*(a + b*Tan[
c + d*x])^(1 + n))/((a + I*b)*d*(1 + n)))/(b*(2 + n)))/(b*(3 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

Maple [F]

$$\int \tan (dx+c)^3 (a+b\tan (dx+c))^n (A+B\tan (dx+c))\,dx$$

input
```
int(tan(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan (dx+c)+A)(b\tan (dx+c)+a)^n \tan (dx+c)^3\,dx$$

input
```
integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*tan(d*x + c)^4 + A*tan(d*x + c)^3)*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^n\tan^3{(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**3*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*tan(c + d*x)**3, x)
```

Maxima [F]

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^n\tan{(dx+c)}^3\,dx$$

input
```
integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Giac [F]

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^n\tan{(dx+c)}^3\,dx$$

input
```
integrate(tan(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^3\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

input
```
int(tan(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \tan^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\left(\int (a+\tan(dx+c)\,b)^n\tan(dx+c)^4\,dx\right)b$$
$$+\left(\int (a+\tan(dx+c)\,b)^n\tan(dx+c)^3\,dx\right)a$$

input
```
int(tan(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*b + a)**n*tan(c + d*x)**4,x)*b + int((tan(c + d*x)*b + a
)**n*tan(c + d*x)**3,x)*a
```

3.496 $\int \tan^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 219

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = -\frac{(aB-Ab(2+n))(a+b\tan(c+dx))^{1+n}}{b^2d(1+n)(2+n)} + \frac{(iA+B)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a-ib)d(1+n)} + \frac{(A+iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+dx))^{1+n}}{2(ia-b)d(1+n)} + \frac{B\tan(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(2+n)}$$

output
```
-(B*a-A*b*(2+n))*(a+b*tan(d*x+c))^(1+n)/b^2/d/(1+n)/(2+n)+1/2*(I*A+B)*hype
rgeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-I*b))*(a+b*tan(d*x+c))^(1+n)/(a-I
*b)/d/(1+n)+1/2*(A+I*B)*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+I*b))
*(a+b*tan(d*x+c))^(1+n)/(I*a-b)/d/(1+n)+B*tan(d*x+c)*(a+b*tan(d*x+c))^(1+n
)/b/d/(2+n)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.77

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\frac{(a+b\tan(c+dx))^{1+n}\left(\frac{4Ab-2aB+2Abn}{b+bn}+\frac{b(iA+B)(2+n)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)}{(a-ib)(1+n)}+\frac{b(-iA+B)(2+n)}{}\right.}{2bd(2+n)}$$

input
```
Integrate[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((a + b*Tan[c + d*x])^(1 + n)*((4*A*b - 2*a*B + 2*A*b*n)/(b + b*n) + (b*(I
*A + B)*(2 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a
 - I*b)])/((a - I*b)*(1 + n)) + (b*((-I)*A + B)*(2 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)])/((a + I*b)*(1 + n)) + 2*
B*Tan[c + d*x]))/(2*b*d*(2 + n))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4090, 25, 3042, 4113, 3042, 4022, 3042, 4020, 25, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^2(A+B\tan(c+dx))(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{ 4090}$$

$$\frac{\int -(a+b\tan(c+dx))^n \left((aB-Ab(n+2))\tan^2(c+dx)+bB(n+2)\tan(c+dx)+aB\right)dx}{b(n+2)}+\frac{B\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}$$

$\downarrow$ 25

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\int (a+b\tan(c+dx))^n \left((aB-Ab(n+2))\tan^2(c+dx)+bB(n+2)\tan(c+dx)+aB\right)dx}{b(n+2)}$$

$\downarrow$ 3042

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\int (a+b\tan(c+dx))^n \left((aB-Ab(n+2))\tan(c+dx)^2+bB(n+2)\tan(c+dx)+aB\right)dx}{b(n+2)}$$

$\downarrow$ 4113

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\int (a+b\tan(c+dx))^n(Ab(n+2)+bB\tan(c+dx)(n+2))dx+\frac{(aB-Ab(n+2))(a+b\tan(c+dx))^{n+1}}{bd(n+1)}}{b(n+2)}$$

$\downarrow$ 3042

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\int (a+b\tan(c+dx))^n(Ab(n+2)+bB\tan(c+dx)(n+2))dx+\frac{(aB-Ab(n+2))(a+b\tan(c+dx))^{n+1}}{bd(n+1)}}{b(n+2)}$$

$\downarrow$ 4022

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\frac{1}{2}b(n+2)(A+iB)\int(1-i\tan(c+dx))(a+b\tan(c+dx))^n dx+\frac{1}{2}b(n+2)(A-iB)\int(i\tan(c+dx)+1)(a+b}{b(n+2)}$$

$\downarrow$ 3042

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\frac{1}{2}b(n+2)(A+iB)\int(1-i\tan(c+dx))(a+b\tan(c+dx))^n dx+\frac{1}{2}b(n+2)(A-iB)\int(i\tan(c+dx)+1)(a+b}{b(n+2)}$$

$\big\downarrow$ 4020

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{\frac{ib(n+2)(A-iB)\int -\frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d}-\frac{ib(n+2)(A+iB)\int -\frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d(-i\tan(c+dx))}{2d}+\frac{(aB-Ab(n+2))(a+b\tan(c+c}{bd(n+1)}}{b(n+2)}$$

$\big\downarrow$ 25

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{-\frac{ib(n+2)(A-iB)\int \frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d}+\frac{ib(n+2)(A+iB)\int \frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d(-i\tan(c+dx))}{2d}+\frac{(aB-Ab(n+2))(a+b\tan(c+dx}{bd(n+1)}}{b(n+2)}$$

$\big\downarrow$ 78

$$\frac{B\tan(c+dx)(a+b\tan(c+dx))^{n+1}}{bd(n+2)}-\frac{-\frac{ib(n+2)(A-iB)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}+\frac{ib(n+2)(A+iB)(a+b\tan(c+dx))^{n+1}\text{Hypergeomet}}{2d(n+1)(a+ib)}}{b(n+2)}$$

input
```
Int[Tan[c + d*x]^2*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(B*Tan[c + d*x]*(a + b*Tan[c + d*x])^(1 + n))/(b*d*(2 + n)) - (((a*B - A*b
*(2 + n))*(a + b*Tan[c + d*x])^(1 + n))/(b*d*(1 + n)) - ((I/2)*b*(A - I*B)
*(2 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)
]*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*d*(1 + n)) + ((I/2)*b*(A + I*B)
*(2 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)
]*(a + b*Tan[c + d*x])^(1 + n))/((a + I*b)*d*(1 + n)))/(b*(2 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [F]

$$\int \tan\left(dx+c\right)^{2}\left(a+b\tan\left(dx+c\right)\right)^{n}\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan\left(dx+c\right)+A)(b\tan\left(dx+c\right)+a)^{n}\tan\left(dx+c\right)^{2}\,dx$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*tan(d*x + c)^3 + A*tan(d*x + c)^2)*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (A+B\tan(c+dx))\,(a+b\tan(c+dx))^n\tan^2(c+dx)\,dx$$

input
```
integrate(tan(d*x+c)**2*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*tan(c + d*x)**2, x)
```

Maxima [F]

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\tan(dx+c)^2\,dx$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Giac [F]

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\tan(dx+c)^2\,dx$$

input
```
integrate(tan(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^2\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

input
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \tan^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\left(\int (a+\tan(dx+c)\,b)^n\tan(dx+c)^3\,dx\right)b$$
$$+\left(\int (a+\tan(dx+c)\,b)^n\tan(dx+c)^2\,dx\right)a$$

input
```
int(tan(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*b + a)**n*tan(c + d*x)**3,x)*b + int((tan(c + d*x)*b + a
)**n*tan(c + d*x)**2,x)*a
```

3.497 $\int \tan(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 168

$$\int \tan(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{B(a+b\tan(c+dx))^{1+n}}{bd(1+n)} - \frac{(A-iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a-ib)d(1+n)} - \frac{(A+iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a+ib)d(1+n)}$$

output
```
B*(a+b*tan(d*x+c))^(1+n)/b/d/(1+n)-1/2*(A-I*B)*hypergeom([1, 1+n],[2+n],(a
+b*tan(d*x+c))/(a-I*b))*(a+b*tan(d*x+c))^(1+n)/(a-I*b)/d/(1+n)-1/2*(A+I*B)
*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+I*b))*(a+b*tan(d*x+c))^(1+n)
/(a+I*b)/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

$$\int \tan(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{\left(\frac{2B}{b}-\frac{(A-iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)}{a-ib}-\frac{(A+iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+ib}\right)}{a+ib}\right)(a+b\tan(c+dx))^{1+n}}{2d(1+n)}$$

input
```
Integrate[Tan[c + d*x]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(((2*B)/b - ((A - I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d
*x])/(a - I*b)])/(a - I*b) - ((A + I*B)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (a + b*Tan[c + d*x])/(a + I*b)])/(a + I*b))*(a + b*Tan[c + d*x])^(1 + n))
/(2*d*(1 + n))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {3042, 4075, 3042, 4022, 3042, 4020, 25, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{4075}$$

$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^n dx+\frac{B(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

$$\Big\downarrow \text{3042}$$

$$\int (A\tan(c+dx)-B)(a+b\tan(c+dx))^n dx+\frac{B(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(-B+iA)\int (1-i\tan(c+dx))(a+b\tan(c+dx))^n dx-\frac{1}{2}(B+iA)\int (i\tan(c+dx)+1)(a+b\tan(c+dx))^n dx+\frac{B(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(-B+iA)\int (1-i\tan(c+dx))(a+b\tan(c+dx))^n dx-\frac{1}{2}(B+iA)\int (i\tan(c+dx)+1)(a+b\tan(c+dx))^n dx+\frac{B(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

$$\Big\downarrow \text{4020}$$

$$-\frac{i(B+iA)\int -\frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d}-\frac{i(-B+iA)\int -\frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d(-i\tan(c+dx))}{2d}+\frac{B(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{i(B+iA)\int \frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d}+\frac{i(-B+iA)\int \frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d(-i\tan(c+dx))}{2d}+\frac{B(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

$$\Big\downarrow \text{78}$$

$$\frac{i(B+iA)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}+\frac{i(-B+iA)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)}+\frac{B(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

input
```
Int[Tan[c + d*x]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
(B*(a + b*Tan[c + d*x])^(1 + n))/(b*d*(1 + n)) + ((I/2)*(I*A + B)*Hypergeo
metric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)]*(a + b*Tan[c +
d*x])^(1 + n))/((a - I*b)*d*(1 + n)) + ((I/2)*(I*A - B)*Hypergeometric2F1[
1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)]*(a + b*Tan[c + d*x])^(1 +
 n))/((a + I*b)*d*(1 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [F]

$$\int \tan (dx+c)\,(a+b\tan (dx+c))^{n}\,(A+B\tan (dx+c))\,dx$$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan (dx+c)+A)(b\tan (dx+c)+a)^{n}\tan (dx+c)\;dx$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
integral((B*tan(d*x + c)^2 + A*tan(d*x + c))*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^n\tan{(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*tan(c + d*x), x)
```

Maxima [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^n\tan{(dx+c)}\ dx$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c), x)
```

Giac [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^n\tan{(dx+c)}\ dx$$

input
```
integrate(tan(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

input
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \tan(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\left(\int (a+\tan(dx+c)\,b)^n\tan(dx+c)^2\,dx\right)b$$
$$+\left(\int (a+\tan(dx+c)\,b)^n\tan(dx+c)\,dx\right)a$$

input
```
int(tan(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*b + a)**n*tan(c + d*x)**2,x)*b + int((tan(c + d*x)*b + a
)**n*tan(c + d*x),x)*a
```

3.498 $\int (a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 143

$$\int (a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{(A-iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(ia+b)d(1+n)} + \frac{(iA-B)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a+ib)d(1+n)}$$

output
```
1/2*(A-I*B)*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-I*b))*(a+b*tan(d*
x+c))^(1+n)/(I*a+b)/d/(1+n)+1/2*(I*A-B)*hypergeom([1, 1+n],[2+n],(a+b*tan(
d*x+c))/(a+I*b))*(a+b*tan(d*x+c))^(1+n)/(a+I*b)/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.84

$$\int (a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{i\left(-\frac{(A-iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)}{a-ib}+\frac{(A+iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+ib}\right)}{a+ib}\right)(a+b\tan}{2d(1+n)}$$

input
```
Integrate[(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((I/2)*(-(((A - I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x
])/(a - I*b)])/(a - I*b)) + ((A + I*B)*Hypergeometric2F1[1, 1 + n, 2 + n,
(a + b*Tan[c + d*x])/(a + I*b)])/(a + I*b))*(a + b*Tan[c + d*x])^(1 + n))/
(d*(1 + n))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4022, 3042, 4020, 25, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (A+B\tan(c+dx))(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{4022}$$

$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))(a+b\tan(c+dx))^n dx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))(a+b\tan(c+dx))^n dx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(A-iB)\int -\frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d}-\frac{i(A+iB)\int -\frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d(-i\tan(c+dx))}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{i(A+iB)\int \frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d(-i\tan(c+dx))}{2d}-\frac{i(A-iB)\int \frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d}$$

$$\Big\downarrow \; 78$$

$$\frac{i(A+iB)(a+b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)}-\frac{i(A-iB)(a+b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}$$

input
```
Int[(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((-1/2*I)*(A - I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x]
)/(a - I*b)]*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*d*(1 + n)) + ((I/2)*
(A + I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b
)]*(a + b*Tan[c + d*x])^(1 + n))/((a + I*b)*d*(1 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int (a+b\tan (dx+c))^n (A+B\tan (dx+c))\,dx$$

input
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int (a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan (dx+c)+A)(b\tan (dx+c)+a)^n\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n, x)
```

Sympy [F]

$$\int (a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^n\,dx$$

input
```
integrate((a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n, x)
```

Maxima [F]

$$\int (a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^n\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan(c+d\,x))\;(a+b\tan(c+d\,x))^n\,dx$$

input
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int (a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\left(\int (a+\tan(dx+c)\,b)^n\,dx\right)a+\left(\int (a+\tan(dx+c)\,b)^n\tan(dx+c)\,dx\right)b$$

input
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*b + a)**n,x)*a + int((tan(c + d*x)*b + a)**n*tan(c + d*x
),x)*b
```

3.499 $\int \cot(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 191

$$\int \cot(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = -\frac{A\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{ad(1+n)} + \frac{(iA+B)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(ia+b)d(1+n)} + \frac{(A+iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a+ib)d(1+n)}$$

output

```
-A*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/a)*(a+b*tan(d*x+c))^(1+n)/a/d
/(1+n)+1/2*(I*A+B)*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-I*b))*(a+b
*tan(d*x+c))^(1+n)/(I*a+b)/d/(1+n)+1/2*(A+I*B)*hypergeom([1, 1+n],[2+n],(a
+b*tan(d*x+c))/(a+I*b))*(a+b*tan(d*x+c))^(1+n)/(a+I*b)/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\int \cot(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{\left(a(a+ib)(A-iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)+(a-ib)\left(a(A+iB)\,\text{Hypergeo}\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((a*(a + I*b)*(A - I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c +
d*x])/(a - I*b)] + (a - I*b)*(a*(A + I*B)*Hypergeometric2F1[1, 1 + n, 2 +
n, (a + b*Tan[c + d*x])/(a + I*b)] - 2*A*(a + I*b)*Hypergeometric2F1[1, 1
+ n, 2 + n, 1 + (b*Tan[c + d*x])/a]))*(a + b*Tan[c + d*x])^(1 + n))/(2*a*(
a - I*b)*(a + I*b)*d*(1 + n))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {3042, 4096, 3042, 4022, 3042, 4020, 25, 78, 4117, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4096}$$

$$\int (B-A\tan(c+dx))(a+b\tan(c+dx))^n dx+A\int \cot(c+dx)(a+b\tan(c+dx))^n\left(\tan^2(c+dx)+1\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (B-A\tan(c+dx))(a+b\tan(c+dx))^n dx + A\int \frac{(a+b\tan(c+dx))^n\left(\tan(c+dx)^2+1\right)}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4022}$$

$$-\frac{1}{2}(-B+iA)\int (1-i\tan(c+dx))(a+b\tan(c+dx))^n dx + \frac{1}{2}(B+iA)\int (i\tan(c+dx)+1)(a+b\tan(c+dx))^n dx + A\int \frac{(a+b\tan(c+dx))^n\left(\tan(c+dx)^2+1\right)}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}(-B+iA)\int (1-i\tan(c+dx))(a+b\tan(c+dx))^n dx + \frac{1}{2}(B+iA)\int (i\tan(c+dx)+1)(a+b\tan(c+dx))^n dx + A\int \frac{(a+b\tan(c+dx))^n\left(\tan(c+dx)^2+1\right)}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{4020}$$

$$\frac{i(B+iA)\int -\frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d} + \frac{i(-B+iA)\int -\frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d(-i\tan(c+dx))}{2d} + A\int \frac{(a+b\tan(c+dx))^n\left(\tan(c+dx)^2+1\right)}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{i(B+iA)\int \frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d} - \frac{i(-B+iA)\int \frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d(-i\tan(c+dx))}{2d} + A\int \frac{(a+b\tan(c+dx))^n\left(\tan(c+dx)^2+1\right)}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{78}$$

$$A\int \frac{(a+b\tan(c+dx))^n\left(\tan(c+dx)^2+1\right)}{\tan(c+dx)}dx - \frac{i(B+iA)(a+b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)} - \frac{i(-B+iA)(a+b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)}$$

$$\Big\downarrow \text{4117}$$

$$\frac{A\int \cot(c+dx)(a+b\tan(c+dx))^n d\tan(c+dx)}{d}-\frac{i(B+iA)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}-\frac{i(-B+iA)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)}$$

$$\Big\downarrow \text{75}$$

$$-\frac{i(B+iA)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}-\frac{i(-B+iA)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a+ib}\right)}{2d(n+1)(a+ib)}-\frac{A(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\tan(c+dx)}{a}+1\right)}{ad(n+1)}$$

input
```
Int[Cot[c + d*x]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((-1/2*I)*(I*A + B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x]
)/(a - I*b)]*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*d*(1 + n)) - ((I/2)*
(I*A - B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b
)]*(a + b*Tan[c + d*x])^(1 + n))/((a + I*b)*d*(1 + n)) - (A*Hypergeometric
2F1[1, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c + d*x])^(1 + n))
/(a*d*(1 + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4096
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)])^(n_))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*Simp[a*A + b*B - (A*b - a*B)*Tan
[e + f*x], x], x], x] + Simp[b*((A*b - a*B)/(a^2 + b^2))   Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [F]

$$\int \cot (dx + c) (a + b \tan (dx + c))^n (A + B \tan (dx + c)) dx$$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot(c + dx)(a + b\tan(c + dx))^n(A + B\tan(c + dx)) \, dx = \int (B \tan (dx + c) + A)(b \tan (dx + c) + a)^n \cot (dx + c) \, dx$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="fri
cas")
```

output
```
integral((B*cot(d*x + c)*tan(d*x + c) + A*cot(d*x + c))*(b*tan(d*x + c) +
a)^n, x)
```

Sympy [F]

$$\int \cot(c + dx)(a + b\tan(c + dx))^n(A + B\tan(c + dx)) \, dx = \int (A + B \tan (c + dx)) (a + b \tan (c + dx))^n \cot (c + dx) \, dx$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*cot(c + d*x), x)
```

Maxima [F]

$$\int \cot(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="max
ima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c), x)
```

Giac [F]

$$\int \cot(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\cot(dx+c)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="gia
c")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

input
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \cot(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)\,(a+\tan(dx+c)\,b)^n\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

3.500 $\int \cot^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 229

$$\begin{aligned}
&\int \cot^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx \\
&= -\frac{A\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{ad} \\
&\quad -\frac{(aB+Abn)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a}\right)(a+b\tan(c+dx))^{1+n}}{a^2d(1+n)} \\
&\quad -\frac{(A-iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(ia+b)d(1+n)} \\
&\quad -\frac{(iA-B)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a+ib)d(1+n)}
\end{aligned}$$

output
```
-A*cot(d*x+c)*(a+b*tan(d*x+c))^(1+n)/a/d-(A*b*n+B*a)*hypergeom([1, 1+n],[2
+n],(a+b*tan(d*x+c))/a)*(a+b*tan(d*x+c))^(1+n)/a^2/d/(1+n)-1/2*(A-I*B)*hyp
ergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-I*b))*(a+b*tan(d*x+c))^(1+n)/(I*
a+b)/d/(1+n)-1/2*(I*A-B)*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+I*b)
)*(a+b*tan(d*x+c))^(1+n)/(a+I*b)/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.88

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{\left(a^2(a+ib)(A-iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)-(a-ib)\left(a^2(A+iB)\,\text{Hyperg}\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((a^2*(a + I*b)*(A - I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c
+ d*x])/(a - I*b)] - (a - I*b)*(a^2*(A + I*B)*Hypergeometric2F1[1, 1 + n,
2 + n, (a + b*Tan[c + d*x])/(a + I*b)] + 2*((-I)*a + b)*(a*B*Hypergeometri
c2F1[1, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a] - A*b*Hypergeometric2F1[2, 1
 + n, 2 + n, 1 + (b*Tan[c + d*x])/a])))*(a + b*Tan[c + d*x])^(1 + n))/(2*a
^2*(a - I*b)*((-I)*a + b)*d*(1 + n))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 4092, 25, 3042, 4136, 25, 3042, 4022, 3042, 4020, 25, 78, 4117, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{\int -\cot(c+dx)(a+b\tan(c+dx))^n \left(Abn\tan^2(c+dx)-aA\tan(c+dx)+aB+Abn\right)dx}{a}-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}$$

$\big\downarrow$ 25

$$\frac{\int \cot(c+dx)(a+b\tan(c+dx))^n \left(Abn\tan^2(c+dx)-aA\tan(c+dx)+aB+Abn\right)dx}{a}-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{(a+b\tan(c+dx))^n\left(Abn\tan(c+dx)^2-aA\tan(c+dx)+aB+Abn\right)}{\tan(c+dx)}dx}{a}-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}$$

$\big\downarrow$ 4136

$$\frac{\int -(a+b\tan(c+dx))^n(aA+aB\tan(c+dx))dx+(aB+Abn)\int \cot(c+dx)(a+b\tan(c+dx))^n \left(\tan^2(c+dx\right.}{a}$$
$$\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}$$

$\big\downarrow$ 25

$$\frac{(aB+Abn)\int \cot(c+dx)(a+b\tan(c+dx))^n \left(\tan^2(c+dx)+1\right)dx-\int (a+b\tan(c+dx))^n(aA+aB\tan(c+\right.}{a}$$
$$\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}$$

$\big\downarrow$ 3042

$$\frac{(aB+Abn)\int \frac{(a+b\tan(c+dx))^n\left(\tan(c+dx)^2+1\right)}{\tan(c+dx)}dx-\int (a+b\tan(c+dx))^n(aA+aB\tan(c+dx))dx}{a}-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}$$

$\big\downarrow$ 4022

$$-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}+$$
$$\frac{(aB+Abn)\int \frac{(a+b\tan(c+dx))^n\left(\tan(c+dx)^2+1\right)}{\tan(c+dx)}dx-\frac{1}{2}a(A+iB)\int (1-i\tan(c+dx))(a+b\tan(c+dx))^ndx-\frac{1}{2}a(A\right.}{a}$$

$\big\downarrow$ 3042

$$-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}+$$
$$\frac{-\frac{1}{2}a(A+iB)\int(1-i\tan(c+dx))(a+b\tan(c+dx))^n dx-\frac{1}{2}a(A-iB)\int(i\tan(c+dx)+1)(a+b\tan(c+dx))}{a}$$

$\Big\downarrow$ 4020

$$-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}+$$
$$\frac{(aB+Abn)\int\frac{(a+b\tan(c+dx))^n(\tan(c+dx)^2+1)}{\tan(c+dx)}dx-\frac{ia(A-iB)\int-\frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d}+\frac{ia(A+iB)\int-\frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}}{2d}}{a}$$

$\Big\downarrow$ 25

$$-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}+$$
$$\frac{(aB+Abn)\int\frac{(a+b\tan(c+dx))^n(\tan(c+dx)^2+1)}{\tan(c+dx)}dx+\frac{ia(A-iB)\int\frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d(i\tan(c+dx))}{2d}-\frac{ia(A+iB)\int\frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d(-}{2d}}{a}$$

$\Big\downarrow$ 78

$$-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}+$$
$$\frac{(aB+Abn)\int\frac{(a+b\tan(c+dx))^n(\tan(c+dx)^2+1)}{\tan(c+dx)}dx+\frac{ia(A-iB)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}-}{a}$$

$\Big\downarrow$ 4117

$$-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}+$$
$$\frac{\frac{(aB+Abn)\int\cot(c+dx)(a+b\tan(c+dx))^n d\tan(c+dx)}{d}+\frac{ia(A-iB)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}-}{a}$$

$\Big\downarrow$ 75

$$-\frac{A\cot(c+dx)(a+b\tan(c+dx))^{n+1}}{ad}+$$
$$\frac{\frac{ia(A-iB)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-ib}\right)}{2d(n+1)(a-ib)}-\frac{ia(A+iB)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1\right.}{2d(n+1)(a+ib)}}{a}$$

input

```
Int[Cot[c + d*x]^2*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
-((A*Cot[c + d*x]*(a + b*Tan[c + d*x])^(1 + n))/(a*d)) + (((I/2)*a*(A - I*
B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - I*b)]*(a +
 b*Tan[c + d*x])^(1 + n))/((a - I*b)*d*(1 + n)) - ((I/2)*a*(A + I*B)*Hyper
geometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)]*(a + b*Tan[c
 + d*x])^(1 + n))/((a + I*b)*d*(1 + n)) - ((a*B + A*b*n)*Hypergeometric2F1
[1, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a]*(a + b*Tan[c + d*x])^(1 + n))/(a
*d*(1 + n)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [F]

$$\int \cot (dx+c)^2 (a+b\tan (dx+c))^n (A+B\tan (dx+c))\,dx$$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan (dx+c)+A)(b\tan (dx+c)+a)^n \cot (dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*cot(d*x + c)^2*tan(d*x + c) + A*cot(d*x + c)^2)*(b*tan(d*x + c
) + a)^n, x)
```

Sympy [F]

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan (c+dx)) (a+b\tan (c+dx))^n \cot^2 (c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*cot(c + d*x)**2, x)
```

Maxima [F]

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Giac [F]

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\cot(dx+c)^2\,dx$$

input
```
integrate(cot(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int \cot(c+dx)^2\,(A+B\tan(c+dx))\,(a+b\tan(c+dx))^n\,dx$$

input
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^2*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \cot^2(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int \cot(dx+c)^2 (a+\tan(dx+c)\,b)^n (A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^2*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

3.501 $\int \cot^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 293

$$
\begin{aligned}
&\int \cot^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx \\
&= -\frac{(2aB-Ab(1-n))\cot(c+dx)(a+b\tan(c+dx))^{1+n}}{2a^2d} \\
&\quad -\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{1+n}}{2ad} \\
&\quad +\frac{(2a^2A-2abBn+Ab^2(1-n)n)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a}\right)(a+b\tan(c+dx)}{2a^3d(1+n)} \\
&\quad -\frac{(iA+B)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)(a+b\tan(c+dx))^{1+n}}{2(ia+b)d(1+n)} \\
&\quad -\frac{(A+iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+ib}\right)(a+b\tan(c+dx))^{1+n}}{2(a+ib)d(1+n)}
\end{aligned}
$$

output
```
-1/2*(2*B*a-A*b*(1-n))*cot(d*x+c)*(a+b*tan(d*x+c))^(1+n)/a^2/d-1/2*A*cot(d
*x+c)^2*(a+b*tan(d*x+c))^(1+n)/a/d+1/2*(2*A*a^2-2*a*b*B*n+A*b^2*(1-n)*n)*h
ypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/a)*(a+b*tan(d*x+c))^(1+n)/a^3/d/(
1+n)-1/2*(I*A+B)*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-I*b))*(a+b*t
an(d*x+c))^(1+n)/(I*a+b)/d/(1+n)-1/2*(A+I*B)*hypergeom([1, 1+n],[2+n],(a+b
*tan(d*x+c))/(a+I*b))*(a+b*tan(d*x+c))^(1+n)/(a+I*b)/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.78

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx =$$
$$-\frac{\left(a^3(a+ib)(A-iB)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-ib}\right)+(a-ib)\left(a^3(A+iB)\,\text{Hype}\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*((a^3*(a + I*b)*(A - I*B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*T
an[c + d*x])/(a - I*b)] + (a - I*b)*(a^3*(A + I*B)*Hypergeometric2F1[1, 1
+ n, 2 + n, (a + b*Tan[c + d*x])/(a + I*b)] - 2*(a + I*b)*(a^2*A*Hypergeom
etric2F1[1, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a] + b*(a*B*Hypergeometric2
F1[2, 1 + n, 2 + n, 1 + (b*Tan[c + d*x])/a] - A*b*Hypergeometric2F1[3, 1 +
 n, 2 + n, 1 + (b*Tan[c + d*x])/a]))))*(a + b*Tan[c + d*x])^(1 + n))/(a^3*
(a - I*b)*(a + I*b)*d*(1 + n))
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 4092, 25, 3042, 4132, 3042, 4136, 27, 3042, 4022, 3042, 4020, 25, 78, 4117, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{4092}$$

$$-\frac{\int -\cot^2(c+dx)(a+b\tan(c+dx))^n\left(-Ab(1-n)\tan^2(c+dx)-2aA\tan(c+dx)+2aB-A(b-bn)\right)dx}{2a}-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \cot^2(c+dx)(a+b\tan(c+dx))^n\left(-Ab(1-n)\tan^2(c+dx)-2aA\tan(c+dx)+2aB-Ab(1-n)\right)dx}{2a}-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a+b\tan(c+dx))^n\left(-Ab(1-n)\tan(c+dx)^2-2aA\tan(c+dx)+2aB-Ab(1-n)\right)}{\tan(c+dx)^2}dx}{2a}-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}$$

$$\Big\downarrow \text{4132}$$

$$\frac{-\frac{\int \cot(c+dx)(a+b\tan(c+dx))^n\left(2Aa^2+2B\tan(c+dx)a^2-2bBna-b(2aB-Ab(1-n))n\tan^2(c+dx)+Ab^2(1-n)n\right)dx}{a}-\frac{\cot(c+dx)(2aB-Ab(1\cdots}{\cdots}}{2a}-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{(a+b\tan(c+dx))^n\left(2Aa^2+2B\tan(c+dx)a^2-2bBna-b(2aB-Ab(1-n))n\tan(c+dx)^2+Ab^2(1-n)n\right)}{\tan(c+dx)}dx}{a}-\frac{\cot(c+dx)(2aB-Ab(1-n))(a+b\tan(c+dx)\cdots}{ad}}{2a}-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}$$

$$\Big\downarrow \text{4136}$$

$$-\frac{\frac{(2a^2A-2abBn+Ab^2(1-n)n)\int\cot(c+dx)(a+b\tan(c+dx))^n(\tan^2(c+dx)+1)dx+\int 2(a^2B-a^2A\tan(c+dx))(a+b\tan(c+dx))^n dx}{a}-\frac{\cot(c+}{}}{2a}$$

$$\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}$$

$\downarrow$ 27

$$-\frac{\frac{(2a^2A-2abBn+Ab^2(1-n)n)\int\cot(c+dx)(a+b\tan(c+dx))^n(\tan^2(c+dx)+1)dx+2\int(a^2B-a^2A\tan(c+dx))(a+b\tan(c+dx))^n dx}{a}-\frac{\cot(c+}{}}{2a}$$

$$\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}$$

$\downarrow$ 3042

$$-\frac{\frac{(2a^2A-2abBn+Ab^2(1-n)n)\int\frac{(a+b\tan(c+dx))^n(\tan(c+dx)^2+1)}{\tan(c+dx)}dx+2\int(a^2B-a^2A\tan(c+dx))(a+b\tan(c+dx))^n dx}{a}-\frac{\cot(c+dx)(2aB-Ab(}{}}{2a}$$

$$\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}$$

$\downarrow$ 4022

$$-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}+$$

$$\frac{-\frac{\cot(c+dx)(2aB-Ab(1-n))(a+b\tan(c+dx))^{n+1}}{ad}-\frac{(2a^2A-2abBn+Ab^2(1-n)n)\int\frac{(a+b\tan(c+dx))^n(\tan(c+dx)^2+1)}{\tan(c+dx)}dx+2\left(\frac{1}{2}a^2(B+iA)\int(i\tan\right.}{}}{2a}$$

$\downarrow$ 3042

$$-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}+$$

$$\frac{-\frac{\cot(c+dx)(2aB-Ab(1-n))(a+b\tan(c+dx))^{n+1}}{ad}-\frac{(2a^2A-2abBn+Ab^2(1-n)n)\int\frac{(a+b\tan(c+dx))^n(\tan(c+dx)^2+1)}{\tan(c+dx)}dx+2\left(\frac{1}{2}a^2(B+iA)\int(i\tan\right.}{}}{2a}$$

$\downarrow$ 4020

$$-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}+$$

$$\frac{-\frac{\cot(c+dx)(2aB-Ab(1-n))(a+b\tan(c+dx))^{n+1}}{ad}-\frac{(2a^2A-2abBn+Ab^2(1-n)n)\int\frac{(a+b\tan(c+dx))^n(\tan(c+dx)^2+1)}{\tan(c+dx)}dx+2\left(\frac{ia^2(B+iA)\int-\frac{(a+}{1}}{}\right.}{a}}{2a}$$

$\downarrow$ 25

$$-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}+\frac{-\frac{\cot(c+dx)(2aB-Ab(1-n))(a+b\tan(c+dx))^{n+1}}{ad}-\frac{(2a^2A-2abBn+Ab^2(1-n)n)\int\frac{(a+b\tan(c+dx))^n\left(\tan(c+dx)^2+1\right)}{\tan(c+dx)}dx+2\left(-\frac{ia^2(B+iA)\int\frac{(a-}{1}}{a}\right.}{2a}}{}$$

$\Big\downarrow$ 78

$$-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}+\frac{-\frac{\cot(c+dx)(2aB-Ab(1-n))(a+b\tan(c+dx))^{n+1}}{ad}-\frac{(2a^2A-2abBn+Ab^2(1-n)n)\int\frac{(a+b\tan(c+dx))^n\left(\tan(c+dx)^2+1\right)}{\tan(c+dx)}dx+2\left(-\frac{ia^2(B+iA)(a+b}{}\right.}{}}{2a}$$

$\Big\downarrow$ 4117

$$-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}+\frac{-\frac{\cot(c+dx)(2aB-Ab(1-n))(a+b\tan(c+dx))^{n+1}}{ad}-\frac{\frac{\left(2a^2A-2abBn+Ab^2(1-n)n\right)\int\cot(c+dx)(a+b\tan(c+dx))^n d\tan(c+dx)}{d}+2\left(-\frac{ia^2(B+iA)(a+b\tan}{}\right.}{}}{2a}$$

$\Big\downarrow$ 75

$$-\frac{A\cot^2(c+dx)(a+b\tan(c+dx))^{n+1}}{2ad}+\frac{-\frac{\cot(c+dx)(2aB-Ab(1-n))(a+b\tan(c+dx))^{n+1}}{ad}-\frac{-\frac{\left(2a^2A-2abBn+Ab^2(1-n)n\right)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b\tan(c+dx)}{a}\right.}{ad(n+1)}}{}}{}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
-1/2*(A*Cot[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n))/(a*d) + (-(((2*a*B -
A*b*(1 - n))*Cot[c + d*x]*(a + b*Tan[c + d*x])^(1 + n))/(a*d)) - (-(((2*a^
2*A - 2*a*b*B*n + A*b^2*(1 - n)*n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 +
(b*Tan[c + d*x])/a]*(a + b*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))) + 2*(((-1
/2*I)*a^2*(I*A + B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x]
)/(a - I*b)]*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*d*(1 + n)) - ((I/2)*
a^2*(I*A - B)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a +
 I*b)]*(a + b*Tan[c + d*x])^(1 + n))/((a + I*b)*d*(1 + n))))/a)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4020
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[c*(d/f)   Subst[Int[(a + (b/d)*x)^m/(d^2 +
c*x), x], x, d*Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && EqQ[c^2 + d^2, 0]
```

rule 4022
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(c + I*d)/2   Int[(a + b*Tan[e + f*x])^m*(
1 - I*Tan[e + f*x]), x], x] + Simp[(c - I*d)/2   Int[(a + b*Tan[e + f*x])^m
*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &&  !IntegerQ[m]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [F]

$$\int \cot (dx + c)^3 (a + b \tan (dx + c))^n (A + B \tan (dx + c)) dx$$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B \tan (dx + c) + A)(b \tan (dx + c) + a)^n \cot (dx + c)^3 \, dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*cot(d*x + c)^3*tan(d*x + c) + A*cot(d*x + c)^3)*(b*tan(d*x + c
) + a)^n, x)
```

Sympy [F]

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (A + B \tan (c + dx)) (a + b \tan (c + dx))^n \cot^3 (c + dx) \, dx$$

input
```
integrate(cot(d*x+c)**3*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*cot(c + d*x)**3, x)
```

Maxima [F]

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Giac [F]

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\cot(dx+c)^3\,dx$$

input
```
integrate(cot(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^3\,(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^3*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \cot^3(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^3(a+\tan(dx+c)\,b)^n(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^3*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

3.502 $\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 175

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{(A+iB)\operatorname{AppellF1}\left(\frac{5}{2},-n,1,\frac{7}{2},-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c}{a}\right.}{5d}$$
$$+\frac{(A-iB)\operatorname{AppellF1}\left(\frac{5}{2},-n,1,\frac{7}{2},-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c}{a}\right.}{5d}$$

output
```
1/5*(A+I*B)*AppellF1(5/2,1,-n,7/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c
)^(5/2)*(a+b*tan(d*x+c))^n/d/(((a+b*tan(d*x+c))/a)^n)+1/5*(A-I*B)*AppellF1
(5/2,1,-n,7/2,I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c)^(5/2)*(a+b*tan(d*x+
c))^n/d/(((a+b*tan(d*x+c))/a)^n)
```

Mathematica [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x
]
```

output
```
Integrate[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),
x]
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 4086, 3042, 4085, 148, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)^{3/2}(A+B\tan(c+dx))(a+b\tan(c+dx))^n dx$$
$$\Big\downarrow \text{4086}$$
$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n dx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2}(A+iB)\int (1-i\tan(c+dx))\tan(c+dx)^{3/2}(a+b\tan(c+dx))^n dx+\frac{1}{2}(A-iB)\int (i\tan(c+dx)+1)\tan(c+dx)^{3/2}(a+b\tan(c+dx))^n dx$$

$\Big\downarrow$ 4085

$$\frac{(A-iB)\int \frac{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d\tan(c+dx)}{2d}$$

$\Big\downarrow$ 148

$$\frac{(A-iB)\int \frac{\tan^2(c+dx)(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)\int \frac{\tan^2(c+dx)(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 395

$$\frac{(A-iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int \frac{\tan^2(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int \frac{\tan^2(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{i\tan(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}$$

$\Big\downarrow$ 394

$$\frac{(A+iB)\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\operatorname{AppellF1}\left(\frac{5}{2},1,-n,\frac{7}{2},-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{5d}+\frac{(A-iB)\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\operatorname{AppellF1}\left(\frac{5}{2},1,-n,\frac{7}{2},i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{5d}$$

input
```
Int[Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((A + I*B)*AppellF1[5/2, 1, -n, 7/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])
/a)]*Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^n)/(5*d*(1 + (b*Tan[c + d*x])
/a)^n) + ((A - I*B)*AppellF1[5/2, 1, -n, 7/2, I*Tan[c + d*x], -((b*Tan[c +
 d*x])/a)]*Tan[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^n)/(5*d*(1 + (b*Tan[c +
 d*x])/a)^n)
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \tan(dx+c)^{\frac{3}{2}} (a+b\tan(dx+c))^n (A+B\tan(dx+c))\, dx$$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\, dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}}\, dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c)^2 + A*tan(d*x + c))*(b*tan(d*x + c) + a)^n*sqrt(t
an(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(tan(d*x+c)**(3/2)*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\tan(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*tan(d*x + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \tan(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\left(\int \sqrt{\tan(dx+c)}\,(a+\tan(dx+c)\,b)^n\tan(dx+c)^2\,dx\right)b$$
$$+\left(\int \sqrt{\tan(dx+c)}\,(a+\tan(dx+c)\,b)^n\tan(dx+c)\,dx\right)a$$

input
```
int(tan(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n*tan(c + d*x)**2,x)*b + int(
sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n*tan(c + d*x),x)*a
```

3.503 $\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 175

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{(A+iB)\,\text{AppellF1}\left(\frac{3}{2},-n,1,\frac{5}{2},-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c+dx)}{a}\right)^{-n}}{3d} + \frac{(A-iB)\,\text{AppellF1}\left(\frac{3}{2},-n,1,\frac{5}{2},-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c+dx)}{a}\right)^{-n}}{3d}$$

output
```
1/3*(A+I*B)*AppellF1(3/2,1,-n,5/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c
)^(3/2)*(a+b*tan(d*x+c))^n/d/(((a+b*tan(d*x+c))/a)^n)+1/3*(A-I*B)*AppellF1
(3/2,1,-n,5/2,I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c)^(3/2)*(a+b*tan(d*x+
c))^n/d/(((a+b*tan(d*x+c))/a)^n)
```

Mathematica [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x
]
```

output
```
Integrate[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),
x]
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 4086, 3042, 4085, 148, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tan(c+dx)}(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\tan(c+dx)}(A+B\tan(c+dx))(a+b\tan(c+dx))^ndx$$
$$\Big\downarrow \text{4086}$$
$$\frac{1}{2}(A+iB)\int(1-i\tan(c+dx))\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^ndx+\frac{1}{2}(A-iB)\int(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^ndx$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2}(A+iB)\int(1-i\tan(c+dx))\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^ndx+\frac{1}{2}(A-iB)\int(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^ndx$$

$$\Big\downarrow \; 4085$$

$$\frac{(A-iB)\int \frac{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d\tan(c+dx)}{2d}$$

$$\Big\downarrow \; 148$$

$$\frac{(A-iB)\int \frac{\tan(c+dx)(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)\int \frac{\tan(c+dx)(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 395$$

$$\frac{(A-iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int \frac{\tan(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int \frac{\tan(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{i\tan(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 394$$

$$\frac{(A+iB)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(\frac{3}{2},1,-n,\frac{5}{2},-i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{3d}+\frac{(A-iB)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(\frac{3}{2},1,-n,\frac{5}{2},i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right)}{3d}$$

input
```
Int[Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
((A + I*B)*AppellF1[3/2, 1, -n, 5/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])
/a)]*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n)/(3*d*(1 + (b*Tan[c + d*x])
/a)^n) + ((A - I*B)*AppellF1[3/2, 1, -n, 5/2, I*Tan[c + d*x], -((b*Tan[c +
 d*x])/a)]*Tan[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n)/(3*d*(1 + (b*Tan[c +
 d*x])/a)^n)
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \sqrt{\tan (dx+c)}\, (a+b\tan (dx+c))^{n} (A+B\tan (dx+c))\, dx$$

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\, dx = \int (B\tan (dx+c)+A)(b\tan (dx+c)+a)^{n}\sqrt{\tan (dx+c)}\, dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*sqrt(tan(d*x + c)), x
)
```

Sympy [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^n\,\sqrt{\tan{(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**(1/2)*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*sqrt(tan(c + d*x)),
x)
```

Maxima [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^n\sqrt{\tan{(dx+c)}}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*sqrt(tan(d*x + c)),
x)
```

Giac [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^n\sqrt{\tan{(dx+c)}}\,dx$$

input
```
integrate(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*sqrt(tan(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int \sqrt{\tan(c+d\,x)}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

input
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(tan(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \left(\int \sqrt{\tan(dx+c)}\,(a+\tan(dx+c)\,b)^n\tan(dx+c)\,dx\right)b$$
$$+ \left(\int \sqrt{\tan(dx+c)}\,(a+\tan(dx+c)\,b)^n\,dx\right)a$$

input
```
int(tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n*tan(c + d*x),x)*b + int(sqr
t(tan(c + d*x))*(tan(c + d*x)*b + a)**n,x)*a
```

3.504 $\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 169

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \frac{(A+iB)\,\mathrm{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n\left(\frac{a+b\tan}{a}\right.}{d} + \frac{(A-iB)\,\mathrm{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n\left(\frac{a+b\tan}{a}\right.}{d}$$

output
```
(A+I*B)*AppellF1(1/2,1,-n,3/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c)^(1
/2)*(a+b*tan(d*x+c))^n/d/(((a+b*tan(d*x+c))/a)^n)+(A-I*B)*AppellF1(1/2,1,-
n,3/2,I*tan(d*x+c),-b*tan(d*x+c)/a)*tan(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n/d/
(((a+b*tan(d*x+c))/a)^n)
```

Mathematica [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$=\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$

input
```
Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]]
,x]
```

output
```
Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]]
, x]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3042, 4086, 3042, 4085, 148, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4086}$$

$$\frac{1}{2}(A+iB)\int \frac{(1-i\tan(c+dx))(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{(i\tan(c+dx)+1)(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}(A+iB)\int \frac{(1-i\tan(c+dx))(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{(i\tan(c+dx)+1)(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4085}$$

$$\frac{(A-iB)\int \frac{(a+b\tan(c+dx))^n}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{(a+b\tan(c+dx))^n}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}}d\tan(c+dx)}{2d}$$

$$\Big\downarrow \text{ 148}$$

$$\frac{(A-iB)\int \frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)\int \frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 334}$$

$$\frac{(A-iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int \frac{\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int \frac{\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{i\tan(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 333}$$

$$\frac{(A+iB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(\frac{1}{2},1,-n,\frac{3}{2},-i\tan(c+dx),-\frac{b\tan(c+d}{a}\right.}{d}$$
$$\frac{(A-iB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(\frac{1}{2},1,-n,\frac{3}{2},i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right.}{d}$$

input
```
Int[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Tan[c + d*x]],x]
```

output
```
((A + I*B)*AppellF1[1/2, 1, -n, 3/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])
/a)]*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a
)^n) + ((A - I*B)*AppellF1[1/2, 1, -n, 3/2, I*Tan[c + d*x], -((b*Tan[c + d
*x])/a)]*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^n)/(d*(1 + (b*Tan[c + d*x
])/a)^n)
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \frac{(a+b\tan(dx+c))^n (A+B\tan(dx+c))}{\sqrt{\tan(dx+c)}}dx$$

input
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^n}{\sqrt{\tan(dx+c)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/sqrt(tan(d*x + c)), x
)
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**n*(A+B*tan(d*x+c))/tan(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n/sqrt(tan(c + d*x)),
x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^n}{\sqrt{\tan\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/tan(c + d*x)^(1/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/tan(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}\,dx$$
$$= \frac{2\sqrt{\tan\left(dx+c\right)}\left(a+\tan\left(dx+c\right)b\right)^n a-4\left(\int \frac{\sqrt{\tan(dx+c)}\,(a+\tan(dx+c)b)^n\tan(dx+c)^2}{2\tan(dx+c)bn+\tan(dx+c)b+2an+a}dx\right)abd\,n^2-4\left(\int \frac{\sqrt{\tan(dx}}{2\tan(}\right.}{}$$

input
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(1/2),x)
```

output
```
(2*sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n*a - 4*int((sqrt(tan(c + d*x)
)*(tan(c + d*x)*b + a)**n*tan(c + d*x)**2)/(2*tan(c + d*x)*b*n + tan(c + d
*x)*b + 2*a*n + a),x)*a*b*d*n**2 - 4*int((sqrt(tan(c + d*x))*(tan(c + d*x)
*b + a)**n*tan(c + d*x)**2)/(2*tan(c + d*x)*b*n + tan(c + d*x)*b + 2*a*n +
 a),x)*a*b*d*n - int((sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n*tan(c + d
*x)**2)/(2*tan(c + d*x)*b*n + tan(c + d*x)*b + 2*a*n + a),x)*a*b*d - 2*int
((sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n*tan(c + d*x))/(2*tan(c + d*x)
*b*n + tan(c + d*x)*b + 2*a*n + a),x)*a**2*d*n - int((sqrt(tan(c + d*x))*(
tan(c + d*x)*b + a)**n*tan(c + d*x))/(2*tan(c + d*x)*b*n + tan(c + d*x)*b
+ 2*a*n + a),x)*a**2*d + 4*int((sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n
)/(2*tan(c + d*x)**2*b*n + tan(c + d*x)**2*b + 2*tan(c + d*x)*a*n + tan(c
+ d*x)*a),x)*a**2*d*n**2 + 2*int((sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)*
*n)/(2*tan(c + d*x)**2*b*n + tan(c + d*x)**2*b + 2*tan(c + d*x)*a*n + tan(
c + d*x)*a),x)*a**2*d*n + 2*int(sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n
,x)*b*d*n + int(sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n,x)*b*d)/(d*(2*n
 + 1))
```

3.505 $\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 171

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = -\frac{(A+iB)\,\text{AppellF1}\left(-\frac{1}{2},-n,1,\frac{1}{2},-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c+dx)}{a}\right)^{-n}}{d\sqrt{\tan(c+dx)}} - \frac{(A-iB)\,\text{AppellF1}\left(-\frac{1}{2},-n,1,\frac{1}{2},-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c+dx)}{a}\right)^{-n}}{d\sqrt{\tan(c+dx)}}$$

output
```
-(A+I*B)*AppellF1(-1/2,1,-n,1/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*
x+c))^n/d/tan(d*x+c)^(1/2)/(((a+b*tan(d*x+c))/a)^n)-(A-I*B)*AppellF1(-1/2,
1,-n,1/2,I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n/d/tan(d*x+c)^(1/
2)/(((a+b*tan(d*x+c))/a)^n)
```

Mathematica [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2)
,x]
```

output
```
Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2)
, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3042, 4086, 3042, 4085, 148, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 4086}$$

$$\frac{1}{2}(A+iB)\int \frac{(1-i\tan(c+dx))(a+b\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}dx+\frac{1}{2}(A-iB)\int \frac{(i\tan(c+dx)+1)(a+b\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}(A+iB)\int \frac{(1-i\tan(c+dx))(a+b\tan(c+dx))^n}{\tan(c+dx)^{3/2}}dx+\frac{1}{2}(A-iB)\int \frac{(i\tan(c+dx)+1)(a+b\tan(c+dx))^n}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \textbf{4085}$$

$$\frac{(A-iB)\int \frac{(a+b\tan(c+dx))^n}{(1-i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{(a+b\tan(c+dx))^n}{(i\tan(c+dx)+1)\tan^{\frac{3}{2}}(c+dx)}d\tan(c+dx)}{2d}$$

$$\Big\downarrow \textbf{148}$$

$$\frac{(A-iB)\int \frac{\cot^2(c+dx)(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)\int \frac{\cot^2(c+dx)(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \textbf{395}$$

$$\frac{(A-iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int \frac{\cot^2(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int \frac{\cot^2(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{i\tan(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}$$

$$\Big\downarrow \textbf{394}$$

$$-\frac{(A+iB)\cot(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(-\frac{1}{2},1,-n,\frac{1}{2},-i\tan(c+dx),-\frac{b\tan(c+}{a}\right.}{d}$$
$$\frac{(A-iB)\cot(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(-\frac{1}{2},1,-n,\frac{1}{2},i\tan(c+dx),-\frac{b\tan(c+dx)}{a}\right.}{d}$$

input
```
Int[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Tan[c + d*x]^(3/2),x]
```

output
```
-(((A + I*B)*AppellF1[-1/2, 1, -n, 1/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*
x])/a)]*Cot[c + d*x]*(a + b*Tan[c + d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a)^n
)) - ((A - I*B)*AppellF1[-1/2, 1, -n, 1/2, I*Tan[c + d*x], -((b*Tan[c + d*
x])/a)]*Cot[c + d*x]*(a + b*Tan[c + d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a)^n
)
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

Maple [F]

$$\int \frac{(a+b\tan (dx+c))^n (A+B\tan (dx+c))}{\tan (dx+c)^{\frac{3}{2}}}dx$$

input
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan (dx+c)+A)(b\tan (dx+c)+a)^n}{\tan (dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/tan(d*x + c)^(3/2), x
)
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**n*(A+B*tan(d*x+c))/tan(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n/tan(c + d*x)**(3/2),
 x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^n}{\tan(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/tan(d*x + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^n}{\tan\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/tan(c + d*x)^(3/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/tan(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}\,dx = \frac{-2\sqrt{\tan\left(dx+c\right)}\left(a+\tan\left(dx+c\right)b\right)^n b^2+2\left(\int \frac{\sqrt{\tan(dx+c)}\,(a+\tan(dx+c)b)^n\tan(dx+c)}{a+\tan(dx+c)b}dx\right)\tan\left(dx+c\right)b^3dn-}{}$$

input
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/tan(d*x+c)^(3/2),x)
```

output
```
( - 2*sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n*b**2 + 2*int((sqrt(tan(c
+ d*x))*(tan(c + d*x)*b + a)**n*tan(c + d*x))/(tan(c + d*x)*b + a),x)*tan(
c + d*x)*b**3*d*n - int((sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n*tan(c
+ d*x))/(tan(c + d*x)*b + a),x)*tan(c + d*x)*b**3*d + int((sqrt(tan(c + d*
x))*(tan(c + d*x)*b + a)**n)/(tan(c + d*x)**3*b + tan(c + d*x)**2*a),x)*ta
n(c + d*x)*a**3*d - int((sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n)/(tan(
c + d*x)**3*b + tan(c + d*x)**2*a),x)*tan(c + d*x)*a*b**2*d + 2*int((sqrt(
tan(c + d*x))*(tan(c + d*x)*b + a)**n)/(tan(c + d*x)**2*b + tan(c + d*x)*a
),x)*tan(c + d*x)*a**2*b*d + 2*int((sqrt(tan(c + d*x))*(tan(c + d*x)*b + a
)**n)/(tan(c + d*x)**2*b + tan(c + d*x)*a),x)*tan(c + d*x)*b**3*d*n - int(
(sqrt(tan(c + d*x))*(tan(c + d*x)*b + a)**n)/(tan(c + d*x)**2*b + tan(c +
d*x)*a),x)*tan(c + d*x)*b**3*d)/(tan(c + d*x)*a*d)
```

3.506 $\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 103

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{2\sqrt[4]{-1}a(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} + \frac{2a(A-iB)\sqrt{\cot(c+dx)}}{d} - \frac{2a(iA+B)\cot^{\frac{3}{2}}(c+dx)}{3d} - \frac{2aA\cot^{\frac{5}{2}}(c+dx)}{5d}$$

output
```
2*(-1)^(1/4)*a*(A-I*B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d+2*a*(A-I*B)*
cot(d*x+c)^(1/2)/d-2/3*a*(I*A+B)*cot(d*x+c)^(3/2)/d-2/5*a*A*cot(d*x+c)^(5/
2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.53

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{2a\cot^{\frac{3}{2}}(c+dx)\left(3A\cot(c+dx)+5(iA+B)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},i\tan(c+dx)\right)\right)}{15d}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x
]
```

output
```
(-2*a*Cot[c + d*x]^(3/2)*(3*A*Cot[c + d*x] + 5*(I*A + B)*Hypergeometric2F1
[-3/2, 1, -1/2, I*Tan[c + d*x]]))/(15*d)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 4064, 3042, 4075, 3042, 4011, 3042, 4011, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{7/2}(a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)(A\cot(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\tan \left(c+dx+\frac{\pi}{2}\right)\right)^{3/2} \left(-a \tan \left(c+dx+\frac{\pi}{2}\right)+ia\right) \left(B-A \tan \left(c+dx+\frac{\pi}{2}\right)\right) dx$$

$$\Big\downarrow \ 4075$$

$$-\frac{2aA\cot^{\frac{5}{2}}(c+dx)}{5d}+\int \cot^{\frac{3}{2}}(c+dx)(a(iA+B)\cot(c+dx)-a(A-iB))dx$$

$$\Big\downarrow \ 3042$$

$$-\frac{2aA\cot^{\frac{5}{2}}(c+dx)}{5d}+\int \left(-\tan \left(c+dx+\frac{\pi}{2}\right)\right)^{3/2} \left(-a(A-iB)-a(iA+B)\tan \left(c+dx+\frac{\pi}{2}\right)\right) dx$$

$$\Big\downarrow \ 4011$$

$$\int \sqrt{\cot(c+dx)}(-a(iA+B)-a(A-iB)\cot(c+dx))dx-\frac{2a(B+iA)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{-\tan \left(c+dx+\frac{\pi}{2}\right)}\left(a(A-iB)\tan \left(c+dx+\frac{\pi}{2}\right)-a(iA+B)\right) dx-\frac{2a(B+iA)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{2aA\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \ 4011$$

$$\int \frac{a(A-iB)-a(iA+B)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx-\frac{2a(B+iA)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(A-iB)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a(A-iB)+a(iA+B)\tan \left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan \left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a(B+iA)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(A-iB)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \ 4016$$

$$\frac{2a^2(A-iB)^2\int \frac{1}{-a(A-iB)-a(iA+B)\cot(c+dx)}d\sqrt{\cot(c+dx)}}{d}-\frac{2a(B+iA)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(A-iB)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{5}{2}}(c+dx)}{5d}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt[4]{-1}a(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a(B+iA)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(A-iB)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{5}{2}}(c+dx)}{5d}$$

input
```
Int[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(2*(-1)^(1/4)*a*(A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d + (2*a
*(A - I*B)*Sqrt[Cot[c + d*x]])/d - (2*a*(I*A + B)*Cot[c + d*x]^(3/2))/(3*d
) - (2*a*A*Cot[c + d*x]^(5/2))/(5*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(84) = 168$.

Time = 0.57 (sec) , antiderivative size = 551, normalized size of antiderivative = 5.35

method	result
derivativedivides	$-\frac{a\left(\frac{1}{\tan(dx+c)}\right)^{\frac{7}{2}}\tan(dx+c)\left(15iB\tan(dx+c)^{\frac{5}{2}}\ln\left(-\frac{\sqrt{2}\sqrt{\tan(dx+c)}-\tan(dx+c)-1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)\sqrt{2}+15iA\tan(dx+c)^{\frac{5}{2}}\ln\left(-\frac{\tan}{\sqrt{2}}\right.\right.}{}$
default	$-\frac{a\left(\frac{1}{\tan(dx+c)}\right)^{\frac{7}{2}}\tan(dx+c)\left(15iB\tan(dx+c)^{\frac{5}{2}}\ln\left(-\frac{\sqrt{2}\sqrt{\tan(dx+c)}-\tan(dx+c)-1}{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}\right)\sqrt{2}+15iA\tan(dx+c)^{\frac{5}{2}}\ln\left(-\frac{\tan}{\sqrt{2}}\right.\right.}{}$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/60*a/d*(1/tan(d*x+c))^(7/2)*tan(d*x+c)*(15*I*B*tan(d*x+c)^(5/2)*ln(-(2^
(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+
1))*2^(1/2)+15*I*A*tan(d*x+c)^(5/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/
2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)+40*I*A*tan(d*x+c)+3
0*I*B*tan(d*x+c)^(5/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)+30*I*A*t
an(d*x+c)^(5/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)+30*I*A*tan(d*x+
c)^(5/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)-30*A*tan(d*x+c)^(5/2)
*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)-30*A*tan(d*x+c)^(5/2)*arctan(-
1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)-15*A*tan(d*x+c)^(5/2)*ln(-(2^(1/2)*tan
(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(1/
2)+15*B*tan(d*x+c)^(5/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1
/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)+30*B*tan(d*x+c)^(5/2)*arctan(1
+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)+30*B*tan(d*x+c)^(5/2)*arctan(-1+2^(1/2)
*tan(d*x+c)^(1/2))*2^(1/2)+30*I*B*tan(d*x+c)^(5/2)*arctan(-1+2^(1/2)*tan(d
*x+c)^(1/2))*2^(1/2)-120*A*tan(d*x+c)^2+120*I*B*tan(d*x+c)^2+40*B*tan(d*x+
c)+24*A)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(81) = 162.

Time = 0.10 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.21

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$

$$-\frac{15\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a^2}{d^2}}\log\left(-\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}-\left(i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/30*(15*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*
A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(-2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) - (I
*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt
((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I
*c)/((I*A + B)*a)) - 15*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) +
 d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(-2*((A - I*B)*a*e^(2*I*d*x
 + 2*I*c) - (-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2
)*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^
(-2*I*d*x - 2*I*c)/((I*A + B)*a)) - 4*((23*A - 20*I*B)*a*e^(4*I*d*x + 4*I*
c) - 6*(4*A - 5*I*B)*a*e^(2*I*d*x + 2*I*c) + (13*A - 10*I*B)*a)*sqrt((I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c)
- 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(81) = 162.

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.84

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{15\left(2\sqrt{2}((i-1)\ A+(i+1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((i-1)\ A+(i+1)\ B)\operatorname{arcta}\right.}{}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/60*(15*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) +
2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqr
t(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*
log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(-(I + 1)*A
 + (I - 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a + 1
20*(A - I*B)*a/sqrt(tan(d*x + c)) + 40*(-I*A - B)*a/tan(d*x + c)^(3/2) - 2
4*A*a/tan(d*x + c)^(5/2))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$

$$-\frac{15\sqrt{2}((i-1)\ Aa+(i+1)\ Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)-\frac{2\left(15\,Aa\tan(dx+c)^2-15i\,Ba\tan(dx\right.}{}}{15\,d}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/15*(15*sqrt(2)*((I - 1)*A*a + (I + 1)*B*a)*arctan(-(1/2*I - 1/2)*sqrt(2
)*sqrt(tan(d*x + c))) - 2*(15*A*a*tan(d*x + c)^2 - 15*I*B*a*tan(d*x + c)^2
 - 5*I*A*a*tan(d*x + c) - 5*B*a*tan(d*x + c) - 3*A*a)/tan(d*x + c)^(5/2))/
d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{a\left(-2\sqrt{\cot(dx+c)}\cot(dx+c)^2 a+10\sqrt{\cot(dx+c)}\,a+5\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)ad+5\left(\int \sqrt{\cot(dx+c)}\right.}{}$$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*( - 2*sqrt(cot(c + d*x))*cot(c + d*x)**2*a + 10*sqrt(cot(c + d*x))*a +
5*int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a*d + 5*int(sqrt(cot(c + d*x))*co
t(c + d*x)**3*tan(c + d*x)**2,x)*b*d*i + 5*int(sqrt(cot(c + d*x))*cot(c +
d*x)**3*tan(c + d*x),x)*a*d*i + 5*int(sqrt(cot(c + d*x))*cot(c + d*x)**3*t
an(c + d*x),x)*b*d))/(5*d)
```

3.507 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 78

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{2\sqrt[4]{-1}a(iA+B)\operatorname{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{2a(iA+B)\sqrt{\cot(c+dx)}}{d} - \frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

output
```
-2*(-1)^(1/4)*a*(I*A+B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2*a*(I*A+B)
*cot(d*x+c)^(1/2)/d-2/3*a*A*cot(d*x+c)^(3/2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{2a\sqrt{\cot(c+dx)}\left(A\cot(c+dx)+3(iA+B)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},i\tan(c+dx)\right)\right)}{3d}$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x
]
```

output
```
(-2*a*Sqrt[Cot[c + d*x]]*(A*Cot[c + d*x] + 3*(I*A + B)*Hypergeometric2F1[-
1/2, 1, 1/2, I*Tan[c + d*x]]))/(3*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4064, 3042, 4075, 3042, 4011, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cot(c+dx)^{5/2}(a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4064}$$

$$\int \sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)(A\cot(c+dx)+B)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$\big\downarrow$ 4075

$$-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}+\int \sqrt{\cot(c+dx)}(a(iA+B)\cot(c+dx)-a(A-iB))dx$$

$\big\downarrow$ 3042

$$-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}+\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-a(A-iB)-a(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$\big\downarrow$ 4011

$$\int \frac{-a(iA+B)-a(A-iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx-\frac{2a(B+iA)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$\big\downarrow$ 3042

$$\int \frac{a(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)-a(iA+B)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a(B+iA)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$\big\downarrow$ 4016

$$\frac{2a^2(B+iA)^2\int \frac{1}{a(iA+B)-a(A-iB)\cot(c+dx)}d\sqrt{\cot(c+dx)}}{d}-\frac{2a(B+iA)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$\big\downarrow$ 221

$$-\frac{2\sqrt[4]{-1}a(B+iA)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a(B+iA)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*(-1)^(1/4)*a*(I*A + B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (2*
a*(I*A + B)*Sqrt[Cot[c + d*x]])/d - (2*a*A*Cot[c + d*x]^(3/2))/(3*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(64) = 128$.

Time = 0.58 (sec) , antiderivative size = 528, normalized size of antiderivative = 6.77

method	result
derivativedivides	$-\frac{a\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}}\tan(dx+c)\left(6iA\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\sqrt{2}\,\tan(dx+c)^{\frac{3}{2}}+6iA\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right.}{}$
default	$-\frac{a\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}}\tan(dx+c)\left(6iA\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\sqrt{2}\,\tan(dx+c)^{\frac{3}{2}}+6iA\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right.}{}$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/12*a/d*(1/tan(d*x+c))^(5/2)*tan(d*x+c)*(6*I*A*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)+6*I*A*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2
))*2^(1/2)*tan(d*x+c)^(3/2)+3*I*A*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)
-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(1/2)*tan(d*x+c)^(3/2)-3*I*
B*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-ta
n(d*x+c)-1))*2^(1/2)*tan(d*x+c)^(3/2)-6*I*B*arctan(1+2^(1/2)*tan(d*x+c)^(1
/2))*2^(1/2)*tan(d*x+c)^(3/2)-6*I*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^
(1/2)*tan(d*x+c)^(3/2)+3*A*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^
(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)*tan(d*x+c)^(3/2)+6*A*arctan(
1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)+6*A*arctan(-1+2^(1/2)
*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)+6*B*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)+6*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*
2^(1/2)*tan(d*x+c)^(3/2)+3*B*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(
tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(1/2)*tan(d*x+c)^(3/2)+24*I*A*ta
n(d*x+c)+24*B*tan(d*x+c)+8*A)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(62) = 124$.

Time = 0.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.90

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$

$$-\frac{3\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{(i\,A^2+2\,AB-i\,B^2)a^2}{d^2}}\log\left(-\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}+\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{(i\,A^2+2\,AB-i\,B^2)a^2}{d^2}}\sqrt{\frac{i\cdots}{e\cdots}}\cdots\right.}{(i\,A+B)a}\cdots\right.}{\cdots}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/6*(3*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)
*log(-2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) + (d*e^(2*I*d*x + 2*I*c) - d)*sqr
t(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) - 3*(d*e^(2*I*d
*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*log(-2*((A - I*B)*
a*e^(2*I*d*x + 2*I*c) - (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B -
 I*B^2)*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) + 4*((4*I*A + 3*B)*a*e^(2*I*d*x +
2*I*c) + (-2*I*A - 3*B)*a)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(62) = 124$.

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.23

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$

$$-\frac{3\left(2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\,a\right.}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/12*(3*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1/2*s
qrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((I - 1)*A + (I + 1)*B)
*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((I - 1)*A
 + (I + 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a - 2
4*(-I*A - B)*a/sqrt(tan(d*x + c)) + 8*A*a/tan(d*x + c)^(3/2))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$=\frac{3\sqrt{2}(-(i+1)\ Aa+(i-1)\ Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan{(dx+c)}}\right)-\frac{2\,(3i\,Aa\tan(dx+c)+3\,Ba\tan(dx+c)+A}{\tan(dx+c)^{\frac{3}{2}}}}{3\,d}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
1/3*(3*sqrt(2)*(-(I + 1)*A*a + (I - 1)*B*a)*arctan(-(1/2*I - 1/2)*sqrt(2)*
sqrt(tan(d*x + c))) - 2*(3*I*A*a*tan(d*x + c) + 3*B*a*tan(d*x + c) + A*a)/
tan(d*x + c)^(3/2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{5/2}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+a\tan(c+d\,x)\,\,1\mathrm{i}\right)\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\begin{aligned}\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx &= a\Bigg(\left(\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)bi\right. \\ &\quad + \left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)ai \\ &\quad + \left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)b \\ &\quad + \left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\,dx\right)a\Bigg)\end{aligned}$$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
a*(int(sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**2,x)*b*i + int(sqr
t(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x),x)*a*i + int(sqrt(cot(c + d*x
))*cot(c + d*x)**2*tan(c + d*x),x)*b + int(sqrt(cot(c + d*x))*cot(c + d*x)
**2,x)*a)
```

3.508 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 53

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{2\sqrt[4]{-1}a(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

output
```
-2*(-1)^(1/4)*a*(A-I*B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2*a*A*cot(d
*x+c)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{2ia\sqrt{\cot(c+dx)}\left(-iA+\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}\right)}{d}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x
]
```

output
```
((-2*I)*a*Sqrt[Cot[c + d*x]]*((-I)*A + (-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3
/4)*Sqrt[Tan[c + d*x]]]*Sqrt[Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 4064, 3042, 4075, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}(a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+ia)(A\cot(c+dx)+B)}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4075}$$

$$-\frac{2aA\sqrt{\cot(c+dx)}}{d}+\int \frac{a(iA+B)\cot(c+dx)-a(A-iB)}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2aA\sqrt{\cot(c+dx)}}{d}+\int \frac{-a(A-iB)-a(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4016}$$

$$-\frac{2aA\sqrt{\cot(c+dx)}}{d}+\frac{2a^2(A-iB)^2\int\frac{1}{a(A-iB)+a(iA+B)\cot(c+dx)}d\sqrt{\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2aA\sqrt{\cot(c+dx)}}{d}-\frac{2\sqrt[4]{-1}a(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*(-1)^(1/4)*a*(A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (2*
a*A*Sqrt[Cot[c + d*x]])/d
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(44) = 88$.

Time = 0.41 (sec) , antiderivative size = 508, normalized size of antiderivative = 9.58

method	result
derivativedivides	$\frac{a\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)\left(iA\sqrt{2}\,\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)\sqrt{\tan(dx+c)}+2iA\sqrt{2}\,\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx}\right.\right.}{}$
default	$\frac{a\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)\left(iA\sqrt{2}\,\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)\sqrt{\tan(dx+c)}+2iA\sqrt{2}\,\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx}\right.\right.}{}$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/4*a/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(I*A*2^(1/2)*ln(-(tan(d*x+c)+2^(1/
2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*tan(d*x+c)
^(1/2)+2*I*A*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(1/2)+2
*I*A*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(1/2)+2*I*B*2^
(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(1/2)+2*I*B*2^(1/2)*ar
ctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(1/2)+I*B*2^(1/2)*ln(-(2^(1/2
)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*
tan(d*x+c)^(1/2)-2*A*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)
^(1/2)-2*A*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(1/2)-A*
2^(1/2)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*ta
n(d*x+c)^(1/2)+1))*tan(d*x+c)^(1/2)+B*2^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(
d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*tan(d*x+c)^(1/2)+
2*B*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(1/2)+2*B*2^(1/2
)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(1/2)-8*A)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 5.96

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$

$$-\frac{4\,Aa\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}-\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a^2}{d^2}}d\log\left(-\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}-(i\,de^{(2i\,dx+2i\,c)}-i\,d)\sqrt{-\frac{(-i\,A^2-2\,AB+i}{d^2}}\right.}{(i\,A+B)a}\right.}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/2*(4*A*a*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) -
sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*d*log(-2*((A - I*B)*a*e^(2*I*d*x +
 2*I*c) - (I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a
^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2
*I*d*x - 2*I*c)/((I*A + B)*a)) + sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*d
*log(-2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) - (-I*d*e^(2*I*d*x + 2*I*c) + I*d
)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)))/d
```

Sympy [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= ia\left(\int\left(-iA\cot^{\frac{3}{2}}(c+dx)\right)dx+\int A\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)\,dx\right.$$
$$\left.+\int B\tan^2(c+dx)\cot^{\frac{3}{2}}(c+dx)\,dx+\int\left(-iB\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)\right)dx\right)$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*A*cot(c + d*x)**(3/2), x) + Integral(A*tan(c + d*x)*cot(c
 + d*x)**(3/2), x) + Integral(B*tan(c + d*x)**2*cot(c + d*x)**(3/2), x) +
Integral(-I*B*tan(c + d*x)*cot(c + d*x)**(3/2), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(43) = 86.

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.92

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$

$$-\frac{\left(2\sqrt{2}((i-1)\ A+(i+1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((i-1)\ A+(i+1)\ B)\arctan\right.}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/4*((2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/s
qrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log
(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(-(I + 1)*A +
(I - 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a + 8*A*
a/sqrt(tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx =$$

$$-\frac{\sqrt{2}(-(i-1)\ Aa-(i+1)\ Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+\frac{2\,Aa}{\sqrt{\tan(dx+c)}}}{d}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-(sqrt(2)*(-(I - 1)*A*a - (I + 1)*B*a)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(
tan(d*x + c))) + 2*A*a/sqrt(tan(d*x + c)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\frac{a\left(-2\sqrt{\cot(dx+c)}\,a-\left(\int\frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)ad+\left(\int\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^2\,dx\right)bdi+\left(\right.}{}$$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
(a*( - 2*sqrt(cot(c + d*x))*a - int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a*d
 + int(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)**2,x)*b*d*i + int(sqrt
(cot(c + d*x))*cot(c + d*x)*tan(c + d*x),x)*a*d*i + int(sqrt(cot(c + d*x))
*cot(c + d*x)*tan(c + d*x),x)*b*d))/d
```

3.509 $\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 55

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{2\sqrt[4]{-1}a(iA+B)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2iaB}{d\sqrt{\cot(c+dx)}}$$

output
```
2*(-1)^(1/4)*a*(I*A+B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d+2*I*a*B/d/co
t(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{2a\left(iB-\frac{\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{\sqrt{\tan(c+dx)}}\right)}{d\sqrt{\cot(c+dx)}}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*a*(I*B - ((-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]])/S
qrt[Tan[c + d*x]]))/(d*Sqrt[Cot[c + d*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {3042, 4064, 3042, 4074, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+ia)(A\cot(c+dx)+B)}{\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4074}$$

$$\int \frac{a(iA+B)+a(A-iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2iaB}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a(iA+B)-a(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2iaB}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4016}$$

$$\frac{2a^2(B+iA)^2\int \frac{1}{a(A-iB)\cot(c+dx)-a(iA+B)}d\sqrt{\cot(c+dx)}}{d}+\frac{2iaB}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt[4]{-1}a(B+iA)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2iaB}{d\sqrt{\cot(c+dx)}}$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(2*(-1)^(1/4)*a*(I*A + B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d + ((2*
I)*a*B)/(d*Sqrt[Cot[c + d*x]])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(45) = 90$.

Time = 0.44 (sec) , antiderivative size = 422, normalized size of antiderivative = 7.67

method	result
derivativedivides	$\frac{a\sqrt{\frac{1}{\tan(dx+c)}}\,\sqrt{\tan(dx+c)}\left(2iA\sqrt{2}\,\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2iA\sqrt{2}\,\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+iA\sqrt{2}\,\ln\left(\right.\right.}{}$
default	$\frac{a\sqrt{\frac{1}{\tan(dx+c)}}\,\sqrt{\tan(dx+c)}\left(2iA\sqrt{2}\,\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+2iA\sqrt{2}\,\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+iA\sqrt{2}\,\ln\left(\right.\right.}{}$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/4*a/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)^(1/2)*(2*I*A*2^(1/2)*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))+2*I*A*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))+I*
A*2^(1/2)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*
tan(d*x+c)^(1/2)+1))-I*B*2^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+
1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))-2*I*B*2^(1/2)*arctan(1+2^(1/2)
*tan(d*x+c)^(1/2))-2*I*B*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))+A*2^(
1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)
-tan(d*x+c)-1))+2*A*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+2*A*2^(1/2)
*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))+2*B*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+
c)^(1/2))+2*B*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))+B*2^(1/2)*ln(-(2
^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)
+1))+8*I*B*tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 364, normalized size of antiderivative = 6.62

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$= \frac{\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{(i\,A^2+2\,AB-i\,B^2)a^2}{d^2}}\log\left(-\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}+\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{\left(i\,A^2+2\,AB-i\,B^2\right)a^2}{d^2}}\sqrt{\frac{i\,e^{(2i\,dx}}{e^{(2i\,dx-}}}}{(i\,A+B)a}\right.}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/2*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*lo
g(-2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) + (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-
(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) - (d*e^(2*I*d*x +
2*I*c) + d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*log(-2*((A - I*B)*a*e^(
2*I*d*x + 2*I*c) - (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^
2)*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e
^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) + 4*(B*a*e^(2*I*d*x + 2*I*c) - B*a)*sqr
t((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(2*I*d*x +
2*I*c) + d)
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$= ia\left(\int\left(-iA\sqrt{\cot(c+dx)}\right)dx + \int A\tan(c+dx)\sqrt{\cot(c+dx)}\,dx + \int B\tan^2(c+dx)\sqrt{\cot(c+dx)}\,dx + \int\left(-iB\tan(c+dx)\sqrt{\cot(c+dx)}\right)dx\right)$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
I*a*(Integral(-I*A*sqrt(cot(c + d*x)), x) + Integral(A*tan(c + d*x)*sqrt(c
ot(c + d*x)), x) + Integral(B*tan(c + d*x)**2*sqrt(cot(c + d*x)), x) + Int
egral(-I*B*tan(c + d*x)*sqrt(cot(c + d*x)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(43) = 86$.

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.82

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{8i\,Ba\sqrt{\tan(dx+c)} + \left(2\sqrt{2}(-(i+1)\ A + (i-1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + \frac{2}{\sqrt{\tan(dx+c)}}\right)\right) + 2\sqrt{2}(-(i+$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/4*(8*I*B*a*sqrt(tan(d*x + c)) + (2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arct
an(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A +
 (I - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2
)*((I - 1)*A + (I + 1)*B)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c)
+ 1) + sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1
/tan(d*x + c) + 1))*a)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = -\frac{\sqrt{2}(-(i+1)\ Aa+(i-1)\ Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)-2i\,Ba\sqrt{\tan(dx+c)}}{d}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-(sqrt(2)*(-(I + 1)*A*a + (I - 1)*B*a)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(
tan(d*x + c))) - 2*I*B*a*sqrt(tan(d*x + c)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx = \int \sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i),x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= a\left(\left(\int \sqrt{\cot(dx+c)}dx\right)a+\left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)^2\,dx\right)bi + \left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)\,dx\right)ai+\left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)\,dx\right)b\right)$$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
a*(int(sqrt(cot(c + d*x)),x)*a + int(sqrt(cot(c + d*x))*tan(c + d*x)**2,x)
*b*i + int(sqrt(cot(c + d*x))*tan(c + d*x),x)*a*i + int(sqrt(cot(c + d*x))
*tan(c + d*x),x)*b)
```

3.510 $\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 80

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{2\sqrt[4]{-1}a(A-iB)\operatorname{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} + \frac{2iaB}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a(iA+B)}{d\sqrt{\cot(c+dx)}}$$

output
```
2*(-1)^(1/4)*a*(A-I*B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d+2/3*I*a*B/d/
cot(d*x+c)^(3/2)+2*a*(I*A+B)/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{2ia\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(3(A-iB)\left(\sqrt[4]{-1}\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}\right)+\right.}{3d}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]]
,x]
```

output
```
(((2*I)/3)*a*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(3*(A - I*B)*((-1)^(1/4
)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + Sqrt[Tan[c + d*x]]) + B*Tan[c +
d*x]^(3/2)))/d
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 4064, 3042, 4074, 3042, 4012, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+ia)(A\cot(c+dx)+B)}{\cot^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4074}$$

$$\int \frac{a(iA+B)+a(A-iB)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2iaB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a(iA+B)-a(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2iaB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4012}$$

$$\int \frac{a(A-iB)-a(iA+B)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2a(B+iA)}{d\sqrt{\cot(c+dx)}}+\frac{2iaB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a(A-iB)+a(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(B+iA)}{d\sqrt{\cot(c+dx)}}+\frac{2iaB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4016}$$

$$\frac{2a^2(A-iB)^2\int \frac{1}{-a(A-iB)-a(iA+B)\cot(c+dx)}d\sqrt{\cot(c+dx)}}{d}+\frac{2a(B+iA)}{d\sqrt{\cot(c+dx)}}+\frac{2iaB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2\sqrt[4]{-1}a(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2a(B+iA)}{d\sqrt{\cot(c+dx)}}+\frac{2iaB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

output
```
(2*(-1)^(1/4)*a*(A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d + (((2
*I)/3)*a*B)/(d*Cot[c + d*x]^(3/2)) + (2*a*(I*A + B))/(d*Sqrt[Cot[c + d*x]]
)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(65) = 130$.

Time = 0.54 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.75

method	result
derivativedivides	$-\frac{a\left(\frac{(-iB+A)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{4}+\frac{(-iA}{}\right.}{}$
default	$-\frac{a\left(\frac{(-iB+A)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{4}+\frac{(-iA}{}\right.}{}$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-a/d*(1/4*(A-I*B)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot
(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+
2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/4*(-I*A-B)*2^(1/2)*(ln((cot(d*x+c
)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*a
rctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))-2
*(I*A+B)/cot(d*x+c)^(1/2)-2/3*I*B/cot(d*x+c)^(3/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(62) = 124$.

Time = 0.09 (sec) , antiderivative size = 429, normalized size of antiderivative = 5.36

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx =$$

$$-\frac{3\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a^2}{d^2}}\log\left(-\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}-\left(i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{-}\right.}{(i}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
-1/6*(3*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^
2 - 2*A*B + I*B^2)*a^2/d^2)*log(-2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) - (I*d
*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c
)/((I*A + B)*a)) - 3*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^2/d^2)*log(-2*((A - I*B)*a*e^(2*I*d*x +
2*I*c) - (-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a
^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2
*I*d*x - 2*I*c)/((I*A + B)*a)) - 4*((3*A - 4*I*B)*a*e^(4*I*d*x + 4*I*c) +
2*I*B*a*e^(2*I*d*x + 2*I*c) - (3*A - 2*I*B)*a)*sqrt((I*e^(2*I*d*x + 2*I*c)
 + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x
+ 2*I*c) + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = ia\left(\int\left(-\frac{iA}{\sqrt{\cot(c+dx)}}\right)dx+\int\frac{A\tan(c+dx)}{\sqrt{\cot(c+dx)}}\,dx+\int\frac{B\tan^2(c+dx)}{\sqrt{\cot(c+dx)}}\,dx+\int\left(-\frac{iB\tan(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
I*a*(Integral(-I*A/sqrt(cot(c + d*x)), x) + Integral(A*tan(c + d*x)/sqrt(c
ot(c + d*x)), x) + Integral(B*tan(c + d*x)**2/sqrt(cot(c + d*x)), x) + Int
egral(-I*B*tan(c + d*x)/sqrt(cot(c + d*x)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(62) = 124$.

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.21

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \frac{8\left(iBa-\frac{3(-iA-B)a}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{3}{2}}+3\left(2\sqrt{2}((i-1)A+(i+1)B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
1/12*(8*(I*B*a - 3*(-I*A - B)*a/tan(d*x + c))*tan(d*x + c)^(3/2) + 3*(2*sq
rt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x
 + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2)
- 2/sqrt(tan(d*x + c)))) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(sqrt(2)/sq
rt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(-(I + 1)*A + (I - 1)*B)*
log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx =$$
$$-\frac{-2iBa\tan(dx+c)^{\frac{3}{2}}+3\sqrt{2}((i-1)Aa+(i+1)Ba)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)-6iA}{3d}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
-1/3*(-2*I*B*a*tan(d*x + c)^(3/2) + 3*sqrt(2)*((I - 1)*A*a + (I + 1)*B*a)*
arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - 6*I*A*a*sqrt(tan(d*x +
 c)) - 6*B*a*sqrt(tan(d*x + c)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\ (a+a\tan(c+d\,x)\ 1\mathrm{i})}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i))/cot(c + d*x)^(1/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i))/cot(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = a\left(\left(\int \frac{\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)}dx\right)a+\left(\int \frac{\sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)^{2}}{\cot\left(dx+c\right)}dx\right)bi + \left(\int \frac{\sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)}{\cot\left(dx+c\right)}dx\right)ai+\left(\int \frac{\sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)}{\cot\left(dx+c\right)}dx\right)b\right)$$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
a*(int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a + int((sqrt(cot(c + d*x))*tan(
c + d*x)**2)/cot(c + d*x),x)*b*i + int((sqrt(cot(c + d*x))*tan(c + d*x))/c
ot(c + d*x),x)*a*i + int((sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*x),x)
*b)
```

3.511 $\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 105

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = -\frac{2\sqrt[4]{-1}a(iA+B)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} + \frac{2iaB}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{2a(iA+B)}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2a(A-iB)}{d\sqrt{\cot(c+dx)}}$$

output
```
-2*(-1)^(1/4)*a*(I*A+B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d+2/5*I*a*B/d
/cot(d*x+c)^(5/2)+2/3*a*(I*A+B)/d/cot(d*x+c)^(3/2)+2*a*(A-I*B)/d/cot(d*x+c
)^(1/2)
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2ia\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(3B\tan^{\frac{5}{2}}(c+dx)+5(A-iB)\left(-3(-1)^{3/4}\arctan\left((-1)^{3/4}\sqrt{\tan(c+d\right.\right.\right.}{15d}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2)
,x]
```

output
```
(((2*I)/15)*a*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(3*B*Tan[c + d*x]^(5/2
) + 5*(A - I*B)*(-3*(-1)^(3/4)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + Sqr
t[Tan[c + d*x]]*(-3*I + Tan[c + d*x]))))/d
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 4064, 3042, 4074, 3042, 4012, 3042, 4012, 25, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\cot(c+dx)^{3/2}}dx$$
$$\Big\downarrow \; 4064$$
$$\int \frac{(a\cot(c+dx)+ia)(A\cot(c+dx)+B)}{\cot^{\frac{7}{2}}(c+dx)}dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4074}$$

$$\int \frac{a(iA+B)+a(A-iB)\cot(c+dx)}{\cot^{\frac{5}{2}}(c+dx)}dx+\frac{2iaB}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a(iA+B)-a(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+\frac{2iaB}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4012}$$

$$\int \frac{a(A-iB)-a(iA+B)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2a(B+iA)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2iaB}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a(A-iB)+a(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2a(B+iA)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2iaB}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4012}$$

$$\int -\frac{a(iA+B)+a(A-iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2a(B+iA)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a(A-iB)}{d\sqrt{\cot(c+dx)}}+\frac{2iaB}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{a(iA+B)+a(A-iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2a(B+iA)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a(A-iB)}{d\sqrt{\cot(c+dx)}}+\frac{2iaB}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{a(iA+B)-a(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(B+iA)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a(A-iB)}{d\sqrt{\cot(c+dx)}}+\frac{2iaB}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4016}$$

$$-\frac{2a^2(B+iA)^2\int \frac{1}{a(A-iB)\cot(c+dx)-a(iA+B)}d\sqrt{\cot(c+dx)}}{d}+\frac{2a(B+iA)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a(A-iB)}{d\sqrt{\cot(c+dx)}}+\frac{2iaB}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\sqrt[4]{-1}a(B+iA)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2a(B+iA)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2a(A-iB)}{d\sqrt{\cot(c+dx)}}+\frac{2iaB}{5d\cot^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2),x]
```

output
```
(-2*(-1)^(1/4)*a*(I*A + B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d + (((
2*I)/5)*a*B)/(d*Cot[c + d*x]^(5/2)) + (2*a*(I*A + B))/(3*d*Cot[c + d*x]^(3
/2)) + (2*a*(A - I*B))/(d*Sqrt[Cot[c + d*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(85) = 170$.

Time = 0.38 (sec) , antiderivative size = 470, normalized size of antiderivative = 4.48

method	result
derivativedivides	$-\frac{a\left(30iA\sqrt{2}\,\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+120iB\sqrt{\tan(dx+c)}+15iA\sqrt{2}\,\ln\left(-\frac{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+30iA\right.}{}$
default	$-\frac{a\left(30iA\sqrt{2}\,\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)+120iB\sqrt{\tan(dx+c)}+15iA\sqrt{2}\,\ln\left(-\frac{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}\right)+30iA\right.}{}$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/60*a/d*(30*I*A*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+120*I*B*tan(d
*x+c)^(1/2)+15*I*A*2^(1/2)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(ta
n(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))+30*I*A*2^(1/2)*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2))-30*I*B*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))-30*I*B*2^
(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))-15*I*B*2^(1/2)*ln(-(tan(d*x+c)+2
^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))-24*I*B
*tan(d*x+c)^(5/2)-40*B*tan(d*x+c)^(3/2)+15*A*2^(1/2)*ln(-(tan(d*x+c)+2^(1/
2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))+30*A*2^(1/
2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+30*A*2^(1/2)*arctan(-1+2^(1/2)*tan(d
*x+c)^(1/2))+30*B*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))+30*B*2^(1/2)*
arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))+15*B*2^(1/2)*ln(-(2^(1/2)*tan(d*x+c)^(
1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))-40*I*A*tan(d*x
+c)^(3/2)-120*A*tan(d*x+c)^(1/2))/(1/tan(d*x+c))^(3/2)/tan(d*x+c)^(3/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(81) = 162$.

Time = 0.11 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.59

$$\int \frac{(a + ia\tan(c + dx))(A + B\tan(c + dx))}{\cot^{\frac{3}{2}}(c + dx)}\,dx =$$

$$-\frac{15\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4i\,c)} + 3\,de^{(2i\,dx+2i\,c)} + d\right)\sqrt{-\frac{(i\,A^2+2\,AB-i\,B^2)a^2}{d^2}}\log\left(-\frac{2\left((A-i\,B)ae^{(2i\,dx+2i\,c)}+(d\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
-1/30*(15*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*
x + 2*I*c) + d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*log(-2*((A - I*B)*a
*e^(2*I*d*x + 2*I*c) + (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B -
I*B^2)*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) - 15*(d*e^(6*I*d*x + 6*I*c) + 3*d*e
^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(I*A^2 + 2*A*B - I
*B^2)*a^2/d^2)*log(-2*((A - I*B)*a*e^(2*I*d*x + 2*I*c) - (d*e^(2*I*d*x + 2
*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^2/d^2)*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((I*A + B)*a)) +
 4*((20*I*A + 23*B)*a*e^(6*I*d*x + 6*I*c) + (10*I*A + B)*a*e^(4*I*d*x + 4*
I*c) + (-20*I*A - 11*B)*a*e^(2*I*d*x + 2*I*c) + (-10*I*A - 13*B)*a)*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(6*I*d*x + 6*I
*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = ia\left(\int\left(-\frac{iA}{\cot^{\frac{3}{2}}(c+dx)}\right)dx+\int\frac{A\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{B\tan^2(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\,dx+\int\left(-\frac{iB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)**(3/2),x)
```

output
```
I*a*(Integral(-I*A/cot(c + d*x)**(3/2), x) + Integral(A*tan(c + d*x)/cot(c
 + d*x)**(3/2), x) + Integral(B*tan(c + d*x)**2/cot(c + d*x)**(3/2), x) +
Integral(-I*B*tan(c + d*x)/cot(c + d*x)**(3/2), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(81) = 162$.

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.82

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{8\left(-3i\,Ba-\frac{5\,(i\,A+B)a}{\tan(dx+c)}-\frac{15\,(A-i\,B)a}{\tan(dx+c)^2}\right)\tan\left(dx+c\right)^{\frac{5}{2}}+15\left(2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
-1/60*(8*(-3*I*B*a - 5*(I*A + B)*a/tan(d*x + c) - 15*(A - I*B)*a/tan(d*x +
 c)^2)*tan(d*x + c)^(5/2) + 15*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(
1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I
 - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(
(I - 1)*A + (I + 1)*B)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1
) + sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/ta
n(d*x + c) + 1))*a)/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx =$$

$$-\frac{-6i\,Ba\tan\left(dx+c\right)^{\frac{5}{2}}-10i\,Aa\tan\left(dx+c\right)^{\frac{3}{2}}-10\,Ba\tan\left(dx+c\right)^{\frac{3}{2}}+15\sqrt{2}((i+1)\ Aa-(i-1)\ Ba}{15\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
-1/15*(-6*I*B*a*tan(d*x + c)^(5/2) - 10*I*A*a*tan(d*x + c)^(3/2) - 10*B*a*
tan(d*x + c)^(3/2) + 15*sqrt(2)*((I + 1)*A*a - (I - 1)*B*a)*arctan(-(1/2*I
 - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - 30*A*a*sqrt(tan(d*x + c)) + 30*I*B*a
*sqrt(tan(d*x + c)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})}{\cot(c+d\,x)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i))/cot(c + d*x)^(3/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i))/cot(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = a\left(\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)^2}dx\right)bi + \left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)^2}dx\right)ai+\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)^2}dx\right)b\right)$$

input
```
int((a+I*a*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
a*(int(sqrt(cot(c + d*x))/cot(c + d*x)**2,x)*a + int((sqrt(cot(c + d*x))*t
an(c + d*x)**2)/cot(c + d*x)**2,x)*b*i + int((sqrt(cot(c + d*x))*tan(c + d
*x))/cot(c + d*x)**2,x)*a*i + int((sqrt(cot(c + d*x))*tan(c + d*x))/cot(c
+ d*x)**2,x)*b)
```

3.512 $\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 128

$$\begin{aligned}\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx &= \frac{4\sqrt[4]{-1}a^2(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{4a^2(A-iB)\sqrt{\cot(c+dx)}}{d}\\ &\quad-\frac{2a^2(7iA+5B)\cot^{\frac{3}{2}}(c+dx)}{15d}-\frac{2A\cot^{\frac{3}{2}}(c+dx)\left(ia^2+a^2\cot(c+dx)\right)}{5d}\end{aligned}$$

output
```
4*(-1)^(1/4)*a^2*(A-I*B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d+4*a^2*(A-I
*B)*cot(d*x+c)^(1/2)/d-2/15*a^2*(7*I*A+5*B)*cot(d*x+c)^(3/2)/d-2/5*A*cot(d
*x+c)^(3/2)*(I*a^2+a^2*cot(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.58

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2a^2\sqrt{\cot(c+dx)}\left(\cot(c+dx)(5(2iA+B)+3A\cot(c+dx))-30(A-iB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}\right.\right.}{15d}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x])
,x]
```

output
```
(-2*a^2*Sqrt[Cot[c + d*x]]*(Cot[c + d*x]*(5*((2*I)*A + B) + 3*A*Cot[c + d*
x]) - 30*(A - I*B)*Hypergeometric2F1[-1/2, 1, 1/2, I*Tan[c + d*x]]))/(15*d
)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4064, 3042, 4077, 27, 3042, 4075, 3042, 4011, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{7/2}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2(A\cot(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^2\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4077}$$

$$-\frac{2}{5}\int \frac{1}{2}\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)(a(3A-5iB)-a(7iA+5B)\cot(c+dx))dx-\frac{2A\cot^{\frac{3}{2}}(c+dx)\left(a^2\cot(c+dx)+ia^2\right)}{5d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}\int \sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)(a(3A-5iB)-a(7iA+5B)\cot(c+dx))dx-\frac{2A\cot^{\frac{3}{2}}(c+dx)\left(a^2\cot(c+dx)+ia^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(3A-5iB)+a(7iA+5B)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2A\cot^{\frac{3}{2}}(c+dx)\left(a^2\cot(c+dx)+ia^2\right)}{5d}$$

$$\Big\downarrow \text{4075}$$

$$\frac{1}{5}\left(-\int \sqrt{\cot(c+dx)}\left(10(iA+B)a^2+10(A-iB)\cot(c+dx)a^2\right)dx-\frac{2a^2(5B+7iA)\cot^{\frac{3}{2}}(c+dx)}{3d}\right)-\frac{2A\cot^{\frac{3}{2}}(c+dx)\left(a^2\cot(c+dx)+ia^2\right)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(10a^2(iA+B)-10a^2(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2a^2(5B+7iA)\cot^{\frac{3}{2}}(c+\right.$$
$$-\frac{2A\cot^{\frac{3}{2}}(c+dx)\left(a^2\cot(c+dx)+ia^2\right)}{5d}$$

$$\Big\downarrow \text{4011}$$

$$\frac{1}{5}\left(-\int \frac{10a^2(iA+B)\cot(c+dx)-10a^2(A-iB)}{\sqrt{\cot(c+dx)}}dx-\frac{2a^2(5B+7iA)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{20a^2(A-iB)\sqrt{\cot(c+\right.$$
$$-\frac{2A\cot^{\frac{3}{2}}(c+dx)\left(a^2\cot(c+dx)+ia^2\right)}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-\int \frac{-10(A-iB)a^2-10(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^2(5B+7iA)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{20a^2(A-iB)\sqrt{}}{d}\right.$$
$$\frac{2A\cot^{\frac{3}{2}}(c+dx)\left(a^2\cot(c+dx)+ia^2\right)}{5d}$$

$\downarrow$ 4016

$$\frac{1}{5}\left(-\frac{200a^4(A-iB)^2\int \frac{1}{10(A-iB)a^2+10(iA+B)\cot(c+dx)a^2}d\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(5B+7iA)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{20a^2(A}{}\right.$$
$$\frac{2A\cot^{\frac{3}{2}}(c+dx)\left(a^2\cot(c+dx)+ia^2\right)}{5d}$$

$\downarrow$ 221

$$\frac{1}{5}\left(\frac{20\sqrt[4]{-1}a^2(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2(5B+7iA)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{20a^2(A-iB)\sqrt{\cot(c}}{d}\right.$$
$$\frac{2A\cot^{\frac{3}{2}}(c+dx)\left(a^2\cot(c+dx)+ia^2\right)}{5d}$$

input
```
Int[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*A*Cot[c + d*x]^(3/2)*(I*a^2 + a^2*Cot[c + d*x]))/(5*d) + ((20*(-1)^(1/
4)*a^2*(A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d + (20*a^2*(A -
I*B)*Sqrt[Cot[c + d*x]])/d - (2*a^2*((7*I)*A + 5*B)*Cot[c + d*x]^(3/2))/(3
*d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(108) = 216$.

Time = 0.73 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.98

method	result
derivativedivides	$-\frac{a^2\left(\frac{2A\cot(dx+c)^{\frac{5}{2}}}{5}+\frac{4iA\cot(dx+c)^{\frac{3}{2}}}{3}+\frac{2B\cot(dx+c)^{\frac{3}{2}}}{3}+4iB\sqrt{\cot(dx+c)}-4A\sqrt{\cot(dx+c)}+\frac{(-2iB+2A)\sqrt{2}\left(\ln\left(\frac{\cot(dx}{\cot(dx}\right.\right.}{}\right.}{}$
default	$-\frac{a^2\left(\frac{2A\cot(dx+c)^{\frac{5}{2}}}{5}+\frac{4iA\cot(dx+c)^{\frac{3}{2}}}{3}+\frac{2B\cot(dx+c)^{\frac{3}{2}}}{3}+4iB\sqrt{\cot(dx+c)}-4A\sqrt{\cot(dx+c)}+\frac{(-2iB+2A)\sqrt{2}\left(\ln\left(\frac{\cot(dx}{\cot(dx}\right.\right.}{}\right.}{}$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
-a^2/d*(2/5*A*cot(d*x+c)^(5/2)+4/3*I*A*cot(d*x+c)^(3/2)+2/3*B*cot(d*x+c)^(
3/2)+4*I*B*cot(d*x+c)^(1/2)-4*A*cot(d*x+c)^(1/2)+1/4*(2*A-2*I*B)*2^(1/2)*(
ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^
(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x
+c)^(1/2)))+1/4*(-2*I*A-2*B)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1
/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x
+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(104) = 208.

Time = 0.10 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.50

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{15\sqrt{-\frac{(-iA^2-2AB+iB^2)a^4}{d^2}}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left((A-iB)a^2e^{(2i\,dx+2i\,c)}-\sqrt{-\frac{(-iA^2-2AB+iB^2)}{d^2}}\right.}{(-i}\right.}{}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/15*(15*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(4*I*d*x + 4*I*c) -
 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) - s
qrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqr
t((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*
I*c)/((-I*A - B)*a^2)) - 15*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(
4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^2*e^(2*
I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(-I*d*e^(2*I*d*x
+ 2*I*c) + I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 2*((43*A - 35*I*B)*a^2*e^(4*I*
d*x + 4*I*c) - 6*(9*A - 10*I*B)*a^2*e^(2*I*d*x + 2*I*c) + (23*A - 25*I*B)*
a^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*
I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.55

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{15\left(2\sqrt{2}(-(i-1)\ A-(i+1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(-(i-1)\ A-(i+1)\ B\right)}{}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/30*(15*(2*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2)
+ 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-1/2*
sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*(-(I + 1)*A + (I - 1)*
B)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*(-(I + 1
)*A + (I - 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^
2 - 120*(A - I*B)*a^2/sqrt(tan(d*x + c)) - 20*(-2*I*A - B)*a^2/tan(d*x + c
)^(3/2) + 12*A*a^2/tan(d*x + c)^(5/2))/d
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(15\sqrt{2}((i-1)\ Aa^2+(i+1)\ Ba^2)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)-\frac{30\,Aa^2\tan(dx+c)^2-30i\,Ba^2}{}\right.}{15\,d}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/15*(15*sqrt(2)*((I - 1)*A*a^2 + (I + 1)*B*a^2)*arctan(-(1/2*I - 1/2)*sq
rt(2)*sqrt(tan(d*x + c))) - (30*A*a^2*tan(d*x + c)^2 - 30*I*B*a^2*tan(d*x
+ c)^2 - 10*I*A*a^2*tan(d*x + c) - 5*B*a^2*tan(d*x + c) - 3*A*a^2)/tan(d*x
 + c)^(5/2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^2\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{a^2\left(-2\sqrt{\cot(dx+c)}\,\cot(dx+c)^2a+10\sqrt{\cot(dx+c)}\,a+5\left(\int\frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)ad-5\left(\int\sqrt{\cot(dx+c)}\right.\right.}{}$$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*( - 2*sqrt(cot(c + d*x))*cot(c + d*x)**2*a + 10*sqrt(cot(c + d*x))*a
 + 5*int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a*d - 5*int(sqrt(cot(c + d*x))
*cot(c + d*x)**3*tan(c + d*x)**3,x)*b*d - 5*int(sqrt(cot(c + d*x))*cot(c +
 d*x)**3*tan(c + d*x)**2,x)*a*d + 10*int(sqrt(cot(c + d*x))*cot(c + d*x)**
3*tan(c + d*x)**2,x)*b*d*i + 10*int(sqrt(cot(c + d*x))*cot(c + d*x)**3*tan
(c + d*x),x)*a*d*i + 5*int(sqrt(cot(c + d*x))*cot(c + d*x)**3*tan(c + d*x)
,x)*b*d))/(5*d)
```

3.513 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 103

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{4\sqrt[4]{-1}a^2(iA+B)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{2a^2(5iA+3B)\sqrt{\cot(c+dx)}}{3d} - \frac{2A\sqrt{\cot(c+dx)}(ia^2+a^2\cot(c+dx))}{3d}$$

output
```
-4*(-1)^(1/4)*a^2*(I*A+B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2/3*a^2*(
5*I*A+3*B)*cot(d*x+c)^(1/2)/d-2/3*A*cot(d*x+c)^(1/2)*(I*a^2+a^2*cot(d*x+c)
)/d
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{2a^2\cot^{\frac{3}{2}}(c+dx)\left(A+3(2iA+B)\tan(c+dx)-6\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\tan\right.}{3d}$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x])
,x]
```

output
```
(-2*a^2*Cot[c + d*x]^(3/2)*(A + 3*((2*I)*A + B)*Tan[c + d*x] - 6*(-1)^(1/4
)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2)))/(3*
d)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4064, 3042, 4077, 27, 3042, 4075, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cot(c+dx)^{5/2}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \ 4064$$

$$\int \frac{(a\cot(c+dx)+ia)^2(A\cot(c+dx)+B)}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^2\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 4077}$$

$$-\frac{2}{3}\int \frac{(\cot(c+dx)a+ia)(a(A-3iB)-a(5iA+3B)\cot(c+dx))}{2\sqrt{\cot(c+dx)}}dx-\frac{2A\sqrt{\cot(c+dx)}\left(a^2\cot(c+dx)+ia^2\right)}{3d}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{3}\int \frac{(\cot(c+dx)a+ia)(a(A-3iB)-a(5iA+3B)\cot(c+dx))}{\sqrt{\cot(c+dx)}}dx-\frac{2A\sqrt{\cot(c+dx)}\left(a^2\cot(c+dx)+ia^2\right)}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{3}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(A-3iB)+a(5iA+3B)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2A\sqrt{\cot(c+dx)}\left(a^2\cot(c+dx)+ia^2\right)}{3d}$$

$$\Big\downarrow \text{ 4075}$$

$$\frac{1}{3}\left(-\int \frac{6(iA+B)a^2+6(A-iB)\cot(c+dx)a^2}{\sqrt{\cot(c+dx)}}dx-\frac{2a^2(3B+5iA)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2A\sqrt{\cot(c+dx)}\left(a^2\cot(c+dx)+ia^2\right)}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(-\int \frac{6a^2(iA+B)-6a^2(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^2(3B+5iA)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2A\sqrt{\cot(c+dx)}\left(a^2\cot(c+dx)+ia^2\right)}{3d}$$

$$\Big\downarrow \text{ 4016}$$

$$\frac{1}{3}\left(-\frac{72a^4(B+iA)^2\int \frac{1}{6a^2(A-iB)\cot(c+dx)-6a^2(iA+B)}d\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(3B+5iA)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2A\sqrt{\cot(c+dx)}\left(a^2\cot(c+dx)+ia^2\right)}{3d}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(-\frac{12\sqrt[4]{-1}a^2(B+iA)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2(3B+5iA)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2A\sqrt{\cot(c+dx)}\left(a^2\cot(c+dx)+ia^2\right)}{3d}$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
((-12*(-1)^(1/4)*a^2*(I*A + B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d -
 (2*a^2*((5*I)*A + 3*B)*Sqrt[Cot[c + d*x]])/d)/3 - (2*A*Sqrt[Cot[c + d*x]]
*(I*a^2 + a^2*Cot[c + d*x]))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(86) = 172$.

Time = 0.76 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.23

method	result
derivativedivides	$-\frac{a^2\left(\frac{2A\cot(dx+c)^{\frac{3}{2}}}{3}+4iA\sqrt{\cot(dx+c)}+2B\sqrt{\cot(dx+c)}+\frac{(-2iA-2B)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(\right.\right.}{4}}{}$
default	$-\frac{a^2\left(\frac{2A\cot(dx+c)^{\frac{3}{2}}}{3}+4iA\sqrt{\cot(dx+c)}+2B\sqrt{\cot(dx+c)}+\frac{(-2iA-2B)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(\right.\right.}{4}}{}$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
-a^2/d*(2/3*A*cot(d*x+c)^(3/2)+4*I*A*cot(d*x+c)^(1/2)+2*B*cot(d*x+c)^(1/2)
+1/4*(-2*I*A-2*B)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot
(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+
2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/4*(-2*A+2*I*B)*2^(1/2)*(ln((cot(d
*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))
+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)
)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(83) = 166$.

Time = 0.11 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.83

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{3\sqrt{-\frac{(iA^2+2AB-iB^2)a^4}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{2\left((A-iB)a^2e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{\left(iA^2+2AB-iB^2\right)a^4}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{ie^{(}}{e^{(2}}}\right.}{(-iA-B)a^2}\right.}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/3*(3*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)
*log(2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*
a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 3*sqrt(-(
I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(2*((A - I*
B)*a^2*e^(2*I*d*x + 2*I*c) - sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(
2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) + 2*((7*I*A + 3*B)*a^2*e^(
2*I*d*x + 2*I*c) + (-5*I*A - 3*B)*a^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) - 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(83) = 166.

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.75

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{3\left(2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\,a}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/6*(3*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) +
2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1/2*sq
rt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((I - 1)*A + (I + 1)*B)*
log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((I - 1)*A
+ (I + 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^2 -
12*(-2*I*A - B)*a^2/sqrt(tan(d*x + c)) + 4*A*a^2/tan(d*x + c)^(3/2))/d
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{2\left(3\sqrt{2}(-(i+1)\ Aa^2+(i-1)\ Ba^2)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)-\frac{6i\,Aa^2\tan(dx+c)+3\,Ba^2\tan(dx}{\tan(dx+c)^{\frac{3}{2}}}\right.}{3\,d}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
2/3*(3*sqrt(2)*(-(I + 1)*A*a^2 + (I - 1)*B*a^2)*arctan(-(1/2*I - 1/2)*sqrt
(2)*sqrt(tan(d*x + c))) - (6*I*A*a^2*tan(d*x + c) + 3*B*a^2*tan(d*x + c) +
 A*a^2)/tan(d*x + c)^(3/2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \int \cot(c+d\,x)^{5/2}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^2\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= a^2\left(-\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^3\,dx\right)b - \left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)a + 2\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)bi + 2\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
a**2*( - int(sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**3,x)*b - int
(sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**2,x)*a + 2*int(sqrt(cot(
c + d*x))*cot(c + d*x)**2*tan(c + d*x)**2,x)*b*i + 2*int(sqrt(cot(c + d*x)
)*cot(c + d*x)**2*tan(c + d*x),x)*a*i + int(sqrt(cot(c + d*x))*cot(c + d*x
)**2*tan(c + d*x),x)*b + int(sqrt(cot(c + d*x))*cot(c + d*x)**2,x)*a)
```

3.514 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 99

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{4\sqrt[4]{-1}a^2(A-iB)\operatorname{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{2a^2(A+iB)\sqrt{\cot(c+dx)}}{d} + \frac{2iB(ia^2+a^2\cot(c+dx))}{d\sqrt{\cot(c+dx)}}$$

output

```
-4*(-1)^(1/4)*a^2*(A-I*B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2*a^2*(A+
I*B)*cot(d*x+c)^(1/2)/d+2*I*B*(I*a^2+a^2*cot(d*x+c))/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2a^2\sqrt{\cot(c+dx)}\left(A+2\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}+B\tan(c+dx\right.}{d}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x])
,x]
```

output
```
(-2*a^2*Sqrt[Cot[c + d*x]]*(A + 2*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*S
qrt[Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] + B*Tan[c + d*x]))/d
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 4064, 3042, 4076, 27, 3042, 4075, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+ia)^2(A\cot(c+dx)+B)}{\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^2\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$2\int \frac{(\cot(c+dx)a+ia)(a(iA+3B)+a(A+iB)\cot(c+dx))}{2\sqrt{\cot(c+dx)}}dx+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(\cot(c+dx)a+ia)(a(iA+3B)+a(A+iB)\cot(c+dx))}{\sqrt{\cot(c+dx)}}dx+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(iA+3B)-a(A+iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4075}$$

$$\int \frac{2a^2(iA+B)\cot(c+dx)-2a^2(A-iB)}{\sqrt{\cot(c+dx)}}dx-\frac{2a^2(A+iB)\sqrt{\cot(c+dx)}}{d}+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-2(A-iB)a^2-2(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^2(A+iB)\sqrt{\cot(c+dx)}}{d}+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4016}$$

$$\frac{8a^4(A-iB)^2\int \frac{1}{2(A-iB)a^2+2(iA+B)\cot(c+dx)a^2}d\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(A+iB)\sqrt{\cot(c+dx)}}{d}+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{4\sqrt[4]{-1}a^2(A-iB)\operatorname{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2(A+iB)\sqrt{\cot(c+dx)}}{d}+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{d\sqrt{\cot(c+dx)}}$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(-4*(-1)^(1/4)*a^2*(A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (
2*a^2*(A + I*B)*Sqrt[Cot[c + d*x]])/d + ((2*I)*B*(I*a^2 + a^2*Cot[c + d*x]
))/(d*Sqrt[Cot[c + d*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(85) = 170$.

Time = 0.44 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.20

method	result
derivativedivides	$-\frac{a^2\left(2A\sqrt{\cot(dx+c)}+\frac{(2iB-2A)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\right.\right.}}{4}}{}$
default	$-\frac{a^2\left(2A\sqrt{\cot(dx+c)}+\frac{(2iB-2A)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\right.\right.}}{4}}{}$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
-a^2/d*(2*A*cot(d*x+c)^(1/2)+1/4*(-2*A+2*I*B)*2^(1/2)*(ln((cot(d*x+c)+2^(1
/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(
1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/4*(2*
I*A+2*B)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2
^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(
-1+2^(1/2)*cot(d*x+c)^(1/2)))+2*B/cot(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(81) = 162$.

Time = 0.09 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.89

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$= \frac{\sqrt{-\frac{(-iA^2-2AB+iB^2)a^4}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left((A-iB)a^2e^{(2i\,dx+2i\,c)}-\sqrt{-\frac{(-iA^2-2AB+iB^2)a^4}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{ie^{(\cdots}}{e^{(\cdots}}}\cdots}{(-iA-B)a^2}\cdots\right)\cdots}{\cdots}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
(sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2
*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d
^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - sqrt(-(-I*
A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)
*a^2*e^(2*I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(-I*d*e
^(2*I*d*x + 2*I*c) + I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 2*((A - I*B)*a^2*e^(
2*I*d*x + 2*I*c) + (A + I*B)*a^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) - 1)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= -a^2\left(\int\left(-A\cot^{\frac{3}{2}}(c+dx)\right)dx + \int A\tan^2(c+dx)\cot^{\frac{3}{2}}(c+dx)\,dx + \int\left(-B\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)\right)dx + \int B\tan^3(c+dx)\cot^{\frac{3}{2}}(c+dx)\,dx + \int\left(-2iA\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)\right)dx + \int\left(-2iB\tan^2(c+dx)\cot^{\frac{3}{2}}(c+dx)\right)dx\right)$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
-a**2*(Integral(-A*cot(c + d*x)**(3/2), x) + Integral(A*tan(c + d*x)**2*co
t(c + d*x)**(3/2), x) + Integral(-B*tan(c + d*x)*cot(c + d*x)**(3/2), x) +
 Integral(B*tan(c + d*x)**3*cot(c + d*x)**(3/2), x) + Integral(-2*I*A*tan(
c + d*x)*cot(c + d*x)**(3/2), x) + Integral(-2*I*B*tan(c + d*x)**2*cot(c +
 d*x)**(3/2), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(81) = 162$.

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.76

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{4Ba^2\sqrt{\tan(dx+c)} - \left(2\sqrt{2}(-(i-1)A-(i+1)B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right) + 2\sqrt{2}(-(i\right.}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/2*(4*B*a^2*sqrt(tan(d*x + c)) - (2*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arc
tan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I - 1)*A
- (I + 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(
2)*(-(I + 1)*A + (I - 1)*B)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c
) + 1) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c))
+ 1/tan(d*x + c) + 1))*a^2 + 4*A*a^2/sqrt(tan(d*x + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.64

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(Ba^2\sqrt{\tan(dx+c)}+\sqrt{2}(-(i-1)\,Aa^2-(i+1)\,Ba^2)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)+\cdots\right.}{d}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2*(B*a^2*sqrt(tan(d*x + c)) + sqrt(2)*(-(I - 1)*A*a^2 - (I + 1)*B*a^2)*ar
ctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) + A*a^2/sqrt(tan(d*x + c))
)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{a^2\left(-2\sqrt{\cot(dx+c)}\,a-\left(\int\frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)ad-\left(\int\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^3\,dx\right)bd-\left(\right.}{}$$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
(a**2*( - 2*sqrt(cot(c + d*x))*a - int(sqrt(cot(c + d*x))/cot(c + d*x),x)*
a*d - int(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)**3,x)*b*d - int(sqr
t(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)**2,x)*a*d + 2*int(sqrt(cot(c + d
*x))*cot(c + d*x)*tan(c + d*x)**2,x)*b*d*i + 2*int(sqrt(cot(c + d*x))*cot(
c + d*x)*tan(c + d*x),x)*a*d*i + int(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c
 + d*x),x)*b*d))/d
```

3.515 $\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 105

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{4\sqrt[4]{-1}a^2(iA+B)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{2a^2(3A-5iB)}{3d\sqrt{\cot(c+dx)}} + \frac{2iB(ia^2+a^2\cot(c+dx))}{3d\cot^{\frac{3}{2}}(c+dx)}$$

output
```
4*(-1)^(1/4)*a^2*(I*A+B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2/3*a^2*(3
*A-5*I*B)/d/cot(d*x+c)^(1/2)+2/3*I*B*(I*a^2+a^2*cot(d*x+c))/d/cot(d*x+c)^(
3/2)
```

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2a^2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(6\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}(3A}{3d}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x])
,x]
```

output
```
(-2*a^2*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(6*(-1)^(1/4)*(A - I*B)*ArcT
an[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + Sqrt[Tan[c + d*x]]*(3*A - (6*I)*B + B*
Tan[c + d*x])))/(3*d)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 4064, 3042, 4076, 27, 3042, 4074, 27, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+ia)^2(A\cot(c+dx)+B)}{\cot^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^2\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{3}\int \frac{(\cot(c+dx)a+ia)(a(3iA+5B)+a(3A-iB)\cot(c+dx))}{2\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(\cot(c+dx)a+ia)(a(3iA+5B)+a(3A-iB)\cot(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(3iA+5B)-a(3A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4074}$$

$$\frac{1}{3}\left(\int \frac{6\left((iA+B)a^2+(A-iB)\cot(c+dx)a^2\right)}{\sqrt{\cot(c+dx)}}dx-\frac{2a^2(3A-5iB)}{d\sqrt{\cot(c+dx)}}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(6\int \frac{(iA+B)a^2+(A-iB)\cot(c+dx)a^2}{\sqrt{\cot(c+dx)}}dx-\frac{2a^2(3A-5iB)}{d\sqrt{\cot(c+dx)}}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(6\int \frac{a^2(iA+B)-a^2(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^2(3A-5iB)}{d\sqrt{\cot(c+dx)}}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4016}$$

$$\frac{1}{3}\left(\frac{12a^4(B+iA)^2\int \frac{1}{a^2(A-iB)\cot(c+dx)-a^2(iA+B)}d\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(3A-5iB)}{d\sqrt{\cot(c+dx)}}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{12\sqrt[4]{-1}a^2(B+iA)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^2(3A-5iB)}{d\sqrt{\cot(c+dx)}}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{3d\cot^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
((12*(-1)^(1/4)*a^2*(I*A + B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d -
(2*a^2*(3*A - (5*I)*B))/(d*Sqrt[Cot[c + d*x]]))/3 + (((2*I)/3)*B*(I*a^2 +
a^2*Cot[c + d*x]))/(d*Cot[c + d*x]^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(87) = 174$.

Time = 0.55 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.14

method	result
derivativedivides	$-\frac{a^2\left(-\frac{2(2iB-A)}{\sqrt{\cot(dx+c)}}+\frac{2B}{3\cot(dx+c)^{\frac{3}{2}}}+\frac{(2iA+2B)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\,\mathrm{arct}}{4}\right.}{}$
default	$-\frac{a^2\left(-\frac{2(2iB-A)}{\sqrt{\cot(dx+c)}}+\frac{2B}{3\cot(dx+c)^{\frac{3}{2}}}+\frac{(2iA+2B)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\,\mathrm{arct}}{4}\right.}{}$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
-a^2/d*(-2*(2*I*B-A)/cot(d*x+c)^(1/2)+2/3*B/cot(d*x+c)^(3/2)+1/4*(2*I*A+2*
B)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)
*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(
1/2)*cot(d*x+c)^(1/2)))+1/4*(2*A-2*I*B)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*co
t(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1
/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(83) = 166$.

Time = 0.09 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.18

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$= \frac{3\sqrt{-\frac{(iA^2+2AB-iB^2)a^4}{d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left((A-i\,B)a^2e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{\left(i\,A^2+2\,AB-i\,B^2\right)a^4}{d^2}}\left(de^{(2i\right.\right.}{(-i\,A-B)a^2}\right.}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/3*(3*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d
*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) + sqrt(
-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I
*A - B)*a^2)) - 3*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(4*I*d*x + 4
*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^2*e^(2*I*d*x + 2*I
*c) - sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^4/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*s
qrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x -
2*I*c)/((-I*A - B)*a^2)) - 2*((-3*I*A - 7*B)*a^2*e^(4*I*d*x + 4*I*c) + 2*B
*a^2*e^(2*I*d*x + 2*I*c) + (3*I*A + 5*B)*a^2)*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x +
 2*I*c) + d)
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -a^2\left(\int\left(-A\sqrt{\cot(c+dx)}\right)dx + \int A\tan^2(c+dx)\sqrt{\cot(c+dx)}\,dx + \int\left(-B\tan(c+dx)\sqrt{\cot(c+dx)}\right)dx + \int B\tan^3(c+dx)\sqrt{\cot(c+dx)}\,dx + \int\left(-2iA\tan(c+dx)\sqrt{\cot(c+dx)}\right)dx + \int\left(-2iB\tan^2(c+dx)\sqrt{\cot(c+dx)}\right)dx\right)$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
-a**2*(Integral(-A*sqrt(cot(c + d*x)), x) + Integral(A*tan(c + d*x)**2*sqr
t(cot(c + d*x)), x) + Integral(-B*tan(c + d*x)*sqrt(cot(c + d*x)), x) + In
tegral(B*tan(c + d*x)**3*sqrt(cot(c + d*x)), x) + Integral(-2*I*A*tan(c +
d*x)*sqrt(cot(c + d*x)), x) + Integral(-2*I*B*tan(c + d*x)**2*sqrt(cot(c +
 d*x)), x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(83) = 166$.

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.71

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{3\left(2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\,\text{arc}\right.}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/6*(3*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2
/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1/2*sqr
t(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((I - 1)*A + (I + 1)*B)*l
og(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((I - 1)*A +
 (I + 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^2 - 4
*(B*a^2 + 3*(A - 2*I*B)*a^2/tan(d*x + c))*tan(d*x + c)^(3/2))/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(Ba^2\tan(dx+c)^{\frac{3}{2}}+3\,Aa^2\sqrt{\tan(dx+c)}-6i\,Ba^2\sqrt{\tan(dx+c)}+3\sqrt{2}(-(i+1)\ Aa^2+(i-1)\ \right.}{3\,d}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/3*(B*a^2*tan(d*x + c)^(3/2) + 3*A*a^2*sqrt(tan(d*x + c)) - 6*I*B*a^2*sq
rt(tan(d*x + c)) + 3*sqrt(2)*(-(I + 1)*A*a^2 + (I - 1)*B*a^2)*arctan(-(1/2
*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\cot(c+dx)}\,(A+B\tan(c+dx))\,(a+a\tan(c+dx)\,1\mathrm{i})^2\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2, x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=a^2\left(\left(\int\sqrt{\cot(dx+c)}dx\right)a-\left(\int\sqrt{\cot(dx+c)}\tan(dx+c)^3\,dx\right)b-\left(\int\sqrt{\cot(dx+c)}\tan(dx+c)^2\,dx\right)a+2\left(\int\sqrt{\cot(dx+c)}\tan(dx+c)^2\,dx\right)bi+2\left(\int\sqrt{\cot(dx+c)}\tan(dx+c)\,dx\right)ai+\left(\int\sqrt{\cot(dx+c)}\tan(dx+c)\,dx\right)b\right)$$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
a**2*(int(sqrt(cot(c + d*x)),x)*a - int(sqrt(cot(c + d*x))*tan(c + d*x)**3
,x)*b - int(sqrt(cot(c + d*x))*tan(c + d*x)**2,x)*a + 2*int(sqrt(cot(c + d
*x))*tan(c + d*x)**2,x)*b*i + 2*int(sqrt(cot(c + d*x))*tan(c + d*x),x)*a*i
 + int(sqrt(cot(c + d*x))*tan(c + d*x),x)*b)
```

3.516 $\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 130

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{4\sqrt[4]{-1}a^2(A-iB)\operatorname{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{2a^2(5A-7iB)}{15d\cot^{\frac{3}{2}}(c+dx)} + \frac{4a^2(iA+B)}{d\sqrt{\cot(c+dx)}} + \frac{2iB(ia^2+a^2\cot(c+dx))}{5d\cot^{\frac{5}{2}}(c+dx)}$$

output
```
4*(-1)^(1/4)*a^2*(A-I*B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-2/15*a^2*(
5*A-7*I*B)/d/cot(d*x+c)^(3/2)+4*a^2*(I*A+B)/d/cot(d*x+c)^(1/2)+2/5*I*B*(I*
a^2+a^2*cot(d*x+c))/d/cot(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 3.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{2a^2\left(30(iA+B)+\frac{30\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)}{\sqrt{\tan(c+dx)}}-5(A-2iB)\tan(c+dx)-3B\tan^2(c+dx)\right)}{15d\sqrt{\cot(c+dx)}}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x
]],x]
```

output
```
(2*a^2*(30*(I*A + B) + (30*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan
[c + d*x]]])/Sqrt[Tan[c + d*x]] - 5*(A - (2*I)*B)*Tan[c + d*x] - 3*B*Tan[c
 + d*x]^2))/(15*d*Sqrt[Cot[c + d*x]])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 4064, 3042, 4076, 27, 3042, 4074, 27, 3042, 4012, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+ia)^2(A\cot(c+dx)+B)}{\cot^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^2\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{5}\int \frac{(\cot(c+dx)a+ia)(a(5iA+7B)+a(5A-3iB)\cot(c+dx))}{2\cot^{\frac{5}{2}}(c+dx)}dx+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \frac{(\cot(c+dx)a+ia)(a(5iA+7B)+a(5A-3iB)\cot(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}dx+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(5iA+7B)-a(5A-3iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4074}$$

$$\frac{1}{5}\left(\int \frac{10\left((iA+B)a^2+(A-iB)\cot(c+dx)a^2\right)}{\cot^{\frac{3}{2}}(c+dx)}dx-\frac{2a^2(5A-7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(10\int \frac{(iA+B)a^2+(A-iB)\cot(c+dx)a^2}{\cot^{\frac{3}{2}}(c+dx)}dx-\frac{2a^2(5A-7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(10\int \frac{a^2(iA+B)-a^2(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2a^2(5A-7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4012}$$

$$\frac{1}{5}\left(10\left(\int \frac{a^2(A-iB)-a^2(iA+B)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2a^2(B+iA)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2a^2(5A-7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(10\left(\int \frac{(A-iB)a^2+(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^2(B+iA)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2a^2(5A-7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4016}$$

$$\frac{1}{5}\left(10\left(\frac{2a^4(A-iB)^2\int \frac{1}{-((A-iB)a^2)-(iA+B)\cot(c+dx)a^2}d\sqrt{\cot(c+dx)}}{d}+\frac{2a^2(B+iA)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2a^2(5A-7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{5}\left(10\left(\frac{2\sqrt[4]{-1}a^2(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2a^2(B+iA)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2a^2(5A-7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)+\frac{2iB\left(a^2\cot(c+dx)+ia^2\right)}{5d\cot^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

output
```
(10*((2*(-1)^(1/4)*a^2*(A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d
 + (2*a^2*(I*A + B))/(d*Sqrt[Cot[c + d*x]])) - (2*a^2*(5*A - (7*I)*B))/(3*
d*Cot[c + d*x]^(3/2)))/5 + (((2*I)/5)*B*(I*a^2 + a^2*Cot[c + d*x]))/(d*Cot
[c + d*x]^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(109) = 218$.

Time = 0.56 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.86

method	result
derivativedivides	$a^2\left(-\frac{(-2iB+2A)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4}-\right.$
default	$a^2\left(-\frac{(-2iB+2A)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4}-\right.$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
a^2/d*(-1/4*(2*A-2*I*B)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1
)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(
1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))-1/4*(-2*I*A-2*B)*2^(1/2)*(ln(
(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/
2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)
^(1/2)))+2*(2*I*A+2*B)/cot(d*x+c)^(1/2)+2/3*(2*I*B-A)/cot(d*x+c)^(3/2)-2/5
*B/cot(d*x+c)^(5/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(104) = 208.

Time = 0.10 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.88

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx =$$

$$-\frac{15\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a^4}{d^2}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left((A-i\,B)a^2e^{(2i\,dx+2i\,c)}-\sqrt{\right.}{}\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algori
thm="fricas")
```

output
```
-1/15*(15*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(d*e^(6*I*d*x + 6*I*c) +
 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a
^2*e^(2*I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(I*d*e^(2
*I*d*x + 2*I*c) - I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^2)) - 15*sqrt(-(-I*A^2 - 2*A*
B + I*B^2)*a^4/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d
*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^2*e^(2*I*d*x + 2*I*c) - sqrt(
-(-I*A^2 - 2*A*B + I*B^2)*a^4/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c
)/((-I*A - B)*a^2)) - 2*((35*A - 43*I*B)*a^2*e^(6*I*d*x + 6*I*c) + (25*A -
 11*I*B)*a^2*e^(4*I*d*x + 4*I*c) - (35*A - 31*I*B)*a^2*e^(2*I*d*x + 2*I*c)
 - (25*A - 23*I*B)*a^2)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) - 1)))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d
*x + 2*I*c) + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= -a^2\left(\int\left(-\frac{A}{\sqrt{\cot(c+dx)}}\right)dx+\int\frac{A\tan^2(c+dx)}{\sqrt{\cot(c+dx)}}\,dx+\int\left(-\frac{B\tan(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx+\int\frac{B\tan^3(c+dx)}{\sqrt{\cot(c+dx)}}\,dx+\int\left(-\frac{2iA\tan(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx+\int\left(-\frac{2iB\tan^2(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**2*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
-a**2*(Integral(-A/sqrt(cot(c + d*x)), x) + Integral(A*tan(c + d*x)**2/sqr
t(cot(c + d*x)), x) + Integral(-B*tan(c + d*x)/sqrt(cot(c + d*x)), x) + In
tegral(B*tan(c + d*x)**3/sqrt(cot(c + d*x)), x) + Integral(-2*I*A*tan(c +
d*x)/sqrt(cot(c + d*x)), x) + Integral(-2*I*B*tan(c + d*x)**2/sqrt(cot(c +
 d*x)), x))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.53

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx =$$

$$-\frac{4\left(3\,Ba^2+\frac{5\,(A-2i\,B)a^2}{\tan(dx+c)}-\frac{30\,(i\,A+B)a^2}{\tan(dx+c)^2}\right)\tan\left(dx+c\right)^{\frac{5}{2}}+15\left(2\sqrt{2}(-(i-1)\;A-(i+1)\;B)\arctan\left(\frac{1}{2}\sqrt{2}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algori
thm="maxima")
```

output
```
-1/30*(4*(3*B*a^2 + 5*(A - 2*I*B)*a^2/tan(d*x + c) - 30*(I*A + B)*a^2/tan(
d*x + c)^2)*tan(d*x + c)^(5/2) + 15*(2*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*ar
ctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I - 1)*A
 - (I + 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt
(2)*(-(I + 1)*A + (I - 1)*B)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x +
c) + 1) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c))
 + 1/tan(d*x + c) + 1))*a^2)/d
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx =$$

$$-\frac{2\left(3\,Ba^2\tan\left(dx+c\right)^{\frac{5}{2}}+5\,Aa^2\tan\left(dx+c\right)^{\frac{3}{2}}-10i\,Ba^2\tan\left(dx+c\right)^{\frac{3}{2}}-30i\,Aa^2\sqrt{\tan\left(dx+c\right)}-30\,B\right.}{15\,d}$$

input
```
integrate((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algori
thm="giac")
```

output
```
-2/15*(3*B*a^2*tan(d*x + c)^(5/2) + 5*A*a^2*tan(d*x + c)^(3/2) - 10*I*B*a^
2*tan(d*x + c)^(3/2) - 30*I*A*a^2*sqrt(tan(d*x + c)) - 30*B*a^2*sqrt(tan(d
*x + c)) + 15*sqrt(2)*((I - 1)*A*a^2 + (I + 1)*B*a^2)*arctan(-(1/2*I - 1/2
)*sqrt(2)*sqrt(tan(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^2}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2)/cot(c + d*x)^(1/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^2)/cot(c + d*x)^(1/2), x
)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = a^2\left(\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a-\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^3}{\cot(dx+c)}dx\right)b-\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)}dx\right)a+2\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)}dx\right)bi+2\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)ai+\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)b\right)$$

input
```
int((a+I*a*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
a**2*(int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a - int((sqrt(cot(c + d*x))*t
an(c + d*x)**3)/cot(c + d*x),x)*b - int((sqrt(cot(c + d*x))*tan(c + d*x)**
2)/cot(c + d*x),x)*a + 2*int((sqrt(cot(c + d*x))*tan(c + d*x)**2)/cot(c +
d*x),x)*b*i + 2*int((sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*x),x)*a*i
+ int((sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*x),x)*b)
```

3.517 $\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 171

$$\begin{aligned}\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= \frac{8\sqrt[4]{-1}a^3(iA+B)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{8a^3(iA+B)\sqrt{\cot(c+dx)}}{d}\\ &\quad+\frac{8a^3(23A-21iB)\cot^{\frac{3}{2}}(c+dx)}{105d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)(ia+a\cot(c+dx))^2}{7d}\\ &\quad-\frac{2(11iA+7B)\cot^{\frac{3}{2}}(c+dx)\left(ia^3+a^3\cot(c+dx)\right)}{35d}\end{aligned}$$

output
```
8*(-1)^(1/4)*a^3*(I*A+B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d+8*a^3*(I*A
+B)*cot(d*x+c)^(1/2)/d+8/105*a^3*(23*A-21*I*B)*cot(d*x+c)^(3/2)/d-2/7*a*A*
cot(d*x+c)^(3/2)*(I*a+a*cot(d*x+c))^2/d-2/35*(11*I*A+7*B)*cot(d*x+c)^(3/2)
*(I*a^3+a^3*cot(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2a^3\cot^{\frac{3}{2}}(c+dx)\left(35iB+21iA\cot(c+dx)+63B\cot(c+dx)+45A\cot^2(c+dx)-60A\cot^2(c+dx)\,\mathrm{H}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(9/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x])
,x]
```

output
```
(2*a^3*Cot[c + d*x]^(3/2)*((35*I)*B + (21*I)*A*Cot[c + d*x] + 63*B*Cot[c +
 d*x] + 45*A*Cot[c + d*x]^2 - 60*A*Cot[c + d*x]^2*Hypergeometric2F1[-7/2,
1, -5/2, I*Tan[c + d*x]] - 84*B*Cot[c + d*x]*Hypergeometric2F1[-5/2, 1, -3
/2, I*Tan[c + d*x]]))/(105*d)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4064, 3042, 4077, 27, 3042, 4077, 27, 3042, 4075, 3042, 4011, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cot(c+dx)^{9/2}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \ 4064$$

$$\int \sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3(A\cot(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4077}$$

$$-\frac{2}{7}\int \frac{1}{2}\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)^2(a(3A-7iB)-a(11iA+7B)\cot(c+dx))dx-\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{7}\int \sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)^2(a(3A-7iB)-a(11iA+7B)\cot(c+dx))dx-\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7}\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(a(3A-7iB)+a(11iA+7B)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{7}\left(\frac{2}{5}\int -2\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)\left(2(6iA+7B)a^2+(23A-21iB)\cot(c+dx)a^2\right)dx-\frac{2(7B+11iA}{}\right.$$
$$-\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(-\frac{4}{5}\int \sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)\left(2(6iA+7B)a^2+(23A-21iB)\cot(c+dx)a^2\right)dx-\frac{2(7B+11iA)}{}\right.$$
$$-\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(-\frac{4}{5}\int\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(2a^2(6iA+7B)-a^2(23A-21iB)\tan\left(c+dx+\frac{\pi}{2}\right.\right.\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$$\Big\downarrow \text{4075}$$

$$\frac{1}{7}\left(-\frac{4}{5}\left(\int\sqrt{\cot(c+dx)}\left(35a^3(iA+B)\cot(c+dx)-35a^3(A-iB)\right)dx-\frac{2a^3(23A-21iB)\cot^{\frac{3}{2}}(c+dx)}{3d}\right)-\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(-\frac{4}{5}\left(\int\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-35(A-iB)a^3-35(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx-\frac{2a^3(23A-21iB)}{3d}\right.\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$$\Big\downarrow \text{4011}$$

$$\frac{1}{7}\left(-\frac{4}{5}\left(\int\frac{-35(iA+B)a^3-35(A-iB)\cot(c+dx)a^3}{\sqrt{\cot(c+dx)}}dx-\frac{2a^3(23A-21iB)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{70a^3(B+iA)\sqrt{}}{d}\right.\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(-\frac{4}{5}\left(\int\frac{35a^3(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)-35a^3(iA+B)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^3(23A-21iB)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{70a^3(B+iA}{}\right.\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$$\Big\downarrow \text{4016}$$

$$\frac{1}{7}\left(-\frac{4}{5}\left(\frac{2450a^6(B+iA)^2\int\frac{1}{35a^3(iA+B)-35a^3(A-iB)\cot(c+dx)}d\sqrt{\cot(c+dx)}}{d}-\frac{2a^3(23A-21iB)\cot^{\frac{3}{2}}(c+dx)}{3d}-\right.\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

$\downarrow$ 221

$$\frac{1}{7}\left(-\frac{4}{5}\left(-\frac{70\sqrt[4]{-1}a^3(B+iA)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{2a^3(23A-21iB)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{70a^3(B+iA\right.\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}{7d}$$

input
```
Int[Cot[c + d*x]^(9/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a*A*Cot[c + d*x]^(3/2)*(I*a + a*Cot[c + d*x])^2)/(7*d) + ((-2*((11*I)*
A + 7*B)*Cot[c + d*x]^(3/2)*(I*a^3 + a^3*Cot[c + d*x]))/(5*d) - (4*((-70*(
-1)^(1/4)*a^3*(I*A + B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (70*a^
3*(I*A + B)*Sqrt[Cot[c + d*x]])/d - (2*a^3*(23*A - (21*I)*B)*Cot[c + d*x]^
(3/2))/(3*d)))/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.61

method	result
derivativedivides	$a^3\left(-\frac{2A\cot(dx+c)^{\frac{7}{2}}}{7}-\frac{6iA\cot(dx+c)^{\frac{5}{2}}}{5}-\frac{2B\cot(dx+c)^{\frac{5}{2}}}{5}-2iB\cot(dx+c)^{\frac{3}{2}}+\frac{8A\cot(dx+c)^{\frac{3}{2}}}{3}+8iA\sqrt{\cot(dx+c)}+8B\sqrt{\cot}\right.$
default	$a^3\left(-\frac{2A\cot(dx+c)^{\frac{7}{2}}}{7}-\frac{6iA\cot(dx+c)^{\frac{5}{2}}}{5}-\frac{2B\cot(dx+c)^{\frac{5}{2}}}{5}-2iB\cot(dx+c)^{\frac{3}{2}}+\frac{8A\cot(dx+c)^{\frac{3}{2}}}{3}+8iA\sqrt{\cot(dx+c)}+8B\sqrt{\cot}\right.$

input
```
int(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
a^3/d*(-2/7*A*cot(d*x+c)^(7/2)-6/5*I*A*cot(d*x+c)^(5/2)-2/5*B*cot(d*x+c)^(
5/2)-2*I*B*cot(d*x+c)^(3/2)+8/3*A*cot(d*x+c)^(3/2)+8*I*A*cot(d*x+c)^(1/2)+
8*B*cot(d*x+c)^(1/2)-1/4*(4*I*A+4*B)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d
*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)
*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))-1/4*(4*A-4*I*B)*
2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*co
t(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2
)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(139) = 278.

Time = 0.11 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.97

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2\left(105\sqrt{-\frac{(iA^2+2AB-iB^2)a^6}{d^2}}\left(de^{(6i\,dx+6i\,c)}-3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{2\left((A-iB)a^3e^{(2i\,dx+2i\,c)}+\cdots\right.}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
2/105*(105*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(6*I*d*x + 6*I*c) -
 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(2*((A - I*B)*a
^3*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(2*I*
d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 105*sqrt(-(I*A^2 + 2*A*B - I
*B^2)*a^6/d^2)*(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2
*I*d*x + 2*I*c) - d)*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt(-(I*A
^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A -
B)*a^3)) - 2*((-319*I*A - 273*B)*a^3*e^(6*I*d*x + 6*I*c) + 2*(323*I*A + 33
6*B)*a^3*e^(4*I*d*x + 4*I*c) + (-551*I*A - 567*B)*a^3*e^(2*I*d*x + 2*I*c)
+ 4*(41*I*A + 42*B)*a^3)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*
I*c) - 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*
d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(9/2)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.27

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{105\left(2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\ a}{}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output

```
1/105*(105*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2)
 + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((I - 1)*A + (I + 1)*
B)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((I - 1)
*A + (I + 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3
 + 840*(I*A + B)*a^3/sqrt(tan(d*x + c)) + 70*(4*A - 3*I*B)*a^3/tan(d*x + c
)^(3/2) + 42*(-3*I*A - B)*a^3/tan(d*x + c)^(5/2) - 30*A*a^3/tan(d*x + c)^(
7/2))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.78

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(210\sqrt{2}(-(i+1)\,Aa^3+(i-1)\,Ba^3)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+\frac{-420i\,Aa^3\tan(dx+c)^3-4}{\cdots}\right.}{105\,d}$$

input

```
integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output

```
-2/105*(210*sqrt(2)*(-(I + 1)*A*a^3 + (I - 1)*B*a^3)*arctan(-(1/2*I - 1/2)
*sqrt(2)*sqrt(tan(d*x + c))) + (-420*I*A*a^3*tan(d*x + c)^3 - 420*B*a^3*ta
n(d*x + c)^3 - 140*A*a^3*tan(d*x + c)^2 + 105*I*B*a^3*tan(d*x + c)^2 + 63*
I*A*a^3*tan(d*x + c) + 21*B*a^3*tan(d*x + c) + 15*A*a^3)/tan(d*x + c)^(7/2
))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{9/2}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^3\,dx$$

input
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$
\begin{aligned}
&\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\
&= a^3\Bigg(-\left(\int\sqrt{\cot(dx+c)}\cot(dx+c)^4\tan(dx+c)^4\,dx\right)bi \\
&\quad-\left(\int\sqrt{\cot(dx+c)}\cot(dx+c)^4\tan(dx+c)^3\,dx\right)ai \\
&\quad-3\left(\int\sqrt{\cot(dx+c)}\cot(dx+c)^4\tan(dx+c)^3\,dx\right)b \\
&\quad-3\left(\int\sqrt{\cot(dx+c)}\cot(dx+c)^4\tan(dx+c)^2\,dx\right)a \\
&\quad+3\left(\int\sqrt{\cot(dx+c)}\cot(dx+c)^4\tan(dx+c)^2\,dx\right)bi \\
&\quad+3\left(\int\sqrt{\cot(dx+c)}\cot(dx+c)^4\tan(dx+c)\,dx\right)ai \\
&\quad+\left(\int\sqrt{\cot(dx+c)}\cot(dx+c)^4\tan(dx+c)\,dx\right)b \\
&\quad+\left(\int\sqrt{\cot(dx+c)}\cot(dx+c)^4\,dx\right)a\Bigg)
\end{aligned}
$$

input
```
int(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
a**3*( - int(sqrt(cot(c + d*x))*cot(c + d*x)**4*tan(c + d*x)**4,x)*b*i - i
nt(sqrt(cot(c + d*x))*cot(c + d*x)**4*tan(c + d*x)**3,x)*a*i - 3*int(sqrt(
cot(c + d*x))*cot(c + d*x)**4*tan(c + d*x)**3,x)*b - 3*int(sqrt(cot(c + d*
x))*cot(c + d*x)**4*tan(c + d*x)**2,x)*a + 3*int(sqrt(cot(c + d*x))*cot(c
+ d*x)**4*tan(c + d*x)**2,x)*b*i + 3*int(sqrt(cot(c + d*x))*cot(c + d*x)**
4*tan(c + d*x),x)*a*i + int(sqrt(cot(c + d*x))*cot(c + d*x)**4*tan(c + d*x
),x)*b + int(sqrt(cot(c + d*x))*cot(c + d*x)**4,x)*a)
```

3.518 $\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 146

$$\begin{aligned}\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= \frac{8\sqrt[4]{-1}a^3(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} \\ &+ \frac{16a^3(6A-5iB)\sqrt{\cot(c+dx)}}{15d} - \frac{2aA\sqrt{\cot(c+dx)}(ia+a\cot(c+dx))^2}{5d} \\ &- \frac{2(9iA+5B)\sqrt{\cot(c+dx)}(ia^3+a^3\cot(c+dx))}{15d}\end{aligned}$$

output
```
8*(-1)^(1/4)*a^3*(A-I*B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d+16/15*a^3*
(6*A-5*I*B)*cot(d*x+c)^(1/2)/d-2/5*a*A*cot(d*x+c)^(1/2)*(I*a+a*cot(d*x+c))
^2/d-2/15*(9*I*A+5*B)*cot(d*x+c)^(1/2)*(I*a^3+a^3*cot(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2a^3\sqrt{\cot(c+dx)}\left(-15iB-5iA\cot(c+dx)-15B\cot(c+dx)-9A\cot^2(c+dx)+12A\cot^2(c+dx)\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x])
,x]
```

output
```
(-2*a^3*Sqrt[Cot[c + d*x]]*((-15*I)*B - (5*I)*A*Cot[c + d*x] - 15*B*Cot[c
+ d*x] - 9*A*Cot[c + d*x]^2 + 12*A*Cot[c + d*x]^2*Hypergeometric2F1[-5/2,
1, -3/2, I*Tan[c + d*x]] + 20*B*Cot[c + d*x]*Hypergeometric2F1[-3/2, 1, -1
/2, I*Tan[c + d*x]]))/(15*d)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 4064, 3042, 4077, 27, 3042, 4077, 27, 3042, 4075, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{7/2}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+ia)^3(A\cot(c+dx)+B)}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4077}$$

$$-\frac{2}{5}\int \frac{(\cot(c+dx)a+ia)^2(a(A-5iB)-a(9iA+5B)\cot(c+dx))}{2\sqrt{\cot(c+dx)}}dx-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}{5d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}\int \frac{(\cot(c+dx)a+ia)^2(a(A-5iB)-a(9iA+5B)\cot(c+dx))}{\sqrt{\cot(c+dx)}}dx-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(a(A-5iB)+a(9iA+5B)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}{5d}$$

$$\Big\downarrow \text{4077}$$

$$\frac{1}{5}\left(\frac{2}{3}\int -\frac{2(\cot(c+dx)a+ia)\left((3iA+5B)a^2+2(6A-5iB)\cot(c+dx)a^2\right)}{\sqrt{\cot(c+dx)}}dx-\frac{2(5B+9iA)\sqrt{\cot(c+dx)}(a}{3d}\right.$$
$$-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(-\frac{4}{3}\int \frac{(\cot(c+dx)a+ia)\left((3iA+5B)a^2+2(6A-5iB)\cot(c+dx)a^2\right)}{\sqrt{\cot(c+dx)}}dx-\frac{2(5B+9iA)\sqrt{\cot(c+dx)}(a^3}{3d}\right.$$
$$-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-\frac{4}{3}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a^2(3iA+5B)-2a^2(6A-5iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(5B+9iA)\sqrt{\cot}}{}\right.$$
$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}{5d}$$

$\Big\downarrow$ 4075

$$\frac{1}{5}\left(-\frac{4}{3}\left(\int \frac{15a^3(iA+B)\cot(c+dx)-15a^3(A-iB)}{\sqrt{\cot(c+dx)}}dx-\frac{4a^3(6A-5iB)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(5B+9iA)\sqrt{\cot}}{}\right.$$
$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}\left(-\frac{4}{3}\left(\int \frac{-15(A-iB)a^3-15(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4a^3(6A-5iB)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(5B+9}{}\right.$$
$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}{5d}$$

$\Big\downarrow$ 4016

$$\frac{1}{5}\left(-\frac{4}{3}\left(\frac{450a^6(A-iB)^2\int \frac{1}{15(A-iB)a^3+15(iA+B)\cot(c+dx)a^3}d\sqrt{\cot(c+dx)}}{d}-\frac{4a^3(6A-5iB)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(}{}\right.$$
$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}{5d}$$

$\Big\downarrow$ 221

$$\frac{1}{5}\left(-\frac{4}{3}\left(-\frac{30\sqrt[4]{-1}a^3(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{4a^3(6A-5iB)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(5B+9iA)}{}\right.$$
$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}{5d}$$

input
```
Int[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a*A*Sqrt[Cot[c + d*x]]*(I*a + a*Cot[c + d*x])^2)/(5*d) + ((-4*((-30*(-
1)^(1/4)*a^3*(A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (4*a^3*
(6*A - (5*I)*B)*Sqrt[Cot[c + d*x]])/d))/3 - (2*((9*I)*A + 5*B)*Sqrt[Cot[c
+ d*x]]*(I*a^3 + a^3*Cot[c + d*x]))/(3*d))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(123) = 246$.

Time = 1.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.73

method	result
derivativedivides	$a^3\left(-\frac{2A\cot(dx+c)^{\frac{5}{2}}}{5}-2iA\cot(dx+c)^{\frac{3}{2}}-\frac{2B\cot(dx+c)^{\frac{3}{2}}}{3}-6iB\sqrt{\cot(dx+c)}+8A\sqrt{\cot(dx+c)}-\frac{(-4iB+4A)\sqrt{2}\left(\ln\left(\frac{\cot(dx}{\cot(dx}\right.\right.}{}\right.$
default	$a^3\left(-\frac{2A\cot(dx+c)^{\frac{5}{2}}}{5}-2iA\cot(dx+c)^{\frac{3}{2}}-\frac{2B\cot(dx+c)^{\frac{3}{2}}}{3}-6iB\sqrt{\cot(dx+c)}+8A\sqrt{\cot(dx+c)}-\frac{(-4iB+4A)\sqrt{2}\left(\ln\left(\frac{\cot(dx}{\cot(dx}\right.\right.}{}\right.$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
a^3/d*(-2/5*A*cot(d*x+c)^(5/2)-2*I*A*cot(d*x+c)^(3/2)-2/3*B*cot(d*x+c)^(3/
2)-6*I*B*cot(d*x+c)^(1/2)+8*A*cot(d*x+c)^(1/2)-1/4*(4*A-4*I*B)*2^(1/2)*(ln
((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1
/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c
)^(1/2)))-1/4*(-4*I*A-4*B)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2
)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c
)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(118) = 236$.

Time = 0.10 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.08

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(15\sqrt{-\frac{(-iA^2-2AB+iB^2)a^6}{d^2}}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left((A-iB)a^3e^{(2i\,dx+2i\,c)}-\sqrt{-\frac{(-iA^2-2AB+\cdots}{d^2}}\cdots\right.}{\cdots}\right.\cdots}{\cdots}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-2/15*(15*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) -
 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - s
qrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqr
t((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*
I*c)/((-I*A - B)*a^3)) - 15*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(
4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^3*e^(2*
I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(-I*d*e^(2*I*d*x
+ 2*I*c) + I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 2*((39*A - 25*I*B)*a^3*e^(4*I*
d*x + 4*I*c) - 3*(19*A - 15*I*B)*a^3*e^(2*I*d*x + 2*I*c) + 4*(6*A - 5*I*B)
*a^3)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4
*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.37

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{15\left(2\sqrt{2}((i-1)\,A+(i+1)\,B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((i-1)\,A+(i+1)\,B)\arctan\right.}{}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/15*(15*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) +
2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqr
t(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*
log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(-(I + 1)*A
 + (I - 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3 +
 30*(4*A - 3*I*B)*a^3/sqrt(tan(d*x + c)) + 10*(-3*I*A - B)*a^3/tan(d*x + c
)^(3/2) - 6*A*a^3/tan(d*x + c)^(5/2))/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{2\left(30\sqrt{2}((i-1)\,Aa^3+(i+1)\,Ba^3)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)-\frac{60\,Aa^3\tan(dx+c)^2-45i\,Ba^3}{}\right.}{15\,d}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/15*(30*sqrt(2)*((I - 1)*A*a^3 + (I + 1)*B*a^3)*arctan(-(1/2*I - 1/2)*sq
rt(2)*sqrt(tan(d*x + c))) - (60*A*a^3*tan(d*x + c)^2 - 45*I*B*a^3*tan(d*x
+ c)^2 - 15*I*A*a^3*tan(d*x + c) - 5*B*a^3*tan(d*x + c) - 3*A*a^3)/tan(d*x
 + c)^(5/2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^3\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\frac{a^3\left(-2\sqrt{\cot(dx+c)}\,\cot(dx+c)^2 a+10\sqrt{\cot(dx+c)}\,a+5\left(\int\frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)ad-5\left(\int\sqrt{\cot(dx+c)}\right.}{}$$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*( - 2*sqrt(cot(c + d*x))*cot(c + d*x)**2*a + 10*sqrt(cot(c + d*x))*a
 + 5*int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a*d - 5*int(sqrt(cot(c + d*x))
*cot(c + d*x)**3*tan(c + d*x)**4,x)*b*d*i - 5*int(sqrt(cot(c + d*x))*cot(c
 + d*x)**3*tan(c + d*x)**3,x)*a*d*i - 15*int(sqrt(cot(c + d*x))*cot(c + d*
x)**3*tan(c + d*x)**3,x)*b*d - 15*int(sqrt(cot(c + d*x))*cot(c + d*x)**3*t
an(c + d*x)**2,x)*a*d + 15*int(sqrt(cot(c + d*x))*cot(c + d*x)**3*tan(c +
d*x)**2,x)*b*d*i + 15*int(sqrt(cot(c + d*x))*cot(c + d*x)**3*tan(c + d*x),
x)*a*d*i + 5*int(sqrt(cot(c + d*x))*cot(c + d*x)**3*tan(c + d*x),x)*b*d))/
(5*d)
```

3.519 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 138

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{8\sqrt[4]{-1}a^3(iA+B)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{16ia^3A\sqrt{\cot(c+dx)}}{3d} + \frac{2iaB(ia+a\cot(c+dx))^2}{d\sqrt{\cot(c+dx)}} - \frac{2(A+3iB)\sqrt{\cot(c+dx)}(ia^3+a^3\cot(c+dx))}{3d}$$

output
```
-8*(-1)^(1/4)*a^3*(I*A+B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-16/3*I*a^
3*A*cot(d*x+c)^(1/2)/d+2*I*a*B*(I*a+a*cot(d*x+c))^2/d/cot(d*x+c)^(1/2)-2/3
*(A+3*I*B)*cot(d*x+c)^(1/2)*(I*a^3+a^3*cot(d*x+c))/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.85 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2a^3\big(3iB-3iA\cot(c+dx)-9B\cot(c+dx)-3A\cot^2(c+dx)+4A\cot^2(c+dx)\,\text{Hypergeometric2F}$$
$$3d\sqrt{\cot(c+$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x])
,x]
```

output
```
(-2*a^3*((3*I)*B - (3*I)*A*Cot[c + d*x] - 9*B*Cot[c + d*x] - 3*A*Cot[c + d
*x]^2 + 4*A*Cot[c + d*x]^2*Hypergeometric2F1[-3/2, 1, -1/2, I*Tan[c + d*x]
] + 12*B*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, I*Tan[c + d*x]]))/(3
*d*Sqrt[Cot[c + d*x]])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 4064, 3042, 4076, 27, 3042, 4077, 27, 3042, 4075, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{5/2}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+ia)^3(A\cot(c+dx)+B)}{\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$2\int \frac{(\cot(c+dx)a+ia)^2(a(iA+5B)+a(A+3iB)\cot(c+dx))}{2\sqrt{\cot(c+dx)}}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(\cot(c+dx)a+ia)^2(a(iA+5B)+a(A+3iB)\cot(c+dx))}{\sqrt{\cot(c+dx)}}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(a(iA+5B)-a(A+3iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4077}$$

$$-\frac{2}{3}\int \frac{2(\cot(c+dx)a+ia)\left(a^2(A-3iB)-2ia^2A\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}}dx-\frac{2(A+3iB)\sqrt{\cot(c+dx)}\left(a^3\cot(c+dx)+ia^3\right)}{3d}+\frac{2iaB(a\cot(c+dx)+ia)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4}{3}\int \frac{(\cot(c+dx)a+ia)\left(a^2(A-3iB)-2ia^2A\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}}dx-\frac{2(A+3iB)\sqrt{\cot(c+dx)}\left(a^3\cot(c+dx)+ia^3\right)}{3d}+\frac{2iaB(a\cot(c+dx)+ia)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4}{3}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left((A-3iB)a^2+2iA\tan\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(A+3iB)\sqrt{\cot(c+dx)}\left(a^3\cot(c+dx)+ia^3\right)}{3d}+\frac{2iaB(a\cot(c+dx)+ia)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4075}$$

$$-\frac{4}{3}\left(\int \frac{3(iA+B)a^3+3(A-iB)\cot(c+dx)a^3}{\sqrt{\cot(c+dx)}}dx+\frac{4ia^3A\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(A+3iB)\sqrt{\cot(c+dx)}\left(a^3\cot(c+dx)+ia^3\right)}{3d}+\frac{2iaB(a\cot(c+dx)+ia)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4}{3}\left(\int \frac{3a^3(iA+B)-3a^3(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{4ia^3A\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(A+3iB)\sqrt{\cot(c+dx)}\left(a^3\cot(c+dx)+ia^3\right)}{3d}+\frac{2iaB(a\cot(c+dx)+ia)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4016}$$

$$-\frac{4}{3}\left(\frac{18a^6(B+iA)^2\int \frac{1}{3a^3(A-iB)\cot(c+dx)-3a^3(iA+B)}d\sqrt{\cot(c+dx)}}{d}+\frac{4ia^3A\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(A+3iB)\sqrt{\cot(c+dx)}\left(a^3\cot(c+dx)+ia^3\right)}{3d}+\frac{2iaB(a\cot(c+dx)+ia)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{4}{3}\left(\frac{6\sqrt[4]{-1}a^3(B+iA)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{4ia^3A\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(A+3iB)\sqrt{\cot(c+dx)}\left(a^3\cot(c+dx)+ia^3\right)}{3d}+\frac{2iaB(a\cot(c+dx)+ia)^2}{d\sqrt{\cot(c+dx)}}$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(-4*((6*(-1)^(1/4)*a^3*(I*A + B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d
 + ((4*I)*a^3*A*Sqrt[Cot[c + d*x]])/d))/3 + ((2*I)*a*B*(I*a + a*Cot[c + d*
x])^2)/(d*Sqrt[Cot[c + d*x]]) - (2*(A + (3*I)*B)*Sqrt[Cot[c + d*x]]*(I*a^3
 + a^3*Cot[c + d*x]))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(116) = 232.

Time = 1.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.75

method	result
derivativedivides	$\frac{a^3\left(-\frac{2A\cot(dx+c)^{\frac{3}{2}}}{3}-6iA\sqrt{\cot(dx+c)}-2B\sqrt{\cot(dx+c)}-\frac{(-4iA-4B)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1\right.\right.}{4}$
default	$\frac{a^3\left(-\frac{2A\cot(dx+c)^{\frac{3}{2}}}{3}-6iA\sqrt{\cot(dx+c)}-2B\sqrt{\cot(dx+c)}-\frac{(-4iA-4B)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1\right.\right.}{4}$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
a^3/d*(-2/3*A*cot(d*x+c)^(3/2)-6*I*A*cot(d*x+c)^(1/2)-2*B*cot(d*x+c)^(1/2)
-1/4*(-4*I*A-4*B)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot
(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+
2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))-1/4*(4*I*B-4*A)*2^(1/2)*(ln((cot(d*
x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+
2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))
)-2*I*B/cot(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(110) = 220$.

Time = 0.09 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.96

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(3\sqrt{-\frac{(iA^2+2AB-iB^2)a^6}{d^2}}\left(de^{(4i\,dx+4i\,c)}-d\right)\log\left(\frac{2\left((A-iB)a^3e^{(2i\,dx+2i\,c)}+\sqrt{-\frac{\left(iA^2+2AB-iB^2\right)a^6}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\cdots\right.}{(-iA-B)a^3}\right.\right.}{\cdots}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-2/3*(3*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) - d)
*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*
a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 3*sqrt(-(
I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) - d)*log(2*((A - I*
B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(
2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c
) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) + 2*((5*I*A + 3*B)*a^3*e^(
4*I*d*x + 4*I*c) + (I*A - 3*B)*a^3*e^(2*I*d*x + 2*I*c) - 4*I*A*a^3)*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I
*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.41

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{-6iBa^3\sqrt{\tan(dx+c)} - 3\left(2\sqrt{2}(-(i+1)A+(i-1)B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(-\right.}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/3*(-6*I*B*a^3*sqrt(tan(d*x + c)) - 3*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)
*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1
)*A + (I - 1)*B)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - s
qrt(2)*((I - 1)*A + (I + 1)*B)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x
+ c) + 1) + sqrt(2)*((I - 1)*A + (I + 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)
) + 1/tan(d*x + c) + 1))*a^3 + 6*(-3*I*A - B)*a^3/sqrt(tan(d*x + c)) - 2*A
*a^3/tan(d*x + c)^(3/2))/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(3i\,Ba^3\sqrt{\tan(dx+c)}-6\sqrt{2}(-(i+1)\,Aa^3+(i-1)\,Ba^3)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)\right.}{3\,d}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/3*(3*I*B*a^3*sqrt(tan(d*x + c)) - 6*sqrt(2)*(-(I + 1)*A*a^3 + (I - 1)*B
*a^3)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - (-9*I*A*a^3*tan(
d*x + c) - 3*B*a^3*tan(d*x + c) - A*a^3)/tan(d*x + c)^(3/2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \int \cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})^3\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= a^3\left(-\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^4\,dx\right)bi - \left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^3\,dx\right)ai - 3\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^3\,dx\right)b - 3\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)a + 3\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)bi + 3\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
a**3*( - int(sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**4,x)*b*i - i
nt(sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**3,x)*a*i - 3*int(sqrt(
cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**3,x)*b - 3*int(sqrt(cot(c + d*
x))*cot(c + d*x)**2*tan(c + d*x)**2,x)*a + 3*int(sqrt(cot(c + d*x))*cot(c
+ d*x)**2*tan(c + d*x)**2,x)*b*i + 3*int(sqrt(cot(c + d*x))*cot(c + d*x)**
2*tan(c + d*x),x)*a*i + int(sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x
),x)*b + int(sqrt(cot(c + d*x))*cot(c + d*x)**2,x)*a)
```

3.520 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 142

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{8\sqrt[4]{-1}a^3(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{16ia^3B\sqrt{\cot(c+dx)}}{3d}+\frac{2iaB(ia+a\cot(c+dx))^2}{3d\cot^{\frac{3}{2}}(c+dx)}-\frac{2(3A-7iB)\left(ia^3+a^3\cot(c+dx)\right)}{3d\sqrt{\cot(c+dx)}}$$

output
```
-8*(-1)^(1/4)*a^3*(A-I*B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-16/3*I*a^
3*B*cot(d*x+c)^(1/2)/d+2/3*I*a*B*(I*a+a*cot(d*x+c))^2/d/cot(d*x+c)^(3/2)-2
/3*(3*A-7*I*B)*(I*a^3+a^3*cot(d*x+c))/d/cot(d*x+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.87 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2a^3\sqrt{\cot(c+dx)}\left(-9A+12A\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},i\tan(c+dx)\right)+12\sqrt[4]{-1}B\arctan\left((-1\right.\right.}{3d}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x])
,x]
```

output
```
(-2*a^3*Sqrt[Cot[c + d*x]]*(-9*A + 12*A*Hypergeometric2F1[-1/2, 1, 1/2, I*
Tan[c + d*x]] + 12*(-1)^(1/4)*B*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sqrt
[Tan[c + d*x]] + (3*I)*A*Tan[c + d*x] + 9*B*Tan[c + d*x] + I*B*Tan[c + d*x
]^2))/(3*d)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 4064, 3042, 4076, 27, 3042, 4076, 27, 3042, 4075, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+ia)^3(A\cot(c+dx)+B)}{\cot^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{3}\int \frac{(\cot(c+dx)a+ia)^2(a(3iA+7B)+a(3A+iB)\cot(c+dx))}{2\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(\cot(c+dx)a+ia)^2(a(3iA+7B)+a(3A+iB)\cot(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(a(3iA+7B)-a(3A+iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{3}\left(2\int \frac{2(\cot(c+dx)a+ia)\left((3iA+5B)a^2+2iB\cot(c+dx)a^2\right)}{\sqrt{\cot(c+dx)}}dx-\frac{2(3A-7iB)\left(a^3\cot(c+dx)+ia^3\right)}{d\sqrt{\cot(c+dx)}}\right)+\frac{2iaB(a\cot(c+dx)+ia)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(4\int \frac{(\cot(c+dx)a+ia)\left((3iA+5B)a^2+2iB\cot(c+dx)a^2\right)}{\sqrt{\cot(c+dx)}}dx-\frac{2(3A-7iB)\left(a^3\cot(c+dx)+ia^3\right)}{d\sqrt{\cot(c+dx)}}\right)+\frac{2iaB(a\cot(c+dx)+ia)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(4\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a^2(3iA+5B)-2ia^2B\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(3A-7iB)\left(a^3\cot(c+dx)-\right.}{d\sqrt{\cot(c+dx)}}\right.$$

$$\frac{2iaB(a\cot(c+dx)+ia)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4075

$$\frac{1}{3}\left(4\left(\int \frac{3a^3(iA+B)\cot(c+dx)-3a^3(A-iB)}{\sqrt{\cot(c+dx)}}dx-\frac{4ia^3B\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(3A-7iB)\left(a^3\cot(c+dx)+\right.}{d\sqrt{\cot(c+dx)}}\right.$$

$$\frac{2iaB(a\cot(c+dx)+ia)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{3}\left(4\left(\int \frac{-3(A-iB)a^3-3(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4ia^3B\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(3A-7iB)\left(a^3\cot(c\right.}{d\sqrt{\cot(c+d}}\right.$$

$$\frac{2iaB(a\cot(c+dx)+ia)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 4016

$$\frac{1}{3}\left(4\left(\frac{18a^6(A-iB)^2\int\frac{1}{3(A-iB)a^3+3(iA+B)\cot(c+dx)a^3}d\sqrt{\cot(c+dx)}}{d}-\frac{4ia^3B\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(3A-7iB)\left(a^3\right.}{d\sqrt{co}}\right.$$

$$\frac{2iaB(a\cot(c+dx)+ia)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 221

$$\frac{1}{3}\left(4\left(-\frac{6\sqrt[4]{-1}a^3(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{4ia^3B\sqrt{\cot(c+dx)}}{d}\right)-\frac{2(3A-7iB)\left(a^3\cot(c+\right.}{d\sqrt{\cot(c+dx}}\right.$$

$$\frac{2iaB(a\cot(c+dx)+ia)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(((2*I)/3)*a*B*(I*a + a*Cot[c + d*x])^2)/(d*Cot[c + d*x]^(3/2)) + (4*((-6*
(-1)^(1/4)*a^3*(A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - ((4*I
)*a^3*B*Sqrt[Cot[c + d*x]])/d) - (2*(3*A - (7*I)*B)*(I*a^3 + a^3*Cot[c + d
*x]))/(d*Sqrt[Cot[c + d*x]]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.66

method	result
derivativedivides	$\frac{a^3\left(-2A\sqrt{\cot(dx+c)}-\frac{(4iB-4A)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{c\right.\right.}{4}}{}$
default	$\frac{a^3\left(-2A\sqrt{\cot(dx+c)}-\frac{(4iB-4A)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{c\right.\right.}{4}}{}$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
a^3/d*(-2*A*cot(d*x+c)^(1/2)-1/4*(4*I*B-4*A)*2^(1/2)*(ln((cot(d*x+c)+2^(1/
2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1
+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))-1/4*(4*I
*A+4*B)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^
(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-
1+2^(1/2)*cot(d*x+c)^(1/2)))+2*(-I*A-3*B)/cot(d*x+c)^(1/2)-2/3*I*B/cot(d*x
+c)^(3/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(112) = 224$.

Time = 0.10 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.11

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= \frac{2\left(3\sqrt{-\frac{(-iA^2-2AB+iB^2)a^6}{d^2}}\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left((A-iB)a^3e^{(2i\,dx+2i\,c)}-\sqrt{-\frac{(-iA^2-2AB+iB^2)}{d^2}}\cdots\right.}{(-i\cdots}\right.\right.}{\cdots}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
2/3*(3*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*
d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt
(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c
)/((-I*A - B)*a^3)) - 3*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(4*I*
d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^3*e^(2*I*d*
x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(-I*d*e^(2*I*d*x + 2*
I*c) + I*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e
^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 2*((3*A - 5*I*B)*a^3*e^(4*I*d*x +
4*I*c) + (3*A + I*B)*a^3*e^(2*I*d*x + 2*I*c) + 4*I*B*a^3)*sqrt((I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) + 2*d*
e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.39

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{3\left(2\sqrt{2}((i-1)\ A+(i+1)\ B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((i-1)\ A+(i+1)\ B)\mathrm{arct}\right.}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/3*(3*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2
/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt
(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*l
og(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(-(I + 1)*A
+ (I - 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3 +
6*A*a^3/sqrt(tan(d*x + c)) - 2*(-I*B*a^3 - 3*(I*A + 3*B)*a^3/tan(d*x + c))
*tan(d*x + c)^(3/2))/d
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(iBa^3\tan\left(dx+c\right)^{\frac{3}{2}}+3iAa^3\sqrt{\tan\left(dx+c\right)}+9Ba^3\sqrt{\tan\left(dx+c\right)}+\frac{3Aa^3}{\sqrt{\tan(dx+c)}}+6\sqrt{2}(-(i-1)\ A\right.}{3d}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/3*(I*B*a^3*tan(d*x + c)^(3/2) + 3*I*A*a^3*sqrt(tan(d*x + c)) + 9*B*a^3*
sqrt(tan(d*x + c)) + 3*A*a^3/sqrt(tan(d*x + c)) + 6*sqrt(2)*(-(I - 1)*A*a^
3 - (I + 1)*B*a^3)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^3\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{a^3\left(-2\sqrt{\cot(dx+c)}\,a-\left(\int\frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)ad-\left(\int\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^4\,dx\right)bdi-\right.}{}$$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
(a**3*( - 2*sqrt(cot(c + d*x))*a - int(sqrt(cot(c + d*x))/cot(c + d*x),x)*
a*d - int(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)**4,x)*b*d*i - int(s
qrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)**3,x)*a*d*i - 3*int(sqrt(cot(c
 + d*x))*cot(c + d*x)*tan(c + d*x)**3,x)*b*d - 3*int(sqrt(cot(c + d*x))*co
t(c + d*x)*tan(c + d*x)**2,x)*a*d + 3*int(sqrt(cot(c + d*x))*cot(c + d*x)*
tan(c + d*x)**2,x)*b*d*i + 3*int(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d
*x),x)*a*d*i + int(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x),x)*b*d))/d
```

3.521 $\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 148

$$\begin{aligned}\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= \frac{8\sqrt[4]{-1}a^3(iA+B)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{16a^3(5A-6iB)}{15d\sqrt{\cot(c+dx)}}\\&\quad+\frac{2iaB(ia+a\cot(c+dx))^2}{5d\cot^{\frac{5}{2}}(c+dx)}-\frac{2(5A-9iB)\left(ia^3+a^3\cot(c+dx)\right)}{15d\cot^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output
```
8*(-1)^(1/4)*a^3*(I*A+B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-16/15*a^3*
(5*A-6*I*B)/d/cot(d*x+c)^(1/2)+2/5*I*a*B*(I*a+a*cot(d*x+c))^2/d/cot(d*x+c)
^(5/2)-2/15*(5*A-9*I*B)*(I*a^3+a^3*cot(d*x+c))/d/cot(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 4.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.74

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2ia^3\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-60\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}\right.}{15d}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x])
,x]
```

output
```
(((-2*I)/15)*a^3*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-60*(-1)^(1/4)*(I*
A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + Sqrt[Tan[c + d*x]]*((-45*I)
*A - 60*B + 5*(A - (3*I)*B)*Tan[c + d*x] + 3*B*Tan[c + d*x]^2)))/d
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4064, 3042, 4076, 27, 3042, 4076, 27, 3042, 4074, 27, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4064}$$

$$\int \frac{(a\cot(c+dx)+ia)^3(A\cot(c+dx)+B)}{\cot^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4076}$$

$$\frac{2}{5}\int \frac{(\cot(c+dx)a+ia)^2(a(5iA+9B)+a(5A-iB)\cot(c+dx))}{2\cot^{\frac{5}{2}}(c+dx)}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int \frac{(\cot(c+dx)a+ia)^2(a(5iA+9B)+a(5A-iB)\cot(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(a(5iA+9B)-a(5A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4076}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{2(\cot(c+dx)a+ia)\left(2(5iA+6B)a^2+(5A-3iB)\cot(c+dx)a^2\right)}{\cot^{\frac{3}{2}}(c+dx)}dx-\frac{2(5A-9iB)\left(a^3\cot(c+dx)+\right.}{3d\cot^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\frac{4}{3}\int \frac{(\cot(c+dx)a+ia)\left(2(5iA+6B)a^2+(5A-3iB)\cot(c+dx)a^2\right)}{\cot^{\frac{3}{2}}(c+dx)}dx-\frac{2(5A-9iB)\left(a^3\cot(c+dx)+i\right.}{3d\cot^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{4}{3}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(2a^2(5iA+6B)-a^2(5A-3iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2(5A-9iB)\left(a^3\cot(\right.}{3d\cot^{\frac{3}{2}}(c-}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4074

$$\frac{1}{5}\left(\frac{4}{3}\left(\int \frac{15\left((iA+B)a^3+(A-iB)\cot(c+dx)a^3\right)}{\sqrt{\cot(c+dx)}}dx-\frac{4a^3(5A-6iB)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2(5A-9iB)\left(a^3\cot(c+dx)+ia^3\right.}{3d\cot^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{5}\left(\frac{4}{3}\left(15\int \frac{(iA+B)a^3+(A-iB)\cot(c+dx)a^3}{\sqrt{\cot(c+dx)}}dx-\frac{4a^3(5A-6iB)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2(5A-9iB)\left(a^3\cot(c+dx)+ia^3\right)}{3d\cot^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{4}{3}\left(15\int \frac{a^3(iA+B)-a^3(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{4a^3(5A-6iB)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2(5A-9iB)\left(a^3\cot(c+dx)\right.}{3d\cot^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 4016

$$\frac{1}{5}\left(\frac{4}{3}\left(\frac{30a^6(B+iA)^2\int \frac{1}{a^3(A-iB)\cot(c+dx)-a^3(iA+B)}d\sqrt{\cot(c+dx)}}{d}-\frac{4a^3(5A-6iB)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2(5A-9iB)\left(a^3\cot(c\right.}{3d\cot^{\frac{3}{2}}(c+}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 221

$$\frac{1}{5}\left(\frac{4}{3}\left(\frac{30\sqrt[4]{-1}a^3(B+iA)\operatorname{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}-\frac{4a^3(5A-6iB)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2(5A-9iB)\left(a^3\cot(c+dx)\right.}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\frac{2iaB(a\cot(c+dx)+ia)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(((2*I)/5)*a*B*(I*a + a*Cot[c + d*x])^2)/(d*Cot[c + d*x]^(5/2)) + ((4*((30
*(-1)^(1/4)*a^3*(I*A + B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d - (4*a
^3*(5*A - (6*I)*B))/(d*Sqrt[Cot[c + d*x]])))/3 - (2*(5*A - (9*I)*B)*(I*a^3
 + a^3*Cot[c + d*x]))/(3*d*Cot[c + d*x]^(3/2)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.64

method	result
derivativedivides	$a^3\left(-\frac{(4iA+4B)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4}-\frac{(-4i}{}\right.$
default	$a^3\left(-\frac{(4iA+4B)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4}-\frac{(-4i}{}\right.$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
a^3/d*(-1/4*(4*I*A+4*B)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1
)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(
1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))-1/4*(4*A-4*I*B)*2^(1/2)*(ln((
cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2
)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^
(1/2)))+2/3*(-I*A-3*B)/cot(d*x+c)^(3/2)+2*(4*I*B-3*A)/cot(d*x+c)^(1/2)-2/5
*I*B/cot(d*x+c)^(5/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(118) = 236.

Time = 0.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.39

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2\left(15\sqrt{-\frac{(iA^2+2AB-iB^2)a^6}{d^2}}\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{2\left((A-i\,B)a^3e^{(2i\,dx+2i\,c)}+\sqrt{\right.}\right.}{}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
2/15*(15*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3
*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(2*((A - I*B)*a^3
*e^(2*I*d*x + 2*I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^6/d^2)*(d*e^(2*I*d*
x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3)) - 15*sqrt(-(I*A^2 + 2*A*B - I*B^
2)*a^6/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*
d*x + 2*I*c) + d)*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt(-(I*A^2
+ 2*A*B - I*B^2)*a^6/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt((I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*
a^3)) - 2*((-25*I*A - 39*B)*a^3*e^(6*I*d*x + 6*I*c) + 2*(-10*I*A - 9*B)*a^
3*e^(4*I*d*x + 4*I*c) + (25*I*A + 33*B)*a^3*e^(2*I*d*x + 2*I*c) + 4*(5*I*A
 + 6*B)*a^3)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/
(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= -ia^3\left(\int iA\sqrt{\cot\left(c+dx\right)}\,dx + \int\left(-3A\tan\left(c+dx\right)\sqrt{\cot\left(c+dx\right)}\right)\,dx + \int A\tan^3\left(c+dx\right)\sqrt{\cot\left(c+dx\right)}\,dx + \int\left(-3B\tan^2\left(c+dx\right)\sqrt{\cot\left(c+dx\right)}\right)\,dx + \int B\tan^4\left(c+dx\right)\sqrt{\cot\left(c+dx\right)}\,dx + \int\left(-3iA\tan^2\left(c+dx\right)\sqrt{\cot\left(c+dx\right)}\right)\,dx + \int iB\tan\left(c+dx\right)\sqrt{\cot\left(c+dx\right)}\,dx + \int\left(-3iB\tan^3\left(c+dx\right)\sqrt{\cot\left(c+dx\right)}\right)\,dx\right)$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
-I*a**3*(Integral(I*A*sqrt(cot(c + d*x)), x) + Integral(-3*A*tan(c + d*x)*
sqrt(cot(c + d*x)), x) + Integral(A*tan(c + d*x)**3*sqrt(cot(c + d*x)), x)
 + Integral(-3*B*tan(c + d*x)**2*sqrt(cot(c + d*x)), x) + Integral(B*tan(c
 + d*x)**4*sqrt(cot(c + d*x)), x) + Integral(-3*I*A*tan(c + d*x)**2*sqrt(c
ot(c + d*x)), x) + Integral(I*B*tan(c + d*x)*sqrt(cot(c + d*x)), x) + Inte
gral(-3*I*B*tan(c + d*x)**3*sqrt(cot(c + d*x)), x))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{15\left(2\sqrt{2}(-(i+1)\,A+(i-1)\,B)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}(-(i+1)\,A+(i-1)\,B)\,\mathrm{ar}\right.}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/15*(15*(2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-1/2*s
qrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((I - 1)*A + (I + 1)*B)
*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((I - 1)*A
 + (I + 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3 -
 2*(3*I*B*a^3 - 5*(-I*A - 3*B)*a^3/tan(d*x + c) + 15*(3*A - 4*I*B)*a^3/tan
(d*x + c)^2)*tan(d*x + c)^(5/2))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\left(3i\,Ba^3\tan(dx+c)^{\frac{5}{2}}+5i\,Aa^3\tan(dx+c)^{\frac{3}{2}}+15\,Ba^3\tan(dx+c)^{\frac{3}{2}}+45\,Aa^3\sqrt{\tan(dx+c)}-60i\right.}{15\,a}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
-2/15*(3*I*B*a^3*tan(d*x + c)^(5/2) + 5*I*A*a^3*tan(d*x + c)^(3/2) + 15*B*
a^3*tan(d*x + c)^(3/2) + 45*A*a^3*sqrt(tan(d*x + c)) - 60*I*B*a^3*sqrt(tan
(d*x + c)) + 30*sqrt(2)*(-(I + 1)*A*a^3 + (I - 1)*B*a^3)*arctan(-(1/2*I -
1/2)*sqrt(2)*sqrt(tan(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+a\tan(c+d\,x)\,\,1\mathrm{i}\right)^3\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3, x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=a^3\Bigg(\left(\left(\int \sqrt{\cot\left(dx+c\right)}dx\right)a-\left(\int \sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)^4dx\right)bi\right.$$
$$-\left(\int \sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)^3dx\right)ai-3\left(\int \sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)^3dx\right)b$$
$$-3\left(\int \sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)^2dx\right)a+3\left(\int \sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)^2dx\right)bi$$
$$+3\left(\int \sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)dx\right)ai+\left(\int \sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)dx\right)b\Bigg)$$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
a**3*(int(sqrt(cot(c + d*x)),x)*a - int(sqrt(cot(c + d*x))*tan(c + d*x)**4
,x)*b*i - int(sqrt(cot(c + d*x))*tan(c + d*x)**3,x)*a*i - 3*int(sqrt(cot(c
 + d*x))*tan(c + d*x)**3,x)*b - 3*int(sqrt(cot(c + d*x))*tan(c + d*x)**2,x
)*a + 3*int(sqrt(cot(c + d*x))*tan(c + d*x)**2,x)*b*i + 3*int(sqrt(cot(c +
 d*x))*tan(c + d*x),x)*a*i + int(sqrt(cot(c + d*x))*tan(c + d*x),x)*b)
```

3.522 $\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 173

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{8\sqrt[4]{-1}a^3(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d} - \frac{8a^3(21A-23iB)}{105d\cot^{\frac{3}{2}}(c+dx)} + \frac{8a^3(iA+B)}{d\sqrt{\cot(c+dx)}} + \frac{2iaB(ia+a\cot(c+dx))^2}{7d\cot^{\frac{7}{2}}(c+dx)} - \frac{2(7A-11iB)\left(ia^3+a^3\cot(c+dx)\right)}{35d\cot^{\frac{5}{2}}(c+dx)}$$

output

```
8*(-1)^(1/4)*a^3*(A-I*B)*arctanh((-1)^(3/4)*cot(d*x+c)^(1/2))/d-8/105*a^3*
(21*A-23*I*B)/d/cot(d*x+c)^(3/2)+8*a^3*(I*A+B)/d/cot(d*x+c)^(1/2)+2/7*I*a*
B*(I*a+a*cot(d*x+c))^2/d/cot(d*x+c)^(7/2)-2/35*(7*A-11*I*B)*(I*a^3+a^3*cot
(d*x+c))/d/cot(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 4.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.76

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \frac{2a^3\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(420\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)+\sqrt{\tan(c+dx)}(420\right.}{105d}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x
]],x]
```

output
```
(2*a^3*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(420*(-1)^(1/4)*(I*A + B)*Arc
Tan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]] + Sqrt[Tan[c + d*x]]*(420*(I*A + B) - 3
5*(3*A - (4*I)*B)*Tan[c + d*x] - (21*I)*(A - (3*I)*B)*Tan[c + d*x]^2 - (15
*I)*B*Tan[c + d*x]^3)))/(105*d)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4064, 3042, 4076, 27, 3042, 4076, 27, 3042, 4074, 27, 3042, 4012, 3042, 4016, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$
$$\Big\downarrow \; 4064$$
$$\int \frac{(a\cot(c+dx)+ia)^3(A\cot(c+dx)+B)}{\cot^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\frac{2}{7}\int \frac{(\cot(c+dx)a+ia)^2(a(7iA+11B)+a(7A-3iB)\cot(c+dx))}{2\cot^{\frac{7}{2}}(c+dx)}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{(\cot(c+dx)a+ia)^2(a(7iA+11B)+a(7A-3iB)\cot(c+dx))}{\cot^{\frac{7}{2}}(c+dx)}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(a(7iA+11B)-a(7A-3iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx+\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4076}$$

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{2(\cot(c+dx)a+ia)\left((21iA+23B)a^2+2(7A-6iB)\cot(c+dx)a^2\right)}{\cot^{\frac{5}{2}}(c+dx)}dx-\frac{2(7A-11iB)\left(a^3\cot(c+dx\right.}{5d\cot^{\frac{5}{2}}(c+dx)}\right.+\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(\frac{4}{5}\int \frac{(\cot(c+dx)a+ia)\left((21iA+23B)a^2+2(7A-6iB)\cot(c+dx)a^2\right)}{\cot^{\frac{5}{2}}(c+dx)}dx-\frac{2(7A-11iB)\left(a^3\cot(c+dx)\right.}{5d\cot^{\frac{5}{2}}(c+dx)}\right.+\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\frac{4}{5}\int \frac{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a^2(21iA+23B)-2a^2(7A-6iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2(7A-11iB)\left(a^3\right.}{5d\cot^{\frac{5}{2}}}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 4074

$$\frac{1}{7}\left(\frac{4}{5}\left(\int \frac{35\left((iA+B)a^3+(A-iB)\cot(c+dx)a^3\right)}{\cot^{\frac{3}{2}}(c+dx)}dx-\frac{2a^3(21A-23iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7A-11iB)\left(a^3\cot(c+dx)+\right.}{5d\cot^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{1}{7}\left(\frac{4}{5}\left(35\int \frac{(iA+B)a^3+(A-iB)\cot(c+dx)a^3}{\cot^{\frac{3}{2}}(c+dx)}dx-\frac{2a^3(21A-23iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7A-11iB)\left(a^3\cot(c+dx)+i\right.}{5d\cot^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{4}{5}\left(35\int \frac{a^3(iA+B)-a^3(A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2a^3(21A-23iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7A-11iB)\left(a^3\cot(c+d\right.}{5d\cot^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 4012

$$\frac{1}{7}\left(\frac{4}{5}\left(35\left(\int \frac{a^3(A-iB)-a^3(iA+B)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2a^3(B+iA)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2a^3(21A-23iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7A-11i}{5}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\left(\frac{4}{5}\left(35\left(\int \frac{(A-iB)a^3+(iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a^3(B+iA)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2a^3(21A-23iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7A}{}\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 4016

$$\frac{1}{7}\left(\frac{4}{5}\left(35\left(\frac{2a^6(A-iB)^2\int\frac{1}{-((A-iB)a^3)-(iA+B)\cot(c+dx)a^3}d\sqrt{\cot(c+dx)}}{d}+\frac{2a^3(B+iA)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2a^3(21A-23iB}{3d\cot^{\frac{3}{2}}(c+dx)}\right.\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 221

$$\frac{1}{7}\left(\frac{4}{5}\left(35\left(\frac{2\sqrt[4]{-1}a^3(A-iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\cot(c+dx)}\right)}{d}+\frac{2a^3(B+iA)}{d\sqrt{\cot(c+dx)}}\right)-\frac{2a^3(21A-23iB)}{3d\cot^{\frac{3}{2}}(c+dx)}\right)-\frac{2(7\right.$$
$$\frac{2iaB(a\cot(c+dx)+ia)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

input
```
Int[((a + I*a*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

output
```
(((2*I)/7)*a*B*(I*a + a*Cot[c + d*x])^2)/(d*Cot[c + d*x]^(7/2)) + ((4*(35*
((2*(-1)^(1/4)*a^3*(A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Cot[c + d*x]]])/d + (
2*a^3*(I*A + B))/(d*Sqrt[Cot[c + d*x]])) - (2*a^3*(21*A - (23*I)*B))/(3*d*
Cot[c + d*x]^(3/2))))/5 - (2*(7*A - (11*I)*B)*(I*a^3 + a^3*Cot[c + d*x]))/
(5*d*Cot[c + d*x]^(5/2)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4016
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2*(c^2/f)   Subst[Int[1/(b*c - d*x^2), x], x, Sqrt[b
*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && EqQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.51

method	result
derivativedivides	$a^3\left(-\frac{(-4iB+4A)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4} - \frac{(-}{}\right.$
default	$a^3\left(-\frac{(-4iB+4A)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4} - \frac{(-}{}\right.$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
a^3/d*(-1/4*(4*A-4*I*B)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1
)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(
1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))-1/4*(-4*I*A-4*B)*2^(1/2)*(ln(
(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/
2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)
^(1/2)))+2/5*(-I*A-3*B)/cot(d*x+c)^(5/2)+2*(4*I*A+4*B)/cot(d*x+c)^(1/2)+2/
3*(4*I*B-3*A)/cot(d*x+c)^(3/2)-2/7*I*B/cot(d*x+c)^(7/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(139) = 278$.

Time = 0.11 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.25

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algori
thm="fricas")
```

output
```
-2/105*(105*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(8*I*d*x + 8*I*c)
 + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*
I*c) + d)*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B
 + I*B^2)*a^6/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt((I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3
)) - 105*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^6/d^2)*(d*e^(8*I*d*x + 8*I*c) +
4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c
) + d)*log(2*((A - I*B)*a^3*e^(2*I*d*x + 2*I*c) - sqrt(-(-I*A^2 - 2*A*B +
I*B^2)*a^6/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((-I*A - B)*a^3))
 - 2*((273*A - 319*I*B)*a^3*e^(8*I*d*x + 8*I*c) + 3*(133*A - 109*I*B)*a^3*
e^(6*I*d*x + 6*I*c) - 5*(21*A - 19*I*B)*a^3*e^(4*I*d*x + 4*I*c) - 3*(133*A
 - 129*I*B)*a^3*e^(2*I*d*x + 2*I*c) - 4*(42*A - 41*I*B)*a^3)*sqrt((I*e^(2*
I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(8*I*d*x + 8*I*c) + 4
*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = -ia^3\left(\int \frac{iA}{\sqrt{\cot(c+dx)}}\,dx + \int \left(-\frac{3A\tan(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx + \int \frac{A\tan^3(c+dx)}{\sqrt{\cot(c+dx)}}\,dx + \int \left(-\frac{3B\tan^2(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx + \int \frac{B\tan^4(c+dx)}{\sqrt{\cot(c+dx)}}\,dx + \int \left(-\frac{3iA\tan^2(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx + \int \frac{iB\tan(c+dx)}{\sqrt{\cot(c+dx)}}\,dx + \int \left(-\frac{3iB\tan^3(c+dx)}{\sqrt{\cot(c+dx)}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(d*x+c))**3*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
-I*a**3*(Integral(I*A/sqrt(cot(c + d*x)), x) + Integral(-3*A*tan(c + d*x)/
sqrt(cot(c + d*x)), x) + Integral(A*tan(c + d*x)**3/sqrt(cot(c + d*x)), x)
 + Integral(-3*B*tan(c + d*x)**2/sqrt(cot(c + d*x)), x) + Integral(B*tan(c
 + d*x)**4/sqrt(cot(c + d*x)), x) + Integral(-3*I*A*tan(c + d*x)**2/sqrt(c
ot(c + d*x)), x) + Integral(I*B*tan(c + d*x)/sqrt(cot(c + d*x)), x) + Inte
gral(-3*I*B*tan(c + d*x)**3/sqrt(cot(c + d*x)), x))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx =$$

$$-\frac{2\left(15i\,Ba^3-\frac{21\,(-i\,A-3\,B)a^3}{\tan(dx+c)}+\frac{35\,(3\,A-4i\,B)a^3}{\tan(dx+c)^2}-\frac{420\,(i\,A+B)a^3}{\tan(dx+c)^3}\right)\tan\left(dx+c\right)^{\frac{7}{2}}-105\left(2\sqrt{2}((i-1)\,A+(i+\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algori
thm="maxima")
```

output
```
-1/105*(2*(15*I*B*a^3 - 21*(-I*A - 3*B)*a^3/tan(d*x + c) + 35*(3*A - 4*I*B
)*a^3/tan(d*x + c)^2 - 420*(I*A + B)*a^3/tan(d*x + c)^3)*tan(d*x + c)^(7/2
) - 105*(2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(1/2*sqrt(2)*(sqrt(2) + 2
/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-1/2*sqrt
(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*l
og(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*(-(I + 1)*A
+ (I - 1)*B)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx =$$

$$-\frac{2\left(15i\,Ba^3\tan\left(dx+c\right)^{\frac{7}{2}}+21i\,Aa^3\tan\left(dx+c\right)^{\frac{5}{2}}+63\,Ba^3\tan\left(dx+c\right)^{\frac{5}{2}}+105\,Aa^3\tan\left(dx+c\right)^{\frac{3}{2}}-1\right.}{}$$

input
```
integrate((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algori
thm="giac")
```

output
```
-2/105*(15*I*B*a^3*tan(d*x + c)^(7/2) + 21*I*A*a^3*tan(d*x + c)^(5/2) + 63
*B*a^3*tan(d*x + c)^(5/2) + 105*A*a^3*tan(d*x + c)^(3/2) - 140*I*B*a^3*tan
(d*x + c)^(3/2) - 420*I*A*a^3*sqrt(tan(d*x + c)) - 420*B*a^3*sqrt(tan(d*x
+ c)) + 210*sqrt(2)*((I - 1)*A*a^3 + (I + 1)*B*a^3)*arctan(-(1/2*I - 1/2)*
sqrt(2)*sqrt(tan(d*x + c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^3}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3)/cot(c + d*x)^(1/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^3)/cot(c + d*x)^(1/2), x
)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= a^3\left(\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a-\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^4}{\cot(dx+c)}dx\right)bi-\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^3}{\cot(dx+c)}dx\right)ai-3\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^3}{\cot(dx+c)}dx\right)b-3\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)}dx\right)a+3\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)}dx\right)bi+3\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)ai+\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)b\right)$$

input
```
int((a+I*a*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
a**3*(int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a - int((sqrt(cot(c + d*x))*t
an(c + d*x)**4)/cot(c + d*x),x)*b*i - int((sqrt(cot(c + d*x))*tan(c + d*x)
**3)/cot(c + d*x),x)*a*i - 3*int((sqrt(cot(c + d*x))*tan(c + d*x)**3)/cot(
c + d*x),x)*b - 3*int((sqrt(cot(c + d*x))*tan(c + d*x)**2)/cot(c + d*x),x)
*a + 3*int((sqrt(cot(c + d*x))*tan(c + d*x)**2)/cot(c + d*x),x)*b*i + 3*in
t((sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*x),x)*a*i + int((sqrt(cot(c
+ d*x))*tan(c + d*x))/cot(c + d*x),x)*b)
```

3.523 $\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 247

$$\begin{aligned}\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx &= -\frac{\left(\frac{1}{4}-\frac{i}{4}\right)((6+i)A+(1+4i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}ad}\\ &+\frac{((7-5i)A+(5+3i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{4\sqrt{2}ad}\\ &+\frac{((-7-5i)A+(5-3i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{4\sqrt{2}ad}+\frac{5(iA-B)\sqrt{\cot(c+dx)}}{2ad}\\ &-\frac{(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{6ad}+\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(ia+a\cot(c+dx))}\end{aligned}$$

output

```
(1/8-1/8*I)*((6+I)*A+(1+4*I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2
)/a/d+1/8*((7-5*I)*A+(5+3*I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)
/a/d+1/8*((-7-5*I)*A+(5-3*I)*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*
x+c)))*2^(1/2)/a/d+5/2*(I*A-B)*cot(d*x+c)^(1/2)/a/d-1/6*(7*A+3*I*B)*cot(d*
x+c)^(3/2)/a/d+1/2*(A+I*B)*cot(d*x+c)^(5/2)/d/(I*a+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.75

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\cot^{\frac{3}{2}}(c+dx)\left(4iA+8A\tan(c+dx)+12iB\tan(c+dx)+15iA\tan^2(c+dx)-15B\tan^2(c+dx)+3\sqrt[4]{\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
,x]
```

output
```
(Cot[c + d*x]^(3/2)*((4*I)*A + 8*A*Tan[c + d*x] + (12*I)*B*Tan[c + d*x] +
(15*I)*A*Tan[c + d*x]^2 - 15*B*Tan[c + d*x]^2 + 3*(-1)^(1/4)*(A - I*B)*Arc
Tan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2)*(-I + Tan[c + d*x])
+ 6*(-1)^(1/4)*(3*A + (2*I)*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Tan[
c + d*x]^(3/2)*(-I + Tan[c + d*x])))/(6*a*d*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.07, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4064, 3042, 4078, 27, 3042, 4011, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cot(c+dx)^{5/2}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$
$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A\cot(c+dx)+B)}{a\cot(c+dx)+ia}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia}dx$$

$\Big\downarrow$ 4078

$$\frac{\int -\frac{1}{2}\cot^{\frac{3}{2}}(c+dx)(5a(iA-B)-a(7A+3iB)\cot(c+dx))dx}{2a^2}+\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}$$

$\Big\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\int \cot^{\frac{3}{2}}(c+dx)(5a(iA-B)-a(7A+3iB)\cot(c+dx))dx}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\int \left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(5a(iA-B)+a(7A+3iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx}{4a^2}$$

$\Big\downarrow$ 4011

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\int \sqrt{\cot(c+dx)}(a(7A+3iB)+5a(iA-B)\cot(c+dx))dx+\frac{2a(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{3d}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a(7A+3iB)-5a(iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{2a(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{3d}}{4a^2}$$

$\Big\downarrow$ 4011

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\int \frac{a(7A+3iB)\cot(c+dx)-5a(iA-B)}{\sqrt{\cot(c+dx)}}dx+\frac{2a(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{10a(-B+iA)\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\int \frac{-5a(iA-B)-a(7A+3iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{10a(-B+iA)\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2\int \frac{a(5(iA-B)-(7A+3iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{10a(-B+iA)\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a\int \frac{5(iA-B)-(7A+3iB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{10a(-B+iA)\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \text{1482}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)((6+i)A+(1+4i)B)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}+\frac{2a(7A+3iB)\cot^{\frac{3}{2}}(c+d}{3d}}{4a^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)((6+i)A+(1+4i)B)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{}{\cot(c-}\right.\right.}{d}}{4a^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)((6+i)A+(1+4i)B)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\right.}{\sqrt{}}\right.\right.}{d}}{4a^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)} - \frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)((6+i)A+(1+4i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}}{4a^2}$$

$\downarrow$ 1479

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)} - \frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((6+i)A+(1+
\cdots}{d}}{4a^2}$$

$\downarrow$ 25

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)} - \frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((6+i)A+(1+4i)B
\cdots}{d}}{4a^2}$$

$\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)} - \frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((6+i)A+(1+
\cdots}{d}}{4a^2}$$

$\downarrow$ 1103

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)} - \frac{\frac{2a\left(\frac{1}{2}((7+5i)A-(5-3i)B)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)((6+i)A+(1+4i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c
\cdots}{\sqrt{2}}\right.}{d}}{4a^2}$$

input
```
Int[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
((A + I*B)*Cot[c + d*x]^(5/2))/(2*d*(I*a + a*Cot[c + d*x])) - ((-10*a*(I*A
 - B)*Sqrt[Cot[c + d*x]])/d + (2*a*(7*A + (3*I)*B)*Cot[c + d*x]^(3/2))/(3*
d) + (2*a*((-1/2 + I/2)*((6 + I)*A + (1 + 4*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sq
rt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2
]) + (((7 + 5*I)*A - (5 - 3*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*
x]]/(2*Sqrt[2])))/2))/d)/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{-\frac{2A\cot(dx+c)^{\frac{3}{2}}}{3}+2iA\sqrt{\cot(dx+c)}-2B\sqrt{\cot(dx+c)}+\frac{4\left(\frac{A}{4}-\frac{iB}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}-\frac{i\left(-\frac{i(iB+A)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4(3i}{}\right.}{}}{ad}$
default	$\frac{-\frac{2A\cot(dx+c)^{\frac{3}{2}}}{3}+2iA\sqrt{\cot(dx+c)}-2B\sqrt{\cot(dx+c)}+\frac{4\left(\frac{A}{4}-\frac{iB}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}-\frac{i\left(-\frac{i(iB+A)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4(3i}{}\right.}{}}{ad}$

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/a/d*(-2/3*A*cot(d*x+c)^(3/2)+2*I*A*cot(d*x+c)^(1/2)-2*B*cot(d*x+c)^(1/2)
+4*(1/4*A-1/4*I*B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)-
I*2^(1/2)))-1/2*I*(-I*(A+I*B)*cot(d*x+c)^(1/2)/(I+cot(d*x+c))+4*(3*I*A-2*B
)/(2^(1/2)+I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(184) = 368$.

Time = 0.12 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.90

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/24*(3*(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((-I*A^2
- 2*A*B + I*B^2)/(a^2*d^2))*log(-2*((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B
 + I*B^2)/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)
/(I*A + B)) - 3*(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((
-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2))*log(2*((a*d*e^(2*I*d*x + 2*I*c) - a*d)*
sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 -
 2*A*B + I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x -
2*I*c)/(I*A + B)) - 6*(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*
sqrt((9*I*A^2 - 12*A*B - 4*I*B^2)/(a^2*d^2))*log(((a*d*e^(2*I*d*x + 2*I*c)
 - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((
9*I*A^2 - 12*A*B - 4*I*B^2)/(a^2*d^2)) + 3*A + 2*I*B)*e^(-2*I*d*x - 2*I*c)
/(a*d)) + 6*(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))*sqrt((9*I*
A^2 - 12*A*B - 4*I*B^2)/(a^2*d^2))*log(-((a*d*e^(2*I*d*x + 2*I*c) - a*d)*s
qrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((9*I*A^2 -
 12*A*B - 4*I*B^2)/(a^2*d^2)) - 3*A - 2*I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) -
 2*((19*I*A - 27*B)*e^(4*I*d*x + 4*I*c) - 2*(19*I*A - 15*B)*e^(2*I*d*x + 2
*I*c) + 3*I*A - 3*B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 - 1)))/(a*d*e^(4*I*d*x + 4*I*c) - a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.51

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{6\sqrt{2}(-(3i-3)\ A+(2i+2)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}((i+1)\ A-(i-1)\ B)}{1}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/12*(6*sqrt(2)*(-(3*I - 3)*A + (2*I + 2)*B)*arctan((1/2*I + 1/2)*sqrt(2)
*sqrt(tan(d*x + c))) + 3*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-(1/2*I -
1/2)*sqrt(2)*sqrt(tan(d*x + c))) + 6*(-I*A*sqrt(tan(d*x + c)) + B*sqrt(tan
(d*x + c)))/(tan(d*x + c) - I) + 8*(-3*I*A*tan(d*x + c) + 3*B*tan(d*x + c)
 + A)/tan(d*x + c)^(3/2))/(a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx=\int \frac{\cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\left(\int \frac{\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)}{\tan(dx+c)i+1}dx\right)b+\left(\int \frac{\sqrt{\cot(dx+c)}\,\cot(dx+c)^2}{\tan(dx+c)i+1}dx\right)a}{a}$$

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(int((sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x))/(tan(c + d*x)*i + 1
),x)*b + int((sqrt(cot(c + d*x))*cot(c + d*x)**2)/(tan(c + d*x)*i + 1),x)*
a)/a
```

3.524 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 214

$$\begin{aligned}\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx &= \frac{((-5-3i)A+(3-i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{4\sqrt{2}ad}\\ &\quad+\frac{((5+3i)A-(3-i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{4\sqrt{2}ad}\\ &\quad+\frac{((5-3i)A+(3+i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{4\sqrt{2}ad}\\ &\quad-\frac{(5A+iB)\sqrt{\cot(c+dx)}}{2ad}+\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(ia+a\cot(c+dx))}\end{aligned}$$

output
```
-1/8*((-5-3*I)*A+(3-I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a/d+
1/8*((5+3*I)*A+(-3+I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a/d+1/
8*((5-3*I)*A+(3+I)*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(
1/2)/a/d-1/2*(5*A+I*B)*cot(d*x+c)^(1/2)/a/d+1/2*(A+I*B)*cot(d*x+c)^(3/2)/d
/(I*a+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\left(-4iA+5A\tan(c+dx)+iB\tan(c+dx)+\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+\right.\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
,x]
```

output
```
-1/2*(Sqrt[Cot[c + d*x]]*((-4*I)*A + 5*A*Tan[c + d*x] + I*B*Tan[c + d*x] +
 (-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sqrt[Tan[c + d
*x]]*(-I + Tan[c + d*x]) + 2*(-1)^(1/4)*((-2*I)*A + B)*ArcTanh[(-1)^(3/4)*
Sqrt[Tan[c + d*x]]]*Sqrt[Tan[c + d*x]]*(-I + Tan[c + d*x])))/(a*d*(-I + Ta
n[c + d*x]))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.09, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4064, 3042, 4078, 27, 3042, 4011, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A\cot(c+dx)+B)}{a\cot(c+dx)+ia}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia}dx$$

$\Big\downarrow$ 4078

$$\frac{\int -\frac{1}{2}\sqrt{\cot(c+dx)}(3a(iA-B)-a(5A+iB)\cot(c+dx))dx}{2a^2}+\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}$$

$\Big\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\int \sqrt{\cot(c+dx)}(3a(iA-B)-a(5A+iB)\cot(c+dx))dx}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(3a(iA-B)+a(5A+iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx}{4a^2}$$

$\Big\downarrow$ 4011

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\int \frac{a(5A+iB)+3a(iA-B)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\int \frac{a(5A+iB)-3a(iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$\Big\downarrow$ 4017

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2\int -\frac{a(5A+iB+3(iA-B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$\Big\downarrow$ 25

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2\int \frac{a(5A+iB+3(iA-B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$\Big\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a\int\frac{5A+iB+3(iA-B)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$\Big\downarrow$ 1482

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a\left(\frac{1}{2}((5-3i)A+(3+i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((5+3i)A-(3-i)B)\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}}{4a^2}$$

$\Big\downarrow$ 1476

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a\left(\frac{1}{2}((5-3i)A+(3+i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\right.\right.}{d}}{4a^2}$$

$\Big\downarrow$ 1082

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a\left(\frac{1}{2}((5-3i)A+(3+i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right.}{\sqrt{2}}\right.\right.}{d}}{4a^2}$$

$\Big\downarrow$ 217

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a\left(\frac{1}{2}((5-3i)A+(3+i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan(1}{}\right.\right.}{d}}{4a^2}$$

$\Big\downarrow$ 1479

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a\left(\frac{1}{2}((5-3i)A+(3+i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+}{2\sqrt{2}}\right.\right.}{d}}{4a^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a\left(\frac{1}{2}((5-3i)A+(3+i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)}{d}}{4a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a\left(\frac{1}{2}((5-3i)A+(3+i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+d}\right.\right.}{d}}{4a^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{2d(a\cot(c+dx)+ia)}-\frac{2a(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a\left(\frac{1}{2}((5+3i)A-(3-i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((5-3i)A+(3+i)B)\left(\frac{\log\left(\cot(c\right.}{}\right.\right.}{d}}{4a^2}$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
((A + I*B)*Cot[c + d*x]^(3/2))/(2*d*(I*a + a*Cot[c + d*x])) - ((2*a*(5*A +
 I*B)*Sqrt[Cot[c + d*x]])/d - (2*a*((((5 + 3*I)*A - (3 - I)*B)*(-(ArcTan[1
 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c +
d*x]]]/Sqrt[2]))/2 + (((5 - 3*I)*A + (3 + I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d)/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.67

method	result	s
derivativedivides	$\frac{-2A\sqrt{\cot(dx+c)}+\frac{i\left(\frac{i(iA-B)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4(iB+2A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{2}+\frac{4\left(-\frac{iA}{4}-\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}}{ad}$	1
default	$\frac{-2A\sqrt{\cot(dx+c)}+\frac{i\left(\frac{i(iA-B)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4(iB+2A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{2}+\frac{4\left(-\frac{iA}{4}-\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}}{ad}$	1

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/a/d*(-2*A*cot(d*x+c)^(1/2)+1/2*I*(I*(I*A-B)*cot(d*x+c)^(1/2)/(I+cot(d*x+
c))+4*(2*A+I*B)/(2^(1/2)+I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2
^(1/2))))+4*(-1/4*I*A-1/4*B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)
/(2^(1/2)-I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 623 vs. 2(163) = 326.

Time = 0.10 (sec) , antiderivative size = 623, normalized size of antiderivative = 2.91

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/8*(a*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(
-2*((I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(
e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) + (A - I
*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a*d*sqrt((I*A^2
 + 2*A*B - I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*((-I*a*d*e^(2*I*d*
x + 2*I*c) + I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
- 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I
*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a*d*sqrt((-4*I*A^2 + 4*A*B + I*B^
2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqr
t((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-4*I*A^2 +
4*A*B + I*B^2)/(a^2*d^2)) + 2*I*A - B)*e^(-2*I*d*x - 2*I*c)/(a*d)) + 2*a*d
*sqrt((-4*I*A^2 + 4*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-((a*d
*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) - 1))*sqrt((-4*I*A^2 + 4*A*B + I*B^2)/(a^2*d^2)) - 2*I*A + B)*e^(-
2*I*d*x - 2*I*c)/(a*d)) + 2*((9*A + I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sq
rt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2
*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{i\left(\int \frac{A\cot^{\frac{3}{2}}(c+dx)}{\tan(c+dx)-i}\,dx + \int \frac{B\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)}{\tan(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*(Integral(A*cot(c + d*x)**(3/2)/(tan(c + d*x) - I), x) + Integral(B*tan
(c + d*x)*cot(c + d*x)**(3/2)/(tan(c + d*x) - I), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.50

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{2\sqrt{2}(-(2i+2)\ A-(i-1)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)-\sqrt{2}(-(i-1)\ A-(i+1)\ B)\operatorname{arc}}{4\,ad}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
1/4*(2*sqrt(2)*(-(2*I + 2)*A - (I - 1)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sqr
t(tan(d*x + c))) - sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-(1/2*I - 1/2)*
sqrt(2)*sqrt(tan(d*x + c))) - 2*(5*A*tan(d*x + c) + I*B*tan(d*x + c) - 4*I
*A)/(tan(d*x + c)^(3/2) - I*sqrt(tan(d*x + c))))/(a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx=\int \frac{\cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{2\sqrt{\cot\left(dx+c\right)}\,bi+\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)\tan(dx+c)-\cot(dx+c)i}dx\right)bd-\left(\int \frac{\sqrt{\cot(dx+c)}\,\cot(dx+c)}{\tan(dx+c)-i}dx\right)adi+\left(\int \frac{\sqrt{\cot(dx+c)}}{\tan(dx\cdots}\right.}{ad}$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(2*sqrt(cot(c + d*x))*b*i + int(sqrt(cot(c + d*x))/(cot(c + d*x)*tan(c + d
*x) - cot(c + d*x)*i),x)*b*d - int((sqrt(cot(c + d*x))*cot(c + d*x))/(tan(
c + d*x) - i),x)*a*d*i + int((sqrt(cot(c + d*x))*cot(c + d*x))/(tan(c + d*
x) - i),x)*b*d + int((sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)*tan(c
 + d*x) - cot(c + d*x)*i),x)*b*d*i)/(a*d)
```

3.525 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 185

$$\begin{aligned}\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx &= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)((2+i)A+B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}ad}\\ &\quad-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)((2+i)A+B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}ad}\\ &\quad+\frac{((3+i)A-(1+i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{4\sqrt{2}ad}+\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))}\end{aligned}$$

output
```
(-1/8+1/8*I)*((2+I)*A+B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a/d+(
-1/8+1/8*I)*((2+I)*A+B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a/d+1/8
*((3+I)*A-(1+I)*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2
)/a/d+1/2*(A+I*B)*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left((-iA+B)\sqrt{\tan(c+dx)}-\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\right.}{2ad(-i+\tan(c+dx}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((-I)*A + B)*Sqrt[Tan[c + d*x]] -
(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(-I + Tan[c + d
*x]) - 2*(-1)^(1/4)*A*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(-I + Tan[c +
 d*x])))/(2*a*d*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4064, 3042, 4078, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A\cot(c+dx)+B)}{a\cot(c+dx)+ia}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{\int -\frac{a(iA-B)-a(3A-iB)\cot(c+dx)}{2\sqrt{\cot(c+dx)}}dx}{2a^2}+\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\int \frac{a(iA-B)-a(3A-iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\int \frac{a(iA-B)+a(3A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\int -\frac{a(iA-B-(3A-iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{2a^2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(iA-B-(3A-iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{2a^2d}+\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{iA-B-(3A-iB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{2ad}+\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{1}{2}((3+i)A-(1+i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)(B+(2+i)A)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{2ad}+\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\frac{1}{2}((3+i)A-(1+i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)(B+(2+i)A)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{}\right.}{2ad}}{\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}}$$

$\Big\downarrow$ 1082

$$\frac{\frac{\frac{1}{2}((3+i)A-(1+i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)(B+(2+i)A)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right.}{2ad}}{\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}}$$

$\Big\downarrow$ 217

$$\frac{\frac{\frac{1}{2}((3+i)A-(1+i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\left(\frac{1}{2}-\frac{i}{2}\right)(B+(2+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan(}{}\right.}{2ad}}{\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}}$$

$\Big\downarrow$ 1479

$$\frac{\frac{\frac{1}{2}((3+i)A-(1+i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\left(\frac{1}{2}-\right.}{2ad}}{\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{1}{2}((3+i)A-(1+i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)(B}{2ad}}{\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{2}((3+i)A-(1+i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)-(}{2ad}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{1}{2}((3+i)A-(1+i)B)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}\right)-\left(\frac{1}{2}-\frac{i}{2}\right)(B+(2+i)}{2ad}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x]),x]
```

output
```
((A + I*B)*Sqrt[Cot[c + d*x]])/(2*d*(I*a + a*Cot[c + d*x])) + ((-1/2 + I/2
)*((2 + I)*A + B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + Arc
Tan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + (((3 + I)*A - (1 + I)*B)*(-
1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + S
qrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2)/(2*a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{\frac{4\left(-\frac{A}{4}+\frac{iB}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{i\left(-\frac{i(iB+A)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4iA\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{2}}{ad}$	125
default	$\frac{\frac{4\left(-\frac{A}{4}+\frac{iB}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{i\left(-\frac{i(iB+A)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4iA\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{2}}{ad}$	125

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/a/d*(4*(-1/4*A+1/4*I*B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2
^(1/2)-I*2^(1/2)))+1/2*I*(-I*(A+I*B)*cot(d*x+c)^(1/2)/(I+cot(d*x+c))+4*I*A
/(2^(1/2)+I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(140) = 280.

Time = 0.10 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.08

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/8*(a*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(
-2*((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) + (A - I*B)
*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a*d*sqrt((-I*A^2 -
 2*A*B + I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(2*((a*d*e^(2*I*d*x + 2*
I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sq
rt((-I*A^2 - 2*A*B + I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^
(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a*d*sqrt(I*A^2/(a^2*d^2))*e^(2*I*d*x + 2
*I*c)*log(-((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(I*A^2/(a^2*d^2)) + A)*e^(-2*I*d*x - 2*I
*c)/(a*d)) + 2*a*d*sqrt(I*A^2/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(((a*d*e^(
2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) - 1))*sqrt(I*A^2/(a^2*d^2)) - A)*e^(-2*I*d*x - 2*I*c)/(a*d)) + 2*((-I*
A + B)*e^(2*I*d*x + 2*I*c) + I*A - B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = -\frac{i\left(\int \frac{A\sqrt{\cot(c+dx)}}{\tan(c+dx)-i}\,dx + \int \frac{B\tan(c+dx)\sqrt{\cot(c+dx)}}{\tan(c+dx)-i}\,dx\right)}{a}$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*(Integral(A*sqrt(cot(c + d*x))/(tan(c + d*x) - I), x) + Integral(B*tan(
c + d*x)*sqrt(cot(c + d*x))/(tan(c + d*x) - I), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{(2i-2)\,\sqrt{2}A\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+\sqrt{2}(-(i+1)\,A+(i-1)\,B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right.\right.}{4\,ad}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/4*((2*I - 2)*sqrt(2)*A*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(tan(d*x + c)))
 + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan
(d*x + c))) + 2*(I*A*sqrt(tan(d*x + c)) - B*sqrt(tan(d*x + c)))/(tan(d*x +
 c) - I))/(a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \int \frac{\sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i),x)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+ia\tan(c+dx)}\,dx = \frac{\left(\int \frac{\sqrt{\cot(dx+c)}}{\tan(dx+c)i+1}dx\right)a+\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\tan(dx+c)i+1}dx\right)b}{a}$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c)),x)
```

output
```
(int(sqrt(cot(c + d*x))/(tan(c + d*x)*i + 1),x)*a + int((sqrt(cot(c + d*x)
)*tan(c + d*x))/(tan(c + d*x)*i + 1),x)*b)/a
```

3.526 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 187

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))}\,dx &= \frac{\left(\frac{1}{4}-\frac{i}{4}\right)(A+(2-i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}ad}\\ &\quad-\frac{\left(\frac{1}{4}-\frac{i}{4}\right)(A+(2-i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}ad}\\ &\quad-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(A-(2+i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}ad}+\frac{(iA-B)\sqrt{\cot(c+dx)}}{2d(ia+a\cot(c+dx))}\end{aligned}$$

output
```
(-1/8+1/8*I)*(A+(2-I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a/d+(
-1/8+1/8*I)*(A+(2-I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a/d-(1/
8+1/8*I)*(A-(2+I)*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1
/2)/a/d+1/2*(I*A-B)*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.73

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left((A+iB)\sqrt{\tan(c+dx)}+\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)(\right.}{2ad(-i+\tan(c+dx)}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x]))
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((A + I*B)*Sqrt[Tan[c + d*x]] + (-1
)^(1/4)*(I*A + B)*ArcTan[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(-I + Tan[c + d*x]
) - 2*(-1)^(1/4)*B*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(-I + Tan[c + d*
x])))/(2*a*d*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 4064, 3042, 4079, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{A\cot(c+dx)+B}{\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(A-3iB)-a(iA-B)\cot(c+dx)}{2\sqrt{\cot(c+dx)}}dx}{2a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A-3iB)-a(iA-B)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{4a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A-3iB)+a(iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{4a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\int -\frac{a(A-3iB-(iA-B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{2a^2d}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\int \frac{a(A-3iB-(iA-B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{2a^2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\int \frac{A-3iB-(iA-B)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{2ad}$$

$$\Big\downarrow \text{1482}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)(A+(2-i)B)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{2ad}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)(A+(2-i)B)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\right.}{2ad}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)(A+(2-i)B)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right.}{2ad}$$

$$\Big\downarrow \text{217}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}-\frac{i}{2}\right)(A+(2-i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan}{}\right.}{2ad}$$

$$\Big\downarrow \text{1479}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}-\right.}{2ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)(\ldots}{2ad}$$

$$\Big\downarrow \text{27}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+(\ldots}{2ad}$$

$\downarrow$ 1103

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{2d(a\cot(c+dx)+ia)}-\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(A+(2-i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)(A-(2+i)B)\left(\frac{\log\left(\cot(c-\right.}{}\right.}{2ad}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

output
```
((I*A - B)*Sqrt[Cot[c + d*x]])/(2*d*(I*a + a*Cot[c + d*x])) - ((1/2 - I/2)
*(A + (2 - I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcT
an[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + (1/2 + I/2)*(A - (2 + I)*B)*
(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 +
 Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/(2*a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{\frac{4\left(\frac{iA}{4}+\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{i\left(-\frac{i(iA-B)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4iB\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{2}}{ad}$	127
default	$\frac{\frac{4\left(\frac{iA}{4}+\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{i\left(-\frac{i(iA-B)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4iB\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{2}}{ad}$	127

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/a/d*(4*(1/4*I*A+1/4*B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^
(1/2)-I*2^(1/2)))+1/2*I*(-I*(I*A-B)*cot(d*x+c)^(1/2)/(I+cot(d*x+c))+4*I*B/
(2^(1/2)+I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(140) = 280$.

Time = 0.10 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.05

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + ia\tan(c + dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/8*(a*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-
2*((I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) + (A - I*
B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a*d*sqrt((I*A^2
+ 2*A*B - I*B^2)/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(-2*((-I*a*d*e^(2*I*d*x
 + 2*I*c) + I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*
c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a*d*sqrt(I*B^2/(a^2*d^2))*e^(2*I*d
*x + 2*I*c)*log(-((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(I*B^2/(a^2*d^2)) + B)*e^(-2*I*d*x
 - 2*I*c)/(a*d)) + 2*a*d*sqrt(I*B^2/(a^2*d^2))*e^(2*I*d*x + 2*I*c)*log(((a
*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) - 1))*sqrt(I*B^2/(a^2*d^2)) - B)*e^(-2*I*d*x - 2*I*c)/(a*d)) + 2
*((A + I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + ia\tan(c + dx))}\,dx = -\frac{i\left(\int \frac{A}{\tan(c+dx)\sqrt{\cot(c+dx)} - i\sqrt{\cot(c+dx)}}\,dx + \int \frac{B\tan(c+dx)}{\tan(c+dx)\sqrt{\cot(c+dx)} - i\sqrt{\cot(c+dx)}}\,dx\right)}{a}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*(Integral(A/(tan(c + d*x)*sqrt(cot(c + d*x)) - I*sqrt(cot(c + d*x))), x
) + Integral(B*tan(c + d*x)/(tan(c + d*x)*sqrt(cot(c + d*x)) - I*sqrt(cot(
c + d*x))), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + ia\tan(c + dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.47

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + ia\tan(c + dx))}\,dx =$$
$$-\frac{(2i - 2)\sqrt{2}B\arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx + c)}\right) + \sqrt{2}((i - 1)A + (i + 1)B)\arctan\left(-\left(\frac{1}{2}i - \frac{1}{2}\right)\right.}{4\,ad}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
-1/4*((2*I - 2)*sqrt(2)*B*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(tan(d*x + c)))
 + sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(tan(
d*x + c))) - 2*(A*sqrt(tan(d*x + c)) + I*B*sqrt(tan(d*x + c)))/(tan(d*x +
c) - I))/(a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))}\,dx=\int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot\left(c+d\,x\right)}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)\tan(dx+c)i+\cot(dx+c)}dx\right)a+\left(\int \frac{\sqrt{\cot(dx+c)}\tan(dx+c)}{\cot(dx+c)\tan(dx+c)i+\cot(dx+c)}dx\right)b}{a}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(int(sqrt(cot(c + d*x))/(cot(c + d*x)*tan(c + d*x)*i + cot(c + d*x)),x)*a
+ int((sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)*tan(c + d*x)*i + cot
(c + d*x)),x)*b)/a
```

3.527 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 222

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx &= \frac{((1-3i)A+(3+5i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{4\sqrt{2}ad} \\ &+ \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((1+2i)A-(4+i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}ad} \\ &+ \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((2+i)A+(1+4i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}ad} \\ &- \frac{A+5iB}{2ad\sqrt{\cot(c+dx)}} + \frac{iA-B}{2d\sqrt{\cot(c+dx)}(ia+a\cot(c+dx))}\end{aligned}$$

output
```
-1/8*((1-3*I)*A+(3+5*I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a/d
+(1/8+1/8*I)*((1+2*I)*A-(4+I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2
)/a/d+(1/8+1/8*I)*((2+I)*A+(1+4*I)*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+
cot(d*x+c)))*2^(1/2)/a/d-1/2*(A+5*I*B)/a/d/cot(d*x+c)^(1/2)+1/2*(I*A-B)/d/
cot(d*x+c)^(1/2)/(I*a+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.69

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)(-i+\tan(c+dx))-2\sqrt[4]{-}\right.}{2ad(-i}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x]))
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(
3/4)*Sqrt[Tan[c + d*x]]]*(-I + Tan[c + d*x]) - 2*(-1)^(1/4)*(A + (2*I)*B)*
ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(-I + Tan[c + d*x]) + I*Sqrt[Tan[c
+ d*x]]*(A + (5*I)*B - 4*B*Tan[c + d*x])))/(2*a*d*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.09, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4064, 3042, 4079, 27, 3042, 4012, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}(a+ia\tan(c+dx))}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{A\cot(c+dx)+B}{\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int -\frac{a(A+5iB)+3a(iA-B)\cot(c+dx)}{2\cot^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\int \frac{a(A+5iB)+3a(iA-B)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\int \frac{a(A+5iB)-3a(iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{4a^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\int \frac{3a(iA-B)-a(A+5iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\int \frac{3a(iA-B)+a(A+5iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}}{4a^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2\int -\frac{a(3(iA-B)-(A+5iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}}{4a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}-\frac{2\int \frac{a(3(iA-B)-(A+5iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a\int\frac{3(iA-B)-(A+5iB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}}{4a^2}$$

$\downarrow$ 1482

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A-(4+i)B)\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}}{4a^2}$$

$\downarrow$ 1476

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A-(4+i)B)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\right.\right.}{d}}{4a^2}$$

$\downarrow$ 1082

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A-(4+i)B)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx}\right.}{\sqrt{2}}\right.\right.}{d}}{4a^2}$$

$\downarrow$ 217

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A-(4+i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.}{d}}{4a^2}$$

$\downarrow$ 1479

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)\right.}{d}}{4a^2}$$

$\downarrow$ 25

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}\right.}{d}}{4a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\right.}{d}}{4a^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-B+iA}{2d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}-\frac{\frac{2a(A+5iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A-(4+i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((2+i)A+(1+4i)B)\left(\frac{\log\left(\cot(c\right.}{}\right.\right.}{d}}{4a^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
(I*A - B)/(2*d*Sqrt[Cot[c + d*x]]*(I*a + a*Cot[c + d*x])) - ((2*a*(A + (5*
I)*B))/(d*Sqrt[Cot[c + d*x]]) - (2*a*((1/2 + I/2)*((1 + 2*I)*A - (4 + I)*B
)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*
Sqrt[Cot[c + d*x]]]/Sqrt[2]) + (1/2 + I/2)*((2 + I)*A + (1 + 4*I)*B)*(-1/2
*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]))))/d)/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$\frac{-\frac{2iB}{\sqrt{\cot(dx+c)}}+\frac{4\left(\frac{A}{4}-\frac{iB}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{i\left(\frac{i(iB+A)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4(iA-2B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{2}}{ad}$	143
default	$\frac{-\frac{2iB}{\sqrt{\cot(dx+c)}}+\frac{4\left(\frac{A}{4}-\frac{iB}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{i\left(\frac{i(iB+A)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4(iA-2B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{2}}{ad}$	143

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/a/d*(-2*I*B/cot(d*x+c)^(1/2)+4*(1/4*A-1/4*I*B)/(2^(1/2)-I*2^(1/2))*arcta
n(2*cot(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2)))+1/2*I*(I*(A+I*B)*cot(d*x+c)^(1/2
)/(I+cot(d*x+c))+4*(I*A-2*B)/(2^(1/2)+I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)
/(2^(1/2)+I*2^(1/2)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 700 vs. 2(161) = 322.

Time = 0.16 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.15

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/8*((a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((-I*A^2 - 2
*A*B + I*B^2)/(a^2*d^2))*log(-2*((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e
^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B +
I*B^2)/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I
*A + B)) - (a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((-I*A^
2 - 2*A*B + I*B^2)/(a^2*d^2))*log(2*((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(
(I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*
B + I*B^2)/(a^2*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c
)/(I*A + B)) + 2*(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt(
(I*A^2 - 4*A*B - 4*I*B^2)/(a^2*d^2))*log(-((a*d*e^(2*I*d*x + 2*I*c) - a*d)
*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 -
 4*A*B - 4*I*B^2)/(a^2*d^2)) + A + 2*I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) - 2*
(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((I*A^2 - 4*A*B -
4*I*B^2)/(a^2*d^2))*log(((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 - 4*A*B - 4*I*B^2)/
(a^2*d^2)) - A - 2*I*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) - 2*((I*A - 9*B)*e^(4*
I*d*x + 4*I*c) + 8*B*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt((I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2
*I*d*x + 2*I*c))
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$= -\frac{i\left(\int \frac{A}{\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)-i\cot^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B\tan(c+dx)}{\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)-i\cot^{\frac{3}{2}}(c+dx)}\,dx\right)}{a}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*(Integral(A/(tan(c + d*x)*cot(c + d*x)**(3/2) - I*cot(c + d*x)**(3/2)),
 x) + Integral(B*tan(c + d*x)/(tan(c + d*x)*cot(c + d*x)**(3/2) - I*cot(c
+ d*x)**(3/2)), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + ia\tan(c + dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.47

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + ia\tan(c + dx))}\,dx$$
$$= \frac{2\sqrt{2}(-(i-1)\ A + (2i+2)\ B)\arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx + c\right)}\right) + \sqrt{2}(-(i+1)\ A + (i-1)\ B)\operatorname{arc}}{4\,ad}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
1/4*(2*sqrt(2)*(-(I - 1)*A + (2*I + 2)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sqr
t(tan(d*x + c))) + sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-(1/2*I - 1/2)*
sqrt(2)*sqrt(tan(d*x + c))) - 8*I*B*sqrt(tan(d*x + c)) - 2*(-I*A*sqrt(tan(
d*x + c)) + B*sqrt(tan(d*x + c)))/(tan(d*x + c) - I))/(a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx=\int \frac{A+B\tan(c+d\,x)}{{\cot\left(c+d\,x\right)}^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \frac{\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)^2\tan(dx+c)i+\cot(dx+c)^2}dx\right)a+\left(\int \frac{\sqrt{\cot(dx+c)}\tan(dx+c)}{\cot(dx+c)^2\tan(dx+c)i+\cot(dx+c)^2}dx\right)b}{a}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(int(sqrt(cot(c + d*x))/(cot(c + d*x)**2*tan(c + d*x)*i + cot(c + d*x)**2)
,x)*a + int((sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)**2*tan(c + d*x
)*i + cot(c + d*x)**2),x)*b)/a
```

3.528 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 257

$$
\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx \\
&= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((4+i)A+(1+6i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}ad} \\
&\quad - \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((4+i)A+(1+6i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}ad} \\
&\quad + \frac{\left(\frac{1}{4}+\frac{i}{4}\right)((1+4i)A-(6+i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}ad} - \frac{3A+7iB}{6ad\cot^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{5(iA-B)}{2ad\sqrt{\cot(c+dx)}} + \frac{iA-B}{2d\cot^{\frac{3}{2}}(c+dx)(ia+a\cot(c+dx))}
\end{aligned}
$$

output

```
(-1/8-1/8*I)*((4+I)*A+(1+6*I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/
2)/a/d-(1/8+1/8*I)*((4+I)*A+(1+6*I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*
2^(1/2)/a/d+(1/8+1/8*I)*((1+4*I)*A-(6+I)*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/
2)/(1+cot(d*x+c)))*2^(1/2)/a/d-1/6*(3*A+7*I*B)/a/d/cot(d*x+c)^(3/2)-5/2*(I
*A-B)/a/d/cot(d*x+c)^(1/2)+1/2*(I*A-B)/d/cot(d*x+c)^(3/2)/(I*a+a*cot(d*x+c
))
```

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.69

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-3(-1)^{3/4}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)(-i+\tan(c+dx))+\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x]))
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-3*(-1)^(3/4)*(A - I*B)*ArcTan[(-1
)^(3/4)*Sqrt[Tan[c + d*x]]]*(-I + Tan[c + d*x]) + 6*(-1)^(1/4)*((-2*I)*A +
 3*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(-I + Tan[c + d*x]) + Sqrt[Ta
n[c + d*x]]*(-15*(A + I*B) + 4*((-3*I)*A + 2*B)*Tan[c + d*x] - (4*I)*B*Tan
[c + d*x]^2)))/(6*a*d*(-I + Tan[c + d*x]))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.04, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3042, 4064, 3042, 4079, 27, 3042, 4012, 3042, 4012, 25, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{5/2}(a+ia\tan(c+dx))}dx$$
$$\Big\downarrow \ 4064$$

$$\int \frac{A\cot(c+dx)+B}{\cot^{\frac{5}{2}}(c+dx)(a\cot(c+dx)+ia)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int -\frac{a(3A+7iB)+5a(iA-B)\cot(c+dx)}{2\cot^{\frac{5}{2}}(c+dx)}dx}{2a^2}+\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\int \frac{a(3A+7iB)+5a(iA-B)\cot(c+dx)}{\cot^{\frac{5}{2}}(c+dx)}dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\int \frac{a(3A+7iB)-5a(iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{4a^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\int \frac{5a(iA-B)-a(3A+7iB)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2a(3A+7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\int \frac{5a(iA-B)+a(3A+7iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2a(3A+7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}}{4a^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\int -\frac{a(3A+7iB)+5a(iA-B)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2a(3A+7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{10a(-B+iA)}{d\sqrt{\cot(c+dx)}}}{4a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{-\int\frac{a(3A+7iB)+5a(iA-B)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2a(3A+7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{10a(-B+iA)}{d\sqrt{\cot(c+dx)}}}{4a^2}}{}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{-\int\frac{a(3A+7iB)-5a(iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a(3A+7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{10a(-B+iA)}{d\sqrt{\cot(c+dx)}}}{4a^2}$$

$$\Big\downarrow \text{ 4017}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{-\frac{2\int-\frac{a(3A+7iB+5(iA-B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a(3A+7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{10a(-B+iA)}{d\sqrt{\cot(c+dx)}}}{4a^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\frac{2\int\frac{a(3A+7iB+5(iA-B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a(3A+7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{10a(-B+iA)}{d\sqrt{\cot(c+dx)}}}{4a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\frac{2a\int\frac{3A+7iB+5(iA-B)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a(3A+7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{10a(-B+iA)}{d\sqrt{\cot(c+dx)}}}{4a^2}$$

$$\Big\downarrow \text{ 1482}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\frac{2a\left(\frac{1}{2}((3-5i)A+(5+7i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B)\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}+\frac{2a(3A+7iB)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{10a}{d\sqrt{}}}{4a^2}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\frac{2a\left(\frac{1}{2}((3-5i)A+(5+7i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int\frac{}{\cot(c-}\right.}{d}}{4a^2}$$

$\downarrow$ 1082

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\frac{2a\left(\frac{1}{2}((3-5i)A+(5+7i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(\right.}{\sqrt{}}\right.}{d}}{4a^2}$$

$\downarrow$ 217

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\frac{2a\left(\frac{1}{2}((3-5i)A+(5+7i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}}{4a^2}$$

$\downarrow$ 1479

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\frac{2a\left(\frac{1}{2}((3-5i)A+(5+7i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+\right.}{d}}{4a^2}$$

$\downarrow$ 25

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{\frac{2a\left(\frac{1}{2}((3-5i)A+(5+7i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B\right.}{d}}{4a^2}$$

$\downarrow$ 27

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{2a\left(\frac{1}{2}((3-5i)A+(5+7i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6\right.}{d}}{4a^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{-B+iA}{2d\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)}-\frac{2a\left(\left(\frac{1}{2}+\frac{i}{2}\right)((4+i)A+(1+6i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((3-5i)A+(5+7i)B)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right.}{2\sqrt{2}}\right.\right.}{d}}{4a^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

output
```
(I*A - B)/(2*d*Cot[c + d*x]^(3/2)*(I*a + a*Cot[c + d*x])) - ((2*a*(3*A + (
7*I)*B))/(3*d*Cot[c + d*x]^(3/2)) + (10*a*(I*A - B))/(d*Sqrt[Cot[c + d*x]]
) + (2*a*((1/2 + I/2)*((4 + I)*A + (1 + 6*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])
 + (((3 - 5*I)*A + (5 + 7*I)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] +
 Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]
]/(2*Sqrt[2])))/2))/d)/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{\frac{4\left(-\frac{iA}{4}-\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}-\frac{i\left(-\frac{i(iA-B)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4(3iB+2A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{2}+\frac{-2iA+2B}{\sqrt{\cot(dx+c)}}-\frac{2i\ldots}{3\cot(d\ldots}}{ad}$
default	$\frac{\frac{4\left(-\frac{iA}{4}-\frac{B}{4}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}-\frac{i\left(-\frac{i(iA-B)\sqrt{\cot(dx+c)}}{i+\cot(dx+c)}+\frac{4(3iB+2A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{2}+\frac{-2iA+2B}{\sqrt{\cot(dx+c)}}-\frac{2i\ldots}{3\cot(d\ldots}}{ad}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/a/d*(4*(-1/4*I*A-1/4*B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2
^(1/2)-I*2^(1/2)))-1/2*I*(-I*(I*A-B)*cot(d*x+c)^(1/2)/(I+cot(d*x+c))+4*(2*
A+3*I*B)/(2^(1/2)+I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))
))+2*(B-I*A)/cot(d*x+c)^(1/2)-2/3*I*B/cot(d*x+c)^(3/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(184) = 368$.

Time = 0.10 (sec) , antiderivative size = 794, normalized size of antiderivative = 3.09

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + ia\tan(c + dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/24*(3*(a*d*e^(6*I*d*x + 6*I*c) + 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I
*d*x + 2*I*c))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2))*log(-2*((I*a*d*e^(2
*I*d*x + 2*I*c) - I*a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*
I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x
+ 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*(a*d*e^(6*I*d*x + 6*I*c) + 2
*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((I*A^2 + 2*A*B -
I*B^2)/(a^2*d^2))*log(-2*((-I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt((I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B
^2)/(a^2*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A
+ B)) + 6*(a*d*e^(6*I*d*x + 6*I*c) + 2*a*d*e^(4*I*d*x + 4*I*c) + a*d*e^(2*
I*d*x + 2*I*c))*sqrt((-4*I*A^2 + 12*A*B + 9*I*B^2)/(a^2*d^2))*log(-((a*d*e
^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) - 1))*sqrt((-4*I*A^2 + 12*A*B + 9*I*B^2)/(a^2*d^2)) + 2*I*A - 3*B)*e
^(-2*I*d*x - 2*I*c)/(a*d)) - 6*(a*d*e^(6*I*d*x + 6*I*c) + 2*a*d*e^(4*I*d*x
 + 4*I*c) + a*d*e^(2*I*d*x + 2*I*c))*sqrt((-4*I*A^2 + 12*A*B + 9*I*B^2)/(a
^2*d^2))*log(((a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-4*I*A^2 + 12*A*B + 9*I*B^2)/(a^2*d^
2)) - 2*I*A + 3*B)*e^(-2*I*d*x - 2*I*c)/(a*d)) + 2*((27*A + 19*I*B)*e^(6*I
*d*x + 6*I*c) + (3*A + 19*I*B)*e^(4*I*d*x + 4*I*c) - (27*A + 35*I*B)*e^(2*
I*d*x + 2*I*c) - 3*A - 3*I*B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*...
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = -\frac{i\left(\int \frac{A}{\tan(c+dx)\cot^{\frac{5}{2}}(c+dx)-i\cot^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{B\tan(c+dx)}{\tan(c+dx)\cot^{\frac{5}{2}}(c+dx)-i\cot^{\frac{5}{2}}(c+dx)}\,dx\right)}{a}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
-I*(Integral(A/(tan(c + d*x)*cot(c + d*x)**(5/2) - I*cot(c + d*x)**(5/2)),
 x) + Integral(B*tan(c + d*x)/(tan(c + d*x)*cot(c + d*x)**(5/2) - I*cot(c
+ d*x)**(5/2)), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.50

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$

$$= \frac{6\sqrt{2}((2i+2)\ A+(3i-3)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}((i-1)\ A+(i+1)\ B)\,\mathrm{arct}}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x, algorith
m="giac")
```

output
```
1/12*(6*sqrt(2)*((2*I + 2)*A + (3*I - 3)*B)*arctan((1/2*I + 1/2)*sqrt(2)*s
qrt(tan(d*x + c))) + 3*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-(1/2*I - 1/
2)*sqrt(2)*sqrt(tan(d*x + c))) - 8*I*B*tan(d*x + c)^(3/2) - 24*I*A*sqrt(ta
n(d*x + c)) + 24*B*sqrt(tan(d*x + c)) - 6*(A*sqrt(tan(d*x + c)) + I*B*sqrt
(tan(d*x + c)))/(tan(d*x + c) - I))/(a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx = \int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))}\,dx$$
$$=\frac{\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)^3\tan(dx+c)i+\cot(dx+c)^3}dx\right)a+\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)^3\tan(dx+c)i+\cot(dx+c)^3}dx\right)b}{a}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c)),x)
```

output
```
(int(sqrt(cot(c + d*x))/(cot(c + d*x)**3*tan(c + d*x)*i + cot(c + d*x)**3)
,x)*a + int((sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)**3*tan(c + d*x
)*i + cot(c + d*x)**3),x)*b)/a
```

3.529 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2} dx$

Optimal result

Integrand size = 36, antiderivative size = 263

$$\begin{aligned}
\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx
&= -\frac{\left(\frac{1}{16}-\frac{i}{16}\right)((2+23i)A-(7+2i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^2d} \\
&\quad+\frac{((25+21i)A-(9-5i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} \\
&\quad+\frac{\left(\frac{1}{16}-\frac{i}{16}\right)((23+2i)A+(2+7i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}a^2d} \\
&\quad-\frac{5(5A+iB)\sqrt{\cot(c+dx)}}{8a^2d}+\frac{(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{8a^2d(i+\cot(c+dx))}+\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2}
\end{aligned}$$

output
```
(1/32-1/32*I)*((2+23*I)*A-(7+2*I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2
^(1/2)/a^2/d+1/32*((25+21*I)*A+(-9+5*I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/
2))*2^(1/2)/a^2/d+(1/32-1/32*I)*((23+2*I)*A+(2+7*I)*B)*arctanh(2^(1/2)*cot
(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/a^2/d-5/8*(5*A+I*B)*cot(d*x+c)^(1/2)
/a^2/d+1/8*(7*A+3*I*B)*cot(d*x+c)^(3/2)/a^2/d/(I+cot(d*x+c))+1/4*(A+I*B)*c
ot(d*x+c)^(5/2)/d/(I*a+a*cot(d*x+c))^2
```

Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.84

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\left(-16A+(-1)^{3/4}(23A+7iB)\text{arctanh}\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^2(c+dx)(\cos(2(c+\cdots}{\cdots}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^2,x]
```

output
```
-1/8*(Sqrt[Cot[c + d*x]]*(-16*A + (-1)^(3/4)*(23*A + (7*I)*B)*ArcTanh[(-1)
^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c +
 d*x)])*Sqrt[Tan[c + d*x]] + 2*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*Sqrt
[Tan[c + d*x]]]*Sec[c + d*x]^2*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*
Sqrt[Tan[c + d*x]] - (43*I)*A*Tan[c + d*x] + 7*B*Tan[c + d*x] + 25*A*Tan[c
 + d*x]^2 + (5*I)*B*Tan[c + d*x]^2))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.09, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4064, 3042, 4078, 27, 3042, 4078, 3042, 4011, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 4064}$$

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A\cot(c+dx)+B)}{(a\cot(c+dx)+ia)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^2}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{\int -\frac{\cot^{\frac{3}{2}}(c+dx)(5a(iA-B)-a(9A+iB)\cot(c+dx))}{2(\cot(c+dx)a+ia)}dx}{4a^2}+\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)(5a(iA-B)-a(9A+iB)\cot(c+dx))}{\cot(c+dx)a+ia}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(5a(iA-B)+a(9A+iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{ia-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{8a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\int \sqrt{\cot(c+dx)}\left(3a^2(7iA-3B)-5a^2(5A+iB)\cot(c+dx)\right)dx}{2a^2}-\frac{(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{d(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(3(7iA-3B)a^2+5(5A+iB)\tan\left(c+dx+\frac{\pi}{2}\right)a^2\right)dx}{2a^2}-\frac{(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{d(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{4011}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\int \frac{5(5A+iB)a^2+3(7iA-3B)\cot(c+dx)a^2}{\sqrt{\cot(c+dx)}}dx+\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}}{2a^2}-\frac{(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{d(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\int \frac{5a^2(5A+iB)-3a^2(7iA-3B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}}{2a^2}-\frac{(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{d(\cot(c+dx)+i)}}{8a^2}$$

$$\downarrow 4017$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\frac{2\int -\frac{a^2(5(5A+iB)+3(7iA-3B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}}{2a^2}-\frac{(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{d(\cot(c+dx)+i)}}{8a^2}$$

$$\downarrow 25$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2\int \frac{a^2(5(5A+iB)+3(7iA-3B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}}{2a^2}-\frac{(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{d(\cot(c+dx)+i)}}{8a^2}$$

$$\downarrow 27$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2\int \frac{5(5A+iB)+3(7iA-3B)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}}{2a^2}-\frac{(7A+3iB)\cot^{\frac{3}{2}}(c+dx)}{d(\cot(c+dx)+i)}}{8a^2}$$

$$\downarrow 1482$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((25+21i)A-(9-5i)B)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}}{2a^2}-(7}{8a^2}$$

$$\downarrow 1476$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{}\right.}{d}}{2a^2}}{8a^2}$$

$\downarrow$ 1082

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx}\right.}{\sqrt{2}}\right.\right.}{d}}{2a^2}}{8a^2}$$

$\downarrow$ 217

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.}{d}}{2a^2}}{8a^2}$$

$\downarrow$ 1479

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c}}{2\sqrt{2}}\right.\right.}{d}}{2a^2}}{8a^2}$$

$\downarrow$ 25

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right.\right.}{d}}{2a^2}}{8a^2}$$

$\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx}\right.\right.}{d}}{2a^2}}{8a^2}$$

$\downarrow$ 1103

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2} - \frac{\frac{\frac{10a^2(5A+iB)\sqrt{\cot(c+dx)}}{d} - \frac{2a^2\left(\frac{1}{2}((25+21i)A-(9-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right) + \left(\frac{1}{2}-\frac{i}{2}\right)((23+2i)A+(2+7i)B)\left(\frac{\log\left(\cot \cdots\right.}{\cdots}\right.\right.}{d}}{2a^2}}{8a^2}$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output
```
((A + I*B)*Cot[c + d*x]^(5/2))/(4*d*(I*a + a*Cot[c + d*x])^2) - (-(((7*A +
 (3*I)*B)*Cot[c + d*x]^(3/2))/(d*(I + Cot[c + d*x]))) + ((10*a^2*(5*A + I*
B)*Sqrt[Cot[c + d*x]])/d - (2*a^2*((((25 + 21*I)*A - (9 - 5*I)*B)*(-(ArcTa
n[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c
 + d*x]]]/Sqrt[2]))/2 + (1/2 - I/2)*((23 + 2*I)*A + (2 + 7*I)*B)*(-1/2*Log
[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*
Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]))))/d)/(2*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.60

method	result
derivativedivides	$\frac{-2A\sqrt{\cot(dx+c)}+\frac{i\left(\frac{\left(-\frac{7iB}{2}-\frac{11A}{2}\right)\cot(dx+c)^{\frac{3}{2}}+\left(\frac{5B}{2}-\frac{9iA}{2}\right)\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^2}+\frac{(7iB+23A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{4}-\frac{i(-iB+\cdots}{\cdots}}{a^2d}$
default	$\frac{-2A\sqrt{\cot(dx+c)}+\frac{i\left(\frac{\left(-\frac{7iB}{2}-\frac{11A}{2}\right)\cot(dx+c)^{\frac{3}{2}}+\left(\frac{5B}{2}-\frac{9iA}{2}\right)\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^2}+\frac{(7iB+23A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{4}-\frac{i(-iB+\cdots}{\cdots}}{a^2d}$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^2/d*(-2*A*cot(d*x+c)^(1/2)+1/4*I*(((-7/2*I*B-11/2*A)*cot(d*x+c)^(3/2)+
(5/2*B-9/2*I*A)*cot(d*x+c)^(1/2))/(I+cot(d*x+c))^2+(7*I*B+23*A)/(2^(1/2)+I
*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))-1/2*I*(A-I*B)/(2
^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(196) = 392.

Time = 0.11 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.54

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
-1/32*(2*a^2*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)
*log(-2*((I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))
 + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a^2*
d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*((-I*
a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) + (A - I*B)
*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a^2*d*sqrt((-529*I
*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(1/8*((a^2*d*
e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) - 1))*sqrt((-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2)) + 23*I*A
- 7*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) + a^2*d*sqrt((-529*I*A^2 + 322*A*B +
49*I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I
*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*s
qrt((-529*I*A^2 + 322*A*B + 49*I*B^2)/(a^4*d^2)) - 23*I*A + 7*B)*e^(-2*I*d
*x - 2*I*c)/(a^2*d)) + 2*(6*(7*A + I*B)*e^(4*I*d*x + 4*I*c) - (9*A + 5*I*B
)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1)))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{A\cot^{\frac{3}{2}}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx + \int \frac{B\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-(Integral(A*cot(c + d*x)**(3/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1),
 x) + Integral(B*tan(c + d*x)*cot(c + d*x)**(3/2)/(tan(c + d*x)**2 - 2*I*t
an(c + d*x) - 1), x))/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.49

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{2}((23i+23)\ A+(7i-7)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan{(dx+c)}}\right)+2\sqrt{2}(-(i-1)\ A-(i+1)\ B$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output
```
-1/16*(sqrt(2)*((23*I + 23)*A + (7*I - 7)*B)*arctan((1/2*I + 1/2)*sqrt(2)*
sqrt(tan(d*x + c))) + 2*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-(1/2*I -
1/2)*sqrt(2)*sqrt(tan(d*x + c))) + 32*A/sqrt(tan(d*x + c)) + 2*(9*A*tan(d*
x + c)^(3/2) + 5*I*B*tan(d*x + c)^(3/2) - 11*I*A*sqrt(tan(d*x + c)) + 7*B*
sqrt(tan(d*x + c)))/(tan(d*x + c) - I)^2)/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{(a+a\tan{(c+d\,x)}\ 1\mathrm{i})^2}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2, x
)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\left(\int \frac{\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\sqrt{\cot(dx+c)}\,\cot(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a}{a^2}$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int((sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x))/(tan(c + d*x)**2 -
 2*tan(c + d*x)*i - 1),x)*b + int((sqrt(cot(c + d*x))*cot(c + d*x))/(tan(c
 + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a))/a**2
```

3.530 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 230

$$\begin{aligned}\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx &= \frac{((9-5i)A+(1-3i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} \\ &\quad+\frac{\left(\frac{1}{16}+\frac{i}{16}\right)((-2+7i)A+(1+2i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^2d} \\ &\quad+\frac{((9+5i)A-(1+3i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{16\sqrt{2}a^2d} \\ &\quad+\frac{(5A+iB)\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))}+\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(ia+a\cot(c+dx))^2}\end{aligned}$$

output
```
-1/32*((9-5*I)*A+(1-3*I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a^
2/d+(1/32+1/32*I)*((-2+7*I)*A+(1+2*I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2)
)*2^(1/2)/a^2/d+1/32*((9+5*I)*A-(1+3*I)*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2
)/(1+cot(d*x+c)))*2^(1/2)/a^2/d+1/8*(5*A+I*B)*cot(d*x+c)^(1/2)/a^2/d/(I+co
t(d*x+c))+1/4*(A+I*B)*cot(d*x+c)^(3/2)/d/(I*a+a*cot(d*x+c))^2
```

Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(2\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^2(c+dx)(\cos(2(c+dx)$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^2,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(2*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)
^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c +
 d*x)]) + (-1)^(1/4)*(7*A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Se
c[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + Sqrt[Tan[c + d*x]]*
(-7*A - (3*I)*B + ((-5*I)*A + B)*Tan[c + d*x])))/(8*a^2*d*(-I + Tan[c + d*
x])^2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.08, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.528$, Rules used = {3042, 4064, 3042, 4078, 27, 3042, 4078, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A\cot(c+dx)+B)}{(a\cot(c+dx)+ia)^2}dx$$
$$\Big\downarrow \textbf{3042}$$
$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^2}dx$$
$$\Big\downarrow \textbf{4078}$$
$$\frac{\int -\frac{\sqrt{\cot(c+dx)}(3a(iA-B)-a(7A-iB)\cot(c+dx))}{2(\cot(c+dx)a+ia)}dx}{4a^2}+\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}$$
$$\Big\downarrow \textbf{27}$$
$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\int \frac{\sqrt{\cot(c+dx)}(3a(iA-B)-a(7A-iB)\cot(c+dx))}{\cot(c+dx)a+ia}dx}{8a^2}$$
$$\Big\downarrow \textbf{3042}$$
$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(3a(iA-B)+a(7A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{ia-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{8a^2}$$
$$\Big\downarrow \textbf{4078}$$
$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\int \frac{a^2(5iA-B)-3a^2(3A-iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{2a^2}-\frac{(5A+iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$
$$\Big\downarrow \textbf{3042}$$
$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\int \frac{(5iA-B)a^2+3(3A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{(5A+iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$
$$\Big\downarrow \textbf{4017}$$
$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\int -\frac{a^2(5iA-B-3(3A-iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{(5A+iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$
$$\Big\downarrow \textbf{25}$$
$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\int \frac{a^2(5iA-B-3(3A-iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{(5A+iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$
$$\Big\downarrow \textbf{27}$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\int \frac{5iA-B-3(3A-iB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{(5A+iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$

$\Big\downarrow$ 1482

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)B-(2-7i)A)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{(5A+iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$

$\Big\downarrow$ 1476

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)B-(2-7i)A)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+$$

$\Big\downarrow$ 1082

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)B-(2-7i)A)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d(\sqrt{}}{\sqrt{2}}$$

$\Big\downarrow$ 217

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)B-(2-7i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)}{d}-$$

$\Big\downarrow$ 1479

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)B-(2-}{d}}{8a^2}$$

$\Big\downarrow$ 25

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)B-(2-7i)A)}{d}}{8a^2}$$

$\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{1}{2}((9+5i)A-(1+3i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)B-(2-7}{d}}{8a^2}$$

$\downarrow$ 1103

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)B-(2-7i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((9+5i)A-(1+3i)B)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}\right.}{d}}{8a^2}$$

input

```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^2,x]
```

output

```
((A + I*B)*Cot[c + d*x]^(3/2))/(4*d*(I*a + a*Cot[c + d*x])^2) - (-(((5*A +
 I*B)*Sqrt[Cot[c + d*x]])/(d*(I + Cot[c + d*x]))) - ((1/2 + I/2)*((-2 + 7*
I)*A + (1 + 2*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + A
rcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + (((9 + 5*I)*A - (1 + 3*I)
*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log
[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2)/d)/(8*a^2
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{\frac{i(iA+B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{2\sqrt{2}-2i\sqrt{2}}+\frac{i\left(\frac{\left(-\frac{7iA}{2}+\frac{3B}{2}\right)\cot(dx+c)^{\frac{3}{2}}+\left(\frac{5A}{2}+\frac{iB}{2}\right)\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^2}+\frac{(7iA+B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{4}}{a^2d}$
default	$\frac{\frac{i(iA+B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{2\sqrt{2}-2i\sqrt{2}}+\frac{i\left(\frac{\left(-\frac{7iA}{2}+\frac{3B}{2}\right)\cot(dx+c)^{\frac{3}{2}}+\left(\frac{5A}{2}+\frac{iB}{2}\right)\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^2}+\frac{(7iA+B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{4}}{a^2d}$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^2/d*(1/2*I*(I*A+B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1
/2)-I*2^(1/2)))+1/4*I*(((-7/2*I*A+3/2*B)*cot(d*x+c)^(3/2)+(5/2*A+1/2*I*B)*
cot(d*x+c)^(1/2))/(I+cot(d*x+c))^2+(7*I*A+B)/(2^(1/2)+I*2^(1/2))*arctan(2*
cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(173) = 346.

Time = 0.12 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.90

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
1/32*(2*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)
*log(-2*((a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)) +
(A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a^2*d*s
qrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(2*((a^2*d*
e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)) - (A - I*B)*e^(2*I
*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - a^2*d*sqrt((49*I*A^2 + 14
*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/8*((a^2*d*e^(2*I*d*x +
 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
))*sqrt((49*I*A^2 + 14*A*B - I*B^2)/(a^4*d^2)) + 7*A - I*B)*e^(-2*I*d*x -
2*I*c)/(a^2*d)) + a^2*d*sqrt((49*I*A^2 + 14*A*B - I*B^2)/(a^4*d^2))*e^(4*I
*d*x + 4*I*c)*log(1/8*((a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((49*I*A^2 + 14*A*B - I*B
^2)/(a^4*d^2)) - 7*A + I*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) - 2*(2*(3*I*A -
B)*e^(4*I*d*x + 4*I*c) - (5*I*A - B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-4*I*d*x - 4*I*c
)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= -\frac{\int \frac{A\sqrt{\cot(c+dx)}}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx + \int \frac{B\tan(c+dx)\sqrt{\cot(c+dx)}}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-(Integral(A*sqrt(cot(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1),
x) + Integral(B*tan(c + d*x)*sqrt(cot(c + d*x))/(tan(c + d*x)**2 - 2*I*tan
(c + d*x) - 1), x))/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{2}((7i-7)\ A+(i+1)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+2\sqrt{2}(-(i+1)\ A+(i-1)\ B)\,\mathrm{ar}}{16\,a^2d}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output
```
-1/16*(sqrt(2)*((7*I - 7)*A + (I + 1)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sqrt
(tan(d*x + c))) + 2*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-(1/2*I - 1/2)
*sqrt(2)*sqrt(tan(d*x + c))) + 2*(5*I*A*tan(d*x + c)^(3/2) - B*tan(d*x + c
)^(3/2) + 7*A*sqrt(tan(d*x + c)) + 3*I*B*sqrt(tan(d*x + c)))/(tan(d*x + c)
 - I)^2)/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)}{{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2,x)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^2, x
)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{-\left(\int \frac{\sqrt{\cot(dx+c)}}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)a-\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\tan(dx+c)^2-2\tan(dx+c)i-1}dx\right)b}{a^2}$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(sqrt(cot(c + d*x))/(tan(c + d*x)**2 - 2*tan(c + d*x)*i - 1),x)*a
+ int((sqrt(cot(c + d*x))*tan(c + d*x))/(tan(c + d*x)**2 - 2*tan(c + d*x)*
i - 1),x)*b))/a**2
```

3.531 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 224

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2}\,dx &= -\frac{((-1+3i)A+(1+3i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} \\ &\quad+\frac{((-1+3i)A+(1+3i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} \\ &\quad-\frac{((1+3i)A+(1-3i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{16\sqrt{2}a^2d} \\ &\quad+\frac{(3iA+B)\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))}+\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2}\end{aligned}$$

output

```
1/32*((-1+3*I)*A+(1+3*I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a^
2/d+1/32*((-1+3*I)*A+(1+3*I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)
/a^2/d-1/32*((1+3*I)*A+(1-3*I)*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(
d*x+c)))*2^(1/2)/a^2/d+1/8*(3*I*A+B)*cot(d*x+c)^(1/2)/a^2/d/(I+cot(d*x+c))
+1/4*(A+I*B)*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))^2
```

Mathematica [A] (verified)

Time = 2.07 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-2\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^2(c+dx)(\cos(2(c+d}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^
2),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-2*(-1)^(1/4)*(I*A + B)*ArcTan[(-1
)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c
+ d*x)]) + (-1)^(1/4)*((-I)*A + B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*
Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + Sqrt[Tan[c + d*x]
]*((-3*I)*A - B + (A - (3*I)*B)*Tan[c + d*x])))/(8*a^2*d*(-I + Tan[c + d*x
])^2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.08, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4064, 3042, 4078, 27, 3042, 4079, 25, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A\cot(c+dx)+B)}{(a\cot(c+dx)+ia)^2}dx$$

$\downarrow$ 3042

$$\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^2}dx$$

$\downarrow$ 4078

$$\frac{\int -\frac{a(iA-B)-a(5A-3iB)\cot(c+dx)}{2\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)}dx}{4a^2}+\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$\downarrow$ 27

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{\int \frac{a(iA-B)-a(5A-3iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)}dx}{8a^2}$$

$\downarrow$ 3042

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{\int \frac{a(iA-B)+a(5A-3iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{8a^2}$$

$\downarrow$ 4079

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\int -\frac{a^2(A-3iB)-a^2(3iA+B)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{2a^2}-\frac{(B+3iA)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$

$\downarrow$ 25

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\int \frac{a^2(A-3iB)-a^2(3iA+B)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{2a^2}-\frac{(B+3iA)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$

$\downarrow$ 3042

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\int \frac{(A-3iB)a^2+(3iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{(B+3iA)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$

$\downarrow$ 4017

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{-\frac{\int -\frac{a^2(A-3iB-(3iA+B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{(B+3iA)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$

$\downarrow$ 25

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\int \frac{a^2(A-3iB-(3iA+B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{(B+3iA)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$

$\downarrow$ 27

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\int \frac{A-3iB-(3iA+B)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{(B+3iA)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$

$\downarrow$ 1482

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((1-3i)A-(1+3i)B)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{(B+3iA)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}$$

$\downarrow$ 1476

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((1-3i)A-(1+3i)B)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)+\sqrt{2}\cdots}\right.}{d}}{8a^2}$$

$\downarrow$ 1082

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((1-3i)A-(1+3i)B)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c\cdots}\right.}{\sqrt{2}}\right.}{d}}{8a^2}$$

$\downarrow$ 217

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}-\frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((1-3i)A-(1+3i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)}{d}-\frac{(B+3iA\cdots}{d(\text{co}\cdots}}{8a^2}$$

$\downarrow$ 1479

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2} - \frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right) + \frac{1}{2}((1-3i)A-(1+3i)B)\Big(}{d}}{8a^2}$$

$\Big\downarrow$ 25

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2} - \frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}} + \frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right) + \frac{1}{2}((1-3i)A-(1+3i)B)\Big(\frac{\arctan}{}}{d}}{8a^2}$$

$\Big\downarrow$ 27

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2} - \frac{\frac{\frac{1}{2}((1+3i)A+(1-3i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}} + \frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right) + \frac{1}{2}((1-3i)A-(1+3i)B)\Big(}{d}}{8a^2}$$

$\Big\downarrow$ 1103

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2} - \frac{\frac{\frac{1}{2}((1-3i)A-(1+3i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right) + \frac{1}{2}((1+3i)A+(1-3i)B)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}} - \frac{\log\Big(}{}\right.}{d}}{8a^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((A + I*B)*Sqrt[Cot[c + d*x]])/(4*d*(I*a + a*Cot[c + d*x])^2) - (-((((3*I)
*A + B)*Sqrt[Cot[c + d*x]])/(d*(I + Cot[c + d*x]))) + ((((1 - 3*I)*A - (1
+ 3*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 + (((1 + 3*I)*A + (1 - 3*I)*B)*(-1
/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sq
rt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2)/d)/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{\frac{i(-iB+A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{2\sqrt{2}-2i\sqrt{2}}+\frac{i\left(\frac{\left(-\frac{iB}{2}+\frac{3A}{2}\right)\cot(dx+c)^{\frac{3}{2}}+\left(\frac{3B}{2}+\frac{iA}{2}\right)\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^2}+\frac{(iB+A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{4}}{a^2d}$
default	$\frac{\frac{i(-iB+A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{2\sqrt{2}-2i\sqrt{2}}+\frac{i\left(\frac{\left(-\frac{iB}{2}+\frac{3A}{2}\right)\cot(dx+c)^{\frac{3}{2}}+\left(\frac{3B}{2}+\frac{iA}{2}\right)\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^2}+\frac{(iB+A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{4}}{a^2d}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^2/d*(1/2*I*(A-I*B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1
/2)-I*2^(1/2)))+1/4*I*(((-1/2*I*B+3/2*A)*cot(d*x+c)^(3/2)+(3/2*B+1/2*I*A)*
cot(d*x+c)^(1/2))/(I+cot(d*x+c))^2+(A+I*B)/(2^(1/2)+I*2^(1/2))*arctan(2*co
t(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(173) = 346$.

Time = 0.10 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.96

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + ia\tan(c + dx))^2}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
1/32*(2*a^2*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*
log(-2*((I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))
+ (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a^2*d
*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-2*((-I*a
^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) + (A - I*B)*
e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + a^2*d*sqrt((-I*A^2
+ 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(1/8*((a^2*d*e^(2*I*d*x
 + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
 1))*sqrt((-I*A^2 + 2*A*B + I*B^2)/(a^4*d^2)) + I*A - B)*e^(-2*I*d*x - 2*I
*c)/(a^2*d)) - a^2*d*sqrt((-I*A^2 + 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x +
 4*I*c)*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 + 2*A*B + I*B^2)/(a^4
*d^2)) - I*A + B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) + 2*(2*(A - I*B)*e^(4*I*d*
x + 4*I*c) - (A - 3*I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt((I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + ia\tan(c + dx))^2}\, dx =$$

$$-\frac{\int \frac{A}{\tan^2(c+dx)\sqrt{\cot(c+dx)}-2i\tan(c+dx)\sqrt{\cot(c+dx)}-\sqrt{\cot(c+dx)}}\,dx + \int \frac{B\tan(c+dx)}{\tan^2(c+dx)\sqrt{\cot(c+dx)}-2i\tan(c+dx)\sqrt{\cot(c+dx)}-}}{a^2}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-(Integral(A/(tan(c + d*x)**2*sqrt(cot(c + d*x)) - 2*I*tan(c + d*x)*sqrt(c
ot(c + d*x)) - sqrt(cot(c + d*x))), x) + Integral(B*tan(c + d*x)/(tan(c +
d*x)**2*sqrt(cot(c + d*x)) - 2*I*tan(c + d*x)*sqrt(cot(c + d*x)) - sqrt(co
t(c + d*x))), x))/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.52

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{2}((i+1)\ A+(i-1)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+2\sqrt{2}((i-1)\ A+(i+1)\ B)\,\text{arcta}}{16\,a^2d}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output
```
-1/16*(sqrt(2)*((I + 1)*A + (I - 1)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(t
an(d*x + c))) + 2*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-(1/2*I - 1/2)*sq
rt(2)*sqrt(tan(d*x + c))) - 2*(A*tan(d*x + c)^(3/2) - 3*I*B*tan(d*x + c)^(
3/2) - 3*I*A*sqrt(tan(d*x + c)) - B*sqrt(tan(d*x + c)))/(tan(d*x + c) - I)
^2)/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2}\,dx=\int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot\left(c+d\,x\right)}\,{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^2), x
)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{-\left(\int \frac{\tan(dx+c)}{\sqrt{\cot(dx+c)}\,\tan(dx+c)^2-2\sqrt{\cot(dx+c)}\,\tan(dx+c)i-\sqrt{\cot(dx+c)}}dx\right)b-\left(\int \frac{1}{\sqrt{\cot(dx+c)}\,\tan(dx+c)^2-2\sqrt{\cot(dx+c)}\,\tan(dx+}\right.}{a^2}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(tan(c + d*x)/(sqrt(cot(c + d*x))*tan(c + d*x)**2 - 2*sqrt(cot(c +
 d*x))*tan(c + d*x)*i - sqrt(cot(c + d*x))),x)*b + int(1/(sqrt(cot(c + d*x
))*tan(c + d*x)**2 - 2*sqrt(cot(c + d*x))*tan(c + d*x)*i - sqrt(cot(c + d*
x))),x)*a))/a**2
```

3.532 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 234

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx &= -\frac{\left(\frac{1}{16}+\frac{i}{16}\right)((2+i)A+(7-2i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^2d} \\ &+\frac{((1+3i)A+(9+5i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} \\ &+\frac{\left(\frac{1}{16}+\frac{i}{16}\right)((1+2i)A+(2-7i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}a^2d} \\ &+\frac{(A+5iB)\sqrt{\cot(c+dx)}}{8a^2d(i+\cot(c+dx))}+\frac{(iA-B)\sqrt{\cot(c+dx)}}{4d(ia+a\cot(c+dx))^2}\end{aligned}$$

output
```
(1/32+1/32*I)*((2+I)*A+(7-2*I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1
/2)/a^2/d+1/32*((1+3*I)*A+(9+5*I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^
(1/2)/a^2/d+(1/32+1/32*I)*((1+2*I)*A+(2-7*I)*B)*arctanh(2^(1/2)*cot(d*x+c)
^(1/2)/(1+cot(d*x+c)))*2^(1/2)/a^2/d+1/8*(A+5*I*B)*cot(d*x+c)^(1/2)/a^2/d/
(I+cot(d*x+c))+1/4*(I*A-B)*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))^2
```

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.84

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-2\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^2(c+dx)(\cos(2(c+d$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^
2),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-2*(-1)^(1/4)*(A - I*B)*ArcTan[(-1
)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c
+ d*x)]) + (-1)^(1/4)*(A - (7*I)*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]
*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - Sqrt[Tan[c + d*x
]]*(A + (5*I)*B + ((3*I)*A - 7*B)*Tan[c + d*x])))/(8*a^2*d*(-I + Tan[c + d
*x])^2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.05, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4064, 3042, 4079, 27, 3042, 4079, 25, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{A\cot(c+dx)+B}{\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^2}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int \frac{a(A-7iB)-3a(iA-B)\cot(c+dx)}{2\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)}dx}{4a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A-7iB)-3a(iA-B)\cot(c+dx)}{\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)}dx}{8a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A-7iB)+3a(iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{8a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{\int -\frac{3(iA+3B)a^2+(A+5iB)\cot(c+dx)a^2}{\sqrt{\cot(c+dx)}}dx}{2a^2}+\frac{(A+5iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(A+5iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}-\frac{\int \frac{3(iA+3B)a^2+(A+5iB)\cot(c+dx)a^2}{\sqrt{\cot(c+dx)}}dx}{2a^2}}{8a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(A+5iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}-\frac{\int \frac{3a^2(iA+3B)-a^2(A+5iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}}{8a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{(A+5iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}-\frac{\int -\frac{a^2(3(iA+3B)+(A+5iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}}{8a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a^2(3(iA+3B)+(A+5iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}+\frac{(A+5iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3(iA+3B)+(A+5iB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{(A+5iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A+(2-7i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((1+3i)A+(9+5i)B)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{(A+5iB)\sqrt{\cot(c+dx)}}{d(\cot(c+dx)+i)}}{8a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A+(2-7i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((1+3i)A+(9+5i)B)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{}{\cot(c+dx}\right.}{d}}{8a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A+(2-7i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((1+3i)A+(9+5i)B)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\right.}{\sqrt{2}}\right.}{d}}{8a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A+(2-7i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((1+3i)A+(9+5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)}{d}+\frac{(A}{}}{8a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A+(2-7i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((1+3i)A+(9+5i)}{d}}{8a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$\downarrow$ 25

$$\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A+(2-7i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((1+3i)A+(9+5i)B)\Big(}{d}}{8a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$\downarrow$ 27

$$\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A+(2-7i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}((1+3i)A+(9+5i)B}{d}}{8a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

$\downarrow$ 1103

$$\frac{\frac{\frac{1}{2}((1+3i)A+(9+5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((1+2i)A+(2-7i)B)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\right.}{d}}{8a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{4d(a\cot(c+dx)+ia)^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
((I*A - B)*Sqrt[Cot[c + d*x]])/(4*d*(I*a + a*Cot[c + d*x])^2) + (((A + (5*
I)*B)*Sqrt[Cot[c + d*x]])/(d*(I + Cot[c + d*x])) + ((((1 + 3*I)*A + (9 + 5
*I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 + (1/2 + I/2)*((1 + 2*I)*A + (2 - 7*I
)*B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Lo
g[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/d)/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{-\frac{i(iA+B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{2\left(\sqrt{2}-i\sqrt{2}\right)}-\frac{\left(-\frac{5iB}{2}-\frac{A}{2}\right)\cot(dx+c)^{\frac{3}{2}}+\left(\frac{7B}{2}-\frac{3iA}{2}\right)\sqrt{\cot(dx+c)}}{4(i+\cot(dx+c))^{2}}-\frac{(-7iB+A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{4\left(\sqrt{2}+i\sqrt{2}\right)}}{a^{2}d}$
default	$\frac{-\frac{i(iA+B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{2\left(\sqrt{2}-i\sqrt{2}\right)}-\frac{\left(-\frac{5iB}{2}-\frac{A}{2}\right)\cot(dx+c)^{\frac{3}{2}}+\left(\frac{7B}{2}-\frac{3iA}{2}\right)\sqrt{\cot(dx+c)}}{4(i+\cot(dx+c))^{2}}-\frac{(-7iB+A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{4\left(\sqrt{2}+i\sqrt{2}\right)}}{a^{2}d}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^2/d*(-1/2*I*(I*A+B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(
1/2)-I*2^(1/2)))-1/4*((-5/2*I*B-1/2*A)*cot(d*x+c)^(3/2)+(7/2*B-3/2*I*A)*co
t(d*x+c)^(1/2))/(I+cot(d*x+c))^2-1/4*(-7*I*B+A)/(2^(1/2)+I*2^(1/2))*arctan
(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(171) = 342.

Time = 0.10 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.85

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
-1/32*(2*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c
)*log(-2*((a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)) +
 (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 2*a^2*d*
sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(2*((a^2*d
*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^4*d^2)) - (A - I*B)*e^(2*
I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + a^2*d*sqrt((I*A^2 + 14*A
*B - 49*I*B^2)/(a^4*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/8*((a^2*d*e^(2*I*d*x
+ 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
1))*sqrt((I*A^2 + 14*A*B - 49*I*B^2)/(a^4*d^2)) + A - 7*I*B)*e^(-2*I*d*x -
 2*I*c)/(a^2*d)) - a^2*d*sqrt((I*A^2 + 14*A*B - 49*I*B^2)/(a^4*d^2))*e^(4*
I*d*x + 4*I*c)*log(1/8*((a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 14*A*B - 49*I*
B^2)/(a^4*d^2)) - A + 7*I*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) + 2*(2*(I*A - 3
*B)*e^(4*I*d*x + 4*I*c) - (3*I*A - 7*B)*e^(2*I*d*x + 2*I*c) + I*A - B)*sqr
t((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-4*I*d*x - 4*
I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{A}{\tan^2(c+dx)\cot^{\frac{3}{2}}(c+dx)-2i\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)-\cot^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B\tan(c+dx)}{\tan^2(c+dx)\cot^{\frac{3}{2}}(c+dx)-2i\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)-\cot^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
-(Integral(A/(tan(c + d*x)**2*cot(c + d*x)**(3/2) - 2*I*tan(c + d*x)*cot(c
 + d*x)**(3/2) - cot(c + d*x)**(3/2)), x) + Integral(B*tan(c + d*x)/(tan(c
 + d*x)**2*cot(c + d*x)**(3/2) - 2*I*tan(c + d*x)*cot(c + d*x)**(3/2) - co
t(c + d*x)**(3/2)), x))/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.50

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{2}((i-1)\ A+(7i+7)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+2\sqrt{2}((i+1)\ A-(i-1)\ B)\,\mathrm{arct}}{16\,a^2d}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output
```
-1/16*(sqrt(2)*((I - 1)*A + (7*I + 7)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sqrt
(tan(d*x + c))) + 2*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-(1/2*I - 1/2)*
sqrt(2)*sqrt(tan(d*x + c))) + 2*(3*I*A*tan(d*x + c)^(3/2) - 7*B*tan(d*x +
c)^(3/2) + A*sqrt(tan(d*x + c)) + 5*I*B*sqrt(tan(d*x + c)))/(tan(d*x + c)
- I)^2)/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \int \frac{A+B\tan(c+d\,x)}{\cot(c+d\,x)^{3/2}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^2}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^2), x
)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{-\left(\int \frac{\tan(dx+c)}{\sqrt{\cot(dx+c)}\cot(dx+c)\tan(dx+c)^2-2\sqrt{\cot(dx+c)}\cot(dx+c)\tan(dx+c)i-\sqrt{\cot(dx+c)}\cot(dx+c)}dx\right)b-\left(\int \frac{}{\sqrt{\cot(dx+c)}\cot(}\right.}{a^2}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(tan(c + d*x)/(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)**2 - 2
*sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)*i - sqrt(cot(c + d*x))*cot(c
 + d*x)),x)*b + int(1/(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)**2 - 2
*sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)*i - sqrt(cot(c + d*x))*cot(c
 + d*x)),x)*a))/a**2
```

3.533 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 265

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx \\
&= -\frac{\left(\frac{1}{16}-\frac{i}{16}\right)((2+7i)A-(23+2i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^2d} \\
&\quad+\frac{((9+5i)A-(25-21i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^2d} \\
&\quad+\frac{\left(\frac{1}{16}-\frac{i}{16}\right)((7+2i)A+(2+23i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}a^2d}+\frac{5(iA-5B)}{8a^2d\sqrt{\cot(c+dx)}} \\
&\quad+\frac{3A+7iB}{8a^2d\sqrt{\cot(c+dx)}(i+\cot(c+dx))}+\frac{iA-B}{4d\sqrt{\cot(c+dx)}(ia+a\cot(c+dx))^2}
\end{aligned}$$

output
```
(1/32-1/32*I)*((2+7*I)*A-(23+2*I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2
^(1/2)/a^2/d+1/32*((9+5*I)*A+(-25+21*I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/
2))*2^(1/2)/a^2/d+(1/32-1/32*I)*((7+2*I)*A+(2+23*I)*B)*arctanh(2^(1/2)*cot
(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/a^2/d+5/8*(I*A-5*B)/a^2/d/cot(d*x+c)
^(1/2)+1/8*(3*A+7*I*B)/a^2/d/cot(d*x+c)^(1/2)/(I+cot(d*x+c))+1/4*(I*A-B)/d
/cot(d*x+c)^(1/2)/(I*a+a*cot(d*x+c))^2
```

Mathematica [A] (verified)

Time = 3.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.81

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-2\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^2(c+dx)(\cos(2(c-$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^
2),x]
```

output
```
-1/8*(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-2*(-1)^(1/4)*(I*A + B)*ArcTa
n[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[
2*(c + d*x)]) + (-1)^(3/4)*(7*A + (23*I)*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c
+ d*x]]]*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + Sqrt[Tan
[c + d*x]]*((5*I)*(A + (5*I)*B) - (7*A + (43*I)*B)*Tan[c + d*x] + 16*B*Tan
[c + d*x]^2)))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.09, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.639$, Rules used = {3042, 4064, 3042, 4079, 27, 3042, 4079, 25, 3042, 4012, 25, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{5/2}(a+ia\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{A\cot(c+dx)+B}{\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^2}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int -\frac{a(A+9iB)+5a(iA-B)\cot(c+dx)}{2\cot^{\frac{3}{2}}(c+dx)(\cot(c+dx)a+ia)}dx}{4a^2}+\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{\int \frac{a(A+9iB)+5a(iA-B)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(\cot(c+dx)a+ia)}dx}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{\int \frac{a(A+9iB)-5a(iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{8a^2}$$

$$\Big\downarrow \text{4079}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{\frac{\int -\frac{5a^2(iA-5B)-3a^2(3A+7iB)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{3A+7iB}{d\sqrt{\cot(c+dx)}(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\int \frac{5a^2(iA-5B)-3a^2(3A+7iB)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{3A+7iB}{d\sqrt{\cot(c+dx)}(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\int \frac{5(iA-5B)a^2+3(3A+7iB)\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a^2}-\frac{3A+7iB}{d\sqrt{\cot(c+dx)}(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\int -\frac{3(3A+7iB)a^2+5(iA-5B)\cot(c+dx)a^2}{\sqrt{\cot(c+dx)}}dx+\frac{10a^2(-5B+iA)}{d\sqrt{\cot(c+dx)}}}{2a^2}-\frac{3A+7iB}{d\sqrt{\cot(c+dx)}(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{10a^2(-5B+iA)}{d\sqrt{\cot(c+dx)}}-\int \frac{3(3A+7iB)a^2+5(iA-5B)\cot(c+dx)a^2}{\sqrt{\cot(c+dx)}}dx}{2a^2}-\frac{3A+7iB}{d\sqrt{\cot(c+dx)}(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{10a^2(-5B+iA)}{d\sqrt{\cot(c+dx)}}-\int \frac{3a^2(3A+7iB)-5a^2(iA-5B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{3A+7iB}{d\sqrt{\cot(c+dx)}(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{10a^2(-5B+iA)}{d\sqrt{\cot(c+dx)}}-\frac{2\int -\frac{a^2(3(3A+7iB)+5(iA-5B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}}{2a^2}-\frac{3A+7iB}{d\sqrt{\cot(c+dx)}(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{2\int \frac{a^2(3(3A+7iB)+5(iA-5B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{10a^2(-5B+iA)}{d\sqrt{\cot(c+dx)}}}{2a^2}-\frac{3A+7iB}{d\sqrt{\cot(c+dx)}(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{2a^2\int\frac{3(3A+7iB)+5(iA-5B)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{10a^2(-5B+iA)}{d\sqrt{\cot(c+dx)}}}{2a^2}-\frac{3A+7iB}{d\sqrt{\cot(c+dx)}(\cot(c+dx)+i)}}{8a^2}$$

$$\Big\downarrow \; 1482$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((9+5i)A-(25-21i)B)\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}+\frac{10a^2(-5B+iA)}{d\sqrt{\cot(c+dx)}}}{2a^2}-\frac{\cdots}{d\sqrt{\cot(c\cdots}}}{8a^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int\frac{\cdots}{\cot(c+dx)\cdots}\cdots\right.\right.}{d}\cdots}{2a^2}\cdots}{8a^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(\cdots\right.}{\sqrt{2}\cdots}\right.\right.}{d}\cdots}{2a^2}\cdots}{8a^2}$$

$$\Big\downarrow \; 217$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{-\frac{\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}+\cdots}{2a^2}\cdots}{8a^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((9+5i)A-(25-\cdots}{d}}{2a^2}}{8a^2}$$

$\Big\downarrow$ 25

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}((9+5i)A-(25-21i)\cdots}{d}}{2a^2}}{8a^2}$$

$\Big\downarrow$ 27

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{\frac{2a^2\left(\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}((9+5i)A-(25-21i\cdots}{d}}{2a^2}}{8a^2}$$

$\Big\downarrow$ 1103

$$\frac{-B+iA}{4d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}-\frac{\frac{2a^2\left(\frac{1}{2}((9+5i)A-(25-21i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)((7+2i)A+(2+23i)B)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+\cdots\right.}{2\sqrt{2}}\cdots}{d}}{2a^2}}{8a^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

output
```
(I*A - B)/(4*d*Sqrt[Cot[c + d*x]]*(I*a + a*Cot[c + d*x])^2) - (-((3*A + (7
*I)*B)/(d*Sqrt[Cot[c + d*x]]*(I + Cot[c + d*x]))) - ((10*a^2*(I*A - 5*B))/
(d*Sqrt[Cot[c + d*x]]) + (2*a^2*((((9 + 5*I)*A - (25 - 21*I)*B)*(-(ArcTan[
1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c +
 d*x]]]/Sqrt[2]))/2 + (1/2 - I/2)*((7 + 2*I)*A + (2 + 23*I)*B)*(-1/2*Log[1
 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]))))/d)/(2*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{-\frac{2B}{\sqrt{\cot(dx+c)}}+\frac{\left(\frac{5iA}{2}-\frac{9B}{2}\right)\cot(dx+c)^{\frac{3}{2}}+\left(-\frac{7A}{2}-\frac{11iB}{2}\right)\sqrt{\cot(dx+c)}}{4(i+\cot(dx+c))^2}+\frac{(7iA-23B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{4\sqrt{2}+4i\sqrt{2}}-\frac{i(-iB+A)\arctan\cdots}{2(\sqrt{2}-\cdots}}{a^2d}$
default	$\frac{-\frac{2B}{\sqrt{\cot(dx+c)}}+\frac{\left(\frac{5iA}{2}-\frac{9B}{2}\right)\cot(dx+c)^{\frac{3}{2}}+\left(-\frac{7A}{2}-\frac{11iB}{2}\right)\sqrt{\cot(dx+c)}}{4(i+\cot(dx+c))^2}+\frac{(7iA-23B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{4\sqrt{2}+4i\sqrt{2}}-\frac{i(-iB+A)\arctan\cdots}{2(\sqrt{2}-\cdots}}{a^2d}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^2/d*(-2*B/cot(d*x+c)^(1/2)+1/4*((5/2*I*A-9/2*B)*cot(d*x+c)^(3/2)+(-7/2
*A-11/2*I*B)*cot(d*x+c)^(1/2))/(I+cot(d*x+c))^2+1/4*(7*I*A-23*B)/(2^(1/2)+
I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))-1/2*I*(A-I*B)/(2
^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(196) = 392$.

Time = 0.11 (sec) , antiderivative size = 761, normalized size of antiderivative = 2.87

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="fricas")
```

output
```
-1/32*(2*(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))*sqrt((I*A
^2 + 2*A*B - I*B^2)/(a^4*d^2))*log(-2*((I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^
2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A
^2 + 2*A*B - I*B^2)/(a^4*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*
x - 2*I*c)/(I*A + B)) - 2*(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*d*x +
4*I*c))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2))*log(-2*((-I*a^2*d*e^(2*I*d
*x + 2*I*c) + I*a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^4*d^2)) + (A - I*B)*e^(2*I*d*x +
2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - (a^2*d*e^(6*I*d*x + 6*I*c) + a^2
*d*e^(4*I*d*x + 4*I*c))*sqrt((-49*I*A^2 + 322*A*B + 529*I*B^2)/(a^4*d^2))*
log(1/8*((a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-49*I*A^2 + 322*A*B + 529*I*B^2)/(a^4
*d^2)) + 7*I*A - 23*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) + (a^2*d*e^(6*I*d*x +
 6*I*c) + a^2*d*e^(4*I*d*x + 4*I*c))*sqrt((-49*I*A^2 + 322*A*B + 529*I*B^2
)/(a^4*d^2))*log(-1/8*((a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt((I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-49*I*A^2 + 322*A*B + 5
29*I*B^2)/(a^4*d^2)) - 7*I*A + 23*B)*e^(-2*I*d*x - 2*I*c)/(a^2*d)) - 2*(6*
(A + 7*I*B)*e^(6*I*d*x + 6*I*c) - (A + 33*I*B)*e^(4*I*d*x + 4*I*c) - 2*(3*
A + 5*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) - 1)))/(a^2*d*e^(6*I*d*x + 6*I*c) + a^2*d*e^(4*I*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.48

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{2}((7i+7)\ A+(23i-23)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+2\sqrt{2}(-(i-1)\ A-(i+1)\ B$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x, algori
thm="giac")
```

output
```
-1/16*(sqrt(2)*((7*I + 7)*A + (23*I - 23)*B)*arctan((1/2*I + 1/2)*sqrt(2)*
sqrt(tan(d*x + c))) + 2*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-(1/2*I -
1/2)*sqrt(2)*sqrt(tan(d*x + c))) + 32*B*sqrt(tan(d*x + c)) - 2*(7*A*tan(d*
x + c)^(3/2) + 11*I*B*tan(d*x + c)^(3/2) - 5*I*A*sqrt(tan(d*x + c)) + 9*B*
sqrt(tan(d*x + c)))/(tan(d*x + c) - I)^2)/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx=\int \frac{A+B\tan(c+d\,x)}{{\cot\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^2), x
)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{-\left(\int \frac{\tan(dx+c)}{\sqrt{\cot(dx+c)}\cot(dx+c)^2\tan(dx+c)^2-2\sqrt{\cot(dx+c)}\cot(dx+c)^2\tan(dx+c)i-\sqrt{\cot(dx+c)}\cot(dx+c)^2}dx\right)b-\left(\int \frac{}{\sqrt{\cot(dx+c)}\ c}\right.}{a^2}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^2,x)
```

output
```
( - (int(tan(c + d*x)/(sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**2
- 2*sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)*i - sqrt(cot(c + d*x))
*cot(c + d*x)**2),x)*b + int(1/(sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c +
 d*x)**2 - 2*sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)*i - sqrt(cot(
c + d*x))*cot(c + d*x)**2),x)*a))/a**2
```

3.534 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 313

$$\begin{aligned}
&\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx \\
&= -\frac{\left(\frac{1}{16}-\frac{i}{16}\right)((1+29i)A-(6+i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^3d} \\
&+\frac{((30+28i)A-(7-5i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} \\
&+\frac{\left(\frac{1}{16}-\frac{i}{16}\right)((29+i)A+(1+6i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}a^3d} \\
&-\frac{5(6A+iB)\sqrt{\cot(c+dx)}}{8a^3d}+\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} \\
&+\frac{(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{12ad(ia+a\cot(c+dx))^2}+\frac{7(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{24d\,(ia^3+a^3\cot(c+dx))}
\end{aligned}$$

output
```
(1/32-1/32*I)*((1+29*I)*A-(6+I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(
1/2)/a^3/d+1/32*((30+28*I)*A+(-7+5*I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2)
)*2^(1/2)/a^3/d+(1/32-1/32*I)*((29+I)*A+(1+6*I)*B)*arctanh(2^(1/2)*cot(d*x
+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/a^3/d-5/8*(6*A+I*B)*cot(d*x+c)^(1/2)/a^3
/d+1/6*(A+I*B)*cot(d*x+c)^(7/2)/d/(I*a+a*cot(d*x+c))^3+1/12*(5*A+2*I*B)*co
t(d*x+c)^(5/2)/a/d/(I*a+a*cot(d*x+c))^2+7/24*(4*A+I*B)*cot(d*x+c)^(3/2)/d/
(I*a^3+a^3*cot(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 3.99 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.79

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\left(-48iA+3\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^3(c+dx)(\cos(3(c+dx))+\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^3,x]
```

output
```
(Sqrt[Cot[c + d*x]]*((-48*I)*A + 3*(-1)^(1/4)*(A - I*B)*ArcTan[(-1)^(3/4)*
Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^3*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])
*Sqrt[Tan[c + d*x]] - 3*(-1)^(1/4)*(29*A + (6*I)*B)*ArcTanh[(-1)^(3/4)*Sqr
t[Tan[c + d*x]]]*Sec[c + d*x]^3*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*Sq
rt[Tan[c + d*x]] + 204*A*Tan[c + d*x] + (27*I)*B*Tan[c + d*x] + (242*I)*A*
Tan[c + d*x]^2 - 38*B*Tan[c + d*x]^2 - 90*A*Tan[c + d*x]^3 - (15*I)*B*Tan[
c + d*x]^3))/(24*a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.09, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.694$, Rules used = {3042, 4064, 3042, 4078, 27, 3042, 4078, 27, 3042, 4078, 27, 3042, 4011, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{7}{2}}(c+dx)(A\cot(c+dx)+B)}{(a\cot(c+dx)+ia)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{\int -\frac{\cot^{\frac{5}{2}}(c+dx)(7a(iA-B)-a(13A+iB)\cot(c+dx))}{2(\cot(c+dx)a+ia)^2}dx}{6a^2}+\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\int \frac{\cot^{\frac{5}{2}}(c+dx)(7a(iA-B)-a(13A+iB)\cot(c+dx))}{(\cot(c+dx)a+ia)^2}dx}{12a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(7a(iA-B)+a(13A+iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{12a^2}$$

$$\Big\downarrow \text{4078}$$

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\int \frac{2\cot^{\frac{3}{2}}(c+dx)\left(5a^2(5iA-2B)-a^2(31A+4iB)\cot(c+dx)\right)}{\cot(c+dx)a+ia}dx}{4a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)\left(5a^2(5iA-2B)-a^2(31A+4iB)\cot(c+dx)\right)}{\cot(c+dx)a+ia}dx}{2a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$\downarrow$ 3042

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(5(5iA-2B)a^2+(31A+4iB)\tan\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{ia-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$\downarrow$ 4078

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\frac{\int 3\sqrt{\cot(c+dx)}\left(7a^3(4iA-B)-5a^3(6A+iB)\cot(c+dx)\right)dx}{2a^2}-\frac{7a^2(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\frac{3\int \sqrt{\cot(c+dx)}\left(7a^3(4iA-B)-5a^3(6A+iB)\cot(c+dx)\right)dx}{2a^2}-\frac{7a^2(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$\downarrow$ 3042

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\frac{3\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(7(4iA-B)a^3+5(6A+iB)\tan\left(c+dx+\frac{\pi}{2}\right)a^3\right)dx}{2a^2}-\frac{7a^2(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$\downarrow$ 4011

$$\frac{\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\int\frac{5(6A+iB)a^3+7(4iA-B)\cot(c+dx)a^3}{\sqrt{\cot(c+dx)}}dx+\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}\right)}{2a^2}-\frac{7a^2(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\int\frac{5a^3(6A+iB)-7a^3(4iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}\right)}{2a^2}-\frac{7a^2(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 4017}$$

$$\frac{\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{2\int-\frac{a^3(5(6A+iB)+7(4iA-B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}\right)}{2a^2}-\frac{7a^2(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2\int\frac{a^3(5(6A+iB)+7(4iA-B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}\right)}{2a^2}-\frac{7a^2(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^3\int\frac{5(6A+iB)+7(4iA-B)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}\right)}{2a^2}-\frac{7a^2(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(5A+2iB)\cot^{\frac{5}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 1482}$$

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\frac{3\left(\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((30+28i)A-(7-5i)B)\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}\right)}{2a^2}-7\cdots}{2a^2}}{12a^2}$$

$$\Big\downarrow \ 1476$$

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\frac{3\left(\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\cdots\right.\right.}{d}\cdots}{2a^2}\cdots}{2a^2}}{12a^2}$$

$$\Big\downarrow \ 1082$$

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\frac{3\left(\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+\cdots}\right.}{\sqrt{2}}\cdots\right.\right.}{d}\cdots}{2a^2}\cdots}{2a^2}}{12a^2}$$

$$\Big\downarrow \ 217$$

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\frac{3\left(\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\cdots}{\cdots}\right.\right.}{d}\cdots}{2a^2}\cdots}{2a^2}}{12a^2}$$

$$\Big\downarrow \ 1479$$

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-$$

$$3\left(\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot}}{2\sqrt{2}}\right.\right.}{d}\right.$$

$$\frac{\quad}{2a^2} \qquad \frac{\quad}{2a^2} \qquad \frac{\quad}{12}$$

$\downarrow$ 25

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-$$

$$3\left(\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+d}}{2\sqrt{2}}\right.\right.}{d}\right.$$

$$\frac{\quad}{2a^2} \qquad \frac{\quad}{2a^2} \qquad \frac{\quad}{12a^2}$$

$\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-$$

$$3\left(\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^3\left(\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+}\right.\right.}{d}\right.$$

$$\frac{\quad}{2a^2} \qquad \frac{\quad}{2a^2} \qquad \frac{\quad}{12a}$$

$\downarrow$ 1103

$$\frac{(A+iB)\cot^{\frac{7}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-$$

$$3\left(\frac{10a^3(6A+iB)\sqrt{\cot(c+dx)}}{d}-\frac{2a^3\left(\frac{1}{2}((30+28i)A-(7-5i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)((29+i)A+(1+6i)B)\left(\frac{\log\left(c\right.}{}\right.\right.}{d}\right.$$

$$\frac{\quad}{2a^2} \qquad \frac{\quad}{2a^2} \qquad \frac{\quad}{12a^2}$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((A + I*B)*Cot[c + d*x]^(7/2))/(6*d*(I*a + a*Cot[c + d*x])^3) - (-((a*(5*A
 + (2*I)*B)*Cot[c + d*x]^(5/2))/(d*(I*a + a*Cot[c + d*x])^2)) + ((-7*a^2*(
4*A + I*B)*Cot[c + d*x]^(3/2))/(d*(I*a + a*Cot[c + d*x])) + (3*((10*a^3*(6
*A + I*B)*Sqrt[Cot[c + d*x]])/d - (2*a^3*(((((30 + 28*I)*A - (7 - 5*I)*B)*(
-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqr
t[Cot[c + d*x]]]/Sqrt[2]))/2 + (1/2 - I/2)*((29 + I)*A + (1 + 6*I)*B)*(-1/
2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqr
t[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]))))/d))/(2*a^2))/(2*a^2
))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.58

method	result
derivativedivides	$\frac{-2A\sqrt{\cot(dx+c)}+\frac{i\left(\frac{i(20iA-9B)\cot(dx+c)^{\frac{5}{2}}+\left(-\frac{98iA}{3}+\frac{38B}{3}\right)\cot(dx+c)^{\frac{3}{2}}+(5iB+14A)\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^{3}}+\frac{2(6iB+29A)\arctan\left(2\ldots\right)}{\sqrt{2}+i\sqrt{2}}\ldots\right.}{8}}{a^{3}d}$
default	$\frac{-2A\sqrt{\cot(dx+c)}+\frac{i\left(\frac{i(20iA-9B)\cot(dx+c)^{\frac{5}{2}}+\left(-\frac{98iA}{3}+\frac{38B}{3}\right)\cot(dx+c)^{\frac{3}{2}}+(5iB+14A)\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^{3}}+\frac{2(6iB+29A)\arctan\left(2\ldots\right)}{\sqrt{2}+i\sqrt{2}}\ldots\right.}{8}}{a^{3}d}$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^3/d*(-2*A*cot(d*x+c)^(1/2)+1/8*I*((I*(20*I*A-9*B)*cot(d*x+c)^(5/2)+(-9
8/3*I*A+38/3*B)*cot(d*x+c)^(3/2)+(14*A+5*I*B)*cot(d*x+c)^(1/2))/(I+cot(d*x
+c))^3+2*(29*A+6*I*B)/(2^(1/2)+I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/
2)+I*2^(1/2))))+4*(-1/16*I*A-1/16*B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+
c)^(1/2)/(2^(1/2)-I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(238) = 476$.

Time = 0.12 (sec) , antiderivative size = 688, normalized size of antiderivative = 2.20

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
-1/96*(3*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)
*log(-2*((I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))
 + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*
d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*((-I*
a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) + (A - I*B)
*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*sqrt((-841
*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*((a^3*
d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) - 1))*sqrt((-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2)) + 29*I*
A - 6*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) + 3*a^3*d*sqrt((-841*I*A^2 + 348*A*
B + 36*I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*((a^3*d*e^(2*I*d*x +
 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
))*sqrt((-841*I*A^2 + 348*A*B + 36*I*B^2)/(a^6*d^2)) - 29*I*A + 6*B)*e^(-2
*I*d*x - 2*I*c)/(a^3*d)) + 2*(2*(73*A + 10*I*B)*e^(6*I*d*x + 6*I*c) - (41*
A + 14*I*B)*e^(4*I*d*x + 4*I*c) - (8*A + 5*I*B)*e^(2*I*d*x + 2*I*c) - A -
I*B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-6*I*
d*x - 6*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\left(\int \frac{A\cot^{\frac{3}{2}}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx+\int \frac{B\tan(c+dx)\cot^{\frac{3}{2}}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx\right)}{a^3}$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*(Integral(A*cot(c + d*x)**(3/2)/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 -
 3*tan(c + d*x) + I), x) + Integral(B*tan(c + d*x)*cot(c + d*x)**(3/2)/(ta
n(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c + d*x) + I), x))/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.48

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{3\sqrt{2}(-(29i+29)\ A-(6i-6)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan(dx+c)}\right)-3\sqrt{2}(-(i-1)\ A-(i+1)\ B}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
1/48*(3*sqrt(2)*(-(29*I + 29)*A - (6*I - 6)*B)*arctan((1/2*I + 1/2)*sqrt(2
)*sqrt(tan(d*x + c))) - 3*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-(1/2*I
- 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - 96*A/sqrt(tan(d*x + c)) - 2*(42*A*tan
(d*x + c)^(5/2) + 15*I*B*tan(d*x + c)^(5/2) - 98*I*A*tan(d*x + c)^(3/2) +
38*B*tan(d*x + c)^(3/2) - 60*A*sqrt(tan(d*x + c)) - 27*I*B*sqrt(tan(d*x +
c)))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{(a+a\tan(c+d\,x)\,1\mathrm{i})^3}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3, x
)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)b-\left(\int \frac{\sqrt{\cot(dx+c)}\,\cot(dx+c)}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)a}{a^3}$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int((sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x))/(tan(c + d*x)**3*i
 + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*b + int((sqrt(cot(c + d*x)
)*cot(c + d*x))/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i
- 1),x)*a))/a**3
```

3.535 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 268

$$\begin{aligned}\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{((-7+5i)A+2iB)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} \\ &\quad+\frac{((-7+5i)A+2iB)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} \\ &\quad+\frac{((7+5i)A-2iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{16\sqrt{2}a^3d}+\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} \\ &\quad+\frac{(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{12ad(ia+a\cot(c+dx))^2}+\frac{5A\sqrt{\cot(c+dx)}}{8d\left(ia^3+a^3\cot(c+dx)\right)}\end{aligned}$$

output
```
1/32*((-7+5*I)*A+2*I*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a^3/d+
1/32*((-7+5*I)*A+2*I*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a^3/d+1
/32*((7+5*I)*A-2*I*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(
1/2)/a^3/d+1/6*(A+I*B)*cot(d*x+c)^(5/2)/d/(I*a+a*cot(d*x+c))^3+1/12*(4*A+I
*B)*cot(d*x+c)^(3/2)/a/d/(I*a+a*cot(d*x+c))^2+5/8*A*cot(d*x+c)^(1/2)/d/(I*
a^3+a^3*cot(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 2.97 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{i\sqrt{\cot(c+dx)}\sec^3(c+dx)\left(12\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)(\cos(3(c+dx))+i\sin$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^3,x]
```

output
```
((-1/96*I)*Sqrt[Cot[c + d*x]]*Sec[c + d*x]^3*(12*(-1)^(1/4)*(A - I*B)*ArcT
an[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])
+ 12*(-1)^(1/4)*(6*A - I*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(Cos[3*
(c + d*x)] + I*Sin[3*(c + d*x)]) - 4*Cos[c + d*x]*(6*A + (3*I)*B + 3*(7*A
+ I*B)*Cos[2*(c + d*x)] + ((19*I)*A - B)*Sin[2*(c + d*x)])*Sqrt[Tan[c + d*
x]])*Sqrt[Tan[c + d*x]])/(a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.09, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3042, 4064, 3042, 4078, 27, 3042, 4078, 27, 3042, 4078, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A\cot(c+dx)+B)}{(a\cot(c+dx)+ia)^3}dx$$

$\Big\downarrow$ 3042

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3}dx$$

$\Big\downarrow$ 4078

$$\frac{\int -\frac{\cot^{\frac{3}{2}}(c+dx)(5a(iA-B)-a(11A-iB)\cot(c+dx))}{2(\cot(c+dx)a+ia)^2}dx}{6a^2}+\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}$$

$\Big\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)(5a(iA-B)-a(11A-iB)\cot(c+dx))}{(\cot(c+dx)a+ia)^2}dx}{12a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(5a(iA-B)+a(11A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{12a^2}$$

$\Big\downarrow$ 4078

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\int \frac{6\sqrt{\cot(c+dx)}\left(a^2(4iA-B)-a^2(6A-iB)\cot(c+dx)\right)}{\cot(c+dx)a+ia}dx}{4a^2}-\frac{a(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$\Big\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\int \frac{\sqrt{\cot(c+dx)}\left(a^2(4iA-B)-a^2(6A-iB)\cot(c+dx)\right)}{\cot(c+dx)a+ia}dx}{2a^2}-\frac{a(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$\Big\downarrow$ 3042

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left((4iA-B)a^2+(6A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)a^2\right)}{ia-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{2a^2}-\frac{a(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$\Big\downarrow$ 4078

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{\int\frac{5ia^3A-a^3(7A-2iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{2a^2}-\frac{5a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{\int\frac{5iAa^3+(7A-2iB)\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{\int-\frac{a^3(5iA-(7A-2iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{5a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(-\frac{\int\frac{a^3(5iA-(7A-2iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{5a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(-\frac{a\int\frac{5iA-(7A-2iB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{5a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(4A+iB)\cot^{\frac{3}{2}}(c+dx)}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{1482}$$

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(-\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2iB-(7-5i)A)\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}-\frac{5a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(4A+iB)\cot^{\frac{3}{2}}(c+d}{d(a\cot(c+dx)+ia)^}}{12a^2}$$

$\downarrow$ 1476

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(-\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2iB-(7-5i)A)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int\frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+}\right.}{d}}{2a^2}}{12a^2}$$

$\downarrow$ 1082

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(-\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2iB-(7-5i)A)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}}{2a^2}}{12a^2}$$

$\downarrow$ 217

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(-\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2iB-(7-5i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}-\frac{5a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right.}{2a^2}}{12a^2}$$

$\downarrow$ 1479

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(-\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(2iB-(7-5i)A)\left(\frac{\arctan\left(\sqrt{}\right.}{}\right.\right.}{d}}{2a^2}}{12a^2}$$

$\Big\downarrow$ 25

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{3\left(-\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(2iB-(7-5i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{c\cdots}\right.}{\sqrt{\cdots}}\right.}{d}\right.}{2a^2}}{12a^2}$$

$\Big\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{3\left(-\frac{a\left(\frac{1}{2}((7+5i)A-2iB)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}(2iB-(7-5i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cdots}\right.}{\cdots}\right.}{d}\right.}{2a^2}}{12a^2}$$

$\Big\downarrow$ 1103

$$\frac{(A+iB)\cot^{\frac{5}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{3\left(-\frac{5a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}-\frac{a\left(\frac{1}{2}(2iB-(7-5i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}((7+5i)A-2iB)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx}\right.}{2\sqrt{2}}\right.}{d}\right.}{2a^2}}{12a^2}$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^3,x]
```

output
```
((A + I*B)*Cot[c + d*x]^(5/2))/(6*d*(I*a + a*Cot[c + d*x])^3) - (-((a*(4*A
 + I*B)*Cot[c + d*x]^(3/2))/(d*(I*a + a*Cot[c + d*x])^2)) + (3*((-5*a^2*A*
Sqrt[Cot[c + d*x]])/(d*(I*a + a*Cot[c + d*x])) - (a*((((-7 + 5*I)*A + (2*I
)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[
2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 + (((7 + 5*I)*A - (2*I)*B)*(-1/2*Log[1
- Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqr
t[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d))/(2*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.60

method	result
derivativedivides	$\frac{\frac{i\left(\frac{-i(2iB+9A)\cot(dx+c)^{\frac{5}{2}}+\left(\frac{2iB}{3}+\frac{38A}{3}\right)\cot(dx+c)^{\frac{3}{2}}+5iA\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^{3}}+\frac{2(6iA+B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{8}+\frac{4\left(-\frac{A}{16}+\frac{iB}{16}\right)a\cdots}{\cdots}}{a^{3}d}$
default	$\frac{\frac{i\left(\frac{-i(2iB+9A)\cot(dx+c)^{\frac{5}{2}}+\left(\frac{2iB}{3}+\frac{38A}{3}\right)\cot(dx+c)^{\frac{3}{2}}+5iA\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^{3}}+\frac{2(6iA+B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{\sqrt{2}+i\sqrt{2}}\right)}{8}+\frac{4\left(-\frac{A}{16}+\frac{iB}{16}\right)a\cdots}{\cdots}}{a^{3}d}$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^3/d*(1/8*I*((-I*(2*I*B+9*A)*cot(d*x+c)^(5/2)+(2/3*I*B+38/3*A)*cot(d*x+
c)^(3/2)+5*I*A*cot(d*x+c)^(1/2))/(I+cot(d*x+c))^3+2*(6*I*A+B)/(2^(1/2)+I*2
^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))+4*(-1/16*A+1/16*I*
B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(209) = 418.

Time = 0.10 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.55

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
1/96*(3*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)
*log(-2*((a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) +
(A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*s
qrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(2*((a^3*d*
e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) - (A - I*B)*e^(2*I
*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*sqrt((36*I*A^2 +
12*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*((a^3*d*e^(2*I*d*x
 + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
 1))*sqrt((36*I*A^2 + 12*A*B - I*B^2)/(a^6*d^2)) + 6*A - I*B)*e^(-2*I*d*x
- 2*I*c)/(a^3*d)) + 3*a^3*d*sqrt((36*I*A^2 + 12*A*B - I*B^2)/(a^6*d^2))*e^
(6*I*d*x + 6*I*c)*log(1/8*((a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((36*I*A^2 + 12*A*B -
 I*B^2)/(a^6*d^2)) - 6*A + I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 2*(2*(10*I
*A - B)*e^(6*I*d*x + 6*I*c) - (14*I*A + B)*e^(4*I*d*x + 4*I*c) - (5*I*A -
2*B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\left(\int \frac{A\sqrt{\cot(c+dx)}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx+\int \frac{B\tan(c+dx)\sqrt{\cot(c+dx)}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx\right)}{a^3}$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*(Integral(A*sqrt(cot(c + d*x))/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 -
3*tan(c + d*x) + I), x) + Integral(B*tan(c + d*x)*sqrt(cot(c + d*x))/(tan(
c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c + d*x) + I), x))/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{3\sqrt{2}((6i-6)\ A+(i+1)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}(-(i+1)\ A+(i-1)\ B)\,a$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*(3*sqrt(2)*((6*I - 6)*A + (I + 1)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sq
rt(tan(d*x + c))) + 3*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-(1/2*I - 1/
2)*sqrt(2)*sqrt(tan(d*x + c))) - 2*(-15*I*A*tan(d*x + c)^(5/2) - 38*A*tan(
d*x + c)^(3/2) - 2*I*B*tan(d*x + c)^(3/2) + 27*I*A*sqrt(tan(d*x + c)) - 6*
B*sqrt(tan(d*x + c)))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)}{{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^3}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3,x)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^3, x
)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^3}\,dx = \frac{-\left(\int \frac{\sqrt{\cot(dx+c)}}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)a-\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\tan(dx+c)^3 i+3\tan(dx+c)^2-3\tan(dx+c)i-1}dx\right)b}{a^3}$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int(sqrt(cot(c + d*x))/(tan(c + d*x)**3*i + 3*tan(c + d*x)**2 - 3*tan
(c + d*x)*i - 1),x)*a + int((sqrt(cot(c + d*x))*tan(c + d*x))/(tan(c + d*x
)**3*i + 3*tan(c + d*x)**2 - 3*tan(c + d*x)*i - 1),x)*b))/a**3
```

3.536 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 266

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3}\,dx &= -\frac{\left(\frac{1}{16}+\frac{i}{16}\right)((1+i)A+B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^3d} \\ &+\frac{\left(\frac{1}{16}+\frac{i}{16}\right)((1+i)A+B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^3d} \\ &-\frac{(2iA+(1-i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{16\sqrt{2}a^3d}+\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(ia+a\cot(c+dx))^3} \\ &+\frac{A\sqrt{\cot(c+dx)}}{4ad(ia+a\cot(c+dx))^2}+\frac{(2iA+B)\sqrt{\cot(c+dx)}}{8d\left(ia^3+a^3\cot(c+dx)\right)}\end{aligned}$$

output
```
(1/32+1/32*I)*((1+I)*A+B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a^3/
d+(1/32+1/32*I)*((1+I)*A+B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a^3
/d-1/32*(2*I*A+(1-I)*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2
^(1/2)/a^3/d+1/6*(A+I*B)*cot(d*x+c)^(3/2)/d/(I*a+a*cot(d*x+c))^3+1/4*A*cot
(d*x+c)^(1/2)/a/d/(I*a+a*cot(d*x+c))^2+1/8*(2*I*A+B)*cot(d*x+c)^(1/2)/d/(I
*a^3+a^3*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 3.80 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.74

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(3\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^3(c+dx)(\cos(3(c+}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^
3),x]
```

output
```
-1/24*(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(3*(-1)^(1/4)*(A - I*B)*ArcTa
n[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^3*(Cos[3*(c + d*x)] + I*Sin[
3*(c + d*x)]) + 3*(-1)^(1/4)*A*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[
c + d*x]^3*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) + Sqrt[Tan[c + d*x]]*(-
3*I + Tan[c + d*x])*((2*I)*A + B + (3*I)*B*Tan[c + d*x])))/(a^3*d*(-I + Ta
n[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.09, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {3042, 4064, 3042, 4078, 27, 3042, 4078, 27, 3042, 4079, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A\cot(c+dx)+B)}{(a\cot(c+dx)+ia)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3}dx$$

$$\Big\downarrow \text{ 4078}$$

$$\frac{\int -\frac{3\sqrt{\cot(c+dx)}(a(iA-B)-a(3A-iB)\cot(c+dx))}{2(\cot(c+dx)a+ia)^2}dx}{6a^2}+\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\int \frac{\sqrt{\cot(c+dx)}(a(iA-B)-a(3A-iB)\cot(c+dx))}{(\cot(c+dx)a+ia)^2}dx}{4a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(a(iA-B)+a(3A-iB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a^2}$$

$$\Big\downarrow \text{ 4078}$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\int \frac{2\left(ia^2A-a^2(3A-2iB)\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)}dx}{4a^2}-\frac{aA\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{4a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\int \frac{ia^2A-a^2(3A-2iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)}dx}{2a^2}-\frac{aA\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{4a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\int \frac{iAa^2+(3A-2iB)\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^2}-\frac{aA\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{4a^2}$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\frac{\int \frac{iBa^3+(2iA+B)\cot(c+dx)a^3}{\sqrt{\cot(c+dx)}}dx}{2a^2}-\frac{a^2(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{aA\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{4a^2}$$

$\downarrow$ 3042

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\frac{\int \frac{ia^3B-a^3(2iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{a^2(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{aA\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{4a^2}$$

$\downarrow$ 4017

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\frac{\int -\frac{a^3(iB+(2iA+B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{a^2(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{aA\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{4a^2}$$

$\downarrow$ 25

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{-\frac{\int \frac{a^3(iB+(2iA+B)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{a^2(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{aA\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{4a^2}$$

$\downarrow$ 27

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{-\frac{a\int \frac{iB+(2iA+B)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{a^2(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{aA\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{4a^2}$$

$\downarrow$ 1482

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{-\frac{a\left(\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(2iA+(1-i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}-\frac{a^2(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}}{2a^2}-\frac{aA\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{4a^2}$$

$\downarrow$ 1476

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{-\frac{a\left(\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)-\frac{1}{2}(2iA+(1-i)B)\int \frac{1-\cot(c+dx}{\cot^2(c+dx)+}\right)}{d}}{2a^2}}{4a^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3} - \frac{-\frac{\frac{a\left(\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}(2iA+(1-i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}}{2a^2}}{4a^2}$$

$$\Big\downarrow \; 217$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3} - \frac{-\frac{\frac{a\left(\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(2iA+(1-i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}}{2a^2}-\frac{a^2(B+2iA)\sqrt{\cot(c+}}{d(a\cot(c+dx)+ia}}{4a^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3} - \frac{-\frac{\frac{a\left(\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(2iA+(1-i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int}{}\right.}{d}}{2a^2}}{4a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3} - \frac{-\frac{\frac{a\left(\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(2iA+(1-i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{}{\cot}}{}\right.}{d}}{2a^2}}{4a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3} - \frac{-\frac{\frac{a\left(\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(2iA+(1-i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{}{c}\right.}{d}}{2a^2}}{4a^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{6d(a\cot(c+dx)+ia)^3}-\frac{-\frac{a^2(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}-\frac{a\left(\left(\frac{1}{2}+\frac{i}{2}\right)(B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(2iA+(1-i)B)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c-\right.}}{2\sqrt{2}}\right.\right.}{d}}{2a^2}}{4a^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
((A + I*B)*Cot[c + d*x]^(3/2))/(6*d*(I*a + a*Cot[c + d*x])^3) - (-((a*A*Sq
rt[Cot[c + d*x]])/(d*(I*a + a*Cot[c + d*x])^2)) + (-((a^2*((2*I)*A + B)*Sq
rt[Cot[c + d*x]])/(d*(I*a + a*Cot[c + d*x]))) - (a*((1/2 + I/2)*((1 + I)*A
 + B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) - (((2*I)*A + (1 - I)*B)*(-1/2*Log[1 - Sq
rt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Co
t[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d)/(2*a^2))/(4*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.58

method	result
derivativedivides	$\frac{\frac{4\left(\frac{iA}{16}+\frac{B}{16}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{i\left(\frac{-i(2iA+B)\cot(dx+c)^{\frac{5}{2}}+\left(\frac{2iA}{3}+\frac{10B}{3}\right)\cot(dx+c)^{\frac{3}{2}}+iB\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^{3}}+\frac{2A\arctan\left(\frac{2\sqrt{\cot}}{\sqrt{2}-}\right.}{\sqrt{2}+i\sqrt{2}}\right.}{8}}{a^{3}d}$
default	$\frac{\frac{4\left(\frac{iA}{16}+\frac{B}{16}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}+\frac{i\left(\frac{-i(2iA+B)\cot(dx+c)^{\frac{5}{2}}+\left(\frac{2iA}{3}+\frac{10B}{3}\right)\cot(dx+c)^{\frac{3}{2}}+iB\sqrt{\cot(dx+c)}}{(i+\cot(dx+c))^{3}}+\frac{2A\arctan\left(\frac{2\sqrt{\cot}}{\sqrt{2}-}\right.}{\sqrt{2}+i\sqrt{2}}\right.}{8}}{a^{3}d}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^3/d*(4*(1/16*I*A+1/16*B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)
/(2^(1/2)-I*2^(1/2)))+1/8*I*((-I*(2*I*A+B)*cot(d*x+c)^(5/2)+(2/3*I*A+10/3*
B)*cot(d*x+c)^(3/2)+I*B*cot(d*x+c)^(1/2))/(I+cot(d*x+c))^3+2*A/(2^(1/2)+I*
2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(209) = 418$.

Time = 0.10 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.39

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + ia\tan(c + dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
1/96*(3*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*
log(-2*((I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))
+ (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d
*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*((-I*a
^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) + (A - I*B)*
e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + 24*a^3*d*sqrt(-1/64
*I*A^2/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*(8*(a^3*d*e^(2*I*d*x + 2*I*c
) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqr
t(-1/64*I*A^2/(a^6*d^2)) + I*A)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 24*a^3*d*s
qrt(-1/64*I*A^2/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*(8*(a^3*d*e^(2*I*d
*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 - 1))*sqrt(-1/64*I*A^2/(a^6*d^2)) - I*A)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) +
2*(2*(A - 2*I*B)*e^(6*I*d*x + 6*I*c) + (A + 4*I*B)*e^(4*I*d*x + 4*I*c) - (
2*A - I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)
/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{A + B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{i\left(\int \frac{A}{\tan^3(c+dx)\sqrt{\cot(c+dx)}-3i\tan^2(c+dx)\sqrt{\cot(c+dx)}-3\tan(c+dx)\sqrt{\cot(c+dx)}+i\sqrt{\cot(c+dx)}}\,dx + \int \frac{}{\tan^3(c+dx)\sqrt{\cot(c+dx)}-}\right.}{a^3}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
I*(Integral(A/(tan(c + d*x)**3*sqrt(cot(c + d*x)) - 3*I*tan(c + d*x)**2*sq
rt(cot(c + d*x)) - 3*tan(c + d*x)*sqrt(cot(c + d*x)) + I*sqrt(cot(c + d*x)
)), x) + Integral(B*tan(c + d*x)/(tan(c + d*x)**3*sqrt(cot(c + d*x)) - 3*I
*tan(c + d*x)**2*sqrt(cot(c + d*x)) - 3*tan(c + d*x)*sqrt(cot(c + d*x)) +
I*sqrt(cot(c + d*x))), x))/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.46

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{(3i+3)\,\sqrt{2}A\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\,\sqrt{2}((i-1)\,A+(i+1)\,B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\right.}{48\,a^3d}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*((3*I + 3)*sqrt(2)*A*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(tan(d*x + c))
) + 3*sqrt(2)*((I - 1)*A + (I + 1)*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(t
an(d*x + c))) - 2*(-3*I*B*tan(d*x + c)^(5/2) - 2*I*A*tan(d*x + c)^(3/2) -
10*B*tan(d*x + c)^(3/2) - 6*A*sqrt(tan(d*x + c)) + 3*I*B*sqrt(tan(d*x + c)
))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot\left(c+d\,x\right)}\,{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^3), x
)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)\tan(dx+c)^3 i+3\cot(dx+c)\tan(dx+c)^2-3\cot(dx+c)\tan(dx+c)i-\cot(dx+c)}dx\right)a-\left(\int \frac{}{\cot(dx+c)\tan(dx+c)^3 i+3\cot(}\right.}{a^3}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int(sqrt(cot(c + d*x))/(cot(c + d*x)*tan(c + d*x)**3*i + 3*cot(c + d*
x)*tan(c + d*x)**2 - 3*cot(c + d*x)*tan(c + d*x)*i - cot(c + d*x)),x)*a +
int((sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)*tan(c + d*x)**3*i + 3*
cot(c + d*x)*tan(c + d*x)**2 - 3*cot(c + d*x)*tan(c + d*x)*i - cot(c + d*x
)),x)*b))/a**3
```

3.537 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 260

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = -\frac{((1+i)A+2B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{((1+i)A+2B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{((-1+i)A+2B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{16\sqrt{2}a^3d} + \frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3} + \frac{(2iA+B)\sqrt{\cot(c+dx)}}{12ad(ia+a\cot(c+dx))^2} + \frac{A\sqrt{\cot(c+dx)}}{8d\left(ia^3+a^3\cot(c+dx)\right)}$$

output
```
1/32*((1+I)*A+2*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a^3/d+1/32*
((1+I)*A+2*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a^3/d+1/32*((-1+I
)*A+2*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/a^3/d+1/
6*(A+I*B)*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))^3+1/12*(2*I*A+B)*cot(d*x+c
)^(1/2)/a/d/(I*a+a*cot(d*x+c))^2+1/8*A*cot(d*x+c)^(1/2)/d/(I*a^3+a^3*cot(d
*x+c))
```

Mathematica [A] (verified)

Time = 2.89 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.78

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(3\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)\sec^3(c+dx)(\cos(3(c+dx)}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^
3),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(3*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)
^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^3*(Cos[3*(c + d*x)] + I*Sin[3*(c +
 d*x)]) - 3*(-1)^(1/4)*B*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*
x]^3*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - I*Sqrt[Tan[c + d*x]]*(-I +
3*Tan[c + d*x])*((-3*I)*A + (A - (2*I)*B)*Tan[c + d*x])))/(24*a^3*d*(-I +
Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.10, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3042, 4064, 3042, 4078, 27, 3042, 4079, 27, 3042, 4079, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A\cot(c+dx)+B)}{(a\cot(c+dx)+ia)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3}dx$$

$$\Big\downarrow \text{4078}$$

$$\frac{\int -\frac{a(iA-B)-a(7A-5iB)\cot(c+dx)}{2\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)^2}dx}{6a^2}+\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\int \frac{a(iA-B)-a(7A-5iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)^2}dx}{12a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\int \frac{a(iA-B)+a(7A-5iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{12a^2}$$

$$\Big\downarrow \text{4079}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{\int \frac{6\left(iBa^2+(2iA+B)\cot(c+dx)a^2\right)}{\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)}dx}{4a^2}-\frac{a(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\int \frac{iBa^2+(2iA+B)\cot(c+dx)a^2}{\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)}dx}{2a^2}-\frac{a(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\int \frac{ia^2B-a^2(2iA+B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^2}-\frac{a(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{4079}$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{\int \frac{(iA+2B)a^3+A\cot(c+dx)a^3}{\sqrt{\cot(c+dx)}}dx}{2a^2}-\frac{a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{\int\frac{a^3(iA+2B)-a^3A\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \; 4017$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{\int-\frac{a^3(\cot(c+dx)A+iA+2B)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(-\frac{\int\frac{a^3(\cot(c+dx)A+iA+2B)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(-\frac{a\int\frac{\cot(c+dx)A+iA+2B}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \; 1482$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(-\frac{a\left(\frac{1}{2}(2B-(1-i)A)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2B+(1+i)A)\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}-\frac{a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}-\frac{a(B+2iA)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2B-(1-i)A)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2B+(1+i)A)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int\frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cdots}\right.\right.}{d}\cdots}{2a^2}\cdots}{12a^2}$$

$\downarrow$ 1082

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2B-(1-i)A)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2B+(1+i)A)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}\cdots}{2a^2}\cdots}{12a^2}$$

$\downarrow$ 217

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2B-(1-i)A)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}-\frac{a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}\cdots}{12a^2}$$

$\downarrow$ 1479

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2B-(1-i)A)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(2B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{\cdots}\right.\right.}{d}\cdots}{2a^2}\cdots}{12a^2}$$

$\downarrow$ 25

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2B-(1-i)A)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(2B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c\right.}{\sqrt{2}}\right.}{d}\right.}{2a^2}}{12a^2}$$

$\downarrow$ 27

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2B-(1-i)A)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}(2B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(\right.}{\sqrt{2}}\right.}{d}\right.}{2a^2}}{12a^2}$$

$\downarrow$ 1103

$$\frac{(A+iB)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}-\frac{3\left(-\frac{a^2A\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}-\frac{a\left(\frac{1}{2}(2B+(1+i)A)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(2B-(1-i)A)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right.}{2\sqrt{2}}\right.}{d}\right.}{2a^2}}{12a^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
((A + I*B)*Sqrt[Cot[c + d*x]])/(6*d*(I*a + a*Cot[c + d*x])^3) - (-((a*((2*
I)*A + B)*Sqrt[Cot[c + d*x]])/(d*(I*a + a*Cot[c + d*x])^2)) + (3*(-((a^2*A
*Sqrt[Cot[c + d*x]])/(d*(I*a + a*Cot[c + d*x]))) - (a*((((1 + I)*A + 2*B)*
(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]]]/Sqrt[2]))/2 + (((-1 + I)*A + 2*B)*(-1/2*Log[1 - Sqrt[2]*
Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d))/(2*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.58

method	result
derivativedivides	$\frac{\frac{4\left(\frac{A}{16}-\frac{iB}{16}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}-\frac{-A\cot(dx+c)^{\frac{5}{2}}+\left(-\frac{10iA}{3}-\frac{2B}{3}\right)\cot(dx+c)^{\frac{3}{2}}+(-2iB+A)\sqrt{\cot(dx+c)}}{8(i+\cot(dx+c))^{3}}+\frac{iB\arctan\left(\frac{2\sqrt{\cot(}}{\sqrt{2}+}\right.}{4\sqrt{2}+4i\sqrt{2}}}{a^{3}d}$
default	$\frac{\frac{4\left(\frac{A}{16}-\frac{iB}{16}\right)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}-i\sqrt{2}}\right)}{\sqrt{2}-i\sqrt{2}}-\frac{-A\cot(dx+c)^{\frac{5}{2}}+\left(-\frac{10iA}{3}-\frac{2B}{3}\right)\cot(dx+c)^{\frac{3}{2}}+(-2iB+A)\sqrt{\cot(dx+c)}}{8(i+\cot(dx+c))^{3}}+\frac{iB\arctan\left(\frac{2\sqrt{\cot(}}{\sqrt{2}+}\right.}{4\sqrt{2}+4i\sqrt{2}}}{a^{3}d}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^3/d*(4*(1/16*A-1/16*I*B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)
/(2^(1/2)-I*2^(1/2)))-1/8*(-A*cot(d*x+c)^(5/2)+(-10/3*I*A-2/3*B)*cot(d*x+c
)^(3/2)+(A-2*I*B)*cot(d*x+c)^(1/2))/(I+cot(d*x+c))^3+1/4*I*B/(2^(1/2)+I*2^
(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(209) = 418.

Time = 0.09 (sec) , antiderivative size = 637, normalized size of antiderivative = 2.45

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
-1/96*(3*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c
)*log(-2*((a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) +
 (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*
sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(2*((a^3*d
*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) - (A - I*B)*e^(2*
I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 24*a^3*d*sqrt(-1/64*I*B^
2/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*(8*(a^3*d*e^(2*I*d*x + 2*I*c) - a
^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-1/
64*I*B^2/(a^6*d^2)) + I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) + 24*a^3*d*sqrt(-
1/64*I*B^2/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*(8*(a^3*d*e^(2*I*d*x +
2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
)*sqrt(-1/64*I*B^2/(a^6*d^2)) - I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) + 2*(2*
(2*I*A + B)*e^(6*I*d*x + 6*I*c) - (4*I*A + 5*B)*e^(4*I*d*x + 4*I*c) - (I*A
 - 4*B)*e^(2*I*d*x + 2*I*c) + I*A - B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) - 1)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)\,(a + ia\tan(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.47

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{(3i+3)\sqrt{2}B\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\tan(dx+c)}\right)+3\sqrt{2}((i+1)A-(i-1)B)\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right.\right.}{48a^3d}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*((3*I + 3)*sqrt(2)*B*arctan((1/2*I + 1/2)*sqrt(2)*sqrt(tan(d*x + c))
) + 3*sqrt(2)*((I + 1)*A - (I - 1)*B)*arctan(-(1/2*I - 1/2)*sqrt(2)*sqrt(t
an(d*x + c))) - 2*(-3*I*A*tan(d*x + c)^(5/2) - 6*B*tan(d*x + c)^(5/2) - 10
*A*tan(d*x + c)^(3/2) + 2*I*B*tan(d*x + c)^(3/2) + 3*I*A*sqrt(tan(d*x + c)
))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \int \frac{A+B\tan\left(c+d\,x\right)}{{\mathrm{cot}\left(c+d\,x\right)}^{3/2}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1\mathrm{i}\right)}^3}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^3), x
)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)^2\tan(dx+c)^3 i+3\cot(dx+c)^2\tan(dx+c)^2-3\cot(dx+c)^2\tan(dx+c)i-\cot(dx+c)^2}dx\right)a-\left(\int \frac{}{\cot(dx+c)^2\tan(dx+c)^3 i+}\right.}{a^3}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int(sqrt(cot(c + d*x))/(cot(c + d*x)**2*tan(c + d*x)**3*i + 3*cot(c +
 d*x)**2*tan(c + d*x)**2 - 3*cot(c + d*x)**2*tan(c + d*x)*i - cot(c + d*x)
**2),x)*a + int((sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)**2*tan(c +
 d*x)**3*i + 3*cot(c + d*x)**2*tan(c + d*x)**2 - 3*cot(c + d*x)**2*tan(c +
 d*x)*i - cot(c + d*x)**2),x)*b))/a**3
```

3.538 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 262

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx &= -\frac{(2A+(5-7i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d}\\ &\quad+\frac{(2A+(5-7i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d}\\ &\quad+\frac{(2A-(5+7i)B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{16\sqrt{2}a^3d}+\frac{(iA-B)\sqrt{\cot(c+dx)}}{6d(ia+a\cot(c+dx))^3}\\ &\quad+\frac{(A+4iB)\sqrt{\cot(c+dx)}}{12ad(ia+a\cot(c+dx))^2}+\frac{5B\sqrt{\cot(c+dx)}}{8d\left(ia^3+a^3\cot(c+dx)\right)}\end{aligned}$$

output
```
1/32*(2*A+(5-7*I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a^3/d+1/3
2*(2*A+(5-7*I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/a^3/d+1/32*(2
*A-(5+7*I)*B)*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/a^3
/d+1/6*(I*A-B)*cot(d*x+c)^(1/2)/d/(I*a+a*cot(d*x+c))^3+1/12*(A+4*I*B)*cot(
d*x+c)^(1/2)/a/d/(I*a+a*cot(d*x+c))^2+5/8*B*cot(d*x+c)^(1/2)/d/(I*a^3+a^3*
cot(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 3.74 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.83

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = -\frac{\sqrt{\cot(c+dx)}\sec^3(c+dx)\left(-3\sqrt[4]{-1}(A-iB)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)(\cos(3(c+dx))+i\sin(\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^
3),x]
```

output
```
-1/24*(Sqrt[Cot[c + d*x]]*Sec[c + d*x]^3*(-3*(-1)^(1/4)*(A - I*B)*ArcTan[(
-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) + 3*
(-1)^(1/4)*(A - (6*I)*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(Cos[3*(c
+ d*x)] + I*Sin[3*(c + d*x)]) + Cos[c + d*x]*(3*(A + (2*I)*B) - 3*(A + (7*
I)*B)*Cos[2*(c + d*x)] + ((-I)*A + 19*B)*Sin[2*(c + d*x)])*Sqrt[Tan[c + d*
x]])*Sqrt[Tan[c + d*x]])/(a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.10, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {3042, 4064, 3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{5/2}(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{A\cot(c+dx)+B}{\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3}dx$$
$$\Big\downarrow \text{ 4079}$$
$$\frac{\int \frac{a(A-11iB)-5a(iA-B)\cot(c+dx)}{2\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)^2}dx}{6a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{a(A-11iB)-5a(iA-B)\cot(c+dx)}{\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)^2}dx}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \frac{a(A-11iB)+5a(iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$
$$\Big\downarrow \text{ 4079}$$
$$\frac{\frac{\int -\frac{6\left((iA+6B)a^2+(A+4iB)\cot(c+dx)a^2\right)}{\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)}dx}{4a^2}+\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\int \frac{(iA+6B)a^2+(A+4iB)\cot(c+dx)a^2}{\sqrt{\cot(c+dx)}(\cot(c+dx)a+ia)}dx}{2a^2}}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\int \frac{a^2(iA+6B)-a^2(A+4iB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^2}}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$
$$\Big\downarrow \text{ 4079}$$
$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(\frac{\int \frac{(2A-7iB)a^3+5B\cot(c+dx)a^3}{\sqrt{\cot(c+dx)}}dx}{2a^2}-\frac{5a^2B\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(\frac{\int \frac{a^3(2A-7iB)-5a^3B\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{2a^2}-\frac{5a^2B\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$$\Big\downarrow \; 4017$$

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(\frac{\int -\frac{a^3(2A-7iB+5B\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{5a^2B\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(-\frac{\int \frac{a^3(2A-7iB+5B\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{a^2d}-\frac{5a^2B\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(-\frac{a\int \frac{2A-7iB+5B\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{5a^2B\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$$\Big\downarrow \; 1482$$

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2A-(5+7i)B)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2A+(5-7i)B)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}-\frac{5a^2B\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}\right)}{2a^2}}{12a^2}+\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2A-(5+7i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2A+(5-7i)B)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int\right.\right.}{d}}{2a^2}}{12a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$\downarrow$ 1082

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2A-(5+7i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2A+(5-7i)B)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\cot(c+}}{}\right.\right.}{d}}{2a^2}}{12a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$\downarrow$ 217

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2A-(5+7i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(2A+(5-7i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+d}\right.}{\sqrt{2}}\right.\right.}{d}}{2a^2}}{12a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$\downarrow$ 1479

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2A-(5+7i)B)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)\right.}{d}}{2a^2}}{12a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$\downarrow$ 25

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2A-(5+7i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(2\cdots}{d}\right.}{2a^2}}{12a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$\downarrow$ 27

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(-\frac{a\left(\frac{1}{2}(2A-(5+7i)B)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}\cdots}{d}\right.}{2a^2}}{12a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

$\downarrow$ 1103

$$\frac{\frac{a(A+4iB)\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)^2}-\frac{3\left(-\frac{5a^2B\sqrt{\cot(c+dx)}}{d(a\cot(c+dx)+ia)}-\frac{a\left(\frac{1}{2}(2A+(5-7i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(2A-(5+7i)B)\left(\cdots\right.}{d}\right.}{2a^2}}{12a^2}$$

$$\frac{(-B+iA)\sqrt{\cot(c+dx)}}{6d(a\cot(c+dx)+ia)^3}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
((I*A - B)*Sqrt[Cot[c + d*x]])/(6*d*(I*a + a*Cot[c + d*x])^3) + ((a*(A + (
4*I)*B)*Sqrt[Cot[c + d*x]])/(d*(I*a + a*Cot[c + d*x])^2) - (3*((-5*a^2*B*S
qrt[Cot[c + d*x]])/(d*(I*a + a*Cot[c + d*x])) - (a*(((2*A + (5 - 7*I)*B)*(
-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqr
t[Cot[c + d*x]]]/Sqrt[2]))/2 + ((2*A - (5 + 7*I)*B)*(-1/2*Log[1 - Sqrt[2]*
Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d))/(2*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.60

method	result
derivativedivides	$\frac{\frac{5B\cot(dx+c)^{\frac{5}{2}}+\left(\frac{38iB}{3}+\frac{2A}{3}\right)\cot(dx+c)^{\frac{3}{2}}+(2iA-9B)\sqrt{\cot(dx+c)}}{8(i+\cot(dx+c))^3}+\frac{(iA+6B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{4\sqrt{2}+4i\sqrt{2}}+\frac{4\left(-\frac{iA}{16}-\frac{B}{16}\right)\arctan\left(\frac{2\sqrt{}}{}\right.}{\sqrt{2}-i\sqrt{2}}}{a^3d}$
default	$\frac{\frac{5B\cot(dx+c)^{\frac{5}{2}}+\left(\frac{38iB}{3}+\frac{2A}{3}\right)\cot(dx+c)^{\frac{3}{2}}+(2iA-9B)\sqrt{\cot(dx+c)}}{8(i+\cot(dx+c))^3}+\frac{(iA+6B)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{4\sqrt{2}+4i\sqrt{2}}+\frac{4\left(-\frac{iA}{16}-\frac{B}{16}\right)\arctan\left(\frac{2\sqrt{}}{}\right.}{\sqrt{2}-i\sqrt{2}}}{a^3d}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^3/d*(1/8*(5*B*cot(d*x+c)^(5/2)+(38/3*I*B+2/3*A)*cot(d*x+c)^(3/2)+(2*I*
A-9*B)*cot(d*x+c)^(1/2))/(I+cot(d*x+c))^3+1/4*(I*A+6*B)/(2^(1/2)+I*2^(1/2)
)*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))+4*(-1/16*I*A-1/16*B)/(2^(
1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)-I*2^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(207) = 414.

Time = 0.10 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.61

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
-1/96*(3*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)
*log(-2*((I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))
 + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*
d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(-2*((-I*
a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^6*d^2)) + (A - I*B)
*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(I*A + B)) - 3*a^3*d*sqrt((-I*A
^2 - 12*A*B + 36*I*B^2)/(a^6*d^2))*e^(6*I*d*x + 6*I*c)*log(1/8*((a^3*d*e^(
2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) - 1))*sqrt((-I*A^2 - 12*A*B + 36*I*B^2)/(a^6*d^2)) + I*A + 6*B)*e^(-
2*I*d*x - 2*I*c)/(a^3*d)) + 3*a^3*d*sqrt((-I*A^2 - 12*A*B + 36*I*B^2)/(a^6
*d^2))*e^(6*I*d*x + 6*I*c)*log(-1/8*((a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*s
qrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 -
12*A*B + 36*I*B^2)/(a^6*d^2)) - I*A - 6*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) +
 2*(2*(A + 10*I*B)*e^(6*I*d*x + 6*I*c) - (5*A + 26*I*B)*e^(4*I*d*x + 4*I*c
) + (4*A + 7*I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.49

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{3\sqrt{2}((i+1)\ A-(6i-6)\ B)\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right)+3\sqrt{2}(-(i-1)\ A-(i+1)\ B)\,\mathrm{a}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*(3*sqrt(2)*((I + 1)*A - (6*I - 6)*B)*arctan((1/2*I + 1/2)*sqrt(2)*sq
rt(tan(d*x + c))) + 3*sqrt(2)*(-(I - 1)*A - (I + 1)*B)*arctan(-(1/2*I - 1/
2)*sqrt(2)*sqrt(tan(d*x + c))) + 2*(6*A*tan(d*x + c)^(5/2) + 27*I*B*tan(d*
x + c)^(5/2) - 2*I*A*tan(d*x + c)^(3/2) + 38*B*tan(d*x + c)^(3/2) - 15*I*B
*sqrt(tan(d*x + c)))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx=\int \frac{A+B\tan\left(c+d\,x\right)}{{\mathrm{cot}\left(c+d\,x\right)}^{5/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^3), x
)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$$
$$=\frac{-\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)^3\tan(dx+c)^3i+3\cot(dx+c)^3\tan(dx+c)^2-3\cot(dx+c)^3\tan(dx+c)i-\cot(dx+c)^3}dx\right)a-\left(\int \frac{}{\cot(dx+c)^3\tan(dx+c)^3i+}\right.}{a^3}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int(sqrt(cot(c + d*x))/(cot(c + d*x)**3*tan(c + d*x)**3*i + 3*cot(c +
 d*x)**3*tan(c + d*x)**2 - 3*cot(c + d*x)**3*tan(c + d*x)*i - cot(c + d*x)
**3),x)*a + int((sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)**3*tan(c +
 d*x)**3*i + 3*cot(c + d*x)**3*tan(c + d*x)**2 - 3*cot(c + d*x)**3*tan(c +
 d*x)*i - cot(c + d*x)**3),x)*b))/a**3
```

3.539 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 313

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx \\
&= \frac{\left(\frac{1}{16}+\frac{i}{16}\right)((1+6i)A-(29+i)B)\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}a^3d} \\
&\quad+\frac{((5-7i)A+(28+30i)B)\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{16\sqrt{2}a^3d} \\
&\quad-\frac{\left(\frac{1}{16}+\frac{i}{16}\right)((6+i)A+(1+29i)B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}a^3d}+\frac{5(A+6iB)}{8a^3d\sqrt{\cot(c+dx)}} \\
&\quad+\frac{iA-B}{6d\sqrt{\cot(c+dx)}(ia+a\cot(c+dx))^3}+\frac{2A+5iB}{12ad\sqrt{\cot(c+dx)}(ia+a\cot(c+dx))^2} \\
&\quad-\frac{7(iA-4B)}{24d\sqrt{\cot(c+dx)}\,(ia^3+a^3\cot(c+dx))}
\end{aligned}$$

output
```
(-1/32-1/32*I)*((1+6*I)*A-(29+I)*B)*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^
(1/2)/a^3/d+1/32*((5-7*I)*A+(28+30*I)*B)*arctan(1+2^(1/2)*cot(d*x+c)^(1/2)
)*2^(1/2)/a^3/d-(1/32+1/32*I)*((6+I)*A+(1+29*I)*B)*arctanh(2^(1/2)*cot(d*x
+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/a^3/d+5/8*(A+6*I*B)/a^3/d/cot(d*x+c)^(1/
2)+1/6*(I*A-B)/d/cot(d*x+c)^(1/2)/(I*a+a*cot(d*x+c))^3+1/12*(2*A+5*I*B)/a/
d/cot(d*x+c)^(1/2)/(I*a+a*cot(d*x+c))^2-7/24*(I*A-4*B)/d/cot(d*x+c)^(1/2)/
(I*a^3+a^3*cot(d*x+c))
```

Mathematica [A] (warning: unable to verify)

Time = 4.66 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.77

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \frac{\sqrt{\cot(c+dx)}\sec^3(c+dx)\left(-12\sqrt[4]{-1}(iA+B)\arctan\left((-1)^{3/4}\sqrt{\tan(c+dx)}\right)(\cos(3(c+dx))+i\sin(3\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^
3),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sec[c + d*x]^3*(-12*(-1)^(1/4)*(I*A + B)*ArcTan[(-1)^(
3/4)*Sqrt[Tan[c + d*x]]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) + 12*(-1)
^(3/4)*(6*A + (29*I)*B)*ArcTanh[(-1)^(3/4)*Sqrt[Tan[c + d*x]]]*(Cos[3*(c +
 d*x)] + I*Sin[3*(c + d*x)]) + (2*I)*((9*A + (33*I)*B)*Cos[c + d*x] + 21*(
A + (7*I)*B)*Cos[3*(c + d*x)] + (2*I)*(19*A + (97*I)*B + (19*A + (145*I)*B
)*Cos[2*(c + d*x)])*Sin[c + d*x])*Sqrt[Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/
(96*a^3*d*(-I + Tan[c + d*x])^3)
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.11, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.694$, Rules used = {3042, 4064, 3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4012, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{7/2}(a+ia\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{A\cot(c+dx)+B}{\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+ia)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(-a\tan\left(c+dx+\frac{\pi}{2}\right)+ia\right)^3}dx$$

$$\Big\downarrow \text{4079}$$

$$\frac{\int -\frac{a(A+13iB)+7a(iA-B)\cot(c+dx)}{2\cot^{\frac{3}{2}}(c+dx)(\cot(c+dx)a+ia)^2}dx}{6a^2}+\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{\int \frac{a(A+13iB)+7a(iA-B)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(\cot(c+dx)a+ia)^2}dx}{12a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{\int \frac{a(A+13iB)-7a(iA-B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{12a^2}$$

$$\Big\downarrow \text{4079}$$

$$\frac{\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{\frac{\int -\frac{2\left(a^2(4iA-31B)-5a^2(2A+5iB)\cot(c+dx)\right)}{\cot^{\frac{3}{2}}(c+dx)(\cot(c+dx)a+ia)}dx}{4a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}}{}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{-\frac{\int \frac{a^2(4iA-31B)-5a^2(2A+5iB)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(\cot(c+dx)a+ia)}dx}{2a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{-\frac{\int \frac{(4iA-31B)a^2+5(2A+5iB)\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(ia-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 4079}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{-\frac{\frac{\int \frac{3\left(5(A+6iB)a^3+7(iA-4B)\cot(c+dx)a^3\right)}{\cot^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{7a^2(-4B+iA)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{-\frac{\frac{3\int \frac{5(A+6iB)a^3+7(iA-4B)\cot(c+dx)a^3}{\cot^{\frac{3}{2}}(c+dx)}dx}{2a^2}-\frac{7a^2(-4B+iA)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{-\frac{\frac{3\int\frac{5a^3(A+6iB)-7a^3(iA-4B)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{2a^2}-\frac{7a^2(-4B+iA)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 4012}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{-\frac{\frac{3\left(\int\frac{7a^3(iA-4B)-5a^3(A+6iB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}\right)}{2a^2}-\frac{7a^2(-4B+iA)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{-\frac{\frac{3\left(\int\frac{7(iA-4B)a^3+5(A+6iB)\tan\left(c+dx+\frac{\pi}{2}\right)a^3}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}\right)}{2a^2}-\frac{7a^2(-4B+iA)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 4017}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{-\frac{\frac{3\left(\frac{2\int-\frac{a^3(7(iA-4B)-5(A+6iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}\right)}{2a^2}-\frac{7a^2(-4B+iA)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{-\frac{\frac{3\left(\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}-\frac{2\int\frac{a^3(7(iA-4B)-5(A+6iB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}\right)}{2a^2}-\frac{7a^2(-4B+iA)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a^3\int\frac{7(iA-4B)-5(A+6iB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}\right)}{2a^2}-\frac{7a^2(-4B+iA)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)}}{2a^2}-\frac{a(2A+5iB)}{d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^2}}{12a^2}$$

$\downarrow$ 1482

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+6i)A-(29+i)B)\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}\right)}{2a^2}-\frac{\ldots}{d\sqrt{\cot\ldots}}}{2a^2}\ldots}{12a^2}$$

$\downarrow$ 1476

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+6i)A-(29+i)B)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(\ldots}\right.\ldots}{d}\ldots}{2a^2}\ldots}{2a^2}\ldots}{12a^2}$$

$\downarrow$ 1082

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+6i)A-(29+i)B)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\ldots\right.\ldots}{d}\ldots}{2a^2}\ldots}{2a^2}\ldots}{12a^2}$$

$\downarrow$ 217

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-\frac{\frac{3\left(\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\left(\frac{1}{2}+\frac{i}{2}\right)((1+6i)A-(29+i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{\ldots}\right)}{\ldots}\right.\ldots}{d}\ldots}{2a^2}\ldots}{2a^2}\ldots}{12a^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-$$

$$-\frac{\frac{3\left(\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)\cdots}{d}\cdots\right)}{2a^2}\cdots}{2a^2}\cdots$$

$$\Big\downarrow \; 25$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-$$

$$-\frac{\frac{3\left(\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\left(\frac{1}{2}\cdots\right.}{d}\cdots\right)}{2a^2}\cdots}{2a^2}\cdots$$

$$\Big\downarrow \; 27$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-$$

$$-\frac{\frac{3\left(\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\left(\frac{1}{2}\cdots\right.}{d}\cdots\right)}{2a^2}\cdots}{2a^2}\cdots$$

$$\Big\downarrow \; 1103$$

$$\frac{-B+iA}{6d\sqrt{\cot(c+dx)}(a\cot(c+dx)+ia)^3}-$$

$$-\frac{\frac{3\left(\frac{10a^3(A+6iB)}{d\sqrt{\cot(c+dx)}}-\frac{2a^3\left(\left(\frac{1}{2}+\frac{i}{2}\right)((1+6i)A-(29+i)B)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)((6+i)A+(1+29i)B)\left(\frac{\log\left(\cot(c+\cdots\right.}{\cdots}\right.}{d}\cdots\right)}{2a^2}\cdots}{2a^2}\cdots}{12a^2}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^3),x]
```

output
```
(I*A - B)/(6*d*Sqrt[Cot[c + d*x]]*(I*a + a*Cot[c + d*x])^3) - (-((a*(2*A +
 (5*I)*B))/(d*Sqrt[Cot[c + d*x]]*(I*a + a*Cot[c + d*x])^2)) - ((-7*a^2*(I*
A - 4*B))/(d*Sqrt[Cot[c + d*x]]*(I*a + a*Cot[c + d*x])) + (3*((10*a^3*(A +
 (6*I)*B))/(d*Sqrt[Cot[c + d*x]]) - (2*a^3*((1/2 + I/2)*((1 + 6*I)*A - (29
 + I)*B)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + S
qrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + (1/2 + I/2)*((6 + I)*A + (1 + 29*I)*
B)*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[
1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]))))/d))/(2*a^2))
/(2*a^2))/(12*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{-i(5iA-14B)\cot(dx+c)^{\frac{5}{2}}+\left(\frac{38iA}{3}-\frac{98B}{3}\right)\cot(dx+c)^{\frac{3}{2}}+(-20iB-9A)\sqrt{\cot(dx+c)}}{8(i+\cot(dx+c))^{3}}+\frac{(29iB+6A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{4\sqrt{2}+4i\sqrt{2}}+\frac{4\left(-\frac{A}{16}\right.}{}}{a^{3}d}$
default	$\frac{\frac{-i(5iA-14B)\cot(dx+c)^{\frac{5}{2}}+\left(\frac{38iA}{3}-\frac{98B}{3}\right)\cot(dx+c)^{\frac{3}{2}}+(-20iB-9A)\sqrt{\cot(dx+c)}}{8(i+\cot(dx+c))^{3}}+\frac{(29iB+6A)\arctan\left(\frac{2\sqrt{\cot(dx+c)}}{\sqrt{2}+i\sqrt{2}}\right)}{4\sqrt{2}+4i\sqrt{2}}+\frac{4\left(-\frac{A}{16}\right.}{}}{a^{3}d}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/a^3/d*(1/8*(-I*(5*I*A-14*B)*cot(d*x+c)^(5/2)+(38/3*I*A-98/3*B)*cot(d*x+c
)^(3/2)+(-9*A-20*I*B)*cot(d*x+c)^(1/2))/(I+cot(d*x+c))^3+1/4*(6*A+29*I*B)/
(2^(1/2)+I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)+I*2^(1/2)))+4*(-1/1
6*A+1/16*I*B)/(2^(1/2)-I*2^(1/2))*arctan(2*cot(d*x+c)^(1/2)/(2^(1/2)-I*2^(
1/2)))+2*I*B/cot(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 783 vs. 2(236) = 472.

Time = 0.15 (sec) , antiderivative size = 783, normalized size of antiderivative = 2.50

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="fricas")
```

output
```
1/96*(3*(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((-I*A
^2 - 2*A*B + I*B^2)/(a^6*d^2))*log(-2*((a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)
*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2
- 2*A*B + I*B^2)/(a^6*d^2)) + (A - I*B)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x -
 2*I*c)/(I*A + B)) - 3*(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^(6*I*d*x + 6*I
*c))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2))*log(2*((a^3*d*e^(2*I*d*x + 2
*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))
*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^6*d^2)) - (A - I*B)*e^(2*I*d*x + 2*I*c))
*e^(-2*I*d*x - 2*I*c)/(I*A + B)) + 3*(a^3*d*e^(8*I*d*x + 8*I*c) + a^3*d*e^
(6*I*d*x + 6*I*c))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2))*log(1/
8*((a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) - 1))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6*d^2))
+ 6*A + 29*I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 3*(a^3*d*e^(8*I*d*x + 8*I*
c) + a^3*d*e^(6*I*d*x + 6*I*c))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^2)/(a^6
*d^2))*log(-1/8*((a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt((I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((36*I*A^2 - 348*A*B - 841*I*B^
2)/(a^6*d^2)) - 6*A - 29*I*B)*e^(-2*I*d*x - 2*I*c)/(a^3*d)) - 2*(2*(10*I*A
 - 73*B)*e^(8*I*d*x + 8*I*c) + 3*(-2*I*A + 35*B)*e^(6*I*d*x + 6*I*c) - (19
*I*A - 49*B)*e^(4*I*d*x + 4*I*c) + 3*(2*I*A - 3*B)*e^(2*I*d*x + 2*I*c) - I
*A + B)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{7}{2}}(c + dx)(a + ia\tan(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(7/2)/(a+I*a*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{7}{2}}(c + dx)(a + ia\tan(c + dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.48

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{7}{2}}(c + dx)(a + ia\tan(c + dx))^3}\,dx =$$

$$-\frac{3\sqrt{2}(-(6i-6)\ A + (29i+29)\ B)\arctan\left(\left(\frac{1}{2}i + \frac{1}{2}\right)\ \sqrt{2}\sqrt{\tan\left(dx+c\right)}\right) + 3\sqrt{2}(-(i+1)\ A + (i-1}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x, algori
thm="giac")
```

output
```
-1/48*(3*sqrt(2)*(-(6*I - 6)*A + (29*I + 29)*B)*arctan((1/2*I + 1/2)*sqrt(
2)*sqrt(tan(d*x + c))) + 3*sqrt(2)*(-(I + 1)*A + (I - 1)*B)*arctan(-(1/2*I
 - 1/2)*sqrt(2)*sqrt(tan(d*x + c))) - 96*I*B*sqrt(tan(d*x + c)) - 2*(-27*I
*A*tan(d*x + c)^(5/2) + 60*B*tan(d*x + c)^(5/2) - 38*A*tan(d*x + c)^(3/2)
- 98*I*B*tan(d*x + c)^(3/2) + 15*I*A*sqrt(tan(d*x + c)) - 42*B*sqrt(tan(d*
x + c)))/(tan(d*x + c) - I)^3)/(a^3*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx=\int \frac{A+B\tan(c+d\,x)}{{\cot\left(c+d\,x\right)}^{7/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^3),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(7/2)*(a + a*tan(c + d*x)*1i)^3), x
)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^3}\,dx$$
$$=\frac{-\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)^4\tan(dx+c)^3 i+3\cot(dx+c)^4\tan(dx+c)^2-3\cot(dx+c)^4\tan(dx+c) i-\cot(dx+c)^4}dx\right)a-\left(\int \frac{}{\cot(dx+c)^4\tan(dx+c)^3 i+}\right.}{a^3}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+I*a*tan(d*x+c))^3,x)
```

output
```
( - (int(sqrt(cot(c + d*x))/(cot(c + d*x)**4*tan(c + d*x)**3*i + 3*cot(c +
 d*x)**4*tan(c + d*x)**2 - 3*cot(c + d*x)**4*tan(c + d*x)*i - cot(c + d*x)
**4),x)*a + int((sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)**4*tan(c +
 d*x)**3*i + 3*cot(c + d*x)**4*tan(c + d*x)**2 - 3*cot(c + d*x)**4*tan(c +
 d*x)*i - cot(c + d*x)**4),x)*b))/a**3
```

3.540 $\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 198

$$\begin{aligned}
&\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx \\
&= -\frac{(1+i)\sqrt{a}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&\quad + \frac{2(13A-5iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{15d} \\
&\quad - \frac{2(iA+5B)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} \\
&\quad - \frac{2A\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}
\end{aligned}$$

output

```
(-1-I)*a^(1/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d
*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+2/15*(13*A-5*I*B)*cot(d*
x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/15*(I*A+5*B)*cot(d*x+c)^(3/2)*(a+I
*a*tan(d*x+c))^(1/2)/d-2/5*A*cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 3.86 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.78

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$

$$-\frac{i\left(\frac{15\sqrt{2}a(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{ia\tan(c+dx)}}+2\cot^3(c+dx)\sqrt{a+ia\tan(c+dx)}(-3iA+(A-5iB)\tan(c-\right)}{15d\sqrt{\cot(c+dx)}}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x
]),x]
```

output
```
((-1/15*I)*((15*Sqrt[2]*a*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]
])/Sqrt[a + I*a*Tan[c + d*x]]])/Sqrt[I*a*Tan[c + d*x]] + 2*Cot[c + d*x]^3*
Sqrt[a + I*a*Tan[c + d*x]]*((-3*I)*A + (A - (5*I)*B)*Tan[c + d*x] + ((13*I
)*A + 5*B)*Tan[c + d*x]^2)))/(d*Sqrt[Cot[c + d*x]])
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 4729, 3042, 4081, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{7/2}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{4081}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+5B)-4aA\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+5B)-4aA\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+5B)-4aA\tan(c+dx))}{\tan(c+dx)^{5/2}}dx}{5a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4081}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((13A-5iB)a^2+2(iA+5B)\tan(c+dx)a^2\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-2A\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((13A-5iB)a^2+2(iA+5B)\tan(c+dx)a^2\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-2A\sqrt{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((13A-5iB)a^2+2(iA+5B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\frac{2A\sqrt{}}{}\right.$$

$\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{\frac{2\int\frac{15a^3(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{15a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{15a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5B+iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{5a}-\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{-\frac{30ia^4(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5B+iA)}{3d\,\mathrm{ta}}}{5a}\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{\frac{(15-15i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(13A-5iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5B+iA)\sqrt{}}{3d\tan}}{5a}\right.$$

input
```
Int[Cot[c + d*x]^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A*Sqrt[a + I*a*Tan[c + d*x]])/(
5*d*Tan[c + d*x]^(5/2)) + ((-2*a*(I*A + 5*B)*Sqrt[a + I*a*Tan[c + d*x]])/(
3*d*Tan[c + d*x]^(3/2)) - (((15 - 15*I)*a^(5/2)*(I*A + B)*ArcTanh[((1 + I)
*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^2*(13*A
 - (5*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*
a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(161) = 322.

Time = 1.11 (sec) , antiderivative size = 634, normalized size of antiderivative = 3.20

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{7}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(15iB\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{7}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(15iB\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right.\right.}{}$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/30/d*(1/tan(d*x+c))^(7/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*(15*I*B
*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-
I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+15*I*A*2^(1/2)*ln((2*2^
(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c
))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-15*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*
a*tan(d*x+c)^4+52*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan
(d*x+c)^3+15*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-40*B*tan(
d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-20*I*B*(-I*a)^
(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^3-16*A*tan(d*x+c)*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-56*I*A*tan(d*x+c)^2*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)+20*I*B*tan(d*x+c)*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)+12*I*A*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1
/2)/(I-tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(150) = 300.

Time = 0.10 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.43

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$

$$-\frac{15\sqrt{2}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a}{d^2}}\log\left(-\frac{4\left((A-i\,B)ae^{(i\,dx+i\,c)}-\left(i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\right.}{}\right.}{}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
-1/30*(15*sqrt(2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sq
rt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) -
(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
 + 2*I*c) - 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(2)*(d*e^(4*I*d*x +
4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2
)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) - (-I*d*e^(2*I*d*x + 2*I*c) + I*d)*s
qrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)
/(I*A + B)) - 4*sqrt(2)*((17*A - 10*I*B)*e^(5*I*d*x + 5*I*c) - 10*(2*A - I
*B)*e^(3*I*d*x + 3*I*c) + 15*A*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^
(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1409 vs. $2(150) = 300$.

Time = 0.58 (sec) , antiderivative size = 1409, normalized size of antiderivative = 7.12

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
1/30*(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*((30*((I + 1)*A - (I - 1)*B)*cos(3*d*x + 3*c) + (-(39*I + 39)*A + (25*I
 - 25)*B)*cos(d*x + c) + 30*((I - 1)*A + (I + 1)*B)*sin(3*d*x + 3*c) + (-(
39*I - 39)*A - (25*I + 25)*B)*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) - 1)) + (30*(-(I - 1)*A - (I + 1)*B)*cos(3*d*x + 3*c)
 + ((39*I - 39)*A + (25*I + 25)*B)*cos(d*x + c) + 30*((I + 1)*A - (I - 1)*
B)*sin(3*d*x + 3*c) + (-(39*I + 39)*A + (25*I - 25)*B)*sin(d*x + c))*sin(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a) + 15*(2*((-(I
 - 1)*A - (I + 1)*B)*cos(2*d*x + 2*c)^2 + (-(I - 1)*A - (I + 1)*B)*sin(2*d
*x + 2*c)^2 + 2*((I - 1)*A + (I + 1)*B)*cos(2*d*x + 2*c) - (I - 1)*A - (I
+ 1)*B)*arctan2(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x +
 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))
+ 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)
) + 2*cos(d*x + c)) + ((-(I + 1)*A + (I - 1)*B)*cos(2*d*x + 2*c)^2 + (-(I
+ 1)*A + (I - 1)*B)*sin(2*d*x + 2*c)^2 + 2*((I + 1)*A - (I - 1)*B)*cos(2*d
*x + 2*c) - (I + 1)*A + (I - 1)*B)*log(4*cos(d*x + c)^2 + 4*sin(d*x + c)^2
 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1
)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c)...
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \sqrt{a}\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^3\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^3\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**3*t
an(c + d*x),x)*b + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c +
 d*x)**3,x)*a)
```

3.541 $\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 155

$$\begin{aligned}
&\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx \\
&= \frac{(1+i)\sqrt{a}(iA+B)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&\quad - \frac{2(iA+3B)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3d} \\
&\quad - \frac{2A\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}
\end{aligned}$$

output

```
(1+I)*a^(1/2)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*
x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2/3*(I*A+3*B)*cot(d*x+c)^
(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/3*A*cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))
^(1/2)/d
```

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.79

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{\sqrt{\cot(c+dx)}\left(3\sqrt{2}(iA+B)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{ia\tan(c+dx)}-2(iA+3B+A\cot(c+dx))\sqrt{}\right.}{3d}$$

input
```
Integrate[Cot[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x
]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*(3*Sqrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c +
 d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*Tan[c + d*x]] - 2*(I*A + 3*B
+ A*Cot[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]))/(3*d)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {3042, 4729, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cot(c+dx)^{5/2}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \ 4729$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$\Big\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+3B)-2aA\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+3B)-2aA\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA+3B)-2aA\tan(c+dx))}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2\int -\frac{3a^2(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-3a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3B+iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-3a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3B+iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\Big\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{6ia^3(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a(3B+iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+i}}{3d\tan}\right.$$

$\Big\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{(3-3i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a(3B+iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+}}{3d\,\text{ta}}\right.$$

input
```
Int[Cot[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A*Sqrt[a + I*a*Tan[c + d*x]])/(
3*d*Tan[c + d*x]^(3/2)) + (((-3 + 3*I)*a^(3/2)*(A - I*B)*ArcTanh[((1 + I)*
Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a*(I*A + 3
*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(126) = 252$.

Time = 0.93 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.59

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,\bigg(3iA\sqrt{2}\,\ln\bigg(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\bigg)}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,\bigg(3iA\sqrt{2}\,\ln\bigg(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\bigg)}{}$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/6/d*(1/tan(d*x+c))^(5/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*(3*I*A*l
n((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*ta
n(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)^3-3*I*B*ln((2*2^(1/2)*(-I*a
)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x
+c)+I))*2^(1/2)*a*tan(d*x+c)^2+3*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*ta
n(d*x+c)^3-12*B*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^
(1/2)+3*A*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)^2-8*A*tan(d*x+c)
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-4*I*A*(-I*a)^(1/2)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+12*I*B*(-I*a)^(1/2)*tan(d*
x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+4*I*A*(-I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1
/2)/(I-tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(117) = 234$.

Time = 0.11 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.77

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$

$$-\frac{3\sqrt{2}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{(iA^2+2\,AB-i\,B^2)a}{d^2}}\log\left(-\frac{4\left((A-i\,B)ae^{(i\,dx+i\,c)}+\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{-\frac{(iA^2+2\,AB-i\,B^2)a}{d^2}}\sqrt{e^{(\cdots}}\right.}{iA+B}\right.}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
-1/6*(3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*
a/d^2)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) + (d*e^(2*I*d*x + 2*I*c) - d)*s
qrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/
(I*A + B)) - 3*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B -
I*B^2)*a/d^2)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) - (d*e^(2*I*d*x + 2*I*c)
 - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x
- I*c)/(I*A + B)) + 4*sqrt(2)*((2*I*A + 3*B)*e^(3*I*d*x + 3*I*c) - 3*B*e^(
I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c
) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1157 vs. $2(117) = 234$.

Time = 0.31 (sec) , antiderivative size = 1157, normalized size of antiderivative = 7.46

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
1/6*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
 1)*((3*(-(I - 1)*A - (I + 1)*B)*cos(3*d*x + 3*c) + (-(I - 1)*A + (3*I + 3
)*B)*cos(d*x + c) + 3*((I + 1)*A - (I - 1)*B)*sin(3*d*x + 3*c) + ((I + 1)*
A + (3*I - 3)*B)*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) - 1)) + (3*(-(I + 1)*A + (I - 1)*B)*cos(3*d*x + 3*c) + (-(I + 1)*A
 - (3*I - 3)*B)*cos(d*x + c) + 3*(-(I - 1)*A - (I + 1)*B)*sin(3*d*x + 3*c)
 + (-(I - 1)*A + (3*I + 3)*B)*sin(d*x + c))*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) - 1)))*sqrt(a) + 3*(2*((-(I + 1)*A + (I - 1)*B)*cos(2
*d*x + 2*c)^2 + (-(I + 1)*A + (I - 1)*B)*sin(2*d*x + 2*c)^2 + 2*((I + 1)*A
 - (I - 1)*B)*cos(2*d*x + 2*c) - (I + 1)*A + (I - 1)*B)*arctan2(2*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*(cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) + (((I
 - 1)*A + (I + 1)*B)*cos(2*d*x + 2*c)^2 + ((I - 1)*A + (I + 1)*B)*sin(2*d*
x + 2*c)^2 + 2*(-(I - 1)*A - (I + 1)*B)*cos(2*d*x + 2*c) + (I - 1)*A + (I
+ 1)*B)*log(4*cos(d*x + c)^2 + 4*sin(d*x + c)^2 + 4*sqrt(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2...
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\sqrt{a}\left(\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\sqrt{\cot\left(dx+c\right)}\,\cot\left(dx+c\right)^{2}\tan\left(dx+c\right)dx\right)b+\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\sqrt{\cot\left(dx+c\right)}\,\cot\left(dx+c\right)^{2}dx\right)a\right)$$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**2*t
an(c + d*x),x)*b + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c +
 d*x)**2,x)*a)
```

3.542 $\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 110

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{(1+i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{2A\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
(1+I)*a^(1/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*
x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2*A*cot(d*x+c)^(1/2)*(a+I
*a*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{\sqrt{\cot(c+dx)}\left(\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{ia\tan(c+dx)}-2A\sqrt{a+ia\tan(c+dx)}\right)}{d}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x
]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*(Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d
*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*Tan[c + d*x]] - 2*A*Sqrt[a + I*
a*Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4729, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \cot(c+dx)^{3/2}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{4729}$$
$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$\Big\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int \frac{a(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left((B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left((B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)$$

$\Big\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2ia^2(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)$$

$\Big\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(1-i)\sqrt{a}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2A\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)$$

input
```
Int[Cot[c + d*x]^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((1 - I)*Sqrt[a]*(I*A + B)*ArcTanh[
((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*A
*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(91) = 182$.

Time = 0.92 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.98

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(-iB\ln\left(\frac{2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{}\right.}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(-iB\ln\left(\frac{2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{}\right.}{}$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/2/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*(-I*B*2^
(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a
+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-I*A*2^(1/2)*ln((2*2^(1/2)*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(ta
n(d*x+c)+I))*a*tan(d*x+c)+A*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*
tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c
)^2-B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+4*I*A*(-I*a)^(1/2)*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-4*A*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I-tan
(d*x+c))/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.36

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$

$$-\frac{4\sqrt{2}A\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}e^{(i\,dx+i\,c)}-\sqrt{2}d\sqrt{-\frac{(-i\,A^2-2\,AB+i\,B^2)a}{d^2}}\log\left(-\frac{4\left((A-i\,B)ae^{(i\,dx+i\,c)}-(i\,de^{(2}\right.}{}\right.}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
-1/2*(4*sqrt(2)*A*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*e^(I*d*x + I*c) - sqrt(2)*d*sqrt(-(-
I*A^2 - 2*A*B + I*B^2)*a/d^2)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) - (I*d*e
^(2*I*d*x + 2*I*c) - I*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) - 1)))*e^(-I*d*x - I*c)/(I*A + B)) + sqrt(2)*d*sqrt(-(-I*A^2 - 2*A*B +
 I*B^2)*a/d^2)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) - (-I*d*e^(2*I*d*x + 2*
I*c) + I*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(
-I*d*x - I*c)/(I*A + B)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(84) = 168$.

Time = 0.22 (sec) , antiderivative size = 558, normalized size of antiderivative = 5.07

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
1/2*((2*((I - 1)*A + (I + 1)*B)*arctan2(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) - (-(I + 1)*A + (I - 1)*B)*log
(4*cos(d*x + c)^2 + 4*sin(d*x + c)^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) - 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) - 1))^2) + 8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2
*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) - 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) - 1)))))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sqrt(a) + 4*((-(I + 1)*A*cos(d*x + c) - (I - 1)*A*sin(d*x + c)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + ((I - 1)*A*co
s(d*x + c) - (I + 1)*A*sin(d*x + c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) - 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 -
2*cos(2*d*x + 2*c) + 1)^(1/4)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \sqrt{a}\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)*tan(
c + d*x),x)*b + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*
x),x)*a)
```

3.543 $\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 152

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{2(-1)^{3/4}\sqrt{a}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(1-i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}$$

output
```
-2*(-1)^(3/4)*a^(1/2)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*
tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(1-I)*a^(1/2)*(A-I*
B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*
x+c)^(1/2)*tan(d*x+c)^(1/2)/d
```

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.06

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \frac{a\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{\sqrt{2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{\sqrt{ia\tan(c+dx)}} + \frac{2\sqrt[4]{-1}B\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\ldots}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x
]),x]
```

output
```
(a*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt
[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]]
)/Sqrt[I*a*Tan[c + d*x]] + (2*(-1)^(1/4)*B*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c +
 d*x]]]*Sqrt[1 + I*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 4729, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{i\tan(c+dx)a+a}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4084}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left((A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{iB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left((A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{iB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}\right)$$

$$\Big\downarrow \text{4027}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{iB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2ia^2(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}\right)$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{iB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}+\frac{(1-i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\right)$$

$$\Big\downarrow \text{4082}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{iaB\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{(1-i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\right)$$

$$\Big\downarrow \text{65}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2iaB\int \frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}+\frac{(1-i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\right)$$

$$\Big\downarrow \text{216}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(1-i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2(-1)^{3/4}\sqrt{a}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}}{\sqrt{a+ia}}\right.}{d}\right.$$

input
```
Int[Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
((-2*(-1)^(3/4)*Sqrt[a]*B*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/S
qrt[a + I*a*Tan[c + d*x]]])/d + ((1 - I)*Sqrt[a]*(A - I*B)*ArcTanh[((1 + I
)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)*Sqrt[Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]]
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(121) = 242$.

Time = 1.03 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.33

method	result
derivativedivides	$\frac{\sqrt{\frac{1}{\tan(dx+c)}}\,\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a\left(iA\sqrt{2}\,\sqrt{ia}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\right.}{}$
default	$\frac{\sqrt{\frac{1}{\tan(dx+c)}}\,\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a\left(iA\sqrt{2}\,\sqrt{ia}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\right.}{}$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
1/2/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(I*A*2^
(1/2)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*tan(d*x+c)-I*B*2^(1/2)*(I*a)^
(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a
+3*a*tan(d*x+c))/(tan(d*x+c)+I))+B*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a
)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x
+c)+I))*tan(d*x+c)-2*I*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*tan(d*x+c)+A*2^(1
/2)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))-2*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*
tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1
/2)))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)/(I-ta
n(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(114) = 228$.

Time = 0.10 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.79

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
1/2*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*log(-4*((A - I*B)*a*e^(I*
d*x + I*c) + (d*e^(2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d
^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 1/2*sqrt(2)*sqrt(-(
I*A^2 + 2*A*B - I*B^2)*a/d^2)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) - (d*e^(
2*I*d*x + 2*I*c) - d)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a/d^2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
 1)))*e^(-I*d*x - I*c)/(I*A + B)) - 1/4*sqrt(4*I*B^2*a/d^2)*log(-16*(3*B*a
^2*e^(2*I*d*x + 2*I*c) - B*a^2 + sqrt(2)*(I*a*d*e^(3*I*d*x + 3*I*c) - I*a*
d*e^(I*d*x + I*c))*sqrt(4*I*B^2*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x -
2*I*c)/B) + 1/4*sqrt(4*I*B^2*a/d^2)*log(-16*(3*B*a^2*e^(2*I*d*x + 2*I*c) -
 B*a^2 + sqrt(2)*(-I*a*d*e^(3*I*d*x + 3*I*c) + I*a*d*e^(I*d*x + I*c))*sqrt
(4*I*B^2*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I
*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/B)
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\left(A+B\tan\left(c+dx\right)\right)\sqrt{\cot\left(c+dx\right)}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))*sqrt(cot(c + d*
x)), x)
```

Maxima [F]

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)\sqrt{ia\tan(dx+c)+a}\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)*sqrt(cot(d*x + c
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int \sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2),
 x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \sqrt{a}\left(\left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)dx\right)b + \left(\int \sqrt{\tan\left(dx+c\right)i+1}\,\sqrt{\cot\left(dx+c\right)}dx\right)a\right)$$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*(int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x),x)*b
 + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x)),x)*a)
```

3.544 $\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 192

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = -\frac{(-1)^{3/4}\sqrt{a}(2A-iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{(1+i)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{B\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}}$$

output
```
-(-1)^(3/4)*a^(1/2)*(2*A-I*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(
a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(1+I)*a^(1/2)
*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*
cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+B*(a+I*a*tan(d*x+c))^(1/2)/d/cot(d*x+c
)^(1/2)
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$=\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}\right.$$
$$\left.+\frac{-\frac{i\sqrt{2}\left(\frac{1}{2}a^2(2A-iB)-\frac{1}{2}ia^2B\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{d\sqrt{ia\tan(c+dx)}}+\frac{\sqrt[4]{-1}a^2(2A-iB)\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{1+i\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{a}\right)$$

input
```
Integrate[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d
*x]],x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((B*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*
Tan[c + d*x]])/d + (((-I)*Sqrt[2]*((a^2*(2*A - I*B))/2 - (I/2)*a^2*B)*ArcT
anh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[Tan[
c + d*x]])/(d*Sqrt[I*a*Tan[c + d*x]]) + ((-1)^(1/4)*a^2*(2*A - I*B)*ArcSin
h[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[1 + I*Tan[c + d*x]])/(d*Sqrt[a + I*a
*Tan[c + d*x]]))/a)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 4729, 3042, 4080, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\downarrow 4729$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(A+B\tan(c+dx))dx$$

$$\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(A+B\tan(c+dx))dx$$

$$\downarrow 4080$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int -\frac{\sqrt{i\tan(c+dx)a+a}(aB-a(2A-iB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx}{a}+\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+da}}{d}\right.$$

$$\downarrow 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(aB-a(2A-iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{2a}\right.$$

$$\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(aB-a(2A-iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{2a}\right.$$

$$\downarrow 4084$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-(B+2i}{2a}\right.$$

$$\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-(B+2i}{2a}\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{-\frac{4ia^3(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+}}}{d}}{}\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{}\right.$$

$\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{}\right.$$

$\downarrow$ 65

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{2}\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{2\sqrt[4]{-1}a^{3/2}(B+2iA)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}}{}\right.$$

input
```
Int[(Sqrt[a + I*a*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-1/2*((2*(-1)^(1/4)*a^(3/2)*((2*I)*
A + B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c +
 d*x]]])/d + ((2 - 2*I)*a^(3/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/a + (B*Sqrt[Tan[c + d*x]]*Sqr
t[a + I*a*Tan[c + d*x]])/d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(155) = 310.

Time = 0.90 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.64

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\left(B\sqrt{-ia}\,\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+}{2\sqrt{ia}}\right.\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\left(B\sqrt{-ia}\,\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+}{2\sqrt{ia}}\right.\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/2/d*(a*(1+I*tan(d*x+c)))^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(B
*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a-2*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)-2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)
*a+2*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I
*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a)/(1/tan(d*x+c))^(1/2)/(1+I*tan(d*
x+c))/tan(d*x+c)/(I*a)^(1/2)/(-I*a)^(1/2)/a
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 793 vs. 2(146) = 292.

Time = 0.10 (sec) , antiderivative size = 793, normalized size of antiderivative = 4.13

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, al
gorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)
*a/d^2)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) - (I*d*e^(2*I*d*x + 2*I*c) - I
*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x -
 I*c)/(I*A + B)) - 2*sqrt(2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-(-I*A^2 - 2
*A*B + I*B^2)*a/d^2)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) - (-I*d*e^(2*I*d*
x + 2*I*c) + I*d)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)
))*e^(-I*d*x - I*c)/(I*A + B)) + 4*sqrt(2)*(I*B*e^(3*I*d*x + 3*I*c) - I*B*
e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*
I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) - (d*e^(2*I*d*x + 2*I*c) + d)*sqrt((4
*I*A^2 + 4*A*B - I*B^2)*a/d^2)*log(-16*(3*(2*I*A + B)*a^2*e^(2*I*d*x + 2*I
*c) + (-2*I*A - B)*a^2 + 2*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x
 + I*c))*sqrt((4*I*A^2 + 4*A*B - I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*
I*d*x - 2*I*c)/(2*I*A + B)) + (d*e^(2*I*d*x + 2*I*c) + d)*sqrt((4*I*A^2 +
4*A*B - I*B^2)*a/d^2)*log(-16*(3*(2*I*A + B)*a^2*e^(2*I*d*x + 2*I*c) + (-2
*I*A - B)*a^2 - 2*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*
sqrt((4*I*A^2 + 4*A*B - I*B^2)*a/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x ...
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\left(A+B\tan\left(c+dx\right)\right)}{\sqrt{\cot\left(c+dx\right)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(c + d*x) - I))*(A + B*tan(c + d*x))/sqrt(cot(c + d*
x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(I*a*tan(d*x + c) + a)/sqrt(cot(d*x + c
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \int \frac{(A+B\tan(c+d\,x))\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/cot(c + d*x)^(1/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(1/2))/cot(c + d*x)^(1/2
), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \sqrt{a}\left(\left(\int \frac{\sqrt{\tan\left(dx+c\right)i+1}\,\sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)}{\cot\left(dx+c\right)}dx\right)b + \left(\int \frac{\sqrt{\tan\left(dx+c\right)i+1}\,\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)}dx\right)a\right)$$

input
```
int((a+I*a*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*(int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x))/co
t(c + d*x),x)*b + int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x)))/cot(c
 + d*x),x)*a)
```

3.545 $\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 245

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\frac{(2-2i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}+\frac{4a(67iA+63B)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{105d}+\frac{4a(19A-21iB)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{105d}-\frac{2a(8iA+7B)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}-\frac{2aA\cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

output
```
(2-2*I)*a^(3/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+4/105*a*(67*I*A+63*B)*c
ot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+4/105*a*(19*A-21*I*B)*cot(d*x+c
)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/35*a*(8*I*A+7*B)*cot(d*x+c)^(5/2)*(a+
I*a*tan(d*x+c))^(1/2)/d-2/7*a*A*cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(1/2)/
d
```

Mathematica [A] (verified)

Time = 8.71 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.31

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{\left(-2i\sqrt{2}(A-iB)e^{-2i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}}\operatorname{arctanh}\left(\frac{e^{i(c+}}{\sqrt{-1+e}}\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(9/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
((((-2*I)*Sqrt[2]*(A - I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[E^(I*(c +
d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x))))/(-1 +
E^((2*I)*(c + d*x)))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x)
)]])/E^((2*I)*(c + d*x)) - (Sqrt[Cot[c + d*x]]*Csc[c + d*x]^3*Sqrt[Sec[c +
 d*x]]*(Cos[c + d*x] - I*Sin[c + d*x])*(7*(A + (6*I)*B)*Cos[c + d*x] + (53
*A - (42*I)*B)*Cos[3*(c + d*x)] + 2*((-110*I)*A - 105*B + ((158*I)*A + 147
*B)*Cos[2*(c + d*x)])*Sin[c + d*x]))/210)*(a + I*a*Tan[c + d*x])^(3/2)*(A
+ B*Tan[c + d*x]))/(d*Sec[c + d*x]^(5/2)*(A*Cos[c + d*x] + B*Sin[c + d*x])
)
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.447$, Rules used = {3042, 4729, 3042, 4076, 27, 3042, 4081, 25, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cot(c+dx)^{9/2}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{ 4076}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{7}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(8iA+7B)-a(6A-7iB)\tan(c+dx))}{2\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2aA\sqrt{a}}{7d\,t}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(8iA+7B)-a(6A-7iB)\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2aA\sqrt{a}}{7d\,t}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(8iA+7B)-a(6A-7iB)\tan(c+dx))}{\tan(c+dx)^{7/2}}dx-\frac{2aA\sqrt{a}}{7d\,t}\right.$$

$$\Big\downarrow \text{ 4081}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}((19A-21iB)a^2+2(8iA+7B)\tan(c+dx)a^2)}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2a(7B+8iA)\sqrt{a}}{5d\tan^{\frac{5}{2}}(}\right.\right.$$

$$\Big\downarrow \text{ 25}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}((19A-21iB)a^2+2(8iA+7B)\tan(c+dx)a^2)}{\tan^{\frac{5}{2}}(c+dx)}dx}{5a}-\frac{2a(7B+8iA)\sqrt{a}}{5d\tan^{\frac{5}{2}}(}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}((19A-21iB)a^2+2(8iA+7B)\tan(c+dx)a^2)}{\tan(c+dx)^{5/2}}dx}{5a}-\frac{2a(7B+8iA)\sqrt{a}}{5d\tan^{\frac{5}{2}}(}\right.\right.$$

$\Big\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\left(\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(67iA+63B)-2a^3(19A-21iB)\tan(c+dx)\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\,\mathrm{ta}}}{3d\tan^{\frac{3}{2}}(c+dx)}\right.}{5a}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(67iA+63B)-2a^3(19A-21iB)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\,\mathrm{tan}}}{3d\tan^{\frac{3}{2}}(c+dx)}\right.}{5a}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(67iA+63B)-2a^3(19A-21iB)\tan(c+dx)\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a^2(19A-21iB)\sqrt{a+ia\,\mathrm{tan}}}{3d\tan^{\frac{3}{2}}(c+dx)}\right.}{5a}\right.\right.$$

$\Big\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\left(\frac{\frac{2\int -\frac{105a^4(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^3(63B+67iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(19A-21iB)\sqrt{a}}{3d\tan^{\frac{3}{2}}(c}\right.}{5a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\left(\frac{-105a^3(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(63B+67iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(19A-21iB)\sqrt{a}}{3d\tan^{\frac{3}{2}}(c}\right.}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\left(\frac{-105a^3(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(63B+67iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a^2(19A-21iB)\sqrt{a}}{3d\tan^{\frac{3}{2}}(c}\right.}{5a}\right.\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\left(\frac{\frac{210ia^5(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^3(63B+67iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a}{}\right.}{5a}\right.\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\left(\frac{-\frac{(105-105i)a^{7/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(63B+67iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\right.}{5a}\right.\right.$$

input
```
Int[Cot[c + d*x]^(9/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*Sqrt[a + I*a*Tan[c + d*x]])
/(7*d*Tan[c + d*x]^(7/2)) + ((-2*a*((8*I)*A + 7*B)*Sqrt[a + I*a*Tan[c + d*
x]])/(5*d*Tan[c + d*x]^(5/2)) - (2*((-2*a^2*(19*A - (21*I)*B)*Sqrt[a + I*a
*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) + (((-105 + 105*I)*a^(7/2)*(A - I
*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]
]])/d - (2*a^3*((67*I)*A + 63*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c
 + d*x]]))/(3*a)))/(5*a))/7)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(200) = 400$.

Time = 0.82 (sec) , antiderivative size = 802, normalized size of antiderivative = 3.27

method	result
derivativedivides	$\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{9}{2}} \tan(dx+c) \sqrt{a(1+i\tan(dx+c))}\, a \left(105i\sqrt{2}\sqrt{ia}\, \ln\left(\frac{2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{9}{2}} \tan(dx+c) \sqrt{a(1+i\tan(dx+c))}\, a \left(105i\sqrt{2}\sqrt{ia}\, \ln\left(\frac{2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+}{\tan(dx+c)+i}\right.\right.}{}$

input
```
int(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
1/210/d*(1/tan(d*x+c))^(9/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(105*
I*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+504*B*(-I*
a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+21
0*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^4+420*A*(-I*a)^(1/2)*ln(1
/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a
)/(I*a)^(1/2))*a*tan(d*x+c)^4+152*A*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)^2*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-96*I*A*(-I*a)^(1/2)*(I*a)^(1/2)*tan(
d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-105*2^(1/2)*(I*a)^(1/2)*ln((2
*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*
x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-168*I*B*(-I*a)^(1/2)*(I*a)^(1/2)*tan(
d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+536*I*A*(-I*a)^(1/2)*(I*a)^
(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+210*(-I*a)^(1/2)*
ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/
2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^4-420*I*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d
*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*
a*tan(d*x+c)^4-84*B*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)-60*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)
*(-I*a)^(1/2))/(-I*a)^(1/2)/(I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(187) = 374$.

Time = 0.09 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.32

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
1/105*(105*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(6*I*d*x +
6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*((A
- I*B)*a^2*e^(I*d*x + I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(
2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)
) - 105*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(6*I*d*x + 6*I
*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*((A - I
*B)*a^2*e^(I*d*x + I*c) - sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(2*I
*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x +
2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) -
 2*sqrt(2)*((-211*I*A - 189*B)*a*e^(7*I*d*x + 7*I*c) + 7*(53*I*A + 57*B)*a
*e^(5*I*d*x + 5*I*c) + 35*(-11*I*A - 9*B)*a*e^(3*I*d*x + 3*I*c) + 105*(I*A
 + B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*
d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(6*I*d*x + 6*I*c) - 3*d
*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(9/2)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3787 vs. $2(187) = 374$.

Time = 2.42 (sec) , antiderivative size = 3787, normalized size of antiderivative = 15.46

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
-1/420*(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
+ 1)*(3*(280*(-(I - 1)*A - (I + 1)*B)*a*cos(7*d*x + 7*c) + 140*((I - 1)*A
+ (3*I + 3)*B)*a*cos(5*d*x + 5*c) + 7*(-(19*I - 19)*A - (29*I + 29)*B)*a*c
os(3*d*x + 3*c) + (-(47*I - 47)*A + (63*I + 63)*B)*a*cos(d*x + c) + 280*((
I + 1)*A - (I - 1)*B)*a*sin(7*d*x + 7*c) + 140*(-(I + 1)*A + (3*I - 3)*B)*
a*sin(5*d*x + 5*c) + 7*((19*I + 19)*A - (29*I - 29)*B)*a*sin(3*d*x + 3*c)
+ ((47*I + 47)*A + (63*I - 63)*B)*a*sin(d*x + c))*cos(7/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) - 1)) + 4*((((141*I - 141)*A + (119*I + 119)*B)
*a*cos(d*x + c) + (-(141*I + 141)*A + (119*I - 119)*B)*a*sin(d*x + c))*cos
(2*d*x + 2*c)^2 + ((141*I - 141)*A + (119*I + 119)*B)*a*cos(d*x + c) + (((
141*I - 141)*A + (119*I + 119)*B)*a*cos(d*x + c) + (-(141*I + 141)*A + (11
9*I - 119)*B)*a*sin(d*x + c))*sin(2*d*x + 2*c)^2 + (-(141*I + 141)*A + (11
9*I - 119)*B)*a*sin(d*x + c) + 210*((-(I - 1)*A - (I + 1)*B)*a*cos(2*d*x +
 2*c)^2 + (-(I - 1)*A - (I + 1)*B)*a*sin(2*d*x + 2*c)^2 + 2*((I - 1)*A + (
I + 1)*B)*a*cos(2*d*x + 2*c) + (-(I - 1)*A - (I + 1)*B)*a)*cos(3*d*x + 3*c
) + 2*((-(141*I - 141)*A - (119*I + 119)*B)*a*cos(d*x + c) + ((141*I + 141
)*A - (119*I - 119)*B)*a*sin(d*x + c))*cos(2*d*x + 2*c) + 210*(((I + 1)*A
- (I - 1)*B)*a*cos(2*d*x + 2*c)^2 + ((I + 1)*A - (I - 1)*B)*a*sin(2*d*x +
2*c)^2 + 2*(-(I + 1)*A + (I - 1)*B)*a*cos(2*d*x + 2*c) + ((I + 1)*A - (I -
 1)*B)*a)*sin(3*d*x + 3*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x...
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{9/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}dx$$

input
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
x)
```

output
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^4\tan(dx+c)^2\,dx\right)bi+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^4\tan(dx+c)\,dx\right)ai+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^4\tan(dx+c)\,dx\right)b+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^4\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**4
*tan(c + d*x)**2,x)*b*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*
cot(c + d*x)**4*tan(c + d*x),x)*a*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(co
t(c + d*x))*cot(c + d*x)**4*tan(c + d*x),x)*b + int(sqrt(tan(c + d*x)*i +
1)*sqrt(cot(c + d*x))*cot(c + d*x)**4,x)*a)
```

3.546 $\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 201

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{(2+2i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{4a(9A-10iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{15d} - \frac{2a(6iA+5B)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} - \frac{2aA\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

output
```
(-2-2*I)*a^(3/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan
(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+4/15*a*(9*A-10*I*B)*co
t(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/15*a*(6*I*A+5*B)*cot(d*x+c)^(3
/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/5*a*A*cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))
^(1/2)/d
```

Mathematica [A] (verified)

Time = 4.58 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.62

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{2\cot^{\frac{3}{2}}(c+dx)\left(-3a^2A(i+\cot(c+dx))^2\tan(c+dx)+a^2(3iA+5B)(-i+\tan(c+\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
(2*Cot[c + d*x]^(3/2)*(-3*a^2*A*(I + Cot[c + d*x])^2*Tan[c + d*x] + a^2*((
3*I)*A + 5*B)*(-I + Tan[c + d*x])^2 + (15*a*(A - I*B)*Tan[c + d*x]*(Sqrt[a
]*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*(1 + I*Tan[c + d*x])*Sqrt[I*a*Ta
n[c + d*x]] - (-1)^(3/4)*a*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[Tan
[c + d*x]]*(-I + Tan[c + d*x]) + Sqrt[1 + I*Tan[c + d*x]]*(a + I*a*Tan[c +
 d*x] - Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[
c + d*x]]]*Sqrt[I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/Sqrt[1 + I
*Tan[c + d*x]]))/(15*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4729, 3042, 4076, 27, 3042, 4081, 25, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{7/2}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{5}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(6iA+5B)-a(4A-5iB)\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA\sqrt{a}}{5d\,\mathrm{t}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(6iA+5B)-a(4A-5iB)\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA\sqrt{a}}{5d\,\mathrm{t}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(6iA+5B)-a(4A-5iB)\tan(c+dx))}{\tan(c+dx)^{5/2}}dx-\frac{2aA\sqrt{a}}{5d\,\mathrm{t}}\right.$$

$$\Big\downarrow \text{4081}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}((9A-10iB)a^2+(6iA+5B)\tan(c+dx)a^2)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5B+6iA)\sqrt{a+}}{3d\tan^{\frac{3}{2}}(c}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}((9A-10iB)a^2+(6iA+5B)\tan(c+dx)a^2)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5B+6iA)\sqrt{a+}}{3d\tan^{\frac{3}{2}}(c}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{2\int \frac{\sqrt{i\tan(c+dx)a+a}((9A-10iB)a^2+(6iA+5B)\tan(c+dx)a^2)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a(5B+6iA)\sqrt{a+}}{3d\tan^{\frac{3}{2}}(c}\right.\right.$$

$\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{2\left(\frac{2\int\frac{15a^3(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^2(9A-10iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)}{3a}-\frac{2a(5B+6i\ldots}{3d\ldots}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{2\left(15a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(9A-10iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)}{3a}-\frac{2a(5B\ldots}{\ldots}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{2\left(15a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(9A-10iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)}{3a}-\frac{2a(5B\ldots}{\ldots}\right.\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{2\left(-\frac{30ia^4(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^2(9A-10iB)\sqrt{a+ia\tan(c+dx\ldots}}{d\sqrt{\tan(c+dx)}}\right.}{3a}\right.\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{2\left(\frac{(15-15i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(9A-10iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right.}{3a}\right.\right.$$

input
```
Int[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*Sqrt[a + I*a*Tan[c + d*x]])
/(5*d*Tan[c + d*x]^(5/2)) + ((-2*a*((6*I)*A + 5*B)*Sqrt[a + I*a*Tan[c + d*
x]])/(3*d*Tan[c + d*x]^(3/2)) - (2*(((15 - 15*I)*a^(5/2)*(I*A + B)*ArcTanh
[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*
a^2*(9*A - (10*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])))/
(3*a))/5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(164) = 328$.

Time = 0.78 (sec) , antiderivative size = 715, normalized size of antiderivative = 3.56

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{7}{2}}\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a\left(-72A\sqrt{-ia}\,\sqrt{ia}\,\tan(dx+c)^2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+60i\right.}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{7}{2}}\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a\left(-72A\sqrt{-ia}\,\sqrt{ia}\,\tan(dx+c)^2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+60i\right.}{}$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/30/d*(1/tan(d*x+c))^(7/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(-72*
A*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)+60*I*A*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^3-15*I*2^(1/2)*(I*
a)^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+80*I*B*tan(d*x+c)^2*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*(I*a)^(1/2)+60*B*(-I*a)^
(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*
a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^3+30*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*ta
n(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2
))*a*tan(d*x+c)^3-15*2^(1/2)*(I*a)^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*ta
n(d*x+c)^3+24*I*A*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^
(1/2)*(I*a)^(1/2)-30*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^3+20*B*(
-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+1
2*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(153) = 306$.

Time = 0.10 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.54

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{15\sqrt{2}\sqrt{-\frac{(-iA^2-2AB+iB^2)a^3}{d^2}}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left((A-iB)a^2e^{(i\,dx+i\,c)}-\sqrt{-\frac{(-iA^2-2AB+iB^2)a^3}{d^2}}\right.}{}\right.}{}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
-1/15*(15*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(4*I*d*x +
4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((A - I*B)*a^2*e^(I*d*x + I*c)
 - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - 15*sqrt(2)*sqrt(-
(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x
+ 2*I*c) + d)*log(4*((A - I*B)*a^2*e^(I*d*x + I*c) - sqrt(-(-I*A^2 - 2*A*B
 + I*B^2)*a^3/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e
^(-I*d*x - I*c)/((-I*A - B)*a)) - 2*sqrt(2)*((27*A - 25*I*B)*a*e^(5*I*d*x
+ 5*I*c) - 10*(3*A - 4*I*B)*a*e^(3*I*d*x + 3*I*c) + 15*(A - I*B)*a*e^(I*d*
x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*x + 2
*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1457 vs. $2(153) = 306$.

Time = 0.45 (sec) , antiderivative size = 1457, normalized size of antiderivative = 7.25

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
1/15*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
+ 1)*((15*((I + 1)*A - (I - 1)*B)*a*cos(3*d*x + 3*c) + (-(16*I + 16)*A + (
15*I - 15)*B)*a*cos(d*x + c) + 15*((I - 1)*A + (I + 1)*B)*a*sin(3*d*x + 3*
c) + (-(16*I - 16)*A - (15*I + 15)*B)*a*sin(d*x + c))*cos(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + (15*(-(I - 1)*A - (I + 1)*B)*a*cos(
3*d*x + 3*c) + ((16*I - 16)*A + (15*I + 15)*B)*a*cos(d*x + c) + 15*((I + 1
)*A - (I - 1)*B)*a*sin(3*d*x + 3*c) + (-(16*I + 16)*A + (15*I - 15)*B)*a*s
in(d*x + c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqr
t(a) + 15*(2*((-(I - 1)*A - (I + 1)*B)*a*cos(2*d*x + 2*c)^2 + (-(I - 1)*A
- (I + 1)*B)*a*sin(2*d*x + 2*c)^2 + 2*((I - 1)*A + (I + 1)*B)*a*cos(2*d*x
+ 2*c) + (-(I - 1)*A - (I + 1)*B)*a)*arctan2(2*(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) + ((-(I + 1)*A + (I - 1)*
B)*a*cos(2*d*x + 2*c)^2 + (-(I + 1)*A + (I - 1)*B)*a*sin(2*d*x + 2*c)^2 +
2*((I + 1)*A - (I - 1)*B)*a*cos(2*d*x + 2*c) + (-(I + 1)*A + (I - 1)*B)*a)
*log(4*cos(d*x + c)^2 + 4*sin(d*x + c)^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d...
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^{3}\tan(dx+c)^{2}dx\right)bi+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^{3}\tan(dx+c)\,dx\right)ai+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^{3}\tan(dx+c)\,dx\right)b+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^{3}dx\right)a\right)$$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**3
*tan(c + d*x)**2,x)*b*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*
cot(c + d*x)**3*tan(c + d*x),x)*a*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(co
t(c + d*x))*cot(c + d*x)**3*tan(c + d*x),x)*b + int(sqrt(tan(c + d*x)*i +
1)*sqrt(cot(c + d*x))*cot(c + d*x)**3,x)*a)
```

3.547 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 157

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{(2+2i)a^{3/2}(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{2a(4iA+3B)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3d} - \frac{2aA\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

output

```
(2+2*I)*a^(3/2)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2/3*a*(4*I*A+3*B)*cot(d
*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/3*a*A*cot(d*x+c)^(3/2)*(a+I*a*tan
(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 4.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.85

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{2a\sqrt{\cot(c+dx)}\left(aA\cot(c+dx)(-i+\tan(c+dx))^2+\frac{3(A-iB)\left(-\sqrt[4]{-1}a\operatorname{arcsinh}\left(\sqrt[4]{-\right.\right.}{}}{}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
(2*a*Sqrt[Cot[c + d*x]]*(a*A*Cot[c + d*x]*(-I + Tan[c + d*x])^2 + (3*(A -
I*B)*(-((-1)^(1/4)*a*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[Tan[c + d
*x]]*(-I + Tan[c + d*x])) + Sqrt[a]*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]
]*Sqrt[I*a*Tan[c + d*x]]*(-I + Tan[c + d*x]) + Sqrt[1 + I*Tan[c + d*x]]*(a
*(-I + Tan[c + d*x]) + I*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/
Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*
x]])))/Sqrt[1 + I*Tan[c + d*x]]))/(3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 4729, 3042, 4076, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cot(c+dx)^{5/2}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4729$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$\downarrow$ 4076

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{3}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA\sqrt{a}}{3d\,\mathrm{t}}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA\sqrt{a}}{3d\,\mathrm{t}}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{\tan(c+dx)^{3/2}}dx-\frac{2aA\sqrt{a}}{3d\,\mathrm{t}}\right.$$

$\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{2\int -\frac{3a^2(A-iB)\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)-\frac{2a}{}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-6a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-6a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right.\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{12ia^3(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right.\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{(6-6i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a(3B+4iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right.\right.$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*Sqrt[a + I*a*Tan[c + d*x]])
/(3*d*Tan[c + d*x]^(3/2)) + (((-6 + 6*I)*a^(3/2)*(A - I*B)*ArcTanh[((1 + I
)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a*((4*I)
*A + 3*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 623 vs. 2(128) = 256.

Time = 0.78 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.97

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}}\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a\left(-12iB\sqrt{-ia}\,\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+}{2\sqrt{ia}}\right.\right.}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}}\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a\left(-12iB\sqrt{-ia}\,\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+}{2\sqrt{ia}}\right.\right.}{}$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/6/d*(1/tan(d*x+c))^(5/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(-12*I
*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^2+3*I*2^(1/2)*(I*a)^(1/2)*
ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*t
an(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+12*A*(-I*a)^(1/2)*ln(1/2*(2*I*a*
tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1
/2))*a*tan(d*x+c)^2+16*I*A*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(-I*a)^(1/2)*(I*a)^(1/2)+6*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c
)^2-3*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+12*B*(
-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+6
*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^2+4*A*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)/(-I*a)^(1/2)/(I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(119) = 238.

Time = 0.10 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.93

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{3\sqrt{2}\sqrt{-\frac{(iA^2+2AB-iB^2)a^3}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\log\left(\frac{4\left((A-iB)a^2e^{(i\,dx+i\,c)}+\sqrt{-\frac{(iA^2+2AB-iB^2)a^3}{d^2}}\left(de^{(2i\,dx+2i\,c)}-d\right)\sqrt{\frac{}{e^{(2i\,c}}}\right.}{(-iA-B)a}\right.}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
-1/3*(3*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I
*c) - d)*log(4*((A - I*B)*a^2*e^(I*d*x + I*c) + sqrt(-(I*A^2 + 2*A*B - I*B
^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x -
I*c)/((-I*A - B)*a)) - 3*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d
*e^(2*I*d*x + 2*I*c) - d)*log(4*((A - I*B)*a^2*e^(I*d*x + I*c) - sqrt(-(I*
A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) -
1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) + 2*sqrt(2)*((5*I*A + 3*B)*a*e^(3*I*
d*x + 3*I*c) + 3*(-I*A - B)*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(
2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1113 vs. 2(119) = 238.

Time = 0.27 (sec) , antiderivative size = 1113, normalized size of antiderivative = 7.09

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
1/3*(2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
 1)*((3*(-(I - 1)*A - (I + 1)*B)*a*cos(3*d*x + 3*c) + ((I - 1)*A + (3*I +
3)*B)*a*cos(d*x + c) + 3*((I + 1)*A - (I - 1)*B)*a*sin(3*d*x + 3*c) + (-(I
 + 1)*A + (3*I - 3)*B)*a*sin(d*x + c))*cos(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) - 1)) + (3*(-(I + 1)*A + (I - 1)*B)*a*cos(3*d*x + 3*c) + (
(I + 1)*A - (3*I - 3)*B)*a*cos(d*x + c) + 3*(-(I - 1)*A - (I + 1)*B)*a*sin
(3*d*x + 3*c) + ((I - 1)*A + (3*I + 3)*B)*a*sin(d*x + c))*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)))*sqrt(a) + 3*(2*((-(I + 1)*A + (I
 - 1)*B)*a*cos(2*d*x + 2*c)^2 + (-(I + 1)*A + (I - 1)*B)*a*sin(2*d*x + 2*c
)^2 + 2*((I + 1)*A - (I - 1)*B)*a*cos(2*d*x + 2*c) + (-(I + 1)*A + (I - 1)
*B)*a)*arctan2(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) +
 2*sin(d*x + c), 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))
 + 2*cos(d*x + c)) + (((I - 1)*A + (I + 1)*B)*a*cos(2*d*x + 2*c)^2 + ((I -
 1)*A + (I + 1)*B)*a*sin(2*d*x + 2*c)^2 + 2*(-(I - 1)*A - (I + 1)*B)*a*cos
(2*d*x + 2*c) + ((I - 1)*A + (I + 1)*B)*a)*log(4*cos(d*x + c)^2 + 4*sin(d*
x + c)^2 + 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x +
2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2) + 8*(cos(2*d*...
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)bi + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**2
*tan(c + d*x)**2,x)*b*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*
cot(c + d*x)**2*tan(c + d*x),x)*a*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(co
t(c + d*x))*cot(c + d*x)**2*tan(c + d*x),x)*b + int(sqrt(tan(c + d*x)*i +
1)*sqrt(cot(c + d*x))*cot(c + d*x)**2,x)*a)
```

3.548 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 186

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{2\sqrt[4]{-1}a^{3/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(2+2i)a^{3/2}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{2aA\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

output
```
2*(-1)^(1/4)*a^(3/2)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*t
an(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(2+2*I)*a^(3/2)*(A-I
*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d
*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2*a*A*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(
1/2)/d
```

Mathematica [A] (verified)

Time = 5.83 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.62

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{2a\sqrt{\cot(c+dx)}\left(\sqrt[4]{-1}aA\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)(1+i\tan(c+dx))\sqrt{\tan(c+dx)}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
(2*a*Sqrt[Cot[c + d*x]]*((-1)^(1/4)*a*A*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*
x]]]*(1 + I*Tan[c + d*x])*Sqrt[Tan[c + d*x]]*Sqrt[I*a*Tan[c + d*x]] + a^(3
/2)*(A - I*B)*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Tan[c + d*x]*(-I + T
an[c + d*x]) + a*Sqrt[1 + I*Tan[c + d*x]]*(A*(-1 - I*Tan[c + d*x])*Sqrt[I*
a*Tan[c + d*x]] + Sqrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]
])/Sqrt[a + I*a*Tan[c + d*x]]]*Tan[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])))/
(d*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*
x]])
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 4729, 3042, 4076, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cot(c+dx)^{3/2}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4729$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow 4076$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(2\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2iA+B)+iaB\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx-\frac{2aA\sqrt{a+ia\tan(c}}{d\sqrt{\tan(c+dx}}\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2iA+B)+iaB\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2aA\sqrt{a+ia\tan(c+}}{d\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2iA+B)+iaB\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2aA\sqrt{a+ia\tan(c+}}{d\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow 4084$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(2a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-B\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a}}{\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(2a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-B\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a}}{\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow 4027$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{4ia^3(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-B\int \frac{(a-ia\tan(c+dx))\sqrt{i}}{\sqrt{\tan(c+}}\right.$$

$$\Big\downarrow 218$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-B\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}}{d}\right.$$

$\Big\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{a^2B\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\Big(\frac{(1}{}}{d}\right.$$

$\Big\downarrow$ 65

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2a^2B\int \frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}+\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\Big(\frac{(1+i)\sqrt{a}\sqrt{}}{\sqrt{a+ia\tan}}}{d}\right.$$

$\Big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(2-2i)a^{3/2}(B+iA)\text{arctanh}\Big(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\Big)}{d}+\frac{2\sqrt[4]{-1}a^{3/2}B\arctan\Big(\frac{(-1)^{3/4}\sqrt{}}{\sqrt{a+ia}}}{d}\right.$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*(-1)^(1/4)*a^(3/2)*B*ArcTan[((-1
)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((2 -
 2*I)*a^(3/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[
a + I*a*Tan[c + d*x]]])/d - (2*a*A*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan
[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(150) = 300.

Time = 0.90 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.83

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a\left(-4iA\sqrt{-ia}\,\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right.\right.}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a\left(-4iA\sqrt{-ia}\,\ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+a}{2\sqrt{ia}}\right.\right.}{}$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/2/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(-4*I*
A*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)+I*(I*a)^(1/2)*2^(1/2)*ln((2
*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*
x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-2*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+
c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*t
an(d*x+c)+(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-2*I*
(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)+4*A*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+2*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x
+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*
tan(d*x+c))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 674 vs. $2(142) = 284$.

Time = 0.10 (sec) , antiderivative size = 674, normalized size of antiderivative = 3.62

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
-1/4*(8*sqrt(2)*A*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x +
 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*e^(I*d*x + I*c) - 4*sqrt(2)*sqrt(-
(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*log(4*((A - I*B)*a^2*e^(I*d*x + I*c) -
 sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) + 4*sqrt(2)*sqrt(-(-I
*A^2 - 2*A*B + I*B^2)*a^3/d^2)*d*log(4*((A - I*B)*a^2*e^(I*d*x + I*c) - sq
rt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) + sqrt(-4*I*B^2*a^3/d^2
)*d*log(-16*(3*B*a^2*e^(2*I*d*x + 2*I*c) - B*a^2 + sqrt(2)*sqrt(-4*I*B^2*a
^3/d^2)*(d*e^(3*I*d*x + 3*I*c) - d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e
^(-2*I*d*x - 2*I*c)/B) - sqrt(-4*I*B^2*a^3/d^2)*d*log(-16*(3*B*a^2*e^(2*I*
d*x + 2*I*c) - B*a^2 - sqrt(2)*sqrt(-4*I*B^2*a^3/d^2)*(d*e^(3*I*d*x + 3*I*
c) - d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/B))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^2\,dx\right)bi+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)\,dx\right)ai+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)\,dx\right)b+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)*ta
n(c + d*x)**2,x)*b*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot
(c + d*x)*tan(c + d*x),x)*a*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c +
d*x))*cot(c + d*x)*tan(c + d*x),x)*b + int(sqrt(tan(c + d*x)*i + 1)*sqrt(c
ot(c + d*x))*cot(c + d*x),x)*a)
```

3.549 $\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 196

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{(-1)^{3/4}a^{3/2}(2iA+3B)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(2-2i)a^{3/2}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{iaB\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}}$$

output
```
-(-1)^(3/4)*a^(3/2)*(2*I*A+3*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)
/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(2-2*I)*a^(
3/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/
2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+I*a*B*(a+I*a*tan(d*x+c))^(1/2)/d/c
ot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 5.74 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.43

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{a\sqrt{\cot(c+dx)}\left(\sqrt[4]{-1}aB\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)(1+i\tan(c+dx))\sqrt{\tan(c+\ldots}{}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
(a*Sqrt[Cot[c + d*x]]*((-1)^(1/4)*a*B*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]
]]*(1 + I*Tan[c + d*x])*Sqrt[Tan[c + d*x]] - 2*Sqrt[a]*(A - I*B)*ArcSinh[S
qrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[I*a*Tan[c + d*x]]*(-I + Tan[c + d*x])
- Sqrt[1 + I*Tan[c + d*x]]*(a*B*Tan[c + d*x]*(-I + Tan[c + d*x]) + 2*Sqrt[
2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c +
 d*x]]]*Sqrt[I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])))/(d*Sqrt[1 + I
*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 4729, 3042, 4077, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \ 4729$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$\downarrow$ 4077

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2A-iB)+a(2iA+3B)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{\tan(c}{}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2A-iB)+a(2iA+3B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{\tan}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}(a(2A-iB)+a(2iA+3B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{\tan}}{}\right.$$

$\downarrow$ 4084

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(4a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-(2A-3iB)\int \frac{(a-ia\tan(c+dx))\sqrt{}}{\sqrt{\tan(c+}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(4a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-(2A-3iB)\int \frac{(a-ia\tan(c+dx))\sqrt{}}{\sqrt{\tan(c+}}\right.\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(-\frac{8ia^3(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-(2A-3iB)\int \frac{(a-ia\tan}{}\right.\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(4-4i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-(2A-3iB)\int\frac{(a-ia\tan\right.\right.$$

$\Big\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(4-4i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2(2A-3iB)\int\frac{}{\sqrt{\tan(c+dx}}}{}\right.\right.$$

$\Big\downarrow$ 65

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(4-4i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(2A-3iB)\int\frac{1}{1-\frac{ia\tan(}{i\tan(c+}}}{d}\right.\right.$$

$\Big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{2\sqrt[4]{-1}a^{3/2}(2A-3iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(4-4i)a^{3/2}(A-iB)\text{ar}}{}\right.\right.$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((2*(-1)^(1/4)*a^(3/2)*(2*A - (3*I)
*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*
x]]])/d + ((4 - 4*I)*a^(3/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/2 + (I*a*B*Sqrt[Tan[c + d*x]]*Sq
rt[a + I*a*Tan[c + d*x]])/d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(158) = 316$.

Time = 0.84 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.51

method	result
derivativedivides	$\frac{\sqrt{\frac{1}{\tan(dx+c)}}\,\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a\left(2iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{-ia}\,\sqrt{ia}-iB\ln\left(\frac{2ia\tan(dx+c)+2}{}\right.\right.}{}$
default	$\frac{\sqrt{\frac{1}{\tan(dx+c)}}\,\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a\left(2iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{-ia}\,\sqrt{ia}-iB\ln\left(\frac{2ia\tan(dx+c)+2}{}\right.\right.}{}$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
1/2/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a*(2*I*B*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*(I*a)^(1/2)-I*B*ln(1/2*
(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(
I*a)^(1/2))*(-I*a)^(1/2)*a+2*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a+2*ln(1/2*
(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(
I*a)^(1/2))*(-I*a)^(1/2)*a+I*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*(I*a)^(1/2
)*a+2*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(
I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I
))*(I*a)^(1/2)*a)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)
^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(148) = 296$.

Time = 0.11 (sec) , antiderivative size = 827, normalized size of antiderivative = 4.22

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
1/4*(4*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*
c) + d)*log(4*((A - I*B)*a^2*e^(I*d*x + I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^
2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I
*c)/((-I*A - B)*a)) - 4*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*
e^(2*I*d*x + 2*I*c) + d)*log(4*((A - I*B)*a^2*e^(I*d*x + I*c) - sqrt(-(I*A
^2 + 2*A*B - I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1
)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) + 4*sqrt(2)*(B*a*e^(3*I*d*x + 3*I*c)
- B*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*
x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) - sqrt((-4*I*A^2 - 12*A*B + 9*I
*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(-16*(3*(2*I*A + 3*B)*a^2*e^
(2*I*d*x + 2*I*c) + (-2*I*A - 3*B)*a^2 + 2*sqrt(2)*sqrt((-4*I*A^2 - 12*A*B
 + 9*I*B^2)*a^3/d^2)*(I*d*e^(3*I*d*x + 3*I*c) - I*d*e^(I*d*x + I*c))*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x
+ 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/(2*I*A + 3*B)) + sqrt((-4*I*A^2 - 12*
A*B + 9*I*B^2)*a^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(-16*(3*(2*I*A + 3*
B)*a^2*e^(2*I*d*x + 2*I*c) + (-2*I*A - 3*B)*a^2 + 2*sqrt(2)*sqrt((-4*I*A^2
 - 12*A*B + 9*I*B^2)*a^3/d^2)*(-I*d*e^(3*I*d*x + 3*I*c) + I*d*e^(I*d*x + I
*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)...
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}(A+B\tan(c+dx))\sqrt{\cot(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))*sqrt(cot(c +
 d*x)), x)
```

Maxima [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x +
 c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{3/2}\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2),
 x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^2\,dx\right)bi + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}dx\right)a\right)$$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a*(int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)**2
,x)*b*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x),x)*
a*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x),x)*b +
int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x)),x)*a)
```

3.550 $\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 244

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = -\frac{(-1)^{3/4}a^{3/2}(12A-11iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{4d} - \frac{(2+2i)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{iaB\sqrt{a+ia\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{a(4iA+5B)\sqrt{a+ia\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}}$$

output
```
-1/4*(-1)^(3/4)*a^(3/2)*(12*A-11*I*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)
^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(2+2*
I)*a^(3/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+1/2*I*a*B*(a+I*a*tan(d*x+c))
^(1/2)/d/cot(d*x+c)^(3/2)+1/4*a*(4*I*A+5*B)*(a+I*a*tan(d*x+c))^(1/2)/d/cot
(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 8.07 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.25

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{a\sqrt{\cot(c+dx)}\left(\sqrt[4]{-1}a(4iA+3B)\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan\right.\right.}{}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Cot[c +
 d*x]],x]
```

output
```
(a*Sqrt[Cot[c + d*x]]*((-1)^(1/4)*a*((4*I)*A + 3*B)*ArcSinh[(-1)^(1/4)*Sqr
t[Tan[c + d*x]]]*Sqrt[Tan[c + d*x]]*(-I + Tan[c + d*x]) + 8*Sqrt[a]*(I*A +
 B)*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[I*a*Tan[c + d*x]]*(-I + T
an[c + d*x]) + Sqrt[1 + I*Tan[c + d*x]]*(-8*Sqrt[2]*(A - I*B)*ArcTanh[(Sqr
t[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*Tan[c +
d*x]]*Sqrt[a + I*a*Tan[c + d*x]] - a*Tan[c + d*x]*(-I + Tan[c + d*x])*(4*A
 - (5*I)*B + 2*B*Tan[c + d*x]))))/(4*d*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4729, 3042, 4077, 27, 3042, 4080, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))dx$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}(A+B\tan(c+dx))dx$$

$\big\downarrow$ 4077

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{1}{2}\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(a(4A-3iB)+a(4iA+5B)\tan(c+dx)$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(a(4A-3iB)+a(4iA+5B)\tan(c+dx))d$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\int \sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}(a(4A-3iB)+a(4iA+5B)\tan(c+dx))d$$

$\big\downarrow$ 4080

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{\int -\frac{\sqrt{i\tan(c+dx)a+a}(a^2(4iA+5B)-a^2(12A-11iB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx}{a}+\frac{a(5B+4iA)\sqrt{\tan(c}{}\right.\right.$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a^2(4iA+5B)}{\sqrt{\tan(c}}}{2a}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a^2(4iA+5B)}{\sqrt{\tan(c}}}{2a}\right.\right.$$

$\big\downarrow$ 4084

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{16a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)}}{\sqrt{\tan(c+dx}}}{}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{16a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)}}{\sqrt{\tan(c+dx}}}{}\right.\right.$$

$$\Big\downarrow \text{ 4027}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{-\frac{32ia^4(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}}}{d}}{}\right.\right.$$

$$\Big\downarrow \text{ 218}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(16-16i)a^{5/2}(B+iA)\text{arctanh}\left(\right.}{d}}{}\right.\right.$$

$$\Big\downarrow \text{ 4082}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(16-16i)a^{5/2}(B+iA)\text{arctanh}\left(\right.}{d}}{}\right.\right.$$

$$\Big\downarrow \text{ 65}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(16-16i)a^{5/2}(B+iA)\text{arctanh}\left(\right.}{d}}{}\right.\right.$$

$$\Big\downarrow \text{ 216}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{a(5B+4iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{2\sqrt[4]{-1}a^{5/2}(11B+12iA)\arctan\left(\right.}{d}\right.\right.$$

input
```
Int[((a + I*a*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]]
,x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((((I/2)*a*B*Tan[c + d*x]^(3/2)*Sqrt[
a + I*a*Tan[c + d*x]])/d + (-1/2*((2*(-1)^(1/4)*a^(5/2)*((12*I)*A + 11*B)*
ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]
)/d + ((16 - 16*I)*a^(5/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/a + (a*((4*I)*A + 5*B)*Sqrt[Tan[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d)/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(194) = 388$.

Time = 0.91 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.86

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\left(-4iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)+\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\left(-4iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c)+\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/8/d*(a*(1+I*tan(d*x+c)))^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-
4*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)*tan(d
*x+c)+4*I*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)-8*I*A*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+5*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*ta
n(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2
))*a-10*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+1
6*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)
^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2)-8*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))
*(I*a)^(1/2)*a)/(1/tan(d*x+c))^(1/2)/(1+I*tan(d*x+c))/tan(d*x+c)/(I*a)^(1/
2)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 913 vs. 2(182) = 364.

Time = 0.10 (sec) , antiderivative size = 913, normalized size of antiderivative = 3.74

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, al
gorithm="fricas")
```

output
```
-1/16*(16*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(4*I*d*x +
4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((A - I*B)*a^2*e^(I*d*x + I*c)
 - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a)) - 16*sqrt(2)*sqrt(-
(-I*A^2 - 2*A*B + I*B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x
+ 2*I*c) + d)*log(4*((A - I*B)*a^2*e^(I*d*x + I*c) - sqrt(-(-I*A^2 - 2*A*B
 + I*B^2)*a^3/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e
^(-I*d*x - I*c)/((-I*A - B)*a)) - 4*sqrt(2)*((4*A - 7*I*B)*a*e^(5*I*d*x +
5*I*c) + 4*I*B*a*e^(3*I*d*x + 3*I*c) - (4*A - 3*I*B)*a*e^(I*d*x + I*c))*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) - 1)) - sqrt((144*I*A^2 + 264*A*B - 121*I*B^2)*a^3/d^2)*(d*e^(
4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-16*(3*(12*I*A + 11*B)
*a^2*e^(2*I*d*x + 2*I*c) + (-12*I*A - 11*B)*a^2 + 2*sqrt(2)*sqrt((144*I*A^
2 + 264*A*B - 121*I*B^2)*a^3/d^2)*(d*e^(3*I*d*x + 3*I*c) - d*e^(I*d*x + I*
c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/(12*I*A + 11*B)) + sqrt((144
*I*A^2 + 264*A*B - 121*I*B^2)*a^3/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I
*d*x + 2*I*c) + d)*log(-16*(3*(12*I*A + 11*B)*a^2*e^(2*I*d*x + 2*I*c) +...
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\,(A+B\tan{(c+dx)})}{\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**(3/2)*(A + B*tan(c + d*x))/sqrt(cot(c +
 d*x)), x)
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(3/2)/sqrt(cot(d*x +
 c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}{\sqrt{\cot(c+d\,x)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/cot(c + d*x)^(1/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(3/2))/cot(c + d*x)^(1/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx=\sqrt{a}\,a\left(\left(\left(\int \frac{\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx\right.\right.\right.}{\cot(dx+c)}$$

$$+\left(\int \frac{\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)ai$$

$$+\left(\int \frac{\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)b$$

$$+\left(\int \frac{\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a\Bigg)$$

input
```
int((a+I*a*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a*(int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)**
2)/cot(c + d*x),x)*b*i + int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*
tan(c + d*x))/cot(c + d*x),x)*a*i + int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot
(c + d*x))*tan(c + d*x))/cot(c + d*x),x)*b + int((sqrt(tan(c + d*x)*i + 1)
*sqrt(cot(c + d*x)))/cot(c + d*x),x)*a)
```

3.551 $\int \cot^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 297

$$\int \cot^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{(4+4i)a^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{8a^2(197A-195iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{315d} + \frac{8a^2(59iA+60B)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{315d} + \frac{2a^2(46A-45iB)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{105d} - \frac{2a^2(4iA+3B)\cot^{\frac{7}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d} - \frac{2aA\cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

output
```
(4+4*I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-8/315*a^2*(197*A-195*I*
B)*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+8/315*a^2*(59*I*A+60*B)*cot
(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d+2/105*a^2*(46*A-45*I*B)*cot(d*x+c
)^(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/21*a^2*(4*I*A+3*B)*cot(d*x+c)^(7/2)*(
a+I*a*tan(d*x+c))^(1/2)/d-2/9*a*A*cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(3/2
)/d
```

Mathematica [A] (verified)

Time = 12.35 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.19

$$\int \cot^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\left(4\sqrt{2}(A-iB)e^{-3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}}\operatorname{arctanh}\left(\frac{e^{i(c+dx)}}{\sqrt{-1+e^{2i(c+dx)}}}\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(11/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c +
d*x]),x]
```

output
```
(((4*Sqrt[2]*(A - I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[E^(I*(c + d*x))
/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x))))/(-1 + E^((2
*I)*(c + d*x)))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])/
E^((3*I)*(c + d*x)) + (Sqrt[Cot[c + d*x]]*Csc[c + d*x]^4*Sqrt[Sec[c + d*x]
]*(Cos[2*c] - I*Sin[2*c])*(-2331*A + (2205*I)*B + 12*(251*A - (260*I)*B)*C
os[2*(c + d*x)] + (-961*A + (915*I)*B)*Cos[4*(c + d*x)] + (282*I)*A*Sin[2*
(c + d*x)] + 390*B*Sin[2*(c + d*x)] - (331*I)*A*Sin[4*(c + d*x)] - 285*B*S
in[4*(c + d*x)]))/(1260*(Cos[d*x] + I*Sin[d*x])^2))*(a + I*a*Tan[c + d*x])
^(5/2)*(A + B*Tan[c + d*x]))/(d*Sec[c + d*x]^(7/2)*(A*Cos[c + d*x] + B*Sin
[c + d*x]))
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.08, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3042, 4729, 3042, 4076, 27, 3042, 4076, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{11/2}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{11/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{9}\int \frac{3(i\tan(c+dx)a+a)^{3/2}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{2\tan^{\frac{9}{2}}(c+dx)}dx-\frac{2aA(\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx-\frac{2aA(a}{9}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4iA+3B)-a(2A-3iB)\tan(c+dx))}{\tan(c+dx)^{9/2}}dx-\frac{2aA(a}{9}\right.$$

$$\Big\downarrow \text{4076}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{2}{7}\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((46A-45iB)a^2+(38iA+39B)\tan(c+dx)a^2\right)}{2\tan^{\frac{7}{2}}(c+dx)}dx\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{1}{7}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((46A-45iB)a^2+(38iA+39B)\tan(c+dx)a^2\right)}{\tan^{\frac{7}{2}}(c+dx)}dx\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{1}{7}\int \frac{\sqrt{i\tan(c+dx)a+a}\left((46A-45iB)a^2+(38iA+39B)\tan(c+dx)a^2\right)}{\tan(c+dx)^{7/2}}dx\right.\right.$$

$\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{2\int \frac{2\sqrt{i\tan(c+dx)a+a}\left(a^3(59iA+60B)-\right.}{\tan^{\frac{5}{2}}(c+dx}}{5a}\right.\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(59iA+60B)-a\right.}{\tan^{\frac{5}{2}}(c+dx)}}{5a}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(59iA+60B)-a\right.}{\tan(c+dx)^{5/}}}{5a}\right.\right.\right.$$

$\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((197A-195iB)a^4\right.}{2\tan^{\frac{3}{2}}(c+}}{3a}\right.}{}\right.\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((197A-195iB)a^4+\right.}{\tan^{\frac{3}{2}}(c+d}}{3a}\right.}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((197A-195iB)a^4+\right.}{\tan(c+dx)^3}}{3a}\right.}{}\right.\right.\right.$$

$\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{\frac{2\int\frac{315a^5(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-}{3a}\right.}{}\right.\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{315a^4(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+}{3a}\right.}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{315a^4(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{3a}\right.}{}\right.\right.\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{-\frac{630ia^6(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-}}{d}}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2a^2(46A-45iB)\sqrt{a+ia\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{4\left(-\frac{2a^3(60B+59iA)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}\right.}{}\right.\right.\right.$$

input
```
Int[Cot[c + d*x]^(11/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),
x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*(a + I*a*Tan[c + d*x])^(3/2
))/(9*d*Tan[c + d*x]^(9/2)) + ((-2*a^2*((4*I)*A + 3*B)*Sqrt[a + I*a*Tan[c
+ d*x]])/(7*d*Tan[c + d*x]^(7/2)) + ((2*a^2*(46*A - (45*I)*B)*Sqrt[a + I*a
*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) - (4*((-2*a^3*((59*I)*A + 60*B)*S
qrt[a + I*a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) - (((315 - 315*I)*a^(9
/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Ta
n[c + d*x]]])/d - (2*a^4*(197*A - (195*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(
d*Sqrt[Tan[c + d*x]]))/(3*a)))/(5*a))/7)/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 892 vs. 2(244) = 488.

Time = 1.83 (sec) , antiderivative size = 893, normalized size of antiderivative = 3.01

method	result
derivativedivides	$\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{11}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(-1576A\sqrt{ia}\,\tan(dx+c)^4\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{-ia}-3\right.}{}$
default	$\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{11}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(-1576A\sqrt{ia}\,\tan(dx+c)^4\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{-ia}-3\right.}{}$

input
```
int(cot(d*x+c)^(11/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_
RETURNVERBOSE)
```

output
```
1/315/d*(1/tan(d*x+c))^(11/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(-
1576*A*(I*a)^(1/2)*tan(d*x+c)^4*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*
a)^(1/2)-315*I*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)
^5+630*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^5+480*B*(-I*a)^(1/2)
*(I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-270*I*B*(I
*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)+
1260*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+
c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^5+1260*I*A*(-I*a)^(1/2
)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(
1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^5-315*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(ta
n(d*x+c)+I))*a*tan(d*x+c)^5+276*A*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)^2*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+1560*I*B*(I*a)^(1/2)*tan(d*x+c)^4*(a*t
an(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-630*(-I*a)^(1/2)*ln(1/2*(2*
I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a
)^(1/2))*a*tan(d*x+c)^5+472*I*A*(I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-190*I*A*(I*a)^(1/2)*tan(d*x+c)*(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-90*B*(-I*a)^(1/2)*(I*a)^(1/2)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(229) = 458.

Time = 0.10 (sec) , antiderivative size = 629, normalized size of antiderivative = 2.12

$$\int \cot^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(11/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, a
lgorithm="fricas")
```

output
```
2/315*(315*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(8*I*d*x +
 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d
*x + 2*I*c) + d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) - sqrt(-(-I*A^2 - 2*
A*B + I*B^2)*a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))
*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 315*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B +
I*B^2)*a^5/d^2)*(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(
4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((A - I*B)*a^3*e^(I*
d*x + I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(-I*d*e^(2*I*d*x + 2*
I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 2*sq
rt(2)*(2*(323*A - 300*I*B)*a^2*e^(9*I*d*x + 9*I*c) - 27*(61*A - 65*I*B)*a^
2*e^(7*I*d*x + 7*I*c) + 63*(37*A - 35*I*B)*a^2*e^(5*I*d*x + 5*I*c) - 1365*
(A - I*B)*a^2*e^(3*I*d*x + 3*I*c) + 315*(A - I*B)*a^2*e^(I*d*x + I*c))*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) - 1)))/(d*e^(8*I*d*x + 8*I*c) - 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e
^(4*I*d*x + 4*I*c) - 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(11/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4543 vs. 2(229) = 458.

Time = 4.79 (sec) , antiderivative size = 4543, normalized size of antiderivative = 15.30

$$\int \cot^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(11/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, a
lgorithm="maxima")
```

output
```
-1/1260*(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
 + 1)*((5040*((I + 1)*A - (I - 1)*B)*a^2*cos(7*d*x + 7*c) + 16800*(-(I + 1
)*A + (I - 1)*B)*a^2*cos(5*d*x + 5*c) + 20496*((I + 1)*A - (I - 1)*B)*a^2*
cos(3*d*x + 3*c) + (-(9071*I + 9071)*A + (8841*I - 8841)*B)*a^2*cos(d*x +
c) + 5040*((I - 1)*A + (I + 1)*B)*a^2*sin(7*d*x + 7*c) + 16800*(-(I - 1)*A
 - (I + 1)*B)*a^2*sin(5*d*x + 5*c) + 20496*((I - 1)*A + (I + 1)*B)*a^2*sin
(3*d*x + 3*c) + (-(9071*I - 9071)*A - (8841*I + 8841)*B)*a^2*sin(d*x + c))
*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 8*((-(121*I +
121)*A + (75*I - 75)*B)*a^2*cos(d*x + c) + (-(121*I - 121)*A - (75*I + 75)
*B)*a^2*sin(d*x + c) + ((-(121*I + 121)*A + (75*I - 75)*B)*a^2*cos(d*x + c
) + (-(121*I - 121)*A - (75*I + 75)*B)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^
2 + ((-(121*I + 121)*A + (75*I - 75)*B)*a^2*cos(d*x + c) + (-(121*I - 121)
*A - (75*I + 75)*B)*a^2*sin(d*x + c))*sin(2*d*x + 2*c)^2 + 630*(((I + 1)*A
 - (I - 1)*B)*a^2*cos(2*d*x + 2*c)^2 + ((I + 1)*A - (I - 1)*B)*a^2*sin(2*d
*x + 2*c)^2 + 2*(-(I + 1)*A + (I - 1)*B)*a^2*cos(2*d*x + 2*c) + ((I + 1)*A
 - (I - 1)*B)*a^2)*cos(3*d*x + 3*c) + 2*(((121*I + 121)*A - (75*I - 75)*B)
*a^2*cos(d*x + c) + ((121*I - 121)*A + (75*I + 75)*B)*a^2*sin(d*x + c))*co
s(2*d*x + 2*c) + 630*(((I - 1)*A + (I + 1)*B)*a^2*cos(2*d*x + 2*c)^2 + ((I
 - 1)*A + (I + 1)*B)*a^2*sin(2*d*x + 2*c)^2 + 2*(-(I - 1)*A - (I + 1)*B)*a
^2*cos(2*d*x + 2*c) + ((I - 1)*A + (I + 1)*B)*a^2)*sin(3*d*x + 3*c))*co...
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(11/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{11/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(11/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2)
,x)
```

output
```
int(cot(c + d*x)^(11/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2)
, x)
```

Reduce [F]

$$\int \cot^{\frac{11}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a^2\Bigg(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^5\tan(dx+c)^3\,dx\right)b$$
$$-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^5\tan(dx+c)^2\,dx\right)a$$
$$+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^5\tan(dx+c)^2\,dx\right)bi$$
$$+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^5\tan(dx+c)\,dx\right)ai$$
$$+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^5\tan(dx+c)\,dx\right)b$$
$$+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^5\,dx\right)a\Bigg)$$

input
```
int(cot(d*x+c)^(11/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d
*x)**5*tan(c + d*x)**3,x)*b - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*
x))*cot(c + d*x)**5*tan(c + d*x)**2,x)*a + 2*int(sqrt(tan(c + d*x)*i + 1)*
sqrt(cot(c + d*x))*cot(c + d*x)**5*tan(c + d*x)**2,x)*b*i + 2*int(sqrt(tan
(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**5*tan(c + d*x),x)*a*i +
int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**5*tan(c + d*
x),x)*b + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**5,
x)*a)
```

3.552 $\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 251

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{(4-4i)a^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{4a^2(130iA+133B)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{105d} + \frac{2a^2(80A-77iB)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{105d} - \frac{2a^2(10iA+7B)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} - \frac{2aA\cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{7d}$$

output
```
(4-4*I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+4/105*a^2*(130*I*A+133*
B)*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+2/105*a^2*(80*A-77*I*B)*cot
(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/35*a^2*(10*I*A+7*B)*cot(d*x+c)^
(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/7*a*A*cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c
))^(3/2)/d
```

Mathematica [A] (verified)

Time = 10.44 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.32

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\left(-4i\sqrt{2}(A-iB)e^{-3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i\left(1+e^{2i(c+dx)}\right)}{-1+e^{2i(c+dx)}}}\operatorname{arctanh}\left(\frac{e^{i(c+}}{\sqrt{-1+e}}\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(9/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
((((-4*I)*Sqrt[2]*(A - I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[E^(I*(c +
d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x))))/(-1 +
E^((2*I)*(c + d*x)))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x)
)]])/E^((3*I)*(c + d*x)) - (Sqrt[Cot[c + d*x]]*Csc[c + d*x]^3*Sqrt[Sec[c +
 d*x]]*(Cos[2*c] - I*Sin[2*c])*((-35*A + (77*I)*B)*Cos[c + d*x] + (95*A -
(77*I)*B)*Cos[3*(c + d*x)] + 2*((-215*I)*A - 245*B + ((305*I)*A + 287*B)*C
os[2*(c + d*x)])*Sin[c + d*x]))/(210*(Cos[d*x] + I*Sin[d*x])^2))*(a + I*a*
Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/(d*Sec[c + d*x]^(7/2)*(A*Cos[c +
 d*x] + B*Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4729, 3042, 4076, 27, 3042, 4076, 27, 3042, 4081, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{9/2}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{ 4076}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{7}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(10iA+7B)-a(4A-7iB)\tan(c+dx))}{2\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2aA(\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(10iA+7B)-a(4A-7iB)\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx-\frac{2aA(\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(10iA+7B)-a(4A-7iB)\tan(c+dx))}{\tan(c+dx)^{7/2}}dx-\frac{2aA(\right.$$

$$\Big\downarrow \text{ 4076}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{2}{5}\int -\frac{\sqrt{i\tan(c+dx)a+a}((80A-77iB)a^2+3(20iA+21B)\tan(c+dx)a^2)}{2\tan^{\frac{5}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{1}{5}\int \frac{\sqrt{i\tan(c+dx)a+a}((80A-77iB)a^2+3(20iA+21B)\tan(c+dx)a^2)}{\tan^{\frac{5}{2}}(c+dx)}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{1}{5}\int \frac{\sqrt{i\tan(c+dx)a+a}((80A-77iB)a^2+3(20iA+21B)\tan(c+dx)a^2)}{\tan(c+dx)^{5/2}}\right.\right.$$

$$\Big\downarrow \text{4081}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\int\frac{\sqrt{i\tan(c+dx)a+a}(a^3(130iA+133B)-}{\tan^{\frac{3}{2}}(c+dx}}{3a}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\int\frac{\sqrt{i\tan(c+dx)a+a}(a^3(130iA+133B)-}{\tan(c+dx)^3}}{3a}\right.\right.\right.$$

$$\Big\downarrow \text{4081}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\left(\frac{2\int-\frac{105a^4(A-iB)\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}\right.}{3}\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\left(-210a^3(A-iB)\int\frac{\sqrt{i\tan(c+dx)}}{\sqrt{\tan(c+dx}}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\left(-210a^3(A-iB)\int\frac{\sqrt{i\tan(c+dx)}}{\sqrt{\tan(c+dx}}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{4027}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\left(\frac{420ia^5(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}}{d}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2a^2(80A-77iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2\left(-\frac{(210-210i)a^{7/2}(A-iB)\text{arctanh}\left(\right.}{d}\right.}{}\right.\right.\right.$$

input
```
Int[Cot[c + d*x]^(9/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*(a + I*a*Tan[c + d*x])^(3/2
))/(7*d*Tan[c + d*x]^(7/2)) + ((-2*a^2*((10*I)*A + 7*B)*Sqrt[a + I*a*Tan[c
 + d*x]])/(5*d*Tan[c + d*x]^(5/2)) + ((2*a^2*(80*A - (77*I)*B)*Sqrt[a + I*
a*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) - (2*(((-210 + 210*I)*a^(7/2)*(A
 - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c +
d*x]]])/d - (2*a^3*((130*I)*A + 133*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt
[Tan[c + d*x]])))/(3*a))/5)/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(206) = 412.

Time = 1.82 (sec) , antiderivative size = 806, normalized size of antiderivative = 3.21

method	result
derivativedivides	$\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{9}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(-154iB\sqrt{-ia}\sqrt{ia}\,\tan(dx+c)^2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+53\right.}{}$
default	$\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{9}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(-154iB\sqrt{-ia}\sqrt{ia}\,\tan(dx+c)^2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+53\right.}{}$

input
```
int(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
1/105/d*(1/tan(d*x+c))^(9/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(-1
54*I*B*(I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*
a)^(1/2)+532*B*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)+520*I*A*(I*a)^(1/2)*tan(d*x+c)^3*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)*(-I*a)^(1/2)+420*A*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x
+c)^4+160*A*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)-420*I*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^4-90*I*
A*(I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2
)-105*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*
tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+210*
I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^4+210*(-I*a)^(1/2)*ln(1/2*(
2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I
*a)^(1/2))*a*tan(d*x+c)^4+105*I*(I*a)^(1/2)*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)
^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+
c)+I))*a*tan(d*x+c)^4-42*B*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)-30*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a
)^(1/2)*(-I*a)^(1/2))/(I*a)^(1/2)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(193) = 386.

Time = 0.09 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.30

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
2/105*(105*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(6*I*d*x +
6*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*((A
- I*B)*a^3*e^(I*d*x + I*c) + sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(
2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^
2)) - 105*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(6*I*d*x + 6
*I*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)*log(4*((A -
 I*B)*a^3*e^(I*d*x + I*c) - sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(2
*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x
+ 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2
)) - 2*sqrt(2)*(2*(-100*I*A - 91*B)*a^2*e^(7*I*d*x + 7*I*c) + 7*(55*I*A +
61*B)*a^2*e^(5*I*d*x + 5*I*c) + 350*(-I*A - B)*a^2*e^(3*I*d*x + 3*I*c) + 1
05*(I*A + B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(6*I*d*x + 6*I
*c) - 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(9/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4087 vs. 2(193) = 386.

Time = 1.96 (sec) , antiderivative size = 4087, normalized size of antiderivative = 16.28

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
-1/105*(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c)
+ 1)*(3*(140*(-(I - 1)*A - (I + 1)*B)*a^2*cos(7*d*x + 7*c) + 140*((I - 1)*
A + (2*I + 2)*B)*a^2*cos(5*d*x + 5*c) + 21*(-(4*I - 4)*A - (9*I + 9)*B)*a^
2*cos(3*d*x + 3*c) + ((4*I - 4)*A + (49*I + 49)*B)*a^2*cos(d*x + c) + 140*
((I + 1)*A - (I - 1)*B)*a^2*sin(7*d*x + 7*c) + 140*(-(I + 1)*A + (2*I - 2)
*B)*a^2*sin(5*d*x + 5*c) + 21*((4*I + 4)*A - (9*I - 9)*B)*a^2*sin(3*d*x +
3*c) + (-(4*I + 4)*A + (49*I - 49)*B)*a^2*sin(d*x + c))*cos(7/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 4*(((65*I - 65)*A + (56*I + 56)*B
)*a^2*cos(d*x + c) + (-(65*I + 65)*A + (56*I - 56)*B)*a^2*sin(d*x + c) + (
((65*I - 65)*A + (56*I + 56)*B)*a^2*cos(d*x + c) + (-(65*I + 65)*A + (56*I
 - 56)*B)*a^2*sin(d*x + c))*cos(2*d*x + 2*c)^2 + (((65*I - 65)*A + (56*I +
 56)*B)*a^2*cos(d*x + c) + (-(65*I + 65)*A + (56*I - 56)*B)*a^2*sin(d*x +
c))*sin(2*d*x + 2*c)^2 + 105*((-(I - 1)*A - (I + 1)*B)*a^2*cos(2*d*x + 2*c
)^2 + (-(I - 1)*A - (I + 1)*B)*a^2*sin(2*d*x + 2*c)^2 + 2*((I - 1)*A + (I
+ 1)*B)*a^2*cos(2*d*x + 2*c) + (-(I - 1)*A - (I + 1)*B)*a^2)*cos(3*d*x + 3
*c) + 2*((-(65*I - 65)*A - (56*I + 56)*B)*a^2*cos(d*x + c) + ((65*I + 65)*
A - (56*I - 56)*B)*a^2*sin(d*x + c))*cos(2*d*x + 2*c) + 105*(((I + 1)*A -
(I - 1)*B)*a^2*cos(2*d*x + 2*c)^2 + ((I + 1)*A - (I - 1)*B)*a^2*sin(2*d*x
+ 2*c)^2 + 2*(-(I + 1)*A + (I - 1)*B)*a^2*cos(2*d*x + 2*c) + ((I + 1)*A -
(I - 1)*B)*a^2)*sin(3*d*x + 3*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos...
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{9/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
x)
```

output
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{9}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^4\tan(dx+c)^3\,dx\right)b - \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^4\tan(dx+c)^2\,dx\right)a + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^4\tan(dx+c)^2\,dx\right)bi + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^4\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^4\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^4\,dx\right)a\right)$$

input
```
int(cot(d*x+c)^(9/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d
*x)**4*tan(c + d*x)**3,x)*b - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*
x))*cot(c + d*x)**4*tan(c + d*x)**2,x)*a + 2*int(sqrt(tan(c + d*x)*i + 1)*
sqrt(cot(c + d*x))*cot(c + d*x)**4*tan(c + d*x)**2,x)*b*i + 2*int(sqrt(tan
(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**4*tan(c + d*x),x)*a*i +
int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**4*tan(c + d*
x),x)*b + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**4,
x)*a)
```

3.553 $\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 205

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{(4+4i)a^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{2a^2(38A-35iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{15d} - \frac{2a^2(8iA+5B)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} - \frac{2aA\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

output
```
(-4-4*I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan
(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+2/15*a^2*(38*A-35*I*B)
*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/15*a^2*(8*I*A+5*B)*cot(d*x+
c)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d-2/5*a*A*cot(d*x+c)^(5/2)*(a+I*a*tan(d*
x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 8.68 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.49

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\left(-4\sqrt{2}(A-iB)e^{-3i(c+dx)}\sqrt{-1+e^{2i(c+dx)}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{\frac{i(1+e^{2i(c+dx)})}{-1+e^{2i(c+dx)}}}\text{arctanh}\left(\frac{e^{i(c+}}{\sqrt{-1+e^{2}}}\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
(((-4*Sqrt[2]*(A - I*B)*Sqrt[-1 + E^((2*I)*(c + d*x))]*Sqrt[E^(I*(c + d*x)
)/(1 + E^((2*I)*(c + d*x)))]*Sqrt[(I*(1 + E^((2*I)*(c + d*x))))/(-1 + E^((
2*I)*(c + d*x)))]*ArcTanh[E^(I*(c + d*x))/Sqrt[-1 + E^((2*I)*(c + d*x))]])
/E^((3*I)*(c + d*x)) - (Sqrt[Cot[c + d*x]]*Csc[c + d*x]^2*Sqrt[Sec[c + d*x
]]*(Cos[2*c] - I*Sin[2*c])*(-35*(A - I*B) + (41*A - (35*I)*B)*Cos[2*(c + d
*x)] + ((11*I)*A + 5*B)*Sin[2*(c + d*x)]))/(15*(Cos[d*x] + I*Sin[d*x])^2))
*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/(d*Sec[c + d*x]^(7/2)*
(A*Cos[c + d*x] + B*Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4729, 3042, 4076, 27, 3042, 4076, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{7/2}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{5}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(8iA+5B)-a(2A-5iB)\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA(a}{5}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(8iA+5B)-a(2A-5iB)\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2aA(a}{5}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(8iA+5B)-a(2A-5iB)\tan(c+dx))}{\tan(c+dx)^{5/2}}dx-\frac{2aA(a}{5}\right.$$

$$\Big\downarrow \text{4076}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{2}{3}\int -\frac{\sqrt{i\tan(c+dx)a+a}((38A-35iB)a^2+(22iA+25B)\tan(c+dx)a^2)}{2\tan^{\frac{3}{2}}(c+dx)}dx\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{1}{3}\int \frac{\sqrt{i\tan(c+dx)a+a}((38A-35iB)a^2+(22iA+25B)\tan(c+dx)a^2)}{\tan^{\frac{3}{2}}(c+dx)}dx\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{1}{3}\int \frac{\sqrt{i\tan(c+dx)a+a}((38A-35iB)a^2+(22iA+25B)\tan(c+dx)a^2)}{\tan(c+dx)^{3/2}}dx\right.\right.$$

$$\Big\downarrow \text{4081}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^2(38A-35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{2\int\frac{30a^3(iA+B)\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{a}\right)\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^2(38A-35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-60a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)a}}{\sqrt{\tan(c+dx)}}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^2(38A-35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-60a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)a}}{\sqrt{\tan(c+dx)}}\right.\right.\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{120ia^4(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}+\frac{2a^2(38A-35iB)\sqrt{a}}{d\sqrt{\tan(c}}\right.\right.\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^2(38A-35iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{(60-60i)a^{5/2}(B+iA)\text{arctanh}\left(\right.}{d}\right.\right.\right.$$

input

```
Int[Cot[c + d*x]^(7/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output

```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*a*A*(a + I*a*Tan[c + d*x])^(3/2
))/(5*d*Tan[c + d*x]^(5/2)) + ((-2*a^2*((8*I)*A + 5*B)*Sqrt[a + I*a*Tan[c
+ d*x]])/(3*d*Tan[c + d*x]^(3/2)) + (((-60 + 60*I)*a^(5/2)*(I*A + B)*ArcTa
nh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + (
2*a^2*(38*A - (35*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
)/3)/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 714 vs. 2(168) = 336.

Time = 1.84 (sec) , antiderivative size = 715, normalized size of antiderivative = 3.49

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{7}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(-76A\sqrt{-ia}\,\sqrt{ia}\,\tan(dx+c)^2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+60\right.}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{7}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(-76A\sqrt{-ia}\,\sqrt{ia}\,\tan(dx+c)^2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+60\right.}{}$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/15/d*(1/tan(d*x+c))^(7/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(-7
6*A*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)+60*I*A*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^3-15*I*2^(1/2)*(
I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+70*I*B*(-I*a)^(1/2)*(
I*a)^(1/2)*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+60*B*(-I*a)^
(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*
a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^3-15*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1/
2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/
(tan(d*x+c)+I))*a*tan(d*x+c)^3+30*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+
2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(
d*x+c)^3+22*I*A*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)-30*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^3+10*B*(-I
*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+6*A
*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/(a*tan(d*
x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(157) = 314$.

Time = 0.10 (sec) , antiderivative size = 515, normalized size of antiderivative = 2.51

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{2\left(15\sqrt{2}\sqrt{-\frac{(-iA^2-2AB+iB^2)a^5}{d^2}}\left(de^{(4i\,dx+4i\,c)}-2\,de^{(2i\,dx+2i\,c)}+d\right)\log\left(\frac{4\left((A-iB)a^3e^{(i\,dx+i\,c)}-\sqrt{-\frac{(-iA^2-2AB+i}{d^2}}\right.}{}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
-2/15*(15*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(4*I*d*x +
4*I*c) - 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c)
 - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 15*sqrt(2)*sqrt
(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I*d*
x + 2*I*c) + d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) - sqrt(-(-I*A^2 - 2*A
*B + I*B^2)*a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))
*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 2*sqrt(2)*(2*(13*A - 10*I*B)*a^2*e^(
5*I*d*x + 5*I*c) - 35*(A - I*B)*a^2*e^(3*I*d*x + 3*I*c) + 15*(A - I*B)*a^2
*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2
*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(2*I
*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1545 vs. $2(157) = 314$.

Time = 0.54 (sec) , antiderivative size = 1545, normalized size of antiderivative = 7.54

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
2/15*(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) +
1)*((30*((I + 1)*A - (I - 1)*B)*a^2*cos(3*d*x + 3*c) + (-(31*I + 31)*A + (
25*I - 25)*B)*a^2*cos(d*x + c) + 30*((I - 1)*A + (I + 1)*B)*a^2*sin(3*d*x
+ 3*c) + (-(31*I - 31)*A - (25*I + 25)*B)*a^2*sin(d*x + c))*cos(3/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + (30*(-(I - 1)*A - (I + 1)*B)*
a^2*cos(3*d*x + 3*c) + ((31*I - 31)*A + (25*I + 25)*B)*a^2*cos(d*x + c) +
30*((I + 1)*A - (I - 1)*B)*a^2*sin(3*d*x + 3*c) + (-(31*I + 31)*A + (25*I
- 25)*B)*a^2*sin(d*x + c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) - 1)))*sqrt(a) + 15*(2*((-(I - 1)*A - (I + 1)*B)*a^2*cos(2*d*x + 2*c)^
2 + (-(I - 1)*A - (I + 1)*B)*a^2*sin(2*d*x + 2*c)^2 + 2*((I - 1)*A + (I +
1)*B)*a^2*cos(2*d*x + 2*c) + (-(I - 1)*A - (I + 1)*B)*a^2)*arctan2(2*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*sin(d*x + c), 2*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1)) + 2*cos(d*x + c)) + (
(-(I + 1)*A + (I - 1)*B)*a^2*cos(2*d*x + 2*c)^2 + (-(I + 1)*A + (I - 1)*B)
*a^2*sin(2*d*x + 2*c)^2 + 2*((I + 1)*A - (I - 1)*B)*a^2*cos(2*d*x + 2*c) +
 (-(I + 1)*A + (I - 1)*B)*a^2)*log(4*cos(d*x + c)^2 + 4*sin(d*x + c)^2 + 4
*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 - 2*cos(2*d*x + 2*c) + 1)*(c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) - 1))^2 + sin(1/2*arc...
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a^2\Bigg(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^3\tan(dx+c)^3\,dx\right)b-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^3\tan(dx+c)^2\,dx\right)a+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^3\tan(dx+c)^2\,dx\right)bi+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^3\tan(dx+c)\,dx\right)ai+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^3\tan(dx+c)\,dx\right)b+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^3\,dx\right)a\Bigg)$$

input
```
int(cot(d*x+c)^(7/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d
*x)**3*tan(c + d*x)**3,x)*b - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*
x))*cot(c + d*x)**3*tan(c + d*x)**2,x)*a + 2*int(sqrt(tan(c + d*x)*i + 1)*
sqrt(cot(c + d*x))*cot(c + d*x)**3*tan(c + d*x)**2,x)*b*i + 2*int(sqrt(tan
(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**3*tan(c + d*x),x)*a*i +
int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**3*tan(c + d*
x),x)*b + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**3,
x)*a)
```

3.554 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 230

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{2(-1)^{3/4}a^{5/2}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(4+4i)a^{5/2}(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{2a^2(2iA+B)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{2aA\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

output
```
2*(-1)^(3/4)*a^(5/2)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*t
an(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(4+4*I)*a^(5/2)*(I*A
+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d
*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2*a^2*(2*I*A+B)*cot(d*x+c)^(1/2)*(a+I*a*tan
(d*x+c))^(1/2)/d-2/3*a*A*cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 575 vs. $2(230) = 460$.

Time = 7.78 (sec) , antiderivative size = 575, normalized size of antiderivative = 2.50

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-\frac{2A(a+ia\tan(c+dx))^{5/2}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{2\left(-\frac{a(5iA+3B)(a+ia\tan(c+dx))^{5/2}}{d\sqrt{\tan(c+dx)}}+\frac{2\left(\frac{ia^4(5iA+3B)\sqrt{a+ia\tan(c+dx)}\left(-\frac{3}{4}(-1)^{3/4}\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)+\frac{5}{4}\sqrt{1+i\tan(c+dx)}\sqrt{\tan(c+dx)}\right.}{d\sqrt{1+i\tan(c+dx)}}\right.}{}\right.}{}\right.$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A*(a + I*a*Tan[c + d*x])^(5/2))
/(3*d*Tan[c + d*x]^(3/2)) + (2*(-((a*((5*I)*A + 3*B)*(a + I*a*Tan[c + d*x]
)^(5/2))/(d*Sqrt[Tan[c + d*x]])) + (2*((I*a^4*((5*I)*A + 3*B)*Sqrt[a + I*a
*Tan[c + d*x]]*((-3*(-1)^(3/4)*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]])/4 +
 (5*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/4 + (I/2)*Sqrt[1 + I*Tan[
c + d*x]]*Tan[c + d*x]^(3/2)))/(d*Sqrt[1 + I*Tan[c + d*x]]) + (a*((-1/4*I)
*a^3*(23*A - (15*I)*B) + a^3*((5*I)*A + 3*B))*(((-4*I)*Sqrt[2]*a*ArcTanh[(
Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[Tan[c + d
*x]])/Sqrt[I*a*Tan[c + d*x]] + ((4*I)*a^(3/2)*ArcSinh[Sqrt[I*a*Tan[c + d*x
]]/Sqrt[a]]*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a*Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + I*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan
[c + d*x]] + (I*Sqrt[a]*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[Tan[c
 + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[1 + I*Tan[c + d*x]]*Sqrt[I*a*Ta
n[c + d*x]]))))/d))/a))/(3*a))
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.92, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4729, 3042, 4076, 27, 3042, 4076, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{5/2}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4076}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{3}\int \frac{3(i\tan(c+dx)a+a)^{3/2}(a(2iA+B)+iaB\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan}{3d\tan^{\frac{3}{2}}(c}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(2iA+B)+iaB\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2aA(a+ia\tan(c}{3d\tan^{\frac{3}{2}}(c+}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(2iA+B)+iaB\tan(c+dx))}{\tan(c+dx)^{3/2}}dx-\frac{2aA(a+ia\tan(c}{3d\tan^{\frac{3}{2}}(c+}\right.$$

$$\Big\downarrow \text{4076}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)}{2\sqrt{\tan(c+dx)}}dx-\frac{2a^2(B+2iA)}{d\sqrt{t}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\int \frac{\sqrt{i\tan(c+dx)a+a}\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(B+2iA)\sqrt{}}{d\sqrt{\tan}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\int \frac{\sqrt{i\tan(c+dx)a+a}\left((4A-3iB)a^2+B\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(B+2iA)\sqrt{}}{d\sqrt{\tan}}\right.$$

$$\Big\downarrow \text{4084}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-4a^2(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-iaB\int \frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+}}{\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-4a^2(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)}}{\sqrt{\tan(c+dx)}}\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{8ia^4(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i}}{\sqrt{\tan(c+}}\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{(4-4i)a^{5/2}(A-iB)\text{arcta}}{d}\right.$$

$\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{ia^3B\int\frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}-\frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\right.}{d}\right.$$

$\downarrow$ 65

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2ia^3B\int\frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}}{\sqrt{a+ia}}\right.}{d}\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{2(-1)^{3/4}a^{5/2}B\arctan\left(\frac{(-1)}{\sqrt{}}\right.}{d}\right.$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*(-1)^(3/4)*a^(5/2)*B*ArcTan[((-1
)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - ((4 -
 4*I)*a^(5/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[
a + I*a*Tan[c + d*x]]])/d - (2*a^2*((2*I)*A + B)*Sqrt[a + I*a*Tan[c + d*x]
])/(d*Sqrt[Tan[c + d*x]]) - (2*a*A*(a + I*a*Tan[c + d*x])^(3/2))/(3*d*Tan[
c + d*x]^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 625 vs. 2(186) = 372.

Time = 1.84 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.72

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(-9iB\sqrt{-ia}\, \ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+}{2\sqrt{ia}}\right.\right.}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(-9iB\sqrt{-ia}\, \ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+}{2\sqrt{ia}}\right.\right.}{}$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/3/d*(1/tan(d*x+c))^(5/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(-9*
I*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^2+3*I*2^(1/2)*(I*a)^(1/2)
*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*
tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+14*I*A*(-I*a)^(1/2)*tan(d*x+c)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+12*A*(-I*a)^(1/2)*ln(1/2
*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/
(I*a)^(1/2))*a*tan(d*x+c)^2-3*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I
))*a*tan(d*x+c)^2+6*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^2+6*B*(
-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+6
*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^2+2*A*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2))/(-I*a)^(1/2)/(a*tan(d*x+c)*(1+I*
tan(d*x+c)))^(1/2)/(I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(176) = 352$.

Time = 0.14 (sec) , antiderivative size = 780, normalized size of antiderivative = 3.39

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
-1/12*(24*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2
*I*c) - d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) + sqrt(-(I*A^2 + 2*A*B - I
*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x
- I*c)/((-I*A - B)*a^2)) - 24*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^
2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) - sqrt
(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*
c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) + 8*sqrt(2)*((8*I*A + 3*B)*a^
2*e^(3*I*d*x + 3*I*c) + 3*(-2*I*A - B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
- 1)) - 3*sqrt(4*I*B^2*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(-16*(3*B*a
^3*e^(2*I*d*x + 2*I*c) - B*a^3 + sqrt(2)*sqrt(4*I*B^2*a^5/d^2)*(I*d*e^(3*I
*d*x + 3*I*c) - I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*
I*c)/(B*a)) + 3*sqrt(4*I*B^2*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*log(-16*
(3*B*a^3*e^(2*I*d*x + 2*I*c) - B*a^3 + sqrt(2)*sqrt(4*I*B^2*a^5/d^2)*(-I*d
*e^(3*I*d*x + 3*I*c) + I*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d
*x - 2*I*c)/(B*a)))/(d*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^3dx\right)b-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2dx\right)a+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2dx\right)bi+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)ai+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)b+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2dx\right)a\right)$$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d
*x)**2*tan(c + d*x)**3,x)*b - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*
x))*cot(c + d*x)**2*tan(c + d*x)**2,x)*a + 2*int(sqrt(tan(c + d*x)*i + 1)*
sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**2,x)*b*i + 2*int(sqrt(tan
(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x),x)*a*i +
int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*
x),x)*b + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**2,
x)*a)
```

3.555 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 236

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{(-1)^{3/4}a^{5/2}(2A-5iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(4+4i)a^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{a^2(2iA-B)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{2aA\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}{d}$$

output
```
(-1)^(3/4)*a^(5/2)*(2*A-5*I*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/
(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(4+4*I)*a^(5
/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+a^2*(2*I*A-B)*(a+I*a*tan(d*x+c))^(1
/2)/d/cot(d*x+c)^(1/2)-2*a*A*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 6.43 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.22

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =$$
$$-\frac{ia^2\sqrt{\cot(c+dx)}\left(-3\sqrt[4]{-1}aA\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}(-i+\tan(c+dx))+5\sqrt{a}(A-\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
((-I)*a^2*Sqrt[Cot[c + d*x]]*(-3*(-1)^(1/4)*a*A*ArcSinh[(-1)^(1/4)*Sqrt[Ta
n[c + d*x]]]*Sqrt[Tan[c + d*x]]*(-I + Tan[c + d*x]) + 5*Sqrt[a]*(A - I*B)*
ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[I*a*Tan[c + d*x]]*(-I + Tan[c
 + d*x]) + Sqrt[1 + I*Tan[c + d*x]]*(4*Sqrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*
Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*Tan[c + d*x]]
*Sqrt[a + I*a*Tan[c + d*x]] + a*(-I + Tan[c + d*x])*(2*A + B*Tan[c + d*x])
)))/(d*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4729, 3042, 4076, 27, 3042, 4077, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$\downarrow$ 4076

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(2\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4iA+B)+a(2A+iB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+}{d\sqrt{}}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4iA+B)+a(2A+iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+ia}{d\sqrt{t}}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4iA+B)+a(2A+iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx-\frac{2aA(a+ia}{d\sqrt{t}}\right.$$

$\downarrow$ 4077

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{\sqrt{i\tan(c+dx)a+a}(3a^2(2iA+B)-a^2(2A-5iB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx+\frac{a^2(-B-}{}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}(3a^2(2iA+B)-a^2(2A-5iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(-B}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}(3a^2(2iA+B)-a^2(2A-5iB)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(-B}{}\right.$$

$\downarrow$ 4084

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(8a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a(5B+2iA)\int\frac{(a-ia\tan(c+dx))}{\sqrt{\tan(}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(8a^2(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a(5B+2iA)\int\frac{(a-ia\tan(c+dx))}{\sqrt{\tan(}}\right.\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(-\frac{16ia^4(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-a(5B+2iA)\int\frac{(a-ia}{}\right.\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(8-8i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-a(5B+2iA)\int\frac{(a-ia\,\text{ta}}{}\right.\right.$$

$\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(8-8i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^3(5B+2iA)\int\frac{}{\sqrt{\tan(c+dx}}}{}\right.\right.$$

$\downarrow$ 65

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(8-8i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(5B+2iA)\int\frac{1}{1-\frac{ia\tan(}{i\tan(c+}}}{d}\right.\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{2\sqrt[4]{-1}a^{5/2}(5B+2iA)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(8-8i)a^{5/2}(B+iA)\text{ar}}{}\right.\right.$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((2*(-1)^(1/4)*a^(5/2)*((2*I)*A + 5
*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*
x]]])/d + ((8 - 8*I)*a^(5/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/2 + (a^2*((2*I)*A - B)*Sqrt[Tan[
c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d - (2*a*A*(a + I*a*Tan[c + d*x])^(3
/2))/(d*Sqrt[Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4076
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-a^2)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(d*f*(b*c + a*d)*(n + 1))), x] - Simp[a/(d*(b*c + a*d)*(n + 1))   Int[
(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*b*d*(m - n
 - 2) - B*(b*c*(m - 1) + a*d*(n + 1)) + (a*A*d*(m + n) - B*(a*c*(m - 1) + b
*d*(n + 1)))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(193) = 386$.

Time = 0.79 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.42

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(-6iA\sqrt{-ia}\, \ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+}{2\sqrt{ia}}\right.\right.}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\, a^2 \left(-6iA\sqrt{-ia}\, \ln\left(\frac{2ia\tan(dx+c)+2\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}+}{2\sqrt{ia}}\right.\right.}{}$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/2/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(-6*
I*A*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)+2*I*2^(1/2)*(I*a)^(1/2)*l
n((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*ta
n(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+2*B*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*
x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-3*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*
tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1
/2))*a*tan(d*x+c)-4*I*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)+2*(I*a)
^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+4*A*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I*a)^(1/2)+4*(-I*a)^(1/2)*ln(1/2*(2*I
*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)
^(1/2))*a*tan(d*x+c))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(I*a)^(1/2)/(-
I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 849 vs. $2(182) = 364$.

Time = 0.10 (sec) , antiderivative size = 849, normalized size of antiderivative = 3.60

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
1/4*(8*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I
*c) + d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) - sqrt(-(-I*A^2 - 2*A*B + I*
B^2)*a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d
*x - I*c)/((-I*A - B)*a^2)) - 8*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5
/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) - s
qrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d
*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 4*sqrt(2)*((2*A -
I*B)*a^2*e^(3*I*d*x + 3*I*c) + (2*A + I*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I
*c) - 1)) - sqrt((4*I*A^2 + 20*A*B - 25*I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*
I*c) + d)*log(-16*(3*(2*I*A + 5*B)*a^3*e^(2*I*d*x + 2*I*c) + (-2*I*A - 5*B
)*a^3 + 2*sqrt(2)*sqrt((4*I*A^2 + 20*A*B - 25*I*B^2)*a^5/d^2)*(d*e^(3*I*d*
x + 3*I*c) - d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*
e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/
((2*I*A + 5*B)*a)) + sqrt((4*I*A^2 + 20*A*B - 25*I*B^2)*a^5/d^2)*(d*e^(2*I
*d*x + 2*I*c) + d)*log(-16*(3*(2*I*A + 5*B)*a^3*e^(2*I*d*x + 2*I*c) + (-2*
I*A - 5*B)*a^3 - 2*sqrt(2)*sqrt((4*I*A^2 + 20*A*B - 25*I*B^2)*a^5/d^2)*(d*
e^(3*I*d*x + 3*I*c) - d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + ...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{3/2}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
 x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\sqrt{a}\,a^2\Bigg(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^3\,dx\right)b-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^2\,dx\right)a+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^2\,dx\right)bi+2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)\,dx\right)ai+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)\,dx\right)b+\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\,dx\right)a\Bigg)$$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d
*x)*tan(c + d*x)**3,x)*b - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))
*cot(c + d*x)*tan(c + d*x)**2,x)*a + 2*int(sqrt(tan(c + d*x)*i + 1)*sqrt(c
ot(c + d*x))*cot(c + d*x)*tan(c + d*x)**2,x)*b*i + 2*int(sqrt(tan(c + d*x)
*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x),x)*a*i + int(sqrt(tan
(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x),x)*b + int(s
qrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x),x)*a)
```

3.556 $\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 38, antiderivative size = 246

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{(-1)^{3/4}a^{5/2}(20iA+23B)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{4d} + \frac{(4-4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{a^2(4A-7iB)\sqrt{a+ia\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{iaB(a+ia\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}}$$

output
```
-1/4*(-1)^(3/4)*a^(5/2)*(20*I*A+23*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)
^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(4-4*
I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-1/4*a^2*(4*A-7*I*B)*(a+I*a*t
an(d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)+1/2*I*a*B*(a+I*a*tan(d*x+c))^(3/2)/d/c
ot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.76 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.24

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx =$$
$$-\frac{ia^2\sqrt{\cot(c+dx)}\left(-3\sqrt[4]{-1}aB\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{\tan(c+dx)}(-i+\tan(c+dx))-20i\sqrt{a}(A\right.}{}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d
*x]),x]
```

output
```
((-1/4*I)*a^2*Sqrt[Cot[c + d*x]]*(-3*(-1)^(1/4)*a*B*ArcSinh[(-1)^(1/4)*Sqr
t[Tan[c + d*x]]]*Sqrt[Tan[c + d*x]]*(-I + Tan[c + d*x]) - (20*I)*Sqrt[a]*(
A - I*B)*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[I*a*Tan[c + d*x]]*(-
I + Tan[c + d*x]) + Sqrt[1 + I*Tan[c + d*x]]*(16*Sqrt[2]*(A - I*B)*ArcTanh
[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[I*a*Tan
[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + a*Tan[c + d*x]*(-I + Tan[c + d*x])
*(4*A - (9*I)*B + 2*B*Tan[c + d*x]))))/(d*Sqrt[1 + I*Tan[c + d*x]]*Sqrt[a
+ I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4729, 3042, 4077, 27, 3042, 4077, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4077}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4A-iB)+a(4iA+7B)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{t}}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4A-iB)+a(4iA+7B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{t}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\int \frac{(i\tan(c+dx)a+a)^{3/2}(a(4A-iB)+a(4iA+7B)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx+\frac{iaB\sqrt{t}}{}\right.$$

$$\Big\downarrow \text{4077}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\int \frac{\sqrt{i\tan(c+dx)a+a}\,(3(4A-3iB)a^2+(20iA+23B)\tan(c+dx)a^2)}{2\sqrt{\tan(c+dx)}}dx-\frac{a}{}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}\,(3(4A-3iB)a^2+(20iA+23B)\tan(c+dx)a^2)}{\sqrt{\tan(c+dx)}}dx-\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{\sqrt{i\tan(c+dx)a+a}\,(3(4A-3iB)a^2+(20iA+23B)\tan(c+dx)a^2)}{\sqrt{\tan(c+dx)}}dx-\right.\right.$$

$$\Big\downarrow \text{4084}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{1}{2}\Bigg(32a^2(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a(20A-23iB)\int\frac{(a-ia\tan(c$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{1}{2}\Bigg(32a^2(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-a(20A-23iB)\int\frac{(a-ia\tan(c$$

$\Big\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{1}{2}\Bigg(-\frac{64ia^4(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-a(20A-23iB)\int\frac{(c$$

$\Big\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{1}{2}\Bigg(\frac{(32-32i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-a(20A-23iB)\int\frac{(c$$

$\Big\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{1}{2}\Bigg(\frac{(32-32i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^3(20A-23iB)\int$$

$\Big\downarrow$ 65

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{1}{2}\Bigg(\frac{(32-32i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(20A-23iB)\int$$

$\Big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\Bigg(\frac{1}{4}\Bigg(\frac{1}{2}\Bigg(\frac{2\sqrt[4]{-1}a^{5/2}(20A-23iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{(32-32i)a^{5/2}(A$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((I/2)*a*B*Sqrt[Tan[c + d*x]]*(a +
I*a*Tan[c + d*x])^(3/2))/d + (((2*(-1)^(1/4)*a^(5/2)*(20*A - (23*I)*B)*Arc
Tan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d
 + ((32 - 32*I)*a^(5/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*
x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/2 - (a^2*(4*A - (7*I)*B)*Sqrt[Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d)/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(196) = 392$.

Time = 0.90 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.33

method	result
derivativedivides	$-\frac{\sqrt{\frac{1}{\tan(dx+c)}}\,\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-18iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{-ia}\,\sqrt{ia}+4B\sqrt{-ia}\,\sqrt{ia}\,\mathrm{ta}\right.}{}$
default	$-\frac{\sqrt{\frac{1}{\tan(dx+c)}}\,\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-18iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{-ia}\,\sqrt{ia}+4B\sqrt{-ia}\,\sqrt{ia}\,\mathrm{ta}\right.}{}$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/8/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(-18
*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*(I*a)^(1/2)+4*B*(-
I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+9*
I*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a-8*I*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(
tan(d*x+c)+I))*a+8*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I
*a)^(1/2)-12*A*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-16*I*ln(1/2*(2*I*a*tan(d*
x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a
*(-I*a)^(1/2)+8*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a-16*ln(1/2
*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/
(I*a)^(1/2))*(-I*a)^(1/2)*a)/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^
(1/2)/(I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 923 vs. 2(184) = 368.

Time = 0.10 (sec) , antiderivative size = 923, normalized size of antiderivative = 3.75

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="fricas")
```

output
```
1/16*(32*sqrt(2)*sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*
I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) +
 sqrt(-(I*A^2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A - B)*a^2)) - 32*sqrt(2)*sqrt(-(I*A^
2 + 2*A*B - I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*
c) + d)*log(4*((A - I*B)*a^3*e^(I*d*x + I*c) - sqrt(-(I*A^2 + 2*A*B - I*B^
2)*a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) - d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I
*c)/((-I*A - B)*a^2)) + 4*sqrt(2)*((4*I*A + 11*B)*a^2*e^(5*I*d*x + 5*I*c)
- 4*B*a^2*e^(3*I*d*x + 3*I*c) + (-4*I*A - 7*B)*a^2*e^(I*d*x + I*c))*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) - 1)) - sqrt((-400*I*A^2 - 920*A*B + 529*I*B^2)*a^5/d^2)*(d*e^(4*I
*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(-16*(3*(20*I*A + 23*B)*a^
3*e^(2*I*d*x + 2*I*c) + (-20*I*A - 23*B)*a^3 + 2*sqrt(2)*sqrt((-400*I*A^2
- 920*A*B + 529*I*B^2)*a^5/d^2)*(I*d*e^(3*I*d*x + 3*I*c) - I*d*e^(I*d*x +
I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(
e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((20*I*A + 23*B)*a)) + sqr
t((-400*I*A^2 - 920*A*B + 529*I*B^2)*a^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d
*e^(2*I*d*x + 2*I*c) + d)*log(-16*(3*(20*I*A + 23*B)*a^3*e^(2*I*d*x + 2...
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2),
 x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \sqrt{a}\,a^2\left(-\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^3\,dx\right)b - \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^2\,dx\right)a + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^2\,dx\right)bi + 2\left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)\,dx\right)ai + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)\,dx\right)b + \left(\int \sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}dx\right)a\right)$$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
sqrt(a)*a**2*( - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d
*x)**3,x)*b - int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)
**2,x)*a + 2*int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)*
*2,x)*b*i + 2*int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)
,x)*a*i + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x),x)*
b + int(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x)),x)*a)
```

3.557 $\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 292

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = -\frac{(-1)^{3/4}a^{5/2}(46A-45iB)\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{8d} - \frac{(4+4i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{a^2(2A-3iB)\sqrt{a+ia\tan(c+dx)}}{4d\cot^{\frac{3}{2}}(c+dx)} + \frac{a^2(18iA+19B)\sqrt{a+ia\tan(c+dx)}}{8d\sqrt{\cot(c+dx)}} + \frac{iaB(a+ia\tan(c+dx))^{3/2}}{3d\cot^{\frac{3}{2}}(c+dx)}$$

output
```
-1/8*(-1)^(3/4)*a^(5/2)*(46*A-45*I*B)*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)
^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(4+4*
I)*a^(5/2)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-1/4*a^2*(2*A-3*I*B)*(a+I*a*t
an(d*x+c))^(1/2)/d/cot(d*x+c)^(3/2)+1/8*a^2*(18*I*A+19*B)*(a+I*a*tan(d*x+c
))^(1/2)/d/cot(d*x+c)^(1/2)+1/3*I*a*B*(a+I*a*tan(d*x+c))^(3/2)/d/cot(d*x+c
)^(3/2)
```

Mathematica [A] (verified)

Time = 11.16 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.79

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx=\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+ia}{3d}\right.$$

$$+\frac{ia^3(6A-5iB)\sqrt{a+ia\tan(c+dx)}\left(-\frac{3}{4}(-1)^{3/4}\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)+\frac{5}{4}\sqrt{1+i\tan(c+dx)}\sqrt{\tan(c+dx)}+\frac{1}{2}i\sqrt{1+i\tan(c+dx)}\tan^{\frac{3}{2}}(c+}{2d\sqrt{1+i\tan(c+dx)}}$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[Cot[c +
 d*x]],x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((B*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[
c + d*x])^(5/2))/(3*d) + (((I/2)*a^3*(6*A - (5*I)*B)*Sqrt[a + I*a*Tan[c +
d*x]]*((-3*(-1)^(3/4)*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]])/4 + (5*Sqrt[
1 + I*Tan[c + d*x]]*Sqrt[Tan[c + d*x]])/4 + (I/2)*Sqrt[1 + I*Tan[c + d*x]]
*Tan[c + d*x]^(3/2)))/(d*Sqrt[1 + I*Tan[c + d*x]]) + (a*((a^2*(6*A - (5*I)
*B))/2 - (I/2)*a^2*B)*(((-4*I)*Sqrt[2]*a*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c +
 d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/Sqrt[I*a*Tan[c + d
*x]] + ((4*I)*a^(3/2)*ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[1 + I*T
an[c + d*x]]*Sqrt[Tan[c + d*x]])/(Sqrt[I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[
c + d*x]]) + I*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + (I*Sqrt[a]*
ArcSinh[Sqrt[I*a*Tan[c + d*x]]/Sqrt[a]]*Sqrt[Tan[c + d*x]]*Sqrt[a + I*a*Ta
n[c + d*x]])/(Sqrt[1 + I*Tan[c + d*x]]*Sqrt[I*a*Tan[c + d*x]])))/d)/(3*a))
```

Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.98, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4729, 3042, 4077, 27, 3042, 4077, 27, 3042, 4080, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{ 4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4077}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{3}{2}\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}(a(2A-iB)+a(2iA+3B)\tan(c+dx\right.$$

$$\Big\downarrow \text{ 27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}(a(2A-iB)+a(2iA+3B)\tan(c+dx)\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}(a(2A-iB)+a(2iA+3B)\tan(c+dx)\right.$$

$$\Big\downarrow \text{ 4077}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{2}\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left((14A-13iB)a^2+(18iA+19B)\tan\right.\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\int\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left((14A-13iB)a^2+(18iA+19B)\tan(c\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\int\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}\left((14A-13iB)a^2+(18iA+19B)\tan(c\right.\right.\right.$$

$\Big\downarrow$ 4080

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int-\frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(18iA+19B)-a^3(46A-45iB)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}}dx}{a}+\frac{a^2(19B+18iA\right.\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(\right.}{}}{}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(\right.}{}}{}\right.\right.\right.$$

$\Big\downarrow$ 4084

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{64a^3(B+iA)\int\frac{\sqrt{i\,\mathrm{ta}}}{\sqrt{}}}{}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{64a^3(B+iA)\int \frac{\sqrt{i\,\mathrm{ta}}}{\sqrt{}}}{}\right.\right.\right.$$

$\big\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{-a^2(45B+46iA)\int}{}\right.\right.\right.$$

$\big\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(64-64i)a^{7/2}(B+iA)\mathrm{arc}}{}}{}\right.\right.\right.$$

$\big\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(64-64i)a^{7/2}(B+iA)\mathrm{arc}}{}}{}\right.\right.\right.$$

$\big\downarrow$ 65

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{(64-64i)a^{7/2}(B+iA)\mathrm{arc}}{}}{}\right.\right.\right.$$

$\big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{a^2(19B+18iA)\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{\frac{2\sqrt[4]{-1}a^{7/2}(45B+46iA)}{}}{}\right.\right.\right.$$

input
```
Int[((a + I*a*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]]
,x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((((I/3)*a*B*Tan[c + d*x]^(3/2)*(a +
I*a*Tan[c + d*x])^(3/2))/d + (-1/2*(a^2*(2*A - (3*I)*B)*Tan[c + d*x]^(3/2)
*Sqrt[a + I*a*Tan[c + d*x]])/d + (-1/2*((2*(-1)^(1/4)*a^(7/2)*((46*I)*A +
45*B)*ArcTan[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c +
d*x]]])/d + ((64 - 64*I)*a^(7/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[T
an[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d)/a + (a^2*((18*I)*A + 19*B)*S
qrt[Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d)/4)/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4077
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan
[e + f*x])^n*Simp[a*A*d*(m + n) + B*(a*c*(m - 1) - b*d*(n + 1)) - (B*(b*c -
 a*d)*(m - 1) - d*(A*b + a*B)*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&
GtQ[m, 1] &&  !LtQ[n, -1]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(234) = 468.

Time = 0.96 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.85

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\left(-52iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,a\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\left(-52iB\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\,\sqrt{ia}\,\sqrt{-ia}\,\tan(dx+c\right.}{}$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/48/d*(a*(1+I*tan(d*x+c)))^(1/2)*a*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
*(-52*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*(I*a)^(1/2)*t
an(d*x+c)+16*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*(I*a)^(1
/2)*tan(d*x+c)^2+54*I*A*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a-108*I*A*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*(I*a)^(1/2)+24*A*(a*tan(d*x+c)*
(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)+57*B*(-I*a)^(1
/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)
^(1/2)+a)/(I*a)^(1/2))*a-114*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)-96*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*(I*a)^(1/2)*a+19
2*I*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)
^(1/2)+a)/(I*a)^(1/2))*a*(-I*a)^(1/2))/(1/tan(d*x+c))^(1/2)/(1+I*tan(d*x+c
))/tan(d*x+c)/(I*a)^(1/2)/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(220) = 440$.

Time = 0.11 (sec) , antiderivative size = 1013, normalized size of antiderivative = 3.47

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, al
gorithm="fricas")
```

output
```
-1/96*(192*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(6*I*d*x +
 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(4*((A
 - I*B)*a^3*e^(I*d*x + I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(I*d
*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((-I*A -
 B)*a^2)) - 192*sqrt(2)*sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)*(d*e^(6*I*
d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(
4*((A - I*B)*a^3*e^(I*d*x + I*c) - sqrt(-(-I*A^2 - 2*A*B + I*B^2)*a^5/d^2)
*(-I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((
I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/((
-I*A - B)*a^2)) - 4*sqrt(2)*((66*A - 91*I*B)*a^2*e^(7*I*d*x + 7*I*c) + 7*(
6*A - I*B)*a^2*e^(5*I*d*x + 5*I*c) - (66*A - 59*I*B)*a^2*e^(3*I*d*x + 3*I*
c) - 3*(14*A - 13*I*B)*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)) - 3*sqrt((
2116*I*A^2 + 4140*A*B - 2025*I*B^2)*a^5/d^2)*(d*e^(6*I*d*x + 6*I*c) + 3*d*
e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*log(-16*(3*(46*I*A + 45
*B)*a^3*e^(2*I*d*x + 2*I*c) + (-46*I*A - 45*B)*a^3 + 2*sqrt(2)*sqrt((2116*
I*A^2 + 4140*A*B - 2025*I*B^2)*a^5/d^2)*(d*e^(3*I*d*x + 3*I*c) - d*e^(I*d*
x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) +
I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-2*I*d*x - 2*I*c)/((46*I*A + 45*B)*a)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^(5/2)/sqrt(cot(d*x +
 c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/cot(c + d*x)^(1/2
),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^(5/2))/cot(c + d*x)^(1/2
), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan\right.\right.$$

$$-\left(\int \frac{\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)}dx\right)a$$

$$+2\left(\int \frac{\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)}dx\right)bi$$

$$+2\left(\int \frac{\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)ai$$

$$+\left(\int \frac{\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)b$$

$$+\left.\left(\int \frac{\sqrt{\tan(dx+c)\,i+1}\,\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a\right)$$

input
```
int((a+I*a*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
sqrt(a)*a**2*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c +
d*x)**3)/cot(c + d*x),x)*b - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*
x))*tan(c + d*x)**2)/cot(c + d*x),x)*a + 2*int((sqrt(tan(c + d*x)*i + 1)*s
qrt(cot(c + d*x))*tan(c + d*x)**2)/cot(c + d*x),x)*b*i + 2*int((sqrt(tan(c
 + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*x),x)*a*i + int(
(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*x),x)
*b + int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x)))/cot(c + d*x),x)*a)
```

3.558 $\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 211

$$\begin{aligned}\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}d} \\ &\quad + \frac{(A+iB)\cot^{\frac{3}{2}}(c+dx)}{d\sqrt{a+ia\tan(c+dx)}} + \frac{(7iA-9B)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3ad} \\ &\quad - \frac{(5A+3iB)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}}{3ad}\end{aligned}$$

output
```
(1/2+1/2*I)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(1/2)/d+(A+I*B)*cot(d*x+c)^
(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)+1/3*(7*I*A-9*B)*cot(d*x+c)^(1/2)*(a+I*a*t
an(d*x+c))^(1/2)/a/d-1/3*(5*A+3*I*B)*cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^(
1/2)/a/d
```

Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.67

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{-\frac{3\sqrt{2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{ia\tan(c+dx)}}-\frac{2(7A+9iB+(-2iA+6B)\cot(c+dx)+2A\cot^2(c+dx))}{\sqrt{a+ia\tan(c+dx)}}}{6d\sqrt{\cot(c+dx)}}$$

input
```
Integrate[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d
*x]],x]
```

output
```
((-3*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]])/Sqrt[I*a*Tan[c + d*x]] - (2*(7*A + (9*I)*B + ((-2*I)*A
+ 6*B)*Cot[c + d*x] + 2*A*Cot[c + d*x]^2))/Sqrt[a + I*a*Tan[c + d*x]])/(6*
d*Sqrt[Cot[c + d*x]])
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4729, 3042, 4079, 27, 3042, 4081, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{5/2}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}\sqrt{i\tan(c+dx)a+a}}dx$$

$\downarrow$ 4079

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(5A+3iB)-4a(iA-B)\tan(c+dx))}{2\tan^{\frac{5}{2}}(c+dx)}dx}{a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+}}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(5A+3iB)-4a(iA-B)\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+}}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(5A+3iB)-4a(iA-B)\tan(c+dx))}{\tan(c+dx)^{5/2}}dx}{2a^2}+\frac{A+iB}{d\tan^{\frac{3}{2}}(c+dx)\sqrt{a+ia\tan(c+}}\right.$$

$\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((7iA-9B)a^2+2(5A+3iB)\tan(c+dx)a^2\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{}{d\,t}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((7iA-9B)a^2+2(5A+3iB)\tan(c+dx)a^2\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{}{d\,\mathrm{ta}}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((7iA-9B)a^2+2(5A+3iB)\tan(c+dx)a^2\right)}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}+\frac{}{d\,\text{ta}}\right.$$

$\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{\frac{2\int \frac{3a^3(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^2(-9B+7iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{3a^2(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(-9B+7iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{3a^2(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(-9B+7iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5A+3iB)\sqrt{a+ia\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}}{2a^2}\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{-\frac{6ia^4(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^2(-9B+7iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5A+3iB}{3d\,\text{ta}}}{2a^2}\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{-\frac{(3-3i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(-9B+7iA)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{3a}-\frac{2a(5A+3iB)\sqrt{\cdot}}{3d\tan^{\frac{3}{2}}}}{2a^2}\right.$$

input
```
Int[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((A + I*B)/(d*Tan[c + d*x]^(3/2)*Sqr
t[a + I*a*Tan[c + d*x]]) + ((-2*a*(5*A + (3*I)*B)*Sqrt[a + I*a*Tan[c + d*x
]])/(3*d*Tan[c + d*x]^(3/2)) - (((3 - 3*I)*a^(5/2)*(A - I*B)*ArcTanh[((1 +
 I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^2*((
7*I)*A - 9*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/(
2*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 747 vs. 2(171) = 342.

Time = 1.04 (sec) , antiderivative size = 748, normalized size of antiderivative = 3.55

method	result
derivativedivides	$\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(-3iA\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\right.}{}$
default	$\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}} \tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(-3iA\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{-ia}\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\right.}{}$

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/12/d*(1/tan(d*x+c))^(5/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)/a*(-3*I*
A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-36*B*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+3*I*A*2^(1/2)*ln((2*2^(1/2)
*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(t
an(d*x+c)+I))*a*tan(d*x+c)^4+3*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(
d*x+c)^4+36*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c
)^2+28*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3
+60*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+6*
A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-6*I*B*2^(1/2)*ln((2*2^
(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c
))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-3*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a
*tan(d*x+c)^2+24*B*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)
^(1/2)+8*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(1/2)/(I-tan(d*x+c))^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(161) = 322$.

Time = 0.13 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.31

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$

$$-\frac{3\sqrt{2}\left(ade^{(3i\,dx+3i\,c)}-ade^{(i\,dx+i\,c)}\right)\sqrt{-\frac{i\,A^2+2\,AB-i\,B^2}{ad^2}}\log\left(-\frac{4\left((A-i\,B)ae^{(i\,dx+i\,c)}+\left(ade^{(2i\,dx+2i\,c)}-ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}}}\right.}{i\,A+}\right.}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="fricas")
```

output
```
-1/12*(3*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*c) - a*d*e^(I*d*x + I*c))*sqrt(-(I*
A^2 + 2*A*B - I*B^2)/(a*d^2))*log(-4*((A - I*B)*a*e^(I*d*x + I*c) + (a*d*e
^(2*I*d*x + 2*I*c) - a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-(I*A^2 + 2*A*B - I*B^2
)/(a*d^2)))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(2)*(a*d*e^(3*I*d*x + 3*I*
c) - a*d*e^(I*d*x + I*c))*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*log(-4*((
A - I*B)*a*e^(I*d*x + I*c) - (a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)
 - 1))*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2)))*e^(-I*d*x - I*c)/(I*A + B))
 + 2*sqrt(2)*((-7*I*A + 15*B)*e^(4*I*d*x + 4*I*c) + 18*(I*A - B)*e^(2*I*d*
x + 2*I*c) - 3*I*A + 3*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I
*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))/(a*d*e^(3*I*d*x + 3*I*c) -
a*d*e^(I*d*x + I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2
),x)
```

output
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2
), x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2}{\tan(dx+c)^2+1}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)}{\tan(dx+c)^2+1}dx\right.\right.}{a}$$

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)
**2*tan(c + d*x)**2)/(tan(c + d*x)**2 + 1),x)*b*i - int((sqrt(tan(c + d*x)
*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x))/(tan(c + d*x)**2
+ 1),x)*a*i + int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x
)**2*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*b + int((sqrt(tan(c + d*x)*i +
 1)*sqrt(cot(c + d*x))*cot(c + d*x)**2)/(tan(c + d*x)**2 + 1),x)*a))/a
```

3.559 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 163

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}d} + \frac{(A+iB)\sqrt{\cot(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{(3A+iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad}$$

output
```
(1/2+1/2*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(1/2)/d+(A+I*B)*cot(d*x+c)^
(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)-(3*A+I*B)*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x
+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.74

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\frac{\sqrt{2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{ia\tan(c+dx)}}+\frac{-6iA+2B-4A\cot(c+dx)}{\sqrt{a+ia\tan(c+dx)}}}{2d\sqrt{\cot(c+dx)}}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d
*x]],x]
```

output
```
((Sqrt[2]*(I*A + B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]])/Sqrt[I*a*Tan[c + d*x]] + ((-6*I)*A + 2*B - 4*A*Cot[c + d*x
])/Sqrt[a + I*a*Tan[c + d*x]])/(2*d*Sqrt[Cot[c + d*x]])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {3042, 4729, 3042, 4079, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \; 4079$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(3A+iB)-2a(iA-B)\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx}{a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+}}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(3A+iB)-2a(iA-B)\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+}}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(3A+iB)-2a(iA-B)\tan(c+dx))}{\tan(c+dx)^{3/2}}dx}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+}}\right.$$

$$\Big\downarrow \; 4081$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2\int \frac{a^2(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3A+iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\,\mathrm{ta}}}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3A+iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+}}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a(3A+iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{A+iB}{d\sqrt{\tan(c+dx)}\sqrt{a+}}\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{2ia^3(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a(3A+iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{}{d\sqrt{\tan(}}\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{(1-i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a(3A+iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{}{d\sqrt{\tan(c+}}\right.$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((A + I*B)/(d*Sqrt[Tan[c + d*x]]*Sqr
t[a + I*a*Tan[c + d*x]]) + (((1 - I)*a^(3/2)*(I*A + B)*ArcTanh[((1 + I)*Sq
rt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a*(3*A + I*B
)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(2*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 708 vs. 2(133) = 266.

Time = 1.02 (sec) , antiderivative size = 709, normalized size of antiderivative = 4.35

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(iB\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(iB\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c}{\tan(dx+c)+i}\right.\right.}{}$

input

```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_R
ETURNVERBOSE)
```

output

```
-1/4/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)/a*(I*B*2
^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+
I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+2*I*A*2^(1/2)*ln(-(-2*2
^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+
c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*
(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*
a*tan(d*x+c)^3+12*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan
(d*x+c)^2-I*B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+2*B*2^(1/2
)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3
*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+4*I*B*(a*tan(d*x+c)*(1+I*tan
(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)
^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+
c)+I))*a*tan(d*x+c)-20*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1
/2)*tan(d*x+c)+4*B*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)
^(1/2)-8*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)/(I-tan(d*x+c))^2/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(125) = 250.

Time = 0.10 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.61

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\left(\sqrt{2}ad\sqrt{-\frac{-iA^2-2AB+iB^2}{ad^2}}e^{(idx+ic)}\log\left(-\frac{4\left((A-iB)ae^{(idx+ic)}+\left(iade^{(2idx+2ic)}-iad\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}}{e^{(2idx+2ic)}}\cdots}\right.}{iA+B}\right.\right.}{\cdots}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="fricas")
```

output
```
-1/4*(sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d*x + I*c)*
log(-4*((A - I*B)*a*e^(I*d*x + I*c) + (I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) - 1))*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2)))*e^(-I*d*x - I
*c)/(I*A + B)) - sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*
d*x + I*c)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) + (-I*a*d*e^(2*I*d*x + 2*I*
c) + I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c)
+ I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2)))*e
^(-I*d*x - I*c)/(I*A + B)) + 2*sqrt(2)*((5*A + I*B)*e^(2*I*d*x + 2*I*c) -
A - I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\cot^{\frac{3}{2}}(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**(3/2)/sqrt(I*a*(tan(c + d*x) -
 I)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{\sqrt{a+a\tan(c+d\,x)\;1\mathrm{i}}}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2
),x)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2
), x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^2}{\tan(dx+c)^2+1}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)\right.}{a}$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)
*tan(c + d*x)**2)/(tan(c + d*x)**2 + 1),x)*b*i - int((sqrt(tan(c + d*x)*i
+ 1)*sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x))/(tan(c + d*x)**2 + 1),x
)*a*i + int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)*tan(
c + d*x))/(tan(c + d*x)**2 + 1),x)*b + int((sqrt(tan(c + d*x)*i + 1)*sqrt(
cot(c + d*x))*cot(c + d*x))/(tan(c + d*x)**2 + 1),x)*a))/a
```

3.560 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 119

$$\begin{aligned}&\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx\\&=\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}d}\\&\quad+\frac{A+iB}{d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

output

```
(1/2-1/2*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(1/2)/d+(A+I*B)/d/cot(d*x+c
)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\frac{\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{ia\tan(c+dx)}}+\frac{2(A+iB)}{\sqrt{a+ia\tan(c+dx)}}}{2d\sqrt{\cot(c+dx)}}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d
*x]],x]
```

output
```
((Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*
Tan[c + d*x]]])/Sqrt[I*a*Tan[c + d*x]] + (2*(A + I*B))/Sqrt[a + I*a*Tan[c
+ d*x]])/(2*d*Sqrt[Cot[c + d*x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4729, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{4079}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx}{2a}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{4027}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{ia(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}\right)$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}\right)$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + I*a*Tan[c + d*x]],x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((1/2 - I/2)*(A - I*B)*ArcTanh[((1
+ I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/(Sqrt[a]*d)
+ ((A + I*B)*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(95) = 190$.

Time = 1.01 (sec) , antiderivative size = 647, normalized size of antiderivative = 5.44

method	result
derivativedivides	$-\frac{\sqrt{\frac{1}{\tan(dx+c)}}\,\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(iA\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\sqrt{\frac{1}{\tan(dx+c)}}\,\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(iA\sqrt{2}\,\ln\left(-\frac{-2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}+ia-3a\tan(dx+c)}{\tan(dx+c)+i}\right.\right.}{}$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/4/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)/a*(I*A*2
^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+
I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-2*I*B*2^(1/2)*ln(-(-2*2
^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+
c))/(tan(d*x+c)+I))*a*tan(d*x+c)+B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*
tan(d*x+c)^2-I*A*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+2*A*2^(1/2)*ln(-(-
2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d
*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+4*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^
(1/2)*(-I*a)^(1/2)*tan(d*x+c)-B*2^(1/2)*ln(-(-2*2^(1/2)*(-I*a)^(1/2)*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+I*a-3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+4*I
*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-4*B*tan(d*x+c)*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)+4*A*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)*(-I*a)^(1/2))/(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)/(-I*a)^(
1/2)/(I-tan(d*x+c))^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(89) = 178.

Time = 0.09 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.55

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(\sqrt{2}ad\sqrt{-\frac{iA^2+2AB-iB^2}{ad^2}}e^{(idx+ic)}\log\left(-\frac{4\left((A-iB)ae^{(idx+ic)}+(ade^{(2idx+2ic)}-ad)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{ie^{(2idx+2ic)}+i}{e^{(2idx+2ic)}-1}}\sqrt{-i\right.}{iA+B}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="fricas")
```

output
```
1/4*(sqrt(2)*a*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x + I*c)*lo
g(-4*((A - I*B)*a*e^(I*d*x + I*c) + (a*d*e^(2*I*d*x + 2*I*c) - a*d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
 2*I*c) - 1))*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2)))*e^(-I*d*x - I*c)/(I*
A + B)) - sqrt(2)*a*d*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2))*e^(I*d*x + I*
c)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) - (a*d*e^(2*I*d*x + 2*I*c) - a*d)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1))*sqrt(-(I*A^2 + 2*A*B - I*B^2)/(a*d^2)))*e^(-I*d*x - I*c
)/(I*A + B)) - 2*sqrt(2)*((I*A - B)*e^(2*I*d*x + 2*I*c) - I*A + B)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1)))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\sqrt{\cot(c+dx)}}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/sqrt(I*a*(tan(c + d*x) -
I)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2
),x)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(1/2
), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\tan(dx+c)^2+1}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\tan(dx+c)^2+1}dx\right)ai+\left(\int \frac{\sqrt{\tan(d}{}\right.\right.}{a}$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)
**2)/(tan(c + d*x)**2 + 1),x)*b*i - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot
(c + d*x))*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*a*i + int((sqrt(tan(c +
d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x))/(tan(c + d*x)**2 + 1),x)*b +
int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x)))/(tan(c + d*x)**2 + 1),x)
*a))/a
```

3.561 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 196

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\sqrt[4]{-1}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}d} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{a}d} + \frac{iA-B}{d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
-2*(-1)^(1/4)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(1/2)/d-(1/2+1/2*I)*(A-I*B)
*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+
c)^(1/2)*tan(d*x+c)^(1/2)/a^(1/2)/d+(I*A-B)/d/cot(d*x+c)^(1/2)/(a+I*a*tan(
d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.97

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left((-2+2i)(-1)^{3/4}\sqrt{a}B\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(c+dx)}\right)\sqrt{1+i\tan(c}{\sqrt{a}d\sqrt{a+ic}}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*
x]]),x]
```

output
```
((1/2 + I/2)*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2 + 2*I)*(-1)^(3/4)*
Sqrt[a]*B*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sqrt[1 + I*Tan[c + d*x]]
+ (1 + I)*Sqrt[a]*(A + I*B)*Sqrt[Tan[c + d*x]] - (A - I*B)*ArcTanh[((1 + I
)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*Sqrt[a + I*a*Tan
[c + d*x]]))/(Sqrt[a]*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.92, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 4729, 3042, 4078, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx$$

$$\Big\downarrow \text{4078}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA-B)+2iaB\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx}{a^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA-B)+2iaB\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{2a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}(a(iA-B)+2iaB\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{2a^2}\right)$$

$$\Big\downarrow \text{4084}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-2B\int \frac{(a-ia\tan(c+dx))\sqrt{}}{\sqrt{\tan(}}}{2a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-2B\int \frac{(a-ia\tan(c+dx))\sqrt{}}{\sqrt{\tan(}}}{2a^2}\right.$$

$$\Big\downarrow \text{4027}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{-\frac{2ia^3(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-2B\int \frac{(}{}}{2a^2}\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-2B\int\frac{(a-ia}{}}{2a^2}\right.$$

$\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2B\int}{\sqrt{\tan}}}{2a^2}\right.$$

$\downarrow$ 65

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{4a^2B\int\frac{}{1-\frac{i}{i\,t}}}{}}{2a^2}\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{4\sqrt[4]{-1}a^{3/2}}{}}{2a^2}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-1/2*((4*(-1)^(1/4)*a^(3/2)*B*ArcTa
n[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d +
 ((1 - I)*a^(3/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/S
qrt[a + I*a*Tan[c + d*x]]])/d)/a^2 + ((I*A - B)*Sqrt[Tan[c + d*x]])/(d*Sqr
t[a + I*a*Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(155) = 310.

Time = 1.06 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.08

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\left(iB\sqrt{2}\,\sqrt{ia}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\,\mathrm{ta}}{\tan(dx+c)+i}\right.\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\left(iB\sqrt{2}\,\sqrt{ia}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\,\mathrm{ta}}{\tan(dx+c)+i}\right.\right.}{}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/4/d*(a*(1+I*tan(d*x+c)))^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I
*B*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+I*A*2^(1/2)
*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-A*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1
/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))
/(tan(d*x+c)+I))*a*tan(d*x+c)+B*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(
1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)
+I))*a+4*I*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)-4*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(
I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a+4*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*t
an(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/
2))*a*tan(d*x+c)+4*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)*(-I
*a)^(1/2))/(1/tan(d*x+c))^(1/2)/a^2/(1+I*tan(d*x+c))/tan(d*x+c)/(I*a)^(1/2
)/(-I*a)^(1/2)/(I-tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(148) = 296$.

Time = 0.09 (sec) , antiderivative size = 736, normalized size of antiderivative = 3.76

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}\sqrt{a + ia\tan(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="fricas")
```

output
```
1/4*(sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d*x + I*c)*l
og(-4*((A - I*B)*a*e^(I*d*x + I*c) + (I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1))*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2)))*e^(-I*d*x - I*
c)/(I*A + B)) - sqrt(2)*a*d*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2))*e^(I*d
*x + I*c)*log(-4*((A - I*B)*a*e^(I*d*x + I*c) + (-I*a*d*e^(2*I*d*x + 2*I*c
) + I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) +
 I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-(-I*A^2 - 2*A*B + I*B^2)/(a*d^2)))*e^
(-I*d*x - I*c)/(I*A + B)) + a*d*sqrt(-4*I*B^2/(a*d^2))*e^(I*d*x + I*c)*log
(-16*(3*B*a^2*e^(2*I*d*x + 2*I*c) - B*a^2 + sqrt(2)*(a^2*d*e^(3*I*d*x + 3*
I*c) - a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^
(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-4*I*B^2/(a*d^2)))*
e^(-2*I*d*x - 2*I*c)/B) - a*d*sqrt(-4*I*B^2/(a*d^2))*e^(I*d*x + I*c)*log(-
16*(3*B*a^2*e^(2*I*d*x + 2*I*c) - B*a^2 - sqrt(2)*(a^2*d*e^(3*I*d*x + 3*I*
c) - a^2*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2
*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-4*I*B^2/(a*d^2)))*e^
(-2*I*d*x - 2*I*c)/B) + 2*sqrt(2)*((A + I*B)*e^(2*I*d*x + 2*I*c) - A - I*B
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1)))*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}\sqrt{a + ia\tan(c + dx)}}\,dx = \int \frac{A + B\tan(c + dx)}{\sqrt{ia(\tan(c + dx) - i)}\sqrt{\cot(c + dx)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(sqrt(I*a*(tan(c + d*x) - I))*sqrt(cot(c + d
*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot\left(c+d\,x\right)}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)\tan(dx+c)^2+\cot(dx+c)}dx\right)bi-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)\tan(dx+c)^2+\cot(dx+c)}dx\right)ai+\left(\int \frac{\sqrt{\tan(d}}{\cot(}\right.\right.}{a}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)
**2)/(cot(c + d*x)*tan(c + d*x)**2 + cot(c + d*x)),x)*b*i - int((sqrt(tan(
c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)*tan(c + d*x
)**2 + cot(c + d*x)),x)*a*i + int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d
*x))*tan(c + d*x))/(cot(c + d*x)*tan(c + d*x)**2 + cot(c + d*x)),x)*b + in
t((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x)))/(cot(c + d*x)*tan(c + d*x)
**2 + cot(c + d*x)),x)*a))/a
```

3.562 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 214

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\frac{1}{4}+\frac{i}{4}\right)(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{3/2}d} + \frac{(A+iB)\sqrt{\cot(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}} + \frac{(11A+5iB)\sqrt{\cot(c+dx)}}{6ad\sqrt{a+ia\tan(c+dx)}} - \frac{(25A+7iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{6a^2d}$$

output
```
(1/4+1/4*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+1/3*(A+I*B)*cot(d*x
+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(3/2)+1/6*(11*A+5*I*B)*cot(d*x+c)^(1/2)/a/d
/(a+I*a*tan(d*x+c))^(1/2)-1/6*(25*A+7*I*B)*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x
+c))^(1/2)/a^2/d
```

Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(3\sqrt{2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)(i+\cot(c+dx))\sqrt{a+\cdots}\right.}{12d\cot^{\frac{5}{2}}(c+dx)(ia\tan(c+dx))^{3/2}(-\cdots}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^(3/2),x]
```

output
```
((I/12)*(3*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt
[a + I*a*Tan[c + d*x]]]*(I + Cot[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]] + (2
*a*((-39*I)*A + 9*B - 12*A*Cot[c + d*x] + (25*A + (7*I)*B)*Tan[c + d*x]))/
Sqrt[I*a*Tan[c + d*x]]))/(d*Cot[c + d*x]^(5/2)*(I*a*Tan[c + d*x])^(3/2)*(-
I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4729, 3042, 4079, 27, 3042, 4079, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(7A+iB)-4a(iA-B)\tan(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(7A+iB)-4a(iA-B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(7A+iB)-4a(iA-B)\tan(c+dx)}{\tan(c+dx)^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{A+iB}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{4079}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(25A+7iB)-2a^2(11iA-5B)\tan(c+dx)\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{}{3d}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(25A+7iB)-2a^2(11iA-5B)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{}{3d}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^2(25A+7iB)-2a^2(11iA-5B)\tan(c+dx)\right)}{\tan(c+dx)^{3/2}}dx}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{}{3d}\right.$$

$\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{2\int \frac{3a^3(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^2(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{3a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{3a^2(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^2(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a(11A+5iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{-\frac{6ia^4(B+iA)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^2(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a(11A}{d\sqrt{\tan(c+dx)}\sqrt{}}}{6a^2}\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{(3-3i)a^{5/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^2(25A+7iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a(11A+5\ldots}{d\sqrt{\tan(c+dx)}\sqrt{a\ldots}}}{6a^2}\right.$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2)
,x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((A + I*B)/(3*d*Sqrt[Tan[c + d*x]]*(
a + I*a*Tan[c + d*x])^(3/2)) + ((a*(11*A + (5*I)*B))/(d*Sqrt[Tan[c + d*x]]
*Sqrt[a + I*a*Tan[c + d*x]]) + (((3 - 3*I)*a^(5/2)*(I*A + B)*ArcTanh[((1 +
 I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (2*a^2*(2
5*A + (7*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(2*a^2)
)/(6*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(172) = 344$.

Time = 0.95 (sec) , antiderivative size = 931, normalized size of antiderivative = 4.35

method	result	size
derivativedivides	Expression too large to display	931
default	Expression too large to display	931

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/24/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*(-3*I*B
*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-
I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4-9*I*A*2^(1/2)*ln((2*2^(
1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c)
)/(tan(d*x+c)+I))*a*tan(d*x+c)^3+3*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*
tan(d*x+c)^4-28*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan
(d*x+c)^3+9*I*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-9*B*2^(1
/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3
*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+256*I*A*(a*tan(d*x+c)*(1+I*t
an(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2-100*A*(a*tan(d*x+c)*(1+I*tan(d
*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+3*I*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)
^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+
c)+I))*a*tan(d*x+c)-9*A*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)^2+
36*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)-64*B*
tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)+3*B*2^(1/2
)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a
*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-48*I*A*(a*tan(d*x+c)*(1+I*tan...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(162) = 324$.

Time = 0.09 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.17

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{\left(3\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{iA^2+2AB-iB^2}{a^3d^2}}e^{(3i\,dx+3i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ia^2de^{(2i\,dx+2i\,c)}-ia^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{ie^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{iA^2+2A}{a^3d}}\right.}{iA+B}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="fricas")
```

output
```
-1/12*(3*sqrt(1/2)*a^2*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^3*d^2))*e^(3*I*d*
x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*
d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(
2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^3*d^2)) + (A - I*B)
*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*a^2*d*sqrt((
I*A^2 + 2*A*B - I*B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt
(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I
*A^2 + 2*A*B - I*B^2)/(a^3*d^2)) + (A - I*B)*a*e^(I*d*x + I*c))*e^(-I*d*x
- I*c)/(I*A + B)) + sqrt(2)*(2*(19*A + 4*I*B)*e^(4*I*d*x + 4*I*c) - (13*A
+ 7*I*B)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-3*I*d*x -
 3*I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2
),x)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2
), x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\int \frac{\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i+\sqrt{\tan(dx+c)i+1}}dx\right)b-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{}}{\tan(dx+c)^3i}\right.}{}$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(int((sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x))/(sqrt(tan(c + d*x)*i +
 1)*tan(c + d*x)*i + sqrt(tan(c + d*x)*i + 1)),x)*b - int((sqrt(tan(c + d*
x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x))/(tan(c + d*x)**3*i
 + tan(c + d*x)**2 + tan(c + d*x)*i + 1),x)*a*i + int((sqrt(tan(c + d*x)*i
 + 1)*sqrt(cot(c + d*x))*cot(c + d*x))/(tan(c + d*x)**3*i + tan(c + d*x)**
2 + tan(c + d*x)*i + 1),x)*a)/(sqrt(a)*a)
```

3.563 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 168

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\frac{1}{4}-\frac{i}{4}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{3/2}d} + \frac{A+iB}{3d\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}} + \frac{7A+iB}{6ad\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
(1/4-1/4*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+1/3*(A+I*B)/d/cot(d
*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2)+1/6*(7*A+I*B)/a/d/cot(d*x+c)^(1/2)/(a
+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.99 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\frac{3\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{ia\tan(c+dx)}} + \frac{6(-3iA+B)+2(7A+iB)\tan(c+dx)}{(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}}{12ad\sqrt{\cot(c+dx)}}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^(3/2),x]
```

output
```
((3*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*
a*Tan[c + d*x]]])/Sqrt[I*a*Tan[c + d*x]] + (6*((-3*I)*A + B) + 2*(7*A + I*
B)*Tan[c + d*x])/((-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]))/(12*a*d
*Sqrt[Cot[c + d*x]])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 4729, 3042, 4079, 27, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(5A-iB)-2a(iA-B)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(5A-iB)-2a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(5A-iB)-2a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}\right)$$

$\Big\downarrow$ 4079

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{3a^2(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}+\frac{a(7A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{3}{2}(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a(7A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{3}{2}(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a(7A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}\right)$$

$\Big\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{a(7A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{3ia^2(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}\right)$$

$\Big\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\left(\frac{3}{2}-\frac{3i}{2}\right)\sqrt{a}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{a(7A+iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}\right)$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(3/2)
,x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((A + I*B)*Sqrt[Tan[c + d*x]])/(3*d
*(a + I*a*Tan[c + d*x])^(3/2)) + (((3/2 - (3*I)/2)*Sqrt[a]*(A - I*B)*ArcTa
nh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + (
a*(7*A + I*B)*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]))/(6*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4729

```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(134) = 268$.

Time = 1.00 (sec) , antiderivative size = 868, normalized size of antiderivative = 5.17

method	result
derivativedivides	$\frac{\sqrt{\frac{1}{\tan(dx+c)}}\,\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(-9iB\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{2}\,a\right.}{}$
default	$\frac{\sqrt{\frac{1}{\tan(dx+c)}}\,\tan(dx+c)\sqrt{a(1+i\tan(dx+c))}\left(-9iB\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right)\sqrt{2}\,a\right.}{}$

input

```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/24/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(-9*
I*B*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3
*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)^2-9*I*A*ln((2*2^(1/2)*
(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(ta
n(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)+3*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(
a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a
*tan(d*x+c)^3-4*B*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a
)^(1/2)+3*I*A*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)^3+9*A*ln((2*
2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x
+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)^2-36*I*A*(-I*a)^(1/2)*(a*tan(d*x
+c)*(1+I*tan(d*x+c)))^(1/2)+3*I*B*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)
*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a-9*B
*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-
I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+28*I*A*(-I*a)^(1/2)*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-3*A*ln((2*2^(1/2)*(-I*a)^(1/
2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I
))*2^(1/2)*a+16*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan
(d*x+c)+64*A*tan(d*x+c)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)
+12*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2))/(a*tan(d*x+c)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(126) = 252$.

Time = 0.10 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.76

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(3\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{-iA^2-2AB+iB^2}{a^3d^2}}e^{(3idx+3ic)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}(a^2de^{(2idx+}\right.}{}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="fricas")
```

output
```
1/12*(3*sqrt(1/2)*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2))*e^(3*I*d*
x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*
d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2)) + (A - I*B)*a*
e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*a^2*d*sqrt((-I*
A^2 - 2*A*B + I*B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2)*sqrt(1/
2)*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 -
2*A*B + I*B^2)/(a^3*d^2)) - (A - I*B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/
(I*A + B)) - sqrt(2)*(2*(4*I*A - B)*e^(4*I*d*x + 4*I*c) - (7*I*A - B)*e^(2
*I*d*x + 2*I*c) - I*A + B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*
I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-3*I*d*x - 3*I*c)/(a^2*
d)
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{\cot{(c+dx)}}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/(I*a*(tan(c + d*x) - I))*
*(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2
),x)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(3/2
), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input

```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2),x)
```

output

```
(sqrt(a)*( - 2*sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*a*i - 8*int(( -
 sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x))/(8
*tan(c + d*x)**3*i + 8*tan(c + d*x)**2 + 8*tan(c + d*x)*i + 8),x)*tan(c +
d*x)**2*a*d - 8*int(( - sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c
+ d*x)*tan(c + d*x))/(8*tan(c + d*x)**3*i + 8*tan(c + d*x)**2 + 8*tan(c +
d*x)*i + 8),x)*a*d + 8*int(( - sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))
*cot(c + d*x))/(8*tan(c + d*x)**3*i + 8*tan(c + d*x)**2 + 8*tan(c + d*x)*i
 + 8),x)*tan(c + d*x)**2*a*d*i + 8*int(( - sqrt(tan(c + d*x)*i + 1)*sqrt(c
ot(c + d*x))*cot(c + d*x))/(8*tan(c + d*x)**3*i + 8*tan(c + d*x)**2 + 8*ta
n(c + d*x)*i + 8),x)*a*d*i - 24*int(( - sqrt(tan(c + d*x)*i + 1)*sqrt(cot(
c + d*x))*tan(c + d*x)**2)/(8*tan(c + d*x)**3*i + 8*tan(c + d*x)**2 + 8*ta
n(c + d*x)*i + 8),x)*tan(c + d*x)**2*a*d - 24*int(( - sqrt(tan(c + d*x)*i
+ 1)*sqrt(cot(c + d*x))*tan(c + d*x)**2)/(8*tan(c + d*x)**3*i + 8*tan(c +
d*x)**2 + 8*tan(c + d*x)*i + 8),x)*a*d - 8*int(( - sqrt(tan(c + d*x)*i + 1
)*sqrt(cot(c + d*x))*tan(c + d*x))/(8*cot(c + d*x)*tan(c + d*x)**3*i + 8*c
ot(c + d*x)*tan(c + d*x)**2 + 8*cot(c + d*x)*tan(c + d*x)*i + 8*cot(c + d*
x)),x)*tan(c + d*x)**2*a*d - 8*int(( - sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c
 + d*x))*tan(c + d*x))/(8*cot(c + d*x)*tan(c + d*x)**3*i + 8*cot(c + d*x)*
tan(c + d*x)**2 + 8*cot(c + d*x)*tan(c + d*x)*i + 8*cot(c + d*x)),x)*a*d +
 32*int(( - sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x))/(...
```

3.564 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 170

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{3/2}d} + \frac{iA-B}{3d\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}} + \frac{iA+5B}{6ad\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
(-1/4-1/4*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x
+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+1/3*(I*A-B)/d/cot(
d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2)+1/6*(I*A+5*B)/a/d/cot(d*x+c)^(1/2)/(
a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{-\frac{3i\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{ia\tan(c+dx)}}+\frac{6(A-iB)+2(iA+5B)\tan(c+dx)}{(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}}{12ad\sqrt{\cot(c+dx)}}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^
(3/2)),x]
```

output
```
(((-3*I)*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]])/Sqrt[I*a*Tan[c + d*x]] + (6*(A - I*B) + 2*(I*A + 5*
B)*Tan[c + d*x])/((-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]))/(12*a*d
*Sqrt[Cot[c + d*x]])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {3042, 4729, 3042, 4078, 27, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx$$

$\Big\downarrow$ 4078

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{a(iA-B)-2a(A-2iB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{a(iA-B)-2a(A-2iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{a(iA-B)-2a(A-2iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}\right)$$

$\Big\downarrow$ 4079

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int \frac{3a^2(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}-\frac{a(5B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}\right)$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{3}{2}(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{a(5B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}\right)$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{3}{2}(B+iA)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{a(5B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}\right)$$

$\Big\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{-\frac{3ia^2(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{a(5B+i}{d\sqrt{a+}}}{6a^2}\right.$$

$\Big\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\left(\frac{3}{2}-\frac{3i}{2}\right)\sqrt{a}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a(5B+iA)\sqrt{}}{d\sqrt{a+ia\tan}}}{6a^2}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2))
,x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((I*A - B)*Sqrt[Tan[c + d*x]])/(3*d
*(a + I*a*Tan[c + d*x])^(3/2)) - (((3/2 - (3*I)/2)*Sqrt[a]*(I*A + B)*ArcTa
nh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d - (
a*(I*A + 5*B)*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]))/(6*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(136) = 272$.

Time = 0.90 (sec) , antiderivative size = 657, normalized size of antiderivative = 3.86

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\left(3iB\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right.\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(dx+c))}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\left(3iB\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan(dx+c)}{\tan(dx+c)+i}\right.\right.}{}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/24/d*(a*(1+I*tan(d*x+c)))^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(3
*I*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1
/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+6*I*A*2^(1/2)*ln((2
*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*
x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-3*A*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*
a*tan(d*x+c)^2-3*I*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a+6*B*2^(1/2)*ln((
2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d
*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-20*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)*tan(d*x+c)+3*A*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a-12
*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+4*A*tan(d*x+c)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-12*B*(-I*a)^(1/2)*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2))/(1/tan(d*x+c))^(1/2)/a^3/(1+I*tan(d*x+c))/t
an(d*x+c)/(I-tan(d*x+c))^2/(-I*a)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(126) = 252.

Time = 0.09 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.71

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(3\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{iA^2+2AB-iB^2}{a^3d^2}}e^{(3i\,dx+3i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ia^2de^{(2i\,dx}\right.\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="fricas")
```

output
```
1/12*(3*sqrt(1/2)*a^2*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^3*d^2))*e^(3*I*d*x
 + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^3*d^2)) + (A - I*B)*
a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*a^2*d*sqrt((I
*A^2 + 2*A*B - I*B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(
1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((I*
A^2 + 2*A*B - I*B^2)/(a^3*d^2)) + (A - I*B)*a*e^(I*d*x + I*c))*e^(-I*d*x -
 I*c)/(I*A + B)) + sqrt(2)*(2*(A - 2*I*B)*e^(4*I*d*x + 4*I*c) - (A - 5*I*B
)*e^(2*I*d*x + 2*I*c) - A - I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I
*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-3*I*d*x - 3*I*c)
/(a^2*d)
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{A+B\tan{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/((I*a*(tan(c + d*x) - I))**(3/2)*sqrt(cot(c
+ d*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot\left(c+d\,x\right)}\,{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\,(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)\tan(dx+c)^3 i+\cot(dx+c)\tan(dx+c)^2+\cot(dx+c)\tan(dx+}\right.\right.}{}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)
**2)/(cot(c + d*x)*tan(c + d*x)**3*i + cot(c + d*x)*tan(c + d*x)**2 + cot(
c + d*x)*tan(c + d*x)*i + cot(c + d*x)),x)*b*i - int((sqrt(tan(c + d*x)*i
+ 1)*sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)*tan(c + d*x)**3*i + co
t(c + d*x)*tan(c + d*x)**2 + cot(c + d*x)*tan(c + d*x)*i + cot(c + d*x)),x
)*a*i + int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x))/(co
t(c + d*x)*tan(c + d*x)**3*i + cot(c + d*x)*tan(c + d*x)**2 + cot(c + d*x)
*tan(c + d*x)*i + cot(c + d*x)),x)*b + int((sqrt(tan(c + d*x)*i + 1)*sqrt(
cot(c + d*x)))/(cot(c + d*x)*tan(c + d*x)**3*i + cot(c + d*x)*tan(c + d*x)
**2 + cot(c + d*x)*tan(c + d*x)*i + cot(c + d*x)),x)*a))/a**2
```

3.565 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 243

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2(-1)^{3/4}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c}{a^{3/2}d}$$
$$+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(iA+B)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{3/2}d}$$
$$+\frac{iA-B}{3d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}+\frac{A+3iB}{2ad\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*(-1)^(3/4)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+(1/4+1/4*I)*(I*A+B)*
arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c
)^(1/2)*tan(d*x+c)^(1/2)/a^(3/2)/d+1/3*(I*A-B)/d/cot(d*x+c)^(3/2)/(a+I*a*t
an(d*x+c))^(3/2)+1/2*(A+3*I*B)/a/d/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/
2)
```

Mathematica [A] (verified)

Time = 3.40 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.95

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\frac{1}{12}+\frac{i}{12}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\Big((12+12i)\sqrt[4]{-1}\sqrt{a}Ba$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^
(3/2)),x]
```

output
```
((1/12 + I/12)*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((12 + 12*I)*(-1)^(1/
4)*Sqrt[a]*B*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*(1 + I*Tan[c + d*x])^(
3/2) + 3*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]]*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]] + (1 + I)
*Sqrt[a]*Sqrt[Tan[c + d*x]]*(-3*(A + (3*I)*B) + ((-5*I)*A + 11*B)*Tan[c +
d*x])))/(a^(3/2)*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.95, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 4729, 3042, 4078, 27, 3042, 4078, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}(a+ia\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{3\sqrt{\tan(c+dx)}(a(iA-B)+2iaB\tan(c+dx))}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{\sqrt{\tan(c+dx)}(a(iA-B)+2iaB\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\int \frac{\sqrt{\tan(c+dx)}(a(iA-B)+2iaB\tan(c+dx))}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}\right)$$

$$\Big\downarrow \text{4078}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{-\frac{\int -\frac{\sqrt{i\tan(c+dx)a+a}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{2\sqrt{\tan(c+dx)}}dx}{a^2}-\frac{a(A+3iB}{d\sqrt{a+i}}}{2a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{a(A+3iB)\sqrt{t}}{d\sqrt{a+ia\tan}}}{2a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{\sqrt{\tan(c+dx)}}dx}{2a^2}-\frac{a(A+3iB)\sqrt{\tan}}{d\sqrt{a+ia\tan}}}{2a^2}\right.$$

$\downarrow$ 4084

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{a^2(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+4iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+d}}{\sqrt{\tan(c+dx)}}}{2a^2}}{2a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{a^2(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+4iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+d}}{\sqrt{\tan(c+dx)}}}{2a^2}}{2a^2}\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{4iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2ia^4(A-iB)\int\frac{1}{-\frac{2\tan(c+}{i\tan(c+d}}}{}}{2a^2}}{2a^2}\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{4iaB\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{(1-i)a^{5/2}(A-iB)\arctan}{}}{2a^2}}{2a^2}\right.$$

$\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{4ia^3B\int\frac{1}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}d\tan(c+dx)}{d}+\frac{(1-i)a^{5/2}(A-iB)\text{ar}}{}}{2a^2}}{2a^2}\right.$$

$$\Big\downarrow \textbf{65}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{8ia^3B\int\frac{1}{1-\frac{ia\tan(c+dx)}{i\tan(c+dx)a+a}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}+\frac{(1-i)a^{5/2}(A-iB)\text{arcta}}{}}{2a^2}}{2a^2}\right.$$

$$\Big\downarrow \textbf{216}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}-\frac{\frac{\frac{(1-i)a^{5/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{8(-1)^{3/4}a^{5/2}B\text{ ar}}{}}{2a^2}}{2a^2}\right.$$

input

```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(3/2))
,x]
```

output

```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((I*A - B)*Tan[c + d*x]^(3/2))/(3*d
*(a + I*a*Tan[c + d*x])^(3/2)) - (((-8*(-1)^(3/4)*a^(5/2)*B*ArcTan[((-1)^(
3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d + ((1 - I)
*a^(5/2)*(A - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I
*a*Tan[c + d*x]]])/d)/(2*a^2) - (a*(A + (3*I)*B)*Sqrt[Tan[c + d*x]])/(d*Sq
rt[a + I*a*Tan[c + d*x]]))/(2*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1224 vs. $2(192) = 384$.

Time = 1.06 (sec) , antiderivative size = 1225, normalized size of antiderivative = 5.04

method	result	size
derivativedivides	Expression too large to display	1225
default	Expression too large to display	1225

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/24*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(72*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/
2)*a*tan(d*x+c)^2+9*I*B*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*(I*a)^(1/2)*a*t
an(d*x+c)+3*A*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*(I*a)^(1/2)*a*tan(d*x+c)^
3-44*I*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*ta
n(d*x+c)^2-3*I*B*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c))
)^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*(I*a)^(1/2)*a*tan(d*x+
c)^3-9*B*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-
I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*(I*a)^(1/2)*a*tan(d*x+c)^2+36*
I*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-24*B*(-
I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^3+3*I*A*ln((2*2^(1/2)*(-I*a)^(1
/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+
I))*2^(1/2)*(I*a)^(1/2)*a-20*A*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)^2*(a*ta
n(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-24*I*B*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*(-I*a)^(1/2)*a-
9*A*2^(1/2)*(I*a)^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+32*I*A*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(181) = 362$.

Time = 0.10 (sec) , antiderivative size = 782, normalized size of antiderivative = 3.22

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + ia\tan(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="fricas")
```

output
```
-1/12*(3*sqrt(1/2)*a^2*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2))*e^(3*I*d
*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I
*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^3*d^2)) + (A - I*B)*a
*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 3*sqrt(1/2)*a^2*d*sqrt((-I
*A^2 - 2*A*B + I*B^2)/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2)*sqrt(1
/2)*(a^2*d*e^(2*I*d*x + 2*I*c) - a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 -
 2*A*B + I*B^2)/(a^3*d^2)) - (A - I*B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
/(I*A + B)) - 3*a^2*d*sqrt(4*I*B^2/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-16*
(3*B*a^2*e^(2*I*d*x + 2*I*c) - B*a^2 + sqrt(2)*(I*a^3*d*e^(3*I*d*x + 3*I*c
) - I*a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(
2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(4*I*B^2/(a^3*d^2)))*
e^(-2*I*d*x - 2*I*c)/B) + 3*a^2*d*sqrt(4*I*B^2/(a^3*d^2))*e^(3*I*d*x + 3*I
*c)*log(-16*(3*B*a^2*e^(2*I*d*x + 2*I*c) - B*a^2 + sqrt(2)*(-I*a^3*d*e^(3*
I*d*x + 3*I*c) + I*a^3*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(4*I*B^2
/(a^3*d^2)))*e^(-2*I*d*x - 2*I*c)/B) + sqrt(2)*(2*(2*I*A - 5*B)*e^(4*I*d*x
 + 4*I*c) - (5*I*A - 11*B)*e^(2*I*d*x + 2*I*c) + I*A - B)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)
), x)
```

Reduce [F]

$$\int \frac{A + B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-\left(\int \frac{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)^2\tan(dx+c)^3 i+\cot(dx+c)^2\tan(dx+c)^2+\cot(dx+c)^2\tan(dx}\right.\right.}{}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*( - int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)
**2)/(cot(c + d*x)**2*tan(c + d*x)**3*i + cot(c + d*x)**2*tan(c + d*x)**2
+ cot(c + d*x)**2*tan(c + d*x)*i + cot(c + d*x)**2),x)*b*i - int((sqrt(tan
(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x))/(cot(c + d*x)**2*tan(c +
 d*x)**3*i + cot(c + d*x)**2*tan(c + d*x)**2 + cot(c + d*x)**2*tan(c + d*x
)*i + cot(c + d*x)**2),x)*a*i + int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c +
 d*x))*tan(c + d*x))/(cot(c + d*x)**2*tan(c + d*x)**3*i + cot(c + d*x)**2*
tan(c + d*x)**2 + cot(c + d*x)**2*tan(c + d*x)*i + cot(c + d*x)**2),x)*b +
 int((sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x)))/(cot(c + d*x)**2*tan(c
+ d*x)**3*i + cot(c + d*x)**2*tan(c + d*x)**2 + cot(c + d*x)**2*tan(c + d*
x)*i + cot(c + d*x)**2),x)*a))/a**2
```

3.566 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 260

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(\frac{1}{8}+\frac{i}{8}\right)(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{5/2}d}$$
$$+\frac{(A+iB)\sqrt{\cot(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}+\frac{(17A+7iB)\sqrt{\cot(c+dx)}}{30ad(a+ia\tan(c+dx))^{3/2}}$$
$$+\frac{(151A+41iB)\sqrt{\cot(c+dx)}}{60a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{(317A+67iB)\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}{60a^3d}$$

output
```
(1/8+1/8*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+1/5*(A+I*B)*cot(d*x
+c)^(1/2)/d/(a+I*a*tan(d*x+c))^(5/2)+1/30*(17*A+7*I*B)*cot(d*x+c)^(1/2)/a/
d/(a+I*a*tan(d*x+c))^(3/2)+1/60*(151*A+41*I*B)*cot(d*x+c)^(1/2)/a^2/d/(a+I
*a*tan(d*x+c))^(1/2)-1/60*(317*A+67*I*B)*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c
))^(1/2)/a^3/d
```

Mathematica [A] (verified)

Time = 5.80 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.86

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-15i\sqrt{2}(A-iB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{ia\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)(i+\cot(c+dx))^2\sqrt{a}}{120a^2d\cot^{\frac{5}{2}}(c+dx)\sqrt{ia\tan}}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^(5/2),x]
```

output
```
((-15*I)*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a
 + I*a*Tan[c + d*x]]]*(I + Cot[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]] + (2
*a*(160*((-5*I)*A + B) - 15*(41*A + (7*I)*B)*Cot[c + d*x] + (120*I)*A*Cot[
c + d*x]^2 + (317*A + (67*I)*B)*Tan[c + d*x]))/Sqrt[I*a*Tan[c + d*x]])/(12
0*a^2*d*Cot[c + d*x]^(5/2)*Sqrt[I*a*Tan[c + d*x]]*(-I + Tan[c + d*x])^2*Sq
rt[a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.447$, Rules used = {3042, 4729, 3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4081, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(i\tan(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \ 4079$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(11A+iB)-6a(iA-B)\tan(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(11A+iB)-6a(iA-B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(11A+iB)-6a(iA-B)\tan(c+dx)}{\tan(c+dx)^{3/2}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{A+iB}{5d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 4079$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{a^2(83A+13iB)-4a^2(17iA-7B)\tan(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{}{5d\sqrt{\tan(c+}}\right.$$

$$\Big\downarrow \ 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{a^2(83A+13iB)-4a^2(17iA-7B)\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{}{5d\sqrt{\tan(c+}}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{a^2(83A+13iB)-4a^2(17iA-7B)\tan(c+dx)}{\tan(c+dx)^{3/2}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(17A+7iB)}{3d\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{}{5d\sqrt{\tan(c+}}\right.$$

$\big\downarrow$ 4079

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(317A+67iB)-2a^3(151iA-41B)\tan(c+dx)\right)}{2\tan^{\frac{3}{2}}(c+dx)}dx}{a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{}{3d\sqrt{t}}}{10a^2}\right.$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(317A+67iB)-2a^3(151iA-41B)\tan(c+dx)\right)}{\tan^{\frac{3}{2}}(c+dx)}dx}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{}{3d\sqrt{t}}}{10a^2}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{\int \frac{\sqrt{i\tan(c+dx)a+a}\left(a^3(317A+67iB)-2a^3(151iA-41B)\tan(c+dx)\right)}{\tan(c+dx)^{3/2}}dx}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{}{3d\sqrt{t}}}{10a^2}\right.$$

$\big\downarrow$ 4081

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{\frac{2\int \frac{15a^4(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a^3(317A+67iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{}{3a}}{10a^2}\right.$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{15a^3(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(317A+67iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{}{3d\sqrt{}}}{10a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{15a^3(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{2a^3(317A+67iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{}{3d\sqrt{}}}{10a^2}\right.$$

$$\Big\downarrow \text{4027}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{-\frac{30ia^5(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{2a^3(317A+67iB)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}}{2a^2}+\frac{a^2(151A}{d\sqrt{\tan(c+dx)}\sqrt{}}}{6a^2}}{10a^2}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{a^2(151A+41iB)}{d\sqrt{\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{\frac{(15-15i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{2a^3(317A+67iB)\sqrt{a+}}{d\sqrt{\tan(c+d}}}{2a^2}}{6a^2}}{10a^2}\right.$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2)
,x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((A + I*B)/(5*d*Sqrt[Tan[c + d*x]]*(
a + I*a*Tan[c + d*x])^(5/2)) + ((a*(17*A + (7*I)*B))/(3*d*Sqrt[Tan[c + d*x
]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((a^2*(151*A + (41*I)*B))/(d*Sqrt[Tan[c
 + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((15 - 15*I)*a^(7/2)*(I*A + B)*Arc
Tanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d -
 (2*a^3*(317*A + (67*I)*B)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x
]]))/(2*a^2))/(6*a^2))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1155 vs. 2(210) = 420.

Time = 0.99 (sec) , antiderivative size = 1156, normalized size of antiderivative = 4.45

method	result	size
derivativedivides	Expression too large to display	1156
default	Expression too large to display	1156

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/240/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(-
1060*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2-6
0*I*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2-15*A*2^(1/2)*ln((2
*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*
x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5+1268*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*tan(d*x+c)^4+268*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*
tan(d*x+c)))^(1/2)*tan(d*x+c)^4+60*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*
tan(d*x+c)^4+908*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(
d*x+c)^3+15*I*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(
d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^5-90*I*B*2
^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*
a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3+90*A*2^(1/2)*ln((2*2^(1/2
)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(
tan(d*x+c)+I))*a*tan(d*x+c)^3-5660*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
*(-I*a)^(1/2)*tan(d*x+c)^2+2940*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)*tan(d*x+c)+15*I*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*
x+c)-60*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(198) = 396$.

Time = 0.10 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.85

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{iA^2+2AB-iB^2}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}-i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{iA^2+2}{a}}\right.}{iA+B}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/120*(15*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(5*I*
d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^
3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2)) + (A - I*
B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*a^3*d*sqr
t((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*s
qrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt
((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2)) + (A - I*B)*a*e^(I*d*x + I*c))*e^(-I*d
*x - I*c)/(I*A + B)) + sqrt(2)*((463*A + 83*I*B)*e^(6*I*d*x + 6*I*c) - 2*(
97*A + 32*I*B)*e^(4*I*d*x + 4*I*c) - 2*(13*A + 8*I*B)*e^(2*I*d*x + 2*I*c)
- 3*A - 3*I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*
c) + I)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2
),x)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2
), x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+1}}\right.}{}$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int((sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x))/(sqrt(tan(c + d*x)
*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt
(tan(c + d*x)*i + 1)),x)*b + int((sqrt(cot(c + d*x))*cot(c + d*x))/(sqrt(t
an(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*
x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a))/(sqrt(a)*a**2)
```

3.567 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 214

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(\frac{1}{8}-\frac{i}{8}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{5/2}d} + \frac{A+iB}{5d\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}} + \frac{13A+3iB}{30ad\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}} + \frac{67A-3iB}{60a^2d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output

```
(1/8-1/8*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+1/5*(A+I*B)/d/cot(d
*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2)+1/30*(13*A+3*I*B)/a/d/cot(d*x+c)^(1/2
)/(a+I*a*tan(d*x+c))^(3/2)+1/60*(67*A-3*I*B)/a^2/d/cot(d*x+c)^(1/2)/(a+I*a
*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\cot^{\frac{3}{2}}(c+dx)\sec^2(c+dx)(ia\tan(c+dx))^{3/2}\left(2(19A+9iB+\right.}{}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])
^(5/2),x]
```

output
```
(Cot[c + d*x]^(3/2)*Sec[c + d*x]^2*(I*a*Tan[c + d*x])^(3/2)*(2*(19*A + (9*
I)*B + (86*A + (6*I)*B)*Cos[2*(c + d*x)] + (80*I)*A*Sin[2*(c + d*x)])*Sqrt
[I*a*Tan[c + d*x]] + 15*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Tan[c
+ d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)
])*Sqrt[a + I*a*Tan[c + d*x]]))/(120*a^4*d*(-I + Tan[c + d*x])^2*Sqrt[a +
I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4729, 3042, 4079, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 4729$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{4079}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(9A-iB)-4a(iA-B)\tan(c+dx)}{2\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(9A-iB)-4a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(9A-iB)-4a(iA-B)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{4079}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{a^2(41A-9iB)-2a^2(13iA-3B)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+d}}{5d(a+ia\tan(c+dx)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{a^2(41A-9iB)-2a^2(13iA-3B)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+d}}{5d(a+ia\tan(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{a^2(41A-9iB)-2a^2(13iA-3B)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB)\sqrt{\tan(c+d}}{5d(a+ia\tan(c+dx)}\right.$$

$$\Big\downarrow \text{4079}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{\int \frac{15a^3(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}+\frac{a^2(67A-3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB}{5d(a+ia}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{15}{2}a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(67A-3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB}{5d(a+i}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{15}{2}a(A-iB)\int \frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(67A-3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}+\frac{(A+iB}{5d(a+i}\right.$$

$\Big\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{a^2(67A-3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{15ia^3(A-iB)\int \frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c+}}{3d(a+ia\tan(c+dx))}}{10a^2}\right.$$

$\Big\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{\left(\frac{15}{2}-\frac{15i}{2}\right)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{a^2(67A-3iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}+\frac{a(13A+3iB)\sqrt{\tan(c}}{3d(a+ia\tan(c+dx)}}{10a^2}\right.$$

input

```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + I*a*Tan[c + d*x])^(5/2)
,x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((A + I*B)*Sqrt[Tan[c + d*x]])/(5*d
*(a + I*a*Tan[c + d*x])^(5/2)) + ((a*(13*A + (3*I)*B)*Sqrt[Tan[c + d*x]])/
(3*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((15/2 - (15*I)/2)*a^(3/2)*(A - I*B)
*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])
/d + (a^2*(67*A - (3*I)*B)*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x
]]))/(6*a^2))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(172) = 344$.

Time = 1.01 (sec) , antiderivative size = 1094, normalized size of antiderivative = 5.11

method	result	size
derivativedivides	Expression too large to display	1094
default	Expression too large to display	1094

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/240/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)*(a*(1+I*tan(d*x+c)))^(1/2)*(-12*I
*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2-90*I*A*
ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*t
an(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)^2+15*B*2^(1/2)*ln((2*2^(1/
2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/
(tan(d*x+c)+I))*a*tan(d*x+c)^4-60*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(-I*a)^(1/2)-60*I*B*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*
x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)^3+60
*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-1060*I*A*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)+60*I*B*ln((2*2^(1/2)*(-I
*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d
*x+c)+I))*2^(1/2)*a*tan(d*x+c)-90*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*
tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*t
an(d*x+c)^2+12*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*
x+c)^3+268*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+
c)^3+15*I*A*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/
2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)^4-60*A*2^(1/2)
*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*
tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+908*A*(a*tan(d*x+c)*(1+I*tan(d...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(162) = 324$.

Time = 0.12 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{-iA^2-2AB+iB^2}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}(a^3de^{(2i\,dx}\right.}{}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
1/120*(15*sqrt(1/2)*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*e^(5*I*
d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*
I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2)) + (A - I*B)*
a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*a^3*d*sqrt((
-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sqrt(2)*sqrt
(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2
 - 2*A*B + I*B^2)/(a^5*d^2)) - (A - I*B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*
c)/(I*A + B)) + sqrt(2)*((-83*I*A + 3*B)*e^(6*I*d*x + 6*I*c) - 2*(-32*I*A
- 3*B)*e^(4*I*d*x + 4*I*c) - 2*(-8*I*A + 3*B)*e^(2*I*d*x + 2*I*c) + 3*I*A
- 3*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/
(e^(2*I*d*x + 2*I*c) - 1)))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\sqrt{\cot(c+dx)}}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/(I*a*(tan(c + d*x) - I))*
*(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)}{{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2
),x)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + a*tan(c + d*x)*1i)^(5/2
), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\sqrt{\cot(dx+c)}}{\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\tan(dx+c)i-\sqrt{\tan(dx+c)i+}}\right.}{}$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(sqrt(cot(c + d*x))/(sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)**2 - 2*
sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)),x)*a +
 int((sqrt(cot(c + d*x))*tan(c + d*x))/(sqrt(tan(c + d*x)*i + 1)*tan(c + d
*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i +
 1)),x)*b))/(sqrt(a)*a**2)
```

3.568 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 216

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(A-iB)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{5/2}d} + \frac{iA-B}{5d\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}} + \frac{3iA+7B}{30ad\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}} - \frac{3iA-13B}{60a^2d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
(-1/8-1/8*I)*(A-I*B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x
+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+1/5*(I*A-B)/d/cot(
d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2)+1/30*(3*I*A+7*B)/a/d/cot(d*x+c)^(1/2
)/(a+I*a*tan(d*x+c))^(3/2)-1/60*(3*I*A-13*B)/a^2/d/cot(d*x+c)^(1/2)/(a+I*a
*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.03

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^2(c+dx)\left(-2i(9A-iB+2(3A-7iB)\cos(2(c+dx))+\right.}{1}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^
(5/2)),x]
```

output
```
(Sec[c + d*x]^2*((-2*I)*(9*A - I*B + 2*(3*A - (7*I)*B)*Cos[2*(c + d*x)] +
20*B*Sin[2*(c + d*x)])*Sqrt[I*a*Tan[c + d*x]] + 15*Sqrt[2]*(I*A + B)*ArcTa
nh[(Sqrt[2]*Sqrt[I*a*Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*(Cos[2*(c
+ d*x)] + I*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]]))/(120*a^2*d*Sqrt
[Cot[c + d*x]]*Sqrt[I*a*Tan[c + d*x]]*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*T
an[c + d*x]])
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4729, 3042, 4078, 27, 3042, 4079, 27, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(i\tan(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(i\tan(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{a(iA-B)-2a(2A-3iB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{a(iA-B)-2a(2A-3iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{a(iA-B)-2a(2A-3iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}\right)$$

$$\Big\downarrow \text{4079}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{a^2(9iA+B)-2a^2(3A-7iB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{a^2(9iA+B)-2a^2(3A-7iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{a^2(9iA+B)-2a^2(3A-7iB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}-\frac{a(7B+3iA)\sqrt{\tan(c+dx)}}{3d(a+ia\tan(c+dx))^{3/2}}}{10a^2}\right.$$

$$\Big\downarrow \text{4079}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{\int\frac{15a^3(iA+B)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}+\frac{a^2(-13B+3iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(7\ldots}{3d(\ldots}}{10a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{15}{2}a(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(-13B+3iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(\ldots}{3a\ldots}}{10a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{15}{2}a(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx+\frac{a^2(-13B+3iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(\ldots}{3a\ldots}}{10a^2}\right.$$

$\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-13B+3iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{15ia^3(B+iA)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\ldots}}{\sqrt{i\,\mathrm{ta}\ldots}}}{d}}{6a^2}\ldots}{10a^2}\right.$$

$\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\sqrt{\tan(c+dx)}}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{\left(\frac{15}{2}-\frac{15i}{2}\right)a^{3/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}+\frac{a^2(-13B+3i\ldots}{d\sqrt{a+ia\ldots}}}{6a^2}\ldots}{10a^2}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2))
,x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((I*A - B)*Sqrt[Tan[c + d*x]])/(5*d
*(a + I*a*Tan[c + d*x])^(5/2)) - (-1/3*(a*((3*I)*A + 7*B)*Sqrt[Tan[c + d*x
]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((15/2 - (15*I)/2)*a^(3/2)*(I*A +
B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]
])/d + (a^2*((3*I)*A - 13*B)*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d
*x]]))/(6*a^2))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(174) = 348$.

Time = 0.96 (sec) , antiderivative size = 848, normalized size of antiderivative = 3.93

method	result
derivativedivides	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\left(15iA\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan\cdots}{\tan(dx+c)+i}\cdots\right.\right.}{\cdots}$
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}\left(15iA\sqrt{2}\,\ln\left(\frac{2\sqrt{2}\,\sqrt{-ia}\,\sqrt{a\tan(dx+c)(1+i\tan(dx+c))}-ia+3a\tan\cdots}{\tan(dx+c)+i}\cdots\right.\right.}{\cdots}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/240*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2
)*(15*I*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)
))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-45*I*B*2^(1/2)
*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*
tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+15*B*2^(1/2)*ln((2*2^(1/2)*(-I*
a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*
x+c)+I))*a*tan(d*x+c)^3-45*I*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d
*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*
x+c)+45*A*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a*tan(d*x+c)^2+12*I*A*(-I*a)^
(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2+15*I*B*2^(1/2)*ln
((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan
(d*x+c))/(tan(d*x+c)+I))*a-45*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(
d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d
*x+c)+160*I*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c
)-52*B*tan(d*x+c)^2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)-15*
A*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a
*tan(d*x+c))/(tan(d*x+c)+I))*2^(1/2)*a+60*I*A*(-I*a)^(1/2)*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)+60*B*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(
1/2))/(1/tan(d*x+c))^(1/2)/a^4/(1+I*tan(d*x+c))/tan(d*x+c)/(-I*a)^(1/2)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(162) = 324$.

Time = 0.09 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.23

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{iA^2+2AB-iB^2}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ia^3de^{(2i}\right.\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
1/120*(15*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(5*I*d
*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3
*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^
(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2)) + (A - I*B
)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*a^3*d*sqrt
((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sq
rt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(
(I*A^2 + 2*A*B - I*B^2)/(a^5*d^2)) + (A - I*B)*a*e^(I*d*x + I*c))*e^(-I*d*
x - I*c)/(I*A + B)) + sqrt(2)*((3*A - 17*I*B)*e^(6*I*d*x + 6*I*c) + 2*(3*A
 + 8*I*B)*e^(4*I*d*x + 4*I*c) - 2*(3*A - 2*I*B)*e^(2*I*d*x + 2*I*c) - 3*A
- 3*I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + ia\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + ia\tan(c + dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot\left(c+d\,x\right)}\,{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\tan(dx+c)}{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}}\right.}{}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(tan(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c +
 d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)*i -
sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))),x)*b + int(1/(sqrt(tan(c + d*
x)*i + 1)*sqrt(cot(c + d*x))*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*
sqrt(cot(c + d*x))*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c +
d*x))),x)*a))/(sqrt(a)*a**2)
```

3.569 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 214

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(\frac{1}{8}+\frac{i}{8}\right)(iA+B)\operatorname{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{}}{a^{5/2}d}$$
$$+\frac{iA-B}{5d\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}+\frac{A+11iB}{30ad\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^{3/2}}$$
$$+\frac{13A-37iB}{60a^2d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
(1/8+1/8*I)*(I*A+B)*arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+1/5*(I*A-B)/d/cot(d
*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2)+1/30*(A+11*I*B)/a/d/cot(d*x+c)^(1/2)/
(a+I*a*tan(d*x+c))^(3/2)+1/60*(13*A-37*I*B)/a^2/d/cot(d*x+c)^(1/2)/(a+I*a*
tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.05

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\cot^{\frac{3}{2}}(c+dx)\sec^2(c+dx)(ia\tan(c+dx))^{3/2}\left(2(A+11iB+2\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^
(5/2)),x]
```

output
```
(Cot[c + d*x]^(3/2)*Sec[c + d*x]^2*(I*a*Tan[c + d*x])^(3/2)*(2*(A + (11*I)
*B + 2*(7*A - (13*I)*B)*Cos[2*(c + d*x)] + 20*(I*A + B)*Sin[2*(c + d*x)])*
Sqrt[I*a*Tan[c + d*x]] - 15*Sqrt[2]*(A - I*B)*ArcTanh[(Sqrt[2]*Sqrt[I*a*Ta
n[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]*(Cos[2*(c + d*x)] + I*Sin[2*(c +
d*x)])*Sqrt[a + I*a*Tan[c + d*x]]))/(120*a^4*d*(-I + Tan[c + d*x])^2*Sqrt[
a + I*a*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 4729, 3042, 4078, 27, 3042, 4078, 27, 3042, 4079, 27, 3042, 4027, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(i\tan(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(i\tan(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\sqrt{\tan(c+dx)}(3a(iA-B)-2a(A-4iB)\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\sqrt{\tan(c+dx)}(3a(iA-B)-2a(A-4iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\sqrt{\tan(c+dx)}(3a(iA-B)-2a(A-4iB)\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{10a^2}\right)$$

$$\Big\downarrow \text{4078}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{\int -\frac{(A+11iB)a^2+2(7iA+13B)\tan(c+dx)a^2}{2\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}-\frac{a(A+11iB)\sqrt{\tan(}}{3d(a+ia\tan(c+da}}{10a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{(A+11iB)a^2+2(7iA+13B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}-\frac{a(A+11iB)\sqrt{\tan(c+d}}{3d(a+ia\tan(c+dx))^{3/}}}{10a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{(A+11iB)a^2+2(7iA+13B)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{i\tan(c+dx)a+a}}dx}{6a^2}-\frac{a(A+11iB)\sqrt{\tan(c+d}}{3d(a+ia\tan(c+dx))^{3/}}}{10a^2}\right.$$

$\Big\downarrow$ 4079

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{\int\frac{15a^3(A-iB)\sqrt{i\tan(c+dx)a+a}}{2\sqrt{\tan(c+dx)}}dx}{a^2}-\frac{a^2(13A-37iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(A}{3d(}}{10a^2}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{15}{2}a(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{a^2(13A-37iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(A}{3d}}{10a^2}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{15}{2}a(A-iB)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-\frac{a^2(13A-37iB)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}}{6a^2}-\frac{a(A}{3d}}{10a^2}\right.$$

$\Big\downarrow$ 4027

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{-\frac{15ia^3(A-iB)\int\frac{1}{-\frac{2\tan(c+dx)a^2}{i\tan(c+dx)a+a}-ia}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{i\tan(c+dx)a+a}}}{d}-\frac{a^2(13A-37i}{d\sqrt{a+ia}}}{6a^2}}{10a^2}\right.$$

$\Big\downarrow$ 218

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{3}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{\left(\frac{15}{2}-\frac{15i}{2}\right)a^{3/2}(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}-\frac{a^2(13A-37i}{d\sqrt{a+ia}}}{6a^2}}{10a^2}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^(5/2))
,x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((I*A - B)*Tan[c + d*x]^(3/2))/(5*d
*(a + I*a*Tan[c + d*x])^(5/2)) - (-1/3*(a*(A + (11*I)*B)*Sqrt[Tan[c + d*x]
])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((15/2 - (15*I)/2)*a^(3/2)*(A - I*B
)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]]
)/d - (a^2*(13*A - (37*I)*B)*Sqrt[Tan[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d
*x]]))/(6*a^2))/(10*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4079
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(a*A + b*B)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(2*f*m*(
b*c - a*d))), x] + Simp[1/(2*a*m*(b*c - a*d))   Int[(a + b*Tan[e + f*x])^(m
 + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(b*c*m - a*d*(2*m + n + 1)) + B*(a*c*m
- b*d*(n + 1)) + d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0]
 && LtQ[m, 0] &&  !GtQ[n, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1095 vs. $2(172) = 344$.

Time = 0.95 (sec) , antiderivative size = 1096, normalized size of antiderivative = 5.12

method	result	size
derivativedivides	Expression too large to display	1096
default	Expression too large to display	1096

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/240/d*(a*(1+I*tan(d*x+c)))^(1/2)*(-308*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c
)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2+90*I*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1
/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+
I))*a*tan(d*x+c)^2-15*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(
1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^4+
60*I*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)+60*I*B*2^(1/2)*l
n((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*ta
n(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^3-60*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)
^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+
c)+I))*a*tan(d*x+c)^3-220*I*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)
^(1/2)*tan(d*x+c)+148*B*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)
*tan(d*x+c)^3-60*I*B*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I
*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)+90*B*
2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I
*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)^2+52*I*A*(a*tan(d*x+c)*(1+
I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^3+212*A*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)*(-I*a)^(1/2)*tan(d*x+c)^2-15*I*A*2^(1/2)*ln((2*2^(1/2)*(-
I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(
d*x+c)+I))*a+60*A*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*ta
n(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x+c)-15*I*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(160) = 320$.

Time = 0.09 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.25

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + ia\tan(c + dx))^{5/2}}\,dx =$$

$$-\frac{\left(15\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{-iA^2-2AB+iB^2}{a^5d^2}}e^{(5i\,dx+5i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}-a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{i\,e^{(2i\,dx+2i\,c)}+i}{e^{(2i\,dx+2i\,c)}-1}}\sqrt{\frac{-i\,A^2-\cdots}{a\cdots}}\cdots\right.}{iA+B}\right.\right.}{\cdots}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/120*(15*sqrt(1/2)*a^3*d*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*e^(5*I
*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2
*I*d*x + 2*I*c) - 1))*sqrt((-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2)) + (A - I*B)
*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*a^3*d*sqrt(
(-I*A^2 - 2*A*B + I*B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sqrt(2)*sqr
t(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) - a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt((-I*A^
2 - 2*A*B + I*B^2)/(a^5*d^2)) - (A - I*B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)/(I*A + B)) - sqrt(2)*((-17*I*A - 23*B)*e^(6*I*d*x + 6*I*c) - 2*(-8*I*A
 - 17*B)*e^(4*I*d*x + 4*I*c) - 2*(-2*I*A + 7*B)*e^(2*I*d*x + 2*I*c) - 3*I*
A + 3*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I
)/(e^(2*I*d*x + 2*I*c) - 1)))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{3/2}\,{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\tan(dx}{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\sqrt{c}}\right.}{}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(tan(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c +
 d*x)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(
c + d*x)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(
c + d*x)),x)*b + int(1/(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c
+ d*x)*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot
(c + d*x)*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot
(c + d*x)),x)*a))/(sqrt(a)*a**2)
```

3.570 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 289

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\sqrt[4]{-1}B\arctan\left(\frac{(-1)^{3/4}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+}{a^{5/2}d}$$
$$+\frac{\left(\frac{1}{8}+\frac{i}{8}\right)(A-iB)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{a^{5/2}d}$$
$$+\frac{iA-B}{5d\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}+\frac{A+3iB}{6ad\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^{3/2}}$$
$$-\frac{iA-7B}{4a^2d\sqrt{\cot(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

output
```
2*(-1)^(1/4)*B*arctan((-1)^(3/4)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+(1/8+1/8*I)*(A-I*B)*
arctanh((1+I)*a^(1/2)*tan(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*cot(d*x+c
)^(1/2)*tan(d*x+c)^(1/2)/a^(5/2)/d+1/5*(I*A-B)/d/cot(d*x+c)^(5/2)/(a+I*a*t
an(d*x+c))^(5/2)+1/6*(A+3*I*B)/a/d/cot(d*x+c)^(3/2)/(a+I*a*tan(d*x+c))^(3/
2)-1/4*(I*A-7*B)/a^2/d/cot(d*x+c)^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 7.14 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.11

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(\frac{1}{120}+\frac{i}{120}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left((-120-120i)\sqrt[4]{-1}\right.$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^
(5/2)),x]
```

output
```
((1/120 + I/120)*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-120 - 120*I)*(-1
)^(1/4)*Sqrt[a]*B*ArcSinh[(-1)^(1/4)*Sqrt[Tan[c + d*x]]]*Sec[c + d*x]^3*(C
os[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - Sec[c + d*x]^2*Sqrt[1 + I*Tan[c +
d*x]]*((-1 - I)*Sqrt[a]*(-11*A - (21*I)*B + 2*(13*A + (63*I)*B)*Cos[2*(c +
 d*x)] + (20*I)*(A + (6*I)*B)*Sin[2*(c + d*x)])*Sqrt[Tan[c + d*x]] + 15*(A
 - I*B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c +
d*x]]]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])
))/(a^(5/2)*d*Sqrt[1 + I*Tan[c + d*x]]*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*
Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.99, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4729, 3042, 4078, 27, 3042, 4078, 27, 3042, 4078, 27, 3042, 4084, 3042, 4027, 218, 4082, 65, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{5/2}(a+ia\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(i\tan(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan(c+dx)^{5/2}(A+B\tan(c+dx))}{(i\tan(c+dx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{4078}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{5\tan^{\frac{3}{2}}(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{2(i\tan(c+dx)a+a)^{3/2}}dx}{5a^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan^{\frac{3}{2}}(c+dx)(a(iA-B)+2iaB\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{2a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\int \frac{\tan(c+dx)^{3/2}(a(iA-B)+2iaB\tan(c+dx))}{(i\tan(c+dx)a+a)^{3/2}}dx}{2a^2}\right)$$

$$\Big\downarrow \text{4078}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{-\frac{\int -\frac{3\sqrt{\tan(c+dx)}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{2\sqrt{i\tan(c+dx)a+a}}dx}{3a^2}-\frac{a(A+3iB)\tan}{3d(a+ia\tan(}}{2a^2}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int \frac{\sqrt{\tan(c+dx)}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c-}{3d(a+ia\tan(c+dx)}}{2a^2}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\int\frac{\sqrt{\tan(c+dx)}\left((A+3iB)a^2+4B\tan(c+dx)a^2\right)}{\sqrt{i\tan(c+dx)a+a}}dx}{2a^2}-\frac{a(A+3iB)\tan^{\frac{3}{2}}(c-}{3d(a+ia\tan(c+dx)}}{2a^2}\right.$$

$\downarrow$ 4078

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((iA-7B)a^3+8iB\tan(c+d\right.}{2\sqrt{\tan(c+dx)}}}{a^2}}{2a^2}}{2a^2}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((iA-7B)a^3+8iB\tan(c+d\right.}{\sqrt{\tan(c+dx)}}}{2a^2}}{2a^2}}{2a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\int\frac{\sqrt{i\tan(c+dx)a+a}\left((iA-7B)a^3+8iB\tan(c+d\right.}{\sqrt{\tan(c+dx)}}}{2a^2}}{2a^2}}{2a^2}\right.$$

$\downarrow$ 4084

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a^3(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-8a^2B\int\frac{(a-}{}}{2a^2}}{2a^2}}{2a^2}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{a^3(B+iA)\int\frac{\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}dx-8a^2B\int\frac{(a-}{}}{2a^2}}{2a^2}}{2a^2}\right.$$

$$\Big\downarrow \text{4027}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{-8a^2B\int\frac{(a-ia\tan(c+dx))\sqrt{i\tan(c+dx)a+a}}{\sqrt{\tan(c+dx)}}d}{2a^2}}{}\right.$$

$$\Big\downarrow \text{218}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c}}{\sqrt{a+ia\tan(c+}}\right.}{d}}{2a^2}}{}\right.$$

$$\Big\downarrow \text{4082}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c}}{\sqrt{a+ia\tan(c+}}\right.}{d}}{2a^2}}{}\right.$$

$$\Big\downarrow \text{65}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c}}{\sqrt{a+ia\tan(c+}}\right.}{d}}{2a^2}}{}\right.$$

$$\Big\downarrow 216$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\tan^{\frac{5}{2}}(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}-\frac{\frac{\frac{a^2(-7B+iA)\sqrt{\tan(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\frac{(1-i)a^{7/2}(B+iA)\text{arctanh}\left(\frac{(1+i)\sqrt{a}\sqrt{\tan(c}}{\sqrt{a+ia\tan(c+}}\right.}{d}}{2a^2}}{}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^(5/2))
,x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((I*A - B)*Tan[c + d*x]^(5/2))/(5*d
*(a + I*a*Tan[c + d*x])^(5/2)) - (-1/3*(a*(A + (3*I)*B)*Tan[c + d*x]^(3/2)
)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (-1/2*((16*(-1)^(1/4)*a^(7/2)*B*ArcTa
n[((-1)^(3/4)*Sqrt[a]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*a*Tan[c + d*x]]])/d +
 ((1 - I)*a^(7/2)*(I*A + B)*ArcTanh[((1 + I)*Sqrt[a]*Sqrt[Tan[c + d*x]])/S
qrt[a + I*a*Tan[c + d*x]]])/d)/a^2 + (a^2*(I*A - 7*B)*Sqrt[Tan[c + d*x]])/
(d*Sqrt[a + I*a*Tan[c + d*x]]))/(2*a^2))/(2*a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4027
```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*tan[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[-2*a*(b/f)   Subst[Int[1/(a*c - b*d - 2*
a^2*x^2), x], x, Sqrt[c + d*Tan[e + f*x]]/Sqrt[a + b*Tan[e + f*x]]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4078
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-(A*b - a*B))*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(2*a*f*m)),
 x] + Simp[1/(2*a^2*m)   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*
x])^(n - 1)*Simp[A*(a*c*m + b*d*n) - B*(b*c*m + a*d*n) - d*(b*B*(m - n) - a
*A*(m + n))*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && LtQ[m, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1541 vs. 2(230) = 460.

Time = 0.99 (sec) , antiderivative size = 1542, normalized size of antiderivative = 5.34

method	result	size
derivativedivides	Expression too large to display	1542
default	Expression too large to display	1542

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/240/d/(1/tan(d*x+c))^(5/2)/tan(d*x+c)^2*(a*(1+I*tan(d*x+c)))^(1/2)/a^3*
(-420*B*(-I*a)^(1/2)*(I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)+240
*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))
^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^4-1440*B*(-I*a)^(1/2)*ln(1
/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a
)/(I*a)^(1/2))*a*tan(d*x+c)^2-15*A*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a
)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x
+c)+I))*a-960*I*B*(-I*a)^(1/2)*ln(1/2*(2*I*a*tan(d*x+c)+2*(a*tan(d*x+c)*(1
+I*tan(d*x+c)))^(1/2)*(I*a)^(1/2)+a)/(I*a)^(1/2))*a*tan(d*x+c)^3+1548*B*(a
*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d*x+c)^2-
220*A*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*(-I*a)^(1/2)*(I*a)^(1/2)*tan(d
*x+c)+148*A*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)
*tan(d*x+c)^3+60*I*A*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan
(d*x+c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(
d*x+c)^3+15*I*B*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+
c)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a-308*I*A*(
I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*tan(d*x+c)^2
-1380*I*B*(I*a)^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c)*(1+I*tan(d*x+c)))^(1/2)*t
an(d*x+c)-15*A*(I*a)^(1/2)*2^(1/2)*ln((2*2^(1/2)*(-I*a)^(1/2)*(a*tan(d*x+c
)*(1+I*tan(d*x+c)))^(1/2)-I*a+3*a*tan(d*x+c))/(tan(d*x+c)+I))*a*tan(d*x...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 799 vs. 2(217) = 434.

Time = 0.11 (sec) , antiderivative size = 799, normalized size of antiderivative = 2.76

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + ia\tan(c + dx))^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/120*(15*sqrt(1/2)*a^3*d*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(5*I*
d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^
3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e
^(2*I*d*x + 2*I*c) - 1))*sqrt((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2)) + (A - I*
B)*a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(I*A + B)) - 15*sqrt(1/2)*a^3*d*sqr
t((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*s
qrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt
((I*A^2 + 2*A*B - I*B^2)/(a^5*d^2)) + (A - I*B)*a*e^(I*d*x + I*c))*e^(-I*d
*x - I*c)/(I*A + B)) + 30*a^3*d*sqrt(-4*I*B^2/(a^5*d^2))*e^(5*I*d*x + 5*I*
c)*log(-16*(3*B*a^2*e^(2*I*d*x + 2*I*c) - B*a^2 + sqrt(2)*(a^4*d*e^(3*I*d*
x + 3*I*c) - a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-4*I*B^2/(a^5
*d^2)))*e^(-2*I*d*x - 2*I*c)/B) - 30*a^3*d*sqrt(-4*I*B^2/(a^5*d^2))*e^(5*I
*d*x + 5*I*c)*log(-16*(3*B*a^2*e^(2*I*d*x + 2*I*c) - B*a^2 - sqrt(2)*(a^4*
d*e^(3*I*d*x + 3*I*c) - a^4*d*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))*sqrt(-4
*I*B^2/(a^5*d^2)))*e^(-2*I*d*x - 2*I*c)/B) + sqrt(2)*((23*A + 123*I*B)*e^(
6*I*d*x + 6*I*c) - 2*(17*A + 72*I*B)*e^(4*I*d*x + 4*I*c) + 2*(7*A + 12*I*B
)*e^(2*I*d*x + 2*I*c) - 3*A - 3*I*B)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + ia\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2)
),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + a*tan(c + d*x)*1i)^(5/2)
), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{-\left(\int \frac{\tan(}{\sqrt{\tan(dx+c)i+1}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2-2\sqrt{\tan(dx+c)i+1}\,\sqrt{}}\right.}{}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

output
```
( - (int(tan(c + d*x)/(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*cot(c +
 d*x)**2*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x))*c
ot(c + d*x)**2*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x)
)*cot(c + d*x)**2),x)*b + int(1/(sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c + d*x
))*cot(c + d*x)**2*tan(c + d*x)**2 - 2*sqrt(tan(c + d*x)*i + 1)*sqrt(cot(c
 + d*x))*cot(c + d*x)**2*tan(c + d*x)*i - sqrt(tan(c + d*x)*i + 1)*sqrt(co
t(c + d*x))*cot(c + d*x)**2),x)*a))/(sqrt(a)*a**2)
```

3.571 $\int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 179

$$\int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{(A-iB)\,\text{AppellF1}(1-m,1-n,1,2-m,-i\tan(c+dx),i\tan(c+dx))\cot^{-1+m}(c+dx)(1+i\tan(c+}{d(1-m)}$$
$$+\frac{iB\cot^{-1+m}(c+dx)\,\text{Hypergeometric2F1}(1-m,1-n,2-m,-i\tan(c+dx))(1+i\tan(c+dx))^{-n}(a}{d(1-m)}$$

output
```
(A-I*B)*AppellF1(1-m,1-n,1,2-m,-I*tan(d*x+c),I*tan(d*x+c))*cot(d*x+c)^(-1+
m)*(a+I*a*tan(d*x+c))^n/d/(1-m)/((1+I*tan(d*x+c))^n)+I*B*cot(d*x+c)^(-1+m)
*hypergeom([1-m, 1-n],[2-m],-I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/(1-m)/((
1+I*tan(d*x+c))^n)
```

Mathematica [F]

$$\int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Cot[c + d*x]^m*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Integrate[Cot[c + d*x]^m*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]), x]
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$, Rules used = {3042, 4729, 3042, 4084, 3042, 4047, 25, 27, 152, 150, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \cot(c+dx)^m(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{4729}$$
$$\tan^m(c+dx)\cot^m(c+dx)\int \tan^{-m}(c+dx)(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{3042}$$
$$\tan^m(c+dx)\cot^m(c+dx)\int \tan(c+dx)^{-m}(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{4084}$$

$$\tan^m(c+dx)\cot^m(c+dx)\left((A-iB)\int\tan^{-m}(c+dx)(i\tan(c+dx)a+a)^ndx+\frac{iB\int\tan^{-m}(c+dx)(a-ia\tan(c+dx))(i\tan(c+dx$$

$\downarrow$ 3042

$$\tan^m(c+dx)\cot^m(c+dx)\left((A-iB)\int\tan(c+dx)^{-m}(i\tan(c+dx)a+a)^ndx+\frac{iB\int\tan(c+dx)^{-m}(a-ia\tan(c+dx))(i\tan(c+dx$$

$\downarrow$ 4047

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{ia^2(A-iB)\int-\frac{\tan^{-m}(c+dx)(i\tan(c+dx)a+a)^{n-1}}{a(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}+\frac{iB\int\tan(c+dx)^{-m}(a-ia\tan(c+dx))($$

$\downarrow$ 25

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{iB\int\tan(c+dx)^{-m}(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^ndx}{a}-\frac{ia^2(A-iB)\int\frac{\tan^{-m}(c+dx)(i\tan(c+dx)a+a}{a(a-ia\tan(c+dx))}}{d}$$

$\downarrow$ 27

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{iB\int\tan(c+dx)^{-m}(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^ndx}{a}-\frac{ia(A-iB)\int\frac{\tan^{-m}(c+dx)(i\tan(c+dx)a+a)}{a-ia\tan(c+dx)}}{d}$$

$\downarrow$ 152

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{iB\int\tan(c+dx)^{-m}(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^ndx}{a}-\frac{i(A-iB)(1+i\tan(c+dx))^{-n}(a+ia}{}\right.$$

$\downarrow$ 150

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{iB\int\tan(c+dx)^{-m}(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^ndx}{a}+\frac{(A-iB)\tan^{1-m}(c+dx)(1+i\tan(c+}{}\right.$$

$\downarrow$ 4082

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{iaB\int\tan^{-m}(c+dx)(i\tan(c+dx)a+a)^{n-1}d\tan(c+dx)}{d}+\frac{(A-iB)\tan^{1-m}(c+dx)(1+i\tan(c+dx))^{-}}{}\right.$$

$\Big\downarrow$ 76

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{iB(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\int(i\tan(c+dx)+1)^{n-1}\tan^{-m}(c+dx)d\tan(c+dx)}{d}+\frac{(A-\right.$$

$\Big\downarrow$ 74

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{(A-iB)\tan^{1-m}(c+dx)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\operatorname{AppellF1}(1-m,1-n,1,2-m,-i\tan\right.}{d(1-m)}$$

input
```
Int[Cot[c + d*x]^m*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Cot[c + d*x]^m*Tan[c + d*x]^m*(((A - I*B)*AppellF1[1 - m, 1 - n, 1, 2 - m,
 (-I)*Tan[c + d*x], I*Tan[c + d*x]]*Tan[c + d*x]^(1 - m)*(a + I*a*Tan[c +
d*x])^n)/(d*(1 - m)*(1 + I*Tan[c + d*x])^n) + (I*B*Hypergeometric2F1[1 - m
, 1 - n, 2 - m, (-I)*Tan[c + d*x]]*Tan[c + d*x]^(1 - m)*(a + I*a*Tan[c + d
*x])^n)/(d*(1 - m)*(1 + I*Tan[c + d*x])^n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \cot\left(dx+c\right)^{m} \left(a+ia\tan\left(dx+c\right)\right)^{n} \left(A+B\tan\left(dx+c\right)\right) dx$$

input
```
int(cot(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan\left(dx+c\right)+A)(i\,a\tan\left(dx+c\right)+a)^{n}\cot\left(dx+c\right)^{m}\,dx$$

input
```
integrate(cot(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
integral(((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c
)/(e^(2*I*d*x + 2*I*c) + 1))^n*((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x +
2*I*c) - 1))^m/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))\cot^m(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**m*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*cot(c + d*x)**m,
 x)
```

Maxima [F]

$$\int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^m\,dx$$

input
```
integrate(cot(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c)^m, x)
```

Giac [F]

$$\int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^m\,dx$$

input
```
integrate(cot(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^m\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int(cot(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cot(c + d*x)^m*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cot^m(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \left(\int (\tan(dx+c)\,ai+a)^n\cot(dx+c)^m\tan(dx+c)\,dx\right)b + \left(\int (\tan(dx+c)\,ai+a)^n\cot(dx+c)^m\,dx\right)a$$

input
```
int(cot(d*x+c)^m*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*cot(c + d*x)**m*tan(c + d*x),x)*b + int((tan
(c + d*x)*a*i + a)**n*cot(c + d*x)**m,x)*a
```

3.572 $\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 247

$$\begin{aligned}
&\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx \\
&= -\frac{2(3B+2iAn)\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n}{3d} \\
&\quad -\frac{2A\cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{3d} \\
&\quad -\frac{2(A-iB)\,\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c-}{d\sqrt{\cot(c+dx)}} \\
&\quad -\frac{2(1-2n)(3iB-2An)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia}{3d\sqrt{\cot(c+dx)}}
\end{aligned}$$

output
```
-2/3*(3*B+2*I*A*n)*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d-2/3*A*cot(d*x+c
)^(3/2)*(a+I*a*tan(d*x+c))^n/d-2*(A-I*B)*AppellF1(1/2,1-n,1,3/2,-I*tan(d*x
+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/cot(d*x+c)^(1/2)/((1+I*tan(d*x+c)
)^n)-2/3*(1-2*n)*(3*I*B-2*A*n)*hypergeom([1/2, 1-n],[3/2],-I*tan(d*x+c))*(
a+I*a*tan(d*x+c))^n/d/cot(d*x+c)^(1/2)/((1+I*tan(d*x+c))^n)
```

Mathematica [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
,x]
```

output
```
Integrate[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.61 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.22, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4729, 3042, 4081, 27, 3042, 4081, 27, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{5/2}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\downarrow \text{4081}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int \frac{(i\tan(c+dx)a+a)^n(a(3B+2iAn)-aA(3-2n)\tan(c+dx))}{2\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{(i\tan(c+dx)a+a)^n(a(3B+2iAn)-aA(3-2n)\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{(i\tan(c+dx)a+a)^n(a(3B+2iAn)-aA(3-2n)\tan(c+dx))}{\tan(c+dx)^{3/2}}dx}{3a}-\frac{2A(a+ia\tan(c+dx))^n}{3d\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\downarrow \text{4081}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2\int \frac{(i\tan(c+dx)a+a)^n\left(a^2\left(6iBn-A\left(4n^2-2n+3\right)\right)-a^2(1-2n)(3B+2iAn)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ic}{d\sqrt{\tan(c+}}}{3a}\right.$$

$$\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{(i\tan(c+dx)a+a)^n\left(a^2\left(6iBn-A\left(4n^2-2n+3\right)\right)-a^2(1-2n)(3B+2iAn)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia\,t}{d\sqrt{\tan(c+d}}}{3a}\right.$$

$$\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\int \frac{(i\tan(c+dx)a+a)^n\left(a^2\left(6iBn-A\left(4n^2-2n+3\right)\right)-a^2(1-2n)(3B+2iAn)\tan(c+dx)\right)}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a(3B+2iAn)(a+ia\,t}{d\sqrt{\tan(c+d}}}{3a}\right.$$

$$\downarrow \text{4084}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{-3a^2(A-iB)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a}{}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{-3a^2(A-iB)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2a}{}}{3a}\right.$$

$\downarrow$ 4047

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{-\frac{3ia^4(A-iB)\int -\frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i t}{\sqrt{\tan(c+}}}{a}}{3a}\right.$$

$\downarrow$ 25

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{3ia^4(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(}{\sqrt{\tan(c+dx)}}}{a}}{3a}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{\frac{3ia^3(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c-}{\sqrt{\tan(c+dx)}}}{a}}{3a}\right.$$

$\downarrow$ 148

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{-\frac{6a^4(A-iB)\int \frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{a\left(ia^2\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{d}-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a}{\sqrt{\tan(c+dx)}}}{a}}{3a}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{-\frac{6a^3(A-iB)\int \frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a}{\sqrt{\tan(c+dx)}}}{a}}{3a}\right.$$

$$\Big\downarrow \; 334$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{-\frac{6a^2(A-iB)\left(a-ia^3\tan^2(c+dx)\right)^n\left(1-ia^2\tan^2(c+dx)\right)^{-n}\int \frac{\left(1-ia^2\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-a(1-2n)(-}{a}}{3a}\right.$$

$$\Big\downarrow \; 333$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{-a(1-2n)(-2An+3iB)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-\frac{6ia^3(A-iB)\tan(c+dx)\left(a-ia^3\tan^2(c+dx)\right)^n}{}}{a}}{}\right.$$

$$\Big\downarrow \; 4082$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{-\frac{a^3(1-2n)(-2An+3iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}}d\tan(c+dx)}{d}-\frac{6ia^3(A-iB)\tan(c+dx)\left(a-ia^3\tan^2(c+dx)\right)^n\left(1-ia\right.}{}}{a}}{3}\right.$$

$$\Big\downarrow \; 76$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{\frac{a^2(1-2n)(-2An+3iB)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\int\frac{(i\tan(c+dx)+1)^{n-1}}{\sqrt{\tan(c+dx)}}d\tan(c+dx)}{d}-\frac{6ia^3(A-iB)\tan}{}}{a}}{}\right.$$

$\downarrow$ 74

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{2a^2(1-2n)(-2An+3iB)\sqrt{\tan(c+dx)}(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c\right.}{d}}{}\right.$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A*(a + I*a*Tan[c + d*x])^n)/(3*
d*Tan[c + d*x]^(3/2)) + ((-2*a*(3*B + (2*I)*A*n)*(a + I*a*Tan[c + d*x])^n)
/(d*Sqrt[Tan[c + d*x]]) + ((-2*a^2*(1 - 2*n)*((3*I)*B - 2*A*n)*Hypergeomet
ric2F1[1/2, 1 - n, 3/2, (-I)*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]*(a + I*a*Tan
[c + d*x])^n)/(d*(1 + I*Tan[c + d*x])^n) - ((6*I)*a^3*(A - I*B)*AppellF1[1
/2, 1, 1 - n, 3/2, (-I)*a^2*Tan[c + d*x]^2, I*a^2*Tan[c + d*x]^2]*Tan[c +
d*x]*(a - I*a^3*Tan[c + d*x]^2)^n)/(d*(1 - I*a^2*Tan[c + d*x]^2)^n))/a)/(3
*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
 /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \cot (dx + c)^{\frac{5}{2}} (a + ia \tan (dx + c))^n (A + B \tan (dx + c)) dx$$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
integral(-((A - I*B)*e^(4*I*d*x + 4*I*c) + 2*A*e^(2*I*d*x + 2*I*c) + A + I
*B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))/(e^(4*I*d*x + 4*I*c) - 2*e^(2*
I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c)^(5/2)
, x)
```

Giac [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c)^(5/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \left(\int (\tan(dx+c)\,ai+a)^n\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)b$$
$$+\left(\int (\tan(dx+c)\,ai+a)^n\sqrt{\cot(dx+c)}\,\cot(dx+c)^2\,dx\right)a$$

input
```
int(cot(d*x+c)^(5/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d
*x),x)*b + int((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x))*cot(c + d*x)**
2,x)*a
```

3.573 $\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 194

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= -\frac{2A\sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n}{d}$$
$$+\frac{2(iA+B)\,\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c-}{d\sqrt{\cot(c+dx)}}$$
$$-\frac{2iA(1-2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx}{d\sqrt{\cot(c+dx)}}$$

output
```
-2*A*cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n/d+2*(I*A+B)*AppellF1(1/2,1-n,1,
3/2,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/cot(d*x+c)^(1/2)/((
1+I*tan(d*x+c))^n)-2*I*A*(1-2*n)*hypergeom([1/2, 1-n],[3/2],-I*tan(d*x+c))
*(a+I*a*tan(d*x+c))^n/d/cot(d*x+c)^(1/2)/((1+I*tan(d*x+c))^n)
```

Mathematica [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
,x]
```

output
```
Integrate[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.24, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4729, 3042, 4081, 27, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4081}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int \frac{(i\tan(c+dx)a+a)^n(a(B+2iAn)-aA(1-2n)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{(i\tan(c+dx)a+a)^n(a(B+2iAn)-aA(1-2n)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{(i\tan(c+dx)a+a)^n(a(B+2iAn)-aA(1-2n)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{2A(a+ia\tan(c+dx))^n}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{4084}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{a(B+iA)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{a(B+iA)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\right.$$

$$\Big\downarrow \text{4047}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{ia^3(B+iA)\int -\frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan}{\sqrt{\tan(c+d}}}{a}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{ia^3(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan}{\sqrt{\tan(c+d}}}{a}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{-\frac{ia^2(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(\cdots}{\sqrt{\tan(c+dx}\cdots}}{a}\right.$$

$\downarrow$ 148

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2a^3(B+iA)\int \frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{a\left(ia^2\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+dx)}}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a\cdots}{\sqrt{\tan(c+dx)}}}{a}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2a^2(B+iA)\int \frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-iA(1-2n)\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a\cdots}{\sqrt{\tan(c+dx)}}}{a}\right.$$

$\downarrow$ 334

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2a(B+iA)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\int \frac{\left(1-ia^2\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-iA(1\cdots}{a}\right.$$

$\downarrow$ 333

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2ia^2(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+\cdots\right.}{d}\cdots}{a}\right.$$

$\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2ia^2(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+\cdots\right.}{d}\cdots}{a}\right.$$

$\Big\downarrow$ 76

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2ia^2(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n \text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+\right.}{d}\right.$$

$\Big\downarrow$ 74

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2ia^2(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n \text{AppellF1}\left(\frac{1}{2},1,1-n,\frac{3}{2},-ia^2\tan^2(c+\right.}{d}\right.$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A*(a + I*a*Tan[c + d*x])^n)/(d*
Sqrt[Tan[c + d*x]]) + (((-2*I)*a*A*(1 - 2*n)*Hypergeometric2F1[1/2, 1 - n,
 3/2, (-I)*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(
1 + I*Tan[c + d*x])^n) + ((2*I)*a^2*(I*A + B)*AppellF1[1/2, 1, 1 - n, 3/2,
 (-I)*a^2*Tan[c + d*x]^2, I*a^2*Tan[c + d*x]^2]*Tan[c + d*x]*(a - I*a^3*Ta
n[c + d*x]^2)^n)/(d*(1 - I*a^2*Tan[c + d*x]^2)^n))/a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4081
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*d - B*c)*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(f*(n +
1)*(c^2 + d^2))), x] - Simp[1/(a*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e +
f*x])^m*(c + d*Tan[e + f*x])^(n + 1)*Simp[A*(b*d*m - a*c*(n + 1)) - B*(b*c*
m + a*d*(n + 1)) - a*(B*c - A*d)*(m + n + 1)*Tan[e + f*x], x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
0] && LtQ[n, -1]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \cot (dx+c)^{\frac{3}{2}} (a+ia\tan (dx+c))^n (A+B\tan (dx+c))\,dx$$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
integral(((I*A + B)*e^(2*I*d*x + 2*I*c) + I*A - B)*(2*a*e^(2*I*d*x + 2*I*c
)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) - 1))/(e^(2*I*d*x + 2*I*c) - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2)
, x)
```

Giac [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\cot(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,(a+a\tan(c+d\,x)\,1\mathrm{i})^n\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \left(\int (\tan(dx+c)\,ai+a)^n\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)\,dx\right)b + \left(\int (\tan(dx+c)\,ai+a)^n\sqrt{\cot(dx+c)}\,\cot(dx+c)\,dx\right)a$$

input
```
int(cot(d*x+c)^(3/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)
,x)*b + int((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x))*cot(c + d*x),x)*a
```

3.574 $\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 36, antiderivative size = 158

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{2(A-iB)\,\text{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+d}{d\sqrt{\cot(c+dx)}}$$
$$+\frac{2iB\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n}{d\sqrt{\cot(c+dx)}}$$

output
```
2*(A-I*B)*AppellF1(1/2,1-n,1,3/2,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*
x+c))^n/d/cot(d*x+c)^(1/2)/((1+I*tan(d*x+c))^n)+2*I*B*hypergeom([1/2, 1-n]
,[3/2],-I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/cot(d*x+c)^(1/2)/((1+I*tan(d*
x+c))^n)
```

Mathematica [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
,x]
```

output
```
Integrate[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x])
, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.25, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4729, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4084}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left((A-iB)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx+\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}\right)$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left((A-iB)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx+\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}\right)$$

$\downarrow$ 4047

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{ia^2(A-iB)\int -\frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(ia\tan(c+dx))}{d}+\frac{iB\int \frac{(a-ia\tan(c+dx))}{\sqrt{\tan}}}{a}\right.$$

$\downarrow$ 25

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{ia^2(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d}{d}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}-\frac{ia(A-iB)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}d(i}{d}\right.$$

$\downarrow$ 148

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a^2(A-iB)\int \frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{a(ia^2\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{d}+\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)}{\sqrt{\tan(c+dx)}}}{a}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(A-iB)\int \frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{iB\int \frac{(a-ia\tan(c+dx))(i\tan(c+dx)a}{\sqrt{\tan(c+dx)}}}{a}\right.$$

$\downarrow$ 334

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2(A-iB)\left(a-ia^3\tan^2(c+dx)\right)^n\left(1-ia^2\tan^2(c+dx)\right)^{-n}\int\frac{\left(1-ia^2\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}}{d}\right.$$

$$\Big\downarrow \text{333}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{iB\int\frac{(a-ia\tan(c+dx))(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{a}+\frac{2ia(A-iB)\tan(c+dx)\left(a-ia^3\tan^2(c+\right.}{}\right.$$

$$\Big\downarrow \text{4082}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{iaB\int\frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{2ia(A-iB)\tan(c+dx)\left(a-ia^3\tan^2(c-\right.}{}\right.$$

$$\Big\downarrow \text{76}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{iB(1+i\tan(c+dx))^{-n}(a+ia\tan(c+dx))^n\int\frac{(i\tan(c+dx)+1)^{n-1}}{\sqrt{\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{2}{}\right.$$

$$\Big\downarrow \text{74}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2ia(A-iB)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan^2(c+dx)\right)^n\text{AppellF}}{d}\right.$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((2*I)*B*Hypergeometric2F1[1/2, 1 -
 n, 3/2, (-I)*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(
d*(1 + I*Tan[c + d*x])^n) + ((2*I)*a*(A - I*B)*AppellF1[1/2, 1, 1 - n, 3/2
, (-I)*a^2*Tan[c + d*x]^2, I*a^2*Tan[c + d*x]^2]*Tan[c + d*x]*(a - I*a^3*T
an[c + d*x]^2)^n)/(d*(1 - I*a^2*Tan[c + d*x]^2)^n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \sqrt{\cot \left(dx+c\right)}\, (a+ia\tan \left(dx+c\right))^{n} \left(A+B\tan \left(dx+c\right)\right) dx$$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
integral(((A - I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c
)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*
x + 2*I*c) - 1))/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))\,\sqrt{\cot(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))*sqrt(cot(c + d*x
)), x)
```

Maxima [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*sqrt(cot(d*x + c))
, x)
```

Giac [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n*sqrt(cot(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n,x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n, x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+ia\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\left(\int (\tan(dx+c)\,ai+a)^n\,\sqrt{\cot(dx+c)}\,\tan(dx+c)\,dx\right)b$$
$$+\left(\int (\tan(dx+c)\,ai+a)^n\,\sqrt{\cot(dx+c)}dx\right)a$$

input
```
int(cot(d*x+c)^(1/2)*(a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x))*tan(c + d*x),x)*b + int((
tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x)),x)*a
```

3.575 $\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 215

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{2B(a+ia\tan(c+dx))^n}{d(1+2n)\sqrt{\cot(c+dx)}}$$
$$-\frac{2(iA+B)\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c-}{d\sqrt{\cot(c+dx)}}$$
$$+\frac{2(2Bn+iA(1+2n))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\,t}{d(1+2n)\sqrt{\cot(c+dx)}}$$

output
```
2*B*(a+I*a*tan(d*x+c))^n/d/(1+2*n)/cot(d*x+c)^(1/2)-2*(I*A+B)*AppellF1(1/2
,1-n,1,3/2,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/cot(d*x+c)^(
1/2)/((1+I*tan(d*x+c))^n)+2*(2*B*n+I*A*(1+2*n))*hypergeom([1/2, 1-n],[3/2]
,-I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/(1+2*n)/cot(d*x+c)^(1/2)/((1+I*tan(
d*x+c))^n)
```

Mathematica [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x
]],x]
```

output
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x
]], x]
```

Rubi [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.25, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4729, 3042, 4080, 27, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))dx$$

$\downarrow$ 4080

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int -\frac{(i\tan(c+dx)a+a)^n(aB-a(2nA+A-2iBn)\tan(c+dx))}{2\sqrt{\tan(c+dx)}}dx}{a(2n+1)}+\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\int \frac{(i\tan(c+dx)a+a)^n(aB-a(2nA+A-2iBn)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\int \frac{(i\tan(c+dx)a+a)^n(aB-a(2nA+A-2iBn)\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx}{a(2n+1)}\right.$$

$\downarrow$ 4084

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a(2n+1)(B+iA)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{a(2n+1)(B+iA)\int \frac{(i\tan(c+dx)a+a)^n}{\sqrt{\tan(c+dx)}}dx}{}\right.$$

$\downarrow$ 4047

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{ia^3(2n+1)(B+iA)\int -\frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}}{d}}{}\right.$$

$\downarrow$ 25

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{-\frac{ia^3(2n+1)(B+iA)\int \frac{(i\tan(c+dx)a+a)^{n-1}}{a\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}}{d}}{}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{-\frac{ia^2(2n+1)(B+iA)\int\frac{(i\tan(c+dx)a+a)^{n-1}}{\sqrt{\tan(c+dx)}(a-ia\tan(c+dx))}}{d}}{}\right.$$

$$\Big\downarrow \; 148$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{2a^3(2n+1)(B+iA)\int\frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{a\left(ia^2\tan^2(c+dx)+1\right)}d\sqrt{\tan}}{d}}{}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{\frac{2a^2(2n+1)(B+iA)\int\frac{\left(a-ia^3\tan^2(c+dx)\right)^{n-1}}{ia^2\tan^2(c+dx)+1}d\sqrt{\tan}}{d}}{}\right.$$

$$\Big\downarrow \; 334$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{2a(2n+1)(B+iA)\left(1-ia^2\tan^2(c+dx)\right)^{-n}\left(a-ia^3\tan\right.}{}\right.$$

$$\Big\downarrow \; 333$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{2ia^2(2n+1)(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)}{}\right.$$

$$\Big\downarrow \; 4082$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{2ia^2(2n+1)(B+iA)\tan(c+dx)\left(1-ia^2\tan^2(c+dx)\right)}{}\right.$$

$$\Big\downarrow \text{76}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{2ia^2(2n+1)(B+iA)\tan(c+dx)(1-ia^2\tan^2(c+dx))}{}\right.$$

$$\Big\downarrow \text{74}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\sqrt{\tan(c+dx)}(a+ia\tan(c+dx))^n}{d(2n+1)}-\frac{2ia^2(2n+1)(B+iA)\tan(c+dx)(1-ia^2\tan^2(c+dx))}{}\right.$$

input
```
Int[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*B*Sqrt[Tan[c + d*x]]*(a + I*a*Ta
n[c + d*x])^n)/(d*(1 + 2*n)) - ((-2*a*(2*B*n + I*A*(1 + 2*n))*Hypergeometr
ic2F1[1/2, 1 - n, 3/2, (-I)*Tan[c + d*x]]*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[
c + d*x])^n)/(d*(1 + I*Tan[c + d*x])^n) + ((2*I)*a^2*(I*A + B)*(1 + 2*n)*A
ppellF1[1/2, 1, 1 - n, 3/2, (-I)*a^2*Tan[c + d*x]^2, I*a^2*Tan[c + d*x]^2]
*Tan[c + d*x]*(a - I*a^3*Tan[c + d*x]^2)^n)/(d*(1 - I*a^2*Tan[c + d*x]^2)^
n))/(a*(1 + 2*n)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n (A+B\tan(dx+c))}{\sqrt{\cot(dx+c)}}dx$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algori
thm="fricas")
```

output
```
integral(((-I*A - B)*e^(4*I*d*x + 4*I*c) + 2*B*e^(2*I*d*x + 2*I*c) + I*A -
 B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2*I*d
*x + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))/(e^(4*I*d*x + 4*I*c) + 2*e^(2*
I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))/sqrt(cot(c + d*x
)), x)
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/sqrt(cot(d*x + c))
, x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/sqrt(cot(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/cot(c + d*x)^(1/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/cot(c + d*x)^(1/2), x
)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \left(\int \frac{(\tan(dx+c)\,ai+a)^n\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)b + \left(\int \frac{(\tan(dx+c)\,ai+a)^n\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
int(((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*
x),x)*b + int(((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x)))/cot(c + d*x),
x)*a
```

3.576 $\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 291

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2B(a+ia\tan(c+dx))^n}{d(3+2n)\cot^{\frac{3}{2}}(c+dx)} - \frac{2(2iBn-A(3+2n))(a+ia\tan(c+dx))^n}{d(1+2n)(3+2n)\sqrt{\cot(c+dx)}}$$
$$- \frac{2(A-iB)\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c-}{d\sqrt{\cot(c+dx)}}$$
$$+ \frac{2(2An(3+2n)-iB(3+6n+4n^2))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right)(1+i\tan(c+}{d(1+2n)(3+2n)\sqrt{\cot(c+dx)}}$$

output
```
2*B*(a+I*a*tan(d*x+c))^n/d/(3+2*n)/cot(d*x+c)^(3/2)-2*(2*I*B*n-A*(3+2*n))*
(a+I*a*tan(d*x+c))^n/d/(1+2*n)/(3+2*n)/cot(d*x+c)^(1/2)-2*(A-I*B)*AppellF1
(1/2,1-n,1,3/2,-I*tan(d*x+c),I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/cot(d*x+
c)^(1/2)/((1+I*tan(d*x+c))^n)+2*(2*A*n*(3+2*n)-I*B*(4*n^2+6*n+3))*hypergeo
m([1/2, 1-n],[3/2],-I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d/(1+2*n)/(3+2*n)/c
ot(d*x+c)^(1/2)/((1+I*tan(d*x+c))^n)
```

Mathematica [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/
2),x]
```

output
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/
2), x]
```

Rubi [A] (warning: unable to verify)

Time = 1.69 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.21, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 4729, 3042, 4080, 27, 3042, 4080, 27, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot(c+dx)^{3/2}}dx$$
$$\Big\downarrow \text{4729}$$
$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{3042}$$
$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan(c+dx)^{3/2}(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4080}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int -\frac{1}{2}\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n(3aB+a(2iBn-A(2n+3))\tan(c+dx)}{a(2n+3)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\int\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n(3a}{a(2n}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\int\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n(3a}{a(2n}\right.$$

$$\Big\downarrow \text{4080}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2\int -\frac{(i\tan(c+dx)a+a)^n\left(a^2(2iBn-A(2n+3))-a^2\left(2iAn\right.\right.}{2\sqrt{\tan(c+dx)}}}{a(2n+1)}}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$$\Big\downarrow \text{4084}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$$\Big\downarrow \text{4047}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$$\Big\downarrow \text{148}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$\downarrow$ 334

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$\downarrow$ 333

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{}\right.$$

$\downarrow$ 76

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{\ }\right.$$

$$\Big\downarrow \; 74$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+3)}-\frac{\frac{2a(-A(2n+3)+2iBn)\sqrt{\tan(c+dx)}(a+ia\tan(c+dx)}{d(2n+1)}}{\ }\right.$$

input
```
Int[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*B*Tan[c + d*x]^(3/2)*(a + I*a*Ta
n[c + d*x])^n)/(d*(3 + 2*n)) - ((2*a*((2*I)*B*n - A*(3 + 2*n))*Sqrt[Tan[c
+ d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 2*n)) - ((2*a^2*(2*A*n*(3 + 2*n)
 - I*B*(3 + 6*n + 4*n^2))*Hypergeometric2F1[1/2, 1 - n, 3/2, (-I)*Tan[c +
d*x]]*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + I*Tan[c + d*x])
^n) - ((2*I)*a^3*(A - I*B)*(3 + 8*n + 4*n^2)*AppellF1[1/2, 1, 1 - n, 3/2,
(-I)*a^2*Tan[c + d*x]^2, I*a^2*Tan[c + d*x]^2]*Tan[c + d*x]*(a - I*a^3*Tan
[c + d*x]^2)^n)/(d*(1 - I*a^2*Tan[c + d*x]^2)^n))/(a*(1 + 2*n)))/(a*(3 + 2
*n)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n (A+B\tan(dx+c))}{\cot(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algori
thm="fricas")
```

output
```
integral(-((A - I*B)*e^(6*I*d*x + 6*I*c) - (A - 3*I*B)*e^(4*I*d*x + 4*I*c)
 - (A + 3*I*B)*e^(2*I*d*x + 2*I*c) + A + I*B)*(2*a*e^(2*I*d*x + 2*I*c)/(e^
(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2*I*d*x + 2*I*c) + I)/(e^(2*I*d*x + 2
*I*c) - 1))/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x +
2*I*c) + 1), x)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(ia(\tan(c+dx)-i))^n\,(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c))/cot(d*x+c)**(3/2),x)
```

output
```
Integral((I*a*(tan(c + d*x) - I))**n*(A + B*tan(c + d*x))/cot(c + d*x)**(3
/2), x)
```

Maxima [F]

$$\int \frac{(a + ia\tan(c + dx))^n (A + B\tan(c + dx))}{\cot^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(B\tan(dx + c) + A)(i\,a\tan(dx + c) + a)^n}{\cot(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/cot(d*x + c)^(3/2)
, x)
```

Giac [F]

$$\int \frac{(a + ia\tan(c + dx))^n (A + B\tan(c + dx))}{\cot^{\frac{3}{2}}(c + dx)}\,dx = \int \frac{(B\tan(dx + c) + A)(i\,a\tan(dx + c) + a)^n}{\cot(dx + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/cot(d*x + c)^(3/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n}{\cot\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/cot(c + d*x)^(3/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/cot(c + d*x)^(3/2), x
)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{(\tan(dx+c)\,ai+a)^n\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)^2}dx\right)b + \left(\int \frac{(\tan(dx+c)\,ai+a)^n\sqrt{\cot(dx+c)}}{\cot(dx+c)^2}dx\right)a$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
int(((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*
x)**2,x)*b + int(((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x)))/cot(c + d*
x)**2,x)*a
```

3.577 $\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 383

$$
\begin{aligned}
&\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2B(a+ia\tan(c+dx))^n}{d(5+2n)\cot^{\frac{5}{2}}(c+dx)} - \frac{2(2iBn-A(5+2n))(a+ia\tan(c+dx))^n}{d(3+2n)(5+2n)\cot^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{2\left(2iAn(5+2n)+B(15+10n+4n^2)\right)(a+ia\tan(c+dx))^n}{d(1+2n)(3+2n)(5+2n)\sqrt{\cot(c+dx)}} \\
&\quad + \frac{2(iA+B)\operatorname{AppellF1}\left(\frac{1}{2},1-n,1,\frac{3}{2},-i\tan(c+dx),i\tan(c+dx)\right)(1+i\tan(c+dx))^{-n}(a+ia\tan(c-}{d\sqrt{\cot(c+dx)}} \\
&\quad - \frac{2(4Bn(9+8n+2n^2)+iA(15+36n+32n^2+8n^3))\operatorname{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-i\tan(c+dx)\right.}{d(1+2n)(3+2n)(5+2n)\sqrt{\cot(c+dx)}}
\end{aligned}
$$

output
```
2*B*(a+I*a*tan(d*x+c))^n/d/(5+2*n)/cot(d*x+c)^(5/2)-2*(2*I*B*n-A*(5+2*n))*
(a+I*a*tan(d*x+c))^n/d/(3+2*n)/(5+2*n)/cot(d*x+c)^(3/2)-2*(2*I*A*n*(5+2*n)
+B*(4*n^2+10*n+15))*(a+I*a*tan(d*x+c))^n/d/(1+2*n)/(3+2*n)/(5+2*n)/cot(d*x
+c)^(1/2)+2*(I*A+B)*AppellF1(1/2,1-n,1,3/2,-I*tan(d*x+c),I*tan(d*x+c))*(a+
I*a*tan(d*x+c))^n/d/cot(d*x+c)^(1/2)/((1+I*tan(d*x+c))^n)-2*(4*B*n*(2*n^2+
8*n+9)+I*A*(8*n^3+32*n^2+36*n+15))*hypergeom([1/2, 1-n],[3/2],-I*tan(d*x+c
))*(a+I*a*tan(d*x+c))^n/d/(1+2*n)/(3+2*n)/(5+2*n)/cot(d*x+c)^(1/2)/((1+I*t
an(d*x+c))^n)
```

Mathematica [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(5/
2),x]
```

output
```
Integrate[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(5/
2), x]
```

Rubi [A] (warning: unable to verify)

Time = 2.31 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.15, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4729, 3042, 4080, 27, 3042, 4080, 27, 3042, 4080, 27, 3042, 4084, 3042, 4047, 25, 27, 148, 27, 334, 333, 4082, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan^{\frac{5}{2}}(c+dx)(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan(c+dx)^{5/2}(i\tan(c+dx)a+a)^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4080}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int -\frac{1}{2}\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^n(5aB+a(2iBn-A(2n+5))\tan(c+dx\right.}{a(2n+5)}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\int\tan^{\frac{3}{2}}(c+dx)(i\tan(c+dx)a+a)^n(5aB\right.}{a(2n}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\int\tan(c+dx)^{3/2}(i\tan(c+dx)a+a)^n(5a\right.}{a(2n}$$

$$\Big\downarrow \text{4080}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{2\int -\frac{1}{2}\sqrt{\tan(c+dx)}(i\tan(c+dx)a+a)^n(3a^2(2iBn-\right.}{}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)\right.}{d(2n+3)}}{}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)\right.}{d(2n+3)}}{}$$

$$\Big\downarrow \text{4080}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)\right.}{d(2n+3)}}{}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$$\Big\downarrow \text{4084}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$$\Big\downarrow \text{4047}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$$\Big\downarrow \text{25}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$\Big\downarrow$ 148

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$\Big\downarrow$ 334

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$\downarrow$ 333

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$\downarrow$ 4082

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$\downarrow$ 76

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

$\downarrow$ 74

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\tan^{\frac{5}{2}}(c+dx)(a+ia\tan(c+dx))^n}{d(2n+5)}-\frac{\frac{2a(-A(2n+5)+2iBn)\tan^{\frac{3}{2}}(c+dx)(a+ia\tan(c+dx)}{d(2n+3)}}{}\right.$$

input
```
Int[((a + I*a*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(5/2),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*B*Tan[c + d*x]^(5/2)*(a + I*a*Ta
n[c + d*x])^n)/(d*(5 + 2*n)) - ((2*a*((2*I)*B*n - A*(5 + 2*n))*Tan[c + d*x
]^(3/2)*(a + I*a*Tan[c + d*x])^n)/(d*(3 + 2*n)) - ((-2*a^2*((2*I)*A*n*(5 +
 2*n) + B*(15 + 10*n + 4*n^2))*Sqrt[Tan[c + d*x]]*(a + I*a*Tan[c + d*x])^n
)/(d*(1 + 2*n)) + ((-2*a^3*(4*B*n*(9 + 8*n + 2*n^2) + I*A*(15 + 36*n + 32*
n^2 + 8*n^3))*Hypergeometric2F1[1/2, 1 - n, 3/2, (-I)*Tan[c + d*x]]*Sqrt[T
an[c + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + I*Tan[c + d*x])^n) + ((2*I)
*a^4*(I*A + B)*(15 + 46*n + 36*n^2 + 8*n^3)*AppellF1[1/2, 1, 1 - n, 3/2, (
-I)*a^2*Tan[c + d*x]^2, I*a^2*Tan[c + d*x]^2]*Tan[c + d*x]*(a - I*a^3*Tan[
c + d*x]^2)^n)/(d*(1 - I*a^2*Tan[c + d*x]^2)^n))/(a*(1 + 2*n)))/(a*(3 + 2*
n)))/(a*(5 + 2*n)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4047
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[a*(b/f)   Subst[Int[(a + x)^(m - 1)*((
c + (d/b)*x)^n/(b^2 + a*x)), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 + d
^2, 0]
```

rule 4080
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp[
1/(a*(m + n))   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^(n - 1)*Sim
p[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (a*A*d*(m + n) - B*(b*d*m - a*c*n))*T
an[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 + b^2, 0] && GtQ[n, 0]
```

rule 4082
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(B/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x, Tan[e + f*x]],
x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 + b^2, 0] && EqQ[A*b + a*B, 0]
```

rule 4084
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b + a*B)/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n, x], x]
 - Simp[B/b   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(a - b*Tan[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[A*b + a*B, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \frac{(a + ia\tan(dx + c))^n (A + B\tan(dx + c))}{\cot(dx + c)^{\frac{5}{2}}} dx$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(5/2),x)
```

output
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\cot(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(5/2),x, algori
thm="fricas")
```

output
```
integral(((I*A + B)*e^(8*I*d*x + 8*I*c) - 2*(I*A + 2*B)*e^(6*I*d*x + 6*I*c
) + 6*B*e^(4*I*d*x + 4*I*c) - 2*(-I*A + 2*B)*e^(2*I*d*x + 2*I*c) - I*A + B
)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt((I*e^(2*I*d*x
 + 2*I*c) + I)/(e^(2*I*d*x + 2*I*c) - 1))/(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*
d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(d*x+c))**n*(A+B*tan(d*x+c))/cot(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\cot(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(5/2),x, algori
thm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/cot(d*x + c)^(5/2)
, x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(i\,a\tan(dx+c)+a)^n}{\cot(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(5/2),x, algori
thm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(I*a*tan(d*x + c) + a)^n/cot(d*x + c)^(5/2)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n}{\cot\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/cot(c + d*x)^(5/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + a*tan(c + d*x)*1i)^n)/cot(c + d*x)^(5/2), x
)
```

Reduce [F]

$$\int \frac{(a+ia\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{5}{2}}(c+dx)}\,dx = \left(\int \frac{(\tan(dx+c)\,ai+a)^n\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)^3}dx\right)b + \left(\int \frac{(\tan(dx+c)\,ai+a)^n\sqrt{\cot(dx+c)}}{\cot(dx+c)^3}dx\right)a$$

input
```
int((a+I*a*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(5/2),x)
```

output
```
int(((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*
x)**3,x)*b + int(((tan(c + d*x)*a*i + a)**n*sqrt(cot(c + d*x)))/cot(c + d*
x)**3,x)*a
```

3.578 $\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 177

$$\begin{aligned}\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx &= -\frac{(b(A-B)+a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}\\ &\quad+\frac{(b(A-B)+a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}\\ &\quad-\frac{(a(A-B)-b(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d}\\ &\quad-\frac{2(Ab+aB)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}\end{aligned}$$

output
```
1/2*(b*(A-B)+a*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*(b
*(A-B)+a*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(a*(A-B)-
b*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/d-2*(A*b
+B*a)*cot(d*x+c)^(1/2)/d-2/3*a*A*cot(d*x+c)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.12

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}\Big(6\sqrt{2}(b(A-B)+a(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*(6*Sqrt[2]*(b*(A - B) + a*(A + B))*(ArcTan[1 - Sqrt[2]
*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + 3*Sqrt[2]
*(a*(A - B) - b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) - (8*a*A)/Tan[c +
 d*x]^(3/2) - (24*(A*b + a*B))/Sqrt[Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/(12
*d)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {3042, 4064, 3042, 4075, 3042, 4011, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cot(c+dx)^{5/2}(a+b\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4064$$

$$\int \sqrt{\cot(c+dx)}(a\cot(c+dx)+b)(A\cot(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4075}$$

$$\int \sqrt{\cot(c+dx)}(-aA+bB+(Ab+aB)\cot(c+dx))dx-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-aA+bB-(Ab+aB)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int \frac{-Ab-aB-(aA-bB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx-\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-Ab-aB-(bB-aA)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{4017}$$

$$\frac{2\int \frac{Ab+aB+(aA-bB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{1482}$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+d}{d}$$

$$\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(}{d}$$

$$\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \; 1082$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)\right.}{d}$$
$$\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \; 217$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1}{\mathrm{co}}\right.}{d}$$
$$\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \; 1479$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)-b(A+B))\left(-\right.\right.}{d}$$
$$\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)-b(A+B))\left(\int\right.\right.}{d}$$
$$\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)-b(A+B))\left(\int\right.\right.}{d}$$
$$\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

$$\Big\downarrow \; 1103$$

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\mathrm{lo}}{-}\right.\right.}{d}$$

$$\frac{2(aB+Ab)\sqrt{\cot(c+dx)}}{d}-\frac{2aA\cot^{\frac{3}{2}}(c+dx)}{3d}$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*(A*b + a*B)*Sqrt[Cot[c + d*x]])/d - (2*a*A*Cot[c + d*x]^(3/2))/(3*d) +
 (2*(((b*(A - B) + a*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sq
rt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 - ((a*(A - B)
- b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt
[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))
/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(153) = 306$.

Time = 0.47 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.02

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}}\tan(dx+c)\left(3A\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)\sqrt{2}\,\tan(dx+c)^{\frac{3}{2}}a+6A\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+\right.\right.}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}}\tan(dx+c)\left(3A\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)\sqrt{2}\,\tan(dx+c)^{\frac{3}{2}}a+6A\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+\right.\right.}{}$

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-1/12/d*(1/tan(d*x+c))^(5/2)*tan(d*x+c)*(3*A*ln(-(tan(d*x+c)+2^(1/2)*tan(d
*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)*tan(d*x+c)
^(3/2)*a+6*A*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)*a
+6*A*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)*b+6*A*arc
tan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)*a+6*A*arctan(-1+
2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)*b+3*A*ln(-(2^(1/2)*tan(
d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(1/2
)*tan(d*x+c)^(3/2)*b-3*B*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1
/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)*tan(d*x+c)^(3/2)*b+6*B*arctan(
1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)*a-6*B*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)*b+6*B*arctan(-1+2^(1/2)*tan(d
*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(3/2)*a-6*B*arctan(-1+2^(1/2)*tan(d*x+c)^(
1/2))*2^(1/2)*tan(d*x+c)^(3/2)*b+3*B*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x
+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(1/2)*tan(d*x+c)^(3/2)*a
+24*A*tan(d*x+c)*b+24*B*tan(d*x+c)*a+8*A*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(153) = 306$.

Time = 0.10 (sec) , antiderivative size = 936, normalized size of antiderivative = 5.29

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/6*(6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^
2 - 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a - (A + B)*b)*d*s
qrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2
)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B
^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*
(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2)*
a^2 + (A^2 - B^2)*b^2))*tan(d*x + c) + 6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B +
B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqr
t(1/2)*((A - B)*a - (A + B)*b)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 -
B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A
^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*
sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^
2)/d^2))/(4*A*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2))*tan(d*x + c) + 3
*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*
A*B + B^2)*b^2)/d^2)*log(2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(
A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A - B
)*a + (A + B)*b - ((A - B)*a - (A + B)*b)*tan(d*x + c))*tan(d*x + c) - 3*s
qrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*
B + B^2)*b^2)/d^2)*log(-2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A
^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A -...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{6\sqrt{2}((A+B)a+(A-B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}((A+B)a+(A-B)b)\arctan\left(\right.}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/12*(6*sqrt(2)*((A + B)*a + (A - B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sq
rt(tan(d*x + c)))) + 6*sqrt(2)*((A + B)*a + (A - B)*b)*arctan(-1/2*sqrt(2)
*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 3*sqrt(2)*((A - B)*a - (A + B)*b)*log
(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 3*sqrt(2)*((A - B)*a -
 (A + B)*b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*A*a/
tan(d*x + c)^(3/2) - 24*(B*a + A*b)/sqrt(tan(d*x + c)))/d
```

Giac [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)\cot(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)*cot(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)), x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$=\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)b^2$$
$$+2\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)ab$$
$$+\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\,dx\right)a^2$$

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**2,x)*b**2 + 2*int(sqr
t(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x),x)*a*b + int(sqrt(cot(c + d*x
))*cot(c + d*x)**2,x)*a**2
```

3.579 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 153

$$\begin{aligned}\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx &= -\frac{(a(A-B)-b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\ &\quad+\frac{(a(A-B)-b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\ &\quad+\frac{(b(A-B)+a(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d}-\frac{2aA\sqrt{\cot(c+dx)}}{d}\end{aligned}$$

output
```
1/2*(a*(A-B)-b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a
*(A-B)-b*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*(b*(A-B)+
a*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/d-2*a*A*
cot(d*x+c)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.17

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\left(2\sqrt{2}(a(A-B)-b(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c\right.\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*(2*Sqrt[2]*(a*(A - B) - b*(A + B))*(ArcTan[1 - Sqrt[2]
*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) - Sqrt[2]*(
b*(A - B) + a*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]
 - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) - (8*a*A)/Sqrt[Tan[
c + d*x]])*Sqrt[Tan[c + d*x]])/(4*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.19, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {3042, 4064, 3042, 4075, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}(a+b\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+b)(A\cot(c+dx)+B)}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$\big\downarrow$ 4075

$$\int \frac{-aA+bB+(Ab+aB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx-\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

$\big\downarrow$ 3042

$$\int \frac{-aA+bB-(Ab+aB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

$\big\downarrow$ 4017

$$\frac{2\int \frac{aA-bB-(Ab+aB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

$\big\downarrow$ 1482

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(a(A-B)-b(A+B))\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+d}{d}$$
$$\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

$\big\downarrow$ 1476

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx}\right.\right.}{d}$$
$$\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

$\big\downarrow$ 1082

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(\right.}{\sqrt{2}}\right.\right.}{d}$$
$$\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

$\big\downarrow$ 217

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{d}$$

$$\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

$\downarrow$ 1479

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\right.}{d}$$

$$\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

$\downarrow$ 25

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}(a(\right.}{d}$$

$$\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)\right.}{d}$$

$$\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\text{lo}}{}\right.\right.}{d}$$

$$\frac{2aA\sqrt{\cot(c+dx)}}{d}$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a*A*Sqrt[Cot[c + d*x]])/d + (2*(((a*(A - B) - b*(A + B))*(-(ArcTan[1 -
 Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*
x]]]/Sqrt[2]))/2 + ((b*(A - B) + a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot
[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] +
Cot[c + d*x]]/(2*Sqrt[2])))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4075
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
*d*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*
x])^m*Simp[A*c - B*d + (B*c + A*d)*Tan[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(134) = 268$.

Time = 0.33 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.35

method	result
derivativedivides	$\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)\left(A\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\sqrt{2}\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)\sqrt{2}\sqrt{\tan(dx+c)}\,b-2A\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)}{}$
default	$\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)\left(A\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\sqrt{2}\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)\sqrt{2}\sqrt{\tan(dx+c)}\,b-2A\arctan\left(1+\sqrt{2}\sqrt{\tan(dx+c)}\right)}{}$

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/4/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(A*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c
)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)*tan(d*x+c)^(1/
2)*b-2*A*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(1/2)*a+2*A
*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(1/2)*b-2*A*arctan(
-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(1/2)*a+2*A*arctan(-1+2^(1
/2)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(1/2)*b-A*ln(-(2^(1/2)*tan(d*x+c)
^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(1/2)*tan(
d*x+c)^(1/2)*a+B*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(
d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)*tan(d*x+c)^(1/2)*a+2*B*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(1/2)*a+2*B*arctan(1+2^(1/2)*tan(d*
x+c)^(1/2))*2^(1/2)*tan(d*x+c)^(1/2)*b+2*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1
/2))*2^(1/2)*tan(d*x+c)^(1/2)*a+2*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^
(1/2)*tan(d*x+c)^(1/2)*b+B*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(ta
n(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(1/2)*tan(d*x+c)^(1/2)*b-8*A*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(134) = 268$.

Time = 0.10 (sec) , antiderivative size = 884, normalized size of antiderivative = 5.78

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
1/2*(2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^
2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A + B)*a + (A - B)*b)*d*s
qrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2
)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B
^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*
(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2)*
a^2 + (A^2 - B^2)*b^2)) + 2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*
(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A +
B)*a + (A - B)*b)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A
^2 + 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B +
B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 -
2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A
*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2)) - sqrt(1/2)*d*sqrt(((A^2 + 2*
A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*log(2*s
qrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*
B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A + B)*a - (A - B)*b - ((A + B)*a
 + (A - B)*b)*tan(d*x + c)) + sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 +
2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*log(-2*sqrt(1/2)*d*sqrt(
((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^
2)*sqrt(tan(d*x + c)) - (A + B)*a - (A - B)*b - ((A + B)*a + (A - B)*b)...
```

Sympy [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})\cot^{\frac{3}{2}}{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))*cot(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.16

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \frac{2\sqrt{2}((A-B)a-(A+B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((A-B)a-(A+B)b)\arctan\left(\right.}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/4*(2*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqr
t(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(-1/2*sqrt(2)*
(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*((A + B)*a + (A - B)*b)*log(sq
rt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*((A + B)*a + (A -
 B)*b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*A*a/sqrt(
tan(d*x + c)))/d
```

Giac [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)\cot(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)*cot(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{3/2}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)), x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \frac{-2\sqrt{\cot(dx+c)}\,a^2-\left(\int\frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a^2d+\left(\int\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^2\,dx\right)b^2d+2(}{d}$$

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
( - 2*sqrt(cot(c + d*x))*a**2 - int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a**
2*d + int(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)**2,x)*b**2*d + 2*in
t(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x),x)*a*b*d)/d
```

3.580 $\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 152

$$\begin{aligned}\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx &= \frac{(b(A-B)+a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\ &\quad - \frac{(b(A-B)+a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\ &\quad + \frac{(a(A-B)-b(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{2bB}{d\sqrt{\cot(c+dx)}}\end{aligned}$$

output
```
-1/2*(b*(A-B)+a*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(
b*(A-B)+a*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a*(A-B)
-b*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/d+2*b*B
/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.17

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\left(2\sqrt{2}(b(A-B)+a(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
-1/4*(Sqrt[Cot[c + d*x]]*(2*Sqrt[2]*(b*(A - B) + a*(A + B))*(ArcTan[1 - Sq
rt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + Sqrt
[2]*(a*(A - B) - b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) - 8*b*B*Sqrt[T
an[c + d*x]])*Sqrt[Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.20, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4064, 3042, 4074, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+b)(A\cot(c+dx)+B)}{\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4074}$$

$$\int \frac{Ab+aB+(aA-bB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ab+aB-(aA-bB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4017}$$

$$\frac{2\int -\frac{Ab+aB+(aA-bB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2bB}{d\sqrt{\cot(c+dx)}}-\frac{2\int \frac{Ab+aB+(aA-bB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}$$

$$\Big\downarrow \text{1482}$$

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A+B)+b(A-B))\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+d}{d}$$

$$\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx}}\right.\right.}{d}$$

$$\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(\right.}{\sqrt{2}}\right.\right.}{d}$$

$$\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

$\downarrow$ 217

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{d}$$

$$\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

$\downarrow$ 1479

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\right.}{d}$$

$$\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

$\downarrow$ 25

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(a(\right.}{d}$$

$$\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)\right.}{d}$$

$$\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(a(A+B)+b}{d}$$

$$\frac{2bB}{d\sqrt{\cot(c+dx)}}$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b*B)/(d*Sqrt[Cot[c + d*x]]) + (2*(-1/2*((b*(A - B) + a*(A + B))*(-(ArcT
an[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[
c + d*x]]]/Sqrt[2])) + ((a*(A - B) - b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt
[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]
] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(133) = 266$.

Time = 0.36 (sec) , antiderivative size = 426, normalized size of antiderivative = 2.80

method	result
derivativedivides	$\sqrt{\frac{1}{\tan(dx+c)}}\,\sqrt{\tan(dx+c)}\left(A\sqrt{2}\,\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)a+2A\sqrt{2}\,\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)a+2A\sqrt{2}\right.$
default	$\sqrt{\frac{1}{\tan(dx+c)}}\,\sqrt{\tan(dx+c)}\left(A\sqrt{2}\,\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)a+2A\sqrt{2}\,\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)a+2A\sqrt{2}\right.$

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/4/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)^(1/2)*(A*2^(1/2)*ln(-(tan(d*x+c)+2^(
1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*a+2*A*2^
(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*a+2*A*2^(1/2)*arctan(1+2^(1/2)*ta
n(d*x+c)^(1/2))*b+2*A*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a+2*A*2^
(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*b+A*2^(1/2)*ln(-(2^(1/2)*tan(d*x
+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*b-B*2^(1/
2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-t
an(d*x+c)-1))*b+2*B*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*a-2*B*2^(1/
2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*b+2*B*2^(1/2)*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2))*a-2*B*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*b+B*2^(1/2
)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+
c)^(1/2)+1))*a+8*B*b*tan(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(133) = 266$.

Time = 0.10 (sec) , antiderivative size = 886, normalized size of antiderivative = 5.83

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
-1/2*(2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A
^2 - 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a - (A + B)*b)*d*
sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^
2)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 -
B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2
*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2)
*a^2 + (A^2 - B^2)*b^2)) + 2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2
*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A -
 B)*a - (A + B)*b)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (
A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B +
 B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 -
 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*
A*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2)) + sqrt(1/2)*d*sqrt(((A^2 - 2
*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*log(2*
sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A
*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A - B)*a + (A + B)*b - ((A - B)*
a - (A + B)*b)*tan(d*x + c)) - sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 -
 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*log(-2*sqrt(1/2)*d*sqrt
(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d
^2)*sqrt(tan(d*x + c)) - (A - B)*a + (A + B)*b - ((A - B)*a - (A + B)*b...
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})\,\sqrt{\cot{(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))*sqrt(cot(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.17

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx =$$
$$-\frac{2\sqrt{2}((A+B)a+(A-B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((A+B)a+(A-B)b)\arctan$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/4*(2*sqrt(2)*((A + B)*a + (A - B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sq
rt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a + (A - B)*b)*arctan(-1/2*sqrt(2)
*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((A - B)*a - (A + B)*b)*log(s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((A - B)*a - (A
+ B)*b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*B*b*sqrt
(tan(d*x + c)))/d
```

Giac [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)*sqrt(cot(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \int \sqrt{\cot(c+dx)}\,(A+B\tan(c+dx))\,(a+b\tan(c+dx))\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)),x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x)), x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))(A+B\tan(c+dx))\,dx = \left(\int \sqrt{\cot(dx+c)}dx\right)a^2 + \left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)^2\,dx\right)b^2 + 2\left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)\,dx\right)ab$$

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x)),x)*a**2 + int(sqrt(cot(c + d*x))*tan(c + d*x)**2,x)
*b**2 + 2*int(sqrt(cot(c + d*x))*tan(c + d*x),x)*a*b
```

3.581 $\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 178

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{(a(A-B)-b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{(a(A-B)-b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{(b(A-B)+a(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+aB)}{d\sqrt{\cot(c+dx)}}$$

output

```
-1/2*(a*(A-B)-b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(
a*(A-B)-b*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(b*(A-B)
+a*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/d+2/3*b
*B/d/cot(d*x+c)^(3/2)+2*(A*b+B*a)/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(6\sqrt{2}(a(A-B)-b(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\right.\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x
]
```

output
```
-1/12*(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(6*Sqrt[2]*(a*(A - B) - b*(A
+ B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Ta
n[c + d*x]]]) - 3*Sqrt[2]*(b*(A - B) + a*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]) - 24*(A*b + a*B)*Sqrt[Tan[c + d*x]] - 8*b*B*Tan[c + d*x]^(3/2)))/d
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {3042, 4064, 3042, 4074, 3042, 4012, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+b)(A\cot(c+dx)+B)}{\cot^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4074}$$

$$\int \frac{Ab+aB+(aA-bB)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ab+aB-(aA-bB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4012}$$

$$\int \frac{aA-bB-(Ab+aB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aA-bB-(-Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4017}$$

$$\frac{2\int -\frac{aA-bB-(Ab+aB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{aA-bB-(Ab+aB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1482}$$

$$\frac{2\left(-\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\left(-\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+ \right.\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2\left(-\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{c\right.}}{\sqrt{2}}\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2\left(-\frac{1}{2}(a(A+B)+b(A-B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{2\left(-\frac{1}{2}(a(A+B)+b(A-B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(a\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(-\frac{1}{2}(a(A+B)+b(A-B))\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx}\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2\left(-\frac{1}{2}(a(A-B)-b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A+B)+b(A-B))\left(\right.\right.}{d}$$

$$\frac{2(aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2bB}{3d\cot^{\frac{3}{2}}(c+dx)}$$

input
```
Int[((a + b*Tan[c + d*x])*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

output
```
(2*b*B)/(3*d*Cot[c + d*x]^(3/2)) + (2*(A*b + a*B))/(d*Sqrt[Cot[c + d*x]])
+ (2*(-1/2*((a*(A - B) - b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x
]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])) - ((b*(A -
 B) + a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/
Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))
/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4074
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b
*c - a*d)*(A*b - a*B)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2
))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*A*c
+ b*B*c + A*b*d - a*B*d - (A*b*c - a*B*c - a*A*d - b*B*d)*Tan[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && LtQ[m
, -1] && NeQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.25

method	result
derivativedivides	$-\frac{-\frac{2(Ab+Ba)}{\sqrt{\cot(dx+c)}}-\frac{2Bb}{3\cot(dx+c)^{\frac{3}{2}}}+\frac{(Aa-Bb)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-\right.\right.}{4}}{}$
default	$-\frac{-\frac{2(Ab+Ba)}{\sqrt{\cot(dx+c)}}-\frac{2Bb}{3\cot(dx+c)^{\frac{3}{2}}}+\frac{(Aa-Bb)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-\right.\right.}{4}}{}$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/d*(-2*(A*b+B*a)/cot(d*x+c)^(1/2)-2/3*B*b/cot(d*x+c)^(3/2)+1/4*(A*a-B*b)
*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*c
ot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/
2)*cot(d*x+c)^(1/2)))+1/4*(-A*b-B*a)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d
*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)
*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(154) = 308$.

Time = 0.10 (sec) , antiderivative size = 910, normalized size of antiderivative = 5.11

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm=
"fricas")
```

output
```
-1/6*(6*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A
^2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A + B)*a + (A - B)*b)*d*
sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^
2)/d^2)*sqrt(tan(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 -
B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2
*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*A*B*a*b - (A^2 - B^2)
*a^2 + (A^2 - B^2)*b^2)) + 6*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2
*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2)*arctan((2*sqrt(1/2)*((A +
 B)*a + (A - B)*b)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (
A^2 + 2*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B +
 B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*sqrt(((A^2 -
 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/d^2))/(4*
A*B*a*b - (A^2 - B^2)*a^2 + (A^2 - B^2)*b^2)) - 3*sqrt(1/2)*d*sqrt(((A^2 +
 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*log(
2*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2
*A*B + B^2)*b^2)/d^2)*sqrt(tan(d*x + c)) - (A + B)*a - (A - B)*b - ((A + B
)*a + (A - B)*b)*tan(d*x + c)) + 3*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a
^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/d^2)*log(-2*sqrt(1/2)*d*
sqrt(((A^2 + 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^
2)/d^2)*sqrt(tan(d*x + c)) - (A + B)*a - (A - B)*b - ((A + B)*a + (A - ...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})}{\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))/sqrt(cot(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx =$$
$$-\frac{6\sqrt{2}((A-B)a-(A+B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}((A-B)a-(A+B)b)\arctan$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/12*(6*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/s
qrt(tan(d*x + c)))) + 6*sqrt(2)*((A - B)*a - (A + B)*b)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 3*sqrt(2)*((A + B)*a + (A - B)*b)*lo
g(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 3*sqrt(2)*((A + B)*a
+ (A - B)*b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*(B*
b + 3*(B*a + A*b)/tan(d*x + c))*tan(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)}{\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)/sqrt(cot(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))}{\sqrt{\cot(c+d\,x)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x)))/cot(c + d*x)^(1/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x)))/cot(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a^2 + \left(\int \frac{\sqrt{\cot(dx+c)}\tan(dx+c)^2}{\cot(dx+c)}dx\right)b^2 + 2\left(\int \frac{\sqrt{\cot(dx+c)}\tan(dx+c)}{\cot(dx+c)}dx\right)ab$$

input
```
int((a+b*tan(d*x+c))*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a**2 + int((sqrt(cot(c + d*x))*tan(
c + d*x)**2)/cot(c + d*x),x)*b**2 + 2*int((sqrt(cot(c + d*x))*tan(c + d*x)
)/cot(c + d*x),x)*a*b
```

3.582 $\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 263

$$\begin{aligned}
&\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx \\
&= \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad -\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad -\frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad +\frac{2(a^2A-Ab^2-2abB)\sqrt{\cot(c+dx)}}{d}-\frac{2a(7Ab+5aB)\cot^{\frac{3}{2}}(c+dx)}{15d} \\
&\quad -\frac{2aA\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))}{5d}
\end{aligned}$$

output
```
-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))
*2^(1/2)/d-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(1+2^(1/2)*cot(d*x+
c)^(1/2))*2^(1/2)/d-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctanh(2^(1/2)*
cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/d+2*(A*a^2-A*b^2-2*B*a*b)*cot(d*x
+c)^(1/2)/d-2/15*a*(7*A*b+5*B*a)*cot(d*x+c)^(3/2)/d-2/5*a*A*cot(d*x+c)^(3/
2)*(b+a*cot(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.97

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{\sqrt{\cot(c+dx)}\Big(30\sqrt{2}(a^2(A-B)+b^2(-A+B)-2ab(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\mathrm{a}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x
]
```

output
```
-1/60*(Sqrt[Cot[c + d*x]]*(30*Sqrt[2]*(a^2*(A - B) + b^2*(-A + B) - 2*a*b*
(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]]) - 15*Sqrt[2]*(2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*
(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]] + Tan[c + d*x]]) + (24*a^2*A)/Tan[c + d*x]^(5/2) + (40*a*(2
*A*b + a*B))/Tan[c + d*x]^(3/2) - (120*(a^2*A - A*b^2 - 2*a*b*B))/Sqrt[Tan
[c + d*x]])*Sqrt[Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.08, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 4064, 3042, 4090, 27, 3042, 4113, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{7/2}(a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2(A\cot(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4090}$$

$$-\frac{2}{5}\int \frac{1}{2}\sqrt{\cot(c+dx)}\left(-a(7Ab+5aB)\cot^2(c+dx)+5\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)+b(3aA-5bB)\right)dx-\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}\int \sqrt{\cot(c+dx)}\left(-a(7Ab+5aB)\cot^2(c+dx)+5\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)+b(3aA-5bB)\right)dx-\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(-a(7Ab+5aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-5\left(Aa^2-2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)+\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{4113}$$

$$\frac{1}{5}\left(-\int \sqrt{\cot(c+dx)}\left(5\left(Ba^2+2Aba-b^2B\right)+5\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)\right)dx-\frac{2a(5aB+7Ab)\cot^{\frac{3}{2}}(c\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(-\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(5\left(Ba^2+2Aba-b^2B\right)-5\left(Aa^2-2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx-\frac{2a(5aI\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\big\downarrow$ 4011

$$\frac{1}{5}\left(-\int \frac{5\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)}{\sqrt{\cot(c+dx)}}dx+\frac{10\left(a^2A-2abB-Ab^2\right)\sqrt{\cot(c+dx)}}{d}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\big\downarrow$ 3042

$$\frac{1}{5}\left(-\int \frac{-5\left(Aa^2-2bBa-Ab^2\right)-5\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{10\left(a^2A-2abB-Ab^2\right)\sqrt{\cot(}}{d}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\big\downarrow$ 4017

$$\frac{1}{5}\left(-\frac{2\int \frac{5\left(Aa^2-2bBa-Ab^2-\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{10\left(a^2A-2abB-Ab^2\right)\sqrt{\cot(c+dx)}}{d}-\frac{2a}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\big\downarrow$ 27

$$\frac{1}{5}\left(-\frac{10\int \frac{Aa^2-2bBa-Ab^2-\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{10\left(a^2A-2abB-Ab^2\right)\sqrt{\cot(c+dx)}}{d}-\frac{2a(}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\big\downarrow$ 1482

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^2(A-B)-2ab(A+B)\right.\right.}{d}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\Big\downarrow$ 1476

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^2(A-B)-2ab(A+B)\right.\right.}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\Big\downarrow$ 1082

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^2(A-B)-2ab(A+B)\right.\right.}{d}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\Big\downarrow$ 217

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^2(A-B)-2ab(A+B)\right.\right.}{d}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\Big\downarrow$ 1479

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+\right.}}{\cot(c+dx)+\sqrt{2}\sqrt{\cot}}}{2}\right.\right.}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\Big\downarrow$ 25

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}}}{2\sqrt{2}}\right.\right.}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\downarrow$ 27

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(}}\right.\right.}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

$\downarrow$ 1103

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(a^2\right.\right.}{d}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}{5d}$$

input
```
Int[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a*A*Cot[c + d*x]^(3/2)*(b + a*Cot[c + d*x]))/(5*d) + ((10*(a^2*A - A*b
^2 - 2*a*b*B)*Sqrt[Cot[c + d*x]])/d - (2*a*(7*A*b + 5*a*B)*Cot[c + d*x]^(3
/2))/(3*d) - (10*(((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*(-(ArcTan[1
 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c +
d*x]]]/Sqrt[2]))/2 + ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(-1/2*Lo
g[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.11

method	result
derivativedivides	$-\frac{\frac{2A\,a^{2}\cot(dx+c)^{\frac{5}{2}}}{5}+\frac{4Aab\cot(dx+c)^{\frac{3}{2}}}{3}+\frac{2B\,a^{2}\cot(dx+c)^{\frac{3}{2}}}{3}-2A\,a^{2}\sqrt{\cot(dx+c)}+2A\,b^{2}\sqrt{\cot(dx+c)}+4Bab\sqrt{\cot(dx+c)}$
default	$-\frac{\frac{2A\,a^{2}\cot(dx+c)^{\frac{5}{2}}}{5}+\frac{4Aab\cot(dx+c)^{\frac{3}{2}}}{3}+\frac{2B\,a^{2}\cot(dx+c)^{\frac{3}{2}}}{3}-2A\,a^{2}\sqrt{\cot(dx+c)}+2A\,b^{2}\sqrt{\cot(dx+c)}+4Bab\sqrt{\cot(dx+c)}$

input
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/d*(2/5*A*a^2*cot(d*x+c)^(5/2)+4/3*A*a*b*cot(d*x+c)^(3/2)+2/3*B*a^2*cot(
d*x+c)^(3/2)-2*A*a^2*cot(d*x+c)^(1/2)+2*A*b^2*cot(d*x+c)^(1/2)+4*B*a*b*cot
(d*x+c)^(1/2)+1/4*(A*a^2-A*b^2-2*B*a*b)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*co
t(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1
/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/4*(-2*A*a*b
-B*a^2+B*b^2)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x
+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*ar
ctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. $2(235) = 470$.

Time = 0.11 (sec) , antiderivative size = 1534, normalized size of antiderivative = 5.83

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/30*(30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b +
 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)
*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2
)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B +
 B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2)*sqrt(t
an(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2
*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b
^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*
A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))/
(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A^
2 - B^2)*b^4))*tan(d*x + c)^2 + 30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a
^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a
*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2
*(A - B)*a*b - (A + B)*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2
)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A
*B + B^2)*b^4)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4
 + 4*(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b
^3 + (A^2 + 2*A*B + B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2
- B^2)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2
- 2*A*B + B^2)*b^4)/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 ...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.06

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$

$$-\frac{30\sqrt{2}((A-B)a^2-2(A+B)ab-(A-B)b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+30\sqrt{2}((A-B)a^2}{}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/60*(30*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*s
qrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 30*sqrt(2)*((A - B)*a^2 - 2*(A
+ B)*a*b - (A - B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)
))) + 15*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)/s
qrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 15*sqrt(2)*((A + B)*a^2 + 2*(A -
 B)*a*b - (A + B)*b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
1) + 24*A*a^2/tan(d*x + c)^(5/2) - 120*(A*a^2 - 2*B*a*b - A*b^2)/sqrt(tan(
d*x + c)) + 40*(B*a^2 + 2*A*a*b)/tan(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^2\cot(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^2*cot(d*x + c)^(7/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^2\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2, x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{-2\sqrt{\cot\left(dx+c\right)}\,\cot\left(dx+c\right)^2a^3+10\sqrt{\cot\left(dx+c\right)}\,a^3+5\left(\int\frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a^3d+5\left(\int\sqrt{\cot\left(dx+c\right)}\right.}{}$$

input
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
( - 2*sqrt(cot(c + d*x))*cot(c + d*x)**2*a**3 + 10*sqrt(cot(c + d*x))*a**3
 + 5*int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a**3*d + 5*int(sqrt(cot(c + d*
x))*cot(c + d*x)**3*tan(c + d*x)**3,x)*b**3*d + 15*int(sqrt(cot(c + d*x))*
cot(c + d*x)**3*tan(c + d*x)**2,x)*a*b**2*d + 15*int(sqrt(cot(c + d*x))*co
t(c + d*x)**3*tan(c + d*x),x)*a**2*b*d)/(5*d)
```

3.583 $\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 229

$$\begin{aligned}\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx\\ &= -\frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}\\ &+\frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}\\ &-\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d}\\ &-\frac{2a(5Ab+3aB)\sqrt{\cot(c+dx)}}{3d}-\frac{2aA\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}{3d}\end{aligned}$$

output
```
1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*
2^(1/2)/d+1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctan(1+2^(1/2)*cot(d*x+c
)^(1/2))*2^(1/2)/d-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctanh(2^(1/2)*c
ot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/d-2/3*a*(5*A*b+3*B*a)*cot(d*x+c)^(
1/2)/d-2/3*a*A*cot(d*x+c)^(1/2)*(b+a*cot(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.99

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\left(6\sqrt{2}(2ab(A-B)+a^2(A+B)-b^2(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x
]
```

output
```
(Sqrt[Cot[c + d*x]]*(6*Sqrt[2]*(2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))
*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]]]) + 3*Sqrt[2]*(a^2*(A - B) + b^2*(-A + B) - 2*a*b*(A + B))*(Log[1 -
 Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]] + Tan[c + d*x]]) - (8*a^2*A)/Tan[c + d*x]^(3/2) - (24*a*(2*A*b + a*
B))/Sqrt[Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/(12*d)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.09, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4064, 3042, 4090, 27, 3042, 4113, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cot(c+dx)^{5/2}(a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4064$$

$$\int \frac{(a\cot(c+dx)+b)^2(A\cot(c+dx)+B)}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4090}$$

$$-\frac{2}{3}\int \frac{-a(5Ab+3aB)\cot^2(c+dx)+3\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)+b(aA-3bB)}{2\sqrt{\cot(c+dx)}}dx-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\int \frac{-a(5Ab+3aB)\cot^2(c+dx)+3\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)+b(aA-3bB)}{\sqrt{\cot(c+dx)}}dx-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int \frac{-a(5Ab+3aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-3\left(Aa^2-2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b(aA-3bB)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{4113}$$

$$\frac{1}{3}\left(-\int \frac{3\left(Ba^2+2Aba-b^2B\right)+3\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx-\frac{2a(3aB+5Ab)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-\int \frac{3\left(Ba^2+2Aba-b^2B\right)-3\left(Aa^2-2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a(3aB+5Ab)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{4017}$$

$$\frac{1}{3}\left(-\frac{2\int -\frac{3\left(Ba^2+2Aba-b^2B+\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{2a(3aB+5Ab)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{6\int \frac{Ba^2+2Aba-b^2B+\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{2a(3aB+5Ab)\sqrt{\cot(c+dx)}}{d}\right)-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{1482}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b\right.\right.}{d}\right.$$

$$-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{}{\cot(c+dx)+\sqrt{}}\right.\right.}{d}\right.$$

$$-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx}\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$$-\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(a^2(A\right.\right.}{d}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$\Big\downarrow$ 1479

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(a^2(A\right.\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$\Big\downarrow$ 25

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(a^2(A\right.\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(a^2(A\right.\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

$\Big\downarrow$ 1103

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(a^2(A\right.\right.}{d}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}{3d}$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a*A*Sqrt[Cot[c + d*x]]*(b + a*Cot[c + d*x]))/(3*d) + ((-2*a*(5*A*b + 3
*a*B)*Sqrt[Cot[c + d*x]])/d + (6*(((2*a*b*(A - B) + a^2*(A + B) - b^2*(A +
 B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[
2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 - ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(
A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] +
 Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{\frac{2Aa^2\cot(dx+c)^{\frac{3}{2}}}{3}+4abA\sqrt{\cot(dx+c)}+2Ba^2\sqrt{\cot(dx+c)}+\frac{\left(-2abA-Ba^2+Bb^2\right)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right.\right.}{}}{}$
default	$-\frac{\frac{2Aa^2\cot(dx+c)^{\frac{3}{2}}}{3}+4abA\sqrt{\cot(dx+c)}+2Ba^2\sqrt{\cot(dx+c)}+\frac{\left(-2abA-Ba^2+Bb^2\right)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right.\right.}{}}{}$

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/d*(2/3*A*a^2*cot(d*x+c)^(3/2)+4*a*b*A*cot(d*x+c)^(1/2)+2*B*a^2*cot(d*x+
c)^(1/2)+1/4*(-2*A*a*b-B*a^2+B*b^2)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*
x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*
cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/4*(-A*a^2+A*b^2
+2*B*a*b)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+
2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan
(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(203) = 406$.

Time = 0.11 (sec) , antiderivative size = 1506, normalized size of antiderivative = 6.58

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/6*(6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2
*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b
^4)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*d
*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B^
2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4)/d^2)*sqrt(tan(
d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(A
^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4)
/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B
 + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))/(8*
A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A^2 -
 B^2)*b^4))*tan(d*x + c) + 6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4
*(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 +
 (A^2 + 2*A*B + B^2)*b^4)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a^2 - 2*(A + B
)*a*b - (A - B)*b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b
 + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^
2)*b^4)/d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A
^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A
^2 + 2*A*B + B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*
a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B
 + B^2)*b^4)/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.10

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \frac{6\sqrt{2}((A+B)a^2+2(A-B)ab-(A+B)b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}((A+B)a^2+2}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/12*(6*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2*sqr
t(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 6*sqrt(2)*((A + B)*a^2 + 2*(A - B
)*a*b - (A + B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))))
 - 3*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*log(sqrt(2)/sqrt(
tan(d*x + c)) + 1/tan(d*x + c) + 1) + 3*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a
*b - (A - B)*b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
8*A*a^2/tan(d*x + c)^(3/2) - 24*(B*a^2 + 2*A*a*b)/sqrt(tan(d*x + c)))/d
```

Giac [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^2\cot(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^2*cot(d*x + c)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+dx)^{5/2}\,(A+B\tan(c+dx))\,(a+b\tan(c+dx))^2\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2, x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^3\,dx\right)b^3$$
$$+3\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)^2\,dx\right)a\,b^2$$
$$+3\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\tan(dx+c)\,dx\right)a^2b$$
$$+\left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\,dx\right)a^3$$

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**3,x)*b**3 + 3*int(sqr
t(cot(c + d*x))*cot(c + d*x)**2*tan(c + d*x)**2,x)*a*b**2 + 3*int(sqrt(cot
(c + d*x))*cot(c + d*x)**2*tan(c + d*x),x)*a**2*b + int(sqrt(cot(c + d*x))
*cot(c + d*x)**2,x)*a**3
```

3.584 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 212

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = -\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{2b^2B}{d\sqrt{\cot(c+dx)}} - \frac{2a^2A\sqrt{\cot(c+dx)}}{d}$$

output
```
1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*
2^(1/2)/d+1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(1+2^(1/2)*cot(d*x+c
)^(1/2))*2^(1/2)/d+1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctanh(2^(1/2)*c
ot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/d+2*b^2*B/d/cot(d*x+c)^(1/2)-2*a^2
*A*cot(d*x+c)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.04

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{\sqrt{\cot(c+dx)}\left(2\sqrt{2}(a^2(A-B)+b^2(-A+B)-2ab(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\text{arcta}\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x
]
```

output
```
(Sqrt[Cot[c + d*x]]*(2*Sqrt[2]*(a^2*(A - B) + b^2*(-A + B) - 2*a*b*(A + B)
)*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c
+ d*x]]]) - Sqrt[2]*(2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(Log[1 - S
qrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]]) - (8*a^2*A)/Sqrt[Tan[c + d*x]] + 8*b^2*B*Sqrt[Tan[c
+ d*x]])*Sqrt[Tan[c + d*x]])/(4*d)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 4064, 3042, 4087, 25, 3042, 4113, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cot(c+dx)^{3/2}(a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4064}$$

$$\int \frac{(a\cot(c+dx)+b)^2(A\cot(c+dx)+B)}{\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4087}$$

$$\frac{2b^2B}{d\sqrt{\cot(c+dx)}}-\int -\frac{a^2A\cot^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(Ab+2aB)}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{a^2A\cot^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(Ab+2aB)}{\sqrt{\cot(c+dx)}}dx+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2A\tan\left(c+dx+\frac{\pi}{2}\right)^2-\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b(Ab+2aB)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4113}$$

$$\int \frac{-Aa^2+2bBa+Ab^2+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx-\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{-Aa^2+2bBa+Ab^2-\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4017}$$

$$\frac{2\int \frac{Aa^2-2bBa-Ab^2-\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \ 1482$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A\right.\right.}{d}$$

$$\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \ 1476$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A\right.\right.}{d}$$

$$\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \ 1082$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A\right.\right.}{d}$$

$$\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \ 217$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A\right.\right.}{d}$$

$$\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \ 1479$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}}{2\sqrt{2}}\right.\right.}{}$$

$$\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{co}}{2\sqrt{2}}\right.\right.}{d}$$
$$\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$\Big\downarrow$ 27

$$\frac{2\left(\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+}\right.\right.}{}$$
$$\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

$\Big\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(a^2(A+B\right.\right.}{d}$$
$$\frac{2a^2A\sqrt{\cot(c+dx)}}{d}+\frac{2b^2B}{d\sqrt{\cot(c+dx)}}$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b^2*B)/(d*Sqrt[Cot[c + d*x]]) - (2*a^2*A*Sqrt[Cot[c + d*x]])/d + (2*(((
a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[
c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 +
 ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[C
ot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
+ Cot[c + d*x]]/(2*Sqrt[2])))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.13

method	result
derivativedivides	$-\frac{2Aa^2\sqrt{\cot(dx+c)}+\frac{(-Aa^2+Ab^2+2Bab)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan(}{4}}{}$
default	$-\frac{2Aa^2\sqrt{\cot(dx+c)}+\frac{(-Aa^2+Ab^2+2Bab)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan(}{4}}{}$

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/d*(2*A*a^2*cot(d*x+c)^(1/2)+1/4*(-A*a^2+A*b^2+2*B*a*b)*2^(1/2)*(ln((cot
(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1
))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/
2)))+1/4*(2*A*a*b+B*a^2-B*b^2)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^
(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d
*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))-2*B*b^2/cot(d*x+c)^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1458 vs. 2(191) = 382.

Time = 0.11 (sec) , antiderivative size = 1458, normalized size of antiderivative = 6.88

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/2*(2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b +
2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*
b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)
*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B +
B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2)*sqrt(ta
n(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*
(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^
4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A
*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))/(
8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A^2
 - B^2)*b^4)) + 2*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2
)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A
*B + B^2)*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*b - (A
 + B)*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2
+ 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^
2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*
a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B
 + B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*
(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^
4)/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a...
```

Sympy [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\int (A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^2\cot^{\frac{3}{2}}{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2*cot(c + d*x)**(3/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.15

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \frac{8\,Bb^2\sqrt{\tan\left(dx+c\right)}+2\,\sqrt{2}((A-B)a^2-2\,(A+B)ab-(A-B)b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/4*(8*B*b^2*sqrt(tan(d*x + c)) + 2*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b -
 (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqr
t(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt
(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A +
 B)*b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*((
A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(-sqrt(2)/sqrt(tan(d*x + c))
+ 1/tan(d*x + c) + 1) - 8*A*a^2/sqrt(tan(d*x + c)))/d
```

Giac [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan\left(dx+c\right)+A)(b\tan\left(dx+c\right)+a)^2\cot\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^2*cot(d*x + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^2\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2, x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$=\frac{-2\sqrt{\cot(dx+c)}\,a^3-\left(\int\frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a^3d+\left(\int\sqrt{\cot(dx+c)}\,\cot(dx+c)\tan(dx+c)^3\,dx\right)b^3d+3(}{}$$

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
( - 2*sqrt(cot(c + d*x))*a**3 - int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a**
3*d + int(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)**3,x)*b**3*d + 3*in
t(sqrt(cot(c + d*x))*cot(c + d*x)*tan(c + d*x)**2,x)*a*b**2*d + 3*int(sqrt
(cot(c + d*x))*cot(c + d*x)*tan(c + d*x),x)*a**2*b*d)/d
```

3.585 $\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 217

$$\begin{aligned}\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx &= \frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\ &\quad - \frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\ &\quad + \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} \\ &\quad + \frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)}{d\sqrt{\cot(c+dx)}}\end{aligned}$$

output

```
-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))
*2^(1/2)/d-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctan(1+2^(1/2)*cot(d*x+
c)^(1/2))*2^(1/2)/d+1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctanh(2^(1/2)*
cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/d+2/3*b^2*B/d/cot(d*x+c)^(3/2)+2*
b*(A*b+2*B*a)/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.04

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(6\sqrt{2}(2ab(A-B)+a^2(A+B)-b^2(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c-\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x
]
```

output
```
-1/12*(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(6*Sqrt[2]*(2*a*b*(A - B) + a
^2*(A + B) - b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + 3*Sqrt[2]*(a^2*(A - B) + b^2*(-A + B)
- 2*a*b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) - 24*b*(A*b + 2*a*B)*Sqrt
[Tan[c + d*x]] - 8*b^2*B*Tan[c + d*x]^(3/2)))/d
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4064, 3042, 4087, 25, 3042, 4111, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+b)^2(A\cot(c+dx)+B)}{\cot^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4087}$$

$$\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}-\int -\frac{a^2A\cot^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(Ab+2aB)}{\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{a^2A\cot^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(Ab+2aB)}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2A\tan\left(c+dx+\frac{\pi}{2}\right)^2-\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b(Ab+2aB)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4111}$$

$$\int \frac{Ba^2+2Aba-b^2B+\left(a^2A-b(Ab+2aB)\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ba^2+2Aba-b^2B-\left(a^2A-b(Ab+2aB)\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4017}$$

$$\frac{2\int -\frac{Ba^2+2Aba-b^2B+\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{Ba^2+2Aba-b^2B+\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1482

$$\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A\right.}{d}$$
$$\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1476

$$\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A\right.}{d}$$
$$\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1082

$$\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A\right.}{d}$$
$$\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 217

$$\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A\right.}{d}$$
$$\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1479

$$2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}}{2\sqrt{2}}\right.\right.$$
$$\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 25

$$\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right.}{d}$$

$$\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+\right.}{}$$

$$\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}\right)-\right.}{d}$$

$$\frac{2b(2aB+Ab)}{d\sqrt{\cot(c+dx)}}+\frac{2b^2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b^2*B)/(3*d*Cot[c + d*x]^(3/2)) + (2*b*(A*b + 2*a*B))/(d*Sqrt[Cot[c + d
*x]]) + (2*(-1/2*((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(-(ArcTan[1
- Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d
*x]]]/Sqrt[2])) + ((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*(-1/2*Log[1
 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.12

method	result
derivativedivides	$-\frac{\frac{\left(2abA+B\,a^{2}-B\,b^{2}\right)\sqrt{2}\,\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{4}}{}$
default	$-\frac{\frac{\left(2abA+B\,a^{2}-B\,b^{2}\right)\sqrt{2}\,\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{4}}{}$

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/d*(1/4*(2*A*a*b+B*a^2-B*b^2)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)
^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(
d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/4*(A*a^2-A*b^2-2*B*
a*b)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/
2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2
^(1/2)*cot(d*x+c)^(1/2)))-2/3*B*b^2/cot(d*x+c)^(3/2)-2*b*(A*b+2*B*a)/cot(d
*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1482 vs. $2(194) = 388$.

Time = 0.11 (sec) , antiderivative size = 1482, normalized size of antiderivative = 6.83

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/6*(6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b +
2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*
b^4)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*
d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(A^2 - 6*A*B + B
^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4)/d^2)*sqrt(tan
(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(
A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4
)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*
B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))/(8
*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A^2
- B^2)*b^4)) + 6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)
*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*
B + B^2)*b^4)/d^2)*arctan((2*sqrt(1/2)*((A - B)*a^2 - 2*(A + B)*a*b - (A -
 B)*b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2*(A^2 -
6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b^4)/d^2)
*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^
3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B +
 B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A
^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)
/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2...
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan(c+dx))(a+b\tan(c+dx))^2\sqrt{\cot(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**2*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2*sqrt(cot(c + d*x)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.17

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx =$$
$$-\frac{6\sqrt{2}((A+B)a^2+2(A-B)ab-(A+B)b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+6\sqrt{2}((A+B)a^2+}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/12*(6*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 6*sqrt(2)*((A + B)*a^2 + 2*(A -
B)*a*b - (A + B)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))
) - 3*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*log(sqrt(2)/sqrt
(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 3*sqrt(2)*((A - B)*a^2 - 2*(A + B)*
a*b - (A - B)*b^2)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
 8*(B*b^2 + 3*(2*B*a*b + A*b^2)/tan(d*x + c))*tan(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^2\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^2*sqrt(cot(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \int \sqrt{\cot(c+dx)}\,(A+B\tan(c+dx))\,(a+b\tan(c+dx))^2\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2,x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2, x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2(A+B\tan(c+dx))\,dx$$
$$= \left(\int \sqrt{\cot(dx+c)}dx\right)a^3 + \left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)^3\,dx\right)b^3$$
$$+ 3\left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)^2\,dx\right)a\,b^2$$
$$+ 3\left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)\,dx\right)a^2b$$

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^2*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x)),x)*a**3 + int(sqrt(cot(c + d*x))*tan(c + d*x)**3,x)
*b**3 + 3*int(sqrt(cot(c + d*x))*tan(c + d*x)**2,x)*a*b**2 + 3*int(sqrt(co
t(c + d*x))*tan(c + d*x),x)*a**2*b
```

3.586 $\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 254

$$\begin{aligned}\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx &= \frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}\\ &-\frac{(a^2(A-B)-b^2(A-B)-2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d}\\ &-\frac{(2ab(A-B)+a^2(A+B)-b^2(A+B))\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d}\\ &+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}+\frac{2b(Ab+2aB)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2(2aAb+a^2B-b^2B)}{d\sqrt{\cot(c+dx)}}\end{aligned}$$

output
```
-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))
*2^(1/2)/d-1/2*(a^2*(A-B)-b^2*(A-B)-2*a*b*(A+B))*arctan(1+2^(1/2)*cot(d*x+
c)^(1/2))*2^(1/2)/d-1/2*(2*a*b*(A-B)+a^2*(A+B)-b^2*(A+B))*arctanh(2^(1/2)*
cot(d*x+c)^(1/2)/(1+cot(d*x+c)))*2^(1/2)/d+2/5*b^2*B/d/cot(d*x+c)^(5/2)+2/
3*b*(A*b+2*B*a)/d/cot(d*x+c)^(3/2)+2*(2*A*a*b+B*a^2-B*b^2)/d/cot(d*x+c)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(30\sqrt{2}(a^2(A-B)+b^2(-A+B)-2ab(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan\right.\right.\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]]
,x]
```

output
```
-1/60*(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(30*Sqrt[2]*(a^2*(A - B) + b^
2*(-A + B) - 2*a*b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcT
an[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) - 15*Sqrt[2]*(2*a*b*(A - B) + a^2*(A +
 B) - b^2*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - L
og[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) - 120*(2*a*A*b + a^2*B
- b^2*B)*Sqrt[Tan[c + d*x]] - 40*b*(A*b + 2*a*B)*Tan[c + d*x]^(3/2) - 24*b
^2*B*Tan[c + d*x]^(5/2)))/d
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.06, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 4064, 3042, 4087, 25, 3042, 4111, 3042, 4012, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+b)^2(A\cot(c+dx)+B)}{\cot^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{4087}$$

$$\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}-\int -\frac{a^2A\cot^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(Ab+2aB)}{\cot^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{a^2A\cot^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(Ab+2aB)}{\cot^{\frac{5}{2}}(c+dx)}dx+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a^2A\tan\left(c+dx+\frac{\pi}{2}\right)^2-\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b(Ab+2aB)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4111}$$

$$\int \frac{Ba^2+2Aba-b^2B+\left(a^2A-b(Ab+2aB)\right)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{Ba^2+2Aba-b^2B-\left(a^2A-b(Ab+2aB)\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4012}$$

$$\int \frac{Aa^2-b(Ab+2aB)-\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\int \frac{Aa^2-b(Ab+2aB)-\left(-Ba^2-2Aba+b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 4017$$

$$\frac{2\int -\frac{Aa^2-2bBa-Ab^2-\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 25$$

$$-\frac{2\int \frac{Aa^2-2bBa-Ab^2-\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 1482$$

$$\frac{2\left(-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(\right.\right.}{d}$$

$$\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 1476$$

$$\frac{2\left(-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(\right.\right.}{d}$$

$$\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow 1082$$

$$\frac{2\left(-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2\right.}{d}$$

$$\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 217

$$\frac{2\left(-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2\right.}{d}$$

$$\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 1479

$$\frac{2\left(-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx}}}{2\sqrt{2}}\right.\right.}{}$$

$$\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 25

$$\frac{2\left(-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{}}{2\sqrt{2}}\right.\right.}{d}$$

$$\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{2\left(-\frac{1}{2}\left(a^2(A+B)+2ab(A-B)-b^2(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}}\right.\right.}{}$$

$$\frac{2\left(a^2B+2aAb-b^2B\right)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\downarrow$ 1103

$$\frac{2\left(-\frac{1}{2}\left(a^2(A-B)-2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(a^2(A+\right.\right.}{d}$$

$$\frac{2(a^2B+2aAb-b^2B)}{d\sqrt{\cot(c+dx)}}+\frac{2b(2aB+Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{2b^2B}{5d\cot^{\frac{5}{2}}(c+dx)}$$

input
```
Int[((a + b*Tan[c + d*x])^2*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

output
```
(2*b^2*B)/(5*d*Cot[c + d*x]^(5/2)) + (2*b*(A*b + 2*a*B))/(3*d*Cot[c + d*x]
^(3/2)) + (2*(2*a*A*b + a^2*B - b^2*B))/(d*Sqrt[Cot[c + d*x]]) + (2*(-1/2*
((a^2*(A - B) - b^2*(A - B) - 2*a*b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Co
t[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])) -
 ((2*a*b*(A - B) + a^2*(A + B) - b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[C
ot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
+ Cot[c + d*x]]/(2*Sqrt[2])))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4087
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(-(B*c - A*d))*(b*c - a*d)^2*((c + d*Tan[e + f*x])^(n + 1)/(f*d^2*(n + 1)*(
c^2 + d^2))), x] + Simp[1/(d*(c^2 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1
)*Simp[B*(b*c - a*d)^2 + A*d*(a^2*c - b^2*c + 2*a*b*d) + d*(B*(a^2*c - b^2*
c + 2*a*b*d) + A*(2*a*b*c - a^2*d + b^2*d))*Tan[e + f*x] + b^2*B*(c^2 + d^2
)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b
*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[n, -1]
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{-\frac{2\left(2abA+B\,a^{2}-B\,b^{2}\right)}{\sqrt{\cot(dx+c)}}-\frac{2B\,b^{2}}{5\cot(dx+c)^{\frac{5}{2}}}-\frac{2b(Ab+2Ba)}{3\cot(dx+c)^{\frac{3}{2}}}+\frac{\left(A\,a^{2}-A\,b^{2}-2Bab\right)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\,\mathrm{arc}\cdots\right.}{4}}{\cdots}$
default	$-\frac{-\frac{2\left(2abA+B\,a^{2}-B\,b^{2}\right)}{\sqrt{\cot(dx+c)}}-\frac{2B\,b^{2}}{5\cot(dx+c)^{\frac{5}{2}}}-\frac{2b(Ab+2Ba)}{3\cot(dx+c)^{\frac{3}{2}}}+\frac{\left(A\,a^{2}-A\,b^{2}-2Bab\right)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\,\mathrm{arc}\cdots\right.}{4}}{\cdots}$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/d*(-2*(2*A*a*b+B*a^2-B*b^2)/cot(d*x+c)^(1/2)-2/5*B*b^2/cot(d*x+c)^(5/2)
-2/3*b*(A*b+2*B*a)/cot(d*x+c)^(3/2)+1/4*(A*a^2-A*b^2-2*B*a*b)*2^(1/2)*(ln(
(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/
2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)
^(1/2)))+1/4*(-2*A*a*b-B*a^2+B*b^2)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*
x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*
cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. $2(226) = 452$.

Time = 0.11 (sec) , antiderivative size = 1502, normalized size of antiderivative = 5.91

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
1/30*(30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b +
 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)
*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2
)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*A*B +
 B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2)*sqrt(t
an(d*x + c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2)*a^3*b + 2
*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A*B + B^2)*b
^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^2 + 6*
A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/d^2))/
(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)*a^2*b^2 - (A^
2 - B^2)*b^4)) + 30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B
^2)*a^3*b + 2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2
*A*B + B^2)*b^4)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^2 + 2*(A - B)*a*b -
(A + B)*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b + 2*(A^
2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*b^4)/
d^2)*sqrt(tan(d*x + c)) - d^2*sqrt(((A^2 + 2*A*B + B^2)*a^4 + 4*(A^2 - B^2
)*a^3*b + 2*(A^2 - 6*A*B + B^2)*a^2*b^2 - 4*(A^2 - B^2)*a*b^3 + (A^2 + 2*A
*B + B^2)*b^4)/d^2)*sqrt(((A^2 - 2*A*B + B^2)*a^4 - 4*(A^2 - B^2)*a^3*b +
2*(A^2 + 6*A*B + B^2)*a^2*b^2 + 4*(A^2 - B^2)*a*b^3 + (A^2 - 2*A*B + B^2)*
b^4)/d^2))/(8*A*B*a^3*b - 8*A*B*a*b^3 - (A^2 - B^2)*a^4 + 6*(A^2 - B^2)...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^2}{\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**2*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**2/sqrt(cot(c + d*x)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \frac{8\left(3\,Bb^2+\frac{5\,(2\,Bab+Ab^2)}{\tan(dx+c)}+\frac{15\,(Ba^2+2\,Aab-Bb^2)}{\tan(dx+c)^2}\right)\tan(dx+c)^{\frac{5}{2}}-30\,\sqrt{2}((A-B)a^2-2\,(A+B)ab-(A-B}{}$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
1/60*(8*(3*B*b^2 + 5*(2*B*a*b + A*b^2)/tan(d*x + c) + 15*(B*a^2 + 2*A*a*b
- B*b^2)/tan(d*x + c)^2)*tan(d*x + c)^(5/2) - 30*sqrt(2)*((A - B)*a^2 - 2*
(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x +
c)))) - 30*sqrt(2)*((A - B)*a^2 - 2*(A + B)*a*b - (A - B)*b^2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 15*sqrt(2)*((A + B)*a^2 + 2*(
A - B)*a*b - (A + B)*b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c)
+ 1) + 15*sqrt(2)*((A + B)*a^2 + 2*(A - B)*a*b - (A + B)*b^2)*log(-sqrt(2)
/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/d
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^2}{\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^2/sqrt(cot(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^2}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2)/cot(c + d*x)^(1/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^2)/cot(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^2(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a^3 + \left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^3}{\cot(dx+c)}dx\right)b^3 + 3\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)}dx\right)a\,b^2 + 3\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)a^2b$$

input
```
int((a+b*tan(d*x+c))^2*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a**3 + int((sqrt(cot(c + d*x))*tan(
c + d*x)**3)/cot(c + d*x),x)*b**3 + 3*int((sqrt(cot(c + d*x))*tan(c + d*x)
**2)/cot(c + d*x),x)*a*b**2 + 3*int((sqrt(cot(c + d*x))*tan(c + d*x))/cot(
c + d*x),x)*a**2*b
```

3.587 $\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 345

$$\begin{aligned}
&\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\
&= \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad - \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad + \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad + \frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\sqrt{\cot(c+dx)}}{d} \\
&\quad + \frac{2a(7a^2A-18Ab^2-21abB)\cot^{\frac{3}{2}}(c+dx)}{21d} \\
&\quad - \frac{2a^2(11Ab+7aB)\cot^{\frac{5}{2}}(c+dx)}{35d} - \frac{2aA\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))^2}{7d}
\end{aligned}$$

output
```
-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctan(-1+2^(1/2)*c
ot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*
(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a^3*(A-B)-3*a*b^2
*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*
x+c)))*2^(1/2)/d+2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*cot(d*x+c)^(1/2)/d+2/
21*a*(7*A*a^2-18*A*b^2-21*B*a*b)*cot(d*x+c)^(3/2)/d-2/35*a^2*(11*A*b+7*B*a
)*cot(d*x+c)^(5/2)/d-2/7*a*A*cot(d*x+c)^(3/2)*(b+a*cot(d*x+c))^2/d
```

Mathematica [A] (verified)

Time = 2.43 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.94

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= \frac{2\sqrt{\cot(c+dx)}\left(-\frac{(3a^2b(A-B)+b^3(-A+B)+a^3(A+B)-3ab^2(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)}{2\sqrt{2}} - \right.}{}$$

input
```
Integrate[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*Sqrt[Cot[c + d*x]]*(-1/2*((3*a^2*b*(A - B) + b^3*(-A + B) + a^3*(A + B)
 - 3*a*b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + S
qrt[2]*Sqrt[Tan[c + d*x]]]))/Sqrt[2] - ((a^3*(A - B) + 3*a*b^2*(-A + B) -
3*a^2*b*(A + B) + b^3*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
 + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(4*Sqrt[2]
) - (a^3*A)/(7*Tan[c + d*x]^(7/2)) - (a^2*(3*A*b + a*B))/(5*Tan[c + d*x]^(
5/2)) + (a*(a^2*A - 3*A*b^2 - 3*a*b*B))/(3*Tan[c + d*x]^(3/2)) + (3*a^2*A*
b - A*b^3 + a^3*B - 3*a*b^2*B)/Sqrt[Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.03, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.697$, Rules used = {3042, 4064, 3042, 4090, 27, 3042, 4120, 27, 3042, 4113, 3042, 4011, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{9/2}(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^3(A\cot(c+dx)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{4090}$$

$$-\frac{2}{7}\int \frac{1}{2}\sqrt{\cot(c+dx)}(b+a\cot(c+dx))\left(-a(11Ab+7aB)\cot^2(c+dx)+7\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)+b(3aA-7bB)\right)dx-\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{7}\int \sqrt{\cot(c+dx)}(b+a\cot(c+dx))\left(-a(11Ab+7aB)\cot^2(c+dx)+7\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)+b(3aA-7bB)\right)dx-\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{7}\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\Big(-a(11Ab+7aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-7\big(Aa^2-2bBa$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\big\downarrow$ 4120

$$\frac{1}{7}\Bigg(-\frac{2}{5}\int \frac{5}{2}\sqrt{\cot(c+dx)}\big((3aA-7bB)b^2+a\big(7Aa^2-21bBa-18Ab^2\big)\cot^2(c+dx)+7\big(Ba^3+3Aba^2-3b^2Ba$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\big\downarrow$ 27

$$\frac{1}{7}\Bigg(-\int \sqrt{\cot(c+dx)}\big((3aA-7bB)b^2+a\big(7Aa^2-21bBa-18Ab^2\big)\cot^2(c+dx)+7\big(Ba^3+3Aba^2-3b^2Ba-$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\Bigg(-\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\Big((3aA-7bB)b^2+a\big(7Aa^2-21bBa-18Ab^2\big)\tan\left(c+dx+\frac{\pi}{2}\right)^2-7\big(Ba^3+3A$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\big\downarrow$ 4113

$$\frac{1}{7}\Bigg(-\int \sqrt{\cot(c+dx)}\big(7\big(Ba^3+3Aba^2-3b^2Ba-Ab^3\big)\cot(c+dx)-7\big(Aa^3-3bBa^2-3Ab^2a+b^3B\big)\big)dx+\frac{2a}{}$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\big\downarrow$ 3042

$$\frac{1}{7}\Bigg(-\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\Big(-7\big(Aa^3-3bBa^2-3Ab^2a+b^3B\big)-7\big(Ba^3+3Aba^2-3b^2Ba-Ab^3\big)\tan\Big(c+d$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\big\downarrow$ 4011

$$\frac{1}{7}\left(-\int \frac{-7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-7\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2a\left(7a^2A-21ab\right.}{}\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}\left(-\int \frac{7\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan\left(c+dx+\frac{\pi}{2}\right)-7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(7a^2A-\right.}{}\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\Big\downarrow$ 4017

$$\frac{1}{7}\left(-\frac{2\int \frac{7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a\left(7a^2A-21abB-18Ab^2\right)}{3d}\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\Big\downarrow$ 27

$$\frac{1}{7}\left(-\frac{14\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a\left(7a^2A-21abB-18Ab^2\right)\mathrm{c}}{3d}\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\Big\downarrow$ 1482

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^3(A-B\right.\right.}{d}\right.$$
$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\Big\downarrow$ 1476

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right.\right.}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\downarrow$ 1082

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{}{-\cot(c}}{}\right.\right.}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\downarrow$ 217

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot}\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\downarrow$ 1479

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot}\right.}{\sqrt{2}}\right.\right.}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\downarrow$ 25

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot\right.}{\sqrt{2}}\right.\right.}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\downarrow$ 27

$$\frac{1}{7}\left(-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot\right.}{\sqrt{2}}\right.\right.}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

$\downarrow$ 1103

$$\frac{1}{7}\left(\frac{2a\left(7a^2A-21abB-18Ab^2\right)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{2a^2(7aB+11Ab)\cot^{\frac{5}{2}}(c+dx)}{5d}-\frac{14\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-\right.\right.}{}\right.$$

$$\frac{2aA\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}{7d}$$

input
```
Int[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a*A*Cot[c + d*x]^(3/2)*(b + a*Cot[c + d*x])^2)/(7*d) + ((14*(3*a^2*A*b
 - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cot[c + d*x]])/d + (2*a*(7*a^2*A - 18*A
*b^2 - 21*a*b*B)*Cot[c + d*x]^(3/2))/(3*d) - (2*a^2*(11*A*b + 7*a*B)*Cot[c
 + d*x]^(5/2))/(5*d) - (14*(((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B)
- 3*a*b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + Ar
cTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 - ((a^3*(A - B) - 3*a*b^2
*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c
 + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Co
t[c + d*x]]/(2*Sqrt[2])))/2))/d)/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. $2(314) = 628$.

Time = 1.10 (sec) , antiderivative size = 1192, normalized size of antiderivative = 3.46

method	result	size
derivativedivides	Expression too large to display	1192
default	Expression too large to display	1192

input
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/420/d*(1/tan(d*x+c))^(9/2)*tan(d*x+c)*(-840*A*tan(d*x+c)^3*b^3-168*B*tan
(d*x+c)*a^3+105*A*tan(d*x+c)^(7/2)*2^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x
+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*a^3+210*A*tan(d*x+c)
^(7/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*a^3-210*A*tan(d*x+c)^(7/
2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*b^3+210*A*tan(d*x+c)^(7/2)*2
^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a^3-210*A*tan(d*x+c)^(7/2)*2^(1
/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*b^3-105*A*tan(d*x+c)^(7/2)*2^(1/2)
*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c
)^(1/2)+1))*b^3+105*B*tan(d*x+c)^(7/2)*2^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan
(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*b^3+210*B*tan(d*
x+c)^(7/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*a^3+210*B*tan(d*x+c)
^(7/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*b^3+210*B*tan(d*x+c)^(7/
2)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a^3+210*B*tan(d*x+c)^(7/2)*
2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*b^3+105*B*tan(d*x+c)^(7/2)*2^(
1/2)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d
*x+c)^(1/2)+1))*a^3+840*B*tan(d*x+c)^3*a^3+280*A*tan(d*x+c)^2*a^3-315*A*ta
n(d*x+c)^(7/2)*2^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2
)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*a*b^2+630*A*tan(d*x+c)^(7/2)*2^(1/2)*arc
tan(1+2^(1/2)*tan(d*x+c)^(1/2))*a^2*b-630*A*tan(d*x+c)^(7/2)*2^(1/2)*arcta
n(1+2^(1/2)*tan(d*x+c)^(1/2))*a*b^2+630*A*tan(d*x+c)^(7/2)*2^(1/2)*arct...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2129 vs. $2(314) = 628$.

Time = 0.17 (sec) , antiderivative size = 2129, normalized size of antiderivative = 6.17

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/210*(210*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b
 + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A
*B + B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*ar
ctan(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A +
 B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 -
10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^
2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x +
 c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 -
10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^
2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2
*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 2
0*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b
^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*
B*a*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^
4 + (A^2 - B^2)*b^6))*tan(d*x + c)^3 + 210*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B
+ B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^
2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 +
(A^2 - 2*A*B + B^2)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B
)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 +
 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(9/2)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.06

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{210\sqrt{2}((A+B)a^3+3(A-B)a^2b-3(A+B)ab^2-(A-B)b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+}{}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/420*(210*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A
- B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 210*sqrt(
2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 105*sqrt(2)*((A - B)*a^3
- 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*log(sqrt(2)/sqrt(tan(d*
x + c)) + 1/tan(d*x + c) + 1) + 105*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b
 - 3*(A - B)*a*b^2 + (A + B)*b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(
d*x + c) + 1) + 120*A*a^3/tan(d*x + c)^(7/2) - 840*(B*a^3 + 3*A*a^2*b - 3*
B*a*b^2 - A*b^3)/sqrt(tan(d*x + c)) - 280*(A*a^3 - 3*B*a^2*b - 3*A*a*b^2)/
tan(d*x + c)^(3/2) + 168*(B*a^3 + 3*A*a^2*b)/tan(d*x + c)^(5/2))/d
```

Giac [F]

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^3\cot(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3*cot(d*x + c)^(9/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{9/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^3\,dx$$

input
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3, x)
```

Reduce [F]

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^{\frac{9}{2}}\,(a+\tan(dx+c)\,b)^3\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

3.588 $\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 304

$$
\begin{aligned}
&\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx \\
&= \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad -\frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad -\frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} \\
&\quad +\frac{2a(5a^2A-14Ab^2-15abB)\sqrt{\cot(c+dx)}}{5d} \\
&\quad -\frac{2a^2(9Ab+5aB)\cot^{\frac{3}{2}}(c+dx)}{15d}-\frac{2aA\sqrt{\cot(c+dx)}(b+a\cot(c+dx))^2}{5d}
\end{aligned}
$$

output
```
-1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctan(-1+2^(1/2)*c
ot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*
(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(3*a^2*b*(A-B)-b^3
*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*
x+c)))*2^(1/2)/d+2/5*a*(5*A*a^2-14*A*b^2-15*B*a*b)*cot(d*x+c)^(1/2)/d-2/15
*a^2*(9*A*b+5*B*a)*cot(d*x+c)^(3/2)/d-2/5*a*A*cot(d*x+c)^(1/2)*(b+a*cot(d*
x+c))^2/d
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.94

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2\sqrt{\cot(c+dx)}\left(-\frac{(a^3(A-B)+3ab^2(-A+B)-3a^2b(A+B)+b^3(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)}{2\sqrt{2}}+\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*Sqrt[Cot[c + d*x]]*(-1/2*((a^3*(A - B) + 3*a*b^2*(-A + B) - 3*a^2*b*(A
+ B) + b^3*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + S
qrt[2]*Sqrt[Tan[c + d*x]]]))/Sqrt[2] + ((3*a^2*b*(A - B) + b^3*(-A + B) +
a^3*(A + B) - 3*a*b^2*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
 + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(4*Sqrt[2]
) - (a^3*A)/(5*Tan[c + d*x]^(5/2)) - (a^2*(3*A*b + a*B))/(3*Tan[c + d*x]^(
3/2)) + (a*(a^2*A - 3*A*b^2 - 3*a*b*B))/Sqrt[Tan[c + d*x]])*Sqrt[Tan[c + d
*x]])/d
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.03, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 4064, 3042, 4090, 27, 3042, 4120, 27, 3042, 4113, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{7/2}(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+b)^3(A\cot(c+dx)+B)}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4090}$$

$$-\frac{2}{5}\int \frac{(b+a\cot(c+dx))\left(-a(9Ab+5aB)\cot^2(c+dx)+5\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)+b(aA-5bB)\right)}{2\sqrt{\cot(c+dx)}}dx$$
$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}\int \frac{(b+a\cot(c+dx))\left(-a(9Ab+5aB)\cot^2(c+dx)+5\left(Aa^2-2bBa-Ab^2\right)\cot(c+dx)+b(aA-5bB)\right)}{\sqrt{\cot(c+dx)}}dx$$
$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(-a(9Ab+5aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-5\left(Aa^2-2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)+\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\downarrow$ 4120

$$\frac{1}{5}\left(-\frac{2}{3}\int \frac{3\left((aA-5bB)b^2+a\left(5Aa^2-15bBa-14Ab^2\right)\cot^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+\right.}{2\sqrt{\cot(c+dx)}}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\downarrow$ 27

$$\frac{1}{5}\left(-\int \frac{(aA-5bB)b^2+a\left(5Aa^2-15bBa-14Ab^2\right)\cot^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-\int \frac{(aA-5bB)b^2+a\left(5Aa^2-15bBa-14Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2-5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan\left(\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\downarrow$ 4113

$$\frac{1}{5}\left(-\int \frac{5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)-5\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)}{\sqrt{\cot(c+dx)}}dx+\frac{2a\left(5a^2A-15abB\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(-\int \frac{-5\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)-5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2a\left(5a^2A-\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\Big\downarrow$ 4017

$$\frac{1}{5}\left(-\frac{2\int \frac{5\left(Aa^3-3bBa^2-3Ab^2a+b^3B-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a\left(5a^2A-15abB-14Ab^2\right)}{d}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(-\frac{10\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2a\left(5a^2A-15abB-14Ab^2\right)\sqrt{}}{d}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\Big\downarrow$ 1482

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^3(A-B\right.\right.}{d}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\Big\downarrow$ 1476

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^3(A-B\right.\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\Big\downarrow$ 1082

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^3(A-\right.\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\Big\downarrow$ 217

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^3(A-B\right.\right.}{d}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\big\downarrow$ 1479

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\right.\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\big\downarrow$ 25

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\ldots}{\ldots}\right.\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\big\downarrow$ 27

$$\frac{1}{5}\left(-\frac{10\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\right.\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

$\big\downarrow$ 1103

$$\frac{1}{5}\left(\frac{2a(5a^2A-15abB-14Ab^2)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(5aB+9Ab)\cot^{\frac{3}{2}}(c+dx)}{3d}-\frac{10\left(\frac{1}{2}(a^3(A-B)-3a^2b(A+\right.}{}\right.$$

$$\frac{2aA\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}{5d}$$

input
```
Int[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a*A*Sqrt[Cot[c + d*x]]*(b + a*Cot[c + d*x])^2)/(5*d) + ((2*a*(5*a^2*A
 - 14*A*b^2 - 15*a*b*B)*Sqrt[Cot[c + d*x]])/d - (2*a^2*(9*A*b + 5*a*B)*Cot[
c + d*x]^(3/2))/(3*d) - (10*(((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A
+ B) + b^3*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + A
rcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 + ((3*a^2*b*(A - B) - b^
3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[
c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]]/(2*Sqrt[2])))/2))/d)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(274) = 548$.

Time = 1.12 (sec) , antiderivative size = 1134, normalized size of antiderivative = 3.73

method	result	size
derivativedivides	Expression too large to display	1134
default	Expression too large to display	1134

input
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/60/d*(1/tan(d*x+c))^(7/2)*tan(d*x+c)*(-30*A*tan(d*x+c)^(5/2)*2^(1/2)*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))*b^3-30*A*tan(d*x+c)^(5/2)*2^(1/2)*arctan(
-1+2^(1/2)*tan(d*x+c)^(1/2))*a^3-30*A*tan(d*x+c)^(5/2)*2^(1/2)*arctan(-1+2
^(1/2)*tan(d*x+c)^(1/2))*b^3-15*A*tan(d*x+c)^(5/2)*2^(1/2)*ln(-(2^(1/2)*ta
n(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*a^3-
15*B*tan(d*x+c)^(5/2)*2^(1/2)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/
(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*b^3+15*B*tan(d*x+c)^(5/2)*2^(1/2)
*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan
(d*x+c)-1))*a^3+30*B*tan(d*x+c)^(5/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^
(1/2))*a^3+40*B*tan(d*x+c)*a^3-30*B*tan(d*x+c)^(5/2)*2^(1/2)*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))*b^3+30*B*tan(d*x+c)^(5/2)*2^(1/2)*arctan(-1+2^(1/2)*
tan(d*x+c)^(1/2))*a^3-30*B*tan(d*x+c)^(5/2)*2^(1/2)*arctan(-1+2^(1/2)*tan(
d*x+c)^(1/2))*b^3-120*A*tan(d*x+c)^2*a^3-15*A*tan(d*x+c)^(5/2)*2^(1/2)*ln(
-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x
+c)-1))*b^3-30*A*tan(d*x+c)^(5/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2
))*a^3+45*B*tan(d*x+c)^(5/2)*2^(1/2)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x
+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*a^2*b-90*B*tan(d*x+c)^(5/2
)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a*b^2+90*B*tan(d*x+c)^(5/2)*
2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a^2*b-45*B*tan(d*x+c)^(5/2)*2^
(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2099 vs. $2(274) = 548$.

Time = 0.13 (sec) , antiderivative size = 2099, normalized size of antiderivative = 6.90

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/30*(30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b +
 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B
 + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*arct
an(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B
)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10
*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*
b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x + c
)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10
*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*
b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2*A
*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*
(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5
 + (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*B*
a*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^4
+ (A^2 - B^2)*b^6))*tan(d*x + c)^2 + 30*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B
^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 -
 B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^
2 + 2*A*B + B^2)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a
^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*
(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.09

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{30\sqrt{2}((A-B)a^3-3(A+B)a^2b-3(A-B)ab^2+(A+B)b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+}{}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/60*(30*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A +
B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 30*sqrt(2)*
((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 15*sqrt(2)*((A + B)*a^3 + 3*
(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*log(sqrt(2)/sqrt(tan(d*x +
c)) + 1/tan(d*x + c) + 1) - 15*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*
(A + B)*a*b^2 - (A - B)*b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x +
 c) + 1) + 24*A*a^3/tan(d*x + c)^(5/2) - 120*(A*a^3 - 3*B*a^2*b - 3*A*a*b^
2)/sqrt(tan(d*x + c)) + 40*(B*a^3 + 3*A*a^2*b)/tan(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^3\cot(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3*cot(d*x + c)^(7/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\,(a+b\tan(c+d\,x))^3\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3, x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^{\frac{7}{2}}\,(a+\tan(dx+c)\,b)^3\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

3.589 $\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 299

$$\begin{aligned}
&\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \\
&-\frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\
&+\frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} \\
&-\frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} \\
&-\frac{2a(3aAb+a^2B+2b^2B)\sqrt{\cot(c+dx)}}{d} \\
&-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(b+a\cot(c+dx))^2}{d\sqrt{\cot(c+dx)}}
\end{aligned}$$

output
```
1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctan(-1+2^(1/2)*co
t(d*x+c)^(1/2))*2^(1/2)/d+1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(
A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(a^3*(A-B)-3*a*b^2*
(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x
+c)))*2^(1/2)/d-2*a*(3*A*a*b+B*a^2+2*B*b^2)*cot(d*x+c)^(1/2)/d-2/3*a^2*(A*
a+3*B*b)*cot(d*x+c)^(3/2)/d+2*b*B*(b+a*cot(d*x+c))^2/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.90

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2\sqrt{\cot(c+dx)}\left(\frac{(3a^2b(A-B)+b^3(-A+B)+a^3(A+B)-3ab^2(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)}{2\sqrt{2}}+\cdots\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*Sqrt[Cot[c + d*x]]*(((3*a^2*b*(A - B) + b^3*(-A + B) + a^3*(A + B) - 3*
a*b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2
]*Sqrt[Tan[c + d*x]]]))/(2*Sqrt[2]) + ((a^3*(A - B) + 3*a*b^2*(-A + B) - 3
*a^2*b*(A + B) + b^3*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(4*Sqrt[2])
 - (a^3*A)/(3*Tan[c + d*x]^(3/2)) - (a^2*(3*A*b + a*B))/Sqrt[Tan[c + d*x]]
 + b^3*B*Sqrt[Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.01, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.606$, Rules used = {3042, 4064, 3042, 4088, 27, 3042, 4120, 27, 3042, 4113, 3042, 4017, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{5/2}(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+b)^3(A\cot(c+dx)+B)}{\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}-2\int -\frac{(b+a\cot(c+dx))\left(a(aA+3bB)\cot^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(Ab+5aB)\right)}{2\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(b+a\cot(c+dx))\left(a(aA+3bB)\cot^2(c+dx)+\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(Ab+5aB)\right)}{\sqrt{\cot(c+dx)}}dx+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(aA+3bB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b(Ab+\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4120}$$

$$\frac{2}{3}\int \frac{3\left((Ab+5aB)b^2+a\left(Ba^2+3Aba+2b^2B\right)\cot^2(c+dx)-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)\right)}{2\sqrt{\cot(c+dx)}}dx-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \; 27$$

$$\int \frac{(Ab+5aB)b^2+a\left(Ba^2+3Aba+2b^2B\right)\cot^2(c+dx)-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(Ab+5aB)b^2+a\left(Ba^2+3Aba+2b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2-\left(-Aa^3+3bBa^2+3Ab^2a-b^3B\right)\tan\left(c+dx+\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}$$

$$\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \; 4113$$

$$\int \frac{-Ba^3-3Aba^2+3b^2Ba+Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx-\frac{2a\left(a^2B+3aAb+2b^2B\right)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\int \frac{-Ba^3-3Aba^2+3b^2Ba+Ab^3-\left(-Aa^3+3bBa^2+3Ab^2a-b^3B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a\left(a^2B+3aAb+2b^2B\right)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \; 4017$$

$$\frac{2\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3+(Aa^3-3bBa^2-3Ab^2a+b^3B)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{2a(a^2B+3aAb+2b^2B)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$\big\downarrow$ 1482

$$\frac{2\left(\frac{1}{2}(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B))\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a^3(A-B)-3a\right.}{d}$$
$$\frac{2a(a^2B+3aAb+2b^2B)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$\big\downarrow$ 1476

$$\frac{2\left(\frac{1}{2}(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B))\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\right.\right.}{}$$
$$\frac{2a(a^2B+3aAb+2b^2B)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$\big\downarrow$ 1082

$$\frac{2\left(\frac{1}{2}(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B))\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d}{}\right.\right.}{c}$$
$$\frac{2a(a^2B+3aAb+2b^2B)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$\big\downarrow$ 217

$$\frac{2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right.\right.}{d}$$

$$\frac{2a\left(a^2B+3aAb+2b^2B\right)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$\downarrow$ 1479

$$\frac{2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right.\right.}{}$$

$$\frac{2a\left(a^2B+3aAb+2b^2B\right)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$\downarrow$ 25

$$\frac{2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right.\right.}{}$$

$$\frac{2a\left(a^2B+3aAb+2b^2B\right)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right.\right.}{}$$

$$\frac{2a\left(a^2B+3aAb+2b^2B\right)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

$\downarrow$ 1103

$$-\frac{2a(a^2B+3aAb+2b^2B)\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(aA+3bB)\cot^{\frac{3}{2}}(c+dx)}{3d}+\frac{2\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right.\right.}{}$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{d\sqrt{\cot(c+dx)}}$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(-2*a*(3*a*A*b + a^2*B + 2*b^2*B)*Sqrt[Cot[c + d*x]])/d - (2*a^2*(a*A + 3*
b*B)*Cot[c + d*x]^(3/2))/(3*d) + (2*b*B*(b + a*Cot[c + d*x])^2)/(d*Sqrt[Co
t[c + d*x]]) + (2*(((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A + B) - 3*a*b^2
*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 +
Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 - ((a^3*(A - B) - 3*a*b^2*(A - B)
- 3*a^2*b*(A + B) + b^3*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
+ Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x
]]/(2*Sqrt[2])))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4120
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])^(n_.)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[b*C*Tan[e + f*x]*((c + d*Tan[e + f*x])^(n +
 1)/(d*f*(n + 2))), x] - Simp[1/(d*(n + 2))   Int[(c + d*Tan[e + f*x])^n*Si
mp[b*c*C - a*A*d*(n + 2) - (A*b + a*B - b*C)*d*(n + 2)*Tan[e + f*x] - (a*C*
d*(n + 2) - b*(c*C - B*d*(n + 2)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] &&
  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(273) = 546$.

Time = 1.12 (sec) , antiderivative size = 1104, normalized size of antiderivative = 3.69

method	result	size
derivativedivides	Expression too large to display	1104
default	Expression too large to display	1104

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/12/d*(1/tan(d*x+c))^(5/2)*tan(d*x+c)*(3*A*2^(1/2)*ln(-(tan(d*x+c)+2^(1/
2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*tan(d*x+c)
^(3/2)*a^3+6*A*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)
*a^3-6*A*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*b^3+6
*A*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*a^3-6*A*2^
(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*b^3-3*A*2^(1/2)
*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c
)^(1/2)+1))*tan(d*x+c)^(3/2)*b^3+3*B*2^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d
*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*tan(d*x+c)^(3/2)*b
^3+6*B*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*a^3+6*B
*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*b^3+6*B*2^(1/
2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*a^3+6*B*2^(1/2)*ar
ctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*b^3+3*B*2^(1/2)*ln(-(2^
(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+
1))*tan(d*x+c)^(3/2)*a^3+24*B*tan(d*x+c)*a^3-18*A*2^(1/2)*arctan(-1+2^(1/2
)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*a*b^2+9*A*2^(1/2)*ln(-(2^(1/2)*tan(d*
x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*tan(d*x+
c)^(3/2)*a^2*b-9*B*2^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^
(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*tan(d*x+c)^(3/2)*a^2*b-18*B*2^(1/2)*
arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*a^2*b-18*B*2^(1/2)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2069 vs. $2(273) = 546$.

Time = 0.13 (sec) , antiderivative size = 2069, normalized size of antiderivative = 6.92

$$\int \cot^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^3(A + B\tan(c + dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/6*(6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b +
3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B
+ B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*arcta
n(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)
*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*
A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*b
^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x + c)
) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*
A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*b
^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2*A*
B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(
A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5
+ (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*B*a
*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^4 +
 (A^2 - B^2)*b^6))*tan(d*x + c) + 6*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*
a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2
)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 -
2*A*B + B^2)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2*b
 - 3*(A - B)*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2
 - B^2)*a^5*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3...
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.05

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{24\,Bb^3\sqrt{\tan(dx+c)}+6\sqrt{2}((A+B)a^3+3\,(A-B)a^2b-3\,(A+B)ab^2-(A-B)b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{\ldots}\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/12*(24*B*b^3*sqrt(tan(d*x + c)) + 6*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2
*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(t
an(d*x + c)))) + 6*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^
2 - (A - B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - 3
*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*l
og(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 3*sqrt(2)*((A - B)*a
^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*log(-sqrt(2)/sqrt(ta
n(d*x + c)) + 1/tan(d*x + c) + 1) - 8*A*a^3/tan(d*x + c)^(3/2) - 24*(B*a^3
 + 3*A*a^2*b)/sqrt(tan(d*x + c)))/d
```

Giac [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^3\cot(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3*cot(d*x + c)^(5/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^3\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3, x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^{\frac{5}{2}}\,(a+\tan(dx+c)\,b)^3\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

3.590 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 295

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = -\frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{2b^2(3Ab+7aB)}{3d\sqrt{\cot(c+dx)}} - \frac{2a^2(3aA+bB)\sqrt{\cot(c+dx)}}{3d} + \frac{2bB(b+a\cot(c+dx))^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

output

```
1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctan(-1+2^(1/2)*co
t(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(
A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*(3*a^2*b*(A-B)-b^3*
(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*x
+c)))*2^(1/2)/d+2/3*b^2*(3*A*b+7*B*a)/d/cot(d*x+c)^(1/2)-2/3*a^2*(3*A*a+B*
b)*cot(d*x+c)^(1/2)/d+2/3*b*B*(b+a*cot(d*x+c))^2/d/cot(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.92

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{(a^3(A-B)+3ab^2(-A+B)-3a^2b(A+B)+b^3(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\right.\right.}{2\sqrt{2}}}{}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((a^3*(A - B) + 3*a*b^2*(-A + B)
 - 3*a^2*b*(A + B) + b^3*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
- ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(2*Sqrt[2]) - ((3*a^2*b*(A - B)
 + b^3*(-A + B) + a^3*(A + B) - 3*a*b^2*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]))/(4*Sqrt[2]) - (a^3*A)/Sqrt[Tan[c + d*x]] + b^2*(A*b + 3*a*B)*Sqrt[
Tan[c + d*x]] + (b^3*B*Tan[c + d*x]^(3/2))/3))/d
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.02, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 4064, 3042, 4088, 27, 3042, 4118, 25, 3042, 4113, 3042, 4017, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+b)^3(A\cot(c+dx)+B)}{\cot^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}-\frac{2}{3}\int -\frac{(b+a\cot(c+dx))\left(a(3aA+bB)\cot^2(c+dx)+3\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(3Ab+7aB)\right)}{2\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(b+a\cot(c+dx))\left(a(3aA+bB)\cot^2(c+dx)+3\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(3Ab+7aB)\right)}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(3aA+bB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-3\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b(3A\right.}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{4118}$$

$$\frac{1}{3}\left(\frac{2b^2(7aB+3Ab)}{d\sqrt{\cot(c+dx)}}-\int -\frac{a^2(3aA+bB)\cot^2(c+dx)+3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)+b\left(10Ba\right.}{\sqrt{\cot(c+dx)}}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\int \frac{a^2(3aA+bB)\cot^2(c+dx)+3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)+b\left(10Ba^2+9Aba-3b^2B\right)}{\sqrt{\cot(c+dx)}}dx\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}\left(\int \frac{a^2(3aA+bB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b\left(10Ba^2+9Aba\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 4113$$

$$\frac{1}{3}\left(\int \frac{3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)-3\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)}{\sqrt{\cot(c+dx)}}dx-\frac{2a^2(3aA+bB)\sqrt{\cot}}{d}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 3042$$

$$\frac{1}{3}\left(\int \frac{-3\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)-3\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2a^2(3aA+b}{}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 4017$$

$$\frac{1}{3}\left(\frac{2\int \frac{3\left(Aa^3-3bBa^2-3Ab^2a+b^3B-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(3aA+bB)\sqrt{\cot(c+dx)}}{d}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 27$$

$$\frac{1}{3}\left(\frac{6\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{2a^2(3aA+bB)\sqrt{\cot(c+dx)}}{d}+\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow 1482$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^3(A-B)\right.\right.}{d}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^3(A-B)\right.\right.}{}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^3(A-B)\right.\right.}{}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(a^3(A-B)\right.\right.}{d}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\ldots}{\ldots}\right.\right.}{\ldots}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 25

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\ldots}{\cot(c+\ldots}}{\ldots}\right.\right.}{\ldots}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{6\left(\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\ldots}{\cot(\ldots}\right.\right.}{\ldots}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1103

$$\frac{1}{3}\left(-\frac{2a^2(3aA+bB)\sqrt{\cot(c+dx)}}{d}+\frac{6\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\ldots\right.}{\ldots}\right.\right.}{\ldots}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{3d\cot^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b*B*(b + a*Cot[c + d*x])^2)/(3*d*Cot[c + d*x]^(3/2)) + ((2*b^2*(3*A*b +
 7*a*B))/(d*Sqrt[Cot[c + d*x]]) - (2*a^2*(3*a*A + b*B)*Sqrt[Cot[c + d*x]])
/d + (6*(((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*
(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]]]/Sqrt[2]))/2 + ((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(A +
 B) - 3*a*b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt
[2]))/2))/d)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(266) = 532$.

Time = 0.38 (sec) , antiderivative size = 1104, normalized size of antiderivative = 3.74

method	result	size
derivativedivides	Expression too large to display	1104
default	Expression too large to display	1104

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/12/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(-3*A*tan(d*x+c)^(1/2)*2^(1/2)*ln(-
(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+
c)-1))*b^3-6*A*tan(d*x+c)^(1/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))
*a^3-6*A*tan(d*x+c)^(1/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*b^3-6
*A*tan(d*x+c)^(1/2)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a^3-6*A*ta
n(d*x+c)^(1/2)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*b^3-3*A*tan(d*x
+c)^(1/2)*2^(1/2)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+
2^(1/2)*tan(d*x+c)^(1/2)+1))*a^3+3*B*tan(d*x+c)^(1/2)*2^(1/2)*ln(-(tan(d*x
+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*a
^3+6*B*tan(d*x+c)^(1/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*a^3-6*B
*tan(d*x+c)^(1/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*b^3+6*B*tan(d
*x+c)^(1/2)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a^3-6*B*tan(d*x+c)
^(1/2)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*b^3-3*B*tan(d*x+c)^(1/2
)*2^(1/2)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*
tan(d*x+c)^(1/2)+1))*b^3+9*A*tan(d*x+c)^(1/2)*2^(1/2)*ln(-(tan(d*x+c)+2^(1
/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*a^2*b+18*
A*tan(d*x+c)^(1/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*a^2*b+18*A*t
an(d*x+c)^(1/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*a*b^2+18*A*tan(
d*x+c)^(1/2)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a^2*b+18*A*tan(d*
x+c)^(1/2)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a*b^2+9*A*tan(d*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2042 vs. $2(266) = 532$.

Time = 0.12 (sec) , antiderivative size = 2042, normalized size of antiderivative = 6.92

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
-1/6*(6*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b +
3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B
+ B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*arcta
n(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)
*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*
A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b
^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x + c)
) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10*
A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*b
^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2*A*
B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(
A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5
+ (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*B*a
*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^4 +
 (A^2 - B^2)*b^6)) + 6*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2
- B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3
*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*
b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*
a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b
 + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 1...
```

Sympy [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \int (A+B\tan(c+dx))\,(a+b\tan(c+dx))^3\cot^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3*cot(c + d*x)**(3/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.07

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx =$$

$$-\frac{\frac{24\,Aa^3}{\sqrt{\tan(dx+c)}} - 6\sqrt{2}((A-B)a^3 - 3\,(A+B)a^2b - 3\,(A-B)ab^2 + (A+B)b^3)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{\cdots}{\sqrt{\tan\cdots}}\right.\right.}{\cdots}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
-1/12*(24*A*a^3/sqrt(tan(d*x + c)) - 6*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^
2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(
tan(d*x + c)))) - 6*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b
^2 + (A + B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) -
3*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*
log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 3*sqrt(2)*((A + B)*
a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*log(-sqrt(2)/sqrt(t
an(d*x + c)) + 1/tan(d*x + c) + 1) - 8*(B*b^3 + 3*(3*B*a*b^2 + A*b^3)/tan(
d*x + c))*tan(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^3\cot(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3*cot(d*x + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\,(a+b\tan(c+d\,x))^3\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3, x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$=\int \cot\left(dx+c\right)^{\frac{3}{2}}\left(a+\tan\left(dx+c\right)b\right)^3\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

3.591 $\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 304

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{2b^2(5Ab+9aB)}{15d\cot^{\frac{3}{2}}(c+dx)} + \frac{2b(15aAb+14a^2B-5b^2B)}{5d\sqrt{\cot(c+dx)}} + \frac{2bB(b+a\cot(c+dx))^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

output
```
-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctan(-1+2^(1/2)*c
ot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(3*a^2*b*(A-B)-b^3*(A-B)+a^3*(A+B)-3*a*b^2*
(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*(a^3*(A-B)-3*a*b^2
*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*
x+c)))*2^(1/2)/d+2/15*b^2*(5*A*b+9*B*a)/d/cot(d*x+c)^(3/2)+2/5*b*(15*A*a*b
+14*B*a^2-5*B*b^2)/d/cot(d*x+c)^(1/2)+2/5*b*B*(b+a*cot(d*x+c))^2/d/cot(d*x
+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.94

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$= \frac{2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-\frac{(3a^2b(A-B)+b^3(-A+B)+a^3(A+B)-3ab^2(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{\right.\right.}{2\sqrt{2}}\right.}{}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x
]
```

output
```
(2*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-1/2*((3*a^2*b*(A - B) + b^3*(-A
 + B) + a^3*(A + B) - 3*a*b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/Sqrt[2] - ((a^3*(A - B) +
3*a*b^2*(-A + B) - 3*a^2*b*(A + B) + b^3*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]))/(4*Sqrt[2]) + b*(3*a*A*b + 3*a^2*B - b^2*B)*Sqrt[Tan[c + d*x]] +
(b^2*(A*b + 3*a*B)*Tan[c + d*x]^(3/2))/3 + (b^3*B*Tan[c + d*x]^(5/2))/5))/
d
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.03, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4064, 3042, 4088, 27, 3042, 4118, 25, 3042, 4111, 27, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{ 4064}$$

$$\int \frac{(a\cot(c+dx)+b)^3(A\cot(c+dx)+B)}{\cot^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 4088}$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}-\frac{2}{5}\int -\frac{(b+a\cot(c+dx))\left(a(5aA-bB)\cot^2(c+dx)+5\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(5Ab+9aB)\right)}{2\cot^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\int \frac{(b+a\cot(c+dx))\left(a(5aA-bB)\cot^2(c+dx)+5\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(5Ab+9aB)\right)}{\cot^{\frac{5}{2}}(c+dx)}dx+\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(5aA-bB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-5\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b(5A\right.}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4118}$$

$$\frac{1}{5}\left(\frac{2b^2(9aB+5Ab)}{3d\cot^{\frac{3}{2}}(c+dx)}-\int -\frac{a^2(5aA-bB)\cot^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)+b\left(14Ba\right.}{\cot^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{5}\left(\int \frac{a^2(5aA-bB)\cot^2(c+dx)+5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)+b\left(14Ba^2+15Aba-5b^2B\right)}{\cot^{\frac{3}{2}}(c+dx)}d\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\left(\int \frac{a^2(5aA-bB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b\left(14Ba^2+15Aba\right.}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{4111}$$

$$\frac{1}{5}\left(\int \frac{5\left(Ba^3+3Aba^2-3b^2Ba-Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}}dx+\frac{2b\left(14a^2B+15aAb-\right.}{d\sqrt{\cot(c+dx)}}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(5\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2b\left(14a^2B+15aAb-5\right.}{d\sqrt{\cot(c+dx)}}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{5}\left(5\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\left(14a^2B+15aA\right.}{d\sqrt{\cot(c+}}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 4017$$

$$\frac{1}{5}\left(\frac{10\int -\frac{Ba^3+3Aba^2-3b^2Ba-Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2b\left(14a^2B+15aAb-5b^2B\right)}{d\sqrt{\cot(c+dx)}}+\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{5}\left(-\frac{10\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2b\left(14a^2B+15aAb-5b^2B\right)}{d\sqrt{\cot(c+dx)}}+\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 1482$$

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^3(A+B)\right.\right.}{d}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 1476$$

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^3(A+B)\right.\right.}{}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^3(A+B)\right.\cdots}{\cdots}\right.\cdots$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 217

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^3(A+B)\right.\cdots}{d}\right.\cdots$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 1479

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int \cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 25

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{1}{5}\left(\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{}{\text{co}}\right.\right.}{}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{5}\left(\frac{2b(14a^2B+15aAb-5b^2B)}{d\sqrt{\cot(c+dx)}}+\frac{10\left(\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\log\left(\cot(c+dx\right.}{}\right.\right.}{}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{5d\cot^{\frac{5}{2}}(c+dx)}$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]),x]
```

output
```
(2*b*B*(b + a*Cot[c + d*x])^2)/(5*d*Cot[c + d*x]^(5/2)) + ((2*b^2*(5*A*b +
 9*a*B))/(3*d*Cot[c + d*x]^(3/2)) + (2*b*(15*a*A*b + 14*a^2*B - 5*b^2*B))/
(d*Sqrt[Cot[c + d*x]]) + (10*(-1/2*((3*a^2*b*(A - B) - b^3*(A - B) + a^3*(
A + B) - 3*a*b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2
]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])) + ((a^3*(A - B) - 3*
a*b^2*(A - B) - 3*a^2*b*(A + B) + b^3*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[
Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d)/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(275) = 550$.

Time = 0.38 (sec) , antiderivative size = 956, normalized size of antiderivative = 3.14

method	result	size
derivativedivides	Expression too large to display	956
default	Expression too large to display	956

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x,method=_RETURNV
ERBOSE)
```

output
```
1/60/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)^(1/2)*(-45*A*ln(-(tan(d*x+c)+2^(1/2
)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)*a*b
^2+90*A*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^2*b-90*A*arctan(1+2^(
1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a*b^2+90*A*arctan(-1+2^(1/2)*tan(d*x+c)^(1/
2))*2^(1/2)*a^2*b-90*A*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a*b^2+4
5*A*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*
x+c)^(1/2)+1))*2^(1/2)*a^2*b-45*B*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)
+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)*a^2*b-90*B*arctan(1+2
^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^2*b-90*B*arctan(1+2^(1/2)*tan(d*x+c)^(1
/2))*2^(1/2)*a*b^2-90*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^2*b-
90*B*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a*b^2-45*B*ln(-(2^(1/2)*t
an(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(
1/2)*a*b^2+40*A*b^3*tan(d*x+c)^(3/2)-120*B*b^3*tan(d*x+c)^(1/2)+24*B*b^3*t
an(d*x+c)^(5/2)+360*A*a*b^2*tan(d*x+c)^(1/2)+360*B*a^2*b*tan(d*x+c)^(1/2)+
15*A*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)
-tan(d*x+c)-1))*2^(1/2)*a^3+30*A*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2
)*a^3-30*A*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*b^3+30*A*arctan(-1+2
^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^3-30*A*arctan(-1+2^(1/2)*tan(d*x+c)^(1/
2))*2^(1/2)*b^3-15*A*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+
c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(1/2)*b^3+15*B*ln(-(tan(d*x+c)+2^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2062 vs. $2(275) = 550$.

Time = 0.12 (sec) , antiderivative size = 2062, normalized size of antiderivative = 6.78

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
1/30*(30*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b +
 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B
 + B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*arct
an(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B
)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10
*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*
b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x + c
)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 - 10
*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^2*
b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2*A
*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*
(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5
 + (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*B*
a*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^4
+ (A^2 - B^2)*b^6)) + 30*sqrt(1/2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^
2 - B^2)*a^5*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 +
 3*(A^2 + 10*A*B + B^2)*a^2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2
)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B
)*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5
*b + 3*(A^2 - 10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 +...
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \int (A+B\tan(c+dx))\,(a+b\tan(c+dx))^3\,\sqrt{\cot(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**3*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3*sqrt(cot(c + d*x)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.10

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \frac{8\left(3\,Bb^3+\frac{5\,(3\,Bab^2+Ab^3)}{\tan(dx+c)}+\frac{15\,(3\,Ba^2b+3\,Aab^2-Bb^3)}{\tan(dx+c)^2}\right)\tan(dx+c)^{\frac{5}{2}}-30\sqrt{2}((A+B)a^3+3\,(A-B)a^2b-3\,(}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
1/60*(8*(3*B*b^3 + 5*(3*B*a*b^2 + A*b^3)/tan(d*x + c) + 15*(3*B*a^2*b + 3*
A*a*b^2 - B*b^3)/tan(d*x + c)^2)*tan(d*x + c)^(5/2) - 30*sqrt(2)*((A + B)*
a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*arctan(1/2*sqrt(2)*
(sqrt(2) + 2/sqrt(tan(d*x + c)))) - 30*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^
2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt
(tan(d*x + c)))) + 15*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a
*b^2 + (A + B)*b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) -
 15*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3
)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/d
```

Giac [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^3\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^3*sqrt(cot(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx = \int \sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^3\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3,x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3, x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3(A+B\tan(c+dx))\,dx &= \left(\int \sqrt{\cot(dx+c)}dx\right)a^4 + \left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)^4\,dx\right)b^4 \\ &+ 4\left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)^3\,dx\right)a\,b^3 \\ &+ 6\left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)^2\,dx\right)a^2b^2 \\ &+ 4\left(\int \sqrt{\cot(dx+c)}\,\tan(dx+c)\,dx\right)a^3b\end{aligned}$$

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^3*(A+B*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x)),x)*a**4 + int(sqrt(cot(c + d*x))*tan(c + d*x)**4,x)
*b**4 + 4*int(sqrt(cot(c + d*x))*tan(c + d*x)**3,x)*a*b**3 + 6*int(sqrt(co
t(c + d*x))*tan(c + d*x)**2,x)*a**2*b**2 + 4*int(sqrt(cot(c + d*x))*tan(c
+ d*x),x)*a**3*b
```

3.592 $\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 345

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{(a^3(A-B)-3ab^2(A-B)-3a^2b(A+B)+b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{(3a^2b(A-B)-b^3(A-B)+a^3(A+B)-3ab^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{2b^2(7Ab+11aB)}{35d\cot^{\frac{5}{2}}(c+dx)} + \frac{2b(21aAb+18a^2B-7b^2B)}{21d\cot^{\frac{3}{2}}(c+dx)} + \frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)}{d\sqrt{\cot(c+dx)}} + \frac{2bB(b+a\cot(c+dx))^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

output
```
-1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*(A+B))*arctan(-1+2^(1/2)*c
ot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(a^3*(A-B)-3*a*b^2*(A-B)-3*a^2*b*(A+B)+b^3*
(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*(3*a^2*b*(A-B)-b^3
*(A-B)+a^3*(A+B)-3*a*b^2*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot(d*
x+c)))*2^(1/2)/d+2/35*b^2*(7*A*b+11*B*a)/d/cot(d*x+c)^(5/2)+2/21*b*(21*A*a
*b+18*B*a^2-7*B*b^2)/d/cot(d*x+c)^(3/2)+2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2
)/d/cot(d*x+c)^(1/2)+2/7*b*B*(b+a*cot(d*x+c))^2/d/cot(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-\frac{(a^3(A-B)+3ab^2(-A+B)-3a^2b(A+B)+b^3(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{\right.\right.}{2\sqrt{2}}}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]]
,x]
```

output
```
(2*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-1/2*((a^3*(A - B) + 3*a*b^2*(-A
 + B) - 3*a^2*b*(A + B) + b^3*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/Sqrt[2] + ((3*a^2*b*(A - B
) + b^3*(-A + B) + a^3*(A + B) - 3*a*b^2*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]))/(4*Sqrt[2]) + (3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Tan[c
+ d*x]] + (b*(3*a*A*b + 3*a^2*B - b^2*B)*Tan[c + d*x]^(3/2))/3 + (b^2*(A*b
 + 3*a*B)*Tan[c + d*x]^(5/2))/5 + (b^3*B*Tan[c + d*x]^(7/2))/7))/d
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.04, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 4064, 3042, 4088, 27, 3042, 4118, 25, 3042, 4111, 27, 3042, 4012, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{(a\cot(c+dx)+b)^3(A\cot(c+dx)+B)}{\cot^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}-\frac{2}{7}\int -\frac{(b+a\cot(c+dx))\left(a(7aA-3bB)\cot^2(c+dx)+7\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(7Ab+11aB)\right)}{2\cot^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\int \frac{(b+a\cot(c+dx))\left(a(7aA-3bB)\cot^2(c+dx)+7\left(Ba^2+2Aba-b^2B\right)\cot(c+dx)+b(7Ab+11aB)\right)}{\cot^{\frac{7}{2}}(c+dx)}dx+\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\int \frac{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)\left(a(7aA-3bB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-7\left(Ba^2+2Aba-b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b(7}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4118}$$

$$\frac{1}{7}\left(\frac{2b^2(11aB+7Ab)}{5d\cot^{\frac{5}{2}}(c+dx)}-\int -\frac{a^2(7aA-3bB)\cot^2(c+dx)+7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)+b(18}{\cot^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{7}\left(\int \frac{a^2(7aA-3bB)\cot^2(c+dx)+7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)+b\left(18Ba^2+21Aba-7b^2B\right)}{\cot^{\frac{5}{2}}(c+dx)}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\left(\int \frac{a^2(7aA-3bB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b\left(18Ba^2+21Ab\right.}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{4111}$$

$$\frac{1}{7}\left(\int \frac{7\left(Ba^3+3Aba^2-3b^2Ba-Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)\right)}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2b\left(18a^2B+21aAb-\right.}{3d\cot^{\frac{3}{2}}(c+dx}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}\left(7\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3+\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)}dx+\frac{2b\left(18a^2B+21aAb-7\right.}{3d\cot^{\frac{3}{2}}(c+dx)}\right.$$
$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(7\int \frac{Ba^3+3Aba^2-3b^2Ba-Ab^3-\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx+\frac{2b\left(18a^2B+21aA\right.}{3d\cot^{\frac{3}{2}}(c+}\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4012$$

$$\frac{1}{7}\left(7\left(\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx+\frac{2\left(a^3B+3a^2Ab-3ab\right.}{d\sqrt{\cot(c+d}}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{7}\left(7\left(\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B-\left(-Ba^3-3Aba^2+3b^2Ba+Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2\left(a^3B+3a^2\right.}{d\sqrt{c}}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 4017$$

$$\frac{1}{7}\left(7\left(\frac{2\int -\frac{Aa^3-3bBa^2-3Ab^2a+b^3B-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2\left(a^3B+3a^2Ab-3ab^2B-\right.}{d\sqrt{\cot(c+dx)}}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{7}\left(7\left(\frac{2\left(a^3B+3a^2Ab-3ab^2B-Ab^3\right)}{d\sqrt{\cot(c+dx)}}-\frac{2\int \frac{Aa^3-3bBa^2-3Ab^2a+b^3B-\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+}}{d}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$$\Big\downarrow \; 1482$$

$$\frac{1}{7}\left(7\left(\frac{2\left(-\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{d}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 1476

$$\frac{1}{7}\left(7\left(\frac{2\left(-\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 1082

$$\frac{1}{7}\left(7\left(\frac{2\left(-\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 217

$$\frac{1}{7}\left(7\left(\frac{2\left(-\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(a^3(A-\right.\right.}{d}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 1479

$$\frac{1}{7}\left(7\left(\frac{2\left(-\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right.\right.}{}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 25

$$\frac{1}{7}\left(7\left(\frac{2\left(-\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\right.\right.}{}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{1}{7}\left(7\left(\frac{2\left(-\frac{1}{2}\left(a^3(A+B)+3a^2b(A-B)-3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\right.\right.}{}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

$\downarrow$ 1103

$$\frac{1}{7}\left(\frac{2b\left(18a^2B+21aAb-7b^2B\right)}{3d\cot^{\frac{3}{2}}(c+dx)}+7\left(\frac{2\left(-\frac{1}{2}\left(a^3(A-B)-3a^2b(A+B)-3ab^2(A-B)+b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.\right.}{}\right.\right.$$

$$\frac{2bB(a\cot(c+dx)+b)^2}{7d\cot^{\frac{7}{2}}(c+dx)}$$

input

```
Int[((a + b*Tan[c + d*x])^3*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

output
```
(2*b*B*(b + a*Cot[c + d*x])^2)/(7*d*Cot[c + d*x]^(7/2)) + ((2*b^2*(7*A*b +
 11*a*B))/(5*d*Cot[c + d*x]^(5/2)) + (2*b*(21*a*A*b + 18*a^2*B - 7*b^2*B))
/(3*d*Cot[c + d*x]^(3/2)) + 7*((2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B))
/(d*Sqrt[Cot[c + d*x]]) + (2*(-1/2*((a^3*(A - B) - 3*a*b^2*(A - B) - 3*a^2
*b*(A + B) + b^3*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2
]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])) - ((3*a^2*b*(A - B)
- b^3*(A - B) + a^3*(A + B) - 3*a*b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[
Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]
 + Cot[c + d*x]]/(2*Sqrt[2])))/2))/d))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4111
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 -
 a*b*B + a^2*C)*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 + b^2))), x
] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
 C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0
]
```

rule 4118
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.
)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(c^2*C - B*c*d + A*d^2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d^2*f*(n + 1)*(c^2 + d^2))), x] + Simp[1/(d*(c^2
 + d^2))   Int[(c + d*Tan[e + f*x])^(n + 1)*Simp[a*d*(A*c - c*C + B*d) + b*
(c^2*C - B*c*d + A*d^2) + d*(A*b*c + a*B*c - b*c*C - a*A*d + b*B*d + a*C*d)
*Tan[e + f*x] + b*C*(c^2 + d^2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[c^2 + d^2, 0] && LtQ[n
, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{-\frac{2\left(3A\,a^{2}b-A\,b^{3}+B\,a^{3}-3Ba\,b^{2}\right)}{\sqrt{\cot(dx+c)}}-\frac{2B\,b^{3}}{7\cot(dx+c)^{\frac{7}{2}}}-\frac{2b\left(3abA+3B\,a^{2}-B\,b^{2}\right)}{3\cot(dx+c)^{\frac{3}{2}}}-\frac{2b^{2}(Ab+3Ba)}{5\cot(dx+c)^{\frac{5}{2}}}+\frac{\left(A\,a^{3}-3Aa\,b^{2}-3B\,a^{2}b+B\,b^{3}\right)}{}}{}$
default	$-\frac{-\frac{2\left(3A\,a^{2}b-A\,b^{3}+B\,a^{3}-3Ba\,b^{2}\right)}{\sqrt{\cot(dx+c)}}-\frac{2B\,b^{3}}{7\cot(dx+c)^{\frac{7}{2}}}-\frac{2b\left(3abA+3B\,a^{2}-B\,b^{2}\right)}{3\cot(dx+c)^{\frac{3}{2}}}-\frac{2b^{2}(Ab+3Ba)}{5\cot(dx+c)^{\frac{5}{2}}}+\frac{\left(A\,a^{3}-3Aa\,b^{2}-3B\,a^{2}b+B\,b^{3}\right)}{}}{}$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/d*(-2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)/cot(d*x+c)^(1/2)-2/7*B*b^3/cot(
d*x+c)^(7/2)-2/3*b*(3*A*a*b+3*B*a^2-B*b^2)/cot(d*x+c)^(3/2)-2/5*b^2*(A*b+3
*B*a)/cot(d*x+c)^(5/2)+1/4*(A*a^3-3*A*a*b^2-3*B*a^2*b+B*b^3)*2^(1/2)*(ln((
cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2
)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^
(1/2)))+1/4*(-3*A*a^2*b+A*b^3-B*a^3+3*B*a*b^2)*2^(1/2)*(ln((cot(d*x+c)-2^(
1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan
(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2104 vs. 2(313) = 626.

Time = 0.13 (sec) , antiderivative size = 2104, normalized size of antiderivative = 6.10

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
1/210*(210*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b
 + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A
*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*ar
ctan(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A -
 B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 +
10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^
2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2)*sqrt(tan(d*x +
 c)) + d^2*sqrt(((A^2 + 2*A*B + B^2)*a^6 + 6*(A^2 - B^2)*a^5*b + 3*(A^2 -
10*A*B + B^2)*a^4*b^2 - 20*(A^2 - B^2)*a^3*b^3 + 3*(A^2 + 10*A*B + B^2)*a^
2*b^4 + 6*(A^2 - B^2)*a*b^5 + (A^2 - 2*A*B + B^2)*b^6)/d^2)*sqrt(((A^2 - 2
*A*B + B^2)*a^6 - 6*(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 2
0*(A^2 - B^2)*a^3*b^3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b
^5 + (A^2 + 2*A*B + B^2)*b^6)/d^2))/(12*A*B*a^5*b - 40*A*B*a^3*b^3 + 12*A*
B*a*b^5 - (A^2 - B^2)*a^6 + 15*(A^2 - B^2)*a^4*b^2 - 15*(A^2 - B^2)*a^2*b^
4 + (A^2 - B^2)*b^6)) + 210*sqrt(1/2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*
(A^2 - B^2)*a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^
3 + 3*(A^2 - 10*A*B + B^2)*a^2*b^4 - 6*(A^2 - B^2)*a*b^5 + (A^2 + 2*A*B +
B^2)*b^6)/d^2)*arctan(-(2*sqrt(1/2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A
+ B)*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^6 - 6*(A^2 - B^2)*
a^5*b + 3*(A^2 + 10*A*B + B^2)*a^4*b^2 + 20*(A^2 - B^2)*a^3*b^3 + 3*(A^...
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^3}{\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**3*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**3/sqrt(cot(c + d*x)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.07

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \frac{8\left(15\,Bb^3+\frac{21\,(3\,Bab^2+Ab^3)}{\tan(dx+c)}+\frac{35\,(3\,Ba^2b+3\,Aab^2-Bb^3)}{\tan(dx+c)^2}+\frac{105\,(Ba^3+3\,Aa^2b-3\,Bab^2-Ab^3)}{\tan(dx+c)^3}\right)\tan\left(dx+c\right)^{\frac{7}{2}}-210\,\sqrt{2}((}{}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
1/420*(8*(15*B*b^3 + 21*(3*B*a*b^2 + A*b^3)/tan(d*x + c) + 35*(3*B*a^2*b +
 3*A*a*b^2 - B*b^3)/tan(d*x + c)^2 + 105*(B*a^3 + 3*A*a^2*b - 3*B*a*b^2 -
A*b^3)/tan(d*x + c)^3)*tan(d*x + c)^(7/2) - 210*sqrt(2)*((A - B)*a^3 - 3*(
A + B)*a^2*b - 3*(A - B)*a*b^2 + (A + B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2)
+ 2/sqrt(tan(d*x + c)))) - 210*sqrt(2)*((A - B)*a^3 - 3*(A + B)*a^2*b - 3*
(A - B)*a*b^2 + (A + B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x
 + c)))) - 105*sqrt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 -
(A - B)*b^3)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 105*sq
rt(2)*((A + B)*a^3 + 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 - (A - B)*b^3)*log(
-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^3}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3)/cot(c + d*x)^(1/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^3)/cot(c + d*x)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{(a+b\tan(c+dx))^3(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx\\&=\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a^4+\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^4}{\cot(dx+c)}dx\right)b^4\\&\quad+4\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^3}{\cot(dx+c)}dx\right)a\,b^3\\&\quad+6\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)^2}{\cot(dx+c)}dx\right)a^2b^2\\&\quad+4\left(\int \frac{\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)a^3b\end{aligned}$$

input
```
int((a+b*tan(d*x+c))^3*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
int(sqrt(cot(c + d*x))/cot(c + d*x),x)*a**4 + int((sqrt(cot(c + d*x))*tan(
c + d*x)**4)/cot(c + d*x),x)*b**4 + 4*int((sqrt(cot(c + d*x))*tan(c + d*x)
**3)/cot(c + d*x),x)*a*b**3 + 6*int((sqrt(cot(c + d*x))*tan(c + d*x)**2)/c
ot(c + d*x),x)*a**2*b**2 + 4*int((sqrt(cot(c + d*x))*tan(c + d*x))/cot(c +
 d*x),x)*a**3*b
```

3.593 $\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 265

$$\begin{aligned}
\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx &= \frac{(b(A-B)-a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\
&\quad - \frac{(b(A-B)-a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\
&\quad - \frac{2b^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}\,(a^2+b^2)\,d} \\
&\quad - \frac{(a(A-B)+b(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\
&\quad + \frac{2(Ab-aB)\sqrt{\cot(c+dx)}}{a^2d} - \frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}
\end{aligned}$$

output
```
-1/2*(b*(A-B)-a*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^
2)/d-1/2*(b*(A-B)-a*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2
+b^2)/d-2*b^(5/2)*(A*b-B*a)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(5/
2)/(a^2+b^2)/d-1/2*(a*(A-B)+b*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+c
ot(d*x+c))*2^(1/2)/(a^2+b^2)/d+2*(A*b-B*a)*cot(d*x+c)^(1/2)/a^2/d-2/3*A*c
ot(d*x+c)^(3/2)/a/d
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{\sqrt{\cot(c+dx)}\left(-\frac{6\sqrt{2}(b(-A+B)+a(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)}{a^2+b^2}+\frac{24b^{5/2}(-Ab+aB)\,\text{arc}}{a^{5/2}(a}\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
-1/12*(Sqrt[Cot[c + d*x]]*((-6*Sqrt[2]*(b*(-A + B) + a*(A + B))*(ArcTan[1
- Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(
a^2 + b^2) + (24*b^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]]
)/Sqrt[a]])/(a^(5/2)*(a^2 + b^2)) - (3*Sqrt[2]*(a*(A - B) + b*(A + B))*(Lo
g[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2) + (8*A)/(a*Tan[c + d*x]^(3/2)) +
 (24*(-(A*b) + a*B))/(a^2*Sqrt[Tan[c + d*x]]))*Sqrt[Tan[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 1.76 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.08, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.758$, Rules used = {3042, 4064, 3042, 4090, 27, 3042, 4130, 27, 3042, 4136, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{5/2}(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A\cot(c+dx)+B)}{a\cot(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{b-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4090}$$

$$-\frac{2\int \frac{3\sqrt{\cot(c+dx)}((Ab-aB)\cot^2(c+dx)+aA\cot(c+dx)+Ab)}{2(b+a\cot(c+dx))}dx}{3a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{\cot(c+dx)}((Ab-aB)\cot^2(c+dx)+aA\cot(c+dx)+Ab)}{b+a\cot(c+dx)}dx}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{-\tan(c+dx+\frac{\pi}{2})}\left((Ab-aB)\tan(c+dx+\frac{\pi}{2})^2-aA\tan(c+dx+\frac{\pi}{2})+Ab\right)}{b-a\tan(c+dx+\frac{\pi}{2})}dx}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{4130}$$

$$-\frac{-\frac{2\int \frac{-B\cot(c+dx)a^2-\left(Aa^2+bBa-Ab^2\right)\cot^2(c+dx)+b(Ab-aB)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a}-\frac{2(Ab-aB)\sqrt{\cot(c+dx)}}{ad}}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{-B\cot(c+dx)a^2-\left(Aa^2+bBa-Ab^2\right)\cot^2(c+dx)+b(Ab-aB)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a}-\frac{2(Ab-aB)\sqrt{\cot(c+dx)}}{ad}}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\int \frac{B\tan\left(c+dx+\frac{\pi}{2}\right)a^2+\left(-Aa^2-bBa+Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2+b(Ab-aB)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{2(Ab-aB)\sqrt{\cot(c+dx)}}{ad}}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 4136

$$-\frac{-\frac{\frac{\int \frac{a^2(Ab-aB)-a^2(aA+bB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{b^3(Ab-aB)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}}{a}-\frac{2(Ab-aB)\sqrt{\cot(c+dx)}}{ad}}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 3042

$$-\frac{-\frac{\frac{\int \frac{(Ab-aB)a^2+(aA+bB)\tan\left(c+dx+\frac{\pi}{2}\right)a^2}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{a}-\frac{2(Ab-aB)\sqrt{\cot(c+dx)}}{ad}}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 4017

$$-\frac{-\frac{\frac{2\int -\frac{a^2(Ab-aB-(aA+bB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{a}-\frac{2(Ab-aB)\sqrt{\cot(c+dx)}}{ad}}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 25

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{a^2(Ab-aB-(aA+bB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}}{a}-\frac{2(Ab-aB)\sqrt{\cot(c+dx)}}{ad}}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a^2\int \frac{Ab-aB-(aA+bB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}}{a}-\frac{2(Ab-aB)\sqrt{\cot(c+dx)}}{ad}}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$\Big\downarrow$ 1482

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(b(A-B)-a(A+B))\int \frac{\cot(c+dx}{\cot^2(c+da}\right.}{d\left(a^2+b^2\right)}}{a}\cdots}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$\Big\downarrow$ 1476

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{1}{2}\int \frac{}{\cot(c+}\right.\right.}{d\left(a^2+\cdots\right.}}{a}\cdots}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$\Big\downarrow$ 1082

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(b(A-B)-a(A+B))\left(\int \frac{1}{-\cot(c-}\right.\right.}{d\left(a^2+b^2\right)}}{a}\cdots}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$\Big\downarrow$ 217

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{}\right.\right.}{d\left(a^2+b^2\right)}}{a}\cdots}{a}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \; 1479$$

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{\ldots}\right)}{\cot(c+d\ldots}\ldots}{\ldots}\right)\ldots\right)}{\ldots}}{a}}{a}\ldots$$

$$\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{c\ldots}\right)}{\cot(c+dx)+\sqrt{\ldots}}\ldots}{\ldots}\right)\ldots\right)}{\ldots}}{a}}{a}\ldots$$

$$\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{c\ldots}}{\cot(c+dx)+\ldots}\ldots\right)\ldots\right)}{\ldots}}{a}}{a}\ldots$$

$$\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \; 1103$$

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a^2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a\ldots\right)}{d\left(c\ldots\right)}}{a}}{a}\ldots$$

$$\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \; 4117$$

$$-\frac{-\frac{\frac{b^3(Ab-aB)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}-\frac{2a^2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a}{d(a}}{a}}{a}$$

$$\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{-\frac{2b^3(Ab-aB)\int \frac{1}{a\cot^2(c+dx)+b}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}-\frac{2a^2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a(A-B)+b(}{d(a^2+b^2)}}{a}}{a}$$

$$\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

$$\Big\downarrow \text{218}$$

$$-\frac{-\frac{\frac{2b^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{a}\cot(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}d(a^2+b^2)}-\frac{2a^2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a(A-B)+b(A+B))\Big(}{d(a^2+b^2)}}{a}}{a}$$

$$\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad}$$

input

```
Int[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output

```
(-2*A*Cot[c + d*x]^(3/2))/(3*a*d) - ((-2*(A*b - a*B)*Sqrt[Cot[c + d*x]])/(
a*d) - ((2*b^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[a]*Cot[c + d*x])/Sqrt[b]])/(Sq
rt[a]*(a^2 + b^2)*d) - (2*a^2*(((b*(A - B) - a*(A + B))*(-(ArcTan[1 - Sqrt
[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/
Sqrt[2]))/2 + ((a*(A - B) + b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c +
d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
 + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/a)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
& !GtQ[n, 0] && !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(231) = 462$.

Time = 0.47 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.91

method	result
derivativedivides	$\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}}\tan(dx+c)\left(-3A\tan(dx+c)^{\frac{3}{2}}\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\sqrt{2}\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)\sqrt{2}\sqrt{ab}\,a^3-6A\tan(dx+c)^{\frac{3}{2}}\arctan\left(\right.\right.$
default	$\left(\frac{1}{\tan(dx+c)}\right)^{\frac{5}{2}}\tan(dx+c)\left(-3A\tan(dx+c)^{\frac{3}{2}}\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\sqrt{\tan(dx+c)}+1}{\sqrt{2}\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)\sqrt{2}\sqrt{ab}\,a^3-6A\tan(dx+c)^{\frac{3}{2}}\arctan\left(\right.\right.$

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/12/d*(1/tan(d*x+c))^(5/2)*tan(d*x+c)*(-3*A*tan(d*x+c)^(3/2)*ln(-(tan(d*x
+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2
^(1/2)*(a*b)^(1/2)*a^3-6*A*tan(d*x+c)^(3/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1
/2))*2^(1/2)*(a*b)^(1/2)*a^3+6*A*tan(d*x+c)^(3/2)*arctan(1+2^(1/2)*tan(d*x
+c)^(1/2))*2^(1/2)*(a*b)^(1/2)*a^2*b-6*A*tan(d*x+c)^(3/2)*arctan(-1+2^(1/2
)*tan(d*x+c)^(1/2))*2^(1/2)*(a*b)^(1/2)*a^3+6*A*tan(d*x+c)^(3/2)*arctan(-1
+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*(a*b)^(1/2)*a^2*b+3*A*tan(d*x+c)^(3/2)*
ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)
^(1/2)+1))*2^(1/2)*(a*b)^(1/2)*a^2*b-3*B*tan(d*x+c)^(3/2)*ln(-(tan(d*x+c)+
2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/
2)*(a*b)^(1/2)*a^2*b-6*B*tan(d*x+c)^(3/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2
))*2^(1/2)*(a*b)^(1/2)*a^3-6*B*tan(d*x+c)^(3/2)*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))*2^(1/2)*(a*b)^(1/2)*a^2*b-6*B*tan(d*x+c)^(3/2)*arctan(-1+2^(1/2)*
tan(d*x+c)^(1/2))*2^(1/2)*(a*b)^(1/2)*a^3-6*B*tan(d*x+c)^(3/2)*arctan(-1+2
^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*(a*b)^(1/2)*a^2*b-3*B*tan(d*x+c)^(3/2)*ln
(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(
1/2)+1))*2^(1/2)*(a*b)^(1/2)*a^3+24*A*tan(d*x+c)^(3/2)*arctan(b*tan(d*x+c)
^(1/2)/(a*b)^(1/2))*b^4-24*B*tan(d*x+c)^(3/2)*arctan(b*tan(d*x+c)^(1/2)/(a
*b)^(1/2))*a*b^3+24*A*tan(d*x+c)*(a*b)^(1/2)*a^2*b+24*A*tan(d*x+c)*(a*b)^(
1/2)*b^3-24*B*tan(d*x+c)*(a*b)^(1/2)*a^3-24*B*tan(d*x+c)*(a*b)^(1/2)*a*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. 2(231) = 462.

Time = 24.79 (sec) , antiderivative size = 2726, normalized size of antiderivative = 10.29

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[-1/6*(6*sqrt(1/2)*(a^4 + a^2*b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^
2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arc
tan(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 -
B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((
A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^
4 + 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A
 - B)*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)
*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*
x + c)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2))*tan(d*x + c) + 6
*sqrt(1/2)*(a^4 + a^2*b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)
*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a
^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*
b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2
*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a
^2*b^2 + b^4)*d^2)) - 2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A - B)*a
*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b +
(A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c))
)/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2))*tan(d*x + c) + 3*sqrt(1
/2)*(a^4 + a^2*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b +
(A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*log(2*sqrt(1/2)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.99

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$= \frac{\frac{24\,(Bab^3-Ab^4)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^4+a^2b^2)\sqrt{ab}} + \frac{3\left(2\sqrt{2}((A+B)a-(A-B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((A+B)a-(A-B)b)\,\mathrm{arct}\right.}{}}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/12*(24*(B*a*b^3 - A*b^4)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^4
+ a^2*b^2)*sqrt(a*b)) + 3*(2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(1/2*sq
rt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a - (A - B)*b
)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((A - B)
*a + (A + B)*b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqr
t(2)*((A - B)*a + (A + B)*b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x +
 c) + 1))/(a^2 + b^2) - 8*(A*a/tan(d*x + c)^(3/2) + 3*(B*a - A*b)/sqrt(tan
(d*x + c)))/a^2)/d
```

Giac [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{5}{2}}}{b\tan(dx+c)+a}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{\cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))}{a+b\tan\left(c+d\,x\right)}\,dx$$

input
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \sqrt{\cot\left(dx+c\right)}\,\cot\left(dx+c\right)^{2}dx$$

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x))*cot(c + d*x)**2,x)
```

3.594 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 236

$$\begin{aligned}\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx &= -\frac{(a(A-B)+b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad+\frac{(a(A-B)+b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} \\ &\quad+\frac{2b^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}\,(a^2+b^2)\,d} \\ &\quad-\frac{(b(A-B)-a(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}-\frac{2A\sqrt{\cot(c+dx)}}{ad}\end{aligned}$$

output

```
1/2*(a*(A-B)+b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2
)/d+1/2*(a*(A-B)+b*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+
b^2)/d+2*b^(3/2)*(A*b-B*a)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(3/2
)/(a^2+b^2)/d-1/2*(b*(A-B)-a*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+co
t(d*x+c)))*2^(1/2)/(a^2+b^2)/d-2*A*cot(d*x+c)^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.06

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}\left(\frac{2\sqrt{2}(a(A-B)+b(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)}{a^2+b^2}+\frac{8b^{3/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}}{}\right.}{a^{3/2}(a^2+b^2)}\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
(Sqrt[Cot[c + d*x]]*((2*Sqrt[2]*(a*(A - B) + b*(A + B))*(ArcTan[1 - Sqrt[2
]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(a^2 + b^
2) + (8*b^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]
])/(a^(3/2)*(a^2 + b^2)) - (Sqrt[2]*(b*(-A + B) + a*(A + B))*(Log[1 - Sqrt
[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]
] + Tan[c + d*x]]))/(a^2 + b^2) - (8*A)/(a*Sqrt[Tan[c + d*x]]))*Sqrt[Tan[c
 + d*x]])/(4*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.42 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.06, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4064, 3042, 4090, 27, 3042, 4136, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A\cot(c+dx)+B)}{a\cot(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{b-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4090}$$

$$-\frac{2\int \frac{(Ab-aB)\cot^2(c+dx)+aA\cot(c+dx)+Ab}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a}-\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(Ab-aB)\cot^2(c+dx)+aA\cot(c+dx)+Ab}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a}-\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2-aA\tan\left(c+dx+\frac{\pi}{2}\right)+Ab}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{4136}$$

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{a(aA+bB)+a(Ab-aB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}}{a}-\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{\int \frac{a(aA+bB)-a(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{a}-\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$$\Big\downarrow \text{4017}$$

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{2\int -\frac{a(aA+bB+(Ab-aB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}}{a}-\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\downarrow$ 25

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{a(aA+bB+(Ab-aB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}}{a}-\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\downarrow$ 27

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a\int \frac{aA+bB+(Ab-aB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}}{a}-\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\downarrow$ 1482

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\int \cdots\right.}{d(a^2+b^2)}}{a}\cdots$$
$$\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\downarrow$ 1476

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \cdots\right.\right.}{\cdots}}{a}\cdots$$
$$\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\downarrow$ 1082

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\int \frac{1}{-\cot(c+dx)-1}\cdots\right.\right.}{d(a^2+b\cdots}}{a}\cdots$$
$$\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\downarrow$ 217

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\cdots}{d(a^2+b^2)}}{a}$$

$$\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\big\downarrow$ 1479

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\cdots}{\cdots}}{a}$$

$$\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\big\downarrow$ 25

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\cdots}{\cdots}}{a}$$

$$\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\big\downarrow$ 27

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\cdots}{\cdots}}{a}$$

$$\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\big\downarrow$ 1103

$$-\frac{\frac{b^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\cdots}{\cdots}}{a}$$

$$\frac{2A\sqrt{\cot(c+dx)}}{ad}$$

$\big\downarrow$ 4117

$$-\frac{\frac{b^2(Ab-aB)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)}{a}}{\frac{2A\sqrt{\cot(c+dx)}}{ad}}$$

$\downarrow$ 73

$$-\frac{-\frac{2b^2(Ab-aB)\int \frac{1}{a\cot^2(c+dx)+b}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(b(A-}{d(a^2+}}{a}}{\frac{2A\sqrt{\cot(c+dx)}}{ad}}$$

$\downarrow$ 218

$$-\frac{\frac{2b^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{a}\cot(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}d(a^2+b^2)}-\frac{2a\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(b(A-B)-a(A}{d(a^2+b^2)}}{a}}{\frac{2A\sqrt{\cot(c+dx)}}{ad}}$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(-2*A*Sqrt[Cot[c + d*x]])/(a*d) - ((2*b^(3/2)*(A*b - a*B)*ArcTan[(Sqrt[a]*
Cot[c + d*x])/Sqrt[b]])/(Sqrt[a]*(a^2 + b^2)*d) - (2*a*(((a*(A - B) + b*(A
 + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 - ((b*(A - B) - a*(A + B))*(-1/2*Log[
1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(206) = 412$.

Time = 0.37 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.91

method	result
derivativedivides	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)\left(A\sqrt{\tan(dx+c)}\,\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)\sqrt{2}\,\sqrt{ab}\,ab+2A\sqrt{\tan(dx+c)}\,\arctan(}{}$
default	$-\frac{\left(\frac{1}{\tan(dx+c)}\right)^{\frac{3}{2}}\tan(dx+c)\left(A\sqrt{\tan(dx+c)}\,\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)\sqrt{2}\,\sqrt{ab}\,ab+2A\sqrt{\tan(dx+c)}\,\arctan(}{}$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
-1/4/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(A*tan(d*x+c)^(1/2)*ln(-(tan(d*x+c)
+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1
/2)*(a*b)^(1/2)*a*b+2*A*tan(d*x+c)^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2)
)*2^(1/2)*(a*b)^(1/2)*a^2+2*A*tan(d*x+c)^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)
^(1/2))*2^(1/2)*(a*b)^(1/2)*a*b+2*A*tan(d*x+c)^(1/2)*arctan(-1+2^(1/2)*tan
(d*x+c)^(1/2))*2^(1/2)*(a*b)^(1/2)*a^2+2*A*tan(d*x+c)^(1/2)*arctan(-1+2^(1
/2)*tan(d*x+c)^(1/2))*2^(1/2)*(a*b)^(1/2)*a*b+A*tan(d*x+c)^(1/2)*ln(-(2^(1
/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)
)*2^(1/2)*(a*b)^(1/2)*a^2-B*tan(d*x+c)^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d
*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)*(a*b)^(1/2
)*a^2-2*B*tan(d*x+c)^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*(a*b
)^(1/2)*a^2+2*B*tan(d*x+c)^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2
)*(a*b)^(1/2)*a*b-2*B*tan(d*x+c)^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))
*2^(1/2)*(a*b)^(1/2)*a^2+2*B*tan(d*x+c)^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)
^(1/2))*2^(1/2)*(a*b)^(1/2)*a*b+B*tan(d*x+c)^(1/2)*ln(-(2^(1/2)*tan(d*x+c)
^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(1/2)*(a*b
)^(1/2)*a*b+8*A*tan(d*x+c)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*b^
3-8*B*tan(d*x+c)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a*b^2+8*A*(a
*b)^(1/2)*a^2+8*A*(a*b)^(1/2)*b^2)/a/(a^2+b^2)/(a*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1271 vs. $2(206) = 412$.

Time = 10.72 (sec) , antiderivative size = 2572, normalized size of antiderivative = 10.90

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[-1/2*(2*sqrt(1/2)*(a^3 + a*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2
- B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arcta
n(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^
2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^
2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4
+ 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A +
 B)*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a
*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x
+ c)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + 2*sqrt(1/2)*(a^3
 + a*b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A
*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 +
b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B
+ B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2*A*B + B^2)*a^2 +
 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2
)) - 2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A + B)*a*b^2 - (A - B)*b^
3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^
2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(4*A*B*a*b + (A
^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) - sqrt(1/2)*(a^3 + a*b^2)*d*sqrt(((A^2 +
 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 + 2*A*B ...
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{(A+B\tan{(c+dx)})\cot^{\frac{3}{2}}{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**(3/2)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.01

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx =$$

$$-\frac{\frac{8\,(Bab^2-Ab^3)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^3+ab^2)\sqrt{ab}} - \frac{2\sqrt{2}((A-B)a+(A+B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((A-B)a+(A+B)b)\,\mathrm{arcta}}{}}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/4*(8*(B*a*b^2 - A*b^3)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^3 +
 a*b^2)*sqrt(a*b)) - (2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(1/2*sqrt(2)
*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a + (A + B)*b)*arc
tan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*((A + B)*a -
(A - B)*b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*
((A + B)*a - (A - B)*b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) +
 1))/(a^2 + b^2) + 8*A/(a*sqrt(tan(d*x + c))))/d
```

Giac [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{3}{2}}}{b\tan(dx+c)+a}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{\cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{a+b\tan\,(c+d\,x)}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{-2\sqrt{\cot\,(dx+c)}-\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)d}{d}$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
( - 2*sqrt(cot(c + d*x)) - int(sqrt(cot(c + d*x))/cot(c + d*x),x)*d)/d
```

3.595 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 217

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(b(A-B)-a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(b(A-B)-a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{2\sqrt{b}(Ab-aB)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\,(a^2+b^2)\,d} + \frac{(a(A-B)+b(A+B))\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}$$

output
```
1/2*(b*(A-B)-a*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2
)/d+1/2*(b*(A-B)-a*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+
b^2)/d-2*b^(1/2)*(A*b-B*a)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(1/2
)/(a^2+b^2)/d+1/2*(a*(A-B)+b*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+co
t(d*x+c)))*2^(1/2)/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}\left(-2\sqrt{2}(b(-A+B)+a(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\text{ta}\cdots}\right.\right.\right.}{}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x
]
```

output
```
(Sqrt[Cot[c + d*x]]*(-2*Sqrt[2]*(b*(-A + B) + a*(A + B))*(ArcTan[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (8*Sqrt
[b]*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[a] - Sq
rt[2]*(a*(A - B) + b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))*Sqrt[Tan[c
+ d*x]])/(4*(a^2 + b^2)*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.576$, Rules used = {3042, 4064, 3042, 4095, 25, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A\cot(c+dx)+B)}{a\cot(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{b-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4095}$$

$$\frac{b(Ab-aB)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}+\frac{\int -\frac{Ab-aB-(aA+bB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b(Ab-aB)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}-\frac{\int \frac{Ab-aB-(aA+bB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{\int \frac{Ab-aB-(-aA-bB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}$$

$$\Big\downarrow \text{4017}$$

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int -\frac{Ab-aB-(aA+bB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{2\int \frac{Ab-aB-(aA+bB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{1482}$$

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\left(-\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+\right.}{d\left(a^2+b^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\left(-\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}\right.\right.}{d\left(a^2+b^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\left(-\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}$$

$\Big\downarrow$ 217

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\left(-\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}$$

$\Big\downarrow$ 1479

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\left(-\frac{1}{2}(a(A-B)+b(A+B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\right.}{d\left(a^2+b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\left(-\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(b\right.}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\left(-\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{b(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\left(-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)+b(A+B))\left(\right.\right.}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{4117}$$

$$\frac{b(Ab-aB)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}-\frac{2\left(-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)+b(A+B))\left(\right.\right.}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{2b(Ab-aB)\int \frac{1}{a\cot^2(c+dx)+b}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{2\left(-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)+b(A+B))\left(\right.\right.}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{b}(Ab-aB)\arctan\left(\frac{\sqrt{a}\cot(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}d\left(a^2+b^2\right)}-\frac{2\left(-\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(a(A-B)+b(A+B))\left(\right.\right.}{d\left(a^2+b^2\right)}$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(2*Sqrt[b]*(A*b - a*B)*ArcTan[(Sqrt[a]*Cot[c + d*x])/Sqrt[b]])/(Sqrt[a]*(a
^2 + b^2)*d) - (2*(-1/2*((b*(A - B) - a*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqr
t[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]
)) - ((a*(A - B) + b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + C
ot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/
(2*Sqrt[2])))/2))/((a^2 + b^2)*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4095
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]])/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[1
/(a^2 + b^2)   Int[Simp[A*(a*c + b*d) + B*(b*c - a*d) - (A*(b*c - a*d) - B*
(a*c + b*d))*Tan[e + f*x], x]/Sqrt[c + d*Tan[e + f*x]], x], x] - Simp[(b*c
- a*d)*((B*a - A*b)/(a^2 + b^2))   Int[(1 + Tan[e + f*x]^2)/((a + b*Tan[e +
 f*x])*Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(190) = 380$.

Time = 0.39 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.45

method	result
derivativedivides	$\sqrt{\frac{1}{\tan(dx+c)}}\,\sqrt{\tan(dx+c)}\,\left(A\sqrt{2}\,\sqrt{ab}\,\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)a+2A\sqrt{2}\,\sqrt{ab}\,\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right.$
default	$\sqrt{\frac{1}{\tan(dx+c)}}\,\sqrt{\tan(dx+c)}\,\left(A\sqrt{2}\,\sqrt{ab}\,\ln\left(-\frac{\tan(dx+c)+\sqrt{2}\,\sqrt{\tan(dx+c)}+1}{\sqrt{2}\,\sqrt{\tan(dx+c)}-\tan(dx+c)-1}\right)a+2A\sqrt{2}\,\sqrt{ab}\,\arctan\left(1+\sqrt{2}\,\sqrt{\tan(dx+c)}\right)\right.$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/4/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)^(1/2)*(A*2^(1/2)*(a*b)^(1/2)*ln(-(ta
n(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-
1))*a+2*A*2^(1/2)*(a*b)^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*a-2*A*2^(
1/2)*(a*b)^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*b+2*A*2^(1/2)*(a*b)^(1
/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a-2*A*2^(1/2)*(a*b)^(1/2)*arctan(-
1+2^(1/2)*tan(d*x+c)^(1/2))*b-A*2^(1/2)*(a*b)^(1/2)*ln(-(2^(1/2)*tan(d*x+c
)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*b+B*2^(1/2)
*(a*b)^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+
c)^(1/2)-tan(d*x+c)-1))*b+2*B*2^(1/2)*(a*b)^(1/2)*arctan(1+2^(1/2)*tan(d*x
+c)^(1/2))*a+2*B*2^(1/2)*(a*b)^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*b+
2*B*2^(1/2)*(a*b)^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a+2*B*2^(1/2)*
(a*b)^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*b+B*2^(1/2)*(a*b)^(1/2)*ln
(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(
1/2)+1))*a+8*A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*b^2-8*B*arctan(b*tan
(d*x+c)^(1/2)/(a*b)^(1/2))*a*b)/(a^2+b^2)/(a*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(190) = 380$.

Time = 3.88 (sec) , antiderivative size = 2488, normalized size of antiderivative = 11.47

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[1/2*(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B
^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan((
(a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*
a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 -
 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A - B)
*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b
+ (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c
)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + 2*sqrt(1/2)*(a^2 +
b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B +
B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 + b^4)*
d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2
)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A
^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) -
2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A - B)*a*b^2 + (A + B)*b^3)*d*
sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^
2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(4*A*B*a*b + (A^2 -
B^2)*a^2 - (A^2 - B^2)*b^2)) + sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 - 2*A*B
+ B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^
2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*...
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{\cot{(c+dx)}}}{a+b\tan{(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{8\,(Bab-Ab^2)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^2+b^2)\sqrt{ab}} - \frac{2\sqrt{2}((A+B)a-(A-B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((A+B)a-(A-B)b)\arctan\left(-\right.}{}}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/4*(8*(B*a*b - A*b^2)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2 + b^
2)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(1/2*sqrt(2)*(sqr
t(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a - (A - B)*b)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((A - B)*a + (A +
B)*b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((A -
 B)*a + (A + B)*b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/
(a^2 + b^2))/d
```

Giac [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{\cot(dx+c)}}{b\tan(dx+c)+a}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(cot(d*x + c))/(b*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{\sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)}{a+b\tan\left(c+d\,x\right)}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \sqrt{\cot\left(dx+c\right)}dx$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x)),x)
```

3.596 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 216

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx &= \frac{(a(A-B)+b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\\ &\quad-\frac{(a(A-B)+b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\\ &\quad+\frac{2\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}\,(a^2+b^2)\,d}\\ &\quad+\frac{(b(A-B)-a(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d}\end{aligned}$$

output
```
-1/2*(a*(A-B)+b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^
2)/d-1/2*(a*(A-B)+b*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2
+b^2)/d+2*a^(1/2)*(A*b-B*a)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/b^(1/
2)/(a^2+b^2)/d+1/2*(b*(A-B)-a*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+c
ot(d*x+c))*2^(1/2)/(a^2+b^2)/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx =$$

$$-\frac{\sqrt{\cot(c+dx)}\left(2\sqrt{2}(a(A-B)+b(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])),x
]
```

output
```
-1/4*(Sqrt[Cot[c + d*x]]*(2*Sqrt[2]*(a*(A - B) + b*(A + B))*(ArcTan[1 - Sq
rt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (8*S
qrt[a]*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[b] -
 Sqrt[2]*(b*(-A + B) + a*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))*Sqrt[Ta
n[c + d*x]])/((a^2 + b^2)*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4064, 3042, 4096, 3042, 4017, 25, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \; 4064$$

$$\int \frac{A\cot(c+dx)+B}{\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 4096$$

$$\frac{\int \frac{aA+bB+(Ab-aB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}-\frac{a(Ab-aB)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{aA+bB-(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}-\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 4017$$

$$\frac{2\int -\frac{aA+bB+(Ab-aB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{aA+bB+(Ab-aB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 1482$$

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A-B)+b(A+B))\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+d}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 1082

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(\right.}{\sqrt{2}}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 217

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 1479

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\downarrow$ 25

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)-\frac{1}{2}(a(}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\Big\downarrow$ 27

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\Big\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(a(A-B)+b\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}$$

$\Big\downarrow$ 4117

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(a(A-B)+b\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}$$

$\Big\downarrow$ 73

$$\frac{2a(Ab-aB)\int \frac{1}{a\cot^2(c+dx)+b}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+$$

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(a(A-B)+b\right.}{d\left(a^2+b^2\right)}$$

$\downarrow$ 218

$$\frac{2\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}\right)-\frac{1}{2}(a(A-B)+b}{d\left(a^2+b^2\right)}$$

$$\frac{2\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{a}\cot(c+dx)}{\sqrt{b}}\right)}{\sqrt{b}d\left(a^2+b^2\right)}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])),x]
```

output
```
(-2*Sqrt[a]*(A*b - a*B)*ArcTan[(Sqrt[a]*Cot[c + d*x])/Sqrt[b]])/(Sqrt[b]*(
a^2 + b^2)*d) + (2*(-1/2*((a*(A - B) + b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sq
rt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2
])) + ((b*(A - B) - a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] +
Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]
/(2*Sqrt[2])))/2))/((a^2 + b^2)*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4096
```
Int[(((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_
.)*(x_)])^(n_))/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[
1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*Simp[a*A + b*B - (A*b - a*B)*Tan
[e + f*x], x], x], x] + Simp[b*((A*b - a*B)/(a^2 + b^2))   Int[(c + d*Tan[e
 + f*x])^n*((1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a,
 b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[c^2 + d^2, 0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{2(Ab-Ba)a\arctan\left(\frac{a\sqrt{\cot(dx+c)}}{\sqrt{ab}}\right)}{\left(a^2+b^2\right)\sqrt{ab}}-\frac{2\left(\frac{(Aa+Bb)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\,\mathrm{arct}}{8}\right.}{}}{}$
default	$\frac{\frac{2(Ab-Ba)a\arctan\left(\frac{a\sqrt{\cot(dx+c)}}{\sqrt{ab}}\right)}{\left(a^2+b^2\right)\sqrt{ab}}-\frac{2\left(\frac{(Aa+Bb)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\,\mathrm{arct}}{8}\right.}{}}{}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(2*(A*b-B*a)*a/(a^2+b^2)/(a*b)^(1/2)*arctan(a*cot(d*x+c)^(1/2)/(a*b)^(
1/2))-2/(a^2+b^2)*(1/8*(A*a+B*b)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c
)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot
(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/8*(A*b-B*a)*2^(1/2
)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+
c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(
d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1241 vs. $2(189) = 378$.

Time = 2.67 (sec) , antiderivative size = 2508, normalized size of antiderivative = 11.61

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[1/2*(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B
^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan((
(a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*
a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 -
 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A + B)
*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b
+ (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c
)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + 2*sqrt(1/2)*(a^2 +
b^2)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B +
B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 + b^4)*
d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2
)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A
^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2)) -
2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A + B)*a*b^2 - (A - B)*b^3)*d*
sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^
2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(4*A*B*a*b + (A^2 -
B^2)*a^2 - (A^2 - B^2)*b^2)) - sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 + 2*A*B
+ B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^
2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 + 2*A*B + B^2)*...
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))}\,dx = \int \frac{A + B\tan{(c + dx)}}{(a + b\tan{(c + dx)})\sqrt{\cot{(c + dx)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))*sqrt(cot(c + d*x))), x
)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.02

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx =$$

$$-\frac{\frac{8\,(Ba^2-Aab)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^2+b^2)\sqrt{ab}}+\frac{2\,\sqrt{2}((A-B)a+(A+B)b)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\,\sqrt{2}((A-B)a+(A+B)b)\arctan}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/4*(8*(B*a^2 - A*a*b)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2 + b
^2)*sqrt(a*b)) + (2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(1/2*sqrt(2)*(sq
rt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(
-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*((A + B)*a - (A -
 B)*b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*((A
+ B)*a - (A - B)*b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))
/(a^2 + b^2))/d
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx = \int \frac{B\tan(dx+c)+A}{(b\tan(dx+c)+a)\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)*sqrt(cot(d*x + c))),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx=\int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot\left(c+d\,x\right)}\,\left(a+b\tan\left(c+d\,x\right)\right)}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx=\int \frac{\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x))/cot(c + d*x),x)
```

3.597 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 237

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{(b(A-B)-a(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{(b(A-B)-a(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} - \frac{2a^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}\,(a^2+b^2)\,d} - \frac{(a(A-B)+b(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{2B}{bd\sqrt{\cot(c+dx)}}$$

output

```
-1/2*(b*(A-B)-a*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^
2)/d-1/2*(b*(A-B)-a*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2
+b^2)/d-2*a^(3/2)*(A*b-B*a)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/b^(3/
2)/(a^2+b^2)/d-1/2*(a*(A-B)+b*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+c
ot(d*x+c)))*2^(1/2)/(a^2+b^2)/d+2*B/b/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = -\frac{\sqrt{\cot(c+dx)}\left(2\sqrt{2}b^{3/2}(b(A-B)-a(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\right.\right.\right.}}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x
]
```

output
```
-1/4*(Sqrt[Cot[c + d*x]]*(2*Sqrt[2]*b^(3/2)*(b*(A - B) - a*(A + B))*(ArcTa
n[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
) + 8*a^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]
- Sqrt[2]*b^(3/2)*(a*(A - B) + b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])
- 8*Sqrt[b]*(a^2 + b^2)*B*Sqrt[Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/(b^(3/2)
*(a^2 + b^2)*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.44 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4064, 3042, 4092, 27, 3042, 4136, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{A\cot(c+dx)+B}{\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{2\int \frac{-aB\cot^2(c+dx)-bB\cot(c+dx)+Ab-aB}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b}+\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-aB\cot^2(c+dx)-bB\cot(c+dx)+Ab-aB}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b}+\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-aB\tan\left(c+dx+\frac{\pi}{2}\right)^2+bB\tan\left(c+dx+\frac{\pi}{2}\right)+Ab-aB}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4136}$$

$$\frac{\frac{a^2(Ab-aB)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}+\frac{\int \frac{b(Ab-aB)-b(aA+bB)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}}{b}+\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{\int \frac{b(Ab-aB)+b(aA+bB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{b}+\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$$\Big\downarrow \text{4017}$$

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{2\int -\frac{b(Ab-aB-(aA+bB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}}{b}+\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\downarrow$ 25

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{b(Ab-aB-(aA+bB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}}{b}+\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\downarrow$ 27

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b\int \frac{Ab-aB-(aA+bB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}}{b}+\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\downarrow$ 1482

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(b(A-B)-a(A+B))\int \frac{\cot}{\cot}\right.}{d(a^2+b^2)}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\downarrow$ 1476

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{1}{2}\int\right.\right.}{d}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\downarrow$ 1082

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\int}{}\right.\right.}{d(a^2+b^2)}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\downarrow$ 217

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\text{arc}}{}\right.\right.}{d(a^2+b^2)}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\downarrow$ 1479

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}}{\cot(c+}}{}\right.\right.}{}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\downarrow$ 25

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{}\right.}{\cot(c+dx)+}}{}\right.\right.}{}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\downarrow$ 27

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}}{\cot(c+dx}\right.\right.}{}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\downarrow$ 1103

$$\frac{\frac{a^2(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\right.}{a}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\Big\downarrow$ 4117

$$\frac{\frac{a^2(Ab-aB)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}-\frac{2b\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\right.}{}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\Big\downarrow$ 73

$$\frac{-\frac{2a^2(Ab-aB)\int \frac{1}{a\cot^2(c+dx)+b}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}-\frac{2b\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a(A-B\right.}{d(a^2+b^2)}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

$\Big\downarrow$ 218

$$\frac{\frac{2a^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{a}\cot(c+dx)}{\sqrt{b}}\right)}{\sqrt{b}d(a^2+b^2)}-\frac{2b\left(\frac{1}{2}(b(A-B)-a(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}(a(A-B)+b(A+\right.}{d(a^2+b^2)}}{b}$$

$$\frac{2B}{bd\sqrt{\cot(c+dx)}}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]
```

output
```
(2*B)/(b*d*Sqrt[Cot[c + d*x]]) + ((2*a^(3/2)*(A*b - a*B)*ArcTan[(Sqrt[a]*C
ot[c + d*x])/Sqrt[b]])/(Sqrt[b]*(a^2 + b^2)*d) - (2*b*(((b*(A - B) - a*(A
+ B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 + ((a*(A - B) + b*(A + B))*(-1/2*Log[1
 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sq
rt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{-\frac{2(Ab-Ba)a^2\arctan\left(\frac{a\sqrt{\cot(dx+c)}}{\sqrt{ab}}\right)}{b\left(a^2+b^2\right)\sqrt{ab}}+\frac{2B}{b\sqrt{\cot(dx+c)}}-\frac{2\left(\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\right.}{8}}{}}{}$
default	$\frac{-\frac{2(Ab-Ba)a^2\arctan\left(\frac{a\sqrt{\cot(dx+c)}}{\sqrt{ab}}\right)}{b\left(a^2+b^2\right)\sqrt{ab}}+\frac{2B}{b\sqrt{\cot(dx+c)}}-\frac{2\left(\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\right.}{8}}{}}{}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(-2/b*(A*b-B*a)*a^2/(a^2+b^2)/(a*b)^(1/2)*arctan(a*cot(d*x+c)^(1/2)/(a
*b)^(1/2))+2*B/b/cot(d*x+c)^(1/2)-2/(a^2+b^2)*(1/8*(A*b-B*a)*2^(1/2)*(ln((
cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2
)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^
(1/2)))+1/8*(-A*a-B*b)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)
/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1
/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1267 vs. $2(207) = 414$.

Time = 8.08 (sec) , antiderivative size = 2560, normalized size of antiderivative = 10.80

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[-1/2*(2*sqrt(1/2)*(a^2*b + b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2
- B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arcta
n(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^
2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^
2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4
+ 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A -
 B)*a*b^2 + (A + B)*b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a
*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x
+ c)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + 2*sqrt(1/2)*(a^2
*b + b^3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A
*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^2 +
b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B
+ B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2*A*B + B^2)*a^2 +
 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2
)) - 2*sqrt(1/2)*((A - B)*a^3 + (A + B)*a^2*b + (A - B)*a*b^2 + (A + B)*b^
3)*d*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^
2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(4*A*B*a*b + (A
^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + sqrt(1/2)*(a^2*b + b^3)*d*sqrt(((A^2 -
 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 - 2*A*B ...
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx=\int \frac{A+B\tan{(c+dx)}}{(a+b\tan{(c+dx)})\cot^{\frac{3}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))*cot(c + d*x)**(3/2)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{\frac{8\,(Ba^3-Aa^2b)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^2b+b^3)\sqrt{ab}} + \frac{8\,B\sqrt{\tan(dx+c)}}{b} + \frac{2\sqrt{2}((A+B)a-(A-B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}((A+B)}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
1/4*(8*(B*a^3 - A*a^2*b)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2*b
+ b^3)*sqrt(a*b)) + 8*B*sqrt(tan(d*x + c))/b + (2*sqrt(2)*((A + B)*a - (A
- B)*b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*(
(A + B)*a - (A - B)*b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c))
)) - sqrt(2)*((A - B)*a + (A + B)*b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/ta
n(d*x + c) + 1) + sqrt(2)*((A - B)*a + (A + B)*b)*log(-sqrt(2)/sqrt(tan(d*
x + c)) + 1/tan(d*x + c) + 1))/(a^2 + b^2))/d
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \int \frac{B\tan(dx+c)+A}{(b\tan(dx+c)+a)\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)*cot(d*x + c)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{3/2}\,\left(a+b\tan\left(c+d\,x\right)\right)}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \int \frac{\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^{2}}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x))/cot(c + d*x)**2,x)
```

3.598 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 264

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = -\frac{(a(A-B)+b(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{(a(A-B)+b(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{2a^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}\,(a^2+b^2)\,d} - \frac{(b(A-B)-a(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)\,d} + \frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)} + \frac{2(Ab-aB)}{b^2d\sqrt{\cot(c+dx)}}$$

output
```
1/2*(a*(A-B)+b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2
)/d+1/2*(a*(A-B)+b*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+
b^2)/d+2*a^(5/2)*(A*b-B*a)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/b^(5/2
)/(a^2+b^2)/d-1/2*(b*(A-B)-a*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+co
t(d*x+c)))*2^(1/2)/(a^2+b^2)/d+2/3*B/b/d/cot(d*x+c)^(3/2)+2*(A*b-B*a)/b^2/
d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{6\sqrt{2}(a(A-B)+b(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)}{a^2+b^2}+\frac{24a^{5/2}(-\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x
]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((6*Sqrt[2]*(a*(A - B) + b*(A + B))
*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c +
 d*x]]]))/(a^2 + b^2) + (24*a^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a]])/(b^(5/2)*(a^2 + b^2)) - (3*Sqrt[2]*(b*(-A + B) + a*
(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2) + (24*(A*b - a*B)*Sq
rt[Tan[c + d*x]])/b^2 + (8*B*Tan[c + d*x]^(3/2))/b))/(12*d)
```

Rubi [A] (warning: unable to verify)

Time = 1.86 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.08, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.758$, Rules used = {3042, 4064, 3042, 4092, 27, 3042, 4132, 27, 3042, 4136, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{5/2}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{ 4064}$$

$$\int \frac{A\cot(c+dx)+B}{\cot^{\frac{5}{2}}(c+dx)(a\cot(c+dx)+b)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 4092}$$

$$\frac{2\int \frac{3(-aB\cot^2(c+dx)-bB\cot(c+dx)+Ab-aB)}{2\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))}dx}{3b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-aB\cot^2(c+dx)-bB\cot(c+dx)+Ab-aB}{\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))}dx}{b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{-aB\tan\left(c+dx+\frac{\pi}{2}\right)^2+bB\tan\left(c+dx+\frac{\pi}{2}\right)+Ab-aB}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 4132}$$

$$\frac{\frac{2\int -\frac{-Ba^2+(Ab-aB)\cot^2(c+dx)a+Aba+b^2B+Ab^2\cot(c+dx)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b}+\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}}{b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\int \frac{-Ba^2+(Ab-aB)\cot^2(c+dx)a+Aba+b^2B+Ab^2\cot(c+dx)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b}}{b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\int \frac{-Ba^2+(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2a+Aba+b^2B-Ab^2\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4136$$

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{\int \frac{(aA+bB)b^2+(Ab-aB)\cot(c+dx)b^2}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{a^3(Ab-aB)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}}{b}}{b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{\int \frac{b^2(aA+bB)-b^2(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{b}}{b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 4017$$

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{2\int -\frac{b^2(aA+bB+(Ab-aB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}+\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{b}}{b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{b^2(aA+bB+(Ab-aB)\cot(c+dx))}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}}{b}}{b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b^2\int \frac{aA+bB+(Ab-aB)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}}{b}}{b}+\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 1482

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}(b(A-B)-a(A+\cdots\right.}{d\left(a^2+b^2\right)}}{b}}{b}$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 1476

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}-\cdots\right.\right.}{\cdots}}{b}}{b}$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 1082

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\int \frac{\cdots}{-\cot\cdots}\right.\right.}{\cdots}}{b}}{b}$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 217

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot\cdots}\right.}{\sqrt{2}}\cdots\right.\right.}{d\left(a^2+b^2\right)\cdots}}{b}}{b}\cdots$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 1479

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot\cdots}\right.}{\sqrt{2}}\cdots\right.\right.}{\cdots}}{\cdots}}{b}\cdots$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 25

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot\cdots}\right.}{\sqrt{2}}\cdots\right.\right.}{\cdots}}{b}}{b}\cdots$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 27

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot\cdots}\right.}{\sqrt{2}}\cdots\right.\right.}{\cdots}}{b}}{b}\cdots$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\big\downarrow$ 1103

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot\cdots}\right)}{\sqrt{2}}\cdots\right.\right.}{\cdots}}{b}}{b}$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 4117

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^3(Ab-aB)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot\cdots}\right)}{\sqrt{2}}\cdots\right.\right.}{\cdots}}{b}}{b}$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 73

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{-\frac{2a^3(Ab-aB)\int \frac{1}{a\cot^2(c+dx)+b}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)\cdots\right.}{\cdots}}{b}}{b}$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 218

$$\frac{\frac{2(Ab-aB)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{2a^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{a}\cot(c+dx)}{\sqrt{b}}\right)}{\sqrt{b}d\left(a^2+b^2\right)}-\frac{2b^2\left(\frac{1}{2}(a(A-B)+b(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}(b(A\cdots\right.}{d\left(a^2+\cdots\right)}}{b}}{b}$$

$$\frac{2B}{3bd\cot^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]
```

output
```
(2*B)/(3*b*d*Cot[c + d*x]^(3/2)) + ((2*(A*b - a*B))/(b*d*Sqrt[Cot[c + d*x]
]) - ((2*a^(5/2)*(A*b - a*B)*ArcTan[(Sqrt[a]*Cot[c + d*x])/Sqrt[b]])/(Sqrt
[b]*(a^2 + b^2)*d) - (2*b^2*(((a*(A - B) + b*(A + B))*(-(ArcTan[1 - Sqrt[2
]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sq
rt[2]))/2 - ((b*(A - B) - a*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*
x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
 d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/b)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.42 (sec), antiderivative size = 286, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{-\frac{2\left(\frac{(-Aa-Bb)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{8}+\frac{(-Ab \cdots}{\cdots}\right)}{a^{2}+b^{2}}}{\cdots}$
default	$-\frac{-\frac{2\left(\frac{(-Aa-Bb)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{8}+\frac{(-Ab \cdots}{\cdots}\right)}{a^{2}+b^{2}}}{\cdots}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETURNVER
BOSE)
```

output
```
1/d*(-2/(a^2+b^2)*(1/8*(-A*a-B*b)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+
c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*co
t(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/8*(-A*b+B*a)*2^(1
/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*
x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*co
t(d*x+c)^(1/2))))+2/b^2*a^3*(A*b-B*a)/(a^2+b^2)/(a*b)^(1/2)*arctan(a*cot(d
*x+c)^(1/2)/(a*b)^(1/2))+2/3/b*B/cot(d*x+c)^(3/2)+2*(A*b-B*a)/b^2/cot(d*x+
c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1332 vs. $2(230) = 460$.

Time = 19.66 (sec), antiderivative size = 2690, normalized size of antiderivative = 10.19

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm=
"fricas")
```

output
```
[-1/6*(6*sqrt(1/2)*(a^2*b^2 + b^4)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^
2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arc
tan(((a^4 + 2*a^2*b^2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 -
B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((
A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^
4 + 2*a^2*b^2 + b^4)*d^2)) + 2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A
 + B)*a*b^2 - (A - B)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)
*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*
x + c)))/(4*A*B*a*b + (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) + 6*sqrt(1/2)*(a
^2*b^2 + b^4)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 +
 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*arctan(-((a^4 + 2*a^2*b^
2 + b^4)*d^2*sqrt(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*
A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(((A^2 - 2*A*B + B^2)*a
^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B + B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)
*d^2)) - 2*sqrt(1/2)*((A + B)*a^3 - (A - B)*a^2*b + (A + B)*a*b^2 - (A - B
)*b^3)*d*sqrt(((A^2 - 2*A*B + B^2)*a^2 + 2*(A^2 - B^2)*a*b + (A^2 + 2*A*B
+ B^2)*b^2)/((a^4 + 2*a^2*b^2 + b^4)*d^2))*sqrt(tan(d*x + c)))/(4*A*B*a*b
+ (A^2 - B^2)*a^2 - (A^2 - B^2)*b^2)) - 3*sqrt(1/2)*(a^2*b^2 + b^4)*d*sqrt
(((A^2 + 2*A*B + B^2)*a^2 - 2*(A^2 - B^2)*a*b + (A^2 - 2*A*B + B^2)*b^2)/(
(a^4 + 2*a^2*b^2 + b^4)*d^2))*log(2*sqrt(1/2)*(a^2 + b^2)*d*sqrt(((A^2 ...
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx=\int \frac{A+B\tan{(c+dx)}}{(a+b\tan{(c+dx)})\cot^{\frac{5}{2}}{(c+dx)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))*cot(c + d*x)**(5/2)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.99

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx =$$

$$-\frac{\frac{24\,(Ba^4-Aa^3b)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^2b^2+b^4)\sqrt{ab}} - \frac{8\left(Bb-\frac{3\,(Ba-Ab)}{\tan(dx+c)}\right)\tan(dx+c)^{\frac{3}{2}}}{b^2} - \frac{3\left(2\sqrt{2}((A-B)a+(A+B)b)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(}}\right.\right.\right.}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm=
"maxima")
```

output
```
-1/12*(24*(B*a^4 - A*a^3*b)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^2
*b^2 + b^4)*sqrt(a*b)) - 8*(B*b - 3*(B*a - A*b)/tan(d*x + c))*tan(d*x + c)
^(3/2)/b^2 - 3*(2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(1/2*sqrt(2)*(sqrt
(2) + 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a + (A + B)*b)*arctan(-1
/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*((A + B)*a - (A - B
)*b)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*((A +
B)*a - (A - B)*b)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(
a^2 + b^2))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algorithm=
"giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx=\int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+b\tan\left(c+d\,x\right)\right)}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx=\int \frac{\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^{3}}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x))/cot(c + d*x)**3,x)
```

3.599 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 366

$$\begin{aligned}
&\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx\\
&= -\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}\\
&\quad+\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}\\
&\quad+\frac{b^{3/2}(7a^2Ab+3Ab^3-5a^3B-ab^2B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{5/2}\,(a^2+b^2)^2\,d}\\
&\quad-\frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d}\\
&\quad-\frac{(2a^2A+3Ab^2-abB)\sqrt{\cot(c+dx)}}{a^2\,(a^2+b^2)\,d}+\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{a\,(a^2+b^2)\,d(b+a\cot(c+dx))}
\end{aligned}$$

output
```
1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*
2^(1/2)/(a^2+b^2)^2/d+1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(1+2^(1/
2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d+b^(3/2)*(7*A*a^2*b+3*A*b^3-5*B*
a^3-B*a*b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(5/2)/(a^2+b^2)^2/
d-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(
1+cot(d*x+c))*2^(1/2)/(a^2+b^2)^2/d-(2*A*a^2+3*A*b^2-B*a*b)*cot(d*x+c)^(1
/2)/a^2/(a^2+b^2)/d+b*(A*b-B*a)*cot(d*x+c)^(3/2)/a/(a^2+b^2)/d/(b+a*cot(d*
x+c))
```

Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.05

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{2\sqrt{2}(a^2(A-B)+b^2(-A+B)+2ab(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right.}{(a^2+b^2)^2}\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*Sqrt[2]*(a^2*(A - B) + b^2*(-A
+ B) + 2*a*b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2)^2 + (4*b^(3/2)*(-(A*b) + a*B)*A
rcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^(5/2)*(a^2 + b^2)) - (8*b^
(3/2)*(3*a^2*A*b + A*b^3 - 2*a^3*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sq
rt[a]])/(a^(5/2)*(a^2 + b^2)^2) - (Sqrt[2]*(2*a*b*(-A + B) + a^2*(A + B) -
 b^2*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2)^2 - (8*A)/(a^2*
Sqrt[Tan[c + d*x]]) + (4*b^2*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]])/(a^2*(a^2
+ b^2)*(a + b*Tan[c + d*x]))))/(4*d)
```

Rubi [A] (warning: unable to verify)

Time = 2.22 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.788$, Rules used = {3042, 4064, 3042, 4088, 27, 3042, 4130, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A\cot(c+dx)+B)}{(a\cot(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}-\frac{\int -\frac{\sqrt{\cot(c+dx)}\left(\left(2Aa^2-bBa+3Ab^2\right)\cot^2(c+dx)-2a(Ab-aB)\cot(c+dx)+3b(Ab-aB)\right)}{2(b+a\cot(c+dx))}dx}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\cot(c+dx)}\left(\left(2Aa^2-bBa+3Ab^2\right)\cot^2(c+dx)-2a(Ab-aB)\cot(c+dx)+3b(Ab-aB)\right)}{b+a\cot(c+dx)}dx}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(\left(2Aa^2-bBa+3Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2+2a(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)+3b(Ab-aB)\right)}{b-a\tan\left(c+dx+\frac{\pi}{2}\right)}dx}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 4130

$$\frac{-\frac{2\int \frac{2(aA+bB)\cot(c+dx)a^2+\left(-2Ba^3+4Aba^2-b^2Ba+3Ab^3\right)\cot^2(c+dx)+b\left(2Aa^2-bBa+3Ab^2\right)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a}-\frac{2\left(2a^2A-abB+3Ab^2\right)\sqrt{\cot(c+dx)}}{ad}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{2(aA+bB)\cot(c+dx)a^2+\left(-2Ba^3+4Aba^2-b^2Ba+3Ab^3\right)\cot^2(c+dx)+b\left(2Aa^2-bBa+3Ab^2\right)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a}-\frac{2\left(2a^2A-abB+3Ab^2\right)\sqrt{\cot(c+dx)}}{ad}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 3042

$$\frac{-\frac{\int \frac{-2(aA+bB)\tan\left(c+dx+\frac{\pi}{2}\right)a^2+\left(-2Ba^3+4Aba^2-b^2Ba+3Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2+b\left(2Aa^2-bBa+3Ab^2\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a}-\frac{2\left(2a^2A-abB+3Ab^2\right)\sqrt{\cot(c+dx)}}{ad}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 4136

$$\frac{-\frac{\frac{\int \frac{2\left(\left(Aa^2+2bBa-Ab^2\right)a^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)a^2\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}}{a}-\frac{2\left(2a^2A-abB\right.}{}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 27

$$-\frac{\frac{\frac{2\int \frac{\left(Aa^2+2bBa-Ab^2\right)a^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)a^2}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}}{a}-\frac{2\left(2a^2A-abB+\ldots\right.}{\ldots}}{2a\left(a^2+b^2\right)}-\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{2\int \frac{a^2\left(Aa^2+2bBa-Ab^2\right)-a^2\left(-Ba^2+2Aba+b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{a}-\ldots}{2a\left(a^2+b^2\right)}-\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 4017

$$-\frac{\frac{\frac{4\int -\frac{a^2\left(Aa^2+2bBa-Ab^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}d\ldots}{a^2+b^2}}{a}\ldots}{2a\left(a^2+b^2\right)}-\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4\int \frac{a^2\left(Aa^2+2bBa-Ab^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}}{a}\ldots}{2a\left(a^2+b^2\right)}-\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2\int \frac{Aa^2+2bBa-Ab^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}}{a}-}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \; 1482$$

$$-\frac{\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\cdots\right.}{d\left(a^2\cdots\right.}}{a}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \; 1476$$

$$-\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+d\cdots}}\right.\right.}{\cdots}}{a}\cdots$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \; 1082$$

$$-\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cdots}\right.}{\sqrt{2}}\right.\right.}{\cdots}}{a}}{2\cdots}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{a}}{2a\left(a^2+b\right.}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 1479

$$-\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{}}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 25

$$-\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{}}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 27

$$-\frac{\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{}}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 1103

$$-\frac{-\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{}}{a}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 4117

$$-\frac{-\frac{b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{}}{a}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 73

$$-\frac{-\frac{2b^2\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\int \frac{1}{a\cot^2(c+dx)+b}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.}{}\right.\right.}{}}{a}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 218

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}+$$

$$-\frac{2(2a^2A-abB+3Ab^2)\sqrt{\cot(c+dx)}}{ad}-\frac{\frac{2b^{3/2}\left(-5a^3B+7a^2Ab-ab^2B+3Ab^3\right)\arctan\left(\frac{\sqrt{a}\cot(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}d\left(a^2+b^2\right)}-\frac{4a^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\text{arcta}}{}\right.\right.}{}}{}$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(A*b - a*B)*Cot[c + d*x]^(3/2))/(a*(a^2 + b^2)*d*(b + a*Cot[c + d*x]))
+ ((-2*(2*a^2*A + 3*A*b^2 - a*b*B)*Sqrt[Cot[c + d*x]])/(a*d) - ((2*b^(3/2)
*(7*a^2*A*b + 3*A*b^3 - 5*a^3*B - a*b^2*B)*ArcTan[(Sqrt[a]*Cot[c + d*x])/S
qrt[b]])/(Sqrt[a]*(a^2 + b^2)*d) - (4*a^2*(((a^2*(A - B) - b^2*(A - B) + 2
*a*b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[
1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 - ((2*a*b*(A - B) - a^2*(A + B
) + b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/
Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))
/2))/((a^2 + b^2)*d))/a)/(2*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2216 vs. $2(334) = 668$.

Time = 0.45 (sec) , antiderivative size = 2217, normalized size of antiderivative = 6.06

method	result	size
derivativedivides	Expression too large to display	2217
default	Expression too large to display	2217

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/4/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(20*A*(a*b)^(1/2)*tan(d*x+c)*a^2*b^
3+28*A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*tan(d*x+c)^(1/2)*a^3*b^3+12*
A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*tan(d*x+c)^(1/2)*a*b^5-4*B*(a*b)^
(1/2)*tan(d*x+c)*a^3*b^2-4*B*(a*b)^(1/2)*tan(d*x+c)*a*b^4-20*B*arctan(b*ta
n(d*x+c)^(1/2)/(a*b)^(1/2))*tan(d*x+c)^(1/2)*a^4*b^2-4*B*arctan(b*tan(d*x+
c)^(1/2)/(a*b)^(1/2))*tan(d*x+c)^(1/2)*a^2*b^4+8*A*(a*b)^(1/2)*a^4*b*tan(d
*x+c)+28*A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*tan(d*x+c)^(3/2)*a^2*b^4
-20*B*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*tan(d*x+c)^(3/2)*a^3*b^3-4*B*
arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*tan(d*x+c)^(3/2)*a*b^5+2*A*(a*b)^(1
/2)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(1/2)*a^5+A*(a*
b)^(1/2)*2^(1/2)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2
^(1/2)*tan(d*x+c)^(1/2)+1))*tan(d*x+c)^(1/2)*a^5-B*(a*b)^(1/2)*2^(1/2)*ln(
-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x
+c)-1))*tan(d*x+c)^(1/2)*a^5-2*B*(a*b)^(1/2)*2^(1/2)*arctan(1+2^(1/2)*tan(
d*x+c)^(1/2))*tan(d*x+c)^(1/2)*a^5-2*B*(a*b)^(1/2)*2^(1/2)*arctan(-1+2^(1/
2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(1/2)*a^5+2*A*(a*b)^(1/2)*2^(1/2)*arctan(1
+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(1/2)*a^5-2*B*(a*b)^(1/2)*2^(1/2)*ar
ctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*a^4*b+2*A*(a*b)^(1/2)*2^
(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*a^4*b+4*A*(a*b)^
(1/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)*a^3*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2431 vs. $2(335) = 670$.

Time = 35.80 (sec) , antiderivative size = 4891, normalized size of antiderivative = 13.36

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{(A+B\tan(c+dx))\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**(3/2)/(a + b*tan(c + d*x))**2,
 x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.05

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{4\,(5\,Ba^3b^2-7\,Aa^2b^3+Bab^4-3\,Ab^5)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^6+2\,a^4b^2+a^2b^4)\sqrt{ab}} - \frac{2\,\sqrt{2}((A-B)a^2+2\,(A+B)ab-(A-B)b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right.\right.}{}}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
-1/4*(4*(5*B*a^3*b^2 - 7*A*a^2*b^3 + B*a*b^4 - 3*A*b^5)*arctan(a/(sqrt(a*b
)*sqrt(tan(d*x + c))))/((a^6 + 2*a^4*b^2 + a^2*b^4)*sqrt(a*b)) - (2*sqrt(2
)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2)
+ 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B
)*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*((A
 + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) +
1/tan(d*x + c) + 1) - sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*
log(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 +
b^4) - 4*(B*a*b^2 - A*b^3)/((a^4*b + a^2*b^3 + (a^5 + a^3*b^2)/tan(d*x + c
))*sqrt(tan(d*x + c))) + 8*A/(a^2*sqrt(tan(d*x + c))))/d
```

Giac [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{3}{2}}}{(b\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\cot(dx+c)}\,\cot(dx+c)}{a+\tan(dx+c)\,b}dx$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
int((sqrt(cot(c + d*x))*cot(c + d*x))/(tan(c + d*x)*b + a),x)
```

3.600 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 317

$$
\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx
&= -\frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad-\frac{\sqrt{b}(5a^2Ab+Ab^3-3a^3B+ab^2B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{a^{3/2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{a\,(a^2+b^2)\,d(b+a\cot(c+dx))}
\end{aligned}
$$

output
```
1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*
2^(1/2)/(a^2+b^2)^2/d+1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctan(1+2^(1/
2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d-b^(1/2)*(5*A*a^2*b+A*b^3-3*B*a^
3+B*a*b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(3/2)/(a^2+b^2)^2/d+
1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+
cot(d*x+c)))*2^(1/2)/(a^2+b^2)^2/d+b*(A*b-B*a)*cot(d*x+c)^(1/2)/a/(a^2+b^2
)/d/(b+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\Big(2\sqrt{2}(2ab(A-B)-a^2(A+B)+b^2(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right.\right.}{}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(2*Sqrt[2]*(2*a*b*(A - B) - a^2*(A
+ B) + b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 + S
qrt[2]*Sqrt[Tan[c + d*x]]]) + (4*Sqrt[b]*(a^2 + b^2)*(A*b - a*B)*ArcTan[(S
qrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/a^(3/2) + (8*Sqrt[b]*(2*a*A*b - a^2*B
 + b^2*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[a] - Sqrt[2]*
(a^2*(A - B) + b^2*(-A + B) + 2*a*b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c +
 d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]
]) + (4*b*(a^2 + b^2)*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(a*(a + b*Tan[c + d*
x])))/(4*(a^2 + b^2)^2*d)
```

Rubi [A] (warning: unable to verify)

Time = 2.80 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.697$, Rules used = {3042, 4064, 3042, 4088, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A\cot(c+dx)+B)}{(a\cot(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}-\frac{\int -\frac{\left(2Aa^2+bBa+Ab^2\right)\cot^2(c+dx)-2a(Ab-aB)\cot(c+dx)+b(Ab-aB)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(2Aa^2+bBa+Ab^2\right)\cot^2(c+dx)-2a(Ab-aB)\cot(c+dx)+b(Ab-aB)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(2Aa^2+bBa+Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2+2a(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)+b(Ab-aB)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \; 4136$$

$$\frac{\frac{\int -\frac{2\left(a\left(-Ba^2+2Aba+b^2B\right)-a\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}-\frac{2\int \frac{a\left(-Ba^2+2Aba+b^2B\right)-a\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{a\left(-Ba^2+2Aba+b^2B\right)+a\left(Aa^2+2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \; 4017$$

$$\frac{\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4\int -\frac{a\left(-Ba^2+2Aba+b^2B-\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+}}{d(a^2+b^2)}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{4\int \frac{a\left(-Ba^2+2Aba+b^2B-\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}+\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}d}{a^2+b^2}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{4a\int \frac{-Ba^2+2Aba+b^2B-\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}+\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 1482

$$\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 1476

$$\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx}}\right.\right.}{d(a^2+b^2)}}{2a\left(a\right.}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 1082

$$\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot\right.}}{\sqrt{2}}\right.\right.}{d(a^2+b^2)}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 217

$$\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\right.\right.}{d(a^2+b^2)}}{2a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 1479

$$\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(-\left(\right.\right.}{d(a^2+b^2)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$\Big\downarrow$ 25

$$\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(-\left(a^2(A\right.\right.}{d(a^2+b^2)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$\Big\downarrow$ 27

$$\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}\left(-\left(a\right.\right.}{d(a^2+b^2)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{4a\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+\right.}}{\sqrt{2}}\right.\right.}{}}{2a\,(a}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$\Big\downarrow$ 4117

$$\frac{\frac{b\left(-3a^3B+5a^2Ab+ab^2B+Ab^3\right)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}+\frac{4a\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c\right.}}{\sqrt{2}}\right.\right.}{}}{2a\,(a}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$\Big\downarrow$ 73

$$\frac{4a\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\mathrm{lo}}{}\right.}{d(a^2+b^2)} \over 2a\left(a^2+\right.}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 218

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}+$$

$$\frac{4a\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\mathrm{lo}}{}\right.}{d(a^2+b^2)} \over 2a\left(a^2+b^2\right.}$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^2,x]
```

output
```
(b*(A*b - a*B)*Sqrt[Cot[c + d*x]])/(a*(a^2 + b^2)*d*(b + a*Cot[c + d*x]))
+ ((2*Sqrt[b]*(5*a^2*A*b + A*b^3 - 3*a^3*B + a*b^2*B)*ArcTan[(Sqrt[a]*Cot[
c + d*x])/Sqrt[b]])/(Sqrt[a]*(a^2 + b^2)*d) + (4*a*(((2*a*b*(A - B) - a^2*
(A + B) + b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])
+ ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 + ((a^2*(A - B) - b^2
*(A - B) + 2*a*b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
 + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*S
qrt[2])))/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(288) = 576$.

Time = 0.44 (sec) , antiderivative size = 1936, normalized size of antiderivative = 6.11

method	result	size
derivativedivides	Expression too large to display	1936
default	Expression too large to display	1936

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/4/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)^(1/2)*(-2*B*(a*b)^(1/2)*arctan(1+2^(
1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^2*b^2-4*B*(a*b)^(1/2)*tan(d*x+c)^(1/2)*a*
b^3+A*(a*b)^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*tan
(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)*a^4+2*A*(a*b)^(1/2)*arctan(1+2^(1/2)*
tan(d*x+c)^(1/2))*2^(1/2)*a^4+2*A*(a*b)^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)
^(1/2))*2^(1/2)*a^4+20*A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^2*b^3*ta
n(d*x+c)+2*B*(a*b)^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^4+2*
B*(a*b)^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^4+B*(a*b)^(1/2
)*ln(-(2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+
c)^(1/2)+1))*2^(1/2)*a^4-12*B*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^3*b
^2*tan(d*x+c)+4*B*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a*b^4*tan(d*x+c)+
4*A*(a*b)^(1/2)*tan(d*x+c)^(1/2)*a^2*b^2-4*B*(a*b)^(1/2)*tan(d*x+c)^(1/2)*
a^3*b-A*(a*b)^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(2^(1/2)*t
an(d*x+c)^(1/2)-tan(d*x+c)-1))*2^(1/2)*a^2*b^2+4*B*(a*b)^(1/2)*arctan(-1+2
^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^3*b-2*B*(a*b)^(1/2)*arctan(-1+2^(1/2)*t
an(d*x+c)^(1/2))*2^(1/2)*a^2*b^2-B*(a*b)^(1/2)*ln(-(2^(1/2)*tan(d*x+c)^(1/
2)-tan(d*x+c)-1)/(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1))*2^(1/2)*a^2*b^2-
4*A*(a*b)^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^3*b-2*A*(a*b)
^(1/2)*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^2*b^2-4*A*(a*b)^(1/2)*
arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*2^(1/2)*a^3*b-2*A*(a*b)^(1/2)*arcta...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(290) = 580$.

Time = 22.17 (sec) , antiderivative size = 4803, normalized size of antiderivative = 15.15

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{\cot{(c+dx)}}}{(a+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/(a + b*tan(c + d*x))**2,
x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{4\,(3\,Ba^3b-5\,Aa^2b^2-Bab^3-Ab^4)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^5+2\,a^3b^2+ab^4)\sqrt{ab}} - \frac{2\sqrt{2}((A+B)a^2-2\,(A-B)ab-(A+B)b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+}{}}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
1/4*(4*(3*B*a^3*b - 5*A*a^2*b^2 - B*a*b^3 - A*b^4)*arctan(a/(sqrt(a*b)*sqr
t(tan(d*x + c))))/((a^5 + 2*a^3*b^2 + a*b^4)*sqrt(a*b)) - (2*sqrt(2)*((A +
 B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqr
t(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((A - B)*a
^2 + 2*(A + B)*a*b - (A - B)*b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d
*x + c) + 1) + sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*log(-sq
rt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4) -
4*(B*a*b - A*b^2)/((a^3*b + a*b^3 + (a^4 + a^2*b^2)/tan(d*x + c))*sqrt(tan
(d*x + c))))/d
```

Giac [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{\cot(dx+c)}}{(b\tan(dx+c)+a)^2}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)^2,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^2}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2,x)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\cot(dx+c)}}{a+\tan(dx+c)\,b}dx$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^2,x)
```

output
```
int(sqrt(cot(c + d*x))/(tan(c + d*x)*b + a),x)
```

3.601 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 317

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2}\,dx \\
&= \frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad + \frac{(3a^2Ab-Ab^3-a^3B+3ab^2B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\,(a^2+b^2)^2\,d} \\
&\quad + \frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{(Ab-aB)\sqrt{\cot(c+dx)}}{(a^2+b^2)\,d(b+a\cot(c+dx))}
\end{aligned}$$

output
```
-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))
*2^(1/2)/(a^2+b^2)^2/d-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(1+2^(1
/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d+(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b
^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(1/2)/b^(1/2)/(a^2+b^2)^2/d
+1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1
+cot(d*x+c)))*2^(1/2)/(a^2+b^2)^2/d-(A*b-B*a)*cot(d*x+c)^(1/2)/(a^2+b^2)/d
/(b+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.06

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(2\sqrt{2}(a^2(A-B)+b^2(-A+B)+2ab(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(\right.\right.\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^2)
,x]
```

output
```
-1/4*(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(2*Sqrt[2]*(a^2*(A - B) + b^2*
(-A + B) + 2*a*b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan
[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) - (4*(a^2 + b^2)*(-(A*b) + a*B)*ArcTan[(
Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (8*Sqrt[b]*(a^2*
A - A*b^2 + 2*a*b*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[a]
 + Sqrt[2]*(2*a*b*(A - B) - a^2*(A + B) + b^2*(A + B))*(Log[1 - Sqrt[2]*Sq
rt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Ta
n[c + d*x]]) - (4*(a^2 + b^2)*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]])/(a + b*Ta
n[c + d*x])))/((a^2 + b^2)^2*d)
```

Rubi [A] (warning: unable to verify)

Time = 2.69 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.697, Rules used = {3042, 4064, 3042, 4091, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 4064}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A\cot(c+dx)+B)}{(a\cot(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 4091}$$

$$\frac{\int -\frac{-a(Ab-aB)\cot^2(c+dx)-2a(aA+bB)\cot(c+dx)+a(Ab-aB)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{-a(Ab-aB)\cot^2(c+dx)-2a(aA+bB)\cot(c+dx)+a(Ab-aB)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{2a\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{-a(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2+2a(aA+bB)\tan\left(c+dx+\frac{\pi}{2}\right)+a(Ab-aB)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{ 4136}$$

$$-\frac{\frac{\int -\frac{2\left(a\left(Aa^2+2bBa-Ab^2\right)+a\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{a\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{a\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}-\frac{2\int \frac{a\left(Aa^2+2bBa-Ab^2\right)+a\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{a\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int \frac{a\left(Aa^2+2bBa-Ab^2\right)-a\left(-Ba^2+2Aba+b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 4017

$$-\frac{\frac{a\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4\int -\frac{a\left(Aa^2+2bBa-Ab^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}}{2a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 25

$$-\frac{\frac{4\int \frac{a\left(Aa^2+2bBa-Ab^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{4a\int \frac{Aa^2+2bBa-Ab^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}+\frac{a\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right.}}{a^2+b^2}}{2a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \text{1482}$$

$$-\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+d}\right.}{d(a^2+b^2)}}{2a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)-\frac{1}{2}\right.}{d(a^2+b^2)}}{2a}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2a\right.\right.}{d(a^2+b^2)}}{2a\left(a^2+\right.}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int\right.}{d(a^2+b^2)}}{2a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\right.}{d(a^2+b^2)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 25

$$-\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\right.}{d(a^2+b^2)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 27

$$-\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\right.}{d(a^2+b^2)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 1103

$$-\frac{\frac{a\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+\right.}{\sqrt{2}}\right.}{}}{2a}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 4117

$$-\frac{\frac{a\left(a^3(-B)+3a^2Ab+3ab^2B-Ab^3\right)\int\frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d(a^2+b^2)}+\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(\right.}{\sqrt{2}}\right.}{}}{2a}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 73

$$-\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\right.}{d(a^2+b^2)} \over 2a\left(a^2\right.}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 218

$$-\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}-$$

$$\frac{\frac{4a\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\mathrm{lo}}{}\right.}{d(a^2+b^2)}}{2a\left(a^2+b^2\right.}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^2),x]
```

output
```
-(((A*b - a*B)*Sqrt[Cot[c + d*x]])/((a^2 + b^2)*d*(b + a*Cot[c + d*x]))) -
 ((2*Sqrt[a]*(3*a^2*A*b - A*b^3 - a^3*B + 3*a*b^2*B)*ArcTan[(Sqrt[a]*Cot[c
 + d*x])/Sqrt[b]])/(Sqrt[b]*(a^2 + b^2)*d) + (4*a*(((a^2*(A - B) - b^2*(A
- B) + 2*a*b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) +
 ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 - ((2*a*b*(A - B) - a^
2*(A + B) + b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sq
rt[2])))/2))/((a^2 + b^2)*d))/(2*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{2\left(-\frac{1}{2}A\,a^2b-\frac{1}{2}A\,b^3+\frac{1}{2}B\,a^3+\frac{1}{2}B a\,b^2\right)\sqrt{\cot(dx+c)}}{b+a\cot(dx+c)}+\frac{\left(3A\,a^2b-A\,b^3-B\,a^3+3Ba\,b^2\right)\arctan\left(\frac{a\sqrt{\cot(dx+c)}}{\sqrt{ab}}\right)}{\sqrt{ab}}}{\left(a^2+b^2\right)^2}-\frac{2\left(\frac{\left(A\,a^2-A\,b^2+2\right.}{}\right.}{}$
default	$\frac{\frac{2\left(-\frac{1}{2}A\,a^2b-\frac{1}{2}A\,b^3+\frac{1}{2}B\,a^3+\frac{1}{2}B a\,b^2\right)\sqrt{\cot(dx+c)}}{b+a\cot(dx+c)}+\frac{\left(3A\,a^2b-A\,b^3-B\,a^3+3Ba\,b^2\right)\arctan\left(\frac{a\sqrt{\cot(dx+c)}}{\sqrt{ab}}\right)}{\sqrt{ab}}}{\left(a^2+b^2\right)^2}-\frac{2\left(\frac{\left(A\,a^2-A\,b^2+2\right.}{}\right.}{}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2/(a^2+b^2)^2*((-1/2*A*a^2*b-1/2*A*b^3+1/2*B*a^3+1/2*B*a*b^2)*cot(d*x
+c)^(1/2)/(b+a*cot(d*x+c))+1/2*(3*A*a^2*b-A*b^3-B*a^3+3*B*a*b^2)/(a*b)^(1/
2)*arctan(a*cot(d*x+c)^(1/2)/(a*b)^(1/2)))-2/(a^2+b^2)^2*(1/8*(A*a^2-A*b^2
+2*B*a*b)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-
2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan
(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/8*(2*A*a*b-B*a^2+B*b^2)*2^(1/2)*(ln((cot(
d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)
)+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2
)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2395 vs. $2(286) = 572$.

Time = 13.48 (sec) , antiderivative size = 4816, normalized size of antiderivative = 15.19

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^2}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^2}\, dx = \int \frac{A + B\tan{(c + dx)}}{(a + b\tan{(c + dx)})^2\sqrt{\cot{(c + dx)}}}\, dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**2*sqrt(cot(c + d*x)))
, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.09

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^2}\, dx = -\frac{\frac{4\,(Ba^3 - 3\,Aa^2b - 3\,Bab^2 + Ab^3)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^4 + 2\,a^2b^2 + b^4)\sqrt{ab}} + \frac{2\sqrt{2}((A-B)a^2 + 2\,(A+B)ab - (A-B)b^2)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + \frac{2}{\sqrt{\tan(dx+c)}}\right)\right) -}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
-1/4*(4*(B*a^3 - 3*A*a^2*b - 3*B*a*b^2 + A*b^3)*arctan(a/(sqrt(a*b)*sqrt(t
an(d*x + c))))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a*b)) + (2*sqrt(2)*((A - B)*a
^2 + 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan
(d*x + c)))) + 2*sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*((A + B)*a^2 -
2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x +
c) + 1) - sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*log(-sqrt(2)
/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4) - 4*(B*
a - A*b)/((a^2*b + b^3 + (a^3 + a*b^2)/tan(d*x + c))*sqrt(tan(d*x + c))))/
d
```

Giac [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^2}\, dx = \int \frac{B\tan(dx + c) + A}{(b\tan(dx + c) + a)^2\sqrt{\cot(dx + c)}}\, dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^2*sqrt(cot(d*x + c)))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^2}\, dx = \int \frac{A + B\tan(c + d\,x)}{\sqrt{\cot(c + d\,x)}\,(a + b\tan(c + d\,x))^2}\, dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^2}\,dx$$
$$= \int \frac{\sqrt{\cot(dx + c)}}{\cot(dx + c)\tan(dx + c)\,b + \cot(dx + c)\,a}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^2,x)
```

output
```
int(sqrt(cot(c + d*x))/(cot(c + d*x)*tan(c + d*x)*b + cot(c + d*x)*a),x)
```

3.602 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 318

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx \\
&= \frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{\sqrt{a}(a^2Ab-3Ab^3+a^3B+5ab^2B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}\,(a^2+b^2)^2\,d} \\
&\quad - \frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad + \frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{b\,(a^2+b^2)\,d(b+a\cot(c+dx))}
\end{aligned}$$

output
```
-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))
*2^(1/2)/(a^2+b^2)^2/d-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctan(1+2^(1
/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d-a^(1/2)*(A*a^2*b-3*A*b^3+B*a^3
+5*B*a*b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/b^(3/2)/(a^2+b^2)^2/d
-1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1
+cot(d*x+c)))*2^(1/2)/(a^2+b^2)^2/d+a*(A*b-B*a)*cot(d*x+c)^(1/2)/b/(a^2+b^
2)/d/(b+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.08

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\Big(-2\sqrt{2}(2ab(A-B)-a^2(A+B)+b^2(A+B))\Big(\arctan\Big(1-\sqrt{2}\sqrt{\tan(c+}}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2)
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-2*Sqrt[2]*(2*a*b*(A - B) - a^2*(A
 + B) + b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (4*Sqrt[a]*(a^2 + b^2)*(A*b - a*B)*ArcTan[(
Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/b^(3/2) + (8*Sqrt[a]*(-2*A*b^3 + a*(
a^2 + 3*b^2)*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/b^(3/2) + Sq
rt[2]*(a^2*(A - B) + b^2*(-A + B) + 2*a*b*(A + B))*(Log[1 - Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]) + (4*a*(a^2 + b^2)*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(b*(a + b*Tan[
c + d*x]))))/(4*(a^2 + b^2)^2*d)
```

Rubi [A] (warning: unable to verify)

Time = 2.76 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.04, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.697, Rules used = {3042, 4064, 3042, 4092, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{A\cot(c+dx)+B}{\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)(a\cot(c+dx)+b)}-\frac{\int -\frac{Ba^2-(Ab-aB)\cot^2(c+dx)a+Aba+2b^2B+2b(Ab-aB)\cot(c+dx)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Ba^2-(Ab-aB)\cot^2(c+dx)a+Aba+2b^2B+2b(Ab-aB)\cot(c+dx)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{2b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{Ba^2-(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2a+Aba+2b^2B-2b(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \; 4136$$

$$\frac{\frac{\int \frac{2\left(b\left(-Ba^2+2Aba+b^2B\right)-b\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{b\left(-Ba^2+2Aba+b^2B\right)-b\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\int \frac{b\left(-Ba^2+2Aba+b^2B\right)+b\left(Aa^2+2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}}{2b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \; 4017$$

$$\frac{\frac{4\int -\frac{b\left(-Ba^2+2Aba+b^2B-\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}+\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}d}{a^2+b^2}}{2b\left(a^2+b^2\right)}$$
$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4\int \frac{b\left(-Ba^2+2Aba+b^2B-\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}}{2b\left(a^2+b^2\right)}$$
$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a(a^3B+a^2Ab+5ab^2B-3Ab^3)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4b\int \frac{-Ba^2+2Aba+b^2B-\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}}{2b\left(a^2+b^2\right)}+$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{a(a^3B+a^2Ab+5ab^2B-3Ab^3)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+}\right.}{}}{2b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{a(a^3B+a^2Ab+5ab^2B-3Ab^3)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+}\right.}{}}{2b\left(a^2\right.}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{a(a^3B+a^2Ab+5ab^2B-3Ab^3)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+}\right.}{}}{2b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{a(a^3B+a^2Ab+5ab^2B-3Ab^3)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+}\right.}{}}{2b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)(a\cot(c+dx)+b)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}}dx}{a^2+b^2} - \frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c-}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot}}}{2\sqrt{}}\right.\right.}{}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 25

$$\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}}dx}{a^2+b^2} - \frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+}}}{2\sqrt{2}}\right.\right.}{}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 27

$$\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}}dx}{a^2+b^2} - \frac{4b\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+}}}{2\sqrt{2}}\right.\right.}{}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 1103

$$\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}}dx}{a^2+b^2} - \frac{4b\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx}\right.}{\sqrt{2}}\right.\right.}{2b\left(a^2\right.}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\Big\downarrow$ 4117

$$\frac{a\left(a^3B+a^2Ab+5ab^2B-3Ab^3\right)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)} - \frac{4b\left(\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+}\right.}{\sqrt{2}}\right.\right.}{2b\left(a\right.}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)}$$

$\downarrow$ 73

$$-\frac{\frac{2a(a^3B+a^2Ab+5ab^2B-3Ab^3)\int\frac{1}{a\cot^2(c+dx)+b}d\sqrt{\cot(c+dx)}}{d(a^2+b^2)}-\frac{4b\left(\frac{1}{2}(-(a^2(A+B))+2ab(A-B)+b^2(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\ldots\right.\right.}{}}{2b(a^2+\ldots}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd(a^2+b^2)(a\cot(c+dx)+b)}$$

$\downarrow$ 218

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{bd(a^2+b^2)(a\cot(c+dx)+b)}+$$

$$\frac{\frac{2\sqrt{a}(a^3B+a^2Ab+5ab^2B-3Ab^3)\arctan\left(\frac{\sqrt{a}\cot(c+dx)}{\sqrt{b}}\right)}{\sqrt{b}d(a^2+b^2)}-\frac{4b\left(\frac{1}{2}(-(a^2(A+B))+2ab(A-B)+b^2(A+B))\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan(1-\ldots}{}\right.\right.}{}}{2b(a^2+b^2)}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^2),x]
```

output
```
(a*(A*b - a*B)*Sqrt[Cot[c + d*x]])/(b*(a^2 + b^2)*d*(b + a*Cot[c + d*x]))
+ ((2*Sqrt[a]*(a^2*A*b - 3*A*b^3 + a^3*B + 5*a*b^2*B)*ArcTan[(Sqrt[a]*Cot[
c + d*x])/Sqrt[b]])/(Sqrt[b]*(a^2 + b^2)*d) - (4*b*(((2*a*b*(A - B) - a^2*
(A + B) + b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])
+ ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 + ((a^2*(A - B) - b^2
*(A - B) + 2*a*b*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
 + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*S
qrt[2])))/2))/((a^2 + b^2)*d))/(2*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{2\left(\frac{\left(2abA-B\,a^{2}+B\,b^{2}\right)\sqrt{2}\,\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{8}\cdots\right)}{(a\cdots}$
default	$-\frac{2\left(\frac{\left(2abA-B\,a^{2}+B\,b^{2}\right)\sqrt{2}\,\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{8}\cdots\right)}{(a\cdots}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(-2/(a^2+b^2)^2*(1/8*(2*A*a*b-B*a^2+B*b^2)*2^(1/2)*(ln((cot(d*x+c)+2^(
1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan
(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/8*(-
A*a^2+A*b^2-2*B*a*b)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(
cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2
))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))-2*a/(a^2+b^2)^2*(-1/2*(A*a^2*b+
A*b^3-B*a^3-B*a*b^2)/b*cot(d*x+c)^(1/2)/(b+a*cot(d*x+c))+1/2*(A*a^2*b-3*A*
b^3+B*a^3+5*B*a*b^2)/b/(a*b)^(1/2)*arctan(a*cot(d*x+c)^(1/2)/(a*b)^(1/2)))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(289) = 578$.

Time = 18.36 (sec) , antiderivative size = 4800, normalized size of antiderivative = 15.09

$$\int \frac{A + B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A + B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \int \frac{A + B\tan{\left(c+dx \right)}}{\left(a+b\tan{\left(c+dx \right)}\right)^2\cot^{\frac{3}{2}}{\left(c+dx \right)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**2,x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**2*cot(c + d*x)**(3/2)
), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.13

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{\frac{4\,(Ba^4+Aa^3b+5\,Ba^2b^2-3\,Aab^3)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^4b+2\,a^2b^3+b^5)\sqrt{ab}} - \frac{2\,\sqrt{2}((A+B)a^2-2\,(A-B)ab-(A+B)b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right.}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
-1/4*(4*(B*a^4 + A*a^3*b + 5*B*a^2*b^2 - 3*A*a*b^3)*arctan(a/(sqrt(a*b)*sq
rt(tan(d*x + c))))/((a^4*b + 2*a^2*b^3 + b^5)*sqrt(a*b)) - (2*sqrt(2)*((A
+ B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sq
rt(tan(d*x + c)))) + 2*sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((A - B)*
a^2 + 2*(A + B)*a*b - (A - B)*b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(
d*x + c) + 1) + sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*log(-s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4) +
 4*(B*a^2 - A*a*b)/((a^2*b^2 + b^4 + (a^3*b + a*b^3)/tan(d*x + c))*sqrt(ta
n(d*x + c))))/d
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \int \frac{B\tan(dx+c)+A}{(b\tan(dx+c)+a)^2\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^2*cot(d*x + c)^(3/2))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx=\int \frac{A+B\tan(c+d\,x)}{{\mathrm{cot}\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^2}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^2),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx=\int \frac{\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^{2}\tan\left(dx+c\right)b+\cot\left(dx+c\right)^{2}a}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^2,x)
```

output
```
int(sqrt(cot(c + d*x))/(cot(c + d*x)**2*tan(c + d*x)*b + cot(c + d*x)**2*a
),x)
```

3.603 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 365

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx \\
&= -\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad+\frac{(a^2(A-B)-b^2(A-B)+2ab(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad-\frac{a^{3/2}(a^2Ab+5Ab^3-3a^3B-7ab^2B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}\,(a^2+b^2)^2\,d} \\
&\quad-\frac{(2ab(A-B)-a^2(A+B)+b^2(A+B))\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^2\,d} \\
&\quad-\frac{aAb-3a^2B-2b^2B}{b^2\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}}+\frac{a(Ab-aB)}{b\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}
\end{aligned}$$

output
```
1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*
2^(1/2)/(a^2+b^2)^2/d+1/2*(a^2*(A-B)-b^2*(A-B)+2*a*b*(A+B))*arctan(1+2^(1/
2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^2/d-a^(3/2)*(A*a^2*b+5*A*b^3-3*B*a^
3-7*B*a*b^2)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/b^(5/2)/(a^2+b^2)^2/
d-1/2*(2*a*b*(A-B)-a^2*(A+B)+b^2*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(
1+cot(d*x+c)))*2^(1/2)/(a^2+b^2)^2/d-(A*a*b-3*B*a^2-2*B*b^2)/b^2/(a^2+b^2)
/d/cot(d*x+c)^(1/2)+a*(A*b-B*a)/b/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(b+a*cot(d*
x+c))
```

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.07

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{2\sqrt{2}(a^2(A-B)+b^2(-A+B)+2ab(A+B))\left(\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\right)}{(a^2+b^2)^2}\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2)
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*Sqrt[2]*(a^2*(A - B) + b^2*(-A
+ B) + 2*a*b*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(a^2 + b^2)^2 + (4*a^(3/2)*(-(A*b) + a*B)*A
rcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(b^(5/2)*(a^2 + b^2)) + (8*a^
(3/2)*(a^2*A*b + 3*A*b^3 - 2*a^3*B - 4*a*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a]])/(b^(5/2)*(a^2 + b^2)^2) - (Sqrt[2]*(2*a*b*(-A + B) + a
^2*(A + B) - b^2*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*
x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(a^2 + b^2)^2 +
 (8*B*Sqrt[Tan[c + d*x]])/b^2 + (4*a^2*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]])/
(b^2*(a^2 + b^2)*(a + b*Tan[c + d*x]))))/(4*d)
```

Rubi [A] (warning: unable to verify)

Time = 3.63 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.788, Rules used = {3042, 4064, 3042, 4092, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{5/2}(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \; 4064$$

$$\int \frac{A\cot(c+dx)+B}{\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 4092$$

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\int \frac{-3Ba^2+3(Ab-aB)\cot^2(c+dx)a+Aba-2b^2B-2b(Ab-aB)\cot(c+dx)}{2\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))}dx}{b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\int \frac{-3Ba^2+3(Ab-aB)\cot^2(c+dx)a+Aba-2b^2B-2b(Ab-aB)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\int \frac{-3Ba^2+3(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2a+Aba-2b^2B+2b(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4132

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\int -\frac{-3Ba^3+Aba^2+\left(-3Ba^2+Aba-2b^2B\right)\cot^2(c+dx)a-4b^2Ba+2Ab^3-2b^2(aA+bB)\cot(c+dx)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b}+\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\int \frac{-3Ba^3+Aba^2+\left(-3Ba^2+Aba-2b^2B\right)\cot^2(c+dx)a-4b^2Ba+2Ab^3-2b^2(aA+bB)\cot(c+dx)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\int \frac{-3Ba^3+Aba^2+\left(-3Ba^2+Aba-2b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2a-4b^2Ba+2Ab^3+2b^2(aA+bB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4136

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{\int -\frac{2\left(\left(Aa^2+2bBa-Ab^2\right)b^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)b^2\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int \frac{\cot^2(c+dx)+1\cdots}{\sqrt{\cot(c+dx)}(b+a\cot\cdots}}{a^2+b^2}}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}-\frac{2\int \frac{\left(Aa^2+2bBa-Ab^2\right)b^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+d\cdots}{\sqrt{\cot(c+dx)}}}{a^2+b^2}}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{2\int\frac{b^2\left(Aa^2+2bBa-Ab^2\right)-b^2\left(-Ba^2+2Aba+\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}}{a^2+b^2}}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 4017

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{4\int-\frac{b^2\left(Aa^2+2bBa-Ab^2+\left(-Ba^2+2Aba+b\right.\right.}{\cot^2(c+dx)+1}}{d\left(a^2+b^2\right)}}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{4\int\frac{b^2\left(Aa^2+2bBa-Ab^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan\left(c\right.}{\sqrt{-\tan\left(c+dx+\right.}}}{a^2+b^2}}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{4b^2\int\frac{Aa^2+2bBa-Ab^2+\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan\left(c+\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}}{a^2+b^2}}{b}}{2b\left(a^2+b^2\right)}$$

$\Big\downarrow$ 1482

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(A-B)+b^2(A+B)\right)\int\frac{1-\cot(c+}{\cot^2(c+dx}\right.}{d\left(a^2+b^2\right)}}{b}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 1476

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-$$

$$\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int\frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}\right.}{d\left(a^2+b^2\right)}}{2}$$

$\downarrow$ 1082

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-$$

$$\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)\right.}{d\left(a^2+b^2\right)}}{2b\left(a^2\right.}$$

$\downarrow$ 217

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-$$

$$\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(\right.\right.}{d\left(a^2+b^2\right)}}{2b\left(a^2+b^2\right)}$$

$\downarrow$ 1479

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-$$

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(\right.\right.}{d\left(a^2+b\right.}$$

$\downarrow$ 25

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(\cdots\right.\right.}{d\left(a^2+b^2\right)\cdots}}{\cdots}$$

$\big\downarrow$ 27

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(\cdots\right.\right.}{d\left(a^2+b^2\right)\cdots}}{\cdots}$$

$\big\downarrow$ 1103

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\big\downarrow$ 4117

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{a^2\left(-3a^3B+a^2Ab-7ab^2B+5Ab^3\right)\int\frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}+\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\big\downarrow$ 73

$$\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(\cdots\right.\right.}{d\left(a^2+b^2\right)}\cdots}{2b(\cdots}$$

$$\Big\downarrow \text{218}$$

$$\frac{a(Ab-aB)}{bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)}-\frac{\frac{2\left(-3a^2B+aAb-2b^2B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{4b^2\left(\frac{1}{2}\left(a^2(A-B)+2ab(A+B)-b^2(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^2(A+B)\right)+2ab(\ldots\right.}{d\left(a^2+b^2\right)}}{\ldots}}{2b\left(a\ldots\right.}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^2),x]
```

output
```
(a*(A*b - a*B))/(b*(a^2 + b^2)*d*Sqrt[Cot[c + d*x]]*(b + a*Cot[c + d*x]))
- ((2*(a*A*b - 3*a^2*B - 2*b^2*B))/(b*d*Sqrt[Cot[c + d*x]]) - ((2*a^(3/2)*
(a^2*A*b + 5*A*b^3 - 3*a^3*B - 7*a*b^2*B)*ArcTan[(Sqrt[a]*Cot[c + d*x])/Sq
rt[b]])/(Sqrt[b]*(a^2 + b^2)*d) + (4*b^2*(((a^2*(A - B) - b^2*(A - B) + 2*
a*b*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1
 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 - ((2*a*b*(A - B) - a^2*(A + B)
 + b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/S
qrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/
2))/((a^2 + b^2)*d))/b)/(2*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.96

method	result
derivativedivides	$-\frac{2a^2\left(\frac{\left(\frac{1}{2}A\,a^2b+\frac{1}{2}A\,b^3-\frac{1}{2}B\,a^3-\frac{1}{2}B\,a\,b^2\right)\sqrt{\cot(dx+c)}}{b+a\cot(dx+c)}+\frac{\left(A\,a^2b+5A\,b^3-3B\,a^3-7B\,a\,b^2\right)\arctan\left(\frac{a\sqrt{\cot(dx+c)}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2\left(a^2+b^2\right)^2}-\frac{2\left(\frac{(-A\,a^2}{}\right.}{}$
default	$-\frac{2a^2\left(\frac{\left(\frac{1}{2}A\,a^2b+\frac{1}{2}A\,b^3-\frac{1}{2}B\,a^3-\frac{1}{2}B\,a\,b^2\right)\sqrt{\cot(dx+c)}}{b+a\cot(dx+c)}+\frac{\left(A\,a^2b+5A\,b^3-3B\,a^3-7B\,a\,b^2\right)\arctan\left(\frac{a\sqrt{\cot(dx+c)}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2\left(a^2+b^2\right)^2}-\frac{2\left(\frac{(-A\,a^2}{}\right.}{}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(-2*a^2/b^2/(a^2+b^2)^2*((1/2*A*a^2*b+1/2*A*b^3-1/2*B*a^3-1/2*B*a*b^2)
*cot(d*x+c)^(1/2)/(b+a*cot(d*x+c))+1/2*(A*a^2*b+5*A*b^3-3*B*a^3-7*B*a*b^2)
/(a*b)^(1/2)*arctan(a*cot(d*x+c)^(1/2)/(a*b)^(1/2)))-2/(a^2+b^2)^2*(1/8*(-
A*a^2+A*b^2-2*B*a*b)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(
cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2
))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/8*(-2*A*a*b+B*a^2-B*b^2)*2^(1/
2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x
+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot
(d*x+c)^(1/2))))+2*B/b^2/cot(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2446 vs. $2(334) = 668$.

Time = 32.89 (sec) , antiderivative size = 4917, normalized size of antiderivative = 13.47

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.10

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \frac{\frac{4\,(3\,Ba^5-Aa^4b+7\,Ba^3b^2-5\,Aa^2b^3)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^4b^2+2\,a^2b^4+b^6)\sqrt{ab}}+\frac{2\,\sqrt{2}((A-B)a^2+2\,(A+B)ab-(A-B)b^2)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)}{\cdots}}{\cdots}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorith
m="maxima")
```

output
```
1/4*(4*(3*B*a^5 - A*a^4*b + 7*B*a^3*b^2 - 5*A*a^2*b^3)*arctan(a/(sqrt(a*b)
*sqrt(tan(d*x + c))))/((a^4*b^2 + 2*a^2*b^4 + b^6)*sqrt(a*b)) + (2*sqrt(2)
*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)*b^2)*arctan(1/2*sqrt(2)*(sqrt(2) +
 2/sqrt(tan(d*x + c)))) + 2*sqrt(2)*((A - B)*a^2 + 2*(A + B)*a*b - (A - B)
*b^2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*((A
+ B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*log(sqrt(2)/sqrt(tan(d*x + c)) + 1
/tan(d*x + c) + 1) - sqrt(2)*((A + B)*a^2 - 2*(A - B)*a*b - (A + B)*b^2)*l
og(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b
^4) + 4*(2*B*a^2*b + 2*B*b^3 + (3*B*a^3 - A*a^2*b + 2*B*a*b^2)/tan(d*x + c
))/((a^2*b^3 + b^5)/sqrt(tan(d*x + c)) + (a^3*b^2 + a*b^4)/tan(d*x + c)^(3
/2)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx=\int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^2}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^2), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^2}\,dx$$
$$=\int \frac{\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^3\tan\left(dx+c\right)b+\cot\left(dx+c\right)^3a}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^2,x)
```

output
```
int(sqrt(cot(c + d*x))/(cot(c + d*x)**3*tan(c + d*x)*b + cot(c + d*x)**3*a
),x)
```

3.604 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 515

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+\frac{b^{3/2}(63a^4Ab+46a^2Ab^3+15Ab^5-35a^5B-6a^3b^2B-3ab^4B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{7/2}\,(a^2+b^2)^3\,d}$$
$$-\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$-\frac{(8a^4A+31a^2Ab^2+15Ab^4-11a^3bB-3ab^3B)\sqrt{\cot(c+dx)}}{4a^3\,(a^2+b^2)^2\,d}$$
$$+\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2}+\frac{b(13a^2Ab+5Ab^3-9a^3B-ab^2B)\cot^{\frac{3}{2}}(c+dx)}{4a^2\,(a^2+b^2)^2\,d(b+a\cot(c+dx))}$$

output
```
1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(-1+2^(1/2)*co
t(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d+1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b
*(A+B)-b^3*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d
+1/4*b^(3/2)*(63*A*a^4*b+46*A*a^2*b^3+15*A*b^5-35*B*a^5-6*B*a^3*b^2-3*B*a*
b^4)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(7/2)/(a^2+b^2)^3/d-1/2*(3
*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctanh(2^(1/2)*cot(d*x+c)
^(1/2)/(1+cot(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d-1/4*(8*A*a^4+31*A*a^2*b^2+15*
A*b^4-11*B*a^3*b-3*B*a*b^3)*cot(d*x+c)^(1/2)/a^3/(a^2+b^2)^2/d+1/2*b*(A*b-
B*a)*cot(d*x+c)^(5/2)/a/(a^2+b^2)/d/(b+a*cot(d*x+c))^2+1/4*b*(13*A*a^2*b+5
*A*b^3-9*B*a^3-B*a*b^2)*cot(d*x+c)^(3/2)/a^2/(a^2+b^2)^2/d/(b+a*cot(d*x+c)
)
```

Mathematica [A] (verified)

Time = 6.34 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.20

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\left(\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\sqrt{2}\arctan\left(1+\right.\right.}{4(a^2+b^2)^3}\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3
,x]
```

output
```
(2*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((a^3*(A - B) - 3*a*b^2*(A - B)
+ 3*a^2*b*(A + B) - b^3*(A + B))*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(4*(a^2 + b^2)^3
) - (3*b^(3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(
8*a^(7/2)*(a^2 + b^2)) - (b^(3/2)*(3*a^2*A*b + A*b^3 - 2*a^3*B)*ArcTan[(Sq
rt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(2*a^(7/2)*(a^2 + b^2)^2) - (b^(3/2)*(
6*a^4*A*b + 3*a^2*A*b^3 + A*b^5 - 3*a^5*B + a^3*b^2*B)*ArcTan[(Sqrt[b]*Sqr
t[Tan[c + d*x]])/Sqrt[a]])/(a^(7/2)*(a^2 + b^2)^3) + ((3*a^2*b*(A - B) - b
^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*(Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[
Tan[c + d*x]] + Tan[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]))/(8*(a^2 + b^2)^3) - A/(a^3*Sqrt[Tan[c + d*x]]) - (b^2*(
A*b - a*B)*Sqrt[Tan[c + d*x]])/(4*a^2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2)
- (3*b^2*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(8*a^3*(a^2 + b^2)*(a + b*Tan[c +
 d*x])) - (b^2*(3*a^2*A*b + A*b^3 - 2*a^3*B)*Sqrt[Tan[c + d*x]])/(2*a^3*(a
^2 + b^2)^2*(a + b*Tan[c + d*x]))))/d
```

Rubi [A] (warning: unable to verify)

Time = 5.00 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.02, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.879$, Rules used = {3042, 4064, 3042, 4088, 27, 3042, 4128, 27, 3042, 4130, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{7}{2}}(c+dx)(A\cot(c+dx)+B)}{(a\cot(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}-\frac{\int -\frac{\cot^{\frac{3}{2}}(c+dx)\left(\left(4Aa^2-bBa+5Ab^2\right)\cot^2(c+dx)-4a(Ab-aB)\cot(c+dx)+5b(Ab-aB)\right)}{2(b+a\cot(c+dx))^2}dx}{2a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cot^{\frac{3}{2}}(c+dx)\left(\left(4Aa^2-bBa+5Ab^2\right)\cot^2(c+dx)-4a(Ab-aB)\cot(c+dx)+5b(Ab-aB)\right)}{(b+a\cot(c+dx))^2}dx}{4a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(\left(4Aa^2-bBa+5Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2+4a(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)+5b(Ab-aB)\right)}{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{4128}$$

$$\frac{\frac{b\left(-9a^3B+13a^2Ab-ab^2B+5Ab^3\right)\cot^{\frac{3}{2}}(c+dx)}{ad\left(a^2+b^2\right)(a\cot(c+dx)+b)}-\frac{\int -\frac{\sqrt{\cot(c+dx)}\left(-8\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)a^2+\left(8Aa^4-11bBa^3+31Ab^2a^2-3b^3Ba+15Ab^4\right)\text{co}\right.}{2(b+a\cot(c+dx))}}{a\left(a^2+b^2\right)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{\cot(c+dx)}\left(-8\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)a^2+\left(8Aa^4-11bBa^3+31Ab^2a^2-3b^3Ba+15Ab^4\right)\cot^2(c+dx)+3b\left(-9Ba^3+13Aba^2-b^2Ba+5Ab^3\right)\right)}{b+a\cot(c+dx)}dx}{2a\left(a^2+b^2\right)}+\frac{b(-}{}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(8\left(-Ba^2+2Aba+b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)a^2+\left(8Aa^4-11bBa^3+31Ab^2a^2-3b^3Ba+15Ab^4\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2+3b\left(-9Ba^3+13Aba^2-b^2Ba+5Ab\right.}{b-a\tan\left(c+dx+\frac{\pi}{2}\right)}}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 4130

$$\frac{-\frac{\frac{2\int \frac{8\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)a^3+\left(-8Ba^5+24Aba^4-3b^2Ba^3+31Ab^3a^2-3b^4Ba+15Ab^5\right)\cot^2(c+dx)+b\left(8Aa^4-11bBa^3+31Ab^2a^2-3b^3Ba+15Ab^4\right)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a}-}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 27

$$\frac{-\frac{\frac{\int \frac{8\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)a^3+\left(-8Ba^5+24Aba^4-3b^2Ba^3+31Ab^3a^2-3b^4Ba+15Ab^5\right)\cot^2(c+dx)+b\left(8Aa^4-11bBa^3+31Ab^2a^2-3b^3Ba+15Ab^4\right)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a}-}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 3042

$$\frac{-\frac{\frac{\int \frac{-8\left(Aa^2+2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)a^3+\left(-8Ba^5+24Aba^4-3b^2Ba^3+31Ab^3a^2-3b^4Ba+15Ab^5\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2+b\left(8Aa^4-11bBa^3+31Ab^2a^2-3b^3Ba+15\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}}{a}}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 4136

$$\frac{-\frac{\frac{\frac{\int \frac{8\left(\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)a^3+\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\cot(c+dx)a^3\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{}{\sqrt{\cot(c-}}}{a^2+b^2}}{a}}{2a(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{8\int \frac{\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)a^3+\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\cot(c+dx)a^3}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{\cot}{\sqrt{\cot(c+d}}}{a^2+b^2}}{a}$$

$$2a(a^2+b^2)$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{8\int \frac{a^3\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-a^3\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{}{\sqrt{-\text{t}}}}{a^2+b^2}}{a}$$

$$2a(a^2+b^2)$$

$$4a\left(a^2 -\right.$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{4017}$$

$$-\frac{\frac{16\int -\frac{a^3\left(Aa^3+3bBa^2-3Ab^2a-b^3B+\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int}{a^2+b^2}}{a}$$

$$2a(a^2+b^2)$$

$$4a\left(a\right.$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16\int \frac{a^3\left(Aa^3+3bBa^2-3Ab^2a-b^3B+\left(-Ba^3+3Ab\right.\right.}{\cot^2(c+dx)+1}}{d\left(a^2+b\right.}}{a}$$

$$2a(a^2+b^2)$$

$$4a\left(a^2\right.$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3\int \frac{Aa^3+3bBa^2-3Ab^2a-b^3B+\left(-Ba^3+3Aba\right.}{\cot^2(c+dx)+1}}{d\left(a^2+b^2\right.}}{a}}{2a(a^2+b^2)}}{4a\,(a^2}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\,(a^2+b^2)\,(a\cot(c+dx)+b)^2}$$

$\big\downarrow$ 1482

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3\right.\right.}{}}{a}}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\,(a^2+b^2)\,(a\cot(c+dx)+b)^2}$$

$\big\downarrow$ 1476

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3\right.\right.}{}}{}}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\,(a^2+b^2)\,(a\cot(c+dx)+b)^2}$$

$\big\downarrow$ 1082

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b\right.\right.}{}}{}}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\,(a^2+b^2)\,(a\cot(c+dx)+b)^2}$$

$\big\downarrow$ 217

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b\right.\right.}{}}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\Big\downarrow$ 1479

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3\left(\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b\right.\right.\right.}{}}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b\right.\right.}{}}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b\right.\right.}{}}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\Big\downarrow$ 1103

$$-\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b\right.\right.}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 4117

$$-\frac{b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}-\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3\right.\right.}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 73

$$-\frac{2b^2\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2Ab^3-3ab^4B+15Ab^5\right)\int\frac{1}{a\cot^2(c+dx)+b}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{16a^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\right.\right.}{}$$

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 218

$$\frac{b(Ab-aB)\cot^{\frac{5}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}+$$

$$\frac{b\left(-9a^3B+13a^2Ab-ab^2B+5Ab^3\right)\cot^{\frac{3}{2}}(c+dx)}{ad(a^2+b^2)(a\cot(c+dx)+b)}+\frac{-\frac{2\left(8a^4A-11a^3bB+31a^2Ab^2-3ab^3B+15Ab^4\right)\sqrt{\cot(c+dx)}}{ad}-\frac{2b^{3/2}\left(-35a^5B+63a^4Ab-6a^3b^2B+46a^2\right.}{\sqrt{ad}}}{}$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(A*b - a*B)*Cot[c + d*x]^(5/2))/(2*a*(a^2 + b^2)*d*(b + a*Cot[c + d*x])
^2) + ((b*(13*a^2*A*b + 5*A*b^3 - 9*a^3*B - a*b^2*B)*Cot[c + d*x]^(3/2))/(
a*(a^2 + b^2)*d*(b + a*Cot[c + d*x])) + ((-2*(8*a^4*A + 31*a^2*A*b^2 + 15*
A*b^4 - 11*a^3*b*B - 3*a*b^3*B)*Sqrt[Cot[c + d*x]])/(a*d) - ((2*b^(3/2)*(6
3*a^4*A*b + 46*a^2*A*b^3 + 15*A*b^5 - 35*a^5*B - 6*a^3*b^2*B - 3*a*b^4*B)*
ArcTan[(Sqrt[a]*Cot[c + d*x])/Sqrt[b]])/(Sqrt[a]*(a^2 + b^2)*d) - (16*a^3*
(((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*(-(ArcTa
n[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c
 + d*x]]]/Sqrt[2]))/2 - ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*
a*b^2*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sq
rt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2
))/((a^2 + b^2)*d))/a)/(2*a*(a^2 + b^2)))/(4*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4580 vs. $2(473) = 946$.

Time = 0.46 (sec) , antiderivative size = 4581, normalized size of antiderivative = 8.90

method	result	size
derivativedivides	Expression too large to display	4581
default	Expression too large to display	4581

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-1/4/d*(1/tan(d*x+c))^(3/2)*tan(d*x+c)*(-3*B*tan(d*x+c)^(1/2)*arctan(b*tan
(d*x+c)^(1/2)/(a*b)^(1/2))*a^3*b^6+16*A*(a*b)^(1/2)*a^7*b*tan(d*x+c)+63*A*
tan(d*x+c)^(5/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^4*b^5+24*A*(a*b)
^(1/2)*a^6*b^2+24*A*(a*b)^(1/2)*a^4*b^4+8*A*(a*b)^(1/2)*a^2*b^6+8*A*(a*b)^
(1/2)*a^8+46*A*tan(d*x+c)^2*(a*b)^(1/2)*a^2*b^6-11*B*tan(d*x+c)^2*(a*b)^(1
/2)*a^5*b^3-14*B*tan(d*x+c)^2*(a*b)^(1/2)*a^3*b^5-3*B*tan(d*x+c)^2*(a*b)^(
1/2)*a*b^7+65*A*tan(d*x+c)*(a*b)^(1/2)*a^5*b^3+74*A*tan(d*x+c)*(a*b)^(1/2)
*a^3*b^5+25*A*tan(d*x+c)*(a*b)^(1/2)*a*b^7+63*A*tan(d*x+c)^(1/2)*arctan(b*
tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^6*b^3+46*A*tan(d*x+c)^(1/2)*arctan(b*tan(d
*x+c)^(1/2)/(a*b)^(1/2))*a^4*b^5+15*A*tan(d*x+c)^(1/2)*arctan(b*tan(d*x+c)
^(1/2)/(a*b)^(1/2))*a^2*b^7+8*A*(a*b)^(1/2)*a^6*b^2*tan(d*x+c)^2-13*B*tan(
d*x+c)*(a*b)^(1/2)*a^6*b^2-18*B*tan(d*x+c)*(a*b)^(1/2)*a^4*b^4-5*B*tan(d*x
+c)*(a*b)^(1/2)*a^2*b^6-35*B*tan(d*x+c)^(1/2)*arctan(b*tan(d*x+c)^(1/2)/(a
*b)^(1/2))*a^7*b^2-3*B*tan(d*x+c)^(5/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1
/2))*a*b^8+126*A*tan(d*x+c)^(3/2)*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a
^5*b^4-2*B*tan(d*x+c)^(1/2)*(a*b)^(1/2)*2^(1/2)*arctan(1+2^(1/2)*tan(d*x+c
)^(1/2))*a^8-2*B*tan(d*x+c)^(1/2)*(a*b)^(1/2)*2^(1/2)*arctan(-1+2^(1/2)*ta
n(d*x+c)^(1/2))*a^8+2*A*tan(d*x+c)^(1/2)*(a*b)^(1/2)*2^(1/2)*arctan(1+2^(1
/2)*tan(d*x+c)^(1/2))*a^8+2*A*tan(d*x+c)^(1/2)*(a*b)^(1/2)*2^(1/2)*arctan(
-1+2^(1/2)*tan(d*x+c)^(1/2))*a^8+A*tan(d*x+c)^(1/2)*(a*b)^(1/2)*2^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3730 vs. $2(470) = 940$.

Time = 174.66 (sec) , antiderivative size = 7491, normalized size of antiderivative = 14.55

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{(A+B\tan{(c+dx)})\cot^{\frac{3}{2}}{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**(3/2)/(a + b*tan(c + d*x))**3,
 x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.12

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{(35\,Ba^5b^2-63\,Aa^4b^3+6\,Ba^3b^4-46\,Aa^2b^5+3\,Bab^6-15\,Ab^7)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6)\sqrt{ab}} - \frac{2\sqrt{2}((A-B)a^3+3\,(A+B)a^2b-3\,(A-B)ab^2-(A}{}}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
-1/4*((35*B*a^5*b^2 - 63*A*a^4*b^3 + 6*B*a^3*b^4 - 46*A*a^2*b^5 + 3*B*a*b^
6 - 15*A*b^7)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^9 + 3*a^7*b^2 +
 3*a^5*b^4 + a^3*b^6)*sqrt(a*b)) - (2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2
*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(t
an(d*x + c)))) + 2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^
2 - (A + B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + s
qrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log
(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*((A + B)*a^3 -
 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(-sqrt(2)/sqrt(tan(d*
x + c)) + 1/tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((11*
B*a^3*b^3 - 15*A*a^2*b^4 + 3*B*a*b^5 - 7*A*b^6)/sqrt(tan(d*x + c)) + (13*B
*a^4*b^2 - 17*A*a^3*b^3 + 5*B*a^2*b^4 - 9*A*a*b^5)/tan(d*x + c)^(3/2))/(a^
7*b^2 + 2*a^5*b^4 + a^3*b^6 + 2*(a^8*b + 2*a^6*b^3 + a^4*b^5)/tan(d*x + c)
 + (a^9 + 2*a^7*b^2 + a^5*b^4)/tan(d*x + c)^2) + 8*A/(a^3*sqrt(tan(d*x + c
))))/d
```

Giac [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{3}{2}}}{(b\tan(dx+c)+a)^3}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Hanged}$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\cot(dx+c)^{\frac{3}{2}}(A+B\tan(dx+c))}{(a+\tan(dx+c)\,b)^3}dx$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

3.605 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 448

$$\begin{aligned}
\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx &= \\
&-\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&+\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&-\frac{\sqrt{b}(35a^4Ab+6a^2Ab^3+3Ab^5-15a^5B+18a^3b^2B+ab^4B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{5/2}\,(a^2+b^2)^3\,d} \\
&+\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&+\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2} \\
&+\frac{b(11a^2Ab+3Ab^3-7a^3B+ab^2B)\sqrt{\cot(c+dx)}}{4a^2\,(a^2+b^2)^2\,d(b+a\cot(c+dx))}
\end{aligned}$$

output
```
1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctan(-1+2^(1/2)*co
t(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d+1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+
B)+3*a*b^2*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d
-1/4*b^(1/2)*(35*A*a^4*b+6*A*a^2*b^3+3*A*b^5-15*B*a^5+18*B*a^3*b^2+B*a*b^4
)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(5/2)/(a^2+b^2)^3/d+1/2*(a^3*
(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1
/2)/(1+cot(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d+1/2*b*(A*b-B*a)*cot(d*x+c)^(3/2)
/a/(a^2+b^2)/d/(b+a*cot(d*x+c))^2+1/4*b*(11*A*a^2*b+3*A*b^3-7*B*a^3+B*a*b^
2)*cot(d*x+c)^(1/2)/a^2/(a^2+b^2)^2/d/(b+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 4.47 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(2\sqrt{2}(3a^2b(A-B)+b^3(-A+B)-a^3(A+B)+3ab^2(A+B))\left(\arctan\left(\right.\right.\right.}{}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(2*Sqrt[2]*(3*a^2*b*(A - B) + b^3*(
-A + B) - a^3*(A + B) + 3*a*b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (3*Sqrt[b]*(a^2 + b^2)^
2*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/a^(5/2) + (4*S
qrt[b]*(a^2 + b^2)*(2*a*A*b - a^2*B + b^2*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c +
d*x]])/Sqrt[a]])/a^(3/2) - (8*Sqrt[b]*(-3*a^2*A*b + A*b^3 + a^3*B - 3*a*b^
2*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[a] - Sqrt[2]*(a^3*
(A - B) + 3*a*b^2*(-A + B) + 3*a^2*b*(A + B) - b^3*(A + B))*(Log[1 - Sqrt[
2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]
 + Tan[c + d*x]]) + (2*b*(a^2 + b^2)^2*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(a*
(a + b*Tan[c + d*x])^2) + (3*b*(a^2 + b^2)^2*(A*b - a*B)*Sqrt[Tan[c + d*x]
])/(a^2*(a + b*Tan[c + d*x])) + (4*b*(a^2 + b^2)*(2*a*A*b - a^2*B + b^2*B)
*Sqrt[Tan[c + d*x]])/(a*(a + b*Tan[c + d*x]))))/(4*(a^2 + b^2)^3*d)
```

Rubi [A] (warning: unable to verify)

Time = 3.85 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.788$, Rules used = {3042, 4064, 3042, 4088, 27, 3042, 4128, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A\cot(c+dx)+B)}{(a\cot(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}-\frac{\int -\frac{\sqrt{\cot(c+dx)}\left(\left(4Aa^2+bBa+3Ab^2\right)\cot^2(c+dx)-4a(Ab-aB)\cot(c+dx)+3b(Ab-aB)\right)}{2(b+a\cot(c+dx))^2}dx}{2a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\cot(c+dx)}\left(\left(4Aa^2+bBa+3Ab^2\right)\cot^2(c+dx)-4a(Ab-aB)\cot(c+dx)+3b(Ab-aB)\right)}{(b+a\cot(c+dx))^2}dx}{4a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{-\tan(c+dx+\frac{\pi}{2})}\left((4Aa^2+bBa+3Ab^2)\tan(c+dx+\frac{\pi}{2})^2+4a(Ab-aB)\tan(c+dx+\frac{\pi}{2})+3b(Ab-aB)\right)}{(b-a\tan(c+dx+\frac{\pi}{2}))^2}dx}{4a(a^2+b^2)}+\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad(a^2+b^2)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 4128

$$\frac{\frac{b(-7a^3B+11a^2Ab+ab^2B+3Ab^3)\sqrt{\cot(c+dx)}}{ad(a^2+b^2)(a\cot(c+dx)+b)}-\frac{\int -\frac{-8\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)a^2+\left(8Aa^4+9bBa^3+3Ab^2a^2+b^3Ba+3Ab^4\right)\cot^2(c+dx)+b\left(-7B\right.}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}}{a(a^2+b^2)}}{4a(a^2+b^2)}$$
$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad(a^2+b^2)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{-8\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)a^2+\left(8Aa^4+9bBa^3+3Ab^2a^2+b^3Ba+3Ab^4\right)\cot^2(c+dx)+b\left(-7Ba^3+11Aba^2+b^2Ba+3Ab^3\right)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{2a(a^2+b^2)}+\frac{b(-7a^3B+11a^2Ab+a}{ad(a^2+b^2)(}}{4a(a^2+b^2)}$$
$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad(a^2+b^2)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{8\left(-Ba^2+2Aba+b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)a^2+\left(8Aa^4+9bBa^3+3Ab^2a^2+b^3Ba+3Ab^4\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2+b\left(-7Ba^3+11Aba^2+b^2Ba+3Ab^3\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2a(a^2+b^2)}+\frac{b(-7a^3B+11}{ad(a}}{4a(a^2+b^2)}$$
$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad(a^2+b^2)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 4136

$$\frac{\frac{\frac{\int -\frac{8\left(a^2\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)-a^2\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int \frac{\cot^2}{\sqrt{\cot(c+dx}}}{a^2+b^2}}{2a(a^2+b^2)}}{4a(a^2+b^2)}$$
$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad(a^2+b^2)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 27

$$\frac{\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}-\frac{8\int\frac{a^2\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)-a^2\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right.}{\sqrt{\cot(c+dx)}}}{a^2+b^2}}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{8\int\frac{\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)a^2+\left(Aa^3+3bBa^2-3\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}}{a^2+b^2}}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 4017

$$\frac{\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16\int-\frac{a^2\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3-\left(Aa^3+3bBa^2-\right.\right.}{\cot^2(c+dx)+1}}{d\left(a^2+b^2\right)}}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{16\int\frac{a^2\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3-\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{}{\sqrt{-\tan}}}{a^2+b^2}}{2a(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 27

$$\frac{16a^2\int\frac{-Ba^3+3Aba^2+3b^2Ba-Ab^3-\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int\frac{}{\sqrt{-\tan(}}}{a^2+b^2}$$
$$2a(a^2+b^2)$$
$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 1482

$$\frac{16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int\frac{\cot(c+dx}{\cot^2(c+dx}}{d\left(a^2+b^2\right)}$$
$$2a(a^2+b^2)$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 1476

$$\frac{16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{1}{2}\int\frac{}{\cot(c+}\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 1082

$$\frac{16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\int\frac{1}{-\cot(c-}}{}\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 217

$$16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.$$
$$\overline{\qquad d\left(a^2+b^2\right)\qquad}$$
$$\overline{\qquad 2a(a^2+\qquad}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 1479

$$16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right.\right.$$
$$\overline{\qquad d\left(a^2+b^2\right)\qquad}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 25

$$16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\right.$$
$$\overline{\qquad d\left(a^2+b^2\right)\qquad}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 27

$$16a^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\right.$$
$$\overline{\qquad d\left(a^2+b^2\right)\qquad}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 1103

$$\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+16a^2\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(\right.\right.$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 4117$$

$$\frac{b\left(-15a^5B+35a^4Ab+18a^3b^2B+6a^2Ab^3+ab^4B+3Ab^5\right)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}+16a^2\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(\right.\right.$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{16a^2\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{b(Ab-aB)\cot^{\frac{3}{2}}(c+dx)}{2ad\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}+\frac{\frac{b\left(-7a^3B+11a^2Ab+ab^2B+3Ab^3\right)\sqrt{\cot(c+dx)}}{ad(a^2+b^2)(a\cot(c+dx)+b)}+\frac{16a^2\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1\right.}{}\right.\right.}{}}{}$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^3,x]
```

output
```
(b*(A*b - a*B)*Cot[c + d*x]^(3/2))/(2*a*(a^2 + b^2)*d*(b + a*Cot[c + d*x])
^2) + ((b*(11*a^2*A*b + 3*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Cot[c + d*x]])/(
a*(a^2 + b^2)*d*(b + a*Cot[c + d*x])) + ((2*Sqrt[b]*(35*a^4*A*b + 6*a^2*A*
b^3 + 3*A*b^5 - 15*a^5*B + 18*a^3*b^2*B + a*b^4*B)*ArcTan[(Sqrt[a]*Cot[c +
 d*x])/Sqrt[b]])/(Sqrt[a]*(a^2 + b^2)*d) + (16*a^2*(((3*a^2*b*(A - B) - b^
3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[
c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 +
 ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*(-1/2*Lo
g[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/(2*
a*(a^2 + b^2)))/(4*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4190 vs. $2(411) = 822$.

Time = 0.44 (sec) , antiderivative size = 4191, normalized size of antiderivative = 9.35

method	result	size
derivativedivides	Expression too large to display	4191
default	Expression too large to display	4191

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/4/d*(1/tan(d*x+c))^(1/2)*tan(d*x+c)^(1/2)*(3*A*(a*b)^(1/2)*tan(d*x+c)^(3
/2)*b^7+3*A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*b^8*tan(d*x+c)^2+35*A*a
rctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^6*b^2+6*A*arctan(b*tan(d*x+c)^(1/2
)/(a*b)^(1/2))*a^4*b^4+3*A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^2*b^6-
15*B*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^7*b+18*B*arctan(b*tan(d*x+c)
^(1/2)/(a*b)^(1/2))*a^5*b^3+B*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^3*b
^5+11*A*(a*b)^(1/2)*tan(d*x+c)^(3/2)*a^4*b^3+14*A*(a*b)^(1/2)*tan(d*x+c)^(
3/2)*a^2*b^5+35*A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^4*b^4*tan(d*x+c
)^2+6*A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^2*b^6*tan(d*x+c)^2-7*B*(a
*b)^(1/2)*tan(d*x+c)^(3/2)*a^5*b^2-6*B*(a*b)^(1/2)*tan(d*x+c)^(3/2)*a^3*b^
4+B*(a*b)^(1/2)*tan(d*x+c)^(3/2)*a*b^6-15*B*arctan(b*tan(d*x+c)^(1/2)/(a*b
)^(1/2))*a^5*b^3*tan(d*x+c)^2+18*B*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*
a^3*b^5*tan(d*x+c)^2+B*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a*b^7*tan(d*
x+c)^2+A*(a*b)^(1/2)*2^(1/2)*ln(-(tan(d*x+c)+2^(1/2)*tan(d*x+c)^(1/2)+1)/(
2^(1/2)*tan(d*x+c)^(1/2)-tan(d*x+c)-1))*a^7+2*A*(a*b)^(1/2)*2^(1/2)*arctan
(1+2^(1/2)*tan(d*x+c)^(1/2))*a^7+2*A*(a*b)^(1/2)*2^(1/2)*arctan(-1+2^(1/2)
*tan(d*x+c)^(1/2))*a^7+70*A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^5*b^3
*tan(d*x+c)+12*A*arctan(b*tan(d*x+c)^(1/2)/(a*b)^(1/2))*a^3*b^5*tan(d*x+c)
-6*A*(a*b)^(1/2)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a^6*b-6*A*(a*
b)^(1/2)*2^(1/2)*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*a^5*b^2+2*A*(a*b)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3674 vs. $2(409) = 818$.

Time = 112.16 (sec) , antiderivative size = 7379, normalized size of antiderivative = 16.47

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{\cot{(c+dx)}}}{(a+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/(a + b*tan(c + d*x))**3,
x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{(15\,Ba^5b-35\,Aa^4b^2-18\,Ba^3b^3-6\,Aa^2b^4-Bab^5-3\,Ab^6)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6)\sqrt{ab}} - \frac{2\sqrt{2}((A+B)a^3-3\,(A-B)a^2b-3\,(A+B)ab^2+(A-B)b^3)}{}}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
1/4*((15*B*a^5*b - 35*A*a^4*b^2 - 18*B*a^3*b^3 - 6*A*a^2*b^4 - B*a*b^5 - 3
*A*b^6)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^8 + 3*a^6*b^2 + 3*a^4
*b^4 + a^2*b^6)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3
*(A + B)*a*b^2 + (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x
 + c)))) + 2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A
 - B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)
*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(sqrt(
2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((A - B)*a^3 + 3*(A
+ B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)
) + 1/tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((7*B*a^3*b
^2 - 11*A*a^2*b^3 - B*a*b^4 - 3*A*b^5)/sqrt(tan(d*x + c)) + (9*B*a^4*b - 1
3*A*a^3*b^2 + B*a^2*b^3 - 5*A*a*b^4)/tan(d*x + c)^(3/2))/(a^6*b^2 + 2*a^4*
b^4 + a^2*b^6 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)/tan(d*x + c) + (a^8 + 2*a^
6*b^2 + a^4*b^4)/tan(d*x + c)^2))/d
```

Giac [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{\cot(dx+c)}}{(b\tan(dx+c)+a)^3}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)^3,
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^3}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3,x)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\cot(dx+c)}}{\tan(dx+c)^2 b^2+2\tan(dx+c)ab+a^2}dx$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^3,x)
```

output
```
int(sqrt(cot(c + d*x))/(tan(c + d*x)**2*b**2 + 2*tan(c + d*x)*a*b + a**2),
x)
```

3.606 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 447

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3}\,dx = \frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$-\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+\frac{(15a^4Ab-18a^2Ab^3-Ab^5-3a^5B+26a^3b^2B-3ab^4B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4a^{3/2}\sqrt{b}\,(a^2+b^2)^3\,d}$$
$$+\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2a\,(a^2+b^2)\,d(b+a\cot(c+dx))^2}-\frac{(9a^2Ab+Ab^3-5a^3B+3ab^2B)\sqrt{\cot(c+dx)}}{4a\,(a^2+b^2)^2\,d(b+a\cot(c+dx))}$$

output
```
-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(-1+2^(1/2)*c
ot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d-1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*
b*(A+B)-b^3*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/
d+1/4*(15*A*a^4*b-18*A*a^2*b^3-A*b^5-3*B*a^5+26*B*a^3*b^2-3*B*a*b^4)*arcta
n(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(3/2)/b^(1/2)/(a^2+b^2)^3/d+1/2*(3*a
^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(
1/2)/(1+cot(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d+1/2*b*(A*b-B*a)*cot(d*x+c)^(1/2
)/a/(a^2+b^2)/d/(b+a*cot(d*x+c))^2-1/4*(9*A*a^2*b+A*b^3-5*B*a^3+3*B*a*b^2)
*cot(d*x+c)^(1/2)/a/(a^2+b^2)^2/d/(b+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 4.11 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.18

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-2\sqrt{2}(a^3(A-B)+3ab^2(-A+B)+3a^2b(A+B)-b^3(A+B))\left(\arctan\right.\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^3)
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-2*Sqrt[2]*(a^3*(A - B) + 3*a*b^2*
(-A + B) + 3*a^2*b*(A + B) - b^3*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
 d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (3*(a^2 + b^2)^2*(-(A*
b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(a^(3/2)*Sqrt[b])
- (4*Sqrt[b]*(a^2 + b^2)*(a^2*A - A*b^2 + 2*a*b*B)*ArcTan[(Sqrt[b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a]])/a^(3/2) + (8*Sqrt[b]*(-(a^3*A) + 3*a*A*b^2 - 3*a^2*
b*B + b^3*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/Sqrt[a] - Sqrt[
2]*(3*a^2*b*(A - B) + b^3*(-A + B) - a^3*(A + B) + 3*a*b^2*(A + B))*(Log[1
 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan[c
 + d*x]] + Tan[c + d*x]]) + (2*(a^2 + b^2)^2*(-(A*b) + a*B)*Sqrt[Tan[c + d
*x]])/(a + b*Tan[c + d*x])^2 + (3*(a^2 + b^2)^2*(-(A*b) + a*B)*Sqrt[Tan[c
+ d*x]])/(a*(a + b*Tan[c + d*x])) - (4*b*(a^2 + b^2)*(a^2*A - A*b^2 + 2*a*
b*B)*Sqrt[Tan[c + d*x]])/(a*(a + b*Tan[c + d*x]))))/(4*(a^2 + b^2)^3*d)
```

Rubi [A] (warning: unable to verify)

Time = 3.82 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.788$, Rules used = {3042, 4064, 3042, 4088, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A\cot(c+dx)+B)}{(a\cot(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4088}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}-\frac{\int -\frac{\left(4Aa^2+3bBa+Ab^2\right)\cot^2(c+dx)-4a(Ab-aB)\cot(c+dx)+b(Ab-aB)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))^2}dx}{2a\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(4Aa^2+3bBa+Ab^2\right)\cot^2(c+dx)-4a(Ab-aB)\cot(c+dx)+b(Ab-aB)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))^2}dx}{4a\left(a^2+b^2\right)}+\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(4Aa^2+3bBa+Ab^2)\tan\left(c+dx+\frac{\pi}{2}\right)^2+4a(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)+b(Ab-aB)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\,(a^2+b^2)}+\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\,(a^2+b^2)\,(a\cot(c+dx)+b)^2}$$

$\downarrow$ 4132

$$\frac{-\frac{\int \frac{-b\left(-5Ba^3+9Aba^2+3b^2Ba+Ab^3\right)\cot^2(c+dx)-8ab\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)+b\left(-3Ba^3+7Aba^2+5b^2Ba-Ab^3\right)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b(a^2+b^2)}-\frac{(-5a^3B+9a^2Ab+3ab^2B+\cdots}{d(a^2+b^2)(a\cot(c+\cdots}}{4a\,(a^2+b^2)}$$
$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\,(a^2+b^2)\,(a\cot(c+dx)+b)^2}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{-b\left(-5Ba^3+9Aba^2+3b^2Ba+Ab^3\right)\cot^2(c+dx)-8ab\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)+b\left(-3Ba^3+7Aba^2+5b^2Ba-Ab^3\right)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{2b(a^2+b^2)}-\frac{(-5a^3B+9a^2Ab+3ab^2B+\cdots}{d(a^2+b^2)(a\cot(c+\cdots}}{4a\,(a^2+b^2)}$$
$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\,(a^2+b^2)\,(a\cot(c+dx)+b)^2}$$

$\downarrow$ 3042

$$\frac{-\frac{\int \frac{-b\left(-5Ba^3+9Aba^2+3b^2Ba+Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2+8ab\left(Aa^2+2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)+b\left(-3Ba^3+7Aba^2+5b^2Ba-Ab^3\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2+b^2)}-\frac{(-5a^3B+9a^2Ab+\cdots}{d(a^2+b^2)\cdots}}{4a\,(a^2+b^2)}$$
$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\,(a^2+b^2)\,(a\cot(c+dx)+b)^2}$$

$\downarrow$ 4136

$$-\frac{\frac{\int -\frac{8\left(ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)+ab\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{\mathrm{co}\cdots}{\sqrt{\cot(c+\cdots}}}{a^2+b^2}}{2b(a^2+b^2)} \cdots$$
$$4a\,(a^2+b^2)$$
$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\,(a^2+b^2)\,(a\cot(c+dx)+b)^2}$$

$\downarrow$ 27

$$-\frac{\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}-\frac{8\int \frac{ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)+ab\left(-Ba^3+3Aba^2+3b^2Ba-\right.}{\sqrt{\cot(c+dx)}}}{a^2+b^2}}{2b(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{8\int \frac{ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-ab\left(-Ba^3+3Ab\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right.}}}{a^2+b^2}}{2b(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 4017

$$-\frac{\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16\int -\frac{ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B+\left(-Ba^3+3Aba\right.\right.}{\cot^2(c+dx)+1}}{d\left(a^2+b^2\right)}}{2b(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{16\int \frac{ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B+\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{}{\sqrt{-}}}{a^2+b^2}}{2b(a^2+b^2)}$$

$$4a\left(a^2+b^2\right)$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{16ab\int \frac{Aa^3+3bBa^2-3Ab^2a-b^3B+\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{}{\sqrt{-\tan \cdots}}}{a^2+b^2}}{2b(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 1482

$$-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int \frac{1-\cot(\cdots}{\cot^2(c+\cdots}\right.}{d\left(a^2+b^2\right)}}{2b(a^2+b^2)}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 1476

$$-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\cdots\right.\right.}{d\left(a^2+b^2\right)}}{\cdots}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 1082

$$-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A\cdots\right.\right.\right.}{d\left(a^2+b^2\right)}}{\cdots}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 217

$$-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2 \cdots\right.\right.}{d\left(a^2+b^2\right)}}{2b(a^2 \cdots}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \; 1479$$

$$-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2 \cdots\right.\right.}{d\left(a^2+b^2\right)}}{\cdots}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2 \cdots\right.\right.}{d\left(a^2+b^2\right)}}{\cdots}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2 \cdots\right.\right.}{d\left(a^2+b^2\right)}}{\cdots}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \; 1103$$

$$-\frac{\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A\right.\right.}{}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 4117

$$-\frac{\frac{b\left(-3a^5B+15a^4Ab+26a^3b^2B-18a^2Ab^3-3ab^4B-Ab^5\right)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}+16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A\right.\right.}{}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 73

$$-\frac{\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 218

$$\frac{b(Ab-aB)\sqrt{\cot(c+dx)}}{2ad\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}+$$

$$\frac{-\frac{\left(-5a^3B+9a^2Ab+3ab^2B+Ab^3\right)\sqrt{\cot(c+dx)}}{d(a^2+b^2)(a\cot(c+dx)+b)}-\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{\right.}{}\right.\right.}{}}{}$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^3),x]
```

output
```
(b*(A*b - a*B)*Sqrt[Cot[c + d*x]])/(2*a*(a^2 + b^2)*d*(b + a*Cot[c + d*x])
^2) + (-(((9*a^2*A*b + A*b^3 - 5*a^3*B + 3*a*b^2*B)*Sqrt[Cot[c + d*x]])/((
a^2 + b^2)*d*(b + a*Cot[c + d*x]))) - ((2*Sqrt[b]*(15*a^4*A*b - 18*a^2*A*b
^3 - A*b^5 - 3*a^5*B + 26*a^3*b^2*B - 3*a*b^4*B)*ArcTan[(Sqrt[a]*Cot[c + d
*x])/Sqrt[b]])/(Sqrt[a]*(a^2 + b^2)*d) + (16*a*b*(((a^3*(A - B) - 3*a*b^2*
(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c
+ d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2 - (
(3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*(-1/2*Log[
1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/(2*b*
(a^2 + b^2)))/(4*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{2\left(\left(-\frac{9}{8}Aa^4b-\frac{5}{4}Aa^2b^3-\frac{1}{8}Ab^5+\frac{5}{8}Ba^5+\frac{1}{4}Ba^3b^2-\frac{3}{8}Bab^4\right)\cot(dx+c)^{\frac{3}{2}}-\frac{b\left(7Aa^4b+6Aa^2b^3-Ab^5-3Ba^5+2Ba^3b^2+5Bab^4\right)}{8a}\right.}{(b+a\cot(dx+c))^2}}{\left(a^2+b^2\right)^3}$
default	$\frac{\frac{2\left(\left(-\frac{9}{8}Aa^4b-\frac{5}{4}Aa^2b^3-\frac{1}{8}Ab^5+\frac{5}{8}Ba^5+\frac{1}{4}Ba^3b^2-\frac{3}{8}Bab^4\right)\cot(dx+c)^{\frac{3}{2}}-\frac{b\left(7Aa^4b+6Aa^2b^3-Ab^5-3Ba^5+2Ba^3b^2+5Bab^4\right)}{8a}\right.}{(b+a\cot(dx+c))^2}}{\left(a^2+b^2\right)^3}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2/(a^2+b^2)^3*(((-9/8*A*a^4*b-5/4*A*a^2*b^3-1/8*A*b^5+5/8*B*a^5+1/4*B
*a^3*b^2-3/8*B*a*b^4)*cot(d*x+c)^(3/2)-1/8*b*(7*A*a^4*b+6*A*a^2*b^3-A*b^5-
3*B*a^5+2*B*a^3*b^2+5*B*a*b^4)/a*cot(d*x+c)^(1/2))/(b+a*cot(d*x+c))^2+1/8*
(15*A*a^4*b-18*A*a^2*b^3-A*b^5-3*B*a^5+26*B*a^3*b^2-3*B*a*b^4)/a/(a*b)^(1/
2)*arctan(a*cot(d*x+c)^(1/2)/(a*b)^(1/2)))-2/(a^2+b^2)^3*(1/8*(A*a^3-3*A*a
*b^2+3*B*a^2*b-B*b^3)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/
(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/
2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/8*(3*A*a^2*b-A*b^3-B*a^3+3*B*
a*b^2)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(
1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1
+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3695 vs. $2(408) = 816$.

Time = 74.27 (sec) , antiderivative size = 7415, normalized size of antiderivative = 16.59

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^3}\, dx = \int \frac{A + B\tan{(c + dx)}}{(a + b\tan{(c + dx)})^3\sqrt{\cot{(c + dx)}}}\, dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**3,x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**3*sqrt(cot(c + d*x)))
, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.22

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^3}\, dx =$$

$$-\frac{\frac{(3\,Ba^5 - 15\,Aa^4b - 26\,Ba^3b^2 + 18\,Aa^2b^3 + 3\,Bab^4 + Ab^5)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6)\sqrt{ab}} + \frac{2\sqrt{2}((A-B)a^3 + 3\,(A+B)a^2b - 3\,(A-B)ab^2 - (A+B)b^3}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
-1/4*((3*B*a^5 - 15*A*a^4*b - 26*B*a^3*b^2 + 18*A*a^2*b^3 + 3*B*a*b^4 + A*
b^5)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6)*sqrt(a*b)) + (2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A -
 B)*a*b^2 - (A + B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)
))) + 2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)
*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*((A
+ B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(sqrt(2)/sq
rt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*((A + B)*a^3 - 3*(A - B)*
a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1
/tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((3*B*a^3*b - 7*
A*a^2*b^2 - 5*B*a*b^3 + A*b^4)/sqrt(tan(d*x + c)) + (5*B*a^4 - 9*A*a^3*b -
 3*B*a^2*b^2 - A*a*b^3)/tan(d*x + c)^(3/2))/(a^5*b^2 + 2*a^3*b^4 + a*b^6 +
 2*(a^6*b + 2*a^4*b^3 + a^2*b^5)/tan(d*x + c) + (a^7 + 2*a^5*b^2 + a^3*b^4
)/tan(d*x + c)^2))/d
```

Giac [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3}\,dx = \int \frac{B\tan\left(dx+c\right)+A}{\left(b\tan\left(dx+c\right)+a\right)^3\sqrt{\cot\left(dx+c\right)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^3*sqrt(cot(d*x + c))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot\left(c+d\,x\right)}\,\left(a+b\tan\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^3),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)\tan\left(dx+c\right)^2b^2+2\cot\left(dx+c\right)\tan\left(dx+c\right)ab+\cot\left(dx+c\right)a^2}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^3,x)
```

output
```
int(sqrt(cot(c + d*x))/(cot(c + d*x)*tan(c + d*x)**2*b**2 + 2*cot(c + d*x)
*tan(c + d*x)*a*b + cot(c + d*x)*a**2),x)
```

3.607 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 445

$$\begin{aligned}
&\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx \\
&= \frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&\quad -\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&\quad -\frac{(3a^4Ab-26a^2Ab^3+3Ab^5+a^5B+18a^3b^2B-15ab^4B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4\sqrt{a}b^{3/2}\,(a^2+b^2)^3\,d} \\
&\quad -\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\
&\quad -\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2\,(a^2+b^2)\,d(b+a\cot(c+dx))^2}+\frac{(5a^2Ab-3Ab^3-a^3B+7ab^2B)\,\sqrt{\cot(c+dx)}}{4b\,(a^2+b^2)^2\,d(b+a\cot(c+dx))}
\end{aligned}$$

output
```
-1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctan(-1+2^(1/2)*c
ot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d-1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A
+B)+3*a*b^2*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/
d-1/4*(3*A*a^4*b-26*A*a^2*b^3+3*A*b^5+B*a^5+18*B*a^3*b^2-15*B*a*b^4)*arcta
n(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/a^(1/2)/b^(3/2)/(a^2+b^2)^3/d-1/2*(a^3
*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(
1/2)/(1+cot(d*x+c))*2^(1/2)/(a^2+b^2)^3/d-1/2*(A*b-B*a)*cot(d*x+c)^(1/2)/
(a^2+b^2)/d/(b+a*cot(d*x+c))^2+1/4*(5*A*a^2*b-3*A*b^3-B*a^3+7*B*a*b^2)*cot
(d*x+c)^(1/2)/b/(a^2+b^2)^2/d/(b+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 4.15 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.20

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-2\sqrt{2}(3a^2b(A-B)+b^3(-A+B)-a^3(A+B)+3ab^2(A+B))\left(\arctan\right.\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3)
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-2*Sqrt[2]*(3*a^2*b*(A - B) + b^3*
(-A + B) - a^3*(A + B) + 3*a*b^2*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
 d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (3*(a^2 + b^2)^2*(A*b
- a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (
8*Sqrt[b]*(-3*a^2*A*b + A*b^3 + a^3*B - 3*a*b^2*B)*ArcTan[(Sqrt[b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a]])/Sqrt[a] + (4*(a^2 + b^2)*(-2*A*b^3 + a*(a^2 + 3*b^2
)*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + Sqr
t[2]*(a^3*(A - B) + 3*a*b^2*(-A + B) + 3*a^2*b*(A + B) - b^3*(A + B))*(Log
[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]] + Tan[c + d*x]]) + (2*a*(a^2 + b^2)^2*(A*b - a*B)*Sqrt[Tan[c +
d*x]])/(b*(a + b*Tan[c + d*x])^2) + (3*(a^2 + b^2)^2*(A*b - a*B)*Sqrt[Tan[
c + d*x]])/(b*(a + b*Tan[c + d*x])) + (4*(a^2 + b^2)*(-2*A*b^3 + a*(a^2 +
3*b^2)*B)*Sqrt[Tan[c + d*x]])/(b*(a + b*Tan[c + d*x]))))/(4*(a^2 + b^2)^3*
d)
```

Rubi [A] (warning: unable to verify)

Time = 3.85 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.788$, Rules used = {3042, 4064, 3042, 4091, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A\cot(c+dx)+B)}{(a\cot(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(B-A\tan\left(c+dx+\frac{\pi}{2}\right)\right)}{\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4091}$$

$$\frac{\int -\frac{-3a(Ab-aB)\cot^2(c+dx)-4a(aA+bB)\cot(c+dx)+a(Ab-aB)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))^2}dx}{2a\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-3a(Ab-aB)\cot^2(c+dx)-4a(aA+bB)\cot(c+dx)+a(Ab-aB)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))^2}dx}{4a\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-3a(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2+4a(aA+bB)\tan\left(c+dx+\frac{\pi}{2}\right)+a(Ab-aB)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4a\left(a^2+b^2\right)}-\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 4132

$$-\frac{-\frac{\int \frac{-a\left(-Ba^3+5Aba^2+7b^2Ba-3Ab^3\right)\cot^2(c+dx)+8ab\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)+a\left(Ba^3+3Aba^2+9b^2Ba-5Ab^3\right)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b(a^2+b^2)}-\frac{a(a^3(-B)+5a^2Ab+7ab^2 \cdots}{bd(a^2+b^2)(a\cot \cdots}}{4a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{-a\left(-Ba^3+5Aba^2+7b^2Ba-3Ab^3\right)\cot^2(c+dx)+8ab\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)+a\left(Ba^3+3Aba^2+9b^2Ba-5Ab^3\right)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{2b(a^2+b^2)}-\frac{a(a^3(-B)+5a^2Ab+7ab^2 \cdots}{bd(a^2+b^2)(a\cot \cdots}}{4a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{\int \frac{-a\left(-Ba^3+5Aba^2+7b^2Ba-3Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2-8ab\left(-Ba^2+2Aba+b^2B\right)\tan\left(c+dx+\frac{\pi}{2}\right)+a\left(Ba^3+3Aba^2+9b^2Ba-5Ab^3\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2+b^2)}-\frac{a(a^3(-B)+5a^2 \cdots}{bd(a^2 \cdots}}{4a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 4136

$$-\frac{-\frac{\frac{\int \frac{8\left(ab\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)-ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int \frac{\cot \cdots}{\sqrt{\cot(c+d \cdots}}}{a^2+b^2}}{2b(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\frac{8\int \frac{ab\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)-ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int \frac{\cot^2(\cdots}{\sqrt{\cot(c+dx) \cdots}}}{a^2+b^2}}{2b(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{\frac{8\int \frac{ab\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)+ab\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx}{a^2+b^2}+\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int \frac{\cdots}{\sqrt{-\tan\cdots}}}{a^2+b^2}}{2b(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 4017

$$-\frac{-\frac{\frac{16\int -\frac{ab\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3-\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int \cdots}{a^2+b^2}}{2b(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 25

$$-\frac{-\frac{\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16\int \frac{ab\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3-\left(Aa^3+3bBa^2\cdots\right.\right.}{\cot^2(c+dx)+1}\cdots}{d\left(a^2+b^2\right)}}{2b(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 27

$$-\frac{-\frac{\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ab\int \frac{-Ba^3+3Aba^2+3b^2Ba-Ab^3-\left(Aa^3+3bBa^2\cdots\right.}{\cot^2(c+dx)+1}\cdots}{d\left(a^2+b^2\right)}}{2b(a^2+b^2)}}{4a\left(a^2+b^2\right)}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 1482

$$-\frac{-\frac{\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(\right.\right.}{}}{2b(a^2+b^2)}}{}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 1476

$$-\frac{-\frac{\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(\right.\right.}{}}{}}{}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 1082

$$-\frac{-\frac{\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(\right.\right.}{}}{}}{}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 217

$$-\frac{-\frac{\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(\right.\right.}{}}{2b(}}{}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 1479

$$-\frac{-\frac{\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(\ldots\right)\right)}{}}{}}{}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(\ldots\right)\right)}{}}{}}{}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ab\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(\ldots\right)\right)}{}}{}}{}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \; 1103$$

$$-\frac{-\frac{\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-\ldots\right)\right)}{}}{}}{}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \; 4117$$

$$-\frac{-\frac{a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int\frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}-\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-\right.\right.}{}}{}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 73

$$-\frac{-\frac{2a\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\int\frac{1}{a\cot^2(c+dx)+b}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}-\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\right.}{}}{}$$

$$\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\Big\downarrow$ 218

$$-\frac{(Ab-aB)\sqrt{\cot(c+dx)}}{2d\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}-$$

$$\frac{-\frac{a\left(a^3(-B)+5a^2Ab+7ab^2B-3Ab^3\right)\sqrt{\cot(c+dx)}}{bd\left(a^2+b^2\right)(a\cot(c+dx)+b)}-\frac{\frac{2\sqrt{a}\left(a^5B+3a^4Ab+18a^3b^2B-26a^2Ab^3-15ab^4B+3Ab^5\right)\arctan\left(\frac{\sqrt{a}\cot(c+dx)}{\sqrt{b}}\right)}{\sqrt{b}d\left(a^2+b^2\right)}-\frac{16ab\left(\frac{1}{2}\left(-\left(a^3(\right.\right.\right.}{}}{}}{}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^3),x]
```

output
```
-1/2*((A*b - a*B)*Sqrt[Cot[c + d*x]])/((a^2 + b^2)*d*(b + a*Cot[c + d*x])^
2) - (-((a*(5*a^2*A*b - 3*A*b^3 - a^3*B + 7*a*b^2*B)*Sqrt[Cot[c + d*x]])/(
b*(a^2 + b^2)*d*(b + a*Cot[c + d*x]))) - ((2*Sqrt[a]*(3*a^4*A*b - 26*a^2*A
*b^3 + 3*A*b^5 + a^5*B + 18*a^3*b^2*B - 15*a*b^4*B)*ArcTan[(Sqrt[a]*Cot[c
+ d*x])/Sqrt[b]])/(Sqrt[b]*(a^2 + b^2)*d) - (16*a*b*(((3*a^2*b*(A - B) - b
^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[Cot
[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/2
+ ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*(-1/2*L
og[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2
]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/(2
*b*(a^2 + b^2)))/(4*a*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{2\left(\frac{-\frac{a\left(5A\,a^4b+2A\,a^2b^3-3A\,b^5-B\,a^5+6B\,a^3b^2+7B\,a\,b^4\right)\cot(dx+c)^{\frac{3}{2}}}{8b}+\left(-\frac{3}{8}A\,a^4b+\frac{5}{8}A\,b^5-\frac{5}{4}B\,a^3b^2-\frac{9}{8}B\,a\,b^4+\frac{1}{4}A\,a^2b^3-\frac{1}{8}B\right.}{(b+a\cot(dx+c))^2}\right.}{\left(a^2+b^2\right)^3}$
default	$-\frac{2\left(\frac{-\frac{a\left(5A\,a^4b+2A\,a^2b^3-3A\,b^5-B\,a^5+6B\,a^3b^2+7B\,a\,b^4\right)\cot(dx+c)^{\frac{3}{2}}}{8b}+\left(-\frac{3}{8}A\,a^4b+\frac{5}{8}A\,b^5-\frac{5}{4}B\,a^3b^2-\frac{9}{8}B\,a\,b^4+\frac{1}{4}A\,a^2b^3-\frac{1}{8}B\right.}{(b+a\cot(dx+c))^2}\right.}{\left(a^2+b^2\right)^3}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(-2/(a^2+b^2)^3*((-1/8*a*(5*A*a^4*b+2*A*a^2*b^3-3*A*b^5-B*a^5+6*B*a^3*
b^2+7*B*a*b^4)/b*cot(d*x+c)^(3/2)+(-3/8*A*a^4*b+5/8*A*b^5-5/4*B*a^3*b^2-9/
8*B*a*b^4+1/4*A*a^2*b^3-1/8*B*a^5)*cot(d*x+c)^(1/2))/(b+a*cot(d*x+c))^2+1/
8*(3*A*a^4*b-26*A*a^2*b^3+3*A*b^5+B*a^5+18*B*a^3*b^2-15*B*a*b^4)/b/(a*b)^(
1/2)*arctan(a*cot(d*x+c)^(1/2)/(a*b)^(1/2)))-2/(a^2+b^2)^3*(1/8*(3*A*a^2*b
-A*b^3-B*a^3+3*B*a*b^2)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1
)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(
1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/8*(-A*a^3+3*A*a*b^2-3*B*a^2
*b+B*b^3)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+
2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan
(-1+2^(1/2)*cot(d*x+c)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3695 vs. $2(402) = 804$.

Time = 68.91 (sec) , antiderivative size = 7418, normalized size of antiderivative = 16.67

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.22

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^3}\,dx =$$

$$-\frac{\frac{(Ba^5+3\,Aa^4b+18\,Ba^3b^2-26\,Aa^2b^3-15\,Bab^4+3\,Ab^5)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sqrt{ab}} - \frac{2\sqrt{2}((A+B)a^3-3\,(A-B)a^2b-3\,(A+B)ab^2+(A-B)b^3}{}}{}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
-1/4*((B*a^5 + 3*A*a^4*b + 18*B*a^3*b^2 - 26*A*a^2*b^3 - 15*B*a*b^4 + 3*A*
b^5)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A +
 B)*a*b^2 + (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)
))) + 2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)
*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*((A
- B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(sqrt(2)/sq
rt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((A - B)*a^3 + 3*(A + B)*
a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(-sqrt(2)/sqrt(tan(d*x + c)) + 1
/tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - ((B*a^3*b + 3*A*
a^2*b^2 + 9*B*a*b^3 - 5*A*b^4)/sqrt(tan(d*x + c)) - (B*a^4 - 5*A*a^3*b - 7
*B*a^2*b^2 + 3*A*a*b^3)/tan(d*x + c)^(3/2))/(a^4*b^3 + 2*a^2*b^5 + b^7 + 2
*(a^5*b^2 + 2*a^3*b^4 + a*b^6)/tan(d*x + c) + (a^6*b + 2*a^4*b^3 + a^2*b^5
)/tan(d*x + c)^2))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx=\int \frac{A+B\tan(c+d\,x)}{{\mathrm{cot}\left(c+d\,x\right)}^{3/2}\,{\left(a+b\,\tan\left(c+d\,x\right)\right)}^3}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^3),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$$
$$=\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)^2\tan(dx+c)^2b^2+2\cot(dx+c)^2\tan(dx+c)\,ab+\cot(dx+c)^2a^2}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^3,x)
```

output
```
int(sqrt(cot(c + d*x))/(cot(c + d*x)**2*tan(c + d*x)**2*b**2 + 2*cot(c + d
*x)**2*tan(c + d*x)*a*b + cot(c + d*x)**2*a**2),x)
```

3.608 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 448

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = -\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$-\frac{\sqrt{a}(a^4Ab+18a^2Ab^3-15Ab^5+3a^5B+6a^3b^2B+35ab^4B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4b^{5/2}\,(a^2+b^2)^3\,d}$$
$$-\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d}$$
$$+\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2b\,(a^2+b^2)\,d(b+a\cot(c+dx))^2}$$
$$-\frac{a(a^2Ab-7Ab^3+3a^3B+11ab^2B)\,\sqrt{\cot(c+dx)}}{4b^2\,(a^2+b^2)^2\,d(b+a\cot(c+dx))}$$

output
```
1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctan(-1+2^(1/2)*co
t(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d+1/2*(a^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b
*(A+B)-b^3*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d
-1/4*a^(1/2)*(A*a^4*b+18*A*a^2*b^3-15*A*b^5+3*B*a^5+6*B*a^3*b^2+35*B*a*b^4
)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/b^(5/2)/(a^2+b^2)^3/d-1/2*(3*a^
2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctanh(2^(1/2)*cot(d*x+c)^(1
/2)/(1+cot(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d+1/2*a*(A*b-B*a)*cot(d*x+c)^(1/2)
/b/(a^2+b^2)/d/(b+a*cot(d*x+c))^2-1/4*a*(A*a^2*b-7*A*b^3+3*B*a^3+11*B*a*b^
2)*cot(d*x+c)^(1/2)/b^2/(a^2+b^2)^2/d/(b+a*cot(d*x+c))
```

Mathematica [A] (verified)

Time = 4.40 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.26

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(2\sqrt{2}(a^3(A-B)+3ab^2(-A+B)+3a^2b(A+B)-b^3(A+B))\left(\arctan\left(\right.\right.\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3)
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(2*Sqrt[2]*(a^3*(A - B) + 3*a*b^2*(
-A + B) + 3*a^2*b*(A + B) - b^3*(A + B))*(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]) + (3*Sqrt[a]*(a^2 + b^2)^
2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/b^(5/2) + (
4*Sqrt[a]*(a^2 + b^2)*(a^2*A*b + 3*A*b^3 - 2*a^3*B - 4*a*b^2*B)*ArcTan[(Sq
rt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/b^(5/2) + (8*Sqrt[a]*(a^2*A*b^3 - 3*A*
b^5 + a^5*B + 3*a^3*b^2*B + 6*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])
/Sqrt[a]])/b^(5/2) + Sqrt[2]*(3*a^2*b*(A - B) + b^3*(-A + B) - a^3*(A + B)
 + 3*a*b^2*(A + B))*(Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] -
Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]) + (2*a^2*(a^2 + b^2)^2
*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]])/(b^2*(a + b*Tan[c + d*x])^2) + (3*a*(a
^2 + b^2)^2*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]])/(b^2*(a + b*Tan[c + d*x]))
+ (4*a*(a^2 + b^2)*(a^2*A*b + 3*A*b^3 - 2*a^3*B - 4*a*b^2*B)*Sqrt[Tan[c +
d*x]])/(b^2*(a + b*Tan[c + d*x]))))/(4*(a^2 + b^2)^3*d)
```

Rubi [A] (warning: unable to verify)

Time = 4.04 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.788, Rules used = {3042, 4064, 3042, 4092, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{5/2}(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4064}$$

$$\int \frac{A\cot(c+dx)+B}{\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{4092}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}-\frac{\int -\frac{3Ba^2-3(Ab-aB)\cot^2(c+dx)a+Aba+4b^2B+4b(Ab-aB)\cot(c+dx)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))^2}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3Ba^2-3(Ab-aB)\cot^2(c+dx)a+Aba+4b^2B+4b(Ab-aB)\cot(c+dx)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))^2}dx}{4b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3Ba^2-3(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2a+Aba+4b^2B-4b(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2+b^2\right)}+\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{ 4132}$$

$$\frac{-\frac{\int -\frac{3Ba^4+Aba^3+3b^2Ba^2+9Ab^3a+\left(3Ba^3+Aba^2+11b^2Ba-7Ab^3\right)\cot^2(c+dx)a+8b^4B-8b^2\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b(a^2+b^2)}-\frac{a\left(3a^3B+a^2Ab+11ab^2B\right.}{bd(a^2+b^2)(a\,\mathrm{co}}}{4b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{3Ba^4+Aba^3+3b^2Ba^2+9Ab^3a+\left(3Ba^3+Aba^2+11b^2Ba-7Ab^3\right)\cot^2(c+dx)a+8b^4B-8b^2\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{2b(a^2+b^2)}-\frac{a\left(3a^3B+a^2Ab+11ab^2B-7A\right.}{bd(a^2+b^2)(a\cot(c+}}{4b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{3Ba^4+Aba^3+3b^2Ba^2+9Ab^3a+\left(3Ba^3+Aba^2+11b^2Ba-7Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2a+8b^4B+8b^2\left(Aa^2+2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2+b^2)}-\frac{a\left(3a^3B+a^2Ab+11a\right.}{bd(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{ 4136}$$

$$\frac{\frac{\frac{\int -\frac{8\left(\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)b^2+\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\cot(c+dx)b^2\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int \frac{\cot^2(c}{\sqrt{\cot(c+dx)}(b}}{a^2+b^2}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int \frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}-\frac{8\int \frac{\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)b^2+\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\mathrm{co}}{\sqrt{\cot(c+dx)}}}{a^2+b^2}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{8\int \frac{b^2\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)-b^2\left(-Ba^3+3Aba^2+\right.}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}}{a^2+b^2}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\big\downarrow$ 4017

$$\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{16\int -\frac{b^2\left(Aa^3+3bBa^2-3Ab^2a-b^3B+\left(-Ba^3+3Aba^2+3\right.\right.}{\cot^2(c+dx)+1}}{d\left(a^2+b^2\right)}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{16\int \frac{b^2\left(Aa^3+3bBa^2-3Ab^2a-b^3B+\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int \frac{}{\sqrt{-\tan}}}{a^2+b^2}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{16b^2\int\frac{Aa^3+3bBa^2-3Ab^2a-b^3B+\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int\frac{}{\sqrt{-\tan(c}}}{a^2+b^2}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 1482

$$\frac{\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\int\frac{1-\cot(c+}{\cot^2(c+dx}}{d\left(a^2+b^2\right)}}{2b(a^2+b^2)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 1476

$$\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int\frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)-}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 1082

$$\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+\right.\right.}{d\left(a^2+b^2\right)}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)(a\cot(c+dx)+b)^2}$$

$\downarrow$ 217

$$\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A\right.}{d\left(a^2+b^2\right)} \cdots$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 1479

$$\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A\right.}{d\left(a^2+b^2\right)} \cdots$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 25

$$\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A\right.}{d\left(a^2+b^2\right)} \cdots$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 27

$$\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A\right.}{d\left(a^2+b^2\right)} \cdots$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\downarrow$ 1103

$$\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int \frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B\right.\right.}{}}{}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\big\downarrow$ 4117

$$\frac{\frac{a\left(3a^5B+a^4Ab+6a^3b^2B+18a^2Ab^3+35ab^4B-15Ab^5\right)\int \frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}+\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B\right.\right.}{}}{}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\big\downarrow$ 73

$$\frac{\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(\right.\right.}{d\left(a^2+b^2\right)}}{}$$

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}$$

$\big\downarrow$ 218

$$\frac{a(Ab-aB)\sqrt{\cot(c+dx)}}{2bd\left(a^2+b^2\right)\left(a\cot(c+dx)+b\right)^2}+$$

$$\frac{\frac{16b^2\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(\right.\right.}{d\left(a^2+b^2\right)}}{}$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^3),x]
```

output
```
(a*(A*b - a*B)*Sqrt[Cot[c + d*x]])/(2*b*(a^2 + b^2)*d*(b + a*Cot[c + d*x])
^2) + (-((a*(a^2*A*b - 7*A*b^3 + 3*a^3*B + 11*a*b^2*B)*Sqrt[Cot[c + d*x]])
/(b*(a^2 + b^2)*d*(b + a*Cot[c + d*x]))) + ((2*Sqrt[a]*(a^4*A*b + 18*a^2*A
*b^3 - 15*A*b^5 + 3*a^5*B + 6*a^3*b^2*B + 35*a*b^4*B)*ArcTan[(Sqrt[a]*Cot[
c + d*x])/Sqrt[b]])/(Sqrt[b]*(a^2 + b^2)*d) + (16*b^2*(((a^3*(A - B) - 3*a
*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*(-(ArcTan[1 - Sqrt[2]*Sqrt[C
ot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]))/
2 - ((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*(-1/2
*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/2))/((a^2 + b^2)*d))/
(2*b*(a^2 + b^2)))/(4*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{2\left(\frac{\left(-A a^{3}+3 A a b^{2}-3 B a^{2} b+B b^{3}\right)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\right.}\right.}{8}\right.}{}$
default	$-\frac{2\left(\frac{\left(-A a^{3}+3 A a b^{2}-3 B a^{2} b+B b^{3}\right)\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\right.}\right.}{8}\right.}{}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(-2/(a^2+b^2)^3*(1/8*(-A*a^3+3*A*a*b^2-3*B*a^2*b+B*b^3)*2^(1/2)*(ln((c
ot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)
+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(
1/2)))+1/8*(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*2^(1/2)*(ln((cot(d*x+c)-2^(1
/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(
1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))-2*a/(a
^2+b^2)^3*((1/8*a*(A*a^4*b-6*A*a^2*b^3-7*A*b^5+3*B*a^5+14*B*a^3*b^2+11*B*a
*b^4)/b^2*cot(d*x+c)^(3/2)-1/8*(A*a^4*b+10*A*a^2*b^3+9*A*b^5-5*B*a^5-18*B*
a^3*b^2-13*B*a*b^4)/b*cot(d*x+c)^(1/2))/(b+a*cot(d*x+c))^2+1/8*(A*a^4*b+18
*A*a^2*b^3-15*A*b^5+3*B*a^5+6*B*a^3*b^2+35*B*a*b^4)/b^2/(a*b)^(1/2)*arctan
(a*cot(d*x+c)^(1/2)/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3674 vs. $2(408) = 816$.

Time = 101.49 (sec) , antiderivative size = 7375, normalized size of antiderivative = 16.46

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^3}\, dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.24

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = $$

$$-\frac{\frac{(3\,Ba^6+Aa^5b+6\,Ba^4b^2+18\,Aa^3b^3+35\,Ba^2b^4-15\,Aab^5)\arctan\left(\frac{a}{\sqrt{ab}\sqrt{\tan(dx+c)}}\right)}{(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)\sqrt{ab}}-\frac{2\sqrt{2}((A-B)a^3+3\,(A+B)a^2b-3\,(A-B)ab^2-(A+B}{}}{}$$

input

```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output

```
-1/4*((3*B*a^6 + A*a^5*b + 6*B*a^4*b^2 + 18*A*a^3*b^3 + 35*B*a^2*b^4 - 15*
A*a*b^5)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^6*b^2 + 3*a^4*b^4 +
3*a^2*b^6 + b^8)*sqrt(a*b)) - (2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b -
3*(A - B)*a*b^2 - (A + B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*
x + c)))) + 2*sqrt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (
A + B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2
)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(sqrt
(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*((A + B)*a^3 - 3*(A
 - B)*a^2*b - 3*(A + B)*a*b^2 + (A - B)*b^3)*log(-sqrt(2)/sqrt(tan(d*x + c
)) + 1/tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((5*B*a^4*
b - A*a^3*b^2 + 13*B*a^2*b^3 - 9*A*a*b^4)/sqrt(tan(d*x + c)) + (3*B*a^5 +
A*a^4*b + 11*B*a^3*b^2 - 7*A*a^2*b^3)/tan(d*x + c)^(3/2))/(a^4*b^4 + 2*a^2
*b^6 + b^8 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)/tan(d*x + c) + (a^6*b^2 + 2*a
^4*b^4 + a^2*b^6)/tan(d*x + c)^2))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input

```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx=\int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^3),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$$
$$=\int \frac{\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^3\tan\left(dx+c\right)^2b^2+2\cot\left(dx+c\right)^3\tan\left(dx+c\right)ab+\cot\left(dx+c\right)^3a^2}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^3,x)
```

output
```
int(sqrt(cot(c + d*x))/(cot(c + d*x)**3*tan(c + d*x)**2*b**2 + 2*cot(c + d
*x)**3*tan(c + d*x)*a*b + cot(c + d*x)**3*a**2),x)
```

3.609 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 514

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx =& -\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\ &+\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\ &-\frac{a^{3/2}(3a^4Ab+6a^2Ab^3+35Ab^5-15a^5B-46a^3b^2B-63ab^4B)\arctan\left(\frac{\sqrt{a}\sqrt{\cot(c+dx)}}{\sqrt{b}}\right)}{4b^{7/2}\,(a^2+b^2)^3\,d} \\ &+\frac{(a^3(A-B)-3ab^2(A-B)+3a^2b(A+B)-b^3(A+B))\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}\,(a^2+b^2)^3\,d} \\ &-\frac{3a^3Ab+11aAb^3-15a^4B-31a^2b^2B-8b^4B}{4b^3\,(a^2+b^2)^2\,d\sqrt{\cot(c+dx)}} \\ &+\frac{a(Ab-aB)}{2b\,(a^2+b^2)\,d\sqrt{\cot(c+dx)}(b+a\cot(c+dx))^2} \\ &+\frac{a(a^2Ab+9Ab^3-5a^3B-13ab^2B)}{4b^2\,(a^2+b^2)^2\,d\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}\end{aligned}$$

output
```
1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+B)+3*a*b^2*(A+B))*arctan(-1+2^(1/2)*co
t(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d+1/2*(3*a^2*b*(A-B)-b^3*(A-B)-a^3*(A+
B)+3*a*b^2*(A+B))*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/(a^2+b^2)^3/d
-1/4*a^(3/2)*(3*A*a^4*b+6*A*a^2*b^3+35*A*b^5-15*B*a^5-46*B*a^3*b^2-63*B*a*
b^4)*arctan(a^(1/2)*cot(d*x+c)^(1/2)/b^(1/2))/b^(7/2)/(a^2+b^2)^3/d+1/2*(a
^3*(A-B)-3*a*b^2*(A-B)+3*a^2*b*(A+B)-b^3*(A+B))*arctanh(2^(1/2)*cot(d*x+c)
^(1/2)/(1+cot(d*x+c)))*2^(1/2)/(a^2+b^2)^3/d-1/4*(3*A*a^3*b+11*A*a*b^3-15*
B*a^4-31*B*a^2*b^2-8*B*b^4)/b^3/(a^2+b^2)^2/d/cot(d*x+c)^(1/2)+1/2*a*(A*b-
B*a)/b/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(b+a*cot(d*x+c))^2+1/4*a*(A*a^2*b+9*A*
b^3-5*B*a^3-13*B*a*b^2)/b^2/(a^2+b^2)^2/d/cot(d*x+c)^(1/2)/(b+a*cot(d*x+c)
)
```

Mathematica [A] (verified)

Time = 6.35 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.24

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \frac{2\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{(3a^2b(A-B)-b^3(A-B)-a^3(A+B)+3ab^2(A+B))\left(\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-\sqrt{2}\arctan\left(1+\right.\right.}{4(a^2+b^2)^3}\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^3)
,x]
```

output
```
(2*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((3*a^2*b*(A - B) - b^3*(A - B)
- a^3*(A + B) + 3*a*b^2*(A + B))*(Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]))/(4*(a^2 + b^2)^3
) + (3*a^(3/2)*(A*b - a*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(
8*b^(7/2)*(a^2 + b^2)) - (a^(3/2)*(2*a^2*A*b + 4*A*b^3 - 3*a^3*B - 5*a*b^2
*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(2*b^(7/2)*(a^2 + b^2)^2
) + (a^(3/2)*(a^4*A*b + 3*a^2*A*b^3 + 6*A*b^5 - 3*a^5*B - 9*a^3*b^2*B - 10
*a*b^4*B)*ArcTan[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]])/(b^(7/2)*(a^2 + b^
2)^3) - ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B) - b^3*(A + B))*(
Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - Sqrt[2]*Log[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]))/(8*(a^2 + b^2)^3) + (B*Sqr
t[Tan[c + d*x]])/b^3 + (a^3*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(4*b^3*(a^2 +
b^2)*(a + b*Tan[c + d*x])^2) + (3*a^2*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(8*b
^3*(a^2 + b^2)*(a + b*Tan[c + d*x])) - (a^2*(2*a^2*A*b + 4*A*b^3 - 3*a^3*B
 - 5*a*b^2*B)*Sqrt[Tan[c + d*x]])/(2*b^3*(a^2 + b^2)^2*(a + b*Tan[c + d*x]
))))/d
```

Rubi [A] (warning: unable to verify)

Time = 5.26 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.02, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.879, Rules used = {3042, 4064, 3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4136, 27, 3042, 4017, 25, 27, 1482, 1476, 1082, 217, 1479, 25, 27, 1103, 4117, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{7/2}(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 4064}$$

$$\int \frac{A\cot(c+dx)+B}{\cot^{\frac{3}{2}}(c+dx)(a\cot(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{B-A\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 4092}$$

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-\frac{\int \frac{-5Ba^2+5(Ab-aB)\cot^2(c+dx)a+Aba-4b^2B-4b(Ab-aB)\cot(c+dx)}{2\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))^2}dx}{2b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-\frac{\int \frac{-5Ba^2+5(Ab-aB)\cot^2(c+dx)a+Aba-4b^2B-4b(Ab-aB)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))^2}dx}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-\frac{\int \frac{-5Ba^2+5(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)^2a+Aba-4b^2B+4b(Ab-aB)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 4132}$$

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-\frac{-\frac{\int -\frac{-15Ba^4+3Aba^3-31b^2Ba^2+11Ab^3a+3\left(-5Ba^3+Aba^2-13b^2Ba+9Ab^3\right)\cot^2(c+dx)a-8b^4B+8b^2\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)}{2\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))}dx}{b(a^2+b^2)}-\frac{a\left(-5a^3B+a\cdots\right.}{bd(a^2+b^2)\sqrt{\cot\cdots}}}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-\frac{\frac{\int \frac{-15Ba^4+3Aba^3-31b^2Ba^2+11Ab^3a+3\left(-5Ba^3+Aba^2-13b^2Ba+9Ab^3\right)\cot^2(c+dx)a-8b^4B+8b^2\left(Aa^2+2bBa-Ab^2\right)\cot(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(b+a\cot(c+dx))}dx}{2b(a^2+b^2)}-\frac{a\left(-5a^3B+a^2Ab\cdots\right.}{bd(a^2+b^2)\sqrt{\cot(c+\cdots}}}{4b\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-\frac{\frac{\int \frac{-15Ba^4+3Aba^3-31b^2Ba^2+11Ab^3a+3\left(-5Ba^3+Aba^2-13b^2Ba+9Ab^3\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2a-8b^4B-8b^2\left(Aa^2+2bBa-Ab^2\right)\tan\left(c+dx+\frac{\pi}{2}\right)}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{2b(a^2+b^2)}-\frac{a(-5a^3\cdots}{bd(a^2+b^2)\cdots}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 4132

$$\frac{\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-\frac{\frac{2\int -\frac{-15Ba^5+3Aba^4-31b^2Ba^3+3Ab^3a^2+\left(-15Ba^4+3Aba^3-31b^2Ba^2+11Ab^3a-8b^4B\right)\cot^2(c+dx)a-24b^4Ba+8Ab^5-8b^3\left(-Ba^2+2Aba+b^2B\right)\cot(c+dx)}{2\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{b}\cdots}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\int \frac{-15Ba^5+3Aba^4-31b^2Ba^3+3Ab^3a^2+\left(-15Ba^4+3Aba^3-31b^2Ba^2+11Ab^3a-8b^4B\right)\cot^2(c+dx)a-24b^4Ba+\cdots}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}\cdots}{b}\cdots}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\int \frac{-15Ba^5+3Aba^4-31b^2Ba^3+3Ab^3a^2+\left(-15Ba^4+3Aba^3-31b^2Ba^2+11Ab^3a-8b^4B\right)\tan\left(c+dx+\frac{\pi}{2}\right)^2a-24b^4\cdots}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}\cdots}{b}\cdots}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 4136

$$\frac{\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{\int -\frac{8\left(b^3\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3\right)-b^3\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\cot(c+dx)\right)}{\sqrt{\cot(c+dx)}}dx}{a^2+b^2}+\frac{a^2\left(-15a^5B+3a^4A\cdots\right.}{\cdots}}{b}\cdots}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^2\left(-15a^5B+3a^4Ab-46a^3b^2B+6a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\cot^2(c+dx)+1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}dx}{a^2+b^2}-\frac{8\int\frac{b^3\left(-Ba^3-\cdots\right.}{\cdots}}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 3042

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^2\left(-15a^5B+3a^4Ab-46a^3b^2B+6a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{8\cdots}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 4017

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^2\left(-15a^5B+3a^4Ab-46a^3b^2B+6a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}-\frac{1\cdots}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 25

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{16\int\frac{b^3\left(-Ba^3+3Aba^2+3b^2Ba-Ab^3-\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\cot(c+dx)\right)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^2\left(-15a^5B+\cdots\right.}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 27

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{16b^3\int\frac{-Ba^3+3Aba^2+3b^2Ba-Ab^3-\left(Aa^3+3bBa^2-3Ab^2a-b^3B\right)\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d\left(a^2+b^2\right)}+\frac{a^2\left(-15a^5B+3a\cdots\right.}{b}}{2b(a^2+b^2)}}{4b\left(a^2+b^2\right)}$$

$\downarrow$ 1482

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^3(A+B)\right)+\cdots\right.\right.}{d\left(a^2+b^2\right)}\cdots}{\cdots}$$

$\downarrow$ 1476

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^3(A+B)\right)+\cdots\right.\right.}{\cdots}\cdots}{\cdots}$$

$\downarrow$ 1082

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^3(A+B)\right)+\cdots\right.\right.}{\cdots}\cdots}{\cdots}$$

$\downarrow$ 217

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(-\left(a^3(A+B)\right)+\right.\right.}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right.\right.}{}$$

$$\Big\downarrow \text{25}$$

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\right.\right.}{}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{16b^3\left(\frac{1}{2}\left(a^3(A-B)+3a^2b(A+B)-3ab^2(A-B)-b^3(A+B)\right)\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\right.\right.}{}$$

$$\Big\downarrow \text{1103}$$

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^2\left(-15a^5B+3a^4Ab-46a^3b^2B+6a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{\tan\left(c+dx+\frac{\pi}{2}\right)^2+1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}\left(b-a\tan\left(c+dx+\frac{\pi}{2}\right)\right)}dx}{a^2+b^2}+\ldots}{\ldots}}{\ldots}$$

$\big\downarrow$ 4117

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{\frac{a^2\left(-15a^5B+3a^4Ab-46a^3b^2B+6a^2Ab^3-63ab^4B+35Ab^5\right)\int\frac{1}{\sqrt{\cot(c+dx)}(b+a\cot(c+dx))}d(-\cot(c+dx))}{d\left(a^2+b^2\right)}+\ldots}{\ldots}}{\ldots}$$

$\big\downarrow$ 73

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{16b^3\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\ldots}\right)}{\sqrt{2}}\ldots\right.\right.}{\ldots}}{\ldots}$$

$\big\downarrow$ 218

$$\frac{a(Ab-aB)}{2bd\left(a^2+b^2\right)\sqrt{\cot(c+dx)}(a\cot(c+dx)+b)^2}-$$

$$\frac{\frac{2\left(-15a^4B+3a^3Ab-31a^2b^2B+11aAb^3-8b^4B\right)}{bd\sqrt{\cot(c+dx)}}-\frac{16b^3\left(\frac{1}{2}\left(-\left(a^3(A+B)\right)+3a^2b(A-B)+3ab^2(A+B)-b^3(A-B)\right)\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\ldots}\right)}{\sqrt{2}}\ldots\right.\right.}{\ldots}}{\ldots}$$

input

```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^3),x]
```

output
```
(a*(A*b - a*B))/(2*b*(a^2 + b^2)*d*Sqrt[Cot[c + d*x]]*(b + a*Cot[c + d*x])
^2) - (-((a*(a^2*A*b + 9*A*b^3 - 5*a^3*B - 13*a*b^2*B))/(b*(a^2 + b^2)*d*S
qrt[Cot[c + d*x]]*(b + a*Cot[c + d*x]))) + ((2*(3*a^3*A*b + 11*a*A*b^3 - 1
5*a^4*B - 31*a^2*b^2*B - 8*b^4*B))/(b*d*Sqrt[Cot[c + d*x]]) - ((2*a^(3/2)*
(3*a^4*A*b + 6*a^2*A*b^3 + 35*A*b^5 - 15*a^5*B - 46*a^3*b^2*B - 63*a*b^4*B
)*ArcTan[(Sqrt[a]*Cot[c + d*x])/Sqrt[b]])/(Sqrt[b]*(a^2 + b^2)*d) + (16*b^
3*(((3*a^2*b*(A - B) - b^3*(A - B) - a^3*(A + B) + 3*a*b^2*(A + B))*(-(Arc
Tan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot
[c + d*x]]]/Sqrt[2]))/2 + ((a^3*(A - B) - 3*a*b^2*(A - B) + 3*a^2*b*(A + B
) - b^3*(A + B))*(-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/
Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2])))
/2))/((a^2 + b^2)*d))/b)/(2*b*(a^2 + b^2)))/(4*b*(a^2 + b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4017
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_
)]], x_Symbol] :> Simp[2/f   Subst[Int[(b*c + d*x^2)/(b^2 + x^4), x], x, Sq
rt[b*Tan[e + f*x]]], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] &
& NeQ[c^2 + d^2, 0]
```

rule 4064
```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp
[g^(m + n)   Int[(g*Cot[e + f*x])^(p - m - n)*(b + a*Cot[e + f*x])^m*(d + c
*Cot[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] &&  !Integer
Q[p] && IntegerQ[m] && IntegerQ[n]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4136
```
Int[(((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2))/((a_.) + (b_.)*tan[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Simp[1/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^
n*Simp[b*B + a*(A - C) + (a*B - b*(A - C))*Tan[e + f*x], x], x], x] + Simp[
(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2)   Int[(c + d*Tan[e + f*x])^n*((1 + Tan[
e + f*x]^2)/(a + b*Tan[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] &
&  !GtQ[n, 0] &&  !LeQ[n, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{2a^2\left(\frac{\left(-\frac{3}{8}A\,a^5b-\frac{7}{4}A\,b^3a^3-\frac{11}{8}Aa\,b^5+\frac{7}{8}B\,a^6+\frac{11}{4}B\,a^4b^2+\frac{15}{8}B\,a^2b^4\right)\cot(dx+c)^{\frac{3}{2}}-\frac{b\left(5A\,a^4b+18A\,a^2b^3+13A\,b^5-9B\,a^5-26B\,a^3\right.}{8}}{(b+a\cot(dx+c))^2}\right.}{b^3\left(a^2+b^2\right)^3}$
default	$\frac{2a^2\left(\frac{\left(-\frac{3}{8}A\,a^5b-\frac{7}{4}A\,b^3a^3-\frac{11}{8}Aa\,b^5+\frac{7}{8}B\,a^6+\frac{11}{4}B\,a^4b^2+\frac{15}{8}B\,a^2b^4\right)\cot(dx+c)^{\frac{3}{2}}-\frac{b\left(5A\,a^4b+18A\,a^2b^3+13A\,b^5-9B\,a^5-26B\,a^3\right.}{8}}{(b+a\cot(dx+c))^2}\right.}{b^3\left(a^2+b^2\right)^3}$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/d*(2*a^2/b^3/(a^2+b^2)^3*(((-3/8*A*a^5*b-7/4*A*b^3*a^3-11/8*A*a*b^5+7/8*
B*a^6+11/4*B*a^4*b^2+15/8*B*a^2*b^4)*cot(d*x+c)^(3/2)-1/8*b*(5*A*a^4*b+18*
A*a^2*b^3+13*A*b^5-9*B*a^5-26*B*a^3*b^2-17*B*a*b^4)*cot(d*x+c)^(1/2))/(b+a
*cot(d*x+c))^2-1/8*(3*A*a^4*b+6*A*a^2*b^3+35*A*b^5-15*B*a^5-46*B*a^3*b^2-6
3*B*a*b^4)/(a*b)^(1/2)*arctan(a*cot(d*x+c)^(1/2)/(a*b)^(1/2)))-2/(a^2+b^2)
^3*(1/8*(-3*A*a^2*b+A*b^3+B*a^3-3*B*a*b^2)*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)
*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2
^(1/2)*cot(d*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2)))+1/8*(A*a^3
-3*A*a*b^2+3*B*a^2*b-B*b^3)*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/
2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+
c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))+2*B/b^3/cot(d*x+c)^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3745 vs. $2(470) = 940$.

Time = 157.34 (sec) , antiderivative size = 7516, normalized size of antiderivative = 14.62

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(7/2)/(a+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.19

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorith
m="maxima")
```

output
```
1/4*((15*B*a^7 - 3*A*a^6*b + 46*B*a^5*b^2 - 6*A*a^4*b^3 + 63*B*a^3*b^4 - 3
5*A*a^2*b^5)*arctan(a/(sqrt(a*b)*sqrt(tan(d*x + c))))/((a^6*b^3 + 3*a^4*b^
5 + 3*a^2*b^7 + b^9)*sqrt(a*b)) - (2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*
b - 3*(A + B)*a*b^2 + (A - B)*b^3)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(ta
n(d*x + c)))) + 2*sqrt(2)*((A + B)*a^3 - 3*(A - B)*a^2*b - 3*(A + B)*a*b^2
 + (A - B)*b^3)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sq
rt(2)*((A - B)*a^3 + 3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(
sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*((A - B)*a^3 +
3*(A + B)*a^2*b - 3*(A - B)*a*b^2 - (A + B)*b^3)*log(-sqrt(2)/sqrt(tan(d*x
 + c)) + 1/tan(d*x + c) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (8*B*a
^4*b^2 + 16*B*a^2*b^4 + 8*B*b^6 + (25*B*a^5*b - 5*A*a^4*b^2 + 49*B*a^3*b^3
 - 13*A*a^2*b^4 + 16*B*a*b^5)/tan(d*x + c) + (15*B*a^6 - 3*A*a^5*b + 31*B*
a^4*b^2 - 11*A*a^3*b^3 + 8*B*a^2*b^4)/tan(d*x + c)^2)/((a^4*b^5 + 2*a^2*b^
7 + b^9)/sqrt(tan(d*x + c)) + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)/tan(d*x + c)
^(3/2) + (a^6*b^3 + 2*a^4*b^5 + a^2*b^7)/tan(d*x + c)^(5/2)))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x, algorith
m="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{7/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^3),x)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(7/2)*(a + b*tan(c + d*x))^3), x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^4\tan\left(dx+c\right)^2b^2+2\cot\left(dx+c\right)^4\tan\left(dx+c\right)ab+\cot\left(dx+c\right)^4a^2}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(7/2)/(a+b*tan(d*x+c))^3,x)
```

output
```
int(sqrt(cot(c + d*x))/(cot(c + d*x)**4*tan(c + d*x)**2*b**2 + 2*cot(c + d
*x)**4*tan(c + d*x)*a*b + cot(c + d*x)**4*a**2),x)
```

3.610 $\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 117

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{B\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{B\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{2B\cot^{\frac{3}{2}}(c+dx)}{3d}$$

output
```
1/2*B*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctan(1+2^(1/2)
*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot
(d*x+c)))*2^(1/2)/d-2/3*B*cot(d*x+c)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{B\left(-3\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(c+dx)}\right)\sqrt[4]{-\cot(c+dx)}\right.}{3d\sqrt[4]{\cot(c+dx)}}$$

input
```
Integrate[(Cot[c + d*x]^(5/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
]),x]
```

output
```
-1/3*(B*(-3*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4) + 3*ArcT
anh[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x])^(1/4) + 2*Cot[c + d*x]^(7/4))
)/(d*Cot[c + d*x]^(1/4))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {2011, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \cot^{\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3954}$$

$$B\left(-\int \sqrt{\cot(c+dx)}dx-\frac{2\cot^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$B\left(-\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{2\cot^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 3957}$$

$$B\left(\frac{\int \frac{\sqrt{\cot(c+dx)}}{\cot^2(c+dx)+1}d\cot(c+dx)}{d}-\frac{2\cot^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 266}$$

$$B\left(\frac{2\int \frac{\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{2\cot^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 826}$$

$$B\left(\frac{2\left(\frac{1}{2}\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}-\frac{2\cot^{\frac{3}{2}}(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)-\frac{1}{2}\int \frac{1-\text{co}}{\cot^2(}\right.}{d}\right.$$

$$\Big\downarrow \text{ 1082}$$

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}\right.$$

$$\Big\downarrow \text{ 217}$$

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}-\frac{2\cot^{\frac{3}{2}}(c+dx}{3d}\right.$$

$\Big\downarrow$ 1479

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$\Big\downarrow$ 25

$$B\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$\Big\downarrow$ 27

$$B\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c\right.}{\sqrt{}}\right.\right.}{d}\right.$$

$\Big\downarrow$ 1103

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)+\sqrt{}\right.}{2}\right.\right.}{d}\right.$$

input
```
Int[(Cot[c + d*x]^(5/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
B*((-2*Cot[c + d*x]^(3/2))/(3*d) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c +
d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])/2 + (Log
[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt
[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]))/2))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{B\left(-\frac{2\cot(dx+c)^{\frac{3}{2}}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4}\right)}{d}$
default	$\frac{B\left(-\frac{2\cot(dx+c)^{\frac{3}{2}}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4}\right)}{d}$

input
```
int(cot(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
B/d*(-2/3*cot(d*x+c)^(3/2)+1/4*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^
(1/2)+1)/(cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d
*x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(95) = 190.

Time = 0.09 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.40

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{6\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2dx+2c)+1}{\sin(2dx+2c)}}\sin(2dx+2c)+\cos(2dx+2c)+1}{\cos(2dx+2c)+1}\right)\sin(2dx+2c)+6\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2dx+}{\sin(2dx-}}}{}\right.}{}$$

input
```
integrate(cot(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/12*(6*sqrt(2)*B*arctan((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x +
 2*c))*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1))*si
n(2*d*x + 2*c) + 6*sqrt(2)*B*arctan((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/s
in(2*d*x + 2*c))*sin(2*d*x + 2*c) - cos(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c
) + 1))*sin(2*d*x + 2*c) + 3*sqrt(2)*B*log((sqrt(2)*sqrt((cos(2*d*x + 2*c)
 + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + sin(2*d*x +
2*c) + 1)/(cos(2*d*x + 2*c) + 1))*sin(2*d*x + 2*c) - 3*sqrt(2)*B*log(-(sqr
t(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) - cos(
2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) + 1))*sin(2*d*x + 2
*c) + 8*(B*cos(2*d*x + 2*c) + B)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2
*c)))/(d*sin(2*d*x + 2*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{3\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx}}\right.\right.}{12\,d}$$

input
```
integrate(cot(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/12*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*
log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2
)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*B - 8*B/tan(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx=\int \frac{(Bb\tan{(dx+c)}+Ba)\cot{(dx+c)}^{\frac{5}{2}}}{b\tan{(dx+c)}+a}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*tan(d*x + c) + B*a)*cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a)
, x)
```

Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx=\frac{(-1)^{1/4}\,B\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\frac{1}{\tan(c+d\,x)}}\right)}{d}-\frac{2\,B\,\left(\frac{1}{\tan(c+d\,x)}\right)^{3/2}}{3\,d}-\frac{(-1)^{1/4}\,B\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\frac{1}{\tan(c+d\,x)}}\right)}{d}$$

input
```
int((cot(c + d*x)^(5/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
((-1)^(1/4)*B*atan((-1)^(1/4)*(1/tan(c + d*x))^(1/2)))/d - (2*B*(1/tan(c +
 d*x))^(3/2))/(3*d) - ((-1)^(1/4)*B*atanh((-1)^(1/4)*(1/tan(c + d*x))^(1/2
)))/d
```

Reduce [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \left(\int \sqrt{\cot(dx+c)}\,\cot(dx+c)^2\,dx\right)b$$

input
```
int(cot(d*x+c)^(5/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x))*cot(c + d*x)**2,x)*b
```

3.611 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 114

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{B\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{B\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{B\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{2B\sqrt{\cot(c+dx)}}{d}$$

output
```
1/2*B*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctan(1+2^(1/2)
*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot
(d*x+c)))*2^(1/2)/d-2*B*cot(d*x+c)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.23

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{B\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}+2\sqrt{\cot(c+dx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\cot(c+dx)\right)}{2\sqrt{2}}-\frac{\text{lo}}{}\right.}{d}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
]),x]
```

output
```
-((B*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*
Sqrt[Cot[c + d*x]]]/Sqrt[2] + 2*Sqrt[Cot[c + d*x]] + Log[1 - Sqrt[2]*Sqrt[
Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Cot[c + d
*x]] + Cot[c + d*x]]/(2*Sqrt[2])))/d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.36, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2011, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{2011}$$
$$B\int \cot^{\frac{3}{2}}(c+dx)dx$$
$$\Big\downarrow \text{3042}$$
$$B\int \left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3954}$$

$$B\left(-\int \frac{1}{\sqrt{\cot(c+dx)}}dx-\frac{2\sqrt{\cot(c+dx)}}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(-\int \frac{1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx-\frac{2\sqrt{\cot(c+dx)}}{d}\right)$$

$$\Big\downarrow \text{3957}$$

$$B\left(\frac{\int \frac{1}{\sqrt{\cot(c+dx)}(\cot^2(c+dx)+1)}d\cot(c+dx)}{d}-\frac{2\sqrt{\cot(c+dx)}}{d}\right)$$

$$\Big\downarrow \text{266}$$

$$B\left(\frac{2\int \frac{1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}-\frac{2\sqrt{\cot(c+dx)}}{d}\right)$$

$$\Big\downarrow \text{755}$$

$$B\left(\frac{2\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}-\frac{2\sqrt{\cot(c+dx)}}{d}\right)$$

$$\Big\downarrow \text{1476}$$

$$B\left(\frac{2\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+}\right.\right.}{d}\right.$$

$$\Big\downarrow \text{1082}$$

$$B\left(\frac{2\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}\right.$$

$$\Big\downarrow \text{217}$$

$$B\left(\frac{2\left(\frac{1}{2}\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}-\frac{2\sqrt{\cot(c+dx}}{d}\right.$$

$\Big\downarrow$ 1479

$$B\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+d}\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$\Big\downarrow$ 25

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}\right.$$

$\Big\downarrow$ 27

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot}\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$\Big\downarrow$ 1103

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{}\right.}{2}\right.\right.}{d}\right.$$

input
```
Int[(Cot[c + d*x]^(3/2)*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
B*((-2*Sqrt[Cot[c + d*x]])/d + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]
]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])/2 + (-1/2*Lo
g[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]
*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]))/2))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{B\left(-2\sqrt{\cot(dx+c)}+\frac{\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4}\right)}{d}$
default	$\frac{B\left(-2\sqrt{\cot(dx+c)}+\frac{\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4}\right)}{d}$

input
```
int(cot(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
B/d*(-2*cot(d*x+c)^(1/2)+1/4*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1
/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x
+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(95) = 190.

Time = 0.08 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.96

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{2\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\sin(2\,dx+2\,c)+\cos(2\,dx+2\,c)+1}{\cos(2\,dx+2\,c)+1}\right)+2\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\sin(2\,dx+2\,c)-}{\cos(2\,dx+2\,c)+1}\right.}{}$$

input
```
integrate(cot(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/4*(2*sqrt(2)*B*arctan((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x +
2*c))*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1)) + 2
*sqrt(2)*B*arctan((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*s
in(2*d*x + 2*c) - cos(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) + 1)) - sqrt(2)*
B*log((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2
*c) + cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1)) + s
qrt(2)*B*log(-(sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2
*d*x + 2*c) - cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) +
 1)) + 8*B*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)))/d
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = B\int \cot^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(cot(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.11

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{2\sqrt{2}B\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}B\log\left(\frac{\cdots}{\sqrt{\tan\cdots}}\right.}{4d}$$

input
```
integrate(cot(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/4*(2*sqrt(2)*B*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*
sqrt(2)*B*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)*
B*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*B*log(-sq
rt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 8*B/sqrt(tan(d*x + c)))/d
```

Giac [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx=\int \frac{(Bb\tan(dx+c)+Ba)\cot(dx+c)^{\frac{3}{2}}}{b\tan(dx+c)+a}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*tan(d*x + c) + B*a)*cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)
, x)
```

Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx=-\frac{2B\sqrt{\frac{1}{\tan(c+dx)}}}{d}-\frac{(-1)^{1/4}B\,\mathrm{atan}\left((-1)^{1/4}\sqrt{\frac{1}{\tan(c+dx)}}\right)1i}{d}-\frac{(-1)^{1/4}B\,\mathrm{atanh}\left((-1)^{1/4}\sqrt{\frac{1}{\tan(c+dx)}}\right)1i}{d}$$

input
```
int((cot(c + d*x)^(3/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
- (2*B*(1/tan(c + d*x))^(1/2))/d - ((-1)^(1/4)*B*atan((-1)^(1/4)*(1/tan(c
+ d*x))^(1/2))*1i)/d - ((-1)^(1/4)*B*atanh((-1)^(1/4)*(1/tan(c + d*x))^(1/
2))*1i)/d
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{b\left(-2\sqrt{\cot(dx+c)}-\left(\int \frac{\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)d\right)}{d}$$

input
```
int(cot(d*x+c)^(3/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
(b*( - 2*sqrt(cot(c + d*x)) - int(sqrt(cot(c + d*x))/cot(c + d*x),x)*d))/d
```

3.612 $\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 98

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{B\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{B\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d}$$

output
```
-1/2*B*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctan(1+2^(1/2
)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+co
t(d*x+c)))*2^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \frac{B\left(-\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(c+dx)}\right)\right)\sqrt[4]{-\cot(c+dx)}}{d\sqrt[4]{\cot(c+dx)}}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
]),x]
```

output
```
(B*(-ArcTan[(-Cot[c + d*x]^2)^(1/4)] + ArcTanh[(-Cot[c + d*x]^2)^(1/4)])*(
-Cot[c + d*x])^(1/4))/(d*Cot[c + d*x]^(1/4))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2011, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \sqrt{\cot(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{B\int \frac{\sqrt{\cot(c+dx)}}{\cot^2(c+dx)+1}d\cot(c+dx)}{d}$$

$$\Big\downarrow 266$$

$$-\frac{2B\int \frac{\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}$$

$$\Big\downarrow 826$$

$$-\frac{2B\left(\frac{1}{2}\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}$$

$$\Big\downarrow 1476$$

$$-\frac{2B\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)-\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}$$

$$\Big\downarrow 1082$$

$$-\frac{2B\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}$$

$$\Big\downarrow 217$$

$$-\frac{2B\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}$$

$$\Big\downarrow 1479$$

$$-\frac{2B\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}\right.}{\sqrt{2}}\right.\right.}{d}$$

$$\Big\downarrow 25$$

$$-\frac{2B\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+\right.}{\sqrt{2}}\right.\right.}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{2B\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c\ldots}\right.}{\sqrt{\ldots}}\right.\right.}{d}$$

$$\Big\downarrow \; 1103$$

$$-\frac{2B\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)+\sqrt{\ldots}\right.}{2\ldots}\right.\right.}{d}$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x]),x]
```

output
```
(-2*B*((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
+ Cot[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c +
 d*x]]/(2*Sqrt[2]))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

method	result	siz
derivativedivides	$-\frac{B\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{4d}$	90
default	$-\frac{B\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{4d}$	90

input
```
int(cot(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
-1/4*B/d*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2
^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(
-1+2^(1/2)*cot(d*x+c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(81) = 162.

Time = 0.08 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.16

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx$$
$$=\frac{2\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\sin(2\,dx+2\,c)+\cos(2\,dx+2\,c)+1}{\cos(2\,dx+2\,c)+1}\right)+2\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\sin(2\,dx+2\,c)-\cos}{\cos(2\,dx+2\,c)+1}\right.}{}$$

input
```
integrate(cot(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/4*(2*sqrt(2)*B*arctan((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2
*c))*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1)) + 2*
sqrt(2)*B*arctan((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*si
n(2*d*x + 2*c) - cos(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) + 1)) + sqrt(2)*B
*log((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*
c) + cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1)) - sq
rt(2)*B*log(-(sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*
d*x + 2*c) - cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) +
1)))/d
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = B\int \sqrt{\cot(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(sqrt(cot(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(d}}\right.\right.}{4d}$$

input
```
integrate(cot(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*log
(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)/s
qrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*B/d
```

Giac [F]

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \int \frac{(Bb\tan{(dx+c)}+Ba)\sqrt{\cot{(dx+c)}}}{b\tan{(dx+c)}+a}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*tan(d*x + c) + B*a)*sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)
, x)
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = -\frac{(-1)^{1/4}B\left(\operatorname{atan}\left((-1)^{1/4}\sqrt{\frac{1}{\tan(c+dx)}}\right)-\operatorname{atanh}\left((-1)^{1/4}\sqrt{\frac{1}{\tan(c+dx)}}\right)\right)}{d}$$

input
```
int((cot(c + d*x)^(1/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x)),x)
```

output
```
-((-1)^(1/4)*B*(atan((-1)^(1/4)*(1/tan(c + d*x))^(1/2)) - atanh((-1)^(1/4)
*(1/tan(c + d*x))^(1/2))))/d
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{a+b\tan(c+dx)}\,dx = \left(\int \sqrt{\cot\left(dx+c\right)}dx\right)b$$

input
```
int(cot(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x)),x)*b
```

3.613 $\int \frac{aB+bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 99

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx = \frac{B\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{B\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{B\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d}$$

output
```
-1/2*B*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctan(1+2^(1/2
)*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+co
t(d*x+c)))*2^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx = \frac{B\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(c+dx)}+\right.\right.}{2\sqrt{2}d}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x]
)),x]
```

output
```
(B*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]] - 2*ArcTan[1 + Sqrt[2]*Sqrt[C
ot[c + d*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]] - Log[1
 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]))/(2*Sqrt[2]*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2011, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{B\int \frac{1}{\sqrt{\cot(c+dx)}(\cot^2(c+dx)+1)}d\cot(c+dx)}{d}$$

$$\Big\downarrow \; 266$$

$$-\frac{2B\int \frac{1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}$$

$$\Big\downarrow \; 755$$

$$-\frac{2B\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}$$

$$\Big\downarrow \; 1476$$

$$-\frac{2B\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+}\right.\right.}{d}$$

$$\Big\downarrow \; 1082$$

$$-\frac{2B\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}$$

$$\Big\downarrow \; 217$$

$$-\frac{2B\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}$$

$$\Big\downarrow \; 1479$$

$$-\frac{2B\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+d}\right.}{\sqrt{2}}\right.\right.}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{2B\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{2B\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot\right.}{\sqrt{2}}\right.}{d}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{2B\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{\right.}{2}\right.}{d}$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])),x]
```

output
```
(-2*B*((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[c + d
*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c
+ d*x]]/(2*Sqrt[2]))/2))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result	si
derivativedivides	$-\frac{B\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{4d}$	90
default	$-\frac{B\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{4d}$	90

input
```
int((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
-1/4*B/d*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)-2
^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(
-1+2^(1/2)*cot(d*x+c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(81) = 162.

Time = 0.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.13

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))}\,dx = \frac{2\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\sin(2\,dx+2\,c)+\cos(2\,dx+2\,c)+1}{\cos(2\,dx+2\,c)+1}\right)+2\sqrt{2}B\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\sin(2\,dx+2\,c)-\cos}{\cos(2\,dx+2\,c)+1}\right.}{}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/4*(2*sqrt(2)*B*arctan((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2
*c))*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1)) + 2*
sqrt(2)*B*arctan((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*si
n(2*d*x + 2*c) - cos(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) + 1)) - sqrt(2)*B
*log((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*
c) + cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(cos(2*d*x + 2*c) + 1)) + sq
rt(2)*B*log(-(sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*
d*x + 2*c) - cos(2*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) +
1)))/d
```

Sympy [F]

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))}\,dx = B\int \frac{1}{\sqrt{\cot(c + dx)}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(1/sqrt(cot(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))}\,dx = -\frac{2\sqrt{2}B\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+\sqrt{2}B\log\left(-\right.}{4d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
-1/4*(2*sqrt(2)*B*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2
*sqrt(2)*B*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + sqrt(2)
*B*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - sqrt(2)*B*log(-s
qrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))/d
```

Giac [F]

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))}\,dx = \int \frac{Bb\tan(dx + c) + Ba}{(b\tan(dx + c) + a)\sqrt{\cot(dx + c)}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*tan(d*x + c) + B*a)/((b*tan(d*x + c) + a)*sqrt(cot(d*x + c)
)), x)
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))}\,dx = \frac{(-1)^{1/4} B \,\mathrm{atan}\left((-1)^{1/4}\sqrt{\frac{1}{\tan(c+dx)}}\right)\,1\mathrm{i}}{d} + \frac{(-1)^{1/4} B \,\mathrm{atanh}\left((-1)^{1/4}\sqrt{\frac{1}{\tan(c+dx)}}\right)\,1\mathrm{i}}{d}$$

input
```
int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))),x)
```

output
```
((-1)^(1/4)*B*atan((-1)^(1/4)*(1/tan(c + d*x))^(1/2))*1i)/d + ((-1)^(1/4)*
B*atanh((-1)^(1/4)*(1/tan(c + d*x))^(1/2))*1i)/d
```

Reduce [F]

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))}\,dx = \left(\int \frac{\sqrt{\cot(dx + c)}}{\cot(dx + c)}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x))/cot(c + d*x),x)*b
```

3.614 $\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 115

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = -\frac{B\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{B\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} - \frac{B\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{2B}{d\sqrt{\cot(c+dx)}}$$

output
```
1/2*B*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctan(1+2^(1/2)
*cot(d*x+c)^(1/2))*2^(1/2)/d-1/2*B*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot
(d*x+c)))*2^(1/2)/d+2*B/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{B\left(2+\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)\sqrt[4]{-\cot^2(c+dx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(c+dx)}\right)\sqrt[4]{-\cot^2(c+dx)}\right)}{d\sqrt{\cot(c+dx)}}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x]
)),x]
```

output
```
(B*(2 + ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x]^2)^(1/4) - ArcTanh[
(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x]^2)^(1/4)))/(d*Sqrt[Cot[c + d*x]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.35, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2011, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$B\left(\frac{2}{d\sqrt{\cot(c+dx)}}-\int\sqrt{\cot(c+dx)}dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$B\left(\frac{2}{d\sqrt{\cot(c+dx)}}-\int\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{ 3957}$$

$$B\left(\frac{\int\frac{\sqrt{\cot(c+dx)}}{\cot^2(c+dx)+1}d\cot(c+dx)}{d}+\frac{2}{d\sqrt{\cot(c+dx)}}\right)$$

$$\Big\downarrow \text{ 266}$$

$$B\left(\frac{2\int\frac{\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2}{d\sqrt{\cot(c+dx)}}\right)$$

$$\Big\downarrow \text{ 826}$$

$$B\left(\frac{2\left(\frac{1}{2}\int\frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}-\frac{1}{2}\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}+\frac{2}{d\sqrt{\cot(c+dx)}}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int\frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)-\frac{1}{2}\int\frac{1-\text{co}}{\cot^2(\text{c}}\right.}{d}\right.$$

$$\Big\downarrow \text{ 1082}$$

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}\right.$$

$$\Big\downarrow \text{ 217}$$

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}+\frac{2}{d\sqrt{\cot(c+dx)}}\right.$$

$\downarrow$ 1479

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$\downarrow$ 25

$$B\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$\downarrow$ 27

$$B\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{c}\right.}{\sqrt{}}\right.\right.}{d}\right.$$

$\downarrow$ 1103

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)+\sqrt{\right.}{2}\right.\right.}{d}\right.$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])),x]
```

output
```
B*(2/(d*Sqrt[Cot[c + d*x]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x]]
]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])/2 + (Log[1 -
Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*S
qrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]))/2))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{B\left(\frac{\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4}+\frac{2}{\sqrt{\cot(dx+c)}}\right)}{d}$
default	$\frac{B\left(\frac{\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)-\sqrt{2}\sqrt{\cot(dx+c)}+1}{\cot(dx+c)+\sqrt{2}\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\cot(dx+c)}\right)\right)}{4}+\frac{2}{\sqrt{\cot(dx+c)}}\right)}{d}$

input
```
int((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
B/d*(1/4*2^(1/2)*(ln((cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1)/(cot(d*x+c)+2
^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*x+c)^(1/2))+2*arctan(
-1+2^(1/2)*cot(d*x+c)^(1/2)))+2/cot(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(95) = 190.

Time = 0.09 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.68

$$\int \frac{aB + bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$$
$$= \frac{8\,B\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\sin\left(2\,dx+2\,c\right) - 2\left(\sqrt{2}B\cos\left(2\,dx+2\,c\right)+\sqrt{2}B\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\sin(2\,dx+2\,c)-}{\cos(2\,dx+2\,c)+1}\right.}{}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
1/4*(8*B*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) -
2*(sqrt(2)*B*cos(2*d*x + 2*c) + sqrt(2)*B)*arctan((sqrt(2)*sqrt((cos(2*d*x
 + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + 1)/(c
os(2*d*x + 2*c) + 1)) - 2*(sqrt(2)*B*cos(2*d*x + 2*c) + sqrt(2)*B)*arctan(
(sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) -
cos(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) + 1)) - (sqrt(2)*B*cos(2*d*x + 2*c
) + sqrt(2)*B)*log((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*
sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(cos(2*d*x + 2
*c) + 1)) + (sqrt(2)*B*cos(2*d*x + 2*c) + sqrt(2)*B)*log(-(sqrt(2)*sqrt((c
os(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) - cos(2*d*x + 2*c)
 - sin(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) + 1)))/(d*cos(2*d*x + 2*c) + d)
```

Sympy [F]

$$\int \frac{aB + bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = B\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(cot(c + d*x)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10

$$\int \frac{aB + bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right)-\sqrt{2}\log\left(\frac{\sqrt{2}}{\sqrt{\tan(dx+}}\right.\right.}{4d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) - sqrt(2)*log
(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)/s
qrt(tan(d*x + c)) + 1/tan(d*x + c) + 1))*B + 8*B*sqrt(tan(d*x + c)))/d
```

Giac [F]

$$\int \frac{aB + bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \int \frac{Bb\tan(dx+c) + Ba}{(b\tan(dx+c)+a)\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
integrate((B*b*tan(d*x + c) + B*a)/((b*tan(d*x + c) + a)*cot(d*x + c)^(3/2
)), x)
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.56

$$\int \frac{aB + bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{2B}{d\sqrt{\frac{1}{\tan(c+dx)}}} + \frac{(-1)^{1/4}B\,\mathrm{atan}\left((-1)^{1/4}\sqrt{\frac{1}{\tan(c+dx)}}\right)}{d} - \frac{(-1)^{1/4}B\,\mathrm{atanh}\left((-1)^{1/4}\sqrt{\frac{1}{\tan(c+dx)}}\right)}{d}$$

input
```
int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))),x)
```

output
```
(2*B)/(d*(1/tan(c + d*x))^(1/2)) + ((-1)^(1/4)*B*atan((-1)^(1/4)*(1/tan(c
+ d*x))^(1/2)))/d - ((-1)^(1/4)*B*atanh((-1)^(1/4)*(1/tan(c + d*x))^(1/2))
)/d
```

Reduce [F]

$$\int \frac{aB + bB\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))}\,dx = \left(\int \frac{\sqrt{\cot (dx + c)}}{\cot (dx + c)^2}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x))/cot(c + d*x)**2,x)*b
```

3.615 $\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 116

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = -\frac{B\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{B\arctan\left(1+\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{B\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(c+dx)}}{1+\cot(c+dx)}\right)}{\sqrt{2}d} + \frac{2B}{3d\cot^{\frac{3}{2}}(c+dx)}$$

output
```
1/2*B*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctan(1+2^(1/2)
*cot(d*x+c)^(1/2))*2^(1/2)/d+1/2*B*arctanh(2^(1/2)*cot(d*x+c)^(1/2)/(1+cot
(d*x+c)))*2^(1/2)/d+2/3*B/d/cot(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx =$$
$$-\frac{B\left(-2+3\arctan\left(\sqrt[4]{-\cot^2(c+dx)}\right)(-\cot^2(c+dx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(c+dx)}\right)(-\cot^2(c\right.}{3d\cot^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x]
)),x]
```

output
```
-1/3*(B*(-2 + 3*ArcTan[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x]^2)^(3/4) +
3*ArcTanh[(-Cot[c + d*x]^2)^(1/4)]*(-Cot[c + d*x]^2)^(3/4)))/(d*Cot[c + d*
x]^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.35, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2011, 3042, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}dx$$
$$\Big\downarrow \text{2011}$$
$$B\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$B\int \frac{1}{\left(-\tan\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$B\left(\frac{2}{3d\cot^{\frac{3}{2}}(c+dx)}-\int \frac{1}{\sqrt{\cot(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$B\left(\frac{2}{3d\cot^{\frac{3}{2}}(c+dx)}-\int \frac{1}{\sqrt{-\tan\left(c+dx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3957}$$

$$B\left(\frac{\int \frac{1}{\sqrt{\cot(c+dx)}(\cot^2(c+dx)+1)}d\cot(c+dx)}{d}+\frac{2}{3d\cot^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{266}$$

$$B\left(\frac{2\int \frac{1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}}{d}+\frac{2}{3d\cot^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{755}$$

$$B\left(\frac{2\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{\cot(c+dx)+1}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}\right)}{d}+\frac{2}{3d\cot^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{1476}$$

$$B\left(\frac{2\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\int \frac{1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+}\right.\right.}{d}\right.$$

$$\Big\downarrow \text{1082}$$

$$B\left(\frac{2\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(c+dx)-1}d\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}\right.$$

$$\Big\downarrow \text{217}$$

$$B\left(\frac{2\left(\frac{1}{2}\int \frac{1-\cot(c+dx)}{\cot^2(c+dx)+1}d\sqrt{\cot(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}+\frac{2}{3d\cot^{\frac{3}{2}}(c+d$$

$$\Big\downarrow \text{1479}$$

$$B\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+d}\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$$\Big\downarrow \text{25}$$

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}\right.$$

$$\Big\downarrow \text{27}$$

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(c+dx)}}{\cot(c+dx)-\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(c+dx)}+1}{\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1}d\sqrt{\cot(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot\right.}{\sqrt{2}}\right.\right.}{d}\right.$$

$$\Big\downarrow \text{1103}$$

$$B\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(c+dx)+\sqrt{2}\sqrt{\cot(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(c+dx)-\sqrt{\right.}{2}\right.\right.}{d}\right.$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])),x]
```

output
```
B*(2/(3*d*Cot[c + d*x]^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[c + d*x
]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[c + d*x]]]/Sqrt[2])/2 + (-1/2*L
og[1 - Sqrt[2]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2
]*Sqrt[Cot[c + d*x]] + Cot[c + d*x]]/(2*Sqrt[2]))/2))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{B\left(\frac{2}{3\cot(dx+c)^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{4}\right)}{d}$
default	$\frac{B\left(\frac{2}{3\cot(dx+c)^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\ln\left(\frac{\cot(dx+c)+\sqrt{2}\,\sqrt{\cot(dx+c)}+1}{\cot(dx+c)-\sqrt{2}\,\sqrt{\cot(dx+c)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(dx+c)}\right)\right)}{4}\right)}{d}$

input
```
int((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x,method=_RETUR
NVERBOSE)
```

output
```
B/d*(2/3/cot(d*x+c)^(3/2)+1/4*2^(1/2)*(ln((cot(d*x+c)+2^(1/2)*cot(d*x+c)^(
1/2)+1)/(cot(d*x+c)-2^(1/2)*cot(d*x+c)^(1/2)+1))+2*arctan(1+2^(1/2)*cot(d*
x+c)^(1/2))+2*arctan(-1+2^(1/2)*cot(d*x+c)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(95) = 190.

Time = 0.11 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.70

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx =$$
$$-\frac{6\left(\sqrt{2}B\cos\left(2\,dx+2\,c\right)+\sqrt{2}B\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\cos(2\,dx+2\,c)+1}{\sin(2\,dx+2\,c)}}\sin(2\,dx+2\,c)+\cos(2\,dx+2\,c)+1}{\cos(2\,dx+2\,c)+1}\right)+6\left(\sqrt{2}B\cos\left(2\,dx\right.\right.}{}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algori
thm="fricas")
```

output
```
-1/12*(6*(sqrt(2)*B*cos(2*d*x + 2*c) + sqrt(2)*B)*arctan((sqrt(2)*sqrt((co
s(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)
+ 1)/(cos(2*d*x + 2*c) + 1)) + 6*(sqrt(2)*B*cos(2*d*x + 2*c) + sqrt(2)*B)*
arctan((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x +
2*c) - cos(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) + 1)) - 3*(sqrt(2)*B*cos(2*
d*x + 2*c) + sqrt(2)*B)*log((sqrt(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x
 + 2*c))*sin(2*d*x + 2*c) + cos(2*d*x + 2*c) + sin(2*d*x + 2*c) + 1)/(cos(
2*d*x + 2*c) + 1)) + 3*(sqrt(2)*B*cos(2*d*x + 2*c) + sqrt(2)*B)*log(-(sqrt
(2)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c))*sin(2*d*x + 2*c) - cos(2
*d*x + 2*c) - sin(2*d*x + 2*c) - 1)/(cos(2*d*x + 2*c) + 1)) + 8*(B*cos(2*d
*x + 2*c) - B)*sqrt((cos(2*d*x + 2*c) + 1)/sin(2*d*x + 2*c)))/(d*cos(2*d*x
 + 2*c) + d)
```

Sympy [F]

$$\int \frac{aB + bB\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = B\int \frac{1}{\cot^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c)),x)
```

output
```
B*Integral(cot(c + d*x)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int \frac{aB + bB\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx = \frac{6\sqrt{2}B\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+\frac{2}{\sqrt{\tan(dx+c)}}\right)\right) + 6\sqrt{2}B\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-\frac{2}{\sqrt{\tan(dx+c)}}\right)\right) + 3\sqrt{2}B\log\left(\frac{}{\sqrt{t}}\right.}{12\,d}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algori
thm="maxima")
```

output
```
1/12*(6*sqrt(2)*B*arctan(1/2*sqrt(2)*(sqrt(2) + 2/sqrt(tan(d*x + c)))) + 6
*sqrt(2)*B*arctan(-1/2*sqrt(2)*(sqrt(2) - 2/sqrt(tan(d*x + c)))) + 3*sqrt(
2)*B*log(sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) - 3*sqrt(2)*B*lo
g(-sqrt(2)/sqrt(tan(d*x + c)) + 1/tan(d*x + c) + 1) + 8*B*tan(d*x + c)^(3/
2))/d
```

Giac [F(-1)]

Timed out.

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx=\text{Timed out}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x, algori
thm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))}\,dx=\frac{2\,B}{3\,d\,\left(\frac{1}{\tan(c+d\,x)}\right)^{3/2}}-\frac{{(-1)}^{1/4}\,B\,\mathrm{atan}\left({(-1)}^{1/4}\,\sqrt{\frac{1}{\tan(c+d\,x)}}\right)\,1{}\mathrm{i}}{d}-\frac{{(-1)}^{1/4}\,B\,\mathrm{atanh}\left({(-1)}^{1/4}\,\sqrt{\frac{1}{\tan(c+d\,x)}}\right)\,1{}\mathrm{i}}{d}$$

input
```
int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))),x)
```

output
```
(2*B)/(3*d*(1/tan(c + d*x))^(3/2)) - ((-1)^(1/4)*B*atan((-1)^(1/4)*(1/tan(
c + d*x))^(1/2))*1i)/d - ((-1)^(1/4)*B*atanh((-1)^(1/4)*(1/tan(c + d*x))^(
1/2))*1i)/d
```

Reduce [F]

$$\int \frac{aB + bB\tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))}\,dx = \left(\int \frac{\sqrt{\cot(dx + c)}}{\cot(dx + c)^3}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c)),x)
```

output
```
int(sqrt(cot(c + d*x))/cot(c + d*x)**3,x)*b
```

3.616 $\int \cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 354

$$\begin{aligned}
&\int \cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx \\
&= -\frac{\sqrt{ia-b}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&\quad-\frac{\sqrt{ia+b}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&\quad+\frac{2(35a^2Ab-8Ab^3+105a^3B+14ab^2B)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{105a^3d} \\
&\quad+\frac{2(35a^2A+4Ab^2-7abB)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{105a^2d} \\
&\quad-\frac{2(Ab+7aB)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{35ad} \\
&\quad-\frac{2A\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{7d}
\end{aligned}$$

output
```
-(I*a-b)^(1/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(I*a+b)^(1/2)*(I*A+B)*arcta
nh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)
*tan(d*x+c)^(1/2)/d+2/105*(35*A*a^2*b-8*A*b^3+105*B*a^3+14*B*a*b^2)*cot(d*
x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/a^3/d+2/105*(35*A*a^2+4*A*b^2-7*B*a*b)*c
ot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/a^2/d-2/35*(A*b+7*B*a)*cot(d*x+c)^(
5/2)*(a+b*tan(d*x+c))^(1/2)/a/d-2/7*A*cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1
/2)/d
```

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.82

$$\int \cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{\cot^{\frac{7}{2}}(c+dx)\left(105\sqrt[4]{-1}a^3\sqrt{-a+ib}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{7}{2}}(c+dx)-105(-1)^{3/}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(9/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x])
,x]
```

output
```
(Cot[c + d*x]^(7/2)*(105*(-1)^(1/4)*a^3*Sqrt[-a + I*b]*(I*A + B)*ArcTan[((
-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan
[c + d*x]^(7/2) - 105*(-1)^(3/4)*a^3*Sqrt[a + I*b]*(A + I*B)*ArcTan[((-1)^
(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c +
d*x]^(7/2) + 2*Sqrt[a + b*Tan[c + d*x]]*(-15*a^3*A - 3*a^2*(A*b + 7*a*B)*T
an[c + d*x] + a*(35*a^2*A + 4*A*b^2 - 7*a*b*B)*Tan[c + d*x]^2 + (35*a^2*A*
b - 8*A*b^3 + 105*a^3*B + 14*a*b^2*B)*Tan[c + d*x]^3)))/(105*a^3*d)
```

Rubi [A] (verified)

Time = 4.14 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.06, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4729, 3042, 4091, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{9/2}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{4091}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2}{7}\int -\frac{-6Ab\tan^2(c+dx)-7(aA-bB)\tan(c+dx)+Ab+7aB}{2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+}}{7d\tan}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\int \frac{-6Ab\tan^2(c+dx)-7(aA-bB)\tan(c+dx)+Ab+7aB}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan}}{7d\tan^{\frac{7}{2}}(c}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\int \frac{-6Ab\tan(c+dx)^2-7(aA-bB)\tan(c+dx)+Ab+7aB}{\tan(c+dx)^{7/2}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan}}{7d\tan^{\frac{7}{2}}(c}\right.$$

$$\Big\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\int\frac{35Aa^2-7bBa+35(Ab+aB)\tan(c+dx)a+4Ab^2+4b(Ab+7aB)\tan^2(c+dx)}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(7aB+}{5a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\int\frac{35Aa^2-7bBa+35(Ab+aB)\tan(c+dx)a+4Ab^2+4b(Ab+7aB)\tan^2(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(7aB+A}{5ad}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\int\frac{35Aa^2-7bBa+35(Ab+aB)\tan(c+dx)a+4Ab^2+4b(Ab+7aB)\tan(c+dx)^2}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2(7aB+A}{5ad}\right.\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{-\frac{2\int-\frac{105Ba^3+35Aba^2-105(aA-bB)\tan(c+dx)a^2+14b^2Ba-8Ab^3-2b\left(35Aa^2-7bBa+4Ab^2\right)\tan^2(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\frac{\int\frac{105Ba^3+35Aba^2-105(aA-bB)\tan(c+dx)a^2+14b^2Ba-8Ab^3-2b\left(35Aa^2-7bBa+4Ab^2\right)\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\frac{\int\frac{105Ba^3+35Aba^2-105(aA-bB)\tan(c+dx)a^2+14b^2Ba-8Ab^3-2b\left(35Aa^2-7bBa+4Ab^2\right)\tan(c+dx)^2}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}-}{5a}\right.\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\frac{-\frac{2\int \frac{105\left((aA-bB)a^3+(Ab+aB)\tan(c+dx)a^3\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\frac{-\frac{105\int \frac{(aA-bB)a^3+(Ab+aB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\frac{-\frac{105\int \frac{(aA-bB)a^3+(Ab+aB)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+35a^2Ab+14ab^2B-8Ab^3\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-}{5a}\right.\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7ab}{3}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7ab}{3}}{}\right.\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7ab}{3}}{}\right.\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7ab}{3}}{}\right.\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7ab}{3}}{}\right.\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+Ab)\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-7ab}{3}}{}\right.\right.$$

input
```
Int[Cot[c + d*x]^(9/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A*Sqrt[a + b*Tan[c + d*x]])/(7*
d*Tan[c + d*x]^(7/2)) + ((-2*(A*b + 7*a*B)*Sqrt[a + b*Tan[c + d*x]])/(5*a*
d*Tan[c + d*x]^(5/2)) - ((-2*(35*a^2*A + 4*A*b^2 - 7*a*b*B)*Sqrt[a + b*Tan
[c + d*x]])/(3*a*d*Tan[c + d*x]^(3/2)) + ((-105*((a^3*(a + I*b)*(A + I*B)*
ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt
[I*a - b]*d) + (a^3*(a - I*b)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c
+ d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(35*a^2*A*b
 - 8*A*b^3 + 105*a^3*B + 14*a*b^2*B)*Sqrt[a + b*Tan[c + d*x]])/(a*d*Sqrt[T
an[c + d*x]]))/(3*a))/(5*a))/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.30 (sec) , antiderivative size = 2185255, normalized size of antiderivative = 6173.04

output too large to display

input
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8179 vs. $2(294) = 588$.

Time = 1.20 (sec) , antiderivative size = 8179, normalized size of antiderivative = 23.10

$$\int \cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(9/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\cot(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(9/2)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{9/2}\,(A+B\tan(c+d\,x))\,\sqrt{a+b\tan(c+d\,x)}\,dx$$

input
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^{\frac{9}{2}}\,\sqrt{a+\tan(dx+c)\,b}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

3.617 $\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 290

$$\begin{aligned}
&\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx \\
&= \frac{\sqrt{ia-b}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&\quad - \frac{\sqrt{ia+b}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&\quad + \frac{2(15a^2A+2Ab^2-5abB)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{15a^2d} \\
&\quad - \frac{2(Ab+5aB)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{15ad} \\
&\quad - \frac{2A\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5d}
\end{aligned}$$

output
```
(I*a-b)^(1/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(I*a+b)^(1/2)*(A-I*B)*arctan
h((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*
tan(d*x+c)^(1/2)/d+2/15*(15*A*a^2+2*A*b^2-5*B*a*b)*cot(d*x+c)^(1/2)*(a+b*t
an(d*x+c))^(1/2)/a^2/d-2/15*(A*b+5*B*a)*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^
(1/2)/a/d-2/5*A*cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.87

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$= \frac{\cot^{\frac{5}{2}}(c+dx)\left(15\sqrt[4]{-1}a^2\sqrt{-a+ib}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{5}{2}}(c+dx)+15\sqrt[4]{-1}a^2\sqrt{}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x])
,x]
```

output
```
(Cot[c + d*x]^(5/2)*(15*(-1)^(1/4)*a^2*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((-
1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[
c + d*x]^(5/2) + 15*(-1)^(1/4)*a^2*Sqrt[a + I*b]*(A + I*B)*ArcTan[((-1)^(1
/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*
x]^(5/2) + 2*Sqrt[a + b*Tan[c + d*x]]*(-3*a^2*A - a*(A*b + 5*a*B)*Tan[c +
d*x] + (15*a^2*A + 2*A*b^2 - 5*a*b*B)*Tan[c + d*x]^2)))/(15*a^2*d)
```

Rubi [A] (verified)

Time = 3.15 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.07, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 4729, 3042, 4091, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$\big\downarrow$ 3042

$$\int \cot(c+dx)^{7/2}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$\big\downarrow$ 4729

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$\big\downarrow$ 4091

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2}{5}\int -\frac{-4Ab\tan^2(c+dx)-5(aA-bB)\tan(c+dx)+Ab+5aB}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+}}{5d\tan}\right.$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\int \frac{-4Ab\tan^2(c+dx)-5(aA-bB)\tan(c+dx)+Ab+5aB}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan}}{5d\tan^{\frac{5}{2}}(c}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\int \frac{-4Ab\tan(c+dx)^2-5(aA-bB)\tan(c+dx)+Ab+5aB}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan}}{5d\tan^{\frac{5}{2}}(c}\right.$$

$\big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{2\int \frac{15Aa^2-5bBa+15(Ab+aB)\tan(c+dx)a+2Ab^2+2b(Ab+5aB)\tan^2(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(5aB+}{3a}\right.\right.$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{\int \frac{15Aa^2-5bBa+15(Ab+aB)\tan(c+dx)a+2Ab^2+2b(Ab+5aB)\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(5aB+A}{3ad}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{\int \frac{15Aa^2-5bBa+15(Ab+aB)\tan(c+dx)a+2Ab^2+2b(Ab+5aB)\tan(c+dx)^2}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(5aB+A}{3ac}\right.\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{-\frac{2\int -\frac{15\left(a^2(Ab+aB)-a^2(aA-bB)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{\frac{15\int \frac{a^2(Ab+aB)-a^2(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(5aE}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{\frac{15\int \frac{a^2(Ab+aB)-a^2(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-5abB+2Ab^2)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(5aE}{}\right.\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5ab}{a}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5ab}{a}}{}\right.\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5ab}{a}}{}\right.\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5ab}{a}}{}\right.\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5ab}{a}}{}\right.\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+Ab)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-5ab}{a}}{}\right.\right.$$

input
```
Int[Cot[c + d*x]^(7/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A*Sqrt[a + b*Tan[c + d*x]])/(5*
d*Tan[c + d*x]^(5/2)) + ((-2*(A*b + 5*a*B)*Sqrt[a + b*Tan[c + d*x]])/(3*a*
d*Tan[c + d*x]^(3/2)) - ((15*(-((a^2*(a + I*b)*(I*A - B)*ArcTan[(Sqrt[I*a
- b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d)) + (
a^2*(a - I*b)*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(15*a^2*A + 2*A*b^2 - 5*a*b
*B)*Sqrt[a + b*Tan[c + d*x]])/(a*d*Sqrt[Tan[c + d*x]]))/(3*a))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.19 (sec) , antiderivative size = 2183123, normalized size of antiderivative = 7528.01

output too large to display

input
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8104 vs. 2(236) = 472.

Time = 1.21 (sec) , antiderivative size = 8104, normalized size of antiderivative = 27.94

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\cot(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(7/2)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\sqrt{a+b\tan(c+d\,x)}\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^{\frac{7}{2}}\sqrt{a+\tan(dx+c)\,b}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

3.618 $\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 239

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \frac{\sqrt{ia-b}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{\sqrt{ia+b}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{2(Ab+3aB)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3ad} - \frac{2A\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

output
```
(I*a-b)^(1/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(1/2)*(I*A+B)*arctan
h((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*
tan(d*x+c)^(1/2)/d-2/3*(A*b+3*B*a)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)
/a/d-2/3*A*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \frac{\cot^{\frac{3}{2}}(c+dx)\left(-3\sqrt[4]{-1}a\sqrt{-a+ib}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\tan^{\frac{3}{2}}(c+dx)+3(-1)^{3/4}a\sqrt{}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x])
,x]
```

output
```
(Cot[c + d*x]^(3/2)*(-3*(-1)^(1/4)*a*Sqrt[-a + I*b]*(I*A + B)*ArcTan[((-1)
^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c
+ d*x]^(3/2) + 3*(-1)^(3/4)*a*Sqrt[a + I*b]*(A + I*B)*ArcTan[((-1)^(1/4)*S
qrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(3
/2) - 2*Sqrt[a + b*Tan[c + d*x]]*(a*A + (A*b + 3*a*B)*Tan[c + d*x])))/(3*a
*d)
```

Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4729, 3042, 4091, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$\downarrow$ 4091

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2}{3}\int -\frac{-2Ab\tan^2(c+dx)-3(aA-bB)\tan(c+dx)+Ab+3aB}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+}}{3d\tan}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{-2Ab\tan^2(c+dx)-3(aA-bB)\tan(c+dx)+Ab+3aB}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan}}{3d\tan^{\frac{3}{2}}(c}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{-2Ab\tan(c+dx)^2-3(aA-bB)\tan(c+dx)+Ab+3aB}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan}}{3d\tan^{\frac{3}{2}}(c}\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{2\int \frac{3(a(aA-bB)+a(Ab+aB)\tan(c+dx))}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)-\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{3\int \frac{a(aA-bB)+a(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)-\frac{2}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{3\int \frac{a(aA-bB)+a(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)-\frac{2}{}\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{1}{2}a(a+ib)\right.}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{1}{2}a(a+ib)\right.}{}\right.\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a-ib)(A-}{}\right.}{}\right.\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a+ib)(A+}{}\right.}{}\right.\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a-ib)(A-}{}\right.}{}\right.\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+Ab)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a+ib)(A+\right.}{}\right.}{}\right.$$

input
```
Int[Cot[c + d*x]^(5/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A*Sqrt[a + b*Tan[c + d*x]])/(3*
d*Tan[c + d*x]^(3/2)) + ((-3*((a*(a + I*b)*(A + I*B)*ArcTan[(Sqrt[I*a - b]
*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a*(a
- I*b)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(A*b + 3*a*B)*Sqrt[a + b*Tan[c + d
*x]])/(a*d*Sqrt[Tan[c + d*x]]))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.18 (sec) , antiderivative size = 2178910, normalized size of antiderivative = 9116.78

output too large to display

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8070 vs. $2(191) = 382$.

Time = 1.18 (sec) , antiderivative size = 8070, normalized size of antiderivative = 33.77

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\cot(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(5/2)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int \cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))\,\sqrt{a+b\tan(c+d\,x)}\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot(dx+c)^{\frac{5}{2}}\sqrt{a+\tan(dx+c)\,b}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

3.619 $\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 194

$$\begin{aligned}
&\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx \\
&= -\frac{\sqrt{ia-b}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&\quad+\frac{\sqrt{ia+b}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&\quad-\frac{2A\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}
\end{aligned}$$

output

```
-(I*a-b)^(1/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(1/2)*(A-I*B)*arcta
nh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)
*tan(d*x+c)^(1/2)/d-2*A*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.97

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx =$$
$$-\frac{\sqrt{\cot(c+dx)}\left(\sqrt[4]{-1}\sqrt{-a+ib}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}+\sqrt[4]{-1}\sqrt{a+}\right.}{d}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x])
,x]
```

output
```
-((Sqrt[Cot[c + d*x]]*((-1)^(1/4)*Sqrt[-a + I*b]*(A - I*B)*ArcTan[((-1)^(1
/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[
c + d*x]] + (-1)^(1/4)*Sqrt[a + I*b]*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a +
 I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] + 2
*A*Sqrt[a + b*Tan[c + d*x]]))/d)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 4729, 3042, 4091, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4091}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-2\int -\frac{Ab+aB-(aA-bB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{Ab+aB-(aA-bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2A\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{4099}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{1}{2}(a+ib)(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(B+iA\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{1}{2}(a+ib)(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(B+iA\right.$$

$$\Big\downarrow \text{4098}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(a-ib)(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}-\frac{(a+ib)(}{}\right.$$

$$\Big\downarrow \text{104}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{(a+ib)(-B+iA)\int \frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(a-ib)(B+iA)\int \frac{}{1-\frac{(ia+}{a+}}}{d}\right.$$

$$\Big\downarrow \text{216}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(a-ib)(B+iA)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{(a+ib)(-B+iA)\arctan\left(\frac{\sqrt{}}{}\right.}{d\sqrt{-b+ia}}\right.$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{(a+ib)(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{(a-ib)(B+iA)\text{arctanh}\left(\frac{\sqrt{b+}}{\sqrt{a}}\right.}{d\sqrt{b+ia}}\right.$$

input
```
Int[Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-(((a + I*b)*(I*A - B)*ArcTan[(Sqrt
[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d)
) + ((a - I*b)*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d) - (2*A*Sqrt[a + b*Tan[c + d*x]])/(d
*Sqrt[Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.28 (sec) , antiderivative size = 2178316, normalized size of antiderivative = 11228.43

output too large to display

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7949 vs. 2(154) = 308.

Time = 1.18 (sec) , antiderivative size = 7949, normalized size of antiderivative = 40.97

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\cot(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(3/2)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\sqrt{a+b\tan\left(c+d\,x\right)}\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \cot\left(dx+c\right)^{\frac{3}{2}}\sqrt{a+\tan\left(dx+c\right)b}\,(A+B\tan\left(dx+c\right))\,dx$$

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

3.620 $\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 229

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = -\frac{\sqrt{ia-b}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{2\sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{\sqrt{ia+b}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}$$

output
```
-(I*a-b)^(1/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+2*b^(1/2)*B*arctanh(b^(1/2)
*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2
)/d-(I*a+b)^(1/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(
d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.04

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\sqrt[4]{-1}\left(\sqrt{-a+ib}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\sqrt{a+ib}(-iA\cdots\right.\right.}{d\sqrt{a-\cdots}}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x])
,x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-1)^(1/4)*(Sqrt[-a + I*b]*(I*A +
B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]] + Sqrt[a + I*b]*((-I)*A + B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqr
t[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])*Sqrt[a + b*Tan[c + d*x]] + 2*S
qrt[a]*Sqrt[b]*B*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b
*Tan[c + d*x])/a]))/(d*Sqrt[a + b*Tan[c + d*x]])
```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.87, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4729, 3042, 4097, 3042, 4099, 3042, 4098, 104, 216, 219, 4117, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \; 4729$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int\frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int\frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$\big\downarrow$ 4097

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int\frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+bB\int\frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int\frac{aA-bB+(Ab+aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+bB\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$\big\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(a+ib)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(A-iB)\int\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(a+ib)(A+iB)\int\frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(a-ib)(A-iB)\int\right.$$

$\big\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(a-ib)(A-iB)\int\frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{(a+ib)(\right.$$

$\big\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(a+ib)(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(a-ib)(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan}{a+b\tan}}}{d}\right.$$

$\big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(a-ib)(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+bB\int\frac{\tan(c+dx)^2 -}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan}}\right.$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(bB\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{(a+ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan}}{\sqrt{a+b\tan(}}\right.}{d\sqrt{-b+ia}}\right.$$

$$\Big\downarrow \text{4117}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{bB\int\frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{(a+ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan}}{\sqrt{a+b\tan(}}\right.}{d\sqrt{-b+ia}}\right.$$

$$\Big\downarrow \text{65}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2bB\int\frac{1}{1-\frac{b\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(a+ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right.}{d\sqrt{-b+ia}}\right.$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(a+ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{(a-ib)(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{t}}{\sqrt{a+b\tan}}\right.}{d\sqrt{b+ia}}\right.$$

input
```
Int[Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]),x]
```

output
```
(((a + I*b)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*Sqrt[Ta
n[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d + ((a - I*b)*(A - I*B)*ArcTanh[(
Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b
]*d))*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4097
```
Int[(Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/Sqrt[(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] :> In
t[Simp[a*A - b*B + (A*b + a*B)*Tan[e + f*x], x]/(Sqrt[a + b*Tan[e + f*x]]*S
qrt[c + d*Tan[e + f*x]]), x] + Simp[b*B   Int[(1 + Tan[e + f*x]^2)/(Sqrt[a
+ b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.18 (sec) , antiderivative size = 2178068, normalized size of antiderivative = 9511.21

output too large to display

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8075 vs. 2(181) = 362.

Time = 2.16 (sec) , antiderivative size = 16183, normalized size of antiderivative = 70.67

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx = \int (A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}\sqrt{\cot{(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))*sqrt(cot(c + d*x)),
 x)
```

Maxima [F]

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)*sqrt(cot(d*x + c))
, x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))\,\sqrt{a+b\tan(c+d\,x)}\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2),x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\cot\left(dx+c\right)}\,\sqrt{a+\tan\left(dx+c\right)b}\,\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c)),x)
```

3.621 $\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 261

$$\begin{aligned}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx &= \frac{\sqrt{ia-b}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\ &+ \frac{(2Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{b}d} \\ &- \frac{\sqrt{ia+b}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\ &+ \frac{B\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}}\end{aligned}$$

output
```
(I*a-b)^(1/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(2*A*b+B*a)*arctanh(b^(1/2)*
tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)
/b^(1/2)/d-(I*a+b)^(1/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a
+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+B*(a+b*tan(d*x+c
))^(1/2)/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\sqrt[4]{-1}\sqrt{-a+ib}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{a+b\tan(c+ \cdots}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x
]],x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-1)^(1/4)*Sqrt[-a + I*b]*(A - I*B
)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]]]*Sqrt[a + b*Tan[c + d*x]] + (-1)^(1/4)*Sqrt[a + I*b]*(A + I*B)*ArcT
an[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]
*Sqrt[a + b*Tan[c + d*x]] + B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]) + ((
2*A*b + a*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*(a + b*Tan[c +
d*x]))/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*Tan[c + d*x])/a])))/(d*Sqrt[a + b*Tan[
c + d*x]])
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4729, 3042, 4093, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$\Big\downarrow$ 4729

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$\Big\downarrow$ 4093

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int -\frac{-\left((2Ab+aB)\tan^2(c+dx)\right)-2(aA-bB)\tan(c+dx)+aB}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{B\sqrt{\tan(c\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{1}{2}\int \frac{-\left((2Ab+aB)\tan^2(c+dx)\right)-2(aA-}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{1}{2}\int \frac{-\left((2Ab+aB)\tan(c+dx)^2\right)-2(aA-}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan}}\right.$$

$\Big\downarrow$ 4138

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int \frac{-\left((2Ab+aB)\tan^2(c+dx)\right)-2(aA-bB)\tan(c+dx)+}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}}{2d}\right.$$

$\Big\downarrow$ 2035

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int \frac{-\left((2Ab+aB)\tan^2(c+dx)\right)-2(aA-bB)\tan(c+dx)+}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}}{d}\right.$$

$\Big\downarrow$ 2257

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int\left(\frac{-2Ab-aB}{\sqrt{a+b\tan(c+dx)}}+\frac{2(Ab+aB-(aA-bB)\tan(c+}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx}\right.}{d}\right.$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{-\sqrt{-b+ia}(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c-}}{\sqrt{a+b\tan(c+d}}\right.}{}\right.$$

input
```
Int[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-((-(Sqrt[I*a - b]*(A + I*B)*ArcTan
[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]) - ((2*A*b +
 a*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt
[b] + Sqrt[I*a + b]*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/S
qrt[a + b*Tan[c + d*x]]])/d) + (B*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*
x]])/d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4093
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[B*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^n/(f*(m + n))), x] + Simp
[1/(m + n)   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^(n - 1)*
Simp[a*A*c*(m + n) - B*(b*c*m + a*d*n) + (A*b*c + a*B*c + a*A*d - b*B*d)*(m
 + n)*Tan[e + f*x] + (A*b*d*(m + n) + B*(a*d*m + b*c*n))*Tan[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[0, m, 1] && LtQ[0, n, 1]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.01 (sec) , antiderivative size = 2180708, normalized size of antiderivative = 8355.20

output too large to display

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8069 vs. $2(209) = 418$.

Time = 3.20 (sec) , antiderivative size = 16171, normalized size of antiderivative = 61.96

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{a+b\tan{(c+dx)}}}{\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))/sqrt(cot(c + d*x)),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}}{\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)/sqrt(cot(d*x + c))
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\sqrt{a+b\tan\left(c+d\,x\right)}}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input

```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/cot(c + d*x)^(1/2),x
)
```

output

```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/cot(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \left(\int \frac{\sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)}{\cot\left(dx+c\right)}dx\right)b + \left(\int \frac{\sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)}dx\right)a$$

input

```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output

```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*x
),x)*b + int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/cot(c + d*x),x)
*a
```

3.622 $\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 324

$$\begin{aligned}\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\sqrt{ia-b}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}\\ &+ \frac{(4aAb-a^2B-8b^2B)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{4b^{3/2}d}\\ &+ \frac{\sqrt{ia+b}(iA+B)\mathrm{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}\\ &+ \frac{(4Ab-aB)\sqrt{a+b\tan(c+dx)}}{4bd\sqrt{\cot(c+dx)}} + \frac{B(a+b\tan(c+dx))^{3/2}}{2bd\sqrt{\cot(c+dx)}}\end{aligned}$$

output

```
(I*a-b)^(1/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+1/4*(4*A*a*b-B*a^2-8*B*b^2)*
arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*
tan(d*x+c)^(1/2)/b^(3/2)/d+(I*a+b)^(1/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan
(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+
1/4*(4*A*b-B*a)*(a+b*tan(d*x+c))^(1/2)/b/d/cot(d*x+c)^(1/2)+1/2*B*(a+b*tan
(d*x+c))^(3/2)/b/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 3.24 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-\left((-4aAb+a^2B+8b^2B)\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)(a+b\tan(c+dx))\right)+\right.}{}$$

input
```
Integrate[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/
2),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-((-4*a*A*b + a^2*B + 8*b^2*B)*Arc
Sinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*(a + b*Tan[c + d*x])) + Sqrt[a]
*Sqrt[b]*Sqrt[1 + (b*Tan[c + d*x])/a]*(-4*(-1)^(1/4)*Sqrt[-a + I*b]*b*(I*A
 + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]*Sqrt[a + b*Tan[c + d*x]] + 4*(-1)^(3/4)*Sqrt[a + I*b]*b*(A + I
*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]]*Sqrt[a + b*Tan[c + d*x]] + Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x]
)*(4*A*b + a*B + 2*b*B*Tan[c + d*x]))))/(4*Sqrt[a]*b^(3/2)*d*Sqrt[a + b*Ta
n[c + d*x]]*Sqrt[1 + (b*Tan[c + d*x])/a])
```

Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.88, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4729, 3042, 4090, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot(c+dx)^{3/2}}dx$$

$\downarrow$ 4729

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))dx$$

$\downarrow$ 4090

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int -\frac{\sqrt{a+b\tan(c+dx)}(-((4Ab-aB)\tan^2(c+dx))+4bB\tan(c+dx)+aB)}{2\sqrt{\tan(c+dx)}}dx}{2b}+\frac{B\sqrt{\tan(c+dx)}(a}{2}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\int \frac{\sqrt{a+b\tan(c+dx)}(-((4Ab-aB)\tan^2(c+dx))+4b}{\sqrt{\tan(c+dx)}}}{4b}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\int \frac{\sqrt{a+b\tan(c+dx)}(-((4Ab-aB)\tan(c+dx)^2)+4b}{\sqrt{\tan(c+dx)}}}{4b}\right.$$

$\downarrow$ 4130

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\int \frac{-((-Ba^2+4Aba-8b^2B)\tan^2(c+dx))+8b(Ab+a}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c}}}{}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\frac{1}{2}\int \frac{-((-Ba^2+4Aba-8b^2B)\tan^2(c+dx))+8b(Ab+}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(}}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\frac{1}{2}\int\frac{-((-Ba^2+4Aba-8b^2B)\tan(c+dx)^2)+8b(Ab+}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(}}}{}\right.$$

$\downarrow$ 4138

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\int\frac{-\left(\left(-Ba^2+4Aba-8b^2B\right)\tan^2(c+dx)\right)+8b(Ab+aB)\tan}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx\right.}}{2d}\right.$$

$\downarrow$ 2035

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\int\frac{-\left(\left(-Ba^2+4Aba-8b^2B\right)\tan^2(c+dx)\right)+8b(Ab+aB)\tan}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}}{d}\right.$$

$\downarrow$ 2257

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{\int\left(\frac{Ba^2-4Aba+8b^2B}{\sqrt{a+b\tan(c+dx)}}+\frac{8(b(aA-bB)+b(Ab+aB)\tan(c+d}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+\right.}\right.}{d}\right.$$

$\downarrow$ 2009

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2bd}-\frac{-\frac{(4Ab-aB)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}+\frac{-\frac{(a}{}}{}}{}\right.$$

input
```
Int[(Sqrt[a + b*Tan[c + d*x]]*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c
+ d*x])^(3/2))/(2*b*d) - ((-4*Sqrt[I*a - b]*b*(I*A - B)*ArcTan[(Sqrt[I*a -
 b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - ((4*a*A*b - a^2*B - 8*
b^2*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqr
t[b] - 4*b*Sqrt[I*a + b]*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x
]])/Sqrt[a + b*Tan[c + d*x]]])/d - ((4*A*b - a*B)*Sqrt[Tan[c + d*x]]*Sqrt[
a + b*Tan[c + d*x]])/d)/(4*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.53 (sec) , antiderivative size = 2183187, normalized size of antiderivative = 6738.23

output too large to display

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8173 vs. $2(260) = 520$.

Time = 3.31 (sec) , antiderivative size = 16383, normalized size of antiderivative = 50.56

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\sqrt{a+b\tan(c+dx)}}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(a + b*tan(c + d*x))/cot(c + d*x)**(3/2)
, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{b\tan(dx+c)+a}}{\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(b*tan(d*x + c) + a)/cot(d*x + c)^(3/2)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\sqrt{a+b\tan\left(c+d\,x\right)}}{\cot\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/cot(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(1/2))/cot(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{a+b\tan(c+dx)}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \left(\int \frac{\sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}\,\tan\left(dx+c\right)}{\cot\left(dx+c\right)^{2}}dx\right)b + \left(\int \frac{\sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^{2}}dx\right)a$$

input
```
int((a+b*tan(d*x+c))^(1/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*x
)**2,x)*b + int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/cot(c + d*x)
**2,x)*a
```

3.623 $\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 422

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{(ia-b)^{3/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}$$
$$-\frac{(ia+b)^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}$$
$$-\frac{2(315a^4A-63a^2Ab^2+8Ab^4-420a^3bB-18ab^3B)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{315a^3d}$$
$$+\frac{2(126a^2Ab+4Ab^3+105a^3B-9ab^2B)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{315a^2d}$$
$$+\frac{2(21a^2A-Ab^2-24abB)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{105ad}$$
$$-\frac{2(10Ab+9aB)\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{63d}$$
$$-\frac{2aA\cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{9d}$$

output
```
(I*a-b)^(3/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(I*a+b)^(3/2)*(I*A+B)*arctan
h((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*
tan(d*x+c)^(1/2)/d-2/315*(315*A*a^4-63*A*a^2*b^2+8*A*b^4-420*B*a^3*b-18*B*
a*b^3)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/a^3/d+2/315*(126*A*a^2*b+4*
A*b^3+105*B*a^3-9*B*a*b^2)*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/a^2/d+2
/105*(21*A*a^2-A*b^2-24*B*a*b)*cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)/a/d
-2/63*(10*A*b+9*B*a)*cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)/d-2/9*a*A*cot
(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 34.79 (sec) , antiderivative size = 5158, normalized size of antiderivative = 12.22

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[c + d*x]^(11/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*
x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 5.33 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.07, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.686$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{11/2}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{11/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{9}\int \frac{-b(8aA-9bB)\tan^2(c+dx)-9\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(10Ab}{2\tan^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\int \frac{-b(8aA-9bB)\tan^2(c+dx)-9\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(10Ab}{\tan^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\int \frac{-b(8aA-9bB)\tan(c+dx)^2-9\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(10Ab}{\tan(c+dx)^{9/2}\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{2\int \frac{3\left(2ab(10Ab+9aB)\tan^2(c+dx)+21a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(21Aa^2-24bBa-Ab^2\right)\right)}{2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{7a}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\int \frac{2ab(10Ab+9aB)\tan^2(c+dx)+21a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(21Aa^2-24bBa-Ab^2\right)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{7a}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\int \frac{2ab(10Ab+9aB)\tan(c+dx)^2+21a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(21Aa^2-24bBa-Ab^2\right)}{\tan(c+dx)^{7/2}\sqrt{a+b\tan(c+dx)}}dx}{7a}\right.\right.$$

$\Big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\left(-\frac{2\int -\frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-4b\left(21Aa^2-24bBa-Ab^2\right)\tan^2(c+dx)a+\left(105Ba^3+126A\right.}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{5a}\right.}{7a}\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\left(\frac{\int \frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-4b\left(21Aa^2-24bBa-Ab^2\right)\tan^2(c+dx)a+\left(105Ba^3+126Aba^2-9\right.}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{5a}\right.}{7a}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\left(\frac{\int \frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-4b\left(21Aa^2-24bBa-Ab^2\right)\tan(c+dx)^2a+\left(105Ba^3+126Aba^2-9\right.}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}}{5a}\right.}{7a}\right.\right.$$

$\Big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\left(-\frac{\frac{2\int \frac{315\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3+2b\left(105Ba^3+126Aba^2-9b^2Ba+4Ab^3\right)\tan^2(c+dx)a+\left(315Aa\right.}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{3a}}{5a}\right.}{}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\left(-\frac{\frac{\int \frac{315\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3+2b\left(105Ba^3+126Aba^2-9b^2Ba+4Ab^3\right)\tan^2(c+dx)a+\left(315Aa^4\right.}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{3a}}{5a}\right.}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\left(-\frac{\frac{\int \frac{315\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3+2b\left(105Ba^3+126Aba^2-9b^2Ba+4Ab^3\right)\tan(c+dx)^2a+\left(315Aa^4\right.}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}}{3a}}{5a}\right.}{}\right.\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\left(\frac{-\frac{-\frac{2\int -\frac{315\left(a^4\left(Ba^2+2Aba-b^2B\right)-a^4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(315a^4A-420a^3bB-63a^2\cdots\right.}{\cdots}}{3a}}{5a}\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\left(\frac{-\frac{\frac{315\int \frac{a^4\left(Ba^2+2Aba-b^2B\right)-a^4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(315a^4A-420a^3bB-63a^2Ab^2-1\cdots\right.}{d\sqrt{\tan(\cdots}}}{3a}}{5a}\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{9}\left(-\frac{3\left(\frac{-\frac{\frac{315\int \frac{a^4\left(Ba^2+2Aba-b^2B\right)-a^4\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(315a^4A-420a^3bB-63a^2Ab^2-1\cdots\right.}{d\sqrt{\tan(\cdots}}}{3a}}{5a}\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(-\frac{2(9aB+10Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2(21\ldots}{\ldots}\right.}{\ldots}\right.\right.$$

input
```
Int[Cot[c + d*x]^(11/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*Sqrt[a + b*Tan[c + d*x]])/(
9*d*Tan[c + d*x]^(9/2)) + ((-2*(10*A*b + 9*a*B)*Sqrt[a + b*Tan[c + d*x]])/
(7*d*Tan[c + d*x]^(7/2)) - (3*((-2*(21*a^2*A - A*b^2 - 24*a*b*B)*Sqrt[a +
b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) + ((-2*(126*a^2*A*b + 4*A*b^3 +
105*a^3*B - 9*a*b^2*B)*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2))
- ((315*(-((a^4*(a + I*b)^2*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d)) + (a^4*(a - I*b)^2*(I*
A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]
])/(Sqrt[I*a + b]*d)))/a - (2*(315*a^4*A - 63*a^2*A*b^2 + 8*A*b^4 - 420*a^
3*b*B - 18*a*b^3*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a
))/(5*a)))/(7*a))/9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.41 (sec) , antiderivative size = 2405378, normalized size of antiderivative = 5699.95

output too large to display

input
```
int(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12342 vs. 2(356) = 712.

Time = 2.10 (sec) , antiderivative size = 12342, normalized size of antiderivative = 29.25

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, alg
orithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(11/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, alg
orithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(11
/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{11/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}\,dx$$

input
```
int(cot(c + d*x)^(11/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^(11/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x
)
```

Reduce [F]

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \cot(dx+c)^{\frac{11}{2}}\,(a+\tan(dx+c)\,b)^{\frac{3}{2}}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

3.624 $\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 351

$$\begin{aligned}
&\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \\
&-\frac{(ia-b)^{3/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&-\frac{(ia+b)^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&+\frac{2(140a^2Ab+6Ab^3+105a^3B-21ab^2B)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{105a^2d} \\
&+\frac{2(35a^2A-3Ab^2-42abB)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{105ad} \\
&-\frac{2(8Ab+7aB)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{35d} \\
&-\frac{2aA\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{7d}
\end{aligned}$$

output
```
-(I*a-b)^(3/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(I*a+b)^(3/2)*(A-I*B)*arcta
nh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)
*tan(d*x+c)^(1/2)/d+2/105*(140*A*a^2*b+6*A*b^3+105*B*a^3-21*B*a*b^2)*cot(d
*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/a^2/d+2/105*(35*A*a^2-3*A*b^2-42*B*a*b)
*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/a/d-2/35*(8*A*b+7*B*a)*cot(d*x+c)
^(5/2)*(a+b*tan(d*x+c))^(1/2)/d-2/7*a*A*cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^
(1/2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 35.13 (sec) , antiderivative size = 5099, normalized size of antiderivative = 14.53

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 4.20 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.06, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{9/2}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{7}\int \frac{-b(6aA-7bB)\tan^2(c+dx)-7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+}{2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\int \frac{-b(6aA-7bB)\tan^2(c+dx)-7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\int \frac{-b(6aA-7bB)\tan(c+dx)^2-7\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(8Ab+}{\tan(c+dx)^{7/2}\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{2\int \frac{4ab(8Ab+7aB)\tan^2(c+dx)+35a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(35Aa^2-42bBa-3Ab^2\right)}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\int \frac{4ab(8Ab+7aB)\tan^2(c+dx)+35a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(35Aa^2-42bBa-3Ab^2\right)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\int\frac{4ab(8Ab+7aB)\tan(c+dx)^2+35a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(35Aa^2-42bBa-3Ab^2\right)}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx}{5a}\right.\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{-\frac{2\int-\frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-2b\left(35Aa^2-42bBa-3Ab^2\right)\tan^2(c+dx)a+\left(105Ba^3+140Aba^2\right.}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\frac{\int\frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-2b\left(35Aa^2-42bBa-3Ab^2\right)\tan^2(c+dx)a+\left(105Ba^3+140Aba^2-21b^2\right.}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\frac{\int\frac{-105\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)a^2-2b\left(35Aa^2-42bBa-3Ab^2\right)\tan(c+dx)^2a+\left(105Ba^3+140Aba^2-21b^2\right.}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{-\frac{2\int\frac{105\left(\left(Aa^2-2bBa-Ab^2\right)a^3+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6A\right.}{d\sqrt{\tan(c+dx)}}}{3a}}{5a}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\frac{-\frac{105\int\frac{\left(Aa^2-2bBa-Ab^2\right)a^3+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)}{d\sqrt{\tan(c+dx)}}}{3a}}{5a}\right.\right.$$

$$\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{\frac{-\frac{105\int\frac{\left(Aa^2-2bBa-Ab^2\right)a^3+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(105a^3B+140a^2Ab-21ab^2B+6Ab^3\right)}{d\sqrt{\tan(c+dx)}}}{3a}}{5a}\right.\right.$$

$$\downarrow \text{4099}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-}{}}{}\right.\right.$$

$$\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-}{}}{}\right.\right.$$

$$\downarrow \text{4098}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-}{}}{}\right.\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-}{}}{}\right.\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-}{}}{}\right.\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2(7aB+8Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(35a^2A-}{}}{}\right.\right.$$

input
```
Int[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*Sqrt[a + b*Tan[c + d*x]])/(
7*d*Tan[c + d*x]^(7/2)) + ((-2*(8*A*b + 7*a*B)*Sqrt[a + b*Tan[c + d*x]])/(
5*d*Tan[c + d*x]^(5/2)) - ((-2*(35*a^2*A - 3*A*b^2 - 42*a*b*B)*Sqrt[a + b*
Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) + ((-105*((a^3*(a + I*b)^2*(A + I*
B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(S
qrt[I*a - b]*d) + (a^3*(a - I*b)^2*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[T
an[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(140*a
^2*A*b + 6*A*b^3 + 105*a^3*B - 21*a*b^2*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Sq
rt[Tan[c + d*x]]))/(3*a))/(5*a))/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 3.37 (sec) , antiderivative size = 2403031, normalized size of antiderivative = 6846.24

output too large to display

input
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12268 vs. 2(291) = 582.

Time = 2.08 (sec) , antiderivative size = 12268, normalized size of antiderivative = 34.95

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(9/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(9/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{9/2}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)^{3/2}\,dx$$

input
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \cot(dx+c)^{\frac{9}{2}}\left(a+\tan(dx+c)\,b\right)^{\frac{3}{2}}\left(A+B\tan(dx+c)\right)dx$$

input
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

3.625 $\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 299

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{(a+ib)^2(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-bd}} + \frac{(ia+b)^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{2(15a^2A-3Ab^2-20abB)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{15ad} - \frac{2(6Ab+5aB)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{15d} - \frac{2aA\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5d}$$

output
```
(a+I*b)^2*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(
1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d+(I*a+b)^(3/2)*(I*A
+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x
+c)^(1/2)*tan(d*x+c)^(1/2)/d+2/15*(15*A*a^2-3*A*b^2-20*B*a*b)*cot(d*x+c)^(
1/2)*(a+b*tan(d*x+c))^(1/2)/a/d-2/15*(6*A*b+5*B*a)*cot(d*x+c)^(3/2)*(a+b*t
an(d*x+c))^(1/2)/d-2/5*a*A*cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx =$$

$$-\frac{\cot^{\frac{5}{2}}(c+dx)\left(30\sqrt[4]{-1}a\left((-a+ib)^{3/2}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)-(a+ib)^{3/2}(A+iB)\,\mathrm{a}\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
-1/30*(Cot[c + d*x]^(5/2)*(30*(-1)^(1/4)*a*((-a + I*b)^(3/2)*(A - I*B)*Arc
Tan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]
]] - (a + I*b)^(3/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + b*Tan[c + d*x]]])*Tan[c + d*x]^(5/2) + 15*a*b*B*Sqrt[a
+ b*Tan[c + d*x]] + 3*a*(4*a*A - 5*b*B)*Sqrt[a + b*Tan[c + d*x]] + 4*a*(6*
A*b + 5*a*B)*Tan[c + d*x]*Sqrt[a + b*Tan[c + d*x]] - 4*(15*a^2*A - 3*A*b^2
 - 20*a*b*B)*Tan[c + d*x]^2*Sqrt[a + b*Tan[c + d*x]]))/(a*d)
```

Rubi [A] (verified)

Time = 3.36 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.04, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{7/2}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{5}\int \frac{-b(4aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(6Ab+}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\int \frac{-b(4aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(6Ab+}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\int \frac{-b(4aA-5bB)\tan(c+dx)^2-5\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(6Ab+}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{2\int \frac{2ab(6Ab+5aB)\tan^2(c+dx)+15a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(15Aa^2-20bBa-3Ab^2\right)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{\int \frac{2ab(6Ab+5aB)\tan^2(c+dx)+15a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(15Aa^2-20bBa-3Ab^2\right)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{\int\frac{2ab(6Ab+5aB)\tan(c+dx)^2+15a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a\left(15Aa^2-20bBa-3Ab^2\right)}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}\right.\right.$$

$\big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{-\frac{2\int-\frac{15\left(a^2\left(Ba^2+2Aba-b^2B\right)-a^2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{}}{d\sqrt{\tan(c+dx}}}{3a}\right.\right.$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{\frac{15\int\frac{a^2\left(Ba^2+2Aba-b^2B\right)-a^2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan}}{d\sqrt{\tan(c+dx)}}}{3a}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{\frac{15\int\frac{a^2\left(Ba^2+2Aba-b^2B\right)-a^2\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(15a^2A-20abB-3Ab^2)\sqrt{a+b\tan}}{d\sqrt{\tan(c+dx)}}}{3a}\right.\right.$$

$\big\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-}{}}{}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-}{}}{}\right.\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-}{}}{}\right.\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-}{}}{}\right.\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-}{}}{}\right.\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2(5aB+6Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(15a^2A-}{}}{}\right.\right.$$

input
```
Int[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*Sqrt[a + b*Tan[c + d*x]])/(
5*d*Tan[c + d*x]^(5/2)) + ((-2*(6*A*b + 5*a*B)*Sqrt[a + b*Tan[c + d*x]])/(
3*d*Tan[c + d*x]^(3/2)) - ((15*(-((a^2*(a + I*b)^2*(I*A - B)*ArcTan[(Sqrt[
I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d))
 + (a^2*(a - I*b)^2*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/S
qrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(15*a^2*A - 3*A*b^2 -
 20*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/(3*a))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.80 (sec) , antiderivative size = 2398647, normalized size of antiderivative = 8022.23

output too large to display

input
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12233 vs. 2(243) = 486.

Time = 2.08 (sec) , antiderivative size = 12233, normalized size of antiderivative = 40.91

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(7/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \cot(dx+c)^{\frac{7}{2}}\left(a+\tan(dx+c)\,b\right)^{\frac{3}{2}}\left(A+B\tan(dx+c)\right)dx$$

input
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

3.626 $\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 236

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{(ia-b)^{3/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(ia+b)^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{2(4Ab+3aB)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3d} - \frac{2aA\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3d}$$

output
```
(I*a-b)^(3/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(3/2)*(A-I*B)*arctan
h((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*
tan(d*x+c)^(1/2)/d-2/3*(4*A*b+3*B*a)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/
2)/d-2/3*a*A*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.03

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{\cot(c+dx)}\left(3\sqrt[4]{-1}\left((-a+ib)^{3/2}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+i(a+\cdots\right.\right.}{\cdots}$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*(3*(-1)^(1/4)*((-a + I*b)^(3/2)*(I*A + B)*ArcTan[((-1)
^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + I*(a
 + I*b)^(3/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]
])/Sqrt[a + b*Tan[c + d*x]]])*Sqrt[Tan[c + d*x]] - 2*(4*A*b + 3*a*B)*Sqrt[
a + b*Tan[c + d*x]] - 3*b*B*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]] + (-2*a*
A + 3*b*B)*Cot[c + d*x]*Sqrt[a + b*Tan[c + d*x]]))/(3*d)
```

Rubi [A] (verified)

Time = 2.56 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{5/2}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$\downarrow$ 4088

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{3}\int \frac{-b(2aA-3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{-b(2aA-3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{-b(2aA-3bB)\tan(c+dx)^2-3\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(4Ab+}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{2\int \frac{3\left(a\left(Aa^2-2bBa-Ab^2\right)+a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+4Ab)\sqrt{a+b}}{d\sqrt{\tan(c+d}}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{3\int \frac{a\left(Aa^2-2bBa-Ab^2\right)+a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+4Ab)\sqrt{a+b\tan}}{d\sqrt{\tan(c+dx)}}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{3\int \frac{a\left(Aa^2-2bBa-Ab^2\right)+a\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(3aB+4Ab)\sqrt{a+b\tan}}{d\sqrt{\tan(c+dx)}}\right.\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{1}{2}a(a-\right.}{}\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{1}{2}a(a-\right.}{}\right.\right.$$

$\big\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a-ib)^2}{}\right.}{}\right.\right.$$

$\big\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a-ib)^2}{}\right.}{}\right.\right.$$

$\big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a-ib)^2}{}\right.}{}\right.\right.$$

$\big\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{3\left(\frac{a(a+ib)^2}{}\right.}{}\right.\right.$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*Sqrt[a + b*Tan[c + d*x]])/(
3*d*Tan[c + d*x]^(3/2)) + ((-3*((a*(a + I*b)^2*(A + I*B)*ArcTan[(Sqrt[I*a
- b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a
*(a - I*b)^2*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(4*A*b + 3*a*B)*Sqrt[a + b*T
an[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.12 (sec) , antiderivative size = 2398803, normalized size of antiderivative = 10164.42

output too large to display

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12148 vs. $2(188) = 376$.

Time = 2.05 (sec) , antiderivative size = 12148, normalized size of antiderivative = 51.47

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(5/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{5/2}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)^{3/2}\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \cot(dx+c)^{\frac{5}{2}}(a+\tan(dx+c)\,b)^{\frac{3}{2}}(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

3.627 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 269

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{(a+ib)^2(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}d} + \frac{2b^{3/2}B\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{(ia+b)^{3/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{2aA\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}$$

output
```
-(a+I*b)^2*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^
(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d+2*b^(3/2)*B*arcta
nh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d
*x+c)^(1/2)/d-(I*a+b)^(3/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)
/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2*a*A*cot(d*x
+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.99

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{\cot(c+dx)}\left(\sqrt[4]{-1}(-a+ib)^{3/2}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+\cdots}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*((-1)^(1/4)*(-a + I*b)^(3/2)*(A - I*B)*ArcTan[((-1)^(1
/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[
c + d*x]] - (-1)^(1/4)*(a + I*b)^(3/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a
 + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] -
 2*a*A*Sqrt[a + b*Tan[c + d*x]] + (2*Sqrt[a]*b^(3/2)*B*ArcSinh[(Sqrt[b]*Sq
rt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[Tan[c + d*x]]*Sqrt[1 + (b*Tan[c + d*x])/a]
)/Sqrt[a + b*Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$\downarrow$ 4088

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(2\int \frac{b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2a\ldots}{}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\ldots}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{2aA\ldots}{}\right.$$

$\downarrow$ 4138

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}-\frac{2aA\sqrt{a+b\tan\ldots}}{d\sqrt{\tan(c\ldots}}\right.$$

$\downarrow$ 2035

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int \frac{b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(2Ab+aB)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{d}-\frac{2aA\sqrt{a+\ldots}}{d\sqrt{\tan\ldots}}\right.$$

$\downarrow$ 2257

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int \left(\frac{Bb^2}{\sqrt{a+b\tan(c+dx)}}+\frac{Ba^2+2Aba-b^2B-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right)d\sqrt{\tan(c+dx)}}{d}-\ldots\right.$$

$\downarrow$ 2009

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}+\frac{2\left(\frac{1}{2}(-b+ia)^{3/2}(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right.\right.}{}\right.$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*(((I*a - b)^(3/2)*(I*A - B)*ArcT
an[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/2 + b^(3/
2)*B*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - ((I*
a + b)^(3/2)*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 b*Tan[c + d*x]]])/2))/d - (2*a*A*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c
+ d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.08 (sec) , antiderivative size = 2397204, normalized size of antiderivative = 8911.54

output too large to display

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12204 vs. 2(215) = 430.

Time = 4.12 (sec) , antiderivative size = 24440, normalized size of antiderivative = 90.86

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx=\int \cot(dx+c)^{\frac{3}{2}}\,(a+\tan(dx+c)\,b)^{\frac{3}{2}}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

3.628 $\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 264

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = -\frac{(ia-b)^{3/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{\sqrt{b}(2Ab+3aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{(ia+b)^{3/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{bB\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}}$$

output

```
-(I*a-b)^(3/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+b^(1/2)*(2*A*b+3*B*a)*arcta
nh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d
*x+c)^(1/2)/d-(I*a+b)^(3/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)
/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+b*B*(a+b*tan(
d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-\sqrt[4]{-1}(-a+ib)^{3/2}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\ldots\right.}{\ldots}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-((-1)^(1/4)*(-a + I*b)^(3/2)*(I*A
 + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]]) - (-1)^(3/4)*(a + I*b)^(3/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqr
t[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + b*B*Sqrt[Tan[c
+ d*x]]*Sqrt[a + b*Tan[c + d*x]] + (Sqrt[a]*Sqrt[b]*(2*A*b + 3*a*B)*ArcSin
h[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/Sqrt
[a + b*Tan[c + d*x]]))/d
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4729, 3042, 4090, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4090}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA-bB}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA-b}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{b(2Ab+3aB)\tan(c+dx)^2+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA-b}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{4138}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA-bB)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{2d}+\frac{bB\sqrt{\text{ta}}}{}\right.$$

$$\Big\downarrow \text{2035}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)+a(2aA-bB)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\sqrt{\tan(c+dx)}}{d}+\frac{bB\sqrt{}}{}\right.$$

$$\Big\downarrow \text{2257}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \left(\frac{b(2Ab+3aB)}{\sqrt{a+b\tan(c+dx)}}+\frac{2\left(Aa^2-2bBa-Ab^2+\left(Ba^2+2Aba-b^2B\right)\tan(c+dx)\right)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right)d\sqrt{\tan(c+dx)}}{d}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{bB\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}+\frac{-\left((-b+ia)^{3/2}(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{}}{\sqrt{a+b\tan}}\right.\right.}{}\right.$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-((I*a - b)^(3/2)*(A + I*B)*ArcTan
[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]) + Sqrt[b]*(
2*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x
]]] - (I*a + b)^(3/2)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])
/Sqrt[a + b*Tan[c + d*x]]])/d + (b*B*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c +
 d*x]])/d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.13 (sec) , antiderivative size = 2396110, normalized size of antiderivative = 9076.17

output too large to display

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12219 vs. $2(212) = 424$.

Time = 4.25 (sec) , antiderivative size = 24471, normalized size of antiderivative = 92.69

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (A + B\tan(c+dx))\,(a+b\tan(c+dx))^{\frac{3}{2}}\sqrt{\cot(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)*sqrt(cot(c + d*x
)), x)
```

Maxima [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x + c
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^{3/2}\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2),x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))\,dx = \int \sqrt{\cot(dx+c)}\,(a+\tan(dx+c)\,b)^{\frac{3}{2}}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c)),x)
```

3.629 $\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 320

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = -\frac{(ia-b)^{3/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(12aAb+3a^2B-8b^2B)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{4\sqrt{b}d} + \frac{(ia+b)^{3/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{bB\sqrt{a+b\tan(c+dx)}}{2d\cot^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+5aB)\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}}$$

output
```
-(I*a-b)^(3/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+1/4*(12*A*a*b+3*B*a^2-8*B*b
^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1
/2)*tan(d*x+c)^(1/2)/b^(1/2)/d+(I*a+b)^(3/2)*(I*A+B)*arctanh((I*a+b)^(1/2)
*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2
)/d+1/2*b*B*(a+b*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(3/2)+1/4*(4*A*b+5*B*a)*(a
+b*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-4\sqrt[4]{-1}(-a+ib)^{3/2}(A-\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d
*x]],x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-4*(-1)^(1/4)*(-a + I*b)^(3/2)*(A
- I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Ta
n[c + d*x]]] + 4*(-1)^(1/4)*(a + I*b)^(3/2)*(A + I*B)*ArcTan[((-1)^(1/4)*S
qrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + (4*A*b + 5*a*
B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 2*b*B*Tan[c + d*x]^(3/2)*
Sqrt[a + b*Tan[c + d*x]] + (Sqrt[a]*(12*a*A*b + 3*a^2*B - 8*b^2*B)*ArcSinh
[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt
[b]*Sqrt[a + b*Tan[c + d*x]])))/(4*d)
```

Rubi [A] (verified)

Time = 2.83 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4729, 3042, 4090, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$\Big\downarrow$ 4090

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{\tan(c+dx)}\left(b(4Ab+5aB)\tan^2(c+dx)+4\left(Ba^2+2Aba-b^2B\right)\tan(c+d\right.}{2\sqrt{a+b\tan(c+dx)}}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\int \frac{\sqrt{\tan(c+dx)}\left(b(4Ab+5aB)\tan^2(c+dx)+4\left(Ba^2+2Aba-b^2B\right)\tan(c+d\right.}{\sqrt{a+b\tan(c+dx)}}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\int \frac{\sqrt{\tan(c+dx)}\left(b(4Ab+5aB)\tan(c+dx)^2+4\left(Ba^2+2Aba-b^2B\right)\tan(c+d\right.}{\sqrt{a+b\tan(c+dx)}}\right.$$

$\Big\downarrow$ 4130

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{\int -\frac{-b\left(3Ba^2+12Aba-8b^2B\right)\tan^2(c+dx)-8b\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+ab(4Ab+5aB)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b}+\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int \frac{-b\left(3Ba^2+12Aba-8b^2B\right)\tan^2(c-}{\sqrt{ta}}}{}\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int \frac{-b\left(3Ba^2+12Aba-8b^2B\right)\tan(c+}{\sqrt{ta}}}{}\right.\right.$$

$\Big\downarrow$ 4138

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int\frac{-b(3Ba^2+12Aba-8b^2B)\tan^2(c-}{\sqrt{\tan(c+dx)}}}{}\right.\right.$$

$\downarrow$ 2035

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int\frac{-b(3Ba^2+12Aba-8b^2B)\tan^2(c-}{\sqrt{a+b}}}{}\right.\right.$$

$\downarrow$ 2257

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{d}-\frac{\int\left(\frac{8(b(Ba^2+2Aba-b^2B)-b(Aa^2-}{\sqrt{a+b\tan(c+dx)}(\text{ta}}\right.}{}\right.\right.$$

$\downarrow$ 2009

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{bB\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{2d}+\frac{1}{4}\left(\frac{(5aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(}}{d}\right.\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((b*B*Tan[c + d*x]^(3/2)*Sqrt[a + b*
Tan[c + d*x]])/(2*d) + (-((4*(I*a - b)^(3/2)*b*(I*A - B)*ArcTan[(Sqrt[I*a
- b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - Sqrt[b]*(12*a*A*b + 3
*a^2*B - 8*b^2*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]] - 4*b*(I*a + b)^(3/2)*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c +
 d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(b*d)) + ((4*A*b + 5*a*B)*Sqrt[Tan[c +
d*x]]*Sqrt[a + b*Tan[c + d*x]])/d)/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12263 vs. 2(258) = 516.

Time = 4.47 (sec) , antiderivative size = 24563, normalized size of antiderivative = 76.76

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^{\frac{3}{2}}}{\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)/sqrt(cot(c + d*x
)), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^{\frac{3}{2}}}{\sqrt{\cot{(dx+c)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)/sqrt(cot(d*x + c
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/cot(c + d*x)^(1/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/cot(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(a+\tan(dx+c)\,b)^{\frac{3}{2}}\,(A+B\tan(dx+c))}{\sqrt{\cot(dx+c)}}dx$$

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

3.630 $\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 383

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \frac{(ia-b)^{3/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+}{d}$$
$$+\frac{(6a^2Ab-16Ab^3-a^3B-24ab^2B)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{8b^{3/2}d}$$
$$+\frac{(ia+b)^{3/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d}$$
$$+\frac{(6aAb-a^2B-8b^2B)\sqrt{a+b\tan(c+dx)}}{8bd\sqrt{\cot(c+dx)}}$$
$$+\frac{(6Ab-aB)(a+b\tan(c+dx))^{3/2}}{12bd\sqrt{\cot(c+dx)}}+\frac{B(a+b\tan(c+dx))^{5/2}}{3bd\sqrt{\cot(c+dx)}}$$

output
```
(I*a-b)^(3/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+1/8*(6*A*a^2*b-16*A*b^3-B*a^
3-24*B*a*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot
(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/b^(3/2)/d+(I*a+b)^(3/2)*(A-I*B)*arctanh((I*
a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d
*x+c)^(1/2)/d+1/8*(6*A*a*b-B*a^2-8*B*b^2)*(a+b*tan(d*x+c))^(1/2)/b/d/cot(d
*x+c)^(1/2)+1/12*(6*A*b-B*a)*(a+b*tan(d*x+c))^(3/2)/b/d/cot(d*x+c)^(1/2)+1
/3*B*(a+b*tan(d*x+c))^(5/2)/b/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 4.06 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\Big(24\sqrt[4]{-1}(-a+ib)^{3/2}b(iA$$

input
```
Integrate[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(
3/2),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(24*(-1)^(1/4)*(-a + I*b)^(3/2)*b*(
I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]] + 24*(-1)^(3/4)*(a + I*b)^(3/2)*b*(A + I*B)*ArcTan[((-1)^(1
/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - 3*(-6*a*
A*b + a^2*B + 8*b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 2*(6*
A*b - a*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2) + 8*B*Sqrt[Tan[c
+ d*x]]*(a + b*Tan[c + d*x])^(5/2) - (3*Sqrt[a]*(-6*a^2*A*b + 16*A*b^3 + a
^3*B + 24*a*b^2*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 +
(b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]])))/(24*b*d)
```

Rubi [A] (verified)

Time = 3.57 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.91, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 4729, 3042, 4090, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan(c+dx)^{3/2}(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int -\frac{(a+b\tan(c+dx))^{3/2}(-((6Ab-aB)\tan^2(c+dx))+6bB\tan(c+dx)+aB)}{2\sqrt{\tan(c+dx)}}dx}{3b}+ B\sqrt{\tan(c+dx)}(\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\int \frac{(a+b\tan(c+dx))^{3/2}(-((6Ab-aB)\tan^2(c+dx))+}{\sqrt{\tan(c+dx)}}}{6b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\int\frac{(a+b\tan(c+dx))^{3/2}(-((6Ab-aB)\tan(c+dx)^2)+}{\sqrt{\tan(c+dx)}}}{6b}\right.$$

$\downarrow$ 4130

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{1}{2}\int\frac{3\sqrt{a+b\tan(c+dx)}(-((-Ba^2+6Aba-8b^2B)\operatorname{ta}}{2\sqrt{\operatorname{ta}}}}{}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{3}{4}\int\frac{\sqrt{a+b\tan(c+dx)}(-((-Ba^2+6Aba-8b^2B)\tan}{\sqrt{\tan}}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{3}{4}\int\frac{\sqrt{a+b\tan(c+dx)}(-((-Ba^2+6Aba-8b^2B)\tan}{\sqrt{\tan}}}{}\right.$$

$\downarrow$ 4130

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{3}{4}\left(\int\frac{-((-Ba^3+6Aba^2-24b^2Ba-16Ab^3)\tan^2(c+}{2\sqrt{\tan}}\right.}{}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{-((-Ba^3+6Aba^2-24b^2Ba-16Ab^3)\tan^2(}{\sqrt{t}}\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{3}{4}\left(\frac{1}{2}\int\frac{-((-Ba^3+6Aba^2-24b^2Ba-16Ab^3)\tan(c}{\sqrt{t}}\right.}{}\right.$$

$\Big\downarrow$ 4138

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{3}{4}\left(\frac{\int\frac{-\left(\left(-Ba^3+6Aba^2-24b^2Ba-16Ab^3\right)\tan^2(c+dx)\right)}{\sqrt{\tan(c+dx)}\sqrt{a+b}}}{}\right.}{}\right.$$

$\Big\downarrow$ 2035

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{3}{4}\left(\frac{\int\frac{-\left(\left(-Ba^3+6Aba^2-24b^2Ba-16Ab^3\right)\tan^2(c+dx)\right)}{\sqrt{a+b\tan(c}}}{}\right.}{}\right.$$

$\Big\downarrow$ 2257

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{\frac{3}{4}\left(\frac{\int\left(\frac{Ba^3-6Aba^2+24b^2Ba+16Ab^3}{\sqrt{a+b\tan(c+dx)}}+\frac{16\left(b\left(Aa^2-2bB\right.\right.}{\sqrt{a}}\right.}{}\right.}{}\right.$$

$\Big\downarrow$ 2009

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3bd}-\frac{-\frac{(6Ab-aB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}+\frac{3}{4}}{}\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(3/2)*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c
+ d*x])^(5/2))/(3*b*d) - (-1/2*((6*A*b - a*B)*Sqrt[Tan[c + d*x]]*(a + b*Ta
n[c + d*x])^(3/2))/d + (3*((-8*(I*a - b)^(3/2)*b*(A + I*B)*ArcTan[(Sqrt[I*
a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - ((6*a^2*A*b - 16*A*
b^3 - a^3*B - 24*a*b^2*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]])/Sqrt[b] - 8*b*(I*a + b)^(3/2)*(A - I*B)*ArcTanh[(Sqrt[I*a
+ b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d - ((6*a*A*b - a^2*B
- 8*b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d))/4)/(6*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12332 vs. $2(312) = 624$.

Time = 4.94 (sec) , antiderivative size = 24697, normalized size of antiderivative = 64.48

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^{\frac{3}{2}}}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**(3/2)/cot(c + d*x)**(3
/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{3}{2}}}{\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(3/2)/cot(d*x + c)^(3/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,9,3]%%%}+%%%{4,[0,7,3]%%%}+%%%{6,[0,5,3]%%%}+%%%{
4,[0,3,3]
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{3/2}}{\cot\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/cot(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(3/2))/cot(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{3/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+\tan(dx+c)\,b)^{\frac{3}{2}}\,(A+B\tan(dx+c))}{\cot(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
int((a+b*tan(d*x+c))^(3/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

3.631 $\int \cot^{\frac{13}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 500

$$\begin{aligned}
&\int \cot^{\frac{13}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \\
&-\frac{(ia-b)^{5/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&-\frac{(ia+b)^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&-\frac{2(8085a^4Ab-495a^2Ab^3+40Ab^5+3465a^5B-5313a^3b^2B-110ab^4B)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3465a^3d} \\
&-\frac{2(1155a^4A-1485a^2Ab^2-20Ab^4-2541a^3bB+55ab^3B)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3465a^2d} \\
&+\frac{2(495a^2Ab-5Ab^3+231a^3B-275ab^2B)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{1155ad} \\
&+\frac{2(99a^2A-113Ab^2-209abB)\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{693d} \\
&-\frac{2a(14Ab+11aB)\cot^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{99d} \\
&-\frac{2aA\cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{11d}
\end{aligned}$$

output
```
-(I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(I*a+b)^(5/2)*(I*A+B)*arcta
nh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)
*tan(d*x+c)^(1/2)/d-2/3465*(8085*A*a^4*b-495*A*a^2*b^3+40*A*b^5+3465*B*a^5
-5313*B*a^3*b^2-110*B*a*b^4)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/a^3/d
-2/3465*(1155*A*a^4-1485*A*a^2*b^2-20*A*b^4-2541*B*a^3*b+55*B*a*b^3)*cot(d
*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/a^2/d+2/1155*(495*A*a^2*b-5*A*b^3+231*B
*a^3-275*B*a*b^2)*cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)/a/d+2/693*(99*A*
a^2-113*A*b^2-209*B*a*b)*cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)/d-2/99*a*
(14*A*b+11*B*a)*cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(1/2)/d-2/11*a*A*cot(d*x
+c)^(11/2)*(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 33.59 (sec) , antiderivative size = 5419, normalized size of antiderivative = 10.84

$$\int \cot^{\frac{13}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[c + d*x]^(13/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*
x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 6.58 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.06, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.771$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{13}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{13/2}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{13/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{11}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(8aA-11bB)\tan^2(c+dx)-11\left(Aa^2-2bBa-Ab^2\right.\right.}{2\tan^{\frac{11}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(8aA-11bB)\tan^2(c+dx)-11\left(Aa^2-2bBa-Ab^2\right.\right.}{\tan^{\frac{11}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(8aA-11bB)\tan(c+dx)^2-11\left(Aa^2-2bBa-Ab^2\right.\right.}{\tan(c+dx)^{11/2}}\right.$$

$$\Big\downarrow \text{4128}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{2}{9}\int -\frac{b\left(88Ba^2+184Aba-99b^2B\right)\tan^2(c+dx)+99\left(Ba^3+3Aba^2-3b^2Ba\right.}{2\tan^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(-\frac{1}{9}\int \frac{b\left(88Ba^2+184Aba-99b^2B\right)\tan^2(c+dx)+99\left(Ba^3+3Aba^2-3b^2Ba\right.}{\tan^{\frac{9}{2}}(c+dx)\sqrt{a+b\tan(}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(-\frac{1}{9}\int \frac{b\left(88Ba^2+184Aba-99b^2B\right)\tan(c+dx)^2+99\left(Ba^3+3Aba^2-3b^2Ba\right.}{\tan(c+dx)^{9/2}\sqrt{a+b\tan}}\right.\right.$$

$\big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\int -\frac{3\left(-2ab\left(99Aa^2-209bBa-113Ab^2\right)\tan^2(c+dx)-231a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(\right.}{2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{7a}\right.\right.\right.$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\int \frac{-2ab\left(99Aa^2-209b\right.}{}}{}\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\int \frac{-2ab\left(99Aa^2-209b\right.}{}}{}\right.\right.\right.$$

$\big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{2\int \frac{1155\left(Ba^3+3Ab\right.}{}}{}\right.}{}\right.\right.\right.$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{\int \frac{1155\left(Ba^3+3Aba^2\right.}{}}{}\right.}{}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{\int\frac{1155\left(Ba^3+3Aba^2\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}\right.\right.\right.$$

$\Big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{-\frac{2\int-\frac{-3465\left(Aa^3\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.}{\cdots}\right.\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{\frac{\int\frac{-3465\left(Aa^3-3bB\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.}{\cdots}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{\int\frac{-3465\left(Aa^3-3bB\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.\right.\right.$$

$\Big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{-\frac{2\int\frac{3465\left(\left(Aa^3-3\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.\right.\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{-\frac{3465\int\frac{\left(Aa^3-3bB\cdots\right.}{\cdots}}{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{11}\left(\frac{1}{9}\left(\frac{2\left(99a^2A-209abB-113Ab^2\right)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}-\frac{3\left(-\frac{-\frac{3465\int\frac{\left(Aa^3-3bB\right.}{}}{}}{}\right.}{}\right.\right.\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\right.\right.\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\right.\right.\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\left(\right.\right.\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\right.\right.$$

$\Big\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{11d\tan^{\frac{11}{2}}(c+dx)}+\frac{1}{11}\left(-\frac{2a(11aB+14Ab)\sqrt{a+b\tan(c+dx)}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{9}\right.\right.$$

input
```
Int[Cot[c + d*x]^(13/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*(a + b*Tan[c + d*x])^(3/2))
/(11*d*Tan[c + d*x]^(11/2)) + ((-2*a*(14*A*b + 11*a*B)*Sqrt[a + b*Tan[c +
d*x]])/(9*d*Tan[c + d*x]^(9/2)) + ((2*(99*a^2*A - 113*A*b^2 - 209*a*b*B)*S
qrt[a + b*Tan[c + d*x]])/(7*d*Tan[c + d*x]^(7/2)) - (3*((-2*(495*a^2*A*b -
 5*A*b^3 + 231*a^3*B - 275*a*b^2*B)*Sqrt[a + b*Tan[c + d*x]])/(5*d*Tan[c +
 d*x]^(5/2)) - ((-2*(1155*a^4*A - 1485*a^2*A*b^2 - 20*A*b^4 - 2541*a^3*b*B
 + 55*a*b^3*B)*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) + ((-346
5*((a^4*(a + I*b)^3*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sq
rt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a^4*(a - I*b)^3*(A - I*B)*Ar
cTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[
I*a + b]*d)))/a - (2*(8085*a^4*A*b - 495*a^2*A*b^3 + 40*A*b^5 + 3465*a^5*B
 - 5313*a^3*b^2*B - 110*a*b^4*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c +
 d*x]]))/(3*a))/(5*a)))/(7*a))/9)/11)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(cot(d*x+c)^(13/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(13/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19121 vs. 2(428) = 856.

Time = 4.38 (sec) , antiderivative size = 19121, normalized size of antiderivative = 38.24

$$\int \cot^{\frac{13}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(13/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, alg
orithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{13}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(13/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{13}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^{\frac{13}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(13/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, alg
orithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(13
/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{13}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(13/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{13}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{13/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(13/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^(13/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x
)
```

Reduce [F]

$$\int \cot^{\frac{13}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(dx+c)^{\frac{13}{2}}\,(a+\tan(dx+c)\,b)^{\frac{5}{2}}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(13/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(13/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.632 $\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 418

$$\begin{aligned}
&\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{(ia-b)^{5/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&-\frac{(ia+b)^{5/2}(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&-\frac{2(315a^4A-483a^2Ab^2-10Ab^4-735a^3bB+45ab^3B)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{315a^2d} \\
&+\frac{2(231a^2Ab-5Ab^3+105a^3B-135ab^2B)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{315ad} \\
&+\frac{2(21a^2A-25Ab^2-45abB)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{105d} \\
&-\frac{2a(4Ab+3aB)\cot^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{21d} \\
&-\frac{2aA\cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{9d}
\end{aligned}$$

output
```
(I*a-b)^(5/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-(I*a+b)^(5/2)*(A-I*B)*arctan
h((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*
tan(d*x+c)^(1/2)/d-2/315*(315*A*a^4-483*A*a^2*b^2-10*A*b^4-735*B*a^3*b+45*
B*a*b^3)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/a^2/d+2/315*(231*A*a^2*b-
5*A*b^3+105*B*a^3-135*B*a*b^2)*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/a/d
+2/105*(21*A*a^2-25*A*b^2-45*B*a*b)*cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2
)/d-2/21*a*(4*A*b+3*B*a)*cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(1/2)/d-2/9*a*A
*cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 33.32 (sec) , antiderivative size = 5348, normalized size of antiderivative = 12.79

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[c + d*x]^(11/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*
x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 5.37 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.07, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.686$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{11/2}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{11/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{9}\int \frac{3\sqrt{a+b\tan(c+dx)}\left(-b(2aA-3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\text{t}\right.}{2\tan^{\frac{9}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(2aA-3bB)\tan^2(c+dx)-3\left(Aa^2-2bBa-Ab^2\right)\text{ta}\right.}{\tan^{\frac{9}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(2aA-3bB)\tan(c+dx)^2-3\left(Aa^2-2bBa-Ab^2\right)\text{ta}\right.}{\tan(c+dx)^{9/2}}\right.$$

$$\Big\downarrow \text{4128}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{2}{7}\int -\frac{b\left(18Ba^2+38Aba-21b^2B\right)\tan^2(c+dx)+21\left(Ba^3+3Aba^2-3b^2Ba-\right.}{2\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{1}{7}\int \frac{b\left(18Ba^2+38Aba-21b^2B\right)\tan^2(c+dx)+21\left(Ba^3+3Aba^2-3b^2Ba-\right.}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(-\frac{1}{7}\int \frac{b\left(18Ba^2+38Aba-21b^2B\right)\tan(c+dx)^2+21\left(Ba^3+3Aba^2-3b^2Ba-\right.}{\tan(c+dx)^{7/2}\sqrt{a+b\tan(c\right.}\right.\right.$$

$\big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\int -\frac{-4ab\left(21Aa^2-45bBa-25Ab^2\right)\tan^2(c+dx)-105a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{5a}\right.\right.\right.$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{\int\frac{-4ab\left(21Aa^2-45bBa-25A\right.}{}}{}\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{\int\frac{-4ab\left(21Aa^2-45bBa-25A\right.}{}}{}\right.\right.\right.$$

$\big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2\int\frac{315\left(Ba^3+3Aba^2-3b^2Ba\right.}{}}{}}{}\right.\right.\right.$$

$\big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{\int\frac{315\left(Ba^3+3Aba^2-3b^2Ba-\right.}{}}{}}{}\right.\right.\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{\int \frac{315\left(Ba^3+3Aba^2-3b^2Ba-\right.}{}}{}}{}\right.\right.\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{-\frac{2\int -\frac{315\left(a^3\left(Ba^3+3Aba^2-\right.\right.}{}}{}}{}}{}\right.\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{\frac{315\int \frac{a^3\left(Ba^3+3Aba^2-3b^2B\right.}{}}{}}{}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\left(\frac{1}{7}\left(\frac{2\left(21a^2A-45abB-25Ab^2\right)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{\frac{315\int \frac{a^3\left(Ba^3+3Aba^2-3b^2B\right.}{}}{}}{}}{}\right.\right.\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2(2\cdots}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2(2\right.\right.\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2(2\right.\right.\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2(2\right.\right.\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2(2\right.\right.\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{9d\tan^{\frac{9}{2}}(c+dx)}+\frac{1}{3}\left(-\frac{2a(3aB+4Ab)\sqrt{a+b\tan(c+dx)}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(\frac{2(2\right.\right.\right.$$

input
```
Int[Cot[c + d*x]^(11/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*(a + b*Tan[c + d*x])^(3/2))
/(9*d*Tan[c + d*x]^(9/2)) + ((-2*a*(4*A*b + 3*a*B)*Sqrt[a + b*Tan[c + d*x]
])/(7*d*Tan[c + d*x]^(7/2)) + ((2*(21*a^2*A - 25*A*b^2 - 45*a*b*B)*Sqrt[a
+ b*Tan[c + d*x]])/(5*d*Tan[c + d*x]^(5/2)) - ((-2*(231*a^2*A*b - 5*A*b^3
+ 105*a^3*B - 135*a*b^2*B)*Sqrt[a + b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/
2)) - ((315*(-((a^3*(a + I*b)^3*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d)) + (a^3*(a - I*b)^3
*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d
*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(315*a^4*A - 483*a^2*A*b^2 - 10*A*b^4 -
735*a^3*b*B + 45*a*b^3*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]])
)/(3*a))/(5*a))/7)/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19062 vs. $2(352) = 704$.

Time = 4.39 (sec) , antiderivative size = 19062, normalized size of antiderivative = 45.60

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, alg
orithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(11/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^{\frac{11}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, alg
orithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(11
/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{11/2}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(11/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^(11/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x
)
```

Reduce [F]

$$\int \cot^{\frac{11}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(dx+c)^{\frac{11}{2}}\left(a+\tan(dx+c)\,b\right)^{\frac{5}{2}}\left(A+B\tan(dx+c)\right)dx$$

input
```
int(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(11/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.633 $\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 349

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{(ia-b)^{5/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(ia+b)^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{2(245a^2Ab-15Ab^3+105a^3B-161ab^2B)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{105ad} + \frac{2(35a^2A-45Ab^2-77abB)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{105d} - \frac{2a(10Ab+7aB)\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{35d} - \frac{2aA\cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{7d}$$

output
```
(I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(5/2)*(I*A+B)*arctan
h((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*
tan(d*x+c)^(1/2)/d+2/105*(245*A*a^2*b-15*A*b^3+105*B*a^3-161*B*a*b^2)*cot(
d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/a/d+2/105*(35*A*a^2-45*A*b^2-77*B*a*b)
*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/d-2/35*a*(10*A*b+7*B*a)*cot(d*x+c
)^(5/2)*(a+b*tan(d*x+c))^(1/2)/d-2/7*a*A*cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))
^(3/2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 33.05 (sec) , antiderivative size = 5276, normalized size of antiderivative = 15.12

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 4.27 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.06, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow 3042$$

$$\int \cot(c+dx)^{9/2}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow 4729$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{9/2}}dx$$

$$\Big\downarrow 4088$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{7}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(4aA-7bB)\tan^2(c+dx)-7\left(Aa^2-2bBa-Ab^2\right)\text{ta}}{2\tan^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(4aA-7bB)\tan^2(c+dx)-7\left(Aa^2-2bBa-Ab^2\right)\text{ta}}{\tan^{\frac{7}{2}}(c+dx)}\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(4aA-7bB)\tan(c+dx)^2-7\left(Aa^2-2bBa-Ab^2\right)\text{ta}}{\tan(c+dx)^{7/2}}\right.$$

$$\Big\downarrow 4128$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{2}{5}\int -\frac{b\left(28Ba^2+60Aba-35b^2B\right)\tan^2(c+dx)+35\left(Ba^3+3Aba^2-3b^2Ba-\right.}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c}}\right.\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{1}{5}\int \frac{b\left(28Ba^2+60Aba-35b^2B\right)\tan^2(c+dx)+35\left(Ba^3+3Aba^2-3b^2Ba-\right.}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+}}\right.\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(-\frac{1}{5}\int \frac{b\left(28Ba^2+60Aba-35b^2B\right)\tan(c+dx)^2+35\left(Ba^3+3Aba^2-3b^2Ba-\right.}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c\right.}\right.\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2\int -\frac{-2ab\left(35Aa^2-77bBa-45Ab^2\right)\tan^2(c+dx)-105a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}}{3a}\right.\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2ab\left(35Aa^2-77bBa-45A\right.}{}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{\int \frac{-2ab\left(35Aa^2-77bBa-45A\right.}{}}{}\right.\right.\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2\int \frac{105\left(\left(Aa^3-3bBa^2-3Ab^2\right.\right.}{}}{}}{}\right.\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{105\int \frac{\left(Aa^3-3bBa^2-3Ab^2a+\right.}{}}{}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{2\left(35a^2A-77abB-45Ab^2\right)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{105\int\frac{\left(Aa^3-3bBa^2-3Ab^2a+\right.}{}}{}}{}\right.\right.\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2(}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2(}{}\right.\right.\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2(}{}\right.\right.\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2(}{}\right.\right.\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(2(\right.\right.\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{7d\tan^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(-\frac{2a(7aB+10Ab)\sqrt{a+b\tan(c+dx)}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(\frac{2(}{}\right.\right.\right.$$

input
```
Int[Cot[c + d*x]^(9/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*(a + b*Tan[c + d*x])^(3/2))
/(7*d*Tan[c + d*x]^(7/2)) + ((-2*a*(10*A*b + 7*a*B)*Sqrt[a + b*Tan[c + d*x
]])/(5*d*Tan[c + d*x]^(5/2)) + ((2*(35*a^2*A - 45*A*b^2 - 77*a*b*B)*Sqrt[a
 + b*Tan[c + d*x]])/(3*d*Tan[c + d*x]^(3/2)) - ((-105*((a^2*(a + I*b)^3*(A
 + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]
])/(Sqrt[I*a - b]*d) + (a^2*(a - I*b)^3*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a - (2*(
245*a^2*A*b - 15*A*b^3 + 105*a^3*B - 161*a*b^2*B)*Sqrt[a + b*Tan[c + d*x]]
)/(d*Sqrt[Tan[c + d*x]]))/(3*a))/5)/7)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18996 vs. $2(289) = 578$.

Time = 4.34 (sec) , antiderivative size = 18996, normalized size of antiderivative = 54.43

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(9/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^{\frac{9}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(9/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{9/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^(9/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cot^{\frac{9}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(dx+c)^{\frac{9}{2}}\,(a+\tan(dx+c)\,b)^{\frac{5}{2}}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(9/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.634 $\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 287

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{(ia-b)^{5/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{(ia+b)^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{2(15a^2A-23Ab^2-35abB)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{15d} - \frac{2a(8Ab+5aB)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{15d} - \frac{2aA\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{5d}$$

output
```
-(I*a-b)^(5/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(5/2)*(A-I*B)*arcta
nh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)
*tan(d*x+c)^(1/2)/d+2/15*(15*A*a^2-23*A*b^2-35*B*a*b)*cot(d*x+c)^(1/2)*(a+
b*tan(d*x+c))^(1/2)/d-2/15*a*(8*A*b+5*B*a)*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c
))^(1/2)/d-2/5*a*A*cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(3/2)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 33.75 (sec) , antiderivative size = 5229, normalized size of antiderivative = 18.22

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.40 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.06, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4128, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{7/2}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{5}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(2aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\text{ta}\right.}{2\tan^{\frac{5}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(2aA-5bB)\tan^2(c+dx)-5\left(Aa^2-2bBa-Ab^2\right)\text{ta}\right.}{\tan^{\frac{5}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b(2aA-5bB)\tan(c+dx)^2-5\left(Aa^2-2bBa-Ab^2\right)\text{ta}\right.}{\tan(c+dx)^{5/2}}\right.$$

$$\Big\downarrow \text{4128}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{2}{3}\int -\frac{b\left(10Ba^2+22Aba-15b^2B\right)\tan^2(c+dx)+15\left(Ba^3+3Aba^2-3b^2Ba-\right.}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{1}{3}\int \frac{b\left(10Ba^2+22Aba-15b^2B\right)\tan^2(c+dx)+15\left(Ba^3+3Aba^2-3b^2Ba-\right.}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(-\frac{1}{3}\int \frac{b\left(10Ba^2+22Aba-15b^2B\right)\tan(c+dx)^2+15\left(Ba^3+3Aba^2-3b^2Ba-\right.}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c}}\right.\right.$$

$$\Big\downarrow 4132$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\int -\frac{15\left(a\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)-a\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}+\frac{2(\cdots}{\cdots}\right.\right.\right.$$

$$\Big\downarrow 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(15a^2A-35abB-23Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{15\int\frac{a\left(Ba^3+3Aba^2-3b^2Ba\cdots\right.}{\sqrt{\cdots}}}{\cdots}\right.\right.\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{2\left(15a^2A-35abB-23Ab^2\right)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\tan(c+dx)}}-\frac{15\int\frac{a\left(Ba^3+3Aba^2-3b^2Ba\cdots\right.}{\sqrt{\cdots}}}{\cdots}\right.\right.\right.$$

$$\Big\downarrow 4099$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2(\cdots}{\cdots}\right.\right.\right.$$

$$\Big\downarrow 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2(\cdots}{\cdots}\right.\right.\right.$$

$$\Big\downarrow 4098$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2(\cdots}{\cdots}\right.\right.\right.$$

$$\Big\downarrow 104$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2(1\right.\right.\right.$$

$\big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2(1\right.\right.\right.$$

$\big\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2aA(a+b\tan(c+dx))^{3/2}}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{1}{5}\left(-\frac{2a(5aB+8Ab)\sqrt{a+b\tan(c+dx)}}{3d\tan^{\frac{3}{2}}(c+dx)}+\frac{1}{3}\left(\frac{2(1\right.\right.\right.$$

input
```
Int[Cot[c + d*x]^(7/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*a*A*(a + b*Tan[c + d*x])^(3/2))
/(5*d*Tan[c + d*x]^(5/2)) + ((-2*a*(8*A*b + 5*a*B)*Sqrt[a + b*Tan[c + d*x]
])/(3*d*Tan[c + d*x]^(3/2)) + ((-15*(-((a*(a + I*b)^3*(I*A - B)*ArcTan[(Sq
rt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*
d)) + (a*(a - I*b)^3*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/
Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/a + (2*(15*a^2*A - 23*A*b^2
 - 35*a*b*B)*Sqrt[a + b*Tan[c + d*x]])/(d*Sqrt[Tan[c + d*x]]))/3)/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18942 vs. $2(233) = 466$.

Time = 4.36 (sec) , antiderivative size = 18942, normalized size of antiderivative = 66.00

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^{\frac{7}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(7/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^(7/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cot^{\frac{7}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(dx+c)^{\frac{7}{2}}\,(a+\tan(dx+c)\,b)^{\frac{5}{2}}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(7/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.635 $\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 300

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = -\frac{(ia-b)^{5/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{2b^{5/2}B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{(ia+b)^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{2a(2Ab+aB)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{d} - \frac{2aA\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}$$

output
```
-(I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+2*b^(5/2)*B*arctanh(b^(1/2)
*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2
)/d-(I*a+b)^(5/2)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(
d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d-2*a*(2*A*b+B*a)*cot(d*x
+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/d-2/3*a*A*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c
))^(3/2)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.60 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.39

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\cot^{\frac{3}{2}}(c+dx)\left(iab(A+iB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{b\tan(c+dx)}{a}\right)\sqrt{a+b\tan(c+dx)}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
(Cot[c + d*x]^(3/2)*(I*a*b*(A + I*B)*Hypergeometric2F1[-3/2, -3/2, -1/2, -
((b*Tan[c + d*x])/a)]*Sqrt[a + b*Tan[c + d*x]] - a*b*(I*A + B)*Hypergeomet
ric2F1[-3/2, -3/2, -1/2, -((b*Tan[c + d*x])/a)]*Sqrt[a + b*Tan[c + d*x]] +
 (I*a + b)*(A - I*B)*Sqrt[1 + (b*Tan[c + d*x])/a]*(3*(-1)^(1/4)*(-a + I*b)
^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Ta
n[c + d*x]]]*Tan[c + d*x]^(3/2) + (I*a + (-3*a + (4*I)*b)*Tan[c + d*x])*Sq
rt[a + b*Tan[c + d*x]]) + (I*a - b)*(A + I*B)*Sqrt[1 + (b*Tan[c + d*x])/a]
*(3*(-1)^(1/4)*(a + I*b)^(3/2)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Tan[c + d*x]^(3/2) + (I*a + (3*a + (4*
I)*b)*Tan[c + d*x])*Sqrt[a + b*Tan[c + d*x]])))/(3*d*Sqrt[1 + (b*Tan[c + d
*x])/a])
```

Rubi [A] (verified)

Time = 2.77 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4128, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{5/2}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2}{3}\int \frac{3\sqrt{a+b\tan(c+dx)}\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+\right.}{2\tan^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{\sqrt{a+b\tan(c+dx)}\left(b^2B\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(\right.}{\tan^{\frac{3}{2}}(c+dx)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{\sqrt{a+b\tan(c+dx)}\left(b^2B\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+a(\right.}{\tan(c+dx)^{3/2}}\right.$$

$$\Big\downarrow \text{4128}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(2\int -\frac{-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-A\right.}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\int \frac{-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\int \frac{-B\tan(c+dx)^2b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$\downarrow$ 4138

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}\right.$$

$\downarrow$ 2035

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int \frac{-B\tan^2(c+dx)b^3+a\left(Aa^2-3bBa-3Ab^2\right)+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\sqrt{\tan(c+}}{d}\right.$$

$\downarrow$ 2257

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int \left(\frac{Aa^3-3bBa^2-3Ab^2a+b^3B+\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}-\frac{b^3B}{\sqrt{a+b\tan(c+dx)}}\right)d\sqrt{}}{d}\right.$$

$\downarrow$ 2009

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\left(\frac{1}{2}(-b+ia)^{5/2}(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+\frac{1}{2}(b+ia)^{5/2}(B+iA)\mathrm{a}\right.}{d}\right.$$

input
```
Int[Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*(((I*a - b)^(5/2)*(I*A - B)*Arc
Tan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/2 - b^(5
/2)*B*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + ((I
*a + b)^(5/2)*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]])/2))/d - (2*a*(2*A*b + a*B)*Sqrt[a + b*Tan[c + d*x]])/(
d*Sqrt[Tan[c + d*x]]) - (2*a*A*(a + b*Tan[c + d*x])^(3/2))/(3*d*Tan[c + d*
x]^(3/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.26 (sec) , antiderivative size = 2654016, normalized size of antiderivative = 8846.72

output too large to display

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17767 vs. 2(242) = 484.

Time = 6.67 (sec) , antiderivative size = 35564, normalized size of antiderivative = 118.55

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\cot(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(5/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^(5/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \cot(dx+c)^{\frac{5}{2}}\,(a+\tan(dx+c)\,b)^{\frac{5}{2}}\,(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.636 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 301

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{(ia-b)^{5/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{b^{3/2}(2Ab+5aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} - \frac{(ia+b)^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{b(2aA+bB)\sqrt{a+b\tan(c+dx)}}{d\sqrt{\cot(c+dx)}} - \frac{2aA\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}{d}$$

output
```
(I*a-b)^(5/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+b^(3/2)*(2*A*b+5*B*a)*arctan
h(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*
x+c)^(1/2)/d-(I*a+b)^(5/2)*(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/
(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+b*(2*A*a+B*b)*
(a+b*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)-2*a*A*cot(d*x+c)^(1/2)*(a+b*tan(
d*x+c))^(3/2)/d
```

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.21

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{\cot(c+dx)}\left((-1)^{3/4}(-a+ib)^{5/2}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(\cdots}{\cdots}$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*((-1)^(3/4)*(-a + I*b)^(5/2)*(I*A + B)*ArcTan[((-1)^(1
/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[
c + d*x]] - (-1)^(1/4)*(a + I*b)^(5/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a
 + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]] -
 (a - I*b)^2*(A - I*B)*Sqrt[a + b*Tan[c + d*x]] - (a + I*b)^2*(A + I*B)*Sq
rt[a + b*Tan[c + d*x]] + b*B*(a + b*Tan[c + d*x])^(3/2) + (b*(2*A*b + 5*a*
B)*Sqrt[a + b*Tan[c + d*x]]*(Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/
Sqrt[a]]*Sqrt[Tan[c + d*x]] - Sqrt[a]*Sqrt[1 + (b*Tan[c + d*x])/a]))/(Sqrt
[a]*Sqrt[1 + (b*Tan[c + d*x])/a])))/d
```

Rubi [A] (verified)

Time = 2.87 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(2\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(2aA+bB)\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+\right.}{2\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(2aA+bB)\tan^2(c+dx)-\left(Aa^2-2bBa-Ab^2\right)\tan(c+\right.}{\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(2aA+bB)\tan(c+dx)^2-\left(Aa^2-2bBa-Ab^2\right)\tan(c+\right.}{\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{4130}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{b^2(2Ab+5aB)\tan^2(c+dx)-2\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)+a}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{b^2(2Ab+5aB)\tan^2(c+dx)-2\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)+}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{b^2(2Ab+5aB)\tan(c+dx)^2-2\left(Aa^3-3bBa^2-3Ab^2a+b^3B\right)\tan(c+dx)+}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{4138}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{b^2(2Ab+5aB)\tan^2(c+dx)-2(Aa^3-3bBa^2-3Ab^2a+b^3B)\tan(c+dx)+a(2Ba^2+6Aba-b^2B)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c\right.$$

$\Big\downarrow$ 2035

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{b^2(2Ab+5aB)\tan^2(c+dx)-2(Aa^3-3bBa^2-3Ab^2a+b^3B)\tan(c+dx)+a(2Ba^2+6Aba-b^2B)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\sqrt{\tan(}{d}\right.$$

$\Big\downarrow$ 2257

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \left(\frac{(2Ab+5aB)b^2}{\sqrt{a+b\tan(c+dx)}}+\frac{2(Ba^3+3Aba^2-3b^2Ba-Ab^3-(Aa^3-3bBa^2-3Ab^2a+b^3B)\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right)d\sqrt{t}}{d}\right.$$

$\Big\downarrow$ 2009

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-b+ia)^{5/2}(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)+b^{3/2}(5aB+2Ab)\text{arctanh}\left(\frac{\sqrt{b}}{\sqrt{a+}}\right.}{d}\right.$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((I*a - b)^(5/2)*(A + I*B)*ArcTan[(
Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + b^(3/2)*(2*A
*b + 5*a*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]
 - (I*a + b)^(5/2)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sq
rt[a + b*Tan[c + d*x]]])/d + (b*(2*a*A + b*B)*Sqrt[Tan[c + d*x]]*Sqrt[a +
b*Tan[c + d*x]])/d - (2*a*A*(a + b*Tan[c + d*x])^(3/2))/(d*Sqrt[Tan[c + d*
x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.37 (sec) , antiderivative size = 2653710, normalized size of antiderivative = 8816.31

output too large to display

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17670 vs. 2(245) = 490.

Time = 7.04 (sec) , antiderivative size = 35373, normalized size of antiderivative = 117.52

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \cot(dx+c)^{\frac{3}{2}}(a+\tan(dx+c)\,b)^{\frac{5}{2}}(A+B\tan(dx+c))\,dx$$

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.637 $\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 320

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{(ia-b)^{5/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{\sqrt{b}(20aAb+15a^2B-8b^2B)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{4d} + \frac{(ia+b)^{5/2}(iA+B)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} + \frac{b(4Ab+7aB)\sqrt{a+b\tan(c+dx)}}{4d\sqrt{\cot(c+dx)}} + \frac{bB(a+b\tan(c+dx))^{3/2}}{2d\sqrt{\cot(c+dx)}}$$

output
```
(I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c
))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+1/4*b^(1/2)*(20*A*a*b+15*B*a
^2-8*B*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d
*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+(I*a+b)^(5/2)*(I*A+B)*arctanh((I*a+b)^(1/2)
*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2
)/d+1/4*b*(4*A*b+7*B*a)*(a+b*tan(d*x+c))^(1/2)/d/cot(d*x+c)^(1/2)+1/2*b*B*
(a+b*tan(d*x+c))^(3/2)/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.97

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(4\sqrt[4]{-1}(-a+ib)^{5/2}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c}}{\sqrt{a+b\tan(c+dx)}}\right.\right.}{}$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x
]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(4*(-1)^(1/4)*(-a + I*b)^(5/2)*(I*A
 + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]] - 4*(-1)^(3/4)*(a + I*b)^(5/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sq
rt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + b*(4*A*b + 7*a
*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 2*b*B*Sqrt[Tan[c + d*x]]
*(a + b*Tan[c + d*x])^(3/2) + (Sqrt[a]*Sqrt[b]*(20*a*A*b + 15*a^2*B - 8*b^
2*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x
])/a])/Sqrt[a + b*Tan[c + d*x]]))/(4*d)
```

Rubi [A] (verified)

Time = 2.91 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.87, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4729, 3042, 4090, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4090}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(4Ab+7aB)\tan^2(c+dx)+4\left(Ba^2+2Aba-b^2B\right)\tan(\right.}{2\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(4Ab+7aB)\tan^2(c+dx)+4\left(Ba^2+2Aba-b^2B\right)\tan(\right.}{\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\int \frac{\sqrt{a+b\tan(c+dx)}\left(b(4Ab+7aB)\tan(c+dx)^2+4\left(Ba^2+2Aba-b^2B\right)\tan(\right.}{\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{4130}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\int \frac{b\left(15Ba^2+20Aba-8b^2B\right)\tan^2(c+dx)+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{b\left(15Ba^2+20Aba-8b^2B\right)\tan^2(c+dx)+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{b\left(15Ba^2+20Aba-8b^2B\right)\tan(c+dx)^2+8\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right.}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx}}\right.\right.$$

$$\Big\downarrow \text{4138}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{\int \frac{b(15Ba^2+20Aba-8b^2B)\tan^2(c+dx)+8(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)+a(8Aa^2-9bBa-}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}}{2d}\right.\right.$$

$$\Big\downarrow \text{2035}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{\int \frac{b(15Ba^2+20Aba-8b^2B)\tan^2(c+dx)+8(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+dx)+a(8Aa^2-9bBa-}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}}{d}\right.\right.$$

$$\Big\downarrow \text{2257}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{4}\left(\frac{\int \left(\frac{b(15Ba^2+20Aba-8b^2B)}{\sqrt{a+b\tan(c+dx)}}+\frac{8(Aa^3-3bBa^2-3Ab^2a+b^3B+(Ba^3+3Aba^2-3b^2Ba-Ab^3)\tan(c+}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right.}{d}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{bB\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}+\frac{1}{4}\left(\frac{b(7aB+4Ab)\sqrt{\tan(c+dx)}\sqrt{a+b}}{d}\right.\right.$$

input
```
Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((b*B*Sqrt[Tan[c + d*x]]*(a + b*Tan[
c + d*x])^(3/2))/(2*d) + ((4*(I*a - b)^(5/2)*(I*A - B)*ArcTan[(Sqrt[I*a -
b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + Sqrt[b]*(20*a*A*b + 15*
a^2*B - 8*b^2*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d
*x]]] + 4*(I*a + b)^(5/2)*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*
x]])/Sqrt[a + b*Tan[c + d*x]]])/d + (b*(4*A*b + 7*a*B)*Sqrt[Tan[c + d*x]]*
Sqrt[a + b*Tan[c + d*x]])/d)/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.25 (sec) , antiderivative size = 2652345, normalized size of antiderivative = 8288.58

output too large to display

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17723 vs. 2(258) = 516.

Time = 7.06 (sec) , antiderivative size = 35483, normalized size of antiderivative = 110.88

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)*sqrt(cot(d*x + c
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx = \int \sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2),x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))\,dx=\int \sqrt{\cot\left(dx+c\right)}\left(a+\tan\left(dx+c\right)b\right)^{\frac{5}{2}}\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c)),x)
```

3.638 $\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 376

$$
\begin{aligned}
&\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \\
&-\frac{(ia-b)^{5/2}(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&+\frac{(30a^2Ab-16Ab^3+5a^3B-40ab^2B)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{8\sqrt{b}d} \\
&+\frac{(ia+b)^{5/2}(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&+\frac{(14aAb+5a^2B-8b^2B)\sqrt{a+b\tan(c+dx)}}{8d\sqrt{\cot(c+dx)}} \\
&+\frac{bB(a+b\tan(c+dx))^{3/2}}{3d\cot^{\frac{3}{2}}(c+dx)}+\frac{(2Ab+3aB)(a+b\tan(c+dx))^{3/2}}{4d\sqrt{\cot(c+dx)}}
\end{aligned}
$$

output
```
-(I*a-b)^(5/2)*(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+1/8*(30*A*a^2*b-16*A*b^3+5*
B*a^3-40*B*a*b^2)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/b^(1/2)/d+(I*a+b)^(5/2)*(A-I*B)*arctanh
((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*t
an(d*x+c)^(1/2)/d+1/8*(14*A*a*b+5*B*a^2-8*B*b^2)*(a+b*tan(d*x+c))^(1/2)/d/
cot(d*x+c)^(1/2)+1/3*b*B*(a+b*tan(d*x+c))^(3/2)/d/cot(d*x+c)^(3/2)+1/4*(2*
A*b+3*B*a)*(a+b*tan(d*x+c))^(3/2)/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 4.16 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(24\sqrt[4]{-1}(-a+ib)^{5/2}(A-\right.}{}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d
*x]],x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(24*(-1)^(1/4)*(-a + I*b)^(5/2)*(A
- I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Ta
n[c + d*x]]] + 24*(-1)^(1/4)*(a + I*b)^(5/2)*(A + I*B)*ArcTan[((-1)^(1/4)*
Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 3*(14*a*A*b
+ 5*a^2*B - 8*b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]] + 6*(2*A*
b + 3*a*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^(3/2) + 8*b*B*Tan[c + d
*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2) + (3*Sqrt[a]*(30*a^2*A*b - 16*A*b^3 +
 5*a^3*B - 40*a*b^2*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[
1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]])))/(24*d)
```

Rubi [A] (verified)

Time = 3.73 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.92, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 4729, 3042, 4090, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{3}\int \frac{3}{2}\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2A\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(b(2Ab+3aB)\tan^2(c+dx)+2\left(Ba^2+2Ab\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(b(2Ab+3aB)\tan(c+dx)^2+2\left(Ba^2+2Ab\right.\right.\right.$$

$$\Big\downarrow \text{4130}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{a+b\tan(c+dx)}\left(-b\left(5Ba^2+14Aba-8b^2B\right)\tan^2(c+dx)-8b\left(Aa^2-2bBa-Ab^2\right)\tan(c+dx)+ab(}{2\sqrt{\tan(c+dx)}}}{2b}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b\left(5Ba^2+1\right.\right.}{}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int \frac{\sqrt{a+b\tan(c+dx)}\left(-b\left(5Ba^2+1\right.\right.}{}}{}\right.\right.$$

$\downarrow$ 4130

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int \frac{-b\left(5Ba^3+30Aba^2-40b^2Ba-\right.}{}}{}\right.\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\frac{1}{2}\int \frac{-b\left(5Ba^3+30Aba^2-40b^2Ba\right.}{}}{}\right.\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\frac{1}{2}\int \frac{-b\left(5Ba^3+30Aba^2-40b^2Ba\right.}{}}{}\right.\right.$$

$\downarrow$ 4138

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int \frac{-b\left(5Ba^3+30Aba^2-40b^2Ba-16A\right.}{}}{}\right.\right.$$

$$\Big\downarrow \text{2035}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int\frac{-b\left(5Ba^3+30Aba^2-40b^2Ba-16A\right.}{}}{}\right.\right.$$

$$\Big\downarrow \text{2257}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}{2d}-\frac{\int\left(\frac{16\left(b\left(Ba^3+3Aba^2-3b^2Ba-Ab^3\right.\right.}{\sqrt{a+b\tan(}}\right.}{}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{bB\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}{3d}+\frac{1}{2}\left(\frac{(3aB+2Ab)\sqrt{\tan(c+dx)}(a+b\tan}{2d}\right.\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((b*B*Tan[c + d*x]^(3/2)*(a + b*Tan[
c + d*x])^(3/2))/(3*d) + (((2*A*b + 3*a*B)*Sqrt[Tan[c + d*x]]*(a + b*Tan[c
 + d*x])^(3/2))/(2*d) - ((8*(I*a - b)^(5/2)*b*(A + I*B)*ArcTan[(Sqrt[I*a -
 b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - Sqrt[b]*(30*a^2*A*b -
16*A*b^3 + 5*a^3*B - 40*a*b^2*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]] - 8*b*(I*a + b)^(5/2)*(A - I*B)*ArcTanh[(Sqrt[I*a +
b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/d - (b*(14*a*A*b + 5*a^2
*B - 8*b^2*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d)/(4*b))/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.22 (sec) , antiderivative size = 2657129, normalized size of antiderivative = 7066.83

output too large to display

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17750 vs. 2(308) = 616.

Time = 7.28 (sec) , antiderivative size = 35533, normalized size of antiderivative = 94.50

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)/sqrt(cot(d*x + c
)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan(c+d\,x))\,{(a+b\tan(c+d\,x))}^{5/2}}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/cot(c + d*x)^(1/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/cot(c + d*x)^(1/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(a+\tan(dx+c)\,b)^{\frac{5}{2}}\,(A+B\tan(dx+c))}{\sqrt{\cot(dx+c)}}dx$$

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

3.639 $\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 457

$$\begin{aligned}
\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx &= \\
&-\frac{(ia-b)^{5/2}(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&+\frac{(40a^3Ab-320aAb^3-5a^4B-240a^2b^2B+128b^4B)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{64b^{3/2}d} \\
&-\frac{(ia+b)^{5/2}(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{d} \\
&+\frac{(40a^2Ab-64Ab^3-5a^3B-112ab^2B)\sqrt{a+b\tan(c+dx)}}{64bd\sqrt{\cot(c+dx)}} \\
&+\frac{(40aAb-5a^2B-48b^2B)(a+b\tan(c+dx))^{3/2}}{96bd\sqrt{\cot(c+dx)}} \\
&+\frac{(8Ab-aB)(a+b\tan(c+dx))^{5/2}}{24bd\sqrt{\cot(c+dx)}}+\frac{B(a+b\tan(c+dx))^{7/2}}{4bd\sqrt{\cot(c+dx)}}
\end{aligned}$$

output
```
-(I*a-b)^(5/2)*(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+1/64*(40*A*a^3*b-320*A*a*b^
3-5*B*a^4-240*B*a^2*b^2+128*B*b^4)*arctanh(b^(1/2)*tan(d*x+c)^(1/2)/(a+b*t
an(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/b^(3/2)/d-(I*a+b)^(5/2
)*(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*c
ot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/d+1/64*(40*A*a^2*b-64*A*b^3-5*B*a^3-112*B
*a*b^2)*(a+b*tan(d*x+c))^(1/2)/b/d/cot(d*x+c)^(1/2)+1/96*(40*A*a*b-5*B*a^2
-48*B*b^2)*(a+b*tan(d*x+c))^(3/2)/b/d/cot(d*x+c)^(1/2)+1/24*(8*A*b-B*a)*(a
+b*tan(d*x+c))^(5/2)/b/d/cot(d*x+c)^(1/2)+1/4*B*(a+b*tan(d*x+c))^(7/2)/b/d
/cot(d*x+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 33.15 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \frac{\cos^3(c+dx)\sqrt{\cot(c+dx)}(A+B\tan(c+dx))\left(\left(64\sqrt[4]{\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(
3/2),x]
```

output
```
(Cos[c + d*x]^3*Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x])*(((64*(-1)^(1/4)*(
a - I*b)^(5/2)*b^(3/2)*(A - I*B)*ArcTan[((-1)^(3/4)*Sqrt[a - I*b]*Sqrt[Tan
[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] + 64*(-1)^(3/4)*(a + I*b)^(5/2)*b^(3
/2)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a
+ b*Tan[c + d*x]]] + (40*a^3*A*b - 320*a*A*b^3 - 5*a^4*B - 240*a^2*b^2*B +
 128*b^4*B)*Log[b*Sqrt[Tan[c + d*x]] + Sqrt[b]*Sqrt[a + b*Tan[c + d*x]]])*
Sqrt[Tan[c + d*x]]*(a + b*Tan[c + d*x])^2)/b^(3/2) + ((a + b*Tan[c + d*x])
^(5/2)*(2*b*(-104*a*A*b - 59*a^2*B + 72*b^2*B) + 48*b^3*B*Sec[c + d*x]^4 +
 (264*a^2*A*b - 256*A*b^3 + 15*a^3*B - 568*a*b^2*B)*Tan[c + d*x] + 2*b*Sec
[c + d*x]^2*(104*a*A*b + 59*a^2*B - 96*b^2*B + 4*b*(8*A*b + 17*a*B)*Tan[c
+ d*x]))/(3*b)))/(64*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2*(A*Cos[c + d*x
] + B*Sin[c + d*x]))
```

Rubi [A] (verified)

Time = 4.60 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.92, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 4729, 3042, 4090, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4130, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan(c+dx)^{3/2}(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{4090}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int -\frac{(a+b\tan(c+dx))^{5/2}(-((8Ab-aB)\tan^2(c+dx))+8bB\tan(c+dx)+aB)}{2\sqrt{\tan(c+dx)}}dx}{4b}+\frac{B\sqrt{\tan(c+dx)}(\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\int \frac{(a+b\tan(c+dx))^{5/2}(-((8Ab-aB)\tan^2(c+dx))+}{\sqrt{\tan(c+dx)}}}{8b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\int\frac{(a+b\tan(c+dx))^{5/2}(-((8Ab-aB)\tan(c+dx)^2)+}{\sqrt{\tan(c+dx)}}}{8b}\right.$$

$\downarrow$ 4130

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{3}\int\frac{(a+b\tan(c+dx))^{3/2}(-((-5Ba^2+40Aba-48b^2}{2}}{}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\int\frac{(a+b\tan(c+dx))^{3/2}(-((-5Ba^2+40Aba-48b^2}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\int\frac{(a+b\tan(c+dx))^{3/2}(-((-5Ba^2+40Aba-48b^2}{}}{}\right.$$

$\downarrow$ 4130

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{1}{2}\int\frac{3\sqrt{a+b\tan(c+dx)}(-((-5Ba^3+40Aba^2-1}{}\right.}{}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{3}{4}\int\frac{\sqrt{a+b\tan(c+dx)}(-((-5Ba^3+40Aba^2-11}{}\right.}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{3}{4}\int\frac{\sqrt{a+b\tan(c+dx)}(-((-5Ba^3+40Aba^2-11}{}\right.}{}\right.$$

$$\Big\downarrow \text{4130}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{3}{4}\left(\int\frac{-((-5Ba^4+40Aba^3-240b^2Ba^2-320Ab^3}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{-((-5Ba^4+40Aba^3-240b^2Ba^2-320A}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{-((-5Ba^4+40Aba^3-240b^2Ba^2-320A}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{4138}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int\frac{-((-5Ba^4+40Aba^3-240b^2Ba^2-320Ab^3a+1}{}}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{2035}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int\frac{-((-5Ba^4+40Aba^3-240b^2Ba^2-320Ab^3a+1}{}}{}\right.\right.}{}\right.$$

$$\Big\downarrow \text{2257}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int\left(\frac{5Ba^4-40Aba^3+240b^2Ba^2+320Ab^3a-128b^4}{\sqrt{a+b\tan(c+dx)}}\right.}{}\right.\right.}{}\right.$$

$\big\downarrow$ 2009

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{7/2}}{4bd}-\frac{-\frac{(8Ab-aB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}{3d}+\frac{1}{6}}{}\right.$$

input
```
Int[((a + b*Tan[c + d*x])^(5/2)*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c
+ d*x])^(7/2))/(4*b*d) - (-1/3*((8*A*b - a*B)*Sqrt[Tan[c + d*x]]*(a + b*Ta
n[c + d*x])^(5/2))/d + (-1/2*((40*a*A*b - 5*a^2*B - 48*b^2*B)*Sqrt[Tan[c +
 d*x]]*(a + b*Tan[c + d*x])^(3/2))/d + (3*((64*(I*a - b)^(5/2)*b*(I*A - B)
*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]] - ((4
0*a^3*A*b - 320*a*A*b^3 - 5*a^4*B - 240*a^2*b^2*B + 128*b^4*B)*ArcTanh[(Sq
rt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[b] + 64*b*(I*a +
 b)^(5/2)*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]])/d - ((40*a^2*A*b - 64*A*b^3 - 5*a^3*B - 112*a*b^2*B)*Sqrt[
Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]])/d))/4)/6)/(8*b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4130
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a
+ b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C
*(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*
m*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
 NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[
c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.27 (sec) , antiderivative size = 2659501, normalized size of antiderivative = 5819.48

output too large to display

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17849 vs. 2(382) = 764.

Time = 8.78 (sec) , antiderivative size = 35731, normalized size of antiderivative = 78.19

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*tan(d*x+c))**(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^{\frac{5}{2}}}{\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^(5/2)/cot(d*x + c)^(3/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[0,14,5]%%%}+%%%{6,[0,12,5]%%%}+%%%{15,[0,10,5]%%%}+
%%%{20,[0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^{5/2}}{\cot\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/cot(c + d*x)^(3/2),x
)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^(5/2))/cot(c + d*x)^(3/2),
x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^{5/2}(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+\tan(dx+c)\,b)^{\frac{5}{2}}\,(A+B\tan(dx+c))}{\cot(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
int((a+b*tan(d*x+c))^(5/2)*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

3.640 $\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 296

$$\begin{aligned}\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx &= \frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}d} \\ &\quad - \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia+b}d} \\ &\quad + \frac{2(15a^2A-8Ab^2+10abB)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{15a^3d} \\ &\quad + \frac{2(4Ab-5aB)\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{15a^2d} \\ &\quad - \frac{2A\cot^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{5ad}\end{aligned}$$

output

```
(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(
d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d-(I*A+B)*arctanh((I*a+b)^(1/2
)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/(I*a+b)^(1/2)/d+2/15*(15*A*a^2-8*A*b^2+10*B*a*b)*cot(d*x+c)^(1/2)*(a+b*
tan(d*x+c))^(1/2)/a^3/d+2/15*(4*A*b-5*B*a)*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c
))^(1/2)/a^2/d-2/5*A*cot(d*x+c)^(5/2)*(a+b*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 4.24 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82

$$\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}\left(-\frac{15\sqrt[4]{-1}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{\sqrt{-a+ib}}+\frac{15\sqrt[4]{-1}(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\ldots}{\sqrt{a+ib}}\ldots\right)}{15d}$$

input
```
Integrate[(Cot[c + d*x]^(7/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x
]],x]
```

output
```
(Sqrt[Cot[c + d*x]]*((-15*(-1)^(1/4)*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a
+ I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/S
qrt[-a + I*b] + (15*(-1)^(1/4)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/Sqrt[a +
 I*b] - (2*(-15*a^2*A + 8*A*b^2 - 10*a*b*B + a*(-4*A*b + 5*a*B)*Cot[c + d*
x] + 3*a^2*A*Cot[c + d*x]^2)*Sqrt[a + b*Tan[c + d*x]])/a^3))/(15*d)
```

Rubi [A] (verified)

Time = 3.06 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 4729, 3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cot(c+dx)^{7/2}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{7}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{7/2}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4092}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int \frac{4Ab\tan^2(c+dx)+5aA\tan(c+dx)+4Ab-5aB}{2\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{4Ab\tan^2(c+dx)+5aA\tan(c+dx)+4Ab-5aB}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{4Ab\tan(c+dx)^2+5aA\tan(c+dx)+4Ab-5aB}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx}{5a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{-\frac{2\int -\frac{15Aa^2+15B\tan(c+dx)a^2+10bBa-8Ab^2-2b(4Ab-5aB)\tan^2(c+dx)}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\int \frac{15Aa^2+15B\tan(c+dx)a^2+10bBa-8Ab^2-2b(4Ab-5aB)\tan^2(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\int\frac{15Aa^2+15B\tan(c+dx)a^2+10bBa-8Ab^2-2b(4Ab-5aB)\tan(c+dx)^2}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\Big\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{-\frac{2\int-\frac{15\left(a^3B-a^3A\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{15\int\frac{a^3B-a^3A\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{15\int\frac{a^3B-a^3A\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}}{5a}\right.$$

$$\Big\downarrow \text{4099}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{5ad\tan^{\frac{5}{2}}(c+dx)}-\frac{-\frac{2(4Ab-5aB)\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}+\frac{-\frac{2\left(15a^2A+10abB-8Ab^2\right)\sqrt{a+b\tan}}{ad\sqrt{\tan(c+dx)}}}{}}{}\right.$$

input
```
Int[(Cot[c + d*x]^(7/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A*Sqrt[a + b*Tan[c + d*x]])/(5*
a*d*Tan[c + d*x]^(5/2)) - ((-2*(4*A*b - 5*a*B)*Sqrt[a + b*Tan[c + d*x]])/(
3*a*d*Tan[c + d*x]^(3/2)) + ((15*(-((a^3*(I*A - B)*ArcTan[(Sqrt[I*a - b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d)) + (a^3*(I
*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]
]])/(Sqrt[I*a + b]*d)))/a - (2*(15*a^2*A - 8*A*b^2 + 10*a*b*B)*Sqrt[a + b*
Tan[c + d*x]])/(a*d*Sqrt[Tan[c + d*x]]))/(3*a))/(5*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.10 (sec) , antiderivative size = 1892755, normalized size of antiderivative = 6394.44

output too large to display

input
```
int(cot(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10265 vs. $2(242) = 484$.

Time = 2.58 (sec) , antiderivative size = 10265, normalized size of antiderivative = 34.68

$$\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{7}{2}}}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(7/2)/sqrt(b*tan(d*x + c) + a)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^{7/2}\,(A+B\tan(c+d\,x))}{\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

input
```
int((cot(c + d*x)^(7/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x
)
```

output
```
int((cot(c + d*x)^(7/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{7}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\cot(dx+c)^{\frac{7}{2}}(A+B\tan(dx+c))}{\sqrt{a+\tan(dx+c)\,b}}dx$$

input
```
int(cot(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(cot(d*x+c)^(7/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

3.641 $\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 243

$$\begin{aligned}\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx &= -\frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d}\\ &\quad-\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia+b}\,d}\\ &\quad+\frac{2(2Ab-3aB)\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{3a^2d}\\ &\quad-\frac{2A\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}{3ad}\end{aligned}$$

output
```
-(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot
(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d-(A-I*B)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1
/2)/(I*a+b)^(1/2)/d+2/3*(2*A*b-3*B*a)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1
/2)/a^2/d-2/3*A*cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.88

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\left(\frac{3\sqrt[4]{-1}(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\tan(c+dx)}}{\sqrt{-a+ib}}+\frac{3(-1)^{3/4}(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\cdots}{\sqrt{a+ib}}\cdots\right)}{3d}$$

input
```
Integrate[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x
]],x]
```

output
```
(Sqrt[Cot[c + d*x]]*((3*(-1)^(1/4)*(I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a +
I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/Sqr
t[-a + I*b] + (3*(-1)^(3/4)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqr
t[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[Tan[c + d*x]])/Sqrt[a + I*
b] - (2*(-2*A*b + 3*a*B + a*A*Cot[c + d*x])*Sqrt[a + b*Tan[c + d*x]])/a^2)
)/(3*d)
```

Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4729, 3042, 4092, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cot(c+dx)^{5/2}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4092}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int \frac{2Ab\tan^2(c+dx)+3aA\tan(c+dx)+2Ab-3aB}{2\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{2Ab\tan^2(c+dx)+3aA\tan(c+dx)+2Ab-3aB}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{2Ab\tan(c+dx)^2+3aA\tan(c+dx)+2Ab-3aB}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{-\frac{2\int -\frac{3\left(Aa^2+B\tan(c+dx)a^2\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{3\int \frac{Aa^2+B\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{3\int\frac{Aa^2+B\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}}{3a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}\right)$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{1}{2}a^2(A+iB)\int\frac{1-i\tan(c}{\sqrt{\tan(c+dx)}\sqrt{a}}\right.}{}}{3a}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{1}{2}a^2(A+iB)\int\frac{1-i\tan(c}{\sqrt{\tan(c+dx)}\sqrt{a}}\right.}{}}{3a}\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{a^2(A-iB)\int\frac{}{(1-i\tan(c+dx))\sqrt{t}}}{}\right.}{}}{}\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{a^2(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+}}{d}\right.}{}}{3}\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{a^2(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}}{d}\right.}{}}{3a}\right.$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{3ad\tan^{\frac{3}{2}}(c+dx)}-\frac{-\frac{2(2Ab-3aB)\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{3\left(\frac{a^2(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan}}{\sqrt{a+b\tan(}}\right.}{d\sqrt{-b+ia}}\right.}{}}{3a}\right.$$

input
```
Int[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A*Sqrt[a + b*Tan[c + d*x]])/(3*
a*d*Tan[c + d*x]^(3/2)) - ((3*((a^2*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[T
an[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a^2*(A - I*B
)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(S
qrt[I*a + b]*d)))/a - (2*(2*A*b - 3*a*B)*Sqrt[a + b*Tan[c + d*x]])/(a*d*Sq
rt[Tan[c + d*x]]))/(3*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.08 (sec) , antiderivative size = 1890726, normalized size of antiderivative = 7780.77

output too large to display

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10191 vs. 2(195) = 390.

Time = 2.63 (sec) , antiderivative size = 10191, normalized size of antiderivative = 41.94

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{5}{2}}}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(5/2)/sqrt(b*tan(d*x + c) + a)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))}{\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

input
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x
)
```

output
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}\,\cot\left(dx+c\right)^{2}dx$$

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x))*cot(c + d*x)**2,x)
```

3.642 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 199

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d} + \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia+b}\,d} - \frac{2A\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}{ad}$$

output

```
-(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot
(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d+(I*A+B)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1
/2)/(I*a+b)^(1/2)/d-2*A*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^(1/2)/a/d
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.97

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = -\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-\frac{\sqrt[4]{-1}(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{\sqrt[4]{-1}(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+i}}{\sqrt{a+b\tan}}\right.}{\sqrt{a+ib}}\right.}{d}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x
]],x]
```

output
```
-((Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-(((-1)^(1/4)*(A - I*B)*ArcTan[(
(-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/S
qrt[-a + I*b]) + ((-1)^(1/4)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sq
rt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b] + (2*A*Sqrt[a +
 b*Tan[c + d*x]])/(a*Sqrt[Tan[c + d*x]])))/d)
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 4729, 3042, 4092, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4092}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int -\frac{aB-aA\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{aB-aA\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{aB-aA\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a}-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{4099}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{1}{2}a(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a(-B+\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{1}{2}a(B+iA)\int \frac{i\tan(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}a(-B+\right.$$

$$\Big\downarrow \text{4098}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{a(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}-\right.$$

$$\Big\downarrow \text{104}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{a(B+iA)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{a(-B+iA)\int\frac{}{(ia}}{}}{a}\right.$$

$\Big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{a(B+iA)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{a(-B+iA)\arctan}{d}}{a}\right.$$

$\Big\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A\sqrt{a+b\tan(c+dx)}}{ad\sqrt{\tan(c+dx)}}+\frac{\frac{a(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}-\frac{a(-B+iA)\arctan\left(\frac{\sqrt{-b}}{\sqrt{}}\right.}{d\sqrt{-b+i}}}{a}\right.$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-((a*(I*A - B)*ArcTan[(Sqrt[I*a -
b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d)) + (a*
(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*
x]]])/(Sqrt[I*a + b]*d))/a - (2*A*Sqrt[a + b*Tan[c + d*x]])/(a*d*Sqrt[Tan[
c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.08 (sec) , antiderivative size = 1888059, normalized size of antiderivative = 9487.73

output too large to display

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10120 vs. $2(156) = 312$.

Time = 2.63 (sec) , antiderivative size = 10120, normalized size of antiderivative = 50.85

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\cot^{\frac{3}{2}}{(c+dx)}}{\sqrt{a+b\tan{(c+dx)}}}\,dx$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**(3/2)/sqrt(a + b*tan(c + d*x))
, x)
```

Maxima [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\cot{(dx+c)}^{\frac{3}{2}}}{\sqrt{b\tan{(dx+c)}+a}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/sqrt(b*tan(d*x + c) + a)
, x)
```

Giac [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{3}{2}}}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/sqrt(b*tan(d*x + c) + a)
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{\sqrt{a+b\tan(c+d\,x)}}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x
)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \sqrt{a+\tan(dx+c)\,b}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)\,dx$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x))*cot(c + d*x),x)
```

3.643 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 163

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d} + \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia+b}\,d}$$

output
```
(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(
d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d+(A-I*B)*arctanh((I*a+b)^(1/2
)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/(I*a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$=\frac{\sqrt[4]{-1}\left(-\frac{(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{(-iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{a+ib}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c}}{d}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x
]],x]
```

output
```
((-1)^(1/4)*(-(((I*A + B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]) + (((-I)*A + B)*ArcTan[((
-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqr
t[a + I*b])*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 4729, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$
$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4099}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{i\tan(c}{\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int \frac{i\tan(c}{\sqrt{\tan(c+dx)}}\right.$$

$$\Big\downarrow \text{4098}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{(A+iB)\int \frac{}{(i\tan(}}{}\right.$$

$$\Big\downarrow \text{104}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A+iB)\int \frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(A-iB)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c}}{\sqrt{a+b\tan}}}{d}\right.$$

$$\Big\downarrow \text{216}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}\right.$$

$$\Big\downarrow \text{219}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}\right)$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/Sqrt[a + b*Tan[c + d*x]],x]
```

output
```
(((A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d
*x]]])/(Sqrt[I*a - b]*d) + ((A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d))*Sqrt[Cot[c + d*x]]*Sq
rt[Tan[c + d*x]]
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.22 (sec) , antiderivative size = 1878877, normalized size of antiderivative = 11526.85

output too large to display

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10053 vs. 2(127) = 254.

Time = 2.78 (sec) , antiderivative size = 10053, normalized size of antiderivative = 61.67

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(A+B\tan(c+dx))\sqrt{\cot(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/sqrt(a + b*tan(c + d*x)),
 x)
```

Maxima [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{\cot(dx+c)}}{\sqrt{b\tan(dx+c)+a}}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(cot(d*x + c))/sqrt(b*tan(d*x + c) + a)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)}{\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),x
)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}\,dx = \int \sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}dx$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int(sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)),x)
```

3.644 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 228

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}d} + \frac{2B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{b}d} - \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia+b}d}$$

output
```
(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(
d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d+2*B*arctanh(b^(1/2)*tan(d*x+
c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/b^(1/2)
/d-(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*
cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.99

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\sqrt[4]{-1}\left(-\frac{(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{(A+iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+}}{\sqrt{a+b\tan(c+dx)}}\right.}{\sqrt{a+ib}}\right.\right.}{d}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]
]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-1)^(1/4)*(-(((A - I*B)*ArcTan[((
-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sq
rt[-a + I*b]) + ((A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d
*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b]) + (2*Sqrt[a]*B*ArcSinh[(Sq
rt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*
Sqrt[a + b*Tan[c + d*x]])))/d
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4729, 3042, 4097, 3042, 4099, 3042, 4098, 104, 216, 219, 4117, 65, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4097}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{A\tan(c+dx)-B}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+B\int \frac{\tan^2(c+dx)+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{A\tan(c+dx)-B}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+B\int \frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\right.$$

$$\Big\downarrow \text{4099}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(B+iA)\int \frac{i\tan}{\sqrt{\tan(c+dx}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(-B+iA)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx-\frac{1}{2}(B+iA)\int \frac{i\tan}{\sqrt{\tan(c+dx}}\right.$$

$$\Big\downarrow \text{4098}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{(B+iA)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{(-B+iA)\int \frac{}{(i}}{}\right.$$

$$\Big\downarrow \text{104}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\int \frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\frac{(B+iA)\int \frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan}}{\sqrt{a+b\,\mathrm{t}}}}{d}\right.$$

$\Big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{(B+iA)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+B\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+$$

$\Big\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(B\int\frac{\tan(c+dx)^2+1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx+\frac{(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right.}{d\sqrt{-b+ia}}\right.$$

$\Big\downarrow$ 4117

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\int\frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)}{d}+\frac{(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right.}{d\sqrt{-b+ia}}\right.$$

$\Big\downarrow$ 65

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2B\int\frac{1}{1-\frac{b\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+\frac{(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{(B}{}\right.$$

$\Big\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(-B+iA)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{(B+iA)\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{b+ia}}+\frac{2}{}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]),x]
```

output
```
(((I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d
*x]]])/(Sqrt[I*a - b]*d) + (2*B*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[
a + b*Tan[c + d*x]]])/(Sqrt[b]*d) - ((I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqrt
[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d))*Sqrt[Cot[c +
 d*x]]*Sqrt[Tan[c + d*x]]
```

Defintions of rubi rules used

rule 65
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2   Sub
st[Int[1/(b - d*x^2), x], x, Sqrt[b*x]/Sqrt[c + d*x]], x] /; FreeQ[{b, c, d
}, x] &&  !GtQ[c, 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4097
```
Int[(Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)]))/Sqrt[(c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] :> In
t[Simp[a*A - b*B + (A*b + a*B)*Tan[e + f*x], x]/(Sqrt[a + b*Tan[e + f*x]]*S
qrt[c + d*Tan[e + f*x]]), x] + Simp[b*B   Int[(1 + Tan[e + f*x]^2)/(Sqrt[a
+ b*Tan[e + f*x]]*Sqrt[c + d*Tan[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2,
0]
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4117
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[A/f   Subst[Int[(a + b*x)^m*(c + d*x)^n, x], x, Tan[e + f*x]], x] /;
FreeQ[{a, b, c, d, e, f, A, C, m, n}, x] && EqQ[A, C]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 0.83 (sec) , antiderivative size = 1885968, normalized size of antiderivative = 8271.79

output too large to display

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10230 vs. 2(180) = 360.

Time = 4.16 (sec) , antiderivative size = 20493, normalized size of antiderivative = 89.88

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{A+B\tan{(c+dx)}}{\sqrt{a+b\tan{(c+dx)}}\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(sqrt(a + b*tan(c + d*x))*sqrt(cot(c + d*x))
), x)
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{B\tan(dx+c)+A}{\sqrt{b\tan(dx+c)+a}\sqrt{\cot(dx+c)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/(sqrt(b*tan(d*x + c) + a)*sqrt(cot(d*x + c)
)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot(c+d\,x)}\,\sqrt{a+b\tan(c+d\,x)}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/cot(c + d*x),x)
```

3.645 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 266

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx &= -\frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d}\\
&\quad+\frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{b^{3/2}d}\\
&\quad-\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia+b}\,d}\\
&\quad+\frac{B\sqrt{a+b\tan(c+dx)}}{bd\sqrt{\cot(c+dx)}}\end{aligned}$$

output

```
-(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot
(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d+(2*A*b-B*a)*arctanh(b^(1/2)
*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2
)/b^(3/2)/d-(A-I*B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c)
)^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(1/2)/d+B*(a+b*tan(d*x+
c))^(1/2)/b/d/cot(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.33

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$$

$$= \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-\sqrt{a}\sqrt{-a+ib}\sqrt{a+ib}(-2Ab+aB)\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]
]),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-(Sqrt[a]*Sqrt[-a + I*b]*Sqrt[a +
I*b]*(-2*A*b + a*B)*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1 +
 (b*Tan[c + d*x])/a]) + Sqrt[b]*((-1)^(1/4)*Sqrt[a + I*b]*b*(I*A + B)*ArcT
an[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]
]*Sqrt[a + b*Tan[c + d*x]] + Sqrt[-a + I*b]*((-1)^(3/4)*b*(A + I*B)*ArcTan
[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*S
qrt[a + b*Tan[c + d*x]] + Sqrt[a + I*b]*B*Sqrt[Tan[c + d*x]]*(a + b*Tan[c
+ d*x])))))/(Sqrt[-a + I*b]*Sqrt[a + I*b]*b^(3/2)*d*Sqrt[a + b*Tan[c + d*x
]])
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4729, 3042, 4090, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 4729$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 4090$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int -\frac{-((2Ab-aB)\tan^2(c+dx))+2bB\tan(c+dx)+aB}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b}+\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+d}}{bd}\right.$$

$$\Big\downarrow \; 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\int \frac{-((2Ab-aB)\tan^2(c+dx))+2bB\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{2b}\right.$$

$$\Big\downarrow \; 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\int \frac{-((2Ab-aB)\tan(c+dx)^2)+2bB\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{2b}\right.$$

$$\Big\downarrow \; 4138$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\int \frac{-((2Ab-aB)\tan^2(c+dx))+2bB\tan(c+dx)+aB}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\,\mathrm{t}}{2bd}\right.$$

$$\Big\downarrow \; 2035$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\int \frac{-((2Ab-aB)\tan^2(c+dx))+2bB\tan(c+dx)+aB}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\sqrt{}}{bd}\right.$$

$$\Big\downarrow \text{2257}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\int\left(\frac{aB-2Ab}{\sqrt{a+b\tan(c+dx)}}+\frac{2(Ab+B\tan(c+dx)b)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx}\right.}{bd}\right.$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{B\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}{bd}-\frac{\frac{b(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}-\frac{(2Ab-aB)a}{}}{}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*Sqrt[a + b*Tan[c + d*x]]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-((((b*(A + I*B)*ArcTan[(Sqrt[I*a -
b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[I*a - b] - ((2*A*b
- a*B)*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqr
t[b] + (b*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]])/Sqrt[I*a + b])/(b*d)) + (B*Sqrt[Tan[c + d*x]]*Sqrt[a + b*T
an[c + d*x]])/(b*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4090
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*B*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m +
 n))), x] + Simp[1/(d*(m + n))   Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Ta
n[e + f*x])^n*Simp[a^2*A*d*(m + n) - b*B*(b*c*(m - 1) + a*d*(n + 1)) + d*(m
 + n)*(2*a*A*b + B*(a^2 - b^2))*Tan[e + f*x] - (b*B*(b*c - a*d)*(m - 1) - b
*(A*b + a*B)*d*(m + n))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2
, 0] && GtQ[m, 1] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 1]
&& ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.27 (sec) , antiderivative size = 1888591, normalized size of antiderivative = 7099.97

output too large to display

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10269 vs. $2(214) = 428$.

Time = 4.86 (sec) , antiderivative size = 20571, normalized size of antiderivative = 77.33

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)\sqrt{a + b\tan(c + dx)}}\,dx = \int \frac{A + B\tan{(c + dx)}}{\sqrt{a + b\tan{(c + dx)}}\cot^{\frac{3}{2}}{(c + dx)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/(sqrt(a + b*tan(c + d*x))*cot(c + d*x)**(3/2
)), x)
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx=\int \frac{B\tan(dx+c)+A}{\sqrt{b\tan(dx+c)+a}\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/(sqrt(b*tan(d*x + c) + a)*cot(d*x + c)^(3/2
)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{3/2}\,\sqrt{a+b\tan\left(c+d\,x\right)}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(1/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\,dx = \int \frac{\sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^{2}}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(1/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/cot(c + d*x)**2,x)
```

3.646 $\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 316

$$\begin{aligned}\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = & -\frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d} \\ & -\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{3/2}d} \\ & +\frac{2b(5a^2Ab+8Ab^3-3a^3B-6ab^2B)}{3a^3(a^2+b^2)d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}} \\ & +\frac{2(4Ab-3aB)\sqrt{\cot(c+dx)}}{3a^2d\sqrt{a+b\tan(c+dx)}}-\frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad\sqrt{a+b\tan(c+dx)}}\end{aligned}$$

output

```
-(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot
(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d-(I*A+B)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1
/2)/(I*a+b)^(3/2)/d+2/3*b*(5*A*a^2*b+8*A*b^3-3*B*a^3-6*B*a*b^2)/a^3/(a^2+b
^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)+2/3*(4*A*b-3*B*a)*cot(d*x+c)
^(1/2)/a^2/d/(a+b*tan(d*x+c))^(1/2)-2/3*A*cot(d*x+c)^(3/2)/a/d/(a+b*tan(d*
x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.95

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cot(c+dx)}\left(\frac{3\sqrt[4]{-1}a\left(\frac{(a+ib)(iA+B)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\cdots\right.}{a^2+b\cdots}\right.}{\cdots}$$

input
```
Integrate[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
3/2),x]
```

output
```
(Sqrt[Cot[c + d*x]]*((3*(-1)^(1/4)*a*(((a + I*b)*(I*A + B)*ArcTan[((-1)^(1
/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a
+ I*b] + ((I*a + b)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c
+ d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b])*Sqrt[Tan[c + d*x]])/(a^
2 + b^2) + (8*A*b - 6*a*B)/(a*Sqrt[a + b*Tan[c + d*x]]) - (2*A*Cot[c + d*x
])/Sqrt[a + b*Tan[c + d*x]] + (2*b*(5*a^2*A*b + 8*A*b^3 - 3*a^3*B - 6*a*b^
2*B)*Tan[c + d*x])/(a^2*(a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]])))/(3*a*d)
```

Rubi [A] (verified)

Time = 3.39 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.07, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 4729, 3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cot(c+dx)^{5/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$\downarrow$ 4729

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}dx$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+b\tan(c+dx))^{3/2}}dx$$

$\downarrow$ 4092

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int \frac{4Ab\tan^2(c+dx)+3aA\tan(c+dx)+4Ab-3aB}{2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}dx}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx}}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{4Ab\tan^2(c+dx)+3aA\tan(c+dx)+4Ab-3aB}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}dx}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{4Ab\tan(c+dx)^2+3aA\tan(c+dx)+4Ab-3aB}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^{3/2}}dx}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{-\frac{2\int -\frac{3Aa^2+3B\tan(c+dx)a^2+6bBa-8Ab^2-2b(4Ab-3aB)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+}}}{3a}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\int \frac{3Aa^2+3B\tan(c+dx)a^2+6bBa-8Ab^2-2b(4Ab-3aB)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{3a}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\int \frac{3Aa^2+3B\tan(c+dx)a^2+6bBa-8Ab^2-2b(4Ab-3aB)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{3a}\right.$$

$\Big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{2\int \frac{3\left(a^3(aA+bB)-a^3(Ab-aB)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2}{ad\sqrt{\tan(c}}}{3a}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{3\int \frac{a^3(aA+bB)-a^3(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2(4A}{ad\sqrt{\tan(c+dx}}}{3a}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{3\int \frac{a^3(aA+bB)-a^3(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2b\left(-3a^3B+5a^2Ab-6ab^2B+8Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2(4A}{ad\sqrt{\tan(c+dx}}}{3a}\right.$$

$\Big\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B\right.}{a}}{}}{}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B\right.}{a}}{}}{}\right.$$

$\big\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B\right.}{a}}{}}{}\right.$$

$\big\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B\right.}{a}}{}}{}\right.$$

$\big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B\right.}{a}}{}}{}\right.$$

$$\downarrow 219$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}-\frac{-\frac{2(4Ab-3aB)}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{-\frac{2b\left(-3a^3B\right.}{a}}{}}{}\right.$$

input
```
Int[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A)/(3*a*d*Tan[c + d*x]^(3/2)*Sq
rt[a + b*Tan[c + d*x]]) - ((-2*(4*A*b - 3*a*B))/(a*d*Sqrt[Tan[c + d*x]]*Sq
rt[a + b*Tan[c + d*x]]) + ((3*((a^3*(a - I*b)*(A + I*B)*ArcTan[(Sqrt[I*a -
 b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a^
3*(a + I*b)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a +
b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/(a*(a^2 + b^2)) - (2*b*(5*a^2*A*b +
8*A*b^3 - 3*a^3*B - 6*a*b^2*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d*Sqrt[a
 + b*Tan[c + d*x]]))/a)/(3*a))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.81 (sec) , antiderivative size = 1562552, normalized size of antiderivative = 4944.78

output too large to display

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18876 vs. $2(259) = 518$.

Time = 6.50 (sec) , antiderivative size = 18876, normalized size of antiderivative = 59.73

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{5}{2}}}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^(3/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^{3/2}}\,dx$$

input
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x
)
```

output
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),
x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\cot\left(dx+c\right)^{\frac{5}{2}}\left(A+B\tan\left(dx+c\right)\right)}{\left(a+\tan\left(dx+c\right)b\right)^{\frac{3}{2}}}dx$$

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

3.647 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 256

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}$$
$$-\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{3/2}d}$$
$$-\frac{2b(a^2A+2Ab^2-abB)}{a^2\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{2A\sqrt{\cot(c+dx)}}{ad\sqrt{a+b\tan(c+dx)}}$$

output
```
(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(
d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d-(A-I*B)*arctanh((I*a+b)^(1/2
)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/(I*a+b)^(3/2)/d-2*b*(A*a^2+2*A*b^2-B*a*b)/a^2/(a^2+b^2)/d/cot(d*x+c)^(1
/2)/(a+b*tan(d*x+c))^(1/2)-2*A*cot(d*x+c)^(1/2)/a/d/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cot(c+dx)}\left(\frac{\sqrt[4]{-1}a\left(\frac{(a+ib)(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}-\cdots\right)}{a^2+b^2}\cdots\right)}{\cdots}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
3/2),x]
```

output
```
(Sqrt[Cot[c + d*x]]*(((-1)^(1/4)*a*(((a + I*b)*(A - I*B)*ArcTan[((-1)^(1/4
)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[-a +
I*b] - ((a - I*b)*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c +
d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b])*Sqrt[Tan[c + d*x]])/(a^2
+ b^2) - (2*A)/Sqrt[a + b*Tan[c + d*x]] - (2*b*(a^2*A + 2*A*b^2 - a*b*B)*T
an[c + d*x])/(a*(a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]])))/(a*d)
```

Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4729, 3042, 4092, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4092}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int \frac{2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{2Ab\tan(c+dx)^2+aA\tan(c+dx)+2Ab-aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{2\int \frac{(Ab-aB)a^2+(aA+bB)\tan(c+dx)a^2}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{}{ad\sqrt{\tan(c+dx}}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\int \frac{(Ab-aB)a^2+(aA+bB)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2}{ad\sqrt{\tan(c+dx)}}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\int \frac{(Ab-aB)a^2+(aA+bB)\tan(c+dx)a^2}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{a}-\frac{2}{ad\sqrt{\tan(c+dx)}}\right.$$

$\big\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}a^2(b+ia)(A}{}}{}\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}a^2(b+ia)(A}{}}{}\right.$$

$\big\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{a^2(b+ia)(A+i}{}}{}\right.$$

$\big\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{a^2(b+ia)(A+i}{}}{}\right.$$

$\big\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{a^2(b+ia)(A+i}{}}{}\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{\frac{2b(a^2A-abB+2Ab^2)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{a^2(b+ia)(A+i}{}}{}\right.$$

input
```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A)/(a*d*Sqrt[Tan[c + d*x]]*Sqrt
[a + b*Tan[c + d*x]]) - (((a^2*(I*a + b)*(A + I*B)*ArcTan[(Sqrt[I*a - b]*S
qrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) - (a^2*(I*
a - b)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/(Sqrt[I*a + b]*d))/(a*(a^2 + b^2)) + (2*b*(a^2*A + 2*A*b^2 -
a*b*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.90 (sec) , antiderivative size = 1561945, normalized size of antiderivative = 6101.35

output too large to display

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18757 vs. 2(212) = 424.

Time = 6.52 (sec) , antiderivative size = 18757, normalized size of antiderivative = 73.27

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan(c+dx))\cot^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*cot(c + d*x)**(3/2)/(a + b*tan(c + d*x))**(3
/2), x)
```

Maxima [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{3}{2}}}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(3/
2), x)
```

Giac [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{3}{2}}}{(b\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(3/
2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^{3/2}}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x
)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),
x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{a+\tan(dx+c)\,b}\,\sqrt{\cot(dx+c)}\,\cot(dx+c)}{a+\tan(dx+c)\,b}dx$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x))*cot(c + d*x))/(tan(c + d*
x)*b + a),x)
```

3.648 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 215

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}$$
$$+\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{3/2}d}$$
$$+\frac{2b(Ab-aB)}{a(a^2+b^2)\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

output
```
(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(
d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d+(I*A+B)*arctanh((I*a+b)^(1/2
)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/(I*a+b)^(3/2)/d+2*b*(A*b-B*a)/a/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d
*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{\sqrt[4]{-1}(-ia+b)(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\ldots}{\sqrt{a+b\ldots}}\right.}{\sqrt{-a+ib}}\ldots\right.}{\ldots}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
3/2),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((-1)^(1/4)*((-I)*a + b)*(A - I*B)
*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]])/Sqrt[-a + I*b] + ((-1)^(1/4)*(a - I*b)*((-I)*A + B)*ArcTan[((-1)^(
1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a +
 I*b] + (2*b*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(a*Sqrt[a + b*Tan[c + d*x]]))
)/((a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 4729, 3042, 4092, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 4092$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int \frac{a(aA+bB)-a(Ab-aB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \ 27$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(aA+bB)-a(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{a(aA+bB)-a(Ab-aB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \ 4099$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}a(a-ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d}{a\,(}\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}a(a-ib)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}d}{a\,(}\right.$$

$$\Big\downarrow \ 4098$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a(a+ib)(A-iB)\int \frac{1}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{2d}}{}\right.$$

$$\Big\downarrow \ 104$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a(a-ib)(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+}{a\left(a^2+\right.}\right.$$

$$\Big\downarrow \text{ 216}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a(a+ib)(A-iB)\int\frac{1}{1-\frac{(ia+b)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}+}{a\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{ 219}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{a(a-ib)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}+\frac{a(a+ib)(}{}}{a\left(a^2+b^2\right)}\right.$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/2),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((a*(a - I*b)*(A + I*B)*ArcTan[(Sqr
t[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d
) + (a*(a + I*b)*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d))/(a*(a^2 + b^2)) + (2*b*(A*b - a*
B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.77 (sec) , antiderivative size = 1561037, normalized size of antiderivative = 7260.64

output too large to display

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18703 vs. $2(175) = 350$.

Time = 6.47 (sec) , antiderivative size = 18703, normalized size of antiderivative = 86.99

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(A+B\tan{(c+dx)})\sqrt{\cot{(c+dx)}}}{(a+b\tan{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/(a + b*tan(c + d*x))**(3/
2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(B\tan{(dx+c)}+A)\sqrt{\cot{(dx+c)}}}{(b\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)^(3/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)}{{\left(a+b\tan\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),x
)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\sqrt{\cot(dx+c)}}{a+\tan(dx+c)\,b}dx$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/(tan(c + d*x)*b + a),x)
```

3.649 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 210

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d} + \frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{3/2}d} - \frac{2(Ab-aB)}{(a^2+b^2)\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

output
```
-(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot
(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d+(A-I*B)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1
/2)/(I*a+b)^(3/2)/d-2*(A*b-B*a)/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+
c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.23

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-\frac{\sqrt[4]{-1}a(a+ib)(A-iB)\arctan\left(\frac{\sqrt[4]{-1}\cdots}{\sqrt{\cdots}}\right)}{\sqrt{-a+ib}}\cdots\right.}{\cdots}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3
/2)),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-(((-1)^(1/4)*a*(a + I*b)*(A - I*B
)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]]])/Sqrt[-a + I*b]) + ((-1)^(1/4)*a*(a - I*b)*(A + I*B)*ArcTan[((-1)^
(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a
+ I*b] + (2*b*(A*b - a*B)*Tan[c + d*x]^(3/2))/Sqrt[a + b*Tan[c + d*x]] + 2
*(-(A*b) + a*B)*Sqrt[Tan[c + d*x]]*Sqrt[a + b*Tan[c + d*x]]))/(a*(a^2 + b^
2)*d)
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 4729, 3042, 4091, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{4091}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int -\frac{b(Ab-aB)+b(aA+bB)\tan(c+dx)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{b(Ab-aB)+b(aA+bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{b(Ab-aB)+b(aA+bB)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}\right)$$

$$\Big\downarrow \text{4099}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}b(b+ia)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{\frac{1}{2}b(b+ia)(A+iB)\int \frac{1-i\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

$$\Big\downarrow \text{4098}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{b(b+ia)(A+iB)\int \frac{1}{(i\tan(c+dx)+1)\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{2d}$$

$$\Big\downarrow \text{104}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,\sqrt{a+b\tan(c+dx)}}+\frac{\frac{b(b+ia)(A+iB)\int\frac{1}{\frac{(ia-b)\tan(c+dx)}{a+b\tan(c+dx)}+1}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}}{d}-\right.}{b\,(a^2+}$$

$$\Big\downarrow 216$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,\sqrt{a+b\tan(c+dx)}}+\frac{\frac{b(b+ia)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{b(-b+ia)}{}}{b\,(a^2+b^2)}\right.$$

$$\Big\downarrow 219$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{d\,(a^2+b^2)\,\sqrt{a+b\tan(c+dx)}}+\frac{\frac{b(b+ia)(A+iB)\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d\sqrt{-b+ia}}-\frac{b(-b+ia)}{}}{b\,(a^2+b^2)}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2)),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((b*(I*a + b)*(A + I*B)*ArcTan[(Sqr
t[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d
) - ((I*a - b)*b*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt
[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d))/(b*(a^2 + b^2)) - (2*(A*b - a*B)
*Sqrt[Tan[c + d*x]])/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.10 (sec) , antiderivative size = 1561051, normalized size of antiderivative = 7433.58

output too large to display

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18672 vs. $2(170) = 340$.

Time = 6.49 (sec) , antiderivative size = 18672, normalized size of antiderivative = 88.91

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan{(c+dx)}}{(a+b\tan{(c+dx)})^{\frac{3}{2}}\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**(3/2)*sqrt(cot(c + d*
x))), x)
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{B\tan{(dx+c)}+A}{(b\tan{(dx+c)}+a)^{\frac{3}{2}}\sqrt{\cot{(dx+c)}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*x +
c))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[3,9,3]%%%}+%%%{4,[3,7,3]%%%}+%%%{6,[3,5,3]%%%}+%%%{
4,[3,3,3]
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot\left(c+d\,x\right)}\,{\left(a+b\tan\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^{3/2}}\,dx = \int \frac{\sqrt{a + \tan(dx + c)\,b}\,\sqrt{\cot(dx + c)}}{\cot(dx + c)\tan(dx + c)\,b + \cot(dx + c)\,a}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/(cot(c + d*x)*tan(c + d*
x)*b + cot(c + d*x)*a),x)
```

3.650 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 279

$$\begin{aligned}\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = &-\frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{3/2}d}\\&+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{b^{3/2}d}\\&-\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{3/2}d}\\&+\frac{2a(Ab-aB)}{b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\end{aligned}$$

output
```
-(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot
(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(3/2)/d+2*B*arctanh(b^(1/2)*tan(d*x
+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/b^(3/2
)/d-(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))
*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(3/2)/d+2*a*(A*b-B*a)/b/(a^2+b^
2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.29

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(2\sqrt{a}\sqrt{-a+ib}\sqrt{a+ib}(a^2+b^2)\,B\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{1+\frac{b\tan(c+dx)}{a}}+\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3
/2)),x]
```

output
```
-((Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(2*Sqrt[a]*Sqrt[-a + I*b]*Sqrt[a
+ I*b]*(a^2 + b^2)*B*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[1
+ (b*Tan[c + d*x])/a] + Sqrt[b]*((-1)^(1/4)*(a + I*b)^(3/2)*b*(I*A + B)*Ar
cTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x
]]]*Sqrt[a + b*Tan[c + d*x]] + Sqrt[-a + I*b]*(2*a*Sqrt[a + I*b]*(A*b - a*
B)*Sqrt[Tan[c + d*x]] + (-1)^(1/4)*b*(I*a + b)*(A + I*B)*ArcTan[((-1)^(1/4
)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]*Sqrt[a + b*T
an[c + d*x]]))))/((-a + I*b)^(3/2)*(a + I*b)^(3/2)*b^(3/2)*d*Sqrt[a + b*Ta
n[c + d*x]]))
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}(a+b\tan(c+dx))^{3/2}}dx$$

$\downarrow$ 4729

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}dx$$

$\downarrow$ 4088

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int -\frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{b(a^2+b^2)}+\frac{2a(Ab-aB)\sqrt{\tan(}}{bd(a^2+b^2)\sqrt{a+b\,\mathrm{ta}}}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{b(a^2+b^2)}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{-((a^2+b^2)B\tan(c+dx)^2)-b(Ab-aB)\tan(c+dx)+a(Ab-}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{b(a^2+b^2)}\right.$$

$\downarrow$ 4138

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{\int \frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(Ab-}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}}{bd(a^2+b^2)}\right.$$

$\downarrow$ 2035

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}-\frac{2\int \frac{-((a^2+b^2)B\tan^2(c+dx))-b(Ab-aB)\tan(c+dx)+a(A}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}}{bd(a^2+b^2)}\right.$$

$\downarrow$ 2257

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{2\int\left(\frac{b(aA+bB)-b(Ab-aB)\tan(c+dx)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}-\frac{(a^2+b^2)B}{\sqrt{a+b\tan(c+d}}\right.}{bd\left(a^2+b^2\right)}\right.$$

$\Big\downarrow$ 2009

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{bd\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}-\frac{2\left(-\frac{B(a^2+b^2)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}}+\frac{b(a-ib)(A}{}\right.}{}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2)),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*(((a - I*b)*b*(A + I*B)*ArcTan[
(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[I*a
- b]) - ((a^2 + b^2)*B*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan
[c + d*x]]])/Sqrt[b] + ((a + I*b)*b*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*Sqrt[
Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[I*a + b])))/(b*(a^2 + b^
2)*d) + (2*a*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(b*(a^2 + b^2)*d*Sqrt[a + b*T
an[c + d*x]]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.80 (sec) , antiderivative size = 1562182, normalized size of antiderivative = 5599.22

output too large to display

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18891 vs. $2(224) = 448$.

Time = 11.16 (sec) , antiderivative size = 37815, normalized size of antiderivative = 135.54

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan(c+dx)}{(a+b\tan(c+dx))^{\frac{3}{2}}\cot^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))/((a + b*tan(c + d*x))**(3/2)*cot(c + d*x)**(
3/2)), x)
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{B\tan(dx+c)+A}{(b\tan(dx+c)+a)^{\frac{3}{2}}\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(3/2)*cot(d*x + c)^(3
/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{3/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^{2}\tan\left(dx+c\right)b+\cot\left(dx+c\right)^{2}a}dx$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/(cot(c + d*x)**2*tan(c +
 d*x)*b + cot(c + d*x)**2*a),x)
```

3.651 $\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 399

$$\begin{aligned}\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx &= \frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d} \\
&+ \frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{5/2}d} \\
&+ \frac{2b(7a^2Ab+8Ab^3-3a^3B-4ab^2B)}{3a^3(a^2+b^2)\,d\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}} \\
&+ \frac{2(2Ab-aB)\sqrt{\cot(c+dx)}}{a^2d(a+b\tan(c+dx))^{3/2}} - \frac{2A\cot^{\frac{3}{2}}(c+dx)}{3ad(a+b\tan(c+dx))^{3/2}} \\
&+ \frac{2b(8a^4Ab+30a^2Ab^3+16Ab^5-3a^5B-17a^3b^2B-8ab^4B)}{3a^4(a^2+b^2)^2\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\end{aligned}$$

output
```
(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(
d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d+(A-I*B)*arctanh((I*a+b)^(1/2
)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/(I*a+b)^(5/2)/d+2/3*b*(7*A*a^2*b+8*A*b^3-3*B*a^3-4*B*a*b^2)/a^3/(a^2+b^
2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2)+2*(2*A*b-B*a)*cot(d*x+c)^(1/2
)/a^2/d/(a+b*tan(d*x+c))^(3/2)-2/3*A*cot(d*x+c)^(3/2)/a/d/(a+b*tan(d*x+c))
^(3/2)+2/3*b*(8*A*a^4*b+30*A*a^2*b^3+16*A*b^5-3*B*a^5-17*B*a^3*b^2-8*B*a*b
^4)/a^4/(a^2+b^2)^2/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.96

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cot(c+dx)}\left(\frac{6(6Ab-3aB)}{a(a+b\tan(c+dx))^{3/2}} - \frac{6A\cot(c+dx)}{(a+b\tan(c+dx))^{3/2}} + \frac{6b(7a^2Ab+8A}{a^2(a^2+}\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
5/2),x]
```

output
```
(Sqrt[Cot[c + d*x]]*((6*(6*A*b - 3*a*B))/(a*(a + b*Tan[c + d*x])^(3/2)) -
(6*A*Cot[c + d*x])/(a + b*Tan[c + d*x])^(3/2) + (6*b*(7*a^2*A*b + 8*A*b^3
- 3*a^3*B - 4*a*b^2*B)*Tan[c + d*x])/(a^2*(a^2 + b^2)*(a + b*Tan[c + d*x])
^(3/2)) + (Sqrt[Tan[c + d*x]]*(9*(-1)^(3/4)*a^4*(((a + I*b)^2*(A - I*B)*Ar
cTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x
]]])/Sqrt[-a + I*b] + ((a - I*b)^2*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I
*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b]) + (6*b*(
8*a^4*A*b + 30*a^2*A*b^3 + 16*A*b^5 - 3*a^5*B - 17*a^3*b^2*B - 8*a*b^4*B)*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]))/(a^3*(a^2 + b^2)^2)))/(9*a*
d)
```

Rubi [A] (verified)

Time = 4.75 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.12, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4729, 3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{5/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{5/2}(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4092}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int \frac{3(2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB)}{2\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}dx}{3a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{2Ab\tan^2(c+dx)+aA\tan(c+dx)+2Ab-aB}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{2Ab\tan(c+dx)^2+aA\tan(c+dx)+2Ab-aB}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{-\frac{2\int -\frac{Aa^2+B\tan(c+dx)a^2+4bBa-8Ab^2-4b(2Ab-aB)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{a}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\int \frac{Aa^2+B\tan(c+dx)a^2+4bBa-8Ab^2-4b(2Ab-aB)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{a}-\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\int \frac{Aa^2+B\tan(c+dx)a^2+4bBa-8Ab^2-4b(2Ab-aB)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{a}-\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{2\int \frac{3Aa^4+9bBa^3-3(Ab-aB)\tan(c+dx)a^3-14Ab^2a^2+8b^3Ba-16Ab^4-2b\left(-3Ba^3+7Aba^2-4b^2Ba+8Ab^3\right)\tan^2(c-}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{3a\left(a^2+b^2\right)}}{a}}{a}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{\int \frac{3Aa^4+9bBa^3-3(Ab-aB)\tan(c+dx)a^3-14Ab^2a^2+8b^3Ba-16Ab^4-2b\left(-3Ba^3+7Aba^2-4b^2Ba+8Ab^3\right)\tan^2(c+\cdots}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{3a\left(a^2+b^2\right)}\cdots}{a}\cdots}{a}\cdots\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{\int \frac{3Aa^4+9bBa^3-3(Ab-aB)\tan(c+dx)a^3-14Ab^2a^2+8b^3Ba-16Ab^4-2b\left(-3Ba^3+7Aba^2-4b^2Ba+8Ab^3\right)\tan(c+d\cdots}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{3a\left(a^2+b^2\right)}\cdots}{a}\cdots}{a}\cdots\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{\frac{2\int \frac{3\left(a^4\left(Aa^2+2bBa-Ab^2\right)-a^4\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8\cdots\right.}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(\cdots}}}{3a\left(a^2+b^2\right)}\cdots}{a}\cdots}{\cdots}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{\frac{3\int \frac{a^4\left(Aa^2+2bBa-Ab^2\right)-a^4\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4\cdots\right.}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+\cdots}}}{3a\left(a^2+b^2\right)}\cdots}{a}\cdots}{\cdots}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{3\int \frac{a^4\left(Aa^2+2bBa-Ab^2\right)-a^4\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}-\frac{2b\left(-3a^5B+8a^4Ab-17a^3b^2B+30a^2Ab^3-8ab^4\right.}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+}}}{3a\left(a^2+b^2\right)}}{a}\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-\right.}{}}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-\right.}{}}{}}{}\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-\right.}{}}{}}{}\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(\cdots}{}}{}}{}\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(\cdots}{}}{}}{}\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{3ad\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(2Ab-aB)}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(\cdots}{}}{}}{}\right.$$

input

```
Int[(Cot[c + d*x]^(5/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A)/(3*a*d*Tan[c + d*x]^(3/2)*(a
 + b*Tan[c + d*x])^(3/2)) - ((-2*(2*A*b - a*B))/(a*d*Sqrt[Tan[c + d*x]]*(a
 + b*Tan[c + d*x])^(3/2)) + ((-2*b*(7*a^2*A*b + 8*A*b^3 - 3*a^3*B - 4*a*b^
2*B)*Sqrt[Tan[c + d*x]])/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) +
((3*((a^4*(a - I*b)^2*(A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/
Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a - b]*d) + (a^4*(a + I*b)^2*(A - I*B)*
ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqr
t[I*a + b]*d)))/(a*(a^2 + b^2)) - (2*b*(8*a^4*A*b + 30*a^2*A*b^3 + 16*A*b^
5 - 3*a^5*B - 17*a^3*b^2*B - 8*a*b^4*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)
*d*Sqrt[a + b*Tan[c + d*x]]))/(3*a*(a^2 + b^2)))/a)/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.51 (sec) , antiderivative size = 2982444, normalized size of antiderivative = 7474.80

output too large to display

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27987 vs. $2(340) = 680$.

Time = 13.20 (sec) , antiderivative size = 27987, normalized size of antiderivative = 70.14

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{5}{2}}}{(b\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(5/2)/(b*tan(d*x + c) + a)^(5/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^{5/2}\,(A+B\tan(c+d\,x))}{(a+b\tan\,(c+d\,x))^{5/2}}\,dx$$

input
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x
)
```

output
```
int((cot(c + d*x)^(5/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\cot\,(dx+c)^{\frac{5}{2}}\,(A+B\tan\,(dx+c))}{(a+\tan\,(dx+c)\,b)^{\frac{5}{2}}}dx$$

input
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int(cot(d*x+c)^(5/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

3.652 $\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 341

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d} - \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{5/2}d} - \frac{2b(3a^2A+4Ab^2-abB)}{3a^2(a^2+b^2)\,d\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}} - \frac{2A\sqrt{\cot(c+dx)}}{ad(a+b\tan(c+dx))^{3/2}} - \frac{2b(3a^4A+17a^2Ab^2+8Ab^4-8a^3bB-2ab^3B)}{3a^3(a^2+b^2)^2\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

output
```
(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(
d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d-(I*A+B)*arctanh((I*a+b)^(1/2
)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/(I*a+b)^(5/2)/d-2/3*b*(3*A*a^2+4*A*b^2-B*a*b)/a^2/(a^2+b^2)/d/cot(d*x+c
)^(1/2)/(a+b*tan(d*x+c))^(3/2)-2*A*cot(d*x+c)^(1/2)/a/d/(a+b*tan(d*x+c))^(
3/2)-2/3*b*(3*A*a^4+17*A*a^2*b^2+8*A*b^4-8*B*a^3*b-2*B*a*b^3)/a^3/(a^2+b^2
)^2/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.98

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cot(c+dx)}\left(-\frac{6A}{(a+b\tan(c+dx))^{3/2}}-\frac{2b(3a^2A+4Ab^2-abB)\tan(c+dx)}{a(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\right.}{}$$

input
```
Integrate[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
5/2),x]
```

output
```
(Sqrt[Cot[c + d*x]]*((-6*A)/(a + b*Tan[c + d*x])^(3/2) - (2*b*(3*a^2*A + 4
*A*b^2 - a*b*B)*Tan[c + d*x])/(a*(a^2 + b^2)*(a + b*Tan[c + d*x])^(3/2)) +
 (Sqrt[Tan[c + d*x]]*(3*(-1)^(1/4)*a^3*(((a + I*b)^2*(A - I*B)*ArcTan[((-1
)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt
[-a + I*b] - ((a - I*b)^2*(A + I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[
Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b]) - (2*b*(3*a^4*A +
 17*a^2*A*b^2 + 8*A*b^4 - 8*a^3*b*B - 2*a*b^3*B)*Sqrt[Tan[c + d*x]])/Sqrt[
a + b*Tan[c + d*x]]))/(a^2*(a^2 + b^2)^2)))/(3*a*d)
```

Rubi [A] (verified)

Time = 3.72 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.12, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 4729, 3042, 4092, 27, 3042, 4132, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\tan(c+dx)^{3/2}(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4092}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int \frac{4Ab\tan^2(c+dx)+aA\tan(c+dx)+4Ab-aB}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\cdots\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{4Ab\tan^2(c+dx)+aA\tan(c+dx)+4Ab-aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\cdots\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\int \frac{4Ab\tan(c+dx)^2+aA\tan(c+dx)+4Ab-aB}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx}{a}-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}\cdots\right.$$

$$\Big\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{2\int \frac{-3Ba^3+9Aba^2+3(aA+bB)\tan(c+dx)a^2-2b^2Ba+8Ab^3+2b\left(3Aa^2-bBa+4Ab^2\right)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a(a^2+b^2)}+\frac{2b(3a^2A\cdots}{3ad(a^2\cdots}}{a}\cdots\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\int\frac{-3Ba^3+9Aba^2+3(aA+bB)\tan(c+dx)a^2-2b^2Ba+8Ab^3+2b\left(3Aa^2-bBa+4Ab^2\right)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a(a^2+b^2)}+\frac{2b(3a^2A-}{3ad(a^2+}}{a}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\int\frac{-3Ba^3+9Aba^2+3(aA+bB)\tan(c+dx)a^2-2b^2Ba+8Ab^3+2b\left(3Aa^2-bBa+4Ab^2\right)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a(a^2+b^2)}+\frac{2b(3a^2A-}{3ad(a^2+}}{a}\right.$$

$\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{2\int\frac{3\left(\left(-Ba^2+2Aba+b^2B\right)a^3+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{3a(a^2+b^2)}}{a}\right.$$

$\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{3\int\frac{\left(-Ba^2+2Aba+b^2B\right)a^3+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{t}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{3a(a^2+b^2)}}{a}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\frac{\frac{3\int\frac{\left(-Ba^2+2Aba+b^2B\right)a^3+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)a^3}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a\left(a^2+b^2\right)}+\frac{2b\left(3a^4A-8a^3bB+17a^2Ab^2-2ab^3B+8Ab^4\right)\sqrt{t}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}}{3a(a^2+b^2)}}{a}\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{2b\left(3a^4A-\right.}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{2b\left(3a^4A-\right.}{}}{}\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{2b\left(3a^4A-\right.}{}}{}\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{2b\left(3a^4A-\right.}{}}{}\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{2b\left(3a^4A-\right.}{}}{}\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2A}{ad\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2b(3a^2A-abB+4Ab^2)\sqrt{\tan(c+dx)}}{3ad(a^2+b^2)(a+b\tan(c+dx))^{3/2}}+\frac{2b\left(3a^4A-\right.}{}}{}\right.$$

input

```
Int[(Cot[c + d*x]^(3/2)*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x
]
```

output

```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*A)/(a*d*Sqrt[Tan[c + d*x]]*(a +
 b*Tan[c + d*x])^(3/2)) - ((2*b*(3*a^2*A + 4*A*b^2 - a*b*B)*Sqrt[Tan[c + d
*x]])/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + ((3*((a^3*(a - I*b)
^2*(I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
d*x]]])/(Sqrt[I*a - b]*d) - (a^3*(a + I*b)^2*(I*A + B)*ArcTanh[(Sqrt[I*a +
 b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/(a*
(a^2 + b^2)) + (2*b*(3*a^4*A + 17*a^2*A*b^2 + 8*A*b^4 - 8*a^3*b*B - 2*a*b^
3*B)*Sqrt[Tan[c + d*x]])/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(3*a*
(a^2 + b^2)))/a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.62 (sec) , antiderivative size = 2976849, normalized size of antiderivative = 8729.76

output too large to display

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27817 vs. 2(289) = 578.

Time = 13.42 (sec) , antiderivative size = 27817, normalized size of antiderivative = 81.57

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx=\int \frac{(B\tan(dx+c)+A)\cot(dx+c)^{\frac{3}{2}}}{(b\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*cot(d*x + c)^(3/2)/(b*tan(d*x + c) + a)^(5/
2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{%%%{
1,[6,14,5]%%%}+%%%{6,[6,12,5]%%%}+%%%{15,[6,10,5]%%%}+%%%{20,[6,8,5]%%%}+%
%%{15,[6,6,5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))}{(a+b\tan\,(c+d\,x))^{5/2}}\,dx$$

input
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x
)
```

output
```
int((cot(c + d*x)^(3/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\cot\,(dx+c)^{\frac{3}{2}}\,(A+B\tan\,(dx+c))}{(a+\tan\,(dx+c)\,b)^{\frac{5}{2}}}dx$$

input
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int(cot(d*x+c)^(3/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

3.653 $\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 287

$$\begin{aligned}\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx =& -\frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}\\ &-\frac{(A-iB)\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{5/2}d}\\ &+\frac{2b(Ab-aB)}{3a(a^2+b^2)\,d\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\\ &+\frac{2b(8a^2Ab+2Ab^3-5a^3B+ab^2B)}{3a^2(a^2+b^2)^2\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}\end{aligned}$$

output
```
-(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot
(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d-(A-I*B)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1
/2)/(I*a+b)^(5/2)/d+2/3*b*(A*b-B*a)/a/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*ta
n(d*x+c))^(3/2)+2/3*b*(8*A*a^2*b+2*A*b^3-5*B*a^3+B*a*b^2)/a^2/(a^2+b^2)^2/
d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-3\sqrt[4]{-1}\left(\frac{(a+ib)^2(iA+B)\arctan\left(\frac{\sqrt[4]{-}}{}\right.}{\sqrt{-a+i}}\right.\right.}{}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(
5/2),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-3*(-1)^(1/4)*(((a + I*b)^2*(I*A +
 B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/Sqrt[-a + I*b] + (I*(a - I*b)^2*(A + I*B)*ArcTan[((-1)^(1/4)*Sq
rt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b])
+ (2*b*(a^2 + b^2)*(A*b - a*B)*Sqrt[Tan[c + d*x]])/(a*(a + b*Tan[c + d*x])
^(3/2)) + (2*b*(8*a^2*A*b + 2*A*b^3 - 5*a^3*B + a*b^2*B)*Sqrt[Tan[c + d*x]
])/(a^2*Sqrt[a + b*Tan[c + d*x]])))/(3*(a^2 + b^2)^2*d)
```

Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4729, 3042, 4092, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{A+B\tan(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4092}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int \frac{3Aa^2+bBa-3(Ab-aB)\tan(c+dx)a+2Ab^2+2b(Ab-aB)\tan^2(c+dx)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)\sqrt{}}{3ad\left(a^2+b^2\right)(a+}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{3Aa^2+bBa-3(Ab-aB)\tan(c+dx)a+2Ab^2+2b(Ab-aB)\tan^2(c+dx)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)\sqrt{t}}{3ad\left(a^2+b^2\right)(a+b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{3Aa^2+bBa-3(Ab-aB)\tan(c+dx)a+2Ab^2+2b(Ab-aB)\tan(c+dx)^2}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3a\left(a^2+b^2\right)}+\frac{2b(Ab-aB)\sqrt{t}}{3ad\left(a^2+b^2\right)(a+b}\right.$$

$$\Big\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2\int \frac{3\left(a^2\left(Aa^2+2bBa-Ab^2\right)-a^2\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{ta}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3a\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{3\int \frac{a^2\left(Aa^2+2bBa-Ab^2\right)-a^2\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3a\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{3\int\frac{a^2\left(Aa^2+2bBa-Ab^2\right)-a^2\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}+\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3a\left(a^2+b^2\right)}\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}\right.}{}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}\right.}{}}{}\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\right.}{}}{}\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\right.}{}}{}\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\right.}{}}{}\right.$$

$\Big\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2b(Ab-aB)\sqrt{\tan(c+dx)}}{3ad\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{\frac{2b\left(-5a^3B+8a^2Ab+ab^2B+2Ab^3\right)\sqrt{\tan(c+dx)}}{ad\left(a^2+b^2\right)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\right.}{}}{}\right.$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(A + B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(5/2),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*b*(A*b - a*B)*Sqrt[Tan[c + d*x]]
)/(3*a*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + ((3*((a^2*(a - I*b)^2*(
A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]
]])/(Sqrt[I*a - b]*d) + (a^2*(a + I*b)^2*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/(a*(a^2
 + b^2)) + (2*b*(8*a^2*A*b + 2*A*b^3 - 5*a^3*B + a*b^2*B)*Sqrt[Tan[c + d*x
]])/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(3*a*(a^2 + b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4092
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[b*(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)
/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^
2 + b^2))   Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[b*
B*(b*c*(m + 1) + a*d*(n + 1)) + A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2
)) - (A*b - a*B)*(b*c - a*d)*(m + 1)*Tan[e + f*x] - b*d*(A*b - a*B)*(m + n
+ 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1]
&& (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m]
 || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.38 (sec) , antiderivative size = 2975320, normalized size of antiderivative = 10366.97

output too large to display

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27790 vs. 2(240) = 480.

Time = 13.43 (sec) , antiderivative size = 27790, normalized size of antiderivative = 96.83

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(A+B\tan(c+dx))\sqrt{\cot(c+dx)}}{(a+b\tan(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*sqrt(cot(c + d*x))/(a + b*tan(c + d*x))**(5/
2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{(B\tan(dx+c)+A)\sqrt{\cot(dx+c)}}{(b\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)^(5/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cot(c+d\,x)}\,(A+B\tan(c+d\,x))}{(a+b\tan(c+d\,x))^{5/2}}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),x
)
```

output
```
int((cot(c + d*x)^(1/2)*(A + B*tan(c + d*x)))/(a + b*tan(c + d*x))^(5/2),
x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{a+\tan(dx+c)\,b}\,\sqrt{\cot(dx+c)}}{\tan(dx+c)^2\,b^2+2\tan(dx+c)\,ab+a^2}dx$$

input
```
int(cot(d*x+c)^(1/2)*(A+B*tan(d*x+c))/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/(tan(c + d*x)**2*b**2 +
2*tan(c + d*x)*a*b + a**2),x)
```

3.654 $\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 284

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx = -\frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d} + \frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{5/2}d} - \frac{2(Ab-aB)}{3(a^2+b^2)\,d\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2a^3B+4ab^2B)}{3a(a^2+b^2)^2\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

output
```
-(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot
(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d+(I*A+B)*arctanh((I*a+b)^(1/
2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1
/2)/(I*a+b)^(5/2)/d-2/3*(A*b-B*a)/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan(d*
x+c))^(3/2)-2/3*(5*A*a^2*b-A*b^3-2*B*a^3+4*B*a*b^2)/a/(a^2+b^2)^2/d/cot(d*
x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 3.13 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.20

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{2b(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{(a+b\tan(c+dx))^{3/2}}+\frac{6b(2aAb-a^2\ldots}{(a^2+b^2)\ldots}\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(5
/2)),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*b*(A*b - a*B)*Tan[c + d*x]^(3/2
))/(a + b*Tan[c + d*x])^(3/2) + (6*b*(2*a*A*b - a^2*B + b^2*B)*Tan[c + d*x
]^(3/2))/((a^2 + b^2)*Sqrt[a + b*Tan[c + d*x]]) + (3*(-(((-1)^(1/4)*a*(a +
 I*b)^2*(A - I*B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sq
rt[a + b*Tan[c + d*x]]])/Sqrt[-a + I*b]) + ((-1)^(1/4)*a*(a - I*b)^2*(A +
I*B)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
 + d*x]]])/Sqrt[a + I*b] + 2*(-2*a*A*b + a^2*B - b^2*B)*Sqrt[Tan[c + d*x]]
*Sqrt[a + b*Tan[c + d*x]]))/(a^2 + b^2)))/(3*a*(a^2 + b^2)*d)
```

Rubi [A] (verified)

Time = 2.90 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4729, 3042, 4091, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}}dx$$

$\Big\downarrow$ 4729

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$\Big\downarrow$ 4091

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2\int -\frac{-2b(Ab-aB)\tan^2(c+dx)+3b(aA+bB)\tan(c+dx)+b(Ab-aB)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\mathrm{ta}}}{3d\left(a^2+b^2\right)\left(a+b\,\mathrm{t}\right.}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{-2b(Ab-aB)\tan^2(c+dx)+3b(aA+bB)\tan(c+dx)+b(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c}}{3d\left(a^2+b^2\right)\left(a+b\tan(c\right.}\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\int \frac{-2b(Ab-aB)\tan(c+dx)^2+3b(aA+bB)\tan(c+dx)+b(Ab-aB)}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}-\frac{2(Ab-aB)\sqrt{\tan(c}}{3d\left(a^2+b^2\right)\left(a+b\tan(c\right.}\right.$$

$\Big\downarrow$ 4132

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{2\int \frac{3\left(ab\left(-Ba^2+2Aba+b^2B\right)+ab\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)\right)}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2b\left(-2a^3B+5a^2Ab+4ab^2B-Ab^3\right)\sqrt{\mathrm{tan}}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3b\left(a^2+b^2\right)}\right.$$

$\Big\downarrow$ 27

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{3\int \frac{ab\left(-Ba^2+2Aba+b^2B\right)+ab\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c-}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3b\left(a^2+b^2\right)}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\frac{3\int \frac{ab\left(-Ba^2+2Aba+b^2B\right)+ab\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx}{a(a^2+b^2)}-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c-}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}}{3b\left(a^2+b^2\right)}\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3}{-}}{}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3}{-}}{}\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b(-2a^3B+5a^2Ab+4ab^2B-Ab^3)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3}{-}}{}\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-2a^3B+5a^2Ab+4ab^2B-Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\ldots}{\ldots}\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-2a^3B+5a^2Ab+4ab^2B-Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\ldots}{\ldots}\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{2(Ab-aB)\sqrt{\tan(c+dx)}}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}+\frac{-\frac{2b\left(-2a^3B+5a^2Ab+4ab^2B-Ab^3\right)\sqrt{\tan(c+dx)}}{ad(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\ldots}{\ldots}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(5/2)),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-2*(A*b - a*B)*Sqrt[Tan[c + d*x]])
/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) + ((3*((a*(a - I*b)^2*b*(I*A
 - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])
/(Sqrt[I*a - b]*d) - (a*(a + I*b)^2*b*(I*A + B)*ArcTanh[(Sqrt[I*a + b]*Sqr
t[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/(a*(a^2 +
b^2)) - (2*b*(5*a^2*A*b - A*b^3 - 2*a^3*B + 4*a*b^2*B)*Sqrt[Tan[c + d*x]])
/(a*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(3*b*(a^2 + b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4091
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^n/(f*(m +
 1)*(a^2 + b^2))), x] + Simp[1/(b*(m + 1)*(a^2 + b^2))   Int[(a + b*Tan[e +
 f*x])^(m + 1)*(c + d*Tan[e + f*x])^(n - 1)*Simp[b*B*(b*c*(m + 1) + a*d*n)
+ A*b*(a*c*(m + 1) - b*d*n) - b*(A*(b*c - a*d) - B*(a*c + b*d))*(m + 1)*Tan
[e + f*x] - b*d*(A*b - a*B)*(m + n + 1)*Tan[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegerQ[m] || Integers
Q[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 1.64 (sec) , antiderivative size = 2978186, normalized size of antiderivative = 10486.57

output too large to display

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27743 vs. $2(232) = 464$.

Time = 13.36 (sec) , antiderivative size = 27743, normalized size of antiderivative = 97.69

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^{5/2}}\,dx = \int \frac{B\tan(dx + c) + A}{(b\tan(dx + c) + a)^{\frac{5}{2}}\sqrt{\cot(dx + c)}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(5/2)*sqrt(cot(d*x +
c))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[6,14,5]%%%}+%%%{6,[6,12,5]%%%}+%%%{15,[6,10,5]%%%}+
%%%{20,[6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\sqrt{\cot\left(c+d\,x\right)}\,\left(a+b\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)\tan\left(dx+c\right)^{2}b^{2}+2\cot\left(dx+c\right)\tan\left(dx+c\right)a$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/(cot(c + d*x)*tan(c + d*
x)**2*b**2 + 2*cot(c + d*x)*tan(c + d*x)*a*b + cot(c + d*x)*a**2),x)
```

3.655 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 284

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(A+iB)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}$$
$$+\frac{(A-iB)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{5/2}d}$$
$$+\frac{2a(Ab-aB)}{3b\left(a^2+b^2\right)d\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}$$
$$+\frac{2(2a^2Ab-4Ab^3+a^3B+7ab^2B)}{3b\left(a^2+b^2\right)^2d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

output
```
(A+I*B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(
d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d+(A-I*B)*arctanh((I*a+b)^(1/2
)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/
2)/(I*a+b)^(5/2)/d+2/3*a*(A*b-B*a)/b/(a^2+b^2)/d/cot(d*x+c)^(1/2)/(a+b*tan
(d*x+c))^(3/2)+2/3*(2*A*a^2*b-4*A*b^3+B*a^3+7*B*a*b^2)/b/(a^2+b^2)^2/d/cot
(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.15

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(-\frac{3B\sqrt{\tan(c+dx)}}{(a+b\tan(c+dx))^{3/2}}+\frac{(2aAb+a^2B+3b}{(a^2+b^2)(a+b}\right.}{}$$

input
```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5
/2)),x]
```

output
```
(Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-3*B*Sqrt[Tan[c + d*x]])/(a + b*T
an[c + d*x])^(3/2) + ((2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Tan[c + d*x]])/((a^
2 + b^2)*(a + b*Tan[c + d*x])^(3/2)) + (3*(-1)^(1/4)*b*(((a + I*b)^2*(I*A
+ B)*ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[
c + d*x]]])/Sqrt[-a + I*b] + (I*(a - I*b)^2*(A + I*B)*ArcTan[((-1)^(1/4)*S
qrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[a + I*b])
 + (2*(2*a^2*A*b - 4*A*b^3 + a^3*B + 7*a*b^2*B)*Sqrt[Tan[c + d*x]])/Sqrt[a
 + b*Tan[c + d*x]])/(a^2 + b^2)^2))/(3*b*d)
```

Rubi [A] (verified)

Time = 2.90 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4132, 27, 3042, 4099, 3042, 4098, 104, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$$

$$\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{3/2}(a+b\tan(c+dx))^{5/2}}dx$$

$$\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan(c+dx)^{3/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\downarrow \text{4088}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int -\frac{-((Ba^2+2Aba+3b^2B)\tan^2(c+dx))-3b(Ab-aB)\tan(c+dx)+a(Ab-aB)}{2\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}dx}{3b(a^2+b^2)}+\frac{2a(Ab-}{3bd(a^2+b^2)}\right.$$

$$\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{\int \frac{-((Ba^2+2Aba+3b^2B)\tan^2(c+dx))-3b(Ab-aB)\tan}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{3b(a^2+b^2)}\right.$$

$$\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{\int \frac{-((Ba^2+2Aba+3b^2B)\tan(c+dx)^2)-3b(Ab-aB)\tan}{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^{3/2}}}{3b(a^2+b^2)}\right.$$

$$\downarrow \text{4132}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd(a^2+b^2)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2\int \frac{3\left(ab\left(Aa^2+2bBa-Ab^2\right)-ab\left(-Ba^2+2Aba+b^2B\right)\tan(c+}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a(a^2+b^2)}}{3b(a^2}\right.$$

$$\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{3\int\frac{ab\left(Aa^2+2bBa-Ab^2\right)-ab\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\ldots}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a(a^2+b^2)}\ldots}{3b\left(a^2+\ldots\right.}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{3\int\frac{ab\left(Aa^2+2bBa-Ab^2\right)-ab\left(-Ba^2+2Aba+b^2B\right)\tan(c+dx)\ldots}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{a(a^2+b^2)}\ldots}{3b\left(a^2+\ldots\right.}\right.$$

$\downarrow$ 4099

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(a^3B+2a^2Ab+7ab^2B-4Ab^3)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}\ldots\right.}{\ldots}}{\ldots}\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(a^3B+2a^2Ab+7ab^2B-4Ab^3)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\frac{1}{2}\ldots\right.}{\ldots}}{\ldots}\right.$$

$\downarrow$ 4098

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2(a^3B+2a^2Ab+7ab^2B-4Ab^3)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\begin{array}{c}a\ldots\\-\ldots\end{array}\right.}{\ldots}}{\ldots}\right.$$

$\downarrow$ 104

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2\left(a^3B+2a^2Ab+7ab^2B-4Ab^3\right)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 216

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2\left(a^3B+2a^2Ab+7ab^2B-4Ab^3\right)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 219

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\sqrt{\tan(c+dx)}}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2\left(a^3B+2a^2Ab+7ab^2B-4Ab^3\right)\sqrt{\tan(c+dx)}}{d(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{3\left(\cdots\right.}{\cdots}}{\cdots}\right.$$

input

```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(5/2)),x
]
```

output

```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*a*(A*b - a*B)*Sqrt[Tan[c + d*x]]
)/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - ((3*((a*(a - I*b)^2*b*(
A + I*B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]
]])/(Sqrt[I*a - b]*d) + (a*(a + I*b)^2*b*(A - I*B)*ArcTanh[(Sqrt[I*a + b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(Sqrt[I*a + b]*d)))/(a*(a^2
 + b^2)) - (2*(2*a^2*A*b - 4*A*b^3 + a^3*B + 7*a*b^2*B)*Sqrt[Tan[c + d*x]]
)/((a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(3*b*(a^2 + b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4098
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*
x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[A^2 + B^2, 0]
```

rule 4099
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[A^2 + B^2,
0]
```

rule 4132
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 + b^2))   Int[(a + b*Tan[e +
f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*
(m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d
)*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Ta
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&
!(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.67 (sec) , antiderivative size = 2978061, normalized size of antiderivative = 10486.13

output too large to display

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27726 vs. $2(236) = 472$.

Time = 13.27 (sec) , antiderivative size = 27726, normalized size of antiderivative = 97.63

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{5/2}}\, dx = \int \frac{B\tan(dx + c) + A}{(b\tan(dx + c) + a)^{\frac{5}{2}}\cot(dx + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(3
/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx=\int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{3/2}\,{\left(a+b\tan\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx=\int \frac{\sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^{2}\tan\left(dx+c\right)^{2}b^{2}+2\cot\left(dx+c\right)^{2}\tan\left(dx+c\right)a}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/(cot(c + d*x)**2*tan(c +
 d*x)**2*b**2 + 2*cot(c + d*x)**2*tan(c + d*x)*a*b + cot(c + d*x)**2*a**2)
,x)
```

3.656 $\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 342

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \frac{(iA-B)\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia-b)^{5/2}d}$$
$$+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{b^{5/2}d}$$
$$-\frac{(iA+B)\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{(ia+b)^{5/2}d}$$
$$+\frac{2a(Ab-aB)}{3b(a^2+b^2)\,d\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}$$
$$+\frac{2a(2Ab^3-a(a^2+3b^2)\,B)}{b^2(a^2+b^2)^2\,d\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}$$

output
```
(I*A-B)*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(
d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(5/2)/d+2*B*arctanh(b^(1/2)*tan(d*x+
c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/b^(5/2)
/d-(I*A+B)*arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*
cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(5/2)/d+2/3*a*(A*b-B*a)/b/(a^2+b
^2)/d/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2)+2*a*(2*A*b^3-a*(a^2+3*b^2)*B
)/b^2/(a^2+b^2)^2/d/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 6.27 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.80

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left(\frac{(A-iB)\left(\frac{3\sqrt[4]{-1}\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-\cdots}}{\sqrt{a+\cdots}}\right)}{(-a+ib)^{5/\cdots}}\cdots\right.}{\cdots}\right.$$

$$-\frac{(A+iB)\left(\frac{3\sqrt[4]{-1}\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{(a+ib)^{5/2}}-\frac{\tan^{\frac{3}{2}}(c+dx)}{(a+ib)(a+b\tan(c+dx))^{3/2}}-\frac{3i\sqrt{\tan(c+dx)}}{(a+ib)^2\sqrt{a+b\tan(c+dx)}}\right)}{3d}$$

$$+\frac{(iA-B)\sqrt{a+b\tan(c+dx)}\left(\frac{b^2\tan^2(c+dx)}{(a+b\tan(c+dx))^2}+\frac{3b\tan(c+dx)}{a+b\tan(c+dx)}-\frac{3\sqrt{b}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{\tan(c+dx)}}{\sqrt{a}\sqrt{1+\frac{b\tan(c+dx)}{a}}}\right)}{3b^3d\sqrt{\tan(c+dx)}}$$

$$\left.-\frac{(iA+B)\sqrt{a+b\tan(c+dx)}\left(\frac{b^2\tan^2(c+dx)}{(a+b\tan(c+dx))^2}+\frac{3b\tan(c+dx)}{a+b\tan(c+dx)}-\frac{3\sqrt{b}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a}}\right)\sqrt{\tan(c+dx)}}{\sqrt{a}\sqrt{1+\frac{b\tan(c+dx)}{a}}}\right)}{3b^3d\sqrt{\tan(c+dx)}}\right)$$

input

```
Integrate[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5
/2)),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((A - I*B)*((3*(-1)^(1/4)*ArcTan[((
-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(-
a + I*b)^(5/2) + Tan[c + d*x]^(3/2)/((a - I*b)*(a + b*Tan[c + d*x])^(3/2))
 - ((3*I)*Sqrt[Tan[c + d*x]])/((a - I*b)^2*Sqrt[a + b*Tan[c + d*x]])))/(3*
d) - ((A + I*B)*((3*(-1)^(1/4)*ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c
 + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(a + I*b)^(5/2) - Tan[c + d*x]^(3/2)/
((a + I*b)*(a + b*Tan[c + d*x])^(3/2)) - ((3*I)*Sqrt[Tan[c + d*x]])/((a +
I*b)^2*Sqrt[a + b*Tan[c + d*x]])))/(3*d) + ((I*A - B)*Sqrt[a + b*Tan[c + d
*x]]*((b^2*Tan[c + d*x]^2)/(a + b*Tan[c + d*x])^2 + (3*b*Tan[c + d*x])/(a
+ b*Tan[c + d*x]) - (3*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a
]]*Sqrt[Tan[c + d*x]])/(Sqrt[a]*Sqrt[1 + (b*Tan[c + d*x])/a])))/(3*b^3*d*S
qrt[Tan[c + d*x]]) - ((I*A + B)*Sqrt[a + b*Tan[c + d*x]]*((b^2*Tan[c + d*x
]^2)/(a + b*Tan[c + d*x])^2 + (3*b*Tan[c + d*x])/(a + b*Tan[c + d*x]) - (3
*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a]]*Sqrt[Tan[c + d*x]])
/(Sqrt[a]*Sqrt[1 + (b*Tan[c + d*x])/a])))/(3*b^3*d*Sqrt[Tan[c + d*x]]))
```

Rubi [A] (verified)

Time = 3.25 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 4729, 3042, 4088, 27, 3042, 4128, 27, 3042, 4138, 2035, 2257, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(c+dx)}{\cot(c+dx)^{5/2}(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan^{\frac{5}{2}}(c+dx)(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan(c+dx)^{5/2}(A+B\tan(c+dx))}{(a+b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{4088}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2\int -\frac{3\sqrt{\tan(c+dx)}\left(-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\tan(c+dx)+a(Ab-aB)\right)}{2(a+b\tan(c+dx))^{3/2}}dx}{3b\left(a^2+b^2\right)}+\frac{2a(A}{3bd\left(a^2+\right.}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int\frac{\sqrt{\tan(c+dx)}\left(-\left(\left(a^2+b^2\right)B\tan^2(c+dx)\right)-b(Ab-aB)\right.}{(a+b\tan(c+dx))^{3/2}}}{b\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\int\frac{\sqrt{\tan(c+dx)}\left(-\left(\left(a^2+b^2\right)B\tan(c+dx)^2\right)-b(Ab-aB)\right.}{(a+b\tan(c+dx))^{3/2}}}{b\left(a^2+b^2\right)}\right.$$

$$\Big\downarrow \text{4128}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2\int\frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx}{2\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}}{b}\right.$$

$$\Big\downarrow \text{27}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx)+}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}}{b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\left(a^2+b^2\right)(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan(c+dx)^2+}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}}{b\left(a^2+b^2\right)}}{b}\right.$$

$$\Big\downarrow \text{4138}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-\frac{\frac{\int\frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx)+}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx\right.}}{bd(a^2+b^2)}}{}\right.$$

$\downarrow$ 2035

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2\int\frac{\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b^2-\left(a^2+b^2\right)^2B\tan^2(c+dx}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right.}}{bd(a^2+b^2)}}{}\right.$$

$\downarrow$ 2257

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-\frac{\frac{2\int\left(\frac{\left(-Ba^2+2Aba+b^2B\right)b^2+\left(Aa^2+2bBa-Ab^2\right)\tan(c+dx)b}{\sqrt{a+b\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}\right.}{bd(a^2+b^2)}}{}\right.$$

$\downarrow$ 2009

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{2a(Ab-aB)\tan^{\frac{3}{2}}(c+dx)}{3bd\,(a^2+b^2)\,(a+b\tan(c+dx))^{3/2}}-\frac{-\frac{2a\left(2Ab^3-aB\left(a^2+3b^2\right)\right)\sqrt{\tan(c+dx)}}{bd(a^2+b^2)\sqrt{a+b\tan(c+dx)}}+\frac{2\left(-\frac{B\left(a^2+\right.}{}\right.}{}}{}\right.$$

input
```
Int[(A + B*Tan[c + d*x])/(Cot[c + d*x]^(5/2)*(a + b*Tan[c + d*x])^(5/2)),x
]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((2*a*(A*b - a*B)*Tan[c + d*x]^(3/2)
)/(3*b*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^(3/2)) - ((2*(((a - I*b)^2*b^2*(
I*A - B)*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]
]])/(2*Sqrt[I*a - b]) - ((a^2 + b^2)^2*B*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x
]])/Sqrt[a + b*Tan[c + d*x]]])/Sqrt[b] - ((a + I*b)^2*b^2*(I*A + B)*ArcTan
h[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]])/(2*Sqrt[I*
a + b])))/(b*(a^2 + b^2)*d) - (2*a*(2*A*b^3 - a*(a^2 + 3*b^2)*B)*Sqrt[Tan[
c + d*x]])/(b*(a^2 + b^2)*d*Sqrt[a + b*Tan[c + d*x]]))/(b*(a^2 + b^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2257
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x] && IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4088
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(b*c - a*d)*(B*c - A*d)*(a + b*Tan[e + f*x])^(m - 1)*((c + d*Tan[e + f*x
])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Simp[1/(d*(n + 1)*(c^2 + d^2))
  Int[(a + b*Tan[e + f*x])^(m - 2)*(c + d*Tan[e + f*x])^(n + 1)*Simp[a*A*d*
(b*d*(m - 1) - a*c*(n + 1)) + (b*B*c - (A*b + a*B)*d)*(b*c*(m - 1) + a*d*(n
 + 1)) - d*((a*A - b*B)*(b*c - a*d) + (A*b + a*B)*(a*c + b*d))*(n + 1)*Tan[
e + f*x] - b*(d*(A*b*c + a*B*c - a*A*d)*(m + n) - b*B*(c^2*(m - 1) - d^2*(n
 + 1)))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 1] &
& LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

rule 4128
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*d^2 + c*(c*C - B*d))*(a + b*Tan[e +
 f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 + d^2))), x] - Sim
p[1/(d*(n + 1)*(c^2 + d^2))   Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e
 + f*x])^(n + 1)*Simp[A*d*(b*d*m - a*c*(n + 1)) + (c*C - B*d)*(b*c*m + a*d*
(n + 1)) - d*(n + 1)*((A - C)*(b*c - a*d) + B*(a*c + b*d))*Tan[e + f*x] - b
*(d*(B*c - A*d)*(m + n + 1) - C*(c^2*m - d^2*(n + 1)))*Tan[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4138
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*(c + d*ff*x)^n*((A + B*ff*x + C*ff^2*x^
2)/(1 + ff^2*x^2)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f
, A, B, C, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 2.69 (sec) , antiderivative size = 2981045, normalized size of antiderivative = 8716.51

output too large to display

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28011 vs. $2(284) = 568$.

Time = 23.00 (sec) , antiderivative size = 56055, normalized size of antiderivative = 163.90

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\tan(c + dx)}{\cot^{\frac{5}{2}}(c + dx)(a + b\tan(c + dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)**(5/2)/(a+b*tan(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx=\int \frac{B\tan\left(dx+c\right)+A}{\left(b\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}\cot\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)/((b*tan(d*x + c) + a)^(5/2)*cot(d*x + c)^(5
/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[6,14,5]%%%}+%%%{6,[6,12,5]%%%}+%%%{15,[6,10,5]%%%}+
%%%{20,[6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{A+B\tan(c+d\,x)}{\cot\left(c+d\,x\right)^{5/2}\,\left(a+b\tan\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2)),x
)
```

output
```
int((A + B*tan(c + d*x))/(cot(c + d*x)^(5/2)*(a + b*tan(c + d*x))^(5/2)),
x)
```

Reduce [F]

$$\int \frac{A+B\tan(c+dx)}{\cot^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}}{\cot\left(dx+c\right)^{3}\tan\left(dx+c\right)^{2}b^{2}+2\cot\left(dx+c\right)^{3}\tan\left(dx+c\right)a}$$

input
```
int((A+B*tan(d*x+c))/cot(d*x+c)^(5/2)/(a+b*tan(d*x+c))^(5/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/(cot(c + d*x)**3*tan(c +
 d*x)**2*b**2 + 2*cot(c + d*x)**3*tan(c + d*x)*a*b + cot(c + d*x)**3*a**2)
,x)
```

3.657 $\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 151

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{B\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d} + \frac{B\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia+b}\,d}$$

output
```
B*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)
^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d+B*arctanh((I*a+b)^(1/2)*tan(d*x+c)
^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(
1/2)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \frac{(-1)^{3/4}B\left(-\frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}} - \frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\cdots}{\sqrt{a+b\tan\cdots}}\right)}{\sqrt{a+ib}}\cdots\right.}{d}$$

input
```
Integrate[(Sqrt[Cot[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x
])^(3/2),x]
```

output
```
((-1)^(3/4)*B*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]]/Sqrt[-a + I*b]) - ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b])*Sqrt[Cot[c +
d*x]]*Sqrt[Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2011, 3042, 4729, 3042, 4058, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{\sqrt{\cot(c+dx)}}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{\sqrt{\cot(c+dx)}}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4729}$$

$$B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 4058}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 615}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \left(\frac{i}{2(i-\tan(c+dx))\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}+\frac{i}{2\sqrt{\tan(c+dx)}(\tan(c+dx)+i)\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}+\frac{\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b+ia}}\right)}{d}$$

input
```
Int[(Sqrt[Cot[c + d*x]]*(a*B + b*B*Tan[c + d*x]))/(a + b*Tan[c + d*x])^(3/
2),x]
```

output
```
(B*(ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sq
rt[I*a - b] + ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]]/Sqrt[I*a + b])*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4058
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
 Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(123) = 246$.

Time = 5.57 (sec) , antiderivative size = 892, normalized size of antiderivative = 5.91

method	result	size
default	Expression too large to display	892

input
```
int(cot(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-1/4*B/d*2^(1/2)/(a^2+b^2)^(1/2)/(-b+(a^2+b^2)^(1/2))^(1/2)*cot(d*x+c)^(1/
2)*(a+b*tan(d*x+c))^(1/2)*((b+(a^2+b^2)^(1/2))^(1/2)*ln(-1/(1-cos(d*x+c))*
(-a*(1-cos(d*x+c))^2*csc(d*x+c)+2*(a^2+b^2)^(1/2)*(1-cos(d*x+c))-2*2^(1/2)
*((a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(b+(a^2+b
^2)^(1/2))^(1/2)*sin(d*x+c)+2*b*(1-cos(d*x+c))+a*sin(d*x+c)))*(-b+(a^2+b^2
)^(1/2))^(1/2)-(b+(a^2+b^2)^(1/2))^(1/2)*(-b+(a^2+b^2)^(1/2))^(1/2)*ln(1/(
1-cos(d*x+c))*(-a*(1-cos(d*x+c))^2*csc(d*x+c)+2*(a^2+b^2)^(1/2)*(1-cos(d*x
+c))+2*2^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*sin(d*x+c)+2*b*(1-cos(d*x+c))+a*sin(d*x+c))
)+2*arctan(1/(-b+(a^2+b^2)^(1/2))^(1/2)*(-(b+(a^2+b^2)^(1/2))^(1/2)*(csc(d
*x+c)-cot(d*x+c))+2^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2))/(1-cos(d*x+c))*sin(d*x+c))*(a^2+b^2)^(1/2)+2*arctan(1/(-b
+(a^2+b^2)^(1/2))^(1/2)*((b+(a^2+b^2)^(1/2))^(1/2)*(csc(d*x+c)-cot(d*x+c))
+2^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))/
(1-cos(d*x+c))*sin(d*x+c))*(a^2+b^2)^(1/2)-2*arctan(1/(-b+(a^2+b^2)^(1/2))
^(1/2)*(-(b+(a^2+b^2)^(1/2))^(1/2)*(csc(d*x+c)-cot(d*x+c))+2^(1/2)*((a*cos
(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))/(1-cos(d*x+c))*s
in(d*x+c))*b-2*arctan(1/(-b+(a^2+b^2)^(1/2))^(1/2)*((b+(a^2+b^2)^(1/2))^(1
/2)*(csc(d*x+c)-cot(d*x+c))+2^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2))/(1-cos(d*x+c))*sin(d*x+c))*b)/((a*cos(d*x+c)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4609 vs. $2(119) = 238$.

Time = 0.56 (sec) , antiderivative size = 4609, normalized size of antiderivative = 30.52

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = B\int \frac{\sqrt{\cot\left(c+dx\right)}}{\sqrt{a+b\tan\left(c+dx\right)}}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))**(3/2),x
)
```

output
```
B*Integral(sqrt(cot(c + d*x))/sqrt(a + b*tan(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{(Bb\tan\left(dx+c\right)+Ba)\sqrt{\cot\left(dx+c\right)}}{\left(b\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*tan(d*x + c) + B*a)*sqrt(cot(d*x + c))/(b*tan(d*x + c) + a)
^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cot\left(c+d\,x\right)}\,\left(B\,a+B\,b\tan(c+d\,x)\right)}{\left(a+b\tan\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((cot(c + d*x)^(1/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/
2),x)
```

output
```
int((cot(c + d*x)^(1/2)*(B*a + B*b*tan(c + d*x)))/(a + b*tan(c + d*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(c+dx)}(aB+bB\tan(c+dx))}{(a+b\tan(c+dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{a+\tan\left(dx+c\right)b}\,\sqrt{\cot\left(dx+c\right)}}{a+\tan\left(dx+c\right)b}dx\right)b$$

input
```
int(cot(d*x+c)^(1/2)*(B*a+b*B*tan(d*x+c))/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/(tan(c + d*x)*b + a),x)*
b
```

3.658 $\int \frac{aB+bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 157

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx=\frac{iB\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d}-\frac{iB\text{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia+b}\,d}$$

output
```
I*B*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+
c)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d-I*B*arctanh((I*a+b)^(1/2)*tan(d*
x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/(I*a+
b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx=\frac{\sqrt[4]{-1}B\left(-\frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-a+ib}}+\frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{a+ib}\sqrt{\tan}}{\sqrt{a+b\tan(c+dx}}\right)}{\sqrt{a+ib}}\right.}{d}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x]
)^(3/2)),x]
```

output
```
((-1)^(1/4)*B*(-(ArcTan[((-1)^(1/4)*Sqrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqr
t[a + b*Tan[c + d*x]]]/Sqrt[-a + I*b]) + ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*
Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[a + I*b])*Sqrt[Cot[c +
d*x]]*Sqrt[Tan[c + d*x]])/d
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {2011, 3042, 4729, 3042, 4058, 613, 104, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2011}$$

$$B\int \frac{1}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\int \frac{1}{\sqrt{\cot(c+dx)}\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4729}$$

$$B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{4058}$$

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}$$

$\Big\downarrow$ 613

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}\int \frac{1}{\sqrt{\tan(c+dx)}(\tan(c+dx)+i)\sqrt{a+b\tan(c+dx)}}d\tan(c+dx)-\frac{1}{2}\int \frac{1}{(i-\tan(c+dx))\sqrt{\tan(c+dx}}\right.}{d}$$

$\Big\downarrow$ 104

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{1}{\frac{(a-ib)\tan(c+dx)}{a+b\tan(c+dx)}+i}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}-\int \frac{1}{i-\frac{(a+ib)\tan(c+dx)}{a+b\tan(c+dx)}}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{d}$$

$\Big\downarrow$ 218

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\int \frac{1}{\frac{(a-ib)\tan(c+dx)}{a+b\tan(c+dx)}+i}d\frac{\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}+\frac{i\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}\right)}{d}$$

$\Big\downarrow$ 221

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{i\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}-\frac{i\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b+ia}}\right)}{d}$$

input

```
Int[(a*B + b*B*Tan[c + d*x])/(Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^(3/2
)),x]
```

output

```
(B*((I*ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]
)/Sqrt[I*a - b] - (I*ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b
*Tan[c + d*x]]])/Sqrt[I*a + b])*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 613
```
Int[Sqrt[(e_.)*(x_)]/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Sym
bol] :> Simp[e/(2*b)   Int[1/(Sqrt[e*x]*Sqrt[c + d*x]*(Rt[-a/b, 2] + x)), x
], x] - Simp[e/(2*b)   Int[1/(Sqrt[e*x]*Sqrt[c + d*x]*(Rt[-a/b, 2] - x)), x
], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4058
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
 Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(127) = 254$.

Time = 4.56 (sec) , antiderivative size = 1030, normalized size of antiderivative = 6.56

method	result	size
default	Expression too large to display	1030

input
```
int((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
-1/8*B/d*2^(1/2)/a/(a^2+b^2)^(1/2)/(-b+(a^2+b^2)^(1/2))^(1/2)*((1-cos(d*x+
c))^2*csc(d*x+c)^2-1)*(a+b*tan(d*x+c))^(1/2)*((a^2+b^2)^(1/2)*ln(-1/(1-cos
(d*x+c))*(a*(1-cos(d*x+c))^2*csc(d*x+c)+2*2^(1/2)*((a*cos(d*x+c)+b*sin(d*x
+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*sin(d*x+
c)-2*(a^2+b^2)^(1/2)*(1-cos(d*x+c))-2*b*(1-cos(d*x+c))-a*sin(d*x+c)))*(b+(
a^2+b^2)^(1/2))^(1/2)*(-b+(a^2+b^2)^(1/2))^(1/2)-(a^2+b^2)^(1/2)*ln(1/(1-c
os(d*x+c))*(-a*(1-cos(d*x+c))^2*csc(d*x+c)+2*(a^2+b^2)^(1/2)*(1-cos(d*x+c)
)+2*2^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2
)*(b+(a^2+b^2)^(1/2))^(1/2)*sin(d*x+c)+2*b*(1-cos(d*x+c))+a*sin(d*x+c)))*(
b+(a^2+b^2)^(1/2))^(1/2)*(-b+(a^2+b^2)^(1/2))^(1/2)-ln(-1/(1-cos(d*x+c))*(
a*(1-cos(d*x+c))^2*csc(d*x+c)+2*2^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))*sin(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*sin(d*x+c)-2*(a^2+
b^2)^(1/2)*(1-cos(d*x+c))-2*b*(1-cos(d*x+c))-a*sin(d*x+c)))*b*(-b+(a^2+b^2
)^(1/2))^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)+ln(1/(1-cos(d*x+c))*(-a*(1-cos(d*
x+c))^2*csc(d*x+c)+2*(a^2+b^2)^(1/2)*(1-cos(d*x+c))+2*2^(1/2)*((a*cos(d*x+
c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(b+(a^2+b^2)^(1/2))^(1
/2)*sin(d*x+c)+2*b*(1-cos(d*x+c))+a*sin(d*x+c)))*b*(-b+(a^2+b^2)^(1/2))^(1
/2)*(b+(a^2+b^2)^(1/2))^(1/2)-2*arctan(1/(-b+(a^2+b^2)^(1/2))^(1/2)*(-(b+(
a^2+b^2)^(1/2))^(1/2)*(csc(d*x+c)-cot(d*x+c))+2^(1/2)*((a*cos(d*x+c)+b*sin
(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))/(1-cos(d*x+c))*sin(d*x+c))...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4649 vs. $2(121) = 242$.

Time = 0.54 (sec) , antiderivative size = 4649, normalized size of antiderivative = 29.61

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{a+b\tan{(c+dx)}}\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)**(1/2)/(a+b*tan(d*x+c))**(3/2),x
)
```

output
```
B*Integral(1/(sqrt(a + b*tan(c + d*x))*sqrt(cot(c + d*x))), x)
```

Maxima [F]

$$\int \frac{aB + bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \int \frac{Bb\tan{(dx+c)} + Ba}{(b\tan{(dx+c)}+a)^{\frac{3}{2}}\sqrt{\cot{(dx+c)}}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
integrate((B*b*tan(d*x + c) + B*a)/((b*tan(d*x + c) + a)^(3/2)*sqrt(cot(d*
x + c))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{1,[3,9,3]%%%}+%%%{4,[3,7,3]%%%}+%%%{6,[3,5,3]%%%}+%%%{
4,[3,3,3]
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\tan(c + dx)}{\sqrt{\cot(c + dx)}(a + b\tan(c + dx))^{3/2}}\,dx = \int \frac{B\,a + B\,b\tan(c + d\,x)}{\sqrt{\cot(c + d\,x)}\,(a + b\tan(c + d\,x))^{3/2}}\,dx$$

input
```
int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2
)),x)
```

output
```
int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(1/2)*(a + b*tan(c + d*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{aB+bB\tan(c+dx)}{\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{a+\tan(dx+c)\,b}\,\sqrt{\cot(dx+c)}}{\cot(dx+c)\tan(dx+c)\,b+\cot(dx+c)\,a}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/(cot(c + d*x)*tan(c + d*
x)*b + cot(c + d*x)*a),x)*b
```

3.659 $\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 215

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = -\frac{B\arctan\left(\frac{\sqrt{ia-b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia-b}\,d} + \frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{b}\,d} - \frac{B\operatorname{arctanh}\left(\frac{\sqrt{ia+b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}}{\sqrt{ia+b}\,d}$$

output
```
-B*arctan((I*a-b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c
)^(1/2)*tan(d*x+c)^(1/2)/(I*a-b)^(1/2)/d+2*B*arctanh(b^(1/2)*tan(d*x+c)^(1
/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^(1/2)*tan(d*x+c)^(1/2)/b^(1/2)/d-B*
arctanh((I*a+b)^(1/2)*tan(d*x+c)^(1/2)/(a+b*tan(d*x+c))^(1/2))*cot(d*x+c)^
(1/2)*tan(d*x+c)^(1/2)/(I*a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.98

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx = \frac{B\sqrt{\cot(c+dx)}\sqrt{\tan(c+dx)}\left((-1)^{3/4}\left(\frac{\arctan\left(\frac{\sqrt[4]{-1}\sqrt{-a+ib}\sqrt{\tan}}{\sqrt{a+b\tan(c+d}}\right)}{\sqrt{-a+ib}}\right.\right.}{}$$

input
```
Integrate[(a*B + b*B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x]
)^(3/2)),x]
```

output
```
(B*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*((-1)^(3/4)*(ArcTan[((-1)^(1/4)*S
qrt[-a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/Sqrt[-a + I*b]
 + ArcTan[((-1)^(1/4)*Sqrt[a + I*b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c +
 d*x]]]/Sqrt[a + I*b]) + (2*Sqrt[a]*ArcSinh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/S
qrt[a]]*Sqrt[1 + (b*Tan[c + d*x])/a])/(Sqrt[b]*Sqrt[a + b*Tan[c + d*x]])))
/d
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2011, 3042, 4729, 3042, 4058, 614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{aB+bB\tan(c+dx)}{\cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2011}$$

$$B\int \frac{1}{\cot^{\frac{3}{2}}(c+dx)\sqrt{a+b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{1}{\cot(c+dx)^{3/2}\sqrt{a+b\tan(c+dx)}}dx$$

$\Big\downarrow$ 4729

$$B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}}dx$$

$\Big\downarrow$ 3042

$$B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan(c+dx)^{3/2}}{\sqrt{a+b\tan(c+dx)}}dx$$

$\Big\downarrow$ 4058

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{\tan^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}d\tan(c+dx)}{d}$$

$\Big\downarrow$ 614

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \left(\frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}}-\frac{1}{\sqrt{\tan(c+dx)}\sqrt{a+b\tan(c+dx)}(\tan^2(c+dx)+1)}\right)d\tan(c+dx}{d}$$

$\Big\downarrow$ 2009

$$\frac{B\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{\arctan\left(\frac{\sqrt{-b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{-b+ia}}+\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b+ia}\sqrt{\tan(c+dx)}}{\sqrt{a+b\tan(c+dx)}}\right)}{\sqrt{b+ia}}\right.}{d}$$

input
```
Int[(a*B + b*B*Tan[c + d*x])/(Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^(3/2
)),x]
```

output
```
(B*(-(ArcTan[(Sqrt[I*a - b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c + d*x]]]/
Sqrt[I*a - b]) + (2*ArcTanh[(Sqrt[b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*Tan[c
+ d*x]]])/Sqrt[b] - ArcTanh[(Sqrt[I*a + b]*Sqrt[Tan[c + d*x]])/Sqrt[a + b*
Tan[c + d*x]]]/Sqrt[I*a + b])*Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]])/d
```

Defintions of rubi rules used

rule 614
```
Int[((e_.)*(x_))^(m_)/(Sqrt[(c_) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^2)), x_Sy
mbol] :> Simp[e^(m + 1/2)   Int[ExpandIntegrand[1/(Sqrt[e*x]*Sqrt[c + d*x])
, x^(m + 1/2)/(a + b*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[
m - 1/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4058
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, S
imp[ff/f   Subst[Int[(a + b*ff*x)^m*((c + d*ff*x)^n/(1 + ff^2*x^2)), x], x,
 Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. $2(175) = 350$.

Time = 4.79 (sec) , antiderivative size = 1026, normalized size of antiderivative = 4.77

method	result	size
default	Expression too large to display	1026

input
```
int((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
1/16*B/d*2^(1/2)/(a^2+b^2)^(1/2)/(-b+(a^2+b^2)^(1/2))^(1/2)/b^(1/2)*((1-co
s(d*x+c))^2*csc(d*x+c)^2-1)^2*(a+b*tan(d*x+c))^(1/2)*(4*2^(1/2)*arctanh(1/
b^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)/(1
-cos(d*x+c))*sin(d*x+c))*(a^2+b^2)^(1/2)*(-b+(a^2+b^2)^(1/2))^(1/2)-2*b^(3
/2)*arctan(1/(-b+(a^2+b^2)^(1/2))^(1/2)*((b+(a^2+b^2)^(1/2))^(1/2)*(csc(d*
x+c)-cot(d*x+c))+2^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2))/(1-cos(d*x+c))*sin(d*x+c))+b^(1/2)*(b+(a^2+b^2)^(1/2))^(1/
2)*(-b+(a^2+b^2)^(1/2))^(1/2)*ln(1/(1-cos(d*x+c))*(a*(1-cos(d*x+c))^2*csc(
d*x+c)+2*2^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*sin(d*x+c)-2*(a^2+b^2)^(1/2)*(1-cos(d*x+c
))-2*b*(1-cos(d*x+c))-a*sin(d*x+c)))-b^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*(-b
+(a^2+b^2)^(1/2))^(1/2)*ln(1/(1-cos(d*x+c))*(-a*(1-cos(d*x+c))^2*csc(d*x+c
)+2*(a^2+b^2)^(1/2)*(1-cos(d*x+c))+2*2^(1/2)*((a*cos(d*x+c)+b*sin(d*x+c))*
sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(b+(a^2+b^2)^(1/2))^(1/2)*sin(d*x+c)+2*
b*(1-cos(d*x+c))+a*sin(d*x+c)))+2*b^(1/2)*arctan(1/(-b+(a^2+b^2)^(1/2))^(1
/2)*(-(b+(a^2+b^2)^(1/2))^(1/2)*(csc(d*x+c)-cot(d*x+c))+2^(1/2)*((a*cos(d*
x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))/(1-cos(d*x+c))*sin(
d*x+c))*(a^2+b^2)^(1/2)+2*(a^2+b^2)^(1/2)*b^(1/2)*arctan(1/(-b+(a^2+b^2)^(
1/2))^(1/2)*((b+(a^2+b^2)^(1/2))^(1/2)*(csc(d*x+c)-cot(d*x+c))+2^(1/2)*((a
*cos(d*x+c)+b*sin(d*x+c))*sin(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))/(1-cos(d*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4702 vs. $2(171) = 342$.

Time = 0.87 (sec) , antiderivative size = 9437, normalized size of antiderivative = 43.89

$$\int \frac{aB + bB\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{aB + bB\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = B\int \frac{1}{\sqrt{a + b\tan{(c + dx)}}\cot^{\frac{3}{2}}{(c + dx)}}\,dx$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)**(3/2)/(a+b*tan(d*x+c))**(3/2),x
)
```

output
```
B*Integral(1/(sqrt(a + b*tan(c + d*x))*cot(c + d*x)**(3/2)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{aB + bB\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{aB + bB\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x,
algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{aB + bB\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \int \frac{B\,a + B\,b\tan(c + d\,x)}{\cot\left(c + d\,x\right)^{3/2}\,\left(a + b\tan\left(c + d\,x\right)\right)^{3/2}}\,dx$$

input
```
int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2
)),x)
```

output
```
int((B*a + B*b*tan(c + d*x))/(cot(c + d*x)^(3/2)*(a + b*tan(c + d*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{aB + bB\tan(c + dx)}{\cot^{\frac{3}{2}}(c + dx)(a + b\tan(c + dx))^{3/2}}\,dx = \left(\int \frac{\sqrt{a + \tan\left(dx + c\right)b}\,\sqrt{\cot\left(dx + c\right)}}{\cot\left(dx + c\right)^{2}\tan\left(dx + c\right)b + \cot\left(dx + c\right)^{2}a}dx\right)b$$

input
```
int((B*a+b*B*tan(d*x+c))/cot(d*x+c)^(3/2)/(a+b*tan(d*x+c))^(3/2),x)
```

output
```
int((sqrt(tan(c + d*x)*b + a)*sqrt(cot(c + d*x)))/(cot(c + d*x)**2*tan(c +
 d*x)*b + cot(c + d*x)**2*a),x)*b
```

3.660 $\int \cot^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 197

$$\int \cot^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \frac{(A+iB)\operatorname{AppellF1}\left(1-m,-n,1,2-m,-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\cot^{-1+m}(c+dx)(a+b\tan(c+d}{2d(1-m)}$$
$$+\frac{(A-iB)\operatorname{AppellF1}\left(1-m,-n,1,2-m,-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)\cot^{-1+m}(c+dx)(a+b\tan(c+d}{2d(1-m)}$$

output
```
1/2*(A+I*B)*AppellF1(1-m,1,-n,2-m,-I*tan(d*x+c),-b*tan(d*x+c)/a)*cot(d*x+c
)^(-1+m)*(a+b*tan(d*x+c))^n/d/(1-m)/(((a+b*tan(d*x+c))/a)^n)+1/2*(A-I*B)*A
ppellF1(1-m,1,-n,2-m,I*tan(d*x+c),-b*tan(d*x+c)/a)*cot(d*x+c)^(-1+m)*(a+b*
tan(d*x+c))^n/d/(1-m)/(((a+b*tan(d*x+c))/a)^n)
```

Mathematica [F]

$$\int \cot^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \cot^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Cot[c + d*x]^m*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Integrate[Cot[c + d*x]^m*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]), x]
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3042, 4729, 3042, 4086, 3042, 4085, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^m(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \cot(c+dx)^m(A+B\tan(c+dx))(a+b\tan(c+dx))^ndx$$
$$\Big\downarrow \text{4729}$$
$$\tan^m(c+dx)\cot^m(c+dx)\int \tan^{-m}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{3042}$$
$$\tan^m(c+dx)\cot^m(c+dx)\int \tan(c+dx)^{-m}(a+b\tan(c+dx))^n(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{4086}$$

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{1}{2}(A+iB)\int(1-i\tan(c+dx))\tan^{-m}(c+dx)(a+b\tan(c+dx))^n dx+\frac{1}{2}(A-iB)\int(i\tan(c+dx)+1)\tan\right.$$

$\big\downarrow$ 3042

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{1}{2}(A+iB)\int(1-i\tan(c+dx))\tan(c+dx)^{-m}(a+b\tan(c+dx))^n dx+\frac{1}{2}(A-iB)\int(i\tan(c+dx)+1)\tan\right.$$

$\big\downarrow$ 4085

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{(A-iB)\int\frac{\tan^{-m}(c+dx)(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int\frac{\tan^{-m}(c+dx)(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d\tan(c+dx)}{2d}\right)$$

$\big\downarrow$ 152

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{(A-iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int\frac{\tan^{-m}(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)(a\right.$$

$\big\downarrow$ 150

$$\tan^m(c+dx)\cot^m(c+dx)\left(\frac{(A+iB)\tan^{1-m}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(1-m,-n,1,2-m,-\frac{b\tan(c+}{a}\right.}{2d(1-m)}\right.$$

input
```
Int[Cot[c + d*x]^m*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Cot[c + d*x]^m*Tan[c + d*x]^m*(((A + I*B)*AppellF1[1 - m, -n, 1, 2 - m, -(
(b*Tan[c + d*x])/a), (-I)*Tan[c + d*x]]*Tan[c + d*x]^(1 - m)*(a + b*Tan[c
+ d*x])^n)/(2*d*(1 - m)*(1 + (b*Tan[c + d*x])/a)^n) + ((A - I*B)*AppellF1[
1 - m, -n, 1, 2 - m, -((b*Tan[c + d*x])/a), I*Tan[c + d*x]]*Tan[c + d*x]^(
1 - m)*(a + b*Tan[c + d*x])^n)/(2*d*(1 - m)*(1 + (b*Tan[c + d*x])/a)^n))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \cot (dx+c)^m (a+b\tan (dx+c))^n (A+B\tan (dx+c))\, dx$$

input
```
int(cot(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\, dx$$
$$= \int (B\tan (dx+c)+A)(b\tan (dx+c)+a)^n \cot (dx+c)^m \, dx$$

input
```
integrate(cot(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="f
ricas")
```

output
```
integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\, dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**m*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\cot(dx+c)^m\,dx$$

input
```
integrate(cot(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="m
axima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^m, x)
```

Giac [F]

$$\int \cot^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\cot(dx+c)^m\,dx$$

input
```
integrate(cot(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorithm="g
iac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$= \int \cot(c+d\,x)^m\,(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^n\,dx$$

input
```
int(cot(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^m*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \cot^m(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \cot\left(dx+c\right)^m\left(a+\tan\left(dx+c\right)b\right)^n\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(cot(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^m*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

3.661 $\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 171

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx =$$
$$-\frac{(A+iB)\,\text{AppellF1}\left(-\frac{1}{2},-n,1,\frac{1}{2},-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n\left(\frac{a+\cdots}{\cdots}\right.}{d}$$
$$-\frac{(A-iB)\,\text{AppellF1}\left(-\frac{1}{2},-n,1,\frac{1}{2},-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)\sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n\left(\frac{a+b\cdots}{\cdots}\right.}{d}$$

output
```
-(A+I*B)*AppellF1(-1/2,1,-n,1/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*cot(d*x+c)^
(1/2)*(a+b*tan(d*x+c))^n/d/(((a+b*tan(d*x+c))/a)^n)-(A-I*B)*AppellF1(-1/2,
1,-n,1/2,I*tan(d*x+c),-b*tan(d*x+c)/a)*cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n
/d/(((a+b*tan(d*x+c))/a)^n)
```

Mathematica [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x
]
```

output
```
Integrate[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),
x]
```

Rubi [A] (warning: unable to verify)

Time = 1.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4729, 3042, 4086, 3042, 4085, 148, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^{\frac{3}{2}}(c+dx)(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(c+dx)^{3/2}(A+B\tan(c+dx))(a+b\tan(c+dx))^n dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\tan(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4086}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(A+iB)\int\frac{(1-i\tan(c+dx))(a+b\tan(c+dx))^n}{\tan^{\frac{3}{2}}(c+dx)}dx+\frac{1}{2}(A-iB)\int\frac{(i\tan(c+\right.$$

$\Big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(A+iB)\int\frac{(1-i\tan(c+dx))(a+b\tan(c+dx))^n}{\tan(c+dx)^{3/2}}dx+\frac{1}{2}(A-iB)\int\frac{(i\tan(c+\right.$$

$\Big\downarrow$ 4085

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int\frac{(a+b\tan(c+dx))^n}{(1-i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int\frac{(a+b\tan(c+dx))^n}{(i\tan(c+dx)+1)\tan^{\frac{3}{2}}(c}}{2d}\right.$$

$\Big\downarrow$ 148

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int\frac{\cot^2(c+dx)(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)\int\frac{\cot^2(c+dx)(a+b\tan(}{i\tan(c+dx)+}}{d}\right.$$

$\Big\downarrow$ 395

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int\frac{\cot^2(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{1-i\tan(c+dx)}d\sqrt{\tan}}{d}\right.$$

$\Big\downarrow$ 394

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(-\frac{(A+iB)\cot(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(-\frac{1}{2},1,-\right.}{d}\right.$$

input
```
Int[Cot[c + d*x]^(3/2)*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(-(((A + I*B)*AppellF1[-1/2, 1, -n,
1/2, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Cot[c + d*x]*(a + b*Tan[c +
 d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a)^n)) - ((A - I*B)*AppellF1[-1/2, 1, -
n, 1/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Cot[c + d*x]*(a + b*Tan[c +
 d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a)^n))
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \cot (dx+c)^{\frac{3}{2}} (a+b\tan (dx+c))^n (A+B\tan (dx+c))\,dx$$

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan (dx+c)+A)(b\tan (dx+c)+a)^n \cot (dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
integral((B*cot(d*x + c)*tan(d*x + c) + A*cot(d*x + c))*(b*tan(d*x + c) +
a)^n*sqrt(cot(d*x + c)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**(3/2)*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\cot(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\cot(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*cot(d*x + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \cot(c+d\,x)^{3/2}\,(A+B\tan(c+d\,x))\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

input
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^(3/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \cot^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \cot\left(dx+c\right)^{\frac{3}{2}}\left(a+\tan\left(dx+c\right)b\right)^n\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(3/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

3.662 $\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 169

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \frac{(A+iB)\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c+dx)}{a}\right)^{-n}}{d\sqrt{\cot(c+dx)}} + \frac{(A-iB)\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c+dx)}{a}\right)^{-n}}{d\sqrt{\cot(c+dx)}}$$

output
```
(A+I*B)*AppellF1(1/2,1,-n,3/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+
c))^n/d/cot(d*x+c)^(1/2)/(((a+b*tan(d*x+c))/a)^n)+(A-I*B)*AppellF1(1/2,1,-
n,3/2,I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n/d/cot(d*x+c)^(1/2)/
(((a+b*tan(d*x+c))/a)^n)
```

Mathematica [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$

input
```
Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x
]
```

output
```
Integrate[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),
x]
```

Rubi [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4729, 3042, 4086, 3042, 4085, 148, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cot(c+dx)}(A+B\tan(c+dx))(a+b\tan(c+dx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{\cot(c+dx)}(A+B\tan(c+dx))(a+b\tan(c+dx))^ndx$$
$$\Big\downarrow \text{4729}$$
$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\tan(c+dx)}}dx$$
$$\Big\downarrow \text{4086}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(A+iB)\int\frac{(1-i\tan(c+dx))(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int\frac{(i\tan(c+\right.$$

$\big\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(A+iB)\int\frac{(1-i\tan(c+dx))(a+b\tan(c+dx))^n}{\sqrt{\tan(c+dx)}}dx+\frac{1}{2}(A-iB)\int\frac{(i\tan(c+\right.$$

$\big\downarrow$ 4085

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int\frac{(a+b\tan(c+dx))^n}{(1-i\tan(c+dx))\sqrt{\tan(c+dx)}}d\tan(c+dx)}{2d}+\frac{(A+iB)\int\frac{(a+b\tan(c+dx))^n}{(i\tan(c+dx)+1)\sqrt{\tan(\right.$$

$\big\downarrow$ 148

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int\frac{(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)\int\frac{(a+b\tan(c+dx))^n}{i\tan(c+dx)+1}d\sqrt{\tan(c+\right.$$

$\big\downarrow$ 334

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int\frac{\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}\right.$$

$\big\downarrow$ 333

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A+iB)\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(\frac{1}{2},1,-\right.}{d}\right.$$

input

```
Int[Sqrt[Cot[c + d*x]]*(a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((A + I*B)*AppellF1[1/2, 1, -n, 3/2
, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Sqrt[Tan[c + d*x]]*(a + b*Tan[
c + d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a)^n) + ((A - I*B)*AppellF1[1/2, 1,
-n, 3/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Sqrt[Tan[c + d*x]]*(a + b*
Tan[c + d*x])^n)/(d*(1 + (b*Tan[c + d*x])/a)^n))
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \sqrt{\cot (dx+c)}\,(a+b\tan (dx+c))^n\,(A+B\tan (dx+c))\,dx$$

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

Fricas [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int (B\tan (dx+c)+A)(b\tan (dx+c)+a)^n\sqrt{\cot (dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)), x
)
```

Sympy [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (A+B\tan(c+dx))\,(a+b\tan(c+dx))^n\,\sqrt{\cot(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**(1/2)*(a+b*tan(d*x+c))**n*(A+B*tan(d*x+c)),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n*sqrt(cot(c + d*x)),
x)
```

Maxima [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)),
x)
```

Giac [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx = \int (B\tan(dx+c)+A)(b\tan(dx+c)+a)^n\sqrt{\cot(dx+c)}\,dx$$

input
```
integrate(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n*sqrt(cot(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\cot\left(c+d\,x\right)}\,\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)^n\,dx$$

input
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n,x)
```

output
```
int(cot(c + d*x)^(1/2)*(A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n, x)
```

Reduce [F]

$$\int \sqrt{\cot(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))\,dx$$
$$=\int \sqrt{\cot\left(dx+c\right)}\,\left(a+\tan\left(dx+c\right)b\right)^n\left(A+B\tan\left(dx+c\right)\right)dx$$

input
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

output
```
int(cot(d*x+c)^(1/2)*(a+b*tan(d*x+c))^n*(A+B*tan(d*x+c)),x)
```

3.663 $\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 175

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \frac{(A+iB)\operatorname{AppellF1}\left(\frac{3}{2},-n,1,\frac{5}{2},-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c+dx)}{a}\right)^{-n}}{3d\cot^{\frac{3}{2}}(c+dx)} + \frac{(A-iB)\operatorname{AppellF1}\left(\frac{3}{2},-n,1,\frac{5}{2},-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c+dx)}{a}\right)^{-n}}{3d\cot^{\frac{3}{2}}(c+dx)}$$

output

```
1/3*(A+I*B)*AppellF1(3/2,1,-n,5/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(
d*x+c))^n/d/cot(d*x+c)^(3/2)/(((a+b*tan(d*x+c))/a)^n)+1/3*(A-I*B)*AppellF1
(3/2,1,-n,5/2,I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n/d/cot(d*x+c
)^(3/2)/(((a+b*tan(d*x+c))/a)^n)
```

Mathematica [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$

input
```
Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]]
,x]
```

output
```
Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]]
, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4729, 3042, 4086, 3042, 4085, 148, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\sqrt{\cot(c+dx)}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\sqrt{\cot(c+dx)}}dx$$
$$\Big\downarrow \text{ 4729}$$
$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))dx$$
$$\Big\downarrow \text{ 3042}$$
$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \ 4086$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(A+iB)\int(1-i\tan(c+dx))\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n dx+\frac{1}{2}(A-iB)\right.$$

$$\Big\downarrow \ 3042$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(A+iB)\int(1-i\tan(c+dx))\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n dx+\frac{1}{2}(A-iB)\right.$$

$$\Big\downarrow \ 4085$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int\frac{\sqrt{\tan(c+dx)}(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int\frac{\sqrt{\tan(c+dx)}(a+b\tan}{i\tan(c+dx)+}}{2d}\right.$$

$$\Big\downarrow \ 148$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int\frac{\tan(c+dx)(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)\int\frac{\tan(c+dx)(a+b\tan(c}{i\tan(c+dx)+1}}{d}\right.$$

$$\Big\downarrow \ 395$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int\frac{\tan(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{1-i\tan(c+dx)}d\sqrt{\tan(}}{d}\right.$$

$$\Big\downarrow \ 394$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A+iB)\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(\frac{3}{2},1,-n\right.}{3d}\right.$$

input
```
Int[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Sqrt[Cot[c + d*x]],x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((A + I*B)*AppellF1[3/2, 1, -n, 5/2
, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan[c + d*x]^(3/2)*(a + b*Tan[
c + d*x])^n)/(3*d*(1 + (b*Tan[c + d*x])/a)^n) + ((A - I*B)*AppellF1[3/2, 1
, -n, 5/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan[c + d*x]^(3/2)*(a +
b*Tan[c + d*x])^n)/(3*d*(1 + (b*Tan[c + d*x])/a)^n))
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \frac{(a+b\tan (dx+c))^{n} (A+B\tan (dx+c))}{\sqrt{\cot (dx+c)}}dx$$

input
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan (dx+c)+A)(b\tan (dx+c)+a)^{n}}{\sqrt{\cot (dx+c)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)), x
)
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(A+B\tan{(c+dx)})\,(a+b\tan{(c+dx)})^n}{\sqrt{\cot{(c+dx)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**n*(A+B*tan(d*x+c))/cot(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n/sqrt(cot(c + d*x)),
x)
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan{(dx+c)}+A)(b\tan{(dx+c)}+a)^n}{\sqrt{\cot{(dx+c)}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)),
x)
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{(B\tan\left(dx+c\right)+A)(b\tan\left(dx+c\right)+a)^n}{\sqrt{\cot\left(dx+c\right)}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/sqrt(cot(d*x + c)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx = \int \frac{\left(A+B\tan(c+d\,x)\right)\,\left(a+b\tan(c+d\,x)\right)^n}{\sqrt{\cot\left(c+d\,x\right)}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/cot(c + d*x)^(1/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/cot(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\sqrt{\cot(c+dx)}}\,dx$$
$$= \left(\int \frac{(a+\tan(dx+c)\,b)^n\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)}dx\right)b$$
$$+ \left(\int \frac{(a+\tan(dx+c)\,b)^n\sqrt{\cot(dx+c)}}{\cot(dx+c)}dx\right)a$$

input
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(1/2),x)
```

output
```
int(((tan(c + d*x)*b + a)**n*sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*x)
,x)*b + int(((tan(c + d*x)*b + a)**n*sqrt(cot(c + d*x)))/cot(c + d*x),x)*a
```

3.664 $\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 175

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \frac{(A+iB)\,\text{AppellF1}\left(\frac{5}{2},-n,1,\frac{7}{2},-\frac{b\tan(c+dx)}{a},-i\tan(c+dx)\right)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c+dx)}{a}\right)^{-n}}{5d\cot^{\frac{5}{2}}(c+dx)} + \frac{(A-iB)\,\text{AppellF1}\left(\frac{5}{2},-n,1,\frac{7}{2},-\frac{b\tan(c+dx)}{a},i\tan(c+dx)\right)(a+b\tan(c+dx))^n\left(\frac{a+b\tan(c+dx)}{a}\right)^{-n}}{5d\cot^{\frac{5}{2}}(c+dx)}$$

output
```
1/5*(A+I*B)*AppellF1(5/2,1,-n,7/2,-I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(
d*x+c))^n/d/cot(d*x+c)^(5/2)/(((a+b*tan(d*x+c))/a)^n)+1/5*(A-I*B)*AppellF1
(5/2,1,-n,7/2,I*tan(d*x+c),-b*tan(d*x+c)/a)*(a+b*tan(d*x+c))^n/d/cot(d*x+c
)^(5/2)/(((a+b*tan(d*x+c))/a)^n)
```

Mathematica [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2)
,x]
```

output
```
Integrate[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2)
, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4729, 3042, 4086, 3042, 4085, 148, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\tan(c+dx))(a+b\tan(c+dx))^n}{\cot(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{4729}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\int \tan(c+dx)^{3/2}(a+b\tan(c+dx))^n(A+B\tan(c+dx))dx$$

$\downarrow$ 4086

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(A+iB)\int(1-i\tan(c+dx))\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^ndx+\frac{1}{2}(A-iB)\right.$$

$\downarrow$ 3042

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{1}{2}(A+iB)\int(1-i\tan(c+dx))\tan(c+dx)^{3/2}(a+b\tan(c+dx))^ndx+\frac{1}{2}(A-iB\right.$$

$\downarrow$ 4085

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int\frac{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\tan(c+dx)}{2d}+\frac{(A+iB)\int\frac{\tan^{\frac{3}{2}}(c+dx)(a+b\tan(c}{i\tan(c+dx)+1}}{2d}\right.$$

$\downarrow$ 148

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)\int\frac{\tan^2(c+dx)(a+b\tan(c+dx))^n}{1-i\tan(c+dx)}d\sqrt{\tan(c+dx)}}{d}+\frac{(A+iB)\int\frac{\tan^2(c+dx)(a+b\tan}{i\tan(c+dx)+}}{d}\right.$$

$\downarrow$ 395

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A-iB)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\int\frac{\tan^2(c+dx)\left(\frac{b\tan(c+dx)}{a}+1\right)^n}{1-i\tan(c+dx)}d\sqrt{\tan}}{d}\right.$$

$\downarrow$ 394

$$\sqrt{\tan(c+dx)}\sqrt{\cot(c+dx)}\left(\frac{(A+iB)\tan^{\frac{5}{2}}(c+dx)(a+b\tan(c+dx))^n\left(\frac{b\tan(c+dx)}{a}+1\right)^{-n}\text{AppellF1}\left(\frac{5}{2},1,-n\right.}{5d}\right.$$

input
```
Int[((a + b*Tan[c + d*x])^n*(A + B*Tan[c + d*x]))/Cot[c + d*x]^(3/2),x]
```

output
```
Sqrt[Cot[c + d*x]]*Sqrt[Tan[c + d*x]]*(((A + I*B)*AppellF1[5/2, 1, -n, 7/2
, (-I)*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan[c + d*x]^(5/2)*(a + b*Tan[
c + d*x])^n)/(5*d*(1 + (b*Tan[c + d*x])/a)^n) + ((A - I*B)*AppellF1[5/2, 1
, -n, 7/2, I*Tan[c + d*x], -((b*Tan[c + d*x])/a)]*Tan[c + d*x]^(5/2)*(a +
b*Tan[c + d*x])^n)/(5*d*(1 + (b*Tan[c + d*x])/a)^n))
```

Defintions of rubi rules used

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4085
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[A^2/f   Subst[Int[(a + b*x)^m*((c + d*x)^n/(A - B*x)), x], x, Tan[e + f*x
]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n
] && EqQ[A^2 + B^2, 0]
```

rule 4086
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A + I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d*Tan[e + f*x])^n*(1 - I*T
an[e + f*x]), x], x] + Simp[(A - I*B)/2   Int[(a + b*Tan[e + f*x])^m*(c + d
*Tan[e + f*x])^n*(1 + I*Tan[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m] &
&  !IntegerQ[n] &&  !IntegersQ[2*m, 2*n] && NeQ[A^2 + B^2, 0]
```

rule 4729
```
Int[(cot[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Simp[(c*Cot[a
+ b*x])^m*(c*Tan[a + b*x])^m   Int[ActivateTrig[u]/(c*Tan[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownTangentIntegrandQ[u,
x]
```

Maple [F]

$$\int \frac{(a+b\tan(dx+c))^n (A+B\tan(dx+c))}{\cot(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^n}{\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorith
m="fricas")
```

output
```
integral((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/cot(d*x + c)^(3/2), x
)
```

Sympy [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\int \frac{(A+B\tan(c+dx))\,(a+b\tan(c+dx))^n}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((a+b*tan(d*x+c))**n*(A+B*tan(d*x+c))/cot(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*tan(c + d*x))*(a + b*tan(c + d*x))**n/cot(c + d*x)**(3/2),
 x)
```

Maxima [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^n}{\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/cot(d*x + c)^(3/2),
x)
```

Giac [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(B\tan(dx+c)+A)(b\tan(dx+c)+a)^n}{\cot(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x, algorith
m="giac")
```

output
```
integrate((B*tan(d*x + c) + A)*(b*tan(d*x + c) + a)^n/cot(d*x + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(A+B\tan(c+d\,x))\,(a+b\tan(c+d\,x))^n}{\cot(c+d\,x)^{3/2}}\,dx$$

input
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/cot(c + d*x)^(3/2),x)
```

output
```
int(((A + B*tan(c + d*x))*(a + b*tan(c + d*x))^n)/cot(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+b\tan(c+dx))^n(A+B\tan(c+dx))}{\cot^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \left(\int \frac{(a+\tan(dx+c)\,b)^n\sqrt{\cot(dx+c)}\,\tan(dx+c)}{\cot(dx+c)^2}dx\right)b$$
$$+\left(\int \frac{(a+\tan(dx+c)\,b)^n\sqrt{\cot(dx+c)}}{\cot(dx+c)^2}dx\right)a$$

input
```
int((a+b*tan(d*x+c))^n*(A+B*tan(d*x+c))/cot(d*x+c)^(3/2),x)
```

output
```
int(((tan(c + d*x)*b + a)**n*sqrt(cot(c + d*x))*tan(c + d*x))/cot(c + d*x)
**2,x)*b + int(((tan(c + d*x)*b + a)**n*sqrt(cot(c + d*x)))/cot(c + d*x)**
2,x)*a
```

3.665 $\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 39, antiderivative size = 63

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \frac{a(iA+B)(c-ic\tan(e+fx))^n}{fn}-\frac{aB(c-ic\tan(e+fx))^{1+n}}{cf(1+n)}$$

output
```
a*(I*A+B)*(c-I*c*tan(f*x+e))^n/f/n-a*B*(c-I*c*tan(f*x+e))^(1+n)/c/f/(1+n)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \frac{ia(c-ic\tan(e+fx))^n(A-iB+An+Bn\tan(e+fx))}{fn(1+n)}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^n,x]
```

output
```
(I*a*(c - I*c*Tan[e + f*x])^n*(A - I*B + A*n + B*n*Tan[e + f*x]))/(f*n*(1
+ n))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3042, 4071, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + ia\tan(e+fx))(A + B\tan(e+fx))(c - ic\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a + ia\tan(e+fx))(A + B\tan(e+fx))(c - ic\tan(e+fx))^n dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int (A + B\tan(e+fx))(c - ic\tan(e+fx))^{n-1} d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{53}$$

$$\frac{ac\int \left((A - iB)(c - ic\tan(e+fx))^{n-1} + \frac{iB(c-ic\tan(e+fx))^n}{c}\right) d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac\left(\frac{(B+iA)(c-ic\tan(e+fx))^n}{cn} - \frac{B(c-ic\tan(e+fx))^{n+1}}{c^2(n+1)}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n,x
]
```

output
```
(a*c*(((I*A + B)*(c - I*c*Tan[e + f*x])^n)/(c*n) - (B*(c - I*c*Tan[e + f*x
])^(1 + n))/(c^2*(1 + n))))/f
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{(iAan+iAa+Ba)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{fn(1+n)}+\frac{iBa\tan(fx+e)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{f(1+n)}$	80
default	$\frac{(iAan+iAa+Ba)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{fn(1+n)}+\frac{iBa\tan(fx+e)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{f(1+n)}$	80
norman	$\frac{(iAan+iAa+Ba)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{fn(1+n)}+\frac{iBa\tan(fx+e)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{f(1+n)}$	80
risch	Expression too large to display	1058

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x,method=_RET
URNVERBOSE)
```

output
```
1/f/n/(1+n)*(I*A*a*n+I*A*a+B*a)*exp(n*ln(c-I*c*tan(f*x+e)))+I*B*a/f/(1+n)*
tan(f*x+e)*exp(n*ln(c-I*c*tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.48

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$
$$=\frac{\left((i\,A-B)an+(i\,A+B)a+((i\,A+B)an+(i\,A+B)a)e^{(2i\,fx+2i\,e)}\right)\left(\frac{2\,c}{e^{(2i\,fx+2i\,e)}+1}\right)^n}{fn^2+fn+(fn^2+fn)e^{(2i\,fx+2i\,e)}}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, algo
rithm="fricas")
```

output
```
((I*A - B)*a*n + (I*A + B)*a + ((I*A + B)*a*n + (I*A + B)*a)*e^(2*I*f*x +
2*I*e))*(2*c/(e^(2*I*f*x + 2*I*e) + 1))^n/(f*n^2 + f*n + (f*n^2 + f*n)*e^(
2*I*f*x + 2*I*e))
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(48) = 96.

Time = 0.58 (sec) , antiderivative size = 394, normalized size of antiderivative = 6.25

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$
$$=\begin{cases} x(A+B\tan(e))\,(ia\tan(e)+a)\,(-ic\tan(e)+c)^n \\ \frac{2Aa}{2cf\tan(e+fx)+2icf}+\frac{2iBafx\tan(e+fx)}{2cf\tan(e+fx)+2icf}-\frac{2Bafx}{2cf\tan(e+fx)+2icf}-\frac{Ba\log\left(\tan^2(e+fx)+1\right)\tan(e+fx)}{2cf\tan(e+fx)+2icf}-\frac{iBa\log\left(\tan^2(e+fx\right.}{2cf\tan(e+fx)+2i} \\ Aax+\frac{iAa\log\left(\tan^2(e+fx)+1\right)}{2f}-iBax+\frac{Ba\log\left(\tan^2(e+fx)+1\right)}{2f}+\frac{iBa\tan(e+fx)}{f} \\ \frac{iAan(-ic\tan(e+fx)+c)^n}{fn^2+fn}+\frac{iAa(-ic\tan(e+fx)+c)^n}{fn^2+fn}+\frac{iBan(-ic\tan(e+fx)+c)^n\tan(e+fx)}{fn^2+fn}+\frac{Ba(-ic\tan(e+fx)+c)^n}{fn^2+fn} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**n,x)
```

output
```
Piecewise((x*(A + B*tan(e))*(I*a*tan(e) + a)*(-I*c*tan(e) + c)**n, Eq(f, 0
)), (2*A*a/(2*c*f*tan(e + f*x) + 2*I*c*f) + 2*I*B*a*f*x*tan(e + f*x)/(2*c*
f*tan(e + f*x) + 2*I*c*f) - 2*B*a*f*x/(2*c*f*tan(e + f*x) + 2*I*c*f) - B*a
*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c*f*tan(e + f*x) + 2*I*c*f) - I*
B*a*log(tan(e + f*x)**2 + 1)/(2*c*f*tan(e + f*x) + 2*I*c*f) - 2*I*B*a/(2*c
*f*tan(e + f*x) + 2*I*c*f), Eq(n, -1)), (A*a*x + I*A*a*log(tan(e + f*x)**2
 + 1)/(2*f) - I*B*a*x + B*a*log(tan(e + f*x)**2 + 1)/(2*f) + I*B*a*tan(e +
 f*x)/f, Eq(n, 0)), (I*A*a*n*(-I*c*tan(e + f*x) + c)**n/(f*n**2 + f*n) + I
*A*a*(-I*c*tan(e + f*x) + c)**n/(f*n**2 + f*n) + I*B*a*n*(-I*c*tan(e + f*x
) + c)**n*tan(e + f*x)/(f*n**2 + f*n) + B*a*(-I*c*tan(e + f*x) + c)**n/(f*
n**2 + f*n), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(57) = 114$.

Time = 0.21 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.94

$$\int (a + ia\tan(e+fx))(A + B\tan(e+fx))(c - ic\tan(e+fx))^n\,dx$$
$$= \frac{((A - iB)ac^n n + (A - iB)ac^n)2^n \cos(-2fx + n\arctan(\sin(2fx+2e), \cos(2fx+2e)+1) - 2e) +}{}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, algo
rithm="maxima")
```

output
```
(((A - I*B)*a*c^n*n + (A - I*B)*a*c^n)*2^n*cos(-2*f*x + n*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e) + ((A + I*B)*a*c^n*n + (A - I*B)*a*
c^n)*2^n*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((I*A +
B)*a*c^n*n + (I*A + B)*a*c^n)*2^n*sin(-2*f*x + n*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1) - 2*e) - ((I*A - B)*a*c^n*n + (I*A + B)*a*c^n)*2^n*
sin(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((-I*n^2 + (-I*n^2
 - I*n)*cos(2*f*x + 2*e) + (n^2 + n)*sin(2*f*x + 2*e) - I*n)*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*f)
```

Giac [F]

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$
$$=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)(-i\,c\tan(fx+e)+c)^n\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, algo
rithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)*(-I*c*tan(f*x + e) +
 c)^n, x)
```

Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.03

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx =$$
$$-\frac{(\cos(e+f\,x)-\sin(e+f\,x)\ 1\mathrm{i})\ \left(c-\frac{c\,\sin(e+f\,x)\,1\mathrm{i}}{\cos(e+f\,x)}\right)^n\left(\frac{a\,(A-B\,1\mathrm{i}+A\,n+B\,n\,1\mathrm{i})}{f\,n\,(n\,1\mathrm{i}+1\mathrm{i})}+\frac{a\,(A-B\,1\mathrm{i})\,(\cos(2\,e+2\,f\,x)+\sin(2\,e\cdots}{f\,n\,(n\,1\mathrm{i}+1\mathrm{i})}\right.}{2\,\cos(e+f\,x)}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^n
,x)
```

output
```
-((cos(e + f*x) - sin(e + f*x)*1i)*(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^
n*((a*(A - B*1i + A*n + B*n*1i))/(f*n*(n*1i + 1i)) + (a*(A - B*1i)*(cos(2*
e + 2*f*x) + sin(2*e + 2*f*x)*1i)*(n + 1))/(f*n*(n*1i + 1i))))/(2*cos(e +
f*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \frac{(-\tan(fx+e)\,ci+c)^n\,a(\tan(fx+e)\,bin+ain+ai+b)}{fn\,(n+1)}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x)
```

output
```
(( - tan(e + f*x)*c*i + c)**n*a*(tan(e + f*x)*b*i*n + a*i*n + a*i + b))/(f
*n*(n + 1))
```

3.666 $\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx$

Optimal result

Integrand size = 39, antiderivative size = 59

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = \frac{a(iA+B)c^4(1-i\tan(e+fx))^4}{4f}-\frac{aBc^4(1-i\tan(e+fx))^5}{5f}$$

output
```
1/4*a*(I*A+B)*c^4*(1-I*tan(f*x+e))^4/f-1/5*a*B*c^4*(1-I*tan(f*x+e))^5/f
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = \frac{iac^4\left(B(i+\tan(e+fx))^5+\frac{5}{4}(A-iB)\tan(e+fx)\left(-4i-6\tan(e+fx)+4i\tan^2(e+fx)+\tan^3(e+\right.\right.}{5f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^4,x]
```

output
```
((I/5)*a*c^4*(B*(I + Tan[e + f*x])^5 + (5*(A - I*B)*Tan[e + f*x]*(-4*I - 6
*Tan[e + f*x] + (4*I)*Tan[e + f*x]^2 + Tan[e + f*x]^3))/4))/f
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3042, 4071, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^4(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^4(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int c^3(1-i\tan(e+fx))^3(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{ac^4\int (1-i\tan(e+fx))^3(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{49}$$

$$\frac{ac^4\int \left(iB(1-i\tan(e+fx))^4+(A-iB)(1-i\tan(e+fx))^3\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac^4\left(\frac{1}{4}(B+iA)(1-i\tan(e+fx))^4-\frac{1}{5}B(1-i\tan(e+fx))^5\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4,x
]
```

output
```
(a*c^4*(((I*A + B)*(1 - I*Tan[e + f*x])^4)/4 - (B*(1 - I*Tan[e + f*x])^5)/
5))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

method	result
risch	$\frac{4 a \,c^{4} \left(5 i A \,{\mathrm e}^{2 i \left(f x +e \right)}+5 B \,{\mathrm e}^{2 i \left(f x +e \right)}+5 i A -3 B \right)}{5 f \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)^{5}}$
derivativedivides	$\frac{i a \,c^{4} \left(\frac{B \tan \left(f x +e \right)^{5}}{5}+\frac{\left(3 i B +A \right) \tan \left(f x +e \right)^{4}}{4}+\frac{\left(3 i A -3 B \right) \tan \left(f x +e \right)^{3}}{3}+\frac{\left(-i B -3 A \right) \tan \left(f x +e \right)^{2}}{2}-i \tan \left(f x +e \right) A \right)}{f}$
default	$\frac{i a \,c^{4} \left(\frac{B \tan \left(f x +e \right)^{5}}{5}+\frac{\left(3 i B +A \right) \tan \left(f x +e \right)^{4}}{4}+\frac{\left(3 i A -3 B \right) \tan \left(f x +e \right)^{3}}{3}+\frac{\left(-i B -3 A \right) \tan \left(f x +e \right)^{2}}{2}-i \tan \left(f x +e \right) A \right)}{f}$
norman	$\frac{A a \,c^{4} \tan \left(f x +e \right)}{f}-\frac{\left(-i A a \,c^{4}+3 B a \,c^{4}\right) \tan \left(f x +e \right)^{4}}{4 f}+\frac{\left(-3 i A a \,c^{4}+B a \,c^{4}\right) \tan \left(f x +e \right)^{2}}{2 f}-\frac{\left(i B a \,c^{4}+A a \,c^{4}\right) \tan \left(fx\right.}{f}$
parallelrisch	$\frac{4 i B a \,c^{4} \tan \left(f x +e \right)^{5}+5 i A \tan \left(f x +e \right)^{4} a \,c^{4}-20 i B \tan \left(f x +e \right)^{3} a \,c^{4}-15 B \tan \left(f x +e \right)^{4} a \,c^{4}-30 i A \tan \left(f x +e \right)^{2} a \,c^{4}-20 A \,t}{20 f}$
parts	$\frac{\left(-3 i A a \,c^{4}+B a \,c^{4}\right) \ln \left(1+\tan \left(f x +e \right)^{2}\right)}{2 f}+\frac{\left(-3 i B a \,c^{4}-2 A a \,c^{4}\right) \left(\tan \left(f x +e \right)-\arctan \left(\tan \left(f x +e \right)\right)\right)}{f}+\frac{\left(-2 i A a \,c^{4}-2 I\right.}{}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x,method=_RET
URNVERBOSE)
```

output
```
4/5*a*c^4*(5*I*A*exp(2*I*(f*x+e))+5*B*exp(2*I*(f*x+e))+5*I*A-3*B)/f/(exp(2
*I*(f*x+e))+1)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = -\frac{4\left(5\left(-i\,A-B\right)ac^4e^{(2i\,fx+2i\,e)}+\left(-5i\,A+3\,B\right)ac^4\right)}{5\left(fe^{(10i\,fx+10i\,e)}+5\,fe^{(8i\,fx+8i\,e)}+10\,fe^{(6i\,fx+6i\,e)}+10\,fe^{(4i\,fx+4i\,e)}+5\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algo
rithm="fricas")
```

output
```
-4/5*(5*(-I*A - B)*a*c^4*e^(2*I*f*x + 2*I*e) + (-5*I*A + 3*B)*a*c^4)/(f*e^
(10*I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) +
 10*f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(46) = 92$.

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.63

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = \frac{20iAac^4-12Bac^4+(20iAac^4e^{2ie}+20Bac^4e^{2ie})\,e^{2ifx}}{5fe^{10ie}e^{10ifx}+25fe^{8ie}e^{8ifx}+50fe^{6ie}e^{6ifx}+50fe^{4ie}e^{4ifx}+25fe^{2ie}e^{2ifx}+5f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**4,x)
```

output
```
(20*I*A*a*c**4 - 12*B*a*c**4 + (20*I*A*a*c**4*exp(2*I*e) + 20*B*a*c**4*exp
(2*I*e))*exp(2*I*f*x))/(5*f*exp(10*I*e)*exp(10*I*f*x) + 25*f*exp(8*I*e)*ex
p(8*I*f*x) + 50*f*exp(6*I*e)*exp(6*I*f*x) + 50*f*exp(4*I*e)*exp(4*I*f*x) +
 25*f*exp(2*I*e)*exp(2*I*f*x) + 5*f)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.61

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = -\frac{-4i\,Bac^4\tan(fx+e)^5+5\,(-i\,A+3\,B)ac^4\tan(fx+e)^4+20\,(A+i\,B)ac^4\tan(fx+e)^3+10\,(3i\,A}{20\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algo
rithm="maxima")
```

output
```
-1/20*(-4*I*B*a*c^4*tan(f*x + e)^5 + 5*(-I*A + 3*B)*a*c^4*tan(f*x + e)^4 +
 20*(A + I*B)*a*c^4*tan(f*x + e)^3 + 10*(3*I*A - B)*a*c^4*tan(f*x + e)^2 -
 20*A*a*c^4*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(49) = 98.

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.10

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = -\frac{-4i\,Bac^4\tan\left(fx+e\right)^5-5i\,Aac^4\tan\left(fx+e\right)^4+15\,Bac^4\tan\left(fx+e\right)^4+20\,Aac^4\tan\left(fx+e\right)^3+20}{20\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, algo
rithm="giac")
```

output
```
-1/20*(-4*I*B*a*c^4*tan(f*x + e)^5 - 5*I*A*a*c^4*tan(f*x + e)^4 + 15*B*a*c
^4*tan(f*x + e)^4 + 20*A*a*c^4*tan(f*x + e)^3 + 20*I*B*a*c^4*tan(f*x + e)^
3 + 30*I*A*a*c^4*tan(f*x + e)^2 - 10*B*a*c^4*tan(f*x + e)^2 - 20*A*a*c^4*t
an(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = \frac{\frac{1\mathrm{i}\,B\,a\,c^4\,\tan(e+f\,x)^5}{5}+\frac{1\mathrm{i}\,a\,(A+B\,3\mathrm{i})\,c^4\,\tan(e+f\,x)^4}{4}+1\mathrm{i}\,a\,(-B+A\,1\mathrm{i})\;c^4\tan(e+f\,x)^3-\frac{1\mathrm{i}\,a\,(3\,A+B\,1\mathrm{i})\,c^4\,\tan(e+f\,x)^2}{2}+}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^4
,x)
```

output
```
(A*a*c^4*tan(e + f*x) + (a*c^4*tan(e + f*x)^4*(A + B*3i)*1i)/4 + (B*a*c^4*
tan(e + f*x)^5*1i)/5 + a*c^4*tan(e + f*x)^3*(A*1i - B)*1i - (a*c^4*tan(e +
 f*x)^2*(3*A + B*1i)*1i)/2)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.63

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx$$
$$=\frac{\tan(fx+e)\,a\,c^4\big(4\tan(fx+e)^4\,bi+5\tan(fx+e)^3\,ai-15\tan(fx+e)^3\,b-20\tan(fx+e)^2\,a-20\,\mathrm{ta}}{20f}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x)
```

output
```
(tan(e + f*x)*a*c**4*(4*tan(e + f*x)**4*b*i + 5*tan(e + f*x)**3*a*i - 15*t
an(e + f*x)**3*b - 20*tan(e + f*x)**2*a - 20*tan(e + f*x)**2*b*i - 30*tan(
e + f*x)*a*i + 10*tan(e + f*x)*b + 20*a))/(20*f)
```

3.667 $\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 39, antiderivative size = 59

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = \frac{a(iA+B)c^3(1-i\tan(e+fx))^3}{3f}-\frac{aBc^3(1-i\tan(e+fx))^4}{4f}$$

output
```
1/3*a*(I*A+B)*c^3*(1-I*tan(f*x+e))^3/f-1/4*a*B*c^3*(1-I*tan(f*x+e))^4/f
```

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = -\frac{ac^3(3B-12A\tan(e+fx)+(12iA-6B)\tan^2(e+fx)+4(A+2iB)\tan^3(e+fx)+3B\tan^4(e+f}{12f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^3,x]
```

output
```
-1/12*(a*c^3*(3*B - 12*A*Tan[e + f*x] + ((12*I)*A - 6*B)*Tan[e + f*x]^2 +
4*(A + (2*I)*B)*Tan[e + f*x]^3 + 3*B*Tan[e + f*x]^4))/f
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 4071, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^3(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^3(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int c^2(1-i\tan(e+fx))^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{ac^3\int (1-i\tan(e+fx))^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{49}$$

$$\frac{ac^3\int \left(iB(1-i\tan(e+fx))^3+(A-iB)(1-i\tan(e+fx))^2\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac^3\left(\frac{1}{3}(B+iA)(1-i\tan(e+fx))^3-\frac{1}{4}B(1-i\tan(e+fx))^4\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3,x
]
```

output
```
(a*c^3*(((I*A + B)*(1 - I*Tan[e + f*x])^3)/3 - (B*(1 - I*Tan[e + f*x])^4)/
4))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

method	result
risch	$\frac{4a\,c^{3}\left(2iA\,{\mathrm e}^{2i(fx+e)}+2B\,{\mathrm e}^{2i(fx+e)}+2iA-B\right)}{3f\left({\mathrm e}^{2i(fx+e)}+1\right)^{4}}$
derivativedivides	$\frac{a\,c^{3}\left(-\frac{B\tan(fx+e)^{4}}{4}-\frac{(2iB+A)\tan(fx+e)^{3}}{3}-\frac{(2iA-B)\tan(fx+e)^{2}}{2}+A\tan(fx+e)\right)}{f}$
default	$\frac{a\,c^{3}\left(-\frac{B\tan(fx+e)^{4}}{4}-\frac{(2iB+A)\tan(fx+e)^{3}}{3}-\frac{(2iA-B)\tan(fx+e)^{2}}{2}+A\tan(fx+e)\right)}{f}$
norman	$\frac{Aa\,c^{3}\tan(fx+e)}{f}-\frac{\left(2iBa\,c^{3}+Aa\,c^{3}\right)\tan(fx+e)^{3}}{3f}+\frac{\left(-2iAa\,c^{3}+Ba\,c^{3}\right)\tan(fx+e)^{2}}{2f}-\frac{Ba\,c^{3}\tan(fx+e)^{4}}{4f}$
parallelrisch	$-\frac{8iB\tan(fx+e)^{3}a\,c^{3}+3B\tan(fx+e)^{4}a\,c^{3}+12iA\tan(fx+e)^{2}a\,c^{3}+4A\tan(fx+e)^{3}a\,c^{3}-6B\tan(fx+e)^{2}a\,c^{3}-12A\tan}{12f}$
parts	$\frac{\left(-2iAa\,c^{3}+Ba\,c^{3}\right)\ln\left(1+\tan(fx+e)^{2}\right)}{2f}+\frac{\left(-2iBa\,c^{3}-Aa\,c^{3}\right)\left(\frac{\tan(fx+e)^{3}}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+A$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x,method=_RET
URNVERBOSE)
```

output
```
4/3*a*c^3*(2*I*A*exp(2*I*(f*x+e))+2*B*exp(2*I*(f*x+e))+2*I*A-B)/f/(exp(2*I
*(f*x+e))+1)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int(a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3}\,dx$$
$$=-\frac{4\left(2\left(-i\,A-B\right)ac^{3}e^{(2i\,fx+2i\,e)}+\left(-2i\,A+B\right)ac^{3}\right)}{3\left(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algo
rithm="fricas")
```

output
```
-4/3*(2*(-I*A - B)*a*c^3*e^(2*I*f*x + 2*I*e) + (-2*I*A + B)*a*c^3)/(f*e^(8
*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*
e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.31

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = \frac{8iAac^3-4Bac^3+(8iAac^3e^{2ie}+8Bac^3e^{2ie})\,e^{2ifx}}{3fe^{8ie}e^{8ifx}+12fe^{6ie}e^{6ifx}+18fe^{4ie}e^{4ifx}+12fe^{2ie}e^{2ifx}+3f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**3,x)
```

output
```
(8*I*A*a*c**3 - 4*B*a*c**3 + (8*I*A*a*c**3*exp(2*I*e) + 8*B*a*c**3*exp(2*I
*e))*exp(2*I*f*x))/(3*f*exp(8*I*e)*exp(8*I*f*x) + 12*f*exp(6*I*e)*exp(6*I*
f*x) + 18*f*exp(4*I*e)*exp(4*I*f*x) + 12*f*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = -\frac{3\,Bac^3\tan(fx+e)^4+4\,(A+2i\,B)ac^3\tan(fx+e)^3-6\,(-2i\,A+B)ac^3\tan(fx+e)^2-12\,Aac^3\tan}{12\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algo
rithm="maxima")
```

output
```
-1/12*(3*B*a*c^3*tan(f*x + e)^4 + 4*(A + 2*I*B)*a*c^3*tan(f*x + e)^3 - 6*(
-2*I*A + B)*a*c^3*tan(f*x + e)^2 - 12*A*a*c^3*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int (a + ia\tan(e + fx))(A + B\tan(e + fx))(c - ic\tan(e + fx))^3 \, dx =$$
$$-\frac{3\,Bac^3\tan(fx+e)^4 + 4\,Aac^3\tan(fx+e)^3 + 8i\,Bac^3\tan(fx+e)^3 + 12i\,Aac^3\tan(fx+e)^2 - 6\,Ba}{12\,f}$$

input

```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, algo
rithm="giac")
```

output

```
-1/12*(3*B*a*c^3*tan(f*x + e)^4 + 4*A*a*c^3*tan(f*x + e)^3 + 8*I*B*a*c^3*t
an(f*x + e)^3 + 12*I*A*a*c^3*tan(f*x + e)^2 - 6*B*a*c^3*tan(f*x + e)^2 - 1
2*A*a*c^3*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int (a + ia\tan(e + fx))(A + B\tan(e + fx))(c - ic\tan(e + fx))^3 \, dx$$
$$= -\frac{\frac{B\,a\,c^3\,\tan(e+f\,x)^4}{4} + \frac{a\,(A+B\,2\mathrm{i})\,c^3\,\tan(e+f\,x)^3}{3} + \frac{a\,(-B+A\,2\mathrm{i})\,c^3\,\tan(e+f\,x)^2}{2} - A\,a\,c^3\tan(e+f\,x)}{f}$$

input

```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^3
,x)
```

output

```
-((a*c^3*tan(e + f*x)^3*(A + B*2i))/3 - A*a*c^3*tan(e + f*x) + (B*a*c^3*ta
n(e + f*x)^4)/4 + (a*c^3*tan(e + f*x)^2*(A*2i - B))/2)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx$$
$$=\frac{\tan(fx+e)\,a\,c^3\left(-3\tan(fx+e)^3 b-4\tan(fx+e)^2 a-8\tan(fx+e)^2 bi-12\tan(fx+e)\,ai+6\tan\right.}{12f}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x)
```

output
```
(tan(e + f*x)*a*c**3*( - 3*tan(e + f*x)**3*b - 4*tan(e + f*x)**2*a - 8*tan
(e + f*x)**2*b*i - 12*tan(e + f*x)*a*i + 6*tan(e + f*x)*b + 12*a))/(12*f)
```

3.668 $\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 39, antiderivative size = 66

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = \frac{aAc^2\tan(e+fx)}{f}-\frac{a(iA-B)c^2\tan^2(e+fx)}{2f}-\frac{iaBc^2\tan^3(e+fx)}{3f}$$

output
```
a*A*c^2*tan(f*x+e)/f-1/2*a*(I*A-B)*c^2*tan(f*x+e)^2/f-1/3*I*a*B*c^2*tan(f*
x+e)^3/f
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = \frac{ac^2(-2B+6A\tan(e+fx)+3(-iA+B)\tan^2(e+fx)-2iB\tan^3(e+fx))}{6f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^2,x]
```

output
```
(a*c^2*(-2*B + 6*A*Tan[e + f*x] + 3*((-I)*A + B)*Tan[e + f*x]^2 - (2*I)*B*
Tan[e + f*x]^3))/(6*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 4071, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^2(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^2(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int c(1-i\tan(e+fx))(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{ac^2\int (1-i\tan(e+fx))(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{49}$$

$$\frac{ac^2\int \left(-iB\tan^2(e+fx)+(B-iA)\tan(e+fx)+A\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac^2\left(-\frac{1}{2}(-B+iA)\tan^2(e+fx)+A\tan(e+fx)-\frac{1}{3}iB\tan^3(e+fx)\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2,x
]
```

output
```
(a*c^2*(A*Tan[e + f*x] - ((I*A - B)*Tan[e + f*x]^2)/2 - (I/3)*B*Tan[e + f*
x]^3))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{i a\,c^{2} \left(\frac{B \tan \left(f x +e \right)^{3}}{3}+\frac{\left(i B +A \right) \tan \left(f x +e \right)^{2}}{2}+i \tan \left(f x +e \right) A \right)}{f}$
default	$-\frac{i a\,c^{2} \left(\frac{B \tan \left(f x +e \right)^{3}}{3}+\frac{\left(i B +A \right) \tan \left(f x +e \right)^{2}}{2}+i \tan \left(f x +e \right) A \right)}{f}$
risch	$\frac{2 a\,c^{2} \left(3 i A\,{\mathrm e}^{2 i \left(f x +e \right)}+3 B\,{\mathrm e}^{2 i \left(f x +e \right)}+3 i A -B \right)}{3 f \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)^{3}}$
norman	$\frac{a A\,c^{2} \tan \left(f x +e \right)}{f}+\frac{\left(-i A a\,c^{2}+B a\,c^{2}\right) \tan \left(f x +e \right)^{2}}{2 f}-\frac{i a B\,c^{2} \tan \left(f x +e \right)^{3}}{3 f}$
parallelrisch	$-\frac{2 i a B\,c^{2} \tan \left(f x +e \right)^{3}+3 i A \tan \left(f x +e \right)^{2} a\,c^{2}-3 B \tan \left(f x +e \right)^{2} a\,c^{2}-6 A \tan \left(f x +e \right) a\,c^{2}}{6 f}$
parts	$\frac{\left(-i A a\,c^{2}+B a\,c^{2}\right) \ln \left(1+\tan \left(f x +e \right)^{2}\right)}{2 f}+\frac{\left(-i A a\,c^{2}+B a\,c^{2}\right) \left(\frac{\tan \left(f x +e \right)^{2}}{2}-\frac{\ln \left(1+\tan \left(f x +e \right)^{2}\right)}{2}\right)}{f}+\frac{\left(-i B a\,c^{2}+A a\,c^{2}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x,method=_RET
URNVERBOSE)
```

output
```
-I/f*a*c^2*(1/3*B*tan(f*x+e)^3+1/2*(A+I*B)*tan(f*x+e)^2+I*A*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$$
$$=-\frac{2\left(3\left(-i\,A-B\right)ac^{2}e^{\left(2i\,fx+2i\,e\right)}+\left(-3i\,A+B\right)ac^{2}\right)}{3\left(fe^{\left(6i\,fx+6i\,e\right)}+3\,fe^{\left(4i\,fx+4i\,e\right)}+3\,fe^{\left(2i\,fx+2i\,e\right)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, algo
rithm="fricas")
```

output
```
-2/3*(3*(-I*A - B)*a*c^2*e^(2*I*f*x + 2*I*e) + (-3*I*A + B)*a*c^2)/(f*e^(6
*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(56) = 112$.

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.77

$$\int (a + ia\tan(e + fx))(A + B\tan(e + fx))(c - ic\tan(e + fx))^2 \, dx = \frac{6iAac^2 - 2Bac^2 + (6iAac^2e^{2ie} + 6Bac^2e^{2ie})\, e^{2ifx}}{3fe^{6ie}e^{6ifx} + 9fe^{4ie}e^{4ifx} + 9fe^{2ie}e^{2ifx} + 3f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**2,x)
```

output
```
(6*I*A*a*c**2 - 2*B*a*c**2 + (6*I*A*a*c**2*exp(2*I*e) + 6*B*a*c**2*exp(2*I
*e))*exp(2*I*f*x))/(3*f*exp(6*I*e)*exp(6*I*f*x) + 9*f*exp(4*I*e)*exp(4*I*f
*x) + 9*f*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int (a + ia\tan(e + fx))(A + B\tan(e + fx))(c - ic\tan(e + fx))^2 \, dx = \frac{-2i\,Bac^2\tan(fx + e)^3 - 3\,(i\,A - B)ac^2\tan(fx + e)^2 + 6\,Aac^2\tan(fx + e)}{6\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, algo
rithm="maxima")
```

output
```
1/6*(-2*I*B*a*c^2*tan(f*x + e)^3 - 3*(I*A - B)*a*c^2*tan(f*x + e)^2 + 6*A*
a*c^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = -\frac{2i\,Bac^2\tan(fx+e)^3+3i\,Aac^2\tan(fx+e)^2-3\,Bac^2\tan(fx+e)^2-6\,Aac^2\tan(fx+e)}{6\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, algo
rithm="giac")
```

output
```
-1/6*(2*I*B*a*c^2*tan(f*x + e)^3 + 3*I*A*a*c^2*tan(f*x + e)^2 - 3*B*a*c^2*
tan(f*x + e)^2 - 6*A*a*c^2*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = \frac{a\,c^2\tan(e+f\,x)\,\left(6\,A-A\tan(e+f\,x)\,3\mathrm{i}+3\,B\tan(e+f\,x)-B\tan(e+f\,x)^2\,2\mathrm{i}\right)}{6\,f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^2
,x)
```

output
```
(a*c^2*tan(e + f*x)*(6*A - A*tan(e + f*x)*3i + 3*B*tan(e + f*x) - B*tan(e
+ f*x)^2*2i))/(6*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = \frac{\tan(fx+e)\,a\,c^2\left(-2\tan(fx+e)^2\,bi-3\tan(fx+e)\,ai+3\tan(fx+e)\,b+6a\right)}{6f}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x)
```

output
```
(tan(e + f*x)*a*c**2*( - 2*tan(e + f*x)**2*b*i - 3*tan(e + f*x)*a*i + 3*ta
n(e + f*x)*b + 6*a))/(6*f)
```

3.669 $\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx$

Optimal result

Integrand size = 37, antiderivative size = 24

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{ac(A+B\tan(e+fx))^2}{2Bf}$$

output
```
1/2*a*c*(A+B*tan(f*x+e))^2/B/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{aBc\sec^2(e+fx)}{2f} + \frac{aAc\tan(e+fx)}{f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
]),x]
```

output
```
(a*B*c*Sec[e + f*x]^2)/(2*f) + (a*A*c*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3042, 4071, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int (A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{17}$$

$$\frac{ac(A+B\tan(e+fx))^2}{2Bf}$$

input
```
Int[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]),x]
```

output
```
(a*c*(A + B*Tan[e + f*x])^2)/(2*B*f)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{ac\left(\frac{B\tan(fx+e)^{2}}{2}+A\tan(fx+e)\right)}{f}$
default	$\frac{ac\left(\frac{B\tan(fx+e)^{2}}{2}+A\tan(fx+e)\right)}{f}$
parallelrisch	$\frac{Bac\tan(fx+e)^{2}+2Aac\tan(fx+e)}{2f}$
norman	$\frac{Aac\tan(fx+e)}{f}+\frac{Bac\tan(fx+e)^{2}}{2f}$
risch	$\frac{2ac\left(iA\,e^{2i(fx+e)}+B\,e^{2i(fx+e)}+iA\right)}{f\left(e^{2i(fx+e)}+1\right)^{2}}$
parts	$Aacx+\frac{Aac(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}+\frac{Bac\ln\left(1+\tan(fx+e)^{2}\right)}{2f}+\frac{Bac\left(\frac{\tan(fx+e)^{2}}{2}-\frac{\ln\left(1+\tan(fx+e\right)}{2}\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x,method=_RETUR
NVERBOSE)
```

output
```
1/f*a*c*(1/2*B*tan(f*x+e)^2+A*tan(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx$$
$$=-\frac{2\left((-i\,A-B)ace^{(2i\,fx+2i\,e)}-i\,Aac\right)}{fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algori
thm="fricas")
```

output
```
-2*((-I*A - B)*a*c*e^(2*I*f*x + 2*I*e) - I*A*a*c)/(f*e^(4*I*f*x + 4*I*e) +
 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.42

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx$$
$$=\frac{2iAac+(2iAace^{2ie}+2Bace^{2ie})\,e^{2ifx}}{fe^{4ie}e^{4ifx}+2fe^{2ie}e^{2ifx}+f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x)
```

output
```
(2*I*A*a*c + (2*I*A*a*c*exp(2*I*e) + 2*B*a*c*exp(2*I*e))*exp(2*I*f*x))/(f*
exp(4*I*e)*exp(4*I*f*x) + 2*f*exp(2*I*e)*exp(2*I*f*x) + f)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{Bac\tan{(fx+e)}^2+2\,Aac\tan{(fx+e)}}{2\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algori
thm="maxima")
```

output
```
1/2*(B*a*c*tan(f*x + e)^2 + 2*A*a*c*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{Bac\tan{(fx+e)}^2+2\,Aac\tan{(fx+e)}}{2\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algori
thm="giac")
```

output
```
1/2*(B*a*c*tan(f*x + e)^2 + 2*A*a*c*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{a\,c\tan(e+f\,x)\,\left(2\,A+B\tan(e+f\,x)\right)}{2\,f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i),x
)
```

output
```
(a*c*tan(e + f*x)*(2*A + B*tan(e + f*x)))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{\tan(fx+e)\,ac(\tan(fx+e)\,b+2a)}{2f}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x)
```

output
```
(tan(e + f*x)*a*c*(tan(e + f*x)*b + 2*a))/(2*f)
```

3.670 $\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\,dx = a(A-iB)x-\frac{a(iA+B)\log(\cos(e+fx))}{f}+\frac{iaB\tan(e+fx)}{f}$$

output
```
a*(A-I*B)*x-a*(I*A+B)*ln(cos(f*x+e))/f+I*a*B*tan(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\,dx = aAx-\frac{iaB\arctan(\tan(e+fx))}{f}-\frac{iaA\log(\cos(e+fx))}{f}-\frac{aB\log(\cos(e+fx))}{f}+\frac{iaB\tan(e+fx)}{f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]),x]
```

output
```
a*A*x - (I*a*B*ArcTan[Tan[e + f*x]])/f - (I*a*A*Log[Cos[e + f*x]])/f - (a*
B*Log[Cos[e + f*x]])/f + (I*a*B*Tan[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4008}$$

$$a(B+iA)\int \tan(e+fx)dx+ax(A-iB)+\frac{iaB\tan(e+fx)}{f}$$

$$\Big\downarrow \text{3042}$$

$$a(B+iA)\int \tan(e+fx)dx+ax(A-iB)+\frac{iaB\tan(e+fx)}{f}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{a(B+iA)\log(\cos(e+fx))}{f}+ax(A-iB)+\frac{iaB\tan(e+fx)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]),x]
```

output
```
a*(A - I*B)*x - (a*(I*A + B)*Log[Cos[e + f*x]])/f + (I*a*B*Tan[e + f*x])/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{a\left(iB\tan(fx+e)+\frac{(iA+B)\ln\left(1+\tan(fx+e)^2\right)}{2}+(-iB+A)\arctan(\tan(fx+e))\right)}{f}$	50
default	$\frac{a\left(iB\tan(fx+e)+\frac{(iA+B)\ln\left(1+\tan(fx+e)^2\right)}{2}+(-iB+A)\arctan(\tan(fx+e))\right)}{f}$	50
norman	$(-iaB+Aa)\,x+\frac{iaB\tan(fx+e)}{f}+\frac{(iAa+Ba)\ln\left(1+\tan(fx+e)^2\right)}{2f}$	52
parts	$Aax+\frac{(iAa+Ba)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{iBa(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}$	55
parallelrisch	$\frac{-2iBxaf+iA\ln\left(1+\tan(fx+e)^2\right)a+2Axaf+2iaB\tan(fx+e)+B\ln\left(1+\tan(fx+e)^2\right)a}{2f}$	61
risch	$\frac{2iaBe}{f}-\frac{2aAe}{f}-\frac{2aB}{f\left(e^{2i(fx+e)}+1\right)}-\frac{a\ln\left(e^{2i(fx+e)}+1\right)B}{f}-\frac{ia\ln\left(e^{2i(fx+e)}+1\right)A}{f}$	78

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*a*(I*B*tan(f*x+e)+1/2*(I*A+B)*ln(1+tan(f*x+e)^2)+(A-I*B)*arctan(tan(f*
x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\,dx = -\frac{2\,Ba-\left(\left(-i\,A-B\right)ae^{\left(2i\,fx+2i\,e\right)}+\left(-i\,A-B\right)a\right)\log\left(e^{\left(2i\,fx+2i\,e\right)}+1\right)}{fe^{\left(2i\,fx+2i\,e\right)}+f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e)),x, algorithm="fricas")
```

output
```
-(2*B*a - ((-I*A - B)*a*e^(2*I*f*x + 2*I*e) + (-I*A - B)*a)*log(e^(2*I*f*x
 + 2*I*e) + 1))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\,dx = -\frac{2Ba}{fe^{2ie}e^{2ifx}+f}-\frac{ia(A-iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e)),x)
```

output
```
-2*B*a/(f*exp(2*I*e)*exp(2*I*f*x) + f) - I*a*(A - I*B)*log(exp(2*I*f*x) +
exp(-2*I*e))/f
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\,dx = \frac{2\,(fx+e)(A-i\,B)a-(-i\,A-B)a\log\left(\tan\left(fx+e\right)^2+1\right)+2i\,Ba\tan\left(fx+e\right)}{2\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*(2*(f*x + e)*(A - I*B)*a - (-I*A - B)*a*log(tan(f*x + e)^2 + 1) + 2*I*
B*a*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\,dx = \frac{i\,Ba\tan\left(fx+e\right)}{f}+\frac{(i\,Aa+Ba)\log\left(\tan\left(fx+e\right)+i\right)}{f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e)),x, algorithm="giac")
```

output
```
I*B*a*tan(f*x + e)/f + (I*A*a + B*a)*log(tan(f*x + e) + I)/f
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\,dx = \frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(B\,a+A\,a\,1\mathrm{i}\right)}{f}+\frac{B\,a\tan(e+f\,x)\,1\mathrm{i}}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i),x)
```

output
```
(log(tan(e + f*x) + 1i)*(A*a*1i + B*a))/f + (B*a*tan(e + f*x)*1i)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int (a + ia\tan(e + fx))(A + B\tan(e + fx))\,dx$$
$$= \frac{a\left(\log\left(\tan\left(fx+e\right)^{2}+1\right)ai+\log\left(\tan\left(fx+e\right)^{2}+1\right)b+2\tan\left(fx+e\right)bi+2afx-2bfix\right)}{2f}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e)),x)
```

output
```
(a*(log(tan(e + f*x)**2 + 1)*a*i + log(tan(e + f*x)**2 + 1)*b + 2*tan(e +
f*x)*b*i + 2*a*f*x - 2*b*f*i*x))/(2*f)
```

3.671 $\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 54

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \frac{iaBx}{c}+\frac{aB\log(\cos(e+fx))}{cf}+\frac{a(A-iB)}{cf(i+\tan(e+fx))}$$

output
```
I*a*B*x/c+a*B*ln(cos(f*x+e))/c/f+a*(A-I*B)/c/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = -\frac{a\left(B\log(i+\tan(e+fx))-\frac{A-iB}{i+\tan(e+fx)}\right)}{cf}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x]),x]
```

output
```
-((a*(B*Log[I + Tan[e + f*x]] - (A - I*B)/(I + Tan[e + f*x])))/(c*f))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 4071, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{c^2(1-i\tan(e+fx))^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^2}d\tan(e+fx)}{cf}$$

$$\Big\downarrow \text{49}$$

$$\frac{a\int \left(\frac{iB-A}{(\tan(e+fx)+i)^2}-\frac{B}{\tan(e+fx)+i}\right)d\tan(e+fx)}{cf}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{A-iB}{\tan(e+fx)+i}-B\log(\tan(e+fx)+i)\right)}{cf}$$

input
```
Int[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x]),x
]
```

output
```
(a*(-(B*Log[I + Tan[e + f*x]]) + (A - I*B)/(I + Tan[e + f*x])))/(c*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

method	result	size
risch	$-\frac{e^{2i(fx+e)}aB}{2cf}-\frac{ie^{2i(fx+e)}Aa}{2cf}-\frac{2iBae}{cf}+\frac{Ba\ln\left(e^{2i(fx+e)}+1\right)}{cf}$	74
derivativedivides	$-\frac{iaB}{fc(i+\tan(fx+e))}+\frac{aA}{fc(i+\tan(fx+e))}-\frac{aB\ln\left(1+\tan(fx+e)^{2}\right)}{2fc}+\frac{iaB\arctan(\tan(fx+e))}{fc}$	83
default	$-\frac{iaB}{fc(i+\tan(fx+e))}+\frac{aA}{fc(i+\tan(fx+e))}-\frac{aB\ln\left(1+\tan(fx+e)^{2}\right)}{2fc}+\frac{iaB\arctan(\tan(fx+e))}{fc}$	83
norman	$\frac{\frac{(-iaB+Aa)\tan(fx+e)}{cf}+\frac{iaBx}{c}+\frac{iaBx\tan(fx+e)^{2}}{c}-\frac{iAa+Ba}{cf}}{1+\tan(fx+e)^{2}}-\frac{aB\ln\left(1+\tan(fx+e)^{2}\right)}{2fc}$	102

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,method=_RETUR
NVERBOSE)
```

output
```
-1/2/c/f*exp(2*I*(f*x+e))*a*B-1/2*I/c/f*exp(2*I*(f*x+e))*A*a-2*I/c/f*B*a*e
+1/c/f*B*a*ln(exp(2*I*(f*x+e))+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int\frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx=\frac{(-i\,A-B)ae^{(2i\,fx+2i\,e)}+2\,Ba\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{2\,cf}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algori
thm="fricas")
```

output
```
1/2*((-I*A - B)*a*e^(2*I*f*x + 2*I*e) + 2*B*a*log(e^(2*I*f*x + 2*I*e) + 1)
)/(c*f)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.63

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \frac{Ba\log\left(e^{2ifx}+e^{-2ie}\right)}{cf} + \begin{cases} \frac{(-iAae^{2ie}-Bae^{2ie})e^{2ifx}}{2cf} & \text{for } cf\neq 0 \\ \frac{x(Aae^{2ie}-iBae^{2ie})}{c} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

output
```
B*a*log(exp(2*I*f*x) + exp(-2*I*e))/(c*f) + Piecewise(((-I*A*a*exp(2*I*e)
- B*a*exp(2*I*e))*exp(2*I*f*x)/(2*c*f), Ne(c*f, 0)), (x*(A*a*exp(2*I*e) -
I*B*a*exp(2*I*e))/c, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = -\frac{Ba\log\left(\tan\left(fx+e\right)+i\right)}{cf}+\frac{Aa-i\,Ba}{cf(\tan\left(fx+e\right)+i)}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algori
thm="giac")
```

output
```
-B*a*log(tan(f*x + e) + I)/(c*f) + (A*a - I*B*a)/(c*f*(tan(f*x + e) + I))
```

Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \frac{\frac{A\,a}{c}-\frac{B\,a\,1\mathrm{i}}{c}}{f\,\left(\tan\left(e+f\,x\right)+1\mathrm{i}\right)}-\frac{B\,a\,\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)}{c\,f}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)
,x)
```

output
```
((A*a)/c - (B*a*1i)/c)/(f*(tan(e + f*x) + 1i)) - (B*a*log(tan(e + f*x) + 1
i))/(c*f)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = -\frac{a\left(\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)i-1}dx\right)bi+\left(\int \frac{\tan(fx+e)}{\tan(fx+e)i-1}dx\right)ai+\left(\int \frac{\tan(fx+e)}{\tan(fx+e)i-1}dx\right)b+\left(\int \frac{1}{\tan(fx+e)i-1}dx\right)a\right)}{c}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

output
```
( - a*(int(tan(e + f*x)**2/(tan(e + f*x)*i - 1),x)*b*i + int(tan(e + f*x)/
(tan(e + f*x)*i - 1),x)*a*i + int(tan(e + f*x)/(tan(e + f*x)*i - 1),x)*b +
 int(1/(tan(e + f*x)*i - 1),x)*a))/c
```

3.672 $\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 46

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx=\frac{a(A+B\tan(e+fx))^2}{2(iA+B)c^2f(1-i\tan(e+fx))^2}$$

output
```
1/2*a*(A+B*tan(f*x+e))^2/(I*A+B)/c^2/f/(1-I*tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx=-\frac{a(A+B\tan(e+fx))^2}{2(iA+B)c^2f(i+\tan(e+fx))^2}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^2,x]
```

output
```
-1/2*(a*(A + B*Tan[e + f*x])^2)/((I*A + B)*c^2*f*(I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3042, 4071, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{c^3(1-i\tan(e+fx))^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^3}d\tan(e+fx)}{c^2f}$$

$$\Big\downarrow \text{48}$$

$$\frac{a(A+B\tan(e+fx))^2}{2c^2f(B+iA)(1-i\tan(e+fx))^2}$$

input
```
Int[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^2
,x]
```

output
```
(a*(A + B*Tan[e + f*x])^2)/(2*(I*A + B)*c^2*f*(1 - I*Tan[e + f*x])^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{a\left(-\frac{-iA-B}{2(i+\tan(fx+e))^2}+\frac{iB}{i+\tan(fx+e)}\right)}{f\,c^2}$	46
default	$\frac{a\left(-\frac{-iA-B}{2(i+\tan(fx+e))^2}+\frac{iB}{i+\tan(fx+e)}\right)}{f\,c^2}$	46
risch	$-\frac{a\,\mathrm{e}^{4i(fx+e)}B}{8f\,c^2}-\frac{ia\,\mathrm{e}^{4i(fx+e)}A}{8f\,c^2}+\frac{a\,\mathrm{e}^{2i(fx+e)}B}{4f\,c^2}-\frac{ia\,\mathrm{e}^{2i(fx+e)}A}{4f\,c^2}$	80
norman	$\frac{\frac{Aa\tan(fx+e)}{cf}+\frac{iaB\tan(fx+e)^3}{cf}+\frac{-iAa+Ba}{2cf}+\frac{(iAa+3Ba)\tan(fx+e)^2}{2cf}}{c\left(1+\tan(fx+e)^2\right)^2}$	95

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/f*a/c^2*(-1/2*(-I*A-B)/(I+tan(f*x+e))^2+I*B/(I+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = \frac{(-iA-B)ae^{(4i\,fx+4i\,e)}-2\,(iA-B)ae^{(2i\,fx+2i\,e)}}{8\,c^2f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algo
rithm="fricas")
```

output
```
1/8*((-I*A - B)*a*e^(4*I*f*x + 4*I*e) - 2*(I*A - B)*a*e^(2*I*f*x + 2*I*e))
/(c^2*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(36) = 72.

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.33

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = \begin{cases} \frac{(-8iAac^2fe^{2ie}+8Bac^2fe^{2ie})e^{2ifx}+(-4iAac^2fe^{4ie}-4Bac^2fe^{4ie})e^{4ifx}}{32c^4f^2} & \text{for } c^4f^2\neq 0 \\ \frac{x(Aae^{4ie}+Aae^{2ie}-iBae^{4ie}+iBae^{2ie})}{2c^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**2,x)
```

output
```
Piecewise((((-8*I*A*a*c**2*f*exp(2*I*e) + 8*B*a*c**2*f*exp(2*I*e))*exp(2*I
*f*x) + (-4*I*A*a*c**2*f*exp(4*I*e) - 4*B*a*c**2*f*exp(4*I*e))*exp(4*I*f*x
))/(32*c**4*f**2), Ne(c**4*f**2, 0)), (x*(A*a*exp(4*I*e) + A*a*exp(2*I*e)
- I*B*a*exp(4*I*e) + I*B*a*exp(2*I*e))/(2*c**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = -\frac{-2i\,Ba\tan(fx+e)-i\,Aa+Ba}{2\,c^2f(\tan(fx+e)+i)^2}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algo
rithm="giac")
```

output
```
-1/2*(-2*I*B*a*tan(f*x + e) - I*A*a + B*a)/(c^2*f*(tan(f*x + e) + I)^2)
```

Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = \frac{\frac{a\,(-B+A\,1\mathrm{i})}{2}+B\,a\tan(e+f\,x)\,1\mathrm{i}}{c^2\,f\,\left(\tan\left(e+f\,x\right)^2+\tan\left(e+f\,x\right)\,2\mathrm{i}-1\right)}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)
^2,x)
```

output
```
((a*(A*1i - B))/2 + B*a*tan(e + f*x)*1i)/(c^2*f*(tan(e + f*x)*2i + tan(e +
 f*x)^2 - 1))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx =$$
$$-\frac{a\left(\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^2+2\tan(fx+e)i-1}dx\right)bi+\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^2+2\tan(fx+e)i-1}dx\right)ai+\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^2+2\tan(fx+e)i-1}dx\right.}{c^2}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x)
```

output
```
( - a*(int(tan(e + f*x)**2/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*b*i
 + int(tan(e + f*x)/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*a*i + int(
tan(e + f*x)/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*b + int(1/(tan(e
+ f*x)**2 + 2*tan(e + f*x)*i - 1),x)*a))/c**2
```

3.673 $\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 55

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = -\frac{a(A-iB)}{3c^3f(i+\tan(e+fx))^3}-\frac{aB}{2c^3f(i+\tan(e+fx))^2}$$

output
```
-1/3*a*(A-I*B)/c^3/f/(I+tan(f*x+e))^3-1/2*a*B/c^3/f/(I+tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = -\frac{a(2A+iB+3B\tan(e+fx))}{6c^3f(i+\tan(e+fx))^3}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^3,x]
```

output
```
-1/6*(a*(2*A + I*B + 3*B*Tan[e + f*x]))/(c^3*f*(I + Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3042, 4071, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{c^4(1-i\tan(e+fx))^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^4}d\tan(e+fx)}{c^3f}$$

$$\Big\downarrow \text{53}$$

$$\frac{a\int \left(\frac{A-iB}{(\tan(e+fx)+i)^4}+\frac{B}{(\tan(e+fx)+i)^3}\right)d\tan(e+fx)}{c^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(-\frac{A-iB}{3(\tan(e+fx)+i)^3}-\frac{B}{2(\tan(e+fx)+i)^2}\right)}{c^3f}$$

input
```
Int[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^3
,x]
```

output
```
(a*(-1/3*(A - I*B)/(I + Tan[e + f*x])^3 - B/(2*(I + Tan[e + f*x])^2)))/(c^
3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{a\left(-\frac{-iB+A}{3(i+\tan(fx+e))^{3}}-\frac{B}{2(i+\tan(fx+e))^{2}}\right)}{f\,c^{3}}$	43
default	$\frac{a\left(-\frac{-iB+A}{3(i+\tan(fx+e))^{3}}-\frac{B}{2(i+\tan(fx+e))^{2}}\right)}{f\,c^{3}}$	43
risch	$-\frac{a\,{\mathrm e}^{6i(fx+e)}B}{24c^{3}f}-\frac{ia\,{\mathrm e}^{6i(fx+e)}A}{24c^{3}f}-\frac{iAa\,{\mathrm e}^{4i(fx+e)}}{8c^{3}f}+\frac{a\,{\mathrm e}^{2i(fx+e)}B}{8c^{3}f}-\frac{ia\,{\mathrm e}^{2i(fx+e)}A}{8c^{3}f}$	100

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,method=_RET
URNVERBOSE)
```

output
```
1/f*a/c^3*(-1/3*(A-I*B)/(I+tan(f*x+e))^3-1/2*B/(I+tan(f*x+e))^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \frac{(-iA-B)ae^{(6i\,fx+6i\,e)}-3i\,Aae^{(4i\,fx+4i\,e)}-3\,(i\,A-B)ae^{(2i\,fx+2i\,e)}}{24\,c^3f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algo
rithm="fricas")
```

output
```
1/24*((-I*A - B)*a*e^(6*I*f*x + 6*I*e) - 3*I*A*a*e^(4*I*f*x + 4*I*e) - 3*(
I*A - B)*a*e^(2*I*f*x + 2*I*e))/(c^3*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(44) = 88.

Time = 0.24 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.65

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \begin{cases} \frac{-192iAac^6f^2e^{4ie}e^{4ifx}+(-192iAac^6f^2e^{2ie}+192Bac^6f^2e^{2ie})e^{2ifx}+(-64iAac^6f^2e^{6ie}-64Bac^6f^2e^{6ie})e^{6ifx}}{1536c^9f^3} & \text{for } c^9f^3\neq 0 \\ \frac{x(Aae^{6ie}+2Aae^{4ie}+Aae^{2ie}-iBae^{6ie}+iBae^{2ie})}{4c^3} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**3,x)
```

output
```
Piecewise(((-192*I*A*a*c**6*f**2*exp(4*I*e)*exp(4*I*f*x) + (-192*I*A*a*c**
6*f**2*exp(2*I*e) + 192*B*a*c**6*f**2*exp(2*I*e))*exp(2*I*f*x) + (-64*I*A*
a*c**6*f**2*exp(6*I*e) - 64*B*a*c**6*f**2*exp(6*I*e))*exp(6*I*f*x))/(1536*
c**9*f**3), Ne(c**9*f**3, 0)), (x*(A*a*exp(6*I*e) + 2*A*a*exp(4*I*e) + A*a
*exp(2*I*e) - I*B*a*exp(6*I*e) + I*B*a*exp(2*I*e))/(4*c**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = -\frac{3\,Ba\tan\left(fx+e\right)+2\,Aa+i\,Ba}{6\,c^3f(\tan\left(fx+e\right)+i)^3}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algo
rithm="giac")
```

output
```
-1/6*(3*B*a*tan(f*x + e) + 2*A*a + I*B*a)/(c^3*f*(tan(f*x + e) + I)^3)
```

Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \frac{\frac{a\,(2\,A+B\,1\mathrm{i})}{6}+\frac{B\,a\tan(e+f\,x)}{2}}{c^3\,f\,\left(-{\tan\left(e+f\,x\right)}^3-{\tan\left(e+f\,x\right)}^2\,3\mathrm{i}+3\tan\left(e+f\,x\right)+1\mathrm{i}\right)}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)
^3,x)
```

output
```
((a*(2*A + B*1i))/6 + (B*a*tan(e + f*x))/2)/(c^3*f*(3*tan(e + f*x) - tan(e
 + f*x)^2*3i - tan(e + f*x)^3 + 1i))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \frac{a\left(\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^3 i-3\tan(fx+e)^2-3\tan(fx+e)i+1}dx\right)bi+\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^3 i-3\tan(fx+e)^2-3\tan(fx+e)i+1}dx\right)ai+\left(\int \frac{}{\tan(}\right.\right.}{c^3}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x)
```

output
```
(a*(int(tan(e + f*x)**2/(tan(e + f*x)**3*i - 3*tan(e + f*x)**2 - 3*tan(e +
 f*x)*i + 1),x)*b*i + int(tan(e + f*x)/(tan(e + f*x)**3*i - 3*tan(e + f*x)
**2 - 3*tan(e + f*x)*i + 1),x)*a*i + int(tan(e + f*x)/(tan(e + f*x)**3*i -
 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b + int(1/(tan(e + f*x)**3*i
 - 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*a))/c**3
```

3.674 $\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 57

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = -\frac{a(iA+B)}{4c^4f(i+\tan(e+fx))^4}-\frac{iaB}{3c^4f(i+\tan(e+fx))^3}$$

output
```
-1/4*a*(I*A+B)/c^4/f/(I+tan(f*x+e))^4-1/3*I*a*B/c^4/f/(I+tan(f*x+e))^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \frac{a(-3iA+B-4iB\tan(e+fx))}{12c^4f(i+\tan(e+fx))^4}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^4,x]
```

output
```
(a*((-3*I)*A + B - (4*I)*B*Tan[e + f*x]))/(12*c^4*f*(I + Tan[e + f*x])^4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3042, 4071, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{c^5(1-i\tan(e+fx))^5}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^5}d\tan(e+fx)}{c^4f}$$

$$\Big\downarrow \text{53}$$

$$\frac{a\int \left(\frac{iB}{(\tan(e+fx)+i)^4}+\frac{iA+B}{(\tan(e+fx)+i)^5}\right)d\tan(e+fx)}{c^4f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(-\frac{B+iA}{4(\tan(e+fx)+i)^4}-\frac{iB}{3(\tan(e+fx)+i)^3}\right)}{c^4f}$$

input
```
Int[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^4
,x]
```

output
```
(a*(-1/4*(I*A + B)/(I + Tan[e + f*x])^4 - ((I/3)*B)/(I + Tan[e + f*x])^3))
/(c^4*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{a\left(-\frac{iB}{3(i+\tan(fx+e))^3}-\frac{iA+B}{4(i+\tan(fx+e))^4}\right)}{f\,c^4}$
default	$\frac{a\left(-\frac{iB}{3(i+\tan(fx+e))^3}-\frac{iA+B}{4(i+\tan(fx+e))^4}\right)}{f\,c^4}$
risch	$-\frac{a\,\mathrm{e}^{8i(fx+e)}B}{64c^4f}-\frac{ia\,\mathrm{e}^{8i(fx+e)}A}{64c^4f}-\frac{\mathrm{e}^{6i(fx+e)}aB}{48c^4f}-\frac{i\mathrm{e}^{6i(fx+e)}Aa}{16c^4f}+\frac{\mathrm{e}^{4i(fx+e)}aB}{32c^4f}-\frac{3i\mathrm{e}^{4i(fx+e)}Aa}{32c^4f}+\frac{a\,\mathrm{e}^{2i(f}}{16c}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,method=_RET
URNVERBOSE)
```

output
```
1/f*a/c^4*(-1/3*I*B/(I+tan(f*x+e))^3-1/4*(I*A+B)/(I+tan(f*x+e))^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = -\frac{3\,(i\,A+B)ae^{(8i\,fx+8i\,e)}+4\,(3i\,A+B)ae^{(6i\,fx+6i\,e)}+6\,(3i\,A-B)ae^{(4i\,fx+4i\,e)}+12\,(i\,A-B)ae^{(2i\,fx+2i\,e)}}{192\,c^4f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algo
rithm="fricas")
```

output
```
-1/192*(3*(I*A + B)*a*e^(8*I*f*x + 8*I*e) + 4*(3*I*A + B)*a*e^(6*I*f*x + 6
*I*e) + 6*(3*I*A - B)*a*e^(4*I*f*x + 4*I*e) + 12*(I*A - B)*a*e^(2*I*f*x +
2*I*e))/(c^4*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(46) = 92.

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 5.33

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \begin{cases} \frac{(-98304iAac^{12}f^3e^{2ie}+98304Bac^{12}f^3e^{2ie})e^{2ifx}+(-147456iAac^{12}f^3e^{4ie}+49152Bac^{12}f^3e^{4ie})e^{4ifx}+(-98304iAac^{12}f^3e^{6ie}-32768Bac^{12}}{1572864c^{16}f^4} \\ \frac{x(Aae^{8ie}+3Aae^{6ie}+3Aae^{4ie}+Aae^{2ie}-iBae^{8ie}-iBae^{6ie}+iBae^{4ie}+iBae^{2ie})}{8c^4} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**4,x)
```

output
```
Piecewise((((-98304*I*A*a*c**12*f**3*exp(2*I*e) + 98304*B*a*c**12*f**3*exp
(2*I*e))*exp(2*I*f*x) + (-147456*I*A*a*c**12*f**3*exp(4*I*e) + 49152*B*a*c
**12*f**3*exp(4*I*e))*exp(4*I*f*x) + (-98304*I*A*a*c**12*f**3*exp(6*I*e) -
 32768*B*a*c**12*f**3*exp(6*I*e))*exp(6*I*f*x) + (-24576*I*A*a*c**12*f**3*
exp(8*I*e) - 24576*B*a*c**12*f**3*exp(8*I*e))*exp(8*I*f*x))/(1572864*c**16
*f**4), Ne(c**16*f**4, 0)), (x*(A*a*exp(8*I*e) + 3*A*a*exp(6*I*e) + 3*A*a*
exp(4*I*e) + A*a*exp(2*I*e) - I*B*a*exp(8*I*e) - I*B*a*exp(6*I*e) + I*B*a*
exp(4*I*e) + I*B*a*exp(2*I*e))/(8*c**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = -\frac{4i\,Ba\tan\left(fx+e\right)+3i\,Aa-Ba}{12\,c^4f(\tan\left(fx+e\right)+i)^4}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algo
rithm="giac")
```

output
```
-1/12*(4*I*B*a*tan(f*x + e) + 3*I*A*a - B*a)/(c^4*f*(tan(f*x + e) + I)^4)
```

Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = -\frac{\frac{a\,(-B+A\,3\mathrm{i})}{12}+\frac{B\,a\,\tan(e+f\,x)\,1\mathrm{i}}{3}}{c^4\,f\,\left(\tan\left(e+f\,x\right)^4+\tan\left(e+f\,x\right)^3\,4\mathrm{i}-6\tan\left(e+f\,x\right)^2-\tan\left(e+f\,x\right)\,4\mathrm{i}+1\right)}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)
^4,x)
```

output
```
-((a*(A*3i - B))/12 + (B*a*tan(e + f*x)*1i)/3)/(c^4*f*(tan(e + f*x)^3*4i -
 6*tan(e + f*x)^2 - tan(e + f*x)*4i + tan(e + f*x)^4 + 1))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \frac{a\left(-\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^4 i-4\tan(fx+e)^3-6\tan(fx+e)^2 i+4\tan(fx+e)+i}dx\right)b-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^4 i-4\tan(fx+e)^3-6\tan(fx+e)^2 i+4\tan(f\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x)
```

output
```
(a*( - int(tan(e + f*x)**2/(tan(e + f*x)**4*i - 4*tan(e + f*x)**3 - 6*tan(
e + f*x)**2*i + 4*tan(e + f*x) + i),x)*b - int(tan(e + f*x)/(tan(e + f*x)*
*4*i - 4*tan(e + f*x)**3 - 6*tan(e + f*x)**2*i + 4*tan(e + f*x) + i),x)*a
+ int(tan(e + f*x)/(tan(e + f*x)**4*i - 4*tan(e + f*x)**3 - 6*tan(e + f*x)
**2*i + 4*tan(e + f*x) + i),x)*b*i + int(1/(tan(e + f*x)**4*i - 4*tan(e +
f*x)**3 - 6*tan(e + f*x)**2*i + 4*tan(e + f*x) + i),x)*a*i))/c**4
```

3.675 $\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 55

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{a(A-iB)}{5c^5f(i+\tan(e+fx))^5}+\frac{aB}{4c^5f(i+\tan(e+fx))^4}$$

output
```
1/5*a*(A-I*B)/c^5/f/(I+tan(f*x+e))^5+1/4*a*B/c^5/f/(I+tan(f*x+e))^4
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{a(4A+iB+5B\tan(e+fx))}{20c^5f(i+\tan(e+fx))^5}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^5,x]
```

output
```
(a*(4*A + I*B + 5*B*Tan[e + f*x]))/(20*c^5*f*(I + Tan[e + f*x])^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3042, 4071, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{c^6(1-i\tan(e+fx))^6}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^6}d\tan(e+fx)}{c^5f}$$

$$\Big\downarrow \text{53}$$

$$\frac{a\int \left(\frac{iB-A}{(\tan(e+fx)+i)^6}-\frac{B}{(\tan(e+fx)+i)^5}\right)d\tan(e+fx)}{c^5f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\left(\frac{A-iB}{5(\tan(e+fx)+i)^5}+\frac{B}{4(\tan(e+fx)+i)^4}\right)}{c^5f}$$

input
```
Int[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^5
,x]
```

output
```
(a*((A - I*B)/(5*(I + Tan[e + f*x])^5) + B/(4*(I + Tan[e + f*x])^4)))/(c^5
*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a\left(-\frac{iB-A}{5(i+\tan(fx+e))^{5}}+\frac{B}{4(i+\tan(fx+e))^{4}}\right)}{f\,c^{5}}$
default	$\frac{a\left(-\frac{iB-A}{5(i+\tan(fx+e))^{5}}+\frac{B}{4(i+\tan(fx+e))^{4}}\right)}{f\,c^{5}}$
risch	$-\frac{a\,e^{10i(fx+e)}B}{160c^{5}f}-\frac{ia\,e^{10i(fx+e)}A}{160c^{5}f}-\frac{e^{8i(fx+e)}aB}{64c^{5}f}-\frac{ie^{8i(fx+e)}Aa}{32c^{5}f}-\frac{iAa\,e^{6i(fx+e)}}{16c^{5}f}+\frac{e^{4i(fx+e)}aB}{32c^{5}f}-\frac{ie^{4i(f}}{16}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x,method=_RET
URNVERBOSE)
```

output
```
1/f*a/c^5*(-1/5*(-A+I*B)/(I+tan(f*x+e))^5+1/4*B/(I+tan(f*x+e))^4)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(45) = 90$.

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.71

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = -\frac{2\,(i\,A+B)ae^{(10i\,fx+10i\,e)}+5\,(2i\,A+B)ae^{(8i\,fx+8i\,e)}+20i\,Aae^{(6i\,fx+6i\,e)}+10\,(2i\,A-B)ae^{(4i\,fx+4i\,e)}+}{320\,c^5 f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, algo
rithm="fricas")
```

output
```
-1/320*(2*(I*A + B)*a*e^(10*I*f*x + 10*I*e) + 5*(2*I*A + B)*a*e^(8*I*f*x +
 8*I*e) + 20*I*A*a*e^(6*I*f*x + 6*I*e) + 10*(2*I*A - B)*a*e^(4*I*f*x + 4*I
*e) + 10*(I*A - B)*a*e^(2*I*f*x + 2*I*e))/(c^5*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(42) = 84$.

Time = 0.38 (sec) , antiderivative size = 348, normalized size of antiderivative = 6.33

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \begin{cases} \frac{-10485760iAac^{20}f^4e^{6ie}e^{6ifx}+(-5242880iAac^{20}f^4e^{2ie}+5242880Bac^{20}f^4e^{2ie})e^{2ifx}+(-10485760iAac^{20}f^4e^{4ie}+5242880Bac^{20}f^4e^{4ie})e^{4}}{167772160c^{25}f^5} \\ \frac{x(Aae^{10ie}+4Aae^{8ie}+6Aae^{6ie}+4Aae^{4ie}+Aae^{2ie}-iBae^{10ie}-2iBae^{8ie}+2iBae^{4ie}+iBae^{2ie})}{16c^5} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**5,x)
```

output
```
Piecewise(((-10485760*I*A*a*c**20*f**4*exp(6*I*e)*exp(6*I*f*x) + (-5242880
*I*A*a*c**20*f**4*exp(2*I*e) + 5242880*B*a*c**20*f**4*exp(2*I*e))*exp(2*I*
f*x) + (-10485760*I*A*a*c**20*f**4*exp(4*I*e) + 5242880*B*a*c**20*f**4*exp
(4*I*e))*exp(4*I*f*x) + (-5242880*I*A*a*c**20*f**4*exp(8*I*e) - 2621440*B*
a*c**20*f**4*exp(8*I*e))*exp(8*I*f*x) + (-1048576*I*A*a*c**20*f**4*exp(10*
I*e) - 1048576*B*a*c**20*f**4*exp(10*I*e))*exp(10*I*f*x))/(167772160*c**25
*f**5), Ne(c**25*f**5, 0)), (x*(A*a*exp(10*I*e) + 4*A*a*exp(8*I*e) + 6*A*a
*exp(6*I*e) + 4*A*a*exp(4*I*e) + A*a*exp(2*I*e) - I*B*a*exp(10*I*e) - 2*I*
B*a*exp(8*I*e) + 2*I*B*a*exp(4*I*e) + I*B*a*exp(2*I*e))/(16*c**5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{5\,Ba\tan\left(fx+e\right)+4\,Aa+i\,Ba}{20\,c^5f(\tan\left(fx+e\right)+i)^5}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, algo
rithm="giac")
```

output
```
1/20*(5*B*a*tan(f*x + e) + 4*A*a + I*B*a)/(c^5*f*(tan(f*x + e) + I)^5)
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{\frac{a\,(4\,A+B\,1\mathrm{i})}{20}+\frac{B\,a\tan(e+f\,x)}{4}}{c^5\,f\,\left({\tan\left(e+f\,x\right)}^5+{\tan\left(e+f\,x\right)}^4\,5{}\mathrm{i}-10\,{\tan\left(e+f\,x\right)}^3-{\tan\left(e+f\,x\right)}^2\,10{}\mathrm{i}+5\,\tan\left(e+f\,x\right)+1{}\mathrm{i}\right)}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)
^5,x)
```

output
```
((a*(4*A + B*1i))/20 + (B*a*tan(e + f*x))/4)/(c^5*f*(5*tan(e + f*x) - tan(
e + f*x)^2*10i - 10*tan(e + f*x)^3 + tan(e + f*x)^4*5i + tan(e + f*x)^5 +
1i))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{a\left(-\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^5+5\tan(fx+e)^4i-10\tan(fx+e)^3-10\tan(fx+e)^2i+5\tan(fx+e)+i}dx\right)b-\left(\int \frac{\mathrm{ta}}{\tan(fx+e)^5+5\tan(fx+e)^4i-10\tan}\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x)
```

output
```
(a*( - int(tan(e + f*x)**2/(tan(e + f*x)**5 + 5*tan(e + f*x)**4*i - 10*tan
(e + f*x)**3 - 10*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*b - int(tan(e
 + f*x)/(tan(e + f*x)**5 + 5*tan(e + f*x)**4*i - 10*tan(e + f*x)**3 - 10*t
an(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*a + int(tan(e + f*x)/(tan(e + f*
x)**5 + 5*tan(e + f*x)**4*i - 10*tan(e + f*x)**3 - 10*tan(e + f*x)**2*i +
5*tan(e + f*x) + i),x)*b*i + int(1/(tan(e + f*x)**5 + 5*tan(e + f*x)**4*i
- 10*tan(e + f*x)**3 - 10*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*a*i))
/c**5
```

3.676 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 41, antiderivative size = 109

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \frac{2a^2(iA+B)(c-ic\tan(e+fx))^n}{fn} - \frac{a^2(iA+3B)(c-ic\tan(e+fx))^{1+n}}{cf(1+n)} + \frac{a^2B(c-ic\tan(e+fx))^{2+n}}{c^2f(2+n)}$$

output
```
2*a^2*(I*A+B)*(c-I*c*tan(f*x+e))^n/f/n-a^2*(I*A+3*B)*(c-I*c*tan(f*x+e))^(1
+n)/c/f/(1+n)+a^2*B*(c-I*c*tan(f*x+e))^(2+n)/c^2/f/(2+n)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = -\frac{a^2(c-ic\tan(e+fx))^n\left(-iA(2+n)^2-B(4+n)+n(A(2+n)-iB(4+n))\tan(e+fx)+Bn(1+\right.}{fn(1+n)(2+n)}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^n,x]
```

output
```
-((a^2*(c - I*c*Tan[e + f*x])^n*((-I)*A*(2 + n)^2 - B*(4 + n) + n*(A*(2 +
n) - I*B*(4 + n))*Tan[e + f*x] + B*n*(1 + n)*Tan[e + f*x]^2))/(f*n*(1 + n)
*(2 + n)))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a(i\tan(e+fx)+1)(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c\int (i\tan(e+fx)+1)(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2c\int \left(2(A-iB)(c-ic\tan(e+fx))^{n-1}+\frac{(3iB-A)(c-ic\tan(e+fx))^n}{c}-\frac{iB(c-ic\tan(e+fx))^{n+1}}{c^2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c\left(-\frac{(3B+iA)(c-ic\tan(e+fx))^{n+1}}{c^2(n+1)}+\frac{2(B+iA)(c-ic\tan(e+fx))^n}{cn}+\frac{B(c-ic\tan(e+fx))^{n+2}}{c^3(n+2)}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n
,x]
```

output
```
(a^2*c*((2*(I*A + B)*(c - I*c*Tan[e + f*x])^n)/(c*n) - ((I*A + 3*B)*(c - I
*c*Tan[e + f*x])^(1 + n))/(c^2*(1 + n)) + (B*(c - I*c*Tan[e + f*x])^(2 + n
))/(c^3*(2 + n))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{(iA\,a^2n^2+4iA\,a^2n+4iA\,a^2+B\,a^2n+4B\,a^2)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{(1+n)fn(2+n)}-\frac{B\,a^2\tan(fx+e)^2\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{f(2+n)}-\frac{a^2(-i}{}$
default	$\frac{(iA\,a^2n^2+4iA\,a^2n+4iA\,a^2+B\,a^2n+4B\,a^2)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{(1+n)fn(2+n)}-\frac{B\,a^2\tan(fx+e)^2\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{f(2+n)}-\frac{a^2(-i}{}$
norman	$\frac{(iA\,a^2n^2+4iA\,a^2n+4iA\,a^2+B\,a^2n+4B\,a^2)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{(1+n)fn(2+n)}-\frac{B\,a^2\tan(fx+e)^2\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{f(2+n)}-\frac{a^2(-i}{}$
risch	Expression too large to display

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x,method=_R
ETURNVERBOSE)
```

output
```
1/(1+n)/f/n*(I*A*a^2*n^2+4*I*A*a^2*n+4*I*A*a^2+B*a^2*n+4*B*a^2)/(2+n)*exp(
n*ln(c-I*c*tan(f*x+e)))-B*a^2/f/(2+n)*tan(f*x+e)^2*exp(n*ln(c-I*c*tan(f*x+
e)))-a^2*(-I*B*n+A*n-4*I*B+2*A)/f/(1+n)/(2+n)*tan(f*x+e)*exp(n*ln(c-I*c*ta
n(f*x+e)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(99) = 198.

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.89

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx =$$
$$-\frac{2\left(\left(-i\,A+B\right)a^2n+2\left(-i\,A-B\right)a^2+\left(\left(-i\,A-B\right)a^2n^2+3\left(-i\,A-B\right)a^2n+2\left(-i\,A-B\right)a^2\right)e^{\left(4i\,fx+\right.}}{fn^3+3\,fn^2+2\,fn+\left(fn^3+3\,fn^2+2\,fn\right)e^{\left(4i\,fx+4i\,e\right)}+}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, al
gorithm="fricas")
```

output
```
-2*((-I*A + B)*a^2*n + 2*(-I*A - B)*a^2 + ((-I*A - B)*a^2*n^2 + 3*(-I*A -
B)*a^2*n + 2*(-I*A - B)*a^2)*e^(4*I*f*x + 4*I*e) + ((-I*A + B)*a^2*n^2 - 4
*I*A*a^2*n + 4*(-I*A - B)*a^2)*e^(2*I*f*x + 2*I*e))*(2*c/(e^(2*I*f*x + 2*I
*e) + 1))^n/(f*n^3 + 3*f*n^2 + 2*f*n + (f*n^3 + 3*f*n^2 + 2*f*n)*e^(4*I*f*
x + 4*I*e) + 2*(f*n^3 + 3*f*n^2 + 2*f*n)*e^(2*I*f*x + 2*I*e))
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1482 vs. $2(87) = 174$.

Time = 1.32 (sec) , antiderivative size = 1482, normalized size of antiderivative = 13.60

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**n,x)
```

output
```
Piecewise((x*(A + B*tan(e))*(I*a*tan(e) + a)**2*(-I*c*tan(e) + c)**n, Eq(f
, 0)), (-2*A*a**2*tan(e + f*x)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(
e + f*x) - 2*c**2*f) - 2*I*B*a**2*f*x*tan(e + f*x)**2/(2*c**2*f*tan(e + f*
x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) + 4*B*a**2*f*x*tan(e + f*x)/(2
*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) + 2*I*B*a**2
*f*x/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) + B*a
**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(2*c**2*f*tan(e + f*x)**2 + 4
*I*c**2*f*tan(e + f*x) - 2*c**2*f) + 2*I*B*a**2*log(tan(e + f*x)**2 + 1)*t
an(e + f*x)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f
) - B*a**2*log(tan(e + f*x)**2 + 1)/(2*c**2*f*tan(e + f*x)**2 + 4*I*c**2*f
*tan(e + f*x) - 2*c**2*f) + 6*I*B*a**2*tan(e + f*x)/(2*c**2*f*tan(e + f*x)
**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f) - 4*B*a**2/(2*c**2*f*tan(e + f*x
)**2 + 4*I*c**2*f*tan(e + f*x) - 2*c**2*f), Eq(n, -2)), (-2*A*a**2*f*x*tan
(e + f*x)/(2*c*f*tan(e + f*x) + 2*I*c*f) - 2*I*A*a**2*f*x/(2*c*f*tan(e + f
*x) + 2*I*c*f) - I*A*a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c*f*tan
(e + f*x) + 2*I*c*f) + A*a**2*log(tan(e + f*x)**2 + 1)/(2*c*f*tan(e + f*x)
 + 2*I*c*f) + 4*A*a**2/(2*c*f*tan(e + f*x) + 2*I*c*f) + 6*I*B*a**2*f*x*tan
(e + f*x)/(2*c*f*tan(e + f*x) + 2*I*c*f) - 6*B*a**2*f*x/(2*c*f*tan(e + f*x
) + 2*I*c*f) - 3*B*a**2*log(tan(e + f*x)**2 + 1)*tan(e + f*x)/(2*c*f*tan(e
 + f*x) + 2*I*c*f) - 3*I*B*a**2*log(tan(e + f*x)**2 + 1)/(2*c*f*tan(e +...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(99) = 198$.

Time = 0.26 (sec) , antiderivative size = 660, normalized size of antiderivative = 6.06

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, al
gorithm="maxima")
```

output
```
2*(((A + I*B)*a^2*c^n*n^2 + 4*A*a^2*c^n*n + 4*(A - I*B)*a^2*c^n)*2^n*cos(-
2*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e) + ((A - I
*B)*a^2*c^n*n^2 + 3*(A - I*B)*a^2*c^n*n + 2*(A - I*B)*a^2*c^n)*2^n*cos(-4*
f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*e) + ((A + I*B
)*a^2*c^n*n + 2*(A - I*B)*a^2*c^n)*2^n*cos(n*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)) - ((I*A - B)*a^2*c^n*n^2 + 4*I*A*a^2*c^n*n + 4*(I*A +
B)*a^2*c^n)*2^n*sin(-2*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1) - 2*e) - ((I*A + B)*a^2*c^n*n^2 + 3*(I*A + B)*a^2*c^n*n + 2*(I*A + B)
*a^2*c^n)*2^n*sin(-4*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1) - 4*e) - ((I*A - B)*a^2*c^n*n + 2*(I*A + B)*a^2*c^n)*2^n*sin(n*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((-I*n^3 - 3*I*n^2 - 2*I*n)*(co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*co
s(4*f*x + 4*e) + (n^3 + 3*n^2 + 2*n)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*sin(4*f*x + 4*e) + (-I*n^3 - 3*I*n^2
 - 2*(I*n^3 + 3*I*n^2 + 2*I*n)*cos(2*f*x + 2*e) + 2*(n^3 + 3*n^2 + 2*n)*si
n(2*f*x + 2*e) - 2*I*n)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/2*n))*f)
```

Giac [F]

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^2(-i\,c\tan(fx+e)+c)^n\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, al
gorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^2*(-I*c*tan(f*x + e)
 + c)^n, x)
```

Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.77

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = -\frac{e^{-e\,2i-f\,x\,2i}\left(c-\frac{c\,\sin(e+f\,x)\,1i}{\cos(e+f\,x)}\right)^n\left(\frac{2\,a^2\,(2\,A-B\,2i+A\,n+B\,n\,1i)}{f\,n\,(n^2\,1i+n\,3i+2i)}+\frac{2\,a^2\,e^{e\,4i+f\,x\,4i}\,(A-B\,1i)\,(n^2+3\,n+2)}{f\,n\,(n^2\,1i+n\,3i+2i)}+\frac{2\,a^2\,e^{e\,2i+f\,x\,2i}\,(n+2}{f\,n\,(n^2}\right.}{4\cos\left(e+f\,x\right)^2}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
^n,x)
```

output
```
-(exp(- e*2i - f*x*2i)*(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^n*((2*a^2*(2
*A - B*2i + A*n + B*n*1i))/(f*n*(n*3i + n^2*1i + 2i)) + (2*a^2*exp(e*4i +
f*x*4i)*(A - B*1i)*(3*n + n^2 + 2))/(f*n*(n*3i + n^2*1i + 2i)) + (2*a^2*ex
p(e*2i + f*x*2i)*(n + 2)*(2*A - B*2i + A*n + B*n*1i))/(f*n*(n*3i + n^2*1i
+ 2i))))/(4*cos(e + f*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.17

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \frac{(-\tan(fx+e)\,ci+c)^n\,a^2\left(-\tan(fx+e)^2\,b\,n^2-\tan(fx+e)^2\,bn-\tan(fx+e)\,a\,n^2-2\tan(fx+e)\right.}{fn\,(n^2+3n+2)}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x)
```

output
```
(( - tan(e + f*x)*c*i + c)**n*a**2*( - tan(e + f*x)**2*b*n**2 - tan(e + f*
x)**2*b*n - tan(e + f*x)*a*n**2 - 2*tan(e + f*x)*a*n + tan(e + f*x)*b*i*n*
*2 + 4*tan(e + f*x)*b*i*n + a*i*n**2 + 4*a*i*n + 4*a*i + b*n + 4*b))/(f*n*
(n**2 + 3*n + 2))
```

3.677 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx$

Optimal result

Integrand size = 41, antiderivative size = 99

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx = \frac{2a^2(iA+B)c^5(1-i\tan(e+fx))^5}{5f} - \frac{a^2(iA+3B)c^5(1-i\tan(e+fx))^6}{6f} + \frac{a^2Bc^5(1-i\tan(e+fx))^7}{7f}$$

output
```
2/5*a^2*(I*A+B)*c^5*(1-I*tan(f*x+e))^5/f-1/6*a^2*(I*A+3*B)*c^5*(1-I*tan(f*
x+e))^6/f+1/7*a^2*B*c^5*(1-I*tan(f*x+e))^7/f
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 270 vs. 2(99) = 198.

Time = 1.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.73

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx = \frac{a^2Ac^5\tan(e+fx)}{f} - \frac{3ia^2Ac^5\tan^2(e+fx)}{2f} + \frac{a^2Bc^5\tan^2(e+fx)}{2f} - \frac{2a^2Ac^5\tan^3(e+fx)}{3f} - \frac{ia^2Bc^5\tan^3(e+fx)}{f} - \frac{ia^2Ac^5\tan^4(e+fx)}{2f} - \frac{a^2Bc^5\tan^4(e+fx)}{2f} - \frac{3a^2Ac^5\tan^5(e+fx)}{5f} - \frac{2ia^2Bc^5\tan^5(e+fx)}{5f} + \frac{ia^2Ac^5\tan^6(e+fx)}{6f} - \frac{a^2Bc^5\tan^6(e+fx)}{2f} + \frac{ia^2Bc^5\tan^7(e+fx)}{7f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^5,x]
```

output
```
(a^2*A*c^5*Tan[e + f*x])/f - (((3*I)/2)*a^2*A*c^5*Tan[e + f*x]^2)/f + (a^2
*B*c^5*Tan[e + f*x]^2)/(2*f) - (2*a^2*A*c^5*Tan[e + f*x]^3)/(3*f) - (I*a^2
*B*c^5*Tan[e + f*x]^3)/f - ((I/2)*a^2*A*c^5*Tan[e + f*x]^4)/f - (a^2*B*c^5
*Tan[e + f*x]^4)/(2*f) - (3*a^2*A*c^5*Tan[e + f*x]^5)/(5*f) - (((2*I)/5)*a
^2*B*c^5*Tan[e + f*x]^5)/f + ((I/6)*a^2*A*c^5*Tan[e + f*x]^6)/f - (a^2*B*c
^5*Tan[e + f*x]^6)/(2*f) + ((I/7)*a^2*B*c^5*Tan[e + f*x]^7)/f
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int ac^4(1-i\tan(e+fx))^4(i\tan(e+fx)+1)(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c^5\int (1-i\tan(e+fx))^4(i\tan(e+fx)+1)(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2c^5\int \left(-iB(1-i\tan(e+fx))^6+(3iB-A)(1-i\tan(e+fx))^5+2(A-iB)(1-i\tan(e+fx))^4\right)d\tan(e+f}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c^5\left(-\frac{1}{6}(3B+iA)(1-i\tan(e+fx))^6+\frac{2}{5}(B+iA)(1-i\tan(e+fx))^5+\frac{1}{7}B(1-i\tan(e+fx))^7\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^5
,x]
```

output
```
(a^2*c^5*((2*(I*A + B)*(1 - I*Tan[e + f*x])^5)/5 - ((I*A + 3*B)*(1 - I*Tan
[e + f*x])^6)/6 + (B*(1 - I*Tan[e + f*x])^7)/7))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
risch	$\frac{32c^5a^2\left(42iA\,e^{4i(fx+e)}+42B\,e^{4i(fx+e)}+49iA\,e^{2i(fx+e)}-21B\,e^{2i(fx+e)}+7iA-3B\right)}{105f\left(e^{2i(fx+e)}+1\right)^7}$
derivativedivides	$-\frac{ic^5a^2\left(-\frac{B\tan(fx+e)^7}{7}+\frac{(-3iB-A)\tan(fx+e)^6}{6}+\frac{(iA+4i(iB-A)+6B)\tan(fx+e)^5}{5}+\frac{(-2iB+2A)\tan(fx+e)^4}{4}+\frac{(-6iA-4i(i\cdots\right.}{f}$
default	$-\frac{ic^5a^2\left(-\frac{B\tan(fx+e)^7}{7}+\frac{(-3iB-A)\tan(fx+e)^6}{6}+\frac{(iA+4i(iB-A)+6B)\tan(fx+e)^5}{5}+\frac{(-2iB+2A)\tan(fx+e)^4}{4}+\frac{(-6iA-4i(i\cdots\right.}{f}$
norman	$\frac{A\,a^2c^5\tan(fx+e)}{f}-\frac{(-iA\,a^2c^5+3B\,a^2c^5)\tan(fx+e)^6}{6f}-\frac{(2iB\,a^2c^5+3A\,a^2c^5)\tan(fx+e)^5}{5f}-\frac{(3iB\,a^2c^5+2A\,a^2\cdots}{3f}$
parallelrisch	$30iB\,a^2c^5\tan(fx+e)^7+35iA\tan(fx+e)^6a^2c^5-84iB\tan(fx+e)^5a^2c^5-105B\tan(fx+e)^6a^2c^5-105iA\tan(fx+e)^4a^2\cdots$
parts	$\frac{(-5iA\,a^2c^5-B\,a^2c^5)\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{(-5iB\,a^2c^5-5A\,a^2c^5)\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan\cdots\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x,method=_R
ETURNVERBOSE)
```

output
```
32/105*c^5*a^2*(42*I*A*exp(4*I*(f*x+e))+42*B*exp(4*I*(f*x+e))+49*I*A*exp(2
*I*(f*x+e))-21*B*exp(2*I*(f*x+e))+7*I*A-3*B)/f/(exp(2*I*(f*x+e))+1)^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.54

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx = -\frac{32\left(42\,(-i\,A-B)a^2c^5e^{(4i\,fx+4i\,e)}+7\,(-7i\,A+3\,B)a^2c^5e^{(2i\,fx+2i\,e)}+(-7i\,A+3\,B\right.}{105\left(fe^{(14i\,fx+14i\,e)}+7\,fe^{(12i\,fx+12i\,e)}+21\,fe^{(10i\,fx+10i\,e)}+35\,fe^{(8i\,fx+8i\,e)}+35\,fe^{(6i\,fx+6i\,e)}+21\,fe^{(4i\,fx\right.}}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x, al
gorithm="fricas")
```

output
```
-32/105*(42*(-I*A - B)*a^2*c^5*e^(4*I*f*x + 4*I*e) + 7*(-7*I*A + 3*B)*a^2*
c^5*e^(2*I*f*x + 2*I*e) + (-7*I*A + 3*B)*a^2*c^5)/(f*e^(14*I*f*x + 14*I*e)
 + 7*f*e^(12*I*f*x + 12*I*e) + 21*f*e^(10*I*f*x + 10*I*e) + 35*f*e^(8*I*f*
x + 8*I*e) + 35*f*e^(6*I*f*x + 6*I*e) + 21*f*e^(4*I*f*x + 4*I*e) + 7*f*e^(
2*I*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(80) = 160.

Time = 0.81 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.45

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx = \frac{224iAa^2c^5-96Ba^2c^5+\left(1568iAa^2c^5e^{2ie}-672Ba^2c^5e^{2ie}\right)e^{2ifx}+\left(1344iAa^2c^5e^{4ie}+1344Ba^2\right.}{105fe^{14ie}e^{14ifx}+735fe^{12ie}e^{12ifx}+2205fe^{10ie}e^{10ifx}+3675fe^{8ie}e^{8ifx}+3675fe^{6ie}e^{6ifx}+2205fe^{4ie}e^{4ifx}}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**5,x)
```

output
```
(224*I*A*a**2*c**5 - 96*B*a**2*c**5 + (1568*I*A*a**2*c**5*exp(2*I*e) - 672
*B*a**2*c**5*exp(2*I*e))*exp(2*I*f*x) + (1344*I*A*a**2*c**5*exp(4*I*e) + 1
344*B*a**2*c**5*exp(4*I*e))*exp(4*I*f*x))/(105*f*exp(14*I*e)*exp(14*I*f*x)
 + 735*f*exp(12*I*e)*exp(12*I*f*x) + 2205*f*exp(10*I*e)*exp(10*I*f*x) + 36
75*f*exp(8*I*e)*exp(8*I*f*x) + 3675*f*exp(6*I*e)*exp(6*I*f*x) + 2205*f*exp
(4*I*e)*exp(4*I*f*x) + 735*f*exp(2*I*e)*exp(2*I*f*x) + 105*f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.53

$$\int (a + ia\tan(e + fx))^2 (A + B\tan(e + fx))(c - ic\tan(e + fx))^5 \, dx$$
$$= \frac{30i\,Ba^2c^5\tan(fx+e)^7 + 35\,(i\,A - 3\,B)a^2c^5\tan(fx+e)^6 - 42\,(3\,A + 2i\,B)a^2c^5\tan(fx+e)^5 + 105\,(-$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x, al
gorithm="maxima")
```

output
```
1/210*(30*I*B*a^2*c^5*tan(f*x + e)^7 + 35*(I*A - 3*B)*a^2*c^5*tan(f*x + e)
^6 - 42*(3*A + 2*I*B)*a^2*c^5*tan(f*x + e)^5 + 105*(-I*A - B)*a^2*c^5*tan(
f*x + e)^4 - 70*(2*A + 3*I*B)*a^2*c^5*tan(f*x + e)^3 + 105*(-3*I*A + B)*a^
2*c^5*tan(f*x + e)^2 + 210*A*a^2*c^5*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(83) = 166.

Time = 0.72 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.10

$$\int (a + ia\tan(e + fx))^2 (A + B\tan(e + fx))(c - ic\tan(e + fx))^5 \, dx =$$
$$-\frac{-30i\,Ba^2c^5\tan(fx+e)^7 - 35i\,Aa^2c^5\tan(fx+e)^6 + 105\,Ba^2c^5\tan(fx+e)^6 + 126\,Aa^2c^5\tan(fx+$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x, al
gorithm="giac")
```

output
```
-1/210*(-30*I*B*a^2*c^5*tan(f*x + e)^7 - 35*I*A*a^2*c^5*tan(f*x + e)^6 + 1
05*B*a^2*c^5*tan(f*x + e)^6 + 126*A*a^2*c^5*tan(f*x + e)^5 + 84*I*B*a^2*c^
5*tan(f*x + e)^5 + 105*I*A*a^2*c^5*tan(f*x + e)^4 + 105*B*a^2*c^5*tan(f*x
+ e)^4 + 140*A*a^2*c^5*tan(f*x + e)^3 + 210*I*B*a^2*c^5*tan(f*x + e)^3 + 3
15*I*A*a^2*c^5*tan(f*x + e)^2 - 105*B*a^2*c^5*tan(f*x + e)^2 - 210*A*a^2*c
^5*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.60

$$\int (a + ia\tan(e + fx))^2(A + B\tan(e + fx))(c - ic\tan(e + fx))^5\,dx$$
$$= \frac{A\,a^2\,c^5\tan(e+f\,x) + \frac{a^2\,c^5\tan(e+f\,x)^3\,(-3\,B+A\,2\mathrm{i})\,1\mathrm{i}}{3} + \frac{a^2\,c^5\tan(e+f\,x)^5\,(-2\,B+A\,3\mathrm{i})\,1\mathrm{i}}{5} - \frac{a^2\,c^5\tan(e+f\,x)^2\,(3\,A+B\,1\mathrm{i})\,1\mathrm{i}}{2} -}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
^5,x)
```

output
```
((a^2*c^5*tan(e + f*x)^3*(A*2i - 3*B)*1i)/3 - (a^2*c^5*tan(e + f*x)^2*(3*A
 + B*1i)*1i)/2 + (a^2*c^5*tan(e + f*x)^5*(A*3i - 2*B)*1i)/5 + A*a^2*c^5*ta
n(e + f*x) - (a^2*c^5*tan(e + f*x)^4*(A - B*1i)*1i)/2 + (a^2*c^5*tan(e + f
*x)^6*(A + B*3i)*1i)/6 + (B*a^2*c^5*tan(e + f*x)^7*1i)/7)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.45

$$\int (a + ia\tan(e + fx))^2(A + B\tan(e + fx))(c - ic\tan(e + fx))^5\,dx$$
$$= \frac{\tan(fx+e)\,a^2c^5\big(30\tan(fx+e)^6\,bi + 35\tan(fx+e)^5\,ai - 105\tan(fx+e)^5\,b - 126\tan(fx+e)^4\,a -}{}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x)
```

output
```
(tan(e + f*x)*a**2*c**5*(30*tan(e + f*x)**6*b*i + 35*tan(e + f*x)**5*a*i -
 105*tan(e + f*x)**5*b - 126*tan(e + f*x)**4*a - 84*tan(e + f*x)**4*b*i -
105*tan(e + f*x)**3*a*i - 105*tan(e + f*x)**3*b - 140*tan(e + f*x)**2*a -
210*tan(e + f*x)**2*b*i - 315*tan(e + f*x)*a*i + 105*tan(e + f*x)*b + 210*
a))/(210*f)
```

3.678 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx$

Optimal result

Integrand size = 41, antiderivative size = 99

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = \frac{a^2(iA+B)c^4(1-i\tan(e+fx))^4}{2f} - \frac{a^2(iA+3B)c^4(1-i\tan(e+fx))^5}{5f} + \frac{a^2Bc^4(1-i\tan(e+fx))^6}{6f}$$

output
```
1/2*a^2*(I*A+B)*c^4*(1-I*tan(f*x+e))^4/f-1/5*a^2*(I*A+3*B)*c^4*(1-I*tan(f*
x+e))^5/f+1/6*a^2*B*c^4*(1-I*tan(f*x+e))^6/f
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = -\frac{a^2c^4(6iA-7B-30A\tan(e+fx)+(30iA-15B)\tan^2(e+fx)+20iB\tan^3(e+fx)+15iA\tan^4(e}{30f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^4,x]
```

output
```
-1/30*(a^2*c^4*((6*I)*A - 7*B - 30*A*Tan[e + f*x] + ((30*I)*A - 15*B)*Tan[
e + f*x]^2 + (20*I)*B*Tan[e + f*x]^3 + (15*I)*A*Tan[e + f*x]^4 + 6*(A + (2
*I)*B)*Tan[e + f*x]^5 + 5*B*Tan[e + f*x]^6))/f
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int ac^3(1-i\tan(e+fx))^3(i\tan(e+fx)+1)(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c^4\int (1-i\tan(e+fx))^3(i\tan(e+fx)+1)(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2c^4\int \left(-iB(1-i\tan(e+fx))^5+(3iB-A)(1-i\tan(e+fx))^4+2(A-iB)(1-i\tan(e+fx))^3\right)d\tan(e+f}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c^4\left(-\frac{1}{5}(3B+iA)(1-i\tan(e+fx))^5+\frac{1}{2}(B+iA)(1-i\tan(e+fx))^4+\frac{1}{6}B(1-i\tan(e+fx))^6\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4
,x]
```

output
```
(a^2*c^4*(((I*A + B)*(1 - I*Tan[e + f*x])^4)/2 - ((I*A + 3*B)*(1 - I*Tan[e
 + f*x])^5)/5 + (B*(1 - I*Tan[e + f*x])^6)/6))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
risch	$\frac{8c^4a^2\left(15iA\,\mathrm{e}^{4i(fx+e)}+15B\,\mathrm{e}^{4i(fx+e)}+18iA\,\mathrm{e}^{2i(fx+e)}-6B\,\mathrm{e}^{2i(fx+e)}+3iA-B\right)}{15f\left(\mathrm{e}^{2i(fx+e)}+1\right)^6}$
derivativedivides	$\frac{c^4a^2\left(-\frac{B\tan(fx+e)^6}{6}+\frac{(-2iB-A)\tan(fx+e)^5}{5}+\frac{(iA+3i(iB-A)+3B)\tan(fx+e)^4}{4}-\frac{2iB\tan(fx+e)^3}{3}+\frac{(-3iA-i(iB-A))\tan(f\cdots}{2}\right.}{f}$
default	$\frac{c^4a^2\left(-\frac{B\tan(fx+e)^6}{6}+\frac{(-2iB-A)\tan(fx+e)^5}{5}+\frac{(iA+3i(iB-A)+3B)\tan(fx+e)^4}{4}-\frac{2iB\tan(fx+e)^3}{3}+\frac{(-3iA-i(iB-A))\tan(f\cdots}{2}\right.}{f}$
norman	$\frac{A\,a^2c^4\tan(fx+e)}{f}-\frac{(2iB\,a^2c^4+A\,a^2c^4)\tan(fx+e)^5}{5f}+\frac{(-2iA\,a^2c^4+B\,a^2c^4)\tan(fx+e)^2}{2f}-\frac{B\,a^2c^4\tan(fx+e)^6}{6f}$
parallelrisch	$-\frac{12iB\tan(fx+e)^5a^2c^4+5B\tan(fx+e)^6a^2c^4+15iA\,a^2c^4\tan(fx+e)^4+6A\tan(fx+e)^5a^2c^4+20iB\,a^2c^4\tan(fx+e)^3\cdots}{30f}$
parts	$\frac{(-4iA\,a^2c^4+B\,a^2c^4)\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{(-4iB\,a^2c^4-A\,a^2c^4)\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\cdots\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x,method=_R
ETURNVERBOSE)
```

output
```
8/15*c^4*a^2*(15*I*A*exp(4*I*(f*x+e))+15*B*exp(4*I*(f*x+e))+18*I*A*exp(2*I
*(f*x+e))-6*B*exp(2*I*(f*x+e))+3*I*A-B)/f/(exp(2*I*(f*x+e))+1)^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = -\frac{8\left(15\,(-i\,A-B)a^2c^4e^{(4i\,fx+4i\,e)}+6\,(-3i\,A+B)a^2c^4e^{(2i\,fx+2i\,e)}+(-3i\,A+B)a^2c^4\right)}{15\left(fe^{(12i\,fx+12i\,e)}+6\,fe^{(10i\,fx+10i\,e)}+15\,fe^{(8i\,fx+8i\,e)}+20\,fe^{(6i\,fx+6i\,e)}+15\,fe^{(4i\,fx+4i\,e)}+6\,fe^{(2i\,fx+2i\,e}\cdots\right.}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, al
gorithm="fricas")
```

output
```
-8/15*(15*(-I*A - B)*a^2*c^4*e^(4*I*f*x + 4*I*e) + 6*(-3*I*A + B)*a^2*c^4*
e^(2*I*f*x + 2*I*e) + (-3*I*A + B)*a^2*c^4)/(f*e^(12*I*f*x + 12*I*e) + 6*f
*e^(10*I*f*x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*
e) + 15*f*e^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(78) = 156$.

Time = 0.52 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.26

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx$$
$$= \frac{24iAa^2c^4 - 8Ba^2c^4 + (144iAa^2c^4e^{2ie} - 48Ba^2c^4e^{2ie})\,e^{2ifx} + (120iAa^2c^4e^{4ie} + 120Ba^2c^4e^{4ie})\,e^{4ifx}}{15fe^{12ie}e^{12ifx} + 90fe^{10ie}e^{10ifx} + 225fe^{8ie}e^{8ifx} + 300fe^{6ie}e^{6ifx} + 225fe^{4ie}e^{4ifx} + 90fe^{2ie}e^{2ifx} + 15f}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**4,x)
```

output
```
(24*I*A*a**2*c**4 - 8*B*a**2*c**4 + (144*I*A*a**2*c**4*exp(2*I*e) - 48*B*a
**2*c**4*exp(2*I*e))*exp(2*I*f*x) + (120*I*A*a**2*c**4*exp(4*I*e) + 120*B*
a**2*c**4*exp(4*I*e))*exp(4*I*f*x))/(15*f*exp(12*I*e)*exp(12*I*f*x) + 90*f
*exp(10*I*e)*exp(10*I*f*x) + 225*f*exp(8*I*e)*exp(8*I*f*x) + 300*f*exp(6*I
*e)*exp(6*I*f*x) + 225*f*exp(4*I*e)*exp(4*I*f*x) + 90*f*exp(2*I*e)*exp(2*I
*f*x) + 15*f)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx =$$
$$-\frac{5\,Ba^2c^4\tan(fx+e)^6 + 6\,(A+2i\,B)a^2c^4\tan(fx+e)^5 + 15i\,Aa^2c^4\tan(fx+e)^4 + 20i\,Ba^2c^4\tan(fx}{30\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, al
gorithm="maxima")
```

output
```
-1/30*(5*B*a^2*c^4*tan(f*x + e)^6 + 6*(A + 2*I*B)*a^2*c^4*tan(f*x + e)^5 +
 15*I*A*a^2*c^4*tan(f*x + e)^4 + 20*I*B*a^2*c^4*tan(f*x + e)^3 + 15*(2*I*A
 - B)*a^2*c^4*tan(f*x + e)^2 - 30*A*a^2*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.41

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = -\frac{5\,Ba^2c^4\tan\left(fx+e\right)^6+6\,Aa^2c^4\tan\left(fx+e\right)^5+12i\,Ba^2c^4\tan\left(fx+e\right)^5+15i\,Aa^2c^4\tan\left(fx+e\right)^4+}{30}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, al
gorithm="giac")
```

output
```
-1/30*(5*B*a^2*c^4*tan(f*x + e)^6 + 6*A*a^2*c^4*tan(f*x + e)^5 + 12*I*B*a^
2*c^4*tan(f*x + e)^5 + 15*I*A*a^2*c^4*tan(f*x + e)^4 + 20*I*B*a^2*c^4*tan(
f*x + e)^3 + 30*I*A*a^2*c^4*tan(f*x + e)^2 - 15*B*a^2*c^4*tan(f*x + e)^2 -
 30*A*a^2*c^4*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = -\frac{\frac{a^2\,c^4\,\tan(e+f\,x)^2\,(-B+A\,2\mathrm{i})}{2}-A\,a^2\,c^4\tan(e+f\,x)+\frac{a^2\,c^4\tan(e+f\,x)^5\,(A+B\,2\mathrm{i})}{5}+\frac{B\,a^2\,c^4\tan(e+f\,x)^6}{6}+\frac{A\,a^2\,c^4\tan(e+f}{2}}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
^4,x)
```

output
```
-((a^2*c^4*tan(e + f*x)^2*(A*2i - B))/2 - A*a^2*c^4*tan(e + f*x) + (A*a^2*
c^4*tan(e + f*x)^4*1i)/2 + (a^2*c^4*tan(e + f*x)^5*(A + B*2i))/5 + (B*a^2*
c^4*tan(e + f*x)^3*2i)/3 + (B*a^2*c^4*tan(e + f*x)^6)/6)/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx$$
$$=\frac{\tan(fx+e)\,a^2c^4\left(-5\tan(fx+e)^5\,b-6\tan(fx+e)^4\,a-12\tan(fx+e)^4\,bi-15\tan(fx+e)^3\,ai-20\right.}{30f}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x)
```

output
```
(tan(e + f*x)*a**2*c**4*( - 5*tan(e + f*x)**5*b - 6*tan(e + f*x)**4*a - 12
*tan(e + f*x)**4*b*i - 15*tan(e + f*x)**3*a*i - 20*tan(e + f*x)**2*b*i - 3
0*tan(e + f*x)*a*i + 15*tan(e + f*x)*b + 30*a))/(30*f)
```

3.679 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 41, antiderivative size = 99

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = \frac{2a^2(iA+B)c^3(1-i\tan(e+fx))^3}{3f} - \frac{a^2(iA+3B)c^3(1-i\tan(e+fx))^4}{4f} + \frac{a^2Bc^3(1-i\tan(e+fx))^5}{5f}$$

output
```
2/3*a^2*(I*A+B)*c^3*(1-I*tan(f*x+e))^3/f-1/4*a^2*(I*A+3*B)*c^3*(1-I*tan(f*
x+e))^4/f+1/5*a^2*B*c^3*(1-I*tan(f*x+e))^5/f
```

Mathematica [A] (verified)

Time = 4.89 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = \frac{a^2c^3(5iA+11B+60A\tan(e+fx)+30(-iA+B)\tan^2(e+fx)+20(A-iB)\tan^3(e+fx)+15(-iA}{60f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^3,x]
```

output
```
(a^2*c^3*((5*I)*A + 11*B + 60*A*Tan[e + f*x] + 30*((-I)*A + B)*Tan[e + f*x
]^2 + 20*(A - I*B)*Tan[e + f*x]^3 + 15*((-I)*A + B)*Tan[e + f*x]^4 - (12*I
)*B*Tan[e + f*x]^5))/(60*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int ac^2(1-i\tan(e+fx))^2(i\tan(e+fx)+1)(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c^3\int (1-i\tan(e+fx))^2(i\tan(e+fx)+1)(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2c^3\int \left(-iB(1-i\tan(e+fx))^4+(3iB-A)(1-i\tan(e+fx))^3+2(A-iB)(1-i\tan(e+fx))^2\right)d\tan(e+f}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c^3\left(-\frac{1}{4}(3B+iA)(1-i\tan(e+fx))^4+\frac{2}{3}(B+iA)(1-i\tan(e+fx))^3+\frac{1}{5}B(1-i\tan(e+fx))^5\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3
,x]
```

output
```
(a^2*c^3*((2*(I*A + B)*(1 - I*Tan[e + f*x])^3)/3 - ((I*A + 3*B)*(1 - I*Tan
[e + f*x])^4)/4 + (B*(1 - I*Tan[e + f*x])^5)/5))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
risch	$\frac{4c^{3}a^{2}\left(20iA\,{\rm e}^{4i(fx+e)}+20B\,{\rm e}^{4i(fx+e)}+25iA\,{\rm e}^{2i(fx+e)}-5B\,{\rm e}^{2i(fx+e)}+5iA-B\right)}{15f\left({\rm e}^{2i(fx+e)}+1\right)^{5}}$
derivativedivides	$\frac{ic^{3}a^{2}\left(-\frac{B\tan(fx+e)^{5}}{5}+\frac{(-iB-A)\tan(fx+e)^{4}}{4}+\frac{(iA+2i(iB-A)+B)\tan(fx+e)^{3}}{3}+\frac{(-iB-A)\tan(fx+e)^{2}}{2}-i\tan(fx+e)A\right)}{f}$
default	$\frac{ic^{3}a^{2}\left(-\frac{B\tan(fx+e)^{5}}{5}+\frac{(-iB-A)\tan(fx+e)^{4}}{4}+\frac{(iA+2i(iB-A)+B)\tan(fx+e)^{3}}{3}+\frac{(-iB-A)\tan(fx+e)^{2}}{2}-i\tan(fx+e)A\right)}{f}$
norman	$\frac{A\,a^{2}c^{3}\tan(fx+e)}{f}+\frac{(-iA\,a^{2}c^{3}+B\,a^{2}c^{3})\tan(fx+e)^{2}}{2f}+\frac{(-iA\,a^{2}c^{3}+B\,a^{2}c^{3})\tan(fx+e)^{4}}{4f}+\frac{(-iB\,a^{2}c^{3}+A\,a^{2}c^{3}) \cdots}{3f}$
parallelrisch	$-\frac{12iB\,a^{2}c^{3}\tan(fx+e)^{5}+15iA\tan(fx+e)^{4}a^{2}c^{3}+20iB\tan(fx+e)^{3}a^{2}c^{3}-15B\tan(fx+e)^{4}a^{2}c^{3}+30iA\tan(fx+e)^{2}a^{2} \cdots}{60f}$
parts	$\frac{(-2iA\,a^{2}c^{3}+2B\,a^{2}c^{3})\left(\frac{\tan(fx+e)^{2}}{2}-\frac{\ln\left(1+\tan(fx+e)^{2}\right)}{2}\right)}{f}+\frac{(-2iB\,a^{2}c^{3}+A\,a^{2}c^{3})\left(\frac{\tan(fx+e)^{3}}{3}-\tan(fx+e)+\arctan \cdots\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
4/15*c^3*a^2*(20*I*A*exp(4*I*(f*x+e))+20*B*exp(4*I*(f*x+e))+25*I*A*exp(2*I
*(f*x+e))-5*B*exp(2*I*(f*x+e))+5*I*A-B)/f/(exp(2*I*(f*x+e))+1)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.25

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = -\frac{4\left(20\left(-iA-B\right)a^{2}c^{3}e^{(4i\,fx+4i\,e)}+5\left(-5i\,A+B\right)a^{2}c^{3}e^{(2i\,fx+2i\,e)}+\left(-5i\,A+B\right)a^{2}c^{3}\right)}{15\left(fe^{(10i\,fx+10i\,e)}+5\,fe^{(8i\,fx+8i\,e)}+10\,fe^{(6i\,fx+6i\,e)}+10\,fe^{(4i\,fx+4i\,e)}+5\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-4/15*(20*(-I*A - B)*a^2*c^3*e^(4*I*f*x + 4*I*e) + 5*(-5*I*A + B)*a^2*c^3*
e^(2*I*f*x + 2*I*e) + (-5*I*A + B)*a^2*c^3)/(f*e^(10*I*f*x + 10*I*e) + 5*f
*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e)
 + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(80) = 160$.

Time = 0.56 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.08

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = \frac{20iAa^2c^3-4Ba^2c^3+(100iAa^2c^3e^{2ie}-20Ba^2c^3e^{2ie})\,e^{2ifx}+(80iAa^2c^3e^{4ie}+80Ba^2c^3e^{4ie})\,e^{4ifx}}{15fe^{10ie}e^{10ifx}+75fe^{8ie}e^{8ifx}+150fe^{6ie}e^{6ifx}+150fe^{4ie}e^{4ifx}+75fe^{2ie}e^{2ifx}+15f}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**3,x)
```

output
```
(20*I*A*a**2*c**3 - 4*B*a**2*c**3 + (100*I*A*a**2*c**3*exp(2*I*e) - 20*B*a
**2*c**3*exp(2*I*e))*exp(2*I*f*x) + (80*I*A*a**2*c**3*exp(4*I*e) + 80*B*a*
*2*c**3*exp(4*I*e))*exp(4*I*f*x))/(15*f*exp(10*I*e)*exp(10*I*f*x) + 75*f*e
xp(8*I*e)*exp(8*I*f*x) + 150*f*exp(6*I*e)*exp(6*I*f*x) + 150*f*exp(4*I*e)*
exp(4*I*f*x) + 75*f*exp(2*I*e)*exp(2*I*f*x) + 15*f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = -\frac{12i\,Ba^2c^3\tan(fx+e)^5-15\,(-i\,A+B)a^2c^3\tan(fx+e)^4-20\,(A-i\,B)a^2c^3\tan(fx+e)^3-30\,(-i}{60\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
-1/60*(12*I*B*a^2*c^3*tan(f*x + e)^5 - 15*(-I*A + B)*a^2*c^3*tan(f*x + e)^
4 - 20*(A - I*B)*a^2*c^3*tan(f*x + e)^3 - 30*(-I*A + B)*a^2*c^3*tan(f*x +
e)^2 - 60*A*a^2*c^3*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.41

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx =$$

$$-\frac{12i\,Ba^2c^3\tan\left(fx+e\right)^5+15i\,Aa^2c^3\tan\left(fx+e\right)^4-15\,Ba^2c^3\tan\left(fx+e\right)^4-20\,Aa^2c^3\tan\left(fx+e\right)^3}{60}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-1/60*(12*I*B*a^2*c^3*tan(f*x + e)^5 + 15*I*A*a^2*c^3*tan(f*x + e)^4 - 15*
B*a^2*c^3*tan(f*x + e)^4 - 20*A*a^2*c^3*tan(f*x + e)^3 + 20*I*B*a^2*c^3*ta
n(f*x + e)^3 + 30*I*A*a^2*c^3*tan(f*x + e)^2 - 30*B*a^2*c^3*tan(f*x + e)^2
 - 60*A*a^2*c^3*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx =$$

$$-\frac{-A\,a^2\,c^3\tan(e+f\,x)+\frac{a^2\,c^3\tan(e+f\,x)^2\,(A+B\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{a^2\,c^3\tan(e+f\,x)^3\,(B+A\,1\mathrm{i})\,1\mathrm{i}}{3}+\frac{a^2\,c^3\tan(e+f\,x)^4\,(A+B\,1\mathrm{i})\,1\mathrm{i}}{4}+\frac{B}{}}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
^3,x)
```

output
```
-((a^2*c^3*tan(e + f*x)^2*(A + B*1i)*1i)/2 - A*a^2*c^3*tan(e + f*x) + (a^2
*c^3*tan(e + f*x)^3*(A*1i + B)*1i)/3 + (a^2*c^3*tan(e + f*x)^4*(A + B*1i)*
1i)/4 + (B*a^2*c^3*tan(e + f*x)^5*1i)/5)/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx$$
$$=\frac{\tan(fx+e)\,a^2c^3\left(-12\tan(fx+e)^4\,bi-15\tan(fx+e)^3\,ai+15\tan(fx+e)^3\,b+20\tan(fx+e)^2\,a-\right.}{60f}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x)
```

output
```
(tan(e + f*x)*a**2*c**3*( - 12*tan(e + f*x)**4*b*i - 15*tan(e + f*x)**3*a*
i + 15*tan(e + f*x)**3*b + 20*tan(e + f*x)**2*a - 20*tan(e + f*x)**2*b*i -
 30*tan(e + f*x)*a*i + 30*tan(e + f*x)*b + 60*a))/(60*f)
```

3.680 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 41, antiderivative size = 62

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = \frac{a^2Bc^2\sec^4(e+fx)}{4f}+\frac{a^2Ac^2\tan(e+fx)}{f}+\frac{a^2Ac^2\tan^3(e+fx)}{3f}$$

output
```
1/4*a^2*B*c^2*sec(f*x+e)^4/f+a^2*A*c^2*tan(f*x+e)/f+1/3*a^2*A*c^2*tan(f*x+
e)^3/f
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = \frac{a^2Bc^2\sec^4(e+fx)}{4f}+\frac{a^2Ac^2\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^2,x]
```

output
```
(a^2*B*c^2*Sec[e + f*x]^4)/(4*f) + (a^2*A*c^2*(Tan[e + f*x] + Tan[e + f*x]
^3/3))/f
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3042, 4071, 27, 82, 455, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int ac(1-i\tan(e+fx))(i\tan(e+fx)+1)(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c^2\int (1-i\tan(e+fx))(i\tan(e+fx)+1)(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{82}$$

$$\frac{a^2c^2\int (A+B\tan(e+fx))\left(\tan^2(e+fx)+1\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{455}$$

$$\frac{a^2c^2\left(A\int \left(\tan^2(e+fx)+1\right)d\tan(e+fx)+\frac{1}{4}B\left(\tan^2(e+fx)+1\right)^2\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c^2\left(A\left(\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)+\frac{1}{4}B\left(\tan^2(e+fx)+1\right)^2\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2
,x]
```

output
```
(a^2*c^2*((B*(1 + Tan[e + f*x]^2)^2)/4 + A*(Tan[e + f*x] + Tan[e + f*x]^3/
3)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{c^2a^2\left(\frac{B\tan(fx+e)^4}{4}+\frac{A\tan(fx+e)^3}{3}+\frac{B\tan(fx+e)^2}{2}+A\tan(fx+e)\right)}{f}$
default	$\frac{c^2a^2\left(\frac{B\tan(fx+e)^4}{4}+\frac{A\tan(fx+e)^3}{3}+\frac{B\tan(fx+e)^2}{2}+A\tan(fx+e)\right)}{f}$
risch	$\frac{4c^2a^2\left(3iA\,e^{4i(fx+e)}+3B\,e^{4i(fx+e)}+4iA\,e^{2i(fx+e)}+iA\right)}{3f\left(e^{2i(fx+e)}+1\right)^4}$
parallelrisch	$\frac{3B\,a^2c^2\tan(fx+e)^4+4A\,a^2c^2\tan(fx+e)^3+6B\,a^2c^2\tan(fx+e)^2+12A\,a^2c^2\tan(fx+e)}{12f}$
norman	$\frac{a^2A\,c^2\tan(fx+e)}{f}+\frac{B\,a^2c^2\tan(fx+e)^2}{2f}+\frac{B\,a^2c^2\tan(fx+e)^4}{4f}+\frac{a^2A\,c^2\tan(fx+e)^3}{3f}$
parts	$A\,a^2c^2x+\frac{A\,a^2c^2\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{B\,a^2c^2\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{B\,a^2c^2\left(\mathrm{ta}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*c^2*a^2*(1/4*B*tan(f*x+e)^4+1/3*A*tan(f*x+e)^3+1/2*B*tan(f*x+e)^2+A*ta
n(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$$
$$= -\frac{4\left(3\left(-iA-B\right)a^2c^2e^{(4i\,fx+4i\,e)}-4i\,Aa^2c^2e^{(2i\,fx+2i\,e)}-i\,Aa^2c^2\right)}{3\left(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-4/3*(3*(-I*A - B)*a^2*c^2*e^(4*I*f*x + 4*I*e) - 4*I*A*a^2*c^2*e^(2*I*f*x
+ 2*I*e) - I*A*a^2*c^2)/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) +
 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.58

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = \frac{16iAa^2c^2e^{2ie}e^{2ifx}+4iAa^2c^2+(12iAa^2c^2e^{4ie}+12Ba^2c^2e^{4ie})\,e^{4ifx}}{3fe^{8ie}e^{8ifx}+12fe^{6ie}e^{6ifx}+18fe^{4ie}e^{4ifx}+12fe^{2ie}e^{2ifx}+3f}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**2,x)
```

output
```
(16*I*A*a**2*c**2*exp(2*I*e)*exp(2*I*f*x) + 4*I*A*a**2*c**2 + (12*I*A*a**2
*c**2*exp(4*I*e) + 12*B*a**2*c**2*exp(4*I*e))*exp(4*I*f*x))/(3*f*exp(8*I*e
)*exp(8*I*f*x) + 12*f*exp(6*I*e)*exp(6*I*f*x) + 18*f*exp(4*I*e)*exp(4*I*f*
x) + 12*f*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = \frac{3\,Ba^2c^2\tan(fx+e)^4+4\,Aa^2c^2\tan(fx+e)^3+6\,Ba^2c^2\tan(fx+e)^2+12\,Aa^2c^2\tan(fx+e)}{12\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
1/12*(3*B*a^2*c^2*tan(f*x + e)^4 + 4*A*a^2*c^2*tan(f*x + e)^3 + 6*B*a^2*c^
2*tan(f*x + e)^2 + 12*A*a^2*c^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$$
$$=\frac{3\,Ba^2c^2\tan\left(fx+e\right)^4+4\,Aa^2c^2\tan\left(fx+e\right)^3+6\,Ba^2c^2\tan\left(fx+e\right)^2+12\,Aa^2c^2\tan\left(fx+e\right)}{12\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/12*(3*B*a^2*c^2*tan(f*x + e)^4 + 4*A*a^2*c^2*tan(f*x + e)^3 + 6*B*a^2*c^
2*tan(f*x + e)^2 + 12*A*a^2*c^2*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$$
$$=\frac{a^2\,c^2\,\sin\left(e+f\,x\right)\,\left(12\,A\cos\left(e+f\,x\right)^3+6\,B\cos\left(e+f\,x\right)^2\,\sin\left(e+f\,x\right)+4\,A\,\cos\left(e+f\,x\right)\,\sin\left(e+f\,x\right.}{12\,f\cos\left(e+f\,x\right)^4}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
^2,x)
```

output
```
(a^2*c^2*sin(e + f*x)*(12*A*cos(e + f*x)^3 + 3*B*sin(e + f*x)^3 + 4*A*cos(
e + f*x)*sin(e + f*x)^2 + 6*B*cos(e + f*x)^2*sin(e + f*x)))/(12*f*cos(e +
f*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = \frac{\tan(fx+e)\,a^2c^2\left(3\tan(fx+e)^3 b+4\tan(fx+e)^2 a+6\tan(fx+e)\,b+12a\right)}{12f}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x)
```

output
```
(tan(e + f*x)*a**2*c**2*(3*tan(e + f*x)**3*b + 4*tan(e + f*x)**2*a + 6*tan
(e + f*x)*b + 12*a))/(12*f)
```

3.681 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx$

Optimal result

Integrand size = 39, antiderivative size = 64

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{a^2Ac\tan(e+fx)}{f}+\frac{a^2(iA+B)c\tan^2(e+fx)}{2f}+\frac{ia^2Bc\tan^3(e+fx)}{3f}$$

output
```
a^2*A*c*tan(f*x+e)/f+1/2*a^2*(I*A+B)*c*tan(f*x+e)^2/f+1/3*I*a^2*B*c*tan(f*
x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{a^2c(-2B+6A\tan(e+fx)+3(iA+B)\tan^2(e+fx)+2iB\tan^3(e+fx))}{6f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x]),x]
```

output
```
(a^2*c*(-2*B + 6*A*Tan[e + f*x] + 3*(I*A + B)*Tan[e + f*x]^2 + (2*I)*B*Tan
[e + f*x]^3))/(6*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 4071, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a(i\tan(e+fx)+1)(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c\int (i\tan(e+fx)+1)(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{49}$$

$$\frac{a^2c\int \left(iB\tan^2(e+fx)+(iA+B)\tan(e+fx)+A\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c\left(\frac{1}{2}(B+iA)\tan^2(e+fx)+A\tan(e+fx)+\frac{1}{3}iB\tan^3(e+fx)\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]),x
]
```

output
```
(a^2*c*(A*Tan[e + f*x] + ((I*A + B)*Tan[e + f*x]^2)/2 + (I/3)*B*Tan[e + f*
x]^3))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{i a^{2} c \left(-\frac{B \tan \left(f x +e \right)^{3}}{3}+\frac{\left(i B -A \right) \tan \left(f x +e \right)^{2}}{2}+i \tan \left(f x +e \right) A \right)}{f}$
default	$-\frac{i a^{2} c \left(-\frac{B \tan \left(f x +e \right)^{3}}{3}+\frac{\left(i B -A \right) \tan \left(f x +e \right)^{2}}{2}+i \tan \left(f x +e \right) A \right)}{f}$
norman	$\frac{a^{2} A c \tan \left(f x +e \right)}{f}+\frac{\left(i a^{2} c A +a^{2} c B \right) \tan \left(f x +e \right)^{2}}{2 f}+\frac{i a^{2} B c \tan \left(f x +e \right)^{3}}{3 f}$
parallelrisch	$\frac{2 i a^{2} B c \tan \left(f x +e \right)^{3}+3 i A \tan \left(f x +e \right)^{2} a^{2} c +3 B \tan \left(f x +e \right)^{2} a^{2} c +6 A \tan \left(f x +e \right) a^{2} c}{6 f}$
risch	$\frac{2 a^{2} c \left(6 i A \,{\mathrm e}^{4 i \left(f x +e \right)}+6 B \,{\mathrm e}^{4 i \left(f x +e \right)}+9 i A \,{\mathrm e}^{2 i \left(f x +e \right)}+3 B \,{\mathrm e}^{2 i \left(f x +e \right)}+3 i A +B \right)}{3 f \left({\mathrm e}^{2 i \left(f x +e \right)}+1\right)^{3}}$
parts	$\frac{\left(i B \,a^{2} c +a^{2} c A \right) \left(\tan \left(f x +e \right)-\arctan \left(\tan \left(f x +e \right)\right)\right)}{f}+\frac{\left(i a^{2} c A +a^{2} c B \right) \ln \left(1+\tan \left(f x +e \right)^{2}\right)}{2 f}+\frac{\left(i a^{2} c A +a^{2} c B \right) \left(\frac{\tan(f\ldots}{2}\ldots\right.}{\ldots}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-I/f*a^2*c*(-1/3*B*tan(f*x+e)^3+1/2*(-A+I*B)*tan(f*x+e)^2+I*A*tan(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.53

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx$$
$$=-\frac{2\left(6\left(-iA-B\right)a^2ce^{(4i\,fx+4i\,e)}+3\left(-3i\,A-B\right)a^2ce^{(2i\,fx+2i\,e)}+\left(-3i\,A-B\right)a^2c\right)}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algo
rithm="fricas")
```

output
```
-2/3*(6*(-I*A - B)*a^2*c*e^(4*I*f*x + 4*I*e) + 3*(-3*I*A - B)*a^2*c*e^(2*I
*f*x + 2*I*e) + (-3*I*A - B)*a^2*c)/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*
x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(56) = 112$.

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.47

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{6iAa^2c+2Ba^2c+\left(18iAa^2ce^{2ie}+6Ba^2ce^{2ie}\right)e^{2ifx}+\left(12iAa^2ce^{4ie}+12Ba^2ce^{4ie}\right)e^{4ifx}}{3fe^{6ie}e^{6ifx}+9fe^{4ie}e^{4ifx}+9fe^{2ie}e^{2ifx}+3f}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x)
```

output
```
(6*I*A*a**2*c + 2*B*a**2*c + (18*I*A*a**2*c*exp(2*I*e) + 6*B*a**2*c*exp(2*
I*e))*exp(2*I*f*x) + (12*I*A*a**2*c*exp(4*I*e) + 12*B*a**2*c*exp(4*I*e))*e
xp(4*I*f*x))/(3*f*exp(6*I*e)*exp(6*I*f*x) + 9*f*exp(4*I*e)*exp(4*I*f*x) +
9*f*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = -\frac{-2i\,Ba^2c\tan(fx+e)^3-3\,(i\,A+B)a^2c\tan(fx+e)^2-6\,Aa^2c\tan(fx+e)}{6\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algo
rithm="maxima")
```

output
```
-1/6*(-2*I*B*a^2*c*tan(f*x + e)^3 - 3*(I*A + B)*a^2*c*tan(f*x + e)^2 - 6*A
*a^2*c*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = -\frac{-2i\,Ba^2c\tan(fx+e)^3-3i\,Aa^2c\tan(fx+e)^2-3\,Ba^2c\tan(fx+e)^2-6\,Aa^2c\tan(fx+e)}{6\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algo
rithm="giac")
```

output
```
-1/6*(-2*I*B*a^2*c*tan(f*x + e)^3 - 3*I*A*a^2*c*tan(f*x + e)^2 - 3*B*a^2*c
*tan(f*x + e)^2 - 6*A*a^2*c*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{a^2\,c\tan(e+f\,x)\,\left(6\,A+A\tan(e+f\,x)\,3\mathrm{i}+3\,B\tan(e+f\,x)+B\tan(e+f\,x)^2\,2\mathrm{i}\right)}{6\,f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
,x)
```

output
```
(a^2*c*tan(e + f*x)*(6*A + A*tan(e + f*x)*3i + 3*B*tan(e + f*x) + B*tan(e
+ f*x)^2*2i))/(6*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{\tan(fx+e)\,a^2c\big(2\tan(fx+e)^2\,bi+3\tan(fx+e)\,ai+3\tan(fx+e)\,b+6a\big)}{6f}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x)
```

output
```
(tan(e + f*x)*a**2*c*(2*tan(e + f*x)**2*b*i + 3*tan(e + f*x)*a*i + 3*tan(e
 + f*x)*b + 6*a))/(6*f)
```

3.682 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\,dx = 2a^2(A-iB)x - \frac{2a^2(iA+B)\log(\cos(e+fx))}{f} - \frac{a^2(A-iB)\tan(e+fx)}{f} + \frac{B(a+ia\tan(e+fx))^2}{2f}$$

output
```
2*a^2*(A-I*B)*x-2*a^2*(I*A+B)*ln(cos(f*x+e))/f-a^2*(A-I*B)*tan(f*x+e)/f+1/
2*B*(a+I*a*tan(f*x+e))^2/f
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\,dx = \frac{a^2(B+4(iA+B)\log(i+\tan(e+fx))-2(A-2iB)\tan(e+fx)-B\tan^2(e+fx))}{2f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]),x]
```

output
```
(a^2*(B + 4*(I*A + B)*Log[I + Tan[e + f*x]] - 2*(A - (2*I)*B)*Tan[e + f*x]
 - B*Tan[e + f*x]^2))/(2*f)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4010, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))dx$$

$$\Big\downarrow \ 4010$$

$$(A-iB)\int (i\tan(e+fx)a+a)^2dx+\frac{B(a+ia\tan(e+fx))^2}{2f}$$

$$\Big\downarrow \ 3042$$

$$(A-iB)\int (i\tan(e+fx)a+a)^2dx+\frac{B(a+ia\tan(e+fx))^2}{2f}$$

$$\Big\downarrow \ 3958$$

$$(A-iB)\left(2ia^2\int \tan(e+fx)dx-\frac{a^2\tan(e+fx)}{f}+2a^2x\right)+\frac{B(a+ia\tan(e+fx))^2}{2f}$$

$$\Big\downarrow \ 3042$$

$$(A-iB)\left(2ia^2\int \tan(e+fx)dx-\frac{a^2\tan(e+fx)}{f}+2a^2x\right)+\frac{B(a+ia\tan(e+fx))^2}{2f}$$

$$\Big\downarrow \ 3956$$

$$(A-iB)\left(-\frac{a^2\tan(e+fx)}{f}-\frac{2ia^2\log(\cos(e+fx))}{f}+2a^2x\right)+\frac{B(a+ia\tan(e+fx))^2}{2f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]),x]
```

output
```
(B*(a + I*a*Tan[e + f*x])^2)/(2*f) + (A - I*B)*(2*a^2*x - ((2*I)*a^2*Log[C
os[e + f*x]])/f - (a^2*Tan[e + f*x])/f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a^2\left(-\frac{B\tan(fx+e)^2}{2}-A\tan(fx+e)+2iB\tan(fx+e)+\frac{(2iA+2B)\ln\left(1+\tan(fx+e)^2\right)}{2}+(-2iB+2A)\arctan(\tan(fx+e))\right)}{f}$
default	$\frac{a^2\left(-\frac{B\tan(fx+e)^2}{2}-A\tan(fx+e)+2iB\tan(fx+e)+\frac{(2iA+2B)\ln\left(1+\tan(fx+e)^2\right)}{2}+(-2iB+2A)\arctan(\tan(fx+e))\right)}{f}$
norman	$(-2iB\,a^2+2A\,a^2)\,x-\frac{(-2iB\,a^2+A\,a^2)\tan(fx+e)}{f}-\frac{B\,a^2\tan(fx+e)^2}{2f}+\frac{(iA\,a^2+B\,a^2)\ln\left(1+\tan(fx+e\right.}{f}$
parallelrisch	$\frac{-4iBx\,a^2f+2iA\ln\left(1+\tan(fx+e)^2\right)a^2+4Ax\,a^2f+4iB\tan(fx+e)a^2-B\tan(fx+e)^2a^2-2A\tan(fx+e)a^2+2B\ln\left(1+\right.}{2f}$
parts	$A\,a^2x+\frac{(2iA\,a^2+B\,a^2)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(2iB\,a^2-A\,a^2)(\tan(fx+e)-\arctan(\tan(fx+e)))}{f}-\frac{B\,a^2\left(\frac{\tan(}{}\right.}{}$
risch	$\frac{4ia^2Be}{f}-\frac{4a^2Ae}{f}-\frac{2a^2\left(iA\,e^{2i(fx+e)}+3B\,e^{2i(fx+e)}+iA+2B\right)}{f\left(e^{2i(fx+e)}+1\right)^2}-\frac{2a^2\ln\left(e^{2i(fx+e)}+1\right)B}{f}-\frac{2ia^2\ln\left(e^{2i(fx+e)}+1\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*a^2*(-1/2*B*tan(f*x+e)^2-A*tan(f*x+e)+2*I*B*tan(f*x+e)+1/2*(2*B+2*I*A)
*ln(1+tan(f*x+e)^2)+(2*A-2*I*B)*arctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.51

$$\int(a+ia\tan(e+fx))^2(A+B\tan(e+fx))\,dx =$$
$$-\frac{2\left(\left(i\,A+3\,B\right)a^2e^{(2i\,fx+2i\,e)}+\left(i\,A+2\,B\right)a^2+\left(\left(i\,A+B\right)a^2e^{(4i\,fx+4i\,e)}+2\,\left(i\,A+B\right)a^2e^{(2i\,fx+2i\,e)}+(i\,A\right.\right.}{fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e)),x, algorithm="fricas")
```

output
```
-2*((I*A + 3*B)*a^2*e^(2*I*f*x + 2*I*e) + (I*A + 2*B)*a^2 + ((I*A + B)*a^2
*e^(4*I*f*x + 4*I*e) + 2*(I*A + B)*a^2*e^(2*I*f*x + 2*I*e) + (I*A + B)*a^2
)*log(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x +
2*I*e) + f)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.52

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\,dx = -\frac{2ia^2(A-iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{f} + \frac{-2iAa^2-4Ba^2+\left(-2iAa^2e^{2ie}-6Ba^2e^{2ie}\right)e^{2ifx}}{fe^{4ie}e^{4ifx}+2fe^{2ie}e^{2ifx}+f}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e)),x)
```

output
```
-2*I*a**2*(A - I*B)*log(exp(2*I*f*x) + exp(-2*I*e))/f + (-2*I*A*a**2 - 4*B
*a**2 + (-2*I*A*a**2*exp(2*I*e) - 6*B*a**2*exp(2*I*e))*exp(2*I*f*x))/(f*ex
p(4*I*e)*exp(4*I*f*x) + 2*f*exp(2*I*e)*exp(2*I*f*x) + f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\,dx = -\frac{Ba^2\tan\left(fx+e\right)^2-4\left(fx+e\right)(A-i\,B)a^2-2\left(i\,A+B\right)a^2\log\left(\tan\left(fx+e\right)^2+1\right)+2\left(A-2i\,B\right)a^2 \mathrm{t}}{2\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e)),x, algorithm="maxima")
```

output
```
-1/2*(B*a^2*tan(f*x + e)^2 - 4*(f*x + e)*(A - I*B)*a^2 - 2*(I*A + B)*a^2*l
og(tan(f*x + e)^2 + 1) + 2*(A - 2*I*B)*a^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int (a + ia\tan(e + fx))^2(A + B\tan(e + fx))\,dx = -\frac{2\left(-i\,Aa^2 - Ba^2\right)\log\left(\tan\left(fx + e\right) + i\right)}{f} - \frac{Ba^2f\tan\left(fx + e\right)^2 + 2\,Aa^2f\tan\left(fx + e\right) - 4i\,Ba^2f\tan\left(fx + e\right)}{2\,f^2}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e)),x, algorithm="giac")
```

output
```
-2*(-I*A*a^2 - B*a^2)*log(tan(f*x + e) + I)/f - 1/2*(B*a^2*f*tan(f*x + e)^
2 + 2*A*a^2*f*tan(f*x + e) - 4*I*B*a^2*f*tan(f*x + e))/f^2
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int (a + ia\tan(e + fx))^2(A + B\tan(e + fx))\,dx = \frac{\ln\left(\tan(e + f\,x) + 1\mathrm{i}\right)\,\left(2\,B\,a^2 + A\,a^2\,2\mathrm{i}\right)}{f} + \frac{\tan(e + f\,x)\,\left(a^2\,\left(B + A\,1\mathrm{i}\right)\,1\mathrm{i} + B\,a^2\,1\mathrm{i}\right)}{f} - \frac{B\,a^2\,\tan(e + f\,x)^2}{2\,f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2,x)
```

output
```
(log(tan(e + f*x) + 1i)*(A*a^2*2i + 2*B*a^2))/f + (tan(e + f*x)*(a^2*(A*1i
 + B)*1i + B*a^2*1i))/f - (B*a^2*tan(e + f*x)^2)/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int (a + ia\tan(e + fx))^2(A + B\tan(e + fx))\,dx$$
$$= \frac{a^2\left(2\log\left(\tan\left(fx+e\right)^2+1\right)ai + 2\log\left(\tan\left(fx+e\right)^2+1\right)b - \tan\left(fx+e\right)^2 b - 2\tan\left(fx+e\right)a + 4\tan\right.}{2f}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e)),x)
```

output
```
(a**2*(2*log(tan(e + f*x)**2 + 1)*a*i + 2*log(tan(e + f*x)**2 + 1)*b - tan
(e + f*x)**2*b - 2*tan(e + f*x)*a + 4*tan(e + f*x)*b*i + 4*a*f*x - 4*b*f*i
*x))/(2*f)
```

3.683 $\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 93

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = -\frac{a^2(A-3iB)x}{c}+\frac{a^2(iA+3B)\log(\cos(e+fx))}{cf} - \frac{ia^2B\tan(e+fx)}{cf}+\frac{2a^2(A-iB)}{cf(i+\tan(e+fx))}$$

output
```
-a^2*(A-3*I*B)*x/c+a^2*(I*A+3*B)*ln(cos(f*x+e))/c/f-I*a^2*B*tan(f*x+e)/c/f
+2*a^2*(A-I*B)/c/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \frac{B(a+ia\tan(e+fx))^2}{f(c-ic\tan(e+fx))} - \frac{a^2(iA+3B)\left(\log(i+\tan(e+fx))+\frac{2i}{i+\tan(e+fx)}\right)}{cf}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x]),x]
```

output
```
(B*(a + I*a*Tan[e + f*x])^2)/(f*(c - I*c*Tan[e + f*x])) - (a^2*(I*A + 3*B)
*(Log[I + Tan[e + f*x]] + (2*I)/(I + Tan[e + f*x])))/(c*f)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{c^2(1-i\tan(e+fx))^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(1-i\tan(e+fx))^2}d\tan(e+fx)}{cf}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2\int \left(-\frac{2(A-iB)}{(\tan(e+fx)+i)^2}-iB-\frac{i(A-3iB)}{\tan(e+fx)+i}\right)d\tan(e+fx)}{cf}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\left(\frac{2(A-iB)}{\tan(e+fx)+i}-(3B+iA)\log(\tan(e+fx)+i)-iB\tan(e+fx)\right)}{cf}$$

input
```
Int[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
,x]
```

output
```
(a^2*(-((I*A + 3*B)*Log[I + Tan[e + f*x]]) - I*B*Tan[e + f*x] + (2*(A - I*
B))/(I + Tan[e + f*x])))/(c*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.57

method	result
risch	$-\frac{e^{2i(fx+e)}a^2B}{cf}-\frac{ie^{2i(fx+e)}Aa^2}{cf}-\frac{6ia^2Be}{cf}+\frac{2a^2Ae}{cf}+\frac{2Ba^2}{fc(e^{2i(fx+e)}+1)}+\frac{3a^2\ln(e^{2i(fx+e)}+1)B}{cf}+\frac{ia^2\text{l}}{}$
derivativedivides	$-\frac{ia^2B\tan(fx+e)}{cf}-\frac{ia^2A\ln\left(1+\tan(fx+e)^2\right)}{2fc}-\frac{3a^2B\ln\left(1+\tan(fx+e)^2\right)}{2fc}-\frac{a^2A\arctan(\tan(fx+e))}{fc}+\frac{3ia^2}{}$
default	$-\frac{ia^2B\tan(fx+e)}{cf}-\frac{ia^2A\ln\left(1+\tan(fx+e)^2\right)}{2fc}-\frac{3a^2B\ln\left(1+\tan(fx+e)^2\right)}{2fc}-\frac{a^2A\arctan(\tan(fx+e))}{fc}+\frac{3ia^2}{}$
norman	$\frac{\frac{(-3iBa^2+2Aa^2)\tan(fx+e)}{cf}-\frac{(-3iBa^2+Aa^2)x}{c}-\frac{2iAa^2+2Ba^2}{cf}-\frac{(-3iBa^2+Aa^2)x\tan(fx+e)^2}{c}-\frac{iBa^2\tan(fx+e)^3}{cf}}{1+\tan(fx+e)^2}-\frac{(i}{}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-1/c/f*exp(2*I*(f*x+e))*a^2*B-I/c/f*exp(2*I*(f*x+e))*A*a^2-6*I*a^2/c/f*B*e
+2*a^2/c/f*A*e+2/f/c*B*a^2/(exp(2*I*(f*x+e))+1)+3*a^2/c/f*ln(exp(2*I*(f*x+
e))+1)*B+I*a^2/c/f*ln(exp(2*I*(f*x+e))+1)*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \frac{(-iA-B)a^2e^{(4i\,fx+4i\,e)}+(-iA-B)a^2e^{(2i\,fx+2i\,e)}+2\,Ba^2+\left((iA+3\,B)a^2e^{(2i\,fx+2i\,e)}+(iA+3\,B)a^2\right)}{cfe^{(2i\,fx+2i\,e)}+cf}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algo
rithm="fricas")
```

output
```
((-I*A - B)*a^2*e^(4*I*f*x + 4*I*e) + (-I*A - B)*a^2*e^(2*I*f*x + 2*I*e) +
 2*B*a^2 + ((I*A + 3*B)*a^2*e^(2*I*f*x + 2*I*e) + (I*A + 3*B)*a^2)*log(e^(
2*I*f*x + 2*I*e) + 1))/(c*f*e^(2*I*f*x + 2*I*e) + c*f)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.44

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \frac{2Ba^2}{cfe^{2ie}e^{2ifx}+cf} + \frac{ia^2(A-3iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{cf} + \begin{cases} \frac{\left(-iAa^2e^{2ie}-Ba^2e^{2ie}\right)e^{2ifx}}{cf} & \text{for } cf\neq 0 \\ \frac{x\left(2Aa^2e^{2ie}-2iBa^2e^{2ie}\right)}{c} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

output
```
2*B*a**2/(c*f*exp(2*I*e)*exp(2*I*f*x) + c*f) + I*a**2*(A - 3*I*B)*log(exp(
2*I*f*x) + exp(-2*I*e))/(c*f) + Piecewise(((-I*A*a**2*exp(2*I*e) - B*a**2*
exp(2*I*e))*exp(2*I*f*x)/(c*f), Ne(c*f, 0)), (x*(2*A*a**2*exp(2*I*e) - 2*I
*B*a**2*exp(2*I*e))/c, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx$$
$$= -\frac{i\,Ba^2\tan(fx+e)}{cf} + \frac{(-i\,Aa^2-3\,Ba^2)\log(\tan(fx+e)+i)}{cf} + \frac{2\,(Aa^2-i\,Ba^2)}{cf(\tan(fx+e)+i)}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algo
rithm="giac")
```

output
```
-I*B*a^2*tan(f*x + e)/(c*f) + (-I*A*a^2 - 3*B*a^2)*log(tan(f*x + e) + I)/(
c*f) + 2*(A*a^2 - I*B*a^2)/(c*f*(tan(f*x + e) + I))
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx$$
$$= -\frac{\ln(\tan(e+f\,x)+1\mathrm{i})\left(\frac{3\,B\,a^2}{c}+\frac{A\,a^2\,1\mathrm{i}}{c}\right)}{f} + \frac{\frac{A\,a^2+B\,a^2\,1\mathrm{i}}{c}+\frac{A\,a^2-B\,a^2\,3\mathrm{i}}{c}}{f\,(\tan(e+f\,x)+1\mathrm{i})} - \frac{B\,a^2\tan(e+f\,x)\,1\mathrm{i}}{c\,f}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1
i),x)
```

output
```
((A*a^2 + B*a^2*1i)/c + (A*a^2 - B*a^2*3i)/c)/(f*(tan(e + f*x) + 1i)) - (l
og(tan(e + f*x) + 1i)*((A*a^2*1i)/c + (3*B*a^2)/c))/f - (B*a^2*tan(e + f*x
)*1i)/(c*f)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx$$
$$=\frac{a^2\left(8\left(\int \frac{1}{\tan(fx+e)+i}dx\right)afi+8\left(\int \frac{1}{\tan(fx+e)+i}dx\right)bf-\log\left(\tan\left(fx+e\right)^2+1\right)ai-3\log\left(\tan\left(fx+e\right)^2+\right.\right.}{2cf}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

output
```
(a**2*(8*int(1/(tan(e + f*x) + i),x)*a*f*i + 8*int(1/(tan(e + f*x) + i),x)
*b*f - log(tan(e + f*x)**2 + 1)*a*i - 3*log(tan(e + f*x)**2 + 1)*b - 2*tan
(e + f*x)*b*i - 6*a*f*x + 10*b*f*i*x))/(2*c*f)
```

3.684 $\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 91

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = -\frac{ia^2Bx}{c^2}-\frac{a^2B\log(\cos(e+fx))}{c^2f}+\frac{a^2(iA+B)}{c^2f(i+\tan(e+fx))^2}-\frac{a^2(A-3iB)}{c^2f(i+\tan(e+fx))}$$

output
```
-I*a^2*B*x/c^2-a^2*B*ln(cos(f*x+e))/c^2/f+a^2*(I*A+B)/c^2/f/(I+tan(f*x+e))
^2-a^2*(A-3*I*B)/c^2/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = \frac{a^2\left(B\log(i+\tan(e+fx))-\frac{2B+(A-3iB)\tan(e+fx)}{(i+\tan(e+fx))^2}\right)}{c^2f}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^2,x]
```

output
```
(a^2*(B*Log[I + Tan[e + f*x]] - (2*B + (A - (3*I)*B)*Tan[e + f*x])/(I + Ta
n[e + f*x])^2))/(c^2*f)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{c^3(1-i\tan(e+fx))^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(1-i\tan(e+fx))^3}d\tan(e+fx)}{c^2f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2\int \left(-\frac{2i(A-iB)}{(\tan(e+fx)+i)^3}+\frac{B}{\tan(e+fx)+i}+\frac{A-3iB}{(\tan(e+fx)+i)^2}\right)d\tan(e+fx)}{c^2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\left(-\frac{A-3iB}{\tan(e+fx)+i}+\frac{B+iA}{(\tan(e+fx)+i)^2}+B\log(\tan(e+fx)+i)\right)}{c^2f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^2,x]
```

output
```
(a^2*(B*Log[I + Tan[e + f*x]] + (I*A + B)/(I + Tan[e + f*x])^2 - (A - (3*I
)*B)/(I + Tan[e + f*x])))/(c^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{e^{4i(fx+e)} a^2 B}{4c^2 f} - \frac{i e^{4i(fx+e)} A a^2}{4c^2 f} + \frac{B a^2 e^{2i(fx+e)}}{c^2 f} + \frac{2iB a^2 e}{c^2 f} - \frac{B a^2 \ln(e^{2i(fx+e)}+1)}{c^2 f}$
derivativedivides	$\frac{i a^2 A}{f c^2 (i+\tan(fx+e))^2} + \frac{a^2 B}{f c^2 (i+\tan(fx+e))^2} + \frac{3i a^2 B}{f c^2 (i+\tan(fx+e))} - \frac{a^2 A}{f c^2 (i+\tan(fx+e))} + \frac{a^2 B \ln\left(1+\tan(fx-\right.}{2f c^2}$
default	$\frac{i a^2 A}{f c^2 (i+\tan(fx+e))^2} + \frac{a^2 B}{f c^2 (i+\tan(fx+e))^2} + \frac{3i a^2 B}{f c^2 (i+\tan(fx+e))} - \frac{a^2 A}{f c^2 (i+\tan(fx+e))} + \frac{a^2 B \ln\left(1+\tan(fx-\right.}{2f c^2}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-1/4/c^2/f*exp(4*I*(f*x+e))*a^2*B-1/4*I/c^2/f*exp(4*I*(f*x+e))*A*a^2+B*a^2
/c^2/f*exp(2*I*(f*x+e))+2*I*B*a^2/c^2/f*e-B*a^2/c^2/f*ln(exp(2*I*(f*x+e))+
1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.68

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = \frac{(-iA-B)a^2e^{(4ifx+4ie)}+4Ba^2e^{(2ifx+2ie)}-4Ba^2\log\left(e^{(2ifx+2ie)}+1\right)}{4c^2f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
1/4*((-I*A - B)*a^2*e^(4*I*f*x + 4*I*e) + 4*B*a^2*e^(2*I*f*x + 2*I*e) - 4*
B*a^2*log(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.76

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx$$
$$= -\frac{Ba^2\log\left(e^{2ifx}+e^{-2ie}\right)}{c^2f} + \begin{cases} \frac{4Ba^2c^2fe^{2ie}e^{2ifx}+\left(-iAa^2c^2fe^{4ie}-Ba^2c^2fe^{4ie}\right)e^{4ifx}}{4c^4f^2} & \text{for } c^4f^2\neq 0 \\ \frac{x\left(Aa^2e^{4ie}-iBa^2e^{4ie}+2iBa^2e^{2ie}\right)}{c^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**2,x)
```

output
```
-B*a**2*log(exp(2*I*f*x) + exp(-2*I*e))/(c**2*f) + Piecewise(((4*B*a**2*c*
*2*f*exp(2*I*e)*exp(2*I*f*x) + (-I*A*a**2*c**2*f*exp(4*I*e) - B*a**2*c**2*
f*exp(4*I*e))*exp(4*I*f*x))/(4*c**4*f**2), Ne(c**4*f**2, 0)), (x*(A*a**2*e
xp(4*I*e) - I*B*a**2*exp(4*I*e) + 2*I*B*a**2*exp(2*I*e))/c**2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx$$
$$=\frac{Ba^2\log\left(\tan\left(fx+e\right)+i\right)}{c^2f}-\frac{2\,Ba^2+\left(Aa^2-3i\,Ba^2\right)\tan\left(fx+e\right)}{c^2f(\tan\left(fx+e\right)+i)^2}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
B*a^2*log(tan(f*x + e) + I)/(c^2*f) - (2*B*a^2 + (A*a^2 - 3*I*B*a^2)*tan(f
*x + e))/(c^2*f*(tan(f*x + e) + I)^2)
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx =$$
$$-\frac{a^2\left(B\,2\mathrm{i}+A\tan(e+f\,x)\;1\mathrm{i}+3\,B\tan(e+f\,x)+B\;\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\;1\mathrm{i}+2\,B\;\ln\left(\tan(e+f\,x)-\right.\right.}{c^2\,f\left(-1+\tan\left(e+f\,x\right)\;1\mathrm{i}\right)^2}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1
i)^2,x)
```

output
```
-(a^2*(B*2i + A*tan(e + f*x)*1i + 3*B*tan(e + f*x) + B*log(tan(e + f*x) +
1i)*1i + 2*B*log(tan(e + f*x) + 1i)*tan(e + f*x) - B*log(tan(e + f*x) + 1i
)*tan(e + f*x)^2*1i)*1i)/(c^2*f*(tan(e + f*x)*1i - 1)^2)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx$$
$$= \frac{a^2\left(\left(\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^2+2\tan(fx+e)i-1}dx\right)b+\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^2+2\tan(fx+e)i-1}dx\right)a-2\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^2+2\tan(fx+e)i-1}dx\right)\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x)
```

output
```
(a**2*(int(tan(e + f*x)**3/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*b +
 int(tan(e + f*x)**2/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*a - 2*int
(tan(e + f*x)**2/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*b*i - 2*int(t
an(e + f*x)/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*a*i - int(tan(e +
f*x)/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*b - int(1/(tan(e + f*x)**
2 + 2*tan(e + f*x)*i - 1),x)*a))/c**2
```

3.685 $\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 93

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = -\frac{2a^2(A-iB)}{3c^3f(i+\tan(e+fx))^3}-\frac{a^2(iA+3B)}{2c^3f(i+\tan(e+fx))^2}-\frac{ia^2B}{c^3f(i+\tan(e+fx))}$$

output
```
-2/3*a^2*(A-I*B)/c^3/f/(I+tan(f*x+e))^3-1/2*a^2*(I*A+3*B)/c^3/f/(I+tan(f*x
+e))^2-I*a^2*B/c^3/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \frac{a^2(-A+iB+3(-iA+B)\tan(e+fx)-6iB\tan^2(e+fx))}{6c^3f(i+\tan(e+fx))^3}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^3,x]
```

output
```
(a^2*(-A + I*B + 3*((-I)*A + B)*Tan[e + f*x] - (6*I)*B*Tan[e + f*x]^2))/(6
*c^3*f*(I + Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{c^4(1-i\tan(e+fx))^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(1-i\tan(e+fx))^4}d\tan(e+fx)}{c^3f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2\int \left(\frac{2(A-iB)}{(\tan(e+fx)+i)^4}+\frac{iB}{(\tan(e+fx)+i)^2}+\frac{iA+3B}{(\tan(e+fx)+i)^3}\right)d\tan(e+fx)}{c^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\left(-\frac{2(A-iB)}{3(\tan(e+fx)+i)^3}-\frac{3B+iA}{2(\tan(e+fx)+i)^2}-\frac{iB}{\tan(e+fx)+i}\right)}{c^3f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^3,x]
```

output
```
(a^2*((-2*(A - I*B))/(3*(I + Tan[e + f*x])^3) - (I*A + 3*B)/(2*(I + Tan[e
+ f*x])^2) - (I*B)/(I + Tan[e + f*x])))/(c^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{a^2\left(-\frac{iA+3B}{2(i+\tan(fx+e))^2}-\frac{-2iB+2A}{3(i+\tan(fx+e))^3}-\frac{iB}{i+\tan(fx+e)}\right)}{f\,c^3}$
default	$\frac{a^2\left(-\frac{iA+3B}{2(i+\tan(fx+e))^2}-\frac{-2iB+2A}{3(i+\tan(fx+e))^3}-\frac{iB}{i+\tan(fx+e)}\right)}{f\,c^3}$
risch	$-\frac{a^2 e^{6i(fx+e)}B}{12c^3f}-\frac{ia^2 e^{6i(fx+e)}A}{12c^3f}+\frac{a^2 e^{4i(fx+e)}B}{8c^3f}-\frac{ia^2 e^{4i(fx+e)}A}{8c^3f}$
norman	$\frac{\frac{2iA\,a^2\tan(fx+e)^2}{cf}+\frac{A\,a^2\tan(fx+e)}{cf}-\frac{iA\,a^2+B\,a^2}{6cf}-\frac{5\left(-iB\,a^2+A\,a^2\right)\tan(fx+e)^3}{3cf}-\frac{\left(iA\,a^2+5B\,a^2\right)\tan(fx+e)^4}{2cf}-\frac{iB\,a^2\tan(fx}{cf}}{c^2\left(1+\tan(fx+e)^2\right)^3}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*a^2/c^3*(-1/2*(I*A+3*B)/(I+tan(f*x+e))^2-1/3*(2*A-2*I*B)/(I+tan(f*x+e)
)^3-I*B/(I+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = -\frac{2\,(i\,A+B)a^2e^{(6i\,fx+6i\,e)}+3\,(i\,A-B)a^2e^{(4i\,fx+4i\,e)}}{24\,c^3f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-1/24*(2*(I*A + B)*a^2*e^(6*I*f*x + 6*I*e) + 3*(I*A - B)*a^2*e^(4*I*f*x +
4*I*e))/(c^3*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.80

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \begin{cases} \frac{(-12iAa^2c^3fe^{4ie}+12Ba^2c^3fe^{4ie})e^{4ifx}+(-8iAa^2c^3fe^{6ie}-8Ba^2c^3fe^{6ie})e^{6ifx}}{96c^6f^2} & \text{for } c^6f^2\neq 0 \\ \frac{x(Aa^2e^{6ie}+Aa^2e^{4ie}-iBa^2e^{6ie}+iBa^2e^{4ie})}{2c^3} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**3,x)
```

output
```
Piecewise((((-12*I*A*a**2*c**3*f*exp(4*I*e) + 12*B*a**2*c**3*f*exp(4*I*e))
*exp(4*I*f*x) + (-8*I*A*a**2*c**3*f*exp(6*I*e) - 8*B*a**2*c**3*f*exp(6*I*e
))*exp(6*I*f*x))/(96*c**6*f**2), Ne(c**6*f**2, 0)), (x*(A*a**2*exp(6*I*e)
+ A*a**2*exp(4*I*e) - I*B*a**2*exp(6*I*e) + I*B*a**2*exp(4*I*e))/(2*c**3),
 True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = -\frac{6i\,Ba^2\tan\left(fx+e\right)^2+3i\,Aa^2\tan\left(fx+e\right)-3\,Ba^2\tan\left(fx+e\right)+Aa^2-i\,Ba^2}{6\,c^3f\left(\tan\left(fx+e\right)+i\right)^3}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-1/6*(6*I*B*a^2*tan(f*x + e)^2 + 3*I*A*a^2*tan(f*x + e) - 3*B*a^2*tan(f*x
+ e) + A*a^2 - I*B*a^2)/(c^3*f*(tan(f*x + e) + I)^3)
```

Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \frac{\frac{a^2\,\left(A-B\,1\mathrm{i}\right)}{6}+\frac{a^2\,\tan\left(e+f\,x\right)\,\left(-3\,B+A\,3\mathrm{i}\right)}{6}+B\,a^2\,\tan\left(e+f\,x\right)^2\,1\mathrm{i}}{c^3\,f\,\left(-{\tan\left(e+f\,x\right)}^3-{\tan\left(e+f\,x\right)}^2\,3\mathrm{i}+3\,\tan\left(e+f\,x\right)+1\mathrm{i}\right)}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1
i)^3,x)
```

output
```
((a^2*(A - B*1i))/6 + (a^2*tan(e + f*x)*(A*3i - 3*B))/6 + B*a^2*tan(e + f*
x)^2*1i)/(c^3*f*(3*tan(e + f*x) - tan(e + f*x)^2*3i - tan(e + f*x)^3 + 1i)
)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx$$
$$= \frac{a^2\left(-\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^3 i-3\tan(fx+e)^2-3\tan(fx+e)i+1}dx\right)b-\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^3 i-3\tan(fx+e)^2-3\tan(fx+e)i+1}dx\right)a+2\left(\int \frac{}{\text{ta}}\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x)
```

output
```
(a**2*( - int(tan(e + f*x)**3/(tan(e + f*x)**3*i - 3*tan(e + f*x)**2 - 3*t
an(e + f*x)*i + 1),x)*b - int(tan(e + f*x)**2/(tan(e + f*x)**3*i - 3*tan(e
 + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*a + 2*int(tan(e + f*x)**2/(tan(e + f
*x)**3*i - 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b*i + 2*int(tan(e
+ f*x)/(tan(e + f*x)**3*i - 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*a
*i + int(tan(e + f*x)/(tan(e + f*x)**3*i - 3*tan(e + f*x)**2 - 3*tan(e + f
*x)*i + 1),x)*b + int(1/(tan(e + f*x)**3*i - 3*tan(e + f*x)**2 - 3*tan(e +
 f*x)*i + 1),x)*a))/c**3
```

3.686 $\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 91

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = -\frac{a^2(iA+B)}{2c^4f(i+\tan(e+fx))^4}+\frac{a^2(A-3iB)}{3c^4f(i+\tan(e+fx))^3}+\frac{a^2B}{2c^4f(i+\tan(e+fx))^2}$$

output
```
-1/2*a^2*(I*A+B)/c^4/f/(I+tan(f*x+e))^4+1/3*a^2*(A-3*I*B)/c^4/f/(I+tan(f*x
+e))^3+1/2*a^2*B/c^4/f/(I+tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.56

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \frac{a^2(-iA+2A\tan(e+fx)+3B\tan^2(e+fx))}{6c^4f(i+\tan(e+fx))^4}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^4,x]
```

output
```
(a^2*((-I)*A + 2*A*Tan[e + f*x] + 3*B*Tan[e + f*x]^2))/(6*c^4*f*(I + Tan[e
 + f*x])^4)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{c^5(1-i\tan(e+fx))^5}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^2\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(1-i\tan(e+fx))^5}d\tan(e+fx)}{c^4f}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{a^2\int \left(\frac{3iB-A}{(\tan(e+fx)+i)^4}-\frac{B}{(\tan(e+fx)+i)^3}+\frac{2(iA+B)}{(\tan(e+fx)+i)^5}\right)d\tan(e+fx)}{c^4f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2\left(\frac{A-3iB}{3(\tan(e+fx)+i)^3}-\frac{B+iA}{2(\tan(e+fx)+i)^4}+\frac{B}{2(\tan(e+fx)+i)^2}\right)}{c^4f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^4,x]
```

output
```
(a^2*(-1/2*(I*A + B)/(I + Tan[e + f*x])^4 + (A - (3*I)*B)/(3*(I + Tan[e +
f*x])^3) + B/(2*(I + Tan[e + f*x])^2)))/(c^4*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{a^2\left(-\frac{2iA+2B}{4(i+\tan(fx+e))^4}-\frac{3iB-A}{3(i+\tan(fx+e))^3}+\frac{B}{2(i+\tan(fx+e))^2}\right)}{f\,c^4}$	68
default	$\frac{a^2\left(-\frac{2iA+2B}{4(i+\tan(fx+e))^4}-\frac{3iB-A}{3(i+\tan(fx+e))^3}+\frac{B}{2(i+\tan(fx+e))^2}\right)}{f\,c^4}$	68
risch	$-\frac{a^2 e^{8i(fx+e)}B}{32c^4 f}-\frac{i a^2 e^{8i(fx+e)}A}{32c^4 f}-\frac{iA\,a^2 e^{6i(fx+e)}}{12c^4 f}+\frac{a^2 e^{4i(fx+e)}B}{16c^4 f}-\frac{i a^2 e^{4i(fx+e)}A}{16c^4 f}$	110

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*a^2/c^4*(-1/4*(2*B+2*I*A)/(I+tan(f*x+e))^4-1/3*(-A+3*I*B)/(I+tan(f*x+e
))^3+1/2*B/(I+tan(f*x+e))^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx$$
$$= -\frac{3\,(i\,A+B)a^2e^{(8i\,fx+8i\,e)}+8i\,Aa^2e^{(6i\,fx+6i\,e)}+6\,(i\,A-B)a^2e^{(4i\,fx+4i\,e)}}{96\,c^4f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, al
gorithm="fricas")
```

output
```
-1/96*(3*(I*A + B)*a^2*e^(8*I*f*x + 8*I*e) + 8*I*A*a^2*e^(6*I*f*x + 6*I*e)
 + 6*(I*A - B)*a^2*e^(4*I*f*x + 4*I*e))/(c^4*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(73) = 146$.

Time = 0.33 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.40

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx$$
$$= \begin{cases} \frac{-512iAa^2c^8f^2e^{6ie}e^{6ifx}+(-384iAa^2c^8f^2e^{4ie}+384Ba^2c^8f^2e^{4ie})e^{4ifx}+(-192iAa^2c^8f^2e^{8ie}-192Ba^2c^8f^2e^{8ie})e^{8ifx}}{6144c^{12}f^3} & \text{for } c^{12}f^3 \neq 0 \\ \frac{x(Aa^2e^{8ie}+2Aa^2e^{6ie}+Aa^2e^{4ie}-iBa^2e^{8ie}+iBa^2e^{4ie})}{4c^4} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**4,x)
```

output
```
Piecewise(((-512*I*A*a**2*c**8*f**2*exp(6*I*e)*exp(6*I*f*x) + (-384*I*A*a*
*2*c**8*f**2*exp(4*I*e) + 384*B*a**2*c**8*f**2*exp(4*I*e))*exp(4*I*f*x) +
(-192*I*A*a**2*c**8*f**2*exp(8*I*e) - 192*B*a**2*c**8*f**2*exp(8*I*e))*exp
(8*I*f*x))/(6144*c**12*f**3), Ne(c**12*f**3, 0)), (x*(A*a**2*exp(8*I*e) +
2*A*a**2*exp(6*I*e) + A*a**2*exp(4*I*e) - I*B*a**2*exp(8*I*e) + I*B*a**2*e
xp(4*I*e))/(4*c**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.56

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \frac{3\,Ba^2\tan\left(fx+e\right)^2+2\,Aa^2\tan\left(fx+e\right)-i\,Aa^2}{6\,c^4f(\tan\left(fx+e\right)+i)^4}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, al
gorithm="giac")
```

output
```
1/6*(3*B*a^2*tan(f*x + e)^2 + 2*A*a^2*tan(f*x + e) - I*A*a^2)/(c^4*f*(tan(
f*x + e) + I)^4)
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \frac{a^2\left(3\,B\tan(e+f\,x)^2+2\,A\tan(e+f\,x)-A\,1\mathrm{i}\right)}{6\,c^4\,f\,\left(\tan\left(e+f\,x\right)^4+\tan\left(e+f\,x\right)^3\,4\mathrm{i}-6\tan\left(e+f\,x\right)^2-\tan\left(e+f\,x\right)\,4\mathrm{i}+1\right)}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1
i)^4,x)
```

output
```
(a^2*(2*A*tan(e + f*x) - A*1i + 3*B*tan(e + f*x)^2))/(6*c^4*f*(tan(e + f*x
)^3*4i - 6*tan(e + f*x)^2 - tan(e + f*x)*4i + tan(e + f*x)^4 + 1))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx$$
$$= \frac{a^2\left(-\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^4 i-4\tan(fx+e)^3-6\tan(fx+e)^2 i+4\tan(fx+e)+i}dx\right)bi-\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^4 i-4\tan(fx+e)^3-6\tan(fx+e)^2 i+4\tan\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x)
```

output
```
(a**2*( - int(tan(e + f*x)**3/(tan(e + f*x)**4*i - 4*tan(e + f*x)**3 - 6*t
an(e + f*x)**2*i + 4*tan(e + f*x) + i),x)*b*i - int(tan(e + f*x)**2/(tan(e
 + f*x)**4*i - 4*tan(e + f*x)**3 - 6*tan(e + f*x)**2*i + 4*tan(e + f*x) +
i),x)*a*i - 2*int(tan(e + f*x)**2/(tan(e + f*x)**4*i - 4*tan(e + f*x)**3 -
 6*tan(e + f*x)**2*i + 4*tan(e + f*x) + i),x)*b - 2*int(tan(e + f*x)/(tan(
e + f*x)**4*i - 4*tan(e + f*x)**3 - 6*tan(e + f*x)**2*i + 4*tan(e + f*x) +
 i),x)*a + int(tan(e + f*x)/(tan(e + f*x)**4*i - 4*tan(e + f*x)**3 - 6*tan
(e + f*x)**2*i + 4*tan(e + f*x) + i),x)*b*i + int(1/(tan(e + f*x)**4*i - 4
*tan(e + f*x)**3 - 6*tan(e + f*x)**2*i + 4*tan(e + f*x) + i),x)*a*i))/c**4
```

3.687 $\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 95

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{2a^2(A-iB)}{5c^5f(i+\tan(e+fx))^5}+\frac{a^2(iA+3B)}{4c^5f(i+\tan(e+fx))^4}+\frac{ia^2B}{3c^5f(i+\tan(e+fx))^3}$$

output
```
2/5*a^2*(A-I*B)/c^5/f/(I+tan(f*x+e))^5+1/4*a^2*(I*A+3*B)/c^5/f/(I+tan(f*x+
e))^4+1/3*I*a^2*B/c^5/f/(I+tan(f*x+e))^3
```

Mathematica [A] (verified)

Time = 5.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{a^2(9A+iB+5(3iA+B)\tan(e+fx)+20iB\tan^2(e+fx))}{60c^5f(i+\tan(e+fx))^5}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^5,x]
```

output
```
(a^2*(9*A + I*B + 5*((3*I)*A + B)*Tan[e + f*x] + (20*I)*B*Tan[e + f*x]^2))
/(60*c^5*f*(I + Tan[e + f*x])^5)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{c^6(1-i\tan(e+fx))^6}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(1-i\tan(e+fx))^6}d\tan(e+fx)}{c^5f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2\int \left(-\frac{2(A-iB)}{(\tan(e+fx)+i)^6}-\frac{iB}{(\tan(e+fx)+i)^4}-\frac{i(A-3iB)}{(\tan(e+fx)+i)^5}\right)d\tan(e+fx)}{c^5f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\left(\frac{2(A-iB)}{5(\tan(e+fx)+i)^5}+\frac{3B+iA}{4(\tan(e+fx)+i)^4}+\frac{iB}{3(\tan(e+fx)+i)^3}\right)}{c^5f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^5,x]
```

output
```
(a^2*((2*(A - I*B))/(5*(I + Tan[e + f*x])^5) + (I*A + 3*B)/(4*(I + Tan[e +
 f*x])^4) + ((I/3)*B)/(I + Tan[e + f*x])^3))/(c^5*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{-iA-3B}{4(i+\tan(fx+e))^{4}}-\frac{2iB-2A}{5(i+\tan(fx+e))^{5}}+\frac{iB}{3(i+\tan(fx+e))^{3}}\right)}{f\,c^{5}}$
default	$\frac{a^{2}\left(-\frac{-iA-3B}{4(i+\tan(fx+e))^{4}}-\frac{2iB-2A}{5(i+\tan(fx+e))^{5}}+\frac{iB}{3(i+\tan(fx+e))^{3}}\right)}{f\,c^{5}}$
risch	$-\frac{a^{2}e^{10i(fx+e)}B}{80c^{5}f}-\frac{ia^{2}e^{10i(fx+e)}A}{80c^{5}f}-\frac{e^{8i(fx+e)}B\,a^{2}}{64c^{5}f}-\frac{3ie^{8i(fx+e)}A\,a^{2}}{64c^{5}f}+\frac{e^{6i(fx+e)}B\,a^{2}}{48c^{5}f}-\frac{ie^{6i(fx+e)}A\,a^{2}}{16c^{5}f}$
norman	$\frac{\frac{A\,a^{2}\tan(fx+e)}{cf}+\frac{-9iA\,a^{2}+B\,a^{2}}{60cf}-\frac{\left(-7iB\,a^{2}+12A\,a^{2}\right)\tan(fx+e)^{3}}{3cf}+\frac{\left(iA\,a^{2}+7B\,a^{2}\right)\tan(fx+e)^{6}}{4cf}+\frac{7\left(-8iB\,a^{2}+3A\,a^{2}\right)\tan(fx+}{15cf}}{\left(1+\tan(fx+e)^{2}\right)^{5}c^{4}}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*a^2/c^5*(-1/4*(-I*A-3*B)/(I+tan(f*x+e))^4-1/5*(-2*A+2*I*B)/(I+tan(f*x+
e))^5+1/3*I*B/(I+tan(f*x+e))^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = -\frac{12\,(i\,A+B)a^2e^{(10i\,fx+10i\,e)}+15\,(3i\,A+B)a^2e^{(8i\,fx+8i\,e)}+20\,(3i\,A-B)a^2e^{(6i\,fx+6i\,e)}+30\,(i\,A-B)a^2}{960\,c^5f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, al
gorithm="fricas")
```

output
```
-1/960*(12*(I*A + B)*a^2*e^(10*I*f*x + 10*I*e) + 15*(3*I*A + B)*a^2*e^(8*I
*f*x + 8*I*e) + 20*(3*I*A - B)*a^2*e^(6*I*f*x + 6*I*e) + 30*(I*A - B)*a^2*
e^(4*I*f*x + 4*I*e))/(c^5*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(76) = 152$.

Time = 0.41 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.49

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \begin{cases} \frac{(-245760iAa^2c^{15}f^3e^{4ie}+245760Ba^2c^{15}f^3e^{4ie})e^{4ifx}+(-491520iAa^2c^{15}f^3e^{6ie}+163840Ba^2c^{15}f^3e^{6ie})e^{6ifx}+(-368640iAa^2c^{15}f^3e^{8ie}-12}{7864320c^{20}f^4} \\ \frac{x(Aa^2e^{10ie}+3Aa^2e^{8ie}+3Aa^2e^{6ie}+Aa^2e^{4ie}-iBa^2e^{10ie}-iBa^2e^{8ie}+iBa^2e^{6ie}+iBa^2e^{4ie})}{8c^5} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**5,x)
```

output
```
Piecewise(((((-245760*I*A*a**2*c**15*f**3*exp(4*I*e) + 245760*B*a**2*c**15*
f**3*exp(4*I*e))*exp(4*I*f*x) + (-491520*I*A*a**2*c**15*f**3*exp(6*I*e) +
163840*B*a**2*c**15*f**3*exp(6*I*e))*exp(6*I*f*x) + (-368640*I*A*a**2*c**1
5*f**3*exp(8*I*e) - 122880*B*a**2*c**15*f**3*exp(8*I*e))*exp(8*I*f*x) + (-
98304*I*A*a**2*c**15*f**3*exp(10*I*e) - 98304*B*a**2*c**15*f**3*exp(10*I*e
))*exp(10*I*f*x))/(7864320*c**20*f**4), Ne(c**20*f**4, 0)), (x*(A*a**2*exp
(10*I*e) + 3*A*a**2*exp(8*I*e) + 3*A*a**2*exp(6*I*e) + A*a**2*exp(4*I*e) -
 I*B*a**2*exp(10*I*e) - I*B*a**2*exp(8*I*e) + I*B*a**2*exp(6*I*e) + I*B*a*
*2*exp(4*I*e))/(8*c**5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = -\frac{-20i\,Ba^2\tan(fx+e)^2-15i\,Aa^2\tan(fx+e)-5\,Ba^2\tan(fx+e)-9\,Aa^2-i\,Ba^2}{60\,c^5f(\tan(fx+e)+i)^5}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, al
gorithm="giac")
```

output
```
-1/60*(-20*I*B*a^2*tan(f*x + e)^2 - 15*I*A*a^2*tan(f*x + e) - 5*B*a^2*tan(
f*x + e) - 9*A*a^2 - I*B*a^2)/(c^5*f*(tan(f*x + e) + I)^5)
```

Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{\frac{a^2\,(9\,A+B\,1\mathrm{i})}{60}+\frac{a^2\,\tan(e+f\,x)\,(5\,B+A\,15\mathrm{i})}{60}+\frac{B\,a^2\,\tan(e+f\,x)^2\,1\mathrm{i}}{3}}{c^5\,f\,\left(\tan\left(e+f\,x\right)^5+\tan\left(e+f\,x\right)^4\,5\mathrm{i}-10\,\tan\left(e+f\,x\right)^3-\tan\left(e+f\,x\right)^2\,10\mathrm{i}+5\,\tan\left(e+f\,x\right)+1\mathrm{i}\right)}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1
i)^5,x)
```

output
```
((a^2*(9*A + B*1i))/60 + (a^2*tan(e + f*x)*(A*15i + 5*B))/60 + (B*a^2*tan(
e + f*x)^2*1i)/3)/(c^5*f*(5*tan(e + f*x) - tan(e + f*x)^2*10i - 10*tan(e +
 f*x)^3 + tan(e + f*x)^4*5i + tan(e + f*x)^5 + 1i))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{a^2\left(-\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^5+5\tan(fx+e)^4i-10\tan(fx+e)^3-10\tan(fx+e)^2i+5\tan(fx+e)+i}dx\right)bi-\left(\int \frac{}{\tan(fx+e)^5+5\tan(fx+e)^4i-10}\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x)
```

output
```
(a**2*( - int(tan(e + f*x)**3/(tan(e + f*x)**5 + 5*tan(e + f*x)**4*i - 10*
tan(e + f*x)**3 - 10*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*b*i - int(
tan(e + f*x)**2/(tan(e + f*x)**5 + 5*tan(e + f*x)**4*i - 10*tan(e + f*x)**
3 - 10*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*a*i - 2*int(tan(e + f*x)
**2/(tan(e + f*x)**5 + 5*tan(e + f*x)**4*i - 10*tan(e + f*x)**3 - 10*tan(e
 + f*x)**2*i + 5*tan(e + f*x) + i),x)*b - 2*int(tan(e + f*x)/(tan(e + f*x)
**5 + 5*tan(e + f*x)**4*i - 10*tan(e + f*x)**3 - 10*tan(e + f*x)**2*i + 5*
tan(e + f*x) + i),x)*a + int(tan(e + f*x)/(tan(e + f*x)**5 + 5*tan(e + f*x
)**4*i - 10*tan(e + f*x)**3 - 10*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x
)*b*i + int(1/(tan(e + f*x)**5 + 5*tan(e + f*x)**4*i - 10*tan(e + f*x)**3
- 10*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*a*i))/c**5
```

3.688 $\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 91

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = \frac{a^2(iA+B)}{3c^6f(i+\tan(e+fx))^6}-\frac{a^2(A-3iB)}{5c^6f(i+\tan(e+fx))^5}-\frac{a^2B}{4c^6f(i+\tan(e+fx))^4}$$

output
```
1/3*a^2*(I*A+B)/c^6/f/(I+tan(f*x+e))^6-1/5*a^2*(A-3*I*B)/c^6/f/(I+tan(f*x+
e))^5-1/4*a^2*B/c^6/f/(I+tan(f*x+e))^4
```

Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = -\frac{a^2(-8iA+B+6(2A-iB)\tan(e+fx)+15B\tan^2(e+fx))}{60c^6f(i+\tan(e+fx))^6}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^6,x]
```

output
```
-1/60*(a^2*((-8*I)*A + B + 6*(2*A - I*B)*Tan[e + f*x] + 15*B*Tan[e + f*x]^
2))/(c^6*f*(I + Tan[e + f*x])^6)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{c^7(1-i\tan(e+fx))^7}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(1-i\tan(e+fx))^7}d\tan(e+fx)}{c^6f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2\int \left(-\frac{2i(A-iB)}{(\tan(e+fx)+i)^7}+\frac{B}{(\tan(e+fx)+i)^5}+\frac{A-3iB}{(\tan(e+fx)+i)^6}\right)d\tan(e+fx)}{c^6f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\left(-\frac{A-3iB}{5(\tan(e+fx)+i)^5}+\frac{B+iA}{3(\tan(e+fx)+i)^6}-\frac{B}{4(\tan(e+fx)+i)^4}\right)}{c^6f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^6,x]
```

output
```
(a^2*((I*A + B)/(3*(I + Tan[e + f*x])^6) - (A - (3*I)*B)/(5*(I + Tan[e + f
*x])^5) - B/(4*(I + Tan[e + f*x])^4)))/(c^6*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{B}{4(i+\tan(fx+e))^{4}}-\frac{-2iA-2B}{6(i+\tan(fx+e))^{6}}-\frac{-3iB+A}{5(i+\tan(fx+e))^{5}}\right)}{f\,c^{6}}$
default	$\frac{a^{2}\left(-\frac{B}{4(i+\tan(fx+e))^{4}}-\frac{-2iA-2B}{6(i+\tan(fx+e))^{6}}-\frac{-3iB+A}{5(i+\tan(fx+e))^{5}}\right)}{f\,c^{6}}$
risch	$-\frac{a^{2}\mathrm{e}^{12i(fx+e)}B}{192c^{6}f}-\frac{ia^{2}\mathrm{e}^{12i(fx+e)}A}{192c^{6}f}-\frac{\mathrm{e}^{10i(fx+e)}B\,a^{2}}{80c^{6}f}-\frac{i\mathrm{e}^{10i(fx+e)}A\,a^{2}}{40c^{6}f}-\frac{3iA\,a^{2}\mathrm{e}^{8i(fx+e)}}{64c^{6}f}+\frac{\mathrm{e}^{6i(fx+e)}B\,a^{2}}{48c^{6}f}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^6,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*a^2/c^6*(-1/4*B/(I+tan(f*x+e))^4-1/6*(-2*I*A-2*B)/(I+tan(f*x+e))^6-1/5
*(A-3*I*B)/(I+tan(f*x+e))^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = -\frac{5\,(i\,A+B)a^2e^{(12i\,fx+12i\,e)}+12\,(2i\,A+B)a^2e^{(10i\,fx+10i\,e)}+45i\,Aa^2e^{(8i\,fx+8i\,e)}+20\,(2i\,A-B)a^2e^{(6i\,fx+}}{960\,c^6f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^6,x, al
gorithm="fricas")
```

output
```
-1/960*(5*(I*A + B)*a^2*e^(12*I*f*x + 12*I*e) + 12*(2*I*A + B)*a^2*e^(10*I
*f*x + 10*I*e) + 45*I*A*a^2*e^(8*I*f*x + 8*I*e) + 20*(2*I*A - B)*a^2*e^(6*
I*f*x + 6*I*e) + 15*(I*A - B)*a^2*e^(4*I*f*x + 4*I*e))/(c^6*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(73) = 146.

Time = 0.51 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.16

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx$$
$$= \begin{cases} \frac{-141557760iAa^2c^{24}f^4e^{8ie}e^{8ifx}+(-47185920iAa^2c^{24}f^4e^{4ie}+47185920Ba^2c^{24}f^4e^{4ie})e^{4ifx}+(-125829120iAa^2c^{24}f^4e^{6ie}+62914560Ba^2c^{\cdots}}{3019898\cdots} \\ \frac{x(Aa^2e^{12ie}+4Aa^2e^{10ie}+6Aa^2e^{8ie}+4Aa^2e^{6ie}+Aa^2e^{4ie}-iBa^2e^{12ie}-2iBa^2e^{10ie}+2iBa^2e^{6ie}+iBa^2e^{4ie})}{16c^6} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**6,x)
```

output
```
Piecewise(((-141557760*I*A*a**2*c**24*f**4*exp(8*I*e)*exp(8*I*f*x) + (-471
85920*I*A*a**2*c**24*f**4*exp(4*I*e) + 47185920*B*a**2*c**24*f**4*exp(4*I*
e))*exp(4*I*f*x) + (-125829120*I*A*a**2*c**24*f**4*exp(6*I*e) + 62914560*B
*a**2*c**24*f**4*exp(6*I*e))*exp(6*I*f*x) + (-75497472*I*A*a**2*c**24*f**4
*exp(10*I*e) - 37748736*B*a**2*c**24*f**4*exp(10*I*e))*exp(10*I*f*x) + (-1
5728640*I*A*a**2*c**24*f**4*exp(12*I*e) - 15728640*B*a**2*c**24*f**4*exp(1
2*I*e))*exp(12*I*f*x))/(3019898880*c**30*f**5), Ne(c**30*f**5, 0)), (x*(A*
a**2*exp(12*I*e) + 4*A*a**2*exp(10*I*e) + 6*A*a**2*exp(8*I*e) + 4*A*a**2*e
xp(6*I*e) + A*a**2*exp(4*I*e) - I*B*a**2*exp(12*I*e) - 2*I*B*a**2*exp(10*I
*e) + 2*I*B*a**2*exp(6*I*e) + I*B*a**2*exp(4*I*e))/(16*c**6), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^6,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx$$
$$= -\frac{15\,Ba^2\tan{(fx+e)}^2+12\,Aa^2\tan{(fx+e)}-6i\,Ba^2\tan{(fx+e)}-8i\,Aa^2+Ba^2}{60\,c^6f(\tan{(fx+e)}+i)^6}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^6,x, al
gorithm="giac")
```

output
```
-1/60*(15*B*a^2*tan(f*x + e)^2 + 12*A*a^2*tan(f*x + e) - 6*I*B*a^2*tan(f*x
 + e) - 8*I*A*a^2 + B*a^2)/(c^6*f*(tan(f*x + e) + I)^6)
```

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx =$$
$$-\frac{\frac{B\,a^2\tan(e+f\,x)^2}{4}+\frac{a^2\tan(e+f\,x)\,(12\,A-B\,6\mathrm{i})}{60}-\frac{a^2\,(-B+A\,8\mathrm{i})}{60}}{c^6\,f\,\left(\tan{(e+f\,x)}^6+\tan{(e+f\,x)}^5\,6\mathrm{i}-15\tan{(e+f\,x)}^4-\tan{(e+f\,x)}^3\,20\mathrm{i}+15\tan{(e+f\,x)}^2+\mathrm{t}\right.}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1
i)^6,x)
```

output
```
-((a^2*tan(e + f*x)*(12*A - B*6i))/60 - (a^2*(A*8i - B))/60 + (B*a^2*tan(e
 + f*x)^2)/4)/(c^6*f*(tan(e + f*x)*6i + 15*tan(e + f*x)^2 - tan(e + f*x)^3
*20i - 15*tan(e + f*x)^4 + tan(e + f*x)^5*6i + tan(e + f*x)^6 - 1))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx$$
$$= \frac{a^2\left(\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^6 i-6\tan(fx+e)^5-15\tan(fx+e)^4 i+20\tan(fx+e)^3+15\tan(fx+e)^2 i-6\tan(fx+e)-i}dx\right)bi+\left(\int \frac{}{\tan(fx+e)^6 i-6\,\mathrm{ta}}\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^6,x)
```

output
```
(a**2*(int(tan(e + f*x)**3/(tan(e + f*x)**6*i - 6*tan(e + f*x)**5 - 15*tan
(e + f*x)**4*i + 20*tan(e + f*x)**3 + 15*tan(e + f*x)**2*i - 6*tan(e + f*x
) - i),x)*b*i + int(tan(e + f*x)**2/(tan(e + f*x)**6*i - 6*tan(e + f*x)**5
 - 15*tan(e + f*x)**4*i + 20*tan(e + f*x)**3 + 15*tan(e + f*x)**2*i - 6*ta
n(e + f*x) - i),x)*a*i + 2*int(tan(e + f*x)**2/(tan(e + f*x)**6*i - 6*tan(
e + f*x)**5 - 15*tan(e + f*x)**4*i + 20*tan(e + f*x)**3 + 15*tan(e + f*x)*
*2*i - 6*tan(e + f*x) - i),x)*b + 2*int(tan(e + f*x)/(tan(e + f*x)**6*i -
6*tan(e + f*x)**5 - 15*tan(e + f*x)**4*i + 20*tan(e + f*x)**3 + 15*tan(e +
 f*x)**2*i - 6*tan(e + f*x) - i),x)*a - int(tan(e + f*x)/(tan(e + f*x)**6*
i - 6*tan(e + f*x)**5 - 15*tan(e + f*x)**4*i + 20*tan(e + f*x)**3 + 15*tan
(e + f*x)**2*i - 6*tan(e + f*x) - i),x)*b*i - int(1/(tan(e + f*x)**6*i - 6
*tan(e + f*x)**5 - 15*tan(e + f*x)**4*i + 20*tan(e + f*x)**3 + 15*tan(e +
f*x)**2*i - 6*tan(e + f*x) - i),x)*a*i))/c**6
```

3.689 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 41, antiderivative size = 151

$$\begin{aligned}\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx &= \frac{4a^3(iA+B)(c-ic\tan(e+fx))^n}{fn}-\frac{4a^3(iA+2B)(c-ic\tan(e+fx))^{1+n}}{cf(1+n)}\\ &\quad+\frac{a^3(iA+5B)(c-ic\tan(e+fx))^{2+n}}{c^2f(2+n)}-\frac{a^3B(c-ic\tan(e+fx))^{3+n}}{c^3f(3+n)}\end{aligned}$$

output
```
4*a^3*(I*A+B)*(c-I*c*tan(f*x+e))^n/f/n-4*a^3*(I*A+2*B)*(c-I*c*tan(f*x+e))^
(1+n)/c/f/(1+n)+a^3*(I*A+5*B)*(c-I*c*tan(f*x+e))^(2+n)/c^2/f/(2+n)-a^3*B*(
c-I*c*tan(f*x+e))^(3+n)/c^3/f/(3+n)
```

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.02

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx =$$

$$-\frac{ia^3(c-ic\tan(e+fx))^n\,(iB(24+9n+n^2)-A(24+23n+8n^2+n^3)-in(2A(3+n)^2-iB(24+9}{fn(1+n)}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^n,x]
```

output
```
((-I)*a^3*(c - I*c*Tan[e + f*x])^n*(I*B*(24 + 9*n + n^2) - A*(24 + 23*n +
8*n^2 + n^3) - I*n*(2*A*(3 + n)^2 - I*B*(24 + 9*n + n^2))*Tan[e + f*x] + n
*(1 + n)*(A*(3 + n) - I*B*(9 + 2*n))*Tan[e + f*x]^2 + B*n*(2 + 3*n + n^2)*
Tan[e + f*x]^3))/(f*n*(1 + n)*(2 + n)*(3 + n))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c\int (i\tan(e+fx)+1)^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 86$$

$$\frac{a^3c\int \left(4(A-iB)(c-ic\tan(e+fx))^{n-1}-\frac{4(A-2iB)(c-ic\tan(e+fx))^n}{c}+\frac{(A-5iB)(c-ic\tan(e+fx))^{n+1}}{c^2}+\frac{iB(c-ic\tan(e+fx}{c^3}\right.}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3c\left(\frac{(5B+iA)(c-ic\tan(e+fx))^{n+2}}{c^3(n+2)}-\frac{4(2B+iA)(c-ic\tan(e+fx))^{n+1}}{c^2(n+1)}+\frac{4(B+iA)(c-ic\tan(e+fx))^n}{cn}-\frac{B(c-ic\tan(e+fx))^{n+3}}{c^4(n+3)}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n
,x]
```

output
```
(a^3*c*((4*(I*A + B)*(c - I*c*Tan[e + f*x])^n)/(c*n) - (4*(I*A + 2*B)*(c -
 I*c*Tan[e + f*x])^(1 + n))/(c^2*(1 + n)) + ((I*A + 5*B)*(c - I*c*Tan[e +
f*x])^(2 + n))/(c^3*(2 + n)) - (B*(c - I*c*Tan[e + f*x])^(3 + n))/(c^4*(3
+ n))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.79

method	result
derivativedivides	$\frac{i(A\,a^3n^3-iB\,a^3n^2+8A\,a^3n^2-9iB\,a^3n+23A\,a^3n-24iB\,a^3+24A\,a^3)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{(n^2+3n+2)fn(3+n)}-\frac{a^3(-iB\,n^2+2A\,n^2-9i}{}$
default	$\frac{i(A\,a^3n^3-iB\,a^3n^2+8A\,a^3n^2-9iB\,a^3n+23A\,a^3n-24iB\,a^3+24A\,a^3)\mathrm{e}^{n\ln(c-ic\tan(fx+e))}}{(n^2+3n+2)fn(3+n)}-\frac{a^3(-iB\,n^2+2A\,n^2-9i}{}$
risch	Expression too large to display

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x,method=_R
ETURNVERBOSE)
```

output
```
I/(n^2+3*n+2)/f/n*(-9*I*B*a^3*n+23*A*a^3*n-24*I*B*a^3+A*a^3*n^3+8*A*a^3*n^
2-I*B*a^3*n^2+24*A*a^3)/(3+n)*exp(n*ln(c-I*c*tan(f*x+e)))-a^3*(-I*B*n^2+2*
A*n^2-9*I*B*n+12*A*n-24*I*B+18*A)/f/(n^2+3*n+2)/(3+n)*tan(f*x+e)*exp(n*ln(
c-I*c*tan(f*x+e)))-(I*A*n+3*I*A+2*B*n+9*B)*a^3/f/(3+n)/(2+n)*tan(f*x+e)^2*
exp(n*ln(c-I*c*tan(f*x+e)))-I/f/(3+n)*B*a^3*tan(f*x+e)^3*exp(n*ln(c-I*c*ta
n(f*x+e)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(137) = 274.

Time = 0.10 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.22

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = -\frac{4\left(2\left(-iA+B\right)a^3n+6\left(-iA-B\right)a^3+\left(\left(-iA-B\right)a^3n^3+6\left(-iA-B\right)a^3n^2+11\left(-iA-B\right)a^3n+\right.\right.}{fn^4+6\,fn^3+11\,fn^2+6\,fn+(}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, al
gorithm="fricas")
```

output
```
-4*(2*(-I*A + B)*a^3*n + 6*(-I*A - B)*a^3 + ((-I*A - B)*a^3*n^3 + 6*(-I*A
- B)*a^3*n^2 + 11*(-I*A - B)*a^3*n + 6*(-I*A - B)*a^3)*e^(6*I*f*x + 6*I*e)
 + ((-I*A + B)*a^3*n^3 + 2*(-4*I*A + B)*a^3*n^2 + 3*(-7*I*A - 3*B)*a^3*n +
 18*(-I*A - B)*a^3)*e^(4*I*f*x + 4*I*e) + 2*((-I*A + B)*a^3*n^2 - 6*I*A*a^
3*n + 9*(-I*A - B)*a^3)*e^(2*I*f*x + 2*I*e))*(2*c/(e^(2*I*f*x + 2*I*e) + 1
))^n/(f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n + (f*n^4 + 6*f*n^3 + 11*f*n^2 + 6
*f*n)*e^(6*I*f*x + 6*I*e) + 3*(f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n)*e^(4*I*
f*x + 4*I*e) + 3*(f*n^4 + 6*f*n^3 + 11*f*n^2 + 6*f*n)*e^(2*I*f*x + 2*I*e))
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3665 vs. 2(122) = 244.

Time = 3.64 (sec) , antiderivative size = 3665, normalized size of antiderivative = 24.27

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**n,x)
```

output
```
Piecewise((x*(A + B*tan(e))*(I*a*tan(e) + a)**3*(-I*c*tan(e) + c)**n, Eq(f
, 0)), (6*A*a**3*tan(e + f*x)**2/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*t
an(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 2*A*a**3/(6*c**3*f
*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) -
6*I*c**3*f) + 6*I*B*a**3*f*x*tan(e + f*x)**3/(6*c**3*f*tan(e + f*x)**3 + 1
8*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 18*B*a
**3*f*x*tan(e + f*x)**2/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*
x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 18*I*B*a**3*f*x*tan(e + f*x
)/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(
e + f*x) - 6*I*c**3*f) + 6*B*a**3*f*x/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**
3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 3*B*a**3*log(
tan(e + f*x)**2 + 1)*tan(e + f*x)**3/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3
*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 9*I*B*a**3*log
(tan(e + f*x)**2 + 1)*tan(e + f*x)**2/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**
3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) + 9*B*a**3*log(
tan(e + f*x)**2 + 1)*tan(e + f*x)/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*
tan(e + f*x)**2 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) + 3*I*B*a**3*log(ta
n(e + f*x)**2 + 1)/(6*c**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2
 - 18*c**3*f*tan(e + f*x) - 6*I*c**3*f) - 30*I*B*a**3*tan(e + f*x)**2/(6*c
**3*f*tan(e + f*x)**3 + 18*I*c**3*f*tan(e + f*x)**2 - 18*c**3*f*tan(e +...
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(137) = 274$.

Time = 0.33 (sec) , antiderivative size = 1056, normalized size of antiderivative = 6.99

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, al
gorithm="maxima")
```

output
```
4*(2*((A + I*B)*a^3*c^n*n^2 + 6*A*a^3*c^n*n + 9*(A - I*B)*a^3*c^n)*2^n*cos
(-2*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e) + ((A +
 I*B)*a^3*c^n*n^3 + 2*(4*A + I*B)*a^3*c^n*n^2 + 3*(7*A - 3*I*B)*a^3*c^n*n
+ 18*(A - I*B)*a^3*c^n)*2^n*cos(-4*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1) - 4*e) + ((A - I*B)*a^3*c^n*n^3 + 6*(A - I*B)*a^3*c^n*n^2
 + 11*(A - I*B)*a^3*c^n*n + 6*(A - I*B)*a^3*c^n)*2^n*cos(-6*f*x + n*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 6*e) + 2*((A + I*B)*a^3*c^n*n
+ 3*(A - I*B)*a^3*c^n)*2^n*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) - 2*((I*A - B)*a^3*c^n*n^2 + 6*I*A*a^3*c^n*n + 9*(I*A + B)*a^3*c^n
)*2^n*sin(-2*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e
) - ((I*A - B)*a^3*c^n*n^3 + 2*(4*I*A - B)*a^3*c^n*n^2 + 3*(7*I*A + 3*B)*a
^3*c^n*n + 18*(I*A + B)*a^3*c^n)*2^n*sin(-4*f*x + n*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1) - 4*e) - ((I*A + B)*a^3*c^n*n^3 + 6*(I*A + B)*a^
3*c^n*n^2 + 11*(I*A + B)*a^3*c^n*n + 6*(I*A + B)*a^3*c^n)*2^n*sin(-6*f*x +
 n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 6*e) - 2*((I*A - B)*a
^3*c^n*n + 3*(I*A + B)*a^3*c^n)*2^n*sin(n*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1)))/(((-I*n^4 - 6*I*n^3 - 11*I*n^2 - 6*I*n)*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*cos(6*f*x + 6*e
) - 3*(I*n^4 + 6*I*n^3 + 11*I*n^2 + 6*I*n)*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*cos(4*f*x + 4*e) + (n^4 + 6...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$
$$=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^3(-i\,c\tan(fx+e)+c)^n\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, al
gorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^3*(-I*c*tan(f*x + e)
 + c)^n, x)
```

Mupad [B] (verification not implemented)

Time = 9.91 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.14

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = -\frac{\left(c+\frac{c\,(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}\right)^n\left(\frac{8\,a^3\,(3\,A-B\,3\mathrm{i}+A\,n+B\,n\,1\mathrm{i})}{f\,n\,(n^3\,1\mathrm{i}+n^2\,6\mathrm{i}+n\,11\mathrm{i}+6\mathrm{i})}+\frac{4\,a^3\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,(n^2+5\,n+6)\,(3\,A-B\,3\mathrm{i}+A\,n+B\,n\,1\mathrm{i})}{f\,n\,(n^3\,1\mathrm{i}+n^2\,6\mathrm{i}+n\,11\mathrm{i}+6\mathrm{i})}+\frac{4\,a^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6}}{f\,n\,(}\right.}{3\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+3\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}+\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}+1}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^n,x)
```

output
```
-((c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^n*((8
*a^3*(3*A - B*3i + A*n + B*n*1i))/(f*n*(n*11i + n^2*6i + n^3*1i + 6i)) + (
4*a^3*exp(e*4i + f*x*4i)*(5*n + n^2 + 6)*(3*A - B*3i + A*n + B*n*1i))/(f*n
*(n*11i + n^2*6i + n^3*1i + 6i)) + (4*a^3*exp(e*6i + f*x*6i)*(A - B*1i)*(1
1*n + 6*n^2 + n^3 + 6))/(f*n*(n*11i + n^2*6i + n^3*1i + 6i)) + (8*a^3*exp(
e*2i + f*x*2i)*(n + 3)*(3*A - B*3i + A*n + B*n*1i))/(f*n*(n*11i + n^2*6i +
 n^3*1i + 6i))))/(3*exp(e*2i + f*x*2i) + 3*exp(e*4i + f*x*4i) + exp(e*6i +
 f*x*6i) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.79

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \frac{(-\tan(fx+e)\,ci+c)^n\,a^3\left(-\tan(fx+e)^3\,bi\,n^3-3\tan(fx+e)^3\,bi\,n^2-2\tan(fx+e)^3\,bin-\tan(fx\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x)
```

output
```
(( - tan(e + f*x)*c*i + c)**n*a**3*( - tan(e + f*x)**3*b*i*n**3 - 3*tan(e
+ f*x)**3*b*i*n**2 - 2*tan(e + f*x)**3*b*i*n - tan(e + f*x)**2*a*i*n**3 -
4*tan(e + f*x)**2*a*i*n**2 - 3*tan(e + f*x)**2*a*i*n - 2*tan(e + f*x)**2*b
*n**3 - 11*tan(e + f*x)**2*b*n**2 - 9*tan(e + f*x)**2*b*n - 2*tan(e + f*x)
*a*n**3 - 12*tan(e + f*x)*a*n**2 - 18*tan(e + f*x)*a*n + tan(e + f*x)*b*i*
n**3 + 9*tan(e + f*x)*b*i*n**2 + 24*tan(e + f*x)*b*i*n + a*i*n**3 + 8*a*i*
n**2 + 23*a*i*n + 24*a*i + b*n**2 + 9*b*n + 24*b))/(f*n*(n**3 + 6*n**2 + 1
1*n + 6))
```

3.690 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^6\,dx$

Optimal result

Integrand size = 41, antiderivative size = 135

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^6\,dx = \frac{2a^3(iA+B)c^6(1-i\tan(e+fx))^6}{3f} - \frac{4a^3(iA+2B)c^6(1-i\tan(e+fx))^7}{7f} + \frac{a^3(iA+5B)c^6(1-i\tan(e+fx))^8}{8f} - \frac{a^3Bc^6(1-i\tan(e+fx))^9}{9f}$$

output
```
2/3*a^3*(I*A+B)*c^6*(1-I*tan(f*x+e))^6/f-4/7*a^3*(I*A+2*B)*c^6*(1-I*tan(f*
x+e))^7/f+1/8*a^3*(I*A+5*B)*c^6*(1-I*tan(f*x+e))^8/f-1/9*a^3*B*c^6*(1-I*ta
n(f*x+e))^9/f
```

Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.37

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^6\,dx$$
$$= \frac{a^3c^6\sec^9(e+fx)(-i\cos(3(e+fx))+\sin(3(e+fx)))(252A+84iB+162(3A+iB)\cos(2(e+fx))+}{}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^6,x]
```

output
```
(a^3*c^6*Sec[e + f*x]^9*((-I)*Cos[3*(e + f*x)] + Sin[3*(e + f*x)])*(252*A
+ (84*I)*B + 162*(3*A + I*B)*Cos[2*(e + f*x)] + 135*(3*A + I*B)*Cos[4*(e +
 f*x)] + 171*A*Cos[6*(e + f*x)] + (169*I)*B*Cos[6*(e + f*x)] - (270*I)*A*S
in[2*(e + f*x)] + 90*B*Sin[2*(e + f*x)] - (351*I)*A*Sin[4*(e + f*x)] + 117
*B*Sin[4*(e + f*x)] - (165*I)*A*Sin[6*(e + f*x)] + 167*B*Sin[6*(e + f*x)])
)/(1008*f)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a^2c^5(1-i\tan(e+fx))^5(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{a^3c^6\int (1-i\tan(e+fx))^5(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 86$$

$$\frac{a^3c^6\int \left(iB(1-i\tan(e+fx))^8+(A-5iB)(1-i\tan(e+fx))^7-4(A-2iB)(1-i\tan(e+fx))^6+4(A-iB)\right.}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3c^6\left(\frac{1}{8}(5B+iA)(1-i\tan(e+fx))^8-\frac{4}{7}(2B+iA)(1-i\tan(e+fx))^7+\frac{2}{3}(B+iA)(1-i\tan(e+fx))^6-\frac{1}{9}B\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^6
,x]
```

output
```
(a^3*c^6*((2*(I*A + B)*(1 - I*Tan[e + f*x])^6)/3 - (4*(I*A + 2*B)*(1 - I*T
an[e + f*x])^7)/7 + ((I*A + 5*B)*(1 - I*Tan[e + f*x])^8)/8 - (B*(1 - I*Tan
[e + f*x])^9)/9))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.80

method	result
risch	$\frac{32c^6a^3\left(84iA\,e^{6i(fx+e)}+84B\,e^{6i(fx+e)}+108iA\,e^{4i(fx+e)}-36B\,e^{4i(fx+e)}+27iA\,e^{2i(fx+e)}-9B\,e^{2i(fx+e)}+3iA-B\right)}{63f\left(e^{2i(fx+e)}+1\right)^9}$
derivativedivides	$\frac{ic^6a^3\left(\frac{B\tan(fx+e)^9}{9}+\frac{(3iB+A)\tan(fx+e)^8}{8}+\frac{(-11B-2iA+5i(-2iB+A))\tan(fx+e)^7}{7}+\frac{(-11A+5i(-2iA-B)+10iB)\tan(fx+e}{6}\right.}{}$
default	$\frac{ic^6a^3\left(\frac{B\tan(fx+e)^9}{9}+\frac{(3iB+A)\tan(fx+e)^8}{8}+\frac{(-11B-2iA+5i(-2iB+A))\tan(fx+e)^7}{7}+\frac{(-11A+5i(-2iA-B)+10iB)\tan(fx+e}{6}\right.}{}$
norman	$\frac{A\,a^3c^6\tan(fx+e)}{f}-\frac{(3iB\,a^3c^6+A\,a^3c^6)\tan(fx+e)^3}{3f}-\frac{(5iA\,a^3c^6+B\,a^3c^6)\tan(fx+e)^4}{4f}-\frac{(-iA\,a^3c^6+3B\,a^3c^6)}{8f}$
parallelrisch	$\frac{56iB\,a^3c^6\tan(fx+e)^9-756iA\tan(fx+e)^2a^3c^6-630iA\tan(fx+e)^4a^3c^6-189B\tan(fx+e)^8a^3c^6+63iA\tan(fx+e)^8a}{}$
parts	$\frac{(-8iB\,a^3c^6-6A\,a^3c^6)\left(\frac{\tan(fx+e)^3}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+\frac{(-6iA\,a^3c^6-6B\,a^3c^6)\left(\frac{\tan(fx+e)^4}{4}-\tan(\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^6,x,method=_R
ETURNVERBOSE)
```

output
```
32/63*c^6*a^3*(84*I*A*exp(6*I*(f*x+e))+84*B*exp(6*I*(f*x+e))+108*I*A*exp(4
*I*(f*x+e))-36*B*exp(4*I*(f*x+e))+27*I*A*exp(2*I*(f*x+e))-9*B*exp(2*I*(f*x
+e))+3*I*A-B)/f/(exp(2*I*(f*x+e))+1)^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.44

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^6\,dx = -\frac{32\left(84\,(-i\,A-B)a^3c^6e^{(6i\,fx+6i\,e)}+36\,(-3i\,A+B)a^3c^6e^{(4i\,fx+4i\,e)}+9\,(-3i\,A\right.}{63\,(fe^{(18i\,fx+18i\,e)}+9\,fe^{(16i\,fx+16i\,e)}+36\,fe^{(14i\,fx+14i\,e)}+84\,fe^{(12i\,fx+12i\,e)}+126\,fe^{(10i\,fx+10i\,e)}+126\,fe}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^6,x, al
gorithm="fricas")
```

output
```
-32/63*(84*(-I*A - B)*a^3*c^6*e^(6*I*f*x + 6*I*e) + 36*(-3*I*A + B)*a^3*c^
6*e^(4*I*f*x + 4*I*e) + 9*(-3*I*A + B)*a^3*c^6*e^(2*I*f*x + 2*I*e) + (-3*I
*A + B)*a^3*c^6)/(f*e^(18*I*f*x + 18*I*e) + 9*f*e^(16*I*f*x + 16*I*e) + 36
*f*e^(14*I*f*x + 14*I*e) + 84*f*e^(12*I*f*x + 12*I*e) + 126*f*e^(10*I*f*x
+ 10*I*e) + 126*f*e^(8*I*f*x + 8*I*e) + 84*f*e^(6*I*f*x + 6*I*e) + 36*f*e^
(4*I*f*x + 4*I*e) + 9*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(110) = 220.

Time = 0.98 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.41

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^6\,dx = \frac{96iAa^3c^6-32Ba^3c^6+\left(864iAa^3c^6e^{2ie}-288Ba^3c^6e^{2ie}\right)e^{2ifx}+\left(3456iAa^3c^6e^{4ie}-1152Ba^3c^6e^{4i}\right.}{63fe^{18ie}e^{18ifx}+567fe^{16ie}e^{16ifx}+2268fe^{14ie}e^{14ifx}+5292fe^{12ie}e^{12ifx}+7938fe^{10ie}e^{10ifx}+7938fe^{8ie}e^{8if}}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**6,x)
```

output
```
(96*I*A*a**3*c**6 - 32*B*a**3*c**6 + (864*I*A*a**3*c**6*exp(2*I*e) - 288*B
*a**3*c**6*exp(2*I*e))*exp(2*I*f*x) + (3456*I*A*a**3*c**6*exp(4*I*e) - 115
2*B*a**3*c**6*exp(4*I*e))*exp(4*I*f*x) + (2688*I*A*a**3*c**6*exp(6*I*e) +
2688*B*a**3*c**6*exp(6*I*e))*exp(6*I*f*x))/(63*f*exp(18*I*e)*exp(18*I*f*x)
 + 567*f*exp(16*I*e)*exp(16*I*f*x) + 2268*f*exp(14*I*e)*exp(14*I*f*x) + 52
92*f*exp(12*I*e)*exp(12*I*f*x) + 7938*f*exp(10*I*e)*exp(10*I*f*x) + 7938*f
*exp(8*I*e)*exp(8*I*f*x) + 5292*f*exp(6*I*e)*exp(6*I*f*x) + 2268*f*exp(4*I
*e)*exp(4*I*f*x) + 567*f*exp(2*I*e)*exp(2*I*f*x) + 63*f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.43

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^6\,dx =$$

$$-\frac{-56i\,Ba^3c^6\tan\left(fx+e\right)^9+63\left(-i\,A+3\,B\right)a^3c^6\tan\left(fx+e\right)^8+72\left(3\,A+i\,B\right)a^3c^6\tan\left(fx+e\right)^7+8\cdots}{\cdots}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^6,x, al
gorithm="maxima")
```

output
```
-1/504*(-56*I*B*a^3*c^6*tan(f*x + e)^9 + 63*(-I*A + 3*B)*a^3*c^6*tan(f*x +
 e)^8 + 72*(3*A + I*B)*a^3*c^6*tan(f*x + e)^7 + 84*(I*A + 5*B)*a^3*c^6*tan
(f*x + e)^6 + 504*(A + I*B)*a^3*c^6*tan(f*x + e)^5 + 126*(5*I*A + B)*a^3*c
^6*tan(f*x + e)^4 + 168*(A + 3*I*B)*a^3*c^6*tan(f*x + e)^3 + 252*(3*I*A -
B)*a^3*c^6*tan(f*x + e)^2 - 504*A*a^3*c^6*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(113) = 226.

Time = 0.93 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.04

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^6\,dx =$$

$$-\frac{-56i\,Ba^3c^6\tan\left(fx+e\right)^9-63i\,Aa^3c^6\tan\left(fx+e\right)^8+189\,Ba^3c^6\tan\left(fx+e\right)^8+216\,Aa^3c^6\tan\left(fx+\cdots\right.}{\cdots}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^6,x, al
gorithm="giac")
```

output
```
-1/504*(-56*I*B*a^3*c^6*tan(f*x + e)^9 - 63*I*A*a^3*c^6*tan(f*x + e)^8 + 1
89*B*a^3*c^6*tan(f*x + e)^8 + 216*A*a^3*c^6*tan(f*x + e)^7 + 72*I*B*a^3*c^
6*tan(f*x + e)^7 + 84*I*A*a^3*c^6*tan(f*x + e)^6 + 420*B*a^3*c^6*tan(f*x +
 e)^6 + 504*A*a^3*c^6*tan(f*x + e)^5 + 504*I*B*a^3*c^6*tan(f*x + e)^5 + 63
0*I*A*a^3*c^6*tan(f*x + e)^4 + 126*B*a^3*c^6*tan(f*x + e)^4 + 168*A*a^3*c^
6*tan(f*x + e)^3 + 504*I*B*a^3*c^6*tan(f*x + e)^3 + 756*I*A*a^3*c^6*tan(f*
x + e)^2 - 252*B*a^3*c^6*tan(f*x + e)^2 - 504*A*a^3*c^6*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.54

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^6\,dx$$
$$= \frac{A\,a^3\,c^6\tan(e+f\,x)+\frac{a^3\,c^6\tan(e+f\,x)^3\,(-3\,B+A\,1\mathrm{i})\,1\mathrm{i}}{3}+a^3\,c^6\tan(e+f\,x)^5\,(-B+A\,1\mathrm{i})\,1\mathrm{i}-\frac{a^3\,c^6\tan(e+f\,x)^4\,(5\,A}{4}}{}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^6,x)
```

output
```
((a^3*c^6*tan(e + f*x)^3*(A*1i - 3*B)*1i)/3 - (a^3*c^6*tan(e + f*x)^2*(3*A
 + B*1i)*1i)/2 + a^3*c^6*tan(e + f*x)^5*(A*1i - B)*1i - (a^3*c^6*tan(e + f
*x)^4*(5*A - B*1i)*1i)/4 + (a^3*c^6*tan(e + f*x)^7*(A*3i - B)*1i)/7 + A*a^
3*c^6*tan(e + f*x) - (a^3*c^6*tan(e + f*x)^6*(A - B*5i)*1i)/6 + (a^3*c^6*t
an(e + f*x)^8*(A + B*3i)*1i)/8 + (B*a^3*c^6*tan(e + f*x)^9*1i)/9)/f
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.41

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^6\,dx$$
$$= \frac{\tan(fx+e)\,a^3c^6\left(56\tan(fx+e)^8\,bi+63\tan(fx+e)^7\,ai-189\tan(fx+e)^7\,b-216\tan(fx+e)^6\,a-\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^6,x)
```

output
```
(tan(e + f*x)*a**3*c**6*(56*tan(e + f*x)**8*b*i + 63*tan(e + f*x)**7*a*i -
 189*tan(e + f*x)**7*b - 216*tan(e + f*x)**6*a - 72*tan(e + f*x)**6*b*i -
84*tan(e + f*x)**5*a*i - 420*tan(e + f*x)**5*b - 504*tan(e + f*x)**4*a - 5
04*tan(e + f*x)**4*b*i - 630*tan(e + f*x)**3*a*i - 126*tan(e + f*x)**3*b -
 168*tan(e + f*x)**2*a - 504*tan(e + f*x)**2*b*i - 756*tan(e + f*x)*a*i +
252*tan(e + f*x)*b + 504*a))/(504*f)
```

3.691 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx$

Optimal result

Integrand size = 41, antiderivative size = 135

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx = \frac{4a^3(iA+B)c^5(1-i\tan(e+fx))^5}{5f} - \frac{2a^3(iA+2B)c^5(1-i\tan(e+fx))^6}{3f} + \frac{a^3(iA+5B)c^5(1-i\tan(e+fx))^7}{7f} - \frac{a^3Bc^5(1-i\tan(e+fx))^8}{8f}$$

output
```
4/5*a^3*(I*A+B)*c^5*(1-I*tan(f*x+e))^5/f-2/3*a^3*(I*A+2*B)*c^5*(1-I*tan(f*
x+e))^6/f+1/7*a^3*(I*A+5*B)*c^5*(1-I*tan(f*x+e))^7/f-1/8*a^3*B*c^5*(1-I*ta
n(f*x+e))^8/f
```

Mathematica [A] (verified)

Time = 5.48 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx$$
$$= \frac{a^3c^5\sec^8(e+fx)(140(-iA+B)\cos(2(e+fx))+84(A-iB)\sin(2(e+fx))+(4A+iB)(-35i+28\,\mathrm{si}}{840f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^5,x]
```

output
```
(a^3*c^5*Sec[e + f*x]^8*(140*((-I)*A + B)*Cos[2*(e + f*x)] + 84*(A - I*B)*
Sin[2*(e + f*x)] + (4*A + I*B)*(-35*I + 28*Sin[4*(e + f*x)] + 8*Sin[6*(e +
 f*x)] + Sin[8*(e + f*x)])))/(840*f)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5(A+B\tan(e+fx))\,dx$$
$$\Big\downarrow \ 3042$$
$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5(A+B\tan(e+fx))dx$$
$$\Big\downarrow \ 4071$$
$$\frac{ac\int a^2c^4(1-i\tan(e+fx))^4(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$
$$\Big\downarrow \ 27$$
$$\frac{a^3c^5\int (1-i\tan(e+fx))^4(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 86$$

$$\frac{a^3c^5\int \left(iB(1-i\tan(e+fx))^7+(A-5iB)(1-i\tan(e+fx))^6-4(A-2iB)(1-i\tan(e+fx))^5+4(A-iB)(\right.}{f}$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3c^5\left(\frac{1}{7}(5B+iA)(1-i\tan(e+fx))^7-\frac{2}{3}(2B+iA)(1-i\tan(e+fx))^6+\frac{4}{5}(B+iA)(1-i\tan(e+fx))^5-\frac{1}{8}B\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^5
,x]
```

output
```
(a^3*c^5*((4*(I*A + B)*(1 - I*Tan[e + f*x])^5)/5 - (2*(I*A + 2*B)*(1 - I*T
an[e + f*x])^6)/3 + ((I*A + 5*B)*(1 - I*Tan[e + f*x])^7)/7 - (B*(1 - I*Tan
[e + f*x])^8)/8))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.80

method	result
risch	$\frac{32c^5a^3\left(84iA\,e^{6i(fx+e)}+84B\,e^{6i(fx+e)}+112iA\,e^{4i(fx+e)}-28B\,e^{4i(fx+e)}+32iA\,e^{2i(fx+e)}-8B\,e^{2i(fx+e)}+4iA-B\right)}{105f\left(e^{2i(fx+e)}+1\right)^8}$
derivativedivides	$\frac{c^5a^3\left(-\frac{B\tan(fx+e)^8}{8}-\frac{(2iB+A)\tan(fx+e)^7}{7}-\frac{(-7B-2iA+4i(-2iB+A))\tan(fx+e)^6}{6}-\frac{(-7A+4i(-2iA-B)+8iB)\tan(fx+e)^5}{5}\right.}{f}$
default	$\frac{c^5a^3\left(-\frac{B\tan(fx+e)^8}{8}-\frac{(2iB+A)\tan(fx+e)^7}{7}-\frac{(-7B-2iA+4i(-2iB+A))\tan(fx+e)^6}{6}-\frac{(-7A+4i(-2iA-B)+8iB)\tan(fx+e)^5}{5}\right.}{f}$
norman	$\frac{A\,a^3c^5\tan(fx+e)}{f}-\frac{\left(2iA\,a^3c^5+B\,a^3c^5\right)\tan(fx+e)^6}{6f}-\frac{\left(2iB\,a^3c^5+A\,a^3c^5\right)\tan(fx+e)^7}{7f}-\frac{\left(4iB\,a^3c^5+A\,a^3c^5\right)\,\mathrm{t}}{5f}$
parallelrisch	$-\frac{240iB\tan(fx+e)^7a^3c^5+105B\tan(fx+e)^8a^3c^5+280iA\tan(fx+e)^6a^3c^5+120A\tan(fx+e)^7a^3c^5+672iB\tan(fx+e}{}$
parts	$\frac{\left(-6iA\,a^3c^5+2B\,a^3c^5\right)\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{\left(-6iB\,a^3c^5-2A\,a^3c^5\right)\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x,method=_R
ETURNVERBOSE)
```

output
```
32/105*c^5*a^3*(84*I*A*exp(6*I*(f*x+e))+84*B*exp(6*I*(f*x+e))+112*I*A*exp(
4*I*(f*x+e))-28*B*exp(4*I*(f*x+e))+32*I*A*exp(2*I*(f*x+e))-8*B*exp(2*I*(f*
x+e))+4*I*A-B)/f/(exp(2*I*(f*x+e))+1)^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.35

$$\int (a + ia\tan(e + fx))^3 (A + B\tan(e + fx))(c - ic\tan(e + fx))^5 \, dx = -\frac{32 \left(84 \left(-i A - B\right) a^3 c^5 e^{(6 i f x + 6 i e)} + 28 \left(-4 i A + B\right) a^3 c^5 e^{(4 i f x + 4 i e)} + 8 \left(-4 i A + B\right) a^3 c^5 e}{105 \left(f e^{(16 i f x + 16 i e)} + 8 f e^{(14 i f x + 14 i e)} + 28 f e^{(12 i f x + 12 i e)} + 56 f e^{(10 i f x + 10 i e)} + 70 f e^{(8 i f x + 8 i e)} + 56 f e^{(6 i}\right.}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x, al
gorithm="fricas")
```

output
```
-32/105*(84*(-I*A - B)*a^3*c^5*e^(6*I*f*x + 6*I*e) + 28*(-4*I*A + B)*a^3*c
^5*e^(4*I*f*x + 4*I*e) + 8*(-4*I*A + B)*a^3*c^5*e^(2*I*f*x + 2*I*e) + (-4*
I*A + B)*a^3*c^5)/(f*e^(16*I*f*x + 16*I*e) + 8*f*e^(14*I*f*x + 14*I*e) + 2
8*f*e^(12*I*f*x + 12*I*e) + 56*f*e^(10*I*f*x + 10*I*e) + 70*f*e^(8*I*f*x +
 8*I*e) + 56*f*e^(6*I*f*x + 6*I*e) + 28*f*e^(4*I*f*x + 4*I*e) + 8*f*e^(2*I
*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(110) = 220.

Time = 0.83 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.27

$$\int (a + ia\tan(e + fx))^3 (A + B\tan(e + fx))(c - ic\tan(e + fx))^5 \, dx = \frac{128iAa^3c^5 - 32Ba^3c^5 + (1024iAa^3c^5e^{2ie} - 256Ba^3c^5e^{2ie})\, e^{2ifx} + (3584iAa^3c^5e^{4ie} - 896Ba^3c^5e^{4ie})\, e^{4ifx}}{105fe^{16ie}e^{16ifx} + 840fe^{14ie}e^{14ifx} + 2940fe^{12ie}e^{12ifx} + 5880fe^{10ie}e^{10ifx} + 7350fe^{8ie}e^{8ifx} + 5880fe^{6ie}e^{6ifx}}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**5,x)
```

output
```
(128*I*A*a**3*c**5 - 32*B*a**3*c**5 + (1024*I*A*a**3*c**5*exp(2*I*e) - 256
*B*a**3*c**5*exp(2*I*e))*exp(2*I*f*x) + (3584*I*A*a**3*c**5*exp(4*I*e) - 8
96*B*a**3*c**5*exp(4*I*e))*exp(4*I*f*x) + (2688*I*A*a**3*c**5*exp(6*I*e) +
 2688*B*a**3*c**5*exp(6*I*e))*exp(6*I*f*x))/(105*f*exp(16*I*e)*exp(16*I*f*
x) + 840*f*exp(14*I*e)*exp(14*I*f*x) + 2940*f*exp(12*I*e)*exp(12*I*f*x) +
5880*f*exp(10*I*e)*exp(10*I*f*x) + 7350*f*exp(8*I*e)*exp(8*I*f*x) + 5880*f
*exp(6*I*e)*exp(6*I*f*x) + 2940*f*exp(4*I*e)*exp(4*I*f*x) + 840*f*exp(2*I*
e)*exp(2*I*f*x) + 105*f)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.23

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx =$$

$$-\frac{105\,Ba^3c^5\tan(fx+e)^8+120\,(A+2i\,B)a^3c^5\tan(fx+e)^7-140\,(-2i\,A-B)a^3c^5\tan(fx+e)^6+1$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x, al
gorithm="maxima")
```

output
```
-1/840*(105*B*a^3*c^5*tan(f*x + e)^8 + 120*(A + 2*I*B)*a^3*c^5*tan(f*x + e
)^7 - 140*(-2*I*A - B)*a^3*c^5*tan(f*x + e)^6 + 168*(A + 4*I*B)*a^3*c^5*ta
n(f*x + e)^5 - 210*(-4*I*A + B)*a^3*c^5*tan(f*x + e)^4 - 280*(A - 2*I*B)*a
^3*c^5*tan(f*x + e)^3 - 420*(-2*I*A + B)*a^3*c^5*tan(f*x + e)^2 - 840*A*a^
3*c^5*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(113) = 226.

Time = 0.91 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.79

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx =$$

$$-\frac{105\,Ba^3c^5\tan(fx+e)^8+120\,Aa^3c^5\tan(fx+e)^7+240i\,Ba^3c^5\tan(fx+e)^7+280i\,Aa^3c^5\tan(fx+$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x, al
gorithm="giac")
```

output
```
-1/840*(105*B*a^3*c^5*tan(f*x + e)^8 + 120*A*a^3*c^5*tan(f*x + e)^7 + 240*
I*B*a^3*c^5*tan(f*x + e)^7 + 280*I*A*a^3*c^5*tan(f*x + e)^6 + 140*B*a^3*c^
5*tan(f*x + e)^6 + 168*A*a^3*c^5*tan(f*x + e)^5 + 672*I*B*a^3*c^5*tan(f*x
+ e)^5 + 840*I*A*a^3*c^5*tan(f*x + e)^4 - 210*B*a^3*c^5*tan(f*x + e)^4 - 2
80*A*a^3*c^5*tan(f*x + e)^3 + 560*I*B*a^3*c^5*tan(f*x + e)^3 + 840*I*A*a^3
*c^5*tan(f*x + e)^2 - 420*B*a^3*c^5*tan(f*x + e)^2 - 840*A*a^3*c^5*tan(f*x
 + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx =$$
$$-\frac{\frac{a^3\,c^5\,\tan(e+f\,x)^2\,(-B+A\,2\mathrm{i})}{2}+\frac{a^3\,c^5\,\tan(e+f\,x)^4\,(-B+A\,4\mathrm{i})}{4}-A\,a^3\,c^5\,\tan(e+f\,x)-\frac{a^3\,c^5\,\tan(e+f\,x)^3\,(A-B\,2\mathrm{i})}{3}+\frac{a^3\,c^5\,\mathrm{ta}}{}}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^5,x)
```

output
```
-((a^3*c^5*tan(e + f*x)^2*(A*2i - B))/2 + (a^3*c^5*tan(e + f*x)^4*(A*4i -
B))/4 - A*a^3*c^5*tan(e + f*x) - (a^3*c^5*tan(e + f*x)^3*(A - B*2i))/3 + (
a^3*c^5*tan(e + f*x)^6*(A*2i + B))/6 + (a^3*c^5*tan(e + f*x)^5*(A + B*4i))
/5 + (a^3*c^5*tan(e + f*x)^7*(A + B*2i))/7 + (B*a^3*c^5*tan(e + f*x)^8)/8)
/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.23

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^5\,dx$$
$$=\frac{\tan(fx+e)\,a^3c^5\left(-105\tan(fx+e)^7\,b-120\tan(fx+e)^6\,a-240\tan(fx+e)^6\,bi-280\tan(fx+e)^5\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5,x)
```

output
```
(tan(e + f*x)*a**3*c**5*( - 105*tan(e + f*x)**7*b - 120*tan(e + f*x)**6*a
- 240*tan(e + f*x)**6*b*i - 280*tan(e + f*x)**5*a*i - 140*tan(e + f*x)**5*
b - 168*tan(e + f*x)**4*a - 672*tan(e + f*x)**4*b*i - 840*tan(e + f*x)**3*
a*i + 210*tan(e + f*x)**3*b + 280*tan(e + f*x)**2*a - 560*tan(e + f*x)**2*
b*i - 840*tan(e + f*x)*a*i + 420*tan(e + f*x)*b + 840*a))/(840*f)
```

3.692 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx$

Optimal result

Integrand size = 41, antiderivative size = 132

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = \frac{a^3(iA+B)c^4(1-i\tan(e+fx))^4}{f} - \frac{4a^3(iA+2B)c^4(1-i\tan(e+fx))^5}{5f} + \frac{a^3(iA+5B)c^4(1-i\tan(e+fx))^6}{6f} - \frac{a^3Bc^4(1-i\tan(e+fx))^7}{7f}$$

output
```
a^3*(I*A+B)*c^4*(1-I*tan(f*x+e))^4/f-4/5*a^3*(I*A+2*B)*c^4*(1-I*tan(f*x+e)
)^5/f+1/6*a^3*(I*A+5*B)*c^4*(1-I*tan(f*x+e))^6/f-1/7*a^3*B*c^4*(1-I*tan(f*
x+e))^7/f
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx$$
$$=\frac{a^3c^4(7iA+29B+210A\tan(e+fx)+105(-iA+B)\tan^2(e+fx)+70(2A-iB)\tan^3(e+fx)+105(\ldots}{21\ldots}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^4,x]
```

output
```
(a^3*c^4*((7*I)*A + 29*B + 210*A*Tan[e + f*x] + 105*((-I)*A + B)*Tan[e + f
*x]^2 + 70*(2*A - I*B)*Tan[e + f*x]^3 + 105*((-I)*A + B)*Tan[e + f*x]^4 +
42*(A - (2*I)*B)*Tan[e + f*x]^5 + 35*((-I)*A + B)*Tan[e + f*x]^6 - (30*I)*
B*Tan[e + f*x]^7))/(210*f)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a^2c^3(1-i\tan(e+fx))^3(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c^4\int(1-i\tan(e+fx))^3(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$\Big\downarrow$ 86

$$\frac{a^3c^4\int\left(iB(1-i\tan(e+fx))^6+(A-5iB)(1-i\tan(e+fx))^5-4(A-2iB)(1-i\tan(e+fx))^4+4(A-iB)(\right.}{f}$$

$\Big\downarrow$ 2009

$$\frac{a^3c^4\left(\frac{1}{6}(5B+iA)(1-i\tan(e+fx))^6-\frac{4}{5}(2B+iA)(1-i\tan(e+fx))^5+(B+iA)(1-i\tan(e+fx))^4-\frac{1}{7}B(1\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4
,x]
```

output
```
(a^3*c^4*((I*A + B)*(1 - I*Tan[e + f*x])^4 - (4*(I*A + 2*B)*(1 - I*Tan[e +
 f*x])^5)/5 + ((I*A + 5*B)*(1 - I*Tan[e + f*x])^6)/6 - (B*(1 - I*Tan[e + f
*x])^7)/7))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

method	result
risch	$\frac{16c^4a^3\left(105iA\,e^{6i(fx+e)}+105B\,e^{6i(fx+e)}+147iA\,e^{4i(fx+e)}-21B\,e^{4i(fx+e)}+49iA\,e^{2i(fx+e)}-7B\,e^{2i(fx+e)}+7iA-B\right)}{105f\left(e^{2i(fx+e)}+1\right)^7}$
derivativedivides	$-\frac{ic^4a^3\left(\frac{B\tan(fx+e)^7}{7}+\frac{(iB+A)\tan(fx+e)^6}{6}+\frac{(-4B-2iA+3i(-2iB+A))\tan(fx+e)^5}{5}+\frac{(-4A+3i(-2iA-B)+5iB)\tan(fx+e)^4}{4}\right.}{f}$
default	$-\frac{ic^4a^3\left(\frac{B\tan(fx+e)^7}{7}+\frac{(iB+A)\tan(fx+e)^6}{6}+\frac{(-4B-2iA+3i(-2iB+A))\tan(fx+e)^5}{5}+\frac{(-4A+3i(-2iA-B)+5iB)\tan(fx+e)^4}{4}\right.}{f}$
norman	$\frac{A\,a^3c^4\tan(fx+e)}{f}+\frac{(-iA\,a^3c^4+B\,a^3c^4)\tan(fx+e)^2}{2f}+\frac{(-iA\,a^3c^4+B\,a^3c^4)\tan(fx+e)^4}{2f}+\frac{(-iA\,a^3c^4+B\,a^3c^4)}{6f}$
parallelrisch	$-\frac{30iB\,a^3c^4\tan(fx+e)^7+35iA\tan(fx+e)^6a^3c^4+84iB\tan(fx+e)^5a^3c^4-35B\tan(fx+e)^6a^3c^4+105iA\tan(fx+e)^4a}{}$
parts	$\frac{(-3iA\,a^3c^4+3B\,a^3c^4)\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln(1+\tan(fx+e)^2)}{2}\right)}{f}+\frac{(-3iA\,a^3c^4+3B\,a^3c^4)\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln(1}{}\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x,method=_R
ETURNVERBOSE)
```

output
```
16/105*c^4*a^3*(105*I*A*exp(6*I*(f*x+e))+105*B*exp(6*I*(f*x+e))+147*I*A*ex
p(4*I*(f*x+e))-21*B*exp(4*I*(f*x+e))+49*I*A*exp(2*I*(f*x+e))-7*B*exp(2*I*(
f*x+e))+7*I*A-B)/f/(exp(2*I*(f*x+e))+1)^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.29

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = -\frac{16\left(105\,(-i\,A-B)a^3c^4e^{(6i\,fx+6i\,e)}+21\,(-7i\,A+B)a^3c^4e^{(4i\,fx+4i\,e)}+7\,(-7i\,A+B)a^3c^4e^{(2i\,fx+2i\,e)}\right.}{105\left(fe^{(14i\,fx+14i\,e)}+7\,fe^{(12i\,fx+12i\,e)}+21\,fe^{(10i\,fx+10i\,e)}+35\,fe^{(8i\,fx+8i\,e)}+35\,fe^{(6i\,fx+6i\,e)}+21\,fe^{(4i\,fx}\right.}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, al
gorithm="fricas")
```

output
```
-16/105*(105*(-I*A - B)*a^3*c^4*e^(6*I*f*x + 6*I*e) + 21*(-7*I*A + B)*a^3*
c^4*e^(4*I*f*x + 4*I*e) + 7*(-7*I*A + B)*a^3*c^4*e^(2*I*f*x + 2*I*e) + (-7
*I*A + B)*a^3*c^4)/(f*e^(14*I*f*x + 14*I*e) + 7*f*e^(12*I*f*x + 12*I*e) +
21*f*e^(10*I*f*x + 10*I*e) + 35*f*e^(8*I*f*x + 8*I*e) + 35*f*e^(6*I*f*x +
6*I*e) + 21*f*e^(4*I*f*x + 4*I*e) + 7*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(107) = 214.

Time = 0.66 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.17

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = \frac{112iAa^3c^4-16Ba^3c^4+\left(784iAa^3c^4e^{2ie}-112Ba^3c^4e^{2ie}\right)e^{2ifx}+\left(2352iAa^3c^4e^{4ie}-336Ba^3c^4e^{4ie}\right)e^{4ifx}+}{105fe^{14ie}e^{14ifx}+735fe^{12ie}e^{12ifx}+2205fe^{10ie}e^{10ifx}+3675fe^{8ie}e^{8ifx}+3675fe^{6ie}e^{6ifx}+2205fe}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**4,x)
```

output
```
(112*I*A*a**3*c**4 - 16*B*a**3*c**4 + (784*I*A*a**3*c**4*exp(2*I*e) - 112*
B*a**3*c**4*exp(2*I*e))*exp(2*I*f*x) + (2352*I*A*a**3*c**4*exp(4*I*e) - 33
6*B*a**3*c**4*exp(4*I*e))*exp(4*I*f*x) + (1680*I*A*a**3*c**4*exp(6*I*e) +
1680*B*a**3*c**4*exp(6*I*e))*exp(6*I*f*x))/(105*f*exp(14*I*e)*exp(14*I*f*x
) + 735*f*exp(12*I*e)*exp(12*I*f*x) + 2205*f*exp(10*I*e)*exp(10*I*f*x) + 3
675*f*exp(8*I*e)*exp(8*I*f*x) + 3675*f*exp(6*I*e)*exp(6*I*f*x) + 2205*f*ex
p(4*I*e)*exp(4*I*f*x) + 735*f*exp(2*I*e)*exp(2*I*f*x) + 105*f)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

$$\int (a + ia\tan(e + fx))^3 (A + B\tan(e + fx))(c - ic\tan(e + fx))^4 \, dx =$$
$$-\frac{30i\,Ba^3c^4\tan(fx+e)^7 + 35\,(i\,A - B)a^3c^4\tan(fx+e)^6 - 42\,(A - 2i\,B)a^3c^4\tan(fx+e)^5 + 105\,(i\,A \cdots}{\cdots}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, al
gorithm="maxima")
```

output
```
-1/210*(30*I*B*a^3*c^4*tan(f*x + e)^7 + 35*(I*A - B)*a^3*c^4*tan(f*x + e)^
6 - 42*(A - 2*I*B)*a^3*c^4*tan(f*x + e)^5 + 105*(I*A - B)*a^3*c^4*tan(f*x
+ e)^4 - 70*(2*A - I*B)*a^3*c^4*tan(f*x + e)^3 + 105*(I*A - B)*a^3*c^4*tan
(f*x + e)^2 - 210*A*a^3*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.58

$$\int (a + ia\tan(e + fx))^3 (A + B\tan(e + fx))(c - ic\tan(e + fx))^4 \, dx =$$
$$-\frac{30i\,Ba^3c^4\tan(fx+e)^7 + 35i\,Aa^3c^4\tan(fx+e)^6 - 35\,Ba^3c^4\tan(fx+e)^6 - 42\,Aa^3c^4\tan(fx+e)^5 \cdots}{\cdots}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x, al
gorithm="giac")
```

output
```
-1/210*(30*I*B*a^3*c^4*tan(f*x + e)^7 + 35*I*A*a^3*c^4*tan(f*x + e)^6 - 35
*B*a^3*c^4*tan(f*x + e)^6 - 42*A*a^3*c^4*tan(f*x + e)^5 + 84*I*B*a^3*c^4*t
an(f*x + e)^5 + 105*I*A*a^3*c^4*tan(f*x + e)^4 - 105*B*a^3*c^4*tan(f*x + e
)^4 - 140*A*a^3*c^4*tan(f*x + e)^3 + 70*I*B*a^3*c^4*tan(f*x + e)^3 + 105*I
*A*a^3*c^4*tan(f*x + e)^2 - 105*B*a^3*c^4*tan(f*x + e)^2 - 210*A*a^3*c^4*t
an(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.18

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = -\frac{\frac{a^3\,c^4\,\tan(e+f\,x)^5\,(2\,B+A\,1\mathrm{i})\,1\mathrm{i}}{5}-A\,a^3\,c^4\tan(e+f\,x)+\frac{a^3\,c^4\tan(e+f\,x)^2\,(A+B\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{a^3\,c^4\tan(e+f\,x)^3\,(B+A\,2\mathrm{i})\,1\mathrm{i}}{3}+\frac{a^3}{}}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^4,x)
```

output
```
-((a^3*c^4*tan(e + f*x)^5*(A*1i + 2*B)*1i)/5 - A*a^3*c^4*tan(e + f*x) + (a
^3*c^4*tan(e + f*x)^2*(A + B*1i)*1i)/2 + (a^3*c^4*tan(e + f*x)^3*(A*2i + B
)*1i)/3 + (a^3*c^4*tan(e + f*x)^4*(A + B*1i)*1i)/2 + (a^3*c^4*tan(e + f*x)
^6*(A + B*1i)*1i)/6 + (B*a^3*c^4*tan(e + f*x)^7*1i)/7)/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.09

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^4\,dx = \frac{\tan(fx+e)\,a^3c^4\left(-30\tan(fx+e)^6\,bi-35\tan(fx+e)^5\,ai+35\tan(fx+e)^5\,b+42\tan(fx+e)^4\,a-\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4,x)
```

output
```
(tan(e + f*x)*a**3*c**4*( - 30*tan(e + f*x)**6*b*i - 35*tan(e + f*x)**5*a*
i + 35*tan(e + f*x)**5*b + 42*tan(e + f*x)**4*a - 84*tan(e + f*x)**4*b*i -
 105*tan(e + f*x)**3*a*i + 105*tan(e + f*x)**3*b + 140*tan(e + f*x)**2*a -
 70*tan(e + f*x)**2*b*i - 105*tan(e + f*x)*a*i + 105*tan(e + f*x)*b + 210*
a))/(210*f)
```

3.693 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 41, antiderivative size = 84

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = \frac{a^3Bc^3\sec^6(e+fx)}{6f}+\frac{a^3Ac^3\tan(e+fx)}{f}+\frac{2a^3Ac^3\tan^3(e+fx)}{3f}+\frac{a^3Ac^3\tan^5(e+fx)}{5f}$$

output
```
1/6*a^3*B*c^3*sec(f*x+e)^6/f+a^3*A*c^3*tan(f*x+e)/f+2/3*a^3*A*c^3*tan(f*x+
e)^3/f+1/5*a^3*A*c^3*tan(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = \frac{a^3Bc^3\sec^6(e+fx)}{6f}+\frac{a^3Ac^3\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^3,x]
```

output
```
(a^3*B*c^3*Sec[e + f*x]^6)/(6*f) + (a^3*A*c^3*(Tan[e + f*x] + (2*Tan[e + f
*x]^3)/3 + Tan[e + f*x]^5/5))/f
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 4071, 27, 82, 455, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a^2c^2(1-i\tan(e+fx))^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c^3\int (1-i\tan(e+fx))^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{82}$$

$$\frac{a^3c^3\int (A+B\tan(e+fx))\left(\tan^2(e+fx)+1\right)^2d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{455}$$

$$\frac{a^3c^3\left(A\int \left(\tan^2(e+fx)+1\right)^2d\tan(e+fx)+\frac{1}{6}B\left(\tan^2(e+fx)+1\right)^3\right)}{f}$$

$$\Big\downarrow \text{210}$$

$$\frac{a^3c^3\left(A\int\left(\tan^4(e+fx)+2\tan^2(e+fx)+1\right)d\tan(e+fx)+\frac{1}{6}B\left(\tan^2(e+fx)+1\right)^3\right)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3c^3\left(A\left(\frac{1}{5}\tan^5(e+fx)+\frac{2}{3}\tan^3(e+fx)+\tan(e+fx)\right)+\frac{1}{6}B\left(\tan^2(e+fx)+1\right)^3\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3
,x]
```

output
```
(a^3*c^3*((B*(1 + Tan[e + f*x]^2)^3)/6 + A*(Tan[e + f*x] + (2*Tan[e + f*x]
^3)/3 + Tan[e + f*x]^5/5)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{c^3a^3\left(\frac{B\tan(fx+e)^6}{6}+\frac{A\tan(fx+e)^5}{5}+\frac{B\tan(fx+e)^4}{2}+\frac{2A\tan(fx+e)^3}{3}+\frac{B\tan(fx+e)^2}{2}+A\tan(fx+e)\right)}{f}$
default	$\frac{c^3a^3\left(\frac{B\tan(fx+e)^6}{6}+\frac{A\tan(fx+e)^5}{5}+\frac{B\tan(fx+e)^4}{2}+\frac{2A\tan(fx+e)^3}{3}+\frac{B\tan(fx+e)^2}{2}+A\tan(fx+e)\right)}{f}$
risch	$\frac{16c^3a^3\left(10iA\,e^{6i(fx+e)}+10B\,e^{6i(fx+e)}+15iA\,e^{4i(fx+e)}+6iA\,e^{2i(fx+e)}+iA\right)}{15f\left(e^{2i(fx+e)}+1\right)^6}$
parallelrisch	$\frac{5B\,a^3c^3\tan(fx+e)^6+6A\,a^3c^3\tan(fx+e)^5+15B\,a^3c^3\tan(fx+e)^4+20A\,a^3c^3\tan(fx+e)^3+15B\,a^3c^3\tan(fx+e)^2+30}{30f}$
norman	$\frac{a^3Ac^3\tan(fx+e)}{f}+\frac{B\,a^3c^3\tan(fx+e)^2}{2f}+\frac{B\,a^3c^3\tan(fx+e)^4}{2f}+\frac{B\,a^3c^3\tan(fx+e)^6}{6f}+\frac{2a^3Ac^3\tan(fx+e)^3}{3f}$
parts	$A\,a^3c^3x+\frac{A\,a^3c^3\left(\frac{\tan(fx+e)^5}{5}-\frac{\tan(fx+e)^3}{3}+\tan(fx+e)-\arctan(\tan(fx+e))\right)}{f}+\frac{B\,a^3c^3\ln\left(1+\tan(fx+e)^2\right)}{2f}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*c^3*a^3*(1/6*B*tan(f*x+e)^6+1/5*A*tan(f*x+e)^5+1/2*B*tan(f*x+e)^4+2/3*
A*tan(f*x+e)^3+1/2*B*tan(f*x+e)^2+A*tan(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.75

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = -\frac{16\left(10(-iA-B)a^3c^3e^{(6i\,fx+6i\,e)}-15i\,Aa^3c^3e^{(4i\,fx+4i\,e)}-6i\,Aa^3c^3e^{(2i\,fx+2i\,e)}-i\,Aa^3c^3\right)}{15\left(fe^{(12i\,fx+12i\,e)}+6\,fe^{(10i\,fx+10i\,e)}+15\,fe^{(8i\,fx+8i\,e)}+20\,fe^{(6i\,fx+6i\,e)}+15\,fe^{(4i\,fx+4i\,e)}+6\,fe^{(2i\,fx+2i\,e}\right.}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-16/15*(10*(-I*A - B)*a^3*c^3*e^(6*I*f*x + 6*I*e) - 15*I*A*a^3*c^3*e^(4*I*
f*x + 4*I*e) - 6*I*A*a^3*c^3*e^(2*I*f*x + 2*I*e) - I*A*a^3*c^3)/(f*e^(12*I
*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20
*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e
) + f)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.67

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = \frac{240iAa^3c^3e^{4ie}e^{4ifx}+96iAa^3c^3e^{2ie}e^{2ifx}+16iAa^3c^3+\left(160iAa^3c^3e^{6ie}+160Ba^3c^3e^{6ie}\right)e^{6ifx}}{15fe^{12ie}e^{12ifx}+90fe^{10ie}e^{10ifx}+225fe^{8ie}e^{8ifx}+300fe^{6ie}e^{6ifx}+225fe^{4ie}e^{4ifx}+90fe^{2ie}e^{2ifx}+15f}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**3,x)
```

output
```
(240*I*A*a**3*c**3*exp(4*I*e)*exp(4*I*f*x) + 96*I*A*a**3*c**3*exp(2*I*e)*e
xp(2*I*f*x) + 16*I*A*a**3*c**3 + (160*I*A*a**3*c**3*exp(6*I*e) + 160*B*a**
3*c**3*exp(6*I*e))*exp(6*I*f*x))/(15*f*exp(12*I*e)*exp(12*I*f*x) + 90*f*ex
p(10*I*e)*exp(10*I*f*x) + 225*f*exp(8*I*e)*exp(8*I*f*x) + 300*f*exp(6*I*e)
*exp(6*I*f*x) + 225*f*exp(4*I*e)*exp(4*I*f*x) + 90*f*exp(2*I*e)*exp(2*I*f*
x) + 15*f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx$$
$$=\frac{5\,Ba^3c^3\tan\left(fx+e\right)^6+6\,Aa^3c^3\tan\left(fx+e\right)^5+15\,Ba^3c^3\tan\left(fx+e\right)^4+20\,Aa^3c^3\tan\left(fx+e\right)^3+15\,B}{30\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
1/30*(5*B*a^3*c^3*tan(f*x + e)^6 + 6*A*a^3*c^3*tan(f*x + e)^5 + 15*B*a^3*c
^3*tan(f*x + e)^4 + 20*A*a^3*c^3*tan(f*x + e)^3 + 15*B*a^3*c^3*tan(f*x + e
)^2 + 30*A*a^3*c^3*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx$$
$$=\frac{5\,Ba^3c^3\tan\left(fx+e\right)^6+6\,Aa^3c^3\tan\left(fx+e\right)^5+15\,Ba^3c^3\tan\left(fx+e\right)^4+20\,Aa^3c^3\tan\left(fx+e\right)^3+15\,B}{30\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
1/30*(5*B*a^3*c^3*tan(f*x + e)^6 + 6*A*a^3*c^3*tan(f*x + e)^5 + 15*B*a^3*c
^3*tan(f*x + e)^4 + 20*A*a^3*c^3*tan(f*x + e)^3 + 15*B*a^3*c^3*tan(f*x + e
)^2 + 30*A*a^3*c^3*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = \frac{a^3\,c^3\,\sin\left(e+f\,x\right)\,\left(30\,A\cos\left(e+f\,x\right)^5+15\,B\cos\left(e+f\,x\right)^4\,\sin\left(e+f\,x\right)+20\,A\cos\left(e+f\,x\right)^3\sin\left(e+f\right.\right.}{30\,f\cos\left(e+\right.}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^3,x)
```

output
```
(a^3*c^3*sin(e + f*x)*(30*A*cos(e + f*x)^5 + 5*B*sin(e + f*x)^5 + 20*A*cos
(e + f*x)^3*sin(e + f*x)^2 + 15*B*cos(e + f*x)^2*sin(e + f*x)^3 + 6*A*cos(
e + f*x)*sin(e + f*x)^4 + 15*B*cos(e + f*x)^4*sin(e + f*x)))/(30*f*cos(e +
 f*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.88

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^3\,dx = \frac{\tan\left(fx+e\right)a^3c^3\left(5\tan\left(fx+e\right)^5b+6\tan\left(fx+e\right)^4a+15\tan\left(fx+e\right)^3b+20\tan\left(fx+e\right)^2a+15\tan\right.}{30f}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3,x)
```

output
```
(tan(e + f*x)*a**3*c**3*(5*tan(e + f*x)**5*b + 6*tan(e + f*x)**4*a + 15*ta
n(e + f*x)**3*b + 20*tan(e + f*x)**2*a + 15*tan(e + f*x)*b + 30*a))/(30*f)
```

3.694 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 41, antiderivative size = 101

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = -\frac{2a^3(iA-B)c^2(1+i\tan(e+fx))^3}{3f} + \frac{a^3(iA-3B)c^2(1+i\tan(e+fx))^4}{4f} + \frac{a^3Bc^2(1+i\tan(e+fx))^5}{5f}$$

output
```
-2/3*a^3*(I*A-B)*c^2*(1+I*tan(f*x+e))^3/f+1/4*a^3*(I*A-3*B)*c^2*(1+I*tan(f
*x+e))^4/f+1/5*a^3*B*c^2*(1+I*tan(f*x+e))^5/f
```

Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = \frac{a^3c^2\sec^5(e+fx)(30(iA+B)\cos(e+fx)+2(25A+7iB+6(5A-iB)\cos(2(e+fx))+(5A-iB)\mathrm{co}}{120f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^2,x]
```

output
```
(a^3*c^2*Sec[e + f*x]^5*(30*(I*A + B)*Cos[e + f*x] + 2*(25*A + (7*I)*B + 6
*(5*A - I*B)*Cos[2*(e + f*x)] + (5*A - I*B)*Cos[4*(e + f*x)])*Sin[e + f*x]
))/(120*f)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a^2c(1-i\tan(e+fx))(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c^2\int (1-i\tan(e+fx))(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3c^2\int \left(iB(i\tan(e+fx)+1)^4+(-A-3iB)(i\tan(e+fx)+1)^3+2(A+iB)(i\tan(e+fx)+1)^2\right)d\tan(e+f}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3c^2\left(\frac{1}{4}(-3B+iA)(1+i\tan(e+fx))^4-\frac{2}{3}(-B+iA)(1+i\tan(e+fx))^3+\frac{1}{5}B(1+i\tan(e+fx))^5\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2
,x]
```

output
```
(a^3*c^2*((-2*(I*A - B)*(1 + I*Tan[e + f*x])^3)/3 + ((I*A - 3*B)*(1 + I*Ta
n[e + f*x])^4)/4 + (B*(1 + I*Tan[e + f*x])^5)/5))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{i c^2 a^3 \left(\frac{B \tan(fx+e)^5}{5}+\frac{(-iB+A)\tan(fx+e)^4}{4}+\frac{(iA-2i(iB+A)-B)\tan(fx+e)^3}{3}+\frac{(-iB+A)\tan(fx+e)^2}{2}-i\tan(fx+e)A\right)}{f}$
default	$\frac{i c^2 a^3 \left(\frac{B \tan(fx+e)^5}{5}+\frac{(-iB+A)\tan(fx+e)^4}{4}+\frac{(iA-2i(iB+A)-B)\tan(fx+e)^3}{3}+\frac{(-iB+A)\tan(fx+e)^2}{2}-i\tan(fx+e)A\right)}{f}$
risch	$\frac{4c^2a^3(30iA\,e^{6i(fx+e)}+30B\,e^{6i(fx+e)}+50iA\,e^{4i(fx+e)}+10B\,e^{4i(fx+e)}+25iA\,e^{2i(fx+e)}+5B\,e^{2i(fx+e)}+5iA+B)}{15f\left(e^{2i(fx+e)}+1\right)^5}$
norman	$\frac{A\,a^3c^2\tan(fx+e)}{f}+\frac{(iA\,a^3c^2+B\,a^3c^2)\tan(fx+e)^2}{2f}+\frac{(iA\,a^3c^2+B\,a^3c^2)\tan(fx+e)^4}{4f}+\frac{(iB\,a^3c^2+A\,a^3c^2)\tan(}{3f}$
parallelrisch	$\frac{12iB\,a^3c^2\tan(fx+e)^5+15iA\tan(fx+e)^4a^3c^2+20iB\tan(fx+e)^3a^3c^2+15B\tan(fx+e)^4a^3c^2+30iA\tan(fx+e)^2a^3c^2}{60f}$
parts	$\frac{(iA\,a^3c^2+B\,a^3c^2)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(iA\,a^3c^2+B\,a^3c^2)\left(\frac{\tan(fx+e)^4}{4}-\frac{\tan(fx+e)^2}{2}+\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{(iB}{}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
I/f*c^2*a^3*(1/5*B*tan(f*x+e)^5+1/4*(A-I*B)*tan(f*x+e)^4+1/3*(I*A-2*I*(A+I
*B)-B)*tan(f*x+e)^3+1/2*(A-I*B)*tan(f*x+e)^2-I*tan(f*x+e)*A)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.50

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx = -\frac{4\left(30\left(-iA-B\right)a^3c^2e^{(6i\,fx+6i\,e)}+10\left(-5iA-B\right)a^3c^2e^{(4i\,fx+4i\,e)}+5\left(-5iA-B\right)a^3c^2e^{(2i\,fx+2i\,e)}+(-\right.}{15\left(fe^{(10i\,fx+10i\,e)}+5\,fe^{(8i\,fx+8i\,e)}+10\,fe^{(6i\,fx+6i\,e)}+10\,fe^{(4i\,fx+4i\,e)}+5\,fe^{(2i\,fx+2i\,e)}+f\right.}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-4/15*(30*(-I*A - B)*a^3*c^2*e^(6*I*f*x + 6*I*e) + 10*(-5*I*A - B)*a^3*c^2
*e^(4*I*f*x + 4*I*e) + 5*(-5*I*A - B)*a^3*c^2*e^(2*I*f*x + 2*I*e) + (-5*I*
A - B)*a^3*c^2)/(f*e^(10*I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*
e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) +
 f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(80) = 160$.

Time = 0.41 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.48

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$$
$$= \frac{20iAa^3c^2+4Ba^3c^2+(100iAa^3c^2e^{2ie}+20Ba^3c^2e^{2ie})\,e^{2ifx}+(200iAa^3c^2e^{4ie}+40Ba^3c^2e^{4ie})\,e^{4ifx}+(120i}{15fe^{10ie}e^{10ifx}+75fe^{8ie}e^{8ifx}+150fe^{6ie}e^{6ifx}+150fe^{4ie}e^{4ifx}+75fe^{2ie}e^{2ifx}+}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**2,x)
```

output
```
(20*I*A*a**3*c**2 + 4*B*a**3*c**2 + (100*I*A*a**3*c**2*exp(2*I*e) + 20*B*a
**3*c**2*exp(2*I*e))*exp(2*I*f*x) + (200*I*A*a**3*c**2*exp(4*I*e) + 40*B*a
**3*c**2*exp(4*I*e))*exp(4*I*f*x) + (120*I*A*a**3*c**2*exp(6*I*e) + 120*B*
a**3*c**2*exp(6*I*e))*exp(6*I*f*x))/(15*f*exp(10*I*e)*exp(10*I*f*x) + 75*f
*exp(8*I*e)*exp(8*I*f*x) + 150*f*exp(6*I*e)*exp(6*I*f*x) + 150*f*exp(4*I*e
)*exp(4*I*f*x) + 75*f*exp(2*I*e)*exp(2*I*f*x) + 15*f)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$$
$$= \frac{12i\,Ba^3c^2\tan{(fx+e)}^5-15\,(-i\,A-B)a^3c^2\tan{(fx+e)}^4+20\,(A+i\,B)a^3c^2\tan{(fx+e)}^3-30\,(-i\,A}{60\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
1/60*(12*I*B*a^3*c^2*tan(f*x + e)^5 - 15*(-I*A - B)*a^3*c^2*tan(f*x + e)^4
 + 20*(A + I*B)*a^3*c^2*tan(f*x + e)^3 - 30*(-I*A - B)*a^3*c^2*tan(f*x + e
)^2 + 60*A*a^3*c^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.39

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx =$$
$$-\frac{-12i\,Ba^3c^2\tan\left(fx+e\right)^5-15i\,Aa^3c^2\tan\left(fx+e\right)^4-15\,Ba^3c^2\tan\left(fx+e\right)^4-20\,Aa^3c^2\tan\left(fx+e\right)}{6}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-1/60*(-12*I*B*a^3*c^2*tan(f*x + e)^5 - 15*I*A*a^3*c^2*tan(f*x + e)^4 - 15
*B*a^3*c^2*tan(f*x + e)^4 - 20*A*a^3*c^2*tan(f*x + e)^3 - 20*I*B*a^3*c^2*t
an(f*x + e)^3 - 30*I*A*a^3*c^2*tan(f*x + e)^2 - 30*B*a^3*c^2*tan(f*x + e)^
2 - 60*A*a^3*c^2*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^2\,dx$$
$$=\frac{A\,a^3\,c^2\tan(e+f\,x)-\frac{a^3\,c^2\tan(e+f\,x)^3\,(-B+A\,1i)\,1i}{3}+\frac{a^3\,c^2\tan(e+f\,x)^2\,(A-B\,1i)\,1i}{2}+\frac{a^3\,c^2\tan(e+f\,x)^4\,(A-B\,1i)\,1i}{4}+\frac{B\,a^3\,c}{}}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^2,x)
```

output
```
(A*a^3*c^2*tan(e + f*x) - (a^3*c^2*tan(e + f*x)^3*(A*1i - B)*1i)/3 + (a^3*
c^2*tan(e + f*x)^2*(A - B*1i)*1i)/2 + (a^3*c^2*tan(e + f*x)^4*(A - B*1i)*1
i)/4 + (B*a^3*c^2*tan(e + f*x)^5*1i)/5)/f
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

$$\int (a + ia\tan(e + fx))^3 (A + B\tan(e + fx))(c - ic\tan(e + fx))^2 \, dx$$
$$= \frac{\tan(fx + e) a^3 c^2 (12\tan(fx + e)^4 bi + 15\tan(fx + e)^3 ai + 15\tan(fx + e)^3 b + 20\tan(fx + e)^2 a + 20}{60f}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2,x)
```

output
```
(tan(e + f*x)*a**3*c**2*(12*tan(e + f*x)**4*b*i + 15*tan(e + f*x)**3*a*i +
 15*tan(e + f*x)**3*b + 20*tan(e + f*x)**2*a + 20*tan(e + f*x)**2*b*i + 30
*tan(e + f*x)*a*i + 30*tan(e + f*x)*b + 60*a))/(60*f)
```

3.695 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx$

Optimal result

Integrand size = 39, antiderivative size = 61

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = -\frac{a^3(iA-B)c(1+i\tan(e+fx))^3}{3f}-\frac{a^3Bc(1+i\tan(e+fx))^4}{4f}$$

output
```
-1/3*a^3*(I*A-B)*c*(1+I*tan(f*x+e))^3/f-1/4*a^3*B*c*(1+I*tan(f*x+e))^4/f
```

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = -\frac{a^3c(3B-12A\tan(e+fx)+(-12iA-6B)\tan^2(e+fx)+4(A-2iB)\tan^3(e+fx)+3B\tan^4(e+}{12f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x]),x]
```

output
```
-1/12*(a^3*c*(3*B - 12*A*Tan[e + f*x] + ((-12*I)*A - 6*B)*Tan[e + f*x]^2 +
 4*(A - (2*I)*B)*Tan[e + f*x]^3 + 3*B*Tan[e + f*x]^4))/f
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 4071, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c\int (i\tan(e+fx)+1)^2(A+B\tan(e+fx))d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{49}$$

$$\frac{a^3c\int \left((A+iB)(i\tan(e+fx)+1)^2-iB(i\tan(e+fx)+1)^3\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3c\left(-\frac{1}{3}(-B+iA)(1+i\tan(e+fx))^3-\frac{1}{4}B(1+i\tan(e+fx))^4\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]),x
]
```

output
```
(a^3*c*(-1/3*((I*A - B)*(1 + I*Tan[e + f*x])^3) - (B*(1 + I*Tan[e + f*x])^
4)/4))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{a^{3}c\left(-\frac{B\tan(fx+e)^{4}}{4}-\frac{(-2iB+A)\tan(fx+e)^{3}}{3}-\frac{(-2iA-B)\tan(fx+e)^{2}}{2}+A\tan(fx+e)\right)}{f}$
default	$\frac{a^{3}c\left(-\frac{B\tan(fx+e)^{4}}{4}-\frac{(-2iB+A)\tan(fx+e)^{3}}{3}-\frac{(-2iA-B)\tan(fx+e)^{2}}{2}+A\tan(fx+e)\right)}{f}$
norman	$\frac{A\,a^{3}c\tan(fx+e)}{f}-\frac{(-2iB\,a^{3}c+A\,a^{3}c)\tan(fx+e)^{3}}{3f}+\frac{(2iA\,a^{3}c+B\,a^{3}c)\tan(fx+e)^{2}}{2f}-\frac{B\,a^{3}c\tan(fx+e)^{4}}{4f}$
parallelrisch	$\frac{8iB\tan(fx+e)^{3}a^{3}c-3B\tan(fx+e)^{4}a^{3}c+12iA\tan(fx+e)^{2}a^{3}c-4A\tan(fx+e)^{3}a^{3}c+6B\tan(fx+e)^{2}a^{3}c+12A\tan(f}{12f}$
risch	$\frac{4a^{3}c(6iA\,e^{6i(fx+e)}+6B\,e^{6i(fx+e)}+12iA\,e^{4i(fx+e)}+6B\,e^{4i(fx+e)}+8iA\,e^{2i(fx+e)}+4B\,e^{2i(fx+e)}+2iA+B)}{3f\left(e^{2i(fx+e)}+1\right)^{4}}$
parts	$\frac{(2iA\,a^{3}c+B\,a^{3}c)\ln\left(1+\tan(fx+e)^{2}\right)}{2f}+\frac{(2iB\,a^{3}c-A\,a^{3}c)\left(\frac{\tan(fx+e)^{3}}{3}-\tan(fx+e)+\arctan(\tan(fx+e))\right)}{f}+A\,a$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
1/f*a^3*c*(-1/4*B*tan(f*x+e)^4-1/3*(A-2*I*B)*tan(f*x+e)^3-1/2*(-B-2*I*A)*t
an(f*x+e)^2+A*tan(f*x+e))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(51) = 102.

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.16

$$\int(a+ia\tan(e+fx))^{3}(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = -\frac{4\left(6\,(-i\,A-B)a^{3}ce^{(6i\,fx+6i\,e)}+6\,(-2i\,A-B)a^{3}ce^{(4i\,fx+4i\,e)}+4\,(-2i\,A-B)a^{3}ce^{(2i\,fx+2i\,e)}+(-2i\,A-\right.}{3\left(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algo
rithm="fricas")
```

output
```
-4/3*(6*(-I*A - B)*a^3*c*e^(6*I*f*x + 6*I*e) + 6*(-2*I*A - B)*a^3*c*e^(4*I
*f*x + 4*I*e) + 4*(-2*I*A - B)*a^3*c*e^(2*I*f*x + 2*I*e) + (-2*I*A - B)*a^
3*c)/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4
*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(48) = 96$.

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.57

$$\int (a + ia\tan(e + fx))^3(A + B\tan(e + fx))(c - ic\tan(e + fx))\,dx = \frac{8iAa^3c + 4Ba^3c + (32iAa^3ce^{2ie} + 16Ba^3ce^{2ie})\,e^{2ifx} + (48iAa^3ce^{4ie} + 24Ba^3ce^{4ie})\,e^{4ifx} + (24iAa^3ce^{6ie} +}{3fe^{8ie}e^{8ifx} + 12fe^{6ie}e^{6ifx} + 18fe^{4ie}e^{4ifx} + 12fe^{2ie}e^{2ifx} + 3f}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x)
```

output
```
(8*I*A*a**3*c + 4*B*a**3*c + (32*I*A*a**3*c*exp(2*I*e) + 16*B*a**3*c*exp(2
*I*e))*exp(2*I*f*x) + (48*I*A*a**3*c*exp(4*I*e) + 24*B*a**3*c*exp(4*I*e))*
exp(4*I*f*x) + (24*I*A*a**3*c*exp(6*I*e) + 24*B*a**3*c*exp(6*I*e))*exp(6*I
*f*x))/(3*f*exp(8*I*e)*exp(8*I*f*x) + 12*f*exp(6*I*e)*exp(6*I*f*x) + 18*f*
exp(4*I*e)*exp(4*I*f*x) + 12*f*exp(2*I*e)*exp(2*I*f*x) + 3*f)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int (a + ia\tan(e + fx))^3(A + B\tan(e + fx))(c - ic\tan(e + fx))\,dx = -\frac{3\,Ba^3c\tan(fx + e)^4 + 4\,(A - 2i\,B)a^3c\tan(fx + e)^3 - 6\,(2i\,A + B)a^3c\tan(fx + e)^2 - 12\,Aa^3c\tan(}{12\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algo
rithm="maxima")
```

output
```
-1/12*(3*B*a^3*c*tan(f*x + e)^4 + 4*(A - 2*I*B)*a^3*c*tan(f*x + e)^3 - 6*(
2*I*A + B)*a^3*c*tan(f*x + e)^2 - 12*A*a^3*c*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.54

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = -\frac{3\,Ba^3c\tan\left(fx+e\right)^4+4\,Aa^3c\tan\left(fx+e\right)^3-8i\,Ba^3c\tan\left(fx+e\right)^3-12i\,Aa^3c\tan\left(fx+e\right)^2-6\,Ba}{12\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x, algo
rithm="giac")
```

output
```
-1/12*(3*B*a^3*c*tan(f*x + e)^4 + 4*A*a^3*c*tan(f*x + e)^3 - 8*I*B*a^3*c*t
an(f*x + e)^3 - 12*I*A*a^3*c*tan(f*x + e)^2 - 6*B*a^3*c*tan(f*x + e)^2 - 1
2*A*a^3*c*tan(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx = \frac{-\frac{B\,c\,a^3\,\tan(e+f\,x)^4}{4}-\frac{c\,(A-B\,2\mathrm{i})\,a^3\,\tan(e+f\,x)^3}{3}+\frac{c\,(B+A\,2\mathrm{i})\,a^3\,\tan(e+f\,x)^2}{2}+A\,c\,a^3\,\tan(e+f\,x)}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
,x)
```

output
```
(A*a^3*c*tan(e + f*x) + (a^3*c*tan(e + f*x)^2*(A*2i + B))/2 - (a^3*c*tan(e
 + f*x)^3*(A - B*2i))/3 - (B*a^3*c*tan(e + f*x)^4)/4)/f
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))\,dx$$
$$= \frac{\tan(fx+e)\,a^3c\big(-3\tan(fx+e)^3\,b-4\tan(fx+e)^2\,a+8\tan(fx+e)^2\,bi+12\tan(fx+e)\,ai+6\tan$$
$$12f$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e)),x)
```

output
```
(tan(e + f*x)*a**3*c*( - 3*tan(e + f*x)**3*b - 4*tan(e + f*x)**2*a + 8*tan
(e + f*x)**2*b*i + 12*tan(e + f*x)*a*i + 6*tan(e + f*x)*b + 12*a))/(12*f)
```

3.696 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\,dx = 4a^3(A-iB)x-\frac{4a^3(iA+B)\log(\cos(e+fx))}{f}-\frac{2a^3(A-iB)\tan(e+fx)}{f}+\frac{a(iA+B)(a+ia\tan(e+fx))^2}{2f}+\frac{B(a+ia\tan(e+fx))^3}{3f}$$

output
```
4*a^3*(A-I*B)*x-4*a^3*(I*A+B)*ln(cos(f*x+e))/f-2*a^3*(A-I*B)*tan(f*x+e)/f+
1/2*a*(I*A+B)*(a+I*a*tan(f*x+e))^2/f+1/3*B*(a+I*a*tan(f*x+e))^3/f
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\,dx = \frac{B(a+ia\tan(e+fx))^3+\frac{3}{2}a^3(iA+B)\left(8\log(i+\tan(e+fx))+6i\tan(e+fx)-\tan^2(e+fx)\right)}{3f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]),x]
```

output
```
(B*(a + I*a*Tan[e + f*x])^3 + (3*a^3*(I*A + B)*(8*Log[I + Tan[e + f*x]] +
(6*I)*Tan[e + f*x] - Tan[e + f*x]^2))/2)/(3*f)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4010, 3042, 3959, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4010}$$

$$(A-iB)\int (i\tan(e+fx)a+a)^3dx+\frac{B(a+ia\tan(e+fx))^3}{3f}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\int (i\tan(e+fx)a+a)^3dx+\frac{B(a+ia\tan(e+fx))^3}{3f}$$

$$\Big\downarrow \text{3959}$$

$$(A-iB)\left(2a\int (i\tan(e+fx)a+a)^2dx+\frac{ia(a+ia\tan(e+fx))^2}{2f}\right)+\frac{B(a+ia\tan(e+fx))^3}{3f}$$

$$\Big\downarrow \text{3042}$$

$$(A-iB)\left(2a\int (i\tan(e+fx)a+a)^2dx+\frac{ia(a+ia\tan(e+fx))^2}{2f}\right)+\frac{B(a+ia\tan(e+fx))^3}{3f}$$

$$\Big\downarrow \text{3958}$$

$$(A-iB)\left(2a\left(2ia^2\int \tan(e+fx)dx-\frac{a^2\tan(e+fx)}{f}+2a^2x\right)+\frac{ia(a+ia\tan(e+fx))^2}{2f}\right)+\frac{B(a+ia\tan(e+fx))^3}{3f}$$

$\Big\downarrow$ 3042

$$(A-iB)\left(2a\left(2ia^2\int \tan(e+fx)dx-\frac{a^2\tan(e+fx)}{f}+2a^2x\right)+\frac{ia(a+ia\tan(e+fx))^2}{2f}\right)+\frac{B(a+ia\tan(e+fx))^3}{3f}$$

$\Big\downarrow$ 3956

$$(A-iB)\left(2a\left(-\frac{a^2\tan(e+fx)}{f}-\frac{2ia^2\log(\cos(e+fx))}{f}+2a^2x\right)+\frac{ia(a+ia\tan(e+fx))^2}{2f}\right)+\frac{B(a+ia\tan(e+fx))^3}{3f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]),x]
```

output
```
(B*(a + I*a*Tan[e + f*x])^3)/(3*f) + (A - I*B)*(((I/2)*a*(a + I*a*Tan[e +
f*x])^2)/f + 2*a*(2*a^2*x - ((2*I)*a^2*Log[Cos[e + f*x]])/f - (a^2*Tan[e +
 f*x])/f))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3959
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[2*a   Int[(a + b*Tan[c + d*
x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n
, 1]
```

rule 4010
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Simp
[(b*c + a*d)/b   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a^3\left(-\frac{iB\tan(fx+e)^3}{3}-\frac{iA\tan(fx+e)^2}{2}+4iB\tan(fx+e)-\frac{3B\tan(fx+e)^2}{2}-3A\tan(fx+e)+\frac{(4iA+4B)\ln\left(1+\tan(fx+e)^2\right)}{2}+(\right.}{f}$
default	$\frac{a^3\left(-\frac{iB\tan(fx+e)^3}{3}-\frac{iA\tan(fx+e)^2}{2}+4iB\tan(fx+e)-\frac{3B\tan(fx+e)^2}{2}-3A\tan(fx+e)+\frac{(4iA+4B)\ln\left(1+\tan(fx+e)^2\right)}{2}+(\right.}{f}$
norman	$(-4iB\,a^3+4A\,a^3)\,x-\frac{(iA\,a^3+3B\,a^3)\tan(fx+e)^2}{2f}-\frac{(-4iB\,a^3+3A\,a^3)\tan(fx+e)}{f}-\frac{iB\,a^3\tan(fx+e)^3}{3f}$
parallelrisch	$\frac{-2iB\,a^3\tan(fx+e)^3-3iA\tan(fx+e)^2a^3-24iBx\,a^3f+12iA\ln\left(1+\tan(fx+e)^2\right)a^3+24Ax\,a^3f+24iB\tan(fx+e)a^3-}{6f}$
risch	$\frac{8ia^3Be}{f}-\frac{8a^3Ae}{f}-\frac{2a^3\left(12iA\,e^{4i(fx+e)}+24B\,e^{4i(fx+e)}+21iA\,e^{2i(fx+e)}+33B\,e^{2i(fx+e)}+9iA+13B\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}-\frac{4a^3\ln(e}{}$
parts	$A\,a^3x+\frac{(-iA\,a^3-3B\,a^3)\left(\frac{\tan(fx+e)^2}{2}-\frac{\ln\left(1+\tan(fx+e)^2\right)}{2}\right)}{f}+\frac{(3iA\,a^3+B\,a^3)\ln\left(1+\tan(fx+e)^2\right)}{2f}+\frac{(3iB\,a^3-}{}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/f*a^3*(-1/3*I*B*tan(f*x+e)^3-1/2*I*A*tan(f*x+e)^2+4*I*B*tan(f*x+e)-3/2*B
*tan(f*x+e)^2-3*A*tan(f*x+e)+1/2*(4*I*A+4*B)*ln(1+tan(f*x+e)^2)+(-4*I*B+4*
A)*arctan(tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.59

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\,dx = -\frac{2\left(12\,(i\,A+2\,B)a^3e^{(4i\,fx+4i\,e)}+3\,(7i\,A+11\,B)a^3e^{(2i\,fx+2i\,e)}+(9i\,A+13\,B)a^3+6\left((i\,A+B)a^3e^{(6i\,fx-}\right.\right.}{3\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e}\right.}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x, algorithm="fricas")
```

output
```
-2/3*(12*(I*A + 2*B)*a^3*e^(4*I*f*x + 4*I*e) + 3*(7*I*A + 11*B)*a^3*e^(2*I
*f*x + 2*I*e) + (9*I*A + 13*B)*a^3 + 6*((I*A + B)*a^3*e^(6*I*f*x + 6*I*e)
+ 3*(I*A + B)*a^3*e^(4*I*f*x + 4*I*e) + 3*(I*A + B)*a^3*e^(2*I*f*x + 2*I*e
) + (I*A + B)*a^3)*log(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(6*I*f*x + 6*I*e) +
3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\,dx = -\frac{4ia^3(A-iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{f} + \frac{-18iAa^3-26Ba^3+\left(-42iAa^3e^{2ie}-66Ba^3e^{2ie}\right)e^{2ifx}+\left(-24iAa^3e^{4ie}-48Ba^3e^{4ie}\right)e^{4ifx}}{3fe^{6ie}e^{6ifx}+9fe^{4ie}e^{4ifx}+9fe^{2ie}e^{2ifx}+3f}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e)),x)
```

output
```
-4*I*a**3*(A - I*B)*log(exp(2*I*f*x) + exp(-2*I*e))/f + (-18*I*A*a**3 - 26
*B*a**3 + (-42*I*A*a**3*exp(2*I*e) - 66*B*a**3*exp(2*I*e))*exp(2*I*f*x) +
(-24*I*A*a**3*exp(4*I*e) - 48*B*a**3*exp(4*I*e))*exp(4*I*f*x))/(3*f*exp(6*
I*e)*exp(6*I*f*x) + 9*f*exp(4*I*e)*exp(4*I*f*x) + 9*f*exp(2*I*e)*exp(2*I*f
*x) + 3*f)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.87

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\,dx = -\frac{2i\,Ba^3\tan(fx+e)^3+3\,(i\,A+3\,B)a^3\tan(fx+e)^2-24\,(fx+e)(A-i\,B)a^3+12\,(-i\,A-B)a^3\log}{6\,f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x, algorithm="maxima")
```

output
```
-1/6*(2*I*B*a^3*tan(f*x + e)^3 + 3*(I*A + 3*B)*a^3*tan(f*x + e)^2 - 24*(f*
x + e)*(A - I*B)*a^3 + 12*(-I*A - B)*a^3*log(tan(f*x + e)^2 + 1) + 6*(3*A
- 4*I*B)*a^3*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\,dx = -\frac{4\,(-i\,Aa^3-Ba^3)\log(\tan(fx+e)+i)}{f} - \frac{2i\,Ba^3f^2\tan(fx+e)^3+3i\,Aa^3f^2\tan(fx+e)^2+9\,Ba^3f^2\tan(fx+e)^2+18\,Aa^3f^2\tan(fx+e)-2}{6\,f^3}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x, algorithm="giac")
```

output
```
-4*(-I*A*a^3 - B*a^3)*log(tan(f*x + e) + I)/f - 1/6*(2*I*B*a^3*f^2*tan(f*x
 + e)^3 + 3*I*A*a^3*f^2*tan(f*x + e)^2 + 9*B*a^3*f^2*tan(f*x + e)^2 + 18*A
*a^3*f^2*tan(f*x + e) - 24*I*B*a^3*f^2*tan(f*x + e))/f^3
```

Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\,dx$$
$$= -\frac{\tan(e+f\,x)^2\,\left(\frac{B\,a^3}{2}+\frac{a^3\,(2\,B+A\,1\mathrm{i})}{2}\right)}{f}+\frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(4\,B\,a^3+A\,a^3\,4\mathrm{i}\right)}{f}$$
$$+\frac{\tan(e+f\,x)\,\left(B\,a^3\,1\mathrm{i}-a^3\,(2\,A-B\,1\mathrm{i})+a^3\,(2\,B+A\,1\mathrm{i})\,1\mathrm{i}\right)}{f}$$
$$-\frac{B\,a^3\tan(e+f\,x)^3\,1\mathrm{i}}{3\,f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3,x)
```

output
```
(log(tan(e + f*x) + 1i)*(A*a^3*4i + 4*B*a^3))/f - (tan(e + f*x)^2*((B*a^3)
/2 + (a^3*(A*1i + 2*B))/2))/f + (tan(e + f*x)*(B*a^3*1i - a^3*(2*A - B*1i)
 + a^3*(A*1i + 2*B)*1i))/f - (B*a^3*tan(e + f*x)^3*1i)/(3*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\,dx$$
$$= \frac{a^3\left(12\log\left(\tan(fx+e)^2+1\right)ai+12\log\left(\tan(fx+e)^2+1\right)b-2\tan(fx+e)^3bi-3\tan(fx+e)^2ai-\right.}{6f}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e)),x)
```

output
```
(a**3*(12*log(tan(e + f*x)**2 + 1)*a*i + 12*log(tan(e + f*x)**2 + 1)*b - 2
*tan(e + f*x)**3*b*i - 3*tan(e + f*x)**2*a*i - 9*tan(e + f*x)**2*b - 18*ta
n(e + f*x)*a + 24*tan(e + f*x)*b*i + 24*a*f*x - 24*b*f*i*x))/(6*f)
```

3.697 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 119

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = -\frac{4a^3(A-2iB)x}{c}+\frac{4a^3(iA+2B)\log(\cos(e+fx))}{cf} + \frac{a^3(A-4iB)\tan(e+fx)}{cf}+\frac{a^3B\tan^2(e+fx)}{2cf}+\frac{4a^3(A-iB)}{cf(i+\tan(e+fx))}$$

output
```
-4*a^3*(A-2*I*B)*x/c+4*a^3*(I*A+2*B)*ln(cos(f*x+e))/c/f+a^3*(A-4*I*B)*tan(
f*x+e)/c/f+1/2*a^3*B*tan(f*x+e)^2/c/f+4*a^3*(A-I*B)/c/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \frac{a^3(14A-27iB+8(A-2iB)\log(i+\tan(e+fx))+(-4iA-11B-8i(A-2iB)\log(i+\tan(e+fx))}{2cf(i+\tan(e+fx))}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x]),x]
```

output
```
(a^3*(14*A - (27*I)*B + 8*(A - (2*I)*B)*Log[I + Tan[e + f*x]] + ((-4*I)*A
- 11*B - (8*I)*(A - (2*I)*B)*Log[I + Tan[e + f*x]])*Tan[e + f*x] + (2*A -
(7*I)*B)*Tan[e + f*x]^2 + B*Tan[e + f*x]^3))/(2*c*f*(I + Tan[e + f*x]))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.68, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{c^2(1-i\tan(e+fx))^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^3\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(1-i\tan(e+fx))^2}d\tan(e+fx)}{cf}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{a^3\int \left(-\frac{4(A-iB)}{(\tan(e+fx)+i)^2}+A\left(1-\frac{4iB}{A}\right)+B\tan(e+fx)-\frac{4i(A-2iB)}{\tan(e+fx)+i}\right)d\tan(e+fx)}{cf}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3\left((A-4iB)\tan(e+fx)+\frac{4(A-iB)}{\tan(e+fx)+i}-4(2B+iA)\log(\tan(e+fx)+i)+\frac{1}{2}B\tan^2(e+fx)\right)}{cf}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
,x]
```

output
```
(a^3*(-4*(I*A + 2*B)*Log[I + Tan[e + f*x]] + (A - (4*I)*B)*Tan[e + f*x] +
(B*Tan[e + f*x]^2)/2 + (4*(A - I*B))/(I + Tan[e + f*x])))/(c*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{2\,\mathrm{e}^{2i(fx+e)} B\,a^3}{cf}-\frac{2i\mathrm{e}^{2i(fx+e)} A\,a^3}{cf}-\frac{16ia^3Be}{fc}+\frac{8a^3Ae}{fc}+\frac{2a^3\left(iA\,\mathrm{e}^{2i(fx+e)}+5B\,\mathrm{e}^{2i(fx+e)}+iA+4B\right)}{cf\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}+\frac{8a}{}$
derivativedivides	$\frac{a^3A\tan(fx+e)}{fc}-\frac{4ia^3\tan(fx+e)B}{fc}+\frac{a^3B\tan(fx+e)^2}{2cf}-\frac{4ia^3B}{fc(i+\tan(fx+e))}+\frac{4a^3A}{fc(i+\tan(fx+e))}-\frac{4a^3A\,\mathrm{arc}}{}$
default	$\frac{a^3A\tan(fx+e)}{fc}-\frac{4ia^3\tan(fx+e)B}{fc}+\frac{a^3B\tan(fx+e)^2}{2cf}-\frac{4ia^3B}{fc(i+\tan(fx+e))}+\frac{4a^3A}{fc(i+\tan(fx+e))}-\frac{4a^3A\,\mathrm{arc}}{}$
norman	$\frac{\frac{\left(-4iB\,a^3+A\,a^3\right)\tan(fx+e)^3}{cf}+\frac{\left(-8iB\,a^3+5A\,a^3\right)\tan(fx+e)}{cf}-\frac{4\left(-2iB\,a^3+A\,a^3\right)x}{c}-\frac{8iA\,a^3+9B\,a^3}{2cf}-\frac{4\left(-2iB\,a^3+A\,a^3\right)x\tan(f}{c}}{1+\tan(fx+e)^2}$

input

```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,method=_RET
URNVERBOSE)
```

output

```
-2/c/f*exp(2*I*(f*x+e))*B*a^3-2*I/c/f*exp(2*I*(f*x+e))*A*a^3-16*I*a^3/f/c*
B*e+8*a^3/f/c*A*e+2*a^3*(I*A*exp(2*I*(f*x+e))+5*B*exp(2*I*(f*x+e))+I*A+4*B
)/c/f/(exp(2*I*(f*x+e))+1)^2+8*a^3/f/c*ln(exp(2*I*(f*x+e))+1)*B+4*I*a^3/f/
c*ln(exp(2*I*(f*x+e))+1)*A
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.38

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx =$$
$$-\frac{2\left((iA+B)a^3e^{(6ifx+6ie)}+2\,(iA+B)a^3e^{(4ifx+4ie)}-4\,Ba^3e^{(2ifx+2ie)}+(-iA-4\,B)a^3+2\left((-iA-\right.\right.}{cfe^{(4ifx+4ie)}+2\,cfe^{(2if}}$$

input

```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algo
rithm="fricas")
```

output
```
-2*((I*A + B)*a^3*e^(6*I*f*x + 6*I*e) + 2*(I*A + B)*a^3*e^(4*I*f*x + 4*I*e
) - 4*B*a^3*e^(2*I*f*x + 2*I*e) + (-I*A - 4*B)*a^3 + 2*((-I*A - 2*B)*a^3*e
^(4*I*f*x + 4*I*e) + 2*(-I*A - 2*B)*a^3*e^(2*I*f*x + 2*I*e) + (-I*A - 2*B)
*a^3)*log(e^(2*I*f*x + 2*I*e) + 1))/(c*f*e^(4*I*f*x + 4*I*e) + 2*c*f*e^(2*
I*f*x + 2*I*e) + c*f)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.73

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \frac{4ia^3(A-2iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{cf} + \frac{2iAa^3+8Ba^3+\left(2iAa^3e^{2ie}+10Ba^3e^{2ie}\right)e^{2ifx}}{cfe^{4ie}e^{4ifx}+2cfe^{2ie}e^{2ifx}+cf} + \begin{cases} \frac{\left(-2iAa^3e^{2ie}-2Ba^3e^{2ie}\right)e^{2ifx}}{cf} & \text{for } cf\neq 0 \\ \frac{x\left(4Aa^3e^{2ie}-4iBa^3e^{2ie}\right)}{c} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

output
```
4*I*a**3*(A - 2*I*B)*log(exp(2*I*f*x) + exp(-2*I*e))/(c*f) + (2*I*A*a**3 +
 8*B*a**3 + (2*I*A*a**3*exp(2*I*e) + 10*B*a**3*exp(2*I*e))*exp(2*I*f*x))/(
c*f*exp(4*I*e)*exp(4*I*f*x) + 2*c*f*exp(2*I*e)*exp(2*I*f*x) + c*f) + Piece
wise(((-2*I*A*a**3*exp(2*I*e) - 2*B*a**3*exp(2*I*e))*exp(2*I*f*x)/(c*f), N
e(c*f, 0)), (x*(4*A*a**3*exp(2*I*e) - 4*I*B*a**3*exp(2*I*e))/c, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx$$
$$= -\frac{4\,(i\,Aa^3+2\,Ba^3)\log{(\tan{(fx+e)}+i)}}{cf} + \frac{4\,(Aa^3-i\,Ba^3)}{cf(\tan{(fx+e)}+i)}$$
$$+ \frac{Ba^3cf\tan{(fx+e)}^2+2\,Aa^3cf\tan{(fx+e)}-8i\,Ba^3cf\tan{(fx+e)}}{2\,c^2f^2}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algo
rithm="giac")
```

output
```
-4*(I*A*a^3 + 2*B*a^3)*log(tan(f*x + e) + I)/(c*f) + 4*(A*a^3 - I*B*a^3)/(
c*f*(tan(f*x + e) + I)) + 1/2*(B*a^3*c*f*tan(f*x + e)^2 + 2*A*a^3*c*f*tan(
f*x + e) - 8*I*B*a^3*c*f*tan(f*x + e))/(c^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx$$
$$= \frac{B\,a^3\tan(e+f\,x)^2}{2\,c\,f} + \frac{\frac{4\,A\,a^3-B\,a^3\,8\mathrm{i}}{c}+\frac{B\,a^3\,4\mathrm{i}}{c}}{f\,\left(\tan(e+f\,x)+1\mathrm{i}\right)}$$
$$- \frac{\tan(e+f\,x)\,\left(\frac{B\,a^3\,2\mathrm{i}}{c}+\frac{a^3\,(2\,B+A\,1\mathrm{i})\,1\mathrm{i}}{c}\right)}{f} - \frac{\ln\left(\tan(e+f\,x)+1\mathrm{i}\right)\,\left(\frac{8\,B\,a^3}{c}+\frac{A\,a^3\,4\mathrm{i}}{c}\right)}{f}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i),x)
```

output
```
((4*A*a^3 - B*a^3*8i)/c + (B*a^3*4i)/c)/(f*(tan(e + f*x) + 1i)) - (log(tan
(e + f*x) + 1i)*((A*a^3*4i)/c + (8*B*a^3)/c))/f - (tan(e + f*x)*((B*a^3*2i
)/c + (a^3*(A*1i + 2*B)*1i)/c))/f + (B*a^3*tan(e + f*x)^2)/(2*c*f)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{c-ic\tan(e+fx)}\,dx$$
$$= \frac{a^3\left(16\left(\int \frac{1}{\tan(fx+e)+i}dx\right)afi+16\left(\int \frac{1}{\tan(fx+e)+i}dx\right)bf-4\log\left(\tan\left(fx+e\right)^2+1\right)ai-8\log\left(\tan\left(fx+e\right)\right.\right.}{2cf}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

output
```
(a**3*(16*int(1/(tan(e + f*x) + i),x)*a*f*i + 16*int(1/(tan(e + f*x) + i),
x)*b*f - 4*log(tan(e + f*x)**2 + 1)*a*i - 8*log(tan(e + f*x)**2 + 1)*b + t
an(e + f*x)**2*b + 2*tan(e + f*x)*a - 8*tan(e + f*x)*b*i - 16*a*f*x + 24*b
*f*i*x))/(2*c*f)
```

3.698 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 123

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = \frac{a^3(A-5iB)x}{c^2}-\frac{a^3(iA+5B)\log(\cos(e+fx))}{c^2f}+\frac{ia^3B\tan(e+fx)}{c^2f}+\frac{2a^3(iA+B)}{c^2f(i+\tan(e+fx))^2}-\frac{4a^3(A-2iB)}{c^2f(i+\tan(e+fx))}$$

output
```
a^3*(A-5*I*B)*x/c^2-a^3*(I*A+5*B)*ln(cos(f*x+e))/c^2/f+I*a^3*B*tan(f*x+e)/
c^2/f+2*a^3*(I*A+B)/c^2/f/(I+tan(f*x+e))^2-4*a^3*(A-2*I*B)/c^2/f/(I+tan(f*
x+e))
```

Mathematica [A] (verified)

Time = 3.88 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = \frac{\frac{B(a+ia\tan(e+fx))^3}{(c-ic\tan(e+fx))^2}+\frac{a^3(iA+5B)\left(\log(i+\tan(e+fx))+\frac{-2+4i\tan(e+fx)}{(i+\tan(e+fx))^2}\right)}{c^2}}{f}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^2,x]
```

output
```
((B*(a + I*a*Tan[e + f*x])^3)/(c - I*c*Tan[e + f*x])^2 + (a^3*(I*A + 5*B)*
(Log[I + Tan[e + f*x]] + (-2 + (4*I)*Tan[e + f*x])/(I + Tan[e + f*x])^2))/
c^2)/f
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{c^3(1-i\tan(e+fx))^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(1-i\tan(e+fx))^3}d\tan(e+fx)}{c^2f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3\int \left(-\frac{4i(A-iB)}{(\tan(e+fx)+i)^3}+iB+\frac{i(A-5iB)}{\tan(e+fx)+i}+\frac{4(A-2iB)}{(\tan(e+fx)+i)^2}\right)d\tan(e+fx)}{c^2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(-\frac{4(A-2iB)}{\tan(e+fx)+i}+\frac{2(B+iA)}{(\tan(e+fx)+i)^2}+(5B+iA)\log(\tan(e+fx)+i)+iB\tan(e+fx)\right)}{c^2f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^2,x]
```

output
```
(a^3*((I*A + 5*B)*Log[I + Tan[e + f*x]] + I*B*Tan[e + f*x] + (2*(I*A + B))
/(I + Tan[e + f*x])^2 - (4*(A - (2*I)*B))/(I + Tan[e + f*x])))/(c^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{e^{4i(fx+e)} B a^3}{2c^2 f} - \frac{i e^{4i(fx+e)} A a^3}{2c^2 f} + \frac{3 e^{2i(fx+e)} B a^3}{c^2 f} + \frac{i e^{2i(fx+e)} A a^3}{c^2 f} + \frac{10 i a^3 B e}{c^2 f} - \frac{2a^3 A e}{c^2 f} - \frac{2B a^3}{f c^2 (e^{2i(fx+}}$
derivativedivides	$\frac{i a^3 B \tan(fx+e)}{c^2 f} + \frac{8 i a^3 B}{f c^2 (i+\tan(fx+e))} - \frac{4a^3 A}{f c^2 (i+\tan(fx+e))} + \frac{i a^3 A \ln\left(1+\tan(fx+e)^2\right)}{2f c^2} + \frac{5a^3 B \ln\left(1+\tan(f\right.}{2f c^2}$
default	$\frac{i a^3 B \tan(fx+e)}{c^2 f} + \frac{8 i a^3 B}{f c^2 (i+\tan(fx+e))} - \frac{4a^3 A}{f c^2 (i+\tan(fx+e))} + \frac{i a^3 A \ln\left(1+\tan(fx+e)^2\right)}{2f c^2} + \frac{5a^3 B \ln\left(1+\tan(f\right.}{2f c^2}$
norman	$\frac{\frac{(-5iB a^3+A a^3)x}{c} + \frac{2iA a^3+6B a^3}{cf} + \frac{(-5iB a^3+A a^3)x \tan(fx+e)^4}{c} + \frac{iB a^3 \tan(fx+e)^5}{cf} + \frac{2(-5iB a^3+A a^3)x \tan(fx+e)^2}{c} - \frac{2(}{}}{c\left(1+\tan(fx+e)^2\right)^2}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-1/2/c^2/f*exp(4*I*(f*x+e))*B*a^3-1/2*I/c^2/f*exp(4*I*(f*x+e))*A*a^3+3/c^2
/f*exp(2*I*(f*x+e))*B*a^3+I/c^2/f*exp(2*I*(f*x+e))*A*a^3+10*I*a^3/c^2/f*B*
e-2*a^3/c^2/f*A*e-2/f/c^2*B*a^3/(exp(2*I*(f*x+e))+1)-5*a^3/c^2/f*ln(exp(2*
I*(f*x+e))+1)*B-I*a^3/c^2/f*ln(exp(2*I*(f*x+e))+1)*A
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx$$
$$= \frac{(-iA-B)a^3e^{(6ifx+6ie)} + (iA+5B)a^3e^{(4ifx+4ie)} - 2(-iA-3B)a^3e^{(2ifx+2ie)} - 4Ba^3 - 2((iA+5B}{2(c^2fe^{(2ifx+2ie)} + c^2f)}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
1/2*((-I*A - B)*a^3*e^(6*I*f*x + 6*I*e) + (I*A + 5*B)*a^3*e^(4*I*f*x + 4*I
*e) - 2*(-I*A - 3*B)*a^3*e^(2*I*f*x + 2*I*e) - 4*B*a^3 - 2*((I*A + 5*B)*a^
3*e^(2*I*f*x + 2*I*e) + (I*A + 5*B)*a^3)*log(e^(2*I*f*x + 2*I*e) + 1))/(c^
2*f*e^(2*I*f*x + 2*I*e) + c^2*f)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.92

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = -\frac{2Ba^3}{c^2fe^{2ie}e^{2ifx}+c^2f} - \frac{ia^3(A-5iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{c^2f} + \begin{cases} \frac{(2iAa^3c^2fe^{2ie}+6Ba^3c^2fe^{2ie})e^{2ifx}+(-iAa^3c^2fe^{4ie}-Ba^3c^2fe^{4ie})e^{4ifx}}{2c^4f^2} & \text{for } c^4f^2\neq 0 \\ \frac{x(2Aa^3e^{4ie}-2Aa^3e^{2ie}-2iBa^3e^{4ie}+6iBa^3e^{2ie})}{c^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**2,x)
```

output
```
-2*B*a**3/(c**2*f*exp(2*I*e)*exp(2*I*f*x) + c**2*f) - I*a**3*(A - 5*I*B)*l
og(exp(2*I*f*x) + exp(-2*I*e))/(c**2*f) + Piecewise((((2*I*A*a**3*c**2*f*e
xp(2*I*e) + 6*B*a**3*c**2*f*exp(2*I*e))*exp(2*I*f*x) + (-I*A*a**3*c**2*f*e
xp(4*I*e) - B*a**3*c**2*f*exp(4*I*e))*exp(4*I*f*x))/(2*c**4*f**2), Ne(c**4
*f**2, 0)), (x*(2*A*a**3*exp(4*I*e) - 2*A*a**3*exp(2*I*e) - 2*I*B*a**3*exp
(4*I*e) + 6*I*B*a**3*exp(2*I*e))/c**2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx$$
$$= \frac{iBa^3\tan(fx+e)}{c^2f} + \frac{(iAa^3+5Ba^3)\log(\tan(fx+e)+i)}{c^2f} - \frac{2(iAa^3+3Ba^3+2(Aa^3-2iBa^3)\tan(fx+e))}{c^2f(\tan(fx+e)+i)^2}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
I*B*a^3*tan(f*x + e)/(c^2*f) + (I*A*a^3 + 5*B*a^3)*log(tan(f*x + e) + I)/(
c^2*f) - 2*(I*A*a^3 + 3*B*a^3 + 2*(A*a^3 - 2*I*B*a^3)*tan(f*x + e))/(c^2*f
*(tan(f*x + e) + I)^2)
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.48

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx =$$
$$-\frac{a^3\left(7B\tan(e+fx)+B\,6\mathrm{i}+A\tan(e+fx)\,4\mathrm{i}-2A+B\tan(e+fx)^2\,2\mathrm{i}+B\tan(e+fx)^3-A\ln\right.}{}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^2,x)
```

output
```
-(a^3*(B*6i - 2*A + A*tan(e + f*x)*4i + 7*B*tan(e + f*x) + B*tan(e + f*x)^
2*2i + B*tan(e + f*x)^3 - A*log(tan(e + f*x) + 1i) + B*log(tan(e + f*x) +
1i)*5i + A*log(tan(e + f*x) + 1i)*tan(e + f*x)*2i + 10*B*log(tan(e + f*x)
+ 1i)*tan(e + f*x) + A*log(tan(e + f*x) + 1i)*tan(e + f*x)^2 - B*log(tan(e
 + f*x) + 1i)*tan(e + f*x)^2*5i)*1i)/(c^2*f*(tan(e + f*x)*1i - 1)^2)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^2}\,dx$$
$$= \frac{a^3\left(\left(\left(\int \frac{\tan(fx+e)^4}{\tan(fx+e)^2+2\tan(fx+e)i-1}dx\right)bi+\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^2+2\tan(fx+e)i-1}dx\right)ai+3\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^2+2\tan(fx+e)i-1}dx\right.\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x)
```

output
```
(a**3*(int(tan(e + f*x)**4/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*b*i
 + int(tan(e + f*x)**3/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*a*i + 3
*int(tan(e + f*x)**3/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*b + 3*int
(tan(e + f*x)**2/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*a - 3*int(tan
(e + f*x)**2/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*b*i - 3*int(tan(e
 + f*x)/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*a*i - int(tan(e + f*x)
/(tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*b - int(1/(tan(e + f*x)**2 +
2*tan(e + f*x)*i - 1),x)*a))/c**2
```

3.699 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 129

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \frac{ia^3Bx}{c^3}+\frac{a^3B\log(\cos(e+fx))}{c^3f}-\frac{a^3(iA+B)(1+i\tan(e+fx))^3}{6c^3f(1-i\tan(e+fx))^3} - \frac{2a^3B}{c^3f(i+\tan(e+fx))^2}-\frac{4ia^3B}{c^3f(i+\tan(e+fx))}$$

output
```
I*a^3*B*x/c^3+a^3*B*ln(cos(f*x+e))/c^3/f-1/6*a^3*(I*A+B)*(1+I*tan(f*x+e))^
3/c^3/f/(1-I*tan(f*x+e))^3-2*a^3*B/c^3/f/(I+tan(f*x+e))^2-4*I*a^3*B/c^3/f/
(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.57

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = -\frac{a^3\left(3B\log(i+\tan(e+fx))+\frac{A-7iB-18B\tan(e+fx)-3(A-5iB)\tan^2(e+fx)}{(i+\tan(e+fx))^3}\right)}{3c^3f}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^3,x]
```

output
```
-1/3*(a^3*(3*B*Log[I + Tan[e + f*x]] + (A - (7*I)*B - 18*B*Tan[e + f*x] -
3*(A - (5*I)*B)*Tan[e + f*x]^2)/(I + Tan[e + f*x])^3))/(c^3*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {3042, 4071, 27, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{c^4(1-i\tan(e+fx))^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^3\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(1-i\tan(e+fx))^4}d\tan(e+fx)}{c^3f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{a^3\left(iB\int \frac{(i\tan(e+fx)+1)^2}{(1-i\tan(e+fx))^3}d\tan(e+fx)-\frac{(B+iA)(1+i\tan(e+fx))^3}{6(1-i\tan(e+fx))^3}\right)}{c^3f}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{a^3\left(iB\int \left(\frac{i}{\tan(e+fx)+i}+\frac{4}{(\tan(e+fx)+i)^2}-\frac{4i}{(\tan(e+fx)+i)^3}\right)d\tan(e+fx)-\frac{(B+iA)(1+i\tan(e+fx))^3}{6(1-i\tan(e+fx))^3}\right)}{c^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(iB\left(-\frac{4}{\tan(e+fx)+i}+\frac{2i}{(\tan(e+fx)+i)^2}+i\log(\tan(e+fx)+i)\right)-\frac{(B+iA)(1+i\tan(e+fx))^3}{6(1-i\tan(e+fx))^3}\right)}{c^3f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^3,x]
```

output
```
(a^3*(-1/6*((I*A + B)*(1 + I*Tan[e + f*x])^3)/(1 - I*Tan[e + f*x])^3 + I*B
*(I*Log[I + Tan[e + f*x]] + (2*I)/(I + Tan[e + f*x])^2 - 4/(I + Tan[e + f*
x]))))/(c^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{e^{6i(fx+e)}B\,a^3}{6c^3f}-\frac{ie^{6i(fx+e)}A\,a^3}{6c^3f}+\frac{B\,a^3e^{4i(fx+e)}}{2c^3f}-\frac{B\,a^3e^{2i(fx+e)}}{c^3f}-\frac{2iB\,a^3e}{c^3f}+\frac{B\,a^3\ln(e^{2i(fx+e)}+1)}{c^3f}$
derivativedivides	$\frac{4ia^3B}{3f\,c^3(i+\tan(fx+e))^3}-\frac{4a^3A}{3f\,c^3(i+\tan(fx+e))^3}-\frac{5ia^3B}{f\,c^3(i+\tan(fx+e))}+\frac{a^3A}{f\,c^3(i+\tan(fx+e))}-\frac{a^3B\ln\left(1+\tan(f\right.}{2f\,c^3}$
default	$\frac{4ia^3B}{3f\,c^3(i+\tan(fx+e))^3}-\frac{4a^3A}{3f\,c^3(i+\tan(fx+e))^3}-\frac{5ia^3B}{f\,c^3(i+\tan(fx+e))}+\frac{a^3A}{f\,c^3(i+\tan(fx+e))}-\frac{a^3B\ln\left(1+\tan(f\right.}{2f\,c^3}$
norman	$\frac{\frac{\left(-iB\,a^3+A\,a^3\right)\tan(fx+e)}{cf}+\frac{\left(-5iB\,a^3+A\,a^3\right)\tan(fx+e)^5}{cf}+\frac{iB\,a^3x}{c}+\frac{iB\,a^3x\tan(fx+e)^6}{c}-\frac{iA\,a^3+7B\,a^3}{3cf}-\frac{2\left(iB\,a^3+5A\,a^3\right)\tan}{3cf}}{c^2\left(1+\tan(fx+\right.}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
-1/6/c^3/f*exp(6*I*(f*x+e))*B*a^3-1/6*I/c^3/f*exp(6*I*(f*x+e))*A*a^3+1/2/c
^3/f*B*a^3*exp(4*I*(f*x+e))-1/c^3/f*B*a^3*exp(2*I*(f*x+e))-2*I/c^3/f*B*a^3
*e+1/c^3/f*B*a^3*ln(exp(2*I*(f*x+e))+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.60

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx$$
$$=\frac{(-iA-B)a^3e^{(6i\,fx+6i\,e)}+3\,Ba^3e^{(4i\,fx+4i\,e)}-6\,Ba^3e^{(2i\,fx+2i\,e)}+6\,Ba^3\log\left(e^{(2i\,fx+2i\,e)}+1\right)}{6\,c^3f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
1/6*((-I*A - B)*a^3*e^(6*I*f*x + 6*I*e) + 3*B*a^3*e^(4*I*f*x + 4*I*e) - 6*
B*a^3*e^(2*I*f*x + 2*I*e) + 6*B*a^3*log(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.64

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx$$
$$=\frac{Ba^3\log\left(e^{2ifx}+e^{-2ie}\right)}{c^3f}+\begin{cases}\frac{6Ba^3c^6f^2e^{4ie}e^{4ifx}-12Ba^3c^6f^2e^{2ie}e^{2ifx}+\left(-2iAa^3c^6f^2e^{6ie}-2Ba^3c^6f^2e^{6ie}\right)e^{6ifx}}{12c^9f^3} & \text{for } c^9f^3\neq 0\\ \frac{x\left(Aa^3e^{6ie}-iBa^3e^{6ie}+2iBa^3e^{4ie}-2iBa^3e^{2ie}\right)}{c^3} & \text{otherwise}\end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**3,x)
```

output
```
B*a**3*log(exp(2*I*f*x) + exp(-2*I*e))/(c**3*f) + Piecewise((((6*B*a**3*c**
6*f**2*exp(4*I*e)*exp(4*I*f*x) - 12*B*a**3*c**6*f**2*exp(2*I*e)*exp(2*I*f*
x) + (-2*I*A*a**3*c**6*f**2*exp(6*I*e) - 2*B*a**3*c**6*f**2*exp(6*I*e))*ex
p(6*I*f*x))/(12*c**9*f**3), Ne(c**9*f**3, 0)), (x*(A*a**3*exp(6*I*e) - I*B
*a**3*exp(6*I*e) + 2*I*B*a**3*exp(4*I*e) - 2*I*B*a**3*exp(2*I*e))/c**3, Tr
ue))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.67

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^3}\,dx = -\frac{Ba^3\log\left(\tan\left(fx+e\right)+i\right)}{c^3f} + \frac{18\,Ba^3\tan\left(fx+e\right)-Aa^3+7i\,Ba^3+3\left(Aa^3-5i\,Ba^3\right)\tan\left(fx+e\right)^2}{3\,c^3f\left(\tan\left(fx+e\right)+i\right)^3}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-B*a^3*log(tan(f*x + e) + I)/(c^3*f) + 1/3*(18*B*a^3*tan(f*x + e) - A*a^3
+ 7*I*B*a^3 + 3*(A*a^3 - 5*I*B*a^3)*tan(f*x + e)^2)/(c^3*f*(tan(f*x + e) +
 I)^3)
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.09

$$\int \frac{(a + ia\tan(e + fx))^3(A + B\tan(e + fx))}{(c - ic\tan(e + fx))^3}\,dx =$$
$$-\frac{a^3\left(15\,B\tan(e+f\,x)^2 - 7\,B + B\tan(e+f\,x)\,18\mathrm{i} + A\tan(e+f\,x)^2\,3\mathrm{i} - A\,1\mathrm{i} - 3\,B\,\ln\left(\tan(e+f\,x)\right.\right.}{}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^3,x)
```

output
```
-(a^3*(B*tan(e + f*x)*18i - 7*B - A*1i + A*tan(e + f*x)^2*3i + 15*B*tan(e
+ f*x)^2 - 3*B*log(tan(e + f*x) + 1i) + B*log(tan(e + f*x) + 1i)*tan(e + f
*x)*9i + 9*B*log(tan(e + f*x) + 1i)*tan(e + f*x)^2 - B*log(tan(e + f*x) +
1i)*tan(e + f*x)^3*3i))/(3*c^3*f*(tan(e + f*x)*1i - 1)^3)
```

Reduce [F]

$$\int \frac{(a + ia\tan(e + fx))^3(A + B\tan(e + fx))}{(c - ic\tan(e + fx))^3}\,dx$$
$$= \frac{a^3\left(-\left(\int \frac{\tan(fx+e)^4}{\tan(fx+e)^3 i - 3\tan(fx+e)^2 - 3\tan(fx+e) i + 1}dx\right) bi - \left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^3 i - 3\tan(fx+e)^2 - 3\tan(fx+e) i + 1}dx\right) ai - 3\left(\int\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x)
```

output
```
(a**3*( - int(tan(e + f*x)**4/(tan(e + f*x)**3*i - 3*tan(e + f*x)**2 - 3*t
an(e + f*x)*i + 1),x)*b*i - int(tan(e + f*x)**3/(tan(e + f*x)**3*i - 3*tan
(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*a*i - 3*int(tan(e + f*x)**3/(tan(e
 + f*x)**3*i - 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b - 3*int(tan(
e + f*x)**2/(tan(e + f*x)**3*i - 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1)
,x)*a + 3*int(tan(e + f*x)**2/(tan(e + f*x)**3*i - 3*tan(e + f*x)**2 - 3*t
an(e + f*x)*i + 1),x)*b*i + 3*int(tan(e + f*x)/(tan(e + f*x)**3*i - 3*tan(
e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*a*i + int(tan(e + f*x)/(tan(e + f*x
)**3*i - 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b + int(1/(tan(e + f
*x)**3*i - 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*a))/c**3
```

3.700 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 99

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = -\frac{a^3(iA+B)(1+i\tan(e+fx))^3}{8c^4f(1-i\tan(e+fx))^4}-\frac{a^3(iA-7B)(1+i\tan(e+fx))^3}{48c^4f(1-i\tan(e+fx))^3}$$

output
```
-1/8*a^3*(I*A+B)*(1+I*tan(f*x+e))^3/c^4/f/(1-I*tan(f*x+e))^4-1/48*a^3*(I*A
-7*B)*(1+I*tan(f*x+e))^3/c^4/f/(1-I*tan(f*x+e))^3
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.80

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \frac{a^3(-iA+B+2(A-2iB)\tan(e+fx)+3i(A+iB)\tan^2(e+fx)+6iB\tan^3(e+fx))}{6c^4f(i+\tan(e+fx))^4}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^4,x]
```

output
```
(a^3*((-I)*A + B + 2*(A - (2*I)*B)*Tan[e + f*x] + (3*I)*(A + I*B)*Tan[e +
f*x]^2 + (6*I)*B*Tan[e + f*x]^3))/(6*c^4*f*(I + Tan[e + f*x])^4)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{c^5(1-i\tan(e+fx))^5}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(1-i\tan(e+fx))^5}d\tan(e+fx)}{c^4f}$$

$$\Big\downarrow \text{87}$$

$$\frac{a^3\left(\frac{1}{8}(A+7iB)\int \frac{(i\tan(e+fx)+1)^2}{(1-i\tan(e+fx))^4}d\tan(e+fx)-\frac{(B+iA)(1+i\tan(e+fx))^3}{8(1-i\tan(e+fx))^4}\right)}{c^4f}$$

$$\Big\downarrow \text{48}$$

$$\frac{a^3\left(-\frac{i(A+7iB)(1+i\tan(e+fx))^3}{48(1-i\tan(e+fx))^3}-\frac{(B+iA)(1+i\tan(e+fx))^3}{8(1-i\tan(e+fx))^4}\right)}{c^4f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^4,x]
```

output
```
(a^3*(-1/8*((I*A + B)*(1 + I*Tan[e + f*x])^3)/(1 - I*Tan[e + f*x])^4 - ((I
/48)*(A + (7*I)*B)*(1 + I*Tan[e + f*x])^3)/(1 - I*Tan[e + f*x])^3))/(c^4*f
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{a^{3} e^{8 i (f x+e)} B}{16 c^{4} f}-\frac{i a^{3} e^{8 i (f x+e)} A}{16 c^{4} f}+\frac{a^{3} e^{6 i (f x+e)} B}{12 c^{4} f}-\frac{i a^{3} e^{6 i (f x+e)} A}{12 c^{4} f}$
derivativedivides	$\frac{a^{3} \left(\frac{i B}{i+\tan (f x+e)}-\frac{4 i A+4 B}{4 (i+\tan (f x+e))^{4}}-\frac{-i A-5 B}{2 (i+\tan (f x+e))^{2}}-\frac{8 i B-4 A}{3 (i+\tan (f x+e))^{3}}\right)}{f\, c^{4}}$
default	$\frac{a^{3} \left(\frac{i B}{i+\tan (f x+e)}-\frac{4 i A+4 B}{4 (i+\tan (f x+e))^{4}}-\frac{-i A-5 B}{2 (i+\tan (f x+e))^{2}}-\frac{8 i B-4 A}{3 (i+\tan (f x+e))^{3}}\right)}{f\, c^{4}}$
norman	$\frac{\frac{a^{3} A \tan (f x+e)}{f c}+\frac{i B\, a^{3} \tan (f x+e)^{7}}{c f}+\frac{-i A\, a^{3}+B\, a^{3}}{6 c f}+\frac{\left(i A\, a^{3}+7 B\, a^{3}\right) \tan (f x+e)^{6}}{2 c f}+\frac{\left(17 i A\, a^{3}+7 B\, a^{3}\right) \tan (f x+e)^{2}}{6 c f}+\frac{7 \left(-2 i B\, a^{3}-\right.}{}}{\left(1+\tan (f x+e)^{2}\right)^{4} c^{3}}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,method=_R
ETURNVERBOSE)
```

output
```
-1/16*a^3/c^4/f*exp(8*I*(f*x+e))*B-1/16*I*a^3/c^4/f*exp(8*I*(f*x+e))*A+1/1
2*a^3/c^4/f*exp(6*I*(f*x+e))*B-1/12*I*a^3/c^4/f*exp(6*I*(f*x+e))*A
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.49

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = -\frac{3\,(i\,A+B)a^3e^{(8i\,fx+8i\,e)}+4\,(i\,A-B)a^3e^{(6i\,fx+6i\,e)}}{48\,c^4f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, al
gorithm="fricas")
```

output
```
-1/48*(3*(I*A + B)*a^3*e^(8*I*f*x + 8*I*e) + 4*(I*A - B)*a^3*e^(6*I*f*x +
6*I*e))/(c^4*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(80) = 160$.

Time = 0.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.69

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \begin{cases} \frac{(-16iAa^3c^4fe^{6ie}+16Ba^3c^4fe^{6ie})e^{6ifx}+(-12iAa^3c^4fe^{8ie}-12Ba^3c^4fe^{8ie})e^{8ifx}}{192c^8f^2} & \text{for } c^8f^2\neq 0 \\ \frac{x(Aa^3e^{8ie}+Aa^3e^{6ie}-iBa^3e^{8ie}+iBa^3e^{6ie})}{2c^4} & \text{otherwise} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**4,x)
```

output
```
Piecewise((((-16*I*A*a**3*c**4*f*exp(6*I*e) + 16*B*a**3*c**4*f*exp(6*I*e))
*exp(6*I*f*x) + (-12*I*A*a**3*c**4*f*exp(8*I*e) - 12*B*a**3*c**4*f*exp(8*I
*e))*exp(8*I*f*x))/(192*c**8*f**2), Ne(c**8*f**2, 0)), (x*(A*a**3*exp(8*I*
e) + A*a**3*exp(6*I*e) - I*B*a**3*exp(8*I*e) + I*B*a**3*exp(6*I*e))/(2*c**
4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = -\frac{-6i\,Ba^3\tan{(fx+e)}^3-3i\,Aa^3\tan{(fx+e)}^2+3\,Ba^3\tan{(fx+e)}^2-2\,Aa^3\tan{(fx+e)}+4i\,Ba^3\,\mathrm{ta}}{6\,c^4f(\tan{(fx+e)}+i)^4}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, al
gorithm="giac")
```

output
```
-1/6*(-6*I*B*a^3*tan(f*x + e)^3 - 3*I*A*a^3*tan(f*x + e)^2 + 3*B*a^3*tan(f
*x + e)^2 - 2*A*a^3*tan(f*x + e) + 4*I*B*a^3*tan(f*x + e) + I*A*a^3 - B*a^
3)/(c^4*f*(tan(f*x + e) + I)^4)
```

Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx = \frac{-\frac{a^3\,\left(-B+A\,1\mathrm{i}\right)}{6}+\frac{a^3\,\tan(e+f\,x)\,\left(2\,A-B\,4\mathrm{i}\right)}{6}+B\,a^3\,{\tan(e+f\,x)}^3\,1\mathrm{i}+\frac{a^3\,{\tan(e+f\,x)}^2\,\left(-3\,B+A\,3\mathrm{i}\right)}{6}}{c^4\,f\,\left({\tan\left(e+f\,x\right)}^4+{\tan\left(e+f\,x\right)}^3\,4\mathrm{i}-6\,{\tan\left(e+f\,x\right)}^2-\tan\left(e+f\,x\right)\,4\mathrm{i}+1\right)}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^4,x)
```

output
```
((a^3*tan(e + f*x)*(2*A - B*4i))/6 - (a^3*(A*1i - B))/6 + B*a^3*tan(e + f*
x)^3*1i + (a^3*tan(e + f*x)^2*(A*3i - 3*B))/6)/(c^4*f*(tan(e + f*x)^3*4i -
 6*tan(e + f*x)^2 - tan(e + f*x)*4i + tan(e + f*x)^4 + 1))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^4}\,dx$$
$$= \frac{a^3\left(4\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^4i-4\tan(fx+e)^3-6\tan(fx+e)^2i+4\tan(fx+e)+i}dx\right)af-28\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^4i-4\tan(fx+e)^3-6\tan(fx+e)^2i+4\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x)
```

output
```
(a**3*(4*int(tan(e + f*x)**3/(tan(e + f*x)**4*i - 4*tan(e + f*x)**3 - 6*ta
n(e + f*x)**2*i + 4*tan(e + f*x) + i),x)*a*f - 28*int(tan(e + f*x)**3/(tan
(e + f*x)**4*i - 4*tan(e + f*x)**3 - 6*tan(e + f*x)**2*i + 4*tan(e + f*x)
+ i),x)*b*f*i - 12*int(tan(e + f*x)**2/(tan(e + f*x)**4*i - 4*tan(e + f*x)
**3 - 6*tan(e + f*x)**2*i + 4*tan(e + f*x) + i),x)*a*f*i + 12*int(tan(e +
f*x)**2/(tan(e + f*x)**4*i - 4*tan(e + f*x)**3 - 6*tan(e + f*x)**2*i + 4*t
an(e + f*x) + i),x)*b*f - 12*int(tan(e + f*x)/(tan(e + f*x)**4*i - 4*tan(e
 + f*x)**3 - 6*tan(e + f*x)**2*i + 4*tan(e + f*x) + i),x)*a*f + 20*int(tan
(e + f*x)/(tan(e + f*x)**4*i - 4*tan(e + f*x)**3 - 6*tan(e + f*x)**2*i + 4
*tan(e + f*x) + i),x)*b*f*i + 4*int(1/(tan(e + f*x)**4*i - 4*tan(e + f*x)*
*3 - 6*tan(e + f*x)**2*i + 4*tan(e + f*x) + i),x)*a*f*i - 4*int(1/(tan(e +
 f*x)**4*i - 4*tan(e + f*x)**3 - 6*tan(e + f*x)**2*i + 4*tan(e + f*x) + i)
,x)*b*f + log(tan(e + f*x)**4 + 4*tan(e + f*x)**3*i - 6*tan(e + f*x)**2 -
4*tan(e + f*x)*i + 1)*b - 2*log(tan(e + f*x)**2 + 1)*b))/(4*c**4*f)
```

3.701 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 122

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{4a^3(A-iB)}{5c^5f(i+\tan(e+fx))^5}+\frac{a^3(iA+2B)}{c^5f(i+\tan(e+fx))^4} - \frac{a^3(A-5iB)}{3c^5f(i+\tan(e+fx))^3}-\frac{a^3B}{2c^5f(i+\tan(e+fx))^2}$$

output
```
4/5*a^3*(A-I*B)/c^5/f/(I+tan(f*x+e))^5+a^3*(I*A+2*B)/c^5/f/(I+tan(f*x+e))^
4-1/3*a^3*(A-5*I*B)/c^5/f/(I+tan(f*x+e))^3-1/2*a^3*B/c^5/f/(I+tan(f*x+e))^
2
```

Mathematica [A] (verified)

Time = 5.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{a^3(4A+iB+5(2iA+B)\tan(e+fx)+(-10A+5iB)\tan^2(e+fx)-15B\tan^3(e+fx))}{30c^5f(i+\tan(e+fx))^5}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^5,x]
```

output
```
(a^3*(4*A + I*B + 5*((2*I)*A + B)*Tan[e + f*x] + (-10*A + (5*I)*B)*Tan[e +
 f*x]^2 - 15*B*Tan[e + f*x]^3))/(30*c^5*f*(I + Tan[e + f*x])^5)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{c^6(1-i\tan(e+fx))^6}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(1-i\tan(e+fx))^6}d\tan(e+fx)}{c^5f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3\int \left(-\frac{4(A-iB)}{(\tan(e+fx)+i)^6}+\frac{B}{(\tan(e+fx)+i)^3}+\frac{A-5iB}{(\tan(e+fx)+i)^4}-\frac{4i(A-2iB)}{(\tan(e+fx)+i)^5}\right)d\tan(e+fx)}{c^5f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(\frac{4(A-iB)}{5(\tan(e+fx)+i)^5}-\frac{A-5iB}{3(\tan(e+fx)+i)^3}+\frac{2B+iA}{(\tan(e+fx)+i)^4}-\frac{B}{2(\tan(e+fx)+i)^2}\right)}{c^5f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^5,x]
```

output
```
(a^3*((4*(A - I*B))/(5*(I + Tan[e + f*x])^5) + (I*A + 2*B)/(I + Tan[e + f*
x])^4 - (A - (5*I)*B)/(3*(I + Tan[e + f*x])^3) - B/(2*(I + Tan[e + f*x])^2
)))/(c^5*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{a^{3}\left(-\frac{B}{2(i+\tan(fx+e))^{2}}-\frac{-4iA-8B}{4(i+\tan(fx+e))^{4}}-\frac{-5iB+A}{3(i+\tan(fx+e))^{3}}-\frac{4iB-4A}{5(i+\tan(fx+e))^{5}}\right)}{f\,c^{5}}$
default	$\frac{a^{3}\left(-\frac{B}{2(i+\tan(fx+e))^{2}}-\frac{-4iA-8B}{4(i+\tan(fx+e))^{4}}-\frac{-5iB+A}{3(i+\tan(fx+e))^{3}}-\frac{4iB-4A}{5(i+\tan(fx+e))^{5}}\right)}{f\,c^{5}}$
risch	$-\frac{a^{3}\mathrm{e}^{10i(fx+e)}B}{40c^{5}f}-\frac{i a^{3}\mathrm{e}^{10i(fx+e)}A}{40c^{5}f}-\frac{iA\,a^{3}\mathrm{e}^{8i(fx+e)}}{16c^{5}f}+\frac{a^{3}\mathrm{e}^{6i(fx+e)}B}{24c^{5}f}-\frac{i a^{3}\mathrm{e}^{6i(fx+e)}A}{24c^{5}f}$
norman	$\frac{\frac{a^{3}A\tan(fx+e)}{fc}+\frac{-4iA\,a^{3}+B\,a^{3}}{30cf}-\frac{\left(-8iB\,a^{3}+19A\,a^{3}\right)\tan(fx+e)^{3}}{3cf}+\frac{7\left(-16iB\,a^{3}+11A\,a^{3}\right)\tan(fx+e)^{5}}{15cf}-\frac{\left(-8iB\,a^{3}+A\,a^{3}\right)\tan(f}{3cf}}{\left(1+\tan(fx+\right.}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*a^3/c^5*(-1/2*B/(I+tan(f*x+e))^2-1/4*(-4*I*A-8*B)/(I+tan(f*x+e))^4-1/3
*(A-5*I*B)/(I+tan(f*x+e))^3-1/5*(-4*A+4*I*B)/(I+tan(f*x+e))^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int \frac{(a+ia\tan(e+fx))^{3}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5}}\,dx = -\frac{6\,(i\,A+B)a^{3}e^{(10i\,fx+10i\,e)}+15i\,Aa^{3}e^{(8i\,fx+8i\,e)}+10\,(i\,A-B)a^{3}e^{(6i\,fx+6i\,e)}}{240\,c^{5}f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, al
gorithm="fricas")
```

output
```
-1/240*(6*(I*A + B)*a^3*e^(10*I*f*x + 10*I*e) + 15*I*A*a^3*e^(8*I*f*x + 8*
I*e) + 10*(I*A - B)*a^3*e^(6*I*f*x + 6*I*e))/(c^5*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(100) = 200.

Time = 0.47 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.79

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx$$
$$= \begin{cases} \frac{-960iAa^3c^{10}f^2e^{8ie}e^{8ifx}+(-640iAa^3c^{10}f^2e^{6ie}+640Ba^3c^{10}f^2e^{6ie})e^{6ifx}+(-384iAa^3c^{10}f^2e^{10ie}-384Ba^3c^{10}f^2e^{10ie})e^{10ifx}}{15360c^{15}f^3} & \text{for } c^{15}f \\ \frac{x(Aa^3e^{10ie}+2Aa^3e^{8ie}+Aa^3e^{6ie}-iBa^3e^{10ie}+iBa^3e^{6ie})}{4c^5} & \text{otherwi} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**5,x)
```

output
```
Piecewise(((-960*I*A*a**3*c**10*f**2*exp(8*I*e)*exp(8*I*f*x) + (-640*I*A*a
**3*c**10*f**2*exp(6*I*e) + 640*B*a**3*c**10*f**2*exp(6*I*e))*exp(6*I*f*x)
 + (-384*I*A*a**3*c**10*f**2*exp(10*I*e) - 384*B*a**3*c**10*f**2*exp(10*I*
e))*exp(10*I*f*x))/(15360*c**15*f**3), Ne(c**15*f**3, 0)), (x*(A*a**3*exp(
10*I*e) + 2*A*a**3*exp(8*I*e) + A*a**3*exp(6*I*e) - I*B*a**3*exp(10*I*e) +
 I*B*a**3*exp(6*I*e))/(4*c**5), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.80

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = -\frac{15\,Ba^3\tan\left(fx+e\right)^3+10\,Aa^3\tan\left(fx+e\right)^2-5i\,Ba^3\tan\left(fx+e\right)^2-10i\,Aa^3\tan\left(fx+e\right)-5\,Ba^3\tan}{30\,c^5f\left(\tan\left(fx+e\right)+i\right)^5}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x, al
gorithm="giac")
```

output
```
-1/30*(15*B*a^3*tan(f*x + e)^3 + 10*A*a^3*tan(f*x + e)^2 - 5*I*B*a^3*tan(f
*x + e)^2 - 10*I*A*a^3*tan(f*x + e) - 5*B*a^3*tan(f*x + e) - 4*A*a^3 - I*B
*a^3)/(c^5*f*(tan(f*x + e) + I)^5)
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx = \frac{\frac{a^3\,\left(4\,A+B\,1{}\mathrm{i}\right)}{30}+\frac{a^3\,\mathrm{tan}\left(e+f\,x\right)\,\left(5\,B+A\,10{}\mathrm{i}\right)}{30}-\frac{B\,a^3\,{\mathrm{tan}\left(e+f\,x\right)}^3}{2}-\frac{a^3\,{\mathrm{tan}\left(e+f\,x\right)}^2\,\left(10\,A-B\,5{}\mathrm{i}\right)}{30}}{c^5\,f\,\left({\mathrm{tan}\left(e+f\,x\right)}^5+{\mathrm{tan}\left(e+f\,x\right)}^4\,5{}\mathrm{i}-10\,{\mathrm{tan}\left(e+f\,x\right)}^3-{\mathrm{tan}\left(e+f\,x\right)}^2\,10{}\mathrm{i}+5\,\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^5,x)
```

output
```
((a^3*(4*A + B*1i))/30 + (a^3*tan(e + f*x)*(A*10i + 5*B))/30 - (B*a^3*tan(
e + f*x)^3)/2 - (a^3*tan(e + f*x)^2*(10*A - B*5i))/30)/(c^5*f*(5*tan(e + f
*x) - tan(e + f*x)^2*10i - 10*tan(e + f*x)^3 + tan(e + f*x)^4*5i + tan(e +
 f*x)^5 + 1i))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{a^3\left(\left(\left(\int \frac{\tan(fx+e)^4}{\tan(fx+e)^5 i-5\tan(fx+e)^4-10\tan(fx+e)^3 i+10\tan(fx+e)^2+5\tan(fx+e) i-1}dx\right)bi+\left(\int \frac{}{\tan(fx+e)^5 i-5\tan(fx+e)^4-10\,\mathrm{ta}}\right.\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^5,x)
```

output
```
(a**3*(int(tan(e + f*x)**4/(tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 10*tan
(e + f*x)**3*i + 10*tan(e + f*x)**2 + 5*tan(e + f*x)*i - 1),x)*b*i + int(t
an(e + f*x)**3/(tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 10*tan(e + f*x)**3
*i + 10*tan(e + f*x)**2 + 5*tan(e + f*x)*i - 1),x)*a*i + 3*int(tan(e + f*x
)**3/(tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 10*tan(e + f*x)**3*i + 10*ta
n(e + f*x)**2 + 5*tan(e + f*x)*i - 1),x)*b + 3*int(tan(e + f*x)**2/(tan(e
+ f*x)**5*i - 5*tan(e + f*x)**4 - 10*tan(e + f*x)**3*i + 10*tan(e + f*x)**
2 + 5*tan(e + f*x)*i - 1),x)*a - 3*int(tan(e + f*x)**2/(tan(e + f*x)**5*i
- 5*tan(e + f*x)**4 - 10*tan(e + f*x)**3*i + 10*tan(e + f*x)**2 + 5*tan(e
+ f*x)*i - 1),x)*b*i - 3*int(tan(e + f*x)/(tan(e + f*x)**5*i - 5*tan(e + f
*x)**4 - 10*tan(e + f*x)**3*i + 10*tan(e + f*x)**2 + 5*tan(e + f*x)*i - 1)
,x)*a*i - int(tan(e + f*x)/(tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 10*tan
(e + f*x)**3*i + 10*tan(e + f*x)**2 + 5*tan(e + f*x)*i - 1),x)*b - int(1/(
tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 10*tan(e + f*x)**3*i + 10*tan(e +
f*x)**2 + 5*tan(e + f*x)*i - 1),x)*a))/c**5
```

3.702 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 127

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = \frac{2a^3(iA+B)}{3c^6f(i+\tan(e+fx))^6} - \frac{4a^3(A-2iB)}{5c^6f(i+\tan(e+fx))^5} - \frac{a^3(iA+5B)}{4c^6f(i+\tan(e+fx))^4} - \frac{ia^3B}{3c^6f(i+\tan(e+fx))^3}$$

output
```
2/3*a^3*(I*A+B)/c^6/f/(I+tan(f*x+e))^6-4/5*a^3*(A-2*I*B)/c^6/f/(I+tan(f*x+
e))^5-1/4*a^3*(I*A+5*B)/c^6/f/(I+tan(f*x+e))^4-1/3*I*a^3*B/c^6/f/(I+tan(f*
x+e))^3
```

Mathematica [A] (verified)

Time = 5.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = -\frac{ia^3(-7A-iB+(-18iA-6B)\tan(e+fx)+15(A-iB)\tan^2(e+fx)+20B\tan^3(e+fx))}{60c^6f(i+\tan(e+fx))^6}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^6,x]
```

output
```
((-1/60*I)*a^3*(-7*A - I*B + ((-18*I)*A - 6*B)*Tan[e + f*x] + 15*(A - I*B)
*Tan[e + f*x]^2 + 20*B*Tan[e + f*x]^3))/(c^6*f*(I + Tan[e + f*x])^6)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{c^7(1-i\tan(e+fx))^7}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(1-i\tan(e+fx))^7}d\tan(e+fx)}{c^6f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3\int \left(-\frac{4i(A-iB)}{(\tan(e+fx)+i)^7}+\frac{iB}{(\tan(e+fx)+i)^4}+\frac{iA+5B}{(\tan(e+fx)+i)^5}+\frac{4(A-2iB)}{(\tan(e+fx)+i)^6}\right)d\tan(e+fx)}{c^6f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(-\frac{4(A-2iB)}{5(\tan(e+fx)+i)^5}-\frac{5B+iA}{4(\tan(e+fx)+i)^4}+\frac{2(B+iA)}{3(\tan(e+fx)+i)^6}-\frac{iB}{3(\tan(e+fx)+i)^3}\right)}{c^6f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^6,x]
```

output
```
(a^3*((2*(I*A + B))/(3*(I + Tan[e + f*x])^6) - (4*(A - (2*I)*B))/(5*(I + T
an[e + f*x])^5) - (I*A + 5*B)/(4*(I + Tan[e + f*x])^4) - ((I/3)*B)/(I + Ta
n[e + f*x])^3))/(c^6*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{a^3\left(-\frac{iB}{3(i+\tan(fx+e))^3}-\frac{-8iB+4A}{5(i+\tan(fx+e))^5}-\frac{-4iA-4B}{6(i+\tan(fx+e))^6}-\frac{iA+5B}{4(i+\tan(fx+e))^4}\right)}{f\,c^6}$
default	$\frac{a^3\left(-\frac{iB}{3(i+\tan(fx+e))^3}-\frac{-8iB+4A}{5(i+\tan(fx+e))^5}-\frac{-4iA-4B}{6(i+\tan(fx+e))^6}-\frac{iA+5B}{4(i+\tan(fx+e))^4}\right)}{f\,c^6}$
risch	$-\frac{a^3 e^{12i(fx+e)}B}{96c^6f}-\frac{ia^3e^{12i(fx+e)}A}{96c^6f}-\frac{e^{10i(fx+e)}B\,a^3}{80c^6f}-\frac{3ie^{10i(fx+e)}A\,a^3}{80c^6f}+\frac{e^{8i(fx+e)}B\,a^3}{64c^6f}-\frac{3ie^{8i(fx+e)}A\,a}{64c^6f}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^6,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*a^3/c^6*(-1/3*I*B/(I+tan(f*x+e))^3-1/5*(-8*I*B+4*A)/(I+tan(f*x+e))^5-1
/6*(-4*B-4*I*A)/(I+tan(f*x+e))^6-1/4*(I*A+5*B)/(I+tan(f*x+e))^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = -\frac{10\,(i\,A+B)a^3e^{(12i\,fx+12i\,e)}+12\,(3i\,A+B)a^3e^{(10i\,fx+10i\,e)}+15\,(3i\,A-B)a^3e^{(8i\,fx+8i\,e)}+20\,(i\,A-B)\epsilon}{960\,c^6f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^6,x, al
gorithm="fricas")
```

output
```
-1/960*(10*(I*A + B)*a^3*e^(12*I*f*x + 12*I*e) + 12*(3*I*A + B)*a^3*e^(10*
I*f*x + 10*I*e) + 15*(3*I*A - B)*a^3*e^(8*I*f*x + 8*I*e) + 20*(I*A - B)*a^
3*e^(6*I*f*x + 6*I*e))/(c^6*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(105) = 210.

Time = 0.54 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.61

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx$$
$$= \begin{cases} \frac{(-491520iAa^3c^{18}f^3e^{6ie}+491520Ba^3c^{18}f^3e^{6ie})e^{6ifx}+(-1105920iAa^3c^{18}f^3e^{8ie}+368640Ba^3c^{18}f^3e^{8ie})e^{8ifx}+(-884736iAa^3c^{18}f^3e^{10ie}-}{23592960c^{24}f^4} \\ \frac{x(Aa^3e^{12ie}+3Aa^3e^{10ie}+3Aa^3e^{8ie}+Aa^3e^{6ie}-iBa^3e^{12ie}-iBa^3e^{10ie}+iBa^3e^{8ie}+iBa^3e^{6ie})}{8c^6} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**6,x)
```

output
```
Piecewise((((-491520*I*A*a**3*c**18*f**3*exp(6*I*e) + 491520*B*a**3*c**18*
f**3*exp(6*I*e))*exp(6*I*f*x) + (-1105920*I*A*a**3*c**18*f**3*exp(8*I*e) +
 368640*B*a**3*c**18*f**3*exp(8*I*e))*exp(8*I*f*x) + (-884736*I*A*a**3*c**
18*f**3*exp(10*I*e) - 294912*B*a**3*c**18*f**3*exp(10*I*e))*exp(10*I*f*x)
+ (-245760*I*A*a**3*c**18*f**3*exp(12*I*e) - 245760*B*a**3*c**18*f**3*exp(
12*I*e))*exp(12*I*f*x))/(23592960*c**24*f**4), Ne(c**24*f**4, 0)), (x*(A*a
**3*exp(12*I*e) + 3*A*a**3*exp(10*I*e) + 3*A*a**3*exp(8*I*e) + A*a**3*exp(
6*I*e) - I*B*a**3*exp(12*I*e) - I*B*a**3*exp(10*I*e) + I*B*a**3*exp(8*I*e)
 + I*B*a**3*exp(6*I*e))/(8*c**6), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^6,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = -\frac{20i\,Ba^3\tan(fx+e)^3+15i\,Aa^3\tan(fx+e)^2+15\,Ba^3\tan(fx+e)^2+18\,Aa^3\tan(fx+e)-6i\,Ba^3}{60\,c^6f(\tan(fx+e)+i)^6}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^6,x, al
gorithm="giac")
```

output
```
-1/60*(20*I*B*a^3*tan(f*x + e)^3 + 15*I*A*a^3*tan(f*x + e)^2 + 15*B*a^3*ta
n(f*x + e)^2 + 18*A*a^3*tan(f*x + e) - 6*I*B*a^3*tan(f*x + e) - 7*I*A*a^3
+ B*a^3)/(c^6*f*(tan(f*x + e) + I)^6)
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = -\frac{-\frac{a^3\,(-B+A\,7\mathrm{i})}{60}+\frac{a^3\tan(e+f\,x)\,(18\,A-B\,6\mathrm{i})}{60}+\frac{B\,a^3\tan(e+f\,x)^3\,1\mathrm{i}}{3}+\frac{a^3\tan(e+f\,x)^2\,(15\,B+A\,15\mathrm{i})}{60}}{c^6\,f\,\left(\tan(e+f\,x)^6+\tan(e+f\,x)^5\,6\mathrm{i}-15\tan(e+f\,x)^4-\tan(e+f\,x)^3\,20\mathrm{i}+15\tan(e+f\,x)^2+\mathrm{t}\right.}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^6,x)
```

output
```
-((a^3*tan(e + f*x)*(18*A - B*6i))/60 - (a^3*(A*7i - B))/60 + (B*a^3*tan(e
 + f*x)^3*1i)/3 + (a^3*tan(e + f*x)^2*(A*15i + 15*B))/60)/(c^6*f*(tan(e +
f*x)*6i + 15*tan(e + f*x)^2 - tan(e + f*x)^3*20i - 15*tan(e + f*x)^4 + tan
(e + f*x)^5*6i + tan(e + f*x)^6 - 1))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^6}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^6,x)
```

output
```
(a**3*( - int(tan(e + f*x)**4/(tan(e + f*x)**6*i - 6*tan(e + f*x)**5 - 15*
tan(e + f*x)**4*i + 20*tan(e + f*x)**3 + 15*tan(e + f*x)**2*i - 6*tan(e +
f*x) - i),x)*b - int(tan(e + f*x)**3/(tan(e + f*x)**6*i - 6*tan(e + f*x)**
5 - 15*tan(e + f*x)**4*i + 20*tan(e + f*x)**3 + 15*tan(e + f*x)**2*i - 6*t
an(e + f*x) - i),x)*a + 3*int(tan(e + f*x)**3/(tan(e + f*x)**6*i - 6*tan(e
 + f*x)**5 - 15*tan(e + f*x)**4*i + 20*tan(e + f*x)**3 + 15*tan(e + f*x)**
2*i - 6*tan(e + f*x) - i),x)*b*i + 3*int(tan(e + f*x)**2/(tan(e + f*x)**6*
i - 6*tan(e + f*x)**5 - 15*tan(e + f*x)**4*i + 20*tan(e + f*x)**3 + 15*tan
(e + f*x)**2*i - 6*tan(e + f*x) - i),x)*a*i + 3*int(tan(e + f*x)**2/(tan(e
 + f*x)**6*i - 6*tan(e + f*x)**5 - 15*tan(e + f*x)**4*i + 20*tan(e + f*x)*
*3 + 15*tan(e + f*x)**2*i - 6*tan(e + f*x) - i),x)*b + 3*int(tan(e + f*x)/
(tan(e + f*x)**6*i - 6*tan(e + f*x)**5 - 15*tan(e + f*x)**4*i + 20*tan(e +
 f*x)**3 + 15*tan(e + f*x)**2*i - 6*tan(e + f*x) - i),x)*a - int(tan(e + f
*x)/(tan(e + f*x)**6*i - 6*tan(e + f*x)**5 - 15*tan(e + f*x)**4*i + 20*tan
(e + f*x)**3 + 15*tan(e + f*x)**2*i - 6*tan(e + f*x) - i),x)*b*i - int(1/(
tan(e + f*x)**6*i - 6*tan(e + f*x)**5 - 15*tan(e + f*x)**4*i + 20*tan(e +
f*x)**3 + 15*tan(e + f*x)**2*i - 6*tan(e + f*x) - i),x)*a*i))/c**6
```

3.703 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7} dx$

Optimal result

Integrand size = 41, antiderivative size = 125

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7} dx = -\frac{4a^3(A-iB)}{7c^7f(i+\tan(e+fx))^7} - \frac{2a^3(iA+2B)}{3c^7f(i+\tan(e+fx))^6} + \frac{a^3(A-5iB)}{5c^7f(i+\tan(e+fx))^5} + \frac{a^3B}{4c^7f(i+\tan(e+fx))^4}$$

output
```
-4/7*a^3*(A-I*B)/c^7/f/(I+tan(f*x+e))^7-2/3*a^3*(I*A+2*B)/c^7/f/(I+tan(f*x
+e))^6+1/5*a^3*(A-5*I*B)/c^7/f/(I+tan(f*x+e))^5+1/4*a^3*B/c^7/f/(I+tan(f*x
+e))^4
```

Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.64

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7} dx = \frac{a^3(-44A-5iB+(-112iA-35B)\tan(e+fx)+21(4A-5iB)\tan^2(e+fx)+105B\tan^3(e+fx))}{420c^7f(i+\tan(e+fx))^7}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^7,x]
```

output
```
(a^3*(-44*A - (5*I)*B + ((-112*I)*A - 35*B)*Tan[e + f*x] + 21*(4*A - (5*I)
*B)*Tan[e + f*x]^2 + 105*B*Tan[e + f*x]^3))/(420*c^7*f*(I + Tan[e + f*x])^
7)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{c^8(1-i\tan(e+fx))^8}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(1-i\tan(e+fx))^8}d\tan(e+fx)}{c^7f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3\int \left(\frac{4(A-iB)}{(\tan(e+fx)+i)^8}-\frac{B}{(\tan(e+fx)+i)^5}+\frac{5iB-A}{(\tan(e+fx)+i)^6}+\frac{4(iA+2B)}{(\tan(e+fx)+i)^7}\right)d\tan(e+fx)}{c^7f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(-\frac{4(A-iB)}{7(\tan(e+fx)+i)^7}+\frac{A-5iB}{5(\tan(e+fx)+i)^5}-\frac{2(2B+iA)}{3(\tan(e+fx)+i)^6}+\frac{B}{4(\tan(e+fx)+i)^4}\right)}{c^7f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^7,x]
```

output
```
(a^3*((-4*(A - I*B))/(7*(I + Tan[e + f*x])^7) - (2*(I*A + 2*B))/(3*(I + Ta
n[e + f*x])^6) + (A - (5*I)*B)/(5*(I + Tan[e + f*x])^5) + B/(4*(I + Tan[e
+ f*x])^4)))/(c^7*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{a^{3}\left(\frac{B}{4(i+\tan(fx+e))^{4}}-\frac{5iB-A}{5(i+\tan(fx+e))^{5}}-\frac{4iA+8B}{6(i+\tan(fx+e))^{6}}-\frac{-4iB+4A}{7(i+\tan(fx+e))^{7}}\right)}{f\,c^{7}}$
default	$\frac{a^{3}\left(\frac{B}{4(i+\tan(fx+e))^{4}}-\frac{5iB-A}{5(i+\tan(fx+e))^{5}}-\frac{4iA+8B}{6(i+\tan(fx+e))^{6}}-\frac{-4iB+4A}{7(i+\tan(fx+e))^{7}}\right)}{f\,c^{7}}$
risch	$-\frac{a^{3}e^{14i(fx+e)}B}{224c^{7}f}-\frac{ia^{3}e^{14i(fx+e)}A}{224c^{7}f}-\frac{e^{12i(fx+e)}B\,a^{3}}{96c^{7}f}-\frac{ie^{12i(fx+e)}A\,a^{3}}{48c^{7}f}-\frac{3iA\,a^{3}e^{10i(fx+e)}}{80c^{7}f}+\frac{e^{8i(fx+e)}B\,a}{64c^{7}f}$
norman	$\frac{a^{3}A\tan(fx+e)}{fc}+\frac{-44iA\,a^{3}+5B\,a^{3}}{420cf}+\frac{B\,a^{3}\tan(fx+e)^{10}}{4cf}+\frac{2\left(-125iB\,a^{3}+139A\,a^{3}\right)\tan(fx+e)^{5}}{15cf}-\frac{6\left(-85iB\,a^{3}+36A\,a^{3}\right)\tan(fx+e}{35cf}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^7,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*a^3/c^7*(1/4*B/(I+tan(f*x+e))^4-1/5*(-A+5*I*B)/(I+tan(f*x+e))^5-1/6*(4
*I*A+8*B)/(I+tan(f*x+e))^6-1/7*(-4*I*B+4*A)/(I+tan(f*x+e))^7)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7}\,dx = -\frac{30\,(i\,A+B)a^3e^{(14i\,fx+14i\,e)}+70\,(2i\,A+B)a^3e^{(12i\,fx+12i\,e)}+252i\,Aa^3e^{(10i\,fx+10i\,e)}+105\,(2i\,A-B)a^3e}{6720\,c^7f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^7,x, al
gorithm="fricas")
```

output
```
-1/6720*(30*(I*A + B)*a^3*e^(14*I*f*x + 14*I*e) + 70*(2*I*A + B)*a^3*e^(12
*I*f*x + 12*I*e) + 252*I*A*a^3*e^(10*I*f*x + 10*I*e) + 105*(2*I*A - B)*a^3
*e^(8*I*f*x + 8*I*e) + 70*(I*A - B)*a^3*e^(6*I*f*x + 6*I*e))/(c^7*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(104) = 208$.

Time = 0.71 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.03

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7}\,dx$$
$$= \begin{cases} \frac{-396361728iAa^3c^{28}f^4e^{10ie}e^{10ifx}+(-110100480iAa^3c^{28}f^4e^{6ie}+110100480Ba^3c^{28}f^4e^{6ie})e^{6ifx}+(-330301440iAa^3c^{28}f^4e^{8ie}+165150720}{1056} \\ \frac{x(Aa^3e^{14ie}+4Aa^3e^{12ie}+6Aa^3e^{10ie}+4Aa^3e^{8ie}+Aa^3e^{6ie}-iBa^3e^{14ie}-2iBa^3e^{12ie}+2iBa^3e^{8ie}+iBa^3e^{6ie})}{16c^7} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**7,x)
```

output
```
Piecewise(((-396361728*I*A*a**3*c**28*f**4*exp(10*I*e)*exp(10*I*f*x) + (-1
10100480*I*A*a**3*c**28*f**4*exp(6*I*e) + 110100480*B*a**3*c**28*f**4*exp(
6*I*e))*exp(6*I*f*x) + (-330301440*I*A*a**3*c**28*f**4*exp(8*I*e) + 165150
720*B*a**3*c**28*f**4*exp(8*I*e))*exp(8*I*f*x) + (-220200960*I*A*a**3*c**2
8*f**4*exp(12*I*e) - 110100480*B*a**3*c**28*f**4*exp(12*I*e))*exp(12*I*f*x
) + (-47185920*I*A*a**3*c**28*f**4*exp(14*I*e) - 47185920*B*a**3*c**28*f**
4*exp(14*I*e))*exp(14*I*f*x))/(10569646080*c**35*f**5), Ne(c**35*f**5, 0))
, (x*(A*a**3*exp(14*I*e) + 4*A*a**3*exp(12*I*e) + 6*A*a**3*exp(10*I*e) + 4
*A*a**3*exp(8*I*e) + A*a**3*exp(6*I*e) - I*B*a**3*exp(14*I*e) - 2*I*B*a**3
*exp(12*I*e) + 2*I*B*a**3*exp(8*I*e) + I*B*a**3*exp(6*I*e))/(16*c**7), Tru
e))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^7,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7}\,dx$$
$$= \frac{105\,Ba^3\tan{(fx+e)}^3+84\,Aa^3\tan{(fx+e)}^2-105i\,Ba^3\tan{(fx+e)}^2-112i\,Aa^3\tan{(fx+e)}-35\,Ba}{420\,c^7f(\tan{(fx+e)}+i)^7}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^7,x, al
gorithm="giac")
```

output
```
1/420*(105*B*a^3*tan(f*x + e)^3 + 84*A*a^3*tan(f*x + e)^2 - 105*I*B*a^3*ta
n(f*x + e)^2 - 112*I*A*a^3*tan(f*x + e) - 35*B*a^3*tan(f*x + e) - 44*A*a^3
 - 5*I*B*a^3)/(c^7*f*(tan(f*x + e) + I)^7)
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.21

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7}\,dx$$
$$= \frac{\frac{a^3\,(44\,A+B\,5\mathrm{i})}{420}+\frac{a^3\tan(e+f\,x)\,(35\,B+A\,112\mathrm{i})}{420}-\frac{B\,a^3\tan(e+f\,x)^3}{4}-\frac{a^3\tan(e+f\,x)^2\,(84\,A}{420}}{c^7\,f\,\left(-\tan{(e+f\,x)}^7-\tan{(e+f\,x)}^6\,7\mathrm{i}+21\tan{(e+f\,x)}^5+\tan{(e+f\,x)}^4\,35\mathrm{i}-35\tan{(e+f\,x)}^3-\mathrm{t}\right.}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^7,x)
```

output
```
((a^3*(44*A + B*5i))/420 + (a^3*tan(e + f*x)*(A*112i + 35*B))/420 - (B*a^3
*tan(e + f*x)^3)/4 - (a^3*tan(e + f*x)^2*(84*A - B*105i))/420)/(c^7*f*(7*t
an(e + f*x) - tan(e + f*x)^2*21i - 35*tan(e + f*x)^3 + tan(e + f*x)^4*35i
+ 21*tan(e + f*x)^5 - tan(e + f*x)^6*7i - tan(e + f*x)^7 + 1i))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^7}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^7,x)
```

output
```
(a**3*( - int(tan(e + f*x)**4/(tan(e + f*x)**7 + 7*tan(e + f*x)**6*i - 21*
tan(e + f*x)**5 - 35*tan(e + f*x)**4*i + 35*tan(e + f*x)**3 + 21*tan(e + f
*x)**2*i - 7*tan(e + f*x) - i),x)*b - int(tan(e + f*x)**3/(tan(e + f*x)**7
 + 7*tan(e + f*x)**6*i - 21*tan(e + f*x)**5 - 35*tan(e + f*x)**4*i + 35*ta
n(e + f*x)**3 + 21*tan(e + f*x)**2*i - 7*tan(e + f*x) - i),x)*a + 3*int(ta
n(e + f*x)**3/(tan(e + f*x)**7 + 7*tan(e + f*x)**6*i - 21*tan(e + f*x)**5
- 35*tan(e + f*x)**4*i + 35*tan(e + f*x)**3 + 21*tan(e + f*x)**2*i - 7*tan
(e + f*x) - i),x)*b*i + 3*int(tan(e + f*x)**2/(tan(e + f*x)**7 + 7*tan(e +
 f*x)**6*i - 21*tan(e + f*x)**5 - 35*tan(e + f*x)**4*i + 35*tan(e + f*x)**
3 + 21*tan(e + f*x)**2*i - 7*tan(e + f*x) - i),x)*a*i + 3*int(tan(e + f*x)
**2/(tan(e + f*x)**7 + 7*tan(e + f*x)**6*i - 21*tan(e + f*x)**5 - 35*tan(e
 + f*x)**4*i + 35*tan(e + f*x)**3 + 21*tan(e + f*x)**2*i - 7*tan(e + f*x)
- i),x)*b + 3*int(tan(e + f*x)/(tan(e + f*x)**7 + 7*tan(e + f*x)**6*i - 21
*tan(e + f*x)**5 - 35*tan(e + f*x)**4*i + 35*tan(e + f*x)**3 + 21*tan(e +
f*x)**2*i - 7*tan(e + f*x) - i),x)*a - int(tan(e + f*x)/(tan(e + f*x)**7 +
 7*tan(e + f*x)**6*i - 21*tan(e + f*x)**5 - 35*tan(e + f*x)**4*i + 35*tan(
e + f*x)**3 + 21*tan(e + f*x)**2*i - 7*tan(e + f*x) - i),x)*b*i - int(1/(t
an(e + f*x)**7 + 7*tan(e + f*x)**6*i - 21*tan(e + f*x)**5 - 35*tan(e + f*x
)**4*i + 35*tan(e + f*x)**3 + 21*tan(e + f*x)**2*i - 7*tan(e + f*x) - i),x
)*a*i))/c**7
```

3.704 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^8}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 127

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^8}\,dx = -\frac{a^3(iA+B)}{2c^8f(i+\tan(e+fx))^8}+\frac{4a^3(A-2iB)}{7c^8f(i+\tan(e+fx))^7}+\frac{a^3(iA+5B)}{6c^8f(i+\tan(e+fx))^6}+\frac{ia^3B}{5c^8f(i+\tan(e+fx))^5}$$

output
```
-1/2*a^3*(I*A+B)/c^8/f/(I+tan(f*x+e))^8+4/7*a^3*(A-2*I*B)/c^8/f/(I+tan(f*x
+e))^7+1/6*a^3*(I*A+5*B)/c^8/f/(I+tan(f*x+e))^6+1/5*I*a^3*B/c^8/f/(I+tan(f
*x+e))^5
```

Mathematica [A] (verified)

Time = 4.72 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.66

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^8}\,dx = \frac{a^3(2(-10iA+B)+2(25A-8iB)\tan(e+fx)+7(5iA+7B)\tan^2(e+fx)+42iB\tan^3(e+fx))}{210c^8f(i+\tan(e+fx))^8}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^8,x]
```

output
```
(a^3*(2*((-10*I)*A + B) + 2*(25*A - (8*I)*B)*Tan[e + f*x] + 7*((5*I)*A + 7
*B)*Tan[e + f*x]^2 + (42*I)*B*Tan[e + f*x]^3))/(210*c^8*f*(I + Tan[e + f*x
])^8)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^8}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{c^9(1-i\tan(e+fx))^9}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(1-i\tan(e+fx))^9}d\tan(e+fx)}{c^8f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3\int \left(-\frac{4(A-2iB)}{(\tan(e+fx)+i)^8}-\frac{iB}{(\tan(e+fx)+i)^6}-\frac{i(A-5iB)}{(\tan(e+fx)+i)^7}+\frac{4(iA+B)}{(\tan(e+fx)+i)^9}\right)d\tan(e+fx)}{c^8f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(\frac{4(A-2iB)}{7(\tan(e+fx)+i)^7}+\frac{5B+iA}{6(\tan(e+fx)+i)^6}-\frac{B+iA}{2(\tan(e+fx)+i)^8}+\frac{iB}{5(\tan(e+fx)+i)^5}\right)}{c^8f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^8,x]
```

output
```
(a^3*(-1/2*(I*A + B)/(I + Tan[e + f*x])^8 + (4*(A - (2*I)*B))/(7*(I + Tan[
e + f*x])^7) + (I*A + 5*B)/(6*(I + Tan[e + f*x])^6) + ((I/5)*B)/(I + Tan[e
 + f*x])^5))/(c^8*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{a^{3}\left(\frac{iB}{5(i+\tan(fx+e))^{5}}-\frac{-iA-5B}{6(i+\tan(fx+e))^{6}}-\frac{4iA+4B}{8(i+\tan(fx+e))^{8}}-\frac{8iB-4A}{7(i+\tan(fx+e))^{7}}\right)}{f\,c^{8}}$
default	$\frac{a^{3}\left(\frac{iB}{5(i+\tan(fx+e))^{5}}-\frac{-iA-5B}{6(i+\tan(fx+e))^{6}}-\frac{4iA+4B}{8(i+\tan(fx+e))^{8}}-\frac{8iB-4A}{7(i+\tan(fx+e))^{7}}\right)}{f\,c^{8}}$
risch	$-\frac{a^{3}\mathrm{e}^{16i(fx+e)}B}{512c^{8}f}-\frac{ia^{3}\mathrm{e}^{16i(fx+e)}A}{512c^{8}f}-\frac{3\,\mathrm{e}^{14i(fx+e)}B\,a^{3}}{448c^{8}f}-\frac{5i\mathrm{e}^{14i(fx+e)}A\,a^{3}}{448c^{8}f}-\frac{\mathrm{e}^{12i(fx+e)}B\,a^{3}}{192c^{8}f}-\frac{5i\mathrm{e}^{12i(fx+e)}}{192c^{8}f}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^8,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*a^3/c^8*(1/5*I*B/(I+tan(f*x+e))^5-1/6*(-I*A-5*B)/(I+tan(f*x+e))^6-1/8*
(4*I*A+4*B)/(I+tan(f*x+e))^8-1/7*(8*I*B-4*A)/(I+tan(f*x+e))^7)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^8}\,dx = -\frac{105\,(i\,A+B)a^{3}e^{(16i\,fx+16i\,e)}+120\,(5i\,A+3\,B)a^{3}e^{(14i\,fx+14i\,e)}+280\,(5i\,A+B)a^{3}e^{(12i\,fx+12i\,e)}+336\,(5}{53760\,c^{8}f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^8,x, al
gorithm="fricas")
```

output
```
-1/53760*(105*(I*A + B)*a^3*e^(16*I*f*x + 16*I*e) + 120*(5*I*A + 3*B)*a^3*
e^(14*I*f*x + 14*I*e) + 280*(5*I*A + B)*a^3*e^(12*I*f*x + 12*I*e) + 336*(5
*I*A - B)*a^3*e^(10*I*f*x + 10*I*e) + 210*(5*I*A - 3*B)*a^3*e^(8*I*f*x + 8
*I*e) + 280*(I*A - B)*a^3*e^(6*I*f*x + 6*I*e))/(c^8*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(104) = 208$.

Time = 0.72 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.91

$$\int \frac{(a + ia\tan(e + fx))^3 (A + B\tan(e + fx))}{(c - ic\tan(e + fx))^8}\,dx$$
$$= \begin{cases} \frac{(-1803886264320iAa^3c^{40}f^5e^{6ie}+1803886264320Ba^3c^{40}f^5e^{6ie})e^{6ifx}+(-6764573491200iAa^3c^{40}f^5e^{8ie}+4058744094720Ba^3c^{40}f^5e^{8ie})e^{8i\cdots}}{\cdots} \\ \frac{x(Aa^3e^{16ie}+5Aa^3e^{14ie}+10Aa^3e^{12ie}+10Aa^3e^{10ie}+5Aa^3e^{8ie}+Aa^3e^{6ie}-iBa^3e^{16ie}-3iBa^3e^{14ie}-2iBa^3e^{12ie}+2iBa^3e^{10ie}+3iBa^3e^{8ie}+iBa\cdots}{32c^8} \end{cases}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**8,x)
```

output
```
Piecewise((((-1803886264320*I*A*a**3*c**40*f**5*exp(6*I*e) + 1803886264320
*B*a**3*c**40*f**5*exp(6*I*e))*exp(6*I*f*x) + (-6764573491200*I*A*a**3*c**
40*f**5*exp(8*I*e) + 4058744094720*B*a**3*c**40*f**5*exp(8*I*e))*exp(8*I*f
*x) + (-10823317585920*I*A*a**3*c**40*f**5*exp(10*I*e) + 2164663517184*B*a
**3*c**40*f**5*exp(10*I*e))*exp(10*I*f*x) + (-9019431321600*I*A*a**3*c**40
*f**5*exp(12*I*e) - 1803886264320*B*a**3*c**40*f**5*exp(12*I*e))*exp(12*I*
f*x) + (-3865470566400*I*A*a**3*c**40*f**5*exp(14*I*e) - 2319282339840*B*a
**3*c**40*f**5*exp(14*I*e))*exp(14*I*f*x) + (-676457349120*I*A*a**3*c**40*
f**5*exp(16*I*e) - 676457349120*B*a**3*c**40*f**5*exp(16*I*e))*exp(16*I*f*
x))/(346346162749440*c**48*f**6), Ne(c**48*f**6, 0)), (x*(A*a**3*exp(16*I*
e) + 5*A*a**3*exp(14*I*e) + 10*A*a**3*exp(12*I*e) + 10*A*a**3*exp(10*I*e)
+ 5*A*a**3*exp(8*I*e) + A*a**3*exp(6*I*e) - I*B*a**3*exp(16*I*e) - 3*I*B*a
**3*exp(14*I*e) - 2*I*B*a**3*exp(12*I*e) + 2*I*B*a**3*exp(10*I*e) + 3*I*B*
a**3*exp(8*I*e) + I*B*a**3*exp(6*I*e))/(32*c**8), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + ia\tan(e + fx))^3 (A + B\tan(e + fx))}{(c - ic\tan(e + fx))^8}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^8,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^8}\,dx = -\frac{-42i\,Ba^3\tan\left(fx+e\right)^3 - 35i\,Aa^3\tan\left(fx+e\right)^2 - 49\,Ba^3\tan\left(fx+e\right)^2 - 50\,Aa^3\tan\left(fx+e\right) + 16i\,B}{210\,c^8f(\tan\left(fx+e\right)+i)^8}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^8,x, al
gorithm="giac")
```

output
```
-1/210*(-42*I*B*a^3*tan(f*x + e)^3 - 35*I*A*a^3*tan(f*x + e)^2 - 49*B*a^3*
tan(f*x + e)^2 - 50*A*a^3*tan(f*x + e) + 16*I*B*a^3*tan(f*x + e) + 20*I*A*
a^3 - 2*B*a^3)/(c^8*f*(tan(f*x + e) + I)^8)
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.26

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^8}\,dx = \frac{-\frac{a^3\,(-2\,B+A\,20\mathrm{i})}{210}+\frac{a^3\tan(e+f\,x)\,(50\,A-B\,16\mathrm{i})}{210}+\frac{B\,a^3\tan(e+f\,x)^3\,1\mathrm{i}}{5}+\frac{a^3\tan}{}}{c^8\,f\,\left(\tan\left(e+f\,x\right)^8+\tan\left(e+f\,x\right)^7\,8\mathrm{i}-28\tan\left(e+f\,x\right)^6-\tan\left(e+f\,x\right)^5\,56\mathrm{i}+70\tan\left(e+f\,x\right)^4+\tan\right.}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^8,x)
```

output
```
((a^3*tan(e + f*x)*(50*A - B*16i))/210 - (a^3*(A*20i - 2*B))/210 + (B*a^3*
tan(e + f*x)^3*1i)/5 + (a^3*tan(e + f*x)^2*(A*35i + 49*B))/210)/(c^8*f*(ta
n(e + f*x)^3*56i - 28*tan(e + f*x)^2 - tan(e + f*x)*8i + 70*tan(e + f*x)^4
 - tan(e + f*x)^5*56i - 28*tan(e + f*x)^6 + tan(e + f*x)^7*8i + tan(e + f*
x)^8 + 1))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^8}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^8,x)
```

output
```
(a**3*(int(tan(e + f*x)**4/(tan(e + f*x)**8*i - 8*tan(e + f*x)**7 - 28*tan
(e + f*x)**6*i + 56*tan(e + f*x)**5 + 70*tan(e + f*x)**4*i - 56*tan(e + f*
x)**3 - 28*tan(e + f*x)**2*i + 8*tan(e + f*x) + i),x)*b + int(tan(e + f*x)
**3/(tan(e + f*x)**8*i - 8*tan(e + f*x)**7 - 28*tan(e + f*x)**6*i + 56*tan
(e + f*x)**5 + 70*tan(e + f*x)**4*i - 56*tan(e + f*x)**3 - 28*tan(e + f*x)
**2*i + 8*tan(e + f*x) + i),x)*a - 3*int(tan(e + f*x)**3/(tan(e + f*x)**8*
i - 8*tan(e + f*x)**7 - 28*tan(e + f*x)**6*i + 56*tan(e + f*x)**5 + 70*tan
(e + f*x)**4*i - 56*tan(e + f*x)**3 - 28*tan(e + f*x)**2*i + 8*tan(e + f*x
) + i),x)*b*i - 3*int(tan(e + f*x)**2/(tan(e + f*x)**8*i - 8*tan(e + f*x)*
*7 - 28*tan(e + f*x)**6*i + 56*tan(e + f*x)**5 + 70*tan(e + f*x)**4*i - 56
*tan(e + f*x)**3 - 28*tan(e + f*x)**2*i + 8*tan(e + f*x) + i),x)*a*i - 3*i
nt(tan(e + f*x)**2/(tan(e + f*x)**8*i - 8*tan(e + f*x)**7 - 28*tan(e + f*x
)**6*i + 56*tan(e + f*x)**5 + 70*tan(e + f*x)**4*i - 56*tan(e + f*x)**3 -
28*tan(e + f*x)**2*i + 8*tan(e + f*x) + i),x)*b - 3*int(tan(e + f*x)/(tan(
e + f*x)**8*i - 8*tan(e + f*x)**7 - 28*tan(e + f*x)**6*i + 56*tan(e + f*x)
**5 + 70*tan(e + f*x)**4*i - 56*tan(e + f*x)**3 - 28*tan(e + f*x)**2*i + 8
*tan(e + f*x) + i),x)*a + int(tan(e + f*x)/(tan(e + f*x)**8*i - 8*tan(e +
f*x)**7 - 28*tan(e + f*x)**6*i + 56*tan(e + f*x)**5 + 70*tan(e + f*x)**4*i
 - 56*tan(e + f*x)**3 - 28*tan(e + f*x)**2*i + 8*tan(e + f*x) + i),x)*b*i
+ int(1/(tan(e + f*x)**8*i - 8*tan(e + f*x)**7 - 28*tan(e + f*x)**6*i +...
```

3.705 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)} dx$

Optimal result

Integrand size = 41, antiderivative size = 115

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx = \frac{(iA(1-n)+B(1+n))\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(c-ic\tan(e+fx))^n}{4afn} + \frac{(iA-B)(c-ic\tan(e+fx))^n}{2af(1+i\tan(e+fx))}$$

output
```
1/4*(I*A*(1-n)+B*(1+n))*hypergeom([1, n],[1+n],1/2-1/2*I*tan(f*x+e))*(c-I*
c*tan(f*x+e))^n/a/f/n+1/2*(I*A-B)*(c-I*c*tan(f*x+e))^n/a/f/(1+I*tan(f*x+e)
)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx = \frac{(c-ic\tan(e+fx))^n\left(2(A+iB)n+(-iA(-1+n)+B(1+n))\,\text{Hypergeometric2F1}\left(1,n,1+n,-\frac{1}{2}i(\right.\right.}{4afn(-i+\tan(e+fx))}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n)/(a + I*a*Tan[e +
 f*x]),x]
```

output
```
((c - I*c*Tan[e + f*x])^n*(2*(A + I*B)*n + ((-I)*A*(-1 + n) + B*(1 + n))*H
ypergeometric2F1[1, n, 1 + n, (-1/2*I)*(I + Tan[e + f*x])]*(-I + Tan[e + f
*x])))/(4*a*f*n*(-I + Tan[e + f*x]))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}}{a^2(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{87}$$

$$\frac{c\left(\frac{1}{2}(A(1-n)-iB(n+1))\int \frac{(c-ic\tan(e+fx))^{n-1}}{i\tan(e+fx)+1}d\tan(e+fx)+\frac{(-B+iA)(c-ic\tan(e+fx))^n}{2c(1+i\tan(e+fx))}\right)}{af}$$

$$\Big\downarrow \text{78}$$

$$\frac{c\left(\frac{i(A(1-n)-iB(n+1))(c-ic\tan(e+fx))^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{4cn}+\frac{(-B+iA)(c-ic\tan(e+fx))^n}{2c(1+i\tan(e+fx))}\right)}{af}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n)/(a + I*a*Tan[e + f*x])
,x]
```

output
```
(c*(((I/4)*(A*(1 - n) - I*B*(1 + n))*Hypergeometric2F1[1, n, 1 + n, (1 - I
*Tan[e + f*x])/2]*(c - I*c*Tan[e + f*x])^n)/(c*n) + ((I*A - B)*(c - I*c*Ta
n[e + f*x])^n)/(2*c*(1 + I*Tan[e + f*x]))))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(A + B\tan(fx + e))(c - ic\tan(fx + e))^n}{a + ia\tan(fx + e)}dx$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x)
```

output
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(A + B\tan(e + fx))(c - ic\tan(e + fx))^n}{a + ia\tan(e + fx)} dx = \int \frac{(B\tan(fx + e) + A)(-i c\tan(fx + e) + c)^n}{i a\tan(fx + e) + a} dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algo
rithm="fricas")
```

output
```
integral(1/2*((A - I*B)*e^(2*I*f*x + 2*I*e) + A + I*B)*(2*c/(e^(2*I*f*x +
2*I*e) + 1))^n*e^(-2*I*f*x - 2*I*e)/a, x)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx = -\frac{i\left(\int \frac{A(-ic\tan(e+fx)+c)^n}{\tan(e+fx)-i}\,dx + \int \frac{B(-ic\tan(e+fx)+c)^n\tan(e+fx)}{\tan(e+fx)-i}\,dx\right)}{a}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**n/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*(Integral(A*(-I*c*tan(e + f*x) + c)**n/(tan(e + f*x) - I), x) + Integra
l(B*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)/(tan(e + f*x) - I), x))/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx = \int \frac{(B\tan(fx+e)+A)(-i\,c\tan(fx+e)+c)^n}{i\,a\tan(fx+e)+a}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x, algo
rithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^n/(I*a*tan(f*x + e)
 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx = \int \frac{(A+B\tan(e+fx))\,(c-c\tan(e+fx)\,1\mathrm{i})^n}{a+a\tan\left(e+fx\right)\,1\mathrm{i}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^n)/(a + a*tan(e + f*x)*1
i),x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^n)/(a + a*tan(e + f*x)*1
i), x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx = \frac{(-\tan(fx+e)ci+c)^n ai + \left(\int \frac{(-\tan(fx+e)ci+c)^n \tan(fx+e)^2}{\tan(fx+e)i+1}dx\right) afn - 2\left(\int \frac{(-\tan(fx+e)ci+c)^n \tan(fx+e)}{\tan(fx+e)i+1}dx\right) a}{afn}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e)),x)
```

output
```
(( - tan(e + f*x)*c*i + c)**n*a*i + int((( - tan(e + f*x)*c*i + c)**n*tan(
e + f*x)**2)/(tan(e + f*x)*i + 1),x)*a*f*n - 2*int((( - tan(e + f*x)*c*i +
 c)**n*tan(e + f*x))/(tan(e + f*x)*i + 1),x)*a*f*i*n + int((( - tan(e + f*
x)*c*i + c)**n*tan(e + f*x))/(tan(e + f*x)*i + 1),x)*b*f*n)/(a*f*n)
```

3.706 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 157

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx = -\frac{4(3A+5iB)c^4x}{a}-\frac{4(3iA-5B)c^4\log(\cos(e+fx))}{af}-\frac{8(A+iB)c^4}{af(i-\tan(e+fx))}+\frac{(5A+12iB)c^4\tan(e+fx)}{af}-\frac{(iA-5B)c^4\tan^2(e+fx)}{2af}-\frac{iBc^4\tan^3(e+fx)}{3af}$$

output

```
-4*(3*A+5*I*B)*c^4*x/a-4*(3*I*A-5*B)*c^4*ln(cos(f*x+e))/a/f-8*(A+I*B)*c^4/
a/f/(I-tan(f*x+e))+(5*A+12*I*B)*c^4*tan(f*x+e)/a/f-1/2*(I*A-5*B)*c^4*tan(f
*x+e)^2/a/f-1/3*I*B*c^4*tan(f*x+e)^3/a/f
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx = \frac{c^4(123A+203iB+24(3A+5iB)\log(i-\tan(e+fx))+(45iA-83B+24i(3A+5iB)\log(i-\tan(e+fx)}{6af(-i+\tan}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4)/(a + I*a*Tan[e +
 f*x]),x]
```

output
```
(c^4*(123*A + (203*I)*B + 24*(3*A + (5*I)*B)*Log[I - Tan[e + f*x]] + ((45*
I)*A - 83*B + (24*I)*(3*A + (5*I)*B)*Log[I - Tan[e + f*x]])*Tan[e + f*x] +
 3*(9*A + (19*I)*B)*Tan[e + f*x]^2 + ((-3*I)*A + 13*B)*Tan[e + f*x]^3 - (2
*I)*B*Tan[e + f*x]^4))/(6*a*f*(-I + Tan[e + f*x]))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^4(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^4(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{c^3(1-i\tan(e+fx))^3(A+B\tan(e+fx))}{a^2(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^4\int \frac{(1-i\tan(e+fx))^3(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^2}d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{86}$$

$$\frac{c^4\int \left(-iB\tan^2(e+fx)+(5B-iA)\tan(e+fx)+5A\left(\frac{12iB}{5A}+1\right)+\frac{4i(3A+5iB)}{\tan(e+fx)-i}-\frac{8(A+iB)}{(\tan(e+fx)-i)^2}\right)d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^4\left(-\frac{1}{2}(-5B+iA)\tan^2(e+fx)+(5A+12iB)\tan(e+fx)-\frac{8(A+iB)}{-\tan(e+fx)+i}+4(-5B+3iA)\log(-\tan(e+fx)\right.}{af}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4)/(a + I*a*Tan[e + f*x])
,x]
```

output
```
(c^4*(4*((3*I)*A - 5*B)*Log[I - Tan[e + f*x]] - (8*(A + I*B))/(I - Tan[e +
 f*x]) + (5*A + (12*I)*B)*Tan[e + f*x] - ((I*A - 5*B)*Tan[e + f*x]^2)/2 -
(I/3)*B*Tan[e + f*x]^3))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{4c^4 e^{-2i(fx+e)} B}{af}+\frac{4ic^4 e^{-2i(fx+e)} A}{af}-\frac{40ic^4 Bx}{a}-\frac{24c^4 Ax}{a}-\frac{40ic^4 Be}{fa}-\frac{24c^4 Ae}{fa}-\frac{2c^4(-12iA\, e^{4i(fx+e)}}{}$
norman	$\frac{\frac{(20ic^4B+13A\,c^4)\tan(fx+e)}{af}-\frac{4(5ic^4B+3A\,c^4)x}{a}-\frac{-17ic^4A+21B\,c^4}{2af}+\frac{5(7ic^4B+3A\,c^4)\tan(fx+e)^3}{3af}-\frac{4(5ic^4B+3A\,c^4)x\tan(f}{a}}{1+\tan(fx+e)^2}$
derivativedivides	$\frac{5c^4B\tan(fx+e)^2}{2fa}-\frac{iB\,c^4\tan(fx+e)^3}{3af}+\frac{5c^4A\tan(fx+e)}{fa}-\frac{ic^4A\tan(fx+e)^2}{2fa}+\frac{12ic^4\tan(fx+e)B}{fa}-12c^4A$
default	$\frac{5c^4B\tan(fx+e)^2}{2fa}-\frac{iB\,c^4\tan(fx+e)^3}{3af}+\frac{5c^4A\tan(fx+e)}{fa}-\frac{ic^4A\tan(fx+e)^2}{2fa}+\frac{12ic^4\tan(fx+e)B}{fa}-12c^4A$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-4*c^4/a/f*exp(-2*I*(f*x+e))*B+4*I*c^4/a/f*exp(-2*I*(f*x+e))*A-40*I*c^4/a*
B*x-24*c^4/a*A*x-40*I*c^4/f/a*B*e-24*c^4/f/a*A*e-2/3*c^4*(-12*I*A*exp(4*I*
(f*x+e))+24*B*exp(4*I*(f*x+e))-27*I*A*exp(2*I*(f*x+e))+57*B*exp(2*I*(f*x+e
))-15*I*A+37*B)/f/a/(exp(2*I*(f*x+e))+1)^3+20*c^4/f/a*ln(exp(2*I*(f*x+e))+
1)*B-12*I*c^4/f/a*ln(exp(2*I*(f*x+e))+1)*A
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(137) = 274.

Time = 0.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.90

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx = -\frac{2\left(12\,(3\,A+5i\,B)c^4fxe^{(8i\,fx+8i\,e)}+6\,(-i\,A+B)c^4+6\,(6\,(3\,A+5i\,B)c^4fx+(-3i\,A+5\,B)c^4)e^{(6i\,fx+}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e)),x, algo
rithm="fricas")
```

output
```
-2/3*(12*(3*A + 5*I*B)*c^4*f*x*e^(8*I*f*x + 8*I*e) + 6*(-I*A + B)*c^4 + 6*
(6*(3*A + 5*I*B)*c^4*f*x + (-3*I*A + 5*B)*c^4)*e^(6*I*f*x + 6*I*e) + 3*(12
*(3*A + 5*I*B)*c^4*f*x + 5*(-3*I*A + 5*B)*c^4)*e^(4*I*f*x + 4*I*e) + (12*(
3*A + 5*I*B)*c^4*f*x + 11*(-3*I*A + 5*B)*c^4)*e^(2*I*f*x + 2*I*e) + 6*((3*
I*A - 5*B)*c^4*e^(8*I*f*x + 8*I*e) + 3*(3*I*A - 5*B)*c^4*e^(6*I*f*x + 6*I*
e) + 3*(3*I*A - 5*B)*c^4*e^(4*I*f*x + 4*I*e) + (3*I*A - 5*B)*c^4*e^(2*I*f*
x + 2*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/(a*f*e^(8*I*f*x + 8*I*e) + 3*a*f
*e^(6*I*f*x + 6*I*e) + 3*a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*e)
)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.09

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx = \frac{30iAc^4-74Bc^4+(54iAc^4e^{2ie}-114Bc^4e^{2ie})\,e^{2ifx}+(24iAc^4e^{4ie}-48Bc^4e^{4ie})\,e^{4ifx}}{3afe^{6ie}e^{6ifx}+9afe^{4ie}e^{4ifx}+9afe^{2ie}e^{2ifx}+3af} + \begin{cases} \frac{(4iAc^4-4Bc^4)e^{-2ie}e^{-2ifx}}{af} & \text{for } afe^{2ie}\neq 0 \\ x\left(-\frac{-24Ac^4-40iBc^4}{a}+\frac{(-24Ac^4e^{2ie}+8Ac^4-40iBc^4e^{2ie}+8iBc^4)e^{-2ie}}{a}\right) & \text{otherwise} \end{cases} - \frac{4ic^4\cdot(3A+5iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{af} + \frac{x(-24Ac^4-40iBc^4)}{a}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**4/(a+I*a*tan(f*x+e)),x)
```

output
```
(30*I*A*c**4 - 74*B*c**4 + (54*I*A*c**4*exp(2*I*e) - 114*B*c**4*exp(2*I*e)
)*exp(2*I*f*x) + (24*I*A*c**4*exp(4*I*e) - 48*B*c**4*exp(4*I*e))*exp(4*I*f
*x))/(3*a*f*exp(6*I*e)*exp(6*I*f*x) + 9*a*f*exp(4*I*e)*exp(4*I*f*x) + 9*a*
f*exp(2*I*e)*exp(2*I*f*x) + 3*a*f) + Piecewise(((4*I*A*c**4 - 4*B*c**4)*ex
p(-2*I*e)*exp(-2*I*f*x)/(a*f), Ne(a*f*exp(2*I*e), 0)), (x*(-(-24*A*c**4 -
40*I*B*c**4)/a + (-24*A*c**4*exp(2*I*e) + 8*A*c**4 - 40*I*B*c**4*exp(2*I*e
) + 8*I*B*c**4)*exp(-2*I*e)/a), True)) - 4*I*c**4*(3*A + 5*I*B)*log(exp(2*
I*f*x) + exp(-2*I*e))/(a*f) + x*(-24*A*c**4 - 40*I*B*c**4)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e)),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx$$
$$= -\frac{4\left(-3i\,Ac^4+5\,Bc^4\right)\log\left(\tan\left(fx+e\right)-i\right)}{af}+\frac{8\left(Ac^4+i\,Bc^4\right)}{af(\tan\left(fx+e\right)-i)}$$
$$-\frac{2i\,Ba^2c^4f^2\tan\left(fx+e\right)^3+3i\,Aa^2c^4f^2\tan\left(fx+e\right)^2-15\,Ba^2c^4f^2\tan\left(fx+e\right)^2-30\,Aa^2c^4f^2\tan\left(fx\right.}{6\,a^3f^3}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e)),x, algo
rithm="giac")
```

output
```
-4*(-3*I*A*c^4 + 5*B*c^4)*log(tan(f*x + e) - I)/(a*f) + 8*(A*c^4 + I*B*c^4
)/(a*f*(tan(f*x + e) - I)) - 1/6*(2*I*B*a^2*c^4*f^2*tan(f*x + e)^3 + 3*I*A
*a^2*c^4*f^2*tan(f*x + e)^2 - 15*B*a^2*c^4*f^2*tan(f*x + e)^2 - 30*A*a^2*c
^4*f^2*tan(f*x + e) - 72*I*B*a^2*c^4*f^2*tan(f*x + e))/(a^3*f^3)
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.31

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx$$
$$= \frac{\ln(\tan(e+fx)-\mathrm{i})\left(-\frac{20\,B\,c^4}{a}+\frac{A\,c^4\,12\mathrm{i}}{a}\right)}{f} - \frac{\tan(e+fx)^2\left(-\frac{B\,c^4}{a}+\frac{c^4\,(A+B\,3\mathrm{i})\,1\mathrm{i}}{2\,a}\right)}{f} - \frac{\frac{(4\,A\,c^4+B\,c^4\,12\mathrm{i})\,1\mathrm{i}}{a}-\frac{(12\,A\,c^4+B\,c^4\,20\mathrm{i})\,1\mathrm{i}}{a}}{f\,(1+\tan(e+fx)\,1\mathrm{i})} + \frac{\tan(e+fx)\left(\frac{2\,c^4\,(A+B\,3\mathrm{i})}{a}+\frac{B\,c^4\,3\mathrm{i}}{a}-\frac{c^4\,(-B+A\,1\mathrm{i})\,3\mathrm{i}}{a}\right)}{f} - \frac{B\,c^4\tan(e+fx)^3\,1\mathrm{i}}{3\,a\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^4)/(a + a*tan(e + f*x)*1
i),x)
```

output
```
(log(tan(e + f*x) - 1i)*((A*c^4*12i)/a - (20*B*c^4)/a))/f - (tan(e + f*x)^
2*((c^4*(A + B*3i)*1i)/(2*a) - (B*c^4)/a))/f - (((4*A*c^4 + B*c^4*12i)*1i)
/a - ((12*A*c^4 + B*c^4*20i)*1i)/a)/(f*(tan(e + f*x)*1i + 1)) + (tan(e + f
*x)*((2*c^4*(A + B*3i))/a + (B*c^4*3i)/a - (c^4*(A*1i - B)*3i)/a))/f - (B*
c^4*tan(e + f*x)^3*1i)/(3*a*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{a+ia\tan(e+fx)}\,dx$$
$$= \frac{c^4\left(24\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)i+1}dx\right)afi - 36\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)i+1}dx\right)bf - 36\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)i+1}dx\right)af - 24\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)i+1}dx\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e)),x)
```

output

```
(c**4*(24*int(tan(e + f*x)**3/(tan(e + f*x)*i + 1),x)*a*f*i - 36*int(tan(e
 + f*x)**3/(tan(e + f*x)*i + 1),x)*b*f - 36*int(tan(e + f*x)**2/(tan(e + f
*x)*i + 1),x)*a*f - 24*int(tan(e + f*x)**2/(tan(e + f*x)*i + 1),x)*b*f*i -
 6*int(( - 1)/(tan(e + f*x) - i),x)*b*f - 24*int(tan(e + f*x)/(tan(e + f*x
)*i + 1),x)*a*f*i + 6*int(tan(e + f*x)/(tan(e + f*x)*i + 1),x)*b*f + 12*in
t(1/(tan(e + f*x)*i + 1),x)*a*f + 24*int(1/(tan(e + f*x)*i + 1),x)*b*f*i +
 6*log(tan(e + f*x)**2 + 1)*a*i - 30*log(tan(e + f*x)**2 + 1)*b - 2*tan(e
+ f*x)**3*b*i - 3*tan(e + f*x)**2*a*i + 15*tan(e + f*x)**2*b + 6*tan(e + f
*x)*a + 36*tan(e + f*x)*b*i - 12*a*f*x - 66*b*f*i*x))/(6*a*f)
```

3.707 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 121

$$\begin{aligned}\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx &= -\frac{4(A+2iB)c^3x}{a}-\frac{4(iA-2B)c^3\log(\cos(e+fx))}{af}\\ &\quad -\frac{4(A+iB)c^3}{af(i-\tan(e+fx))}+\frac{(A+4iB)c^3\tan(e+fx)}{af}+\frac{Bc^3\tan^2(e+fx)}{2af}\end{aligned}$$

output
```
-4*(A+2*I*B)*c^3*x/a-4*(I*A-2*B)*c^3*ln(cos(f*x+e))/a/f-4*(A+I*B)*c^3/a/f/
(I-tan(f*x+e))+(A+4*I*B)*c^3*tan(f*x+e)/a/f+1/2*B*c^3*tan(f*x+e)^2/a/f
```

Mathematica [A] (verified)

Time = 3.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx = \frac{c^3(14A+27iB+8(A+2iB)\log(i-\tan(e+fx))+(4iA-11B+8i(A+2iB)\log(i-\tan(e+fx)))\,t}{2af(-i+\tan(e+fx))}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3)/(a + I*a*Tan[e +
 f*x]),x]
```

output
```
(c^3*(14*A + (27*I)*B + 8*(A + (2*I)*B)*Log[I - Tan[e + f*x]] + ((4*I)*A -
 11*B + (8*I)*(A + (2*I)*B)*Log[I - Tan[e + f*x]])*Tan[e + f*x] + (2*A + (
7*I)*B)*Tan[e + f*x]^2 + B*Tan[e + f*x]^3))/(2*a*f*(-I + Tan[e + f*x]))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^3(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^3(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{c^2(1-i\tan(e+fx))^2(A+B\tan(e+fx))}{a^2(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\int \frac{(1-i\tan(e+fx))^2(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^2}d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{86}$$

$$\frac{c^3\int \left(-\frac{4(A+iB)}{(\tan(e+fx)-i)^2}+A\left(\frac{4iB}{A}+1\right)+B\tan(e+fx)+\frac{4i(A+2iB)}{\tan(e+fx)-i}\right)d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left((A+4iB)\tan(e+fx)-\frac{4(A+iB)}{-\tan(e+fx)+i}+4(-2B+iA)\log(-\tan(e+fx)+i)+\frac{1}{2}B\tan^2(e+fx)\right)}{af}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3)/(a + I*a*Tan[e + f*x])
,x]
```

output
```
(c^3*(4*(I*A - 2*B)*Log[I - Tan[e + f*x]] - (4*(A + I*B))/(I - Tan[e + f*x
]) + (A + (4*I)*B)*Tan[e + f*x] + (B*Tan[e + f*x]^2)/2))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.58

method	result
derivativedivides	$-\frac{4c^{3}A\arctan(\tan(fx+e))}{fa}+\frac{2ic^{3}A\ln\left(1+\tan(fx+e)^{2}\right)}{fa}-\frac{8ic^{3}B\arctan(\tan(fx+e))}{fa}-\frac{4c^{3}B\ln\left(1+\tan(fx+e)^{2}\right)}{fa}$
default	$-\frac{4c^{3}A\arctan(\tan(fx+e))}{fa}+\frac{2ic^{3}A\ln\left(1+\tan(fx+e)^{2}\right)}{fa}-\frac{8ic^{3}B\arctan(\tan(fx+e))}{fa}-\frac{4c^{3}B\ln\left(1+\tan(fx+e)^{2}\right)}{fa}$
norman	$\frac{\frac{(4ic^{3}B+Ac^{3})\tan(fx+e)^{3}}{af}+\frac{(8ic^{3}B+5Ac^{3})\tan(fx+e)}{af}-\frac{4(2ic^{3}B+Ac^{3})x}{a}-\frac{-8ic^{3}A+9Bc^{3}}{2af}-\frac{4(2ic^{3}B+Ac^{3})x\tan(fx+e)^{2}}{a}+}{1+\tan(fx+e)^{2}}$
risch	$-\frac{2c^{3}e^{-2i(fx+e)}B}{af}+\frac{2ic^{3}e^{-2i(fx+e)}A}{af}-\frac{16ic^{3}Bx}{a}-\frac{8c^{3}Ax}{a}-\frac{16ic^{3}Be}{fa}-\frac{8c^{3}Ae}{fa}-\frac{2c^{3}(-iAe^{2i(fx+e)}+3B}{af(e^{2i(fx+}}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-4/f*c^3/a*A*arctan(tan(f*x+e))+2*I/f*c^3/a*A*ln(1+tan(f*x+e)^2)-8*I/f*c^3
/a*B*arctan(tan(f*x+e))-4/f*c^3/a*B*ln(1+tan(f*x+e)^2)+4*I/f*c^3/a/(-I+tan
(f*x+e))*B+4/f*c^3/a/(-I+tan(f*x+e))*A+1/f*c^3/a*A*tan(f*x+e)+4*I/f*c^3/a*
tan(f*x+e)*B+1/2*B*c^3*tan(f*x+e)^2/a/f
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(107) = 214.

Time = 0.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.81

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3}}{a+ia\tan(e+fx)}\,dx =$$
$$-\frac{2\left(4\left(A+2iB\right)c^{3}fxe^{(6ifx+6ie)}+(-iA+B)c^{3}+2\left(4\left(A+2iB\right)c^{3}fx+(-iA+2B)c^{3}\right)e^{(4ifx+4ie)}+(\ldots}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algo
rithm="fricas")
```

output
```
-2*(4*(A + 2*I*B)*c^3*f*x*e^(6*I*f*x + 6*I*e) + (-I*A + B)*c^3 + 2*(4*(A +
 2*I*B)*c^3*f*x + (-I*A + 2*B)*c^3)*e^(4*I*f*x + 4*I*e) + (4*(A + 2*I*B)*c
^3*f*x + 3*(-I*A + 2*B)*c^3)*e^(2*I*f*x + 2*I*e) + 2*((I*A - 2*B)*c^3*e^(6
*I*f*x + 6*I*e) + 2*(I*A - 2*B)*c^3*e^(4*I*f*x + 4*I*e) + (I*A - 2*B)*c^3*
e^(2*I*f*x + 2*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/(a*f*e^(6*I*f*x + 6*I*e
) + 2*a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*e))
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.19

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx = \frac{2iAc^3-8Bc^3+\left(2iAc^3e^{2ie}-6Bc^3e^{2ie}\right)e^{2ifx}}{afe^{4ie}e^{4ifx}+2afe^{2ie}e^{2ifx}+af} + \begin{cases} \frac{(2iAc^3-2Bc^3)e^{-2ie}e^{-2ifx}}{af} & \text{for } afe^{2ie}\neq 0 \\ x\left(-\frac{-8Ac^3-16iBc^3}{a}+\frac{(-8Ac^3e^{2ie}+4Ac^3-16iBc^3e^{2ie}+4iBc^3)e^{-2ie}}{a}\right) & \text{otherwise} \end{cases} - \frac{4ic^3(A+2iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{af} + \frac{x(-8Ac^3-16iBc^3)}{a}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**3/(a+I*a*tan(f*x+e)),x)
```

output
```
(2*I*A*c**3 - 8*B*c**3 + (2*I*A*c**3*exp(2*I*e) - 6*B*c**3*exp(2*I*e))*exp
(2*I*f*x))/(a*f*exp(4*I*e)*exp(4*I*f*x) + 2*a*f*exp(2*I*e)*exp(2*I*f*x) +
a*f) + Piecewise(((2*I*A*c**3 - 2*B*c**3)*exp(-2*I*e)*exp(-2*I*f*x)/(a*f),
 Ne(a*f*exp(2*I*e), 0)), (x*(-(-8*A*c**3 - 16*I*B*c**3)/a + (-8*A*c**3*exp
(2*I*e) + 4*A*c**3 - 16*I*B*c**3*exp(2*I*e) + 4*I*B*c**3)*exp(-2*I*e)/a),
True)) - 4*I*c**3*(A + 2*I*B)*log(exp(2*I*f*x) + exp(-2*I*e))/(a*f) + x*(-
8*A*c**3 - 16*I*B*c**3)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx\\&= -\frac{4\left(-i\,Ac^3+2\,Bc^3\right)\log\left(\tan\left(fx+e\right)-i\right)}{af}+\frac{4\left(Ac^3+i\,Bc^3\right)}{af(\tan\left(fx+e\right)-i)}\\&\quad+\frac{Bac^3f\tan\left(fx+e\right)^2+2\,Aac^3f\tan\left(fx+e\right)+8i\,Bac^3f\tan\left(fx+e\right)}{2\,a^2f^2}\end{aligned}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x, algo
rithm="giac")
```

output
```
-4*(-I*A*c^3 + 2*B*c^3)*log(tan(f*x + e) - I)/(a*f) + 4*(A*c^3 + I*B*c^3)/
(a*f*(tan(f*x + e) - I)) + 1/2*(B*a*c^3*f*tan(f*x + e)^2 + 2*A*a*c^3*f*tan
(f*x + e) + 8*I*B*a*c^3*f*tan(f*x + e))/(a^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx$$
$$= \frac{\ln(\tan(e+f\,x)-\mathrm{i})\,\left(-\frac{8\,B\,c^3}{a}+\frac{A\,c^3\,4\mathrm{i}}{a}\right)}{f}+\frac{\tan(e+f\,x)\,\left(\frac{c^3\,(A+B\,2\mathrm{i})}{a}+\frac{B\,c^3\,2\mathrm{i}}{a}\right)}{f}$$
$$+\frac{\frac{4\,B\,c^3}{a}+\frac{(4\,A\,c^3+B\,c^3\,8\mathrm{i})\,1\mathrm{i}}{a}}{f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}+\frac{B\,c^3\,\tan(e+f\,x)^2}{2\,a\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^3)/(a + a*tan(e + f*x)*1
i),x)
```

output
```
(log(tan(e + f*x) - 1i)*((A*c^3*4i)/a - (8*B*c^3)/a))/f + (tan(e + f*x)*((
c^3*(A + B*2i))/a + (B*c^3*2i)/a))/f + (((4*A*c^3 + B*c^3*8i)*1i)/a + (4*B
*c^3)/a)/(f*(tan(e + f*x)*1i + 1)) + (B*c^3*tan(e + f*x)^2)/(2*a*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{a+ia\tan(e+fx)}\,dx$$
$$= \frac{c^3\left(-16\left(\int \frac{1}{\tan(fx+e)-i}dx\right)afi+16\left(\int \frac{1}{\tan(fx+e)-i}dx\right)bf+4\log\left(\tan\left(fx+e\right)^2+1\right)ai-8\log\left(\tan\left(fx+\right.\right.\right.}{2af}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e)),x)
```

output
```
(c**3*( - 16*int(1/(tan(e + f*x) - i),x)*a*f*i + 16*int(1/(tan(e + f*x) -
i),x)*b*f + 4*log(tan(e + f*x)**2 + 1)*a*i - 8*log(tan(e + f*x)**2 + 1)*b
+ tan(e + f*x)**2*b + 2*tan(e + f*x)*a + 8*tan(e + f*x)*b*i - 16*a*f*x - 2
4*b*f*i*x))/(2*a*f)
```

3.708 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 96

$$\begin{aligned}\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx &= -\frac{(A+3iB)c^2x}{a}-\frac{(iA-3B)c^2\log(\cos(e+fx))}{af}\\&\quad-\frac{2(A+iB)c^2}{af(i-\tan(e+fx))}+\frac{iBc^2\tan(e+fx)}{af}\end{aligned}$$

output
```
-(A+3*I*B)*c^2*x/a-(I*A-3*B)*c^2*ln(cos(f*x+e))/a/f-2*(A+I*B)*c^2/a/f/(I-t
an(f*x+e))+I*B*c^2*tan(f*x+e)/a/f
```

Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx = \frac{\frac{B(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}+\frac{(iA-3B)c^2\left(\log(i-\tan(e+fx))-\frac{2i}{-i+\tan(e+fx)}\right)}{a}}{f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2)/(a + I*a*Tan[e +
 f*x]),x]
```

output
```
((B*(c - I*c*Tan[e + f*x])^2)/(a + I*a*Tan[e + f*x]) + ((I*A - 3*B)*c^2*(L
og[I - Tan[e + f*x]] - (2*I)/(-I + Tan[e + f*x])))/a)/f
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^2(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^2(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{c(1-i\tan(e+fx))(A+B\tan(e+fx))}{a^2(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{(1-i\tan(e+fx))(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^2}d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{86}$$

$$\frac{c^2\int \left(-\frac{2(A+iB)}{(\tan(e+fx)-i)^2}+iB+\frac{i(A+3iB)}{\tan(e+fx)-i}\right)d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\left(-\frac{2(A+iB)}{-\tan(e+fx)+i}+(-3B+iA)\log(-\tan(e+fx)+i)+iB\tan(e+fx)\right)}{af}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2)/(a + I*a*Tan[e + f*x])
,x]
```

output
```
(c^2*((I*A - 3*B)*Log[I - Tan[e + f*x]] - (2*(A + I*B))/(I - Tan[e + f*x])
 + I*B*Tan[e + f*x]))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{iB\,c^2\tan(fx+e)}{af}+\frac{ic^2A\ln\left(1+\tan(fx+e)^2\right)}{2fa}-\frac{3c^2B\ln\left(1+\tan(fx+e)^2\right)}{2fa}-\frac{c^2A\arctan(\tan(fx+e))}{fa}-\frac{3ic^2B\,a}{}$
default	$\frac{iB\,c^2\tan(fx+e)}{af}+\frac{ic^2A\ln\left(1+\tan(fx+e)^2\right)}{2fa}-\frac{3c^2B\ln\left(1+\tan(fx+e)^2\right)}{2fa}-\frac{c^2A\arctan(\tan(fx+e))}{fa}-\frac{3ic^2B\,a}{}$
norman	$\frac{\frac{\left(3ic^2B+2A\,c^2\right)\tan(fx+e)}{af}+\frac{ic^2B\tan(fx+e)^3}{af}-\frac{\left(3ic^2B+A\,c^2\right)x}{a}-\frac{-2iA\,c^2+2B\,c^2}{af}-\frac{\left(3ic^2B+A\,c^2\right)x\tan(fx+e)^2}{a}}{1+\tan(fx+e)^2}-\frac{(-iA\,c^2}{}$
risch	$-\frac{c^2\mathrm{e}^{-2i(fx+e)}B}{af}+\frac{ic^2\mathrm{e}^{-2i(fx+e)}A}{af}-\frac{6ic^2Bx}{a}-\frac{2c^2Ax}{a}-\frac{6ic^2Be}{af}-\frac{2c^2Ae}{af}-\frac{2c^2B}{fa\left(\mathrm{e}^{2i(fx+e)}+1\right)}+\frac{3c^2\ln(}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
I*B*c^2*tan(f*x+e)/a/f+1/2*I/f*c^2/a*A*ln(1+tan(f*x+e)^2)-3/2/f*c^2/a*B*ln
(1+tan(f*x+e)^2)-1/f*c^2/a*A*arctan(tan(f*x+e))-3*I/f*c^2/a*B*arctan(tan(f
*x+e))+2*I/f*c^2/a/(-I+tan(f*x+e))*B+2/f*c^2/a/(-I+tan(f*x+e))*A
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.59

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx = -\frac{2\,(A+3i\,B)c^2fxe^{(4i\,fx+4i\,e)}-(i\,A-B)c^2+(2\,(A+3i\,B)c^2fx-(i\,A-3\,B)c^2)e^{(2i\,fx+2i\,e)}-((-i\,A}{afe^{(4i\,fx+4i\,e)}+afe^{(2i\,fx+2i\,e)}}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algo
rithm="fricas")
```

output
```
-(2*(A + 3*I*B)*c^2*f*x*e^(4*I*f*x + 4*I*e) - (I*A - B)*c^2 + (2*(A + 3*I*
B)*c^2*f*x - (I*A - 3*B)*c^2)*e^(2*I*f*x + 2*I*e) - ((-I*A + 3*B)*c^2*e^(4
*I*f*x + 4*I*e) + (-I*A + 3*B)*c^2*e^(2*I*f*x + 2*I*e))*log(e^(2*I*f*x + 2
*I*e) + 1))/(a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*e))
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.02

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx$$
$$= -\frac{2Bc^2}{afe^{2ie}e^{2ifx}+af} + \begin{cases} \frac{(iAc^2-Bc^2)e^{-2ie}e^{-2ifx}}{af} & \text{for } afe^{2ie}\neq 0 \\ x\left(-\frac{-2Ac^2-6iBc^2}{a}+\frac{(-2Ac^2e^{2ie}+2Ac^2-6iBc^2e^{2ie}+2iBc^2)e^{-2ie}}{a}\right) & \text{otherwise} \end{cases} - \frac{ic^2(A+3iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{af} + \frac{x(-2Ac^2-6iBc^2)}{a}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**2/(a+I*a*tan(f*x+e)),x)
```

output
```
-2*B*c**2/(a*f*exp(2*I*e)*exp(2*I*f*x) + a*f) + Piecewise((((I*A*c**2 - B*c
**2)*exp(-2*I*e)*exp(-2*I*f*x)/(a*f), Ne(a*f*exp(2*I*e), 0)), (x*(-(-2*A*c
**2 - 6*I*B*c**2)/a + (-2*A*c**2*exp(2*I*e) + 2*A*c**2 - 6*I*B*c**2*exp(2*
I*e) + 2*I*B*c**2)*exp(-2*I*e)/a), True)) - I*c**2*(A + 3*I*B)*log(exp(2*I
*f*x) + exp(-2*I*e))/(a*f) + x*(-2*A*c**2 - 6*I*B*c**2)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.81

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx$$
$$=\frac{iBc^2\tan(fx+e)}{af}+\frac{(iAc^2-3Bc^2)\log(\tan(fx+e)-i)}{af}+\frac{2(Ac^2+iBc^2)}{af(\tan(fx+e)-i)}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x, algo
rithm="giac")
```

output
```
I*B*c^2*tan(f*x + e)/(a*f) + (I*A*c^2 - 3*B*c^2)*log(tan(f*x + e) - I)/(a*
f) + 2*(A*c^2 + I*B*c^2)/(a*f*(tan(f*x + e) - I))
```

Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx$$
$$=\frac{\ln(\tan(e+f\,x)-\mathrm{i})\,\left(-\frac{3\,B\,c^2}{a}+\frac{A\,c^2\,1\mathrm{i}}{a}\right)}{f}$$
$$+\frac{\frac{(A\,c^2-B\,c^2\,1\mathrm{i})\,1\mathrm{i}}{a}+\frac{(A\,c^2+B\,c^2\,3\mathrm{i})\,1\mathrm{i}}{a}}{f\,\left(1+\tan(e+f\,x)\,1\mathrm{i}\right)}+\frac{B\,c^2\tan(e+f\,x)\,1\mathrm{i}}{a\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^2)/(a + a*tan(e + f*x)*1
i),x)
```

output
```
(log(tan(e + f*x) - 1i)*((A*c^2*1i)/a - (3*B*c^2)/a))/f + (((A*c^2 - B*c^2
*1i)*1i)/a + ((A*c^2 + B*c^2*3i)*1i)/a)/(f*(tan(e + f*x)*1i + 1)) + (B*c^2
*tan(e + f*x)*1i)/(a*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{a+ia\tan(e+fx)}\,dx$$
$$= \frac{c^2\left(-8\left(\int \frac{1}{\tan(fx+e)-i}dx\right)afi+8\left(\int \frac{1}{\tan(fx+e)-i}dx\right)bf+\log\left(\tan(fx+e)^2+1\right)ai-3\log\left(\tan(fx+e)^2\right.\right.}{2af}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e)),x)
```

output
```
(c**2*( - 8*int(1/(tan(e + f*x) - i),x)*a*f*i + 8*int(1/(tan(e + f*x) - i)
,x)*b*f + log(tan(e + f*x)**2 + 1)*a*i - 3*log(tan(e + f*x)**2 + 1)*b + 2*
tan(e + f*x)*b*i - 6*a*f*x - 10*b*f*i*x))/(2*a*f)
```

3.709 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 57

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{a+ia\tan(e+fx)}\,dx = -\frac{iBcx}{a}+\frac{Bc\log(\cos(e+fx))}{af}-\frac{(A+iB)c}{af(i-\tan(e+fx))}$$

output
```
-I*B*c*x/a+B*c*ln(cos(f*x+e))/a/f-(A+I*B)*c/a/f/(I-tan(f*x+e))
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{a+ia\tan(e+fx)}\,dx = -\frac{c\left(B\log(i-\tan(e+fx))+\frac{A+iB}{i-\tan(e+fx)}\right)}{af}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]))/(a + I*a*Tan[e + f
*x]),x]
```

output
```
-((c*(B*Log[I - Tan[e + f*x]] + (A + I*B)/(I - Tan[e + f*x])))/(a*f))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 4071, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^2(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^2}d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{49}$$

$$\frac{c\int \left(\frac{-A-iB}{(\tan(e+fx)-i)^2}-\frac{B}{\tan(e+fx)-i}\right)d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(-\frac{A+iB}{-\tan(e+fx)+i}-B\log(-\tan(e+fx)+i)\right)}{af}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]))/(a + I*a*Tan[e + f*x]),x
]
```

output
```
(c*(-(B*Log[I - Tan[e + f*x]]) - (A + I*B)/(I - Tan[e + f*x])))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

method	result	size
derivativedivides	$-\frac{cB\ln\left(1+\tan(fx+e)^2\right)}{2fa}-\frac{icB\arctan(\tan(fx+e))}{fa}+\frac{icB}{fa(-i+\tan(fx+e))}+\frac{cA}{fa(-i+\tan(fx+e))}$	83
default	$-\frac{cB\ln\left(1+\tan(fx+e)^2\right)}{2fa}-\frac{icB\arctan(\tan(fx+e))}{fa}+\frac{icB}{fa(-i+\tan(fx+e))}+\frac{cA}{fa(-i+\tan(fx+e))}$	83
risch	$-\frac{c\,\mathrm{e}^{-2i(fx+e)}B}{2af}+\frac{ic\,\mathrm{e}^{-2i(fx+e)}A}{2af}-\frac{2iBcx}{a}-\frac{2iBce}{af}+\frac{Bc\ln(\mathrm{e}^{2i(fx+e)}+1)}{af}$	83
norman	$\frac{\frac{(icB+cA)\tan(fx+e)}{af}-\frac{-icA+Bc}{af}-\frac{iBcx}{a}-\frac{icBx\tan(fx+e)^2}{a}}{1+\tan(fx+e)^2}-\frac{cB\ln\left(1+\tan(fx+e)^2\right)}{2fa}$	102

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x,method=_RETUR
NVERBOSE)
```

output
```
-1/2/f*c/a*B*ln(1+tan(f*x+e)^2)-I/f*c/a*B*arctan(tan(f*x+e))+I/f*c/a/(-I+t
an(f*x+e))*B+1/f*c/a/(-I+tan(f*x+e))*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{a+ia\tan(e+fx)}\,dx$$
$$=\frac{\left(-4i\,Bcfxe^{(2i\,fx+2i\,e)}+2\,Bce^{(2i\,fx+2i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)+(i\,A-B)c\right)e^{(-2i\,fx-2i\,e)}}{2\,af}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algori
thm="fricas")
```

output
```
1/2*(-4*I*B*c*f*x*e^(2*I*f*x + 2*I*e) + 2*B*c*e^(2*I*f*x + 2*I*e)*log(e^(2
*I*f*x + 2*I*e) + 1) + (I*A - B)*c)*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.96

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{a+ia\tan(e+fx)}\,dx$$
$$= -\frac{2iBcx}{a} + \frac{Bc\log\left(e^{2ifx}+e^{-2ie}\right)}{af} + \begin{cases} \frac{(iAc-Bc)e^{-2ie}e^{-2ifx}}{2af} & \text{for } afe^{2ie}\neq 0 \\ x\left(\frac{2iBc}{a} + \frac{(Ac-2iBce^{2ie}+iBc)e^{-2ie}}{a}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x)
```

output
```
-2*I*B*c*x/a + B*c*log(exp(2*I*f*x) + exp(-2*I*e))/(a*f) + Piecewise(((I*A
*c - B*c)*exp(-2*I*e)*exp(-2*I*f*x)/(2*a*f), Ne(a*f*exp(2*I*e), 0)), (x*(2
*I*B*c/a + (A*c - 2*I*B*c*exp(2*I*e) + I*B*c)*exp(-2*I*e)/a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{a+ia\tan(e+fx)}\,dx$$
$$=-\frac{Bc\log\left(\tan\left(fx+e\right)-i\right)}{af}+\frac{Ac+i\,Bc}{af(\tan\left(fx+e\right)-i)}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algori
thm="giac")
```

output
```
-B*c*log(tan(f*x + e) - I)/(a*f) + (A*c + I*B*c)/(a*f*(tan(f*x + e) - I))
```

Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{a+ia\tan(e+fx)}\,dx$$
$$=\frac{-\frac{B\,c}{a}+\frac{A\,c\,1\mathrm{i}}{a}}{f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}-\frac{B\,c\,\ln\left(\tan(e+f\,x)-\mathrm{i}\right)}{a\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i))/(a + a*tan(e + f*x)*1i)
,x)
```

output
```
((A*c*1i)/a - (B*c)/a)/(f*(tan(e + f*x)*1i + 1)) - (B*c*log(tan(e + f*x) -
 1i))/(a*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{a+ia\tan(e+fx)}\,dx = \frac{c\left(-\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)i+1}dx\right)bi-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)i+1}dx\right)ai+\left(\int \frac{\tan(fx+e)}{\tan(fx+e)i+1}dx\right)b+\left(\int \frac{1}{\tan(fx+e)i+1}dx\right)a\right)}{a}$$

input

```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e)),x)
```

output

```
(c*( - int(tan(e + f*x)**2/(tan(e + f*x)*i + 1),x)*b*i - int(tan(e + f*x)/
(tan(e + f*x)*i + 1),x)*a*i + int(tan(e + f*x)/(tan(e + f*x)*i + 1),x)*b +
 int(1/(tan(e + f*x)*i + 1),x)*a))/a
```

3.710 $\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}\,dx=\frac{(A-iB)x}{2a}+\frac{iA-B}{2f(a+ia\tan(e+fx))}$$

output
```
1/2*(A-I*B)*x/a+1/2*(I*A-B)/f/(a+I*a*tan(f*x+e))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}\,dx=\frac{(A-iB)\arctan(\tan(e+fx))}{2af}-\frac{A+iB}{2af(i-\tan(e+fx))}$$

input
```
Integrate[(A + B*Tan[e + f*x])/(a + I*a*Tan[e + f*x]),x]
```

output
```
((A - I*B)*ArcTan[Tan[e + f*x]])/(2*a*f) - (A + I*B)/(2*a*f*(I - Tan[e + f
*x]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}dx$$
$$\Big\downarrow \text{4009}$$
$$\frac{(A-iB)\int 1dx}{2a}+\frac{-B+iA}{2f(a+ia\tan(e+fx))}$$
$$\Big\downarrow \text{24}$$
$$\frac{-B+iA}{2f(a+ia\tan(e+fx))}+\frac{x(A-iB)}{2a}$$

input
```
Int[(A + B*Tan[e + f*x])/(a + I*a*Tan[e + f*x]),x]
```

output
```
((A - I*B)*x)/(2*a) + (I*A - B)/(2*f*(a + I*a*Tan[e + f*x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

method	result	size
risch	$-\frac{ixB}{2a}+\frac{xA}{2a}-\frac{\mathrm{e}^{-2i(fx+e)}B}{4af}+\frac{i\mathrm{e}^{-2i(fx+e)}A}{4af}$	54
derivativedivides	$-\frac{iB\arctan(\tan(fx+e))}{2fa}+\frac{A\arctan(\tan(fx+e))}{2fa}+\frac{A}{2fa(-i+\tan(fx+e))}+\frac{iB}{2fa(-i+\tan(fx+e))}$	76
default	$-\frac{iB\arctan(\tan(fx+e))}{2fa}+\frac{A\arctan(\tan(fx+e))}{2fa}+\frac{A}{2fa(-i+\tan(fx+e))}+\frac{iB}{2fa(-i+\tan(fx+e))}$	76
norman	$\frac{\frac{(-iB+A)x}{2a}-\frac{-iA+B}{2af}+\frac{(iB+A)\tan(fx+e)}{2af}+\frac{(-iB+A)x\tan(fx+e)^2}{2a}}{1+\tan(fx+e)^2}$	81

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*x/a*B+1/2*x/a*A-1/4/a/f*exp(-2*I*(f*x+e))*B+1/4*I/a/f*exp(-2*I*(f*x
+e))*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}\,dx=\frac{\left(2\,(A-i\,B)fxe^{(2i\,fx+2i\,e)}+i\,A-B\right)e^{(-2i\,fx-2i\,e)}}{4\,af}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(2*(A - I*B)*f*x*e^(2*I*f*x + 2*I*e) + I*A - B)*e^(-2*I*f*x - 2*I*e)/(
a*f)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

$$\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}\,dx = \begin{cases} \frac{(iA-B)e^{-2ie}e^{-2ifx}}{4af} & \text{for } afe^{2ie}\neq 0 \\ x\left(-\frac{A-iB}{2a}+\frac{(Ae^{2ie}+A-iBe^{2ie}+iB)e^{-2ie}}{2a}\right) & \text{otherwise} \end{cases} + \frac{x(A-iB)}{2a}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e)),x)
```

output
```
Piecewise((((I*A - B)*exp(-2*I*e)*exp(-2*I*f*x)/(4*a*f), Ne(a*f*exp(2*I*e),
 0)), (x*(-(A - I*B)/(2*a) + (A*exp(2*I*e) + A - I*B*exp(2*I*e) + I*B)*exp
(-2*I*e)/(2*a)), True)) + x*(A - I*B)/(2*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}\,dx = -\frac{(-iA-B)\log(\tan(fx+e)+i)}{4\,af} - \frac{(iA+B)\log(\tan(fx+e)-i)}{4\,af} + \frac{A+iB}{2\,af(\tan(fx+e)-i)}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

output
```
-1/4*(-I*A - B)*log(tan(f*x + e) + I)/(a*f) - 1/4*(I*A + B)*log(tan(f*x +
e) - I)/(a*f) + 1/2*(A + I*B)/(a*f*(tan(f*x + e) - I))
```

Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}\,dx = -\frac{x\,(B+A\,1\mathrm{i})\,1\mathrm{i}}{2\,a} + \frac{-\frac{B}{2\,a}+\frac{A\,1\mathrm{i}}{2\,a}}{f\,\left(1+\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}$$

input
```
int((A + B*tan(e + f*x))/(a + a*tan(e + f*x)*1i),x)
```

output
```
((A*1i)/(2*a) - B/(2*a))/(f*(tan(e + f*x)*1i + 1)) - (x*(A*1i + B)*1i)/(2*
a)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{a+ia\tan(e+fx)}\,dx = \frac{\left(\int \frac{\tan(fx+e)}{\tan(fx+e)i+1}dx\right)b+\left(\int \frac{1}{\tan(fx+e)i+1}dx\right)a}{a}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e)),x)
```

output
```
(int(tan(e + f*x)/(tan(e + f*x)*i + 1),x)*b + int(1/(tan(e + f*x)*i + 1),x
)*a)/a
```

3.711 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 45

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx = \frac{Ax}{2ac}-\frac{\cos^2(e+fx)(B-A\tan(e+fx))}{2acf}$$

output
```
1/2*A*x/a/c-1/2*cos(f*x+e)^2*(B-A*tan(f*x+e))/a/c/f
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx = \frac{-2B\cos^2(e+fx)+A(2(e+fx)+\sin(2(e+fx)))}{4acf}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*
x])),x]
```

output
```
(-2*B*Cos[e + f*x]^2 + A*(2*(e + f*x) + Sin[2*(e + f*x)]))/(4*a*c*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {3042, 4071, 27, 82, 454, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^2c^2(1-i\tan(e+fx))^2(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^2(i\tan(e+fx)+1)^2}d\tan(e+fx)}{acf}$$

$$\Big\downarrow \text{82}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{acf}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{1}{2}A\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{B-A\tan(e+fx)}{2(\tan^2(e+fx)+1)}}{acf}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}A\arctan(\tan(e+fx))-\frac{B-A\tan(e+fx)}{2(\tan^2(e+fx)+1)}}{acf}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])),x
]
```

output
```
((A*ArcTan[Tan[e + f*x]])/2 - (B - A*Tan[e + f*x])/(2*(1 + Tan[e + f*x]^2)
))/(a*c*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

method	result
risch	$\frac{Ax}{2ac}-\frac{B\cos(2fx+2e)}{4acf}+\frac{A\sin(2fx+2e)}{4acf}$
norman	$\frac{\frac{Ax}{2ac}-\frac{B}{2acf}+\frac{A\tan(fx+e)}{2acf}+\frac{Ax\tan(fx+e)^2}{2ac}}{1+\tan(fx+e)^2}$
derivativedivides	$\frac{A\arctan(\tan(fx+e))}{2fac}+\frac{A}{4fac(i+\tan(fx+e))}-\frac{iB}{4fac(i+\tan(fx+e))}+\frac{iB}{4fac(-i+\tan(fx+e))}+\frac{A}{4fac(-i+\tan($
default	$\frac{A\arctan(\tan(fx+e))}{2fac}+\frac{A}{4fac(i+\tan(fx+e))}-\frac{iB}{4fac(i+\tan(fx+e))}+\frac{iB}{4fac(-i+\tan(fx+e))}+\frac{A}{4fac(-i+\tan($

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x,method=_RETUR
NVERBOSE)
```

output
```
1/2*A*x/a/c-1/4*B/a/c/f*cos(2*f*x+2*e)+1/4*A/a/c/f*sin(2*f*x+2*e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx = \frac{\left(4\,Afxe^{(2i\,fx+2i\,e)}+(-i\,A-B)e^{(4i\,fx+4i\,e)}+i\,A-B\right)e^{(-2i\,fx-2i\,e)}}{8\,acf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algori
thm="fricas")
```

output
```
1/8*(4*A*f*x*e^(2*I*f*x + 2*I*e) + (-I*A - B)*e^(4*I*f*x + 4*I*e) + I*A -
B)*e^(-2*I*f*x - 2*I*e)/(a*c*f)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.67

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx$$
$$=\frac{Ax}{2ac}+\begin{cases}\frac{((8iAacf-8Bacf)e^{-2ifx}+(-8iAacfe^{4ie}-8Bacfe^{4ie})e^{2ifx})e^{-2ie}}{64a^2c^2f^2} & \text{for } a^2c^2f^2e^{2ie}\neq 0\\ x\left(-\frac{A}{2ac}+\frac{(Ae^{4ie}+2Ae^{2ie}+A-iBe^{4ie}+iB)e^{-2ie}}{4ac}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

output
```
A*x/(2*a*c) + Piecewise((((8*I*A*a*c*f - 8*B*a*c*f)*exp(-2*I*f*x) + (-8*I*
A*a*c*f*exp(4*I*e) - 8*B*a*c*f*exp(4*I*e))*exp(2*I*f*x))*exp(-2*I*e)/(64*a
**2*c**2*f**2), Ne(a**2*c**2*f**2*exp(2*I*e), 0)), (x*(-A/(2*a*c) + (A*exp
(4*I*e) + 2*A*exp(2*I*e) + A - I*B*exp(4*I*e) + I*B)*exp(-2*I*e)/(4*a*c)),
 True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx = \frac{(fx+e)A}{2\,acf} + \frac{A\tan{(fx+e)}-B}{2\left(\tan{(fx+e)}^2+1\right)acf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x, algori
thm="giac")
```

output
```
1/2*(f*x + e)*A/(a*c*f) + 1/2*(A*tan(f*x + e) - B)/((tan(f*x + e)^2 + 1)*a
*c*f)
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx = \frac{\frac{A\,\sin(2\,e+2\,f\,x)}{2}-\frac{B\,\cos(2\,e+2\,f\,x)}{2}+A\,f\,x}{2\,a\,c\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i))
,x)
```

output
```
((A*sin(2*e + 2*f*x))/2 - (B*cos(2*e + 2*f*x))/2 + A*f*x)/(2*a*c*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))}\,dx = \frac{\tan{(fx+e)}^2\,afx+\tan{(fx+e)}^2\,b+\tan{(fx+e)}\,a+afx}{2acf\left(\tan{(fx+e)}^2+1\right)}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e)),x)
```

output
```
(tan(e + f*x)**2*a*f*x + tan(e + f*x)**2*b + tan(e + f*x)*a + a*f*x)/(2*a*
c*f*(tan(e + f*x)**2 + 1))
```

3.712 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 113

$$\begin{aligned}\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}\,dx &= \frac{(3A+iB)x}{8ac^2}-\frac{A+iB}{8ac^2f(i-\tan(e+fx))}\\ &\quad+\frac{iA+B}{8ac^2f(i+\tan(e+fx))^2}+\frac{A}{4ac^2f(i+\tan(e+fx))}\end{aligned}$$

output
```
1/8*(3*A+I*B)*x/a/c^2-1/8*(A+I*B)/a/c^2/f/(I-tan(f*x+e))+1/8*(I*A+B)/a/c^2
/f/(I+tan(f*x+e))^2+1/4*A/a/c^2/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 4.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}\,dx = \frac{\sec^2(e+fx)(5A-iB-(A+3iB)\cos(2(e+fx))+3iA\sin(2(e+fx))-B\sin(2(e+fx))+2(3A+i}{16ac^2f(-i+\tan(e+fx))(i+\tan(e+fx))^2}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*
x])^2),x]
```

output
```
(Sec[e + f*x]^2*(5*A - I*B - (A + (3*I)*B)*Cos[2*(e + f*x)] + (3*I)*A*Sin[
2*(e + f*x)] - B*Sin[2*(e + f*x)] + 2*(3*A + I*B)*ArcTan[Tan[e + f*x]]*(I
+ Tan[e + f*x])))/(16*a*c^2*f*(-I + Tan[e + f*x])*(I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^2c^3(1-i\tan(e+fx))^3(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^3(i\tan(e+fx)+1)^2}d\tan(e+fx)}{ac^2f}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(-\frac{A}{4(\tan(e+fx)+i)^2}+\frac{3A+iB}{8(\tan^2(e+fx)+1)}+\frac{-A-iB}{8(\tan(e+fx)-i)^2}-\frac{i(A-iB)}{4(\tan(e+fx)+i)^3}\right)d\tan(e+fx)}{ac^2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}(3A+iB)\arctan(\tan(e+fx))-\frac{A+iB}{8(-\tan(e+fx)+i)}+\frac{B+iA}{8(\tan(e+fx)+i)^2}+\frac{A}{4(\tan(e+fx)+i)}}{ac^2f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2)
,x]
```

output
```
(((3*A + I*B)*ArcTan[Tan[e + f*x]])/8 - (A + I*B)/(8*(I - Tan[e + f*x])) +
 (I*A + B)/(8*(I + Tan[e + f*x])^2) + A/(4*(I + Tan[e + f*x])))/(a*c^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15

method	result
risch	$\frac{ixB}{8a\,c^2}+\frac{3xA}{8a\,c^2}-\frac{\mathrm{e}^{4i(fx+e)}B}{32a\,c^2f}-\frac{i\mathrm{e}^{4i(fx+e)}A}{32a\,c^2f}-\frac{\cos(2fx+2e)B}{8a\,c^2f}-\frac{i\cos(2fx+2e)A}{8a\,c^2f}+\frac{A\sin(2fx+2e)}{4a\,c^2f}$
norman	$\frac{\frac{(iB+3A)x}{8ac}-\frac{iA+B}{4acf}+\frac{(iB+3A)\tan(fx+e)^3}{8acf}+\frac{(iB+3A)x\tan(fx+e)^2}{4ac}+\frac{(iB+3A)x\tan(fx+e)^4}{8ac}+\frac{(-iB+5A)\tan(fx+e)}{8acf}}{c\left(1+\tan(fx+e)^2\right)^2}$
derivativedivides	$\frac{A}{4a\,c^2f(i+\tan(fx+e))}+\frac{3A\arctan(\tan(fx+e))}{8fa\,c^2}+\frac{iB\arctan(\tan(fx+e))}{8fa\,c^2}+\frac{iA}{8fa\,c^2(i+\tan(fx+e))^2}+\frac{}{8fa\,c^2(i-}$
default	$\frac{A}{4a\,c^2f(i+\tan(fx+e))}+\frac{3A\arctan(\tan(fx+e))}{8fa\,c^2}+\frac{iB\arctan(\tan(fx+e))}{8fa\,c^2}+\frac{iA}{8fa\,c^2(i+\tan(fx+e))^2}+\frac{}{8fa\,c^2(i-}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/8*I*x/a/c^2*B+3/8*x/a/c^2*A-1/32/a/c^2/f*exp(4*I*(f*x+e))*B-1/32*I/a/c^2
/f*exp(4*I*(f*x+e))*A-1/8/a/c^2/f*cos(2*f*x+2*e)*B-1/8*I/a/c^2/f*cos(2*f*x
+2*e)*A+1/4*A/a/c^2/f*sin(2*f*x+2*e)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}\,dx$$
$$=\frac{\left(4\,(3\,A+i\,B)fxe^{(2i\,fx+2i\,e)}+(-i\,A-B)e^{(6i\,fx+6i\,e)}-2\,(3i\,A+B)e^{(4i\,fx+4i\,e)}+2i\,A-2\,B\right)e^{(-2i\,fx-2i\,e)}}{32\,ac^2f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algo
rithm="fricas")
```

output
```
1/32*(4*(3*A + I*B)*f*x*e^(2*I*f*x + 2*I*e) + (-I*A - B)*e^(6*I*f*x + 6*I*
e) - 2*(3*I*A + B)*e^(4*I*f*x + 4*I*e) + 2*I*A - 2*B)*e^(-2*I*f*x - 2*I*e)
/(a*c^2*f)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.51

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}\,dx$$
$$= \begin{cases} \frac{((512iAa^2c^4f^2-512Ba^2c^4f^2)e^{-2ifx}+(-1536iAa^2c^4f^2e^{4ie}-512Ba^2c^4f^2e^{4ie})e^{2ifx}+(-256iAa^2c^4f^2e^{6ie}-256Ba^2c^4f^2e^{6ie})e^{4ifx})e^{-2ie}}{8192a^3c^6f^3} \\ x\left(-\frac{3A+iB}{8ac^2}+\frac{(Ae^{6ie}+3Ae^{4ie}+3Ae^{2ie}+A-iBe^{6ie}-iBe^{4ie}+iBe^{2ie}+iB)e^{-2ie}}{8ac^2}\right) \end{cases}$$
$$+\frac{x(3A+iB)}{8ac^2}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**2,x)
```

output
```
Piecewise((((512*I*A*a**2*c**4*f**2 - 512*B*a**2*c**4*f**2)*exp(-2*I*f*x)
+ (-1536*I*A*a**2*c**4*f**2*exp(4*I*e) - 512*B*a**2*c**4*f**2*exp(4*I*e))*
exp(2*I*f*x) + (-256*I*A*a**2*c**4*f**2*exp(6*I*e) - 256*B*a**2*c**4*f**2*
exp(6*I*e))*exp(4*I*f*x))*exp(-2*I*e)/(8192*a**3*c**6*f**3), Ne(a**3*c**6*
f**3*exp(2*I*e), 0)), (x*(-(3*A + I*B)/(8*a*c**2) + (A*exp(6*I*e) + 3*A*ex
p(4*I*e) + 3*A*exp(2*I*e) + A - I*B*exp(6*I*e) - I*B*exp(4*I*e) + I*B*exp(
2*I*e) + I*B)*exp(-2*I*e)/(8*a*c**2)), True)) + x*(3*A + I*B)/(8*a*c**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}\,dx$$
$$= -\frac{(-3i\,A+B)\log\left(\tan\left(fx+e\right)+i\right)}{16\,ac^2f} - \frac{(3i\,A-B)\log\left(\tan\left(fx+e\right)-i\right)}{16\,ac^2f}$$
$$+ \frac{(3\,A+i\,B)\tan\left(fx+e\right)^2-(-3i\,A+B)\tan\left(fx+e\right)+2\,A-2i\,B}{8\,ac^2f(\tan\left(fx+e\right)+i)^2(\tan\left(fx+e\right)-i)}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x, algo
rithm="giac")
```

output
```
-1/16*(-3*I*A + B)*log(tan(f*x + e) + I)/(a*c^2*f) - 1/16*(3*I*A - B)*log(
tan(f*x + e) - I)/(a*c^2*f) + 1/8*((3*A + I*B)*tan(f*x + e)^2 - (-3*I*A +
B)*tan(f*x + e) + 2*A - 2*I*B)/(a*c^2*f*(tan(f*x + e) + I)^2*(tan(f*x + e)
 - I))
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^2}\,dx$$
$$= \frac{\tan(e+f\,x)\,\left(-\frac{B}{8\,a\,c^2}+\frac{A\,3{}\mathrm{i}}{8\,a\,c^2}\right)+\tan(e+f\,x)^2\,\left(\frac{3\,A}{8\,a\,c^2}+\frac{B\,1{}\mathrm{i}}{8\,a\,c^2}\right)+\frac{A}{4\,a\,c^2}-\frac{B\,1{}\mathrm{i}}{4\,a\,c^2}}{f\,\left({\mathrm{tan}\left(e+f\,x\right)}^3+{\mathrm{tan}\left(e+f\,x\right)}^2\,1{}\mathrm{i}+\mathrm{tan}\left(e+f\,x\right)+1{}\mathrm{i}\right)}$$
$$-\frac{x\,\left(-B+A\,3{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,a\,c^2}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^
2),x)
```

output
```
(tan(e + f*x)*((A*3i)/(8*a*c^2) - B/(8*a*c^2)) + tan(e + f*x)^2*((3*A)/(8*
a*c^2) + (B*1i)/(8*a*c^2)) + A/(4*a*c^2) - (B*1i)/(4*a*c^2))/(f*(tan(e + f
*x) + tan(e + f*x)^2*1i + tan(e + f*x)^3 + 1i)) - (x*(A*3i - B)*1i)/(8*a*c
^2)
```

Reduce [F]

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))(c - ic\tan(e + fx))^2}\, dx$$
$$= \frac{-2\left(\int \frac{1}{\tan(fx+e)^3 i - \tan(fx+e)^2 + \tan(fx+e) i - 1} dx\right) \tan(fx+e)^2 af + 2\left(\int \frac{1}{\tan(fx+e)^3 i - \tan(fx+e)^2 + \tan(fx+e) i - 1} dx\right) \mathrm{t}}{}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^2,x)
```

output
```
( - 2*int(1/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*
tan(e + f*x)**2*a*f + 2*int(1/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e
 + f*x)*i - 1),x)*tan(e + f*x)**2*b*f*i - 2*int(1/(tan(e + f*x)**3*i - tan
(e + f*x)**2 + tan(e + f*x)*i - 1),x)*a*f + 2*int(1/(tan(e + f*x)**3*i - t
an(e + f*x)**2 + tan(e + f*x)*i - 1),x)*b*f*i + tan(e + f*x)**2*b*e*i + ta
n(e + f*x)**2*b*f*i*x + tan(e + f*x)*b*i + b*e*i + b*f*i*x)/(2*a*c**2*f*(t
an(e + f*x)**2 + 1))
```

3.713 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 149

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx = \frac{(2A+iB)x}{8ac^3}-\frac{A+iB}{16ac^3f(i-\tan(e+fx))}-\frac{A-iB}{12ac^3f(i+\tan(e+fx))^3}+\frac{iA}{8ac^3f(i+\tan(e+fx))^2}+\frac{3A+iB}{16ac^3f(i+\tan(e+fx))}$$

output
```
1/8*(2*A+I*B)*x/a/c^3-1/16*(A+I*B)/a/c^3/f/(I-tan(f*x+e))-1/12*(A-I*B)/a/c
^3/f/(I+tan(f*x+e))^3+1/8*I*A/a/c^3/f/(I+tan(f*x+e))^2+1/16*(3*A+I*B)/a/c^
3/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 4.01 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx = \frac{\sec^3(e+fx)(9iA\cos(e+fx)+(-1+2\cos(2(e+fx)))((-iA+2B)\cos(e+fx)-(2A+iB)\sin(e+}{24ac^3f(-i+\tan(e+fx))(i+}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*
x])^3),x]
```

output
```
(Sec[e + f*x]^3*((9*I)*A*Cos[e + f*x] + (-1 + 2*Cos[2*(e + f*x)])*(((-I)*A
 + 2*B)*Cos[e + f*x] - (2*A + I*B)*Sin[e + f*x]) - 3*(2*A + I*B)*ArcTan[Ta
n[e + f*x]]*Sec[e + f*x]*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)])))/(24*a*c
^3*f*(-I + Tan[e + f*x])*(I + Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^2c^4(1-i\tan(e+fx))^4(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^4(i\tan(e+fx)+1)^2}d\tan(e+fx)}{ac^3f}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(-\frac{iA}{4(\tan(e+fx)+i)^3}+\frac{2A+iB}{8(\tan^2(e+fx)+1)}+\frac{-A-iB}{16(\tan(e+fx)-i)^2}+\frac{-3A-iB}{16(\tan(e+fx)+i)^2}+\frac{A-iB}{4(\tan(e+fx)+i)^4}\right)d\tan(e+fx)}{ac^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}(2A+iB)\arctan(\tan(e+fx))-\frac{A+iB}{16(-\tan(e+fx)+i)}+\frac{3A+iB}{16(\tan(e+fx)+i)}-\frac{A-iB}{12(\tan(e+fx)+i)^3}+\frac{iA}{8(\tan(e+fx)+i)^2}}{ac^3f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3)
,x]
```

output
```
(((2*A + I*B)*ArcTan[Tan[e + f*x]])/8 - (A + I*B)/(16*(I - Tan[e + f*x]))
- (A - I*B)/(12*(I + Tan[e + f*x])^3) + ((I/8)*A)/(I + Tan[e + f*x])^2 + (
3*A + I*B)/(16*(I + Tan[e + f*x])))/(a*c^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.31

method	result
risch	$\frac{ixB}{8a\,c^3}+\frac{xA}{4a\,c^3}-\frac{\mathrm{e}^{6i(fx+e)}B}{96a\,c^3f}-\frac{i\mathrm{e}^{6i(fx+e)}A}{96a\,c^3f}-\frac{\mathrm{e}^{4i(fx+e)}B}{32a\,c^3f}-\frac{i\mathrm{e}^{4i(fx+e)}A}{16a\,c^3f}-\frac{\cos(2fx+2e)B}{32a\,c^3f}-\frac{5i\cos(2fx+}{32a\,c^3f}$
derivativedivides	$\frac{iB\arctan(\tan(fx+e))}{8fa\,c^3}+\frac{A\arctan(\tan(fx+e))}{4fa\,c^3}+\frac{iB}{16fa\,c^3(-i+\tan(fx+e))}+\frac{A}{16fa\,c^3(-i+\tan(fx+e))}+\frac{}{8a\,c^3}$
default	$\frac{iB\arctan(\tan(fx+e))}{8fa\,c^3}+\frac{A\arctan(\tan(fx+e))}{4fa\,c^3}+\frac{iB}{16fa\,c^3(-i+\tan(fx+e))}+\frac{A}{16fa\,c^3(-i+\tan(fx+e))}+\frac{}{8a\,c^3}$
norman	$\frac{\frac{(iB+2A)x}{8ac}-\frac{4iA+B}{12acf}+\frac{B\tan(fx+e)^2}{4acf}+\frac{(-iB+6A)\tan(fx+e)}{8acf}+\frac{(iB+2A)\tan(fx+e)^3}{3acf}+\frac{(iB+2A)\tan(fx+e)^5}{8acf}+\frac{3(iB+2A)x\tan(f}{8ac}}{\left(1+\tan(fx+e)^2\right)^3c^2}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x,method=_RET
URNVERBOSE)
```

output
```
1/8*I*x/a/c^3*B+1/4*x/a/c^3*A-1/96/a/c^3/f*exp(6*I*(f*x+e))*B-1/96*I/a/c^3
/f*exp(6*I*(f*x+e))*A-1/32/a/c^3/f*exp(4*I*(f*x+e))*B-1/16*I/a/c^3/f*exp(4
*I*(f*x+e))*A-1/32/a/c^3/f*cos(2*f*x+2*e)*B-5/32*I/a/c^3/f*cos(2*f*x+2*e)*
A+1/32*I/a/c^3/f*sin(2*f*x+2*e)*B+7/32/a/c^3/f*sin(2*f*x+2*e)*A
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.62

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx = \frac{(12\,(2\,A+i\,B)fxe^{(2i\,fx+2i\,e)}+(-i\,A-B)e^{(8i\,fx+8i\,e)}-3\,(2i\,A+B)e^{(6i\,fx+6i\,e)}-18i\,Ae^{(4i\,fx+4i\,e)}+3i\,A}{96\,ac^3f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algo
rithm="fricas")
```

output
```
1/96*(12*(2*A + I*B)*f*x*e^(2*I*f*x + 2*I*e) + (-I*A - B)*e^(8*I*f*x + 8*I
*e) - 3*(2*I*A + B)*e^(6*I*f*x + 6*I*e) - 18*I*A*e^(4*I*f*x + 4*I*e) + 3*I
*A - 3*B)*e^(-2*I*f*x - 2*I*e)/(a*c^3*f)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.20

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx$$
$$= \begin{cases} \frac{(-294912iAa^3c^9f^3e^{4ie}e^{2ifx}+(49152iAa^3c^9f^3-49152Ba^3c^9f^3)e^{-2ifx}+(-98304iAa^3c^9f^3e^{6ie}-49152Ba^3c^9f^3e^{6ie})e^{4ifx}+(-16384iAa^3c}{1572864a^4c^{12}f^4} \\ x\left(-\frac{2A+iB}{8ac^3}+\frac{(Ae^{8ie}+4Ae^{6ie}+6Ae^{4ie}+4Ae^{2ie}+A-iBe^{8ie}-2iBe^{6ie}+2iBe^{2ie}+iB)e^{-2ie}}{16ac^3}\right) \end{cases}$$
$$+\frac{x(2A+iB)}{8ac^3}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**3,x)
```

output
```
Piecewise(((-294912*I*A*a**3*c**9*f**3*exp(4*I*e)*exp(2*I*f*x) + (49152*I*
A*a**3*c**9*f**3 - 49152*B*a**3*c**9*f**3)*exp(-2*I*f*x) + (-98304*I*A*a**
3*c**9*f**3*exp(6*I*e) - 49152*B*a**3*c**9*f**3*exp(6*I*e))*exp(4*I*f*x) +
 (-16384*I*A*a**3*c**9*f**3*exp(8*I*e) - 16384*B*a**3*c**9*f**3*exp(8*I*e)
)*exp(6*I*f*x))*exp(-2*I*e)/(1572864*a**4*c**12*f**4), Ne(a**4*c**12*f**4*
exp(2*I*e), 0)), (x*(-(2*A + I*B)/(8*a*c**3) + (A*exp(8*I*e) + 4*A*exp(6*I
*e) + 6*A*exp(4*I*e) + 4*A*exp(2*I*e) + A - I*B*exp(8*I*e) - 2*I*B*exp(6*I
*e) + 2*I*B*exp(2*I*e) + I*B)*exp(-2*I*e)/(16*a*c**3)), True)) + x*(2*A +
I*B)/(8*a*c**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx$$
$$= -\frac{(-2i\,A+B)\log\left(\tan\left(fx+e\right)+i\right)}{16\,ac^3f} - \frac{(2i\,A-B)\log\left(\tan\left(fx+e\right)-i\right)}{16\,ac^3f}$$
$$+ \frac{3\,(2\,A+i\,B)\tan\left(fx+e\right)^3 - 6\,(-2i\,A+B)\tan\left(fx+e\right)^2 - (2\,A+i\,B)\tan\left(fx+e\right) + 8i\,A + 2\,B}{24\,ac^3f(\tan\left(fx+e\right)+i)^3(\tan\left(fx+e\right)-i)}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x, algo
rithm="giac")
```

output
```
-1/16*(-2*I*A + B)*log(tan(f*x + e) + I)/(a*c^3*f) - 1/16*(2*I*A - B)*log(
tan(f*x + e) - I)/(a*c^3*f) + 1/24*(3*(2*A + I*B)*tan(f*x + e)^3 - 6*(-2*I
*A + B)*tan(f*x + e)^2 - (2*A + I*B)*tan(f*x + e) + 8*I*A + 2*B)/(a*c^3*f*
(tan(f*x + e) + I)^3*(tan(f*x + e) - I))
```

Mupad [B] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.08

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx$$
$$= \frac{\frac{B}{12\,a\,c^3} + \mathrm{tan}(e+f\,x)^2\,\left(-\frac{B}{4\,a\,c^3}+\frac{A\,1\mathrm{i}}{2\,a\,c^3}\right) + \mathrm{tan}(e+f\,x)^3\,\left(\frac{A}{4\,a\,c^3}+\frac{B\,1\mathrm{i}}{8\,a\,c^3}\right) - \mathrm{tan}(e+f\,x)\,\left(\frac{A}{12\,a\,c^3}+\frac{B\,1\mathrm{i}}{24\,a\,c^3}\right) + }{f\,\left({\mathrm{tan}\left(e+f\,x\right)}^4+{\mathrm{tan}\left(e+f\,x\right)}^3\,2\mathrm{i}+\mathrm{tan}\left(e+f\,x\right)\,2\mathrm{i}-1\right)}$$
$$- \frac{x\,\left(-B+A\,2\mathrm{i}\right)\,1\mathrm{i}}{8\,a\,c^3}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^
3),x)
```

output
```
(tan(e + f*x)^2*((A*1i)/(2*a*c^3) - B/(4*a*c^3)) - tan(e + f*x)*(A/(12*a*c
^3) + (B*1i)/(24*a*c^3)) + tan(e + f*x)^3*(A/(4*a*c^3) + (B*1i)/(8*a*c^3))
 + (A*1i)/(3*a*c^3) + B/(12*a*c^3))/(f*(tan(e + f*x)*2i + tan(e + f*x)^3*2
i + tan(e + f*x)^4 - 1)) - (x*(A*2i - B)*1i)/(8*a*c^3)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^3}\,dx$$
$$= \frac{-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^4+2\tan(fx+e)^3 i+2\tan(fx+e)i-1}dx\right)b-\left(\int \frac{1}{\tan(fx+e)^4+2\tan(fx+e)^3 i+2\tan(fx+e)i-1}dx\right)a}{a\,c^3}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^3,x)
```

output
```
( - (int(tan(e + f*x)/(tan(e + f*x)**4 + 2*tan(e + f*x)**3*i + 2*tan(e + f
*x)*i - 1),x)*b + int(1/(tan(e + f*x)**4 + 2*tan(e + f*x)**3*i + 2*tan(e +
 f*x)*i - 1),x)*a))/(a*c**3)
```

3.714 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 181

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx = \frac{(5A+3iB)x}{32ac^4} - \frac{A+iB}{32ac^4f(i-\tan(e+fx))} - \frac{iA+B}{16ac^4f(i+\tan(e+fx))^4} - \frac{A}{12ac^4f(i+\tan(e+fx))^3} + \frac{3iA-B}{32ac^4f(i+\tan(e+fx))^2} + \frac{2A+iB}{16ac^4f(i+\tan(e+fx))}$$

output
```
1/32*(5*A+3*I*B)*x/a/c^4-1/32*(A+I*B)/a/c^4/f/(I-tan(f*x+e))-1/16*(I*A+B)/
a/c^4/f/(I+tan(f*x+e))^4-1/12*A/a/c^4/f/(I+tan(f*x+e))^3+1/32*(3*I*A-B)/a/
c^4/f/(I+tan(f*x+e))^2+1/16*(2*A+I*B)/a/c^4/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx = \frac{i\left(3(-5iA+3B)\arctan(\tan(e+fx)) + \frac{32iA+3(5A+3iB)\tan(e+fx)+7i(5A+3iB)\tan^2(e+fx)+9(5A+3iB)\tan^3(e+fx)+3(}{(-i+\tan(e+fx))(i+\tan(e+fx))^4}\right.}{96ac^4f}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*
x])^4),x]
```

output
```
((I/96)*(3*((-5*I)*A + 3*B)*ArcTan[Tan[e + f*x]] + ((32*I)*A + 3*(5*A + (3
*I)*B)*Tan[e + f*x] + (7*I)*(5*A + (3*I)*B)*Tan[e + f*x]^2 + 9*(5*A + (3*I
)*B)*Tan[e + f*x]^3 + 3*((-5*I)*A + 3*B)*Tan[e + f*x]^4)/((-I + Tan[e + f*
x])*(I + Tan[e + f*x])^4)))/(a*c^4*f)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^2c^5(1-i\tan(e+fx))^5(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^5(i\tan(e+fx)+1)^2}d\tan(e+fx)}{ac^4f}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{A}{4(\tan(e+fx)+i)^4}+\frac{5A+3iB}{32(\tan^2(e+fx)+1)}+\frac{-A-iB}{32(\tan(e+fx)-i)^2}+\frac{-2A-iB}{16(\tan(e+fx)+i)^2}+\frac{B-3iA}{16(\tan(e+fx)+i)^3}+\frac{iA+B}{4(\tan(e+fx)+i)^5}\right)d\,\text{ta}}{ac^4f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{32}(5A+3iB)\arctan(\tan(e+fx))-\frac{A+iB}{32(-\tan(e+fx)+i)}+\frac{2A+iB}{16(\tan(e+fx)+i)}+\frac{-B+3iA}{32(\tan(e+fx)+i)^2}-\frac{B+iA}{16(\tan(e+fx)+i)^4}-\frac{}{12(}}{ac^4f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4)
,x]
```

output
```
(((5*A + (3*I)*B)*ArcTan[Tan[e + f*x]])/32 - (A + I*B)/(32*(I - Tan[e + f*
x])) - (I*A + B)/(16*(I + Tan[e + f*x])^4) - A/(12*(I + Tan[e + f*x])^3) +
 ((3*I)*A - B)/(32*(I + Tan[e + f*x])^2) + (2*A + I*B)/(16*(I + Tan[e + f*
x])))/(a*c^4*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.31

method	result
risch	$\frac{3ixB}{32a\,c^4}+\frac{5xA}{32a\,c^4}-\frac{e^{8i(fx+e)}B}{256a\,c^4f}-\frac{ie^{8i(fx+e)}A}{256a\,c^4f}-\frac{e^{6i(fx+e)}B}{64a\,c^4f}-\frac{5ie^{6i(fx+e)}A}{192a\,c^4f}-\frac{e^{4i(fx+e)}B}{64a\,c^4f}-\frac{5ie^{4i(fx+e)}A}{64a\,c^4f}$
derivativedivides	$\frac{3iB\arctan(\tan(fx+e))}{32fa\,c^4}+\frac{5A\arctan(\tan(fx+e))}{32fa\,c^4}+\frac{3iA}{32fa\,c^4(i+\tan(fx+e))^2}+\frac{A}{32fa\,c^4(-i+\tan(fx+e))}-$
default	$\frac{3iB\arctan(\tan(fx+e))}{32fa\,c^4}+\frac{5A\arctan(\tan(fx+e))}{32fa\,c^4}+\frac{3iA}{32fa\,c^4(i+\tan(fx+e))^2}+\frac{A}{32fa\,c^4(-i+\tan(fx+e))}-$
norman	$\frac{\frac{(3iB+5A)x}{32ac}-\frac{iA}{3acf}+\frac{(iA+3B)\tan(fx+e)^2}{6acf}+\frac{11(3iB+5A)\tan(fx+e)^5}{96acf}+\frac{(3iB+5A)\tan(fx+e)^7}{32acf}+\frac{(3iB+5A)x\tan(fx+e)^2}{8ac}+\frac{3(3iB}{}}{(1+\tan(fx+}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x,method=_RET
URNVERBOSE)
```

output
```
3/32*I*x/a/c^4*B+5/32*x/a/c^4*A-1/256/a/c^4/f*exp(8*I*(f*x+e))*B-1/256*I/a
/c^4/f*exp(8*I*(f*x+e))*A-1/64/a/c^4/f*exp(6*I*(f*x+e))*B-5/192*I/a/c^4/f*
exp(6*I*(f*x+e))*A-1/64/a/c^4/f*exp(4*I*(f*x+e))*B-5/64*I/a/c^4/f*exp(4*I*
(f*x+e))*A+1/64/a/c^4/f*cos(2*f*x+2*e)*B-9/64*I/a/c^4/f*cos(2*f*x+2*e)*A+3
/64*I/a/c^4/f*sin(2*f*x+2*e)*B+11/64/a/c^4/f*sin(2*f*x+2*e)*A
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.64

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx = \frac{(24\,(5\,A+3i\,B)fxe^{(2i\,fx+2i\,e)}-3\,(i\,A+B)e^{(10i\,fx+10i\,e)}-4\,(5i\,A+3\,B)e^{(8i\,fx+8i\,e)}-12\,(5i\,A+B)e^{(6i\,fx}}{768\,ac^4f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algo
rithm="fricas")
```

output
```
1/768*(24*(5*A + 3*I*B)*f*x*e^(2*I*f*x + 2*I*e) - 3*(I*A + B)*e^(10*I*f*x
+ 10*I*e) - 4*(5*I*A + 3*B)*e^(8*I*f*x + 8*I*e) - 12*(5*I*A + B)*e^(6*I*f*
x + 6*I*e) - 24*(5*I*A - B)*e^(4*I*f*x + 4*I*e) + 12*I*A - 12*B)*e^(-2*I*f
*x - 2*I*e)/(a*c^4*f)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.43

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx$$
$$=\begin{cases} \frac{((100663296iAa^4c^{16}f^4-100663296Ba^4c^{16}f^4)e^{-2ifx}+(-1006632960iAa^4c^{16}f^4e^{4ie}+201326592Ba^4c^{16}f^4e^{4ie})e^{2ifx}+(-503316480iAa^4c^{1}\cdots}{\cdots} \\ x\left(-\frac{5A+3iB}{32ac^4}+\frac{(Ae^{10ie}+5Ae^{8ie}+10Ae^{6ie}+10Ae^{4ie}+5Ae^{2ie}+A-iBe^{10ie}-3iBe^{8ie}-2iBe^{6ie}+2iBe^{4ie}+3iBe^{2ie}+iB)e^{-2ie}}{32ac^4}\right) \end{cases}$$
$$+\frac{x(5A+3iB)}{32ac^4}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**4,x)
```

output
```
Piecewise((((100663296*I*A*a**4*c**16*f**4 - 100663296*B*a**4*c**16*f**4)*
exp(-2*I*f*x) + (-1006632960*I*A*a**4*c**16*f**4*exp(4*I*e) + 201326592*B*
a**4*c**16*f**4*exp(4*I*e))*exp(2*I*f*x) + (-503316480*I*A*a**4*c**16*f**4
*exp(6*I*e) - 100663296*B*a**4*c**16*f**4*exp(6*I*e))*exp(4*I*f*x) + (-167
772160*I*A*a**4*c**16*f**4*exp(8*I*e) - 100663296*B*a**4*c**16*f**4*exp(8*
I*e))*exp(6*I*f*x) + (-25165824*I*A*a**4*c**16*f**4*exp(10*I*e) - 25165824
*B*a**4*c**16*f**4*exp(10*I*e))*exp(8*I*f*x))*exp(-2*I*e)/(6442450944*a**5
*c**20*f**5), Ne(a**5*c**20*f**5*exp(2*I*e), 0)), (x*(-(5*A + 3*I*B)/(32*a
*c**4) + (A*exp(10*I*e) + 5*A*exp(8*I*e) + 10*A*exp(6*I*e) + 10*A*exp(4*I*
e) + 5*A*exp(2*I*e) + A - I*B*exp(10*I*e) - 3*I*B*exp(8*I*e) - 2*I*B*exp(6
*I*e) + 2*I*B*exp(4*I*e) + 3*I*B*exp(2*I*e) + I*B)*exp(-2*I*e)/(32*a*c**4)
), True)) + x*(5*A + 3*I*B)/(32*a*c**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.86

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx$$
$$= -\frac{(-5i\,A+3\,B)\log\left(\tan\left(fx+e\right)+i\right)}{64\,ac^4f} - \frac{(5i\,A-3\,B)\log\left(\tan\left(fx+e\right)-i\right)}{64\,ac^4f}$$
$$+ \frac{3\,(5\,A+3i\,B)\tan\left(fx+e\right)^4 - 9\,(-5i\,A+3\,B)\tan\left(fx+e\right)^3 - 7\,(5\,A+3i\,B)\tan\left(fx+e\right)^2 - 3\,(-5i}{96\,ac^4f(\tan\left(fx+e\right)+i)^4(\tan\left(fx+e\right)-i)}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x, algo
rithm="giac")
```

output
```
-1/64*(-5*I*A + 3*B)*log(tan(f*x + e) + I)/(a*c^4*f) - 1/64*(5*I*A - 3*B)*
log(tan(f*x + e) - I)/(a*c^4*f) + 1/96*(3*(5*A + 3*I*B)*tan(f*x + e)^4 - 9
*(-5*I*A + 3*B)*tan(f*x + e)^3 - 7*(5*A + 3*I*B)*tan(f*x + e)^2 - 3*(-5*I*
A + 3*B)*tan(f*x + e) - 32*A)/(a*c^4*f*(tan(f*x + e) + I)^4*(tan(f*x + e)
- I))
```

Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.13

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx =$$
$$-\frac{\tan(e+f\,x)\,\left(-\frac{3\,B}{32\,a\,c^4}+\frac{A\,5\mathrm{i}}{32\,a\,c^4}\right)+\tan(e+f\,x)^4\,\left(\frac{5\,A}{32\,a\,c^4}+\frac{B\,3\mathrm{i}}{32\,a\,c^4}\right)+\tan(e+f\,x)^3\,\left(-\frac{9\,B}{32\,a\,c^4}+\frac{A\,15\mathrm{i}}{32\,a\,c^4}\right)-\mathrm{t}}{f\,\left(-\tan\left(e+f\,x\right)^5-\tan\left(e+f\,x\right)^4\,3\mathrm{i}+2\tan\left(e+f\,x\right)^3-\tan\left(e+f\,x\right)^2\,2\mathrm{i}+3\,\mathrm{ta}\right.}$$
$$-\frac{x\,\left(-3\,B+A\,5\mathrm{i}\right)\,1\mathrm{i}}{32\,a\,c^4}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^
4),x)
```

output
```
- (tan(e + f*x)*((A*5i)/(32*a*c^4) - (3*B)/(32*a*c^4)) + tan(e + f*x)^4*((
5*A)/(32*a*c^4) + (B*3i)/(32*a*c^4)) + tan(e + f*x)^3*((A*15i)/(32*a*c^4)
- (9*B)/(32*a*c^4)) - tan(e + f*x)^2*((35*A)/(96*a*c^4) + (B*7i)/(32*a*c^4
)) - A/(3*a*c^4))/(f*(3*tan(e + f*x) - tan(e + f*x)^2*2i + 2*tan(e + f*x)^
3 - tan(e + f*x)^4*3i - tan(e + f*x)^5 + 1i)) - (x*(A*5i - 3*B)*1i)/(32*a*
c^4)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^4}\,dx$$
$$=\frac{\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^5 i-3\tan(fx+e)^4-2\tan(fx+e)^3 i-2\tan(fx+e)^2-3\tan(fx+e)i+1}dx\right)b+\left(\int \frac{1}{\tan(fx+e)^5 i-3\tan(fx+e)^4-2\tan(fx+e)^{3}}\right.}{a\,c^4}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^4,x)
```

output
```
(int(tan(e + f*x)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)*
*3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b + int(1/(tan(e + f*x
)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*t
an(e + f*x)*i + 1),x)*a)/(a*c**4)
```

3.715 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 115

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx = \frac{(iA(2-n)+B(2+n))\,\text{Hypergeometric2F1}\left(2,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(c-ic\tan(e+fx))^n}{16a^2fn} + \frac{(iA-B)(c-ic\tan(e+fx))^n}{4a^2f(1+i\tan(e+fx))^2}$$

output

```
1/16*(I*A*(2-n)+B*(2+n))*hypergeom([2, n],[1+n],1/2-1/2*I*tan(f*x+e))*(c-I
*c*tan(f*x+e))^n/a^2/f/n+1/4*(I*A-B)*(c-I*c*tan(f*x+e))^n/a^2/f/(1+I*tan(f
*x+e))^2
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.32

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx = \frac{(c-ic\tan(e+fx))^n\left(-iA(-3+n)+B(1+n)+\frac{i(-1+n)(A(-2+n)+iB(2+n))\,\text{Hypergeometric2F1}\left(1,n,1+n,-\frac{1}{2}i(i+\text{t}\right.}{n}\right.}{16a^2f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n)/(a + I*a*Tan[e +
 f*x])^2,x]
```

output
```
((c - I*c*Tan[e + f*x])^n*((-I)*A*(-3 + n) + B*(1 + n) + (I*(-1 + n)*(A*(-
2 + n) + I*B*(2 + n))*Hypergeometric2F1[1, n, 1 + n, (-1/2*I)*(I + Tan[e +
 f*x])])/n + ((-I)*A*(-3 + n) + B*(1 + n))*(Cos[2*(e + f*x)] - I*Sin[2*(e
+ f*x)]) - (2*(A + I*B)*(I + Tan[e + f*x]))/(-I + Tan[e + f*x])^2))/(16*a^
2*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}}{a^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}}{(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{c\left(\frac{1}{4}(A(2-n)-iB(n+2))\int \frac{(c-ic\tan(e+fx))^{n-1}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)+\frac{(-B+iA)(c-ic\tan(e+fx))^n}{4c(1+i\tan(e+fx))^2}\right)}{a^2f}$$

$$\Big\downarrow \text{78}$$

$$\frac{c\left(\frac{i(A(2-n)-iB(n+2))(c-ic\tan(e+fx))^n \operatorname{Hypergeometric2F1}\left(2,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{16cn}+\frac{(-B+iA)(c-ic\tan(e+fx))^n}{4c(1+i\tan(e+fx))^2}\right)}{a^2f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n)/(a + I*a*Tan[e + f*x])
^2,x]
```

output
```
(c*(((I/16)*(A*(2 - n) - I*B*(2 + n))*Hypergeometric2F1[2, n, 1 + n, (1 -
I*Tan[e + f*x])/2]*(c - I*c*Tan[e + f*x])^n)/(c*n) + ((I*A - B)*(c - I*c*T
an[e + f*x])^n)/(4*c*(1 + I*Tan[e + f*x])^2)))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(A+B\tan(fx+e))\,(c-ic\tan(fx+e))^n}{(a+ia\tan(fx+e))^2}dx$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
```

output
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(B\tan(fx+e)+A)(-i\,c\tan(fx+e)+c)^n}{(i\,a\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
integral(1/4*((A - I*B)*e^(4*I*f*x + 4*I*e) + 2*A*e^(2*I*f*x + 2*I*e) + A
+ I*B)*(2*c/(e^(2*I*f*x + 2*I*e) + 1))^n*e^(-4*I*f*x - 4*I*e)/a^2, x)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{A(-ic\tan(e+fx)+c)^n}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx + \int \frac{B(-ic\tan(e+fx)+c)^n\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**2,x)
```

output
```
-(Integral(A*(-I*c*tan(e + f*x) + c)**n/(tan(e + f*x)**2 - 2*I*tan(e + f*x
) - 1), x) + Integral(B*(-I*c*tan(e + f*x) + c)**n*tan(e + f*x)/(tan(e + f
*x)**2 - 2*I*tan(e + f*x) - 1), x))/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(B\tan(fx+e)+A)(-ic\tan(fx+e)+c)^n}{(ia\tan(fx+e)+a)^2}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^n/(I*a*tan(f*x + e)
 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(A+B\tan(e+f\,x))\,{(c-c\tan(e+f\,x)\;1\mathrm{i})}^n}{{(a+a\tan\left(e+f\,x\right)\;1\mathrm{i})}^2}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^n)/(a + a*tan(e + f*x)*1
i)^2,x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^n)/(a + a*tan(e + f*x)*1
i)^2, x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx = \frac{-\left(\int \frac{(-\tan(fx+e)ci+c)^n}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)a-\left(\int \frac{(-\tan(fx+e)ci+c)^n\tan(fx+e)}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)b}{a^2}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^2,x)
```

output
```
( - (int(( - tan(e + f*x)*c*i + c)**n/(tan(e + f*x)**2 - 2*tan(e + f*x)*i
- 1),x)*a + int((( - tan(e + f*x)*c*i + c)**n*tan(e + f*x))/(tan(e + f*x)*
*2 - 2*tan(e + f*x)*i - 1),x)*b))/a**2
```

3.716 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 194

$$\begin{aligned}\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^2}\,dx &= \frac{8(3A+7iB)c^5x}{a^2}+\frac{8(3iA-7B)c^5\log(\cos(e+fx))}{a^2f}-\frac{8(iA-B)c^5}{a^2f(i-\tan(e+fx))^2}\\ &+\frac{16(2A+3iB)c^5}{a^2f(i-\tan(e+fx))}-\frac{(7A+24iB)c^5\tan(e+fx)}{a^2f}\\ &+\frac{(iA-7B)c^5\tan^2(e+fx)}{2a^2f}+\frac{iBc^5\tan^3(e+fx)}{3a^2f}\end{aligned}$$

output
```
8*(3*A+7*I*B)*c^5*x/a^2+8*(3*I*A-7*B)*c^5*ln(cos(f*x+e))/a^2/f-8*(I*A-B)*c
^5/a^2/f/(I-tan(f*x+e))^2+16*(2*A+3*I*B)*c^5/a^2/f/(I-tan(f*x+e))-(7*A+24*
I*B)*c^5*tan(f*x+e)/a^2/f+1/2*(I*A-7*B)*c^5*tan(f*x+e)^2/a^2/f+1/3*I*B*c^5
*tan(f*x+e)^3/a^2/f
```

Mathematica [A] (verified)

Time = 4.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.02

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{c^5(315iA-737B+48i(3A+7iB)\log(i-\tan(e+fx))-2(246A+569iB+48(3A+7iB)\log(i-\tan$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^5)/(a + I*a*Tan[e +
 f*x])^2,x]
```

output
```
(c^5*((315*I)*A - 737*B + (48*I)*(3*A + (7*I)*B)*Log[I - Tan[e + f*x]] - 2
*(246*A + (569*I)*B + 48*(3*A + (7*I)*B)*Log[I - Tan[e + f*x]])*Tan[e + f*
x] + 2*(5*((-9*I)*A + 23*B) + 24*((-3*I)*A + 7*B)*Log[I - Tan[e + f*x]])*T
an[e + f*x]^2 - 4*(9*A + (26*I)*B)*Tan[e + f*x]^3 + ((3*I)*A - 17*B)*Tan[e
 + f*x]^4 + (2*I)*B*Tan[e + f*x]^5))/(6*a^2*f*(-I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^5(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c-ic\tan(e+fx))^5(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{c^4(1-i\tan(e+fx))^4(A+B\tan(e+fx))}{a^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^5 \int \frac{(1-i\tan(e+fx))^4(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2 f}$$

$$\Big\downarrow \text{86}$$

$$\frac{c^5 \int \left(iB\tan^2(e+fx)+i(A+7iB)\tan(e+fx)-7A\left(\frac{24iB}{7A}+1\right)-\frac{8i(3A+7iB)}{\tan(e+fx)-i}+\frac{16(2A+3iB)}{(\tan(e+fx)-i)^2}+\frac{16i(A+iB)}{(\tan(e+fx)-i)^3}\right.}{a^2 f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^5 \left(\frac{1}{2}(-7B+iA)\tan^2(e+fx)-(7A+24iB)\tan(e+fx)+\frac{16(2A+3iB)}{-\tan(e+fx)+i}-\frac{8(-B+iA)}{(-\tan(e+fx)+i)^2}-8(-7B+3iA)\log\right.}{a^2 f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^5)/(a + I*a*Tan[e + f*x])
^2,x]
```

output
```
(c^5*(-8*((3*I)*A - 7*B)*Log[I - Tan[e + f*x]] - (8*(I*A - B))/(I - Tan[e
+ f*x])^2 + (16*(2*A + (3*I)*B))/(I - Tan[e + f*x]) - (7*A + (24*I)*B)*Tan
[e + f*x] + ((I*A - 7*B)*Tan[e + f*x]^2)/2 + (I/3)*B*Tan[e + f*x]^3))/(a^2
*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.38

method	result
risch	$\frac{20c^5 e^{-2i(fx+e)} B}{a^2 f} - \frac{12ic^5 e^{-2i(fx+e)} A}{a^2 f} - \frac{2c^5 e^{-4i(fx+e)} B}{a^2 f} + \frac{2ic^5 e^{-4i(fx+e)} A}{a^2 f} + \frac{112ic^5 Bx}{a^2} + \frac{48c^5 Ax}{a^2} + 11$
derivativedivides	$\frac{iB c^5 \tan(fx+e)^3}{3a^2 f} + \frac{ic^5 A \tan(fx+e)^2}{2f a^2} - \frac{24ic^5 B \tan(fx+e)}{f a^2} - \frac{7c^5 B \tan(fx+e)^2}{2f a^2} - \frac{7c^5 A \tan(fx+e)}{f a^2} - \frac{}{f a^2(-}$
default	$\frac{iB c^5 \tan(fx+e)^3}{3a^2 f} + \frac{ic^5 A \tan(fx+e)^2}{2f a^2} - \frac{24ic^5 B \tan(fx+e)}{f a^2} - \frac{7c^5 B \tan(fx+e)^2}{2f a^2} - \frac{7c^5 A \tan(fx+e)}{f a^2} - \frac{}{f a^2(-}$
norman	$\frac{\frac{-25ic^5 A+47B c^5}{af} + \frac{8(7ic^5 B+3A c^5)x}{a} - \frac{(-ic^5 A+7B c^5)\tan(fx+e)^6}{2af} - \frac{7(10ic^5 B+3A c^5)\tan(fx+e)^5}{3af} + \frac{(-83ic^5 A+133B c^5)\tan}{2af}}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
20*c^5/a^2/f*exp(-2*I*(f*x+e))*B-12*I*c^5/a^2/f*exp(-2*I*(f*x+e))*A-2*c^5/
a^2/f*exp(-4*I*(f*x+e))*B+2*I*c^5/a^2/f*exp(-4*I*(f*x+e))*A+112*I*c^5/a^2*
B*x+48*c^5/a^2*A*x+112*I*c^5/f/a^2*B*e+48*c^5/f/a^2*A*e+2/3*c^5*(-18*I*A*e
xp(4*I*(f*x+e))+54*B*exp(4*I*(f*x+e))-39*I*A*exp(2*I*(f*x+e))+123*B*exp(2*
I*(f*x+e))-21*I*A+73*B)/f/a^2/(exp(2*I*(f*x+e))+1)^3-56*c^5/f/a^2*ln(exp(2
*I*(f*x+e))+1)*B+24*I*c^5/f/a^2*ln(exp(2*I*(f*x+e))+1)*A
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.70

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{2\left(24\left(3A+7iB\right)c^5fxe^{(10ifx+10ie)}-3\left(3iA-7B\right)c^5e^{(2ifx+2ie)}-3\left(-iA+B\right)c^5+12\left(6\left(3A+7iB\right)c^5\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
2/3*(24*(3*A + 7*I*B)*c^5*f*x*e^(10*I*f*x + 10*I*e) - 3*(3*I*A - 7*B)*c^5*
e^(2*I*f*x + 2*I*e) - 3*(-I*A + B)*c^5 + 12*(6*(3*A + 7*I*B)*c^5*f*x - (3*
I*A - 7*B)*c^5)*e^(8*I*f*x + 8*I*e) + 6*(12*(3*A + 7*I*B)*c^5*f*x - 5*(3*I
*A - 7*B)*c^5)*e^(6*I*f*x + 6*I*e) + 2*(12*(3*A + 7*I*B)*c^5*f*x - 11*(3*I
*A - 7*B)*c^5)*e^(4*I*f*x + 4*I*e) - 12*((-3*I*A + 7*B)*c^5*e^(10*I*f*x +
10*I*e) + 3*(-3*I*A + 7*B)*c^5*e^(8*I*f*x + 8*I*e) + 3*(-3*I*A + 7*B)*c^5*
e^(6*I*f*x + 6*I*e) + (-3*I*A + 7*B)*c^5*e^(4*I*f*x + 4*I*e))*log(e^(2*I*f
*x + 2*I*e) + 1))/(a^2*f*e^(10*I*f*x + 10*I*e) + 3*a^2*f*e^(8*I*f*x + 8*I*
e) + 3*a^2*f*e^(6*I*f*x + 6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e))
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.29

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{-42iAc^5+146Bc^5+\left(-78iAc^5e^{2ie}+246Bc^5e^{2ie}\right)e^{2ifx}+\left(-36iAc^5e^{4ie}+108Bc^5e^{4ie}\right)e^{4ifx}}{3a^2fe^{6ie}e^{6ifx}+9a^2fe^{4ie}e^{4ifx}+9a^2fe^{2ie}e^{2ifx}+3a^2f}$$
$$+\begin{cases}\frac{\left(\left(2iAa^2c^5fe^{2ie}-2Ba^2c^5fe^{2ie}\right)e^{-4ifx}+\left(-12iAa^2c^5fe^{4ie}+20Ba^2c^5fe^{4ie}\right)e^{-2ifx}\right)e^{-6ie}}{a^4f^2} & \text{for } a^4f^2e^{6ie}\neq 0\\ x\left(-\frac{48Ac^5+112iBc^5}{a^2}+\frac{\left(48Ac^5e^{4ie}-24Ac^5e^{2ie}+8Ac^5+112iBc^5e^{4ie}-40iBc^5e^{2ie}+8iBc^5\right)e^{-4ie}}{a^2}\right) & \text{otherwise}\end{cases}$$
$$+\frac{8ic^5\cdot(3A+7iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{a^2f}+\frac{x(48Ac^5+112iBc^5)}{a^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**5/(a+I*a*tan(f*x+e))**2,x)
```

output
```
(-42*I*A*c**5 + 146*B*c**5 + (-78*I*A*c**5*exp(2*I*e) + 246*B*c**5*exp(2*I
*e))*exp(2*I*f*x) + (-36*I*A*c**5*exp(4*I*e) + 108*B*c**5*exp(4*I*e))*exp(
4*I*f*x))/(3*a**2*f*exp(6*I*e)*exp(6*I*f*x) + 9*a**2*f*exp(4*I*e)*exp(4*I*
f*x) + 9*a**2*f*exp(2*I*e)*exp(2*I*f*x) + 3*a**2*f) + Piecewise((((2*I*A*a
**2*c**5*f*exp(2*I*e) - 2*B*a**2*c**5*f*exp(2*I*e))*exp(-4*I*f*x) + (-12*I
*A*a**2*c**5*f*exp(4*I*e) + 20*B*a**2*c**5*f*exp(4*I*e))*exp(-2*I*f*x))*ex
p(-6*I*e)/(a**4*f**2), Ne(a**4*f**2*exp(6*I*e), 0)), (x*(-(48*A*c**5 + 112
*I*B*c**5)/a**2 + (48*A*c**5*exp(4*I*e) - 24*A*c**5*exp(2*I*e) + 8*A*c**5
+ 112*I*B*c**5*exp(4*I*e) - 40*I*B*c**5*exp(2*I*e) + 8*I*B*c**5)*exp(-4*I*
e)/a**2), True)) + 8*I*c**5*(3*A + 7*I*B)*log(exp(2*I*f*x) + exp(-2*I*e))/
(a**2*f) + x*(48*A*c**5 + 112*I*B*c**5)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.97

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^2}\,dx$$
$$= -\frac{8\,(3i\,Ac^5-7\,Bc^5)\log\left(\tan\left(fx+e\right)-i\right)}{a^2f}$$
$$-\frac{8\left(-3i\,Ac^5+5\,Bc^5+2\left(2\,Ac^5+3i\,Bc^5\right)\tan\left(fx+e\right)\right)}{a^2f\left(\tan\left(fx+e\right)-i\right)^2}$$
$$-\frac{-2i\,Ba^4c^5f^2\tan\left(fx+e\right)^3-3i\,Aa^4c^5f^2\tan\left(fx+e\right)^2+21\,Ba^4c^5f^2\tan\left(fx+e\right)^2+42\,Aa^4c^5f^2\tan\left(\right.}{6\,a^6f^3}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-8*(3*I*A*c^5 - 7*B*c^5)*log(tan(f*x + e) - I)/(a^2*f) - 8*(-3*I*A*c^5 + 5
*B*c^5 + 2*(2*A*c^5 + 3*I*B*c^5)*tan(f*x + e))/(a^2*f*(tan(f*x + e) - I)^2
) - 1/6*(-2*I*B*a^4*c^5*f^2*tan(f*x + e)^3 - 3*I*A*a^4*c^5*f^2*tan(f*x + e
)^2 + 21*B*a^4*c^5*f^2*tan(f*x + e)^2 + 42*A*a^4*c^5*f^2*tan(f*x + e) + 14
4*I*B*a^4*c^5*f^2*tan(f*x + e))/(a^6*f^3)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.45

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^2}\,dx$$
$$= -\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\left(-\frac{56\,B\,c^5}{a^2}+\frac{A\,c^5\,24\mathrm{i}}{a^2}\right)}{f}+\frac{\tan(e+f\,x)^2\left(-\frac{3\,B\,c^5}{2\,a^2}+\frac{c^5\,(A+B\,4\mathrm{i})\,1\mathrm{i}}{2\,a^2}\right)}{f}$$
$$-\frac{\tan(e+f\,x)\left(\frac{3\,c^5\,(A+B\,4\mathrm{i})}{a^2}+\frac{B\,c^5\,6\mathrm{i}}{a^2}-\frac{c^5\,(-3\,B+A\,2\mathrm{i})\,2\mathrm{i}}{a^2}\right)}{f}$$
$$+\frac{-\frac{(-24\,B\,c^5+A\,c^5\,8\mathrm{i})\,1\mathrm{i}}{2\,a^2}+\frac{16\,A\,c^5+B\,c^5\,64\mathrm{i}}{2\,a^2}+\frac{(-56\,B\,c^5+A\,c^5\,24\mathrm{i})\,3\mathrm{i}}{2\,a^2}+\tan(e+f\,x)\left(\frac{(16\,A\,c^5+B\,c^5\,64\mathrm{i})\,1\mathrm{i}}{a^2}-\frac{2\,(-56\,B\,c^5+A}{a^2}\right.}{f\,\left(\tan\left(e+f\,x\right)^2\,1\mathrm{i}+2\tan\left(e+f\,x\right)-\mathrm{i}\right)}$$
$$+\frac{B\,c^5\tan(e+f\,x)^3\,1\mathrm{i}}{3\,a^2\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^5)/(a + a*tan(e + f*x)*1
i)^2,x)
```

output
```
(tan(e + f*x)^2*((c^5*(A + B*4i)*1i)/(2*a^2) - (3*B*c^5)/(2*a^2)))/f - (lo
g(tan(e + f*x) - 1i)*((A*c^5*24i)/a^2 - (56*B*c^5)/a^2))/f - (tan(e + f*x)
*((3*c^5*(A + B*4i))/a^2 + (B*c^5*6i)/a^2 - (c^5*(A*2i - 3*B)*2i)/a^2))/f
+ ((16*A*c^5 + B*c^5*64i)/(2*a^2) - ((A*c^5*8i - 24*B*c^5)*1i)/(2*a^2) + (
(A*c^5*24i - 56*B*c^5)*3i)/(2*a^2) + tan(e + f*x)*(((16*A*c^5 + B*c^5*64i)
*1i)/a^2 - (2*(A*c^5*24i - 56*B*c^5))/a^2))/(f*(2*tan(e + f*x) + tan(e + f
*x)^2*1i - 1i)) + (B*c^5*tan(e + f*x)^3*1i)/(3*a^2*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{c^5\left(\left(\left(\int \frac{\tan(fx+e)^6}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)bi + \left(\int \frac{\tan(fx+e)^5}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)ai - 5\left(\int \frac{\tan(fx+e)^5}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right.\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^2,x)
```

output
```
(c**5*(int(tan(e + f*x)**6/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i
 + int(tan(e + f*x)**5/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i - 5
*int(tan(e + f*x)**5/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b - 5*int
(tan(e + f*x)**4/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a - 10*int(ta
n(e + f*x)**4/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i - 10*int(tan
(e + f*x)**3/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i + 10*int(tan(
e + f*x)**3/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b + 10*int(tan(e +
 f*x)**2/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a + 5*int(tan(e + f*x
)**2/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i + 5*int(tan(e + f*x)/
(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i - int(tan(e + f*x)/(tan(e
+ f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b - int(1/(tan(e + f*x)**2 - 2*tan(e
+ f*x)*i - 1),x)*a))/a**2
```

3.717 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 158

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}\,dx = \frac{6(A+3iB)c^4x}{a^2}+\frac{6(iA-3B)c^4\log(\cos(e+fx))}{a^2f}-\frac{4(iA-B)c^4}{a^2f(i-\tan(e+fx))^2}+\frac{4(3A+5iB)c^4}{a^2f(i-\tan(e+fx))}-\frac{(A+6iB)c^4\tan(e+fx)}{a^2f}-\frac{Bc^4\tan^2(e+fx)}{2a^2f}$$

output
```
6*(A+3*I*B)*c^4*x/a^2+6*(I*A-3*B)*c^4*ln(cos(f*x+e))/a^2/f-4*(I*A-B)*c^4/a
^2/f/(I-tan(f*x+e))^2+4*(3*A+5*I*B)*c^4/a^2/f/(I-tan(f*x+e))-(A+6*I*B)*c^4
*tan(f*x+e)/a^2/f-1/2*B*c^4*tan(f*x+e)^2/a^2/f
```

Mathematica [A] (verified)

Time = 5.64 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}\,dx = \frac{c^4\left(\frac{2(A+3iB)(i+\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}+\frac{B(i+\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}+\frac{12(-iA+3B)\left(\log(i-\tan(e+fx))+\frac{-2-4i\tan(e+fx)}{(-i+\tan(e+fx))^2}\right)}{a^2}\right)}{2f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4)/(a + I*a*Tan[e +
 f*x])^2,x]
```

output
```
(c^4*((2*(A + (3*I)*B)*(I + Tan[e + f*x])^3)/(a + I*a*Tan[e + f*x])^2 + (B
*(I + Tan[e + f*x])^4)/(a + I*a*Tan[e + f*x])^2 + (12*((-I)*A + 3*B)*(Log[
I - Tan[e + f*x]] + (-2 - (4*I)*Tan[e + f*x])/(-I + Tan[e + f*x])^2))/a^2)
)/(2*f)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^4(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^4(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{c^3(1-i\tan(e+fx))^3(A+B\tan(e+fx))}{a^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^4\int \frac{(1-i\tan(e+fx))^3(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \text{86}$$

$$\frac{c^4\int \left(\frac{8i(A+iB)}{(\tan(e+fx)-i)^3}-A\left(\frac{6iB}{A}+1\right)-B\tan(e+fx)-\frac{6i(A+3iB)}{\tan(e+fx)-i}+\frac{4(3A+5iB)}{(\tan(e+fx)-i)^2}\right)d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^4\left(-(A+6iB)\tan(e+fx)+\frac{4(3A+5iB)}{-\tan(e+fx)+i}-\frac{4(-B+iA)}{(-\tan(e+fx)+i)^2}-6(-3B+iA)\log(-\tan(e+fx)+i)-\frac{1}{2}B\tan^2\right.}{a^2f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4)/(a + I*a*Tan[e + f*x])
^2,x]
```

output
```
(c^4*(-6*(I*A - 3*B)*Log[I - Tan[e + f*x]] - (4*(I*A - B))/(I - Tan[e + f*
x])^2 + (4*(3*A + (5*I)*B))/(I - Tan[e + f*x]) - (A + (6*I)*B)*Tan[e + f*x
] - (B*Tan[e + f*x]^2)/2))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.51

method	result
derivativedivides	$-\frac{B c^4 \tan(fx+e)^2}{2a^2 f} - \frac{c^4 A \tan(fx+e)}{f a^2} - \frac{6ic^4 B \tan(fx+e)}{f a^2} + \frac{6c^4 A \arctan(\tan(fx+e))}{f a^2} - \frac{3ic^4 A \ln\left(1+\tan(fx+\right.}{f a^2}$
default	$-\frac{B c^4 \tan(fx+e)^2}{2a^2 f} - \frac{c^4 A \tan(fx+e)}{f a^2} - \frac{6ic^4 B \tan(fx+e)}{f a^2} + \frac{6c^4 A \arctan(\tan(fx+e))}{f a^2} - \frac{3ic^4 A \ln\left(1+\tan(fx+\right.}{f a^2}$
risch	$\frac{8c^4 e^{-2i(fx+e)} B}{a^2 f} - \frac{4ic^4 e^{-2i(fx+e)} A}{a^2 f} - \frac{c^4 e^{-4i(fx+e)} B}{a^2 f} + \frac{ic^4 e^{-4i(fx+e)} A}{a^2 f} + \frac{36ic^4 Bx}{a^2} + \frac{12c^4 Ax}{a^2} + \frac{36ic^4 Be}{f a^2}$
norman	$\frac{\frac{-8ic^4 A+17B c^4}{af} + \frac{6(3ic^4 B+A c^4)x}{a} - \frac{(6ic^4 B+A c^4)\tan(fx+e)^5}{af} + \frac{(-32ic^4 A+51B c^4)\tan(fx+e)^2}{2af} + \frac{12(3ic^4 B+A c^4)x\tan(fx-}{a}}{a\left(1+\tan(fx+\right.}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-1/2*B*c^4*tan(f*x+e)^2/a^2/f-1/f*c^4/a^2*A*tan(f*x+e)-6*I/f*c^4/a^2*B*tan
(f*x+e)+6/f*c^4/a^2*A*arctan(tan(f*x+e))-3*I/f*c^4/a^2*A*ln(1+tan(f*x+e)^2
)+18*I/f*c^4/a^2*B*arctan(tan(f*x+e))+9/f*c^4/a^2*B*ln(1+tan(f*x+e)^2)-20*
I/f*c^4/a^2/(-I+tan(f*x+e))*B-12/f*c^4/a^2/(-I+tan(f*x+e))*A-4*I/f*c^4/a^2
/(-I+tan(f*x+e))^2*A+4/f*c^4/a^2/(-I+tan(f*x+e))^2*B
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.58

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}\,dx = \frac{12(A+3iB)c^4fxe^{(8ifx+8ie)} - 2(iA-3B)c^4e^{(2ifx+2ie)} + (iA-B)c^4 + 6(4(A+3iB)c^4fx - (iA-3}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
(12*(A + 3*I*B)*c^4*f*x*e^(8*I*f*x + 8*I*e) - 2*(I*A - 3*B)*c^4*e^(2*I*f*x
 + 2*I*e) + (I*A - B)*c^4 + 6*(4*(A + 3*I*B)*c^4*f*x - (I*A - 3*B)*c^4)*e^
(6*I*f*x + 6*I*e) + 3*(4*(A + 3*I*B)*c^4*f*x - 3*(I*A - 3*B)*c^4)*e^(4*I*f
*x + 4*I*e) - 6*((-I*A + 3*B)*c^4*e^(8*I*f*x + 8*I*e) + 2*(-I*A + 3*B)*c^4
*e^(6*I*f*x + 6*I*e) + (-I*A + 3*B)*c^4*e^(4*I*f*x + 4*I*e))*log(e^(2*I*f*
x + 2*I*e) + 1))/(a^2*f*e^(8*I*f*x + 8*I*e) + 2*a^2*f*e^(6*I*f*x + 6*I*e)
+ a^2*f*e^(4*I*f*x + 4*I*e))
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.39

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}\,dx = \frac{-2iAc^4+12Bc^4+\left(-2iAc^4e^{2ie}+10Bc^4e^{2ie}\right)e^{2ifx}}{a^2fe^{4ie}e^{4ifx}+2a^2fe^{2ie}e^{2ifx}+a^2f} + \begin{cases} \frac{\left(\left(iAa^2c^4fe^{2ie}-Ba^2c^4fe^{2ie}\right)e^{-4ifx}+\left(-4iAa^2c^4fe^{4ie}+8Ba^2c^4fe^{4ie}\right)e^{-2ifx}\right)e^{-6ie}}{a^4f^2} & \text{for } a^4f^2e^{6ie}\neq 0 \\ x\left(-\frac{12Ac^4+36iBc^4}{a^2}+\frac{\left(12Ac^4e^{4ie}-8Ac^4e^{2ie}+4Ac^4+36iBc^4e^{4ie}-16iBc^4e^{2ie}+4iBc^4\right)e^{-4ie}}{a^2}\right) & \text{otherwise} \end{cases} + \frac{6ic^4(A+3iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{a^2f} + \frac{x(12Ac^4+36iBc^4)}{a^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**4/(a+I*a*tan(f*x+e))**2,x)
```

output
```
(-2*I*A*c**4 + 12*B*c**4 + (-2*I*A*c**4*exp(2*I*e) + 10*B*c**4*exp(2*I*e))
*exp(2*I*f*x))/(a**2*f*exp(4*I*e)*exp(4*I*f*x) + 2*a**2*f*exp(2*I*e)*exp(2
*I*f*x) + a**2*f) + Piecewise((((I*A*a**2*c**4*f*exp(2*I*e) - B*a**2*c**4*
f*exp(2*I*e))*exp(-4*I*f*x) + (-4*I*A*a**2*c**4*f*exp(4*I*e) + 8*B*a**2*c*
*4*f*exp(4*I*e))*exp(-2*I*f*x))*exp(-6*I*e)/(a**4*f**2), Ne(a**4*f**2*exp(
6*I*e), 0)), (x*(-(12*A*c**4 + 36*I*B*c**4)/a**2 + (12*A*c**4*exp(4*I*e) -
 8*A*c**4*exp(2*I*e) + 4*A*c**4 + 36*I*B*c**4*exp(4*I*e) - 16*I*B*c**4*exp
(2*I*e) + 4*I*B*c**4)*exp(-4*I*e)/a**2), True)) + 6*I*c**4*(A + 3*I*B)*log
(exp(2*I*f*x) + exp(-2*I*e))/(a**2*f) + x*(12*A*c**4 + 36*I*B*c**4)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.89

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}\,dx = -\frac{6\,(i\,Ac^4-3\,Bc^4)\log\left(\tan\left(fx+e\right)-i\right)}{a^2f} - \frac{4\,(-2i\,Ac^4+4\,Bc^4+(3\,Ac^4+5i\,Bc^4)\tan\left(fx+e\right))}{a^2f(\tan\left(fx+e\right)-i)^2} - \frac{Ba^2c^4f\tan\left(fx+e\right)^2+2\,Aa^2c^4f\tan\left(fx+e\right)+12i\,Ba^2c^4f\tan\left(fx+e\right)}{2\,a^4f^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-6*(I*A*c^4 - 3*B*c^4)*log(tan(f*x + e) - I)/(a^2*f) - 4*(-2*I*A*c^4 + 4*B
*c^4 + (3*A*c^4 + 5*I*B*c^4)*tan(f*x + e))/(a^2*f*(tan(f*x + e) - I)^2) -
1/2*(B*a^2*c^4*f*tan(f*x + e)^2 + 2*A*a^2*c^4*f*tan(f*x + e) + 12*I*B*a^2*
c^4*f*tan(f*x + e))/(a^4*f^2)
```

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.31

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}\,dx$$
$$= -\frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\left(-\frac{18\,B\,c^4}{a^2}+\frac{A\,c^4\,6\mathrm{i}}{a^2}\right)}{f}-\frac{\tan(e+f\,x)\left(\frac{c^4\,(A+B\,3\mathrm{i})}{a^2}+\frac{B\,c^4\,3\mathrm{i}}{a^2}\right)}{f}$$
$$-\frac{\frac{(-6\,B\,c^4+A\,c^4\,2\mathrm{i})\,1\mathrm{i}}{2\,a^2}-\frac{(-18\,B\,c^4+A\,c^4\,6\mathrm{i})\,3\mathrm{i}}{2\,a^2}+\tan(e+f\,x)\left(\frac{2\,(-18\,B\,c^4+A\,c^4\,6\mathrm{i})}{a^2}+\frac{16\,B\,c^4}{a^2}\right)-\frac{B\,c^4\,8\mathrm{i}}{a^2}}{f\,\left(\tan\left(e+f\,x\right)^2\,1\mathrm{i}+2\tan\left(e+f\,x\right)-\mathrm{i}\right)}$$
$$-\frac{B\,c^4\tan(e+f\,x)^2}{2\,a^2\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^4)/(a + a*tan(e + f*x)*1
i)^2,x)
```

output
```
- (log(tan(e + f*x) - 1i)*((A*c^4*6i)/a^2 - (18*B*c^4)/a^2))/f - (tan(e +
f*x)*((c^4*(A + B*3i))/a^2 + (B*c^4*3i)/a^2))/f - (((A*c^4*2i - 6*B*c^4)*1
i)/(2*a^2) - ((A*c^4*6i - 18*B*c^4)*3i)/(2*a^2) + tan(e + f*x)*((2*(A*c^4*
6i - 18*B*c^4))/a^2 + (16*B*c^4)/a^2) - (B*c^4*8i)/a^2)/(f*(2*tan(e + f*x)
 + tan(e + f*x)^2*1i - 1i)) - (B*c^4*tan(e + f*x)^2)/(2*a^2*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{c^4\left(-\left(\int\frac{\tan(fx+e)^5}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)b-\left(\int\frac{\tan(fx+e)^4}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)a-4\left(\int\frac{\tan(fx+e)^4}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^2,x)
```

output

```
(c**4*( - int(tan(e + f*x)**5/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*
b - int(tan(e + f*x)**4/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a - 4*
int(tan(e + f*x)**4/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i - 4*in
t(tan(e + f*x)**3/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i + 6*int(
tan(e + f*x)**3/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b + 6*int(tan(
e + f*x)**2/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a + 4*int(tan(e +
f*x)**2/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i + 4*int(tan(e + f*
x)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i - int(tan(e + f*x)/(tan
(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b - int(1/(tan(e + f*x)**2 - 2*tan
(e + f*x)*i - 1),x)*a))/a**2
```

3.718 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 128

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx = \frac{(A+5iB)c^3x}{a^2}+\frac{(iA-5B)c^3\log(\cos(e+fx))}{a^2f} - \frac{2(iA-B)c^3}{a^2f(i-\tan(e+fx))^2}+\frac{4(A+2iB)c^3}{a^2f(i-\tan(e+fx))}-\frac{iBc^3\tan(e+fx)}{a^2f}$$

output
```
(A+5*I*B)*c^3*x/a^2+(I*A-5*B)*c^3*ln(cos(f*x+e))/a^2/f-2*(I*A-B)*c^3/a^2/f
/(I-tan(f*x+e))^2+4*(A+2*I*B)*c^3/a^2/f/(I-tan(f*x+e))-I*B*c^3*tan(f*x+e)/
a^2/f
```

Mathematica [A] (verified)

Time = 5.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx = \frac{\frac{B(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}+\frac{(-iA+5B)c^3\left(\log(i-\tan(e+fx))+\frac{-2-4i\tan(e+fx)}{(-i+\tan(e+fx))^2}\right)}{a^2}}{f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3)/(a + I*a*Tan[e +
 f*x])^2,x]
```

output
```
((B*(c - I*c*Tan[e + f*x])^3)/(a + I*a*Tan[e + f*x])^2 + (((-I)*A + 5*B)*c
^3*(Log[I - Tan[e + f*x]] + (-2 - (4*I)*Tan[e + f*x])/(-I + Tan[e + f*x])^
2))/a^2)/f
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^3(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^3(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{c^2(1-i\tan(e+fx))^2(A+B\tan(e+fx))}{a^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\int \frac{(1-i\tan(e+fx))^2(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \text{86}$$

$$\frac{c^3\int \left(\frac{4i(A+iB)}{(\tan(e+fx)-i)^3}-iB-\frac{i(A+5iB)}{\tan(e+fx)-i}+\frac{4(A+2iB)}{(\tan(e+fx)-i)^2}\right)d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(-\frac{2(-B+iA)}{(-\tan(e+fx)+i)^2}+\frac{4(A+2iB)}{-\tan(e+fx)+i}-(-5B+iA)\log(-\tan(e+fx)+i)-iB\tan(e+fx)\right)}{a^2f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3)/(a + I*a*Tan[e + f*x])
^2,x]
```

output
```
(c^3*(-((I*A - 5*B)*Log[I - Tan[e + f*x]]) - (2*(I*A - B))/(I - Tan[e + f*
x])^2 + (4*(A + (2*I)*B))/(I - Tan[e + f*x]) - I*B*Tan[e + f*x]))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.56

method	result
derivativedivides	$-\frac{iB\,c^3\tan(fx+e)}{a^2f}-\frac{2ic^3A}{f\,a^2(-i+\tan(fx+e))^2}+\frac{2c^3B}{f\,a^2(-i+\tan(fx+e))^2}-\frac{ic^3A\ln\left(1+\tan(fx+e)^2\right)}{2f\,a^2}+\frac{5c^3B\ln\left(\right.}{}$
default	$-\frac{iB\,c^3\tan(fx+e)}{a^2f}-\frac{2ic^3A}{f\,a^2(-i+\tan(fx+e))^2}+\frac{2c^3B}{f\,a^2(-i+\tan(fx+e))^2}-\frac{ic^3A\ln\left(1+\tan(fx+e)^2\right)}{2f\,a^2}+\frac{5c^3B\ln\left(\right.}{}$
risch	$\frac{3c^3\mathrm{e}^{-2i(fx+e)}B}{a^2f}-\frac{ic^3\mathrm{e}^{-2i(fx+e)}A}{a^2f}-\frac{c^3\mathrm{e}^{-4i(fx+e)}B}{2a^2f}+\frac{ic^3\mathrm{e}^{-4i(fx+e)}A}{2a^2f}+\frac{10ic^3Bx}{a^2}+\frac{2c^3Ax}{a^2}+\frac{10ic^3Be}{a^2f}+$
norman	$\frac{\frac{\left(5ic^3B+A\,c^3\right)x}{a}+\frac{-2ic^3A+6B\,c^3}{af}+\frac{\left(5ic^3B+A\,c^3\right)x\tan(fx+e)^4}{a}+\frac{2\left(5ic^3B+A\,c^3\right)x\tan(fx+e)^2}{a}-\frac{2\left(5ic^3B+2A\,c^3\right)\tan(fx+e)^3}{af}}{a\left(1+\tan(fx+e)^2\right)^2}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-I*B*c^3*tan(f*x+e)/a^2/f-2*I/f*c^3/a^2/(-I+tan(f*x+e))^2*A+2/f*c^3/a^2/(-
I+tan(f*x+e))^2*B-1/2*I/f*c^3/a^2*A*ln(1+tan(f*x+e)^2)+5/2/f*c^3/a^2*B*ln(
1+tan(f*x+e)^2)+1/f*c^3/a^2*A*arctan(tan(f*x+e))+5*I/f*c^3/a^2*B*arctan(ta
n(f*x+e))-8*I/f*c^3/a^2/(-I+tan(f*x+e))*B-4/f*c^3/a^2/(-I+tan(f*x+e))*A
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.38

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx$$
$$=\frac{4\,(A+5i\,B)c^3fxe^{(6i\,fx+6i\,e)}+(-i\,A+5\,B)c^3e^{(2i\,fx+2i\,e)}+(i\,A-B)c^3+2\,(2\,(A+5i\,B)c^3fx-(i\,A-5}{2\,(a^2fe^{(6i\,fx+6i\,e)}+a^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
1/2*(4*(A + 5*I*B)*c^3*f*x*e^(6*I*f*x + 6*I*e) + (-I*A + 5*B)*c^3*e^(2*I*f
*x + 2*I*e) + (I*A - B)*c^3 + 2*(2*(A + 5*I*B)*c^3*f*x - (I*A - 5*B)*c^3)*
e^(4*I*f*x + 4*I*e) - 2*((-I*A + 5*B)*c^3*e^(6*I*f*x + 6*I*e) + (-I*A + 5*
B)*c^3*e^(4*I*f*x + 4*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/(a^2*f*e^(6*I*f*
x + 6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e))
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.41

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx = \frac{2Bc^3}{a^2fe^{2ie}e^{2ifx}+a^2f} + \begin{cases} \frac{((iAa^2c^3fe^{2ie}-Ba^2c^3fe^{2ie})e^{-4ifx}+(-2iAa^2c^3fe^{4ie}+6Ba^2c^3fe^{4ie})e^{-2ifx})e^{-6ie}}{2a^4f^2} & \text{for } a^4f^2e^{6ie}\neq 0 \\ x\left(-\frac{2Ac^3+10iBc^3}{a^2}+\frac{(2Ac^3e^{4ie}-2Ac^3e^{2ie}+2Ac^3+10iBc^3e^{4ie}-6iBc^3e^{2ie}+2iBc^3)e^{-4ie}}{a^2}\right) & \text{otherwise} \end{cases} + \frac{ic^3(A+5iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{a^2f} + \frac{x(2Ac^3+10iBc^3)}{a^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**3/(a+I*a*tan(f*x+e))**2,x)
```

output
```
2*B*c**3/(a**2*f*exp(2*I*e)*exp(2*I*f*x) + a**2*f) + Piecewise((((I*A*a**2
*c**3*f*exp(2*I*e) - B*a**2*c**3*f*exp(2*I*e))*exp(-4*I*f*x) + (-2*I*A*a**
2*c**3*f*exp(4*I*e) + 6*B*a**2*c**3*f*exp(4*I*e))*exp(-2*I*f*x))*exp(-6*I*
e)/(2*a**4*f**2), Ne(a**4*f**2*exp(6*I*e), 0)), (x*(-(2*A*c**3 + 10*I*B*c*
*3)/a**2 + (2*A*c**3*exp(4*I*e) - 2*A*c**3*exp(2*I*e) + 2*A*c**3 + 10*I*B*
c**3*exp(4*I*e) - 6*I*B*c**3*exp(2*I*e) + 2*I*B*c**3)*exp(-4*I*e)/a**2), T
rue)) + I*c**3*(A + 5*I*B)*log(exp(2*I*f*x) + exp(-2*I*e))/(a**2*f) + x*(2
*A*c**3 + 10*I*B*c**3)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx = -\frac{i\,Bc^3\tan(fx+e)}{a^2f} + \frac{(-i\,Ac^3+5\,Bc^3)\log(\tan(fx+e)-i)}{a^2f} - \frac{2\left(-i\,Ac^3+3\,Bc^3+2\left(Ac^3+2i\,Bc^3\right)\tan(fx+e)\right)}{a^2f(\tan(fx+e)-i)^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-I*B*c^3*tan(f*x + e)/(a^2*f) + (-I*A*c^3 + 5*B*c^3)*log(tan(f*x + e) - I)
/(a^2*f) - 2*(-I*A*c^3 + 3*B*c^3 + 2*(A*c^3 + 2*I*B*c^3)*tan(f*x + e))/(a^
2*f*(tan(f*x + e) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.52

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{c^3\left(6B-A\,2\mathrm{i}+4A\tan(e+f\,x)+B\tan(e+f\,x)\,7\mathrm{i}-A\,\ln\left(-1-\tan(e+f\,x)\,1\mathrm{i}\right)1\mathrm{i}+5B\,\ln\left(-1-\right.\right.}{}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^3)/(a + a*tan(e + f*x)*1
i)^2,x)
```

output
```
(c^3*(6*B - A*2i + 4*A*tan(e + f*x) + B*tan(e + f*x)*7i - A*log(- tan(e +
f*x)*1i - 1)*1i + 5*B*log(- tan(e + f*x)*1i - 1) + 2*B*tan(e + f*x)^2 + B*
tan(e + f*x)^3*1i + A*tan(e + f*x)^2*log(- tan(e + f*x)*1i - 1)*1i - 5*B*t
an(e + f*x)^2*log(- tan(e + f*x)*1i - 1) + 2*A*tan(e + f*x)*log(- tan(e +
f*x)*1i - 1) + B*tan(e + f*x)*log(- tan(e + f*x)*1i - 1)*10i))/(a^2*f*(tan
(e + f*x)*1i + 1)^2)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{c^3\left(-\left(\int \frac{\tan(fx+e)^4}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)bi-\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)ai+3\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^2-2\tan(fx+e)i-1}d\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^2,x)
```

output
```
(c**3*( - int(tan(e + f*x)**4/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*
b*i - int(tan(e + f*x)**3/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i
+ 3*int(tan(e + f*x)**3/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b + 3*
int(tan(e + f*x)**2/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a + 3*int(
tan(e + f*x)**2/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i + 3*int(ta
n(e + f*x)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i - int(tan(e + f
*x)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b - int(1/(tan(e + f*x)**2
 - 2*tan(e + f*x)*i - 1),x)*a))/a**2
```

3.719 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 97

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx = \frac{iBc^2x}{a^2}-\frac{Bc^2\log(\cos(e+fx))}{a^2f}-\frac{(iA-B)c^2}{a^2f(i-\tan(e+fx))^2}+\frac{(A+3iB)c^2}{a^2f(i-\tan(e+fx))}$$

output
```
I*B*c^2*x/a^2-B*c^2*ln(cos(f*x+e))/a^2/f-(I*A-B)*c^2/a^2/f/(I-tan(f*x+e))^
2+(A+3*I*B)*c^2/a^2/f/(I-tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx = \frac{c^2\left(B\log(i-\tan(e+fx))-\frac{2B+(A+3iB)\tan(e+fx)}{(-i+\tan(e+fx))^2}\right)}{a^2f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2)/(a + I*a*Tan[e +
 f*x])^2,x]
```

output
```
(c^2*(B*Log[I - Tan[e + f*x]] - (2*B + (A + (3*I)*B)*Tan[e + f*x])/(-I + T
an[e + f*x])^2))/(a^2*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^2(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c-ic\tan(e+fx))^2(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{c(1-i\tan(e+fx))(A+B\tan(e+fx))}{a^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{27}$$
$$\frac{c^2\int \frac{(1-i\tan(e+fx))(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2f}$$
$$\Big\downarrow \text{86}$$
$$\frac{c^2\int \left(\frac{2i(A+iB)}{(\tan(e+fx)-i)^3}+\frac{B}{\tan(e+fx)-i}+\frac{A+3iB}{(\tan(e+fx)-i)^2}\right)d\tan(e+fx)}{a^2f}$$
$$\Big\downarrow \text{2009}$$
$$\frac{c^2\left(-\frac{-B+iA}{(-\tan(e+fx)+i)^2}+\frac{A+3iB}{-\tan(e+fx)+i}+B\log(-\tan(e+fx)+i)\right)}{a^2f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2)/(a + I*a*Tan[e + f*x])
^2,x]
```

output
```
(c^2*(B*Log[I - Tan[e + f*x]] - (I*A - B)/(I - Tan[e + f*x])^2 + (A + (3*I
)*B)/(I - Tan[e + f*x])))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

method	result
risch	$\frac{B\,c^{2} \mathrm{e}^{-2 i (f x +e)}}{a^{2} f}-\frac{c^{2} \mathrm{e}^{-4 i (f x +e)} B}{4 a^{2} f}+\frac{i c^{2} \mathrm{e}^{-4 i (f x +e)} A}{4 a^{2} f}+\frac{2 i B\,c^{2} x}{a^{2}}+\frac{2 i B\,c^{2} e}{a^{2} f}-\frac{B\,c^{2} \ln \left(\mathrm{e}^{2 i (f x +e)}+1\right)}{a^{2} f}$
derivativedivides	$-\frac{3 i c^{2} B}{f\,a^{2} \left(-i+\tan \left(f x +e \right)\right)}-\frac{c^{2} A}{f\,a^{2} \left(-i+\tan \left(f x +e \right)\right)}-\frac{i c^{2} A}{f\,a^{2} \left(-i+\tan \left(f x +e \right)\right)^{2}}+\frac{c^{2} B}{f\,a^{2} \left(-i+\tan \left(f x +e \right)\right)^{2}}+\frac{c^{2} B \ln \left(\right.}{}$
default	$-\frac{3 i c^{2} B}{f\,a^{2} \left(-i+\tan \left(f x +e \right)\right)}-\frac{c^{2} A}{f\,a^{2} \left(-i+\tan \left(f x +e \right)\right)}-\frac{i c^{2} A}{f\,a^{2} \left(-i+\tan \left(f x +e \right)\right)^{2}}+\frac{c^{2} B}{f\,a^{2} \left(-i+\tan \left(f x +e \right)\right)^{2}}+\frac{c^{2} B \ln \left(\right.}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
B*c^2/a^2/f*exp(-2*I*(f*x+e))-1/4*c^2/a^2/f*exp(-4*I*(f*x+e))*B+1/4*I*c^2/
a^2/f*exp(-4*I*(f*x+e))*A+2*I*B*c^2*x/a^2+2*I*B*c^2/a^2/f*e-B*c^2/a^2/f*ln
(exp(2*I*(f*x+e))+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx = \frac{\left(8i\,Bc^2fxe^{(4i\,fx+4i\,e)} - 4\,Bc^2e^{(4i\,fx+4i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right) + 4\,Bc^2e^{(2i\,fx+2i\,e)} + (i\,A-B)c^2\right)e^{(-4i\,fx-4i\,e}}{4\,a^2f}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
1/4*(8*I*B*c^2*f*x*e^(4*I*f*x + 4*I*e) - 4*B*c^2*e^(4*I*f*x + 4*I*e)*log(e
^(2*I*f*x + 2*I*e) + 1) + 4*B*c^2*e^(2*I*f*x + 2*I*e) + (I*A - B)*c^2)*e^(
-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.12

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{2iBc^2x}{a^2} - \frac{Bc^2\log\left(e^{2ifx}+e^{-2ie}\right)}{a^2f} + \begin{cases} \frac{(4Ba^2c^2fe^{4ie}e^{-2ifx}+(iAa^2c^2fe^{2ie}-Ba^2c^2fe^{2ie})e^{-4ifx})e^{-6ie}}{4a^4f^2} & \text{for } a^4f^2e^{6ie}\neq 0 \\ x\left(-\frac{2iBc^2}{a^2}+\frac{(Ac^2+2iBc^2e^{4ie}-2iBc^2e^{2ie}+iBc^2)e^{-4ie}}{a^2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**2/(a+I*a*tan(f*x+e))**2,x)
```

output
```
2*I*B*c**2*x/a**2 - B*c**2*log(exp(2*I*f*x) + exp(-2*I*e))/(a**2*f) + Piec
ewise(((4*B*a**2*c**2*f*exp(4*I*e)*exp(-2*I*f*x) + (I*A*a**2*c**2*f*exp(2*
I*e) - B*a**2*c**2*f*exp(2*I*e))*exp(-4*I*f*x))*exp(-6*I*e)/(4*a**4*f**2),
 Ne(a**4*f**2*exp(6*I*e), 0)), (x*(-2*I*B*c**2/a**2 + (A*c**2 + 2*I*B*c**2
*exp(4*I*e) - 2*I*B*c**2*exp(2*I*e) + I*B*c**2)*exp(-4*I*e)/a**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx = \frac{Bc^2\log\left(\tan\left(fx+e\right)-i\right)}{a^2f} - \frac{2\,Bc^2+\left(Ac^2+3i\,Bc^2\right)\tan\left(fx+e\right)}{a^2f(\tan\left(fx+e\right)-i)^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
B*c^2*log(tan(f*x + e) - I)/(a^2*f) - (2*B*c^2 + (A*c^2 + 3*I*B*c^2)*tan(f
*x + e))/(a^2*f*(tan(f*x + e) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx = \frac{c^2\left(2\,B+A\tan(e+f\,x)+B\tan(e+f\,x)\;3\mathrm{i}+B\;\ln\left(-1-\tan(e+f\,x)\;1\mathrm{i}\right)-B\tan(e+f\,x)^2\;\ln\left(-1-\right.\right.}{a^2\,f\left(1+\tan\left(e+f\,x\right)\;1\mathrm{i}\right)^2}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^2)/(a + a*tan(e + f*x)*1
i)^2,x)
```

output
```
(c^2*(2*B + A*tan(e + f*x) + B*tan(e + f*x)*3i + B*log(- tan(e + f*x)*1i -
 1) - B*tan(e + f*x)^2*log(- tan(e + f*x)*1i - 1) + B*tan(e + f*x)*log(- t
an(e + f*x)*1i - 1)*2i))/(a^2*f*(tan(e + f*x)*1i + 1)^2)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{c^2\left(\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)b+\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)a+2\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)b\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^2,x)
```

output
```
(c**2*(int(tan(e + f*x)**3/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b +
 int(tan(e + f*x)**2/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a + 2*int
(tan(e + f*x)**2/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i + 2*int(t
an(e + f*x)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i - int(tan(e +
f*x)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b - int(1/(tan(e + f*x)**
2 - 2*tan(e + f*x)*i - 1),x)*a))/a**2
```

3.720 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^2} dx$

Optimal result

Integrand size = 39, antiderivative size = 48

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^2} dx = -\frac{c(A+B\tan(e+fx))^2}{2a^2(iA-B)f(1+i\tan(e+fx))^2}$$

output
```
-1/2*c*(A+B*tan(f*x+e))^2/a^2/(I*A-B)/f/(1+I*tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^2} dx = \frac{c(A\cos(e+fx)+B\sin(e+fx))^2(i\cos(2(e+fx))+\sin(2(e+fx)))}{2a^2(A+iB)f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]))/(a + I*a*Tan[e + f
*x])^2,x]
```

output
```
(c*(A*Cos[e + f*x] + B*Sin[e + f*x])^2*(I*Cos[2*(e + f*x)] + Sin[2*(e + f*
x)]))/(2*a^2*(A + I*B)*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3042, 4071, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \text{48}$$

$$-\frac{c(A+B\tan(e+fx))^2}{2a^2f(-B+iA)(1+i\tan(e+fx))^2}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]))/(a + I*a*Tan[e + f*x])^2
,x]
```

output
```
-1/2*(c*(A + B*Tan[e + f*x])^2)/(a^2*(I*A - B)*f*(1 + I*Tan[e + f*x])^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{c\left(-\frac{iA-B}{2(-i+\tan(fx+e))^2}-\frac{iB}{-i+\tan(fx+e)}\right)}{f\,a^2}$	46
default	$\frac{c\left(-\frac{iA-B}{2(-i+\tan(fx+e))^2}-\frac{iB}{-i+\tan(fx+e)}\right)}{f\,a^2}$	46
risch	$\frac{c\,{\rm e}^{-2i(fx+e)}B}{4a^2f}+\frac{ic\,{\rm e}^{-2i(fx+e)}A}{4a^2f}-\frac{c\,{\rm e}^{-4i(fx+e)}B}{8a^2f}+\frac{ic\,{\rm e}^{-4i(fx+e)}A}{8a^2f}$	80
norman	$\frac{\frac{cA\tan(fx+e)}{af}+\frac{icA+Bc}{2af}+\frac{(-icA+3Bc)\tan(fx+e)^2}{2af}-\frac{icB\tan(fx+e)^3}{af}}{a\left(1+\tan(fx+e)^2\right)^2}$	95

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x,method=_RET
URNVERBOSE)
```

output
```
1/f*c/a^2*(-1/2*(I*A-B)/(-I+tan(f*x+e))^2-I*B/(-I+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = -\frac{\left(2\left(-iA-B\right)ce^{\left(2ifx+2ie\right)}-\left(iA-B\right)c\right)e^{\left(-4ifx-4ie\right)}}{8a^2f}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algo
rithm="fricas")
```

output
```
-1/8*(2*(-I*A - B)*c*e^(2*I*f*x + 2*I*e) - (I*A - B)*c)*e^(-4*I*f*x - 4*I*
e)/(a^2*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(37) = 74.

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.29

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = \begin{cases} \frac{\left(\left(4iAa^2cfe^{2ie}-4Ba^2cfe^{2ie}\right)e^{-4ifx}+\left(8iAa^2cfe^{4ie}+8Ba^2cfe^{4ie}\right)e^{-2ifx}\right)e^{-6ie}}{32a^4f^2} & \text{for } a^4f^2e^{6ie}\neq 0 \\ \frac{x\left(Ace^{2ie}+Ac-iBce^{2ie}+iBc\right)e^{-4ie}}{2a^2} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))**2,x)
```

output
```
Piecewise((((4*I*A*a**2*c*f*exp(2*I*e) - 4*B*a**2*c*f*exp(2*I*e))*exp(-4*I
*f*x) + (8*I*A*a**2*c*f*exp(4*I*e) + 8*B*a**2*c*f*exp(4*I*e))*exp(-2*I*f*x
))*exp(-6*I*e)/(32*a**4*f**2), Ne(a**4*f**2*exp(6*I*e), 0)), (x*(A*c*exp(2
*I*e) + A*c - I*B*c*exp(2*I*e) + I*B*c)*exp(-4*I*e)/(2*a**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = -\frac{2i\,Bc\tan(fx+e)+i\,Ac+Bc}{2\,a^2f(\tan(fx+e)-i)^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algo
rithm="giac")
```

output
```
-1/2*(2*I*B*c*tan(f*x + e) + I*A*c + B*c)/(a^2*f*(tan(f*x + e) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 5.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = \frac{\frac{c\,(A-B\,1\mathrm{i})}{2}+B\,c\tan(e+f\,x)}{a^2\,f\,\left(\tan\left(e+f\,x\right)^2\,1\mathrm{i}+2\tan\left(e+f\,x\right)-\mathrm{i}\right)}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i))/(a + a*tan(e + f*x)*1i)
^2,x)
```

output
```
((c*(A - B*1i))/2 + B*c*tan(e + f*x))/(a^2*f*(2*tan(e + f*x) + tan(e + f*x
)^2*1i - 1i))
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{c\left(\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)bi+\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)ai-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)b}{a^2}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x)
```

output
```
(c*(int(tan(e + f*x)**2/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i +
int(tan(e + f*x)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i - int(tan
(e + f*x)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b - int(1/(tan(e + f
*x)**2 - 2*tan(e + f*x)*i - 1),x)*a))/a**2
```

3.721 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx=\frac{(A-iB)x}{4a^2}+\frac{iA-B}{4f(a+ia\tan(e+fx))^2}+\frac{iA+B}{4f\left(a^2+ia^2\tan(e+fx)\right)}$$

output
```
1/4*(A-I*B)*x/a^2+1/4*(I*A-B)/f/(a+I*a*tan(f*x+e))^2+1/4*(I*A+B)/f/(a^2+I*
a^2*tan(f*x+e))
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx=\frac{(A-iB)\arctan(\tan(e+fx))}{4a^2f}-\frac{iA-B}{4a^2f(i-\tan(e+fx))^2}-\frac{A-iB}{4a^2f(i-\tan(e+fx))}$$

input
```
Integrate[(A + B*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^2,x]
```

output
```
((A - I*B)*ArcTan[Tan[e + f*x]])/(4*a^2*f) - (I*A - B)/(4*a^2*f*(I - Tan[e
 + f*x])^2) - (A - I*B)/(4*a^2*f*(I - Tan[e + f*x]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 4009, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{4009}$$
$$\frac{(A-iB)\int \frac{1}{i\tan(e+fx)a+a}dx}{2a}+\frac{-B+iA}{4f(a+ia\tan(e+fx))^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{(A-iB)\int \frac{1}{i\tan(e+fx)a+a}dx}{2a}+\frac{-B+iA}{4f(a+ia\tan(e+fx))^2}$$
$$\Big\downarrow \text{3960}$$
$$\frac{(A-iB)\left(\frac{\int 1dx}{2a}+\frac{i}{2f(a+ia\tan(e+fx))}\right)}{2a}+\frac{-B+iA}{4f(a+ia\tan(e+fx))^2}$$
$$\Big\downarrow \text{24}$$
$$\frac{-B+iA}{4f(a+ia\tan(e+fx))^2}+\frac{(A-iB)\left(\frac{x}{2a}+\frac{i}{2f(a+ia\tan(e+fx))}\right)}{2a}$$

input
```
Int[(A + B*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^2,x]
```

output
```
(I*A - B)/(4*f*(a + I*a*Tan[e + f*x])^2) + ((A - I*B)*(x/(2*a) + (I/2)/(f*
(a + I*a*Tan[e + f*x]))))/(2*a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{ixB}{4a^2}+\frac{xA}{4a^2}+\frac{iA\,\mathrm{e}^{-2i(fx+e)}}{4a^2f}-\frac{\mathrm{e}^{-4i(fx+e)}B}{16a^2f}+\frac{i\mathrm{e}^{-4i(fx+e)}A}{16a^2f}$
derivativedivides	$\frac{A\arctan(\tan(fx+e))}{4f\,a^2}-\frac{iB\arctan(\tan(fx+e))}{4f\,a^2}+\frac{B}{4f\,a^2(-i+\tan(fx+e))^2}-\frac{iA}{4f\,a^2(-i+\tan(fx+e))^2}+\frac{}{4f\,a^2(-}$
default	$\frac{A\arctan(\tan(fx+e))}{4f\,a^2}-\frac{iB\arctan(\tan(fx+e))}{4f\,a^2}+\frac{B}{4f\,a^2(-i+\tan(fx+e))^2}-\frac{iA}{4f\,a^2(-i+\tan(fx+e))^2}+\frac{}{4f\,a^2(-}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*x/a^2*B+1/4*x/a^2*A+1/4*I*A/a^2/f*exp(-2*I*(f*x+e))-1/16/a^2/f*exp(
-4*I*(f*x+e))*B+1/16*I/a^2/f*exp(-4*I*(f*x+e))*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx = \frac{\left(4\,(A-i\,B)fxe^{(4i\,fx+4i\,e)}+4i\,Ae^{(2i\,fx+2i\,e)}+i\,A-B\right)e^{(-4i\,fx-4i\,e)}}{16\,a^2f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/16*(4*(A - I*B)*f*x*e^(4*I*f*x + 4*I*e) + 4*I*A*e^(2*I*f*x + 2*I*e) + I*
A - B)*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.02

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx = \begin{cases} \frac{\left(16iAa^2fe^{4ie}e^{-2ifx}+\left(4iAa^2fe^{2ie}-4Ba^2fe^{2ie}\right)e^{-4ifx}\right)e^{-6ie}}{64a^4f^2} & \text{for } a^4f^2e^{6ie}\neq 0 \\ x\left(-\frac{A-iB}{4a^2}+\frac{\left(Ae^{4ie}+2Ae^{2ie}+A-iBe^{4ie}+iB\right)e^{-4ie}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(A-iB)}{4a^2}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**2,x)
```

output
```
Piecewise(((16*I*A*a**2*f*exp(4*I*e)*exp(-2*I*f*x) + (4*I*A*a**2*f*exp(2*I
*e) - 4*B*a**2*f*exp(2*I*e))*exp(-4*I*f*x))*exp(-6*I*e)/(64*a**4*f**2), Ne
(a**4*f**2*exp(6*I*e), 0)), (x*(-(A - I*B)/(4*a**2) + (A*exp(4*I*e) + 2*A*
exp(2*I*e) + A - I*B*exp(4*I*e) + I*B)*exp(-4*I*e)/(4*a**2)), True)) + x*(
A - I*B)/(4*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx = -\frac{(-iA-B)\log(\tan(fx+e)+i)}{8a^2f} + \frac{(-iA-B)\log(\tan(fx+e)-i)}{8a^2f} + \frac{(A-iB)\tan(fx+e)-2iA}{4a^2f(\tan(fx+e)-i)^2}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/8*(-I*A - B)*log(tan(f*x + e) + I)/(a^2*f) + 1/8*(-I*A - B)*log(tan(f*x
 + e) - I)/(a^2*f) + 1/4*((A - I*B)*tan(f*x + e) - 2*I*A)/(a^2*f*(tan(f*x
+ e) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx = \frac{\frac{A}{2\,a^2}+\tan(e+f\,x)\,\left(\frac{B}{4\,a^2}+\frac{A\,1\mathrm{i}}{4\,a^2}\right)}{f\,\left(\tan\left(e+f\,x\right)^2\,1\mathrm{i}+2\tan\left(e+f\,x\right)-\mathrm{i}\right)}-\frac{x\,\left(B+A\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,a^2}$$

input
```
int((A + B*tan(e + f*x))/(a + a*tan(e + f*x)*1i)^2,x)
```

output
```
(A/(2*a^2) + tan(e + f*x)*((A*1i)/(4*a^2) + B/(4*a^2)))/(f*(2*tan(e + f*x)
 + tan(e + f*x)^2*1i - 1i)) - (x*(A*1i + B)*1i)/(4*a^2)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2}\,dx = \frac{-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)b-\left(\int \frac{1}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)a}{a^2}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x)
```

output
```
( - (int(tan(e + f*x)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b + int(
1/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a))/a**2
```

3.722 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 117

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}\,dx = \frac{(3A-iB)x}{8a^2c}-\frac{iA-B}{8a^2cf(i-\tan(e+fx))^2} - \frac{A}{4a^2cf(i-\tan(e+fx))}+\frac{A-iB}{8a^2cf(i+\tan(e+fx))}$$

output
```
1/8*(3*A-I*B)*x/a^2/c-1/8*(I*A-B)/a^2/c/f/(I-tan(f*x+e))^2-1/4*A/a^2/c/f/(
I-tan(f*x+e))+1/8*(A-I*B)/a^2/c/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.44 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}\,dx = \frac{\sec^2(e+fx)(5A+iB-(A-3iB)\cos(2(e+fx))-3iA\sin(2(e+fx))-B\sin(2(e+fx))+2(3A-i}{16a^2cf(-i+\tan(e+fx))^2(i+\tan(e+fx))}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e +
f*x])),x]
```

output
```
(Sec[e + f*x]^2*(5*A + I*B - (A - (3*I)*B)*Cos[2*(e + f*x)] - (3*I)*A*Sin[
2*(e + f*x)] - B*Sin[2*(e + f*x)] + 2*(3*A - I*B)*ArcTan[Tan[e + f*x]]*(-I
 + Tan[e + f*x])))/(16*a^2*c*f*(-I + Tan[e + f*x])^2*(I + Tan[e + f*x]))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^3c^2(1-i\tan(e+fx))^2(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^2(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2cf}$$
$$\Big\downarrow \text{86}$$
$$\frac{\int \left(-\frac{A}{4(\tan(e+fx)-i)^2}+\frac{3A-iB}{8(\tan^2(e+fx)+1)}+\frac{iB-A}{8(\tan(e+fx)+i)^2}+\frac{i(A+iB)}{4(\tan(e+fx)-i)^3}\right)d\tan(e+fx)}{a^2cf}$$
$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}(3A - iB)\arctan(\tan(e + fx)) + \frac{A-iB}{8(\tan(e+fx)+i)} - \frac{-B+iA}{8(-\tan(e+fx)+i)^2} - \frac{A}{4(-\tan(e+fx)+i)}}{a^2cf}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x]))
,x]
```

output
```
(((3*A - I*B)*ArcTan[Tan[e + f*x]])/8 - (I*A - B)/(8*(I - Tan[e + f*x])^2)
 - A/(4*(I - Tan[e + f*x])) + (A - I*B)/(8*(I + Tan[e + f*x])))/(a^2*c*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{ixB}{8a^2c}+\frac{3xA}{8a^2c}-\frac{e^{-4i(fx+e)}B}{32a^2cf}+\frac{ie^{-4i(fx+e)}A}{32a^2cf}-\frac{\cos(2fx+2e)B}{8a^2cf}+\frac{i\cos(2fx+2e)A}{8a^2cf}+\frac{A\sin(2fx+2e)}{4a^2cf}$
norman	$\frac{\frac{(-iB+3A)x}{8ac}-\frac{-iA+B}{4acf}+\frac{(-iB+3A)\tan(fx+e)^3}{8acf}+\frac{(-iB+3A)x\tan(fx+e)^2}{4ac}+\frac{(-iB+3A)x\tan(fx+e)^4}{8ac}+\frac{(iB+5A)\tan(fx+e)}{8acf}}{a\left(1+\tan(fx+e)^2\right)^2}$
derivativedivides	$\frac{B}{8f\,a^2c(-i+\tan(fx+e))^2}-\frac{iA}{8f\,a^2c(-i+\tan(fx+e))^2}+\frac{3A\arctan(\tan(fx+e))}{8f\,a^2c}-\frac{iB\arctan(\tan(fx+e))}{8f\,a^2c}+\frac{}{4f\,a}$
default	$\frac{B}{8f\,a^2c(-i+\tan(fx+e))^2}-\frac{iA}{8f\,a^2c(-i+\tan(fx+e))^2}+\frac{3A\arctan(\tan(fx+e))}{8f\,a^2c}-\frac{iB\arctan(\tan(fx+e))}{8f\,a^2c}+\frac{}{4f\,a}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-1/8*I*x/a^2/c*B+3/8*x/a^2/c*A-1/32/a^2/c/f*exp(-4*I*(f*x+e))*B+1/32*I/a^2
/c/f*exp(-4*I*(f*x+e))*A-1/8/a^2/c/f*cos(2*f*x+2*e)*B+1/8*I/a^2/c/f*cos(2*
f*x+2*e)*A+1/4*A/a^2/c/f*sin(2*f*x+2*e)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}\,dx = \frac{\left(4\,(3\,A-i\,B)fxe^{(4i\,fx+4i\,e)}-2\,(i\,A+B)e^{(6i\,fx+6i\,e)}-2\,(-3i\,A+B)e^{(2i\,fx+2i\,e)}+i\,A-B\right)e^{(-4i\,fx-4i\,e)}}{32\,a^2cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x, algo
rithm="fricas")
```

output
```
1/32*(4*(3*A - I*B)*f*x*e^(4*I*f*x + 4*I*e) - 2*(I*A + B)*e^(6*I*f*x + 6*I
*e) - 2*(-3*I*A + B)*e^(2*I*f*x + 2*I*e) + I*A - B)*e^(-4*I*f*x - 4*I*e)/(
a^2*c*f)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.53

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}\,dx$$
$$= \begin{cases} \frac{((256iAa^4c^2f^2e^{2ie}-256Ba^4c^2f^2e^{2ie})e^{-4ifx}+(1536iAa^4c^2f^2e^{4ie}-512Ba^4c^2f^2e^{4ie})e^{-2ifx}+(-512iAa^4c^2f^2e^{8ie}-512Ba^4c^2f^2e^{8ie})e^{2ifx}}{8192a^6c^3f^3} \\ x\left(-\frac{3A-iB}{8a^2c}+\frac{(Ae^{6ie}+3Ae^{4ie}+3Ae^{2ie}+A-iBe^{6ie}-iBe^{4ie}+iBe^{2ie}+iB)e^{-4ie}}{8a^2c}\right) \end{cases}$$
$$+\frac{x(3A-iB)}{8a^2c}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e)),x)
```

output
```
Piecewise((((256*I*A*a**4*c**2*f**2*exp(2*I*e) - 256*B*a**4*c**2*f**2*exp(
2*I*e))*exp(-4*I*f*x) + (1536*I*A*a**4*c**2*f**2*exp(4*I*e) - 512*B*a**4*c
**2*f**2*exp(4*I*e))*exp(-2*I*f*x) + (-512*I*A*a**4*c**2*f**2*exp(8*I*e) -
 512*B*a**4*c**2*f**2*exp(8*I*e))*exp(2*I*f*x))*exp(-6*I*e)/(8192*a**6*c**
3*f**3), Ne(a**6*c**3*f**3*exp(6*I*e), 0)), (x*(-(3*A - I*B)/(8*a**2*c) +
(A*exp(6*I*e) + 3*A*exp(4*I*e) + 3*A*exp(2*I*e) + A - I*B*exp(6*I*e) - I*B
*exp(4*I*e) + I*B*exp(2*I*e) + I*B)*exp(-4*I*e)/(8*a**2*c)), True)) + x*(3
*A - I*B)/(8*a**2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}\,dx$$
$$= -\frac{(-3i\,A-B)\log\left(\tan\left(fx+e\right)+i\right)}{16\,a^2cf} - \frac{(3i\,A+B)\log\left(\tan\left(fx+e\right)-i\right)}{16\,a^2cf} + \frac{(3\,A-i\,B)\tan\left(fx+e\right)^2-(3i\,A+B)\tan\left(fx+e\right)+2\,A+2i\,B}{8\,a^2cf(\tan\left(fx+e\right)+i)(\tan\left(fx+e\right)-i)^2}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x, algo
rithm="giac")
```

output
```
-1/16*(-3*I*A - B)*log(tan(f*x + e) + I)/(a^2*c*f) - 1/16*(3*I*A + B)*log(
tan(f*x + e) - I)/(a^2*c*f) + 1/8*((3*A - I*B)*tan(f*x + e)^2 - (3*I*A + B
)*tan(f*x + e) + 2*A + 2*I*B)/(a^2*c*f*(tan(f*x + e) + I)*(tan(f*x + e) -
I)^2)
```

Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))}\,dx$$
$$= \frac{\tan(e+f\,x)\,\left(\frac{3\,A}{8\,a^2\,c}-\frac{B\,1\mathrm{i}}{8\,a^2\,c}\right)+\tan(e+f\,x)^2\left(\frac{B}{8\,a^2\,c}+\frac{A\,3\mathrm{i}}{8\,a^2\,c}\right)-\frac{B}{4\,a^2\,c}+\frac{A\,1\mathrm{i}}{4\,a^2\,c}}{f\,\left(\tan\left(e+f\,x\right)^3\,1\mathrm{i}+\tan\left(e+f\,x\right)^2+\tan\left(e+f\,x\right)\,1\mathrm{i}+1\right)} - \frac{x\,(B+A\,3\mathrm{i})\,1\mathrm{i}}{8\,a^2\,c}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i
)),x)
```

output
```
(tan(e + f*x)*((3*A)/(8*a^2*c) - (B*1i)/(8*a^2*c)) + tan(e + f*x)^2*((A*3i
)/(8*a^2*c) + B/(8*a^2*c)) + (A*1i)/(4*a^2*c) - B/(4*a^2*c))/(f*(tan(e + f
*x)*1i + tan(e + f*x)^2 + tan(e + f*x)^3*1i + 1)) - (x*(A*3i + B)*1i)/(8*a
^2*c)
```

Reduce [F]

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))^2(c - ic\tan(e + fx))}\,dx$$
$$= \frac{2\left(\int \frac{1}{\tan(fx+e)^3 i+\tan(fx+e)^2+\tan(fx+e)i+1}dx\right)\tan(fx+e)^2 af + 2\left(\int \frac{1}{\tan(fx+e)^3 i+\tan(fx+e)^2+\tan(fx+e)i+1}dx\right)\tan}{}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e)),x)
```

output
```
(2*int(1/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan
(e + f*x)**2*a*f + 2*int(1/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e +
f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i + 2*int(1/(tan(e + f*x)**3*i + tan(e
+ f*x)**2 + tan(e + f*x)*i + 1),x)*a*f + 2*int(1/(tan(e + f*x)**3*i + tan(
e + f*x)**2 + tan(e + f*x)*i + 1),x)*b*f*i - tan(e + f*x)**2*b*e*i - tan(e
 + f*x)**2*b*f*i*x - tan(e + f*x)*b*i - b*e*i - b*f*i*x)/(2*a**2*c*f*(tan(
e + f*x)**2 + 1))
```

3.723 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 71

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx = \frac{3Ax}{8a^2c^2}+\frac{3A\cos(e+fx)\sin(e+fx)}{8a^2c^2f}-\frac{\cos^4(e+fx)(B-A\tan(e+fx))}{4a^2c^2f}$$

output
```
3/8*A*x/a^2/c^2+3/8*A*cos(f*x+e)*sin(f*x+e)/a^2/c^2/f-1/4*cos(f*x+e)^4*(B-
A*tan(f*x+e))/a^2/c^2/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx = \frac{-8B\cos^4(e+fx)+A(12(e+fx)+8\sin(2(e+fx))+\sin(4(e+fx)))}{32a^2c^2f}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e +
f*x])^2),x]
```

output
```
(-8*B*Cos[e + f*x]^4 + A*(12*(e + f*x) + 8*Sin[2*(e + f*x)] + Sin[4*(e + f
*x)]))/(32*a^2*c^2*f)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 4071, 27, 82, 454, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^3c^3(1-i\tan(e+fx))^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2c^2f}$$

$$\Big\downarrow \text{82}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{a^2c^2f}$$

$$\Big\downarrow \text{454}$$

$$\frac{\frac{3}{4}A\int \frac{1}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)-\frac{B-A\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}}{a^2c^2f}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3}{4}A\left(\frac{1}{2}\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)-\frac{B-A\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}}{a^2c^2f}$$

$$\Big\downarrow 216$$

$$\frac{\frac{3}{4}A\left(\frac{1}{2}\arctan(\tan(e+fx))+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)-\frac{B-A\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}}{a^2c^2f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^
2),x]
```

output
```
(-1/4*(B - A*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^2 + (3*A*(ArcTan[Tan[e + f
*x]]/2 + Tan[e + f*x]/(2*(1 + Tan[e + f*x]^2))))/4)/(a^2*c^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.35

method	result
risch	$\frac{3Ax}{8a^2c^2}-\frac{B\cos(4fx+4e)}{32a^2c^2f}+\frac{A\sin(4fx+4e)}{32a^2c^2f}-\frac{B\cos(2fx+2e)}{8a^2c^2f}+\frac{A\sin(2fx+2e)}{4a^2c^2f}$
norman	$\frac{\frac{3Ax}{8ac}-\frac{B}{4acf}+\frac{5A\tan(fx+e)}{8acf}+\frac{3A\tan(fx+e)^3}{8acf}+\frac{3Ax\tan(fx+e)^2}{4ac}+\frac{3Ax\tan(fx+e)^4}{8ac}}{\left(1+\tan(fx+e)^2\right)^2ac}$
derivativedivides	$-\frac{iA}{16f\,a^2c^2(-i+\tan(fx+e))^2}+\frac{B}{16f\,a^2c^2(-i+\tan(fx+e))^2}+\frac{3A\arctan(\tan(fx+e))}{8f\,a^2c^2}+\frac{3A}{16f\,a^2c^2(-i+\tan(fx+e$
default	$-\frac{iA}{16f\,a^2c^2(-i+\tan(fx+e))^2}+\frac{B}{16f\,a^2c^2(-i+\tan(fx+e))^2}+\frac{3A\arctan(\tan(fx+e))}{8f\,a^2c^2}+\frac{3A}{16f\,a^2c^2(-i+\tan(fx+e$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
3/8*A*x/a^2/c^2-1/32*B/a^2/c^2/f*cos(4*f*x+4*e)+1/32*A/a^2/c^2/f*sin(4*f*x
+4*e)-1/8*B/a^2/c^2/f*cos(2*f*x+2*e)+1/4*A/a^2/c^2/f*sin(2*f*x+2*e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx = \frac{(24\,Afxe^{(4i\,fx+4i\,e)}+(-i\,A-B)e^{(8i\,fx+8i\,e)}-4\,(2i\,A+B)e^{(6i\,fx+6i\,e)}-4\,(-2i\,A+B)e^{(2i\,fx+2i\,e)}+i\,A-}{64\,a^2c^2f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
1/64*(24*A*f*x*e^(4*I*f*x + 4*I*e) + (-I*A - B)*e^(8*I*f*x + 8*I*e) - 4*(2
*I*A + B)*e^(6*I*f*x + 6*I*e) - 4*(-2*I*A + B)*e^(2*I*f*x + 2*I*e) + I*A -
 B)*e^(-4*I*f*x - 4*I*e)/(a^2*c^2*f)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 360, normalized size of antiderivative = 5.07

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx = \frac{3Ax}{8a^2c^2} + \begin{cases} \frac{((16384iAa^6c^6f^3e^{2ie}-16384Ba^6c^6f^3e^{2ie})e^{-4ifx}+(131072iAa^6c^6f^3e^{4ie}-65536Ba^6c^6f^3e^{4ie})e^{-2ifx}+(-131072iAa^6c^6f^3e^{8ie}-65536}{1048576a^8c^8f^4} \\ x\left(-\frac{3A}{8a^2c^2}+\frac{(Ae^{8ie}+4Ae^{6ie}+6Ae^{4ie}+4Ae^{2ie}+A-iBe^{8ie}-2iBe^{6ie}+2iBe^{2ie}+iB)e^{-4ie}}{16a^2c^2}\right) \end{cases}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**2,x)
```

output
```
3*A*x/(8*a**2*c**2) + Piecewise((((16384*I*A*a**6*c**6*f**3*exp(2*I*e) - 1
6384*B*a**6*c**6*f**3*exp(2*I*e))*exp(-4*I*f*x) + (131072*I*A*a**6*c**6*f*
*3*exp(4*I*e) - 65536*B*a**6*c**6*f**3*exp(4*I*e))*exp(-2*I*f*x) + (-13107
2*I*A*a**6*c**6*f**3*exp(8*I*e) - 65536*B*a**6*c**6*f**3*exp(8*I*e))*exp(2
*I*f*x) + (-16384*I*A*a**6*c**6*f**3*exp(10*I*e) - 16384*B*a**6*c**6*f**3*
exp(10*I*e))*exp(4*I*f*x))*exp(-6*I*e)/(1048576*a**8*c**8*f**4), Ne(a**8*c
**8*f**4*exp(6*I*e), 0)), (x*(-3*A/(8*a**2*c**2) + (A*exp(8*I*e) + 4*A*exp
(6*I*e) + 6*A*exp(4*I*e) + 4*A*exp(2*I*e) + A - I*B*exp(8*I*e) - 2*I*B*exp
(6*I*e) + 2*I*B*exp(2*I*e) + I*B)*exp(-4*I*e)/(16*a**2*c**2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx = \frac{3\,(fx+e)A}{8\,a^2c^2f} + \frac{3\,A\tan\left(fx+e\right)^3+5\,A\tan\left(fx+e\right)-2\,B}{8\,\left(\tan\left(fx+e\right)^2+1\right)^2a^2c^2f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
3/8*(f*x + e)*A/(a^2*c^2*f) + 1/8*(3*A*tan(f*x + e)^3 + 5*A*tan(f*x + e) -
 2*B)/((tan(f*x + e)^2 + 1)^2*a^2*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx = \frac{3\,A\,x}{8\,a^2\,c^2}+\frac{\cos\left(e+f\,x\right)^4\,\left(\frac{3\,A\,\tan(e+f\,x)^3}{8}+\frac{5\,A\,\tan(e+f\,x)}{8}-\frac{B}{4}\right)}{a^2\,c^2\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i
)^2),x)
```

output
```
(3*A*x)/(8*a^2*c^2) + (cos(e + f*x)^4*((5*A*tan(e + f*x))/8 - B/4 + (3*A*t
an(e + f*x)^3)/8))/(a^2*c^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.51

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^2}\,dx = \frac{3\tan\left(fx+e\right)^4afx+2\tan\left(fx+e\right)^4b+3\tan\left(fx+e\right)^3a+6\tan\left(fx+e\right)^2afx+4\tan\left(fx+e\right)^2b+5}{8a^2c^2f\left(\tan\left(fx+e\right)^4+2\tan\left(fx+e\right)^2+1\right)}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^2,x)
```

output
```
(3*tan(e + f*x)**4*a*f*x + 2*tan(e + f*x)**4*b + 3*tan(e + f*x)**3*a + 6*t
an(e + f*x)**2*a*f*x + 4*tan(e + f*x)**2*b + 5*tan(e + f*x)*a + 3*a*f*x)/(
8*a**2*c**2*f*(tan(e + f*x)**4 + 2*tan(e + f*x)**2 + 1))
```

3.724 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 183

$$\begin{aligned}\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx &= \frac{(5A+iB)x}{16a^2c^3}-\frac{iA-B}{32a^2c^3f(i-\tan(e+fx))^2}\\&\quad-\frac{2A+iB}{16a^2c^3f(i-\tan(e+fx))}-\frac{A-iB}{24a^2c^3f(i+\tan(e+fx))^3}\\&\quad+\frac{3iA+B}{32a^2c^3f(i+\tan(e+fx))^2}+\frac{3A}{16a^2c^3f(i+\tan(e+fx))}\end{aligned}$$

output
```
1/16*(5*A+I*B)*x/a^2/c^3-1/32*(I*A-B)/a^2/c^3/f/(I-tan(f*x+e))^2-1/16*(2*A
+I*B)/a^2/c^3/f/(I-tan(f*x+e))-1/24*(A-I*B)/a^2/c^3/f/(I+tan(f*x+e))^3+1/3
2*(3*I*A+B)/a^2/c^3/f/(I+tan(f*x+e))^2+3/16*A/a^2/c^3/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.63 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.95

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx$$
$$= \frac{\sec^4(e+fx)(47A-5iB-2(7A+11iB)\cos(2(e+fx))-A\cos(4(e+fx))-5iB\cos(4(e+fx))+40}{192a^2c^3}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e +
f*x])^3),x]
```

output
```
(Sec[e + f*x]^4*(47*A - (5*I)*B - 2*(7*A + (11*I)*B)*Cos[2*(e + f*x)] - A*
Cos[4*(e + f*x)] - (5*I)*B*Cos[4*(e + f*x)] + (40*I)*A*Sin[2*(e + f*x)] -
8*B*Sin[2*(e + f*x)] + (5*I)*A*Sin[4*(e + f*x)] - B*Sin[4*(e + f*x)] + 12*
(5*A + I*B)*ArcTan[Tan[e + f*x]]*(I + Tan[e + f*x])))/(192*a^2*c^3*f*(-I +
 Tan[e + f*x])^2*(I + Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^3c^4(1-i\tan(e+fx))^4(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^4(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2c^3f}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(-\frac{3A}{16(\tan(e+fx)+i)^2}+\frac{5A+iB}{16(\tan^2(e+fx)+1)}+\frac{-2A-iB}{16(\tan(e+fx)-i)^2}+\frac{i(A+iB)}{16(\tan(e+fx)-i)^3}-\frac{i(3A-iB)}{16(\tan(e+fx)+i)^3}+\frac{A-iB}{8(\tan(e+fx)+i)^4}\right)}{a^2c^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{16}(5A+iB)\arctan(\tan(e+fx))-\frac{2A+iB}{16(-\tan(e+fx)+i)}-\frac{-B+iA}{32(-\tan(e+fx)+i)^2}+\frac{B+3iA}{32(\tan(e+fx)+i)^2}-\frac{A-iB}{24(\tan(e+fx)+i)^3}+}{a^2c^3f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^
3),x]
```

output
```
(((5*A + I*B)*ArcTan[Tan[e + f*x]])/16 - (I*A - B)/(32*(I - Tan[e + f*x])^
2) - (2*A + I*B)/(16*(I - Tan[e + f*x])) - (A - I*B)/(24*(I + Tan[e + f*x]
)^3) + ((3*I)*A + B)/(32*(I + Tan[e + f*x])^2) + (3*A)/(16*(I + Tan[e + f*
x])))/(a^2*c^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.14

method	result
norman	$\frac{\frac{(iB+5A)x}{16ac}-\frac{iA+B}{6acf}+\frac{(iB+5A)\tan(fx+e)^3}{6acf}+\frac{(iB+5A)\tan(fx+e)^5}{16acf}+\frac{3(iB+5A)x\tan(fx+e)^2}{16ac}+\frac{3(iB+5A)x\tan(fx+e)^4}{16ac}+\frac{(iB+5A}{}}{\left(1+\tan(fx+e)^2\right)^3 a\, c^2}$
risch	$\frac{ixB}{16a^2c^3}+\frac{5xA}{16a^2c^3}-\frac{e^{6i(fx+e)}B}{192a^2c^3f}-\frac{ie^{6i(fx+e)}A}{192a^2c^3f}-\frac{\cos(4fx+4e)B}{32a^2c^3f}-\frac{i\cos(4fx+4e)A}{32a^2c^3f}-\frac{i\sin(4fx+4e)B}{64a^2c^3f}+$
derivativedivides	$-\frac{iA}{32f\,a^2c^3(-i+\tan(fx+e))^2}+\frac{B}{32f\,a^2c^3(i+\tan(fx+e))^2}-\frac{A}{24f\,a^2c^3(i+\tan(fx+e))^3}+\frac{iB}{24f\,a^2c^3(i+\tan(fx+e)}$
default	$-\frac{iA}{32f\,a^2c^3(-i+\tan(fx+e))^2}+\frac{B}{32f\,a^2c^3(i+\tan(fx+e))^2}-\frac{A}{24f\,a^2c^3(i+\tan(fx+e))^3}+\frac{iB}{24f\,a^2c^3(i+\tan(fx+e)}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
(1/16*(5*A+I*B)/a/c*x-1/6*(I*A+B)/a/c/f+1/6*(5*A+I*B)/a/c/f*tan(f*x+e)^3+1
/16*(5*A+I*B)/a/c/f*tan(f*x+e)^5+3/16*(5*A+I*B)/a/c*x*tan(f*x+e)^2+3/16*(5
*A+I*B)/a/c*x*tan(f*x+e)^4+1/16*(5*A+I*B)/a/c*x*tan(f*x+e)^6+1/16*(11*A-I*
B)/a/c/f*tan(f*x+e))/(1+tan(f*x+e)^2)^3/a/c^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.63

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx = \frac{\left(24\,(5\,A+i\,B)fxe^{(4i\,fx+4i\,e)}-2\,(i\,A+B)e^{(10i\,fx+10i\,e)}-3\,(5i\,A+3\,B)e^{(8i\,fx+8i\,e)}-12\,(5i\,A+B)e^{(6i\,fx+\cdots}\right.}{384\,a^2c^3f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
1/384*(24*(5*A + I*B)*f*x*e^(4*I*f*x + 4*I*e) - 2*(I*A + B)*e^(10*I*f*x +
10*I*e) - 3*(5*I*A + 3*B)*e^(8*I*f*x + 8*I*e) - 12*(5*I*A + B)*e^(6*I*f*x
+ 6*I*e) - 6*(-5*I*A + 3*B)*e^(2*I*f*x + 2*I*e) + 3*I*A - 3*B)*e^(-4*I*f*x
 - 4*I*e)/(a^2*c^3*f)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.48

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx = \begin{cases} \frac{((50331648iAa^8c^{12}f^4e^{2ie}-50331648Ba^8c^{12}f^4e^{2ie})e^{-4ifx}+(503316480iAa^8c^{12}f^4e^{4ie}-301989888Ba^8c^{12}f^4e^{4ie})e^{-2ifx}+(-1006632960i\cdots}{\cdots} \\ x\left(-\frac{5A+iB}{16a^2c^3}+\frac{(Ae^{10ie}+5Ae^{8ie}+10Ae^{6ie}+10Ae^{4ie}+5Ae^{2ie}+A-iBe^{10ie}-3iBe^{8ie}-2iBe^{6ie}+2iBe^{4ie}+3iBe^{2ie}+iB)e^{-4ie}}{32a^2c^3}\right) \end{cases} + \frac{x(5A+iB)}{16a^2c^3}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**3,x)
```

output
```
Piecewise((((50331648*I*A*a**8*c**12*f**4*exp(2*I*e) - 50331648*B*a**8*c**
12*f**4*exp(2*I*e))*exp(-4*I*f*x) + (503316480*I*A*a**8*c**12*f**4*exp(4*I
*e) - 301989888*B*a**8*c**12*f**4*exp(4*I*e))*exp(-2*I*f*x) + (-1006632960
*I*A*a**8*c**12*f**4*exp(8*I*e) - 201326592*B*a**8*c**12*f**4*exp(8*I*e))*
exp(2*I*f*x) + (-251658240*I*A*a**8*c**12*f**4*exp(10*I*e) - 150994944*B*a
**8*c**12*f**4*exp(10*I*e))*exp(4*I*f*x) + (-33554432*I*A*a**8*c**12*f**4*
exp(12*I*e) - 33554432*B*a**8*c**12*f**4*exp(12*I*e))*exp(6*I*f*x))*exp(-6
*I*e)/(6442450944*a**10*c**15*f**5), Ne(a**10*c**15*f**5*exp(6*I*e), 0)),
(x*(-(5*A + I*B)/(16*a**2*c**3) + (A*exp(10*I*e) + 5*A*exp(8*I*e) + 10*A*e
xp(6*I*e) + 10*A*exp(4*I*e) + 5*A*exp(2*I*e) + A - I*B*exp(10*I*e) - 3*I*B
*exp(8*I*e) - 2*I*B*exp(6*I*e) + 2*I*B*exp(4*I*e) + 3*I*B*exp(2*I*e) + I*B
)*exp(-4*I*e)/(32*a**2*c**3)), True)) + x*(5*A + I*B)/(16*a**2*c**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.84

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx = -\frac{(-5i\,A+B)\log\left(\tan\left(fx+e\right)+i\right)}{32\,a^2c^3f} - \frac{(5i\,A-B)\log\left(\tan\left(fx+e\right)-i\right)}{32\,a^2c^3f} + \frac{3\,(5\,A+i\,B)\tan\left(fx+e\right)^4 - 3\,(-5i\,A+B)\tan\left(fx+e\right)^3 + 5\,(5\,A+i\,B)\tan\left(fx+e\right)^2 - 5\,(-5i\,A+}{48\,a^2c^3f(\tan\left(fx+e\right)+i)^3(\tan\left(fx+e\right)-i)^2}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-1/32*(-5*I*A + B)*log(tan(f*x + e) + I)/(a^2*c^3*f) - 1/32*(5*I*A - B)*lo
g(tan(f*x + e) - I)/(a^2*c^3*f) + 1/48*(3*(5*A + I*B)*tan(f*x + e)^4 - 3*(
-5*I*A + B)*tan(f*x + e)^3 + 5*(5*A + I*B)*tan(f*x + e)^2 - 5*(-5*I*A + B)
*tan(f*x + e) + 8*A - 8*I*B)/(a^2*c^3*f*(tan(f*x + e) + I)^3*(tan(f*x + e)
 - I)^2)
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.14

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^3}\,dx = \frac{\tan(e+f\,x)\,\left(-\frac{5\,B}{48\,a^2\,c^3}+\frac{A\,25\mathrm{i}}{48\,a^2\,c^3}\right)+\tan(e+f\,x)^3\,\left(-\frac{B}{16\,a^2\,c^3}+\frac{A\,5\mathrm{i}}{16\,a^2\,c^3}\right)+\tan(e+f\,x)^4\,\left(\frac{5\,A}{16\,a^2\,c^3}+\frac{B\,1\mathrm{i}}{16\,a^2\,c^3}\right)-}{f\,\left({\tan\left(e+f\,x\right)}^5+{\tan\left(e+f\,x\right)}^4\,1\mathrm{i}+2\,{\tan\left(e+f\,x\right)}^3+{\tan\left(e+f\,x\right)}^2\,2\mathrm{i}+\right.} - \frac{x\,\left(-B+A\,5\mathrm{i}\right)\,1\mathrm{i}}{16\,a^2\,c^3}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i
)^3),x)
```

output
```
(tan(e + f*x)*((A*25i)/(48*a^2*c^3) - (5*B)/(48*a^2*c^3)) + tan(e + f*x)^3
*((A*5i)/(16*a^2*c^3) - B/(16*a^2*c^3)) + tan(e + f*x)^4*((5*A)/(16*a^2*c^
3) + (B*1i)/(16*a^2*c^3)) + tan(e + f*x)^2*((25*A)/(48*a^2*c^3) + (B*5i)/(
48*a^2*c^3)) + A/(6*a^2*c^3) - (B*1i)/(6*a^2*c^3))/(f*(tan(e + f*x) + tan(
e + f*x)^2*2i + 2*tan(e + f*x)^3 + tan(e + f*x)^4*1i + tan(e + f*x)^5 + 1i
)) - (x*(A*5i - B)*1i)/(16*a^2*c^3)
```

Reduce [F]

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))^2(c - ic\tan(e + fx))^3}\,dx$$
$$= \frac{8\left(\int \frac{1}{\tan(fx+e)^5+\tan(fx+e)^4 i+2\tan(fx+e)^3+2\tan(fx+e)^2 i+\tan(fx+e)+i}dx\right)\tan(fx+e)^4 afi + 8\left(\int \frac{}{\tan(fx+e)^5+\tan(fx-}\right.}{}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^3,x)
```

output
```
(8*int(1/(tan(e + f*x)**5 + tan(e + f*x)**4*i + 2*tan(e + f*x)**3 + 2*tan(
e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**4*a*f*i + 8*int(1/(tan(
e + f*x)**5 + tan(e + f*x)**4*i + 2*tan(e + f*x)**3 + 2*tan(e + f*x)**2*i
+ tan(e + f*x) + i),x)*tan(e + f*x)**4*b*f + 16*int(1/(tan(e + f*x)**5 + t
an(e + f*x)**4*i + 2*tan(e + f*x)**3 + 2*tan(e + f*x)**2*i + tan(e + f*x)
+ i),x)*tan(e + f*x)**2*a*f*i + 16*int(1/(tan(e + f*x)**5 + tan(e + f*x)**
4*i + 2*tan(e + f*x)**3 + 2*tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e
 + f*x)**2*b*f + 8*int(1/(tan(e + f*x)**5 + tan(e + f*x)**4*i + 2*tan(e +
f*x)**3 + 2*tan(e + f*x)**2*i + tan(e + f*x) + i),x)*a*f*i + 8*int(1/(tan(
e + f*x)**5 + tan(e + f*x)**4*i + 2*tan(e + f*x)**3 + 2*tan(e + f*x)**2*i
+ tan(e + f*x) + i),x)*b*f + 3*tan(e + f*x)**4*b*e*i + 3*tan(e + f*x)**4*b
*f*i*x + 3*tan(e + f*x)**3*b*i + 6*tan(e + f*x)**2*b*e*i + 6*tan(e + f*x)*
*2*b*f*i*x + 5*tan(e + f*x)*b*i + 3*b*e*i + 3*b*f*i*x)/(8*a**2*c**3*f*(tan
(e + f*x)**4 + 2*tan(e + f*x)**2 + 1))
```

3.725 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 221

$$\begin{aligned}\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx &= \frac{5(3A+iB)x}{64a^2c^4}-\frac{iA-B}{64a^2c^4f(i-\tan(e+fx))^2}-\frac{5A+3iB}{64a^2c^4f(i-\tan(e+fx))}\\&\quad-\frac{iA+B}{32a^2c^4f(i+\tan(e+fx))^4}-\frac{3A-iB}{48a^2c^4f(i+\tan(e+fx))^3}\\&\quad+\frac{3iA}{32a^2c^4f(i+\tan(e+fx))^2}+\frac{5A+iB}{32a^2c^4f(i+\tan(e+fx))}\end{aligned}$$

output
```
5/64*(3*A+I*B)*x/a^2/c^4-1/64*(I*A-B)/a^2/c^4/f/(I-tan(f*x+e))^2-1/64*(5*A
+3*I*B)/a^2/c^4/f/(I-tan(f*x+e))-1/32*(I*A+B)/a^2/c^4/f/(I+tan(f*x+e))^4-1
/48*(3*A-I*B)/a^2/c^4/f/(I+tan(f*x+e))^3+3/32*I*A/a^2/c^4/f/(I+tan(f*x+e))
^2+1/32*(5*A+I*B)/a^2/c^4/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 4.15 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.96

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx$$
$$= \frac{\sec^5(e+fx)(240iA\cos(e+fx)+5(-9iA+11B)\cos(3(e+fx))-3iA\cos(5(e+fx))+9B\cos(5(e+}{}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e +
f*x])^4),x]
```

output
```
(Sec[e + f*x]^5*((240*I)*A*Cos[e + f*x] + 5*((-9*I)*A + 11*B)*Cos[3*(e + f
*x)] - (3*I)*A*Cos[5*(e + f*x)] + 9*B*Cos[5*(e + f*x)] + 102*A*Sin[e + f*x
] + (34*I)*B*Sin[e + f*x] - 60*(3*A + I*B)*ArcTan[Tan[e + f*x]]*Sec[e + f*
x]*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)]) - 87*A*Sin[3*(e + f*x)] - (29*I
)*B*Sin[3*(e + f*x)] - 9*A*Sin[5*(e + f*x)] - (3*I)*B*Sin[5*(e + f*x)]))/(
768*a^2*c^4*f*(-I + Tan[e + f*x])^2*(I + Tan[e + f*x])^4)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}dx$$
$$\Big\downarrow \; 4071$$
$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^3c^5(1-i\tan(e+fx))^5(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\downarrow 27$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^5(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2c^4f}$$

$$\downarrow 86$$

$$\frac{\int \left(-\frac{3iA}{16(\tan(e+fx)+i)^3}+\frac{5(3A+iB)}{64(\tan^2(e+fx)+1)}+\frac{-5A-3iB}{64(\tan(e+fx)-i)^2}+\frac{-5A-iB}{32(\tan(e+fx)+i)^2}+\frac{i(A+iB)}{32(\tan(e+fx)-i)^3}+\frac{3A-iB}{16(\tan(e+fx)+i)^4}\right.}{a^2c^4f}$$

$$\downarrow 2009$$

$$\frac{\frac{5}{64}(3A+iB)\arctan(\tan(e+fx))-\frac{5A+3iB}{64(-\tan(e+fx)+i)}+\frac{5A+iB}{32(\tan(e+fx)+i)}-\frac{-B+iA}{64(-\tan(e+fx)+i)^2}-\frac{3A-iB}{48(\tan(e+fx)+i)^3}-}{a^2c^4f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^
4),x]
```

output
```
((5*(3*A + I*B)*ArcTan[Tan[e + f*x]])/64 - (I*A - B)/(64*(I - Tan[e + f*x]
)^2) - (5*A + (3*I)*B)/(64*(I - Tan[e + f*x])) - (I*A + B)/(32*(I + Tan[e
+ f*x])^4) - (3*A - I*B)/(48*(I + Tan[e + f*x])^3) + (((3*I)/32)*A)/(I + T
an[e + f*x])^2 + (5*A + I*B)/(32*(I + Tan[e + f*x])))/(a^2*c^4*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.27

method	result
norman	$\frac{\frac{5(iB+3A)x}{64ac}-\frac{3iA+B}{12acf}+\frac{B\tan(fx+e)^2}{6acf}+\frac{73(iB+3A)\tan(fx+e)^3}{192acf}+\frac{55(iB+3A)\tan(fx+e)^5}{192acf}+\frac{5(iB+3A)\tan(fx+e)^7}{64acf}+\frac{5(iB+3A)x}{16}}{a\,c^3\left(1+\tan(fx+\right.}$
risch	$\frac{5ixB}{64a^2c^4}+\frac{15xA}{64a^2c^4}-\frac{e^{8i(fx+e)}B}{512a^2c^4f}-\frac{ie^{8i(fx+e)}A}{512a^2c^4f}-\frac{e^{6i(fx+e)}B}{96a^2c^4f}-\frac{ie^{6i(fx+e)}A}{64a^2c^4f}-\frac{3\cos(4fx+4e)B}{128a^2c^4f}-\frac{7i\cos(}{128}$
derivativedivides	$-\frac{iA}{32f\,a^2c^4(i+\tan(fx+e))^4}+\frac{iB}{48f\,a^2c^4(i+\tan(fx+e))^3}+\frac{15A\arctan(\tan(fx+e))}{64f\,a^2c^4}+\frac{B}{64f\,a^2c^4(-i+\tan(fx+e))}$
default	$-\frac{iA}{32f\,a^2c^4(i+\tan(fx+e))^4}+\frac{iB}{48f\,a^2c^4(i+\tan(fx+e))^3}+\frac{15A\arctan(\tan(fx+e))}{64f\,a^2c^4}+\frac{B}{64f\,a^2c^4(-i+\tan(fx+e))}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x,method=_R
ETURNVERBOSE)
```

output
```
(5/64*(3*A+I*B)/a/c*x-1/12*(3*I*A+B)/a/c/f+1/6/a/c/f*B*tan(f*x+e)^2+73/192
*(3*A+I*B)/a/c/f*tan(f*x+e)^3+55/192*(3*A+I*B)/a/c/f*tan(f*x+e)^5+5/64*(3*
A+I*B)/a/c/f*tan(f*x+e)^7+5/16*(3*A+I*B)/a/c*x*tan(f*x+e)^2+15/32*(3*A+I*B
)/a/c*x*tan(f*x+e)^4+5/16*(3*A+I*B)/a/c*x*tan(f*x+e)^6+5/64*(3*A+I*B)/a/c*
x*tan(f*x+e)^8+1/64*(49*A-5*I*B)/a/c/f*tan(f*x+e))/a/c^3/(1+tan(f*x+e)^2)^
4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.57

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx = \frac{(120\,(3\,A+i\,B)fxe^{(4i\,fx+4i\,e)} - 3\,(i\,A+B)e^{(12i\,fx+12i\,e)} - 8\,(3i\,A+2\,B)e^{(10i\,fx+10i\,e)} - 30\,(3i\,A+B)e^{(8i}}{1536\,a^2c^4f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, al
gorithm="fricas")
```

output
```
1/1536*(120*(3*A + I*B)*f*x*e^(4*I*f*x + 4*I*e) - 3*(I*A + B)*e^(12*I*f*x
+ 12*I*e) - 8*(3*I*A + 2*B)*e^(10*I*f*x + 10*I*e) - 30*(3*I*A + B)*e^(8*I*
f*x + 8*I*e) - 240*I*A*e^(6*I*f*x + 6*I*e) - 24*(-3*I*A + 2*B)*e^(2*I*f*x
+ 2*I*e) + 6*I*A - 6*B)*e^(-4*I*f*x - 4*I*e)/(a^2*c^4*f)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.25

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx = \begin{cases} \frac{(-2061584302080iAa^{10}c^{20}f^5e^{8ie}e^{2ifx}+(51539607552iAa^{10}c^{20}f^5e^{2ie}-51539607552Ba^{10}c^{20}f^5e^{2ie})e^{-4ifx}+(618475290624iAa^{10}c^{20}f^5e^{4i}}{} \\ x\left(-\frac{15A+5iB}{64a^2c^4}+\frac{(Ae^{12ie}+6Ae^{10ie}+15Ae^{8ie}+20Ae^{6ie}+15Ae^{4ie}+6Ae^{2ie}+A-iBe^{12ie}-4iBe^{10ie}-5iBe^{8ie}+5iBe^{4ie}+4iBe^{2ie}+iB)e^{-4i}}{64a^2c^4}\right. \end{cases} + \frac{x(15A+5iB)}{64a^2c^4}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**4,x)
```

output
```
Piecewise(((-2061584302080*I*A*a**10*c**20*f**5*exp(8*I*e)*exp(2*I*f*x) +
(51539607552*I*A*a**10*c**20*f**5*exp(2*I*e) - 51539607552*B*a**10*c**20*f
**5*exp(2*I*e))*exp(-4*I*f*x) + (618475290624*I*A*a**10*c**20*f**5*exp(4*I
*e) - 412316860416*B*a**10*c**20*f**5*exp(4*I*e))*exp(-2*I*f*x) + (-773094
113280*I*A*a**10*c**20*f**5*exp(10*I*e) - 257698037760*B*a**10*c**20*f**5*
exp(10*I*e))*exp(4*I*f*x) + (-206158430208*I*A*a**10*c**20*f**5*exp(12*I*e
) - 137438953472*B*a**10*c**20*f**5*exp(12*I*e))*exp(6*I*f*x) + (-25769803
776*I*A*a**10*c**20*f**5*exp(14*I*e) - 25769803776*B*a**10*c**20*f**5*exp(
14*I*e))*exp(8*I*f*x))*exp(-6*I*e)/(13194139533312*a**12*c**24*f**6), Ne(a
**12*c**24*f**6*exp(6*I*e), 0)), (x*(-(15*A + 5*I*B)/(64*a**2*c**4) + (A*e
xp(12*I*e) + 6*A*exp(10*I*e) + 15*A*exp(8*I*e) + 20*A*exp(6*I*e) + 15*A*ex
p(4*I*e) + 6*A*exp(2*I*e) + A - I*B*exp(12*I*e) - 4*I*B*exp(10*I*e) - 5*I*
B*exp(8*I*e) + 5*I*B*exp(4*I*e) + 4*I*B*exp(2*I*e) + I*B)*exp(-4*I*e)/(64*
a**2*c**4)), True)) + x*(15*A + 5*I*B)/(64*a**2*c**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.78

$$\int \frac{A + B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx$$
$$= \frac{5\,(3i\,A - B)\log\left(\tan\left(fx+e\right)+i\right)}{128\,a^2c^4f} - \frac{5\,(3i\,A - B)\log\left(\tan\left(fx+e\right)-i\right)}{128\,a^2c^4f}$$
$$+ \frac{15\,(3\,A + i\,B)\tan\left(fx+e\right)^5 - 30\,(-3i\,A + B)\tan\left(fx+e\right)^4 + 10\,(3\,A + i\,B)\tan\left(fx+e\right)^3 - 50\,(-3i}{192\,a^2c^4f(\tan\left(fx+e\right)+i)^4(\tan\left(fx+e\right.}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x, al
gorithm="giac")
```

output
```
5/128*(3*I*A - B)*log(tan(f*x + e) + I)/(a^2*c^4*f) - 5/128*(3*I*A - B)*lo
g(tan(f*x + e) - I)/(a^2*c^4*f) + 1/192*(15*(3*A + I*B)*tan(f*x + e)^5 - 3
0*(-3*I*A + B)*tan(f*x + e)^4 + 10*(3*A + I*B)*tan(f*x + e)^3 - 50*(-3*I*A
 + B)*tan(f*x + e)^2 - 17*(3*A + I*B)*tan(f*x + e) + 48*I*A + 16*B)/(a^2*c
^4*f*(tan(f*x + e) + I)^4*(tan(f*x + e) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 6.89 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.12

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx = \frac{\frac{B}{12\,a^2\,c^4}+\tan(e+f\,x)^4\,\left(-\frac{5\,B}{32\,a^2\,c^4}+\frac{A\,15\mathrm{i}}{32\,a^2\,c^4}\right)+\tan(e+f\,x)^3\,\left(\frac{5\,A}{32\,a^2\,c^4}+\frac{B\,5\mathrm{i}}{96\,a^2\,c^4}\right)+\tan(e+f\,x)^5\,\left(\frac{15\,A}{64\,a^2\,c^4}+\frac{}{6}\right.}{f\,\left(\tan(e+f\,x)^6+\tan(e+f\,x)^5\,2\mathrm{i}+\tan(e+f\,x)^4+\tan(e+f\right.} + \frac{5\,x\,(3\,A+B\,1\mathrm{i})}{64\,a^2\,c^4}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i
)^4),x)
```

output
```
(tan(e + f*x)^4*((A*15i)/(32*a^2*c^4) - (5*B)/(32*a^2*c^4)) - tan(e + f*x)
*((17*A)/(64*a^2*c^4) + (B*17i)/(192*a^2*c^4)) + tan(e + f*x)^3*((5*A)/(32
*a^2*c^4) + (B*5i)/(96*a^2*c^4)) + tan(e + f*x)^5*((15*A)/(64*a^2*c^4) + (
B*5i)/(64*a^2*c^4)) + tan(e + f*x)^2*((A*25i)/(32*a^2*c^4) - (25*B)/(96*a^
2*c^4)) + (A*1i)/(4*a^2*c^4) + B/(12*a^2*c^4))/(f*(tan(e + f*x)*2i - tan(e
 + f*x)^2 + tan(e + f*x)^3*4i + tan(e + f*x)^4 + tan(e + f*x)^5*2i + tan(e
 + f*x)^6 - 1)) + (5*x*(3*A + B*1i))/(64*a^2*c^4)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^4}\,dx$$
$$= \frac{-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^6+2\tan(fx+e)^5 i+\tan(fx+e)^4+4\tan(fx+e)^3 i-\tan(fx+e)^2+2\tan(fx+e) i-1}dx\right)b-\left(\int \frac{}{\tan(fx+e)^6+2\tan(fx+e)^5 i}\right.}{a^2c^4}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^4,x)
```

output
```
( - (int(tan(e + f*x)/(tan(e + f*x)**6 + 2*tan(e + f*x)**5*i + tan(e + f*x
)**4 + 4*tan(e + f*x)**3*i - tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*b
+ int(1/(tan(e + f*x)**6 + 2*tan(e + f*x)**5*i + tan(e + f*x)**4 + 4*tan(e
 + f*x)**3*i - tan(e + f*x)**2 + 2*tan(e + f*x)*i - 1),x)*a))/(a**2*c**4)
```

3.726 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 251

$$\begin{aligned}\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}\,dx &= \frac{3(7A+3iB)x}{128a^2c^5}-\frac{iA-B}{128a^2c^5f(i-\tan(e+fx))^2}\\ &\quad-\frac{3A+2iB}{64a^2c^5f(i-\tan(e+fx))}+\frac{A-iB}{40a^2c^5f(i+\tan(e+fx))^5}\\ &\quad-\frac{3iA+B}{64a^2c^5f(i+\tan(e+fx))^4}-\frac{A}{16a^2c^5f(i+\tan(e+fx))^3}\\ &\quad+\frac{5iA-B}{64a^2c^5f(i+\tan(e+fx))^2}+\frac{5(3A+iB)}{128a^2c^5f(i+\tan(e+fx))}\end{aligned}$$

output
```
3/128*(7*A+3*I*B)*x/a^2/c^5-1/128*(I*A-B)/a^2/c^5/f/(I-tan(f*x+e))^2-1/64*
(3*A+2*I*B)/a^2/c^5/f/(I-tan(f*x+e))+1/40*(A-I*B)/a^2/c^5/f/(I+tan(f*x+e))
^5-1/64*(3*I*A+B)/a^2/c^5/f/(I+tan(f*x+e))^4-1/16*A/a^2/c^5/f/(I+tan(f*x+e
))^3+1/64*(5*I*A-B)/a^2/c^5/f/(I+tan(f*x+e))^2+5/128*(3*A+I*B)/a^2/c^5/f/(
I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.43 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.93

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{\sec^6(e+fx)(-241A+11iB-8(71A+9iB)\cos(2(e+fx))+3(33A+37iB)\cos(4(e+fx))+6A\cos}{}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e +
f*x])^5),x]
```

output
```
(Sec[e + f*x]^6*(-241*A + (11*I)*B - 8*(71*A + (9*I)*B)*Cos[2*(e + f*x)] +
 3*(33*A + (37*I)*B)*Cos[4*(e + f*x)] + 6*A*Cos[6*(e + f*x)] + (14*I)*B*Co
s[6*(e + f*x)] + (350*I)*A*Sin[2*(e + f*x)] - 150*B*Sin[2*(e + f*x)] + 60*
((-7*I)*A + 3*B)*ArcTan[Tan[e + f*x]]*Sec[e + f*x]*(Cos[3*(e + f*x)] - I*S
in[3*(e + f*x)]) - (161*I)*A*Sin[4*(e + f*x)] + 69*B*Sin[4*(e + f*x)] - (1
4*I)*A*Sin[6*(e + f*x)] + 6*B*Sin[6*(e + f*x)]))/(2560*a^2*c^5*f*(-I + Tan
[e + f*x])^2*(I + Tan[e + f*x])^5)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}dx$$
$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^3c^6(1-i\tan(e+fx))^6(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^6(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2c^5f}$$

$\Big\downarrow$ 86

$$\frac{\int \left(\frac{3A}{16(\tan(e+fx)+i)^4}+\frac{3(7A+3iB)}{128(\tan^2(e+fx)+1)}+\frac{-3A-2iB}{64(\tan(e+fx)-i)^2}-\frac{5(3A+iB)}{128(\tan(e+fx)+i)^2}+\frac{i(A+iB)}{64(\tan(e+fx)-i)^3}+\frac{B-5iA}{32(\tan(e+fx)+i)^3}\right.}{a^2c^5f}$$

$\Big\downarrow$ 2009

$$\frac{\frac{3}{128}(7A+3iB)\arctan(\tan(e+fx))-\frac{3A+2iB}{64(-\tan(e+fx)+i)}+\frac{5(3A+iB)}{128(\tan(e+fx)+i)}-\frac{-B+iA}{128(-\tan(e+fx)+i)^2}+\frac{-B+5iA}{64(\tan(e+fx)+i)^2}}{a^2c^5f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^
5),x]
```

output
```
((3*(7*A + (3*I)*B)*ArcTan[Tan[e + f*x]])/128 - (I*A - B)/(128*(I - Tan[e
+ f*x])^2) - (3*A + (2*I)*B)/(64*(I - Tan[e + f*x])) + (A - I*B)/(40*(I +
Tan[e + f*x])^5) - ((3*I)*A + B)/(64*(I + Tan[e + f*x])^4) - A/(16*(I + Ta
n[e + f*x])^3) + ((5*I)*A - B)/(64*(I + Tan[e + f*x])^2) + (5*(3*A + I*B))
/(128*(I + Tan[e + f*x])))/(a^2*c^5*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.21

method	result
risch	$\frac{9ixB}{128a^2c^5}+\frac{21xA}{128a^2c^5}-\frac{e^{10i(fx+e)}B}{1280a^2c^5f}-\frac{ie^{10i(fx+e)}A}{1280a^2c^5f}-\frac{5\,e^{8i(fx+e)}B}{1024a^2c^5f}-\frac{7ie^{8i(fx+e)}A}{1024a^2c^5f}-\frac{3\,e^{6i(fx+e)}B}{256a^2c^5f}-\frac{7ie^{6}}{25}$
norman	$\frac{3(3iB+7A)x}{128ac}-\frac{11iA+B}{40acf}+\frac{(-9iB+107A)\tan(fx+e)}{128acf}+\frac{(3iB+7A)\tan(fx+e)^5}{5acf}+\frac{7(3iB+7A)\tan(fx+e)^7}{64acf}+\frac{3(3iB+7A)\tan(fx+e)^9}{128acf}$
derivativedivides	$-\frac{A}{16a^2c^5f(i+\tan(fx+e))^3}-\frac{iB}{40f\,a^2c^5(i+\tan(fx+e))^5}+\frac{15A}{128f\,a^2c^5(i+\tan(fx+e))}-\frac{iA}{128f\,a^2c^5(-i+\tan(fx+e}$
default	$-\frac{A}{16a^2c^5f(i+\tan(fx+e))^3}-\frac{iB}{40f\,a^2c^5(i+\tan(fx+e))^5}+\frac{15A}{128f\,a^2c^5(i+\tan(fx+e))}-\frac{iA}{128f\,a^2c^5(-i+\tan(fx+e}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^5,x,method=_R
ETURNVERBOSE)
```

output
```
9/128*I*x/a^2/c^5*B+21/128*x/a^2/c^5*A-1/1280/a^2/c^5/f*exp(10*I*(f*x+e))*
B-1/1280*I/a^2/c^5/f*exp(10*I*(f*x+e))*A-5/1024/a^2/c^5/f*exp(8*I*(f*x+e))
*B-7/1024*I/a^2/c^5/f*exp(8*I*(f*x+e))*A-3/256/a^2/c^5/f*exp(6*I*(f*x+e))*
B-7/256*I/a^2/c^5/f*exp(6*I*(f*x+e))*A-3/256/a^2/c^5/f*cos(4*f*x+4*e)*B-17
/256*I/a^2/c^5/f*cos(4*f*x+4*e)*A-1/128*I/a^2/c^5/f*sin(4*f*x+4*e)*B+9/128
/a^2/c^5/f*sin(4*f*x+4*e)*A-7/64*I*A/a^2/c^5/f*cos(2*f*x+2*e)+5/128*I/a^2/
c^5/f*sin(2*f*x+2*e)*B+21/128/a^2/c^5/f*sin(2*f*x+2*e)*A
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{\left(120\,(7\,A+3i\,B)fxe^{(4i\,fx+4i\,e)}-4\,(i\,A+B)e^{(14i\,fx+14i\,e)}-5\,(7i\,A+5\,B)e^{(12i\,fx+12i\,e)}-20\,(7i\,A+3\,B)e\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^5,x, al
gorithm="fricas")
```

output
```
1/5120*(120*(7*A + 3*I*B)*f*x*e^(4*I*f*x + 4*I*e) - 4*(I*A + B)*e^(14*I*f*
x + 14*I*e) - 5*(7*I*A + 5*B)*e^(12*I*f*x + 12*I*e) - 20*(7*I*A + 3*B)*e^(
10*I*f*x + 10*I*e) - 50*(7*I*A + B)*e^(8*I*f*x + 8*I*e) - 100*(7*I*A - B)*
e^(6*I*f*x + 6*I*e) - 20*(-7*I*A + 5*B)*e^(2*I*f*x + 2*I*e) + 10*I*A - 10*
B)*e^(-4*I*f*x - 4*I*e)/(a^2*c^5*f)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.41

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}\,dx$$
$$= \begin{cases} \frac{((1125899906842624 0iAa^{12}c^{30}f^6e^{2ie}-11258999068426240Ba^{12}c^{30}f^6e^{2ie})e^{-4ifx}+(157625986957967360iAa^{12}c^{30}f^6e^{4ie}-1125899906842}{} \\ x\left(-\frac{21A+9iB}{128a^2c^5}+\frac{(Ae^{14ie}+7Ae^{12ie}+21Ae^{10ie}+35Ae^{8ie}+35Ae^{6ie}+21Ae^{4ie}+7Ae^{2ie}+A-iBe^{14ie}-5iBe^{12ie}-9iBe^{10ie}-5iBe^{8ie}+5iBe^{6}}{128a^2c^5}\right. \end{cases}$$
$$+\frac{x(21A+9iB)}{128a^2c^5}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**5,x)
```

output
```
Piecewise((((112589990684262400*I*A*a**12*c**30*f**6*exp(2*I*e) - 112589990
68426240*B*a**12*c**30*f**6*exp(2*I*e))*exp(-4*I*f*x) + (15762598695796736
0*I*A*a**12*c**30*f**6*exp(4*I*e) - 112589990684262400*B*a**12*c**30*f**6*
exp(4*I*e))*exp(-2*I*f*x) + (-788129934789836800*I*A*a**12*c**30*f**6*exp(
8*I*e) + 112589990684262400*B*a**12*c**30*f**6*exp(8*I*e))*exp(2*I*f*x) +
(-394064967394918400*I*A*a**12*c**30*f**6*exp(10*I*e) - 56294995342131200*
B*a**12*c**30*f**6*exp(10*I*e))*exp(4*I*f*x) + (-157625986957967360*I*A*a*
*12*c**30*f**6*exp(12*I*e) - 67553994410557440*B*a**12*c**30*f**6*exp(12*I
*e))*exp(6*I*f*x) + (-39406496739491840*I*A*a**12*c**30*f**6*exp(14*I*e) -
 28147497671065600*B*a**12*c**30*f**6*exp(14*I*e))*exp(8*I*f*x) + (-450359
9627370496*I*A*a**12*c**30*f**6*exp(16*I*e) - 4503599627370496*B*a**12*c**
30*f**6*exp(16*I*e))*exp(10*I*f*x))*exp(-6*I*e)/(5764607523034234880*a**14
*c**35*f**7), Ne(a**14*c**35*f**7*exp(6*I*e), 0)), (x*(-(21*A + 9*I*B)/(12
8*a**2*c**5) + (A*exp(14*I*e) + 7*A*exp(12*I*e) + 21*A*exp(10*I*e) + 35*A*
exp(8*I*e) + 35*A*exp(6*I*e) + 21*A*exp(4*I*e) + 7*A*exp(2*I*e) + A - I*B*
exp(14*I*e) - 5*I*B*exp(12*I*e) - 9*I*B*exp(10*I*e) - 5*I*B*exp(8*I*e) + 5
*I*B*exp(6*I*e) + 9*I*B*exp(4*I*e) + 5*I*B*exp(2*I*e) + I*B)*exp(-4*I*e)/(
128*a**2*c**5)), True)) + x*(21*A + 9*I*B)/(128*a**2*c**5)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^5,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.77

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}\,dx$$
$$= -\frac{3\,(-7i\,A+3\,B)\log\left(\tan\left(fx+e\right)+i\right)}{256\,a^2c^5f} - \frac{3\,(7i\,A-3\,B)\log\left(\tan\left(fx+e\right)-i\right)}{256\,a^2c^5f}$$
$$+ \frac{15\,(7\,A+3i\,B)\tan\left(fx+e\right)^6 - 45\,(-7i\,A+3\,B)\tan\left(fx+e\right)^5 - 20\,(7\,A+3i\,B)\tan\left(fx+e\right)^4 - 60\,(}{640\,a^2c^5f(\tan\left(fx+e\right)}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^5,x, al
gorithm="giac")
```

output
```
-3/256*(-7*I*A + 3*B)*log(tan(f*x + e) + I)/(a^2*c^5*f) - 3/256*(7*I*A - 3
*B)*log(tan(f*x + e) - I)/(a^2*c^5*f) + 1/640*(15*(7*A + 3*I*B)*tan(f*x +
e)^6 - 45*(-7*I*A + 3*B)*tan(f*x + e)^5 - 20*(7*A + 3*I*B)*tan(f*x + e)^4
- 60*(-7*I*A + 3*B)*tan(f*x + e)^3 - 67*(7*A + 3*I*B)*tan(f*x + e)^2 - (7*
I*A - 3*B)*tan(f*x + e) - 176*A + 16*I*B)/(a^2*c^5*f*(tan(f*x + e) + I)^5*
(tan(f*x + e) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.16

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{\tan(e+f\,x)\,\left(-\frac{3\,B}{640\,a^2\,c^5}+\frac{A\,7\mathrm{i}}{640\,a^2\,c^5}\right)+\tan(e+f\,x)^4\,\left(\frac{7\,A}{32\,a^2\,c^5}+\frac{B\,3\mathrm{i}}{32\,a^2\,c^5}\right)-\tan(e+f\,x)^3\,\left(-\frac{9\,B}{32\,a^2\,c^5}+\frac{A\,21\mathrm{i}}{32\,a^2\,c^5}\right)}{f\,\left(-\tan\left(e+f\,x\right)^7-\tan\left(e+f\,x\right)^6\,3\mathrm{i}+\tan\left(e+f\,x\right)^5\right.}$$
$$+ \frac{3\,x\,(7\,A+B\,3\mathrm{i})}{128\,a^2\,c^5}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i
)^5),x)
```

output
```
(tan(e + f*x)*((A*7i)/(640*a^2*c^5) - (3*B)/(640*a^2*c^5)) + tan(e + f*x)^
4*((7*A)/(32*a^2*c^5) + (B*3i)/(32*a^2*c^5)) - tan(e + f*x)^3*((A*21i)/(32
*a^2*c^5) - (9*B)/(32*a^2*c^5)) - tan(e + f*x)^6*((21*A)/(128*a^2*c^5) + (
B*9i)/(128*a^2*c^5)) - tan(e + f*x)^5*((A*63i)/(128*a^2*c^5) - (27*B)/(128
*a^2*c^5)) + tan(e + f*x)^2*((469*A)/(640*a^2*c^5) + (B*201i)/(640*a^2*c^5
)) + (11*A)/(40*a^2*c^5) - (B*1i)/(40*a^2*c^5))/(f*(3*tan(e + f*x) - tan(e
 + f*x)^2*1i + 5*tan(e + f*x)^3 - tan(e + f*x)^4*5i + tan(e + f*x)^5 - tan
(e + f*x)^6*3i - tan(e + f*x)^7 + 1i)) + (3*x*(7*A + B*3i))/(128*a^2*c^5)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^7 i-3\tan(fx+e)^6-\tan(fx+e)^5 i-5\tan(fx+e)^4-5\tan(fx+e)^3 i-\tan(fx+e)^2-3\tan(fx+e) i+1}dx\right)b+\left(\int \frac{}{\tan(fx+e)^7 i-}\right.}{a^2c^5}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^5,x)
```

output
```
(int(tan(e + f*x)/(tan(e + f*x)**7*i - 3*tan(e + f*x)**6 - tan(e + f*x)**5
*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)**3*i - tan(e + f*x)**2 - 3*tan(e +
 f*x)*i + 1),x)*b + int(1/(tan(e + f*x)**7*i - 3*tan(e + f*x)**6 - tan(e +
 f*x)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)**3*i - tan(e + f*x)**2 - 3
*tan(e + f*x)*i + 1),x)*a)/(a**2*c**5)
```

3.727 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 115

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx = \frac{(iA(3-n)+B(3+n))\,\text{Hypergeometric2F1}\left(3,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(c-ic\tan(e+fx))^n}{48a^3fn} + \frac{(iA-B)(c-ic\tan(e+fx))^n}{6a^3f(1+i\tan(e+fx))^3}$$

output
```
1/48*(I*A*(3-n)+B*(3+n))*hypergeom([3, n],[1+n],1/2-1/2*I*tan(f*x+e))*(c-I
*c*tan(f*x+e))^n/a^3/f/n+1/6*(I*A-B)*(c-I*c*tan(f*x+e))^n/a^3/f/(1+I*tan(f
*x+e))^3
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.83

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx = \frac{(c-ic\tan(e+fx))^n\left(-B(-7+n^2)+iA(11-6n+n^2)+\frac{(2-3n+n^2)(-iA(-3+n)+B(3+n))\,\text{Hypergeometric2F1}(1}{n}\right.}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n)/(a + I*a*Tan[e +
 f*x])^3,x]
```

output
```
((c - I*c*Tan[e + f*x])^n*(-(B*(-7 + n^2)) + I*A*(11 - 6*n + n^2) + ((2 -
3*n + n^2)*((-I)*A*(-3 + n) + B*(3 + n))*Hypergeometric2F1[1, n, 1 + n, (-
1/2*I)*(I + Tan[e + f*x])])/n + ((8*I)*(A + I*B)*(I + Tan[e + f*x]))/(-I +
 Tan[e + f*x])^3 + (2*(A*(-5 + n) + I*B*(1 + n))*(I + Tan[e + f*x]))/(-I +
 Tan[e + f*x])^2 + ((B*(-7 + n^2) - I*A*(11 - 6*n + n^2))*(I + Tan[e + f*x
]))/(-I + Tan[e + f*x])))/(96*a^3*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}}{a^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}}{(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \text{87}$$

$$\frac{c\left(\frac{1}{6}(A(3-n)-iB(n+3))\int \frac{(c-ic\tan(e+fx))^{n-1}}{(i\tan(e+fx)+1)^3}d\tan(e+fx)+\frac{(-B+iA)(c-ic\tan(e+fx))^n}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

$$\Big\downarrow 78$$

$$\frac{c\left(\frac{i(A(3-n)-iB(n+3))(c-ic\tan(e+fx))^n \text{Hypergeometric2F1}\left(3,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{48cn}+\frac{(-B+iA)(c-ic\tan(e+fx))^n}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n)/(a + I*a*Tan[e + f*x])
^3,x]
```

output
```
(c*(((I/48)*(A*(3 - n) - I*B*(3 + n))*Hypergeometric2F1[3, n, 1 + n, (1 -
I*Tan[e + f*x])/2]*(c - I*c*Tan[e + f*x])^n)/(c*n) + ((I*A - B)*(c - I*c*T
an[e + f*x])^n)/(6*c*(1 + I*Tan[e + f*x])^3)))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int \frac{(A+B\tan(fx+e))\,(c-ic\tan(fx+e))^n}{(a+ia\tan(fx+e))^3}dx$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
```

output
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
```

Fricas [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx = \int \frac{(B\tan(fx+e)+A)(-i\,c\tan(fx+e)+c)^n}{(i\,a\tan(fx+e)+a)^3}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
integral(1/8*((A - I*B)*e^(6*I*f*x + 6*I*e) + (3*A - I*B)*e^(4*I*f*x + 4*I
*e) + (3*A + I*B)*e^(2*I*f*x + 2*I*e) + A + I*B)*(2*c/(e^(2*I*f*x + 2*I*e)
 + 1))^n*e^(-6*I*f*x - 6*I*e)/a^3, x)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx = \frac{i\left(\int \frac{A(-ic\tan(e+fx)+c)^n}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx + \int \frac{B(-ic\tan(e+fx)+c)^n\tan(e+fx)}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx\right)}{a^3}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**n/(a+I*a*tan(f*x+e))**3,x)
```

output
```
I*(Integral(A*(-I*c*tan(e + f*x) + c)**n/(tan(e + f*x)**3 - 3*I*tan(e + f*
x)**2 - 3*tan(e + f*x) + I), x) + Integral(B*(-I*c*tan(e + f*x) + c)**n*ta
n(e + f*x)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x
))/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx = \int \frac{(B\tan(fx+e)+A)(-i\,c\tan(fx+e)+c)^n}{(i\,a\tan(fx+e)+a)^3}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(-I*c*tan(f*x + e) + c)^n/(I*a*tan(f*x + e)
 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx = \int \frac{(A+B\tan(e+f\,x))\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^n}{\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^3}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^n)/(a + a*tan(e + f*x)*1
i)^3,x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^n)/(a + a*tan(e + f*x)*1
i)^3, x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^n}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \frac{(-\tan(fx+e)\,ci+c)^n\,ai+\left(\int \frac{(-\tan(fx+e)ci+c)^n\tan(fx+e)^4}{\tan(fx+e)^3i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right)afn-4\left(\int \frac{(-\tan(fx+e)ci+c)^n\,\mathrm{t}}{\tan(fx+e)^3i+3\tan(fx+e)^2}\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(( - tan(e + f*x)*c*i + c)**n*a*i + int((( - tan(e + f*x)*c*i + c)**n*tan(
e + f*x)**4)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1
),x)*a*f*n - 4*int((( - tan(e + f*x)*c*i + c)**n*tan(e + f*x)**3)/(tan(e +
 f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*i*n - 6*int(
(( - tan(e + f*x)*c*i + c)**n*tan(e + f*x)**2)/(tan(e + f*x)**3*i + 3*tan(
e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*n + 4*int((( - tan(e + f*x)*c*i
 + c)**n*tan(e + f*x))/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e +
f*x)*i - 1),x)*a*f*i*n - int((( - tan(e + f*x)*c*i + c)**n*tan(e + f*x))/(
tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*f*n)/(a
**3*f*n)
```

3.728 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 191

$$\begin{aligned}\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^3}\,dx &= -\frac{8(A+4iB)c^5x}{a^3}-\frac{8(iA-4B)c^5\log(\cos(e+fx))}{a^3f}\\ &\quad+\frac{16(A+iB)c^5}{3a^3f(i-\tan(e+fx))^3}+\frac{8(2iA-3B)c^5}{a^3f(i-\tan(e+fx))^2}\\ &\quad-\frac{8(3A+7iB)c^5}{a^3f(i-\tan(e+fx))}+\frac{(A+8iB)c^5\tan(e+fx)}{a^3f}+\frac{Bc^5\tan^2(e+fx)}{2a^3f}\end{aligned}$$

output
```
-8*(A+4*I*B)*c^5*x/a^3-8*(I*A-4*B)*c^5*ln(cos(f*x+e))/a^3/f+16/3*(A+I*B)*c
^5/a^3/f/(I-tan(f*x+e))^3+8*(2*I*A-3*B)*c^5/a^3/f/(I-tan(f*x+e))^2-8*(3*A+
7*I*B)*c^5/a^3/f/(I-tan(f*x+e))+(A+8*I*B)*c^5*tan(f*x+e)/a^3/f+1/2*B*c^5*t
an(f*x+e)^2/a^3/f
```

Mathematica [A] (verified)

Time = 3.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.80

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^3}\,dx$$
$$=\frac{\frac{2(A+4iB)c^5(i+\tan(e+fx))^4}{a^3(-i+\tan(e+fx))^3}+\frac{B(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^3}-\frac{16i(A+4iB)c^5\left(-\log(i-\tan(e+fx))+\frac{2\left(-4i+9\tan(e+fx)+9i\tan^2(e+fx)\right)}{3(-i+\tan(e+fx))^3}\right)}{a^3}}{2f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^5)/(a + I*a*Tan[e +
 f*x])^3,x]
```

output
```
((2*(A + (4*I)*B)*c^5*(I + Tan[e + f*x])^4)/(a^3*(-I + Tan[e + f*x])^3) +
(B*(c - I*c*Tan[e + f*x])^5)/(a + I*a*Tan[e + f*x])^3 - ((16*I)*(A + (4*I)
*B)*c^5*(-Log[I - Tan[e + f*x]] + (2*(-4*I + 9*Tan[e + f*x] + (9*I)*Tan[e
+ f*x]^2))/(3*(-I + Tan[e + f*x])^3)))/a^3)/(2*f)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^5(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c-ic\tan(e+fx))^5(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{c^4(1-i\tan(e+fx))^4(A+B\tan(e+fx))}{a^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{c^5\int\frac{(1-i\tan(e+fx))^4(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \; 86$$

$$\frac{c^5\int\left(\frac{16(A+iB)}{(\tan(e+fx)-i)^4}+A\left(\frac{8iB}{A}+1\right)+B\tan(e+fx)+\frac{8i(A+4iB)}{\tan(e+fx)-i}-\frac{8(3A+7iB)}{(\tan(e+fx)-i)^2}+\frac{16(3B-2iA)}{(\tan(e+fx)-i)^3}\right)d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^5\left((A+8iB)\tan(e+fx)-\frac{8(3A+7iB)}{-\tan(e+fx)+i}+\frac{8(-3B+2iA)}{(-\tan(e+fx)+i)^2}+\frac{16(A+iB)}{3(-\tan(e+fx)+i)^3}+8(-4B+iA)\log(-\tan(e+fx\right.}{a^3f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^5)/(a + I*a*Tan[e + f*x])
^3,x]
```

output
```
(c^5*(8*(I*A - 4*B)*Log[I - Tan[e + f*x]] + (16*(A + I*B))/(3*(I - Tan[e +
 f*x])^3) + (8*((2*I)*A - 3*B))/(I - Tan[e + f*x])^2 - (8*(3*A + (7*I)*B))
/(I - Tan[e + f*x]) + (A + (8*I)*B)*Tan[e + f*x] + (B*Tan[e + f*x]^2)/2))/
(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{c^5 A \tan(fx+e)}{f\,a^3}+\frac{8ic^5 \tan(fx+e) B}{f\,a^3}+\frac{B\,c^5 \tan(fx+e)^2}{2a^3 f}+\frac{16ic^5 A}{f\,a^3(-i+\tan(fx+e))^2}-\frac{24c^5 B}{f\,a^3(-i+\tan(fx+e))^2}-$
default	$\frac{c^5 A \tan(fx+e)}{f\,a^3}+\frac{8ic^5 \tan(fx+e) B}{f\,a^3}+\frac{B\,c^5 \tan(fx+e)^2}{2a^3 f}+\frac{16ic^5 A}{f\,a^3(-i+\tan(fx+e))^2}-\frac{24c^5 B}{f\,a^3(-i+\tan(fx+e))^2}-$
risch	$-\frac{18c^5 e^{-2i(fx+e)} B}{a^3 f}+\frac{6ic^5 e^{-2i(fx+e)} A}{a^3 f}+\frac{4c^5 e^{-4i(fx+e)} B}{a^3 f}-\frac{2ic^5 e^{-4i(fx+e)} A}{a^3 f}-\frac{2c^5 e^{-6i(fx+e)} B}{3a^3 f}+\frac{2ic^5 e^{-}}{3}$
norman	$\frac{\frac{(8ic^5 B+A\,c^5)\tan(fx+e)^7}{af}+\frac{(32ic^5 B+9A\,c^5)\tan(fx+e)}{af}+\frac{(80ic^5 B+27A\,c^5)\tan(fx+e)^5}{af}-\frac{8(4ic^5 B+A\,c^5)x}{a}-\frac{-80ic^5 A+233B}{6af}}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
1/f*c^5/a^3*A*tan(f*x+e)+8*I/f*c^5/a^3*tan(f*x+e)*B+1/2*B*c^5*tan(f*x+e)^2
/a^3/f+16*I/f*c^5/a^3/(-I+tan(f*x+e))^2*A-24/f*c^5/a^3/(-I+tan(f*x+e))^2*B
-16/3/f*c^5/a^3/(-I+tan(f*x+e))^3*A-16/3*I/f*c^5/a^3/(-I+tan(f*x+e))^3*B-8
/f*c^5/a^3*A*arctan(tan(f*x+e))+4*I/f*c^5/a^3*A*ln(1+tan(f*x+e)^2)-32*I/f*
c^5/a^3*B*arctan(tan(f*x+e))-16/f*c^5/a^3*B*ln(1+tan(f*x+e)^2)+56*I/f*c^5/
a^3/(-I+tan(f*x+e))*B+24/f*c^5/a^3/(-I+tan(f*x+e))*A
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^3}\,dx = -\frac{2\left(24\,(A+4i\,B)c^5fxe^{(10i\,fx+10i\,e)}+4\,(-i\,A+4\,B)c^5e^{(4i\,fx+4i\,e)}+(i\,A-4\,B)c^5e^{(2i\,fx+2i\,e)}+(-i\,A+B\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-2/3*(24*(A + 4*I*B)*c^5*f*x*e^(10*I*f*x + 10*I*e) + 4*(-I*A + 4*B)*c^5*e^
(4*I*f*x + 4*I*e) + (I*A - 4*B)*c^5*e^(2*I*f*x + 2*I*e) + (-I*A + B)*c^5 +
 12*(4*(A + 4*I*B)*c^5*f*x + (-I*A + 4*B)*c^5)*e^(8*I*f*x + 8*I*e) + 6*(4*
(A + 4*I*B)*c^5*f*x + 3*(-I*A + 4*B)*c^5)*e^(6*I*f*x + 6*I*e) + 12*((I*A -
 4*B)*c^5*e^(10*I*f*x + 10*I*e) + 2*(I*A - 4*B)*c^5*e^(8*I*f*x + 8*I*e) +
(I*A - 4*B)*c^5*e^(6*I*f*x + 6*I*e))*log(e^(2*I*f*x + 2*I*e) + 1))/(a^3*f*
e^(10*I*f*x + 10*I*e) + 2*a^3*f*e^(8*I*f*x + 8*I*e) + a^3*f*e^(6*I*f*x + 6
*I*e))
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.48

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^3}\,dx = \frac{2iAc^5-16Bc^5+(2iAc^5e^{2ie}-14Bc^5e^{2ie})\,e^{2ifx}}{a^3fe^{4ie}e^{4ifx}+2a^3fe^{2ie}e^{2ifx}+a^3f} + \begin{cases} \frac{\left((2iAa^6c^5f^2e^{6ie}-2Ba^6c^5f^2e^{6ie})e^{-6ifx}+(-6iAa^6c^5f^2e^{8ie}+12Ba^6c^5f^2e^{8ie})e^{-4ifx}+(18iAa^6c^5f^2e^{10ie}-54Ba^6c^5f^2e^{10ie})e^{-2ifx}\right)e^{-1}}{3a^9f^3} \\ x\left(-\frac{-16Ac^5-64iBc^5}{a^3}+\frac{(-16Ac^5e^{6ie}+12Ac^5e^{4ie}-8Ac^5e^{2ie}+4Ac^5-64iBc^5e^{6ie}+36iBc^5e^{4ie}-16iBc^5e^{2ie}+4iBc^5)e^{-6ie}}{a^3}\right) \end{cases} - \frac{8ic^5(A+4iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{a^3f}+\frac{x(-16Ac^5-64iBc^5)}{a^3}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**5/(a+I*a*tan(f*x+e))**3,x)
```

output
```
(2*I*A*c**5 - 16*B*c**5 + (2*I*A*c**5*exp(2*I*e) - 14*B*c**5*exp(2*I*e))*e
xp(2*I*f*x))/(a**3*f*exp(4*I*e)*exp(4*I*f*x) + 2*a**3*f*exp(2*I*e)*exp(2*I
*f*x) + a**3*f) + Piecewise((((2*I*A*a**6*c**5*f**2*exp(6*I*e) - 2*B*a**6*
c**5*f**2*exp(6*I*e))*exp(-6*I*f*x) + (-6*I*A*a**6*c**5*f**2*exp(8*I*e) +
12*B*a**6*c**5*f**2*exp(8*I*e))*exp(-4*I*f*x) + (18*I*A*a**6*c**5*f**2*exp
(10*I*e) - 54*B*a**6*c**5*f**2*exp(10*I*e))*exp(-2*I*f*x))*exp(-12*I*e)/(3
*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0)), (x*(-(-16*A*c**5 - 64*I*B*c**5
)/a**3 + (-16*A*c**5*exp(6*I*e) + 12*A*c**5*exp(4*I*e) - 8*A*c**5*exp(2*I*
e) + 4*A*c**5 - 64*I*B*c**5*exp(6*I*e) + 36*I*B*c**5*exp(4*I*e) - 16*I*B*c
**5*exp(2*I*e) + 4*I*B*c**5)*exp(-6*I*e)/a**3), True)) - 8*I*c**5*(A + 4*I
*B)*log(exp(2*I*f*x) + exp(-2*I*e))/(a**3*f) + x*(-16*A*c**5 - 64*I*B*c**5
)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.86

$$\begin{aligned}
&\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^3}\,dx \\
&= -\frac{8\left(-i\,Ac^5+4\,Bc^5\right)\log\left(\tan\left(fx+e\right)-i\right)}{a^3f} \\
&\quad -\frac{8\left(5\,Ac^5+14i\,Bc^5-3\left(3\,Ac^5+7i\,Bc^5\right)\tan\left(fx+e\right)^2+3\left(4i\,Ac^5-11\,Bc^5\right)\tan\left(fx+e\right)\right)}{3\,a^3f(\tan\left(fx+e\right)-i)^3} \\
&\quad +\frac{Ba^3c^5f\tan\left(fx+e\right)^2+2\,Aa^3c^5f\tan\left(fx+e\right)+16i\,Ba^3c^5f\tan\left(fx+e\right)}{2\,a^6f^2}
\end{aligned}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-8*(-I*A*c^5 + 4*B*c^5)*log(tan(f*x + e) - I)/(a^3*f) - 8/3*(5*A*c^5 + 14*
I*B*c^5 - 3*(3*A*c^5 + 7*I*B*c^5)*tan(f*x + e)^2 + 3*(4*I*A*c^5 - 11*B*c^5
)*tan(f*x + e))/(a^3*f*(tan(f*x + e) - I)^3) + 1/2*(B*a^3*c^5*f*tan(f*x +
e)^2 + 2*A*a^3*c^5*f*tan(f*x + e) + 16*I*B*a^3*c^5*f*tan(f*x + e))/(a^6*f^
2)
```

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.22

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \frac{\ln\left(\tan(e+f\,x)-\mathrm{i}\right)\,\left(-\frac{32\,B\,c^5}{a^3}+\frac{A\,c^5\,8\mathrm{i}}{a^3}\right)}{f}+\frac{\tan(e+f\,x)\,\left(\frac{c^5\,(A+B\,4\mathrm{i})}{a^3}+\frac{B\,c^5\,4\mathrm{i}}{a^3}\right)}{f}$$
$$+\frac{\frac{5\,\left(-32\,B\,c^5+A\,c^5\,8\mathrm{i}\right)}{3\,a^3}+\tan(e+f\,x)\,\left(\frac{\left(-32\,B\,c^5+A\,c^5\,8\mathrm{i}\right)\,4\mathrm{i}}{a^3}+\frac{B\,c^5\,40\mathrm{i}}{a^3}\right)-\tan(e+f\,x)^2\,\left(\frac{3\,\left(-32\,B\,c^5+A\,c^5\,8\mathrm{i}\right)}{a^3}+\frac{40\,B\,c^5}{a^3}\right.}{f\,\left(-\tan\left(e+f\,x\right)^3\,1\mathrm{i}-3\tan\left(e+f\,x\right)^2+\tan\left(e+f\,x\right)\,3\mathrm{i}+1\right)}$$
$$+\frac{B\,c^5\tan(e+f\,x)^2}{2\,a^3\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^5)/(a + a*tan(e + f*x)*1
i)^3,x)
```

output
```
(log(tan(e + f*x) - 1i)*((A*c^5*8i)/a^3 - (32*B*c^5)/a^3))/f + (tan(e + f*
x)*((c^5*(A + B*4i))/a^3 + (B*c^5*4i)/a^3))/f + ((5*(A*c^5*8i - 32*B*c^5))
/(3*a^3) + tan(e + f*x)*(((A*c^5*8i - 32*B*c^5)*4i)/a^3 + (B*c^5*40i)/a^3)
 - tan(e + f*x)^2*((3*(A*c^5*8i - 32*B*c^5))/a^3 + (40*B*c^5)/a^3) + (16*B
*c^5)/a^3)/(f*(tan(e + f*x)*3i - 3*tan(e + f*x)^2 - tan(e + f*x)^3*1i + 1)
) + (B*c^5*tan(e + f*x)^2)/(2*a^3*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^5}{(a+ia\tan(e+fx))^3}\,dx$$
$$=\frac{c^5\left(160\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^3 i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right)af+320\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^3 i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right)bfi\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^5/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(c**5*(160*int(tan(e + f*x)**2/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*
tan(e + f*x)*i - 1),x)*a*f + 320*int(tan(e + f*x)**2/(tan(e + f*x)**3*i +
3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*f*i - 160*int(tan(e + f*x)/
(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*i +
416*int(tan(e + f*x)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*
x)*i - 1),x)*b*f - 64*int(1/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan
(e + f*x)*i - 1),x)*a*f - 160*int(1/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2
 - 3*tan(e + f*x)*i - 1),x)*b*f*i + 8*log(tan(e + f*x)**2 + 1)*a*i - 32*lo
g(tan(e + f*x)**2 + 1)*b + tan(e + f*x)**2*b + 2*tan(e + f*x)*a + 16*tan(e
 + f*x)*b*i - 64*a*f*x - 176*b*f*i*x))/(2*a**3*f)
```

3.729 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 164

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx = -\frac{(A+7iB)c^4x}{a^3}-\frac{(iA-7B)c^4\log(\cos(e+fx))}{a^3f}+\frac{8(A+iB)c^4}{3a^3f(i-\tan(e+fx))^3}+\frac{2(3iA-5B)c^4}{a^3f(i-\tan(e+fx))^2}-\frac{6(A+3iB)c^4}{a^3f(i-\tan(e+fx))}+\frac{iBc^4\tan(e+fx)}{a^3f}$$

output
```
-(A+7*I*B)*c^4*x/a^3-(I*A-7*B)*c^4*ln(cos(f*x+e))/a^3/f+8/3*(A+I*B)*c^4/a^
3/f/(I-tan(f*x+e))^3+2*(3*I*A-5*B)*c^4/a^3/f/(I-tan(f*x+e))^2-6*(A+3*I*B)*
c^4/a^3/f/(I-tan(f*x+e))+I*B*c^4*tan(f*x+e)/a^3/f
```

Mathematica [A] (verified)

Time = 5.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.65

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx = \frac{c^4\left(\frac{B(i+\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}-\frac{i(A+7iB)\left(-\log(i-\tan(e+fx))+\frac{2\left(-4i+9\tan(e+fx)+9i\tan^2(e+fx)\right)}{3(-i+\tan(e+fx))^3}\right)}{a^3}\right)}{f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4)/(a + I*a*Tan[e +
 f*x])^3,x]
```

output
```
(c^4*((B*(I + Tan[e + f*x])^4)/(a + I*a*Tan[e + f*x])^3 - (I*(A + (7*I)*B)
*(-Log[I - Tan[e + f*x]] + (2*(-4*I + 9*Tan[e + f*x] + (9*I)*Tan[e + f*x]^
2))/(3*(-I + Tan[e + f*x])^3)))/a^3))/f
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^4(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^4(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{c^3(1-i\tan(e+fx))^3(A+B\tan(e+fx))}{a^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^4\int \frac{(1-i\tan(e+fx))^3(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \text{86}$$

$$\frac{c^4\int \left(\frac{8(A+iB)}{(\tan(e+fx)-i)^4}+iB+\frac{i(A+7iB)}{\tan(e+fx)-i}-\frac{6(A+3iB)}{(\tan(e+fx)-i)^2}+\frac{4(5B-3iA)}{(\tan(e+fx)-i)^3}\right)d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^4\left(\frac{2(-5B+3iA)}{(-\tan(e+fx)+i)^2}-\frac{6(A+3iB)}{-\tan(e+fx)+i}+\frac{8(A+iB)}{3(-\tan(e+fx)+i)^3}+(-7B+iA)\log(-\tan(e+fx)+i)+iB\tan(e+fx)\right)}{a^3f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^4)/(a + I*a*Tan[e + f*x])
^3,x]
```

output
```
(c^4*((I*A - 7*B)*Log[I - Tan[e + f*x]] + (8*(A + I*B))/(3*(I - Tan[e + f*
x])^3) + (2*((3*I)*A - 5*B))/(I - Tan[e + f*x])^2 - (6*(A + (3*I)*B))/(I -
 Tan[e + f*x]) + I*B*Tan[e + f*x]))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{iB\,c^4\tan(fx+e)}{a^3f}+\frac{ic^4A\ln\left(1+\tan(fx+e)^2\right)}{2f\,a^3}-\frac{7c^4B\ln\left(1+\tan(fx+e)^2\right)}{2f\,a^3}-\frac{c^4A\arctan(\tan(fx+e))}{f\,a^3}-\frac{7ic^4B\,a}{}$
default	$\frac{iB\,c^4\tan(fx+e)}{a^3f}+\frac{ic^4A\ln\left(1+\tan(fx+e)^2\right)}{2f\,a^3}-\frac{7c^4B\ln\left(1+\tan(fx+e)^2\right)}{2f\,a^3}-\frac{c^4A\arctan(\tan(fx+e))}{f\,a^3}-\frac{7ic^4B\,a}{}$
risch	$-\frac{5c^4\mathrm{e}^{-2i(fx+e)}B}{a^3f}+\frac{ic^4\mathrm{e}^{-2i(fx+e)}A}{a^3f}+\frac{3c^4\mathrm{e}^{-4i(fx+e)}B}{2a^3f}-\frac{ic^4\mathrm{e}^{-4i(fx+e)}A}{2a^3f}-\frac{c^4\mathrm{e}^{-6i(fx+e)}B}{3a^3f}+\frac{ic^4\mathrm{e}^{-6i(fx+}}{3a^3f}$
norman	$\frac{\frac{\left(7ic^4B+2A\,c^4\right)\tan(fx+e)}{af}+\frac{ic^4B\tan(fx+e)^7}{af}-\frac{\left(7ic^4B+A\,c^4\right)x}{a}-\frac{-8ic^4A+32B\,c^4}{3af}-\frac{3\left(7ic^4B+A\,c^4\right)x\tan(fx+e)^2}{a}-\frac{3\left(7ic^4B+\right.}{}}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
I*B*c^4*tan(f*x+e)/a^3/f+1/2*I/f*c^4/a^3*A*ln(1+tan(f*x+e)^2)-7/2/f*c^4/a^
3*B*ln(1+tan(f*x+e)^2)-1/f*c^4/a^3*A*arctan(tan(f*x+e))-7*I/f*c^4/a^3*B*ar
ctan(tan(f*x+e))+6*I/f*c^4/a^3/(-I+tan(f*x+e))^2*A-10/f*c^4/a^3/(-I+tan(f*
x+e))^2*B-8/3/f*c^4/a^3/(-I+tan(f*x+e))^3*A-8/3*I/f*c^4/a^3/(-I+tan(f*x+e)
)^3*B+18*I/f*c^4/a^3/(-I+tan(f*x+e))*B+6/f*c^4/a^3/(-I+tan(f*x+e))*A
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.20

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx = -\frac{12\,(A+7i\,B)c^4fxe^{(8i\,fx+8i\,e)}+3\,(-i\,A+7\,B)c^4e^{(4i\,fx+4i\,e)}-(-i\,A+7\,B)c^4e^{(2i\,fx+2i\,e)}+2\,(-i\,A+B}{6\,(}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-1/6*(12*(A + 7*I*B)*c^4*f*x*e^(8*I*f*x + 8*I*e) + 3*(-I*A + 7*B)*c^4*e^(4
*I*f*x + 4*I*e) - (-I*A + 7*B)*c^4*e^(2*I*f*x + 2*I*e) + 2*(-I*A + B)*c^4
+ 6*(2*(A + 7*I*B)*c^4*f*x + (-I*A + 7*B)*c^4)*e^(6*I*f*x + 6*I*e) + 6*((I
*A - 7*B)*c^4*e^(8*I*f*x + 8*I*e) + (I*A - 7*B)*c^4*e^(6*I*f*x + 6*I*e))*l
og(e^(2*I*f*x + 2*I*e) + 1))/(a^3*f*e^(8*I*f*x + 8*I*e) + a^3*f*e^(6*I*f*x
 + 6*I*e))
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.46

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx = -\frac{2Bc^4}{a^3fe^{2ie}e^{2ifx}+a^3f} + \begin{cases} \frac{((2iAa^6c^4f^2e^{6ie}-2Ba^6c^4f^2e^{6ie})e^{-6ifx}+(-3iAa^6c^4f^2e^{8ie}+9Ba^6c^4f^2e^{8ie})e^{-4ifx}+(6iAa^6c^4f^2e^{10ie}-30Ba^6c^4f^2e^{10ie})e^{-2ifx})e^{-12ie}}{6a^9f^3} \\ x\left(-\frac{-2Ac^4-14iBc^4}{a^3}+\frac{(-2Ac^4e^{6ie}+2Ac^4e^{4ie}-2Ac^4e^{2ie}+2Ac^4-14iBc^4e^{6ie}+10iBc^4e^{4ie}-6iBc^4e^{2ie}+2iBc^4)e^{-6ie}}{a^3}\right) \end{cases} - \frac{ic^4(A+7iB)\log\left(e^{2ifx}+e^{-2ie}\right)}{a^3f} + \frac{x(-2Ac^4-14iBc^4)}{a^3}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**4/(a+I*a*tan(f*x+e))**3,x)
```

output
```
-2*B*c**4/(a**3*f*exp(2*I*e)*exp(2*I*f*x) + a**3*f) + Piecewise((((2*I*A*a
**6*c**4*f**2*exp(6*I*e) - 2*B*a**6*c**4*f**2*exp(6*I*e))*exp(-6*I*f*x) +
(-3*I*A*a**6*c**4*f**2*exp(8*I*e) + 9*B*a**6*c**4*f**2*exp(8*I*e))*exp(-4*
I*f*x) + (6*I*A*a**6*c**4*f**2*exp(10*I*e) - 30*B*a**6*c**4*f**2*exp(10*I*
e))*exp(-2*I*f*x))*exp(-12*I*e)/(6*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0
)), (x*(-(-2*A*c**4 - 14*I*B*c**4)/a**3 + (-2*A*c**4*exp(6*I*e) + 2*A*c**4
*exp(4*I*e) - 2*A*c**4*exp(2*I*e) + 2*A*c**4 - 14*I*B*c**4*exp(6*I*e) + 10
*I*B*c**4*exp(4*I*e) - 6*I*B*c**4*exp(2*I*e) + 2*I*B*c**4)*exp(-6*I*e)/a**
3), True)) - I*c**4*(A + 7*I*B)*log(exp(2*I*f*x) + exp(-2*I*e))/(a**3*f) +
 x*(-2*A*c**4 - 14*I*B*c**4)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.74

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \frac{i\,Bc^4\tan(fx+e)}{a^3f} + \frac{(i\,Ac^4-7\,Bc^4)\log(\tan(fx+e)-i)}{a^3f} - \frac{2\left(4\,Ac^4+16i\,Bc^4-9\,(Ac^4+3i\,Bc^4)\tan(fx+e)^2+3\,(3i\,Ac^4-13\,Bc^4)\tan(fx+e)\right)}{3\,a^3f(\tan(fx+e)-i)^3}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
I*B*c^4*tan(f*x + e)/(a^3*f) + (I*A*c^4 - 7*B*c^4)*log(tan(f*x + e) - I)/(
a^3*f) - 2/3*(4*A*c^4 + 16*I*B*c^4 - 9*(A*c^4 + 3*I*B*c^4)*tan(f*x + e)^2
+ 3*(3*I*A*c^4 - 13*B*c^4)*tan(f*x + e))/(a^3*f*(tan(f*x + e) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 7.51 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.62

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx = -\frac{c^4\left(25\,B\tan(e+f\,x)-\frac{B\,32\mathrm{i}}{3}-A\tan(e+f\,x)\,6\mathrm{i}-\frac{8\,A}{3}-A\,\ln\left(-1-\tan(e+f\,x)\,1\mathrm{i}\right)-B\,\ln\left(-1-\mathrm{t}\right.\right.}{}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^4)/(a + a*tan(e + f*x)*1
i)^3,x)
```

output
```
-(c^4*(25*B*tan(e + f*x) - (B*32i)/3 - A*tan(e + f*x)*6i - (8*A)/3 - A*log
(- tan(e + f*x)*1i - 1) - B*log(- tan(e + f*x)*1i - 1)*7i + 6*A*tan(e + f*
x)^2 + B*tan(e + f*x)^2*15i + 3*B*tan(e + f*x)^3 + B*tan(e + f*x)^4*1i + 3
*A*tan(e + f*x)^2*log(- tan(e + f*x)*1i - 1) + A*tan(e + f*x)^3*log(- tan(
e + f*x)*1i - 1)*1i + B*tan(e + f*x)^2*log(- tan(e + f*x)*1i - 1)*21i - 7*
B*tan(e + f*x)^3*log(- tan(e + f*x)*1i - 1) - A*tan(e + f*x)*log(- tan(e +
 f*x)*1i - 1)*3i + 21*B*tan(e + f*x)*log(- tan(e + f*x)*1i - 1))*1i)/(a^3*
f*(tan(e + f*x)*1i + 1)^3)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^4}{(a+ia\tan(e+fx))^3}\,dx = \frac{c^4\left(-48\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^3-3\tan(fx+e)^2 i-3\tan(fx+e)+i}dx\right)afi+112\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^3-3\tan(fx+e)^2 i-3\tan(fx+e)+i}dx\right)bf\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^4/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(c**4*( - 48*int(tan(e + f*x)**2/(tan(e + f*x)**3 - 3*tan(e + f*x)**2*i -
3*tan(e + f*x) + i),x)*a*f*i + 112*int(tan(e + f*x)**2/(tan(e + f*x)**3 -
3*tan(e + f*x)**2*i - 3*tan(e + f*x) + i),x)*b*f - 32*int(tan(e + f*x)/(ta
n(e + f*x)**3 - 3*tan(e + f*x)**2*i - 3*tan(e + f*x) + i),x)*a*f - 128*int
(tan(e + f*x)/(tan(e + f*x)**3 - 3*tan(e + f*x)**2*i - 3*tan(e + f*x) + i)
,x)*b*f*i + 16*int(1/(tan(e + f*x)**3 - 3*tan(e + f*x)**2*i - 3*tan(e + f*
x) + i),x)*a*f*i - 48*int(1/(tan(e + f*x)**3 - 3*tan(e + f*x)**2*i - 3*tan
(e + f*x) + i),x)*b*f + log(tan(e + f*x)**2 + 1)*a*i - 7*log(tan(e + f*x)*
*2 + 1)*b + 2*tan(e + f*x)*b*i - 14*a*f*x - 50*b*f*i*x))/(2*a**3*f)
```

3.730 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 135

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx = -\frac{iBc^3x}{a^3}+\frac{Bc^3\log(\cos(e+fx))}{a^3f}-\frac{2Bc^3}{a^3f(i-\tan(e+fx))^2} - \frac{4iBc^3}{a^3f(i-\tan(e+fx))}+\frac{(iA-B)c^3(1-i\tan(e+fx))^3}{6a^3f(1+i\tan(e+fx))^3}$$

output
```
-I*B*c^3*x/a^3+B*c^3*ln(cos(f*x+e))/a^3/f-2*B*c^3/a^3/f/(I-tan(f*x+e))^2-4
*I*B*c^3/a^3/f/(I-tan(f*x+e))+1/6*(I*A-B)*c^3*(1-I*tan(f*x+e))^3/a^3/f/(1+
I*tan(f*x+e))^3
```

Mathematica [A] (verified)

Time = 5.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.56

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx = -\frac{c^3\left(3B\log(i-\tan(e+fx))+\frac{A+7iB-18B\tan(e+fx)-3(A+5iB)\tan^2(e+fx)}{(-i+\tan(e+fx))^3}\right)}{3a^3f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3)/(a + I*a*Tan[e +
 f*x])^3,x]
```

output
```
-1/3*(c^3*(3*B*Log[I - Tan[e + f*x]] + (A + (7*I)*B - 18*B*Tan[e + f*x] -
3*(A + (5*I)*B)*Tan[e + f*x]^2)/(-I + Tan[e + f*x])^3))/(a^3*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {3042, 4071, 27, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^3(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^3(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{c^2(1-i\tan(e+fx))^2(A+B\tan(e+fx))}{a^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\int \frac{(1-i\tan(e+fx))^2(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \text{87}$$

$$\frac{c^3\left(\frac{(-B+iA)(1-i\tan(e+fx))^3}{6(1+i\tan(e+fx))^3}-iB\int \frac{(1-i\tan(e+fx))^2}{(i\tan(e+fx)+1)^3}d\tan(e+fx)\right)}{a^3f}$$

$$\Big\downarrow \text{49}$$

$$\frac{c^3\left(\frac{(-B+iA)(1-i\tan(e+fx))^3}{6(1+i\tan(e+fx))^3}-iB\int\left(-\frac{i}{\tan(e+fx)-i}+\frac{4}{(\tan(e+fx)-i)^2}+\frac{4i}{(\tan(e+fx)-i)^3}\right)d\tan(e+fx)\right)}{a^3f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^3\left(\frac{(-B+iA)(1-i\tan(e+fx))^3}{6(1+i\tan(e+fx))^3}-iB\left(\frac{4}{-\tan(e+fx)+i}-\frac{2i}{(-\tan(e+fx)+i)^2}-i\log(-\tan(e+fx)+i)\right)\right)}{a^3f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^3)/(a + I*a*Tan[e + f*x])
^3,x]
```

output
```
(c^3*((-I)*B*((-I)*Log[I - Tan[e + f*x]] - (2*I)/(I - Tan[e + f*x])^2 + 4/
(I - Tan[e + f*x])) + ((I*A - B)*(1 - I*Tan[e + f*x])^3)/(6*(1 + I*Tan[e +
 f*x])^3)))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{B\,c^3 \mathrm{e}^{-2i(fx+e)}}{a^3 f}+\frac{B\,c^3 \mathrm{e}^{-4i(fx+e)}}{2a^3 f}-\frac{c^3 \mathrm{e}^{-6i(fx+e)} B}{6a^3 f}+\frac{i c^3 \mathrm{e}^{-6i(fx+e)} A}{6a^3 f}-\frac{2iB\,c^3 x}{a^3}-\frac{2iB\,c^3 e}{a^3 f}+\frac{B\,c^3 \ln(\mathrm{e}}{}$
derivativedivides	$-\frac{4c^3 A}{3f\,a^3(-i+\tan(fx+e))^3}-\frac{4ic^3 B}{3f\,a^3(-i+\tan(fx+e))^3}+\frac{2ic^3 A}{f\,a^3(-i+\tan(fx+e))^2}-\frac{4c^3 B}{f\,a^3(-i+\tan(fx+e))^2}-\frac{c^3 B}{}$
default	$-\frac{4c^3 A}{3f\,a^3(-i+\tan(fx+e))^3}-\frac{4ic^3 B}{3f\,a^3(-i+\tan(fx+e))^3}+\frac{2ic^3 A}{f\,a^3(-i+\tan(fx+e))^2}-\frac{4c^3 B}{f\,a^3(-i+\tan(fx+e))^2}-\frac{c^3 B}{}$
norman	$\frac{\frac{\left(ic^3 B+A\,c^3\right)\tan(fx+e)}{af}+\frac{\left(5ic^3 B+A\,c^3\right)\tan(fx+e)^5}{af}-\frac{-ic^3 A+7B\,c^3}{3af}-\frac{2\left(-ic^3 B+5A\,c^3\right)\tan(fx+e)^3}{3af}-\frac{3\left(-ic^3 A+3B\,c^3\right)\tan(f}{af}}{a^2\left(1+\tan(fx+e)^2\right.}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
-1/a^3/f*B*c^3*exp(-2*I*(f*x+e))+1/2/a^3/f*B*c^3*exp(-4*I*(f*x+e))-1/6*c^3
/a^3/f*exp(-6*I*(f*x+e))*B+1/6*I*c^3/a^3/f*exp(-6*I*(f*x+e))*A-2*I/a^3*B*c
^3*x-2*I/a^3/f*B*c^3*e+1/a^3/f*B*c^3*ln(exp(2*I*(f*x+e))+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.76

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \frac{\left(-12i\,Bc^3fxe^{(6i\,fx+6i\,e)}+6\,Bc^3e^{(6i\,fx+6i\,e)}\log\left(e^{(2i\,fx+2i\,e)}+1\right)-6\,Bc^3e^{(4i\,fx+4i\,e)}+3\,Bc^3e^{(2i\,fx+2i\,e)}+(i\right.}{6\,a^3f}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
1/6*(-12*I*B*c^3*f*x*e^(6*I*f*x + 6*I*e) + 6*B*c^3*e^(6*I*f*x + 6*I*e)*log
(e^(2*I*f*x + 2*I*e) + 1) - 6*B*c^3*e^(4*I*f*x + 4*I*e) + 3*B*c^3*e^(2*I*f
*x + 2*I*e) + (I*A - B)*c^3)*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.91

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx = -\frac{2iBc^3x}{a^3}+\frac{Bc^3\log\left(e^{2ifx}+e^{-2ie}\right)}{a^3f}$$
$$+\begin{cases}\frac{\left(-12Ba^6c^3f^2e^{10ie}e^{-2ifx}+6Ba^6c^3f^2e^{8ie}e^{-4ifx}+\left(2iAa^6c^3f^2e^{6ie}-2Ba^6c^3f^2e^{6ie}\right)e^{-6ifx}\right)e^{-12ie}}{12a^9f^3} & \text{for } a^9f^3e^{12ie}\neq 0\\ x\left(\frac{2iBc^3}{a^3}+\frac{\left(Ac^3-2iBc^3e^{6ie}+2iBc^3e^{4ie}-2iBc^3e^{2ie}+iBc^3\right)e^{-6ie}}{a^3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**3/(a+I*a*tan(f*x+e))**3,x)
```

output
```
-2*I*B*c**3*x/a**3 + B*c**3*log(exp(2*I*f*x) + exp(-2*I*e))/(a**3*f) + Pie
cewise(((-12*B*a**6*c**3*f**2*exp(10*I*e)*exp(-2*I*f*x) + 6*B*a**6*c**3*f*
*2*exp(8*I*e)*exp(-4*I*f*x) + (2*I*A*a**6*c**3*f**2*exp(6*I*e) - 2*B*a**6*
c**3*f**2*exp(6*I*e))*exp(-6*I*f*x))*exp(-12*I*e)/(12*a**9*f**3), Ne(a**9*
f**3*exp(12*I*e), 0)), (x*(2*I*B*c**3/a**3 + (A*c**3 - 2*I*B*c**3*exp(6*I*
e) + 2*I*B*c**3*exp(4*I*e) - 2*I*B*c**3*exp(2*I*e) + I*B*c**3)*exp(-6*I*e)
/a**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx = -\frac{Bc^3\log\left(\tan\left(fx+e\right)-i\right)}{a^3f} + \frac{18\,Bc^3\tan\left(fx+e\right)-Ac^3-7i\,Bc^3+3\left(Ac^3+5i\,Bc^3\right)\tan\left(fx+e\right)^2}{3\,a^3f\left(\tan\left(fx+e\right)-i\right)^3}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-B*c^3*log(tan(f*x + e) - I)/(a^3*f) + 1/3*(18*B*c^3*tan(f*x + e) - A*c^3
- 7*I*B*c^3 + 3*(A*c^3 + 5*I*B*c^3)*tan(f*x + e)^2)/(a^3*f*(tan(f*x + e) -
 I)^3)
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx = -\frac{c^3\left(18\,B\tan(e+f\,x)-B\,7\mathrm{i}-A-B\,\ln\left(-1-\tan(e+f\,x)\,1\mathrm{i}\right)\,3\mathrm{i}+3\,A\tan(e+f\,x)^2+B\tan(e+f$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^3)/(a + a*tan(e + f*x)*1
i)^3,x)
```

output
```
-(c^3*(18*B*tan(e + f*x) - B*7i - A - B*log(- tan(e + f*x)*1i - 1)*3i + 3*
A*tan(e + f*x)^2 + B*tan(e + f*x)^2*15i + B*tan(e + f*x)^2*log(- tan(e + f
*x)*1i - 1)*9i - 3*B*tan(e + f*x)^3*log(- tan(e + f*x)*1i - 1) + 9*B*tan(e
 + f*x)*log(- tan(e + f*x)*1i - 1))*1i)/(3*a^3*f*(tan(e + f*x)*1i + 1)^3)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^3}{(a+ia\tan(e+fx))^3}\,dx = \frac{c^3\left(-\left(\int \frac{\tan(fx+e)^4}{\tan(fx+e)^3 i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right)bi-\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^3 i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right)ai+3\left(\int\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^3/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(c**3*( - int(tan(e + f*x)**4/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*t
an(e + f*x)*i - 1),x)*b*i - int(tan(e + f*x)**3/(tan(e + f*x)**3*i + 3*tan
(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*i + 3*int(tan(e + f*x)**3/(tan(e
 + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b + 3*int(tan(
e + f*x)**2/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1)
,x)*a + 3*int(tan(e + f*x)**2/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*t
an(e + f*x)*i - 1),x)*b*i + 3*int(tan(e + f*x)/(tan(e + f*x)**3*i + 3*tan(
e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*i - int(tan(e + f*x)/(tan(e + f*x
)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b - int(1/(tan(e + f
*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a))/a**3
```

3.731 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 99

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx = \frac{2(A+iB)c^2}{3a^3f(i-\tan(e+fx))^3}+\frac{(iA-3B)c^2}{2a^3f(i-\tan(e+fx))^2}-\frac{iBc^2}{a^3f(i-\tan(e+fx))}$$

output
```
2/3*(A+I*B)*c^2/a^3/f/(I-tan(f*x+e))^3+1/2*(I*A-3*B)*c^2/a^3/f/(I-tan(f*x+
e))^2-I*B*c^2/a^3/f/(I-tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx = \frac{c^2(-A-iB+3(iA+B)\tan(e+fx)+6iB\tan^2(e+fx))}{6a^3f(-i+\tan(e+fx))^3}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2)/(a + I*a*Tan[e +
 f*x])^3,x]
```

output
```
(c^2*(-A - I*B + 3*(I*A + B)*Tan[e + f*x] + (6*I)*B*Tan[e + f*x]^2))/(6*a^
3*f*(-I + Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^2(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^2(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{c(1-i\tan(e+fx))(A+B\tan(e+fx))}{a^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{(1-i\tan(e+fx))(A+B\tan(e+fx))}{(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \text{86}$$

$$\frac{c^2\int \left(\frac{2(A+iB)}{(\tan(e+fx)-i)^4}-\frac{iB}{(\tan(e+fx)-i)^2}+\frac{3B-iA}{(\tan(e+fx)-i)^3}\right)d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\left(\frac{-3B+iA}{2(-\tan(e+fx)+i)^2}+\frac{2(A+iB)}{3(-\tan(e+fx)+i)^3}-\frac{iB}{-\tan(e+fx)+i}\right)}{a^3f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^2)/(a + I*a*Tan[e + f*x])
^3,x]
```

output
```
(c^2*((2*(A + I*B))/(3*(I - Tan[e + f*x])^3) + (I*A - 3*B)/(2*(I - Tan[e +
 f*x])^2) - (I*B)/(I - Tan[e + f*x])))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{c^{2}\left(-\frac{-iA+3B}{2(-i+\tan(fx+e))^{2}}-\frac{2iB+2A}{3(-i+\tan(fx+e))^{3}}+\frac{iB}{-i+\tan(fx+e)}\right)}{f\,a^{3}}$
default	$\frac{c^{2}\left(-\frac{-iA+3B}{2(-i+\tan(fx+e))^{2}}-\frac{2iB+2A}{3(-i+\tan(fx+e))^{3}}+\frac{iB}{-i+\tan(fx+e)}\right)}{f\,a^{3}}$
risch	$\frac{c^{2}\mathrm{e}^{-4i(fx+e)}B}{8a^{3}f}+\frac{ic^{2}\mathrm{e}^{-4i(fx+e)}A}{8a^{3}f}-\frac{c^{2}\mathrm{e}^{-6i(fx+e)}B}{12a^{3}f}+\frac{ic^{2}\mathrm{e}^{-6i(fx+e)}A}{12a^{3}f}$
norman	$\frac{-\frac{2iA\,c^{2}\tan(fx+e)^{2}}{af}+\frac{A\,c^{2}\tan(fx+e)}{af}+\frac{ic^{2}B\tan(fx+e)^{5}}{af}-\frac{-iA\,c^{2}+B\,c^{2}}{6af}-\frac{5\left(ic^{2}B+A\,c^{2}\right)\tan(fx+e)^{3}}{3af}-\frac{\left(-iA\,c^{2}+5B\,c^{2}\right)\tan(f}{2af}}{a^{2}\left(1+\tan(fx+e)^{2}\right)^{3}}$

input

```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output

```
1/f*c^2/a^3*(-1/2*(-I*A+3*B)/(-I+tan(f*x+e))^2-1/3*(2*A+2*I*B)/(-I+tan(f*x
+e))^3+I*B/(-I+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.49

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx = -\frac{\left(3\,(-i\,A-B)c^2e^{(2i\,fx+2i\,e)}+2\,(-i\,A+B)c^2\right)e^{(-6i\,fx-6i\,e)}}{24\,a^3f}$$

input

```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output

```
-1/24*(3*(-I*A - B)*c^2*e^(2*I*f*x + 2*I*e) + 2*(-I*A + B)*c^2)*e^(-6*I*f*
x - 6*I*e)/(a^3*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(78) = 156$.

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx = \begin{cases} \frac{((8iAa^3c^2fe^{4ie}-8Ba^3c^2fe^{4ie})e^{-6ifx}+(12iAa^3c^2fe^{6ie}+12Ba^3c^2fe^{6ie})e^{-4ifx})e^{-10ie}}{96a^6f^2} & \text{for } a^6f^2e^{10ie}\neq 0 \\ \frac{x(Ac^2e^{2ie}+Ac^2-iBc^2e^{2ie}+iBc^2)e^{-6ie}}{2a^3} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**2/(a+I*a*tan(f*x+e))**3,x)
```

output
```
Piecewise((((8*I*A*a**3*c**2*f*exp(4*I*e) - 8*B*a**3*c**2*f*exp(4*I*e))*ex
p(-6*I*f*x) + (12*I*A*a**3*c**2*f*exp(6*I*e) + 12*B*a**3*c**2*f*exp(6*I*e)
)*exp(-4*I*f*x))*exp(-10*I*e)/(96*a**6*f**2), Ne(a**6*f**2*exp(10*I*e), 0)
), (x*(A*c**2*exp(2*I*e) + A*c**2 - I*B*c**2*exp(2*I*e) + I*B*c**2)*exp(-6
*I*e)/(2*a**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.69

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx$$
$$= -\frac{-6i\,Bc^2\tan(fx+e)^2-3i\,Ac^2\tan(fx+e)-3\,Bc^2\tan(fx+e)+Ac^2+i\,Bc^2}{6\,a^3f(\tan(fx+e)-i)^3}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-1/6*(-6*I*B*c^2*tan(f*x + e)^2 - 3*I*A*c^2*tan(f*x + e) - 3*B*c^2*tan(f*x
 + e) + A*c^2 + I*B*c^2)/(a^3*f*(tan(f*x + e) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \frac{\frac{c^2\,(-B+A\,1i)}{6}+\frac{c^2\tan(e+f\,x)\,(3\,A-B\,3i)}{6}+B\,c^2\tan(e+f\,x)^2}{a^3\,f\,\left(-\tan\left(e+f\,x\right)^3\,1i-3\tan\left(e+f\,x\right)^2+\tan\left(e+f\,x\right)\,3i+1\right)}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^2)/(a + a*tan(e + f*x)*1
i)^3,x)
```

output
```
((c^2*(A*1i - B))/6 + (c^2*tan(e + f*x)*(3*A - B*3i))/6 + B*c^2*tan(e + f*
x)^2)/(a^3*f*(tan(e + f*x)*3i - 3*tan(e + f*x)^2 - tan(e + f*x)^3*1i + 1))
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^2}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \frac{c^2\left(\left(\left(\int \frac{\tan(fx+e)^3}{\tan(fx+e)^3 i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right)b+\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^3 i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right)a+2\left(\int \frac{}{\tan(}\right.\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^2/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(c**2*(int(tan(e + f*x)**3/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(
e + f*x)*i - 1),x)*b + int(tan(e + f*x)**2/(tan(e + f*x)**3*i + 3*tan(e +
f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a + 2*int(tan(e + f*x)**2/(tan(e + f*x)
**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*i + 2*int(tan(e + f
*x)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*i
- int(tan(e + f*x)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)
*i - 1),x)*b - int(1/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*
x)*i - 1),x)*a))/a**3
```

3.732 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 59

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^3}\,dx = \frac{(A+iB)c}{3a^3f(i-\tan(e+fx))^3} - \frac{Bc}{2a^3f(i-\tan(e+fx))^2}$$

output
```
1/3*(A+I*B)*c/a^3/f/(I-tan(f*x+e))^3-1/2*B*c/a^3/f/(I-tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 5.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^3}\,dx = \frac{c(-2A+iB-3B\tan(e+fx))}{6a^3f(-i+\tan(e+fx))^3}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]))/(a + I*a*Tan[e + f
*x])^3,x]
```

output
```
(c*(-2*A + I*B - 3*B*Tan[e + f*x]))/(6*a^3*f*(-I + Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 4071, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \text{53}$$

$$\frac{c\int \left(\frac{A+iB}{(\tan(e+fx)-i)^4}+\frac{B}{(\tan(e+fx)-i)^3}\right)d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\frac{A+iB}{3(-\tan(e+fx)+i)^3}-\frac{B}{2(-\tan(e+fx)+i)^2}\right)}{a^3f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x]))/(a + I*a*Tan[e + f*x])^3
,x]
```

output
```
(c*((A + I*B)/(3*(I - Tan[e + f*x])^3) - B/(2*(I - Tan[e + f*x])^2)))/(a^3
*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{c\left(-\frac{iB+A}{3(-i+\tan(fx+e))^3}-\frac{B}{2(-i+\tan(fx+e))^2}\right)}{f\,a^3}$	43
default	$\frac{c\left(-\frac{iB+A}{3(-i+\tan(fx+e))^3}-\frac{B}{2(-i+\tan(fx+e))^2}\right)}{f\,a^3}$	43
risch	$\frac{c\,\mathrm{e}^{-2i(fx+e)}B}{8a^3f}+\frac{ic\,\mathrm{e}^{-2i(fx+e)}A}{8a^3f}+\frac{icA\,\mathrm{e}^{-4i(fx+e)}}{8a^3f}-\frac{c\,\mathrm{e}^{-6i(fx+e)}B}{24a^3f}+\frac{ic\,\mathrm{e}^{-6i(fx+e)}A}{24a^3f}$	100

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x,method=_RET
URNVERBOSE)
```

output
```
1/f*c/a^3*(-1/3*(A+I*B)/(-I+tan(f*x+e))^3-1/2*B/(-I+tan(f*x+e))^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^3}\,dx = -\frac{\left(3\left(-iA-B\right)ce^{(4i\,fx+4i\,e)}-3i\,Ace^{(2i\,fx+2i\,e)}-\left(i\,A-B\right)c\right)e^{(-6i\,fx-6i\,e)}}{24\,a^3 f}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algo
rithm="fricas")
```

output
```
-1/24*(3*(-I*A - B)*c*e^(4*I*f*x + 4*I*e) - 3*I*A*c*e^(2*I*f*x + 2*I*e) -
(I*A - B)*c)*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.49

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^3}\,dx = \begin{cases} \frac{\left(192iAa^6cf^2e^{8ie}e^{-4ifx}+\left(64iAa^6cf^2e^{6ie}-64Ba^6cf^2e^{6ie}\right)e^{-6ifx}+\left(192iAa^6cf^2e^{10ie}+192Ba^6cf^2e^{10ie}\right)e^{-2ifx}\right)e^{-12ie}}{1536a^9f^3} & \text{for } a^9f^3e^{12ie} \\ \frac{x\left(Ace^{4ie}+2Ace^{2ie}+Ac-iBce^{4ie}+iBc\right)e^{-6ie}}{4a^3} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))**3,x)
```

output
```
Piecewise(((192*I*A*a**6*c*f**2*exp(8*I*e)*exp(-4*I*f*x) + (64*I*A*a**6*c*
f**2*exp(6*I*e) - 64*B*a**6*c*f**2*exp(6*I*e))*exp(-6*I*f*x) + (192*I*A*a*
*6*c*f**2*exp(10*I*e) + 192*B*a**6*c*f**2*exp(10*I*e))*exp(-2*I*f*x))*exp(
-12*I*e)/(1536*a**9*f**3), Ne(a**9*f**3*exp(12*I*e), 0)), (x*(A*c*exp(4*I*
e) + 2*A*c*exp(2*I*e) + A*c - I*B*c*exp(4*I*e) + I*B*c)*exp(-6*I*e)/(4*a**
3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))}{(a+ia\tan(e+fx))^3}\,dx = -\frac{3\,Bc\tan\left(fx+e\right)+2\,Ac-i\,Bc}{6\,a^3f(\tan\left(fx+e\right)-i)^3}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algo
rithm="giac")
```

output
```
-1/6*(3*B*c*tan(f*x + e) + 2*A*c - I*B*c)/(a^3*f*(tan(f*x + e) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{(A + B\tan(e + fx))(c - ic\tan(e + fx))}{(a + ia\tan(e + fx))^3}\,dx = \frac{\frac{c\,(B+A\,2\mathrm{i})}{6} + \frac{B\,c\tan(e+f\,x)\,1\mathrm{i}}{2}}{a^3\,f\,\left(-\tan\left(e+f\,x\right)^3\,1\mathrm{i} - 3\tan\left(e+f\,x\right)^2 + \tan\left(e+f\,x\right)\,3\mathrm{i} + 1\right)}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i))/(a + a*tan(e + f*x)*1i)
^3,x)
```

output
```
((c*(A*2i + B))/6 + (B*c*tan(e + f*x)*1i)/2)/(a^3*f*(tan(e + f*x)*3i - 3*t
an(e + f*x)^2 - tan(e + f*x)^3*1i + 1))
```

Reduce [F]

$$\int \frac{(A + B\tan(e + fx))(c - ic\tan(e + fx))}{(a + ia\tan(e + fx))^3}\,dx = \frac{c\left(\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^3 i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right)bi + \left(\int \frac{\tan(fx+e)}{\tan(fx+e)^3 i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right)ai - \left(\int \frac{\tan(f}{}\right.\right.}{a^3}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(c*(int(tan(e + f*x)**2/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e +
 f*x)*i - 1),x)*b*i + int(tan(e + f*x)/(tan(e + f*x)**3*i + 3*tan(e + f*x)
**2 - 3*tan(e + f*x)*i - 1),x)*a*i - int(tan(e + f*x)/(tan(e + f*x)**3*i +
 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b - int(1/(tan(e + f*x)**3*i
 + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a))/a**3
```

3.733 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 112

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx = \frac{(A-iB)x}{8a^3}+\frac{iA-B}{6f(a+ia\tan(e+fx))^3} + \frac{iA+B}{8af(a+ia\tan(e+fx))^2}+\frac{iA+B}{8f\,(a^3+ia^3\tan(e+fx))}$$

output
```
1/8*(A-I*B)*x/a^3+1/6*(I*A-B)/f/(a+I*a*tan(f*x+e))^3+1/8*(I*A+B)/a/f/(a+I*
a*tan(f*x+e))^2+1/8*(I*A+B)/f/(a^3+I*a^3*tan(f*x+e))
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx = \frac{-10A+2iB+3(iA+B)\arctan(\tan(e+fx))\sec^3(e+fx)(\cos(3(e+fx))+i\sin(3(e+fx)))+(-9i\cdots}{24a^3f(-i+\tan(e+fx))^3}$$

input
```
Integrate[(A + B*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^3,x]
```

output
```
(-10*A + (2*I)*B + 3*(I*A + B)*ArcTan[Tan[e + f*x]]*Sec[e + f*x]^3*(Cos[3*
(e + f*x)] + I*Sin[3*(e + f*x)]) + ((-9*I)*A - 9*B)*Tan[e + f*x] + 3*(A -
I*B)*Tan[e + f*x]^2)/(24*a^3*f*(-I + Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 4009, 3042, 3960, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4009}$$

$$\frac{(A-iB)\int \frac{1}{(i\tan(e+fx)a+a)^2}dx}{2a}+\frac{-B+iA}{6f(a+ia\tan(e+fx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\int \frac{1}{(i\tan(e+fx)a+a)^2}dx}{2a}+\frac{-B+iA}{6f(a+ia\tan(e+fx))^3}$$

$$\Big\downarrow \text{3960}$$

$$\frac{(A-iB)\left(\frac{\int \frac{1}{i\tan(e+fx)a+a}dx}{2a}+\frac{i}{4f(a+ia\tan(e+fx))^2}\right)}{2a}+\frac{-B+iA}{6f(a+ia\tan(e+fx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-iB)\left(\frac{\int \frac{1}{i\tan(e+fx)a+a}dx}{2a}+\frac{i}{4f(a+ia\tan(e+fx))^2}\right)}{2a}+\frac{-B+iA}{6f(a+ia\tan(e+fx))^3}$$

$$\Big\downarrow \text{3960}$$

$$\frac{(A-iB)\left(\frac{\frac{\int 1dx}{2a}+\frac{i}{2f(a+ia\tan(e+fx))}}{2a}+\frac{i}{4f(a+ia\tan(e+fx))^2}\right)}{2a}+\frac{-B+iA}{6f(a+ia\tan(e+fx))^3}$$

$$\Big\downarrow \; 24$$

$$\frac{-B+iA}{6f(a+ia\tan(e+fx))^3}+\frac{(A-iB)\left(\frac{\frac{x}{2a}+\frac{i}{2f(a+ia\tan(e+fx))}}{2a}+\frac{i}{4f(a+ia\tan(e+fx))^2}\right)}{2a}$$

input
```
Int[(A + B*Tan[e + f*x])/(a + I*a*Tan[e + f*x])^3,x]
```

output
```
(I*A - B)/(6*f*(a + I*a*Tan[e + f*x])^3) + ((A - I*B)*((I/4)/(f*(a + I*a*T
an[e + f*x])^2) + (x/(2*a) + (I/2)/(f*(a + I*a*Tan[e + f*x])))/(2*a)))/(2*
a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{ixB}{8a^3}+\frac{xA}{8a^3}+\frac{e^{-2i(fx+e)}B}{16a^3f}+\frac{3ie^{-2i(fx+e)}A}{16a^3f}-\frac{e^{-4i(fx+e)}B}{32a^3f}+\frac{3ie^{-4i(fx+e)}A}{32a^3f}-\frac{e^{-6i(fx+e)}B}{48a^3f}+\frac{ie^{-6i(}}{48}$
derivativedivides	$-\frac{iA}{8f\,a^3(-i+\tan(fx+e))^2}-\frac{B}{8f\,a^3(-i+\tan(fx+e))^2}+\frac{A}{8f\,a^3(-i+\tan(fx+e))}-\frac{iB}{8f\,a^3(-i+\tan(fx+e))}+\frac{A\,a}{}$
default	$-\frac{iA}{8f\,a^3(-i+\tan(fx+e))^2}-\frac{B}{8f\,a^3(-i+\tan(fx+e))^2}+\frac{A}{8f\,a^3(-i+\tan(fx+e))}-\frac{iB}{8f\,a^3(-i+\tan(fx+e))}+\frac{A\,a}{}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*x/a^3*B+1/8*x/a^3*A+1/16/a^3/f*exp(-2*I*(f*x+e))*B+3/16*I/a^3/f*exp
(-2*I*(f*x+e))*A-1/32/a^3/f*exp(-4*I*(f*x+e))*B+3/32*I/a^3/f*exp(-4*I*(f*x
+e))*A-1/48/a^3/f*exp(-6*I*(f*x+e))*B+1/48*I/a^3/f*exp(-6*I*(f*x+e))*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx = \frac{\left(12\,(A-i\,B)fxe^{(6i\,fx+6i\,e)}-6\,(-3i\,A-B)e^{(4i\,fx+4i\,e)}-3\,(-3i\,A+B)e^{(2i\,fx+2i\,e)}+2i\,A-2\,B\right)e^{(-6i\,fx-}}{96\,a^3f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/96*(12*(A - I*B)*f*x*e^(6*I*f*x + 6*I*e) - 6*(-3*I*A - B)*e^(4*I*f*x + 4
*I*e) - 3*(-3*I*A + B)*e^(2*I*f*x + 2*I*e) + 2*I*A - 2*B)*e^(-6*I*f*x - 6*
I*e)/(a^3*f)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.30

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \begin{cases} \frac{((512iAa^6f^2e^{6ie}-512Ba^6f^2e^{6ie})e^{-6ifx}+(2304iAa^6f^2e^{8ie}-768Ba^6f^2e^{8ie})e^{-4ifx}+(4608iAa^6f^2e^{10ie}+1536Ba^6f^2e^{10ie})e^{-2ifx})e^{-12ie}}{24576a^9f^3} \\ x\left(-\frac{A-iB}{8a^3}+\frac{(Ae^{6ie}+3Ae^{4ie}+3Ae^{2ie}+A-iBe^{6ie}-iBe^{4ie}+iBe^{2ie}+iB)e^{-6ie}}{8a^3}\right) \end{cases}$$
$$+\frac{x(A-iB)}{8a^3}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3,x)
```

output
```
Piecewise((((512*I*A*a**6*f**2*exp(6*I*e) - 512*B*a**6*f**2*exp(6*I*e))*ex
p(-6*I*f*x) + (2304*I*A*a**6*f**2*exp(8*I*e) - 768*B*a**6*f**2*exp(8*I*e))
*exp(-4*I*f*x) + (4608*I*A*a**6*f**2*exp(10*I*e) + 1536*B*a**6*f**2*exp(10
*I*e))*exp(-2*I*f*x))*exp(-12*I*e)/(24576*a**9*f**3), Ne(a**9*f**3*exp(12*
I*e), 0)), (x*(-(A - I*B)/(8*a**3) + (A*exp(6*I*e) + 3*A*exp(4*I*e) + 3*A*
exp(2*I*e) + A - I*B*exp(6*I*e) - I*B*exp(4*I*e) + I*B*exp(2*I*e) + I*B)*e
xp(-6*I*e)/(8*a**3)), True)) + x*(A - I*B)/(8*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx$$
$$= -\frac{(-iA-B)\log(\tan(fx+e)+i)}{16a^3f} - \frac{(iA+B)\log(\tan(fx+e)-i)}{16a^3f}$$
$$+ \frac{3(A-iB)\tan(fx+e)^2 - 9(iA+B)\tan(fx+e) - 10A + 2iB}{24a^3f(\tan(fx+e)-i)^3}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x, algorithm="giac")
```

output
```
-1/16*(-I*A - B)*log(tan(f*x + e) + I)/(a^3*f) - 1/16*(I*A + B)*log(tan(f*
x + e) - I)/(a^3*f) + 1/24*(3*(A - I*B)*tan(f*x + e)^2 - 9*(I*A + B)*tan(f
*x + e) - 10*A + 2*I*B)/(a^3*f*(tan(f*x + e) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3}\,dx$$
$$= -\frac{\tan(e+fx)^2\left(\frac{B}{8a^3}+\frac{A\,1i}{8a^3}\right)-\frac{A\,5i}{12a^3}-\frac{B}{12a^3}+\tan(e+fx)\left(\frac{3A}{8a^3}-\frac{B\,3i}{8a^3}\right)}{f\left(-\tan(e+fx)^3\,1i-3\tan(e+fx)^2+\tan(e+fx)\,3i+1\right)}$$
$$-\frac{x(B+A\,1i)\,1i}{8a^3}$$

input
```
int((A + B*tan(e + f*x))/(a + a*tan(e + f*x)*1i)^3,x)
```

output
```
- (tan(e + f*x)^2*((A*1i)/(8*a^3) + B/(8*a^3)) - (A*5i)/(12*a^3) - B/(12*a
^3) + tan(e + f*x)*((3*A)/(8*a^3) - (B*3i)/(8*a^3)))/(f*(tan(e + f*x)*3i -
 3*tan(e + f*x)^2 - tan(e + f*x)^3*1i + 1)) - (x*(A*1i + B)*1i)/(8*a^3)
```

Reduce [F]

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))^3} \, dx$$
$$= \frac{-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^3 i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right) b - \left(\int \frac{1}{\tan(fx+e)^3 i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right) a}{a^3}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3,x)
```

output
```
( - (int(tan(e + f*x)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f
*x)*i - 1),x)*b + int(1/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e +
 f*x)*i - 1),x)*a))/a**3
```

3.734 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 153

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}\,dx = \frac{(2A-iB)x}{8a^3c}+\frac{A+iB}{12a^3cf(i-\tan(e+fx))^3}-\frac{iA}{8a^3cf(i-\tan(e+fx))^2}-\frac{3A-iB}{16a^3cf(i-\tan(e+fx))}+\frac{A-iB}{16a^3cf(i+\tan(e+fx))}$$

output
```
1/8*(2*A-I*B)*x/a^3/c+1/12*(A+I*B)/a^3/c/f/(I-tan(f*x+e))^3-1/8*I*A/a^3/c/
f/(I-tan(f*x+e))^2-1/16*(3*A-I*B)/a^3/c/f/(I-tan(f*x+e))+1/16*(A-I*B)/a^3/
c/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.55 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}\,dx = \frac{\sec^3(e+fx)(-9iA\cos(e+fx)+(-1+2\cos(2(e+fx)))((iA+2B)\cos(e+fx)+(-2A+iB)\sin(e-}{24a^3cf(-i+\tan(e+fx))^3(i}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e +
f*x])),x]
```

output
```
(Sec[e + f*x]^3*((-9*I)*A*Cos[e + f*x] + (-1 + 2*Cos[2*(e + f*x)])*((I*A +
 2*B)*Cos[e + f*x] + (-2*A + I*B)*Sin[e + f*x]) - 3*(2*A - I*B)*ArcTan[Tan
[e + f*x]]*Sec[e + f*x]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])))/(24*a^3*
c*f*(-I + Tan[e + f*x])^3*(I + Tan[e + f*x]))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4c^2(1-i\tan(e+fx))^2(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^2(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3cf}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{iA}{4(\tan(e+fx)-i)^3}+\frac{2A-iB}{8(\tan^2(e+fx)+1)}+\frac{iB-3A}{16(\tan(e+fx)-i)^2}+\frac{iB-A}{16(\tan(e+fx)+i)^2}+\frac{A+iB}{4(\tan(e+fx)-i)^4}\right)d\tan(e+fx)}{a^3cf}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}(2A-iB)\arctan(\tan(e+fx))-\frac{3A-iB}{16(-\tan(e+fx)+i)}+\frac{A-iB}{16(\tan(e+fx)+i)}+\frac{A+iB}{12(-\tan(e+fx)+i)^3}-\frac{iA}{8(-\tan(e+fx)+i)^2}}{a^3cf}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x]))
,x]
```

output
```
(((2*A - I*B)*ArcTan[Tan[e + f*x]])/8 + (A + I*B)/(12*(I - Tan[e + f*x])^3
) - ((I/8)*A)/(I - Tan[e + f*x])^2 - (3*A - I*B)/(16*(I - Tan[e + f*x])) +
 (A - I*B)/(16*(I + Tan[e + f*x])))/(a^3*c*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{ixB}{8a^3c}+\frac{xA}{4a^3c}-\frac{e^{-4i(fx+e)}B}{32a^3cf}+\frac{ie^{-4i(fx+e)}A}{16a^3cf}-\frac{e^{-6i(fx+e)}B}{96a^3cf}+\frac{ie^{-6i(fx+e)}A}{96a^3cf}-\frac{\cos(2fx+2e)B}{32a^3cf}+\frac{5i\cos}{}$
derivativedivides	$\frac{A}{16f\,a^3c(i+\tan(fx+e))}-\frac{iB}{16f\,a^3c(i+\tan(fx+e))}-\frac{iB\arctan(\tan(fx+e))}{8f\,a^3c}+\frac{A\arctan(\tan(fx+e))}{4f\,a^3c}-\frac{}{8f\,a^3c(-}$
default	$\frac{A}{16f\,a^3c(i+\tan(fx+e))}-\frac{iB}{16f\,a^3c(i+\tan(fx+e))}-\frac{iB\arctan(\tan(fx+e))}{8f\,a^3c}+\frac{A\arctan(\tan(fx+e))}{4f\,a^3c}-\frac{}{8f\,a^3c(-}$
norman	$\frac{\frac{(-iB+2A)x}{8ac}-\frac{-4iA+B}{12acf}+\frac{B\tan(fx+e)^2}{4acf}+\frac{(-iB+2A)\tan(fx+e)^3}{3acf}+\frac{(-iB+2A)\tan(fx+e)^5}{8acf}+\frac{3(-iB+2A)x\tan(fx+e)^2}{8ac}+\frac{3(-iB}{}}{\left(1+\tan(fx+e)^2\right)^3a^2}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-1/8*I*x/a^3/c*B+1/4*x/a^3/c*A-1/32/a^3/c/f*exp(-4*I*(f*x+e))*B+1/16*I/a^3
/c/f*exp(-4*I*(f*x+e))*A-1/96/a^3/c/f*exp(-6*I*(f*x+e))*B+1/96*I/a^3/c/f*e
xp(-6*I*(f*x+e))*A-1/32/a^3/c/f*cos(2*f*x+2*e)*B+5/32*I/a^3/c/f*cos(2*f*x+
2*e)*A-1/32*I/a^3/c/f*sin(2*f*x+2*e)*B+7/32/a^3/c/f*sin(2*f*x+2*e)*A
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.59

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}\,dx$$
$$=\frac{(12\,(2\,A-i\,B)fxe^{(6i\,fx+6i\,e)}-3\,(i\,A+B)e^{(8i\,fx+8i\,e)}+18i\,Ae^{(4i\,fx+4i\,e)}-3\,(-2i\,A+B)e^{(2i\,fx+2i\,e)}+i\,A}{96\,a^3cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x, algo
rithm="fricas")
```

output
```
1/96*(12*(2*A - I*B)*f*x*e^(6*I*f*x + 6*I*e) - 3*(I*A + B)*e^(8*I*f*x + 8*
I*e) + 18*I*A*e^(4*I*f*x + 4*I*e) - 3*(-2*I*A + B)*e^(2*I*f*x + 2*I*e) + I
*A - B)*e^(-6*I*f*x - 6*I*e)/(a^3*c*f)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.22

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))^3(c - ic\tan(e + fx))}\, dx$$
$$= \begin{cases} \frac{(294912iAa^9c^3f^3e^{10ie}e^{-2ifx} + (16384iAa^9c^3f^3e^{6ie} - 16384Ba^9c^3f^3e^{6ie})e^{-6ifx} + (98304iAa^9c^3f^3e^{8ie} - 49152Ba^9c^3f^3e^{8ie})e^{-4ifx} + (-491}{1572864a^{12}c^4f^4} \\ x\left(-\frac{2A-iB}{8a^3c} + \frac{(Ae^{8ie}+4Ae^{6ie}+6Ae^{4ie}+4Ae^{2ie}+A-iBe^{8ie}-2iBe^{6ie}+2iBe^{2ie}+iB)e^{-6ie}}{16a^3c}\right) \end{cases}$$
$$+ \frac{x(2A - iB)}{8a^3c}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e)),x)
```

output
```
Piecewise(((294912*I*A*a**9*c**3*f**3*exp(10*I*e)*exp(-2*I*f*x) + (16384*I
*A*a**9*c**3*f**3*exp(6*I*e) - 16384*B*a**9*c**3*f**3*exp(6*I*e))*exp(-6*I
*f*x) + (98304*I*A*a**9*c**3*f**3*exp(8*I*e) - 49152*B*a**9*c**3*f**3*exp(
8*I*e))*exp(-4*I*f*x) + (-49152*I*A*a**9*c**3*f**3*exp(14*I*e) - 49152*B*a
**9*c**3*f**3*exp(14*I*e))*exp(2*I*f*x))*exp(-12*I*e)/(1572864*a**12*c**4*
f**4), Ne(a**12*c**4*f**4*exp(12*I*e), 0)), (x*(-(2*A - I*B)/(8*a**3*c) +
(A*exp(8*I*e) + 4*A*exp(6*I*e) + 6*A*exp(4*I*e) + 4*A*exp(2*I*e) + A - I*B
*exp(8*I*e) - 2*I*B*exp(6*I*e) + 2*I*B*exp(2*I*e) + I*B)*exp(-6*I*e)/(16*a
**3*c)), True)) + x*(2*A - I*B)/(8*a**3*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))^3(c - ic\tan(e + fx))}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}\,dx$$
$$= -\frac{(-2i\,A-B)\log\left(\tan\left(fx+e\right)+i\right)}{16\,a^3cf} - \frac{(2i\,A+B)\log\left(\tan\left(fx+e\right)-i\right)}{16\,a^3cf}$$
$$+ \frac{3\,(2\,A-i\,B)\tan\left(fx+e\right)^3-6\,(2i\,A+B)\tan\left(fx+e\right)^2-(2\,A-i\,B)\tan\left(fx+e\right)-8i\,A+2\,B}{24\,a^3cf(\tan\left(fx+e\right)+i)(\tan\left(fx+e\right)-i)^3}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x, algo
rithm="giac")
```

output
```
-1/16*(-2*I*A - B)*log(tan(f*x + e) + I)/(a^3*c*f) - 1/16*(2*I*A + B)*log(
tan(f*x + e) - I)/(a^3*c*f) + 1/24*(3*(2*A - I*B)*tan(f*x + e)^3 - 6*(2*I*
A + B)*tan(f*x + e)^2 - (2*A - I*B)*tan(f*x + e) - 8*I*A + 2*B)/(a^3*c*f*(
tan(f*x + e) + I)*(tan(f*x + e) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}\,dx$$
$$= \frac{\frac{A}{3\,a^3\,c}+\tan(e+f\,x)^2\,\left(\frac{A}{2\,a^3\,c}-\frac{B\,1\mathrm{i}}{4\,a^3\,c}\right)+\tan(e+f\,x)^3\,\left(\frac{B}{8\,a^3\,c}+\frac{A\,1\mathrm{i}}{4\,a^3\,c}\right)-\tan(e+f\,x)\,\left(\frac{B}{24\,a^3\,c}+\frac{A\,1\mathrm{i}}{12\,a^3\,c}\right)+\frac{B\,1}{12\,a}}{f\,\left(\tan\left(e+f\,x\right)^4\,1\mathrm{i}+2\tan\left(e+f\,x\right)^3+2\tan\left(e+f\,x\right)-\mathrm{i}\right)}$$
$$-\frac{x\,\left(B+A\,2\mathrm{i}\right)\,1\mathrm{i}}{8\,a^3\,c}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i
)),x)
```

output
```
(tan(e + f*x)^2*(A/(2*a^3*c) - (B*1i)/(4*a^3*c)) - tan(e + f*x)*((A*1i)/(1
2*a^3*c) + B/(24*a^3*c)) + tan(e + f*x)^3*((A*1i)/(4*a^3*c) + B/(8*a^3*c))
 + A/(3*a^3*c) + (B*1i)/(12*a^3*c))/(f*(2*tan(e + f*x) + 2*tan(e + f*x)^3
+ tan(e + f*x)^4*1i - 1i)) - (x*(A*2i + B)*1i)/(8*a^3*c)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))}\,dx$$
$$=\frac{-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^4-2\tan(fx+e)^3 i-2\tan(fx+e)i-1}dx\right)b-\left(\int \frac{1}{\tan(fx+e)^4-2\tan(fx+e)^3 i-2\tan(fx+e)i-1}dx\right)a}{a^3c}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e)),x)
```

output
```
( - (int(tan(e + f*x)/(tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f
*x)*i - 1),x)*b + int(1/(tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e +
 f*x)*i - 1),x)*a))/(a**3*c)
```

3.735 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 185

$$\begin{aligned}&\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2}\,dx\\&=\frac{(5A-iB)x}{16a^3c^2}+\frac{A+iB}{24a^3c^2f(i-\tan(e+fx))^3}\\&\quad-\frac{3iA-B}{32a^3c^2f(i-\tan(e+fx))^2}-\frac{3A}{16a^3c^2f(i-\tan(e+fx))}\\&\quad+\frac{iA+B}{32a^3c^2f(i+\tan(e+fx))^2}+\frac{2A-iB}{16a^3c^2f(i+\tan(e+fx))}\end{aligned}$$

output
```
1/16*(5*A-I*B)*x/a^3/c^2+1/24*(A+I*B)/a^3/c^2/f/(I-tan(f*x+e))^3-1/32*(3*I
*A-B)/a^3/c^2/f/(I-tan(f*x+e))^2-3/16*A/a^3/c^2/f/(I-tan(f*x+e))+1/32*(I*A
+B)/a^3/c^2/f/(I+tan(f*x+e))^2+1/16*(2*A-I*B)/a^3/c^2/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.66 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.93

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2}\,dx$$
$$= \frac{\sec^4(e+fx)(47A+5iB+(-14A+22iB)\cos(2(e+fx))-A\cos(4(e+fx))+5iB\cos(4(e+fx))-}{192a^3c}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e +
f*x])^2),x]
```

output
```
(Sec[e + f*x]^4*(47*A + (5*I)*B + (-14*A + (22*I)*B)*Cos[2*(e + f*x)] - A*
Cos[4*(e + f*x)] + (5*I)*B*Cos[4*(e + f*x)] - (40*I)*A*Sin[2*(e + f*x)] -
8*B*Sin[2*(e + f*x)] - (5*I)*A*Sin[4*(e + f*x)] - B*Sin[4*(e + f*x)] + 12*
(5*A - I*B)*ArcTan[Tan[e + f*x]]*(-I + Tan[e + f*x])))/(192*a^3*c^2*f*(-I
+ Tan[e + f*x])^3*(I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4c^3(1-i\tan(e+fx))^3(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^3(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3c^2f}$$

$$\Big\downarrow \; 86$$

$$\frac{\int \left(-\frac{3A}{16(\tan(e+fx)-i)^2}+\frac{5A-iB}{16(\tan^2(e+fx)+1)}+\frac{iB-2A}{16(\tan(e+fx)+i)^2}+\frac{i(3A+iB)}{16(\tan(e+fx)-i)^3}-\frac{i(A-iB)}{16(\tan(e+fx)+i)^3}+\frac{A+iB}{8(\tan(e+fx)-i)^4}\right) \ldots}{a^3c^2f}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{16}(5A-iB)\arctan(\tan(e+fx))+\frac{2A-iB}{16(\tan(e+fx)+i)}-\frac{-B+3iA}{32(-\tan(e+fx)+i)^2}+\frac{B+iA}{32(\tan(e+fx)+i)^2}+\frac{A+iB}{24(-\tan(e+fx)+i)^3}- \ldots}{a^3c^2f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^
2),x]
```

output
```
(((5*A - I*B)*ArcTan[Tan[e + f*x]])/16 + (A + I*B)/(24*(I - Tan[e + f*x])^
3) - ((3*I)*A - B)/(32*(I - Tan[e + f*x])^2) - (3*A)/(16*(I - Tan[e + f*x]
)) + (I*A + B)/(32*(I + Tan[e + f*x])^2) + (2*A - I*B)/(16*(I + Tan[e + f*
x])))/(a^3*c^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.13

method	result
norman	$\frac{\frac{(-iB+5A)x}{16ac}-\frac{-iA+B}{6acf}+\frac{(-iB+5A)\tan(fx+e)^3}{6acf}+\frac{(-iB+5A)\tan(fx+e)^5}{16acf}+\frac{3(-iB+5A)x\tan(fx+e)^2}{16ac}+\frac{3(-iB+5A)x\tan(fx+e)^4}{16ac}}{\left(1+\tan(fx+e)^2\right)^3a^2c}$
risch	$-\frac{ixB}{16a^3c^2}+\frac{5xA}{16a^3c^2}-\frac{e^{-6i(fx+e)}B}{192a^3c^2f}+\frac{ie^{-6i(fx+e)}A}{192a^3c^2f}-\frac{\cos(4fx+4e)B}{32a^3c^2f}+\frac{i\cos(4fx+4e)A}{32a^3c^2f}+\frac{i\sin(4fx+4e)B}{64a^3c^2f}$
derivativedivides	$-\frac{3iA}{32f\,a^3c^2(-i+\tan(fx+e))^2}+\frac{5A\arctan(\tan(fx+e))}{16f\,a^3c^2}-\frac{iB\arctan(\tan(fx+e))}{16f\,a^3c^2}-\frac{iB}{16f\,a^3c^2(i+\tan(fx+e))}+$
default	$-\frac{3iA}{32f\,a^3c^2(-i+\tan(fx+e))^2}+\frac{5A\arctan(\tan(fx+e))}{16f\,a^3c^2}-\frac{iB\arctan(\tan(fx+e))}{16f\,a^3c^2}-\frac{iB}{16f\,a^3c^2(i+\tan(fx+e))}+$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
(1/16*(5*A-I*B)/a/c*x-1/6*(-I*A+B)/a/c/f+1/6*(5*A-I*B)/a/c/f*tan(f*x+e)^3+
1/16*(5*A-I*B)/a/c/f*tan(f*x+e)^5+3/16*(5*A-I*B)/a/c*x*tan(f*x+e)^2+3/16*(
5*A-I*B)/a/c*x*tan(f*x+e)^4+1/16*(5*A-I*B)/a/c*x*tan(f*x+e)^6+1/16*(I*B+11
*A)/a/c/f*tan(f*x+e))/(1+tan(f*x+e)^2)^3/a^2/c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.62

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2}\,dx$$
$$= \frac{\left(24\,(5\,A-i\,B)fxe^{(6i\,fx+6i\,e)}-3\,(i\,A+B)e^{(10i\,fx+10i\,e)}-6\,(5i\,A+3\,B)e^{(8i\,fx+8i\,e)}-12\,(-5i\,A+B)e^{(4i\,f}\right.}{384\,a^3c^2f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
1/384*(24*(5*A - I*B)*f*x*e^(6*I*f*x + 6*I*e) - 3*(I*A + B)*e^(10*I*f*x +
10*I*e) - 6*(5*I*A + 3*B)*e^(8*I*f*x + 8*I*e) - 12*(-5*I*A + B)*e^(4*I*f*x
 + 4*I*e) - 3*(-5*I*A + 3*B)*e^(2*I*f*x + 2*I*e) + 2*I*A - 2*B)*e^(-6*I*f*
x - 6*I*e)/(a^3*c^2*f)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.44

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2}\,dx$$
$$= \begin{cases} \frac{((33554432iAa^{12}c^8f^4e^{6ie}-33554432Ba^{12}c^8f^4e^{6ie})e^{-6ifx}+(251658240iAa^{12}c^8f^4e^{8ie}-150994944Ba^{12}c^8f^4e^{8ie})e^{-4ifx}+(1006632960iAa}{} \\ x\left(-\frac{5A-iB}{16a^3c^2}+\frac{(Ae^{10ie}+5Ae^{8ie}+10Ae^{6ie}+10Ae^{4ie}+5Ae^{2ie}+A-iBe^{10ie}-3iBe^{8ie}-2iBe^{6ie}+2iBe^{4ie}+3iBe^{2ie}+iB)e^{-6ie}}{32a^3c^2}\right) \end{cases}$$
$$+\frac{x(5A-iB)}{16a^3c^2}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**2,x)
```

output
```
Piecewise((((33554432*I*A*a**12*c**8*f**4*exp(6*I*e) - 33554432*B*a**12*c*
*8*f**4*exp(6*I*e))*exp(-6*I*f*x) + (251658240*I*A*a**12*c**8*f**4*exp(8*I
*e) - 150994944*B*a**12*c**8*f**4*exp(8*I*e))*exp(-4*I*f*x) + (1006632960*
I*A*a**12*c**8*f**4*exp(10*I*e) - 201326592*B*a**12*c**8*f**4*exp(10*I*e))
*exp(-2*I*f*x) + (-503316480*I*A*a**12*c**8*f**4*exp(14*I*e) - 301989888*B
*a**12*c**8*f**4*exp(14*I*e))*exp(2*I*f*x) + (-50331648*I*A*a**12*c**8*f**
4*exp(16*I*e) - 50331648*B*a**12*c**8*f**4*exp(16*I*e))*exp(4*I*f*x))*exp(
-12*I*e)/(6442450944*a**15*c**10*f**5), Ne(a**15*c**10*f**5*exp(12*I*e), 0
)), (x*(-(5*A - I*B)/(16*a**3*c**2) + (A*exp(10*I*e) + 5*A*exp(8*I*e) + 10
*A*exp(6*I*e) + 10*A*exp(4*I*e) + 5*A*exp(2*I*e) + A - I*B*exp(10*I*e) - 3
*I*B*exp(8*I*e) - 2*I*B*exp(6*I*e) + 2*I*B*exp(4*I*e) + 3*I*B*exp(2*I*e) +
 I*B)*exp(-6*I*e)/(32*a**3*c**2)), True)) + x*(5*A - I*B)/(16*a**3*c**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))^3(c - ic\tan(e + fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))^3(c - ic\tan(e + fx))^2}\,dx$$
$$= -\frac{(-5i\,A - B)\log(\tan(fx + e) + i)}{32\,a^3c^2f} - \frac{(5i\,A + B)\log(\tan(fx + e) - i)}{32\,a^3c^2f}$$
$$+ \frac{3\,(5\,A - i\,B)\tan(fx + e)^4 - 3\,(5i\,A + B)\tan(fx + e)^3 + 5\,(5\,A - i\,B)\tan(fx + e)^2 - 5\,(5i\,A + B)\,\mathrm{t}}{48\,a^3c^2f(\tan(fx + e) + i)^2(\tan(fx + e) - i)^3}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-1/32*(-5*I*A - B)*log(tan(f*x + e) + I)/(a^3*c^2*f) - 1/32*(5*I*A + B)*lo
g(tan(f*x + e) - I)/(a^3*c^2*f) + 1/48*(3*(5*A - I*B)*tan(f*x + e)^4 - 3*(
5*I*A + B)*tan(f*x + e)^3 + 5*(5*A - I*B)*tan(f*x + e)^2 - 5*(5*I*A + B)*t
an(f*x + e) + 8*A + 8*I*B)/(a^3*c^2*f*(tan(f*x + e) + I)^2*(tan(f*x + e) -
 I)^3)
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.12

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))^3 (c - ic\tan(e + fx))^2} dx$$
$$= \frac{\tan(e + f\,x)\,\left(\frac{25\,A}{48\,a^3\,c^2} - \frac{B\,5\mathrm{i}}{48\,a^3\,c^2}\right) + \tan(e + f\,x)^3\,\left(\frac{5\,A}{16\,a^3\,c^2} - \frac{B\,1\mathrm{i}}{16\,a^3\,c^2}\right) + \tan(e + f\,x)^4\,\left(\frac{B}{16\,a^3\,c^2} + \frac{A\,5\mathrm{i}}{16\,a^3\,c^2}\right) + \mathrm{ta}}{f\,\left(\tan\left(e + f\,x\right)^5\,1\mathrm{i} + \tan\left(e + f\,x\right)^4 + \tan\left(e + f\,x\right)^3\,2\mathrm{i} + 2\tan\left(e + f\,x\right)^2 + \mathrm{ta}\right.}$$
$$- \frac{x\,(B + A\,5\mathrm{i})\,1\mathrm{i}}{16\,a^3\,c^2}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i
)^2),x)
```

output
```
(tan(e + f*x)*((25*A)/(48*a^3*c^2) - (B*5i)/(48*a^3*c^2)) + tan(e + f*x)^3
*((5*A)/(16*a^3*c^2) - (B*1i)/(16*a^3*c^2)) + tan(e + f*x)^4*((A*5i)/(16*a
^3*c^2) + B/(16*a^3*c^2)) + tan(e + f*x)^2*((A*25i)/(48*a^3*c^2) + (5*B)/(
48*a^3*c^2)) + (A*1i)/(6*a^3*c^2) - B/(6*a^3*c^2))/(f*(tan(e + f*x)*1i + 2
*tan(e + f*x)^2 + tan(e + f*x)^3*2i + tan(e + f*x)^4 + tan(e + f*x)^5*1i +
 1)) - (x*(A*5i + B)*1i)/(16*a^3*c^2)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^2}\,dx$$
$$= \frac{-8\left(\int \frac{1}{\tan(fx+e)^5-\tan(fx+e)^4i+2\tan(fx+e)^3-2\tan(fx+e)^2i+\tan(fx+e)-i}dx\right)\tan(fx+e)^4 afi+8\left(\int \frac{}{\tan(fx+e)^5-\tan(}\right.}{}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^2,x)
```

output
```
( - 8*int(1/(tan(e + f*x)**5 - tan(e + f*x)**4*i + 2*tan(e + f*x)**3 - 2*t
an(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*x)**4*a*f*i + 8*int(1/(t
an(e + f*x)**5 - tan(e + f*x)**4*i + 2*tan(e + f*x)**3 - 2*tan(e + f*x)**2
*i + tan(e + f*x) - i),x)*tan(e + f*x)**4*b*f - 16*int(1/(tan(e + f*x)**5
- tan(e + f*x)**4*i + 2*tan(e + f*x)**3 - 2*tan(e + f*x)**2*i + tan(e + f*
x) - i),x)*tan(e + f*x)**2*a*f*i + 16*int(1/(tan(e + f*x)**5 - tan(e + f*x
)**4*i + 2*tan(e + f*x)**3 - 2*tan(e + f*x)**2*i + tan(e + f*x) - i),x)*ta
n(e + f*x)**2*b*f - 8*int(1/(tan(e + f*x)**5 - tan(e + f*x)**4*i + 2*tan(e
 + f*x)**3 - 2*tan(e + f*x)**2*i + tan(e + f*x) - i),x)*a*f*i + 8*int(1/(t
an(e + f*x)**5 - tan(e + f*x)**4*i + 2*tan(e + f*x)**3 - 2*tan(e + f*x)**2
*i + tan(e + f*x) - i),x)*b*f - 3*tan(e + f*x)**4*b*e*i - 3*tan(e + f*x)**
4*b*f*i*x - 3*tan(e + f*x)**3*b*i - 6*tan(e + f*x)**2*b*e*i - 6*tan(e + f*
x)**2*b*f*i*x - 5*tan(e + f*x)*b*i - 3*b*e*i - 3*b*f*i*x)/(8*a**3*c**2*f*(
tan(e + f*x)**4 + 2*tan(e + f*x)**2 + 1))
```

3.736 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 99

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx = \frac{5Ax}{16a^3c^3}+\frac{5A\cos(e+fx)\sin(e+fx)}{16a^3c^3f} + \frac{5A\cos^3(e+fx)\sin(e+fx)}{24a^3c^3f}-\frac{\cos^6(e+fx)(B-A\tan(e+fx))}{6a^3c^3f}$$

output
```
5/16*A*x/a^3/c^3+5/16*A*cos(f*x+e)*sin(f*x+e)/a^3/c^3/f+5/24*A*cos(f*x+e)^
3*sin(f*x+e)/a^3/c^3/f-1/6*cos(f*x+e)^6*(B-A*tan(f*x+e))/a^3/c^3/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.64

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx = \frac{-32B\cos^6(e+fx)+A(60e+60fx+45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx)))}{192a^3c^3f}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e +
f*x])^3),x]
```

output
```
(-32*B*Cos[e + f*x]^6 + A*(60*e + 60*f*x + 45*Sin[2*(e + f*x)] + 9*Sin[4*(
e + f*x)] + Sin[6*(e + f*x)]))/(192*a^3*c^3*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {3042, 4071, 27, 82, 454, 215, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4c^4(1-i\tan(e+fx))^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3c^3f}$$

$$\Big\downarrow \text{ 82}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(\tan^2(e+fx)+1)^4}d\tan(e+fx)}{a^3c^3f}$$

$$\Big\downarrow \text{ 454}$$

$$\frac{\frac{5}{6}A\int \frac{1}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)-\frac{B-A\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}}{a^3c^3f}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{5}{6}A\left(\frac{3}{4}\int \frac{1}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)+\frac{\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)-\frac{B-A\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}}{a^3c^3f}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{5}{6}A\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)-\frac{B-A\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}}{a^3c^3f}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{5}{6}A\left(\frac{3}{4}\left(\frac{1}{2}\arctan(\tan(e+fx))+\frac{\tan(e+fx)}{2(\tan^2(e+fx)+1)}\right)+\frac{\tan(e+fx)}{4(\tan^2(e+fx)+1)^2}\right)-\frac{B-A\tan(e+fx)}{6(\tan^2(e+fx)+1)^3}}{a^3c^3f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^
3),x]
```

output
```
(-1/6*(B - A*Tan[e + f*x])/(1 + Tan[e + f*x]^2)^3 + (5*A*(Tan[e + f*x]/(4*
(1 + Tan[e + f*x]^2)^2) + (3*(ArcTan[Tan[e + f*x]]/2 + Tan[e + f*x]/(2*(1
+ Tan[e + f*x]^2))))/4))/6)/(a^3*c^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 82
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b, c, d,
 e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.39

method	result
risch	$\frac{5Ax}{16a^3c^3} - \frac{B\cos(6fx+6e)}{192f\,c^3a^3} + \frac{A\sin(6fx+6e)}{192f\,c^3a^3} - \frac{B\cos(4fx+4e)}{32f\,c^3a^3} + \frac{3A\sin(4fx+4e)}{64f\,c^3a^3} - \frac{5B\cos(2fx+2e)}{64f\,c^3a^3} + 15A$
norman	$\frac{\frac{5Ax}{16ac} - \frac{B}{6acf} + \frac{11A\tan(fx+e)}{16acf} + \frac{5A\tan(fx+e)^3}{6acf} + \frac{5A\tan(fx+e)^5}{16acf} + \frac{15Ax\tan(fx+e)^2}{16ac} + \frac{15Ax\tan(fx+e)^4}{16ac} + \frac{5Ax\tan(fx+e)^6}{16ac}}{\left(1+\tan(fx+e)^2\right)^3 a^2c^2}$
derivativedivides	$\frac{iB}{48f\,a^3c^3(i+\tan(fx+e))^3} + \frac{5A\arctan(\tan(fx+e))}{16f\,a^3c^3} - \frac{A}{48f\,a^3c^3(i+\tan(fx+e))^3} - \frac{iB}{32f\,a^3c^3(i+\tan(fx+e))} + 3$
default	$\frac{iB}{48f\,a^3c^3(i+\tan(fx+e))^3} + \frac{5A\arctan(\tan(fx+e))}{16f\,a^3c^3} - \frac{A}{48f\,a^3c^3(i+\tan(fx+e))^3} - \frac{iB}{32f\,a^3c^3(i+\tan(fx+e))} + 3$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
5/16*A*x/a^3/c^3-1/192*B/f/c^3/a^3*cos(6*f*x+6*e)+1/192*A/f/c^3/a^3*sin(6*
f*x+6*e)-1/32*B/f/c^3/a^3*cos(4*f*x+4*e)+3/64*A/f/c^3/a^3*sin(4*f*x+4*e)-5
/64*B/f/c^3/a^3*cos(2*f*x+2*e)+15/64*A/f/c^3/a^3*sin(2*f*x+2*e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.27

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx = \frac{\left(120\,Afxe^{(6i\,fx+6i\,e)}+(-i\,A-B)e^{(12i\,fx+12i\,e)}-3\,(3i\,A+2\,B)e^{(10i\,fx+10i\,e)}-15\,(3i\,A+B)e^{(8i\,fx+8i\,e)}-\right.}{384\,a^3c^3f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
1/384*(120*A*f*x*e^(6*I*f*x + 6*I*e) + (-I*A - B)*e^(12*I*f*x + 12*I*e) -
3*(3*I*A + 2*B)*e^(10*I*f*x + 10*I*e) - 15*(3*I*A + B)*e^(8*I*f*x + 8*I*e)
 - 15*(-3*I*A + B)*e^(4*I*f*x + 4*I*e) - 3*(-3*I*A + 2*B)*e^(2*I*f*x + 2*I
*e) + I*A - B)*e^(-6*I*f*x - 6*I*e)/(a^3*c^3*f)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 508, normalized size of antiderivative = 5.13

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx = \frac{5Ax}{16a^3c^3} + \begin{cases} \frac{((103079215104iAa^{15}c^{15}f^5e^{6ie}-103079215104Ba^{15}c^{15}f^5e^{6ie})e^{-6ifx}+(927712935936iAa^{15}c^{15}f^5e^{8ie}-618475290624Ba^{15}c^{15}f^5e^{8ie})e}{} \\ x\left(-\frac{5A}{16a^3c^3}+\frac{(Ae^{12ie}+6Ae^{10ie}+15Ae^{8ie}+20Ae^{6ie}+15Ae^{4ie}+6Ae^{2ie}+A-iBe^{12ie}-4iBe^{10ie}-5iBe^{8ie}+5iBe^{4ie}+4iBe^{2ie}+iB)e^{-6i}}{64a^3c^3}\right. \end{cases}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**3,x)
```

output
```
5*A*x/(16*a**3*c**3) + Piecewise((((103079215104*I*A*a**15*c**15*f**5*exp(
6*I*e) - 103079215104*B*a**15*c**15*f**5*exp(6*I*e))*exp(-6*I*f*x) + (9277
12935936*I*A*a**15*c**15*f**5*exp(8*I*e) - 618475290624*B*a**15*c**15*f**5
*exp(8*I*e))*exp(-4*I*f*x) + (4638564679680*I*A*a**15*c**15*f**5*exp(10*I*
e) - 1546188226560*B*a**15*c**15*f**5*exp(10*I*e))*exp(-2*I*f*x) + (-46385
64679680*I*A*a**15*c**15*f**5*exp(14*I*e) - 1546188226560*B*a**15*c**15*f*
*5*exp(14*I*e))*exp(2*I*f*x) + (-927712935936*I*A*a**15*c**15*f**5*exp(16*
I*e) - 618475290624*B*a**15*c**15*f**5*exp(16*I*e))*exp(4*I*f*x) + (-10307
9215104*I*A*a**15*c**15*f**5*exp(18*I*e) - 103079215104*B*a**15*c**15*f**5
*exp(18*I*e))*exp(6*I*f*x))*exp(-12*I*e)/(39582418599936*a**18*c**18*f**6)
, Ne(a**18*c**18*f**6*exp(12*I*e), 0)), (x*(-5*A/(16*a**3*c**3) + (A*exp(1
2*I*e) + 6*A*exp(10*I*e) + 15*A*exp(8*I*e) + 20*A*exp(6*I*e) + 15*A*exp(4*
I*e) + 6*A*exp(2*I*e) + A - I*B*exp(12*I*e) - 4*I*B*exp(10*I*e) - 5*I*B*ex
p(8*I*e) + 5*I*B*exp(4*I*e) + 4*I*B*exp(2*I*e) + I*B)*exp(-6*I*e)/(64*a**3
*c**3)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx = \frac{5\,(fx+e)A}{16\,a^3c^3f} + \frac{15\,A\tan\left(fx+e\right)^5 + 40\,A\tan\left(fx+e\right)^3 + 33\,A\tan\left(fx+e\right) - 8\,B}{48\,\left(\tan\left(fx+e\right)^2+1\right)^3a^3c^3f}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x, al
gorithm="giac")
```

output
```
5/16*(f*x + e)*A/(a^3*c^3*f) + 1/48*(15*A*tan(f*x + e)^5 + 40*A*tan(f*x +
e)^3 + 33*A*tan(f*x + e) - 8*B)/((tan(f*x + e)^2 + 1)^3*a^3*c^3*f)
```

Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx = \frac{5\,A\,x}{16\,a^3\,c^3} + \frac{\cos\left(e+f\,x\right)^6\,\left(\frac{5\,A\tan\left(e+f\,x\right)^5}{16}+\frac{5\,A\tan\left(e+f\,x\right)^3}{6}+\frac{11\,A\tan\left(e+f\,x\right)}{16}-\frac{B}{6}\right)}{a^3\,c^3\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i
)^3),x)
```

output
```
(5*A*x)/(16*a^3*c^3) + (cos(e + f*x)^6*((11*A*tan(e + f*x))/16 - B/6 + (5*
A*tan(e + f*x)^3)/6 + (5*A*tan(e + f*x)^5)/16))/(a^3*c^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.54

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^3}\,dx$$
$$= \frac{15\tan(fx+e)^6 afx+8\tan(fx+e)^6 b+15\tan(fx+e)^5 a+45\tan(fx+e)^4 afx+24\tan(fx+e)^4 \cdots}{48a^3c^3f\left(\tan(fx+e)^6+3\tan(fx+\cdots\right.}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^3,x)
```

output
```
(15*tan(e + f*x)**6*a*f*x + 8*tan(e + f*x)**6*b + 15*tan(e + f*x)**5*a + 4
5*tan(e + f*x)**4*a*f*x + 24*tan(e + f*x)**4*b + 40*tan(e + f*x)**3*a + 45
*tan(e + f*x)**2*a*f*x + 24*tan(e + f*x)**2*b + 33*tan(e + f*x)*a + 15*a*f
*x)/(48*a**3*c**3*f*(tan(e + f*x)**6 + 3*tan(e + f*x)**4 + 3*tan(e + f*x)*
*2 + 1))
```

3.737 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 251

$$\begin{aligned}\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx &= \frac{5(7A+iB)x}{128a^3c^4}+\frac{A+iB}{96a^3c^4f(i-\tan(e+fx))^3}\\&\quad-\frac{5iA-3B}{128a^3c^4f(i-\tan(e+fx))^2}-\frac{5(3A+iB)}{128a^3c^4f(i-\tan(e+fx))}\\&\quad-\frac{iA+B}{64a^3c^4f(i+\tan(e+fx))^4}-\frac{2A-iB}{48a^3c^4f(i+\tan(e+fx))^3}\\&\quad+\frac{5iA+B}{64a^3c^4f(i+\tan(e+fx))^2}+\frac{5A}{32a^3c^4f(i+\tan(e+fx))}\end{aligned}$$

output
```
5/128*(7*A+I*B)*x/a^3/c^4+1/96*(A+I*B)/a^3/c^4/f/(I-tan(f*x+e))^3-1/128*(5
*I*A-3*B)/a^3/c^4/f/(I-tan(f*x+e))^2-5/128*(3*A+I*B)/a^3/c^4/f/(I-tan(f*x+
e))-1/64*(I*A+B)/a^3/c^4/f/(I+tan(f*x+e))^4-1/48*(2*A-I*B)/a^3/c^4/f/(I+ta
n(f*x+e))^3+1/64*(5*I*A+B)/a^3/c^4/f/(I+tan(f*x+e))^2+5/32*A/a^3/c^4/f/(I+
tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.91 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.88

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx = \frac{\sec^6(e+fx)(319A-23iB-(113A+119iB)\cos(2(e+fx))-13A\cos(4(e+fx))-43iB\cos(4(e+f}{}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e +
f*x])^4),x]
```

output
```
(Sec[e + f*x]^6*(319*A - (23*I)*B - (113*A + (119*I)*B)*Cos[2*(e + f*x)] -
 13*A*Cos[4*(e + f*x)] - (43*I)*B*Cos[4*(e + f*x)] - A*Cos[6*(e + f*x)] -
(7*I)*B*Cos[6*(e + f*x)] + (315*I)*A*Sin[2*(e + f*x)] - 45*B*Sin[2*(e + f*
x)] + (63*I)*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)] + (7*I)*A*Sin[6*(e
+ f*x)] - B*Sin[6*(e + f*x)] + 60*(7*A + I*B)*ArcTan[Tan[e + f*x]]*(I + Ta
n[e + f*x])))/(1536*a^3*c^4*f*(-I + Tan[e + f*x])^3*(I + Tan[e + f*x])^4)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4c^5(1-i\tan(e+fx))^5(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^5(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3c^4f}$$

$$\Big\downarrow 86$$

$$\frac{\int \left(-\frac{5A}{32(\tan(e+fx)+i)^2}+\frac{5(7A+iB)}{128(\tan^2(e+fx)+1)}-\frac{5(3A+iB)}{128(\tan(e+fx)-i)^2}+\frac{i(5A+3iB)}{64(\tan(e+fx)-i)^3}-\frac{i(5A-iB)}{32(\tan(e+fx)+i)^3}+\frac{A+iB}{32(\tan(e+fx)-i)^4}\right.}{a^3c^4f}$$

$$\Big\downarrow 2009$$

$$\frac{\frac{5}{128}(7A+iB)\arctan(\tan(e+fx))-\frac{5(3A+iB)}{128(-\tan(e+fx)+i)}-\frac{-3B+5iA}{128(-\tan(e+fx)+i)^2}+\frac{B+5iA}{64(\tan(e+fx)+i)^2}+\frac{A+iB}{96(-\tan(e+fx)+i)^3}}{a^3c^4f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^
4),x]
```

output
```
((5*(7*A + I*B)*ArcTan[Tan[e + f*x]])/128 + (A + I*B)/(96*(I - Tan[e + f*x
])^3) - ((5*I)*A - 3*B)/(128*(I - Tan[e + f*x])^2) - (5*(3*A + I*B))/(128*
(I - Tan[e + f*x])) - (I*A + B)/(64*(I + Tan[e + f*x])^4) - (2*A - I*B)/(4
8*(I + Tan[e + f*x])^3) + ((5*I)*A + B)/(64*(I + Tan[e + f*x])^2) + (5*A)/
(32*(I + Tan[e + f*x]))))/(a^3*c^4*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.04

method	result
norman	$\frac{\frac{5(iB+7A)x}{128ac}-\frac{iA+B}{8acf}+\frac{(-5iB+93A)\tan(fx+e)}{128acf}+\frac{73(iB+7A)\tan(fx+e)^3}{384acf}+\frac{55(iB+7A)\tan(fx+e)^5}{384acf}+\frac{5(iB+7A)\tan(fx+e)^7}{128acf}+\cdots}{a^2c^3\left(1+\tan(fx+e)^2\right)^4}$
risch	$-\frac{i\sin(6fx+6e)B}{192a^3c^4f}+\frac{35xA}{128a^3c^4}-\frac{e^{8i(fx+e)}B}{1024a^3c^4f}-\frac{i\sin(4fx+4e)B}{128a^3c^4f}-\frac{\cos(6fx+6e)B}{128a^3c^4f}-\frac{i\cos(6fx+6e)A}{128a^3c^4f}-\cdots$
derivativedivides	$-\frac{5iA}{128f\,a^3c^4(-i+\tan(fx+e))^2}+\frac{35A\arctan(\tan(fx+e))}{128f\,a^3c^4}-\frac{iA}{64f\,a^3c^4(i+\tan(fx+e))^4}+\cdots$
default	$-\frac{5iA}{128f\,a^3c^4(-i+\tan(fx+e))^2}+\frac{35A\arctan(\tan(fx+e))}{128f\,a^3c^4}-\frac{iA}{64f\,a^3c^4(i+\tan(fx+e))^4}+\cdots$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x,method=_R
ETURNVERBOSE)
```

output
```
(5/128*(7*A+I*B)/a/c*x-1/8*(I*A+B)/a/c/f+1/128*(-5*I*B+93*A)/a/c/f*tan(f*x
+e)+73/384*(7*A+I*B)/a/c/f*tan(f*x+e)^3+55/384*(7*A+I*B)/a/c/f*tan(f*x+e)^
5+5/128*(7*A+I*B)/a/c/f*tan(f*x+e)^7+5/32*(7*A+I*B)/a/c*x*tan(f*x+e)^2+15/
64*(7*A+I*B)/a/c*x*tan(f*x+e)^4+5/32*(7*A+I*B)/a/c*x*tan(f*x+e)^6+5/128*(7
*A+I*B)/a/c*x*tan(f*x+e)^8)/a^2/c^3/(1+tan(f*x+e)^2)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx = \frac{\left(120\,(7\,A+i\,B)fxe^{(6i\,fx+6i\,e)}-3\,(i\,A+B)e^{(14i\,fx+14i\,e)}-4\,(7i\,A+5\,B)e^{(12i\,fx+12i\,e)}-18\,(7i\,A+3\,B)e^{(}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, al
gorithm="fricas")
```

output
```
1/3072*(120*(7*A + I*B)*f*x*e^(6*I*f*x + 6*I*e) - 3*(I*A + B)*e^(14*I*f*x
+ 14*I*e) - 4*(7*I*A + 5*B)*e^(12*I*f*x + 12*I*e) - 18*(7*I*A + 3*B)*e^(10
*I*f*x + 10*I*e) - 60*(7*I*A + B)*e^(8*I*f*x + 8*I*e) - 36*(-7*I*A + 3*B)*
e^(4*I*f*x + 4*I*e) - 6*(-7*I*A + 5*B)*e^(2*I*f*x + 2*I*e) + 4*I*A - 4*B)*
e^(-6*I*f*x - 6*I*e)/(a^3*c^4*f)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.41

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx = \begin{cases} \frac{((135107988821114888iAa^{18}c^{24}f^6e^{6ie}-135107988821114888Ba^{18}c^{24}f^6e^{6ie})e^{-6ifx}+(1418633882621706624iAa^{18}c^{24}f^6e^{8ie}-10133099161583}{} \\ x\left(-\frac{35A+5iB}{128a^3c^4}+\frac{(Ae^{14ie}+7Ae^{12ie}+21Ae^{10ie}+35Ae^{8ie}+35Ae^{6ie}+21Ae^{4ie}+7Ae^{2ie}+A-iBe^{14ie}-5iBe^{12ie}-9iBe^{10ie}-5iBe^{8ie}+5iBe^{6}}{128a^3c^4}\right. \end{cases} + \frac{x(35A+5iB)}{128a^3c^4}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**4,x)
```

output
```
Piecewise((((13510798882111488*I*A*a**18*c**24*f**6*exp(6*I*e) - 135107988
82111488*B*a**18*c**24*f**6*exp(6*I*e))*exp(-6*I*f*x) + (14186338826217062
4*I*A*a**18*c**24*f**6*exp(8*I*e) - 101330991615836160*B*a**18*c**24*f**6*
exp(8*I*e))*exp(-4*I*f*x) + (851180329573023744*I*A*a**18*c**24*f**6*exp(1
0*I*e) - 364791569817010176*B*a**18*c**24*f**6*exp(10*I*e))*exp(-2*I*f*x)
+ (-1418633882621706240*I*A*a**18*c**24*f**6*exp(14*I*e) - 202661983231672
320*B*a**18*c**24*f**6*exp(14*I*e))*exp(2*I*f*x) + (-425590164786511872*I*
A*a**18*c**24*f**6*exp(16*I*e) - 182395784908505088*B*a**18*c**24*f**6*exp
(16*I*e))*exp(4*I*f*x) + (-94575592174780416*I*A*a**18*c**24*f**6*exp(18*I
*e) - 67553994410557440*B*a**18*c**24*f**6*exp(18*I*e))*exp(6*I*f*x) + (-1
0133099161583616*I*A*a**18*c**24*f**6*exp(20*I*e) - 10133099161583616*B*a*
*18*c**24*f**6*exp(20*I*e))*exp(8*I*f*x))*exp(-12*I*e)/(103762935414616227
84*a**21*c**28*f**7), Ne(a**21*c**28*f**7*exp(12*I*e), 0)), (x*(-(35*A + 5
*I*B)/(128*a**3*c**4) + (A*exp(14*I*e) + 7*A*exp(12*I*e) + 21*A*exp(10*I*e
) + 35*A*exp(8*I*e) + 35*A*exp(6*I*e) + 21*A*exp(4*I*e) + 7*A*exp(2*I*e) +
 A - I*B*exp(14*I*e) - 5*I*B*exp(12*I*e) - 9*I*B*exp(10*I*e) - 5*I*B*exp(8
*I*e) + 5*I*B*exp(6*I*e) + 9*I*B*exp(4*I*e) + 5*I*B*exp(2*I*e) + I*B)*exp(
-6*I*e)/(128*a**3*c**4)), True)) + x*(35*A + 5*I*B)/(128*a**3*c**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.74

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx$$
$$= -\frac{5(-7iA+B)\log(\tan(fx+e)+i)}{256a^3c^4f} - \frac{5(7iA-B)\log(\tan(fx+e)-i)}{256a^3c^4f}$$
$$+\frac{15(7A+iB)\tan(fx+e)^6 - 15(-7iA+B)\tan(fx+e)^5 + 40(7A+iB)\tan(fx+e)^4 - 40(-7i}{384a^3c^4f(\tan(fx+e)+}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x, al
gorithm="giac")
```

output
```
-5/256*(-7*I*A + B)*log(tan(f*x + e) + I)/(a^3*c^4*f) - 5/256*(7*I*A - B)*
log(tan(f*x + e) - I)/(a^3*c^4*f) + 1/384*(15*(7*A + I*B)*tan(f*x + e)^6 -
 15*(-7*I*A + B)*tan(f*x + e)^5 + 40*(7*A + I*B)*tan(f*x + e)^4 - 40*(-7*I
*A + B)*tan(f*x + e)^3 + 33*(7*A + I*B)*tan(f*x + e)^2 - 33*(-7*I*A + B)*t
an(f*x + e) + 48*A - 48*I*B)/(a^3*c^4*f*(tan(f*x + e) + I)^4*(tan(f*x + e)
 - I)^3)
```

Mupad [B] (verification not implemented)

Time = 6.85 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.14

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx$$
$$= \frac{\tan(e+f\,x)\,\left(-\frac{11\,B}{128\,a^3\,c^4}+\frac{A\,77\mathrm{i}}{128\,a^3\,c^4}\right)+\tan(e+f\,x)^3\,\left(-\frac{5\,B}{48\,a^3\,c^4}+\frac{A\,35\mathrm{i}}{48\,a^3\,c^4}\right)+\tan(e+f\,x)^4\,\left(\frac{35\,A}{48\,a^3\,c^4}+\frac{B\,5\mathrm{i}}{48\,a^3\,c^4}\right)}{f\,\left(\tan(e+f\,x)^7+\tan(e+f\,x)^6\,1\mathrm{i}+3\tan(e+f\,x)^5\right.}$$
$$+\frac{5\,x\,(7\,A+B\,1\mathrm{i})}{128\,a^3\,c^4}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i
)^4),x)
```

output
```
(tan(e + f*x)*((A*77i)/(128*a^3*c^4) - (11*B)/(128*a^3*c^4)) + tan(e + f*x
)^3*((A*35i)/(48*a^3*c^4) - (5*B)/(48*a^3*c^4)) + tan(e + f*x)^4*((35*A)/(
48*a^3*c^4) + (B*5i)/(48*a^3*c^4)) + tan(e + f*x)^5*((A*35i)/(128*a^3*c^4)
 - (5*B)/(128*a^3*c^4)) + tan(e + f*x)^6*((35*A)/(128*a^3*c^4) + (B*5i)/(1
28*a^3*c^4)) + tan(e + f*x)^2*((77*A)/(128*a^3*c^4) + (B*11i)/(128*a^3*c^4
)) + A/(8*a^3*c^4) - (B*1i)/(8*a^3*c^4))/(f*(tan(e + f*x) + tan(e + f*x)^2
*3i + 3*tan(e + f*x)^3 + tan(e + f*x)^4*3i + 3*tan(e + f*x)^5 + tan(e + f*
x)^6*1i + tan(e + f*x)^7 + 1i)) + (5*x*(7*A + B*1i))/(128*a^3*c^4)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^4,x)
```

output
```
(48*int(1/(tan(e + f*x)**7 + tan(e + f*x)**6*i + 3*tan(e + f*x)**5 + 3*tan
(e + f*x)**4*i + 3*tan(e + f*x)**3 + 3*tan(e + f*x)**2*i + tan(e + f*x) +
i),x)*tan(e + f*x)**6*a*f*i + 48*int(1/(tan(e + f*x)**7 + tan(e + f*x)**6*
i + 3*tan(e + f*x)**5 + 3*tan(e + f*x)**4*i + 3*tan(e + f*x)**3 + 3*tan(e
+ f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**6*b*f + 144*int(1/(tan(e
+ f*x)**7 + tan(e + f*x)**6*i + 3*tan(e + f*x)**5 + 3*tan(e + f*x)**4*i +
3*tan(e + f*x)**3 + 3*tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x
)**4*a*f*i + 144*int(1/(tan(e + f*x)**7 + tan(e + f*x)**6*i + 3*tan(e + f*
x)**5 + 3*tan(e + f*x)**4*i + 3*tan(e + f*x)**3 + 3*tan(e + f*x)**2*i + ta
n(e + f*x) + i),x)*tan(e + f*x)**4*b*f + 144*int(1/(tan(e + f*x)**7 + tan(
e + f*x)**6*i + 3*tan(e + f*x)**5 + 3*tan(e + f*x)**4*i + 3*tan(e + f*x)**
3 + 3*tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**2*a*f*i + 144
*int(1/(tan(e + f*x)**7 + tan(e + f*x)**6*i + 3*tan(e + f*x)**5 + 3*tan(e
+ f*x)**4*i + 3*tan(e + f*x)**3 + 3*tan(e + f*x)**2*i + tan(e + f*x) + i),
x)*tan(e + f*x)**2*b*f + 48*int(1/(tan(e + f*x)**7 + tan(e + f*x)**6*i + 3
*tan(e + f*x)**5 + 3*tan(e + f*x)**4*i + 3*tan(e + f*x)**3 + 3*tan(e + f*x
)**2*i + tan(e + f*x) + i),x)*a*f*i + 48*int(1/(tan(e + f*x)**7 + tan(e +
f*x)**6*i + 3*tan(e + f*x)**5 + 3*tan(e + f*x)**4*i + 3*tan(e + f*x)**3 +
3*tan(e + f*x)**2*i + tan(e + f*x) + i),x)*b*f + 15*tan(e + f*x)**6*b*e*i
+ 15*tan(e + f*x)**6*b*f*i*x + 15*tan(e + f*x)**5*b*i + 45*tan(e + f*x)...
```

3.738 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 287

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}\,dx = \frac{7(4A+iB)x}{128a^3c^5}+\frac{A+iB}{192a^3c^5f(i-\tan(e+fx))^3}-\frac{3iA-2B}{128a^3c^5f(i-\tan(e+fx))^2} - \frac{3(7A+3iB)}{256a^3c^5f(i-\tan(e+fx))}+\frac{A-iB}{80a^3c^5f(i+\tan(e+fx))^5} - \frac{2iA+B}{64a^3c^5f(i+\tan(e+fx))^4}-\frac{5A-iB}{96a^3c^5f(i+\tan(e+fx))^3} + \frac{5iA}{64a^3c^5f(i+\tan(e+fx))^2}+\frac{5(7A+iB)}{256a^3c^5f(i+\tan(e+fx))}$$

output
```
7/128*(4*A+I*B)*x/a^3/c^5+1/192*(A+I*B)/a^3/c^5/f/(I-tan(f*x+e))^3-1/128*(
3*I*A-2*B)/a^3/c^5/f/(I-tan(f*x+e))^2-3/256*(7*A+3*I*B)/a^3/c^5/f/(I-tan(f
*x+e))+1/80*(A-I*B)/a^3/c^5/f/(I+tan(f*x+e))^5-1/64*(2*I*A+B)/a^3/c^5/f/(I
+tan(f*x+e))^4-1/96*(5*A-I*B)/a^3/c^5/f/(I+tan(f*x+e))^3+5/64*I*A/a^3/c^5/
f/(I+tan(f*x+e))^2+5/256*(7*A+I*B)/a^3/c^5/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 6.13 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.91

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{\sec^7(e+fx)(2100iA\cos(e+fx)+63(-8iA+7B)\cos(3(e+fx))-56iA\cos(5(e+fx))+119B\cos(5}{}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e +
f*x])^5),x]
```

output
```
(Sec[e + f*x]^7*((2100*I)*A*Cos[e + f*x] + 63*((-8*I)*A + 7*B)*Cos[3*(e +
f*x)] - (56*I)*A*Cos[5*(e + f*x)] + 119*B*Cos[5*(e + f*x)] - (4*I)*A*Cos[7
*(e + f*x)] + 16*B*Cos[7*(e + f*x)] + 872*A*Sin[e + f*x] + (218*I)*B*Sin[e
 + f*x] - 420*(4*A + I*B)*ArcTan[Tan[e + f*x]]*Sec[e + f*x]*(Cos[2*(e + f*
x)] - I*Sin[2*(e + f*x)]) - 956*A*Sin[3*(e + f*x)] - (239*I)*B*Sin[3*(e +
f*x)] - 164*A*Sin[5*(e + f*x)] - (41*I)*B*Sin[5*(e + f*x)] - 16*A*Sin[7*(e
 + f*x)] - (4*I)*B*Sin[7*(e + f*x)]))/(7680*a^3*c^5*f*(-I + Tan[e + f*x])^
3*(I + Tan[e + f*x])^5)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}dx$$
$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4c^6(1-i\tan(e+fx))^6(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^6(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3c^5f}$$

$\Big\downarrow$ 86

$$\frac{\int \left(-\frac{5iA}{32(\tan(e+fx)+i)^3}+\frac{7(4A+iB)}{128(\tan^2(e+fx)+1)}-\frac{3(7A+3iB)}{256(\tan(e+fx)-i)^2}-\frac{5(7A+iB)}{256(\tan(e+fx)+i)^2}+\frac{i(3A+2iB)}{64(\tan(e+fx)-i)^3}+\frac{A+iB}{64(\tan(e+fx)-i)}\right.}{a^3c^5f}$$

$\Big\downarrow$ 2009

$$\frac{\frac{7}{128}(4A+iB)\arctan(\tan(e+fx))-\frac{3(7A+3iB)}{256(-\tan(e+fx)+i)}+\frac{5(7A+iB)}{256(\tan(e+fx)+i)}-\frac{-2B+3iA}{128(-\tan(e+fx)+i)^2}+\frac{A+iB}{192(-\tan(e+fx)+i)}}{a^3c^5f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^
5),x]
```

output
```
((7*(4*A + I*B)*ArcTan[Tan[e + f*x]])/128 + (A + I*B)/(192*(I - Tan[e + f*
x])^3) - ((3*I)*A - 2*B)/(128*(I - Tan[e + f*x])^2) - (3*(7*A + (3*I)*B))/
(256*(I - Tan[e + f*x])) + (A - I*B)/(80*(I + Tan[e + f*x])^5) - ((2*I)*A
+ B)/(64*(I + Tan[e + f*x])^4) - (5*A - I*B)/(96*(I + Tan[e + f*x])^3) + (
((5*I)/64)*A)/(I + Tan[e + f*x])^2 + (5*(7*A + I*B))/(256*(I + Tan[e + f*x
])))/(a^3*c^5*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.17

method	result
norman	$\frac{\frac{7(iB+4A)x}{128ac}-\frac{8iA+3B}{40acf}+\frac{B\tan(fx+e)^2}{8acf}+\frac{79(iB+4A)\tan(fx+e)^3}{192acf}+\frac{7(iB+4A)\tan(fx+e)^5}{15acf}+\frac{49(iB+4A)\tan(fx+e)^7}{192acf}+}{}$
risch	$-\frac{i e^{8i(fx+e)}A}{256a^3c^5f}+\frac{7xA}{32a^3c^5}-\frac{e^{10i(fx+e)}B}{2560a^3c^5f}-\frac{21i\cos(2fx+2e)A}{256a^3c^5f}-\frac{3 e^{8i(fx+e)}B}{1024a^3c^5f}-\frac{13i\sin(6fx+6e)B}{1536a^3c^5f}-$
derivativedivides	$\frac{5iB}{256f a^3c^5(i+\tan(fx+e))}-\frac{A}{192f a^3c^5(-i+\tan(fx+e))^3}-\frac{3iA}{128f a^3c^5(-i+\tan(fx+e))^2}+$
default	$\frac{5iB}{256f a^3c^5(i+\tan(fx+e))}-\frac{A}{192f a^3c^5(-i+\tan(fx+e))^3}-\frac{3iA}{128f a^3c^5(-i+\tan(fx+e))^2}+$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^5,x,method=_R
ETURNVERBOSE)
```

output
```
(7/128*(4*A+I*B)/a/c*x-1/40*(3*B+8*I*A)/a/c/f+1/8/a/c/f*B*tan(f*x+e)^2+79/
192*(4*A+I*B)/a/c/f*tan(f*x+e)^3+7/15*(4*A+I*B)/a/c/f*tan(f*x+e)^5+49/192*
(4*A+I*B)/a/c/f*tan(f*x+e)^7+7/128*(4*A+I*B)/a/c/f*tan(f*x+e)^9+35/128*(4*
A+I*B)/a/c*x*tan(f*x+e)^2+35/64*(4*A+I*B)/a/c*x*tan(f*x+e)^4+35/64*(4*A+I*
B)/a/c*x*tan(f*x+e)^6+35/128*(4*A+I*B)/a/c*x*tan(f*x+e)^8+7/128*(4*A+I*B)/
a/c*x*tan(f*x+e)^10+1/128*(100*A-7*I*B)/a/c/f*tan(f*x+e))/(1+tan(f*x+e)^2)
^5/a^2/c^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.55

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{(840\,(4\,A+i\,B)fxe^{(6i\,fx+6i\,e)}-6\,(i\,A+B)e^{(16i\,fx+16i\,e)}-15\,(4i\,A+3\,B)e^{(14i\,fx+14i\,e)}-140\,(2i\,A+B)e}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^5,x, al
gorithm="fricas")
```

output
```
1/15360*(840*(4*A + I*B)*f*x*e^(6*I*f*x + 6*I*e) - 6*(I*A + B)*e^(16*I*f*x
 + 16*I*e) - 15*(4*I*A + 3*B)*e^(14*I*f*x + 14*I*e) - 140*(2*I*A + B)*e^(1
2*I*f*x + 12*I*e) - 210*(4*I*A + B)*e^(10*I*f*x + 10*I*e) - 2100*I*A*e^(8*
I*f*x + 8*I*e) - 420*(-2*I*A + B)*e^(4*I*f*x + 4*I*e) - 30*(-4*I*A + 3*B)*
e^(2*I*f*x + 2*I*e) + 10*I*A - 10*B)*e^(-6*I*f*x - 6*I*e)/(a^3*c^5*f)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 646, normalized size of antiderivative = 2.25

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}\,dx$$
$$= \begin{cases} \frac{(-726340547902313594880 0iAa^{21}c^{35}f^7e^{14ie}e^{2ifx}+(345876451382054092 80iAa^{21}c^{35}f^7e^{6ie}-345876451382054092 80Ba^{21}c^{35}f^7e^{6ie})e^{-6if}}{} \\ x\left(-\frac{28A+7iB}{128a^3c^5}+\frac{(Ae^{16ie}+8Ae^{14ie}+28Ae^{12ie}+56Ae^{10ie}+70Ae^{8ie}+56Ae^{6ie}+28Ae^{4ie}+8Ae^{2ie}+A-iBe^{16ie}-6iBe^{14ie}-14iBe^{12ie}-14i}{256a^3c^5}\right. \end{cases}$$
$$+\frac{x(28A+7iB)}{128a^3c^5}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**5,x)
```

output
```
Piecewise(((-7263405479023135948800*I*A*a**21*c**35*f**7*exp(14*I*e)*exp(2
*I*f*x) + (345876451382054092 80*I*A*a**21*c**35*f**7*exp(6*I*e) - 34587645
138205409280*B*a**21*c**35*f**7*exp(6*I*e))*exp(-6*I*f*x) + (4150517416584
64911360*I*A*a**21*c**35*f**7*exp(8*I*e) - 311288806243848683520*B*a**21*c
**35*f**7*exp(8*I*e))*exp(-4*I*f*x) + (290536219160925437952 0*I*A*a**21*c*
*35*f**7*exp(10*I*e) - 1452681095804627189760*B*a**21*c**35*f**7*exp(10*I*
e))*exp(-2*I*f*x) + (-2905362191609254379520*I*A*a**21*c**35*f**7*exp(16*I
*e) - 726340547902313594880*B*a**21*c**35*f**7*exp(16*I*e))*exp(4*I*f*x) +
 (-968454063869751459840*I*A*a**21*c**35*f**7*exp(18*I*e) - 48422703193487
5729920*B*a**21*c**35*f**7*exp(18*I*e))*exp(6*I*f*x) + (-20752587082923245
5680*I*A*a**21*c**35*f**7*exp(20*I*e) - 155644403121924341760*B*a**21*c**3
5*f**7*exp(20*I*e))*exp(8*I*f*x) + (-20752587082923245568*I*A*a**21*c**35*
f**7*exp(22*I*e) - 20752587082923245568*B*a**21*c**35*f**7*exp(22*I*e))*ex
p(10*I*f*x))*exp(-12*I*e)/(531266229322835086540 80*a**24*c**40*f**8), Ne(a
**24*c**40*f**8*exp(12*I*e), 0)), (x*(-(28*A + 7*I*B)/(128*a**3*c**5) + (A
*exp(16*I*e) + 8*A*exp(14*I*e) + 28*A*exp(12*I*e) + 56*A*exp(10*I*e) + 70*
A*exp(8*I*e) + 56*A*exp(6*I*e) + 28*A*exp(4*I*e) + 8*A*exp(2*I*e) + A - I*
B*exp(16*I*e) - 6*I*B*exp(14*I*e) - 14*I*B*exp(12*I*e) - 14*I*B*exp(10*I*e
) + 14*I*B*exp(6*I*e) + 14*I*B*exp(4*I*e) + 6*I*B*exp(2*I*e) + I*B)*exp(-6
*I*e)/(256*a**3*c**5)), True)) + x*(28*A + 7*I*B)/(128*a**3*c**5)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^5,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.70

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}\,dx$$
$$= -\frac{7\,(-4i\,A+B)\log\left(\tan\left(fx+e\right)+i\right)}{256\,a^3c^5f} - \frac{7\,(4i\,A-B)\log\left(\tan\left(fx+e\right)-i\right)}{256\,a^3c^5f}$$
$$+ \frac{105\,(4\,A+i\,B)\tan\left(fx+e\right)^7 - 210\,(-4i\,A+B)\tan\left(fx+e\right)^6 + 175\,(4\,A+i\,B)\tan\left(fx+e\right)^5 - 560}{1920}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^5,x, al
gorithm="giac")
```

output
```
-7/256*(-4*I*A + B)*log(tan(f*x + e) + I)/(a^3*c^5*f) - 7/256*(4*I*A - B)*
log(tan(f*x + e) - I)/(a^3*c^5*f) + 1/1920*(105*(4*A + I*B)*tan(f*x + e)^7
 - 210*(-4*I*A + B)*tan(f*x + e)^6 + 175*(4*A + I*B)*tan(f*x + e)^5 - 560*
(-4*I*A + B)*tan(f*x + e)^4 - 49*(4*A + I*B)*tan(f*x + e)^3 - 462*(-4*I*A
+ B)*tan(f*x + e)^2 - 183*(4*A + I*B)*tan(f*x + e) + 384*I*A + 144*B)/(a^3
*c^5*f*(tan(f*x + e) + I)^5*(tan(f*x + e) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.11

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{\frac{3B}{40a^3c^5}+\tan(e+fx)^4\left(-\frac{7B}{24a^3c^5}+\frac{A\,7\mathrm{i}}{6a^3c^5}\right)+\tan(e+fx)^6\left(-\frac{7B}{64a^3c^5}+\frac{A\,7\mathrm{i}}{16a^3c^5}\right)+\tan(e+fx)^7\left(\frac{7A}{32a^3c^5}+\right.}{f\left(\tan(e+fx)^8+\tan(e+fx)^7\,2\mathrm{i}+\right.}$$
$$+\frac{7x(4A+B\,1\mathrm{i})}{128a^3c^5}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i
)^5),x)
```

output
```
(tan(e + f*x)^4*((A*7i)/(6*a^3*c^5) - (7*B)/(24*a^3*c^5)) - tan(e + f*x)*(
(61*A)/(160*a^3*c^5) + (B*61i)/(640*a^3*c^5)) + tan(e + f*x)^6*((A*7i)/(16
*a^3*c^5) - (7*B)/(64*a^3*c^5)) + tan(e + f*x)^7*((7*A)/(32*a^3*c^5) + (B*
7i)/(128*a^3*c^5)) + tan(e + f*x)^5*((35*A)/(96*a^3*c^5) + (B*35i)/(384*a^
3*c^5)) + tan(e + f*x)^2*((A*77i)/(80*a^3*c^5) - (77*B)/(320*a^3*c^5)) - t
an(e + f*x)^3*((49*A)/(480*a^3*c^5) + (B*49i)/(1920*a^3*c^5)) + (A*1i)/(5*
a^3*c^5) + (3*B)/(40*a^3*c^5))/(f*(tan(e + f*x)*2i - 2*tan(e + f*x)^2 + ta
n(e + f*x)^3*6i + tan(e + f*x)^5*6i + 2*tan(e + f*x)^6 + tan(e + f*x)^7*2i
 + tan(e + f*x)^8 - 1)) + (7*x*(4*A + B*1i))/(128*a^3*c^5)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^5}\,dx$$
$$= \frac{-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^8+2\tan(fx+e)^7 i+2\tan(fx+e)^6+6\tan(fx+e)^5 i+6\tan(fx+e)^3 i-2\tan(fx+e)^2+2\tan(fx+e)i-1}dx\right)b-\left(\int \frac{}{\tan(fx+}\right.}{a^3c^5}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^5,x)
```

output
```
( - (int(tan(e + f*x)/(tan(e + f*x)**8 + 2*tan(e + f*x)**7*i + 2*tan(e + f
*x)**6 + 6*tan(e + f*x)**5*i + 6*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 + 2
*tan(e + f*x)*i - 1),x)*b + int(1/(tan(e + f*x)**8 + 2*tan(e + f*x)**7*i +
 2*tan(e + f*x)**6 + 6*tan(e + f*x)**5*i + 6*tan(e + f*x)**3*i - 2*tan(e +
 f*x)**2 + 2*tan(e + f*x)*i - 1),x)*a))/(a**3*c**5)
```

3.739 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 319

$$\begin{aligned}\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}\,dx &= \frac{7(3A+iB)x}{128a^3c^6}+\frac{A+iB}{384a^3c^6f(i-\tan(e+fx))^3} \\ &\quad -\frac{7iA-5B}{512a^3c^6f(i-\tan(e+fx))^2}-\frac{7(2A+iB)}{256a^3c^6f(i-\tan(e+fx))} \\ &\quad +\frac{iA+B}{96a^3c^6f(i+\tan(e+fx))^6}+\frac{2A-iB}{80a^3c^6f(i+\tan(e+fx))^5} \\ &\quad -\frac{5iA+B}{128a^3c^6f(i+\tan(e+fx))^4}-\frac{5A}{96a^3c^6f(i+\tan(e+fx))^3} \\ &\quad +\frac{5(7iA-B)}{512a^3c^6f(i+\tan(e+fx))^2}+\frac{7(4A+iB)}{256a^3c^6f(i+\tan(e+fx))}\end{aligned}$$

output
```
7/128*(3*A+I*B)*x/a^3/c^6+1/384*(A+I*B)/a^3/c^6/f/(I-tan(f*x+e))^3-1/512*(
7*I*A-5*B)/a^3/c^6/f/(I-tan(f*x+e))^2-7/256*(2*A+I*B)/a^3/c^6/f/(I-tan(f*x
+e))+1/96*(I*A+B)/a^3/c^6/f/(I+tan(f*x+e))^6+1/80*(2*A-I*B)/a^3/c^6/f/(I+t
an(f*x+e))^5-1/128*(5*I*A+B)/a^3/c^6/f/(I+tan(f*x+e))^4-5/96*A/a^3/c^6/f/(
I+tan(f*x+e))^3+5/512*(7*I*A-B)/a^3/c^6/f/(I+tan(f*x+e))^2+7/256*(4*A+I*B)
/a^3/c^6/f/(I+tan(f*x+e))
```

Mathematica [A] (verified)

Time = 6.41 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.87

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}\,dx$$
$$= \frac{\sec^8(e+fx)(-1271A+43iB-8(391A+37iB)\cos(2(e+fx))+(734A+618iB)\cos(4(e+fx))+76}{}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e +
f*x])^6),x]
```

output
```
(Sec[e + f*x]^8*(-1271*A + (43*I)*B - 8*(391*A + (37*I)*B)*Cos[2*(e + f*x)
] + (734*A + (618*I)*B)*Cos[4*(e + f*x)] + 76*A*Cos[6*(e + f*x)] + (132*I)
*B*Cos[6*(e + f*x)] + 5*A*Cos[8*(e + f*x)] + (15*I)*B*Cos[8*(e + f*x)] + (
1890*I)*A*Sin[2*(e + f*x)] - 630*B*Sin[2*(e + f*x)] + 840*((-3*I)*A + B)*A
rcTan[Tan[e + f*x]]*Sec[e + f*x]*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)]) -
 (1176*I)*A*Sin[4*(e + f*x)] + 392*B*Sin[4*(e + f*x)] - (174*I)*A*Sin[6*(e
 + f*x)] + 58*B*Sin[6*(e + f*x)] - (15*I)*A*Sin[8*(e + f*x)] + 5*B*Sin[8*(
e + f*x)]))/(15360*a^3*c^6*f*(-I + Tan[e + f*x])^3*(I + Tan[e + f*x])^6)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}dx$$
$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4c^7(1-i\tan(e+fx))^7(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{A+B\tan(e+fx)}{(1-i\tan(e+fx))^7(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3c^6f}$$

$$\Big\downarrow \textbf{86}$$

$$\frac{\int \left(\frac{5A}{32(\tan(e+fx)+i)^4}+\frac{7(3A+iB)}{128(\tan^2(e+fx)+1)}-\frac{7(2A+iB)}{256(\tan(e+fx)-i)^2}-\frac{7(4A+iB)}{256(\tan(e+fx)+i)^2}+\frac{i(7A+5iB)}{256(\tan(e+fx)-i)^3}+\frac{5(B-7iA)}{256(\tan(e+fx)+i)}\right.}{a^3c^6f}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{7}{128}(3A+iB)\arctan(\tan(e+fx))-\frac{7(2A+iB)}{256(-\tan(e+fx)+i)}+\frac{7(4A+iB)}{256(\tan(e+fx)+i)}-\frac{-5B+7iA}{512(-\tan(e+fx)+i)^2}+\frac{5(-B+7iA)}{512(\tan(e+fx)+i)^2}}{a^3c^6f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^
6),x]
```

output
```
((7*(3*A + I*B)*ArcTan[Tan[e + f*x]])/128 + (A + I*B)/(384*(I - Tan[e + f*
x])^3) - ((7*I)*A - 5*B)/(512*(I - Tan[e + f*x])^2) - (7*(2*A + I*B))/(256
*(I - Tan[e + f*x])) + (I*A + B)/(96*(I + Tan[e + f*x])^6) + (2*A - I*B)/(
80*(I + Tan[e + f*x])^5) - ((5*I)*A + B)/(128*(I + Tan[e + f*x])^4) - (5*A
)/(96*(I + Tan[e + f*x])^3) + (5*((7*I)*A - B))/(512*(I + Tan[e + f*x])^2)
 + (7*(4*A + I*B))/(256*(I + Tan[e + f*x])))/(a^3*c^6*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.23

method	result
norman	$\frac{7(iB+3A)x}{128ac}-\frac{7iA+B}{30acf}+\frac{281(iB+3A)\tan(fx+e)^5}{320acf}+\frac{231(iB+3A)\tan(fx+e)^7}{320acf}+\frac{119(iB+3A)\tan(fx+e)^9}{384acf}+\frac{7(iB+3A)\tan(fx+e)^{11}}{128acf}$
risch	$-\frac{7i\sin(4fx+4e)B}{2048a^3c^6f}+\frac{21xA}{128a^3c^6}-\frac{e^{12i(fx+e)}B}{6144a^3c^6f}-\frac{9i e^{8i(fx+e)}A}{1024a^3c^6f}-\frac{7e^{10i(fx+e)}B}{5120a^3c^6f}-\frac{9i\sin(6fx+6e)B}{1024a^3c^6f}-\frac{5e^{8i}}{102}$
derivativedivides	$\frac{7A}{128f\,a^3c^6(-i+\tan(fx+e))}+\frac{A}{40f\,a^3c^6(i+\tan(fx+e))^5}-\frac{A}{384f\,a^3c^6(-i+\tan(fx+e))^3}+\frac{21A\arctan(\tan(fx+e)}{128f\,a^3c^6}$
default	$\frac{7A}{128f\,a^3c^6(-i+\tan(fx+e))}+\frac{A}{40f\,a^3c^6(i+\tan(fx+e))^5}-\frac{A}{384f\,a^3c^6(-i+\tan(fx+e))^3}+\frac{21A\arctan(\tan(fx+e)}{128f\,a^3c^6}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^6,x,method=_R
ETURNVERBOSE)
```

output
```
(7/128*(3*A+I*B)/a/c*x-1/30*(7*I*A+B)/a/c/f+281/320*(3*A+I*B)/a/c/f*tan(f*
x+e)^5+231/320*(3*A+I*B)/a/c/f*tan(f*x+e)^7+119/384*(3*A+I*B)/a/c/f*tan(f*
x+e)^9+7/128*(3*A+I*B)/a/c/f*tan(f*x+e)^11+21/64*(3*A+I*B)/a/c*x*tan(f*x+e
)^2+105/128*(3*A+I*B)/a/c*x*tan(f*x+e)^4+35/32*(3*A+I*B)/a/c*x*tan(f*x+e)^
6+105/128*(3*A+I*B)/a/c*x*tan(f*x+e)^8+21/64*(3*A+I*B)/a/c*x*tan(f*x+e)^10
+7/128*(3*A+I*B)/a/c*x*tan(f*x+e)^12+1/128*(107*A-7*I*B)/a/c/f*tan(f*x+e)+
1/384*(667*A+265*I*B)/a/c/f*tan(f*x+e)^3+1/10/a/c/f*(I*A+3*B)*tan(f*x+e)^2
)/(1+tan(f*x+e)^2)^6/a^2/c^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.58

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}\,dx$$
$$= \frac{(1680\,(3\,A+i\,B)fxe^{(6i\,fx+6i\,e)} - 5\,(i\,A+B)e^{(18i\,fx+18i\,e)} - 6\,(9i\,A+7\,B)e^{(16i\,fx+16i\,e)} - 30\,(9i\,A+5\,B)e}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^6,x, al
gorithm="fricas")
```

output
```
1/30720*(1680*(3*A + I*B)*f*x*e^(6*I*f*x + 6*I*e) - 5*(I*A + B)*e^(18*I*f*
x + 18*I*e) - 6*(9*I*A + 7*B)*e^(16*I*f*x + 16*I*e) - 30*(9*I*A + 5*B)*e^(
14*I*f*x + 14*I*e) - 280*(3*I*A + B)*e^(12*I*f*x + 12*I*e) - 210*(9*I*A +
B)*e^(10*I*f*x + 10*I*e) - 420*(9*I*A - B)*e^(8*I*f*x + 8*I*e) - 120*(-9*I
*A + 5*B)*e^(4*I*f*x + 4*I*e) - 15*(-9*I*A + 7*B)*e^(2*I*f*x + 2*I*e) + 10
*I*A - 10*B)*e^(-6*I*f*x - 6*I*e)/(a^3*c^6*f)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.36

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**6,x)
```

output
```
Piecewise((((68002077353322891077222240*I*A*a**24*c**48*f**8*exp(6*I*e) - 6
8002077353322891077222240*B*a**24*c**48*f**8*exp(6*I*e))*exp(-6*I*f*x) + (9
18028044269859029542502 40*I*A*a**24*c**48*f**8*exp(8*I*e) - 71402181220989
035631083520*B*a**24*c**48*f**8*exp(8*I*e))*exp(-4*I*f*x) + (7344224354158
87223634001920*I*A*a**24*c**48*f**8*exp(10*I*e) - 408012464119937346463334
400*B*a**24*c**48*f**8*exp(10*I*e))*exp(-2*I*f*x) + (-257047852395560528 27
19006720*I*A*a**24*c**48*f**8*exp(14*I*e) + 285608724883956142524334080*B*
a**24*c**48*f**8*exp(14*I*e))*exp(2*I*f*x) + (-128523926197780264135950336
0*I*A*a**24*c**48*f**8*exp(16*I*e) - 142804362441978071262167040*B*a**24*c
**48*f**8*exp(16*I*e))*exp(4*I*f*x) + (-5712174497679122850486681 60*I*A*a*
*24*c**48*f**8*exp(18*I*e) - 190405816589304095016222720*B*a**24*c**48*f**
8*exp(18*I*e))*exp(6*I*f*x) + (-183605608853971805908500480*I*A*a**24*c**4
8*f**8*exp(20*I*e) - 102003116029984336615833600*B*a**24*c**48*f**8*exp(20
*I*e))*exp(8*I*f*x) + (-36721121770794361181700096*I*A*a**24*c**48*f**8*ex
p(22*I*e) - 28560872488395614252433408*B*a**24*c**48*f**8*exp(22*I*e))*exp
(10*I*f*x) + (-3400103867666144553861120*I*A*a**24*c**48*f**8*exp(24*I*e)
- 3400103867666144553861120*B*a**24*c**48*f**8*exp(24*I*e))*exp(12*I*f*x))
*exp(-12*I*e)/(2089023816294079213892272l280*a**27*c**54*f**9), Ne(a**27*c
**54*f**9*exp(12*I*e), 0)), (x*(-(21*A + 7*I*B)/(128*a**3*c**6) + (A*exp(1
8*I*e) + 9*A*exp(16*I*e) + 36*A*exp(14*I*e) + 84*A*exp(12*I*e) + 126*A*...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^6,x, al
gorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.69

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}\,dx$$
$$= -\frac{7(-3iA+B)\log(\tan(fx+e)+i)}{256a^3c^6f} - \frac{7(3iA-B)\log(\tan(fx+e)-i)}{256a^3c^6f}$$
$$+\frac{105(3A+iB)\tan(fx+e)^8 - 315(-3iA+B)\tan(fx+e)^7 - 35(3A+iB)\tan(fx+e)^6 - 735(\cdot}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^6,x, al
gorithm="giac")
```

output
```
-7/256*(-3*I*A + B)*log(tan(f*x + e) + I)/(a^3*c^6*f) - 7/256*(3*I*A - B)*
log(tan(f*x + e) - I)/(a^3*c^6*f) + 1/1920*(105*(3*A + I*B)*tan(f*x + e)^8
 - 315*(-3*I*A + B)*tan(f*x + e)^7 - 35*(3*A + I*B)*tan(f*x + e)^6 - 735*(
-3*I*A + B)*tan(f*x + e)^5 - 609*(3*A + I*B)*tan(f*x + e)^4 - 413*(-3*I*A
+ B)*tan(f*x + e)^3 - 645*(3*A + I*B)*tan(f*x + e)^2 - 87*(3*I*A - B)*tan(
f*x + e) - 448*A + 64*I*B)/(a^3*c^6*f*(tan(f*x + e) + I)^6*(tan(f*x + e) -
 I)^3)
```

Mupad [B] (verification not implemented)

Time = 7.69 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.10

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}\,dx$$
$$= \frac{\tan(e+fx)\left(-\frac{29B}{640a^3c^6}+\frac{A\,87\mathrm{i}}{640a^3c^6}\right) - \tan(e+fx)^8\left(\frac{21A}{128a^3c^6}+\frac{B\,7\mathrm{i}}{128a^3c^6}\right) - \tan(e+fx)^7\left(-\frac{21B}{128a^3c^6}+\frac{A\,6}{128a}\right.}{f\,(-\tan(e+fx}$$
$$+\frac{7x(3A+B\,1\mathrm{i})}{128a^3c^6}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i
)^6),x)
```

output
```
(tan(e + f*x)*((A*87i)/(640*a^3*c^6) - (29*B)/(640*a^3*c^6)) - tan(e + f*x
)^8*((21*A)/(128*a^3*c^6) + (B*7i)/(128*a^3*c^6)) - tan(e + f*x)^7*((A*63i
)/(128*a^3*c^6) - (21*B)/(128*a^3*c^6)) + tan(e + f*x)^2*((129*A)/(128*a^3
*c^6) + (B*43i)/(128*a^3*c^6)) - tan(e + f*x)^5*((A*147i)/(128*a^3*c^6) -
(49*B)/(128*a^3*c^6)) + tan(e + f*x)^6*((7*A)/(128*a^3*c^6) + (B*7i)/(384*
a^3*c^6)) + tan(e + f*x)^4*((609*A)/(640*a^3*c^6) + (B*203i)/(640*a^3*c^6)
) - tan(e + f*x)^3*((A*413i)/(640*a^3*c^6) - (413*B)/(1920*a^3*c^6)) + (7*
A)/(30*a^3*c^6) - (B*1i)/(30*a^3*c^6))/(f*(3*tan(e + f*x) + 8*tan(e + f*x)
^3 - tan(e + f*x)^4*6i + 6*tan(e + f*x)^5 - tan(e + f*x)^6*8i - tan(e + f*
x)^8*3i - tan(e + f*x)^9 + 1i)) + (7*x*(3*A + B*1i))/(128*a^3*c^6)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^6}\,dx$$
$$= \frac{\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^9 i-3\tan(fx+e)^8-8\tan(fx+e)^6-6\tan(fx+e)^5 i-6\tan(fx+e)^4-8\tan(fx+e)^3 i-3\tan(fx+e) i+1}dx\right)b+\left(\int \frac{}{\tan(fx+e)^9\cdots}\right.}{a^3c^6}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^6,x)
```

output
```
(int(tan(e + f*x)/(tan(e + f*x)**9*i - 3*tan(e + f*x)**8 - 8*tan(e + f*x)*
*6 - 6*tan(e + f*x)**5*i - 6*tan(e + f*x)**4 - 8*tan(e + f*x)**3*i - 3*tan
(e + f*x)*i + 1),x)*b + int(1/(tan(e + f*x)**9*i - 3*tan(e + f*x)**8 - 8*t
an(e + f*x)**6 - 6*tan(e + f*x)**5*i - 6*tan(e + f*x)**4 - 8*tan(e + f*x)*
*3*i - 3*tan(e + f*x)*i + 1),x)*a)/(a**3*c**6)
```

3.740 $\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 62

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{2a(iA+B)(c-ic\tan(e+fx))^{7/2}}{7f} - \frac{2aB(c-ic\tan(e+fx))^{9/2}}{9cf}$$

output
```
2/7*a*(I*A+B)*(c-I*c*tan(f*x+e))^(7/2)/f-2/9*a*B*(c-I*c*tan(f*x+e))^(9/2)/
c/f
```

Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = -\frac{2ac^3(i+\tan(e+fx))^3(9A-2iB+7B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{63f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(7/2),x]
```

output
```
(-2*a*c^3*(I + Tan[e + f*x])^3*(9*A - (2*I)*B + 7*B*Tan[e + f*x])*Sqrt[c -
 I*c*Tan[e + f*x]])/(63*f)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {3042, 4071, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int (A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{53}$$

$$\frac{ac\int \left(\frac{iB(c-ic\tan(e+fx))^{7/2}}{c}+(A-iB)(c-ic\tan(e+fx))^{5/2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac\left(\frac{2(B+iA)(c-ic\tan(e+fx))^{7/2}}{7c}-\frac{2B(c-ic\tan(e+fx))^{9/2}}{9c^2}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/
2),x]
```

output
```
(a*c*((2*(I*A + B)*(c - I*c*Tan[e + f*x])^(7/2))/(7*c) - (2*B*(c - I*c*Tan
[e + f*x])^(9/2))/(9*c^2)))/f
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2ia\left(\frac{iB(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{(-iBc+cA)(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}\right)}{fc}$
default	$\frac{2ia\left(\frac{iB(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{(-iBc+cA)(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}\right)}{fc}$
parts	$\frac{2iAac\left(-\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-4\sqrt{c-ic\tan(fx+e)}\,c^{2}+4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{f}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f*a/c*(1/9*I*B*(c-I*c*tan(f*x+e))^(9/2)+1/7*(-I*B*c+c*A)*(c-I*c*tan(f*
x+e))^(7/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(48) = 96$.

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.74

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = -\frac{16\sqrt{2}\left(9\,(-i\,A-B)ac^3e^{(2i\,fx+2i\,e)}+(-9i\,A+5\,B)ac^3\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{63\left(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="fricas")
```

output
```
-16/63*sqrt(2)*(9*(-I*A - B)*a*c^3*e^(2*I*f*x + 2*I*e) + (-9*I*A + 5*B)*a*
c^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I
*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = ia\left(\int \left(-iAc^3\sqrt{-ic\tan(e+fx)+c}\right)dx + \int \left(-2Ac^3\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\right)dx + \int \left(-2Ac^3\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)\right)dx + \int \left(-2Bc^3\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)\right)dx + \int \left(-2Bc^3\sqrt{-ic\tan(e+fx)+c}\tan^4(e+fx)\right)dx + \int iAc^3\sqrt{-ic\tan(e+fx)+c}\tan^4(e+fx)\,dx + \int \left(-iBc^3\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\right)dx + \int iBc^3\sqrt{-ic\tan(e+fx)+c}\tan^5(e+fx)\,dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2),x)
```

output
```
I*a*(Integral(-I*A*c**3*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-2*A*c*
*3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(-2*A*c**3*sqrt(
-I*c*tan(e + f*x) + c)*tan(e + f*x)**3, x) + Integral(-2*B*c**3*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(-2*B*c**3*sqrt(-I*c*tan(e
 + f*x) + c)*tan(e + f*x)**4, x) + Integral(I*A*c**3*sqrt(-I*c*tan(e + f*x
) + c)*tan(e + f*x)**4, x) + Integral(-I*B*c**3*sqrt(-I*c*tan(e + f*x) + c
)*tan(e + f*x), x) + Integral(I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
 f*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{2i\left(7i\,(-i\,c\tan(fx+e)+c)^{\frac{9}{2}}Ba+9\,(-i\,c\tan(fx+e)+c)^{\frac{7}{2}}(A-i\,B)ac\right)}{63\,cf}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="maxima")
```

output
```
2/63*I*(7*I*(-I*c*tan(f*x + e) + c)^(9/2)*B*a + 9*(-I*c*tan(f*x + e) + c)^
(7/2)*(A - I*B)*a*c)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{16\,a\,c^3\,\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,\left(A\,9\mathrm{i}-5\,B+A\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,9\mathrm{i}+9\,B\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)}{63\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(
7/2),x)
```

output
```
(16*a*c^3*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1
))^(1/2)*(A*9i - 5*B + A*exp(e*2i + f*x*2i)*9i + 9*B*exp(e*2i + f*x*2i)))/
(63*f*(exp(e*2i + f*x*2i) + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.71

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{2\sqrt{c}\,\sqrt{-\tan\left(fx+e\right)i+1}\,a\,c^3\left(-7\tan\left(fx+e\right)^4b-9\tan\left(fx+e\right)^3a-19\tan\left(fx+e\right)^3bi\right.}{63}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(2*sqrt(c)*sqrt( - tan(e + f*x)*i + 1)*a*c**3*( - 7*tan(e + f*x)**4*b - 9*
tan(e + f*x)**3*a - 19*tan(e + f*x)**3*b*i - 27*tan(e + f*x)**2*a*i + 15*t
an(e + f*x)**2*b + 27*tan(e + f*x)*a + tan(e + f*x)*b*i + 9*a*i + 2*b))/(6
3*f)
```

3.741 $\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 62

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\frac{2a(iA+B)(c-ic\tan(e+fx))^{5/2}}{5f}-\frac{2aB(c-ic\tan(e+fx))^{7/2}}{7cf}$$

output
```
2/5*a*(I*A+B)*(c-I*c*tan(f*x+e))^(5/2)/f-2/7*a*B*(c-I*c*tan(f*x+e))^(7/2)/
c/f
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=-\frac{2iac^2(i+\tan(e+fx))^2(7A-2iB+5B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{35f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(5/2),x]
```

output
```
(((-2*I)/35)*a*c^2*(I + Tan[e + f*x])^2*(7*A - (2*I)*B + 5*B*Tan[e + f*x])
*Sqrt[c - I*c*Tan[e + f*x]])/f
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {3042, 4071, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int (A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{53}$$

$$\frac{ac\int \left(\frac{iB(c-ic\tan(e+fx))^{5/2}}{c}+(A-iB)(c-ic\tan(e+fx))^{3/2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac\left(\frac{2(B+iA)(c-ic\tan(e+fx))^{5/2}}{5c}-\frac{2B(c-ic\tan(e+fx))^{7/2}}{7c^2}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/
2),x]
```

output
```
(a*c*((2*(I*A + B)*(c - I*c*Tan[e + f*x])^(5/2))/(5*c) - (2*B*(c - I*c*Tan
[e + f*x])^(7/2))/(7*c^2)))/f
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2ia\left(\frac{iB(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{(-iBc+cA)(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}\right)}{fc}$
default	$\frac{2ia\left(\frac{iB(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{(-iBc+cA)(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}\right)}{fc}$
parts	$\frac{2iAac\left(-\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-2c\sqrt{c-ic\tan(fx+e)}+2c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{f}+\frac{a(iA+B)\left(\frac{2(c-ic\,\mathrm{ta}}{}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f*a/c*(1/7*I*B*(c-I*c*tan(f*x+e))^(7/2)+1/5*(-I*B*c+c*A)*(c-I*c*tan(f*
x+e))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = -\frac{8\sqrt{2}\left(7\left(-iA-B\right)ac^2e^{(2i\,fx+2i\,e)}+\left(-7i\,A+3\,B\right)ac^2\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{35\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="fricas")
```

output
```
-8/35*sqrt(2)*(7*(-I*A - B)*a*c^2*e^(2*I*f*x + 2*I*e) + (-7*I*A + 3*B)*a*c
^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*
f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = ia\left(\int\left(-iAc^2\sqrt{-ic\tan{(e+fx)}+c}\right)dx + \int\left(-Ac^2\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\right)dx + \int\left(-Ac^2\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}\right)dx + \int\left(-Bc^2\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\right)dx + \int\left(-Bc^2\sqrt{-ic\tan{(e+fx)}+c}\tan^4{(e+fx)}\right)dx + \int\left(-iAc^2\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\right)dx + \int\left(-iBc^2\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\right)dx + \int\left(-iBc^2\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2),x)
```

output
```
I*a*(Integral(-I*A*c**2*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-A*c**2
*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(-A*c**2*sqrt(-I*c
*tan(e + f*x) + c)*tan(e + f*x)**3, x) + Integral(-B*c**2*sqrt(-I*c*tan(e
+ f*x) + c)*tan(e + f*x)**2, x) + Integral(-B*c**2*sqrt(-I*c*tan(e + f*x)
+ c)*tan(e + f*x)**4, x) + Integral(-I*A*c**2*sqrt(-I*c*tan(e + f*x) + c)*
tan(e + f*x)**2, x) + Integral(-I*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e
 + f*x), x) + Integral(-I*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)*
*3, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\frac{2i\left(5i\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{7}{2}}Ba+7\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}(A-i\,B)ac\right)}{35\,cf}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="maxima")
```

output
```
2/35*I*(5*I*(-I*c*tan(f*x + e) + c)^(7/2)*B*a + 7*(-I*c*tan(f*x + e) + c)^
(5/2)*(A - I*B)*a*c)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 10.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int (a + ia\tan(e + fx))(A + B\tan(e + fx))(c - ic\tan(e + fx))^{5/2}\,dx = \frac{8\,a\,c^2\,\sqrt{c + \frac{c\,(\mathrm{e}^{e\,2\mathrm{i} + f\,x\,2\mathrm{i}}\,1\mathrm{i} - \mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i} + f\,x\,2\mathrm{i}} + 1}}\,\left(A\,7\mathrm{i} - 3\,B + A\,\mathrm{e}^{e\,2\mathrm{i} + f\,x\,2\mathrm{i}}\,7\mathrm{i} + 7\,B\,\mathrm{e}^{e\,2\mathrm{i} + f\,x\,2\mathrm{i}}\right)}{35\,f\,\left(\mathrm{e}^{e\,2\mathrm{i} + f\,x\,2\mathrm{i}} + 1\right)^3}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(
5/2),x)
```

output
```
(8*a*c^2*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1)
)^(1/2)*(A*7i - 3*B + A*exp(e*2i + f*x*2i)*7i + 7*B*exp(e*2i + f*x*2i)))/(
35*f*(exp(e*2i + f*x*2i) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.35

$$\int (a + ia\tan(e + fx))(A + B\tan(e + fx))(c - ic\tan(e + fx))^{5/2}\,dx = \frac{2\sqrt{c}\,\sqrt{-\tan(fx+e)\,i + 1}\,a\,c^2\left(-5\tan(fx+e)^3\,bi - 7\tan(fx+e)^2\,ai + 8\tan(fx+e)^2\,b\right.}{35f}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt( - tan(e + f*x)*i + 1)*a*c**2*( - 5*tan(e + f*x)**3*b*i -
7*tan(e + f*x)**2*a*i + 8*tan(e + f*x)**2*b + 14*tan(e + f*x)*a + tan(e +
f*x)*b*i + 7*a*i + 2*b))/(35*f)
```

3.742 $\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 62

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\frac{2a(iA+B)(c-ic\tan(e+fx))^{3/2}}{3f}-\frac{2aB(c-ic\tan(e+fx))^{5/2}}{5cf}$$

output
```
2/3*a*(I*A+B)*(c-I*c*tan(f*x+e))^(3/2)/f-2/5*a*B*(c-I*c*tan(f*x+e))^(5/2)/
c/f
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\frac{2ac(i+\tan(e+fx))(5A-2iB+3B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{15f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(3/2),x]
```

output
```
(2*a*c*(I + Tan[e + f*x])*(5*A - (2*I)*B + 3*B*Tan[e + f*x])*Sqrt[c - I*c*
Tan[e + f*x]])/(15*f)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {3042, 4071, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int (A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{53}$$

$$\frac{ac\int \left(\frac{iB(c-ic\tan(e+fx))^{3/2}}{c}+(A-iB)\sqrt{c-ic\tan(e+fx)}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac\left(\frac{2(B+iA)(c-ic\tan(e+fx))^{3/2}}{3c}-\frac{2B(c-ic\tan(e+fx))^{5/2}}{5c^2}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/
2),x]
```

output
```
(a*c*((2*(I*A + B)*(c - I*c*Tan[e + f*x])^(3/2))/(3*c) - (2*B*(c - I*c*Tan
[e + f*x])^(5/2))/(5*c^2)))/f
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2ia\left(\frac{iB(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{(-iBc+cA)(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}\right)}{fc}$
default	$\frac{2ia\left(\frac{iB(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{(-iBc+cA)(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}\right)}{fc}$
parts	$\frac{2iAac\left(-\sqrt{c-ic\tan(fx+e)}+\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{f}+\frac{a(iA+B)\left(\frac{2(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+2c\sqrt{c-ic\,t}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f*a/c*(1/5*I*B*(c-I*c*tan(f*x+e))^(5/2)+1/3*(-I*B*c+c*A)*(c-I*c*tan(f*
x+e))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = -\frac{4\sqrt{2}\left(5\,(-i\,A-B)ace^{(2i\,fx+2i\,e)}+(-5i\,A+B)ac\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{15\,\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="fricas")
```

output
```
-4/15*sqrt(2)*(5*(-I*A - B)*a*c*e^(2*I*f*x + 2*I*e) + (-5*I*A + B)*a*c)*sq
rt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x +
2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = ia\left(\int\left(-iAc\sqrt{-ic\tan(e+fx)+c}\right)dx + \int\left(-iAc\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)\right)dx + \int\left(-iBc\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\right)dx + \int\left(-iBc\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2),x)
```

output
```
I*a*(Integral(-I*A*c*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-I*A*c*sqr
t(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(-I*B*c*sqrt(-I*c*t
an(e + f*x) + c)*tan(e + f*x), x) + Integral(-I*B*c*sqrt(-I*c*tan(e + f*x)
 + c)*tan(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{2i\left(3i\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}Ba+5\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}(A-i\,B)ac\right)}{15\,cf}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="maxima")
```

output
```
2/15*I*(3*I*(-I*c*tan(f*x + e) + c)^(5/2)*B*a + 5*(-I*c*tan(f*x + e) + c)^
(3/2)*(A - I*B)*a*c)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.96 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.60

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\frac{4\,a\,c\,\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,\left(A\,5\mathrm{i}-B+A\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,5\mathrm{i}+5\,B\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right)}{15\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(
3/2),x)
```

output
```
(4*a*c*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^
(1/2)*(A*5i - B + A*exp(e*2i + f*x*2i)*5i + 5*B*exp(e*2i + f*x*2i)))/(15*f
*(exp(e*2i + f*x*2i) + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\frac{2\sqrt{c}\,\sqrt{-\tan(fx+e)\,i+1}\,ac\big(3\tan(fx+e)^2\,b+5\tan(fx+e)\,a+\tan(fx+e)\,bi+5ai+$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt( - tan(e + f*x)*i + 1)*a*c*(3*tan(e + f*x)**2*b + 5*tan(e
+ f*x)*a + tan(e + f*x)*b*i + 5*a*i + 2*b))/(15*f)
```

3.743 $\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 60

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \frac{2a(iA+B)\sqrt{c-ic\tan(e+fx)}}{f}-\frac{2aB(c-ic\tan(e+fx))^{3/2}}{3cf}$$

output
```
2*a*(I*A+B)*(c-I*c*tan(f*x+e))^(1/2)/f-2/3*a*B*(c-I*c*tan(f*x+e))^(3/2)/c/
f
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \frac{2a(3iA+2B+iB\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{3f}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e +
 f*x]],x]
```

output
```
(2*a*((3*I)*A + 2*B + I*B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(3*f)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {3042, 4071, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{53}$$

$$\frac{ac\int \left(\frac{A-iB}{\sqrt{c-ic\tan(e+fx)}}+\frac{iB\sqrt{c-ic\tan(e+fx)}}{c}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac\left(\frac{2(B+iA)\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2B(c-ic\tan(e+fx))^{3/2}}{3c^2}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]
,x]
```

output
```
(a*c*((2*(I*A + B)*Sqrt[c - I*c*Tan[e + f*x]])/c - (2*B*(c - I*c*Tan[e + f
*x])^(3/2))/(3*c^2)))/f
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{2ia\left(\frac{iB(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-i\sqrt{c-ic\tan(fx+e)}\,Bc+\sqrt{c-ic\tan(fx+e)}\,cA\right)}{fc}$
default	$\frac{2ia\left(\frac{iB(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-i\sqrt{c-ic\tan(fx+e)}\,Bc+\sqrt{c-ic\tan(fx+e)}\,cA\right)}{fc}$
parts	$\frac{iAa\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{f}+\frac{a(iA+B)\left(2\sqrt{c-ic\tan(fx+e)}-\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{f}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f*a/c*(1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)-I*(c-I*c*tan(f*x+e))^(1/2)*B*c
+(c-I*c*tan(f*x+e))^(1/2)*c*A)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = -\frac{2\sqrt{2}\left(3\,(-i\,A-B)ae^{(2i\,fx+2i\,e)}+(-3i\,A-B)a\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{3\left(fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x,
algorithm="fricas")
```

output
```
-2/3*sqrt(2)*(3*(-I*A - B)*a*e^(2*I*f*x + 2*I*e) + (-3*I*A - B)*a)*sqrt(c/
(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = ia\left(\int \left(-iA\sqrt{-ic\tan(e+fx)+c}\right)dx + \int A\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\,dx + \int B\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)\,dx + \int \left(-iB\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(1/2),x)
```

output
```
I*a*(Integral(-I*A*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(A*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x), x) + Integral(B*sqrt(-I*c*tan(e + f*x) + c
)*tan(e + f*x)**2, x) + Integral(-I*B*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
f*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int (a + ia\tan(e + fx))(A + B\tan(e + fx))\sqrt{c - ic\tan(e + fx)}\,dx = \frac{2i\left(i\left(-i\,c\tan\left(fx + e\right) + c\right)^{\frac{3}{2}}Ba + 3\sqrt{-i\,c\tan\left(fx + e\right) + c}(A - i\,B)ac\right)}{3\,cf}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x,
algorithm="maxima")
```

output
```
2/3*I*(I*(-I*c*tan(f*x + e) + c)^(3/2)*B*a + 3*sqrt(-I*c*tan(f*x + e) + c)
*(A - I*B)*a*c)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a + ia\tan(e + fx))(A + B\tan(e + fx))\sqrt{c - ic\tan(e + fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.70

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \frac{a\sqrt{-\frac{c\left(-2\cos(e+f\,x)^2+\sin(2\,e+2\,f\,x)\,1\mathrm{i}\right)}{2\cos(e+f\,x)^2}}\,\left(A\,3\mathrm{i}+2\,B+A\left(2\cos\left(e+f\,x\right)^2-1\right)\,3\mathrm{i}+2\,B\left(2\cos\left(e+f\,x\right)^2-1\right)\right.}{3\,f\cos\left(e+f\,x\right)^2}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^(
1/2),x)
```

output
```
(a*(-(c*(sin(2*e + 2*f*x)*1i - 2*cos(e + f*x)^2))/(2*cos(e + f*x)^2))^(1/2
)*(A*3i + 2*B + A*(2*cos(e + f*x)^2 - 1)*3i + 2*B*(2*cos(e + f*x)^2 - 1) +
 B*sin(2*e + 2*f*x)*1i))/(3*f*cos(e + f*x)^2)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \frac{\sqrt{c}\,a\left(2\sqrt{-\tan\left(fx+e\right)i+1}\,ai+\left(\int\sqrt{-\tan\left(fx+e\right)i+1}\,\tan\left(fx+e\right)^2dx\right)bfi+\left(\int\sqrt{-\tan\left(fx+\right.\right.\right.}{f}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*a*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int(sqrt( - tan(e + f*x)*i
 + 1)*tan(e + f*x)**2,x)*b*f*i + int(sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x),x)*b*f))/f
```

3.744 $\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 58

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{2a(iA+B)}{f\sqrt{c-ic\tan(e+fx)}}-\frac{2aB\sqrt{c-ic\tan(e+fx)}}{cf}$$

output
```
-2*a*(I*A+B)/f/(c-I*c*tan(f*x+e))^(1/2)-2*a*B*(c-I*c*tan(f*x+e))^(1/2)/c/f
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{2ia(-A+2iB+B\tan(e+fx))}{f\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e
 + f*x]],x]
```

output
```
((2*I)*a*(-A + (2*I)*B + B*Tan[e + f*x]))/(f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {3042, 4071, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{53}$$

$$\frac{ac\int \left(\frac{A-iB}{(c-ic\tan(e+fx))^{3/2}}+\frac{iB}{c\sqrt{c-ic\tan(e+fx)}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac\left(-\frac{2(B+iA)}{c\sqrt{c-ic\tan(e+fx)}}-\frac{2B\sqrt{c-ic\tan(e+fx)}}{c^2}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e + f*x
]],x]
```

output
```
(a*c*((-2*(I*A + B))/(c*Sqrt[c - I*c*Tan[e + f*x]]) - (2*B*Sqrt[c - I*c*Ta
n[e + f*x]])/c^2))/f
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{2ia\left(i\sqrt{c-ic\tan(fx+e)}\,B-\frac{c(-iB+A)}{\sqrt{c-ic\tan(fx+e)}}\right)}{fc}$
default	$\frac{2ia\left(i\sqrt{c-ic\tan(fx+e)}\,B-\frac{c(-iB+A)}{\sqrt{c-ic\tan(fx+e)}}\right)}{fc}$
parts	$\frac{2iAac\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}-\frac{1}{2c\sqrt{c-ic\tan(fx+e)}}\right)}{f}+\frac{a(iA+B)\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{2\sqrt{c}}-\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f*a/c*(I*(c-I*c*tan(f*x+e))^(1/2)*B-c*(A-I*B)/(c-I*c*tan(f*x+e))^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{\sqrt{2}\left((-i\,A-B)ae^{(2i\,fx+2i\,e)}+(-i\,A-3\,B)a\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{cf}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,
algorithm="fricas")
```

output
```
sqrt(2)*((-I*A - B)*a*e^(2*I*f*x + 2*I*e) + (-I*A - 3*B)*a)*sqrt(c/(e^(2*I
*f*x + 2*I*e) + 1))/(c*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = ia\left(\int\left(-\frac{iA}{\sqrt{-ic\tan(e+fx)+c}}\right)dx+\int\frac{A\tan(e+fx)}{\sqrt{-ic\tan(e+fx)+c}}\,dx + \int\frac{B\tan^2(e+fx)}{\sqrt{-ic\tan(e+fx)+c}}\,dx+\int\left(-\frac{iB\tan(e+fx)}{\sqrt{-ic\tan(e+fx)+c}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(1/2),x)
```

output
```
I*a*(Integral(-I*A/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(A*tan(e + f*
x)/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(B*tan(e + f*x)**2/sqrt(-I*c*
tan(e + f*x) + c), x) + Integral(-I*B*tan(e + f*x)/sqrt(-I*c*tan(e + f*x)
+ c), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{2i\left(i\sqrt{-i\,c\tan(fx+e)+c}Ba - \frac{(A-i\,B)ac}{\sqrt{-i\,c\tan(fx+e)+c}}\right)}{cf}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,
algorithm="maxima")
```

output
```
2*I*(I*sqrt(-I*c*tan(f*x + e) + c)*B*a - (A - I*B)*a*c/sqrt(-I*c*tan(f*x +
 e) + c))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 5.68 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.83

$$\int \frac{(a + ia\tan(e + fx))(A + B\tan(e + fx))}{\sqrt{c - ic\tan(e + fx)}}\,dx =$$

$$-\frac{a\sqrt{\frac{2c}{e^{e\,2i+f\,x\,2i}+1}}\left(A\,1i + 3B + A\left(\frac{e^{-e\,2i-f\,x\,2i}}{2} + \frac{e^{e\,2i+f\,x\,2i}}{2}\right)1i - A\left(\frac{e^{-e\,2i-f\,x\,2i}\,1i}{2} - \frac{e^{e\,2i+f\,x\,2i}\,1i}{2}\right) + B\left(\frac{e^{-e\,2i-f}}{2}\right.\right.}{c\,f}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)
^(1/2),x)
```

output
```
-(a*((2*c)/(exp(e*2i + f*x*2i) + 1))^(1/2)*(A*1i + 3*B + A*(exp(- e*2i - f
*x*2i)/2 + exp(e*2i + f*x*2i)/2)*1i - A*((exp(- e*2i - f*x*2i)*1i)/2 - (ex
p(e*2i + f*x*2i)*1i)/2) + B*(exp(- e*2i - f*x*2i)/2 + exp(e*2i + f*x*2i)/2
) + B*((exp(- e*2i - f*x*2i)*1i)/2 - (exp(e*2i + f*x*2i)*1i)/2)*1i))/(c*f)
```

Reduce [F]

$$\int \frac{(a + ia\tan(e + fx))(A + B\tan(e + fx))}{\sqrt{c - ic\tan(e + fx)}}\,dx$$

$$= \frac{\sqrt{c}\,a\left(2\sqrt{-\tan(fx+e)\,i+1}\,ai - \left(\int \frac{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^3}{\tan(fx+e)^2+1}dx\right)\tan(fx+e)^2\,bf - \left(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\tan(fx+e)}\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*a*(2*sqrt( - tan(e + f*x)*i + 1)*a*i - int((sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f - int
((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**2 + 1),x)*b*
f - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1
),x)*tan(e + f*x)**2*a*f + 2*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)
**2)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f*i - int((sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*a*f + 2*int((sqrt(
- tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*b*f*i + 5*
int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**2 + 1),x)*ta
n(e + f*x)**2*a*f*i + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(
e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f + 5*int((sqrt( - tan(e + f*x)*i +
1)*tan(e + f*x))/(tan(e + f*x)**2 + 1),x)*a*f*i + int((sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x))/(tan(e + f*x)**2 + 1),x)*b*f))/(c*f*(tan(e + f*x)**
2 + 1))
```

3.745 $\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 60

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{2a(iA+B)}{3f(c-ic\tan(e+fx))^{3/2}}+\frac{2aB}{cf\sqrt{c-ic\tan(e+fx)}}$$

output
```
-2/3*a*(I*A+B)/f/(c-I*c*tan(f*x+e))^(3/2)+2*a*B/c/f/(c-I*c*tan(f*x+e))^(1/
2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{2a\left(\frac{iA+B}{(c-ic\tan(e+fx))^{3/2}}-\frac{3B}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3f}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(3/2),x]
```

output
```
(-2*a*((I*A + B)/(c - I*c*Tan[e + f*x])^(3/2) - (3*B)/(c*Sqrt[c - I*c*Tan[
e + f*x]])))/(3*f)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {3042, 4071, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{53}$$

$$\frac{ac\int \left(\frac{A-iB}{(c-ic\tan(e+fx))^{5/2}}+\frac{iB}{c(c-ic\tan(e+fx))^{3/2}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac\left(\frac{2B}{c^2\sqrt{c-ic\tan(e+fx)}}-\frac{2(B+iA)}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(
3/2),x]
```

output
```
(a*c*((-2*(I*A + B))/(3*c*(c - I*c*Tan[e + f*x])^(3/2)) + (2*B)/(c^2*Sqrt[
c - I*c*Tan[e + f*x]])))/f
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{2ia\left(-\frac{c(-iB+A)}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{iB}{\sqrt{c-ic\tan(fx+e)}}\right)}{fc}$
default	$\frac{2ia\left(-\frac{c(-iB+A)}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{iB}{\sqrt{c-ic\tan(fx+e)}}\right)}{fc}$
risch	$-\frac{a\left(iA\,e^{2i(fx+e)}+B\,e^{2i(fx+e)}+iA-5B\right)\sqrt{2}}{6c\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
parts	$\frac{2iAac\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{5}{2}}}-\frac{1}{4c^{2}\sqrt{c-ic\tan(fx+e)}}-\frac{1}{6c(c-ic\tan(fx+e))^{\frac{3}{2}}}\right)}{f}+\frac{a(iA+B)\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\right.}{}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f*a/c*(-1/3*c*(A-I*B)/(c-I*c*tan(f*x+e))^(3/2)-I*B/(c-I*c*tan(f*x+e))^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.25

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}((-iA-B)ae^{(4i\,fx+4i\,e)} - 2\,(i\,A-2\,B)ae^{(2i\,fx+2i\,e)}}{6\,c^2f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="fricas")
```

output
```
1/6*sqrt(2)*((-I*A - B)*a*e^(4*I*f*x + 4*I*e) - 2*(I*A - 2*B)*a*e^(2*I*f*x
 + 2*I*e) + (-I*A + 5*B)*a)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = ia\left(\int \left(-\frac{iA}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{}}\right.\right.$$
$$+\int \frac{A\tan(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\,dx$$
$$+\int \frac{B\tan^2(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\,dx$$
$$\left.+\int \left(-\frac{iB\tan(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(3/2),x)
```

output
```
I*a*(Integral(-I*A/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt
(-I*c*tan(e + f*x) + c)), x) + Integral(A*tan(e + f*x)/(-I*c*sqrt(-I*c*tan
(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integra
l(B*tan(e + f*x)**2/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqr
t(-I*c*tan(e + f*x) + c)), x) + Integral(-I*B*tan(e + f*x)/(-I*c*sqrt(-I*c
*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{2i\,(3i\,(-i\,c\tan{(fx+e)}+c)Ba+(A-i\,B)ac)}{3\,(-i\,c\tan{(fx+e)}+c)^{\frac{3}{2}}cf}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="maxima")
```

output
```
-2/3*I*(3*I*(-I*c*tan(f*x + e) + c)*B*a + (A - I*B)*a*c)/((-I*c*tan(f*x +
e) + c)^(3/2)*c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.83

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{a\sqrt{-\frac{c\left(-2\cos(e+fx)^2+\sin(2e+2fx)\,1\mathrm{i}\right)}{2\cos(e+fx)^2}}\left(B\left(2\cos\left(2e+2fx\right)^2-1\right)-4B\left(2\cos\left(e+fx\right)^2-1\right)-2A\sin\left(2e\right.\right.}{}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)
^(3/2),x)
```

output
```
-(a*(-(c*(sin(2*e + 2*f*x)*1i - 2*cos(e + f*x)^2))/(2*cos(e + f*x)^2))^(1/
2)*(A*1i - 5*B + A*(2*cos(e + f*x)^2 - 1)*2i - 4*B*(2*cos(e + f*x)^2 - 1)
- 2*A*sin(2*e + 2*f*x) - A*sin(4*e + 4*f*x) - B*sin(2*e + 2*f*x)*4i + B*si
n(4*e + 4*f*x)*1i + A*(2*cos(2*e + 2*f*x)^2 - 1)*1i + B*(2*cos(2*e + 2*f*x
)^2 - 1)))/(6*c^2*f)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*a*( - 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*b*i - int(sqrt(
- tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*
i - 1),x)*tan(e + f*x)**2*a*f - 2*int(sqrt( - tan(e + f*x)*i + 1)/(tan(e +
 f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f*
i - int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i - tan(e + f*x)**2 +
 tan(e + f*x)*i - 1),x)*a*f - 2*int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f
*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*b*f*i - 5*int((sqrt( -
 tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i - tan(e + f*x)**2
 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*a*f - 2*int((sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f
*x)*i - 1),x)*tan(e + f*x)**2*b*f*i - 5*int((sqrt( - tan(e + f*x)*i + 1)*t
an(e + f*x)**2)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1)
,x)*a*f - 2*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x
)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*b*f*i - 4*int((sqrt( - t
an(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**3*i - tan(e + f*x)**2 + ta
n(e + f*x)*i - 1),x)*tan(e + f*x)**2*a*f*i - 4*int((sqrt( - tan(e + f*x)*i
 + 1)*tan(e + f*x))/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i
- 1),x)*a*f*i))/(c**2*f*(tan(e + f*x)**2 + 1))
```

3.746 $\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 62

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{2a(iA+B)}{5f(c-ic\tan(e+fx))^{5/2}}+\frac{2aB}{3cf(c-ic\tan(e+fx))^{3/2}}$$

output
```
-2/5*a*(I*A+B)/f/(c-I*c*tan(f*x+e))^(5/2)+2/3*a*B/c/f/(c-I*c*tan(f*x+e))^(
3/2)
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{2a\left(\frac{3(iA+B)}{(c-ic\tan(e+fx))^{5/2}}-\frac{5B}{c(c-ic\tan(e+fx))^{3/2}}\right)}{15f}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(5/2),x]
```

output
```
(-2*a*((3*(I*A + B))/(c - I*c*Tan[e + f*x])^(5/2) - (5*B)/(c*(c - I*c*Tan[
e + f*x])^(3/2))))/(15*f)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {3042, 4071, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{A+B\tan(e+fx)}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{53}$$
$$\frac{ac\int \left(\frac{A-iB}{(c-ic\tan(e+fx))^{7/2}}+\frac{iB}{c(c-ic\tan(e+fx))^{5/2}}\right)d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{2009}$$
$$\frac{ac\left(\frac{2B}{3c^2(c-ic\tan(e+fx))^{3/2}}-\frac{2(B+iA)}{5c(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(
5/2),x]
```

output
```
(a*c*((-2*(I*A + B))/(5*c*(c - I*c*Tan[e + f*x])^(5/2)) + (2*B)/(3*c^2*(c
- I*c*Tan[e + f*x])^(3/2))))/f
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2ia\left(-\frac{iB}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{c(-iB+A)}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}\right)}{fc}$
default	$\frac{2ia\left(-\frac{iB}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{c(-iB+A)}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}\right)}{fc}$
risch	$-\frac{a\left(3iA\,\mathrm{e}^{4i(fx+e)}+3B\,\mathrm{e}^{4i(fx+e)}+6iA\,\mathrm{e}^{2i(fx+e)}-4B\,\mathrm{e}^{2i(fx+e)}+3iA-7B\right)\sqrt{2}}{60c^2\sqrt{\frac{c}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$
parts	$\frac{2iAac\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{16c^{\frac{7}{2}}}-\frac{1}{8c^3\sqrt{c-ic\tan(fx+e)}}-\frac{1}{12c^2(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{10c(c-ic\tan(fx+e))^{\frac{5}{2}}}\right)}{f}+$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f*a/c*(-1/3*I*B/(c-I*c*tan(f*x+e))^(3/2)-1/5*c*(A-I*B)/(c-I*c*tan(f*x+
e))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{\sqrt{2}(3\,(i\,A+B)ae^{(6i\,fx+6i\,e)}-(-9i\,A+B)ae^{(4i\,fx+4i\,e)}-(-9i\,A+11\,B)ae^{(2i\,fx+2i\,e)}-(-3i\,A+7\,B)a)\sqrt{}}{60\,c^3f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="fricas")
```

output
```
-1/60*sqrt(2)*(3*(I*A + B)*a*e^(6*I*f*x + 6*I*e) - (-9*I*A + B)*a*e^(4*I*f
*x + 4*I*e) - (-9*I*A + 11*B)*a*e^(2*I*f*x + 2*I*e) - (-3*I*A + 7*B)*a)*sq
rt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = ia\left(\int\left(-\frac{}{-c^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-2i}\right.\right.$$

$$+\int\frac{A\tan(e+fx)}{-c^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-2ic^2\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c^2\sqrt{-ic\tan(e}}$$

$$+\int\frac{B\tan^2(e+fx)}{-c^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-2ic^2\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c^2\sqrt{-ic\tan(e}}$$

$$+\int\left(-\frac{iB\tan(e+fx)}{-c^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-2ic^2\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c^2\sqrt{-ic\tan}}\right.$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(5/2),x)
```

output
```
I*a*(Integral(-I*A/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*
I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f
*x) + c)), x) + Integral(A*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x) + c)
*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**
2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(B*tan(e + f*x)**2/(-c**2*sqr
t(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x)
 + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-I*B
*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**
2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) +
 c)), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{2i\,(5i\,(-i\,c\tan{(fx+e)}+c)Ba+3\,(A-i\,B)ac)}{15\,(-i\,c\tan{(fx+e)}+c)^{\frac{5}{2}}cf}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="maxima")
```

output
```
-2/15*I*(5*I*(-I*c*tan(f*x + e) + c)*B*a + 3*(A - I*B)*a*c)/((-I*c*tan(f*x
 + e) + c)^(5/2)*c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.74

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{a\sqrt{-\frac{c\left(-2\cos(e+f\,x)^2+\sin(2\,e+2\,f\,x)\,1\mathrm{i}\right)}{2\cos(e+f\,x)^2}}\,\left(7\,B+11\,B\,\left(2\cos\left(e\right.\right.\right.}{}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)
^(5/2),x)
```

output
```
(a*(-(c*(sin(2*e + 2*f*x)*1i - 2*cos(e + f*x)^2))/(2*cos(e + f*x)^2))^(1/2
)*(7*B - A*3i - A*(2*cos(e + f*x)^2 - 1)*9i + 11*B*(2*cos(e + f*x)^2 - 1)
+ 9*A*sin(2*e + 2*f*x) + 9*A*sin(4*e + 4*f*x) + 3*A*sin(6*e + 6*f*x) + B*s
in(2*e + 2*f*x)*11i + B*sin(4*e + 4*f*x)*1i - B*sin(6*e + 6*f*x)*3i - A*(2
*cos(2*e + 2*f*x)^2 - 1)*9i - A*(2*cos(3*e + 3*f*x)^2 - 1)*3i + B*(2*cos(2
*e + 2*f*x)^2 - 1) - 3*B*(2*cos(3*e + 3*f*x)^2 - 1)))/(60*c^3*f)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{a\left(\left(\int \frac{\tan(fx+e)^2}{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^2+2\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)i-\sqrt{-\tan(fx+e)i+1}}dx\right)bi+\left(\int \frac{}{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
( - a*(int(tan(e + f*x)**2/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 +
2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)
),x)*b*i + int(tan(e + f*x)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 +
 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1
)),x)*a*i + int(tan(e + f*x)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2
+ 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i +
1)),x)*b + int(1/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*sqrt( -
tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*a))/(
sqrt(c)*c**2)
```

3.747 $\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 62

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{2a(iA+B)}{7f(c-ic\tan(e+fx))^{7/2}}+\frac{2aB}{5cf(c-ic\tan(e+fx))^{5/2}}$$

output
```
-2/7*a*(I*A+B)/f/(c-I*c*tan(f*x+e))^(7/2)+2/5*a*B/c/f/(c-I*c*tan(f*x+e))^(
5/2)
```

Mathematica [A] (verified)

Time = 3.59 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{2a\left(\frac{5(iA+B)}{(c-ic\tan(e+fx))^{7/2}}-\frac{7B}{c(c-ic\tan(e+fx))^{5/2}}\right)}{35f}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(7/2),x]
```

output
```
(-2*a*((5*(I*A + B))/(c - I*c*Tan[e + f*x])^(7/2) - (7*B)/(c*(c - I*c*Tan[
e + f*x])^(5/2))))/(35*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {3042, 4071, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{53}$$

$$\frac{ac\int \left(\frac{A-iB}{(c-ic\tan(e+fx))^{9/2}}+\frac{iB}{c(c-ic\tan(e+fx))^{7/2}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ac\left(\frac{2B}{5c^2(c-ic\tan(e+fx))^{5/2}}-\frac{2(B+iA)}{7c(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])^(
7/2),x]
```

output
```
(a*c*((-2*(I*A + B))/(7*c*(c - I*c*Tan[e + f*x])^(7/2)) + (2*B)/(5*c^2*(c
- I*c*Tan[e + f*x])^(5/2))))/f
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2ia\left(-\frac{iB}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}-\frac{c(-iB+A)}{7(c-ic\tan(fx+e))^{\frac{7}{2}}}\right)}{fc}$
default	$\frac{2ia\left(-\frac{iB}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}-\frac{c(-iB+A)}{7(c-ic\tan(fx+e))^{\frac{7}{2}}}\right)}{fc}$
risch	$-\frac{a\left(5iA\,e^{6i(fx+e)}+5B\,e^{6i(fx+e)}+15iA\,e^{4i(fx+e)}+B\,e^{4i(fx+e)}+15iA\,e^{2i(fx+e)}-13B\,e^{2i(fx+e)}+5iA-9B\right)\sqrt{2}}{280c^{3}\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
parts	$\frac{2iAac\left(\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{32c^{\frac{9}{2}}}-\frac{1}{16c^{4}\sqrt{c-ic\tan(fx+e)}}-\frac{1}{24c^{3}(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{20c^{2}(c-ic\tan(fx+e))^{\frac{5}{2}}}-\frac{}{14c\ldots}\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f*a/c*(-1/5*I*B/(c-I*c*tan(f*x+e))^(5/2)-1/7*c*(A-I*B)/(c-I*c*tan(f*x+
e))^(7/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(48) = 96$.

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\int \frac{(a + ia\tan(e + fx))(A + B\tan(e + fx))}{(c - ic\tan(e + fx))^{7/2}}\,dx =$$
$$-\frac{\sqrt{2}(5\,(i\,A + B)ae^{(8i\,fx+8i\,e)} + 2\,(10i\,A + 3\,B)ae^{(6i\,fx+6i\,e)} + 6\,(5i\,A - 2\,B)ae^{(4i\,fx+4i\,e)} + 2\,(10i\,A - 11\,B)}{280\,c^4 f}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="fricas")
```

output
```
-1/280*sqrt(2)*(5*(I*A + B)*a*e^(8*I*f*x + 8*I*e) + 2*(10*I*A + 3*B)*a*e^(
6*I*f*x + 6*I*e) + 6*(5*I*A - 2*B)*a*e^(4*I*f*x + 4*I*e) + 2*(10*I*A - 11*
B)*a*e^(2*I*f*x + 2*I*e) - (-5*I*A + 9*B)*a)*sqrt(c/(e^(2*I*f*x + 2*I*e) +
 1))/(c^4*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = ia\left(\int\left(-\frac{}{ic^3\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)-3c^3}\right.\right.$$
$$+\int\frac{A\tan(e+fx)}{ic^3\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)-3c^3\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-3ic^3\sqrt{-ic\tan(e}$$
$$+\int\frac{B\tan^2(e+fx)}{ic^3\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)-3c^3\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-3ic^3\sqrt{-ic\tan(e}$$
$$+\int\left(-\frac{iB\tan(e+fx)}{ic^3\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)-3c^3\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-3ic^3\sqrt{-ic\tan}\right.$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(7/2),x)
```

output
```
I*a*(Integral(-I*A/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - 3
*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-I*c*tan
(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) + Inte
gral(A*tan(e + f*x)/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 -
3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-I*c*ta
n(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) + Int
egral(B*tan(e + f*x)**2/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**
3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-I*
c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) +
 Integral(-I*B*tan(e + f*x)/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt
(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)),
x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{2i\,(7i\,(-i\,c\tan(fx+e)+c)Ba+5\,(A-i\,B)ac)}{35\,(-i\,c\tan(fx+e)+c)^{\frac{7}{2}}cf}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="maxima")
```

output
```
-2/35*I*(7*I*(-I*c*tan(f*x + e) + c)*B*a + 5*(A - I*B)*a*c)/((-I*c*tan(f*x
 + e) + c)^(7/2)*c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.53

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\sqrt{c-\frac{c\,\sin\left(e+f\,x\right)\,1{}\mathrm{i}}{\cos\left(e+f\,x\right)}}\,\left(\frac{a\,\left(5\,A+B\,9{}\mathrm{i}\right)\,1{}\mathrm{i}}{280\,c^4\,f}+\frac{a\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\left(A-B\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{56\,c^4\,f}+\frac{a\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\left(5\,A+B\,2{}\mathrm{i}\right)\,3{}\mathrm{i}}{140\,c^4\,f}+\frac{a\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,\left(10\,A+B\,11{}\mathrm{i}\right)\,1{}\mathrm{i}}{140\,c^4\,f}+\frac{a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\left(10\,A-B\,3{}\mathrm{i}\right)\,1{}\mathrm{i}}{140\,c^4\,f}\right)$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i))/(c - c*tan(e + f*x)*1i)
^(7/2),x)
```

output
```
-(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)*((a*(5*A + B*9i)*1i)/(280*c^
4*f) + (a*exp(e*8i + f*x*8i)*(A - B*1i)*1i)/(56*c^4*f) + (a*exp(e*4i + f*x
*4i)*(5*A + B*2i)*3i)/(140*c^4*f) + (a*exp(e*2i + f*x*2i)*(10*A + B*11i)*1
i)/(140*c^4*f) + (a*exp(e*6i + f*x*6i)*(10*A - B*3i)*1i)/(140*c^4*f))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{too large to display}$$

input
```
int((a+I*a*tan(f*x+e))*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*a*(130*sqrt( - tan(e + f*x)*i + 1)*a*i + 32*sqrt( - tan(e + f*x)*
i + 1)*b - 5*int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**5*i - 3*tan(e
+ f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1
),x)*tan(e + f*x)**2*a*f + 16*int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x
)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*t
an(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i - 5*int(sqrt( - tan(e + f*x)*i
 + 1)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan
(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*a*f + 16*int(sqrt( - tan(e + f*x)*
i + 1)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*ta
n(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b*f*i - 195*int((sqrt( - tan(e +
f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*ta
n(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*tan(e + f*x
)**2*a*f + 48*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f
*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3
*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i - 195*int((sqrt( - tan(e + f
*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan
(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*a*f + 48*int
((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**5*i - 3*tan(
e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i +
 1),x)*b*f*i - 650*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(t...
```

3.748 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 105

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx=\frac{4a^2(iA+B)(c-ic\tan(e+fx))^{7/2}}{7f}-\frac{2a^2(iA+3B)(c-ic\tan(e+fx))^{9/2}}{9cf}+\frac{2a^2B(c-ic\tan(e+fx))^{11/2}}{11c^2f}$$

output
```
4/7*a^2*(I*A+B)*(c-I*c*tan(f*x+e))^(7/2)/f-2/9*a^2*(I*A+3*B)*(c-I*c*tan(f*
x+e))^(9/2)/c/f+2/11*a^2*B*(c-I*c*tan(f*x+e))^(11/2)/c^2/f
```

Mathematica [A] (verified)

Time = 5.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx=\frac{a^2c^4\sec^6(e+fx)(121iA-33B+(121iA+93B)\cos(2(e+fx))+(-77A+105iB)\sin(2(e}{693f\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^(7/2),x]
```

output
```
(a^2*c^4*Sec[e + f*x]^6*((121*I)*A - 33*B + ((121*I)*A + 93*B)*Cos[2*(e +
f*x)] + (-77*A + (105*I)*B)*Sin[2*(e + f*x)])*(Cos[4*(e + f*x)] - I*Sin[4*
(e + f*x)]))/(693*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int a(i\tan(e+fx)+1)(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^2c\int (i\tan(e+fx)+1)(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{a^2c\int \left(-\frac{iB(c-ic\tan(e+fx))^{9/2}}{c^2}+\frac{(3iB-A)(c-ic\tan(e+fx))^{7/2}}{c}+2(A-iB)(c-ic\tan(e+fx))^{5/2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2c\left(-\frac{2(3B+iA)(c-ic\tan(e+fx))^{9/2}}{9c^2}+\frac{4(B+iA)(c-ic\tan(e+fx))^{7/2}}{7c}+\frac{2B(c-ic\tan(e+fx))^{11/2}}{11c^3}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(
7/2),x]
```

output
```
(a^2*c*((4*(I*A + B)*(c - I*c*Tan[e + f*x])^(7/2))/(7*c) - (2*(I*A + 3*B)*
(c - I*c*Tan[e + f*x])^(9/2))/(9*c^2) + (2*B*(c - I*c*Tan[e + f*x])^(11/2)
)/(11*c^3)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{2ia^{2}\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{(3iBc-cA)(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{2(-iBc+cA)c(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}\right)}{f\,c^{2}}$
default	$\frac{2ia^{2}\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{(3iBc-cA)(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{2(-iBc+cA)c(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}\right)}{f\,c^{2}}$
parts	$\frac{2iA\,a^{2}c\left(-\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-4\sqrt{c-ic\tan(fx+e)}\,c^{2}+4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{f}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^2/c^2*(-1/11*I*B*(c-I*c*tan(f*x+e))^(11/2)+1/9*(3*I*B*c-c*A)*(c-I*
c*tan(f*x+e))^(9/2)+2/7*(-I*B*c+c*A)*c*(c-I*c*tan(f*x+e))^(7/2))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.42

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx =$$
$$-\frac{32\sqrt{2}\left(99\,(-i\,A-B)a^{2}c^{3}e^{(4i\,fx+4i\,e)}+11\,(-11i\,A+3\,B)a^{2}c^{3}e^{(2i\,fx+2i\,e)}+2\,(-11i\,A+3\,B)a^{2}c^{3}\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{693\left(fe^{(10i\,fx+10i\,e)}+5\,fe^{(8i\,fx+8i\,e)}+10\,fe^{(6i\,fx+6i\,e)}+10\,fe^{(4i\,fx+4i\,e)}+5\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
-32/693*sqrt(2)*(99*(-I*A - B)*a^2*c^3*e^(4*I*f*x + 4*I*e) + 11*(-11*I*A +
 3*B)*a^2*c^3*e^(2*I*f*x + 2*I*e) + 2*(-11*I*A + 3*B)*a^2*c^3)*sqrt(c/(e^(
2*I*f*x + 2*I*e) + 1))/(f*e^(10*I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e)
+ 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2
*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx =$$

$$-a^2\left(\int \left(-Ac^3\sqrt{-ic\tan{(e+fx)}+c}\right)\,dx\right.$$
$$+\int \left(-2Ac^3\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\right)\,dx$$
$$+\int \left(-Ac^3\sqrt{-ic\tan{(e+fx)}+c}\tan^4{(e+fx)}\right)\,dx$$
$$+\int \left(-Bc^3\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\right)\,dx$$
$$+\int \left(-2Bc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}\right)\,dx$$
$$+\int \left(-Bc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^5{(e+fx)}\right)\,dx$$
$$+\int iAc^3\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\,dx$$
$$+\int 2iAc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}\,dx$$
$$+\int iAc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^5{(e+fx)}\,dx$$
$$+\int iBc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\,dx$$
$$+\int 2iBc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^4{(e+fx)}\,dx$$
$$\left.+\int iBc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^6{(e+fx)}\,dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2)
,x)
```

output
```
-a**2*(Integral(-A*c**3*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-2*A*c*
*3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(-A*c**3*sqrt
(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4, x) + Integral(-B*c**3*sqrt(-I*c*t
an(e + f*x) + c)*tan(e + f*x), x) + Integral(-2*B*c**3*sqrt(-I*c*tan(e + f
*x) + c)*tan(e + f*x)**3, x) + Integral(-B*c**3*sqrt(-I*c*tan(e + f*x) + c
)*tan(e + f*x)**5, x) + Integral(I*A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(
e + f*x), x) + Integral(2*I*A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x
)**3, x) + Integral(I*A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**5,
x) + Integral(I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + I
ntegral(2*I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4, x) + Integ
ral(I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**6, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx =$$

$$-\frac{2i\left(63i\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{11}{2}}Ba^2+77\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{9}{2}}(A-3i\,B)a^2c-198\left(-i\,c\tan\left(fx+e\right.\right.\right.}{693\,c^2f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
-2/693*I*(63*I*(-I*c*tan(f*x + e) + c)^(11/2)*B*a^2 + 77*(-I*c*tan(f*x + e
) + c)^(9/2)*(A - 3*I*B)*a^2*c - 198*(-I*c*tan(f*x + e) + c)^(7/2)*(A - I*
B)*a^2*c^2)/(c^2*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.26

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{32\,a^2\,c^3\,\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,\left(A\,22\mathrm{i}-6\,B+A\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,121\mathrm{i}+A\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,99\mathrm{i}-33\,B\,\mathrm{e}^{e\,2\mathrm{i}+\dots}\right.}{693\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^5}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
^(7/2),x)
```

output
```
(32*a^2*c^3*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) +
 1))^(1/2)*(A*22i - 6*B + A*exp(e*2i + f*x*2i)*121i + A*exp(e*4i + f*x*4i)
*99i - 33*B*exp(e*2i + f*x*2i) + 99*B*exp(e*4i + f*x*4i)))/(693*f*(exp(e*2
i + f*x*2i) + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.27

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{2\sqrt{c}\,\sqrt{-\tan\left(fx+e\right)i+1}\,a^2c^3\left(-63\tan\left(fx+e\right)^5bi-77\tan\left(fx+e\right)^4ai+84\tan\left(fx+e\dots\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(2*sqrt(c)*sqrt( - tan(e + f*x)*i + 1)*a**2*c**3*( - 63*tan(e + f*x)**5*b*
i - 77*tan(e + f*x)**4*a*i + 84*tan(e + f*x)**4*b + 110*tan(e + f*x)**3*a
- 96*tan(e + f*x)**3*b*i - 132*tan(e + f*x)**2*a*i + 162*tan(e + f*x)**2*b
 + 286*tan(e + f*x)*a + 15*tan(e + f*x)*b*i + 121*a*i + 30*b))/(693*f)
```

3.749 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 105

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\frac{4a^2(iA+B)(c-ic\tan(e+fx))^{5/2}}{5f}-\frac{2a^2(iA+3B)(c-ic\tan(e+fx))^{7/2}}{7cf}+\frac{2a^2B(c-ic\tan(e+fx))^{9/2}}{9c^2f}$$

output
```
4/5*a^2*(I*A+B)*(c-I*c*tan(f*x+e))^(5/2)/f-2/7*a^2*(I*A+3*B)*(c-I*c*tan(f*
x+e))^(7/2)/c/f+2/9*a^2*B*(c-I*c*tan(f*x+e))^(9/2)/c^2/f
```

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\frac{a^2c^3\sec^5(e+fx)(81iA-9B+(81iA+61B)\cos(2(e+fx))+(-45A+65iB)\sin(2(e+f}{315f\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^(5/2),x]
```

output
```
(a^2*c^3*Sec[e + f*x]^5*((81*I)*A - 9*B + ((81*I)*A + 61*B)*Cos[2*(e + f*x
)] + (-45*A + (65*I)*B)*Sin[2*(e + f*x)])*(Cos[3*(e + f*x)] - I*Sin[3*(e +
 f*x)]))/(315*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a(i\tan(e+fx)+1)(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c\int (i\tan(e+fx)+1)(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2c\int \left(-\frac{iB(c-ic\tan(e+fx))^{7/2}}{c^2}+\frac{(3iB-A)(c-ic\tan(e+fx))^{5/2}}{c}+2(A-iB)(c-ic\tan(e+fx))^{3/2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c\left(-\frac{2(3B+iA)(c-ic\tan(e+fx))^{7/2}}{7c^2}+\frac{4(B+iA)(c-ic\tan(e+fx))^{5/2}}{5c}+\frac{2B(c-ic\tan(e+fx))^{9/2}}{9c^3}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(
5/2),x]
```

output
```
(a^2*c*((4*(I*A + B)*(c - I*c*Tan[e + f*x])^(5/2))/(5*c) - (2*(I*A + 3*B)*
(c - I*c*Tan[e + f*x])^(7/2))/(7*c^2) + (2*B*(c - I*c*Tan[e + f*x])^(9/2))
/(9*c^3)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{2ia^2\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{(3iBc-cA)(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2(-iBc+cA)c(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}\right)}{f\,c^2}$
default	$\frac{2ia^2\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{(3iBc-cA)(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2(-iBc+cA)c(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}\right)}{f\,c^2}$
parts	$\frac{2iA\,a^2c\left(-\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-2c\sqrt{c-ic\tan(fx+e)}+2c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{f}+\frac{a^2(2iA+B)\left(\frac{2(c-\cdots\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^2/c^2*(-1/9*I*B*(c-I*c*tan(f*x+e))^(9/2)+1/7*(3*I*B*c-c*A)*(c-I*c*
tan(f*x+e))^(7/2)+2/5*(-I*B*c+c*A)*c*(c-I*c*tan(f*x+e))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.27

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx =$$
$$-\frac{16\sqrt{2}\left(63\,(-i\,A-B)a^2c^2e^{(4i\,fx+4i\,e)}+9\,(-9i\,A+B)a^2c^2e^{(2i\,fx+2i\,e)}+2\,(-9i\,A+B)a^2c^2\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{315\left(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
-16/315*sqrt(2)*(63*(-I*A - B)*a^2*c^2*e^(4*I*f*x + 4*I*e) + 9*(-9*I*A + B
)*a^2*c^2*e^(2*I*f*x + 2*I*e) + 2*(-9*I*A + B)*a^2*c^2)*sqrt(c/(e^(2*I*f*x
 + 2*I*e) + 1))/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(
4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx =$$
$$-a^2\left(\int\left(-Ac^2\sqrt{-ic\tan(e+fx)+c}\right)dx + \int\left(-2Ac^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)\right)dx + \int\left(-Ac^2\sqrt{-ic\tan(e+fx)+c}\tan^4(e+fx)\right)dx + \int\left(-Bc^2\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\right)dx + \int\left(-2Bc^2\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)\right)dx + \int\left(-Bc^2\sqrt{-ic\tan(e+fx)+c}\tan^5(e+fx)\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2)
,x)
```

output
```
-a**2*(Integral(-A*c**2*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-2*A*c*
*2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(-A*c**2*sqrt
(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4, x) + Integral(-B*c**2*sqrt(-I*c*t
an(e + f*x) + c)*tan(e + f*x), x) + Integral(-2*B*c**2*sqrt(-I*c*tan(e + f
*x) + c)*tan(e + f*x)**3, x) + Integral(-B*c**2*sqrt(-I*c*tan(e + f*x) + c
)*tan(e + f*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx =$$
$$-\frac{2i\left(35i\left(-i\,c\tan(fx+e)+c\right)^{\frac{9}{2}}Ba^2+45\left(-i\,c\tan(fx+e)+c\right)^{\frac{7}{2}}(A-3i\,B)a^2c-126\left(-i\,c\tan(fx+e)\right.}{315\,c^2f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
-2/315*I*(35*I*(-I*c*tan(f*x + e) + c)^(9/2)*B*a^2 + 45*(-I*c*tan(f*x + e)
 + c)^(7/2)*(A - 3*I*B)*a^2*c - 126*(-I*c*tan(f*x + e) + c)^(5/2)*(A - I*B
)*a^2*c^2)/(c^2*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.26

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \frac{16\,a^2\,c^2\,\sqrt{c+\frac{c\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,\left(A\,18\mathrm{i}-2\,B+A\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,81\mathrm{i}+A\,{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,63\mathrm{i}-9\,B\,{\mathrm{e}}^{e\,2\mathrm{i}+f\,x}\right.}{315\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
^(5/2),x)
```

output
```
(16*a^2*c^2*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) +
 1))^(1/2)*(A*18i - 2*B + A*exp(e*2i + f*x*2i)*81i + A*exp(e*4i + f*x*4i)*
63i - 9*B*exp(e*2i + f*x*2i) + 63*B*exp(e*4i + f*x*4i)))/(315*f*(exp(e*2i
+ f*x*2i) + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \frac{2\sqrt{c}\,\sqrt{-\tan(fx+e)\,i+1}\,a^2c^2\left(35\tan(fx+e)^4 b+45\tan(fx+e)^3 a+5\tan(fx+e)^3 bi\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt( - tan(e + f*x)*i + 1)*a**2*c**2*(35*tan(e + f*x)**4*b + 4
5*tan(e + f*x)**3*a + 5*tan(e + f*x)**3*b*i + 9*tan(e + f*x)**2*a*i + 69*t
an(e + f*x)**2*b + 117*tan(e + f*x)*a + 13*tan(e + f*x)*b*i + 81*a*i + 26*
b))/(315*f)
```

3.750 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 105

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{4a^2(iA+B)(c-ic\tan(e+fx))^{3/2}}{3f} - \frac{2a^2(iA+3B)(c-ic\tan(e+fx))^{5/2}}{5cf} + \frac{2a^2B(c-ic\tan(e+fx))^{7/2}}{7c^2f}$$

output
```
4/3*a^2*(I*A+B)*(c-I*c*tan(f*x+e))^(3/2)/f-2/5*a^2*(I*A+3*B)*(c-I*c*tan(f*
x+e))^(5/2)/c/f+2/7*a^2*B*(c-I*c*tan(f*x+e))^(7/2)/c^2/f
```

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{a^2c^2\sec^4(e+fx)(\cos(2(e+fx))-i\sin(2(e+fx)))(7(7iA+B)+(49iA+37B)\cos(2(e+}{105f\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^(3/2),x]
```

output
```
(a^2*c^2*Sec[e + f*x]^4*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)])*(7*((7*I)*
A + B) + ((49*I)*A + 37*B)*Cos[2*(e + f*x)] + (-21*A + (33*I)*B)*Sin[2*(e
+ f*x)]))/(105*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int a(i\tan(e+fx)+1)(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^2c\int (i\tan(e+fx)+1)(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{a^2c\int \left(-\frac{iB(c-ic\tan(e+fx))^{5/2}}{c^2}+\frac{(3iB-A)(c-ic\tan(e+fx))^{3/2}}{c}+2(A-iB)\sqrt{c-ic\tan(e+fx)}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2c\left(-\frac{2(3B+iA)(c-ic\tan(e+fx))^{5/2}}{5c^2}+\frac{4(B+iA)(c-ic\tan(e+fx))^{3/2}}{3c}+\frac{2B(c-ic\tan(e+fx))^{7/2}}{7c^3}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(
3/2),x]
```

output
```
(a^2*c*((4*(I*A + B)*(c - I*c*Tan[e + f*x])^(3/2))/(3*c) - (2*(I*A + 3*B)*
(c - I*c*Tan[e + f*x])^(5/2))/(5*c^2) + (2*B*(c - I*c*Tan[e + f*x])^(7/2))
/(7*c^3)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{2ia^2\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{(3iBc-cA)(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2(-iBc+cA)c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}\right)}{f\,c^2}$
default	$\frac{2ia^2\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{(3iBc-cA)(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{2(-iBc+cA)c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}\right)}{f\,c^2}$
parts	$\frac{2iA\,a^2c\left(-\sqrt{c-ic\tan(fx+e)}+\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{f}+\frac{a^2(2iA+B)\left(\frac{2(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+2c\sqrt{c-\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^2/c^2*(-1/7*I*B*(c-I*c*tan(f*x+e))^(7/2)+1/5*(3*I*B*c-c*A)*(c-I*c*
tan(f*x+e))^(5/2)+2/3*(-I*B*c+c*A)*c*(c-I*c*tan(f*x+e))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = -\frac{8\sqrt{2}\left(35\,(-i\,A-B)a^2ce^{(4i\,fx+4i\,e)}+7\,(-7i\,A-B)a^2ce^{(2i\,fx+2i\,e)}+2\,(-7i\,A-B)a^2c\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{105\left(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
-8/105*sqrt(2)*(35*(-I*A - B)*a^2*c*e^(4*I*f*x + 4*I*e) + 7*(-7*I*A - B)*a
^2*c*e^(2*I*f*x + 2*I*e) + 2*(-7*I*A - B)*a^2*c)*sqrt(c/(e^(2*I*f*x + 2*I*
e) + 1))/(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x
 + 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = -a^2\left(\int \left(-Ac\sqrt{-ic\tan{(e+fx)}+c}\right)dx + \int \left(-Ac\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\right)dx + \int \left(-Bc\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\right)dx + \int \left(-Bc\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}\right)dx + \int \left(-iAc\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\right)dx + \int \left(-iAc\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}\right)dx + \int \left(-iBc\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\right)dx + \int \left(-iBc\sqrt{-ic\tan{(e+fx)}+c}\tan^4{(e+fx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)
,x)
```

output
```
-a**2*(Integral(-A*c*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-A*c*sqrt(
-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(-B*c*sqrt(-I*c*tan(e
 + f*x) + c)*tan(e + f*x), x) + Integral(-B*c*sqrt(-I*c*tan(e + f*x) + c)*
tan(e + f*x)**3, x) + Integral(-I*A*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
f*x), x) + Integral(-I*A*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3, x)
 + Integral(-I*B*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integ
ral(-I*B*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx =$$
$$-\frac{2i\left(15i\,(-i\,c\tan\left(fx+e\right)+c)^{\frac{7}{2}}Ba^2+21\,(-i\,c\tan\left(fx+e\right)+c)^{\frac{5}{2}}(A-3i\,B)a^2c-70\,(-i\,c\tan\left(fx+e\right)\right.}{105\,c^2f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
-2/105*I*(15*I*(-I*c*tan(f*x + e) + c)^(7/2)*B*a^2 + 21*(-I*c*tan(f*x + e)
 + c)^(5/2)*(A - 3*I*B)*a^2*c - 70*(-I*c*tan(f*x + e) + c)^(3/2)*(A - I*B)
*a^2*c^2)/(c^2*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.24

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{8\,a^2\,c\,\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,\left(A\,14\mathrm{i}+2\,B+A\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,49\mathrm{i}+A\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,35\mathrm{i}+7\,B\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\right.}{105\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
^(3/2),x)
```

output
```
(8*a^2*c*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1)
)^(1/2)*(A*14i + 2*B + A*exp(e*2i + f*x*2i)*49i + A*exp(e*4i + f*x*4i)*35i
 + 7*B*exp(e*2i + f*x*2i) + 35*B*exp(e*4i + f*x*4i)))/(105*f*(exp(e*2i + f
*x*2i) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{2\sqrt{c}\,\sqrt{-\tan(fx+e)\,i+1}\,a^2c\big(15\tan(fx+e)^3\,bi+21\tan(fx+e)^2\,ai+18\tan(fx+e)^2\,b}{105f}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt( - tan(e + f*x)*i + 1)*a**2*c*(15*tan(e + f*x)**3*b*i + 21
*tan(e + f*x)**2*a*i + 18*tan(e + f*x)**2*b + 28*tan(e + f*x)*a + 11*tan(e
 + f*x)*b*i + 49*a*i + 22*b))/(105*f)
```

3.751 $\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}$

Optimal result

Integrand size = 43, antiderivative size = 103

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \frac{4a^2(iA+B)\sqrt{c-ic\tan(e+fx)}}{f} - \frac{2a^2(iA+3B)(c-ic\tan(e+fx))^{3/2}}{3cf} + \frac{2a^2B(c-ic\tan(e+fx))^{5/2}}{5c^2f}$$

output
```
4*a^2*(I*A+B)*(c-I*c*tan(f*x+e))^(1/2)/f-2/3*a^2*(I*A+3*B)*(c-I*c*tan(f*x+
e))^(3/2)/c/f+2/5*a^2*B*(c-I*c*tan(f*x+e))^(5/2)/c^2/f
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \frac{2a^2c(i+\tan(e+fx))\left(25A-18iB+(5iA+9B)\tan(e+fx)+3iB\tan^2(e+fx)\right)}{15f\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e
 + f*x]],x]
```

output
```
(2*a^2*c*(I + Tan[e + f*x])*(25*A - (18*I)*B + ((5*I)*A + 9*B)*Tan[e + f*x
] + (3*I)*B*Tan[e + f*x]^2))/(15*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2c\int \left(-\frac{iB(c-ic\tan(e+fx))^{3/2}}{c^2}+\frac{(3iB-A)\sqrt{c-ic\tan(e+fx)}}{c}+\frac{2(A-iB)}{\sqrt{c-ic\tan(e+fx)}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c\left(-\frac{2(3B+iA)(c-ic\tan(e+fx))^{3/2}}{3c^2}+\frac{4(B+iA)\sqrt{c-ic\tan(e+fx)}}{c}+\frac{2B(c-ic\tan(e+fx))^{5/2}}{5c^3}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x
]],x]
```

output
```
(a^2*c*((4*(I*A + B)*Sqrt[c - I*c*Tan[e + f*x]])/c - (2*(I*A + 3*B)*(c - I
*c*Tan[e + f*x])^(3/2))/(3*c^2) + (2*B*(c - I*c*Tan[e + f*x])^(5/2))/(5*c^
3)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{2 i a^{2}\left(-\frac{i B (c-i c \tan(f x+e))^{\frac{5}{2}}}{5}+\frac{(3 i B c-c A)(c-i c \tan(f x+e))^{\frac{3}{2}}}{3}+2 \sqrt{c-i c \tan(f x+e)}\,(-i B c+c A) c\right)}{f\,c^{2}}$
default	$\frac{2 i a^{2}\left(-\frac{i B (c-i c \tan(f x+e))^{\frac{5}{2}}}{5}+\frac{(3 i B c-c A)(c-i c \tan(f x+e))^{\frac{3}{2}}}{3}+2 \sqrt{c-i c \tan(f x+e)}\,(-i B c+c A) c\right)}{f\,c^{2}}$
parts	$\frac{i A\,a^{2} \sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-i c \tan(f x+e)}\,\sqrt{2}}{2 \sqrt{c}}\right)}{f}+\frac{a^{2}(2 i A+B)\left(2 \sqrt{c-i c \tan(f x+e)}-\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-i c \tan(f x+e)}\,\sqrt{2}}{2 \sqrt{c}}\right.\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^2/c^2*(-1/5*I*B*(c-I*c*tan(f*x+e))^(5/2)+1/3*(3*I*B*c-c*A)*(c-I*c*
tan(f*x+e))^(3/2)+2*(c-I*c*tan(f*x+e))^(1/2)*(-I*B*c+c*A)*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = -\frac{4\sqrt{2}\left(15(-iA-B)a^2e^{(4i\,fx+4i\,e)}+5(-5i\,A-3\,B)a^2e^{(2i\,fx+2i\,e)}+2(-5i\,A-3\,B)a^2\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{15\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
-4/15*sqrt(2)*(15*(-I*A - B)*a^2*e^(4*I*f*x + 4*I*e) + 5*(-5*I*A - 3*B)*a^
2*e^(2*I*f*x + 2*I*e) + 2*(-5*I*A - 3*B)*a^2)*sqrt(c/(e^(2*I*f*x + 2*I*e)
+ 1))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int (a + ia\tan(e+fx))^2(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= -a^2\left(\int \left(-A\sqrt{-ic\tan(e+fx)+c}\right)dx + \int A\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)\,dx + \int\left(-B\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\right)dx + \int B\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)\,dx + \int\left(-2iA\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\right)dx + \int\left(-2iB\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(1/2)
,x)
```

output
```
-a**2*(Integral(-A*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(A*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(-B*sqrt(-I*c*tan(e + f*x)
 + c)*tan(e + f*x), x) + Integral(B*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x)**3, x) + Integral(-2*I*A*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) +
 Integral(-2*I*B*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx =$$
$$-\frac{2i\left(3i\,(-i\,c\tan(fx+e)+c)^{\frac{5}{2}}Ba^2 + 5\,(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}(A-3i\,B)a^2c - 30\,\sqrt{-i\,c\tan(fx+e}\right.}{15\,c^2f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
-2/15*I*(3*I*(-I*c*tan(f*x + e) + c)^(5/2)*B*a^2 + 5*(-I*c*tan(f*x + e) +
c)^(3/2)*(A - 3*I*B)*a^2*c - 30*sqrt(-I*c*tan(f*x + e) + c)*(A - I*B)*a^2*
c^2)/(c^2*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.34

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= \frac{2\,a^2\,\sqrt{\frac{c\,\left(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+2\,f\,x)\,1\mathrm{i}\right)}{\cos(2\,e+2\,f\,x)+1}}\,\left(A\,250\mathrm{i}+174\,B+A\,\cos\left(2\,e+2\,f\,x\right)\,375\mathrm{i}+A\,\cos\left(4\,e+4\,f\,x\right)\,150\mathrm{i}\right.}{}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i)
^(1/2),x)
```

output
```
(2*a^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
 + 1))^(1/2)*(A*250i + 174*B + A*cos(2*e + 2*f*x)*375i + A*cos(4*e + 4*f*x
)*150i + A*cos(6*e + 6*f*x)*25i + 267*B*cos(2*e + 2*f*x) + 114*B*cos(4*e +
 4*f*x) + 21*B*cos(6*e + 6*f*x) - 25*A*sin(2*e + 2*f*x) - 20*A*sin(4*e + 4
*f*x) - 5*A*sin(6*e + 6*f*x) + B*sin(2*e + 2*f*x)*45i + B*sin(4*e + 4*f*x)
*36i + B*sin(6*e + 6*f*x)*9i))/(15*f*(15*cos(2*e + 2*f*x) + 6*cos(4*e + 4*
f*x) + cos(6*e + 6*f*x) + 10))
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^2(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$=\frac{\sqrt{c}\,a^2\left(2\sqrt{-\tan(fx+e)\,i+1}\,ai-\left(\int\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^3\,dx\right)bf-\left(\int\sqrt{-\tan(fx+\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*a**2*(2*sqrt( - tan(e + f*x)*i + 1)*a*i - int(sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**3,x)*b*f - int(sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x)**2,x)*a*f + 2*int(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)*b*f*
i + int(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x),x)*a*f*i + int(sqrt( - ta
n(e + f*x)*i + 1)*tan(e + f*x),x)*b*f))/f
```

3.752 $\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 101

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{4a^2(iA+B)}{f\sqrt{c-ic\tan(e+fx)}}-\frac{2a^2(iA+3B)\sqrt{c-ic\tan(e+fx)}}{cf}+\frac{2a^2B(c-ic\tan(e+fx))^{3/2}}{3c^2f}$$

output
```
-4*a^2*(I*A+B)/f/(c-I*c*tan(f*x+e))^(1/2)-2*a^2*(I*A+3*B)*(c-I*c*tan(f*x+e
))^(1/2)/c/f+2/3*a^2*B*(c-I*c*tan(f*x+e))^(3/2)/c^2/f
```

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{2a^2(9iA+14B+(3A-7iB)\tan(e+fx)+B\tan^2(e+fx))}{3f\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan
[e + f*x]],x]
```

output
```
(-2*a^2*((9*I)*A + 14*B + (3*A - (7*I)*B)*Tan[e + f*x] + B*Tan[e + f*x]^2)
)/(3*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^2c\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{a^2c\int \left(\frac{2(A-iB)}{(c-ic\tan(e+fx))^{3/2}}-\frac{iB\sqrt{c-ic\tan(e+fx)}}{c^2}+\frac{3iB-A}{c\sqrt{c-ic\tan(e+fx)}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2c\left(-\frac{2(3B+iA)\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{4(B+iA)}{c\sqrt{c-ic\tan(e+fx)}}+\frac{2B(c-ic\tan(e+fx))^{3/2}}{3c^3}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e + f
*x]],x]
```

output
```
(a^2*c*((-4*(I*A + B))/(c*Sqrt[c - I*c*Tan[e + f*x]]) - (2*(I*A + 3*B)*Sqr
t[c - I*c*Tan[e + f*x]])/c^2 + (2*B*(c - I*c*Tan[e + f*x])^(3/2))/(3*c^3))
)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2ia^2\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+3i\sqrt{c-ic\tan(fx+e)}\,Bc-\sqrt{c-ic\tan(fx+e)}\,cA-\frac{2c^2(-iB+A)}{\sqrt{c-ic\tan(fx+e)}}\right)}{f\,c^2}$
default	$\frac{2ia^2\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+3i\sqrt{c-ic\tan(fx+e)}\,Bc-\sqrt{c-ic\tan(fx+e)}\,cA-\frac{2c^2(-iB+A)}{\sqrt{c-ic\tan(fx+e)}}\right)}{f\,c^2}$
parts	$\frac{2iA\,a^2c\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}-\frac{1}{2c\sqrt{c-ic\tan(fx+e)}}\right)}{f}+\frac{a^2(2iA+B)\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right.}{2\sqrt{c}}\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^2/c^2*(-1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)+3*I*(c-I*c*tan(f*x+e))^(1
/2)*B*c-(c-I*c*tan(f*x+e))^(1/2)*c*A-2*c^2*(A-I*B)/(c-I*c*tan(f*x+e))^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int\frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx=-\frac{2\sqrt{2}\left(3\,(i\,A+B)a^2e^{(4i\,fx+4i\,e)}+3\,(3i\,A+5\,B)a^2e^{(2i\,fx+2i\,e)}+2\,(3i\,A+5\,B)a^2\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{3\,(cfe^{(2i\,fx+2i\,e)}+cf)}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
-2/3*sqrt(2)*(3*(I*A + B)*a^2*e^(4*I*f*x + 4*I*e) + 3*(3*I*A + 5*B)*a^2*e^
(2*I*f*x + 2*I*e) + 2*(3*I*A + 5*B)*a^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))
/(c*f*e^(2*I*f*x + 2*I*e) + c*f)
```

Sympy [F]

$$\int \frac{(a + ia\tan(e + fx))^2(A + B\tan(e + fx))}{\sqrt{c - ic\tan(e + fx)}}\,dx$$
$$= -a^2\left(\int\left(-\frac{A}{\sqrt{-ic\tan(e+fx)+c}}\right)dx + \int\frac{A\tan^2(e+fx)}{\sqrt{-ic\tan(e+fx)+c}}\,dx + \int\left(-\frac{B\tan(e+fx)}{\sqrt{-ic\tan(e+fx)+c}}\right)dx + \int\frac{B\tan^3(e+fx)}{\sqrt{-ic\tan(e+fx)+c}}\,dx + \int\left(-\frac{2iA\tan(e+fx)}{\sqrt{-ic\tan(e+fx)+c}}\right)dx + \int\left(-\frac{2iB\tan^2(e+fx)}{\sqrt{-ic\tan(e+fx)+c}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(1/2)
,x)
```

output
```
-a**2*(Integral(-A/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(A*tan(e + f*
x)**2/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-B*tan(e + f*x)/sqrt(-I*c
*tan(e + f*x) + c), x) + Integral(B*tan(e + f*x)**3/sqrt(-I*c*tan(e + f*x)
 + c), x) + Integral(-2*I*A*tan(e + f*x)/sqrt(-I*c*tan(e + f*x) + c), x) +
 Integral(-2*I*B*tan(e + f*x)**2/sqrt(-I*c*tan(e + f*x) + c), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{(a + ia\tan(e + fx))^2(A + B\tan(e + fx))}{\sqrt{c - ic\tan(e + fx)}}\,dx = -\frac{2i\left(\frac{6\,(A-i\,B)a^2c}{\sqrt{-i\,c\tan(fx+e)+c}} + \frac{i\,(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}\,Ba^2+3\,\sqrt{-i\,c\tan(fx+e)+c}(A-3i\,B)a^2c}{c}\right)}{3\,cf}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
-2/3*I*(6*(A - I*B)*a^2*c/sqrt(-I*c*tan(f*x + e) + c) + (I*(-I*c*tan(f*x +
 e) + c)^(3/2)*B*a^2 + 3*sqrt(-I*c*tan(f*x + e) + c)*(A - 3*I*B)*a^2*c)/c)
/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.74

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{2\sqrt{2}\,a^2\sqrt{\frac{c}{\cos(2e+2fx)+1+\sin(2e+2fx)\,1i}}\,(A\,6i+10\,B+A\,\cos(2\,e+2\,f\,x)\,9i+A\,\cos(4\,e+4\,f\,x)\,3i+15}{3\,c\,f\,(}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1
i)^(1/2),x)
```

output
```
-(2*2^(1/2)*a^2*(c/(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))^(1/2)*(A*
6i + 10*B + A*cos(2*e + 2*f*x)*9i + A*cos(4*e + 4*f*x)*3i + 15*B*cos(2*e +
 2*f*x) + 3*B*cos(4*e + 4*f*x) - 9*A*sin(2*e + 2*f*x) - 3*A*sin(4*e + 4*f*
x) + B*sin(2*e + 2*f*x)*15i + B*sin(4*e + 4*f*x)*3i))/(3*c*f*(cos(2*e + 2*
f*x) + sin(2*e + 2*f*x)*1i + 1))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*a**2*(2*sqrt( - tan(e + f*x)*i + 1)*a*i - int((sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f*i
- int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2 + 1),
x)*b*f*i - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)
**2 + 1),x)*tan(e + f*x)**2*a*f*i - 3*int((sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)**3)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f - int((sqrt( -
tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**2 + 1),x)*a*f*i - 3*in
t((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**2 + 1),x)*b
*f - 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2
+ 1),x)*tan(e + f*x)**2*a*f + 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x)**2)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f*i - 3*int((sqrt( - ta
n(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*a*f + 3*int((s
qrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*b*f*i
 + 6*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**2 + 1),
x)*tan(e + f*x)**2*a*f*i + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/
(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f + 6*int((sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x))/(tan(e + f*x)**2 + 1),x)*a*f*i + int((sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**2 + 1),x)*b*f))/(c*f*(tan(e + f
*x)**2 + 1))
```

3.753 $\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 101

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{4a^2(iA+B)}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{2a^2(iA+3B)}{cf\sqrt{c-ic\tan(e+fx)}} + \frac{2a^2B\sqrt{c-ic\tan(e+fx)}}{c^2f}$$

output
```
-4/3*a^2*(I*A+B)/f/(c-I*c*tan(f*x+e))^(3/2)+2*a^2*(I*A+3*B)/c/f/(c-I*c*tan
(f*x+e))^(1/2)+2*a^2*B*(c-I*c*tan(f*x+e))^(1/2)/c^2/f
```

Mathematica [A] (verified)

Time = 4.58 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{2a^2(-A+10iB+3(iA+5B)\tan(e+fx)-3iB\tan^2}{3cf(i+\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^(3/2),x]
```

output
```
(2*a^2*(-A + (10*I)*B + 3*(I*A + 5*B)*Tan[e + f*x] - (3*I)*B*Tan[e + f*x]^
2))/(3*c*f*(I + Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2c\int \left(\frac{2(A-iB)}{(c-ic\tan(e+fx))^{5/2}}-\frac{iB}{c^2\sqrt{c-ic\tan(e+fx)}}+\frac{3iB-A}{c(c-ic\tan(e+fx))^{3/2}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c\left(\frac{2(3B+iA)}{c^2\sqrt{c-ic\tan(e+fx)}}-\frac{4(B+iA)}{3c(c-ic\tan(e+fx))^{3/2}}+\frac{2B\sqrt{c-ic\tan(e+fx)}}{c^3}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^(3/2),x]
```

output
```
(a^2*c*((-4*(I*A + B))/(3*c*(c - I*c*Tan[e + f*x])^(3/2)) + (2*(I*A + 3*B)
)/(c^2*Sqrt[c - I*c*Tan[e + f*x]]) + (2*B*Sqrt[c - I*c*Tan[e + f*x]])/c^3)
)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{2ia^{2}\left(-iB\sqrt{c-ic\tan(fx+e)}-\frac{2c^{2}(-iB+A)}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{c(-3iB+A)}{\sqrt{c-ic\tan(fx+e)}}\right)}{f\,c^{2}}$
default	$\frac{2ia^{2}\left(-iB\sqrt{c-ic\tan(fx+e)}-\frac{2c^{2}(-iB+A)}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{c(-3iB+A)}{\sqrt{c-ic\tan(fx+e)}}\right)}{f\,c^{2}}$
parts	$\frac{2iA\,a^{2}c\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{8c^{\frac{5}{2}}}-\frac{1}{4c^{2}\sqrt{c-ic\tan(fx+e)}}-\frac{1}{6c(c-ic\tan(fx+e))^{\frac{3}{2}}}\right)}{f}+\frac{a^{2}(2iA+B)\left(-\frac{\sqrt{2}\,\operatorname{arcta}}{}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^2/c^2*(-I*B*(c-I*c*tan(f*x+e))^(1/2)-2/3*c^2*(A-I*B)/(c-I*c*tan(f*
x+e))^(3/2)+c*(A-3*I*B)/(c-I*c*tan(f*x+e))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx=\frac{\sqrt{2}\left((-iA-B)a^2e^{(4i\,fx+4i\,e)}+(iA+7\,B)a^2e^{(2i\,fx+2i\,e)}\right.}{3\,c^2f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
1/3*sqrt(2)*((-I*A - B)*a^2*e^(4*I*f*x + 4*I*e) + (I*A + 7*B)*a^2*e^(2*I*f
*x + 2*I*e) - 2*(-I*A - 7*B)*a^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f
)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -a^2\left(\int \left(-\frac{A}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\right)dx + \int \frac{A\tan^2(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\,dx + \int \left(-\frac{B\tan(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\right)dx + \int \frac{B\tan^3(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\,dx + \int \left(-\frac{2iA\tan(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\right)dx + \int \left(-\frac{2iB\tan^2(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(3/2)
,x)
```

output
```
-a**2*(Integral(-A/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt
(-I*c*tan(e + f*x) + c)), x) + Integral(A*tan(e + f*x)**2/(-I*c*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Inte
gral(-B*tan(e + f*x)/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sq
rt(-I*c*tan(e + f*x) + c)), x) + Integral(B*tan(e + f*x)**3/(-I*c*sqrt(-I*
c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + In
tegral(-2*I*A*tan(e + f*x)/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)
+ c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-2*I*B*tan(e + f*x)**2/(-I
*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c
)), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{2i\left(\frac{3i\sqrt{-ic\tan(fx+e)+c}Ba^2}{c}-\frac{3(-ic\tan(fx+e)+c)(A-3iB)a^2-2(A-iB)a^2c}{(-ic\tan(fx+e)+c)^{\frac{3}{2}}}\right)}{3cf}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
-2/3*I*(3*I*sqrt(-I*c*tan(f*x + e) + c)*B*a^2/c - (3*(-I*c*tan(f*x + e) +
c)*(A - 3*I*B)*a^2 - 2*(A - I*B)*a^2*c)/(-I*c*tan(f*x + e) + c)^(3/2))/(c*
f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.56

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{a^2\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(A\,2\mathrm{i}+14\,B+A\cos$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1
i)^(3/2),x)
```

output
```
(a^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
 1))^(1/2)*(A*2i + 14*B + A*cos(2*e + 2*f*x)*1i - A*cos(4*e + 4*f*x)*1i +
7*B*cos(2*e + 2*f*x) - B*cos(4*e + 4*f*x) - A*sin(2*e + 2*f*x) + A*sin(4*e
 + 4*f*x) + B*sin(2*e + 2*f*x)*7i - B*sin(4*e + 4*f*x)*1i))/(3*c^2*f)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*a**2*(2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*a - 6*sqrt( - ta
n(e + f*x)*i + 1)*tan(e + f*x)*b*i + 2*sqrt( - tan(e + f*x)*i + 1)*b + int
(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e
+ f*x)*i - 1),x)*tan(e + f*x)**2*a*f - 7*int(sqrt( - tan(e + f*x)*i + 1)/(
tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)*
*2*b*f*i + int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i - tan(e + f*
x)**2 + tan(e + f*x)*i - 1),x)*a*f - 7*int(sqrt( - tan(e + f*x)*i + 1)/(ta
n(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*b*f*i + int((sq
rt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i - tan(e + f*
x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f*i + int((sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(
e + f*x)*i - 1),x)*b*f*i - 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)
**2)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e +
 f*x)**2*a*f - 6*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e
+ f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f
*i - 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*
i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*a*f - 6*int((sqrt( - tan(e +
f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e
+ f*x)*i - 1),x)*b*f*i - 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/
(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f...
```

3.754 $\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 103

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{4a^2(iA+B)}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{2a^2(iA+3B)}{3cf(c-ic\tan(e+fx))^{3/2}} - \frac{2a^2B}{c^2f\sqrt{c-ic\tan(e+fx)}}$$

output
```
-4/5*a^2*(I*A+B)/f/(c-I*c*tan(f*x+e))^(5/2)+2/3*a^2*(I*A+3*B)/c/f/(c-I*c*t
an(f*x+e))^(3/2)-2*a^2*B/c^2/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 5.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{2a^2(-iA-6B+5(A+3iB)\tan(e+fx)+15B\tan^2(e+fx))}{15c^2f(i+\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^(5/2),x]
```

output
```
(-2*a^2*((-I)*A - 6*B + 5*(A + (3*I)*B)*Tan[e + f*x] + 15*B*Tan[e + f*x]^2
))/(15*c^2*f*(I + Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2c\int \left(\frac{2(A-iB)}{(c-ic\tan(e+fx))^{7/2}}-\frac{iB}{c^2(c-ic\tan(e+fx))^{3/2}}+\frac{3iB-A}{c(c-ic\tan(e+fx))^{5/2}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c\left(\frac{2(3B+iA)}{3c^2(c-ic\tan(e+fx))^{3/2}}-\frac{4(B+iA)}{5c(c-ic\tan(e+fx))^{5/2}}-\frac{2B}{c^3\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^(5/2),x]
```

output
```
(a^2*c*((-4*(I*A + B))/(5*c*(c - I*c*Tan[e + f*x])^(5/2)) + (2*(I*A + 3*B)
)/(3*c^2*(c - I*c*Tan[e + f*x])^(3/2)) - (2*B)/(c^3*Sqrt[c - I*c*Tan[e + f
*x]])))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{2ia^2\left(-\frac{2c^2(-iB+A)}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{c(-3iB+A)}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{iB}{\sqrt{c-ic\tan(fx+e)}}\right)}{f\,c^2}$
default	$\frac{2ia^2\left(-\frac{2c^2(-iB+A)}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{c(-3iB+A)}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{iB}{\sqrt{c-ic\tan(fx+e)}}\right)}{f\,c^2}$
risch	$-\frac{a^2\left(3iA\,e^{4i(fx+e)}+3B\,e^{4i(fx+e)}+iA\,e^{2i(fx+e)}-9B\,e^{2i(fx+e)}-2iA+18B\right)\sqrt{2}}{30c^2\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
parts	$\frac{2iA\,a^2c\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{16c^{\frac{7}{2}}}-\frac{1}{8c^3\sqrt{c-ic\tan(fx+e)}}-\frac{1}{12c^2(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{10c(c-ic\tan(fx+e))^{\frac{5}{2}}}\right)}{f}+$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^2/c^2*(-2/5*c^2*(A-I*B)/(c-I*c*tan(f*x+e))^(5/2)+1/3*c*(A-3*I*B)/(
c-I*c*tan(f*x+e))^(3/2)+I*B/(c-I*c*tan(f*x+e))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{2}\left(3\,(i\,A+B)a^2e^{(6i\,fx+6i\,e)}+2\,(2i\,A-3\,B)a^2e^{(4i\,fx+4i\,e)}-(i\,A-9\,B)a^2e^{(2i\,fx+2i\,e)}+2\,(-i\,A+9\,B)a^2\right)}{30\,c^3f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
-1/30*sqrt(2)*(3*(I*A + B)*a^2*e^(6*I*f*x + 6*I*e) + 2*(2*I*A - 3*B)*a^2*e
^(4*I*f*x + 4*I*e) - (I*A - 9*B)*a^2*e^(2*I*f*x + 2*I*e) + 2*(-I*A + 9*B)*
a^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx =$$

$$-a^2\Bigg(\int\Bigg(-\frac{A}{-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan^2\left(e+fx\right)-2ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)+c^2\sqrt{-i}$$

$$+\int\frac{A\tan^2\left(e+fx\right)}{-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan^2\left(e+fx\right)-2ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)+c^2\sqrt{-ic\tan\left(e-\right.}$$

$$+\int\Bigg(-\frac{B\tan\left(e+fx\right)}{-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan^2\left(e+fx\right)-2ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)+c^2\sqrt{-ic\tan}$$

$$+\int\frac{B\tan^3\left(e+fx\right)}{-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan^2\left(e+fx\right)-2ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)+c^2\sqrt{-ic\tan\left(e-\right.}$$

$$+\int\Bigg(-\frac{2iA\tan\left(e+fx\right)}{-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan^2\left(e+fx\right)-2ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)+c^2\sqrt{-ic\tan}$$

$$+\int\Bigg(-\frac{2iB\tan^2\left(e+fx\right)}{-c^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan^2\left(e+fx\right)-2ic^2\sqrt{-ic\tan\left(e+fx\right)+c}\tan\left(e+fx\right)+c^2\sqrt{-ic\tan}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(5/2)
,x)
```

output
```
-a**2*(Integral(-A/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*
I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f
*x) + c)), x) + Integral(A*tan(e + f*x)**2/(-c**2*sqrt(-I*c*tan(e + f*x) +
 c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) +
c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-B*tan(e + f*x)/(-c**2*sq
rt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x
) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(B*t
an(e + f*x)**3/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c*
*2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x)
+ c)), x) + Integral(-2*I*A*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x) + c
)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*
*2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-2*I*B*tan(e + f*x)**2/(-c*
*2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e
+ f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{2i\left(15i\left(-i\,c\tan\left(fx+e\right)+c\right)^2Ba^2+5\left(-i\,c\tan\left(fx+\right.\right.}{15\left(-i\,c\tan\left(fx+e\right)\right.}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
2/15*I*(15*I*(-I*c*tan(f*x + e) + c)^2*B*a^2 + 5*(-I*c*tan(f*x + e) + c)*(
A - 3*I*B)*a^2*c - 6*(A - I*B)*a^2*c^2)/((-I*c*tan(f*x + e) + c)^(5/2)*c^2
*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.02

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{a^2\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(18\,B-A\,2\mathrm{i}-A\,\cos(2\,e+2\,f\,x)\,1\mathrm{i}+A\,\cos(4\,e+4\,f\,x)\,4\mathrm{i}+A\,\cos(}{}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1
i)^(5/2),x)
```

output
```
-(a^2*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(18*B - A*2i - A*cos(2*e + 2*f*x)*1i + A*cos(4*e + 4*f*x)*4i +
 A*cos(6*e + 6*f*x)*3i + 9*B*cos(2*e + 2*f*x) - 6*B*cos(4*e + 4*f*x) + 3*B
*cos(6*e + 6*f*x) + A*sin(2*e + 2*f*x) - 4*A*sin(4*e + 4*f*x) - 3*A*sin(6*
e + 6*f*x) + B*sin(2*e + 2*f*x)*9i - B*sin(4*e + 4*f*x)*6i + B*sin(6*e + 6
*f*x)*3i))/(30*c^3*f)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{a^2\left(\left(\int \frac{\tan(fx+e)^3}{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^2+2\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e}\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(a**2*(int(tan(e + f*x)**3/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 +
2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)
),x)*b + int(tan(e + f*x)**2/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2
+ 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i +
1)),x)*a - 2*int(tan(e + f*x)**2/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)
**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*
i + 1)),x)*b*i - 2*int(tan(e + f*x)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*
x)*i + 1)),x)*a*i - int(tan(e + f*x)/(sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f
*x)*i + 1)),x)*b - int(1/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*
sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),
x)*a))/(sqrt(c)*c**2)
```

3.755 $\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 105

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{4a^2(iA+B)}{7f(c-ic\tan(e+fx))^{7/2}} + \frac{2a^2(iA+3B)}{5cf(c-ic\tan(e+fx))^{5/2}} - \frac{2a^2B}{3c^2f(c-ic\tan(e+fx))^{3/2}}$$

output
```
-4/7*a^2*(I*A+B)/f/(c-I*c*tan(f*x+e))^(7/2)+2/5*a^2*(I*A+3*B)/c/f/(c-I*c*t
an(f*x+e))^(5/2)-2/3*a^2*B/c^2/f/(c-I*c*tan(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 5.65 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \frac{2a^2(-9A+2iB+7(-3iA+B)\tan(e+fx)-35iB\text{ta}}{105c^3f(i+\tan(e+fx))^3\sqrt{c-ic\tan(e+f}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^(7/2),x]
```

output
```
(2*a^2*(-9*A + (2*I)*B + 7*((-3*I)*A + B)*Tan[e + f*x] - (35*I)*B*Tan[e +
f*x]^2))/(105*c^3*f*(I + Tan[e + f*x])^3*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2c\int \frac{(i\tan(e+fx)+1)(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^2c\int \left(\frac{2(A-iB)}{(c-ic\tan(e+fx))^{9/2}}-\frac{iB}{c^2(c-ic\tan(e+fx))^{5/2}}+\frac{3iB-A}{c(c-ic\tan(e+fx))^{7/2}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2c\left(\frac{2(3B+iA)}{5c^2(c-ic\tan(e+fx))^{5/2}}-\frac{4(B+iA)}{7c(c-ic\tan(e+fx))^{7/2}}-\frac{2B}{3c^3(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^2*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^(7/2),x]
```

output
```
(a^2*c*((-4*(I*A + B))/(7*c*(c - I*c*Tan[e + f*x])^(7/2)) + (2*(I*A + 3*B)
)/(5*c^2*(c - I*c*Tan[e + f*x])^(5/2)) - (2*B)/(3*c^3*(c - I*c*Tan[e + f*x
])^(3/2))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{2ia^2\left(-\frac{2c^2(-iB+A)}{7(c-ic\tan(fx+e))^{\frac{7}{2}}}+\frac{iB}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{c(-3iB+A)}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}\right)}{f\,c^2}$
default	$\frac{2ia^2\left(-\frac{2c^2(-iB+A)}{7(c-ic\tan(fx+e))^{\frac{7}{2}}}+\frac{iB}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{c(-3iB+A)}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}\right)}{f\,c^2}$
risch	$-\frac{a^2\left(15iA\,e^{6i(fx+e)}+15B\,e^{6i(fx+e)}+24iA\,e^{4i(fx+e)}-18B\,e^{4i(fx+e)}+3iA\,e^{2i(fx+e)}-11B\,e^{2i(fx+e)}-6iA+22B\right)\sqrt{2}}{420c^3\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
parts	$\frac{2iA\,a^2c\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{32c^{\frac{9}{2}}}-\frac{1}{16c^4\sqrt{c-ic\tan(fx+e)}}-\frac{1}{24c^3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{20c^2(c-ic\tan(fx+e))^{\frac{5}{2}}}-\cdots\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^2/c^2*(-2/7*c^2*(A-I*B)/(c-I*c*tan(f*x+e))^(7/2)+1/3*I*B/(c-I*c*ta
n(f*x+e))^(3/2)+1/5*c*(A-3*I*B)/(c-I*c*tan(f*x+e))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.17

$$\int\frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx=$$
$$-\frac{\sqrt{2}\left(15\,(i\,A+B)a^2e^{(8i\,fx+8i\,e)}+3\,(13i\,A-B)a^2e^{(6i\,fx+6i\,e)}-(-27i\,A+29\,B)a^2e^{(4i\,fx+4i\,e)}-(3i\,A-11\,B\cdots\right.}{420\,c^4f}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
-1/420*sqrt(2)*(15*(I*A + B)*a^2*e^(8*I*f*x + 8*I*e) + 3*(13*I*A - B)*a^2*
e^(6*I*f*x + 6*I*e) - (-27*I*A + 29*B)*a^2*e^(4*I*f*x + 4*I*e) - (3*I*A -
11*B)*a^2*e^(2*I*f*x + 2*I*e) + 2*(-3*I*A + 11*B)*a^2)*sqrt(c/(e^(2*I*f*x
+ 2*I*e) + 1))/(c^4*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))**2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(7/2)
,x)
```

output
```
-a**2*(Integral(-A/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - 3
*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-I*c*tan
(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) + Inte
gral(A*tan(e + f*x)**2/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3
 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-I*c
*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) +
Integral(-B*tan(e + f*x)/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)*
*3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-I
*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x)
+ Integral(B*tan(e + f*x)**3/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f
*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqr
t(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)),
 x) + Integral(-2*I*A*tan(e + f*x)/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan
(e + f*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c*
*3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x)
+ c)), x) + Integral(-2*I*B*tan(e + f*x)**2/(I*c**3*sqrt(-I*c*tan(e + f*x)
 + c)*tan(e + f*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2
 - 3*I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(
e + f*x) + c)), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \frac{2i\left(35i\left(-i\,c\tan\left(fx+e\right)+c\right)^2Ba^2+21\left(-i\,c\tan\left(fx\right.\right.\right.}{105\left(-i\,c\tan\left(fx+e\right)\right.}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
2/105*I*(35*I*(-I*c*tan(f*x + e) + c)^2*B*a^2 + 21*(-I*c*tan(f*x + e) + c)
*(A - 3*I*B)*a^2*c - 30*(A - I*B)*a^2*c^2)/((-I*c*tan(f*x + e) + c)^(7/2)*
c^2*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.59

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\sqrt{c-\frac{c\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}}\left(-\frac{a^2\,(3\,A+B\,11\mathrm{i})\,1\mathrm{i}}{210\,c^4\,f} - \frac{a^2\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,(3\,A+B\,11\mathrm{i})\,1\mathrm{i}}{420\,c^4\,f} + \frac{a^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,(13\,A+B\,1\mathrm{i})\,1\mathrm{i}}{140\,c^4\,f} + \frac{a^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,(27\,A+B\,29\mathrm{i})\,1\mathrm{i}}{420\,c^4\,f} + \frac{a^2\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,(A-B\,1\mathrm{i})\,1\mathrm{i}}{28\,c^4\,f}\right)$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^2)/(c - c*tan(e + f*x)*1
i)^(7/2),x)
```

output
```
-(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i)*(13
*A + B*1i)*1i)/(140*c^4*f) - (a^2*exp(e*2i + f*x*2i)*(3*A + B*11i)*1i)/(42
0*c^4*f) - (a^2*(3*A + B*11i)*1i)/(210*c^4*f) + (a^2*exp(e*4i + f*x*4i)*(2
7*A + B*29i)*1i)/(420*c^4*f) + (a^2*exp(e*8i + f*x*8i)*(A - B*1i)*1i)/(28*
c^4*f))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^2*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x)
```

output

```
(sqrt(c)*a**2*(46*sqrt( - tan(e + f*x)*i + 1)*a*i - 2*sqrt( - tan(e + f*x)
*i + 1)*b - 2*int(( - sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**5*i - 3*
tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)
*i + 1),x)*tan(e + f*x)**2*a*f + 6*int(( - sqrt( - tan(e + f*x)*i + 1))/(t
an(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x
)**2 - 3*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i - 2*int(( - sqrt( -
tan(e + f*x)*i + 1))/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*
x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*a*f + 6*int(( - sqr
t( - tan(e + f*x)*i + 1))/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e
 + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b*f*i - 6*int(
( - sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**5*i - 3*ta
n(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i
 + 1),x)*tan(e + f*x)**2*a*f + 18*int(( - sqrt( - tan(e + f*x)*i + 1)*tan(
e + f*x)**4)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i
- 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i - 6*i
nt(( - sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**5*i - 3
*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x
)*i + 1),x)*a*f + 18*int(( - sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/
(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f
*x)**2 - 3*tan(e + f*x)*i + 1),x)*b*f*i + 24*int(( - sqrt( - tan(e + f*...
```

3.756 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 144

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx=\frac{8a^3(iA+B)(c-ic\tan(e+fx))^{7/2}}{7f}-\frac{8a^3(iA+2B)(c-ic\tan(e+fx))^{9/2}}{9cf}+\frac{2a^3(iA+5B)(c-ic\tan(e+fx))^{11/2}}{11c^2f}-\frac{2a^3B(c-ic\tan(e+fx))^{13/2}}{13c^3f}$$

output
```
8/7*a^3*(I*A+B)*(c-I*c*tan(f*x+e))^(7/2)/f-8/9*a^3*(I*A+2*B)*(c-I*c*tan(f*
x+e))^(9/2)/c/f+2/11*a^3*(I*A+5*B)*(c-I*c*tan(f*x+e))^(11/2)/c^2/f-2/13*a^
3*B*(c-I*c*tan(f*x+e))^(13/2)/c^3/f
```

Mathematica [A] (verified)

Time = 5.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{a^3c^4\sec^6(e+fx)(i\cos(4(e+fx))+\sin(4(e+fx)))(2(572A+737iB+7(169iA+86B)\text{ta}}{9009f\sqrt{c-ic\tan}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^(7/2),x]
```

output
```
(a^3*c^4*Sec[e + f*x]^6*(I*Cos[4*(e + f*x)] + Sin[4*(e + f*x)])*(2*(572*A
+ (737*I)*B + 7*((169*I)*A + 86*B)*Tan[e + f*x]) + Cos[2*(e + f*x)]*(2782*
A - (2558*I)*B + 14*((169*I)*A + 185*B)*Tan[e + f*x])))/(9009*f*Sqrt[c - I
*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c\int (i\tan(e+fx)+1)^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3c\int \left(\frac{iB(c-ic\tan(e+fx))^{11/2}}{c^3}+\frac{(A-5iB)(c-ic\tan(e+fx))^{9/2}}{c^2}-\frac{4(A-2iB)(c-ic\tan(e+fx))^{7/2}}{c}+4(A-iB)(c-ic\tan(e+f\right.}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3c\left(\frac{2(5B+iA)(c-ic\tan(e+fx))^{11/2}}{11c^3}-\frac{8(2B+iA)(c-ic\tan(e+fx))^{9/2}}{9c^2}+\frac{8(B+iA)(c-ic\tan(e+fx))^{7/2}}{7c}-\frac{2B(c-ic\tan(e+fx))^{13/2}}{13c^4}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(
7/2),x]
```

output
```
(a^3*c*((8*(I*A + B)*(c - I*c*Tan[e + f*x])^(7/2))/(7*c) - (8*(I*A + 2*B)*
(c - I*c*Tan[e + f*x])^(9/2))/(9*c^2) + (2*(I*A + 5*B)*(c - I*c*Tan[e + f*
x])^(11/2))/(11*c^3) - (2*B*(c - I*c*Tan[e + f*x])^(13/2))/(13*c^4)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{2ia^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{13}{2}}}{13}+\frac{(-5iBc+cA)(c-ic\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{\left(-4(-iBc+cA)c+4iB\,c^2\right)(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{4(-iBc+cA}{}\right.}{f\,c^3}$
default	$\frac{2ia^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{13}{2}}}{13}+\frac{(-5iBc+cA)(c-ic\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{\left(-4(-iBc+cA)c+4iB\,c^2\right)(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{4(-iBc+cA}{}\right.}{f\,c^3}$
parts	$\frac{2iA\,a^3c\left(-\frac{(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{2c(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-4\sqrt{c-ic\tan(fx+e)}\,c^2+4c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{f}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^3/c^3*(1/13*I*B*(c-I*c*tan(f*x+e))^(13/2)+1/11*(-5*I*B*c+c*A)*(c-I
*c*tan(f*x+e))^(11/2)+1/9*(-4*(-I*B*c+c*A)*c+4*I*B*c^2)*(c-I*c*tan(f*x+e))
^(9/2)+4/7*(-I*B*c+c*A)*c^2*(c-I*c*tan(f*x+e))^(7/2))
```

Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.24

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx =$$
$$-\frac{64\sqrt{2}\left(1287\,(-i\,A-B)a^3c^3e^{(6i\,fx+6i\,e)}+143\,(-13i\,A+B)a^3c^3e^{(4i\,fx+4i\,e)}+52\,(-13i\,A+B)a^3c^3e^{(2i\,fx+2}\right.}{9009\,(fe^{(12i\,fx+12i\,e)}+6\,fe^{(10i\,fx+10i\,e)}+15\,fe^{(8i\,fx+8i\,e)}+20\,fe^{(6i\,fx+6i\,e)}+15\,fe^{(4i\,fx+4i}}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
-64/9009*sqrt(2)*(1287*(-I*A - B)*a^3*c^3*e^(6*I*f*x + 6*I*e) + 143*(-13*I
*A + B)*a^3*c^3*e^(4*I*f*x + 4*I*e) + 52*(-13*I*A + B)*a^3*c^3*e^(2*I*f*x
+ 2*I*e) + 8*(-13*I*A + B)*a^3*c^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e
^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e
) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x +
 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx =$$

$$-ia^3\left(\int iAc^3\sqrt{-ic\tan{(e+fx)}+c}\,dx + \int 3iAc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\,dx + \int 3iAc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^4{(e+fx)}\,dx + \int iAc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^6{(e+fx)}\,dx + \int iBc^3\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\,dx + \int 3iBc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}\,dx + \int 3iBc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^5{(e+fx)}\,dx + \int iBc^3\sqrt{-ic\tan{(e+fx)}+c}\tan^7{(e+fx)}\,dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2)
,x)
```

output
```
-I*a**3*(Integral(I*A*c**3*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(3*I*
A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + Integral(3*I*A*c*
*3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4, x) + Integral(I*A*c**3*sqr
t(-I*c*tan(e + f*x) + c)*tan(e + f*x)**6, x) + Integral(I*B*c**3*sqrt(-I*c
*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(3*I*B*c**3*sqrt(-I*c*tan(e
+ f*x) + c)*tan(e + f*x)**3, x) + Integral(3*I*B*c**3*sqrt(-I*c*tan(e + f*
x) + c)*tan(e + f*x)**5, x) + Integral(I*B*c**3*sqrt(-I*c*tan(e + f*x) + c
)*tan(e + f*x)**7, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{2i\left(693i\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{13}{2}}Ba^3+819\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{11}{2}}(A-5i\,B)a^3c-4004\,\cdots\right.}{9009\,c^3\cdots}$$

input

```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="maxima")
```

output

```
2/9009*I*(693*I*(-I*c*tan(f*x + e) + c)^(13/2)*B*a^3 + 819*(-I*c*tan(f*x +
 e) + c)^(11/2)*(A - 5*I*B)*a^3*c - 4004*(-I*c*tan(f*x + e) + c)^(9/2)*(A
- 2*I*B)*a^3*c^2 + 5148*(-I*c*tan(f*x + e) + c)^(7/2)*(A - I*B)*a^3*c^3)/(
c^3*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.42

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = -\frac{\left(\frac{a^3\,c^3\,(A-B\,1\mathrm{i})\,64\mathrm{i}}{9\,f}+\frac{a^3\,c^3\,(A-B\,3\mathrm{i})\,64\mathrm{i}}{9\,f}\right)\,\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}}{{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4} - \frac{\left(\frac{a^3\,c^3\,(A-B\,1\mathrm{i})\,64\mathrm{i}}{13\,f}-\frac{a^3\,c^3\,(A+B\,1\mathrm{i})\,64\mathrm{i}}{13\,f}\right)\,\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}}{{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^6} + \frac{\left(\frac{256\,B\,a^3\,c^3}{11\,f}+\frac{a^3\,c^3\,(A-B\,1\mathrm{i})\,64\mathrm{i}}{11\,f}\right)\,\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}}{{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^5} + \frac{a^3\,c^3\,(A-B\,1\mathrm{i})\,\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{7\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^(7/2),x)
```

output
```
(((a^3*c^3*(A - B*1i)*64i)/(11*f) + (256*B*a^3*c^3)/(11*f))*(c + (c*(exp(e
*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2))/(exp(e*2i + f*
x*2i) + 1)^5 - (((a^3*c^3*(A - B*1i)*64i)/(13*f) - (a^3*c^3*(A + B*1i)*64i
)/(13*f))*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1
))^(1/2))/(exp(e*2i + f*x*2i) + 1)^6 - (((a^3*c^3*(A - B*1i)*64i)/(9*f) +
(a^3*c^3*(A - B*3i)*64i)/(9*f))*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(
exp(e*2i + f*x*2i) + 1))^(1/2))/(exp(e*2i + f*x*2i) + 1)^4 + (a^3*c^3*(A -
 B*1i)*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^
(1/2)*64i)/(7*f*(exp(e*2i + f*x*2i) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{2\sqrt{c}\,\sqrt{-\tan(fx+e)\,i+1}\,a^3c^3\left(693\tan(fx+e)^6\,b+819\tan(fx+e)^5\,a+63\tan(fx+e)\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(2*sqrt(c)*sqrt( - tan(e + f*x)*i + 1)*a**3*c**3*(693*tan(e + f*x)**6*b +
819*tan(e + f*x)**5*a + 63*tan(e + f*x)**5*b*i + 91*tan(e + f*x)**4*a*i +
2072*tan(e + f*x)**4*b + 2678*tan(e + f*x)**3*a + 206*tan(e + f*x)**3*b*i
+ 390*tan(e + f*x)**2*a*i + 2049*tan(e + f*x)**2*b + 3523*tan(e + f*x)*a +
 271*tan(e + f*x)*b*i + 1963*a*i + 542*b))/(9009*f)
```

3.757 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 144

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\frac{8a^3(iA+B)(c-ic\tan(e+fx))^{5/2}}{5f}-\frac{8a^3(iA+2B)(c-ic\tan(e+fx))^{7/2}}{7cf}+\frac{2a^3(iA+5B)(c-ic\tan(e+fx))^{9/2}}{9c^2f}-\frac{2a^3B(c-ic\tan(e+fx))^{11/2}}{11c^3f}$$

output

```
8/5*a^3*(I*A+B)*(c-I*c*tan(f*x+e))^(5/2)/f-8/7*a^3*(I*A+2*B)*(c-I*c*tan(f*
x+e))^(7/2)/c/f+2/9*a^3*(I*A+5*B)*(c-I*c*tan(f*x+e))^(9/2)/c^2/f-2/11*a^3*
B*(c-I*c*tan(f*x+e))^(11/2)/c^3/f
```

Mathematica [A] (verified)

Time = 4.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \frac{a^3c^3\sec^6(e+fx)(143(11iA+B)\cos(e+fx)+(781iA+701B)\cos(3(e+fx))-10(121A}{3465f\sqrt{c-i}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^(5/2),x]
```

output
```
(a^3*c^3*Sec[e + f*x]^6*(143*((11*I)*A + B)*Cos[e + f*x] + ((781*I)*A + 70
1*B)*Cos[3*(e + f*x)] - 10*(121*A - (74*I)*B + (121*A - (137*I)*B)*Cos[2*(
e + f*x)])*Sin[e + f*x])*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)]))/(3465*f*
Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c\int (i\tan(e+fx)+1)^2(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3c\int \left(\frac{iB(c-ic\tan(e+fx))^{9/2}}{c^3}+\frac{(A-5iB)(c-ic\tan(e+fx))^{7/2}}{c^2}-\frac{4(A-2iB)(c-ic\tan(e+fx))^{5/2}}{c}+4(A-iB)(c-ic\tan(e+fx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3c\left(\frac{2(5B+iA)(c-ic\tan(e+fx))^{9/2}}{9c^3}-\frac{8(2B+iA)(c-ic\tan(e+fx))^{7/2}}{7c^2}+\frac{8(B+iA)(c-ic\tan(e+fx))^{5/2}}{5c}-\frac{2B(c-ic\tan(e+fx))^{11/2}}{11c^4}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(
5/2),x]
```

output
```
(a^3*c*((8*(I*A + B)*(c - I*c*Tan[e + f*x])^(5/2))/(5*c) - (8*(I*A + 2*B)*
(c - I*c*Tan[e + f*x])^(7/2))/(7*c^2) + (2*(I*A + 5*B)*(c - I*c*Tan[e + f*
x])^(9/2))/(9*c^3) - (2*B*(c - I*c*Tan[e + f*x])^(11/2))/(11*c^4)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{2ia^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{(-5iBc+cA)(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{\left(-4(-iBc+cA)c+4iB\,c^2\right)(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{4(-iBc+cA)}{}\right.}{f\,c^3}$
default	$\frac{2ia^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{(-5iBc+cA)(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{\left(-4(-iBc+cA)c+4iB\,c^2\right)(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{4(-iBc+cA)}{}\right.}{f\,c^3}$
parts	$\frac{2iA\,a^3c\left(-\frac{(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-2c\sqrt{c-ic\tan(fx+e)}+2c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{f}+\frac{a^3(3iA+B)\left(\frac{2(c-}{}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^3/c^3*(1/11*I*B*(c-I*c*tan(f*x+e))^(11/2)+1/9*(-5*I*B*c+c*A)*(c-I*
c*tan(f*x+e))^(9/2)+1/7*(-4*(-I*B*c+c*A)*c+4*I*B*c^2)*(c-I*c*tan(f*x+e))^(
7/2)+4/5*(-I*B*c+c*A)*c^2*(c-I*c*tan(f*x+e))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.20

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx =$$

$$-\frac{32\sqrt{2}\left(693\,(-i\,A-B)a^3c^2e^{(6i\,fx+6i\,e)}+99\,(-11i\,A-B)a^3c^2e^{(4i\,fx+4i\,e)}+44\,(-11i\,A-B)a^3c^2e^{(2i\,fx+2i\,e)}\right.}{3465\left(fe^{(10i\,fx+10i\,e)}+5\,fe^{(8i\,fx+8i\,e)}+10\,fe^{(6i\,fx+6i\,e)}+10\,fe^{(4i\,fx+4i\,e)}+5\,fe^{(2i\right.}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
-32/3465*sqrt(2)*(693*(-I*A - B)*a^3*c^2*e^(6*I*f*x + 6*I*e) + 99*(-11*I*A
 - B)*a^3*c^2*e^(4*I*f*x + 4*I*e) + 44*(-11*I*A - B)*a^3*c^2*e^(2*I*f*x +
2*I*e) + 8*(-11*I*A - B)*a^3*c^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(
10*I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) +
10*f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx =$$

$$-ia^3\left(\int iAc^2\sqrt{-ic\tan(e+fx)+c}\,dx + \int\left(-Ac^2\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\right)dx + \int\left(-2Ac^2\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)\right)dx + \int\left(-Ac^2\sqrt{-ic\tan(e+fx)+c}\tan^5(e+fx)\right)dx + \int\left(-Bc^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)\right)dx + \int\left(-2Bc^2\sqrt{-ic\tan(e+fx)+c}\tan^4(e+fx)\right)dx + \int\left(-Bc^2\sqrt{-ic\tan(e+fx)+c}\tan^6(e+fx)\right)dx + \int 2iAc^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)\,dx + \int iAc^2\sqrt{-ic\tan(e+fx)+c}\tan^4(e+fx)\,dx + \int iBc^2\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)\,dx + \int 2iBc^2\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)\,dx + \int iBc^2\sqrt{-ic\tan(e+fx)+c}\tan^5(e+fx)\,dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2)
,x)
```

output
```
-I*a**3*(Integral(I*A*c**2*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-A*c
**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(-2*A*c**2*sqrt
(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3, x) + Integral(-A*c**2*sqrt(-I*c*t
an(e + f*x) + c)*tan(e + f*x)**5, x) + Integral(-B*c**2*sqrt(-I*c*tan(e +
f*x) + c)*tan(e + f*x)**2, x) + Integral(-2*B*c**2*sqrt(-I*c*tan(e + f*x)
+ c)*tan(e + f*x)**4, x) + Integral(-B*c**2*sqrt(-I*c*tan(e + f*x) + c)*ta
n(e + f*x)**6, x) + Integral(2*I*A*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e
+ f*x)**2, x) + Integral(I*A*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)
**4, x) + Integral(I*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) +
 Integral(2*I*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3, x) + Int
egral(I*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \frac{2i\left(315i\,(-i\,c\tan\left(fx+e\right)+c)^{\frac{11}{2}}Ba^3+385\,(-i\,c\tan\left(fx+e\right)+c)^{\frac{9}{2}}(A-5i\,B)a^3c-1980\,(\right.}{3465\,c^3}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
2/3465*I*(315*I*(-I*c*tan(f*x + e) + c)^(11/2)*B*a^3 + 385*(-I*c*tan(f*x +
 e) + c)^(9/2)*(A - 5*I*B)*a^3*c - 1980*(-I*c*tan(f*x + e) + c)^(7/2)*(A -
 2*I*B)*a^3*c^2 + 2772*(-I*c*tan(f*x + e) + c)^(5/2)*(A - I*B)*a^3*c^3)/(c
^3*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.42

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = -\frac{\left(\frac{a^3\,c^2\,(A-B\,1\mathrm{i})\,32\mathrm{i}}{7\,f}+\frac{a^3\,c^2\,(A-B\,3\mathrm{i})\,32\mathrm{i}}{7\,f}\right)\sqrt{c+\frac{c\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3} - \frac{\left(\frac{a^3\,c^2\,(A-B\,1\mathrm{i})\,32\mathrm{i}}{11\,f}-\frac{a^3\,c^2\,(A+B\,1\mathrm{i})\,32\mathrm{i}}{11\,f}\right)\sqrt{c+\frac{c\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^5} + \frac{\left(\frac{128\,B\,a^3\,c^2}{9\,f}+\frac{a^3\,c^2\,(A-B\,1\mathrm{i})\,32\mathrm{i}}{9\,f}\right)\sqrt{c+\frac{c\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}}{{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4} + \frac{a^3\,c^2\,(A-B\,1\mathrm{i})\,\sqrt{c+\frac{c\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{5\,f\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^(5/2),x)
```

output
```
(((a^3*c^2*(A - B*1i)*32i)/(9*f) + (128*B*a^3*c^2)/(9*f))*(c + (c*(exp(e*2
i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2))/(exp(e*2i + f*x*
2i) + 1)^4 - (((a^3*c^2*(A - B*1i)*32i)/(11*f) - (a^3*c^2*(A + B*1i)*32i)/
(11*f))*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))
^(1/2))/(exp(e*2i + f*x*2i) + 1)^5 - (((a^3*c^2*(A - B*1i)*32i)/(7*f) + (a
^3*c^2*(A - B*3i)*32i)/(7*f))*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(ex
p(e*2i + f*x*2i) + 1))^(1/2))/(exp(e*2i + f*x*2i) + 1)^3 + (a^3*c^2*(A - B
*1i)*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1
/2)*32i)/(5*f*(exp(e*2i + f*x*2i) + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \frac{2\sqrt{c}\,\sqrt{-\tan(fx+e)\,i+1}\,a^3c^2\left(315\tan(fx+e)^5\,bi+385\tan(fx+e)^4\,ai+350\tan(fx+\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt( - tan(e + f*x)*i + 1)*a**3*c**2*(315*tan(e + f*x)**5*b*i
+ 385*tan(e + f*x)**4*a*i + 350*tan(e + f*x)**4*b + 440*tan(e + f*x)**3*a
+ 590*tan(e + f*x)**3*b*i + 858*tan(e + f*x)**2*a*i + 708*tan(e + f*x)**2*
b + 1144*tan(e + f*x)*a + 211*tan(e + f*x)*b*i + 1177*a*i + 422*b))/(3465*
f)
```

3.758 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 144

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{8a^3(iA+B)(c-ic\tan(e+fx))^{3/2}}{3f} - \frac{8a^3(iA+2B)(c-ic\tan(e+fx))^{5/2}}{5cf} + \frac{2a^3(iA+5B)(c-ic\tan(e+fx))^{7/2}}{7c^2f} - \frac{2a^3B(c-ic\tan(e+fx))^{9/2}}{9c^3f}$$

output
```
8/3*a^3*(I*A+B)*(c-I*c*tan(f*x+e))^(3/2)/f-8/5*a^3*(I*A+2*B)*(c-I*c*tan(f*
x+e))^(5/2)/c/f+2/7*a^3*(I*A+5*B)*(c-I*c*tan(f*x+e))^(7/2)/c^2/f-2/9*a^3*B
*(c-I*c*tan(f*x+e))^(9/2)/c^3/f
```

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int (a + ia\tan(e + fx))^3(A + B\tan(e + fx))(c - ic\tan(e + fx))^{3/2}\,dx = \frac{a^3c^2\sec^5(e + fx)(99(3iA + B)\cos(e + fx) + (129iA + 113B)\cos(3(e + fx)) - 2(81A - 62}{315f\sqrt{c - ic\tan}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^(3/2),x]
```

output
```
(a^3*c^2*Sec[e + f*x]^5*(99*((3*I)*A + B)*Cos[e + f*x] + ((129*I)*A + 113*
B)*Cos[3*(e + f*x)] - 2*(81*A - (62*I)*B + (81*A - (97*I)*B)*Cos[2*(e + f*
x)])*Sin[e + f*x])*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x)]))/(315*f*Sqrt[c
- I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + ia\tan(e + fx))^3(c - ic\tan(e + fx))^{3/2}(A + B\tan(e + fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a + ia\tan(e + fx))^3(c - ic\tan(e + fx))^{3/2}(A + B\tan(e + fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int a^2(i\tan(e + fx) + 1)^2(A + B\tan(e + fx))\sqrt{c - ic\tan(e + fx)}d\tan(e + fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c\int(i\tan(e+fx)+1)^2(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)}{f}$$

$\big\downarrow$ 86

$$\frac{a^3c\int\left(\frac{iB(c-ic\tan(e+fx))^{7/2}}{c^3}+\frac{(A-5iB)(c-ic\tan(e+fx))^{5/2}}{c^2}-\frac{4(A-2iB)(c-ic\tan(e+fx))^{3/2}}{c}+4(A-iB)\sqrt{c-ic\tan(e+f}\right.}{f}$$

$\big\downarrow$ 2009

$$\frac{a^3c\left(\frac{2(5B+iA)(c-ic\tan(e+fx))^{7/2}}{7c^3}-\frac{8(2B+iA)(c-ic\tan(e+fx))^{5/2}}{5c^2}+\frac{8(B+iA)(c-ic\tan(e+fx))^{3/2}}{3c}-\frac{2B(c-ic\tan(e+fx))^{9/2}}{9c^4}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(
3/2),x]
```

output
```
(a^3*c*((8*(I*A + B)*(c - I*c*Tan[e + f*x])^(3/2))/(3*c) - (8*(I*A + 2*B)*
(c - I*c*Tan[e + f*x])^(5/2))/(5*c^2) + (2*(I*A + 5*B)*(c - I*c*Tan[e + f*
x])^(7/2))/(7*c^3) - (2*B*(c - I*c*Tan[e + f*x])^(9/2))/(9*c^4)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{2ia^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{(-5iBc+cA)(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{\left(-4(-iBc+cA)c+4iB\,c^2\right)(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4(-iBc+cA)c^{2}}{}\right.}{f\,c^3}$
default	$\frac{2ia^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{9}{2}}}{9}+\frac{(-5iBc+cA)(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{\left(-4(-iBc+cA)c+4iB\,c^2\right)(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{4(-iBc+cA)c^{2}}{}\right.}{f\,c^3}$
parts	$\frac{2iA\,a^3c\left(-\sqrt{c-ic\tan(fx+e)}+\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\right)}{f}+\frac{a^3(3iA+B)\left(\frac{2(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+2c\sqrt{c-}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^3/c^3*(1/9*I*B*(c-I*c*tan(f*x+e))^(9/2)+1/7*(-5*I*B*c+c*A)*(c-I*c*
tan(f*x+e))^(7/2)+1/5*(-4*(-I*B*c+c*A)*c+4*I*B*c^2)*(c-I*c*tan(f*x+e))^(5/
2)+4/3*(-I*B*c+c*A)*c^2*(c-I*c*tan(f*x+e))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx =$$
$$-\frac{16\sqrt{2}\left(105\,(-i\,A-B)a^3ce^{(6i\,fx+6i\,e)}+63\,(-3i\,A-B)a^3ce^{(4i\,fx+4i\,e)}+36\,(-3i\,A-B)a^3ce^{(2i\,fx+2i\,e)}+8\,(\right.}{315\,(fe^{(8i\,fx+8i\,e)}+4\,fe^{(6i\,fx+6i\,e)}+6\,fe^{(4i\,fx+4i\,e)}+4\,fe^{(2i\,fx+2i\,e)}+f)}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
-16/315*sqrt(2)*(105*(-I*A - B)*a^3*c*e^(6*I*f*x + 6*I*e) + 63*(-3*I*A - B
)*a^3*c*e^(4*I*f*x + 4*I*e) + 36*(-3*I*A - B)*a^3*c*e^(2*I*f*x + 2*I*e) +
8*(-3*I*A - B)*a^3*c)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(8*I*f*x + 8*
I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x
+ 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx =$$
$$-ia^3\left(\int iAc\sqrt{-ic\tan{(e+fx)}+c}\,dx\right.$$
$$+\int\left(-2Ac\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\right)\,dx$$
$$+\int\left(-2Ac\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}\right)\,dx$$
$$+\int\left(-2Bc\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\right)\,dx$$
$$+\int\left(-2Bc\sqrt{-ic\tan{(e+fx)}+c}\tan^4{(e+fx)}\right)\,dx$$
$$+\int\left(-iAc\sqrt{-ic\tan{(e+fx)}+c}\tan^4{(e+fx)}\right)\,dx$$
$$+\int iBc\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\,dx$$
$$\left.+\int\left(-iBc\sqrt{-ic\tan{(e+fx)}+c}\tan^5{(e+fx)}\right)\,dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)
,x)
```

output
```
-I*a**3*(Integral(I*A*c*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-2*A*c*
sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(-2*A*c*sqrt(-I*c*t
an(e + f*x) + c)*tan(e + f*x)**3, x) + Integral(-2*B*c*sqrt(-I*c*tan(e + f
*x) + c)*tan(e + f*x)**2, x) + Integral(-2*B*c*sqrt(-I*c*tan(e + f*x) + c)
*tan(e + f*x)**4, x) + Integral(-I*A*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
 f*x)**4, x) + Integral(I*B*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x)
 + Integral(-I*B*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**5, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{2i\left(35i\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{9}{2}}Ba^3+45\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{7}{2}}(A-5i\,B)a^3c-252\left(-i\,c\right.\right.}{315\,c^3f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
2/315*I*(35*I*(-I*c*tan(f*x + e) + c)^(9/2)*B*a^3 + 45*(-I*c*tan(f*x + e)
+ c)^(7/2)*(A - 5*I*B)*a^3*c - 252*(-I*c*tan(f*x + e) + c)^(5/2)*(A - 2*I*
B)*a^3*c^2 + 420*(-I*c*tan(f*x + e) + c)^(3/2)*(A - I*B)*a^3*c^3)/(c^3*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.33

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = -\frac{\left(\frac{a^3\,c\,(A-B\,1\mathrm{i})\,16\mathrm{i}}{5\,f}+\frac{a^3\,c\,(A-B\,3\mathrm{i})\,16\mathrm{i}}{5\,f}\right)\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}}{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2} - \frac{\left(\frac{a^3\,c\,(A-B\,1\mathrm{i})\,16\mathrm{i}}{9\,f}-\frac{a^3\,c\,(A+B\,1\mathrm{i})\,16\mathrm{i}}{9\,f}\right)\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}}{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4} + \frac{\left(\frac{64\,B\,a^3\,c}{7\,f}+\frac{a^3\,c\,(A-B\,1\mathrm{i})\,16\mathrm{i}}{7\,f}\right)\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}}{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3} + \frac{a^3\,c\,(A-B\,1\mathrm{i})\,\sqrt{c+\frac{c\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{3\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^(3/2),x)
```

output
```
(((a^3*c*(A - B*1i)*16i)/(7*f) + (64*B*a^3*c)/(7*f))*(c + (c*(exp(e*2i + f
*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2))/(exp(e*2i + f*x*2i) +
 1)^3 - (((a^3*c*(A - B*1i)*16i)/(9*f) - (a^3*c*(A + B*1i)*16i)/(9*f))*(c
+ (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2))/(ex
p(e*2i + f*x*2i) + 1)^4 - (((a^3*c*(A - B*1i)*16i)/(5*f) + (a^3*c*(A - B*3
i)*16i)/(5*f))*(c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i
) + 1))^(1/2))/(exp(e*2i + f*x*2i) + 1)^2 + (a^3*c*(A - B*1i)*(c + (c*(exp
(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2)*16i)/(3*f*(ex
p(e*2i + f*x*2i) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.74

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{2\sqrt{c}\,\sqrt{-\tan(fx+e)\,i+1}\,a^3c\big(-35\tan(fx+e)^4\,b-45\tan(fx+e)^3\,a+85\tan(fx+e)^3\,\cdots}{\cdots}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt( - tan(e + f*x)*i + 1)*a**3*c*( - 35*tan(e + f*x)**4*b - 4
5*tan(e + f*x)**3*a + 85*tan(e + f*x)**3*b*i + 117*tan(e + f*x)**2*a*i + 3
9*tan(e + f*x)**2*b + 51*tan(e + f*x)*a + 53*tan(e + f*x)*b*i + 213*a*i +
106*b))/(315*f)
```

3.759 $\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\sqrt{c-ic\tan$

Optimal result

Integrand size = 43, antiderivative size = 142

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \frac{8a^3(iA+B)\sqrt{c-ic\tan(e+fx)}}{f} - \frac{8a^3(iA+2B)(c-ic\tan(e+fx))^{3/2}}{3cf} + \frac{2a^3(iA+5B)(c-ic\tan(e+fx))^{5/2}}{5c^2f} - \frac{2a^3B(c-ic\tan(e+fx))^{7/2}}{7c^3f}$$

output
```
8*a^3*(I*A+B)*(c-I*c*tan(f*x+e))^(1/2)/f-8/3*a^3*(I*A+2*B)*(c-I*c*tan(f*x+
e))^(3/2)/c/f+2/5*a^3*(I*A+5*B)*(c-I*c*tan(f*x+e))^(5/2)/c^2/f-2/7*a^3*B*(
c-I*c*tan(f*x+e))^(7/2)/c^3/f
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = -\frac{2a^3c(i+\tan(e+fx))\left(-301A+230iB+(-98iA-115B)\tan(e+fx)+3(7A-20iB)\tan^2(e+fx\right.}{105f\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e
 + f*x]],x]
```

output
```
(-2*a^3*c*(I + Tan[e + f*x])*(-301*A + (230*I)*B + ((-98*I)*A - 115*B)*Tan
[e + f*x] + 3*(7*A - (20*I)*B)*Tan[e + f*x]^2 + 15*B*Tan[e + f*x]^3))/(105
*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^3c\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{a^3c\int \left(\frac{iB(c-ic\tan(e+fx))^{5/2}}{c^3}+\frac{(A-5iB)(c-ic\tan(e+fx))^{3/2}}{c^2}-\frac{4(A-2iB)\sqrt{c-ic\tan(e+fx)}}{c}+\frac{4(A-iB)}{\sqrt{c-ic\tan(e+fx)}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3c\left(\frac{2(5B+iA)(c-ic\tan(e+fx))^{5/2}}{5c^3}-\frac{8(2B+iA)(c-ic\tan(e+fx))^{3/2}}{3c^2}+\frac{8(B+iA)\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2B(c-ic\tan(e+fx))^{7/2}}{7c^4}\right)}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x
]],x]
```

output
```
(a^3*c*((8*(I*A + B)*Sqrt[c - I*c*Tan[e + f*x]])/c - (8*(I*A + 2*B)*(c - I
*c*Tan[e + f*x])^(3/2))/(3*c^2) + (2*(I*A + 5*B)*(c - I*c*Tan[e + f*x])^(5
/2))/(5*c^3) - (2*B*(c - I*c*Tan[e + f*x])^(7/2))/(7*c^4)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2ia^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{(-5iBc+cA)(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{\left(-4(-iBc+cA)c+4iB\,c^2\right)(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+4\sqrt{c-ic\tan(\ldots}{f\,c^3}$
default	$\frac{2ia^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{(-5iBc+cA)(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+\frac{\left(-4(-iBc+cA)c+4iB\,c^2\right)(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+4\sqrt{c-ic\tan(\ldots}{f\,c^3}$
parts	$\frac{iA\,a^3\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{f}+\frac{a^3(3iA+B)\left(2\sqrt{c-ic\tan(fx+e)}-\sqrt{c}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\ldots}{2\sqrt{c}}\right.\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^3/c^3*(1/7*I*B*(c-I*c*tan(f*x+e))^(7/2)+1/5*(-5*I*B*c+c*A)*(c-I*c*
tan(f*x+e))^(5/2)+1/3*(-4*(-I*B*c+c*A)*c+4*I*B*c^2)*(c-I*c*tan(f*x+e))^(3/
2)+4*(c-I*c*tan(f*x+e))^(1/2)*(-I*B*c+c*A)*c^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

$$\int(a+ia\tan(e+fx))^3(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx =$$

$$-\frac{8\sqrt{2}(105\,(-i\,A-B)a^3e^{(6i\,fx+6i\,e)}+35\,(-7i\,A-5\,B)a^3e^{(4i\,fx+4i\,e)}+28\,(-7i\,A-5\,B)a^3e^{(2i\,fx+2i\,e)}+\ldots}{105\,(fe^{(6i\,fx+6i\,e)}+3\,fe^{(4i\,fx+4i\,e)}+3\,fe^{(2i\,fx+2i\,e)}+f)}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
-8/105*sqrt(2)*(105*(-I*A - B)*a^3*e^(6*I*f*x + 6*I*e) + 35*(-7*I*A - 5*B)
*a^3*e^(4*I*f*x + 4*I*e) + 28*(-7*I*A - 5*B)*a^3*e^(2*I*f*x + 2*I*e) + 8*(
-7*I*A - 5*B)*a^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(f*e^(6*I*f*x + 6*I*e
) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= -ia^3\left(\int iA\sqrt{-ic\tan{(e+fx)}+c}\,dx + \int\left(-3A\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\right)dx + \int A\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}\,dx + \int\left(-3B\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\right)dx + \int B\sqrt{-ic\tan{(e+fx)}+c}\tan^4{(e+fx)}\,dx + \int\left(-3iA\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}\right)dx + \int iB\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}\,dx + \int\left(-3iB\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(1/2)
,x)
```

output
```
-I*a**3*(Integral(I*A*sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-3*A*sqrt
(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(A*sqrt(-I*c*tan(e + f*
x) + c)*tan(e + f*x)**3, x) + Integral(-3*B*sqrt(-I*c*tan(e + f*x) + c)*ta
n(e + f*x)**2, x) + Integral(B*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4
, x) + Integral(-3*I*A*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2, x) + I
ntegral(I*B*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x), x) + Integral(-3*I*B
*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= \frac{2i\left(15i\,(-i\,c\tan\left(fx+e\right)+c)^{\frac{7}{2}}Ba^3+21\,(-i\,c\tan\left(fx+e\right)+c)^{\frac{5}{2}}(A-5i\,B)a^3c-140\,(-i\,c\tan\left(fx+e\right)\right.}{105\,c^3f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
2/105*I*(15*I*(-I*c*tan(f*x + e) + c)^(7/2)*B*a^3 + 21*(-I*c*tan(f*x + e)
+ c)^(5/2)*(A - 5*I*B)*a^3*c - 140*(-I*c*tan(f*x + e) + c)^(3/2)*(A - 2*I*
B)*a^3*c^2 + 420*sqrt(-I*c*tan(f*x + e) + c)*(A - I*B)*a^3*c^3)/(c^3*f)
```

Giac [F(-2)]

Exception generated.

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.20

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= -\frac{\sqrt{c+\frac{c\,(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\left(\frac{a^3\,(A-B\,1\mathrm{i})\,8\mathrm{i}}{3\,f}+\frac{a^3\,(A-B\,3\mathrm{i})\,8\mathrm{i}}{3\,f}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}$$
$$-\frac{\sqrt{c+\frac{c\,(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\left(\frac{a^3\,(A-B\,1\mathrm{i})\,8\mathrm{i}}{7\,f}-\frac{a^3\,(A+B\,1\mathrm{i})\,8\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3}$$
$$+\frac{\sqrt{c+\frac{c\,(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\left(\frac{32\,B\,a^3}{5\,f}+\frac{a^3\,(A-B\,1\mathrm{i})\,8\mathrm{i}}{5\,f}\right)}{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2}$$
$$+\frac{a^3\,(A-B\,1\mathrm{i})\,\sqrt{c+\frac{c\,(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,8\mathrm{i}}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i)
^(1/2),x)
```

output
```
((c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2)*
((a^3*(A - B*1i)*8i)/(5*f) + (32*B*a^3)/(5*f)))/(exp(e*2i + f*x*2i) + 1)^2
 - ((c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/
2)*((a^3*(A - B*1i)*8i)/(7*f) - (a^3*(A + B*1i)*8i)/(7*f)))/(exp(e*2i + f*
x*2i) + 1)^3 - ((c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2
i) + 1))^(1/2)*((a^3*(A - B*1i)*8i)/(3*f) + (a^3*(A - B*3i)*8i)/(3*f)))/(e
xp(e*2i + f*x*2i) + 1) + (a^3*(A - B*1i)*(c + (c*(exp(e*2i + f*x*2i)*1i -
1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2)*8i)/f
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^3(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= \frac{\sqrt{c}\,a^3\left(2\sqrt{-\tan(fx+e)\,i+1}\,ai-\left(\int\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^4\,dx\right)bfi-\left(\int\sqrt{-\tan(fx+\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*a**3*(2*sqrt( - tan(e + f*x)*i + 1)*a*i - int(sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**4,x)*b*f*i - int(sqrt( - tan(e + f*x)*i + 1)*tan(e
+ f*x)**3,x)*a*f*i - 3*int(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3,x)*
b*f - 3*int(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)*a*f + 3*int(sqr
t( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)*b*f*i + 2*int(sqrt( - tan(e +
f*x)*i + 1)*tan(e + f*x),x)*a*f*i + int(sqrt( - tan(e + f*x)*i + 1)*tan(e
+ f*x),x)*b*f))/f
```

3.760 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 140

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{8a^3(iA+B)}{f\sqrt{c-ic\tan(e+fx)}}-\frac{8a^3(iA+2B)\sqrt{c-ic\tan(e+fx)}}{cf}+\frac{2a^3(iA+5B)(c-ic\tan(e+fx))^{3/2}}{3c^2f}-\frac{2a^3B(c-ic\tan(e+fx))^{5/2}}{5c^3f}$$

output

```
-8*a^3*(I*A+B)/f/(c-I*c*tan(f*x+e))^(1/2)-8*a^3*(I*A+2*B)*(c-I*c*tan(f*x+e
))^(1/2)/c/f+2/3*a^3*(I*A+5*B)*(c-I*c*tan(f*x+e))^(3/2)/c^2/f-2/5*a^3*B*(c
-I*c*tan(f*x+e))^(5/2)/c^3/f
```

Mathematica [A] (verified)

Time = 3.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{2ia^3(115A-158iB+(-50iA-79B)\tan(e+fx)+(5A-16iB)\tan^2(e+fx)+3B\tan^3(e+fx))}{15f\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan
[e + f*x]],x]
```

output
```
(((-2*I)/15)*a^3*(115*A - (158*I)*B + ((-50*I)*A - 79*B)*Tan[e + f*x] + (5
*A - (16*I)*B)*Tan[e + f*x]^2 + 3*B*Tan[e + f*x]^3))/(f*Sqrt[c - I*c*Tan[e
 + f*x]])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3c\int \left(\frac{iB(c-ic\tan(e+fx))^{3/2}}{c^3}+\frac{(A-5iB)\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{4(A-2iB)}{c\sqrt{c-ic\tan(e+fx)}}+\frac{4(A-iB)}{(c-ic\tan(e+fx))^{3/2}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3c\left(\frac{2(5B+iA)(c-ic\tan(e+fx))^{3/2}}{3c^3}-\frac{8(2B+iA)\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{8(B+iA)}{c\sqrt{c-ic\tan(e+fx)}}-\frac{2B(c-ic\tan(e+fx))^{5/2}}{5c^4}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e + f
*x]],x]
```

output
```
(a^3*c*((-8*(I*A + B))/(c*Sqrt[c - I*c*Tan[e + f*x]]) - (8*(I*A + 2*B)*Sqr
t[c - I*c*Tan[e + f*x]])/c^2 + (2*(I*A + 5*B)*(c - I*c*Tan[e + f*x])^(3/2)
)/(3*c^3) - (2*B*(c - I*c*Tan[e + f*x])^(5/2))/(5*c^4)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{2ia^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{5iBc(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+\frac{Ac(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+8i\sqrt{c-ic\tan(fx+e)}\,B\,c^2-4\sqrt{c-ic\tan(f x}{f\,c^3}$
default	$\frac{2ia^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{5iBc(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+\frac{Ac(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+8i\sqrt{c-ic\tan(fx+e)}\,B\,c^2-4\sqrt{c-ic\tan(f x}{f\,c^3}$
parts	$\frac{2iA\,a^3c\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}-\frac{1}{2c\sqrt{c-ic\tan(fx+e)}}\right)}{f}+\frac{a^3(3iA+B)\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right.}{2\sqrt{c}}\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^3/c^3*(1/5*I*B*(c-I*c*tan(f*x+e))^(5/2)-5/3*I*B*c*(c-I*c*tan(f*x+e
))^(3/2)+1/3*A*c*(c-I*c*tan(f*x+e))^(3/2)+8*I*(c-I*c*tan(f*x+e))^(1/2)*B*c
^2-4*(c-I*c*tan(f*x+e))^(1/2)*A*c^2-4*c^3*(A-I*B)/(c-I*c*tan(f*x+e))^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx =$$
$$-\frac{4\sqrt{2}\left(15\,(i\,A+B)a^3e^{(6i\,fx+6i\,e)}+15\,(5i\,A+7\,B)a^3e^{(4i\,fx+4i\,e)}+20\,(5i\,A+7\,B)a^3e^{(2i\,fx+2i\,e)}+8\,(5i\,A\right.}{15\left(cfe^{(4i\,fx+4i\,e)}+2\,cfe^{(2i\,fx+2i\,e)}+cf\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
-4/15*sqrt(2)*(15*(I*A + B)*a^3*e^(6*I*f*x + 6*I*e) + 15*(5*I*A + 7*B)*a^3
*e^(4*I*f*x + 4*I*e) + 20*(5*I*A + 7*B)*a^3*e^(2*I*f*x + 2*I*e) + 8*(5*I*A
 + 7*B)*a^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c*f*e^(4*I*f*x + 4*I*e) +
2*c*f*e^(2*I*f*x + 2*I*e) + c*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$= -ia^3\left(\int \frac{iA}{\sqrt{-ic\tan{(e+fx)}+c}}\,dx+\int\left(-\frac{3A\tan{(e+fx)}}{\sqrt{-ic\tan{(e+fx)}+c}}\right)dx\right.$$
$$+\int \frac{A\tan^3{(e+fx)}}{\sqrt{-ic\tan{(e+fx)}+c}}\,dx+\int\left(-\frac{3B\tan^2{(e+fx)}}{\sqrt{-ic\tan{(e+fx)}+c}}\right)dx$$
$$+\int \frac{B\tan^4{(e+fx)}}{\sqrt{-ic\tan{(e+fx)}+c}}\,dx+\int\left(-\frac{3iA\tan^2{(e+fx)}}{\sqrt{-ic\tan{(e+fx)}+c}}\right)dx$$
$$\left.+\int \frac{iB\tan{(e+fx)}}{\sqrt{-ic\tan{(e+fx)}+c}}\,dx+\int\left(-\frac{3iB\tan^3{(e+fx)}}{\sqrt{-ic\tan{(e+fx)}+c}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(1/2)
,x)
```

output
```
-I*a**3*(Integral(I*A/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-3*A*tan(
e + f*x)/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(A*tan(e + f*x)**3/sqrt
(-I*c*tan(e + f*x) + c), x) + Integral(-3*B*tan(e + f*x)**2/sqrt(-I*c*tan(
e + f*x) + c), x) + Integral(B*tan(e + f*x)**4/sqrt(-I*c*tan(e + f*x) + c)
, x) + Integral(-3*I*A*tan(e + f*x)**2/sqrt(-I*c*tan(e + f*x) + c), x) + I
ntegral(I*B*tan(e + f*x)/sqrt(-I*c*tan(e + f*x) + c), x) + Integral(-3*I*B
*tan(e + f*x)**3/sqrt(-I*c*tan(e + f*x) + c), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx =$$
$$-\frac{2i\left(\frac{60\,(A-i\,B)a^3c}{\sqrt{-i\,c\tan(fx+e)+c}}-\frac{3i\,(-i\,c\tan(fx+e)+c)^{\frac{5}{2}}Ba^3+5\,(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}(A-5i\,B)a^3c-60\,\sqrt{-i\,c\tan(fx+e)+c}(A-2i\,B)a^3c^2}{c^2}\right)}{15\,cf}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
-2/15*I*(60*(A - I*B)*a^3*c/sqrt(-I*c*tan(f*x + e) + c) - (3*I*(-I*c*tan(f
*x + e) + c)^(5/2)*B*a^3 + 5*(-I*c*tan(f*x + e) + c)^(3/2)*(A - 5*I*B)*a^3
*c - 60*sqrt(-I*c*tan(f*x + e) + c)*(A - 2*I*B)*a^3*c^2)/c^2)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.51

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$= -\sqrt{c+\frac{c\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)1i}{e^{e\,2i+f\,x\,2i}+1}}\left(\frac{a^3\,(A-B\,1i)\,4i}{c\,f}+\frac{a^3\,e^{e\,2i+f\,x\,2i}\,(A-B\,1i)\,4i}{c\,f}\right)$$
$$-\left(\frac{a^3\,(A-B\,1i)\,4i}{c\,f}+\frac{a^3\,(A-B\,3i)\,4i}{c\,f}\right)\sqrt{c+\frac{c\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)1i}{e^{e\,2i+f\,x\,2i}+1}}$$
$$-\frac{\left(\frac{a^3\,(A-B\,1i)\,4i}{5\,c\,f}-\frac{a^3\,(A+B\,1i)\,4i}{5\,c\,f}\right)\sqrt{c+\frac{c\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)1i}{e^{e\,2i+f\,x\,2i}+1}}}{\left(e^{e\,2i+f\,x\,2i}+1\right)^2}$$
$$+\frac{\left(\frac{16\,B\,a^3}{3\,c\,f}+\frac{a^3\,(A-B\,1i)\,4i}{3\,c\,f}\right)\sqrt{c+\frac{c\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)1i}{e^{e\,2i+f\,x\,2i}+1}}}{e^{e\,2i+f\,x\,2i}+1}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^(1/2),x)
```

output
```
(((a^3*(A - B*1i)*4i)/(3*c*f) + (16*B*a^3)/(3*c*f))*(c + (c*(exp(e*2i + f*
x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2))/(exp(e*2i + f*x*2i) +
1) - ((a^3*(A - B*1i)*4i)/(c*f) + (a^3*(A - B*3i)*4i)/(c*f))*(c + (c*(exp(
e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2) - (((a^3*(A -
B*1i)*4i)/(5*c*f) - (a^3*(A + B*1i)*4i)/(5*c*f))*(c + (c*(exp(e*2i + f*x*2
i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/2))/(exp(e*2i + f*x*2i) + 1)^
2 - (c + (c*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(1/
2)*((a^3*(A - B*1i)*4i)/(c*f) + (a^3*exp(e*2i + f*x*2i)*(A - B*1i)*4i)/(c*
f))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Too large to display}$$

input

```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x)
```

output

```
(sqrt(c)*a**3*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int((sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f +
int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**2 + 1),x)
*b*f + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2
+ 1),x)*tan(e + f*x)**2*a*f - 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x)**4)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f*i + int((sqrt( - tan(
e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2 + 1),x)*a*f - 4*int((sqr
t( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2 + 1),x)*b*f*i -
 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**2 + 1)
,x)*tan(e + f*x)**2*a*f*i - 6*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x
)**3)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f - 4*int((sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**2 + 1),x)*a*f*i - 6*int((sqr
t( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**2 + 1),x)*b*f - 6
*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x
)*tan(e + f*x)**2*a*f + 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2
)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f*i - 6*int((sqrt( - tan(e +
f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*a*f + 4*int((sqrt( -
 tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*b*f*i + 7*i
nt((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**2 + 1),x)*tan
(e + f*x)**2*a*f*i + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(ta...
```

3.761 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 140

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{8a^3(iA+B)}{3f(c-ic\tan(e+fx))^{3/2}}+\frac{8a^3(iA+2B)}{cf\sqrt{c-ic\tan(e+fx)}} +\frac{2a^3(iA+5B)\sqrt{c-ic\tan(e+fx)}}{c^2f}-\frac{2a^3B(c-ic\tan(e+fx))^{3/2}}{3c^3f}$$

output
```
-8/3*a^3*(I*A+B)/f/(c-I*c*tan(f*x+e))^(3/2)+8*a^3*(I*A+2*B)/c/f/(c-I*c*tan
(f*x+e))^(1/2)+2*a^3*(I*A+5*B)*(c-I*c*tan(f*x+e))^(1/2)/c^2/f-2/3*a^3*B*(c
-I*c*tan(f*x+e))^(3/2)/c^3/f
```

Mathematica [A] (verified)

Time = 5.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{2a^3(-11A+34iB+3(6iA+17B)\tan(e+fx)+3(A}{3cf(i+\tan(e+fx))\sqrt{c-i}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^(3/2),x]
```

output
```
(2*a^3*(-11*A + (34*I)*B + 3*((6*I)*A + 17*B)*Tan[e + f*x] + 3*(A - (4*I)*
B)*Tan[e + f*x]^2 + B*Tan[e + f*x]^3))/(3*c*f*(I + Tan[e + f*x])*Sqrt[c -
I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3c\int \left(\frac{4(A-iB)}{(c-ic\tan(e+fx))^{5/2}}+\frac{iB\sqrt{c-ic\tan(e+fx)}}{c^3}+\frac{A-5iB}{c^2\sqrt{c-ic\tan(e+fx)}}-\frac{4(A-2iB)}{c(c-ic\tan(e+fx))^{3/2}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3c\left(\frac{2(5B+iA)\sqrt{c-ic\tan(e+fx)}}{c^3}+\frac{8(2B+iA)}{c^2\sqrt{c-ic\tan(e+fx)}}-\frac{8(B+iA)}{3c(c-ic\tan(e+fx))^{3/2}}-\frac{2B(c-ic\tan(e+fx))^{3/2}}{3c^4}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^(3/2),x]
```

output
```
(a^3*c*((-8*(I*A + B))/(3*c*(c - I*c*Tan[e + f*x])^(3/2)) + (8*(I*A + 2*B)
)/(c^2*Sqrt[c - I*c*Tan[e + f*x]]) + (2*(I*A + 5*B)*Sqrt[c - I*c*Tan[e + f
*x]])/c^3 - (2*B*(c - I*c*Tan[e + f*x])^(3/2))/(3*c^4)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{2 i a^{3} \left(\frac{i B (c-i c \tan(f x+e))^{\frac{3}{2}}}{3}-5 i \sqrt{c-i c \tan(f x+e)}\, B c+\sqrt{c-i c \tan(f x+e)}\, c A+\frac{4 c^{2} (-2 i B+A)}{\sqrt{c-i c \tan(f x+e)}}-\frac{4 c^{3} (-i B+A)}{3 (c-i c \tan(f x+e))^{\frac{3}{2}}}\right)}{f c^{3}}$
default	$\frac{2 i a^{3} \left(\frac{i B (c-i c \tan(f x+e))^{\frac{3}{2}}}{3}-5 i \sqrt{c-i c \tan(f x+e)}\, B c+\sqrt{c-i c \tan(f x+e)}\, c A+\frac{4 c^{2} (-2 i B+A)}{\sqrt{c-i c \tan(f x+e)}}-\frac{4 c^{3} (-i B+A)}{3 (c-i c \tan(f x+e))^{\frac{3}{2}}}\right)}{f c^{3}}$
parts	$\frac{2 i A a^{3} c \left(\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c-i c \tan(f x+e)}\, \sqrt{2}}{2 \sqrt{c}}\right)}{8 c^{\frac{5}{2}}}-\frac{1}{4 c^{2} \sqrt{c-i c \tan(f x+e)}}-\frac{1}{6 c (c-i c \tan(f x+e))^{\frac{3}{2}}}\right)}{f}+\frac{a^{3} (3 i A+B) \left(-\frac{\sqrt{2}\, \operatorname{arctan}}{}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^3/c^3*(1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)-5*I*(c-I*c*tan(f*x+e))^(1/
2)*B*c+(c-I*c*tan(f*x+e))^(1/2)*c*A+4*c^2*(A-2*I*B)/(c-I*c*tan(f*x+e))^(1/
2)-4/3*c^3*(A-I*B)/(c-I*c*tan(f*x+e))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\sqrt{2}\left((iA+B)a^3e^{(6ifx+6ie)}+3(-iA-3B)a^3e^{(4ifx+4ie)}+12(-iA-3B)a^3e^{(2ifx+2ie)}+8(-iA-3\right.}{3\left(c^2fe^{(2ifx+2ie)}+c^2f\right)}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
-2/3*sqrt(2)*((I*A + B)*a^3*e^(6*I*f*x + 6*I*e) + 3*(-I*A - 3*B)*a^3*e^(4*
I*f*x + 4*I*e) + 12*(-I*A - 3*B)*a^3*e^(2*I*f*x + 2*I*e) + 8*(-I*A - 3*B)*
a^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^2*f*e^(2*I*f*x + 2*I*e) + c^2*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx =$$
$$-ia^3\left(\int \frac{iA}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\,dx\right.$$
$$+\int\left(-\frac{3A\tan(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\right)dx$$
$$+\int \frac{A\tan^3(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\,dx$$
$$+\int\left(-\frac{3B\tan^2(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\right)dx$$
$$+\int \frac{B\tan^4(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\,dx$$
$$+\int\left(-\frac{3iA\tan^2(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\right)dx$$
$$+\int \frac{iB\tan(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\,dx$$
$$\left.+\int\left(-\frac{3iB\tan^3(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+c\sqrt{-ic\tan(e+fx)+c}}\right)dx\right)$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(3/2)
,x)
```

output
```
-I*a**3*(Integral(I*A/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*s
qrt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*A*tan(e + f*x)/(-I*c*sqrt(-I
*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + I
ntegral(A*tan(e + f*x)**3/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) +
 c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*B*tan(e + f*x)**2/(-I*c*
sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)),
 x) + Integral(B*tan(e + f*x)**4/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
 f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*I*A*tan(e + f*x)*
*2/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*
x) + c)), x) + Integral(I*B*tan(e + f*x)/(-I*c*sqrt(-I*c*tan(e + f*x) + c)
*tan(e + f*x) + c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*I*B*tan(e
 + f*x)**3/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c*sqrt(-I*c*ta
n(e + f*x) + c)), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{2i\left(\frac{4\,(3\,(-i\,c\tan(fx+e)+c)(A-2i\,B)a^3-(A-i\,B)a^3c)}{(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}}+\frac{i\,(-i\,c\tan(fx}{}\right.}{3\,cf}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
2/3*I*(4*(3*(-I*c*tan(f*x + e) + c)*(A - 2*I*B)*a^3 - (A - I*B)*a^3*c)/(-I
*c*tan(f*x + e) + c)^(3/2) + (I*(-I*c*tan(f*x + e) + c)^(3/2)*B*a^3 + 3*sq
rt(-I*c*tan(f*x + e) + c)*(A - 5*I*B)*a^3*c)/c^2)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.82 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.58

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{a^3\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(A\,20\mathrm{i}+60\,B+A\,\mathrm{co}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^(3/2),x)
```

output
```
(a^3*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
 1))^(1/2)*(A*20i + 60*B + A*cos(2*e + 2*f*x)*23i + A*cos(4*e + 4*f*x)*2i
- A*cos(6*e + 6*f*x)*1i + 69*B*cos(2*e + 2*f*x) + 8*B*cos(4*e + 4*f*x) - B
*cos(6*e + 6*f*x) - 7*A*sin(2*e + 2*f*x) - 2*A*sin(4*e + 4*f*x) + A*sin(6*
e + 6*f*x) + B*sin(2*e + 2*f*x)*21i + B*sin(4*e + 4*f*x)*8i - B*sin(6*e +
6*f*x)*1i))/(3*c^2*f*(cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*a**3*(16*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*a - 24*sqrt( -
tan(e + f*x)*i + 1)*tan(e + f*x)*b*i + 4*sqrt( - tan(e + f*x)*i + 1)*a*i +
 10*sqrt( - tan(e + f*x)*i + 1)*b + 16*int(sqrt( - tan(e + f*x)*i + 1)/(ta
n(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2
*a*f - 29*int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i - tan(e + f*x
)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f*i + 16*int(sqrt( - tan(e
 + f*x)*i + 1)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),
x)*a*f - 29*int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i - tan(e + f
*x)**2 + tan(e + f*x)*i - 1),x)*b*f*i - 2*int((sqrt( - tan(e + f*x)*i + 1)
*tan(e + f*x)**5)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i -
1),x)*tan(e + f*x)**2*b*f - 2*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x
)**5)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*b*f -
2*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i - t
an(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*a*f + 8*int((sqrt(
 - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i - tan(e + f*x)*
*2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f*i - 2*int((sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e
 + f*x)*i - 1),x)*a*f + 8*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4
)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*b*f*i + 6*
int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i - ...
```

3.762 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 140

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{8a^3(iA+B)}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{8a^3(iA+2B)}{3cf(c-ic\tan(e+fx))^{3/2}} - \frac{2a^3(iA+5B)}{c^2f\sqrt{c-ic\tan(e+fx)}} - \frac{2a^3B\sqrt{c-ic\tan(e+fx)}}{c^3f}$$

output
```
-8/5*a^3*(I*A+B)/f/(c-I*c*tan(f*x+e))^(5/2)+8/3*a^3*(I*A+2*B)/c/f/(c-I*c*t
an(f*x+e))^(3/2)-2*a^3*(I*A+5*B)/c^2/f/(c-I*c*tan(f*x+e))^(1/2)-2*a^3*B*(c
-I*c*tan(f*x+e))^(1/2)/c^3/f
```

Mathematica [A] (verified)

Time = 5.67 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{2a^3(7iA+62B+5(2A-31iB)\tan(e+fx)-15i(A-}{15c^2f(i+\tan(e+fx))^2\sqrt{c-}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^(5/2),x]
```

output
```
(2*a^3*((7*I)*A + 62*B + 5*(2*A - (31*I)*B)*Tan[e + f*x] - (15*I)*(A - (8*
I)*B)*Tan[e + f*x]^2 + (15*I)*B*Tan[e + f*x]^3))/(15*c^2*f*(I + Tan[e + f*
x])^2*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3c\int \left(\frac{4(A-iB)}{(c-ic\tan(e+fx))^{7/2}}+\frac{iB}{c^3\sqrt{c-ic\tan(e+fx)}}+\frac{A-5iB}{c^2(c-ic\tan(e+fx))^{3/2}}-\frac{4(A-2iB)}{c(c-ic\tan(e+fx))^{5/2}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3c\left(-\frac{2(5B+iA)}{c^3\sqrt{c-ic\tan(e+fx)}}+\frac{8(2B+iA)}{3c^2(c-ic\tan(e+fx))^{3/2}}-\frac{8(B+iA)}{5c(c-ic\tan(e+fx))^{5/2}}-\frac{2B\sqrt{c-ic\tan(e+fx)}}{c^4}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^(5/2),x]
```

output
```
(a^3*c*((-8*(I*A + B))/(5*c*(c - I*c*Tan[e + f*x])^(5/2)) + (8*(I*A + 2*B)
)/(3*c^2*(c - I*c*Tan[e + f*x])^(3/2)) - (2*(I*A + 5*B))/(c^3*Sqrt[c - I*c
*Tan[e + f*x]]) - (2*B*Sqrt[c - I*c*Tan[e + f*x]])/c^4))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{2 i a^{3} \left(i \sqrt{c -i c \tan \left(f x +e \right)}\, B +\frac{4 c^{2} \left(-2 i B +A \right)}{3 \left(c -i c \tan \left(f x +e \right)\right)^{\frac{3}{2}}}-\frac{4 c^{3} \left(-i B +A \right)}{5 \left(c -i c \tan \left(f x +e \right)\right)^{\frac{5}{2}}}-\frac{c \left(-5 i B +A \right)}{\sqrt{c -i c \tan \left(f x +e \right)}}\right)}{f \,c^{3}}$
default	$\frac{2 i a^{3} \left(i \sqrt{c -i c \tan \left(f x +e \right)}\, B +\frac{4 c^{2} \left(-2 i B +A \right)}{3 \left(c -i c \tan \left(f x +e \right)\right)^{\frac{3}{2}}}-\frac{4 c^{3} \left(-i B +A \right)}{5 \left(c -i c \tan \left(f x +e \right)\right)^{\frac{5}{2}}}-\frac{c \left(-5 i B +A \right)}{\sqrt{c -i c \tan \left(f x +e \right)}}\right)}{f \,c^{3}}$
parts	$\frac{2 i A \,a^{3} c \left(\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c -i c \tan \left(f x +e \right)}\, \sqrt{2}}{2 \sqrt{c}}\right)}{16 c^{\frac{7}{2}}}-\frac{1}{8 c^{3} \sqrt{c -i c \tan \left(f x +e \right)}}-\frac{1}{12 c^{2} \left(c -i c \tan \left(f x +e \right)\right)^{\frac{3}{2}}}-\frac{1}{10 c \left(c -i c \tan \left(f x +e \right)\right)^{\frac{5}{2}}}\right)}{f}+$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^3/c^3*(I*(c-I*c*tan(f*x+e))^(1/2)*B+4/3*c^2*(A-2*I*B)/(c-I*c*tan(f
*x+e))^(3/2)-4/5*c^3*(A-I*B)/(c-I*c*tan(f*x+e))^(5/2)-c*(A-5*I*B)/(c-I*c*t
an(f*x+e))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{2}\big(3\,(i\,A+B)a^3e^{(6i\,fx+6i\,e)}-(i\,A+11\,B)a^3e^{(4i\,fx+4i\,e)}+4\,(i\,A+11\,B)a^3e^{(2i\,fx+2i\,e)}+8\,(i\,A+11\,B)a^3}{15\,c^3f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
-1/15*sqrt(2)*(3*(I*A + B)*a^3*e^(6*I*f*x + 6*I*e) - (I*A + 11*B)*a^3*e^(4
*I*f*x + 4*I*e) + 4*(I*A + 11*B)*a^3*e^(2*I*f*x + 2*I*e) + 8*(I*A + 11*B)*
a^3)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(5/2)
,x)
```

output
```
-I*a**3*(Integral(I*A/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 -
 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e
+ f*x) + c)), x) + Integral(-3*A*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x
) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)
 + c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(A*tan(e + f*x)**3/(-c*
*2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e
+ f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integra
l(-3*B*tan(e + f*x)**2/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2
- 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e
 + f*x) + c)), x) + Integral(B*tan(e + f*x)**4/(-c**2*sqrt(-I*c*tan(e + f*
x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x
) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(-3*I*A*tan(e + f*x)**
2/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + I
ntegral(I*B*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**
2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**2*sqrt(-I*c*tan
(e + f*x) + c)), x) + Integral(-3*I*B*tan(e + f*x)**3/(-c**2*sqrt(-I*c*tan
(e + f*x) + c)*tan(e + f*x)**2 - 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(
e + f*x) + c**2*sqrt(-I*c*tan(e + f*x) + c)), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{2i\left(-\frac{15i\sqrt{-ic\tan(fx+e)+c}Ba^3}{c^2}+\frac{15\,(-ic\tan(fx+e)+c)^2(A-5iB)a^3-20\,(-ic\tan(fx+e)+c)(A-2iB)a^3c+12\,(A-iB)a^3c^2}{(-ic\tan(fx+e)+c)^{\frac{5}{2}}c}\right)}{15\,cf}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
-2/15*I*(-15*I*sqrt(-I*c*tan(f*x + e) + c)*B*a^3/c^2 + (15*(-I*c*tan(f*x +
 e) + c)^2*(A - 5*I*B)*a^3 - 20*(-I*c*tan(f*x + e) + c)*(A - 2*I*B)*a^3*c
+ 12*(A - I*B)*a^3*c^2)/((-I*c*tan(f*x + e) + c)^(5/2)*c))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.49

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{a^3\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,\left(A\,8\mathrm{i}+88\,B+A\,\cos\left(2\,e+2\,f\,x\right)\,4\mathrm{i}-A\,\cos\left(4\,e+4\,f\,x\right)\,1\mathrm{i}+A\,\cos\left(\right.\right.}{}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^(5/2),x)
```

output
```
-(a^3*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(A*8i + 88*B + A*cos(2*e + 2*f*x)*4i - A*cos(4*e + 4*f*x)*1i +
 A*cos(6*e + 6*f*x)*3i + 44*B*cos(2*e + 2*f*x) - 11*B*cos(4*e + 4*f*x) + 3
*B*cos(6*e + 6*f*x) - 4*A*sin(2*e + 2*f*x) + A*sin(4*e + 4*f*x) - 3*A*sin(
6*e + 6*f*x) + B*sin(2*e + 2*f*x)*44i - B*sin(4*e + 4*f*x)*11i + B*sin(6*e
 + 6*f*x)*3i))/(15*c^3*f)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{a^3\left(\left(\int \frac{\tan(fx+e)^4}{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^2+2\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e}\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(a**3*(int(tan(e + f*x)**4/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 +
2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)
),x)*b*i + int(tan(e + f*x)**3/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**
2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i
+ 1)),x)*a*i + 3*int(tan(e + f*x)**3/(sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f
*x)*i + 1)),x)*b + 3*int(tan(e + f*x)**2/(sqrt( - tan(e + f*x)*i + 1)*tan(
e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e
 + f*x)*i + 1)),x)*a - 3*int(tan(e + f*x)**2/(sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - t
an(e + f*x)*i + 1)),x)*b*i - 3*int(tan(e + f*x)/(sqrt( - tan(e + f*x)*i +
1)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(
- tan(e + f*x)*i + 1)),x)*a*i - int(tan(e + f*x)/(sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(
 - tan(e + f*x)*i + 1)),x)*b - int(1/(sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f
*x)*i + 1)),x)*a))/(sqrt(c)*c**2)
```

3.763 $\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 142

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{8a^3(iA+B)}{7f(c-ic\tan(e+fx))^{7/2}}+\frac{8a^3(iA+2B)}{5cf(c-ic\tan(e+fx))^{5/2}} -\frac{2a^3(iA+5B)}{3c^2f(c-ic\tan(e+fx))^{3/2}}+\frac{2a^3B}{c^3f\sqrt{c-ic\tan(e+fx)}}$$

output
```
-8/7*a^3*(I*A+B)/f/(c-I*c*tan(f*x+e))^(7/2)+8/5*a^3*(I*A+2*B)/c/f/(c-I*c*t
an(f*x+e))^(5/2)-2/3*a^3*(I*A+5*B)/c^2/f/(c-I*c*tan(f*x+e))^(3/2)+2*a^3*B/
c^3/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 5.85 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \frac{2a^3(-11A-38iB+(-14iA-133B)\tan(e+fx)+3}{105c^3f(i+\tan(e+fx))^3\sqrt{}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e +
 f*x])^(7/2),x]
```

output
```
(2*a^3*(-11*A - (38*I)*B + ((-14*I)*A - 133*B)*Tan[e + f*x] + 35*(A + (4*I
)*B)*Tan[e + f*x]^2 + 105*B*Tan[e + f*x]^3))/(105*c^3*f*(I + Tan[e + f*x])
^3*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 4071, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{a^2(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^3c\int \frac{(i\tan(e+fx)+1)^2(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{86}$$

$$\frac{a^3c\int \left(\frac{4(A-iB)}{(c-ic\tan(e+fx))^{9/2}}+\frac{iB}{c^3(c-ic\tan(e+fx))^{3/2}}+\frac{A-5iB}{c^2(c-ic\tan(e+fx))^{5/2}}-\frac{4(A-2iB)}{c(c-ic\tan(e+fx))^{7/2}}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3c\left(-\frac{2(5B+iA)}{3c^3(c-ic\tan(e+fx))^{3/2}}+\frac{8(2B+iA)}{5c^2(c-ic\tan(e+fx))^{5/2}}-\frac{8(B+iA)}{7c(c-ic\tan(e+fx))^{7/2}}+\frac{2B}{c^4\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^3*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x])
^(7/2),x]
```

output
```
(a^3*c*((-8*(I*A + B))/(7*c*(c - I*c*Tan[e + f*x])^(7/2)) + (8*(I*A + 2*B)
)/(5*c^2*(c - I*c*Tan[e + f*x])^(5/2)) - (2*(I*A + 5*B))/(3*c^3*(c - I*c*T
an[e + f*x])^(3/2)) + (2*B)/(c^4*Sqrt[c - I*c*Tan[e + f*x]])))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{2ia^3\left(-\frac{c(-5iB+A)}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{4c^2(-2iB+A)}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}-\frac{iB}{\sqrt{c-ic\tan(fx+e)}}-\frac{4c^3(-iB+A)}{7(c-ic\tan(fx+e))^{\frac{7}{2}}}\right)}{f\,c^3}$
default	$\frac{2ia^3\left(-\frac{c(-5iB+A)}{3(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{4c^2(-2iB+A)}{5(c-ic\tan(fx+e))^{\frac{5}{2}}}-\frac{iB}{\sqrt{c-ic\tan(fx+e)}}-\frac{4c^3(-iB+A)}{7(c-ic\tan(fx+e))^{\frac{7}{2}}}\right)}{f\,c^3}$
risch	$-\frac{a^3\left(15iA\,e^{6i(fx+e)}+15B\,e^{6i(fx+e)}+3iA\,e^{4i(fx+e)}-39B\,e^{4i(fx+e)}-4iA\,e^{2i(fx+e)}+52B\,e^{2i(fx+e)}+8iA-104B\right)\sqrt{2}}{210c^3\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
parts	$\frac{2iA\,a^3c\left(\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{32c^{\frac{9}{2}}}-\frac{1}{16c^4\sqrt{c-ic\tan(fx+e)}}-\frac{1}{24c^3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{1}{20c^2(c-ic\tan(fx+e))^{\frac{5}{2}}}-\frac{}{14}\right.}{f}$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f*a^3/c^3*(-1/3*c*(A-5*I*B)/(c-I*c*tan(f*x+e))^(3/2)+4/5*c^2*(A-2*I*B)
/(c-I*c*tan(f*x+e))^(5/2)-I*B/(c-I*c*tan(f*x+e))^(1/2)-4/7*c^3*(A-I*B)/(c-
I*c*tan(f*x+e))^(7/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx =$$
$$-\frac{\sqrt{2}\left(15\,(i\,A+B)a^3e^{(8i\,fx+8i\,e)}+6\,(3i\,A-4\,B)a^3e^{(6i\,fx+6i\,e)}-(i\,A-13\,B)a^3e^{(4i\,fx+4i\,e)}+4\,(i\,A-13\,B)a\right.}{210\,c^4f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
-1/210*sqrt(2)*(15*(I*A + B)*a^3*e^(8*I*f*x + 8*I*e) + 6*(3*I*A - 4*B)*a^3
*e^(6*I*f*x + 6*I*e) - (I*A - 13*B)*a^3*e^(4*I*f*x + 4*I*e) + 4*(I*A - 13*
B)*a^3*e^(2*I*f*x + 2*I*e) + 8*(I*A - 13*B)*a^3)*sqrt(c/(e^(2*I*f*x + 2*I*
e) + 1))/(c^4*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))**3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(7/2)
,x)
```

output
```
-I*a**3*(Integral(I*A/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3
- 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x) + I
ntegral(-3*A*tan(e + f*x)/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)
**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sqrt(-
I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c)), x)
 + Integral(A*tan(e + f*x)**3/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
f*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*c**3*sq
rt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x) + c))
, x) + Integral(-3*B*tan(e + f*x)**2/(I*c**3*sqrt(-I*c*tan(e + f*x) + c)*t
an(e + f*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*I*
c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e + f*x
) + c)), x) + Integral(B*tan(e + f*x)**4/(I*c**3*sqrt(-I*c*tan(e + f*x) +
c)*tan(e + f*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 -
3*I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*c*tan(e +
 f*x) + c)), x) + Integral(-3*I*A*tan(e + f*x)**2/(I*c**3*sqrt(-I*c*tan(e
+ f*x) + c)*tan(e + f*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f
*x)**2 - 3*I*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + c**3*sqrt(-I*
c*tan(e + f*x) + c)), x) + Integral(I*B*tan(e + f*x)/(I*c**3*sqrt(-I*c*tan
(e + f*x) + c)*tan(e + f*x)**3 - 3*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx =$$
$$-\frac{2i\left(105i\,(-i\,c\tan\left(fx+e\right)+c)^3Ba^3+35\,(-i\,c\tan\left(fx+e\right)+c)^2(A-5i\,B)a^3c-84\,(-i\,c\tan\left(fx+e\right)\cdots\right.}{105\,(-i\,c\tan\left(fx+e\right)+c)^{\frac{7}{2}}c^3f}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
-2/105*I*(105*I*(-I*c*tan(f*x + e) + c)^3*B*a^3 + 35*(-I*c*tan(f*x + e) +
c)^2*(A - 5*I*B)*a^3*c - 84*(-I*c*tan(f*x + e) + c)*(A - 2*I*B)*a^3*c^2 +
60*(A - I*B)*a^3*c^3)/((-I*c*tan(f*x + e) + c)^(7/2)*c^3*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\sqrt{c-\frac{c\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}}\left(\frac{a^3\,(A+B\,13\mathrm{i})\,4\mathrm{i}}{105\,c^4\,f}+\frac{a^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,(3\,A+B\,4\mathrm{i})\,1\mathrm{i}}{35\,c^4\,f}+\frac{a^3\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,(A+B\,13\mathrm{i})\,2\mathrm{i}}{105\,c^4\,f}+\frac{a^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,(A-B\,1\mathrm{i})\,1\mathrm{i}}{14\,c^4\,f}-\frac{a^3\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,(A+B\,13\mathrm{i})\,1\mathrm{i}}{210\,c^4\,f}\right)$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^3)/(c - c*tan(e + f*x)*1
i)^(7/2),x)
```

output
```
-(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)*((a^3*(A + B*13i)*4i)/(105*c
^4*f) + (a^3*exp(e*6i + f*x*6i)*(3*A + B*4i)*1i)/(35*c^4*f) + (a^3*exp(e*2
i + f*x*2i)*(A + B*13i)*2i)/(105*c^4*f) + (a^3*exp(e*8i + f*x*8i)*(A - B*1
i)*1i)/(14*c^4*f) - (a^3*exp(e*4i + f*x*4i)*(A + B*13i)*1i)/(210*c^4*f))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^3(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^3*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*a**3*(66*sqrt( - tan(e + f*x)*i + 1)*a*i - 16*sqrt( - tan(e + f*x
)*i + 1)*b - 12*int(( - sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**5*i -
3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*
x)*i + 1),x)*tan(e + f*x)**2*a*f + 18*int(( - sqrt( - tan(e + f*x)*i + 1))
/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e +
f*x)**2 - 3*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i - 12*int(( - sqrt
( - tan(e + f*x)*i + 1))/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e
+ f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*a*f + 18*int((
- sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*
tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b*f*i - 3
6*int(( - sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**5*i
- 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e +
f*x)*i + 1),x)*tan(e + f*x)**2*a*f + 54*int(( - sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)**4)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)
**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i
 - 36*int(( - sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**
5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(
e + f*x)*i + 1),x)*a*f + 54*int(( - sqrt( - tan(e + f*x)*i + 1)*tan(e + f*
x)**4)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*ta
n(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b*f*i + 144*int(( - sqrt( - ta...
```

3.764 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 220

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx = -\frac{2\sqrt{2}(5iA-9B)c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{af} + \frac{2(5iA-9B)c^3\sqrt{c-ic\tan(e+fx)}}{af} + \frac{(5iA-9B)c^2(c-ic\tan(e+fx))^{3/2}}{3af} + \frac{(5iA-9B)c(c-ic\tan(e+fx))^{5/2}}{10af} + \frac{(iA-B)(c-ic\tan(e+fx))^{7/2}}{2af(1+i\tan(e+fx))}$$

output
```
-2*2^(1/2)*(5*I*A-9*B)*c^(7/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2
)/c^(1/2))/a/f+2*(5*I*A-9*B)*c^3*(c-I*c*tan(f*x+e))^(1/2)/a/f+1/3*(5*I*A-9
*B)*c^2*(c-I*c*tan(f*x+e))^(3/2)/a/f+1/10*(5*I*A-9*B)*c*(c-I*c*tan(f*x+e))
^(5/2)/a/f+1/2*(I*A-B)*(c-I*c*tan(f*x+e))^(7/2)/a/f/(1+I*tan(f*x+e))
```

Mathematica [A] (verified)

Time = 5.85 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.67

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx = \frac{2c^3\left(15\sqrt{2}(-5iA+9B)\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)+\right.}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2))/(a + I*a*Tan
[e + f*x]),x]
```

output
```
(2*c^3*(15*Sqrt[2]*((-5*I)*A + 9*B)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f
*x]]/(Sqrt[2]*Sqrt[c])] + (Sqrt[c - I*c*Tan[e + f*x]]*(95*A + (168*I)*B +
((60*I)*A - 117*B)*Tan[e + f*x] + (5*A + (18*I)*B)*Tan[e + f*x]^2 + 3*B*Ta
n[e + f*x]^3))/(-I + Tan[e + f*x])))/(15*a*f)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.209$, Rules used = {3042, 4071, 27, 87, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{a+ia\tan(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a^2(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{87}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(5A+9iB)\int \frac{(c-ic\tan(e+fx))^{5/2}}{i\tan(e+fx)+1}d\tan(e+fx)\right)}{af}$$

$$\Big\downarrow \text{60}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(5A+9iB)\left(2c\int \frac{(c-ic\tan(e+fx))^{3/2}}{i\tan(e+fx)+1}d\tan(e+fx)-\frac{2}{5}i(c-ic\tan(e+fx))^{5/2}\right)\right)}{af}$$

$$\Big\downarrow \text{60}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(5A+9iB)\left(2c\left(2c\int \frac{\sqrt{c-ic\tan(e+fx)}}{i\tan(e+fx)+1}d\tan(e+fx)-\frac{2}{3}i(c-ic\tan(e+fx))^{3/2}\right)-\right.\right.}{af}$$

$$\Big\downarrow \text{60}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(5A+9iB)\left(2c\left(2c\left(2c\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)-2i\sqrt{c-ic\mathrm{t}}\right.\right.\right.\right.}{af}$$

$$\Big\downarrow \text{73}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(5A+9iB)\left(2c\left(2c\left(4i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}-2i\sqrt{c-ic\tan(e}\right.\right.\right.\right.}{af}$$

$$\Big\downarrow \text{219}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(5A+9iB)\left(2c\left(2c\left(2i\sqrt{2}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)-2i\sqrt{c-ic\tan(e+fx)}\right)\right.\right.\right.}{af}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2))/(a + I*a*Tan[e + f
*x]),x]
```

output
```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(7/2))/(2*c*(1 + I*Tan[e + f*x])) -
((5*A + (9*I)*B)*(((-2*I)/5)*(c - I*c*Tan[e + f*x])^(5/2) + 2*c*(((-2*I)/3
)*(c - I*c*Tan[e + f*x])^(3/2) + 2*c*((2*I)*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c
 - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])] - (2*I)*Sqrt[c - I*c*Tan[e + f*x]]
))))/4))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{2ic\left(\frac{iB(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+iBc(c-ic\tan(fx+e))^{\frac{3}{2}}+\frac{Ac(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+8i\sqrt{c-ic\tan(fx+e)}\,B\,c^{2}+4\sqrt{c-ic\tan(fa}{fa}$
default	$\frac{2ic\left(\frac{iB(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}+iBc(c-ic\tan(fx+e))^{\frac{3}{2}}+\frac{Ac(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+8i\sqrt{c-ic\tan(fx+e)}\,B\,c^{2}+4\sqrt{c-ic\tan(fa}{fa}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f/a*c*(1/5*I*B*(c-I*c*tan(f*x+e))^(5/2)+I*B*c*(c-I*c*tan(f*x+e))^(3/2)
+1/3*A*c*(c-I*c*tan(f*x+e))^(3/2)+8*I*(c-I*c*tan(f*x+e))^(1/2)*B*c^2+4*(c-
I*c*tan(f*x+e))^(1/2)*A*c^2-4*c^3*((-1/4*I*B-1/4*A)*(c-I*c*tan(f*x+e))^(1/
2)/(1/2*c+1/2*I*c*tan(f*x+e))+1/2*(9/2*I*B+5/2*A)*2^(1/2)/c^(1/2)*arctanh(
1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(173) = 346$.

Time = 0.11 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.12

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx =$$

$$-\frac{15\sqrt{2}\sqrt{-\frac{(25A^2+90iAB-81B^2)c^7}{a^2f^2}}\left(afe^{(6ifx+6ie)}+2afe^{(4ifx+4ie)}+afe^{(2ifx+2ie)}\right)\log\left(-\frac{8\left((5iA-9B)c^4+\sqrt{-\frac{(2}{}}\right.}{}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x,
algorithm="fricas")
```

output
```
-1/15*(15*sqrt(2)*sqrt(-(25*A^2 + 90*I*A*B - 81*B^2)*c^7/(a^2*f^2))*(a*f*e
^(6*I*f*x + 6*I*e) + 2*a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*e))*
log(-8*((5*I*A - 9*B)*c^4 + sqrt(-(25*A^2 + 90*I*A*B - 81*B^2)*c^7/(a^2*f^
2))*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(
-I*f*x - I*e)/(a*f)) - 15*sqrt(2)*sqrt(-(25*A^2 + 90*I*A*B - 81*B^2)*c^7/(
a^2*f^2))*(a*f*e^(6*I*f*x + 6*I*e) + 2*a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*
I*f*x + 2*I*e))*log(-8*((5*I*A - 9*B)*c^4 - sqrt(-(25*A^2 + 90*I*A*B - 81*
B^2)*c^7/(a^2*f^2))*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(c/(e^(2*I*f*x + 2
*I*e) + 1)))*e^(-I*f*x - I*e)/(a*f)) + 2*sqrt(2)*(15*(-5*I*A + 9*B)*c^3*e^
(6*I*f*x + 6*I*e) + 35*(-5*I*A + 9*B)*c^3*e^(4*I*f*x + 4*I*e) + 23*(-5*I*A
 + 9*B)*c^3*e^(2*I*f*x + 2*I*e) + 15*(-I*A + B)*c^3)*sqrt(c/(e^(2*I*f*x +
2*I*e) + 1)))/(a*f*e^(6*I*f*x + 6*I*e) + 2*a*f*e^(4*I*f*x + 4*I*e) + a*f*e
^(2*I*f*x + 2*I*e))
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx = -\frac{i\left(\int \frac{Ac^3\sqrt{-ic\tan{(e+fx)}+c}}{\tan{(e+fx)}-i}\,dx + \int \left(-\frac{3Ac^3\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}}{\tan{(e+fx)}-i}\right)dx + \int \frac{Bc^3\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}}{\tan{(e+fx)}-i}\,dx + \cdots$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*(Integral(A*c**3*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x) - I), x) + I
ntegral(-3*A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x
) - I), x) + Integral(B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan
(e + f*x) - I), x) + Integral(-3*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e
+ f*x)**3/(tan(e + f*x) - I), x) + Integral(-3*I*A*c**3*sqrt(-I*c*tan(e +
f*x) + c)*tan(e + f*x)/(tan(e + f*x) - I), x) + Integral(I*A*c**3*sqrt(-I*
c*tan(e + f*x) + c)*tan(e + f*x)**3/(tan(e + f*x) - I), x) + Integral(-3*I
*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x) - I), x)
 + Integral(I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4/(tan(e +
f*x) - I), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.86

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx = \frac{i\left(\frac{15\sqrt{2}(5A+9iB)c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a} - \frac{60\sqrt{-ict\cdots}}{(-icta\cdots}\cdots\right.}{\cdots}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x,
algorithm="maxima")
```

output
```
1/15*I*(15*sqrt(2)*(5*A + 9*I*B)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c
*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a - 6
0*sqrt(-I*c*tan(f*x + e) + c)*(A + I*B)*c^5/((-I*c*tan(f*x + e) + c)*a - 2
*a*c) + 2*(3*I*(-I*c*tan(f*x + e) + c)^(5/2)*B*c^2 + 5*(-I*c*tan(f*x + e)
+ c)^(3/2)*(A + 3*I*B)*c^3 + 60*sqrt(-I*c*tan(f*x + e) + c)*(A + 2*I*B)*c^
4)/a)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx &= \frac{4\,B\,c^4\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{a\,f\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)-2\,a\,c\,f} \\ &+ \frac{A\,c^3\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,8\mathrm{i}}{a\,f}+\frac{A\,c^2\,{\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)}^{3/2}\,2\mathrm{i}}{3\,a\,f} \\ &- \frac{16\,B\,c^3\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{a\,f}-\frac{2\,B\,c^2\,{\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)}^{3/2}}{a\,f} \\ &- \frac{2\,B\,c\,{\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)}^{5/2}}{5\,a\,f}+\frac{\sqrt{2}\,A\,{\left(-c\right)}^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,10\mathrm{i}}{a\,f} \\ &- \frac{\sqrt{2}\,B\,c^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,18\mathrm{i}}{a\,f}+\frac{A\,c^4\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,4\mathrm{i}}{a\,f\,\left(c+c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}\end{aligned}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*
x)*1i),x)
```

output
```
(4*B*c^4*(c - c*tan(e + f*x)*1i)^(1/2))/(a*f*(c - c*tan(e + f*x)*1i) - 2*a
*c*f) + (A*c^3*(c - c*tan(e + f*x)*1i)^(1/2)*8i)/(a*f) + (A*c^2*(c - c*tan
(e + f*x)*1i)^(3/2)*2i)/(3*a*f) - (16*B*c^3*(c - c*tan(e + f*x)*1i)^(1/2))
/(a*f) - (2*B*c^2*(c - c*tan(e + f*x)*1i)^(3/2))/(a*f) - (2*B*c*(c - c*tan
(e + f*x)*1i)^(5/2))/(5*a*f) + (2^(1/2)*A*(-c)^(7/2)*atan((2^(1/2)*(c - c*
tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*10i)/(a*f) - (2^(1/2)*B*c^(7/2)*at
an((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(2*c^(1/2)))*18i)/(a*f) + (A
*c^4*(c - c*tan(e + f*x)*1i)^(1/2)*4i)/(a*f*(c + c*tan(e + f*x)*1i))
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a+ia\tan(e+fx)}\,dx = \frac{\sqrt{c}\,c^3\left(2\sqrt{-\tan(fx+e)\,i+1}\,ai+\left(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\tan(fx+e)}\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e)),x)
```

output
```
(sqrt(c)*c**3*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int((sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)*i + 1),x)*b*f*i + int((sqrt( - ta
n(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)*i + 1),x)*a*f*i - 3*int((
sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)*i + 1),x)*b*f -
 2*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)*i + 1),
x)*a*f - 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)
*i + 1),x)*b*f*i - 5*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e
 + f*x)*i + 1),x)*a*f*i + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(
tan(e + f*x)*i + 1),x)*b*f))/(a*f)
```

3.765 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 180

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx = -\frac{\sqrt{2}(3iA-7B)c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{af} + \frac{(3iA-7B)c^2\sqrt{c-ic\tan(e+fx)}}{af} + \frac{(3iA-7B)c(c-ic\tan(e+fx))^{3/2}}{6af} + \frac{(iA-B)(c-ic\tan(e+fx))^{5/2}}{2af(1+i\tan(e+fx))}$$

output
```
-2^(1/2)*(3*I*A-7*B)*c^(5/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/
c^(1/2))/a/f+(3*I*A-7*B)*c^2*(c-I*c*tan(f*x+e))^(1/2)/a/f+1/6*(3*I*A-7*B)*
c*(c-I*c*tan(f*x+e))^(3/2)/a/f+1/2*(I*A-B)*(c-I*c*tan(f*x+e))^(5/2)/a/f/(1
+I*tan(f*x+e))
```

Mathematica [A] (verified)

Time = 3.77 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.74

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx = \frac{c^2\left(3\sqrt{2}(-3iA+7B)\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)+\frac{2\ldots}{\ldots}\right)}{3a\ldots}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan
[e + f*x]),x]
```

output
```
(c^2*(3*Sqrt[2]*((-3*I)*A + 7*B)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]
]/(Sqrt[2]*Sqrt[c])] + (2*Sqrt[c - I*c*Tan[e + f*x]]*(6*A + (13*I)*B + (3*
I)*(A + (3*I)*B)*Tan[e + f*x] + I*B*Tan[e + f*x]^2))/(-I + Tan[e + f*x])))
/(3*a*f)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {3042, 4071, 27, 87, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{a+ia\tan(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a^2(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)}{af}$$

$\Big\downarrow$ 87

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(3A+7iB)\int \frac{(c-ic\tan(e+fx))^{3/2}}{i\tan(e+fx)+1}d\tan(e+fx)\right)}{af}$$

$\Big\downarrow$ 60

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(3A+7iB)\left(2c\int \frac{\sqrt{c-ic\tan(e+fx)}}{i\tan(e+fx)+1}d\tan(e+fx)-\frac{2}{3}i(c-ic\tan(e+fx))^{3/2}\right)\right)}{af}$$

$\Big\downarrow$ 60

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(3A+7iB)\left(2c\left(2c\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)-2i\sqrt{c-ic\tan(\right.\right.\right.}{af}$$

$\Big\downarrow$ 73

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(3A+7iB)\left(2c\left(4i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}-2i\sqrt{c-ic\tan(e+\right.\right.\right.}{af}$$

$\Big\downarrow$ 219

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(3A+7iB)\left(2c\left(2i\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)-2i\sqrt{c-ic\tan(e+fx)}\right)-\right.\right.}{af}$$

input

```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan[e + f
*x]),x]
```

output

```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(5/2))/(2*c*(1 + I*Tan[e + f*x])) -
((3*A + (7*I)*B)*(((-2*I)/3)*(c - I*c*Tan[e + f*x])^(3/2) + 2*c*((2*I)*Sqr
t[2]*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])] - (2*I)
*Sqrt[c - I*c*Tan[e + f*x]])))/4))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2ic\left(\frac{iB(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+3i\sqrt{c-ic\tan(fx+e)}\,Bc+\sqrt{c-ic\tan(fx+e)}\,cA-4c^{2}\left(\frac{\left(-\frac{iB}{8}-\frac{A}{8}\right)\sqrt{c-ic\tan(fx+e)}}{\frac{c}{2}+\frac{ic\tan(fx+e)}{2}}+\frac{\left(\frac{7iB}{2}+\right.}{}\right.\right.}{fa}$
default	$\frac{2ic\left(\frac{iB(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+3i\sqrt{c-ic\tan(fx+e)}\,Bc+\sqrt{c-ic\tan(fx+e)}\,cA-4c^{2}\left(\frac{\left(-\frac{iB}{8}-\frac{A}{8}\right)\sqrt{c-ic\tan(fx+e)}}{\frac{c}{2}+\frac{ic\tan(fx+e)}{2}}+\frac{\left(\frac{7iB}{2}+\right.}{}\right.\right.}{fa}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f/a*c*(1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)+3*I*(c-I*c*tan(f*x+e))^(1/2)*B
*c+(c-I*c*tan(f*x+e))^(1/2)*c*A-4*c^2*((-1/8*I*B-1/8*A)*(c-I*c*tan(f*x+e))
^(1/2)/(1/2*c+1/2*I*c*tan(f*x+e))+1/4*(7/2*I*B+3/2*A)*2^(1/2)/c^(1/2)*arct
anh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(141) = 282.

Time = 0.11 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.26

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx = -\frac{3\sqrt{2}\left(afe^{(4i\,fx+4i\,e)}+afe^{(2i\,fx+2i\,e)}\right)\sqrt{-\frac{(9\,A^2+42i\,AB-49\,B^2)c^5}{a^2f^2}}\log\left(-\frac{4\left((3i\,A-7\,B)c^3+\left(afe^{(2i\,fx+2i\,e)}+af\right)\sqrt{-\frac{(9\,A^2+\cdots}{\cdots}}\cdots\right.}{af\cdots}\right.\cdots}{\cdots}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x,
algorithm="fricas")
```

output
```
-1/6*(3*sqrt(2)*(a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*e))*sqrt(-
(9*A^2 + 42*I*A*B - 49*B^2)*c^5/(a^2*f^2))*log(-4*((3*I*A - 7*B)*c^3 + (a*
f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(-(9*A^2 + 42*I*A*B - 49*B^2)*c^5/(a^2*f^
2))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a*f)) - 3*sqrt(2)
*(a*f*e^(4*I*f*x + 4*I*e) + a*f*e^(2*I*f*x + 2*I*e))*sqrt(-(9*A^2 + 42*I*A
*B - 49*B^2)*c^5/(a^2*f^2))*log(-4*((3*I*A - 7*B)*c^3 - (a*f*e^(2*I*f*x +
2*I*e) + a*f)*sqrt(-(9*A^2 + 42*I*A*B - 49*B^2)*c^5/(a^2*f^2))*sqrt(c/(e^(
2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a*f)) + 2*sqrt(2)*(3*(-3*I*A + 7
*B)*c^2*e^(4*I*f*x + 4*I*e) + 4*(-3*I*A + 7*B)*c^2*e^(2*I*f*x + 2*I*e) + 3
*(-I*A + B)*c^2)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(a*f*e^(4*I*f*x + 4*I*
e) + a*f*e^(2*I*f*x + 2*I*e))
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx = -\frac{i\left(\int \frac{Ac^2\sqrt{-ic\tan{(e+fx)}+c}}{\tan{(e+fx)}-i}\,dx+\int\left(-\frac{Ac^2\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}}{\tan{(e+fx)}-i}\right)dx+\int \frac{Bc^2\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}}{\tan{(e+fx)}-i}\,dx+\cdots\right.}{\cdots}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*(Integral(A*c**2*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x) - I), x) + I
ntegral(-A*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x)
- I), x) + Integral(B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e
 + f*x) - I), x) + Integral(-B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x)**3/(tan(e + f*x) - I), x) + Integral(-2*I*A*c**2*sqrt(-I*c*tan(e + f*x)
 + c)*tan(e + f*x)/(tan(e + f*x) - I), x) + Integral(-2*I*B*c**2*sqrt(-I*c
*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x) - I), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx = \frac{i\left(\frac{3\sqrt{2}(3A+7iB)c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a} - \frac{12\sqrt{-ic\,\mathrm{ta}}}{(-ic\,\mathrm{tan}}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x,
algorithm="maxima")
```

output
```
1/6*I*(3*sqrt(2)*(3*A + 7*I*B)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*t
an(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a - 12*
sqrt(-I*c*tan(f*x + e) + c)*(A + I*B)*c^4/((-I*c*tan(f*x + e) + c)*a - 2*a
*c) + 4*(I*(-I*c*tan(f*x + e) + c)^(3/2)*B*c^2 + 3*sqrt(-I*c*tan(f*x + e)
+ c)*(A + 3*I*B)*c^3)/a)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.36

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx = \frac{2\,B\,c^3\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{a\,f\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)-2\,a\,c\,f} + \frac{A\,c^2\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,2\mathrm{i}}{a\,f} - \frac{6\,B\,c^2\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{a\,f} - \frac{2\,B\,c\,{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^{3/2}}{3\,a\,f} - \frac{\sqrt{2}\,A\,{\left(-c\right)}^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,3\mathrm{i}}{a\,f} - \frac{\sqrt{2}\,B\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,7\mathrm{i}}{a\,f} + \frac{A\,c^3\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,2\mathrm{i}}{a\,f\,\left(c+c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*
x)*1i),x)
```

output
```
(2*B*c^3*(c - c*tan(e + f*x)*1i)^(1/2))/(a*f*(c - c*tan(e + f*x)*1i) - 2*a
*c*f) + (A*c^2*(c - c*tan(e + f*x)*1i)^(1/2)*2i)/(a*f) - (6*B*c^2*(c - c*t
an(e + f*x)*1i)^(1/2))/(a*f) - (2*B*c*(c - c*tan(e + f*x)*1i)^(3/2))/(3*a*
f) - (2^(1/2)*A*(-c)^(5/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2
*(-c)^(1/2)))*3i)/(a*f) - (2^(1/2)*B*c^(5/2)*atan((2^(1/2)*(c - c*tan(e +
f*x)*1i)^(1/2)*1i)/(2*c^(1/2)))*7i)/(a*f) + (A*c^3*(c - c*tan(e + f*x)*1i)
^(1/2)*2i)/(a*f*(c + c*tan(e + f*x)*1i))
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a+ia\tan(e+fx)}\,dx = \frac{\sqrt{c}\,c^2\Big(2\sqrt{-\tan\left(fx+e\right)i+1}\,ai-\Big(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\tan(fx+e)}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e)),x)
```

output
```
(sqrt(c)*c**2*(2*sqrt( - tan(e + f*x)*i + 1)*a*i - int((sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)*i + 1),x)*b*f - 2*int((sqrt( - ta
n(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)*i + 1),x)*b*f*i - 4*int((
sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)*i + 1),x)*a*f*i +
int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)*i + 1),x)*b*f
))/(a*f)
```

3.766 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 144

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx = -\frac{(iA-5B)c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}af} + \frac{(iA-5B)c\sqrt{c-ic\tan(e+fx)}}{2af} + \frac{(iA-B)(c-ic\tan(e+fx))^{3/2}}{2af(1+i\tan(e+fx))}$$

output
```
-1/2*(I*A-5*B)*c^(3/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2
))*2^(1/2)/a/f+1/2*(I*A-5*B)*c*(c-I*c*tan(f*x+e))^(1/2)/a/f+1/2*(I*A-B)*(c
-I*c*tan(f*x+e))^(3/2)/a/f/(1+I*tan(f*x+e))
```

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.75

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx = \frac{c\left(\sqrt{2}(-iA+5B)\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)+\frac{2(A+3}{}\right.}{2af}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan
[e + f*x]),x]
```

output
```
(c*(Sqrt[2]*((-I)*A + 5*B)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqr
t[2]*Sqrt[c])] + (2*(A + (3*I)*B - 2*B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e +
f*x]])/(-I + Tan[e + f*x])))/(2*a*f)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3042, 4071, 27, 87, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{a+ia\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a^2(i\tan(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)}{af}$$

$\downarrow$ 87

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(A+5iB)\int\frac{\sqrt{c-ic\tan(e+fx)}}{i\tan(e+fx)+1}d\tan(e+fx)\right)}{af}$$

$\downarrow$ 60

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(A+5iB)\left(2c\int\frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)-2i\sqrt{c-ic\tan(e+f}\right.\right.}{af}$$

$\downarrow$ 73

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(A+5iB)\left(4i\int\frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}-2i\sqrt{c-ic\tan(e+fx)}\right)\right)}{af}$$

$\downarrow$ 219

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{2c(1+i\tan(e+fx))}-\frac{1}{4}(A+5iB)\left(2i\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)-2i\sqrt{c-ic\tan(e+fx)}\right)\right)}{af}$$

input

```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan[e + f
*x]),x]
```

output

```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(3/2))/(2*c*(1 + I*Tan[e + f*x])) -
((A + (5*I)*B)*((2*I)*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(
Sqrt[2]*Sqrt[c])] - (2*I)*Sqrt[c - I*c*Tan[e + f*x]]))/4))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2ic\left(i\sqrt{c-ic\tan(fx+e)}\,B+c\left(\frac{\left(\frac{iB}{4}+\frac{A}{4}\right)\sqrt{c-ic\tan(fx+e)}}{\frac{c}{2}+\frac{ic\tan(fx+e)}{2}}-\frac{\left(\frac{A}{2}+\frac{5iB}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{2\sqrt{c}}\right)\right)}{fa}$	111
default	$\frac{2ic\left(i\sqrt{c-ic\tan(fx+e)}\,B+c\left(\frac{\left(\frac{iB}{4}+\frac{A}{4}\right)\sqrt{c-ic\tan(fx+e)}}{\frac{c}{2}+\frac{ic\tan(fx+e)}{2}}-\frac{\left(\frac{A}{2}+\frac{5iB}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{2\sqrt{c}}\right)\right)}{fa}$	111

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f/a*c*(I*(c-I*c*tan(f*x+e))^(1/2)*B+c*((1/4*I*B+1/4*A)*(c-I*c*tan(f*x+
e))^(1/2)/(1/2*c+1/2*I*c*tan(f*x+e))-1/2*(1/2*A+5/2*I*B)*2^(1/2)/c^(1/2)*a
rctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(111) = 222.

Time = 0.10 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.29

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx = -\frac{\left(\sqrt{2}af\sqrt{-\frac{(A^2+10iAB-25B^2)c^3}{a^2f^2}}e^{(2ifx+2ie)}\log\left(-\frac{2\left((iA-5B)c^2+\left(afe^{(2ifx+2ie)}+af\right)\sqrt{-\frac{\left(A^2+10iAB-25B^2\right)c^3}{a^2f^2}}\sqrt{\frac{c}{e^{(2ifx+2ie)}}}\right.}{af}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x,
algorithm="fricas")
```

output
```
-1/4*(sqrt(2)*a*f*sqrt(-(A^2 + 10*I*A*B - 25*B^2)*c^3/(a^2*f^2))*e^(2*I*f*
x + 2*I*e)*log(-2*((I*A - 5*B)*c^2 + (a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(
-(A^2 + 10*I*A*B - 25*B^2)*c^3/(a^2*f^2))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)
))*e^(-I*f*x - I*e)/(a*f)) - sqrt(2)*a*f*sqrt(-(A^2 + 10*I*A*B - 25*B^2)*c
^3/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(-2*((I*A - 5*B)*c^2 - (a*f*e^(2*I*f*
x + 2*I*e) + a*f)*sqrt(-(A^2 + 10*I*A*B - 25*B^2)*c^3/(a^2*f^2))*sqrt(c/(e
^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a*f)) + 2*sqrt(2)*((-I*A + 5*B
)*c*e^(2*I*f*x + 2*I*e) + (-I*A + B)*c)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))
*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx = -\frac{i\left(\int \frac{Ac\sqrt{-ic\tan(e+fx)+c}}{\tan(e+fx)-i}\,dx + \int \frac{Bc\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan(e+fx)-i}\,dx + \int \left(-\frac{iAc\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan(e+fx)-i}\right)dx + \int \left(\right.\right.}{a}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*(Integral(A*c*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x) - I), x) + Inte
gral(B*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x) - I), x) +
 Integral(-I*A*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x) -
I), x) + Integral(-I*B*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(
e + f*x) - I), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx = \frac{i\left(\frac{\sqrt{2}(A+5i\,B)c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-i\,c\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-i\,c\tan(fx+e)+c}}\right)}{a}-\frac{4\sqrt{-i\,c\tan(f}{(-i\,c\tan(f}\right.}{4\,cf}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x,
algorithm="maxima")
```

output
```
1/4*I*(sqrt(2)*(A + 5*I*B)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f
*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a - 4*sqrt(
-I*c*tan(f*x + e) + c)*(A + I*B)*c^3/((-I*c*tan(f*x + e) + c)*a - 2*a*c) +
 8*I*sqrt(-I*c*tan(f*x + e) + c)*B*c^2/a)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.31

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx = \frac{B\,c^2\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{a\,f\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)-2\,a\,c\,f} - \frac{2\,B\,c\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{a\,f} + \frac{\sqrt{2}\,A\,{\left(-c\right)}^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,1\mathrm{i}}{2\,a\,f} + \frac{5\,\sqrt{2}\,B\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{2\,a\,f} + \frac{A\,c^2\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,1\mathrm{i}}{a\,f\,\left(c+c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*
x)*1i),x)
```

output
```
(B*c^2*(c - c*tan(e + f*x)*1i)^(1/2))/(a*f*(c - c*tan(e + f*x)*1i) - 2*a*c
*f) - (2*B*c*(c - c*tan(e + f*x)*1i)^(1/2))/(a*f) + (2^(1/2)*A*(-c)^(3/2)*
atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*1i)/(2*a*f) +
 (5*2^(1/2)*B*c^(3/2)*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(
1/2))))/(2*a*f) + (A*c^2*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(a*f*(c + c*tan
(e + f*x)*1i))
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a+ia\tan(e+fx)}\,dx = \frac{\sqrt{c}\,c\left(2\sqrt{-\tan\left(fx+e\right)i+1}\,ai+\left(\int\frac{\sqrt{-\tan(fx+e)i+1}\,t}{\tan(fx+e)i}\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e)),x)
```

output
```
(sqrt(c)*c*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int((sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**2)/(tan(e + f*x)*i + 1),x)*a*f - int((sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)*i + 1),x)*b*f*i - 3*int((sqrt(
 - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)*i + 1),x)*a*f*i + int((
sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)*i + 1),x)*b*f))/(a
*f)
```

3.767 $\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 109

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx = \frac{(iA+3B)\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}af}+\frac{(iA-B)\sqrt{c-ic\tan(e+fx)}}{2af(1+i\tan(e+fx))}$$

output
```
1/4*(I*A+3*B)*c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
)*2^(1/2)/a/f+1/2*(I*A-B)*(c-I*c*tan(f*x+e))^(1/2)/a/f/(1+I*tan(f*x+e))
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx = \frac{\sqrt{2}(iA+3B)\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)+\frac{2(A+iB)\sqrt{c-ic\tan(e+fx)}}{-i+\tan(e+fx)}}{4af}$$

input
```
Integrate[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e
 + f*x]),x]
```

output
```
(Sqrt[2]*(I*A + 3*B)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*S
qrt[c])] + (2*(A + I*B)*Sqrt[c - I*c*Tan[e + f*x]])/(-I + Tan[e + f*x]))/(
4*a*f)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {3042, 4071, 27, 87, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{a+ia\tan(e+fx)}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^2(i\tan(e+fx)+1)^2\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^2\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{87}$$

$$\frac{c\left(\frac{1}{4}(A-3iB)\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)}{af}$$

$$\Big\downarrow \text{73}$$

$$\frac{c\left(\frac{i(A-3iB)\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{2c}+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)}{af}$$

$$\Big\downarrow \; 219$$

$$\frac{c\left(\frac{i(A-3iB)\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\sqrt{c}}+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)}{af}$$

input
```
Int[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e + f*x
]),x]
```

output
```
(c*(((I/2)*(A - (3*I)*B)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[
c])])/(Sqrt[2]*Sqrt[c]) + ((I*A - B)*Sqrt[c - I*c*Tan[e + f*x]])/(2*c*(1 +
 I*Tan[e + f*x]))))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2ic\left(\frac{\left(\frac{iB}{8}+\frac{A}{8}\right)\sqrt{c-ic\tan(fx+e)}}{\frac{c}{2}+\frac{ic\tan(fx+e)}{2}}+\frac{\left(-\frac{3iB}{2}+\frac{A}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}}\right)}{fa}$	90
default	$\frac{2ic\left(\frac{\left(\frac{iB}{8}+\frac{A}{8}\right)\sqrt{c-ic\tan(fx+e)}}{\frac{c}{2}+\frac{ic\tan(fx+e)}{2}}+\frac{\left(-\frac{3iB}{2}+\frac{A}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}}\right)}{fa}$	90

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f/a*c*((1/8*I*B+1/8*A)*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*c*tan(f*x
+e))+1/4*(-3/2*I*B+1/2*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(
1/2)*2^(1/2)/c^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(82) = 164.

Time = 0.08 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.99

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx = \frac{\left(\sqrt{\frac{1}{2}}af\sqrt{-\frac{(A^2-6i\,AB-9\,B^2)c}{a^2f^2}}e^{(2i\,fx+2i\,e)}\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(afe^{(2i\,fx+2i\,e)}+af\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{(A^2-6i\,AB-9\,B^2)c}{a^2f^2}}+(i\,A+3\,B\right.}{af}\right.\right.}{}$$

input

```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x,
algorithm="fricas")
```

output

```
1/4*(sqrt(1/2)*a*f*sqrt(-(A^2 - 6*I*A*B - 9*B^2)*c/(a^2*f^2))*e^(2*I*f*x +
 2*I*e)*log((sqrt(2)*sqrt(1/2)*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(c/(e^(
2*I*f*x + 2*I*e) + 1))*sqrt(-(A^2 - 6*I*A*B - 9*B^2)*c/(a^2*f^2)) + (I*A +
 3*B)*c)*e^(-I*f*x - I*e)/(a*f)) - sqrt(1/2)*a*f*sqrt(-(A^2 - 6*I*A*B - 9*
B^2)*c/(a^2*f^2))*e^(2*I*f*x + 2*I*e)*log(-(sqrt(2)*sqrt(1/2)*(a*f*e^(2*I*
f*x + 2*I*e) + a*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(A^2 - 6*I*A*B
 - 9*B^2)*c/(a^2*f^2)) - (I*A + 3*B)*c)*e^(-I*f*x - I*e)/(a*f)) + sqrt(2)*
((I*A - B)*e^(2*I*f*x + 2*I*e) + I*A - B)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)
))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx = -\frac{i\left(\int \frac{A\sqrt{-ic\tan(e+fx)+c}}{\tan(e+fx)-i}\,dx+\int \frac{B\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan(e+fx)-i}\,dx\right)}{a}$$

input

```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e)),x)
```

output
```
-I*(Integral(A*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x) - I), x) + Integr
al(B*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x) - I), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx = -\frac{i\left(\frac{\sqrt{2}(A-3i\,B)c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-i\,c\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-i\,c\tan(fx+e)+c}}\right)}{a}+\frac{4\sqrt{-i\,c\tan(fx+e)+c}(A+i\,B)c^2}{(-i\,c\tan(fx+e)+c)a-2\,ac}\right)}{8\,cf}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x,
algorithm="maxima")
```

output
```
-1/8*I*(sqrt(2)*(A - 3*I*B)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(
f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a + 4*sqrt
(-I*c*tan(f*x + e) + c)*(A + I*B)*c^2/((-I*c*tan(f*x + e) + c)*a - 2*a*c))
/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.46

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx$$
$$= -\frac{B\,c\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{2\,\left(a\,c\,f+a\,c\,f\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}+\frac{A\,c\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,1\mathrm{i}}{2\,a\,f\,\left(c+c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}$$
$$+\frac{\sqrt{2}\,A\,\sqrt{-c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,1\mathrm{i}}{4\,a\,f}+\frac{3\,\sqrt{2}\,B\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{4\,a\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(1/2))/(a + a*tan(e + f*
x)*1i),x)
```

output
```
(A*c*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(2*a*f*(c + c*tan(e + f*x)*1i)) - (
B*c*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(a*c*f + a*c*f*tan(e + f*x)*1i)) + (
2^(1/2)*A*(-c)^(1/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^
(1/2)))*1i)/(4*a*f) + (3*2^(1/2)*B*c^(1/2)*atanh((2^(1/2)*(c - c*tan(e + f
*x)*1i)^(1/2))/(2*c^(1/2))))/(4*a*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx$$
$$= \frac{\sqrt{c}\left(2\sqrt{-\tan\left(fx+e\right)i+1}\,ai+\left(\int\frac{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^2}{\tan(fx+e)i+1}dx\right)af-2\left(\int\frac{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)}{\tan(fx+e)i+1}dx\right)a\right.}{af}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e)),x)
```

output
```
(sqrt(c)*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int((sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2)/(tan(e + f*x)*i + 1),x)*a*f - 2*int((sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)*i + 1),x)*a*f*i + int((sqrt( - ta
n(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)*i + 1),x)*b*f))/(a*f)
```

3.768 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 141

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{(3iA+B)\,\mathrm{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}a\sqrt{c}f} - \frac{3iA+B}{4af\sqrt{c-ic\tan(e+fx)}} + \frac{iA-B}{2af(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}$$

output
```
1/8*(3*I*A+B)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2
)/a/c^(1/2)/f-1/4*(3*I*A+B)/a/f/(c-I*c*tan(f*x+e))^(1/2)+1/2*(I*A-B)/a/f/(
1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$=\frac{\frac{\sqrt{2}(3iA+B)\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}+\frac{-2A+6iB+(-6iA-2B)\tan(e+fx)}{(-i+\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}}{8af}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*Sqrt[c - I*c*Tan[e
+ f*x]]),x]
```

output
```
((Sqrt[2]*((3*I)*A + B)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c
])]])/Sqrt[c] + (-2*A + (6*I)*B + ((-6*I)*A - 2*B)*Tan[e + f*x])/((-I + Tan
[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]))/(8*a*f)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {3042, 4071, 27, 87, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^2(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{af}$$

$$\Big\downarrow \text{87}$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)+\frac{-B+iA}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)}{af}$$

$$\Big\downarrow \text{61}$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\left(\frac{\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{2c}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}\right)+\frac{-B+iA}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)}{af}$$

$$\Big\downarrow \text{73}$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\left(\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}\right)+\frac{-B+iA}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)}{af}$$

$$\Big\downarrow \text{219}$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\left(\frac{i\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}c^{3/2}}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}\right)+\frac{-B+iA}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)}{af}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]
]),x]
```

output
```
(c*((I*A - B)/(2*c*(1 + I*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]) + ((3*
A - I*B)*((I*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[
2]*c^(3/2)) - I/(c*Sqrt[c - I*c*Tan[e + f*x]])))/4))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2ic\left(\frac{\frac{\left(\frac{iB}{4}+\frac{A}{4}\right)\sqrt{c-ic\tan\left(fx+e\right)}}{\frac{c}{2}+\frac{ic\tan\left(fx+e\right)}{2}}+\frac{\left(\frac{3A}{2}-\frac{iB}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{c}}\right)}{2\sqrt{c}}}{4c}-\frac{-iB+A}{4c\sqrt{c-ic\tan\left(fx+e\right)}}\right)}{fa}$	121
default	$\frac{2ic\left(\frac{\frac{\left(\frac{iB}{4}+\frac{A}{4}\right)\sqrt{c-ic\tan\left(fx+e\right)}}{\frac{c}{2}+\frac{ic\tan\left(fx+e\right)}{2}}+\frac{\left(\frac{3A}{2}-\frac{iB}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan\left(fx+e\right)}\,\sqrt{2}}{2\sqrt{c}}\right)}{2\sqrt{c}}}{4c}-\frac{-iB+A}{4c\sqrt{c-ic\tan\left(fx+e\right)}}\right)}{fa}$	121

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f/a*c*(1/4/c*((1/4*I*B+1/4*A)*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*c*
tan(f*x+e))+1/2*(3/2*A-1/2*I*B)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x
+e))^(1/2)*2^(1/2)/c^(1/2)))-1/4/c*(A-I*B)/(c-I*c*tan(f*x+e))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(110) = 220.

Time = 0.11 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.52

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{\left(\sqrt{\frac{1}{2}}acf\sqrt{-\frac{9A^2-6iAB-B^2}{a^2cf^2}}e^{(2ifx+2ie)}\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(afe^{(2ifx+2ie)}+af\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{9A^2-6iAB-B^2}{a^2cf^2}}+3iA+B\right)e^{(-i}}{2af}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,
algorithm="fricas")
```

output
```
1/8*(sqrt(1/2)*a*c*f*sqrt(-(9*A^2 - 6*I*A*B - B^2)/(a^2*c*f^2))*e^(2*I*f*x
 + 2*I*e)*log(1/2*(sqrt(2)*sqrt(1/2)*(a*f*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(
c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(9*A^2 - 6*I*A*B - B^2)/(a^2*c*f^2)) +
3*I*A + B)*e^(-I*f*x - I*e)/(a*f)) - sqrt(1/2)*a*c*f*sqrt(-(9*A^2 - 6*I*A*
B - B^2)/(a^2*c*f^2))*e^(2*I*f*x + 2*I*e)*log(-1/2*(sqrt(2)*sqrt(1/2)*(a*f
*e^(2*I*f*x + 2*I*e) + a*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(9*A^2
 - 6*I*A*B - B^2)/(a^2*c*f^2)) - 3*I*A - B)*e^(-I*f*x - I*e)/(a*f)) - sqrt
(2)*(2*(I*A + B)*e^(4*I*f*x + 4*I*e) - (-I*A - 3*B)*e^(2*I*f*x + 2*I*e) -
I*A + B)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a*c*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{i\left(\int \frac{A}{\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)-i\sqrt{-ic\tan(e+fx)+c}}\,dx + \int \frac{B\tan(e+fx)}{\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)-i\sqrt{-ic\tan(e+fx)+c}}\,dx\right)}{a}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**(1/2),x)
```

output
```
-I*(Integral(A/(sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) - I*sqrt(-I*c*tan
(e + f*x) + c)), x) + Integral(B*tan(e + f*x)/(sqrt(-I*c*tan(e + f*x) + c)
*tan(e + f*x) - I*sqrt(-I*c*tan(e + f*x) + c)), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{i\left(\frac{\sqrt{2}(3A-iB)\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a}+\frac{4\left((-ic\tan(fx+e)+c)(3A-iB)c-4(A-iB)c^2\right)}{(-ic\tan(fx+e)+c)^{\frac{3}{2}}a-2\sqrt{-ic\tan(fx+e)+c}ac}\right)}{16cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,
algorithm="maxima")
```

output
```
-1/16*I*(sqrt(2)*(3*A - I*B)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan
(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a + 4*((-
I*c*tan(f*x + e) + c)*(3*A - I*B)*c - 4*(A - I*B)*c^2)/((-I*c*tan(f*x + e)
 + c)^(3/2)*a - 2*sqrt(-I*c*tan(f*x + e) + c)*a*c))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.50

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))\sqrt{c - ic\tan(e + fx)}}\,dx$$
$$= \frac{B\,c - \frac{B\,(c - c\tan(e + f\,x)\,1i)}{4}}{a\,f\,(c - c\tan{(e + f\,x)}\,1i)^{3/2} - 2\,a\,c\,f\,\sqrt{c - c\tan{(e + f\,x)}\,1i}} + \frac{\frac{A\,(c - c\tan(e + f\,x)\,1i)\,3i}{4\,a\,f} - \frac{A\,c\,1i}{a\,f}}{2\,c\,\sqrt{c - c\tan{(e + f\,x)}\,1i} - (c - c\tan{(e + f\,x)}\,1i)^{3/2}} - \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c - c\tan(e + f\,x)\,1i}}{2\,\sqrt{-c}}\right)\,3i}{8\,a\,\sqrt{-c}\,f} + \frac{\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c - c\tan(e + f\,x)\,1i}}{2\,\sqrt{c}}\right)}{8\,a\,\sqrt{c}\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^
(1/2)),x)
```

output
```
(B*c - (B*(c - c*tan(e + f*x)*1i))/4)/(a*f*(c - c*tan(e + f*x)*1i)^(3/2) -
 2*a*c*f*(c - c*tan(e + f*x)*1i)^(1/2)) + ((A*(c - c*tan(e + f*x)*1i)*3i)/
(4*a*f) - (A*c*1i)/(a*f))/(2*c*(c - c*tan(e + f*x)*1i)^(1/2) - (c - c*tan(
e + f*x)*1i)^(3/2)) - (2^(1/2)*A*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/
2))/(2*(-c)^(1/2)))*3i)/(8*a*(-c)^(1/2)*f) + (2^(1/2)*B*atanh((2^(1/2)*(c
- c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(8*a*c^(1/2)*f)
```

Reduce [F]

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))\sqrt{c - ic\tan(e + fx)}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt( - tan(e + f*x)*i + 1)*a*i - 2*sqrt( - tan(e + f*x)*i + 1
)*b + int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i + tan(e + f*x)**2
 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*f - int(sqrt( - tan(e + f*x)*i
 + 1)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e
+ f*x)**2*b*f*i + int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i + tan
(e + f*x)**2 + tan(e + f*x)*i + 1),x)*a*f - int(sqrt( - tan(e + f*x)*i + 1
)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*b*f*i + 9*
int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i + tan
(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*f - int((sqrt( - t
an(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i + tan(e + f*x)**2 +
 tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i + 9*int((sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f
*x)*i + 1),x)*a*f - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan
(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*b*f*i + 8*int((s
qrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**3*i + tan(e + f*x)
**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*f*i + 8*int((sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e +
 f*x)*i + 1),x)*a*f*i))/(2*a*c*f*(tan(e + f*x)**2 + 1))
```

3.769 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 184

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(5iA-B)\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}ac^{3/2}f} - \frac{5iA-B}{12af(c-ic\tan(e+fx))^{3/2}} + \frac{iA-B}{2af(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}} - \frac{5iA-B}{8acf\sqrt{c-ic\tan(e+fx)}}$$

output
```
1/16*(5*I*A-B)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/
2)/a/c^(3/2)/f-1/12*(5*I*A-B)/a/f/(c-I*c*tan(f*x+e))^(3/2)+1/2*(I*A-B)/a/f
/(1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2)-1/8*(5*I*A-B)/a/c/f/(c-I*c*tan(
f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.91 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.56

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{3(-5iA+B)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\frac{1}{2}i(i+t\right.}{}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*
x])^(3/2)),x]
```

output
```
(3*((-5*I)*A + B)*Hypergeometric2F1[-1/2, 1, 1/2, (-1/2*I)*(I + Tan[e + f*
x])] + (2*I)*Cos[e + f*x]*((A + (5*I)*B)*Cos[e + f*x] + ((-5*I)*A + B)*Sin
[e + f*x]))/(24*a*c*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {3042, 4071, 27, 87, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^2(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{af}$$

$\downarrow$ 87

$$\frac{c\left(\frac{1}{4}(5A+iB)\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)+\frac{-B+iA}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\right)}{af}$$

$\downarrow$ 61

$$\frac{c\left(\frac{1}{4}(5A+iB)\left(\frac{\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{-B+iA}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\right)}{af}$$

$\downarrow$ 61

$$\frac{c\left(\frac{1}{4}(5A+iB)\left(\frac{\frac{\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{2c}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{-B+iA}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\right)}{af}$$

$\downarrow$ 73

$$\frac{c\left(\frac{1}{4}(5A+iB)\left(\frac{\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{-B+iA}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\right)}{af}$$

$\downarrow$ 219

$$\frac{c\left(\frac{1}{4}(5A+iB)\left(\frac{\frac{i\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}c^{3/2}}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{-B+iA}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\right)}{af}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3
/2)),x]
```

output
```
(c*((I*A - B)/(2*c*(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2)) + ((
5*A + I*B)*((-1/3*I)/(c*(c - I*c*Tan[e + f*x])^(3/2)) + ((I*ArcTanh[Sqrt[c
 - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[2]*c^(3/2)) - I/(c*Sqrt[c -
 I*c*Tan[e + f*x]]))/(2*c)))/4))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{2ic\left(\frac{\frac{\left(\frac{iB}{8}+\frac{A}{8}\right)\sqrt{c-ic\tan(fx+e)}}{\frac{c}{2}+\frac{ic\tan(fx+e)}{2}}+\frac{\left(\frac{iB}{2}+\frac{5A}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}}}{4c^2}-\frac{A}{4c^2\sqrt{c-ic\tan(fx+e)}}-\frac{-iB+A}{12c(c-ic\tan(fx+e)}\right.}{fa}$
default	$\frac{2ic\left(\frac{\frac{\left(\frac{iB}{8}+\frac{A}{8}\right)\sqrt{c-ic\tan(fx+e)}}{\frac{c}{2}+\frac{ic\tan(fx+e)}{2}}+\frac{\left(\frac{iB}{2}+\frac{5A}{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}}}{4c^2}-\frac{A}{4c^2\sqrt{c-ic\tan(fx+e)}}-\frac{-iB+A}{12c(c-ic\tan(fx+e)}\right.}{fa}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f/a*c*(1/4/c^2*((1/8*I*B+1/8*A)*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*
c*tan(f*x+e))+1/4*(1/2*I*B+5/2*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f
*x+e))^(1/2)*2^(1/2)/c^(1/2)))-1/4/c^2*A/(c-I*c*tan(f*x+e))^(1/2)-1/12/c*(
A-I*B)/(c-I*c*tan(f*x+e))^(3/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(135) = 270.

Time = 0.09 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.11

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\left(3\sqrt{\frac{1}{2}}ac^2f\sqrt{-\frac{25A^2+10iAB-B^2}{a^2c^3f^2}}e^{(2ifx+2ie)}\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="fricas")
```

output
```
1/48*(3*sqrt(1/2)*a*c^2*f*sqrt(-(25*A^2 + 10*I*A*B - B^2)/(a^2*c^3*f^2))*e
^(2*I*f*x + 2*I*e)*log(1/4*(sqrt(2)*sqrt(1/2)*(a*c*f*e^(2*I*f*x + 2*I*e) +
 a*c*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(25*A^2 + 10*I*A*B - B^2)/
(a^2*c^3*f^2)) + 5*I*A - B)*e^(-I*f*x - I*e)/(a*c*f)) - 3*sqrt(1/2)*a*c^2*
f*sqrt(-(25*A^2 + 10*I*A*B - B^2)/(a^2*c^3*f^2))*e^(2*I*f*x + 2*I*e)*log(-
1/4*(sqrt(2)*sqrt(1/2)*(a*c*f*e^(2*I*f*x + 2*I*e) + a*c*f)*sqrt(c/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(-(25*A^2 + 10*I*A*B - B^2)/(a^2*c^3*f^2)) - 5*I*A
+ B)*e^(-I*f*x - I*e)/(a*c*f)) - sqrt(2)*(2*(I*A + B)*e^(6*I*f*x + 6*I*e)
+ 4*(4*I*A + B)*e^(4*I*f*x + 4*I*e) - (-11*I*A - 5*B)*e^(2*I*f*x + 2*I*e)
- 3*I*A + 3*B)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a*
c^2*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{i\left(\int \frac{A}{-ic\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-ic\sqrt{-ic\tan(e+fx)+c}}\,dx + \int \frac{B\tan(e+fx)}{-ic\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-ic\sqrt{-ic\tan(e+fx)+c}}\,dx\right.}{a}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**(3/2),x)
```

output
```
-I*(Integral(A/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - I*c*sqr
t(-I*c*tan(e + f*x) + c)), x) + Integral(B*tan(e + f*x)/(-I*c*sqrt(-I*c*ta
n(e + f*x) + c)*tan(e + f*x)**2 - I*c*sqrt(-I*c*tan(e + f*x) + c)), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{i\left(\frac{3\sqrt{2}(5A+iB)\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a\sqrt{c}}+\frac{4\left(3(-ic\tan(fx+e)+c)^2(5A+iB)-4(-ic\tan(fx+e)+c)(5A+iB)c-8(A-iB)c^2\right)}{(-ic\tan(fx+e)+c)^{\frac{5}{2}}a-2(-ic\tan(fx+e)+c)^{\frac{3}{2}}ac}\right)}{96cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="maxima")
```

output
```
-1/96*I*(3*sqrt(2)*(5*A + I*B)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x +
 e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/(a*sqrt(c)) + 4
*(3*(-I*c*tan(f*x + e) + c)^2*(5*A + I*B) - 4*(-I*c*tan(f*x + e) + c)*(5*A
 + I*B)*c - 8*(A - I*B)*c^2)/((-I*c*tan(f*x + e) + c)^(5/2)*a - 2*(-I*c*ta
n(f*x + e) + c)^(3/2)*a*c))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.89 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.42

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\frac{B\,c}{3}-\frac{B\,(c-c\tan(e+f\,x)\,1\mathrm{i})}{6}+\frac{B\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2}{8\,c}}{a\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}-2\,a\,c\,f\,(c-c\tan(e+\ldots}$$
$$-\frac{\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})\,5\mathrm{i}}{6\,a\,f}+\frac{A\,c\,1\mathrm{i}}{3\,a\,f}-\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2\,5\mathrm{i}}{8\,a\,c\,f}}{2\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}-(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}}$$
$$+\frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,5\mathrm{i}}{16\,a\,(-c)^{3/2}\,f}-\frac{\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{16\,a\,c^{3/2}\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^
(3/2)),x)
```

output
```
((B*c)/3 - (B*(c - c*tan(e + f*x)*1i))/6 + (B*(c - c*tan(e + f*x)*1i)^2)/(
8*c))/(a*f*(c - c*tan(e + f*x)*1i)^(5/2) - 2*a*c*f*(c - c*tan(e + f*x)*1i)
^(3/2)) - ((A*(c - c*tan(e + f*x)*1i)*5i)/(6*a*f) + (A*c*1i)/(3*a*f) - (A*
(c - c*tan(e + f*x)*1i)^2*5i)/(8*a*c*f))/(2*c*(c - c*tan(e + f*x)*1i)^(3/2
) - (c - c*tan(e + f*x)*1i)^(5/2)) + (2^(1/2)*A*atan((2^(1/2)*(c - c*tan(e
 + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*5i)/(16*a*(-c)^(3/2)*f) - (2^(1/2)*B*at
anh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(16*a*c^(3/2)*f)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{-\tan(fx+e)\,i+1}\,ai+\left(\int\frac{\sqrt{-\tan(fx+e)i+1}\,\tan\ldots}{\tan(fx+e)^4+2\tan(f\ldots}\right.\right.}{\ldots}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x)
```

output

```
(sqrt(c)*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int((sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2)/(tan(e + f*x)**4 + 2*tan(e + f*x)**2 + 1),x)*tan(e +
 f*x)**4*b*f*i + 2*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(
e + f*x)**4 + 2*tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f*i + int((sqrt(
 - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**4 + 2*tan(e + f*x)*
*2 + 1),x)*b*f*i + 8*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e
 + f*x)**4 + 2*tan(e + f*x)**2 + 1),x)*tan(e + f*x)**4*a*f*i + int((sqrt(
- tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**4 + 2*tan(e + f*x)**2 +
 1),x)*tan(e + f*x)**4*b*f + 16*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x))/(tan(e + f*x)**4 + 2*tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*a*f*i +
2*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**4 + 2*tan(
e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f + 8*int((sqrt( - tan(e + f*x)*i +
1)*tan(e + f*x))/(tan(e + f*x)**4 + 2*tan(e + f*x)**2 + 1),x)*a*f*i + int(
(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**4 + 2*tan(e + f*
x)**2 + 1),x)*b*f))/(a*c**2*f*(tan(e + f*x)**4 + 2*tan(e + f*x)**2 + 1))
```

3.770 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 223

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\,dx=\frac{(7iA-3B)\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}ac^{5/2}f} - \frac{7iA-3B}{20af(c-ic\tan(e+fx))^{5/2}} + \frac{iA-B}{2af(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5/2}} - \frac{7iA-3B}{24acf(c-ic\tan(e+fx))^{3/2}} - \frac{7iA-3B}{16ac^2f\sqrt{c-ic\tan(e+fx)}}$$

output
```
1/32*(7*I*A-3*B)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(
1/2)/a/c^(5/2)/f-1/20*(7*I*A-3*B)/a/f/(c-I*c*tan(f*x+e))^(5/2)+1/2*(I*A-B)
/a/f/(1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2)-1/24*(7*I*A-3*B)/a/c/f/(c-I
*c*tan(f*x+e))^(3/2)-1/16*(7*I*A-3*B)/a/c^2/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.92 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.57

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{5(7A+3iB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\frac{1}{2}i(i+\right.}{120ac^2f(-i+\tan(e+f}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*
x])^(5/2)),x]
```

output
```
(5*(7*A + (3*I)*B)*Hypergeometric2F1[-3/2, 1, -1/2, (-1/2*I)*(I + Tan[e +
f*x])]*Sec[e + f*x]^2 + (6*I)*((3*I)*A - 7*B + (7*A + (3*I)*B)*Tan[e + f*x
]))/(120*a*c^2*f*(-I + Tan[e + f*x])*(I + Tan[e + f*x])^2*Sqrt[c - I*c*Tan
[e + f*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.209$, Rules used = {3042, 4071, 27, 87, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^2(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{af}$$

$$\Big\downarrow \; 87$$

$$\frac{c\left(\frac{1}{4}(7A+3iB)\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)+\frac{-B+iA}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\right)}{af}$$

$$\Big\downarrow \; 61$$

$$\frac{c\left(\frac{1}{4}(7A+3iB)\left(\frac{\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{-B+iA}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5\ldots}}\right.}{af}$$

$$\Big\downarrow \; 61$$

$$\frac{c\left(\frac{1}{4}(7A+3iB)\left(\frac{\frac{\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{\ldots}{2c(1+i\tan(e+\ldots}\right.}{af}$$

$$\Big\downarrow \; 61$$

$$\frac{c\left(\frac{1}{4}(7A+3iB)\left(\frac{\frac{\frac{\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{2c}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\ldots\right.}{af}$$

$$\Big\downarrow \; 73$$

$$\frac{c\left(\frac{1}{4}(7A+3iB)\left(\frac{\frac{\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{\ldots}{2c(1\ldots}\right.}{af}$$

$$\Big\downarrow \; 219$$

$$\frac{c\left(\frac{1}{4}(7A+3iB)\left(\frac{\frac{\frac{i\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}c^{3/2}}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{}{2c(1+i\tan(}\right.}{af}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5
/2)),x]
```

output
```
(c*((I*A - B)/(2*c*(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2)) + ((
7*A + (3*I)*B)*((-1/5*I)/(c*(c - I*c*Tan[e + f*x])^(5/2)) + ((-1/3*I)/(c*(
c - I*c*Tan[e + f*x])^(3/2)) + ((I*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqr
t[2]*Sqrt[c])])/(Sqrt[2]*c^(3/2)) - I/(c*Sqrt[c - I*c*Tan[e + f*x]]))/(2*c
))/(2*c)))/4))/(a*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{2ic\left(-\frac{iB+3A}{16c^3\sqrt{c-ic\tan(fx+e)}}-\frac{A}{12c^2(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{-iB+A}{20c(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{\frac{\left(\frac{iB}{4}+\frac{A}{4}\right)\sqrt{c-ic\tan(fx+e)}}{\frac{c}{2}+\frac{ic\tan(fx+e)}{2}}+\left(\frac{7A}{2}+\frac{3iB}{2}\right)\sqrt{}}{16c^3}\right.}{fa}$
default	$\frac{2ic\left(-\frac{iB+3A}{16c^3\sqrt{c-ic\tan(fx+e)}}-\frac{A}{12c^2(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{-iB+A}{20c(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{\frac{\left(\frac{iB}{4}+\frac{A}{4}\right)\sqrt{c-ic\tan(fx+e)}}{\frac{c}{2}+\frac{ic\tan(fx+e)}{2}}+\left(\frac{7A}{2}+\frac{3iB}{2}\right)\sqrt{}}{16c^3}\right.}{fa}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,method=
_RETURNVERBOSE)
```

output
```
2*I/f/a*c*(-1/16/c^3*(3*A+I*B)/(c-I*c*tan(f*x+e))^(1/2)-1/12/c^2*A/(c-I*c*
tan(f*x+e))^(3/2)-1/20/c*(A-I*B)/(c-I*c*tan(f*x+e))^(5/2)+1/16/c^3*((1/4*I
*B+1/4*A)*(c-I*c*tan(f*x+e))^(1/2)/(1/2*c+1/2*I*c*tan(f*x+e))+1/2*(7/2*A+3
/2*I*B)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(172) = 344.

Time = 0.11 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.87

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\left(15\sqrt{\frac{1}{2}}ac^3f\sqrt{-\frac{49A^2+42iAB-9B^2}{a^2c^5f^2}}e^{(2ifx+2ie)}\log\left(\frac{\left(\sqrt{2}\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="fricas")
```

output
```
1/480*(15*sqrt(1/2)*a*c^3*f*sqrt(-(49*A^2 + 42*I*A*B - 9*B^2)/(a^2*c^5*f^2
))*e^(2*I*f*x + 2*I*e)*log(1/8*(sqrt(2)*sqrt(1/2)*(a*c^2*f*e^(2*I*f*x + 2*
I*e) + a*c^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(49*A^2 + 42*I*A*B
 - 9*B^2)/(a^2*c^5*f^2)) + 7*I*A - 3*B)*e^(-I*f*x - I*e)/(a*c^2*f)) - 15*s
qrt(1/2)*a*c^3*f*sqrt(-(49*A^2 + 42*I*A*B - 9*B^2)/(a^2*c^5*f^2))*e^(2*I*f
*x + 2*I*e)*log(-1/8*(sqrt(2)*sqrt(1/2)*(a*c^2*f*e^(2*I*f*x + 2*I*e) + a*c
^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(49*A^2 + 42*I*A*B - 9*B^2)/
(a^2*c^5*f^2)) - 7*I*A + 3*B)*e^(-I*f*x - I*e)/(a*c^2*f)) - sqrt(2)*(6*(I*
A + B)*e^(8*I*f*x + 8*I*e) + 2*(19*I*A + 9*B)*e^(6*I*f*x + 6*I*e) + 4*(37*
I*A - 3*B)*e^(4*I*f*x + 4*I*e) - (-101*I*A + 9*B)*e^(2*I*f*x + 2*I*e) - 15
*I*A + 15*B)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-2*I*f*x - 2*I*e)/(a*c^
3*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{i\Big(\int \frac{A}{-c^2\sqrt{-ic\tan{(e+fx)}+c}\tan^3{(e+fx)}-ic^2\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}-c^2\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}-ic^2\sqrt{-ic\tan{(e+fx)}+c}}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))**(5/2),x)
```

output
```
-I*(Integral(A/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - I*c**2
*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - c**2*sqrt(-I*c*tan(e + f*x)
 + c)*tan(e + f*x) - I*c**2*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(B*
tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - I*c**2*s
qrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - c**2*sqrt(-I*c*tan(e + f*x) +
 c)*tan(e + f*x) - I*c**2*sqrt(-I*c*tan(e + f*x) + c)), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.87

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{i\left(\frac{4\left(15(-ic\tan(fx+e)+c)^3(7A+3iB)-20(-ic\tan(fx+e)+c)^2(7A+3iB)c-8(-ic\tan(fx+e)+c)(7A+3iB)c^2-48(A-iB)c^3\right)}{(-ic\tan(fx+e)+c)^{\frac{7}{2}}ac-2(-ic\tan(fx+e)+c)^{\frac{5}{2}}ac^2}+\frac{15\sqrt{}}{}\right.}{960\,cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="maxima")
```

output
```
-1/960*I*(4*(15*(-I*c*tan(f*x + e) + c)^3*(7*A + 3*I*B) - 20*(-I*c*tan(f*x
 + e) + c)^2*(7*A + 3*I*B)*c - 8*(-I*c*tan(f*x + e) + c)*(7*A + 3*I*B)*c^2
 - 48*(A - I*B)*c^3)/((-I*c*tan(f*x + e) + c)^(7/2)*a*c - 2*(-I*c*tan(f*x
+ e) + c)^(5/2)*a*c^2) + 15*sqrt(2)*(7*A + 3*I*B)*log(-(sqrt(2)*sqrt(c) -
sqrt(-I*c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c
)))/(a*c^(3/2)))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,
algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.09 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.38

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\frac{B\,c}{5}-\frac{B\,(c-c\tan(e+f\,x)\,1\mathrm{i})}{10}-\frac{B\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2}{4\,c}+\frac{3\,B\,(c-c\,\mathrm{ta}}{16}}{a\,f\,(c-c\tan(e+f\,x)\;1\mathrm{i})^{7/2}-2\,a\,c\,f\,(c-c\tan(e+}$$
$$-\frac{\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})\,7\mathrm{i}}{30\,a\,f}+\frac{A\,c\,1\mathrm{i}}{5\,a\,f}+\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2\,7\mathrm{i}}{12\,a\,c\,f}-\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^3\,7\mathrm{i}}{16\,a\,c^2\,f}}{2\,c\,(c-c\tan(e+f\,x)\;1\mathrm{i})^{5/2}-(c-c\tan(e+f\,x)\;1\mathrm{i})^{7/2}}$$
$$-\frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,7\mathrm{i}}{32\,a\,(-c)^{5/2}\,f}-\frac{3\,\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{32\,a\,c^{5/2}\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)*(c - c*tan(e + f*x)*1i)^
(5/2)),x)
```

output
```
((B*c)/5 - (B*(c - c*tan(e + f*x)*1i))/10 - (B*(c - c*tan(e + f*x)*1i)^2)/
(4*c) + (3*B*(c - c*tan(e + f*x)*1i)^3)/(16*c^2))/(a*f*(c - c*tan(e + f*x)
*1i)^(7/2) - 2*a*c*f*(c - c*tan(e + f*x)*1i)^(5/2)) - ((A*(c - c*tan(e + f
*x)*1i)*7i)/(30*a*f) + (A*c*1i)/(5*a*f) + (A*(c - c*tan(e + f*x)*1i)^2*7i)
/(12*a*c*f) - (A*(c - c*tan(e + f*x)*1i)^3*7i)/(16*a*c^2*f))/(2*c*(c - c*t
an(e + f*x)*1i)^(5/2) - (c - c*tan(e + f*x)*1i)^(7/2)) - (2^(1/2)*A*atan((
2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*7i)/(32*a*(-c)^(5/2
)*f) - (3*2^(1/2)*B*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/
2))))/(32*a*c^(5/2)*f)
```

Reduce [F]

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))(c - ic\tan(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
( - sqrt( - tan(e + f*x)*i + 1)*a*i + 165888*int(( - sqrt( - tan(e + f*x)*
i + 1))/(36864*tan(e + f*x)**5*i - 36864*tan(e + f*x)**4 + 73728*tan(e + f
*x)**3*i - 73728*tan(e + f*x)**2 + 36864*tan(e + f*x)*i - 36864),x)*tan(e
+ f*x)**4*a*f + 331776*int(( - sqrt( - tan(e + f*x)*i + 1))/(36864*tan(e +
 f*x)**5*i - 36864*tan(e + f*x)**4 + 73728*tan(e + f*x)**3*i - 73728*tan(e
 + f*x)**2 + 36864*tan(e + f*x)*i - 36864),x)*tan(e + f*x)**2*a*f + 165888
*int(( - sqrt( - tan(e + f*x)*i + 1))/(36864*tan(e + f*x)**5*i - 36864*tan
(e + f*x)**4 + 73728*tan(e + f*x)**3*i - 73728*tan(e + f*x)**2 + 36864*tan
(e + f*x)*i - 36864),x)*a*f - 129024*int(( - sqrt( - tan(e + f*x)*i + 1)*t
an(e + f*x)**2)/(36864*tan(e + f*x)**5*i - 36864*tan(e + f*x)**4 + 73728*t
an(e + f*x)**3*i - 73728*tan(e + f*x)**2 + 36864*tan(e + f*x)*i - 36864),x
)*tan(e + f*x)**4*a*f - 258048*int(( - sqrt( - tan(e + f*x)*i + 1)*tan(e +
 f*x)**2)/(36864*tan(e + f*x)**5*i - 36864*tan(e + f*x)**4 + 73728*tan(e +
 f*x)**3*i - 73728*tan(e + f*x)**2 + 36864*tan(e + f*x)*i - 36864),x)*tan(
e + f*x)**2*a*f - 129024*int(( - sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*
*2)/(36864*tan(e + f*x)**5*i - 36864*tan(e + f*x)**4 + 73728*tan(e + f*x)*
*3*i - 73728*tan(e + f*x)**2 + 36864*tan(e + f*x)*i - 36864),x)*a*f - 4*in
t(tan(e + f*x)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3*i - sqrt( - ta
n(e + f*x)*i + 1)*tan(e + f*x)**2 + sqrt( - tan(e + f*x)*i + 1)*tan(e + f*
x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*tan(e + f*x)**4*b*f - 8*int(tan(...
```

3.771 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 275

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{7(5iA-13B)c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}a^2f} - \frac{7(5iA-13B)c^4\sqrt{c-ic\tan(e+fx)}}{2a^2f} - \frac{7(5iA-13B)c^3(c-ic\tan(e+fx))^{3/2}}{12a^2f} - \frac{7(5iA-13B)c^2(c-ic\tan(e+fx))^{5/2}}{40a^2f} - \frac{(5iA-13B)c(c-ic\tan(e+fx))^{7/2}}{8a^2f(1+i\tan(e+fx))} + \frac{(iA-B)(c-ic\tan(e+fx))^{9/2}}{4a^2f(1+i\tan(e+fx))^2}$$

output
```
7/2*(5*I*A-13*B)*c^(9/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1
/2))*2^(1/2)/a^2/f-7/2*(5*I*A-13*B)*c^4*(c-I*c*tan(f*x+e))^(1/2)/a^2/f-7/1
2*(5*I*A-13*B)*c^3*(c-I*c*tan(f*x+e))^(3/2)/a^2/f-7/40*(5*I*A-13*B)*c^2*(c
-I*c*tan(f*x+e))^(5/2)/a^2/f-1/8*(5*I*A-13*B)*c*(c-I*c*tan(f*x+e))^(7/2)/a
^2/f/(1+I*tan(f*x+e))+1/4*(I*A-B)*(c-I*c*tan(f*x+e))^(9/2)/a^2/f/(1+I*tan(
f*x+e))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.59

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^2}\,dx =$$
$$-\frac{2c^4\sqrt{c-ic\tan(e+fx)}\left(-775iA+2018B+5(109A+281iB)\tan(e+fx)+(85iA-239B)\tan^2(e+f\right.}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(9/2))/(a + I*a*Tan
[e + f*x])^2,x]
```

output
```
(-2*c^4*Sqrt[c - I*c*Tan[e + f*x]]*((-775*I)*A + 2018*B + 5*(109*A + (281*
I)*B)*Tan[e + f*x] + ((85*I)*A - 239*B)*Tan[e + f*x]^2 + 5*(A + (5*I)*B)*T
an[e + f*x]^3 + 3*B*Tan[e + f*x]^4 + 70*((-5*I)*A + 13*B)*Hypergeometric2F
1[1/2, 3, 3/2, (-1/2*I)*(I + Tan[e + f*x])]*(-I + Tan[e + f*x])^2))/(15*a^
2*f*(-I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.83, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 4071, 27, 87, 51, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{9/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{9/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \text{87}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(5A+13iB)\int \frac{(c-ic\tan(e+fx))^{7/2}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)\right)}{a^2f}$$

$$\Big\downarrow \text{51}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(5A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{7/2}}{1+i\tan(e+fx)}-\frac{7}{2}c\int \frac{(c-ic\tan(e+fx))^{5/2}}{i\tan(e+fx)+1}d\tan(e+fx)\right)\right)}{a^2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(5A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{7/2}}{1+i\tan(e+fx)}-\frac{7}{2}c\left(2c\int \frac{(c-ic\tan(e+fx))^{3/2}}{i\tan(e+fx)+1}d\tan(e+fx)-\frac{2}{5}i(c\right.\right.\right.}{a^2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(5A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{7/2}}{1+i\tan(e+fx)}-\frac{7}{2}c\left(2c\left(2c\int \frac{\sqrt{c-ic\tan(e+fx)}}{i\tan(e+fx)+1}d\tan(e+fx)-\frac{2}{3}i\right.\right.\right.\right.}{a^2f}$$

$$\Big\downarrow \text{60}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(5A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{7/2}}{1+i\tan(e+fx)}-\frac{7}{2}c\left(2c\left(2c\left(2c\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\,t\right.\right.\right.\right.\right.}{a^2f}$$

$$\Big\downarrow \text{73}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(5A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{7/2}}{1+i\tan(e+fx)}-\frac{7}{2}c\left(2c\left(2c\left(4i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e\right.\right.\right.\right.\right.}{a^2f}$$

$$\Big\downarrow \text{219}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(5A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{7/2}}{1+i\tan(e+fx)}-\frac{7}{2}c\left(2c\left(2c\left(2i\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)-\right.\right.\right.\right.\right.}{a^2f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(9/2))/(a + I*a*Tan[e + f
*x])^2,x]
```

output
```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(9/2))/(4*c*(1 + I*Tan[e + f*x])^2)
- ((5*A + (13*I)*B)*((I*(c - I*c*Tan[e + f*x])^(7/2))/(1 + I*Tan[e + f*x])
 - (7*c*(((-2*I)/5)*(c - I*c*Tan[e + f*x])^(5/2) + 2*c*(((-2*I)/3)*(c - I*
c*Tan[e + f*x])^(3/2) + 2*c*((2*I)*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Ta
n[e + f*x]]/(Sqrt[2]*Sqrt[c])] - (2*I)*Sqrt[c - I*c*Tan[e + f*x]]))))/2))/
8))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{2ic^2\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{5iBc(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{Ac(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-18i\sqrt{c-ic\tan(fx+e)}\,Bc^2-6\sqrt{c-ic\tan}\right.}{}$
default	$\frac{2ic^2\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{5}{2}}}{5}-\frac{5iBc(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{Ac(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-18i\sqrt{c-ic\tan(fx+e)}\,Bc^2-6\sqrt{c-ic\tan}\right.}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^2*c^2*(-1/5*I*B*(c-I*c*tan(f*x+e))^(5/2)-5/3*I*B*c*(c-I*c*tan(f*x+
e))^(3/2)-1/3*A*c*(c-I*c*tan(f*x+e))^(3/2)-18*I*(c-I*c*tan(f*x+e))^(1/2)*B
*c^2-6*(c-I*c*tan(f*x+e))^(1/2)*A*c^2+8*c^3*(4*((21/64*I*B+13/64*A)*(c-I*c
*tan(f*x+e))^(3/2)+(-19/32*I*B*c-11/32*c*A)*(c-I*c*tan(f*x+e))^(1/2))/(c+I
*c*tan(f*x+e))^2+7/8*(13/4*I*B+5/4*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*t
an(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(216) = 432.

Time = 0.11 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.87

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^2}\,dx =$$

$$-\frac{105\sqrt{2}\sqrt{-\frac{(25A^2+130iAB-169B^2)c^9}{a^4f^2}}\left(a^2fe^{(8ifx+8ie)}+2a^2fe^{(6ifx+6ie)}+a^2fe^{(4ifx+4ie)}\right)\log\left(-\frac{14\left((-5iA+13B\right.}{}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="fricas")
```

output
```
-1/60*(105*sqrt(2)*sqrt(-(25*A^2 + 130*I*A*B - 169*B^2)*c^9/(a^4*f^2))*(a^
2*f*e^(8*I*f*x + 8*I*e) + 2*a^2*f*e^(6*I*f*x + 6*I*e) + a^2*f*e^(4*I*f*x +
 4*I*e))*log(-14*((-5*I*A + 13*B)*c^5 + sqrt(-(25*A^2 + 130*I*A*B - 169*B^
2)*c^9/(a^4*f^2))*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*x +
 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^2*f)) - 105*sqrt(2)*sqrt(-(25*A^2 + 130
*I*A*B - 169*B^2)*c^9/(a^4*f^2))*(a^2*f*e^(8*I*f*x + 8*I*e) + 2*a^2*f*e^(6
*I*f*x + 6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e))*log(-14*((-5*I*A + 13*B)*c^5
- sqrt(-(25*A^2 + 130*I*A*B - 169*B^2)*c^9/(a^4*f^2))*(a^2*f*e^(2*I*f*x +
2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^2*f
)) + 2*sqrt(2)*(105*(5*I*A - 13*B)*c^4*e^(8*I*f*x + 8*I*e) + 245*(5*I*A -
13*B)*c^4*e^(6*I*f*x + 6*I*e) + 161*(5*I*A - 13*B)*c^4*e^(4*I*f*x + 4*I*e)
 + 15*(5*I*A - 13*B)*c^4*e^(2*I*f*x + 2*I*e) + 30*(-I*A + B)*c^4)*sqrt(c/(
e^(2*I*f*x + 2*I*e) + 1)))/(a^2*f*e^(8*I*f*x + 8*I*e) + 2*a^2*f*e^(6*I*f*x
 + 6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\tan(e + fx))(c - ic\tan(e + fx))^{9/2}}{(a + ia\tan(e + fx))^2} dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(9/2)/(a+I*a*tan(f*x+e))**2
,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.89

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^2}\,dx =$$
$$-\frac{i\left(\frac{105\sqrt{2}(5A+13iB)c^{\frac{11}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2} - \frac{60\left((-ic\tan(fx+e)+c)^{\frac{3}{2}}(13A+21iB)c^6-2\sqrt{-ic\tan(fx+e)+c}(11A+19i\right.}{(-ic\tan(fx+e)+c)^2a^2-4(-ic\tan(fx+e)+c)a^2c+4a^2c^2}}{60\,cf}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="maxima")
```

output
```
-1/60*I*(105*sqrt(2)*(5*A + 13*I*B)*c^(11/2)*log(-(sqrt(2)*sqrt(c) - sqrt(
-I*c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a
^2 - 60*((-I*c*tan(f*x + e) + c)^(3/2)*(13*A + 21*I*B)*c^6 - 2*sqrt(-I*c*t
an(f*x + e) + c)*(11*A + 19*I*B)*c^7)/((-I*c*tan(f*x + e) + c)^2*a^2 - 4*(
-I*c*tan(f*x + e) + c)*a^2*c + 4*a^2*c^2) + 8*(3*I*(-I*c*tan(f*x + e) + c)
^(5/2)*B*c^3 + 5*(-I*c*tan(f*x + e) + c)^(3/2)*(A + 5*I*B)*c^4 + 90*sqrt(-
I*c*tan(f*x + e) + c)*(A + 3*I*B)*c^5)/a^2)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 6.92 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.46

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{38\,B\,c^6\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}-21\,B\,c^5\,\left(c-c\tan\right.}{4\,a^2\,c^2\,f+a^2\,f\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2-4\,a^2\,c\,f\,\left(c-\right.}$$
$$-\frac{\frac{A\,c^6\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,22\mathrm{i}}{a^2\,f}-\frac{A\,c^5\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\,13\mathrm{i}}{a^2\,f}}{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2-4\,c\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)+4\,c^2}$$
$$-\frac{A\,c^4\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,12\mathrm{i}}{a^2\,f}-\frac{A\,c^3\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\,2\mathrm{i}}{3\,a^2\,f}$$
$$+\frac{36\,B\,c^4\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{a^2\,f}+\frac{10\,B\,c^3\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}}{3\,a^2\,f}$$
$$+\frac{2\,B\,c^2\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}}{5\,a^2\,f}+\frac{\sqrt{2}\,A\,\left(-c\right)^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,35\mathrm{i}}{2\,a^2\,f}$$
$$+\frac{\sqrt{2}\,B\,c^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,91\mathrm{i}}{2\,a^2\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(9/2))/(a + a*tan(e + f*
x)*1i)^2,x)
```

output
```
(38*B*c^6*(c - c*tan(e + f*x)*1i)^(1/2) - 21*B*c^5*(c - c*tan(e + f*x)*1i)
^(3/2))/(4*a^2*c^2*f + a^2*f*(c - c*tan(e + f*x)*1i)^2 - 4*a^2*c*f*(c - c*
tan(e + f*x)*1i)) - ((A*c^6*(c - c*tan(e + f*x)*1i)^(1/2)*22i)/(a^2*f) - (
A*c^5*(c - c*tan(e + f*x)*1i)^(3/2)*13i)/(a^2*f))/((c - c*tan(e + f*x)*1i)
^2 - 4*c*(c - c*tan(e + f*x)*1i) + 4*c^2) - (A*c^4*(c - c*tan(e + f*x)*1i)
^(1/2)*12i)/(a^2*f) - (A*c^3*(c - c*tan(e + f*x)*1i)^(3/2)*2i)/(3*a^2*f) +
 (36*B*c^4*(c - c*tan(e + f*x)*1i)^(1/2))/(a^2*f) + (10*B*c^3*(c - c*tan(e
 + f*x)*1i)^(3/2))/(3*a^2*f) + (2*B*c^2*(c - c*tan(e + f*x)*1i)^(5/2))/(5*
a^2*f) + (2^(1/2)*A*(-c)^(9/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2)
)/(2*(-c)^(1/2)))*35i)/(2*a^2*f) + (2^(1/2)*B*c^(9/2)*atan((2^(1/2)*(c - c
*tan(e + f*x)*1i)^(1/2)*1i)/(2*c^(1/2)))*91i)/(2*a^2*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{\sqrt{c}\,c^4\left(-\left(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)a-\left(\int \frac{\sqrt{-\tan(}}{\tan(fx+}\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
(sqrt(c)*c**4*( - int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**2 - 2*tan
(e + f*x)*i - 1),x)*a - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**5)/
(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b - int((sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a - 4*
int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2 - 2*tan
(e + f*x)*i - 1),x)*b*i - 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*
*3)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i + 6*int((sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x
)*b + 6*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2
 - 2*tan(e + f*x)*i - 1),x)*a + 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e +
 f*x)**2)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i + 4*int((sqrt( -
 tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1
),x)*a*i - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**2
 - 2*tan(e + f*x)*i - 1),x)*b))/a**2
```

3.772 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 238

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{5(3iA-11B)c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}a^2f} - \frac{5(3iA-11B)c^3\sqrt{c-ic\tan(e+fx)}}{4a^2f} - \frac{5(3iA-11B)c^2(c-ic\tan(e+fx))^{3/2}}{24a^2f} - \frac{(3iA-11B)c(c-ic\tan(e+fx))^{5/2}}{8a^2f(1+i\tan(e+fx))} + \frac{(iA-B)(c-ic\tan(e+fx))^{7/2}}{4a^2f(1+i\tan(e+fx))^2}$$

output
```
5/4*(3*I*A-11*B)*c^(7/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1
/2))*2^(1/2)/a^2/f-5/4*(3*I*A-11*B)*c^3*(c-I*c*tan(f*x+e))^(1/2)/a^2/f-5/2
4*(3*I*A-11*B)*c^2*(c-I*c*tan(f*x+e))^(3/2)/a^2/f-1/8*(3*I*A-11*B)*c*(c-I*
c*tan(f*x+e))^(5/2)/a^2/f/(1+I*tan(f*x+e))+1/4*(I*A-B)*(c-I*c*tan(f*x+e))^
(7/2)/a^2/f/(1+I*tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 6.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.76

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{15\sqrt{2}(-3iA+11B)c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\sec^2(}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2))/(a + I*a*Tan
[e + f*x])^2,x]
```

output
```
(15*Sqrt[2]*((-3*I)*A + 11*B)*c^(7/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(
Sqrt[2]*Sqrt[c])]*Sec[e + f*x]^2*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]) +
 2*c^3*Sqrt[c - I*c*Tan[e + f*x]]*((27*I)*A - 103*B - (51*A + (175*I)*B)*T
an[e + f*x] + 4*((-3*I)*A + 14*B)*Tan[e + f*x]^2 - (4*I)*B*Tan[e + f*x]^3)
)/(12*a^2*f*(-I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.209$, Rules used = {3042, 4071, 27, 87, 51, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2f}$$

$\big\downarrow$ 87

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(3A+11iB)\int \frac{(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)\right)}{a^2f}$$

$\big\downarrow$ 51

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(3A+11iB)\left(\frac{i(c-ic\tan(e+fx))^{5/2}}{1+i\tan(e+fx)}-\frac{5}{2}c\int \frac{(c-ic\tan(e+fx))^{3/2}}{i\tan(e+fx)+1}d\tan(e+fx)\right)\right)}{a^2f}$$

$\big\downarrow$ 60

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(3A+11iB)\left(\frac{i(c-ic\tan(e+fx))^{5/2}}{1+i\tan(e+fx)}-\frac{5}{2}c\left(2c\int \frac{\sqrt{c-ic\tan(e+fx)}}{i\tan(e+fx)+1}d\tan(e+fx)-\frac{2}{3}i(c-\right.\right.\right.}{a^2f}$$

$\big\downarrow$ 60

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(3A+11iB)\left(\frac{i(c-ic\tan(e+fx))^{5/2}}{1+i\tan(e+fx)}-\frac{5}{2}c\left(2c\left(2c\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(\right.\right.\right.\right.}{a^2f}$$

$\big\downarrow$ 73

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(3A+11iB)\left(\frac{i(c-ic\tan(e+fx))^{5/2}}{1+i\tan(e+fx)}-\frac{5}{2}c\left(2c\left(4i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+\right.\right.\right.\right.}{a^2f}$$

$\big\downarrow$ 219

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(3A+11iB)\left(\frac{i(c-ic\tan(e+fx))^{5/2}}{1+i\tan(e+fx)}-\frac{5}{2}c\left(2c\left(2i\sqrt{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)-2i\right.\right.\right.\right.}{a^2f}$$

input

```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2))/(a + I*a*Tan[e + f
*x])^2,x]
```

output
```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(7/2))/(4*c*(1 + I*Tan[e + f*x])^2)
- ((3*A + (11*I)*B)*((I*(c - I*c*Tan[e + f*x])^(5/2))/(1 + I*Tan[e + f*x])
 - (5*c*(((-2*I)/3)*(c - I*c*Tan[e + f*x])^(3/2) + 2*c*((2*I)*Sqrt[2]*Sqrt
[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])] - (2*I)*Sqrt[c -
 I*c*Tan[e + f*x]])))/2))/8))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{2ic^2\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-5i\sqrt{c-ic\tan(fx+e)}\,Bc-\sqrt{c-ic\tan(fx+e)}\,cA+2c^2\left(\frac{4\left(\frac{17iB}{32}+\frac{9A}{32}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+\cdots}{(c+ic\tan\cdots}\right.\right.}{f\,a^2}$
default	$\frac{2ic^2\left(-\frac{iB(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}-5i\sqrt{c-ic\tan(fx+e)}\,Bc-\sqrt{c-ic\tan(fx+e)}\,cA+2c^2\left(\frac{4\left(\frac{17iB}{32}+\frac{9A}{32}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+\cdots}{(c+ic\tan\cdots}\right.\right.}{f\,a^2}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^2*c^2*(-1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)-5*I*(c-I*c*tan(f*x+e))^(1
/2)*B*c-(c-I*c*tan(f*x+e))^(1/2)*c*A+2*c^2*(4*((17/32*I*B+9/32*A)*(c-I*c*t
an(f*x+e))^(3/2)+(-15/16*I*B*c-7/16*c*A)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*
tan(f*x+e))^2+5/4*(11/4*I*B+3/4*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(
f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(185) = 370$.

Time = 0.11 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.92

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx =$$

$$-\frac{15\sqrt{\frac{1}{2}}\left(a^2fe^{(6i\,fx+6i\,e)}+a^2fe^{(4i\,fx+4i\,e)}\right)\sqrt{-\frac{(9\,A^2+66i\,AB-121\,B^2)c^7}{a^4f^2}}\log\left(-\frac{5\left((-3i\,A+11\,B)c^4+\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2fe^{(2i\,fx+2i\,e)}\right.\right.}{}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="fricas")
```

output
```
-1/12*(15*sqrt(1/2)*(a^2*f*e^(6*I*f*x + 6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e)
)*sqrt(-(9*A^2 + 66*I*A*B - 121*B^2)*c^7/(a^4*f^2))*log(-5*((-3*I*A + 11*B
)*c^4 + sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(-(9*A^2
 + 66*I*A*B - 121*B^2)*c^7/(a^4*f^2))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e
^(-I*f*x - I*e)/(a^2*f)) - 15*sqrt(1/2)*(a^2*f*e^(6*I*f*x + 6*I*e) + a^2*f
*e^(4*I*f*x + 4*I*e))*sqrt(-(9*A^2 + 66*I*A*B - 121*B^2)*c^7/(a^4*f^2))*lo
g(-5*((-3*I*A + 11*B)*c^4 - sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) +
 a^2*f)*sqrt(-(9*A^2 + 66*I*A*B - 121*B^2)*c^7/(a^4*f^2))*sqrt(c/(e^(2*I*f
*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^2*f)) + sqrt(2)*(15*(3*I*A - 11*B)*
c^3*e^(6*I*f*x + 6*I*e) + 20*(3*I*A - 11*B)*c^3*e^(4*I*f*x + 4*I*e) + 3*(3
*I*A - 11*B)*c^3*e^(2*I*f*x + 2*I*e) + 6*(-I*A + B)*c^3)*sqrt(c/(e^(2*I*f*
x + 2*I*e) + 1)))/(a^2*f*e^(6*I*f*x + 6*I*e) + a^2*f*e^(4*I*f*x + 4*I*e))
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{Ac^3\sqrt{-ic\tan{(e+fx)}+c}}{\tan^2{(e+fx)}-2i\tan{(e+fx)}-1}\,dx + \int \left(-\frac{3Ac^3\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}}{\tan^2{(e+fx)}-2i\tan{(e+fx)}-1}\right)dx + \int \frac{Bc^3\sqrt{-ic\tan{(e+fx)}+c}\tan{(e+fx)}}{\tan^2{(e+fx)}-2i\tan{(e+fx)}-1}\,dx}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2)/(a+I*a*tan(f*x+e))**2
,x)
```

output
```
-(Integral(A*c**3*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**2 - 2*I*tan(e
 + f*x) - 1), x) + Integral(-3*A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
f*x)**2/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x) + Integral(B*c**3*sqr
t(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x)**2 - 2*I*tan(e + f*x)
- 1), x) + Integral(-3*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3/
(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x) + Integral(-3*I*A*c**3*sqrt(-
I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1
), x) + Integral(I*A*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3/(tan
(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x) + Integral(-3*I*B*c**3*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1)
, x) + Integral(I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4/(tan(
e + f*x)**2 - 2*I*tan(e + f*x) - 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.93

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{i\left(\frac{15\sqrt{2}(3A+11iB)c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2} - \frac{12\left((-ic\tan(fx+e)+c)^{\frac{3}{2}}(9A+17iB)c^5-2\sqrt{-ic\tan(fx+e)+c}(7A+15iB)c^6\right.}{(-ic\tan(fx+e)+c)^2a^2-4(-ic\tan(fx+e)+c)a^2c+4a^2c^2}\right.}{24cf}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="maxima")
```

output
```
-1/24*I*(15*sqrt(2)*(3*A + 11*I*B)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I
*c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^2
 - 12*((-I*c*tan(f*x + e) + c)^(3/2)*(9*A + 17*I*B)*c^5 - 2*sqrt(-I*c*tan(
f*x + e) + c)*(7*A + 15*I*B)*c^6)/((-I*c*tan(f*x + e) + c)^2*a^2 - 4*(-I*c
*tan(f*x + e) + c)*a^2*c + 4*a^2*c^2) + 16*(I*(-I*c*tan(f*x + e) + c)^(3/2
)*B*c^3 + 3*sqrt(-I*c*tan(f*x + e) + c)*(A + 5*I*B)*c^4)/a^2)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.47

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx=\frac{15\,B\,c^5\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}-\frac{17\,B\,c^4\,(c-c\tan\dots}{2\dots}}{4\,a^2\,c^2\,f+a^2\,f\,{\left(c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2-4\,a^2\,c\,f\,\left(c-\dots\right)}$$
$$-\frac{\frac{A\,c^5\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,7{}\mathrm{i}}{a^2\,f}-\frac{A\,c^4\,{\left(c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,9{}\mathrm{i}}{2\,a^2\,f}}{{\left(c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2-4\,c\,\left(c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)+4\,c^2}$$
$$-\frac{A\,c^3\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,2{}\mathrm{i}}{a^2\,f}+\frac{10\,B\,c^3\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}}{a^2\,f}$$
$$+\frac{2\,B\,c^2\,{\left(c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}}{3\,a^2\,f}-\frac{\sqrt{2}\,A\,{\left(-c\right)}^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-c}}\right)\,15{}\mathrm{i}}{4\,a^2\,f}$$
$$+\frac{\sqrt{2}\,B\,c^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\tan\left(e+f\,x\right)\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,\sqrt{c}}\right)\,55{}\mathrm{i}}{4\,a^2\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*
x)*1i)^2,x)
```

output
```
(15*B*c^5*(c - c*tan(e + f*x)*1i)^(1/2) - (17*B*c^4*(c - c*tan(e + f*x)*1i
)^(3/2))/2)/(4*a^2*c^2*f + a^2*f*(c - c*tan(e + f*x)*1i)^2 - 4*a^2*c*f*(c
- c*tan(e + f*x)*1i)) - ((A*c^5*(c - c*tan(e + f*x)*1i)^(1/2)*7i)/(a^2*f)
- (A*c^4*(c - c*tan(e + f*x)*1i)^(3/2)*9i)/(2*a^2*f))/((c - c*tan(e + f*x)
*1i)^2 - 4*c*(c - c*tan(e + f*x)*1i) + 4*c^2) - (A*c^3*(c - c*tan(e + f*x)
*1i)^(1/2)*2i)/(a^2*f) + (10*B*c^3*(c - c*tan(e + f*x)*1i)^(1/2))/(a^2*f)
+ (2*B*c^2*(c - c*tan(e + f*x)*1i)^(3/2))/(3*a^2*f) - (2^(1/2)*A*(-c)^(7/2
)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*15i)/(4*a^2
*f) + (2^(1/2)*B*c^(7/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(
2*c^(1/2)))*55i)/(4*a^2*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{\sqrt{c}\,c^3\left(-\left(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)a-\left(\int \frac{\sqrt{-\tan(}}{\tan(fx+}\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
(sqrt(c)*c**3*( - int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**2 - 2*tan
(e + f*x)*i - 1),x)*a - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/
(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i - int((sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i
+ 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**2 - 2
*tan(e + f*x)*i - 1),x)*b + 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x
)**2)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a + 3*int((sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x
)*b*i + 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**2
- 2*tan(e + f*x)*i - 1),x)*a*i - int((sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x))/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b))/a**2
```

3.773 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 199

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3(iA-9B)c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}a^2f} - \frac{3(iA-9B)c^2\sqrt{c-ic\tan(e+fx)}}{8a^2f} - \frac{(iA-9B)c(c-ic\tan(e+fx))^{3/2}}{8a^2f(1+i\tan(e+fx))} + \frac{(iA-B)(c-ic\tan(e+fx))^{5/2}}{4a^2f(1+i\tan(e+fx))^2}$$

output
```
3/8*(I*A-9*B)*c^(5/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
)*2^(1/2)/a^2/f-3/8*(I*A-9*B)*c^2*(c-I*c*tan(f*x+e))^(1/2)/a^2/f-1/8*(I*A-
9*B)*c*(c-I*c*tan(f*x+e))^(3/2)/a^2/f/(1+I*tan(f*x+e))+1/4*(I*A-B)*(c-I*c*
tan(f*x+e))^(5/2)/a^2/f/(1+I*tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 5.76 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.84

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{c^2\sec^2(e+fx)\left(3\sqrt{2}(-iA+9B)\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-ic\tan}}{\sqrt{2}\sqrt{}}\right.\right.}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan
[e + f*x])^2,x]
```

output
```
(c^2*Sec[e + f*x]^2*(3*Sqrt[2]*((-I)*A + 9*B)*Sqrt[c]*ArcTanh[Sqrt[c - I*c
*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])
+ I*(A + (9*I)*B + (A + (25*I)*B)*Cos[2*(e + f*x)] + ((5*I)*A - 29*B)*Sin[
2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]))/(8*a^2*f*(-I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {3042, 4071, 27, 87, 51, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \; 87$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(A+9iB)\int \frac{(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)\right)}{a^2f}$$

$$\Big\downarrow \; 51$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(A+9iB)\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{1+i\tan(e+fx)}-\frac{3}{2}c\int \frac{\sqrt{c-ic\tan(e+fx)}}{i\tan(e+fx)+1}d\tan(e+fx)\right)\right)}{a^2f}$$

$$\Big\downarrow \; 60$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(A+9iB)\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{1+i\tan(e+fx)}-\frac{3}{2}c\left(2c\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx\right.\right.\right.}{a^2f}$$

$$\Big\downarrow \; 73$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(A+9iB)\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{1+i\tan(e+fx)}-\frac{3}{2}c\left(4i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}-\right.\right.\right.}{a^2f}$$

$$\Big\downarrow \; 219$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{4c(1+i\tan(e+fx))^2}-\frac{1}{8}(A+9iB)\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{1+i\tan(e+fx)}-\frac{3}{2}c\left(2i\sqrt{2}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)-2i\sqrt{c-\right.\right.\right.}{a^2f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan[e + f
*x])^2,x]
```

output
```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(5/2))/(4*c*(1 + I*Tan[e + f*x])^2)
- ((A + (9*I)*B)*((I*(c - I*c*Tan[e + f*x])^(3/2))/(1 + I*Tan[e + f*x]) -
(3*c*((2*I)*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sq
rt[c])] - (2*I)*Sqrt[c - I*c*Tan[e + f*x]]))/2))/8))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{2ic^2\left(-i\sqrt{c-ic\tan(fx+e)}\,B+c\left(\frac{4\left(\frac{13iB}{32}+\frac{5A}{32}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+4\left(-\frac{11}{16}iBc-\frac{3}{16}cA\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^2}+\frac{3\left(\frac{9iB}{4}+\frac{A}{4}\right)\sqrt{2}}{}\right.\right.}{f\,a^2}$
default	$\frac{2ic^2\left(-i\sqrt{c-ic\tan(fx+e)}\,B+c\left(\frac{4\left(\frac{13iB}{32}+\frac{5A}{32}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+4\left(-\frac{11}{16}iBc-\frac{3}{16}cA\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^2}+\frac{3\left(\frac{9iB}{4}+\frac{A}{4}\right)\sqrt{2}}{}\right.\right.}{f\,a^2}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^2*c^2*(-I*B*(c-I*c*tan(f*x+e))^(1/2)+c*(4*((13/32*I*B+5/32*A)*(c-I
*c*tan(f*x+e))^(3/2)+(-11/16*I*B*c-3/16*c*A)*(c-I*c*tan(f*x+e))^(1/2))/(c+
I*c*tan(f*x+e))^2+3/4*(9/4*I*B+1/4*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*t
an(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(154) = 308$.

Time = 0.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.91

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx =$$

$$-\frac{\left(3\sqrt{\frac{1}{2}}a^2f\sqrt{-\frac{(A^2+18i\,AB-81\,B^2)c^5}{a^4f^2}}e^{(4i\,fx+4i\,e)}\log\left(-\frac{3\left((-i\,A+9\,B)c^3+\sqrt{2}\sqrt{\frac{1}{2}}(a^2fe^{(2i\,fx+2i\,e)}+a^2f)\sqrt{-\frac{(A^2+18i\,AB-81\,B^2}{a^4f^2}}\right.}{2\,a^2f}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(1/2)*a^2*f*sqrt(-(A^2 + 18*I*A*B - 81*B^2)*c^5/(a^4*f^2))*e^(
4*I*f*x + 4*I*e)*log(-3/2*((-I*A + 9*B)*c^3 + sqrt(2)*sqrt(1/2)*(a^2*f*e^(
2*I*f*x + 2*I*e) + a^2*f)*sqrt(-(A^2 + 18*I*A*B - 81*B^2)*c^5/(a^4*f^2))*s
qrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^2*f)) - 3*sqrt(1/2)*
a^2*f*sqrt(-(A^2 + 18*I*A*B - 81*B^2)*c^5/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*l
og(-3/2*((-I*A + 9*B)*c^3 - sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) +
 a^2*f)*sqrt(-(A^2 + 18*I*A*B - 81*B^2)*c^5/(a^4*f^2))*sqrt(c/(e^(2*I*f*x
+ 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^2*f)) + sqrt(2)*(3*(I*A - 9*B)*c^2*e^(
4*I*f*x + 4*I*e) - (-I*A + 9*B)*c^2*e^(2*I*f*x + 2*I*e) + 2*(-I*A + B)*c^2
)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx =$$

$$-\frac{\int \frac{Ac^2\sqrt{-ic\tan(e+fx)+c}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx+\int\left(-\frac{Ac^2\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\right)dx+\int\frac{Bc^2\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx+}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**2
,x)
```

output
```
-(Integral(A*c**2*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**2 - 2*I*tan(e
 + f*x) - 1), x) + Integral(-A*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x)**2/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x) + Integral(B*c**2*sqrt(
-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) -
1), x) + Integral(-B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3/(tan
(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x) + Integral(-2*I*A*c**2*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x
) + Integral(-2*I*B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(
e + f*x)**2 - 2*I*tan(e + f*x) - 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.97

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx =$$

$$-\frac{i\left(\frac{3\sqrt{2}(A+9iB)c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2}+\frac{32i\sqrt{-ic\tan(fx+e)+c}Bc^3}{a^2}-\frac{4\left((-ic\tan(fx+e)+c)^{\frac{3}{2}}(5A+13iB)c^4-2\sqrt{-ic\tan\cdots}\right.}{(-ic\tan(fx+e)+c)^2a^2-4(-ic\tan(fx\cdots}\right.}{16\,cf}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="maxima")
```

output
```
-1/16*I*(3*sqrt(2)*(A + 9*I*B)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*t
an(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^2 + 3
2*I*sqrt(-I*c*tan(f*x + e) + c)*B*c^3/a^2 - 4*((-I*c*tan(f*x + e) + c)^(3/
2)*(5*A + 13*I*B)*c^4 - 2*sqrt(-I*c*tan(f*x + e) + c)*(3*A + 11*I*B)*c^5)/
((-I*c*tan(f*x + e) + c)^2*a^2 - 4*(-I*c*tan(f*x + e) + c)*a^2*c + 4*a^2*c
^2))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.48

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx=\frac{\frac{11\,B\,c^4\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2}-\frac{13\,B\,c^3\,(c-c\tan(e+f\,x}{4}}{4\,a^2\,c^2\,f+a^2\,f\,{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^2-4\,a^2\,c\,f\,\left(c-\right.}$$
$$-\frac{\frac{A\,c^4\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,3\mathrm{i}}{2\,a^2\,f}-\frac{A\,c^3\,{(c-c\tan(e+f\,x)\,1\mathrm{i})}^{3/2}\,5\mathrm{i}}{4\,a^2\,f}}{{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^2-4\,c\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)+4\,c^2}$$
$$+\frac{2\,B\,c^2\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{a^2\,f}+\frac{\sqrt{2}\,A\,{\left(-c\right)}^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,3\mathrm{i}}{8\,a^2\,f}$$
$$-\frac{27\,\sqrt{2}\,B\,c^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{8\,a^2\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*
x)*1i)^2,x)
```

output
```
((11*B*c^4*(c - c*tan(e + f*x)*1i)^(1/2))/2 - (13*B*c^3*(c - c*tan(e + f*x
)*1i)^(3/2))/4)/(4*a^2*c^2*f + a^2*f*(c - c*tan(e + f*x)*1i)^2 - 4*a^2*c*f
*(c - c*tan(e + f*x)*1i)) - ((A*c^4*(c - c*tan(e + f*x)*1i)^(1/2)*3i)/(2*a
^2*f) - (A*c^3*(c - c*tan(e + f*x)*1i)^(3/2)*5i)/(4*a^2*f))/((c - c*tan(e
+ f*x)*1i)^2 - 4*c*(c - c*tan(e + f*x)*1i) + 4*c^2) + (2*B*c^2*(c - c*tan(
e + f*x)*1i)^(1/2))/(a^2*f) + (2^(1/2)*A*(-c)^(5/2)*atan((2^(1/2)*(c - c*t
an(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*3i)/(8*a^2*f) - (27*2^(1/2)*B*c^(5/
2)*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(8*a^2*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{\sqrt{c}\,c^2\left(-\left(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)a+\left(\int \frac{\sqrt{-\tan(}}{\tan(fx+}\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
(sqrt(c)*c**2*( - int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**2 - 2*tan
(e + f*x)*i - 1),x)*a + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/
(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b + int((sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a + 2*
int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 - 2*tan
(e + f*x)*i - 1),x)*b*i + 2*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))
/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i - int((sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b))/a*
*2
```

3.774 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 160

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{(iA+7B)c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}a^2f} + \frac{(iA+7B)c\sqrt{c-ic\tan(e+fx)}}{8a^2f(1+i\tan(e+fx))} + \frac{(iA-B)(c-ic\tan(e+fx))^{3/2}}{4a^2f(1+i\tan(e+fx))^2}$$

output
```
-1/16*(I*A+7*B)*c^(3/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*2^(1/2)/a^2/f+1/8*(I*A+7*B)*c*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(1+I*tan(
f*x+e))+1/4*(I*A-B)*(c-I*c*tan(f*x+e))^(3/2)/a^2/f/(1+I*tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 5.79 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.90

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{\sqrt{2}(iA+7B)c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\sec^2(e+fx)$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan
[e + f*x])^2,x]
```

output
```
(Sqrt[2]*(I*A + 7*B)*c^(3/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*S
qrt[c])]*Sec[e + f*x]^2*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]) - 2*c*((3*
I)*A + 5*B + (A + (9*I)*B)*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(16*a
^2*f*(-I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3042, 4071, 27, 87, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a^3(i\tan(e+fx)+1)^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)+1)^3}d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \textbf{87}$$

$$\frac{c\left(\frac{1}{8}(A-7iB)\int \frac{\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{4c(1+i\tan(e+fx))^2}\right)}{a^2f}$$

$$\Big\downarrow \textbf{51}$$

$$\frac{c\left(\frac{1}{8}(A-7iB)\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{1+i\tan(e+fx)}-\frac{1}{2}c\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)\right)+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{4c(1+i\tan(e+fx))^2}\right)}{a^2f}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{c\left(\frac{1}{8}(A-7iB)\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{1+i\tan(e+fx)}-i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}\right)+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{4c(1+i\tan(e+fx))^2}\right)}{a^2f}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{c\left(\frac{1}{8}(A-7iB)\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{1+i\tan(e+fx)}-\frac{i\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}}\right)+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{4c(1+i\tan(e+fx))^2}\right)}{a^2f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan[e + f
*x])^2,x]
```

output
```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(3/2))/(4*c*(1 + I*Tan[e + f*x])^2)
+ ((A - (7*I)*B)*(((-I)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2
]*Sqrt[c])])/Sqrt[2] + (I*Sqrt[c - I*c*Tan[e + f*x]])/(1 + I*Tan[e + f*x])
))/8))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{2ic^2\left(\frac{4\left(\frac{9iB}{64}+\frac{A}{64}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+4\left(-\frac{7}{32}iBc+\frac{1}{32}cA\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^2}-\frac{\left(-\frac{7iB}{4}+\frac{A}{4}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{8\sqrt{c}}\right)}{f\,a^2}$
default	$\frac{2ic^2\left(\frac{4\left(\frac{9iB}{64}+\frac{A}{64}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+4\left(-\frac{7}{32}iBc+\frac{1}{32}cA\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^2}-\frac{\left(-\frac{7iB}{4}+\frac{A}{4}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{8\sqrt{c}}\right)}{f\,a^2}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^2*c^2*(4*((9/64*I*B+1/64*A)*(c-I*c*tan(f*x+e))^(3/2)+(-7/32*I*B*c+
1/32*c*A)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^2-1/8*(-7/4*I*B+1/4
*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(123) = 246.

Time = 0.08 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.32

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{\left(\sqrt{\frac{1}{2}}a^2f\sqrt{-\frac{(A^2-14iAB-49B^2)c^3}{a^4f^2}}e^{(4ifx+4ie)}\log\left(\left(\frac{\left((-iA\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="fricas")
```

output
```
1/16*(sqrt(1/2)*a^2*f*sqrt(-(A^2 - 14*I*A*B - 49*B^2)*c^3/(a^4*f^2))*e^(4*
I*f*x + 4*I*e)*log(1/4*((-I*A - 7*B)*c^2 + sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I
*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(A^2 - 14*I
*A*B - 49*B^2)*c^3/(a^4*f^2)))*e^(-I*f*x - I*e)/(a^2*f)) - sqrt(1/2)*a^2*f
*sqrt(-(A^2 - 14*I*A*B - 49*B^2)*c^3/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(1/
4*((-I*A - 7*B)*c^2 - sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f
)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(A^2 - 14*I*A*B - 49*B^2)*c^3/(a
^4*f^2)))*e^(-I*f*x - I*e)/(a^2*f)) + sqrt(2)*((I*A + 7*B)*c*e^(4*I*f*x +
4*I*e) + (3*I*A + 5*B)*c*e^(2*I*f*x + 2*I*e) - 2*(-I*A + B)*c)*sqrt(c/(e^(
2*I*f*x + 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{Ac\sqrt{-ic\tan(e+fx)+c}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx + \int \frac{Bc\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx + \int\left(-\frac{iAc\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\right)dx +}{a^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**2
,x)
```

output
```
-(Integral(A*c*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**2 - 2*I*tan(e +
f*x) - 1), x) + Integral(B*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan
(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x) + Integral(-I*A*c*sqrt(-I*c*tan(e
 + f*x) + c)*tan(e + f*x)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1), x) + I
ntegral(-I*B*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x)**
2 - 2*I*tan(e + f*x) - 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{i\left(\frac{\sqrt{2}(A-7iB)c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2}+\frac{4\left((-ic\tan(f\cdots\right.}{(-ic\tan\cdots}\right.}{32\,cf}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="maxima")
```

output
```
1/32*I*(sqrt(2)*(A - 7*I*B)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(
f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^2 + 4*((
-I*c*tan(f*x + e) + c)^(3/2)*(A + 9*I*B)*c^3 + 2*sqrt(-I*c*tan(f*x + e) +
c)*(A - 7*I*B)*c^4)/((-I*c*tan(f*x + e) + c)^2*a^2 - 4*(-I*c*tan(f*x + e)
+ c)*a^2*c + 4*a^2*c^2))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.67

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{\frac{7\,B\,c^3\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{4}-\frac{9\,B\,c^2\,(c-c\tan(e+f\,x)}{8}}{4\,a^2\,c^2\,f+a^2\,f\,(c-c\tan\left(e+f\,x\right)\,1\mathrm{i})^2-4\,a^2\,c\,f\,(c-} + \frac{\frac{A\,c^3\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,1\mathrm{i}}{4\,a^2\,f}+\frac{A\,c^2\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}\,1\mathrm{i}}{8\,a^2\,f}}{(c-c\tan\left(e+f\,x\right)\,1\mathrm{i})^2-4\,c\,(c-c\tan\left(e+f\,x\right)\,1\mathrm{i})+4\,c^2} + \frac{\sqrt{2}\,A\,(-c)^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,1\mathrm{i}}{16\,a^2\,f} - \frac{7\,\sqrt{2}\,B\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{16\,a^2\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*
x)*1i)^2,x)
```

output
```
((7*B*c^3*(c - c*tan(e + f*x)*1i)^(1/2))/4 - (9*B*c^2*(c - c*tan(e + f*x)*
1i)^(3/2))/8)/(4*a^2*c^2*f + a^2*f*(c - c*tan(e + f*x)*1i)^2 - 4*a^2*c*f*(
c - c*tan(e + f*x)*1i)) + ((A*c^3*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(4*a^2
*f) + (A*c^2*(c - c*tan(e + f*x)*1i)^(3/2)*1i)/(8*a^2*f))/((c - c*tan(e +
f*x)*1i)^2 - 4*c*(c - c*tan(e + f*x)*1i) + 4*c^2) + (2^(1/2)*A*(-c)^(3/2)*
atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*1i)/(16*a^2*f
) - (7*2^(1/2)*B*c^(3/2)*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*
c^(1/2))))/(16*a^2*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^2}\,dx = \frac{\sqrt{c}\,c\left(-\left(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)a+\left(\int \frac{\sqrt{-\tan(f}}{\tan(fx+e}\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
(sqrt(c)*c*( - int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**2 - 2*tan(e
+ f*x)*i - 1),x)*a + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(ta
n(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*i + int((sqrt( - tan(e + f*x)*i
 + 1)*tan(e + f*x))/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*i - int(
(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**2 - 2*tan(e + f*
x)*i - 1),x)*b))/a**2
```

3.775 $\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 159

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{(3iA+5B)\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}a^2f} + \frac{(iA-B)\sqrt{c-ic\tan(e+fx)}}{4a^2f(1+i\tan(e+fx))^2} + \frac{(3iA+5B)\sqrt{c-ic\tan(e+fx)}}{16a^2f(1+i\tan(e+fx))}$$

output
```
1/32*(3*I*A+5*B)*c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1
/2))*2^(1/2)/a^2/f+1/4*(I*A-B)*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(1+I*tan(f*x
+e))^2+1/16*(3*I*A+5*B)*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(1+I*tan(f*x+e))
```

Mathematica [A] (verified)

Time = 3.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$$
$$= \frac{\sqrt{2}(3A-5iB)\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\sec^2(e+fx)(-i\cos(2(e+fx))+\sin(2(e+fx)))+2(-7iA}{32a^2f(-i+\tan(e+fx))^2}$$

input
```
Integrate[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e
 + f*x])^2,x]
```

output
```
(Sqrt[2]*(3*A - (5*I)*B)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[
2]*Sqrt[c])]*Sec[e + f*x]^2*((-I)*Cos[2*(e + f*x)] + Sin[2*(e + f*x)]) + 2
*((-7*I)*A - B + (3*A - (5*I)*B)*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])
/(32*a^2*f*(-I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {3042, 4071, 27, 87, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^3(i\tan(e+fx)+1)^3\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^3\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \; 87$$

$$\frac{c\left(\frac{1}{8}(3A-5iB)\int \frac{1}{(i\tan(e+fx)+1)^2\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{4c(1+i\tan(e+fx))^2}\right)}{a^2f}$$

$$\Big\downarrow \; 52$$

$$\frac{c\left(\frac{1}{8}(3A-5iB)\left(\frac{1}{4}\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{4c(1+i\tan(e+fx))^2}\right)}{a^2f}$$

$$\Big\downarrow \; 73$$

$$\frac{c\left(\frac{1}{8}(3A-5iB)\left(\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{2c}+\frac{i\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{4c(1+i\tan(e+fx))^2}\right)}{a^2f}$$

$$\Big\downarrow \; 219$$

$$\frac{c\left(\frac{1}{8}(3A-5iB)\left(\frac{i\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\sqrt{c}}+\frac{i\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{4c(1+i\tan(e+fx))^2}\right)}{a^2f}$$

input

```
Int[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e + f*x
])^2,x]
```

output

```
(c*(((I*A - B)*Sqrt[c - I*c*Tan[e + f*x]])/(4*c*(1 + I*Tan[e + f*x])^2) +
((3*A - (5*I)*B)*(((I/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[
c])])/(Sqrt[2]*Sqrt[c]) + ((I/2)*Sqrt[c - I*c*Tan[e + f*x]])/(c*(1 + I*Tan
[e + f*x]))))/8))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{2ic^2\left(\frac{-\frac{(-5iB+3A)(c-ic\tan(fx+e))^{\frac{3}{2}}}{32c}+4\left(\frac{5A}{64}-\frac{3iB}{64}\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^2}+\frac{(-5iB+3A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{64c^{\frac{3}{2}}}\right)}{f\,a^2}$
default	$\frac{2ic^2\left(\frac{-\frac{(-5iB+3A)(c-ic\tan(fx+e))^{\frac{3}{2}}}{32c}+4\left(\frac{5A}{64}-\frac{3iB}{64}\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^2}+\frac{(-5iB+3A)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)}{64c^{\frac{3}{2}}}\right)}{f\,a^2}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^2*c^2*(4*(-1/128/c*(3*A-5*I*B)*(c-I*c*tan(f*x+e))^(3/2)+(5/64*A-3/
64*I*B)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^2+1/64/c^(3/2)*(3*A-5
*I*B)*2^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(122) = 244$.

Time = 0.09 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.28

$$\int \frac{(A + B\tan(e + fx))\sqrt{c - ic\tan(e + fx)}}{(a + ia\tan(e + fx))^2}\,dx$$
$$= \frac{\left(\sqrt{\frac{1}{2}}a^2 f\sqrt{-\frac{(9A^2 - 30iAB - 25B^2)c}{a^4f^2}}e^{(4ifx+4ie)}\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}(a^2fe^{(2ifx+2ie)} + a^2f)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{(9A^2-30iAB-25B^2)c}{a^4f^2}}\right.}{8a^2f}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="fricas")
```

output
```
1/32*(sqrt(1/2)*a^2*f*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)*c/(a^4*f^2))*e^(4*
I*f*x + 4*I*e)*log(1/8*(sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2
*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)*c/
(a^4*f^2)) + (3*I*A + 5*B)*c)*e^(-I*f*x - I*e)/(a^2*f)) - sqrt(1/2)*a^2*f*
sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)*c/(a^4*f^2))*e^(4*I*f*x + 4*I*e)*log(-1/
8*(sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)*c/(a^4*f^2)) - (3*I*A +
 5*B)*c)*e^(-I*f*x - I*e)/(a^2*f)) + sqrt(2)*((5*I*A + 3*B)*e^(4*I*f*x + 4
*I*e) + (7*I*A + B)*e^(2*I*f*x + 2*I*e) + 2*I*A - 2*B)*sqrt(c/(e^(2*I*f*x
+ 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]

$$\int \frac{(A + B\tan(e + fx))\sqrt{c - ic\tan(e + fx)}}{(a + ia\tan(e + fx))^2}\,dx$$
$$= -\frac{\int \frac{A\sqrt{-ic\tan(e+fx)+c}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx + \int \frac{B\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**2
,x)
```

output
```
-(Integral(A*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**2 - 2*I*tan(e + f*
x) - 1), x) + Integral(B*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e +
 f*x)**2 - 2*I*tan(e + f*x) - 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.08

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{i\left(\frac{\sqrt{2}(3A-5iB)c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2}+\frac{4\left((-ic\tan(fx+e)+c)^{\frac{3}{2}}(3A-5iB)c^2-2\sqrt{-ic\tan(fx+e)+c}(5A-3iB)c^3\right)}{(-ic\tan(fx+e)+c)^2a^2-4(-ic\tan(fx+e)+c)a^2c+4a^2c^2}\right)}{64cf}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="maxima")
```

output
```
-1/64*I*(sqrt(2)*(3*A - 5*I*B)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*t
an(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^2 + 4
*((-I*c*tan(f*x + e) + c)^(3/2)*(3*A - 5*I*B)*c^2 - 2*sqrt(-I*c*tan(f*x +
e) + c)*(5*A - 3*I*B)*c^3)/((-I*c*tan(f*x + e) + c)^2*a^2 - 4*(-I*c*tan(f*
x + e) + c)*a^2*c + 4*a^2*c^2))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.66

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$$
$$= -\frac{\frac{5\,B\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}}{16}-\frac{3\,B\,c^2\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{8}}{4\,a^2\,c^2\,f+a^2\,f\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2-4\,a^2\,c\,f\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}$$
$$+\frac{\frac{A\,c^2\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,5\mathrm{i}}{8\,a^2\,f}-\frac{A\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}\,3\mathrm{i}}{16\,a^2\,f}}{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2-4\,c\,\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)+4\,c^2}$$
$$+\frac{\sqrt{2}\,A\,\sqrt{-c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,3\mathrm{i}}{32\,a^2\,f}+\frac{5\,\sqrt{2}\,B\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{32\,a^2\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(1/2))/(a + a*tan(e + f*
x)*1i)^2,x)
```

output
```
((A*c^2*(c - c*tan(e + f*x)*1i)^(1/2)*5i)/(8*a^2*f) - (A*c*(c - c*tan(e +
f*x)*1i)^(3/2)*3i)/(16*a^2*f))/((c - c*tan(e + f*x)*1i)^2 - 4*c*(c - c*tan
(e + f*x)*1i) + 4*c^2) - ((5*B*c*(c - c*tan(e + f*x)*1i)^(3/2))/16 - (3*B*
c^2*(c - c*tan(e + f*x)*1i)^(1/2))/8)/(4*a^2*c^2*f + a^2*f*(c - c*tan(e +
f*x)*1i)^2 - 4*a^2*c*f*(c - c*tan(e + f*x)*1i)) + (2^(1/2)*A*(-c)^(1/2)*at
an((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*3i)/(32*a^2*f)
+ (5*2^(1/2)*B*c^(1/2)*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^
(1/2))))/(32*a^2*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$$
$$= -\frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)a+\left(\int \frac{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)b\right)}{a^2}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^2,x)
```

output
```
( - sqrt(c)*(int(sqrt( - tan(e + f*x)*i + 1)/(tan(e + f*x)**2 - 2*tan(e +
f*x)*i - 1),x)*a + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e +
 f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b))/a**2
```

3.776 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 195

$$\begin{aligned}\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx &= \frac{3(5iA+3B)\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}a^2\sqrt{c}f} - \frac{3(5iA+3B)}{32a^2f\sqrt{c-ic\tan(e+fx)}} \\ &\quad + \frac{iA-B}{4a^2f(1+i\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}} \\ &\quad + \frac{5iA+3B}{16a^2f(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\end{aligned}$$

output
```
3/64*(5*I*A+3*B)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(
1/2)/a^2/c^(1/2)/f-3/32*(5*I*A+3*B)/a^2/f/(c-I*c*tan(f*x+e))^(1/2)+1/4*(I*
A-B)/a^2/f/(1+I*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2)+1/16*(5*I*A+3*B)/a^
2/f/(1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$=\frac{\frac{3\sqrt{2}a(5iA+3B)\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c}}+\frac{2a\sqrt{c-ic\tan(e+fx)}(3A+11iB+(-20iA-12B)\tan(e+fx)+3(5A-3iB)\tan^2(e+fx))}{c(-i+\tan(e+fx))^2(i+\tan(e+fx))}}{64a^3f}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[
e + f*x]]),x]
```

output
```
((3*Sqrt[2]*a*((5*I)*A + 3*B)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*
Sqrt[c])])/Sqrt[c] + (2*a*Sqrt[c - I*c*Tan[e + f*x]]*(3*A + (11*I)*B + ((-
20*I)*A - 12*B)*Tan[e + f*x] + 3*(5*A - (3*I)*B)*Tan[e + f*x]^2))/(c*(-I +
 Tan[e + f*x])^2*(I + Tan[e + f*x])))/(64*a^3*f)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {3042, 4071, 27, 87, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^3(i\tan(e+fx)+1)^3(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^3(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{a^2 f}$$

$$\Big\downarrow \; 87$$

$$\frac{c\left(\frac{1}{8}(5A-3iB)\int \frac{1}{(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)+\frac{-B+iA}{4c(1+i\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\right)}{a^2 f}$$

$$\Big\downarrow \; 52$$

$$\frac{c\left(\frac{1}{8}(5A-3iB)\left(\frac{3}{4}\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)+\frac{i}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)+\frac{}{4c(1+i\tan(e-}\right.}{a^2 f}$$

$$\Big\downarrow \; 61$$

$$\frac{c\left(\frac{1}{8}(5A-3iB)\left(\frac{3}{4}\left(\frac{\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{2c}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}\right)+\frac{i}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)\right.}{a^2 f}$$

$$\Big\downarrow \; 73$$

$$\frac{c\left(\frac{1}{8}(5A-3iB)\left(\frac{3}{4}\left(\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}\right)+\frac{i}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)+\frac{}{4}\right.}{a^2 f}$$

$$\Big\downarrow \; 219$$

$$\frac{c\left(\frac{1}{8}(5A-3iB)\left(\frac{3}{4}\left(\frac{i\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}c^{3/2}}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}\right)+\frac{i}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)+\frac{}{4c(1+i\,\text{ta}}\right.}{a^2 f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*
x]]),x]
```

output
```
(c*((I*A - B)/(4*c*(1 + I*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]]) + ((
5*A - (3*I)*B)*((I/2)/(c*(1 + I*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])
+ (3*((I*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[2]*c
^(3/2)) - I/(c*Sqrt[c - I*c*Tan[e + f*x]])))/4))/8))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{2ic^2\left(\frac{\frac{4\left(\frac{iB}{32}-\frac{7A}{32}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+4\left(\frac{9}{16}cA+\frac{1}{16}iBc\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^2}+\frac{3\left(-\frac{3iB}{4}+\frac{5A}{4}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}}}{8c^2}\cdots\right.}{f\,a^2}$
default	$\frac{2ic^2\left(\frac{\frac{4\left(\frac{iB}{32}-\frac{7A}{32}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+4\left(\frac{9}{16}cA+\frac{1}{16}iBc\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^2}+\frac{3\left(-\frac{3iB}{4}+\frac{5A}{4}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(fx+e)}\sqrt{2}}{2\sqrt{c}}\right)}{4\sqrt{c}}}{8c^2}\cdots\right.}{f\,a^2}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^2*c^2*(1/8/c^2*(4*((1/32*I*B-7/32*A)*(c-I*c*tan(f*x+e))^(3/2)+(9/1
6*c*A+1/16*I*B*c)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^2+3/4*(-3/4
*I*B+5/4*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c
^(1/2)))-1/8/c^2*(A-I*B)/(c-I*c*tan(f*x+e))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(150) = 300.

Time = 0.09 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.00

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$= \frac{\left(3\sqrt{\frac{1}{2}}a^2cf\sqrt{-\frac{25A^2-30iAB-9B^2}{a^4cf^2}}e^{(4ifx+4ie)}\log\left(\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}(a^2fe^{(2ifx+2ie)}+a^2f)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{25A^2-30iAB-9B^2}{a^4cf^2}}+\right.}{16a^2f}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
1/64*(3*sqrt(1/2)*a^2*c*f*sqrt(-(25*A^2 - 30*I*A*B - 9*B^2)/(a^4*c*f^2))*e
^(4*I*f*x + 4*I*e)*log(3/16*(sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e)
+ a^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(25*A^2 - 30*I*A*B - 9*B^
2)/(a^4*c*f^2)) + 5*I*A + 3*B)*e^(-I*f*x - I*e)/(a^2*f)) - 3*sqrt(1/2)*a^2
*c*f*sqrt(-(25*A^2 - 30*I*A*B - 9*B^2)/(a^4*c*f^2))*e^(4*I*f*x + 4*I*e)*lo
g(-3/16*(sqrt(2)*sqrt(1/2)*(a^2*f*e^(2*I*f*x + 2*I*e) + a^2*f)*sqrt(c/(e^(
2*I*f*x + 2*I*e) + 1))*sqrt(-(25*A^2 - 30*I*A*B - 9*B^2)/(a^4*c*f^2)) - 5*
I*A - 3*B)*e^(-I*f*x - I*e)/(a^2*f)) - sqrt(2)*(8*(I*A + B)*e^(6*I*f*x + 6
*I*e) - (I*A - 9*B)*e^(4*I*f*x + 4*I*e) - (11*I*A - 3*B)*e^(2*I*f*x + 2*I*
e) - 2*I*A + 2*B)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/
(a^2*c*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx =$$
$$-\frac{\int \frac{A}{\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-2i\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)-\sqrt{-ic\tan(e+fx)+c}}\,dx+\int \frac{}{\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)}}{a^2}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**(1/2)
,x)
```

output
```
-(Integral(A/(sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*sqrt(-I*c*
tan(e + f*x) + c)*tan(e + f*x) - sqrt(-I*c*tan(e + f*x) + c)), x) + Integr
al(B*tan(e + f*x)/(sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - 2*I*sqrt(
-I*c*tan(e + f*x) + c)*tan(e + f*x) - sqrt(-I*c*tan(e + f*x) + c)), x))/a*
*2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{i\left(\frac{3\sqrt{2}(5A-3iB)\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^2}+\frac{4\left(3(-ic\tan(fx+e)+c)^2(5A-3iB)c-10(-ic\tan(fx+e)+c)(5A-3iB)c^2+\dots\right.}{(-ic\tan(fx+e)+c)^{\frac{5}{2}}a^2-4(-ic\tan(fx+e)+c)^{\frac{3}{2}}a^2c+4\sqrt{-ic\tan(fx+\dots}}\right.}{128\,cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
-1/128*I*(3*sqrt(2)*(5*A - 3*I*B)*sqrt(c)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*
c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^2
+ 4*(3*(-I*c*tan(f*x + e) + c)^2*(5*A - 3*I*B)*c - 10*(-I*c*tan(f*x + e) +
 c)*(5*A - 3*I*B)*c^2 + 32*(A - I*B)*c^3)/((-I*c*tan(f*x + e) + c)^(5/2)*a
^2 - 4*(-I*c*tan(f*x + e) + c)^(3/2)*a^2*c + 4*sqrt(-I*c*tan(f*x + e) + c)
*a^2*c^2))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.56

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx =$$

$$-\frac{\frac{A\,{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2\,15{}\mathrm{i}}{32\,a^2\,f}+\frac{A\,c^2\,1{}\mathrm{i}}{a^2\,f}-\frac{A\,c\,\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)\,25{}\mathrm{i}}{16\,a^2\,f}}{-4\,c\,{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}+{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{5/2}+4\,c^2\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}}-\frac{B\,c^2+\frac{9\,B\,{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^2}{32}-\frac{15\,B\,c\,\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}{16}}{a^2\,f\,{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{5/2}-4\,a^2\,c\,f\,{\left(c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}\right)}^{3/2}+4\,a^2\,c^2\,f\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}}-\frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{-c}}\right)\,15{}\mathrm{i}}{64\,a^2\,\sqrt{-c}\,f}+\frac{9\,\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\mathrm{tan}\left(e+f\,x\right)\,1{}\mathrm{i}}}{2\,\sqrt{c}}\right)}{64\,a^2\,\sqrt{c}\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i
)^(1/2)),x)
```

output
```
(9*2^(1/2)*B*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(
64*a^2*c^(1/2)*f) - (B*c^2 + (9*B*(c - c*tan(e + f*x)*1i)^2)/32 - (15*B*c*
(c - c*tan(e + f*x)*1i))/16)/(a^2*f*(c - c*tan(e + f*x)*1i)^(5/2) - 4*a^2*
c*f*(c - c*tan(e + f*x)*1i)^(3/2) + 4*a^2*c^2*f*(c - c*tan(e + f*x)*1i)^(1
/2)) - (2^(1/2)*A*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/
2)))*15i)/(64*a^2*(-c)^(1/2)*f) - ((A*(c - c*tan(e + f*x)*1i)^2*15i)/(32*a
^2*f) + (A*c^2*1i)/(a^2*f) - (A*c*(c - c*tan(e + f*x)*1i)*25i)/(16*a^2*f))
/((c - c*tan(e + f*x)*1i)^(5/2) - 4*c*(c - c*tan(e + f*x)*1i)^(3/2) + 4*c^
2*(c - c*tan(e + f*x)*1i)^(1/2))
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$= \frac{-\left(\left(\int \frac{\tan(fx+e)}{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^2-2\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)i-\sqrt{-\tan(fx+e)i+1}}dx\right)b-\left(\int \frac{}{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)} \right.\right.}{\sqrt{c}\,a^2}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
( - (int(tan(e + f*x)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqr
t( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*
b + int(1/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*a))/(sqrt(c)
*a**2)
```

3.777 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 226

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{5(7iA+B)\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2}a^2c^{3/2}f} - \frac{5(7iA+B)}{96a^2f(c-ic\tan(e+fx))^{3/2}} + \frac{iA-B}{4a^2f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}} + \frac{7iA+B}{16a^2f(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}} - \frac{5(7iA+B)}{64a^2cf\sqrt{c-ic\tan(e+fx)}}$$

output

```
5/128*(7*I*A+B)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1
/2)/a^2/c^(3/2)/f-5/96*(7*I*A+B)/a^2/f/(c-I*c*tan(f*x+e))^(3/2)+1/4*(I*A-B
)/a^2/f/(1+I*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2)+1/16*(7*I*A+B)/a^2/f/(
1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2)-5/64*(7*I*A+B)/a^2/c/f/(c-I*c*tan
(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.76 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.69

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{\sec^2(e+fx)\left(-66A-18iB+4(A-7iB)\cos(2(e+fx))+28iA\sin(2(e+fx))+4B\sin(2(e+fx))+\ldots\right.}{192a^2cf(-i+\tan(e+fx))^2(i+\tan(e+\ldots}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e +
f*x])^(3/2)),x]
```

output
```
-1/192*(Sec[e + f*x]^2*(-66*A - (18*I)*B + 4*(A - (7*I)*B)*Cos[2*(e + f*x)
] + (28*I)*A*Sin[2*(e + f*x)] + 4*B*Sin[2*(e + f*x)] + 15*((7*I)*A + B)*Hy
pergeometric2F1[-1/2, 1, 1/2, (-1/2*I)*(I + Tan[e + f*x])]*(-I + Tan[e + f
*x])))/(a^2*c*f*(-I + Tan[e + f*x])^2*(I + Tan[e + f*x])*Sqrt[c - I*c*Tan[
e + f*x]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.209$, Rules used = {3042, 4071, 27, 87, 52, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^3(i\tan(e+fx)+1)^3(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 27

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^3(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{a^2f}$$

$\Big\downarrow$ 87

$$\frac{c\left(\frac{1}{8}(7A-iB)\int \frac{1}{(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)+\frac{-B+iA}{4c(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\right)}{a^2f}$$

$\Big\downarrow$ 52

$$\frac{c\left(\frac{1}{8}(7A-iB)\left(\frac{5}{4}\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)+\frac{i}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\right)+\frac{}{4c(1+i\tan(e}\right.}{a^2f}$$

$\Big\downarrow$ 61

$$\frac{c\left(\frac{1}{8}(7A-iB)\left(\frac{5}{4}\left(\frac{\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{i}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx)}\right.\right.}{a^2f}$$

$\Big\downarrow$ 61

$$\frac{c\left(\frac{1}{8}(7A-iB)\left(\frac{5}{4}\left(\frac{\frac{\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{2c}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{}{2c(1+i\tan(e+fx))}\right.\right.}{a^2f}$$

$\Big\downarrow$ 73

$$\frac{c\left(\frac{1}{8}(7A-iB)\left(\frac{5}{4}\left(\frac{\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{i}{2c(1+i\tan(e+fx))(c-ic}\right.\right.}{a^2f}$$

$\Big\downarrow$ 219

$$\frac{c\left(\frac{1}{8}(7A-iB)\left(\frac{5}{4}\left(\frac{\frac{i\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}c^{3/2}}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{i}{2c(1+i\tan(e+fx))(c-ic\tan(e+\right.\right.}{a^2f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^
(3/2)),x]
```

output
```
(c*((I*A - B)/(4*c*(1 + I*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(3/2)) +
((7*A - I*B)*((I/2)/(c*(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))
+ (5*((-1/3*I)/(c*(c - I*c*Tan[e + f*x])^(3/2)) + ((I*ArcTanh[Sqrt[c - I*c
*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[2]*c^(3/2)) - I/(c*Sqrt[c - I*c*T
an[e + f*x]]))/(2*c)))/4))/8))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{2ic^2\left(-\frac{-iB+3A}{16c^3\sqrt{c-ic\tan(fx+e)}}-\frac{-iB+A}{24c^2(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{\frac{4\left(-\frac{3iB}{32}-\frac{11A}{32}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+4\left(\frac{13}{16}cA+\frac{5}{16}iBc\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^2}}{16c^3}\right)}{f\,a^2}$
default	$\frac{2ic^2\left(-\frac{-iB+3A}{16c^3\sqrt{c-ic\tan(fx+e)}}-\frac{-iB+A}{24c^2(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{\frac{4\left(-\frac{3iB}{32}-\frac{11A}{32}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+4\left(\frac{13}{16}cA+\frac{5}{16}iBc\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^2}}{16c^3}\right)}{f\,a^2}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^2*c^2*(-1/16/c^3*(3*A-I*B)/(c-I*c*tan(f*x+e))^(1/2)-1/24/c^2*(A-I*
B)/(c-I*c*tan(f*x+e))^(3/2)+1/16/c^3*(4*((-3/32*I*B-11/32*A)*(c-I*c*tan(f*
x+e))^(3/2)+(13/16*c*A+5/16*I*B*c)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*
x+e))^2+5/4*(7/4*A-1/4*I*B)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))
^(1/2)*2^(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(181) = 362.

Time = 0.09 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.86

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\left(15\sqrt{\frac{1}{2}}a^2c^2f\sqrt{-\frac{49A^2-14iAB-B^2}{a^4c^3f^2}}e^{(4ifx+4ie)}\log\left(\frac{5\left(\sqrt{\cdots}\right.}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
1/384*(15*sqrt(1/2)*a^2*c^2*f*sqrt(-(49*A^2 - 14*I*A*B - B^2)/(a^4*c^3*f^2
))*e^(4*I*f*x + 4*I*e)*log(5/32*(sqrt(2)*sqrt(1/2)*(a^2*c*f*e^(2*I*f*x + 2
*I*e) + a^2*c*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(49*A^2 - 14*I*A*
B - B^2)/(a^4*c^3*f^2)) + 7*I*A + B)*e^(-I*f*x - I*e)/(a^2*c*f)) - 15*sqrt
(1/2)*a^2*c^2*f*sqrt(-(49*A^2 - 14*I*A*B - B^2)/(a^4*c^3*f^2))*e^(4*I*f*x
+ 4*I*e)*log(-5/32*(sqrt(2)*sqrt(1/2)*(a^2*c*f*e^(2*I*f*x + 2*I*e) + a^2*c
*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(49*A^2 - 14*I*A*B - B^2)/(a^4
*c^3*f^2)) - 7*I*A - B)*e^(-I*f*x - I*e)/(a^2*c*f)) - sqrt(2)*(8*(I*A + B)
*e^(8*I*f*x + 8*I*e) + 8*(11*I*A + 5*B)*e^(6*I*f*x + 6*I*e) - (-41*I*A - 4
7*B)*e^(4*I*f*x + 4*I*e) + 3*(-15*I*A + 7*B)*e^(2*I*f*x + 2*I*e) - 6*I*A +
 6*B)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*c^2*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{\int \frac{A}{-ic\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)-c\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-ic\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)-c\sqrt{-ic\tan(e+fx)+c}}\,dx+}{a^2}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**(3/2)
,x)
```

output
```
-(Integral(A/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - c*sqrt(-I
*c*tan(e + f*x) + c)*tan(e + f*x)**2 - I*c*sqrt(-I*c*tan(e + f*x) + c)*tan
(e + f*x) - c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(B*tan(e + f*x)/(
-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - c*sqrt(-I*c*tan(e + f*x
) + c)*tan(e + f*x)**2 - I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) - c*
sqrt(-I*c*tan(e + f*x) + c)), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{i\left(\frac{4\left(15\,(-i\,c\tan(fx+e)+c)^3(7\,A-i\,B)-50\,(-i\,c\tan(fx+e)+c)^2(7\,A-i\,B)c+32\,(-i\,c\tan(fx+e)+c)(7\,A-i\,B)c^2+64\,(A-i\,B)c^3\right)}{(-i\,c\tan(fx+e)+c)^{\frac{7}{2}}a^2-4\,(-i\,c\tan(fx+e)+c)^{\frac{5}{2}}a^2c+4\,(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}a^2c^2}+\frac{15\sqrt{2}(\cdots}{\cdots}\right.}{768\,cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
-1/768*I*(4*(15*(-I*c*tan(f*x + e) + c)^3*(7*A - I*B) - 50*(-I*c*tan(f*x +
 e) + c)^2*(7*A - I*B)*c + 32*(-I*c*tan(f*x + e) + c)*(7*A - I*B)*c^2 + 64
*(A - I*B)*c^3)/((-I*c*tan(f*x + e) + c)^(7/2)*a^2 - 4*(-I*c*tan(f*x + e)
+ c)^(5/2)*a^2*c + 4*(-I*c*tan(f*x + e) + c)^(3/2)*a^2*c^2) + 15*sqrt(2)*(
7*A - I*B)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c))/(sqrt(2)*s
qrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/(a^2*sqrt(c)))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.56

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{\frac{B\,c^2}{3}-\frac{25\,B\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2}{96}+\frac{5\,B\,(c-c\tan(e+f\,x)\,1\mathrm{i})^3}{64\,c}+\frac{B\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})}{6}}{a^2\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{7/2}-4\,a^2\,c\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}+4\,a^2\,c^2\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}} - \frac{-\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2\,175\mathrm{i}}{96\,a^2\,f}+\frac{A\,c^2\,1\mathrm{i}}{3\,a^2\,f}+\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^3\,35\mathrm{i}}{64\,a^2\,c\,f}+\frac{A\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})\,7\mathrm{i}}{6\,a^2\,f}}{-4\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}+(c-c\tan(e+f\,x)\,1\mathrm{i})^{7/2}+4\,c^2\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}} + \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,35\mathrm{i}}{128\,a^2\,(-c)^{3/2}\,f} + \frac{5\,\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{128\,a^2\,c^{3/2}\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i
)^(3/2)),x)
```

output
```
(2^(1/2)*A*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*35
i)/(128*a^2*(-c)^(3/2)*f) - ((A*c^2*1i)/(3*a^2*f) - (A*(c - c*tan(e + f*x)
*1i)^2*175i)/(96*a^2*f) + (A*(c - c*tan(e + f*x)*1i)^3*35i)/(64*a^2*c*f) +
 (A*c*(c - c*tan(e + f*x)*1i)*7i)/(6*a^2*f))/((c - c*tan(e + f*x)*1i)^(7/2
) - 4*c*(c - c*tan(e + f*x)*1i)^(5/2) + 4*c^2*(c - c*tan(e + f*x)*1i)^(3/2
)) - ((B*c^2)/3 - (25*B*(c - c*tan(e + f*x)*1i)^2)/96 + (5*B*(c - c*tan(e
+ f*x)*1i)^3)/(64*c) + (B*c*(c - c*tan(e + f*x)*1i))/6)/(a^2*f*(c - c*tan(
e + f*x)*1i)^(7/2) - 4*a^2*c*f*(c - c*tan(e + f*x)*1i)^(5/2) + 4*a^2*c^2*f
*(c - c*tan(e + f*x)*1i)^(3/2)) + (5*2^(1/2)*B*atanh((2^(1/2)*(c - c*tan(e
 + f*x)*1i)^(1/2))/(2*c^(1/2))))/(128*a^2*c^(3/2)*f)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
( - sqrt( - tan(e + f*x)*i + 1)*a*i + 7*int(sqrt( - tan(e + f*x)*i + 1)/(2
*tan(e + f*x)**5*i + 2*tan(e + f*x)**4 + 4*tan(e + f*x)**3*i + 4*tan(e + f
*x)**2 + 2*tan(e + f*x)*i + 2),x)*tan(e + f*x)**4*a*f + 14*int(sqrt( - tan
(e + f*x)*i + 1)/(2*tan(e + f*x)**5*i + 2*tan(e + f*x)**4 + 4*tan(e + f*x)
**3*i + 4*tan(e + f*x)**2 + 2*tan(e + f*x)*i + 2),x)*tan(e + f*x)**2*a*f +
 7*int(sqrt( - tan(e + f*x)*i + 1)/(2*tan(e + f*x)**5*i + 2*tan(e + f*x)**
4 + 4*tan(e + f*x)**3*i + 4*tan(e + f*x)**2 + 2*tan(e + f*x)*i + 2),x)*a*f
 + 3*int(tan(e + f*x)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3*i + sqr
t( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 + sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)*i + sqrt( - tan(e + f*x)*i + 1)),x)*tan(e + f*x)**4*b*f + 6*int(
tan(e + f*x)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3*i + sqrt( - tan(
e + f*x)*i + 1)*tan(e + f*x)**2 + sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)
*i + sqrt( - tan(e + f*x)*i + 1)),x)*tan(e + f*x)**2*b*f + 3*int(tan(e + f
*x)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3*i + sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**2 + sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i + sqrt
( - tan(e + f*x)*i + 1)),x)*b*f + 7*int((sqrt( - tan(e + f*x)*i + 1)*tan(e
 + f*x)**2)/(2*tan(e + f*x)**5*i + 2*tan(e + f*x)**4 + 4*tan(e + f*x)**3*i
 + 4*tan(e + f*x)**2 + 2*tan(e + f*x)*i + 2),x)*tan(e + f*x)**4*a*f + 14*i
nt((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(2*tan(e + f*x)**5*i + 2*
tan(e + f*x)**4 + 4*tan(e + f*x)**3*i + 4*tan(e + f*x)**2 + 2*tan(e + f...
```

3.778 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 273

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{7(9iA-B)\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{128\sqrt{2}a^2c^{5/2}f} - \frac{7(9iA-B)}{160a^2f(c-ic\tan(e+fx))^{5/2}} + \frac{iA-B}{4a^2f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}} + \frac{9iA-B}{16a^2f(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5/2}} - \frac{7(9iA-B)}{192a^2cf(c-ic\tan(e+fx))^{3/2}} - \frac{7(9iA-B)}{128a^2c^2f\sqrt{c-ic\tan(e+fx)}}$$

output
```
7/256*(9*I*A-B)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1
/2)/a^2/c^(5/2)/f-7/160*(9*I*A-B)/a^2/f/(c-I*c*tan(f*x+e))^(5/2)+1/4*(I*A-
B)/a^2/f/(1+I*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2)+1/16*(9*I*A-B)/a^2/f/
(1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2)-7/192*(9*I*A-B)/a^2/c/f/(c-I*c*t
an(f*x+e))^(3/2)-7/128*(9*I*A-B)/a^2/c^2/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.52

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\cos(e+fx)\left(35(-9iA+B)\,\text{Hypergeometric2F1}\left(-\frac{3}{2}\right.\right.}{}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e +
f*x])^(5/2)),x]
```

output
```
(Cos[e + f*x]*(35*((-9*I)*A + B)*Hypergeometric2F1[-3/2, 1, -1/2, (-1/2*I)
*(I + Tan[e + f*x])]*(Cos[e + f*x] + I*Sin[e + f*x]) + 6*Cos[e + f*x]*((65
*I)*A - 25*B + (2*I)*(A + (9*I)*B)*Cos[2*(e + f*x)] + 2*(9*A + I*B)*Sin[2*
(e + f*x)])))/(960*a^2*c^2*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4071, 27, 87, 52, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^3(i\tan(e+fx)+1)^3(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^3(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{a^2f}$$

$$\Big\downarrow \ 87$$

$$\frac{c\left(\frac{1}{8}(9A+iB)\int \frac{1}{(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)+\frac{-B+iA}{4c(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\right)}{a^2f}$$

$$\Big\downarrow \ 52$$

$$\frac{c\left(\frac{1}{8}(9A+iB)\left(\frac{7}{4}\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)+\frac{i}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\right)+\frac{\cdots}{4c(1+i\tan(e\cdots}\right.}{a^2f}$$

$$\Big\downarrow \ 61$$

$$\frac{c\left(\frac{1}{8}(9A+iB)\left(\frac{7}{4}\left(\frac{\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{i}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx\cdots}\right.\right.}{a^2f}$$

$$\Big\downarrow \ 61$$

$$\frac{c\left(\frac{1}{8}(9A+iB)\left(\frac{7}{4}\left(\frac{\frac{\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{\cdots}{2c(1+i\tan(\cdots}\right.\right.}{a^2f}$$

$$\Big\downarrow \ 61$$

$$\frac{c\left(\frac{1}{8}(9A+iB)\left(\frac{7}{4}\left(\frac{\frac{\frac{\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{2c}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)\cdots\right.\right.}{a^2f}$$

$$\Big\downarrow \ 73$$

$$\frac{c\left(\frac{1}{8}(9A+iB)\left(\frac{7}{4}\left(\frac{\frac{\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{\cdots}{2\cdots}\right.\right.}{a^2f}$$

$$\Big\downarrow \text{219}$$

$$\frac{c\left(\frac{1}{8}(9A+iB)\left(\frac{7}{4}\left(\frac{\frac{\frac{i\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}c^{3/2}}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{}{2c(1+i\,\text{ta}}\right.\right.}{a^2f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^
(5/2)),x]
```

output
```
(c*((I*A - B)/(4*c*(1 + I*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(5/2)) +
((9*A + I*B)*((I/2)/(c*(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))
+ (7*((-1/5*I)/(c*(c - I*c*Tan[e + f*x])^(5/2)) + ((-1/3*I)/(c*(c - I*c*Ta
n[e + f*x])^(3/2)) + ((I*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[
c])])/(Sqrt[2]*c^(3/2)) - I/(c*Sqrt[c - I*c*Tan[e + f*x]]))/(2*c))/(2*c)))
/4))/8))/(a^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{2ic^2\left(-\frac{3A}{16c^4\sqrt{c-ic\tan(fx+e)}}-\frac{-iB+3A}{48c^3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{-iB+A}{40c^2(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{4\left(-\frac{7iB}{64}-\frac{15A}{64}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+4\left(\frac{9}{32}\right.}{(c+ic\tan(fx+e}\right.}{f\,a^2}$
default	$\frac{2ic^2\left(-\frac{3A}{16c^4\sqrt{c-ic\tan(fx+e)}}-\frac{-iB+3A}{48c^3(c-ic\tan(fx+e))^{\frac{3}{2}}}-\frac{-iB+A}{40c^2(c-ic\tan(fx+e))^{\frac{5}{2}}}+\frac{4\left(-\frac{7iB}{64}-\frac{15A}{64}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+4\left(\frac{9}{32}\right.}{(c+ic\tan(fx+e}\right.}{f\,a^2}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^2*c^2*(-3/16/c^4*A/(c-I*c*tan(f*x+e))^(1/2)-1/48/c^3*(3*A-I*B)/(c-
I*c*tan(f*x+e))^(3/2)-1/40/c^2*(A-I*B)/(c-I*c*tan(f*x+e))^(5/2)+1/16/c^4*(
4*((-7/64*I*B-15/64*A)*(c-I*c*tan(f*x+e))^(3/2)+(9/32*I*B*c+17/32*c*A)*(c-
I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^2+7/8*(1/4*I*B+9/4*A)*2^(1/2)/c^
(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(202) = 404.

Time = 0.11 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.63

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\left(105\sqrt{\frac{1}{2}}a^2c^3f\sqrt{-\frac{81A^2+18iAB-B^2}{a^4c^5f^2}}e^{(4ifx+4ie)}\log\left(\frac{7\left(\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="fricas")
```

output

```
1/3840*(105*sqrt(1/2)*a^2*c^3*f*sqrt(-(81*A^2 + 18*I*A*B - B^2)/(a^4*c^5*f
^2))*e^(4*I*f*x + 4*I*e)*log(7/64*(sqrt(2)*sqrt(1/2)*(a^2*c^2*f*e^(2*I*f*x
 + 2*I*e) + a^2*c^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(81*A^2 + 1
8*I*A*B - B^2)/(a^4*c^5*f^2)) + 9*I*A - B)*e^(-I*f*x - I*e)/(a^2*c^2*f)) -
 105*sqrt(1/2)*a^2*c^3*f*sqrt(-(81*A^2 + 18*I*A*B - B^2)/(a^4*c^5*f^2))*e^
(4*I*f*x + 4*I*e)*log(-7/64*(sqrt(2)*sqrt(1/2)*(a^2*c^2*f*e^(2*I*f*x + 2*I
*e) + a^2*c^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(81*A^2 + 18*I*A*
B - B^2)/(a^4*c^5*f^2)) - 9*I*A + B)*e^(-I*f*x - I*e)/(a^2*c^2*f)) - sqrt(
2)*(24*(I*A + B)*e^(10*I*f*x + 10*I*e) + 16*(12*I*A + 7*B)*e^(8*I*f*x + 8*
I*e) + 8*(129*I*A + 19*B)*e^(6*I*f*x + 6*I*e) - (-609*I*A - 199*B)*e^(4*I*
f*x + 4*I*e) + 15*(-19*I*A + 11*B)*e^(2*I*f*x + 2*I*e) - 30*I*A + 30*B)*sq
rt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-4*I*f*x - 4*I*e)/(a^2*c^3*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{\int \frac{A}{-c^2\sqrt{-ic\tan{(e+fx)}+c}\tan^4{(e+fx)}-2c^2\sqrt{-ic\tan{(e+fx)}+c}\tan^2{(e+fx)}-c^2\sqrt{-ic\tan{(e+fx)}+c}}\,dx+\int \frac{}{-c^2\sqrt{-ic\tan{(e+fx)}+c}\tan}}{a^2}$$

input

```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**2/(c-I*c*tan(f*x+e))**(5/2)
,x)
```

output

```
-(Integral(A/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4 - 2*c**2*s
qrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - c**2*sqrt(-I*c*tan(e + f*x) +
 c)), x) + Integral(B*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(
e + f*x)**4 - 2*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2 - c**2*sq
rt(-I*c*tan(e + f*x) + c)), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.90

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{i\left(\frac{4\left(105(-ic\tan(fx+e)+c)^4(9A+iB)-350(-ic\tan(fx+e)+c)^3(9A+iB)c+224(-ic\tan(fx+e)+c)^2(9A+iB)c^2+64(-ic\tan(fx+e)+c\right.}{(-ic\tan(fx+e)+c)^{\frac{9}{2}}a^2c-4(-ic\tan(fx+e)+c)^{\frac{7}{2}}a^2c^2+4(-ic\tan(fx+e)+c)^{\frac{5}{2}}a^2c^3}\right.}{7680\,cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
-1/7680*I*(4*(105*(-I*c*tan(f*x + e) + c)^4*(9*A + I*B) - 350*(-I*c*tan(f*
x + e) + c)^3*(9*A + I*B)*c + 224*(-I*c*tan(f*x + e) + c)^2*(9*A + I*B)*c^
2 + 64*(-I*c*tan(f*x + e) + c)*(9*A + I*B)*c^3 + 384*(A - I*B)*c^4)/((-I*c
*tan(f*x + e) + c)^(9/2)*a^2*c - 4*(-I*c*tan(f*x + e) + c)^(7/2)*a^2*c^2 +
 4*(-I*c*tan(f*x + e) + c)^(5/2)*a^2*c^3) + 105*sqrt(2)*(9*A + I*B)*log(-(
sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*
c*tan(f*x + e) + c)))/(a^2*c^(3/2)))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.46

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{-\frac{B\,c^2}{5}+\frac{7\,B\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2}{60}-\frac{35\,B\,(c-c\tan(e+f\,x)\,1\mathrm{i}}{192\,c}}{a^2\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{9/2}-4\,a^2\,c\,f\,(c-c\tan(e}$$

$$-\frac{\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2\,21\mathrm{i}}{20\,a^2\,f}+\frac{A\,c^2\,1\mathrm{i}}{5\,a^2\,f}-\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^3\,105\mathrm{i}}{64\,a^2\,c\,f}+\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^4\,63\mathrm{i}}{128\,a^2\,c^2\,f}+\frac{A\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})\,3\mathrm{i}}{10\,a^2\,f}}{-4\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{7/2}+(c-c\tan(e+f\,x)\,1\mathrm{i})^{9/2}+4\,c^2\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}}$$

$$-\frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,63\mathrm{i}}{256\,a^2\,(-c)^{5/2}\,f}-\frac{7\,\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{256\,a^2\,c^{5/2}\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^2*(c - c*tan(e + f*x)*1i
)^(5/2)),x)
```

output
```
((7*B*(c - c*tan(e + f*x)*1i)^2)/60 - (B*c^2)/5 - (35*B*(c - c*tan(e + f*x
)*1i)^3)/(192*c) + (7*B*(c - c*tan(e + f*x)*1i)^4)/(128*c^2) + (B*c*(c - c
*tan(e + f*x)*1i))/30)/(a^2*f*(c - c*tan(e + f*x)*1i)^(9/2) - 4*a^2*c*f*(c
 - c*tan(e + f*x)*1i)^(7/2) + 4*a^2*c^2*f*(c - c*tan(e + f*x)*1i)^(5/2)) -
 ((A*(c - c*tan(e + f*x)*1i)^2*21i)/(20*a^2*f) + (A*c^2*1i)/(5*a^2*f) - (A
*(c - c*tan(e + f*x)*1i)^3*105i)/(64*a^2*c*f) + (A*(c - c*tan(e + f*x)*1i)
^4*63i)/(128*a^2*c^2*f) + (A*c*(c - c*tan(e + f*x)*1i)*3i)/(10*a^2*f))/((c
 - c*tan(e + f*x)*1i)^(9/2) - 4*c*(c - c*tan(e + f*x)*1i)^(7/2) + 4*c^2*(c
 - c*tan(e + f*x)*1i)^(5/2)) - (2^(1/2)*A*atan((2^(1/2)*(c - c*tan(e + f*x
)*1i)^(1/2))/(2*(-c)^(1/2)))*63i)/(256*a^2*(-c)^(5/2)*f) - (7*2^(1/2)*B*at
anh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(256*a^2*c^(5/2)
*f)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int((sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2)/(tan(e + f*x)**6 + 3*tan(e + f*x)**4 + 3*tan(e + f*x
)**2 + 1),x)*tan(e + f*x)**6*b*f*i + 3*int((sqrt( - tan(e + f*x)*i + 1)*ta
n(e + f*x)**2)/(tan(e + f*x)**6 + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**2 +
1),x)*tan(e + f*x)**4*b*f*i + 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x)**2)/(tan(e + f*x)**6 + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**2 + 1),x)*t
an(e + f*x)**2*b*f*i + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(
tan(e + f*x)**6 + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**2 + 1),x)*b*f*i + 12
*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**6 + 3*tan(e
 + f*x)**4 + 3*tan(e + f*x)**2 + 1),x)*tan(e + f*x)**6*a*f*i + int((sqrt(
- tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**6 + 3*tan(e + f*x)**4 +
 3*tan(e + f*x)**2 + 1),x)*tan(e + f*x)**6*b*f + 36*int((sqrt( - tan(e + f
*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**6 + 3*tan(e + f*x)**4 + 3*tan(e +
f*x)**2 + 1),x)*tan(e + f*x)**4*a*f*i + 3*int((sqrt( - tan(e + f*x)*i + 1)
*tan(e + f*x))/(tan(e + f*x)**6 + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**2 +
1),x)*tan(e + f*x)**4*b*f + 36*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*
x))/(tan(e + f*x)**6 + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**2 + 1),x)*tan(e
 + f*x)**2*a*f*i + 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e
 + f*x)**6 + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2
*b*f + 12*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)*...
```

3.779 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 291

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx = -\frac{35(iA-5B)c^{9/2}\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{4\sqrt{2}a^3f} + \frac{35(iA-5B)c^4\sqrt{c-ic\tan(e+fx)}}{8a^3f} + \frac{35(iA-5B)c^3(c-ic\tan(e+fx))^{3/2}}{48a^3f} + \frac{7(iA-5B)c^2(c-ic\tan(e+fx))^{5/2}}{16a^3f(1+i\tan(e+fx))} - \frac{(iA-5B)c(c-ic\tan(e+fx))^{7/2}}{8a^3f(1+i\tan(e+fx))^2} + \frac{(iA-B)(c-ic\tan(e+fx))^{9/2}}{6a^3f(1+i\tan(e+fx))^3}$$

output
```
-35/8*(I*A-5*B)*c^(9/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*2^(1/2)/a^3/f+35/8*(I*A-5*B)*c^4*(c-I*c*tan(f*x+e))^(1/2)/a^3/f+35/48*
(I*A-5*B)*c^3*(c-I*c*tan(f*x+e))^(3/2)/a^3/f+7/16*(I*A-5*B)*c^2*(c-I*c*tan
(f*x+e))^(5/2)/a^3/f/(1+I*tan(f*x+e))-1/8*(I*A-5*B)*c*(c-I*c*tan(f*x+e))^(
7/2)/a^3/f/(1+I*tan(f*x+e))^2+1/6*(I*A-B)*(c-I*c*tan(f*x+e))^(9/2)/a^3/f/(
1+I*tan(f*x+e))^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.82 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.65

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{2\left(a^4B(c-ic\tan(e+fx))^{9/2}-(iA-5B)\left(3a^4c(c-\right.\right.}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(9/2))/(a + I*a*Tan
[e + f*x])^3,x]
```

output
```
(2*(a^4*B*(c - I*c*Tan[e + f*x])^(9/2) - (I*A - 5*B)*(3*a^4*c*(c - I*c*Tan
[e + f*x])^(7/2) + 7*c^4*Sqrt[c - I*c*Tan[e + f*x]]*(20*a^4*(1 - I*Tan[e +
 f*x]) + 6*a^4*(I + Tan[e + f*x])^2 - 3*(8*a^4 - a*Hypergeometric2F1[1/2,
4, 3/2, (-1/2*I)*(I + Tan[e + f*x])]*(a + I*a*Tan[e + f*x])^3)))))/(3*a^4*
f*(a + I*a*Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3042, 4071, 27, 87, 51, 51, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{9/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{9/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{a^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \; 87$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{4}(A+5iB)\int \frac{(c-ic\tan(e+fx))^{7/2}}{(i\tan(e+fx)+1)^3}d\tan(e+fx)\right)}{a^3f}$$

$$\Big\downarrow \; 51$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{4}(A+5iB)\left(\frac{i(c-ic\tan(e+fx))^{7/2}}{2(1+i\tan(e+fx))^2}-\frac{7}{4}c\int \frac{(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)\right)\right)}{a^3f}$$

$$\Big\downarrow \; 51$$

$$c\Big(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{4}(A+5iB)\Big(\frac{i(c-ic\tan(e+fx))^{7/2}}{2(1+i\tan(e+fx))^2}-\frac{7}{4}c\Big(\frac{i(c-ic\tan(e+fx))^{5/2}}{1+i\tan(e+fx)}-\frac{5}{2}c\int \frac{(c-ic\tan(e+fx))^{3/2}}{i\tan(e+fx)+1}d\,\mathrm{t}$$
$$a^3f$$

$$\Big\downarrow \; 60$$

$$c\Big(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{4}(A+5iB)\Big(\frac{i(c-ic\tan(e+fx))^{7/2}}{2(1+i\tan(e+fx))^2}-\frac{7}{4}c\Big(\frac{i(c-ic\tan(e+fx))^{5/2}}{1+i\tan(e+fx)}-\frac{5}{2}c\Big(2c\int \frac{\sqrt{c-ic\tan(e+fx)}}{i\tan(e+fx)+1}d$$
$$a^3f$$

$$\Big\downarrow \; 60$$

$$c\Big(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{4}(A+5iB)\Big(\frac{i(c-ic\tan(e+fx))^{7/2}}{2(1+i\tan(e+fx))^2}-\frac{7}{4}c\Big(\frac{i(c-ic\tan(e+fx))^{5/2}}{1+i\tan(e+fx)}-\frac{5}{2}c\Big(2c\Big(2c\int \frac{}{(i\tan(e+fx)+}$$
$$a^3f$$

$$\Big\downarrow \; 73$$

$$c\Big(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{4}(A+5iB)\Big(\frac{i(c-ic\tan(e+fx))^{7/2}}{2(1+i\tan(e+fx))^2}-\frac{7}{4}c\Big(\frac{i(c-ic\tan(e+fx))^{5/2}}{1+i\tan(e+fx)}-\frac{5}{2}c\Big(2c\Big(4i\int \frac{1}{2-\frac{c-ic\tan(e-}{c}}$$
$$a^3f$$

$$\Big\downarrow \; 219$$

$$c\Big(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{4}(A+5iB)\Big(\frac{i(c-ic\tan(e+fx))^{7/2}}{2(1+i\tan(e+fx))^2}-\frac{7}{4}c\Big(\frac{i(c-ic\tan(e+fx))^{5/2}}{1+i\tan(e+fx)}-\frac{5}{2}c\Big(2c\Big(2i\sqrt{2}\sqrt{c}\,\mathrm{arctanh}\Big($$
$$a^3f$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(9/2))/(a + I*a*Tan[e + f
*x])^3,x]
```

output
```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(9/2))/(6*c*(1 + I*Tan[e + f*x])^3)
- ((A + (5*I)*B)*(((I/2)*(c - I*c*Tan[e + f*x])^(7/2))/(1 + I*Tan[e + f*x]
)^2 - (7*c*((I*(c - I*c*Tan[e + f*x])^(5/2))/(1 + I*Tan[e + f*x]) - (5*c*(
((-2*I)/3)*(c - I*c*Tan[e + f*x])^(3/2) + 2*c*((2*I)*Sqrt[2]*Sqrt[c]*ArcTa
nh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])] - (2*I)*Sqrt[c - I*c*Tan[
e + f*x]])))/2))/4))/4))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{2ic^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+7i\sqrt{c-ic\tan(fx+e)}\,Bc+\sqrt{c-ic\tan(fx+e)}\,cA-8c^2\left(\frac{8\left(-\frac{81iB}{512}-\frac{29A}{512}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+\cdots}{\cdots}\right.\right.}{\cdots}$
default	$\frac{2ic^3\left(\frac{iB(c-ic\tan(fx+e))^{\frac{3}{2}}}{3}+7i\sqrt{c-ic\tan(fx+e)}\,Bc+\sqrt{c-ic\tan(fx+e)}\,cA-8c^2\left(\frac{8\left(-\frac{81iB}{512}-\frac{29A}{512}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+\cdots}{\cdots}\right.\right.}{\cdots}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^3*c^3*(1/3*I*B*(c-I*c*tan(f*x+e))^(3/2)+7*I*(c-I*c*tan(f*x+e))^(1/
2)*B*c+(c-I*c*tan(f*x+e))^(1/2)*c*A-8*c^2*(8*((-81/512*I*B-29/512*A)*(c-I*
c*tan(f*x+e))^(5/2)+(53/96*I*B*c+17/96*c*A)*(c-I*c*tan(f*x+e))^(3/2)+(-63/
128*I*B*c^2-19/128*A*c^2)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3+3
5/16*(1/8*A+5/8*I*B)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*
2^(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(228) = 456$.

Time = 0.11 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.62

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx =$$

$$-\frac{105\sqrt{\frac{1}{2}}\left(a^3fe^{(8ifx+8ie)}+a^3fe^{(6ifx+6ie)}\right)\sqrt{-\frac{(A^2+10iAB-25B^2)c^9}{a^6f^2}}\log\left(-\frac{35\left((iA-5B)c^5+\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3fe^{(2ifx+2ie)}+a^3\right.\right.}{}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="fricas")
```

output
```
-1/24*(105*sqrt(1/2)*(a^3*f*e^(8*I*f*x + 8*I*e) + a^3*f*e^(6*I*f*x + 6*I*e
))*sqrt(-(A^2 + 10*I*A*B - 25*B^2)*c^9/(a^6*f^2))*log(-35/2*((I*A - 5*B)*c
^5 + sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(-(A^2 + 10
*I*A*B - 25*B^2)*c^9/(a^6*f^2))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f
*x - I*e)/(a^3*f)) - 105*sqrt(1/2)*(a^3*f*e^(8*I*f*x + 8*I*e) + a^3*f*e^(6
*I*f*x + 6*I*e))*sqrt(-(A^2 + 10*I*A*B - 25*B^2)*c^9/(a^6*f^2))*log(-35/2*
((I*A - 5*B)*c^5 - sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*s
qrt(-(A^2 + 10*I*A*B - 25*B^2)*c^9/(a^6*f^2))*sqrt(c/(e^(2*I*f*x + 2*I*e)
+ 1)))*e^(-I*f*x - I*e)/(a^3*f)) + sqrt(2)*(105*(-I*A + 5*B)*c^4*e^(8*I*f*
x + 8*I*e) + 140*(-I*A + 5*B)*c^4*e^(6*I*f*x + 6*I*e) + 21*(-I*A + 5*B)*c^
4*e^(4*I*f*x + 4*I*e) + 6*(I*A - 5*B)*c^4*e^(2*I*f*x + 2*I*e) + 8*(-I*A +
B)*c^4)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(a^3*f*e^(8*I*f*x + 8*I*e) + a^
3*f*e^(6*I*f*x + 6*I*e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(9/2)/(a+I*a*tan(f*x+e))**3
,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.91

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{i\left(\frac{105\sqrt{2}(A+5iB)c^{\frac{11}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^3} - \frac{4\left(3\left(-ic\right.\right.}{}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="maxima")
```

output
```
1/48*I*(105*sqrt(2)*(A + 5*I*B)*c^(11/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c
*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^3 -
 4*(3*(-I*c*tan(f*x + e) + c)^(5/2)*(29*A + 81*I*B)*c^6 - 16*(-I*c*tan(f*x
 + e) + c)^(3/2)*(17*A + 53*I*B)*c^7 + 12*sqrt(-I*c*tan(f*x + e) + c)*(19*
A + 63*I*B)*c^8)/((-I*c*tan(f*x + e) + c)^3*a^3 - 6*(-I*c*tan(f*x + e) + c
)^2*a^3*c + 12*(-I*c*tan(f*x + e) + c)*a^3*c^2 - 8*a^3*c^3) + 32*(I*(-I*c*
tan(f*x + e) + c)^(3/2)*B*c^4 + 3*sqrt(-I*c*tan(f*x + e) + c)*(A + 7*I*B)*
c^5)/a^3)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.52

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\frac{A\,c^7\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,19\mathrm{i}}{a^3\,f}-\frac{A\,c^6\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}}{3\,a^3\,f}}{6\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2-12\,c^2\,(c-c\tan(e+f\,x}$$
$$-\frac{63\,B\,c^7\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}-\frac{212\,B\,c^6\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}}{3}+\frac{81\,B\,c^5\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}}{4}}{8\,a^3\,c^3\,f-a^3\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^3+6\,a^3\,c\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2-12\,a^3\,c^2\,f\,(c-c\tan(e+f\,x}$$
$$+\frac{A\,c^4\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,2\mathrm{i}}{a^3\,f}-\frac{14\,B\,c^4\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{a^3\,f}$$
$$-\frac{2\,B\,c^3\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}}{3\,a^3\,f}-\frac{\sqrt{2}\,A\,(-c)^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,35\mathrm{i}}{8\,a^3\,f}$$
$$-\frac{\sqrt{2}\,B\,c^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,1\mathrm{i}}{2\,\sqrt{c}}\right)\,175\mathrm{i}}{8\,a^3\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(9/2))/(a + a*tan(e + f*
x)*1i)^3,x)
```

output
```
((A*c^7*(c - c*tan(e + f*x)*1i)^(1/2)*19i)/(a^3*f) - (A*c^6*(c - c*tan(e +
 f*x)*1i)^(3/2)*68i)/(3*a^3*f) + (A*c^5*(c - c*tan(e + f*x)*1i)^(5/2)*29i)
/(4*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^2 - 12*c^2*(c - c*tan(e + f*x)*1i
) - (c - c*tan(e + f*x)*1i)^3 + 8*c^3) - (63*B*c^7*(c - c*tan(e + f*x)*1i)
^(1/2) - (212*B*c^6*(c - c*tan(e + f*x)*1i)^(3/2))/3 + (81*B*c^5*(c - c*ta
n(e + f*x)*1i)^(5/2))/4)/(8*a^3*c^3*f - a^3*f*(c - c*tan(e + f*x)*1i)^3 +
6*a^3*c*f*(c - c*tan(e + f*x)*1i)^2 - 12*a^3*c^2*f*(c - c*tan(e + f*x)*1i)
) + (A*c^4*(c - c*tan(e + f*x)*1i)^(1/2)*2i)/(a^3*f) - (14*B*c^4*(c - c*ta
n(e + f*x)*1i)^(1/2))/(a^3*f) - (2*B*c^3*(c - c*tan(e + f*x)*1i)^(3/2))/(3
*a^3*f) - (2^(1/2)*A*(-c)^(9/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2
))/(2*(-c)^(1/2)))*35i)/(8*a^3*f) - (2^(1/2)*B*c^(9/2)*atan((2^(1/2)*(c -
c*tan(e + f*x)*1i)^(1/2)*1i)/(2*c^(1/2)))*175i)/(8*a^3*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\sqrt{c}\,c^4\left(2\sqrt{-\tan(fx+e)\,i+1}\,ai-\left(\int \frac{\sqrt{-\tan(fx}}{\tan(fx+e)^3 i+3\tan}\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(sqrt(c)*c**4*(2*sqrt( - tan(e + f*x)*i + 1)*a*i - int((sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(
e + f*x)*i - 1),x)*b*f - 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**
4)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*f*i
 - 8*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i
+ 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*i + 6*int((sqrt( - tan(
e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 -
3*tan(e + f*x)*i - 1),x)*b*f + 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x)**2)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)
*b*f*i + 8*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**3
*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*i - int((sqrt( - tan
(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*
tan(e + f*x)*i - 1),x)*b*f))/(a**3*f)
```

3.780 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 252

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx = -\frac{5(iA-13B)c^{7/2}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{8\sqrt{2}a^3f} + \frac{5(iA-13B)c^3\sqrt{c-ic\tan(e+fx)}}{16a^3f} + \frac{5(iA-13B)c^2(c-ic\tan(e+fx))^{3/2}}{48a^3f(1+i\tan(e+fx))} - \frac{(iA-13B)c(c-ic\tan(e+fx))^{5/2}}{24a^3f(1+i\tan(e+fx))^2} + \frac{(iA-B)(c-ic\tan(e+fx))^{7/2}}{6a^3f(1+i\tan(e+fx))^3}$$

output
```
-5/16*(I*A-13*B)*c^(7/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1
/2))*2^(1/2)/a^3/f+5/16*(I*A-13*B)*c^3*(c-I*c*tan(f*x+e))^(1/2)/a^3/f+5/48
*(I*A-13*B)*c^2*(c-I*c*tan(f*x+e))^(3/2)/a^3/f/(1+I*tan(f*x+e))-1/24*(I*A-
13*B)*c*(c-I*c*tan(f*x+e))^(5/2)/a^3/f/(1+I*tan(f*x+e))^2+1/6*(I*A-B)*(c-I
*c*tan(f*x+e))^(7/2)/a^3/f/(1+I*tan(f*x+e))^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.63 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.64

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx =$$

$$-\frac{c^3\sqrt{c-ic\tan(e+fx)}\left(-5(A+13iB)\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},-\frac{1}{2}i(i+\tan(e+fx))\right)\sec^3(e+fx)(\right.}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2))/(a + I*a*Tan
[e + f*x])^3,x]
```

output
```
-1/6*(c^3*Sqrt[c - I*c*Tan[e + f*x]]*(-5*(A + (13*I)*B)*Hypergeometric2F1[
1/2, 3, 3/2, (-1/2*I)*(I + Tan[e + f*x])]*Sec[e + f*x]^3*(Cos[3*(e + f*x)]
 + I*Sin[3*(e + f*x)]) - 4*(-3*(A + (12*I)*B) + ((-4*I)*A + 61*B)*Tan[e +
f*x] + 3*(A + (10*I)*B)*Tan[e + f*x]^2 - 3*B*Tan[e + f*x]^3)))/(a^3*f*(-I
+ Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.209$, Rules used = {3042, 4071, 27, 87, 51, 51, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{a^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \textbf{87}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{12}(A+13iB)\int \frac{(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)+1)^3}d\tan(e+fx)\right)}{a^3f}$$

$$\Big\downarrow \textbf{51}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{12}(A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{5/2}}{2(1+i\tan(e+fx))^2}-\frac{5}{4}c\int \frac{(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)\right)\right)}{a^3f}$$

$$\Big\downarrow \textbf{51}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{12}(A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{5/2}}{2(1+i\tan(e+fx))^2}-\frac{5}{4}c\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{1+i\tan(e+fx)}-\frac{3}{2}c\int \frac{\sqrt{c-ic\tan(e+fx)}}{i\tan(e+fx)+1}d\tan\right.\right.\right.}{a^3f}$$

$$\Big\downarrow \textbf{60}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{12}(A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{5/2}}{2(1+i\tan(e+fx))^2}-\frac{5}{4}c\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{1+i\tan(e+fx)}-\frac{3}{2}c\left(2c\int \frac{}{(i\tan(e+fx)+1)}\right.\right.\right.\right.}{a^3f}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{12}(A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{5/2}}{2(1+i\tan(e+fx))^2}-\frac{5}{4}c\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{1+i\tan(e+fx)}-\frac{3}{2}c\left(4i\int \frac{1}{2-\frac{c-ic\tan(e+f}{c}}\right.\right.\right.\right.}{a^3f}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{c\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{6c(1+i\tan(e+fx))^3}-\frac{1}{12}(A+13iB)\left(\frac{i(c-ic\tan(e+fx))^{5/2}}{2(1+i\tan(e+fx))^2}-\frac{5}{4}c\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{1+i\tan(e+fx)}-\frac{3}{2}c\left(2i\sqrt{2}\sqrt{c}\text{arctanh}\left(\right.\right.\right.\right.\right.}{a^3f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2))/(a + I*a*Tan[e + f
*x])^3,x]
```

output
```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(7/2))/(6*c*(1 + I*Tan[e + f*x])^3)
- ((A + (13*I)*B)*(((I/2)*(c - I*c*Tan[e + f*x])^(5/2))/(1 + I*Tan[e + f*x
])^2 - (5*c*((I*(c - I*c*Tan[e + f*x])^(3/2))/(1 + I*Tan[e + f*x]) - (3*c*
((2*I)*Sqrt[2]*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c]
)] - (2*I)*Sqrt[c - I*c*Tan[e + f*x]]))/2))/4))/12))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{2ic^3\left(i\sqrt{c-ic\tan(fx+e)}\,B+c\left(\frac{8\left(\frac{47iB}{128}+\frac{11A}{128}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+8\left(-\frac{29}{24}iBc-\frac{5}{24}cA\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+8\left(\frac{33}{32}iB\,c^2+\frac{5}{32}\cdots\right.}{(c+ic\tan(fx+e))^3}\right.\right.}{f\,a^3}$
default	$\frac{2ic^3\left(i\sqrt{c-ic\tan(fx+e)}\,B+c\left(\frac{8\left(\frac{47iB}{128}+\frac{11A}{128}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+8\left(-\frac{29}{24}iBc-\frac{5}{24}cA\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+8\left(\frac{33}{32}iB\,c^2+\frac{5}{32}\cdots\right.}{(c+ic\tan(fx+e))^3}\right.\right.}{f\,a^3}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^3*c^3*(I*(c-I*c*tan(f*x+e))^(1/2)*B+c*(8*((47/128*I*B+11/128*A)*(c
-I*c*tan(f*x+e))^(5/2)+(-29/24*I*B*c-5/24*c*A)*(c-I*c*tan(f*x+e))^(3/2)+(3
3/32*I*B*c^2+5/32*A*c^2)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3-5/
4*(13/8*I*B+1/8*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^
(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(197) = 394$.

Time = 0.09 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.60

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx = -\frac{\left(15\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{(A^2+26iAB-169B^2)c^7}{a^6f^2}}e^{(6ifx+6ie)}\log\left(-\frac{5\left((iA-13B)c^4+\sqrt{2}\sqrt{\frac{1}{2}}(a^3fe^{(2ifx+2ie)}+a^3f)\sqrt{-\frac{(A^2+26iAB-169}{a^6f^2}}\right.}{4a^3f}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="fricas")
```

output
```
-1/48*(15*sqrt(1/2)*a^3*f*sqrt(-(A^2 + 26*I*A*B - 169*B^2)*c^7/(a^6*f^2))*
e^(6*I*f*x + 6*I*e)*log(-5/4*((I*A - 13*B)*c^4 + sqrt(2)*sqrt(1/2)*(a^3*f*
e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(-(A^2 + 26*I*A*B - 169*B^2)*c^7/(a^6*f^2
))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^3*f)) - 15*sqrt(
1/2)*a^3*f*sqrt(-(A^2 + 26*I*A*B - 169*B^2)*c^7/(a^6*f^2))*e^(6*I*f*x + 6*
I*e)*log(-5/4*((I*A - 13*B)*c^4 - sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*
I*e) + a^3*f)*sqrt(-(A^2 + 26*I*A*B - 169*B^2)*c^7/(a^6*f^2))*sqrt(c/(e^(2
*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a^3*f)) + sqrt(2)*(15*(-I*A + 13*
B)*c^3*e^(6*I*f*x + 6*I*e) + 5*(-I*A + 13*B)*c^3*e^(4*I*f*x + 4*I*e) + 2*(
I*A - 13*B)*c^3*e^(2*I*f*x + 2*I*e) + 8*(-I*A + B)*c^3)*sqrt(c/(e^(2*I*f*x
 + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{i\left(\int \frac{Ac^3\sqrt{-ic\tan(e+fx)+c}}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx + \int \left(-\frac{3}{\tan^3}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2)/(a+I*a*tan(f*x+e))**3
,x)
```

output
```
I*(Integral(A*c**3*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**3 - 3*I*tan(
e + f*x)**2 - 3*tan(e + f*x) + I), x) + Integral(-3*A*c**3*sqrt(-I*c*tan(e
 + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*ta
n(e + f*x) + I), x) + Integral(B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
f*x)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x) + In
tegral(-3*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3/(tan(e + f*x)
**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x) + Integral(-3*I*A*c**3
*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x)**3 - 3*I*tan(e + f
*x)**2 - 3*tan(e + f*x) + I), x) + Integral(I*A*c**3*sqrt(-I*c*tan(e + f*x
) + c)*tan(e + f*x)**3/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e +
f*x) + I), x) + Integral(-3*I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f
*x)**2/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x) +
Integral(I*B*c**3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4/(tan(e + f*x
)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.95

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{i\left(\frac{15\sqrt{2}(A+13iB)c^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^3} + \frac{192i\sqrt{-ic}}{}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="maxima")
```

output
```
1/96*I*(15*sqrt(2)*(A + 13*I*B)*c^(9/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*
tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^3 +
192*I*sqrt(-I*c*tan(f*x + e) + c)*B*c^4/a^3 - 4*(3*(-I*c*tan(f*x + e) + c)
^(5/2)*(11*A + 47*I*B)*c^5 - 16*(-I*c*tan(f*x + e) + c)^(3/2)*(5*A + 29*I*
B)*c^6 + 12*sqrt(-I*c*tan(f*x + e) + c)*(5*A + 33*I*B)*c^7)/((-I*c*tan(f*x
 + e) + c)^3*a^3 - 6*(-I*c*tan(f*x + e) + c)^2*a^3*c + 12*(-I*c*tan(f*x +
e) + c)*a^3*c^2 - 8*a^3*c^3))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.53

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\frac{A\,c^6\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,5\mathrm{i}}{2\,a^3\,f}-\frac{A\,c^5\,{(c-c\tan(e+f\,x)\,1\mathrm{i})}^{3/2}}{3\,a^3\,f}}{6\,c\,{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^2-12\,c^2\,\left(c-c\tan\left(e+f\,x\right.\right.}$$
$$-\frac{\frac{33\,B\,c^6\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2}-\frac{58\,B\,c^5\,{(c-c\tan(e+f\,x)\,1\mathrm{i})}^{3/2}}{3}+\frac{47\,B\,c^4\,{(c-c\tan(e+f\,x)\,1\mathrm{i})}^{5/2}}{8}}{8\,a^3\,c^3\,f-a^3\,f\,{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^3+6\,a^3\,c\,f\,{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^2-12\,a^3\,c^2\,f\,\left(c-c\tan\left(e+f\,x\right.\right.}$$
$$-\frac{2\,B\,c^3\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{a^3\,f}+\frac{\sqrt{2}\,A\,{\left(-c\right)}^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,5\mathrm{i}}{16\,a^3\,f}$$
$$+\frac{65\,\sqrt{2}\,B\,c^{7/2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{16\,a^3\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*
x)*1i)^3,x)
```

output
```
((A*c^6*(c - c*tan(e + f*x)*1i)^(1/2)*5i)/(2*a^3*f) - (A*c^5*(c - c*tan(e
+ f*x)*1i)^(3/2)*10i)/(3*a^3*f) + (A*c^4*(c - c*tan(e + f*x)*1i)^(5/2)*11i
)/(8*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^2 - 12*c^2*(c - c*tan(e + f*x)*1
i) - (c - c*tan(e + f*x)*1i)^3 + 8*c^3) - ((33*B*c^6*(c - c*tan(e + f*x)*1
i)^(1/2))/2 - (58*B*c^5*(c - c*tan(e + f*x)*1i)^(3/2))/3 + (47*B*c^4*(c -
c*tan(e + f*x)*1i)^(5/2))/8)/(8*a^3*c^3*f - a^3*f*(c - c*tan(e + f*x)*1i)^
3 + 6*a^3*c*f*(c - c*tan(e + f*x)*1i)^2 - 12*a^3*c^2*f*(c - c*tan(e + f*x)
*1i)) - (2*B*c^3*(c - c*tan(e + f*x)*1i)^(1/2))/(a^3*f) + (2^(1/2)*A*(-c)^
(7/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*5i)/(16
*a^3*f) + (65*2^(1/2)*B*c^(7/2)*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/
2))/(2*c^(1/2))))/(16*a^3*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\sqrt{c}\,c^3\Big(2\sqrt{-\tan(fx+e)\,i+1}\,ai+\Big(\int \frac{\sqrt{-\tan(fx}}{\tan(fx+e)^3 i+3\tan}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(sqrt(c)*c**3*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int((sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(
e + f*x)*i - 1),x)*a*f - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)
/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*f*i -
 5*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i +
3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*i + 3*int((sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*
tan(e + f*x)*i - 1),x)*b*f - 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*
x)**2)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a
*f + 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*
i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*f*i + 7*int((sqrt( - ta
n(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3
*tan(e + f*x)*i - 1),x)*a*f*i - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x))/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*f
))/(a**3*f)
```

3.781 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 213

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx=\frac{(iA+11B)c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{16\sqrt{2}a^3f}-\frac{(iA+11B)c^2\sqrt{c-ic\tan(e+fx)}}{16a^3f(1+i\tan(e+fx))}+\frac{(iA+11B)c(c-ic\tan(e+fx))^{3/2}}{24a^3f(1+i\tan(e+fx))^2}+\frac{(iA-B)(c-ic\tan(e+fx))^{5/2}}{6a^3f(1+i\tan(e+fx))^3}$$

output
```
1/32*(I*A+11*B)*c^(5/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*2^(1/2)/a^3/f-1/16*(I*A+11*B)*c^2*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(1+I*
tan(f*x+e))+1/24*(I*A+11*B)*c*(c-I*c*tan(f*x+e))^(3/2)/a^3/f/(1+I*tan(f*x+
e))^2+1/6*(I*A-B)*(c-I*c*tan(f*x+e))^(5/2)/a^3/f/(1+I*tan(f*x+e))^3
```

Mathematica [A] (verified)

Time = 6.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx = -\frac{c^2\sec^3(e+fx)\left(3\sqrt{2}(A-11iB)\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)(\cos(3(e+fx))+i\sin(3(e+fx)))+2\cos}{96a^3f(-i}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan
[e + f*x])^3,x]
```

output
```
-1/96*(c^2*Sec[e + f*x]^3*(3*Sqrt[2]*(A - (11*I)*B)*Sqrt[c]*ArcTanh[Sqrt[c
 - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f
*x)]) + 2*Cos[e + f*x]*(2*(A - (11*I)*B) + (5*A + (41*I)*B)*Cos[2*(e + f*x
)] + ((-11*I)*A - 25*B)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]))/(a^
3*f*(-I + Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {3042, 4071, 27, 87, 51, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{a^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \; 87$$

$$\frac{c\left(\frac{1}{12}(A-11iB)\int \frac{(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)+1)^3}d\tan(e+fx)+\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

$$\Big\downarrow \; 51$$

$$\frac{c\left(\frac{1}{12}(A-11iB)\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{2(1+i\tan(e+fx))^2}-\frac{3}{4}c\int \frac{\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)+1)^2}d\tan(e+fx)\right)+\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

$$\Big\downarrow \; 51$$

$$\frac{c\left(\frac{1}{12}(A-11iB)\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{2(1+i\tan(e+fx))^2}-\frac{3}{4}c\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{1+i\tan(e+fx)}-\frac{1}{2}c\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)\right)\right)+\right.}{a^3f}$$

$$\Big\downarrow \; 73$$

$$\frac{c\left(\frac{1}{12}(A-11iB)\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{2(1+i\tan(e+fx))^2}-\frac{3}{4}c\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{1+i\tan(e+fx)}-i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}\right)\right)+\frac{(-B+}{6}\right.}{a^3f}$$

$$\Big\downarrow \; 219$$

$$\frac{c\left(\frac{1}{12}(A-11iB)\left(\frac{i(c-ic\tan(e+fx))^{3/2}}{2(1+i\tan(e+fx))^2}-\frac{3}{4}c\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{1+i\tan(e+fx)}-\frac{i\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}}\right)\right)+\frac{(-B+iA)(c-ic\tan(}{6c(1+i\tan(e+}\right.}{a^3f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan[e + f
*x])^3,x]
```

output
```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(5/2))/(6*c*(1 + I*Tan[e + f*x])^3)
+ ((A - (11*I)*B)*(((I/2)*(c - I*c*Tan[e + f*x])^(3/2))/(1 + I*Tan[e + f*x
])^2 - (3*c*(((-I)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqr
t[c])])/Sqrt[2] + (I*Sqrt[c - I*c*Tan[e + f*x]])/(1 + I*Tan[e + f*x])))/4)
)/12))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2ic^3\left(\frac{8\left(\frac{21iB}{256}+\frac{A}{256}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+8\left(-\frac{11}{48}iBc+\frac{1}{48}cA\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+8\left(\frac{11}{64}iB\,c^2-\frac{1}{64}A\,c^2\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^3}+\frac{(-1}{}}{f\,a^3}$
default	$\frac{2ic^3\left(\frac{8\left(\frac{21iB}{256}+\frac{A}{256}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+8\left(-\frac{11}{48}iBc+\frac{1}{48}cA\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+8\left(\frac{11}{64}iB\,c^2-\frac{1}{64}A\,c^2\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^3}+\frac{(-1}{}}{f\,a^3}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^3*c^3*(8*((21/256*I*B+1/256*A)*(c-I*c*tan(f*x+e))^(5/2)+(-11/48*I*
B*c+1/48*c*A)*(c-I*c*tan(f*x+e))^(3/2)+(11/64*I*B*c^2-1/64*A*c^2)*(c-I*c*t
an(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3+1/8*(-11/8*I*B+1/8*A)*2^(1/2)/c^(1/
2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(166) = 332$.

Time = 0.09 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.89

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\left(3\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{(A^2-22i\,AB-121\,B^2)c^5}{a^6f^2}}e^{(6i\,fx+6i\,e)}\log\left(\left(\frac{(i}{}\right.\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="fricas")
```

output
```
1/96*(3*sqrt(1/2)*a^3*f*sqrt(-(A^2 - 22*I*A*B - 121*B^2)*c^5/(a^6*f^2))*e^
(6*I*f*x + 6*I*e)*log(1/8*((I*A + 11*B)*c^3 + sqrt(2)*sqrt(1/2)*(a^3*f*e^(
2*I*f*x + 2*I*e) + a^3*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(A^2 - 2
2*I*A*B - 121*B^2)*c^5/(a^6*f^2)))*e^(-I*f*x - I*e)/(a^3*f)) - 3*sqrt(1/2)
*a^3*f*sqrt(-(A^2 - 22*I*A*B - 121*B^2)*c^5/(a^6*f^2))*e^(6*I*f*x + 6*I*e)
*log(1/8*((I*A + 11*B)*c^3 - sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e)
+ a^3*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(A^2 - 22*I*A*B - 121*B^2
)*c^5/(a^6*f^2)))*e^(-I*f*x - I*e)/(a^3*f)) - sqrt(2)*(3*(I*A + 11*B)*c^2*
e^(6*I*f*x + 6*I*e) - (-I*A - 11*B)*c^2*e^(4*I*f*x + 4*I*e) + 2*(-5*I*A -
7*B)*c^2*e^(2*I*f*x + 2*I*e) + 8*(-I*A + B)*c^2)*sqrt(c/(e^(2*I*f*x + 2*I*
e) + 1)))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{i\left(\int \frac{Ac^2\sqrt{-ic\tan(e+fx)+c}}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx + \int\left(-\frac{}{\tan^3}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**3
,x)
```

output
```
I*(Integral(A*c**2*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**3 - 3*I*tan(
e + f*x)**2 - 3*tan(e + f*x) + I), x) + Integral(-A*c**2*sqrt(-I*c*tan(e +
 f*x) + c)*tan(e + f*x)**2/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(
e + f*x) + I), x) + Integral(B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x) + Inte
gral(-B*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3/(tan(e + f*x)**3
- 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x) + Integral(-2*I*A*c**2*sqr
t(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)*
*2 - 3*tan(e + f*x) + I), x) + Integral(-2*I*B*c**2*sqrt(-I*c*tan(e + f*x)
 + c)*tan(e + f*x)**2/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f
*x) + I), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx =$$

$$-\frac{i\left(\frac{3\sqrt{2}(A-11iB)c^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^3}+\frac{4\left(3(-ic\tan(fx+e)+c)^{\frac{5}{2}}(A+21iB)c^4+16(-ic\tan(fx+e)+c)^{\frac{3}{2}}(A-11iB)c^5-1\right.}{(-ic\tan(fx+e)+c)^3a^3-6(-ic\tan(fx+e)+c)^2a^3c+12(-ic\tan(f}\right.}{192\,cf}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="maxima")
```

output
```
-1/192*I*(3*sqrt(2)*(A - 11*I*B)*c^(7/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c
*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^3 +
 4*(3*(-I*c*tan(f*x + e) + c)^(5/2)*(A + 21*I*B)*c^4 + 16*(-I*c*tan(f*x +
e) + c)^(3/2)*(A - 11*I*B)*c^5 - 12*sqrt(-I*c*tan(f*x + e) + c)*(A - 11*I*
B)*c^6)/((-I*c*tan(f*x + e) + c)^3*a^3 - 6*(-I*c*tan(f*x + e) + c)^2*a^3*c
 + 12*(-I*c*tan(f*x + e) + c)*a^3*c^2 - 8*a^3*c^3))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.69

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{-\frac{A\,c^5\,\sqrt{c-c\tan(e+f\,x)\,1i}\,1i}{4\,a^3\,f}+\frac{A\,c^4\,(c-c\tan(e+f\,x)\,1i)^{3/}}{3\,a^3\,f}}{6\,c\,(c-c\tan\left(e+f\,x\right)\,1i)^2-12\,c^2\,(c-c\tan\left(e+f\,x\right.} - \frac{\frac{11\,B\,c^5\,\sqrt{c-c\tan(e+f\,x)\,1i}}{4}-\frac{11\,B\,c^4\,(c-c\tan(e+f\,x)\,1i)^{3/2}}{3}+\frac{21\,B\,c^3\,(c-c\tan(e+f\,x)\,1i)^{5/2}}{16}}{8\,a^3\,c^3\,f-a^3\,f\,(c-c\tan\left(e+f\,x\right)\,1i)^3+6\,a^3\,c\,f\,(c-c\tan\left(e+f\,x\right)\,1i)^2-12\,a^3\,c^2\,f\,(c-c\tan\left(e+f\,x\right.} + \frac{\sqrt{2}\,A\,(-c)^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1i}}{2\,\sqrt{-c}}\right)\,1i}{32\,a^3\,f} + \frac{11\,\sqrt{2}\,B\,c^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1i}}{2\,\sqrt{c}}\right)}{32\,a^3\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*
x)*1i)^3,x)
```

output
```
((A*c^4*(c - c*tan(e + f*x)*1i)^(3/2)*1i)/(3*a^3*f) - (A*c^5*(c - c*tan(e
+ f*x)*1i)^(1/2)*1i)/(4*a^3*f) + (A*c^3*(c - c*tan(e + f*x)*1i)^(5/2)*1i)/
(16*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^2 - 12*c^2*(c - c*tan(e + f*x)*1i
) - (c - c*tan(e + f*x)*1i)^3 + 8*c^3) - ((11*B*c^5*(c - c*tan(e + f*x)*1i
)^(1/2))/4 - (11*B*c^4*(c - c*tan(e + f*x)*1i)^(3/2))/3 + (21*B*c^3*(c - c
*tan(e + f*x)*1i)^(5/2))/16)/(8*a^3*c^3*f - a^3*f*(c - c*tan(e + f*x)*1i)^
3 + 6*a^3*c*f*(c - c*tan(e + f*x)*1i)^2 - 12*a^3*c^2*f*(c - c*tan(e + f*x)
*1i)) + (2^(1/2)*A*(-c)^(5/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))
/(2*(-c)^(1/2)))*1i)/(32*a^3*f) + (11*2^(1/2)*B*c^(5/2)*atanh((2^(1/2)*(c
- c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(32*a^3*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\sqrt{c}\,c^2\left(2\sqrt{-\tan(fx+e)\,i+1}\,ai+\left(\int \frac{\sqrt{-\tan(fx}}{\tan(fx+e)^3i+3\tan}\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(sqrt(c)*c**2*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int((sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(
e + f*x)*i - 1),x)*a*f - 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**
3)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*i
 + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i +
3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*f - 5*int((sqrt( - tan(e +
f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*ta
n(e + f*x)*i - 1),x)*a*f + 2*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)
**2)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*f
*i + 6*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**3*i +
 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*i - int((sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(
e + f*x)*i - 1),x)*b*f))/(a**3*f)
```

3.782 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 211

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = -\frac{(iA+3B)c^{3/2}\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{32\sqrt{2}a^3f}+\frac{(iA+3B)c\sqrt{c-ic\tan(e+fx)}}{8a^3f(1+i\tan(e+fx))^2} - \frac{(iA+3B)c\sqrt{c-ic\tan(e+fx)}}{32a^3f(1+i\tan(e+fx))}+\frac{(iA-B)(c-ic\tan(e+fx))^{3/2}}{6a^3f(1+i\tan(e+fx))^3}$$

output

```
-1/64*(I*A+3*B)*c^(3/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*2^(1/2)/a^3/f+1/8*(I*A+3*B)*c*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(1+I*tan(
f*x+e))^2-1/32*(I*A+3*B)*c*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(1+I*tan(f*x+e))
+1/6*(I*A-B)*(c-I*c*tan(f*x+e))^(3/2)/a^3/f/(1+I*tan(f*x+e))^3
```

Mathematica [A] (verified)

Time = 5.94 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{c\sec^3(e+fx)\left(3\sqrt{2}(A-3iB)\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+}}{\sqrt{2}\sqrt{c}}\right.\right.}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan
[e + f*x])^3,x]
```

output
```
(c*Sec[e + f*x]^3*(3*Sqrt[2]*(A - (3*I)*B)*Sqrt[c]*ArcTanh[Sqrt[c - I*c*Ta
n[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)]) - 2
*Cos[e + f*x]*(2*(7*A - (5*I)*B) + (11*A - I*B)*Cos[2*(e + f*x)] + ((-5*I)
*A + 17*B)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]))/(192*a^3*f*(-I +
 Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {3042, 4071, 27, 87, 51, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{a^4(i\tan(e+fx)+1)^4}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)+1)^4}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \ 87$$

$$\frac{c\left(\frac{1}{4}(A-3iB)\int \frac{\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)+1)^3}d\tan(e+fx)+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

$$\Big\downarrow \ 51$$

$$\frac{c\left(\frac{1}{4}(A-3iB)\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{2(1+i\tan(e+fx))^2}-\frac{1}{4}c\int \frac{1}{(i\tan(e+fx)+1)^2\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)\right)+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

$$\Big\downarrow \ 52$$

$$\frac{c\left(\frac{1}{4}(A-3iB)\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{2(1+i\tan(e+fx))^2}-\frac{1}{4}c\left(\frac{1}{4}\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)\right)+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

$$\Big\downarrow \ 73$$

$$\frac{c\left(\frac{1}{4}(A-3iB)\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{2(1+i\tan(e+fx))^2}-\frac{1}{4}c\left(\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{2c}+\frac{i\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)\right)+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

$$\Big\downarrow \ 219$$

$$\frac{c\left(\frac{1}{4}(A-3iB)\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{2(1+i\tan(e+fx))^2}-\frac{1}{4}c\left(\frac{i\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\sqrt{c}}+\frac{i\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)\right)+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan[e + f
*x])^3,x]
```

output
```
(c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(3/2))/(6*c*(1 + I*Tan[e + f*x])^3)
+ ((A - (3*I)*B)*(((I/2)*Sqrt[c - I*c*Tan[e + f*x]])/(1 + I*Tan[e + f*x])^
2 - (c*(((I/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqr
t[2]*Sqrt[c]) + ((I/2)*Sqrt[c - I*c*Tan[e + f*x]])/(c*(1 + I*Tan[e + f*x])
)))/4))/4))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{2ic^3\left(\frac{-\frac{(-3iB+A)(c-ic\tan(fx+e))^{\frac{5}{2}}}{64c}+8\left(\frac{iB}{96}+\frac{A}{96}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+\frac{c(-3iB+A)\sqrt{c-ic\tan(fx+e)}}{16}}{(c+ic\tan(fx+e))^3}-\frac{(-3iB+A)\sqrt{2}\,\operatorname{arctanh}(}{12}\right.}{f\,a^3}$
default	$\frac{2ic^3\left(\frac{-\frac{(-3iB+A)(c-ic\tan(fx+e))^{\frac{5}{2}}}{64c}+8\left(\frac{iB}{96}+\frac{A}{96}\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+\frac{c(-3iB+A)\sqrt{c-ic\tan(fx+e)}}{16}}{(c+ic\tan(fx+e))^3}-\frac{(-3iB+A)\sqrt{2}\,\operatorname{arctanh}(}{12}\right.}{f\,a^3}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^3*c^3*(8*(-1/512/c*(A-3*I*B)*(c-I*c*tan(f*x+e))^(5/2)+(1/96*I*B+1/
96*A)*(c-I*c*tan(f*x+e))^(3/2)+1/128*c*(A-3*I*B)*(c-I*c*tan(f*x+e))^(1/2))
/(c+I*c*tan(f*x+e))^3-1/128/c^(3/2)*(A-3*I*B)*2^(1/2)*arctanh(1/2*(c-I*c*t
an(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(164) = 328.

Time = 0.09 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.87

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\left(3\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{(A^2-6i\,AB-9\,B^2)c^3}{a^6f^2}}e^{(6i\,fx+6i\,e)}\log\left(\left(\frac{\left((-i\,A\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="fricas")
```

output
```
1/192*(3*sqrt(1/2)*a^3*f*sqrt(-(A^2 - 6*I*A*B - 9*B^2)*c^3/(a^6*f^2))*e^(6
*I*f*x + 6*I*e)*log(1/16*((-I*A - 3*B)*c^2 + sqrt(2)*sqrt(1/2)*(a^3*f*e^(2
*I*f*x + 2*I*e) + a^3*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(A^2 - 6*
I*A*B - 9*B^2)*c^3/(a^6*f^2)))*e^(-I*f*x - I*e)/(a^3*f)) - 3*sqrt(1/2)*a^3
*f*sqrt(-(A^2 - 6*I*A*B - 9*B^2)*c^3/(a^6*f^2))*e^(6*I*f*x + 6*I*e)*log(1/
16*((-I*A - 3*B)*c^2 - sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*
f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(A^2 - 6*I*A*B - 9*B^2)*c^3/(a^
6*f^2)))*e^(-I*f*x - I*e)/(a^3*f)) - sqrt(2)*(3*(-I*A - 3*B)*c*e^(6*I*f*x
+ 6*I*e) - (17*I*A + 19*B)*c*e^(4*I*f*x + 4*I*e) + 2*(-11*I*A - B)*c*e^(2*
I*f*x + 2*I*e) + 8*(-I*A + B)*c)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-6*
I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{i\left(\int \frac{Ac\sqrt{-ic\tan(e+fx)+c}}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx + \int \frac{Bc\sqrt{}}{\tan^3(e+}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**3
,x)
```

output
```
I*(Integral(A*c*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**3 - 3*I*tan(e +
 f*x)**2 - 3*tan(e + f*x) + I), x) + Integral(B*c*sqrt(-I*c*tan(e + f*x) +
 c)*tan(e + f*x)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) +
 I), x) + Integral(-I*A*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)/(tan(e
+ f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I), x) + Integral(-I*B*
c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**2/(tan(e + f*x)**3 - 3*I*tan(e
 + f*x)**2 - 3*tan(e + f*x) + I), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{i\left(\frac{3\sqrt{2}(A-3iB)c^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^3} + \frac{4\left(3(-ic\tan\right.}{(}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="maxima")
```

output
```
1/384*I*(3*sqrt(2)*(A - 3*I*B)*c^(5/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*t
an(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^3 + 4
*(3*(-I*c*tan(f*x + e) + c)^(5/2)*(A - 3*I*B)*c^3 - 16*(-I*c*tan(f*x + e)
+ c)^(3/2)*(A + I*B)*c^4 - 12*sqrt(-I*c*tan(f*x + e) + c)*(A - 3*I*B)*c^5)
/((-I*c*tan(f*x + e) + c)^3*a^3 - 6*(-I*c*tan(f*x + e) + c)^2*a^3*c + 12*(
-I*c*tan(f*x + e) + c)*a^3*c^2 - 8*a^3*c^3))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.71

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\frac{A\,c^4\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,1\mathrm{i}}{8\,a^3\,f}+\frac{A\,c^3\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}}{6\,a^3\,f}\cdots}{6\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2-12\,c^2\,(c-c\tan(e+f\,x\cdots}$$
$$-\frac{-\frac{3\,B\,c^4\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{8}+\frac{B\,c^3\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}}{6}+\frac{3\,B\,c^2\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}}{32}}{8\,a^3\,c^3\,f-a^3\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^3+6\,a^3\,c\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2-12\,a^3\,c^2\,f\,(c-c\tan(e+f\,x\cdots}$$
$$+\frac{\sqrt{2}\,A\,(-c)^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,1\mathrm{i}}{64\,a^3\,f}-\frac{3\,\sqrt{2}\,B\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{64\,a^3\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*
x)*1i)^3,x)
```

output
```
((A*c^4*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(8*a^3*f) + (A*c^3*(c - c*tan(e
+ f*x)*1i)^(3/2)*1i)/(6*a^3*f) - (A*c^2*(c - c*tan(e + f*x)*1i)^(5/2)*1i)/
(32*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^2 - 12*c^2*(c - c*tan(e + f*x)*1i
) - (c - c*tan(e + f*x)*1i)^3 + 8*c^3) - ((B*c^3*(c - c*tan(e + f*x)*1i)^(
3/2))/6 - (3*B*c^4*(c - c*tan(e + f*x)*1i)^(1/2))/8 + (3*B*c^2*(c - c*tan(
e + f*x)*1i)^(5/2))/32)/(8*a^3*c^3*f - a^3*f*(c - c*tan(e + f*x)*1i)^3 + 6
*a^3*c*f*(c - c*tan(e + f*x)*1i)^2 - 12*a^3*c^2*f*(c - c*tan(e + f*x)*1i))
 + (2^(1/2)*A*(-c)^(3/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(
-c)^(1/2)))*1i)/(64*a^3*f) - (3*2^(1/2)*B*c^(3/2)*atanh((2^(1/2)*(c - c*ta
n(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(64*a^3*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^3}\,dx = \frac{\sqrt{c}\,c\Big(2\sqrt{-\tan(fx+e)\,i+1}\,ai+\Big(\int \frac{\sqrt{-\tan(fx+}}{\tan(fx+e)^3 i+3\tan(}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(sqrt(c)*c*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int((sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e +
 f*x)*i - 1),x)*a*f - 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/
(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*i -
6*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i + 3
*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f + int((sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e
 + f*x)*i - 1),x)*b*f*i + 5*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))
/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*i -
 int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**3*i + 3*tan
(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*f))/(a**3*f)
```

3.783 $\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 209

$$\begin{aligned}\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx &= \frac{(5iA+7B)\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{64\sqrt{2}a^3f}+\frac{(iA-B)\sqrt{c-ic\tan(e+fx)}}{6a^3f(1+i\tan(e+fx))^3}\\ &\quad+\frac{(5iA+7B)\sqrt{c-ic\tan(e+fx)}}{48a^3f(1+i\tan(e+fx))^2}+\frac{(5iA+7B)\sqrt{c-ic\tan(e+fx)}}{64a^3f(1+i\tan(e+fx))}\end{aligned}$$

output
```
1/128*(5*I*A+7*B)*c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(
1/2))*2^(1/2)/a^3/f+1/6*(I*A-B)*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(1+I*tan(f*
x+e))^3+1/48*(5*I*A+7*B)*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(1+I*tan(f*x+e))^2
+1/64*(5*I*A+7*B)*(c-I*c*tan(f*x+e))^(1/2)/a^3/f/(1+I*tan(f*x+e))
```

Mathematica [A] (verified)

Time = 3.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.83

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx =$$
$$-\frac{\sec^3(e+fx)\left(3\sqrt{2}(5A-7iB)\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)(\cos(3(e+fx))+i\sin(3(e+fx)))+2\cos\right.}{384a^3f(-i}$$

input
```
Integrate[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e
 + f*x])^3,x]
```

output
```
-1/384*(Sec[e + f*x]^3*(3*Sqrt[2]*(5*A - (7*I)*B)*Sqrt[c]*ArcTanh[Sqrt[c -
 I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x
)]) + 2*Cos[e + f*x]*(26*A + (2*I)*B + (41*A - (19*I)*B)*Cos[2*(e + f*x)]
+ 5*((5*I)*A + 7*B)*Sin[2*(e + f*x)])*Sqrt[c - I*c*Tan[e + f*x]]))/(a^3*f*
(-I + Tan[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.186$, Rules used = {3042, 4071, 27, 87, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^3}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4(i\tan(e+fx)+1)^4\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^4\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \ 87$$

$$\frac{c\left(\frac{1}{12}(5A-7iB)\int \frac{1}{(i\tan(e+fx)+1)^3\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

$$\Big\downarrow \ 52$$

$$\frac{c\left(\frac{1}{12}(5A-7iB)\left(\frac{3}{8}\int \frac{1}{(i\tan(e+fx)+1)^2\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{c-ic\tan(e+fx)}}{4c(1+i\tan(e+fx))^2}\right)+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{6c(1+i\tan(e+fx))^3}\right)}{a^3f}$$

$$\Big\downarrow \ 52$$

$$\frac{c\left(\frac{1}{12}(5A-7iB)\left(\frac{3}{8}\left(\frac{1}{4}\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)+\frac{i\sqrt{c-ic\tan(e+fx)}}{4c(1+i\tan(e+fx))^2}\right)+\frac{(-\ldots}{\ldots}\right.}{a^3f}$$

$$\Big\downarrow \ 73$$

$$\frac{c\left(\frac{1}{12}(5A-7iB)\left(\frac{3}{8}\left(\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{2c}+\frac{i\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)+\frac{i\sqrt{c-ic\tan(e+fx)}}{4c(1+i\tan(e+fx))^2}\right)+\frac{(-B+iA)\sqrt{c-ic\ldots}}{6c(1+i\tan(e\ldots}\right.}{a^3f}$$

$$\Big\downarrow \ 219$$

$$\frac{c\left(\frac{1}{12}(5A-7iB)\left(\frac{3}{8}\left(\frac{i\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{2\sqrt{2}\sqrt{c}}+\frac{i\sqrt{c-ic\tan(e+fx)}}{2c(1+i\tan(e+fx))}\right)+\frac{i\sqrt{c-ic\tan(e+fx)}}{4c(1+i\tan(e+fx))^2}\right)+\frac{(-B+iA)\sqrt{c-ic\tan(e+f\ldots}}{6c(1+i\tan(e+fx))^3}\right.}{a^3f}$$

input
```
Int[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e + f*x
])^3,x]
```

output
```
(c*(((I*A - B)*Sqrt[c - I*c*Tan[e + f*x]])/(6*c*(1 + I*Tan[e + f*x])^3) +
((5*A - (7*I)*B)*(((I/4)*Sqrt[c - I*c*Tan[e + f*x]])/(c*(1 + I*Tan[e + f*x
])^2) + (3*(((I/2)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/
(Sqrt[2]*Sqrt[c]) + ((I/2)*Sqrt[c - I*c*Tan[e + f*x]])/(c*(1 + I*Tan[e + f
*x]))))/8))/12))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{2ic^3\left(\frac{\frac{(-7iB+5A)(c-ic\tan(fx+e))^{\frac{5}{2}}}{128c^2}-\frac{(-7iB+5A)(c-ic\tan(fx+e))^{\frac{3}{2}}}{24c}+8\left(\frac{11A}{256}-\frac{9iB}{256}\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^3}+\frac{(-7iB+5A)\sqrt{2}\,\mathrm{arcta}}{}\right.}{f\,a^3}$
default	$\frac{2ic^3\left(\frac{\frac{(-7iB+5A)(c-ic\tan(fx+e))^{\frac{5}{2}}}{128c^2}-\frac{(-7iB+5A)(c-ic\tan(fx+e))^{\frac{3}{2}}}{24c}+8\left(\frac{11A}{256}-\frac{9iB}{256}\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^3}+\frac{(-7iB+5A)\sqrt{2}\,\mathrm{arcta}}{}\right.}{f\,a^3}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^3*c^3*(8*(1/1024/c^2*(5*A-7*I*B)*(c-I*c*tan(f*x+e))^(5/2)-1/192/c*
(5*A-7*I*B)*(c-I*c*tan(f*x+e))^(3/2)+(11/256*A-9/256*I*B)*(c-I*c*tan(f*x+e
))^(1/2))/(c+I*c*tan(f*x+e))^3+1/256/c^(5/2)*(5*A-7*I*B)*2^(1/2)*arctanh(1
/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(162) = 324$.

Time = 0.09 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \frac{\left(3\sqrt{\frac{1}{2}}a^3f\sqrt{-\frac{(25A^2-70iAB-49B^2)c}{a^6f^2}}e^{(6ifx+6ie)}\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3fe^{(2ifx+2ie)}+a^3f\right)\sqrt{\frac{c}{e^{(2ifx+2ie)}+1}}\sqrt{-\frac{(25A^2-70iAB-49}{a^6f^2}}\right.}{32a^3f}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="fricas")
```

output
```
1/384*(3*sqrt(1/2)*a^3*f*sqrt(-(25*A^2 - 70*I*A*B - 49*B^2)*c/(a^6*f^2))*e
^(6*I*f*x + 6*I*e)*log(1/32*(sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e)
+ a^3*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(25*A^2 - 70*I*A*B - 49*B
^2)*c/(a^6*f^2)) + (5*I*A + 7*B)*c)*e^(-I*f*x - I*e)/(a^3*f)) - 3*sqrt(1/2
)*a^3*f*sqrt(-(25*A^2 - 70*I*A*B - 49*B^2)*c/(a^6*f^2))*e^(6*I*f*x + 6*I*e
)*log(-1/32*(sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(c/
(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(25*A^2 - 70*I*A*B - 49*B^2)*c/(a^6*f^2))
 - (5*I*A + 7*B)*c)*e^(-I*f*x - I*e)/(a^3*f)) - sqrt(2)*(3*(-11*I*A - 9*B)
*e^(6*I*f*x + 6*I*e) - (59*I*A + 25*B)*e^(4*I*f*x + 4*I*e) + 2*(-17*I*A +
5*B)*e^(2*I*f*x + 2*I*e) - 8*I*A + 8*B)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))
*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \frac{i\left(\int \frac{A\sqrt{-ic\tan(e+fx)+c}}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx + \int \frac{B\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)}{\tan^3(e+fx)-3i\tan^2(e+fx)-3\tan(e+fx)+i}\,dx\right)}{a^3}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**3
,x)
```

output
```
I*(Integral(A*sqrt(-I*c*tan(e + f*x) + c)/(tan(e + f*x)**3 - 3*I*tan(e + f
*x)**2 - 3*tan(e + f*x) + I), x) + Integral(B*sqrt(-I*c*tan(e + f*x) + c)*
tan(e + f*x)/(tan(e + f*x)**3 - 3*I*tan(e + f*x)**2 - 3*tan(e + f*x) + I),
 x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.05

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx =$$

$$-\frac{i\left(\frac{3\sqrt{2}(5A-7iB)c^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{c}-\sqrt{-ic\tan(fx+e)+c}}{\sqrt{2}\sqrt{c}+\sqrt{-ic\tan(fx+e)+c}}\right)}{a^3}+\frac{4\left(3\,(-ic\tan(fx+e)+c)^{\frac{5}{2}}(5A-7iB)c^2-16\,(-ic\tan(fx+e)+c)^{\frac{3}{2}}(5A-7iB)\right.}{(-ic\tan(fx+e)+c)^3a^3-6\,(-ic\tan(fx+e)+c)^2a^3c+12\,(-ic}\right.}{768\,cf}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="maxima")
```

output
```
-1/768*I*(3*sqrt(2)*(5*A - 7*I*B)*c^(3/2)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*
c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/a^3
+ 4*(3*(-I*c*tan(f*x + e) + c)^(5/2)*(5*A - 7*I*B)*c^2 - 16*(-I*c*tan(f*x
+ e) + c)^(3/2)*(5*A - 7*I*B)*c^3 + 12*sqrt(-I*c*tan(f*x + e) + c)*(11*A -
 9*I*B)*c^4)/((-I*c*tan(f*x + e) + c)^3*a^3 - 6*(-I*c*tan(f*x + e) + c)^2*
a^3*c + 12*(-I*c*tan(f*x + e) + c)*a^3*c^2 - 8*a^3*c^3))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeError: Bad Ar
gument Ty
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.70

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \frac{\frac{7\,B\,c\,{\left(c-c\,\tan(e+f\,x)\,1\mathrm{i}\right)}^{5/2}}{64}+\frac{9\,B\,c^3\,\sqrt{c-c\,\tan(e+f\,x)\,1\mathrm{i}}}{16}-\frac{7\,B\,c^2\,{\left(c-c\,\tan(e+f\,x)\,1\mathrm{i}\right)}^{3/2}}{12}}{8\,a^3\,c^3\,f-a^3\,f\,{\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^3+6\,a^3\,c\,f\,{\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^2-12\,a^3\,c^2\,f\,\left(c-c\,\tan\left(e+f\,x\right.\right.}$$
$$+\frac{\frac{A\,c^3\,\sqrt{c-c\,\tan(e+f\,x)\,1\mathrm{i}}\,11\mathrm{i}}{16\,a^3\,f}-\frac{A\,c^2\,{\left(c-c\,\tan(e+f\,x)\,1\mathrm{i}\right)}^{3/2}\,5\mathrm{i}}{12\,a^3\,f}+\frac{A\,c\,{\left(c-c\,\tan(e+f\,x)\,1\mathrm{i}\right)}^{5/2}\,5\mathrm{i}}{64\,a^3\,f}}{6\,c\,{\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^2-12\,c^2\,\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)-{\left(c-c\,\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}^3+8\,c^3}$$
$$+\frac{\sqrt{2}\,A\,\sqrt{-c}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,5\mathrm{i}}{128\,a^3\,f}+\frac{7\,\sqrt{2}\,B\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\,\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{128\,a^3\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(1/2))/(a + a*tan(e + f*
x)*1i)^3,x)
```

output
```
((7*B*c*(c - c*tan(e + f*x)*1i)^(5/2))/64 + (9*B*c^3*(c - c*tan(e + f*x)*1
i)^(1/2))/16 - (7*B*c^2*(c - c*tan(e + f*x)*1i)^(3/2))/12)/(8*a^3*c^3*f -
a^3*f*(c - c*tan(e + f*x)*1i)^3 + 6*a^3*c*f*(c - c*tan(e + f*x)*1i)^2 - 12
*a^3*c^2*f*(c - c*tan(e + f*x)*1i)) + ((A*c^3*(c - c*tan(e + f*x)*1i)^(1/2
)*11i)/(16*a^3*f) - (A*c^2*(c - c*tan(e + f*x)*1i)^(3/2)*5i)/(12*a^3*f) +
(A*c*(c - c*tan(e + f*x)*1i)^(5/2)*5i)/(64*a^3*f))/(6*c*(c - c*tan(e + f*x
)*1i)^2 - 12*c^2*(c - c*tan(e + f*x)*1i) - (c - c*tan(e + f*x)*1i)^3 + 8*c
^3) + (2^(1/2)*A*(-c)^(1/2)*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(
2*(-c)^(1/2)))*5i)/(128*a^3*f) + (7*2^(1/2)*B*c^(1/2)*atanh((2^(1/2)*(c -
c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(128*a^3*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^3}\,dx$$
$$= \frac{\sqrt{c}\left(2\sqrt{-\tan(fx+e)i+1}\,ai+\left(\int \frac{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^4}{\tan(fx+e)^3i+3\tan(fx+e)^2-3\tan(fx+e)i-1}dx\right)af-4\left(\int \frac{\sqrt{-\tan(fx+e)i-}}{\tan(fx+e)^3i+3\tan(fx-}\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^3,x)
```

output
```
(sqrt(c)*(2*sqrt( - tan(e + f*x)*i + 1)*a*i + int((sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f
*x)*i - 1),x)*a*f - 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(t
an(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f*i - 6*
int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i + 3*t
an(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f + 4*int((sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x))/(tan(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e +
f*x)*i - 1),x)*a*f*i - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan
(e + f*x)**3*i + 3*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*b*f))/(a**3*
f)
```

3.784 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 245

$$\begin{aligned}\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx &= \frac{5(7iA+5B)\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{128\sqrt{2}a^3\sqrt{c}f}-\frac{5(7iA+5B)}{128a^3f\sqrt{c-ic\tan(e+fx)}}\\&\quad+\frac{iA-B}{6a^3f(1+i\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\\&\quad+\frac{7iA+5B}{48a^3f(1+i\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\\&\quad+\frac{5(7iA+5B)}{192a^3f(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\end{aligned}$$

output
```
5/256*(7*I*A+5*B)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^
(1/2)/a^3/c^(1/2)/f-5/128*(7*I*A+5*B)/a^3/f/(c-I*c*tan(f*x+e))^(1/2)+1/6*(
I*A-B)/a^3/f/(1+I*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(1/2)+1/48*(7*I*A+5*B)/
a^3/f/(1+I*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(1/2)+5/192*(7*I*A+5*B)/a^3/f/
(1+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.86

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$=\frac{\sec^3(e+fx)\left(2\sqrt{c}((-125A+7iB)\cos(e+fx)+8(5A-7iB)\cos(3(e+fx))-i(7A-5iB)(7\sin(e+\right.}{768a^3\sqrt{c}f(-i}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*Sqrt[c - I*c*Tan[
e + f*x]]),x]
```

output
```
(Sec[e + f*x]^3*(2*Sqrt[c]*((-125*A + (7*I)*B)*Cos[e + f*x] + 8*(5*A - (7*
I)*B)*Cos[3*(e + f*x)] - I*(7*A - (5*I)*B)*(7*Sin[e + f*x] - 8*Sin[3*(e +
f*x)])) - 15*Sqrt[2]*(7*A - (5*I)*B)*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(S
qrt[2]*Sqrt[c])]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)])*Sqrt[c - I*c*Tan[
e + f*x]]))/(768*a^3*Sqrt[c]*f*(-I + Tan[e + f*x])^3*Sqrt[c - I*c*Tan[e +
f*x]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {3042, 4071, 27, 87, 52, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}dx$$
$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4(i\tan(e+fx)+1)^4(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^4(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \ 87$$

$$\frac{c\left(\frac{1}{12}(7A-5iB)\int \frac{1}{(i\tan(e+fx)+1)^3(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)+\frac{-B+iA}{6c(1+i\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\right)}{a^3f}$$

$$\Big\downarrow \ 52$$

$$\frac{c\left(\frac{1}{12}(7A-5iB)\left(\frac{5}{8}\int \frac{1}{(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)+\frac{i}{4c(1+i\tan(e+fx))^2\sqrt{c-ic\tan(e+fx)}}\right)+\frac{}{6c(1+i\tan}\right.}{a^3f}$$

$$\Big\downarrow \ 52$$

$$\frac{c\left(\frac{1}{12}(7A-5iB)\left(\frac{5}{8}\left(\frac{3}{4}\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)+\frac{i}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)+\frac{}{4c(1+i\tan}\right.\right.}{a^3f}$$

$$\Big\downarrow \ 61$$

$$\frac{c\left(\frac{1}{12}(7A-5iB)\left(\frac{5}{8}\left(\frac{3}{4}\left(\frac{\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{2c}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}\right)+\frac{i}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+f}}\right.\right.\right.}{a^3f}$$

$$\Big\downarrow \ 73$$

$$\frac{c\left(\frac{1}{12}(7A-5iB)\left(\frac{5}{8}\left(\frac{3}{4}\left(\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}\right)+\frac{i}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)\right.\right.}{a^3f}$$

$$\Big\downarrow \ 219$$

$$\frac{c\left(\frac{1}{12}(7A-5iB)\left(\frac{5}{8}\left(\frac{3}{4}\left(\frac{i\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}c^{3/2}}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}\right)+\frac{i}{2c(1+i\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}\right)+\frac{}{4c(1}\right.\right.}{a^3f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*Sqrt[c - I*c*Tan[e + f*
x]]),x]
```

output
```
(c*((I*A - B)/(6*c*(1 + I*Tan[e + f*x])^3*Sqrt[c - I*c*Tan[e + f*x]]) + ((
7*A - (5*I)*B)*((I/4)/(c*(1 + I*Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]]
) + (5*((I/2)/(c*(1 + I*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]]) + (3*((I
*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[2]*c^(3/2))
- I/(c*Sqrt[c - I*c*Tan[e + f*x]])))/4))/8))/12))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2ic^3\left(-\frac{-iB+A}{16c^3\sqrt{c-ic\tan(fx+e)}}+\frac{\frac{8\left(-\frac{9iB}{128}+\frac{19A}{128}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+8\left(\frac{7}{24}iBc-\frac{17}{24}cA\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+8\left(-\frac{7}{32}iBc^2+\frac{29}{32}A \cdots}{(c+ic\tan(fx+e))^3}}{16c^3}\cdots}{f a^3}$
default	$\frac{2ic^3\left(-\frac{-iB+A}{16c^3\sqrt{c-ic\tan(fx+e)}}+\frac{\frac{8\left(-\frac{9iB}{128}+\frac{19A}{128}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+8\left(\frac{7}{24}iBc-\frac{17}{24}cA\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+8\left(-\frac{7}{32}iBc^2+\frac{29}{32}A \cdots}{(c+ic\tan(fx+e))^3}}{16c^3}\cdots}{f a^3}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^3*c^3*(-1/16/c^3*(A-I*B)/(c-I*c*tan(f*x+e))^(1/2)+1/16/c^3*(8*((-9
/128*I*B+19/128*A)*(c-I*c*tan(f*x+e))^(5/2)+(7/24*I*B*c-17/24*c*A)*(c-I*c*
tan(f*x+e))^(3/2)+(-7/32*I*B*c^2+29/32*A*c^2)*(c-I*c*tan(f*x+e))^(1/2))/(c
+I*c*tan(f*x+e))^3+5/4*(-5/8*I*B+7/8*A)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c
*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(190) = 380$.

Time = 0.12 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.67

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{\left(15\sqrt{\frac{1}{2}}a^3cf\sqrt{-\frac{49\,A^2-70i\,AB-25\,B^2}{a^6cf^2}}e^{(6i\,fx+6i\,e)}\log\left(\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3fe^{(2i\,fx+2i\,e)}+a^3f\right)\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{-\frac{49\,A^2-70i\,AB-25\,\ldots}{a^6cf^2}}\right.}{64\,a^3f}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
1/768*(15*sqrt(1/2)*a^3*c*f*sqrt(-(49*A^2 - 70*I*A*B - 25*B^2)/(a^6*c*f^2)
)*e^(6*I*f*x + 6*I*e)*log(5/64*(sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*
e) + a^3*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(49*A^2 - 70*I*A*B - 2
5*B^2)/(a^6*c*f^2)) + 7*I*A + 5*B)*e^(-I*f*x - I*e)/(a^3*f)) - 15*sqrt(1/2
)*a^3*c*f*sqrt(-(49*A^2 - 70*I*A*B - 25*B^2)/(a^6*c*f^2))*e^(6*I*f*x + 6*I
*e)*log(-5/64*(sqrt(2)*sqrt(1/2)*(a^3*f*e^(2*I*f*x + 2*I*e) + a^3*f)*sqrt(
c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(49*A^2 - 70*I*A*B - 25*B^2)/(a^6*c*f^2
)) - 7*I*A - 5*B)*e^(-I*f*x - I*e)/(a^3*f)) - sqrt(2)*(48*(I*A + B)*e^(8*I
*f*x + 8*I*e) + 3*(-13*I*A + 9*B)*e^(6*I*f*x + 6*I*e) - (125*I*A + 7*B)*e^
(4*I*f*x + 4*I*e) + 2*(-23*I*A + 11*B)*e^(2*I*f*x + 2*I*e) - 8*I*A + 8*B)*
sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)/(a^3*c*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{i\left(\int \frac{A}{\sqrt{-ic\tan(e+fx)+c}\tan^3(e+fx)-3i\sqrt{-ic\tan(e+fx)+c}\tan^2(e+fx)-3\sqrt{-ic\tan(e+fx)+c}\tan(e+fx)+i\sqrt{-ic\tan(e+fx)+c}}\,dx + \right.}{a^3}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**(1/2)
,x)
```

output
```
I*(Integral(A/(sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - 3*I*sqrt(-I*c
*tan(e + f*x) + c)*tan(e + f*x)**2 - 3*sqrt(-I*c*tan(e + f*x) + c)*tan(e +
 f*x) + I*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(B*tan(e + f*x)/(sqrt
(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - 3*I*sqrt(-I*c*tan(e + f*x) + c)*
tan(e + f*x)**2 - 3*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + I*sqrt(-I*c
*tan(e + f*x) + c)), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{i\left(\frac{4\left(15(-ic\tan(fx+e)+c)^3(7A-5iB)c-80(-ic\tan(fx+e)+c)^2(7A-5iB)c^2+132(-ic\tan(fx+e)+c)(7A-5iB)c^3-384(A-iB)c^4\right)}{(-ic\tan(fx+e)+c)^{\frac{7}{2}}a^3-6(-ic\tan(fx+e)+c)^{\frac{5}{2}}a^3c+12(-ic\tan(fx+e)+c)^{\frac{3}{2}}a^3c^2-8\sqrt{-ic\tan(fx+e)+c}a^3c^3}\right.}{1536cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
-1/1536*I*(4*(15*(-I*c*tan(f*x + e) + c)^3*(7*A - 5*I*B)*c - 80*(-I*c*tan(
f*x + e) + c)^2*(7*A - 5*I*B)*c^2 + 132*(-I*c*tan(f*x + e) + c)*(7*A - 5*I
*B)*c^3 - 384*(A - I*B)*c^4)/((-I*c*tan(f*x + e) + c)^(7/2)*a^3 - 6*(-I*c*
tan(f*x + e) + c)^(5/2)*a^3*c + 12*(-I*c*tan(f*x + e) + c)^(3/2)*a^3*c^2 -
 8*sqrt(-I*c*tan(f*x + e) + c)*a^3*c^3) + 15*sqrt(2)*(7*A - 5*I*B)*sqrt(c)
*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c))/(sqrt(2)*sqrt(c) + s
qrt(-I*c*tan(f*x + e) + c)))/a^3)/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.61

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$= \frac{B\,c^3-\frac{25\,B\,(c-c\tan(e+f\,x)\,1\mathrm{i})^3}{128}+\frac{25\,B\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2}{24}-\frac{55\,B\,c^2\,(c-c\tan(e+f\,x)\,1}{32}}{a^3\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{7/2}-6\,a^3\,c\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}-8\,a^3\,c^3\,f\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}+1}$$
$$+\frac{\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^3\,35\mathrm{i}}{128\,a^3\,f}-\frac{A\,c^3\,1\mathrm{i}}{a^3\,f}-\frac{A\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2\,35\mathrm{i}}{24\,a^3\,f}+\frac{A\,c^2\,(c-c\tan(e+f\,x)\,1\mathrm{i})\,77\mathrm{i}}{32\,a^3\,f}}{6\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}-(c-c\tan(e+f\,x)\,1\mathrm{i})^{7/2}+8\,c^3\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}-12\,c^2\,(c-c}$$
$$-\frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,35\mathrm{i}}{256\,a^3\,\sqrt{-c}\,f}+\frac{25\,\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{256\,a^3\,\sqrt{c}\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i
)^(1/2)),x)
```

output
```
(B*c^3 - (25*B*(c - c*tan(e + f*x)*1i)^3)/128 + (25*B*c*(c - c*tan(e + f*x
)*1i)^2)/24 - (55*B*c^2*(c - c*tan(e + f*x)*1i))/32)/(a^3*f*(c - c*tan(e +
 f*x)*1i)^(7/2) - 6*a^3*c*f*(c - c*tan(e + f*x)*1i)^(5/2) - 8*a^3*c^3*f*(c
 - c*tan(e + f*x)*1i)^(1/2) + 12*a^3*c^2*f*(c - c*tan(e + f*x)*1i)^(3/2))
+ ((A*(c - c*tan(e + f*x)*1i)^3*35i)/(128*a^3*f) - (A*c^3*1i)/(a^3*f) - (A
*c*(c - c*tan(e + f*x)*1i)^2*35i)/(24*a^3*f) + (A*c^2*(c - c*tan(e + f*x)*
1i)*77i)/(32*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^(5/2) - (c - c*tan(e + f
*x)*1i)^(7/2) + 8*c^3*(c - c*tan(e + f*x)*1i)^(1/2) - 12*c^2*(c - c*tan(e
+ f*x)*1i)^(3/2)) - (2^(1/2)*A*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2)
)/(2*(-c)^(1/2)))*35i)/(256*a^3*(-c)^(1/2)*f) + (25*2^(1/2)*B*atanh((2^(1/
2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(256*a^3*c^(1/2)*f)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3\sqrt{c-ic\tan(e+fx)}}\,dx = \text{too large to display}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(1/2),x)
```

output

```
(sqrt(c)*(12*sqrt( - tan(e + f*x)*i + 1)*a*i + 2*sqrt( - tan(e + f*x)*i +
1)*b + int(( - sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**5*i + 3*tan(e +
 f*x)**4 - 2*tan(e + f*x)**3*i + 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1)
,x)*tan(e + f*x)**2*b*f*i + int(( - sqrt( - tan(e + f*x)*i + 1))/(tan(e +
f*x)**5*i + 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i + 2*tan(e + f*x)**2 -
3*tan(e + f*x)*i - 1),x)*b*f*i - 3*int(( - sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)**4)/(tan(e + f*x)**5*i + 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i
 + 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f*i - 3*
int(( - sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**5*i +
3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i + 2*tan(e + f*x)**2 - 3*tan(e + f*
x)*i - 1),x)*b*f*i + 6*int(( - sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2
)/(tan(e + f*x)**5*i + 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i + 2*tan(e +
 f*x)**2 - 3*tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f*i + 6*int(( - sqrt
( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**5*i + 3*tan(e + f*
x)**4 - 2*tan(e + f*x)**3*i + 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)
*b*f*i - 18*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x
)**5*i + 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i + 2*tan(e + f*x)**2 - 3*t
an(e + f*x)*i - 1),x)*tan(e + f*x)**2*a*f - 18*int((sqrt( - tan(e + f*x)*i
 + 1)*tan(e + f*x)**4)/(tan(e + f*x)**5*i + 3*tan(e + f*x)**4 - 2*tan(e +
f*x)**3*i + 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i - 1),x)*a*f + 48*int((...
```

3.785 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 274

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{35(3iA+B)\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{256\sqrt{2}a^3c^{3/2}f} - \frac{35(3iA+B)}{384a^3f(c-ic\tan(e+fx))^{3/2}} + \frac{iA-B}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}} + \frac{3iA+B}{16a^3f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}} + \frac{7(3iA+B)}{64a^3f(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}} - \frac{35(3iA+B)}{256a^3cf\sqrt{c-ic\tan(e+fx)}}$$

output
```
35/512*(3*I*A+B)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(
1/2)/a^3/c^(3/2)/f-35/384*(3*I*A+B)/a^3/f/(c-I*c*tan(f*x+e))^(3/2)+1/6*(I*
A-B)/a^3/f/(1+I*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2)+1/16*(3*I*A+B)/a^3/
f/(1+I*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(3/2)+7/64*(3*I*A+B)/a^3/f/(1+I*ta
n(f*x+e))/(c-I*c*tan(f*x+e))^(3/2)-35/256*(3*I*A+B)/a^3/c/f/(c-I*c*tan(f*x
+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.50 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.70

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\sec^4(e+fx)\left(105(3iA+B)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e +
f*x])^(3/2)),x]
```

output
```
(Sec[e + f*x]^4*(105*((3*I)*A + B)*Hypergeometric2F1[-1/2, 1, 1/2, (-1/2*I
)*(I + Tan[e + f*x])]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]) + 2*Cos[e +
f*x]*(3*((-55*I)*A + 3*B)*Cos[e + f*x] + 8*(I*A + 3*B)*Cos[3*(e + f*x)] +
(3*A - I*B)*(27*Sin[e + f*x] - 8*Sin[3*(e + f*x)]))))/(768*a^3*c*f*(-I + T
an[e + f*x])^3*(I + Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4071, 27, 87, 52, 52, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4(i\tan(e+fx)+1)^4(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^4(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \; 87$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\int \frac{1}{(i\tan(e+fx)+1)^3(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)+\frac{-B+iA}{6c(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\right)}{a^3f}$$

$$\Big\downarrow \; 52$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\left(\frac{7}{8}\int \frac{1}{(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)+\frac{i}{4c(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{3/2}}\right)+\frac{}{6c(1+i\tan}\right.}{a^3f}$$

$$\Big\downarrow \; 52$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\left(\frac{7}{8}\left(\frac{5}{4}\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)+\frac{i}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}\right)+\frac{}{4c(1+i\tan}\right.\right.}{a^3f}$$

$$\Big\downarrow \; 61$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\left(\frac{7}{8}\left(\frac{5}{4}\left(\frac{\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{i}{2c(1+i\tan(e+fx))(c-ic\tan(e+}\right.\right.\right.}{a^3f}$$

$$\Big\downarrow \; 61$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\left(\frac{7}{8}\left(\frac{5}{4}\left(\frac{\frac{\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{2c}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{}{2c(1+i\tan(e+}\right.\right.\right.}{a^3f}$$

$$\Big\downarrow \; 73$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\left(\frac{7}{8}\left(\frac{5}{4}\left(\frac{\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{i}{2c(1+i\tan(e+fx))(}\right.\right.\right.}{a^3f}$$

$$\Big\downarrow \; 219$$

$$\frac{c\left(\frac{1}{4}(3A-iB)\left(\frac{7}{8}\left(\frac{5}{4}\left(\frac{\frac{i\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}c^{3/2}}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}\right)+\frac{i}{2c(1+i\tan(e+fx))(c-ic\tan\ldots}\right.\right.\right.}{a^3f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^
(3/2)),x]
```

output
```
(c*((I*A - B)/(6*c*(1 + I*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(3/2)) +
((3*A - I*B)*((I/4)/(c*(1 + I*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(3/2)
) + (7*((I/2)/(c*(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2)) + (5*(
(-1/3*I)/(c*(c - I*c*Tan[e + f*x])^(3/2)) + ((I*ArcTanh[Sqrt[c - I*c*Tan[e
 + f*x]]/(Sqrt[2]*Sqrt[c])])/(Sqrt[2]*c^(3/2)) - I/(c*Sqrt[c - I*c*Tan[e +
 f*x]]))/(2*c)))/4))/8))/4))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{2ic^3\left(-\frac{-iB+2A}{16c^4\sqrt{c-ic\tan(fx+e)}}-\frac{-iB+A}{48c^3(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{8\left(-\frac{3iB}{256}+\frac{41A}{256}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+8\left(\frac{1}{48}iBc-\frac{35}{48}cA\right)(c-ic\tan(fx-}{(c+ic\tan(fx+e))^3}\right.}{f\,a^3}$
default	$\frac{2ic^3\left(-\frac{-iB+2A}{16c^4\sqrt{c-ic\tan(fx+e)}}-\frac{-iB+A}{48c^3(c-ic\tan(fx+e))^{\frac{3}{2}}}+\frac{8\left(-\frac{3iB}{256}+\frac{41A}{256}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+8\left(\frac{1}{48}iBc-\frac{35}{48}cA\right)(c-ic\tan(fx-}{(c+ic\tan(fx+e))^3}\right.}{f\,a^3}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^3*c^3*(-1/16/c^4*(2*A-I*B)/(c-I*c*tan(f*x+e))^(1/2)-1/48/c^3*(A-I*
B)/(c-I*c*tan(f*x+e))^(3/2)+1/16/c^4*(8*((-3/256*I*B+41/256*A)*(c-I*c*tan(
f*x+e))^(5/2)+(1/48*I*B*c-35/48*c*A)*(c-I*c*tan(f*x+e))^(3/2)+(3/64*I*B*c^
2+55/64*A*c^2)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3+35/8*(3/8*A-
1/8*I*B)*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1
/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.60

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\left(105\sqrt{\frac{1}{2}}a^3c^2f\sqrt{-\frac{9A^2-6iAB-B^2}{a^6c^3f^2}}e^{(6ifx+6ie)}\log\left(\frac{35\left(\sqrt{\right.}}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
1/1536*(105*sqrt(1/2)*a^3*c^2*f*sqrt(-(9*A^2 - 6*I*A*B - B^2)/(a^6*c^3*f^2
))*e^(6*I*f*x + 6*I*e)*log(35/128*(sqrt(2)*sqrt(1/2)*(a^3*c*f*e^(2*I*f*x +
 2*I*e) + a^3*c*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(9*A^2 - 6*I*A*
B - B^2)/(a^6*c^3*f^2)) + 3*I*A + B)*e^(-I*f*x - I*e)/(a^3*c*f)) - 105*sqr
t(1/2)*a^3*c^2*f*sqrt(-(9*A^2 - 6*I*A*B - B^2)/(a^6*c^3*f^2))*e^(6*I*f*x +
 6*I*e)*log(-35/128*(sqrt(2)*sqrt(1/2)*(a^3*c*f*e^(2*I*f*x + 2*I*e) + a^3*
c*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(9*A^2 - 6*I*A*B - B^2)/(a^6*
c^3*f^2)) - 3*I*A - B)*e^(-I*f*x - I*e)/(a^3*c*f)) - sqrt(2)*(16*(I*A + B)
*e^(10*I*f*x + 10*I*e) + 32*(7*I*A + 4*B)*e^(8*I*f*x + 8*I*e) - (-43*I*A -
 121*B)*e^(6*I*f*x + 6*I*e) + 5*(-43*I*A + 7*B)*e^(4*I*f*x + 4*I*e) + 2*(-
29*I*A + 17*B)*e^(2*I*f*x + 2*I*e) - 8*I*A + 8*B)*sqrt(c/(e^(2*I*f*x + 2*I
*e) + 1)))*e^(-6*I*f*x - 6*I*e)/(a^3*c^2*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{i\left(\int \frac{}{-ic\sqrt{-ic\tan(e+fx)+c}\tan^4(e+fx)-2c\sqrt{-ic\tan(e+fx)+c}\tan^3}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**(3/2)
,x)
```

output
```
I*(Integral(A/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4 - 2*c*sqrt
(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 - 2*c*sqrt(-I*c*tan(e + f*x) + c)*
tan(e + f*x) + I*c*sqrt(-I*c*tan(e + f*x) + c)), x) + Integral(B*tan(e + f
*x)/(-I*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4 - 2*c*sqrt(-I*c*tan(
e + f*x) + c)*tan(e + f*x)**3 - 2*c*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*
x) + I*c*sqrt(-I*c*tan(e + f*x) + c)), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.96

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{i\left(\frac{4\left(105\,(-i\,c\tan(fx+e)+c)^4(3\,A-i\,B)-560\,(-i\,c\tan(fx+e)+c)^3(3\,A-i\,B)c+924\,(-i\,c\tan(fx+e)+c)^2(3\,A-i\,B)c^2-384\,(-i\,c\tan(fx+e)+\cdots\right.}{(-i\,c\tan(fx+e)+c)^{\frac{9}{2}}a^3-6\,(-i\,c\tan(fx+e)+c)^{\frac{7}{2}}a^3c+12\,(-i\,c\tan(fx+e)+c)^{\frac{5}{2}}a^3c^2-8\,(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}a\cdots}\right.}{3072\,cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
-1/3072*I*(4*(105*(-I*c*tan(f*x + e) + c)^4*(3*A - I*B) - 560*(-I*c*tan(f*
x + e) + c)^3*(3*A - I*B)*c + 924*(-I*c*tan(f*x + e) + c)^2*(3*A - I*B)*c^
2 - 384*(-I*c*tan(f*x + e) + c)*(3*A - I*B)*c^3 - 256*(A - I*B)*c^4)/((-I*
c*tan(f*x + e) + c)^(9/2)*a^3 - 6*(-I*c*tan(f*x + e) + c)^(7/2)*a^3*c + 12
*(-I*c*tan(f*x + e) + c)^(5/2)*a^3*c^2 - 8*(-I*c*tan(f*x + e) + c)^(3/2)*a
^3*c^3) + 105*sqrt(2)*(3*A - I*B)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*
x + e) + c))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/(a^3*sqrt(c)
))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.31 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.62

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^3\,35\mathrm{i}}{16\,a^3\,f}+\frac{A\,c^3\,1\mathrm{i}}{3\,a^3\,f}-\frac{A\,(c-c\tan(e+f\,x)\,1\mathrm{i})^4\,105\mathrm{i}}{256\,a^3\,c\,f}-\frac{A\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2\,231\mathrm{i}}{64\,a^3\,f}+\frac{A\,c^2\,(c-c\tan(e+f}{6\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{7/2}-(c-c\tan(e+f\,x)\,1\mathrm{i})^{9/2}+8\,c^3\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}-12\,c^2\,(c-c\,\mathrm{t}} + \frac{\frac{B\,c^3}{3}+\frac{35\,B\,(c-c\tan(e+f\,x)\,1\mathrm{i})^3}{48}-\frac{77\,B\,c\,(c-c\tan(e+f\,x)\,1\mathrm{i})^2}{64}+\frac{B\,c^2\,(c-c\tan(e+f\,x)\,1\mathrm{i})}{2}-\frac{35\,B\,(c-c}{a^3\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{9/2}-6\,a^3\,c\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{7/2}-8\,a^3\,c^3\,f\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}+} + \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-c}}\right)\,105\mathrm{i}}{512\,a^3\,(-c)^{3/2}\,f} + \frac{35\,\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{c}}\right)}{512\,a^3\,c^{3/2}\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i
)^(3/2)),x)
```

output
```
((B*c^3)/3 + (35*B*(c - c*tan(e + f*x)*1i)^3)/48 - (77*B*c*(c - c*tan(e +
f*x)*1i)^2)/64 + (B*c^2*(c - c*tan(e + f*x)*1i))/2 - (35*B*(c - c*tan(e +
f*x)*1i)^4)/(256*c))/(a^3*f*(c - c*tan(e + f*x)*1i)^(9/2) - 6*a^3*c*f*(c -
 c*tan(e + f*x)*1i)^(7/2) - 8*a^3*c^3*f*(c - c*tan(e + f*x)*1i)^(3/2) + 12
*a^3*c^2*f*(c - c*tan(e + f*x)*1i)^(5/2)) - ((A*(c - c*tan(e + f*x)*1i)^3*
35i)/(16*a^3*f) + (A*c^3*1i)/(3*a^3*f) - (A*(c - c*tan(e + f*x)*1i)^4*105i
)/(256*a^3*c*f) - (A*c*(c - c*tan(e + f*x)*1i)^2*231i)/(64*a^3*f) + (A*c^2
*(c - c*tan(e + f*x)*1i)*3i)/(2*a^3*f))/(6*c*(c - c*tan(e + f*x)*1i)^(7/2)
 - (c - c*tan(e + f*x)*1i)^(9/2) + 8*c^3*(c - c*tan(e + f*x)*1i)^(3/2) - 1
2*c^2*(c - c*tan(e + f*x)*1i)^(5/2)) + (2^(1/2)*A*atan((2^(1/2)*(c - c*tan
(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*105i)/(512*a^3*(-c)^(3/2)*f) + (35*2^
(1/2)*B*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*c^(1/2))))/(512*a
^3*c^(3/2)*f)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{-\left(\int \frac{\tan(fx+}{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^4-2\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e}\right.}{}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
( - (int(tan(e + f*x)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4 - 2*sqr
t( - tan(e + f*x)*i + 1)*tan(e + f*x)**3*i - 2*sqrt( - tan(e + f*x)*i + 1)
*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*b + int(1/(sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**4 - 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x
)**3*i - 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*
x)*i + 1)),x)*a))/(sqrt(c)*a**3*c)
```

3.786 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 311

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{21(11iA+B)\operatorname{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{512\sqrt{2}a^3c^{5/2}f} - \frac{21(11iA+B)}{640a^3f(c-ic\tan(e+fx))^{5/2}} + \frac{iA-B}{6a^3f(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}} + \frac{11iA+B}{48a^3f(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}} + \frac{3(11iA+B)}{64a^3f(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5/2}} - \frac{7(11iA+B)}{256a^3cf(c-ic\tan(e+fx))^{3/2}} - \frac{21(11iA+B)}{512a^3c^2f\sqrt{c-ic\tan(e+fx)}}$$

output
```
21/1024*(11*I*A+B)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)/a^3/c^(5/2)/f-21/640*(11*I*A+B)/a^3/f/(c-I*c*tan(f*x+e))^(5/2)+1/6*
(I*A-B)/a^3/f/(1+I*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2)+1/48*(11*I*A+B)/
a^3/f/(1+I*tan(f*x+e))^2/(c-I*c*tan(f*x+e))^(5/2)+3/64*(11*I*A+B)/a^3/f/(1
+I*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2)-7/256*(11*I*A+B)/a^3/c/f/(c-I*c*ta
n(f*x+e))^(3/2)-21/512*(11*I*A+B)/a^3/c^2/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.79 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.60

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\sec^3(e+fx)\left(30(71A+11iB)\cos(e+fx)-16(A-\right.}{}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e +
f*x])^(5/2)),x]
```

output
```
(Sec[e + f*x]^3*(30*(71*A + (11*I)*B)*Cos[e + f*x] - 16*(A - (11*I)*B)*Cos
[3*(e + f*x)] - 105*(11*A - I*B)*Hypergeometric2F1[-3/2, 1, -1/2, (-1/2*I)
*(I + Tan[e + f*x])]*Sec[e + f*x]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])
+ 2*((11*I)*A + B)*(55*Sin[e + f*x] - 8*Sin[3*(e + f*x)])))/(3840*a^3*c^2*
f*(-I + Tan[e + f*x])^3*(I + Tan[e + f*x])^2*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {3042, 4071, 27, 87, 52, 52, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{a^4(i\tan(e+fx)+1)^4(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)+1)^4(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{a^3f}$$

$$\Big\downarrow \; 87$$

$$\frac{c\left(\frac{1}{12}(11A-iB)\int \frac{1}{(i\tan(e+fx)+1)^3(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)+\frac{-B+iA}{6c(1+i\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\right)}{a^3f}$$

$$\Big\downarrow \; 52$$

$$\frac{c\left(\frac{1}{12}(11A-iB)\left(\frac{9}{8}\int \frac{1}{(i\tan(e+fx)+1)^2(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)+\frac{i}{4c(1+i\tan(e+fx))^2(c-ic\tan(e+fx))^{5/2}}\right)+\frac{}{6c(1+i}\right.}{a^3f}$$

$$\Big\downarrow \; 52$$

$$\frac{c\left(\frac{1}{12}(11A-iB)\left(\frac{9}{8}\left(\frac{7}{4}\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)+\frac{i}{2c(1+i\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}\right)+\frac{}{4c(1+}\right.\right.}{a^3f}$$

$$\Big\downarrow \; 61$$

$$\frac{c\left(\frac{1}{12}(11A-iB)\left(\frac{9}{8}\left(\frac{7}{4}\left(\frac{\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{i}{2c(1+i\tan(e+fx))(c-ic\tan}\right.\right.\right.}{a^3f}$$

$$\Big\downarrow \; 61$$

$$\frac{c\left(\frac{1}{12}(11A-iB)\left(\frac{9}{8}\left(\frac{7}{4}\left(\frac{\frac{\int \frac{1}{(i\tan(e+fx)+1)(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{}{2c(1}\right.\right.\right.}{a^3f}$$

$$\Big\downarrow \; 61$$

$$c\left(\frac{1}{12}(11A-iB)\left(\frac{9}{8}\left(\frac{7}{4}\left(\frac{\frac{\frac{\int \frac{1}{(i\tan(e+fx)+1)\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{2c}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx}\right.\right.\right.\right.$$

$$\Big\downarrow \; 73$$

$$\frac{c\left(\frac{1}{12}(11A-iB)\left(\frac{9}{8}\left(\frac{7}{4}\left(\frac{\frac{\frac{i\int \frac{1}{2-\frac{c-ic\tan(e+fx)}{c}}d\sqrt{c-ic\tan(e+fx)}}{c^2}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right.\right.\right.\right.}{a}$$

$$\Big\downarrow \; 219$$

$$\frac{c\left(\frac{1}{12}(11A-iB)\left(\frac{9}{8}\left(\frac{7}{4}\left(\frac{\frac{\frac{i\,\text{arctanh}\left(\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}c^{3/2}}-\frac{i}{c\sqrt{c-ic\tan(e+fx)}}}{2c}-\frac{i}{3c(c-ic\tan(e+fx))^{3/2}}}{2c}-\frac{i}{5c(c-ic\tan(e+fx))^{5/2}}\right)+\frac{}{2}\right.\right.\right.}{a^3 f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^
(5/2)),x]
```

output
```
(c*((I*A - B)/(6*c*(1 + I*Tan[e + f*x])^3*(c - I*c*Tan[e + f*x])^(5/2)) +
((11*A - I*B)*((I/4)/(c*(1 + I*Tan[e + f*x])^2*(c - I*c*Tan[e + f*x])^(5/2
)) + (9*((I/2)/(c*(1 + I*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2)) + (7*
((-1/5*I)/(c*(c - I*c*Tan[e + f*x])^(5/2)) + ((-1/3*I)/(c*(c - I*c*Tan[e +
 f*x])^(3/2)) + ((I*ArcTanh[Sqrt[c - I*c*Tan[e + f*x]]/(Sqrt[2]*Sqrt[c])])
/(Sqrt[2]*c^(3/2)) - I/(c*Sqrt[c - I*c*Tan[e + f*x]]))/(2*c))/(2*c)))/4))/
8))/12))/(a^3*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{2ic^3\left(\frac{\frac{8\left(\frac{11iB}{256}+\frac{71A}{256}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+8\left(-\frac{59}{48}cA-\frac{11}{48}iBc\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+8\left(\frac{21}{64}iBc^2+\frac{89}{64}Ac^2\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^3}+\frac{21(}{}}{32c^5}\right.}{f}$
default	$\frac{2ic^3\left(\frac{\frac{8\left(\frac{11iB}{256}+\frac{71A}{256}\right)(c-ic\tan(fx+e))^{\frac{5}{2}}+8\left(-\frac{59}{48}cA-\frac{11}{48}iBc\right)(c-ic\tan(fx+e))^{\frac{3}{2}}+8\left(\frac{21}{64}iBc^2+\frac{89}{64}Ac^2\right)\sqrt{c-ic\tan(fx+e)}}{(c+ic\tan(fx+e))^3}+\frac{21(}{}}{32c^5}\right.}{f}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2*I/f/a^3*c^3*(1/32/c^5*(8*((11/256*I*B+71/256*A)*(c-I*c*tan(f*x+e))^(5/2)
+(-59/48*c*A-11/48*I*B*c)*(c-I*c*tan(f*x+e))^(3/2)+(21/64*I*B*c^2+89/64*A*
c^2)*(c-I*c*tan(f*x+e))^(1/2))/(c+I*c*tan(f*x+e))^3+21/8*(-1/8*I*B+11/8*A)
*2^(1/2)/c^(1/2)*arctanh(1/2*(c-I*c*tan(f*x+e))^(1/2)*2^(1/2)/c^(1/2)))-1/
32/c^5*(5*A-I*B)/(c-I*c*tan(f*x+e))^(1/2)-1/48/c^4*(2*A-I*B)/(c-I*c*tan(f*
x+e))^(3/2)-1/80/c^3*(A-I*B)/(c-I*c*tan(f*x+e))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.49

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\left(315\sqrt{\frac{1}{2}}a^3c^3f\sqrt{-\frac{121A^2-22iAB-B^2}{a^6c^5f^2}}e^{(6ifx+6ie)}\log\left(\frac{21}{-}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
1/15360*(315*sqrt(1/2)*a^3*c^3*f*sqrt(-(121*A^2 - 22*I*A*B - B^2)/(a^6*c^5
*f^2))*e^(6*I*f*x + 6*I*e)*log(21/256*(sqrt(2)*sqrt(1/2)*(a^3*c^2*f*e^(2*I
*f*x + 2*I*e) + a^3*c^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(121*A^
2 - 22*I*A*B - B^2)/(a^6*c^5*f^2)) + 11*I*A + B)*e^(-I*f*x - I*e)/(a^3*c^2
*f)) - 315*sqrt(1/2)*a^3*c^3*f*sqrt(-(121*A^2 - 22*I*A*B - B^2)/(a^6*c^5*f
^2))*e^(6*I*f*x + 6*I*e)*log(-21/256*(sqrt(2)*sqrt(1/2)*(a^3*c^2*f*e^(2*I*
f*x + 2*I*e) + a^3*c^2*f)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-(121*A^2
 - 22*I*A*B - B^2)/(a^6*c^5*f^2)) - 11*I*A - B)*e^(-I*f*x - I*e)/(a^3*c^2*
f)) - sqrt(2)*(48*(I*A + B)*e^(12*I*f*x + 12*I*e) + 16*(29*I*A + 19*B)*e^(
10*I*f*x + 10*I*e) + 16*(199*I*A + 59*B)*e^(8*I*f*x + 8*I*e) - (-1433*I*A
- 1003*B)*e^(6*I*f*x + 6*I*e) + 5*(-329*I*A + 101*B)*e^(4*I*f*x + 4*I*e) +
 10*(-35*I*A + 23*B)*e^(2*I*f*x + 2*I*e) - 40*I*A + 40*B)*sqrt(c/(e^(2*I*f
*x + 2*I*e) + 1)))*e^(-6*I*f*x - 6*I*e)/(a^3*c^3*f)
```

Sympy [F]

$$\int \frac{A + B\tan(e + fx)}{(a + ia\tan(e + fx))^3 (c - ic\tan(e + fx))^{5/2}}\,dx = \frac{i\left(\int \frac{}{-c^2\sqrt{-ic\tan(e+fx)+c}\tan^5(e+fx)+ic^2\sqrt{-ic\tan(e+fx)+c}\tan\cdots}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**3/(c-I*c*tan(f*x+e))**(5/2)
,x)
```

output
```
I*(Integral(A/(-c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**5 + I*c**2*
sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**4 - 2*c**2*sqrt(-I*c*tan(e + f*x
) + c)*tan(e + f*x)**3 + 2*I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)
**2 - c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x) + I*c**2*sqrt(-I*c*tan
(e + f*x) + c)), x) + Integral(B*tan(e + f*x)/(-c**2*sqrt(-I*c*tan(e + f*x
) + c)*tan(e + f*x)**5 + I*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**
4 - 2*c**2*sqrt(-I*c*tan(e + f*x) + c)*tan(e + f*x)**3 + 2*I*c**2*sqrt(-I*
c*tan(e + f*x) + c)*tan(e + f*x)**2 - c**2*sqrt(-I*c*tan(e + f*x) + c)*tan
(e + f*x) + I*c**2*sqrt(-I*c*tan(e + f*x) + c)), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.94

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{i\left(\frac{4\left(315\,(-i\,c\tan(fx+e)+c)^5(11\,A-i\,B)-1680\,(-i\,c\tan(fx+e)+c)^4(11\,A-i\,B)c+2772\,(-i\,c\tan(fx+e)+c)^3(11\,A-i\,B)c^2-1152\,(-i\,c\tan(f}{(-i\,c\tan(fx+e)+c)^{\frac{11}{2}}a^3c-6\,(-i\,c\tan(fx+e)+c)^{\frac{9}{2}}a^3c^2+12\,(-i\,c\tan(fx+e)+c)^{\frac{7}{2}}a^3c^3}\right.}{3072}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
-1/30720*I*(4*(315*(-I*c*tan(f*x + e) + c)^5*(11*A - I*B) - 1680*(-I*c*tan
(f*x + e) + c)^4*(11*A - I*B)*c + 2772*(-I*c*tan(f*x + e) + c)^3*(11*A - I
*B)*c^2 - 1152*(-I*c*tan(f*x + e) + c)^2*(11*A - I*B)*c^3 - 256*(-I*c*tan(
f*x + e) + c)*(11*A - I*B)*c^4 - 1536*(A - I*B)*c^5)/((-I*c*tan(f*x + e) +
 c)^(11/2)*a^3*c - 6*(-I*c*tan(f*x + e) + c)^(9/2)*a^3*c^2 + 12*(-I*c*tan(
f*x + e) + c)^(7/2)*a^3*c^3 - 8*(-I*c*tan(f*x + e) + c)^(5/2)*a^3*c^4) + 3
15*sqrt(2)*(11*A - I*B)*log(-(sqrt(2)*sqrt(c) - sqrt(-I*c*tan(f*x + e) + c
))/(sqrt(2)*sqrt(c) + sqrt(-I*c*tan(f*x + e) + c)))/(a^3*c^(3/2)))/(c*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.58

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^3*(c - c*tan(e + f*x)*1i
)^(5/2)),x)
```

output
```
((B*c^3)/5 - (231*B*(c - c*tan(e + f*x)*1i)^3)/640 + (3*B*c*(c - c*tan(e +
 f*x)*1i)^2)/20 + (B*c^2*(c - c*tan(e + f*x)*1i))/30 + (7*B*(c - c*tan(e +
 f*x)*1i)^4)/(32*c) - (21*B*(c - c*tan(e + f*x)*1i)^5)/(512*c^2))/(a^3*f*(
c - c*tan(e + f*x)*1i)^(11/2) - 6*a^3*c*f*(c - c*tan(e + f*x)*1i)^(9/2) -
8*a^3*c^3*f*(c - c*tan(e + f*x)*1i)^(5/2) + 12*a^3*c^2*f*(c - c*tan(e + f*
x)*1i)^(7/2)) - ((A*c^3*1i)/(5*a^3*f) - (A*(c - c*tan(e + f*x)*1i)^3*2541i
)/(640*a^3*f) + (A*(c - c*tan(e + f*x)*1i)^4*77i)/(32*a^3*c*f) - (A*(c - c
*tan(e + f*x)*1i)^5*231i)/(512*a^3*c^2*f) + (A*c*(c - c*tan(e + f*x)*1i)^2
*33i)/(20*a^3*f) + (A*c^2*(c - c*tan(e + f*x)*1i)*11i)/(30*a^3*f))/(6*c*(c
 - c*tan(e + f*x)*1i)^(9/2) - (c - c*tan(e + f*x)*1i)^(11/2) + 8*c^3*(c -
c*tan(e + f*x)*1i)^(5/2) - 12*c^2*(c - c*tan(e + f*x)*1i)^(7/2)) - (2^(1/2
)*A*atan((2^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(2*(-c)^(1/2)))*231i)/(10
24*a^3*(-c)^(5/2)*f) + (21*2^(1/2)*B*atanh((2^(1/2)*(c - c*tan(e + f*x)*1i
)^(1/2))/(2*c^(1/2))))/(1024*a^3*c^(5/2)*f)
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^3(c-ic\tan(e+fx))^{5/2}}\,dx = \text{too large to display}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^3/(c-I*c*tan(f*x+e))^(5/2),x)
```

output

```
(sqrt(c)*(18*sqrt( - tan(e + f*x)*i + 1)*a*i - 2*sqrt( - tan(e + f*x)*i +
1)*b - int(( - sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**7*i + tan(e + f
*x)**6 + 3*tan(e + f*x)**5*i + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**3*i + 3
*tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**6*a*f + int(( - sq
rt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**7*i + tan(e + f*x)**6 + 3*tan(e
+ f*x)**5*i + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**3*i + 3*tan(e + f*x)**2
+ tan(e + f*x)*i + 1),x)*tan(e + f*x)**6*b*f*i - 3*int(( - sqrt( - tan(e +
 f*x)*i + 1))/(tan(e + f*x)**7*i + tan(e + f*x)**6 + 3*tan(e + f*x)**5*i +
 3*tan(e + f*x)**4 + 3*tan(e + f*x)**3*i + 3*tan(e + f*x)**2 + tan(e + f*x
)*i + 1),x)*tan(e + f*x)**4*a*f + 3*int(( - sqrt( - tan(e + f*x)*i + 1))/(
tan(e + f*x)**7*i + tan(e + f*x)**6 + 3*tan(e + f*x)**5*i + 3*tan(e + f*x)
**4 + 3*tan(e + f*x)**3*i + 3*tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan
(e + f*x)**4*b*f*i - 3*int(( - sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)*
*7*i + tan(e + f*x)**6 + 3*tan(e + f*x)**5*i + 3*tan(e + f*x)**4 + 3*tan(e
 + f*x)**3*i + 3*tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*
a*f + 3*int(( - sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**7*i + tan(e +
f*x)**6 + 3*tan(e + f*x)**5*i + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**3*i +
3*tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i - int(( -
 sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**7*i + tan(e + f*x)**6 + 3*tan
(e + f*x)**5*i + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**3*i + 3*tan(e + f*...
```

3.787 $\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 272

$$\begin{aligned}
&\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \\
&-\frac{5\sqrt{a}(4iA-3B)c^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{4f} \\
&-\frac{5(4iA-3B)c^3\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{8f} \\
&-\frac{5(4iA-3B)c^2\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{24f} \\
&-\frac{(4iA-3B)c\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}{12f} \\
&+\frac{B\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{7/2}}{4f}
\end{aligned}$$

output
```
-5/4*a^(1/2)*(4*I*A-3*B)*c^(7/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a
^(1/2)/(c-I*c*tan(f*x+e))^(1/2))/f-5/8*(4*I*A-3*B)*c^3*(a+I*a*tan(f*x+e))^
(1/2)*(c-I*c*tan(f*x+e))^(1/2)/f-5/24*(4*I*A-3*B)*c^2*(a+I*a*tan(f*x+e))^(
1/2)*(c-I*c*tan(f*x+e))^(3/2)/f-1/12*(4*I*A-3*B)*c*(a+I*a*tan(f*x+e))^(1/2
)*(c-I*c*tan(f*x+e))^(5/2)/f+1/4*B*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x
+e))^(7/2)/f
```

Mathematica [A] (verified)

Time = 5.58 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.63

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{30\sqrt{a}(-4iA+3B)c^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)+c^3\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+}$$

input
```
Integrate[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*(c - I*c*Tan[e +
 f*x])^(7/2),x]
```

output
```
(30*Sqrt[a]*((-4*I)*A + 3*B)*c^(7/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e +
f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + c^3*Sqrt[a + I*a*Tan[e + f*
x]]*Sqrt[c - I*c*Tan[e + f*x]]*((-88*I)*A + 72*B - 9*(4*A + (5*I)*B)*Tan[e
 + f*x] + (8*I)*(A + (3*I)*B)*Tan[e + f*x]^2 + (6*I)*B*Tan[e + f*x]^3))/(2
4*f)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 90, 60, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \ 4071$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 90$$

$$\frac{ac\left(\frac{1}{4}(4A+3iB)\int \frac{(c-ic\tan(e+fx))^{5/2}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)+\frac{B\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{7/2}}{4ac}\right)}{f}$$

$$\Big\downarrow \ 60$$

$$\frac{ac\left(\frac{1}{4}(4A+3iB)\left(\frac{5}{3}c\int \frac{(c-ic\tan(e+fx))^{3/2}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}{3a}\right)+\frac{B\sqrt{a+ia\tan(e+fx)}(c-}{4ac}\right.}{f}$$

$$\Big\downarrow \ 60$$

$$\frac{ac\left(\frac{1}{4}(4A+3iB)\left(\frac{5}{3}c\left(\frac{3}{2}c\int \frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{2a}\right)-\frac{i\sqrt{a+ia\tan(e+fx)}(c}{3a}\right.\right.}{f}$$

$$\Big\downarrow \ 60$$

$$\frac{ac\left(\frac{1}{4}(4A+3iB)\left(\frac{5}{3}c\left(\frac{3}{2}c\left(c\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)-\frac{i\sqrt{}}{}\right.\right.\right.}{f}$$

$$\Big\downarrow \ 45$$

$$\frac{ac\left(\frac{1}{4}(4A+3iB)\left(\frac{5}{3}c\left(\frac{3}{2}c\left(2c\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)-\frac{i\sqrt{a+ia\,\mathrm{ta}}}{}\right.\right.\right.}{f}$$

$$\Big\downarrow \ 218$$

$$\frac{ac\left(\frac{1}{4}(4A+3iB)\left(\frac{5}{3}c\left(\frac{3}{2}c\left(-\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)-\frac{i\sqrt{a+ia\tan(e+fx)}(}{2}\right.\right.\right.}{f}$$

input
```
Int[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])
^(7/2),x]
```

output
```
(a*c*((B*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(7/2))/(4*a*c)
+ ((4*A + (3*I)*B)*(((-1/3*I)*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e +
f*x])^(5/2))/a + (5*c*(((-1/2*I)*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e
 + f*x])^(3/2))/a + (3*c*(((-2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan
[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[a] - (I*Sqrt[a + I
*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/a))/2))/3))/4))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^3\left(6iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+8iA\sqrt{ac}\,\sqrt{ac\left(1+\tan(f\right.\right.}}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^3\left(6iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+8iA\sqrt{ac}\,\sqrt{ac\left(1+\tan(f\right.\right.}}{}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^3\left(2i\tan(fx+e)^2\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}-22i\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx\right.\right.}}{6f\sqrt{ac\left(1+\tan(fx+e)^2\right.}}$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/24/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*c^3*(6*I*B*(
a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3+8*I*A*(a*c)^(1/2)*(a*
c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+45*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/
2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-45*I*B*(a*c)^(1/2)*(a*c*
(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-24*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))
^(1/2)*tan(f*x+e)^2-88*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)+60*A*l
n((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a
*c-36*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+72*B*(a*c)^(1/
2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(206) = 412$.

Time = 0.10 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.22

$$\int \sqrt{a + ia\tan(e + fx)}(A + B\tan(e + fx))(c - ic\tan(e + fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="fricas")
```

output
```
1/48*(15*sqrt((16*A^2 + 24*I*A*B - 9*B^2)*a*c^7/f^2)*(f*e^(6*I*f*x + 6*I*e
) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*((4*I*
A - 3*B)*c^3*e^(3*I*f*x + 3*I*e) + (4*I*A - 3*B)*c^3*e^(I*f*x + I*e))*sqrt
(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((16
*A^2 + 24*I*A*B - 9*B^2)*a*c^7/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((4*I*A -
 3*B)*c^3*e^(2*I*f*x + 2*I*e) + (4*I*A - 3*B)*c^3)) - 15*sqrt((16*A^2 + 24
*I*A*B - 9*B^2)*a*c^7/f^2)*(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e
) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*((4*I*A - 3*B)*c^3*e^(3*I*f*x +
3*I*e) + (4*I*A - 3*B)*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) +
1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((16*A^2 + 24*I*A*B - 9*B^2)*a
*c^7/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((4*I*A - 3*B)*c^3*e^(2*I*f*x + 2*I
*e) + (4*I*A - 3*B)*c^3)) - 4*(15*(4*I*A - 3*B)*c^3*e^(7*I*f*x + 7*I*e) +
55*(4*I*A - 3*B)*c^3*e^(5*I*f*x + 5*I*e) + 73*(4*I*A - 3*B)*c^3*e^(3*I*f*x
 + 3*I*e) + 3*(44*I*A - 49*B)*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(6*I*f*x + 6*I*e) + 3*f
*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1343 vs. 2(206) = 412.

Time = 0.77 (sec) , antiderivative size = 1343, normalized size of antiderivative = 4.94

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="maxima")
```

output
```
-96*(60*(4*A + 3*I*B)*c^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 220*(4*A + 3*I*B)*c^3*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 292*(4*A + 3*I*B)*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 12*(44*A + 49*I*B)*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 60*(4*I*A - 3*B)*c^3*sin(7/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 220*(4*I*A - 3*B)*c^3*sin(5/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 292*(4*I*A - 3*B)*c^3*sin(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 12*(44*I*A - 49*B)*c^3*sin(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 30*((4*A + 3*I*B)*c^3*cos(8*f*x + 8*e) + 4*(4
*A + 3*I*B)*c^3*cos(6*f*x + 6*e) + 6*(4*A + 3*I*B)*c^3*cos(4*f*x + 4*e) +
4*(4*A + 3*I*B)*c^3*cos(2*f*x + 2*e) + (4*I*A - 3*B)*c^3*sin(8*f*x + 8*e)
+ 4*(4*I*A - 3*B)*c^3*sin(6*f*x + 6*e) + 6*(4*I*A - 3*B)*c^3*sin(4*f*x + 4
*e) + 4*(4*I*A - 3*B)*c^3*sin(2*f*x + 2*e) + (4*A + 3*I*B)*c^3)*arctan2(co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 30*((4*A + 3*I*B)*c^3*cos(8*f*x + 8*
e) + 4*(4*A + 3*I*B)*c^3*cos(6*f*x + 6*e) + 6*(4*A + 3*I*B)*c^3*cos(4*f*x
+ 4*e) + 4*(4*A + 3*I*B)*c^3*cos(2*f*x + 2*e) + (4*I*A - 3*B)*c^3*sin(8*f*
x + 8*e) + 4*(4*I*A - 3*B)*c^3*sin(6*f*x + 6*e) + 6*(4*I*A - 3*B)*c^3*sin(
4*f*x + 4*e) + 4*(4*I*A - 3*B)*c^3*sin(2*f*x + 2*e) + (4*A + 3*I*B)*c^3)*a
rctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*a...
```

Giac [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \int (B\tan(fx+e)+A)\sqrt{i\,a\tan(fx+e)+a}(-i\,c\tan(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*sqrt(I*a*tan(f*x + e) + a)*(-I*c*tan(f*x +
e) + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+iа\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx=\int (A+B\tan(e+f\,x))\,\sqrt{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{7/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)
*1i)^(7/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)
*1i)^(7/2), x)
```

Reduce [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c^3\left(\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+1}\,\tan\left(fx+e\right)^2ai-3\sqrt{\tan\left(fx+e\right)i+\cdots}\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**3*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)**2*a*i - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2*b - 11*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*a*i + 9*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b + 3*
int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4,x
)*b*f*i - 9*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e
 + f*x)**2,x)*a*f - 9*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2,x)*b*f*i + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan
(e + f*x)*i + 1),x)*a*f))/(3*f)
```

3.788 $\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 217

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = -\frac{\sqrt{a}(3iA-2B)c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f} - \frac{(3iA-2B)c^2\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2f} - \frac{(3iA-2B)c\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{6f} + \frac{B\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}{3f}$$

output

```
-a^(1/2)*(3*I*A-2*B)*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/
2)/(c-I*c*tan(f*x+e))^(1/2))/f-1/2*(3*I*A-2*B)*c^2*(a+I*a*tan(f*x+e))^(1/2
)*(c-I*c*tan(f*x+e))^(1/2)/f-1/6*(3*I*A-2*B)*c*(a+I*a*tan(f*x+e))^(1/2)*(c
-I*c*tan(f*x+e))^(3/2)/f+1/3*B*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))
^(5/2)/f
```

Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.69

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \frac{6\sqrt{a}(-3iA+2B)c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)-c^2\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+}{6f}$$

input
```
Integrate[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*(c - I*c*Tan[e +
 f*x])^(5/2),x]
```

output
```
(6*Sqrt[a]*((-3*I)*A + 2*B)*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f
*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] - c^2*Sqrt[a + I*a*Tan[e + f*x
]]*Sqrt[c - I*c*Tan[e + f*x]]*((12*I)*A - 10*B + 3*(A + (2*I)*B)*Tan[e + f
*x] + 2*B*Tan[e + f*x]^2))/(6*f)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 90, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{90}$$

$$\frac{ac\left(\frac{1}{3}(3A+2iB)\int \frac{(c-ic\tan(e+fx))^{3/2}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)+\frac{B\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}{3ac}\right)}{f}$$

$$\Big\downarrow \textbf{60}$$

$$\frac{ac\left(\frac{1}{3}(3A+2iB)\left(\frac{3}{2}c\int \frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{2a}\right)+\frac{B\sqrt{a+ia\tan(e+fx)}(c-ic}{3ac}\right.}{f}$$

$$\Big\downarrow \textbf{60}$$

$$\frac{ac\left(\frac{1}{3}(3A+2iB)\left(\frac{3}{2}c\left(c\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)-\frac{i\sqrt{a+ia}}{}\right.\right.}{f}$$

$$\Big\downarrow \textbf{45}$$

$$\frac{ac\left(\frac{1}{3}(3A+2iB)\left(\frac{3}{2}c\left(2c\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)-\frac{i\sqrt{a+ia\tan(e+}}{}\right.\right.}{f}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{ac\left(\frac{1}{3}(3A+2iB)\left(\frac{3}{2}c\left(-\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)-\frac{i\sqrt{a+ia\tan(e+fx)}(c-ic\tan}{2a}\right.\right.}{f}$$

input
```
Int[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])
^(5/2),x]
```

output
```
(a*c*((B*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(5/2))/(3*a*c)
+ ((3*A + (2*I)*B)*(((-1/2*I)*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e +
f*x])^(3/2))/a + (3*c*(((-2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e
+ f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[a] - (I*Sqrt[a + I*a*
Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/a))/2))/3))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.31

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^2\left(-6iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac+6iB\sqrt{ac}\,\sqrt{ac}\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^2\left(-6iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac+6iB\sqrt{ac}\,\sqrt{ac}\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^2\left(3\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac-4i\sqrt{ac}\,\sqrt{ac\left(1+\tan\right.}\right.}{2f\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}}$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/6/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*c^2*(-6*I*B*
ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*
a*c+6*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+2*B*(a*c)^(1
/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+12*I*A*(a*c)^(1/2)*(a*c*(1+t
an(f*x+e)^2))^(1/2)-9*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^
2))^(1/2))/(a*c)^(1/2))*a*c+3*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*t
an(f*x+e)-10*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2)/(a*c*
(1+tan(f*x+e)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(165) = 330.

Time = 0.11 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.52

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt((9*A^2 + 12*I*A*B - 4*B^2)*a*c^5/f^2)*(f*e^(4*I*f*x + 4*I*e)
+ 2*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((3*I*A - 2*B)*c^2*e^(3*I*f*x + 3
*I*e) + (3*I*A - 2*B)*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1
))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((9*A^2 + 12*I*A*B - 4*B^2)*a*c
^5/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-3*I*A + 2*B)*c^2*e^(2*I*f*x + 2*I*
e) + (-3*I*A + 2*B)*c^2)) - 3*sqrt((9*A^2 + 12*I*A*B - 4*B^2)*a*c^5/f^2)*(
f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((3*I*A - 2
*B)*c^2*e^(3*I*f*x + 3*I*e) + (3*I*A - 2*B)*c^2*e^(I*f*x + I*e))*sqrt(a/(e
^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((9*A^2 +
 12*I*A*B - 4*B^2)*a*c^5/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-3*I*A + 2*B)
*c^2*e^(2*I*f*x + 2*I*e) + (-3*I*A + 2*B)*c^2)) - 4*(3*(3*I*A - 2*B)*c^2*e
^(5*I*f*x + 5*I*e) + 8*(3*I*A - 2*B)*c^2*e^(3*I*f*x + 3*I*e) + 3*(5*I*A -
6*B)*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I
*f*x + 2*I*e) + 1)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\int\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}(-ic(\tan\left(e+fx\right)+i))^{\frac{5}{2}}\left(A+B\tan\left(e+fx\right)\right)\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Integral(sqrt(I*a*(tan(e + f*x) - I))*(-I*c*(tan(e + f*x) + I))**(5/2)*(A
+ B*tan(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(165) = 330$.

Time = 0.40 (sec) , antiderivative size = 1085, normalized size of antiderivative = 5.00

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
-6*(12*(3*A + 2*I*B)*c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 32*(3*A + 2*I*B)*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 12*(5*A + 6*I*B)*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 12*(3*I*A - 2*B)*c^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) + 32*(3*I*A - 2*B)*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 12*(5*I*A - 6*B)*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 6*((3*A + 2*I*B)*c^2*cos(6*f*x + 6*e) + 3*(3*A + 2*I*B)*c^
2*cos(4*f*x + 4*e) + 3*(3*A + 2*I*B)*c^2*cos(2*f*x + 2*e) + (3*I*A - 2*B)*
c^2*sin(6*f*x + 6*e) + 3*(3*I*A - 2*B)*c^2*sin(4*f*x + 4*e) + 3*(3*I*A - 2
*B)*c^2*sin(2*f*x + 2*e) + (3*A + 2*I*B)*c^2)*arctan2(cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 1) + 6*((3*A + 2*I*B)*c^2*cos(6*f*x + 6*e) + 3*(3*A + 2*I*B
)*c^2*cos(4*f*x + 4*e) + 3*(3*A + 2*I*B)*c^2*cos(2*f*x + 2*e) + (3*I*A - 2
*B)*c^2*sin(6*f*x + 6*e) + 3*(3*I*A - 2*B)*c^2*sin(4*f*x + 4*e) + 3*(3*I*A
 - 2*B)*c^2*sin(2*f*x + 2*e) + (3*A + 2*I*B)*c^2)*arctan2(cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 1) + 3*((3*I*A - 2*B)*c^2*cos(6*f*x + 6*e) + 3*(3*I*A
- 2*B)*c^2*cos(4*f*x + 4*e) + 3*(3*I*A - 2*B)*c^2*cos(2*f*x + 2*e) - (3*A
+ 2*I*B)*c^2*sin(6*f*x + 6*e) - 3*(3*A + 2*I*B)*c^2*sin(4*f*x + 4*e) - 3*(
3*A + 2*I*B)*c^2*sin(2*f*x + 2*e) + (3*I*A - 2*B)*c^2)*log(cos(1/2*arct...
```

Giac [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\int (B\tan(fx+e)+A)\sqrt{i\,a\tan(fx+e)+a}(-i\,c\tan(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*sqrt(I*a*tan(f*x + e) + a)*(-I*c*tan(f*x +
e) + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\int (A+B\tan(e+f\,x))\,\sqrt{a+a\tan(e+f\,x)\,1\mathrm{i}}\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{5/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)
*1i)^(5/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)
*1i)^(5/2), x)
```

Reduce [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^2\Big(-\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^2 b-6\sqrt{\tan(fx+e)\,i}$$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2*b - 6*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
 + 1)*a*i + 5*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - 3*i
nt(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)
*a*f - 6*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x)**2,x)*b*f*i + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1),x)*a*f))/(3*f)
```

3.789 $\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 164

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = -\frac{\sqrt{a}(2iA-B)c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f} - \frac{(2iA-B)c\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2f} + \frac{B\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{2f}$$

output
```
-a^(1/2)*(2*I*A-B)*c^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)
/(c-I*c*tan(f*x+e))^(1/2))/f-1/2*(2*I*A-B)*c*(a+I*a*tan(f*x+e))^(1/2)*(c-I
*c*tan(f*x+e))^(1/2)/f+1/2*B*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(
3/2)/f
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.79

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{2\sqrt{a}(-2iA+B)c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)+c\sqrt{a+ia\tan(e+fx)}(-2iA+2B-iB\,\mathrm{t}}{2f}$$

input
```
Integrate[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*(c - I*c*Tan[e +
 f*x])^(3/2),x]
```

output
```
(2*Sqrt[a]*((-2*I)*A + B)*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x
]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + c*Sqrt[a + I*a*Tan[e + f*x]]*(
(-2*I)*A + 2*B - I*B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(2*f)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 90, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 90$$

$$\frac{ac\left(\frac{1}{2}(2A+iB)\int \frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)+\frac{B\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{2ac}\right)}{f}$$

$$\Big\downarrow \; 60$$

$$\frac{ac\left(\frac{1}{2}(2A+iB)\left(c\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)+\frac{B\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{2ac}\right)}{f}$$

$$\Big\downarrow \; 45$$

$$\frac{ac\left(\frac{1}{2}(2A+iB)\left(2c\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)+\frac{B\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{2ac}\right)}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{ac\left(\frac{1}{2}(2A+iB)\left(-\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)+\frac{B\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{2ac}\right)}{f}$$

input
```
Int[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])
^(3/2),x]
```

output
```
(a*c*((B*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(3/2))/(2*a*c)
+ ((2*A + I*B)*(((-2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]]
)/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[a] - (I*Sqrt[a + I*a*Tan[e +
 f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/a))/2))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.36

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c\left(-iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac+iB\sqrt{ac}\,\sqrt{ac\left(1-\right.}\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c\left(-iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac+iB\sqrt{ac}\,\sqrt{ac\left(1-\right.}\right.}{}$
parts	$\frac{A\left(-i\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}+\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\right)\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(f}}{f\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}}$

input

```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output

```
-1/2/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*c*(-I*B*ln((
a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+
I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+2*I*A*(a*c*(1+tan(
f*x+e)^2))^(1/2)*(a*c)^(1/2)-2*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+ta
n(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-2*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2)
)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(122) = 244$.

Time = 0.11 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.80

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{\sqrt{\frac{(4A^2+4iAB-B^2)ac^3}{f^2}}\left(fe^{(2ifx+2ie)}+f\right)\log\left(-\frac{4\left(2\left((2iA-B)ce^{(3ifx+3ie)}+(2iA-B)ce^{(ifx+ie)}\right)\sqrt{\frac{a}{e^{(2ifx+2}}}\right.}{(-2iA+B)ce^{(2i}}\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
1/4*(sqrt((4*A^2 + 4*I*A*B - B^2)*a*c^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*l
og(-4*(2*((2*I*A - B)*c*e^(3*I*f*x + 3*I*e) + (2*I*A - B)*c*e^(I*f*x + I*e
))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + s
qrt((4*A^2 + 4*I*A*B - B^2)*a*c^3/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-2*I
*A + B)*c*e^(2*I*f*x + 2*I*e) + (-2*I*A + B)*c)) - sqrt((4*A^2 + 4*I*A*B -
 B^2)*a*c^3/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((2*I*A - B)*c*e^(3
*I*f*x + 3*I*e) + (2*I*A - B)*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*
e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((4*A^2 + 4*I*A*B - B^2)*
a*c^3/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-2*I*A + B)*c*e^(2*I*f*x + 2*I*e
) + (-2*I*A + B)*c)) - 4*((2*I*A - B)*c*e^(3*I*f*x + 3*I*e) + (2*I*A - 3*B
)*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x
+ 2*I*e) + 1)))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \int \sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{3}{2}}\left(A+B\tan\left(e+fx\right)\right)\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Integral(sqrt(I*a*(tan(e + f*x) - I))*(-I*c*(tan(e + f*x) + I))**(3/2)*(A
+ B*tan(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(122) = 244$.

Time = 0.24 (sec) , antiderivative size = 771, normalized size of antiderivative = 4.70

$$\int \sqrt{a + ia\tan(e + fx)}(A + B\tan(e + fx))(c - ic\tan(e + fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
-4*(4*(2*A + I*B)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 4*(2*A + 3*I*B)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
4*(2*I*A - B)*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(
2*I*A - 3*B)*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*((
2*A + I*B)*c*cos(4*f*x + 4*e) + 2*(2*A + I*B)*c*cos(2*f*x + 2*e) + (2*I*A
- B)*c*sin(4*f*x + 4*e) + 2*(2*I*A - B)*c*sin(2*f*x + 2*e) + (2*A + I*B)*c
)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 2*((2*A + I*B)*c*cos(4*f
*x + 4*e) + 2*(2*A + I*B)*c*cos(2*f*x + 2*e) + (2*I*A - B)*c*sin(4*f*x + 4
*e) + 2*(2*I*A - B)*c*sin(2*f*x + 2*e) + (2*A + I*B)*c)*arctan2(cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 1) + ((2*I*A - B)*c*cos(4*f*x + 4*e) + 2*(2*I*A
- B)*c*cos(2*f*x + 2*e) - (2*A + I*B)*c*sin(4*f*x + 4*e) - 2*(2*A + I*B)*c
*sin(2*f*x + 2*e) + (2*I*A - B)*c)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + ((-2*I*
A + B)*c*cos(4*f*x + 4*e) + 2*(-2*I*A + B)*c*cos(2*f*x + 2*e) + (2*A + I*B
)*c*sin(4*f*x + 4*e) + 2*(2*A + I*B)*c*sin(2*f*x + 2*e) + (-2*I*A + B)*c)*
log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x...
```

Giac [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \int (B\tan(fx+e)+A)\sqrt{i\,a\tan(fx+e)+a}(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*sqrt(I*a*tan(f*x + e) + a)*(-I*c*tan(f*x +
e) + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \int (A+B\tan(e+f\,x))\,\sqrt{a+a\tan(e+f\,x)\,1\mathrm{i}}\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{3/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)
*1i)^(3/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)
*1i)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c\Big(-\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,ai+\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(f}$$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - int(sqrt(
tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)*b*f*i +
 int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1),x)*a*f))/f
```

3.790 $\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 104

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = -\frac{2i\sqrt{a}A\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f} + \frac{B\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{f}$$

output
```
-2*I*a^(1/2)*A*c^(1/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-
I*c*tan(f*x+e))^(1/2))/f+B*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/
2)/f
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= -\frac{2i\sqrt{a}A\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f} + \frac{B\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{f}$$

input
```
Integrate[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan
[e + f*x]],x]
```

output
```
((-2*I)*Sqrt[a]*A*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqr
t[a]*Sqrt[c - I*c*Tan[e + f*x]])])/f + (B*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[
c - I*c*Tan[e + f*x]])/f
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4071, 90, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))dx$$
$$\Big\downarrow \text{4071}$$
$$\frac{ac\int \frac{A+B\tan(e+fx)}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 90$$

$$\frac{ac\left(A\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{B\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{ac}\right)}{f}$$

$$\Big\downarrow \ 45$$

$$\frac{ac\left(2A\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{B\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{ac}\right)}{f}$$

$$\Big\downarrow \ 218$$

$$\frac{ac\left(\frac{B\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{ac}-\frac{2iA\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}\sqrt{c}}\right)}{f}$$

input
```
Int[Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f
*x]],x]
```

output
```
(a*c*(((-2*I)*A*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[
c - I*c*Tan[e + f*x]])])/(Sqrt[a]*Sqrt[c]) + (B*Sqrt[a + I*a*Tan[e + f*x]]
*Sqrt[c - I*c*Tan[e + f*x]])/(a*c)))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\left(A\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac+B\sqrt{ac}\,\sqrt{ac\left(1+\tan(f\right.}\right.}{f\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\left(A\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac+B\sqrt{ac}\,\sqrt{ac\left(1+\tan(f\right.}\right.}{f\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,ac\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)}{f\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}}+\frac{B\sqrt{a(1+i\tan(fx+e}}{}$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*(A*ln((a*c*tan(
f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+B*(a*c)^
(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/
2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(78) = 156$.

Time = 0.10 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.82

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= \frac{4\,B\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}e^{(i\,fx+i\,e)}+\sqrt{\frac{A^2ac}{f^2}}f\log\left(\frac{4\left(2\left(Ae^{(3i\,fx+3i\,e)}+Ae^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e}}}\right.}{Ae^{(2i\,fx+2i\,e)}+A}\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
1/2*(4*B*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1
))*e^(I*f*x + I*e) + sqrt(A^2*a*c/f^2)*f*log(4*(2*(A*e^(3*I*f*x + 3*I*e) +
 A*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x +
 2*I*e) + 1)) - sqrt(A^2*a*c/f^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*f))/(A*e^(2
*I*f*x + 2*I*e) + A)) - sqrt(A^2*a*c/f^2)*f*log(4*(2*(A*e^(3*I*f*x + 3*I*e
) + A*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*
x + 2*I*e) + 1)) - sqrt(A^2*a*c/f^2)*(-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(A*
e^(2*I*f*x + 2*I*e) + A)))/f
```

Sympy [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$=\int \sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}(A+B\tan\left(e+fx\right))\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(e + f*x) - I))*sqrt(-I*c*(tan(e + f*x) + I))*(A + B
*tan(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(78) = 156$.

Time = 0.20 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.30

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
-(2*(A*cos(2*f*x + 2*e) + I*A*sin(2*f*x + 2*e) + A)*arctan2(cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 1) + 2*(A*cos(2*f*x + 2*e) + I*A*sin(2*f*x + 2*e) + A
)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*I*B*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - (-I*A*cos(2*f*x + 2*e) + A*sin(2*f*x
 + 2*e) - I*A)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - (I*A*cos(2*f*x + 2*e) - A*s
in(2*f*x + 2*e) + I*A)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 4*B*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/(f*(-2*I*cos(2*f*x
+ 2*e) + 2*sin(2*f*x + 2*e) - 2*I))
```

Giac [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$=\int (B\tan{(fx+e)}+A)\sqrt{i\,a\tan{(fx+e)}+a}\sqrt{-i\,c\tan{(fx+e)}+c}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*sqrt(I*a*tan(f*x + e) + a)*sqrt(-I*c*tan(f*
x + e) + c), x)
```

Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.28

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= -\frac{A\sqrt{a}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\left(\sqrt{a+a\tan(e+f\,x)\,1\mathrm{i}}-\sqrt{a}\right)}{\sqrt{a}\left(\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}-\sqrt{c}\right)}\right)\,4\mathrm{i}}{f}$$
$$+\frac{\sqrt{2}\,B\,\sqrt{\frac{c}{2\cos(e+f\,x)^2+\sin(2\,e+2\,f\,x)\,1\mathrm{i}}}\,\sqrt{\frac{a\left(2\cos(e+f\,x)^2+\sin(2\,e+2\,f\,x)\,1\mathrm{i}\right)}{2\cos(e+f\,x)^2}}}{f}$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)
*1i)^(1/2),x)
```

output
```
(2^(1/2)*B*(c/(sin(2*e + 2*f*x)*1i + 2*cos(e + f*x)^2))^(1/2)*((a*(sin(2*e
 + 2*f*x)*1i + 2*cos(e + f*x)^2))/(2*cos(e + f*x)^2))^(1/2))/f - (A*a^(1/2
)*c^(1/2)*atan((c^(1/2)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/(a^(1/2
)*((c - c*tan(e + f*x)*1i)^(1/2) - c^(1/2))))*4i)/f
```

Reduce [F]

$$\int \sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$$
$$= \frac{\sqrt{c}\sqrt{a}\left(\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,b+\left(\int\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}d\right.\right.}{f}$$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b +
 int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1),x)*a*f))/f
```

3.791 $\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 109

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{2\sqrt{a}B\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}f} - \frac{(iA+B)\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}$$

output
```
2*a^(1/2)*B*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))/c^(1/2)/f-(I*A+B)*(a+I*a*tan(f*x+e))^(1/2)/f/(c-I*c*tan(f*x+e)
)^(1/2)
```

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{2\sqrt{a}B\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}f} + \frac{(-iA-B)\sqrt{a+ia\tan(e+fx)}}{f\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*T
an[e + f*x]],x]
```

output
```
(2*Sqrt[a]*B*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c -
 I*c*Tan[e + f*x]])])/(Sqrt[c]*f) + (((-I)*A - B)*Sqrt[a + I*a*Tan[e + f*x
]])/(f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4071, 87, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{iB\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{45}$$

$$\frac{ac\left(\frac{2iB\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}}{c}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{ac\left(\frac{2B\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}c^{3/2}}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

input
```
Int[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e +
 f*x]],x]
```

output
```
(a*c*((2*B*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I
*c*Tan[e + f*x]])])/(Sqrt[a]*c^(3/2)) - ((I*A + B)*Sqrt[a + I*a*Tan[e + f*
x]])/(a*c*Sqrt[c - I*c*Tan[e + f*x]])))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(88) = 176$.

Time = 0.66 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.94

method	result
derivativedivides	$-\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(-2iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)-B\ln\right.}{}$
default	$-\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(-2iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)-B\ln\right.}{}$
parts	$-\frac{iA\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(i\tan(fx+e)-1)}{fc(i+\tan(fx+e))^2}+\frac{iB\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,(}{}$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/c*(-2*I*B*ln((
a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*
tan(f*x+e)-B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/
(a*c)^(1/2))*a*c*tan(f*x+e)^2+I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)
*tan(f*x+e)+I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)+B*ln((a*c*tan(f*x
+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+B*(a*c)^(1/
2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e
)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I+tan(f*x+e))^2/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(83) = 166$.

Time = 0.11 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.05

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx =$$

$$-\frac{cf\sqrt{-\frac{B^2a}{cf^2}}\log\left(\frac{4\left(2\left(Be^{(3i\,fx+3i\,e)}+Be^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}+\left(cfe^{(2i\,fx+2i\,e)}-cf\right)\sqrt{-\frac{B^2a}{cf^2}}\right)}{Be^{(2i\,fx+2i\,e)}+B}\right)-cf\sqrt{\cdots}}{\cdots}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
-1/2*(c*f*sqrt(-B^2*a/(c*f^2))*log(4*(2*(B*e^(3*I*f*x + 3*I*e) + B*e^(I*f*
x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) +
1)) + (c*f*e^(2*I*f*x + 2*I*e) - c*f)*sqrt(-B^2*a/(c*f^2)))/(B*e^(2*I*f*x
+ 2*I*e) + B)) - c*f*sqrt(-B^2*a/(c*f^2))*log(4*(2*(B*e^(3*I*f*x + 3*I*e)
+ B*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x
+ 2*I*e) + 1)) - (c*f*e^(2*I*f*x + 2*I*e) - c*f)*sqrt(-B^2*a/(c*f^2)))/(B*
e^(2*I*f*x + 2*I*e) + B)) + 2*((I*A + B)*e^(3*I*f*x + 3*I*e) + (I*A + B)*e
^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I
*e) + 1)))/(c*f)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\left(A+B\tan\left(e+fx\right)\right)}{\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Integral(sqrt(I*a*(tan(e + f*x) - I))*(A + B*tan(e + f*x))/sqrt(-I*c*(tan(
e + f*x) + I)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$= \frac{4\,B\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\left(\sqrt{a+a\tan(e+f\,x)\,1\mathrm{i}}-\sqrt{a}\right)}{\sqrt{a}\,\left(\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}-\sqrt{c}\right)}\right)}{\sqrt{c}\,f} + \frac{A\,\sqrt{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}{c\,f\,\left(\tan\left(e+f\,x\right)+1\mathrm{i}\right)} - \frac{4\,B\,a\,\left(\sqrt{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}-\sqrt{a}\right)}{c\,f\,\left(\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}-\sqrt{c}\right)\,\left(\frac{a}{c}-\frac{\left(\sqrt{a+a\tan(e+f\,x)\,1\mathrm{i}}-\sqrt{a}\right)^2}{\left(\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}-\sqrt{c}\right)^2}+\frac{2\,\sqrt{a}\,\left(\sqrt{a+a\tan(e+f\,x)\,1\mathrm{i}}-\sqrt{a}\right)}{\sqrt{c}\,\left(\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}-\sqrt{c}\right)}\right)}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2))/(c - c*tan(e + f*
x)*1i)^(1/2),x)
```

output
```
(4*B*a^(1/2)*atan((c^(1/2)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/(a^(
1/2)*((c - c*tan(e + f*x)*1i)^(1/2) - c^(1/2)))))/(c^(1/2)*f) + (A*(a + a*
tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x)*1i)^(1/2))/(c*f*(tan(e + f*x) +
 1i)) - (4*B*a*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/(c*f*((c - c*tan
(e + f*x)*1i)^(1/2) - c^(1/2))*(a/c - ((a + a*tan(e + f*x)*1i)^(1/2) - a^(
1/2))^2/((c - c*tan(e + f*x)*1i)^(1/2) - c^(1/2))^2 + (2*a^(1/2)*((a + a*t
an(e + f*x)*1i)^(1/2) - a^(1/2)))/(c^(1/2)*((c - c*tan(e + f*x)*1i)^(1/2)
- c^(1/2)))))
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$$
$$= \frac{\sqrt{c}\,\sqrt{a}\,\left(\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+1}\,\tan\left(fx+e\right)a-\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+\right.}\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)*a - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*a*i - s
qrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b + int((sqrt(tan(e +
f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2
+ 1),x)*tan(e + f*x)**2*b*f*i + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(
e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*b*f*i))/(c*f*(ta
n(e + f*x)**2 + 1))
```

3.792 $\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 102

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{(iA+B)\sqrt{a+ia\tan(e+fx)}}{3f(c-ic\tan(e+fx))^{3/2}} - \frac{(iA-2B)\sqrt{a+ia\tan(e+fx)}}{3cf\sqrt{c-ic\tan(e+fx)}}$$

output
```
-1/3*(I*A+B)*(a+I*a*tan(f*x+e))^(1/2)/f/(c-I*c*tan(f*x+e))^(3/2)-1/3*(I*A-
2*B)*(a+I*a*tan(f*x+e))^(1/2)/c/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{a+ia\tan(e+fx)}(2A+iB+(-iA+2B)\tan(e+}{3cf(i+\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e
 + f*x])^(3/2),x]
```

output
```
(Sqrt[a + I*a*Tan[e + f*x]]*(2*A + I*B + ((-I)*A + 2*B)*Tan[e + f*x]))/(3*
c*f*(I + Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {3042, 4071, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{ac\left(\frac{(A+2iB)\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3c}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{ac\left(-\frac{i(A+2iB)\sqrt{a+ia\tan(e+fx)}}{3ac^2\sqrt{c-ic\tan(e+fx)}}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

input
```
Int[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x
])^(3/2),x]
```

output
```
(a*c*(-1/3*((I*A + B)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*(c - I*c*Tan[e + f*
x])^(3/2)) - ((I/3)*(A + (2*I)*B)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c^2*Sqrt[
c - I*c*Tan[e + f*x]])))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,(iA\,e^{2i(fx+e)}+B\,e^{2i(fx+e)}+3iA-3B)}{6c\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(2iB\tan(fx+e)^2+3i\tan(fx+e)A+A\tan(fx+e)^2-iB-3B\tan(fx+e)-2\right.}{3f\,c^2(i+\tan(fx+e))^3}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(2iB\tan(fx+e)^2+3i\tan(fx+e)A+A\tan(fx+e)^2-iB-3B\tan(fx+e)-2\right.}{3f\,c^2(i+\tan(fx+e))^3}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(3i\tan(fx+e)+\tan(fx+e)^2-2\right)}{3f\,c^2(i+\tan(fx+e))^3}+\frac{iB\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan}{3f\,c^2}$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/6/c*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(2*I*(f*x+e)
)+1))^(1/2)*(I*A*exp(2*I*(f*x+e))+B*exp(2*I*(f*x+e))+3*I*A-3*B)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{((-i\,A-B)e^{(5i\,fx+5i\,e)}-2\,(2i\,A-B)e^{(3i\,fx+3i\,e)}-3\,(i}{6\,c^2f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
1/6*((-I*A - B)*e^(5*I*f*x + 5*I*e) - 2*(2*I*A - B)*e^(3*I*f*x + 3*I*e) -
3*(I*A - B)*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(
2*I*f*x + 2*I*e) + 1))/(c^2*f)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{ia(\tan(e+fx)-i)}(A+B\tan(e+fx))}{(-ic(\tan(e+fx)+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Integral(sqrt(I*a*(tan(e + f*x) - I))*(A + B*tan(e + f*x))/(-I*c*(tan(e +
f*x) + I))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,\left(A\,3\mathrm{i}-3\,B+A\,\cos\left(2\,e+2\,f\,x\right)\,1\mathrm{i}+B\,\cos\left(2\,e+2\,f\,x\right)-A\,\sin\left(2\,e+2\right.\right.}{6\,c\,f\,\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2))/(c - c*tan(e + f*
x)*1i)^(3/2),x)
```

output
```
-(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1)
)^(1/2)*(A*3i - 3*B + A*cos(2*e + 2*f*x)*1i + B*cos(2*e + 2*f*x) - A*sin(2
*e + 2*f*x) + B*sin(2*e + 2*f*x)*1i))/(6*c*f*((c*(cos(2*e + 2*f*x) - sin(2
*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(-\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+1}\;\mathrm{t}\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)*a + 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*ta
n(e + f*x)*b*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b +
2*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)
**3 + tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**2*a*f*i + 2*i
nt((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3
 + tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**2*b*f + 2*int((s
qrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 + ta
n(e + f*x)**2*i + tan(e + f*x) + i),x)*a*f*i + 2*int((sqrt(tan(e + f*x)*i
+ 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 + tan(e + f*x)**2*i + t
an(e + f*x) + i),x)*b*f))/(c**2*f*(tan(e + f*x)**2 + 1))
```

3.793 $\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}} \, dx$

Optimal result

Integrand size = 45, antiderivative size = 155

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}} \, dx = -\frac{(iA+B)\sqrt{a+ia\tan(e+fx)}}{5f(c-ic\tan(e+fx))^{5/2}} - \frac{(2iA-3B)\sqrt{a+ia\tan(e+fx)}}{15cf(c-ic\tan(e+fx))^{3/2}} - \frac{(2iA-3B)\sqrt{a+ia\tan(e+fx)}}{15c^2f\sqrt{c-ic\tan(e+fx)}}$$

output
```
-1/5*(I*A+B)*(a+I*a*tan(f*x+e))^(1/2)/f/(c-I*c*tan(f*x+e))^(5/2)-1/15*(2*I
*A-3*B)*(a+I*a*tan(f*x+e))^(1/2)/c/f/(c-I*c*tan(f*x+e))^(3/2)-1/15*(2*I*A-
3*B)*(a+I*a*tan(f*x+e))^(1/2)/c^2/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 4.63 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}} \, dx = \frac{a(-i+\tan(e+fx))\left(-7A-3iB+(6iA-9B)\tan(e\cdot\right.}{15c^2f(i+\tan(e+fx))^2\sqrt{a+ia\tan(e+f}}$$

input
```
Integrate[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e
 + f*x])^(5/2),x]
```

output
```
(a*(-I + Tan[e + f*x])*(-7*A - (3*I)*B + ((6*I)*A - 9*B)*Tan[e + f*x] + (2
*A + (3*I)*B)*Tan[e + f*x]^2))/(15*c^2*f*(I + Tan[e + f*x])^2*Sqrt[a + I*a
*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4071, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(2A+3iB)\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{5c}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{(2A+3iB)\left(\frac{\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{5c}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(\frac{(2A+3iB)\left(-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac^2\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{5c}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

input
```
Int[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x
])^(5/2),x]
```

output
```
(a*c*(-1/5*((I*A + B)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*(c - I*c*Tan[e + f*
x])^(5/2)) + ((2*A + (3*I)*B)*(((-1/3*I)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*
(c - I*c*Tan[e + f*x])^(3/2)) - ((I/3)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c^2*
Sqrt[c - I*c*Tan[e + f*x]])))/(5*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{\sqrt{\frac{a\,{\mathrm e}^{2 i (f x +e)}}{{\mathrm e}^{2 i (f x +e)}+1}}\, \left(3 i A \,{\mathrm e}^{4 i (f x +e)}+3 B \,{\mathrm e}^{4 i (f x +e)}+10 i A \,{\mathrm e}^{2 i (f x +e)}+15 i A -15 B \right)}{60 c^{2} \sqrt{\frac{c}{{\mathrm e}^{2 i (f x +e)}+1}}\, f}$
derivativedivides	$-\frac{i \sqrt{a \left(1+i \tan \left(f x +e \right)\right)}\, \sqrt{-c \left(i \tan \left(f x +e \right)-1\right)}\, \left(2 i A \tan \left(f x +e \right)^{3}-12 i B \tan \left(f x +e \right)^{2}-3 B \tan \left(f x +e \right)^{3}-13 i \tan \left(f x +e \right)\right.}{15 f \,c^{3} \left(i+\tan \left(f x +e \right)\right)^{4}}$
default	$-\frac{i \sqrt{a \left(1+i \tan \left(f x +e \right)\right)}\, \sqrt{-c \left(i \tan \left(f x +e \right)-1\right)}\, \left(2 i A \tan \left(f x +e \right)^{3}-12 i B \tan \left(f x +e \right)^{2}-3 B \tan \left(f x +e \right)^{3}-13 i \tan \left(f x +e \right)\right.}{15 f \,c^{3} \left(i+\tan \left(f x +e \right)\right)^{4}}$
parts	$\frac{A \sqrt{a \left(1+i \tan \left(f x +e \right)\right)}\, \sqrt{-c \left(i \tan \left(f x +e \right)-1\right)}\, \left(8 i \tan \left(f x +e \right)^{2}+2 \tan \left(f x +e \right)^{3}-7 i -13 \tan \left(f x +e \right)\right)}{15 f \,c^{3} \left(i+\tan \left(f x +e \right)\right)^{4}}+\frac{i B \sqrt{a \left(1+i \tan \left(\right.\right.}}{}$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/60/c^2*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(2*I*(f*x
+e))+1))^(1/2)*(3*I*A*exp(4*I*(f*x+e))+3*B*exp(4*I*(f*x+e))+10*I*A*exp(2*I
*(f*x+e))+15*I*A-15*B)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{\left(3\,(i\,A+B)e^{(7i\,fx+7i\,e)}-(-13i\,A-3\,B)e^{(5i\,fx+5i\,e)}+5\,(5i\,A-3\,B)e^{(3i\,fx+3i\,e)}+15\,(i\,A-B)e^{(i\,fx+i\,e)}\right)\sqrt{}}{60\,c^3 f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/60*(3*(I*A + B)*e^(7*I*f*x + 7*I*e) - (-13*I*A - 3*B)*e^(5*I*f*x + 5*I*
e) + 5*(5*I*A - 3*B)*e^(3*I*f*x + 3*I*e) + 15*(I*A - B)*e^(I*f*x + I*e))*s
qrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{ia\,(\tan{(e+fx)}-i)}(A+B\tan{(e+fx)})}{(-ic\,(\tan{(e+fx)}+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Integral(sqrt(I*a*(tan(e + f*x) - I))*(A + B*tan(e + f*x))/(-I*c*(tan(e +
f*x) + I))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,\left(A\,15\mathrm{i}-15\,B+A\,\cos\left(2\,e+2\,f\,x\right)\,10\mathrm{i}+A\,\cos\left(4\,e+4\,f\,x\right)\,3\mathrm{i}+3\,B\,\cos\right.}{60\,c^2\,f\,\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2}{\cos(2\,e+2\,f\,x)+}}}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2))/(c - c*tan(e + f*
x)*1i)^(5/2),x)
```

output
```
-(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1)
)^(1/2)*(A*15i - 15*B + A*cos(2*e + 2*f*x)*10i + A*cos(4*e + 4*f*x)*3i + 3
*B*cos(4*e + 4*f*x) - 10*A*sin(2*e + 2*f*x) - 3*A*sin(4*e + 4*f*x) + B*sin
(4*e + 4*f*x)*3i))/(60*c^2*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i +
 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\tan(fx+e)i+1}}{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^2+2\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)i-\sqrt{-\tan(fx+e)i+1}}dx\right)a+\left(\int \frac{}{\sqrt{-\tan(fx+e)i+1}\,\tan(f}\right.\right.}{\sqrt{c}\,c^2}$$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
( - sqrt(a)*(int(sqrt(tan(e + f*x)*i + 1)/(sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(
e + f*x)*i + 1)),x)*a + int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x))/(sqrt(
 - tan(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*b))/(sqrt(c)*c**2)
```

3.794 $\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 208

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{(iA+B)\sqrt{a+ia\tan(e+fx)}}{7f(c-ic\tan(e+fx))^{7/2}} - \frac{(3iA-4B)\sqrt{a+ia\tan(e+fx)}}{35cf(c-ic\tan(e+fx))^{5/2}} - \frac{2(3iA-4B)\sqrt{a+ia\tan(e+fx)}}{105c^2f(c-ic\tan(e+fx))^{3/2}} - \frac{2(3iA-4B)\sqrt{a+ia\tan(e+fx)}}{105c^3f\sqrt{c-ic\tan(e+fx)}}$$

output
```
-1/7*(I*A+B)*(a+I*a*tan(f*x+e))^(1/2)/f/(c-I*c*tan(f*x+e))^(7/2)-1/35*(3*I
*A-4*B)*(a+I*a*tan(f*x+e))^(1/2)/c/f/(c-I*c*tan(f*x+e))^(5/2)-2/105*(3*I*A
-4*B)*(a+I*a*tan(f*x+e))^(1/2)/c^2/f/(c-I*c*tan(f*x+e))^(3/2)-2/105*(3*I*A
-4*B)*(a+I*a*tan(f*x+e))^(1/2)/c^3/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 6.00 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \frac{a(-i+\tan(e+fx))\,(-36iA+13B-13(3A+4iB)\,\text{ta}}{105c^3f(i+\tan(e+fx))^3\sqrt{}}$$

input
```
Integrate[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e
 + f*x])^(7/2),x]
```

output
```
(a*(-I + Tan[e + f*x])*((-36*I)*A + 13*B - 13*(3*A + (4*I)*B)*Tan[e + f*x]
 + (8*I)*(3*A + (4*I)*B)*Tan[e + f*x]^2 + (6*A + (8*I)*B)*Tan[e + f*x]^3))
/(105*c^3*f*(I + Tan[e + f*x])^3*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*T
an[e + f*x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 4071, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 87$$

$$\frac{ac\left(\frac{(3A+4iB)\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{7c}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

$\big\downarrow$ 55

$$\frac{ac\left(\frac{(3A+4iB)\left(\frac{2\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{5c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{7c}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

$\big\downarrow$ 55

$$\frac{ac\left(\frac{(3A+4iB)\left(\frac{2\left(\frac{\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{5c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{7c}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

$\big\downarrow$ 48

$$\frac{ac\left(\frac{(3A+4iB)\left(\frac{2\left(-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac^2\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{5c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{7c}-\frac{(B+iA)\sqrt{a+ia\tan(e+fx)}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

input
```
Int[(Sqrt[a + I*a*Tan[e + f*x]]*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f*x
])^(7/2),x]
```

output
```
(a*c*(-1/7*((I*A + B)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*(c - I*c*Tan[e + f*
x])^(7/2)) + ((3*A + (4*I)*B)*(((-1/5*I)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*
(c - I*c*Tan[e + f*x])^(5/2)) + (2*(((-1/3*I)*Sqrt[a + I*a*Tan[e + f*x]])/
(a*c*(c - I*c*Tan[e + f*x])^(3/2)) - ((I/3)*Sqrt[a + I*a*Tan[e + f*x]])/(a
*c^2*Sqrt[c - I*c*Tan[e + f*x]])))/(5*c)))/(7*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,\left(15iA\,e^{6i(fx+e)}+15B\,e^{6i(fx+e)}+63iA\,e^{4i(fx+e)}+21B\,e^{4i(fx+e)}+105iA\,e^{2i(fx+e)}-35B\,e^{2i(fx+e)}+\right.}{840c^3\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(8iB\tan(fx+e)^4+30iA\tan(fx+e)^3+6A\tan(fx+e)^4-84iB\tan(fx+e)^2\right.}{105f\,c^4(i+\tan(fx+e))^5}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(8iB\tan(fx+e)^4+30iA\tan(fx+e)^3+6A\tan(fx+e)^4-84iB\tan(fx+e)^2\right.}{105f\,c^4(i+\tan(fx+e))^5}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(10i\tan(fx+e)^3+2\tan(fx+e)^4-25i\tan(fx+e)-21\tan(fx+e)^2+12\right)}{35f\,c^4(i+\tan(fx+e))^5}$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/840/c^3*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(2*I*(f*
x+e))+1))^(1/2)*(15*I*A*exp(6*I*(f*x+e))+15*B*exp(6*I*(f*x+e))+63*I*A*exp(
4*I*(f*x+e))+21*B*exp(4*I*(f*x+e))+105*I*A*exp(2*I*(f*x+e))-35*B*exp(2*I*(
f*x+e))+105*I*A-105*B)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx =$$
$$-\frac{(15\,(i\,A+B)e^{(9i\,fx+9i\,e)}+6\,(13i\,A+6\,B)e^{(7i\,fx+7i\,e)}+14\,(12i\,A-B)e^{(5i\,fx+5i\,e)}+70\,(3i\,A-2\,B)e^{(3i\,fx+}}{840\,c^4f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="fricas")
```

output
```
-1/840*(15*(I*A + B)*e^(9*I*f*x + 9*I*e) + 6*(13*I*A + 6*B)*e^(7*I*f*x + 7
*I*e) + 14*(12*I*A - B)*e^(5*I*f*x + 5*I*e) + 70*(3*I*A - 2*B)*e^(3*I*f*x
+ 3*I*e) + 105*(I*A - B)*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)
)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^4*f)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\left(A+B\tan\left(e+fx\right)\right)}{\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
7/2),x)
```

output
```
Integral(sqrt(I*a*(tan(e + f*x) - I))*(A + B*tan(e + f*x))/(-I*c*(tan(e +
f*x) + I))**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx =$$

$$-\frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1{}\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,\left(A\,105{}\mathrm{i}-105\,B+A\,\cos\left(2\,e+2\,f\,x\right)\,105{}\mathrm{i}+A\,\cos\left(4\,e+4\,f\,x\right)\,63{}\mathrm{i}+A\right.}{}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(1/2))/(c - c*tan(e + f*
x)*1i)^(7/2),x)
```

output
```
-(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1)
)^(1/2)*(A*105i - 105*B + A*cos(2*e + 2*f*x)*105i + A*cos(4*e + 4*f*x)*63i
 + A*cos(6*e + 6*f*x)*15i - 35*B*cos(2*e + 2*f*x) + 21*B*cos(4*e + 4*f*x)
+ 15*B*cos(6*e + 6*f*x) - 105*A*sin(2*e + 2*f*x) - 63*A*sin(4*e + 4*f*x) -
 15*A*sin(6*e + 6*f*x) - B*sin(2*e + 2*f*x)*35i + B*sin(4*e + 4*f*x)*21i +
 B*sin(6*e + 6*f*x)*15i))/(840*c^3*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f
*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+ia\tan(e+fx)}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \frac{\sqrt{c}\sqrt{a}\,\Big(-\Big(\int -\frac{\sqrt{\tan(fx+e)i+1}\,\sqrt{-\tan(fx+e)i+1}\ \mathrm{ta}}{\tan(fx+e)^5 i-3\tan(fx+e)^4-2\tan(fx+e)^3 i-2\tan(f}}{}$$

input
```
int((a+I*a*tan(f*x+e))^(1/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*( - int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e
+ f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b*i + int((sqrt
(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*
x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*
tan(e + f*x)*i + 1),x)*a*i + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(
e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*b + int((sqrt
(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**5*i - 3*t
an(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*
i + 1),x)*a))/c**4
```

3.795 $\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 279

$$\begin{aligned}\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx &= \\ -\frac{a^{3/2}(5iA-2B)c^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{4f} \\ +\frac{a(5A+2iB)c^3\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{8f} \\ -\frac{(5iA-2B)c^2(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{12f} \\ -\frac{(5iA-2B)c(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}{20f} \\ +\frac{B(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{7/2}}{5f}\end{aligned}$$

output
```
-1/4*a^(3/2)*(5*I*A-2*B)*c^(7/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a
^(1/2)/(c-I*c*tan(f*x+e))^(1/2))/f+1/8*a*(5*A+2*I*B)*c^3*tan(f*x+e)*(a+I*a
*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)/f-1/12*(5*I*A-2*B)*c^2*(a+I*a*
tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2)/f-1/20*(5*I*A-2*B)*c*(a+I*a*tan
(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(5/2)/f+1/5*B*(a+I*a*tan(f*x+e))^(3/2)*(
c-I*c*tan(f*x+e))^(7/2)/f
```

Mathematica [A] (verified)

Time = 8.77 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.85

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx =$$

$$-\frac{a^{3/2}c^4(i+\tan(e+fx))\left(30(5A+2iB)\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\sqrt{a+ia\tan(e+fx)}+\sqrt{a}\sqrt{1-i\tan(e}{120f\sqrt{1}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e
 + f*x])^(7/2),x]
```

output
```
-1/120*(a^(3/2)*c^4*(I + Tan[e + f*x])*(30*(5*A + (2*I)*B)*ArcSin[Sqrt[a +
 I*a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[a + I*a*Tan[e + f*x]] + Sqrt[a]
*Sqrt[1 - I*Tan[e + f*x]]*(-I + Tan[e + f*x])*((80*I)*A - 56*B + (-45*A +
(30*I)*B)*Tan[e + f*x] + ((80*I)*A - 32*B)*Tan[e + f*x]^2 + 30*(A + (2*I)*
B)*Tan[e + f*x]^3 + 24*B*Tan[e + f*x]^4)))/(f*Sqrt[1 - I*Tan[e + f*x]]*Sqr
t[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 90, 59, 59, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \sqrt{i\tan(e+fx)a+a}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 90$$

$$\frac{ac\Big(\frac{1}{5}(5A+2iB)\int \sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{7}}{5ac}}{f}$$

$$\Big\downarrow \; 59$$

$$\frac{ac\Big(\frac{1}{5}(5A+2iB)\left(\frac{5}{4}c\int \sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)-\frac{i(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+f}{4a}\right.}{f}$$

$$\Big\downarrow \; 59$$

$$\frac{ac\Big(\frac{1}{5}(5A+2iB)\left(\frac{5}{4}c\Big(c\int \sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)-\frac{i(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+f}{3a}\right.}{f}$$

$$\Big\downarrow \; 40$$

$$ac\Big(\frac{1}{5}(5A+2iB)\left(\frac{5}{4}c\Big(c\Big(\frac{1}{2}ac\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\right.$$

$$\Big\downarrow \; 45$$

$$ac\Big(\frac{1}{5}(5A+2iB)\left(\frac{5}{4}c\Big(c\Big(ac\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\,\mathrm{ta}\right.$$

$$\Big\downarrow \; 218$$

$$ac\Big(\frac{1}{5}(5A+2iB)\left(\frac{5}{4}c\Big(c\Big(\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}-i\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e-}}{\sqrt{a}\sqrt{c-ic\tan(e-}}\right.\right.$$

input
```
Int[(a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(7/2),x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(7/2))/(5*a*c
) + ((5*A + (2*I)*B)*(((-1/4*I)*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[
e + f*x])^(5/2))/a + (5*c*(((-1/3*I)*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c
*Tan[e + f*x])^(3/2))/a + c*((-I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a +
 I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + (Tan[e + f*x]*
Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/2)))/4))/5))/f
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.48

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^3a\left(60iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+24B\sqrt{ac\left(1+\tan(f\right.}\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^3a\left(60iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+24B\sqrt{ac\left(1+\tan(f\right.}\right.}{}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^3a\left(16i\tan(fx+e)^2\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+6\tan(fx+e)^3\sqrt{ac\left(\right.}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/120/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*c^3*a*(60*
I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+24*B*(a*c*(1+tan
(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4+80*I*A*(a*c*(1+tan(f*x+e)^2))^(
1/2)*(a*c)^(1/2)*tan(f*x+e)^2+30*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2
)*tan(f*x+e)^3-30*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2)
)^(1/2))/(a*c)^(1/2))*a*c+30*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*
tan(f*x+e)-32*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+80*I
*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)-75*A*ln((a*c*tan(f*x+e)+(a*c)^
(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-45*A*(a*c)^(1/2)*(a*c
*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-56*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2)
)^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(213) = 426$.

Time = 0.10 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.44

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="fricas")
```

output
```
1/240*(15*sqrt((25*A^2 + 20*I*A*B - 4*B^2)*a^3*c^7/f^2)*(f*e^(8*I*f*x + 8*
I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x
+ 2*I*e) + f)*log(-4*(2*((5*I*A - 2*B)*a*c^3*e^(3*I*f*x + 3*I*e) + (5*I*A
- 2*B)*a*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^
(2*I*f*x + 2*I*e) + 1)) + sqrt((25*A^2 + 20*I*A*B - 4*B^2)*a^3*c^7/f^2)*(f
*e^(2*I*f*x + 2*I*e) - f))/((-5*I*A + 2*B)*a*c^3*e^(2*I*f*x + 2*I*e) + (-5
*I*A + 2*B)*a*c^3)) - 15*sqrt((25*A^2 + 20*I*A*B - 4*B^2)*a^3*c^7/f^2)*(f*
e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) +
4*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((5*I*A - 2*B)*a*c^3*e^(3*I*f*x + 3
*I*e) + (5*I*A - 2*B)*a*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) +
 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((25*A^2 + 20*I*A*B - 4*B^2)*
a^3*c^7/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-5*I*A + 2*B)*a*c^3*e^(2*I*f*x
 + 2*I*e) + (-5*I*A + 2*B)*a*c^3)) - 4*(15*(5*I*A - 2*B)*a*c^3*e^(9*I*f*x
+ 9*I*e) + 70*(5*I*A - 2*B)*a*c^3*e^(7*I*f*x + 7*I*e) + 128*(5*I*A - 2*B)*
a*c^3*e^(5*I*f*x + 5*I*e) + 10*(29*I*A - 50*B)*a*c^3*e^(3*I*f*x + 3*I*e) +
 15*(-5*I*A + 2*B)*a*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)
)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f
*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1655 vs. $2(213) = 426$.

Time = 2.55 (sec) , antiderivative size = 1655, normalized size of antiderivative = 5.93

$$\int (a + ia\tan(e + fx))^{3/2}(A + B\tan(e + fx))(c - ic\tan(e + fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="maxima")
```

output
```
-480*(60*(5*A + 2*I*B)*a*c^3*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 280*(5*A + 2*I*B)*a*c^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 512*(5*A + 2*I*B)*a*c^3*cos(5/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 40*(29*A + 50*I*B)*a*c^3*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 60*(5*A + 2*I*B)*a*c^3*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 60*(5*I*A - 2*B)*a*c^3*sin(9/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 280*(5*I*A - 2*B)*a*c^3*sin(7/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 512*(5*I*A - 2*B)*a*c^3*sin(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 40*(29*I*A - 50*B)*a*c^3*sin(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 60*(-5*I*A + 2*B)*a*c^3*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*((5*A + 2*I*B)*a
*c^3*cos(10*f*x + 10*e) + 5*(5*A + 2*I*B)*a*c^3*cos(8*f*x + 8*e) + 10*(5*A
 + 2*I*B)*a*c^3*cos(6*f*x + 6*e) + 10*(5*A + 2*I*B)*a*c^3*cos(4*f*x + 4*e)
 + 5*(5*A + 2*I*B)*a*c^3*cos(2*f*x + 2*e) + (5*I*A - 2*B)*a*c^3*sin(10*f*x
 + 10*e) + 5*(5*I*A - 2*B)*a*c^3*sin(8*f*x + 8*e) + 10*(5*I*A - 2*B)*a*c^3
*sin(6*f*x + 6*e) + 10*(5*I*A - 2*B)*a*c^3*sin(4*f*x + 4*e) + 5*(5*I*A - 2
*B)*a*c^3*sin(2*f*x + 2*e) + (5*A + 2*I*B)*a*c^3)*arctan2(cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 1) + 30*((5*A + 2*I*B)*a*c^3*cos(10*f*x + 10*e) + 5*(5*
A + 2*I*B)*a*c^3*cos(8*f*x + 8*e) + 10*(5*A + 2*I*B)*a*c^3*cos(6*f*x + ...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{\frac{3}{2}}(-i\,c\tan(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(3/2)*(-I*c*tan(f*x
+ e) + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)
*1i)^(7/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)
*1i)^(7/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a\,c^3\Big(-3\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^4 b-10\sqrt{}}{}$$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*c**3*( - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**4*b - 10*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f
*x)*i + 1)*tan(e + f*x)**2*a*i + 4*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x)**2*b - 10*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(
e + f*x)*i + 1)*a*i + 7*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
1)*b - 15*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
 f*x)**4,x)*a*f - 30*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)**4,x)*b*f*i - 30*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan
(e + f*x)*i + 1)*tan(e + f*x)**2,x)*b*f*i + 15*int(sqrt(tan(e + f*x)*i + 1
)*sqrt( - tan(e + f*x)*i + 1),x)*a*f))/(15*f)
```

3.796 $\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 226

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = -\frac{a^{3/2}(4iA-B)c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{4f} + \frac{a(4A+iB)c^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{8f} - \frac{(4iA-B)c(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{12f} + \frac{B(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}{4f}$$

output
```
-1/4*a^(3/2)*(4*I*A-B)*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(
1/2)/(c-I*c*tan(f*x+e))^(1/2))/f+1/8*a*(4*A+I*B)*c^2*tan(f*x+e)*(a+I*a*tan
(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)/f-1/12*(4*I*A-B)*c*(a+I*a*tan(f*x+
e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2)/f+1/4*B*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c
*tan(f*x+e))^(5/2)/f
```

Mathematica [A] (verified)

Time = 5.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.91

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \frac{a^{3/2}c^3\sqrt{1-i\tan(e+fx)}\left(6(-4iA+B)\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\sqrt{a+ia\tan(e+\cdots}\right.}{}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e
 + f*x])^(5/2),x]
```

output
```
(a^(3/2)*c^3*Sqrt[1 - I*Tan[e + f*x]]*(6*((-4*I)*A + B)*ArcSin[Sqrt[a + I*
a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[a + I*a*Tan[e + f*x]] + Sqrt[a]*Sq
rt[1 - I*Tan[e + f*x]]*(-I + Tan[e + f*x])*(8*(A + I*B) + 3*((4*I)*A + B)*
Tan[e + f*x] + 8*(A + I*B)*Tan[e + f*x]^2 + 6*B*Tan[e + f*x]^3)))/(24*f*Sq
rt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 90, 59, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \sqrt{i\tan(e+fx)a+a}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 90

$$\frac{ac\left(\frac{1}{4}(4A+iB)\int\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/}}{4ac}\right.}{f}$$

$\Big\downarrow$ 59

$$\frac{ac\left(\frac{1}{4}(4A+iB)\left(c\int\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)-\frac{i(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{3a}\right.\right.}{f}$$

$\Big\downarrow$ 40

$$\frac{ac\left(\frac{1}{4}(4A+iB)\left(c\left(\frac{1}{2}ac\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-}\right.\right.\right.}{f}$$

$\Big\downarrow$ 45

$$\frac{ac\left(\frac{1}{4}(4A+iB)\left(c\left(ac\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e}\right.\right.\right.}{f}$$

$\Big\downarrow$ 218

$$\frac{ac\left(\frac{1}{4}(4A+iB)\left(c\left(\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}-i\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)\right.\right.\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(5/2),x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(5/2))/(4*a*c
) + ((4*A + I*B)*(((-1/3*I)*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e +
f*x])^(3/2))/a + c*((-I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[
e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + (Tan[e + f*x]*Sqrt[a +
I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/2)))/4))/f
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.55

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^2a\left(6iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+8iA\sqrt{ac}\,\sqrt{ac\left(1+\tan\right.}\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^2a\left(6iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+8iA\sqrt{ac}\,\sqrt{ac\left(1+\tan\right.}\right.}{}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,c^2a\left(2i\tan(fx+e)^2\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+2i\sqrt{ac}\,\sqrt{ac\left(1+\tan(\right.}\right.}{6f\sqrt{ac\left(1+\tan(fx+e\right.}}$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/24/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*c^2*a*(6*I*
B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3+8*I*A*(a*c)^(1/2)*
(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2-3*I*B*ln((a*c*tan(f*x+e)+(a*c)^(
1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+3*I*B*(a*c)^(1/2)*(a*c
*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-8*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))
^(1/2)*tan(f*x+e)^2+8*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)-12*A*ln
((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*
c-12*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-8*B*(a*c)^(1/2)
*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(170) = 340$.

Time = 0.11 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.72

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
1/48*(3*sqrt((16*A^2 + 8*I*A*B - B^2)*a^3*c^5/f^2)*(f*e^(6*I*f*x + 6*I*e)
+ 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((4*I*A
 - B)*a*c^2*e^(3*I*f*x + 3*I*e) + (4*I*A - B)*a*c^2*e^(I*f*x + I*e))*sqrt(
a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((16*
A^2 + 8*I*A*B - B^2)*a^3*c^5/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-4*I*A +
B)*a*c^2*e^(2*I*f*x + 2*I*e) + (-4*I*A + B)*a*c^2)) - 3*sqrt((16*A^2 + 8*I
*A*B - B^2)*a^3*c^5/f^2)*(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e)
+ 3*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((4*I*A - B)*a*c^2*e^(3*I*f*x + 3
*I*e) + (4*I*A - B)*a*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1
))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((16*A^2 + 8*I*A*B - B^2)*a^3*c
^5/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-4*I*A + B)*a*c^2*e^(2*I*f*x + 2*I*
e) + (-4*I*A + B)*a*c^2)) - 4*(3*(4*I*A - B)*a*c^2*e^(7*I*f*x + 7*I*e) + 1
1*(4*I*A - B)*a*c^2*e^(5*I*f*x + 5*I*e) - (-20*I*A + 53*B)*a*c^2*e^(3*I*f*
x + 3*I*e) + 3*(-4*I*A + B)*a*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(6*I*f*x + 6*I*e) + 3*f
*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1375 vs. $2(170) = 340$.

Time = 0.81 (sec) , antiderivative size = 1375, normalized size of antiderivative = 6.08

$$\int (a + ia\tan(e + fx))^{3/2}(A + B\tan(e + fx))(c - ic\tan(e + fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
-96*(12*(4*A + I*B)*a*c^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 44*(4*A + I*B)*a*c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 4*(20*A + 53*I*B)*a*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 12*(4*A + I*B)*a*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 12*(4*I*A - B)*a*c^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 44*(4*I*A - B)*a*c^2*sin(5/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*(20*I*A - 53*B)*a*c^2*sin(3/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 12*(-4*I*A + B)*a*c^2*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 6*((4*A + I*B)*a*c^2*cos(8*f*x + 8*e) + 4*(4*A
 + I*B)*a*c^2*cos(6*f*x + 6*e) + 6*(4*A + I*B)*a*c^2*cos(4*f*x + 4*e) + 4*
(4*A + I*B)*a*c^2*cos(2*f*x + 2*e) + (4*I*A - B)*a*c^2*sin(8*f*x + 8*e) +
4*(4*I*A - B)*a*c^2*sin(6*f*x + 6*e) + 6*(4*I*A - B)*a*c^2*sin(4*f*x + 4*e
) + 4*(4*I*A - B)*a*c^2*sin(2*f*x + 2*e) + (4*A + I*B)*a*c^2)*arctan2(cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 1) + 6*((4*A + I*B)*a*c^2*cos(8*f*x + 8*e)
+ 4*(4*A + I*B)*a*c^2*cos(6*f*x + 6*e) + 6*(4*A + I*B)*a*c^2*cos(4*f*x + 4
*e) + 4*(4*A + I*B)*a*c^2*cos(2*f*x + 2*e) + (4*I*A - B)*a*c^2*sin(8*f*x +
 8*e) + 4*(4*I*A - B)*a*c^2*sin(6*f*x + 6*e) + 6*(4*I*A - B)*a*c^2*sin(4*f
*x + 4*e) + 4*(4*I*A - B)*a*c^2*sin(2*f*x + 2*e) + (4*A + I*B)*a*c^2)*arct
an2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arct...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\int (B\tan\left(fx+e\right)+A)(i\,a\tan\left(fx+e\right)+a)^{\frac{3}{2}}(-i\,c\tan\left(fx+e\right)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(3/2)*(-I*c*tan(f*x
+ e) + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input

```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)
*1i)^(5/2),x)
```

output

```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)
*1i)^(5/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a\,c^2\left(-\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^2\,ai+\sqrt{\mathrm{ta}}\right.}{}$$

input

```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a*c**2*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**2*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**2*b - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - 3*in
t(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4,x)*
b*f*i + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
 f*x)**2,x)*a*f - 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
1)*tan(e + f*x)**2,x)*b*f*i + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1),x)*a*f))/(3*f)
```

3.797 $\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 157

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = -\frac{ia^{3/2}Ac^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f} + \frac{aAc\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2f} + \frac{B(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{3f}$$

output
```
-I*a^(3/2)*A*c^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*
c*tan(f*x+e))^(1/2))/f+1/2*a*A*c*tan(f*x+e)*(a+I*a*tan(f*x+e))^(1/2)*(c-I*
c*tan(f*x+e))^(1/2)/f+1/3*B*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3
/2)/f
```

Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{a^{3/2}c^2(i+\tan(e+fx))\left(-6A\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\sqrt{a+ia\tan(e+fx)}+\sqrt{a}\right.}{6f\sqrt{1-i\tan(e+fx)}\sqrt{a+ia}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e
 + f*x])^(3/2),x]
```

output
```
(a^(3/2)*c^2*(I + Tan[e + f*x])*(-6*A*ArcSin[Sqrt[a + I*a*Tan[e + f*x]]/(S
qrt[2]*Sqrt[a])]*Sqrt[a + I*a*Tan[e + f*x]] + Sqrt[a]*Sqrt[1 - I*Tan[e + f
*x]]*(-I + Tan[e + f*x])*(2*B + 3*A*Tan[e + f*x] + 2*B*Tan[e + f*x]^2)))/(
6*f*Sqrt[1 - I*Tan[e + f*x]]*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e
 + f*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 90, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \sqrt{i\tan(e+fx)a+a}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)}{f}$$

$$\Big\downarrow 90$$

$$\frac{ac\left(A\int\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{3ac}\right)}{f}$$

$$\Big\downarrow 40$$

$$\frac{ac\left(A\left(\frac{1}{2}ac\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+f}\right.\right.}{f}$$

$$\Big\downarrow 45$$

$$\frac{ac\left(A\left(ac\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}\right)+\frac{B(a}{}\right.}{f}$$

$$\Big\downarrow 218$$

$$\frac{ac\left(A\left(\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}-i\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)\right)+\frac{B(a+ia\,\mathrm{ta}}{}\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(3/2),x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(3/2))/(3*a*c
) + A*((-I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(S
qrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + (Tan[e + f*x]*Sqrt[a + I*a*Tan[e + f
*x]]*Sqrt[c - I*c*Tan[e + f*x]])/2)))/f
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.14

method	result
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,ac\left(\tan(fx+e)\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{a\cdots}}{\sqrt{ac}}\right.\right.}{2f\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,ac\left(2B\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^2+3A\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\cdots}{\cdots}\right.\right.}{6f\sqrt{ac\left(1+\tan(fx+e)\cdots\right.}}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,ac\left(2B\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^2+3A\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\cdots}{\cdots}\right.\right.}{6f\sqrt{ac\left(1+\tan(fx+e)\cdots\right.}}$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/2*A/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a*c*(tan(f
*x+e)*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+ln((a*c*tan(f*x+e)+(a*c)^(1
/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c)/(a*c*(1+tan(f*x+e)^2))
^(1/2)/(a*c)^(1/2)-1/3*B/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1)
)^(1/2)*a*c*(1+tan(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(119) = 238$.

Time = 0.11 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.76

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{3\sqrt{\frac{A^2a^3c^3}{f^2}}\left(fe^{(4i\,fx+4i\,e)}+2\,fe^{(2i\,fx+2i\,e)}+f\right)\log\left(\frac{4\left(2\left(Aace^{(3i\,fx+3i\,e)}+Aace^{(i\,fx+i\,e)}\right)\sqrt{\frac{\cdots}{e^{\cdots}}}\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
1/12*(3*sqrt(A^2*a^3*c^3/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*
I*e) + f)*log(4*(2*(A*a*c*e^(3*I*f*x + 3*I*e) + A*a*c*e^(I*f*x + I*e))*sqr
t(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(A^
2*a^3*c^3/f^2)*(I*f*e^(2*I*f*x + 2*I*e) - I*f))/(A*a*c*e^(2*I*f*x + 2*I*e)
 + A*a*c)) - 3*sqrt(A^2*a^3*c^3/f^2)*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f
*x + 2*I*e) + f)*log(4*(2*(A*a*c*e^(3*I*f*x + 3*I*e) + A*a*c*e^(I*f*x + I*
e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) -
sqrt(A^2*a^3*c^3/f^2)*(-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(A*a*c*e^(2*I*f*x
+ 2*I*e) + A*a*c)) - 4*(3*I*A*a*c*e^(5*I*f*x + 5*I*e) - 8*B*a*c*e^(3*I*f*x
 + 3*I*e) - 3*I*A*a*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*s
qrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x
+ 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\int \left(ia(\tan{(e+fx)}-i)\right)^{\frac{3}{2}}\left(-ic(\tan{(e+fx)}+i)\right)^{\frac{3}{2}}\left(A+B\tan{(e+fx)}\right)\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(3/2)*(-I*c*(tan(e + f*x) + I))**(3/2)*
(A + B*tan(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(119) = 238$.

Time = 0.26 (sec) , antiderivative size = 853, normalized size of antiderivative = 5.43

$$\int (a + ia \tan(e + fx))^{3/2}(A + B \tan(e + fx))(c - ic \tan(e + fx))^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
-(12*A*a*c*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 32*I*B*a
*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*A*a*c*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12*I*A*a*c*sin(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*B*a*c*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 12*I*A*a*c*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 6*(A*a*c*cos(6*f*x + 6*e) + 3*A*a*c*cos(4*f*x + 4*e)
 + 3*A*a*c*cos(2*f*x + 2*e) + I*A*a*c*sin(6*f*x + 6*e) + 3*I*A*a*c*sin(4*f
*x + 4*e) + 3*I*A*a*c*sin(2*f*x + 2*e) + A*a*c)*arctan2(cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 1) + 6*(A*a*c*cos(6*f*x + 6*e) + 3*A*a*c*cos(4*f*x + 4*e)
 + 3*A*a*c*cos(2*f*x + 2*e) + I*A*a*c*sin(6*f*x + 6*e) + 3*I*A*a*c*sin(4*f
*x + 4*e) + 3*I*A*a*c*sin(2*f*x + 2*e) + A*a*c)*arctan2(cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 1) + 3*(I*A*a*c*cos(6*f*x + 6*e) + 3*I*A*a*c*cos(4*f*x +
 4*e) + 3*I*A*a*c*cos(2*f*x + 2*e) - A*a*c*sin(6*f*x + 6*e) - 3*A*a*c*sin(
4*f*x + 4*e) - 3*A*a*c*sin(2*f*x + 2*e) + I*A*a*c)*log(cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + 1) + 3*(-I*A*a*c*cos(6*f*x + 6*e) - 3*I*A*a*c*cos(4*f*x + 4*e) - 3*I*A
*a*c*cos(2*f*x + 2*e) + A*a*c*sin(6*f*x + 6*e) + 3*A*a*c*sin(4*f*x + 4*...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{\frac{3}{2}}(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(3/2)*(-I*c*tan(f*x
+ e) + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)
*1i)^(3/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)
*1i)^(3/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,ac\Big(\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^2\,b+\sqrt{\tan(f}$$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*c*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)
*tan(e + f*x)**2*b + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
b + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x
)**2,x)*a*f + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1),x
)*a*f))/(3*f)
```

3.798 $\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 160

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = -\frac{a^{3/2}(2iA+B)\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f} + \frac{a(2iA+B)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2f} + \frac{B(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2f}$$

output
```
-a^(3/2)*(2*I*A+B)*c^(1/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)
/(c-I*c*tan(f*x+e))^(1/2))/f+1/2*a*(2*I*A+B)*(a+I*a*tan(f*x+e))^(1/2)*(c-I
*c*tan(f*x+e))^(1/2)/f+1/2*B*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(
1/2)/f
```

Mathematica [A] (verified)

Time = 2.69 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx=\frac{a^{3/2}c(i+\tan(e+fx))\left(-2(2A-iB)\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\sqrt{\ }\right.}{2f\sqrt{1-i\tan(e}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*Sqrt[c - I*c*T
an[e + f*x]],x]
```

output
```
(a^(3/2)*c*(I + Tan[e + f*x])*(-2*(2*A - I*B)*ArcSin[Sqrt[a + I*a*Tan[e +
f*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[a + I*a*Tan[e + f*x]] + Sqrt[a]*Sqrt[1 - I*T
an[e + f*x]]*(1 + I*Tan[e + f*x])*(2*A - (2*I)*B + B*Tan[e + f*x])))/(2*f*
Sqrt[1 - I*Tan[e + f*x]]*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f
*x]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 4071, 90, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{\sqrt{i\tan(e+fx)a+a}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{90}$$

$$\frac{ac\left(\frac{1}{2}(2A-iB)\int \frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2ac}\right)}{f}$$

$$\Big\downarrow \text{60}$$

$$\frac{ac\left(\frac{1}{2}(2A-iB)\left(a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{B(a+ia\tan(e}{}\right.}{f}$$

$$\Big\downarrow \text{45}$$

$$\frac{ac\left(\frac{1}{2}(2A-iB)\left(2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{B(a+ia\tan(e+fx))^{3/2}}{2ac}\right.}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{ac\left(\frac{1}{2}(2A-iB)\left(\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}}\right)+\frac{B(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e}{2ac}\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e +
 f*x]],x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f*x]])/(2*a*c)
+ ((2*A - I*B)*(((-2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]]
)/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[c] + (I*Sqrt[a + I*a*Tan[e +
 f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/c))/2))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(-iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac+iB\sqrt{ac}\,\sqrt{ac\left(1+\tan\right.}\right.}{2f}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(-iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac+iB\sqrt{ac}\,\sqrt{ac\left(1+\tan\right.}\right.}{2f}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(i\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}+\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)\right.}}{\sqrt{ac}}\right.\right.}{f\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}}$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/2/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a*(-I*B*ln((a
*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+I
*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+2*I*A*(a*c*(1+tan(f
*x+e)^2))^(1/2)*(a*c)^(1/2)+2*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan
(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+2*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))
^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(120) = 240.

Time = 0.10 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.87

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx =$$

$$-\frac{\sqrt{\frac{(4A^2-4iAB-B^2)a^3c}{f^2}}\left(fe^{(2ifx+2ie)}+f\right)\log\left(-\frac{4\left(2\left((-2iA-B)ae^{(3ifx+3ie)}+(-2iA-B)ae^{(ifx+ie)}\right)\sqrt{\frac{a}{e^{(2ifx+2ie)}+1}}\sqrt{\frac{c}{e^{(2i}}}\right.}{(2iA+B)ae^{(2ifx+2ie)}+(2iA}\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt((4*A^2 - 4*I*A*B - B^2)*a^3*c/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*
log(-4*(2*((-2*I*A - B)*a*e^(3*I*f*x + 3*I*e) + (-2*I*A - B)*a*e^(I*f*x +
I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))
+ sqrt((4*A^2 - 4*I*A*B - B^2)*a^3*c/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((2
*I*A + B)*a*e^(2*I*f*x + 2*I*e) + (2*I*A + B)*a)) - sqrt((4*A^2 - 4*I*A*B
- B^2)*a^3*c/f^2)*(f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((-2*I*A - B)*a*e^
(3*I*f*x + 3*I*e) + (-2*I*A - B)*a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((4*A^2 - 4*I*A*B - B^
2)*a^3*c/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((2*I*A + B)*a*e^(2*I*f*x + 2*I
*e) + (2*I*A + B)*a)) + 4*((-2*I*A - 3*B)*a*e^(3*I*f*x + 3*I*e) + (-2*I*A
- B)*a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f
*x + 2*I*e) + 1)))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \int (ia(\tan(e+fx)-i))^{\frac{3}{2}}\sqrt{-ic(\tan(e+fx)+i)}(A+B\tan(e+fx))\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(3/2)*sqrt(-I*c*(tan(e + f*x) + I))*(A
+ B*tan(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 771 vs. 2(120) = 240.

Time = 0.24 (sec) , antiderivative size = 771, normalized size of antiderivative = 4.82

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
4*(4*(2*A - 3*I*B)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 4*(2*A - I*B)*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4
*(2*I*A + 3*B)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*
(2*I*A + B)*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*((2
*A - I*B)*a*cos(4*f*x + 4*e) + 2*(2*A - I*B)*a*cos(2*f*x + 2*e) - (-2*I*A
- B)*a*sin(4*f*x + 4*e) - 2*(-2*I*A - B)*a*sin(2*f*x + 2*e) + (2*A - I*B)*
a)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 2*((2*A - I*B)*a*cos(4*
f*x + 4*e) + 2*(2*A - I*B)*a*cos(2*f*x + 2*e) - (-2*I*A - B)*a*sin(4*f*x +
 4*e) - 2*(-2*I*A - B)*a*sin(2*f*x + 2*e) + (2*A - I*B)*a)*arctan2(cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 1) + ((-2*I*A - B)*a*cos(4*f*x + 4*e) + 2*(-2
*I*A - B)*a*cos(2*f*x + 2*e) + (2*A - I*B)*a*sin(4*f*x + 4*e) + 2*(2*A - I
*B)*a*sin(2*f*x + 2*e) + (-2*I*A - B)*a)*log(cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (
(2*I*A + B)*a*cos(4*f*x + 4*e) + 2*(2*I*A + B)*a*cos(2*f*x + 2*e) - (2*A -
 I*B)*a*sin(4*f*x + 4*e) - 2*(2*A - I*B)*a*sin(2*f*x + 2*e) + (2*I*A + B)*
a)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{\frac{3}{2}}\sqrt{-i\,c\tan(fx+$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(3/2)*sqrt(-I*c*tan(
f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\,\sqrt{c-c\tan(e+f\,x)\,1\mathrm{i}}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)
*1i)^(1/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x)
*1i)^(1/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a\left(\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,ai+\sqrt{\tan\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*a
*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b + int(sqrt(tan
(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)*b*f*i + in
t(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1),x)*a*f))/f
```

3.799 $\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 169

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{2a^{3/2}(iA+2B)\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}f} - \frac{(iA+B)(a+ia\tan(e+fx))^{3/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{a(iA+2B)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{cf}$$

output
```
2*a^(3/2)*(I*A+2*B)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c
*tan(f*x+e))^(1/2))/c^(1/2)/f-(I*A+B)*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*ta
n(f*x+e))^(1/2)-a*(I*A+2*B)*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1
/2)/c/f
```

Mathematica [A] (verified)

Time = 3.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.88

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{2a^{3/2}(iA+2B)\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\sqrt{1-i\tan(e}{f\sqrt{a+}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/Sqrt[c - I*c
*Tan[e + f*x]],x]
```

output
```
(2*a^(3/2)*(I*A + 2*B)*ArcSin[Sqrt[a + I*a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[a])
]*Sqrt[1 - I*Tan[e + f*x]]*Sqrt[a + I*a*Tan[e + f*x]] - a^2*(-I + Tan[e +
f*x])*(-2*A + (3*I)*B + B*Tan[e + f*x]))/(f*Sqrt[a + I*a*Tan[e + f*x]]*Sqr
t[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 87, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{\sqrt{i\tan(e+fx)a+a}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{ac\left(-\frac{(A-2iB)\int \frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{ac\left(-\frac{(A-2iB)\left(a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{ 45}$$

$$\frac{ac\left(-\frac{(A-2iB)\left(2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{ac\left(-\frac{(A-2iB)\left(\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e
 + f*x]],x]
```

output
```
(a*c*(-(((I*A + B)*(a + I*a*Tan[e + f*x])^(3/2))/(a*c*Sqrt[c - I*c*Tan[e +
 f*x]])) - ((A - (2*I)*B)*(((-2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Ta
n[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[c] + (I*Sqrt[a +
I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/c))/c))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(140) = 280$.

Time = 0.63 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.94

method	result
derivativedivides	$\frac{\left(2iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^2-2iA\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\cdots\right.}{}$
default	$\frac{\left(2iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^2-2iA\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\cdots\right.}{}$
parts	$\frac{iA\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(i\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)\tan(fx+e)^2ac-i\ln\left(\cdots\right.\right.}{}$

input

```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output

```
1/f*(2*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a
*c)^(1/2))*a*c*tan(f*x+e)^2-2*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+t
an(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-A*ln((a*c*tan(f*x+e)+(a*c
)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2-2*I*B*
ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*
a*c-4*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-4*B*ln((a*c*
tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(
f*x+e)-B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+2*I*A*(a*c*
(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*
(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+2*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+
e)^2))^(1/2)*tan(f*x+e)+3*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))*(a*(
1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c/(a*c*(1+tan(f*x+e)^
2))^(1/2)/(I+tan(f*x+e))^2/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(131) = 262.

Time = 0.10 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.59

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{c\sqrt{\frac{(A^2-4i\,AB-4\,B^2)a^3}{cf^2}}\,f\log\left(-\frac{4\left(2\left((-i\,A-2\,B)ae^{(3i\,fx+3i\,e)}+\right.\right.}{}\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
1/2*(c*sqrt((A^2 - 4*I*A*B - 4*B^2)*a^3/(c*f^2))*f*log(-4*(2*((-I*A - 2*B)
*a*e^(3*I*f*x + 3*I*e) + (-I*A - 2*B)*a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (c*f*e^(2*I*f*x + 2*I
*e) - c*f)*sqrt((A^2 - 4*I*A*B - 4*B^2)*a^3/(c*f^2)))/((I*A + 2*B)*a*e^(2*
I*f*x + 2*I*e) + (I*A + 2*B)*a)) - c*sqrt((A^2 - 4*I*A*B - 4*B^2)*a^3/(c*f
^2))*f*log(-4*(2*((-I*A - 2*B)*a*e^(3*I*f*x + 3*I*e) + (-I*A - 2*B)*a*e^(I
*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e)
 + 1)) - (c*f*e^(2*I*f*x + 2*I*e) - c*f)*sqrt((A^2 - 4*I*A*B - 4*B^2)*a^3/
(c*f^2)))/((I*A + 2*B)*a*e^(2*I*f*x + 2*I*e) + (I*A + 2*B)*a)) - 4*((I*A +
 B)*a*e^(3*I*f*x + 3*I*e) + (I*A + 2*B)*a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \int \frac{(ia(\tan(e+fx)-i))^{\frac{3}{2}}\,(A+B\tan(e+fx))}{\sqrt{-ic\,(\tan(e+fx)+i)}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(3/2)*(A + B*tan(e + f*x))/sqrt(-I*c*(t
an(e + f*x) + I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(131) = 262$.

Time = 0.22 (sec) , antiderivative size = 599, normalized size of antiderivative = 3.54

$$\int \frac{(a + ia\tan(e + fx))^{3/2}(A + B\tan(e + fx))}{\sqrt{c - ic\tan(e + fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
(2*((A - 2*I*B)*a*cos(2*f*x + 2*e) - (-I*A - 2*B)*a*sin(2*f*x + 2*e) + (A
- 2*I*B)*a)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))),
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 2*((A - 2*I*B)
*a*cos(2*f*x + 2*e) - (-I*A - 2*B)*a*sin(2*f*x + 2*e) + (A - 2*I*B)*a)*arc
tan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 4*((A - I*B)*a*cos(2*f*x + 2
*e) + (I*A + B)*a*sin(2*f*x + 2*e) + (A - 2*I*B)*a)*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + ((I*A + 2*B)*a*cos(2*f*x + 2*e) - (A - 2*
I*B)*a*sin(2*f*x + 2*e) + (I*A + 2*B)*a)*log(cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (
(-I*A - 2*B)*a*cos(2*f*x + 2*e) + (A - 2*I*B)*a*sin(2*f*x + 2*e) + (-I*A -
 2*B)*a)*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 4*((I*A + B)*a*cos(2*f*x + 2*e) -
 (A - I*B)*a*sin(2*f*x + 2*e) + (I*A + 2*B)*a)*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((-2*I*c*cos(2*f*x + 2*e) + 2*c
*sin(2*f*x + 2*e) - 2*I*c)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \int \frac{(A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}}{\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*
x)*1i)^(1/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*
x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a\left(-\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+}\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)**2*b + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)
*tan(e + f*x)*a - 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*a
*i - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - int((sqrt(
tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e +
f*x)**2 + 1),x)*tan(e + f*x)**2*a*f + 2*int((sqrt(tan(e + f*x)*i + 1)*sqrt
( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*tan(e +
f*x)**2*b*f*i - int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*a*f + 2*int((sqrt(tan(e + f*x)*i
 + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x
)*b*f*i))/(c*f*(tan(e + f*x)**2 + 1))
```

3.800 $\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 155

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{2a^{3/2}B\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f} - \frac{(iA+B)(a+ia\tan(e+fx))^{3/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{2aB\sqrt{a+ia\tan(e+fx)}}{cf\sqrt{c-ic\tan(e+fx)}}$$

output
```
-2*a^(3/2)*B*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*
x+e))^(1/2))/c^(3/2)/f-1/3*(I*A+B)*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*tan(f
*x+e))^(3/2)+2*a*B*(a+I*a*tan(f*x+e))^(1/2)/c/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 5.02 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.41

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{a^{3/2}\sec^2(e+fx)\left(\sqrt{a}(6iB+(A-iB)\cos(2(e+fx)\right.}{}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(3/2),x]
```

output
```
(a^(3/2)*Sec[e + f*x]^2*(Sqrt[a]*((6*I)*B + (A - I*B)*Cos[2*(e + f*x)] + (
I*A + B)*Sin[2*(e + f*x)])*Sqrt[1 - I*Tan[e + f*x]] - (6*I)*B*ArcSin[Sqrt[
a + I*a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[a])]*(Cos[2*(e + f*x)] - I*Sin[2*(e +
f*x)])*Sqrt[a + I*a*Tan[e + f*x]]))/(3*c*f*Sqrt[1 - I*Tan[e + f*x]]*(I + T
an[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 87, 57, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{\sqrt{i\tan(e+fx)a+a}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{iB\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow 57$$

$$\frac{ac\left(\frac{iB\left(-\frac{a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow 45$$

$$\frac{ac\left(\frac{iB\left(-\frac{2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}}{c}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow 218$$

$$\frac{ac\left(\frac{iB\left(\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(3/2),x]
```

output
```
(a*c*(-1/3*((I*A + B)*(a + I*a*Tan[e + f*x])^(3/2))/(a*c*(c - I*c*Tan[e +
f*x])^(3/2)) + (I*B*(((2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f
*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/c^(3/2) - ((2*I)*Sqrt[a + I*a
*Tan[e + f*x]])/(c*Sqrt[c - I*c*Tan[e + f*x]])))/c))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(126) = 252$.

Time = 0.68 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.62

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(3iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^3-9iB\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(3iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^3-9iB\right.}{}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^2\right)}{3f\,c^2(i+\tan(fx+e))^3}-\frac{iB\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}}{}$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/3/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c^2*(3*I*B
*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))
*a*c*tan(f*x+e)^3-9*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^
2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-7*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*
(a*c)^(1/2)*tan(f*x+e)^2-9*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*
x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+A*(a*c*(1+tan(f*x+e)^2))^(1/
2)*(a*c)^(1/2)*tan(f*x+e)^2+5*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)
+3*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1
/2))*a*c+12*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+A*(a*c)^
(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I+tan(f*
x+e))^3/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(121) = 242$.

Time = 0.12 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.45

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{3\,c^2f\sqrt{-\frac{B^2a^3}{c^3f^2}}\log\left(\frac{4\left(2\left(Bae^{(3i\,fx+3i\,e)}+Bae^{(i\,fx+i\,e)}\right)\sqrt{\frac{}{e^{(2i\,fx}}}\cdots\right.}{B\cdots}\right.\cdots}{\cdots}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
1/6*(3*c^2*f*sqrt(-B^2*a^3/(c^3*f^2))*log(4*(2*(B*a*e^(3*I*f*x + 3*I*e) +
B*a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x
+ 2*I*e) + 1)) + (c^2*f*e^(2*I*f*x + 2*I*e) - c^2*f)*sqrt(-B^2*a^3/(c^3*f^
2)))/(B*a*e^(2*I*f*x + 2*I*e) + B*a)) - 3*c^2*f*sqrt(-B^2*a^3/(c^3*f^2))*l
og(4*(2*(B*a*e^(3*I*f*x + 3*I*e) + B*a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x
 + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (c^2*f*e^(2*I*f*x + 2*
I*e) - c^2*f)*sqrt(-B^2*a^3/(c^3*f^2)))/(B*a*e^(2*I*f*x + 2*I*e) + B*a)) -
 2*((I*A + B)*a*e^(5*I*f*x + 5*I*e) + (I*A - 5*B)*a*e^(3*I*f*x + 3*I*e) -
6*B*a*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*
x + 2*I*e) + 1)))/(c^2*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \int \frac{(ia(\tan{(e+fx)}-i))^{\frac{3}{2}}\,(A+B\tan{(e+fx)})}{(-ic\,(\tan{(e+fx)}+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(3/2)*(A + B*tan(e + f*x))/(-I*c*(tan(e
 + f*x) + I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{(6\,Ba\arctan(\cos(fx+e),\sin(fx+e)+1)+6\,Ba\arctan(\cos(fx+e),-\sin(fx+e)+1)-2\,(-i\,A}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
-1/6*(6*B*a*arctan2(cos(f*x + e), sin(f*x + e) + 1) + 6*B*a*arctan2(cos(f*
x + e), -sin(f*x + e) + 1) - 2*(-I*A - B)*a*cos(3*f*x + 3*e) - 12*B*a*cos(
f*x + e) + 3*I*B*a*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) +
1) - 3*I*B*a*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 2
*(A - I*B)*a*sin(3*f*x + 3*e) - 12*I*B*a*sin(f*x + e))*sqrt(a)/(c^(3/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx=\int \frac{(A+B\tan(e+f\,x))\ (a+a\tan(e+f\,x)\ 1\mathrm{i})^{3/2}}{(c-c\tan\left(e+f\,x\right)\ 1\mathrm{i})^{3/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*
x)*1i)^(3/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*
x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*( - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)*a + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)*b*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
a*i + 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - 2*int(( -
 sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 +
tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**2*a*f*i - int(( - s
qrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 + ta
n(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**2*b*f - 2*int(( - sqr
t(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 + tan(
e + f*x)**2*i + tan(e + f*x) + i),x)*a*f*i - int(( - sqrt(tan(e + f*x)*i +
 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 + tan(e + f*x)**2*i + ta
n(e + f*x) + i),x)*b*f + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f
*x)*i - 1),x)*tan(e + f*x)**2*b*f + int((sqrt(tan(e + f*x)*i + 1)*sqrt( -
tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i - tan(e + f*x)**2
+ tan(e + f*x)*i - 1),x)*b*f - 2*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan
(e + f*x)*i + 1))/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i -
1),x)*tan(e + f*x)**2*a*f + 2*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
+ f*x)*i + 1))/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(e + f*x)*i - 1),
x)*tan(e + f*x)**2*b*f*i - 2*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(...
```

3.801 $\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 102

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{3/2}}{5f(c-ic\tan(e+fx))^{5/2}}-\frac{(iA-4B)(a+ia\tan(e+fx))^{3/2}}{15cf(c-ic\tan(e+fx))^{3/2}}$$

output
```
-1/5*(I*A+B)*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*tan(f*x+e))^(5/2)-1/15*(I*A
-4*B)*(a+I*a*tan(f*x+e))^(3/2)/c/f/(c-I*c*tan(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 5.68 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{a^2\sec^2(e+fx)(\cos(2(e+fx))+i\sin(2(e+fx)))(4a \cdots}{15c^2f(i+\tan(e+fx))^2\sqrt{a+ia\tan(e+f \cdots}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(5/2),x]
```

output
```
(a^2*Sec[e + f*x]^2*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*((4*I)*A - B +
 (A + (4*I)*B)*Tan[e + f*x]))/(15*c^2*f*(I + Tan[e + f*x])^2*Sqrt[a + I*a*
Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {3042, 4071, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{\sqrt{i\tan(e+fx)a+a}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(A+4iB)\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{5c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(-\frac{i(A+4iB)(a+ia\tan(e+fx))^{3/2}}{15ac^2(c-ic\tan(e+fx))^{3/2}}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(5/2),x]
```

output
```
(a*c*(-1/5*((I*A + B)*(a + I*a*Tan[e + f*x])^(3/2))/(a*c*(c - I*c*Tan[e +
f*x])^(5/2)) - ((I/15)*(A + (4*I)*B)*(a + I*a*Tan[e + f*x])^(3/2))/(a*c^2*
(c - I*c*Tan[e + f*x])^(3/2))))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^2\right)(-4A+i\tan(fx+e)A-iB-4B\tan(fx+e))}{15f\,c^3(i+\tan(fx+e))^4}$
default	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^2\right)(-4A+i\tan(fx+e)A-iB-4B\tan(fx+e))}{15f\,c^3(i+\tan(fx+e))^4}$
risch	$-\frac{a\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,(3iA\,e^{4i(fx+e)}+3B\,e^{4i(fx+e)}+5iA\,e^{2i(fx+e)}-5B\,e^{2i(fx+e)})}{30c^2\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^2\right)(4i+\tan(fx+e))}{15f\,c^3(i+\tan(fx+e))^4}-\frac{iB\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c}}{}$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/15*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c^3*(1+t
an(f*x+e)^2)*(-4*A+I*A*tan(f*x+e)-I*B-4*B*tan(f*x+e))/(I+tan(f*x+e))^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{\left(3\,(i\,A+B)ae^{(7i\,fx+7i\,e)}+2\,(4i\,A-B)ae^{(5i\,fx+5i\,e)}+5\,(i\,A-B)ae^{(3i\,fx+3i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}}{30\,c^3f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/30*(3*(I*A + B)*a*e^(7*I*f*x + 7*I*e) + 2*(4*I*A - B)*a*e^(5*I*f*x + 5*
I*e) + 5*(I*A - B)*a*e^(3*I*f*x + 3*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)
)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^3*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \int \frac{(ia(\tan(e+fx)-i))^{\frac{3}{2}}(A+B\tan(e+fx))}{(-ic(\tan(e+fx)+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(3/2)*(A + B*tan(e + f*x))/(-I*c*(tan(e
 + f*x) + I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.51

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{30\,(3\,(A-i\,B)a\cos(7\,fx+7\,e)+2\,(4\,A+i\,B)a\cos(5\,fx+5\,e)+5\,(A+i\,B)a\cos(3\,fx+3\,e)-3\,(-i}{-900\,(i\,c^3\cos(2\,fx+2\,e)-c^3}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
-30*(3*(A - I*B)*a*cos(7*f*x + 7*e) + 2*(4*A + I*B)*a*cos(5*f*x + 5*e) + 5
*(A + I*B)*a*cos(3*f*x + 3*e) - 3*(-I*A - B)*a*sin(7*f*x + 7*e) - 2*(-4*I*
A + B)*a*sin(5*f*x + 5*e) - 5*(-I*A + B)*a*sin(3*f*x + 3*e))*sqrt(a)*sqrt(
c)/((-900*I*c^3*cos(2*f*x + 2*e) + 900*c^3*sin(2*f*x + 2*e) - 900*I*c^3)*f
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.86

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{a\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,\left(A\,\cos\left(2\,e+2\,f\,x\right)\,5\mathrm{i}+A\,\cos\left(4\,e+4\,f\,x\right)\,3\mathrm{i}-5\,B\,\cos\left(2\,e+2\,f\,x\right)+\right.}{30\,c^2\,f\,\sqrt{\frac{c\,(\cos}{}}}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*
x)*1i)^(5/2),x)
```

output
```
-(a*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*(A*cos(2*e + 2*f*x)*5i + A*cos(4*e + 4*f*x)*3i - 5*B*cos(2*e + 2
*f*x) + 3*B*cos(4*e + 4*f*x) - 5*A*sin(2*e + 2*f*x) - 3*A*sin(4*e + 4*f*x)
 - B*sin(2*e + 2*f*x)*5i + B*sin(4*e + 4*f*x)*3i))/(30*c^2*f*((c*(cos(2*e
+ 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\tan(fx+e)i+1}}{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^{2}+2\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)i-\sqrt{-\tan(fx+e)i+1}}dx\right)a+\left(\int \frac{}{\sqrt{-\tan(fx+e)i+1}\,\tan(\cdots}\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
( - sqrt(a)*a*(int(sqrt(tan(e + f*x)*i + 1)/(sqrt( - tan(e + f*x)*i + 1)*t
an(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - ta
n(e + f*x)*i + 1)),x)*a + int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(
sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*b*i + int((sqrt(tan(e +
 f*x)*i + 1)*tan(e + f*x))/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 +
2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)
),x)*a*i + int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x))/(sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i -
 sqrt( - tan(e + f*x)*i + 1)),x)*b))/(sqrt(c)*c**2)
```

3.802 $\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 155

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{3/2}}{7f(c-ic\tan(e+fx))^{7/2}}-\frac{(2iA-5B)(a+ia\tan(e+fx))^{3/2}}{35cf(c-ic\tan(e+fx))^{5/2}}-\frac{(2iA-5B)(a+ia\tan(e+fx))^{3/2}}{105c^2f(c-ic\tan(e+fx))^{3/2}}$$

output
```
-1/7*(I*A+B)*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*tan(f*x+e))^(7/2)-1/35*(2*I
*A-5*B)*(a+I*a*tan(f*x+e))^(3/2)/c/f/(c-I*c*tan(f*x+e))^(5/2)-1/105*(2*I*A
-5*B)*(a+I*a*tan(f*x+e))^(3/2)/c^2/f/(c-I*c*tan(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 13.94 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \frac{a\cos(e+fx)(\cos(fx)-i\sin(fx))(-21iA+5(-5iA}{}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(7/2),x]
```

output
```
(a*Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*((-21*I)*A + 5*((-5*I)*A + 2*B)*Co
s[2*(e + f*x)] - 5*(2*A + (5*I)*B)*Sin[2*(e + f*x)])*(Cos[5*e + 6*f*x] + I
*Sin[5*e + 6*f*x])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/
(210*c^4*f)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3042, 4071, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{\sqrt{i\tan(e+fx)a+a}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(2A+5iB)\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{7c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{(2A+5iB)\left(\frac{\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{5c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{7c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(\frac{(2A+5iB)\left(-\frac{i(a+ia\tan(e+fx))^{3/2}}{15ac^2(c-ic\tan(e+fx))^{3/2}}-\frac{i(a+ia\tan(e+fx))^{3/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{7c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(7/2),x]
```

output
```
(a*c*(-1/7*((I*A + B)*(a + I*a*Tan[e + f*x])^(3/2))/(a*c*(c - I*c*Tan[e +
f*x])^(7/2)) + ((2*A + (5*I)*B)*(((-1/5*I)*(a + I*a*Tan[e + f*x])^(3/2))/(
a*c*(c - I*c*Tan[e + f*x])^(5/2)) - ((I/15)*(a + I*a*Tan[e + f*x])^(3/2))/
(a*c^2*(c - I*c*Tan[e + f*x])^(3/2))))/(7*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^2\right)\left(5B-25iB\tan(fx+e)-5B\tan(fx+e)^2-23iA-10A\tan\right.}{105f\,c^4(i+\tan(fx+e))^5}$
default	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^2\right)\left(5B-25iB\tan(fx+e)-5B\tan(fx+e)^2-23iA-10A\tan\right.}{105f\,c^4(i+\tan(fx+e))^5}$
risch	$-\frac{a\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\left(15iA\,e^{6i(fx+e)}+15B\,e^{6i(fx+e)}+42iA\,e^{4i(fx+e)}+35iA\,e^{2i(fx+e)}-35B\,e^{2i(fx+e)}\right)}{420c^3\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^2\right)\left(-23+10i\tan(fx+e)+2\tan(fx+e)^2\right)}{105f\,c^4(i+\tan(fx+e))^5}-\frac{iB\sqrt{a(}}{}$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/105*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c^4*(1+
tan(f*x+e)^2)*(5*B-25*I*B*tan(f*x+e)-5*B*tan(f*x+e)^2-23*I*A-10*A*tan(f*x+
e)+2*I*A*tan(f*x+e)^2)/(I+tan(f*x+e))^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{(15\,(i\,A+B)ae^{(9i\,fx+9i\,e)}+3\,(19i\,A+5\,B)ae^{(7i\,fx+7i\,e)}+7\,(11i\,A-5\,B)ae^{(5i\,fx+5i\,e)}+35\,(i\,A-B)ae^{(3i\,\cdots}}{420\,c^4 f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="fricas")
```

output
```
-1/420*(15*(I*A + B)*a*e^(9*I*f*x + 9*I*e) + 3*(19*I*A + 5*B)*a*e^(7*I*f*x
 + 7*I*e) + 7*(11*I*A - 5*B)*a*e^(5*I*f*x + 5*I*e) + 35*(I*A - B)*a*e^(3*I
*f*x + 3*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*
e) + 1))/(c^4*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \int \frac{(ia(\tan{(e+fx)}-i))^{\frac{3}{2}}\,(A+B\tan{(e+fx)})}{(-ic\,(\tan{(e+fx)}+i))^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
7/2),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(3/2)*(A + B*tan(e + f*x))/(-I*c*(tan(e
 + f*x) + I))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.20

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \frac{\left(15\left(-iA-B\right)a\cos\left(\frac{7}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="maxima")
```

output
```
1/420*(15*(-I*A - B)*a*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 42*I*A*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 35*(-I
*A + B)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 15*(A - I
*B)*a*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 42*A*a*sin(5/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 35*(A + I*B)*a*sin(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(7/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.39

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx =$$

$$-\frac{a\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(A\,\cos\left(2\,e+2\,f\,x\right)\,35\mathrm{i}+A\,\cos\left(4\,e+4\,f\,x\right)\,42\mathrm{i}+A\,\cos\left(6\,e+6\,f\,x\right)\,1$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*
x)*1i)^(7/2),x)
```

output
```
-(a*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*(A*cos(2*e + 2*f*x)*35i + A*cos(4*e + 4*f*x)*42i + A*cos(6*e + 6
*f*x)*15i - 35*B*cos(2*e + 2*f*x) + 15*B*cos(6*e + 6*f*x) - 35*A*sin(2*e +
 2*f*x) - 42*A*sin(4*e + 4*f*x) - 15*A*sin(6*e + 6*f*x) - B*sin(2*e + 2*f*
x)*35i + B*sin(6*e + 6*f*x)*15i))/(420*c^3*f*((c*(cos(2*e + 2*f*x) - sin(2
*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*t
an(e + f*x)*a - 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)*b*i + int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
1)*tan(e + f*x)**3)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x
)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*f*
i + 5*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
 f*x)**3)/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2
*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f + int(( -
sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan
(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)*
*2 - 3*tan(e + f*x)*i + 1),x)*a*f*i + 5*int(( - sqrt(tan(e + f*x)*i + 1)*s
qrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**5*i - 3*tan(e +
 f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1)
,x)*b*f + 9*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(
e + f*x))/(tan(e + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2
*tan(e + f*x)**2 - 3*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*f*i + 9*int(
(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e
 + f*x)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2
 - 3*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f + 9*int((sqrt(tan(e + f*x)
*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**5*i - ...
```

3.803 $\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 208

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{3/2}}{9f(c-ic\tan(e+fx))^{9/2}} - \frac{(iA-2B)(a+ia\tan(e+fx))^{3/2}}{21cf(c-ic\tan(e+fx))^{7/2}} - \frac{2(iA-2B)(a+ia\tan(e+fx))^{3/2}}{105c^2f(c-ic\tan(e+fx))^{5/2}} - \frac{2(iA-2B)(a+ia\tan(e+fx))^{3/2}}{315c^3f(c-ic\tan(e+fx))^{3/2}}$$

output
```
-1/9*(I*A+B)*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*tan(f*x+e))^(9/2)-1/21*(I*A
-2*B)*(a+I*a*tan(f*x+e))^(3/2)/c/f/(c-I*c*tan(f*x+e))^(7/2)-2/105*(I*A-2*B
)*(a+I*a*tan(f*x+e))^(3/2)/c^2/f/(c-I*c*tan(f*x+e))^(5/2)-2/315*(I*A-2*B)*
(a+I*a*tan(f*x+e))^(3/2)/c^3/f/(c-I*c*tan(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 13.56 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.71

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \frac{a\cos(e+fx)(\cos(fx)-i\sin(fx))(9(-18iA+B)\text{co}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(9/2),x]
```

output
```
(a*Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*(9*((-18*I)*A + B)*Cos[e + f*x] +
35*((-2*I)*A + B)*Cos[3*(e + f*x)] - (A + (2*I)*B)*(27*Sin[e + f*x] + 35*S
in[3*(e + f*x)]))*(Cos[6*e + 7*f*x] + I*Sin[6*e + 7*f*x])*Sqrt[a + I*a*Tan
[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(1260*c^5*f)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{\sqrt{i\tan(e+fx)a+a}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(A+2iB)\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{3c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{f}$$

$$\Big\downarrow \textbf{55}$$

$$\frac{ac\left(\frac{(A+2iB)\left(\frac{2\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{7c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{3c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{f}$$

$$\Big\downarrow \textbf{55}$$

$$\frac{ac\left(\frac{(A+2iB)\left(\frac{2\left(\frac{\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{5c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{7c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{3c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{f}$$

$$\Big\downarrow \textbf{48}$$

$$\frac{ac\left(\frac{(A+2iB)\left(\frac{2\left(-\frac{i(a+ia\tan(e+fx))^{3/2}}{15ac^2(c-ic\tan(e+fx))^{3/2}}-\frac{i(a+ia\tan(e+fx))^{3/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{7c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{3c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(9/2),x]
```

output
```
(a*c*(-1/9*((I*A + B)*(a + I*a*Tan[e + f*x])^(3/2))/(a*c*(c - I*c*Tan[e +
f*x])^(9/2)) + ((A + (2*I)*B)*(((-1/7*I)*(a + I*a*Tan[e + f*x])^(3/2))/(a*
c*(c - I*c*Tan[e + f*x])^(7/2)) + (2*(((-1/5*I)*(a + I*a*Tan[e + f*x])^(3/
2))/(a*c*(c - I*c*Tan[e + f*x])^(5/2)) - ((I/15)*(a + I*a*Tan[e + f*x])^(3
/2))/(a*c^2*(c - I*c*Tan[e + f*x])^(3/2))))/(7*c)))/(3*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^{2}\right)\left(2iA\tan(fx+e)^{3}-24iB\tan(fx+e)^{2}-4B\tan(fx+e)^{3}\right.}{315f\,c^{5}(i+\tan(fx+e))^{6}}$
default	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^{2}\right)\left(2iA\tan(fx+e)^{3}-24iB\tan(fx+e)^{2}-4B\tan(fx+e)^{3}\right.}{315f\,c^{5}(i+\tan(fx+e))^{6}}$
risch	$-\frac{a\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,(35iA\,e^{8i(fx+e)}+35B\,e^{8i(fx+e)}+135iA\,e^{6i(fx+e)}+45B\,e^{6i(fx+e)}+189iA\,e^{4i(fx+e)}-63B\,e^{4i(fx+e}}{2520c^{4}\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
parts	$\frac{iA\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^{2}\right)\left(58-33i\tan(fx+e)-12\tan(fx+e)^{2}+2i\tan(fx+e)\right.}{315f\,c^{5}(i+\tan(fx+e))^{6}}$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/315*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c^5*(1+
tan(f*x+e)^2)*(2*I*A*tan(f*x+e)^3-24*I*B*tan(f*x+e)^2-4*B*tan(f*x+e)^3-33*
I*A*tan(f*x+e)-12*A*tan(f*x+e)^2+11*I*B+66*B*tan(f*x+e)+58*A)/(I+tan(f*x+e
))^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.65

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = -\frac{(35\,(iA+B)ae^{(11i\,fx+11i\,e)}+10\,(17iA+8\,B)ae^{(9i\,fx+9i\,e)}+18\,(18iA-B)ae^{(7i\,fx+7i\,e)}+42\,(7iA-4\,B)}{2520\,c^5 f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="fricas")
```

output
```
-1/2520*(35*(I*A + B)*a*e^(11*I*f*x + 11*I*e) + 10*(17*I*A + 8*B)*a*e^(9*I
*f*x + 9*I*e) + 18*(18*I*A - B)*a*e^(7*I*f*x + 7*I*e) + 42*(7*I*A - 4*B)*a
*e^(5*I*f*x + 5*I*e) + 105*(I*A - B)*a*e^(3*I*f*x + 3*I*e))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.25

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \frac{\left(35\left(-iA-B\right)a\cos\left(\frac{9}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(\right.\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="maxima")
```

output
```
1/2520*(35*(-I*A - B)*a*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 45*(-3*I*A - B)*a*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + 63*(-3*I*A + B)*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 105*(-I*A + B)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 35*(A - I*B)*a*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 45*
(3*A - I*B)*a*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 63*(3
*A + I*B)*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 105*(A
+ I*B)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^
(9/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.39

$$\int \frac{(a + ia\tan(e + fx))^{3/2}(A + B\tan(e + fx))}{(c - ic\tan(e + fx))^{9/2}}\,dx = -\frac{a\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)\,1\mathrm{i})}{\cos(2e+2fx)+1}}\,(A\,\cos(2e+2fx)\,105\mathrm{i} + A\,\cos(4e+4fx)\,189\mathrm{i} + A\,\cos(6e+6fx)$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*
x)*1i)^(9/2),x)
```

output
```
-(a*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*(A*cos(2*e + 2*f*x)*105i + A*cos(4*e + 4*f*x)*189i + A*cos(6*e +
 6*f*x)*135i + A*cos(8*e + 8*f*x)*35i - 105*B*cos(2*e + 2*f*x) - 63*B*cos(
4*e + 4*f*x) + 45*B*cos(6*e + 6*f*x) + 35*B*cos(8*e + 8*f*x) - 105*A*sin(2
*e + 2*f*x) - 189*A*sin(4*e + 4*f*x) - 135*A*sin(6*e + 6*f*x) - 35*A*sin(8
*e + 8*f*x) - B*sin(2*e + 2*f*x)*105i - B*sin(4*e + 4*f*x)*63i + B*sin(6*e
 + 6*f*x)*45i + B*sin(8*e + 8*f*x)*35i))/(2520*c^4*f*((c*(cos(2*e + 2*f*x)
 - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{(a + ia\tan(e + fx))^{3/2}(A + B\tan(e + fx))}{(c - ic\tan(e + fx))^{9/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a\left(\left(\int -\frac{\sqrt{\tan(fx+e)i+1}\,\sqrt{-\tan(fx+e)i+1}\,\mathrm{t}}{\tan(fx+e)^6+4\tan(fx+e)^5 i-5\tan(fx+e)^4-5\tan(}\right.\right.$$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e +
 f*x)**4 - 5*tan(e + f*x)**2 - 4*tan(e + f*x)*i + 1),x)*b - 2*int(( - sqrt
(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*
x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)**2 - 4*ta
n(e + f*x)*i + 1),x)*a*i - int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan
(e + f*x)**4 - 5*tan(e + f*x)**2 - 4*tan(e + f*x)*i + 1),x)*b - int(( - sq
rt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**6 + 4*t
an(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)**2 - 4*tan(e + f*x)*
i + 1),x)*a - int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*ta
n(e + f*x)**2)/(tan(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4
- 5*tan(e + f*x)**2 - 4*tan(e + f*x)*i + 1),x)*a + 2*int((sqrt(tan(e + f*x
)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**6 + 4
*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)**2 - 4*tan(e + f*x
)*i + 1),x)*b*i))/c**5
```

3.804 $\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 261

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{3/2}}{11f(c-ic\tan(e+fx))^{11/2}} - \frac{(4iA-7B)(a+ia\tan(e+fx))^{3/2}}{99cf(c-ic\tan(e+fx))^{9/2}} - \frac{(4iA-7B)(a+ia\tan(e+fx))^{3/2}}{231c^2f(c-ic\tan(e+fx))^{7/2}} - \frac{2(4iA-7B)(a+ia\tan(e+fx))^{3/2}}{1155c^3f(c-ic\tan(e+fx))^{5/2}} - \frac{2(4iA-7B)(a+ia\tan(e+fx))^{3/2}}{3465c^4f(c-ic\tan(e+fx))^{3/2}}$$

output

```
-1/11*(I*A+B)*(a+I*a*tan(f*x+e))^(3/2)/f/(c-I*c*tan(f*x+e))^(11/2)-1/99*(4
*I*A-7*B)*(a+I*a*tan(f*x+e))^(3/2)/c/f/(c-I*c*tan(f*x+e))^(9/2)-1/231*(4*I
*A-7*B)*(a+I*a*tan(f*x+e))^(3/2)/c^2/f/(c-I*c*tan(f*x+e))^(7/2)-2/1155*(4*
I*A-7*B)*(a+I*a*tan(f*x+e))^(3/2)/c^3/f/(c-I*c*tan(f*x+e))^(5/2)-2/3465*(4
*I*A-7*B)*(a+I*a*tan(f*x+e))^(3/2)/c^4/f/(c-I*c*tan(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 14.88 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.69

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx =$$
$$-\frac{ia\cos(e+fx)(\cos(fx)-i\sin(fx))(1485A+308(7A+iB)\cos(2(e+fx))+105(7A+4iB)\cos(4(e+$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(11/2),x]
```

output
```
((-1/27720*I)*a*Cos[e + f*x]*(Cos[f*x] - I*Sin[f*x])*(1485*A + 308*(7*A +
I*B)*Cos[2*(e + f*x)] + 105*(7*A + (4*I)*B)*Cos[4*(e + f*x)] - (616*I)*A*S
in[2*(e + f*x)] + 1078*B*Sin[2*(e + f*x)] - (420*I)*A*Sin[4*(e + f*x)] + 7
35*B*Sin[4*(e + f*x)])*(Cos[7*e + 8*f*x] + I*Sin[7*e + 8*f*x])*Sqrt[a + I*
a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(c^6*f)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}dx$$
$$\Big\downarrow \ 4071$$
$$\frac{ac\int \frac{\sqrt{i\tan(e+fx)a+a}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 87

$$\frac{ac\left(\frac{(4A+7iB)\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{11/2}}d\tan(e+fx)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

$\Big\downarrow$ 55

$$\frac{ac\left(\frac{(4A+7iB)\left(\frac{\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{3c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

$\Big\downarrow$ 55

$$\frac{ac\left(\frac{(4A+7iB)\left(\frac{\frac{2\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{7c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{7ac(c-ic\tan(e+fx))^{7/2}}}{3c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

$\Big\downarrow$ 55

$$\frac{ac\left(\frac{(4A+7iB)\left(\frac{\frac{2\left(\frac{\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{5c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{7c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{7ac(c-ic\tan(e+fx))^{7/2}}}{3c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{3/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

$\Big\downarrow$ 48

$$\frac{ac\left(\frac{(4A+7iB)\left(\frac{\frac{2\left(-\frac{i(a+ia\tan(e+fx))^{3/2}}{15ac^2(c-ic\tan(e+fx))^{3/2}}-\frac{i(a+ia\tan(e+fx))^{3/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{7c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{7ac(c-ic\tan(e+fx))^{7/2}}}{3c}-\frac{i(a+ia\tan(e+fx))^{3/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{(B+iA)(a+ia\,\mathrm{t}}{11ac(c-ic\,\mathrm{tan}}\right.}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(3/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(11/2),x]
```

output
```
(a*c*(-1/11*((I*A + B)*(a + I*a*Tan[e + f*x])^(3/2))/(a*c*(c - I*c*Tan[e +
 f*x])^(11/2)) + ((4*A + (7*I)*B)*(((-1/9*I)*(a + I*a*Tan[e + f*x])^(3/2))
/(a*c*(c - I*c*Tan[e + f*x])^(9/2)) + (((-1/7*I)*(a + I*a*Tan[e + f*x])^(3
/2))/(a*c*(c - I*c*Tan[e + f*x])^(7/2)) + (2*(((-1/5*I)*(a + I*a*Tan[e + f
*x])^(3/2))/(a*c*(c - I*c*Tan[e + f*x])^(5/2)) - ((I/15)*(a + I*a*Tan[e +
f*x])^(3/2))/(a*c^2*(c - I*c*Tan[e + f*x])^(3/2))))/(7*c))/(3*c)))/(11*c))
)/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.61

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^2\right)\left(14iB\tan(fx+e)^4+56iA\tan(fx+e)^3+8A\tan(fx+\right.}{3465f\,c^6(i+}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^2\right)\left(14iB\tan(fx+e)^4+56iA\tan(fx+e)^3+8A\tan(fx+\right.}{3465f\,c^6(i+}$
risch	$-\frac{a\sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,(315iA\,e^{10i(fx+e)}+315B\,e^{10i(fx+e)}+1540iA\,e^{8i(fx+e)}+770B\,e^{8i(fx+e)}+2970iA\,e^{6i(fx+e)}+2772}{55440c^5\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a\left(1+\tan(fx+e)^2\right)\left(56i\tan(fx+e)^3+8\tan(fx+e)^4-364i\tan(fx+e)\right.}{3465f\,c^6(i+\tan(fx+e))^7}$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/3465/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a/c^6*(1+
tan(f*x+e)^2)*(14*I*B*tan(f*x+e)^4+56*I*A*tan(f*x+e)^3+8*A*tan(f*x+e)^4-31
5*I*B*tan(f*x+e)^2-98*B*tan(f*x+e)^3-364*I*A*tan(f*x+e)-180*A*tan(f*x+e)^2
+91*I*B+637*B*tan(f*x+e)+547*A)/(I+tan(f*x+e))^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.59

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx =$$
$$-\frac{(315\,(i\,A+B)ae^{(13i\,fx+13i\,e)}+35\,(53i\,A+31\,B)ae^{(11i\,fx+11i\,e)}+110\,(41i\,A+7\,B)ae^{(9i\,fx+9i\,e)}+198\,(29i}{5}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11
/2),x, algorithm="fricas")
```

output
```
-1/55440*(315*(I*A + B)*a*e^(13*I*f*x + 13*I*e) + 35*(53*I*A + 31*B)*a*e^(
11*I*f*x + 11*I*e) + 110*(41*I*A + 7*B)*a*e^(9*I*f*x + 9*I*e) + 198*(29*I*
A - 7*B)*a*e^(7*I*f*x + 7*I*e) + 231*(17*I*A - 11*B)*a*e^(5*I*f*x + 5*I*e)
 + 1155*(I*A - B)*a*e^(3*I*f*x + 3*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))
*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^6*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.20

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \frac{\left(315\left(-iA-B\right)a\cos\left(\frac{11}{2}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\right.\right.\right.$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11
/2),x, algorithm="maxima")
```

output
```
1/55440*(315*(-I*A - B)*a*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 770*(-2*I*A - B)*a*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 2970*I*A*a*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
1386*(-2*I*A + B)*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 1155*(-I*A + B)*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
315*(A - I*B)*a*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 77
0*(2*A - I*B)*a*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 297
0*A*a*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1386*(2*A + I
*B)*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1155*(A + I*B
)*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(11/2
)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.21

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx =$$

$$-\frac{a\sqrt{\frac{a(\cos(2e+2fx)+1+\sin(2e+2fx)\,1i)}{\cos(2e+2fx)+1}}\,(A\cos(2e+2fx)\,1155i+A\cos(4e+4fx)\,2772i+A\cos(6e+6f$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(3/2))/(c - c*tan(e + f*
x)*1i)^(11/2),x)
```

output
```
-(a*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) +
1))^(1/2)*(A*cos(2*e + 2*f*x)*1155i + A*cos(4*e + 4*f*x)*2772i + A*cos(6*e
 + 6*f*x)*2970i + A*cos(8*e + 8*f*x)*1540i + A*cos(10*e + 10*f*x)*315i - 1
155*B*cos(2*e + 2*f*x) - 1386*B*cos(4*e + 4*f*x) + 770*B*cos(8*e + 8*f*x)
+ 315*B*cos(10*e + 10*f*x) - 1155*A*sin(2*e + 2*f*x) - 2772*A*sin(4*e + 4*
f*x) - 2970*A*sin(6*e + 6*f*x) - 1540*A*sin(8*e + 8*f*x) - 315*A*sin(10*e
+ 10*f*x) - B*sin(2*e + 2*f*x)*1155i - B*sin(4*e + 4*f*x)*1386i + B*sin(8*
e + 8*f*x)*770i + B*sin(10*e + 10*f*x)*315i))/(55440*c^5*f*((c*(cos(2*e +
2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(3/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a*(int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)**3)/(tan(e + f*x)**7*i - 5*tan(e + f*x)**6 - 9*tan(e + f*
x)**5*i + 5*tan(e + f*x)**4 - 5*tan(e + f*x)**3*i + 9*tan(e + f*x)**2 + 5*
tan(e + f*x)*i - 1),x)*b + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f
*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**7*i - 5*tan(e + f*x)**6 - 9*tan
(e + f*x)**5*i + 5*tan(e + f*x)**4 - 5*tan(e + f*x)**3*i + 9*tan(e + f*x)*
*2 + 5*tan(e + f*x)*i - 1),x)*a - 2*int((sqrt(tan(e + f*x)*i + 1)*sqrt( -
tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**7*i - 5*tan(e + f*x)**
6 - 9*tan(e + f*x)**5*i + 5*tan(e + f*x)**4 - 5*tan(e + f*x)**3*i + 9*tan(
e + f*x)**2 + 5*tan(e + f*x)*i - 1),x)*b*i - 2*int((sqrt(tan(e + f*x)*i +
1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**7*i - 5*tan(e
+ f*x)**6 - 9*tan(e + f*x)**5*i + 5*tan(e + f*x)**4 - 5*tan(e + f*x)**3*i
+ 9*tan(e + f*x)**2 + 5*tan(e + f*x)*i - 1),x)*a*i - int((sqrt(tan(e + f*x
)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**7*i - 5*
tan(e + f*x)**6 - 9*tan(e + f*x)**5*i + 5*tan(e + f*x)**4 - 5*tan(e + f*x)
**3*i + 9*tan(e + f*x)**2 + 5*tan(e + f*x)*i - 1),x)*b - int((sqrt(tan(e +
 f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**7*i - 5*tan(e + f
*x)**6 - 9*tan(e + f*x)**5*i + 5*tan(e + f*x)**4 - 5*tan(e + f*x)**3*i + 9
*tan(e + f*x)**2 + 5*tan(e + f*x)*i - 1),x)*a))/c**6
```

3.805 $\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 288

$$\begin{aligned}\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx &= \\ -\frac{a^{5/2}(6iA-B)c^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{8f} \\ +\frac{a^2(6A+iB)c^3\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{16f} \\ +\frac{a(6A+iB)c^2\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{24f} \\ -\frac{(6iA-B)c(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}{30f} \\ +\frac{B(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{7/2}}{6f}\end{aligned}$$

output
```
-1/8*a^(5/2)*(6*I*A-B)*c^(7/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(
1/2)/(c-I*c*tan(f*x+e))^(1/2))/f+1/16*a^2*(6*A+I*B)*c^3*tan(f*x+e)*(a+I*a*
tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)/f+1/24*a*(6*A+I*B)*c^2*tan(f*x+
e)*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2)/f-1/30*(6*I*A-B)*c*(a
+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(5/2)/f+1/6*B*(a+I*a*tan(f*x+e))
^(5/2)*(c-I*c*tan(f*x+e))^(7/2)/f
```

Mathematica [A] (warning: unable to verify)

Time = 18.78 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.97

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{(-6iA+B)c^4e^{-i(3e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\arctan\left(e^{i(e+fx)}\right)(a+ia\tan(e+fx))^{5/2}}{8\sqrt{\frac{c}{1+e^{2i(e+fx)}}}f\sec^{\frac{7}{2}}(e+fx)(\cos(fx)+i\sin(fx))^{5/2}(A\cos(e+fx)+B\sin} + \frac{\cos^3(e+fx)\sqrt{\sec(e+fx)(c\cos(e+fx)-ic\sin(e+fx))}\left(\sec(e)\sec^4(e+fx)(-6iA\cos(e)+6B\cos(e)\right.}{}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e
 + f*x])^(7/2),x]
```

output
```
(((-6*I)*A + B)*c^4*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e
+ f*x)))]*ArcTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[
e + f*x]))/(8*E^(I*(3*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e
+ f*x]^(7/2)*(Cos[f*x] + I*Sin[f*x])^(5/2)*(A*Cos[e + f*x] + B*Sin[e + f*x
])) + (Cos[e + f*x]^3*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x]
)]*(Sec[e]*Sec[e + f*x]^4*((-6*I)*A*Cos[e] + 6*B*Cos[e] - (5*I)*B*Sin[e])*
((c^3*Cos[2*e])/30 - (I/30)*c^3*Sin[2*e]) - I*B*c^3*Sec[e]*Sec[e + f*x]^5*
(Cos[2*e]/6 - (I/6)*Sin[2*e])*Sin[f*x] + Sec[e]*Sec[e + f*x]^3*(Cos[2*e]/2
4 - (I/24)*Sin[2*e])*(6*A*c^3*Sin[f*x] + I*B*c^3*Sin[f*x]) + Sec[e]*Sec[e
+ f*x]*(Cos[2*e]/16 - (I/16)*Sin[2*e])*(6*A*c^3*Sin[f*x] + I*B*c^3*Sin[f*x
]) + (6*A + I*B)*Sec[e + f*x]^2*((c^3*Cos[2*e])/24 - (I/24)*c^3*Sin[2*e])*
Tan[e] + (6*A + I*B)*((c^3*Cos[2*e])/16 - (I/16)*c^3*Sin[2*e])*Tan[e])*(a
+ I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(f*(Cos[f*x] + I*Sin[f*x])
^2*(A*Cos[e + f*x] + B*Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {3042, 4071, 90, 59, 40, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int(i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 90$$

$$\frac{ac\left(\frac{1}{6}(6A+iB)\int(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))}{6ac}\right.}{f}$$

$$\Big\downarrow \; 59$$

$$\frac{ac\left(\frac{1}{6}(6A+iB)\left(c\int(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)-\frac{i(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+f}{5a}\right.\right.}{f}$$

$$\Big\downarrow \; 40$$

$$\frac{ac\left(\frac{1}{6}(6A+iB)\left(c\left(\frac{3}{4}ac\int\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{1}{4}\tan(e+fx)(a+ia\tan(e+\right.\right.\right.}{f}$$

$$\Big\downarrow \; 40$$

$$\frac{ac\left(\frac{1}{6}(6A+iB)\left(c\left(\frac{3}{4}ac\left(\frac{1}{2}ac\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\right.\right.\right.\right.}{}$$

$$\Big\downarrow \; 45$$

$$ac\left(\frac{1}{6}(6A+iB)\left(c\left(\frac{3}{4}ac\left(ac\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan\right.\right.\right.\right.$$

$\downarrow$ 218

$$ac\left(\frac{1}{6}(6A+iB)\left(c\left(\frac{3}{4}ac\left(\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}-i\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e}}{\sqrt{a}\sqrt{c-ic\tan(e}}\right.\right.\right.\right.\right.$$

input
```
Int[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(7/2),x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(7/2))/(6*a*c
) + ((6*A + I*B)*(((-1/5*I)*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e +
f*x])^(5/2))/a + c*((Tan[e + f*x]*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Ta
n[e + f*x])^(3/2))/4 + (3*a*c*((-I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
 + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + (Tan[e + f*x
]*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/2))/4)))/6))/f
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(236) = 472$.

Time = 0.88 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.66

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2c^3\left(40iB\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}\,\tan(fx+e)^5+48iA\sqrt{ac\left(1+\tan\right.}\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2c^3\left(40iB\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}\,\tan(fx+e)^5+48iA\sqrt{ac\left(1+\tan\right.}\right.}{}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2c^3\left(8i\tan(fx+e)^4\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+16i\tan(fx+e)^2\sqrt{a}\right.}{}$

input

```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,m
ethod=_RETURNVERBOSE)
```

output

```
-1/240/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2*c^3*(4
0*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^5+48*I*A*(a*c*(1
+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4+70*I*B*(a*c*(1+tan(f*x+e)^2
))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-48*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^
(1/2)*tan(f*x+e)^4+96*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x
+e)^2-60*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-15*I*B*ln
((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*
c+15*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)-96*B*(a*c)^(1
/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+48*I*A*(a*c*(1+tan(f*x+e)^2)
)^(1/2)*(a*c)^(1/2)-90*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)
^2))^(1/2))/(a*c)^(1/2))*a*c-150*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2
)*tan(f*x+e)-48*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2)/(a
*c*(1+tan(f*x+e)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 756 vs. 2(220) = 440.

Time = 0.11 (sec) , antiderivative size = 756, normalized size of antiderivative = 2.62

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="fricas")
```

output
```
1/480*(15*sqrt((36*A^2 + 12*I*A*B - B^2)*a^5*c^7/f^2)*(f*e^(10*I*f*x + 10*
I*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*
x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((6*I*A - B)*a^2*c^3*e
^(3*I*f*x + 3*I*e) + (6*I*A - B)*a^2*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f
*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((36*A^2 + 12*I*
A*B - B^2)*a^5*c^7/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-6*I*A + B)*a^2*c^3
*e^(2*I*f*x + 2*I*e) + (-6*I*A + B)*a^2*c^3)) - 15*sqrt((36*A^2 + 12*I*A*B
 - B^2)*a^5*c^7/f^2)*(f*e^(10*I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e) +
10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I
*e) + f)*log(-4*(2*((6*I*A - B)*a^2*c^3*e^(3*I*f*x + 3*I*e) + (6*I*A - B)*
a^2*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*
f*x + 2*I*e) + 1)) - sqrt((36*A^2 + 12*I*A*B - B^2)*a^5*c^7/f^2)*(f*e^(2*I
*f*x + 2*I*e) - f))/((-6*I*A + B)*a^2*c^3*e^(2*I*f*x + 2*I*e) + (-6*I*A +
B)*a^2*c^3)) - 4*(15*(6*I*A - B)*a^2*c^3*e^(11*I*f*x + 11*I*e) + 85*(6*I*A
 - B)*a^2*c^3*e^(9*I*f*x + 9*I*e) + 198*(6*I*A - B)*a^2*c^3*e^(7*I*f*x + 7
*I*e) + 6*(58*I*A - 223*B)*a^2*c^3*e^(5*I*f*x + 5*I*e) + 85*(-6*I*A + B)*a
^2*c^3*e^(3*I*f*x + 3*I*e) + 15*(-6*I*A + B)*a^2*c^3*e^(I*f*x + I*e))*sqrt
(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(10*
I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*
f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2033 vs. 2(220) = 440.

Time = 4.05 (sec) , antiderivative size = 2033, normalized size of antiderivative = 7.06

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c - ic\tan(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="maxima")
```

output
```
-3840*(60*(6*A + I*B)*a^2*c^3*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) + 340*(6*A + I*B)*a^2*c^3*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 792*(6*A + I*B)*a^2*c^3*cos(7/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 24*(58*A + 223*I*B)*a^2*c^3*cos(5/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 340*(6*A + I*B)*a^2*c^3*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*(6*A + I*B)*a^2*c^3*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 60*(6*I*A - B)*a^2*c^3*sin(11/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 340*(6*I*A - B)*a^2*c^3*sin(9
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 792*(6*I*A - B)*a^2*c^3*
sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 24*(58*I*A - 223*B)
*a^2*c^3*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 340*(-6*I*
A + B)*a^2*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 60*(
-6*I*A + B)*a^2*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 30*((6*A + I*B)*a^2*c^3*cos(12*f*x + 12*e) + 6*(6*A + I*B)*a^2*c^3*cos(10
*f*x + 10*e) + 15*(6*A + I*B)*a^2*c^3*cos(8*f*x + 8*e) + 20*(6*A + I*B)*a^
2*c^3*cos(6*f*x + 6*e) + 15*(6*A + I*B)*a^2*c^3*cos(4*f*x + 4*e) + 6*(6*A
+ I*B)*a^2*c^3*cos(2*f*x + 2*e) + (6*I*A - B)*a^2*c^3*sin(12*f*x + 12*e) +
 6*(6*I*A - B)*a^2*c^3*sin(10*f*x + 10*e) + 15*(6*I*A - B)*a^2*c^3*sin(8*f
*x + 8*e) + 20*(6*I*A - B)*a^2*c^3*sin(6*f*x + 6*e) + 15*(6*I*A - B)*a^2*c
^3*sin(4*f*x + 4*e) + 6*(6*I*A - B)*a^2*c^3*sin(2*f*x + 2*e) + (6*A + I...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{\frac{5}{2}}(-i\,c\tan(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(5/2)*(-I*c*tan(f*x
+ e) + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \int (A+B\tan(e+f\,x))\,(a+a\tan(e+f\,x)\,1\mathrm{i})^{5/2}\,(c-c\tan(e+f\,x)\,1\mathrm{i})^{7/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)
*1i)^(7/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)
*1i)^(7/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2c^3\left(-\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^4\,ai+\sqrt{\tan\cdots}\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*c**3*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**4*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f
*x)*i + 1)*tan(e + f*x)**4*b - 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e +
f*x)*i + 1)*tan(e + f*x)**2*a*i + 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**2*b - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
+ f*x)*i + 1)*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b
 - 5*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)
**6,x)*b*f*i + 5*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)**4,x)*a*f - 10*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f
*x)*i + 1)*tan(e + f*x)**4,x)*b*f*i + 10*int(sqrt(tan(e + f*x)*i + 1)*sqrt
( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)*a*f - 5*int(sqrt(tan(e + f*x)*i
 + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)*b*f*i + 5*int(sqrt(ta
n(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1),x)*a*f))/(5*f)
```

3.806 $\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 213

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = -\frac{3ia^{5/2}Ac^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{4f} + \frac{3a^2Ac^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{8f} + \frac{aAc\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{4f} + \frac{B(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}{5f}$$

output
```
-3/4*I*a^(5/2)*A*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(
c-I*c*tan(f*x+e))^(1/2))/f+3/8*a^2*A*c^2*tan(f*x+e)*(a+I*a*tan(f*x+e))^(1/
2)*(c-I*c*tan(f*x+e))^(1/2)/f+1/4*a*A*c*tan(f*x+e)*(a+I*a*tan(f*x+e))^(3/2
)*(c-I*c*tan(f*x+e))^(3/2)/f+1/5*B*(a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x
+e))^(5/2)/f
```

Mathematica [A] (verified)

Time = 5.86 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.69

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\frac{a^{5/2}c^3\left(-\frac{240A\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)(i+\tan(e+fx))}{\sqrt{1-i\tan(e+fx)}}+\frac{\sqrt{a}\sec^6(e+fx)(64B+70A\sin(2(e+fx))+15A}{\sqrt{a+ia\tan(e+fx)}}\right.}{320f\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e
 + f*x])^(5/2),x]
```

output
```
(a^(5/2)*c^3*((-240*A*ArcSin[Sqrt[a + I*a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[a])]
*(I + Tan[e + f*x]))/Sqrt[1 - I*Tan[e + f*x]] + (Sqrt[a]*Sec[e + f*x]^6*(6
4*B + 70*A*Sin[2*(e + f*x)] + 15*A*Sin[4*(e + f*x)]))/Sqrt[a + I*a*Tan[e +
 f*x]]))/(320*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3042, 4071, 90, 40, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int (i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 90$$

$$\frac{ac\left(A\int (i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}{5ac}\right)}{f}$$

$$\Big\downarrow \; 40$$

$$\frac{ac\left(A\left(\frac{3}{4}ac\int \sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{1}{4}\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-\right.\right.}{f}$$

$$\Big\downarrow \; 40$$

$$\frac{ac\left(A\left(\frac{3}{4}ac\left(\frac{1}{2}ac\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e\right.\right.\right.}{f}$$

$$\Big\downarrow \; 45$$

$$\frac{ac\left(A\left(\frac{3}{4}ac\left(ac\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}\right)\right.\right.}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{ac\left(A\left(\frac{3}{4}ac\left(\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}-i\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)\right)+\frac{1}{4}\text{ta}\right.\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(5/2),x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(5/2))/(5*a*c
) + A*((Tan[e + f*x]*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])^(
3/2))/4 + (3*a*c*((-I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e
+ f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + (Tan[e + f*x]*Sqrt[a + I*
a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/2))/4)))/f
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.04

method	result
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2c^2\left(2\tan(fx+e)^3\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+3\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\cdots}{\cdots}\right.\right.\cdots}{8f\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2c^2\left(8B\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}\,\tan(fx+e)^4+10A\sqrt{ac\left(1+\tan(fx+\cdots\right.}\right.\cdots}{\cdots}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2c^2\left(8B\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}\,\tan(fx+e)^4+10A\sqrt{ac\left(1+\tan(fx+\cdots\right.}\right.\cdots}{\cdots}$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/8*A/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2*c^2*(2*
tan(f*x+e)^3*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+3*ln((a*c*tan(f*x+e)
+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+5*tan(f*x+e)*(
a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c
)^(1/2)+1/5*B/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2
*c^2*(1+tan(f*x+e)^2)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(163) = 326.

Time = 0.11 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.75

$$\int(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
1/80*(15*sqrt(A^2*a^5*c^5/f^2)*(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6
*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*(A
*a^2*c^2*e^(3*I*f*x + 3*I*e) + A*a^2*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f
*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(A^2*a^5*c^5/f^2
)*(I*f*e^(2*I*f*x + 2*I*e) - I*f))/(A*a^2*c^2*e^(2*I*f*x + 2*I*e) + A*a^2*
c^2)) - 15*sqrt(A^2*a^5*c^5/f^2)*(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x +
 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*
(A*a^2*c^2*e^(3*I*f*x + 3*I*e) + A*a^2*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I
*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(A^2*a^5*c^5/f
^2)*(-I*f*e^(2*I*f*x + 2*I*e) + I*f))/(A*a^2*c^2*e^(2*I*f*x + 2*I*e) + A*a
^2*c^2)) + 4*(-15*I*A*a^2*c^2*e^(9*I*f*x + 9*I*e) - 70*I*A*a^2*c^2*e^(7*I*
f*x + 7*I*e) + 128*B*a^2*c^2*e^(5*I*f*x + 5*I*e) + 70*I*A*a^2*c^2*e^(3*I*f
*x + 3*I*e) + 15*I*A*a^2*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e)
+ 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6
*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1445 vs. $2(163) = 326$.

Time = 0.66 (sec) , antiderivative size = 1445, normalized size of antiderivative = 6.78

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
-(60*A*a^2*c^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 280*
A*a^2*c^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 512*I*B*a
^2*c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 280*A*a^2*c^
2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*A*a^2*c^2*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 60*I*A*a^2*c^2*sin(9/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 280*I*A*a^2*c^2*sin(7/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 512*B*a^2*c^2*sin(5/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 280*I*A*a^2*c^2*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*I*A*a^2*c^2*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 30*(A*a^2*c^2*cos(10*f*x + 10*e) + 5*A*a^2
*c^2*cos(8*f*x + 8*e) + 10*A*a^2*c^2*cos(6*f*x + 6*e) + 10*A*a^2*c^2*cos(4
*f*x + 4*e) + 5*A*a^2*c^2*cos(2*f*x + 2*e) + I*A*a^2*c^2*sin(10*f*x + 10*e
) + 5*I*A*a^2*c^2*sin(8*f*x + 8*e) + 10*I*A*a^2*c^2*sin(6*f*x + 6*e) + 10*
I*A*a^2*c^2*sin(4*f*x + 4*e) + 5*I*A*a^2*c^2*sin(2*f*x + 2*e) + A*a^2*c^2)
*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 30*(A*a^2*c^2*cos(10*f*x
+ 10*e) + 5*A*a^2*c^2*cos(8*f*x + 8*e) + 10*A*a^2*c^2*cos(6*f*x + 6*e) + 1
0*A*a^2*c^2*cos(4*f*x + 4*e) + 5*A*a^2*c^2*cos(2*f*x + 2*e) + I*A*a^2*c^2*
sin(10*f*x + 10*e) + 5*I*A*a^2*c^2*sin(8*f*x + 8*e) + 10*I*A*a^2*c^2*sin(6
*f*x + 6*e) + 10*I*A*a^2*c^2*sin(4*f*x + 4*e) + 5*I*A*a^2*c^2*sin(2*f*x...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{\frac{5}{2}}(-i\,c\tan(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(5/2)*(-I*c*tan(f*x
+ e) + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1i\right)^{5/2}\left(c-c\tan(e+f\,x)\,1i\right)^{5/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)
*1i)^(5/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)
*1i)^(5/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2c^2\Big(\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+1}\,\tan\left(fx+e\right)^4 b+2\sqrt{\tan}}{}$$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*c**2*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**4*b + 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**2*b + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*b + 5*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(
e + f*x)**4,x)*a*f + 10*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**2,x)*a*f + 5*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan
(e + f*x)*i + 1),x)*a*f))/(5*f)
```

3.807 $\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 222

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = -\frac{a^{5/2}(4iA+B)c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{4f} + \frac{a^2(4A-iB)c\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{8f} + \frac{a(4iA+B)(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{12f} + \frac{B(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}{4f}$$

output
```
-1/4*a^(5/2)*(4*I*A+B)*c^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(
1/2)/(c-I*c*tan(f*x+e))^(1/2))/f+1/8*a^2*(4*A-I*B)*c*tan(f*x+e)*(a+I*a*tan
(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)/f+1/12*a*(4*I*A+B)*(a+I*a*tan(f*x+
e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2)/f+1/4*B*(a+I*a*tan(f*x+e))^(5/2)*(c-I*c
*tan(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 5.58 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{a^{5/2}c^2(i+\tan(e+fx))\left(-6(4A-iB)\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\sqrt{a+ia\tan(e+f}\right.}{24f\sqrt{}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e
 + f*x])^(3/2),x]
```

output
```
(a^(5/2)*c^2*(I + Tan[e + f*x])*(-6*(4*A - I*B)*ArcSin[Sqrt[a + I*a*Tan[e
+ f*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[a + I*a*Tan[e + f*x]] + Sqrt[a]*Sqrt[1 - I
*Tan[e + f*x]]*(-I + Tan[e + f*x])*(8*(I*A + B) + 3*(4*A + I*B)*Tan[e + f*
x] + 8*(I*A + B)*Tan[e + f*x]^2 + (6*I)*B*Tan[e + f*x]^3)))/(24*f*Sqrt[1 -
 I*Tan[e + f*x]]*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 90, 59, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int (i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)}{f}$$

$$\downarrow \text{90}$$

$$\frac{ac\left(\frac{1}{4}(4A-iB)\int(i\tan(e+fx)a+a)^{3/2}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}{4ac}}{f}$$

$$\downarrow \text{59}$$

$$\frac{ac\left(\frac{1}{4}(4A-iB)\left(a\int\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{3c}\right.\right.}{f}$$

$$\downarrow \text{40}$$

$$\frac{ac\left(\frac{1}{4}(4A-iB)\left(a\left(\frac{1}{2}ac\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-\right.\right.\right.}{f}$$

$$\downarrow \text{45}$$

$$\frac{ac\left(\frac{1}{4}(4A-iB)\left(a\left(ac\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e\right.\right.\right.}{f}$$

$$\downarrow \text{218}$$

$$\frac{ac\left(\frac{1}{4}(4A-iB)\left(a\left(\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}-i\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)\right.\right.\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(3/2),x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(3/2))/(4*a*c
) + ((4*A - I*B)*(((I/3)*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x
])^(3/2))/c + a*((-I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e +
 f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + (Tan[e + f*x]*Sqrt[a + I*a
*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/2))/4))/f
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2c\left(6iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+8iA\sqrt{ac}\,\sqrt{ac\left(1+\tan(\right.}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2c\left(6iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+8iA\sqrt{ac}\,\sqrt{ac\left(1+\tan(\right.}\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2c\left(2i\tan(fx+e)^2\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+2i\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx\right.}\right.}{6f\sqrt{ac\left(1+\tan(fx+e)^2\right.}}$

input

```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output

```
1/24/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2*c*(6*I*B
*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3+8*I*A*(a*c)^(1/2)*(
a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2-3*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1
/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+3*I*B*(a*c)^(1/2)*(a*c*
(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+8*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^
(1/2)*tan(f*x+e)^2+8*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)+12*A*ln(
(a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c
+12*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+8*B*(a*c)^(1/2)*
(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(168) = 336$.

Time = 0.11 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.77

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
-1/48*(3*sqrt((16*A^2 - 8*I*A*B - B^2)*a^5*c^3/f^2)*(f*e^(6*I*f*x + 6*I*e)
 + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((-4*I
*A - B)*a^2*c*e^(3*I*f*x + 3*I*e) + (-4*I*A - B)*a^2*c*e^(I*f*x + I*e))*sq
rt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((
16*A^2 - 8*I*A*B - B^2)*a^5*c^3/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((4*I*A
+ B)*a^2*c*e^(2*I*f*x + 2*I*e) + (4*I*A + B)*a^2*c)) - 3*sqrt((16*A^2 - 8*
I*A*B - B^2)*a^5*c^3/f^2)*(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e)
 + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((-4*I*A - B)*a^2*c*e^(3*I*f*x +
 3*I*e) + (-4*I*A - B)*a^2*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e)
+ 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((16*A^2 - 8*I*A*B - B^2)*a^
5*c^3/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((4*I*A + B)*a^2*c*e^(2*I*f*x + 2*
I*e) + (4*I*A + B)*a^2*c)) + 4*(3*(4*I*A + B)*a^2*c*e^(7*I*f*x + 7*I*e) -
(20*I*A + 53*B)*a^2*c*e^(5*I*f*x + 5*I*e) + 11*(-4*I*A - B)*a^2*c*e^(3*I*f
*x + 3*I*e) + 3*(-4*I*A - B)*a^2*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(6*I*f*x + 6*I*e) + 3*
f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1375 vs. $2(168) = 336$.

Time = 0.81 (sec) , antiderivative size = 1375, normalized size of antiderivative = 6.19

$$\int (a + ia\tan(e + fx))^{5/2}(A + B\tan(e + fx))(c - ic\tan(e + fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
-96*(12*(4*A - I*B)*a^2*c*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 4*(20*A - 53*I*B)*a^2*c*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 44*(4*A - I*B)*a^2*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 12*(4*A - I*B)*a^2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - 12*(-4*I*A - B)*a^2*c*sin(7/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - 4*(20*I*A + 53*B)*a^2*c*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 44*(4*I*A + B)*a^2*c*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 12*(4*I*A + B)*a^2*c*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 6*((4*A - I*B)*a^2*c*cos(8*f*x + 8*e) + 4*(4*A
 - I*B)*a^2*c*cos(6*f*x + 6*e) + 6*(4*A - I*B)*a^2*c*cos(4*f*x + 4*e) + 4*
(4*A - I*B)*a^2*c*cos(2*f*x + 2*e) - (-4*I*A - B)*a^2*c*sin(8*f*x + 8*e) -
 4*(-4*I*A - B)*a^2*c*sin(6*f*x + 6*e) - 6*(-4*I*A - B)*a^2*c*sin(4*f*x +
4*e) - 4*(-4*I*A - B)*a^2*c*sin(2*f*x + 2*e) + (4*A - I*B)*a^2*c)*arctan2(
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 6*((4*A - I*B)*a^2*c*cos(8*f*x + 8
*e) + 4*(4*A - I*B)*a^2*c*cos(6*f*x + 6*e) + 6*(4*A - I*B)*a^2*c*cos(4*f*x
 + 4*e) + 4*(4*A - I*B)*a^2*c*cos(2*f*x + 2*e) - (-4*I*A - B)*a^2*c*sin(8*
f*x + 8*e) - 4*(-4*I*A - B)*a^2*c*sin(6*f*x + 6*e) - 6*(-4*I*A - B)*a^2*c*
sin(4*f*x + 4*e) - 4*(-4*I*A - B)*a^2*c*sin(2*f*x + 2*e) + (4*A - I*B)*a^2
*c)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{\frac{5}{2}}(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(5/2)*(-I*c*tan(f*x
+ e) + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)
*1i)^(3/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)
*1i)^(3/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a^2c\Big(\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^2\,ai+\sqrt{\tan(}$$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*c*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)**2*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2*b + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b + 3*int(s
qrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4,x)*b*f
*i + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*
x)**2,x)*a*f + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)**2,x)*b*f*i + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e +
f*x)*i + 1),x)*a*f))/(3*f)
```

3.808 $\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 217

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = -\frac{a^{5/2}(3iA+2B)\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{f} + \frac{a^2(3iA+2B)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2f} + \frac{a(3iA+2B)(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{6f} + \frac{B(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}{3f}$$

output
```
-a^(5/2)*(3*I*A+2*B)*c^(1/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/
2)/(c-I*c*tan(f*x+e))^(1/2))/f+1/2*a^2*(3*I*A+2*B)*(a+I*a*tan(f*x+e))^(1/2
)*(c-I*c*tan(f*x+e))^(1/2)/f+1/6*a*(3*I*A+2*B)*(a+I*a*tan(f*x+e))^(3/2)*(c
-I*c*tan(f*x+e))^(1/2)/f+1/3*B*(a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))
^(1/2)/f
```

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.91

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx =$$
$$-\frac{a^{5/2}c(i+\tan(e+fx))\left(6(3A-2iB)\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\sqrt{a+ia\tan(e+fx)}+\sqrt{a}\sqrt{1-i\tan(e+}\right.}{6f\sqrt{1-i\tan(e+fx)}\sqrt{a+ia\tan(e+}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*Sqrt[c - I*c*T
an[e + f*x]],x]
```

output
```
-1/6*(a^(5/2)*c*(I + Tan[e + f*x])*(6*(3*A - (2*I)*B)*ArcSin[Sqrt[a + I*a*
Tan[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[a + I*a*Tan[e + f*x]] + Sqrt[a]*Sqrt
[1 - I*Tan[e + f*x]]*(-I + Tan[e + f*x])*((-12*I)*A - 10*B + 3*(A - (2*I)*
B)*Tan[e + f*x] + 2*B*Tan[e + f*x]^2)))/(f*Sqrt[1 - I*Tan[e + f*x]]*Sqrt[a
 + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 90, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 90$$

$$\frac{ac\left(\frac{1}{3}(3A-2iB)\int \frac{(i\tan(e+fx)a+a)^{3/2}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}{3ac}\right)}{f}$$

$$\Big\downarrow \; 60$$

$$\frac{ac\left(\frac{1}{3}(3A-2iB)\left(\frac{3}{2}a\int \frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2c}\right)+\frac{B(a+ia\tan(e+fx))^{5/2}\sqrt{c}}{3ac}\right.}{f}$$

$$\Big\downarrow \; 60$$

$$\frac{ac\left(\frac{1}{3}(3A-2iB)\left(\frac{3}{2}a\left(a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{i(a+ia}{}\right.\right.}{f}$$

$$\Big\downarrow \; 45$$

$$\frac{ac\left(\frac{1}{3}(3A-2iB)\left(\frac{3}{2}a\left(2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{i(a+ia\tan(e+f}{}\right.\right.}{f}$$

$$\Big\downarrow \; 218$$

$$\frac{ac\left(\frac{1}{3}(3A-2iB)\left(\frac{3}{2}a\left(\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-i}}{2c}\right.\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e +
 f*x]],x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e + f*x]])/(3*a*c)
+ ((3*A - (2*I)*B)*(((I/2)*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e
 + f*x]])/c + (3*a*(((-2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f
*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[c] + (I*Sqrt[a + I*a*Tan
[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/c))/2))/3))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(-6iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac+6iB\sqrt{ac}\,\sqrt{ac\left(1\right.}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(-6iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac+6iB\sqrt{ac}\,\sqrt{ac\left(1\right.}\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(4i\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}-\tan(fx+e)\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{a}\right.}{2f\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}}$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/6/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2*(-6*I*B*l
n((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a
*c+6*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-2*B*(a*c)^(1/
2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+12*I*A*(a*c)^(1/2)*(a*c*(1+ta
n(f*x+e)^2))^(1/2)+9*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2
))^(1/2))/(a*c)^(1/2))*a*c-3*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*ta
n(f*x+e)+10*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2)/(a*c*(
1+tan(f*x+e)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(165) = 330.

Time = 0.12 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.52

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
-1/12*(3*sqrt((9*A^2 - 12*I*A*B - 4*B^2)*a^5*c/f^2)*(f*e^(4*I*f*x + 4*I*e)
 + 2*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((-3*I*A - 2*B)*a^2*e^(3*I*f*x +
 3*I*e) + (-3*I*A - 2*B)*a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e)
+ 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((9*A^2 - 12*I*A*B - 4*B^2)*
a^5*c/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((3*I*A + 2*B)*a^2*e^(2*I*f*x + 2*
I*e) + (3*I*A + 2*B)*a^2)) - 3*sqrt((9*A^2 - 12*I*A*B - 4*B^2)*a^5*c/f^2)*
(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((-3*I*A -
 2*B)*a^2*e^(3*I*f*x + 3*I*e) + (-3*I*A - 2*B)*a^2*e^(I*f*x + I*e))*sqrt(a
/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((9*A^
2 - 12*I*A*B - 4*B^2)*a^5*c/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((3*I*A + 2*
B)*a^2*e^(2*I*f*x + 2*I*e) + (3*I*A + 2*B)*a^2)) + 4*(3*(-5*I*A - 6*B)*a^2
*e^(5*I*f*x + 5*I*e) + 8*(-3*I*A - 2*B)*a^2*e^(3*I*f*x + 3*I*e) + 3*(-3*I*
A - 2*B)*a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^
(2*I*f*x + 2*I*e) + 1)))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e)
+ f)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx=\int (ia(\tan{(e+fx)}-i))^{\frac{5}{2}}\sqrt{-ic(\tan{(e+fx)}+i)}(A+B\tan{(e+fx)})\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(5/2)*sqrt(-I*c*(tan(e + f*x) + I))*(A
+ B*tan(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(165) = 330$.

Time = 0.38 (sec) , antiderivative size = 1087, normalized size of antiderivative = 5.01

$$\int (a + ia\tan(e + fx))^{5/2}(A + B\tan(e + fx))\sqrt{c - ic\tan(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
6*(12*(5*A - 6*I*B)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 32*(3*A - 2*I*B)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 12*(3*A - 2*I*B)*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 12*(5*I*A + 6*B)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 32*(3*I*A + 2*B)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 12*(3*I*A + 2*B)*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - 6*((3*A - 2*I*B)*a^2*cos(6*f*x + 6*e) + 3*(3*A - 2*I*B)*a^2
*cos(4*f*x + 4*e) + 3*(3*A - 2*I*B)*a^2*cos(2*f*x + 2*e) - (-3*I*A - 2*B)*
a^2*sin(6*f*x + 6*e) - 3*(-3*I*A - 2*B)*a^2*sin(4*f*x + 4*e) - 3*(-3*I*A -
 2*B)*a^2*sin(2*f*x + 2*e) + (3*A - 2*I*B)*a^2)*arctan2(cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 1) - 6*((3*A - 2*I*B)*a^2*cos(6*f*x + 6*e) + 3*(3*A - 2*I
*B)*a^2*cos(4*f*x + 4*e) + 3*(3*A - 2*I*B)*a^2*cos(2*f*x + 2*e) - (-3*I*A
- 2*B)*a^2*sin(6*f*x + 6*e) - 3*(-3*I*A - 2*B)*a^2*sin(4*f*x + 4*e) - 3*(-
3*I*A - 2*B)*a^2*sin(2*f*x + 2*e) + (3*A - 2*I*B)*a^2)*arctan2(cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 1) + 3*((-3*I*A - 2*B)*a^2*cos(6*f*x + 6*e) + 3*(
-3*I*A - 2*B)*a^2*cos(4*f*x + 4*e) + 3*(-3*I*A - 2*B)*a^2*cos(2*f*x + 2*e)
 + (3*A - 2*I*B)*a^2*sin(6*f*x + 6*e) + 3*(3*A - 2*I*B)*a^2*sin(4*f*x + 4*
e) + 3*(3*A - 2*I*B)*a^2*sin(2*f*x + 2*e) + (-3*I*A - 2*B)*a^2)*log(cos...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx=\int (B\tan(fx+e)+A)(ia\tan(fx+e)+a)^{\frac{5}{2}}\sqrt{-ic\tan(fx+$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(5/2)*sqrt(-I*c*tan(
f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx=\int (A+B\tan(e+fx))\,\left(a+a\tan(e+fx)\,1\mathrm{i}\right)^{5/2}\,\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)
*1i)^(1/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x)
*1i)^(1/2), x)
```

Reduce [F]

$$\int (a + ia\tan(e + fx))^{5/2}(A + B\tan(e+fx))\sqrt{c - ic\tan(e + fx)}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(-\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx$$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2*b + 6*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
 + 1)*a*i + 5*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - 3*i
nt(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)
*a*f + 6*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x)**2,x)*b*f*i + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1),x)*a*f))/(3*f)
```

3.809 $\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 227

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{3a^{5/2}(2iA+3B)\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}f} - \frac{(iA+B)(a+ia\tan(e+fx))^{5/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{3a^2(2iA+3B)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2cf} - \frac{a(2iA+3B)(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2cf}$$

output

```
3*a^(5/2)*(2*I*A+3*B)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I
*c*tan(f*x+e))^(1/2))/c^(1/2)/f-(I*A+B)*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I*c*
tan(f*x+e))^(1/2)-3/2*a^2*(2*I*A+3*B)*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(
f*x+e))^(1/2)/c/f-1/2*a*(2*I*A+3*B)*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*
x+e))^(1/2)/c/f
```

Mathematica [A] (verified)

Time = 6.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{a^{5/2}\left(\frac{6(2A-3iB)\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)(i+\tan(e+fx))}{\sqrt{1-i\tan(e+fx)}} - \frac{i\sqrt{a}(-}{}\right.}{2f\sqrt{c-ict}}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/Sqrt[c - I*c
*Tan[e + f*x]],x]
```

output
```
(a^(5/2)*((6*(2*A - (3*I)*B)*ArcSin[Sqrt[a + I*a*Tan[e + f*x]]/(Sqrt[2]*Sq
rt[a])]*(I + Tan[e + f*x]))/Sqrt[1 - I*Tan[e + f*x]] - (I*Sqrt[a]*(-I + Ta
n[e + f*x])*(2*((5*I)*A + 7*B) + (2*A - (5*I)*B)*Tan[e + f*x] + B*Tan[e +
f*x]^2))/Sqrt[a + I*a*Tan[e + f*x]]))/(2*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 87, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 87$$

$$\frac{ac\left(-\frac{(2A-3iB)\int \frac{(i\tan(e+fx)a+a)^{3/2}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$\downarrow$ 60

$$\frac{ac\left(-\frac{(2A-3iB)\left(\frac{3}{2}a\int \frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2c}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$\downarrow$ 60

$$\frac{ac\left(-\frac{(2A-3iB)\left(\frac{3}{2}a\left(a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e}{2c}\right.}{c}\right.}{f}$$

$\downarrow$ 45

$$\frac{ac\left(-\frac{(2A-3iB)\left(\frac{3}{2}a\left(2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+f}{2c}\right.}{c}\right.}{f}$$

$\downarrow$ 218

$$\frac{ac\left(-\frac{(2A-3iB)\left(\frac{3}{2}a\left(\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2c}\right)}{c}-\frac{(B}{}\right.}{f}$$

input

```
Int[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e
 + f*x]],x]
```

output
```
(a*c*(-(((I*A + B)*(a + I*a*Tan[e + f*x])^(5/2))/(a*c*Sqrt[c - I*c*Tan[e +
 f*x]])) - ((2*A - (3*I)*B)*(((I/2)*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c -
I*c*Tan[e + f*x]])/c + (3*a*(((-2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*
Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[c] + (I*Sqrt[a
+ I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/c))/2))/c))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(187) = 374.

Time = 0.64 (sec) , antiderivative size = 565, normalized size of antiderivative = 2.49

method	result
derivativedivides	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(6iA\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^2+18iB\right.}{}$
default	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(6iA\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^2+18iB\right.}{}$
parts	$\frac{iA\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(3i\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)\tan(fx+e)^2ac-3i\ln\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/2*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c*(6*I*
A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2)
)*a*c*tan(f*x+e)^2+18*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e
)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+9*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2
)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+4*I*B*(a*c)^
(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2-B*(a*c)^(1/2)*(a*c*(1+tan(
f*x+e)^2))^(1/2)*tan(f*x+e)^3-6*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1
+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-12*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2
)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-12*I*A*(a*c)^(
1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-2*A*(a*c*(1+tan(f*x+e)^2))^(1
/2)*(a*c)^(1/2)*tan(f*x+e)^2-9*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+ta
n(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-14*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)
^2))^(1/2)-19*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+10*A*(
a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I+t
an(f*x+e))^2/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(175) = 350$.

Time = 0.11 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.33

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{3\sqrt{\frac{(4A^2-12iAB-9B^2)a^5}{cf^2}}\left(cfe^{(2ifx+2ie)}+cf\right)\log\left(\frac{4\left(2\left(\right.\right.}{}\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
1/4*(3*sqrt((4*A^2 - 12*I*A*B - 9*B^2)*a^5/(c*f^2))*(c*f*e^(2*I*f*x + 2*I*
e) + c*f)*log(4*(2*((-2*I*A - 3*B)*a^2*e^(3*I*f*x + 3*I*e) + (-2*I*A - 3*B
)*a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*
x + 2*I*e) + 1)) + sqrt((4*A^2 - 12*I*A*B - 9*B^2)*a^5/(c*f^2))*(c*f*e^(2*
I*f*x + 2*I*e) - c*f))/((-2*I*A - 3*B)*a^2*e^(2*I*f*x + 2*I*e) + (-2*I*A -
 3*B)*a^2)) - 3*sqrt((4*A^2 - 12*I*A*B - 9*B^2)*a^5/(c*f^2))*(c*f*e^(2*I*f
*x + 2*I*e) + c*f)*log(4*(2*((-2*I*A - 3*B)*a^2*e^(3*I*f*x + 3*I*e) + (-2*
I*A - 3*B)*a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(
e^(2*I*f*x + 2*I*e) + 1)) - sqrt((4*A^2 - 12*I*A*B - 9*B^2)*a^5/(c*f^2))*(
c*f*e^(2*I*f*x + 2*I*e) - c*f))/((-2*I*A - 3*B)*a^2*e^(2*I*f*x + 2*I*e) +
(-2*I*A - 3*B)*a^2)) - 4*(4*(I*A + B)*a^2*e^(5*I*f*x + 5*I*e) + 5*(2*I*A +
 3*B)*a^2*e^(3*I*f*x + 3*I*e) + 3*(2*I*A + 3*B)*a^2*e^(I*f*x + I*e))*sqrt(
a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c*f*e^(2*
I*f*x + 2*I*e) + c*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx=\int \frac{(ia(\tan{(e+fx)}-i))^{\frac{5}{2}}\,(A+B\tan{(e+fx)})}{\sqrt{-ic\,(\tan{(e+fx)}+i)}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(5/2)*(A + B*tan(e + f*x))/sqrt(-I*c*(t
an(e + f*x) + I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 993 vs. $2(175) = 350$.

Time = 0.27 (sec) , antiderivative size = 993, normalized size of antiderivative = 4.37

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx=\text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
-4*(4*(2*A - 7*I*B)*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 4*(2*I*A + 7*B)*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) - 6*((2*A - 3*I*B)*a^2*cos(4*f*x + 4*e) + 2*(2*A - 3*I*B)*a^2*cos(2*f*
x + 2*e) - (-2*I*A - 3*B)*a^2*sin(4*f*x + 4*e) - 2*(-2*I*A - 3*B)*a^2*sin(
2*f*x + 2*e) + (2*A - 3*I*B)*a^2)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 1) - 6*((2*A - 3*I*B)*a^2*cos(4*f*x + 4*e) + 2*(2*A - 3*I*B)*a^2*cos(2*
f*x + 2*e) - (-2*I*A - 3*B)*a^2*sin(4*f*x + 4*e) - 2*(-2*I*A - 3*B)*a^2*si
n(2*f*x + 2*e) + (2*A - 3*I*B)*a^2)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 1) + 4*(4*(A - I*B)*a^2*cos(4*f*x + 4*e) + 8*(A - I*B)*a^2*cos(2*f*x
 + 2*e) + 4*(I*A + B)*a^2*sin(4*f*x + 4*e) + 8*(I*A + B)*a^2*sin(2*f*x + 2
*e) + 3*(2*A - 3*I*B)*a^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 3*((-2*I*A - 3*B)*a^2*cos(4*f*x + 4*e) + 2*(-2*I*A - 3*B)*a^2*cos(
2*f*x + 2*e) + (2*A - 3*I*B)*a^2*sin(4*f*x + 4*e) + 2*(2*A - 3*I*B)*a^2*si
n(2*f*x + 2*e) + (-2*I*A - 3*B)*a^2)*log(cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 3*((2
*I*A + 3*B)*a^2*cos(4*f*x + 4*e) + 2*(2*I*A + 3*B)*a^2*cos(2*f*x + 2*e) -
(2*A - 3*I*B)*a^2*sin(4*f*x + 4*e) - 2*(2*A - 3*I*B)*a^2*sin(2*f*x + 2*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \int \frac{(A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}}{\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*
x)*1i)^(1/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*
x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(-\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2*a*i - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**2*b + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)*a - 5*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*a*i - 7*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - int(
(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(ta
n(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f*i - int((sqrt(tan(e + f*x)*i + 1
)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2 + 1),x)*b*
f*i - 3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x)**2)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*a*f + 3*int((sqrt(tan(e
+ f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**
2 + 1),x)*tan(e + f*x)**2*b*f*i - 3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( -
tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*a*f + 3*int(
(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(ta
n(e + f*x)**2 + 1),x)*b*f*i))/(c*f*(tan(e + f*x)**2 + 1))
```

3.810 $\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 226

$$\begin{aligned}\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx &= -\frac{2a^{5/2}(iA+4B)\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f}\\ &\quad-\frac{(iA+B)(a+ia\tan(e+fx))^{5/2}}{3f(c-ic\tan(e+fx))^{3/2}}+\frac{2a(iA+4B)(a+ia\tan(e+fx))^{3/2}}{3cf\sqrt{c-ic\tan(e+fx)}}\\ &\quad+\frac{a^2(iA+4B)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c^2f}\end{aligned}$$

output

```
-2*a^(5/2)*(I*A+4*B)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*
c*tan(f*x+e))^(1/2))/c^(3/2)/f-1/3*(I*A+B)*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I
*c*tan(f*x+e))^(3/2)+2/3*a*(I*A+4*B)*(a+I*a*tan(f*x+e))^(3/2)/c/f/(c-I*c*t
an(f*x+e))^(1/2)+a^2*(I*A+4*B)*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))
^(1/2)/c^2/f
```

Mathematica [A] (verified)

Time = 7.21 (sec), antiderivative size = 179, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{a^2\cos^2(e+fx)\sqrt{a+ia\tan(e+fx)}\Big((-3-3i)(A-$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(3/2),x]
```

output
```
(a^2*Cos[e + f*x]^2*Sqrt[a + I*a*Tan[e + f*x]]*((-3 - 3*I)*(A - (4*I)*B)*A
rcSin[(1/2 + I/2)*Sqrt[-I + Tan[e + f*x]]]*Sqrt[2 - (2*I)*Tan[e + f*x]]*Sq
rt[-I + Tan[e + f*x]]*(I + Tan[e + f*x]) + (-I + Tan[e + f*x])*(-4*A + (19
*I)*B + ((8*I)*A + 26*B)*Tan[e + f*x] - (3*I)*B*Tan[e + f*x]^2)))/(3*c*f*S
qrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec), antiderivative size = 222, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 87, 57, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(-\frac{(A-4iB)\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$\downarrow$ 57

$$\frac{ac\left(-\frac{(A-4iB)\left(-\frac{3a\int \frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$\downarrow$ 60

$$\frac{ac\left(-\frac{(A-4iB)\left(-\frac{3a\left(a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{(B+i}{3ac}\right.}{f}$$

$\downarrow$ 45

$$\frac{ac\left(-\frac{(A-4iB)\left(-\frac{3a\left(2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{(B+iA)}{3ac(}\right.}{f}$$

$\downarrow$ 218

$$ac\left(-\frac{(A-4iB)\left(-\frac{3a\left(\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{(B+iA)(a+ia\tan(}{3ac(c-ic\tan(e-}\right.$$
$$\overline{\qquad f \qquad}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(3/2),x]
```

output
```
(a*c*(-1/3*((I*A + B)*(a + I*a*Tan[e + f*x])^(5/2))/(a*c*(c - I*c*Tan[e +
f*x])^(3/2)) - ((A - (4*I)*B)*(((-2*I)*(a + I*a*Tan[e + f*x])^(3/2))/(c*Sq
rt[c - I*c*Tan[e + f*x]]) - (3*a*(((-2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a +
 I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[c] + (I*Sq
rt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/c))/c))/(3*c)))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(186) = 372$.

Time = 0.70 (sec) , antiderivative size = 667, normalized size of antiderivative = 2.95

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(-12iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^3+9i\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(-12iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^3+9i\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(9i\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)\tan(fx+e)^2ac+3\ln\left(\right.\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/3/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c^2*(-12*
I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/
2))*a*c*tan(f*x+e)^3+9*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+
e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+3*A*ln((a*c*tan(f*x+e)+(a*c)^(
1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+36*I*B*ln
((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*
c*tan(f*x+e)+29*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+
36*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1
/2))*a*c*tan(f*x+e)^2+3*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x
+e)^3-3*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(
a*c)^(1/2))*a*c-12*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)
-9*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1
/2))*a*c*tan(f*x+e)-8*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e
)^2-19*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)-12*B*ln((a*c*tan(f*x+e
)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-45*B*(a*c)^(1
/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+4*A*(a*c)^(1/2)*(a*c*(1+tan(f*
x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)/(I+tan(f*x+e))^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(176) = 352$.

Time = 0.10 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.18

$$\int \frac{(a + ia\tan(e + fx))^{5/2}(A + B\tan(e + fx))}{(c - ic\tan(e + fx))^{3/2}}\,dx =$$

$$-\frac{3\,c^2\sqrt{\frac{(A^2-8i\,AB-16\,B^2)a^5}{c^3f^2}}f\log\left(-\frac{4\left(2\left((-i\,A-4\,B)a^2e^{(3i\,fx+3i\,e)}+(-i\,A-4\,B)a^2e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}+(c\right.}{(i\,A+4\,B)a^2e^{(2i\,fx+2i\,e)}+(i\,A+4\,B)a^2}\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
-1/6*(3*c^2*sqrt((A^2 - 8*I*A*B - 16*B^2)*a^5/(c^3*f^2))*f*log(-4*(2*((-I*
A - 4*B)*a^2*e^(3*I*f*x + 3*I*e) + (-I*A - 4*B)*a^2*e^(I*f*x + I*e))*sqrt(
a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (c^2*f*e^
(2*I*f*x + 2*I*e) - c^2*f)*sqrt((A^2 - 8*I*A*B - 16*B^2)*a^5/(c^3*f^2)))/(
(I*A + 4*B)*a^2*e^(2*I*f*x + 2*I*e) + (I*A + 4*B)*a^2)) - 3*c^2*sqrt((A^2
- 8*I*A*B - 16*B^2)*a^5/(c^3*f^2))*f*log(-4*(2*((-I*A - 4*B)*a^2*e^(3*I*f*
x + 3*I*e) + (-I*A - 4*B)*a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e)
 + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (c^2*f*e^(2*I*f*x + 2*I*e) - c^
2*f)*sqrt((A^2 - 8*I*A*B - 16*B^2)*a^5/(c^3*f^2)))/((I*A + 4*B)*a^2*e^(2*I
*f*x + 2*I*e) + (I*A + 4*B)*a^2)) + 4*((I*A + B)*a^2*e^(5*I*f*x + 5*I*e) +
 2*(-I*A - 4*B)*a^2*e^(3*I*f*x + 3*I*e) + 3*(-I*A - 4*B)*a^2*e^(I*f*x + I*
e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(
c^2*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \int \frac{(ia(\tan{(e+fx)}-i))^{\frac{5}{2}}\,(A+B\tan{(e+fx)})}{(-ic\,(\tan{(e+fx)}+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(5/2)*(A + B*tan(e + f*x))/(-I*c*(tan(e
 + f*x) + I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(176) = 352$.

Time = 0.24 (sec) , antiderivative size = 795, normalized size of antiderivative = 3.52

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
-3*(6*((A - 4*I*B)*a^2*cos(2*f*x + 2*e) - (-I*A - 4*B)*a^2*sin(2*f*x + 2*e
) + (A - 4*I*B)*a^2)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 6*((A
 - 4*I*B)*a^2*cos(2*f*x + 2*e) - (-I*A - 4*B)*a^2*sin(2*f*x + 2*e) + (A -
4*I*B)*a^2)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))),
-sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*((A - I*B)*
a^2*cos(2*f*x + 2*e) - (-I*A - B)*a^2*sin(2*f*x + 2*e) + (A - I*B)*a^2)*co
s(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*((A - 3*I*B)*a^2*c
os(2*f*x + 2*e) + (I*A + 3*B)*a^2*sin(2*f*x + 2*e) + (A - 4*I*B)*a^2)*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*((-I*A - 4*B)*a^2*cos
(2*f*x + 2*e) + (A - 4*I*B)*a^2*sin(2*f*x + 2*e) + (-I*A - 4*B)*a^2)*log(c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 1) - 3*((I*A + 4*B)*a^2*cos(2*f*x + 2*e) - (A - 4*I*
B)*a^2*sin(2*f*x + 2*e) + (I*A + 4*B)*a^2)*log(cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) -
 4*((-I*A - B)*a^2*cos(2*f*x + 2*e) + (A - I*B)*a^2*sin(2*f*x + 2*e) + (-I
*A - B)*a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12*((I
*A + 3*B)*a^2*cos(2*f*x + 2*e) - (A - 3*I*B)*a^2*sin(2*f*x + 2*e) + (I*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx=\int \frac{(A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}}{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*
x)*1i)^(3/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*
x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - 6*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)*a + 4*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)*b*i + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*a*i + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - 3*in
t(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)*
*3 + tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**2*a*f*i - int(
( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3
 + tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**2*b*f - 3*int((
- sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 +
 tan(e + f*x)**2*i + tan(e + f*x) + i),x)*a*f*i - int(( - sqrt(tan(e + f*x
)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 + tan(e + f*x)**2*i
 + tan(e + f*x) + i),x)*b*f + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan(
e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f*i + int((sqrt(tan(e + f*x)*i + 1)*s
qrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i - tan(e + f
*x)**2 + tan(e + f*x)*i - 1),x)*b*f*i + int((sqrt(tan(e + f*x)*i + 1)*sqrt
( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i - tan(e + f*x)
**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*a*f*i + 3*int((sqrt(tan(e + f
*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i
 - tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f + int((...
```

3.811 $\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 203

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{2a^{5/2}B\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f} - \frac{(iA+B)(a+ia\tan(e+fx))^{5/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{2aB(a+ia\tan(e+fx))^{3/2}}{3cf(c-ic\tan(e+fx))^{3/2}} - \frac{2a^2B\sqrt{a+ia\tan(e+fx)}}{c^2f\sqrt{c-ic\tan(e+fx)}}$$

output

```
2*a^(5/2)*B*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))/c^(5/2)/f-1/5*(I*A+B)*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I*c*tan(f*
x+e))^(5/2)+2/3*a*B*(a+I*a*tan(f*x+e))^(3/2)/c/f/(c-I*c*tan(f*x+e))^(3/2)-
2*a^2*B*(a+I*a*tan(f*x+e))^(1/2)/c^2/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 15.68 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{a^2\cos^2(e+fx)\left(\cos\left(\frac{1}{2}(e-2fx)\right)-i\sin\left(\frac{1}{2}(e-2fx\right)\right.}{}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(5/2),x]
```

output
```
(a^2*Cos[e + f*x]^2*(Cos[(e - 2*f*x)/2] - I*Sin[(e - 2*f*x)/2])*(Cos[(e -
2*f*x)/2] + I*Sin[(e - 2*f*x)/2])*(-10*B + ((3*I)*A + 33*B)*Cos[2*(e + f*x
)] - 3*A*Sin[2*(e + f*x)] - (27*I)*B*Sin[2*(e + f*x)] - 30*B*ArcTan[E^(I*(
e + f*x))]*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)]))*(-I + Tan[e + f*x])^2*
Sqrt[a + I*a*Tan[e + f*x]])/(15*c^2*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 87, 57, 57, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{iB\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{57}$$

$$\frac{ac\left(\frac{iB\left(-\frac{a\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{57}$$

$$\frac{ac\left(\frac{iB\left(-\frac{a\left(-\frac{a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{45}$$

$$\frac{ac\left(\frac{iB\left(-\frac{a\left(-\frac{2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}}{c}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{ac\left(\frac{iB\left(-\frac{a\left(\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(5/2),x]
```

output
```
(a*c*(-1/5*((I*A + B)*(a + I*a*Tan[e + f*x])^(5/2))/(a*c*(c - I*c*Tan[e +
f*x])^(5/2)) + (I*B*((((-2*I)/3)*(a + I*a*Tan[e + f*x])^(3/2))/(c*(c - I*c
*Tan[e + f*x])^(3/2)) - (a*(((2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Ta
n[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/c^(3/2) - ((2*I)*Sqrt[
a + I*a*Tan[e + f*x]])/(c*Sqrt[c - I*c*Tan[e + f*x]])))/c))/c))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(166) = 332$.

Time = 0.62 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.50

method	result
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{2}\left(1+\tan(fx+e)^{2}\right)(i-\tan(fx+e))}{5f\,c^{3}(i+\tan(fx+e))^{4}}+\frac{iB\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c}}{}$
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{2}\left(-15iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^{2}\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^{4}+\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{2}\left(-15iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^{2}\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^{4}+\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/5*A/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c^3*(1
+tan(f*x+e)^2)*(I-tan(f*x+e))/(I+tan(f*x+e))^4+1/15*I*B/f*(a*(1+I*tan(f*x+
e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^2/c^3*(60*I*ln((a*c*tan(f*x+e)+(a
*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*tan(f*x+e)^3*a*c+15*l
n((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*t
an(f*x+e)^4*a*c-60*I*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))
^(1/2))/(a*c)^(1/2))*tan(f*x+e)*a*c-97*I*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2)
)^(1/2)*tan(f*x+e)^2-90*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^
2))^(1/2))/(a*c)^(1/2))*tan(f*x+e)^2*a*c-43*tan(f*x+e)^3*(a*c*(1+tan(f*x+e
)^2))^(1/2)*(a*c)^(1/2)+23*I*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)+15*l
n((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a
*c+77*tan(f*x+e)*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x
+e)^2))^(1/2)/(I+tan(f*x+e))^4/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(159) = 318.

Time = 0.10 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.06

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{15\,c^3f\sqrt{-\frac{B^2a^5}{c^5f^2}}\log\left(\frac{4\left(2\left(Ba^2e^{(3i\,fx+3i\,e)}+Ba^2e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}+\left(c^3fe^{(2i\,fx+2i\,e)}-c^3f\right)\sqrt{-\frac{B^2a^5}{c^5f^2}}\right)}{Ba^2e^{(2i\,fx+2i\,e)}+Ba^2}\right)}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/30*(15*c^3*f*sqrt(-B^2*a^5/(c^5*f^2))*log(4*(2*(B*a^2*e^(3*I*f*x + 3*I*
e) + B*a^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2
*I*f*x + 2*I*e) + 1)) + (c^3*f*e^(2*I*f*x + 2*I*e) - c^3*f)*sqrt(-B^2*a^5/
(c^5*f^2)))/(B*a^2*e^(2*I*f*x + 2*I*e) + B*a^2)) - 15*c^3*f*sqrt(-B^2*a^5/
(c^5*f^2))*log(4*(2*(B*a^2*e^(3*I*f*x + 3*I*e) + B*a^2*e^(I*f*x + I*e))*sq
rt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (c^3*f
*e^(2*I*f*x + 2*I*e) - c^3*f)*sqrt(-B^2*a^5/(c^5*f^2)))/(B*a^2*e^(2*I*f*x
+ 2*I*e) + B*a^2)) + 2*(3*(I*A + B)*a^2*e^(7*I*f*x + 7*I*e) + (3*I*A - 7*B
)*a^2*e^(5*I*f*x + 5*I*e) + 20*B*a^2*e^(3*I*f*x + 3*I*e) + 30*B*a^2*e^(I*f
*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) +
 1)))/(c^3*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \int \frac{(ia(\tan(e+fx)-i))^{\frac{5}{2}}\,(A+B\tan(e+fx))}{(-ic\,(\tan(e+fx)+i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(5/2)*(A + B*tan(e + f*x))/(-I*c*(tan(e
 + f*x) + I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.05

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{(30\,Ba^2\arctan\left(\cos\left(fx+e\right),\sin\left(fx+e\right)+1\right)+30\,\ldots}{\ldots}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
1/30*(30*B*a^2*arctan2(cos(f*x + e), sin(f*x + e) + 1) + 30*B*a^2*arctan2(
cos(f*x + e), -sin(f*x + e) + 1) - 6*(I*A + B)*a^2*cos(5*f*x + 5*e) + 20*B
*a^2*cos(3*f*x + 3*e) - 60*B*a^2*cos(f*x + e) + 15*I*B*a^2*log(cos(f*x + e
)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 15*I*B*a^2*log(cos(f*x + e)^2
 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) + 6*(A - I*B)*a^2*sin(5*f*x + 5*e)
 + 20*I*B*a^2*sin(3*f*x + 3*e) - 60*I*B*a^2*sin(f*x + e))*sqrt(a)/(c^(5/2)
*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx=\int \frac{(A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}}{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*
x)*1i)^(5/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*
x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx=\frac{\sqrt{a}\,a^2\Big(-\Big(\int \frac{\sqrt{\tan(fx+e)i+1}}{\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^2+2\sqrt{-\tan(fx+e)i+1}\,\cdots}$$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*a**2*( - int(sqrt(tan(e + f*x)*i + 1)/(sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( -
 tan(e + f*x)*i + 1)),x)*a + int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**3
)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*b + int((sqrt(tan(e
+ f*x)*i + 1)*tan(e + f*x)**2)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**
2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i
+ 1)),x)*a - 2*int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(sqrt( - tan
(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*b*i - 2*int((sqrt(tan(e + f*x)*i +
 1)*tan(e + f*x))/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*sqrt( -
 tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*a*i
- int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x))/(sqrt( - tan(e + f*x)*i + 1)
*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( -
tan(e + f*x)*i + 1)),x)*b))/(sqrt(c)*c**2)
```

3.812 $\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 102

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{5/2}}{7f(c-ic\tan(e+fx))^{7/2}}-\frac{(iA-6B)(a+ia\tan(e+fx))^{5/2}}{35cf(c-ic\tan(e+fx))^{5/2}}$$

output
```
-1/7*(I*A+B)*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I*c*tan(f*x+e))^(7/2)-1/35*(I*A
-6*B)*(a+I*a*tan(f*x+e))^(5/2)/c/f/(c-I*c*tan(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 13.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.19

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \frac{a^2\cos(e+fx)((-6iA+B)\cos(e+fx)-(A+6iB)}{}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(7/2),x]
```

output
```
(a^2*Cos[e + f*x]*(((-6*I)*A + B)*Cos[e + f*x] - (A + (6*I)*B)*Sin[e + f*x
])*(Cos[6*e + 8*f*x] + I*Sin[6*e + 8*f*x])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt
[c - I*c*Tan[e + f*x]])/(35*c^4*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {3042, 4071, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(A+6iB)\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{7c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(-\frac{i(A+6iB)(a+ia\tan(e+fx))^{5/2}}{35ac^2(c-ic\tan(e+fx))^{5/2}}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(7/2),x]
```

output
```
(a*c*(-1/7*((I*A + B)*(a + I*a*Tan[e + f*x])^(5/2))/(a*c*(c - I*c*Tan[e +
f*x])^(7/2)) - ((I/35)*(A + (6*I)*B)*(a + I*a*Tan[e + f*x])^(5/2))/(a*c^2*
(c - I*c*Tan[e + f*x])^(5/2))))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{a^{2} \sqrt{\frac{a\,{\mathrm e}^{2 i (f x+e)}}{{\mathrm e}^{2 i (f x+e)}+1}}\, \left(5 i A\,{\mathrm e}^{6 i (f x+e)}+5 B\,{\mathrm e}^{6 i (f x+e)}+7 i A\,{\mathrm e}^{4 i (f x+e)}-7 B\,{\mathrm e}^{4 i (f x+e)}\right)}{70 c^{3} \sqrt{\frac{c}{{\mathrm e}^{2 i (f x+e)}+1}}\, f}$
derivativedivides	$-\frac{i \sqrt{a (1+i \tan (f x+e))}\, \sqrt{-c (i \tan (f x+e)-1)}\, a^{2} \left(1+\tan (f x+e)^{2}\right) \left(i A \tan (f x+e)^{2}+5 i B \tan (f x+e)-6 B \tan (f x+e)^{2}\right.}{35 f\,c^{4} (i+\tan (f x+e))^{5}}$
default	$-\frac{i \sqrt{a (1+i \tan (f x+e))}\, \sqrt{-c (i \tan (f x+e)-1)}\, a^{2} \left(1+\tan (f x+e)^{2}\right) \left(i A \tan (f x+e)^{2}+5 i B \tan (f x+e)-6 B \tan (f x+e)^{2}\right.}{35 f\,c^{4} (i+\tan (f x+e))^{5}}$
parts	$\frac{A \sqrt{a (1+i \tan (f x+e))}\, \sqrt{-c (i \tan (f x+e)-1)}\, a^{2} \left(1+\tan (f x+e)^{2}\right) \left(5 i \tan (f x+e)+\tan (f x+e)^{2}+6\right)}{35 f\,c^{4} (i+\tan (f x+e))^{5}}-\frac{i B \sqrt{a (1+i \tan}{}$

input

```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,m
ethod=_RETURNVERBOSE)
```

output

```
-1/70*a^2/c^3*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(2*I*
(f*x+e))+1))^(1/2)/f*(5*I*A*exp(6*I*(f*x+e))+5*B*exp(6*I*(f*x+e))+7*I*A*ex
p(4*I*(f*x+e))-7*B*exp(4*I*(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{\left(5\,(i\,A+B)a^2e^{(9i\,fx+9i\,e)}+2\,(6i\,A-B)a^2e^{(7i\,fx+7i\,e)}+7\,(i\,A-B)a^2e^{(5i\,fx+5i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e}}}}{70\,c^4\,f}$$

input

```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="fricas")
```

output
```
-1/70*(5*(I*A + B)*a^2*e^(9*I*f*x + 9*I*e) + 2*(6*I*A - B)*a^2*e^(7*I*f*x
+ 7*I*e) + 7*(I*A - B)*a^2*e^(5*I*f*x + 5*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e
) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^4*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(78) = 156$.

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.63

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx=$$
$$-\frac{70\,(5\,(A-i\,B)a^2\cos(9\,fx+9\,e)+2\,(6\,A+i\,B)a^2\cos(7\,fx+7\,e)+7\,(A+i\,B)a^2\cos(5\,fx+5\,e)-5\,(\cdots}{-4900\,(i\,c^4\cos(2\,fx+2\,e)\cdots}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="maxima")
```

output
```
-70*(5*(A - I*B)*a^2*cos(9*f*x + 9*e) + 2*(6*A + I*B)*a^2*cos(7*f*x + 7*e)
 + 7*(A + I*B)*a^2*cos(5*f*x + 5*e) - 5*(-I*A - B)*a^2*sin(9*f*x + 9*e) -
2*(-6*I*A + B)*a^2*sin(7*f*x + 7*e) - 7*(-I*A + B)*a^2*sin(5*f*x + 5*e))*s
qrt(a)*sqrt(c)/((-4900*I*c^4*cos(2*f*x + 2*e) + 4900*c^4*sin(2*f*x + 2*e)
- 4900*I*c^4)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 8.54 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.88

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{a^2\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,\left(A\,\cos\left(4\,e+4\,f\,x\right)\,7\mathrm{i}+A\,\cos\left(6\,e+6\,f\,x\right)\,5\mathrm{i}-7\,B\,\cos\left(4\,e+4\,f\,x\right)-\cdots\right.}{70\,c^3\,f\,\sqrt{\frac{c\,(\cos\cdots}{\cdots}}}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*
x)*1i)^(7/2),x)
```

output
```
-(a^2*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(A*cos(4*e + 4*f*x)*7i + A*cos(6*e + 6*f*x)*5i - 7*B*cos(4*e +
 4*f*x) + 5*B*cos(6*e + 6*f*x) - 7*A*sin(4*e + 4*f*x) - 5*A*sin(6*e + 6*f*
x) - B*sin(4*e + 4*f*x)*7i + B*sin(6*e + 6*f*x)*5i))/(70*c^3*f*((c*(cos(2*
e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)*a + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b
+ 2*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x)**3)/(tan(e + f*x)**5 + 3*tan(e + f*x)**4*i - 2*tan(e + f*x)**3 + 2*tan
(e + f*x)**2*i - 3*tan(e + f*x) - i),x)*tan(e + f*x)**2*a*f - 6*int(( - sq
rt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e
 + f*x)**5 + 3*tan(e + f*x)**4*i - 2*tan(e + f*x)**3 + 2*tan(e + f*x)**2*i
 - 3*tan(e + f*x) - i),x)*tan(e + f*x)**2*b*f*i + 2*int(( - sqrt(tan(e + f
*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**5 +
 3*tan(e + f*x)**4*i - 2*tan(e + f*x)**3 + 2*tan(e + f*x)**2*i - 3*tan(e +
 f*x) - i),x)*a*f - 6*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**5 + 3*tan(e + f*x)**4*i - 2*tan(e
 + f*x)**3 + 2*tan(e + f*x)**2*i - 3*tan(e + f*x) - i),x)*b*f*i + 6*int((s
qrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e +
 f*x)**5 + 3*tan(e + f*x)**4*i - 2*tan(e + f*x)**3 + 2*tan(e + f*x)**2*i -
 3*tan(e + f*x) - i),x)*tan(e + f*x)**2*a*f - 2*int((sqrt(tan(e + f*x)*i +
 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**5 + 3*tan(e +
 f*x)**4*i - 2*tan(e + f*x)**3 + 2*tan(e + f*x)**2*i - 3*tan(e + f*x) - i)
,x)*tan(e + f*x)**2*b*f*i + 6*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**5 + 3*tan(e + f*x)**4*i - 2*...
```

3.813 $\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 155

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{5/2}}{9f(c-ic\tan(e+fx))^{9/2}} - \frac{(2iA-7B)(a+ia\tan(e+fx))^{5/2}}{63cf(c-ic\tan(e+fx))^{7/2}} - \frac{(2iA-7B)(a+ia\tan(e+fx))^{5/2}}{315c^2f(c-ic\tan(e+fx))^{5/2}}$$

output
```
-1/9*(I*A+B)*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I*c*tan(f*x+e))^(9/2)-1/63*(2*I
*A-7*B)*(a+I*a*tan(f*x+e))^(5/2)/c/f/(c-I*c*tan(f*x+e))^(7/2)-1/315*(2*I*A
-7*B)*(a+I*a*tan(f*x+e))^(5/2)/c^2/f/(c-I*c*tan(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 13.85 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \frac{a^2\cos(e+fx)(-45iA+7(-7iA+2B)\cos(2(e+fx$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(9/2),x]
```

output
```
(a^2*Cos[e + f*x]*((-45*I)*A + 7*((-7*I)*A + 2*B)*Cos[2*(e + f*x)] - 7*(2*
A + (7*I)*B)*Sin[2*(e + f*x)])*(Cos[7*e + 9*f*x] + I*Sin[7*e + 9*f*x])*Sqr
t[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(630*c^5*f*(Cos[f*x] +
 I*Sin[f*x])^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4071, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(2A+7iB)\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{9c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{(2A+7iB)\left(\frac{\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{7c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{9c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(\frac{(2A+7iB)\left(-\frac{i(a+ia\tan(e+fx))^{5/2}}{35ac^2(c-ic\tan(e+fx))^{5/2}}-\frac{i(a+ia\tan(e+fx))^{5/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{9c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(9/2),x]
```

output
```
(a*c*(-1/9*((I*A + B)*(a + I*a*Tan[e + f*x])^(5/2))/(a*c*(c - I*c*Tan[e +
f*x])^(9/2)) + ((2*A + (7*I)*B)*(((-1/7*I)*(a + I*a*Tan[e + f*x])^(5/2))/(
a*c*(c - I*c*Tan[e + f*x])^(7/2)) - ((I/35)*(a + I*a*Tan[e + f*x])^(5/2))/
(a*c^2*(c - I*c*Tan[e + f*x])^(5/2))))/(9*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{a^{2} \sqrt{\frac{a\,\mathrm{e}^{2 i (f x+e)}}{\mathrm{e}^{2 i (f x+e)}+1}}\, \left(35 i A\,\mathrm{e}^{8 i (f x+e)}+35 B\,\mathrm{e}^{8 i (f x+e)}+90 i A\,\mathrm{e}^{6 i (f x+e)}+63 i A\,\mathrm{e}^{4 i (f x+e)}-63 B\,\mathrm{e}^{4 i (f x+e)}\right)}{1260 c^{4} \sqrt{\frac{c}{\mathrm{e}^{2 i (f x+e)}+1}}\, f}$
derivativedivides	$-\frac{i \sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{2}\left(1+\tan(fx+e)^{2}\right)\left(-47A-33i\tan(fx+e)A-12A\tan(fx+e)^{2}+2iA\right.}{315 f\,c^{5}\left(i+\tan(fx+e)\right)^{6}}$
default	$-\frac{i \sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{2}\left(1+\tan(fx+e)^{2}\right)\left(-47A-33i\tan(fx+e)A-12A\tan(fx+e)^{2}+2iA\right.}{315 f\,c^{5}\left(i+\tan(fx+e)\right)^{6}}$
parts	$-\frac{iA \sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{2}\left(1+\tan(fx+e)^{2}\right)\left(-47-33i\tan(fx+e)-12\tan(fx+e)^{2}+2i\tan(f\right.}{315 f\,c^{5}\left(i+\tan(fx+e)\right)^{6}}$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/1260*a^2/c^4*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(2*
I*(f*x+e))+1))^(1/2)/f*(35*I*A*exp(8*I*(f*x+e))+35*B*exp(8*I*(f*x+e))+90*I
*A*exp(6*I*(f*x+e))+63*I*A*exp(4*I*(f*x+e))-63*B*exp(4*I*(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.81

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = -\frac{(35\,(i\,A+B)a^2e^{(11i\,fx+11i\,e)}+5\,(25i\,A+7\,B)a^2e^{(9i\,fx+9i\,e)}+9\,(17i\,A-7\,B)a^2e^{(7i\,fx+7i\,e)}+63\,(i\,A-B)a}{1260\,c^5f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="fricas")
```

output
```
-1/1260*(35*(I*A + B)*a^2*e^(11*I*f*x + 11*I*e) + 5*(25*I*A + 7*B)*a^2*e^(
9*I*f*x + 9*I*e) + 9*(17*I*A - 7*B)*a^2*e^(7*I*f*x + 7*I*e) + 63*(I*A - B)
*a^2*e^(5*I*f*x + 5*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I
*f*x + 2*I*e) + 1))/(c^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.28

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \frac{\left(35\left(-iA-B\right)a^2\cos\left(\frac{9}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="maxima")
```

output
```
1/1260*(35*(-I*A - B)*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 90*I*A*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6
3*(-I*A + B)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 35
*(A - I*B)*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 90*A
*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 63*(A + I*B)*a
^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(9/2)*
f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.40

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx =$$
$$-\frac{a^2\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(A\,\cos\left(4\,e+4\,f\,x\right)\,63\mathrm{i}+A\,\cos\left(6\,e+6\,f\,x\right)\,90\mathrm{i}+A\,\cos\left(8\,e+8\,f\,x\right)}{}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*
x)*1i)^(9/2),x)
```

output
```
-(a^2*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(A*cos(4*e + 4*f*x)*63i + A*cos(6*e + 6*f*x)*90i + A*cos(8*e +
 8*f*x)*35i - 63*B*cos(4*e + 4*f*x) + 35*B*cos(8*e + 8*f*x) - 63*A*sin(4*e
 + 4*f*x) - 90*A*sin(6*e + 6*f*x) - 35*A*sin(8*e + 8*f*x) - B*sin(4*e + 4*
f*x)*63i + B*sin(8*e + 8*f*x)*35i))/(1260*c^4*f*((c*(cos(2*e + 2*f*x) - si
n(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \text{too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)*a + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
1)*tan(e + f*x)*b*i - int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e
 + f*x)**4 - 5*tan(e + f*x)**2 - 4*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*
a*f + 7*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e
 + f*x)**4)/(tan(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5
*tan(e + f*x)**2 - 4*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i - int((
- sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(t
an(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)*
*2 - 4*tan(e + f*x)*i + 1),x)*a*f + 7*int(( - sqrt(tan(e + f*x)*i + 1)*sqr
t( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**6 + 4*tan(e + f*x
)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)**2 - 4*tan(e + f*x)*i + 1),x)*
b*f*i - 8*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x))/(tan(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*
tan(e + f*x)**2 - 4*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*f*i + 8*int((
 - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan
(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)**2
 - 4*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f - 8*int(( - sqrt(tan(e + f
*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**6 +...
```

3.814 $\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 208

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{5/2}}{11f(c-ic\tan(e+fx))^{11/2}} - \frac{(3iA-8B)(a+ia\tan(e+fx))^{5/2}}{99cf(c-ic\tan(e+fx))^{9/2}} - \frac{2(3iA-8B)(a+ia\tan(e+fx))^{5/2}}{693c^2f(c-ic\tan(e+fx))^{7/2}} - \frac{2(3iA-8B)(a+ia\tan(e+fx))^{5/2}}{3465c^3f(c-ic\tan(e+fx))^{5/2}}$$

output

```
-1/11*(I*A+B)*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I*c*tan(f*x+e))^(11/2)-1/99*(3
*I*A-8*B)*(a+I*a*tan(f*x+e))^(5/2)/c/f/(c-I*c*tan(f*x+e))^(9/2)-2/693*(3*I
*A-8*B)*(a+I*a*tan(f*x+e))^(5/2)/c^2/f/(c-I*c*tan(f*x+e))^(7/2)-2/3465*(3*
I*A-8*B)*(a+I*a*tan(f*x+e))^(5/2)/c^3/f/(c-I*c*tan(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 14.87 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \frac{a^2\cos(e+fx)(55(-24iA+B)\cos(e+fx)+63(-8a$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(11/2),x]
```

output
```
(a^2*Cos[e + f*x]*(55*((-24*I)*A + B)*Cos[e + f*x] + 63*((-8*I)*A + 3*B)*C
os[3*(e + f*x)] - (3*A + (8*I)*B)*(55*Sin[e + f*x] + 63*Sin[3*(e + f*x)]))
*(Cos[8*e + 10*f*x] + I*Sin[8*e + 10*f*x])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt
[c - I*c*Tan[e + f*x]])/(13860*c^6*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 4071, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(3A+8iB)\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{11/2}}d\tan(e+fx)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

$\big\downarrow$ 55

$$\frac{ac\left(\frac{(3A+8iB)\left(\frac{2\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{9c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

$\big\downarrow$ 55

$$\frac{ac\left(\frac{(3A+8iB)\left(\frac{2\left(\frac{\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{7c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{9c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

$\big\downarrow$ 48

$$\frac{ac\left(\frac{(3A+8iB)\left(\frac{2\left(-\frac{i(a+ia\tan(e+fx))^{5/2}}{35ac^2(c-ic\tan(e+fx))^{5/2}}-\frac{i(a+ia\tan(e+fx))^{5/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{9c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(11/2),x]
```

output
```
(a*c*(-1/11*((I*A + B)*(a + I*a*Tan[e + f*x])^(5/2))/(a*c*(c - I*c*Tan[e +
 f*x])^(11/2)) + ((3*A + (8*I)*B)*(((-1/9*I)*(a + I*a*Tan[e + f*x])^(5/2))
/(a*c*(c - I*c*Tan[e + f*x])^(9/2)) + (2*(((-1/7*I)*(a + I*a*Tan[e + f*x])
^(5/2))/(a*c*(c - I*c*Tan[e + f*x])^(7/2)) - ((I/35)*(a + I*a*Tan[e + f*x]
)^(5/2))/(a*c^2*(c - I*c*Tan[e + f*x])^(5/2))))/(9*c)))/(11*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{a^2\sqrt{\frac{a\,\mathrm{e}^{2i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\,(315iA\,\mathrm{e}^{10i(fx+e)}+315B\,\mathrm{e}^{10i(fx+e)}+1155iA\,\mathrm{e}^{8i(fx+e)}+385B\,\mathrm{e}^{8i(fx+e)}+1485iA\,\mathrm{e}^{6i(fx+e)}-495}{27720c^5\sqrt{\frac{c}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$
derivativedivides	$-\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(1+\tan(fx+e)^2\right)\left(6iA\tan(fx+e)^4-112iB\tan(fx+e)^3-16B\tan(f\right.}{3465f\,c^6(i}$
default	$-\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(1+\tan(fx+e)^2\right)\left(6iA\tan(fx+e)^4-112iB\tan(fx+e)^3-16B\tan(f\right.}{3465f\,c^6(i}$
parts	$-\frac{iA\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^2\left(1+\tan(fx+e)^2\right)\left(2i\tan(fx+e)^4-45i\tan(fx+e)^2-14\tan(fx+e)\right.}{1155f\,c^6(i+\tan(fx+e))^7}$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/27720*a^2/c^5*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(2
*I*(f*x+e))+1))^(1/2)/f*(315*I*A*exp(10*I*(f*x+e))+315*B*exp(10*I*(f*x+e))
+1155*I*A*exp(8*I*(f*x+e))+385*B*exp(8*I*(f*x+e))+1485*I*A*exp(6*I*(f*x+e)
)-495*B*exp(6*I*(f*x+e))+693*I*A*exp(4*I*(f*x+e))-693*B*exp(4*I*(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.70

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = -\frac{(315\,(i\,A+B)a^2e^{(13i\,fx+13i\,e)}+70\,(21i\,A+10\,B)a^2e^{(11i\,fx+11i\,e)}+110\,(24i\,A-B)a^2e^{(9i\,fx+9i\,e)}+198\,(11}{27720\,c^6 f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11
/2),x, algorithm="fricas")
```

output
```
-1/27720*(315*(I*A + B)*a^2*e^(13*I*f*x + 13*I*e) + 70*(21*I*A + 10*B)*a^2
*e^(11*I*f*x + 11*I*e) + 110*(24*I*A - B)*a^2*e^(9*I*f*x + 9*I*e) + 198*(1
1*I*A - 6*B)*a^2*e^(7*I*f*x + 7*I*e) + 693*(I*A - B)*a^2*e^(5*I*f*x + 5*I*
e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c
^6*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.33

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \frac{\left(315\left(-iA-B\right)a^2\cos\left(\frac{11}{2}\arctan\left(\sin\left(2\,fx+2\,e\right),\text{co}\right.\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11
/2),x, algorithm="maxima")
```

output
```
1/27720*(315*(-I*A - B)*a^2*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 385*(-3*I*A - B)*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) + 495*(-3*I*A + B)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 693*(-I*A + B)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 315*(A - I*B)*a^2*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 385*(3*A - I*B)*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 495*(3*A + I*B)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 693*(A + I*B)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))))*sqrt(a)/(c^(11/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.40

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = -\frac{a^2\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(A\,\cos(4\,e+4\,f\,x)\,693\mathrm{i}+A\,\cos(6\,e+6\,f\,x)\,1485\mathrm{i}+A\,\cos(8\,e+8\,f}{}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*
x)*1i)^(11/2),x)
```

output
```
-(a^2*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(A*cos(4*e + 4*f*x)*693i + A*cos(6*e + 6*f*x)*1485i + A*cos(8*
e + 8*f*x)*1155i + A*cos(10*e + 10*f*x)*315i - 693*B*cos(4*e + 4*f*x) - 49
5*B*cos(6*e + 6*f*x) + 385*B*cos(8*e + 8*f*x) + 315*B*cos(10*e + 10*f*x) -
 693*A*sin(4*e + 4*f*x) - 1485*A*sin(6*e + 6*f*x) - 1155*A*sin(8*e + 8*f*x
) - 315*A*sin(10*e + 10*f*x) - B*sin(4*e + 4*f*x)*693i - B*sin(6*e + 6*f*x
)*495i + B*sin(8*e + 8*f*x)*385i + B*sin(10*e + 10*f*x)*315i))/(27720*c^5*
f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1)
)^(1/2))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**7 + 5*tan(e + f*x)**6*i - 9*t
an(e + f*x)**5 - 5*tan(e + f*x)**4*i - 5*tan(e + f*x)**3 - 9*tan(e + f*x)*
*2*i + 5*tan(e + f*x) + i),x)*b + 3*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt(
 - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**7 + 5*tan(e + f*x)*
*6*i - 9*tan(e + f*x)**5 - 5*tan(e + f*x)**4*i - 5*tan(e + f*x)**3 - 9*tan
(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*a*i + 3*int(( - sqrt(tan(e + f*x)*
i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**7 + 5*t
an(e + f*x)**6*i - 9*tan(e + f*x)**5 - 5*tan(e + f*x)**4*i - 5*tan(e + f*x
)**3 - 9*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*b + int((sqrt(tan(e +
f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**7
+ 5*tan(e + f*x)**6*i - 9*tan(e + f*x)**5 - 5*tan(e + f*x)**4*i - 5*tan(e
+ f*x)**3 - 9*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*a - 3*int((sqrt(t
an(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f
*x)**7 + 5*tan(e + f*x)**6*i - 9*tan(e + f*x)**5 - 5*tan(e + f*x)**4*i - 5
*tan(e + f*x)**3 - 9*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*b*i - 3*in
t((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan
(e + f*x)**7 + 5*tan(e + f*x)**6*i - 9*tan(e + f*x)**5 - 5*tan(e + f*x)**4
*i - 5*tan(e + f*x)**3 - 9*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*a +
int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))...
```

3.815 $\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 261

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{5/2}}{13f(c-ic\tan(e+fx))^{13/2}} - \frac{(4iA-9B)(a+ia\tan(e+fx))^{5/2}}{143cf(c-ic\tan(e+fx))^{11/2}} - \frac{(4iA-9B)(a+ia\tan(e+fx))^{5/2}}{429c^2f(c-ic\tan(e+fx))^{9/2}} - \frac{2(4iA-9B)(a+ia\tan(e+fx))^{5/2}}{3003c^3f(c-ic\tan(e+fx))^{7/2}} - \frac{2(4iA-9B)(a+ia\tan(e+fx))^{5/2}}{15015c^4f(c-ic\tan(e+fx))^{5/2}}$$

output
```
-1/13*(I*A+B)*(a+I*a*tan(f*x+e))^(5/2)/f/(c-I*c*tan(f*x+e))^(13/2)-1/143*(
4*I*A-9*B)*(a+I*a*tan(f*x+e))^(5/2)/c/f/(c-I*c*tan(f*x+e))^(11/2)-1/429*(4
*I*A-9*B)*(a+I*a*tan(f*x+e))^(5/2)/c^2/f/(c-I*c*tan(f*x+e))^(9/2)-2/3003*(
4*I*A-9*B)*(a+I*a*tan(f*x+e))^(5/2)/c^3/f/(c-I*c*tan(f*x+e))^(7/2)-2/15015
*(4*I*A-9*B)*(a+I*a*tan(f*x+e))^(5/2)/c^4/f/(c-I*c*tan(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 16.34 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.70

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx =$$
$$-\frac{ia^2\cos(e+fx)(5005A+780(9A+iB)\cos(2(e+fx))+231(9A+4iB)\cos(4(e+fx))-1560iA\sin(2($$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(13/2),x]
```

output
```
((-1/120120*I)*a^2*Cos[e + f*x]*(5005*A + 780*(9*A + I*B)*Cos[2*(e + f*x)]
 + 231*(9*A + (4*I)*B)*Cos[4*(e + f*x)] - (1560*I)*A*Sin[2*(e + f*x)] + 35
10*B*Sin[2*(e + f*x)] - (924*I)*A*Sin[4*(e + f*x)] + 2079*B*Sin[4*(e + f*x
)])*(Cos[9*e + 11*f*x] + I*Sin[9*e + 11*f*x])*Sqrt[a + I*a*Tan[e + f*x]]*S
qrt[c - I*c*Tan[e + f*x]])/(c^7*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{3/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 87$$

$$\frac{ac\left(\frac{(4A+9iB)\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{13/2}}d\tan(e+fx)}{13c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{f}$$

$$\Big\downarrow \ 55$$

$$\frac{ac\left(\frac{(4A+9iB)\left(\frac{3\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{11/2}}d\tan(e+fx)}{11c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{f}$$

$$\Big\downarrow \ 55$$

$$\frac{ac\left(\frac{(4A+9iB)\left(\frac{3\left(\frac{2\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{9c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{(B+iA)(a+ia\tan(e+fx))^{5/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{f}$$

$$\Big\downarrow \ 55$$

$$ac\left(\frac{(4A+9iB)\left(\frac{3\left(\frac{2\left(\frac{\int\frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{7c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{9c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{(B\ldots}{\ldots}\right)$$
$$\overline{\qquad\qquad f\qquad\qquad}$$

$\downarrow$ 48

$$ac\left(\frac{(4A+9iB)\left(\frac{3\left(\frac{2\left(-\frac{i(a+ia\tan(e+fx))^{5/2}}{35ac^2(c-ic\tan(e+fx))^{5/2}}-\frac{i(a+ia\tan(e+fx))^{5/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{9c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{i(a+ia\tan(e+fx))^{5/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{(B+iA)(\ldots}{13ac(c\ldots}\right)$$
$$\overline{\qquad\qquad f\qquad\qquad}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(5/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(13/2),x]
```

output
```
(a*c*(-1/13*((I*A + B)*(a + I*a*Tan[e + f*x])^(5/2))/(a*c*(c - I*c*Tan[e +
 f*x])^(13/2)) + ((4*A + (9*I)*B)*(((-1/11*I)*(a + I*a*Tan[e + f*x])^(5/2)
)/(a*c*(c - I*c*Tan[e + f*x])^(11/2)) + (3*(((-1/9*I)*(a + I*a*Tan[e + f*x
])^(5/2))/(a*c*(c - I*c*Tan[e + f*x])^(9/2)) + (2*(((-1/7*I)*(a + I*a*Tan[
e + f*x])^(5/2))/(a*c*(c - I*c*Tan[e + f*x])^(7/2)) - ((I/35)*(a + I*a*Tan
[e + f*x])^(5/2))/(a*c^2*(c - I*c*Tan[e + f*x])^(5/2))))/(9*c)))/(11*c)))/
(13*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{a^{2} \sqrt{\frac{a\,e^{2i(fx+e)}}{e^{2i(fx+e)}+1}}\,\left(1155iA\,e^{12i(fx+e)}+1155B\,e^{12i(fx+e)}+5460iA\,e^{10i(fx+e)}+2730B\,e^{10i(fx+e)}+10010iA\,e^{8i(fx+e}\right.}{240240c^{6}\sqrt{\frac{c}{e^{2i(fx+e)}+1}}\,f}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{2}\left(1+\tan(fx+e)^{2}\right)\left(18iB\tan(fx+e)^{5}+64iA\tan(fx+e)^{4}+8A\tan(fx+e)\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{2}\left(1+\tan(fx+e)^{2}\right)\left(18iB\tan(fx+e)^{5}+64iA\tan(fx+e)^{4}+8A\tan(fx+e)\right.}{}$
parts	$-\frac{iA\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{2}\left(1+\tan(fx+e)^{2}\right)\left(8i\tan(fx+e)^{5}-236i\tan(fx+e)^{3}-64\tan(fx+\right.}{15015f\,c^{7}(i+\tan(fx+e))^{8}}$

input
```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/240240*a^2/c^6*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(
2*I*(f*x+e))+1))^(1/2)/f*(1155*I*A*exp(12*I*(f*x+e))+1155*B*exp(12*I*(f*x+
e))+5460*I*A*exp(10*I*(f*x+e))+2730*B*exp(10*I*(f*x+e))+10010*I*A*exp(8*I*
(f*x+e))+8580*I*A*exp(6*I*(f*x+e))-4290*B*exp(6*I*(f*x+e))+3003*I*A*exp(4*
I*(f*x+e))-3003*B*exp(4*I*(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.64

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = -\frac{(1155\,(iA+B)a^2e^{(15i\,fx+15i\,e)}+105\,(63iA+37\,B)a^2e^{(13i\,fx+13i\,e)}+910\,(17iA+3\,B)a^2e^{(11i\,fx+11i\,e)}+1}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13
/2),x, algorithm="fricas")
```

output
```
-1/240240*(1155*(I*A + B)*a^2*e^(15*I*f*x + 15*I*e) + 105*(63*I*A + 37*B)*
a^2*e^(13*I*f*x + 13*I*e) + 910*(17*I*A + 3*B)*a^2*e^(11*I*f*x + 11*I*e) +
 1430*(13*I*A - 3*B)*a^2*e^(9*I*f*x + 9*I*e) + 429*(27*I*A - 17*B)*a^2*e^(
7*I*f*x + 7*I*e) + 3003*(I*A - B)*a^2*e^(5*I*f*x + 5*I*e))*sqrt(a/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^7*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
13/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.27

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = \frac{(1155\,(-i\,A-B)a^2\cos\left(\frac{13}{2}\arctan\left(\sin\left(2\,fx+2\,e\right),\right.\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13
/2),x, algorithm="maxima")
```

output
```
1/240240*(1155*(-I*A - B)*a^2*cos(13/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) + 2730*(-2*I*A - B)*a^2*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - 10010*I*A*a^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 4290*(-2*I*A + B)*a^2*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 3003*(-I*A + B)*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 1155*(A - I*B)*a^2*sin(13/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 2730*(2*A - I*B)*a^2*sin(11/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 10010*A*a^2*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) + 4290*(2*A + I*B)*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 3003*(A + I*B)*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))))*sqrt(a)/(c^(13/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx =$$

$$-\frac{\sqrt{a+\frac{a\sin(e+fx)\,1i}{\cos(e+fx)}}\left(\frac{a^2\,e^{e\,6i+f\,x\,6i}\,(2\,A+B\,1i)\,1i}{56\,c^6\,f}+\frac{a^2\,e^{e\,10i+f\,x\,10i}\,(2\,A-B\,1i)\,1i}{88\,c^6\,f}+\frac{A\,a^2\,e^{e\,8i+f\,x\,8i}\,1i}{24\,c^6\,f}+\frac{a^2\,e^{e\,4i+f\,x\,4i}\,(A+B\,1i)\,1i}{80\,c^6\,f}+\right.}{\sqrt{c-\frac{c\sin(e+fx)\,1i}{\cos(e+fx)}}}$$

input

```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(5/2))/(c - c*tan(e + f*
x)*1i)^(13/2),x)
```

output

```
-((a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i)*(2
*A + B*1i)*1i)/(56*c^6*f) + (a^2*exp(e*10i + f*x*10i)*(2*A - B*1i)*1i)/(88
*c^6*f) + (A*a^2*exp(e*8i + f*x*8i)*1i)/(24*c^6*f) + (a^2*exp(e*4i + f*x*4
i)*(A + B*1i)*1i)/(80*c^6*f) + (a^2*exp(e*12i + f*x*12i)*(A - B*1i)*1i)/(2
08*c^6*f)))/(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = \text{Too large to display}$$

input

```
int((a+I*a*tan(f*x+e))^(5/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**8 + 6*tan(e + f*x)**7*i - 14*
tan(e + f*x)**6 - 14*tan(e + f*x)**5*i - 14*tan(e + f*x)**3*i + 14*tan(e +
 f*x)**2 + 6*tan(e + f*x)*i - 1),x)*a*i - 3*int(( - sqrt(tan(e + f*x)*i +
1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**8 + 6*tan(e
 + f*x)**7*i - 14*tan(e + f*x)**6 - 14*tan(e + f*x)**5*i - 14*tan(e + f*x)
**3*i + 14*tan(e + f*x)**2 + 6*tan(e + f*x)*i - 1),x)*b + 3*int(( - sqrt(t
an(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)
**8 + 6*tan(e + f*x)**7*i - 14*tan(e + f*x)**6 - 14*tan(e + f*x)**5*i - 14
*tan(e + f*x)**3*i + 14*tan(e + f*x)**2 + 6*tan(e + f*x)*i - 1),x)*a*i + i
nt(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/
(tan(e + f*x)**8 + 6*tan(e + f*x)**7*i - 14*tan(e + f*x)**6 - 14*tan(e + f
*x)**5*i - 14*tan(e + f*x)**3*i + 14*tan(e + f*x)**2 + 6*tan(e + f*x)*i -
1),x)*b + int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(t
an(e + f*x)**8 + 6*tan(e + f*x)**7*i - 14*tan(e + f*x)**6 - 14*tan(e + f*x
)**5*i - 14*tan(e + f*x)**3*i + 14*tan(e + f*x)**2 + 6*tan(e + f*x)*i - 1)
,x)*a + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x)**4)/(tan(e + f*x)**8 + 6*tan(e + f*x)**7*i - 14*tan(e + f*x)**6 - 14*
tan(e + f*x)**5*i - 14*tan(e + f*x)**3*i + 14*tan(e + f*x)**2 + 6*tan(e +
f*x)*i - 1),x)*b*i + 3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*...
```

3.816 $\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 350

$$\begin{aligned}
&\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}\,dx = \\
&-\frac{5a^{7/2}(8iA-B)c^{9/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{64f} \\
&+\frac{5a^3(8A+iB)c^4\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{128f} \\
&+\frac{5a^2(8A+iB)c^3\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{192f} \\
&+\frac{a(8A+iB)c^2\tan(e+fx)(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}{48f} \\
&-\frac{(8iA-B)c(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{7/2}}{56f} \\
&+\frac{B(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{9/2}}{8f}
\end{aligned}$$

output
```
-5/64*a^(7/2)*(8*I*A-B)*c^(9/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^
(1/2)/(c-I*c*tan(f*x+e))^(1/2))/f+5/128*a^3*(8*A+I*B)*c^4*tan(f*x+e)*(a+I*
a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)/f+5/192*a^2*(8*A+I*B)*c^3*tan
(f*x+e)*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2)/f+1/48*a*(8*A+I*
B)*c^2*tan(f*x+e)*(a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(5/2)/f-1/56
*(8*I*A-B)*c*(a+I*a*tan(f*x+e))^(7/2)*(c-I*c*tan(f*x+e))^(7/2)/f+1/8*B*(a+
I*a*tan(f*x+e))^(7/2)*(c-I*c*tan(f*x+e))^(9/2)/f
```

Mathematica [A] (verified)

Time = 7.73 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.61

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}\,dx = \frac{a^{7/2}c^5\left(\frac{1}{64}\sqrt{a}\sec^9(e+fx)(24576(-iA+B)\cos(e+fx)+7(2264A-2789iB)\sin$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e
 + f*x])^(9/2),x]
```

output
```
(a^(7/2)*c^5*((Sqrt[a]*Sec[e + f*x]^9*(24576*((-I)*A + B)*Cos[e + f*x] + 7
*(2264*A - (2789*I)*B)*Sin[e + f*x] + 7*(8*A + I*B)*(383*Sin[3*(e + f*x)]
+ 115*Sin[5*(e + f*x)] + 15*Sin[7*(e + f*x)])))/64 + 210*((-8*I)*A + B)*Ar
cSin[Sqrt[a + I*a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[1 - I*Tan[e + f*x]
]*Sqrt[a + I*a*Tan[e + f*x]]))/(2688*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c -
 I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4071, 90, 59, 40, 40, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{9/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{9/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int (i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{90}$$

$$\frac{ac\left(\frac{1}{8}(8A+iB)\int (i\tan(e+fx)a+a)^{5/2}(c-ic\tan(e+fx))^{7/2}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))}{8ac}\right.}{f}$$

$$\Big\downarrow \text{59}$$

$$\frac{ac\left(\frac{1}{8}(8A+iB)\left(c\int (i\tan(e+fx)a+a)^{5/2}(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)-\frac{i(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+f}{7a}\right.\right.}{f}$$

$$\Big\downarrow \text{40}$$

$$ac\left(\frac{1}{8}(8A+iB)\left(c\left(\frac{5}{6}ac\int (i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)+\frac{1}{6}\tan(e+fx)(a+ia\tan\right.\right.\right.$$

$$\Big\downarrow \text{40}$$

$$ac\left(\frac{1}{8}(8A+iB)\left(c\left(\frac{5}{6}ac\left(\frac{3}{4}ac\int \sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{1}{4}\tan(e+fx)(a+ia\tan\right.\right.\right.\right.$$

$$\Big\downarrow \text{40}$$

$$ac\left(\frac{1}{8}(8A+iB)\left(c\left(\frac{5}{6}ac\left(\frac{3}{4}ac\left(\frac{1}{2}ac\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{45}$$

$$ac\left(\frac{1}{8}(8A+iB)\left(c\left(\frac{5}{6}ac\left(\frac{3}{4}ac\left(ac\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c\right.\right.\right.\right.\right.$$

$$\Big\downarrow 218$$

$$ac\left(\frac{1}{8}(8A+iB)\left(c\left(\frac{5}{6}ac\left(\frac{3}{4}ac\left(\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}-i\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia}}{\sqrt{a}\sqrt{c-ic}}\right.\right.\right.\right.\right.\right.$$

input
```
Int[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(9/2),x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e + f*x])^(9/2))/(8*a*c
) + ((8*A + I*B)*(((-1/7*I)*(a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e +
f*x])^(7/2))/a + c*((Tan[e + f*x]*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Ta
n[e + f*x])^(5/2))/6 + (5*a*c*((Tan[e + f*x]*(a + I*a*Tan[e + f*x])^(3/2)*
(c - I*c*Tan[e + f*x])^(3/2))/4 + (3*a*c*((-I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqr
t[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + (
Tan[e + f*x]*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/2))/4)
)/6)))/8))/f
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(289) = 578$.

Time = 0.83 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.73

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c^4\left(826iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+336iB\sqrt{ac\left(1+\mathrm{ta}\right.}\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c^4\left(826iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+336iB\sqrt{ac\left(1+\mathrm{ta}\right.}\right.}{}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c^4\left(48i\tan(fx+e)^6\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+144i\tan(fx+e)^4\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/2688/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3*c^4*(
826*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+336*I*B*(a*c
*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^7+384*I*A*(a*c*(1+tan(f*x+
e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^6-384*B*tan(f*x+e)^6*(a*c*(1+tan(f*x+e
)^2))^(1/2)*(a*c)^(1/2)+105*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*t
an(f*x+e)-448*A*tan(f*x+e)^5*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+952*
I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^5-1152*B*(a*c*(1+t
an(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4-105*I*B*ln((a*c*tan(f*x+e)+(a
*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-1456*A*(a*c*(1+ta
n(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+1152*I*A*(a*c*(1+tan(f*x+e)^2)
)^(1/2)*(a*c)^(1/2)*tan(f*x+e)^2+384*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c
)^(1/2)-1152*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+1152*
I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4-840*A*ln((a*c*ta
n(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-1848*A
*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-384*B*(a*c)^(1/2)*(a*
c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(270) = 540$.

Time = 0.12 (sec) , antiderivative size = 882, normalized size of antiderivative = 2.52

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="fricas")
```

output
```
1/5376*(105*sqrt((64*A^2 + 16*I*A*B - B^2)*a^7*c^9/f^2)*(f*e^(14*I*f*x + 1
4*I*e) + 7*f*e^(12*I*f*x + 12*I*e) + 21*f*e^(10*I*f*x + 10*I*e) + 35*f*e^(
8*I*f*x + 8*I*e) + 35*f*e^(6*I*f*x + 6*I*e) + 21*f*e^(4*I*f*x + 4*I*e) + 7
*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*((8*I*A - B)*a^3*c^4*e^(3*I*f*x + 3*I
*e) + (8*I*A - B)*a^3*c^4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1
))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((64*A^2 + 16*I*A*B - B^2)*a^7*
c^9/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((8*I*A - B)*a^3*c^4*e^(2*I*f*x + 2*
I*e) + (8*I*A - B)*a^3*c^4)) - 105*sqrt((64*A^2 + 16*I*A*B - B^2)*a^7*c^9/
f^2)*(f*e^(14*I*f*x + 14*I*e) + 7*f*e^(12*I*f*x + 12*I*e) + 21*f*e^(10*I*f
*x + 10*I*e) + 35*f*e^(8*I*f*x + 8*I*e) + 35*f*e^(6*I*f*x + 6*I*e) + 21*f*
e^(4*I*f*x + 4*I*e) + 7*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*((8*I*A - B)*a
^3*c^4*e^(3*I*f*x + 3*I*e) + (8*I*A - B)*a^3*c^4*e^(I*f*x + I*e))*sqrt(a/(
e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((64*A^2
 + 16*I*A*B - B^2)*a^7*c^9/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((8*I*A - B)*
a^3*c^4*e^(2*I*f*x + 2*I*e) + (8*I*A - B)*a^3*c^4)) - 4*(105*(8*I*A - B)*a
^3*c^4*e^(15*I*f*x + 15*I*e) + 805*(8*I*A - B)*a^3*c^4*e^(13*I*f*x + 13*I*
e) + 2681*(8*I*A - B)*a^3*c^4*e^(11*I*f*x + 11*I*e) + 5053*(8*I*A - B)*a^3
*c^4*e^(9*I*f*x + 9*I*e) - (-8728*I*A + 44099*B)*a^3*c^4*e^(7*I*f*x + 7*I*
e) + 2681*(-8*I*A + B)*a^3*c^4*e^(5*I*f*x + 5*I*e) + 805*(-8*I*A + B)*a^3*
c^4*e^(3*I*f*x + 3*I*e) + 105*(-8*I*A + B)*a^3*c^4*e^(I*f*x + I*e))*sqr...
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2603 vs. $2(270) = 540$.

Time = 16.54 (sec) , antiderivative size = 2603, normalized size of antiderivative = 7.44

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="maxima")
```

output
```
-86016*(420*(8*A + I*B)*a^3*c^4*cos(15/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 3220*(8*A + I*B)*a^3*c^4*cos(13/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 10724*(8*A + I*B)*a^3*c^4*cos(11/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 20212*(8*A + I*B)*a^3*c^4*cos(9/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(8728*A + 44099*I*B)*a^3*c^4*cos(7/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10724*(8*A + I*B)*a^3*c^4*
cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3220*(8*A + I*B)*a^
3*c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 420*(8*A + I*
B)*a^3*c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 420*(8*I
*A - B)*a^3*c^4*sin(15/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 32
20*(8*I*A - B)*a^3*c^4*sin(13/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 10724*(8*I*A - B)*a^3*c^4*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 20212*(8*I*A - B)*a^3*c^4*sin(9/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*(8728*I*A - 44099*B)*a^3*c^4*sin(7/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + 10724*(-8*I*A + B)*a^3*c^4*sin(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3220*(-8*I*A + B)*a^3*c^4*sin(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 420*(-8*I*A + B)*a^3*c^4*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 210*((8*A + I*B)*a^3*
c^4*cos(16*f*x + 16*e) + 8*(8*A + I*B)*a^3*c^4*cos(14*f*x + 14*e) + 28*(8*
A + I*B)*a^3*c^4*cos(12*f*x + 12*e) + 56*(8*A + I*B)*a^3*c^4*cos(10*f*x...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}\,dx=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{\frac{7}{2}}(-i\,c\tan(fx+e)+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(7/2)*(-I*c*tan(f*x
+ e) + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}\,dx=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{9/2}dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)
*1i)^(9/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)
*1i)^(9/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}\,dx=\text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*c**4*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**6*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f
*x)*i + 1)*tan(e + f*x)**6*b - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e +
f*x)*i + 1)*tan(e + f*x)**4*a*i + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**4*b - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(
e + f*x)*i + 1)*tan(e + f*x)**2*a*i + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - t
an(e + f*x)*i + 1)*tan(e + f*x)**2*b - sqrt(tan(e + f*x)*i + 1)*sqrt( - ta
n(e + f*x)*i + 1)*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
1)*b - 7*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
f*x)**8,x)*b*f*i + 7*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)**6,x)*a*f - 21*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**6,x)*b*f*i + 21*int(sqrt(tan(e + f*x)*i + 1)*
sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4,x)*a*f - 21*int(sqrt(tan(e + f
*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4,x)*b*f*i + 21*int(s
qrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)*a*f
 - 7*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)
**2,x)*b*f*i + 7*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1),
x)*a*f))/(7*f)
```

3.817 $\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 267

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = -\frac{5ia^{7/2}Ac^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{8f} + \frac{5a^3Ac^3\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{16f} + \frac{5a^2Ac^2\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{24f} + \frac{aAc\tan(e+fx)(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}{6f} + \frac{B(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{7/2}}{7f}$$

output
```
-5/8*I*a^(7/2)*A*c^(7/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(
c-I*c*tan(f*x+e))^(1/2))/f+5/16*a^3*A*c^3*tan(f*x+e)*(a+I*a*tan(f*x+e))^(1
/2)*(c-I*c*tan(f*x+e))^(1/2)/f+5/24*a^2*A*c^2*tan(f*x+e)*(a+I*a*tan(f*x+e)
)^(3/2)*(c-I*c*tan(f*x+e))^(3/2)/f+1/6*a*A*c*tan(f*x+e)*(a+I*a*tan(f*x+e))
^(5/2)*(c-I*c*tan(f*x+e))^(5/2)/f+1/7*B*(a+I*a*tan(f*x+e))^(7/2)*(c-I*c*ta
n(f*x+e))^(7/2)/f
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 535 vs. 2(267) = 534.

Time = 18.84 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.00

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx =$$

$$-\frac{5iAc^4e^{-i(4e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\arctan\left(e^{i(e+fx)}\right)(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{8\sqrt{\frac{c}{1+e^{2i(e+fx)}}}f\sec^{\frac{9}{2}}(e+fx)(\cos(fx)+i\sin(fx))^{7/2}(A\cos(e+fx)+B\sin(e+fx))}$$
$$+\frac{\cos^4(e+fx)\sqrt{\sec(e+fx)(c\cos(e+fx)-ic\sin(e+fx))}\left(\sec^6(e+fx)\left(\frac{1}{7}Bc^3\cos(3e)-\frac{1}{7}iBc^3\sin(3e)\right)\right.}{}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e
 + f*x])^(7/2),x]
```

output
```
(((-5*I)/8)*A*c^4*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e +
f*x)))]*ArcTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e
+ f*x]))/(E^(I*(4*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f*
x]^(9/2)*(Cos[f*x] + I*Sin[f*x])^(7/2)*(A*Cos[e + f*x] + B*Sin[e + f*x]))
+ (Cos[e + f*x]^4*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]*(
Sec[e + f*x]^6*((B*c^3*Cos[3*e])/7 - (I/7)*B*c^3*Sin[3*e]) + A*c^3*Sec[e]*
Sec[e + f*x]^5*(Cos[3*e]/6 - (I/6)*Sin[3*e])*Sin[f*x] + A*c^3*Sec[e]*Sec[e
 + f*x]^3*((5*Cos[3*e])/24 - ((5*I)/24)*Sin[3*e])*Sin[f*x] + A*c^3*Sec[e]*
Sec[e + f*x]*((5*Cos[3*e])/16 - ((5*I)/16)*Sin[3*e])*Sin[f*x] + Sec[e + f*
x]^4*((A*c^3*Cos[3*e])/6 - (I/6)*A*c^3*Sin[3*e])*Tan[e] + Sec[e + f*x]^2*(
(5*A*c^3*Cos[3*e])/24 - ((5*I)/24)*A*c^3*Sin[3*e])*Tan[e] + ((5*A*c^3*Cos[
3*e])/16 - ((5*I)/16)*A*c^3*Sin[3*e])*Tan[e])*(a + I*a*Tan[e + f*x])^(7/2)
*(A + B*Tan[e + f*x]))/(f*(Cos[f*x] + I*Sin[f*x])^3*(A*Cos[e + f*x] + B*Si
n[e + f*x]))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 90, 40, 40, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int (i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{ac\left(A\int (i\tan(e+fx)a+a)^{5/2}(c-ic\tan(e+fx))^{5/2}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{7/2}}{7ac}\right)}{f}$$

$$\downarrow 40$$

$$\frac{ac\Big(A\big(\frac{5}{6}ac\int(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)+\frac{1}{6}\tan(e+fx)(a+ia\tan(e+fx))^{5/2}}{f}$$

$$\downarrow 40$$

$$ac\Big(A\Big(\frac{5}{6}ac\Big(\frac{3}{4}ac\int\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{1}{4}\tan(e+fx)(a+ia\tan(e+fx))^{3/}$$

$$\downarrow 40$$

$$ac\Big(A\Big(\frac{5}{6}ac\Big(\frac{3}{4}ac\Big(\frac{1}{2}ac\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic}$$

$$\downarrow 45$$

$$ac\Bigg(A\Bigg(\frac{5}{6}ac\Bigg(\frac{3}{4}ac\Bigg(ac\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+}$$

$$\downarrow 218$$

$$ac\Big(A\Big(\frac{5}{6}ac\Big(\frac{3}{4}ac\Big(\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}-i\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)\Big)$$

input
```
Int[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(7/2),x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e + f*x])^(7/2))/(7*a*c
) + A*((Tan[e + f*x]*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x])^(
5/2))/6 + (5*a*c*((Tan[e + f*x]*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[
e + f*x])^(3/2))/4 + (3*a*c*((-I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a +
 I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + (Tan[e + f*x]*
Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/2))/4))/6)))/f
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.01

method	result
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c^3\left(8\tan(fx+e)^5\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+26\tan(fx+e)^3\sqrt{ac\left(1\right.}\right.}{48f\sqrt{ac\left(1+\tan(\right.}}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c^3\left(48B\tan(fx+e)^6\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+56A\tan(fx+e)^5\sqrt{ac}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c^3\left(48B\tan(fx+e)^6\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+56A\tan(fx+e)^5\sqrt{ac}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/48*A/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3*c^3*(8
*tan(f*x+e)^5*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+26*tan(f*x+e)^3*(a*
c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+15*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a
*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+33*tan(f*x+e)*(a*c*(1+tan(f*x
+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)+1/7*B/
f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3*c^3*(1+tan(f*
x+e)^2)*(tan(f*x+e)^4+2*tan(f*x+e)^2+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(207) = 414.

Time = 0.11 (sec) , antiderivative size = 693, normalized size of antiderivative = 2.60

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="fricas")
```

output
```
1/672*(105*sqrt(A^2*a^7*c^7/f^2)*(f*e^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*
x + 10*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e
^(4*I*f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*(A*a^3*c^3*e^(3
*I*f*x + 3*I*e) + A*a^3*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) +
 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(A^2*a^7*c^7/f^2)*(I*f*e^(2*I
*f*x + 2*I*e) - I*f))/(A*a^3*c^3*e^(2*I*f*x + 2*I*e) + A*a^3*c^3)) - 105*s
qrt(A^2*a^7*c^7/f^2)*(f*e^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10*I*e)
+ 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*f*x +
4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*(A*a^3*c^3*e^(3*I*f*x + 3*I
*e) + A*a^3*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/
(e^(2*I*f*x + 2*I*e) + 1)) - sqrt(A^2*a^7*c^7/f^2)*(-I*f*e^(2*I*f*x + 2*I*
e) + I*f))/(A*a^3*c^3*e^(2*I*f*x + 2*I*e) + A*a^3*c^3)) + 4*(-105*I*A*a^3*
c^3*e^(13*I*f*x + 13*I*e) - 700*I*A*a^3*c^3*e^(11*I*f*x + 11*I*e) - 1981*I
*A*a^3*c^3*e^(9*I*f*x + 9*I*e) + 3072*B*a^3*c^3*e^(7*I*f*x + 7*I*e) + 1981
*I*A*a^3*c^3*e^(5*I*f*x + 5*I*e) + 700*I*A*a^3*c^3*e^(3*I*f*x + 3*I*e) + 1
05*I*A*a^3*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(
e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(12*I*f*x + 12*I*e) + 6*f*e^(10*I*f*x + 10
*I*e) + 15*f*e^(8*I*f*x + 8*I*e) + 20*f*e^(6*I*f*x + 6*I*e) + 15*f*e^(4*I*
f*x + 4*I*e) + 6*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1901 vs. 2(207) = 414.

Time = 2.30 (sec) , antiderivative size = 1901, normalized size of antiderivative = 7.12

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="maxima")
```

output
```
-(420*A*a^3*c^3*cos(13/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 28
00*A*a^3*c^3*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 7924*
A*a^3*c^3*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 12288*I*B
*a^3*c^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 7924*A*a^3
*c^3*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2800*A*a^3*c^3
*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 420*A*a^3*c^3*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 420*I*A*a^3*c^3*sin(13/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2800*I*A*a^3*c^3*sin(11/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 7924*I*A*a^3*c^3*sin(9/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 12288*B*a^3*c^3*sin(7/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 7924*I*A*a^3*c^3*sin(5/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2800*I*A*a^3*c^3*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 420*I*A*a^3*c^3*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 210*(A*a^3*c^3*cos(14*f*x + 14*e) + 7*
A*a^3*c^3*cos(12*f*x + 12*e) + 21*A*a^3*c^3*cos(10*f*x + 10*e) + 35*A*a^3*
c^3*cos(8*f*x + 8*e) + 35*A*a^3*c^3*cos(6*f*x + 6*e) + 21*A*a^3*c^3*cos(4*
f*x + 4*e) + 7*A*a^3*c^3*cos(2*f*x + 2*e) + I*A*a^3*c^3*sin(14*f*x + 14*e)
 + 7*I*A*a^3*c^3*sin(12*f*x + 12*e) + 21*I*A*a^3*c^3*sin(10*f*x + 10*e) +
35*I*A*a^3*c^3*sin(8*f*x + 8*e) + 35*I*A*a^3*c^3*sin(6*f*x + 6*e) + 21*I*A
*a^3*c^3*sin(4*f*x + 4*e) + 7*I*A*a^3*c^3*sin(2*f*x + 2*e) + A*a^3*c^3)...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx=\int (B\tan(fx+e)+A)(ia\tan(fx+e)+a)^{\frac{7}{2}}(-ic\tan(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(7/2)*(-I*c*tan(f*x
+ e) + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}\,dx=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)
*1i)^(7/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)
*1i)^(7/2), x)
```

Reduce [F]

$$\int (a + ia\tan(e + fx))^{7/2}(A + B\tan(e + fx))(c - ic\tan(e+fx))^{7/2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^3c^3\left(\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^6\,b + 3\sqrt{\tan}\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*c**3*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**6*b + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**4*b + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**2*b + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*b + 7*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)**6,x)*a*f + 21*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**4,x)*a*f + 21*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - t
an(e + f*x)*i + 1)*tan(e + f*x)**2,x)*a*f + 7*int(sqrt(tan(e + f*x)*i + 1)
*sqrt( - tan(e + f*x)*i + 1),x)*a*f))/(7*f)
```

3.818 $\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 284

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = -\frac{a^{7/2}(6iA+B)c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{8f} + \frac{a^3(6A-iB)c^2\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{16f} + \frac{a^2(6A-iB)c\tan(e+fx)(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{24f} + \frac{a(6iA+B)(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}{30f} + \frac{B(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}}{6f}$$

output
```
-1/8*a^(7/2)*(6*I*A+B)*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(
1/2)/(c-I*c*tan(f*x+e))^(1/2))/f+1/16*a^3*(6*A-I*B)*c^2*tan(f*x+e)*(a+I*a*
tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)/f+1/24*a^2*(6*A-I*B)*c*tan(f*x+
e)*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2)/f+1/30*a*(6*I*A+B)*(a
+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(5/2)/f+1/6*B*(a+I*a*tan(f*x+e))
^(7/2)*(c-I*c*tan(f*x+e))^(5/2)/f
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 572 vs. $2(284) = 568$.

Time = 18.59 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.01

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = -\frac{i(6A-iB)c^3e^{-i(4e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\arctan\left(e^{i(e+fx)}\right)(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{8\sqrt{\frac{c}{1+e^{2i(e+fx)}}}f\sec^{\frac{9}{2}}(e+fx)(\cos(fx)+i\sin(fx))^{7/2}(A\cos(e+fx)+B\sin(e+fx))} + \frac{\cos^4(e+fx)\sqrt{\sec(e+fx)(c\cos(e+fx)-ic\sin(e+fx))}\left(\sec(e)\sec^4(e+fx)(6iA\cos(e)+6B\cos(e)+\right.}{}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e
 + f*x])^(5/2),x]
```

output
```
((-1/8*I)*(6*A - I*B)*c^3*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*
I)*(e + f*x)))]*ArcTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(7/2)*(A +
B*Tan[e + f*x]))/(E^(I*(4*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Se
c[e + f*x]^(9/2)*(Cos[f*x] + I*Sin[f*x])^(7/2)*(A*Cos[e + f*x] + B*Sin[e +
 f*x])) + (Cos[e + f*x]^4*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e +
f*x])]*(Sec[e]*Sec[e + f*x]^4*((6*I)*A*Cos[e] + 6*B*Cos[e] + (5*I)*B*Sin[e
])*((c^2*Cos[3*e])/30 - (I/30)*c^2*Sin[3*e]) + I*B*c^2*Sec[e]*Sec[e + f*x]
^5*(Cos[3*e]/6 - (I/6)*Sin[3*e])*Sin[f*x] + Sec[e]*Sec[e + f*x]^3*(Cos[3*e
]/24 - (I/24)*Sin[3*e])*(6*A*c^2*Sin[f*x] - I*B*c^2*Sin[f*x]) + Sec[e]*Sec
[e + f*x]*(Cos[3*e]/16 - (I/16)*Sin[3*e])*(6*A*c^2*Sin[f*x] - I*B*c^2*Sin[
f*x]) + (6*A - I*B)*Sec[e + f*x]^2*((c^2*Cos[3*e])/24 - (I/24)*c^2*Sin[3*e
])*Tan[e] + (6*A - I*B)*((c^2*Cos[3*e])/16 - (I/16)*c^2*Sin[3*e])*Tan[e])*
(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(f*(Cos[f*x] + I*Sin[f*
x])^3*(A*Cos[e + f*x] + B*Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 90, 59, 40, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int (i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 90$$

$$\frac{ac\left(\frac{1}{6}(6A-iB)\int (i\tan(e+fx)a+a)^{5/2}(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))}{6ac}\right.}{f}$$

$$\Big\downarrow \text{ 59}$$

$$\frac{ac\left(\frac{1}{6}(6A-iB)\left(a\int(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{3/2}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+f}{5c}\right.\right.}{f}$$

$$\Big\downarrow \text{ 40}$$

$$\frac{ac\left(\frac{1}{6}(6A-iB)\left(a\left(\frac{3}{4}ac\int\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{1}{4}\tan(e+fx)(a+ia\tan(e+\right.\right.\right.}{f}$$

$$\Big\downarrow \text{ 40}$$

$$ac\left(\frac{1}{6}(6A-iB)\left(a\left(\frac{3}{4}ac\left(\frac{1}{2}ac\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 45}$$

$$ac\left(\frac{1}{6}(6A-iB)\left(a\left(\frac{3}{4}ac\left(ac\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\,t}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 218}$$

$$ac\left(\frac{1}{6}(6A-iB)\left(a\left(\frac{3}{4}ac\left(\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}-i\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e}}{\sqrt{a}\sqrt{c-ic\tan(e}}\right.\right.\right.\right.\right.$$

input
```
Int[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(5/2),x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e + f*x])^(5/2))/(6*a*c
) + ((6*A - I*B)*(((I/5)*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*x
])^(5/2))/c + a*((Tan[e + f*x]*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e
 + f*x])^(3/2))/4 + (3*a*c*((-I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a +
I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + (Tan[e + f*x]*S
qrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/2))/4)))/6))/f
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(232) = 464$.

Time = 0.75 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.68

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c^2\left(40iB\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}\,\tan(fx+e)^5+48iA\sqrt{ac\left(1+\tan(fx\right.}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c^2\left(40iB\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}\,\tan(fx+e)^5+48iA\sqrt{ac\left(1+\tan(fx\right.}\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c^2\left(8i\tan(fx+e)^4\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+16i\tan(fx+e)^2\sqrt{ac\left(\right.}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/240/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3*c^2*(40
*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^5+48*I*A*(a*c*(1+
tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4+70*I*B*(a*c*(1+tan(f*x+e)^2)
)^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+48*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(
1/2)*tan(f*x+e)^4+96*I*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+
e)^2+60*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-15*I*B*ln(
(a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c
+15*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)+96*B*(a*c)^(1/
2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+90*A*ln((a*c*tan(f*x+e)+(a*c)
^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+48*I*A*(a*c*(1+tan(f
*x+e)^2))^(1/2)*(a*c)^(1/2)+150*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)
*tan(f*x+e)+48*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2)/(a*
c*(1+tan(f*x+e)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 756 vs. $2(218) = 436$.

Time = 0.11 (sec) , antiderivative size = 756, normalized size of antiderivative = 2.66

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/480*(15*sqrt((36*A^2 - 12*I*A*B - B^2)*a^7*c^5/f^2)*(f*e^(10*I*f*x + 10
*I*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f
*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((-6*I*A - B)*a^3*c^2
*e^(3*I*f*x + 3*I*e) + (-6*I*A - B)*a^3*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*
I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((36*A^2 - 12
*I*A*B - B^2)*a^7*c^5/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((6*I*A + B)*a^3*c
^2*e^(2*I*f*x + 2*I*e) + (6*I*A + B)*a^3*c^2)) - 15*sqrt((36*A^2 - 12*I*A*
B - B^2)*a^7*c^5/f^2)*(f*e^(10*I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e) +
 10*f*e^(6*I*f*x + 6*I*e) + 10*f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*
I*e) + f)*log(-4*(2*((-6*I*A - B)*a^3*c^2*e^(3*I*f*x + 3*I*e) + (-6*I*A -
B)*a^3*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2
*I*f*x + 2*I*e) + 1)) - sqrt((36*A^2 - 12*I*A*B - B^2)*a^7*c^5/f^2)*(f*e^(
2*I*f*x + 2*I*e) - f))/((6*I*A + B)*a^3*c^2*e^(2*I*f*x + 2*I*e) + (6*I*A +
 B)*a^3*c^2)) + 4*(15*(6*I*A + B)*a^3*c^2*e^(11*I*f*x + 11*I*e) + 85*(6*I*
A + B)*a^3*c^2*e^(9*I*f*x + 9*I*e) + 6*(-58*I*A - 223*B)*a^3*c^2*e^(7*I*f*
x + 7*I*e) + 198*(-6*I*A - B)*a^3*c^2*e^(5*I*f*x + 5*I*e) + 85*(-6*I*A - B
)*a^3*c^2*e^(3*I*f*x + 3*I*e) + 15*(-6*I*A - B)*a^3*c^2*e^(I*f*x + I*e))*s
qrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(
10*I*f*x + 10*I*e) + 5*f*e^(8*I*f*x + 8*I*e) + 10*f*e^(6*I*f*x + 6*I*e) +
10*f*e^(4*I*f*x + 4*I*e) + 5*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2033 vs. $2(218) = 436$.

Time = 4.91 (sec) , antiderivative size = 2033, normalized size of antiderivative = 7.16

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
-3840*(60*(6*A - I*B)*a^3*c^2*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) + 340*(6*A - I*B)*a^3*c^2*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 24*(58*A - 223*I*B)*a^3*c^2*cos(7/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 792*(6*A - I*B)*a^3*c^2*cos(5/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 340*(6*A - I*B)*a^3*c^2*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*(6*A - I*B)*a^3*c^2*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*(-6*I*A - B)*a^3*c^2*sin(11/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 340*(-6*I*A - B)*a^3*c^2*sin
(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 24*(58*I*A + 223*B)*a^
3*c^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 792*(6*I*A +
B)*a^3*c^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 340*(6*I
*A + B)*a^3*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*
(6*I*A + B)*a^3*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 30*((6*A - I*B)*a^3*c^2*cos(12*f*x + 12*e) + 6*(6*A - I*B)*a^3*c^2*cos(10
*f*x + 10*e) + 15*(6*A - I*B)*a^3*c^2*cos(8*f*x + 8*e) + 20*(6*A - I*B)*a^
3*c^2*cos(6*f*x + 6*e) + 15*(6*A - I*B)*a^3*c^2*cos(4*f*x + 4*e) + 6*(6*A
- I*B)*a^3*c^2*cos(2*f*x + 2*e) - (-6*I*A - B)*a^3*c^2*sin(12*f*x + 12*e)
- 6*(-6*I*A - B)*a^3*c^2*sin(10*f*x + 10*e) - 15*(-6*I*A - B)*a^3*c^2*sin(
8*f*x + 8*e) - 20*(-6*I*A - B)*a^3*c^2*sin(6*f*x + 6*e) - 15*(-6*I*A - B)*
a^3*c^2*sin(4*f*x + 4*e) - 6*(-6*I*A - B)*a^3*c^2*sin(2*f*x + 2*e) + (6...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{\frac{7}{2}}(-i\,c\tan(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(7/2)*(-I*c*tan(f*x
+ e) + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)
*1i)^(5/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)
*1i)^(5/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a^3c^2\left(\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^4\,ai+\sqrt{\tan}\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*c**2*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**4*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**4*b + 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**2*a*i + 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e +
f*x)*i + 1)*tan(e + f*x)**2*b + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f
*x)*i + 1)*a*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b +
5*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**6
,x)*b*f*i + 5*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)**4,x)*a*f + 10*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**4,x)*b*f*i + 10*int(sqrt(tan(e + f*x)*i + 1)*sqrt( -
 tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)*a*f + 5*int(sqrt(tan(e + f*x)*i +
1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2,x)*b*f*i + 5*int(sqrt(tan(e
 + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1),x)*a*f))/(5*f)
```

3.819 $\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 279

$$
\begin{aligned}
\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx &= \\
-\frac{a^{7/2}(5iA+2B)c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{4f} \\
+\frac{a^3(5A-2iB)c\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{8f} \\
+\frac{a^2(5iA+2B)(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}{12f} \\
+\frac{a(5iA+2B)(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}{20f} \\
+\frac{B(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}}{5f}
\end{aligned}
$$

output
```
-1/4*a^(7/2)*(5*I*A+2*B)*c^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a
^(1/2)/(c-I*c*tan(f*x+e))^(1/2))/f+1/8*a^3*(5*A-2*I*B)*c*tan(f*x+e)*(a+I*a
*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))^(1/2)/f+1/12*a^2*(5*I*A+2*B)*(a+I*a*
tan(f*x+e))^(3/2)*(c-I*c*tan(f*x+e))^(3/2)/f+1/20*a*(5*I*A+2*B)*(a+I*a*tan
(f*x+e))^(5/2)*(c-I*c*tan(f*x+e))^(3/2)/f+1/5*B*(a+I*a*tan(f*x+e))^(7/2)*(
c-I*c*tan(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 7.84 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.85

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx =$$
$$-\frac{a^{7/2}c^2(i+\tan(e+fx))\left(30(5A-2iB)\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\sqrt{a+ia\tan(e+fx)}+\sqrt{a}\sqrt{1-i\tan(e}\right.}{120f\sqrt{}}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e
 + f*x])^(3/2),x]
```

output
```
-1/120*(a^(7/2)*c^2*(I + Tan[e + f*x])*(30*(5*A - (2*I)*B)*ArcSin[Sqrt[a +
 I*a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[a + I*a*Tan[e + f*x]] + Sqrt[a]
*Sqrt[1 - I*Tan[e + f*x]]*(-I + Tan[e + f*x])*((-80*I)*A - 56*B - 15*(3*A
+ (2*I)*B)*Tan[e + f*x] + ((-80*I)*A - 32*B)*Tan[e + f*x]^2 + 30*(A - (2*I
)*B)*Tan[e + f*x]^3 + 24*B*Tan[e + f*x]^4)))/(f*Sqrt[1 - I*Tan[e + f*x]]*S
qrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 90, 59, 59, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))\,dx$$

$\Big\downarrow$ 3042

$$\int (a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))dx$$

$\Big\downarrow$ 4071

$$\frac{ac\int (i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 90

$$\frac{ac\Big(\frac{1}{5}(5A-2iB)\int (i\tan(e+fx)a+a)^{5/2}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{7/2}(c-ic\tan(e+fx))^{3}}{5ac}}{f}$$

$\Big\downarrow$ 59

$$\frac{ac\Big(\frac{1}{5}(5A-2iB)\left(\frac{5}{4}a\int (i\tan(e+fx)a+a)^{3/2}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+f}{4c}\right.}{f}$$

$\Big\downarrow$ 59

$$\frac{ac\Big(\frac{1}{5}(5A-2iB)\left(\frac{5}{4}a\left(a\int \sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+}{3c}\right.\right.}{f}$$

$\Big\downarrow$ 40

$$ac\Big(\frac{1}{5}(5A-2iB)\left(\frac{5}{4}a\left(a\left(\frac{1}{2}ac\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\right.\right.\right.$$

$\Big\downarrow$ 45

$$ac\left(\frac{1}{5}(5A-2iB)\left(\frac{5}{4}a\left(a\left(ac\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ict}\right.\right.\right.\right.$$

$\Big\downarrow$ 218

$$ac\Big(\frac{1}{5}(5A-2iB)\left(\frac{5}{4}a\left(a\left(\frac{1}{2}\tan(e+fx)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}-i\sqrt{a}\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e}}{\sqrt{a}\sqrt{c-ic\tan(e}}\right.\right.\right.\right.$$

input
```
Int[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x
])^(3/2),x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(7/2)*(c - I*c*Tan[e + f*x])^(3/2))/(5*a*c
) + ((5*A - (2*I)*B)*(((I/4)*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e +
 f*x])^(3/2))/c + (5*a*(((I/3)*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e
 + f*x])^(3/2))/c + a*((-I)*Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*T
an[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])] + (Tan[e + f*x]*Sqrt[a
 + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/2)))/4))/5))/f
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 59
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[2*c*(n/(m + n + 1)
)   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c\left(60iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3-24B\sqrt{ac\left(1+\tan(fx+\right.}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c\left(60iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3-24B\sqrt{ac\left(1+\tan(fx+\right.}\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3c\left(16i\tan(fx+e)^2\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}-6\tan(fx+e)^3\sqrt{ac\left(1+\right.}\right.}{24}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/120/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3*c*(60*I
*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3-24*B*(a*c*(1+tan(
f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4+80*I*A*(a*c*(1+tan(f*x+e)^2))^(1
/2)*(a*c)^(1/2)*tan(f*x+e)^2-30*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)
*tan(f*x+e)^3-30*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))
^(1/2))/(a*c)^(1/2))*a*c+30*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*t
an(f*x+e)+32*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+80*I*
A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+75*A*ln((a*c*tan(f*x+e)+(a*c)^(
1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+45*A*(a*c)^(1/2)*(a*c*
(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+56*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))
^(1/2))/(a*c)^(1/2)/(a*c*(1+tan(f*x+e)^2))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(213) = 426$.

Time = 0.11 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.44

$$\int (a + ia\tan(e + fx))^{7/2}(A + B\tan(e + fx))(c - ic\tan(e + fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
-1/240*(15*sqrt((25*A^2 - 20*I*A*B - 4*B^2)*a^7*c^3/f^2)*(f*e^(8*I*f*x + 8
*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x
 + 2*I*e) + f)*log(-4*(2*((-5*I*A - 2*B)*a^3*c*e^(3*I*f*x + 3*I*e) + (-5*I
*A - 2*B)*a^3*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/
(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((25*A^2 - 20*I*A*B - 4*B^2)*a^7*c^3/f^2)
*(f*e^(2*I*f*x + 2*I*e) - f))/((5*I*A + 2*B)*a^3*c*e^(2*I*f*x + 2*I*e) + (
5*I*A + 2*B)*a^3*c)) - 15*sqrt((25*A^2 - 20*I*A*B - 4*B^2)*a^7*c^3/f^2)*(f
*e^(8*I*f*x + 8*I*e) + 4*f*e^(6*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) +
 4*f*e^(2*I*f*x + 2*I*e) + f)*log(-4*(2*((-5*I*A - 2*B)*a^3*c*e^(3*I*f*x +
 3*I*e) + (-5*I*A - 2*B)*a^3*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e
) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((25*A^2 - 20*I*A*B - 4*B^
2)*a^7*c^3/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((5*I*A + 2*B)*a^3*c*e^(2*I*f
*x + 2*I*e) + (5*I*A + 2*B)*a^3*c)) + 4*(15*(5*I*A + 2*B)*a^3*c*e^(9*I*f*x
 + 9*I*e) + 10*(-29*I*A - 50*B)*a^3*c*e^(7*I*f*x + 7*I*e) + 128*(-5*I*A -
2*B)*a^3*c*e^(5*I*f*x + 5*I*e) + 70*(-5*I*A - 2*B)*a^3*c*e^(3*I*f*x + 3*I*
e) + 15*(-5*I*A - 2*B)*a^3*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e)
+ 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(8*I*f*x + 8*I*e) + 4*f*e^(6
*I*f*x + 6*I*e) + 6*f*e^(4*I*f*x + 4*I*e) + 4*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1657 vs. $2(213) = 426$.

Time = 1.87 (sec) , antiderivative size = 1657, normalized size of antiderivative = 5.94

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
-480*(60*(5*A - 2*I*B)*a^3*c*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) - 40*(29*A - 50*I*B)*a^3*c*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - 512*(5*A - 2*I*B)*a^3*c*cos(5/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 280*(5*A - 2*I*B)*a^3*c*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - 60*(5*A - 2*I*B)*a^3*c*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 60*(-5*I*A - 2*B)*a^3*c*sin(9/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(29*I*A + 50*B)*a^3*c*sin(7/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 512*(5*I*A + 2*B)*a^3*c*sin(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 280*(5*I*A + 2*B)*a^3*c*sin(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*(5*I*A + 2*B)*a^3*c*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*((5*A - 2*I*B)*a
^3*c*cos(10*f*x + 10*e) + 5*(5*A - 2*I*B)*a^3*c*cos(8*f*x + 8*e) + 10*(5*A
 - 2*I*B)*a^3*c*cos(6*f*x + 6*e) + 10*(5*A - 2*I*B)*a^3*c*cos(4*f*x + 4*e)
 + 5*(5*A - 2*I*B)*a^3*c*cos(2*f*x + 2*e) - (-5*I*A - 2*B)*a^3*c*sin(10*f*
x + 10*e) - 5*(-5*I*A - 2*B)*a^3*c*sin(8*f*x + 8*e) - 10*(-5*I*A - 2*B)*a^
3*c*sin(6*f*x + 6*e) - 10*(-5*I*A - 2*B)*a^3*c*sin(4*f*x + 4*e) - 5*(-5*I*
A - 2*B)*a^3*c*sin(2*f*x + 2*e) + (5*A - 2*I*B)*a^3*c)*arctan2(cos(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 1) + 30*((5*A - 2*I*B)*a^3*c*cos(10*f*x + 10*e) +
5*(5*A - 2*I*B)*a^3*c*cos(8*f*x + 8*e) + 10*(5*A - 2*I*B)*a^3*c*cos(6*f...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{\frac{7}{2}}(-i\,c\tan(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(7/2)*(-I*c*tan(f*x
+ e) + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)
*1i)^(3/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)
*1i)^(3/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^3c\Big(-3\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)^4\,b+10\sqrt{}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*c*( - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**4*b + 10*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f
*x)*i + 1)*tan(e + f*x)**2*a*i + 4*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x)**2*b + 10*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(
e + f*x)*i + 1)*a*i + 7*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
1)*b - 15*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
 f*x)**4,x)*a*f + 30*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)**4,x)*b*f*i + 30*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan
(e + f*x)*i + 1)*tan(e + f*x)**2,x)*b*f*i + 15*int(sqrt(tan(e + f*x)*i + 1
)*sqrt( - tan(e + f*x)*i + 1),x)*a*f))/(15*f)
```

3.820 $\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 272

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = -\frac{5a^{7/2}(4iA+3B)\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{4f} + \frac{5a^3(4iA+3B)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{8f} + \frac{5a^2(4iA+3B)(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{24f} + \frac{a(4iA+3B)(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}{12f} + \frac{B(a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}}{4f}$$

output
```
-5/4*a^(7/2)*(4*I*A+3*B)*c^(1/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a
^(1/2)/(c-I*c*tan(f*x+e))^(1/2))/f+5/8*a^3*(4*I*A+3*B)*(a+I*a*tan(f*x+e))^
(1/2)*(c-I*c*tan(f*x+e))^(1/2)/f+5/24*a^2*(4*I*A+3*B)*(a+I*a*tan(f*x+e))^(
3/2)*(c-I*c*tan(f*x+e))^(1/2)/f+1/12*a*(4*I*A+3*B)*(a+I*a*tan(f*x+e))^(5/2
)*(c-I*c*tan(f*x+e))^(1/2)/f+1/4*B*(a+I*a*tan(f*x+e))^(7/2)*(c-I*c*tan(f*x
+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.81

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \frac{a^{7/2}c(i+\tan(e+fx))\left(-30(4A-3iB)\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)\right.}{}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*Sqrt[c - I*c*T
an[e + f*x]],x]
```

output
```
(a^(7/2)*c*(I + Tan[e + f*x])*(-30*(4*A - (3*I)*B)*ArcSin[Sqrt[a + I*a*Tan
[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[a + I*a*Tan[e + f*x]] + Sqrt[a]*Sqrt[1
- I*Tan[e + f*x]]*(-I + Tan[e + f*x])*((88*I)*A + 72*B - 9*(4*A - (5*I)*B)
*Tan[e + f*x] - (8*I)*(A - (3*I)*B)*Tan[e + f*x]^2 - (6*I)*B*Tan[e + f*x]^
3)))/(24*f*Sqrt[1 - I*Tan[e + f*x]]*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*
c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 90, 60, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{90}$$

$$\frac{ac\left(\frac{1}{4}(4A-3iB)\int \frac{(i\tan(e+fx)a+a)^{5/2}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{B(a+ia\tan(e+fx))^{7/2}\sqrt{c-ic\tan(e+fx)}}{4ac}\right)}{f}$$

$$\Big\downarrow \text{60}$$

$$\frac{ac\left(\frac{1}{4}(4A-3iB)\left(\frac{5}{3}a\int \frac{(i\tan(e+fx)a+a)^{3/2}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}{3c}\right)+\frac{B(a+ia\tan(e+fx))^{7/2}}{4ac}\right.}{f}$$

$$\Big\downarrow \text{60}$$

$$\frac{ac\left(\frac{1}{4}(4A-3iB)\left(\frac{5}{3}a\left(\frac{3}{2}a\int \frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2c}\right)+\frac{i(a+ia\tan(e+fx))^{5/}}{3c}\right.\right.}{f}$$

$$\Big\downarrow \text{60}$$

$$\frac{ac\left(\frac{1}{4}(4A-3iB)\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{i(}{}\right.\right.\right.}{f}$$

$$\Big\downarrow \text{45}$$

$$\frac{ac\left(\frac{1}{4}(4A-3iB)\left(\frac{5}{3}a\left(\frac{3}{2}a\left(2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{i(a+ia\tan}{}\right.\right.\right.}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{ac\left(\frac{1}{4}(4A-3iB)\left(\frac{5}{3}a\left(\frac{3}{2}a\left(\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}}{2c}\right.\right.\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e +
 f*x]],x]
```

output
```
(a*c*((B*(a + I*a*Tan[e + f*x])^(7/2)*Sqrt[c - I*c*Tan[e + f*x]])/(4*a*c)
+ ((4*A - (3*I)*B)*(((I/3)*(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e
 + f*x]])/c + (5*a*(((I/2)*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e
 + f*x]])/c + (3*a*(((-2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f
*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[c] + (I*Sqrt[a + I*a*Tan
[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/c))/2))/3))/4))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.28

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(6iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+8iA\sqrt{ac}\,\sqrt{ac\left(1+\tan\right.}\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(6iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+8iA\sqrt{ac}\,\sqrt{ac\left(1+\tan\right.}\right.}{}$
parts	$\frac{A\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,a^3\left(-2i\tan(fx+e)^2\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}+15\ln\left(\frac{ac\tan(fx+e)+\sqrt{}}{}\right.\right.}{6f\sqrt{ac\left(1+\tan(fx+e)\right.}}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/24/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3*(6*I*B*
(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3+8*I*A*(a*c)^(1/2)*(a
*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+45*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1
/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-45*I*B*(a*c)^(1/2)*(a*c
*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+24*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2)
)^(1/2)*tan(f*x+e)^2-88*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)-60*A*
ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*
a*c+36*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-72*B*(a*c)^(1
/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(206) = 412$.

Time = 0.11 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.22

$$\int (a + ia\tan(e + fx))^{7/2}(A + B\tan(e + fx))\sqrt{c - ic\tan(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
-1/48*(15*sqrt((16*A^2 - 24*I*A*B - 9*B^2)*a^7*c/f^2)*(f*e^(6*I*f*x + 6*I*
e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*((-4*
I*A - 3*B)*a^3*e^(3*I*f*x + 3*I*e) + (-4*I*A - 3*B)*a^3*e^(I*f*x + I*e))*s
qrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt(
(16*A^2 - 24*I*A*B - 9*B^2)*a^7*c/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-4*I
*A - 3*B)*a^3*e^(2*I*f*x + 2*I*e) + (-4*I*A - 3*B)*a^3)) - 15*sqrt((16*A^2
 - 24*I*A*B - 9*B^2)*a^7*c/f^2)*(f*e^(6*I*f*x + 6*I*e) + 3*f*e^(4*I*f*x +
4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)*log(4*(2*((-4*I*A - 3*B)*a^3*e^(3*I*
f*x + 3*I*e) + (-4*I*A - 3*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((16*A^2 - 24*I*A*B - 9
*B^2)*a^7*c/f^2)*(f*e^(2*I*f*x + 2*I*e) - f))/((-4*I*A - 3*B)*a^3*e^(2*I*f
*x + 2*I*e) + (-4*I*A - 3*B)*a^3)) + 4*(3*(-44*I*A - 49*B)*a^3*e^(7*I*f*x
+ 7*I*e) + 73*(-4*I*A - 3*B)*a^3*e^(5*I*f*x + 5*I*e) + 55*(-4*I*A - 3*B)*a
^3*e^(3*I*f*x + 3*I*e) + 15*(-4*I*A - 3*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^
(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(f*e^(6*I*f*x +
 6*I*e) + 3*f*e^(4*I*f*x + 4*I*e) + 3*f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1345 vs. 2(206) = 412.

Time = 0.78 (sec) , antiderivative size = 1345, normalized size of antiderivative = 4.94

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
96*(12*(44*A - 49*I*B)*a^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 292*(4*A - 3*I*B)*a^3*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 220*(4*A - 3*I*B)*a^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 60*(4*A - 3*I*B)*a^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 12*(44*I*A + 49*B)*a^3*sin(7/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 292*(4*I*A + 3*B)*a^3*sin(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 220*(4*I*A + 3*B)*a^3*sin(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 60*(4*I*A + 3*B)*a^3*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - 30*((4*A - 3*I*B)*a^3*cos(8*f*x + 8*e) + 4*(4*
A - 3*I*B)*a^3*cos(6*f*x + 6*e) + 6*(4*A - 3*I*B)*a^3*cos(4*f*x + 4*e) + 4
*(4*A - 3*I*B)*a^3*cos(2*f*x + 2*e) - (-4*I*A - 3*B)*a^3*sin(8*f*x + 8*e)
- 4*(-4*I*A - 3*B)*a^3*sin(6*f*x + 6*e) - 6*(-4*I*A - 3*B)*a^3*sin(4*f*x +
 4*e) - 4*(-4*I*A - 3*B)*a^3*sin(2*f*x + 2*e) + (4*A - 3*I*B)*a^3)*arctan2
(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 30*((4*A - 3*I*B)*a^3*cos(8*f*x +
 8*e) + 4*(4*A - 3*I*B)*a^3*cos(6*f*x + 6*e) + 6*(4*A - 3*I*B)*a^3*cos(4*f
*x + 4*e) + 4*(4*A - 3*I*B)*a^3*cos(2*f*x + 2*e) - (-4*I*A - 3*B)*a^3*sin(
8*f*x + 8*e) - 4*(-4*I*A - 3*B)*a^3*sin(6*f*x + 6*e) - 6*(-4*I*A - 3*B)*a^
3*sin(4*f*x + 4*e) - 4*(-4*I*A - 3*B)*a^3*sin(2*f*x + 2*e) + (4*A - 3*I*B)
*a^3)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -si...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{\frac{7}{2}}\sqrt{-i\,c\tan(fx+$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(7/2)*sqrt(-I*c*tan(
f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\;1\mathrm{i}\right)^{7/2}\,\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)
*1i)^(1/2),x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2)*(c - c*tan(e + f*x)
*1i)^(1/2), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a^3\Big(-\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+1}\,\tan\left(fx\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2*a*i - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**2*b + 11*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*
x)*i + 1)*a*i + 9*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b -
 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**
4,x)*b*f*i - 9*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*ta
n(e + f*x)**2,x)*a*f + 9*int(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**2,x)*b*f*i + 3*int(sqrt(tan(e + f*x)*i + 1)*sqrt( -
tan(e + f*x)*i + 1),x)*a*f))/(3*f)
```

3.821 $\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 283

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{5a^{7/2}(3iA+4B)\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}f} - \frac{(iA+B)(a+ia\tan(e+fx))^{7/2}}{f\sqrt{c-ic\tan(e+fx)}} - \frac{5a^3(3iA+4B)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2cf} - \frac{5a^2(3iA+4B)(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{6cf} - \frac{a(3iA+4B)(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}{3cf}$$

output
```
5*a^(7/2)*(3*I*A+4*B)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I
*c*tan(f*x+e))^(1/2))/c^(1/2)/f-(I*A+B)*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*c*
tan(f*x+e))^(1/2)-5/2*a^3*(3*I*A+4*B)*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(
f*x+e))^(1/2)/c/f-5/6*a^2*(3*I*A+4*B)*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c*tan(
f*x+e))^(1/2)/c/f-1/3*a*(3*I*A+4*B)*(a+I*a*tan(f*x+e))^(5/2)*(c-I*c*tan(f*
x+e))^(1/2)/c/f
```

Mathematica [A] (verified)

Time = 6.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.65

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{a^{7/2}\left(\frac{30(3A-4iB)\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)(i+\tan(e+fx))}{\sqrt{1-i\tan(e+fx)}}+\frac{\sqrt{a}(-\cdots}{\cdots}\right.}{6\cdots}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/Sqrt[c - I*c
*Tan[e + f*x]],x]
```

output
```
(a^(7/2)*((30*(3*A - (4*I)*B)*ArcSin[Sqrt[a + I*a*Tan[e + f*x]]/(Sqrt[2]*S
qrt[a])]*(I + Tan[e + f*x]))/Sqrt[1 - I*Tan[e + f*x]] + (Sqrt[a]*(-I + Tan
[e + f*x])*(72*A - (94*I)*B + ((-21*I)*A - 34*B)*Tan[e + f*x] + (3*A - (10
*I)*B)*Tan[e + f*x]^2 + 2*B*Tan[e + f*x]^3))/Sqrt[a + I*a*Tan[e + f*x]]))/
(6*f*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {3042, 4071, 87, 60, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 87

$$\frac{ac\left(-\frac{(3A-4iB)\int \frac{(i\tan(e+fx)a+a)^{5/2}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$\Big\downarrow$ 60

$$\frac{ac\left(-\frac{(3A-4iB)\left(\frac{5}{3}a\int \frac{(i\tan(e+fx)a+a)^{3/2}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}{3c}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{ac\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$\Big\downarrow$ 60

$$\frac{ac\left(-\frac{(3A-4iB)\left(\frac{5}{3}a\left(\frac{3}{2}a\int \frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2c}\right)+\frac{i(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}{3c}\right)}{c}-\right.}{f}$$

$\Big\downarrow$ 60

$$\frac{ac\left(-\frac{(3A-4iB)\left(\frac{5}{3}a\left(\frac{3}{2}a\left(a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic}}{2c}\right.\right.}{c}\right.}{f}$$

$\Big\downarrow$ 45

$$\frac{ac\left(-\frac{(3A-4iB)\left(\frac{5}{3}a\left(\frac{3}{2}a\left(2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan}}{2c}\right.\right.}{c}\right.}{f}$$

$\Big\downarrow$ 218

$$\frac{ac\left(-\frac{(3A-4iB)\left(\frac{5}{3}a\left(\frac{3}{2}a\left(\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2c}\right)+\right.}{c}\right.}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/Sqrt[c - I*c*Tan[e
 + f*x]],x]
```

output
```
(a*c*(-(((I*A + B)*(a + I*a*Tan[e + f*x])^(7/2))/(a*c*Sqrt[c - I*c*Tan[e +
 f*x]])) - ((3*A - (4*I)*B)*(((I/3)*(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c -
I*c*Tan[e + f*x]])/c + (5*a*(((I/2)*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c -
I*c*Tan[e + f*x]])/c + (3*a*(((-2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*
Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[c] + (I*Sqrt[a
+ I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/c))/2))/3))/c))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(234) = 468.

Time = 0.64 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.22

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(-60iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^2+\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(-60iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^2+\right.}{}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(30i\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)\tan(fx+e)ac+6i\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/6/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c*(-60*I
*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2
))*a*c*tan(f*x+e)^2+8*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x
+e)^3-2*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4+90*I*A*ln(
(a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c
*tan(f*x+e)+45*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/
2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+18*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2)
)^(1/2)*tan(f*x+e)^2-3*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+
e)^3+60*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(
a*c)^(1/2))*a*c+120*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2)
)^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+128*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e
)^2))^(1/2)*tan(f*x+e)+24*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f
*x+e)^2-45*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/
(a*c)^(1/2))*a*c-72*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)-93*A*(a*c
)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-94*B*(a*c)^(1/2)*(a*c*(1+t
an(f*x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I+tan(f*x+e))^2/(a*c)^(
1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(219) = 438$.

Time = 0.10 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.08

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
1/12*(15*sqrt((9*A^2 - 24*I*A*B - 16*B^2)*a^7/(c*f^2))*(c*f*e^(4*I*f*x + 4
*I*e) + 2*c*f*e^(2*I*f*x + 2*I*e) + c*f)*log(4*(2*((-3*I*A - 4*B)*a^3*e^(3
*I*f*x + 3*I*e) + (-3*I*A - 4*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x +
 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((9*A^2 - 24*I*A*B -
 16*B^2)*a^7/(c*f^2))*(c*f*e^(2*I*f*x + 2*I*e) - c*f))/((-3*I*A - 4*B)*a^3
*e^(2*I*f*x + 2*I*e) + (-3*I*A - 4*B)*a^3)) - 15*sqrt((9*A^2 - 24*I*A*B -
16*B^2)*a^7/(c*f^2))*(c*f*e^(4*I*f*x + 4*I*e) + 2*c*f*e^(2*I*f*x + 2*I*e)
+ c*f)*log(4*(2*((-3*I*A - 4*B)*a^3*e^(3*I*f*x + 3*I*e) + (-3*I*A - 4*B)*a
^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x +
 2*I*e) + 1)) - sqrt((9*A^2 - 24*I*A*B - 16*B^2)*a^7/(c*f^2))*(c*f*e^(2*I*
f*x + 2*I*e) - c*f))/((-3*I*A - 4*B)*a^3*e^(2*I*f*x + 2*I*e) + (-3*I*A - 4
*B)*a^3)) - 4*(24*(I*A + B)*a^3*e^(7*I*f*x + 7*I*e) + 33*(3*I*A + 4*B)*a^3
*e^(5*I*f*x + 5*I*e) + 40*(3*I*A + 4*B)*a^3*e^(3*I*f*x + 3*I*e) + 15*(3*I*
A + 4*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^
(2*I*f*x + 2*I*e) + 1)))/(c*f*e^(4*I*f*x + 4*I*e) + 2*c*f*e^(2*I*f*x + 2*I
*e) + c*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1329 vs. $2(219) = 438$.

Time = 0.48 (sec) , antiderivative size = 1329, normalized size of antiderivative = 4.70

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
-6*(12*(9*A - 20*I*B)*a^3*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 32*(6*A - 11*I*B)*a^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 12*(9*I*A + 20*B)*a^3*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 32*(6*I*A + 11*B)*a^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - 30*((3*A - 4*I*B)*a^3*cos(6*f*x + 6*e) + 3*(3*A - 4*I*B)*a
^3*cos(4*f*x + 4*e) + 3*(3*A - 4*I*B)*a^3*cos(2*f*x + 2*e) - (-3*I*A - 4*B
)*a^3*sin(6*f*x + 6*e) - 3*(-3*I*A - 4*B)*a^3*sin(4*f*x + 4*e) - 3*(-3*I*A
 - 4*B)*a^3*sin(2*f*x + 2*e) + (3*A - 4*I*B)*a^3)*arctan2(cos(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 1) - 30*((3*A - 4*I*B)*a^3*cos(6*f*x + 6*e) + 3*(3*A -
4*I*B)*a^3*cos(4*f*x + 4*e) + 3*(3*A - 4*I*B)*a^3*cos(2*f*x + 2*e) - (-3*I
*A - 4*B)*a^3*sin(6*f*x + 6*e) - 3*(-3*I*A - 4*B)*a^3*sin(4*f*x + 4*e) - 3
*(-3*I*A - 4*B)*a^3*sin(2*f*x + 2*e) + (3*A - 4*I*B)*a^3)*arctan2(cos(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 1) + 12*(8*(A - I*B)*a^3*cos(6*f*x + 6*e) + 24
*(A - I*B)*a^3*cos(4*f*x + 4*e) + 24*(A - I*B)*a^3*cos(2*f*x + 2*e) + 8*(I
*A + B)*a^3*sin(6*f*x + 6*e) + 24*(I*A + B)*a^3*sin(4*f*x + 4*e) + 24*(I*A
 + B)*a^3*sin(2*f*x + 2*e) + 5*(3*A - 4*I*B)*a^3)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 15*((-3*I*A - 4*B)*a^3*cos(6*f*x + 6*e) + 3
*(-3*I*A - 4*B)*a^3*cos(4*f*x + 4*e) + 3*(-3*I*A - 4*B)*a^3*cos(2*f*x +...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \int \frac{(A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}}{\sqrt{c-c\tan\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(1/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)**4*b - 12*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)**2*a*i - 22*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)**2*b + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)*a - 36*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*a*i - 47*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b +
3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**
4)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*a*f - 12*int((sqrt(tan(e + f*x
)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2 + 1
),x)*tan(e + f*x)**2*b*f*i + 3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2 + 1),x)*a*f - 12*int((sqr
t(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e
+ f*x)**2 + 1),x)*b*f*i - 18*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*a*f
 + 12*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*
x)**2)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f*i - 18*int((sqrt(tan(e
 + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)*
*2 + 1),x)*a*f + 12*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*b*f*i))/(3*c*f*(tan(e + f*x)
**2 + 1))
```

3.822 $\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 285

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = -\frac{5a^{7/2}(2iA+5B)\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}f} - \frac{(iA+B)(a+ia\tan(e+fx))^{7/2}}{3f(c-ic\tan(e+fx))^{3/2}} + \frac{2a(2iA+5B)(a+ia\tan(e+fx))^{5/2}}{3cf\sqrt{c-ic\tan(e+fx)}} + \frac{5a^3(2iA+5B)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2c^2f} + \frac{5a^2(2iA+5B)(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{6c^2f}$$

output
```
-5*a^(7/2)*(2*I*A+5*B)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-
I*c*tan(f*x+e))^(1/2))/c^(3/2)/f-1/3*(I*A+B)*(a+I*a*tan(f*x+e))^(7/2)/f/(c
-I*c*tan(f*x+e))^(3/2)+2/3*a*(2*I*A+5*B)*(a+I*a*tan(f*x+e))^(5/2)/c/f/(c-I
*c*tan(f*x+e))^(1/2)+5/2*a^3*(2*I*A+5*B)*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*t
an(f*x+e))^(1/2)/c^2/f+5/6*a^2*(2*I*A+5*B)*(a+I*a*tan(f*x+e))^(3/2)*(c-I*c
*tan(f*x+e))^(1/2)/c^2/f
```

Mathematica [A] (warning: unable to verify)

Time = 19.22 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.81

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{5i(2A-5iB)e^{-i(4e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\arctan\left(e^{i(e+fx)}\right)(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{c\sqrt{\frac{c}{1+e^{2i(e+fx)}}}f\sec^{\frac{9}{2}}(e+fx)(\cos(fx)+i\sin(fx))^{7/2}(A\cos(e+fx)+B\sin(e+fx))}$$

$$+\frac{\cos^4(e+fx)\left((5iA+11B)\cos(2fx)\left(\frac{2\cos(e)}{3c^2}-\frac{2i\sin(e)}{3c^2}\right)+(A-iB)\cos(4fx)\left(-\frac{2i\cos(e)}{3c^2}+\frac{2\sin(e)}{3c^2}\right)+\sec(\right.}{}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(3/2),x]
```

output
```
((-5*I)*(2*A - (5*I)*B)*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)
*(e + f*x)))]*ArcTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*
Tan[e + f*x]))/(c*E^(I*(4*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Se
c[e + f*x]^(9/2)*(Cos[f*x] + I*Sin[f*x])^(7/2)*(A*Cos[e + f*x] + B*Sin[e +
 f*x])) + (Cos[e + f*x]^4*(((5*I)*A + 11*B)*Cos[2*f*x]*((2*Cos[e])/(3*c^2)
 - (((2*I)/3)*Sin[e])/c^2) + (A - I*B)*Cos[4*f*x]*((((-2*I)/3)*Cos[e])/c^2
 + (2*Sin[e])/(3*c^2)) + Sec[e]*((10*I)*A*Cos[e] + 26*B*Cos[e] + I*B*Sin[e
])*(Cos[3*e]/(2*c^2) - ((I/2)*Sin[3*e])/c^2) + I*B*Sec[e]*Sec[e + f*x]*(Co
s[3*e]/(2*c^2) - ((I/2)*Sin[3*e])/c^2)*Sin[f*x] + (5*A - (11*I)*B)*((-2*Co
s[e])/(3*c^2) + (((2*I)/3)*Sin[e])/c^2)*Sin[2*f*x] + (A - I*B)*((2*Cos[e])
/(3*c^2) + (((2*I)/3)*Sin[e])/c^2)*Sin[4*f*x])*Sqrt[Sec[e + f*x]*(c*Cos[e
+ f*x] - I*c*Sin[e + f*x])]*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*
x]))/(f*(Cos[f*x] + I*Sin[f*x])^3*(A*Cos[e + f*x] + B*Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 87, 57, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 4071$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 87$$

$$\frac{ac\left(-\frac{(2A-5iB)\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \ 57$$

$$\frac{ac\left(-\frac{(2A-5iB)\left(-\frac{5a\int \frac{(i\tan(e+fx)a+a)^{3/2}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{2i(a+ia\tan(e+fx))^{5/2}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \ 60$$

$$\frac{ac\left(-\frac{(2A-5iB)\left(-\frac{5a\left(\frac{3}{2}a\int \frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2c}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{5/2}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{(B+iA)(a+ia\,\mathrm{t}}{3ac(c-ic\tan}\right.}{f}$$

$$\Big\downarrow \ 60$$

$$ac\left(-\frac{(2A-5iB)\left(-\frac{5a\left(\frac{3}{2}a\left(a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2c}}{c}\right.}{3c}\right.}{f}$$

$$\Big\downarrow \; 45$$

$$ac\left(-\frac{(2A-5iB)\left(-\frac{5a\left(\frac{3}{2}a\left(2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(}}{2c}}{c}\right.}{3c}\right.}{f}$$

$$\Big\downarrow \; 218$$

$$ac\left(-\frac{(2A-5iB)\left(-\frac{5a\left(\frac{3}{2}a\left(\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}}\right)+\frac{i(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}{2c}\right)}{c}-\frac{2}{}\right.}{3c}\right.}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(3/2),x]
```

output
```
(a*c*(-1/3*((I*A + B)*(a + I*a*Tan[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e +
f*x])^(3/2)) - ((2*A - (5*I)*B)*(((-2*I)*(a + I*a*Tan[e + f*x])^(5/2))/(c*
Sqrt[c - I*c*Tan[e + f*x]]) - (5*a*(((I/2)*(a + I*a*Tan[e + f*x])^(3/2)*Sq
rt[c - I*c*Tan[e + f*x]])/c + (3*a*(((-2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a
 + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[c] + (I*
Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/c))/2))/c))/(3*c)))
/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(234) = 468$.

Time = 0.77 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.56

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(185iB\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}\,\tan(fx+e)^2+6iA\sqrt{ac}\,\sqrt{ac\left(1+\tan\right.}\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(185iB\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}\,\tan(fx+e)^2+6iA\sqrt{ac}\,\sqrt{ac\left(1+\tan\right.}\right.}{}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(45i\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)\tan(fx+e)^2ac+3i\,\mathrm{ta}\right.}{}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/6/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c^2*(185*
I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+6*I*A*(a*c)^(1/2
)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3-75*I*B*ln((a*c*tan(f*x+e)+(a*c
)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3-118*I*
B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)+30*A*ln((a*c*tan(f*x+e)+(a*c)^(
1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+90*I*A*ln
((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*
c*tan(f*x+e)^2-114*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)
+225*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^
(1/2))*a*c*tan(f*x+e)^2+21*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(
f*x+e)^3+3*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^4+225*I
*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2
))*a*c*tan(f*x+e)-90*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2
))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-74*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a
*c)^(1/2)*tan(f*x+e)^2-30*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f
*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-75*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c
*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-279*B*(a*c)^(1/2)*(a*c*(1+tan(f
*x+e)^2))^(1/2)*tan(f*x+e)+46*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/
(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)/(I+tan(f*x+e))^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(221) = 442$.

Time = 0.11 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.00

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
-1/12*(15*(c^2*f*e^(2*I*f*x + 2*I*e) + c^2*f)*sqrt((4*A^2 - 20*I*A*B - 25*
B^2)*a^7/(c^3*f^2))*log(4*(2*((-2*I*A - 5*B)*a^3*e^(3*I*f*x + 3*I*e) + (-2
*I*A - 5*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/
(e^(2*I*f*x + 2*I*e) + 1)) + (c^2*f*e^(2*I*f*x + 2*I*e) - c^2*f)*sqrt((4*A
^2 - 20*I*A*B - 25*B^2)*a^7/(c^3*f^2)))/((-2*I*A - 5*B)*a^3*e^(2*I*f*x + 2
*I*e) + (-2*I*A - 5*B)*a^3)) - 15*(c^2*f*e^(2*I*f*x + 2*I*e) + c^2*f)*sqrt
((4*A^2 - 20*I*A*B - 25*B^2)*a^7/(c^3*f^2))*log(4*(2*((-2*I*A - 5*B)*a^3*e
^(3*I*f*x + 3*I*e) + (-2*I*A - 5*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (c^2*f*e^(2*I*f*x + 2
*I*e) - c^2*f)*sqrt((4*A^2 - 20*I*A*B - 25*B^2)*a^7/(c^3*f^2)))/((-2*I*A -
 5*B)*a^3*e^(2*I*f*x + 2*I*e) + (-2*I*A - 5*B)*a^3)) + 4*(4*(I*A + B)*a^3*
e^(7*I*f*x + 7*I*e) + 8*(-2*I*A - 5*B)*a^3*e^(5*I*f*x + 5*I*e) + 25*(-2*I*
A - 5*B)*a^3*e^(3*I*f*x + 3*I*e) + 15*(-2*I*A - 5*B)*a^3*e^(I*f*x + I*e))*
sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c^2*
f*e^(2*I*f*x + 2*I*e) + c^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1176 vs. $2(221) = 442$.

Time = 0.31 (sec) , antiderivative size = 1176, normalized size of antiderivative = 4.13

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
-2*(30*((2*A - 5*I*B)*a^3*cos(4*f*x + 4*e) + 2*(2*A - 5*I*B)*a^3*cos(2*f*x
 + 2*e) - (-2*I*A - 5*B)*a^3*sin(4*f*x + 4*e) - 2*(-2*I*A - 5*B)*a^3*sin(2
*f*x + 2*e) + (2*A - 5*I*B)*a^3)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 1) + 30*((2*A - 5*I*B)*a^3*cos(4*f*x + 4*e) + 2*(2*A - 5*I*B)*a^3*cos(2*
f*x + 2*e) - (-2*I*A - 5*B)*a^3*sin(4*f*x + 4*e) - 2*(-2*I*A - 5*B)*a^3*si
n(2*f*x + 2*e) + (2*A - 5*I*B)*a^3)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + 1) + 4*(4*(A - I*B)*a^3*cos(4*f*x + 4*e) + 8*(A - I*B)*a^3*cos(2*f*x
 + 2*e) - 4*(-I*A - B)*a^3*sin(4*f*x + 4*e) - 8*(-I*A - B)*a^3*sin(2*f*x +
 2*e) - (2*A - 29*I*B)*a^3)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 12*(8*(A - 2*I*B)*a^3*cos(4*f*x + 4*e) + 16*(A - 2*I*B)*a^3*cos(2
*f*x + 2*e) + 8*(I*A + 2*B)*a^3*sin(4*f*x + 4*e) + 16*(I*A + 2*B)*a^3*sin(
2*f*x + 2*e) + 5*(2*A - 5*I*B)*a^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 15*((-2*I*A - 5*B)*a^3*cos(4*f*x + 4*e) + 2*(-2*I*A - 5*B
)*a^3*cos(2*f*x + 2*e) + (2*A - 5*I*B)*a^3*sin(4*f*x + 4*e) + 2*(2*A - 5*I
*B)*a^3*sin(2*f*x + 2*e) + (-2*I*A - 5*B)*a^3)*log(cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
1) - 15*((2*I*A + 5*B)*a^3*cos(4*f*x + 4*e) + 2*(2*I*A + 5*B)*a^3*cos(2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx=\int \frac{(A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}}{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(3/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*( - 10*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)*a + 5*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
 + 1)*tan(e + f*x)*b*i + 6*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
 + 1)*a*i + 4*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - 4*i
nt(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)
**3 + tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**2*a*f*i - int
(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**
3 + tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**2*b*f - 4*int((
 - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3
+ tan(e + f*x)**2*i + tan(e + f*x) + i),x)*a*f*i - int(( - sqrt(tan(e + f*
x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 + tan(e + f*x)**2*
i + tan(e + f*x) + i),x)*b*f - int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**3*i - tan(e + f*x)**2 + tan
(e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f - int((sqrt(tan(e + f*x)*i + 1)*sq
rt( - tan(e + f*x)*i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**3*i - tan(e + f*
x)**2 + tan(e + f*x)*i - 1),x)*b*f - int((sqrt(tan(e + f*x)*i + 1)*sqrt( -
 tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i - tan(e + f*x)**2
 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*a*f + 4*int((sqrt(tan(e + f*x)*i
 + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i - ta
n(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2*b*f*i - int((sqr...
```

3.823 $\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 283

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{2a^{7/2}(iA+6B)\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{5/2}f} - \frac{(iA+B)(a+ia\tan(e+fx))^{7/2}}{5f(c-ic\tan(e+fx))^{5/2}} + \frac{2a(iA+6B)(a+ia\tan(e+fx))^{5/2}}{15cf(c-ic\tan(e+fx))^{3/2}} - \frac{2a^2(iA+6B)(a+ia\tan(e+fx))^{3/2}}{3c^2f\sqrt{c-ic\tan(e+fx)}} - \frac{a^3(iA+6B)\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c^3f}$$

output
```
2*a^(7/2)*(I*A+6*B)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c
*tan(f*x+e))^(1/2))/c^(5/2)/f-1/5*(I*A+B)*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*
c*tan(f*x+e))^(5/2)+2/15*a*(I*A+6*B)*(a+I*a*tan(f*x+e))^(5/2)/c/f/(c-I*c*t
an(f*x+e))^(3/2)-2/3*a^2*(I*A+6*B)*(a+I*a*tan(f*x+e))^(3/2)/c^2/f/(c-I*c*t
an(f*x+e))^(1/2)-a^3*(I*A+6*B)*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*tan(f*x+e))
^(1/2)/c^3/f
```

Mathematica [A] (warning: unable to verify)

Time = 19.16 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.87

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{2(iA+6B)e^{-i(4e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\arctan\left(e^{i(e+f\cdots\right.}{c^2\sqrt{\frac{c}{1+e^{2i(e+fx)}}}f\sec^{\frac{9}{2}}(e+fx)(\cos(fx)+i\sin(\cdots}$$
$$+\frac{\cos^4(e+fx)\left((A-6iB)\cos(2fx)\left(-\frac{2i\cos(e)}{3c^3}-\frac{2\sin(e)}{3c^3}\right)+(iA+6B)\cos(4fx)\left(\frac{2\cos(e)}{15c^3}+\frac{2i\sin(e)}{15c^3}\right)+(A-\cdots\right.}{\cdots}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(5/2),x]
```

output
```
(2*(I*A + 6*B)*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x
)))]*ArcTan[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f
*x]))/(c^2*E^(I*(4*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f
*x]^(9/2)*(Cos[f*x] + I*Sin[f*x])^(7/2)*(A*Cos[e + f*x] + B*Sin[e + f*x]))
 + (Cos[e + f*x]^4*((A - (6*I)*B)*Cos[2*f*x]*((((-2*I)/3)*Cos[e])/c^3 - (2
*Sin[e])/(3*c^3)) + (I*A + 6*B)*Cos[4*f*x]*((2*Cos[e])/(15*c^3) + (((2*I)/
15)*Sin[e])/c^3) + (A - (6*I)*B)*(((-I)*Cos[3*e])/c^3 - Sin[3*e]/c^3) + (A
 - I*B)*Cos[6*f*x]*(((-1/5*I)*Cos[3*e])/c^3 + Sin[3*e]/(5*c^3)) + (A - (6*
I)*B)*((2*Cos[e])/(3*c^3) - (((2*I)/3)*Sin[e])/c^3)*Sin[2*f*x] + (A - (6*I
)*B)*((-2*Cos[e])/(15*c^3) - (((2*I)/15)*Sin[e])/c^3)*Sin[4*f*x] + (A - I*
B)*(Cos[3*e]/(5*c^3) + ((I/5)*Sin[3*e])/c^3)*Sin[6*f*x])*Sqrt[Sec[e + f*x]
*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*
Tan[e + f*x]))/(f*(Cos[f*x] + I*Sin[f*x])^3*(A*Cos[e + f*x] + B*Sin[e + f*
x]))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 87, 57, 57, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(-\frac{(A-6iB)\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{5c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{57}$$

$$\frac{ac\left(-\frac{(A-6iB)\left(-\frac{5a\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3c}-\frac{2i(a+ia\tan(e+fx))^{5/2}}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{5c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{57}$$

$$\frac{ac\left(-\frac{(A-6iB)\left(-\frac{5a\left(-\frac{3a\int \frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{2i(a+ia\tan(e+fx))^{5/2}}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{5c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{60}$$

$$\frac{ac\left(-\frac{(A-6iB)\left(-\frac{5a\left(-\frac{3a\left(a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{2i(\cdots}{3c(\cdots}\right.}{5c}\right.}{f}$$

$\downarrow$ 45

$$\frac{ac\left(-\frac{(A-6iB)\left(-\frac{5a\left(-\frac{3a\left(2a\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}+\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{2i(a\cdots}{3c(c\cdots}\right.}{5c}\right.}{f}$$

$\downarrow$ 218

$$\frac{ac\left(-\frac{(A-6iB)\left(-\frac{5a\left(-\frac{3a\left(\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{c}-\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{c}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{2i(a+ia\tan(e+f}{3c(c-ic\tan(e+f}}{5c}$$

$$f$$

input

```
Int[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(5/2),x]
```

output

```
(a*c*(-1/5*((I*A + B)*(a + I*a*Tan[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e +
f*x])^(5/2)) - ((A - (6*I)*B)*((((-2*I)/3)*(a + I*a*Tan[e + f*x])^(5/2))/(
c*(c - I*c*Tan[e + f*x])^(3/2)) - (5*a*(((-2*I)*(a + I*a*Tan[e + f*x])^(3/
2))/(c*Sqrt[c - I*c*Tan[e + f*x]]) - (3*a*(((-2*I)*Sqrt[a]*ArcTan[(Sqrt[c]
*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[c
] + (I*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/c))/c))/(3*c
)))/(5*c)))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 832 vs. 2(234) = 468.

Time = 0.70 (sec) , antiderivative size = 833, normalized size of antiderivative = 2.94

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(246iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+60iA\ln\left(\frac{ac\tan(fx\cdots}{\cdots}\right.\right.}{\cdots}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(246iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+60iA\ln\left(\frac{ac\tan(fx\cdots}{\cdots}\right.\right.}{\cdots}$
parts	Expression too large to display

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c^3*(24
6*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3+60*I*A*ln((a*c
*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan
(f*x+e)^3+15*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)
)/(a*c)^(1/2))*a*c*tan(f*x+e)^4-474*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))
^(1/2)*tan(f*x+e)-90*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)
^2))^(1/2))/(a*c)^(1/2))*a*c+360*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+
tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+15*B*(a*c*(1+tan(f*x+e
)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4-94*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)
^2))^(1/2)*tan(f*x+e)^2+26*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)-90
*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2
))*a*c*tan(f*x+e)^2-46*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+
e)^3+540*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/
(a*c)^(1/2))*a*c*tan(f*x+e)^2-90*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(
1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^4-360*B*ln((a*c*tan(f*
x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)
-564*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2-60*I*A*ln((a*
c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*ta
n(f*x+e)+15*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))
/(a*c)^(1/2))*a*c+74*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(221) = 442$.

Time = 0.12 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.82

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{15\,c^3\sqrt{\frac{(A^2-12i\,AB-36\,B^2)a^7}{c^5f^2}}\,f\log\left(-\frac{4\left(2\left((-i\,A-6\,B)a^3e^{(3i\,f}\right.\right.}{}\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
1/30*(15*c^3*sqrt((A^2 - 12*I*A*B - 36*B^2)*a^7/(c^5*f^2))*f*log(-4*(2*((-
I*A - 6*B)*a^3*e^(3*I*f*x + 3*I*e) + (-I*A - 6*B)*a^3*e^(I*f*x + I*e))*sqr
t(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (c^3*f*
e^(2*I*f*x + 2*I*e) - c^3*f)*sqrt((A^2 - 12*I*A*B - 36*B^2)*a^7/(c^5*f^2))
)/((I*A + 6*B)*a^3*e^(2*I*f*x + 2*I*e) + (I*A + 6*B)*a^3)) - 15*c^3*sqrt((
A^2 - 12*I*A*B - 36*B^2)*a^7/(c^5*f^2))*f*log(-4*(2*((-I*A - 6*B)*a^3*e^(3
*I*f*x + 3*I*e) + (-I*A - 6*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (c^3*f*e^(2*I*f*x + 2*I*e)
 - c^3*f)*sqrt((A^2 - 12*I*A*B - 36*B^2)*a^7/(c^5*f^2)))/((I*A + 6*B)*a^3*
e^(2*I*f*x + 2*I*e) + (I*A + 6*B)*a^3)) - 4*(3*(I*A + B)*a^3*e^(7*I*f*x +
7*I*e) + 2*(-I*A - 6*B)*a^3*e^(5*I*f*x + 5*I*e) + 10*(I*A + 6*B)*a^3*e^(3*
I*f*x + 3*I*e) + 15*(I*A + 6*B)*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x +
2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(221) = 442$.

Time = 0.26 (sec) , antiderivative size = 936, normalized size of antiderivative = 3.31

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
15*(30*((A - 6*I*B)*a^3*cos(2*f*x + 2*e) - (-I*A - 6*B)*a^3*sin(2*f*x + 2*
e) + (A - 6*I*B)*a^3)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 30*(
(A - 6*I*B)*a^3*cos(2*f*x + 2*e) - (-I*A - 6*B)*a^3*sin(2*f*x + 2*e) + (A
- 6*I*B)*a^3)*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
, -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 12*((A - I*
B)*a^3*cos(2*f*x + 2*e) + (I*A + B)*a^3*sin(2*f*x + 2*e) + (A - I*B)*a^3)*
cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*((A - 3*I*B)*a^3
*cos(2*f*x + 2*e) - (-I*A - 3*B)*a^3*sin(2*f*x + 2*e) + (A - 3*I*B)*a^3)*c
os(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 60*((A - 5*I*B)*a^3*
cos(2*f*x + 2*e) + (I*A + 5*B)*a^3*sin(2*f*x + 2*e) + (A - 6*I*B)*a^3)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 15*((-I*A - 6*B)*a^3*c
os(2*f*x + 2*e) + (A - 6*I*B)*a^3*sin(2*f*x + 2*e) + (-I*A - 6*B)*a^3)*log
(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 1) - 15*((I*A + 6*B)*a^3*cos(2*f*x + 2*e) - (A - 6
*I*B)*a^3*sin(2*f*x + 2*e) + (I*A + 6*B)*a^3)*log(cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 - 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1
) - 12*((I*A + B)*a^3*cos(2*f*x + 2*e) - (A - I*B)*a^3*sin(2*f*x + 2*e)...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx=\int \frac{(A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}}{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(5/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*a**3*( - int(sqrt(tan(e + f*x)*i + 1)/(sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( -
 tan(e + f*x)*i + 1)),x)*a + int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**4
)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*b*i + int((sqrt(tan(
e + f*x)*i + 1)*tan(e + f*x)**3)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)
**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*
i + 1)),x)*a*i + 3*int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(sqrt( -
 tan(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e
 + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*b + 3*int((sqrt(tan(e + f*x)*i
 + 1)*tan(e + f*x)**2)/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*sq
rt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)
*a - 3*int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i -
sqrt( - tan(e + f*x)*i + 1)),x)*b*i - 3*int((sqrt(tan(e + f*x)*i + 1)*tan(
e + f*x))/(sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2 + 2*sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e + f*x)*i + 1)),x)*a*i - int((s
qrt(tan(e + f*x)*i + 1)*tan(e + f*x))/(sqrt( - tan(e + f*x)*i + 1)*tan(e +
 f*x)**2 + 2*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt( - tan(e +
f*x)*i + 1)),x)*b))/(sqrt(c)*c**2)
```

3.824 $\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}} dx$

Optimal result

Integrand size = 45, antiderivative size = 251

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}} dx = -\frac{2a^{7/2}B\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{7/2}f} - \frac{(iA+B)(a+ia\tan(e+fx))^{7/2}}{7f(c-ic\tan(e+fx))^{7/2}} + \frac{2aB(a+ia\tan(e+fx))^{5/2}}{5cf(c-ic\tan(e+fx))^{5/2}} - \frac{2a^2B(a+ia\tan(e+fx))^{3/2}}{3c^2f(c-ic\tan(e+fx))^{3/2}} + \frac{2a^3B\sqrt{a+ia\tan(e+fx)}}{c^3f\sqrt{c-ic\tan(e+fx)}}$$

output
```
-2*a^(7/2)*B*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*
x+e))^(1/2))/c^(7/2)/f-1/7*(I*A+B)*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*c*tan(f
*x+e))^(7/2)+2/5*a*B*(a+I*a*tan(f*x+e))^(5/2)/c/f/(c-I*c*tan(f*x+e))^(5/2)
-2/3*a^2*B*(a+I*a*tan(f*x+e))^(3/2)/c^2/f/(c-I*c*tan(f*x+e))^(3/2)+2*a^3*B
*(a+I*a*tan(f*x+e))^(1/2)/c^3/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 570 vs. 2(251) = 502.

Time = 19.24 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.27

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = -\frac{2Be^{-i(4e+fx)}\sqrt{e^{ifx}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\arctan\left(e^{i(e+fx)}\right)(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{c^3\sqrt{\frac{c}{1+e^{2i(e+fx)}}}f\sec^{\frac{9}{2}}(e+fx)(\cos(fx)+i\sin(fx))^{7/2}(A\cos(e+fx)+B\sin(e+fx))} + \frac{\cos^4(e+fx)\left(\frac{B\cos(3e)}{c^4}+\cos(4fx)\left(-\frac{2B\cos(e)}{15c^4}-\frac{2iB\sin(e)}{15c^4}\right)+\cos(2fx)\left(\frac{2B\cos(e)}{3c^4}-\frac{2iB\sin(e)}{3c^4}\right)-\frac{iB\sin(3e)}{c^4}+(\cdots}{\cdots}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(7/2),x]
```

output
```
(-2*B*Sqrt[E^(I*f*x)]*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*ArcT
an[E^(I*(e + f*x))]*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c^
3*E^(I*(4*e + f*x))*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*f*Sec[e + f*x]^(9/2)
*(Cos[f*x] + I*Sin[f*x])^(7/2)*(A*Cos[e + f*x] + B*Sin[e + f*x])) + (Cos[e
 + f*x]^4*((B*Cos[3*e])/c^4 + Cos[4*f*x]*((-2*B*Cos[e])/(15*c^4) - (((2*I)
/15)*B*Sin[e])/c^4) + Cos[2*f*x]*((2*B*Cos[e])/(3*c^4) - (((2*I)/3)*B*Sin[
e])/c^4) - (I*B*Sin[3*e])/c^4 + ((-5*I)*A + 9*B)*Cos[6*f*x]*(Cos[3*e]/(70*
c^4) + ((I/70)*Sin[3*e])/c^4) + (A - I*B)*Cos[8*f*x]*(((-1/14*I)*Cos[5*e])
/c^4 + Sin[5*e]/(14*c^4)) + ((((2*I)/3)*B*Cos[e])/c^4 + (2*B*Sin[e])/(3*c^
4))*Sin[2*f*x] + ((((-2*I)/15)*B*Cos[e])/c^4 + (2*B*Sin[e])/(15*c^4))*Sin[
4*f*x] + (5*A + (9*I)*B)*(Cos[3*e]/(70*c^4) + ((I/70)*Sin[3*e])/c^4)*Sin[6
*f*x] + (A - I*B)*(Cos[5*e]/(14*c^4) + ((I/14)*Sin[5*e])/c^4)*Sin[8*f*x])*
Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]*(a + I*a*Tan[e + f*
x])^(7/2)*(A + B*Tan[e + f*x]))/(f*(Cos[f*x] + I*Sin[f*x])^3*(A*Cos[e + f*
x] + B*Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.178, Rules used = {3042, 4071, 87, 57, 57, 57, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 87$$

$$\frac{ac\left(\frac{iB\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

$$\Big\downarrow \; 57$$

$$\frac{ac\left(\frac{iB\left(-\frac{a\int \frac{(i\tan(e+fx)a+a)^{3/2}}{(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{c}-\frac{2i(a+ia\tan(e+fx))^{5/2}}{5c(c-ic\tan(e+fx))^{5/2}}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

$$\Big\downarrow \; 57$$

$$\frac{ac\left(\frac{iB\left(-\frac{a\left(-\frac{a\int \frac{\sqrt{i\tan(e+fx)a+a}}{(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{5/2}}{5c(c-ic\tan(e+fx))^{5/2}}\right)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{7ac(c-ic\tan(e+fx))^{7/2}}\right)}{f}$$

$\downarrow$ 57

$$\frac{ac\left(\frac{iB\left(-\frac{a\left(-\frac{a\left(-\frac{a\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{5/2}}{5c(c-ic\tan(e+fx))^{5/2}}\right)}{c}-\right.}{f}$$

$\downarrow$ 45

$$\frac{ac\left(\frac{iB\left(-\frac{a\left(-\frac{a\left(-\frac{2a\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}}{c}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{5/2}}{5c(c-ic\tan(e+fx))^{5/2}}\right)}{c}-\right.}{f}$$

$\Big\downarrow$ 218

$$\frac{ac\left(\frac{iB\left(-\frac{a\left(-\frac{a\left(\frac{2i\sqrt{a}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{c^{3/2}}-\frac{2i\sqrt{a+ia\tan(e+fx)}}{c\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{3/2}}{3c(c-ic\tan(e+fx))^{3/2}}\right)}{c}-\frac{2i(a+ia\tan(e+fx))^{5/2}}{5c(c-ic\tan(e+fx))^{5/2}}\right)}{c}-\frac{(B+iA)(a+i\cdots}{7ac(c-ic\,\mathrm{ta}\cdots}\right.}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(7/2),x]
```

output
```
(a*c*(-1/7*((I*A + B)*(a + I*a*Tan[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e +
f*x])^(7/2)) + (I*B*((((-2*I)/5)*(a + I*a*Tan[e + f*x])^(5/2))/(c*(c - I*c
*Tan[e + f*x])^(5/2)) - (a*((((-2*I)/3)*(a + I*a*Tan[e + f*x])^(3/2))/(c*(
c - I*c*Tan[e + f*x])^(3/2)) - (a*(((2*I)*Sqrt[a]*ArcTan[(Sqrt[c]*Sqrt[a +
 I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/c^(3/2) - ((2*I
)*Sqrt[a + I*a*Tan[e + f*x]])/(c*Sqrt[c - I*c*Tan[e + f*x]])))/c))/c))/c))
/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(206) = 412$.

Time = 0.75 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.38

method	result
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(1+\tan(fx+e)^2\right)\left(2i\tan(fx+e)-\tan(fx+e)^2+1\right)}{7f\,c^4(i+\tan(fx+e))^5} - \frac{iB\sqrt{a(1+i\tan}}{}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(-105iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^5+1\right.}{}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(-105iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^5+1\right.}{}$

input

```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x,m
ethod=_RETURNVERBOSE)
```

output

```
1/7*A/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c^4*(1+
tan(f*x+e)^2)*(2*I*tan(f*x+e)-tan(f*x+e)^2+1)/(I+tan(f*x+e))^5-1/105*I*B/f
*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)*a^3/c^4*(525*I*ln(
(a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*tan
(f*x+e)^4*a*c+105*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1
/2))/(a*c)^(1/2))*tan(f*x+e)^5*a*c-1050*I*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(
a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*tan(f*x+e)^2*a*c-950*I*(a*c)^(1/
2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3-1050*ln((a*c*tan(f*x+e)+(a*c)
^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*tan(f*x+e)^3*a*c-337*tan
(f*x+e)^4*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)+105*I*ln((a*c*tan(f*x+e
)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+730*I*(a*c)^(
1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+525*ln((a*c*tan(f*x+e)+(a*c)^
(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*tan(f*x+e)*a*c+1176*tan(f
*x+e)^2*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)-167*(a*c)^(1/2)*(a*c*(1+t
an(f*x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I+tan(f*x+e))^5/(a*c)^(
1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(197) = 394$.

Time = 0.10 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.73

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \frac{105\,c^4 f\sqrt{-\frac{B^2a^7}{c^7f^2}}\log\left(\frac{4\left(2\left(Ba^3e^{(3i\,fx+3i\,e)}+Ba^3e^{(i\,fx+i\,e)}\right)\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="fricas")
```

output
```
1/210*(105*c^4*f*sqrt(-B^2*a^7/(c^7*f^2))*log(4*(2*(B*a^3*e^(3*I*f*x + 3*I
*e) + B*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(
2*I*f*x + 2*I*e) + 1)) + (c^4*f*e^(2*I*f*x + 2*I*e) - c^4*f)*sqrt(-B^2*a^7
/(c^7*f^2)))/(B*a^3*e^(2*I*f*x + 2*I*e) + B*a^3)) - 105*c^4*f*sqrt(-B^2*a^
7/(c^7*f^2))*log(4*(2*(B*a^3*e^(3*I*f*x + 3*I*e) + B*a^3*e^(I*f*x + I*e))*
sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (c^4
*f*e^(2*I*f*x + 2*I*e) - c^4*f)*sqrt(-B^2*a^7/(c^7*f^2)))/(B*a^3*e^(2*I*f*
x + 2*I*e) + B*a^3)) - 2*(15*(I*A + B)*a^3*e^(9*I*f*x + 9*I*e) + 3*(5*I*A
- 9*B)*a^3*e^(7*I*f*x + 7*I*e) + 28*B*a^3*e^(5*I*f*x + 5*I*e) - 140*B*a^3*
e^(3*I*f*x + 3*I*e) - 210*B*a^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*
e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(c^4*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.98

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx =$$
$$-\frac{(210\,Ba^3\arctan\left(\cos\left(fx+e\right),\sin\left(fx+e\right)+1\right)+210\,Ba^3\arctan\left(\cos\left(fx+e\right),-\sin\left(fx+e\right)+1\right)-3}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="maxima")
```

output
```
-1/210*(210*B*a^3*arctan2(cos(f*x + e), sin(f*x + e) + 1) + 210*B*a^3*arct
an2(cos(f*x + e), -sin(f*x + e) + 1) - 30*(-I*A - B)*a^3*cos(7*f*x + 7*e)
- 84*B*a^3*cos(5*f*x + 5*e) + 140*B*a^3*cos(3*f*x + 3*e) - 420*B*a^3*cos(f
*x + e) + 105*I*B*a^3*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e)
 + 1) - 105*I*B*a^3*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) +
 1) - 30*(A - I*B)*a^3*sin(7*f*x + 7*e) - 84*I*B*a^3*sin(5*f*x + 5*e) + 14
0*I*B*a^3*sin(3*f*x + 3*e) - 420*I*B*a^3*sin(f*x + e))*sqrt(a)/(c^(7/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx=\int \frac{(A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}}{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(7/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{7/2}}\,dx=\text{too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)*a + 4*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
1)*tan(e + f*x)*b*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)
*a*i + 4*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b + 2*int((
- sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(t
an(e + f*x)**5 + 3*tan(e + f*x)**4*i - 2*tan(e + f*x)**3 + 2*tan(e + f*x)*
*2*i - 3*tan(e + f*x) - i),x)*tan(e + f*x)**2*a*f - 8*int(( - sqrt(tan(e +
 f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**5
 + 3*tan(e + f*x)**4*i - 2*tan(e + f*x)**3 + 2*tan(e + f*x)**2*i - 3*tan(e
 + f*x) - i),x)*tan(e + f*x)**2*b*f*i + 2*int(( - sqrt(tan(e + f*x)*i + 1)
*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**5 + 3*tan(e +
 f*x)**4*i - 2*tan(e + f*x)**3 + 2*tan(e + f*x)**2*i - 3*tan(e + f*x) - i)
,x)*a*f - 8*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*t
an(e + f*x)**3)/(tan(e + f*x)**5 + 3*tan(e + f*x)**4*i - 2*tan(e + f*x)**3
 + 2*tan(e + f*x)**2*i - 3*tan(e + f*x) - i),x)*b*f*i + 4*int(( - sqrt(tan
(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x
)**5*i - 3*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*t
an(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*f*i + 6*int(( - sqrt(tan(e + f*x)*
i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**5*i - 3
*tan(e + f*x)**4 - 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 - 3*tan(e + ...
```

3.825 $\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 102

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{7/2}}{9f(c-ic\tan(e+fx))^{9/2}}-\frac{(iA-8B)(a+ia\tan(e+fx))^{7/2}}{63cf(c-ic\tan(e+fx))^{7/2}}$$

output
```
-1/9*(I*A+B)*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*c*tan(f*x+e))^(9/2)-1/63*(I*A
-8*B)*(a+I*a*tan(f*x+e))^(7/2)/c/f/(c-I*c*tan(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 14.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.19

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \frac{a^3\cos(e+fx)((-8iA+B)\cos(e+fx)-(A+8iB)$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(9/2),x]
```

output
```
(a^3*Cos[e + f*x]*(((-8*I)*A + B)*Cos[e + f*x] - (A + (8*I)*B)*Sin[e + f*x
])*(Cos[8*e + 11*f*x] + I*Sin[8*e + 11*f*x])*Sqrt[a + I*a*Tan[e + f*x]]*Sq
rt[c - I*c*Tan[e + f*x]])/(63*c^5*f*(Cos[f*x] + I*Sin[f*x])^3)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {3042, 4071, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(A+8iB)\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{9c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(-\frac{i(A+8iB)(a+ia\tan(e+fx))^{7/2}}{63ac^2(c-ic\tan(e+fx))^{7/2}}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(9/2),x]
```

output
```
(a*c*(-1/9*((I*A + B)*(a + I*a*Tan[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e +
f*x])^(9/2)) - ((I/63)*(A + (8*I)*B)*(a + I*a*Tan[e + f*x])^(7/2))/(a*c^2*
(c - I*c*Tan[e + f*x])^(7/2))))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{a^{3}\sqrt{\frac{a\,\mathrm{e}^{2i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(7iA\,\mathrm{e}^{8i(fx+e)}+7B\,\mathrm{e}^{8i(fx+e)}+9iA\,\mathrm{e}^{6i(fx+e)}-9B\,\mathrm{e}^{6i(fx+e)}\right)}{126c^{4}\sqrt{\frac{c}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{3}\left(1+\tan(fx+e)^{2}\right)\left(8iB\tan(fx+e)^{3}+6iA\tan(fx+e)^{2}+A\tan(fx+e)\right.}{63f\,c^{5}(i+\tan(fx+e))^{6}}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{3}\left(1+\tan(fx+e)^{2}\right)\left(8iB\tan(fx+e)^{3}+6iA\tan(fx+e)^{2}+A\tan(fx+e)\right.}{63f\,c^{5}(i+\tan(fx+e))^{6}}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^{3}\left(1+\tan(fx+e)^{2}\right)\left(6i\tan(fx+e)^{2}+\tan(fx+e)^{3}-8i+15\tan(fx+e)\right.}{63f\,c^{5}(i+\tan(fx+e))^{6}}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/126*a^3/c^4*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(2*I
*(f*x+e))+1))^(1/2)/f*(7*I*A*exp(8*I*(f*x+e))+7*B*exp(8*I*(f*x+e))+9*I*A*e
xp(6*I*(f*x+e))-9*B*exp(6*I*(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int\frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx=$$
$$-\frac{\left(7\,(i\,A+B)a^{3}e^{(11i\,fx+11i\,e)}+2\,(8i\,A-B)a^{3}e^{(9i\,fx+9i\,e)}+9\,(i\,A-B)a^{3}e^{(7i\,fx+7i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2}}}}{126\,c^{5}f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="fricas")
```

output
```
-1/126*(7*(I*A + B)*a^3*e^(11*I*f*x + 11*I*e) + 2*(8*I*A - B)*a^3*e^(9*I*f
*x + 9*I*e) + 9*(I*A - B)*a^3*e^(7*I*f*x + 7*I*e))*sqrt(a/(e^(2*I*f*x + 2*
I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
9/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(78) = 156$.

Time = 0.23 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.63

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx =$$
$$-\frac{126\,(7\,(A-i\,B)a^3\cos(11\,fx+11\,e)+2\,(8\,A+i\,B)a^3\cos(9\,fx+9\,e)+9\,(A+i\,B)a^3\cos(7\,fx+7\,e)-}{-15876\,(i\,c^5\cos(2\,fx+2}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="maxima")
```

output
```
-126*(7*(A - I*B)*a^3*cos(11*f*x + 11*e) + 2*(8*A + I*B)*a^3*cos(9*f*x + 9
*e) + 9*(A + I*B)*a^3*cos(7*f*x + 7*e) - 7*(-I*A - B)*a^3*sin(11*f*x + 11*
e) - 2*(-8*I*A + B)*a^3*sin(9*f*x + 9*e) - 9*(-I*A + B)*a^3*sin(7*f*x + 7*
e))*sqrt(a)*sqrt(c)/((-15876*I*c^5*cos(2*f*x + 2*e) + 15876*c^5*sin(2*f*x
+ 2*e) - 15876*I*c^5)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.83 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.88

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx =$$

$$-\frac{a^3\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,\left(A\,\cos\left(6\,e+6\,f\,x\right)\,9\mathrm{i}+A\,\cos\left(8\,e+8\,f\,x\right)\,7\mathrm{i}-9\,B\,\cos\left(6\,e+6\,f\,x\right)\cdots\right.}{126\,c^4\,f\,\sqrt{\frac{c\,(\mathrm{co}\cdots}{}}}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(9/2),x)
```

output
```
-(a^3*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(A*cos(6*e + 6*f*x)*9i + A*cos(8*e + 8*f*x)*7i - 9*B*cos(6*e +
 6*f*x) + 7*B*cos(8*e + 8*f*x) - 9*A*sin(6*e + 6*f*x) - 7*A*sin(8*e + 8*f*
x) - B*sin(6*e + 6*f*x)*9i + B*sin(8*e + 8*f*x)*7i))/(126*c^4*f*((c*(cos(2
*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(9/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)*a - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)
*b - 2*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e
+ f*x)**4)/(tan(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*
tan(e + f*x)**2 - 4*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*f + 8*int(( -
 sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(ta
n(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)**
2 - 4*tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i - 2*int(( - sqrt(tan(e
+ f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**
6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)**2 - 4*tan(e
+ f*x)*i + 1),x)*a*f + 8*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e +
f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*ta
n(e + f*x)**4 - 5*tan(e + f*x)**2 - 4*tan(e + f*x)*i + 1),x)*b*f*i - 2*int
(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**
6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x)**2 - 4*tan(e
+ f*x)*i + 1),x)*tan(e + f*x)**2*a*f - 2*int(( - sqrt(tan(e + f*x)*i + 1)*
sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*ta
n(e + f*x)**4 - 5*tan(e + f*x)**2 - 4*tan(e + f*x)*i + 1),x)*a*f - 12*int(
(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(ta
n(e + f*x)**6 + 4*tan(e + f*x)**5*i - 5*tan(e + f*x)**4 - 5*tan(e + f*x...
```

3.826 $\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 155

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{7/2}}{11f(c-ic\tan(e+fx))^{11/2}} - \frac{(2iA-9B)(a+ia\tan(e+fx))^{7/2}}{99cf(c-ic\tan(e+fx))^{9/2}} - \frac{(2iA-9B)(a+ia\tan(e+fx))^{7/2}}{693c^2f(c-ic\tan(e+fx))^{7/2}}$$

output
```
-1/11*(I*A+B)*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*c*tan(f*x+e))^(11/2)-1/99*(2
*I*A-9*B)*(a+I*a*tan(f*x+e))^(7/2)/c/f/(c-I*c*tan(f*x+e))^(9/2)-1/693*(2*I
*A-9*B)*(a+I*a*tan(f*x+e))^(7/2)/c^2/f/(c-I*c*tan(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 15.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \frac{a^3\cos(e+fx)(-77iA+9(-9iA+2B)\cos(2(e+fx$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(11/2),x]
```

output
```
(a^3*Cos[e + f*x]*((-77*I)*A + 9*((-9*I)*A + 2*B)*Cos[2*(e + f*x)] - 9*(2*
A + (9*I)*B)*Sin[2*(e + f*x)])*(Cos[9*e + 12*f*x] + I*Sin[9*e + 12*f*x])*S
qrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/(1386*c^6*f*(Cos[f*x
] + I*Sin[f*x])^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4071, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(2A+9iB)\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{11/2}}d\tan(e+fx)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{(2A+9iB)\left(\frac{\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{9c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

$$\Big\downarrow \ 48$$

$$\frac{ac\left(\frac{(2A+9iB)\left(-\frac{i(a+ia\tan(e+fx))^{7/2}}{63ac^2(c-ic\tan(e+fx))^{7/2}}-\frac{i(a+ia\tan(e+fx))^{7/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(11/2),x]
```

output
```
(a*c*(-1/11*((I*A + B)*(a + I*a*Tan[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e +
 f*x])^(11/2)) + ((2*A + (9*I)*B)*(((-1/9*I)*(a + I*a*Tan[e + f*x])^(7/2))
/(a*c*(c - I*c*Tan[e + f*x])^(9/2)) - ((I/63)*(a + I*a*Tan[e + f*x])^(7/2)
)/(a*c^2*(c - I*c*Tan[e + f*x])^(7/2))))/(11*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{a^{3} \sqrt{\frac{a\,\mathrm{e}^{2 i (f x+e)}}{\mathrm{e}^{2 i (f x+e)}+1}}\, \left(63 i A\,\mathrm{e}^{10 i (f x+e)}+63 B\,\mathrm{e}^{10 i (f x+e)}+154 i A\,\mathrm{e}^{8 i (f x+e)}+99 i A\,\mathrm{e}^{6 i (f x+e)}-99 B\,\mathrm{e}^{6 i (f x+e)}\right)}{2772 c^{5} \sqrt{\frac{c}{\mathrm{e}^{2 i (f x+e)}+1}}\, f}$
derivativedivides	$\frac{i \sqrt{a (1+i \tan (f x+e))}\, \sqrt{-c (i \tan (f x+e)-1)}\, a^{3} \left(1+\tan (f x+e)^{2}\right) \left(2 i A \tan (f x+e)^{4}-63 i B \tan (f x+e)^{3}-9 B \tan (f x+e)\right.}{693 f\,c^{6} (i+\tan (f}$
default	$\frac{i \sqrt{a (1+i \tan (f x+e))}\, \sqrt{-c (i \tan (f x+e)-1)}\, a^{3} \left(1+\tan (f x+e)^{2}\right) \left(2 i A \tan (f x+e)^{4}-63 i B \tan (f x+e)^{3}-9 B \tan (f x+e)\right.}{693 f\,c^{6} (i+\tan (f}$
parts	$\frac{i A \sqrt{a (1+i \tan (f x+e))}\, \sqrt{-c (i \tan (f x+e)-1)}\, a^{3} \left(1+\tan (f x+e)^{2}\right) \left(2 i \tan (f x+e)^{4}-45 i \tan (f x+e)^{2}-14 \tan (f x+e)^{3}+\right.}{693 f\,c^{6} (i+\tan (f x+e))^{7}}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/2772*a^3/c^5*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(2*
I*(f*x+e))+1))^(1/2)/f*(63*I*A*exp(10*I*(f*x+e))+63*B*exp(10*I*(f*x+e))+15
4*I*A*exp(8*I*(f*x+e))+99*I*A*exp(6*I*(f*x+e))-99*B*exp(6*I*(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.81

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = -\frac{(63\,(i\,A+B)a^3e^{(13i\,fx+13i\,e)}+7\,(31i\,A+9\,B)a^3e^{(11i\,fx+11i\,e)}+11\,(23i\,A-9\,B)a^3e^{(9i\,fx+9i\,e)}+99\,(i\,A-}{2772\,c^6f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11
/2),x, algorithm="fricas")
```

output
```
-1/2772*(63*(I*A + B)*a^3*e^(13*I*f*x + 13*I*e) + 7*(31*I*A + 9*B)*a^3*e^(
11*I*f*x + 11*I*e) + 11*(23*I*A - 9*B)*a^3*e^(9*I*f*x + 9*I*e) + 99*(I*A -
 B)*a^3*e^(7*I*f*x + 7*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(
2*I*f*x + 2*I*e) + 1))/(c^6*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.28

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \frac{\left(63\left(-iA-B\right)a^3\cos\left(\frac{11}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11
/2),x, algorithm="maxima")
```

output
```
1/2772*(63*(-I*A - B)*a^3*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 154*I*A*a^3*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 99*(-I*A + B)*a^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
63*(A - I*B)*a^3*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1
54*A*a^3*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 99*(A + I*
B)*a^3*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)/(c^(1
1/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 8.37 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.40

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx =$$

$$-\frac{a^3\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(A\,\cos\left(6\,e+6\,f\,x\right)\,99\mathrm{i}+A\,\cos\left(8\,e+8\,f\,x\right)\,154\mathrm{i}+A\,\cos\left(10\,e+10\,f\right.}{}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(11/2),x)
```

output
```
-(a^3*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
+ 1))^(1/2)*(A*cos(6*e + 6*f*x)*99i + A*cos(8*e + 8*f*x)*154i + A*cos(10*e
 + 10*f*x)*63i - 99*B*cos(6*e + 6*f*x) + 63*B*cos(10*e + 10*f*x) - 99*A*si
n(6*e + 6*f*x) - 154*A*sin(8*e + 8*f*x) - 63*A*sin(10*e + 10*f*x) - B*sin(
6*e + 6*f*x)*99i + B*sin(10*e + 10*f*x)*63i))/(2772*c^5*f*((c*(cos(2*e + 2
*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{11/2}}\,dx = \text{too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(11/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**3*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)*a - 4*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)*b*i + int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
 + 1)*tan(e + f*x)**5)/(tan(e + f*x)**7 + 5*tan(e + f*x)**6*i - 9*tan(e +
f*x)**5 - 5*tan(e + f*x)**4*i - 5*tan(e + f*x)**3 - 9*tan(e + f*x)**2*i +
5*tan(e + f*x) + i),x)*tan(e + f*x)**2*a*f - 9*int(( - sqrt(tan(e + f*x)*i
 + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**7 + 5*ta
n(e + f*x)**6*i - 9*tan(e + f*x)**5 - 5*tan(e + f*x)**4*i - 5*tan(e + f*x)
**3 - 9*tan(e + f*x)**2*i + 5*tan(e + f*x) + i),x)*tan(e + f*x)**2*b*f*i +
 int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)
**5)/(tan(e + f*x)**7 + 5*tan(e + f*x)**6*i - 9*tan(e + f*x)**5 - 5*tan(e
+ f*x)**4*i - 5*tan(e + f*x)**3 - 9*tan(e + f*x)**2*i + 5*tan(e + f*x) + i
),x)*a*f - 9*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)**5)/(tan(e + f*x)**7 + 5*tan(e + f*x)**6*i - 9*tan(e + f*x)**
5 - 5*tan(e + f*x)**4*i - 5*tan(e + f*x)**3 - 9*tan(e + f*x)**2*i + 5*tan(
e + f*x) + i),x)*b*f*i + 20*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**7 + 5*tan(e + f*x)**6*i - 9
*tan(e + f*x)**5 - 5*tan(e + f*x)**4*i - 5*tan(e + f*x)**3 - 9*tan(e + f*x
)**2*i + 5*tan(e + f*x) + i),x)*tan(e + f*x)**2*a*f*i - 20*int(( - sqrt(ta
n(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e +...
```

3.827 $\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 208

$$\begin{aligned}\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx &= \\ &-\frac{(iA+B)(a+ia\tan(e+fx))^{7/2}}{13f(c-ic\tan(e+fx))^{13/2}}-\frac{(3iA-10B)(a+ia\tan(e+fx))^{7/2}}{143cf(c-ic\tan(e+fx))^{11/2}} \\ &-\frac{2(3iA-10B)(a+ia\tan(e+fx))^{7/2}}{1287c^2f(c-ic\tan(e+fx))^{9/2}}-\frac{2(3iA-10B)(a+ia\tan(e+fx))^{7/2}}{9009c^3f(c-ic\tan(e+fx))^{7/2}}\end{aligned}$$

output
```
-1/13*(I*A+B)*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*c*tan(f*x+e))^(13/2)-1/143*(
3*I*A-10*B)*(a+I*a*tan(f*x+e))^(7/2)/c/f/(c-I*c*tan(f*x+e))^(11/2)-2/1287*
(3*I*A-10*B)*(a+I*a*tan(f*x+e))^(7/2)/c^2/f/(c-I*c*tan(f*x+e))^(9/2)-2/900
9*(3*I*A-10*B)*(a+I*a*tan(f*x+e))^(7/2)/c^3/f/(c-I*c*tan(f*x+e))^(7/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 495 vs. 2(208) = 416.

Time = 18.70 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.38

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = \frac{\cos^4(e+fx)\left((-iA+B)\cos(6fx)\left(\frac{\cos(3e)}{112c^7}+\frac{i\sin(3e)}{112c^7}\right)\right.}{}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(13/2),x]
```

output
```
(Cos[e + f*x]^4*(((-I)*A + B)*Cos[6*f*x]*(Cos[3*e]/(112*c^7) + ((I/112)*Si
n[3*e])/c^7) + ((-15*I)*A + 8*B)*Cos[8*f*x]*(Cos[5*e]/(504*c^7) + ((I/504)
*Sin[5*e])/c^7) + ((-30*I)*A + B)*Cos[10*f*x]*(Cos[7*e]/(792*c^7) + ((I/79
2)*Sin[7*e])/c^7) + (25*A - (12*I)*B)*Cos[12*f*x]*(((-1/1144*I)*Cos[9*e])/
c^7 + Sin[9*e]/(1144*c^7)) + (A - I*B)*Cos[14*f*x]*(((-1/208*I)*Cos[11*e])
/c^7 + Sin[11*e]/(208*c^7)) + (A + I*B)*(Cos[3*e]/(112*c^7) + ((I/112)*Sin
[3*e])/c^7)*Sin[6*f*x] + (15*A + (8*I)*B)*(Cos[5*e]/(504*c^7) + ((I/504)*S
in[5*e])/c^7)*Sin[8*f*x] + (30*A + I*B)*(Cos[7*e]/(792*c^7) + ((I/792)*Sin
[7*e])/c^7)*Sin[10*f*x] + (25*A - (12*I)*B)*(Cos[9*e]/(1144*c^7) + ((I/114
4)*Sin[9*e])/c^7)*Sin[12*f*x] + (A - I*B)*(Cos[11*e]/(208*c^7) + ((I/208)*
Sin[11*e])/c^7)*Sin[14*f*x])*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e
 + f*x])]*(a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(f*(Cos[f*x]
+ I*Sin[f*x])^3*(A*Cos[e + f*x] + B*Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(3A+10iB)\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{13/2}}d\tan(e+fx)}{13c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{(3A+10iB)\left(\frac{2\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{11/2}}d\tan(e+fx)}{11c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{(3A+10iB)\left(\frac{2\left(\frac{\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{9c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(\frac{(3A+10iB)\left(\frac{2\left(-\frac{i(a+ia\tan(e+fx))^{7/2}}{63ac^2(c-ic\tan(e+fx))^{7/2}}-\frac{i(a+ia\tan(e+fx))^{7/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(13/2),x]
```

output
```
(a*c*(-1/13*((I*A + B)*(a + I*a*Tan[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e +
 f*x])^(13/2)) + ((3*A + (10*I)*B)*(((-1/11*I)*(a + I*a*Tan[e + f*x])^(7/2
))/(a*c*(c - I*c*Tan[e + f*x])^(11/2)) + (2*(((-1/9*I)*(a + I*a*Tan[e + f*
x])^(7/2))/(a*c*(c - I*c*Tan[e + f*x])^(9/2)) - ((I/63)*(a + I*a*Tan[e + f
*x])^(7/2))/(a*c^2*(c - I*c*Tan[e + f*x])^(7/2))))/(11*c)))/(13*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{a^{3} \sqrt{\frac{a\,\mathrm{e}^{2 i (f x+e)}}{\mathrm{e}^{2 i (f x+e)}+1}}\, \left(693 i A\,\mathrm{e}^{12 i (f x+e)}+693 B\,\mathrm{e}^{12 i (f x+e)}+2457 i A\,\mathrm{e}^{10 i (f x+e)}+819 B\,\mathrm{e}^{10 i (f x+e)}+3003 i A\,\mathrm{e}^{8 i (f x+e)}-10\right.}{72072 c^{6} \sqrt{\frac{c}{\mathrm{e}^{2 i (f x+e)}+1}}\, f}$
derivativedivides	$\frac{i \sqrt{a (1+i \tan (f x+e))}\, \sqrt{-c (i \tan (f x+e)-1)}\, a^{3} \left(1+\tan (f x+e)^{2}\right) \left(6 i A \tan (f x+e)^{5}-160 i B \tan (f x+e)^{4}-20 B \tan (f x+\right.}{}$
default	$\frac{i \sqrt{a (1+i \tan (f x+e))}\, \sqrt{-c (i \tan (f x+e)-1)}\, a^{3} \left(1+\tan (f x+e)^{2}\right) \left(6 i A \tan (f x+e)^{5}-160 i B \tan (f x+e)^{4}-20 B \tan (f x+\right.}{}$
parts	$\frac{i A \sqrt{a (1+i \tan (f x+e))}\, \sqrt{-c (i \tan (f x+e)-1)}\, a^{3} \left(1+\tan (f x+e)^{2}\right) \left(2 i \tan (f x+e)^{5}-59 i \tan (f x+e)^{3}-16 \tan (f x+e)^{4}-\right.}{3003 f\, c^{7} (i+\tan (f x+e))^{8}}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/72072*a^3/c^6*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(2
*I*(f*x+e))+1))^(1/2)/f*(693*I*A*exp(12*I*(f*x+e))+693*B*exp(12*I*(f*x+e))
+2457*I*A*exp(10*I*(f*x+e))+819*B*exp(10*I*(f*x+e))+3003*I*A*exp(8*I*(f*x+
e))-1001*B*exp(8*I*(f*x+e))+1287*I*A*exp(6*I*(f*x+e))-1287*B*exp(6*I*(f*x+
e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.70

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = -\frac{\left(693\,(i\,A+B)a^3e^{(15i\,fx+15i\,e)}+126\,(25i\,A+12\,B)a^3e^{(13i\,fx+13i\,e)}+182\,(30i\,A-B)a^3e^{(11i\,fx+11i\,e)}+286\right.}{72072\,c^7f}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13
/2),x, algorithm="fricas")
```

output
```
-1/72072*(693*(I*A + B)*a^3*e^(15*I*f*x + 15*I*e) + 126*(25*I*A + 12*B)*a^
3*e^(13*I*f*x + 13*I*e) + 182*(30*I*A - B)*a^3*e^(11*I*f*x + 11*I*e) + 286
*(15*I*A - 8*B)*a^3*e^(9*I*f*x + 9*I*e) + 1287*(I*A - B)*a^3*e^(7*I*f*x +
7*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)
)/(c^7*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
13/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.33

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = \frac{\left(693\,(-i\,A-B)a^3\cos\left(\frac{13}{2}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13
/2),x, algorithm="maxima")
```

output
```
1/72072*(693*(-I*A - B)*a^3*cos(13/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 819*(-3*I*A - B)*a^3*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 1001*(-3*I*A + B)*a^3*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 1287*(-I*A + B)*a^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 693*(A - I*B)*a^3*sin(13/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 819*(3*A - I*B)*a^3*sin(11/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) + 1001*(3*A + I*B)*a^3*sin(9/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 1287*(A + I*B)*a^3*sin(7/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sqrt(a)/(c^(13/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.80

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = -\frac{\sqrt{a+\frac{a\sin(e+fx)\,1i}{\cos(e+fx)}}\left(\frac{a^3\,e^{e\,8i+f\,x\,8i}\,(3\,A+B\,1i)\,1i}{72\,c^6\,f}+\frac{a^3\,e^{e\,10i+f\,x\,10i}\,(3\,A-B\,1i)\,1i}{88\,c^6\,f}+\frac{a^3\,e^{e\,6i+f\,x\,6i}\,(A+B\,1i)\,1i}{56\,c^6\,f}+\frac{a^3\,e^{e\,12i+f\,x\,12i}\,(A-}{104\,c^6\,f}\right.}{\sqrt{c-\frac{c\sin(e+fx)\,1i}{\cos(e+fx)}}}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(13/2),x)
```

output
```
-((a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)*((a^3*exp(e*8i + f*x*8i)*(3
*A + B*1i)*1i)/(72*c^6*f) + (a^3*exp(e*10i + f*x*10i)*(3*A - B*1i)*1i)/(88
*c^6*f) + (a^3*exp(e*6i + f*x*6i)*(A + B*1i)*1i)/(56*c^6*f) + (a^3*exp(e*1
2i + f*x*12i)*(A - B*1i)*1i)/(104*c^6*f)))/(c - (c*sin(e + f*x)*1i)/cos(e
+ f*x))^(1/2)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{13/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(13/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**3*(int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**8 + 6*tan(e + f*x)**7*i - 14*tan
(e + f*x)**6 - 14*tan(e + f*x)**5*i - 14*tan(e + f*x)**3*i + 14*tan(e + f*
x)**2 + 6*tan(e + f*x)*i - 1),x)*b - 4*int(( - sqrt(tan(e + f*x)*i + 1)*sq
rt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**8 + 6*tan(e + f*
x)**7*i - 14*tan(e + f*x)**6 - 14*tan(e + f*x)**5*i - 14*tan(e + f*x)**3*i
 + 14*tan(e + f*x)**2 + 6*tan(e + f*x)*i - 1),x)*a*i - 6*int(( - sqrt(tan(
e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)
**8 + 6*tan(e + f*x)**7*i - 14*tan(e + f*x)**6 - 14*tan(e + f*x)**5*i - 14
*tan(e + f*x)**3*i + 14*tan(e + f*x)**2 + 6*tan(e + f*x)*i - 1),x)*b + 4*i
nt(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/
(tan(e + f*x)**8 + 6*tan(e + f*x)**7*i - 14*tan(e + f*x)**6 - 14*tan(e + f
*x)**5*i - 14*tan(e + f*x)**3*i + 14*tan(e + f*x)**2 + 6*tan(e + f*x)*i -
1),x)*a*i + int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*t
an(e + f*x))/(tan(e + f*x)**8 + 6*tan(e + f*x)**7*i - 14*tan(e + f*x)**6 -
 14*tan(e + f*x)**5*i - 14*tan(e + f*x)**3*i + 14*tan(e + f*x)**2 + 6*tan(
e + f*x)*i - 1),x)*b + int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*
x)*i + 1))/(tan(e + f*x)**8 + 6*tan(e + f*x)**7*i - 14*tan(e + f*x)**6 - 1
4*tan(e + f*x)**5*i - 14*tan(e + f*x)**3*i + 14*tan(e + f*x)**2 + 6*tan(e
+ f*x)*i - 1),x)*a - int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)...
```

3.828 $\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 261

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{7/2}}{15f(c-ic\tan(e+fx))^{15/2}} - \frac{(4iA-11B)(a+ia\tan(e+fx))^{7/2}}{195cf(c-ic\tan(e+fx))^{13/2}} - \frac{(4iA-11B)(a+ia\tan(e+fx))^{7/2}}{715c^2f(c-ic\tan(e+fx))^{11/2}} - \frac{2(4iA-11B)(a+ia\tan(e+fx))^{7/2}}{6435c^3f(c-ic\tan(e+fx))^{9/2}} - \frac{2(4iA-11B)(a+ia\tan(e+fx))^{7/2}}{45045c^4f(c-ic\tan(e+fx))^{7/2}}$$

output
```
-1/15*(I*A+B)*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*c*tan(f*x+e))^(15/2)-1/195*(
4*I*A-11*B)*(a+I*a*tan(f*x+e))^(7/2)/c/f/(c-I*c*tan(f*x+e))^(13/2)-1/715*(
4*I*A-11*B)*(a+I*a*tan(f*x+e))^(7/2)/c^2/f/(c-I*c*tan(f*x+e))^(11/2)-2/643
5*(4*I*A-11*B)*(a+I*a*tan(f*x+e))^(7/2)/c^3/f/(c-I*c*tan(f*x+e))^(9/2)-2/4
5045*(4*I*A-11*B)*(a+I*a*tan(f*x+e))^(7/2)/c^4/f/(c-I*c*tan(f*x+e))^(7/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 577 vs. 2(261) = 522.

Time = 18.86 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.21

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}\,dx = \frac{\cos^4(e+fx)\left((-iA+B)\cos(6fx)\left(\frac{\cos(3e)}{224c^8}+\frac{i\sin(3e)}{224c^8}\right)\right.}{}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(15/2),x]
```

output
```
(Cos[e + f*x]^4*(((-I)*A + B)*Cos[6*f*x]*(Cos[3*e]/(224*c^8) + ((I/224)*Si
n[3*e])/c^8) + ((-37*I)*A + 23*B)*Cos[8*f*x]*(Cos[5*e]/(2016*c^8) + ((I/20
16)*Sin[5*e])/c^8) + ((-49*I)*A + 11*B)*Cos[10*f*x]*(Cos[7*e]/(1584*c^8) +
 ((I/1584)*Sin[7*e])/c^8) + (61*A - (11*I)*B)*Cos[12*f*x]*(((-1/2288*I)*Co
s[9*e])/c^8 + Sin[9*e]/(2288*c^8)) + (73*A - (43*I)*B)*Cos[14*f*x]*(((-1/6
240*I)*Cos[11*e])/c^8 + Sin[11*e]/(6240*c^8)) + (A - I*B)*Cos[16*f*x]*(((-
1/480*I)*Cos[13*e])/c^8 + Sin[13*e]/(480*c^8)) + (A + I*B)*(Cos[3*e]/(224*
c^8) + ((I/224)*Sin[3*e])/c^8)*Sin[6*f*x] + (37*A + (23*I)*B)*(Cos[5*e]/(2
016*c^8) + ((I/2016)*Sin[5*e])/c^8)*Sin[8*f*x] + (49*A + (11*I)*B)*(Cos[7*
e]/(1584*c^8) + ((I/1584)*Sin[7*e])/c^8)*Sin[10*f*x] + (61*A - (11*I)*B)*(
Cos[9*e]/(2288*c^8) + ((I/2288)*Sin[9*e])/c^8)*Sin[12*f*x] + (73*A - (43*I
)*B)*(Cos[11*e]/(6240*c^8) + ((I/6240)*Sin[11*e])/c^8)*Sin[14*f*x] + (A -
I*B)*(Cos[13*e]/(480*c^8) + ((I/480)*Sin[13*e])/c^8)*Sin[16*f*x])*Sqrt[Sec
[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]*(a + I*a*Tan[e + f*x])^(7/2
)*(A + B*Tan[e + f*x]))/(f*(Cos[f*x] + I*Sin[f*x])^3*(A*Cos[e + f*x] + B*S
in[e + f*x]))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{ac\left(\frac{(4A+11iB)\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{15/2}}d\tan(e+fx)}{15c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{15ac(c-ic\tan(e+fx))^{15/2}}\right)}{f}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{ac\left(\frac{(4A+11iB)\left(\frac{3\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{13/2}}d\tan(e+fx)}{13c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{15c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{15ac(c-ic\tan(e+fx))^{15/2}}\right)}{f}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{ac\left(\frac{(4A+11iB)\left(\frac{3\left(\frac{2\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{11/2}}d\tan(e+fx)}{11c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{15c}-\frac{(B+iA)(a+ia\tan(e+fx)}{15ac(c-ic\tan(e+fx))^{15}}\right.}{f}$$

$$\Big\downarrow \text{ 55}$$

$$ac\left(\frac{(4A+11iB)\left(\frac{3\left(\frac{2\left(\frac{\int\frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{9c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{15c}-\right.$$
$$\overline{\qquad f \qquad}$$

$\downarrow$ 48

$$ac\left(\frac{(4A+11iB)\left(\frac{3\left(\frac{2\left(-\frac{i(a+ia\tan(e+fx))^{7/2}}{63ac^2(c-ic\tan(e+fx))^{7/2}}-\frac{i(a+ia\tan(e+fx))^{7/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{15c}-\frac{(B+i}{15ac}\right.$$
$$\overline{\qquad f \qquad}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(15/2),x]
```

output
```
(a*c*(-1/15*((I*A + B)*(a + I*a*Tan[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e +
 f*x])^(15/2)) + ((4*A + (11*I)*B)*(((-1/13*I)*(a + I*a*Tan[e + f*x])^(7/2
))/(a*c*(c - I*c*Tan[e + f*x])^(13/2)) + (3*(((-1/11*I)*(a + I*a*Tan[e + f
*x])^(7/2))/(a*c*(c - I*c*Tan[e + f*x])^(11/2)) + (2*(((-1/9*I)*(a + I*a*T
an[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e + f*x])^(9/2)) - ((I/63)*(a + I*a*
Tan[e + f*x])^(7/2))/(a*c^2*(c - I*c*Tan[e + f*x])^(7/2))))/(11*c)))/(13*c
)))/(15*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{a^3\sqrt{\frac{a\,\mathrm{e}^{2i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\left(3003iA\,\mathrm{e}^{14i(fx+e)}+3003B\,\mathrm{e}^{14i(fx+e)}+13860iA\,\mathrm{e}^{12i(fx+e)}+6930B\,\mathrm{e}^{12i(fx+e)}+24570iA\,\mathrm{e}^{10i(fx}\right.}{720720c^7\sqrt{\frac{c}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(1+\tan(fx+e)^2\right)\left(22iB\tan(fx+e)^6+72iA\tan(fx+e)^5+8A\tan(fx\right.}{}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(1+\tan(fx+e)^2\right)\left(22iB\tan(fx+e)^6+72iA\tan(fx+e)^5+8A\tan(fx\right.}{}$
parts	$\frac{iA\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(1+\tan(fx+e)^2\right)\left(-4243i+6858\tan(fx+e)+8i\tan(fx+e)^6+780\tan\right.}{45045f\,c^8(i+\tan(fx+e))^9}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(15/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/720720*a^3/c^7*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(
2*I*(f*x+e))+1))^(1/2)/f*(3003*I*A*exp(14*I*(f*x+e))+3003*B*exp(14*I*(f*x+
e))+13860*I*A*exp(12*I*(f*x+e))+6930*B*exp(12*I*(f*x+e))+24570*I*A*exp(10*
I*(f*x+e))+20020*I*A*exp(8*I*(f*x+e))-10010*B*exp(8*I*(f*x+e))+6435*I*A*ex
p(6*I*(f*x+e))-6435*B*exp(6*I*(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.64

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}\,dx =$$
$$-\frac{\left(3003\,(i\,A+B)a^3e^{(17i\,fx+17i\,e)}+231\,(73i\,A+43\,B)a^3e^{(15i\,fx+15i\,e)}+630\,(61i\,A+11\,B)a^3e^{(13i\,fx+13i\,e)}+\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(15
/2),x, algorithm="fricas")
```

output
```
-1/720720*(3003*(I*A + B)*a^3*e^(17*I*f*x + 17*I*e) + 231*(73*I*A + 43*B)*
a^3*e^(15*I*f*x + 15*I*e) + 630*(61*I*A + 11*B)*a^3*e^(13*I*f*x + 13*I*e)
+ 910*(49*I*A - 11*B)*a^3*e^(11*I*f*x + 11*I*e) + 715*(37*I*A - 23*B)*a^3*
e^(9*I*f*x + 9*I*e) + 6435*(I*A - B)*a^3*e^(7*I*f*x + 7*I*e))*sqrt(a/(e^(2
*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^8*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
15/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.27

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}\,dx = \frac{\left(3003\left(-iA-B\right)a^3\cos\left(\frac{15}{2}\arctan\left(\sin\left(2fx+2e\right),\right.\right.\right.}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(15
/2),x, algorithm="maxima")
```

output
```
1/720720*(3003*(-I*A - B)*a^3*cos(15/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) + 6930*(-2*I*A - B)*a^3*cos(13/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - 24570*I*A*a^3*cos(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) + 10010*(-2*I*A + B)*a^3*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 6435*(-I*A + B)*a^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 3003*(A - I*B)*a^3*sin(15/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 6930*(2*A - I*B)*a^3*sin(13/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 24570*A*a^3*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 10010*(2*A + I*B)*a^3*sin(9/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 6435*(A + I*B)*a^3*sin(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*sqrt(a)/(c^(15/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(15
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}\,dx = -\frac{\sqrt{a+\frac{a\sin(e+fx)\,1i}{\cos(e+fx)}}\left(\frac{a^3\,e^{e\,8i+f\,x\,8i}\,(2\,A+B\,1i)\,1i}{72\,c^7\,f}+\frac{a^3\,e^{e\,12i+f\,x\,12i}\,(2\,A-B\,1i)\,1i}{104\,c^7\,f}+\frac{A\,a^3\,e^{e\,10i+f\,x\,10i}\,3i}{88\,c^7\,f}+\frac{a^3\,e^{e\,6i+f\,x\,6i}\,(A+B\,1i)\,1i}{112\,c^7\,f}\right.}{\sqrt{c-\frac{c\sin(e+fx)\,1i}{\cos(e+fx)}}}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(15/2),x)
```

output
```
-((a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)*((a^3*exp(e*8i + f*x*8i)*(2
*A + B*1i)*1i)/(72*c^7*f) + (a^3*exp(e*12i + f*x*12i)*(2*A - B*1i)*1i)/(10
4*c^7*f) + (A*a^3*exp(e*10i + f*x*10i)*3i)/(88*c^7*f) + (a^3*exp(e*6i + f*
x*6i)*(A + B*1i)*1i)/(112*c^7*f) + (a^3*exp(e*14i + f*x*14i)*(A - B*1i)*1i
)/(240*c^7*f)))/(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{15/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(15/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*(int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**9*i - 7*tan(e + f*x)**8 - 20*tan(e
+ f*x)**7*i + 28*tan(e + f*x)**6 + 14*tan(e + f*x)**5*i + 14*tan(e + f*x)*
*4 + 28*tan(e + f*x)**3*i - 20*tan(e + f*x)**2 - 7*tan(e + f*x)*i + 1),x)*
b + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)
**4)/(tan(e + f*x)**9*i - 7*tan(e + f*x)**8 - 20*tan(e + f*x)**7*i + 28*ta
n(e + f*x)**6 + 14*tan(e + f*x)**5*i + 14*tan(e + f*x)**4 + 28*tan(e + f*x
)**3*i - 20*tan(e + f*x)**2 - 7*tan(e + f*x)*i + 1),x)*a - 4*int((sqrt(tan
(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x
)**9*i - 7*tan(e + f*x)**8 - 20*tan(e + f*x)**7*i + 28*tan(e + f*x)**6 + 1
4*tan(e + f*x)**5*i + 14*tan(e + f*x)**4 + 28*tan(e + f*x)**3*i - 20*tan(e
 + f*x)**2 - 7*tan(e + f*x)*i + 1),x)*b*i - 4*int((sqrt(tan(e + f*x)*i + 1
)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**9*i - 7*tan(
e + f*x)**8 - 20*tan(e + f*x)**7*i + 28*tan(e + f*x)**6 + 14*tan(e + f*x)*
*5*i + 14*tan(e + f*x)**4 + 28*tan(e + f*x)**3*i - 20*tan(e + f*x)**2 - 7*
tan(e + f*x)*i + 1),x)*a*i - 6*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**9*i - 7*tan(e + f*x)**8 - 2
0*tan(e + f*x)**7*i + 28*tan(e + f*x)**6 + 14*tan(e + f*x)**5*i + 14*tan(e
 + f*x)**4 + 28*tan(e + f*x)**3*i - 20*tan(e + f*x)**2 - 7*tan(e + f*x)*i
+ 1),x)*b - 6*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)...
```

3.829 $\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 314

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}\,dx = -\frac{(iA+B)(a+ia\tan(e+fx))^{7/2}}{17f(c-ic\tan(e+fx))^{17/2}} - \frac{(5iA-12B)(a+ia\tan(e+fx))^{7/2}}{255cf(c-ic\tan(e+fx))^{15/2}} - \frac{4(5iA-12B)(a+ia\tan(e+fx))^{7/2}}{3315c^2f(c-ic\tan(e+fx))^{13/2}} - \frac{4(5iA-12B)(a+ia\tan(e+fx))^{7/2}}{12155c^3f(c-ic\tan(e+fx))^{11/2}} - \frac{8(5iA-12B)(a+ia\tan(e+fx))^{7/2}}{109395c^4f(c-ic\tan(e+fx))^{9/2}} - \frac{8(5iA-12B)(a+ia\tan(e+fx))^{7/2}}{765765c^5f(c-ic\tan(e+fx))^{7/2}}$$

output
```
-1/17*(I*A+B)*(a+I*a*tan(f*x+e))^(7/2)/f/(c-I*c*tan(f*x+e))^(17/2)-1/255*(
5*I*A-12*B)*(a+I*a*tan(f*x+e))^(7/2)/c/f/(c-I*c*tan(f*x+e))^(15/2)-4/3315*
(5*I*A-12*B)*(a+I*a*tan(f*x+e))^(7/2)/c^2/f/(c-I*c*tan(f*x+e))^(13/2)-4/12
155*(5*I*A-12*B)*(a+I*a*tan(f*x+e))^(7/2)/c^3/f/(c-I*c*tan(f*x+e))^(11/2)-
8/109395*(5*I*A-12*B)*(a+I*a*tan(f*x+e))^(7/2)/c^4/f/(c-I*c*tan(f*x+e))^(9
/2)-8/765765*(5*I*A-12*B)*(a+I*a*tan(f*x+e))^(7/2)/c^5/f/(c-I*c*tan(f*x+e)
)^(7/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 655 vs. $2(314) = 628$.

Time = 19.10 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.09

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}\,dx = \frac{\cos^4(e+fx)\left((-iA+B)\cos(6fx)\left(\frac{\cos(3e)}{448c^9}+\frac{i\sin(3e)}{448c^9}\right)\right.}{}$$

input
```
Integrate[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan
[e + f*x])^(17/2),x]
```

output
```
(Cos[e + f*x]^4*(((-I)*A + B)*Cos[6*f*x]*(Cos[3*e]/(448*c^9) + ((I/448)*Si
n[3*e])/c^9) + ((-22*I)*A + 15*B)*Cos[8*f*x]*(Cos[5*e]/(2016*c^9) + ((I/20
16)*Sin[5*e])/c^9) + ((-145*I)*A + 51*B)*Cos[10*f*x]*(Cos[7*e]/(6336*c^9)
+ ((I/6336)*Sin[7*e])/c^9) + ((-60*I)*A + B)*Cos[12*f*x]*(Cos[9*e]/(2288*c
^9) + ((I/2288)*Sin[9*e])/c^9) + (215*A - (69*I)*B)*Cos[14*f*x]*(((-1/1248
0*I)*Cos[11*e])/c^9 + Sin[11*e]/(12480*c^9)) + (50*A - (33*I)*B)*Cos[16*f*
x]*(((-1/8160*I)*Cos[13*e])/c^9 + Sin[13*e]/(8160*c^9)) + (A - I*B)*Cos[18
*f*x]*(((-1/1088*I)*Cos[15*e])/c^9 + Sin[15*e]/(1088*c^9)) + (A + I*B)*(Co
s[3*e]/(448*c^9) + ((I/448)*Sin[3*e])/c^9)*Sin[6*f*x] + (22*A + (15*I)*B)*
(Cos[5*e]/(2016*c^9) + ((I/2016)*Sin[5*e])/c^9)*Sin[8*f*x] + (145*A + (51*
I)*B)*(Cos[7*e]/(6336*c^9) + ((I/6336)*Sin[7*e])/c^9)*Sin[10*f*x] + (60*A
+ I*B)*(Cos[9*e]/(2288*c^9) + ((I/2288)*Sin[9*e])/c^9)*Sin[12*f*x] + (215*
A - (69*I)*B)*(Cos[11*e]/(12480*c^9) + ((I/12480)*Sin[11*e])/c^9)*Sin[14*f
*x] + (50*A - (33*I)*B)*(Cos[13*e]/(8160*c^9) + ((I/8160)*Sin[13*e])/c^9)*
Sin[16*f*x] + (A - I*B)*(Cos[15*e]/(1088*c^9) + ((I/1088)*Sin[15*e])/c^9)*
Sin[18*f*x])*Sqrt[Sec[e + f*x]*(c*Cos[e + f*x] - I*c*Sin[e + f*x])]*(a + I
*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(f*(Cos[f*x] + I*Sin[f*x])^3*
(A*Cos[e + f*x] + B*Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 87, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(i\tan(e+fx)a+a)^{5/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{19/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(5A+12iB)\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{17/2}}d\tan(e+fx)}{17c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{17ac(c-ic\tan(e+fx))^{17/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{(5A+12iB)\left(\frac{4\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{15/2}}d\tan(e+fx)}{15c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{15ac(c-ic\tan(e+fx))^{15/2}}\right)}{17c}-\frac{(B+iA)(a+ia\tan(e+fx))^{7/2}}{17ac(c-ic\tan(e+fx))^{17/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$ac\left(\frac{(5A+12iB)\left(\frac{4\left(\frac{3\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{13/2}}d\tan(e+fx)}{13c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{15c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{15ac(c-ic\tan(e+fx))^{15/2}}\right)}{17c}-\frac{(B+iA)(a+ia\tan(e+fx)}{17ac(c-ic\tan(e+fx))^{17}}\right.$$
$$\overline{\qquad f \qquad}$$

$\downarrow$ 55

$$ac\left(\frac{(5A+12iB)\left(\frac{4\left(\frac{3\left(\frac{2\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{11/2}}d\tan(e+fx)}{11c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{15c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{15ac(c-ic\tan(e+fx))^{15/2}}\right)}{17c}\right.$$
$$\overline{\qquad f \qquad}$$

$\downarrow$ 55

$$\frac{ac\left(\frac{(5A+12iB)\left(\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int \frac{(i\tan(e+fx)a+a)^{5/2}}{(c-ic\tan(e+fx))^{9/2}}d\tan(e+fx)}{9c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{15c}}{17c}\right.}{f}$$

$\downarrow$ 48

$$\frac{ac\left(\frac{(5A+12iB)\left(\frac{4\left(\frac{3\left(\frac{2\left(-\frac{i(a+ia\tan(e+fx))^{7/2}}{63ac^2(c-ic\tan(e+fx))^{7/2}}-\frac{i(a+ia\tan(e+fx))^{7/2}}{9ac(c-ic\tan(e+fx))^{9/2}}\right)}{11c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{11ac(c-ic\tan(e+fx))^{11/2}}\right)}{13c}-\frac{i(a+ia\tan(e+fx))^{7/2}}{13ac(c-ic\tan(e+fx))^{13/2}}\right)}{15c}-\frac{i(a}{15ac(}\right.}{17c}\right.}{f}$$

input
```
Int[((a + I*a*Tan[e + f*x])^(7/2)*(A + B*Tan[e + f*x]))/(c - I*c*Tan[e + f
*x])^(17/2),x]
```

output
```
(a*c*(-1/17*((I*A + B)*(a + I*a*Tan[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e +
 f*x])^(17/2)) + ((5*A + (12*I)*B)*(((-1/15*I)*(a + I*a*Tan[e + f*x])^(7/2
))/(a*c*(c - I*c*Tan[e + f*x])^(15/2)) + (4*(((-1/13*I)*(a + I*a*Tan[e + f
*x])^(7/2))/(a*c*(c - I*c*Tan[e + f*x])^(13/2)) + (3*(((-1/11*I)*(a + I*a*
Tan[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e + f*x])^(11/2)) + (2*(((-1/9*I)*(
a + I*a*Tan[e + f*x])^(7/2))/(a*c*(c - I*c*Tan[e + f*x])^(9/2)) - ((I/63)*
(a + I*a*Tan[e + f*x])^(7/2))/(a*c^2*(c - I*c*Tan[e + f*x])^(7/2))))/(11*c
)))/(13*c)))/(15*c)))/(17*c)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{a^3\sqrt{\frac{a\,\mathrm{e}^{2i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(45045iA\,\mathrm{e}^{16i(fx+e)}+45045B\,\mathrm{e}^{16i(fx+e)}+255255iA\,\mathrm{e}^{14i(fx+e)}+153153B\,\mathrm{e}^{14i(fx+e)}+589050iA\right.}{}$
derivativedivides	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(1+\tan(fx+e)^2\right)\left(12960iB\tan(fx+e)^4-960iB\tan(fx+e)^6-96B\tan\right.}{}$
default	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(1+\tan(fx+e)^2\right)\left(12960iB\tan(fx+e)^4-960iB\tan(fx+e)^6-96B\tan\right.}{}$
parts	$\frac{iA\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,a^3\left(1+\tan(fx+e)^2\right)\left(8i\tan(fx+e)^7-372i\tan(fx+e)^5-80\tan(fx+e)^6\right.}{153153f\,c^9(i+\tan(fx+e))^{10}}$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(17/2),x,
method=_RETURNVERBOSE)
```

output
```
-1/24504480*a^3/c^8*(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(ex
p(2*I*(f*x+e))+1))^(1/2)/f*(45045*I*A*exp(16*I*(f*x+e))+45045*B*exp(16*I*(
f*x+e))+255255*I*A*exp(14*I*(f*x+e))+153153*B*exp(14*I*(f*x+e))+589050*I*A
*exp(12*I*(f*x+e))+117810*B*exp(12*I*(f*x+e))+696150*I*exp(10*I*(f*x+e))*A
-139230*B*exp(10*I*(f*x+e))+425425*I*A*exp(8*I*(f*x+e))-255255*B*exp(8*I*(
f*x+e))+109395*I*A*exp(6*I*(f*x+e))-109395*B*exp(6*I*(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.60

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}\,dx =$$
$$-\frac{(45045\,(i\,A+B)a^3e^{(19i\,fx+19i\,e)}+6006\,(50i\,A+33\,B)a^3e^{(17i\,fx+17i\,e)}+3927\,(215i\,A+69\,B)a^3e^{(15i\,fx+15i}}{}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(17
/2),x, algorithm="fricas")
```

output
```
-1/24504480*(45045*(I*A + B)*a^3*e^(19*I*f*x + 19*I*e) + 6006*(50*I*A + 33
*B)*a^3*e^(17*I*f*x + 17*I*e) + 3927*(215*I*A + 69*B)*a^3*e^(15*I*f*x + 15
*I*e) + 21420*(60*I*A - B)*a^3*e^(13*I*f*x + 13*I*e) + 7735*(145*I*A - 51*
B)*a^3*e^(11*I*f*x + 11*I*e) + 24310*(22*I*A - 15*B)*a^3*e^(9*I*f*x + 9*I*
e) + 109395*(I*A - B)*a^3*e^(7*I*f*x + 7*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e)
 + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))/(c^9*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+I*a*tan(f*x+e))**(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))**(
17/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.31

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}\,dx=\text{Too large to display}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(17
/2),x, algorithm="maxima")
```

output
```
1/24504480*(45045*(-I*A - B)*a^3*cos(17/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 51051*(-5*I*A - 3*B)*a^3*cos(15/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 117810*(-5*I*A - B)*a^3*cos(13/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 139230*(-5*I*A + B)*a^3*cos(11/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 85085*(-5*I*A + 3*B)*a^3*cos(9/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 109395*(-I*A + B)*a^3*cos(7/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 45045*(A - I*B)*a^3*sin(17/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 51051*(5*A - 3*I*B)*a^3*sin(1
5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 117810*(5*A - I*B)*a^3*
sin(13/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 139230*(5*A + I*B)
*a^3*sin(11/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 85085*(5*A +
3*I*B)*a^3*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 109395*(
A + I*B)*a^3*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)
/(c^(17/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(17
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}\,dx =$$
$$-\frac{\sqrt{a+\frac{a\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}}\left(\frac{a^3\,\mathrm{e}^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,(5\,A+B\,3\mathrm{i})\,1\mathrm{i}}{288\,c^8\,f}+\frac{a^3\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,(5\,A+B\,1\mathrm{i})\,1\mathrm{i}}{176\,c^8\,f}+\frac{a^3\,\mathrm{e}^{e\,12\mathrm{i}+f\,x\,12\mathrm{i}}\,(5\,A-B\,1\mathrm{i})\,1\mathrm{i}}{208\,c^8\,f}+\frac{a^3\,\mathrm{e}^{e\,14\mathrm{i}+f\,x\,14\mathrm{i}}\,(5}{480\,c^8}\right.}{\sqrt{c-\frac{c\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}}}$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(7/2))/(c - c*tan(e + f*
x)*1i)^(17/2),x)
```

output
```
-((a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)*((a^3*exp(e*8i + f*x*8i)*(5
*A + B*3i)*1i)/(288*c^8*f) + (a^3*exp(e*10i + f*x*10i)*(5*A + B*1i)*1i)/(1
76*c^8*f) + (a^3*exp(e*12i + f*x*12i)*(5*A - B*1i)*1i)/(208*c^8*f) + (a^3*
exp(e*14i + f*x*14i)*(5*A - B*3i)*1i)/(480*c^8*f) + (a^3*exp(e*6i + f*x*6i
)*(A + B*1i)*1i)/(224*c^8*f) + (a^3*exp(e*16i + f*x*16i)*(A - B*1i)*1i)/(5
44*c^8*f)))/(c - (c*sin(e + f*x)*1i)/cos(e + f*x))^(1/2)
```

Reduce [F]

$$\int \frac{(a+ia\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(c-ic\tan(e+fx))^{17/2}}\,dx = \text{Too large to display}$$

input
```
int((a+I*a*tan(f*x+e))^(7/2)*(A+B*tan(f*x+e))/(c-I*c*tan(f*x+e))^(17/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*( - int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**10 + 8*tan(e + f*x)**9*i - 27
*tan(e + f*x)**8 - 48*tan(e + f*x)**7*i + 42*tan(e + f*x)**6 + 42*tan(e +
f*x)**4 + 48*tan(e + f*x)**3*i - 27*tan(e + f*x)**2 - 8*tan(e + f*x)*i + 1
),x)*b + 4*int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*ta
n(e + f*x)**3)/(tan(e + f*x)**10 + 8*tan(e + f*x)**9*i - 27*tan(e + f*x)**
8 - 48*tan(e + f*x)**7*i + 42*tan(e + f*x)**6 + 42*tan(e + f*x)**4 + 48*ta
n(e + f*x)**3*i - 27*tan(e + f*x)**2 - 8*tan(e + f*x)*i + 1),x)*a*i + 6*in
t(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3
)/(tan(e + f*x)**10 + 8*tan(e + f*x)**9*i - 27*tan(e + f*x)**8 - 48*tan(e
+ f*x)**7*i + 42*tan(e + f*x)**6 + 42*tan(e + f*x)**4 + 48*tan(e + f*x)**3
*i - 27*tan(e + f*x)**2 - 8*tan(e + f*x)*i + 1),x)*b - 4*int(( - sqrt(tan(
e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**1
0 + 8*tan(e + f*x)**9*i - 27*tan(e + f*x)**8 - 48*tan(e + f*x)**7*i + 42*t
an(e + f*x)**6 + 42*tan(e + f*x)**4 + 48*tan(e + f*x)**3*i - 27*tan(e + f*
x)**2 - 8*tan(e + f*x)*i + 1),x)*a*i - int(( - sqrt(tan(e + f*x)*i + 1)*sq
rt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**10 + 8*tan(e + f*x)
**9*i - 27*tan(e + f*x)**8 - 48*tan(e + f*x)**7*i + 42*tan(e + f*x)**6 + 4
2*tan(e + f*x)**4 + 48*tan(e + f*x)**3*i - 27*tan(e + f*x)**2 - 8*tan(e +
f*x)*i + 1),x)*b - int(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x...
```

3.830 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 228

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx=\frac{3(2iA-3B)c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}f}$$
$$+\frac{3(2iA-3B)c^2\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2af}$$
$$+\frac{(2iA-3B)c\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{2af}$$
$$+\frac{(iA-B)(c-ic\tan(e+fx))^{5/2}}{f\sqrt{a+ia\tan(e+fx)}}$$

output
```
3*(2*I*A-3*B)*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I
*c*tan(f*x+e))^(1/2))/a^(1/2)/f+3/2*(2*I*A-3*B)*c^2*(a+I*a*tan(f*x+e))^(1/
2)*(c-I*c*tan(f*x+e))^(1/2)/a/f+1/2*(2*I*A-3*B)*c*(a+I*a*tan(f*x+e))^(1/2)
*(c-I*c*tan(f*x+e))^(3/2)/a/f+(I*A-B)*(c-I*c*tan(f*x+e))^(5/2)/f/(a+I*a*ta
n(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 4.59 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.71

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \frac{c^2\sqrt{c-ic\tan(e+fx)}\left(-((2A+3iB)(i+\tan(e+f\right.}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/Sqrt[a + I*a
*Tan[e + f*x]],x]
```

output
```
(c^2*Sqrt[c - I*c*Tan[e + f*x]]*(-((2*A + (3*I)*B)*(I + Tan[e + f*x])) - B
*(I + Tan[e + f*x])^2 + 6*((2*I)*A - 3*B)*(1 + (ArcSin[Sqrt[a + I*a*Tan[e
+ f*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[1 - I
*Tan[e + f*x]]))))/(2*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 87, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{a+ia\tan(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{\sqrt{a+ia\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)a+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{(2A+3iB)\int\frac{(c-ic\tan(e+fx))^{3/2}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)}{a}\right)}{f}$$

$\downarrow$ 60

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{(2A+3iB)\left(\frac{3}{2}c\int\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{2a}\right)}{a}\right)}{f}$$

$\downarrow$ 60

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{(2A+3iB)\left(\frac{3}{2}c\left(c\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)-\right.}{a}\right.}{f}$$

$\downarrow$ 45

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{(2A+3iB)\left(\frac{3}{2}c\left(2c\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)-\frac{i}{}\right.}{a}\right.}{f}$$

$\downarrow$ 218

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{(2A+3iB)\left(\frac{3}{2}c\left(-\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)-\frac{i\sqrt{a+ia\tan(}}{}\right.}{a}\right.}{f}$$

input

```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/Sqrt[a + I*a*Tan[e
 + f*x]],x]
```

output
```
(a*c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(5/2))/(a*c*Sqrt[a + I*a*Tan[e + f
*x]]) - ((2*A + (3*I)*B)*(((-1/2*I)*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Ta
n[e + f*x])^(3/2))/a + (3*c*(((-2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*
Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[a] - (I*Sqrt[a
+ I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/a))/2))/a))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(188) = 376$.

Time = 0.84 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.48

method	result
derivativedivides	$\frac{i\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^2\left(6iA\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^2+18iB\right.}{}$
default	$\frac{i\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^2\left(6iA\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^2+18iB\right.}{}$
parts	$\frac{iA\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^2\left(3i\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)\tan(fx+e)^2ac-3i\ln\right.}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/2*I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a*(6*I*
A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2)
)*a*c*tan(f*x+e)^2+18*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e
)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+4*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x
+e)^2))^(1/2)*tan(f*x+e)^2-9*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(
f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+B*(a*c)^(1/2)*(a*c*(1+tan(
f*x+e)^2))^(1/2)*tan(f*x+e)^3-6*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1
+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-12*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x
+e)^2))^(1/2)*tan(f*x+e)+12*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f
*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+2*A*(a*c*(1+tan(f*x+e)^2))^(1
/2)*(a*c)^(1/2)*tan(f*x+e)^2-14*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/
2)+9*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^
(1/2))*a*c+19*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-10*A*(
a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I-t
an(f*x+e))^2/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(176) = 352$.

Time = 0.10 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.40

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
-1/4*(3*sqrt((4*A^2 + 12*I*A*B - 9*B^2)*c^5/(a*f^2))*(a*f*e^(3*I*f*x + 3*I
*e) + a*f*e^(I*f*x + I*e))*log(4*(2*((2*I*A - 3*B)*c^2*e^(3*I*f*x + 3*I*e)
 + (2*I*A - 3*B)*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sq
rt(c/(e^(2*I*f*x + 2*I*e) + 1)) + sqrt((4*A^2 + 12*I*A*B - 9*B^2)*c^5/(a*f
^2))*(a*f*e^(2*I*f*x + 2*I*e) - a*f))/((2*I*A - 3*B)*c^2*e^(2*I*f*x + 2*I*
e) + (2*I*A - 3*B)*c^2)) - 3*sqrt((4*A^2 + 12*I*A*B - 9*B^2)*c^5/(a*f^2))*
(a*f*e^(3*I*f*x + 3*I*e) + a*f*e^(I*f*x + I*e))*log(4*(2*((2*I*A - 3*B)*c^
2*e^(3*I*f*x + 3*I*e) + (2*I*A - 3*B)*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - sqrt((4*A^2 + 12*I*
A*B - 9*B^2)*c^5/(a*f^2))*(a*f*e^(2*I*f*x + 2*I*e) - a*f))/((2*I*A - 3*B)*
c^2*e^(2*I*f*x + 2*I*e) + (2*I*A - 3*B)*c^2)) + 4*(3*(-2*I*A + 3*B)*c^2*e^
(4*I*f*x + 4*I*e) + 5*(-2*I*A + 3*B)*c^2*e^(2*I*f*x + 2*I*e) + 4*(-I*A + B
)*c^2)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))
)/(a*f*e^(3*I*f*x + 3*I*e) + a*f*e^(I*f*x + I*e))
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \int \frac{(-ic(\tan(e+fx)+i))^{\frac{5}{2}}\,(A+B\tan(e+fx))}{\sqrt{ia\,(\tan(e+fx)-i)}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(
1/2),x)
```

output
```
Integral((-I*c*(tan(e + f*x) + I))**(5/2)*(A + B*tan(e + f*x))/sqrt(I*a*(t
an(e + f*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1319 vs. $2(176) = 352$.

Time = 0.44 (sec) , antiderivative size = 1319, normalized size of antiderivative = 5.79

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
4*(12*(2*A + 3*I*B)*c^2*cos(4*f*x + 4*e) + 20*(2*A + 3*I*B)*c^2*cos(2*f*x
+ 2*e) + 12*(2*I*A - 3*B)*c^2*sin(4*f*x + 4*e) + 20*(2*I*A - 3*B)*c^2*sin(
2*f*x + 2*e) + 16*(A + I*B)*c^2 + 6*((2*A + 3*I*B)*c^2*cos(5/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*A + 3*I*B)*c^2*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + (2*A + 3*I*B)*c^2*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + (2*I*A - 3*B)*c^2*sin(5/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*I*A - 3*B)*c^2*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + (2*I*A - 3*B)*c^2*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 1) + 6*((2*A + 3*I*B)*c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 2*(2*A + 3*I*B)*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + (2*A + 3*I*B)*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + (2*I*A - 3*B)*c^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 2*(2*I*A - 3*B)*c^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + (2*I*A - 3*B)*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 3*((2*I*A - 3*B)*
c^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*I*A - 3*B)
*c^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (2*I*A - 3*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx=\int \frac{(A+B\tan(e+f\,x))\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}}{\sqrt{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*
x)*1i)^(1/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*
x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c^2\left(\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+1}\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)**2*a*i - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2*b + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)*a + 5*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*a*i - 7*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b + int((sq
rt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e
 + f*x)**2 + 1),x)*tan(e + f*x)**2*b*f*i + int((sqrt(tan(e + f*x)*i + 1)*s
qrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**2 + 1),x)*b*f*i
 - 3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x
)**2)/(tan(e + f*x)**2 + 1),x)*tan(e + f*x)**2*a*f - 3*int((sqrt(tan(e + f
*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 +
 1),x)*tan(e + f*x)**2*b*f*i - 3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan
(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*a*f - 3*int((sq
rt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e
 + f*x)**2 + 1),x)*b*f*i))/(a*f*(tan(e + f*x)**2 + 1))
```

3.831 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 169

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \frac{2(iA-2B)c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}f} + \frac{(iA-2B)c\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{af} + \frac{(iA-B)(c-ic\tan(e+fx))^{3/2}}{f\sqrt{a+ia\tan(e+fx)}}$$

output
```
2*(I*A-2*B)*c^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c
*tan(f*x+e))^(1/2))/a^(1/2)/f+(I*A-2*B)*c*(a+I*a*tan(f*x+e))^(1/2)*(c-I*c*
tan(f*x+e))^(1/2)/a/f+(I*A-B)*(c-I*c*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e)
)^(1/2)
```

Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \frac{\sqrt{c-ic\tan(e+fx)}\Big(Bc(1-i\tan(e+fx))+2(iA \cdots}{f\sqrt{a+ia\tan \cdots}}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/Sqrt[a + I*a
*Tan[e + f*x]],x]
```

output
```
(Sqrt[c - I*c*Tan[e + f*x]]*(B*c*(1 - I*Tan[e + f*x]) + 2*(I*A - 2*B)*c*(1
 + (ArcSin[Sqrt[a + I*a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[a])]*Sqrt[a + I*a*Tan[
e + f*x]])/(Sqrt[a]*Sqrt[1 - I*Tan[e + f*x]]))))/(f*Sqrt[a + I*a*Tan[e + f
*x]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 87, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{a+ia\tan(e+fx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{\sqrt{a+ia\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)a+a)^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{(A+2iB)\int\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)}{a}\right)}{f}$$

$$\Big\downarrow \text{60}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{(A+2iB)\left(c\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)}{a}\right)}{f}$$

$$\Big\downarrow \text{45}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{(A+2iB)\left(2c\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)}{a}\right)}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{(A+2iB)\left(-\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a}\right)}{a}\right)}{f}$$

input

```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/Sqrt[a + I*a*Tan[e
 + f*x]],x]
```

output

```
(a*c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(3/2))/(a*c*Sqrt[a + I*a*Tan[e + f
*x]]) - ((A + (2*I)*B)*(((-2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e
 + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[a] - (I*Sqrt[a + I*a
*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/a))/a))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(140) = 280$.

Time = 0.84 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.95

method	result
derivativedivides	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c\left(-2iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^2+2iA\right.}{}$
default	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c\left(-2iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^2+2iA\right.}{}$
parts	$\frac{iA\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c\left(i\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)\tan(fx+e)^2ac-i\ln\right.}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a*c*(-2*I*B*ln(
(a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c
*tan(f*x+e)^2+2*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^
(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(
1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+2*I*B*ln((a*c*tan(f*
x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+4*I*B*(a*c
)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-4*B*ln((a*c*tan(f*x+e)+(a*
c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)-B*(a*c)
^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2-2*I*A*(a*c)^(1/2)*(a*c*(1
+tan(f*x+e)^2))^(1/2)+A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^
2))^(1/2))/(a*c)^(1/2))*a*c+2*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*t
an(f*x+e)+3*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)
^2))^(1/2)/(I-tan(f*x+e))^2/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(131) = 262$.

Time = 0.11 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.70

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx =$$

$$-\frac{\left(a\sqrt{\frac{(A^2+4i\,AB-4\,B^2)c^3}{af^2}}fe^{(i\,fx+i\,e)}\log\left(-\frac{4\left(2\left((i\,A-2\,B)ce^{(3i\,fx+3i\,e)}+(i\,A-2\,B)ce^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}\right.}{(-i\,A+2\,B)ce^{(2i\,fx+2i\,e)}+(-i\,A+2\,B)c}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
-1/2*(a*sqrt((A^2 + 4*I*A*B - 4*B^2)*c^3/(a*f^2))*f*e^(I*f*x + I*e)*log(-4
*(2*((I*A - 2*B)*c*e^(3*I*f*x + 3*I*e) + (I*A - 2*B)*c*e^(I*f*x + I*e))*sq
rt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (a*f*e
^(2*I*f*x + 2*I*e) - a*f)*sqrt((A^2 + 4*I*A*B - 4*B^2)*c^3/(a*f^2)))/((-I*
A + 2*B)*c*e^(2*I*f*x + 2*I*e) + (-I*A + 2*B)*c)) - a*sqrt((A^2 + 4*I*A*B
- 4*B^2)*c^3/(a*f^2))*f*e^(I*f*x + I*e)*log(-4*(2*((I*A - 2*B)*c*e^(3*I*f*
x + 3*I*e) + (I*A - 2*B)*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) +
1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (a*f*e^(2*I*f*x + 2*I*e) - a*f)*sq
rt((A^2 + 4*I*A*B - 4*B^2)*c^3/(a*f^2)))/((-I*A + 2*B)*c*e^(2*I*f*x + 2*I*
e) + (-I*A + 2*B)*c)) + 4*((-I*A + 2*B)*c*e^(2*I*f*x + 2*I*e) + (-I*A + B)
*c)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e
^(-I*f*x - I*e)/(a*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \int \frac{(-ic(\tan(e+fx)+i))^{\frac{3}{2}}\,(A+B\tan(e+fx))}{\sqrt{ia\,(\tan(e+fx)-i)}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(
1/2),x)
```

output
```
Integral((-I*c*(tan(e + f*x) + I))**(3/2)*(A + B*tan(e + f*x))/sqrt(I*a*(t
an(e + f*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(131) = 262$.

Time = 0.25 (sec) , antiderivative size = 898, normalized size of antiderivative = 5.31

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
(4*(A + 2*I*B)*c*cos(2*f*x + 2*e) + 4*(I*A - 2*B)*c*sin(2*f*x + 2*e) + 4*(
A + I*B)*c + 2*((A + 2*I*B)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + (A + 2*I*B)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))) + (I*A - 2*B)*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 (I*A - 2*B)*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*arcta
n2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 2*((A + 2*I*B)*c*cos(3/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (A + 2*I*B)*c*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + (I*A - 2*B)*c*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + (I*A - 2*B)*c*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))))*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) -
((-I*A + 2*B)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (-I
*A + 2*B)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (A + 2*
I*B)*c*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (A + 2*I*B)*
c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*log(cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e)))^2 + 2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 1) - ((I*A - 2*B)*c*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + (I*A - 2*B)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx=\int \frac{(A+B\tan(e+f\,x))\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}}{\sqrt{a+a\tan\left(e+f\,x\right)\,1\mathrm{i}}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*
x)*1i)^(1/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*
x)*1i)^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{\sqrt{a+ia\tan(e+fx)}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c\Big(-\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)**2*b + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)
*tan(e + f*x)*a + 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*a
*i - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - int((sqrt(
tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e +
f*x)**2 + 1),x)*tan(e + f*x)**2*a*f - 2*int((sqrt(tan(e + f*x)*i + 1)*sqrt
( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*tan(e +
f*x)**2*b*f*i - int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*a*f - 2*int((sqrt(tan(e + f*x)*i
 + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x
)*b*f*i))/(a*f*(tan(e + f*x)**2 + 1))
```

3.832 $\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 110

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx = -\frac{2B\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}f}+\frac{(iA-B)\sqrt{c-ic\tan(e+fx)}}{f\sqrt{a+ia\tan(e+fx)}}$$

output
```
-2*B*c^(1/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*
x+e))^(1/2))/a^(1/2)/f+(I*A-B)*(c-I*c*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e
))^(1/2)
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx = -\frac{2B\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}f}+\frac{(A+iB)c(i+\tan(e+fx))}{f\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/Sqrt[a + I*a*T
an[e + f*x]],x]
```

output
```
(-2*B*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c
- I*c*Tan[e + f*x]])])/(Sqrt[a]*f) + ((A + I*B)*c*(I + Tan[e + f*x]))/(f*S
qrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4071, 87, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{a+ia\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{\sqrt{a+ia\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{3/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{ac\left(\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{iB\int \frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{a}\right)}{f}$$

$$\Big\downarrow \text{ 45}$$

$$\frac{ac\left(\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{2iB\int \frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}}{a}\right)}{f}$$

$$\Big\downarrow \text{218}$$

$$\frac{ac\left(\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{ac\sqrt{a+ia\tan(e+fx)}}-\frac{2B\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{3/2}\sqrt{c}}\right)}{f}$$

input
```
Int[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/Sqrt[a + I*a*Tan[e +
 f*x]],x]
```

output
```
(a*c*((-2*B*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c -
I*c*Tan[e + f*x]])])/(a^(3/2)*Sqrt[c]) + ((I*A - B)*Sqrt[c - I*c*Tan[e + f
*x]])/(a*c*Sqrt[a + I*a*Tan[e + f*x]])))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(89) = 178$.

Time = 0.74 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.94

method	result
derivativedivides	$-\frac{i\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\left(-2iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^{2}\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)+B\ln\right.}{}$
default	$-\frac{i\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\left(-2iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^{2}\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)+B\ln\right.}{}$
parts	$-\frac{iA\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,(1+i\tan(fx+e))}{fa(i-\tan(fx+e))^{2}}+\frac{iB\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a*(-2*I*B*ln((
a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*
tan(f*x+e)+B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/
(a*c)^(1/2))*a*c*tan(f*x+e)^2+I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)
*tan(f*x+e)-B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))
/(a*c)^(1/2))*a*c+I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)-B*(a*c)^(1/
2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e
)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I-tan(f*x+e))^2/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(84) = 168.

Time = 0.10 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.16

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx$$
$$= \frac{\left(af\sqrt{-\frac{B^2c}{af^2}}e^{(ifx+ie)}\log\left(\frac{4\left(2\,(Be^{(3i\,fx+3i\,e)}+Be^{(i\,fx+i\,e)})\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}+(afe^{(2i\,fx+2i\,e)}-af)\sqrt{-\frac{B^2c}{af^2}}\right)}{Be^{(2i\,fx+2i\,e)}+B}\right)\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
1/2*(a*f*sqrt(-B^2*c/(a*f^2))*e^(I*f*x + I*e)*log(4*(2*(B*e^(3*I*f*x + 3*I
*e) + B*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*
f*x + 2*I*e) + 1)) + (a*f*e^(2*I*f*x + 2*I*e) - a*f)*sqrt(-B^2*c/(a*f^2)))
/(B*e^(2*I*f*x + 2*I*e) + B)) - a*f*sqrt(-B^2*c/(a*f^2))*e^(I*f*x + I*e)*l
og(4*(2*(B*e^(3*I*f*x + 3*I*e) + B*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) - (a*f*e^(2*I*f*x + 2*I*e) -
 a*f)*sqrt(-B^2*c/(a*f^2)))/(B*e^(2*I*f*x + 2*I*e) + B)) - 2*((-I*A + B)*e
^(2*I*f*x + 2*I*e) - I*A + B)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^
(2*I*f*x + 2*I*e) + 1)))*e^(-I*f*x - I*e)/(a*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx$$
$$= \int \frac{\sqrt{-ic\,(\tan{(e+fx)}+i)}\,(A+B\tan{(e+fx)})}{\sqrt{ia\,(\tan{(e+fx)}-i)}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(
1/2),x)
```

output
```
Integral(sqrt(-I*c*(tan(e + f*x) + I))*(A + B*tan(e + f*x))/sqrt(I*a*(tan(
e + f*x) - I)), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx =$$
$$-\frac{(2\,B\arctan\left(\cos\left(fx+e\right),\sin\left(fx+e\right)+1\right)+2\,B\arctan\left(\cos\left(fx+e\right),-\sin\left(fx+e\right)+1\right)-2\left(i\,A-\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
-1/2*(2*B*arctan2(cos(f*x + e), sin(f*x + e) + 1) + 2*B*arctan2(cos(f*x +
e), -sin(f*x + e) + 1) - 2*(I*A - B)*cos(f*x + e) + I*B*log(cos(f*x + e)^2
 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - I*B*log(cos(f*x + e)^2 + sin(f*x
 + e)^2 - 2*sin(f*x + e) + 1) - 2*(A + I*B)*sin(f*x + e))*sqrt(c)/(sqrt(a)
*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 8.51 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.27

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx$$
$$=\frac{A\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}\,1\mathrm{i}}{f\sqrt{a+a\tan(e+fx)\,1\mathrm{i}}}-\frac{4B\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\left(\sqrt{a+a\tan(e+fx)\,1\mathrm{i}}-\sqrt{a}\right)}{\sqrt{a}\left(\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}-\sqrt{c}\right)}\right)}{\sqrt{a}\,f}$$
$$-\frac{4B\left(\sqrt{a+a\tan(e+fx)\,1\mathrm{i}}-\sqrt{a}\right)}{f\left(\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}-\sqrt{c}\right)\left(-\frac{a}{c}+\frac{\left(\sqrt{a+a\tan(e+fx)\,1\mathrm{i}}-\sqrt{a}\right)^2}{\left(\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}-\sqrt{c}\right)^2}+\frac{2\sqrt{a}\left(\sqrt{a+a\tan(e+fx)\,1\mathrm{i}}-\sqrt{a}\right)}{\sqrt{c}\left(\sqrt{c-c\tan(e+fx)\,1\mathrm{i}}-\sqrt{c}\right)}\right)}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(1/2))/(a + a*tan(e + f*
x)*1i)^(1/2),x)
```

output
```
(A*(c - c*tan(e + f*x)*1i)^(1/2)*1i)/(f*(a + a*tan(e + f*x)*1i)^(1/2)) - (
4*B*c^(1/2)*atan((c^(1/2)*((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/(a^(1
/2)*((c - c*tan(e + f*x)*1i)^(1/2) - c^(1/2)))))/(a^(1/2)*f) - (4*B*((a +
a*tan(e + f*x)*1i)^(1/2) - a^(1/2)))/(f*((c - c*tan(e + f*x)*1i)^(1/2) - c
^(1/2))*(((a + a*tan(e + f*x)*1i)^(1/2) - a^(1/2))^2/((c - c*tan(e + f*x)*
1i)^(1/2) - c^(1/2))^2 - a/c + (2*a^(1/2)*((a + a*tan(e + f*x)*1i)^(1/2) -
 a^(1/2)))/(c^(1/2)*((c - c*tan(e + f*x)*1i)^(1/2) - c^(1/2)))))
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{\sqrt{a+ia\tan(e+fx)}}\,dx$$
$$=\frac{\sqrt{c}\sqrt{a}\left(\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,\tan(fx+e)\,a+\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)*a + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*a*i - s
qrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b - int((sqrt(tan(e +
f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2
+ 1),x)*tan(e + f*x)**2*b*f*i - int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(
e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**2 + 1),x)*b*f*i))/(a*f*(ta
n(e + f*x)**2 + 1))
```

3.833 $\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 92

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{iA+B}{f\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{iA\sqrt{c-ic\tan(e+fx)}}{cf\sqrt{a+ia\tan(e+fx)}}$$

output
```
-(I*A+B)/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)+I*A*(c-I*c*ta
n(f*x+e))^(1/2)/c/f/(a+I*a*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{-B+A\tan(e+fx)}{f\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[(A + B*Tan[e + f*x])/(Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Ta
n[e + f*x]]),x]
```

output
```
(-B + A*Tan[e + f*x])/(f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f
*x]])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {3042, 4071, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{A\int \frac{1}{(i\tan(e+fx)a+a)^{3/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{B+iA}{ac\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(\frac{iA\sqrt{c-ic\tan(e+fx)}}{ac^2\sqrt{a+ia\tan(e+fx)}}-\frac{B+iA}{ac\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

input
```
Int[(A + B*Tan[e + f*x])/(Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e +
f*x]]),x]
```

output
```
(a*c*(-((I*A + B)/(a*c*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x
]])) + (I*A*Sqrt[c - I*c*Tan[e + f*x]])/(a*c^2*Sqrt[a + I*a*Tan[e + f*x]])
))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{i A\,\mathrm{e}^{2 i (f x+e)}+B\,\mathrm{e}^{2 i (f x+e)}-i A+B}{2 \sqrt{\frac{a\,\mathrm{e}^{2 i (f x+e)}}{\mathrm{e}^{2 i (f x+e)}+1}}\,\left(\mathrm{e}^{2 i (f x+e)}+1\right) \sqrt{\frac{c}{\mathrm{e}^{2 i (f x+e)}+1}}\, f}$
derivativedivides	$\frac{\sqrt{a (1+i \tan (f x+e))}\,\sqrt{-c (i \tan (f x+e)-1)}\,\left(A \tan (f x+e)^{3}-B \tan (f x+e)^{2}+A \tan (f x+e)-B\right)}{f a c (i+\tan (f x+e))^{2} (i-\tan (f x+e))^{2}}$
default	$\frac{\sqrt{a (1+i \tan (f x+e))}\,\sqrt{-c (i \tan (f x+e)-1)}\,\left(A \tan (f x+e)^{3}-B \tan (f x+e)^{2}+A \tan (f x+e)-B\right)}{f a c (i+\tan (f x+e))^{2} (i-\tan (f x+e))^{2}}$
parts	$\frac{A \sqrt{a (1+i \tan (f x+e))}\,\sqrt{-c (i \tan (f x+e)-1)}\,\left(1+\tan (f x+e)^{2}\right) \tan (f x+e)}{f a c (i-\tan (f x+e))^{2} (i+\tan (f x+e))^{2}}+\frac{B \left(-1-\tan (f x+e)^{2}\right) \sqrt{-c (i \tan (f x+\cdots}}{f c a (i-\tan (f x+e))^{2} (i+\cdots}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/2/(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(exp(2*I*(f*x+e))+1)/
(c/(exp(2*I*(f*x+e))+1))^(1/2)*(I*A*exp(2*I*(f*x+e))+B*exp(2*I*(f*x+e))-I*
A+B)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{\left((-iA-B)e^{(4i fx+4ie)}+2\,Be^{(3i fx+3ie)}-2\,Be^{(2i fx+2ie)}+2\,Be^{(i fx+ie)}+iA-B\right)\sqrt{\frac{a}{e^{(2i fx+2ie)}+1}}\sqrt{\frac{\cdots}{e^{(2i fx\cdots}}}}{2\,acf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
1/2*((-I*A - B)*e^(4*I*f*x + 4*I*e) + 2*B*e^(3*I*f*x + 3*I*e) - 2*B*e^(2*I
*f*x + 2*I*e) + 2*B*e^(I*f*x + I*e) + I*A - B)*sqrt(a/(e^(2*I*f*x + 2*I*e)
 + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-I*f*x - I*e)/(a*c*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}\,dx = \int \frac{A+B\tan{(e+fx)}}{\sqrt{ia\left(\tan{(e+fx)}-i\right)}\sqrt{-ic\left(\tan{(e+fx)}+i\right)}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(1/2)/(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Integral((A + B*tan(e + f*x))/(sqrt(I*a*(tan(e + f*x) - I))*sqrt(-I*c*(tan
(e + f*x) + I))), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.35

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{2\left((A-iB)\cos{(4fx+4e)}-2iB\cos{(2fx+2e)}-(-iA-B)\sin{(4fx+4e)}+2B\sin{(2fx+2e}\right.}{-4\left(iac\cos{(3fx+3e)}+iac\cos{(fx+e)}-ac\sin{(3fx+3e)}-ac\sin{(fx+e)}\right)}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
-2*((A - I*B)*cos(4*f*x + 4*e) - 2*I*B*cos(2*f*x + 2*e) - (-I*A - B)*sin(4
*f*x + 4*e) + 2*B*sin(2*f*x + 2*e) - A - I*B)*sqrt(a)*sqrt(c)/((-4*I*a*c*c
os(3*f*x + 3*e) - 4*I*a*c*cos(f*x + e) + 4*a*c*sin(3*f*x + 3*e) + 4*a*c*si
n(f*x + e))*f)
```

Giac [F]

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}\,dx = \int \frac{B\tan(fx+e)+A}{\sqrt{i\,a\tan(fx+e)+a}\sqrt{-i\,c\tan(fx+e)+c}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)/(sqrt(I*a*tan(f*x + e) + a)*sqrt(-I*c*tan(f
*x + e) + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.55

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(A\,1\mathrm{i}+B-A\,\cos(2\,e+2\,f\,x)\,1\mathrm{i}+B\,\cos(2\,e+2\,f\,x)-A\,\sin(2\,e+}{2\,a\,f\,\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x
)*1i)^(1/2)),x)
```

output
```
-(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1)
)^(1/2)*(A*1i + B - A*cos(2*e + 2*f*x)*1i + B*cos(2*e + 2*f*x) - A*sin(2*e
 + 2*f*x) - B*sin(2*e + 2*f*x)*1i))/(2*a*f*((c*(cos(2*e + 2*f*x) - sin(2*e
 + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{A + B\tan(e + fx)}{\sqrt{a + ia\tan(e + fx)}\sqrt{c - ic\tan(e + fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,\sqrt{\tan(fx + e)\,i + 1}\,\sqrt{-\tan(fx + e)\,i + 1}\,(\tan(fx + e)\,a - b)}{acf\left(\tan(fx + e)^2 + 1\right)}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*(tan
(e + f*x)*a - b))/(a*c*f*(tan(e + f*x)**2 + 1))
```

3.834 $\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 157

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{iA-B}{f\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}} - \frac{(2iA-B)\sqrt{a+ia\tan(e+fx)}}{3af(c-ic\tan(e+fx))^{3/2}} - \frac{(2iA-B)\sqrt{a+ia\tan(e+fx)}}{3acf\sqrt{c-ic\tan(e+fx)}}$$

output
```
(I*A-B)/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2)-1/3*(2*I*A-B)*
(a+I*a*tan(f*x+e))^(1/2)/a/f/(c-I*c*tan(f*x+e))^(3/2)-1/3*(2*I*A-B)*(a+I*a
*tan(f*x+e))^(1/2)/a/c/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.88 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.62

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{A-iB+(2iA-B)\tan(e+fx)+(2A+iB)\tan^2(}{3cf(i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\,\mathrm{ta}}}$$

input
```
Integrate[(A + B*Tan[e + f*x])/(Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e
+ f*x])^(3/2)),x]
```

output
```
(A - I*B + ((2*I)*A - B)*Tan[e + f*x] + (2*A + I*B)*Tan[e + f*x]^2)/(3*c*f
*(I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4071, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(2A+iB)\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{a}+\frac{-B+iA}{ac\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{(2A+iB)\left(\frac{\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{a}+\frac{-B+iA}{ac\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(\frac{(2A+iB)\left(-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac^2\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{a}+\frac{-B+iA}{ac\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

input
```
Int[(A + B*Tan[e + f*x])/(Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x]
)^(3/2)),x]
```

output
```
(a*c*((I*A - B)/(a*c*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(3/
2)) + ((2*A + I*B)*(((-1/3*I)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*(c - I*c*Ta
n[e + f*x])^(3/2)) - ((I/3)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c^2*Sqrt[c - I*
c*Tan[e + f*x]])))/a))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{iA\,{\mathrm e}^{4i(fx+e)}+B\,{\mathrm e}^{4i(fx+e)}+6iA\,{\mathrm e}^{2i(fx+e)}-3iA+3B}{12c\sqrt{\frac{a\,{\mathrm e}^{2i(fx+e)}}{{\mathrm e}^{2i(fx+e)}+1}}\left({\mathrm e}^{2i(fx+e)}+1\right)\sqrt{\frac{c}{{\mathrm e}^{2i(fx+e)}+1}}\,f}$
derivativedivides	$-\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(2iA\tan(fx+e)^{4}-iB\tan(fx+e)^{3}-B\tan(fx+e)^{4}+3iA\tan(fx+e)^{2}-\right.}{3fa\,c^{2}(i-\tan(fx+e))^{2}(i+\tan(fx+e))^{3}}$
default	$-\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(2iA\tan(fx+e)^{4}-iB\tan(fx+e)^{3}-B\tan(fx+e)^{4}+3iA\tan(fx+e)^{2}-\right.}{3fa\,c^{2}(i-\tan(fx+e))^{2}(i+\tan(fx+e))^{3}}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(2i\tan(fx+e)^{3}+2\tan(fx+e)^{4}+2i\tan(fx+e)+3\tan(fx+e)^{2}+1\right)}{3fa\,c^{2}(i+\tan(fx+e))^{3}(i-\tan(fx+e))^{2}}+$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/12/c/(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(exp(2*I*(f*x+e))+
1)/(c/(exp(2*I*(f*x+e))+1))^(1/2)*(I*A*exp(4*I*(f*x+e))+B*exp(4*I*(f*x+e))
+6*I*A*exp(2*I*(f*x+e))-3*I*A+3*B)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{((-iA-B)e^{(6ifx+6ie)}+(-7iA-B)e^{(4ifx+4ie)}-4}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
1/12*((-I*A - B)*e^(6*I*f*x + 6*I*e) + (-7*I*A - B)*e^(4*I*f*x + 4*I*e) -
4*(-I*A - B)*e^(3*I*f*x + 3*I*e) - 3*(I*A + B)*e^(2*I*f*x + 2*I*e) - 4*(-I
*A - B)*e^(I*f*x + I*e) + 3*I*A - 3*B)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*s
qrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-I*f*x - I*e)/(a*c^2*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\,dx = \int \frac{A+B\tan(e+fx)}{\sqrt{ia(\tan(e+fx)-i)}\,(-ic(\tan(e+fx)+i))^{\frac{3}{2}}}\,d$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(1/2)/(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Integral((A + B*tan(e + f*x))/(sqrt(I*a*(tan(e + f*x) - I))*(-I*c*(tan(e +
 f*x) + I))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\,dx = \int \frac{B\tan(fx+e)+A}{\sqrt{ia\tan(fx+e)+a}(-ic\tan(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)/(sqrt(I*a*tan(f*x + e) + a)*(-I*c*tan(f*x +
 e) + c)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.93

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1i)}{\cos(2\,e+2\,f\,x)+1}}\,(2\,A\,\sin(2\,e+2\,f\,x)\,-}{6\,a\,c\,f}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x
)*1i)^(3/2)),x)
```

output
```
(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*(A*cos(2*e + 2*f*x)*1i - A*3i - 2*B*cos(2*e + 2*f*x) + 2*A*sin(2*e
+ 2*f*x) + B*sin(2*e + 2*f*x)*1i))/(6*a*c*f*((c*(cos(2*e + 2*f*x) - sin(2*
e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{A + B\tan(e + fx)}{\sqrt{a + ia\tan(e + fx)}(c - ic\tan(e + fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\sqrt{\tan(fx + e)\,i + 1}\,\sqrt{-\tan(fx + e)\,i + 1}\ \mathrm{t}\right.}{}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan
(e + f*x)*b*i + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))
/(tan(e + f*x)**3 + tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)*
*2*a*f*i + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan
(e + f*x)**3 + tan(e + f*x)**2*i + tan(e + f*x) + i),x)*tan(e + f*x)**2*b*
f + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*
x)**3 + tan(e + f*x)**2*i + tan(e + f*x) + i),x)*a*f*i + int((sqrt(tan(e +
 f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 + tan(e + f*x)*
*2*i + tan(e + f*x) + i),x)*b*f))/(a*c**2*f*(tan(e + f*x)**2 + 1))
```

3.835 $\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 213

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{iA-B}{f\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}} - \frac{(3iA-2B)\sqrt{a+ia\tan(e+fx)}}{5af(c-ic\tan(e+fx))^{5/2}} - \frac{2(3iA-2B)\sqrt{a+ia\tan(e+fx)}}{15acf(c-ic\tan(e+fx))^{3/2}} - \frac{2(3iA-2B)\sqrt{a+ia\tan(e+fx)}}{15ac^2f\sqrt{c-ic\tan(e+fx)}}$$

output
```
(I*A-B)/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2)-1/5*(3*I*A-2*B
)*(a+I*a*tan(f*x+e))^(1/2)/a/f/(c-I*c*tan(f*x+e))^(5/2)-2/15*(3*I*A-2*B)*(
a+I*a*tan(f*x+e))^(1/2)/a/c/f/(c-I*c*tan(f*x+e))^(3/2)-2/15*(3*I*A-2*B)*(a
+I*a*tan(f*x+e))^(1/2)/a/c^2/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 4.76 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.54

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{6iA+B-(3A+2iB)\tan(e+fx)+(12iA-8B)\,\text{ta}}{15c^2f(i+\tan(e+fx))^2\sqrt{a+ia\tan(e}}$$

input
```
Integrate[(A + B*Tan[e + f*x])/(Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e
+ f*x])^(5/2)),x]
```

output
```
((6*I)*A + B - (3*A + (2*I)*B)*Tan[e + f*x] + ((12*I)*A - 8*B)*Tan[e + f*x
]^2 + (6*A + (4*I)*B)*Tan[e + f*x]^3)/(15*c^2*f*(I + Tan[e + f*x])^2*Sqrt[
a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(3A+2iB)\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{a}+\frac{-B+iA}{ac\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$\Big\downarrow$ 55

$$\frac{ac\left(\frac{(3A+2iB)\left(\frac{2\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{5c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{a}+\frac{-B+iA}{ac\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$\Big\downarrow$ 55

$$\frac{ac\left(\frac{(3A+2iB)\left(\frac{2\left(\frac{\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{5c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{a}+\frac{}{ac\sqrt{a+ia\tan(e+}}\right.}{f}$$

$\Big\downarrow$ 48

$$\frac{ac\left(\frac{(3A+2iB)\left(\frac{2\left(-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac^2\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{5c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{a}+\frac{-B+iA}{ac\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

input
```
Int[(A + B*Tan[e + f*x])/(Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x]
)^(5/2)),x]
```

output
```
(a*c*((I*A - B)/(a*c*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e + f*x])^(5/
2)) + ((3*A + (2*I)*B)*(((-1/5*I)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*(c - I*
c*Tan[e + f*x])^(5/2)) + (2*(((-1/3*I)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*(c
 - I*c*Tan[e + f*x])^(3/2)) - ((I/3)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c^2*Sq
rt[c - I*c*Tan[e + f*x]])))/(5*c)))/a))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{3iA\,\mathrm{e}^{6i(fx+e)}+3B\,\mathrm{e}^{6i(fx+e)}+15iA\,\mathrm{e}^{4i(fx+e)}+5B\,\mathrm{e}^{4i(fx+e)}+45iA\,\mathrm{e}^{2i(fx+e)}-15B\,\mathrm{e}^{2i(fx+e)}-15iA+15B}{120c^{2}\sqrt{\frac{a\,\mathrm{e}^{2i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)f}$
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(4iB\tan(fx+e)^{5}+12iA\tan(fx+e)^{4}+6A\tan(fx+e)^{5}+2iB\tan(fx+e)^{3}-\right.}{15fa\,c^{3}(i+\tan(fx+e)}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(4iB\tan(fx+e)^{5}+12iA\tan(fx+e)^{4}+6A\tan(fx+e)^{5}+2iB\tan(fx+e)^{3}-\right.}{15fa\,c^{3}(i+\tan(fx+e)}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(4i\tan(fx+e)^{4}+2\tan(fx+e)^{5}+6i\tan(fx+e)^{2}+\tan(fx+e)^{3}+2i-\tan(\right.}{5fa\,c^{3}(i+\tan(fx+e))^{4}(i-\tan(fx+e))^{2}}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/120/c^2/(a*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^(1/2)/(c/(exp(2*I*(f*
x+e))+1))^(1/2)/(exp(2*I*(f*x+e))+1)*(3*I*A*exp(6*I*(f*x+e))+3*B*exp(6*I*(
f*x+e))+15*I*A*exp(4*I*(f*x+e))+5*B*exp(4*I*(f*x+e))+45*I*A*exp(2*I*(f*x+e
))-15*B*exp(2*I*(f*x+e))-15*I*A+15*B)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.76

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(3\,(i\,A+B)e^{(8i\,fx+8i\,e)}+2\,(9i\,A+4\,B)e^{(6i\,fx+6i\,e)}+10\,(6i\,A-B)e^{(4i\,fx+4i\,e)}+8\,(-6i\,A-B)e^{(3i\,fx+3i\,e)}\right.}{120\,ac^{3}}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/120*(3*(I*A + B)*e^(8*I*f*x + 8*I*e) + 2*(9*I*A + 4*B)*e^(6*I*f*x + 6*I
*e) + 10*(6*I*A - B)*e^(4*I*f*x + 4*I*e) + 8*(-6*I*A - B)*e^(3*I*f*x + 3*I
*e) + 30*I*A*e^(2*I*f*x + 2*I*e) + 8*(-6*I*A - B)*e^(I*f*x + I*e) - 15*I*A
 + 15*B)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1
))*e^(-I*f*x - I*e)/(a*c^3*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\,dx=\int \frac{A+B\tan\left(e+fx\right)}{\sqrt{ia\left(\tan\left(e+fx\right)-i\right)}\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)^{\frac{5}{2}}}\,d$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(1/2)/(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Integral((A + B*tan(e + f*x))/(sqrt(I*a*(tan(e + f*x) - I))*(-I*c*(tan(e +
 f*x) + I))**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\,dx=\int \frac{B\tan(fx+e)+A}{\sqrt{ia\tan(fx+e)+a}(-ic\tan(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)/(sqrt(I*a*tan(f*x + e) + a)*(-I*c*tan(f*x +
 e) + c)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\,dx=-\frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(A\,45\mathrm{i}-15\,B+A\,\cos(4\,e+4\,f\,x)\,3\mathrm{i}+20\,B\,\cos(2\,e+2\,f\,x)+3\,B\,\cos}{120\,a\,c^2\,f\,\sqrt{\frac{c\,(\cos(2}{}}}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(1/2)*(c - c*tan(e + f*x
)*1i)^(5/2)),x)
```

output
```
-(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1)
)^(1/2)*(A*45i - 15*B + A*cos(4*e + 4*f*x)*3i + 20*B*cos(2*e + 2*f*x) + 3*
B*cos(4*e + 4*f*x) - 30*A*sin(2*e + 2*f*x) - 3*A*sin(4*e + 4*f*x) - B*sin(
2*e + 2*f*x)*10i + B*sin(4*e + 4*f*x)*3i))/(120*a*c^2*f*((c*(cos(2*e + 2*f
*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{-\left(\int \frac{\mathrm{t}}{\sqrt{\tan(fx+e)i+1}\,\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^2+2\sqrt{\tan(fx+e)}}\right.}{}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
( - (int(tan(e + f*x)/(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)**2 + 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)
*tan(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)),x)
*b + int(1/(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x)**2 + 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*
x)*i - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)),x)*a))/(sqrt(
c)*sqrt(a)*c**2)
```

3.836 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 287

$$
\begin{aligned}
\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx =& -\frac{5(2iA-5B)c^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{3/2}f} \\
&-\frac{5(2iA-5B)c^3\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2a^2f} \\
&-\frac{5(2iA-5B)c^2\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{6a^2f} \\
&-\frac{2(2iA-5B)c(c-ic\tan(e+fx))^{5/2}}{3af\sqrt{a+ia\tan(e+fx)}}+\frac{(iA-B)(c-ic\tan(e+fx))^{7/2}}{3f(a+ia\tan(e+fx))^{3/2}}
\end{aligned}
$$

output
```
-5*(2*I*A-5*B)*c^(7/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-
I*c*tan(f*x+e))^(1/2))/a^(3/2)/f-5/2*(2*I*A-5*B)*c^3*(a+I*a*tan(f*x+e))^(1
/2)*(c-I*c*tan(f*x+e))^(1/2)/a^2/f-5/6*(2*I*A-5*B)*c^2*(a+I*a*tan(f*x+e))^
(1/2)*(c-I*c*tan(f*x+e))^(3/2)/a^2/f-2/3*(2*I*A-5*B)*c*(c-I*c*tan(f*x+e))^
(5/2)/a/f/(a+I*a*tan(f*x+e))^(1/2)+1/3*(I*A-B)*(c-I*c*tan(f*x+e))^(7/2)/f/
(a+I*a*tan(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.56

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{c^3\sec(e+fx)\sqrt{c-ic\tan(e+fx)}\Big(20i(2A+5iB)\,\mathrm{co}}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2))/(a + I*a*Tan
[e + f*x])^(3/2),x]
```

output
```
(c^3*Sec[e + f*x]*Sqrt[c - I*c*Tan[e + f*x]]*((20*I)*(2*A + (5*I)*B)*Cos[e
 + f*x]^3*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 + I*Tan[e + f*x])/2]*Sqrt
[2 - (2*I)*Tan[e + f*x]] + 3*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)])*((-2*
I)*A + 6*B - I*B*Tan[e + f*x])))/(6*a*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 87, 57, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c-ic\tan(e+fx))^{7/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)a+a)^{5/2}}d\tan(e+fx)}{f}$$

$\Big\downarrow$ 87

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(2A+5iB)\int\frac{(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)a+a)^{3/2}}d\tan(e+fx)}{3a}\right)}{f}$$

$\Big\downarrow$ 57

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(2A+5iB)\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{5c\int\frac{(c-ic\tan(e+fx))^{3/2}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)}{a}\right)}{3a}\right)}{f}$$

$\Big\downarrow$ 60

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(2A+5iB)\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{5c\left(\frac{3}{2}c\int\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+f}{2a}\right.}{a}\right.}{3a}\right.}{f}$$

$\Big\downarrow$ 60

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(2A+5iB)\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{5c\left(\frac{3}{2}c\left(c\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(}{}\right.\right.}{a}\right.}{3a}\right.}{f}$$

$\Big\downarrow$ 45

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(2A+5iB)\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{5c\left(\frac{3}{2}c\left(2c\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+\cdots}}{\cdots}\right)\right)}{a}\right)}{3a}\right)}{f}$$

$\big\downarrow$ 218

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(2A+5iB)\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{5c\left(\frac{3}{2}c\left(-\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-i\cdots}}{a}\right)\cdots\right)}{a}\right)}{3a}\right)}{f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2))/(a + I*a*Tan[e + f
*x])^(3/2),x]
```

output
```
(a*c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(7/2))/(3*a*c*(a + I*a*Tan[e + f*x
])^(3/2)) - ((2*A + (5*I)*B)*(((2*I)*(c - I*c*Tan[e + f*x])^(5/2))/(a*Sqrt
[a + I*a*Tan[e + f*x]]) - (5*c*(((-1/2*I)*Sqrt[a + I*a*Tan[e + f*x]]*(c -
I*c*Tan[e + f*x])^(3/2))/a + (3*c*(((-2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
+ I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[a] - (I*S
qrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/a))/2))/a))/(3*a)))/
f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(236) = 472$.

Time = 0.87 (sec), antiderivative size = 733, normalized size of antiderivative = 2.55

method	result
derivativedivides	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^3\left(185iB\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}\,\tan(fx+e)^2+6iA\sqrt{ac}\,\sqrt{ac\left(1+\tan(\right.}\right.}{}$
default	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^3\left(185iB\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\sqrt{ac}\,\tan(fx+e)^2+6iA\sqrt{ac}\,\sqrt{ac\left(1+\tan(\right.}\right.}{}$
parts	$\frac{A\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^3\left(45i\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)\tan(fx+e)^2ac+3i\,\mathrm{ta}\right.}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/6/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^3/a^2*(185*
I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+6*I*A*(a*c)^(1/2
)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3-75*I*B*ln((a*c*tan(f*x+e)+(a*c
)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3-118*I*
B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)-30*A*ln((a*c*tan(f*x+e)+(a*c)^(
1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+90*I*A*ln
((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*
c*tan(f*x+e)^2-114*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)
-225*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^
(1/2))*a*c*tan(f*x+e)^2-21*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(
f*x+e)^3+3*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^4+225*I
*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2
))*a*c*tan(f*x+e)+90*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2
))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+74*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a
*c)^(1/2)*tan(f*x+e)^2-30*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f
*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+75*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c
*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+279*B*(a*c)^(1/2)*(a*c*(1+tan(f
*x+e)^2))^(1/2)*tan(f*x+e)-46*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/
(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)/(I-tan(f*x+e))^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(219) = 438$.

Time = 0.14 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.05

$$\int \frac{(A + B\tan(e + fx))(c - ic\tan(e + fx))^{7/2}}{(a + ia\tan(e + fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
1/12*(15*(a^2*f*e^(5*I*f*x + 5*I*e) + a^2*f*e^(3*I*f*x + 3*I*e))*sqrt((4*A
^2 + 20*I*A*B - 25*B^2)*c^7/(a^3*f^2))*log(4*(2*((2*I*A - 5*B)*c^3*e^(3*I*
f*x + 3*I*e) + (2*I*A - 5*B)*c^3*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I
*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (a^2*f*e^(2*I*f*x + 2*I*e) -
 a^2*f)*sqrt((4*A^2 + 20*I*A*B - 25*B^2)*c^7/(a^3*f^2)))/((2*I*A - 5*B)*c^
3*e^(2*I*f*x + 2*I*e) + (2*I*A - 5*B)*c^3)) - 15*(a^2*f*e^(5*I*f*x + 5*I*e
) + a^2*f*e^(3*I*f*x + 3*I*e))*sqrt((4*A^2 + 20*I*A*B - 25*B^2)*c^7/(a^3*f
^2))*log(4*(2*((2*I*A - 5*B)*c^3*e^(3*I*f*x + 3*I*e) + (2*I*A - 5*B)*c^3*e
^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I
*e) + 1)) - (a^2*f*e^(2*I*f*x + 2*I*e) - a^2*f)*sqrt((4*A^2 + 20*I*A*B - 2
5*B^2)*c^7/(a^3*f^2)))/((2*I*A - 5*B)*c^3*e^(2*I*f*x + 2*I*e) + (2*I*A - 5
*B)*c^3)) - 4*(15*(2*I*A - 5*B)*c^3*e^(6*I*f*x + 6*I*e) + 25*(2*I*A - 5*B)
*c^3*e^(4*I*f*x + 4*I*e) + 8*(2*I*A - 5*B)*c^3*e^(2*I*f*x + 2*I*e) + 4*(-I
*A + B)*c^3)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e)
 + 1)))/(a^2*f*e^(5*I*f*x + 5*I*e) + a^2*f*e^(3*I*f*x + 3*I*e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2)/(a+I*a*tan(f*x+e))**(
3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(219) = 438$.

Time = 0.60 (sec) , antiderivative size = 1373, normalized size of antiderivative = 4.78

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
-12*(60*(2*A + 5*I*B)*c^3*cos(6*f*x + 6*e) + 100*(2*A + 5*I*B)*c^3*cos(4*f
*x + 4*e) + 32*(2*A + 5*I*B)*c^3*cos(2*f*x + 2*e) + 60*(2*I*A - 5*B)*c^3*s
in(6*f*x + 6*e) + 100*(2*I*A - 5*B)*c^3*sin(4*f*x + 4*e) + 32*(2*I*A - 5*B
)*c^3*sin(2*f*x + 2*e) - 16*(A + I*B)*c^3 + 30*((2*A + 5*I*B)*c^3*cos(7/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*A + 5*I*B)*c^3*cos(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (2*A + 5*I*B)*c^3*cos(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (2*I*A - 5*B)*c^3*sin(7/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*(2*I*A - 5*B)*c^3*sin(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (2*I*A - 5*B)*c^3*sin(3/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*arctan2(cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1) + 30*((2*A + 5*I*B)*c^3*cos(7/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + 2*(2*A + 5*I*B)*c^3*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + (2*A + 5*I*B)*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + (2*I*A - 5*B)*c^3*sin(7/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 2*(2*I*A - 5*B)*c^3*sin(5/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + (2*I*A - 5*B)*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))), -sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 15*((
2*I*A - 5*B)*c^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \int \frac{(A+B\tan(e+f\,x))\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*
x)*1i)^(3/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*
x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**3*( - 10*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)
*i + 1)*tan(e + f*x)*a - 5*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
 + 1)*tan(e + f*x)*b*i - 6*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
 + 1)*a*i + 4*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b + 4*i
nt(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)
**3 - tan(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*x)**2*a*f*i - int
(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**
3 - tan(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*x)**2*b*f + 4*int((
 - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3
- tan(e + f*x)**2*i + tan(e + f*x) - i),x)*a*f*i - int(( - sqrt(tan(e + f*
x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 - tan(e + f*x)**2*
i + tan(e + f*x) - i),x)*b*f + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan
(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f + int((sqrt(tan(e + f*x)*i + 1)*sq
rt( - tan(e + f*x)*i + 1)*tan(e + f*x)**5)/(tan(e + f*x)**3*i + tan(e + f*
x)**2 + tan(e + f*x)*i + 1),x)*b*f + int((sqrt(tan(e + f*x)*i + 1)*sqrt( -
 tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i + tan(e + f*x)**2
 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*f + 4*int((sqrt(tan(e + f*x)*i
 + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i + ta
n(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i + int((sqr...
```

3.837 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 229

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = -\frac{2(iA-4B)c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{3/2}f} - \frac{(iA-4B)c^2\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a^2f} - \frac{2(iA-4B)c(c-ic\tan(e+fx))^{3/2}}{3af\sqrt{a+ia\tan(e+fx)}} + \frac{(iA-B)(c-ic\tan(e+fx))^{5/2}}{3f(a+ia\tan(e+fx))^{3/2}}$$

output

```
-2*(I*A-4*B)*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*
c*tan(f*x+e))^(1/2))/a^(3/2)/f-(I*A-4*B)*c^2*(a+I*a*tan(f*x+e))^(1/2)*(c-I
*c*tan(f*x+e))^(1/2)/a^2/f-2/3*(I*A-4*B)*c*(c-I*c*tan(f*x+e))^(3/2)/a/f/(a
+I*a*tan(f*x+e))^(1/2)+1/3*(I*A-B)*(c-I*c*tan(f*x+e))^(5/2)/f/(a+I*a*tan(f
*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.63 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{c^2\cos^2(e+fx)\left(3B\sec^3(e+fx)(\cos(3(e+fx))-i\,\mathrm{s}\right.}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan
[e + f*x])^(3/2),x]
```

output
```
(c^2*Cos[e + f*x]^2*(3*B*Sec[e + f*x]^3*(Cos[3*(e + f*x)] - I*Sin[3*(e + f
*x)]) + (4*I)*(A + (4*I)*B)*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 + I*Tan
[e + f*x])/2]*Sqrt[2 - (2*I)*Tan[e + f*x]])*Sqrt[c - I*c*Tan[e + f*x]])/(3
*a*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 87, 57, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)a+a)^{5/2}}d\tan(e+fx)}{f}$$

$\downarrow$ 87

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(A+4iB)\int\frac{(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)a+a)^{3/2}}d\tan(e+fx)}{3a}\right)}{f}$$

$\downarrow$ 57

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(A+4iB)\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{3c\int\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)}{a}\right)}{3a}\right)}{f}$$

$\downarrow$ 60

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(A+4iB)\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{3c\left(c\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)-\frac{i\sqrt{a+ia\tan(e+fx)}}{a}\right.}{a}\right.}{3a}\right.}{f}$$

$\downarrow$ 45

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(A+4iB)\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{3c\left(2c\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{}}{a}\right.}{a}\right.}{3a}\right.}{f}$$

$\downarrow$ 218

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{(A+4iB)\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{3c\left(-\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}}-\frac{i\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e}{a}\right.}{a}\right.}{3a}\right.}{f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan[e + f
*x])^(3/2),x]
```

output
```
(a*c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(5/2))/(3*a*c*(a + I*a*Tan[e + f*x
])^(3/2)) - ((A + (4*I)*B)*(((2*I)*(c - I*c*Tan[e + f*x])^(3/2))/(a*Sqrt[a
 + I*a*Tan[e + f*x]]) - (3*c*(((-2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a
*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[a] - (I*Sqrt[a
 + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/a))/a))/(3*a)))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(189) = 378$.

Time = 0.80 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.92

method	result
derivativedivides	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^2\left(-12iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^3+9i\right.}{}$
default	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^2\left(-12iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^3+9i\right.}{}$
parts	$\frac{A\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^2\left(9i\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)\tan(fx+e)^2ac-3\ln\left(\right.\right.}{}$

input

```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output

```
1/3/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a^2*(-12*
I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/
2))*a*c*tan(f*x+e)^3+9*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+
e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2-3*A*ln((a*c*tan(f*x+e)+(a*c)^(
1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3+36*I*B*ln
((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*
c*tan(f*x+e)+29*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2-
36*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1
/2))*a*c*tan(f*x+e)^2-3*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x
+e)^3-3*I*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(
a*c)^(1/2))*a*c-12*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)
+9*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1
/2))*a*c*tan(f*x+e)+8*A*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e
)^2-19*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)+12*B*ln((a*c*tan(f*x+e
)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c+45*B*(a*c)^(1
/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-4*A*(a*c)^(1/2)*(a*c*(1+tan(f*
x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(a*c)^(1/2)/(I-tan(f*x+e))^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(175) = 350.

Time = 0.10 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.23

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{\left(3\,a^2\sqrt{\frac{(A^2+8i\,AB-16\,B^2)c^5}{a^3f^2}}fe^{(3i\,fx+3i\,e)}\log\left(-\frac{4\left(2\left((i\,A-\right.\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
1/6*(3*a^2*sqrt((A^2 + 8*I*A*B - 16*B^2)*c^5/(a^3*f^2))*f*e^(3*I*f*x + 3*I
*e)*log(-4*(2*((I*A - 4*B)*c^2*e^(3*I*f*x + 3*I*e) + (I*A - 4*B)*c^2*e^(I*
f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e)
+ 1)) + (a^2*f*e^(2*I*f*x + 2*I*e) - a^2*f)*sqrt((A^2 + 8*I*A*B - 16*B^2)*
c^5/(a^3*f^2)))/((-I*A + 4*B)*c^2*e^(2*I*f*x + 2*I*e) + (-I*A + 4*B)*c^2))
 - 3*a^2*sqrt((A^2 + 8*I*A*B - 16*B^2)*c^5/(a^3*f^2))*f*e^(3*I*f*x + 3*I*e
)*log(-4*(2*((I*A - 4*B)*c^2*e^(3*I*f*x + 3*I*e) + (I*A - 4*B)*c^2*e^(I*f*
x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) +
1)) - (a^2*f*e^(2*I*f*x + 2*I*e) - a^2*f)*sqrt((A^2 + 8*I*A*B - 16*B^2)*c^
5/(a^3*f^2)))/((-I*A + 4*B)*c^2*e^(2*I*f*x + 2*I*e) + (-I*A + 4*B)*c^2)) -
 4*(3*(I*A - 4*B)*c^2*e^(4*I*f*x + 4*I*e) + 2*(I*A - 4*B)*c^2*e^(2*I*f*x +
 2*I*e) + (-I*A + B)*c^2)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I
*f*x + 2*I*e) + 1)))*e^(-3*I*f*x - 3*I*e)/(a^2*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \int \frac{(-ic(\tan(e+fx)+i))^{\frac{5}{2}}\,(A+B\tan(e+fx))}{(ia(\tan(e+fx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(
3/2),x)
```

output
```
Integral((-I*c*(tan(e + f*x) + I))**(5/2)*(A + B*tan(e + f*x))/(I*a*(tan(e
 + f*x) - I))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A + B\tan(e + fx))(c - ic\tan(e + fx))^{5/2}}{(a + ia\tan(e + fx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A + B\tan(e + fx))(c - ic\tan(e + fx))^{5/2}}{(a + ia\tan(e + fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx=\int \frac{(A+B\tan(e+f\,x))\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*
x)*1i)^(3/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*
x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*( - 6*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)*a - 4*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)*b*i - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i
+ 1)*a*i + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b + 3*in
t(( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)*
*3 - tan(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*x)**2*a*f*i - int(
( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3
 - tan(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*x)**2*b*f + 3*int((
- sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 -
 tan(e + f*x)**2*i + tan(e + f*x) - i),x)*a*f*i - int(( - sqrt(tan(e + f*x
)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 - tan(e + f*x)**2*i
 + tan(e + f*x) - i),x)*b*f + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
+ f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(
e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f*i + int((sqrt(tan(e + f*x)*i + 1)*s
qrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(tan(e + f*x)**3*i + tan(e + f
*x)**2 + tan(e + f*x)*i + 1),x)*b*f*i + int((sqrt(tan(e + f*x)*i + 1)*sqrt
( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i + tan(e + f*x)
**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*f*i - 3*int((sqrt(tan(e + f
*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i
 + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*f + int((...
```

3.838 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 157

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{2Bc^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{3/2}f} + \frac{2Bc\sqrt{c-ic\tan(e+fx)}}{af\sqrt{a+ia\tan(e+fx)}} + \frac{(iA-B)(c-ic\tan(e+fx))^{3/2}}{3f(a+ia\tan(e+fx))^{3/2}}$$

output
```
2*B*c^(3/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*x
+e))^(1/2))/a^(3/2)/f+2*B*c*(c-I*c*tan(f*x+e))^(1/2)/a/f/(a+I*a*tan(f*x+e)
)^(1/2)+1/3*(I*A-B)*(c-I*c*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 3.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.87

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{c\sqrt{c-ic\tan(e+fx)}\left(\frac{6B\arcsin\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{1-i\tan(e+fx)}} + \frac{\sqrt{a}(A\cdots}{(-i+\cdots}\right.}{3a^{3/2}f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan
[e + f*x])^(3/2),x]
```

output
```
(c*Sqrt[c - I*c*Tan[e + f*x]]*((6*B*ArcSin[Sqrt[a + I*a*Tan[e + f*x]]/(Sqr
t[2]*Sqrt[a])])/Sqrt[1 - I*Tan[e + f*x]] + (Sqrt[a]*(A - (5*I)*B + ((-I)*A
 + 7*B)*Tan[e + f*x]))/((-I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]])))/
(3*a^(3/2)*f)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 87, 57, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)a+a)^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{iB\int \frac{\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)a+a)^{3/2}}d\tan(e+fx)}{a}\right)}{f}$$

$$\Big\downarrow \text{57}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{iB\left(\frac{2i\sqrt{c-ic\tan(e+fx)}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{c\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{a}\right)}{a}\right)}{f}$$

$\Big\downarrow$ 45

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{iB\left(\frac{2i\sqrt{c-ic\tan(e+fx)}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{2c\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}}{a}\right)}{a}\right)}{f}$$

$\Big\downarrow$ 218

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{3ac(a+ia\tan(e+fx))^{3/2}}-\frac{iB\left(\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{3/2}}+\frac{2i\sqrt{c-ic\tan(e+fx)}}{a\sqrt{a+ia\tan(e+fx)}}\right)}{a}\right)}{f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan[e + f
*x])^(3/2),x]
```

output
```
(a*c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(3/2))/(3*a*c*(a + I*a*Tan[e + f*x
])^(3/2)) - (I*B*(((2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e + f*x]
])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/a^(3/2) + ((2*I)*Sqrt[c - I*c*Ta
n[e + f*x]])/(a*Sqrt[a + I*a*Tan[e + f*x]])))/a))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(128) = 256$.

Time = 0.86 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.60

method	result
derivativedivides	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c\left(-3iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^3+9iB\right.}{}$
default	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c\left(-3iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^3+9iB\right.}{}$
parts	$\frac{A\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c\left(1+\tan(fx+e)^2\right)}{3f\,a^2(i-\tan(fx+e))^3}+\frac{iB\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c\left(\right.}{}$

input

```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output

```
1/3/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^2*c*(-3*I*B
*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))
*a*c*tan(f*x+e)^3+9*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^
2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+7*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*
(a*c)^(1/2)*tan(f*x+e)^2-9*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*
x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+A*(a*c*(1+tan(f*x+e)^2))^(1/
2)*(a*c)^(1/2)*tan(f*x+e)^2-5*I*B*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)
+3*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1
/2))*a*c+12*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)+A*(a*c)^
(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c*(1+tan(f*x+e)^2))^(1/2)/(I-tan(f*
x+e))^3/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(119) = 238.

Time = 0.10 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.55

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = -\frac{\left(3\,a^2f\sqrt{-\frac{B^2c^3}{a^3f^2}}e^{(3i\,fx+3i\,e)}\log\left(\frac{4\left(2\left(Bce^{(3i\,fx+3i\,e)}+Bce^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}+\left(a^2fe^{(2i\,fx+2i\,e)}-a^2f\right)\sqrt{\cdots}\right.}{Bce^{(2i\,fx+2i\,e)}+Bc}\right.\right.}{\cdots}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
-1/6*(3*a^2*f*sqrt(-B^2*c^3/(a^3*f^2))*e^(3*I*f*x + 3*I*e)*log(4*(2*(B*c*e
^(3*I*f*x + 3*I*e) + B*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)
)*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (a^2*f*e^(2*I*f*x + 2*I*e) - a^2*f)*
sqrt(-B^2*c^3/(a^3*f^2)))/(B*c*e^(2*I*f*x + 2*I*e) + B*c)) - 3*a^2*f*sqrt(
-B^2*c^3/(a^3*f^2))*e^(3*I*f*x + 3*I*e)*log(4*(2*(B*c*e^(3*I*f*x + 3*I*e)
+ B*c*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*
x + 2*I*e) + 1)) - (a^2*f*e^(2*I*f*x + 2*I*e) - a^2*f)*sqrt(-B^2*c^3/(a^3*
f^2)))/(B*c*e^(2*I*f*x + 2*I*e) + B*c)) - 2*(6*B*c*e^(4*I*f*x + 4*I*e) - (
-I*A - 5*B)*c*e^(2*I*f*x + 2*I*e) - (-I*A + B)*c)*sqrt(a/(e^(2*I*f*x + 2*I
*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-3*I*f*x - 3*I*e)/(a^2*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(-ic(\tan{(e+fx)}+i)\right)^{\frac{3}{2}}\left(A+B\tan{(e+fx)}\right)}{\left(ia\left(\tan{(e+fx)}-i\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(
3/2),x)
```

output
```
Integral((-I*c*(tan(e + f*x) + I))**(3/2)*(A + B*tan(e + f*x))/(I*a*(tan(e
 + f*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.07

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{(6\,Bc\arctan\left(\cos\left(fx+e\right),\sin\left(fx+e\right)+1\right)+6\,Bc\,\text{a}}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
1/6*(6*B*c*arctan2(cos(f*x + e), sin(f*x + e) + 1) + 6*B*c*arctan2(cos(f*x
 + e), -sin(f*x + e) + 1) - 2*(-I*A + B)*c*cos(3*f*x + 3*e) + 12*B*c*cos(f
*x + e) + 3*I*B*c*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1
) - 3*I*B*c*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) + 2*
(A + I*B)*c*sin(3*f*x + 3*e) - 12*I*B*c*sin(f*x + e))*sqrt(c)/(a^(3/2)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \int \frac{(A+B\tan(e+f\,x))\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{3/2}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*
x)*1i)^(3/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*
x)*1i)^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x)*a - 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)*b*i - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
a*i + 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b + 2*int(( -
 sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 -
tan(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*x)**2*a*f*i - int(( - s
qrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 - ta
n(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*x)**2*b*f + 2*int(( - sqr
t(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 - tan(
e + f*x)**2*i + tan(e + f*x) - i),x)*a*f*i - int(( - sqrt(tan(e + f*x)*i +
 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 - tan(e + f*x)**2*i + ta
n(e + f*x) - i),x)*b*f - int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x
)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f
*x)*i + 1),x)*tan(e + f*x)**2*b*f - int((sqrt(tan(e + f*x)*i + 1)*sqrt( -
tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(tan(e + f*x)**3*i + tan(e + f*x)**2
+ tan(e + f*x)*i + 1),x)*b*f + 2*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan
(e + f*x)*i + 1))/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x)*i +
1),x)*tan(e + f*x)**2*a*f + 2*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
+ f*x)*i + 1))/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),
x)*tan(e + f*x)**2*b*f*i + 2*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(...
```

3.839 $\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 104

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{(iA-B)\sqrt{c-ic\tan(e+fx)}}{3f(a+ia\tan(e+fx))^{3/2}} + \frac{(iA+2B)\sqrt{c-ic\tan(e+fx)}}{3af\sqrt{a+ia\tan(e+fx)}}$$

output
```
1/3*(I*A-B)*(c-I*c*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^(3/2)+1/3*(I*A+2
*B)*(c-I*c*tan(f*x+e))^(1/2)/a/f/(a+I*a*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{(2A-iB+(iA+2B)\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{3af(-i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}}$$

input
```
Integrate[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e
 + f*x])^(3/2),x]
```

output
```
((2*A - I*B + (I*A + 2*B)*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(3*a*f
*(-I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {3042, 4071, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{5/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(A-2iB)\int \frac{1}{(i\tan(e+fx)a+a)^{3/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{3a}+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{3ac(a+ia\tan(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(\frac{i(A-2iB)\sqrt{c-ic\tan(e+fx)}}{3a^2c\sqrt{a+ia\tan(e+fx)}}+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{3ac(a+ia\tan(e+fx))^{3/2}}\right)}{f}$$

input
```
Int[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e + f*x
])^(3/2),x]
```

output
```
(a*c*(((I*A - B)*Sqrt[c - I*c*Tan[e + f*x]])/(3*a*c*(a + I*a*Tan[e + f*x])
^(3/2)) + ((I/3)*(A - (2*I)*B)*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*c*Sqrt[a +
 I*a*Tan[e + f*x]])))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,\left(2iB\tan(fx+e)^2+3i\tan(fx+e)A-A\tan(fx+e)^2-iB+3B\tan(fx+e)+2\right.}{3f\,a^2(i-\tan(fx+e))^3}$
default	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,\left(2iB\tan(fx+e)^2+3i\tan(fx+e)A-A\tan(fx+e)^2-iB+3B\tan(fx+e)+2\right.}{3f\,a^2(i-\tan(fx+e))^3}$
parts	$\frac{A\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,\left(3i\tan(fx+e)-\tan(fx+e)^2+2\right)}{3f\,a^2(i-\tan(fx+e))^3}-\frac{iB\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i}}{3f\,a^2}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/3/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^2*(2*I*B*ta
n(f*x+e)^2+3*I*A*tan(f*x+e)-A*tan(f*x+e)^2-I*B+3*B*tan(f*x+e)+2*A)/(I-tan(
f*x+e))^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx = -\frac{\left(3\,(-i\,A-B)e^{(4i\,fx+4i\,e)}+2\,(-2i\,A-B)e^{(2i\,fx+2i\,e)}-i\,A+B\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}e^{(-3i\,fx-3i\,e)}}{6\,a^2f}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
-1/6*(3*(-I*A - B)*e^(4*I*f*x + 4*I*e) + 2*(-2*I*A - B)*e^(2*I*f*x + 2*I*e
) - I*A + B)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e)
 + 1))*e^(-3*I*f*x - 3*I*e)/(a^2*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{-ic\,(\tan{(e+fx)}+i)}(A+B\tan{(e+fx)})}{(ia\,(\tan{(e+fx)}-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(
3/2),x)
```

output
```
Integral(sqrt(-I*c*(tan(e + f*x) + I))*(A + B*tan(e + f*x))/(I*a*(tan(e +
f*x) - I))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.88

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+}{\cos(2\,e+2\,f\,x)+1}}}{}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(1/2))/(a + a*tan(e + f*
x)*1i)^(3/2),x)
```

output
```
(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
 + 1))^(1/2)*(A*3i + 3*B + A*cos(2*e + 2*f*x)*4i + A*cos(4*e + 4*f*x)*1i +
 2*B*cos(2*e + 2*f*x) - B*cos(4*e + 4*f*x) + 4*A*sin(2*e + 2*f*x) + A*sin(
4*e + 4*f*x) - B*sin(2*e + 2*f*x)*2i + B*sin(4*e + 4*f*x)*1i))/(12*a^2*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,\left(-\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\ \mathrm{ta}\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(3/2),x)
```

output

```
(sqrt(c)*sqrt(a)*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)*a - 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*ta
n(e + f*x)*b*i + sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*b -
2*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)
**3 - tan(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*x)**2*a*f*i + 2*i
nt((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3
 - tan(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*x)**2*b*f - 2*int((s
qrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 - ta
n(e + f*x)**2*i + tan(e + f*x) - i),x)*a*f*i + 2*int((sqrt(tan(e + f*x)*i
+ 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 - tan(e + f*x)**2*i + t
an(e + f*x) - i),x)*b*f))/(a**2*f*(tan(e + f*x)**2 + 1))
```

3.840 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 152

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{iA+B}{f(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}} + \frac{(2iA+B)\sqrt{c-ic\tan(e+fx)}}{3cf(a+ia\tan(e+fx))^{3/2}} + \frac{(2iA+B)\sqrt{c-ic\tan(e+fx)}}{3acf\sqrt{a+ia\tan(e+fx)}}$$

output
```
-(I*A+B)/f/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/2)+1/3*(2*I*A+B)
*(c-I*c*tan(f*x+e))^(1/2)/c/f/(a+I*a*tan(f*x+e))^(3/2)+1/3*(2*I*A+B)*(c-I*
c*tan(f*x+e))^(1/2)/a/c/f/(a+I*a*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{A+iB+(-2iA-B)\tan(e+fx)+(2A-iB)\tan}{3af(-i+\tan(e+fx))\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic}}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*
Tan[e + f*x]]),x]
```

output
```
(A + I*B + ((-2*I)*A - B)*Tan[e + f*x] + (2*A - I*B)*Tan[e + f*x]^2)/(3*a*
f*(-I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]
])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4071, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{5/2}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(2A-iB)\int \frac{1}{(i\tan(e+fx)a+a)^{5/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{B+iA}{ac(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{(2A-iB)\left(\frac{\int \frac{1}{(i\tan(e+fx)a+a)^{3/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{3a}+\frac{i\sqrt{c-ic\tan(e+fx)}}{3ac(a+ia\tan(e+fx))^{3/2}}\right)}{c}-\frac{B+iA}{ac(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(\frac{(2A-iB)\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{3a^2c\sqrt{a+ia\tan(e+fx)}}+\frac{i\sqrt{c-ic\tan(e+fx)}}{3ac(a+ia\tan(e+fx))^{3/2}}\right)}{c}-\frac{B+iA}{ac(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e
+ f*x]]),x]
```

output
```
(a*c*(-((I*A + B)/(a*c*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[e + f
*x]])) + ((2*A - I*B)*(((I/3)*Sqrt[c - I*c*Tan[e + f*x]])/(a*c*(a + I*a*Ta
n[e + f*x])^(3/2)) + ((I/3)*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*c*Sqrt[a + I*
a*Tan[e + f*x]])))/c))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(2iA\tan(fx+e)^4-iB\tan(fx+e)^3+B\tan(fx+e)^4+3iA\tan(fx+e)^2+2A\right.}{3f\,a^2c(i+\tan(fx+e))^2(i-\tan(fx+e))^3}$
default	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(2iA\tan(fx+e)^4-iB\tan(fx+e)^3+B\tan(fx+e)^4+3iA\tan(fx+e)^2+2A\right.}{3f\,a^2c(i+\tan(fx+e))^2(i-\tan(fx+e))^3}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(2i\tan(fx+e)^3-2\tan(fx+e)^4+2i\tan(fx+e)-3\tan(fx+e)^2-1\right)}{3f\,a^2c(i-\tan(fx+e))^3(i+\tan(fx+e))^2}-$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/3*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^2/c*(2*I*
A*tan(f*x+e)^4-I*B*tan(f*x+e)^3+B*tan(f*x+e)^4+3*I*A*tan(f*x+e)^2+2*A*tan(
f*x+e)^3-I*B*tan(f*x+e)+I*A+2*A*tan(f*x+e)-B)/(I+tan(f*x+e))^2/(I-tan(f*x+
e))^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx =$$
$$-\frac{\left(3\,(i\,A+B)e^{(6i\,fx+6i\,e)}+4\,(i\,A-B)e^{(5i\,fx+5i\,e)}+3\,(-i\,A+B)e^{(4i\,fx+4i\,e)}+4\,(i\,A-B)e^{(3i\,fx+3i\,e)}-(7i\,A}{12\,a^2cf}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
-1/12*(3*(I*A + B)*e^(6*I*f*x + 6*I*e) + 4*(I*A - B)*e^(5*I*f*x + 5*I*e) +
 3*(-I*A + B)*e^(4*I*f*x + 4*I*e) + 4*(I*A - B)*e^(3*I*f*x + 3*I*e) - (7*I
*A - B)*e^(2*I*f*x + 2*I*e) - I*A + B)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*s
qrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-3*I*f*x - 3*I*e)/(a^2*c*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \int \frac{A+B\tan\left(e+fx\right)}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}}\,d$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(3/2)/(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Integral((A + B*tan(e + f*x))/((I*a*(tan(e + f*x) - I))**(3/2)*sqrt(-I*c*(
tan(e + f*x) + I))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \int \frac{B\tan(fx+e)+A}{(i\,a\tan(fx+e)+a)^{\frac{3}{2}}\sqrt{-i\,c\tan(fx+e)+c}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)/((I*a*tan(f*x + e) + a)^(3/2)*sqrt(-I*c*tan
(f*x + e) + c)), x)
```

Mupad [B] (verification not implemented)

Time = 6.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1i)}{\cos(2\,e+2\,f\,x)+1}}\,(6\,A\,\sin(2\,e+2\,f\,x) -$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x
)*1i)^(1/2)),x)
```

output
```
(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*(A*cos(2*e + 2*f*x)*6i - 3*B - A*3i + A*cos(4*e + 4*f*x)*1i - B*cos
(4*e + 4*f*x) + 6*A*sin(2*e + 2*f*x) + A*sin(4*e + 4*f*x) + B*sin(4*e + 4*
f*x)*1i))/(12*a^2*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos
(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(-\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+1}\right.}{}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*( - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)*b*i - int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
1))/(tan(e + f*x)**3 - tan(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*
x)**2*a*f*i + int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(
tan(e + f*x)**3 - tan(e + f*x)**2*i + tan(e + f*x) - i),x)*tan(e + f*x)**2
*b*f - int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e +
 f*x)**3 - tan(e + f*x)**2*i + tan(e + f*x) - i),x)*a*f*i + int((sqrt(tan(
e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1))/(tan(e + f*x)**3 - tan(e + f*
x)**2*i + tan(e + f*x) - i),x)*b*f))/(a**2*c*f*(tan(e + f*x)**2 + 1))
```

3.841 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 152

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{-iA-B}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}} + \frac{iA}{3cf(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}} + \frac{2A\tan(e+fx)}{3acf\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}$$

output
```
1/3*(-I*A-B)/f/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2)+1/3*I*A/c
/f/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/2)+2/3*A*tan(f*x+e)/a/c/
f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 4.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.52

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\cos^2(e+fx)\left(-B+3A\tan(e+fx)+2A\tan^3(e+\right.}{3acf\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[
e + f*x])^(3/2)),x]
```

output
```
(Cos[e + f*x]^2*(-B + 3*A*Tan[e + f*x] + 2*A*Tan[e + f*x]^3))/(3*a*c*f*Sqr
t[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4071, 87, 55, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{5/2}(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{A\int \frac{1}{(i\tan(e+fx)a+a)^{5/2}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{c}-\frac{B+iA}{3ac(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{A\left(\frac{2\int \frac{1}{(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3a}+\frac{i}{3ac(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{B+iA}{3ac(a+ia\tan(e+fx))^{3/2}(c-ic\tan$$

$$\Big\downarrow \ 41$$

$$\frac{ac\left(\frac{A\left(\frac{2\tan(e+fx)}{3a^2c\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{i}{3ac(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\right)}{c}-\frac{B+iA}{3ac(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*
x])^(3/2)),x]
```

output
```
(a*c*(-1/3*(I*A + B)/(a*c*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*
x])^(3/2)) + (A*((I/3)/(a*c*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c - I*c*Tan[
e + f*x]]) + (2*Tan[e + f*x])/(3*a^2*c*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c -
 I*c*Tan[e + f*x]])))/c))/f
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(2A\tan(fx+e)^5+5A\tan(fx+e)^3-B\tan(fx+e)^2+3A\tan(fx+e)-B\right)}{3f\,a^2c^2(i+\tan(fx+e))^3(i-\tan(fx+e))^3}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(2A\tan(fx+e)^5+5A\tan(fx+e)^3-B\tan(fx+e)^2+3A\tan(fx+e)-B\right)}{3f\,a^2c^2(i+\tan(fx+e))^3(i-\tan(fx+e))^3}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(1+\tan(fx+e)^2\right)\tan(fx+e)\left(2\tan(fx+e)^2+3\right)}{3f\,a^2c^2(i+\tan(fx+e))^3(i-\tan(fx+e))^3}+\frac{B\left(1+\tan(fx+e\right.}{3f\,c^2}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/3/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^2/c^2*(2*A
*tan(f*x+e)^5+5*A*tan(f*x+e)^3-B*tan(f*x+e)^2+3*A*tan(f*x+e)-B)/(I+tan(f*x
+e))^3/(I-tan(f*x+e))^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{((-iA-B)e^{(8i\,fx+8i\,e)}-2\,(5i\,A+2\,B)e^{(6i\,fx+6i\,e)}-$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
1/24*((-I*A - B)*e^(8*I*f*x + 8*I*e) - 2*(5*I*A + 2*B)*e^(6*I*f*x + 6*I*e)
 + 8*B*e^(5*I*f*x + 5*I*e) - 6*B*e^(4*I*f*x + 4*I*e) + 8*B*e^(3*I*f*x + 3*
I*e) - 2*(-5*I*A + 2*B)*e^(2*I*f*x + 2*I*e) + I*A - B)*sqrt(a/(e^(2*I*f*x
+ 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-3*I*f*x - 3*I*e)/(a^2
*c^2*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \int \frac{A+B\tan(e+fx)}{(ia(\tan(e+fx)-i))^{\frac{3}{2}}(-ic(\tan(e+fx)+i))}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(3/2)/(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Integral((A + B*tan(e + f*x))/((I*a*(tan(e + f*x) - I))**(3/2)*(-I*c*(tan(
e + f*x) + I))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.32

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{(3\,(3i\,A-B)\cos\left(2\,fx+2\,e\right)-3\,(3\,A+i\,B)\sin\left(2\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
1/24*((3*(3*I*A - B)*cos(2*f*x + 2*e) - 3*(3*A + I*B)*sin(2*f*x + 2*e) - 2
*B)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(-3*I*A - B)*
cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*(3*A + I*B)*cos(
2*f*x + 2*e) + 3*(3*I*A - B)*sin(2*f*x + 2*e) + 2*A)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(3*A - I*B)*sin(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))))/(a^(3/2)*c^(3/2)*f)
```

Giac [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \int \frac{B\tan\left(fx+e\right)+A}{\left(i\,a\tan\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,d$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)/((I*a*tan(f*x + e) + a)^(3/2)*(-I*c*tan(f*x
 + e) + c)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.30

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(10\,A\,\sin\left(2\,e+2\,f\right.}{}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x
)*1i)^(3/2)),x)
```

output
```
(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*(A*cos(2*e + 2*f*x)*8i - 3*B - A*9i + A*cos(4*e + 4*f*x)*1i - 4*B*c
os(2*e + 2*f*x) - B*cos(4*e + 4*f*x) + 10*A*sin(2*e + 2*f*x) + A*sin(4*e +
 4*f*x) + B*sin(2*e + 2*f*x)*2i + B*sin(4*e + 4*f*x)*1i))/(24*a^2*c*f*((c*
(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.55

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,\sqrt{\tan\left(fx+e\right)i+1}\,\sqrt{-\tan\left(fx+e\right)i+1}\,(}{3a^2c^2f\left(\tan\left(fx+e\right)^4+2\,\mathrm{ta}\right.}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*(2*t
an(e + f*x)**3*a + 3*tan(e + f*x)*a - b))/(3*a**2*c**2*f*(tan(e + f*x)**4
+ 2*tan(e + f*x)**2 + 1))
```

3.842 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 269

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{iA-B}{3f(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}} + \frac{4iA-B}{3af\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}} - \frac{(4iA-B)\sqrt{a+ia\tan(e+fx)}}{5a^2f(c-ic\tan(e+fx))^{5/2}} - \frac{2(4iA-B)\sqrt{a+ia\tan(e+fx)}}{15a^2cf(c-ic\tan(e+fx))^{3/2}} - \frac{2(4iA-B)\sqrt{a+ia\tan(e+fx)}}{15a^2c^2f\sqrt{c-ic\tan(e+fx)}}$$

output

```
1/3*(I*A-B)/f/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2)+1/3*(4*I*A
-B)/a/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(5/2)-1/5*(4*I*A-B)*(a
+I*a*tan(f*x+e))^(1/2)/a^2/f/(c-I*c*tan(f*x+e))^(5/2)-2/15*(4*I*A-B)*(a+I*
a*tan(f*x+e))^(1/2)/a^2/c/f/(c-I*c*tan(f*x+e))^(3/2)-2/15*(4*I*A-B)*(a+I*a
*tan(f*x+e))^(1/2)/a^2/c^2/f/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 6.70 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{3(A-iB)+(12iA-3B)\tan(e+fx)+3(4A+iB\cdots}{15ac^2f(-i+\tan(e+fx))(i+\tan\cdots}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[
e + f*x])^(5/2)),x]
```

output
```
(3*(A - I*B) + ((12*I)*A - 3*B)*Tan[e + f*x] + 3*(4*A + I*B)*Tan[e + f*x]^
2 + ((8*I)*A - 2*B)*Tan[e + f*x]^3 + (8*A + (2*I)*B)*Tan[e + f*x]^4)/(15*a
*c^2*f*(-I + Tan[e + f*x])*(I + Tan[e + f*x])^2*Sqrt[a + I*a*Tan[e + f*x]]
*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 87, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{5/2}(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{ac\left(\frac{(4A+iB)\int \frac{1}{(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{3a}+\frac{-B+iA}{3ac(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$\big\downarrow$ 55

$$\frac{ac\left(\frac{(4A+iB)\left(\frac{3\int \frac{1}{\sqrt{i\tan(e+fx)a+a}(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{a}+\frac{i}{ac\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\right)}{3a}+\frac{-B+iA}{3ac(a+ia\tan(e+fx))^{3/2}(c-ic

$$\Big\downarrow \; 48$$

$$\frac{ac\left(\frac{(4A+iB)\left(\frac{3\left(\frac{2\left(-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac^2\sqrt{c-ic\tan(e+fx)}}-\frac{i\sqrt{a+ia\tan(e+fx)}}{3ac(c-ic\tan(e+fx))^{3/2}}\right)}{5c}-\frac{i\sqrt{a+ia\tan(e+fx)}}{5ac(c-ic\tan(e+fx))^{5/2}}\right)}{a}+\frac{i}{ac\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{5/2}}\right)}{3a}+\frac{}{3}\right.}{f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*
x])^(5/2)),x]
```

output
```
(a*c*((I*A - B)/(3*a*c*(a + I*a*Tan[e + f*x])^(3/2)*(c - I*c*Tan[e + f*x])
^(5/2)) + ((4*A + I*B)*(I/(a*c*Sqrt[a + I*a*Tan[e + f*x]]*(c - I*c*Tan[e +
 f*x])^(5/2)) + (3*(((-1/5*I)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*(c - I*c*Ta
n[e + f*x])^(5/2)) + (2*(((-1/3*I)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c*(c - I
*c*Tan[e + f*x])^(3/2)) - ((I/3)*Sqrt[a + I*a*Tan[e + f*x]])/(a*c^2*Sqrt[c
 - I*c*Tan[e + f*x]])))/(5*c))/a))/(3*a)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(8iA\tan(fx+e)^6-2iB\tan(fx+e)^5-2B\tan(fx+e)^6+20iA\tan(fx+e)^4\right.}{15f\,a^2c^3(i+}$
default	$\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(8iA\tan(fx+e)^6-2iB\tan(fx+e)^5-2B\tan(fx+e)^6+20iA\tan(fx+e)^4\right.}{15f\,a^2c^3(i+}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(8i\tan(fx+e)^5+8\tan(fx+e)^6+20i\tan(fx+e)^3+20\tan(fx+e)^4+12\right.}{15f\,a^2c^3(i-\tan(fx+e))^3(i+\tan(fx+e))^4}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/15*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^2/c^3*(8
*I*A*tan(f*x+e)^6-2*I*B*tan(f*x+e)^5-2*B*tan(f*x+e)^6+20*I*A*tan(f*x+e)^4-
8*A*tan(f*x+e)^5-5*I*B*tan(f*x+e)^3-5*B*tan(f*x+e)^4+15*I*A*tan(f*x+e)^2-2
0*A*tan(f*x+e)^3-3*I*B*tan(f*x+e)+3*I*A-12*A*tan(f*x+e)+3*B)/(I+tan(f*x+e)
)^4/(I-tan(f*x+e))^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.67

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx = -\frac{\left(3\,(i\,A+B)e^{(10i\,fx+10i\,e)}-(-23i\,A-13\,B)e^{(8i\,fx+8i\,e)}+10\,(11i\,A+B)e^{(6i\,fx+6i\,e)}+48\,(-i\,A-B)e^{(5i\,fx}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/240*(3*(I*A + B)*e^(10*I*f*x + 10*I*e) - (-23*I*A - 13*B)*e^(8*I*f*x +
8*I*e) + 10*(11*I*A + B)*e^(6*I*f*x + 6*I*e) + 48*(-I*A - B)*e^(5*I*f*x +
5*I*e) + 30*(I*A + B)*e^(4*I*f*x + 4*I*e) + 48*(-I*A - B)*e^(3*I*f*x + 3*I
*e) + 5*(-13*I*A + 7*B)*e^(2*I*f*x + 2*I*e) - 5*I*A + 5*B)*sqrt(a/(e^(2*I*
f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-3*I*f*x - 3*I*e)/
(a^2*c^3*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \int \frac{A+B\tan\left(e+fx\right)}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{3}{2}}\left(-ic\left(\tan\left(e+fx\right)+i\right)\right)}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(3/2)/(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Integral((A + B*tan(e + f*x))/((I*a*(tan(e + f*x) - I))**(3/2)*(-I*c*(tan(
e + f*x) + I))**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \int \frac{B\tan\left(fx+e\right)+A}{\left(i\,a\tan\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,d$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)/((I*a*tan(f*x + e) + a)^(3/2)*(-I*c*tan(f*x
 + e) + c)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.73

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(40\,A\,\sin\,(2\,e+2\,f}{}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(3/2)*(c - c*tan(e + f*x
)*1i)^(5/2)),x)
```

output
```
(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*(A*cos(2*e + 2*f*x)*20i - A*45i + A*cos(4*e + 4*f*x)*1i - 20*B*cos(
2*e + 2*f*x) - 4*B*cos(4*e + 4*f*x) + 40*A*sin(2*e + 2*f*x) + 4*A*sin(4*e
+ 4*f*x) + B*sin(2*e + 2*f*x)*10i + B*sin(4*e + 4*f*x)*1i))/(120*a^2*c^2*f
*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2))
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*t
an(e + f*x)**3*a + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)*a - 3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
1)*tan(e + f*x))/(tan(e + f*x)**5*i - tan(e + f*x)**4 + 2*tan(e + f*x)**3*
i - 2*tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**4*a*f*i - 3*i
nt((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(ta
n(e + f*x)**5*i - tan(e + f*x)**4 + 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**
2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**4*b*f - 6*int((sqrt(tan(e + f*x)*
i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**5*i - tan(
e + f*x)**4 + 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 + tan(e + f*x)*i - 1
),x)*tan(e + f*x)**2*a*f*i - 6*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**5*i - tan(e + f*x)**4 + 2*tan(
e + f*x)**3*i - 2*tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)*tan(e + f*x)**2
*b*f - 3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
 f*x))/(tan(e + f*x)**5*i - tan(e + f*x)**4 + 2*tan(e + f*x)**3*i - 2*tan(
e + f*x)**2 + tan(e + f*x)*i - 1),x)*a*f*i - 3*int((sqrt(tan(e + f*x)*i +
1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**5*i - tan(e +
f*x)**4 + 2*tan(e + f*x)**3*i - 2*tan(e + f*x)**2 + tan(e + f*x)*i - 1),x)
*b*f))/(3*a**2*c**3*f*(tan(e + f*x)**4 + 2*tan(e + f*x)**2 + 1))
```

3.843 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 343

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{7(2iA-7B)c^{9/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{5/2}f} + \frac{7(2iA-7B)c^4\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{2a^3f} + \frac{7(2iA-7B)c^3\sqrt{a+ia\tan(e+fx)}(c-ic\tan(e+fx))^{3/2}}{6a^3f} + \frac{14(2iA-7B)c^2(c-ic\tan(e+fx))^{5/2}}{15a^2f\sqrt{a+ia\tan(e+fx)}} - \frac{2(2iA-7B)c(c-ic\tan(e+fx))^{7/2}}{15af(a+ia\tan(e+fx))^{3/2}} + \frac{(iA-B)(c-ic\tan(e+fx))^{9/2}}{5f(a+ia\tan(e+fx))^{5/2}}$$

output
```
7*(2*I*A-7*B)*c^(9/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I
*c*tan(f*x+e))^(1/2))/a^(5/2)/f+7/2*(2*I*A-7*B)*c^4*(a+I*a*tan(f*x+e))^(1/
2)*(c-I*c*tan(f*x+e))^(1/2)/a^3/f+7/6*(2*I*A-7*B)*c^3*(a+I*a*tan(f*x+e))^(
1/2)*(c-I*c*tan(f*x+e))^(3/2)/a^3/f+14/15*(2*I*A-7*B)*c^2*(c-I*c*tan(f*x+e
))^(5/2)/a^2/f/(a+I*a*tan(f*x+e))^(1/2)-2/15*(2*I*A-7*B)*c*(c-I*c*tan(f*x+
e))^(7/2)/a/f/(a+I*a*tan(f*x+e))^(3/2)+1/5*(I*A-B)*(c-I*c*tan(f*x+e))^(9/2
)/f/(a+I*a*tan(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 15.64 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.72

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{2}c^4e^{-4i(e+fx)}\sqrt{\frac{c}{1+e^{2i(e+fx)}}}\left(-2iA\left(6-8e^{2i(e+fx)}+56e^{4i(e+fx)}+175e^{6i(e+fx)}+105e^{8i(e+fx)}\right)+B\left(12-5\right.\right.}{15a^2\left(1+e^{2i(e+\right.}}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(9/2))/(a + I*a*Tan
[e + f*x])^(5/2),x]
```

output
```
-1/15*(Sqrt[2]*c^4*Sqrt[c/(1 + E^((2*I)*(e + f*x)))]*((-2*I)*A*(6 - 8*E^((
2*I)*(e + f*x)) + 56*E^((4*I)*(e + f*x)) + 175*E^((6*I)*(e + f*x)) + 105*E
^((8*I)*(e + f*x))) + B*(12 - 56*E^((2*I)*(e + f*x)) + 392*E^((4*I)*(e + f
*x)) + 1225*E^((6*I)*(e + f*x)) + 735*E^((8*I)*(e + f*x))) + 105*((-2*I)*A
 + 7*B)*E^((5*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^2*ArcTan[E^(I*(e + f
*x))]))/(a^2*E^((4*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^2*f*Sqrt[a + I*
a*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4071, 87, 57, 57, 60, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{9/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{9/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(i\tan(e+fx)a+a)^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(2A+7iB)\int \frac{(c-ic\tan(e+fx))^{7/2}}{(i\tan(e+fx)a+a)^{5/2}}d\tan(e+fx)}{5a}\right)}{f}$$

$$\Big\downarrow \text{57}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(2A+7iB)\left(\frac{2i(c-ic\tan(e+fx))^{7/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{7c\int \frac{(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)a+a)^{3/2}}d\tan(e+fx)}{3a}\right)}{5a}\right)}{f}$$

$$\Big\downarrow \text{57}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(2A+7iB)\left(\frac{2i(c-ic\tan(e+fx))^{7/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{7c\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{5c\int \frac{(c-ic\tan(e+fx))^{3/2}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)}{a}\right)}{3a}\right)}{5a}\right)}{f}$$

$$\Big\downarrow \text{60}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(2A+7iB)\left(\frac{2i(c-ic\tan(e+fx))^{7/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{7c\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{5c\left(\frac{3}{2}c\int\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)-\frac{i\sqrt{\cdots}}{\cdots}\right)}{a}\cdots\right)}{3a}\cdots\right)}{5a}\cdots\right)}{f}$$

$\downarrow$ 60

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(2A+7iB)\left(\frac{2i(c-ic\tan(e+fx))^{7/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{7c\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{5c\left(\frac{3}{2}c\left(c\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+\cdots}}\cdots\right)\cdots\right)}{\cdots}\right)}{\cdots}\right)}{5a}\cdots\right)}{f}$$

$\downarrow$ 45

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(2A+7iB)\left(\frac{2i(c-ic\tan(e+fx))^{7/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{7c\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{5c\left(\frac{3}{2}c\left(2c\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan\cdots}}{\sqrt{c-\cdots}}\right.\right.}{\cdots}\right.}{\cdots}\right.}{5a\cdots}\right.}{f\cdots}$$

$\downarrow$ 218

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{9/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(2A+7iB)\left(\frac{2i(c-ic\tan(e+fx))^{7/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{7c\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{5c\left(\frac{3}{2}c\left(-\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx\cdots}}{\sqrt{a}\sqrt{c-ic\tan(e+fx\cdots}}\right)}{\sqrt{a}}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{5a\cdots}\right.}{f\cdots}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(9/2))/(a + I*a*Tan[e + f
*x])^(5/2),x]
```

output
```
(a*c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(9/2))/(5*a*c*(a + I*a*Tan[e + f*x
])^(5/2)) - ((2*A + (7*I)*B)*((((2*I)/3)*(c - I*c*Tan[e + f*x])^(7/2))/(a*
(a + I*a*Tan[e + f*x])^(3/2)) - (7*c*(((2*I)*(c - I*c*Tan[e + f*x])^(5/2))
/(a*Sqrt[a + I*a*Tan[e + f*x]]) - (5*c*(((-1/2*I)*Sqrt[a + I*a*Tan[e + f*x
]]*(c - I*c*Tan[e + f*x])^(3/2))/a + (3*c*(((-2*I)*Sqrt[c]*ArcTan[(Sqrt[c]
*Sqrt[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[a
] - (I*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/a))/2))/a))/
(3*a)))/(5*a)))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(283) = 566$.

Time = 0.77 (sec) , antiderivative size = 899, normalized size of antiderivative = 2.62

method	result	size
derivativedivides	Expression too large to display	899
default	Expression too large to display	899
parts	Expression too large to display	957

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/30/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^4/a^3*(201
4*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3+840*I*A*ln((a*
c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*ta
n(f*x+e)^3+30*I*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^4-73
5*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(
1/2))*a*c*tan(f*x+e)^4-210*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*
x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^4-3881*I*B*(a*c)^(1/2)*(a*c*(1
+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-735*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a
*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-2940*B*ln((a*c*tan(f*x+e)+(a*
c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3-150*B
*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4+15*I*B*(a*c)^(1/2)*
(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^5-1316*I*A*(a*c)^(1/2)*(a*c*(1+tan
(f*x+e)^2))^(1/2)*tan(f*x+e)^2+1260*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*
(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+584*A*(a*c*(1+tan(f
*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+334*I*A*(a*c)^(1/2)*(a*c*(1+tan(f
*x+e)^2))^(1/2)+4410*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)
^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+2940*B*ln((a*c*tan(f*x+e)+(a*c)^
(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+4576*B*(a*
c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2-840*I*A*ln((a*c*tan(f*x
+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(263) = 526$.

Time = 0.11 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.78

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/60*(105*(a^3*f*e^(7*I*f*x + 7*I*e) + a^3*f*e^(5*I*f*x + 5*I*e))*sqrt((4
*A^2 + 28*I*A*B - 49*B^2)*c^9/(a^5*f^2))*log(4*(2*((2*I*A - 7*B)*c^4*e^(3*
I*f*x + 3*I*e) + (2*I*A - 7*B)*c^4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2
*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (a^3*f*e^(2*I*f*x + 2*I*e)
 - a^3*f)*sqrt((4*A^2 + 28*I*A*B - 49*B^2)*c^9/(a^5*f^2)))/((2*I*A - 7*B)*
c^4*e^(2*I*f*x + 2*I*e) + (2*I*A - 7*B)*c^4)) - 105*(a^3*f*e^(7*I*f*x + 7*
I*e) + a^3*f*e^(5*I*f*x + 5*I*e))*sqrt((4*A^2 + 28*I*A*B - 49*B^2)*c^9/(a^
5*f^2))*log(4*(2*((2*I*A - 7*B)*c^4*e^(3*I*f*x + 3*I*e) + (2*I*A - 7*B)*c^
4*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x +
2*I*e) + 1)) - (a^3*f*e^(2*I*f*x + 2*I*e) - a^3*f)*sqrt((4*A^2 + 28*I*A*B
- 49*B^2)*c^9/(a^5*f^2)))/((2*I*A - 7*B)*c^4*e^(2*I*f*x + 2*I*e) + (2*I*A
- 7*B)*c^4)) + 4*(105*(-2*I*A + 7*B)*c^4*e^(8*I*f*x + 8*I*e) + 175*(-2*I*A
 + 7*B)*c^4*e^(6*I*f*x + 6*I*e) + 56*(-2*I*A + 7*B)*c^4*e^(4*I*f*x + 4*I*e
) + 8*(2*I*A - 7*B)*c^4*e^(2*I*f*x + 2*I*e) + 12*(-I*A + B)*c^4)*sqrt(a/(e
^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))/(a^3*f*e^(7*I*
f*x + 7*I*e) + a^3*f*e^(5*I*f*x + 5*I*e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(9/2)/(a+I*a*tan(f*x+e))**(
5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \int \frac{(A+B\tan(e+f\,x))\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{9/2}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(9/2))/(a + a*tan(e + f*
x)*1i)^(5/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(9/2))/(a + a*tan(e + f*
x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{9/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(9/2)/(a+I*a*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*c**4*( - int(sqrt( - tan(e + f*x)*i + 1)/(sqrt(tan(e + f*x)*i + 1
)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e
 + f*x)*i + 1)),x)*a - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**5)/(
sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(
e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*b - int((sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**4)/(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(t
an(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*a - 4*int
((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(sqrt(tan(e + f*x)*i + 1)*t
an(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e +
f*x)*i + 1)),x)*b*i - 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/
(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan
(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*a*i + 6*int((sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**3)/(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*s
qrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*b +
6*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(sqrt(tan(e + f*x)*i +
 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan
(e + f*x)*i + 1)),x)*a + 4*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**
2)/(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*
tan(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*b*i + 4*int((sqrt( - tan(e +
 f*x)*i + 1)*tan(e + f*x))/(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - ...
```

3.844 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 284

$$\begin{aligned}\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx &= \frac{2(iA-6B)c^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{5/2}f} \\ &+ \frac{(iA-6B)c^3\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}{a^3f} \\ &+ \frac{2(iA-6B)c^2(c-ic\tan(e+fx))^{3/2}}{3a^2f\sqrt{a+ia\tan(e+fx)}} \\ &- \frac{2(iA-6B)c(c-ic\tan(e+fx))^{5/2}}{15af(a+ia\tan(e+fx))^{3/2}} + \frac{(iA-B)(c-ic\tan(e+fx))^{7/2}}{5f(a+ia\tan(e+fx))^{5/2}}\end{aligned}$$

output
```
2*(I*A-6*B)*c^(7/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c
*tan(f*x+e))^(1/2))/a^(5/2)/f+(I*A-6*B)*c^3*(a+I*a*tan(f*x+e))^(1/2)*(c-I*
c*tan(f*x+e))^(1/2)/a^3/f+2/3*(I*A-6*B)*c^2*(c-I*c*tan(f*x+e))^(3/2)/a^2/f
/(a+I*a*tan(f*x+e))^(1/2)-2/15*(I*A-6*B)*c*(c-I*c*tan(f*x+e))^(5/2)/a/f/(a
+I*a*tan(f*x+e))^(3/2)+1/5*(I*A-B)*(c-I*c*tan(f*x+e))^(7/2)/f/(a+I*a*tan(f
*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 14.91 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.72

$$\int \frac{(A + B\tan(e + fx))(c - ic\tan(e + fx))^{7/2}}{(a + ia\tan(e + fx))^{5/2}}\,dx = \frac{2\sqrt{2}c^2e^{-4i(e+fx)}\left(\frac{c}{1+e^{2i(e+fx)}}\right)^{3/2}\left(iA\left(3 - 2e^{2i(e+fx)} + 1\right.\right.}{}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2))/(a + I*a*Tan
[e + f*x])^(5/2),x]
```

output
```
(2*Sqrt[2]*c^2*(c/(1 + E^((2*I)*(e + f*x))))^(3/2)*(I*A*(3 - 2*E^((2*I)*(e
 + f*x)) + 10*E^((4*I)*(e + f*x)) + 15*E^((6*I)*(e + f*x))) - 3*B*(1 - 4*E
^((2*I)*(e + f*x)) + 20*E^((4*I)*(e + f*x)) + 30*E^((6*I)*(e + f*x))) + (1
5*I)*(A + (6*I)*B)*E^((5*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))*ArcTan[E^
(I*(e + f*x))]))/(15*a^2*E^((4*I)*(e + f*x))*f*Sqrt[a + I*a*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.178$, Rules used = {3042, 4071, 87, 57, 57, 60, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - ic\tan(e + fx))^{7/2}(A + B\tan(e + fx))}{(a + ia\tan(e + fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c - ic\tan(e + fx))^{7/2}(A + B\tan(e + fx))}{(a + ia\tan(e + fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)a+a)^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(A+6iB)\int\frac{(c-ic\tan(e+fx))^{5/2}}{(i\tan(e+fx)a+a)^{5/2}}d\tan(e+fx)}{5a}\right)}{f}$$

$\downarrow$ 57

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(A+6iB)\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{5c\int\frac{(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)a+a)^{3/2}}d\tan(e+fx)}{3a}\right)}{5a}\right)}{f}$$

$\downarrow$ 57

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(A+6iB)\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{5c\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{3c\int\frac{\sqrt{c-ic\tan(e+fx)}}{\sqrt{i\tan(e+fx)a+a}}d\tan(e+fx)}{a}\right)}{3a}\right)}{5a}\right)}{f}$$

$\downarrow$ 60

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(A+6iB)\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{5c\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{3c\left(c\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan\right.}{}\right.}{3a}\right.}{5a}\right.}{f}$$

$\downarrow$ 45

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(A+6iB)\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{5c\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{3c\left(2c\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+\ldots}}{\sqrt{c-ic\tan\ldots}}\right.}{\ldots}\right.}{3a}\right.}{5a}\right.}{f}$$

$\downarrow$ 218

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{7/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{(A+6iB)\left(\frac{2i(c-ic\tan(e+fx))^{5/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{5c\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{3c\left(-\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{\sqrt{a}}-\ldots\right.}{a}\right.}{3a}\right.}{5a}\right.}{f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(7/2))/(a + I*a*Tan[e + f
*x])^(5/2),x]
```

output
```
(a*c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(7/2))/(5*a*c*(a + I*a*Tan[e + f*x
])^(5/2)) - ((A + (6*I)*B)*((((2*I)/3)*(c - I*c*Tan[e + f*x])^(5/2))/(a*(a
 + I*a*Tan[e + f*x])^(3/2)) - (5*c*(((2*I)*(c - I*c*Tan[e + f*x])^(3/2))/(
a*Sqrt[a + I*a*Tan[e + f*x]]) - (3*c*(((-2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt
[a + I*a*Tan[e + f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/Sqrt[a] - (
I*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])/a))/a))/(3*a)))/(
5*a)))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(235) = 470$.

Time = 0.81 (sec) , antiderivative size = 835, normalized size of antiderivative = 2.94

method	result
derivativedivides	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^3\left(246iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+60iA\ln\left(\frac{ac\tan(fx+e)}{}\right.\right.}{}$
default	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^3\left(246iB\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}\,\tan(fx+e)^3+60iA\ln\left(\frac{ac\tan(fx+e)}{}\right.\right.}{}$
parts	Expression too large to display

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/15/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^3/a^3*(246
*I*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^3+60*I*A*ln((a*c*
tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(
f*x+e)^3-15*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))
/(a*c)^(1/2))*a*c*tan(f*x+e)^4-90*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*
(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^4-474*I*B*(a*c)^(1/2)
*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)-360*B*ln((a*c*tan(f*x+e)+(a*c)^(1
/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^3-15*B*(a*c*
(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^4-90*I*B*ln((a*c*tan(f*x+e)
+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c-94*I*A*(a*c)^(
1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2+90*A*ln((a*c*tan(f*x+e)+(a*
c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+46*A*
(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2)*tan(f*x+e)^3+26*I*A*(a*c)^(1/2)*(
a*c*(1+tan(f*x+e)^2))^(1/2)+540*I*B*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1
+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)^2+360*B*ln((a*c*tan(f*x
+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan(f*x+e)+
564*B*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+e)^2-60*I*A*ln((a*c
*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*c*tan
(f*x+e)-15*A*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/
(a*c)^(1/2))*a*c-74*A*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)*tan(f*x+...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(218) = 436.

Time = 0.11 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.88

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{\left(15\,a^3\sqrt{\frac{(A^2+12i\,AB-36\,B^2)c^7}{a^5f^2}}fe^{(5i\,fx+5i\,e)}\log\left(-\frac{4\left(2\left((i\,A-6\,B)c^3e^{(3i\,fx+3i\,e)}+(i\,A-6\,B)c^3e^{(i\,fx+i\,e)}\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{\cdots}{e^{(2i\cdots}}}\cdots\right.}{(-i\,A+6\,B)c^3e^{(2i\,fx+2i\,e)}+(\cdots}\right.\right.}{\cdots}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/30*(15*a^3*sqrt((A^2 + 12*I*A*B - 36*B^2)*c^7/(a^5*f^2))*f*e^(5*I*f*x +
 5*I*e)*log(-4*(2*((I*A - 6*B)*c^3*e^(3*I*f*x + 3*I*e) + (I*A - 6*B)*c^3*e
^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I
*e) + 1)) + (a^3*f*e^(2*I*f*x + 2*I*e) - a^3*f)*sqrt((A^2 + 12*I*A*B - 36*
B^2)*c^7/(a^5*f^2)))/((-I*A + 6*B)*c^3*e^(2*I*f*x + 2*I*e) + (-I*A + 6*B)*
c^3)) - 15*a^3*sqrt((A^2 + 12*I*A*B - 36*B^2)*c^7/(a^5*f^2))*f*e^(5*I*f*x
+ 5*I*e)*log(-4*(2*((I*A - 6*B)*c^3*e^(3*I*f*x + 3*I*e) + (I*A - 6*B)*c^3*
e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*
I*e) + 1)) - (a^3*f*e^(2*I*f*x + 2*I*e) - a^3*f)*sqrt((A^2 + 12*I*A*B - 36
*B^2)*c^7/(a^5*f^2)))/((-I*A + 6*B)*c^3*e^(2*I*f*x + 2*I*e) + (-I*A + 6*B)
*c^3)) + 4*(15*(-I*A + 6*B)*c^3*e^(6*I*f*x + 6*I*e) + 10*(-I*A + 6*B)*c^3*
e^(4*I*f*x + 4*I*e) + 2*(I*A - 6*B)*c^3*e^(2*I*f*x + 2*I*e) + 3*(-I*A + B)
*c^3)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)))
*e^(-5*I*f*x - 5*I*e)/(a^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(7/2)/(a+I*a*tan(f*x+e))**(
5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1025 vs. $2(218) = 436$.

Time = 0.36 (sec) , antiderivative size = 1025, normalized size of antiderivative = 3.61

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
15*(60*(A + 6*I*B)*c^3*cos(6*f*x + 6*e) + 40*(A + 6*I*B)*c^3*cos(4*f*x + 4
*e) - 8*(A + 6*I*B)*c^3*cos(2*f*x + 2*e) + 60*(I*A - 6*B)*c^3*sin(6*f*x +
6*e) + 40*(I*A - 6*B)*c^3*sin(4*f*x + 4*e) + 8*(-I*A + 6*B)*c^3*sin(2*f*x
+ 2*e) + 12*(A + I*B)*c^3 + 30*((A + 6*I*B)*c^3*cos(7/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + (A + 6*I*B)*c^3*cos(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + (I*A - 6*B)*c^3*sin(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + (I*A - 6*B)*c^3*sin(5/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))))*arctan2(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))), sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 30*((A
+ 6*I*B)*c^3*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (A + 6
*I*B)*c^3*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (I*A - 6*
B)*c^3*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (I*A - 6*B)*
c^3*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*arctan2(cos(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), -sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + 1) + 15*((I*A - 6*B)*c^3*cos(7/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + (I*A - 6*B)*c^3*cos(5/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - (A + 6*I*B)*c^3*sin(7/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - (A + 6*I*B)*c^3*sin(5/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))))*log(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*s...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx=\int \frac{(A+B\tan(e+f\,x))\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{7/2}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*
x)*1i)^(5/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(7/2))/(a + a*tan(e + f*
x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{7/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(7/2)/(a+I*a*tan(f*x+e))^(5/2),x)
```

output

```
(sqrt(c)*c**3*( - int(sqrt( - tan(e + f*x)*i + 1)/(sqrt(tan(e + f*x)*i + 1
)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e
 + f*x)*i + 1)),x)*a - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**4)/(
sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(
e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*b*i - int((sqrt( - tan(e + f*x)*
i + 1)*tan(e + f*x)**3)/(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt
(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*a*i + 3
*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(sqrt(tan(e + f*x)*i +
1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(
e + f*x)*i + 1)),x)*b + 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2
)/(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*t
an(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*a + 3*int((sqrt( - tan(e + f*
x)*i + 1)*tan(e + f*x)**2)/(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*s
qrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*b*i
 + 3*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(sqrt(tan(e + f*x)*i +
1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(
e + f*x)*i + 1)),x)*a*i - int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(
sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(
e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*b))/(sqrt(a)*a**2)
```

3.845 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 205

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = -\frac{2Bc^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{5/2}f} - \frac{2Bc^2\sqrt{c-ic\tan(e+fx)}}{a^2f\sqrt{a+ia\tan(e+fx)}} + \frac{2Bc(c-ic\tan(e+fx))^{3/2}}{3af(a+ia\tan(e+fx))^{3/2}} + \frac{(iA-B)(c-ic\tan(e+fx))^{5/2}}{5f(a+ia\tan(e+fx))^{5/2}}$$

output
```
-2*B*c^(5/2)*arctan(c^(1/2)*(a+I*a*tan(f*x+e))^(1/2)/a^(1/2)/(c-I*c*tan(f*
x+e))^(1/2))/a^(5/2)/f-2*B*c^2*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(a+I*a*tan(f
*x+e))^(1/2)+2/3*B*c*(c-I*c*tan(f*x+e))^(3/2)/a/f/(a+I*a*tan(f*x+e))^(3/2)
+1/5*(I*A-B)*(c-I*c*tan(f*x+e))^(5/2)/f/(a+I*a*tan(f*x+e))^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{c^3(i+\tan(e+fx))\left(-20\sqrt{2}B\,\text{Hypergeometric2F1}\,(\cdots\right.}{15a^2f\sqrt{1-i\tan \cdots}}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan
[e + f*x])^(5/2),x]
```

output
```
(c^3*(I + Tan[e + f*x])*(-20*Sqrt[2]*B*Hypergeometric2F1[-3/2, -3/2, -1/2,
 (1 + I*Tan[e + f*x])/2]*(-I + Tan[e + f*x]) + 3*(A + I*B)*Sqrt[1 - I*Tan[
e + f*x]]*(I + Tan[e + f*x])^2))/(15*a^2*f*Sqrt[1 - I*Tan[e + f*x]]*(-I +
Tan[e + f*x])^2*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 4071, 87, 57, 57, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c-ic\tan(e+fx))^{5/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)a+a)^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow 87$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{iB\int\frac{(c-ic\tan(e+fx))^{3/2}}{(i\tan(e+fx)a+a)^{5/2}}d\tan(e+fx)}{a}\right)}{f}$$

$$\Big\downarrow 57$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{iB\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{c\int\frac{\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)a+a)^{3/2}}d\tan(e+fx)}{a}\right)}{a}\right)}{f}$$

$$\Big\downarrow 57$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{iB\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{c\left(\frac{2i\sqrt{c-ic\tan(e+fx)}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{c\int\frac{1}{\sqrt{i\tan(e+fx)a+a}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{a}\right)}{a}\right)}{a}\right)}{f}$$

$$\Big\downarrow 45$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{iB\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{c\left(\frac{2i\sqrt{c-ic\tan(e+fx)}}{a\sqrt{a+ia\tan(e+fx)}}-\frac{2c\int\frac{1}{ia+\frac{ic(i\tan(e+fx)a+a)}{c-ic\tan(e+fx)}}d\frac{\sqrt{i\tan(e+fx)a+a}}{\sqrt{c-ic\tan(e+fx)}}}{a}\right)}{a}\right)}{a}\right)}{f}$$

$$\Big\downarrow 218$$

$$\frac{ac\left(\frac{(-B+iA)(c-ic\tan(e+fx))^{5/2}}{5ac(a+ia\tan(e+fx))^{5/2}}-\frac{iB\left(\frac{2i(c-ic\tan(e+fx))^{3/2}}{3a(a+ia\tan(e+fx))^{3/2}}-\frac{c\left(\frac{2i\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+ia\tan(e+fx)}}{\sqrt{a}\sqrt{c-ic\tan(e+fx)}}\right)}{a^{3/2}}+\frac{2i\sqrt{c-ic\tan(e+fx)}}{a\sqrt{a+ia\tan(e+fx)}}\right)}{a}\right)}{a}\right)}{f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(5/2))/(a + I*a*Tan[e + f
*x])^(5/2),x]
```

output
```
(a*c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(5/2))/(5*a*c*(a + I*a*Tan[e + f*x
])^(5/2)) - (I*B*((((2*I)/3)*(c - I*c*Tan[e + f*x])^(3/2))/(a*(a + I*a*Tan
[e + f*x])^(3/2)) - (c*(((2*I)*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + I*a*Tan[e
+ f*x]])/(Sqrt[a]*Sqrt[c - I*c*Tan[e + f*x]])])/a^(3/2) + ((2*I)*Sqrt[c -
I*c*Tan[e + f*x]])/(a*Sqrt[a + I*a*Tan[e + f*x]])))/a))/a))/f
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(168) = 336$.

Time = 0.71 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.48

method	result
parts	$\frac{A\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^2\left(1+\tan(fx+e)^2\right)(i+\tan(fx+e))}{5f\,a^3(i-\tan(fx+e))^4}+\frac{iB\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+}}{}$
derivativedivides	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^2\left(-15iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^4+90}{}$
default	$\frac{\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c^2\left(-15iB\ln\left(\frac{ac\tan(fx+e)+\sqrt{ac}\,\sqrt{ac\left(1+\tan(fx+e)^2\right)}}{\sqrt{ac}}\right)ac\tan(fx+e)^4+90}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/5*A/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a^3*(1+
tan(f*x+e)^2)*(I+tan(f*x+e))/(I-tan(f*x+e))^4+1/15*I*B/f*(-c*(I*tan(f*x+e)
-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)*c^2/a^3*(60*I*ln((a*c*tan(f*x+e)+(a*
c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*tan(f*x+e)^3*a*c-15*ln
((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*ta
n(f*x+e)^4*a*c-60*I*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^
(1/2))/(a*c)^(1/2))*tan(f*x+e)*a*c-97*I*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))
^(1/2)*tan(f*x+e)^2+90*ln((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2
))^(1/2))/(a*c)^(1/2))*tan(f*x+e)^2*a*c+43*tan(f*x+e)^3*(a*c*(1+tan(f*x+e)
^2))^(1/2)*(a*c)^(1/2)+23*I*(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2)-15*ln
((a*c*tan(f*x+e)+(a*c)^(1/2)*(a*c*(1+tan(f*x+e)^2))^(1/2))/(a*c)^(1/2))*a*
c-77*tan(f*x+e)*(a*c*(1+tan(f*x+e)^2))^(1/2)*(a*c)^(1/2))/(a*c*(1+tan(f*x+
e)^2))^(1/2)/(I-tan(f*x+e))^4/(a*c)^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(157) = 314$.

Time = 0.10 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.13

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{\left(15\,a^3 f\sqrt{-\frac{B^2c^5}{a^5f^2}}e^{(5i\,fx+5i\,e)}\log\left(\frac{4\left(2\left(Bc^2e^{(3i\,fx+3i\,e)}+Bc^2\right.\right.}{}\right.\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
1/30*(15*a^3*f*sqrt(-B^2*c^5/(a^5*f^2))*e^(5*I*f*x + 5*I*e)*log(4*(2*(B*c^
2*e^(3*I*f*x + 3*I*e) + B*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e)
 + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1)) + (a^3*f*e^(2*I*f*x + 2*I*e) - a^
3*f)*sqrt(-B^2*c^5/(a^5*f^2)))/(B*c^2*e^(2*I*f*x + 2*I*e) + B*c^2)) - 15*a
^3*f*sqrt(-B^2*c^5/(a^5*f^2))*e^(5*I*f*x + 5*I*e)*log(4*(2*(B*c^2*e^(3*I*f
*x + 3*I*e) + B*c^2*e^(I*f*x + I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqr
t(c/(e^(2*I*f*x + 2*I*e) + 1)) - (a^3*f*e^(2*I*f*x + 2*I*e) - a^3*f)*sqrt(
-B^2*c^5/(a^5*f^2)))/(B*c^2*e^(2*I*f*x + 2*I*e) + B*c^2)) - 2*(30*B*c^2*e^
(6*I*f*x + 6*I*e) + 20*B*c^2*e^(4*I*f*x + 4*I*e) + (-3*I*A - 7*B)*c^2*e^(2
*I*f*x + 2*I*e) + 3*(-I*A + B)*c^2)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt
(c/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-5*I*f*x - 5*I*e)/(a^3*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \int \frac{(-ic(\tan(e+fx)+i))^{\frac{5}{2}}\,(A+B\tan(e+fx))}{(ia(\tan(e+fx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(5/2)/(a+I*a*tan(f*x+e))**(
5/2),x)
```

output
```
Integral((-I*c*(tan(e + f*x) + I))**(5/2)*(A + B*tan(e + f*x))/(I*a*(tan(e
 + f*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.05

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{(30\,Bc^2\arctan\left(\cos\left(fx+e\right),\sin\left(fx+e\right)+1\right)+30\,Bc^2\arctan\left(\cos\left(fx+e\right),-\sin\left(fx+e\right)+1\right)-6\,(i$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
-1/30*(30*B*c^2*arctan2(cos(f*x + e), sin(f*x + e) + 1) + 30*B*c^2*arctan2
(cos(f*x + e), -sin(f*x + e) + 1) - 6*(I*A - B)*c^2*cos(5*f*x + 5*e) - 20*
B*c^2*cos(3*f*x + 3*e) + 60*B*c^2*cos(f*x + e) + 15*I*B*c^2*log(cos(f*x +
e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 15*I*B*c^2*log(cos(f*x + e)^
2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 6*(A + I*B)*c^2*sin(5*f*x + 5*e
) + 20*I*B*c^2*sin(3*f*x + 3*e) - 60*I*B*c^2*sin(f*x + e))*sqrt(c)/(a^(5/2
)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \int \frac{(A+B\tan(e+f\,x))\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^{5/2}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*
x)*1i)^(5/2),x)
```

output
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(5/2))/(a + a*tan(e + f*
x)*1i)^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{5/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,c^2\left(-\left(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\sqrt{\tan(fx+e)i+1}\,\tan(fx+e)^2-2\sqrt{\tan(fx+e)i+1}\,\tan(f}\right.\right.}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(5/2)/(a+I*a*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*c**2*( - int(sqrt( - tan(e + f*x)*i + 1)/(sqrt(tan(e + f*x)*i + 1
)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e
 + f*x)*i + 1)),x)*a + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**3)/(
sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(
e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*b + int((sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x)**2)/(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(t
an(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*a + 2*int
((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(sqrt(tan(e + f*x)*i + 1)*t
an(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e +
f*x)*i + 1)),x)*b*i + 2*int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(sq
rt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e
+ f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*a*i - int((sqrt( - tan(e + f*x)*i
+ 1)*tan(e + f*x))/(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(
e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*b))/(sqrt(a)
*a**2)
```

3.846 $\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 104

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{(iA-B)(c-ic\tan(e+fx))^{3/2}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{(iA+4B)(c-ic\tan(e+fx))^{3/2}}{15af(a+ia\tan(e+fx))^{3/2}}$$

output
```
1/5*(I*A-B)*(c-I*c*tan(f*x+e))^(3/2)/f/(a+I*a*tan(f*x+e))^(5/2)+1/15*(I*A+
4*B)*(c-I*c*tan(f*x+e))^(3/2)/a/f/(a+I*a*tan(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = -\frac{ic(i+\tan(e+fx))(-4iA-B+(A-4iB)\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{15a^2f(-i+\tan(e+fx))^2\sqrt{a+ia\tan(e+fx)}}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan
[e + f*x])^(5/2),x]
```

output
```
((-1/15*I)*c*(I + Tan[e + f*x])*((-4*I)*A - B + (A - (4*I)*B)*Tan[e + f*x]
)*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*f*(-I + Tan[e + f*x])^2*Sqrt[a + I*a*Ta
n[e + f*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {3042, 4071, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-ic\tan(e+fx))^{3/2}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)a+a)^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(A-4iB)\int \frac{\sqrt{c-ic\tan(e+fx)}}{(i\tan(e+fx)a+a)^{5/2}}d\tan(e+fx)}{5a}+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{5ac(a+ia\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(\frac{i(A-4iB)(c-ic\tan(e+fx))^{3/2}}{15a^2c(a+ia\tan(e+fx))^{3/2}}+\frac{(-B+iA)(c-ic\tan(e+fx))^{3/2}}{5ac(a+ia\tan(e+fx))^{5/2}}\right)}{f}$$

input
```
Int[((A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^(3/2))/(a + I*a*Tan[e + f
*x])^(5/2),x]
```

output
```
(a*c*(((I*A - B)*(c - I*c*Tan[e + f*x])^(3/2))/(5*a*c*(a + I*a*Tan[e + f*x
])^(5/2)) + ((I/15)*(A - (4*I)*B)*(c - I*c*Tan[e + f*x])^(3/2))/(a^2*c*(a
+ I*a*Tan[e + f*x])^(3/2))))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{i\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c\left(1+\tan(fx+e)^2\right)(i\tan(fx+e)A-iB+4B\tan(fx+e)+4A)}{15f\,a^3(i-\tan(fx+e))^4}$
default	$\frac{i\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c\left(1+\tan(fx+e)^2\right)(i\tan(fx+e)A-iB+4B\tan(fx+e)+4A)}{15f\,a^3(i-\tan(fx+e))^4}$
parts	$\frac{A\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,c\left(1+\tan(fx+e)^2\right)(4i-\tan(fx+e))}{15f\,a^3(i-\tan(fx+e))^4}-\frac{iB\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1-}{15}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/15*I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^3*c*(1+t
an(f*x+e)^2)*(I*A*tan(f*x+e)-I*B+4*B*tan(f*x+e)+4*A)/(I-tan(f*x+e))^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{\left(5\,(-i\,A-B)ce^{(4i\,fx+4i\,e)}+2\,(-4i\,A-B)ce^{(2i\,fx+2i\,e)}+3\,(-i\,A+B)c\right)\sqrt{\frac{a}{e^{(2i\,fx+2i\,e)}+1}}\sqrt{\frac{c}{e^{(2i\,fx+2i\,e)}+1}}e^{(-5i}}{30\,a^3f}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/30*(5*(-I*A - B)*c*e^(4*I*f*x + 4*I*e) + 2*(-4*I*A - B)*c*e^(2*I*f*x +
2*I*e) + 3*(-I*A + B)*c)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*
f*x + 2*I*e) + 1))*e^(-5*I*f*x - 5*I*e)/(a^3*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \int \frac{(-ic(\tan(e+fx)+i))^{\frac{3}{2}}\,(A+B\tan(e+fx))}{(ia(\tan(e+fx)-i))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(3/2)/(a+I*a*tan(f*x+e))**(
5/2),x)
```

output
```
Integral((-I*c*(tan(e + f*x) + I))**(3/2)*(A + B*tan(e + f*x))/(I*a*(tan(e
 + f*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.47

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{30\,(5\,(A-i\,B)c\cos(4\,fx+4\,e)+2\,(4\,A-i\,B)c\cos(}{-900\,(i\,a^3\cos(7\,fx+}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
30*(5*(A - I*B)*c*cos(4*f*x + 4*e) + 2*(4*A - I*B)*c*cos(2*f*x + 2*e) - 5*
(-I*A - B)*c*sin(4*f*x + 4*e) - 2*(-4*I*A - B)*c*sin(2*f*x + 2*e) + 3*(A +
 I*B)*c)*sqrt(a)*sqrt(c)/((-900*I*a^3*cos(7*f*x + 7*e) - 900*I*a^3*cos(5*f
*x + 5*e) + 900*a^3*sin(7*f*x + 7*e) + 900*a^3*sin(5*f*x + 5*e))*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.55 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.31

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{c\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(}{\cos(2\,e+2\,f\,x)}}}{}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(3/2))/(a + a*tan(e + f*
x)*1i)^(5/2),x)
```

output
```
(c*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1
))^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*
x) + 1))^(1/2)*(A*cos(2*e + 2*f*x)*5i + A*cos(4*e + 4*f*x)*8i + A*cos(6*e
+ 6*f*x)*3i + 5*B*cos(2*e + 2*f*x) + 2*B*cos(4*e + 4*f*x) - 3*B*cos(6*e +
6*f*x) + 5*A*sin(2*e + 2*f*x) + 8*A*sin(4*e + 4*f*x) + 3*A*sin(6*e + 6*f*x
) - B*sin(2*e + 2*f*x)*5i - B*sin(4*e + 4*f*x)*2i + B*sin(6*e + 6*f*x)*3i)
)/(60*a^3*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c-ic\tan(e+fx))^{3/2}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,c\Big(-\Big(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\sqrt{\tan(fx+e)i+1}\,\tan(fx+e)^2-2\sqrt{\tan(fx+e)i+1}\,\tan(fx}}{}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(3/2)/(a+I*a*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*c*( - int(sqrt( - tan(e + f*x)*i + 1)/(sqrt(tan(e + f*x)*i + 1)*t
an(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e +
f*x)*i + 1)),x)*a + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(sqr
t(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e +
 f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*b*i + int((sqrt( - tan(e + f*x)*i +
 1)*tan(e + f*x))/(sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e
 + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)),x)*a*i - int((sq
rt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(sqrt(tan(e + f*x)*i + 1)*tan(e +
f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e + f*x)*i
+ 1)),x)*b))/(sqrt(a)*a**2)
```

3.847 $\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 157

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{(iA-B)\sqrt{c-ic\tan(e+fx)}}{5f(a+ia\tan(e+fx))^{5/2}} + \frac{(2iA+3B)\sqrt{c-ic\tan(e+fx)}}{15af(a+ia\tan(e+fx))^{3/2}} + \frac{(2iA+3B)\sqrt{c-ic\tan(e+fx)}}{15a^2f\sqrt{a+ia\tan(e+fx)}}$$

output

```
1/5*(I*A-B)*(c-I*c*tan(f*x+e))^(1/2)/f/(a+I*a*tan(f*x+e))^(5/2)+1/15*(2*I*
A+3*B)*(c-I*c*tan(f*x+e))^(1/2)/a/f/(a+I*a*tan(f*x+e))^(3/2)+1/15*(2*I*A+3
*B)*(c-I*c*tan(f*x+e))^(1/2)/a^2/f/(a+I*a*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.63

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{c-ic\tan(e+fx)}(-7iA-3B+(6A-9iB)\tan(e+}{15a^2f(-i+\tan(e+fx))^2\sqrt{a+ia}}$$

input

```
Integrate[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e
 + f*x])^(5/2),x]
```

output
```
(Sqrt[c - I*c*Tan[e + f*x]]*((-7*I)*A - 3*B + (6*A - (9*I)*B)*Tan[e + f*x]
 + ((2*I)*A + 3*B)*Tan[e + f*x]^2))/(15*a^2*f*(-I + Tan[e + f*x])^2*Sqrt[a
 + I*a*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3042, 4071, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-ic\tan(e+fx)}(A+B\tan(e+fx))}{(a+ia\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{7/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(2A-3iB)\int \frac{1}{(i\tan(e+fx)a+a)^{5/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{5a}+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{5ac(a+ia\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{55}$$

$$\frac{ac\left(\frac{(2A-3iB)\left(\frac{\int \frac{1}{(i\tan(e+fx)a+a)^{3/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{3a}+\frac{i\sqrt{c-ic\tan(e+fx)}}{3ac(a+ia\tan(e+fx))^{3/2}}\right)}{5a}+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{5ac(a+ia\tan(e+fx))^{5/2}}\right)}{f}$$

$$\Big\downarrow \text{48}$$

$$\frac{ac\left(\frac{(2A-3iB)\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{3a^2c\sqrt{a+ia\tan(e+fx)}}+\frac{i\sqrt{c-ic\tan(e+fx)}}{3ac(a+ia\tan(e+fx))^{3/2}}\right)}{5a}+\frac{(-B+iA)\sqrt{c-ic\tan(e+fx)}}{5ac(a+ia\tan(e+fx))^{5/2}}\right)}{f}$$

input
```
Int[((A + B*Tan[e + f*x])*Sqrt[c - I*c*Tan[e + f*x]])/(a + I*a*Tan[e + f*x
])^(5/2),x]
```

output
```
(a*c*(((I*A - B)*Sqrt[c - I*c*Tan[e + f*x]])/(5*a*c*(a + I*a*Tan[e + f*x])
^(5/2)) + ((2*A - (3*I)*B)*(((I/3)*Sqrt[c - I*c*Tan[e + f*x]])/(a*c*(a + I
*a*Tan[e + f*x])^(3/2)) + ((I/3)*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*c*Sqrt[a
 + I*a*Tan[e + f*x]])))/(5*a)))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\frac{i\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,\left(2iA\tan(fx+e)^3-12iB\tan(fx+e)^2+3B\tan(fx+e)^3-13i\tan(fx+e)\right.}{15f\,a^3(i-\tan(fx+e))^4}$
default	$-\frac{i\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,\left(2iA\tan(fx+e)^3-12iB\tan(fx+e)^2+3B\tan(fx+e)^3-13i\tan(fx+e)\right.}{15f\,a^3(i-\tan(fx+e))^4}$
parts	$-\frac{A\sqrt{-c(i\tan(fx+e)-1)}\,\sqrt{a(1+i\tan(fx+e))}\,\left(8i\tan(fx+e)^2-2\tan(fx+e)^3-7i+13\tan(fx+e)\right)}{15f\,a^3(i-\tan(fx+e))^4}+\frac{iB\sqrt{-c(i\tan}{}$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/15*I/f*(-c*(I*tan(f*x+e)-1))^(1/2)*(a*(1+I*tan(f*x+e)))^(1/2)/a^3*(2*I*
A*tan(f*x+e)^3-12*I*B*tan(f*x+e)^2+3*B*tan(f*x+e)^3-13*I*A*tan(f*x+e)+8*A*
tan(f*x+e)^2+3*I*B-12*B*tan(f*x+e)-7*A)/(I-tan(f*x+e))^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.71

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx = -\frac{\left(15\left(-iA-B\right)e^{(6i\,fx+6i\,e)}+5\left(-5i\,A-3\,B\right)e^{(4i\,fx+4i\,e)}-\left(13i\,A-3\,B\right)e^{(2i\,fx+2i\,e)}-3i\,A+3\,B\right)\sqrt{\frac{a}{e^{(2i\,fx+}}}}{60\,a^3 f}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/60*(15*(-I*A - B)*e^(6*I*f*x + 6*I*e) + 5*(-5*I*A - 3*B)*e^(4*I*f*x + 4
*I*e) - (13*I*A - 3*B)*e^(2*I*f*x + 2*I*e) - 3*I*A + 3*B)*sqrt(a/(e^(2*I*f
*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-5*I*f*x - 5*I*e)/(
a^3*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}\left(A+B\tan\left(e+fx\right)\right)}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(1/2)/(a+I*a*tan(f*x+e))**(
5/2),x)
```

output
```
Integral(sqrt(-I*c*(tan(e + f*x) + I))*(A + B*tan(e + f*x))/(I*a*(tan(e +
f*x) - I))**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeError: Bad Argument TypeError: Bad Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 7.66 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.57

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,\sqrt{\frac{c\,(\cos(2\,e+2\,f\,x)+1-\sin(2\,e+}{\cos(2\,e+2\,f\,x)+1}}}{}$$

input
```
int(((A + B*tan(e + f*x))*(c - c*tan(e + f*x)*1i)^(1/2))/(a + a*tan(e + f*
x)*1i)^(5/2),x)
```

output
```
(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x)
 + 1))^(1/2)*(A*15i + 15*B + A*cos(2*e + 2*f*x)*25i + A*cos(4*e + 4*f*x)*1
3i + A*cos(6*e + 6*f*x)*3i + 15*B*cos(2*e + 2*f*x) - 3*B*cos(4*e + 4*f*x)
- 3*B*cos(6*e + 6*f*x) + 25*A*sin(2*e + 2*f*x) + 13*A*sin(4*e + 4*f*x) + 3
*A*sin(6*e + 6*f*x) - B*sin(2*e + 2*f*x)*15i + B*sin(4*e + 4*f*x)*3i + B*s
in(6*e + 6*f*x)*3i))/(120*a^3*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))\sqrt{c-ic\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\tan(fx+e)i+1}}{\sqrt{\tan(fx+e)i+1}\,\tan(fx+e)^2-2\sqrt{\tan(fx+e)i+1}\,\tan(fx+e)i-\sqrt{\tan(fx+e)i+1}}dx\right)a+\left(\int \frac{\sqrt{-\tan}}{\sqrt{\tan(fx+e)i+1}\,\tan(fx+e)^2-2}\right.\right.}{\sqrt{a}\,a^2}$$

input
```
int((A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(1/2)/(a+I*a*tan(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*(int(sqrt( - tan(e + f*x)*i + 1)/(sqrt(tan(e + f*x)*i + 1)*tan
(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*i - sqrt(tan(e + f*
x)*i + 1)),x)*a + int((sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(sqrt(tan
(e + f*x)*i + 1)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)
*i - sqrt(tan(e + f*x)*i + 1)),x)*b))/(sqrt(a)*a**2)
```

3.848 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 212

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx = -\frac{iA+B}{f(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}} + \frac{(3iA+2B)\sqrt{c-ic\tan(e+fx)}}{5cf(a+ia\tan(e+fx))^{5/2}} + \frac{2(3iA+2B)\sqrt{c-ic\tan(e+fx)}}{15acf(a+ia\tan(e+fx))^{3/2}} + \frac{2(3iA+2B)\sqrt{c-ic\tan(e+fx)}}{15a^2cf\sqrt{a+ia\tan(e+fx)}}$$

output

```
-(I*A+B)/f/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/2)+1/5*(3*I*A+2*
B)*(c-I*c*tan(f*x+e))^(1/2)/c/f/(a+I*a*tan(f*x+e))^(5/2)+2/15*(3*I*A+2*B)*
(c-I*c*tan(f*x+e))^(1/2)/a/c/f/(a+I*a*tan(f*x+e))^(3/2)+2/15*(3*I*A+2*B)*(
c-I*c*tan(f*x+e))^(1/2)/a^2/c/f/(a+I*a*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.54

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{-6iA+B+(-3A+2iB)\tan(e+fx)+(-12iA-8}{15a^2f(-i+\tan(e+fx))^2\sqrt{a+ia\tan}}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*
Tan[e + f*x]]),x]
```

output
```
((-6*I)*A + B + (-3*A + (2*I)*B)*Tan[e + f*x] + ((-12*I)*A - 8*B)*Tan[e +
f*x]^2 + (6*A - (4*I)*B)*Tan[e + f*x]^3)/(15*a^2*f*(-I + Tan[e + f*x])^2*S
qrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}dx$$

$$\Big\downarrow \; 4071$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{7/2}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 87$$

$$\frac{ac\left(\frac{(3A-2iB)\int \frac{1}{(i\tan(e+fx)a+a)^{7/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{c}-\frac{B+iA}{ac(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$\big\downarrow$ 55

$$\frac{ac\left(\frac{(3A-2iB)\left(\frac{2\int \frac{1}{(i\tan(e+fx)a+a)^{5/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{5a}+\frac{i\sqrt{c-ic\tan(e+fx)}}{5ac(a+ia\tan(e+fx))^{5/2}}\right)}{c}-\frac{B+iA}{ac(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

$\big\downarrow$ 55

$$\frac{ac\left(\frac{(3A-2iB)\left(\frac{2\left(\frac{\int \frac{1}{(i\tan(e+fx)a+a)^{3/2}\sqrt{c-ic\tan(e+fx)}}d\tan(e+fx)}{3a}+\frac{i\sqrt{c-ic\tan(e+fx)}}{3ac(a+ia\tan(e+fx))^{3/2}}\right)}{5a}+\frac{i\sqrt{c-ic\tan(e+fx)}}{5ac(a+ia\tan(e+fx))^{5/2}}\right)}{c}-\frac{\cdots}{ac(a+ia\tan(e+\cdots}\right.}{f}$$

$\big\downarrow$ 48

$$\frac{ac\left(\frac{(3A-2iB)\left(\frac{2\left(\frac{i\sqrt{c-ic\tan(e+fx)}}{3a^2c\sqrt{a+ia\tan(e+fx)}}+\frac{i\sqrt{c-ic\tan(e+fx)}}{3ac(a+ia\tan(e+fx))^{3/2}}\right)}{5a}+\frac{i\sqrt{c-ic\tan(e+fx)}}{5ac(a+ia\tan(e+fx))^{5/2}}\right)}{c}-\frac{B+iA}{ac(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\right)}{f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e
+ f*x]]),x]
```

output
```
(a*c*(-((I*A + B)/(a*c*(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c - I*c*Tan[e + f
*x]])) + ((3*A - (2*I)*B)*(((I/5)*Sqrt[c - I*c*Tan[e + f*x]])/(a*c*(a + I*
a*Tan[e + f*x])^(5/2)) + (2*(((I/3)*Sqrt[c - I*c*Tan[e + f*x]])/(a*c*(a +
I*a*Tan[e + f*x])^(3/2)) + ((I/3)*Sqrt[c - I*c*Tan[e + f*x]])/(a^2*c*Sqrt[
a + I*a*Tan[e + f*x]])))/(5*a)))/c))/f
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(4iB\tan(fx+e)^5+12iA\tan(fx+e)^4-6A\tan(fx+e)^5+2iB\tan(fx+e)\right.}{15f\,a^3c(i-\tan(fx+}$
default	$-\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(4iB\tan(fx+e)^5+12iA\tan(fx+e)^4-6A\tan(fx+e)^5+2iB\tan(fx+e)\right.}{15f\,a^3c(i-\tan(fx+}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(4i\tan(fx+e)^4-2\tan(fx+e)^5+6i\tan(fx+e)^2-\tan(fx+e)^3+2i+\tan\right.}{5f\,a^3c(i-\tan(fx+e))^4(i+\tan(fx+e))^2}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^3/c*(4*I*
B*tan(f*x+e)^5+12*I*A*tan(f*x+e)^4-6*A*tan(f*x+e)^5+2*I*B*tan(f*x+e)^3+8*B
*tan(f*x+e)^4+18*I*A*tan(f*x+e)^2-3*A*tan(f*x+e)^3-2*I*B*tan(f*x+e)+7*B*ta
n(f*x+e)^2+6*I*A+3*A*tan(f*x+e)-B)/(I-tan(f*x+e))^4/(I+tan(f*x+e))^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.76

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx =$$

$$-\frac{(15\,(i\,A+B)e^{(8i\,fx+8i\,e)}+8\,(6i\,A-B)e^{(7i\,fx+7i\,e)}-30i\,Ae^{(6i\,fx+6i\,e)}+8\,(6i\,A-B)e^{(5i\,fx+5i\,e)}+10\,(-6i\,}{120\,a^3}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="fricas")
```

output
```
-1/120*(15*(I*A + B)*e^(8*I*f*x + 8*I*e) + 8*(6*I*A - B)*e^(7*I*f*x + 7*I*
e) - 30*I*A*e^(6*I*f*x + 6*I*e) + 8*(6*I*A - B)*e^(5*I*f*x + 5*I*e) + 10*(
-6*I*A - B)*e^(4*I*f*x + 4*I*e) + 2*(-9*I*A + 4*B)*e^(2*I*f*x + 2*I*e) - 3
*I*A + 3*B)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e)
+ 1))*e^(-5*I*f*x - 5*I*e)/(a^3*c*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx=\int \frac{A+B\tan\left(e+fx\right)}{\left(ia\left(\tan\left(e+fx\right)-i\right)\right)^{\frac{5}{2}}\sqrt{-ic\left(\tan\left(e+fx\right)+i\right)}}\,d$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(5/2)/(c-I*c*tan(f*x+e))**(
1/2),x)
```

output
```
Integral((A + B*tan(e + f*x))/((I*a*(tan(e + f*x) - I))**(5/2)*sqrt(-I*c*(
tan(e + f*x) + I))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx=\int \frac{B\tan\left(fx+e\right)+A}{\left(i\,a\tan\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-i\,c\tan\left(fx+e\right)+c}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)/((I*a*tan(f*x + e) + a)^(5/2)*sqrt(-I*c*tan
(f*x + e) + c)), x)
```

Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.16

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx=\frac{\sqrt{\frac{a\,\left(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1{}\mathrm{i}\right)}{\cos(2\,e+2\,f\,x)+1}}\,\left(15\,B\,\cos\left(2\,e+2\,f\,x\right)\right.}{}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x
)*1i)^(1/2)),x)
```

output
```
(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*(A*cos(2*e + 2*f*x)*45i - 15*B - A*15i + A*cos(4*e + 4*f*x)*15i + A
*cos(6*e + 6*f*x)*3i + 15*B*cos(2*e + 2*f*x) - 5*B*cos(4*e + 4*f*x) - 3*B*
cos(6*e + 6*f*x) + 45*A*sin(2*e + 2*f*x) + 15*A*sin(4*e + 4*f*x) + 3*A*sin
(6*e + 6*f*x) - B*sin(2*e + 2*f*x)*15i + B*sin(4*e + 4*f*x)*5i + B*sin(6*e
 + 6*f*x)*3i))/(120*a^3*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i + 1)
)/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\,dx = \frac{-\left(\int \frac{\mathrm{t}}{\sqrt{\tan(fx+e)i+1}\,\sqrt{-\tan(fx+e)i+1}\,\tan(fx+e)^{2}-2\sqrt{\tan(fx+e)}}\right.}{}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/2),x)
```

output
```
( - (int(tan(e + f*x)/(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1
)*tan(e + f*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)
*tan(e + f*x)*i - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)),x)
*b + int(1/(sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f
*x)**2 - 2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*
x)*i - sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)),x)*a))/(sqrt(
c)*sqrt(a)*a**2)
```

3.849 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 218

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{-iA-B}{3f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}} + \frac{4iA+B}{15cf(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}} + \frac{4iA+B}{15acf(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}} + \frac{2(4A-iB)\tan(e+fx)}{15a^2cf\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}$$

output
```
1/3*(-I*A-B)/f/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2)+1/15*(4*I
*A+B)/c/f/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(1/2)+1/15*(4*I*A+B)
/a/c/f/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(1/2)+2/15*(4*A-I*B)*ta
n(f*x+e)/a^2/c/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.70

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \frac{3(A+iB)+(-12iA-3B)\tan(e+fx)+3(4A-}{15a^2cf(-i+\tan(e+fx))^2(i+}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[
e + f*x])^(3/2)),x]
```

output
```
(3*(A + I*B) + ((-12*I)*A - 3*B)*Tan[e + f*x] + 3*(4*A - I*B)*Tan[e + f*x]
^2 + ((-8*I)*A - 2*B)*Tan[e + f*x]^3 + (8*A - (2*I)*B)*Tan[e + f*x]^4)/(15
*a^2*c*f*(-I + Tan[e + f*x])^2*(I + Tan[e + f*x])*Sqrt[a + I*a*Tan[e + f*x
]]*Sqrt[c - I*c*Tan[e + f*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 87, 55, 55, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{7/2}(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{(4A-iB)\int \frac{1}{(i\tan(e+fx)a+a)^{7/2}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3c}-\frac{B+iA}{3ac(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\right)}{f}$$

$\Big\downarrow$ 55

$$\frac{ac\left(\frac{(4A-iB)\left(\frac{3\int \frac{1}{(i\tan(e+fx)a+a)^{5/2}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{5a}+\frac{i}{5ac(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\frac{B+iA}{3ac(a+ia\tan(e+fx))^{5/2}(c}\right.}{f}$$

$\Big\downarrow$ 55

$$\frac{ac\left(\frac{(4A-iB)\left(\frac{3\left(\frac{2\int \frac{1}{(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3a}+\frac{i}{3ac(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\right)}{5a}+\frac{i}{5ac(a+ia\tan(e+fx))^{5/2}\sqrt{c-}}\right.}{3c}\right.}{f}$$

$\Big\downarrow$ 41

$$\frac{ac\left(\frac{(4A-iB)\left(\frac{3\left(\frac{2\tan(e+fx)}{3a^2c\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{i}{3ac(a+ia\tan(e+fx))^{3/2}\sqrt{c-ic\tan(e+fx)}}\right)}{5a}+\frac{i}{5ac(a+ia\tan(e+fx))^{5/2}\sqrt{c-ic\tan(e+fx)}}\right)}{3c}-\right.}{f}$$

input
```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*
x])^(3/2)),x]
```

output
```
(a*c*(-1/3*(I*A + B)/(a*c*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*
x])^(3/2)) + ((4*A - I*B)*((I/5)/(a*c*(a + I*a*Tan[e + f*x])^(5/2)*Sqrt[c
- I*c*Tan[e + f*x]]) + (3*((I/3)/(a*c*(a + I*a*Tan[e + f*x])^(3/2)*Sqrt[c
- I*c*Tan[e + f*x]]) + (2*Tan[e + f*x])/(3*a^2*c*Sqrt[a + I*a*Tan[e + f*x]
]*Sqrt[c - I*c*Tan[e + f*x]])))/(5*a)))/(3*c)))/f
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.91

method	result
derivativedivides	$-\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(8iA\tan(fx+e)^{6}-2iB\tan(fx+e)^{5}+2B\tan(fx+e)^{6}+20iA\tan(fx+e\right.}{15f\,a^{3}c^{2}\left(\right.}$
default	$-\frac{i\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(8iA\tan(fx+e)^{6}-2iB\tan(fx+e)^{5}+2B\tan(fx+e)^{6}+20iA\tan(fx+e\right.}{15f\,a^{3}c^{2}\left(\right.}$
parts	$-\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(8i\tan(fx+e)^{5}-8\tan(fx+e)^{6}+20i\tan(fx+e)^{3}-20\tan(fx+e)^{4}+12\right.}{15f\,a^{3}c^{2}(i+\tan(fx+e))^{3}(i-\tan(fx+e))^{4}}$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/15*I/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^3/c^2*(
8*I*A*tan(f*x+e)^6-2*I*B*tan(f*x+e)^5+2*B*tan(f*x+e)^6+20*I*A*tan(f*x+e)^4
+8*A*tan(f*x+e)^5-5*I*B*tan(f*x+e)^3+5*B*tan(f*x+e)^4+15*I*A*tan(f*x+e)^2+
20*A*tan(f*x+e)^3-3*I*B*tan(f*x+e)+3*I*A+12*A*tan(f*x+e)-3*B)/(I-tan(f*x+e
))^4/(I+tan(f*x+e))^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.83

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(5\,(i\,A+B)e^{(10i\,fx+10i\,e)}+5\,(13i\,A+7\,B)e^{(8i\,fx+8i\,e)}+48\,(i\,A-B)e^{(7i\,fx+7i\,e)}+30\,(-i\,A+B)e^{(6i\,fx+6i\,e}\right.}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="fricas")
```

output
```
-1/240*(5*(I*A + B)*e^(10*I*f*x + 10*I*e) + 5*(13*I*A + 7*B)*e^(8*I*f*x +
8*I*e) + 48*(I*A - B)*e^(7*I*f*x + 7*I*e) + 30*(-I*A + B)*e^(6*I*f*x + 6*I
*e) + 48*(I*A - B)*e^(5*I*f*x + 5*I*e) + 10*(-11*I*A + B)*e^(4*I*f*x + 4*I
*e) - (23*I*A - 13*B)*e^(2*I*f*x + 2*I*e) - 3*I*A + 3*B)*sqrt(a/(e^(2*I*f*
x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-5*I*f*x - 5*I*e)/(a
^3*c^2*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \int \frac{A+B\tan(e+fx)}{(ia(\tan(e+fx)-i))^{\frac{5}{2}}(-ic(\tan(e+fx)+i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(5/2)/(c-I*c*tan(f*x+e))**(
3/2),x)
```

output
```
Integral((A + B*tan(e + f*x))/((I*a*(tan(e + f*x) - I))**(5/2)*(-I*c*(tan(
e + f*x) + I))**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx=\int \frac{B\tan\left(fx+e\right)+A}{\left(i\,a\tan\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,d$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)/((I*a*tan(f*x + e) + a)^(5/2)*(-I*c*tan(f*x
 + e) + c)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.14

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx=\frac{\sqrt{\frac{a\,\left(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i}\right)}{\cos(2\,e+2\,f\,x)+1}}\,\left(95\,A\,\sin\left(2\,e+2\,f\right.\right.}{}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x
)*1i)^(3/2)),x)
```

output
```
(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*(A*cos(2*e + 2*f*x)*85i - 30*B - A*60i + A*cos(4*e + 4*f*x)*20i + A
*cos(6*e + 6*f*x)*3i - 5*B*cos(2*e + 2*f*x) - 10*B*cos(4*e + 4*f*x) - 3*B*
cos(6*e + 6*f*x) + 95*A*sin(2*e + 2*f*x) + 20*A*sin(4*e + 4*f*x) + 3*A*sin
(6*e + 6*f*x) - B*sin(2*e + 2*f*x)*5i + B*sin(4*e + 4*f*x)*10i + B*sin(6*e
 + 6*f*x)*3i))/(240*a^3*c*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x)*1i +
1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(2*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*t
an(e + f*x)**3*a + 3*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*
tan(e + f*x)*a - 3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i +
1)*tan(e + f*x))/(tan(e + f*x)**5*i + tan(e + f*x)**4 + 2*tan(e + f*x)**3*
i + 2*tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**4*a*f*i + 3*i
nt((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(ta
n(e + f*x)**5*i + tan(e + f*x)**4 + 2*tan(e + f*x)**3*i + 2*tan(e + f*x)**
2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**4*b*f - 6*int((sqrt(tan(e + f*x)*
i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**5*i + tan(
e + f*x)**4 + 2*tan(e + f*x)**3*i + 2*tan(e + f*x)**2 + tan(e + f*x)*i + 1
),x)*tan(e + f*x)**2*a*f*i + 6*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e
 + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**5*i + tan(e + f*x)**4 + 2*tan(
e + f*x)**3*i + 2*tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2
*b*f - 3*int((sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*tan(e +
 f*x))/(tan(e + f*x)**5*i + tan(e + f*x)**4 + 2*tan(e + f*x)**3*i + 2*tan(
e + f*x)**2 + tan(e + f*x)*i + 1),x)*a*f*i + 3*int((sqrt(tan(e + f*x)*i +
1)*sqrt( - tan(e + f*x)*i + 1)*tan(e + f*x))/(tan(e + f*x)**5*i + tan(e +
f*x)**4 + 2*tan(e + f*x)**3*i + 2*tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)
*b*f))/(3*a**3*c**2*f*(tan(e + f*x)**4 + 2*tan(e + f*x)**2 + 1))
```

3.850 $\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 206

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{-iA-B}{5f(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}} + \frac{iA}{5cf(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}} + \frac{4A\tan(e+fx)}{15acf(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}} + \frac{8A\tan(e+fx)}{15a^2c^2f\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}$$

output
```
1/5*(-I*A-B)/f/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2)+1/5*I*A/c
/f/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(3/2)+4/15*A*tan(f*x+e)/a/c
/f/(a+I*a*tan(f*x+e))^(3/2)/(c-I*c*tan(f*x+e))^(3/2)+8/15*A*tan(f*x+e)/a^2
/c^2/f/(a+I*a*tan(f*x+e))^(1/2)/(c-I*c*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 6.96 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.44

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\cos^4(e+fx)\,(-3B+15A\tan(e+fx)+20A\tan^3(\ldots}{15a^2c^2f\sqrt{a+ia\tan(e+fx)}\sqrt{c-i\ldots}}$$

input
```
Integrate[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[
e + f*x])^(5/2)),x]
```

output
```
(Cos[e + f*x]^4*(-3*B + 15*A*Tan[e + f*x] + 20*A*Tan[e + f*x]^3 + 8*A*Tan[
e + f*x]^5))/(15*a^2*c^2*f*Sqrt[a + I*a*Tan[e + f*x]]*Sqrt[c - I*c*Tan[e +
 f*x]])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 4071, 87, 55, 42, 41}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int \frac{A+B\tan(e+fx)}{(i\tan(e+fx)a+a)^{7/2}(c-ic\tan(e+fx))^{7/2}}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{ac\left(\frac{A\int \frac{1}{(i\tan(e+fx)a+a)^{7/2}(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{c}-\frac{B+iA}{5ac(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\right)}{f}$$

$\downarrow$ 55

$$\frac{ac\left(\frac{A\left(\frac{4\int \frac{1}{(i\tan(e+fx)a+a)^{5/2}(c-ic\tan(e+fx))^{5/2}}d\tan(e+fx)}{5a}+\frac{i}{5ac(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\right)}{c}-\frac{B+iA}{5ac(a+ia\tan(e+fx))^{5/2}(c-ic\tan\ldots}\right.}{f}$$

$\downarrow$ 42

$$\frac{ac\left(\frac{A\left(\frac{4\left(\frac{2\int \frac{1}{(i\tan(e+fx)a+a)^{3/2}(c-ic\tan(e+fx))^{3/2}}d\tan(e+fx)}{3ac}+\frac{\tan(e+fx)}{3ac(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\right)}{5a}+\frac{i}{5ac(a+ia\tan(e+fx))^{5/2}(c-ic\tan\ldots}\right.}{c}\right.}{f}$$

$\downarrow$ 41

$$\frac{ac\left(\frac{A\left(\frac{4\left(\frac{2\tan(e+fx)}{3a^2c^2\sqrt{a+ia\tan(e+fx)}\sqrt{c-ic\tan(e+fx)}}+\frac{\tan(e+fx)}{3ac(a+ia\tan(e+fx))^{3/2}(c-ic\tan(e+fx))^{3/2}}\right)}{5a}+\frac{i}{5ac(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{3/2}}\right)}{c}-\ldots\right.}{f}$$

input

```
Int[(A + B*Tan[e + f*x])/((a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*
x])^(5/2)),x]
```

output

```
(a*c*(-1/5*(I*A + B)/(a*c*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e + f*
x])^(5/2)) + (A*((I/5)/(a*c*(a + I*a*Tan[e + f*x])^(5/2)*(c - I*c*Tan[e +
f*x])^(3/2)) + (4*(Tan[e + f*x]/(3*a*c*(a + I*a*Tan[e + f*x])^(3/2)*(c - I
*c*Tan[e + f*x])^(3/2)) + (2*Tan[e + f*x])/(3*a^2*c^2*Sqrt[a + I*a*Tan[e +
 f*x]]*Sqrt[c - I*c*Tan[e + f*x]])))/(5*a)))/c))/f
```

Defintions of rubi rules used

rule 41
```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

rule 42
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Simp[(2*m +
3)/(2*a*c*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.60

method	result
derivativedivides	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(8A\tan(fx+e)^7+28A\tan(fx+e)^5+35A\tan(fx+e)^3-3B\tan(fx+e)^2+1\right.}{15f\,a^3c^3(i+\tan(fx+e))^4(i-\tan(fx+e))^4}$
default	$\frac{\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(8A\tan(fx+e)^7+28A\tan(fx+e)^5+35A\tan(fx+e)^3-3B\tan(fx+e)^2+1\right.}{15f\,a^3c^3(i+\tan(fx+e))^4(i-\tan(fx+e))^4}$
parts	$\frac{A\sqrt{a(1+i\tan(fx+e))}\,\sqrt{-c(i\tan(fx+e)-1)}\,\left(1+\tan(fx+e)^2\right)\tan(fx+e)\left(8\tan(fx+e)^4+20\tan(fx+e)^2+15\right)}{15f\,a^3c^3(i-\tan(fx+e))^4(i+\tan(fx+e))^4}-$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/15/f*(a*(1+I*tan(f*x+e)))^(1/2)*(-c*(I*tan(f*x+e)-1))^(1/2)/a^3/c^3*(8*A
*tan(f*x+e)^7+28*A*tan(f*x+e)^5+35*A*tan(f*x+e)^3-3*B*tan(f*x+e)^2+15*A*ta
n(f*x+e)-3*B)/(I+tan(f*x+e))^4/(I-tan(f*x+e))^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.90

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx =$$
$$-\frac{(3\,(i\,A+B)e^{(12i\,fx+12i\,e)}+2\,(14i\,A+9\,B)e^{(10i\,fx+10i\,e)}+5\,(35i\,A+9\,B)e^{(8i\,fx+8i\,e)}-96\,Be^{(7i\,fx+7i\,e)}+6}{}$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="fricas")
```

output
```
-1/480*(3*(I*A + B)*e^(12*I*f*x + 12*I*e) + 2*(14*I*A + 9*B)*e^(10*I*f*x +
 10*I*e) + 5*(35*I*A + 9*B)*e^(8*I*f*x + 8*I*e) - 96*B*e^(7*I*f*x + 7*I*e)
 + 60*B*e^(6*I*f*x + 6*I*e) - 96*B*e^(5*I*f*x + 5*I*e) + 5*(-35*I*A + 9*B)
*e^(4*I*f*x + 4*I*e) + 2*(-14*I*A + 9*B)*e^(2*I*f*x + 2*I*e) - 3*I*A + 3*B
)*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(c/(e^(2*I*f*x + 2*I*e) + 1))*e^(-
5*I*f*x - 5*I*e)/(a^3*c^3*f)
```

Sympy [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx=\int \frac{A+B\tan{(e+fx)}}{(ia(\tan{(e+fx)}-i))^{\frac{5}{2}}(-ic(\tan{(e+fx)}+i))$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))**(5/2)/(c-I*c*tan(f*x+e))**(
5/2),x)
```

output
```
Integral((A + B*tan(e + f*x))/((I*a*(tan(e + f*x) - I))**(5/2)*(-I*c*(tan(
e + f*x) + I))**(5/2)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(160) = 320.

Time = 0.22 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.62

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx=\frac{(30(5iA-B)\cos(4fx+4e)+5(5iA-3B)\cos$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="maxima")
```

output
```
1/480*((30*(5*I*A - B)*cos(4*f*x + 4*e) + 5*(5*I*A - 3*B)*cos(2*f*x + 2*e)
 - 30*(5*A + I*B)*sin(4*f*x + 4*e) - 5*(5*A + 3*I*B)*sin(2*f*x + 2*e) - 6*
B)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 5*(-5*I*A - 3*B)
*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(-5*I*A - B)*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (30*(5*A + I*B)*cos(4
*f*x + 4*e) + 5*(5*A + 3*I*B)*cos(2*f*x + 2*e) + 30*(5*I*A - B)*sin(4*f*x
+ 4*e) + 5*(5*I*A - 3*B)*sin(2*f*x + 2*e) + 6*A)*sin(5/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 5*(5*A - 3*I*B)*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 30*(5*A - I*B)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))/(a^(5/2)*c^(5/2)*f)
```

Giac [F]

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx=\int \frac{B\tan\left(fx+e\right)+A}{\left(i\,a\tan\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-i\,c\tan\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,d$$

input
```
integrate((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/
2),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)/((I*a*tan(f*x + e) + a)^(5/2)*(-I*c*tan(f*x
 + e) + c)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 7.22 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.21

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx=\frac{\sqrt{\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}}\,(175\,A\,\sin\left(2\,e+2\,f\right.}{}$$

input
```
int((A + B*tan(e + f*x))/((a + a*tan(e + f*x)*1i)^(5/2)*(c - c*tan(e + f*x
)*1i)^(5/2)),x)
```

output
```
(((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))
^(1/2)*(A*cos(2*e + 2*f*x)*125i - 30*B - A*150i + A*cos(4*e + 4*f*x)*22i +
 A*cos(6*e + 6*f*x)*3i - 45*B*cos(2*e + 2*f*x) - 18*B*cos(4*e + 4*f*x) - 3
*B*cos(6*e + 6*f*x) + 175*A*sin(2*e + 2*f*x) + 28*A*sin(4*e + 4*f*x) + 3*A
*sin(6*e + 6*f*x) + B*sin(2*e + 2*f*x)*15i + B*sin(4*e + 4*f*x)*12i + B*si
n(6*e + 6*f*x)*3i))/(480*a^3*c^2*f*((c*(cos(2*e + 2*f*x) - sin(2*e + 2*f*x
)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.51

$$\int \frac{A+B\tan(e+fx)}{(a+ia\tan(e+fx))^{5/2}(c-ic\tan(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,\sqrt{\tan(fx+e)\,i+1}\,\sqrt{-\tan(fx+e)\,i+1}\,(}{15a^3c^3f\left(\tan(fx+e)^6+\right.}$$

input
```
int((A+B*tan(f*x+e))/(a+I*a*tan(f*x+e))^(5/2)/(c-I*c*tan(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*sqrt(tan(e + f*x)*i + 1)*sqrt( - tan(e + f*x)*i + 1)*(8*t
an(e + f*x)**5*a + 20*tan(e + f*x)**3*a + 15*tan(e + f*x)*a - 3*b))/(15*a*
*3*c**3*f*(tan(e + f*x)**6 + 3*tan(e + f*x)**4 + 3*tan(e + f*x)**2 + 1))
```

3.851 $\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 41, antiderivative size = 150

$$\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx = \frac{(iA+B)(a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^n}{2fn} - \frac{2^{-1+n}(B(m-n)+iA(m+n))\,\text{Hypergeometric2F1}\left(m,-n,1+m,\frac{1}{2}(1+i\tan(e+fx))\right)(1-i\tan(}{fmn}$$

output
```
1/2*(I*A+B)*(a+I*a*tan(f*x+e))^m*(c-I*c*tan(f*x+e))^n/f/n-2^(-1+n)*(B*(m-n
)+I*A*(m+n))*hypergeom([m, -n],[1+m],1/2+1/2*I*tan(f*x+e))*(a+I*a*tan(f*x+
e))^m*(c-I*c*tan(f*x+e))^n/f/m/n/((1-I*tan(f*x+e))^n)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$
$$=\frac{2^{-1+n}\left((-iA-B)\,\text{Hypergeometric2F1}\left(m,1-n,1+m,\frac{1}{2}(1+i\tan(e+fx))\right)+2B\,\text{Hypergeometric2F}\ldots\right.}{\ldots}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^m*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^n,x]
```

output
```
(2^(-1 + n)*(((-I)*A - B)*Hypergeometric2F1[m, 1 - n, 1 + m, (1 + I*Tan[e
+ f*x])/2] + 2*B*Hypergeometric2F1[m, -n, 1 + m, (1 + I*Tan[e + f*x])/2])*
(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^n)/(f*m*(1 - I*Tan[e + f*x
])^n)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {3042, 4071, 88, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int (i\tan(e+fx)a+a)^{m-1}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{n-1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{88}$$

$$\frac{ac\left(\frac{(B+iA)(a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^n}{2acn}-\frac{(-A(m+n)+iB(m-n))\int(i\tan(e+fx)a+a)^{m-1}(c-ic\tan(e+fx))^nd\tan(e+fx)}{2cn}\right)}{f}$$

$\Big\downarrow$ 80

$$\frac{ac\left(\frac{(B+iA)(a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^n}{2acn}-\frac{2^{n-1}(-A(m+n)+iB(m-n))(1-i\tan(e+fx))^{-n}(c-ic\tan(e+fx))^n\int\left(\frac{1}{2}-\frac{1}{2}i\tan(e+f\right.}{cn}\right.}{f}$$

$\Big\downarrow$ 79

$$\frac{ac\left(\frac{i2^{n-1}(-A(m+n)+iB(m-n))(1-i\tan(e+fx))^{-n}(a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^n\text{Hypergeometric2F1}\left(m,-n,m+1,\frac{1}{2}(i\tan(e+fx)-\right.}{acmn}\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^m*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f*x])^n
,x]
```

output
```
(a*c*(((I*A + B)*(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x])^n)/(2*a*c
*n) + (I*2^(-1 + n)*(I*B*(m - n) - A*(m + n))*Hypergeometric2F1[m, -n, 1 +
 m, (1 + I*Tan[e + f*x])/2]*(a + I*a*Tan[e + f*x])^m*(c - I*c*Tan[e + f*x]
)^n)/(a*c*m*n*(1 - I*Tan[e + f*x])^n)))/f
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (a+ia\tan (fx+e))^m (A+B\tan (fx+e)) (c-ic\tan (fx+e))^n \,dx$$

input
```
int((a+I*a*tan(f*x+e))^m*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x)
```

output
```
int((a+I*a*tan(f*x+e))^m*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx \\ = \int (B\tan (fx+e)+A)(i\,a\tan (fx+e)+a)^m(-i\,c\tan (fx+e)+c)^n\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^m*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, al
gorithm="fricas")
```

output
```
integral(((A - I*B)*e^(2*I*f*x + 2*I*e) + A + I*B)*(2*c/(e^(2*I*f*x + 2*I*
e) + 1))^n*e^(2*I*f*m*x + 2*I*e*m + m*log(a/c) + m*log(2*c/(e^(2*I*f*x + 2
*I*e) + 1)))/(e^(2*I*f*x + 2*I*e) + 1), x)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$
$$=\int (ia(\tan(e+fx)-i))^m\,(-ic(\tan(e+fx)+i))^n\,(A+B\tan(e+fx))\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))**m*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**n,x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**m*(-I*c*(tan(e + f*x) + I))**n*(A + B*t
an(e + f*x)), x)
```

Maxima [F]

$$\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$
$$=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^m(-i\,c\tan(fx+e)+c)^n\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^m*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, al
gorithm="maxima")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^m*(-I*c*tan(f*x + e)
 + c)^n, x)
```

Giac [F]

$$\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$
$$=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^m(-i\,c\tan(fx+e)+c)^n\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^m*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x, al
gorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^m*(-I*c*tan(f*x + e)
 + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$
$$=\int (A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^m\,\left(c-c\tan(e+f\,x)\,1\mathrm{i}\right)^n\,dx$$

input
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)
^n,x)
```

output
```
int((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^m*(c - c*tan(e + f*x)*1i)
^n, x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^n\,dx$$
$$= \frac{-(\tan(fx+e)\,ai+a)^m\,(-\tan(fx+e)\,ci+c)^n\,ai+\left(\int(\tan(fx+e)\,ai+a)^m\,(-\tan(fx+e)\,ci+c)^n\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^m*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^n,x)
```

output
```
( - (tan(e + f*x)*a*i + a)**m*( - tan(e + f*x)*c*i + c)**n*a*i + int((tan(
e + f*x)*a*i + a)**m*( - tan(e + f*x)*c*i + c)**n*tan(e + f*x),x)*a*f*i*m
+ int((tan(e + f*x)*a*i + a)**m*( - tan(e + f*x)*c*i + c)**n*tan(e + f*x),
x)*a*f*i*n + int((tan(e + f*x)*a*i + a)**m*( - tan(e + f*x)*c*i + c)**n*ta
n(e + f*x),x)*b*f*m - int((tan(e + f*x)*a*i + a)**m*( - tan(e + f*x)*c*i +
 c)**n*tan(e + f*x),x)*b*f*n)/(f*(m - n))
```

3.852 $\int (a+ia\tan(e+fx))^{1+m}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-1-m}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 151

$$\int (a+ia\tan(e+fx))^{1+m}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-1-m}\,dx$$
$$= -\frac{(iA+B)(a+ia\tan(e+fx))^{1+m}(c-ic\tan(e+fx))^{-1-m}}{2f(1+m)} + \frac{2^{-1-m}B\,\text{Hypergeometric2F1}\left(1+m,1+m,2+m,\frac{1}{2}(1+i\tan(e+fx))\right)(1-i\tan(e+fx))^{1+m}(a+ \ldots}{f(1+m)}$$

output
```
-1/2*(I*A+B)*(a+I*a*tan(f*x+e))^(1+m)*(c-I*c*tan(f*x+e))^(-1-m)/f/(1+m)+2^
(-1-m)*B*hypergeom([1+m, 1+m],[2+m],1/2+1/2*I*tan(f*x+e))*(1-I*tan(f*x+e))
^(1+m)*(a+I*a*tan(f*x+e))^(1+m)*(c-I*c*tan(f*x+e))^(-1-m)/f/(1+m)
```

Mathematica [A] (verified)

Time = 5.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

$$\int (a+ia\tan(e+fx))^{1+m}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-1-m}\,dx$$
$$=\frac{a(a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^{-m}\left(\frac{2^{1+m}B\,\text{Hypergeometric2F1}\left(-m,-m,1-m,-\frac{1}{2}i(i+\tan(e+fx))\right)(1+i\tan(e-\right.}{m}}{2cf}$$

input
```
Integrate[(a + I*a*Tan[e + f*x])^(1 + m)*(A + B*Tan[e + f*x])*(c - I*c*Tan
[e + f*x])^(-1 - m),x]
```

output
```
(a*(a + I*a*Tan[e + f*x])^m*((2^(1 + m)*B*Hypergeometric2F1[-m, -m, 1 - m,
 (-1/2*I)*(I + Tan[e + f*x])])/(m*(1 + I*Tan[e + f*x])^m) + ((I*A + B)*(-I
 + Tan[e + f*x]))/((1 + m)*(I + Tan[e + f*x]))))/(2*c*f*(c - I*c*Tan[e + f
*x])^m)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.106, Rules used = {3042, 4071, 88, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+ia\tan(e+fx))^{m+1}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-m-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+ia\tan(e+fx))^{m+1}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-m-1}dx$$

$$\Big\downarrow \text{4071}$$

$$\frac{ac\int (i\tan(e+fx)a+a)^m(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-m-2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{88}$$

$$\frac{ac\left(\frac{iB\int (i\tan(e+fx)a+a)^m(c-ic\tan(e+fx))^{-m-1}d\tan(e+fx)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{m+1}(c-ic\tan(e+fx))^{-m-1}}{2ac(m+1)}\right)}{f}$$

$\Big\downarrow$ 80

$$\frac{ac\left(\frac{iB2^m(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m\int\left(\frac{1}{2}i\tan(e+fx)+\frac{1}{2}\right)^m(c-ic\tan(e+fx))^{-m-1}d\tan(e+fx)}{c}-\frac{(B+iA)(a+ia\tan(e+fx))^{m}}{2ac(m}\right.}{f}$$

$\Big\downarrow$ 79

$$\frac{ac\left(\frac{B2^m(1+i\tan(e+fx))^{-m}(a+ia\tan(e+fx))^m(c-ic\tan(e+fx))^{-m}\text{Hypergeometric2F1}\left(-m,-m,1-m,\frac{1}{2}(1-i\tan(e+fx))\right)}{c^2m}-\frac{(B+iA)(a+}{}\right.}{f}$$

input
```
Int[(a + I*a*Tan[e + f*x])^(1 + m)*(A + B*Tan[e + f*x])*(c - I*c*Tan[e + f
*x])^(-1 - m),x]
```

output
```
(a*c*(-1/2*((I*A + B)*(a + I*a*Tan[e + f*x])^(1 + m)*(c - I*c*Tan[e + f*x]
)^(-1 - m))/(a*c*(1 + m)) + (2^m*B*Hypergeometric2F1[-m, -m, 1 - m, (1 - I
*Tan[e + f*x])/2]*(a + I*a*Tan[e + f*x])^m)/(c^2*m*(1 + I*Tan[e + f*x])^m*
(c - I*c*Tan[e + f*x])^m)))/f
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [F]

$$\int (a+ia\tan(fx+e))^{1+m}(A+B\tan(fx+e))(c-ic\tan(fx+e))^{-1-m}\,dx$$

input
```
int((a+I*a*tan(f*x+e))^(1+m)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(-1-m),x)
```

output
```
int((a+I*a*tan(f*x+e))^(1+m)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(-1-m),x)
```

Fricas [F]

$$\int (a+ia\tan(e+fx))^{1+m}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-1-m}\,dx$$
$$=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{m+1}(-i\,c\tan(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(1+m)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(-1
-m),x, algorithm="fricas")
```

output
```
integral(((A - I*B)*e^(2*I*f*x + 2*I*e) + A + I*B)*(2*c/(e^(2*I*f*x + 2*I*
e) + 1))^(-m - 1)*e^(2*I*e*m - 2*(-I*f*m - I*f)*x + (m + 1)*log(a/c) + (m
+ 1)*log(2*c/(e^(2*I*f*x + 2*I*e) + 1)) + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1)
, x)
```

Sympy [F]

$$\int (a+ia\tan(e+fx))^{1+m}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-1-m}\,dx$$
$$=\int (ia(\tan(e+fx)-i))^{m+1}\,(-ic(\tan(e+fx)+i))^{-m-1}\,(A+B\tan(e+fx))\;dx$$

input
```
integrate((a+I*a*tan(f*x+e))**(1+m)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))**(
-1-m),x)
```

output
```
Integral((I*a*(tan(e + f*x) - I))**(m + 1)*(-I*c*(tan(e + f*x) + I))**(-m
- 1)*(A + B*tan(e + f*x)), x)
```

Maxima [F]

$$\int (a+ia\tan(e+fx))^{1+m}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-1-m}\,dx$$
$$=\int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{m+1}(-i\,c\tan(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(1+m)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(-1
-m),x, algorithm="maxima")
```

output
```
-(2*(-I*B*a^(m + 1)*m - I*B*a^(m + 1))*cos(2*f*m*x + 2*e*m) + ((A - I*B)*a
^(m + 1)*m^2 - (A - I*B)*a^(m + 1)*m)*cos(2*e*m + 2*(f*m + 2*f)*x + 4*e) +
 ((A + I*B)*a^(m + 1)*m^2 - (A - I*B)*a^(m + 1)*m - 2*I*B*a^(m + 1))*cos(2
*e*m + 2*(f*m + f)*x + 2*e) - 4*(B*a^(m + 1)*c^(m + 1)*f*m^3 - B*a^(m + 1)
*c^(m + 1)*f*m + (B*a^(m + 1)*c^(m + 1)*f*m^3 - B*a^(m + 1)*c^(m + 1)*f*m)
*cos(2*f*x + 2*e) - (-I*B*a^(m + 1)*c^(m + 1)*f*m^3 + I*B*a^(m + 1)*c^(m +
 1)*f*m)*sin(2*f*x + 2*e))*integrate(((cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*
e) + 1)*cos(2*f*m*x + 2*e*m) + (sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin
(2*f*m*x + 2*e*m))/((c^(m + 1)*m - c^(m + 1))*cos(4*f*x + 4*e)^2 + 4*(c^(m
 + 1)*m - c^(m + 1))*cos(2*f*x + 2*e)^2 + (c^(m + 1)*m - c^(m + 1))*sin(4*
f*x + 4*e)^2 + 4*(c^(m + 1)*m - c^(m + 1))*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) + 4*(c^(m + 1)*m - c^(m + 1))*sin(2*f*x + 2*e)^2 + c^(m + 1)*m + 2*(c^(
m + 1)*m + 2*(c^(m + 1)*m - c^(m + 1))*cos(2*f*x + 2*e) - c^(m + 1))*cos(4
*f*x + 4*e) + 4*(c^(m + 1)*m - c^(m + 1))*cos(2*f*x + 2*e) - c^(m + 1)), x
) + 4*(-I*B*a^(m + 1)*c^(m + 1)*f*m^3 + I*B*a^(m + 1)*c^(m + 1)*f*m + (-I*
B*a^(m + 1)*c^(m + 1)*f*m^3 + I*B*a^(m + 1)*c^(m + 1)*f*m)*cos(2*f*x + 2*e
) + (B*a^(m + 1)*c^(m + 1)*f*m^3 - B*a^(m + 1)*c^(m + 1)*f*m)*sin(2*f*x +
2*e))*integrate(-((sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(2*f*m*x + 2*
e*m) - (cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*m*x + 2*e*m))/(
(c^(m + 1)*m - c^(m + 1))*cos(4*f*x + 4*e)^2 + 4*(c^(m + 1)*m - c^(m + ...
```

Giac [F]

$$\int (a+ia\tan(e+fx))^{1+m}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-1-m}\,dx = \int (B\tan(fx+e)+A)(i\,a\tan(fx+e)+a)^{m+1}(-i\,c\tan(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a+I*a*tan(f*x+e))^(1+m)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(-1
-m),x, algorithm="giac")
```

output
```
integrate((B*tan(f*x + e) + A)*(I*a*tan(f*x + e) + a)^(m + 1)*(-I*c*tan(f*
x + e) + c)^(-m - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\tan(e+fx))^{1+m}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-1-m}\,dx$$
$$=\int \frac{(A+B\tan(e+f\,x))\,\left(a+a\tan(e+f\,x)\,1\mathrm{i}\right)^{m+1}}{\left(c-c\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{m+1}}\,dx$$

input
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(m + 1))/(c - c*tan(e +
f*x)*1i)^(m + 1),x)
```

output
```
int(((A + B*tan(e + f*x))*(a + a*tan(e + f*x)*1i)^(m + 1))/(c - c*tan(e +
f*x)*1i)^(m + 1), x)
```

Reduce [F]

$$\int (a+ia\tan(e+fx))^{1+m}(A+B\tan(e+fx))(c-ic\tan(e+fx))^{-1-m}\,dx =$$
$$-\frac{a\left(\left(\int \frac{(\tan(fx+e)ai+a)^m}{(-\tan(fx+e)ci+c)^m\tan(fx+e)i-(-\tan(fx+e)ci+c)^m}dx\right)a+\left(\int \frac{(\tan(fx+e)ai+a)^m\tan(fx+e)^2}{(-\tan(fx+e)ci+c)^m\tan(fx+e)i-(-\tan(fx+e)ci+c)^m}\right.\right.}{}$$

input
```
int((a+I*a*tan(f*x+e))^(1+m)*(A+B*tan(f*x+e))*(c-I*c*tan(f*x+e))^(-1-m),x)
```

output
```
( - a*(int((tan(e + f*x)*a*i + a)**m/(( - tan(e + f*x)*c*i + c)**m*tan(e +
 f*x)*i - ( - tan(e + f*x)*c*i + c)**m),x)*a + int(((tan(e + f*x)*a*i + a)
**m*tan(e + f*x)**2)/(( - tan(e + f*x)*c*i + c)**m*tan(e + f*x)*i - ( - ta
n(e + f*x)*c*i + c)**m),x)*b*i + int(((tan(e + f*x)*a*i + a)**m*tan(e + f*
x))/(( - tan(e + f*x)*c*i + c)**m*tan(e + f*x)*i - ( - tan(e + f*x)*c*i +
c)**m),x)*a*i + int(((tan(e + f*x)*a*i + a)**m*tan(e + f*x))/(( - tan(e +
f*x)*c*i + c)**m*tan(e + f*x)*i - ( - tan(e + f*x)*c*i + c)**m),x)*b))/c
```

3.853 $\int \frac{(c-ic\tan(e+fx))^n(-i(2+n)+(-2+n)\tan(e+fx))}{(-i+\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 33

$$\int \frac{(c-ic\tan(e+fx))^n(-i(2+n)+(-2+n)\tan(e+fx))}{(-i+\tan(e+fx))^2}\,dx = \frac{(c-ic\tan(e+fx))^n}{f(i-\tan(e+fx))^2}$$

output
```
(c-I*c*tan(f*x+e))^n/f/(I-tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 5.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{(c-ic\tan(e+fx))^n(-i(2+n)+(-2+n)\tan(e+fx))}{(-i+\tan(e+fx))^2}\,dx = \frac{(c-ic\tan(e+fx))^n}{f(-i+\tan(e+fx))^2}$$

input
```
Integrate[((c - I*c*Tan[e + f*x])^n*((-I)*(2 + n) + (-2 + n)*Tan[e + f*x])
)/(-I + Tan[e + f*x])^2,x]
```

output
```
(c - I*c*Tan[e + f*x])^n/(f*(-I + Tan[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {3042, 4071, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{((n-2)\tan(e+fx)-i(n+2))(c-ic\tan(e+fx))^n}{(\tan(e+fx)-i)^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{((n-2)\tan(e+fx)-i(n+2))(c-ic\tan(e+fx))^n}{(\tan(e+fx)-i)^2}dx$$

$$\Big\downarrow \text{ 4071}$$

$$-\frac{ic\int \frac{(c-ic\tan(e+fx))^{n-1}(i(n+2)+(2-n)\tan(e+fx))}{(i-\tan(e+fx))^3}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 83}$$

$$\frac{(c-ic\tan(e+fx))^n}{f(-\tan(e+fx)+i)^2}$$

input
```
Int[((c - I*c*Tan[e + f*x])^n*((-I)*(2 + n) + (-2 + n)*Tan[e + f*x]))/(-I
+ Tan[e + f*x])^2,x]
```

output
```
(c - I*c*Tan[e + f*x])^n/(f*(I - Tan[e + f*x])^2)
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4071
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a*(c/f)   Subst[Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(A + B*x), x], x
, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(31) = 62$.

Time = 1.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.79

method	result	size
derivativedivides	$\frac{\frac{\tan(fx+e)^2 e^{n\ln(c-ic\tan(fx+e))}}{f}-\frac{e^{n\ln(c-ic\tan(fx+e))}}{f}+\frac{2i\tan(fx+e)e^{n\ln(c-ic\tan(fx+e))}}{f}}{\left(1+\tan(fx+e)^2\right)^2}$	92
default	$\frac{\frac{\tan(fx+e)^2 e^{n\ln(c-ic\tan(fx+e))}}{f}-\frac{e^{n\ln(c-ic\tan(fx+e))}}{f}+\frac{2i\tan(fx+e)e^{n\ln(c-ic\tan(fx+e))}}{f}}{\left(1+\tan(fx+e)^2\right)^2}$	92

input
```
int((c-I*c*tan(f*x+e))^n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e))^2,x,
method=_RETURNVERBOSE)
```

output
```
(1/f*tan(f*x+e)^2*exp(n*ln(c-I*c*tan(f*x+e)))-1/f*exp(n*ln(c-I*c*tan(f*x+e
)))+2*I/f*tan(f*x+e)*exp(n*ln(c-I*c*tan(f*x+e))))/(1+tan(f*x+e)^2)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int \frac{(c-ic\tan(e+fx))^n(-i(2+n)+(-2+n)\tan(e+fx))}{(-i+\tan(e+fx))^2}\,dx = -\frac{\left(\frac{2c}{e^{(2i\,fx+2i\,e)}+1}\right)^n\left(e^{(4i\,fx+4i\,e)}+2\,e^{(2i\,fx+2i\,e)}+1\right)e^{(-4i\,fx-4i\,e)}}{4\,f}$$

input
```
integrate((c-I*c*tan(f*x+e))^n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e)
)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*c/(e^(2*I*f*x + 2*I*e) + 1))^n*(e^(4*I*f*x + 4*I*e) + 2*e^(2*I*f*x
 + 2*I*e) + 1)*e^(-4*I*f*x - 4*I*e)/f
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(24) = 48.

Time = 0.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{(c-ic\tan(e+fx))^n(-i(2+n)+(-2+n)\tan(e+fx))}{(-i+\tan(e+fx))^2}\,dx = \begin{cases} \frac{(-ic\tan(e+fx)+c)^n}{f\tan^2(e+fx)-2if\tan(e+fx)-f} & \text{for } f\neq 0 \\ \frac{x((n-2)\tan(e)-i(n+2))(-ic\tan(e)+c)^n}{(\tan(e)-i)^2} & \text{otherwise} \end{cases}$$

input
```
integrate((c-I*c*tan(f*x+e))**n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e
))**2,x)
```

output
```
Piecewise(((-I*c*tan(e + f*x) + c)**n/(f*tan(e + f*x)**2 - 2*I*f*tan(e + f
*x) - f), Ne(f, 0)), (x*((n - 2)*tan(e) - I*(n + 2))*(-I*c*tan(e) + c)**n/
(tan(e) - I)**2, True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c-ic\tan(e+fx))^n(-i(2+n)+(-2+n)\tan(e+fx))}{(-i+\tan(e+fx))^2}\,dx$$
$$= \text{Exception raised: RuntimeError}$$

input
```
integrate((c-I*c*tan(f*x+e))^n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e)
)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int \frac{(c-ic\tan(e+fx))^n(-i(2+n)+(-2+n)\tan(e+fx))}{(-i+\tan(e+fx))^2}\,dx$$
$$= \int \frac{((n-2)\tan(fx+e)-i\,n-2i)(-i\,c\tan(fx+e)+c)^n}{(\tan(fx+e)-i)^2}\,dx$$

input
```
integrate((c-I*c*tan(f*x+e))^n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e)
)^2,x, algorithm="giac")
```

output
```
integrate(((n - 2)*tan(f*x + e) - I*n - 2*I)*(-I*c*tan(f*x + e) + c)^n/(ta
n(f*x + e) - I)^2, x)
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.73

$$\int \frac{(c-ic\tan(e+fx))^n(-i(2+n)+(-2+n)\tan(e+fx))}{(-i+\tan(e+fx))^2}\,dx$$
$$=\frac{\left(-\frac{c\left(-2\cos(e+f\,x)^2+\sin(2\,e+2\,f\,x)\,1\mathrm{i}\right)}{2\cos(e+f\,x)^2}\right)^n\left(-4\cos\left(e+f\,x\right)^2-2\cos\left(2\,e+2\,f\,x\right)^2+\sin\left(2\,e+2\,f\,x\right)\,2\mathrm{i}+\sin\left(4\right.\right.}{4\,f}$$

input
```
int(-((c - c*tan(e + f*x)*1i)^n*(n*1i - tan(e + f*x)*(n - 2) + 2i))/(tan(e
 + f*x) - 1i)^2,x)
```

output
```
((-(c*(sin(2*e + 2*f*x)*1i - 2*cos(e + f*x)^2))/(2*cos(e + f*x)^2))^n*(sin
(2*e + 2*f*x)*2i + sin(4*e + 4*f*x)*1i - 2*cos(2*e + 2*f*x)^2 - 4*cos(e +
f*x)^2 + 2))/(4*f)
```

Reduce [F]

$$\int \frac{(c-ic\tan(e+fx))^n(-i(2+n)+(-2+n)\tan(e+fx))}{(-i+\tan(e+fx))^2}\,dx$$
$$=-\left(\int \frac{(-\tan(fx+e)\,ci+c)^n}{\tan(fx+e)^2-2\tan(fx+e)\,i-1}dx\right)in$$
$$-2\left(\int \frac{(-\tan(fx+e)\,ci+c)^n}{\tan(fx+e)^2-2\tan(fx+e)\,i-1}dx\right)i$$
$$+\left(\int \frac{(-\tan(fx+e)\,ci+c)^n\tan(fx+e)}{\tan(fx+e)^2-2\tan(fx+e)\,i-1}dx\right)n$$
$$-2\left(\int \frac{(-\tan(fx+e)\,ci+c)^n\tan(fx+e)}{\tan(fx+e)^2-2\tan(fx+e)\,i-1}dx\right)$$

input
```
int((c-I*c*tan(f*x+e))^n*(-I*(2+n)+(-2+n)*tan(f*x+e))/(-I+tan(f*x+e))^2,x)
```

output

```
 - int(( - tan(e + f*x)*c*i + c)**n/(tan(e + f*x)**2 - 2*tan(e + f*x)*i -
1),x)*i*n - 2*int(( - tan(e + f*x)*c*i + c)**n/(tan(e + f*x)**2 - 2*tan(e
+ f*x)*i - 1),x)*i + int((( - tan(e + f*x)*c*i + c)**n*tan(e + f*x))/(tan(
e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*n - 2*int((( - tan(e + f*x)*c*i + c
)**n*tan(e + f*x))/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)
```

3.854 $\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 104

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = \frac{(A-iB)(c-id)x}{4a^2}+\frac{B(c+3id)+A(ic+d)}{4a^2f(1+i\tan(e+fx))}+\frac{(iA-B)(c+id)}{4f(a+ia\tan(e+fx))^2}$$

output
```
1/4*(A-I*B)*(c-I*d)*x/a^2+1/4*(B*(c+3*I*d)+A*(I*c+d))/a^2/f/(1+I*tan(f*x+e
))+1/4*(I*A-B)*(c+I*d)/f/(a+I*a*tan(f*x+e))^2
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = \frac{(A-iB)(c-id)\arctan(\tan(e+fx))+\frac{(A+iB)(-ic+d)}{(-i+\tan(e+fx))^2}+\frac{Ac-iBc-iAd+3Bd}{-i+\tan(e+fx)}}{4a^2f}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c + d*Tan[e + f*x]))/(a + I*a*Tan[e + f*x
])^2,x]
```

output
```
((A - I*B)*(c - I*d)*ArcTan[Tan[e + f*x]] + ((A + I*B)*((-I)*c + d))/(-I +
 Tan[e + f*x])^2 + (A*c - I*B*c - I*A*d + 3*B*d)/(-I + Tan[e + f*x]))/(4*a
^2*f)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3042, 4073, 3042, 4009, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}dx$$

$$\Big\downarrow \text{4073}$$

$$\frac{(-B+iA)(c+id)}{4f(a+ia\tan(e+fx))^2}-\frac{i\int \frac{a(B(c+id)+A(ic+d))+2aBd\tan(e+fx)}{i\tan(e+fx)a+a}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)(c+id)}{4f(a+ia\tan(e+fx))^2}-\frac{i\int \frac{a(B(c+id)+A(ic+d))+2aBd\tan(e+fx)}{i\tan(e+fx)a+a}dx}{2a^2}$$

$$\Big\downarrow \text{4009}$$

$$\frac{(-B+iA)(c+id)}{4f(a+ia\tan(e+fx))^2}-\frac{i\left(\frac{1}{2}(B+iA)(c-id)\int 1dx-\frac{Ac-iAd-iBc+3Bd}{2f(1+i\tan(e+fx))}\right)}{2a^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{(-B+iA)(c+id)}{4f(a+ia\tan(e+fx))^2}-\frac{i\left(\frac{1}{2}x(B+iA)(c-id)-\frac{Ac-iAd-iBc+3Bd}{2f(1+i\tan(e+fx))}\right)}{2a^2}$$

input
```
Int[((A + B*Tan[e + f*x])*(c + d*Tan[e + f*x]))/(a + I*a*Tan[e + f*x])^2,x
]
```

output
```
((-1/2*I)*(((I*A + B)*(c - I*d)*x)/2 - (A*c - I*B*c - I*A*d + 3*B*d)/(2*f*
(1 + I*Tan[e + f*x]))))/a^2 + ((I*A - B)*(c + I*d))/(4*f*(a + I*a*Tan[e +
f*x])^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{ixAd}{4a^2}-\frac{ixBc}{4a^2}+\frac{xcA}{4a^2}-\frac{xBd}{4a^2}+\frac{i\mathrm{e}^{-2i(fx+e)}cA}{4f\,a^2}+\frac{i\mathrm{e}^{-2i(fx+e)}Bd}{4f\,a^2}-\frac{\mathrm{e}^{-4i(fx+e)}Ad}{16f\,a^2}-\frac{\mathrm{e}^{-4i(fx+e)}Bc}{16f\,a^2}+ i$
derivativedivides	$-\frac{iAd\arctan(\tan(fx+e))}{4f\,a^2}+\frac{Ac\arctan(\tan(fx+e))}{4f\,a^2}-\frac{iAc}{4f\,a^2(-i+\tan(fx+e))^2}-\frac{Bd\arctan(\tan(fx+e))}{4f\,a^2}-\frac{iBc}{}$
default	$-\frac{iAd\arctan(\tan(fx+e))}{4f\,a^2}+\frac{Ac\arctan(\tan(fx+e))}{4f\,a^2}-\frac{iAc}{4f\,a^2(-i+\tan(fx+e))^2}-\frac{Bd\arctan(\tan(fx+e))}{4f\,a^2}-\frac{iBc}{}$

input
```
int((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
-1/4*I*x/a^2*A*d-1/4*I*x/a^2*B*c+1/4*x/a^2*c*A-1/4*x/a^2*B*d+1/4*I/f/a^2*e
xp(-2*I*(f*x+e))*c*A+1/4*I/f/a^2*exp(-2*I*(f*x+e))*B*d-1/16/f/a^2*exp(-4*I
*(f*x+e))*A*d-1/16/f/a^2*exp(-4*I*(f*x+e))*B*c+1/16*I/f/a^2*exp(-4*I*(f*x+
e))*c*A-1/16*I/f/a^2*exp(-4*I*(f*x+e))*B*d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int\frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx$$
$$=\frac{(4((A-iB)c-(iA+B)d)fxe^{(4i\,fx+4i\,e)}+(iA-B)c-(A+iB)d-4(-iAc-iBd)e^{(2i\,fx+2i\,e)})e^{(-4}}{16\,a^2f}$$

input
```
integrate((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algori
thm="fricas")
```

output
```
1/16*(4*((A - I*B)*c - (I*A + B)*d)*f*x*e^(4*I*f*x + 4*I*e) + (I*A - B)*c
- (A + I*B)*d - 4*(-I*A*c - I*B*d)*e^(2*I*f*x + 2*I*e))*e^(-4*I*f*x - 4*I*
e)/(a^2*f)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.85

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = \begin{cases} \frac{((16iAa^2cfe^{4ie}+16iBa^2dfe^{4ie})e^{-2ifx}+(4iAa^2cfe^{2ie}-4Aa^2dfe^{2ie}-4Ba^2cfe^{2ie}-4iBa^2dfe^{2ie})e^{-4ifx})e^{-6ie}}{64a^4f^2} & \text{for } a^4f^2e^{6ie} \neq 0 \\ x\left(-\frac{Ac-iAd-iBc-Bd}{4a^2}+\frac{(Ace^{4ie}+2Ace^{2ie}+Ac-iAde^{4ie}+iAd-iBce^{4ie}+iBc-Bde^{4ie}+2Bde^{2ie}-Bd)e^{-4ie}}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x(Ac-iAd-iBc-Bd)}{4a^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))**2,x)
```

output
```
Piecewise((((16*I*A*a**2*c*f*exp(4*I*e) + 16*I*B*a**2*d*f*exp(4*I*e))*exp(
-2*I*f*x) + (4*I*A*a**2*c*f*exp(2*I*e) - 4*A*a**2*d*f*exp(2*I*e) - 4*B*a**
2*c*f*exp(2*I*e) - 4*I*B*a**2*d*f*exp(2*I*e))*exp(-4*I*f*x))*exp(-6*I*e)/(
64*a**4*f**2), Ne(a**4*f**2*exp(6*I*e), 0)), (x*(-(A*c - I*A*d - I*B*c - B
*d)/(4*a**2) + (A*c*exp(4*I*e) + 2*A*c*exp(2*I*e) + A*c - I*A*d*exp(4*I*e)
 + I*A*d - I*B*c*exp(4*I*e) + I*B*c - B*d*exp(4*I*e) + 2*B*d*exp(2*I*e) -
B*d)*exp(-4*I*e)/(4*a**2)), True)) + x*(A*c - I*A*d - I*B*c - B*d)/(4*a**2
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = -\frac{(-iAc-Bc-Ad+iBd)\log(\tan(fx+e)+i)}{8a^2f} - \frac{(iAc+Bc+Ad-iBd)\log(\tan(fx+e)-i)}{8a^2f} - \frac{2iAc+2iBd+i(iAc+Bc+Ad+3iBd)\tan(fx+e)}{4a^2f(\tan(fx+e)-i)^2}$$

input
```
integrate((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/8*(-I*A*c - B*c - A*d + I*B*d)*log(tan(f*x + e) + I)/(a^2*f) - 1/8*(I*A
*c + B*c + A*d - I*B*d)*log(tan(f*x + e) - I)/(a^2*f) - 1/4*(2*I*A*c + 2*I
*B*d + I*(I*A*c + B*c + A*d + 3*I*B*d)*tan(f*x + e))/(a^2*f*(tan(f*x + e)
- I)^2)
```

Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.53

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = -\frac{B\,d\,f\,x-A\,c\,f\,x+A\,d\,f\,x\,1\mathrm{i}+B\,c\,f\,x\,1\mathrm{i}}{4\,a^2\,f} + \frac{(A\,c+3\,B\,d-A\,d\,1\mathrm{i}-B\,c\,1\mathrm{i})\,\tan(e+f\,x)^3+(2\,A\,d+2\,B\,c+B\,d\,4\mathrm{i})\,\tan(e+f\,x)^2+(3\,A\,c+B\,d+\cdots}{f\,\left(4\,a^2\tan\left(e+f\,x\right)^4+8\,a^2\tan\left(e+f\,x\right)^2+4\,a^2\right)}$$

input
```
int(((A + B*tan(e + f*x))*(c + d*tan(e + f*x)))/(a + a*tan(e + f*x)*1i)^2,
x)
```

output
```
(A*c*2i + B*d*2i + tan(e + f*x)*(3*A*c + A*d*1i + B*c*1i + B*d) + tan(e +
f*x)^2*(2*A*d + 2*B*c + B*d*4i) + tan(e + f*x)^3*(A*c - A*d*1i - B*c*1i +
3*B*d))/(f*(4*a^2 + 8*a^2*tan(e + f*x)^2 + 4*a^2*tan(e + f*x)^4)) - (A*d*f
*x*1i - A*c*f*x + B*c*f*x*1i + B*d*f*x)/(4*a^2*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^2}\,dx = \frac{-\left(\int \frac{\tan(fx+e)^2}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)bd-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)ad-\left(\int \frac{\tan(fx+e)}{\tan(fx+e)^2-2\tan(fx+e)i-1}dx\right)b}{a^2}$$

input
```
int((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^2,x)
```

output
```
( - (int(tan(e + f*x)**2/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*d +
 int(tan(e + f*x)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*d + int(ta
n(e + f*x)/(tan(e + f*x)**2 - 2*tan(e + f*x)*i - 1),x)*b*c + int(1/(tan(e
+ f*x)**2 - 2*tan(e + f*x)*i - 1),x)*a*c))/a**2
```

3.855 $\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 147

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx = -\frac{(iA+B)(c-id)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{(iA-B)(c+id)}{3f(a+ia\tan(e+fx))^{3/2}} + \frac{B(c+3id)+A(ic+d)}{2af\sqrt{a+ia\tan(e+fx)}}$$

output
```
-1/4*(I*A+B)*(c-I*d)*arctanh(1/2*(a+I*a*tan(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*2^(1/2)/a^(3/2)/f+1/3*(I*A-B)*(c+I*d)/f/(a+I*a*tan(f*x+e))^(3/2)+1/2*(B*(
c+3*I*d)+A*(I*c+d))/a/f/(a+I*a*tan(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx = -\frac{i\left(\frac{3\sqrt{2}(A-iB)(c-id)\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{a}}-\frac{4a(A+iB)(c+id)}{(a+ia\tan(e+fx))^{3/2}}-\frac{6(Ac-iBc-iAd+3Bd)}{\sqrt{a+ia\tan(e+fx)}}\right)}{12af}$$

input
```
Integrate[((A + B*Tan[e + f*x])*(c + d*Tan[e + f*x]))/(a + I*a*Tan[e + f*x
])^(3/2),x]
```

output
```
((-1/12*I)*((3*Sqrt[2]*(A - I*B)*(c - I*d)*ArcTanh[Sqrt[a + I*a*Tan[e + f*
x]]/(Sqrt[2]*Sqrt[a])])/Sqrt[a] - (4*a*(A + I*B)*(c + I*d))/(a + I*a*Tan[e
 + f*x])^(3/2) - (6*(A*c - I*B*c - I*A*d + 3*B*d))/Sqrt[a + I*a*Tan[e + f*
x]]))/(a*f)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 4073, 3042, 4009, 3042, 3961, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4073}$$

$$\frac{(-B+iA)(c+id)}{3f(a+ia\tan(e+fx))^{3/2}}-\frac{i\int \frac{a(B(c+id)+A(ic+d))+2aBd\tan(e+fx)}{\sqrt{i\tan(e+fx)a+a}}dx}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)(c+id)}{3f(a+ia\tan(e+fx))^{3/2}}-\frac{i\int \frac{a(B(c+id)+A(ic+d))+2aBd\tan(e+fx)}{\sqrt{i\tan(e+fx)a+a}}dx}{2a^2}$$

$$\Big\downarrow \text{4009}$$

$$\frac{(-B+iA)(c+id)}{3f(a+ia\tan(e+fx))^{3/2}}-\frac{i\left(\frac{1}{2}(B+iA)(c-id)\int \sqrt{i\tan(e+fx)a+a}dx-\frac{a(Ac-iAd-iBc+3Bd)}{f\sqrt{a+ia\tan(e+fx)}}\right)}{2a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(-B+iA)(c+id)}{3f(a+ia\tan(e+fx))^{3/2}}-\frac{i\left(\frac{1}{2}(B+iA)(c-id)\int \sqrt{i\tan(e+fx)a+a}dx-\frac{a(Ac-iAd-iBc+3Bd)}{f\sqrt{a+ia\tan(e+fx)}}\right)}{2a^2}$$

$$\Big\downarrow \text{3961}$$

$$\frac{(-B+iA)(c+id)}{3f(a+ia\tan(e+fx))^{3/2}}-\frac{i\left(-\frac{ia(B+iA)(c-id)\int \frac{1}{a-ia\tan(e+fx)}d\sqrt{i\tan(e+fx)a+a}}{f}-\frac{a(Ac-iAd-iBc+3Bd)}{f\sqrt{a+ia\tan(e+fx)}}\right)}{2a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{(-B+iA)(c+id)}{3f(a+ia\tan(e+fx))^{3/2}}-\frac{i\left(-\frac{i\sqrt{a}(B+iA)(c-id)\text{arctanh}\left(\frac{\sqrt{a+ia\tan(e+fx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}f}-\frac{a(Ac-iAd-iBc+3Bd)}{f\sqrt{a+ia\tan(e+fx)}}\right)}{2a^2}$$

input

```
Int[((A + B*Tan[e + f*x])*(c + d*Tan[e + f*x]))/(a + I*a*Tan[e + f*x])^(3/
2),x]
```

output

```
((I*A - B)*(c + I*d))/(3*f*(a + I*a*Tan[e + f*x])^(3/2)) - ((I/2)*(((-I)*S
qrt[a]*(I*A + B)*(c - I*d)*ArcTanh[Sqrt[a + I*a*Tan[e + f*x]]/(Sqrt[2]*Sqr
t[a])])/(Sqrt[2]*f) - (a*(A*c - I*B*c - I*A*d + 3*B*d))/(f*Sqrt[a + I*a*Ta
n[e + f*x]])))/a^2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3961
```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a
, b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

rule 4009
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Simp[(b*c + a*d)/(2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2
, 0] && LtQ[m, 0]
```

rule 4073
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(
A*b - a*B))*(a*c + b*d)*((a + b*Tan[e + f*x])^m/(2*a^2*f*m)), x] + Simp[1/(
2*a*b)   Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[A*b*c + a*B*c + a*A*d + b*B*
d + 2*a*B*d*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2i\left(-\frac{\frac{1}{4}iAd+\frac{1}{4}iBc-\frac{1}{4}cA-\frac{3}{4}Bd}{\sqrt{a+ia\tan(fx+e)}}+\frac{a(iAd+iBc+cA-Bd)}{6(a+ia\tan(fx+e))^{\frac{3}{2}}}-\frac{\left(-\frac{1}{4}iAd-\frac{1}{4}iBc+\frac{1}{4}cA-\frac{1}{4}Bd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}\right)}{fa}$
default	$\frac{2i\left(-\frac{\frac{1}{4}iAd+\frac{1}{4}iBc-\frac{1}{4}cA-\frac{3}{4}Bd}{\sqrt{a+ia\tan(fx+e)}}+\frac{a(iAd+iBc+cA-Bd)}{6(a+ia\tan(fx+e))^{\frac{3}{2}}}-\frac{\left(-\frac{1}{4}iAd-\frac{1}{4}iBc+\frac{1}{4}cA-\frac{1}{4}Bd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)}{2\sqrt{a}}\right)}{fa}$
parts	$\frac{2icAa\left(\frac{1}{4a^{2}\sqrt{a+ia\tan(fx+e)}}+\frac{1}{6a(a+ia\tan(fx+e))^{\frac{3}{2}}}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)}{8a^{\frac{5}{2}}}\right)}{f}+\frac{(Ad+Bc)\left(-\frac{1}{3(a+ia\tan(\right.}}{}$

input
```
int((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
2*I/f/a*(-(1/4*I*A*d+1/4*I*B*c-1/4*c*A-3/4*B*d)/(a+I*a*tan(f*x+e))^(1/2)+1
/6*a*(-B*d+I*A*d+I*B*c+c*A)/(a+I*a*tan(f*x+e))^(3/2)-1/2*(-1/4*I*A*d-1/4*I
*B*c+1/4*c*A-1/4*B*d)*2^(1/2)/a^(1/2)*arctanh(1/2*(a+I*a*tan(f*x+e))^(1/2)
*2^(1/2)/a^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(108) = 216.

Time = 0.12 (sec) , antiderivative size = 619, normalized size of antiderivative = 4.21

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
1/12*(3*sqrt(1/2)*a^2*f*sqrt(-((A^2 - 2*I*A*B - B^2)*c^2 + 2*(-I*A^2 - 2*A
*B + I*B^2)*c*d - (A^2 - 2*I*A*B - B^2)*d^2)/(a^3*f^2))*e^(3*I*f*x + 3*I*e
)*log(4*(sqrt(2)*sqrt(1/2)*(I*a^2*f*e^(2*I*f*x + 2*I*e) + I*a^2*f)*sqrt(a/
(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-((A^2 - 2*I*A*B - B^2)*c^2 + 2*(-I*A^2 -
2*A*B + I*B^2)*c*d - (A^2 - 2*I*A*B - B^2)*d^2)/(a^3*f^2)) + ((A - I*B)*a*
c + (-I*A - B)*a*d)*e^(I*f*x + I*e))*e^(-I*f*x - I*e)/((A - I*B)*c - (I*A
+ B)*d)) - 3*sqrt(1/2)*a^2*f*sqrt(-((A^2 - 2*I*A*B - B^2)*c^2 + 2*(-I*A^2
- 2*A*B + I*B^2)*c*d - (A^2 - 2*I*A*B - B^2)*d^2)/(a^3*f^2))*e^(3*I*f*x +
3*I*e)*log(4*(sqrt(2)*sqrt(1/2)*(-I*a^2*f*e^(2*I*f*x + 2*I*e) - I*a^2*f)*s
qrt(a/(e^(2*I*f*x + 2*I*e) + 1))*sqrt(-((A^2 - 2*I*A*B - B^2)*c^2 + 2*(-I*
A^2 - 2*A*B + I*B^2)*c*d - (A^2 - 2*I*A*B - B^2)*d^2)/(a^3*f^2)) + ((A - I
*B)*a*c + (-I*A - B)*a*d)*e^(I*f*x + I*e))*e^(-I*f*x - I*e)/((A - I*B)*c -
 (I*A + B)*d)) + sqrt(2)*((I*A - B)*c - (A + I*B)*d - 2*((-2*I*A - B)*c -
(A + 4*I*B)*d)*e^(4*I*f*x + 4*I*e) + ((5*I*A + B)*c + (A + 7*I*B)*d)*e^(2*
I*f*x + 2*I*e))*sqrt(a/(e^(2*I*f*x + 2*I*e) + 1)))*e^(-3*I*f*x - 3*I*e)/(a
^2*f)
```

Sympy [F]

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx = \int \frac{(A+B\tan(e+fx))\,(c+d\tan(e+fx))}{(ia(\tan(e+fx)-i))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*tan(e + f*x))*(c + d*tan(e + f*x))/(I*a*(tan(e + f*x) - I)
)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{i\left(\frac{3\sqrt{2}(A-iB)(c-id)\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(fx+e)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(fx+e)+a}}\right)}{\sqrt{a}} + \frac{4(2(A+iB)ac-}{}\right.}{24af}$$

input
```
integrate((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
1/24*I*(3*sqrt(2)*(A - I*B)*(c - I*d)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan
(f*x + e) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(f*x + e) + a)))/sqrt(a) +
4*(2*(A + I*B)*a*c + 2*(I*A - B)*a*d + 3*((A - I*B)*c + (-I*A + 3*B)*d)*(I
*a*tan(f*x + e) + a))/(I*a*tan(f*x + e) + a)^(3/2))/(a*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument TypeError: Bad
Argument TypeDone
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.67

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx = \frac{\frac{(A\,c+A\,d\,1\mathrm{i})\,1\mathrm{i}}{3\,f}+\frac{(A\,c-A\,d\,1\mathrm{i})\,(a+a\tan(e+f\,x)\,1\mathrm{i})\,1\mathrm{i}}{2\,a\,f}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^{3/2}} - \frac{\frac{B\,c+B\,d\,1\mathrm{i}}{3\,f}-\frac{(B\,c+B\,d\,3\mathrm{i})\,(a+a\tan(e+f\,x)\,1\mathrm{i})}{2\,a\,f}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^{3/2}} + \frac{\sqrt{2}\,B\,\mathrm{atanh}\left(\frac{\sqrt{2}\,B\,(d+c\,1\mathrm{i})\,\sqrt{a+a\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{-a}\,(B\,c-B\,d\,1\mathrm{i})}\right)\,(d+c\,1\mathrm{i})}{4\,(-a)^{3/2}\,f} + \frac{\sqrt{2}\,A\,\mathrm{atan}\left(\frac{\sqrt{2}\,A\,(d+c\,1\mathrm{i})\,\sqrt{a+a\tan(e+f\,x)\,1\mathrm{i}}}{2\,\sqrt{a}\,(A\,c-A\,d\,1\mathrm{i})}\right)\,(d+c\,1\mathrm{i})\,1\mathrm{i}}{4\,a^{3/2}\,f}$$

input
```
int(((A + B*tan(e + f*x))*(c + d*tan(e + f*x)))/(a + a*tan(e + f*x)*1i)^(3
/2),x)
```

output
```
(((A*c + A*d*1i)*1i)/(3*f) + ((A*c - A*d*1i)*(a + a*tan(e + f*x)*1i)*1i)/(
2*a*f))/(a + a*tan(e + f*x)*1i)^(3/2) - ((B*c + B*d*1i)/(3*f) - ((B*c + B*
d*3i)*(a + a*tan(e + f*x)*1i))/(2*a*f))/(a + a*tan(e + f*x)*1i)^(3/2) + (2
^(1/2)*B*atanh((2^(1/2)*B*(c*1i + d)*(a + a*tan(e + f*x)*1i)^(1/2))/(2*(-a
)^(1/2)*(B*c - B*d*1i)))*(c*1i + d))/(4*(-a)^(3/2)*f) + (2^(1/2)*A*atan((2
^(1/2)*A*(c*1i + d)*(a + a*tan(e + f*x)*1i)^(1/2))/(2*a^(1/2)*(A*c - A*d*1
i)))*(c*1i + d)*1i)/(4*a^(3/2)*f)
```

Reduce [F]

$$\int \frac{(A+B\tan(e+fx))(c+d\tan(e+fx))}{(a+ia\tan(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*tan(f*x+e))*(c+d*tan(f*x+e))/(a+I*a*tan(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*( - 4*sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)*b*d - 2*sqrt(tan(e +
f*x)*i + 1)*a*c*i - 2*sqrt(tan(e + f*x)*i + 1)*a*d - 2*sqrt(tan(e + f*x)*i
 + 1)*b*c + 2*sqrt(tan(e + f*x)*i + 1)*b*d*i + int(sqrt(tan(e + f*x)*i + 1
)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*
x)**2*a*c*f + int(sqrt(tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i + tan(e + f*
x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*d*f*i + int(sqrt(tan(e +
f*x)*i + 1)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*
tan(e + f*x)**2*b*c*f*i + 5*int(sqrt(tan(e + f*x)*i + 1)/(tan(e + f*x)**3*
i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*d*f + int(s
qrt(tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x
)*i + 1),x)*a*c*f + int(sqrt(tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i + tan(
e + f*x)**2 + tan(e + f*x)*i + 1),x)*a*d*f*i + int(sqrt(tan(e + f*x)*i + 1
)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*b*c*f*i +
5*int(sqrt(tan(e + f*x)*i + 1)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(
e + f*x)*i + 1),x)*b*d*f + 9*int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2
)/(tan(e + f*x)**3*i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*
x)**2*a*c*f + int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)
**3*i + tan(e + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*a*d*f*i +
 int((sqrt(tan(e + f*x)*i + 1)*tan(e + f*x)**2)/(tan(e + f*x)**3*i + tan(e
 + f*x)**2 + tan(e + f*x)*i + 1),x)*tan(e + f*x)**2*b*c*f*i + 5*int((sq...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file